Computer algebra independent integration tests

Summer 2022 edition

6-Hyperbolic-functions/6.6-Hyperbolic-cosecant/183-6.6.3-Hyperbolic-cosecant-functions

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [175]. This is test number [183].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (175)	0.00 (0)
Mathematica	100.00 (175)	0.00 (0)
Fricas	94.86 (166)	5.14 (9)
Maple	77.71 (136)	22.29 (39)
Maxima	62.29 (109)	37.71 (66)
Giac	61.14 (107)	38.86 (68)
Mupad	52.00 (91)	48.00 (84)
Sympy	0.00 (0)	100.00 (175)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

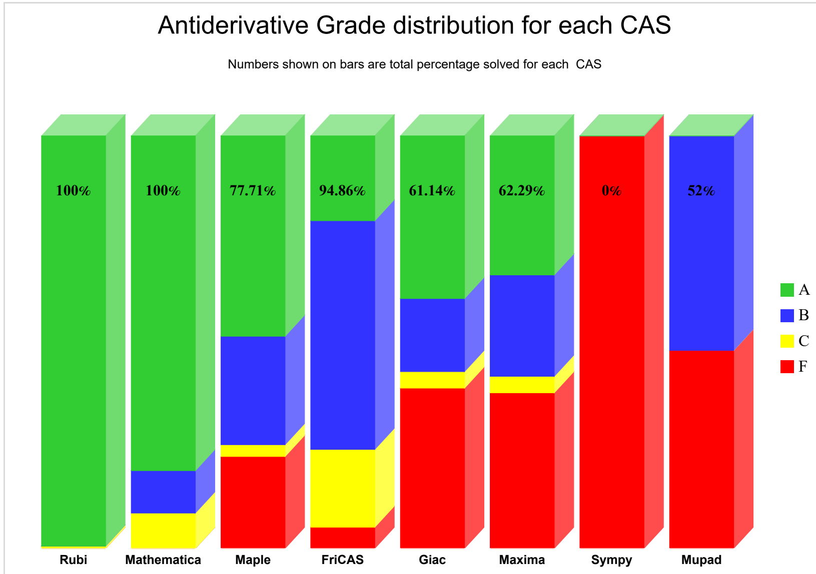
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

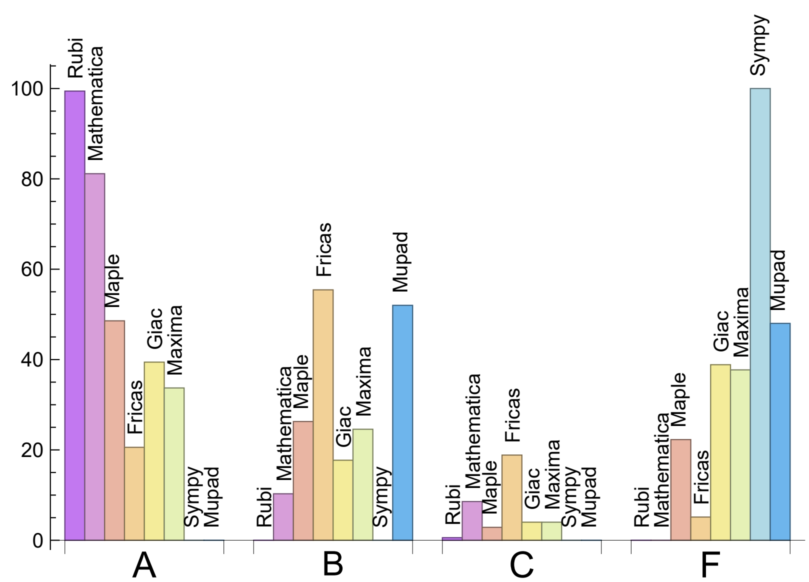
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.43	0.00	0.57	0.00
Mathematica	81.14	10.29	8.57	0.00
Maple	48.57	26.29	2.86	22.29
Giac	39.43	17.71	4.00	38.86
Maxima	33.71	24.57	4.00	37.71
Fricas	20.57	55.43	18.86	5.14
Mupad	N/A	52.00	0.00	48.00
Sympy	0.00	0.00	0.00	100.00

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	39	100.00 %	0.00 %	0.00 %
Fricas	9	77.78 %	0.00 %	22.22 %
Giac	68	61.76 %	33.82 %	4.41 %
Maxima	66	96.97 %	0.00 %	3.03 %
Sympy	175	100.00 %	0.00 %	0.00 %
Mupad	84	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

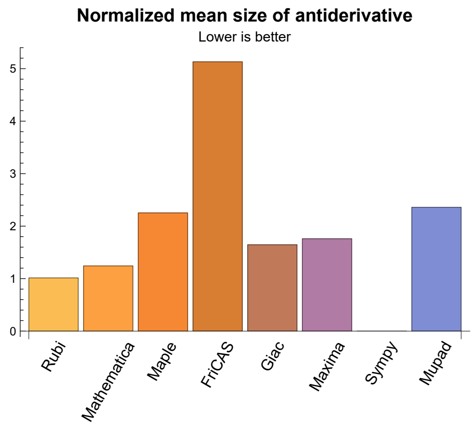
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

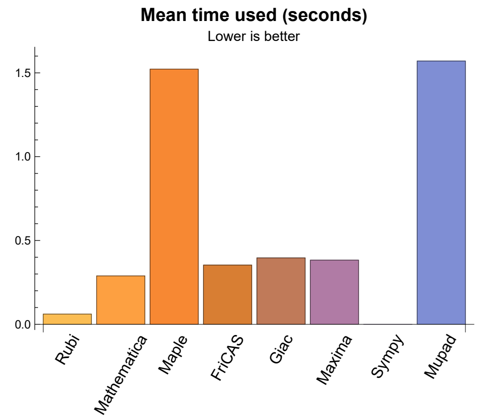
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.06	68.71	1.01	62.00	1.00
Mathematica	0.29	72.43	1.24	61.00	1.00
Maple	1.52	128.13	2.25	101.00	1.70
Maxima	0.38	105.02	1.76	75.00	1.55
Fricas	0.35	420.99	5.13	156.50	3.25
Sympy	0.00	0.00	0.00	0.00	0.00
Giac	0.40	92.25	1.65	62.00	1.47
Mupad	1.57	151.23	2.36	81.00	2.16

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
1
2
3
4
5
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0.0
0.5
1.0
1.5
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {164}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

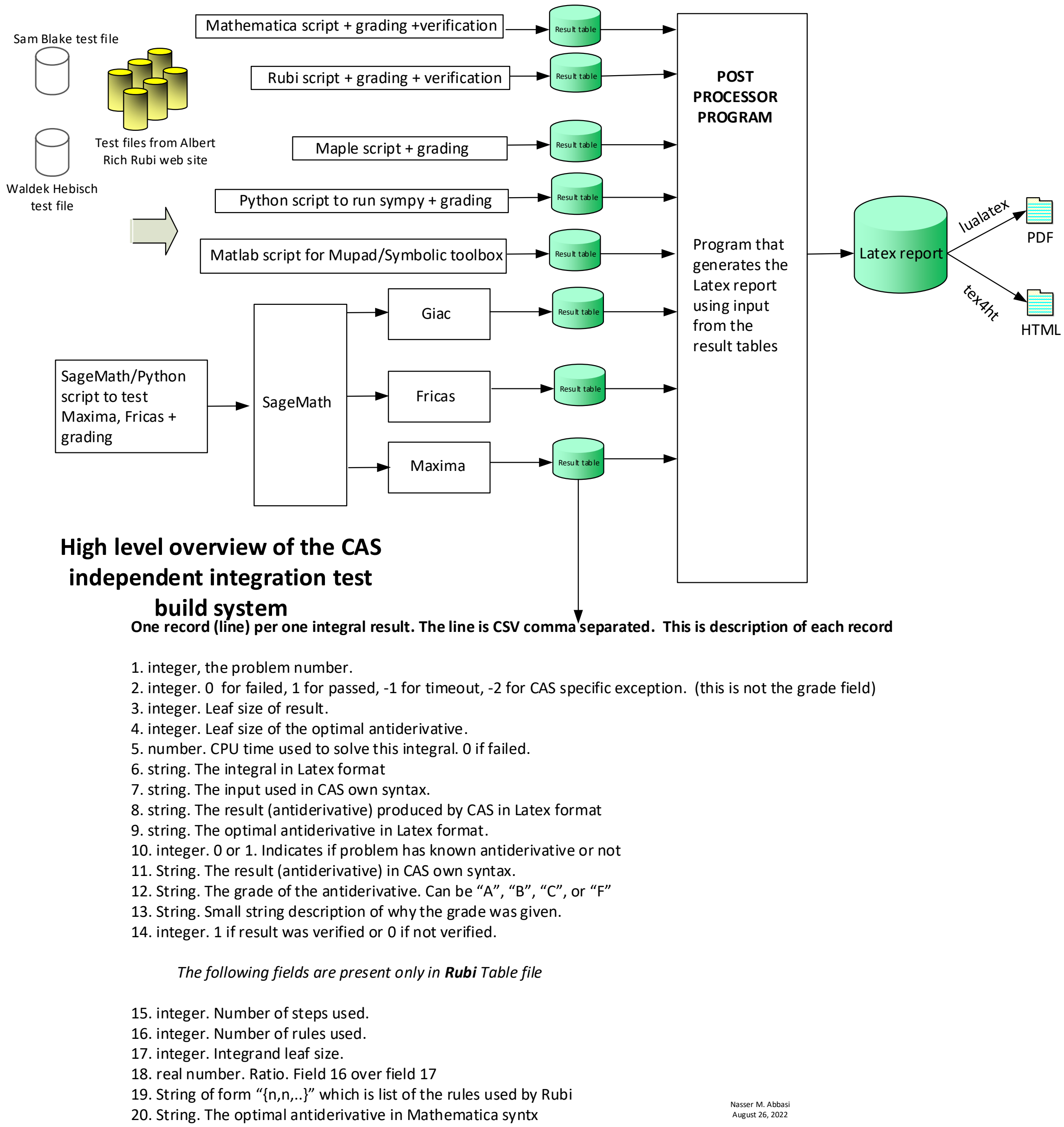
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175 }

B grade: { }

C grade: { 160 }

F grade: { }

2.1.2 Mathematica

A grade: { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 104, 105, 106, 107, 108, 109, 111, 114, 116, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 133, 135, 137, 138, 139, 141, 143, 145, 147, 149, 152, 153, 156, 157, 158, 160, 163, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175 }

B grade: { 1, 24, 55, 67, 68, 69, 70, 73, 89, 91, 103, 110, 112, 159, 161, 162, 164, 165 }

C grade: { 113, 115, 117, 132, 134, 136, 140, 142, 144, 146, 148, 150, 151, 154, 155 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 42, 43, 44, 45, 49, 50, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 79, 80, 81, 82, 83, 85, 87, 96, 97, 98, 99, 100, 102, 104, 106, 107, 109, 113, 114, 115, 116, 117, 118, 119, 121, 123, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 138, 140, 141, 142, 143, 144, 145, 146, 147, 148, 150, 151, 152, 165, 170, 171, 172, 173, 174, 175 }

B grade: { 3, 17, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 46, 47, 48, 62, 63, 64, 65, 76, 77, 78, 84, 86, 88, 89, 90, 91, 92, 93, 94, 95, 101, 103, 105, 108, 110, 111, 112, 120, 122, 124 }

C grade: { 160, 166, 167, 168, 169 }

F grade: { 13, 14, 15, 16, 18, 19, 20, 21, 36, 37, 38, 39, 40, 41, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 137, 139, 149, 153, 154, 155, 156, 157, 158, 159, 161, 162, 163, 164 }

2.1.4 Maxima

A grade: { 1, 2, 29, 30, 31, 32, 33, 34, 35, 45, 46, 47, 48, 49, 50, 62, 63, 64, 65, 66, 67, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 84, 87, 93, 95, 97, 98, 105, 106, 107, 114, 115, 116, 117, 118, 119, 123, 126, 127, 128, 129, 130, 131, 133, 145, 162, 165, 166 }

B grade: { 3, 4, 5, 6, 42, 43, 44, 68, 69, 70, 76, 83, 85, 86, 88, 89, 92, 94, 96, 99, 100, 101, 102, 103, 104, 108, 109, 110, 111, 112, 113, 120, 121, 122, 124, 125, 141, 153, 160, 161, 167, 168, 169 }

C grade: { 22, 23, 24, 25, 26, 27, 28 }

F grade: { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 36, 37, 38, 39, 40, 41, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 90, 91, 132, 134, 135, 136, 137, 138, 139, 140, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 154, 155, 156, 157, 158, 159, 163, 164, 170, 171, 172, 173, 174, 175 }

2.1.5 FriCAS

A grade: { 49, 50, 62, 63, 66, 84, 97, 107, 108, 117, 118, 128, 129, 130, 131, 132, 133, 135, 136, 137, 139, 140, 141, 142, 143, 144, 147, 148, 149, 151, 152, 153, 154, 155, 161, 162 }

B grade: { 1, 2, 3, 4, 5, 6, 29, 30, 31, 32, 33, 34, 35, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 109, 110, 111, 112, 113, 114, 115, 116, 119, 120, 121, 122, 123, 124, 125, 126, 127, 145, 160, 163, 164, 165, 166, 167, 168, 169 }

C grade: { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 36, 37, 38, 39, 40, 41, 170, 171, 172, 173, 174, 175 }

F grade: { 21, 134, 138, 146, 150, 156, 157, 158, 159 }

2.1.6 Sympy

A grade: { }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175 }

2.1.7 Giac

A grade: { 2, 4, 6, 29, 30, 31, 33, 34, 35, 42, 43, 44, 45, 46, 47, 48, 49, 50, 62, 63, 64, 65, 66, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 93, 94, 95, 96, 97, 98, 100, 102, 103, 105, 107, 108, 114, 116, 117, 118, 119, 123, 125, 126, 127, 128, 129, 130, 131, 161, 162, 166, 168 }

B grade: { 1, 3, 5, 32, 68, 85, 86, 87, 88, 89, 90, 91, 92, 99, 101, 104, 106, 109, 110, 111, 112, 113, 115, 120, 121, 122, 124, 160, 165, 167, 169 }

C grade: { 22, 23, 24, 25, 26, 27, 28 }

F grade: { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 36, 37, 38, 39, 40, 41, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 163, 164, 170, 171, 172, 173, 174, 175 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 25, 32, 42, 43, 44, 45, 49, 50, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 133, 141, 145, 153, 160, 161, 162, 165, 166, 167, 168, 169 }

C grade: { }

F grade: { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 40, 41, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 76, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 154, 155, 156, 157, 158, 159, 163, 164, 170, 171, 172, 173, 174, 175 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	38	15	14	38	0	27	27
N.S.	1	1.00	3.17	1.25	1.17	3.17	0.00	2.25	2.25
time (sec)	N/A	0.005	0.014	0.181	0.261	0.410	0.000	0.390	0.111

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	19	18	43	0	18	18
N.S.	1	1.00	1.00	1.73	1.64	3.91	0.00	1.64	1.64
time (sec)	N/A	0.007	0.009	0.825	0.262	0.442	0.000	0.383	0.073

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	57	65	84	387	0	84	86
N.S.	1	1.00	1.68	1.91	2.47	11.38	0.00	2.47	2.53
time (sec)	N/A	0.015	0.012	1.145	0.265	0.415	0.000	0.378	1.403

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	35	32	90	164	0	31	31
N.S.	1	1.00	1.35	1.23	3.46	6.31	0.00	1.19	1.19
time (sec)	N/A	0.011	0.012	0.995	0.270	0.346	0.000	0.397	1.455

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	95	89	133	1114	0	110	193
N.S.	1	1.00	1.73	1.62	2.42	20.25	0.00	2.00	3.51
time (sec)	N/A	0.033	0.018	0.980	0.273	0.403	0.000	0.393	0.062

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	56	43	205	344	0	42	42
N.S.	1	1.00	1.33	1.02	4.88	8.19	0.00	1.00	1.00
time (sec)	N/A	0.019	0.015	0.990	0.262	0.418	0.000	0.401	0.077

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	61	101	0	194	0	0	-1
N.S.	1	1.00	0.76	1.26	0.00	2.42	0.00	0.00	-0.01
time (sec)	N/A	0.025	0.091	1.576	0.000	0.099	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	57	154	0	96	0	0	-1
N.S.	1	1.00	0.75	2.03	0.00	1.26	0.00	0.00	-0.01
time (sec)	N/A	0.024	0.157	1.410	0.000	0.103	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	48	87	0	24	0	0	-1
N.S.	1	1.00	0.89	1.61	0.00	0.44	0.00	0.00	-0.02
time (sec)	N/A	0.015	0.146	0.924	0.000	0.083	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	50	108	0	150	0	0	-1
N.S.	1	1.00	0.93	2.00	0.00	2.78	0.00	0.00	-0.02
time (sec)	N/A	0.016	0.027	1.185	0.000	0.081	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	63	100	0	223	0	0	-1
N.S.	1	1.00	0.79	1.25	0.00	2.79	0.00	0.00	-0.01
time (sec)	N/A	0.025	0.051	1.303	0.000	0.126	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	67	164	0	370	0	0	-1
N.S.	1	1.00	0.84	2.05	0.00	4.62	0.00	0.00	-0.01
time (sec)	N/A	0.025	0.092	1.376	0.000	0.098	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	79	0	0	484	0	0	-1
N.S.	1	1.00	0.68	0.00	0.00	4.17	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.112	1.307	0.000	0.102	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	66	0	0	217	0	0	-1
N.S.	1	1.00	0.75	0.00	0.00	2.47	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.078	1.103	0.000	0.102	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	60	0	0	107	0	0	-1
N.S.	1	1.00	0.71	0.00	0.00	1.27	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.042	1.124	0.000	0.105	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	54	0	0	27	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.48	0.00	0.00	-0.02
time (sec)	N/A	0.017	0.027	1.311	0.000	0.130	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	52	227	0	154	0	0	-1
N.S.	1	1.00	0.93	4.05	0.00	2.75	0.00	0.00	-0.02
time (sec)	N/A	0.018	0.035	1.954	0.000	0.114	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	73	0	0	232	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	2.58	0.00	0.00	-0.01
time (sec)	N/A	0.030	0.066	0.888	0.000	0.105	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	68	0	0	378	0	0	-1
N.S.	1	1.00	0.76	0.00	0.00	4.20	0.00	0.00	-0.01
time (sec)	N/A	0.030	0.097	0.944	0.000	0.113	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	80	0	0	483	0	0	-1
N.S.	1	1.00	0.68	0.00	0.00	4.09	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.114	0.863	0.000	0.105	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	67	0	0	0	0	0	-1
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.027	0.075	1.017	0.000	0.000	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	C	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	41	114	74	117	0	72	-1
N.S.	1	1.00	1.02	2.85	1.85	2.92	0.00	1.80	-0.02
time (sec)	N/A	0.015	0.084	0.886	0.470	0.355	0.000	0.384	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	C	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	41	99	49	67	0	57	-1
N.S.	1	1.00	1.71	4.12	2.04	2.79	0.00	2.38	-0.04
time (sec)	N/A	0.009	0.042	0.799	0.480	0.453	0.000	0.401	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	C	C	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	20	67	19	15	0	27	-1
N.S.	1	1.00	6.67	22.33	6.33	5.00	0.00	9.00	-0.33
time (sec)	N/A	0.006	0.005	0.886	0.478	0.498	0.000	0.390	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	C	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	58	11	14	0	25	31
N.S.	1	1.00	1.00	4.46	0.85	1.08	0.00	1.92	2.38
time (sec)	N/A	0.008	0.004	0.807	0.491	0.378	0.000	0.382	1.715

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	C	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	27	118	23	26	0	38	-1
N.S.	1	1.00	0.82	3.58	0.70	0.79	0.00	1.15	-0.03
time (sec)	N/A	0.010	0.011	0.809	0.478	0.425	0.000	0.381	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	C	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	33	178	35	38	0	50	-1
N.S.	1	1.00	0.67	3.63	0.71	0.78	0.00	1.02	-0.02
time (sec)	N/A	0.014	0.019	0.797	0.479	0.447	0.000	0.382	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	C	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	39	238	47	50	0	62	-1
N.S.	1	1.00	0.60	3.66	0.72	0.77	0.00	0.95	-0.02
time (sec)	N/A	0.018	0.027	0.802	0.469	0.415	0.000	0.381	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	41	123	92	1128	0	75	-1
N.S.	1	1.00	0.63	1.89	1.42	17.35	0.00	1.15	-0.02
time (sec)	N/A	0.024	0.074	0.933	0.472	0.494	0.000	0.394	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	30	103	60	340	0	58	-1
N.S.	1	1.00	0.65	2.24	1.30	7.39	0.00	1.26	-0.02
time (sec)	N/A	0.018	0.049	0.809	0.486	0.493	0.000	0.404	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	20	67	24	97	0	29	-1
N.S.	1	1.00	0.77	2.58	0.92	3.73	0.00	1.12	-0.04
time (sec)	N/A	0.013	0.005	1.050	0.475	0.383	0.000	0.384	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	58	17	83	0	24	33
N.S.	1	1.00	1.00	4.46	1.31	6.38	0.00	1.85	2.54
time (sec)	N/A	0.010	0.006	0.976	0.462	0.395	0.000	0.390	1.603

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	27	130	35	285	0	41	-1
N.S.	1	1.00	0.75	3.61	0.97	7.92	0.00	1.14	-0.03
time (sec)	N/A	0.016	0.017	0.804	0.479	0.452	0.000	0.384	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	36	196	53	590	0	53	-1
N.S.	1	1.00	0.65	3.56	0.96	10.73	0.00	0.96	-0.02
time (sec)	N/A	0.022	0.021	0.835	0.496	0.584	0.000	0.387	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	42	262	71	984	0	65	-1
N.S.	1	1.00	0.57	3.54	0.96	13.30	0.00	0.88	-0.01
time (sec)	N/A	0.028	0.039	0.882	0.485	0.440	0.000	0.385	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	68	0	0	1389	0	0	-1
N.S.	1	1.00	0.50	0.00	0.00	10.29	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.122	0.954	0.000	0.134	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	56	0	0	395	0	0	-1
N.S.	1	1.00	0.69	0.00	0.00	4.88	0.00	0.00	-0.01
time (sec)	N/A	0.038	0.084	0.851	0.000	0.083	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	60	42	0	0	60	0	0	-1
N.S.	1	1.07	0.75	0.00	0.00	1.07	0.00	0.00	-0.02
time (sec)	N/A	0.024	0.022	0.951	0.000	0.101	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	43	0	0	127	0	0	-1
N.S.	1	1.00	0.69	0.00	0.00	2.05	0.00	0.00	-0.02
time (sec)	N/A	0.026	0.054	1.207	0.000	0.108	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	57	0	0	407	0	0	-1
N.S.	1	1.00	0.64	0.00	0.00	4.57	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.055	0.849	0.000	0.089	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	71	0	0	718	0	0	-1
N.S.	1	1.00	0.53	0.00	0.00	5.32	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.087	0.846	0.000	0.106	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	59	72	620	2825	0	51	498
N.S.	1	1.00	0.36	0.44	3.78	17.23	0.00	0.31	3.04
time (sec)	N/A	0.029	0.034	1.032	0.486	0.541	0.000	0.387	1.485

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	47	60	322	1493	0	39	356
N.S.	1	1.00	0.40	0.51	2.73	12.65	0.00	0.33	3.02
time (sec)	N/A	0.025	0.022	0.891	0.489	0.526	0.000	0.389	1.440

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	33	46	120	529	0	27	46
N.S.	1	1.00	0.53	0.74	1.94	8.53	0.00	0.44	0.74
time (sec)	N/A	0.017	0.014	0.885	0.479	0.425	0.000	0.400	1.457

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	29	13	81	0	13	71
N.S.	1	1.00	1.00	1.81	0.81	5.06	0.00	0.81	4.44
time (sec)	N/A	0.012	0.005	1.036	0.471	0.443	0.000	0.390	1.456

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	24	89	22	253	0	26	-1
N.S.	1	1.00	0.67	2.47	0.61	7.03	0.00	0.72	-0.03
time (sec)	N/A	0.011	0.018	1.108	0.478	0.560	0.000	0.402	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	38	230	46	1141	0	50	-1
N.S.	1	1.00	0.44	2.67	0.53	13.27	0.00	0.58	-0.01
time (sec)	N/A	0.028	0.029	0.965	0.471	0.354	0.000	0.384	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	55	362	72	2600	0	76	-1
N.S.	1	1.00	0.42	2.74	0.55	19.70	0.00	0.58	-0.01
time (sec)	N/A	0.036	0.078	0.977	0.470	0.393	0.000	0.415	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	54	54	35	32	0	29	26
N.S.	1	1.00	1.69	1.69	1.09	1.00	0.00	0.91	0.81
time (sec)	N/A	0.011	0.080	1.972	0.267	0.362	0.000	0.389	1.553

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	54	54	35	32	0	29	26
N.S.	1	1.00	1.69	1.69	1.09	1.00	0.00	0.91	0.81
time (sec)	N/A	0.012	0.076	2.112	0.286	0.383	0.000	0.394	1.501

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	136	0	0	566	0	0	-1
N.S.	1	1.00	1.27	0.00	0.00	5.29	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.953	4.454	0.000	0.471	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	100	0	0	461	0	0	-1
N.S.	1	1.00	1.39	0.00	0.00	6.40	0.00	0.00	-0.01
time (sec)	N/A	0.031	0.773	3.123	0.000	0.490	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	80	0	0	383	0	0	-1
N.S.	1	1.00	2.00	0.00	0.00	9.58	0.00	0.00	-0.02
time (sec)	N/A	0.015	0.577	4.770	0.000	0.374	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	118	0	0	551	0	0	-1
N.S.	1	1.00	1.30	0.00	0.00	6.05	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.840	2.751	0.000	0.483	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	327	0	0	873	0	0	-1
N.S.	1	1.00	2.66	0.00	0.00	7.10	0.00	0.00	-0.01
time (sec)	N/A	0.113	1.679	2.609	0.000	0.447	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	80	0	0	383	0	0	-1
N.S.	1	1.00	2.00	0.00	0.00	9.58	0.00	0.00	-0.02
time (sec)	N/A	0.021	0.613	5.970	0.000	0.434	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	117	0	0	551	0	0	-1
N.S.	1	1.00	1.29	0.00	0.00	6.05	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.784	3.190	0.000	0.487	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	46	0	0	212	0	0	-1
N.S.	1	1.00	2.00	0.00	0.00	9.22	0.00	0.00	-0.04
time (sec)	N/A	0.014	0.488	1.497	0.000	0.388	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	46	0	0	212	0	0	-1
N.S.	1	1.00	2.00	0.00	0.00	9.22	0.00	0.00	-0.04
time (sec)	N/A	0.015	0.463	1.420	0.000	0.380	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	46	0	0	218	0	0	-1
N.S.	1	1.00	2.00	0.00	0.00	9.48	0.00	0.00	-0.04
time (sec)	N/A	0.013	0.455	1.722	0.000	0.377	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	46	0	0	218	0	0	-1
N.S.	1	1.00	2.00	0.00	0.00	9.48	0.00	0.00	-0.04
time (sec)	N/A	0.015	0.445	1.777	0.000	0.417	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	63	128	71	79	0	66	64
N.S.	1	1.00	1.09	2.21	1.22	1.36	0.00	1.14	1.10
time (sec)	N/A	0.063	0.109	1.217	0.261	0.416	0.000	0.395	1.589

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	56	101	59	67	0	50	52
N.S.	1	1.00	1.22	2.20	1.28	1.46	0.00	1.09	1.13
time (sec)	N/A	0.054	0.096	1.122	0.269	0.389	0.000	0.383	0.176

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	46	80	47	55	0	40	38
N.S.	1	1.00	1.28	2.22	1.31	1.53	0.00	1.11	1.06
time (sec)	N/A	0.042	0.091	1.201	0.273	0.362	0.000	0.398	1.474

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	35	51	31	39	0	26	24
N.S.	1	1.00	1.75	2.55	1.55	1.95	0.00	1.30	1.20
time (sec)	N/A	0.036	0.038	1.113	0.269	0.363	0.000	0.394	1.460

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	27	12	12	8	0	8	10
N.S.	1	1.00	1.93	0.86	0.86	0.57	0.00	0.57	0.71
time (sec)	N/A	0.017	0.014	0.680	0.268	0.343	0.000	0.395	0.066

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	37	19	27	31	0	22	26
N.S.	1	1.00	2.18	1.12	1.59	1.82	0.00	1.29	1.53
time (sec)	N/A	0.041	0.025	0.624	0.306	0.354	0.000	0.371	0.219

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	70	35	53	77	0	46	60
N.S.	1	1.00	2.69	1.35	2.04	2.96	0.00	1.77	2.31
time (sec)	N/A	0.066	0.081	0.790	0.266	0.365	0.000	0.404	1.645

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	81	53	77	120	0	50	63
N.S.	1	1.00	2.19	1.43	2.08	3.24	0.00	1.35	1.70
time (sec)	N/A	0.048	0.235	0.724	0.276	0.347	0.000	0.397	1.685

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	508	176	234	1440	0	169	239
N.S.	1	1.00	4.66	1.61	2.15	13.21	0.00	1.55	2.19
time (sec)	N/A	0.092	6.176	2.356	0.292	0.391	0.000	0.401	0.210

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	118	133	136	769	0	122	170
N.S.	1	1.00	1.57	1.77	1.81	10.25	0.00	1.63	2.27
time (sec)	N/A	0.039	0.596	2.408	0.270	0.382	0.000	0.400	0.150

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	61	60	44	222	0	59	74
N.S.	1	1.00	1.79	1.76	1.29	6.53	0.00	1.74	2.18
time (sec)	N/A	0.024	0.162	2.133	0.260	0.492	0.000	0.382	1.480

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	43	20	19	44	0	32	42
N.S.	1	1.00	2.53	1.18	1.12	2.59	0.00	1.88	2.47
time (sec)	N/A	0.009	0.009	0.573	0.270	0.393	0.000	0.389	0.066

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	64	82	85	186	0	84	121
N.S.	1	1.00	1.19	1.52	1.57	3.44	0.00	1.56	2.24
time (sec)	N/A	0.045	0.080	1.463	0.472	0.443	0.000	0.405	0.328

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	142	176	187	645	0	161	269
N.S.	1	1.00	1.41	1.74	1.85	6.39	0.00	1.59	2.66
time (sec)	N/A	0.123	0.284	2.064	0.493	0.405	0.000	0.397	1.948

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	213	328	373	2094	0	293	-1
N.S.	1	1.00	1.31	2.01	2.29	12.85	0.00	1.80	-0.01
time (sec)	N/A	0.226	0.653	2.128	0.500	0.451	0.000	0.406	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	104	198	157	807	0	155	199
N.S.	1	1.00	0.97	1.85	1.47	7.54	0.00	1.45	1.86
time (sec)	N/A	0.343	0.293	0.856	0.467	0.412	0.000	0.403	1.826

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	82	152	116	456	0	115	157
N.S.	1	1.00	1.02	1.90	1.45	5.70	0.00	1.44	1.96
time (sec)	N/A	0.217	0.093	0.833	0.477	0.442	0.000	0.390	1.668

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	61	92	84	238	0	86	129
N.S.	1	1.00	1.07	1.61	1.47	4.18	0.00	1.51	2.26
time (sec)	N/A	0.078	0.073	0.848	0.473	0.364	0.000	0.386	1.579

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	45	35	54	111	0	56	49
N.S.	1	1.00	1.22	0.95	1.46	3.00	0.00	1.51	1.32
time (sec)	N/A	0.049	0.019	0.430	0.487	0.366	0.000	0.394	1.516

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	58	49	83	156	0	82	287
N.S.	1	1.00	1.16	0.98	1.66	3.12	0.00	1.64	5.74
time (sec)	N/A	0.082	0.036	0.552	0.466	0.384	0.000	0.384	1.685

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	71	73	100	345	0	98	292
N.S.	1	1.00	1.20	1.24	1.69	5.85	0.00	1.66	4.95
time (sec)	N/A	0.119	0.201	0.561	0.468	0.378	0.000	0.391	1.728

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	124	108	158	947	0	141	617
N.S.	1	1.00	1.49	1.30	1.90	11.41	0.00	1.70	7.43
time (sec)	N/A	0.208	0.333	0.677	0.469	0.418	0.000	0.393	2.094

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	32	116	42	43	0	38	41
N.S.	1	1.00	0.84	3.05	1.11	1.13	0.00	1.00	1.08
time (sec)	N/A	0.091	0.025	0.821	0.265	0.369	0.000	0.396	1.551

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	15	39	36	0	35	39
N.S.	1	1.00	1.00	0.79	2.05	1.89	0.00	1.84	2.05
time (sec)	N/A	0.070	0.009	0.498	0.263	0.417	0.000	0.383	0.142

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	68	30	31	0	26	29
N.S.	1	1.00	1.00	3.40	1.50	1.55	0.00	1.30	1.45
time (sec)	N/A	0.066	0.022	0.773	0.261	0.368	0.000	0.384	1.463

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	21	20	21	28	0	25	12
N.S.	1	1.00	1.31	1.25	1.31	1.75	0.00	1.56	0.75
time (sec)	N/A	0.044	0.012	0.991	0.273	0.365	0.000	0.392	0.093

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	20	43	41	52	0	53	46
N.S.	1	1.00	0.71	1.54	1.46	1.86	0.00	1.89	1.64
time (sec)	N/A	0.057	0.022	1.274	0.257	0.366	0.000	0.388	0.229

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	64	49	81	32	0	27	31
N.S.	1	1.00	3.37	2.58	4.26	1.68	0.00	1.42	1.63
time (sec)	N/A	0.080	0.040	1.093	0.260	0.356	0.000	0.401	1.573

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	60	89	0	142	0	94	122
N.S.	1	1.00	1.50	2.22	0.00	3.55	0.00	2.35	3.05
time (sec)	N/A	0.091	0.049	1.129	0.000	0.398	0.000	0.380	1.950

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	96	93	0	68	0	55	207
N.S.	1	1.00	3.31	3.21	0.00	2.34	0.00	1.90	7.14
time (sec)	N/A	0.087	0.078	1.121	0.000	0.356	0.000	0.395	1.924

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	124	378	242	1398	0	194	228
N.S.	1	1.00	1.22	3.71	2.37	13.71	0.00	1.90	2.24
time (sec)	N/A	0.142	0.266	0.681	0.265	0.434	0.000	0.405	2.067

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	180	311	217	924	0	221	247
N.S.	1	1.00	1.44	2.49	1.74	7.39	0.00	1.77	1.98
time (sec)	N/A	0.265	0.823	0.638	0.486	0.416	0.000	0.396	2.024

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	67	191	127	476	0	97	121
N.S.	1	1.00	1.18	3.35	2.23	8.35	0.00	1.70	2.12
time (sec)	N/A	0.110	0.097	0.680	0.268	0.379	0.000	0.389	1.684

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	80	150	122	304	0	121	159
N.S.	1	1.00	1.04	1.95	1.58	3.95	0.00	1.57	2.06
time (sec)	N/A	0.143	0.154	0.695	0.474	0.370	0.000	0.388	1.693

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	19	31	48	80	0	39	20
N.S.	1	1.00	0.95	1.55	2.40	4.00	0.00	1.95	1.00
time (sec)	N/A	0.053	0.009	0.646	0.260	0.360	0.000	0.397	0.084

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	36	72	66	57	0	89	93
N.S.	1	1.00	0.56	1.12	1.03	0.89	0.00	1.39	1.45
time (sec)	N/A	0.081	0.041	0.762	0.488	0.390	0.000	0.387	2.357

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	67	81	91	256	0	85	133
N.S.	1	1.00	1.12	1.35	1.52	4.27	0.00	1.42	2.22
time (sec)	N/A	0.104	0.130	0.817	0.470	0.408	0.000	0.396	1.659

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	78	143	161	675	0	218	256
N.S.	1	1.00	0.82	1.51	1.69	7.11	0.00	2.29	2.69
time (sec)	N/A	0.158	0.108	0.927	0.487	0.483	0.000	0.396	2.994

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	114	170	226	1155	0	174	269
N.S.	1	1.00	1.10	1.63	2.17	11.11	0.00	1.67	2.59
time (sec)	N/A	0.190	0.454	0.868	0.480	0.387	0.000	0.399	1.793

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	138	316	348	2778	0	374	513
N.S.	1	1.00	0.93	2.12	2.34	18.64	0.00	2.51	3.44
time (sec)	N/A	0.265	0.198	1.076	0.484	0.460	0.000	0.402	5.507

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	75	155	144	304	0	120	274
N.S.	1	1.00	0.69	1.42	1.32	2.79	0.00	1.10	2.51
time (sec)	N/A	0.060	0.142	1.562	0.325	0.381	0.000	0.403	4.105

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	126	99	96	124	0	62	237
N.S.	1	1.00	2.42	1.90	1.85	2.38	0.00	1.19	4.56
time (sec)	N/A	0.078	0.092	1.490	0.270	0.389	0.000	0.388	2.218

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	61	109	96	186	0	98	140
N.S.	1	1.00	0.79	1.42	1.25	2.42	0.00	1.27	1.82
time (sec)	N/A	0.048	0.089	1.537	0.290	0.433	0.000	0.386	0.577

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	71	67	42	50	0	38	85
N.S.	1	1.00	1.97	1.86	1.17	1.39	0.00	1.06	2.36
time (sec)	N/A	0.062	0.060	1.365	0.271	0.442	0.000	0.388	1.634

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	39	65	45	71	0	55	50
N.S.	1	1.00	0.87	1.44	1.00	1.58	0.00	1.22	1.11
time (sec)	N/A	0.032	0.030	1.414	0.262	0.413	0.000	0.397	0.225

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	17	15	11	0	13	14
N.S.	1	1.00	1.00	1.31	1.15	0.85	0.00	1.00	1.08
time (sec)	N/A	0.016	0.007	0.927	0.260	0.380	0.000	0.396	1.463

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	13	27	20	16	0	17	21
N.S.	1	1.00	1.18	2.45	1.82	1.45	0.00	1.55	1.91
time (sec)	N/A	0.027	0.025	0.798	0.257	0.457	0.000	0.391	0.178

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	12	36	40	0	39	27
N.S.	1	1.00	1.00	1.00	3.00	3.33	0.00	3.25	2.25
time (sec)	N/A	0.030	0.009	0.848	0.260	0.464	0.000	0.387	1.571

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	65	61	55	86	0	43	56
N.S.	1	1.00	2.41	2.26	2.04	3.19	0.00	1.59	2.07
time (sec)	N/A	0.044	0.027	1.082	0.260	0.396	0.000	0.389	1.640

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	78	75	100	0	65	81
N.S.	1	1.00	1.00	2.60	2.50	3.33	0.00	2.17	2.70
time (sec)	N/A	0.035	0.012	1.434	0.261	0.408	0.000	0.392	1.682

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	129	95	96	157	0	71	106
N.S.	1	1.00	3.00	2.21	2.23	3.65	0.00	1.65	2.47
time (sec)	N/A	0.055	0.030	1.389	0.261	0.455	0.000	0.389	1.862

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	253	364	383	4025	0	432	611
N.S.	1	1.00	1.30	1.88	1.97	20.75	0.00	2.23	3.15
time (sec)	N/A	0.197	0.346	1.655	0.494	0.461	0.000	0.392	7.041

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	141	207	261	1746	0	215	707
N.S.	1	1.00	0.77	1.13	1.43	9.54	0.00	1.17	3.86
time (sec)	N/A	0.286	0.519	1.124	0.490	0.398	0.000	0.401	3.429

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	191	182	172	965	0	234	335
N.S.	1	1.00	1.69	1.61	1.52	8.54	0.00	2.07	2.96
time (sec)	N/A	0.117	0.138	1.044	0.478	0.571	0.000	0.390	4.011

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	82	95	108	349	0	102	376
N.S.	1	1.00	0.82	0.95	1.08	3.49	0.00	1.02	3.76
time (sec)	N/A	0.151	0.239	0.898	0.468	0.382	0.000	0.397	2.471

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	63	97	74	75	0	89	132
N.S.	1	1.00	1.03	1.59	1.21	1.23	0.00	1.46	2.16
time (sec)	N/A	0.070	0.044	0.917	0.465	0.461	0.000	0.391	2.526

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	11	21	28	27	0	22	25
N.S.	1	1.00	0.58	1.11	1.47	1.42	0.00	1.16	1.32
time (sec)	N/A	0.021	0.007	0.618	0.289	0.361	0.000	0.396	0.099

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	65	84	90	141	0	89	316
N.S.	1	1.00	1.14	1.47	1.58	2.47	0.00	1.56	5.54
time (sec)	N/A	0.125	0.060	0.708	0.469	0.520	0.000	0.405	0.344

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	37	95	82	199	0	80	261
N.S.	1	1.00	1.16	2.97	2.56	6.22	0.00	2.50	8.16
time (sec)	N/A	0.050	0.037	0.727	0.263	0.378	0.000	0.406	1.840

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	151	150	178	831	0	161	378
N.S.	1	1.00	1.72	1.70	2.02	9.44	0.00	1.83	4.30
time (sec)	N/A	0.233	0.415	0.907	0.507	0.444	0.000	0.404	2.623

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	83	181	190	1288	0	170	155
N.S.	1	1.00	1.19	2.59	2.71	18.40	0.00	2.43	2.21
time (sec)	N/A	0.067	0.090	0.749	0.267	0.383	0.000	0.399	1.975

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	269	260	330	3160	0	305	543
N.S.	1	1.00	1.47	1.42	1.80	17.27	0.00	1.67	2.97
time (sec)	N/A	0.253	1.077	0.973	0.484	0.626	0.000	0.413	2.898

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	130	314	364	4024	0	295	317
N.S.	1	1.00	1.09	2.64	3.06	33.82	0.00	2.48	2.66
time (sec)	N/A	0.100	0.178	0.777	0.283	0.525	0.000	0.405	2.182

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	84	91	386	592	0	90	413
N.S.	1	1.00	0.42	0.46	1.94	2.97	0.00	0.45	2.08
time (sec)	N/A	0.214	0.058	3.608	0.481	0.456	0.000	0.411	1.595

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	72	80	209	315	0	77	91
N.S.	1	1.00	0.49	0.54	1.42	2.14	0.00	0.52	0.62
time (sec)	N/A	0.121	0.048	3.563	0.481	0.387	0.000	0.412	1.530

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	56	69	84	121	0	64	78
N.S.	1	1.00	0.97	1.19	1.45	2.09	0.00	1.10	1.34
time (sec)	N/A	0.082	0.032	3.507	0.480	0.389	0.000	0.404	1.549

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	44	68	39	42	0	48	-1
N.S.	1	1.00	0.96	1.48	0.85	0.91	0.00	1.04	-0.02
time (sec)	N/A	0.065	0.027	3.769	0.470	0.377	0.000	0.394	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	48	106	36	66	0	71	-1
N.S.	1	1.00	0.65	1.43	0.49	0.89	0.00	0.96	-0.01
time (sec)	N/A	0.081	0.036	3.266	0.482	0.478	0.000	0.399	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	76	216	62	126	0	204	-1
N.S.	1	1.00	0.47	1.33	0.38	0.78	0.00	1.26	-0.01
time (sec)	N/A	0.111	0.046	3.246	0.485	0.400	0.000	0.412	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	106	326	90	218	0	278	-1
N.S.	1	1.00	0.42	1.30	0.36	0.87	0.00	1.11	-0.00
time (sec)	N/A	0.139	0.071	3.174	0.482	0.417	0.000	0.411	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	80	125	0	71	0	0	-1
N.S.	1	1.00	0.99	1.54	0.00	0.88	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.121	1.164	0.000	0.095	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	44	39	46	48	0	0	42
N.S.	1	1.00	1.47	1.30	1.53	1.60	0.00	0.00	1.40
time (sec)	N/A	0.029	0.034	0.898	0.522	0.382	0.000	0.000	2.153

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	60	127	0	0	0	0	-1
N.S.	1	1.00	0.50	1.07	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.079	0.987	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	74	97	0	92	0	0	-1
N.S.	1	1.00	1.07	1.41	0.00	1.33	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.097	1.189	0.000	0.394	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	57	109	0	62	0	0	-1
N.S.	1	1.00	0.95	1.82	0.00	1.03	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.076	0.964	0.000	0.100	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	77	0	0	86	0	0	-1
N.S.	1	1.00	1.28	0.00	0.00	1.43	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.065	0.761	0.000	0.481	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	43	90	0	0	0	0	-1
N.S.	1	1.00	0.93	1.96	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.022	0.055	2.908	0.000	0.000	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	54	0	0	43	0	0	-1
N.S.	1	1.00	1.32	0.00	0.00	1.05	0.00	0.00	-0.02
time (sec)	N/A	0.033	0.083	0.493	0.000	0.432	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	58	126	0	36	0	0	-1
N.S.	1	1.00	0.78	1.70	0.00	0.49	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.067	0.888	0.000	0.109	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	33	38	89	37	0	0	58
N.S.	1	1.00	1.32	1.52	3.56	1.48	0.00	0.00	2.32
time (sec)	N/A	0.029	0.028	0.822	0.485	0.392	0.000	0.000	1.471

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	60	112	0	37	0	0	-1
N.S.	1	1.00	0.94	1.75	0.00	0.58	0.00	0.00	-0.02
time (sec)	N/A	0.041	0.069	0.890	0.000	0.081	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	95	121	0	110	0	0	-1
N.S.	1	1.00	0.74	0.95	0.00	0.86	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.142	1.188	0.000	0.389	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	80	133	0	79	0	0	-1
N.S.	1	1.00	0.68	1.13	0.00	0.67	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.120	0.973	0.000	0.098	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	44	47	46	56	0	0	42
N.S.	1	1.00	1.47	1.57	1.53	1.87	0.00	0.00	1.40
time (sec)	N/A	0.030	0.035	0.887	0.506	0.445	0.000	0.000	1.555

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	63	140	0	0	0	0	-1
N.S.	1	1.00	0.39	0.86	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.089	0.987	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	87	113	0	102	0	0	-1
N.S.	1	1.00	0.91	1.18	0.00	1.06	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.121	1.149	0.000	0.421	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	65	124	0	70	0	0	-1
N.S.	1	1.00	0.76	1.44	0.00	0.81	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.091	0.970	0.000	0.109	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	88	0	0	94	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	1.03	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.109	0.569	0.000	0.448	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	60	152	0	0	0	0	-1
N.S.	1	1.00	0.46	1.17	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.080	0.968	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	63	130	0	106	0	0	-1
N.S.	1	1.00	0.66	1.35	0.00	1.10	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.066	1.087	0.000	0.398	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	54	163	0	31	0	0	-1
N.S.	1	1.00	0.81	2.43	0.00	0.46	0.00	0.00	-0.01
time (sec)	N/A	0.025	0.071	2.562	0.000	0.100	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	33	0	87	29	0	0	29
N.S.	1	1.00	1.22	0.00	3.22	1.07	0.00	0.00	1.07
time (sec)	N/A	0.031	0.024	0.572	0.484	0.367	0.000	0.000	1.474

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	66	0	0	28	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.41	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.075	0.575	0.000	0.118	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	53	0	0	78	0	0	-1
N.S.	1	1.00	0.77	0.00	0.00	1.13	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.074	0.563	0.000	0.396	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	62	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.048	0.790	1.361	0.000	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	126	0	0	0	0	0	-1
N.S.	1	1.00	1.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	2.886	1.284	0.000	0.000	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	101	0	0	0	0	0	-1
N.S.	1	1.00	1.46	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.059	4.069	1.922	0.000	0.000	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	200	0	0	0	0	0	-1
N.S.	1	1.00	2.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	5.940	1.615	0.000	0.000	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	C	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	137	30	509	95	187	0	215	65
N.S.	1	3.26	0.71	12.12	2.26	4.45	0.00	5.12	1.55
time (sec)	N/A	0.113	0.310	7.541	0.485	0.382	0.000	0.589	1.520

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	62	0	76	48	0	38	48
N.S.	1	1.00	2.38	0.00	2.92	1.85	0.00	1.46	1.85
time (sec)	N/A	0.033	0.088	2.720	0.278	0.390	0.000	0.403	1.647

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	65	0	49	49	0	39	36
N.S.	1	1.00	2.50	0.00	1.88	1.88	0.00	1.50	1.38
time (sec)	N/A	0.041	0.073	3.839	0.280	0.394	0.000	0.393	1.592

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	115	0	0	475	0	0	-1
N.S.	1	1.00	1.28	0.00	0.00	5.28	0.00	0.00	-0.01
time (sec)	N/A	0.075	3.567	2.202	0.000	0.482	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	140	0	0	539	0	0	-1
N.S.	1	1.00	2.12	0.00	0.00	8.17	0.00	0.00	-0.02
time (sec)	N/A	0.065	3.355	2.127	0.000	0.376	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	54	23	22	65	0	145	43
N.S.	1	1.00	2.70	1.15	1.10	3.25	0.00	7.25	2.15
time (sec)	N/A	0.014	0.048	3.017	0.264	0.422	0.000	0.418	1.620

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	116	29	71	0	28	25
N.S.	1	1.00	1.00	6.11	1.53	3.74	0.00	1.47	1.32
time (sec)	N/A	0.022	0.051	3.663	0.288	0.351	0.000	0.390	1.438

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	81	534	150	643	0	210	140
N.S.	1	1.00	1.47	9.71	2.73	11.69	0.00	3.82	2.55
time (sec)	N/A	0.038	0.047	4.173	0.284	0.368	0.000	0.410	1.489

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	56	222	92	272	0	47	55
N.S.	1	1.00	1.33	5.29	2.19	6.48	0.00	1.12	1.31
time (sec)	N/A	0.028	0.043	3.980	0.284	0.362	0.000	0.393	1.454

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	135	744	232	1806	0	248	318
N.S.	1	1.00	1.52	8.36	2.61	20.29	0.00	2.79	3.57
time (sec)	N/A	0.054	0.046	4.309	0.291	0.448	0.000	0.422	1.477

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	84	144	0	318	0	0	-1
N.S.	1	1.00	0.76	1.30	0.00	2.86	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.153	5.414	0.000	0.097	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	80	212	0	159	0	0	-1
N.S.	1	1.00	0.75	1.98	0.00	1.49	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.082	5.325	0.000	0.097	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	66	120	0	39	0	0	-1
N.S.	1	1.00	0.92	1.67	0.00	0.54	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.069	4.678	0.000	0.096	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	68	146	0	248	0	0	-1
N.S.	1	1.00	0.94	2.03	0.00	3.44	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.060	4.984	0.000	0.145	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	86	143	0	370	0	0	-1
N.S.	1	1.00	0.77	1.29	0.00	3.33	0.00	0.00	-0.01
time (sec)	N/A	0.047	0.093	5.157	0.000	0.137	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	95	227	0	602	0	0	-1
N.S.	1	1.00	0.86	2.05	0.00	5.42	0.00	0.00	-0.01
time (sec)	N/A	0.047	0.132	5.468	0.000	0.102	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [160] had the largest ratio of [45]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	6	0.167
2	A	2	2	1.00	8	0.250
3	A	2	2	1.00	8	0.250
4	A	2	1	1.00	8	0.125
5	A	3	2	1.00	8	0.250
6	A	2	1	1.00	8	0.125
7	A	3	3	1.00	10	0.300
8	A	3	3	1.00	10	0.300
9	A	2	2	1.00	10	0.200
10	A	2	2	1.00	10	0.200
11	A	3	3	1.00	10	0.300
12	A	3	3	1.00	10	0.300
13	A	4	3	1.00	12	0.250
14	A	3	3	1.00	12	0.250
15	A	3	3	1.00	12	0.250
16	A	2	2	1.00	12	0.167
17	A	2	2	1.00	12	0.167
18	A	3	3	1.00	12	0.250
19	A	3	3	1.00	12	0.250
20	A	4	3	1.00	12	0.250
21	A	2	2	1.00	10	0.200
22	A	4	3	1.00	10	0.300
23	A	3	3	1.00	10	0.300
24	A	2	2	1.00	10	0.200
25	A	2	2	1.00	10	0.200

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	3	3	1.00	10	0.300
27	A	4	3	1.00	10	0.300
28	A	5	3	1.00	10	0.300
29	A	5	4	1.00	10	0.400
30	A	4	4	1.00	10	0.400
31	A	3	3	1.00	10	0.300
32	A	2	2	1.00	10	0.200
33	A	3	3	1.00	10	0.300
34	A	4	3	1.00	10	0.300
35	A	5	3	1.00	10	0.300
36	A	7	4	1.00	10	0.400
37	A	5	4	1.00	10	0.400
38	A	4	4	1.07	10	0.400
39	A	4	4	1.00	10	0.400
40	A	5	4	1.00	10	0.400
41	A	7	4	1.00	10	0.400
42	A	3	2	1.00	10	0.200
43	A	3	2	1.00	10	0.200
44	A	3	2	1.00	10	0.200
45	A	3	3	1.00	10	0.300
46	A	3	3	1.00	10	0.300
47	A	5	3	1.00	10	0.300
48	A	7	3	1.00	10	0.300
49	A	2	2	1.00	15	0.133
50	A	2	2	1.00	15	0.133
51	A	5	5	1.00	17	0.294
52	A	4	4	1.00	17	0.235
53	A	2	2	1.00	17	0.118
54	A	5	4	1.00	17	0.235
55	A	6	5	1.00	17	0.294
56	A	2	2	1.00	17	0.118
57	A	5	4	1.00	17	0.235
58	A	2	2	1.00	12	0.167
59	A	2	2	1.00	12	0.167
60	A	2	2	1.00	12	0.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	2	2	1.00	12	0.167
62	A	7	5	1.00	13	0.385
63	A	6	5	1.00	13	0.385
64	A	5	5	1.00	13	0.385
65	A	4	4	1.00	11	0.364
66	A	1	1	1.00	11	0.091
67	A	3	3	1.00	13	0.231
68	A	4	4	1.00	13	0.308
69	A	6	6	1.00	13	0.462
70	A	6	5	1.00	12	0.417
71	A	5	4	1.00	12	0.333
72	A	4	4	1.00	12	0.333
73	A	2	1	1.00	10	0.100
74	A	4	4	1.00	12	0.333
75	A	6	6	1.00	12	0.500
76	A	7	7	1.00	12	0.583
77	A	8	7	1.00	13	0.538
78	A	7	7	1.00	13	0.538
79	A	6	6	1.00	11	0.546
80	A	4	4	1.00	11	0.364
81	A	6	6	1.00	13	0.462
82	A	7	7	1.00	13	0.538
83	A	8	8	1.00	13	0.615
84	A	7	7	1.00	13	0.538
85	A	6	4	1.00	13	0.308
86	A	5	5	1.00	13	0.385
87	A	4	3	1.00	11	0.273
88	A	6	6	1.00	11	0.546
89	A	6	5	1.00	13	0.385
90	A	7	7	1.00	13	0.538
91	A	7	6	1.00	13	0.462
92	A	5	4	1.00	13	0.308
93	A	7	6	1.00	13	0.462
94	A	5	4	1.00	13	0.308
95	A	6	6	1.00	13	0.462

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	5	4	1.00	11	0.364
97	A	4	3	1.00	11	0.273
98	A	6	6	1.00	13	0.462
99	A	6	5	1.00	13	0.385
100	A	7	6	1.00	13	0.462
101	A	7	5	1.00	13	0.385
102	A	3	2	1.00	13	0.154
103	A	5	3	1.00	13	0.231
104	A	3	2	1.00	13	0.154
105	A	4	3	1.00	13	0.231
106	A	3	2	1.00	11	0.182
107	A	2	2	1.00	11	0.182
108	A	3	2	1.00	13	0.154
109	A	3	2	1.00	13	0.154
110	A	4	3	1.00	13	0.231
111	A	3	2	1.00	13	0.154
112	A	5	3	1.00	13	0.231
113	A	11	7	1.00	13	0.538
114	A	16	9	1.00	13	0.692
115	A	8	6	1.00	13	0.462
116	A	10	9	1.00	13	0.692
117	A	6	5	1.00	11	0.454
118	A	4	4	1.00	11	0.364
119	A	8	8	1.00	13	0.615
120	A	3	2	1.00	13	0.154
121	A	7	7	1.00	13	0.538
122	A	3	2	1.00	13	0.154
123	A	16	9	1.00	13	0.692
124	A	3	2	1.00	13	0.154
125	A	6	5	1.00	25	0.200
126	A	6	5	1.00	25	0.200
127	A	4	4	1.00	25	0.160
128	A	4	4	1.00	25	0.160
129	A	5	4	1.00	25	0.160
130	A	6	5	1.00	25	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	6	5	1.00	25	0.200
132	A	6	6	1.00	15	0.400
133	A	3	3	1.00	15	0.200
134	A	9	9	1.00	15	0.600
135	A	6	6	1.00	15	0.400
136	A	5	5	1.00	13	0.385
137	A	6	6	1.00	11	0.546
138	A	3	2	1.00	15	0.133
139	A	5	5	1.00	15	0.333
140	A	7	7	1.00	15	0.467
141	A	3	3	1.00	15	0.200
142	A	5	5	1.00	15	0.333
143	A	8	7	1.00	15	0.467
144	A	7	6	1.00	15	0.400
145	A	3	3	1.00	15	0.200
146	A	10	9	1.00	15	0.600
147	A	7	6	1.00	15	0.400
148	A	6	5	1.00	15	0.333
149	A	7	6	1.00	15	0.400
150	A	9	8	1.00	13	0.615
151	A	7	7	1.00	11	0.636
152	A	4	3	1.00	15	0.200
153	A	3	3	1.00	15	0.200
154	A	5	5	1.00	15	0.333
155	A	6	6	1.00	15	0.400
156	A	4	4	1.00	11	0.364
157	A	4	4	1.00	13	0.308
158	A	4	4	1.00	13	0.308
159	A	4	4	1.00	13	0.308
160	C	9	4	3.26	45	0.089
161	A	3	3	1.00	15	0.200
162	A	4	4	1.00	15	0.267
163	A	3	3	1.00	20	0.150
164	A	3	3	1.00	21	0.143
165	A	2	1	1.00	15	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	3	2	1.00	17	0.118
167	A	3	2	1.00	17	0.118
168	A	3	1	1.00	17	0.059
169	A	4	2	1.00	17	0.118
170	A	4	3	1.00	19	0.158
171	A	4	3	1.00	19	0.158
172	A	3	2	1.00	19	0.105
173	A	3	2	1.00	19	0.105
174	A	4	3	1.00	19	0.158
175	A	4	3	1.00	19	0.158

Chapter 3

Listing of integrals

Local contents

3.1 $\int \operatorname{csch}(a+bx)\,dx$

Optimal. Leaf size=12

$$-\frac{\tanh^{-1}(\cosh(a+bx))}{b}$$

[Out] -arctanh(cosh(b*x+a))/b

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3855}

$$-\frac{\tanh^{-1}(\cosh(a+bx))}{b}$$

Antiderivative was successfully verified.

[In] Int[Csch[a + b*x],x]

[Out] -(ArcTanh[Cosh[a + b*x]]/b)

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \operatorname{csch}(a+bx)\,dx = -\frac{\tanh^{-1}(\cosh(a+bx))}{b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 38 vs. 2(12) = 24.
time = 0.01, size = 38, normalized size = 3.17

$$-\frac{\log\left(\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}+\frac{\log\left(\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[a + b*x],x]

[Out] -(Log[Cosh[a/2 + (b*x)/2]]/b) + Log[Sinh[a/2 + (b*x)/2]]/b

Maple [A]
time = 0.18, size = 15, normalized size = 1.25

method	result	size
derivativedivides	$\frac{\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}$	15
default	$\frac{\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}$	15
risch	$-\frac{\ln\left(e^{bx+a}+1\right)}{b}+\frac{\ln\left(e^{bx+a}-1\right)}{b}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*ln(tanh(1/2*b*x+1/2*a))
```

Maxima [A]
time = 0.26, size = 14, normalized size = 1.17

$$\frac{\log\left(\tanh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a),x, algorithm="maxima")
```

```
[Out] log(tanh(1/2*b*x + 1/2*a))/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(12) = 24$.
time = 0.41, size = 38, normalized size = 3.17

$$-\frac{\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)-\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a),x, algorithm="fricas")
```

```
[Out] -(log(cosh(b*x + a) + sinh(b*x + a) + 1) - log(cosh(b*x + a) + sinh(b*x + a
) - 1))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\operatorname{csch}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a),x)
```

```
[Out] Integral(csch(a + b*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(12) = 24$.
time = 0.39, size = 27, normalized size = 2.25

$$-\frac{\log\left(e^{(bx+a)}+1\right)-\log\left(\left|e^{(bx+a)}-1\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a),x, algorithm="giac")
```

```
[Out] -(log(e^(b*x + a) + 1) - log(abs(e^(b*x + a) - 1)))/b
```

Mupad [B]
time = 0.11, size = 27, normalized size = 2.25

$$-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(a + b*x),x)
```

```
[Out] -(2*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2)
```

3.2 $\int \operatorname{csch}^2(a+bx)\,dx$

Optimal. Leaf size=11

$$-\frac{\coth(a+bx)}{b}$$

```
[Out] -coth(b*x+a)/b
```

Rubi [A]
time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3852, 8}

$$-\frac{\coth(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^2,x]
```

```
[Out] -(Coth[a + b*x]/b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^2(a+bx)\,dx &= -\frac{i\text{Subst}\left(\int 1\,dx, x, -i\coth(a+bx)\right)}{b}\\ &= -\frac{\coth(a+bx)}{b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 11, normalized size = 1.00

$$-\frac{\coth(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^2,x]
```

```
[Out] -(Coth[a + b*x]/b)
```

Maple [A]
time = 0.82, size = 19, normalized size = 1.73

method	result	size
risch	$-\frac{2}{b\left(\mathrm{e}^{2bx+2a}-1\right)}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/b/(exp(2*b*x+2*a)-1)
```

Maxima [A]
time = 0.26, size = 18, normalized size = 1.64

$$\frac{2}{b\left(e^{(-2\,bx-2\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 2/(b*(e^(-2*b*x - 2*a) - 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(11) = 22$.
time = 0.44, size = 43, normalized size = 3.91

$$-\frac{2}{b\cosh\left(bx+a\right)^2+2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^2-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -2/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2
 - b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**2,x)
```

```
[Out] Integral(csch(a + b*x)**2, x)
```

Giac [A]
time = 0.38, size = 18, normalized size = 1.64

$$-\frac{2}{b(e^{(2\,bx+2\,a)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -2/(b*(e^(2*b*x + 2*a) - 1))
```

Mupad [B]
time = 0.07, size = 18, normalized size = 1.64

$$-\frac{2}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(a + b*x)^2,x)
```

```
[Out] -2/(b*(exp(2*a + 2*b*x) - 1))
```

3.3 $\int \text{csch}^3(a+bx)\,dx$

Optimal. Leaf size=34

$$\frac{\tanh^{-1}(\cosh(a+bx))}{2b}-\frac{\coth(a+bx)\text{csch}(a+bx)}{2b}$$

```
[Out] 1/2*arctanh(cosh(b*x+a))/b-1/2*coth(b*x+a)*csch(b*x+a)/b
```

Rubi [A]
time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3853, 3855}

$$\frac{\tanh^{-1}(\cosh(a+bx))}{2b}-\frac{\coth(a+bx)\text{csch}(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^3,x]
```

```
[Out] ArcTanh[Cosh[a + b*x]]/(2*b) - (Coth[a + b*x]*Csch[a + b*x])/(2*b)
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \text{csch}^3(a+bx)\,dx&=-\frac{\coth(a+bx)\text{csch}(a+bx)}{2b}-\frac{1}{2}\int \text{csch}(a+bx)\,dx\\&=\frac{\tanh^{-1}(\cosh(a+bx))}{2b}-\frac{\coth(a+bx)\text{csch}(a+bx)}{2b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 57, normalized size = 1.68

$$-\frac{\text{csch}^2\left(\frac{1}{2}(a+bx)\right)}{8b}-\frac{\log\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)}{2b}-\frac{\text{sech}^2\left(\frac{1}{2}(a+bx)\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^3,x]
```

```
[Out] -1/8*Csch[(a + b*x)/2]^2/b - Log[Tanh[(a + b*x)/2]]/(2*b) - Sech[(a + b*x)/
2]^2/(8*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(30) = 60$.
time = 1.14, size = 65, normalized size = 1.91

method	result	size
risch	$-\frac{e^{bx+a}\left(e^{2bx+2a}+1\right)}{b\left(e^{2bx+2a}-1\right)^{2}}+\frac{\ln\left(e^{bx+a}+1\right)}{2b}-\frac{\ln\left(e^{bx+a}-1\right)}{2b}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -exp(b*x+a)*(exp(2*b*x+2*a)+1)/b/(exp(2*b*x+2*a)-1)^2+1/2/b*ln(exp(b*x+a)+1
)-1/2/b*ln(exp(b*x+a)-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(30) = 60$.
time = 0.27, size = 84, normalized size = 2.47

$$\frac{\log\left(e^{(-bx-a)}+1\right)}{2\,b}-\frac{\log\left(e^{(-bx-a)}-1\right)}{2\,b}+\frac{e^{(-bx-a)}+e^{(-3\,bx-3\,a)}}{b(2\,e^{(-2\,bx-2\,a)}-e^{(-4\,bx-4\,a)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*log(e^(-b*x - a) + 1)/b - 1/2*log(e^(-b*x - a) - 1)/b + (e^(-b*x - a) +
 e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(30) = 60$.
time = 0.41, size = 387, normalized size = 11.38

$$-\frac{2\,\cosh\left(bx+a\right)^{3}+6\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}+2\,\sinh\left(bx+a\right)^{3}-\left(\cosh\left(bx+a\right)^{4}+4\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3}+\sinh\left(bx+a\right)^{4}+2\left(3\,\cosh\left(bx+a\right)^{2}-1\right)\sinh\left(bx+a\right)^{2}-2\,\cosh\left(bx+a\right)^{2}+4\left(\cosh\left(bx+a\right)^{3}-\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right)+1\right)\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)+\left(\cosh\left(bx+a\right)^{4}+4\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3}+\sinh\left(bx+a\right)^{4}+2\left(3\,\cosh\left(bx+a\right)^{2}-1\right)\sinh\left(bx+a\right)^{2}-2\,\cosh\left(bx+a\right)^{2}+4\left(\cosh\left(bx+a\right)^{3}-\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right)+1\right)\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)-1\right)+2\left(3\,\cosh\left(bx+a\right)^{2}+1\right)\sinh\left(bx+a\right)+2\,\cosh\left(bx+a\right)}{2\left(b\cosh\left(bx+a\right)^{4}+4\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3}+b\sinh\left(bx+a\right)^{4}-2\,b\cosh\left(bx+a\right)^{2}+2\left(3\,b\cosh\left(bx+a\right)^{2}-b\right)\sinh\left(bx+a\right)^{2}+4\left(b\cosh\left(bx+a\right)^{3}-b\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right)+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*cosh(b*x + a)^3 + 6*cosh(b*x + a)*sinh(b*x + a)^2 + 2*sinh(b*x + a)
^3 - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 +
 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*
x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x +
```

```
a) + 1) + (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x +
a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(c
osh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh
(b*x + a) - 1) + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + 2*cosh(b*x + a))
/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4
 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 4*(b
*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**3,x)
```

```
[Out] Integral(csch(a + b*x)**3, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(30) = 60.
time = 0.38, size = 84, normalized size = 2.47

$$-\frac{\frac{4\left(e^{(bx+a)}+e^{(-bx-a)}\right)}{\left(e^{(bx+a)}+e^{(-bx-a)}\right)^2-4}-\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right)+\log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/4*(4*(e^(b*x + a) + e^(-b*x - a))/((e^(b*x + a) + e^(-b*x - a))^2 - 4) -
 log(e^(b*x + a) + e^(-b*x - a) + 2) + log(e^(b*x + a) + e^(-b*x - a) - 2))
/b
```

Mupad [**B**]
time = 1.40, size = 86, normalized size = 2.53

$$\frac{\operatorname{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}-\frac{2\,e^{a+b\,x}}{b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)}-\frac{e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(a + b*x)^3,x)
```

```
[Out] atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b)/(-b^2)^(1/2) - (2*exp(a + b*x))/(b*(
exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - exp(a + b*x)/(b*(exp(2*a + 2*
b*x) - 1))
```

3.4 $\int \operatorname{csch}^4(a+bx)\,dx$

Optimal. Leaf size=26

$$\frac{\coth(a+bx)}{b}-\frac{\coth^3(a+bx)}{3b}$$

```
[Out] coth(b*x+a)/b-1/3*coth(b*x+a)^3/b
```

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3852}

$$\frac{\coth(a+bx)}{b}-\frac{\coth^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^4,x]
```

```
[Out] Coth[a + b*x]/b - Coth[a + b*x]^3/(3*b)
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^4(a+bx)\,dx &= \frac{i\text{Subst}\left(\int \left(1+x^2\right)\,dx, x, -i\coth(a+bx)\right)}{b}\\ &= \frac{\coth(a+bx)}{b}-\frac{\coth^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 1.35

$$\frac{2\coth(a+bx)}{3b}-\frac{\coth(a+bx)\operatorname{csch}^2(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^4,x]
```

```
[Out] (2*Coth[a + b*x])/(3*b) - (Coth[a + b*x]*Csch[a + b*x]^2)/(3*b)
```

Maple [A]
time = 1.00, size = 32, normalized size = 1.23

method	result	size
risch	$-\frac{4\left(3\,\mathrm{e}^{2bx+2a}-1\right)}{3b\left(\mathrm{e}^{2bx+2a}-1\right)^{3}}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -4/3*(3*exp(2*b*x+2*a)-1)/b/(exp(2*b*x+2*a)-1)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(24) = 48$.
time = 0.27, size = 90, normalized size = 3.46

$$\frac{4\,e^{(-2\,bx-2\,a)}}{b\left(3\,e^{(-2\,bx-2\,a)}-3\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}-1\right)}-\frac{4}{3\,b\left(3\,e^{(-2\,bx-2\,a)}-3\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 4*e^(-2*b*x - 2*a)/(b*(3*e^(-2*b*x - 2*a) - 3*e^(-4*b*x - 4*a) + e^(-6*b*x
- 6*a) - 1)) - 4/3/(b*(3*e^(-2*b*x - 2*a) - 3*e^(-4*b*x - 4*a) + e^(-6*b*x
- 6*a) - 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(24) = 48$.
time = 0.35, size = 164, normalized size = 6.31

$$-\frac{8\,\left(\cosh\left(bx+a\right)+2\,\sinh\left(bx+a\right)\right)}{3\,\left(b\cosh\left(bx+a\right)^{5}+5\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{4}+b\sinh\left(bx+a\right)^{5}-3\,b\cosh\left(bx+a\right)^{3}+\left(10\,b\cosh\left(bx+a\right)^{2}-3\,b\right)\sinh\left(bx+a\right)^{3}+\left(10\,b\cosh\left(bx+a\right)^{3}-9\,b\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right)^{2}+2\,b\cosh\left(bx+a\right)+\left(5\,b\cosh\left(bx+a\right)^{4}-9\,b\cosh\left(bx+a\right)^{2}+4\,b\right)\sinh\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -8/3*(cosh(b*x + a) + 2*sinh(b*x + a))/(b*cosh(b*x + a)^5 + 5*b*cosh(b*x +
a)*sinh(b*x + a)^4 + b*sinh(b*x + a)^5 - 3*b*cosh(b*x + a)^3 + (10*b*cosh(b
*x + a)^2 - 3*b)*sinh(b*x + a)^3 + (10*b*cosh(b*x + a)^3 - 9*b*cosh(b*x + a
))*sinh(b*x + a)^2 + 2*b*cosh(b*x + a) + (5*b*cosh(b*x + a)^4 - 9*b*cosh(b*
x + a)^2 + 4*b)*sinh(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\operatorname{csch}^{4}\left(a+bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**4,x)
```

```
[Out] Integral(csch(a + b*x)**4, x)
```

Giac [A]
time = 0.40, size = 31, normalized size = 1.19

$$-\frac{4\left(3\,e^{(2\,bx+2\,a)}-1\right)}{3\,b\left(e^{(2\,bx+2\,a)}-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -4/3*(3*e^(2*b*x + 2*a) - 1)/(b*(e^(2*b*x + 2*a) - 1)^3)
```

Mupad [B]
time = 1.46, size = 31, normalized size = 1.19

$$-\frac{4\left(3\,\mathrm{e}^{2\,a+2\,b\,x}-1\right)}{3\,b\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(a + b*x)^4,x)
```

```
[Out] -(4*(3*exp(2*a + 2*b*x) - 1))/(3*b*(exp(2*a + 2*b*x) - 1)^3)
```

3.5 $\int \operatorname{csch}^5(a+bx)\,dx$

Optimal. Leaf size=55

$$-\frac{3\tanh^{-1}(\cosh(a+bx))}{8b}+\frac{3\coth(a+bx)\operatorname{csch}(a+bx)}{8b}-\frac{\coth(a+bx)\operatorname{csch}^3(a+bx)}{4b}$$

```
[Out] -3/8*arctanh(cosh(b*x+a))/b+3/8*coth(b*x+a)*csch(b*x+a)/b-1/4*coth(b*x+a)*c
sch(b*x+a)^3/b
```

Rubi [A]
time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3853, 3855}

$$-\frac{3\tanh^{-1}(\cosh(a+bx))}{8b}-\frac{\coth(a+bx)\operatorname{csch}^3(a+bx)}{4b}+\frac{3\coth(a+bx)\operatorname{csch}(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^5,x]
```

```
[Out] (-3*ArcTanh[Cosh[a + b*x]])/(8*b) + (3*Coth[a + b*x]*Csch[a + b*x])/(8*b) -
 (Coth[a + b*x]*Csch[a + b*x]^3)/(4*b)
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^5(a+bx)\,dx &= -\frac{\coth(a+bx)\operatorname{csch}^3(a+bx)}{4b}-\frac{3}{4}\int \operatorname{csch}^3(a+bx)\,dx\\
&= \frac{3\coth(a+bx)\operatorname{csch}(a+bx)}{8b}-\frac{\coth(a+bx)\operatorname{csch}^3(a+bx)}{4b}+\frac{3}{8}\int \operatorname{csch}(a+bx)\,dx\\
&= -\frac{3\tanh^{-1}(\cosh(a+bx))}{8b}+\frac{3\coth(a+bx)\operatorname{csch}(a+bx)}{8b}-\frac{\coth(a+bx)\operatorname{csch}^3(a+bx)}{4b}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 95, normalized size = 1.73

$$\frac{3\text{csch}^2\left(\frac{1}{2}(a+bx)\right)}{32b}-\frac{\text{csch}^4\left(\frac{1}{2}(a+bx)\right)}{64b}+\frac{3\log\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)}{8b}+\frac{3\text{sech}^2\left(\frac{1}{2}(a+bx)\right)}{32b}+\frac{\text{sech}^4\left(\frac{1}{2}(a+bx)\right)}{64b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^5,x]
```

```
[Out] (3*Csch[(a + b*x)/2]^2)/(32*b) - Csch[(a + b*x)/2]^4/(64*b) + (3*Log[Tanh[(
a + b*x)/2]])/(8*b) + (3*Sech[(a + b*x)/2]^2)/(32*b) + Sech[(a + b*x)/2]^4/
(64*b)
```

Maple [A]
time = 0.98, size = 89, normalized size = 1.62

method	result	size
risch	$\frac{e^{bx+a}\left(3\,e^{6bx+6a}-11\,e^{4bx+4a}-11\,e^{2bx+2a}+3\right)}{4b\left(e^{2bx+2a}-1\right)^4}+\frac{3\ln\left(e^{bx+a}-1\right)}{8b}-\frac{3\ln\left(e^{bx+a}+1\right)}{8b}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*exp(b*x+a)*(3*exp(6*b*x+6*a)-11*exp(4*b*x+4*a)-11*exp(2*b*x+2*a)+3)/b/(
exp(2*b*x+2*a)-1)^4+3/8/b*ln(exp(b*x+a)-1)-3/8/b*ln(exp(b*x+a)+1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(49) = 98$.
time = 0.27, size = 133, normalized size = 2.42

$$-\frac{3\,\log\left(e^{(-bx-a)}+1\right)}{8\,b}+\frac{3\,\log\left(e^{(-bx-a)}-1\right)}{8\,b}-\frac{3\,e^{(-bx-a)}-11\,e^{(-3\,bx-3\,a)}-11\,e^{(-5\,bx-5\,a)}+3\,e^{(-7\,bx-7\,a)}}{4\,b(4\,e^{(-2\,bx-2\,a)}-6\,e^{(-4\,bx-4\,a)}+4\,e^{(-6\,bx-6\,a)}-e^{(-8\,bx-8\,a)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -3/8*log(e^(-b*x - a) + 1)/b + 3/8*log(e^(-b*x - a) - 1)/b - 1/4*(3*e^(-b*x
 - a) - 11*e^(-3*b*x - 3*a) - 11*e^(-5*b*x - 5*a) + 3*e^(-7*b*x - 7*a))/(b*
(4*e^(-2*b*x - 2*a) - 6*e^(-4*b*x - 4*a) + 4*e^(-6*b*x - 6*a) - e^(-8*b*x -
 8*a) - 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1114 vs. $2(49) = 98$.
time = 0.40, size = 1114, normalized size = 20.25

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(b*x+a)^5,x, algorithm="fricas")

[Out] 1/8*(6*cosh(b*x + a)^7 + 42*cosh(b*x + a)*sinh(b*x + a)^6 + 6*sinh(b*x + a)^7 + 2*(63*cosh(b*x + a)^2 - 11)*sinh(b*x + a)^5 - 22*cosh(b*x + a)^5 + 10*(21*cosh(b*x + a)^3 - 11*cosh(b*x + a))*sinh(b*x + a)^4 + 2*(105*cosh(b*x + a)^4 - 110*cosh(b*x + a)^2 - 11)*sinh(b*x + a)^3 - 22*cosh(b*x + a)^3 + 2*(63*cosh(b*x + a)^5 - 110*cosh(b*x + a)^3 - 33*cosh(b*x + a))*sinh(b*x + a)^2 - 3*(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4*(7*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 8*(7*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 - 30*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 - 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 4*cosh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 3*(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4*(7*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 + 8*(7*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 - 30*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5 - 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a)^6 - 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 4*cosh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 2*(21*cosh(b*x + a)^6 - 55*cosh(b*x + a)^4 - 33*cosh(b*x + a)^2 + 3)*sinh(b*x + a) + 6*cosh(b*x + a))/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*sinh(b*x + a)^7 + b*sinh(b*x + a)^8 - 4*b*cosh(b*x + a)^6 + 4*(7*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^6 + 8*(7*b*cosh(b*x + a)^3 - 3*b*cosh(b*x + a))*sinh(b*x + a)^5 + 6*b*cosh(b*x + a)^4 + 2*(35*b*cosh(b*x + a)^4 - 30*b*cosh(b*x + a)^2 + 3*b)*sinh(b*x + a)^4 + 8*(7*b*cosh(b*x + a)^5 - 10*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^3 - 4*b*cosh(b*x + a)^2 + 4*(7*b*cosh(b*x + a)^6 - 15*b*cosh(b*x + a)^4 + 9*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 8*(b*cosh(b*x + a)^7 - 3*b*cosh(b*x + a)^5 + 3*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^5(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(b*x+a)**5,x)

[Out] Integral(csch(a + b*x)**5, x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(49) = 98.
time = 0.39, size = 110, normalized size = 2.00

$$\frac{\frac{4\left(3\left(e^{(bx+a)}+e^{(-bx-a)}\right)^{3}-20\,e^{(bx+a)}-20\,e^{(-bx-a)}\right)}{\left(\left(e^{(bx+a)}+e^{(-bx-a)}\right)^{2}-4\right)^{2}}-3\,\log\left(e^{(bx+a)}+e^{(-bx-a)}+2\right)+3\,\log\left(e^{(bx+a)}+e^{(-bx-a)}-2\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/16*(4*(3*(e^(b*x + a) + e^(-b*x - a))^3 - 20*e^(b*x + a) - 20*e^(-b*x - a
))/((e^(b*x + a) + e^(-b*x - a))^2 - 4)^2 - 3*log(e^(b*x + a) + e^(-b*x - a
) + 2) + 3*log(e^(b*x + a) + e^(-b*x - a) - 2))/b
```

Mupad [B]
time = 0.06, size = 193, normalized size = 3.51

$$\frac{3\,e^{a+b\,x}}{4\,b\,\left(e^{2\,a+2\,b\,x}-1\right)}-\frac{e^{a+b\,x}}{2\,b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)}-\frac{2\,e^{a+b\,x}}{b\,\left(3\,e^{2\,a+2\,b\,x}-3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}-1\right)}-\frac{4\,e^{3\,a+3\,b\,x}}{b\,\left(6\,e^{4\,a+4\,b\,x}-4\,e^{2\,a+2\,b\,x}-4\,e^{6\,a+6\,b\,x}+e^{8\,a+8\,b\,x}+1\right)}-\frac{3\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{4\,\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(a + b*x)^5,x)
```

```
[Out] (3*exp(a + b*x))/(4*b*(exp(2*a + 2*b*x) - 1)) - exp(a + b*x)/(2*b*(exp(4*a
+ 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (2*exp(a + b*x))/(b*(3*exp(2*a + 2*b*
x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1)) - (4*exp(3*a + 3*b*x))/(b*
(6*exp(4*a + 4*b*x) - 4*exp(2*a + 2*b*x) - 4*exp(6*a + 6*b*x) + exp(8*a + 8
*b*x) + 1)) - (3*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(4*(-b^2)^(1/2))
```

3.6 $\int \operatorname{csch}^6(a+bx)\,dx$

Optimal. Leaf size=42

$$-\frac{\coth(a+bx)}{b}+\frac{2\coth^3(a+bx)}{3b}-\frac{\coth^5(a+bx)}{5b}$$

```
[Out] -coth(b*x+a)/b+2/3*coth(b*x+a)^3/b-1/5*coth(b*x+a)^5/b
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3852}

$$-\frac{\coth^5(a+bx)}{5b}+\frac{2\coth^3(a+bx)}{3b}-\frac{\coth(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^6,x]
```

```
[Out] -(Coth[a + b*x]/b) + (2*Coth[a + b*x]^3)/(3*b) - Coth[a + b*x]^5/(5*b)
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^6(a+bx)\,dx &= -\frac{i\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx, x, -i\coth(a+bx)\right)}{b}\\ &= -\frac{\coth(a+bx)}{b}+\frac{2\coth^3(a+bx)}{3b}-\frac{\coth^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 56, normalized size = 1.33

$$-\frac{8\coth(a+bx)}{15b}+\frac{4\coth(a+bx)\operatorname{csch}^2(a+bx)}{15b}-\frac{\coth(a+bx)\operatorname{csch}^4(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^6,x]
```

```
[Out] (-8*Coth[a + b*x])/(15*b) + (4*Coth[a + b*x]*Csch[a + b*x]^2)/(15*b) - (Cot
h[a + b*x]*Csch[a + b*x]^4)/(5*b)
```

Maple [A]
time = 0.99, size = 43, normalized size = 1.02

method	result	size
risch	$-\frac{16\left(10\,{e}^{4 b x +4 a}-5\,{e}^{2 b x +2 a}+1\right)}{15 b \left({e}^{2 b x +2 a}-1\right)^{5}}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^6,x,method=_RETURNVERBOSE)
```

```
[Out] -16/15*(10*exp(4*b*x+4*a)-5*exp(2*b*x+2*a)+1)/b/(exp(2*b*x+2*a)-1)^5
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(38) = 76.
time = 0.26, size = 205, normalized size = 4.88

$$-\frac{16\,e^{\left(-2\,bx-2\,a\right)}}{3\,b{\left(5\,e^{\left(-2\,bx-2\,a\right)}-10\,e^{\left(-4\,bx-4\,a\right)}+10\,e^{\left(-6\,bx-6\,a\right)}-5\,e^{\left(-8\,bx-8\,a\right)}+e^{\left(-10\,bx-10\,a\right)}-1\right)}}+\frac{32\,e^{\left(-4\,bx-4\,a\right)}}{3\,b{\left(5\,e^{\left(-2\,bx-2\,a\right)}-10\,e^{\left(-4\,bx-4\,a\right)}+10\,e^{\left(-6\,bx-6\,a\right)}-5\,e^{\left(-8\,bx-8\,a\right)}+e^{\left(-10\,bx-10\,a\right)}-1\right)}}+\frac{16}{15\,b{\left(5\,e^{\left(-2\,bx-2\,a\right)}-10\,e^{\left(-4\,bx-4\,a\right)}+10\,e^{\left(-6\,bx-6\,a\right)}-5\,e^{\left(-8\,bx-8\,a\right)}+e^{\left(-10\,bx-10\,a\right)}-1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] -16/3*e^(-2*b*x - 2*a)/(b*(5*e^(-2*b*x - 2*a) - 10*e^(-4*b*x - 4*a) + 10*e^
(-6*b*x - 6*a) - 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) - 1)) + 32/3*e^(-4
*b*x - 4*a)/(b*(5*e^(-2*b*x - 2*a) - 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x - 6
*a) - 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) - 1)) + 16/15/(b*(5*e^(-2*b*x
 - 2*a) - 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x - 6*a) - 5*e^(-8*b*x - 8*a) +
e^(-10*b*x - 10*a) - 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 344 vs. 2(38) = 76.
time = 0.42, size = 344, normalized size = 8.19

$$-\frac{16\,{\left(11\,\cosh\left(bx+a\right)^{2}+18\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+11\,\sinh\left(bx+a\right)^{2}-5\right)}}{15\,{\left(b\cosh\left(bx+a\right)^{8}+8\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{7}+b\sinh\left(bx+a\right)^{8}-5\,b\cosh\left(bx+a\right)^{6}+{\left(28\,b\cosh\left(bx+a\right)^{2}-5\,b\right)}\sinh\left(bx+a\right)^{6}+2\,{\left(28\,b\cosh\left(bx+a\right)^{3}-15\,b\cosh\left(bx+a\right)\right)}\sinh\left(bx+a\right)^{5}+10\,b\cosh\left(bx+a\right)^{4}+5\,{\left(14\,b\cosh\left(bx+a\right)^{4}-15\,b\cosh\left(bx+a\right)^{2}+2\,b\right)}\sinh\left(bx+a\right)^{4}+4\,{\left(14\,b\cosh\left(bx+a\right)^{5}-25\,b\cosh\left(bx+a\right)^{3}+10\,b\cosh\left(bx+a\right)\right)}\sinh\left(bx+a\right)^{3}-11\,b\cosh\left(bx+a\right)^{2}+{\left(28\,b\cosh\left(bx+a\right)^{6}-75\,b\cosh\left(bx+a\right)^{4}+60\,b\cosh\left(bx+a\right)^{2}-11\,b\right)}\sinh\left(bx+a\right)^{2}+2\,{\left(4\,b\cosh\left(bx+a\right)^{7}-15\,b\cosh\left(bx+a\right)^{5}+20\,b\cosh\left(bx+a\right)^{3}-9\,b\cosh\left(bx+a\right)\right)}\sinh\left(bx+a\right)+5\,b\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] -16/15*(11*cosh(b*x + a)^2 + 18*cosh(b*x + a)*sinh(b*x + a) + 11*sinh(b*x +
 a)^2 - 5)/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*sinh(b*x + a)^7 + b*sinh(
b*x + a)^8 - 5*b*cosh(b*x + a)^6 + (28*b*cosh(b*x + a)^2 - 5*b)*sinh(b*x +
a)^6 + 2*(28*b*cosh(b*x + a)^3 - 15*b*cosh(b*x + a))*sinh(b*x + a)^5 + 10*b
*cosh(b*x + a)^4 + 5*(14*b*cosh(b*x + a)^4 - 15*b*cosh(b*x + a)^2 + 2*b)*si
nh(b*x + a)^4 + 4*(14*b*cosh(b*x + a)^5 - 25*b*cosh(b*x + a)^3 + 10*b*cosh(
```

```
b*x + a))*sinh(b*x + a)^3 - 11*b*cosh(b*x + a)^2 + (28*b*cosh(b*x + a)^6 -
75*b*cosh(b*x + a)^4 + 60*b*cosh(b*x + a)^2 - 11*b)*sinh(b*x + a)^2 + 2*(4*
b*cosh(b*x + a)^7 - 15*b*cosh(b*x + a)^5 + 20*b*cosh(b*x + a)^3 - 9*b*cosh(
b*x + a))*sinh(b*x + a) + 5*b)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^6(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**6,x)
```

```
[Out] Integral(csch(a + b*x)**6, x)
```

Giac [**A**]
time = 0.40, size = 42, normalized size = 1.00

$$-\frac{16\left(10\,e^{(4\,bx+4\,a)}-5\,e^{(2\,bx+2\,a)}+1\right)}{15\,b\left(e^{(2\,bx+2\,a)}-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^6,x, algorithm="giac")
```

```
[Out] -16/15*(10*e^(4*b*x + 4*a) - 5*e^(2*b*x + 2*a) + 1)/(b*(e^(2*b*x + 2*a) - 1
)^5)
```

Mupad [**B**]
time = 0.08, size = 42, normalized size = 1.00

$$-\frac{16\left(10\,\mathrm{e}^{4\,a+4\,b\,x}-5\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)}{15\,b\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(a + b*x)^6,x)
```

```
[Out] -(16*(10*exp(4*a + 4*b*x) - 5*exp(2*a + 2*b*x) + 1))/(15*b*(exp(2*a + 2*b*x
) - 1)^5)
```

3.7 $\int \operatorname{csch}^{\frac{5}{2}}(a+bx)\,dx$

<u>**Optimal**</u>. Leaf size=80

$$-\frac{2\cosh(a+bx)\operatorname{csch}^{\frac{3}{2}}(a+bx)}{3b}+\frac{2i\sqrt{\operatorname{csch}(a+bx)}\,F\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|\,2\right)\sqrt{i\sinh(a+bx)}}{3b}$$

```
[Out] -2/3*cosh(b*x+a)*csch(b*x+a)^(3/2)/b-2/3*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2
)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*EllipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*
x),2^(1/2))*csch(b*x+a)^(1/2)*(I*sinh(b*x+a))^(1/2)/b
```

<u>**Rubi**</u> [A]
time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.300$, Rules used = {3853, 3856, 2720}

$$-\frac{2\cosh(a+bx)\operatorname{csch}^{\frac{3}{2}}(a+bx)}{3b}+\frac{2i\sqrt{i\sinh(a+bx)}\,\sqrt{\operatorname{csch}(a+bx)}\,F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|\,2\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^(5/2),x]
```

```
[Out] (-2*Cosh[a + b*x]*Csch[a + b*x]^(3/2))/(3*b) + (((2*I)/3)*Sqrt[Csch[a + b*x
]]*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/b
```

<u>Rule 2720</u>

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

<u>Rule 3853</u>

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

<u>Rule 3856</u>

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

<u>Rubi steps</u>

$$\int \operatorname{csch}^{\frac{5}{2}}(a+bx)\,dx = -\frac{2\cosh(a+bx)\operatorname{csch}^{\frac{3}{2}}(a+bx)}{3b} - \frac{1}{3}\int \sqrt{\operatorname{csch}(a+bx)}\,dx$$

$$= -\frac{2\cosh(a+bx)\operatorname{csch}^{\frac{3}{2}}(a+bx)}{3b} - \frac{1}{3}\left(\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}\right)\int \frac{1}{\sqrt{i\sinh(a}}$$

$$= -\frac{2\cosh(a+bx)\operatorname{csch}^{\frac{3}{2}}(a+bx)}{3b} + \frac{2i\sqrt{\operatorname{csch}(a+bx)}\,F\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{i\sinh(a}}{3b}$$

Mathematica [**A**]
time = 0.09, size = 61, normalized size = 0.76

$$-\frac{2\sqrt{\operatorname{csch}(a+bx)}\left(\coth(a+bx)+iF\left(\frac{1}{4}(-2ia+\pi-2ibx)\middle|2\right)\sqrt{i\sinh(a+bx)}\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^(5/2),x]
```

```
[Out] (-2*Sqrt[Csch[a + b*x]]*(Coth[a + b*x] + I*EllipticF[((-2*I)*a + Pi - (2*I)
*b*x)/4, 2]*Sqrt[I*Sinh[a + b*x]]))/(3*b)
```

Maple [**A**]
time = 1.58, size = 101, normalized size = 1.26

method	result
default	$-\frac{i\sqrt{1-i\sinh(bx+a)}\,\sqrt{2}\,\sqrt{1+i\sinh(bx+a)}\,\sqrt{i\sinh(bx+a)}\,\text{EllipticF}\left(\sqrt{1-i\sinh(bx+a}\right.}{3\sinh(bx+a)^{\frac{3}{2}}\cosh(bx+a)b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/sinh(b*x+a)^(3/2)*(I*(1-I*sinh(b*x+a))^(1/2)*2^(1/2)*(1+I*sinh(b*x+a))
^(1/2)*(I*sinh(b*x+a))^(1/2)*EllipticF((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))
*sinh(b*x+a)+2*cosh(b*x+a)^2)/cosh(b*x+a)/b
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csch(b*x + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 194, normalized size = 2.42

$$-\frac{2\left(\sqrt{2}\left(\cosh\left(bx+a\right)^{2}+2\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^{2}+1\right)\sqrt{\frac{\cosh\left(bx+a\right)+\sinh\left(bx+a\right)}{\cosh\left(bx+a\right)^{2}+2\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^{2}-1}}+\left(\sqrt{2}\cosh\left(bx+a\right)^{2}+2\sqrt{2}\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sqrt{2}\sinh\left(bx+a\right)^{2}-\sqrt{2}\right)\mathrm{weierstrassPInverse}\left(4,0,\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)}{3\left(b\cosh\left(bx+a\right)^{2}+2b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^{2}-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(sqrt(2)*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x +
 a)^2 + 1)*sqrt((cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b
*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) + (sqrt(2)*cosh(b*x + a)^2 +
2*sqrt(2)*cosh(b*x + a)*sinh(b*x + a) + sqrt(2)*sinh(b*x + a)^2 - sqrt(2))*
weierstrassPInverse(4, 0, cosh(b*x + a) + sinh(b*x + a)))/(b*cosh(b*x + a)^
2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^{\frac{5}{2}}\left(a+bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**(5/2),x)
```

```
[Out] Integral(csch(a + b*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{1}{\sinh\left(a+b\,x\right)}\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sinh(a + b*x))^(5/2),x)
```

```
[Out] int((1/sinh(a + b*x))^(5/2), x)
```

3.8 $\int \operatorname{csch}^{\frac{3}{2}}(a+bx)\,dx$

Optimal. Leaf size=76

$$-\frac{2\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{b}-\frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)}{b\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}$$

```
[Out] -2*cosh(b*x+a)*csch(b*x+a)^(1/2)/b+2*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1
/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*EllipticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2
^(1/2))/b/csch(b*x+a)^(1/2)/(I*sinh(b*x+a))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3853, 3856, 2719}

$$-\frac{2\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{b}-\frac{2iE\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^(3/2),x]
```

```
[Out] (-2*Cosh[a + b*x]*Sqrt[Csch[a + b*x]])/b - ((2*I)*EllipticE[(I*a - Pi/2 + I
*b*x)/2, 2])/(b*Sqrt[Csch[a + b*x]]*Sqrt[I*Sinh[a + b*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \operatorname{csch}^{\frac{3}{2}}(a+bx)\,dx = -\frac{2\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{b} + \int \frac{1}{\sqrt{\operatorname{csch}(a+bx)}}\,dx$$
$$= -\frac{2\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{b} + \frac{\int \sqrt{i\sinh(a+bx)}\,dx}{\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}$$
$$= -\frac{2\cosh(a+bx)\sqrt{\operatorname{csch}(a+bx)}}{b} - \frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)}{b\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}$$

Mathematica [**A**]
time = 0.16, size = 57, normalized size = 0.75

$$-\frac{2\sqrt{\operatorname{csch}(a+bx)}\left(\cosh(a+bx)-E\left(\frac{1}{4}(-2ia+\pi-2ibx)\middle|2\right)\sqrt{i\sinh(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^(3/2),x]
```

```
[Out] (-2*Sqrt[Csch[a + b*x]]*(Cosh[a + b*x] - EllipticE[((-2*I)*a + Pi - (2*I)*b
*x)/4, 2]*Sqrt[I*Sinh[a + b*x]]))/b
```

Maple [**A**]
time = 1.41, size = 154, normalized size = 2.03

method	result
default	$\frac{2\sqrt{1-i\sinh(bx+a)}\sqrt{2}\sqrt{1+i\sinh(bx+a)}\sqrt{i\sinh(bx+a)}\,\mathrm{EllipticE}\left(\sqrt{1-i\sinh(bx+a)},\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (2*(1-I*sinh(b*x+a))^(1/2)*2^(1/2)*(1+I*sinh(b*x+a))^(1/2)*(I*sinh(b*x+a))^
(1/2)*EllipticE((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))-(1-I*sinh(b*x+a))^(1/2
)*2^(1/2)*(1+I*sinh(b*x+a))^(1/2)*(I*sinh(b*x+a))^(1/2)*EllipticF((1-I*sinh
(b*x+a))^(1/2),1/2*2^(1/2))-2*cosh(b*x+a)^2)/cosh(b*x+a)/sinh(b*x+a)^(1/2)/
b
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csch(b*x + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 96, normalized size = 1.26

$$-\frac{2\left(\sqrt{2}\sqrt{\frac{\cosh(bx+a)+\sinh(bx+a)}{\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2-1}}(\cosh(bx+a)+\sinh(bx+a))+\sqrt{2}\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cosh(bx+a)+\sinh(bx+a)))\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(sqrt(2)*sqrt((cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(
b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1))*(cosh(b*x + a) + sinh(b*x +
a)) + sqrt(2)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cosh(b*x + a)
 + sinh(b*x + a))))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^{\frac{3}{2}}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**(3/2),x)
```

```
[Out] Integral(csch(a + b*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{1}{\sinh\left(a+b\,x\right)}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sinh(a + b*x))^(3/2),x)
```

```
[Out] int((1/sinh(a + b*x))^(3/2), x)
```

3.9 $\int \sqrt{\operatorname{csch}(a+bx)}\, dx$

Optimal. Leaf size=54

$$-\frac{2i\sqrt{\operatorname{csch}(a+bx)}\, F\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|\, 2\right) \sqrt{i\sinh(a+bx)}}{b}$$

```
[Out] 2*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*E
llipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))*csch(b*x+a)^(1/2)*(I*sinh(b
*x+a))^(1/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3856, 2720}

$$-\frac{2i\sqrt{i\sinh(a+bx)}\, \sqrt{\operatorname{csch}(a+bx)}\, F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|\, 2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Csch[a + b*x]],x]
```

```
[Out] ((-2*I)*Sqrt[Csch[a + b*x]]*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sin
h[a + b*x]])/b
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\operatorname{csch}(a+bx)}\, dx &= \left(\sqrt{\operatorname{csch}(a+bx)}\, \sqrt{i\sinh(a+bx)}\right)\int \frac{1}{\sqrt{i\sinh(a+bx)}}\, dx\\ &= -\frac{2i\sqrt{\operatorname{csch}(a+bx)}\, F\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|\, 2\right) \sqrt{i\sinh(a+bx)}}{b}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 48, normalized size = 0.89

$$\frac{2\text{csch}^{\frac{3}{2}}(a+bx)F\left(\frac{1}{4}(-2ia+\pi-2ibx)\middle|2\right)(i\sinh(a+bx))^{3/2}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Csch[a + b*x]],x]
```

```
[Out] (2*Csch[a + b*x]^(3/2)*EllipticF[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*(I*Sinh[
a + b*x])^(3/2))/b
```

Maple [A]
time = 0.92, size = 87, normalized size = 1.61

method	result
default	$\frac{i\sqrt{-i(\sinh(bx+a)+i)}\,\sqrt{2}\,\sqrt{-i(-\sinh(bx+a)+i)}\,\sqrt{i\sinh(bx+a)}\,\text{EllipticF}\left(\sqrt{-i(\sinh(\right.}{\cosh(bx+a)\sqrt{\sinh(bx+a)}\,b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] I*(-I*(sinh(b*x+a)+I))^(1/2)*2^(1/2)*(-I*(-sinh(b*x+a)+I))^(1/2)*(I*sinh(b*
x+a))^(1/2)*EllipticF((-I*(sinh(b*x+a)+I))^(1/2),1/2*2^(1/2))/cosh(b*x+a)/s
inh(b*x+a)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(csch(b*x + a)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 24, normalized size = 0.44

$$\frac{2\sqrt{2}\,\text{weierstrassPInverse}(4,0,\cosh(bx+a)+\sinh(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*weierstrassPInverse(4, 0, cosh(b*x + a) + sinh(b*x + a))/b
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\operatorname{csch}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(csch(a + b*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(csch(b*x + a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\frac{1}{\sinh\left(a+b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sinh(a + b*x))^(1/2),x)
```

```
[Out] int((1/sinh(a + b*x))^(1/2), x)
```

3.10 $\int \frac{1}{\sqrt{\operatorname{csch}(a+bx)}}\,dx$

Optimal. Leaf size=54

$$-\frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)}{b\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}$$

```
[Out] 2*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*E
llipticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))/b/csch(b*x+a)^(1/2)/(I*sinh
(b*x+a))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3856, 2719}

$$-\frac{2iE\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Csch[a + b*x]],x]
```

```
[Out] ((-2*I)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2])/(b*Sqrt[Csch[a + b*x]]*Sqrt[I
*Sinh[a + b*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\operatorname{csch}(a+bx)}}\,dx &= \frac{\int \sqrt{i\sinh(a+bx)}\,dx}{\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}\\ &= -\frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)}{b\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 50, normalized size = 0.93

$$\frac{2\sqrt{\operatorname{csch}(a+bx)}\,E\left(\frac{1}{2}\left(\frac{\pi}{2}-i(a+bx)\right)\middle|\,2\right)\sqrt{i\sinh(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Csch[a + b*x]],x]
```

```
[Out] (2*Sqrt[Csch[a + b*x]]*EllipticE[(Pi/2 - I*(a + b*x))/2, 2]*Sqrt[I*Sinh[a +
 b*x]])/b
```

Maple [A]
time = 1.18, size = 108, normalized size = 2.00

method	result
default	$\frac{\sqrt{-i\left(\sinh\left(bx+a\right)+i\right)}\,\sqrt{2}\,\sqrt{-i\left(-\sinh\left(bx+a\right)+i\right)}\,\sqrt{i\sinh\left(bx+a\right)}\,\left(2\,\mathrm{EllipticE}\left(\sqrt{1-i\sinh\right.\right.}{\cosh\left(bx+a\right)\sqrt{\sinh\left(bx+a\right)}\,b}$
risch	$\frac{\sqrt{2}}{b\sqrt{\frac{\mathrm{e}^{bx+a}}{\mathrm{e}^{2bx+2a}-1}}}-\frac{\left(\frac{2\,\mathrm{e}^{2bx+2a}-2}{\sqrt{\left(\mathrm{e}^{2bx+2a}-1\right)\mathrm{e}^{bx+a}}}-\frac{\sqrt{\mathrm{e}^{bx+a}+1}\,\sqrt{-2\,\mathrm{e}^{bx+a}+2}\,\sqrt{-\mathrm{e}^{bx+a}}\,\left(-2\,\mathrm{EllipticE}\left(\sqrt{\mathrm{e}^{bx+a}}\right.\right.}{\sqrt{\mathrm{e}^{3bx+3a}-\mathrm{e}^{bx+a}}}\right.}{b\sqrt{\frac{\mathrm{e}^{bx+a}}{\mathrm{e}^{2bx+2a}-1}}\,\left(\mathrm{e}^{2bx+2a}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/csch(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-I*(sinh(b*x+a)+I))^(1/2)*2^(1/2)*(-I*(-sinh(b*x+a)+I))^(1/2)*(I*sinh(b*x+
a))^(1/2)*(2*EllipticE((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))-EllipticF((1-I*
sinh(b*x+a))^(1/2),1/2*2^(1/2)))/cosh(b*x+a)/sinh(b*x+a)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(csch(b*x + a)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 150, normalized size = 2.78

$$-\frac{\sqrt{2}\left(\cosh\left(bx+a\right)^{2}+2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^{2}-1\right)\sqrt{\frac{\cosh\left(bx+a\right)+\sinh\left(bx+a\right)}{\cosh\left(bx+a\right)^{2}+2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^{2}-1}}+2\left(\sqrt{2}\cosh\left(bx+a\right)+\sqrt{2}\sinh\left(bx+a\right)\right)\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cosh\left(bx+a\right)+\sinh\left(bx+a\right)))}{b\cosh\left(bx+a\right)+b\sinh\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/csch(b*x+a)^(1/2),x, algorithm="fricas")

[Out] -(sqrt(2)*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*sqrt((cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) + 2*(sqrt(2)*cosh(b*x + a) + sqrt(2)*sinh(b*x + a))*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cosh(b*x + a) + sinh(b*x + a))))/(b*cosh(b*x + a) + b*sinh(b*x + a))

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\operatorname{csch}(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/csch(b*x+a)**(1/2),x)

[Out] Integral(1/sqrt(csch(a + b*x)), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/csch(b*x+a)^(1/2),x, algorithm="giac")

[Out] integrate(1/sqrt(csch(b*x + a)), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\frac{1}{\sinh(a+b\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1/sinh(a + b*x))^(1/2),x)

[Out] int(1/(1/sinh(a + b*x))^(1/2), x)

3.11 $\int \frac{1}{\mathbf{csch}^{\frac{3}{2}}(a+bx)}\,dx$

Optimal. Leaf size=80

$$\frac{2\cosh(a+bx)}{3b\sqrt{\operatorname{csch}(a+bx)}}+\frac{2i\sqrt{\operatorname{csch}(a+bx)}\,F\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{i\sinh(a+bx)}}{3b}$$

```
[Out] 2/3*cosh(b*x+a)/b/csch(b*x+a)^(1/2)-2/3*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)
^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*EllipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*x
),2^(1/2))*csch(b*x+a)^(1/2)*(I*sinh(b*x+a))^(1/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3854, 3856, 2720}

$$\frac{2\cosh(a+bx)}{3b\sqrt{\operatorname{csch}(a+bx)}}+\frac{2i\sqrt{i\sinh(a+bx)}\,\sqrt{\operatorname{csch}(a+bx)}\,F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^(-3/2),x]
```

```
[Out] (2*Cosh[a + b*x])/(3*b*Sqrt[Csch[a + b*x]]) + (((2*I)/3)*Sqrt[Csch[a + b*x]
]*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/b
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(a+bx)}\,dx = \frac{2\cosh(a+bx)}{3b\sqrt{\operatorname{csch}(a+bx)}} - \frac{1}{3}\int \sqrt{\operatorname{csch}(a+bx)}\,dx$$
$$= \frac{2\cosh(a+bx)}{3b\sqrt{\operatorname{csch}(a+bx)}} - \frac{1}{3}\left(\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}\right)\int \frac{1}{\sqrt{i\sinh(a+bx)}}\,dx$$
$$= \frac{2\cosh(a+bx)}{3b\sqrt{\operatorname{csch}(a+bx)}} + \frac{2i\sqrt{\operatorname{csch}(a+bx)}\,F\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|\,2\right)\sqrt{i\sinh(a+bx)}}{3b}$$

Mathematica [A]
time = 0.05, size = 63, normalized size = 0.79

$$\frac{\sqrt{\operatorname{csch}(a+bx)}\left(-2iF\left(\frac{1}{4}(-2ia+\pi-2ibx)\middle|\,2\right)\sqrt{i\sinh(a+bx)}+\sinh(2(a+bx))\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^(-3/2),x]
```

```
[Out] (Sqrt[Csch[a + b*x]]*((-2*I)*EllipticF[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sq
rt[I*Sinh[a + b*x]] + Sinh[2*(a + b*x)]))/(3*b)
```

Maple [A]
time = 1.30, size = 100, normalized size = 1.25

method	result
default	$-\frac{\frac{i\sqrt{1-i\sinh(bx+a)}\,\sqrt{2}\,\sqrt{1+i\sinh(bx+a)}\,\sqrt{i\sinh(bx+a)}\,\mathrm{EllipticF}\left(\sqrt{1-i\sinh(bx+a)}\cdots\right.}{3}}{\cosh(bx+a)\sqrt{\sinh(bx+a)}\,b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/csch(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/3*I*(1-I*sinh(b*x+a))^(1/2)*2^(1/2)*(1+I*sinh(b*x+a))^(1/2)*(I*sinh(b*x
+a))^(1/2)*EllipticF((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))+2/3*cosh(b*x+a)^2
*sinh(b*x+a))/cosh(b*x+a)/sinh(b*x+a)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(b*x+a)^(3/2),x, algorithm="maxima")
[Out] integrate(csch(b*x + a)^(-3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 223, normalized size = 2.79

$$\frac{\sqrt{2}\left(\cosh\left(bx+a\right)^{4}+4\,\cosh\left(bx+a\right)^{3}\sinh\left(bx+a\right)+6\,\cosh\left(bx+a\right)^{2}\sinh\left(bx+a\right)^{2}+4\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3}+\sinh\left(bx+a\right)^{4}-1\right)\sqrt{\frac{\cosh\left(bx+a\right)+\sinh\left(bx+a\right)}{\cosh\left(bx+a\right)^{2}+2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^{2}-1}}-4\left(\sqrt{2}\,\cosh\left(bx+a\right)^{2}+2\,\sqrt{2}\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sqrt{2}\,\sinh\left(bx+a\right)^{2}\right)\mathrm{weierstrassPInverse}\left(4,0,\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)}{6\left(b\cosh\left(bx+a\right)^{2}+2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(b*x+a)^(3/2),x, algorithm="fricas")
[Out] 1/6*(sqrt(2)*(cosh(b*x + a)^4 + 4*cosh(b*x + a)^3*sinh(b*x + a) + 6*cosh(b*
x + a)^2*sinh(b*x + a)^2 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^
4 - 1)*sqrt((cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b*x +
 a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) - 4*(sqrt(2)*cosh(b*x + a)^2 + 2*
sqrt(2)*cosh(b*x + a)*sinh(b*x + a) + sqrt(2)*sinh(b*x + a)^2)*weierstrassP
Inverse(4, 0, cosh(b*x + a) + sinh(b*x + a)))/(b*cosh(b*x + a)^2 + 2*b*cosh
(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(b*x+a)**(3/2),x)
[Out] Integral(csch(a + b*x)**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(b*x+a)^(3/2),x, algorithm="giac")
[Out] integrate(csch(b*x + a)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(\frac{1}{\mathrm{sinh}\left(a+b\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/sinh(a + b*x))^(3/2),x)
[Out] int(1/(1/sinh(a + b*x))^(3/2), x)
```

3.12 $\int \frac{1}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}\,dx$

Optimal. Leaf size=80

$$\frac{2\cosh(a+bx)}{5b\operatorname{csch}^{\frac{3}{2}}(a+bx)}+\frac{6iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)}{5b\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}$$

```
[Out] 2/5*cosh(b*x+a)/b/csch(b*x+a)^(3/2)-6/5*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)
^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*EllipticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*x
),2^(1/2))/b/csch(b*x+a)^(1/2)/(I*sinh(b*x+a))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3854, 3856, 2719}

$$\frac{2\cosh(a+bx)}{5b\operatorname{csch}^{\frac{3}{2}}(a+bx)}+\frac{6iE\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{5b\sqrt{i\sinh(a+bx)}\sqrt{\operatorname{csch}(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*x]^(-5/2),x]
```

```
[Out] (2*Cosh[a + b*x])/(5*b*Csch[a + b*x]^(3/2)) + (((6*I)/5)*EllipticE[(I*a - P
i/2 + I*b*x)/2, 2])/(b*Sqrt[Csch[a + b*x]]*Sqrt[I*Sinh[a + b*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{\operatorname{csch}^{\frac{5}{2}}(a+bx)}\,dx = \frac{2\cosh(a+bx)}{5b\operatorname{csch}^{\frac{3}{2}}(a+bx)} - \frac{3}{5}\int \frac{1}{\sqrt{\operatorname{csch}(a+bx)}}\,dx$$
$$= \frac{2\cosh(a+bx)}{5b\operatorname{csch}^{\frac{3}{2}}(a+bx)} - \frac{3\int \sqrt{i\sinh(a+bx)}\,dx}{5\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}$$
$$= \frac{2\cosh(a+bx)}{5b\operatorname{csch}^{\frac{3}{2}}(a+bx)} + \frac{6iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)}{5b\sqrt{\operatorname{csch}(a+bx)}\sqrt{i\sinh(a+bx)}}$$

Mathematica [A]
time = 0.09, size = 67, normalized size = 0.84

$$\frac{2\left(\cosh(a+bx) - 3\operatorname{csch}^2(a+bx)E\left(\frac{1}{4}(-2ia+\pi-2ibx)\middle|2\right)\sqrt{i\sinh(a+bx)}\right)}{5b\operatorname{csch}^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*x]^(-5/2),x]
[Out] (2*(Cosh[a + b*x] - 3*Csch[a + b*x]^2*EllipticE[((-2*I)*a + Pi - (2*I)*b*x)
/4, 2]*Sqrt[I*Sinh[a + b*x]]))/(5*b*Csch[a + b*x]^(3/2))
```

Maple [A]
time = 1.38, size = 164, normalized size = 2.05

method	result
default	$-\frac{6\sqrt{1-i\sinh(bx+a)}\,\sqrt{2}\,\sqrt{1+i\sinh(bx+a)}\,\sqrt{i\sinh(bx+a)}\,\mathrm{EllipticE}\left(\sqrt{1-i\sinh(bx+a)},\right.}{5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/csch(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
[Out] (-6/5*(1-I*sinh(b*x+a))^(1/2)*2^(1/2)*(1+I*sinh(b*x+a))^(1/2)*(I*sinh(b*x+a
))^(1/2)*EllipticE((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))+3/5*(1-I*sinh(b*x+a
))^(1/2)*2^(1/2)*(1+I*sinh(b*x+a))^(1/2)*(I*sinh(b*x+a))^(1/2)*EllipticF((1
-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))+2/5*cosh(b*x+a)^4-2/5*cosh(b*x+a)^2)/cos
h(b*x+a)/sinh(b*x+a)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csch(b*x + a)^(-5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 370, normalized size = 4.62

$$\frac{\sqrt{2}\left(\cosh\left(bx+a\right)^{6}+6\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{5}+\sinh\left(bx+a\right)^{6}+\left(15\,\cosh\left(bx+a\right)^{2}+11\right)\sinh\left(bx+a\right)^{4}+11\,\cosh\left(bx+a\right)^{4}+4\left(5\,\cosh\left(bx+a\right)^{3}+11\,\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right)^{3}+\left(15\,\cosh\left(bx+a\right)^{4}+66\,\cosh\left(bx+a\right)^{2}-13\right)\sinh\left(bx+a\right)^{2}-13\,\cosh\left(bx+a\right)^{2}+2\left(3\,\cosh\left(bx+a\right)^{5}+22\,\cosh\left(bx+a\right)^{3}-13\,\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right)+1\right)\sqrt{\frac{\cosh\left(bx+a\right)+\sinh\left(bx+a\right)}{\cosh\left(bx+a\right)^{2}+2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^{2}-1}}+24\left(\sqrt{2}\,\cosh\left(bx+a\right)^{3}+3\,\sqrt{2}\,\cosh\left(bx+a\right)^{2}\sinh\left(bx+a\right)+3\,\sqrt{2}\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}+\sqrt{2}\,\sinh\left(bx+a\right)^{3}\right){\rm weierstrassZeta}(4,0,{\rm weierstrassPInverse}(4,0,\cosh\left(bx+a\right)+\sinh\left(bx+a\right)))}{20\left(b\cosh\left(bx+a\right)^{3}+3\,b\cosh\left(bx+a\right)^{2}\sinh\left(bx+a\right)+3\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}+b\sinh\left(bx+a\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/20*(sqrt(2)*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x
 + a)^6 + (15*cosh(b*x + a)^2 + 11)*sinh(b*x + a)^4 + 11*cosh(b*x + a)^4 +
4*(5*cosh(b*x + a)^3 + 11*cosh(b*x + a))*sinh(b*x + a)^3 + (15*cosh(b*x + a
)^4 + 66*cosh(b*x + a)^2 - 13)*sinh(b*x + a)^2 - 13*cosh(b*x + a)^2 + 2*(3*
cosh(b*x + a)^5 + 22*cosh(b*x + a)^3 - 13*cosh(b*x + a))*sinh(b*x + a) + 1)
*sqrt((cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*si
nh(b*x + a) + sinh(b*x + a)^2 - 1)) + 24*(sqrt(2)*cosh(b*x + a)^3 + 3*sqrt(
2)*cosh(b*x + a)^2*sinh(b*x + a) + 3*sqrt(2)*cosh(b*x + a)*sinh(b*x + a)^2
+ sqrt(2)*sinh(b*x + a)^3)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0,
cosh(b*x + a) + sinh(b*x + a))))/(b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)^2*s
inh(b*x + a) + 3*b*cosh(b*x + a)*sinh(b*x + a)^2 + b*sinh(b*x + a)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{csch}^{\frac{5}{2}}{\left(a + b x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(b*x+a)**(5/2),x)
```

```
[Out] Integral(csch(a + b*x)**(-5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^(-5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{1}{\sinh(a+b\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/sinh(a + b*x))^(5/2),x)
[Out] int(1/(1/sinh(a + b*x))^(5/2), x)
```

3.13 $\int (b\text{csch}(c+dx))^{7/2}\,dx$

Optimal. Leaf size=116

$$\frac{6b^3\cosh(c+dx)\sqrt{b\text{csch}(c+dx)}}{5d}-\frac{2b\cosh(c+dx)(b\text{csch}(c+dx))^{5/2}}{5d}+\frac{6ib^4E\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{5d\sqrt{b\text{csch}(c+dx)}\sqrt{i\sinh(c+dx)}}$$

```
[Out] -2/5*b*cosh(d*x+c)*(b*csch(d*x+c))^(5/2)/d+6/5*b^3*cosh(d*x+c)*(b*csch(d*x+
c))^(1/2)/d-6/5*I*b^4*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1
/4*Pi+1/2*I*d*x)*EllipticE(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))/d/(b*csch
(d*x+c))^(1/2)/(I*sinh(d*x+c))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3853, 3856, 2719}

$$\frac{6ib^4E\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{5d\sqrt{i\sinh(c+dx)}\sqrt{b\text{csch}(c+dx)}}+\frac{6b^3\cosh(c+dx)\sqrt{b\text{csch}(c+dx)}}{5d}-\frac{2b\cosh(c+dx)(b\text{csch}(c+dx))^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csch[c + d*x])^(7/2),x]
```

```
[Out] (6*b^3*Cosh[c + d*x]*Sqrt[b*Csch[c + d*x]])/(5*d) - (2*b*Cosh[c + d*x]*(b*C
sch[c + d*x])^(5/2))/(5*d) + (((6*I)/5)*b^4*EllipticE[(I*c - Pi/2 + I*d*x)/
2, 2])/(d*Sqrt[b*Csch[c + d*x]]*Sqrt[I*Sinh[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int (b\text{csch}(c+dx))^{7/2}\,dx = -\frac{2b\cosh(c+dx)(b\text{csch}(c+dx))^{5/2}}{5d} - \frac{1}{5}\left(3b^2\right)\int (b\text{csch}(c+dx))^{3/2}\,dx$$
$$= \frac{6b^3\cosh(c+dx)\sqrt{b\text{csch}(c+dx)}}{5d} - \frac{2b\cosh(c+dx)(b\text{csch}(c+dx))^{5/2}}{5d} - \frac{1}{5}\left(3b^4\right)\int$$
$$= \frac{6b^3\cosh(c+dx)\sqrt{b\text{csch}(c+dx)}}{5d} - \frac{2b\cosh(c+dx)(b\text{csch}(c+dx))^{5/2}}{5d} - \frac{(3b^4}{5\sqrt{b\text{csch}}}$$
$$= \frac{6b^3\cosh(c+dx)\sqrt{b\text{csch}(c+dx)}}{5d} - \frac{2b\cosh(c+dx)(b\text{csch}(c+dx))^{5/2}}{5d} + \frac{6ib^4}{5d\sqrt{b\text{csc}}}$$

Mathematica [A]
time = 0.11, size = 79, normalized size = 0.68

$$-\frac{2b^3\sqrt{b\text{csch}(c+dx)}\left(-3\cosh(c+dx)+\coth(c+dx)\text{csch}(c+dx)+3E\left(\frac{1}{4}(-2ic+\pi-2idx)\middle|2\right)\sqrt{i\sinh(c+dx)}\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csch[c + d*x])^(7/2),x]
```

```
[Out] (-2*b^3*Sqrt[b*Csch[c + d*x]]*(-3*Cosh[c + d*x] + Coth[c + d*x]*Csch[c + d*
x] + 3*EllipticE[((-2*I)*c + Pi - (2*I)*d*x)/4, 2]*Sqrt[I*Sinh[c + d*x]]))/
(5*d)
```

Maple [F]
time = 1.31, size = 0, normalized size = 0.00

$$\int (b\,\text{csch}(dx+c))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csch(d*x+c))^(7/2),x)
```

```
[Out] int((b*csch(d*x+c))^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*csch(d*x + c))^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 484, normalized size = 4.17

$$\frac{2\left(3\sqrt{2}\left(b^3\cosh\left(dx+c\right)^4+4\,b^3\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+b^3\sinh\left(dx+c\right)^4-2\,b^3\cosh\left(dx+c\right)^2+b^3+2\left(3\,b^3\cosh\left(dx+c\right)^2-b^3\right)\sinh\left(dx+c\right)^2+4\left(b^3\cosh\left(dx+c\right)^3-b^3\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)\right)\sqrt{b}\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cosh\left(dx+c\right)+\sinh\left(dx+c\right)))+\sqrt{2}\left(3\,b^3\cosh\left(dx+c\right)^5+15\,b^3\cosh\left(dx+c\right)\sinh\left(dx+c\right)^4+3\,b^3\sinh\left(dx+c\right)^5-8\,b^3\cosh\left(dx+c\right)^3+b^3\cosh\left(dx+c\right)+2\left(15\,b^3\cosh\left(dx+c\right)^2-4\,b^3\right)\sinh\left(dx+c\right)^3+6\left(5\,b^3\cosh\left(dx+c\right)^3-4\,b^3\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^2+\left(15\,b^3\cosh\left(dx+c\right)^4-24\,b^3\cosh\left(dx+c\right)^2+b^3\right)\sinh\left(dx+c\right)\right)\sqrt{\frac{b\cosh\left(dx+c\right)+b\sinh\left(dx+c\right)}{\cosh\left(dx+c\right)^2+2\cosh\left(dx+c\right)\sinh\left(dx+c\right)+\sinh\left(dx+c\right)^2-1}}\right)}{5\left(d\cosh\left(dx+c\right)^4+4\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+d\sinh\left(dx+c\right)^4-2\,d\cosh\left(dx+c\right)^2+2\left(3\,d\cosh\left(dx+c\right)^2-d\right)\sinh\left(dx+c\right)^2+4\left(d\cosh\left(dx+c\right)^3-d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/5*(3*sqrt(2)*(b^3*cosh(d*x + c)^4 + 4*b^3*cosh(d*x + c)*sinh(d*x + c)^3 +
 b^3*sinh(d*x + c)^4 - 2*b^3*cosh(d*x + c)^2 + b^3 + 2*(3*b^3*cosh(d*x + c)
^2 - b^3)*sinh(d*x + c)^2 + 4*(b^3*cosh(d*x + c)^3 - b^3*cosh(d*x + c))*sin
h(d*x + c))*sqrt(b)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cosh(d*
x + c) + sinh(d*x + c))) + sqrt(2)*(3*b^3*cosh(d*x + c)^5 + 15*b^3*cosh(d*x
 + c)*sinh(d*x + c)^4 + 3*b^3*sinh(d*x + c)^5 - 8*b^3*cosh(d*x + c)^3 + b^3
*cosh(d*x + c) + 2*(15*b^3*cosh(d*x + c)^2 - 4*b^3)*sinh(d*x + c)^3 + 6*(5*
b^3*cosh(d*x + c)^3 - 4*b^3*cosh(d*x + c))*sinh(d*x + c)^2 + (15*b^3*cosh(d
*x + c)^4 - 24*b^3*cosh(d*x + c)^2 + b^3)*sinh(d*x + c))*sqrt((b*cosh(d*x +
 c) + b*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + s
inh(d*x + c)^2 - 1)))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^
3 + d*sinh(d*x + c)^4 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 - d)*s
inh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \operatorname{csch}\left(c+dx\right)\right)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))**(7/2),x)
```

```
[Out] Integral((b*csch(c + d*x))**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*csch(d*x + c))^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{b}{\sinh\left(c+d\,x\right)}\right)^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/sinh(c + d*x))^(7/2),x)
[Out] int((b/sinh(c + d*x))^(7/2), x)
```

3.14 $\int (b\text{csch}(c+dx))^{5/2}\,dx$

Optimal. Leaf size=88

$$-\frac{2b\cosh(c+dx)(b\text{csch}(c+dx))^{3/2}}{3d}+\frac{2ib^2\sqrt{b\text{csch}(c+dx)}\,F\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{i\sinh(c+dx)}}{3d}$$

```
[Out] -2/3*b*cosh(d*x+c)*(b*csch(d*x+c))^(3/2)/d-2/3*I*b^2*(sin(1/2*I*c+1/4*Pi+1/
2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)*EllipticF(cos(1/2*I*c+1/4*P
i+1/2*I*d*x),2^(1/2))*(b*csch(d*x+c))^(1/2)*(I*sinh(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.03, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3853, 3856, 2720}

$$-\frac{2b\cosh(c+dx)(b\text{csch}(c+dx))^{3/2}}{3d}+\frac{2ib^2\sqrt{i\sinh(c+dx)}\,F\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\text{csch}(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csch[c + d*x])^(5/2),x]
```

```
[Out] (-2*b*Cosh[c + d*x]*(b*Csch[c + d*x])^(3/2))/(3*d) + (((2*I)/3)*b^2*Sqrt[b*
Csch[c + d*x]]*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*Sinh[c + d*x]])/
d
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\int (b\operatorname{csch}(c+dx))^{5/2}\,dx &= -\frac{2b\cosh(c+dx)(b\operatorname{csch}(c+dx))^{3/2}}{3d} - \frac{1}{3}b^2\int \sqrt{b\operatorname{csch}(c+dx)}\,dx \\
&= -\frac{2b\cosh(c+dx)(b\operatorname{csch}(c+dx))^{3/2}}{3d} - \frac{1}{3}\left(b^2\sqrt{b\operatorname{csch}(c+dx)}\,\sqrt{i\sinh(c+dx)}\right)\int \\
&= -\frac{2b\cosh(c+dx)(b\operatorname{csch}(c+dx))^{3/2}}{3d} + \frac{2ib^2\sqrt{b\operatorname{csch}(c+dx)}\,F\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|\,2\right)}{3d}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 66, normalized size = 0.75

$$
-\frac{2b^2\sqrt{b\operatorname{csch}(c+dx)}\left(\coth(c+dx)+iF\left(\frac{1}{4}(-2ic+\pi-2idx)\middle|\,2\right)\sqrt{i\sinh(c+dx)}\right)}{3d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csch[c + d*x])^(5/2),x]
```

```
[Out] (-2*b^2*Sqrt[b*Csch[c + d*x]]*(Coth[c + d*x] + I*EllipticF[((-2*I)*c + Pi -
 (2*I)*d*x)/4, 2]*Sqrt[I*Sinh[c + d*x]]))/(3*d)
```

Maple [F]
time = 1.10, size = 0, normalized size = 0.00

$$
\int \left(b\operatorname{csch}(dx+c)\right)^{\frac{5}{2}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csch(d*x+c))^(5/2),x)
```

```
[Out] int((b*csch(d*x+c))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*csch(d*x + c))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 217, normalized size = 2.47

$$
-\frac{2\left(\sqrt{2}\left(b^2\cosh(dx+c)^2+2b^2\cosh(dx+c)\sinh(dx+c)+b^2\sinh(dx+c)^2-b^2\right)\sqrt{b}\,\mathrm{weierstrassPInverse}(4,0,\cosh(dx+c)+\sinh(dx+c))+\sqrt{2}\left(b^2\cosh(dx+c)^2+2b^2\cosh(dx+c)\sinh(dx+c)+b^2\sinh(dx+c)^2+b^2\right)\sqrt{\frac{b\cosh(dx+c)+b\sinh(dx+c)}{\cosh(dx+c)^2+2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2-1}}\right)}{3\left(d\cosh(dx+c)^2+2d\cosh(dx+c)\sinh(dx+c)+d\sinh(dx+c)^2-d\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(sqrt(2)*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^
2*sinh(d*x + c)^2 - b^2)*sqrt(b)*weierstrassPInverse(4, 0, cosh(d*x + c) +
sinh(d*x + c)) + sqrt(2)*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*
x + c) + b^2*sinh(d*x + c)^2 + b^2)*sqrt((b*cosh(d*x + c) + b*sinh(d*x + c)
)/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)))
/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x + c)^2 -
 d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\operatorname{csch}(c+dx))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))**(5/2),x)
```

```
[Out] Integral((b*csch(c + d*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*csch(d*x + c))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{b}{\mathrm{sinh}\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/sinh(c + d*x))^(5/2),x)
```

```
[Out] int((b/sinh(c + d*x))^(5/2), x)
```

3.15 $\int (b\text{csch}(c+dx))^{3/2}\,dx$

Optimal. Leaf size=84

$$-\frac{2b\cosh(c+dx)\sqrt{b\text{csch}(c+dx)}}{d}-\frac{2ib^2E\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{d\sqrt{b\text{csch}(c+dx)}\sqrt{i\sinh(c+dx)}}$$

```
[Out] -2*b*cosh(d*x+c)*(b*csch(d*x+c))^(1/2)/d+2*I*b^2*(sin(1/2*I*c+1/4*Pi+1/2*I*
d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)*EllipticE(cos(1/2*I*c+1/4*Pi+1/
2*I*d*x),2^(1/2))/d/(b*csch(d*x+c))^(1/2)/(I*sinh(d*x+c))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3853, 3856, 2719}

$$-\frac{2b\cosh(c+dx)\sqrt{b\text{csch}(c+dx)}}{d}-\frac{2ib^2E\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{d\sqrt{i\sinh(c+dx)}\sqrt{b\text{csch}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csch[c + d*x])^(3/2),x]
```

```
[Out] (-2*b*Cosh[c + d*x]*Sqrt[b*Csch[c + d*x]])/d - ((2*I)*b^2*EllipticE[(I*c -
Pi/2 + I*d*x)/2, 2])/(d*Sqrt[b*Csch[c + d*x]]*Sqrt[I*Sinh[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int (b\text{csch}(c+dx))^{3/2}\,dx = -\frac{2b\cosh(c+dx)\sqrt{b\text{csch}(c+dx)}}{d} + b^2\int \frac{1}{\sqrt{b\text{csch}(c+dx)}}\,dx$$
$$= -\frac{2b\cosh(c+dx)\sqrt{b\text{csch}(c+dx)}}{d} + \frac{b^2\int \sqrt{i\sinh(c+dx)}\,dx}{\sqrt{b\text{csch}(c+dx)}\sqrt{i\sinh(c+dx)}}$$
$$= -\frac{2b\cosh(c+dx)\sqrt{b\text{csch}(c+dx)}}{d} - \frac{2ib^2E\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{d\sqrt{b\text{csch}(c+dx)}\sqrt{i\sinh(c+dx)}}$$

Mathematica [A]
time = 0.04, size = 60, normalized size = 0.71

$$-\frac{2b\sqrt{b\text{csch}(c+dx)}\left(\cosh(c+dx)-E\left(\frac{1}{4}(-2ic+\pi-2idx)\middle|2\right)\sqrt{i\sinh(c+dx)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csch[c + d*x])^(3/2),x]
[Out] (-2*b*Sqrt[b*Csch[c + d*x]]*(Cosh[c + d*x] - EllipticE[((-2*I)*c + Pi - (2*
I)*d*x)/4, 2]*Sqrt[I*Sinh[c + d*x]]))/d
```

Maple [F]
time = 1.12, size = 0, normalized size = 0.00

$$\int (b\,\text{csch}(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csch(d*x+c))^(3/2),x)
[Out] int((b*csch(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*csch(d*x + c))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 107, normalized size = 1.27

$$-\frac{2\left(\sqrt{2}\,b^{\frac{3}{2}}\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cosh\left(dx+c\right)+\sinh\left(dx+c\right)))+\sqrt{2}\left(b\cosh\left(dx+c\right)+b\sinh\left(dx+c\right)\right)\sqrt{\frac{b\cosh\left(dx+c\right)+b\sinh\left(dx+c\right)}{\cosh\left(dx+c\right)^{2}+2\cosh\left(dx+c\right)\sinh\left(dx+c\right)+\sinh\left(dx+c\right)^{2}-1}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(sqrt(2)*b^(3/2)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cosh(d*
x + c) + sinh(d*x + c))) + sqrt(2)*(b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt
((b*cosh(d*x + c) + b*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sin
h(d*x + c) + sinh(d*x + c)^2 - 1)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\operatorname{csch}\left(c+dx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*csch(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*csch(d*x + c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{b}{\sinh\left(c+d\,x\right)}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/sinh(c + d*x))^(3/2),x)
```

```
[Out] int((b/sinh(c + d*x))^(3/2), x)
```

3.16 $\int \sqrt{b\operatorname{csch}(c+dx)}\,dx$

Optimal. Leaf size=56

$$-\frac{2i\sqrt{b\operatorname{csch}(c+dx)}\,F\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{i\sinh(c+dx)}}{d}$$

```
[Out] 2*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)*E
llipticF(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))*(b*csch(d*x+c))^(1/2)*(I*si
nh(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3856, 2720}

$$-\frac{2i\sqrt{i\sinh(c+dx)}\,F\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\operatorname{csch}(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Csch[c + d*x]],x]
```

```
[Out] ((-2*I)*Sqrt[b*Csch[c + d*x]]*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*S
inh[c + d*x]])/d
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \sqrt{b\operatorname{csch}(c+dx)}\,dx &= \left(\sqrt{b\operatorname{csch}(c+dx)}\,\sqrt{i\sinh(c+dx)}\right)\int \frac{1}{\sqrt{i\sinh(c+dx)}}\,dx\\ &= -\frac{2i\sqrt{b\operatorname{csch}(c+dx)}\,F\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{i\sinh(c+dx)}}{d}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 54, normalized size = 0.96

$$\frac{2i\sqrt{b\text{csch}(c+dx)}\,F\left(\frac{1}{2}\left(\frac{\pi}{2}-i(c+dx)\right)\middle|2\right)\sqrt{i\sinh(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Csch[c + d*x]],x]
```

```
[Out] ((2*I)*Sqrt[b*Csch[c + d*x]]*EllipticF[(Pi/2 - I*(c + d*x))/2, 2]*Sqrt[I*Si
nh[c + d*x]])/d
```

Maple [F]
time = 1.31, size = 0, normalized size = 0.00

$$\int \sqrt{b\,\mathrm{csch}\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csch(d*x+c))^(1/2),x)
```

```
[Out] int((b*csch(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*csch(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 27, normalized size = 0.48

$$\frac{2\,\sqrt{2}\,\sqrt{b}\,\mathrm{weierstrassPInverse}(4,0,\cosh\left(dx+c\right)+\sinh\left(dx+c\right))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*sqrt(b)*weierstrassPInverse(4, 0, cosh(d*x + c) + sinh(d*x + c))/
d
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\operatorname{csch}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(b*csch(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*csch(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\frac{b}{\sinh(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/sinh(c + d*x))^(1/2),x)
```

```
[Out] int((b/sinh(c + d*x))^(1/2), x)
```

3.17 $\int \frac{1}{\sqrt{b\text{csch}(c+dx)}}\,dx$

Optimal. Leaf size=56

$$-\frac{2iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{d\sqrt{b\text{csch}(c+dx)}\sqrt{i\sinh(c+dx)}}$$

```
[Out] 2*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)*E
llipticE(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))/d/(b*csch(d*x+c))^(1/2)/(I*
sinh(d*x+c))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3856, 2719}

$$-\frac{2iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{d\sqrt{i\sinh(c+dx)}\sqrt{b\text{csch}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Csch[c + d*x]],x]
```

```
[Out] ((-2*I)*EllipticE[(I*c - Pi/2 + I*d*x)/2, 2])/(d*Sqrt[b*Csch[c + d*x]]*Sqrt
[I*Sinh[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{b\text{csch}(c+dx)}}\,dx &= \frac{\int \sqrt{i\sinh(c+dx)}\,dx}{\sqrt{b\text{csch}(c+dx)}\sqrt{i\sinh(c+dx)}}\\ &= -\frac{2iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{d\sqrt{b\text{csch}(c+dx)}\sqrt{i\sinh(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 52, normalized size = 0.93

$$\frac{2iE\left(\frac{1}{4}(-2ic+\pi-2idx)\middle|2\right)}{d\sqrt{b\text{csch}(c+dx)}\sqrt{i\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Csch[c + d*x]],x]
```

```
[Out] ((2*I)*EllipticE[((-2*I)*c + Pi - (2*I)*d*x)/4, 2])/(d*Sqrt[b*Csch[c + d*x]
]*Sqrt[I*Sinh[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(79) = 158$.
time = 1.95, size = 227, normalized size = 4.05

method	result
risch	$\frac{\sqrt{2}}{d\sqrt{\frac{b\,\mathrm{e}^{dx+c}}{\mathrm{e}^{2dx+2c}-1}}}-\frac{\left(\frac{2b\,\mathrm{e}^{2dx+2c}-2b}{b\sqrt{\mathrm{e}^{dx+c}\left(b\,\mathrm{e}^{2dx+2c}-b\right)}}-\frac{\sqrt{\mathrm{e}^{dx+c}+1}\,\sqrt{-2\,\mathrm{e}^{dx+c}+2}\,\sqrt{-\mathrm{e}^{dx+c}}\left(-2\,\mathrm{EllipticE}\left(\sqrt{\mathrm{e}^{d}}\right.\right.}{\sqrt{b\,\mathrm{e}^{3dx+3c}-b\,\mathrm{e}}}\right.}{d\sqrt{\frac{b\,\mathrm{e}^{dx+c}}{\mathrm{e}^{2dx+2c}-1}}\;(\mathrm{e}^{2d}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*csch(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*2^(1/2)/(b*exp(d*x+c)/(exp(d*x+c)^2-1))^(1/2)-1/d*(2*(b*exp(d*x+c)^2-b)
/b/(exp(d*x+c)*(b*exp(d*x+c)^2-b))^(1/2)-(exp(d*x+c)+1)^(1/2)*(-2*exp(d*x+c
)+2)^(1/2)*(-exp(d*x+c))^(1/2)/(b*exp(d*x+c)^3-b*exp(d*x+c))^(1/2)*(-2*Elli
pticE((exp(d*x+c)+1)^(1/2),1/2*2^(1/2))+EllipticF((exp(d*x+c)+1)^(1/2),1/2*
2^(1/2))))*2^(1/2)/(b*exp(d*x+c)/(exp(d*x+c)^2-1))^(1/2)*(b*(exp(d*x+c)^2-1
)*exp(d*x+c))^(1/2)/(exp(d*x+c)^2-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*csch(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 154, normalized size = 2.75

$$-\frac{2\sqrt{2}\sqrt{b}\left(\cosh(dx+c)+\sinh(dx+c)\right){\rm weierstrassZeta}\left(4,0,{\rm weierstrassPInverse}\left(4,0,\cosh(dx+c)+\sinh(dx+c)\right)\right)+\sqrt{2}\left(\cosh(dx+c)^2+2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2-1\right)\sqrt{\frac{b\cosh(dx+c)+b\sinh(dx+c)}{\cosh(dx+c)^2+2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2-1}}}{bd\cosh(dx+c)+bd\sinh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*sqrt(2)*sqrt(b)*(cosh(d*x + c) + sinh(d*x + c))*weierstrassZeta(4, 0, w
eierstrassPInverse(4, 0, cosh(d*x + c) + sinh(d*x + c))) + sqrt(2)*(cosh(d*
x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt((b*cos
h(d*x + c) + b*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x +
 c) + sinh(d*x + c)^2 - 1)))/(b*d*cosh(d*x + c) + b*d*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\operatorname{csch}(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*csch(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*csch(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\frac{b}{\sinh(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/sinh(c + d*x))^(1/2),x)
```

```
[Out] int(1/(b/sinh(c + d*x))^(1/2), x)
```

3.18 $\int \frac{1}{(b\operatorname{csch}(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=90

$$\frac{2\cosh(c+dx)}{3bd\sqrt{b\operatorname{csch}(c+dx)}}+\frac{2i\sqrt{b\operatorname{csch}(c+dx)}\,F\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{i\sinh(c+dx)}}{3b^2d}$$

[Out] 2/3*cosh(d*x+c)/b/d/(b*csch(d*x+c))^(1/2)-2/3*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)*EllipticF(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))*(b*csch(d*x+c))^(1/2)*(I*sinh(d*x+c))^(1/2)/b^2/d

Rubi [A]
time = 0.03, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3854, 3856, 2720}

$$\frac{2\cosh(c+dx)}{3bd\sqrt{b\operatorname{csch}(c+dx)}}+\frac{2i\sqrt{i\sinh(c+dx)}\,F\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\operatorname{csch}(c+dx)}}{3b^2d}$$

Antiderivative was successfully verified.

[In] Int[(b*Csch[c + d*x])^(-3/2),x]

[Out] (2*Cosh[c + d*x])/(3*b*d*Sqrt[b*Csch[c + d*x]]) + (((2*I)/3)*Sqrt[b*Csch[c + d*x]]*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*Sinh[c + d*x]])/(b^2*d)

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{(b\text{csch}(c+dx))^{3/2}}\,dx = \frac{2\cosh(c+dx)}{3bd\sqrt{b\text{csch}(c+dx)}} - \frac{\int \sqrt{b\text{csch}(c+dx)}\,dx}{3b^2}$$

$$= \frac{2\cosh(c+dx)}{3bd\sqrt{b\text{csch}(c+dx)}} - \frac{\left(\sqrt{b\text{csch}(c+dx)}\sqrt{i\sinh(c+dx)}\right)\int \frac{1}{\sqrt{i\sinh(c+dx)}}}{3b^2}$$

$$= \frac{2\cosh(c+dx)}{3bd\sqrt{b\text{csch}(c+dx)}} + \frac{2i\sqrt{b\text{csch}(c+dx)}\,F\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{i\sinh(c+dx}}{3b^2d}$$

Mathematica [A]
time = 0.07, size = 73, normalized size = 0.81

$$\frac{\text{csch}^2(c+dx)\left(-2iF\left(\frac{1}{4}(-2ic+\pi-2idx)\middle|2\right)\sqrt{i\sinh(c+dx)}+\sinh(2(c+dx))\right)}{3d(b\text{csch}(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csch[c + d*x])^(-3/2),x]
```

```
[Out] (Csch[c + d*x]^2*((-2*I)*EllipticF[((-2*I)*c + Pi - (2*I)*d*x)/4, 2]*Sqrt[I
*Sinh[c + d*x]] + Sinh[2*(c + d*x)]))/(3*d*(b*Csch[c + d*x])^(3/2))
```

Maple [F]
time = 0.89, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,\text{csch}\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*csch(d*x+c))^(3/2),x)
```

```
[Out] int(1/(b*csch(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*csch(d*x + c))^(-3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 232, normalized size = 2.58

$$-\frac{4\sqrt{2}\left(\cosh\left(dx+c\right)^{2}+2\cosh\left(dx+c\right)\sinh\left(dx+c\right)+\sinh\left(dx+c\right)^{2}\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(4,0,\cosh\left(dx+c\right)+\sinh\left(dx+c\right)\right)-\sqrt{2}\left(\cosh\left(dx+c\right)^{4}+4\cosh\left(dx+c\right)^{3}\sinh\left(dx+c\right)+6\cosh\left(dx+c\right)^{2}\sinh\left(dx+c\right)^{2}+4\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3}+\sinh\left(dx+c\right)^{4}-1\right)\sqrt{\frac{b\cosh\left(dx+c\right)+b\sinh\left(dx+c\right)}{\cosh\left(dx+c\right)^{2}+2\cosh\left(dx+c\right)\sinh\left(dx+c\right)+\sinh\left(dx+c\right)^{2}-1}}}{6\left(b^{2}d\cosh\left(dx+c\right)^{2}+2\,b^{2}d\cosh\left(dx+c\right)\sinh\left(dx+c\right)+b^{2}d\sinh\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6*(4*sqrt(2)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x
 + c)^2)*sqrt(b)*weierstrassPInverse(4, 0, cosh(d*x + c) + sinh(d*x + c)) -
 sqrt(2)*(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x +
c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 -
1)*sqrt((b*cosh(d*x + c) + b*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x +
 c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)))/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d
*cosh(d*x + c)*sinh(d*x + c) + b^2*d*sinh(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\operatorname{csch}\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*csch(c + d*x))**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*csch(d*x + c))^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(\frac{b}{\sinh\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/sinh(c + d*x))^(3/2),x)
```

```
[Out] int(1/(b/sinh(c + d*x))^(3/2), x)
```

3.19 $\int \frac{1}{(b\operatorname{csch}(c+dx))^{5/2}} dx$

Optimal. Leaf size=90

$$\frac{2\cosh(c+dx)}{5bd(b\operatorname{csch}(c+dx))^{3/2}}+\frac{6iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{5b^2d\sqrt{b\operatorname{csch}(c+dx)}\sqrt{i\sinh(c+dx)}}$$

```
[Out] 2/5*cosh(d*x+c)/b/d/(b*csch(d*x+c))^(3/2)-6/5*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d
*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)*EllipticE(cos(1/2*I*c+1/4*Pi+1/2
*I*d*x),2^(1/2))/b^2/d/(b*csch(d*x+c))^(1/2)/(I*sinh(d*x+c))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3854, 3856, 2719}

$$\frac{2\cosh(c+dx)}{5bd(b\operatorname{csch}(c+dx))^{3/2}}+\frac{6iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{5b^2d\sqrt{i\sinh(c+dx)}\sqrt{b\operatorname{csch}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csch[c + d*x])^(-5/2),x]
```

```
[Out] (2*Cosh[c + d*x])/(5*b*d*(b*Csch[c + d*x])^(3/2)) + (((6*I)/5)*EllipticE[(I
*c - Pi/2 + I*d*x)/2, 2])/(b^2*d*Sqrt[b*Csch[c + d*x]]*Sqrt[I*Sinh[c + d*x]
])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{(b\text{csch}(c+dx))^{5/2}}\,dx = \frac{2\cosh(c+dx)}{5bd(b\text{csch}(c+dx))^{3/2}} - \frac{3\int \frac{1}{\sqrt{b\text{csch}(c+dx)}}\,dx}{5b^2}$$
$$= \frac{2\cosh(c+dx)}{5bd(b\text{csch}(c+dx))^{3/2}} - \frac{3\int \sqrt{i\sinh(c+dx)}\,dx}{5b^2\sqrt{b\text{csch}(c+dx)}\sqrt{i\sinh(c+dx)}}$$
$$= \frac{2\cosh(c+dx)}{5bd(b\text{csch}(c+dx))^{3/2}} + \frac{6iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|\,2\right)}{5b^2d\sqrt{b\text{csch}(c+dx)}\sqrt{i\sinh(c+dx)}}$$

Mathematica [A]
time = 0.10, size = 68, normalized size = 0.76

$$\frac{-\frac{6iE\left(\frac{1}{4}(-2ic+\pi-2idx)|2\right)}{\sqrt{i\sinh(c+dx)}}+\sinh(2(c+dx))}{5b^2d\sqrt{b\text{csch}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csch[c + d*x])^(-5/2),x]
[Out] (((-6*I)*EllipticE[((-2*I)*c + Pi - (2*I)*d*x)/4, 2])/Sqrt[I*Sinh[c + d*x]]
 + Sinh[2*(c + d*x)])/(5*b^2*d*Sqrt[b*Csch[c + d*x]])
```

Maple [F]
time = 0.94, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,\text{csch}\,(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*csch(d*x+c))^(5/2),x)
[Out] int(1/(b*csch(d*x+c))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*csch(d*x + c))^(-5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 378, normalized size = 4.20

$$\frac{24\sqrt{2}\left(\cosh\left(dx+c\right)^3+3\cosh\left(dx+c\right)^2\sinh\left(dx+c\right)+3\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2+\sinh\left(dx+c\right)^3\right)\sqrt{b}\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cosh\left(dx+c\right)+\sinh\left(dx+c\right)))+\sqrt{2}\left(\cosh\left(dx+c\right)^6+6\cosh\left(dx+c\right)\sinh\left(dx+c\right)^5+\sinh\left(dx+c\right)^6+\left(15\cosh\left(dx+c\right)^2+11\right)\sinh\left(dx+c\right)^4+11\cosh\left(dx+c\right)^4+4\left(5\cosh\left(dx+c\right)^3+11\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^3+\left(15\cosh\left(dx+c\right)^4+66\cosh\left(dx+c\right)^2-13\right)\sinh\left(dx+c\right)^2-13\cosh\left(dx+c\right)^2+2\left(3\cosh\left(dx+c\right)^5+22\cosh\left(dx+c\right)^3-13\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)+1\right)\sqrt{\frac{b\cosh\left(dx+c\right)+b\sinh\left(dx+c\right)}{\cosh\left(dx+c\right)^2+2\cosh\left(dx+c\right)\sinh\left(dx+c\right)+\sinh\left(dx+c\right)^2-1}}}{20\left(b^3d\cosh\left(dx+c\right)^3+3\,b^3d\cosh\left(dx+c\right)^2\sinh\left(dx+c\right)+3\,b^3d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2+b^3d\sinh\left(dx+c\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/20*(24*sqrt(2)*(cosh(d*x + c)^3 + 3*cosh(d*x + c)^2*sinh(d*x + c) + 3*cos
h(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3)*sqrt(b)*weierstrassZeta(4, 0,
 weierstrassPInverse(4, 0, cosh(d*x + c) + sinh(d*x + c))) + sqrt(2)*(cosh(
d*x + c)^6 + 6*cosh(d*x + c)*sinh(d*x + c)^5 + sinh(d*x + c)^6 + (15*cosh(d
*x + c)^2 + 11)*sinh(d*x + c)^4 + 11*cosh(d*x + c)^4 + 4*(5*cosh(d*x + c)^3
 + 11*cosh(d*x + c))*sinh(d*x + c)^3 + (15*cosh(d*x + c)^4 + 66*cosh(d*x +
c)^2 - 13)*sinh(d*x + c)^2 - 13*cosh(d*x + c)^2 + 2*(3*cosh(d*x + c)^5 + 22
*cosh(d*x + c)^3 - 13*cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt((b*cosh(d*x +
c) + b*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + si
nh(d*x + c)^2 - 1)))/(b^3*d*cosh(d*x + c)^3 + 3*b^3*d*cosh(d*x + c)^2*sinh(
d*x + c) + 3*b^3*d*cosh(d*x + c)*sinh(d*x + c)^2 + b^3*d*sinh(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\operatorname{csch}\left(c+dx\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))**(5/2),x)
```

```
[Out] Integral((b*csch(c + d*x))**(-5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*csch(d*x + c))^(-5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(\frac{b}{\mathrm{sinh}\left(c+d\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/sinh(c + d*x))^(5/2),x)
[Out] int(1/(b/sinh(c + d*x))^(5/2), x)
```

3.20 $\int \frac{1}{(b\operatorname{csch}(c+dx))^{7/2}} dx$

Optimal. Leaf size=118

$$\frac{2\cosh(c+dx)}{7bd(b\operatorname{csch}(c+dx))^{5/2}}-\frac{10\cosh(c+dx)}{21b^3d\sqrt{b\operatorname{csch}(c+dx)}}-\frac{10i\sqrt{b\operatorname{csch}(c+dx)}\,F\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{i\sinh(c+dx)}}{21b^4d}$$

```
[Out] 2/7*cosh(d*x+c)/b/d/(b*csch(d*x+c))^(5/2)-10/21*cosh(d*x+c)/b^3/d/(b*csch(d
*x+c))^(1/2)+10/21*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/
4*Pi+1/2*I*d*x)*EllipticF(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))*(b*csch(d*
x+c))^(1/2)*(I*sinh(d*x+c))^(1/2)/b^4/d
```

Rubi [A]
time = 0.04, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3854, 3856, 2720}

$$-\frac{10i\sqrt{i\sinh(c+dx)}\,F\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\operatorname{csch}(c+dx)}}{21b^4d}-\frac{10\cosh(c+dx)}{21b^3d\sqrt{b\operatorname{csch}(c+dx)}}+\frac{2\cosh(c+dx)}{7bd(b\operatorname{csch}(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csch[c + d*x])^(-7/2),x]
```

```
[Out] (2*Cosh[c + d*x])/(7*b*d*(b*Csch[c + d*x])^(5/2)) - (10*Cosh[c + d*x])/(21*
b^3*d*Sqrt[b*Csch[c + d*x]]) - (((10*I)/21)*Sqrt[b*Csch[c + d*x]]*EllipticF
[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*Sinh[c + d*x]])/(b^4*d)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{(b\text{csch}(c+dx))^{7/2}}\,dx = \frac{2\cosh(c+dx)}{7bd(b\text{csch}(c+dx))^{5/2}} - \frac{5\int \frac{1}{(b\text{csch}(c+dx))^{3/2}}\,dx}{7b^2}$$
$$= \frac{2\cosh(c+dx)}{7bd(b\text{csch}(c+dx))^{5/2}} - \frac{10\cosh(c+dx)}{21b^3d\sqrt{b\text{csch}(c+dx)}} + \frac{5\int \sqrt{b\text{csch}(c+dx)}\,dx}{21b^4}$$
$$= \frac{2\cosh(c+dx)}{7bd(b\text{csch}(c+dx))^{5/2}} - \frac{10\cosh(c+dx)}{21b^3d\sqrt{b\text{csch}(c+dx)}} + \frac{\left(5\sqrt{b\text{csch}(c+dx)}\sqrt{i\sinh(\ldots}{2\ldots}$$
$$= \frac{2\cosh(c+dx)}{7bd(b\text{csch}(c+dx))^{5/2}} - \frac{10\cosh(c+dx)}{21b^3d\sqrt{b\text{csch}(c+dx)}} - \frac{10i\sqrt{b\text{csch}(c+dx)}F\left(\frac{1}{2}(ic\ldots}{2\ldots}$$

Mathematica [A]
time = 0.11, size = 80, normalized size = 0.68

$$\frac{\sqrt{b\text{csch}(c+dx)}\left(40iF\left(\frac{1}{4}(-2ic+\pi-2idx)\middle|2\right)\sqrt{i\sinh(c+dx)} - 26\sinh(2(c+dx)) + 3\sinh(4(c+dx))\right)}{84b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csch[c + d*x])^(-7/2),x]
[Out] (Sqrt[b*Csch[c + d*x]]*((40*I)*EllipticF[((-2*I)*c + Pi - (2*I)*d*x)/4, 2]*
Sqrt[I*Sinh[c + d*x]] - 26*Sinh[2*(c + d*x)] + 3*Sinh[4*(c + d*x)]))/(84*b^
4*d)
```

Maple [F]
time = 0.86, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,\text{csch}\,(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*csch(d*x+c))^(7/2),x)
[Out] int(1/(b*csch(d*x+c))^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*csch(d*x + c))^(-7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 483, normalized size = 4.09

$$\frac{80 \sqrt{2} \left(\cosh\left(dx+c\right)^{4}+4 \cosh\left(dx+c\right)^{3} \sinh\left(dx+c\right)+6 \cosh\left(dx+c\right)^{2} \sinh\left(dx+c\right)^{2}+4 \cosh\left(dx+c\right) \sinh\left(dx+c\right)^{3}+\sinh\left(dx+c\right)^{4}\right) \sqrt{b}\, \mathrm{weierstrassPInverse}\left(4, 0, \cosh\left(dx+c\right)+\sinh\left(dx+c\right)\right)+\sqrt{2} \left(3 \cosh\left(dx+c\right)^{8}+24 \cosh\left(dx+c\right) \sinh\left(dx+c\right)^{7}+3 \sinh\left(dx+c\right)^{8}+2 \left(42 \cosh\left(dx+c\right)^{2}-13\right) \sinh\left(dx+c\right)^{6}-26 \cosh\left(dx+c\right)^{6}+12 \left(14 \cosh\left(dx+c\right)^{3}-13 \cosh\left(dx+c\right)\right) \sinh\left(dx+c\right)^{5}+30 \left(7 \cosh\left(dx+c\right)^{4}-13 \cosh\left(dx+c\right)^{2}\right) \sinh\left(dx+c\right)^{4}+8 \left(21 \cosh\left(dx+c\right)^{5}-65 \cosh\left(dx+c\right)^{3}\right) \sinh\left(dx+c\right)^{3}+2 \left(42 \cosh\left(dx+c\right)^{6}-195 \cosh\left(dx+c\right)^{4}+13\right) \sinh\left(dx+c\right)^{2}+26 \cosh\left(dx+c\right)^{2}+4 \left(6 \cosh\left(dx+c\right)^{7}-39 \cosh\left(dx+c\right)^{5}+13 \cosh\left(dx+c\right)\right) \sinh\left(dx+c\right)-3\right) \sqrt{\frac{b \cosh\left(dx+c\right)+b \sinh\left(dx+c\right)}{\cosh\left(dx+c\right)^{2}+2 \cosh\left(dx+c\right) \sinh\left(dx+c\right)+\sinh\left(dx+c\right)^{2}-1}}}{168 \left(b^{4} d \cosh\left(dx+c\right)^{4}+4 b^{4} d \cosh\left(dx+c\right)^{3} \sinh\left(dx+c\right)+6 b^{4} d \cosh\left(dx+c\right)^{2} \sinh\left(dx+c\right)^{2}+4 b^{4} d \cosh\left(dx+c\right) \sinh\left(dx+c\right)^{3}+b^{4} d \sinh\left(dx+c\right)^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/168*(80*sqrt(2)*(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*co
sh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x
+ c)^4)*sqrt(b)*weierstrassPInverse(4, 0, cosh(d*x + c) + sinh(d*x + c)) +
sqrt(2)*(3*cosh(d*x + c)^8 + 24*cosh(d*x + c)*sinh(d*x + c)^7 + 3*sinh(d*x
+ c)^8 + 2*(42*cosh(d*x + c)^2 - 13)*sinh(d*x + c)^6 - 26*cosh(d*x + c)^6 +
 12*(14*cosh(d*x + c)^3 - 13*cosh(d*x + c))*sinh(d*x + c)^5 + 30*(7*cosh(d*
x + c)^4 - 13*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(21*cosh(d*x + c)^5 - 65
*cosh(d*x + c)^3)*sinh(d*x + c)^3 + 2*(42*cosh(d*x + c)^6 - 195*cosh(d*x +
c)^4 + 13)*sinh(d*x + c)^2 + 26*cosh(d*x + c)^2 + 4*(6*cosh(d*x + c)^7 - 39
*cosh(d*x + c)^5 + 13*cosh(d*x + c))*sinh(d*x + c) - 3)*sqrt((b*cosh(d*x +
c) + b*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + si
nh(d*x + c)^2 - 1)))/(b^4*d*cosh(d*x + c)^4 + 4*b^4*d*cosh(d*x + c)^3*sinh(
d*x + c) + 6*b^4*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b^4*d*cosh(d*x + c)*
sinh(d*x + c)^3 + b^4*d*sinh(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \operatorname{csch}\left(c+dx\right)\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))**(7/2),x)
```

```
[Out] Integral((b*csch(c + d*x))**(-7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*csch(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*csch(d*x + c))^(-7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{b}{\sinh(c+d\,x)}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/sinh(c + d*x))^(7/2),x)
[Out] int(1/(b/sinh(c + d*x))^(7/2), x)
```

3.21 $\int (b\operatorname{csch}(c+dx))^n\,dx$

Optimal. Leaf size=74

$$\frac{b\cosh(c+dx)(b\operatorname{csch}(c+dx))^{-1+n}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};-\sinh^2(c+dx)\right)}{d(1-n)\sqrt{\cosh^2(c+dx)}}$$

```
[Out] b*cosh(d*x+c)*(b*csch(d*x+c))^(-1+n)*hypergeom([1/2, 1/2-1/2*n],[3/2-1/2*n]
,-sinh(d*x+c)^2)/d/(1-n)/(cosh(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3857, 2722}

$$\frac{b\cosh(c+dx)(b\operatorname{csch}(c+dx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};-\sinh^2(c+dx)\right)}{d(1-n)\sqrt{\cosh^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csch[c + d*x])^n,x]
```

```
[Out] (b*Cosh[c + d*x]*(b*Csch[c + d*x])^(-1 + n)*Hypergeometric2F1[1/2, (1 - n)/
2, (3 - n)/2, -Sinh[c + d*x]^2])/(d*(1 - n)*Sqrt[Cosh[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int (b\operatorname{csch}(c+dx))^n\,dx &= (b\operatorname{csch}(c+dx))^n\left(\frac{\sinh(c+dx)}{b}\right)^n\int\left(\frac{\sinh(c+dx)}{b}\right)^{-n}dx\\ &= \frac{\cosh(c+dx)(b\operatorname{csch}(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};-\sinh^2(c+dx)\right)\sinh(c+dx)}{d(1-n)\sqrt{\cosh^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 67, normalized size = 0.91

$$-\frac{\cosh(c+dx)(b\operatorname{csch}(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3}{2};\cosh^2(c+dx)\right)\sinh(c+dx)\left(-\sinh^2(c+dx)\right)^{\frac{1}{2}(-1+n)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csch[c + d*x])^n,x]
[Out] -((Cosh[c + d*x]*(b*Csch[c + d*x])^n*Hypergeometric2F1[1/2, (1 + n)/2, 3/2,
 Cosh[c + d*x]^2]*Sinh[c + d*x]*(-Sinh[c + d*x]^2)^((-1 + n)/2))/d)
```

Maple [F]
time = 1.02, size = 0, normalized size = 0.00

$$\int (b\operatorname{csch}(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csch(d*x+c))^n,x)
[Out] int((b*csch(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*csch(d*x + c))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*csch(d*x + c))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\operatorname{csch}(c+dx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))**n,x)
```

```
[Out] Integral((b*csch(c + d*x))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csch(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*csch(d*x + c))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{b}{\sinh\left(c+d\,x\right)}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/sinh(c + d*x))^n,x)
```

```
[Out] int((b/sinh(c + d*x))^n, x)
```

3.22 $\int \left(-\mathbf{csch}^2(x)\right)^{5/2}\,dx$

Optimal. Leaf size=40

$$\frac{3}{8}\text{ArcSin}(\coth(x))+\frac{3}{8}\coth(x)\sqrt{-\text{csch}^2(x)}+\frac{1}{4}\coth(x)\left(-\text{csch}^2(x)\right)^{3/2}$$

```
[Out] 3/8*arcsin(coth(x))+1/4*coth(x)*(-csch(x)^2)^(3/2)+3/8*coth(x)*(-csch(x)^2)
^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4207, 201, 222}

$$\frac{3}{8}\text{ArcSin}(\coth(x))+\frac{1}{4}\coth(x)\left(-\text{csch}^2(x)\right)^{3/2}+\frac{3}{8}\coth(x)\sqrt{-\text{csch}^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(-Csch[x]^2)^(5/2),x]
```

```
[Out] (3*ArcSin[Coth[x]])/8 + (3*Coth[x]*Sqrt[-Csch[x]^2])/8 + (Coth[x]*(-Csch[x]
^2)^(3/2))/4
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \left(-\operatorname{csch}^2(x)\right)^{5/2}\,dx = \text{Subst}\left(\int \left(1-x^2\right)^{3/2}\,dx, x, \coth(x)\right)$$
$$= \frac{1}{4}\coth(x)\left(-\operatorname{csch}^2(x)\right)^{3/2} + \frac{3}{4}\text{Subst}\left(\int \sqrt{1-x^2}\,dx, x, \coth(x)\right)$$
$$= \frac{3}{8}\coth(x)\sqrt{-\operatorname{csch}^2(x)} + \frac{1}{4}\coth(x)\left(-\operatorname{csch}^2(x)\right)^{3/2} + \frac{3}{8}\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \text{co}\right.$$
$$= \frac{3}{8}\sin^{-1}(\coth(x)) + \frac{3}{8}\coth(x)\sqrt{-\operatorname{csch}^2(x)} + \frac{1}{4}\coth(x)\left(-\operatorname{csch}^2(x)\right)^{3/2}$$

Mathematica [A]
time = 0.08, size = 41, normalized size = 1.02

$$\frac{1}{64}\left(-\operatorname{csch}^2(x)\right)^{5/2}\sinh(x)\left(-22\cosh(x)+6\left(\cosh(3x)+4\log\left(\tanh\left(\frac{x}{2}\right)\right)\sinh^4(x)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Csch[x]^2)^(5/2),x]
```

```
[Out] ((-Csch[x]^2)^(5/2)*Sinh[x]*(-22*Cosh[x] + 6*(Cosh[3*x] + 4*Log[Tanh[x/2]]*
Sinh[x]^4)))/64
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(30) = 60$.
time = 0.89, size = 114, normalized size = 2.85

method	result
risch	$\frac{\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}\,(3\,e^{6x}-11\,e^{4x}-11\,e^{2x}+3)}{4(e^{2x}-1)^3} + \frac{3\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}\,e^{-x}(e^{2x}-1)\ln(e^x-1)}{8} - \frac{3\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}\,e^{-x}(e^{2x}-1)\ln(e^x+1)}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-csch(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/(exp(2*x)-1)^3*(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)*(3*exp(6*x)-11*exp(4*x)
-11*exp(2*x)+3)+3/8*(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)*exp(-x)*(exp(2*x)-1)*l
n(exp(x)-1)-3/8*(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)*exp(-x)*(exp(2*x)-1)*ln(ex
p(x)+1)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.47, size = 74, normalized size = 1.85

$$\frac{3i\,e^{(-x)} - 11i\,e^{(-3\,x)} - 11i\,e^{(-5\,x)} + 3i\,e^{(-7\,x)}}{4\left(4\,e^{(-2\,x)} - 6\,e^{(-4\,x)} + 4\,e^{(-6\,x)} - e^{(-8\,x)} - 1\right)} + \frac{3}{8}i\,\log\left(e^{(-x)}+1\right) - \frac{3}{8}i\,\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*(3*I*e^(-x) - 11*I*e^(-3*x) - 11*I*e^(-5*x) + 3*I*e^(-7*x))/(4*e^(-2*x)
 - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) + 3/8*I*log(e^(-x) + 1) - 3/8*I*
log(e^(-x) - 1)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.36, size = 117, normalized size = 2.92

$$-\frac{3\left(i\,e^{(8\,x)}-4i\,e^{(6\,x)}+6i\,e^{(4\,x)}-4i\,e^{(2\,x)}+i\right)\log\left(e^{x}+1\right)+3\left(-i\,e^{(8\,x)}+4i\,e^{(6\,x)}-6i\,e^{(4\,x)}+4i\,e^{(2\,x)}-i\right)\log\left(e^{x}-1\right)-6i\,e^{(7\,x)}+22i\,e^{(5\,x)}+22i\,e^{(3\,x)}-6i\,e^{x}}{8\left(e^{(8\,x)}-4\,e^{(6\,x)}+6\,e^{(4\,x)}-4\,e^{(2\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/8*(3*(I*e^(8*x) - 4*I*e^(6*x) + 6*I*e^(4*x) - 4*I*e^(2*x) + I)*log(e^x +
 1) + 3*(-I*e^(8*x) + 4*I*e^(6*x) - 6*I*e^(4*x) + 4*I*e^(2*x) - I)*log(e^x
- 1) - 6*I*e^(7*x) + 22*I*e^(5*x) + 22*I*e^(3*x) - 6*I*e^x)/(e^(8*x) - 4*e^
(6*x) + 6*e^(4*x) - 4*e^(2*x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-\operatorname{csch}^{2}\left(x\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(x)**2)**(5/2),x)
```

```
[Out] Integral((-csch(x)**2)**(5/2), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.38, size = 72, normalized size = 1.80

$$-\frac{1}{16}\left(\frac{4i\left(3\left(e^{(-x)}+e^{x}\right)^{3}-20\,e^{(-x)}-20\,e^{x}\right)}{\left(\left(e^{(-x)}+e^{x}\right)^{2}-4\right)^{2}}-3i\,\log\left(e^{(-x)}+e^{x}+2\right)+3i\,\log\left(e^{(-x)}+e^{x}-2\right)\right)\operatorname{sgn}\left(-e^{(3\,x)}+e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/16*(4*I*(3*(e^(-x) + e^x)^3 - 20*e^(-x) - 20*e^x)/((e^(-x) + e^x)^2 - 4)
^2 - 3*I*log(e^(-x) + e^x + 2) + 3*I*log(e^(-x) + e^x - 2))*sgn(-e^(3*x) +
e^x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \left(-\frac{1}{\sinh\left(x\right)^2}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1/sinh(x)^2)^(5/2),x)
```

```
[Out] int((-1/sinh(x)^2)^(5/2), x)
```

3.23 $\int \left(-\text{csch}^2(x)\right)^{3/2} dx$

Optimal. Leaf size=24

$$\frac{1}{2}\text{ArcSin}(\coth(x))+\frac{1}{2}\coth(x)\sqrt{-\text{csch}^2(x)}$$

```
[Out] 1/2*arcsin(coth(x))+1/2*coth(x)*(-csch(x)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4207, 201, 222}

$$\frac{1}{2}\text{ArcSin}(\coth(x))+\frac{1}{2}\coth(x)\sqrt{-\text{csch}^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(-Csch[x]^2)^(3/2),x]
```

```
[Out] ArcSin[Coth[x]]/2 + (Coth[x]*Sqrt[-Csch[x]^2])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \left(-\operatorname{csch}^2(x)\right)^{3/2} dx = \text{Subst}\left(\int \sqrt{1-x^2}\, dx, x, \coth(x)\right)$$
$$= \frac{1}{2}\coth(x)\sqrt{-\operatorname{csch}^2(x)} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\, dx, x, \coth(x)\right)$$
$$= \frac{1}{2}\sin^{-1}(\coth(x)) + \frac{1}{2}\coth(x)\sqrt{-\operatorname{csch}^2(x)}$$

Mathematica [A]
time = 0.04, size = 41, normalized size = 1.71

$$\frac{1}{4}\operatorname{csch}\left(\frac{x}{2}\right)\sqrt{-\operatorname{csch}^2(x)}\operatorname{sech}\left(\frac{x}{2}\right)\left(\cosh(x) + \log\left(\tanh\left(\frac{x}{2}\right)\right)\sinh^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Csch[x]^2)^(3/2),x]
```

```
[Out] (Csch[x/2]*Sqrt[-Csch[x]^2]*Sech[x/2]*(Cosh[x] + Log[Tanh[x/2]]*Sinh[x]^2))
/4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(18) = 36$.
time = 0.80, size = 99, normalized size = 4.12

method	result	size
risch	$\frac{\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}\,(1+e^{2x})}{e^{2x}-1}+\frac{\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}\,e^{-x}(e^{2x}-1)\ln(e^x-1)}{2}-\frac{\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}\,e^{-x}(e^{2x}-1)\ln(e^x+1)}{2}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-csch(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(exp(2*x)-1)*(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)*(1+exp(2*x))+1/2*(-exp(2*x)
/(exp(2*x)-1)^2)^(1/2)*exp(-x)*(exp(2*x)-1)*ln(exp(x)-1)-1/2*(-exp(2*x)/(ex
p(2*x)-1)^2)^(1/2)*exp(-x)*(exp(2*x)-1)*ln(exp(x)+1)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.48, size = 49, normalized size = 2.04

$$\frac{i\,e^{(-x)} + i\,e^{(-3\,x)}}{2\,e^{(-2\,x)} - e^{(-4\,x)} - 1} + \frac{1}{2}i\log\left(e^{(-x)} + 1\right) - \frac{1}{2}i\log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] (I*e^(-x) + I*e^(-3*x))/(2*e^(-2*x) - e^(-4*x) - 1) + 1/2*I*log(e^(-x) + 1)
 - 1/2*I*log(e^(-x) - 1)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.45, size = 67, normalized size = 2.79

$$\frac{\left(-i\,e^{(4\,x)}+2i\,e^{(2\,x)}-i\right)\log\left(e^{x}+1\right)+\left(i\,e^{(4\,x)}-2i\,e^{(2\,x)}+i\right)\log\left(e^{x}-1\right)+2i\,e^{(3\,x)}+2i\,e^{x}}{2\left(e^{(4\,x)}-2\,e^{(2\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((-I*e^(4*x) + 2*I*e^(2*x) - I)*log(e^x + 1) + (I*e^(4*x) - 2*I*e^(2*x)
 + I)*log(e^x - 1) + 2*I*e^(3*x) + 2*I*e^x)/(e^(4*x) - 2*e^(2*x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-\operatorname{csch}^{2}\left(x\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(x)**2)**(3/2),x)
```

```
[Out] Integral((-csch(x)**2)**(3/2), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.40, size = 57, normalized size = 2.38

$$-\frac{1}{4}\left(\frac{4i\left(e^{(-x)}+e^{x}\right)}{\left(e^{(-x)}+e^{x}\right)^{2}-4}-i\,\log\left(e^{(-x)}+e^{x}+2\right)+i\,\log\left(e^{(-x)}+e^{x}-2\right)\right)\operatorname{sgn}\left(-e^{(3\,x)}+e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*(4*I*(e^(-x) + e^x)/((e^(-x) + e^x)^2 - 4) - I*log(e^(-x) + e^x + 2) +
 I*log(e^(-x) + e^x - 2))*sgn(-e^(3*x) + e^x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int\left(-\frac{1}{\sinh\left(x\right)^{2}}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1/sinh(x)^2)^(3/2),x)
```

```
[Out] int((-1/sinh(x)^2)^(3/2), x)
```

3.24 $\int \sqrt{-\operatorname{csch}^2(x)}\, dx$

Optimal. Leaf size=3

$$\text{ArcSin}(\coth(x))$$

```
[Out] arcsin(coth(x))
```

Rubi [A]
time = 0.01, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4207, 222}

$$\text{ArcSin}(\coth(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-Csch[x]^2],x]
```

```
[Out] ArcSin[Coth[x]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \sqrt{-\operatorname{csch}^2(x)}\, dx &= \text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\, dx, x, \coth(x)\right)\\ &= \sin^{-1}(\coth(x))\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 20 vs. 2(3) = 6.
time = 0.00, size = 20, normalized size = 6.67

$$\sqrt{-\operatorname{csch}^2(x)}\, \log\left(\tanh\left(\frac{x}{2}\right)\right) \sinh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-Csch[x]^2],x]
```

```
[Out] Sqrt[-Csch[x]^2]*Log[Tanh[x/2]]*Sinh[x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(3) = 6$.
time = 0.89, size = 67, normalized size = 22.33

method	result	size
risch	$-\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^{2}}}\, e^{-x}\left(e^{2x}-1\right)\ln\left(e^{x}+1\right)+\sqrt{-\frac{e^{2x}}{\left(e^{2x}-1\right)^{2}}}\, e^{-x}\left(e^{2x}-1\right)\ln\left(e^{x}-1\right)$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-csch(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)*exp(-x)*(exp(2*x)-1)*ln(exp(x)+1)+(-exp(2
*x)/(exp(2*x)-1)^2)^(1/2)*exp(-x)*(exp(2*x)-1)*ln(exp(x)-1)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.48, size = 19, normalized size = 6.33

$$i\,\log\left(e^{(-x)}+1\right)-i\,\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] I*log(e^(-x) + 1) - I*log(e^(-x) - 1)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.50, size = 15, normalized size = 5.00

$$-i\,\log\left(e^{x}+1\right)+i\,\log\left(e^{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -I*log(e^x + 1) + I*log(e^x - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-\operatorname{csch}^{2}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-csch(x)**2), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.39, size = 27, normalized size = 9.00

$$(i \log (e^x + 1) - i \log (|e^x - 1|))\text{sgn}\left(-e^{(3x)} + e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csch(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] (I*log(e^x + 1) - I*log(abs(e^x - 1)))*sgn(-e^(3*x) + e^x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.33

$$\int \sqrt{-\frac{1}{\sinh (x)^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1/sinh(x)^2)^(1/2),x)
```

```
[Out] int((-1/sinh(x)^2)^(1/2), x)
```

3.25 $\int \frac{1}{\sqrt{-\operatorname{csch}^2(x)}} \, dx$

Optimal. Leaf size=13

$$\frac{\coth(x)}{\sqrt{-\operatorname{csch}^2(x)}}$$

```
[Out] coth(x)/(-csch(x)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4207, 197}

$$\frac{\coth(x)}{\sqrt{-\operatorname{csch}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-Csch[x]^2],x]
```

```
[Out] Coth[x]/Sqrt[-Csch[x]^2]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{-\operatorname{csch}^2(x)}} \, dx &= \text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^{3/2}} \, dx, x, \coth(x)\right) \\ &= \frac{\coth(x)}{\sqrt{-\operatorname{csch}^2(x)}} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$\frac{\coth(x)}{\sqrt{-\operatorname{csch}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-Csch[x]^2],x]
```

```
[Out] Coth[x]/Sqrt[-Csch[x]^2]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(11) = 22$.
time = 0.81, size = 58, normalized size = 4.46

method	result	size
risch	$\frac{e^{2x}}{2\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}\,(e^{2x}-1)}+\frac{1}{2(e^{2x}-1)\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-csch(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)/(exp(2*x)-1)*exp(2*x)+1/2/(exp(2*x)-1)
/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.49, size = 11, normalized size = 0.85

$$\frac{1}{2}i\,e^{(-x)}+\frac{1}{2}i\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*I*e^(-x) + 1/2*I*e^x
```

Fricas [C] Result contains complex when optimal does not.
time = 0.38, size = 14, normalized size = 1.08

$$\frac{1}{2}\left(-i\,e^{(2\,x)}-i\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*e^(2*x) - I)*e^(-x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\operatorname{csch}^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)**2)**(1/2),x)
[Out] Integral(1/sqrt(-csch(x)**2), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.38, size = 25, normalized size = 1.92

$$-\frac{-i\,e^{(-x)}-i\,e^{x}}{2\,\operatorname{sgn}\left(-e^{(3\,x)}+e^{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)^2)^(1/2),x, algorithm="giac")
[Out] -1/2*(-I*e^(-x) - I*e^x)/sgn(-e^(3*x) + e^x)
```

Mupad [B]
time = 1.72, size = 31, normalized size = 2.38

$$-\mathrm{e}^{-2\,x}\,\sqrt{-\frac{1}{{\left(\frac{\mathrm{e}^{-x}}{2}-\frac{\mathrm{e}^{x}}{2}\right)}^2}}\,\left(\frac{\mathrm{e}^{4\,x}}{4}-\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1/sinh(x)^2)^(1/2),x)
[Out] -exp(-2*x)*(-1/(exp(-x)/2 - exp(x)/2)^2)^(1/2)*(exp(4*x)/4 - 1/4)
```

3.26 $\int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=33

$$\frac{\coth(x)}{3\left(-\operatorname{csch}^2(x)\right)^{3/2}}+\frac{2\coth(x)}{3\sqrt{-\operatorname{csch}^2(x)}}$$

[Out] 1/3*coth(x)/(-csch(x)^2)^(3/2)+2/3*coth(x)/(-csch(x)^2)^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4207, 198, 197}

$$\frac{2\coth(x)}{3\sqrt{-\operatorname{csch}^2(x)}}+\frac{\coth(x)}{3\left(-\operatorname{csch}^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(-Csch[x]^2)^(-3/2),x]

[Out] Coth[x]/(3*(-Csch[x]^2)^(3/2)) + (2*Coth[x])/(3*Sqrt[-Csch[x]^2])

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{3/2}}\,dx &= \operatorname{Subst}\left(\int \frac{1}{(1-x^2)^{5/2}}\,dx, x, \coth(x)\right)\\ &= \frac{\coth(x)}{3\left(-\operatorname{csch}^2(x)\right)^{3/2}} + \frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{(1-x^2)^{3/2}}\,dx, x, \coth(x)\right)\\ &= \frac{\coth(x)}{3\left(-\operatorname{csch}^2(x)\right)^{3/2}} + \frac{2\coth(x)}{3\sqrt{-\operatorname{csch}^2(x)}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 0.82

$$\frac{9\coth(x) - \cosh(3x)\operatorname{csch}(x)}{12\sqrt{-\operatorname{csch}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Csch[x]^2)^(-3/2),x]
```

```
[Out] (9*Coth[x] - Cosh[3*x]*Csch[x])/(12*Sqrt[-Csch[x]^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(25) = 50$.
time = 0.81, size = 118, normalized size = 3.58

method	result	si
risch	$-\frac{e^{4x}}{24(e^{2x}-1)\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}} + \frac{3\,e^{2x}}{8\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}\,(e^{2x}-1)} + \frac{3}{8(e^{2x}-1)\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}} - \frac{e^{-2x}}{24(e^{2x}-1)\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-csch(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*exp(4*x)/(exp(2*x)-1)/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)+3/8/(-exp(2*x)
/(exp(2*x)-1)^2)^(1/2)/(exp(2*x)-1)*exp(2*x)+3/8/(exp(2*x)-1)/(-exp(2*x)/(e
xp(2*x)-1)^2)^(1/2)-1/24*exp(-2*x)/(exp(2*x)-1)/(-exp(2*x)/(exp(2*x)-1)^2)^
(1/2)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.48, size = 23, normalized size = 0.70

$$-\frac{1}{24}i\,e^{(3\,x)} + \frac{3}{8}i\,e^{(-x)} - \frac{1}{24}i\,e^{(-3\,x)} + \frac{3}{8}i\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/24*I*e^(3*x) + 3/8*I*e^(-x) - 1/24*I*e^(-3*x) + 3/8*I*e^x
```

Fricas [C] Result contains complex when optimal does not.
time = 0.43, size = 26, normalized size = 0.79

$$\frac{1}{24}\left(i\,e^{(6\,x)}-9i\,e^{(4\,x)}-9i\,e^{(2\,x)}+i\right)e^{(-3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*(I*e^(6*x) - 9*I*e^(4*x) - 9*I*e^(2*x) + I)*e^(-3*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(-\operatorname{csch}^{2}(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)**2)**(3/2),x)
```

```
[Out] Integral((-csch(x)**2)**(-3/2), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.38, size = 38, normalized size = 1.15

$$\frac{\left(9i\,e^{(2\,x)}-i\right)e^{(-3\,x)}-i\,e^{(3\,x)}+9i\,e^{x}}{24\,\operatorname{sgn}\left(-e^{(3\,x)}+e^{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/24*((9*I*e^(2*x) - I)*e^(-3*x) - I*e^(3*x) + 9*I*e^x)/sgn(-e^(3*x) + e^x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{1}{{\left(-\frac{1}{{\mathrm{sinh}\left(x\right)}^2}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1/sinh(x)^2)^(3/2),x)
```

```
[Out] int(1/(-1/sinh(x)^2)^(3/2), x)
```

3.27 $\int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=49

$$\frac{\coth(x)}{5\left(-\operatorname{csch}^2(x)\right)^{5/2}}+\frac{4\coth(x)}{15\left(-\operatorname{csch}^2(x)\right)^{3/2}}+\frac{8\coth(x)}{15\sqrt{-\operatorname{csch}^2(x)}}$$

```
[Out] 1/5*coth(x)/(-csch(x)^2)^(5/2)+4/15*coth(x)/(-csch(x)^2)^(3/2)+8/15*coth(x)
/(-csch(x)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.300$, Rules used = {4207, 198, 197}

$$\frac{8\coth(x)}{15\sqrt{-\operatorname{csch}^2(x)}}+\frac{4\coth(x)}{15\left(-\operatorname{csch}^2(x)\right)^{3/2}}+\frac{\coth(x)}{5\left(-\operatorname{csch}^2(x)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Csch[x]^2)^(-5/2),x]
```

```
[Out] Coth[x]/(5*(-Csch[x]^2)^(5/2)) + (4*Coth[x])/(15*(-Csch[x]^2)^(3/2)) + (8*C
oth[x])/(15*Sqrt[-Csch[x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{1}{(1-x^2)^{7/2}}\,dx, x, \coth(x)\right)$$
$$= \frac{\coth(x)}{5\left(-\operatorname{csch}^2(x)\right)^{5/2}} + \frac{4}{5}\text{Subst}\left(\int \frac{1}{(1-x^2)^{5/2}}\,dx, x, \coth(x)\right)$$
$$= \frac{\coth(x)}{5\left(-\operatorname{csch}^2(x)\right)^{5/2}} + \frac{4\coth(x)}{15\left(-\operatorname{csch}^2(x)\right)^{3/2}} + \frac{8}{15}\text{Subst}\left(\int \frac{1}{(1-x^2)^{3/2}}\,dx, x, \coth(x)\right)$$
$$= \frac{\coth(x)}{5\left(-\operatorname{csch}^2(x)\right)^{5/2}} + \frac{4\coth(x)}{15\left(-\operatorname{csch}^2(x)\right)^{3/2}} + \frac{8\coth(x)}{15\sqrt{-\operatorname{csch}^2(x)}}$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 0.67

$$\frac{(150\cosh(x) - 25\cosh(3x) + 3\cosh(5x))\operatorname{csch}(x)}{240\sqrt{-\operatorname{csch}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Csch[x]^2)^(-5/2),x]
[Out] ((150*Cosh[x] - 25*Cosh[3*x] + 3*Cosh[5*x])*Csch[x])/(240*Sqrt[-Csch[x]^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(37) = 74.
time = 0.80, size = 178, normalized size = 3.63

method	result
risch	$\frac{e^{6x}}{160(e^{2x}-1)\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}} - \frac{5\,e^{4x}}{96(e^{2x}-1)\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}} + \frac{5\,e^{2x}}{16\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}\,(e^{2x}-1)} + \frac{5}{16(e^{2x}-1)\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}} - \frac{}{96(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-csch(x)^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/160*exp(6*x)/(exp(2*x)-1)/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)-5/96*exp(4*x)/
(exp(2*x)-1)/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)+5/16/(-exp(2*x)/(exp(2*x)-1)^
2)^(1/2)/(exp(2*x)-1)*exp(2*x)+5/16/(exp(2*x)-1)/(-exp(2*x)/(exp(2*x)-1)^2)
^(1/2)-5/96*exp(-2*x)/(exp(2*x)-1)/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)+1/160*e
xp(-4*x)/(exp(2*x)-1)/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.48, size = 35, normalized size = 0.71

$$\frac{1}{160}i\,e^{(5\,x)}-\frac{5}{96}i\,e^{(3\,x)}+\frac{5}{16}i\,e^{(-x)}-\frac{5}{96}i\,e^{(-3\,x)}+\frac{1}{160}i\,e^{(-5\,x)}+\frac{5}{16}i\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/160*I*e^(5*x) - 5/96*I*e^(3*x) + 5/16*I*e^(-x) - 5/96*I*e^(-3*x) + 1/160*
I*e^(-5*x) + 5/16*I*e^x
```

Fricas [C] Result contains complex when optimal does not.
time = 0.45, size = 38, normalized size = 0.78

$$\frac{1}{480}\left(-3i\,e^{(10\,x)}+25i\,e^{(8\,x)}-150i\,e^{(6\,x)}-150i\,e^{(4\,x)}+25i\,e^{(2\,x)}-3i\right)e^{(-5\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/480*(-3*I*e^(10*x) + 25*I*e^(8*x) - 150*I*e^(6*x) - 150*I*e^(4*x) + 25*I*
e^(2*x) - 3*I)*e^(-5*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(-\operatorname{csch}^{2}\left(x\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)**2)**(5/2),x)
```

```
[Out] Integral((-csch(x)**2)**(-5/2), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.38, size = 50, normalized size = 1.02

$$-\frac{\left(-150i\,e^{(4\,x)}+25i\,e^{(2\,x)}-3i\right)e^{(-5\,x)}-3i\,e^{(5\,x)}+25i\,e^{(3\,x)}-150i\,e^{x}}{480\,\mathrm{sgn}\left(-e^{(3\,x)}+e^{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/480*((-150*I*e^(4*x) + 25*I*e^(2*x) - 3*I)*e^(-5*x) - 3*I*e^(5*x) + 25*I
*e^(3*x) - 150*I*e^x)/sgn(-e^(3*x) + e^x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(-\frac{1}{\sinh(x)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1/sinh(x)^2)^(5/2),x)
[Out] int(1/(-1/sinh(x)^2)^(5/2), x)
```

3.28 $\int \frac{1}{\left(-\text{csch}^2(x)\right)^{7/2}}\,dx$

Optimal. Leaf size=65

$$\frac{\coth(x)}{7\left(-\text{csch}^2(x)\right)^{7/2}}+\frac{6\coth(x)}{35\left(-\text{csch}^2(x)\right)^{5/2}}+\frac{8\coth(x)}{35\left(-\text{csch}^2(x)\right)^{3/2}}+\frac{16\coth(x)}{35\sqrt{-\text{csch}^2(x)}}$$

```
[Out] 1/7*coth(x)/(-csch(x)^2)^(7/2)+6/35*coth(x)/(-csch(x)^2)^(5/2)+8/35*coth(x)
/(-csch(x)^2)^(3/2)+16/35*coth(x)/(-csch(x)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4207, 198, 197}

$$\frac{16\coth(x)}{35\sqrt{-\text{csch}^2(x)}}+\frac{8\coth(x)}{35\left(-\text{csch}^2(x)\right)^{3/2}}+\frac{6\coth(x)}{35\left(-\text{csch}^2(x)\right)^{5/2}}+\frac{\coth(x)}{7\left(-\text{csch}^2(x)\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Csch[x]^2)^(-7/2),x]
```

```
[Out] Coth[x]/(7*(-Csch[x]^2)^(7/2)) + (6*Coth[x])/(35*(-Csch[x]^2)^(5/2)) + (8*C
oth[x])/(35*(-Csch[x]^2)^(3/2)) + (16*Coth[x])/(35*Sqrt[-Csch[x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(-\operatorname{csch}^2(x)\right)^{7/2}}\,dx = \text{Subst}\left(\int \frac{1}{(1-x^2)^{9/2}}\,dx, x, \coth(x)\right)$$
$$= \frac{\coth(x)}{7\left(-\operatorname{csch}^2(x)\right)^{7/2}} + \frac{6}{7}\text{Subst}\left(\int \frac{1}{(1-x^2)^{7/2}}\,dx, x, \coth(x)\right)$$
$$= \frac{\coth(x)}{7\left(-\operatorname{csch}^2(x)\right)^{7/2}} + \frac{6\coth(x)}{35\left(-\operatorname{csch}^2(x)\right)^{5/2}} + \frac{24}{35}\text{Subst}\left(\int \frac{1}{(1-x^2)^{5/2}}\,dx, x, \coth(x)\right)$$
$$= \frac{\coth(x)}{7\left(-\operatorname{csch}^2(x)\right)^{7/2}} + \frac{6\coth(x)}{35\left(-\operatorname{csch}^2(x)\right)^{5/2}} + \frac{8\coth(x)}{35\left(-\operatorname{csch}^2(x)\right)^{3/2}} + \frac{16}{35}\text{Subst}\left(\int \frac{1}{(1-x^2)\right.$$
$$= \frac{\coth(x)}{7\left(-\operatorname{csch}^2(x)\right)^{7/2}} + \frac{6\coth(x)}{35\left(-\operatorname{csch}^2(x)\right)^{5/2}} + \frac{8\coth(x)}{35\left(-\operatorname{csch}^2(x)\right)^{3/2}} + \frac{16\coth(x)}{35\sqrt{-\operatorname{csch}^2(x)}}$$

Mathematica [**A**]
time = 0.03, size = 39, normalized size = 0.60

$$\frac{(1225\cosh(x) - 245\cosh(3x) + 49\cosh(5x) - 5\cosh(7x))\operatorname{csch}(x)}{2240\sqrt{-\operatorname{csch}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Csch[x]^2)^(-7/2),x]
[Out] ((1225*Cosh[x] - 245*Cosh[3*x] + 49*Cosh[5*x] - 5*Cosh[7*x])*Csch[x])/(2240
*Sqrt[-Csch[x]^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(49) = 98.
time = 0.80, size = 238, normalized size = 3.66

method	result
risch	$-\frac{e^{8x}}{896(e^{2x}-1)\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}} + \frac{7\,e^{6x}}{640(e^{2x}-1)\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}} - \frac{7\,e^{4x}}{128(e^{2x}-1)\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}} + \frac{35\,e^{2x}}{128\sqrt{-\frac{e^{2x}}{(e^{2x}-1)^2}}\,(e^{2x}-1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-csch(x)^2)^(7/2),x,method=_RETURNVERBOSE)
[Out] -1/896*exp(8*x)/(exp(2*x)-1)/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)+7/640*exp(6*x
)/(exp(2*x)-1)/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)-7/128*exp(4*x)/(exp(2*x)-1)
/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)+35/128/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)/(
```

```
exp(2*x)-1)*exp(2*x)+35/128/(exp(2*x)-1)/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)-7
/128*exp(-2*x)/(exp(2*x)-1)/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)+7/640*exp(-4*x
)/(exp(2*x)-1)/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)-1/896*exp(-6*x)/(exp(2*x)-1
)/(-exp(2*x)/(exp(2*x)-1)^2)^(1/2)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.47, size = 47, normalized size = 0.72

$$-\frac{1}{896} i e^{(7x)} + \frac{7}{640} i e^{(5x)} - \frac{7}{128} i e^{(3x)} + \frac{35}{128} i e^{(-x)} - \frac{7}{128} i e^{(-3x)} + \frac{7}{640} i e^{(-5x)} - \frac{1}{896} i e^{(-7x)} + \frac{35}{128} i e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/896*I*e^(7*x) + 7/640*I*e^(5*x) - 7/128*I*e^(3*x) + 35/128*I*e^(-x) - 7/
128*I*e^(-3*x) + 7/640*I*e^(-5*x) - 1/896*I*e^(-7*x) + 35/128*I*e^x
```

Fricas [C] Result contains complex when optimal does not.
time = 0.42, size = 50, normalized size = 0.77

$$\frac{1}{4480} \left(5i e^{(14x)} - 49i e^{(12x)} + 245i e^{(10x)} - 1225i e^{(8x)} - 1225i e^{(6x)} + 245i e^{(4x)} - 49i e^{(2x)} + 5i\right) e^{(-7x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/4480*(5*I*e^(14*x) - 49*I*e^(12*x) + 245*I*e^(10*x) - 1225*I*e^(8*x) - 12
25*I*e^(6*x) + 245*I*e^(4*x) - 49*I*e^(2*x) + 5*I)*e^(-7*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\operatorname{csch}^{2}(x)\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)**2)**(7/2),x)
```

```
[Out] Integral((-csch(x)**2)**(-7/2), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.38, size = 62, normalized size = 0.95

$$-\frac{\left(-1225i e^{(6x)} + 245i e^{(4x)} - 49i e^{(2x)} + 5i\right) e^{(-7x)} + 5i e^{(7x)} - 49i e^{(5x)} + 245i e^{(3x)} - 1225i e^{x}}{4480 \operatorname{sgn}\left(-e^{(3x)} + e^{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-csch(x)^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/4480*((-1225*I*e^(6*x) + 245*I*e^(4*x) - 49*I*e^(2*x) + 5*I)*e^(-7*x) +
5*I*e^(7*x) - 49*I*e^(5*x) + 245*I*e^(3*x) - 1225*I*e^x)/sgn(-e^(3*x) + e^x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(-\frac{1}{\sinh(x)^2}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1/sinh(x)^2)^(7/2),x)
```

```
[Out] int(1/(-1/sinh(x)^2)^(7/2), x)
```

3.29 $\int \left(a\text{csch}^2(x)\right)^{5/2}\,dx$

Optimal. Leaf size=65

$$-\frac{3}{8}a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\ \coth(x)}{\sqrt{a\text{csch}^2(x)}}\right)+\frac{3}{8}a^2\coth(x)\sqrt{a\text{csch}^2(x)}-\frac{1}{4}a\coth(x)\left(a\text{csch}^2(x)\right)^{3/2}$$

```
[Out] -3/8*a^(5/2)*arctanh(coth(x)*a^(1/2)/(a*csch(x)^2)^(1/2))-1/4*a*coth(x)*(a*
csch(x)^2)^(3/2)+3/8*a^2*coth(x)*(a*csch(x)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4207, 201, 223, 212}

$$-\frac{3}{8}a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\ \coth(x)}{\sqrt{a\text{csch}^2(x)}}\right)+\frac{3}{8}a^2\coth(x)\sqrt{a\text{csch}^2(x)}-\frac{1}{4}a\coth(x)\left(a\text{csch}^2(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Csch[x]^2)^(5/2),x]
```

```
[Out] (-3*a^(5/2)*ArcTanh[(Sqrt[a]*Coth[x])/Sqrt[a*Csch[x]^2]])/8 + (3*a^2*Coth[x
]*Sqrt[a*Csch[x]^2])/8 - (a*Coth[x]*(a*Csch[x]^2)^(3/2))/4
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \left(a\text{csch}^2(x)\right)^{5/2}\,dx &= -\left(a\text{Subst}\left(\int \left(-a+ax^2\right)^{3/2}\,dx,x,\coth(x)\right)\right)\\
&= -\frac{1}{4}a\coth(x)\left(a\text{csch}^2(x)\right)^{3/2}+\frac{1}{4}\left(3a^2\right)\text{Subst}\left(\int \sqrt{-a+ax^2}\,dx,x,\coth(x)\right)\\
&= \frac{3}{8}a^2\coth(x)\sqrt{a\text{csch}^2(x)}-\frac{1}{4}a\coth(x)\left(a\text{csch}^2(x)\right)^{3/2}-\frac{1}{8}\left(3a^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{-a+ax}}\right.\\
&= \frac{3}{8}a^2\coth(x)\sqrt{a\text{csch}^2(x)}-\frac{1}{4}a\coth(x)\left(a\text{csch}^2(x)\right)^{3/2}-\frac{1}{8}\left(3a^3\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,d\right.\\
&= -\frac{3}{8}a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\coth(x)}{\sqrt{a\text{csch}^2(x)}}\right)+\frac{3}{8}a^2\coth(x)\sqrt{a\text{csch}^2(x)}-\frac{1}{4}a\coth(x)\left(a\text{csch}^2(x\right.
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 41, normalized size = 0.63

$$\frac{1}{64}\left(a\text{csch}^2(x)\right)^{5/2}\sinh(x)\left(-22\cosh(x)+6\left(\cosh(3x)+4\log\left(\tanh\left(\frac{x}{2}\right)\right)\sinh^4(x)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Csch[x]^2)^(5/2),x]
```

```
[Out] ((a*Csch[x]^2)^(5/2)*Sinh[x]*(-22*Cosh[x] + 6*(Cosh[3*x] + 4*Log[Tanh[x/2]]
*Sinh[x]^4)))/64
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(49) = 98.
time = 0.93, size = 123, normalized size = 1.89

method	result
risch	$\frac{a^2\sqrt{\frac{a\,e^{2x}}{(e^{2x}-1)^2}}\,(3\,e^{6x}-11\,e^{4x}-11\,e^{2x}+3)}{4(e^{2x}-1)^3}-\frac{3a^2e^{-x}(e^{2x}-1)\sqrt{\frac{a\,e^{2x}}{(e^{2x}-1)^2}}\,\ln(e^x+1)}{8}+\frac{3a^2e^{-x}(e^{2x}-1)\sqrt{\frac{a\,e^{2x}}{(e^{2x}-1)^2}}\,\ln(e^x-1)}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*csch(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^2/(exp(2*x)-1)^3*(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)*(3*exp(6*x)-11*exp
(4*x)-11*exp(2*x)+3)-3/8*a^2*exp(-x)*(exp(2*x)-1)*(a*exp(2*x)/(exp(2*x)-1)^
2)^(1/2)*ln(exp(x)+1)+3/8*a^2*exp(-x)*(exp(2*x)-1)*(a*exp(2*x)/(exp(2*x)-1)
^2)^(1/2)*ln(exp(x)-1)
```

Maxima [A]
time = 0.47, size = 92, normalized size = 1.42

$$\frac{3}{8}\,a^{\frac{5}{2}}\log\left(e^{(-x)}+1\right)-\frac{3}{8}\,a^{\frac{5}{2}}\log\left(e^{(-x)}-1\right)+\frac{3\,a^{\frac{5}{2}}e^{(-x)}-11\,a^{\frac{5}{2}}e^{(-3\,x)}-11\,a^{\frac{5}{2}}e^{(-5\,x)}+3\,a^{\frac{5}{2}}e^{(-7\,x)}}{4\,\left(4\,e^{(-2\,x)}-6\,e^{(-4\,x)}+4\,e^{(-6\,x)}-e^{(-8\,x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 3/8*a^(5/2)*log(e^(-x) + 1) - 3/8*a^(5/2)*log(e^(-x) - 1) + 1/4*(3*a^(5/2)*
e^(-x) - 11*a^(5/2)*e^(-3*x) - 11*a^(5/2)*e^(-5*x) + 3*a^(5/2)*e^(-7*x))/(4
*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1128 vs. $2(49) = 98$.
time = 0.49, size = 1128, normalized size = 17.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/8*(6*a^2*cosh(x)^7 - 6*(a^2*e^(2*x) - a^2)*sinh(x)^7 - 22*a^2*cosh(x)^5
- 42*(a^2*cosh(x)*e^(2*x) - a^2*cosh(x))*sinh(x)^6 + 2*(63*a^2*cosh(x)^2 -
11*a^2 - (63*a^2*cosh(x)^2 - 11*a^2)*e^(2*x))*sinh(x)^5 - 22*a^2*cosh(x)^3
+ 10*(21*a^2*cosh(x)^3 - 11*a^2*cosh(x) - (21*a^2*cosh(x)^3 - 11*a^2*cosh(x
))*e^(2*x))*sinh(x)^4 + 2*(105*a^2*cosh(x)^4 - 110*a^2*cosh(x)^2 - 11*a^2 -
 (105*a^2*cosh(x)^4 - 110*a^2*cosh(x)^2 - 11*a^2)*e^(2*x))*sinh(x)^3 + 6*a^
2*cosh(x) + 2*(63*a^2*cosh(x)^5 - 110*a^2*cosh(x)^3 - 33*a^2*cosh(x) - (63*
a^2*cosh(x)^5 - 110*a^2*cosh(x)^3 - 33*a^2*cosh(x))*e^(2*x))*sinh(x)^2 - 2*
(3*a^2*cosh(x)^7 - 11*a^2*cosh(x)^5 - 11*a^2*cosh(x)^3 + 3*a^2*cosh(x))*e^(
2*x) + 3*(a^2*cosh(x)^8 - (a^2*e^(2*x) - a^2)*sinh(x)^8 - 4*a^2*cosh(x)^6 -
 8*(a^2*cosh(x)*e^(2*x) - a^2*cosh(x))*sinh(x)^7 + 4*(7*a^2*cosh(x)^2 - a^2
 - (7*a^2*cosh(x)^2 - a^2)*e^(2*x))*sinh(x)^6 + 6*a^2*cosh(x)^4 + 8*(7*a^2*
cosh(x)^3 - 3*a^2*cosh(x) - (7*a^2*cosh(x)^3 - 3*a^2*cosh(x))*e^(2*x))*sinh
(x)^5 + 2*(35*a^2*cosh(x)^4 - 30*a^2*cosh(x)^2 + 3*a^2 - (35*a^2*cosh(x)^4
- 30*a^2*cosh(x)^2 + 3*a^2)*e^(2*x))*sinh(x)^4 - 4*a^2*cosh(x)^2 + 8*(7*a^2
*cosh(x)^5 - 10*a^2*cosh(x)^3 + 3*a^2*cosh(x) - (7*a^2*cosh(x)^5 - 10*a^2*c
osh(x)^3 + 3*a^2*cosh(x))*e^(2*x))*sinh(x)^3 + 4*(7*a^2*cosh(x)^6 - 15*a^2*
```

```
cosh(x)^4 + 9*a^2*cosh(x)^2 - a^2 - (7*a^2*cosh(x)^6 - 15*a^2*cosh(x)^4 + 9
*a^2*cosh(x)^2 - a^2)*e^(2*x))*sinh(x)^2 + a^2 - (a^2*cosh(x)^8 - 4*a^2*cos
h(x)^6 + 6*a^2*cosh(x)^4 - 4*a^2*cosh(x)^2 + a^2)*e^(2*x) + 8*(a^2*cosh(x)^
7 - 3*a^2*cosh(x)^5 + 3*a^2*cosh(x)^3 - a^2*cosh(x) - (a^2*cosh(x)^7 - 3*a^
2*cosh(x)^5 + 3*a^2*cosh(x)^3 - a^2*cosh(x))*e^(2*x))*sinh(x))*log((cosh(x)
 + sinh(x) - 1)/(cosh(x) + sinh(x) + 1)) + 2*(21*a^2*cosh(x)^6 - 55*a^2*cos
h(x)^4 - 33*a^2*cosh(x)^2 + 3*a^2 - (21*a^2*cosh(x)^6 - 55*a^2*cosh(x)^4 -
33*a^2*cosh(x)^2 + 3*a^2)*e^(2*x))*sinh(x))*sqrt(a/(e^(4*x) - 2*e^(2*x) + 1
))*e^x/(8*cosh(x)*e^x*sinh(x)^7 + e^x*sinh(x)^8 + 4*(7*cosh(x)^2 - 1)*e^x*s
inh(x)^6 + 8*(7*cosh(x)^3 - 3*cosh(x))*e^x*sinh(x)^5 + 2*(35*cosh(x)^4 - 30
*cosh(x)^2 + 3)*e^x*sinh(x)^4 + 8*(7*cosh(x)^5 - 10*cosh(x)^3 + 3*cosh(x))*
e^x*sinh(x)^3 + 4*(7*cosh(x)^6 - 15*cosh(x)^4 + 9*cosh(x)^2 - 1)*e^x*sinh(x
)^2 + 8*(cosh(x)^7 - 3*cosh(x)^5 + 3*cosh(x)^3 - cosh(x))*e^x*sinh(x) + (co
sh(x)^8 - 4*cosh(x)^6 + 6*cosh(x)^4 - 4*cosh(x)^2 + 1)*e^x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \operatorname{csch}^{2}\left(x\right)\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)**2)**(5/2),x)
```

```
[Out] Integral((a*csch(x)**2)**(5/2), x)
```

Giac [**A**]
time = 0.39, size = 75, normalized size = 1.15

$$\frac{1}{16}\, a^{\frac{5}{2}} \left(\frac{4\left(3\left(e^{(-x)}+e^{x}\right)^{3}-20\,e^{(-x)}-20\,e^{x}\right)}{\left(\left(e^{(-x)}+e^{x}\right)^{2}-4\right)^{2}}-3\,\log\left(e^{(-x)}+e^{x}+2\right)+3\,\log\left(e^{(-x)}+e^{x}-2\right)\right)\operatorname{sgn}\left(e^{(3\,x)}-e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/16*a^(5/2)*(4*(3*(e^(-x) + e^x)^3 - 20*e^(-x) - 20*e^x)/((e^(-x) + e^x)^2
 - 4)^2 - 3*log(e^(-x) + e^x + 2) + 3*log(e^(-x) + e^x - 2))*sgn(e^(3*x) -
e^x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(\frac{a}{{\mathrm{sinh}\left(x\right)}^2}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/sinh(x)^2)^(5/2),x)
```

```
[Out] int((a/sinh(x)^2)^(5/2), x)
```

3.30 $\int \left(a\operatorname{csch}^2(x)\right)^{3/2} dx$

Optimal. Leaf size=46

$$\frac{1}{2}a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\ \coth(x)}{\sqrt{a\operatorname{csch}^2(x)}}\right)-\frac{1}{2}a\coth(x)\sqrt{a\operatorname{csch}^2(x)}$$

[Out] 1/2*a^(3/2)*arctanh(coth(x)*a^(1/2)/(a*csch(x)^2)^(1/2))-1/2*a*coth(x)*(a*csch(x)^2)^(1/2)

Rubi [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4207, 201, 223, 212}

$$\frac{1}{2}a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\ \coth(x)}{\sqrt{a\operatorname{csch}^2(x)}}\right)-\frac{1}{2}a\coth(x)\sqrt{a\operatorname{csch}^2(x)}$$

Antiderivative was successfully verified.

[In] Int[(a*Csch[x]^2)^(3/2),x]

[Out] (a^(3/2)*ArcTanh[(Sqrt[a]*Coth[x])/Sqrt[a*Csch[x]^2]])/2 - (a*Coth[x]*Sqrt[a*Csch[x]^2])/2

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(a\operatorname{csch}^2(x)\right)^{3/2} dx &= -\left(a\text{Subst}\left(\int \sqrt{-a+ax^2}\, dx, x, \coth(x)\right)\right)\\ &= -\frac{1}{2}a\coth(x)\sqrt{a\operatorname{csch}^2(x)} + \frac{1}{2}a^2\text{Subst}\left(\int \frac{1}{\sqrt{-a+ax^2}}\, dx, x, \coth(x)\right)\\ &= -\frac{1}{2}a\coth(x)\sqrt{a\operatorname{csch}^2(x)} + \frac{1}{2}a^2\text{Subst}\left(\int \frac{1}{1-ax^2}\, dx, x, \frac{\coth(x)}{\sqrt{a\operatorname{csch}^2(x)}}\right)\\ &= \frac{1}{2}a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\coth(x)}{\sqrt{a\operatorname{csch}^2(x)}}\right) - \frac{1}{2}a\coth(x)\sqrt{a\operatorname{csch}^2(x)}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 30, normalized size = 0.65

$$-\frac{1}{2}a\sqrt{a\operatorname{csch}^2(x)}\left(\coth(x)\operatorname{csch}(x) + \log\left(\tanh\left(\frac{x}{2}\right)\right)\right)\sinh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Csch[x]^2)^(3/2),x]
[Out] -1/2*(a*Sqrt[a*Csch[x]^2]*(Coth[x]*Csch[x] + Log[Tanh[x/2]])*Sinh[x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(34) = 68.
time = 0.81, size = 103, normalized size = 2.24

method	result	size
risch	$-\frac{a\sqrt{\frac{a\,e^{2x}}{(e^{2x}-1)^2}}\,(1+e^{2x})}{e^{2x}-1} + \frac{a\,e^{-x}(e^{2x}-1)\sqrt{\frac{a\,e^{2x}}{(e^{2x}-1)^2}}\,\ln(e^x+1)}{2} - \frac{a\,e^{-x}(e^{2x}-1)\sqrt{\frac{a\,e^{2x}}{(e^{2x}-1)^2}}\,\ln(e^x-1)}{2}$	103

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*csch(x)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -a/(exp(2*x)-1)*(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)*(1+exp(2*x))+1/2*a*exp(-x
)*(exp(2*x)-1)*(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)*ln(exp(x)+1)-1/2*a*exp(-x)
*(exp(2*x)-1)*(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)*ln(exp(x)-1)
```

Maxima [A]
time = 0.49, size = 60, normalized size = 1.30

$$-\frac{1}{2}\,a^{\frac{3}{2}}\log\left(e^{(-x)}+1\right)+\frac{1}{2}\,a^{\frac{3}{2}}\log\left(e^{(-x)}-1\right)-\frac{a^{\frac{3}{2}}e^{(-x)}+a^{\frac{3}{2}}e^{(-3\,x)}}{2\,e^{(-2\,x)}-e^{(-4\,x)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*a^(3/2)*log(e^(-x) + 1) + 1/2*a^(3/2)*log(e^(-x) - 1) - (a^(3/2)*e^(-x
) + a^(3/2)*e^(-3*x))/(2*e^(-2*x) - e^(-4*x) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(34) = 68$.
time = 0.49, size = 340, normalized size = 7.39

$$\frac{\left(2\,a\cosh\left(x\right)^{3}-2\,\left(ae^{(2\,x)}-a\right)\sinh\left(x\right)^{3}-6\,\left(a\cosh\left(x\right)e^{(2\,x)}-a\cosh\left(x\right)\right)\sinh\left(x\right)^{2}+2\,a\cosh\left(x\right)-2\,\left(a\cosh\left(x\right)^{3}+a\cosh\left(x\right)\right)e^{(2\,x)}-\left(a\cosh\left(x\right)^{4}-\left(ae^{(2\,x)}-a\right)\sinh\left(x\right)^{4}-4\,\left(a\cosh\left(x\right)e^{(2\,x)}-a\cosh\left(x\right)\right)\sinh\left(x\right)^{3}-2\,a\cosh\left(x\right)^{2}+2\,\left(3\,a\cosh\left(x\right)^{2}-\left(3\,a\cosh\left(x\right)^{2}-a\right)e^{(2\,x)}-a\right)\sinh\left(x\right)^{2}-\left(a\cosh\left(x\right)^{4}-2\,a\cosh\left(x\right)^{2}+a\right)e^{(2\,x)}+4\,\left(a\cosh\left(x\right)^{3}-a\cosh\left(x\right)-\left(a\cosh\left(x\right)^{3}-a\cosh\left(x\right)\right)e^{(2\,x)}\right)\sinh\left(x\right)+a\right)\log\left(\frac{\cosh(x)+\sinh(x)+1}{\cosh(x)+\sinh(x)-1}\right)+2\,\left(3\,a\cosh\left(x\right)^{2}-\left(3\,a\cosh\left(x\right)^{2}+a\right)e^{(2\,x)}+a\right)\sinh\left(x\right)\right)\sqrt{\frac{a}{e^{(4\,x)}-2\,e^{(2\,x)}+1}}\,e^{x}}{2\,\left(4\,\cosh\left(x\right)e^{x}\sinh\left(x\right)^{3}+e^{x}\sinh\left(x\right)^{4}+2\,\left(3\,\cosh\left(x\right)^{2}-1\right)e^{x}\sinh\left(x\right)^{2}+4\,\left(\cosh\left(x\right)^{3}-\cosh\left(x\right)\right)e^{x}\sinh\left(x\right)+\left(\cosh\left(x\right)^{4}-2\,\cosh\left(x\right)^{2}+1\right)e^{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*cosh(x)^3 - 2*(a*e^(2*x) - a)*sinh(x)^3 - 6*(a*cosh(x)*e^(2*x) - a
*cosh(x))*sinh(x)^2 + 2*a*cosh(x) - 2*(a*cosh(x)^3 + a*cosh(x))*e^(2*x) - (
a*cosh(x)^4 - (a*e^(2*x) - a)*sinh(x)^4 - 4*(a*cosh(x)*e^(2*x) - a*cosh(x))
*sinh(x)^3 - 2*a*cosh(x)^2 + 2*(3*a*cosh(x)^2 - (3*a*cosh(x)^2 - a)*e^(2*x)
 - a)*sinh(x)^2 - (a*cosh(x)^4 - 2*a*cosh(x)^2 + a)*e^(2*x) + 4*(a*cosh(x)^
3 - a*cosh(x) - (a*cosh(x)^3 - a*cosh(x))*e^(2*x))*sinh(x) + a)*log((cosh(x
) + sinh(x) + 1)/(cosh(x) + sinh(x) - 1)) + 2*(3*a*cosh(x)^2 - (3*a*cosh(x)
^2 + a)*e^(2*x) + a)*sinh(x))*sqrt(a/(e^(4*x) - 2*e^(2*x) + 1))*e^x/(4*cosh
(x)*e^x*sinh(x)^3 + e^x*sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*e^x*sinh(x)^2 + 4*(
cosh(x)^3 - cosh(x))*e^x*sinh(x) + (cosh(x)^4 - 2*cosh(x)^2 + 1)*e^x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\operatorname{csch}^{2}\left(x\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)**2)**(3/2),x)
```

```
[Out] Integral((a*csch(x)**2)**(3/2), x)
```

Giac [A]
time = 0.40, size = 58, normalized size = 1.26

$$-\frac{1}{4}\,a^{\frac{3}{2}}\left(\frac{4\,\left(e^{(-x)}+e^{x}\right)}{\left(e^{(-x)}+e^{x}\right)^{2}-4}-\log\left(e^{(-x)}+e^{x}+2\right)+\log\left(e^{(-x)}+e^{x}-2\right)\right)\operatorname{sgn}\left(e^{(3\,x)}-e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*a^(3/2)*(4*(e^(-x) + e^x)/((e^(-x) + e^x)^2 - 4) - log(e^(-x) + e^x +
2) + log(e^(-x) + e^x - 2))*sgn(e^(3*x) - e^x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \left(\frac{a}{\sinh(x)^2}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/sinh(x)^2)^(3/2),x)
```

```
[Out] int((a/sinh(x)^2)^(3/2), x)
```

3.31 $\int \sqrt{a\operatorname{csch}^2(x)}\,dx$

Optimal. Leaf size=26

$$-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\coth(x)}{\sqrt{a\operatorname{csch}^2(x)}}\right)$$

```
[Out] -arctanh(coth(x)*a^(1/2)/(a*csch(x)^2)^(1/2))*a^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4207, 223, 212}

$$-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\coth(x)}{\sqrt{a\operatorname{csch}^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Csch[x]^2],x]
```

```
[Out] -(Sqrt[a]*ArcTanh[(Sqrt[a]*Coth[x])/Sqrt[a*Csch[x]^2]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sqrt{a\text{csch}^2(x)}\, dx = -\left(a\text{Subst}\left(\int \frac{1}{\sqrt{-a+ax^2}}\, dx, x, \coth(x)\right)\right)$$
$$= -\left(a\text{Subst}\left(\int \frac{1}{1-ax^2}\, dx, x, \frac{\coth(x)}{\sqrt{a\text{csch}^2(x)}}\right)\right)$$
$$= -\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\coth(x)}{\sqrt{a\text{csch}^2(x)}}\right)$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 0.77

$$\sqrt{a\text{csch}^2(x)}\log\left(\tanh\left(\frac{x}{2}\right)\right)\sinh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Csch[x]^2],x]
[Out] Sqrt[a*Csch[x]^2]*Log[Tanh[x/2]]*Sinh[x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(20) = 40.
time = 1.05, size = 67, normalized size = 2.58

method	result	size
risch	$\sqrt{\frac{a\,\mathrm{e}^{2x}}{(\mathrm{e}^{2x}-1)^2}}\;\mathrm{e}^{-x}(\mathrm{e}^{2x}-1)\ln(\mathrm{e}^{x}-1)-\sqrt{\frac{a\,\mathrm{e}^{2x}}{(\mathrm{e}^{2x}-1)^2}}\;\mathrm{e}^{-x}(\mathrm{e}^{2x}-1)\ln(\mathrm{e}^{x}+1)$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*csch(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] (a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)*exp(-x)*(exp(2*x)-1)*ln(exp(x)-1)-(a*exp(
2*x)/(exp(2*x)-1)^2)^(1/2)*exp(-x)*(exp(2*x)-1)*ln(exp(x)+1)
```

Maxima [A]
time = 0.48, size = 24, normalized size = 0.92

$$\sqrt{a}\log\left(e^{(-x)}+1\right)-\sqrt{a}\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(a)*log(e^(-x) + 1) - sqrt(a)*log(e^(-x) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(20) = 40$.
time = 0.38, size = 97, normalized size = 3.73

$$\left[\sqrt{\frac{a}{e^{(4x)} - 2e^{(2x)} + 1}}\left(e^{(2x)} - 1\right)\log\left(\frac{\cosh(x) + \sinh(x) - 1}{\cosh(x) + \sinh(x) + 1}\right), 2\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{a}{e^{(4x)} - 2e^{(2x)} + 1}}\left(e^{(2x)} - 1\right)e^{x}}{a\cosh(x)e^{x} + ae^{x}\sinh(x)}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(a/(e^(4*x) - 2*e^(2*x) + 1))*(e^(2*x) - 1)*log((cosh(x) + sinh(x) - 1
)/(cosh(x) + sinh(x) + 1)), 2*sqrt(-a)*arctan(sqrt(-a)*sqrt(a/(e^(4*x) - 2*
e^(2*x) + 1))*(e^(2*x) - 1)*e^x/(a*cosh(x)*e^x + a*e^x*sinh(x)))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\operatorname{csch}^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a*csch(x)**2), x)
```

Giac [A]
time = 0.38, size = 29, normalized size = 1.12

$$-\sqrt{a}\left(\log\left(e^{x} + 1\right) - \log\left(|e^{x} - 1|\right)\right)\operatorname{sgn}\left(e^{(3x)} - e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(a)*(log(e^x + 1) - log(abs(e^x - 1)))*sgn(e^(3*x) - e^x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int\sqrt{\frac{a}{\sinh(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/sinh(x)^2)^(1/2),x)
```

```
[Out] int((a/sinh(x)^2)^(1/2), x)
```

3.32 $\int \frac{1}{\sqrt{a\operatorname{csch}^2(x)}}\,dx$

Optimal. Leaf size=13

$$\frac{\coth(x)}{\sqrt{a\operatorname{csch}^2(x)}}$$

```
[Out] coth(x)/(a*csch(x)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4207, 197}

$$\frac{\coth(x)}{\sqrt{a\operatorname{csch}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Csch[x]^2],x]
```

```
[Out] Coth[x]/Sqrt[a*Csch[x]^2]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\sqrt{a\operatorname{csch}^2(x)}}\,dx = -\left(a\operatorname{Subst}\left(\int \frac{1}{(-a+ax^2)^{3/2}}\,dx, x, \coth(x)\right)\right)$$
$$= \frac{\coth(x)}{\sqrt{a\operatorname{csch}^2(x)}}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$\frac{\coth(x)}{\sqrt{a\mathrm{csch}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Csch[x]^2],x]
```

```
[Out] Coth[x]/Sqrt[a*Csch[x]^2]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(11) = 22$.
time = 0.98, size = 58, normalized size = 4.46

method	result	size
risch	$\frac{e^{2x}}{2\sqrt{\frac{a\,e^{2x}}{(e^{2x}-1)^2}}\,(e^{2x}-1)}+\frac{1}{2(e^{2x}-1)\sqrt{\frac{a\,e^{2x}}{(e^{2x}-1)^2}}}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*csch(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)/(exp(2*x)-1)*exp(2*x)+1/2/(exp(2*x)-1
)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)
```

Maxima [A]
time = 0.46, size = 17, normalized size = 1.31

$$-\frac{e^{(-x)}}{2\sqrt{a}}-\frac{e^{x}}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*e^(-x)/sqrt(a) - 1/2*e^x/sqrt(a)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(11) = 22$.
time = 0.40, size = 83, normalized size = 6.38

$$\frac{\left(\left(e^{(2x)}-1\right)\sinh\left(x\right)^2-\cosh\left(x\right)^2+\left(\cosh\left(x\right)^2+1\right)e^{(2x)}+2\left(\cosh\left(x\right)e^{(2x)}-\cosh\left(x\right)\right)\sinh\left(x\right)-1\right)\sqrt{\frac{a}{e^{(4x)}-2\,e^{(2x)}+1}}\,e^{x}}{2\left(a\cosh\left(x\right)e^{x}+ae^{x}\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*((e^(2*x) - 1)*sinh(x)^2 - cosh(x)^2 + (cosh(x)^2 + 1)*e^(2*x) + 2*(cos
h(x)*e^(2*x) - cosh(x))*sinh(x) - 1)*sqrt(a/(e^(4*x) - 2*e^(2*x) + 1))*e^x/
(a*cosh(x)*e^x + a*e^x*sinh(x))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a \operatorname{csch}^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*csch(x)**2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 24 vs. 2(11) = 22.
time = 0.39, size = 24, normalized size = 1.85

$$\frac{e^{(-x)} + e^x}{2\sqrt{a}\operatorname{sgn}\left(e^{(3x)} - e^x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(e^(-x) + e^x)/(sqrt(a)*sgn(e^(3*x) - e^x))
```

Mupad [**B**]
time = 1.60, size = 33, normalized size = 2.54

$$-\frac{\left(\frac{\mathrm{e}^{-2x}}{2} - \frac{\mathrm{e}^{2x}}{2}\right)\sqrt{\frac{1}{\left(\frac{\mathrm{e}^{-x}}{2} - \frac{\mathrm{e}^{x}}{2}\right)^2}}}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/sinh(x)^2)^(1/2),x)
```

```
[Out] -((exp(-2*x)/2 - exp(2*x)/2)*(1/(exp(-x)/2 - exp(x)/2)^2)^(1/2))/(2*a^(1/2)
)
```

3.33 $\int \frac{1}{\left(a\operatorname{csch}^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=36

$$\frac{\coth(x)}{3\left(a\operatorname{csch}^2(x)\right)^{3/2}}-\frac{2\coth(x)}{3a\sqrt{a\operatorname{csch}^2(x)}}$$

```
[Out] 1/3*coth(x)/(a*csch(x)^2)^(3/2)-2/3*coth(x)/a/(a*csch(x)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4207, 198, 197}

$$\frac{\coth(x)}{3\left(a\operatorname{csch}^2(x)\right)^{3/2}}-\frac{2\coth(x)}{3a\sqrt{a\operatorname{csch}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Csch[x]^2)^(-3/2),x]
```

```
[Out] Coth[x]/(3*(a*Csch[x]^2)^(3/2)) - (2*Coth[x])/(3*a*Sqrt[a*Csch[x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a\operatorname{csch}^2(x)\right)^{3/2}}\,dx = -\left(a\text{Subst}\left(\int \frac{1}{\left(-a+ax^2\right)^{5/2}}\,dx, x, \coth(x)\right)\right)$$
$$= \frac{\coth(x)}{3\left(a\operatorname{csch}^2(x)\right)^{3/2}} + \frac{2}{3}\text{Subst}\left(\int \frac{1}{\left(-a+ax^2\right)^{3/2}}\,dx, x, \coth(x)\right)$$
$$= \frac{\coth(x)}{3\left(a\operatorname{csch}^2(x)\right)^{3/2}} - \frac{2\coth(x)}{3a\sqrt{a\operatorname{csch}^2(x)}}$$

Mathematica [A]
time = 0.02, size = 27, normalized size = 0.75

$$\frac{(-9\cosh(x)+\cosh(3x))\operatorname{csch}^3(x)}{12\left(a\operatorname{csch}^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Csch[x]^2)^(-3/2),x]
```

```
[Out] ((-9*Cosh[x] + Cosh[3*x])*Csch[x]^3)/(12*(a*Csch[x]^2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(28) = 56$.
time = 0.80, size = 130, normalized size = 3.61

method	result	size
risch	$\frac{e^{4x}}{24a(e^{2x}-1)\sqrt{\frac{a\,e^{2x}}{(e^{2x}-1)^2}}} - \frac{3\,e^{2x}}{8a(e^{2x}-1)\sqrt{\frac{a\,e^{2x}}{(e^{2x}-1)^2}}} - \frac{3}{8\sqrt{\frac{a\,e^{2x}}{(e^{2x}-1)^2}}\,(e^{2x}-1)a} + \frac{e^{-2x}}{24a(e^{2x}-1)\sqrt{\frac{a\,e^{2x}}{(e^{2x}-1)^2}}}$	130

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*csch(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/a*exp(4*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)-3/8/a*exp(2*
x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)-3/8/(a*exp(2*x)/(exp(2*x)
-1)^2)^(1/2)/(exp(2*x)-1)/a+1/24/a*exp(-2*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(
2*x)-1)^2)^(1/2)
```

Maxima [A]
time = 0.48, size = 35, normalized size = 0.97

$$-\frac{e^{(3\,x)}}{24\,a^{\frac{3}{2}}} + \frac{3\,e^{(-x)}}{8\,a^{\frac{3}{2}}} - \frac{e^{(-3\,x)}}{24\,a^{\frac{3}{2}}} + \frac{3\,e^{x}}{8\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/24*e^(3*x)/a^(3/2) + 3/8*e^(-x)/a^(3/2) - 1/24*e^(-3*x)/a^(3/2) + 3/8*e^
x/a^(3/2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(28) = 56$.
time = 0.45, size = 285, normalized size = 7.92

$$\frac{\left(\left(e^{(2x)}-1\right)\sinh(x)^6-\cosh(x)^6+6\left(\cosh(x)e^{(2x)}-\cosh(x)\right)\sinh(x)^5-3\left(5\cosh(x)^2-\left(5\cosh(x)^2-3\right)e^{(2x)}-3\right)\sinh(x)^4+9\cosh(x)^4-4\left(5\cosh(x)^3-\left(5\cosh(x)^3-9\cosh(x)\right)e^{(2x)}-9\cosh(x)\right)\sinh(x)^3-3\left(5\cosh(x)^4-18\cosh(x)^2-\left(5\cosh(x)^4-18\cosh(x)^2-3\right)e^{(2x)}-3\right)\sinh(x)^2+9\cosh(x)^2+\left(\cosh(x)^6-9\cosh(x)^4-9\cosh(x)^2+1\right)e^{(2x)}-6\left(\cosh(x)^5-6\cosh(x)^3-\left(\cosh(x)^5-6\cosh(x)^3-3\cosh(x)\right)e^{(2x)}-3\cosh(x)\right)\sinh(x)-1\right)\sqrt{\frac{a}{e^{(4x)}-2e^{(2x)}+1}}\,e^x}{24\left(a^2\cosh(x)^3e^x+3a^2\cosh(x)^2e^x\sinh(x)+3a^2\cosh(x)e^x\sinh(x)^2+a^2e^x\sinh(x)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*((e^(2*x) - 1)*sinh(x)^6 - cosh(x)^6 + 6*(cosh(x)*e^(2*x) - cosh(x))*s
inh(x)^5 - 3*(5*cosh(x)^2 - (5*cosh(x)^2 - 3)*e^(2*x) - 3)*sinh(x)^4 + 9*co
sh(x)^4 - 4*(5*cosh(x)^3 - (5*cosh(x)^3 - 9*cosh(x))*e^(2*x) - 9*cosh(x))*s
inh(x)^3 - 3*(5*cosh(x)^4 - 18*cosh(x)^2 - (5*cosh(x)^4 - 18*cosh(x)^2 - 3)
*e^(2*x) - 3)*sinh(x)^2 + 9*cosh(x)^2 + (cosh(x)^6 - 9*cosh(x)^4 - 9*cosh(x
)^2 + 1)*e^(2*x) - 6*(cosh(x)^5 - 6*cosh(x)^3 - (cosh(x)^5 - 6*cosh(x)^3 -
3*cosh(x))*e^(2*x) - 3*cosh(x))*sinh(x) - 1)*sqrt(a/(e^(4*x) - 2*e^(2*x) +
1))*e^x/(a^2*cosh(x)^3*e^x + 3*a^2*cosh(x)^2*e^x*sinh(x) + 3*a^2*cosh(x)*e^
x*sinh(x)^2 + a^2*e^x*sinh(x)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\operatorname{csch}^2(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)**2)**(3/2),x)
```

```
[Out] Integral((a*csch(x)**2)**(-3/2), x)
```

Giac [A]
time = 0.38, size = 41, normalized size = 1.14

$$-\frac{\left(9\,e^{(2x)}-1\right)e^{(-3x)}-e^{(3x)}+9\,e^x}{24\,a^{\frac{3}{2}}\operatorname{sgn}\left(e^{(3x)}-e^x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*((9*e^(2*x) - 1)*e^(-3*x) - e^(3*x) + 9*e^x)/(a^(3/2)*sgn(e^(3*x) - e
^x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\left(\frac{a}{\sinh(x)^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/sinh(x)^2)^(3/2),x)
[Out] int(1/(a/sinh(x)^2)^(3/2), x)
```

3.34 $\int \frac{1}{\left(a\,\mathbf{csch}^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=55

$$\frac{\coth(x)}{5\left(a\,\text{csch}^2(x)\right)^{5/2}}-\frac{4\coth(x)}{15a\left(a\,\text{csch}^2(x)\right)^{3/2}}+\frac{8\coth(x)}{15a^2\sqrt{a\,\text{csch}^2(x)}}$$

```
[Out] 1/5*coth(x)/(a*csch(x)^2)^(5/2)-4/15*coth(x)/a/(a*csch(x)^2)^(3/2)+8/15*cot
h(x)/a^2/(a*csch(x)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4207, 198, 197}

$$\frac{8\coth(x)}{15a^2\sqrt{a\,\text{csch}^2(x)}}-\frac{4\coth(x)}{15a\left(a\,\text{csch}^2(x)\right)^{3/2}}+\frac{\coth(x)}{5\left(a\,\text{csch}^2(x)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Csch[x]^2)^(-5/2),x]
```

```
[Out] Coth[x]/(5*(a*Csch[x]^2)^(5/2)) - (4*Coth[x])/(15*a*(a*Csch[x]^2)^(3/2)) +
(8*Coth[x])/(15*a^2*Sqrt[a*Csch[x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a\operatorname{csch}^2(x)\right)^{5/2}}\,dx &= -\left(a\text{Subst}\left(\int \frac{1}{\left(-a+ax^2\right)^{7/2}}\,dx,x,\coth(x)\right)\right)\\
&= \frac{\coth(x)}{5\left(a\operatorname{csch}^2(x)\right)^{5/2}}+\frac{4}{5}\text{Subst}\left(\int \frac{1}{\left(-a+ax^2\right)^{5/2}}\,dx,x,\coth(x)\right)\\
&= \frac{\coth(x)}{5\left(a\operatorname{csch}^2(x)\right)^{5/2}}-\frac{4\coth(x)}{15a\left(a\operatorname{csch}^2(x)\right)^{3/2}}-\frac{8\text{Subst}\left(\int \frac{1}{\left(-a+ax^2\right)^{3/2}}\,dx,x,\coth(x)\right)}{15a}\\
&= \frac{\coth(x)}{5\left(a\operatorname{csch}^2(x)\right)^{5/2}}-\frac{4\coth(x)}{15a\left(a\operatorname{csch}^2(x)\right)^{3/2}}+\frac{8\coth(x)}{15a^2\sqrt{a\operatorname{csch}^2(x)}}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 36, normalized size = 0.65

$$
\frac{\left(150\cosh(x)-25\cosh(3x)+3\cosh(5x)\right)\sqrt{a\operatorname{csch}^2(x)}\,\sinh(x)}{240a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Csch[x]^2)^(-5/2),x]
```

```
[Out] ((150*Cosh[x] - 25*Cosh[3*x] + 3*Cosh[5*x])*Sqrt[a*Csch[x]^2]*Sinh[x])/(240
*a^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(43) = 86$.
time = 0.84, size = 196, normalized size = 3.56

method	result
risch	$\frac{e^{6x}}{160a^2(e^{2x}-1)\sqrt{\frac{a\,e^{2x}}{(e^{2x}-1)^2}}}-\frac{5\,e^{4x}}{96a^2(e^{2x}-1)\sqrt{\frac{a\,e^{2x}}{(e^{2x}-1)^2}}}+\frac{5\,e^{2x}}{16a^2(e^{2x}-1)\sqrt{\frac{a\,e^{2x}}{(e^{2x}-1)^2}}}+\frac{5}{16\sqrt{\frac{a\,e^{2x}}{(e^{2x}-1)^2}}\,(e^{2x}-1)a^2}-$ 96a

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*csch(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/160/a^2*exp(6*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)-5/96/a^2*
exp(4*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)+5/16/a^2*exp(2*x)/(
exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)+5/16/(a*exp(2*x)/(exp(2*x)-1)
^2)^(1/2)/(exp(2*x)-1)/a^2-5/96/a^2*exp(-2*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp
(2*x)-1)^2)^(1/2)+1/160/a^2*exp(-4*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)
^2)^(1/2)
```

Maxima [A]
time = 0.50, size = 53, normalized size = 0.96

$$-\frac{e^{(5\,x)}}{160\,a^{\frac{5}{2}}}+\frac{5\,e^{(3\,x)}}{96\,a^{\frac{5}{2}}}-\frac{5\,e^{(-x)}}{16\,a^{\frac{5}{2}}}+\frac{5\,e^{(-3\,x)}}{96\,a^{\frac{5}{2}}}-\frac{e^{(-5\,x)}}{160\,a^{\frac{5}{2}}}-\frac{5\,e^{x}}{16\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/160*e^(5*x)/a^(5/2) + 5/96*e^(3*x)/a^(5/2) - 5/16*e^(-x)/a^(5/2) + 5/96*
e^(-3*x)/a^(5/2) - 1/160*e^(-5*x)/a^(5/2) - 5/16*e^x/a^(5/2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 590 vs. 2(43) = 86.
time = 0.58, size = 590, normalized size = 10.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/480*(3*(e^(2*x) - 1)*sinh(x)^10 - 3*cosh(x)^10 + 30*(cosh(x)*e^(2*x) - co
sh(x))*sinh(x)^9 - 5*(27*cosh(x)^2 - (27*cosh(x)^2 - 5)*e^(2*x) - 5)*sinh(x
)^8 + 25*cosh(x)^8 - 40*(9*cosh(x)^3 - (9*cosh(x)^3 - 5*cosh(x))*e^(2*x) -
5*cosh(x))*sinh(x)^7 - 10*(63*cosh(x)^4 - 70*cosh(x)^2 - (63*cosh(x)^4 - 70
*cosh(x)^2 + 15)*e^(2*x) + 15)*sinh(x)^6 - 150*cosh(x)^6 - 4*(189*cosh(x)^5
 - 350*cosh(x)^3 - (189*cosh(x)^5 - 350*cosh(x)^3 + 225*cosh(x))*e^(2*x) +
225*cosh(x))*sinh(x)^5 - 10*(63*cosh(x)^6 - 175*cosh(x)^4 + 225*cosh(x)^2 -
 (63*cosh(x)^6 - 175*cosh(x)^4 + 225*cosh(x)^2 + 15)*e^(2*x) + 15)*sinh(x)^
4 - 150*cosh(x)^4 - 40*(9*cosh(x)^7 - 35*cosh(x)^5 + 75*cosh(x)^3 - (9*cosh
(x)^7 - 35*cosh(x)^5 + 75*cosh(x)^3 + 15*cosh(x))*e^(2*x) + 15*cosh(x))*sin
h(x)^3 - 5*(27*cosh(x)^8 - 140*cosh(x)^6 + 450*cosh(x)^4 + 180*cosh(x)^2 -
(27*cosh(x)^8 - 140*cosh(x)^6 + 450*cosh(x)^4 + 180*cosh(x)^2 - 5)*e^(2*x)
- 5)*sinh(x)^2 + 25*cosh(x)^2 + (3*cosh(x)^10 - 25*cosh(x)^8 + 150*cosh(x)^
6 + 150*cosh(x)^4 - 25*cosh(x)^2 + 3)*e^(2*x) - 10*(3*cosh(x)^9 - 20*cosh(x
)^7 + 90*cosh(x)^5 + 60*cosh(x)^3 - (3*cosh(x)^9 - 20*cosh(x)^7 + 90*cosh(x
)^5 + 60*cosh(x)^3 - 5*cosh(x))*e^(2*x) - 5*cosh(x))*sinh(x) - 3)*sqrt(a/(e
^(4*x) - 2*e^(2*x) + 1))*e^x/(a^3*cosh(x)^5*e^x + 5*a^3*cosh(x)^4*e^x*sinh(
x) + 10*a^3*cosh(x)^3*e^x*sinh(x)^2 + 10*a^3*cosh(x)^2*e^x*sinh(x)^3 + 5*a^
3*cosh(x)*e^x*sinh(x)^4 + a^3*e^x*sinh(x)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\operatorname{csch}^{2}\left(x\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)**2)**(5/2),x)
```

```
[Out] Integral((a*csch(x)**2)**(-5/2), x)
```

Giac [A]
time = 0.39, size = 53, normalized size = 0.96

$$\frac{\left(150\,e^{(4\,x)} - 25\,e^{(2\,x)} + 3\right)e^{(-5\,x)} + 3\,e^{(5\,x)} - 25\,e^{(3\,x)} + 150\,e^{x}}{480\,a^{\frac{5}{2}}\operatorname{sgn}\left(e^{(3\,x)} - e^{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/480*((150*e^(4*x) - 25*e^(2*x) + 3)*e^(-5*x) + 3*e^(5*x) - 25*e^(3*x) + 1
50*e^x)/(a^(5/2)*sgn(e^(3*x) - e^x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(\frac{a}{{\mathrm{sinh}\left(x\right)}^2}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/sinh(x)^2)^(5/2),x)
```

```
[Out] int(1/(a/sinh(x)^2)^(5/2), x)
```

3.35 $\int \frac{1}{\left(a\mathrm{csch}^2(x)\right)^{7/2}}\,dx$

Optimal. Leaf size=74

$$\frac{\coth(x)}{7\left(a\mathrm{csch}^2(x)\right)^{7/2}}-\frac{6\coth(x)}{35a\left(a\mathrm{csch}^2(x)\right)^{5/2}}+\frac{8\coth(x)}{35a^2\left(a\mathrm{csch}^2(x)\right)^{3/2}}-\frac{16\coth(x)}{35a^3\sqrt{a\mathrm{csch}^2(x)}}$$

```
[Out] 1/7*coth(x)/(a*csch(x)^2)^(7/2)-6/35*coth(x)/a/(a*csch(x)^2)^(5/2)+8/35*cot
h(x)/a^2/(a*csch(x)^2)^(3/2)-16/35*coth(x)/a^3/(a*csch(x)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4207, 198, 197}

$$-\frac{16\coth(x)}{35a^3\sqrt{a\mathrm{csch}^2(x)}}+\frac{8\coth(x)}{35a^2\left(a\mathrm{csch}^2(x)\right)^{3/2}}-\frac{6\coth(x)}{35a\left(a\mathrm{csch}^2(x)\right)^{5/2}}+\frac{\coth(x)}{7\left(a\mathrm{csch}^2(x)\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Csch[x]^2)^(-7/2),x]
```

```
[Out] Coth[x]/(7*(a*Csch[x]^2)^(7/2)) - (6*Coth[x])/(35*a*(a*Csch[x]^2)^(5/2)) +
(8*Coth[x])/(35*a^2*(a*Csch[x]^2)^(3/2)) - (16*Coth[x])/(35*a^3*Sqrt[a*Csch
[x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{(a\text{csch}^2(x))^{7/2}}\,dx = -\left(a\text{Subst}\left(\int \frac{1}{(-a+ax^2)^{9/2}}\,dx, x, \coth(x)\right)\right)$$

$$= \frac{\coth(x)}{7\left(a\text{csch}^2(x)\right)^{7/2}} + \frac{6}{7}\text{Subst}\left(\int \frac{1}{(-a+ax^2)^{7/2}}\,dx, x, \coth(x)\right)$$

$$= \frac{\coth(x)}{7\left(a\text{csch}^2(x)\right)^{7/2}} - \frac{6\coth(x)}{35a\left(a\text{csch}^2(x)\right)^{5/2}} - \frac{24\text{Subst}\left(\int \frac{1}{(-a+ax^2)^{5/2}}\,dx, x, \coth(x)\right)}{35a}$$

$$= \frac{\coth(x)}{7\left(a\text{csch}^2(x)\right)^{7/2}} - \frac{6\coth(x)}{35a\left(a\text{csch}^2(x)\right)^{5/2}} + \frac{8\coth(x)}{35a^2\left(a\text{csch}^2(x)\right)^{3/2}} + \frac{16\text{Subst}\left(\int \frac{1}{(-a+ax^2)^{3/2}}\right.}{35a^2}$$

$$= \frac{\coth(x)}{7\left(a\text{csch}^2(x)\right)^{7/2}} - \frac{6\coth(x)}{35a\left(a\text{csch}^2(x)\right)^{5/2}} + \frac{8\coth(x)}{35a^2\left(a\text{csch}^2(x)\right)^{3/2}} - \frac{16\coth(x)}{35a^3\sqrt{a\text{csch}^2(x)}}$$

Mathematica [A]
time = 0.04, size = 42, normalized size = 0.57

$$\frac{(-1225\cosh(x)+245\cosh(3x)-49\cosh(5x)+5\cosh(7x))\sqrt{a\text{csch}^2(x)}\,\sinh(x)}{2240a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Csch[x]^2)^(-7/2),x]
```

```
[Out] ((-1225*Cosh[x] + 245*Cosh[3*x] - 49*Cosh[5*x] + 5*Cosh[7*x])*Sqrt[a*Csch[x
]^2]*Sinh[x])/(2240*a^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 261 vs. 2(58) = 116.
time = 0.88, size = 262, normalized size = 3.54

method	result
risch	$\frac{e^{8x}}{896a^3(e^{2x}-1)\sqrt{\frac{a\,e^{2x}}{(e^{2x}-1)^2}}} - \frac{7\,e^{6x}}{640a^3(e^{2x}-1)\sqrt{\frac{a\,e^{2x}}{(e^{2x}-1)^2}}} + \frac{7\,e^{4x}}{128a^3(e^{2x}-1)\sqrt{\frac{a\,e^{2x}}{(e^{2x}-1)^2}}} - \frac{35\,e^{2x}}{128a^3(e^{2x}-1)\sqrt{\frac{a\,e^{2x}}{(e^{2x}-1)^2}}} -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*csch(x)^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/896/a^3*exp(8*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)-7/640/a^3
*exp(6*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)+7/128/a^3*exp(4*x)
```

```
/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)-35/128/a^3*exp(2*x)/(exp(2*
x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)-35/128/(a*exp(2*x)/(exp(2*x)-1)^2)^
(1/2)/(exp(2*x)-1)/a^3+7/128/a^3*exp(-2*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*
x)-1)^2)^(1/2)-7/640/a^3*exp(-4*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)
^(1/2)+1/896/a^3*exp(-6*x)/(exp(2*x)-1)/(a*exp(2*x)/(exp(2*x)-1)^2)^(1/2)
```

Maxima [**A**]
time = 0.48, size = 71, normalized size = 0.96

$$-\frac{e^{(7x)}}{896\,a^{\frac{7}{2}}}+\frac{7\,e^{(5x)}}{640\,a^{\frac{7}{2}}}-\frac{7\,e^{(3x)}}{128\,a^{\frac{7}{2}}}+\frac{35\,e^{(-x)}}{128\,a^{\frac{7}{2}}}-\frac{7\,e^{(-3x)}}{128\,a^{\frac{7}{2}}}+\frac{7\,e^{(-5x)}}{640\,a^{\frac{7}{2}}}-\frac{e^{(-7x)}}{896\,a^{\frac{7}{2}}}+\frac{35\,e^{x}}{128\,a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/896*e^(7*x)/a^(7/2) + 7/640*e^(5*x)/a^(7/2) - 7/128*e^(3*x)/a^(7/2) + 35
/128*e^(-x)/a^(7/2) - 7/128*e^(-3*x)/a^(7/2) + 7/640*e^(-5*x)/a^(7/2) - 1/8
96*e^(-7*x)/a^(7/2) + 35/128*e^x/a^(7/2)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 984 vs. $2(58) = 116$.
time = 0.44, size = 984, normalized size = 13.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/4480*(5*(e^(2*x) - 1)*sinh(x)^14 - 5*cosh(x)^14 + 70*(cosh(x)*e^(2*x) - c
osh(x))*sinh(x)^13 - 7*(65*cosh(x)^2 - (65*cosh(x)^2 - 7)*e^(2*x) - 7)*sinh
(x)^12 + 49*cosh(x)^12 - 28*(65*cosh(x)^3 - (65*cosh(x)^3 - 21*cosh(x))*e^(
2*x) - 21*cosh(x))*sinh(x)^11 - 7*(715*cosh(x)^4 - 462*cosh(x)^2 - (715*cos
h(x)^4 - 462*cosh(x)^2 + 35)*e^(2*x) + 35)*sinh(x)^10 - 245*cosh(x)^10 - 70
*(143*cosh(x)^5 - 154*cosh(x)^3 - (143*cosh(x)^5 - 154*cosh(x)^3 + 35*cosh(
x))*e^(2*x) + 35*cosh(x))*sinh(x)^9 - 35*(429*cosh(x)^6 - 693*cosh(x)^4 + 3
15*cosh(x)^2 - (429*cosh(x)^6 - 693*cosh(x)^4 + 315*cosh(x)^2 - 35)*e^(2*x)
 - 35)*sinh(x)^8 + 1225*cosh(x)^8 - 8*(2145*cosh(x)^7 - 4851*cosh(x)^5 + 36
75*cosh(x)^3 - (2145*cosh(x)^7 - 4851*cosh(x)^5 + 3675*cosh(x)^3 - 1225*cos
h(x))*e^(2*x) - 1225*cosh(x))*sinh(x)^7 - 7*(2145*cosh(x)^8 - 6468*cosh(x)^
6 + 7350*cosh(x)^4 - 4900*cosh(x)^2 - (2145*cosh(x)^8 - 6468*cosh(x)^6 + 73
50*cosh(x)^4 - 4900*cosh(x)^2 - 175)*e^(2*x) - 175)*sinh(x)^6 + 1225*cosh(x
)^6 - 14*(715*cosh(x)^9 - 2772*cosh(x)^7 + 4410*cosh(x)^5 - 4900*cosh(x)^3
- (715*cosh(x)^9 - 2772*cosh(x)^7 + 4410*cosh(x)^5 - 4900*cosh(x)^3 - 525*c
osh(x))*e^(2*x) - 525*cosh(x))*sinh(x)^5 - 35*(143*cosh(x)^10 - 693*cosh(x)
^8 + 1470*cosh(x)^6 - 2450*cosh(x)^4 - 525*cosh(x)^2 - (143*cosh(x)^10 - 69
3*cosh(x)^8 + 1470*cosh(x)^6 - 2450*cosh(x)^4 - 525*cosh(x)^2 + 7)*e^(2*x)
```

```
+ 7)*sinh(x)^4 - 245*cosh(x)^4 - 140*(13*cosh(x)^11 - 77*cosh(x)^9 + 210*co
sh(x)^7 - 490*cosh(x)^5 - 175*cosh(x)^3 - (13*cosh(x)^11 - 77*cosh(x)^9 + 2
10*cosh(x)^7 - 490*cosh(x)^5 - 175*cosh(x)^3 + 7*cosh(x))*e^(2*x) + 7*cosh(
x))*sinh(x)^3 - 7*(65*cosh(x)^12 - 462*cosh(x)^10 + 1575*cosh(x)^8 - 4900*c
osh(x)^6 - 2625*cosh(x)^4 + 210*cosh(x)^2 - (65*cosh(x)^12 - 462*cosh(x)^10
 + 1575*cosh(x)^8 - 4900*cosh(x)^6 - 2625*cosh(x)^4 + 210*cosh(x)^2 - 7)*e^
(2*x) - 7)*sinh(x)^2 + 49*cosh(x)^2 + (5*cosh(x)^14 - 49*cosh(x)^12 + 245*c
osh(x)^10 - 1225*cosh(x)^8 - 1225*cosh(x)^6 + 245*cosh(x)^4 - 49*cosh(x)^2
+ 5)*e^(2*x) - 14*(5*cosh(x)^13 - 42*cosh(x)^11 + 175*cosh(x)^9 - 700*cosh(
x)^7 - 525*cosh(x)^5 + 70*cosh(x)^3 - (5*cosh(x)^13 - 42*cosh(x)^11 + 175*c
osh(x)^9 - 700*cosh(x)^7 - 525*cosh(x)^5 + 70*cosh(x)^3 - 7*cosh(x))*e^(2*x
) - 7*cosh(x))*sinh(x) - 5)*sqrt(a/(e^(4*x) - 2*e^(2*x) + 1))*e^x/(a^4*cosh
(x)^7*e^x + 7*a^4*cosh(x)^6*e^x*sinh(x) + 21*a^4*cosh(x)^5*e^x*sinh(x)^2 +
35*a^4*cosh(x)^4*e^x*sinh(x)^3 + 35*a^4*cosh(x)^3*e^x*sinh(x)^4 + 21*a^4*co
sh(x)^2*e^x*sinh(x)^5 + 7*a^4*cosh(x)*e^x*sinh(x)^6 + a^4*e^x*sinh(x)^7)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a \operatorname{csch}^{2}(x)\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)**2)**(7/2),x)
[Out] Integral((a*csch(x)**2)**(-7/2), x)
```

Giac [A]
time = 0.39, size = 65, normalized size = 0.88

$$-\frac{\left(1225\, e^{(6\, x)} - 245\, e^{(4\, x)} + 49\, e^{(2\, x)} - 5\right)e^{(-7\, x)} - 5\, e^{(7\, x)} + 49\, e^{(5\, x)} - 245\, e^{(3\, x)} + 1225\, e^{x}}{4480\, a^{\frac{7}{2}} \operatorname{sgn}\left(e^{(3\, x)} - e^{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^2)^(7/2),x, algorithm="giac")
[Out] -1/4480*((1225*e^(6*x) - 245*e^(4*x) + 49*e^(2*x) - 5)*e^(-7*x) - 5*e^(7*x)
 + 49*e^(5*x) - 245*e^(3*x) + 1225*e^x)/(a^(7/2)*sgn(e^(3*x) - e^x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(\frac{a}{{\mathrm{sinh}\left(x\right)}^2}\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/sinh(x)^2)^(7/2),x)
[Out] int(1/(a/sinh(x)^2)^(7/2), x)
```

3.36 $\int \left(a\mathrm{csch}^3(x)\right)^{5/2} dx$

Optimal. Leaf size=135

$$-\frac{154}{585}a^2\coth(x)\sqrt{a\mathrm{csch}^3(x)}+\frac{22}{117}a^2\coth(x)\mathrm{csch}^2(x)\sqrt{a\mathrm{csch}^3(x)}-\frac{2}{13}a^2\coth(x)\mathrm{csch}^4(x)\sqrt{a\mathrm{csch}^3(x)}+\frac{15}{19}$$

```
[Out] -154/585*a^2*coth(x)*(a*csch(x)^3)^(1/2)+22/117*a^2*coth(x)*csch(x)^2*(a*cs
ch(x)^3)^(1/2)-2/13*a^2*coth(x)*csch(x)^4*(a*csch(x)^3)^(1/2)+154/195*a^2*c
osh(x)*sinh(x)*(a*csch(x)^3)^(1/2)-154/195*I*a^2*(sin(1/4*Pi+1/2*I*x)^2)^(1
/2)/sin(1/4*Pi+1/2*I*x)*EllipticE(cos(1/4*Pi+1/2*I*x),2^(1/2))*sinh(x)^2*(a
*csch(x)^3)^(1/2)/(I*sinh(x))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4208, 3853, 3856, 2719}

$$-\frac{154}{585}a^2\coth(x)\sqrt{a\mathrm{csch}^3(x)}-\frac{2}{13}a^2\coth(x)\mathrm{csch}^4(x)\sqrt{a\mathrm{csch}^3(x)}+\frac{22}{117}a^2\coth(x)\mathrm{csch}^2(x)\sqrt{a\mathrm{csch}^3(x)}+\frac{154}{195}a^2\sinh(x)\cosh(x)\sqrt{a\mathrm{csch}^3(x)}-\frac{154ia^2\sinh^2(x)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{a\mathrm{csch}^3(x)}}{195\sqrt{i\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Csch[x]^3)^(5/2),x]
```

```
[Out] (-154*a^2*Coth[x]*Sqrt[a*Csch[x]^3])/585 + (22*a^2*Coth[x]*Csch[x]^2*Sqrt[a
*Csch[x]^3])/117 - (2*a^2*Coth[x]*Csch[x]^4*Sqrt[a*Csch[x]^3])/13 + (154*a^
2*Cosh[x]*Sqrt[a*Csch[x]^3]*Sinh[x])/195 - (((154*I)/195)*a^2*Sqrt[a*Csch[x
]^3]*EllipticE[Pi/4 - (I/2)*x, 2]*Sinh[x]^2)/Sqrt[I*Sinh[x]]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\int \left(a\text{csch}^3(x)\right)^{5/2}\,dx = -\frac{\left(a^2\sqrt{a\text{csch}^3(x)}\right)\int (i\text{csch}(x))^{15/2}\,dx}{(i\text{csch}(x))^{3/2}}$$

$$= -\frac{2}{13}a^2\coth(x)\text{csch}^4(x)\sqrt{a\text{csch}^3(x)} - \frac{\left(11a^2\sqrt{a\text{csch}^3(x)}\right)\int (i\text{csch}(x))^{11/2}\,dx}{13(i\text{csch}(x))^{3/2}}$$

$$= \frac{22}{117}a^2\coth(x)\text{csch}^2(x)\sqrt{a\text{csch}^3(x)} - \frac{2}{13}a^2\coth(x)\text{csch}^4(x)\sqrt{a\text{csch}^3(x)} - \frac{\left(77a^2\sqrt{\ }\right.}{}$$

$$= -\frac{154}{585}a^2\coth(x)\sqrt{a\text{csch}^3(x)} + \frac{22}{117}a^2\coth(x)\text{csch}^2(x)\sqrt{a\text{csch}^3(x)} - \frac{2}{13}a^2\coth(x)\text{c}$$

$$= -\frac{154}{585}a^2\coth(x)\sqrt{a\text{csch}^3(x)} + \frac{22}{117}a^2\coth(x)\text{csch}^2(x)\sqrt{a\text{csch}^3(x)} - \frac{2}{13}a^2\coth(x)\text{c}$$

$$= -\frac{154}{585}a^2\coth(x)\sqrt{a\text{csch}^3(x)} + \frac{22}{117}a^2\coth(x)\text{csch}^2(x)\sqrt{a\text{csch}^3(x)} - \frac{2}{13}a^2\coth(x)\text{c}$$

$$= -\frac{154}{585}a^2\coth(x)\sqrt{a\text{csch}^3(x)} + \frac{22}{117}a^2\coth(x)\text{csch}^2(x)\sqrt{a\text{csch}^3(x)} - \frac{2}{13}a^2\coth(x)\text{c}$$

Mathematica [A]
time = 0.12, size = 68, normalized size = 0.50

$$-\frac{2}{585}a^2\sqrt{a\text{csch}^3(x)}\left(-231\cosh(x)+\coth(x)\text{csch}(x)\left(77-55\text{csch}^2(x)+45\text{csch}^4(x)\right)+231E\left(\frac{1}{4}(\pi-2ix)\middle|2\right)\sqrt{i\sinh(x)}\right)\sinh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Csch[x]^3)^(5/2),x]
[Out] (-2*a^2*Sqrt[a*Csch[x]^3]*(-231*Cosh[x] + Coth[x]*Csch[x]*(77 - 55*Csch[x]^
2 + 45*Csch[x]^4) + 231*EllipticE[(Pi - (2*I)*x)/4, 2]*Sqrt[I*Sinh[x]])*Sin
h[x])/585
```

Maple [F]
time = 0.95, size = 0, normalized size = 0.00

$$\int \left(a\operatorname{csch}(x)^3\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*csch(x)^3)^(5/2),x)
```

```
[Out] int((a*csch(x)^3)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*csch(x)^3)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 1389, normalized size = 10.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/585*(231*sqrt(2)*(a^2*cosh(x)^12 + 12*a^2*cosh(x)*sinh(x)^11 + a^2*sinh(x
)^12 - 6*a^2*cosh(x)^10 + 6*(11*a^2*cosh(x)^2 - a^2)*sinh(x)^10 + 15*a^2*co
sh(x)^8 + 20*(11*a^2*cosh(x)^3 - 3*a^2*cosh(x))*sinh(x)^9 + 15*(33*a^2*cosh
(x)^4 - 18*a^2*cosh(x)^2 + a^2)*sinh(x)^8 - 20*a^2*cosh(x)^6 + 24*(33*a^2*c
osh(x)^5 - 30*a^2*cosh(x)^3 + 5*a^2*cosh(x))*sinh(x)^7 + 4*(231*a^2*cosh(x)
^6 - 315*a^2*cosh(x)^4 + 105*a^2*cosh(x)^2 - 5*a^2)*sinh(x)^6 + 15*a^2*cosh
(x)^4 + 24*(33*a^2*cosh(x)^7 - 63*a^2*cosh(x)^5 + 35*a^2*cosh(x)^3 - 5*a^2*
cosh(x))*sinh(x)^5 + 15*(33*a^2*cosh(x)^8 - 84*a^2*cosh(x)^6 + 70*a^2*cosh(
x)^4 - 20*a^2*cosh(x)^2 + a^2)*sinh(x)^4 - 6*a^2*cosh(x)^2 + 20*(11*a^2*cos
h(x)^9 - 36*a^2*cosh(x)^7 + 42*a^2*cosh(x)^5 - 20*a^2*cosh(x)^3 + 3*a^2*cos
h(x))*sinh(x)^3 + 6*(11*a^2*cosh(x)^10 - 45*a^2*cosh(x)^8 + 70*a^2*cosh(x)^
6 - 50*a^2*cosh(x)^4 + 15*a^2*cosh(x)^2 - a^2)*sinh(x)^2 + a^2 + 12*(a^2*co
sh(x)^11 - 5*a^2*cosh(x)^9 + 10*a^2*cosh(x)^7 - 10*a^2*cosh(x)^5 + 5*a^2*co
sh(x)^3 - a^2*cosh(x))*sinh(x))*sqrt(a)*weierstrassZeta(4, 0, weierstrassPI
nverse(4, 0, cosh(x) + sinh(x))) + sqrt(2)*(231*a^2*cosh(x)^13 + 3003*a^2*c
osh(x)*sinh(x)^12 + 231*a^2*sinh(x)^13 - 1540*a^2*cosh(x)^11 + 154*(117*a^2
*cosh(x)^2 - 10*a^2)*sinh(x)^11 + 4367*a^2*cosh(x)^9 + 1694*(39*a^2*cosh(x)
```

```
^3 - 10*a^2*cosh(x))*sinh(x)^10 + 11*(15015*a^2*cosh(x)^4 - 7700*a^2*cosh(x
)^2 + 397*a^2)*sinh(x)^9 - 6808*a^2*cosh(x)^7 + 33*(9009*a^2*cosh(x)^5 - 77
00*a^2*cosh(x)^3 + 1191*a^2*cosh(x))*sinh(x)^8 + 4*(99099*a^2*cosh(x)^6 - 1
27050*a^2*cosh(x)^4 + 39303*a^2*cosh(x)^2 - 1702*a^2)*sinh(x)^7 + 1277*a^2*
cosh(x)^5 + 28*(14157*a^2*cosh(x)^7 - 25410*a^2*cosh(x)^5 + 13101*a^2*cosh(
x)^3 - 1702*a^2*cosh(x))*sinh(x)^6 + (297297*a^2*cosh(x)^8 - 711480*a^2*cos
h(x)^6 + 550242*a^2*cosh(x)^4 - 142968*a^2*cosh(x)^2 + 1277*a^2)*sinh(x)^5
- 484*a^2*cosh(x)^3 + (165165*a^2*cosh(x)^9 - 508200*a^2*cosh(x)^7 + 550242
*a^2*cosh(x)^5 - 238280*a^2*cosh(x)^3 + 6385*a^2*cosh(x))*sinh(x)^4 + 2*(33
033*a^2*cosh(x)^10 - 127050*a^2*cosh(x)^8 + 183414*a^2*cosh(x)^6 - 119140*a
^2*cosh(x)^4 + 6385*a^2*cosh(x)^2 - 242*a^2)*sinh(x)^3 + 77*a^2*cosh(x) + 2
*(9009*a^2*cosh(x)^11 - 42350*a^2*cosh(x)^9 + 78606*a^2*cosh(x)^7 - 71484*a
^2*cosh(x)^5 + 6385*a^2*cosh(x)^3 - 726*a^2*cosh(x))*sinh(x)^2 + (3003*a^2*
cosh(x)^12 - 16940*a^2*cosh(x)^10 + 39303*a^2*cosh(x)^8 - 47656*a^2*cosh(x)
^6 + 6385*a^2*cosh(x)^4 - 1452*a^2*cosh(x)^2 + 77*a^2)*sinh(x))*sqrt((a*cos
h(x) + a*sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)))/(cosh(x
)^12 + 12*cosh(x)*sinh(x)^11 + sinh(x)^12 + 6*(11*cosh(x)^2 - 1)*sinh(x)^10
 - 6*cosh(x)^10 + 20*(11*cosh(x)^3 - 3*cosh(x))*sinh(x)^9 + 15*(33*cosh(x)^
4 - 18*cosh(x)^2 + 1)*sinh(x)^8 + 15*cosh(x)^8 + 24*(33*cosh(x)^5 - 30*cosh
(x)^3 + 5*cosh(x))*sinh(x)^7 + 4*(231*cosh(x)^6 - 315*cosh(x)^4 + 105*cosh(
x)^2 - 5)*sinh(x)^6 - 20*cosh(x)^6 + 24*(33*cosh(x)^7 - 63*cosh(x)^5 + 35*c
osh(x)^3 - 5*cosh(x))*sinh(x)^5 + 15*(33*cosh(x)^8 - 84*cosh(x)^6 + 70*cosh
(x)^4 - 20*cosh(x)^2 + 1)*sinh(x)^4 + 15*cosh(x)^4 + 20*(11*cosh(x)^9 - 36*
cosh(x)^7 + 42*cosh(x)^5 - 20*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 6*(11*cosh
(x)^10 - 45*cosh(x)^8 + 70*cosh(x)^6 - 50*cosh(x)^4 + 15*cosh(x)^2 - 1)*sin
h(x)^2 - 6*cosh(x)^2 + 12*(cosh(x)^11 - 5*cosh(x)^9 + 10*cosh(x)^7 - 10*cos
h(x)^5 + 5*cosh(x)^3 - cosh(x))*sinh(x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\operatorname{csch}^{3}\left(x\right)\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)**3)**(5/2),x)
```

```
[Out] Integral((a*csch(x)**3)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^3)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*csch(x)^3)^(5/2), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{a}{\sinh(x)^3}\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/sinh(x)^3)^(5/2),x)
[Out] int((a/sinh(x)^3)^(5/2), x)
```

3.37 $\int \left(a\operatorname{csch}^3(x)\right)^{3/2} dx$

Optimal. Leaf size=81

$$\frac{10}{21}a\cosh(x)\sqrt{a\operatorname{csch}^3(x)}-\frac{2}{7}a\coth(x)\operatorname{csch}(x)\sqrt{a\operatorname{csch}^3(x)}+\frac{10}{21}ia\sqrt{a\operatorname{csch}^3(x)}\,F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{i\sinh(x)}\,\sin$$

```
[Out] 10/21*a*cosh(x)*(a*csch(x)^3)^(1/2)-2/7*a*coth(x)*csch(x)*(a*csch(x)^3)^(1/
2)+10/21*I*a*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticF(co
s(1/4*Pi+1/2*I*x),2^(1/2))*sinh(x)*(a*csch(x)^3)^(1/2)*(I*sinh(x))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4208, 3853, 3856, 2720}

$$\frac{10}{21}a\cosh(x)\sqrt{a\operatorname{csch}^3(x)}-\frac{2}{7}a\coth(x)\operatorname{csch}(x)\sqrt{a\operatorname{csch}^3(x)}+\frac{10}{21}ia\sqrt{i\sinh(x)}\sinh(x)F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{a\operatorname{csch}^3(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Csch[x]^3)^(3/2),x]
```

```
[Out] (10*a*Cosh[x]*Sqrt[a*Csch[x]^3])/21 - (2*a*Coth[x]*Csch[x]*Sqrt[a*Csch[x]^3
])/7 + ((10*I)/21)*a*Sqrt[a*Csch[x]^3]*EllipticF[Pi/4 - (I/2)*x, 2]*Sqrt[I*
Sinh[x]]*Sinh[x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
```

```
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\int \left(a\operatorname{csch}^3(x)\right)^{3/2}\,dx = \frac{\left(ia\sqrt{a\operatorname{csch}^3(x)}\right)\int (i\operatorname{csch}(x))^{9/2}\,dx}{(i\operatorname{csch}(x))^{3/2}}$$

$$= -\frac{2}{7}a\coth(x)\operatorname{csch}(x)\sqrt{a\operatorname{csch}^3(x)} + \frac{\left(5ia\sqrt{a\operatorname{csch}^3(x)}\right)\int (i\operatorname{csch}(x))^{5/2}\,dx}{7(i\operatorname{csch}(x))^{3/2}}$$

$$= \frac{10}{21}a\cosh(x)\sqrt{a\operatorname{csch}^3(x)} - \frac{2}{7}a\coth(x)\operatorname{csch}(x)\sqrt{a\operatorname{csch}^3(x)} + \frac{\left(5ia\sqrt{a\operatorname{csch}^3(x)}\right)}{21(i\operatorname{csch}(}$$

$$= \frac{10}{21}a\cosh(x)\sqrt{a\operatorname{csch}^3(x)} - \frac{2}{7}a\coth(x)\operatorname{csch}(x)\sqrt{a\operatorname{csch}^3(x)} + \frac{1}{21}\left(5a\sqrt{a\operatorname{csch}^3(x)}\right.$$

$$= \frac{10}{21}a\cosh(x)\sqrt{a\operatorname{csch}^3(x)} - \frac{2}{7}a\coth(x)\operatorname{csch}(x)\sqrt{a\operatorname{csch}^3(x)} + \frac{10}{21}ia\sqrt{a\operatorname{csch}^3(x)}\,F\Big($$

Mathematica [A]
time = 0.08, size = 56, normalized size = 0.69

$$-\frac{2}{21}a\sqrt{a\operatorname{csch}^3(x)}\left(\coth(x)\left(-5+3\operatorname{csch}^2(x)\right) - 5iF\left(\frac{1}{4}(\pi-2ix)\middle|2\right)\sqrt{i\sinh(x)}\right)\sinh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Csch[x]^3)^(3/2),x]
[Out] (-2*a*Sqrt[a*Csch[x]^3]*(Coth[x]*(-5 + 3*Csch[x]^2) - (5*I)*EllipticF[(Pi -
 (2*I)*x)/4, 2]*Sqrt[I*Sinh[x]])*Sinh[x])/21
```

Maple [F]
time = 0.85, size = 0, normalized size = 0.00

$$\int \left(a\operatorname{csch}(x)^3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*csch(x)^3)^(3/2),x)
[Out] int((a*csch(x)^3)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*csch(x)^3)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 395, normalized size = 4.88

$$\frac{2\left(5\sqrt{2}\left(a\cosh\left(x\right)^{6}+6\,a\cosh\left(x\right)\sinh\left(x\right)^{5}+a\sinh\left(x\right)^{6}-3\,a\cosh\left(x\right)^{4}+3\left(5\,a\cosh\left(x\right)^{2}-a\right)\sinh\left(x\right)^{4}+4\left(5\,a\cosh\left(x\right)^{3}-3\,a\cosh\left(x\right)\right)\sinh\left(x\right)^{3}+3\,a\cosh\left(x\right)^{2}+3\left(5\,a\cosh\left(x\right)^{4}-6\,a\cosh\left(x\right)^{2}+a\right)\sinh\left(x\right)^{2}+6\left(a\cosh\left(x\right)^{5}-2\,a\cosh\left(x\right)^{3}+a\cosh\left(x\right)\right)\sinh\left(x\right)-a\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(4,0,\cosh\left(x\right)+\sinh\left(x\right)\right)+\sqrt{2}\left(5\,a\cosh\left(x\right)^{6}+30\,a\cosh\left(x\right)\sinh\left(x\right)^{5}+5\,a\sinh\left(x\right)^{6}-17\,a\cosh\left(x\right)^{4}+\left(75\,a\cosh\left(x\right)^{2}-17\,a\right)\sinh\left(x\right)^{4}+4\left(25\,a\cosh\left(x\right)^{3}-17\,a\cosh\left(x\right)\right)\sinh\left(x\right)^{3}-17\,a\cosh\left(x\right)^{2}+\left(75\,a\cosh\left(x\right)^{4}-102\,a\cosh\left(x\right)^{2}-17\,a\right)\sinh\left(x\right)^{2}+2\left(15\,a\cosh\left(x\right)^{5}-34\,a\cosh\left(x\right)^{3}-17\,a\cosh\left(x\right)\right)\sinh\left(x\right)+5\,a\right)\sqrt{\frac{a\cosh\left(x\right)+a\sinh\left(x\right)}{\cosh\left(x\right)^{2}+2\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}-1}}\right)}{21\left(\cosh\left(x\right)^{6}+6\cosh\left(x\right)\sinh\left(x\right)^{5}+\sinh\left(x\right)^{6}+3\left(5\cosh\left(x\right)^{2}-1\right)\sinh\left(x\right)^{4}-3\cosh\left(x\right)^{4}+4\left(5\cosh\left(x\right)^{3}-3\cosh\left(x\right)\right)\sinh\left(x\right)^{3}+3\left(5\cosh\left(x\right)^{4}-6\cosh\left(x\right)^{2}+1\right)\sinh\left(x\right)^{2}+3\cosh\left(x\right)^{2}+6\left(\cosh\left(x\right)^{5}-2\cosh\left(x\right)^{3}+\cosh\left(x\right)\right)\sinh\left(x\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/21*(5*sqrt(2)*(a*cosh(x)^6 + 6*a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 - 3*a*co
sh(x)^4 + 3*(5*a*cosh(x)^2 - a)*sinh(x)^4 + 4*(5*a*cosh(x)^3 - 3*a*cosh(x))
*sinh(x)^3 + 3*a*cosh(x)^2 + 3*(5*a*cosh(x)^4 - 6*a*cosh(x)^2 + a)*sinh(x)^
2 + 6*(a*cosh(x)^5 - 2*a*cosh(x)^3 + a*cosh(x))*sinh(x) - a)*sqrt(a)*weiers
trassPInverse(4, 0, cosh(x) + sinh(x)) + sqrt(2)*(5*a*cosh(x)^6 + 30*a*cosh
(x)*sinh(x)^5 + 5*a*sinh(x)^6 - 17*a*cosh(x)^4 + (75*a*cosh(x)^2 - 17*a)*si
nh(x)^4 + 4*(25*a*cosh(x)^3 - 17*a*cosh(x))*sinh(x)^3 - 17*a*cosh(x)^2 + (7
5*a*cosh(x)^4 - 102*a*cosh(x)^2 - 17*a)*sinh(x)^2 + 2*(15*a*cosh(x)^5 - 34*
a*cosh(x)^3 - 17*a*cosh(x))*sinh(x) + 5*a)*sqrt((a*cosh(x) + a*sinh(x))/(co
sh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)))/(cosh(x)^6 + 6*cosh(x)*sinh(
x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 - 1)*sinh(x)^4 - 3*cosh(x)^4 + 4*(5*cosh(
x)^3 - 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4 - 6*cosh(x)^2 + 1)*sinh(x)^2 +
 3*cosh(x)^2 + 6*(cosh(x)^5 - 2*cosh(x)^3 + cosh(x))*sinh(x) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\operatorname{csch}^{3}\left(x\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)**3)**(3/2),x)
```

```
[Out] Integral((a*csch(x)**3)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*csch(x)^3)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{a}{\sinh\left(x\right)^3}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/sinh(x)^3)^(3/2),x)
```

```
[Out] int((a/sinh(x)^3)^(3/2), x)
```

3.38 $\int \sqrt{a\operatorname{csch}^3(x)}\,dx$

Optimal. Leaf size=56

$$-2i\sqrt{a\operatorname{csch}^3(x)}\,E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)(i\sinh(x))^{3/2}-2\cosh(x)\sqrt{a\operatorname{csch}^3(x)}\,\sinh(x)$$

```
[Out] -2*I*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticE(cos(1/4*Pi
+1/2*I*x),2^(1/2))*(I*sinh(x))^(3/2)*(a*csch(x)^3)^(1/2)-2*cosh(x)*sinh(x)*
(a*csch(x)^3)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4208, 3853, 3856, 2719}

$$-2\sinh(x)\cosh(x)\sqrt{a\operatorname{csch}^3(x)}+\frac{2i\sinh^2(x)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{a\operatorname{csch}^3(x)}}{\sqrt{i\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Csch[x]^3],x]
```

```
[Out] -2*Cosh[x]*Sqrt[a*Csch[x]^3]*Sinh[x] + ((2*I)*Sqrt[a*Csch[x]^3]*EllipticE[P
i/4 - (I/2)*x, 2]*Sinh[x]^2)/Sqrt[I*Sinh[x]]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
```

```
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a\text{csch}^3(x)}\,dx &= \frac{\sqrt{a\text{csch}^3(x)}\int (i\text{csch}(x))^{3/2}\,dx}{(i\text{csch}(x))^{3/2}}\\
&= -2\cosh(x)\sqrt{a\text{csch}^3(x)}\sinh(x) - \frac{\sqrt{a\text{csch}^3(x)}\int \frac{1}{\sqrt{i\text{csch}(x)}}\,dx}{(i\text{csch}(x))^{3/2}}\\
&= -2\cosh(x)\sqrt{a\text{csch}^3(x)}\sinh(x) + \frac{\left(\sqrt{a\text{csch}^3(x)}\sinh^2(x)\right)\int \sqrt{i\sinh(x)}\,dx}{\sqrt{i\sinh(x)}}\\
&= -2\cosh(x)\sqrt{a\text{csch}^3(x)}\sinh(x) + \frac{2i\sqrt{a\text{csch}^3(x)}\,E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sinh^2(x)}{\sqrt{i\sinh(x)}}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 42, normalized size = 0.75

$$-2\sqrt{a\text{csch}^3(x)}\left(\cosh(x) - E\left(\frac{1}{4}(\pi-2ix)\middle|2\right)\sqrt{i\sinh(x)}\right)\sinh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Csch[x]^3],x]
[Out] -2*Sqrt[a*Csch[x]^3]*(Cosh[x] - EllipticE[(Pi - (2*I)*x)/4, 2]*Sqrt[I*Sinh[
x]])*Sinh[x]
```

Maple [F]
time = 0.95, size = 0, normalized size = 0.00

$$\int \sqrt{a\,\text{csch}\,(x)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*csch(x)^3)^(1/2),x)
[Out] int((a*csch(x)^3)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*csch(x)^3), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 60, normalized size = 1.07

$$-2\sqrt{2}\sqrt{\frac{a\cosh(x)+a\sinh(x)}{\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2-1}}(\cosh(x)+\sinh(x))-2\sqrt{2}\sqrt{a}\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cosh(x)+\sinh(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(2)*sqrt((a*cosh(x) + a*sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + si
nh(x)^2 - 1))*(cosh(x) + sinh(x)) - 2*sqrt(2)*sqrt(a)*weierstrassZeta(4, 0,
 weierstrassPInverse(4, 0, cosh(x) + sinh(x)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\operatorname{csch}^3(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)**3)**(1/2),x)
```

```
[Out] Integral(sqrt(a*csch(x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*csch(x)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\sqrt{\frac{a}{\sinh(x)^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/sinh(x)^3)^(1/2),x)
```

```
[Out] int((a/sinh(x)^3)^(1/2), x)
```

3.39 $\int \frac{1}{\sqrt{a\text{csch}^3(x)}}\,dx$

Optimal. Leaf size=62

$$\frac{2\coth(x)}{3\sqrt{a\text{csch}^3(x)}}-\frac{2i\text{csch}^2(x)F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{i\sinh(x)}}{3\sqrt{a\text{csch}^3(x)}}$$

```
[Out] 2/3*coth(x)/(a*csch(x)^3)^(1/2)-2/3*I*csch(x)^2*(sin(1/4*Pi+1/2*I*x)^2)^(1/
2)/sin(1/4*Pi+1/2*I*x)*EllipticF(cos(1/4*Pi+1/2*I*x),2^(1/2))*(I*sinh(x))^(
1/2)/(a*csch(x)^3)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4208, 3854, 3856, 2720}

$$\frac{2\coth(x)}{3\sqrt{a\text{csch}^3(x)}}-\frac{2i\sqrt{i\sinh(x)}\,\text{csch}^2(x)F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)}{3\sqrt{a\text{csch}^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Csch[x]^3],x]
```

```
[Out] (2*Coth[x])/(3*Sqrt[a*Csch[x]^3]) - (((2*I)/3)*Csch[x]^2*EllipticF[Pi/4 - (
I/2)*x, 2]*Sqrt[I*Sinh[x]])/Sqrt[a*Csch[x]^3]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a\operatorname{csch}^3(x)}}\,dx &= \frac{(i\operatorname{csch}(x))^{3/2}\int \frac{1}{(i\operatorname{csch}(x))^{3/2}}\,dx}{\sqrt{a\operatorname{csch}^3(x)}}\\ &= \frac{2\coth(x)}{3\sqrt{a\operatorname{csch}^3(x)}}+\frac{(i\operatorname{csch}(x))^{3/2}\int \sqrt{i\operatorname{csch}(x)}\,dx}{3\sqrt{a\operatorname{csch}^3(x)}}\\ &= \frac{2\coth(x)}{3\sqrt{a\operatorname{csch}^3(x)}}-\frac{\left(\operatorname{csch}^2(x)\sqrt{i\sinh(x)}\right)\int \frac{1}{\sqrt{i\sinh(x)}}\,dx}{3\sqrt{a\operatorname{csch}^3(x)}}\\ &= \frac{2\coth(x)}{3\sqrt{a\operatorname{csch}^3(x)}}-\frac{2i\operatorname{csch}^2(x)F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{i\sinh(x)}}{3\sqrt{a\operatorname{csch}^3(x)}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 43, normalized size = 0.69

$$\frac{2\left(\coth(x)+\frac{\operatorname{csch}(x)F\left(\frac{1}{4}(\pi-2ix)\middle|2\right)}{\sqrt{i\sinh(x)}}\right)}{3\sqrt{a\operatorname{csch}^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Csch[x]^3],x]
[Out] (2*(Coth[x] + (Csch[x]*EllipticF[(Pi - (2*I)*x)/4, 2])/Sqrt[I*Sinh[x]]))/(3
*Sqrt[a*Csch[x]^3])
```

Maple [F]
time = 1.21, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\operatorname{csch}(x)^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*csch(x)^3)^(1/2),x)
```

```
[Out] int(1/(a*csch(x)^3)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*csch(x)^3), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 127, normalized size = 2.05

$$-\frac{4\sqrt{2}\left(\cosh\left(x\right)^{2}+2\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}\right)\sqrt{a}\,\mathrm{weierstrassPInverse}(4,0,\cosh\left(x\right)+\sinh\left(x\right))-\sqrt{2}\left(\cosh\left(x\right)^{4}+4\cosh\left(x\right)^{3}\sinh\left(x\right)+6\cosh\left(x\right)^{2}\sinh\left(x\right)^{2}+4\cosh\left(x\right)\sinh\left(x\right)^{3}+\sinh\left(x\right)^{4}-1\right)\sqrt{\frac{a\cosh\left(x\right)+a\sinh\left(x\right)}{\cosh\left(x\right)^{2}+2\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}-1}}}{6\left(a\cosh\left(x\right)^{2}+2a\cosh\left(x\right)\sinh\left(x\right)+a\sinh\left(x\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(4*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)*sqrt(a)*weierst
rassPInverse(4, 0, cosh(x) + sinh(x)) - sqrt(2)*(cosh(x)^4 + 4*cosh(x)^3*si
nh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 - 1)*sqrt((
a*cosh(x) + a*sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)))/(a
*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{a\operatorname{csch}^{3}\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)**3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*csch(x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(a*csch(x)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\frac{a}{\sinh(x)^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/sinh(x)^3)^(1/2),x)
[Out] int(1/(a/sinh(x)^3)^(1/2), x)
```

3.40 $\int \frac{1}{\left(a\operatorname{csch}^3(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=89

$$-\frac{14\cosh(x)}{45a\sqrt{a\operatorname{csch}^3(x)}}+\frac{14i\operatorname{csch}(x)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)}{15a\sqrt{a\operatorname{csch}^3(x)}\sqrt{i\sinh(x)}}+\frac{2\cosh(x)\sinh^2(x)}{9a\sqrt{a\operatorname{csch}^3(x)}}$$

```
[Out] -14/45*cosh(x)/a/(a*csch(x)^3)^(1/2)+2/9*cosh(x)*sinh(x)^2/a/(a*csch(x)^3)^
(1/2)+14/15*I*csch(x)*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*Ell
ipticE(cos(1/4*Pi+1/2*I*x),2^(1/2))/a/(a*csch(x)^3)^(1/2)/(I*sinh(x))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4208, 3854, 3856, 2719}

$$-\frac{14\cosh(x)}{45a\sqrt{a\operatorname{csch}^3(x)}}+\frac{2\sinh^2(x)\cosh(x)}{9a\sqrt{a\operatorname{csch}^3(x)}}+\frac{14i\operatorname{csch}(x)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)}{15a\sqrt{i\sinh(x)}\sqrt{a\operatorname{csch}^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Csch[x]^3)^(-3/2),x]
```

```
[Out] (-14*Cosh[x])/(45*a*Sqrt[a*Csch[x]^3]) + (((14*I)/15)*Csch[x]*EllipticE[Pi/
4 - (I/2)*x, 2])/(a*Sqrt[a*Csch[x]^3]*Sqrt[I*Sinh[x]]) + (2*Cosh[x]*Sinh[x]
^2)/(9*a*Sqrt[a*Csch[x]^3])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
& !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a\operatorname{csch}^3(x)\right)^{3/2}}\,dx &= -\frac{\left(i(i\operatorname{csch}(x))^{3/2}\right)\int \frac{1}{(i\operatorname{csch}(x))^{9/2}}\,dx}{a\sqrt{a\operatorname{csch}^3(x)}}\\
&= \frac{2\cosh(x)\sinh^2(x)}{9a\sqrt{a\operatorname{csch}^3(x)}} - \frac{\left(7i(i\operatorname{csch}(x))^{3/2}\right)\int \frac{1}{(i\operatorname{csch}(x))^{5/2}}\,dx}{9a\sqrt{a\operatorname{csch}^3(x)}}\\
&= -\frac{14\cosh(x)}{45a\sqrt{a\operatorname{csch}^3(x)}} + \frac{2\cosh(x)\sinh^2(x)}{9a\sqrt{a\operatorname{csch}^3(x)}} - \frac{\left(7i(i\operatorname{csch}(x))^{3/2}\right)\int \frac{1}{\sqrt{i\operatorname{csch}(x)}}\,dx}{15a\sqrt{a\operatorname{csch}^3(x)}}\\
&= -\frac{14\cosh(x)}{45a\sqrt{a\operatorname{csch}^3(x)}} + \frac{2\cosh(x)\sinh^2(x)}{9a\sqrt{a\operatorname{csch}^3(x)}} + \frac{(7\operatorname{csch}(x))\int \sqrt{i\sinh(x)}\,dx}{15a\sqrt{a\operatorname{csch}^3(x)}\sqrt{i\sinh(x)}}\\
&= -\frac{14\cosh(x)}{45a\sqrt{a\operatorname{csch}^3(x)}} + \frac{14i\operatorname{csch}(x)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)}{15a\sqrt{a\operatorname{csch}^3(x)}\sqrt{i\sinh(x)}} + \frac{2\cosh(x)\sinh^2(x)}{9a\sqrt{a\operatorname{csch}^3(x)}}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 57, normalized size = 0.64

$$\frac{-33\cosh(x)+5\cosh(3x)+84\operatorname{csch}^2(x)E\left(\frac{1}{4}(\pi-2ix)\middle|2\right)\sqrt{i\sinh(x)}}{90a\sqrt{a\operatorname{csch}^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Csch[x]^3)^(-3/2),x]
```

```
[Out] (-33*Cosh[x] + 5*Cosh[3*x] + 84*Csch[x]^2*EllipticE[(Pi - (2*I)*x)/4, 2]*Sq
rt[I*Sinh[x]])/(90*a*Sqrt[a*Csch[x]^3])
```

Maple [F]
time = 0.85, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\operatorname{csch}(x)^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*csch(x)^3)^(3/2),x)
```

```
[Out] int(1/(a*csch(x)^3)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*csch(x)^3)^(-3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 407, normalized size = 4.57

$$-\frac{672\sqrt{2}\left(\cosh(x)^5+5\cosh(x)^4\sinh(x)+10\cosh(x)^3\sinh(x)^2+10\cosh(x)^2\sinh(x)^3+5\cosh(x)\sinh(x)^4+\sinh(x)^5\right)\sqrt{a}\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cosh(x)+\sinh(x)))-\sqrt{2}\left(5\cosh(x)^{10}+50\cosh(x)\sinh(x)^9+5\sinh(x)^{10}+(225\cosh(x)^2-43)\sinh(x)^8-43\cosh(x)^8+8(75\cosh(x)^3-43\cosh(x))\sinh(x)^7+2(525\cosh(x)^4-602\cosh(x)^2-149)\sinh(x)^6-298\cosh(x)^6+4(315\cosh(x)^5-602\cosh(x)^3-447\cosh(x))\sinh(x)^5+2(525\cosh(x)^6-1505\cosh(x)^4-2235\cosh(x)^2+187)\sinh(x)^4+374\cosh(x)^4+8(75\cosh(x)^7-301\cosh(x)^5-745\cosh(x)^3+187\cosh(x))\sinh(x)^3+(225\cosh(x)^8-1204\cosh(x)^6-4470\cosh(x)^4+2244\cosh(x)^2-43)\sinh(x)^2-43\cosh(x)^2+2(25\cosh(x)^9-172\cosh(x)^7-894\cosh(x)^5+748\cosh(x)^3-43\cosh(x))\sinh(x)+5\right)\sqrt{\frac{a\cosh(x)+a\sinh(x)}{\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2-1}}}{720\left(a^2\cosh(x)^5+5a^2\cosh(x)^4\sinh(x)+10a^2\cosh(x)^3\sinh(x)^2+10a^2\cosh(x)^2\sinh(x)^3+5a^2\cosh(x)\sinh(x)^4+a^2\sinh(x)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/720*(672*sqrt(2)*(cosh(x)^5 + 5*cosh(x)^4*sinh(x) + 10*cosh(x)^3*sinh(x)
^2 + 10*cosh(x)^2*sinh(x)^3 + 5*cosh(x)*sinh(x)^4 + sinh(x)^5)*sqrt(a)*weie
rstrassZeta(4, 0, weierstrassPInverse(4, 0, cosh(x) + sinh(x))) - sqrt(2)*(
5*cosh(x)^10 + 50*cosh(x)*sinh(x)^9 + 5*sinh(x)^10 + (225*cosh(x)^2 - 43)*s
inh(x)^8 - 43*cosh(x)^8 + 8*(75*cosh(x)^3 - 43*cosh(x))*sinh(x)^7 + 2*(525*
cosh(x)^4 - 602*cosh(x)^2 - 149)*sinh(x)^6 - 298*cosh(x)^6 + 4*(315*cosh(x)
^5 - 602*cosh(x)^3 - 447*cosh(x))*sinh(x)^5 + 2*(525*cosh(x)^6 - 1505*cosh(
x)^4 - 2235*cosh(x)^2 + 187)*sinh(x)^4 + 374*cosh(x)^4 + 8*(75*cosh(x)^7 -
301*cosh(x)^5 - 745*cosh(x)^3 + 187*cosh(x))*sinh(x)^3 + (225*cosh(x)^8 - 1
204*cosh(x)^6 - 4470*cosh(x)^4 + 2244*cosh(x)^2 - 43)*sinh(x)^2 - 43*cosh(x
)^2 + 2*(25*cosh(x)^9 - 172*cosh(x)^7 - 894*cosh(x)^5 + 748*cosh(x)^3 - 43*
cosh(x))*sinh(x) + 5)*sqrt((a*cosh(x) + a*sinh(x))/(cosh(x)^2 + 2*cosh(x)*s
inh(x) + sinh(x)^2 - 1)))/(a^2*cosh(x)^5 + 5*a^2*cosh(x)^4*sinh(x) + 10*a^2
*cosh(x)^3*sinh(x)^2 + 10*a^2*cosh(x)^2*sinh(x)^3 + 5*a^2*cosh(x)*sinh(x)^4
 + a^2*sinh(x)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\operatorname{csch}^{3}(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)**3)**(3/2),x)
```

```
[Out] Integral((a*csch(x)**3)**(-3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*csch(x)^3)^(-3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{a}{\sinh(x)^3}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/sinh(x)^3)^(3/2),x)
```

```
[Out] int(1/(a/sinh(x)^3)^(3/2), x)
```

3.41 $\int \frac{1}{\left(a\operatorname{csch}^3(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=135

$$-\frac{26\coth(x)}{77a^2\sqrt{a\operatorname{csch}^3(x)}}+\frac{26i\operatorname{csch}^2(x)F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{i\sinh(x)}}{77a^2\sqrt{a\operatorname{csch}^3(x)}}+\frac{78\cosh(x)\sinh(x)}{385a^2\sqrt{a\operatorname{csch}^3(x)}}-\frac{26\cosh(x)\sinh^3(x)}{165a^2\sqrt{a\operatorname{csch}^3(x)}}+\frac{2\,\mathrm{co}}{15a}$$

```
[Out] -26/77*coth(x)/a^2/(a*csch(x)^3)^(1/2)+78/385*cosh(x)*sinh(x)/a^2/(a*csch(x
)^3)^(1/2)-26/165*cosh(x)*sinh(x)^3/a^2/(a*csch(x)^3)^(1/2)+2/15*cosh(x)*si
nh(x)^5/a^2/(a*csch(x)^3)^(1/2)+26/77*I*csch(x)^2*(sin(1/4*Pi+1/2*I*x)^2)^(
1/2)/sin(1/4*Pi+1/2*I*x)*EllipticF(cos(1/4*Pi+1/2*I*x),2^(1/2))*(I*sinh(x))
^(1/2)/a^2/(a*csch(x)^3)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4208, 3854, 3856, 2720}

$$-\frac{26\coth(x)}{77a^2\sqrt{a\operatorname{csch}^3(x)}}+\frac{2\sinh^5(x)\cosh(x)}{15a^2\sqrt{a\operatorname{csch}^3(x)}}-\frac{26\sinh^3(x)\cosh(x)}{165a^2\sqrt{a\operatorname{csch}^3(x)}}+\frac{78\sinh(x)\cosh(x)}{385a^2\sqrt{a\operatorname{csch}^3(x)}}+\frac{26i\sqrt{i\sinh(x)}\operatorname{csch}^2(x)F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)}{77a^2\sqrt{a\operatorname{csch}^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Csch[x]^3)^(-5/2),x]
```

```
[Out] (-26*Coth[x])/(77*a^2*Sqrt[a*Csch[x]^3]) + (((26*I)/77)*Csch[x]^2*EllipticF
[Pi/4 - (I/2)*x, 2]*Sqrt[I*Sinh[x]])/(a^2*Sqrt[a*Csch[x]^3]) + (78*Cosh[x]*
Sinh[x])/(385*a^2*Sqrt[a*Csch[x]^3]) - (26*Cosh[x]*Sinh[x]^3)/(165*a^2*Sqrt
[a*Csch[x]^3]) + (2*Cosh[x]*Sinh[x]^5)/(15*a^2*Sqrt[a*Csch[x]^3])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a\operatorname{csch}^3(x)\right)^{5/2}}\,dx &= -\frac{(i\operatorname{csch}(x))^{3/2}\int \frac{1}{(i\operatorname{csch}(x))^{15/2}}\,dx}{a^2\sqrt{a\operatorname{csch}^3(x)}} \\
&= \frac{2\cosh(x)\sinh^5(x)}{15a^2\sqrt{a\operatorname{csch}^3(x)}} - \frac{\left(13(i\operatorname{csch}(x))^{3/2}\right)\int \frac{1}{(i\operatorname{csch}(x))^{11/2}}\,dx}{15a^2\sqrt{a\operatorname{csch}^3(x)}} \\
&= -\frac{26\cosh(x)\sinh^3(x)}{165a^2\sqrt{a\operatorname{csch}^3(x)}} + \frac{2\cosh(x)\sinh^5(x)}{15a^2\sqrt{a\operatorname{csch}^3(x)}} - \frac{\left(39(i\operatorname{csch}(x))^{3/2}\right)\int \frac{1}{(i\operatorname{csch}(x))^{7/2}}\,dx}{55a^2\sqrt{a\operatorname{csch}^3(x)}} \\
&= \frac{78\cosh(x)\sinh(x)}{385a^2\sqrt{a\operatorname{csch}^3(x)}} - \frac{26\cosh(x)\sinh^3(x)}{165a^2\sqrt{a\operatorname{csch}^3(x)}} + \frac{2\cosh(x)\sinh^5(x)}{15a^2\sqrt{a\operatorname{csch}^3(x)}} - \frac{\left(39(i\operatorname{csch}(x))^{3/2}\right)\int}{77a^2\sqrt{acs}} \\
&= -\frac{26\coth(x)}{77a^2\sqrt{a\operatorname{csch}^3(x)}} + \frac{78\cosh(x)\sinh(x)}{385a^2\sqrt{a\operatorname{csch}^3(x)}} - \frac{26\cosh(x)\sinh^3(x)}{165a^2\sqrt{a\operatorname{csch}^3(x)}} + \frac{2\cosh(x)\sinh^5(x}{15a^2\sqrt{a\operatorname{csch}^3(x)}} \\
&= -\frac{26\coth(x)}{77a^2\sqrt{a\operatorname{csch}^3(x)}} + \frac{78\cosh(x)\sinh(x)}{385a^2\sqrt{a\operatorname{csch}^3(x)}} - \frac{26\cosh(x)\sinh^3(x)}{165a^2\sqrt{a\operatorname{csch}^3(x)}} + \frac{2\cosh(x)\sinh^5(x}{15a^2\sqrt{a\operatorname{csch}^3(x)}} \\
&= -\frac{26\coth(x)}{77a^2\sqrt{a\operatorname{csch}^3(x)}} + \frac{26i\operatorname{csch}^2(x)F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{i\sinh(x)}}{77a^2\sqrt{a\operatorname{csch}^3(x)}} + \frac{78\cosh(x)\sinh(x)}{385a^2\sqrt{a\operatorname{csch}^3(x)}} -
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 71, normalized size = 0.53

$$\frac{\sqrt{a\operatorname{csch}^3(x)}\,\sinh(x)\left(24960iF\left(\frac{1}{4}(\pi-2ix)\middle|2\right)\sqrt{i\sinh(x)}-19122\sinh(2x)+4406\sinh(4x)-826\sinh(6x)+77\sinh(8x)\right)}{73920a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Csch[x]^3)^(-5/2),x]
```

```
[Out] (Sqrt[a*Csch[x]^3]*Sinh[x]*((24960*I)*EllipticF[(Pi - (2*I)*x)/4, 2]*Sqrt[I
*Sinh[x]] - 19122*Sinh[2*x] + 4406*Sinh[4*x] - 826*Sinh[6*x] + 77*Sinh[8*x]
))/(73920*a^3)
```

Maple [F]
time = 0.85, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\operatorname{csch}\left(x\right)^{3}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*csch(x)^3)^(5/2),x)
```

```
[Out] int(1/(a*csch(x)^3)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*csch(x)^3)^(-5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 718, normalized size = 5.32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/147840*(49920*sqrt(2)*(cosh(x)^8 + 8*cosh(x)^7*sinh(x) + 28*cosh(x)^6*sin
h(x)^2 + 56*cosh(x)^5*sinh(x)^3 + 70*cosh(x)^4*sinh(x)^4 + 56*cosh(x)^3*sin
h(x)^5 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8)*sqrt(a)*
weierstrassPInverse(4, 0, cosh(x) + sinh(x)) + sqrt(2)*(77*cosh(x)^16 + 123
2*cosh(x)*sinh(x)^15 + 77*sinh(x)^16 + 14*(660*cosh(x)^2 - 59)*sinh(x)^14 -
 826*cosh(x)^14 + 196*(220*cosh(x)^3 - 59*cosh(x))*sinh(x)^13 + 2*(70070*co
sh(x)^4 - 37583*cosh(x)^2 + 2203)*sinh(x)^12 + 4406*cosh(x)^12 + 8*(42042*c
osh(x)^5 - 37583*cosh(x)^3 + 6609*cosh(x))*sinh(x)^11 + 2*(308308*cosh(x)^6
 - 413413*cosh(x)^4 + 145398*cosh(x)^2 - 9561)*sinh(x)^10 - 19122*cosh(x)^1
0 + 4*(220220*cosh(x)^7 - 413413*cosh(x)^5 + 242330*cosh(x)^3 - 47805*cosh(
x))*sinh(x)^9 + 6*(165165*cosh(x)^8 - 413413*cosh(x)^6 + 363495*cosh(x)^4 -
```

```
 143415*cosh(x)^2)*sinh(x)^8 + 16*(55055*cosh(x)^9 - 177177*cosh(x)^7 + 218
097*cosh(x)^5 - 143415*cosh(x)^3)*sinh(x)^7 + 2*(308308*cosh(x)^10 - 124023
9*cosh(x)^8 + 2035572*cosh(x)^6 - 2007810*cosh(x)^4 + 9561)*sinh(x)^6 + 191
22*cosh(x)^6 + 4*(84084*cosh(x)^11 - 413413*cosh(x)^9 + 872388*cosh(x)^7 -
1204686*cosh(x)^5 + 28683*cosh(x))*sinh(x)^5 + 2*(70070*cosh(x)^12 - 413413
*cosh(x)^10 + 1090485*cosh(x)^8 - 2007810*cosh(x)^6 + 143415*cosh(x)^2 - 22
03)*sinh(x)^4 - 4406*cosh(x)^4 + 8*(5390*cosh(x)^13 - 37583*cosh(x)^11 + 12
1165*cosh(x)^9 - 286830*cosh(x)^7 + 47805*cosh(x)^3 - 2203*cosh(x))*sinh(x)
^3 + 2*(4620*cosh(x)^14 - 37583*cosh(x)^12 + 145398*cosh(x)^10 - 430245*cos
h(x)^8 + 143415*cosh(x)^4 - 13218*cosh(x)^2 + 413)*sinh(x)^2 + 826*cosh(x)^
2 + 4*(308*cosh(x)^15 - 2891*cosh(x)^13 + 13218*cosh(x)^11 - 47805*cosh(x)^
9 + 28683*cosh(x)^5 - 4406*cosh(x)^3 + 413*cosh(x))*sinh(x) - 77)*sqrt((a*c
osh(x) + a*sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)))/(a^3*
cosh(x)^8 + 8*a^3*cosh(x)^7*sinh(x) + 28*a^3*cosh(x)^6*sinh(x)^2 + 56*a^3*c
osh(x)^5*sinh(x)^3 + 70*a^3*cosh(x)^4*sinh(x)^4 + 56*a^3*cosh(x)^3*sinh(x)^
5 + 28*a^3*cosh(x)^2*sinh(x)^6 + 8*a^3*cosh(x)*sinh(x)^7 + a^3*sinh(x)^8)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\operatorname{csch}^3(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)**3)**(5/2),x)
[Out] Integral((a*csch(x)**3)**(-5/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^3)^(5/2),x, algorithm="giac")
[Out] integrate((a*csch(x)^3)^(-5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{a}{\sinh(x)^3}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/sinh(x)^3)^(5/2),x)
[Out] int(1/(a/sinh(x)^3)^(5/2), x)
```

3.42 $\int \left(a\text{csch}^4(x)\right)^{7/2}\,dx$

Optimal. Leaf size=164

$$2a^3\cosh^2(x)\coth(x)\sqrt{a\text{csch}^4(x)}-3a^3\cosh^2(x)\coth^3(x)\sqrt{a\text{csch}^4(x)}+\frac{20}{7}a^3\cosh^2(x)\coth^5(x)\sqrt{a\text{csch}^4(x)}$$

```
[Out] 2*a^3*cosh(x)^2*coth(x)*(a*csch(x)^4)^(1/2)-3*a^3*cosh(x)^2*coth(x)^3*(a*cs
ch(x)^4)^(1/2)+20/7*a^3*cosh(x)^2*coth(x)^5*(a*csch(x)^4)^(1/2)-5/3*a^3*cos
h(x)^2*coth(x)^7*(a*csch(x)^4)^(1/2)+6/11*a^3*cosh(x)^2*coth(x)^9*(a*csch(x
)^4)^(1/2)-1/13*a^3*cosh(x)^2*coth(x)^11*(a*csch(x)^4)^(1/2)-a^3*cosh(x)*si
nh(x)*(a*csch(x)^4)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4208, 3852}

$$-\frac{1}{13}a^3\cosh^2(x)\coth^{11}(x)\sqrt{a\text{csch}^4(x)}+\frac{6}{11}a^3\cosh^2(x)\coth^9(x)\sqrt{a\text{csch}^4(x)}-\frac{5}{3}a^3\cosh^2(x)\coth^7(x)\sqrt{a\text{csch}^4(x)}+\frac{20}{7}a^3\cosh^2(x)\coth^5(x)\sqrt{a\text{csch}^4(x)}-3a^3\cosh^2(x)\coth^3(x)\sqrt{a\text{csch}^4(x)}+2a^3\cosh^2(x)\coth(x)\sqrt{a\text{csch}^4(x)}-a^3\sinh(x)\cosh(x)\sqrt{a\text{csch}^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Csch[x]^4)^(7/2),x]
```

```
[Out] 2*a^3*Cosh[x]^2*Coth[x]*Sqrt[a*Csch[x]^4] - 3*a^3*Cosh[x]^2*Coth[x]^3*Sqrt[
a*Csch[x]^4] + (20*a^3*Cosh[x]^2*Coth[x]^5*Sqrt[a*Csch[x]^4])/7 - (5*a^3*Co
sh[x]^2*Coth[x]^7*Sqrt[a*Csch[x]^4])/3 + (6*a^3*Cosh[x]^2*Coth[x]^9*Sqrt[a*
Csch[x]^4])/11 - (a^3*Cosh[x]^2*Coth[x]^11*Sqrt[a*Csch[x]^4])/13 - a^3*Cosh
[x]*Sqrt[a*Csch[x]^4]*Sinh[x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\int \left(a\text{csch}^4(x)\right)^{7/2} dx = \left(a^3\sqrt{a\text{csch}^4(x)}\,\sinh^2(x)\right)\int \text{csch}^{14}(x)\,dx$$
$$= -\left(\left(ia^3\sqrt{a\text{csch}^4(x)}\,\sinh^2(x)\right)\text{Subst}\left(\int \left(1+6x^2+15x^4+20x^6+15x^8+6x^{10}+x\right.\right.\right.$$
$$= 2a^3\cosh^2(x)\coth(x)\sqrt{a\text{csch}^4(x)} - 3a^3\cosh^2(x)\coth^3(x)\sqrt{a\text{csch}^4(x)} + \frac{20}{7}a^3\cosh^2($$

Mathematica [A]
time = 0.03, size = 59, normalized size = 0.36

$$-\frac{a^3\cosh(x)\sqrt{a\text{csch}^4(x)}\,(1024-512\text{csch}^2(x)+384\text{csch}^4(x)-320\text{csch}^6(x)+280\text{csch}^8(x)-252\text{csch}^{10}(x)+231\text{csch}^{12}(x))\sinh(x)}{3003}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Csch[x]^4)^(7/2),x]
```

```
[Out] -1/3003*(a^3*Cosh[x]*Sqrt[a*Csch[x]^4]*(1024 - 512*Csch[x]^2 + 384*Csch[x]^
4 - 320*Csch[x]^6 + 280*Csch[x]^8 - 252*Csch[x]^10 + 231*Csch[x]^12)*Sinh[x
])
```

Maple [A]
time = 1.03, size = 72, normalized size = 0.44

method	result	size
risch	$-\frac{2048a^3\mathrm{e}^{-2x}\sqrt{\frac{a\,\mathrm{e}^{4x}}{(\mathrm{e}^{2x}-1)^4}}\,(1716\,\mathrm{e}^{12x}-1287\,\mathrm{e}^{10x}+715\,\mathrm{e}^{8x}-286\,\mathrm{e}^{6x}+78\,\mathrm{e}^{4x}-13\,\mathrm{e}^{2x}+1)}{3003(\mathrm{e}^{2x}-1)^{11}}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*csch(x)^4)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2048/3003*a^3*exp(-2*x)/(exp(2*x)-1)^11*(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)*
(1716*exp(12*x)-1287*exp(10*x)+715*exp(8*x)-286*exp(6*x)+78*exp(4*x)-13*exp
(2*x)+1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 620 vs. 2(142) = 284.
time = 0.49, size = 620, normalized size = 3.78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^4)^(7/2),x, algorithm="maxima")
```

```
[Out] -2048/231*a^(7/2)*e^(-2*x)/(13*e^(-2*x) - 78*e^(-4*x) + 286*e^(-6*x) - 715*
e^(-8*x) + 1287*e^(-10*x) - 1716*e^(-12*x) + 1716*e^(-14*x) - 1287*e^(-16*x
) + 715*e^(-18*x) - 286*e^(-20*x) + 78*e^(-22*x) - 13*e^(-24*x) + e^(-26*x)
 - 1) + 4096/77*a^(7/2)*e^(-4*x)/(13*e^(-2*x) - 78*e^(-4*x) + 286*e^(-6*x)
- 715*e^(-8*x) + 1287*e^(-10*x) - 1716*e^(-12*x) + 1716*e^(-14*x) - 1287*e^
(-16*x) + 715*e^(-18*x) - 286*e^(-20*x) + 78*e^(-22*x) - 13*e^(-24*x) + e^(
-26*x) - 1) - 4096/21*a^(7/2)*e^(-6*x)/(13*e^(-2*x) - 78*e^(-4*x) + 286*e^(
-6*x) - 715*e^(-8*x) + 1287*e^(-10*x) - 1716*e^(-12*x) + 1716*e^(-14*x) - 1
287*e^(-16*x) + 715*e^(-18*x) - 286*e^(-20*x) + 78*e^(-22*x) - 13*e^(-24*x)
 + e^(-26*x) - 1) + 10240/21*a^(7/2)*e^(-8*x)/(13*e^(-2*x) - 78*e^(-4*x) +
286*e^(-6*x) - 715*e^(-8*x) + 1287*e^(-10*x) - 1716*e^(-12*x) + 1716*e^(-14
*x) - 1287*e^(-16*x) + 715*e^(-18*x) - 286*e^(-20*x) + 78*e^(-22*x) - 13*e^
(-24*x) + e^(-26*x) - 1) - 6144/7*a^(7/2)*e^(-10*x)/(13*e^(-2*x) - 78*e^(-4
*x) + 286*e^(-6*x) - 715*e^(-8*x) + 1287*e^(-10*x) - 1716*e^(-12*x) + 1716*
e^(-14*x) - 1287*e^(-16*x) + 715*e^(-18*x) - 286*e^(-20*x) + 78*e^(-22*x) -
 13*e^(-24*x) + e^(-26*x) - 1) + 8192/7*a^(7/2)*e^(-12*x)/(13*e^(-2*x) - 78
*e^(-4*x) + 286*e^(-6*x) - 715*e^(-8*x) + 1287*e^(-10*x) - 1716*e^(-12*x) +
 1716*e^(-14*x) - 1287*e^(-16*x) + 715*e^(-18*x) - 286*e^(-20*x) + 78*e^(-2
2*x) - 13*e^(-24*x) + e^(-26*x) - 1) + 2048/3003*a^(7/2)/(13*e^(-2*x) - 78*
e^(-4*x) + 286*e^(-6*x) - 715*e^(-8*x) + 1287*e^(-10*x) - 1716*e^(-12*x) +
1716*e^(-14*x) - 1287*e^(-16*x) + 715*e^(-18*x) - 286*e^(-20*x) + 78*e^(-22
*x) - 13*e^(-24*x) + e^(-26*x) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2825 vs. $2(142) = 284$.
time = 0.54, size = 2825, normalized size = 17.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^4)^(7/2),x, algorithm="fricas")
```

```
[Out] -2048/3003*(1716*a^3*cosh(x)^12 - 1287*a^3*cosh(x)^10 + 1716*(a^3*e^(4*x) -
 2*a^3*e^(2*x) + a^3)*sinh(x)^12 + 20592*(a^3*cosh(x)*e^(4*x) - 2*a^3*cosh(
x)*e^(2*x) + a^3*cosh(x))*sinh(x)^11 + 715*a^3*cosh(x)^8 + 1287*(88*a^3*cos
h(x)^2 - a^3 + (88*a^3*cosh(x)^2 - a^3)*e^(4*x) - 2*(88*a^3*cosh(x)^2 - a^3
)*e^(2*x))*sinh(x)^10 + 4290*(88*a^3*cosh(x)^3 - 3*a^3*cosh(x) + (88*a^3*co
sh(x)^3 - 3*a^3*cosh(x))*e^(4*x) - 2*(88*a^3*cosh(x)^3 - 3*a^3*cosh(x))*e^(
2*x))*sinh(x)^9 - 286*a^3*cosh(x)^6 + 715*(1188*a^3*cosh(x)^4 - 81*a^3*cosh
(x)^2 + a^3 + (1188*a^3*cosh(x)^4 - 81*a^3*cosh(x)^2 + a^3)*e^(4*x) - 2*(11
88*a^3*cosh(x)^4 - 81*a^3*cosh(x)^2 + a^3)*e^(2*x))*sinh(x)^8 + 1144*(1188*
a^3*cosh(x)^5 - 135*a^3*cosh(x)^3 + 5*a^3*cosh(x) + (1188*a^3*cosh(x)^5 - 1
35*a^3*cosh(x)^3 + 5*a^3*cosh(x))*e^(4*x) - 2*(1188*a^3*cosh(x)^5 - 135*a^3
*cosh(x)^3 + 5*a^3*cosh(x))*e^(2*x))*sinh(x)^7 + 78*a^3*cosh(x)^4 + 286*(55
44*a^3*cosh(x)^6 - 945*a^3*cosh(x)^4 + 70*a^3*cosh(x)^2 - a^3 + (5544*a^3*c
```

```
osh(x)^6 - 945*a^3*cosh(x)^4 + 70*a^3*cosh(x)^2 - a^3)*e^(4*x) - 2*(5544*a^
3*cosh(x)^6 - 945*a^3*cosh(x)^4 + 70*a^3*cosh(x)^2 - a^3)*e^(2*x))*sinh(x)^
6 + 572*(2376*a^3*cosh(x)^7 - 567*a^3*cosh(x)^5 + 70*a^3*cosh(x)^3 - 3*a^3*
cosh(x) + (2376*a^3*cosh(x)^7 - 567*a^3*cosh(x)^5 + 70*a^3*cosh(x)^3 - 3*a^
3*cosh(x))*e^(4*x) - 2*(2376*a^3*cosh(x)^7 - 567*a^3*cosh(x)^5 + 70*a^3*cos
h(x)^3 - 3*a^3*cosh(x))*e^(2*x))*sinh(x)^5 - 13*a^3*cosh(x)^2 + 26*(32670*a
^3*cosh(x)^8 - 10395*a^3*cosh(x)^6 + 1925*a^3*cosh(x)^4 - 165*a^3*cosh(x)^2
 + 3*a^3 + (32670*a^3*cosh(x)^8 - 10395*a^3*cosh(x)^6 + 1925*a^3*cosh(x)^4
- 165*a^3*cosh(x)^2 + 3*a^3)*e^(4*x) - 2*(32670*a^3*cosh(x)^8 - 10395*a^3*c
osh(x)^6 + 1925*a^3*cosh(x)^4 - 165*a^3*cosh(x)^2 + 3*a^3)*e^(2*x))*sinh(x)
^4 + 104*(3630*a^3*cosh(x)^9 - 1485*a^3*cosh(x)^7 + 385*a^3*cosh(x)^5 - 55*
a^3*cosh(x)^3 + 3*a^3*cosh(x) + (3630*a^3*cosh(x)^9 - 1485*a^3*cosh(x)^7 +
385*a^3*cosh(x)^5 - 55*a^3*cosh(x)^3 + 3*a^3*cosh(x))*e^(4*x) - 2*(3630*a^3
*cosh(x)^9 - 1485*a^3*cosh(x)^7 + 385*a^3*cosh(x)^5 - 55*a^3*cosh(x)^3 + 3*
a^3*cosh(x))*e^(2*x))*sinh(x)^3 + a^3 + 13*(8712*a^3*cosh(x)^10 - 4455*a^3*
cosh(x)^8 + 1540*a^3*cosh(x)^6 - 330*a^3*cosh(x)^4 + 36*a^3*cosh(x)^2 - a^3
 + (8712*a^3*cosh(x)^10 - 4455*a^3*cosh(x)^8 + 1540*a^3*cosh(x)^6 - 330*a^3
*cosh(x)^4 + 36*a^3*cosh(x)^2 - a^3)*e^(4*x) - 2*(8712*a^3*cosh(x)^10 - 445
5*a^3*cosh(x)^8 + 1540*a^3*cosh(x)^6 - 330*a^3*cosh(x)^4 + 36*a^3*cosh(x)^2
 - a^3)*e^(2*x))*sinh(x)^2 + (1716*a^3*cosh(x)^12 - 1287*a^3*cosh(x)^10 + 7
15*a^3*cosh(x)^8 - 286*a^3*cosh(x)^6 + 78*a^3*cosh(x)^4 - 13*a^3*cosh(x)^2
+ a^3)*e^(4*x) - 2*(1716*a^3*cosh(x)^12 - 1287*a^3*cosh(x)^10 + 715*a^3*cos
h(x)^8 - 286*a^3*cosh(x)^6 + 78*a^3*cosh(x)^4 - 13*a^3*cosh(x)^2 + a^3)*e^(
2*x) + 26*(792*a^3*cosh(x)^11 - 495*a^3*cosh(x)^9 + 220*a^3*cosh(x)^7 - 66*
a^3*cosh(x)^5 + 12*a^3*cosh(x)^3 - a^3*cosh(x) + (792*a^3*cosh(x)^11 - 495*
a^3*cosh(x)^9 + 220*a^3*cosh(x)^7 - 66*a^3*cosh(x)^5 + 12*a^3*cosh(x)^3 - a
^3*cosh(x))*e^(4*x) - 2*(792*a^3*cosh(x)^11 - 495*a^3*cosh(x)^9 + 220*a^3*c
osh(x)^7 - 66*a^3*cosh(x)^5 + 12*a^3*cosh(x)^3 - a^3*cosh(x))*e^(2*x))*sinh
(x))*sqrt(a/(e^(8*x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*x) + 1))*e^(2*x)/(26*
cosh(x)*e^(2*x)*sinh(x)^25 + e^(2*x)*sinh(x)^26 + 13*(25*cosh(x)^2 - 1)*e^(
2*x)*sinh(x)^24 + 104*(25*cosh(x)^3 - 3*cosh(x))*e^(2*x)*sinh(x)^23 + 26*(5
75*cosh(x)^4 - 138*cosh(x)^2 + 3)*e^(2*x)*sinh(x)^22 + 572*(115*cosh(x)^5 -
 46*cosh(x)^3 + 3*cosh(x))*e^(2*x)*sinh(x)^21 + 286*(805*cosh(x)^6 - 483*co
sh(x)^4 + 63*cosh(x)^2 - 1)*e^(2*x)*sinh(x)^20 + 1144*(575*cosh(x)^7 - 483*
cosh(x)^5 + 105*cosh(x)^3 - 5*cosh(x))*e^(2*x)*sinh(x)^19 + 143*(10925*cosh
(x)^8 - 12236*cosh(x)^6 + 3990*cosh(x)^4 - 380*cosh(x)^2 + 5)*e^(2*x)*sinh(
x)^18 + 286*(10925*cosh(x)^9 - 15732*cosh(x)^7 + 7182*cosh(x)^5 - 1140*cosh
(x)^3 + 45*cosh(x))*e^(2*x)*sinh(x)^17 + 143*(37145*cosh(x)^10 - 66861*cosh
(x)^8 + 40698*cosh(x)^6 - 9690*cosh(x)^4 + 765*cosh(x)^2 - 9)*e^(2*x)*sinh(
x)^16 + 208*(37145*cosh(x)^11 - 81719*cosh(x)^9 + 63954*cosh(x)^7 - 21318*c
osh(x)^5 + 2805*cosh(x)^3 - 99*cosh(x))*e^(2*x)*sinh(x)^15 + 52*(185725*cos
h(x)^12 - 490314*cosh(x)^10 + 479655*cosh(x)^8 - 213180*cosh(x)^6 + 42075*c
osh(x)^4 - 2970*cosh(x)^2 + 33)*e^(2*x)*sinh(x)^14 + 8*(1300075*cosh(x)^13
- 4056234*cosh(x)^11 + 4849845*cosh(x)^9 - 2771340*cosh(x)^7 + 765765*cosh(
x)^5 - 90090*cosh(x)^3 + 3003*cosh(x))*e^(2*x)*sinh(x)^13 + 52*(185725*cosh
```

```
(x)^14 - 676039*cosh(x)^12 + 969969*cosh(x)^10 - 692835*cosh(x)^8 + 255255*
cosh(x)^6 - 45045*cosh(x)^4 + 3003*cosh(x)^2 - 33)*e^(2*x)*sinh(x)^12 + 208
*(37145*cosh(x)^15 - 156009*cosh(x)^13 + 264537*cosh(x)^11 - 230945*cosh(x)
^9 + 109395*cosh(x)^7 - 27027*cosh(x)^5 + 3003*cosh(x)^3 - 99*cosh(x))*e^(2
*x)*sinh(x)^11 + 143*(37145*cosh(x)^16 - 178296*cosh(x)^14 + 352716*cosh(x)
^12 - 369512*cosh(x)^10 + 218790*cosh(x)^8 - 72072*cosh(x)^6 + 12012*cosh(x
)^4 - 792*cosh(x)^2 + 9)*e^(2*x)*sinh(x)^10 + 286*(10925*cosh(x)^17 - 59432
*cosh(x)^15 + 135660*cosh(x)^13 - 167960*cosh(x)^11 + 121550*cosh(x)^9 - 51
480*cosh(x)^7 + 12012*cosh(x)^5 - 1320*cosh(x)^...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\operatorname{csch}^4(x)\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)**4)**(7/2),x)
```

```
[Out] Integral((a*csch(x)**4)**(7/2), x)
```

Giac [A]
time = 0.39, size = 51, normalized size = 0.31

$$-\frac{2048\,a^{\frac{7}{2}}\left(1716\,e^{(12\,x)}-1287\,e^{(10\,x)}+715\,e^{(8\,x)}-286\,e^{(6\,x)}+78\,e^{(4\,x)}-13\,e^{(2\,x)}+1\right)}{3003\,\left(e^{(2\,x)}-1\right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^4)^(7/2),x, algorithm="giac")
```

```
[Out] -2048/3003*a^(7/2)*(1716*e^(12*x) - 1287*e^(10*x) + 715*e^(8*x) - 286*e^(6*
x) + 78*e^(4*x) - 13*e^(2*x) + 1)/(e^(2*x) - 1)^13
```

Mupad [B]
time = 1.48, size = 498, normalized size = 3.04

$$-\frac{2048\,a^3\,\sqrt{\frac{a}{{\left(\frac{{\mathrm{e}}^{-x}}{2}-\frac{{\mathrm{e}}^x}{2}\right)}^4}}\,\left(6\,{\mathrm{e}}^{4\,x}-4\,{\mathrm{e}}^{2\,x}-4\,{\mathrm{e}}^{6\,x}+{\mathrm{e}}^{8\,x}+1\right)}{7\,{\left({\mathrm{e}}^{2\,x}-1\right)}^7\,\left({\mathrm{e}}^{2\,x}-2\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{6\,x}\right)}-\frac{1536\,a^3\,\sqrt{\frac{a}{{\left(\frac{{\mathrm{e}}^{-x}}{2}-\frac{{\mathrm{e}}^x}{2}\right)}^4}}\,\left(6\,{\mathrm{e}}^{4\,x}-4\,{\mathrm{e}}^{2\,x}-4\,{\mathrm{e}}^{6\,x}+{\mathrm{e}}^{8\,x}+1\right)}{{\left({\mathrm{e}}^{2\,x}-1\right)}^8\,\left({\mathrm{e}}^{2\,x}-2\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{6\,x}\right)}-\frac{10240\,a^3\,\sqrt{\frac{a}{{\left(\frac{{\mathrm{e}}^{-x}}{2}-\frac{{\mathrm{e}}^x}{2}\right)}^4}}\,\left(6\,{\mathrm{e}}^{4\,x}-4\,{\mathrm{e}}^{2\,x}-4\,{\mathrm{e}}^{6\,x}+{\mathrm{e}}^{8\,x}+1\right)}{3\,{\left({\mathrm{e}}^{2\,x}-1\right)}^9\,\left({\mathrm{e}}^{2\,x}-2\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{6\,x}\right)}-\frac{4096\,a^3\,\sqrt{\frac{a}{{\left(\frac{{\mathrm{e}}^{-x}}{2}-\frac{{\mathrm{e}}^x}{2}\right)}^4}}\,\left(6\,{\mathrm{e}}^{4\,x}-4\,{\mathrm{e}}^{2\,x}-4\,{\mathrm{e}}^{6\,x}+{\mathrm{e}}^{8\,x}+1\right)}{{\left({\mathrm{e}}^{2\,x}-1\right)}^{10}\,\left({\mathrm{e}}^{2\,x}-2\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{6\,x}\right)}-\frac{30720\,a^3\,\sqrt{\frac{a}{{\left(\frac{{\mathrm{e}}^{-x}}{2}-\frac{{\mathrm{e}}^x}{2}\right)}^4}}\,\left(6\,{\mathrm{e}}^{4\,x}-4\,{\mathrm{e}}^{2\,x}-4\,{\mathrm{e}}^{6\,x}+{\mathrm{e}}^{8\,x}+1\right)}{11\,{\left({\mathrm{e}}^{2\,x}-1\right)}^{11}\,\left({\mathrm{e}}^{2\,x}-2\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{6\,x}\right)}-\frac{1024\,a^3\,\sqrt{\frac{a}{{\left(\frac{{\mathrm{e}}^{-x}}{2}-\frac{{\mathrm{e}}^x}{2}\right)}^4}}\,\left(6\,{\mathrm{e}}^{4\,x}-4\,{\mathrm{e}}^{2\,x}-4\,{\mathrm{e}}^{6\,x}+{\mathrm{e}}^{8\,x}+1\right)}{{\left({\mathrm{e}}^{2\,x}-1\right)}^{12}\,\left({\mathrm{e}}^{2\,x}-2\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{6\,x}\right)}-\frac{2048\,a^3\,\sqrt{\frac{a}{{\left(\frac{{\mathrm{e}}^{-x}}{2}-\frac{{\mathrm{e}}^x}{2}\right)}^4}}\,\left(6\,{\mathrm{e}}^{4\,x}-4\,{\mathrm{e}}^{2\,x}-4\,{\mathrm{e}}^{6\,x}+{\mathrm{e}}^{8\,x}+1\right)}{13\,{\left({\mathrm{e}}^{2\,x}-1\right)}^{13}\,\left({\mathrm{e}}^{2\,x}-2\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{6\,x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/sinh(x)^4)^(7/2),x)
```

```
[Out] - (2048*a^3*(a/(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(6*exp(4*x) - 4*exp(2*x) - 4
*exp(6*x) + exp(8*x) + 1))/(7*(exp(2*x) - 1)^7*(exp(2*x) - 2*exp(4*x) + exp
(6*x))) - (1536*a^3*(a/(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(6*exp(4*x) - 4*exp(
2*x) - 4*exp(6*x) + exp(8*x) + 1))/((exp(2*x) - 1)^8*(exp(2*x) - 2*exp(4*x)
 + exp(6*x))) - (10240*a^3*(a/(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(6*exp(4*x) -
```

```
 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1))/(3*(exp(2*x) - 1)^9*(exp(2*x) - 2
*exp(4*x) + exp(6*x))) - (4096*a^3*(a/(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(6*ex
p(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1))/((exp(2*x) - 1)^10*(exp(2
*x) - 2*exp(4*x) + exp(6*x))) - (30720*a^3*(a/(exp(-x)/2 - exp(x)/2)^4)^(1/
2)*(6*exp(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1))/(11*(exp(2*x) - 1
)^11*(exp(2*x) - 2*exp(4*x) + exp(6*x))) - (1024*a^3*(a/(exp(-x)/2 - exp(x)
/2)^4)^(1/2)*(6*exp(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1))/((exp(2
*x) - 1)^12*(exp(2*x) - 2*exp(4*x) + exp(6*x))) - (2048*a^3*(a/(exp(-x)/2 -
 exp(x)/2)^4)^(1/2)*(6*exp(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1))/
(13*(exp(2*x) - 1)^13*(exp(2*x) - 2*exp(4*x) + exp(6*x)))
```

3.43 $\int \left(a\text{csch}^4(x)\right)^{5/2}\,dx$

Optimal. Leaf size=118

$$\frac{4}{3}a^2\cosh^2(x)\coth(x)\sqrt{a\text{csch}^4(x)}-\frac{6}{5}a^2\cosh^2(x)\coth^3(x)\sqrt{a\text{csch}^4(x)}+\frac{4}{7}a^2\cosh^2(x)\coth^5(x)\sqrt{a\text{csch}^4(x)}$$

```
[Out] 4/3*a^2*cosh(x)^2*coth(x)*(a*csch(x)^4)^(1/2)-6/5*a^2*cosh(x)^2*coth(x)^3*(
a*csch(x)^4)^(1/2)+4/7*a^2*cosh(x)^2*coth(x)^5*(a*csch(x)^4)^(1/2)-1/9*a^2*
cosh(x)^2*coth(x)^7*(a*csch(x)^4)^(1/2)-a^2*cosh(x)*sinh(x)*(a*csch(x)^4)^(
1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4208, 3852}

$$-\frac{1}{9}a^2\cosh^2(x)\coth^7(x)\sqrt{a\text{csch}^4(x)}+\frac{4}{7}a^2\cosh^2(x)\coth^5(x)\sqrt{a\text{csch}^4(x)}-\frac{6}{5}a^2\cosh^2(x)\coth^3(x)\sqrt{a\text{csch}^4(x)}+\frac{4}{3}a^2\cosh^2(x)\coth(x)\sqrt{a\text{csch}^4(x)}-a^2\sinh(x)\cosh(x)\sqrt{a\text{csch}^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Csch[x]^4)^(5/2),x]
```

```
[Out] (4*a^2*Cosh[x]^2*Coth[x]*Sqrt[a*Csch[x]^4])/3 - (6*a^2*Cosh[x]^2*Coth[x]^3*
Sqrt[a*Csch[x]^4])/5 + (4*a^2*Cosh[x]^2*Coth[x]^5*Sqrt[a*Csch[x]^4])/7 - (a
^2*Cosh[x]^2*Coth[x]^7*Sqrt[a*Csch[x]^4])/9 - a^2*Cosh[x]*Sqrt[a*Csch[x]^4]
*Sinh[x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\int \left(a\text{csch}^4(x)\right)^{5/2}\,dx = \left(a^2\sqrt{a\text{csch}^4(x)}\,\sinh^2(x)\right)\int \text{csch}^{10}(x)\,dx$$
$$= -\left(\left(ia^2\sqrt{a\text{csch}^4(x)}\,\sinh^2(x)\right)\text{Subst}\left(\int \left(1+4x^2+6x^4+4x^6+x^8\right)\,dx, x, -i\coth$$
$$= \frac{4}{3}a^2\cosh^2(x)\coth(x)\sqrt{a\text{csch}^4(x)} - \frac{6}{5}a^2\cosh^2(x)\coth^3(x)\sqrt{a\text{csch}^4(x)} + \frac{4}{7}a^2\cosh^2($$

Mathematica [A]
time = 0.02, size = 47, normalized size = 0.40

$$-\frac{1}{315}a^2\cosh(x)\sqrt{a\text{csch}^4(x)}\left(128-64\text{csch}^2(x)+48\text{csch}^4(x)-40\text{csch}^6(x)+35\text{csch}^8(x)\right)\sinh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Csch[x]^4)^(5/2),x]
```

```
[Out] -1/315*(a^2*Cosh[x]*Sqrt[a*Csch[x]^4]*(128 - 64*Csch[x]^2 + 48*Csch[x]^4 -
40*Csch[x]^6 + 35*Csch[x]^8)*Sinh[x])
```

Maple [A]
time = 0.89, size = 60, normalized size = 0.51

method	result	size
risch	$-\frac{256a^2\mathrm{e}^{-2x}\sqrt{\frac{a\,\mathrm{e}^{4x}}{(\mathrm{e}^{2x}-1)^4}}\,\left(126\,\mathrm{e}^{8x}-84\,\mathrm{e}^{6x}+36\,\mathrm{e}^{4x}-9\,\mathrm{e}^{2x}+1\right)}{315(\mathrm{e}^{2x}-1)^7}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*csch(x)^4)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -256/315*a^2*exp(-2*x)/(exp(2*x)-1)^7*(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)*(12
6*exp(8*x)-84*exp(6*x)+36*exp(4*x)-9*exp(2*x)+1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 322 vs. 2(100) = 200.
time = 0.49, size = 322, normalized size = 2.73

$$-\frac{256\,a^{\frac{5}{2}}e^{(-2x)}}{35\left(9e^{(-2x)}-36e^{(-4x)}+84e^{(-6x)}-126e^{(-8x)}+126e^{(-10x)}-84e^{(-12x)}+36e^{(-14x)}-9e^{(-16x)}+e^{(-18x)}-1\right)}+\frac{1024\,a^{\frac{5}{2}}e^{(-4x)}}{35\left(9e^{(-2x)}-36e^{(-4x)}+84e^{(-6x)}-126e^{(-8x)}+126e^{(-10x)}-84e^{(-12x)}+36e^{(-14x)}-9e^{(-16x)}+e^{(-18x)}-1\right)}-\frac{1024\,a^{\frac{5}{2}}e^{(-6x)}}{15\left(9e^{(-2x)}-36e^{(-4x)}+84e^{(-6x)}-126e^{(-8x)}+126e^{(-10x)}-84e^{(-12x)}+36e^{(-14x)}-9e^{(-16x)}+e^{(-18x)}-1\right)}+\frac{512\,a^{\frac{5}{2}}e^{(-8x)}}{5\left(9e^{(-2x)}-36e^{(-4x)}+84e^{(-6x)}-126e^{(-8x)}+126e^{(-10x)}-84e^{(-12x)}+36e^{(-14x)}-9e^{(-16x)}+e^{(-18x)}-1\right)}+\frac{256\,a^{\frac{5}{2}}}{315\left(9e^{(-2x)}-36e^{(-4x)}+84e^{(-6x)}-126e^{(-8x)}+126e^{(-10x)}-84e^{(-12x)}+36e^{(-14x)}-9e^{(-16x)}+e^{(-18x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] -256/35*a^(5/2)*e^(-2*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) - 126*e^(-
8*x) + 126*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) + e^(-18*x
) - 1) + 1024/35*a^(5/2)*e^(-4*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e^(-6*x) -
 126*e^(-8*x) + 126*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^(-16*x) +
 e^(-18*x) - 1) - 1024/15*a^(5/2)*e^(-6*x)/(9*e^(-2*x) - 36*e^(-4*x) + 84*e
^(-6*x) - 126*e^(-8*x) + 126*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x) - 9*e^
(-16*x) + e^(-18*x) - 1) + 512/5*a^(5/2)*e^(-8*x)/(9*e^(-2*x) - 36*e^(-4*x)
 + 84*e^(-6*x) - 126*e^(-8*x) + 126*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x)
 - 9*e^(-16*x) + e^(-18*x) - 1) + 256/315*a^(5/2)/(9*e^(-2*x) - 36*e^(-4*x)
 + 84*e^(-6*x) - 126*e^(-8*x) + 126*e^(-10*x) - 84*e^(-12*x) + 36*e^(-14*x)
 - 9*e^(-16*x) + e^(-18*x) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1493 vs. $2(100) = 200$.
time = 0.53, size = 1493, normalized size = 12.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] -256/315*(126*a^2*cosh(x)^8 + 126*(a^2*e^(4*x) - 2*a^2*e^(2*x) + a^2)*sinh(
x)^8 - 84*a^2*cosh(x)^6 + 1008*(a^2*cosh(x)*e^(4*x) - 2*a^2*cosh(x)*e^(2*x)
 + a^2*cosh(x))*sinh(x)^7 + 84*(42*a^2*cosh(x)^2 - a^2 + (42*a^2*cosh(x)^2
- a^2)*e^(4*x) - 2*(42*a^2*cosh(x)^2 - a^2)*e^(2*x))*sinh(x)^6 + 36*a^2*cos
h(x)^4 + 504*(14*a^2*cosh(x)^3 - a^2*cosh(x) + (14*a^2*cosh(x)^3 - a^2*cosh
(x))*e^(4*x) - 2*(14*a^2*cosh(x)^3 - a^2*cosh(x))*e^(2*x))*sinh(x)^5 + 36*(
245*a^2*cosh(x)^4 - 35*a^2*cosh(x)^2 + a^2 + (245*a^2*cosh(x)^4 - 35*a^2*co
sh(x)^2 + a^2)*e^(4*x) - 2*(245*a^2*cosh(x)^4 - 35*a^2*cosh(x)^2 + a^2)*e^(
2*x))*sinh(x)^4 - 9*a^2*cosh(x)^2 + 48*(147*a^2*cosh(x)^5 - 35*a^2*cosh(x)^
3 + 3*a^2*cosh(x) + (147*a^2*cosh(x)^5 - 35*a^2*cosh(x)^3 + 3*a^2*cosh(x))*
e^(4*x) - 2*(147*a^2*cosh(x)^5 - 35*a^2*cosh(x)^3 + 3*a^2*cosh(x))*e^(2*x))
*sinh(x)^3 + 9*(392*a^2*cosh(x)^6 - 140*a^2*cosh(x)^4 + 24*a^2*cosh(x)^2 -
a^2 + (392*a^2*cosh(x)^6 - 140*a^2*cosh(x)^4 + 24*a^2*cosh(x)^2 - a^2)*e^(4
*x) - 2*(392*a^2*cosh(x)^6 - 140*a^2*cosh(x)^4 + 24*a^2*cosh(x)^2 - a^2)*e^
(2*x))*sinh(x)^2 + a^2 + (126*a^2*cosh(x)^8 - 84*a^2*cosh(x)^6 + 36*a^2*cos
h(x)^4 - 9*a^2*cosh(x)^2 + a^2)*e^(4*x) - 2*(126*a^2*cosh(x)^8 - 84*a^2*cos
h(x)^6 + 36*a^2*cosh(x)^4 - 9*a^2*cosh(x)^2 + a^2)*e^(2*x) + 18*(56*a^2*cos
h(x)^7 - 28*a^2*cosh(x)^5 + 8*a^2*cosh(x)^3 - a^2*cosh(x) + (56*a^2*cosh(x)
^7 - 28*a^2*cosh(x)^5 + 8*a^2*cosh(x)^3 - a^2*cosh(x))*e^(4*x) - 2*(56*a^2*
cosh(x)^7 - 28*a^2*cosh(x)^5 + 8*a^2*cosh(x)^3 - a^2*cosh(x))*e^(2*x))*sinh
(x))*sqrt(a/(e^(8*x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*x) + 1))*e^(2*x)/(18*
cosh(x)*e^(2*x)*sinh(x)^17 + e^(2*x)*sinh(x)^18 + 9*(17*cosh(x)^2 - 1)*e^(2
*x)*sinh(x)^16 + 48*(17*cosh(x)^3 - 3*cosh(x))*e^(2*x)*sinh(x)^15 + 36*(85*
cosh(x)^4 - 30*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^14 + 504*(17*cosh(x)^5 - 10*c
```

```
osh(x)^3 + cosh(x))*e^(2*x)*sinh(x)^13 + 84*(221*cosh(x)^6 - 195*cosh(x)^4
+ 39*cosh(x)^2 - 1)*e^(2*x)*sinh(x)^12 + 144*(221*cosh(x)^7 - 273*cosh(x)^5
 + 91*cosh(x)^3 - 7*cosh(x))*e^(2*x)*sinh(x)^11 + 18*(2431*cosh(x)^8 - 4004
*cosh(x)^6 + 2002*cosh(x)^4 - 308*cosh(x)^2 + 7)*e^(2*x)*sinh(x)^10 + 4*(12
155*cosh(x)^9 - 25740*cosh(x)^7 + 18018*cosh(x)^5 - 4620*cosh(x)^3 + 315*co
sh(x))*e^(2*x)*sinh(x)^9 + 18*(2431*cosh(x)^10 - 6435*cosh(x)^8 + 6006*cosh
(x)^6 - 2310*cosh(x)^4 + 315*cosh(x)^2 - 7)*e^(2*x)*sinh(x)^8 + 144*(221*co
sh(x)^11 - 715*cosh(x)^9 + 858*cosh(x)^7 - 462*cosh(x)^5 + 105*cosh(x)^3 -
7*cosh(x))*e^(2*x)*sinh(x)^7 + 84*(221*cosh(x)^12 - 858*cosh(x)^10 + 1287*c
osh(x)^8 - 924*cosh(x)^6 + 315*cosh(x)^4 - 42*cosh(x)^2 + 1)*e^(2*x)*sinh(x
)^6 + 504*(17*cosh(x)^13 - 78*cosh(x)^11 + 143*cosh(x)^9 - 132*cosh(x)^7 +
63*cosh(x)^5 - 14*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x)^5 + 36*(85*cosh(x)^1
4 - 455*cosh(x)^12 + 1001*cosh(x)^10 - 1155*cosh(x)^8 + 735*cosh(x)^6 - 245
*cosh(x)^4 + 35*cosh(x)^2 - 1)*e^(2*x)*sinh(x)^4 + 48*(17*cosh(x)^15 - 105*
cosh(x)^13 + 273*cosh(x)^11 - 385*cosh(x)^9 + 315*cosh(x)^7 - 147*cosh(x)^5
 + 35*cosh(x)^3 - 3*cosh(x))*e^(2*x)*sinh(x)^3 + 9*(17*cosh(x)^16 - 120*cos
h(x)^14 + 364*cosh(x)^12 - 616*cosh(x)^10 + 630*cosh(x)^8 - 392*cosh(x)^6 +
 140*cosh(x)^4 - 24*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^2 + 18*(cosh(x)^17 - 8*c
osh(x)^15 + 28*cosh(x)^13 - 56*cosh(x)^11 + 70*cosh(x)^9 - 56*cosh(x)^7 + 2
8*cosh(x)^5 - 8*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x) + (cosh(x)^18 - 9*cosh
(x)^16 + 36*cosh(x)^14 - 84*cosh(x)^12 + 126*cosh(x)^10 - 126*cosh(x)^8 + 8
4*cosh(x)^6 - 36*cosh(x)^4 + 9*cosh(x)^2 - 1)*e^(2*x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\operatorname{csch}^4(x)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)**4)**(5/2),x)
```

```
[Out] Integral((a*csch(x)**4)**(5/2), x)
```

Giac [A]
time = 0.39, size = 39, normalized size = 0.33

$$-\frac{256\,a^{\frac{5}{2}}\left(126\,e^{(8\,x)}-84\,e^{(6\,x)}+36\,e^{(4\,x)}-9\,e^{(2\,x)}+1\right)}{315\,\left(e^{(2\,x)}-1\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] -256/315*a^(5/2)*(126*e^(8*x) - 84*e^(6*x) + 36*e^(4*x) - 9*e^(2*x) + 1)/(e
^(2*x) - 1)^9
```

Mupad [B]
time = 1.44, size = 356, normalized size = 3.02

$$-\frac{128\,a^2\,\sqrt{\frac{a}{{\left(\frac{{\mathrm{e}}^{-x}}{2}-\frac{{\mathrm{e}}^x}{2}\right)}^4}}\,\left(6\,{\mathrm{e}}^{4\,x}-4\,{\mathrm{e}}^{2\,x}-4\,{\mathrm{e}}^{6\,x}+{\mathrm{e}}^{8\,x}+1\right)}{5\,{\left({\mathrm{e}}^{2\,x}-1\right)}^5\,\left({\mathrm{e}}^{2\,x}-2\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{6\,x}\right)}-\frac{256\,a^2\,\sqrt{\frac{a}{{\left(\frac{{\mathrm{e}}^{-x}}{2}-\frac{{\mathrm{e}}^x}{2}\right)}^4}}\,\left(6\,{\mathrm{e}}^{4\,x}-4\,{\mathrm{e}}^{2\,x}-4\,{\mathrm{e}}^{6\,x}+{\mathrm{e}}^{8\,x}+1\right)}{3\,{\left({\mathrm{e}}^{2\,x}-1\right)}^6\,\left({\mathrm{e}}^{2\,x}-2\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{6\,x}\right)}-\frac{768\,a^2\,\sqrt{\frac{a}{{\left(\frac{{\mathrm{e}}^{-x}}{2}-\frac{{\mathrm{e}}^x}{2}\right)}^4}}\,\left(6\,{\mathrm{e}}^{4\,x}-4\,{\mathrm{e}}^{2\,x}-4\,{\mathrm{e}}^{6\,x}+{\mathrm{e}}^{8\,x}+1\right)}{7\,{\left({\mathrm{e}}^{2\,x}-1\right)}^7\,\left({\mathrm{e}}^{2\,x}-2\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{6\,x}\right)}-\frac{64\,a^2\,\sqrt{\frac{a}{{\left(\frac{{\mathrm{e}}^{-x}}{2}-\frac{{\mathrm{e}}^x}{2}\right)}^4}}\,\left(6\,{\mathrm{e}}^{4\,x}-4\,{\mathrm{e}}^{2\,x}-4\,{\mathrm{e}}^{6\,x}+{\mathrm{e}}^{8\,x}+1\right)}{{\left({\mathrm{e}}^{2\,x}-1\right)}^8\,\left({\mathrm{e}}^{2\,x}-2\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{6\,x}\right)}-\frac{128\,a^2\,\sqrt{\frac{a}{{\left(\frac{{\mathrm{e}}^{-x}}{2}-\frac{{\mathrm{e}}^x}{2}\right)}^4}}\,\left(6\,{\mathrm{e}}^{4\,x}-4\,{\mathrm{e}}^{2\,x}-4\,{\mathrm{e}}^{6\,x}+{\mathrm{e}}^{8\,x}+1\right)}{9\,{\left({\mathrm{e}}^{2\,x}-1\right)}^9\,\left({\mathrm{e}}^{2\,x}-2\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{6\,x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/sinh(x)^4)^(5/2),x)
```

```
[Out] - (128*a^2*(a/(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(6*exp(4*x) - 4*exp(2*x) - 4*
exp(6*x) + exp(8*x) + 1))/(5*(exp(2*x) - 1)^5*(exp(2*x) - 2*exp(4*x) + exp(
6*x))) - (256*a^2*(a/(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(6*exp(4*x) - 4*exp(2*
x) - 4*exp(6*x) + exp(8*x) + 1))/(3*(exp(2*x) - 1)^6*(exp(2*x) - 2*exp(4*x)
 + exp(6*x))) - (768*a^2*(a/(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(6*exp(4*x) - 4
*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1))/(7*(exp(2*x) - 1)^7*(exp(2*x) - 2*e
xp(4*x) + exp(6*x))) - (64*a^2*(a/(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(6*exp(4*
x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1))/((exp(2*x) - 1)^8*(exp(2*x) -
 2*exp(4*x) + exp(6*x))) - (128*a^2*(a/(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(6*e
xp(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1))/(9*(exp(2*x) - 1)^9*(exp
(2*x) - 2*exp(4*x) + exp(6*x)))
```

3.44 $\int \left(a\text{csch}^4(x)\right)^{3/2} dx$

Optimal. Leaf size=62

$$\frac{2}{3}a\cosh^2(x)\coth(x)\sqrt{a\text{csch}^4(x)}-\frac{1}{5}a\cosh^2(x)\coth^3(x)\sqrt{a\text{csch}^4(x)}-a\cosh(x)\sqrt{a\text{csch}^4(x)}\,\sinh(x)$$

```
[Out] 2/3*a*cosh(x)^2*coth(x)*(a*csch(x)^4)^(1/2)-1/5*a*cosh(x)^2*coth(x)^3*(a*cs
ch(x)^4)^(1/2)-a*cosh(x)*sinh(x)*(a*csch(x)^4)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4208, 3852}

$$-\frac{1}{5}a\cosh^2(x)\coth^3(x)\sqrt{a\text{csch}^4(x)}+\frac{2}{3}a\cosh^2(x)\coth(x)\sqrt{a\text{csch}^4(x)}-a\sinh(x)\cosh(x)\sqrt{a\text{csch}^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Csch[x]^4)^(3/2),x]
```

```
[Out] (2*a*Cosh[x]^2*Coth[x]*Sqrt[a*Csch[x]^4])/3 - (a*Cosh[x]^2*Coth[x]^3*Sqrt[a
*Csch[x]^4])/5 - a*Cosh[x]*Sqrt[a*Csch[x]^4]*Sinh[x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(a\text{csch}^4(x)\right)^{3/2} dx &= \left(a\sqrt{a\text{csch}^4(x)}\,\sinh^2(x)\right)\int \text{csch}^6(x)\,dx\\ &= -\left(\left(ia\sqrt{a\text{csch}^4(x)}\,\sinh^2(x)\right)\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx,x,-i\coth(x)\right)\right)\\ &= \frac{2}{3}a\cosh^2(x)\coth(x)\sqrt{a\text{csch}^4(x)}-\frac{1}{5}a\cosh^2(x)\coth^3(x)\sqrt{a\text{csch}^4(x)}-a\cosh(x)\sqrt{a}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 0.53

$$-\frac{1}{15}a\cosh(x)\sqrt{a\text{csch}^4(x)}\,\left(8-4\text{csch}^2(x)+3\text{csch}^4(x)\right)\sinh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Csch[x]^4)^(3/2),x]
```

```
[Out] -1/15*(a*Cosh[x]*Sqrt[a*Csch[x]^4]*(8 - 4*Csch[x]^2 + 3*Csch[x]^4)*Sinh[x])
```

Maple [A]
time = 0.88, size = 46, normalized size = 0.74

method	result	size
risch	$-\frac{16a\,\mathrm{e}^{-2x}\sqrt{\frac{a\,\mathrm{e}^{4x}}{(\mathrm{e}^{2x}-1)^4}}\,\left(10\,\mathrm{e}^{4x}-5\,\mathrm{e}^{2x}+1\right)}{15(\mathrm{e}^{2x}-1)^3}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*csch(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -16/15*a*exp(-2*x)/(exp(2*x)-1)^3*(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)*(10*exp
(4*x)-5*exp(2*x)+1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(52) = 104$.
time = 0.48, size = 120, normalized size = 1.94

$$-\frac{16\,a^{\frac{3}{2}}e^{(-2\,x)}}{3\,(5\,e^{(-2\,x)}-10\,e^{(-4\,x)}+10\,e^{(-6\,x)}-5\,e^{(-8\,x)}+e^{(-10\,x)}-1)}+\frac{32\,a^{\frac{3}{2}}e^{(-4\,x)}}{3\,(5\,e^{(-2\,x)}-10\,e^{(-4\,x)}+10\,e^{(-6\,x)}-5\,e^{(-8\,x)}+e^{(-10\,x)}-1)}+\frac{16\,a^{\frac{3}{2}}}{15\,(5\,e^{(-2\,x)}-10\,e^{(-4\,x)}+10\,e^{(-6\,x)}-5\,e^{(-8\,x)}+e^{(-10\,x)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] -16/3*a^(3/2)*e^(-2*x)/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^(-6*x) - 5*e^(-8*x)
 + e^(-10*x) - 1) + 32/3*a^(3/2)*e^(-4*x)/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^
(-6*x) - 5*e^(-8*x) + e^(-10*x) - 1) + 16/15*a^(3/2)/(5*e^(-2*x) - 10*e^(-4
*x) + 10*e^(-6*x) - 5*e^(-8*x) + e^(-10*x) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(52) = 104$.
time = 0.42, size = 529, normalized size = 8.53

$$-\frac{16\,\left(10\,a\cosh\left(x\right)^{4}+10\,\left(ae^{(4\,x)}-2\,ae^{(2\,x)}+a\right)\sinh\left(x\right)^{4}+40\,\left(a\cosh\left(x\right)e^{(4\,x)}-2\,a\cosh\left(x\right)e^{(2\,x)}+a\cosh\left(x\right)\right)\sinh\left(x\right)^{3}-5\,a\cosh\left(x\right)^{2}+5\,\left(12\,a\cosh\left(x\right)^{2}+\left(12\,a\cosh\left(x\right)^{2}-a\right)e^{(4\,x)}-2\,\left(12\,a\cosh\left(x\right)^{2}-a\right)e^{(2\,x)}-a\right)\sinh\left(x\right)^{2}+\left(10\,a\cosh\left(x\right)^{4}-5\,a\cosh\left(x\right)^{2}+a\right)e^{(4\,x)}-2\,\left(10\,a\cosh\left(x\right)^{4}-5\,a\cosh\left(x\right)^{2}+a\right)e^{(2\,x)}+10\,\left(4\,a\cosh\left(x\right)^{3}-a\cosh\left(x\right)+\left(4\,a\cosh\left(x\right)^{3}-a\cosh\left(x\right)\right)e^{(4\,x)}-2\,\left(4\,a\cosh\left(x\right)^{3}-a\cosh\left(x\right)\right)e^{(2\,x)}\right)\sinh\left(x\right)+a\right)\sqrt{\frac{a}{e^{(8\,x)}-4\,e^{(6\,x)}+6\,e^{(4\,x)}-4\,e^{(2\,x)}+1}}\,e^{(2\,x)}}{15\,\left(10\,\cosh\left(x\right)e^{(2\,x)}\sinh\left(x\right)^{9}+e^{(2\,x)}\sinh\left(x\right)^{10}+5\,\left(9\,\cosh\left(x\right)^{2}-1\right)e^{(2\,x)}\sinh\left(x\right)^{8}+40\,\left(3\,\cosh\left(x\right)^{3}-\cosh\left(x\right)\right)e^{(2\,x)}\sinh\left(x\right)^{7}+10\,\left(21\,\cosh\left(x\right)^{4}-14\,\cosh\left(x\right)^{2}+1\right)e^{(2\,x)}\sinh\left(x\right)^{6}+4\,\left(63\,\cosh\left(x\right)^{5}-70\,\cosh\left(x\right)^{3}+15\,\cosh\left(x\right)\right)e^{(2\,x)}\sinh\left(x\right)^{5}+10\,\left(21\,\cosh\left(x\right)^{6}-35\,\cosh\left(x\right)^{4}+15\,\cosh\left(x\right)^{2}-1\right)e^{(2\,x)}\sinh\left(x\right)^{4}+40\,\left(3\,\cosh\left(x\right)^{7}-7\,\cosh\left(x\right)^{5}+5\,\cosh\left(x\right)^{3}-\cosh\left(x\right)\right)e^{(2\,x)}\sinh\left(x\right)^{3}+5\,\left(9\,\cosh\left(x\right)^{8}-28\,\cosh\left(x\right)^{6}+30\,\cosh\left(x\right)^{4}-12\,\cosh\left(x\right)^{2}+1\right)e^{(2\,x)}\sinh\left(x\right)^{2}+10\,\left(\cosh\left(x\right)^{9}-4\,\cosh\left(x\right)^{7}+6\,\cosh\left(x\right)^{5}-4\,\cosh\left(x\right)^{3}+\cosh\left(x\right)\right)e^{(2\,x)}\sinh\left(x\right)+\left(\cosh\left(x\right)^{10}-5\,\cosh\left(x\right)^{8}+10\,\cosh\left(x\right)^{6}-10\,\cosh\left(x\right)^{4}+5\,\cosh\left(x\right)^{2}-1\right)e^{(2\,x)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] -16/15*(10*a*cosh(x)^4 + 10*(a*e^(4*x) - 2*a*e^(2*x) + a)*sinh(x)^4 + 40*(a
*cosh(x)*e^(4*x) - 2*a*cosh(x)*e^(2*x) + a*cosh(x))*sinh(x)^3 - 5*a*cosh(x)
^2 + 5*(12*a*cosh(x)^2 + (12*a*cosh(x)^2 - a)*e^(4*x) - 2*(12*a*cosh(x)^2 -
 a)*e^(2*x) - a)*sinh(x)^2 + (10*a*cosh(x)^4 - 5*a*cosh(x)^2 + a)*e^(4*x) -
 2*(10*a*cosh(x)^4 - 5*a*cosh(x)^2 + a)*e^(2*x) + 10*(4*a*cosh(x)^3 - a*cos
h(x) + (4*a*cosh(x)^3 - a*cosh(x))*e^(4*x) - 2*(4*a*cosh(x)^3 - a*cosh(x))*
e^(2*x))*sinh(x) + a)*sqrt(a/(e^(8*x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*x) +
 1))*e^(2*x)/(10*cosh(x)*e^(2*x)*sinh(x)^9 + e^(2*x)*sinh(x)^10 + 5*(9*cosh
(x)^2 - 1)*e^(2*x)*sinh(x)^8 + 40*(3*cosh(x)^3 - cosh(x))*e^(2*x)*sinh(x)^7
 + 10*(21*cosh(x)^4 - 14*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^6 + 4*(63*cosh(x)^5
 - 70*cosh(x)^3 + 15*cosh(x))*e^(2*x)*sinh(x)^5 + 10*(21*cosh(x)^6 - 35*cos
h(x)^4 + 15*cosh(x)^2 - 1)*e^(2*x)*sinh(x)^4 + 40*(3*cosh(x)^7 - 7*cosh(x)^
5 + 5*cosh(x)^3 - cosh(x))*e^(2*x)*sinh(x)^3 + 5*(9*cosh(x)^8 - 28*cosh(x)^
6 + 30*cosh(x)^4 - 12*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^2 + 10*(cosh(x)^9 - 4*
cosh(x)^7 + 6*cosh(x)^5 - 4*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x) + (cosh(x)
^10 - 5*cosh(x)^8 + 10*cosh(x)^6 - 10*cosh(x)^4 + 5*cosh(x)^2 - 1)*e^(2*x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\operatorname{csch}^4(x)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)**4)**(3/2),x)
```

```
[Out] Integral((a*csch(x)**4)**(3/2), x)
```

Giac [A]
time = 0.40, size = 27, normalized size = 0.44

$$-\frac{16\, a^{\frac{3}{2}} \left(10\, e^{(4\, x)} - 5\, e^{(2\, x)} + 1\right)}{15\, \left(e^{(2\, x)} - 1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] -16/15*a^(3/2)*(10*e^(4*x) - 5*e^(2*x) + 1)/(e^(2*x) - 1)^5
```

Mupad [B]
time = 1.46, size = 46, normalized size = 0.74

$$-\frac{4\, a\, \mathrm{e}^{-2\,x}\, \sqrt{\frac{a}{{\left(\frac{\mathrm{e}^{-x}}{2}-\frac{\mathrm{e}^{x}}{2}\right)}^4}}\, \left(10\,\mathrm{e}^{4\,x}-5\,\mathrm{e}^{2\,x}+1\right)}{15\,{\left(\mathrm{e}^{2\,x}-1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/sinh(x)^4)^(3/2),x)
```

```
[Out] -(4*a*exp(-2*x)*(a/(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(10*exp(4*x) - 5*exp(2*x
) + 1))/(15*(exp(2*x) - 1)^3)
```

3.45 $\int \sqrt{a\operatorname{csch}^4(x)}\,dx$

Optimal. Leaf size=16

$$-\cosh(x)\sqrt{a\operatorname{csch}^4(x)}\,\sinh(x)$$

```
[Out] -cosh(x)*sinh(x)*(a*csch(x)^4)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4208, 3852, 8}

$$\sinh(x)(-\cosh(x))\sqrt{a\operatorname{csch}^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Csch[x]^4],x]
```

```
[Out] -(Cosh[x]*Sqrt[a*Csch[x]^4]*Sinh[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a\operatorname{csch}^4(x)}\,dx &= \left(\sqrt{a\operatorname{csch}^4(x)}\,\sinh^2(x)\right)\int \operatorname{csch}^2(x)\,dx\\
&= -\left(\left(i\sqrt{a\operatorname{csch}^4(x)}\,\sinh^2(x)\right)\operatorname{Subst}\left(\int 1\,dx, x, -i\coth(x)\right)\right)\\
&= -\cosh(x)\sqrt{a\operatorname{csch}^4(x)}\,\sinh(x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$-\cosh(x)\sqrt{a\operatorname{csch}^4(x)}\sinh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Csch[x]^4],x]
[Out] -(Cosh[x]*Sqrt[a*Csch[x]^4]*Sinh[x])
```

Maple [A]
time = 1.04, size = 29, normalized size = 1.81

method	result	size
risch	$-2\sqrt{\frac{a\,\mathrm{e}^{4x}}{(\mathrm{e}^{2x}-1)^4}}\,\mathrm{e}^{-2x}(\mathrm{e}^{2x}-1)$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*csch(x)^4)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)*exp(-2*x)*(exp(2*x)-1)
```

Maxima [A]
time = 0.47, size = 13, normalized size = 0.81

$$\frac{2\sqrt{a}}{e^{(-2x)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^4)^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(a)/(e^(-2*x) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(14) = 28.
time = 0.44, size = 81, normalized size = 5.06

$$-\frac{2\sqrt{\frac{a}{e^{(8x)}-4e^{(6x)}+6e^{(4x)}-4e^{(2x)}+1}}\left(e^{(4x)}-2e^{(2x)}+1\right)e^{(2x)}}{2\cosh(x)e^{(2x)}\sinh(x)+e^{(2x)}\sinh(x)^2+\left(\cosh(x)^2-1\right)e^{(2x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(a/(e^(8*x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*x) + 1))*(e^(4*x) - 2*e
^(2*x) + 1)*e^(2*x)/(2*cosh(x)*e^(2*x)*sinh(x) + e^(2*x)*sinh(x)^2 + (cosh(
x)^2 - 1)*e^(2*x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \operatorname{csch}^4(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)**4)**(1/2),x)
```

```
[Out] Integral(sqrt(a*csch(x)**4), x)
```

Giac [A]
time = 0.39, size = 13, normalized size = 0.81

$$-\frac{2\sqrt{a}}{e^{(2x)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csch(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(a)/(e^(2*x) - 1)
```

Mupad [B]
time = 1.46, size = 71, normalized size = 4.44

$$-\frac{\sqrt{a}\,\sqrt{\frac{1}{\left(\frac{e^{-x}}{2}-\frac{e^{x}}{2}\right)^4}}\,\left(3\,e^{4x}-2\,e^{2x}-2\,e^{6x}+\frac{e^{8x}}{2}+\frac{1}{2}\right)}{\left(e^{2x}-1\right)\,\left(e^{2x}-2\,e^{4x}+e^{6x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/sinh(x)^4)^(1/2),x)
```

```
[Out] -(a^(1/2)*(1/(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(3*exp(4*x) - 2*exp(2*x) - 2*e
xp(6*x) + exp(8*x)/2 + 1/2))/((exp(2*x) - 1)*(exp(2*x) - 2*exp(4*x) + exp(6
*x)))
```

3.46 $\int \frac{1}{\sqrt{a\operatorname{csch}^4(x)}}\,dx$

Optimal. Leaf size=36

$$\frac{\coth(x)}{2\sqrt{a\operatorname{csch}^4(x)}}-\frac{x\operatorname{csch}^2(x)}{2\sqrt{a\operatorname{csch}^4(x)}}$$

```
[Out] 1/2*coth(x)/(a*csch(x)^4)^(1/2)-1/2*x*csch(x)^2/(a*csch(x)^4)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4208, 2715, 8}

$$\frac{\coth(x)}{2\sqrt{a\operatorname{csch}^4(x)}}-\frac{x\operatorname{csch}^2(x)}{2\sqrt{a\operatorname{csch}^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Csch[x]^4],x]
```

```
[Out] Coth[x]/(2*Sqrt[a*Csch[x]^4]) - (x*Csch[x]^2)/(2*Sqrt[a*Csch[x]^4])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\sqrt{a\operatorname{csch}^4(x)}}\,dx = \frac{\operatorname{csch}^2(x)\int \sinh^2(x)\,dx}{\sqrt{a\operatorname{csch}^4(x)}}$$
$$= \frac{\coth(x)}{2\sqrt{a\operatorname{csch}^4(x)}} - \frac{\operatorname{csch}^2(x)\int 1\,dx}{2\sqrt{a\operatorname{csch}^4(x)}}$$
$$= \frac{\coth(x)}{2\sqrt{a\operatorname{csch}^4(x)}} - \frac{x\operatorname{csch}^2(x)}{2\sqrt{a\operatorname{csch}^4(x)}}$$

Mathematica [A]
time = 0.02, size = 24, normalized size = 0.67

$$\frac{\coth(x) - x\operatorname{csch}^2(x)}{2\sqrt{a\operatorname{csch}^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Csch[x]^4],x]
```

```
[Out] (Coth[x] - x*Csch[x]^2)/(2*Sqrt[a*Csch[x]^4])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(28) = 56$.
time = 1.11, size = 89, normalized size = 2.47

method	result	size
risch	$-\frac{e^{2x}x}{2\sqrt{\frac{a\,e^{4x}}{(e^{2x}-1)^4}}\,(e^{2x}-1)^2}+\frac{e^{4x}}{8\sqrt{\frac{a\,e^{4x}}{(e^{2x}-1)^4}}\,(e^{2x}-1)^2}-\frac{1}{8(e^{2x}-1)^2\sqrt{\frac{a\,e^{4x}}{(e^{2x}-1)^4}}}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*csch(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)/(exp(2*x)-1)^2*exp(2*x)*x+1/8/(a*exp
(4*x)/(exp(2*x)-1)^4)^(1/2)/(exp(2*x)-1)^2*exp(4*x)-1/8/(exp(2*x)-1)^2/(a*e
xp(4*x)/(exp(2*x)-1)^4)^(1/2)
```

Maxima [A]
time = 0.48, size = 22, normalized size = 0.61

$$-\frac{\left(e^{(-4\,x)} - 1\right)e^{(2\,x)}}{8\,\sqrt{a}} - \frac{x}{2\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*(e^(-4*x) - 1)*e^(2*x)/sqrt(a) - 1/2*x/sqrt(a)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(28) = 56$.
time = 0.56, size = 253, normalized size = 7.03

$$\frac{\left(\left(e^{(4x)} - 2e^{(2x)} + 1\right)\sinh(x)^4 + \cosh(x)^4 + 4\left(\cosh(x)e^{(4x)} - 2\cosh(x)e^{(2x)} + \cosh(x)\right)\sinh(x)^3 - 4x\cosh(x)^2 + 2\left(3\cosh(x)^2 + (3\cosh(x)^2 - 2x)e^{(4x)} - 2\left(3\cosh(x)^2 - 2x\right)e^{(2x)} - 2x\right)\sinh(x)^2 + \left(\cosh(x)^4 - 4x\cosh(x)^2 - 1\right)e^{(4x)} - 2\left(\cosh(x)^4 - 4x\cosh(x)^2 - 1\right)e^{(2x)} + 4\left(\cosh(x)^3 - 2x\cosh(x) + \left(\cosh(x)^3 - 2x\cosh(x)\right)e^{(4x)} - 2\left(\cosh(x)^3 - 2x\cosh(x)\right)e^{(2x)}\right)\sinh(x) - 1\right)\sqrt{\frac{a}{e^{(8x)} - 4e^{(6x)} + 6e^{(4x)} - 4e^{(2x)} + 1}}\, e^{(2x)}}{8\left(a\cosh(x)^2e^{(2x)} + 2a\cosh(x)e^{(2x)}\sinh(x) + ae^{(2x)}\sinh(x)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*((e^(4*x) - 2*e^(2*x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(4*x) -
 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^3 - 4*x*cosh(x)^2 + 2*(3*cosh(x)^2 +
(3*cosh(x)^2 - 2*x)*e^(4*x) - 2*(3*cosh(x)^2 - 2*x)*e^(2*x) - 2*x)*sinh(x)^
2 + (cosh(x)^4 - 4*x*cosh(x)^2 - 1)*e^(4*x) - 2*(cosh(x)^4 - 4*x*cosh(x)^2
- 1)*e^(2*x) + 4*(cosh(x)^3 - 2*x*cosh(x) + (cosh(x)^3 - 2*x*cosh(x))*e^(4*
x) - 2*(cosh(x)^3 - 2*x*cosh(x))*e^(2*x))*sinh(x) - 1)*sqrt(a/(e^(8*x) - 4*
e^(6*x) + 6*e^(4*x) - 4*e^(2*x) + 1))*e^(2*x)/(a*cosh(x)^2*e^(2*x) + 2*a*co
sh(x)*e^(2*x)*sinh(x) + a*e^(2*x)*sinh(x)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\operatorname{csch}^4(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)**4)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*csch(x)**4), x)
```

Giac [A]
time = 0.40, size = 26, normalized size = 0.72

$$\frac{\left(2e^{(2x)} - 1\right)e^{(-2x)} - 4x + e^{(2x)}}{8\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*((2*e^(2*x) - 1)*e^(-2*x) - 4*x + e^(2*x))/sqrt(a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{\frac{a}{\sinh\left(x\right)^4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/sinh(x)^4)^(1/2),x)
[Out] int(1/(a/sinh(x)^4)^(1/2), x)
```

3.47 $\int \frac{1}{\left(a\,\mathrm{csch}^4(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=86

$$\frac{5\coth(x)}{16a\sqrt{a\mathrm{csch}^4(x)}}-\frac{5x\mathrm{csch}^2(x)}{16a\sqrt{a\mathrm{csch}^4(x)}}-\frac{5\cosh(x)\sinh(x)}{24a\sqrt{a\mathrm{csch}^4(x)}}+\frac{\cosh(x)\sinh^3(x)}{6a\sqrt{a\mathrm{csch}^4(x)}}$$

```
[Out] 5/16*coth(x)/a/(a*csch(x)^4)^(1/2)-5/16*x*csch(x)^2/a/(a*csch(x)^4)^(1/2)-5
/24*cosh(x)*sinh(x)/a/(a*csch(x)^4)^(1/2)+1/6*cosh(x)*sinh(x)^3/a/(a*csch(x
)^4)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4208, 2715, 8}

$$-\frac{5x\mathrm{csch}^2(x)}{16a\sqrt{a\mathrm{csch}^4(x)}}+\frac{5\coth(x)}{16a\sqrt{a\mathrm{csch}^4(x)}}+\frac{\sinh^3(x)\cosh(x)}{6a\sqrt{a\mathrm{csch}^4(x)}}-\frac{5\sinh(x)\cosh(x)}{24a\sqrt{a\mathrm{csch}^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Csch[x]^4)^(-3/2),x]
```

```
[Out] (5*Coth[x])/(16*a*Sqrt[a*Csch[x]^4]) - (5*x*Csch[x]^2)/(16*a*Sqrt[a*Csch[x]
^4]) - (5*Cosh[x]*Sinh[x])/(24*a*Sqrt[a*Csch[x]^4]) + (Cosh[x]*Sinh[x]^3)/(
6*a*Sqrt[a*Csch[x]^4])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
& !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a\text{csch}^4(x)\right)^{3/2}}\,dx &= \frac{\text{csch}^2(x)\int \sinh^6(x)\,dx}{a\sqrt{a\text{csch}^4(x)}}\\
&= \frac{\cosh(x)\sinh^3(x)}{6a\sqrt{a\text{csch}^4(x)}} - \frac{\left(5\text{csch}^2(x)\right)\int \sinh^4(x)\,dx}{6a\sqrt{a\text{csch}^4(x)}}\\
&= -\frac{5\cosh(x)\sinh(x)}{24a\sqrt{a\text{csch}^4(x)}} + \frac{\cosh(x)\sinh^3(x)}{6a\sqrt{a\text{csch}^4(x)}} + \frac{\left(5\text{csch}^2(x)\right)\int \sinh^2(x)\,dx}{8a\sqrt{a\text{csch}^4(x)}}\\
&= \frac{5\coth(x)}{16a\sqrt{a\text{csch}^4(x)}} - \frac{5\cosh(x)\sinh(x)}{24a\sqrt{a\text{csch}^4(x)}} + \frac{\cosh(x)\sinh^3(x)}{6a\sqrt{a\text{csch}^4(x)}} - \frac{\left(5\text{csch}^2(x)\right)\int 1\,dx}{16a\sqrt{a\text{csch}^4(x)}}\\
&= \frac{5\coth(x)}{16a\sqrt{a\text{csch}^4(x)}} - \frac{5x\text{csch}^2(x)}{16a\sqrt{a\text{csch}^4(x)}} - \frac{5\cosh(x)\sinh(x)}{24a\sqrt{a\text{csch}^4(x)}} + \frac{\cosh(x)\sinh^3(x)}{6a\sqrt{a\text{csch}^4(x)}}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 38, normalized size = 0.44

$$\frac{\text{csch}^6(x)(-60x + 45\sinh(2x) - 9\sinh(4x) + \sinh(6x))}{192\left(a\text{csch}^4(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Csch[x]^4)^(-3/2),x]
```

```
[Out] (Csch[x]^6*(-60*x + 45*Sinh[2*x] - 9*Sinh[4*x] + Sinh[6*x]))/(192*(a*Csch[x
]^4)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(70) = 140.
time = 0.96, size = 230, normalized size = 2.67

method	result
risch	$-\frac{5\,e^{2x}x}{16a(e^{2x}-1)^2\sqrt{\frac{a\,e^{4x}}{(e^{2x}-1)^4}}} + \frac{e^{8x}}{384a(e^{2x}-1)^2\sqrt{\frac{a\,e^{4x}}{(e^{2x}-1)^4}}} - \frac{3\,e^{6x}}{128a(e^{2x}-1)^2\sqrt{\frac{a\,e^{4x}}{(e^{2x}-1)^4}}} + \frac{15\,e^{4x}}{128a(e^{2x}-1)^2\sqrt{\frac{a\,e^{4x}}{(e^{2x}-1)^4}}} -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*csch(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -5/16/a*exp(2*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)*x+1/384/a
*exp(8*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)-3/128/a*exp(6*x)
```

```
/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)+15/128/a*exp(4*x)/(exp(2*
x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)-15/128/(a*exp(4*x)/(exp(2*x)-1)^4
)^(1/2)/(exp(2*x)-1)^2/a+3/128/a*exp(-2*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(
2*x)-1)^4)^(1/2)-1/384/a*exp(-4*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^
4)^(1/2)
```

Maxima [A]
time = 0.47, size = 46, normalized size = 0.53

$$-\frac{\left(9\,e^{(-2\,x)}-45\,e^{(-4\,x)}+45\,e^{(-8\,x)}-9\,e^{(-10\,x)}+e^{(-12\,x)}-1\right)e^{(6\,x)}}{384\,a^{\frac{3}{2}}}-\frac{5\,x}{16\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/384*(9*e^(-2*x) - 45*e^(-4*x) + 45*e^(-8*x) - 9*e^(-10*x) + e^(-12*x) -
1)*e^(6*x)/a^(3/2) - 5/16*x/a^(3/2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1141 vs. $2(70) = 140$.
time = 0.35, size = 1141, normalized size = 13.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/384*((e^(4*x) - 2*e^(2*x) + 1)*sinh(x)^12 + cosh(x)^12 + 12*(cosh(x)*e^(4
*x) - 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^11 + 3*(22*cosh(x)^2 + (22*cosh(
x)^2 - 3)*e^(4*x) - 2*(22*cosh(x)^2 - 3)*e^(2*x) - 3)*sinh(x)^10 - 9*cosh(x
)^10 + 10*(22*cosh(x)^3 + (22*cosh(x)^3 - 9*cosh(x))*e^(4*x) - 2*(22*cosh(x
)^3 - 9*cosh(x))*e^(2*x) - 9*cosh(x))*sinh(x)^9 + 45*(11*cosh(x)^4 - 9*cosh
(x)^2 + (11*cosh(x)^4 - 9*cosh(x)^2 + 1)*e^(4*x) - 2*(11*cosh(x)^4 - 9*cosh
(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^8 + 45*cosh(x)^8 + 72*(11*cosh(x)^5 - 15*co
sh(x)^3 + (11*cosh(x)^5 - 15*cosh(x)^3 + 5*cosh(x))*e^(4*x) - 2*(11*cosh(x)
^5 - 15*cosh(x)^3 + 5*cosh(x))*e^(2*x) + 5*cosh(x))*sinh(x)^7 - 120*x*cosh(
x)^6 + 6*(154*cosh(x)^6 - 315*cosh(x)^4 + 210*cosh(x)^2 + (154*cosh(x)^6 -
315*cosh(x)^4 + 210*cosh(x)^2 - 20*x)*e^(4*x) - 2*(154*cosh(x)^6 - 315*cosh
(x)^4 + 210*cosh(x)^2 - 20*x)*e^(2*x) - 20*x)*sinh(x)^6 + 36*(22*cosh(x)^7
- 63*cosh(x)^5 + 70*cosh(x)^3 - 20*x*cosh(x) + (22*cosh(x)^7 - 63*cosh(x)^5
 + 70*cosh(x)^3 - 20*x*cosh(x))*e^(4*x) - 2*(22*cosh(x)^7 - 63*cosh(x)^5 +
70*cosh(x)^3 - 20*x*cosh(x))*e^(2*x))*sinh(x)^5 + 45*(11*cosh(x)^8 - 42*cos
h(x)^6 + 70*cosh(x)^4 - 40*x*cosh(x)^2 + (11*cosh(x)^8 - 42*cosh(x)^6 + 70*
cosh(x)^4 - 40*x*cosh(x)^2 - 1)*e^(4*x) - 2*(11*cosh(x)^8 - 42*cosh(x)^6 +
70*cosh(x)^4 - 40*x*cosh(x)^2 - 1)*e^(2*x) - 1)*sinh(x)^4 - 45*cosh(x)^4 +
20*(11*cosh(x)^9 - 54*cosh(x)^7 + 126*cosh(x)^5 - 120*x*cosh(x)^3 + (11*cos
```

```
h(x)^9 - 54*cosh(x)^7 + 126*cosh(x)^5 - 120*x*cosh(x)^3 - 9*cosh(x))*e^(4*x
) - 2*(11*cosh(x)^9 - 54*cosh(x)^7 + 126*cosh(x)^5 - 120*x*cosh(x)^3 - 9*co
sh(x))*e^(2*x) - 9*cosh(x))*sinh(x)^3 + 3*(22*cosh(x)^10 - 135*cosh(x)^8 +
420*cosh(x)^6 - 600*x*cosh(x)^4 - 90*cosh(x)^2 + (22*cosh(x)^10 - 135*cosh(
x)^8 + 420*cosh(x)^6 - 600*x*cosh(x)^4 - 90*cosh(x)^2 + 3)*e^(4*x) - 2*(22*
cosh(x)^10 - 135*cosh(x)^8 + 420*cosh(x)^6 - 600*x*cosh(x)^4 - 90*cosh(x)^2
 + 3)*e^(2*x) + 3)*sinh(x)^2 + 9*cosh(x)^2 + (cosh(x)^12 - 9*cosh(x)^10 + 4
5*cosh(x)^8 - 120*x*cosh(x)^6 - 45*cosh(x)^4 + 9*cosh(x)^2 - 1)*e^(4*x) - 2
*(cosh(x)^12 - 9*cosh(x)^10 + 45*cosh(x)^8 - 120*x*cosh(x)^6 - 45*cosh(x)^4
 + 9*cosh(x)^2 - 1)*e^(2*x) + 6*(2*cosh(x)^11 - 15*cosh(x)^9 + 60*cosh(x)^7
 - 120*x*cosh(x)^5 - 30*cosh(x)^3 + (2*cosh(x)^11 - 15*cosh(x)^9 + 60*cosh(
x)^7 - 120*x*cosh(x)^5 - 30*cosh(x)^3 + 3*cosh(x))*e^(4*x) - 2*(2*cosh(x)^1
1 - 15*cosh(x)^9 + 60*cosh(x)^7 - 120*x*cosh(x)^5 - 30*cosh(x)^3 + 3*cosh(x
))*e^(2*x) + 3*cosh(x))*sinh(x) - 1)*sqrt(a/(e^(8*x) - 4*e^(6*x) + 6*e^(4*x
) - 4*e^(2*x) + 1))*e^(2*x)/(a^2*cosh(x)^6*e^(2*x) + 6*a^2*cosh(x)^5*e^(2*x
)*sinh(x) + 15*a^2*cosh(x)^4*e^(2*x)*sinh(x)^2 + 20*a^2*cosh(x)^3*e^(2*x)*s
inh(x)^3 + 15*a^2*cosh(x)^2*e^(2*x)*sinh(x)^4 + 6*a^2*cosh(x)*e^(2*x)*sinh(
x)^5 + a^2*e^(2*x)*sinh(x)^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\operatorname{csch}^4\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)**4)**(3/2),x)
```

```
[Out] Integral((a*csch(x)**4)**(-3/2), x)
```

Giac [A]
time = 0.38, size = 50, normalized size = 0.58

$$\frac{\left(110\,e^{(6\,x)}-45\,e^{(4\,x)}+9\,e^{(2\,x)}-1\right)e^{(-6\,x)}-120\,x+e^{(6\,x)}-9\,e^{(4\,x)}+45\,e^{(2\,x)}}{384\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] 1/384*((110*e^(6*x) - 45*e^(4*x) + 9*e^(2*x) - 1)*e^(-6*x) - 120*x + e^(6*x
) - 9*e^(4*x) + 45*e^(2*x))/a^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(\frac{a}{{\mathrm{sinh}\left(x\right)}^4}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/sinh(x)^4)^(3/2),x)
[Out] int(1/(a/sinh(x)^4)^(3/2), x)
```

3.48 $\int \frac{1}{\left(a\operatorname{csch}^4(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=132

$$\frac{63\coth(x)}{256a^2\sqrt{a\operatorname{csch}^4(x)}}-\frac{63x\operatorname{csch}^2(x)}{256a^2\sqrt{a\operatorname{csch}^4(x)}}-\frac{21\cosh(x)\sinh(x)}{128a^2\sqrt{a\operatorname{csch}^4(x)}}+\frac{21\cosh(x)\sinh^3(x)}{160a^2\sqrt{a\operatorname{csch}^4(x)}}-\frac{9\cosh(x)\sinh^5(x)}{80a^2\sqrt{a\operatorname{csch}^4(x)}}+\frac{\cos}{10a}$$

```
[Out] 63/256*coth(x)/a^2/(a*csch(x)^4)^(1/2)-63/256*x*csch(x)^2/a^2/(a*csch(x)^4)
^(1/2)-21/128*cosh(x)*sinh(x)/a^2/(a*csch(x)^4)^(1/2)+21/160*cosh(x)*sinh(x
)^3/a^2/(a*csch(x)^4)^(1/2)-9/80*cosh(x)*sinh(x)^5/a^2/(a*csch(x)^4)^(1/2)+
1/10*cosh(x)*sinh(x)^7/a^2/(a*csch(x)^4)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4208, 2715, 8}

$$-\frac{63x\operatorname{csch}^2(x)}{256a^2\sqrt{a\operatorname{csch}^4(x)}}+\frac{63\coth(x)}{256a^2\sqrt{a\operatorname{csch}^4(x)}}+\frac{\sinh^7(x)\cosh(x)}{10a^2\sqrt{a\operatorname{csch}^4(x)}}-\frac{9\sinh^5(x)\cosh(x)}{80a^2\sqrt{a\operatorname{csch}^4(x)}}+\frac{21\sinh^3(x)\cosh(x)}{160a^2\sqrt{a\operatorname{csch}^4(x)}}-\frac{21\sinh(x)\cosh(x)}{128a^2\sqrt{a\operatorname{csch}^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Csch[x]^4)^(-5/2),x]
```

```
[Out] (63*Coth[x])/(256*a^2*Sqrt[a*Csch[x]^4]) - (63*x*Csch[x]^2)/(256*a^2*Sqrt[a
*Csch[x]^4]) - (21*Cosh[x]*Sinh[x])/(128*a^2*Sqrt[a*Csch[x]^4]) + (21*Cosh[
x]*Sinh[x]^3)/(160*a^2*Sqrt[a*Csch[x]^4]) - (9*Cosh[x]*Sinh[x]^5)/(80*a^2*S
qrt[a*Csch[x]^4]) + (Cosh[x]*Sinh[x]^7)/(10*a^2*Sqrt[a*Csch[x]^4])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
```

```
&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a\text{csch}^4(x)\right)^{5/2}}\,dx = \frac{\text{csch}^2(x)\int \sinh^{10}(x)\,dx}{a^2\sqrt{a\text{csch}^4(x)}}$$
$$= \frac{\cosh(x)\sinh^7(x)}{10a^2\sqrt{a\text{csch}^4(x)}} - \frac{\left(9\text{csch}^2(x)\right)\int \sinh^8(x)\,dx}{10a^2\sqrt{a\text{csch}^4(x)}}$$
$$= -\frac{9\cosh(x)\sinh^5(x)}{80a^2\sqrt{a\text{csch}^4(x)}} + \frac{\cosh(x)\sinh^7(x)}{10a^2\sqrt{a\text{csch}^4(x)}} + \frac{\left(63\text{csch}^2(x)\right)\int \sinh^6(x)\,dx}{80a^2\sqrt{a\text{csch}^4(x)}}$$
$$= \frac{21\cosh(x)\sinh^3(x)}{160a^2\sqrt{a\text{csch}^4(x)}} - \frac{9\cosh(x)\sinh^5(x)}{80a^2\sqrt{a\text{csch}^4(x)}} + \frac{\cosh(x)\sinh^7(x)}{10a^2\sqrt{a\text{csch}^4(x)}} - \frac{\left(21\text{csch}^2(x)\right)\int \sin}{32a^2\sqrt{a\text{csch}^4}}$$
$$= -\frac{21\cosh(x)\sinh(x)}{128a^2\sqrt{a\text{csch}^4(x)}} + \frac{21\cosh(x)\sinh^3(x)}{160a^2\sqrt{a\text{csch}^4(x)}} - \frac{9\cosh(x)\sinh^5(x)}{80a^2\sqrt{a\text{csch}^4(x)}} + \frac{\cosh(x)\sinh^7(}{10a^2\sqrt{a\text{csch}^4(}}$$
$$= \frac{63\coth(x)}{256a^2\sqrt{a\text{csch}^4(x)}} - \frac{21\cosh(x)\sinh(x)}{128a^2\sqrt{a\text{csch}^4(x)}} + \frac{21\cosh(x)\sinh^3(x)}{160a^2\sqrt{a\text{csch}^4(x)}} - \frac{9\cosh(x)\sinh^5(}{80a^2\sqrt{a\text{csch}^4(x}}$$
$$= \frac{63\coth(x)}{256a^2\sqrt{a\text{csch}^4(x)}} - \frac{63x\text{csch}^2(x)}{256a^2\sqrt{a\text{csch}^4(x)}} - \frac{21\cosh(x)\sinh(x)}{128a^2\sqrt{a\text{csch}^4(x)}} + \frac{21\cosh(x)\sinh^3(}{160a^2\sqrt{a\text{csch}^4(}}$$

Mathematica [A]
time = 0.08, size = 55, normalized size = 0.42

$$\frac{\sqrt{a\text{csch}^4(x)}\,\sinh^2(x)(-2520x + 2100\sinh(2x) - 600\sinh(4x) + 150\sinh(6x) - 25\sinh(8x) + 2\sinh(10x))}{10240a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Csch[x]^4)^(-5/2),x]
[Out] (Sqrt[a*Csch[x]^4]*Sinh[x]^2*(-2520*x + 2100*Sinh[2*x] - 600*Sinh[4*x] + 15
0*Sinh[6*x] - 25*Sinh[8*x] + 2*Sinh[10*x]))/(10240*a^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(108) = 216$.
time = 0.98, size = 362, normalized size = 2.74

method	result
risch	$-\frac{63\,\mathrm{e}^{2x}x}{256a^2(\mathrm{e}^{2x}-1)^2\sqrt{\frac{a\,\mathrm{e}^{4x}}{(\mathrm{e}^{2x}-1)^4}}}+\frac{\mathrm{e}^{12x}}{10240a^2(\mathrm{e}^{2x}-1)^2\sqrt{\frac{a\,\mathrm{e}^{4x}}{(\mathrm{e}^{2x}-1)^4}}}-\frac{5\,\mathrm{e}^{10x}}{4096a^2(\mathrm{e}^{2x}-1)^2\sqrt{\frac{a\,\mathrm{e}^{4x}}{(\mathrm{e}^{2x}-1)^4}}}+\frac{15\,\mathrm{e}^{8x}}{2048a^2(\mathrm{e}^{2x}-1)^2\sqrt{\frac{a}{(\mathrm{e}^{2x}}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*csch(x)^4)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -63/256/a^2*exp(2*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)*x+1/1
0240/a^2*exp(12*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)-5/4096/
a^2*exp(10*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)+15/2048/a^2*
exp(8*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)-15/512/a^2*exp(6*
x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)+105/1024/a^2*exp(4*x)/(
exp(2*x)-1)^2/(a*exp(4*x)/(exp(2*x)-1)^4)^(1/2)-105/1024/(a*exp(4*x)/(exp(2
*x)-1)^4)^(1/2)/(exp(2*x)-1)^2/a^2+15/512/a^2*exp(-2*x)/(exp(2*x)-1)^2/(a*e
xp(4*x)/(exp(2*x)-1)^4)^(1/2)-15/2048/a^2*exp(-4*x)/(exp(2*x)-1)^2/(a*exp(4
*x)/(exp(2*x)-1)^4)^(1/2)+5/4096/a^2*exp(-6*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(
exp(2*x)-1)^4)^(1/2)-1/10240/a^2*exp(-8*x)/(exp(2*x)-1)^2/(a*exp(4*x)/(exp(
2*x)-1)^4)^(1/2)
```

Maxima [A]
time = 0.47, size = 72, normalized size = 0.55

$$-\frac{\left(25\,e^{(-2\,x)}-150\,e^{(-4\,x)}+600\,e^{(-6\,x)}-2100\,e^{(-8\,x)}+2100\,e^{(-12\,x)}-600\,e^{(-14\,x)}+150\,e^{(-16\,x)}-25\,e^{(-18\,x)}+2\,e^{(-20\,x)}-2\right)e^{(10\,x)}}{20480\,a^{\frac{5}{2}}}-\frac{63\,x}{256\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/20480*(25*e^(-2*x) - 150*e^(-4*x) + 600*e^(-6*x) - 2100*e^(-8*x) + 2100*
e^(-12*x) - 600*e^(-14*x) + 150*e^(-16*x) - 25*e^(-18*x) + 2*e^(-20*x) - 2)
*e^(10*x)/a^(5/2) - 63/256*x/a^(5/2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2600 vs. 2(108) = 216.
time = 0.39, size = 2600, normalized size = 19.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/20480*(2*(e^(4*x) - 2*e^(2*x) + 1)*sinh(x)^20 + 2*cosh(x)^20 + 40*(cosh(x
)*e^(4*x) - 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^19 + 5*(76*cosh(x)^2 + (76
*cosh(x)^2 - 5)*e^(4*x) - 2*(76*cosh(x)^2 - 5)*e^(2*x) - 5)*sinh(x)^18 - 25
```

```
*cosh(x)^18 + 30*(76*cosh(x)^3 + (76*cosh(x)^3 - 15*cosh(x))*e^(4*x) - 2*(7
6*cosh(x)^3 - 15*cosh(x))*e^(2*x) - 15*cosh(x))*sinh(x)^17 + 15*(646*cosh(x
)^4 - 255*cosh(x)^2 + (646*cosh(x)^4 - 255*cosh(x)^2 + 10)*e^(4*x) - 2*(646
*cosh(x)^4 - 255*cosh(x)^2 + 10)*e^(2*x) + 10)*sinh(x)^16 + 150*cosh(x)^16
 + 48*(646*cosh(x)^5 - 425*cosh(x)^3 + (646*cosh(x)^5 - 425*cosh(x)^3 + 50*c
osh(x))*e^(4*x) - 2*(646*cosh(x)^5 - 425*cosh(x)^3 + 50*cosh(x))*e^(2*x) +
50*cosh(x))*sinh(x)^15 + 60*(1292*cosh(x)^6 - 1275*cosh(x)^4 + 300*cosh(x)^
2 + (1292*cosh(x)^6 - 1275*cosh(x)^4 + 300*cosh(x)^2 - 10)*e^(4*x) - 2*(129
2*cosh(x)^6 - 1275*cosh(x)^4 + 300*cosh(x)^2 - 10)*e^(2*x) - 10)*sinh(x)^14
 - 600*cosh(x)^14 + 120*(1292*cosh(x)^7 - 1785*cosh(x)^5 + 700*cosh(x)^3 +
(1292*cosh(x)^7 - 1785*cosh(x)^5 + 700*cosh(x)^3 - 70*cosh(x))*e^(4*x) - 2*
(1292*cosh(x)^7 - 1785*cosh(x)^5 + 700*cosh(x)^3 - 70*cosh(x))*e^(2*x) - 70
*cosh(x))*sinh(x)^13 + 60*(4199*cosh(x)^8 - 7735*cosh(x)^6 + 4550*cosh(x)^4
 - 910*cosh(x)^2 + (4199*cosh(x)^8 - 7735*cosh(x)^6 + 4550*cosh(x)^4 - 910*
cosh(x)^2 + 35)*e^(4*x) - 2*(4199*cosh(x)^8 - 7735*cosh(x)^6 + 4550*cosh(x)
^4 - 910*cosh(x)^2 + 35)*e^(2*x) + 35)*sinh(x)^12 + 2100*cosh(x)^12 + 80*(4
199*cosh(x)^9 - 9945*cosh(x)^7 + 8190*cosh(x)^5 - 2730*cosh(x)^3 + (4199*co
sh(x)^9 - 9945*cosh(x)^7 + 8190*cosh(x)^5 - 2730*cosh(x)^3 + 315*cosh(x))*e
^(4*x) - 2*(4199*cosh(x)^9 - 9945*cosh(x)^7 + 8190*cosh(x)^5 - 2730*cosh(x)
^3 + 315*cosh(x))*e^(2*x) + 315*cosh(x))*sinh(x)^11 - 5040*x*cosh(x)^10 + 2
*(184756*cosh(x)^10 - 546975*cosh(x)^8 + 600600*cosh(x)^6 - 300300*cosh(x)^
4 + 69300*cosh(x)^2 + (184756*cosh(x)^10 - 546975*cosh(x)^8 + 600600*cosh(x
)^6 - 300300*cosh(x)^4 + 69300*cosh(x)^2 - 2520*x)*e^(4*x) - 2*(184756*cosh
(x)^10 - 546975*cosh(x)^8 + 600600*cosh(x)^6 - 300300*cosh(x)^4 + 69300*cos
h(x)^2 - 2520*x)*e^(2*x) - 2520*x)*sinh(x)^10 + 20*(16796*cosh(x)^11 - 6077
5*cosh(x)^9 + 85800*cosh(x)^7 - 60060*cosh(x)^5 + 23100*cosh(x)^3 - 2520*x*
cosh(x) + (16796*cosh(x)^11 - 60775*cosh(x)^9 + 85800*cosh(x)^7 - 60060*cos
h(x)^5 + 23100*cosh(x)^3 - 2520*x*cosh(x))*e^(4*x) - 2*(16796*cosh(x)^11 -
60775*cosh(x)^9 + 85800*cosh(x)^7 - 60060*cosh(x)^5 + 23100*cosh(x)^3 - 252
0*x*cosh(x))*e^(2*x))*sinh(x)^9 + 30*(8398*cosh(x)^12 - 36465*cosh(x)^10 +
64350*cosh(x)^8 - 60060*cosh(x)^6 + 34650*cosh(x)^4 - 7560*x*cosh(x)^2 + (8
398*cosh(x)^12 - 36465*cosh(x)^10 + 64350*cosh(x)^8 - 60060*cosh(x)^6 + 346
50*cosh(x)^4 - 7560*x*cosh(x)^2 - 70)*e^(4*x) - 2*(8398*cosh(x)^12 - 36465*
cosh(x)^10 + 64350*cosh(x)^8 - 60060*cosh(x)^6 + 34650*cosh(x)^4 - 7560*x*c
osh(x)^2 - 70)*e^(2*x) - 70)*sinh(x)^8 - 2100*cosh(x)^8 + 240*(646*cosh(x)^
13 - 3315*cosh(x)^11 + 7150*cosh(x)^9 - 8580*cosh(x)^7 + 6930*cosh(x)^5 - 2
520*x*cosh(x)^3 + (646*cosh(x)^13 - 3315*cosh(x)^11 + 7150*cosh(x)^9 - 8580
*cosh(x)^7 + 6930*cosh(x)^5 - 2520*x*cosh(x)^3 - 70*cosh(x))*e^(4*x) - 2*(6
46*cosh(x)^13 - 3315*cosh(x)^11 + 7150*cosh(x)^9 - 8580*cosh(x)^7 + 6930*co
sh(x)^5 - 2520*x*cosh(x)^3 - 70*cosh(x))*e^(2*x) - 70*cosh(x))*sinh(x)^7 +
60*(1292*cosh(x)^14 - 7735*cosh(x)^12 + 20020*cosh(x)^10 - 30030*cosh(x)^8
 + 32340*cosh(x)^6 - 17640*x*cosh(x)^4 - 980*cosh(x)^2 + (1292*cosh(x)^14 -
7735*cosh(x)^12 + 20020*cosh(x)^10 - 30030*cosh(x)^8 + 32340*cosh(x)^6 - 17
640*x*cosh(x)^4 - 980*cosh(x)^2 + 10)*e^(4*x) - 2*(1292*cosh(x)^14 - 7735*c
osh(x)^12 + 20020*cosh(x)^10 - 30030*cosh(x)^8 + 32340*cosh(x)^6 - 17640*x*
```

```
cosh(x)^4 - 980*cosh(x)^2 + 10)*e^(2*x) + 10)*sinh(x)^6 + 600*cosh(x)^6 + 2
4*(1292*cosh(x)^15 - 8925*cosh(x)^13 + 27300*cosh(x)^11 - 50050*cosh(x)^9 +
 69300*cosh(x)^7 - 52920*x*cosh(x)^5 - 4900*cosh(x)^3 + (1292*cosh(x)^15 -
8925*cosh(x)^13 + 27300*cosh(x)^11 - 50050*cosh(x)^9 + 69300*cosh(x)^7 - 52
920*x*cosh(x)^5 - 4900*cosh(x)^3 + 150*cosh(x))*e^(4*x) - 2*(1292*cosh(x)^1
5 - 8925*cosh(x)^13 + 27300*cosh(x)^11 - 50050*cosh(x)^9 + 69300*cosh(x)^7
- 52920*x*cosh(x)^5 - 4900*cosh(x)^3 + 150*cosh(x))*e^(2*x) + 150*cosh(x))*
sinh(x)^5 + 30*(323*cosh(x)^16 - 2550*cosh(x)^14 + 9100*cosh(x)^12 - 20020*
cosh(x)^10 + 34650*cosh(x)^8 - 35280*x*cosh(x)^6 - 4900*cosh(x)^4 + 300*cos
h(x)^2 + (323*cosh(x)^16 - 2550*cosh(x)^14 + 9100*cosh(x)^12 - 20020*cosh(x
)^10 + 34650*cosh(x)^8 - 35280*x*cosh(x)^6 - 4900*cosh(x)^4 + 300*cosh(x)^2
 - 5)*e^(4*x) - 2*(323*cosh(x)^16 - 2550*cosh(x)^14 + 9100*cosh(x)^12 - 200
20*cosh(x)^10 + 34650*cosh(x)^8 - 35280*x*cosh(x)^6 - 4900*cosh(x)^4 + 300*
cosh(x)^2 - 5)*e^(2*x) - 5)*sinh(x)^4 - 150*cosh(x)^4 + 120*(19*cosh(x)^17
- 170*cosh(x)^15 + 700*cosh(x)^13 - 1820*cosh(x)^11 + 3850*cosh(x)^9 - 5040
*x*cosh(x)^7 - 980*cosh(x)^5 + 100*cosh(x)^3 + (19*cosh(x)^17 - 170*cosh(x)
^15 + 700*cosh(x)^13 - 1820*cosh(x)^11 + 3850*cosh(x)^9 - 5040*x*cosh(x)^7
- 980*cosh(x)^5 + 100*cosh(x)^3 - 5*cosh(x))*e^(4*x) - 2*(19*cosh(x)^17 - 1
70*cosh(x)^15 + 700*cosh(x)^13 - 1820*cosh(x)^1...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\operatorname{csch}^4\left(x\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)**4)**(5/2),x)
```

```
[Out] Integral((a*csch(x)**4)**(-5/2), x)
```

Giac [A]
time = 0.41, size = 76, normalized size = 0.58

$$\frac{\left(5754\,e^{(10\,x)}-2100\,e^{(8\,x)}+600\,e^{(6\,x)}-150\,e^{(4\,x)}+25\,e^{(2\,x)}-2\right)e^{(-10\,x)}-5040\,x+2\,e^{(10\,x)}-25\,e^{(8\,x)}+150\,e^{(6\,x)}-600\,e^{(4\,x)}+2100\,e^{(2\,x)}}{20480\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*csch(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] 1/20480*((5754*e^(10*x) - 2100*e^(8*x) + 600*e^(6*x) - 150*e^(4*x) + 25*e^(
2*x) - 2)*e^(-10*x) - 5040*x + 2*e^(10*x) - 25*e^(8*x) + 150*e^(6*x) - 600*
e^(4*x) + 2100*e^(2*x))/a^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(\frac{a}{{\mathrm{sinh}\left(x\right)}^4}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/sinh(x)^4)^(5/2),x)
[Out] int(1/(a/sinh(x)^4)^(5/2), x)
```

3.49 $\int \frac{1}{a+ia\,\mathrm{csch}(a+bx)}\,dx$

Optimal. Leaf size=32

$$\frac{x}{a}-\frac{\coth(a+bx)}{b(a+ia\,\mathrm{csch}(a+bx))}$$

```
[Out] x/a-coth(b*x+a)/b/(a+I*a*csch(b*x+a))
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3862, 8}

$$\frac{x}{a}-\frac{\coth(a+bx)}{b(a+ia\,\mathrm{csch}(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Csch[a + b*x])^(-1),x]
```

```
[Out] x/a - Coth[a + b*x]/(b*(a + I*a*Csch[a + b*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+ia\,\mathrm{csch}(a+bx)}\,dx &= -\frac{\coth(a+bx)}{b(a+ia\,\mathrm{csch}(a+bx))}+\frac{\int a\,dx}{a^2}\\ &= \frac{x}{a}-\frac{\coth(a+bx)}{b(a+ia\,\mathrm{csch}(a+bx))}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 54, normalized size = 1.69

$$\frac{1}{b}+\frac{x}{a}-\frac{2\sinh\left(\frac{1}{2}(a+bx)\right)}{ab\left(\cosh\left(\frac{1}{2}(a+bx)\right)-i\sinh\left(\frac{1}{2}(a+bx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Csch[a + b*x])^(-1),x]
```

```
[Out] b^(-1) + x/a - (2*Sinh[(a + b*x)/2])/(a*b*(Cosh[(a + b*x)/2] - I*Sinh[(a + b*x)/2]))
```

Maple [A]
time = 1.97, size = 54, normalized size = 1.69

method	result	size
risch	$\frac{x}{a}+\frac{2i}{ba\left(e^{bx+a}+i\right)}$	27
derivativedivides	$\frac{-\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)-\frac{2}{\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+i}}{ab}$	54
default	$\frac{-\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)-\frac{2}{\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+i}}{ab}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*csch(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 4/b/a*(-1/4*ln(tanh(1/2*b*x+1/2*a)-1)+1/4*ln(tanh(1/2*b*x+1/2*a)+1)-1/2/(tanh(1/2*b*x+1/2*a)+I))
```

Maxima [A]
time = 0.27, size = 35, normalized size = 1.09

$$\frac{bx+a}{ab}+\frac{2i}{\left(ae^{(-bx-a)}-i\,a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*csch(b*x+a)),x, algorithm="maxima")
```

```
[Out] (b*x + a)/(a*b) + 2*I/((a*e^(-b*x - a) - I*a)*b)
```

Fricas [A]
time = 0.36, size = 32, normalized size = 1.00

$$\frac{bxe^{(bx+a)}+i\,bx+2i}{abe^{(bx+a)}+i\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*csch(b*x+a)),x, algorithm="fricas")
```

```
[Out] (b*x*e^(b*x + a) + I*b*x + 2*I)/(a*b*e^(b*x + a) + I*a*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int \frac{1}{\operatorname{csch}(a+bx)-i}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*csch(b*x+a)),x)
```

```
[Out] -I*Integral(1/(csch(a + b*x) - I), x)/a
```

Giac [A]
time = 0.39, size = 29, normalized size = 0.91

$$\frac{\frac{bx+a}{a}+\frac{2i}{a\left(e^{(bx+a)}+i\right)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*csch(b*x+a)),x, algorithm="giac")
```

```
[Out] ((b*x + a)/a + 2*I/(a*(e^(b*x + a) + I)))/b
```

Mupad [B]
time = 1.55, size = 26, normalized size = 0.81

$$\frac{x}{a}+\frac{2\mathrm{i}}{a\,b\,\left(\mathrm{e}^{a+b\,x}+1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + (a*1i)/sinh(a + b*x)),x)
```

```
[Out] x/a + 2i/(a*b*(exp(a + b*x) + 1i))
```

3.50 $\int \frac{1}{a-ia\text{csch}(a+bx)}\,dx$

Optimal. Leaf size=32

$$\frac{x}{a}-\frac{\coth(a+bx)}{b(a-ia\text{csch}(a+bx))}$$

```
[Out] x/a-coth(b*x+a)/b/(a-I*a*csch(b*x+a))
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3862, 8}

$$\frac{x}{a}-\frac{\coth(a+bx)}{b(a-ia\text{csch}(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*Csch[a + b*x])^(-1),x]
```

```
[Out] x/a - Coth[a + b*x]/(b*(a - I*a*Csch[a + b*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a-ia\text{csch}(a+bx)}\,dx&=-\frac{\coth(a+bx)}{b(a-ia\text{csch}(a+bx))}+\frac{\int a\,dx}{a^2}\\&=\frac{x}{a}-\frac{\coth(a+bx)}{b(a-ia\text{csch}(a+bx))}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 54, normalized size = 1.69

$$\frac{1}{b}+\frac{x}{a}-\frac{2\sinh\left(\frac{1}{2}(a+bx)\right)}{ab\left(\cosh\left(\frac{1}{2}(a+bx)\right)+i\sinh\left(\frac{1}{2}(a+bx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*Csch[a + b*x])^(-1),x]
```

```
[Out] b^(-1) + x/a - (2*Sinh[(a + b*x)/2])/(a*b*(Cosh[(a + b*x)/2] + I*Sinh[(a + b*x)/2]))
```

Maple [A]
time = 2.11, size = 54, normalized size = 1.69

method	result	size
risch	$\frac{x}{a}-\frac{2i}{ba\left(e^{bx+a}-i\right)}$	27
derivativedivides	$\frac{-\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)-\frac{2}{\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)-i}+\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{ab}$	54
default	$\frac{-\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)-\frac{2}{\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)-i}+\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{ab}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*csch(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 4/b/a*(-1/4*ln(tanh(1/2*b*x+1/2*a)-1)-1/2/(tanh(1/2*b*x+1/2*a)-I)+1/4*ln(tanh(1/2*b*x+1/2*a)+1))
```

Maxima [A]
time = 0.29, size = 35, normalized size = 1.09

$$\frac{bx+a}{ab}-\frac{2i}{\left(ae^{(-bx-a)}+i\,a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*csch(b*x+a)),x, algorithm="maxima")
```

```
[Out] (b*x + a)/(a*b) - 2*I/((a*e^(-b*x - a) + I*a)*b)
```

Fricas [A]
time = 0.38, size = 32, normalized size = 1.00

$$\frac{bxe^{(bx+a)}-i\,bx-2i}{abe^{(bx+a)}-i\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*csch(b*x+a)),x, algorithm="fricas")
```

```
[Out] (b*x*e^(b*x + a) - I*b*x - 2*I)/(a*b*e^(b*x + a) - I*a*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{i\int \frac{1}{\operatorname{csch}\,(a+bx)+i}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*csch(b*x+a)),x)
```

```
[Out] I*Integral(1/(csch(a + b*x) + I), x)/a
```

Giac [A]
time = 0.39, size = 29, normalized size = 0.91

$$\frac{\frac{bx+a}{a}-\frac{2i}{a\left(e^{(bx+a)}-i\right)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*csch(b*x+a)),x, algorithm="giac")
```

```
[Out] ((b*x + a)/a - 2*I/(a*(e^(b*x + a) - I)))/b
```

Mupad [B]
time = 1.50, size = 26, normalized size = 0.81

$$\frac{x}{a}-\frac{2\mathrm{i}}{a\,b\,\left({\mathrm{e}}^{a+b\,x}-\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - (a*1i)/sinh(a + b*x)),x)
```

```
[Out] x/a - 2i/(a*b*(exp(a + b*x) - 1i))
```

3.51 $\int (a+ia\text{csch}(c+dx))^{5/2}\,dx$

Optimal. Leaf size=107

$$\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{d}+\frac{14a^3\coth(c+dx)}{3d\sqrt{a+ia\text{csch}(c+dx)}}+\frac{2a^2\coth(c+dx)\sqrt{a+ia\text{csch}(c+dx)}}{3d}$$

```
[Out] 2*a^(5/2)*arctanh(coth(d*x+c)*a^(1/2)/(a+I*a*csch(d*x+c))^(1/2))/d+14/3*a^3
*coth(d*x+c)/d/(a+I*a*csch(d*x+c))^(1/2)+2/3*a^2*coth(d*x+c)*(a+I*a*csch(d*
x+c))^(1/2)/d
```

Rubi [A]
time = 0.09, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$,
Rules used = {3860, 4000, 3859, 209, 3877}

$$\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{d}+\frac{14a^3\coth(c+dx)}{3d\sqrt{a+ia\text{csch}(c+dx)}}+\frac{2a^2\coth(c+dx)\sqrt{a+ia\text{csch}(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Csch[c + d*x])^(5/2),x]
```

```
[Out] (2*a^(5/2)*ArcTanh[(Sqrt[a]*Coth[c + d*x])/Sqrt[a + I*a*Csch[c + d*x]]])/d
+ (14*a^3*Coth[c + d*x])/(3*d*Sqrt[a + I*a*Csch[c + d*x]]) + (2*a^2*Coth[c
+ d*x]*Sqrt[a + I*a*Csch[c + d*x]])/(3*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3860

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Dist[a/(n - 1)
, Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Integ
```

```
erQ[2*n]
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4000

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dis
t[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int (a+ia\operatorname{csch}(c+dx))^{5/2}\,dx &= \frac{2a^2\coth(c+dx)\sqrt{a+ia\operatorname{csch}(c+dx)}}{3d}+\frac{1}{3}(2a)\int \sqrt{a+ia\operatorname{csch}(c+dx)}\,\Big(\\
&= \frac{2a^2\coth(c+dx)\sqrt{a+ia\operatorname{csch}(c+dx)}}{3d}+\frac{1}{3}\left(7ia^2\right)\int \operatorname{csch}(c+dx)\sqrt{a+iac\\
&= \frac{14a^3\coth(c+dx)}{3d\sqrt{a+ia\operatorname{csch}(c+dx)}}+\frac{2a^2\coth(c+dx)\sqrt{a+ia\operatorname{csch}(c+dx)}}{3d}-\frac{(2ia^3}{}\\
&= \frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\operatorname{csch}(c+dx)}}\right)}{d}+\frac{14a^3\coth(c+dx)}{3d\sqrt{a+ia\operatorname{csch}(c+dx)}}+\frac{2a^2c}{}
\end{aligned}$$

Mathematica [A]
time = 0.95, size = 136, normalized size = 1.27

$$\frac{2a^2\sqrt{a+ia\operatorname{csch}(c+dx)}\left(-7i+\coth(c+dx)+\frac{3(-1)^{3/4}\tanh^{-1}\left((-1)^{3/4}\sqrt{i+\operatorname{csch}(c+dx)}\right)\coth(c+dx)}{(-i+\operatorname{csch}(c+dx))\sqrt{i+\operatorname{csch}(c+dx)}}+\frac{14\sinh\left(\frac{1}{2}(c+dx)\right)}{\cosh\left(\frac{1}{2}(c+dx)\right)-i\sinh\left(\frac{1}{2}(c+dx)\right)}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Csch[c + d*x])^(5/2),x]
```

```
[Out] (2*a^2*Sqrt[a + I*a*Csch[c + d*x]]*(-7*I + Coth[c + d*x] + (3*(-1)^(3/4)*Ar
cTanh[(-1)^(3/4)*Sqrt[I + Csch[c + d*x]]]*Coth[c + d*x])/((-I + Csch[c + d*
```

```
x])*Sqrt[I + Csch[c + d*x]]) + (14*Sinh[(c + d*x)/2])/(Cosh[(c + d*x)/2] -
I*Sinh[(c + d*x)/2])))/(3*d)
```

Maple [**F**]
time = 4.45, size = 0, normalized size = 0.00

$$\int \left(a + i a \,\mathrm{csch}\left(d x + c\right)\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*csch(d*x+c))^(5/2),x)
```

```
[Out] int((a+I*a*csch(d*x+c))^(5/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*csch(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*csch(d*x + c) + a)^(5/2), x)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 566 vs. 2(87) = 174.
time = 0.47, size = 566, normalized size = 5.29

$$\frac{3 \sqrt{\frac{a^5}{d^2}} \left(d e^{(2dx+2c)} - d\right) \log\left(\frac{2 \left(a^3 e^{(dx+c)} + i a^3 + \sqrt{\frac{a^5}{d^2}} \left(d e^{(2dx+2c)} - d\right) \sqrt{\frac{a}{e^{(2dx+2c)} - 1}}\right) e^{(-dx-c)}}{d}\right) - 3 \sqrt{\frac{a^5}{d^2}} \left(d e^{(2dx+2c)} - d\right) \log\left(\frac{2 \left(a^3 e^{(dx+c)} + i a^3 - \sqrt{\frac{a^5}{d^2}} \left(d e^{(2dx+2c)} - d\right) \sqrt{\frac{a}{e^{(2dx+2c)} - 1}}\right) e^{(-dx-c)}}{d}\right) + 3 \sqrt{\frac{a^5}{d^2}} \left(d e^{(2dx+2c)} - d\right) \log\left(\frac{2 \left(\sqrt{\frac{a^5}{d^2}} \left(a d e^{(2dx+2c)} - i a d e^{(dx+c)} - 2 a d\right) + \left(a^3 e^{(3dx+3c)} - 2 i a^3 e^{(2dx+2c)} - a^3 e^{(dx+c)} + 2 i a^3\right) \sqrt{\frac{a}{e^{(2dx+2c)} - 1}}\right) e^{(-2dx-2c)}}{d}\right) - 3 \sqrt{\frac{a^5}{d^2}} \left(d e^{(2dx+2c)} - d\right) \log\left(-\frac{2 \left(\sqrt{\frac{a^5}{d^2}} \left(a d e^{(2dx+2c)} - i a d e^{(dx+c)} - 2 a d\right) - \left(a^3 e^{(3dx+3c)} - 2 i a^3 e^{(2dx+2c)} - a^3 e^{(dx+c)} + 2 i a^3\right) \sqrt{\frac{a}{e^{(2dx+2c)} - 1}}\right) e^{(-2dx-2c)}}{d}\right) + 8 \left(4 a^2 e^{(3dx+3c)} - 3 i a^2 e^{(2dx+2c)} - 3 a^2 e^{(dx+c)} + 4 i a^2\right) \sqrt{\frac{a}{e^{(2dx+2c)} - 1}}}{6 \left(d e^{(2dx+2c)} - d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*csch(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(a^5/d^2)*(d*e^(2*d*x + 2*c) - d)*log(2*(a^3*e^(d*x + c) + I*a^3
 + sqrt(a^5/d^2)*(d*e^(2*d*x + 2*c) - d)*sqrt(a/(e^(2*d*x + 2*c) - 1)))*e^(
-d*x - c)/d) - 3*sqrt(a^5/d^2)*(d*e^(2*d*x + 2*c) - d)*log(2*(a^3*e^(d*x +
c) + I*a^3 - sqrt(a^5/d^2)*(d*e^(2*d*x + 2*c) - d)*sqrt(a/(e^(2*d*x + 2*c)
- 1)))*e^(-d*x - c)/d) + 3*sqrt(a^5/d^2)*(d*e^(2*d*x + 2*c) - d)*log(2*(sqr
t(a^5/d^2)*(a*d*e^(2*d*x + 2*c) - I*a*d*e^(d*x + c) - 2*a*d) + (a^3*e^(3*d*
x + 3*c) - 2*I*a^3*e^(2*d*x + 2*c) - a^3*e^(d*x + c) + 2*I*a^3)*sqrt(a/(e^(
2*d*x + 2*c) - 1)))*e^(-2*d*x - 2*c)/d) - 3*sqrt(a^5/d^2)*(d*e^(2*d*x + 2*c
) - d)*log(-2*(sqrt(a^5/d^2)*(a*d*e^(2*d*x + 2*c) - I*a*d*e^(d*x + c) - 2*a
*d) - (a^3*e^(3*d*x + 3*c) - 2*I*a^3*e^(2*d*x + 2*c) - a^3*e^(d*x + c) + 2*
I*a^3)*sqrt(a/(e^(2*d*x + 2*c) - 1)))*e^(-2*d*x - 2*c)/d) + 8*(4*a^2*e^(3*d
*x + 3*c) - 3*I*a^2*e^(2*d*x + 2*c) - 3*a^2*e^(d*x + c) + 4*I*a^2)*sqrt(a/(
e^(2*d*x + 2*c) - 1)))/(d*e^(2*d*x + 2*c) - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia \operatorname{csch}(c+dx)+a)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*csch(d*x+c))**(5/2),x)
[Out] Integral((I*a*csch(c + d*x) + a)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*csch(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((I*a*csch(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a\,1\mathrm{i}}{\sinh\left(c+d\,x\right)}\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + (a*1i)/sinh(c + d*x))^(5/2),x)
[Out] int((a + (a*1i)/sinh(c + d*x))^(5/2), x)
```

3.52 $\int (a + ia\,\mathbf{csch}(c + dx))^{3/2}\, dx$

Optimal. Leaf size=72

$$\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+iacsch(c+dx)}}\right)}{d}+\frac{2a^2\coth(c+dx)}{d\sqrt{a+iacsch(c+dx)}}$$

[Out] 2*a^(3/2)*arctanh(coth(d*x+c)*a^(1/2)/(a+I*a*csch(d*x+c))^(1/2))/d+2*a^2*coth(d*x+c)/d/(a+I*a*csch(d*x+c))^(1/2)

Rubi [A]
time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3860, 21, 3859, 209}

$$\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+iacsch(c+dx)}}\right)}{d}+\frac{2a^2\coth(c+dx)}{d\sqrt{a+iacsch(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(a + I*a*Csch[c + d*x])^(3/2),x]

[Out] (2*a^(3/2)*ArcTanh[(Sqrt[a]*Coth[c + d*x])/Sqrt[a + I*a*Csch[c + d*x]]])/d + (2*a^2*Coth[c + d*x])/(d*Sqrt[a + I*a*Csch[c + d*x]])

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3860

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Dist[a/(n - 1)
, Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Integ
erQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (a+ia\operatorname{csch}(c+dx))^{3/2}\,dx &= \frac{2a^2\coth(c+dx)}{d\sqrt{a+ia\operatorname{csch}(c+dx)}}+(2a)\int \frac{\frac{a}{2}+\frac{1}{2}ia\operatorname{csch}(c+dx)}{\sqrt{a+ia\operatorname{csch}(c+dx)}}\,dx\\ &= \frac{2a^2\coth(c+dx)}{d\sqrt{a+ia\operatorname{csch}(c+dx)}}+a\int \sqrt{a+ia\operatorname{csch}(c+dx)}\,dx\\ &= \frac{2a^2\coth(c+dx)}{d\sqrt{a+ia\operatorname{csch}(c+dx)}}-\frac{(2ia^2)\operatorname{Subst}\left(\int \frac{1}{a+x^2}\,dx,x,\frac{ia\coth(c+dx)}{\sqrt{a+ia\operatorname{csch}(c+dx)}}\right)}{d}\\ &= \frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\operatorname{csch}(c+dx)}}\right)}{d}+\frac{2a^2\coth(c+dx)}{d\sqrt{a+ia\operatorname{csch}(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.77, size = 100, normalized size = 1.39

$$-\frac{2ia\coth(c+dx)\sqrt{a+ia\operatorname{csch}(c+dx)}\left(-\sqrt[4]{-1}\tanh^{-1}\left((-1)^{3/4}\sqrt{i+\operatorname{csch}(c+dx)}\right)+\sqrt{i+\operatorname{csch}(c+dx)}\right)}{d(-i+\operatorname{csch}(c+dx))\sqrt{i+\operatorname{csch}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Csch[c + d*x])^(3/2),x]
```

```
[Out] ((-2*I)*a*Coth[c + d*x]*Sqrt[a + I*a*Csch[c + d*x]]*(-((-1)^(1/4)*ArcTanh[(
-1)^(3/4)*Sqrt[I + Csch[c + d*x]]]) + Sqrt[I + Csch[c + d*x]]))/(d*(-I + Cs
ch[c + d*x])*Sqrt[I + Csch[c + d*x]])
```

Maple [F]
time = 3.12, size = 0, normalized size = 0.00

$$\int (a+ia\operatorname{csch}(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*csch(d*x+c))^(3/2),x)
```

```
[Out] int((a+I*a*csch(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*csch(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*csch(d*x + c) + a)^(3/2), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 461 vs. 2(60) = 120.
time = 0.49, size = 461, normalized size = 6.40

$$\frac{\sqrt{\frac{a^3}{d^2}}\,d\log\left(\frac{2\left(a^2e^{(dx+c)}+i\,a^2+\left(de^{(2\,dx+2\,c)}-d\right)\sqrt{\frac{a^3}{d^2}}\sqrt{\frac{a}{e^{(2\,dx+2\,c)}-1}}\right)e^{(-dx-c)}}{d}\right)-\sqrt{\frac{a^3}{d^2}}\,d\log\left(\frac{2\left(a^2e^{(dx+c)}+i\,a^2-\left(de^{(2\,dx+2\,c)}-d\right)\sqrt{\frac{a^3}{d^2}}\sqrt{\frac{a}{e^{(2\,dx+2\,c)}-1}}\right)e^{(-dx-c)}}{d}\right)+\sqrt{\frac{a^3}{d^2}}\,d\log\left(\frac{2\left(\left(ade^{(2\,dx+2\,c)}-i\,ade^{(dx+c)}-2\,ad\right)\sqrt{\frac{a^3}{d^2}}+\left(a^2e^{(3\,dx+3\,c)}-2i\,a^2e^{(2\,dx+2\,c)}-a^2e^{(dx+c)}+2i\,a^2\right)\sqrt{\frac{a}{e^{(2\,dx+2\,c)}-1}}\right)e^{(-2\,dx-2\,c)}}{d}\right)-\sqrt{\frac{a^3}{d^2}}\,d\log\left(-\frac{2\left(\left(ade^{(2\,dx+2\,c)}-i\,ade^{(dx+c)}-2\,ad\right)\sqrt{\frac{a^3}{d^2}}-\left(a^2e^{(3\,dx+3\,c)}-2i\,a^2e^{(2\,dx+2\,c)}-a^2e^{(dx+c)}+2i\,a^2\right)\sqrt{\frac{a}{e^{(2\,dx+2\,c)}-1}}\right)e^{(-2\,dx-2\,c)}}{d}\right)+4\left(ae^{(dx+c)}-i\,a\right)\sqrt{\frac{a}{e^{(2\,dx+2\,c)}-1}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*csch(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a^3/d^2)*d*log(2*(a^2*e^(d*x + c) + I*a^2 + (d*e^(2*d*x + 2*c) -
d)*sqrt(a^3/d^2)*sqrt(a/(e^(2*d*x + 2*c) - 1)))*e^(-d*x - c)/d) - sqrt(a^3/
d^2)*d*log(2*(a^2*e^(d*x + c) + I*a^2 - (d*e^(2*d*x + 2*c) - d)*sqrt(a^3/d^
2)*sqrt(a/(e^(2*d*x + 2*c) - 1)))*e^(-d*x - c)/d) + sqrt(a^3/d^2)*d*log(2*(
(a*d*e^(2*d*x + 2*c) - I*a*d*e^(d*x + c) - 2*a*d)*sqrt(a^3/d^2) + (a^2*e^(3
*d*x + 3*c) - 2*I*a^2*e^(2*d*x + 2*c) - a^2*e^(d*x + c) + 2*I*a^2)*sqrt(a/(
e^(2*d*x + 2*c) - 1)))*e^(-2*d*x - 2*c)/d) - sqrt(a^3/d^2)*d*log(-2*((a*d*e
^(2*d*x + 2*c) - I*a*d*e^(d*x + c) - 2*a*d)*sqrt(a^3/d^2) - (a^2*e^(3*d*x +
 3*c) - 2*I*a^2*e^(2*d*x + 2*c) - a^2*e^(d*x + c) + 2*I*a^2)*sqrt(a/(e^(2*d
*x + 2*c) - 1)))*e^(-2*d*x - 2*c)/d) + 4*(a*e^(d*x + c) - I*a)*sqrt(a/(e^(2
*d*x + 2*c) - 1)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ia\operatorname{csch}\left(c+dx\right)+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*csch(d*x+c))**(3/2),x)
```

```
[Out] Integral((I*a*csch(c + d*x) + a)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*csch(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*csch(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a\,1\mathrm{i}}{\sinh\left(c+d\,x\right)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + (a*1i)/sinh(c + d*x))^(3/2),x)
```

```
[Out] int((a + (a*1i)/sinh(c + d*x))^(3/2), x)
```

3.53 $\int \sqrt{a+ia\,\mathrm{csch}(c+dx)}\,dx$

Optimal. Leaf size=40

$$\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\coth(c+dx)}{\sqrt{a+ia\,\mathrm{csch}(c+dx)}}\right)}{d}$$

```
[Out] 2*arctanh(coth(d*x+c)*a^(1/2)/(a+I*a*csch(d*x+c))^(1/2))*a^(1/2)/d
```

Rubi [A]
time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3859, 209}

$$\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\coth(c+dx)}{\sqrt{a+ia\,\mathrm{csch}(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Csch[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[(Sqrt[a]*Coth[c + d*x])/Sqrt[a + I*a*Csch[c + d*x]]])/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+ia\,\mathrm{csch}(c+dx)}\,dx &= -\frac{(2ia)\mathrm{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, \frac{ia\coth(c+dx)}{\sqrt{a+ia\,\mathrm{csch}(c+dx)}}\right)}{d}\\ &= \frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\coth(c+dx)}{\sqrt{a+ia\,\mathrm{csch}(c+dx)}}\right)}{d}\end{aligned}$$

Mathematica [**A**]
time = 0.58, size = 80, normalized size = 2.00

$$\frac{2(-1)^{3/4}\tanh^{-1}\left((-1)^{3/4}\sqrt{i+\operatorname{csch}(c+dx)}\right)\coth(c+dx)\sqrt{a+ia\operatorname{csch}(c+dx)}}{d(-i+\operatorname{csch}(c+dx))\sqrt{i+\operatorname{csch}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Csch[c + d*x]],x]
```

```
[Out] (2*(-1)^(3/4)*ArcTanh[(-1)^(3/4)*Sqrt[I + Csch[c + d*x]]]*Coth[c + d*x]*Sqr
t[a + I*a*Csch[c + d*x]])/(d*(-I + Csch[c + d*x])*Sqrt[I + Csch[c + d*x]])
```

Maple [**F**]
time = 4.77, size = 0, normalized size = 0.00

$$\int \sqrt{a+ia\operatorname{csch}\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*csch(d*x+c))^(1/2),x)
```

```
[Out] int((a+I*a*csch(d*x+c))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*csch(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(I*a*csch(d*x + c) + a), x)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 383 vs. 2(32) = 64.
time = 0.37, size = 383, normalized size = 9.58

$$\frac{1}{2}\sqrt{\frac{a}{d^2}}\log\left(\frac{2\left(\left(de^{(2dx+2c)}-d\right)\sqrt{\frac{a}{e^{(2dx+2c)}-1}}\sqrt{\frac{a}{d^2}}+ae^{(dx+c)}+ia\right)e^{(-dx-c)}}{d}\right)-\frac{1}{2}\sqrt{\frac{a}{d^2}}\log\left(-\frac{2\left(\left(de^{(2dx+2c)}-d\right)\sqrt{\frac{a}{e^{(2dx+2c)}-1}}\sqrt{\frac{a}{d^2}}-ae^{(dx+c)}-ia\right)e^{(-dx-c)}}{d}\right)+\frac{1}{2}\sqrt{\frac{a}{d^2}}\log\left(\frac{2\left(\left(ae^{(3dx+3c)}-2iae^{(2dx+2c)}-ae^{(dx+c)}+2ia\right)\sqrt{\frac{a}{e^{(2dx+2c)}-1}}+\left(ade^{(2dx+2c)}-iade^{(dx+c)}-2ad\right)\sqrt{\frac{a}{d^2}}\right)e^{(-2dx-2c)}}{d}\right)-\frac{1}{2}\sqrt{\frac{a}{d^2}}\log\left(\frac{2\left(\left(ae^{(3dx+3c)}-2iae^{(2dx+2c)}-ae^{(dx+c)}+2ia\right)\sqrt{\frac{a}{e^{(2dx+2c)}-1}}-\left(ade^{(2dx+2c)}-iade^{(dx+c)}-2ad\right)\sqrt{\frac{a}{d^2}}\right)e^{(-2dx-2c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*csch(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(a/d^2)*log(2*((d*e^(2*d*x + 2*c) - d)*sqrt(a/(e^(2*d*x + 2*c) - 1)
)*sqrt(a/d^2) + a*e^(d*x + c) + I*a)*e^(-d*x - c)/d) - 1/2*sqrt(a/d^2)*log(
-2*((d*e^(2*d*x + 2*c) - d)*sqrt(a/(e^(2*d*x + 2*c) - 1))*sqrt(a/d^2) - a*e
```

```
^(d*x + c) - I*a)*e^(-d*x - c)/d) + 1/2*sqrt(a/d^2)*log(2*((a*e^(3*d*x + 3*
c) - 2*I*a*e^(2*d*x + 2*c) - a*e^(d*x + c) + 2*I*a)*sqrt(a/(e^(2*d*x + 2*c)
 - 1)) + (a*d*e^(2*d*x + 2*c) - I*a*d*e^(d*x + c) - 2*a*d)*sqrt(a/d^2))*e^(
-2*d*x - 2*c)/d) - 1/2*sqrt(a/d^2)*log(2*((a*e^(3*d*x + 3*c) - 2*I*a*e^(2*d
*x + 2*c) - a*e^(d*x + c) + 2*I*a)*sqrt(a/(e^(2*d*x + 2*c) - 1)) - (a*d*e^(
2*d*x + 2*c) - I*a*d*e^(d*x + c) - 2*a*d)*sqrt(a/d^2))*e^(-2*d*x - 2*c)/d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia\operatorname{csch}(c + dx) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*csch(d*x+c))**(1/2),x)
[Out] Integral(sqrt(I*a*csch(c + d*x) + a), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*csch(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(I*a*csch(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{a + \frac{a\,1\mathrm{i}}{\sinh(c + d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + (a*1i)/sinh(c + d*x))^(1/2),x)
[Out] int((a + (a*1i)/sinh(c + d*x))^(1/2), x)
```

3.54 $\int \frac{1}{\sqrt{a+ia\textbf{csch}(c+dx)}}\,dx$

Optimal. Leaf size=91

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\coth(c+dx)}{\sqrt{2}\,\sqrt{a+ia\text{csch}(c+dx)}}\right)}{\sqrt{a}\,d}$$

```
[Out] 2*arctanh(coth(d*x+c)*a^(1/2)/(a+I*a*csch(d*x+c))^(1/2))/d/a^(1/2)-arctanh(
1/2*coth(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*csch(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2
)
```

Rubi [A]
time = 0.06, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3861, 3859, 209, 3880}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\coth(c+dx)}{\sqrt{2}\,\sqrt{a+ia\text{csch}(c+dx)}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + I*a*Csch[c + d*x]],x]
```

```
[Out] (2*ArcTanh[(Sqrt[a]*Coth[c + d*x])/Sqrt[a + I*a*Csch[c + d*x]]])/(Sqrt[a]*d
) - (Sqrt[2]*ArcTanh[(Sqrt[a]*Coth[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Csch[c +
 d*x]])])/(Sqrt[a]*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3861

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[1/a, In
t[Sqrt[a + b*Csc[c + d*x]], x], x] - Dist[b/a, Int[Csc[c + d*x]/Sqrt[a + b*
Csc[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+ia\text{csch}(c+dx)}}\,dx = -\left(i\int \frac{\text{csch}(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\,dx\right)+\frac{\int \sqrt{a+ia\text{csch}(c+dx)}\,dx}{a}$$

$$= -\frac{(2i)\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, \frac{ia\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{d}+\frac{(2i)\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x\right.}{}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{2}\sqrt{a+ia\text{csch}(c+dx}}\right.}{\sqrt{a}\,d}$$

Mathematica [**A**]
time = 0.84, size = 118, normalized size = 1.30

$$\frac{\sqrt{a}\left(2\text{ArcTan}\left(\frac{\sqrt{ia(i+\text{csch}(c+dx))}}{\sqrt{a}}\right)-\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{ia(i+\text{csch}(c+dx))}}{\sqrt{2}\sqrt{a}}\right)\right)\coth(c+dx)}{d\sqrt{ia(i+\text{csch}(c+dx))}\,\sqrt{a+ia\text{csch}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + I*a*Csch[c + d*x]],x]
[Out] (Sqrt[a]*(2*ArcTan[Sqrt[I*a*(I + Csch[c + d*x])]/Sqrt[a]] - Sqrt[2]*ArcTan[
Sqrt[I*a*(I + Csch[c + d*x])]/(Sqrt[2]*Sqrt[a])])*Coth[c + d*x])/(d*Sqrt[I*
a*(I + Csch[c + d*x])]*Sqrt[a + I*a*Csch[c + d*x]])
```

Maple [**F**]
time = 2.75, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+ia\,\text{csch}\,(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*csch(d*x+c))^(1/2),x)
[Out] int(1/(a+I*a*csch(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*csch(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(I*a*csch(d*x + c) + a), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 551 vs. 2(72) = 144.
time = 0.48, size = 551, normalized size = 6.05

$$-\frac{1}{2}\sqrt{2}\sqrt{\frac{1}{ad^2}}\log\left(2\left(\sqrt{2}\left(ade^{(2dx+2c)}-ad\right)\sqrt{\frac{a}{e^{(2dx+2c)}-1}}\sqrt{\frac{1}{ad^2}}+ae^{(dx+c)}-ia\right)e^{(-dx-c)}\right)+\frac{1}{2}\sqrt{2}\sqrt{\frac{1}{ad^2}}\log\left(-2\left(\sqrt{2}\left(ade^{(2dx+2c)}-ad\right)\sqrt{\frac{a}{e^{(2dx+2c)}-1}}\sqrt{\frac{1}{ad^2}}-ae^{(dx+c)}+ia\right)e^{(-dx-c)}\right)+\frac{1}{2}\sqrt{\frac{1}{ad^2}}\log\left(\frac{2\left(\left(de^{(2dx+2c)}-d\right)\sqrt{\frac{a}{e^{(2dx+2c)}-1}}\sqrt{\frac{1}{ad^2}}+e^{(dx+c)}+i\right)e^{(-dx-c)}}{d}\right)-\frac{1}{2}\sqrt{\frac{1}{ad^2}}\log\left(-\frac{2\left(\left(de^{(2dx+2c)}-d\right)\sqrt{\frac{a}{e^{(2dx+2c)}-1}}\sqrt{\frac{1}{ad^2}}-e^{(dx+c)}-i\right)e^{(-dx-c)}}{d}\right)+\frac{1}{2}\sqrt{\frac{1}{ad^2}}\log\left(\frac{2\left(\left(ade^{(2dx+2c)}-iade^{(dx+c)}-2ad\right)\sqrt{\frac{1}{ad^2}}+\sqrt{\frac{a}{e^{(2dx+2c)}-1}}\left(e^{(3dx+3c)}-2ie^{(2dx+2c)}-e^{(dx+c)}+2i\right)\right)e^{(-2dx-2c)}}{d}\right)-\frac{1}{2}\sqrt{\frac{1}{ad^2}}\log\left(-\frac{2\left(\left(ade^{(2dx+2c)}-iade^{(dx+c)}-2ad\right)\sqrt{\frac{1}{ad^2}}-\sqrt{\frac{a}{e^{(2dx+2c)}-1}}\left(e^{(3dx+3c)}-2ie^{(2dx+2c)}-e^{(dx+c)}+2i\right)\right)e^{(-2dx-2c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*csch(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*sqrt(1/(a*d^2))*log(2*(sqrt(2)*(a*d*e^(2*d*x + 2*c) - a*d)*sqr
t(a/(e^(2*d*x + 2*c) - 1))*sqrt(1/(a*d^2)) + a*e^(d*x + c) - I*a)*e^(-d*x -
 c)) + 1/2*sqrt(2)*sqrt(1/(a*d^2))*log(-2*(sqrt(2)*(a*d*e^(2*d*x + 2*c) - a
*d)*sqrt(a/(e^(2*d*x + 2*c) - 1))*sqrt(1/(a*d^2)) - a*e^(d*x + c) + I*a)*e^
(-d*x - c)) + 1/2*sqrt(1/(a*d^2))*log(2*((d*e^(2*d*x + 2*c) - d)*sqrt(a/(e^
(2*d*x + 2*c) - 1))*sqrt(1/(a*d^2)) + e^(d*x + c) + I)*e^(-d*x - c)/d) - 1/
2*sqrt(1/(a*d^2))*log(-2*((d*e^(2*d*x + 2*c) - d)*sqrt(a/(e^(2*d*x + 2*c) -
 1))*sqrt(1/(a*d^2)) - e^(d*x + c) - I)*e^(-d*x - c)/d) + 1/2*sqrt(1/(a*d^2
))*log(2*((a*d*e^(2*d*x + 2*c) - I*a*d*e^(d*x + c) - 2*a*d)*sqrt(1/(a*d^2))
 + sqrt(a/(e^(2*d*x + 2*c) - 1))*(e^(3*d*x + 3*c) - 2*I*e^(2*d*x + 2*c) - e
^(d*x + c) + 2*I))*e^(-2*d*x - 2*c)/d) - 1/2*sqrt(1/(a*d^2))*log(-2*((a*d*e
^(2*d*x + 2*c) - I*a*d*e^(d*x + c) - 2*a*d)*sqrt(1/(a*d^2)) - sqrt(a/(e^(2*
d*x + 2*c) - 1))*(e^(3*d*x + 3*c) - 2*I*e^(2*d*x + 2*c) - e^(d*x + c) + 2*I
))*e^(-2*d*x - 2*c)/d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ia\operatorname{csch}(c+dx)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*csch(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(I*a*csch(c + d*x) + a), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*csch(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(I*a*csch(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+\frac{a\,1\mathrm{i}}{\sinh\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + (a*1i)/sinh(c + d*x))^(1/2),x)
[Out] int(1/(a + (a*1i)/sinh(c + d*x))^(1/2), x)
```

3.55 $\int \frac{1}{(a+ia\text{csch}(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=123

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{a^{3/2}d}-\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\,\coth(c+dx)}{\sqrt{2}\,\sqrt{a+ia\text{csch}(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{\coth(c+dx)}{2d(a+ia\text{csch}(c+dx))^{3/2}}$$

```
[Out] 2*arctanh(coth(d*x+c)*a^(1/2)/(a+I*a*csch(d*x+c))^(1/2))/a^(3/2)/d-1/2*coth
(d*x+c)/d/(a+I*a*csch(d*x+c))^(3/2)-5/4*arctanh(1/2*coth(d*x+c)*a^(1/2)*2^(
1/2)/(a+I*a*csch(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3862, 4005, 3859, 209, 3880}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,\coth(c+dx)}{\sqrt{a+ia\text{csch}(c+dx)}}\right)}{a^{3/2}d}-\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\,\coth(c+dx)}{\sqrt{2}\,\sqrt{a+ia\text{csch}(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{\coth(c+dx)}{2d(a+ia\text{csch}(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Csch[c + d*x])^(-3/2),x]
```

```
[Out] (2*ArcTanh[(Sqrt[a]*Coth[c + d*x])/Sqrt[a + I*a*Csch[c + d*x]]])/(a^(3/2)*d
) - (5*ArcTanh[(Sqrt[a]*Coth[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Csch[c + d*x]]
)])/(2*Sqrt[2]*a^(3/2)*d) - Coth[c + d*x]/(2*d*(a + I*a*Csch[c + d*x])^(3/2
))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
```

```
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a+ia\operatorname{csch}(c+dx))^{3/2}}\,dx &= -\frac{\coth(c+dx)}{2d(a+ia\operatorname{csch}(c+dx))^{3/2}}-\frac{\int \frac{-2a+\frac{1}{2}ia\operatorname{csch}(c+dx)}{\sqrt{a+ia\operatorname{csch}(c+dx)}}\,dx}{2a^2}\\
&= -\frac{\coth(c+dx)}{2d(a+ia\operatorname{csch}(c+dx))^{3/2}}+\frac{\int \sqrt{a+ia\operatorname{csch}(c+dx)}\,dx}{a^2}-\frac{(5i)\int \frac{\mathrm{cs}}{\sqrt{a+i\mathrm{c}}}}{}\\
&= -\frac{\coth(c+dx)}{2d(a+ia\operatorname{csch}(c+dx))^{3/2}}-\frac{(2i)\operatorname{Subst}\left(\int \frac{1}{a+x^2}\,dx,x,\frac{ia\coth(c+dx)}{\sqrt{a+ia\operatorname{csch}(c+dx}}\right.}{ad}\\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a+ia\operatorname{csch}(c+dx)}}\right)}{a^{3/2}d}-\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{2}\sqrt{a+ia\operatorname{csch}(c+dx)}}\right.}{2\sqrt{2}\,a^{3/2}d}
\end{aligned}
$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 327 vs. 2(123) = 246.
time = 1.68, size = 327, normalized size = 2.66

$$
\frac{\left(-2\sqrt{a}-8\operatorname{ArcTan}\left(\frac{\sqrt{ia(i+\operatorname{csch}(c+dx))}}{\sqrt{a}}\right)\sqrt{ia(i+\operatorname{csch}(c+dx))}+5\sqrt{2}\operatorname{ArcTan}\left(\frac{\sqrt{ia(i+\operatorname{csch}(c+dx))}}{\sqrt{2}\sqrt{a}}\right)\sqrt{ia(i+\operatorname{csch}(c+dx))}+i\operatorname{csch}(c+dx)\left(2\sqrt{a}-8\operatorname{ArcTan}\left(\frac{\sqrt{ia(i+\operatorname{csch}(c+dx))}}{\sqrt{a}}\right)\sqrt{ia(i+\operatorname{csch}(c+dx))}+5\sqrt{2}\operatorname{ArcTan}\left(\frac{\sqrt{ia(i+\operatorname{csch}(c+dx))}}{\sqrt{2}\sqrt{a}}\right)\sqrt{ia(i+\operatorname{csch}(c+dx))}\right)\right)\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{4a^{3/2}d(i+\operatorname{csch}(c+dx))\sqrt{a+ia\operatorname{csch}(c+dx)}\left(\cosh\left(\frac{1}{2}(c+dx)\right)-i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Csch[c + d*x])^(-3/2),x]
[Out] ((-2*Sqrt[a] - 8*ArcTan[Sqrt[I*a*(I + Csch[c + d*x])]/Sqrt[a]]*Sqrt[I*a*(I
+ Csch[c + d*x])] + 5*Sqrt[2]*ArcTan[Sqrt[I*a*(I + Csch[c + d*x])]/(Sqrt[2]
```

```
*Sqrt[a])]*Sqrt[I*a*(I + Csch[c + d*x])] + I*Csch[c + d*x]*(2*Sqrt[a] - 8*A
rcTan[Sqrt[I*a*(I + Csch[c + d*x])]/Sqrt[a]]*Sqrt[I*a*(I + Csch[c + d*x])]
+ 5*Sqrt[2]*ArcTan[Sqrt[I*a*(I + Csch[c + d*x])]/(Sqrt[2]*Sqrt[a])]*Sqrt[I*
a*(I + Csch[c + d*x])]))*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]))/(4*a^(3
/2)*d*(I + Csch[c + d*x])*Sqrt[a + I*a*Csch[c + d*x]]*(Cosh[(c + d*x)/2] -
I*Sinh[(c + d*x)/2]))
```

Maple [F]
time = 2.61, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + ia\operatorname{csch}(dx + c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*csch(d*x+c))^(3/2),x)
```

```
[Out] int(1/(a+I*a*csch(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*csch(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*csch(d*x + c) + a)^(-3/2), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 873 vs. 2(96) = 192.
time = 0.45, size = 873, normalized size = 7.10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*csch(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(5*sqrt(1/2)*(a^2*d*e^(2*d*x + 2*c) + 2*I*a^2*d*e^(d*x + c) - a^2*d)*s
qrt(1/(a^3*d^2))*log(2*(2*sqrt(1/2)*(a^2*d*e^(2*d*x + 2*c) - a^2*d)*sqrt(a/
(e^(2*d*x + 2*c) - 1))*sqrt(1/(a^3*d^2)) + a*e^(d*x + c) - I*a)*e^(-d*x - c
)) - 5*sqrt(1/2)*(a^2*d*e^(2*d*x + 2*c) + 2*I*a^2*d*e^(d*x + c) - a^2*d)*sq
rt(1/(a^3*d^2))*log(-2*(2*sqrt(1/2)*(a^2*d*e^(2*d*x + 2*c) - a^2*d)*sqrt(a/
(e^(2*d*x + 2*c) - 1))*sqrt(1/(a^3*d^2)) - a*e^(d*x + c) + I*a)*e^(-d*x - c
)) - 2*(a^2*d*e^(2*d*x + 2*c) + 2*I*a^2*d*e^(d*x + c) - a^2*d)*sqrt(1/(a^3*
d^2))*log(2*((a*d*e^(2*d*x + 2*c) - a*d)*sqrt(a/(e^(2*d*x + 2*c) - 1))*sqrt
(1/(a^3*d^2)) + e^(d*x + c) + I)*e^(-d*x - c)/(a*d)) + 2*(a^2*d*e^(2*d*x +
```

```
2*c) + 2*I*a^2*d*e^(d*x + c) - a^2*d)*sqrt(1/(a^3*d^2))*log(-2*((a*d*e^(2*d
*x + 2*c) - a*d)*sqrt(a/(e^(2*d*x + 2*c) - 1))*sqrt(1/(a^3*d^2)) - e^(d*x +
 c) - I)*e^(-d*x - c)/(a*d)) - 2*(a^2*d*e^(2*d*x + 2*c) + 2*I*a^2*d*e^(d*x
+ c) - a^2*d)*sqrt(1/(a^3*d^2))*log(2*((a^2*d*e^(2*d*x + 2*c) - I*a^2*d*e^(
d*x + c) - 2*a^2*d)*sqrt(1/(a^3*d^2)) + sqrt(a/(e^(2*d*x + 2*c) - 1))*(e^(3
*d*x + 3*c) - 2*I*e^(2*d*x + 2*c) - e^(d*x + c) + 2*I))*e^(-2*d*x - 2*c)/(a
*d)) + 2*(a^2*d*e^(2*d*x + 2*c) + 2*I*a^2*d*e^(d*x + c) - a^2*d)*sqrt(1/(a^
3*d^2))*log(-2*((a^2*d*e^(2*d*x + 2*c) - I*a^2*d*e^(d*x + c) - 2*a^2*d)*sqr
t(1/(a^3*d^2)) - sqrt(a/(e^(2*d*x + 2*c) - 1))*(e^(3*d*x + 3*c) - 2*I*e^(2*
d*x + 2*c) - e^(d*x + c) + 2*I))*e^(-2*d*x - 2*c)/(a*d)) + 2*sqrt(a/(e^(2*d
*x + 2*c) - 1))*(e^(3*d*x + 3*c) - I*e^(2*d*x + 2*c) - e^(d*x + c) + I))/(a
^2*d*e^(2*d*x + 2*c) + 2*I*a^2*d*e^(d*x + c) - a^2*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ia\operatorname{csch}(c+dx)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*csch(d*x+c))**(3/2),x)
[Out] Integral((I*a*csch(c + d*x) + a)**(-3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*csch(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((I*a*csch(d*x + c) + a)^(-3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{a\,1\mathrm{i}}{\sinh(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + (a*1i)/sinh(c + d*x))^(3/2),x)
[Out] int(1/(a + (a*1i)/sinh(c + d*x))^(3/2), x)
```

3.56 $\int \sqrt{a - ia\text{csch}(c + dx)}\, dx$

Optimal. Leaf size=40

$$\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\coth(c+dx)}{\sqrt{a-ia\text{csch}(c+dx)}}\right)}{d}$$

```
[Out] 2*arctanh(coth(d*x+c)*a^(1/2)/(a-I*a*csch(d*x+c))^(1/2))*a^(1/2)/d
```

Rubi [A]
time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3859, 209}

$$\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\coth(c+dx)}{\sqrt{a-ia\text{csch}(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - I*a*Csch[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[(Sqrt[a]*Coth[c + d*x])/Sqrt[a - I*a*Csch[c + d*x]]])/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a-ia\text{csch}(c+dx)}\,dx &= \frac{(2ia)\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{ia\coth(c+dx)}{\sqrt{a-ia\text{csch}(c+dx)}}\right)}{d}\\ &= \frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\coth(c+dx)}{\sqrt{a-ia\text{csch}(c+dx)}}\right)}{d}\end{aligned}$$

Mathematica [A]
time = 0.61, size = 80, normalized size = 2.00

$$-\frac{2(-1)^{3/4}\text{ArcTan}\left((-1)^{3/4}\sqrt{-i+\text{csch}(c+dx)}\right)\coth(c+dx)\sqrt{a-ia\text{csch}(c+dx)}}{d\sqrt{-i+\text{csch}(c+dx)}\,(i+\text{csch}(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - I*a*Csch[c + d*x]],x]
```

```
[Out] (-2*(-1)^(3/4)*ArcTan[(-1)^(3/4)*Sqrt[-I + Csch[c + d*x]]]*Coth[c + d*x]*Sq
rt[a - I*a*Csch[c + d*x]])/(d*Sqrt[-I + Csch[c + d*x]]*(I + Csch[c + d*x]))
```

Maple [F]
time = 5.97, size = 0, normalized size = 0.00

$$\int\sqrt{a-ia\,\text{csch}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*csch(d*x+c))^(1/2),x)
```

```
[Out] int((a-I*a*csch(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*csch(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-I*a*csch(d*x + c) + a), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 383 vs. 2(32) = 64.
time = 0.43, size = 383, normalized size = 9.58

$$\frac{1}{2}\sqrt{\frac{a}{d^2}}\log\left(\frac{2\left(\left(de^{(2\,dx+2\,c)}-d\right)\sqrt{\frac{a}{e^{(2\,dx+2\,c)}-1}}\sqrt{\frac{a}{d^2}}+ae^{(dx+c)}-i\,a\right)e^{(-dx-c)}}{d}\right)-\frac{1}{2}\sqrt{\frac{a}{d^2}}\log\left(-\frac{2\left(\left(de^{(2\,dx+2\,c)}-d\right)\sqrt{\frac{a}{e^{(2\,dx+2\,c)}-1}}\sqrt{\frac{a}{d^2}}-ae^{(dx+c)}+i\,a\right)e^{(-dx-c)}}{d}\right)+\frac{1}{2}\sqrt{\frac{a}{d^2}}\log\left(\frac{2\left(\left(ae^{(3\,dx+3\,c)}+2i\,ae^{(2\,dx+2\,c)}-ae^{(dx+c)}-2i\,a\right)\sqrt{\frac{a}{e^{(2\,dx+2\,c)}-1}}+\left(ade^{(2\,dx+2\,c)}+i\,ade^{(dx+c)}-2\,ad\right)\sqrt{\frac{a}{d^2}}\right)e^{(-2\,dx-2\,c)}}{d}\right)-\frac{1}{2}\sqrt{\frac{a}{d^2}}\log\left(\frac{2\left(\left(ae^{(3\,dx+3\,c)}+2i\,ae^{(2\,dx+2\,c)}-ae^{(dx+c)}-2i\,a\right)\sqrt{\frac{a}{e^{(2\,dx+2\,c)}-1}}-\left(ade^{(2\,dx+2\,c)}+i\,ade^{(dx+c)}-2\,ad\right)\sqrt{\frac{a}{d^2}}\right)e^{(-2\,dx-2\,c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*csch(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(a/d^2)*log(2*((d*e^(2*d*x + 2*c) - d)*sqrt(a/(e^(2*d*x + 2*c) - 1)
)*sqrt(a/d^2) + a*e^(d*x + c) - I*a)*e^(-d*x - c)/d) - 1/2*sqrt(a/d^2)*log(
-2*((d*e^(2*d*x + 2*c) - d)*sqrt(a/(e^(2*d*x + 2*c) - 1))*sqrt(a/d^2) - a*e
```

```
^(d*x + c) + I*a)*e^(-d*x - c)/d) + 1/2*sqrt(a/d^2)*log(2*((a*e^(3*d*x + 3*
c) + 2*I*a*e^(2*d*x + 2*c) - a*e^(d*x + c) - 2*I*a)*sqrt(a/(e^(2*d*x + 2*c)
 - 1)) + (a*d*e^(2*d*x + 2*c) + I*a*d*e^(d*x + c) - 2*a*d)*sqrt(a/d^2))*e^(
-2*d*x - 2*c)/d) - 1/2*sqrt(a/d^2)*log(2*((a*e^(3*d*x + 3*c) + 2*I*a*e^(2*d
*x + 2*c) - a*e^(d*x + c) - 2*I*a)*sqrt(a/(e^(2*d*x + 2*c) - 1)) - (a*d*e^(
2*d*x + 2*c) + I*a*d*e^(d*x + c) - 2*a*d)*sqrt(a/d^2))*e^(-2*d*x - 2*c)/d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-ia\operatorname{csch}(c+dx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*csch(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(-I*a*csch(c + d*x) + a), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*csch(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-I*a*csch(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{a-\frac{a\,1{}\mathrm{i}}{\sinh\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - (a*1i)/sinh(c + d*x))^(1/2),x)
```

```
[Out] int((a - (a*1i)/sinh(c + d*x))^(1/2), x)
```

3.57 $\int \frac{1}{\sqrt{a-ia\mathbf{csch}(c+dx)}}\,dx$

Optimal. Leaf size=91

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,\coth(c+dx)}{\sqrt{a-ia\text{csch}(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\coth(c+dx)}{\sqrt{2}\,\sqrt{a-ia\text{csch}(c+dx)}}\right)}{\sqrt{a}\,d}$$

```
[Out] 2*arctanh(coth(d*x+c)*a^(1/2)/(a-I*a*csch(d*x+c))^(1/2))/d/a^(1/2)-arctanh(
1/2*coth(d*x+c)*a^(1/2)*2^(1/2)/(a-I*a*csch(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2
)
```

Rubi [A]
time = 0.07, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3861, 3859, 209, 3880}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,\coth(c+dx)}{\sqrt{a-ia\text{csch}(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\coth(c+dx)}{\sqrt{2}\,\sqrt{a-ia\text{csch}(c+dx)}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a - I*a*Csch[c + d*x]],x]
```

```
[Out] (2*ArcTanh[(Sqrt[a]*Coth[c + d*x])/Sqrt[a - I*a*Csch[c + d*x]]])/(Sqrt[a]*d
) - (Sqrt[2]*ArcTanh[(Sqrt[a]*Coth[c + d*x])/(Sqrt[2]*Sqrt[a - I*a*Csch[c +
 d*x]])])/(Sqrt[a]*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3861

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[1/a, In
t[Sqrt[a + b*Csc[c + d*x]], x], x] - Dist[b/a, Int[Csc[c + d*x]/Sqrt[a + b*
Csc[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a-ia\text{csch}(c+dx)}}\,dx = i\int \frac{\text{csch}(c+dx)}{\sqrt{a-ia\text{csch}(c+dx)}}\,dx + \frac{\int \sqrt{a-ia\text{csch}(c+dx)}\,dx}{a}$$
$$= \frac{(2i)\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{ia\coth(c+dx)}{\sqrt{a-ia\text{csch}(c+dx)}}\right)}{d} - \frac{(2i)\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx\right.}{}$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{a-ia\text{csch}(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\coth(c+dx)}{\sqrt{2}\sqrt{a-ia\text{csch}(c+d}\right.}{\sqrt{a}\,d}$$

Mathematica [A]
time = 0.78, size = 117, normalized size = 1.29

$$\frac{\sqrt{a}\left(2\text{ArcTan}\left(\frac{\sqrt{-ia(-i+\text{csch}(c+dx))}}{\sqrt{a}}\right)-\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{-ia(-i+\text{csch}(c+dx))}}{\sqrt{2}\sqrt{a}}\right)\right)\coth(c+dx)}{d\sqrt{a(-1-i\text{csch}(c+dx))}\sqrt{a-ia\text{csch}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a - I*a*Csch[c + d*x]],x]
[Out] (Sqrt[a]*(2*ArcTan[Sqrt[(-I)*a*(-I + Csch[c + d*x])]/Sqrt[a]] - Sqrt[2]*Arc
Tan[Sqrt[(-I)*a*(-I + Csch[c + d*x])]/(Sqrt[2]*Sqrt[a])])*Coth[c + d*x])/(d
*Sqrt[a*(-1 - I*Csch[c + d*x])]*Sqrt[a - I*a*Csch[c + d*x]])
```

Maple [F]
time = 3.19, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a-ia\,\text{csch}\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*csch(d*x+c))^(1/2),x)
[Out] int(1/(a-I*a*csch(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*csch(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-I*a*csch(d*x + c) + a), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 551 vs. 2(72) = 144.
time = 0.49, size = 551, normalized size = 6.05

$$-\frac{1}{2}\sqrt{2}\sqrt{\frac{1}{ad^2}}\log\left(2\left(\sqrt{2}\left(ade^{(2dx+2c)}-ad\right)\sqrt{\frac{a}{e^{(2dx+2c)}-1}}\sqrt{\frac{1}{ad^2}}+ae^{(dx+c)}+ia\right)e^{(-dx-c)}\right)+\frac{1}{2}\sqrt{2}\sqrt{\frac{1}{ad^2}}\log\left(-2\left(\sqrt{2}\left(ade^{(2dx+2c)}-ad\right)\sqrt{\frac{a}{e^{(2dx+2c)}-1}}\sqrt{\frac{1}{ad^2}}-ae^{(dx+c)}-ia\right)e^{(-dx-c)}\right)+\frac{1}{2}\sqrt{\frac{1}{ad^2}}\log\left(\frac{2\left(\left(de^{(2dx+2c)}-d\right)\sqrt{\frac{a}{e^{(2dx+2c)}-1}}\sqrt{\frac{1}{ad^2}}+e^{(dx+c)}-i\right)e^{(-dx-c)}}{d}\right)-\frac{1}{2}\sqrt{\frac{1}{ad^2}}\log\left(-\frac{2\left(\left(de^{(2dx+2c)}-d\right)\sqrt{\frac{a}{e^{(2dx+2c)}-1}}\sqrt{\frac{1}{ad^2}}-e^{(dx+c)}+i\right)e^{(-dx-c)}}{d}\right)+\frac{1}{2}\sqrt{\frac{1}{ad^2}}\log\left(\frac{2\left(\left(ade^{(2dx+2c)}+iade^{(dx+c)}-2ad\right)\sqrt{\frac{1}{ad^2}}+\sqrt{\frac{a}{e^{(2dx+2c)}-1}}\left(e^{(3dx+3c)}+2ie^{(2dx+2c)}-e^{(dx+c)}-2i\right)\right)e^{(-2dx-2c)}}{d}\right)-\frac{1}{2}\sqrt{\frac{1}{ad^2}}\log\left(-\frac{2\left(\left(ade^{(2dx+2c)}+iade^{(dx+c)}-2ad\right)\sqrt{\frac{1}{ad^2}}-\sqrt{\frac{a}{e^{(2dx+2c)}-1}}\left(e^{(3dx+3c)}+2ie^{(2dx+2c)}-e^{(dx+c)}-2i\right)\right)e^{(-2dx-2c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*csch(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*sqrt(1/(a*d^2))*log(2*(sqrt(2)*(a*d*e^(2*d*x + 2*c) - a*d)*sqr
t(a/(e^(2*d*x + 2*c) - 1))*sqrt(1/(a*d^2)) + a*e^(d*x + c) + I*a)*e^(-d*x -
 c)) + 1/2*sqrt(2)*sqrt(1/(a*d^2))*log(-2*(sqrt(2)*(a*d*e^(2*d*x + 2*c) - a
*d)*sqrt(a/(e^(2*d*x + 2*c) - 1))*sqrt(1/(a*d^2)) - a*e^(d*x + c) - I*a)*e^
(-d*x - c)) + 1/2*sqrt(1/(a*d^2))*log(2*((d*e^(2*d*x + 2*c) - d)*sqrt(a/(e^
(2*d*x + 2*c) - 1))*sqrt(1/(a*d^2)) + e^(d*x + c) - I)*e^(-d*x - c)/d) - 1/
2*sqrt(1/(a*d^2))*log(-2*((d*e^(2*d*x + 2*c) - d)*sqrt(a/(e^(2*d*x + 2*c) -
 1))*sqrt(1/(a*d^2)) - e^(d*x + c) + I)*e^(-d*x - c)/d) + 1/2*sqrt(1/(a*d^2
))*log(2*((a*d*e^(2*d*x + 2*c) + I*a*d*e^(d*x + c) - 2*a*d)*sqrt(1/(a*d^2))
 + sqrt(a/(e^(2*d*x + 2*c) - 1))*(e^(3*d*x + 3*c) + 2*I*e^(2*d*x + 2*c) - e
^(d*x + c) - 2*I))*e^(-2*d*x - 2*c)/d) - 1/2*sqrt(1/(a*d^2))*log(-2*((a*d*e
^(2*d*x + 2*c) + I*a*d*e^(d*x + c) - 2*a*d)*sqrt(1/(a*d^2)) - sqrt(a/(e^(2*
d*x + 2*c) - 1))*(e^(3*d*x + 3*c) + 2*I*e^(2*d*x + 2*c) - e^(d*x + c) - 2*I
))*e^(-2*d*x - 2*c)/d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-ia\operatorname{csch}(c+dx)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*csch(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(-I*a*csch(c + d*x) + a), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*csch(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-I*a*csch(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a-\frac{a\,1{}\mathrm{i}}{\mathrm{sinh}\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - (a*1i)/sinh(c + d*x))^(1/2),x)
```

```
[Out] int(1/(a - (a*1i)/sinh(c + d*x))^(1/2), x)
```

3.58 $\int \sqrt{3+3i\text{csch}(x)}\,dx$

Optimal. Leaf size=23

$$2\sqrt{3}\,\tanh^{-1}\left(\frac{\coth(x)}{\sqrt{1+i\text{csch}(x)}}\right)$$

```
[Out] 2*arctanh(coth(x)/(1+I*csch(x))^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3859, 209}

$$2\sqrt{3}\,\tanh^{-1}\left(\frac{\coth(x)}{\sqrt{1+i\text{csch}(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 + (3*I)*Csch[x]],x]
```

```
[Out] 2*Sqrt[3]*ArcTanh[Coth[x]/Sqrt[1 + I*Csch[x]]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{3+3i\text{csch}(x)}\,dx &= -\left(6i\text{Subst}\left(\int \frac{1}{3+x^2}\,dx, x, \frac{3i\coth(x)}{\sqrt{3+3i\text{csch}(x)}}\right)\right)\\ &= 2\sqrt{3}\,\tanh^{-1}\left(\frac{\coth(x)}{\sqrt{1+i\text{csch}(x)}}\right)\end{aligned}$$

Mathematica [A]
time = 0.49, size = 46, normalized size = 2.00

$$\frac{2\sqrt{3}\,\mathrm{ArcTan}\left(\sqrt{-1+i\,\mathrm{csch}(x)}\right)\coth(x)}{\sqrt{-1+i\,\mathrm{csch}(x)}\,\sqrt{1+i\,\mathrm{csch}(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 + (3*I)*Csch[x]],x]
```

```
[Out] (2*Sqrt[3]*ArcTan[Sqrt[-1 + I*Csch[x]]]*Coth[x])/(Sqrt[-1 + I*Csch[x]]*Sqrt
[1 + I*Csch[x]])
```

Maple [F]
time = 1.50, size = 0, normalized size = 0.00

$$\int \sqrt{3+3i\,\mathrm{csch}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+3*I*csch(x))^(1/2),x)
```

```
[Out] int((3+3*I*csch(x))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+3*I*csch(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(3*I*csch(x) + 3), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(17) = 34.
time = 0.39, size = 212, normalized size = 9.22

$$\frac{1}{2}\sqrt{3}\log\left(2\left(\frac{\sqrt{3}\left(\sqrt{3}e^{(2x)}-\sqrt{3}\right)}{\sqrt{e^{(2x)}-1}}+3e^{x}+3i\right)e^{(-x)}\right)-\frac{1}{2}\sqrt{3}\log\left(-2\left(\frac{\sqrt{3}\left(\sqrt{3}e^{(2x)}-\sqrt{3}\right)}{\sqrt{e^{(2x)}-1}}-3e^{x}-3i\right)e^{(-x)}\right)+\frac{1}{2}\sqrt{3}\log\left(6\left(\sqrt{3}e^{(2x)}-i\sqrt{3}e^{x}+\frac{\sqrt{3}\left(e^{(3x)}-2ie^{(2x)}-e^{x}+2i\right)}{\sqrt{e^{(2x)}-1}}-2\sqrt{3}\right)e^{(-2x)}\right)-\frac{1}{2}\sqrt{3}\log\left(-6\left(\sqrt{3}e^{(2x)}-i\sqrt{3}e^{x}-\frac{\sqrt{3}\left(e^{(3x)}-2ie^{(2x)}-e^{x}+2i\right)}{\sqrt{e^{(2x)}-1}}-2\sqrt{3}\right)e^{(-2x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+3*I*csch(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*log(2*(sqrt(3)*(sqrt(3)*e^(2*x) - sqrt(3))/sqrt(e^(2*x) - 1) +
3*e^x + 3*I)*e^(-x)) - 1/2*sqrt(3)*log(-2*(sqrt(3)*(sqrt(3)*e^(2*x) - sqrt(
3))/sqrt(e^(2*x) - 1) - 3*e^x - 3*I)*e^(-x)) + 1/2*sqrt(3)*log(6*(sqrt(3)*e
```

```
^(2*x) - I*sqrt(3)*e^x + sqrt(3)*(e^(3*x) - 2*I*e^(2*x) - e^x + 2*I)/sqrt(e
^(2*x) - 1) - 2*sqrt(3))*e^(-2*x)) - 1/2*sqrt(3)*log(-6*(sqrt(3)*e^(2*x) -
I*sqrt(3)*e^x - sqrt(3)*(e^(3*x) - 2*I*e^(2*x) - e^x + 2*I)/sqrt(e^(2*x) -
1) - 2*sqrt(3))*e^(-2*x))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\sqrt{3}\int\sqrt{i\operatorname{csch}(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+3*I*csch(x))**(1/2),x)
```

```
[Out] sqrt(3)*Integral(sqrt(I*csch(x) + 1), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+3*I*csch(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(3*I*csch(x) + 3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.04

$$\int\sqrt{3+\frac{3\mathrm{i}}{\sinh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3i/sinh(x) + 3)^(1/2),x)
```

```
[Out] int((3i/sinh(x) + 3)^(1/2), x)
```

3.59 $\int \sqrt{3-3i\text{csch}(x)}\,dx$

Optimal. Leaf size=23

$$2\sqrt{3}\tanh^{-1}\left(\frac{\coth(x)}{\sqrt{1-i\text{csch}(x)}}\right)$$

```
[Out] 2*arctanh(coth(x)/(1-I*csch(x))^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3859, 209}

$$2\sqrt{3}\tanh^{-1}\left(\frac{\coth(x)}{\sqrt{1-i\text{csch}(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - (3*I)*Csch[x]],x]
```

```
[Out] 2*Sqrt[3]*ArcTanh[Coth[x]/Sqrt[1 - I*Csch[x]]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{3-3i\text{csch}(x)}\,dx &= 6i\text{Subst}\left(\int \frac{1}{3+x^2}\,dx, x, -\frac{3i\coth(x)}{\sqrt{3-3i\text{csch}(x)}}\right)\\ &= 2\sqrt{3}\tanh^{-1}\left(\frac{\coth(x)}{\sqrt{1-i\text{csch}(x)}}\right)\end{aligned}$$

Mathematica [A]
time = 0.46, size = 46, normalized size = 2.00

$$\frac{2\sqrt{3}\,\text{ArcTan}\left(\sqrt{-1-i\text{csch}(x)}\right)\coth(x)}{\sqrt{-1-i\text{csch}(x)}\,\sqrt{1-i\text{csch}(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - (3*I)*Csch[x]],x]
```

```
[Out] (2*Sqrt[3]*ArcTan[Sqrt[-1 - I*Csch[x]]]*Coth[x])/(Sqrt[-1 - I*Csch[x]]*Sqrt
[1 - I*Csch[x]])
```

Maple [F]
time = 1.42, size = 0, normalized size = 0.00

$$\int \sqrt{3-3i\text{csch}\left(x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-3*I*csch(x))^(1/2),x)
```

```
[Out] int((3-3*I*csch(x))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*I*csch(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-3*I*csch(x) + 3), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(17) = 34.
time = 0.38, size = 212, normalized size = 9.22

$$\frac{1}{2}\sqrt{3}\log\left(2\left(\frac{\sqrt{3}\left(\sqrt{3}e^{(2x)}-\sqrt{3}\right)}{\sqrt{e^{(2x)}-1}}+3e^{x}-3i\right)e^{(-x)}\right)-\frac{1}{2}\sqrt{3}\log\left(-2\left(\frac{\sqrt{3}\left(\sqrt{3}e^{(2x)}-\sqrt{3}\right)}{\sqrt{e^{(2x)}-1}}-3e^{x}+3i\right)e^{(-x)}\right)+\frac{1}{2}\sqrt{3}\log\left(6\left(\sqrt{3}e^{(2x)}+i\sqrt{3}e^{x}+\frac{\sqrt{3}\left(e^{(3x)}+2ie^{(2x)}-e^{x}-2i\right)}{\sqrt{e^{(2x)}-1}}-2\sqrt{3}\right)e^{(-2x)}\right)-\frac{1}{2}\sqrt{3}\log\left(-6\left(\sqrt{3}e^{(2x)}+i\sqrt{3}e^{x}-\frac{\sqrt{3}\left(e^{(3x)}+2ie^{(2x)}-e^{x}-2i\right)}{\sqrt{e^{(2x)}-1}}-2\sqrt{3}\right)e^{(-2x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*I*csch(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*log(2*(sqrt(3)*(sqrt(3)*e^(2*x) - sqrt(3))/sqrt(e^(2*x) - 1) +
3*e^x - 3*I)*e^(-x)) - 1/2*sqrt(3)*log(-2*(sqrt(3)*(sqrt(3)*e^(2*x) - sqrt(
3))/sqrt(e^(2*x) - 1) - 3*e^x + 3*I)*e^(-x)) + 1/2*sqrt(3)*log(6*(sqrt(3)*e
```

```
^(2*x) + I*sqrt(3)*e^x + sqrt(3)*(e^(3*x) + 2*I*e^(2*x) - e^x - 2*I)/sqrt(e
^(2*x) - 1) - 2*sqrt(3))*e^(-2*x)) - 1/2*sqrt(3)*log(-6*(sqrt(3)*e^(2*x) +
I*sqrt(3)*e^x - sqrt(3)*(e^(3*x) + 2*I*e^(2*x) - e^x - 2*I)/sqrt(e^(2*x) -
1) - 2*sqrt(3))*e^(-2*x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\sqrt{3}\int\sqrt{-i\operatorname{csch}(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*I*csch(x))**(1/2),x)
```

```
[Out] sqrt(3)*Integral(sqrt(-I*csch(x) + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*I*csch(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-3*I*csch(x) + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int\sqrt{3-\frac{3\mathrm{i}}{\sinh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3 - 3i/sinh(x))^(1/2),x)
```

```
[Out] int((3 - 3i/sinh(x))^(1/2), x)
```

3.60 $\int \sqrt{-3+3i\text{csch}(x)}\,dx$

Optimal. Leaf size=23

$$-2\sqrt{3}\,\text{ArcTan}\left(\frac{\coth(x)}{\sqrt{-1+i\text{csch}(x)}}\right)$$

```
[Out] -2*arctan(coth(x)/(-1+I*csch(x))^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3859, 213}

$$-2\sqrt{3}\,\text{ArcTan}\left(\frac{\coth(x)}{\sqrt{-1+i\text{csch}(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-3 + (3*I)*Csch[x]],x]
```

```
[Out] -2*Sqrt[3]*ArcTan[Coth[x]/Sqrt[-1 + I*Csch[x]]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d), Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{-3+3i\text{csch}(x)}\,dx &= -\left(6i\text{Subst}\left(\int \frac{1}{-3+x^2}\,dx, x, \frac{3i\coth(x)}{\sqrt{-3+3i\text{csch}(x)}}\right)\right)\\ &= -2\sqrt{3}\,\tan^{-1}\left(\frac{\coth(x)}{\sqrt{-1+i\text{csch}(x)}}\right)\end{aligned}$$

Mathematica [A]
time = 0.45, size = 46, normalized size = 2.00

$$-\frac{2\sqrt{3}\tanh^{-1}\left(\sqrt{1+i\operatorname{csch}(x)}\right)\coth(x)}{\sqrt{-1+i\operatorname{csch}(x)}\sqrt{1+i\operatorname{csch}(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-3 + (3*I)*Csch[x]],x]
```

```
[Out] (-2*Sqrt[3]*ArcTanh[Sqrt[1 + I*Csch[x]]]*Coth[x])/(Sqrt[-1 + I*Csch[x]]*Sqr
t[1 + I*Csch[x]])
```

Maple [F]
time = 1.72, size = 0, normalized size = 0.00

$$\int\sqrt{-3+3i\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+3*I*csch(x))^(1/2),x)
```

```
[Out] int((-3+3*I*csch(x))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+3*I*csch(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(3*I*csch(x) - 3), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(17) = 34.
time = 0.38, size = 218, normalized size = 9.48

$$-\frac{1}{2}i\sqrt{3}\log\left(-2\left(\frac{\sqrt{3}\left(i\sqrt{3}e^{(2x)}-i\sqrt{3}\right)}{\sqrt{e^{(2x)}-1}}-3ie^{x}-3\right)e^{(-x)}\right)+\frac{1}{2}i\sqrt{3}\log\left(-2\left(\frac{\sqrt{3}\left(-i\sqrt{3}e^{(2x)}+i\sqrt{3}\right)}{\sqrt{e^{(2x)}-1}}-3ie^{x}-3\right)e^{(-x)}\right)-\frac{1}{2}i\sqrt{3}\log\left(-6\left(i\sqrt{3}e^{(2x)}-\sqrt{3}e^{x}+\frac{\sqrt{3}\left(-ie^{(3x)}+2e^{(2x)}+ie^{x}-2\right)}{\sqrt{e^{(2x)}-1}}-2i\sqrt{3}\right)e^{(-2x)}\right)+\frac{1}{2}i\sqrt{3}\log\left(-6\left(-i\sqrt{3}e^{(2x)}+\sqrt{3}e^{x}+\frac{\sqrt{3}\left(-ie^{(3x)}+2e^{(2x)}+ie^{x}-2\right)}{\sqrt{e^{(2x)}-1}}+2i\sqrt{3}\right)e^{(-2x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+3*I*csch(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*I*sqrt(3)*log(-2*(sqrt(3)*(I*sqrt(3)*e^(2*x) - I*sqrt(3))/sqrt(e^(2*x)
 - 1) - 3*I*e^x - 3)*e^(-x)) + 1/2*I*sqrt(3)*log(-2*(sqrt(3)*(-I*sqrt(3)*e^
(2*x) + I*sqrt(3))/sqrt(e^(2*x) - 1) - 3*I*e^x - 3)*e^(-x)) - 1/2*I*sqrt(3)
```

```
*log(-6*(I*sqrt(3)*e^(2*x) - sqrt(3)*e^x + sqrt(3)*(-I*e^(3*x) + 2*e^(2*x)
+ I*e^x - 2)/sqrt(e^(2*x) - 1) - 2*I*sqrt(3))*e^(-2*x)) + 1/2*I*sqrt(3)*log
(-6*(-I*sqrt(3)*e^(2*x) + sqrt(3)*e^x + sqrt(3)*(-I*e^(3*x) + 2*e^(2*x) + I
*e^x - 2)/sqrt(e^(2*x) - 1) + 2*I*sqrt(3))*e^(-2*x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\sqrt{3}\int\sqrt{i\operatorname{csch}(x)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+3*I*csch(x))**(1/2),x)
```

```
[Out] sqrt(3)*Integral(sqrt(I*csch(x) - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+3*I*csch(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(3*I*csch(x) - 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int\sqrt{-3+\frac{3\mathrm{i}}{\sinh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3i/sinh(x) - 3)^(1/2),x)
```

```
[Out] int((3i/sinh(x) - 3)^(1/2), x)
```

3.61 $\int \sqrt{-3-3i\text{csch}(x)}\,dx$

Optimal. Leaf size=23

$$-2\sqrt{3}\,\text{ArcTan}\left(\frac{\coth(x)}{\sqrt{-1-i\text{csch}(x)}}\right)$$

[Out] -2*arctan(coth(x)/(-1-I*csch(x))^(1/2))*3^(1/2)

Rubi [A]
time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3859, 213}

$$-2\sqrt{3}\,\text{ArcTan}\left(\frac{\coth(x)}{\sqrt{-1-i\text{csch}(x)}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-3 - (3*I)*Csch[x]],x]

[Out] -2*Sqrt[3]*ArcTan[Coth[x]/Sqrt[-1 - I*Csch[x]]]

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d), Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{-3-3i\text{csch}(x)}\,dx &= 6i\text{Subst}\left(\int \frac{1}{-3+x^2}\,dx, x, -\frac{3i\coth(x)}{\sqrt{-3-3i\text{csch}(x)}}\right)\\ &= -2\sqrt{3}\,\tan^{-1}\left(\frac{\coth(x)}{\sqrt{-1-i\text{csch}(x)}}\right)\end{aligned}$$

Mathematica [A]
time = 0.45, size = 46, normalized size = 2.00

$$-\frac{2\sqrt{3}\tanh^{-1}\left(\sqrt{1-i\text{csch}(x)}\right)\coth(x)}{\sqrt{-1-i\text{csch}(x)}\sqrt{1-i\text{csch}(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-3 - (3*I)*Csch[x]],x]
```

```
[Out] (-2*Sqrt[3]*ArcTanh[Sqrt[1 - I*Csch[x]]]*Coth[x])/(Sqrt[-1 - I*Csch[x]]*Sqr
t[1 - I*Csch[x]])
```

Maple [F]
time = 1.78, size = 0, normalized size = 0.00

$$\int\sqrt{-3-3i\text{csch}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3-3*I*csch(x))^(1/2),x)
```

```
[Out] int((-3-3*I*csch(x))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-3*I*csch(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-3*I*csch(x) - 3), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(17) = 34.
time = 0.42, size = 218, normalized size = 9.48

$$-\frac{1}{2}i\sqrt{3}\log\left(-2\left(\frac{\sqrt{3}\left(i\sqrt{3}e^{(2x)}-i\sqrt{3}\right)}{\sqrt{e^{(2x)}-1}}-3ie^{x}+3\right)e^{(-x)}\right)+\frac{1}{2}i\sqrt{3}\log\left(-2\left(\frac{\sqrt{3}\left(-i\sqrt{3}e^{(2x)}+i\sqrt{3}\right)}{\sqrt{e^{(2x)}-1}}-3ie^{x}+3\right)e^{(-x)}\right)-\frac{1}{2}i\sqrt{3}\log\left(-6\left(i\sqrt{3}e^{(2x)}+\sqrt{3}e^{x}+\frac{\sqrt{3}\left(-ie^{(3x)}-2e^{(2x)}+ie^{x}+2\right)}{\sqrt{e^{(2x)}-1}}-2i\sqrt{3}\right)e^{(-2x)}\right)+\frac{1}{2}i\sqrt{3}\log\left(-6\left(-i\sqrt{3}e^{(2x)}-\sqrt{3}e^{x}+\frac{\sqrt{3}\left(-ie^{(3x)}-2e^{(2x)}+ie^{x}+2\right)}{\sqrt{e^{(2x)}-1}}+2i\sqrt{3}\right)e^{(-2x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-3*I*csch(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*I*sqrt(3)*log(-2*(sqrt(3)*(I*sqrt(3)*e^(2*x) - I*sqrt(3))/sqrt(e^(2*x)
 - 1) - 3*I*e^x + 3)*e^(-x)) + 1/2*I*sqrt(3)*log(-2*(sqrt(3)*(-I*sqrt(3)*e^
(2*x) + I*sqrt(3))/sqrt(e^(2*x) - 1) - 3*I*e^x + 3)*e^(-x)) - 1/2*I*sqrt(3)
```

```
*log(-6*(I*sqrt(3)*e^(2*x) + sqrt(3)*e^x + sqrt(3)*(-I*e^(3*x) - 2*e^(2*x)
+ I*e^x + 2)/sqrt(e^(2*x) - 1) - 2*I*sqrt(3))*e^(-2*x)) + 1/2*I*sqrt(3)*log
(-6*(-I*sqrt(3)*e^(2*x) - sqrt(3)*e^x + sqrt(3)*(-I*e^(3*x) - 2*e^(2*x) + I
*e^x + 2)/sqrt(e^(2*x) - 1) + 2*I*sqrt(3))*e^(-2*x))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\sqrt{3}\int\sqrt{-i\operatorname{csch}(x)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-3*I*csch(x))**(1/2),x)
[Out] sqrt(3)*Integral(sqrt(-I*csch(x) - 1), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3-3*I*csch(x))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-3*I*csch(x) - 3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.04

$$\int\sqrt{-3-\frac{3\mathrm{i}}{\sinh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- 3i/sinh(x) - 3)^(1/2),x)
[Out] int((- 3i/sinh(x) - 3)^(1/2), x)
```

3.62 $\int \frac{\sinh^4(x)}{i+\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=58

$$-\frac{15ix}{8}-4\cosh(x)+\frac{4\cosh^3(x)}{3}+\frac{15}{8}i\cosh(x)\sinh(x)-\frac{5}{4}i\cosh(x)\sinh^3(x)-\frac{\cosh(x)\sinh^3(x)}{i+\operatorname{csch}(x)}$$

```
[Out] -15/8*I*x-4*cosh(x)+4/3*cosh(x)^3+15/8*I*cosh(x)*sinh(x)-5/4*I*cosh(x)*sinh
(x)^3-cosh(x)*sinh(x)^3/(I+csch(x))
```

Rubi [A]
time = 0.06, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3904, 3872, 2715, 8, 2713}

$$-\frac{15ix}{8}+\frac{4\cosh^3(x)}{3}-4\cosh(x)-\frac{5}{4}i\sinh^3(x)\cosh(x)+\frac{15}{8}i\sinh(x)\cosh(x)-\frac{\sinh^3(x)\cosh(x)}{\operatorname{csch}(x)+i}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^4/(I + Csch[x]),x]
```

```
[Out] ((-15*I)/8)*x - 4*Cosh[x] + (4*Cosh[x]^3)/3 + ((15*I)/8)*Cosh[x]*Sinh[x] -
((5*I)/4)*Cosh[x]*Sinh[x]^3 - (Cosh[x]*Sinh[x]^3)/(I + Csch[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^4(x)}{i+\operatorname{csch}(x)}\,dx &= -\frac{\cosh(x)\sinh^3(x)}{i+\operatorname{csch}(x)} + \int (-5i+4\operatorname{csch}(x))\sinh^4(x)\,dx \\
&= -\frac{\cosh(x)\sinh^3(x)}{i+\operatorname{csch}(x)} - 5i\int \sinh^4(x)\,dx + 4\int \sinh^3(x)\,dx \\
&= -\frac{5}{4}i\cosh(x)\sinh^3(x) - \frac{\cosh(x)\sinh^3(x)}{i+\operatorname{csch}(x)} + \frac{15}{4}i\int \sinh^2(x)\,dx - 4\text{Subst}\left(\int \left(1-x^2\right)\, d\right. \\
&= -4\cosh(x) + \frac{4\cosh^3(x)}{3} + \frac{15}{8}i\cosh(x)\sinh(x) - \frac{5}{4}i\cosh(x)\sinh^3(x) - \frac{\cosh(x)\sinh^3(}{i+\operatorname{csch}(x)} \\
&= -\frac{15ix}{8} - 4\cosh(x) + \frac{4\cosh^3(x)}{3} + \frac{15}{8}i\cosh(x)\sinh(x) - \frac{5}{4}i\cosh(x)\sinh^3(x) - \frac{\cosh(x}{i+}
\end{aligned}
$$

Mathematica [A]
time = 0.11, size = 63, normalized size = 1.09

$$
\frac{1}{96}\left(-180ix - 168\cosh(x) + 8\cosh(3x) + \frac{192\sinh\left(\frac{x}{2}\right)}{-i\cosh\left(\frac{x}{2}\right)+\sinh\left(\frac{x}{2}\right)} + 48i\sinh(2x) - 3i\sinh(4x)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^4/(I + Csch[x]),x]
[Out] ((-180*I)*x - 168*Cosh[x] + 8*Cosh[3*x] + (192*Sinh[x/2])/((-I)*Cosh[x/2] +
 Sinh[x/2]) + (48*I)*Sinh[2*x] - (3*I)*Sinh[4*x])/96
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(46) = 92.
time = 1.22, size = 128, normalized size = 2.21

method	result
risch	$-\frac{15ix}{8}-\frac{i e^{4x}}{64}+\frac{e^{3x}}{24}+\frac{i e^{2x}}{4}-\frac{7 e^{x}}{8}-\frac{7 e^{-x}}{8}-\frac{i e^{-2x}}{4}+\frac{e^{-3x}}{24}+\frac{i e^{-4x}}{64}-\frac{2}{e^{x}-i}$
default	$-\frac{i}{4\left(\tanh\left(\frac{x}{2}\right)-1\right)^4}+\frac{15i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{8}+\frac{\frac{3}{2}+\frac{7i}{8}}{\tanh\left(\frac{x}{2}\right)-1}+\frac{-\frac{1}{2}+\frac{5i}{8}}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}+\frac{-\frac{1}{3}-\frac{i}{2}}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^3}+\frac{2i}{-i+\tanh\left(\frac{x}{2}\right)}-\frac{15i\ln(\tanh}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(I+csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*I/(tanh(1/2*x)-1)^4+15/8*I*ln(tanh(1/2*x)-1)+(3/2+7/8*I)/(tanh(1/2*x)-
1)+(-1/2+5/8*I)/(tanh(1/2*x)-1)^2-(1/3+1/2*I)/(tanh(1/2*x)-1)^3+2*I/(-I+tan
h(1/2*x))-15/8*I*ln(tanh(1/2*x)+1)+1/4*I/(tanh(1/2*x)+1)^4+(1/3-1/2*I)/(tan
h(1/2*x)+1)^3-(1/2+5/8*I)/(tanh(1/2*x)+1)^2+(-3/2+7/8*I)/(tanh(1/2*x)+1)
```

Maxima [A]
time = 0.26, size = 71, normalized size = 1.22

$$-\frac{15}{8}i\,x-\frac{-5i\,e^{(-x)}+40\,e^{(-2\,x)}+120i\,e^{(-3\,x)}+552\,e^{(-4\,x)}-3}{16\,(12i\,e^{(-4\,x)}+12\,e^{(-5\,x)})}-\frac{7}{8}\,e^{(-x)}-\frac{1}{4}i\,e^{(-2\,x)}+\frac{1}{24}\,e^{(-3\,x)}+\frac{1}{64}i\,e^{(-4\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -15/8*I*x - 1/16*(-5*I*e^(-x) + 40*e^(-2*x) + 120*I*e^(-3*x) + 552*e^(-4*x)
 - 3)/(12*I*e^(-4*x) + 12*e^(-5*x)) - 7/8*e^(-x) - 1/4*I*e^(-2*x) + 1/24*e^
(-3*x) + 1/64*I*e^(-4*x)
```

Fricas [A]
time = 0.42, size = 79, normalized size = 1.36

$$-\frac{24\,(15i\,x-7i)e^{(5\,x)}+24\,(15\,x+23)e^{(4\,x)}+3i\,e^{(9\,x)}-5\,e^{(8\,x)}-40i\,e^{(7\,x)}+120\,e^{(6\,x)}-120i\,e^{(3\,x)}+40\,e^{(2\,x)}+5i\,e^{x}-3}{192\,(e^{(5\,x)}-i\,e^{(4\,x)})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] -1/192*(24*(15*I*x - 7*I)*e^(5*x) + 24*(15*x + 23)*e^(4*x) + 3*I*e^(9*x) -
5*e^(8*x) - 40*I*e^(7*x) + 120*e^(6*x) - 120*I*e^(3*x) + 40*e^(2*x) + 5*I*e
^x - 3)/(e^(5*x) - I*e^(4*x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sinh^4(x)}{\operatorname{csch}(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**4/(I+csch(x)),x)
```

```
[Out] Integral(sinh(x)**4/(csch(x) + I), x)
```

Giac [A]
time = 0.39, size = 66, normalized size = 1.14

$$-\frac{\left(552\,e^{(4\,x)} - 120i\,e^{(3\,x)} + 40\,e^{(2\,x)} + 5i\,e^{x} - 3\right)e^{(-4\,x)}}{192\,(e^{x} - i)} - \frac{1}{64}i\,e^{(4\,x)} + \frac{1}{24}\,e^{(3\,x)} + \frac{1}{4}i\,e^{(2\,x)} - \frac{7}{8}\,e^{x} - \frac{15}{8}i\,\log\left(i\,e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(I+csch(x)),x, algorithm="giac")
```

```
[Out] -1/192*(552*e^(4*x) - 120*I*e^(3*x) + 40*e^(2*x) + 5*I*e^x - 3)*e^(-4*x)/(e
^x - I) - 1/64*I*e^(4*x) + 1/24*e^(3*x) + 1/4*I*e^(2*x) - 7/8*e^x - 15/8*I*
log(I*e^x)
```

Mupad [B]
time = 1.59, size = 64, normalized size = 1.10

$$\frac{\mathrm{e}^{-3\,x}}{24} - \frac{7\,\mathrm{e}^{-x}}{8} - \frac{\mathrm{e}^{-2\,x}\,1\mathrm{i}}{4} + \frac{\mathrm{e}^{2\,x}\,1\mathrm{i}}{4} - \frac{x\,15\mathrm{i}}{8} + \frac{\mathrm{e}^{3\,x}}{24} + \frac{\mathrm{e}^{-4\,x}\,1\mathrm{i}}{64} - \frac{\mathrm{e}^{4\,x}\,1\mathrm{i}}{64} - \frac{7\,\mathrm{e}^{x}}{8} - \frac{2}{\mathrm{e}^{x} - \mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(1/sinh(x) + 1i),x)
```

```
[Out] (exp(2*x)*1i)/4 - (7*exp(-x))/8 - (exp(-2*x)*1i)/4 - (x*15i)/8 + exp(-3*x)/
24 + exp(3*x)/24 + (exp(-4*x)*1i)/64 - (exp(4*x)*1i)/64 - (7*exp(x))/8 - 2/
(exp(x) - 1i)
```

3.63 $\int \frac{\sinh^3(x)}{i+\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=46

$$-\frac{3x}{2}+4i\cosh(x)-\frac{4}{3}i\cosh^3(x)+\frac{3}{2}\cosh(x)\sinh(x)-\frac{\cosh(x)\sinh^2(x)}{i+\operatorname{csch}(x)}$$

[Out] -3/2*x+4*I*cosh(x)-4/3*I*cosh(x)^3+3/2*cosh(x)*sinh(x)-cosh(x)*sinh(x)^2/(I +csch(x))

Rubi [A]
time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3904, 3872, 2713, 2715, 8}

$$-\frac{3x}{2}-\frac{4}{3}i\cosh^3(x)+4i\cosh(x)+\frac{3}{2}\sinh(x)\cosh(x)-\frac{\sinh^2(x)\cosh(x)}{\operatorname{csch}(x)+i}$$

Antiderivative was successfully verified.

[In] Int[Sinh[x]^3/(I + Csch[x]),x]

[Out] (-3*x)/2 + (4*I)*Cosh[x] - ((4*I)/3)*Cosh[x]^3 + (3*Cosh[x]*Sinh[x])/2 - (Cosh[x]*Sinh[x]^2)/(I + Csch[x])

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^3(x)}{i+\operatorname{csch}(x)}\,dx &= -\frac{\cosh(x)\sinh^2(x)}{i+\operatorname{csch}(x)}+\int(-4i+3\operatorname{csch}(x))\sinh^3(x)\,dx\\
&= -\frac{\cosh(x)\sinh^2(x)}{i+\operatorname{csch}(x)}-4i\int\sinh^3(x)\,dx+3\int\sinh^2(x)\,dx\\
&= \frac{3}{2}\cosh(x)\sinh(x)-\frac{\cosh(x)\sinh^2(x)}{i+\operatorname{csch}(x)}+4i\text{Subst}\left(\int\left(1-x^2\right)\,dx,x,\cosh(x)\right)-\frac{3\int 1\,d
}{2}\\
&= -\frac{3x}{2}+4i\cosh(x)-\frac{4}{3}i\cosh^3(x)+\frac{3}{2}\cosh(x)\sinh(x)-\frac{\cosh(x)\sinh^2(x)}{i+\operatorname{csch}(x)}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 56, normalized size = 1.22

$$\frac{1}{12}\left(21i\cosh(x)-i\cosh(3x)+3\left(-6x+\frac{8\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)}+\sinh(2x)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(I + Csch[x]),x]
[Out] ((21*I)*Cosh[x] - I*Cosh[3*x] + 3*(-6*x + (8*Sinh[x/2])/(Cosh[x/2] + I*Sinh
[x/2]) + Sinh[2*x]))/12
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(37) = 74.
time = 1.12, size = 101, normalized size = 2.20

method	result
risch	$-\frac{3x}{2}-\frac{i e^{3x}}{24}+\frac{e^{2x}}{8}+\frac{7i e^{x}}{8}+\frac{7i e^{-x}}{8}-\frac{e^{-2x}}{8}-\frac{i e^{-3x}}{24}+\frac{2i}{e^{x}-i}$
default	$-\frac{i}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}+\frac{\frac{1}{2}+\frac{3i}{2}}{\tanh\left(\frac{x}{2}\right)+1}+\frac{-\frac{1}{2}+\frac{i}{2}}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}-\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2}+\frac{2}{-i+\tanh\left(\frac{x}{2}\right)}+\frac{i}{3\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}+\frac{\frac{1}{2}+\frac{i}{2}}{\left(\tanh\left(\frac{x}{2}\right)-\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(I+csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*I/(tanh(1/2*x)+1)^3+(1/2+3/2*I)/(tanh(1/2*x)+1)+(-1/2+1/2*I)/(tanh(1/2
*x)+1)^2-3/2*ln(tanh(1/2*x)+1)+2/(-I+tanh(1/2*x))+1/3*I/(tanh(1/2*x)-1)^3+(
1/2+1/2*I)/(tanh(1/2*x)-1)^2+(1/2-3/2*I)/(tanh(1/2*x)-1)+3/2*ln(tanh(1/2*x)
-1)
```

Maxima [A]
time = 0.27, size = 59, normalized size = 1.28

$$-\frac{3}{2}\,x+\frac{2i\,e^{(-x)}-18\,e^{(-2\,x)}+69i\,e^{(-3\,x)}+1}{8\,(3i\,e^{(-3\,x)}+3\,e^{(-4\,x)})}+\frac{7}{8}i\,e^{(-x)}-\frac{1}{8}\,e^{(-2\,x)}-\frac{1}{24}i\,e^{(-3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -3/2*x + 1/8*(2*I*e^(-x) - 18*e^(-2*x) + 69*I*e^(-3*x) + 1)/(3*I*e^(-3*x) +
 3*e^(-4*x)) + 7/8*I*e^(-x) - 1/8*e^(-2*x) - 1/24*I*e^(-3*x)
```

Fricas [A]
time = 0.39, size = 67, normalized size = 1.46

$$-\frac{3\,(12\,x-7)e^{(4\,x)}+3\,(-12i\,x-23i)e^{(3\,x)}+i\,e^{(7\,x)}-2\,e^{(6\,x)}-18i\,e^{(5\,x)}-18\,e^{(2\,x)}-2i\,e^{x}+1}{24\,(e^{(4\,x)}-i\,e^{(3\,x)})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] -1/24*(3*(12*x - 7)*e^(4*x) + 3*(-12*I*x - 23*I)*e^(3*x) + I*e^(7*x) - 2*e^
(6*x) - 18*I*e^(5*x) - 18*e^(2*x) - 2*I*e^x + 1)/(e^(4*x) - I*e^(3*x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3(x)}{\operatorname{csch}(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(I+csch(x)),x)
```

```
[Out] Integral(sinh(x)**3/(csch(x) + I), x)
```

Giac [A]
time = 0.38, size = 50, normalized size = 1.09

$$-\frac{3}{2}x + \frac{i\left(69e^{(3x)} - 18i e^{(2x)} + 2e^{x} + i\right)e^{(-3x)}}{24\left(e^{x} - i\right)} - \frac{1}{24}i e^{(3x)} + \frac{1}{8}e^{(2x)} + \frac{7}{8}i e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(I+csch(x)),x, algorithm="giac")
```

```
[Out] -3/2*x + 1/24*I*(69*e^(3*x) - 18*I*e^(2*x) + 2*e^x + I)*e^(-3*x)/(e^x - I)
- 1/24*I*e^(3*x) + 1/8*e^(2*x) + 7/8*I*e^x
```

Mupad [B]
time = 0.18, size = 52, normalized size = 1.13

$$\frac{e^{2x}}{8} + \frac{e^{-x}\,7i}{8} - \frac{e^{-2x}}{8} - \frac{3x}{2} - \frac{e^{-3x}\,1i}{24} - \frac{e^{3x}\,1i}{24} + \frac{e^{x}\,7i}{8} + \frac{2i}{e^{x} - i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(1/sinh(x) + 1i),x)
```

```
[Out] (exp(-x)*7i)/8 - (3*x)/2 - exp(-2*x)/8 + exp(2*x)/8 - (exp(-3*x)*1i)/24 - (
exp(3*x)*1i)/24 + (exp(x)*7i)/8 + 2i/(exp(x) - 1i)
```

3.64 $\int \frac{\sinh^2(x)}{i+\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=36

$$\frac{3ix}{2}+2\cosh(x)-\frac{3}{2}i\cosh(x)\sinh(x)-\frac{\cosh(x)\sinh(x)}{i+\operatorname{csch}(x)}$$

```
[Out] 3/2*I*x+2*cosh(x)-3/2*I*cosh(x)*sinh(x)-cosh(x)*sinh(x)/(I+csch(x))
```

Rubi [A]
time = 0.04, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3904, 3872, 2715, 8, 2718}

$$\frac{3ix}{2}+2\cosh(x)-\frac{3}{2}i\sinh(x)\cosh(x)-\frac{\sinh(x)\cosh(x)}{\operatorname{csch}(x)+i}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(I + Csch[x]),x]
```

```
[Out] ((3*I)/2)*x + 2*Cosh[x] - ((3*I)/2)*Cosh[x]*Sinh[x] - (Cosh[x]*Sinh[x])/(I
+ Csch[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^2(x)}{i+\operatorname{csch}(x)}\,dx &= -\frac{\cosh(x)\sinh(x)}{i+\operatorname{csch}(x)}+\int (-3i+2\operatorname{csch}(x))\sinh^2(x)\,dx\\
&= -\frac{\cosh(x)\sinh(x)}{i+\operatorname{csch}(x)}-3i\int \sinh^2(x)\,dx+2\int \sinh(x)\,dx\\
&= 2\cosh(x)-\frac{3}{2}i\cosh(x)\sinh(x)-\frac{\cosh(x)\sinh(x)}{i+\operatorname{csch}(x)}+\frac{3}{2}i\int 1\,dx\\
&= \frac{3ix}{2}+2\cosh(x)-\frac{3}{2}i\cosh(x)\sinh(x)-\frac{\cosh(x)\sinh(x)}{i+\operatorname{csch}(x)}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 46, normalized size = 1.28

$$\cosh(x)+\frac{1}{4}i\left(6x-\frac{8\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)}-\sinh(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(I + Csch[x]),x]
[Out] Cosh[x] + (I/4)*(6*x - (8*Sinh[x/2])/(Cosh[x/2] + I*Sinh[x/2]) - Sinh[2*x])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(29) = 58.
time = 1.20, size = 80, normalized size = 2.22

method	result
risch	$\frac{3ix}{2}-\frac{i e^{2x}}{8}+\frac{e^{x}}{2}+\frac{e^{-x}}{2}+\frac{i e^{-2x}}{8}+\frac{2}{e^{x}-i}$
default	$-\frac{2i}{-i+\tanh\left(\frac{x}{2}\right)}-\frac{3i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2}-\frac{i}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{-1-\frac{i}{2}}{\tanh\left(\frac{x}{2}\right)-1}+\frac{i}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{3i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2}+\frac{1-\frac{i}{2}}{\tanh\left(\frac{x}{2}\right)+1}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(I+csch(x)),x,method=_RETURNVERBOSE)
[Out] -2*I/(-I+tanh(1/2*x))-3/2*I*ln(tanh(1/2*x)-1)-1/2*I/(tanh(1/2*x)-1)^2-(1+1/
2*I)/(tanh(1/2*x)-1)+1/2*I/(tanh(1/2*x)+1)^2+3/2*I*ln(tanh(1/2*x)+1)+(1-1/2
*I)/(tanh(1/2*x)+1)
```

Maxima [A]
time = 0.27, size = 47, normalized size = 1.31

$$\frac{3}{2} i\, x + \frac{3i\, e^{(-x)} + 20\, e^{(-2\, x)} + 1}{4\, \left(2i\, e^{(-2\, x)} + 2\, e^{(-3\, x)}\right)} + \frac{1}{2}\, e^{(-x)} + \frac{1}{8} i\, e^{(-2\, x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] 3/2*I*x + 1/4*(3*I*e^(-x) + 20*e^(-2*x) + 1)/(2*I*e^(-2*x) + 2*e^(-3*x)) +
1/2*e^(-x) + 1/8*I*e^(-2*x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(26) = 52.
time = 0.36, size = 55, normalized size = 1.53

$$-\frac{4\, \left(-3i\, x + i\right) e^{(3\, x)} - 4\, \left(3\, x + 5\right) e^{(2\, x)} + i\, e^{(5\, x)} - 3\, e^{(4\, x)} + 3i\, e^{x} - 1}{8\, \left(e^{(3\, x)} - i\, e^{(2\, x)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] -1/8*(4*(-3*I*x + I)*e^(3*x) - 4*(3*x + 5)*e^(2*x) + I*e^(5*x) - 3*e^(4*x)
+ 3*I*e^x - 1)/(e^(3*x) - I*e^(2*x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{2}\left(x\right)}{\operatorname{csch}\left(x\right) + i}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(I+csch(x)),x)
```

```
[Out] Integral(sinh(x)**2/(csch(x) + I), x)
```

Giac [A]
time = 0.40, size = 40, normalized size = 1.11

$$\frac{3}{2} i\, x + \frac{\left(-20i\, e^{(2\, x)} - 3\, e^{x} - i\right) e^{(-2\, x)}}{8\, \left(-i\, e^{x} - 1\right)} - \frac{1}{8} i\, e^{(2\, x)} + \frac{1}{2}\, e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(I+csch(x)),x, algorithm="giac")
```

```
[Out] 3/2*I*x + 1/8*(-20*I*e^(2*x) - 3*e^x - I)*e^(-2*x)/(-I*e^x - 1) - 1/8*I*e^(
2*x) + 1/2*e^x
```

Mupad [B]
time = 1.47, size = 38, normalized size = 1.06

$$\frac{x\,3\mathrm{i}}{2}+\frac{\mathrm{e}^{-x}}{2}+\frac{\mathrm{e}^{-2\,x}\,1\mathrm{i}}{8}-\frac{\mathrm{e}^{2\,x}\,1\mathrm{i}}{8}+\frac{\mathrm{e}^{x}}{2}+\frac{2}{\mathrm{e}^{x}-\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(1/sinh(x) + 1i),x)
[Out] (x*3i)/2 + exp(-x)/2 + (exp(-2*x)*1i)/8 - (exp(2*x)*1i)/8 + exp(x)/2 + 2/(e
xp(x) - 1i)
```

3.65 $\int \frac{\sinh(x)}{i+\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=20

$$x - 2i\cosh(x) - \frac{\cosh(x)}{i + \operatorname{csch}(x)}$$

[Out] x-2*I*cosh(x)-cosh(x)/(I+csch(x))

Rubi [A]

time = 0.04, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3904, 3872, 2718, 8}

$$x - 2i\cosh(x) - \frac{\cosh(x)}{\operatorname{csch}(x) + i}$$

Antiderivative was successfully verified.

[In] Int[Sinh[x]/(I + Csch[x]),x]

[Out] x - (2*I)*Cosh[x] - Cosh[x]/(I + Csch[x])

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh(x)}{i+\operatorname{csch}(x)}\,dx &= -\frac{\cosh(x)}{i+\operatorname{csch}(x)} + \int (-2i+\operatorname{csch}(x))\sinh(x)\,dx \\
&= -\frac{\cosh(x)}{i+\operatorname{csch}(x)} - 2i\int \sinh(x)\,dx + \int 1\,dx \\
&= x - 2i\cosh(x) - \frac{\cosh(x)}{i+\operatorname{csch}(x)}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 35, normalized size = 1.75

$$
x - i\cosh(x) - \frac{2\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right) + i\sinh\left(\frac{x}{2}\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(I + Csch[x]),x]
```

```
[Out] x - I*Cosh[x] - (2*Sinh[x/2])/(Cosh[x/2] + I*Sinh[x/2])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(18) = 36.
time = 1.11, size = 51, normalized size = 2.55

method	result	size
risch	$x - \frac{i e^{x}}{2} - \frac{i e^{-x}}{2} - \frac{2i}{e^{x}-i}$	25
default	$\frac{i}{\tanh\left(\frac{x}{2}\right)-1} - \ln\left(\tanh\left(\frac{x}{2}\right)-1\right) - \frac{2}{-i+\tanh\left(\frac{x}{2}\right)} - \frac{i}{\tanh\left(\frac{x}{2}\right)+1} + \ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(I+csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] I/(tanh(1/2*x)-1)-ln(tanh(1/2*x)-1)-2/(-I+tanh(1/2*x))-I/(tanh(1/2*x)+1)+ln
(tanh(1/2*x)+1)
```

Maxima [A]
time = 0.27, size = 31, normalized size = 1.55

$$
x - \frac{5i\,e^{(-x)} - 1}{2\left(i\,e^{(-x)} + e^{(-2\,x)}\right)} - \frac{1}{2}i\,e^{(-x)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] x - 1/2*(5*I*e^(-x) - 1)/(I*e^(-x) + e^(-2*x)) - 1/2*I*e^(-x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(16) = 32$.
time = 0.36, size = 39, normalized size = 1.95

$$\frac{(2x-1)e^{(2x)} + (-2ix-5i)e^{x} - i\,e^{(3x)} - 1}{2\left(e^{(2x)} - i\,e^{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(I+csch(x)),x, algorithm="fricas")
[Out] 1/2*((2*x - 1)*e^(2*x) + (-2*I*x - 5*I)*e^x - I*e^(3*x) - 1)/(e^(2*x) - I*e
^x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(x)}{\operatorname{csch}(x) + i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(I+csch(x)),x)
[Out] Integral(sinh(x)/(csch(x) + I), x)
```

Giac [A]
time = 0.39, size = 26, normalized size = 1.30

$$x + \frac{(5\,e^{x} - i)e^{(-x)}}{2\left(i\,e^{x} + 1\right)} - \frac{1}{2}i\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(I+csch(x)),x, algorithm="giac")
[Out] x + 1/2*(5*e^x - I)*e^(-x)/(I*e^x + 1) - 1/2*I*e^x
```

Mupad [B]
time = 1.46, size = 24, normalized size = 1.20

$$x - \frac{\mathrm{e}^{-x}\,1\mathrm{i}}{2} - \frac{\mathrm{e}^{x}\,1\mathrm{i}}{2} - \frac{2\mathrm{i}}{\mathrm{e}^{x} - \mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(1/sinh(x) + 1i),x)
[Out] x - (exp(-x)*1i)/2 - (exp(x)*1i)/2 - 2i/(exp(x) - 1i)
```

3.66 $\int \frac{\operatorname{csch}(x)}{i+\operatorname{csch}(x)} \, dx$

Optimal. Leaf size=14

$$\frac{i\coth(x)}{i+\operatorname{csch}(x)}$$

[Out] I*coth(x)/(I+csch(x))

Rubi [A]
time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3879}

$$\frac{i\coth(x)}{\operatorname{csch}(x)+i}$$

Antiderivative was successfully verified.

[In] Int[Csch[x]/(I + Csch[x]),x]

[Out] (I*Coth[x])/(I + Csch[x])

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\operatorname{csch}(x)}{i+\operatorname{csch}(x)} \, dx = \frac{i\coth(x)}{i+\operatorname{csch}(x)}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 1.93

$$\frac{2\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[x]/(I + Csch[x]),x]

[Out] (2*Sinh[x/2])/(Cosh[x/2] + I*Sinh[x/2])

Maple [A]
time = 0.68, size = 12, normalized size = 0.86

method	result	size
risch	$\frac{2i}{e^x - i}$	11
default	$\frac{2}{-i+\tanh\left(\frac{x}{2}\right)}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)/(I+csch(x)),x,method=_RETURNVERBOSE)
[Out] 2/(-I+tanh(1/2*x))
```

Maxima [A]
time = 0.27, size = 12, normalized size = 0.86

$$-\frac{2}{i\,e^{(-x)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(I+csch(x)),x, algorithm="maxima")
[Out] -2/(I*e^(-x) - 1)
```

Fricas [A]
time = 0.34, size = 8, normalized size = 0.57

$$\frac{2i}{e^x - i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(I+csch(x)),x, algorithm="fricas")
[Out] 2*I/(e^x - I)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(x)}{\operatorname{csch}(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(I+csch(x)),x)
[Out] Integral(csch(x)/(csch(x) + I), x)
```

Giac [A]
time = 0.39, size = 8, normalized size = 0.57

$$\frac{2i}{e^x - i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(I+csch(x)),x, algorithm="giac")
```

```
[Out] 2*I/(e^x - I)
```

Mupad [B]
time = 0.07, size = 10, normalized size = 0.71

$$\frac{2\mathrm{i}}{\mathrm{e}^x - \mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)*(1/sinh(x) + 1i)),x)
```

```
[Out] 2i/(exp(x) - 1i)
```

3.67 $\int \frac{\operatorname{csch}^2(x)}{i+\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=17

$$-\tanh^{-1}(\cosh(x))+\frac{\coth(x)}{i+\operatorname{csch}(x)}$$

```
[Out] -arctanh(cosh(x))+coth(x)/(I+csch(x))
```

Rubi [A]
time = 0.04, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3874, 3855, 3879}

$$-\tanh^{-1}(\cosh(x))+\frac{\coth(x)}{\operatorname{csch}(x)+i}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^2/(I + Csch[x]),x]
```

```
[Out] -ArcTanh[Cosh[x]] + Coth[x]/(I + Csch[x])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3874

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^2(x)}{i+\operatorname{csch}(x)}\,dx &= -\left(i\int \frac{\operatorname{csch}(x)}{i+\operatorname{csch}(x)}\,dx\right)+\int \operatorname{csch}(x)\,dx\\ &= -\tanh^{-1}(\cosh(x))+\frac{\coth(x)}{i+\operatorname{csch}(x)}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 37 vs. $2(17) = 34$.
time = 0.02, size = 37, normalized size = 2.18

$$\log\left(\tanh\left(\frac{x}{2}\right)\right) - \frac{2i\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right) + i\sinh\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^2/(I + Csch[x]),x]
```

```
[Out] Log[Tanh[x/2]] - ((2*I)*Sinh[x/2])/(Cosh[x/2] + I*Sinh[x/2])
```

Maple [A]
time = 0.62, size = 19, normalized size = 1.12

method	result	size
default	$-\frac{2i}{-i+\tanh\left(\frac{x}{2}\right)} + \ln\left(\tanh\left(\frac{x}{2}\right)\right)$	19
risch	$\frac{2}{e^{x}-i} + \ln\left(e^{x} - 1\right) - \ln\left(e^{x} + 1\right)$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^2/(I+csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*I/(-I+tanh(1/2*x))+ln(tanh(1/2*x))
```

Maxima [A]
time = 0.31, size = 27, normalized size = 1.59

$$\frac{2}{e^{(-x)} + i} - \log\left(e^{(-x)} + 1\right) + \log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] 2/(e^(-x) + I) - log(e^(-x) + 1) + log(e^(-x) - 1)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(15) = 30$.
time = 0.35, size = 31, normalized size = 1.82

$$-\frac{(e^{x} - i)\log\left(e^{x} + 1\right) - (e^{x} - i)\log\left(e^{x} - 1\right) - 2}{e^{x} - i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] -((e^x - I)*log(e^x + 1) - (e^x - I)*log(e^x - 1) - 2)/(e^x - I)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(x)}{\operatorname{csch}(x) + i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2/(I+csch(x)),x)
```

```
[Out] Integral(csch(x)**2/(csch(x) + I), x)
```

Giac [A]
time = 0.37, size = 22, normalized size = 1.29

$$\frac{2}{e^x - i} - \log(e^x + 1) + \log(|e^x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(I+csch(x)),x, algorithm="giac")
```

```
[Out] 2/(e^x - I) - log(e^x + 1) + log(abs(e^x - 1))
```

Mupad [B]
time = 0.22, size = 26, normalized size = 1.53

$$\ln(2 - 2\,e^x) - \ln(-2\,e^x - 2) + \frac{2}{e^x - i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^2*(1/sinh(x) + 1i)),x)
```

```
[Out] log(2 - 2*exp(x)) - log(- 2*exp(x) - 2) + 2/(exp(x) - 1i)
```

3.68 $\int \frac{\text{csch}^3(x)}{i+\text{csch}(x)} \, dx$

Optimal. Leaf size=26

$$i \tanh^{-1}(\cosh(x)) - \coth(x) - \frac{i \coth(x)}{i + \text{csch}(x)}$$

[Out] I*arctanh(cosh(x))-coth(x)-I*coth(x)/(I+csch(x))

Rubi [A]
time = 0.07, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3875, 3874, 3855, 3879}

$$-\coth(x) + i \tanh^{-1}(\cosh(x)) - \frac{i \coth(x)}{\text{csch}(x) + i}$$

Antiderivative was successfully verified.

[In] Int[Csch[x]^3/(I + Csch[x]),x]

[Out] I*ArcTanh[Cosh[x]] - Coth[x] - (I*Coth[x])/(I + Csch[x])

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3874

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3875

```
Int[csc[(e_.) + (f_.)*(x_)]^3/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Simp[-Cot[e + f*x]/(b*f), x] - Dist[a/b, Int[Csc[e + f*x]^2/(a + b*
Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^3(x)}{i+\operatorname{csch}(x)}\,dx &= -\coth(x) - i\int \frac{\operatorname{csch}^2(x)}{i+\operatorname{csch}(x)}\,dx \\ &= -\coth(x) - i\int \operatorname{csch}(x)\,dx - \int \frac{\operatorname{csch}(x)}{i+\operatorname{csch}(x)}\,dx \\ &= i\tanh^{-1}(\cosh(x)) - \coth(x) - \frac{i\coth(x)}{i+\operatorname{csch}(x)}\end{aligned}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 70 vs. $2(26) = 52$.
time = 0.08, size = 70, normalized size = 2.69

$$-\frac{1}{2}\coth\left(\frac{x}{2}\right) + i\log\left(\cosh\left(\frac{x}{2}\right)\right) - i\log\left(\sinh\left(\frac{x}{2}\right)\right) - \frac{2\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right) + i\sinh\left(\frac{x}{2}\right)} - \frac{1}{2}\tanh\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^3/(I + Csch[x]),x]
[Out] -1/2*Coth[x/2] + I*Log[Cosh[x/2]] - I*Log[Sinh[x/2]] - (2*Sinh[x/2])/(Cosh[
x/2] + I*Sinh[x/2]) - Tanh[x/2]/2
```

Maple [**A**]
time = 0.79, size = 35, normalized size = 1.35

method	result	size
default	$-\frac{\tanh\left(\frac{x}{2}\right)}{2} - \frac{2}{-i+\tanh\left(\frac{x}{2}\right)} - \frac{1}{2\tanh\left(\frac{x}{2}\right)} - i\ln\left(\tanh\left(\frac{x}{2}\right)\right)$	35
risch	$-\frac{2i\left(e^{2x}-2-ie^{x}\right)}{\left(e^{2x}-1\right)\left(e^{x}-i\right)} - i\ln\left(e^{x}-1\right) + i\ln\left(e^{x}+1\right)$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^3/(I+csch(x)),x,method=_RETURNVERBOSE)
[Out] -1/2*tanh(1/2*x)-2/(-I+tanh(1/2*x))-1/2/tanh(1/2*x)-I*ln(tanh(1/2*x))
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(20) = 40$.
time = 0.27, size = 53, normalized size = 2.04

$$-\frac{2\left(e^{(-x)} - i\,e^{(-2\,x)} + 2i\right)}{e^{(-x)} - i\,e^{(-2\,x)} - e^{(-3\,x)} + i} + i\,\log\left(e^{(-x)} + 1\right) - i\,\log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -2*(e^(-x) - I*e^(-2*x) + 2*I)/(e^(-x) - I*e^(-2*x) - e^(-3*x) + I) + I*log
(e^(-x) + 1) - I*log(e^(-x) - 1)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(20) = 40$.
time = 0.37, size = 77, normalized size = 2.96

$$\frac{\left(i\,e^{(3\,x)} + e^{(2\,x)} - i\,e^{x} - 1\right)\log\left(e^{x} + 1\right) + \left(-i\,e^{(3\,x)} - e^{(2\,x)} + i\,e^{x} + 1\right)\log\left(e^{x} - 1\right) - 2i\,e^{(2\,x)} - 2\,e^{x} + 4i}{e^{(3\,x)} - i\,e^{(2\,x)} - e^{x} + i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] ((I*e^(3*x) + e^(2*x) - I*e^x - 1)*log(e^x + 1) + (-I*e^(3*x) - e^(2*x) + I
*e^x + 1)*log(e^x - 1) - 2*I*e^(2*x) - 2*e^x + 4*I)/(e^(3*x) - I*e^(2*x) -
e^x + I)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^{3}\left(x\right)}{\operatorname{csch}\left(x\right) + i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**3/(I+csch(x)),x)
```

```
[Out] Integral(csch(x)**3/(csch(x) + I), x)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(20) = 40$.
time = 0.40, size = 46, normalized size = 1.77

$$\frac{2\left(e^{(2\,x)} - i\,e^{x} - 2\right)}{i\,e^{(3\,x)} + e^{(2\,x)} - i\,e^{x} - 1} + i\,\log\left(e^{x} + 1\right) - i\,\log\left(\left|e^{x} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(I+csch(x)),x, algorithm="giac")
```

```
[Out] 2*(e^(2*x) - I*e^x - 2)/(I*e^(3*x) + e^(2*x) - I*e^x - 1) + I*log(e^x + 1)
- I*log(abs(e^x - 1))
```

Mupad [B]
time = 1.64, size = 60, normalized size = 2.31

$$-\ln\left(\mathrm{e}^{x}\,2\mathrm{i} - 2\mathrm{i}\right)\,1\mathrm{i} + \ln\left(\mathrm{e}^{x}\,2\mathrm{i} + 2\mathrm{i}\right)\,1\mathrm{i} + \frac{\mathrm{e}^{2\,x}\,2\mathrm{i} + 2\,\mathrm{e}^{x} - 4\mathrm{i}}{\mathrm{e}^{2\,x}\,1\mathrm{i} - \mathrm{e}^{3\,x} + \mathrm{e}^{x} - \mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^3*(1/sinh(x) + 1i)),x)
```

```
[Out] log(exp(x)*2i + 2i)*1i - log(exp(x)*2i - 2i)*1i + (exp(2*x)*2i + 2*exp(x) -
 4i)/(exp(2*x)*1i - exp(3*x) + exp(x) - 1i)
```

3.69 $\int \frac{\operatorname{csch}^4(x)}{i+\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=37

$$\frac{3}{2}\tanh^{-1}(\cosh(x))+2i\coth(x)-\frac{3}{2}\coth(x)\operatorname{csch}(x)+\frac{\coth(x)\operatorname{csch}^2(x)}{i+\operatorname{csch}(x)}$$

```
[Out] 3/2*arctanh(cosh(x))+2*I*coth(x)-3/2*coth(x)*csch(x)+coth(x)*csch(x)^2/(I+c
sch(x))
```

Rubi [A]
time = 0.05, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3903, 3872, 3852, 8, 3853, 3855}

$$2i\coth(x)+\frac{3}{2}\tanh^{-1}(\cosh(x))+\frac{\coth(x)\operatorname{csch}^2(x)}{\operatorname{csch}(x)+i}-\frac{3}{2}\coth(x)\operatorname{csch}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^4/(I + Csch[x]),x]
```

```
[Out] (3*ArcTanh[Cosh[x]])/2 + (2*I)*Coth[x] - (3*Coth[x]*Csch[x])/2 + (Coth[x]*C
sch[x]^2)/(I + Csch[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3903

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(f*(a +
b*Csc[e + f*x]))), x] - Dist[d^2/(a*b), Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
- 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^4(x)}{i+\operatorname{csch}(x)}\,dx &= \frac{\coth(x)\operatorname{csch}^2(x)}{i+\operatorname{csch}(x)} - \int (2i-3\operatorname{csch}(x))\operatorname{csch}^2(x)\,dx\\ &= \frac{\coth(x)\operatorname{csch}^2(x)}{i+\operatorname{csch}(x)} - 2i\int \operatorname{csch}^2(x)\,dx + 3\int \operatorname{csch}^3(x)\,dx\\ &= -\frac{3}{2}\coth(x)\operatorname{csch}(x) + \frac{\coth(x)\operatorname{csch}^2(x)}{i+\operatorname{csch}(x)} - \frac{3}{2}\int \operatorname{csch}(x)\,dx - 2\mathrm{Subst}(\int 1\,dx, x, -i\coth(x\\ &= \frac{3}{2}\tanh^{-1}(\cosh(x)) + 2i\coth(x) - \frac{3}{2}\coth(x)\operatorname{csch}(x) + \frac{\coth(x)\operatorname{csch}^2(x)}{i+\operatorname{csch}(x)}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 81 vs. $2(37) = 74$.
time = 0.23, size = 81, normalized size = 2.19

$$\frac{1}{8}\left(4i\coth\left(\frac{x}{2}\right) - \operatorname{csch}^2\left(\frac{x}{2}\right) - 12\log\left(\tanh\left(\frac{x}{2}\right)\right) - \operatorname{sech}^2\left(\frac{x}{2}\right) + \frac{16\sinh\left(\frac{x}{2}\right)}{-i\cosh\left(\frac{x}{2}\right) + \sinh\left(\frac{x}{2}\right)} + 4i\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^4/(I + Csch[x]),x]
```

```
[Out] ((4*I)*Coth[x/2] - Csch[x/2]^2 - 12*Log[Tanh[x/2]] - Sech[x/2]^2 + (16*Sinh
[x/2])/((-I)*Cosh[x/2] + Sinh[x/2]) + (4*I)*Tanh[x/2])/8
```

Maple [A]
time = 0.72, size = 53, normalized size = 1.43

method	result	size
default	$\frac{i\tanh\left(\frac{x}{2}\right)}{2}+\frac{\left(\tanh^{2}\left(\frac{x}{2}\right)\right)}{8}-\frac{1}{8\tanh\left(\frac{x}{2}\right)^{2}}+\frac{i}{2\tanh\left(\frac{x}{2}\right)}-\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2}+\frac{2i}{-i+\tanh\left(\frac{x}{2}\right)}$	53
risch	$-\frac{-5\,\mathrm{e}^{2x}-3i\mathrm{e}^{3x}+3\,\mathrm{e}^{4x}+4+i\mathrm{e}^{x}}{\left(\mathrm{e}^{2x}-1\right)^{2}\left(\mathrm{e}^{x}-i\right)}-\frac{3\ln\left(\mathrm{e}^{x}-1\right)}{2}+\frac{3\ln\left(\mathrm{e}^{x}+1\right)}{2}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^4/(I+csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*tanh(1/2*x)+1/8*tanh(1/2*x)^2-1/8/tanh(1/2*x)^2+1/2*I/tanh(1/2*x)-3/2
*ln(tanh(1/2*x))+2*I/(-I+tanh(1/2*x))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(29) = 58.
time = 0.28, size = 77, normalized size = 2.08

$$-\frac{-i\,e^{(-x)}-5\,e^{(-2\,x)}+3i\,e^{(-3\,x)}+3\,e^{(-4\,x)}+4}{e^{(-x)}-2i\,e^{(-2\,x)}-2\,e^{(-3\,x)}+i\,e^{(-4\,x)}+e^{(-5\,x)}+i}+\frac{3}{2}\log\left(e^{(-x)}+1\right)-\frac{3}{2}\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -(-I*e^(-x) - 5*e^(-2*x) + 3*I*e^(-3*x) + 3*e^(-4*x) + 4)/(e^(-x) - 2*I*e^(
-2*x) - 2*e^(-3*x) + I*e^(-4*x) + e^(-5*x) + I) + 3/2*log(e^(-x) + 1) - 3/2
*log(e^(-x) - 1)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(29) = 58.
time = 0.35, size = 120, normalized size = 3.24

$$\frac{3\left(e^{(5\,x)}-i\,e^{(4\,x)}-2\,e^{(3\,x)}+2i\,e^{(2\,x)}+e^{x}-i\right)\log\left(e^{x}+1\right)-3\left(e^{(5\,x)}-i\,e^{(4\,x)}-2\,e^{(3\,x)}+2i\,e^{(2\,x)}+e^{x}-i\right)\log\left(e^{x}-1\right)-6\,e^{(4\,x)}+6i\,e^{(3\,x)}+10\,e^{(2\,x)}-2i\,e^{x}-8}{2\left(e^{(5\,x)}-i\,e^{(4\,x)}-2\,e^{(3\,x)}+2i\,e^{(2\,x)}+e^{x}-i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(3*(e^(5*x) - I*e^(4*x) - 2*e^(3*x) + 2*I*e^(2*x) + e^x - I)*log(e^x +
1) - 3*(e^(5*x) - I*e^(4*x) - 2*e^(3*x) + 2*I*e^(2*x) + e^x - I)*log(e^x -
1) - 6*e^(4*x) + 6*I*e^(3*x) + 10*e^(2*x) - 2*I*e^x - 8)/(e^(5*x) - I*e^(4*
x) - 2*e^(3*x) + 2*I*e^(2*x) + e^x - I)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{csch}^{4}\left(x\right)}{\operatorname{csch}\left(x\right)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**4/(I+csch(x)),x)
```

```
[Out] Integral(csch(x)**4/(csch(x) + I), x)
```

Giac [A]
time = 0.40, size = 50, normalized size = 1.35

$$-\frac{e^{(3x)} - 2i e^{(2x)} + e^x + 2i}{\left(e^{(2x)} - 1\right)^2} - \frac{2i}{i e^x + 1} + \frac{3}{2} \log\left(e^x + 1\right) - \frac{3}{2} \log\left(\left|e^x - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(I+csch(x)),x, algorithm="giac")
```

```
[Out] -(e^(3*x) - 2*I*e^(2*x) + e^x + 2*I)/(e^(2*x) - 1)^2 - 2*I/(I*e^x + 1) + 3/
2*log(e^x + 1) - 3/2*log(abs(e^x - 1))
```

Mupad [B]
time = 1.69, size = 63, normalized size = 1.70

$$\frac{3\ln\left(3\,\mathrm{e}^x + 3\right)}{2} - \frac{3\ln\left(3\,\mathrm{e}^x - 3\right)}{2} - \frac{\mathrm{e}^x}{\mathrm{e}^{2x} - 1} - \frac{2\,\mathrm{e}^x}{\left(\mathrm{e}^{2x} - 1\right)^2} - \frac{2}{\mathrm{e}^x - \mathrm{i}} + \frac{2\mathrm{i}}{\mathrm{e}^{2x} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^4*(1/sinh(x) + 1i)),x)
```

```
[Out] (3*log(3*exp(x) + 3))/2 - (3*log(3*exp(x) - 3))/2 - exp(x)/(exp(2*x) - 1) -
 (2*exp(x))/(exp(2*x) - 1)^2 - 2/(exp(x) - 1i) + 2i/(exp(2*x) - 1)
```

3.70 $\int (a + b\text{csch}(c + dx))^4 \, dx$

Optimal. Leaf size=109

$$a^4x - \frac{2ab(2a^2-b^2)\tanh^{-1}(\cosh(c+dx))}{d} - \frac{b^2(17a^2-2b^2)\coth(c+dx)}{3d} - \frac{4ab^3\coth(c+dx)\text{csch}(c+dx)}{3d} - b^2$$

```
[Out] a^4*x-2*a*b*(2*a^2-b^2)*arctanh(cosh(d*x+c))/d-1/3*b^2*(17*a^2-2*b^2)*coth(
d*x+c)/d-4/3*a*b^3*coth(d*x+c)*csch(d*x+c)/d-1/3*b^2*coth(d*x+c)*(a+b*csch(
d*x+c))^2/d
```

Rubi [A]
time = 0.09, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3867, 4133, 3855, 3852, 8}

$$a^4x - \frac{b^2(17a^2-2b^2)\coth(c+dx)}{3d} - \frac{2ab(2a^2-b^2)\tanh^{-1}(\cosh(c+dx))}{d} - \frac{4ab^3\coth(c+dx)\text{csch}(c+dx)}{3d} - \frac{b^2\coth(c+dx)(a+b\text{csch}(c+dx))^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Csch[c + d*x])^4,x]
```

```
[Out] a^4*x - (2*a*b*(2*a^2 - b^2)*ArcTanh[Cosh[c + d*x]])/d - (b^2*(17*a^2 - 2*b
^2)*Coth[c + d*x])/(3*d) - (4*a*b^3*Coth[c + d*x]*Csch[c + d*x])/(3*d) - (b
^2*Coth[c + d*x]*(a + b*Csch[c + d*x])^2)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3867

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Dist[1/(n - 1)
, Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a
^2*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /;
```

```
FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 4133

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(-b)*C*Csc[e +
 f*x]*(Cot[e + f*x]/(2*f)), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A
 + C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x]
```

Rubi steps

$$\int (a+b\operatorname{csch}(c+dx))^4\,dx = -\frac{b^2\coth(c+dx)(a+b\operatorname{csch}(c+dx))^2}{3d}+\frac{1}{3}\int (a+b\operatorname{csch}(c+dx))\left(3a^3+b\left(9a^2\right.\right.$$

$$= -\frac{4ab^3\coth(c+dx)\operatorname{csch}(c+dx)}{3d}-\frac{b^2\coth(c+dx)(a+b\operatorname{csch}(c+dx))^2}{3d}+\frac{1}{6}\int$$

$$= a^4x-\frac{4ab^3\coth(c+dx)\operatorname{csch}(c+dx)}{3d}-\frac{b^2\coth(c+dx)(a+b\operatorname{csch}(c+dx))^2}{3d}+$$

$$= a^4x-\frac{2ab(2a^2-b^2)\tanh^{-1}(\cosh(c+dx))}{d}-\frac{4ab^3\coth(c+dx)\operatorname{csch}(c+dx)}{3d}-$$

$$= a^4x-\frac{2ab(2a^2-b^2)\tanh^{-1}(\cosh(c+dx))}{d}-\frac{b^2(17a^2-2b^2)\coth(c+dx)}{3d}-$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 508 vs. 2(109) = 218.
time = 6.18, size = 508, normalized size = 4.66

$$\frac{a^4(c+dx)(a+b\operatorname{csch}(c+dx))^4\sinh^4(c+dx)}{d(b+a\sinh(c+dx))^4}+\frac{\left(-9a^2b^2\cosh\left(\frac{1}{2}(c+dx)\right)+b^4\cosh\left(\frac{1}{2}(c+dx)\right)\right)\operatorname{csch}\left(\frac{1}{2}(c+dx)\right)(a+b\operatorname{csch}(c+dx))^4\sinh^4(c+dx)}{3d(b+a\sinh(c+dx))^4}-\frac{ab^3\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)(a+b\operatorname{csch}(c+dx))^4\sinh^4(c+dx)}{2d(b+a\sinh(c+dx))^4}-\frac{b^4\coth\left(\frac{1}{2}(c+dx)\right)\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)(a+b\operatorname{csch}(c+dx))^4\sinh^4(c+dx)}{24d(b+a\sinh(c+dx))^4}+\frac{2ab(2a^2-b^2)(a+b\operatorname{csch}(c+dx))^4\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)\sinh^4(c+dx)}{d(b+a\sinh(c+dx))^4}-\frac{ab^3(a+b\operatorname{csch}(c+dx))^4\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)\sinh^4(c+dx)}{2d(b+a\sinh(c+dx))^4}+\frac{(a+b\operatorname{csch}(c+dx))^4\operatorname{sech}\left(\frac{1}{2}(c+dx)\right)\left(-9a^2b^2\sinh\left(\frac{1}{2}(c+dx)\right)+b^4\sinh\left(\frac{1}{2}(c+dx)\right)\right)\sinh^4(c+dx)}{3d(b+a\sinh(c+dx))^4}+\frac{b^4(a+b\operatorname{csch}(c+dx))^4\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)\sinh^4(c+dx)\tanh\left(\frac{1}{2}(c+dx)\right)}{24d(b+a\sinh(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Csch[c + d*x])^4,x]
```

```
[Out] (a^4*(c + d*x)*(a + b*Csch[c + d*x])^4*Sinh[c + d*x]^4)/(d*(b + a*Sinh[c +
d*x])^4) + ((-9*a^2*b^2*Cosh[(c + d*x)/2] + b^4*Cosh[(c + d*x)/2])*Csch[(c
+ d*x)/2]*(a + b*Csch[c + d*x])^4*Sinh[c + d*x]^4)/(3*d*(b + a*Sinh[c + d*x
])^4) - (a*b^3*Csch[(c + d*x)/2]^2*(a + b*Csch[c + d*x])^4*Sinh[c + d*x]^4)
/(2*d*(b + a*Sinh[c + d*x])^4) - (b^4*Coth[(c + d*x)/2]*Csch[(c + d*x)/2]^2
*(a + b*Csch[c + d*x])^4*Sinh[c + d*x]^4)/(24*d*(b + a*Sinh[c + d*x])^4) +
(2*a*b*(2*a^2 - b^2)*(a + b*Csch[c + d*x])^4*Log[Tanh[(c + d*x)/2]]*Sinh[c
+ d*x]^4)/(d*(b + a*Sinh[c + d*x])^4) - (a*b^3*(a + b*Csch[c + d*x])^4*Sech
[(c + d*x)/2]^2*Sinh[c + d*x]^4)/(2*d*(b + a*Sinh[c + d*x])^4) + ((a + b*Cs
```

```
ch[c + d*x])^4*Sech[(c + d*x)/2]*(-9*a^2*b^2*Sinh[(c + d*x)/2] + b^4*Sinh[(
c + d*x)/2])*Sinh[c + d*x]^4)/(3*d*(b + a*Sinh[c + d*x])^4) + (b^4*(a + b*C
sch[c + d*x])^4*Sech[(c + d*x)/2]^2*Sinh[c + d*x]^4*Tanh[(c + d*x)/2])/(24*
d*(b + a*Sinh[c + d*x])^4)
```

Maple [A]
time = 2.36, size = 176, normalized size = 1.61

method	result
risch	$x\,a^4-\frac{4b^2\left(3ab\,\mathrm{e}^{5dx+5c}+9a^2\mathrm{e}^{4dx+4c}-18a^2\mathrm{e}^{2dx+2c}+3b^2\mathrm{e}^{2dx+2c}-3ab\,\mathrm{e}^{dx+c}+9a^2-b^2\right)}{3d\left(\mathrm{e}^{2dx+2c}-1\right)^3}-\frac{4a^3b\ln\left(\mathrm{e}^{dx+c}+1\right)}{d}+\frac{2a\,b^3\ln\left(\mathrm{e}^{dx+c}+\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*csch(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] x*a^4-4/3*b^2*(3*a*b*exp(5*d*x+5*c)+9*a^2*exp(4*d*x+4*c)-18*a^2*exp(2*d*x+2
*c)+3*b^2*exp(2*d*x+2*c)-3*a*b*exp(d*x+c)+9*a^2-b^2)/d/(exp(2*d*x+2*c)-1)^3
-4*a^3*b/d*ln(exp(d*x+c)+1)+2*a*b^3/d*ln(exp(d*x+c)+1)+4*a^3*b/d*ln(exp(d*x
+c)-1)-2*a*b^3/d*ln(exp(d*x+c)-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 234 vs. 2(103) = 206.
time = 0.29, size = 234, normalized size = 2.15

$$a^4x+2\,ab^3\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)+\frac{4}{3}\,b^4\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}-\frac{1}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}\right)+\frac{4\,a^3b\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}+\frac{12\,a^2b^2}{d(e^{(-2\,dx-2\,c)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] a^4*x + 2*a*b^3*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(
-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1
))) + 4/3*b^4*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*
c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c)
 + e^(-6*d*x - 6*c) - 1))) + 4*a^3*b*log(tanh(1/2*d*x + 1/2*c))/d + 12*a^2*
b^2/(d*(e^(-2*d*x - 2*c) - 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1440 vs. 2(103) = 206.
time = 0.39, size = 1440, normalized size = 13.21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^4*d*x*cosh(d*x + c)^6 + 3*a^4*d*x*sinh(d*x + c)^6 - 12*a*b^3*cosh(
d*x + c)^5 - 3*a^4*d*x + 6*(3*a^4*d*x*cosh(d*x + c) - 2*a*b^3)*sinh(d*x + c
```

```
)^5 + 12*a*b^3*cosh(d*x + c) - 9*(a^4*d*x + 4*a^2*b^2)*cosh(d*x + c)^4 + 3*
(15*a^4*d*x*cosh(d*x + c)^2 - 3*a^4*d*x - 20*a*b^3*cosh(d*x + c) - 12*a^2*b
^2)*sinh(d*x + c)^4 - 36*a^2*b^2 + 4*b^4 + 12*(5*a^4*d*x*cosh(d*x + c)^3 -
10*a*b^3*cosh(d*x + c)^2 - 3*(a^4*d*x + 4*a^2*b^2)*cosh(d*x + c))*sinh(d*x
+ c)^3 + 3*(3*a^4*d*x + 24*a^2*b^2 - 4*b^4)*cosh(d*x + c)^2 + 3*(15*a^4*d*x
*cosh(d*x + c)^4 - 40*a*b^3*cosh(d*x + c)^3 + 3*a^4*d*x + 24*a^2*b^2 - 4*b^
4 - 18*(a^4*d*x + 4*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 6*((2*a^3*b
 - a*b^3)*cosh(d*x + c)^6 + 6*(2*a^3*b - a*b^3)*cosh(d*x + c)*sinh(d*x + c)
^5 + (2*a^3*b - a*b^3)*sinh(d*x + c)^6 - 3*(2*a^3*b - a*b^3)*cosh(d*x + c)^
4 - 3*(2*a^3*b - a*b^3 - 5*(2*a^3*b - a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)
^4 - 2*a^3*b + a*b^3 + 4*(5*(2*a^3*b - a*b^3)*cosh(d*x + c)^3 - 3*(2*a^3*b
- a*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(2*a^3*b - a*b^3)*cosh(d*x + c)
^2 + 3*(5*(2*a^3*b - a*b^3)*cosh(d*x + c)^4 + 2*a^3*b - a*b^3 - 6*(2*a^3*b
- a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 6*((2*a^3*b - a*b^3)*cosh(d*x +
 c)^5 - 2*(2*a^3*b - a*b^3)*cosh(d*x + c)^3 + (2*a^3*b - a*b^3)*cosh(d*x +
c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 6*((2*a^3*b - a
*b^3)*cosh(d*x + c)^6 + 6*(2*a^3*b - a*b^3)*cosh(d*x + c)*sinh(d*x + c)^5 +
 (2*a^3*b - a*b^3)*sinh(d*x + c)^6 - 3*(2*a^3*b - a*b^3)*cosh(d*x + c)^4 -
3*(2*a^3*b - a*b^3 - 5*(2*a^3*b - a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 -
 2*a^3*b + a*b^3 + 4*(5*(2*a^3*b - a*b^3)*cosh(d*x + c)^3 - 3*(2*a^3*b - a*
b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(2*a^3*b - a*b^3)*cosh(d*x + c)^2 +
 3*(5*(2*a^3*b - a*b^3)*cosh(d*x + c)^4 + 2*a^3*b - a*b^3 - 6*(2*a^3*b - a*
b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 6*((2*a^3*b - a*b^3)*cosh(d*x + c)^
5 - 2*(2*a^3*b - a*b^3)*cosh(d*x + c)^3 + (2*a^3*b - a*b^3)*cosh(d*x + c))*
sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 6*(3*a^4*d*x*cosh(d
*x + c)^5 - 10*a*b^3*cosh(d*x + c)^4 + 2*a*b^3 - 6*(a^4*d*x + 4*a^2*b^2)*co
sh(d*x + c)^3 + (3*a^4*d*x + 24*a^2*b^2 - 4*b^4)*cosh(d*x + c))*sinh(d*x +
c))/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*x + c
)^6 - 3*d*cosh(d*x + c)^4 + 3*(5*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^4 + 4
*(5*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*d*cosh(d*x +
 c)^2 + 3*(5*d*cosh(d*x + c)^4 - 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 +
 6*(d*cosh(d*x + c)^5 - 2*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c
) - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\operatorname{csch}(c + dx))^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(d*x+c))**4,x)
```

```
[Out] Integral((a + b*csch(c + d*x))**4, x)
```

Giac [A]

time = 0.40, size = 169, normalized size = 1.55

$$\frac{3\,(dx+c)a^4-6\,(2\,a^3b-ab^3)\log\left(e^{(dx+c)}+1\right)+6\,(2\,a^3b-ab^3)\log\left(\left|e^{(dx+c)}-1\right|\right)-\frac{4\,(3\,ab^3e^{(5\,dx+5\,c)}+9\,a^2b^2e^{(4\,dx+4\,c)}-18\,a^2b^2e^{(2\,dx+2\,c)}+3\,b^4e^{(2\,dx+2\,c)}-3\,ab^3e^{(dx+c)}+9\,a^2b^2-b^4)}{\left(e^{(2\,dx+2\,c)}-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*a^4 - 6*(2*a^3*b - a*b^3)*log(e^(d*x + c) + 1) + 6*(2*a^3*
b - a*b^3)*log(abs(e^(d*x + c) - 1)) - 4*(3*a*b^3*e^(5*d*x + 5*c) + 9*a^2*b
^2*e^(4*d*x + 4*c) - 18*a^2*b^2*e^(2*d*x + 2*c) + 3*b^4*e^(2*d*x + 2*c) - 3
*a*b^3*e^(d*x + c) + 9*a^2*b^2 - b^4)/(e^(2*d*x + 2*c) - 1)^3)/d
```

Mupad [B]
time = 0.21, size = 239, normalized size = 2.19

$$a^4\,x-\frac{\frac{12\,a^2\,b^2}{d}+\frac{4\,a\,b^3\,{\mathrm{e}}^{c+d\,x}}{d}}{{\mathrm{e}}^{2\,c+2\,d\,x}-1}-\frac{\frac{4\,b^4}{d}+\frac{8\,a\,b^3\,{\mathrm{e}}^{c+d\,x}}{d}}{{\mathrm{e}}^{4\,c+4\,d\,x}-2\,{\mathrm{e}}^{2\,c+2\,d\,x}+1}-\frac{8\,b^4}{3\,d\,\left(3\,{\mathrm{e}}^{2\,c+2\,d\,x}-3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}-1\right)}+\frac{4\,\mathrm{atan}\left(\frac{{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^c\,\left(a\,b^3\,\sqrt{-d^2}-2\,a^3\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{4\,a^6\,b^2-4\,a^4\,b^4+a^2\,b^6}}\right)\,\sqrt{4\,a^6\,b^2-4\,a^4\,b^4+a^2\,b^6}}{\sqrt{-d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/sinh(c + d*x))^4,x)
```

```
[Out] a^4*x - ((12*a^2*b^2)/d + (4*a*b^3*exp(c + d*x))/d)/(exp(2*c + 2*d*x) - 1)
- ((4*b^4)/d + (8*a*b^3*exp(c + d*x))/d)/(exp(4*c + 4*d*x) - 2*exp(2*c + 2*
d*x) + 1) - (8*b^4)/(3*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c
 + 6*d*x) - 1)) + (4*atan((exp(d*x)*exp(c)*(a*b^3*(-d^2)^(1/2) - 2*a^3*b*(-
d^2)^(1/2)))/(d*(a^2*b^6 - 4*a^4*b^4 + 4*a^6*b^2)^(1/2)))*(a^2*b^6 - 4*a^4*
b^4 + 4*a^6*b^2)^(1/2))/(-d^2)^(1/2)
```

3.71 $\int (a+b\text{csch}(c+dx))^3 \, dx$

Optimal. Leaf size=75

$$a^3x-\frac{b(6a^2-b^2)\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{5ab^2\coth(c+dx)}{2d}-\frac{b^2\coth(c+dx)(a+b\text{csch}(c+dx))}{2d}$$

```
[Out] a^3*x-1/2*b*(6*a^2-b^2)*arctanh(cosh(d*x+c))/d-5/2*a*b^2*coth(d*x+c)/d-1/2*
b^2*coth(d*x+c)*(a+b*csch(d*x+c))/d
```

Rubi [A]
time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3867, 3855, 3852, 8}

$$a^3x-\frac{b(6a^2-b^2)\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{5ab^2\coth(c+dx)}{2d}-\frac{b^2\coth(c+dx)(a+b\text{csch}(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Csch[c + d*x])^3,x]
```

```
[Out] a^3*x - (b*(6*a^2 - b^2)*ArcTanh[Cosh[c + d*x]])/(2*d) - (5*a*b^2*Coth[c +
d*x])/(2*d) - (b^2*Coth[c + d*x]*(a + b*Csch[c + d*x]))/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3867

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Dist[1/(n - 1)
, Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a
^2*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rubi steps

$$\int (a+b\text{csch}(c+dx))^3\,dx = -\frac{b^2\coth(c+dx)(a+b\text{csch}(c+dx))}{2d}+\frac{1}{2}\int \left(2a^3+b\left(6a^2-b^2\right)\text{csch}(c+dx)+\right.$$
$$= a^3x-\frac{b^2\coth(c+dx)(a+b\text{csch}(c+dx))}{2d}+\frac{1}{2}\left(5ab^2\right)\int \text{csch}^2(c+dx)\,dx+\frac{1}{2}($$
$$= a^3x-\frac{b\left(6a^2-b^2\right)\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{b^2\coth(c+dx)(a+b\text{csch}(c+dx))}{2d}$$
$$= a^3x-\frac{b\left(6a^2-b^2\right)\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{5ab^2\coth(c+dx)}{2d}-\frac{b^2\coth(c+dx}{}$$

Mathematica [A]
time = 0.60, size = 118, normalized size = 1.57

$$-\frac{-8a^3c-8a^3dx+12ab^2\coth\left(\frac{1}{2}(c+dx)\right)+b^3\text{csch}^2\left(\frac{1}{2}(c+dx)\right)-24a^2b\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+4b^3\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+b^3\text{sech}^2\left(\frac{1}{2}(c+dx)\right)+12ab^2\tanh\left(\frac{1}{2}(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Csch[c + d*x])^3,x]
```

```
[Out] -1/8*(-8*a^3*c - 8*a^3*d*x + 12*a*b^2*Coth[(c + d*x)/2] + b^3*Csch[(c + d*x
)/2]^2 - 24*a^2*b*Log[Tanh[(c + d*x)/2]] + 4*b^3*Log[Tanh[(c + d*x)/2]] + b
^3*Sech[(c + d*x)/2]^2 + 12*a*b^2*Tanh[(c + d*x)/2])/d
```

Maple [A]
time = 2.41, size = 133, normalized size = 1.77

method	result
risch	$a^3x-\frac{b^2\left(b\,e^{3dx+3c}+6a\,e^{2dx+2c}+b\,e^{dx+c}-6a\right)}{d\left(e^{2dx+2c}-1\right)^2}+\frac{3a^2b\ln\left(e^{dx+c}-1\right)}{d}-\frac{b^3\ln\left(e^{dx+c}-1\right)}{2d}-\frac{3a^2b\ln\left(e^{dx+c}+1\right)}{d}+\frac{b^3\ln\left(e^{dx+c}+\right.}{2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*csch(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*x-b^2*(b*exp(3*d*x+3*c)+6*a*exp(2*d*x+2*c)+b*exp(d*x+c)-6*a)/d/(exp(2*d
*x+2*c)-1)^2+3*a^2*b/d*ln(exp(d*x+c)-1)-1/2*b^3/d*ln(exp(d*x+c)-1)-3*a^2*b/
d*ln(exp(d*x+c)+1)+1/2*b^3/d*ln(exp(d*x+c)+1)
```

Maxima [A]
time = 0.27, size = 136, normalized size = 1.81

$$a^3x+\frac{1}{2}b^3\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d\left(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1\right)}\right)+\frac{3\,a^2b\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}+\frac{6\,ab^2}{d\left(e^{(-2\,dx-2\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] a^3*x + 1/2*b^3*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(
-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1
))) + 3*a^2*b*log(tanh(1/2*d*x + 1/2*c))/d + 6*a*b^2/(d*(e^(-2*d*x - 2*c) -
 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 769 vs. $2(69) = 138$.
time = 0.38, size = 769, normalized size = 10.25

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^3*d*x*cosh(d*x + c)^4 + 2*a^3*d*x*sinh(d*x + c)^4 - 2*b^3*cosh(d*x
 + c)^3 + 2*a^3*d*x - 2*b^3*cosh(d*x + c) + 2*(4*a^3*d*x*cosh(d*x + c) - b^
3)*sinh(d*x + c)^3 + 12*a*b^2 - 4*(a^3*d*x + 3*a*b^2)*cosh(d*x + c)^2 + 2*(
6*a^3*d*x*cosh(d*x + c)^2 - 2*a^3*d*x - 3*b^3*cosh(d*x + c) - 6*a*b^2)*sinh
(d*x + c)^2 - ((6*a^2*b - b^3)*cosh(d*x + c)^4 + 4*(6*a^2*b - b^3)*cosh(d*x
 + c)*sinh(d*x + c)^3 + (6*a^2*b - b^3)*sinh(d*x + c)^4 + 6*a^2*b - b^3 - 2
*(6*a^2*b - b^3)*cosh(d*x + c)^2 - 2*(6*a^2*b - b^3 - 3*(6*a^2*b - b^3)*cos
h(d*x + c)^2)*sinh(d*x + c)^2 + 4*((6*a^2*b - b^3)*cosh(d*x + c)^3 - (6*a^2
*b - b^3)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) +
 1) + ((6*a^2*b - b^3)*cosh(d*x + c)^4 + 4*(6*a^2*b - b^3)*cosh(d*x + c)*si
nh(d*x + c)^3 + (6*a^2*b - b^3)*sinh(d*x + c)^4 + 6*a^2*b - b^3 - 2*(6*a^2*
b - b^3)*cosh(d*x + c)^2 - 2*(6*a^2*b - b^3 - 3*(6*a^2*b - b^3)*cosh(d*x +
c)^2)*sinh(d*x + c)^2 + 4*((6*a^2*b - b^3)*cosh(d*x + c)^3 - (6*a^2*b - b^3
)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*
(4*a^3*d*x*cosh(d*x + c)^3 - 3*b^3*cosh(d*x + c)^2 - b^3 - 4*(a^3*d*x + 3*a
*b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*
sinh(d*x + c)^3 + d*sinh(d*x + c)^4 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x
 + c)^2 - d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh
(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{csch}(c + dx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(d*x+c))**3,x)
```

```
[Out] Integral((a + b*csch(c + d*x))**3, x)
```

Giac [A]
time = 0.40, size = 122, normalized size = 1.63

$$\frac{2\,(dx+c)a^3-(6\,a^2b-b^3)\log\left(e^{(dx+c)}+1\right)+(6\,a^2b-b^3)\log\left(\left|e^{(dx+c)}-1\right|\right)-\frac{2\,(b^3e^{(3\,dx+3\,c)}+6\,ab^2e^{(2\,dx+2\,c)}+b^3e^{(dx+c)}-6\,ab^2)}{\left(e^{(2\,dx+2\,c)}-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)*a^3 - (6*a^2*b - b^3)*log(e^(d*x + c) + 1) + (6*a^2*b - b^
3)*log(abs(e^(d*x + c) - 1)) - 2*(b^3*e^(3*d*x + 3*c) + 6*a*b^2*e^(2*d*x +
2*c) + b^3*e^(d*x + c) - 6*a*b^2)/(e^(2*d*x + 2*c) - 1)^2)/d
```

Mupad [B]
time = 0.15, size = 170, normalized size = 2.27

$$a^3\,x-\frac{\frac{6\,a\,b^2}{d}+\frac{b^3\,\mathrm{e}^{c+d\,x}}{d}}{\mathrm{e}^{2\,c+2\,d\,x}-1}+\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(b^3\,\sqrt{-d^2}-6\,a^2\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{36\,a^4\,b^2-12\,a^2\,b^4+b^6}}\right)\,\sqrt{36\,a^4\,b^2-12\,a^2\,b^4+b^6}}{\sqrt{-d^2}}-\frac{2\,b^3\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/sinh(c + d*x))^3,x)
```

```
[Out] a^3*x - ((6*a*b^2)/d + (b^3*exp(c + d*x))/d)/(exp(2*c + 2*d*x) - 1) + (atan
((exp(d*x)*exp(c)*(b^3*(-d^2)^(1/2) - 6*a^2*b*(-d^2)^(1/2)))/(d*(b^6 - 12*a
^2*b^4 + 36*a^4*b^2)^(1/2)))*(b^6 - 12*a^2*b^4 + 36*a^4*b^2)^(1/2))/(-d^2)^
(1/2) - (2*b^3*exp(c + d*x))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)
)
```

3.72 $\int (a+b\text{csch}(c+dx))^2 \, dx$

Optimal. Leaf size=34

$$a^2x-\frac{2ab\tanh^{-1}(\cosh(c+dx))}{d}-\frac{b^2\coth(c+dx)}{d}$$

```
[Out] a^2*x-2*a*b*arctanh(cosh(d*x+c))/d-b^2*coth(d*x+c)/d
```

Rubi [A]
time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3858, 3855, 3852, 8}

$$a^2x-\frac{2ab\tanh^{-1}(\cosh(c+dx))}{d}-\frac{b^2\coth(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Csch[c + d*x])^2,x]
```

```
[Out] a^2*x - (2*a*b*ArcTanh[Cosh[c + d*x]])/d - (b^2*Coth[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3858

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Dist[2*a*b, Int[Csc[c + d*x], x], x] + Dist[b^2, Int[Csc[c + d*x]^2, x],
x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int (a+b\operatorname{csch}(c+dx))^2\,dx = a^2x+(2ab)\int \operatorname{csch}(c+dx)\,dx+b^2\int \operatorname{csch}^2(c+dx)\,dx$$
$$= a^2x-\frac{2ab\tanh^{-1}(\cosh(c+dx))}{d}-\frac{(ib^2)\operatorname{Subst}(\int 1\,dx,x,-i\coth(c+dx))}{d}$$
$$= a^2x-\frac{2ab\tanh^{-1}(\cosh(c+dx))}{d}-\frac{b^2\coth(c+dx)}{d}$$

Mathematica [A]
time = 0.16, size = 61, normalized size = 1.79

$$-\frac{b^2\coth\left(\frac{1}{2}(c+dx)\right)-2a\left(ac+adx+2b\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)\right)+b^2\tanh\left(\frac{1}{2}(c+dx)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Csch[c + d*x])^2,x]
```

```
[Out] -1/2*(b^2*Coth[(c + d*x)/2] - 2*a*(a*c + a*d*x + 2*b*Log[Tanh[(c + d*x)/2]]
) + b^2*Tanh[(c + d*x)/2])/d
```

Maple [A]
time = 2.13, size = 60, normalized size = 1.76

method	result	size
risch	$a^2x-\frac{2b^2}{d(e^{2dx+2c}-1)}-\frac{2ab\ln(e^{dx+c}+1)}{d}+\frac{2ab\ln(e^{dx+c}-1)}{d}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*csch(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*x-2*b^2/d/(exp(2*d*x+2*c)-1)-2*a*b/d*ln(exp(d*x+c)+1)+2*a*b/d*ln(exp(d*
x+c)-1)
```

Maxima [A]
time = 0.26, size = 44, normalized size = 1.29

$$a^2x+\frac{2\,ab\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}+\frac{2\,b^2}{d(e^{(-2\,dx-2\,c)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 2*a*b*log(tanh(1/2*d*x + 1/2*c))/d + 2*b^2/(d*(e^(-2*d*x - 2*c) - 1
))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(34) = 68$.

time = 0.49, size = 222, normalized size = 6.53

$$\frac{a^2 dx \cosh(dx+c)^2 + 2\,a^2 dx \cosh(dx+c)\sinh(dx+c) + a^2 dx \sinh(dx+c)^2 - a^2 dx - 2\,b^2 - 2\,(ab\cosh(dx+c)^2 + 2\,ab\cosh(dx+c)\sinh(dx+c) + ab\sinh(dx+c)^2 - ab)\log(\cosh(dx+c) + \sinh(dx+c) + 1) + 2\,(ab\cosh(dx+c)^2 + 2\,ab\cosh(dx+c)\sinh(dx+c) + ab\sinh(dx+c)^2 - ab)\log(\cosh(dx+c) + \sinh(dx+c) - 1)}{d\cosh(dx+c)^2 + 2\,d\cosh(dx+c)\sinh(dx+c) + d\sinh(dx+c)^2 - d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(d*x+c))^2,x, algorithm="fricas")
[Out] (a^2*d*x*cosh(d*x + c)^2 + 2*a^2*d*x*cosh(d*x + c)*sinh(d*x + c) + a^2*d*x*
sinh(d*x + c)^2 - a^2*d*x - 2*b^2 - 2*(a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*x
 + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 - a*b)*log(cosh(d*x + c) + sinh(d
*x + c) + 1) + 2*(a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d*x + c) +
 a*b*sinh(d*x + c)^2 - a*b)*log(cosh(d*x + c) + sinh(d*x + c) - 1))/(d*cosh
(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x + c)^2 - d)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\operatorname{csch}(c+dx))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(d*x+c))**2,x)
[Out] Integral((a + b*csch(c + d*x))**2, x)
```

Giac [A]

time = 0.38, size = 59, normalized size = 1.74

$$\frac{(dx+c)a^2 - 2\,ab\log\left(e^{(dx+c)} + 1\right) + 2\,ab\log\left(\left|e^{(dx+c)} - 1\right|\right) - \frac{2\,b^2}{e^{(2\,dx+2\,c)} - 1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csch(d*x+c))^2,x, algorithm="giac")
[Out] ((d*x + c)*a^2 - 2*a*b*log(e^(d*x + c) + 1) + 2*a*b*log(abs(e^(d*x + c) - 1
)) - 2*b^2/(e^(2*d*x + 2*c) - 1))/d
```

Mupad [B]

time = 1.48, size = 74, normalized size = 2.18

$$a^2\,x - \frac{2\,b^2}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x} - 1\right)} - \frac{4\,\mathrm{atan}\left(\frac{a\,b\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{-d^2}}{d\,\sqrt{a^2\,b^2}}\right)\,\sqrt{a^2\,b^2}}{\sqrt{-d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/sinh(c + d*x))^2,x)
[Out] a^2*x - (2*b^2)/(d*(exp(2*c + 2*d*x) - 1)) - (4*atan((a*b*exp(d*x)*exp(c)*(
-d^2)^(1/2))/(d*(a^2*b^2)^(1/2)))*(a^2*b^2)^(1/2))/(-d^2)^(1/2)
```

3.73 $\int (a+b\text{csch}(c+dx))\,dx$

Optimal. Leaf size=17

$$ax-\frac{b\tanh^{-1}(\cosh(c+dx))}{d}$$

[Out] a*x-b*arctanh(cosh(d*x+c))/d

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3855}

$$ax-\frac{b\tanh^{-1}(\cosh(c+dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[a + b*Csch[c + d*x],x]

[Out] a*x - (b*ArcTanh[Cosh[c + d*x]])/d

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\text{csch}(c+dx))\,dx &= ax+b\int \text{csch}(c+dx)\,dx\\ &= ax-\frac{b\tanh^{-1}(\cosh(c+dx))}{d}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 43 vs. 2(17) = 34.
time = 0.01, size = 43, normalized size = 2.53

$$ax-\frac{b\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{b\log\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[a + b*Csch[c + d*x],x]

[Out] a*x - (b*Log[Cosh[c/2 + (d*x)/2]])/d + (b*Log[Sinh[c/2 + (d*x)/2]])/d

Maple [A]
time = 0.57, size = 20, normalized size = 1.18

method	result	size
default	$ax+\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}$	20
derivativedivides	$\frac{(dx+c)a+b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}$	25
risch	$ax-\frac{\ln\left(\mathrm{e}^{dx+c}+1\right)b}{d}+\frac{\ln\left(\mathrm{e}^{dx+c}-1\right)b}{d}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*csch(d*x+c),x,method=_RETURNVERBOSE)
[Out] a*x+b/d*ln(tanh(1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.27, size = 19, normalized size = 1.12

$$ax+\frac{b\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*csch(d*x+c),x, algorithm="maxima")
[Out] a*x + b*log(tanh(1/2*d*x + 1/2*c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(17)=34$.
time = 0.39, size = 44, normalized size = 2.59

$$\frac{adx-b\log\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)+1\right)+b\log\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)-1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*csch(d*x+c),x, algorithm="fricas")
[Out] (a*d*x - b*log(cosh(d*x + c) + sinh(d*x + c) + 1) + b*log(cosh(d*x + c) + s
inh(d*x + c) - 1))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\operatorname{csch}\left(c+dx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*csch(d*x+c),x)
```

```
[Out] Integral(a + b*csch(c + d*x), x)
```

Giac [A]
time = 0.39, size = 32, normalized size = 1.88

$$ax - \frac{b\left(\log\left(e^{(dx+c)} + 1\right) - \log\left(\left|e^{(dx+c)} - 1\right|\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*csch(d*x+c),x, algorithm="giac")
```

```
[Out] a*x - b*(log(e^(d*x + c) + 1) - log(abs(e^(d*x + c) - 1)))/d
```

Mupad [B]
time = 0.07, size = 42, normalized size = 2.47

$$a\,x - \frac{2\,\mathrm{atan}\left(\frac{b\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{-d^2}}{d\,\sqrt{b^2}}\right)\,\sqrt{b^2}}{\sqrt{-d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b/sinh(c + d*x),x)
```

```
[Out] a*x - (2*atan((b*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(b^2)^(1/2)))*(b^2)^(1/2)
)/(-d^2)^(1/2)
```

3.74 $\int \frac{1}{a+b\operatorname{csch}(c+dx)}\,dx$

Optimal. Leaf size=54

$$\frac{x}{a}+\frac{2b\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}\,d}$$

```
[Out] x/a+2*b*arctanh((a-b*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/a/d/(a^2+b^2)^(1
/2)
```

Rubi [A]
time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3868, 2739, 632, 210}

$$\frac{2b\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{ad\sqrt{a^2+b^2}}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Csch[c + d*x])^(-1),x]
```

```
[Out] x/a + (2*b*ArcTanh[(a - b*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a*Sqrt[a^2
+ b^2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3868

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{a+b\operatorname{csch}(c+dx)}\,dx &= \frac{x}{a}-\frac{\int \frac{1}{1+\frac{a\sinh(c+dx)}{b}}\,dx}{a}\\
&= \frac{x}{a}+\frac{(2i)\mathrm{Subst}\left(\int \frac{1}{1-\frac{2iax}{b}+x^2}\,dx,x,\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{ad}\\
&= \frac{x}{a}-\frac{(4i)\mathrm{Subst}\left(\int \frac{1}{-4\left(1+\frac{a^2}{b^2}\right)-x^2}\,dx,x,-\frac{2ia}{b}+2\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{ad}\\
&= \frac{x}{a}+\frac{2b\tanh^{-1}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}\,d}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 64, normalized size = 1.19

$$\frac{\frac{c}{d}+x-\frac{2b\operatorname{ArcTan}\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}\,d}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Csch[c + d*x])^(-1),x]
```

```
[Out] (c/d + x - (2*b*ArcTan[(a - b*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/(Sqrt[-
a^2 - b^2]*d))/a
```

Maple [A]
time = 1.46, size = 82, normalized size = 1.52

method	result	size
derivativedivides	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a}+\frac{2b\,\mathrm{arctanh}\left(\frac{-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2a}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}}{d}$	82

default	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a}+\frac{2b\,\mathrm{arctanh}\left(\frac{-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2a}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}}{d}$	82
risch	$\frac{x}{a}+\frac{b\ln\left(\mathrm{e}^{dx+c}+\frac{b\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,a}\right)}{\sqrt{a^2+b^2}\,da}-\frac{b\ln\left(\mathrm{e}^{dx+c}+\frac{b\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,a}\right)}{\sqrt{a^2+b^2}\,da}$	124

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*csch(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a*ln(tanh(1/2*d*x+1/2*c)+1)-1/a*ln(tanh(1/2*d*x+1/2*c)-1)+2/a*b/(a^2
+b^2)^(1/2)*arctanh(1/2*(-2*b*tanh(1/2*d*x+1/2*c)+2*a)/(a^2+b^2)^(1/2)))
```

Maxima [A]
time = 0.47, size = 85, normalized size = 1.57

$$-\frac{b\log\left(\frac{ae^{(-dx-c)}-b-\sqrt{a^2+b^2}}{ae^{(-dx-c)}-b+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,ad}+\frac{dx+c}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csch(d*x+c)),x, algorithm="maxima")
```

```
[Out] -b*log((a*e^(-d*x - c) - b - sqrt(a^2 + b^2))/(a*e^(-d*x - c) - b + sqrt(a^
2 + b^2)))/(sqrt(a^2 + b^2)*a*d) + (d*x + c)/(a*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(51) = 102$.
time = 0.44, size = 186, normalized size = 3.44

$$\frac{(a^2+b^2)dx+\sqrt{a^2+b^2}\,b\log\left(\frac{a^2\cosh(dx+c)^2+a^2\sinh(dx+c)^2+2\,ab\cosh(dx+c)+a^2+2\,b^2+2\,(a^2\cosh(dx+c)+ab)\sinh(dx+c)+2\sqrt{a^2+b^2}\,(a\cosh(dx+c)+a\sinh(dx+c)+b)}{a\cosh(dx+c)^2+a\sinh(dx+c)^2+2\,b\cosh(dx+c)+2\,(a\cosh(dx+c)+b)\sinh(dx+c)-a}\right)}{(a^3+ab^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csch(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((a^2 + b^2)*d*x + sqrt(a^2 + b^2)*b*log((a^2*cosh(d*x + c)^2 + a^2*sinh(d*
x + c)^2 + 2*a*b*cosh(d*x + c) + a^2 + 2*b^2 + 2*(a^2*cosh(d*x + c) + a*b)*
sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(a*cosh(d*x + c) + a*sinh(d*x + c) + b))/
(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*(a*cosh(d*x
+ c) + b)*sinh(d*x + c) - a)))/((a^3 + a*b^2)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{a+b\operatorname{csch}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csch(d*x+c)),x)
```

```
[Out] Integral(1/(a + b*csch(c + d*x)), x)
```

Giac [A]
time = 0.41, size = 84, normalized size = 1.56

$$-\frac{\frac{b\log\left(\left|\frac{2\,ae^{(dx+c)}+2\,b-2\,\sqrt{a^2+b^2}}{2\,ae^{(dx+c)}+2\,b+2\,\sqrt{a^2+b^2}}\right|\right)}{\sqrt{a^2+b^2}\,a}-\frac{dx+c}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csch(d*x+c)),x, algorithm="giac")
```

```
[Out] -(b*log(abs(2*a*e^(d*x + c) + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^(d*x + c)
+ 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a) - (d*x + c)/a)/d
```

Mupad [B]
time = 0.33, size = 121, normalized size = 2.24

$$\frac{x}{a}-\frac{b\,\ln\left(\frac{2\,b\,{\mathrm{e}}^{c+d\,x}}{a^2}-\frac{2\,b\,\left(a-b\,{\mathrm{e}}^{c+d\,x}\right)}{a^2\,\sqrt{a^2+b^2}}\right)}{a\,d\,\sqrt{a^2+b^2}}+\frac{b\,\ln\left(\frac{2\,b\,{\mathrm{e}}^{c+d\,x}}{a^2}+\frac{2\,b\,\left(a-b\,{\mathrm{e}}^{c+d\,x}\right)}{a^2\,\sqrt{a^2+b^2}}\right)}{a\,d\,\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/sinh(c + d*x)),x)
```

```
[Out] x/a - (b*log((2*b*exp(c + d*x))/a^2 - (2*b*(a - b*exp(c + d*x)))/(a^2*(a^2
+ b^2)^(1/2))))/(a*d*(a^2 + b^2)^(1/2)) + (b*log((2*b*exp(c + d*x))/a^2 + (
2*b*(a - b*exp(c + d*x)))/(a^2*(a^2 + b^2)^(1/2))))/(a*d*(a^2 + b^2)^(1/2))
```

3.75 $\int \frac{1}{(a+b\,\mathrm{csch}(c+dx))^2}\,dx$

Optimal. Leaf size=101

$$\frac{x}{a^2}+\frac{2b(2a^2+b^2)\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}d}-\frac{b^2\coth(c+dx)}{a\left(a^2+b^2\right)d(a+b\mathrm{csch}(c+dx))}$$

```
[Out] x/a^2+2*b*(2*a^2+b^2)*arctanh((a-b*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/a^
2/(a^2+b^2)^(3/2)/d-b^2*coth(d*x+c)/a/(a^2+b^2)/d/(a+b*csch(d*x+c))
```

Rubi [A]
time = 0.12, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3870, 4004, 3916, 2739, 632, 210}

$$\frac{2b(2a^2+b^2)\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2d\left(a^2+b^2\right)^{3/2}}-\frac{b^2\coth(c+dx)}{ad\left(a^2+b^2\right)(a+b\mathrm{csch}(c+dx))}+\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Csch[c + d*x])^(-2),x]
```

```
[Out] x/a^2 + (2*b*(2*a^2 + b^2)*ArcTanh[(a - b*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2
]])/(a^2*(a^2 + b^2)^(3/2)*d) - (b^2*Coth[c + d*x])/(a*(a^2 + b^2)*d*(a + b
*Csch[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\text{csch}(c+dx))^2}\,dx = -\frac{b^2\coth(c+dx)}{a\left(a^2+b^2\right)d(a+b\text{csch}(c+dx))} - \frac{\int \frac{-a^2-b^2+ab\text{csch}(c+dx)}{a+b\text{csch}(c+dx)}\,dx}{a\left(a^2+b^2\right)}$$

$$= \frac{x}{a^2} - \frac{b^2\coth(c+dx)}{a\left(a^2+b^2\right)d(a+b\text{csch}(c+dx))} - \frac{\left(b\left(2a^2+b^2\right)\right)\int \frac{\text{csch}(c+dx)}{a+b\text{csch}(c+dx)}\,dx}{a^2\left(a^2+b^2\right)}$$

$$= \frac{x}{a^2} - \frac{b^2\coth(c+dx)}{a\left(a^2+b^2\right)d(a+b\text{csch}(c+dx))} - \frac{\left(2a^2+b^2\right)\int \frac{1}{1+\frac{a\sinh(c+dx)}{b}}\,dx}{a^2\left(a^2+b^2\right)}$$

$$= \frac{x}{a^2} - \frac{b^2\coth(c+dx)}{a\left(a^2+b^2\right)d(a+b\text{csch}(c+dx))} + \frac{\left(2i\left(2a^2+b^2\right)\right)\text{Subst}\left(\int \frac{1}{1-\frac{2iax}{b}+x^2}\,dx,\right.}{a^2\left(a^2+b^2\right)d}$$

$$= \frac{x}{a^2} - \frac{b^2\coth(c+dx)}{a\left(a^2+b^2\right)d(a+b\text{csch}(c+dx))} - \frac{\left(4i\left(2a^2+b^2\right)\right)\text{Subst}\left(\int \frac{1}{-4\left(1+\frac{a^2}{b^2}\right)-x^2}\right.}{a^2\left(a^2+\right.}$$

$$= \frac{x}{a^2} + \frac{2b\left(2a^2+b^2\right)\tanh^{-1}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}d} - \frac{b^2\coth(c+dx)}{a\left(a^2+b^2\right)d(a+b\text{csch}(c+\right.}$$

Mathematica [A]
time = 0.28, size = 142, normalized size = 1.41

$$\frac{\operatorname{csch}(c+dx)\left(-\frac{ab^2\coth(c+dx)}{a^2+b^2}+(c+dx)(a+b\operatorname{csch}(c+dx))+\frac{2b(2a^2+b^2)\operatorname{ArcTan}\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)(a+b\operatorname{csch}(c+dx))}{(-a^2-b^2)^{3/2}}\right)(b+a\sinh(c+dx))}{a^2d(a+b\operatorname{csch}(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Csch[c + d*x])^(-2),x]
```

```
[Out] (Csch[c + d*x]*(-((a*b^2*Coth[c + d*x])/(a^2 + b^2)) + (c + d*x)*(a + b*Csc
h[c + d*x]) + (2*b*(2*a^2 + b^2)*ArcTan[(a - b*Tanh[(c + d*x)/2])/Sqrt[-a^2
 - b^2]]*(a + b*Csch[c + d*x]))/(-a^2 - b^2)^(3/2))*(b + a*Sinh[c + d*x]))/
(a^2*d*(a + b*Csch[c + d*x])^2)
```

Maple [A]
time = 2.06, size = 176, normalized size = 1.74

method	result
derivativedivides	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^2}-\frac{2b\left(\frac{\frac{a^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2+2b^2}+\frac{ab}{2a^2+2b^2}}{-\frac{b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{b}{2}}-\frac{2\left(2a^2+b^2\right)\operatorname{arctanh}\left(\frac{-2b\tanh\left(\frac{dx}{2}\right.}{2\sqrt{a^2+}}\right.}{\left(2a^2+2b^2\right)\sqrt{a^2+b^2}}\right.}{a^2}}{d}$
default	$\frac{-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^2}-\frac{2b\left(\frac{\frac{a^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2+2b^2}+\frac{ab}{2a^2+2b^2}}{-\frac{b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{b}{2}}-\frac{2\left(2a^2+b^2\right)\operatorname{arctanh}\left(\frac{-2b\tanh\left(\frac{dx}{2}\right.}{2\sqrt{a^2+}}\right.}{\left(2a^2+2b^2\right)\sqrt{a^2+b^2}}\right.}{a^2}}{d}$
risch	$\frac{x}{a^2}-\frac{2b^2\left(-b\,e^{dx+c}+a\right)}{d\,a^2(a^2+b^2)\left(a\,e^{2dx+2c}+2b\,e^{dx+c}-a\right)}+\frac{2b\ln\left(e^{dx+c}+\frac{\left(a^2+b^2\right)^{\frac{3}{2}}b+a^4+2a^2b^2+b^4}{a\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}d}+\frac{b^3\ln\left(e^{dx+c}+\frac{\left(a^2+b^2\right)^{\frac{3}{2}}}{a\left(a\right.}\right.}{(a^2+b^2)^{\frac{3}{2}}d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*csch(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a^2*ln(tanh(1/2*d*x+1/2*c)-1)+1/a^2*ln(tanh(1/2*d*x+1/2*c)+1)-2*b/a
^2*((1/2*a^2/(a^2+b^2)*tanh(1/2*d*x+1/2*c)+1/2*a*b/(a^2+b^2))/(-1/2*b*tanh(
1/2*d*x+1/2*c)^2+a*tanh(1/2*d*x+1/2*c)+1/2*b)-2*(2*a^2+b^2)/(2*a^2+2*b^2)/(
a^2+b^2)^(1/2)*arctanh(1/2*(-2*b*tanh(1/2*d*x+1/2*c)+2*a)/(a^2+b^2)^(1/2)))
)
```

Maxima [A]
time = 0.49, size = 187, normalized size = 1.85

$$-\frac{(2a^2b+b^3)\log\left(\frac{ae^{(-dx-c)}-b-\sqrt{a^2+b^2}}{ae^{(-dx-c)}-b+\sqrt{a^2+b^2}}\right)}{(a^4+a^2b^2)\sqrt{a^2+b^2}\,d}-\frac{2\left(b^3e^{(-dx-c)}+ab^2\right)}{(a^5+a^3b^2+2(a^4b+a^2b^3)e^{(-dx-c)}-(a^5+a^3b^2)e^{(-2dx-2c)})d}+\frac{dx+c}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csch(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(2*a^2*b + b^3)*log((a*e^(-d*x - c) - b - sqrt(a^2 + b^2))/(a*e^(-d*x - c)
 - b + sqrt(a^2 + b^2)))/((a^4 + a^2*b^2)*sqrt(a^2 + b^2)*d) - 2*(b^3*e^(-d
*x - c) + a*b^2)/((a^5 + a^3*b^2 + 2*(a^4*b + a^2*b^3)*e^(-d*x - c) - (a^5
+ a^3*b^2)*e^(-2*d*x - 2*c))*d) + (d*x + c)/(a^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 645 vs. $2(98) = 196$.
time = 0.40, size = 645, normalized size = 6.39

$$-\frac{2\,a^{3}b^{2}+2\,ab^{4}-(a^{5}+2\,a^{3}b^{2}+ab^{4})dx\cosh\left(dx+c\right)^{2}-(a^{5}+2\,a^{3}b^{2}+ab^{4})dx\sinh\left(dx+c\right)^{2}+(a^{5}+2\,a^{3}b^{2}+ab^{4})dx+\left(2\,a^{3}b+ab^{3}-(2\,a^{3}b+ab^{3})\cosh\left(dx+c\right)^{2}-(2\,a^{3}b+ab^{3})\sinh\left(dx+c\right)^{2}-2\,(2\,a^{2}b^{2}+b^{4})\cosh\left(dx+c\right)-2\,(2\,a^{2}b^{2}+b^{4}+(2\,a^{3}b+ab^{3})\cosh\left(dx+c\right))\sinh\left(dx+c\right)\right)\sqrt{a^{2}+b^{2}}\log\left(\frac{a^{2}\cosh(dx+c)^{2}+a^{2}\sinh(dx+c)^{2}+2\,ab\cosh(dx+c)+a^{2}+2\,b^{2}+2\,(a^{2}\cosh(dx+c)+ab)\sinh(dx+c)+2\,\sqrt{a^{2}+b^{2}}\,(a\cosh(dx+c)+a\sinh(dx+c)+b)}{a\cosh(dx+c)^{2}+a\sinh(dx+c)^{2}+2\,b\cosh(dx+c)+2\,(a\cosh(dx+c)+b)\sinh(dx+c)-a}\right)-2\,(a^{2}b^{3}+b^{5}+(a^{4}b+2\,a^{2}b^{3}+b^{5})dx)\cosh\left(dx+c\right)-2\,(a^{2}b^{3}+b^{5}+(a^{5}+2\,a^{3}b^{2}+ab^{4})dx\cosh\left(dx+c\right)+(a^{4}b+2\,a^{2}b^{3}+b^{5})dx)\sinh\left(dx+c\right)}{(a^{7}+2\,a^{5}b^{2}+a^{3}b^{4})d\cosh\left(dx+c\right)^{2}+(a^{7}+2\,a^{5}b^{2}+a^{3}b^{4})d\sinh\left(dx+c\right)^{2}+2\,(a^{6}b+2\,a^{4}b^{3}+a^{2}b^{5})d\cosh\left(dx+c\right)-(a^{7}+2\,a^{5}b^{2}+a^{3}b^{4})d+2\,((a^{7}+2\,a^{5}b^{2}+a^{3}b^{4})d\cosh\left(dx+c\right)+(a^{6}b+2\,a^{4}b^{3}+a^{2}b^{5})d)\sinh\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csch(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(2*a^3*b^2 + 2*a*b^4 - (a^5 + 2*a^3*b^2 + a*b^4)*d*x*cosh(d*x + c)^2 - (a^
5 + 2*a^3*b^2 + a*b^4)*d*x*sinh(d*x + c)^2 + (a^5 + 2*a^3*b^2 + a*b^4)*d*x
+ (2*a^3*b + a*b^3 - (2*a^3*b + a*b^3)*cosh(d*x + c)^2 - (2*a^3*b + a*b^3)*
sinh(d*x + c)^2 - 2*(2*a^2*b^2 + b^4)*cosh(d*x + c) - 2*(2*a^2*b^2 + b^4 +
(2*a^3*b + a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + b^2)*log((a^2*co
sh(d*x + c)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + a^2 + 2*b^2 + 2
*(a^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(a*cosh(d*x +
c) + a*sinh(d*x + c) + b))/(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + 2*b*cos
h(d*x + c) + 2*(a*cosh(d*x + c) + b)*sinh(d*x + c) - a)) - 2*(a^2*b^3 + b^5
 + (a^4*b + 2*a^2*b^3 + b^5)*d*x)*cosh(d*x + c) - 2*(a^2*b^3 + b^5 + (a^5 +
 2*a^3*b^2 + a*b^4)*d*x*cosh(d*x + c) + (a^4*b + 2*a^2*b^3 + b^5)*d*x)*sinh
(d*x + c))/((a^7 + 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x + c)^2 + (a^7 + 2*a^5*b^
2 + a^3*b^4)*d*sinh(d*x + c)^2 + 2*(a^6*b + 2*a^4*b^3 + a^2*b^5)*d*cosh(d*x
 + c) - (a^7 + 2*a^5*b^2 + a^3*b^4)*d + 2*((a^7 + 2*a^5*b^2 + a^3*b^4)*d*co
sh(d*x + c) + (a^6*b + 2*a^4*b^3 + a^2*b^5)*d)*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\operatorname{csch}\left(c+dx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csch(d*x+c))**2,x)
```

```
[Out] Integral((a + b*csch(c + d*x))**(-2), x)
```

Giac [A]
time = 0.40, size = 161, normalized size = 1.59

$$-\frac{\frac{(2a^2b+b^3)\log\left(\frac{\left|2ae^{(dx+c)}+2b-2\sqrt{a^2+b^2}\right|}{\left|2ae^{(dx+c)}+2b+2\sqrt{a^2+b^2}\right|}\right)}{(a^4+a^2b^2)\sqrt{a^2+b^2}}-\frac{2(b^3e^{(dx+c)}-ab^2)}{(a^4+a^2b^2)(ae^{(2dx+2c)}+2be^{(dx+c)}-a)}-\frac{dx+c}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csch(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -((2*a^2*b + b^3)*log(abs(2*a*e^(d*x + c) + 2*b - 2*sqrt(a^2 + b^2))/abs(2*
a*e^(d*x + c) + 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + a^2*b^2)*sqrt(a^2 + b^2))
 - 2*(b^3*e^(d*x + c) - a*b^2)/((a^4 + a^2*b^2)*(a*e^(2*d*x + 2*c) + 2*b*e^
(d*x + c) - a)) - (d*x + c)/a^2)/d
```

Mupad [B]
time = 1.95, size = 269, normalized size = 2.66

$$\frac{x}{a^2}-\frac{\frac{2b^2}{d(a^3+ab^2)}-\frac{2b^3e^{c+dx}}{ad(a^3+ab^2)}}{2be^{c+dx}-a+ae^{2c+2dx}}-\frac{b\ln\left(\frac{2e^{c+dx}(2a^2b+b^3)}{a^3(a^2+b^2)}-\frac{2b(2a^2+b^2)(a-be^{c+dx})}{a^3(a^2+b^2)^{3/2}}\right)(2a^2+b^2)}{a^2d(a^2+b^2)^{3/2}}+\frac{b\ln\left(\frac{2e^{c+dx}(2a^2b+b^3)}{a^3(a^2+b^2)}+\frac{2b(2a^2+b^2)(a-be^{c+dx})}{a^3(a^2+b^2)^{3/2}}\right)(2a^2+b^2)}{a^2d(a^2+b^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/sinh(c + d*x))^2,x)
```

```
[Out] x/a^2 - ((2*b^2)/(d*(a*b^2 + a^3)) - (2*b^3*exp(c + d*x))/(a*d*(a*b^2 + a^3
)))/(2*b*exp(c + d*x) - a + a*exp(2*c + 2*d*x)) - (b*log((2*exp(c + d*x)*(2
*a^2*b + b^3))/(a^3*(a^2 + b^2)) - (2*b*(2*a^2 + b^2)*(a - b*exp(c + d*x)))
/(a^3*(a^2 + b^2)^(3/2)))*(2*a^2 + b^2))/(a^2*d*(a^2 + b^2)^(3/2)) + (b*log
((2*exp(c + d*x)*(2*a^2*b + b^3))/(a^3*(a^2 + b^2)) + (2*b*(2*a^2 + b^2)*(a
 - b*exp(c + d*x)))/(a^3*(a^2 + b^2)^(3/2)))*(2*a^2 + b^2))/(a^2*d*(a^2 + b
^2)^(3/2))
```

3.76 $\int \frac{1}{(a+b\,\mathbf{csch}(c+dx))^3}\,dx$

Optimal. Leaf size=163

$$\frac{x}{a^3}+\frac{b(6a^4+5a^2b^2+2b^4)\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^3\,(a^2+b^2)^{5/2}\,d}-\frac{b^2\coth(c+dx)}{2a\,(a^2+b^2)\,d(a+b\text{csch}(c+dx))^2}-\frac{b^2(5a^2+2b^2)}{2a^2\,(a^2+b^2)^2\,d(a}$$

```
[Out] x/a^3+b*(6*a^4+5*a^2*b^2+2*b^4)*arctanh((a-b*tanh(1/2*d*x+1/2*c))/(a^2+b^2)
^(1/2))/a^3/(a^2+b^2)^(5/2)/d-1/2*b^2*coth(d*x+c)/a/(a^2+b^2)/d/(a+b*csch(d
*x+c))^2-1/2*b^2*(5*a^2+2*b^2)*coth(d*x+c)/a^2/(a^2+b^2)^2/d/(a+b*csch(d*x+
c))
```

Rubi [A]
time = 0.23, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {3870, 4145, 4004, 3916, 2739, 632, 210}

$$\frac{x}{a^3}-\frac{b^2(5a^2+2b^2)\coth(c+dx)}{2a^2d\,(a^2+b^2)^2\,(a+b\text{csch}(c+dx))}-\frac{b^2\coth(c+dx)}{2ad\,(a^2+b^2)\,(a+b\text{csch}(c+dx))^2}+\frac{b(6a^4+5a^2b^2+2b^4)\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^3d\,(a^2+b^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Csch[c + d*x])^(-3),x]
```

```
[Out] x/a^3 + (b*(6*a^4 + 5*a^2*b^2 + 2*b^4)*ArcTanh[(a - b*Tanh[(c + d*x)/2])/Sq
rt[a^2 + b^2]])/(a^3*(a^2 + b^2)^(5/2)*d) - (b^2*Coth[c + d*x])/(2*a*(a^2 +
 b^2)*d*(a + b*Csch[c + d*x])^2) - (b^2*(5*a^2 + 2*b^2)*Coth[c + d*x])/(2*a
^2*(a^2 + b^2)^2*d*(a + b*Csch[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4145

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+b\text{csch}(c+dx))^3}\,dx = -\frac{b^2\coth(c+dx)}{2a\,(a^2+b^2)\,d(a+b\text{csch}(c+dx))^2} - \frac{\int \frac{-2(a^2+b^2)+2ab\text{csch}(c+dx)-b^2\text{csch}^2(c+dx)}{(a+b\text{csch}(c+dx))^2}}{2a\,(a^2+b^2)}$$

$$= -\frac{b^2\coth(c+dx)}{2a\,(a^2+b^2)\,d(a+b\text{csch}(c+dx))^2} - \frac{b^2(5a^2+2b^2)\coth(c+dx)}{2a^2\,(a^2+b^2)^2\,d(a+b\text{csch}(c+dx))} +$$

$$= \frac{x}{a^3} - \frac{b^2\coth(c+dx)}{2a\,(a^2+b^2)\,d(a+b\text{csch}(c+dx))^2} - \frac{b^2(5a^2+2b^2)\coth(c+dx)}{2a^2\,(a^2+b^2)^2\,d(a+b\text{csch}(c+dx))}$$

$$= \frac{x}{a^3} - \frac{b^2\coth(c+dx)}{2a\,(a^2+b^2)\,d(a+b\text{csch}(c+dx))^2} - \frac{b^2(5a^2+2b^2)\coth(c+dx)}{2a^2\,(a^2+b^2)^2\,d(a+b\text{csch}(c+dx))}$$

$$= \frac{x}{a^3} - \frac{b^2\coth(c+dx)}{2a\,(a^2+b^2)\,d(a+b\text{csch}(c+dx))^2} - \frac{b^2(5a^2+2b^2)\coth(c+dx)}{2a^2\,(a^2+b^2)^2\,d(a+b\text{csch}(c+dx))}$$

$$= \frac{x}{a^3} - \frac{b^2\coth(c+dx)}{2a\,(a^2+b^2)\,d(a+b\text{csch}(c+dx))^2} - \frac{b^2(5a^2+2b^2)\coth(c+dx)}{2a^2\,(a^2+b^2)^2\,d(a+b\text{csch}(c+dx))}$$

$$= \frac{x}{a^3} + \frac{b(6a^4+5a^2b^2+2b^4)\tanh^{-1}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2+b^2}}\right)}{a^3\,(a^2+b^2)^{5/2}\,d} - \frac{b^2\coth(c-}{2a\,(a^2+b^2)\,d(a+b}$$

Mathematica [A]
time = 0.65, size = 213, normalized size = 1.31

$$\frac{\text{csch}^2(c+dx)(b+a\sinh(c+dx))\left(\frac{ab^3\coth(c+dx)}{a^2+b^2} - \frac{3ab^2(2a^2+b^2)\coth(c+dx)(b+a\sinh(c+dx))}{(a^2+b^2)^2} + 2(c+dx)\text{csch}(c+dx)(b+a\sinh(c+dx))^2 - \frac{2b(6a^4+5a^2b^2+2b^4)\text{ArcTan}\left(\frac{a-b\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)\text{csch}(c+dx)(b+a\sinh(c+dx))^2}{(-a^2-b^2)^{5/2}}\right)}{2a^3d(a+b\text{csch}(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Csch[c + d*x])^(-3),x]
```

```
[Out] (Csch[c + d*x]^2*(b + a*Sinh[c + d*x])*((a*b^3*Coth[c + d*x])/(a^2 + b^2) -
 (3*a*b^2*(2*a^2 + b^2)*Coth[c + d*x]*(b + a*Sinh[c + d*x]))/(a^2 + b^2)^2
+ 2*(c + d*x)*Csch[c + d*x]*(b + a*Sinh[c + d*x])^2 - (2*b*(6*a^4 + 5*a^2*b
^2 + 2*b^4)*ArcTan[(a - b*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]]*Csch[c + d*x
]*(b + a*Sinh[c + d*x])^2)/(-a^2 - b^2)^(5/2)))/(2*a^3*d*(a + b*Csch[c + d*
x])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(154) = 308$.
time = 2.13, size = 328, normalized size = 2.01

method	result
derivativedivides	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^3}-\frac{2b\left(\frac{-\frac{b\,a^2\left(4a^2+b^2\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2\left(a^4+2a^2b^2+b^4\right)}+\frac{4a\left(10a^4-a^2b^2-2b^4\right)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^4+16a^2b^2+8b^4}+\frac{4a^2b\left(16a^2+7b^2\right)\tanh\left(\frac{dx}{2}+\cdots\right)}{8a^4+16a^2b^2+8b^4}\cdots}{\left(-b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+b\right)^2}\cdots\right)}{a^3}\cdots}{d}$
default	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^3}-\frac{2b\left(\frac{-\frac{b\,a^2\left(4a^2+b^2\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2\left(a^4+2a^2b^2+b^4\right)}+\frac{4a\left(10a^4-a^2b^2-2b^4\right)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^4+16a^2b^2+8b^4}+\frac{4a^2b\left(16a^2+7b^2\right)\tanh\left(\frac{dx}{2}+\cdots\right)}{8a^4+16a^2b^2+8b^4}\cdots}{\left(-b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+b\right)^2}\cdots\right)}{a^3}\cdots}{d}$
risch	$\frac{x}{a^3}-\frac{b^2\left(-7a^3b\,e^{3dx+3c}-4a\,b^3e^{3dx+3c}+6a^4e^{2dx+2c}-9a^2b^2e^{2dx+2c}-6b^4e^{2dx+2c}+17a^3b\,e^{dx+c}+8b^3e^{dx+c}a-6a^4-3a^2b^2\cdots\right)}{a^3d\left(a^2+b^2\right)^2\left(a\,e^{2dx+2c}+2b\,e^{dx+c}-a\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*csch(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a^3*ln(tanh(1/2*d*x+1/2*c)+1)-2/a^3*b*(4*(-1/8*b*a^2*(4*a^2+b^2)/(a^
4+2*a^2*b^2+b^4)*tanh(1/2*d*x+1/2*c)^3+1/8*a*(10*a^4-a^2*b^2-2*b^4)/(a^4+2*
a^2*b^2+b^4)*tanh(1/2*d*x+1/2*c)^2+1/8*a^2*b*(16*a^2+7*b^2)/(a^4+2*a^2*b^2+
b^4)*tanh(1/2*d*x+1/2*c)+1/8*a*b^2*(5*a^2+2*b^2)/(a^4+2*a^2*b^2+b^4))/(-b*t
anh(1/2*d*x+1/2*c)^2+2*a*tanh(1/2*d*x+1/2*c)+b)^2-2*(6*a^4+5*a^2*b^2+2*b^4)
/(4*a^4+8*a^2*b^2+4*b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(-2*b*tanh(1/2*d*x+1/2
*c)+2*a)/(a^2+b^2)^(1/2)))-1/a^3*ln(tanh(1/2*d*x+1/2*c)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(156) = 312$.
time = 0.50, size = 373, normalized size = 2.29

$$-\frac{\left(6\,a^4b+5\,a^2b^3+2\,b^5\right)\log\left(\frac{ae^{(-dx-c)}-b-\sqrt{a^2+b^2}}{ae^{(-dx-c)}-b+\sqrt{a^2+b^2}}\right)}{2\left(a^7+2\,a^5b^2+a^3b^4\right)\sqrt{a^2+b^2}\,d}-\frac{6\,a^4b^2+3\,a^2b^4+\left(17\,a^3b^3+8\,ab^5\right)e^{(-dx-c)}-3\left(2\,a^4b^2-3\,a^2b^4-2\,b^6\right)e^{(-2\,dx-2\,c)}-\left(7\,a^3b^3+4\,ab^5\right)e^{(-3\,dx-3\,c)}}{\left(a^9+2\,a^7b^2+a^5b^4+4\left(a^8b+2\,a^6b^3+a^4b^5\right)e^{(-dx-c)}-2\left(a^9-3\,a^5b^4-2\,a^3b^6\right)e^{(-2\,dx-2\,c)}-4\left(a^8b+2\,a^6b^3+a^4b^5\right)e^{(-3\,dx-3\,c)}+\left(a^9+2\,a^7b^2+a^5b^4\right)e^{(-4\,dx-4\,c)}\right)d}+\frac{dx+c}{a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csch(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(6*a^4*b + 5*a^2*b^3 + 2*b^5)*log((a*e^(-d*x - c) - b - sqrt(a^2 + b^2
))/(a*e^(-d*x - c) - b + sqrt(a^2 + b^2)))/((a^7 + 2*a^5*b^2 + a^3*b^4)*sqr
t(a^2 + b^2)*d) - (6*a^4*b^2 + 3*a^2*b^4 + (17*a^3*b^3 + 8*a*b^5)*e^(-d*x -
 c) - 3*(2*a^4*b^2 - 3*a^2*b^4 - 2*b^6)*e^(-2*d*x - 2*c) - (7*a^3*b^3 + 4*a
*b^5)*e^(-3*d*x - 3*c))/((a^9 + 2*a^7*b^2 + a^5*b^4 + 4*(a^8*b + 2*a^6*b^3
+ a^4*b^5)*e^(-d*x - c) - 2*(a^9 - 3*a^5*b^4 - 2*a^3*b^6)*e^(-2*d*x - 2*c)
- 4*(a^8*b + 2*a^6*b^3 + a^4*b^5)*e^(-3*d*x - 3*c) + (a^9 + 2*a^7*b^2 + a^5
*b^4)*e^(-4*d*x - 4*c))*d) + (d*x + c)/(a^3*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2094 vs. $2(156) = 312$.
time = 0.45, size = 2094, normalized size = 12.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csch(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(12*a^6*b^2 + 18*a^4*b^4 + 6*a^2*b^6 + 2*(a^8 + 3*a^6*b^2 + 3*a^4*b^4 +
 a^2*b^6)*d*x*cosh(d*x + c)^4 + 2*(a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*d
*x*sinh(d*x + c)^4 + 2*(7*a^5*b^3 + 11*a^3*b^5 + 4*a*b^7 + 4*(a^7*b + 3*a^5
*b^3 + 3*a^3*b^5 + a*b^7)*d*x)*cosh(d*x + c)^3 + 2*(7*a^5*b^3 + 11*a^3*b^5
+ 4*a*b^7 + 4*(a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*d*x*cosh(d*x + c) + 4
*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*x)*sinh(d*x + c)^3 + 2*(a^8 + 3*
a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*d*x - 2*(6*a^6*b^2 - 3*a^4*b^4 - 15*a^2*b^6
- 6*b^8 + 2*(a^8 + a^6*b^2 - 3*a^4*b^4 - 5*a^2*b^6 - 2*b^8)*d*x)*cosh(d*x +
 c)^2 - 2*(6*a^6*b^2 - 3*a^4*b^4 - 15*a^2*b^6 - 6*b^8 - 6*(a^8 + 3*a^6*b^2
+ 3*a^4*b^4 + a^2*b^6)*d*x*cosh(d*x + c)^2 + 2*(a^8 + a^6*b^2 - 3*a^4*b^4 -
 5*a^2*b^6 - 2*b^8)*d*x - 3*(7*a^5*b^3 + 11*a^3*b^5 + 4*a*b^7 + 4*(a^7*b +
3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*x)*cosh(d*x + c))*sinh(d*x + c)^2 + (6*a^6
*b + 5*a^4*b^3 + 2*a^2*b^5 + (6*a^6*b + 5*a^4*b^3 + 2*a^2*b^5)*cosh(d*x + c
)^4 + (6*a^6*b + 5*a^4*b^3 + 2*a^2*b^5)*sinh(d*x + c)^4 + 4*(6*a^5*b^2 + 5*
a^3*b^4 + 2*a*b^6)*cosh(d*x + c)^3 + 4*(6*a^5*b^2 + 5*a^3*b^4 + 2*a*b^6 + (
6*a^6*b + 5*a^4*b^3 + 2*a^2*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 - 2*(6*a^6*
b - 7*a^4*b^3 - 8*a^2*b^5 - 4*b^7)*cosh(d*x + c)^2 - 2*(6*a^6*b - 7*a^4*b^3
 - 8*a^2*b^5 - 4*b^7 - 3*(6*a^6*b + 5*a^4*b^3 + 2*a^2*b^5)*cosh(d*x + c)^2
- 6*(6*a^5*b^2 + 5*a^3*b^4 + 2*a*b^6)*cosh(d*x + c))*sinh(d*x + c)^2 - 4*(6
*a^5*b^2 + 5*a^3*b^4 + 2*a*b^6)*cosh(d*x + c) - 4*(6*a^5*b^2 + 5*a^3*b^4 +
2*a*b^6 - (6*a^6*b + 5*a^4*b^3 + 2*a^2*b^5)*cosh(d*x + c)^3 - 3*(6*a^5*b^2
+ 5*a^3*b^4 + 2*a*b^6)*cosh(d*x + c)^2 + (6*a^6*b - 7*a^4*b^3 - 8*a^2*b^5 -
 4*b^7)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + b^2)*log((a^2*cosh(d*x + c
)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + a^2 + 2*b^2 + 2*(a^2*cosh
(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(a*cosh(d*x + c) + a*sin
h(d*x + c) + b))/(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + 2*b*cosh(d*x + c)
 + 2*(a*cosh(d*x + c) + b)*sinh(d*x + c) - a)) - 2*(17*a^5*b^3 + 25*a^3*b^5
 + 8*a*b^7 + 4*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*x)*cosh(d*x + c) -
 2*(17*a^5*b^3 + 25*a^3*b^5 + 8*a*b^7 - 4*(a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^
2*b^6)*d*x*cosh(d*x + c)^3 + 4*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*x
- 3*(7*a^5*b^3 + 11*a^3*b^5 + 4*a*b^7 + 4*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 +
a*b^7)*d*x)*cosh(d*x + c)^2 + 2*(6*a^6*b^2 - 3*a^4*b^4 - 15*a^2*b^6 - 6*b^8
 + 2*(a^8 + a^6*b^2 - 3*a^4*b^4 - 5*a^2*b^6 - 2*b^8)*d*x)*cosh(d*x + c))*si
nh(d*x + c))/((a^11 + 3*a^9*b^2 + 3*a^7*b^4 + a^5*b^6)*d*cosh(d*x + c)^4 +
(a^11 + 3*a^9*b^2 + 3*a^7*b^4 + a^5*b^6)*d*sinh(d*x + c)^4 + 4*(a^10*b + 3*
a^8*b^3 + 3*a^6*b^5 + a^4*b^7)*d*cosh(d*x + c)^3 - 2*(a^11 + a^9*b^2 - 3*a^
```

```
7*b^4 - 5*a^5*b^6 - 2*a^3*b^8)*d*cosh(d*x + c)^2 + 4*((a^11 + 3*a^9*b^2 + 3
*a^7*b^4 + a^5*b^6)*d*cosh(d*x + c) + (a^10*b + 3*a^8*b^3 + 3*a^6*b^5 + a^4
*b^7)*d)*sinh(d*x + c)^3 - 4*(a^10*b + 3*a^8*b^3 + 3*a^6*b^5 + a^4*b^7)*d*c
osh(d*x + c) + 2*(3*(a^11 + 3*a^9*b^2 + 3*a^7*b^4 + a^5*b^6)*d*cosh(d*x + c
)^2 + 6*(a^10*b + 3*a^8*b^3 + 3*a^6*b^5 + a^4*b^7)*d*cosh(d*x + c) - (a^11
+ a^9*b^2 - 3*a^7*b^4 - 5*a^5*b^6 - 2*a^3*b^8)*d)*sinh(d*x + c)^2 + (a^11 +
 3*a^9*b^2 + 3*a^7*b^4 + a^5*b^6)*d + 4*((a^11 + 3*a^9*b^2 + 3*a^7*b^4 + a^
5*b^6)*d*cosh(d*x + c)^3 + 3*(a^10*b + 3*a^8*b^3 + 3*a^6*b^5 + a^4*b^7)*d*c
osh(d*x + c)^2 - (a^11 + a^9*b^2 - 3*a^7*b^4 - 5*a^5*b^6 - 2*a^3*b^8)*d*cos
h(d*x + c) - (a^10*b + 3*a^8*b^3 + 3*a^6*b^5 + a^4*b^7)*d)*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{csch}\left(c + dx\right)\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csch(d*x+c))**3,x)
```

```
[Out] Integral((a + b*csch(c + d*x))**(-3), x)
```

Giac [A]
time = 0.41, size = 293, normalized size = 1.80

$$-\frac{\frac{(6\,a^4b+5\,a^2b^3+2\,b^5)\log\left(\left|\frac{2\,ae^{(dx+c)}+2\,b-2\,\sqrt{a^2+b^2}}{2\,ae^{(dx+c)}+2\,b+2\,\sqrt{a^2+b^2}}\right|\right)}{(a^7+2\,a^5b^2+a^3b^4)\sqrt{a^2+b^2}}-\frac{2\,(7\,a^3b^3e^{(3\,dx+3\,c)}+4\,ab^5e^{(3\,dx+3\,c)}-6\,a^4b^2e^{(2\,dx+2\,c)}+9\,a^2b^4e^{(2\,dx+2\,c)}+6\,b^6e^{(2\,dx+2\,c)}-17\,a^3b^3e^{(dx+c)}-8\,ab^5e^{(dx+c)}+6\,a^4b^2+3\,a^2b^4)}{(a^7+2\,a^5b^2+a^3b^4)(ae^{(2\,dx+2\,c)}+2\,be^{(dx+c)}-a)^2}-\frac{2\,(dx+c)}{a^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csch(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*((6*a^4*b + 5*a^2*b^3 + 2*b^5)*log(abs(2*a*e^(d*x + c) + 2*b - 2*sqrt(
a^2 + b^2))/abs(2*a*e^(d*x + c) + 2*b + 2*sqrt(a^2 + b^2)))/((a^7 + 2*a^5*b
^2 + a^3*b^4)*sqrt(a^2 + b^2)) - 2*(7*a^3*b^3*e^(3*d*x + 3*c) + 4*a*b^5*e^(
3*d*x + 3*c) - 6*a^4*b^2*e^(2*d*x + 2*c) + 9*a^2*b^4*e^(2*d*x + 2*c) + 6*b^
6*e^(2*d*x + 2*c) - 17*a^3*b^3*e^(d*x + c) - 8*a*b^5*e^(d*x + c) + 6*a^4*b^
2 + 3*a^2*b^4)/((a^7 + 2*a^5*b^2 + a^3*b^4)*(a*e^(2*d*x + 2*c) + 2*b*e^(d*x
 + c) - a)^2) - 2*(d*x + c)/a^3)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+\frac{b}{\mathrm{sinh}\left(c+d\,x\right)}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/sinh(c + d*x))^3,x)
[Out] int(1/(a + b/sinh(c + d*x))^3, x)
```

3.77 $\int \frac{\sinh^3(x)}{a+b\,\mathrm{csch}(x)}\,dx$

Optimal. Leaf size=107

$$\frac{b(a^2-2b^2)\,x}{2a^4}-\frac{2b^4\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4\sqrt{a^2+b^2}}-\frac{(2a^2-3b^2)\cosh(x)}{3a^3}-\frac{b\cosh(x)\sinh(x)}{2a^2}+\frac{\cosh(x)\sinh^2(x)}{3a}$$

```
[Out] 1/2*b*(a^2-2*b^2)*x/a^4-1/3*(2*a^2-3*b^2)*cosh(x)/a^3-1/2*b*cosh(x)*sinh(x)
/a^2+1/3*cosh(x)*sinh(x)^2/a-2*b^4*arctanh((a-b*tanh(1/2*x))/(a^2+b^2)^(1/2
))/a^4/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3938, 4189, 4004, 3916, 2739, 632, 212}

$$-\frac{b\sinh(x)\cosh(x)}{2a^2}+\frac{bx(a^2-2b^2)}{2a^4}-\frac{2b^4\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4\sqrt{a^2+b^2}}-\frac{(2a^2-3b^2)\cosh(x)}{3a^3}+\frac{\sinh^2(x)\cosh(x)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(a + b*Csch[x]),x]
```

```
[Out] (b*(a^2 - 2*b^2)*x)/(2*a^4) - (2*b^4*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b
^2]])/(a^4*Sqrt[a^2 + b^2]) - ((2*a^2 - 3*b^2)*Cosh[x])/(3*a^3) - (b*Cosh[x
]*Sinh[x])/(2*a^2) + (Cosh[x]*Sinh[x]^2)/(3*a)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3938

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n -
 a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\sinh^3(x)}{a+b\text{csch}(x)}\,dx = \frac{\cosh(x)\sinh^2(x)}{3a} - \frac{i\int \frac{\left(-3ib-2ia\text{csch}(x)-2ib\text{csch}^2(x)\right)\sinh^2(x)}{a+b\text{csch}(x)}\,dx}{3a}$$

$$= -\frac{b\cosh(x)\sinh(x)}{2a^2} + \frac{\cosh(x)\sinh^2(x)}{3a} + \frac{\int \frac{\left(-2(2a^2-3b^2)-ab\text{csch}(x)+3b^2\text{csch}^2(x)\right)\sinh(x)}{a+b\text{csch}(x)}\,dx}{6a^2}$$

$$= -\frac{(2a^2-3b^2)\cosh(x)}{3a^3} - \frac{b\cosh(x)\sinh(x)}{2a^2} + \frac{\cosh(x)\sinh^2(x)}{3a} + \frac{i\int \frac{-3ib(a^2-2b^2)-3iab^2\text{csc}}{a+b\text{csch}(x)}}{6a^3}$$

$$= \frac{b(a^2-2b^2)\,x}{2a^4} - \frac{(2a^2-3b^2)\cosh(x)}{3a^3} - \frac{b\cosh(x)\sinh(x)}{2a^2} + \frac{\cosh(x)\sinh^2(x)}{3a} + \frac{b^4\int \frac{\text{cs}}{a+b}}{}$$

$$= \frac{b(a^2-2b^2)\,x}{2a^4} - \frac{(2a^2-3b^2)\cosh(x)}{3a^3} - \frac{b\cosh(x)\sinh(x)}{2a^2} + \frac{\cosh(x)\sinh^2(x)}{3a} + \frac{b^3\int \frac{}{1+}}{a}$$

$$= \frac{b(a^2-2b^2)\,x}{2a^4} - \frac{(2a^2-3b^2)\cosh(x)}{3a^3} - \frac{b\cosh(x)\sinh(x)}{2a^2} + \frac{\cosh(x)\sinh^2(x)}{3a} + \frac{(2b^3)\,\text{Su}}{}$$

$$= \frac{b(a^2-2b^2)\,x}{2a^4} - \frac{(2a^2-3b^2)\cosh(x)}{3a^3} - \frac{b\cosh(x)\sinh(x)}{2a^2} + \frac{\cosh(x)\sinh^2(x)}{3a} - \frac{(4b^3)\,\text{Su}}{}$$

$$= \frac{b(a^2-2b^2)\,x}{2a^4} - \frac{2b^4\tanh^{-1}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{a^2+b^2}}\right)}{a^4\sqrt{a^2+b^2}} - \frac{(2a^2-3b^2)\cosh(x)}{3a^3} - \frac{b\cosh(x)\sinh(x)}{2a^2}$$

Mathematica [A]
time = 0.29, size = 104, normalized size = 0.97

$$\frac{(-9a^3+12ab^2)\cosh(x)+a^3\cosh(3x)+3b\left(2a^2x-4b^2x+\frac{8b^3\text{ArcTan}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-a^2\sinh(2x)\right)}{12a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(a + b*Csch[x]),x]
```

```
[Out] ((-9*a^3 + 12*a*b^2)*Cosh[x] + a^3*Cosh[3*x] + 3*b*(2*a^2*x - 4*b^2*x + (8*
b^3*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - a^2*Sinh
[2*x]))/(12*a^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(93) = 186$.
time = 0.86, size = 198, normalized size = 1.85

method	result
default	$\frac{1}{3a\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}-\frac{a-b}{2a^{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}-\frac{a^{2}+ab-2b^{2}}{2a^{3}\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{b\left(a^{2}-2b^{2}\right)\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2a^{4}}-\frac{2b^{4}\,\mathrm{arctanh}\left(\frac{-2b\tanh\left(\frac{x}{2}\right)+2a}{2\sqrt{a^{2}+b^{2}}}\right)}{a^{4}\sqrt{a^{2}+b^{2}}}$
risch	$\frac{bx}{2a^{2}}-\frac{b^{3}x}{a^{4}}+\frac{e^{3x}}{24a}-\frac{b\,e^{2x}}{8a^{2}}-\frac{3\,e^{x}}{8a}+\frac{e^{x}b^{2}}{2a^{3}}-\frac{3\,e^{-x}}{8a}+\frac{e^{-x}b^{2}}{2a^{3}}+\frac{b\,e^{-2x}}{8a^{2}}+\frac{e^{-3x}}{24a}+\frac{b^{4}\ln\left(e^{x}+\frac{b\sqrt{a^{2}+b^{2}}-a^{2}-b^{2}}{\sqrt{a^{2}+b^{2}}\,a}\right)}{\sqrt{a^{2}+b^{2}}\,a^{4}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/a/(tanh(1/2*x)+1)^3-1/2*(a-b)/a^2/(tanh(1/2*x)+1)^2-1/2*(a^2+a*b-2*b^2)
/a^3/(tanh(1/2*x)+1)+1/2*b*(a^2-2*b^2)/a^4*ln(tanh(1/2*x)+1)-2*b^4/a^4/(a^2
+b^2)^(1/2)*arctanh(1/2*(-2*b*tanh(1/2*x)+2*a)/(a^2+b^2)^(1/2))-1/3/a/(tanh
(1/2*x)-1)^3-1/2*(a+b)/a^2/(tanh(1/2*x)-1)^2-1/2*(-a^2+a*b+2*b^2)/a^3/(tanh
(1/2*x)-1)-1/2*b*(a^2-2*b^2)/a^4*ln(tanh(1/2*x)-1)
```

Maxima [A]
time = 0.47, size = 157, normalized size = 1.47

$$\frac{b^{4}\log\left(\frac{ae^{(-x)}-b-\sqrt{a^{2}+b^{2}}}{ae^{(-x)}-b+\sqrt{a^{2}+b^{2}}}\right)}{\sqrt{a^{2}+b^{2}}\,a^{4}}-\frac{\left(3\,abe^{(-x)}-a^{2}+3\,(3\,a^{2}-4\,b^{2})e^{(-2\,x)}\right)e^{(3\,x)}}{24\,a^{3}}+\frac{3\,abe^{(-2\,x)}+a^{2}e^{(-3\,x)}-3\,(3\,a^{2}-4\,b^{2})e^{(-x)}}{24\,a^{3}}+\frac{(a^{2}b-2\,b^{3})x}{2\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] b^4*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))/
(sqrt(a^2 + b^2)*a^4) - 1/24*(3*a*b*e^(-x) - a^2 + 3*(3*a^2 - 4*b^2)*e^(-2*
x))*e^(3*x)/a^3 + 1/24*(3*a*b*e^(-2*x) + a^2*e^(-3*x) - 3*(3*a^2 - 4*b^2)*e
^(-x))/a^3 + 1/2*(a^2*b - 2*b^3)*x/a^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 807 vs. $2(95) = 190$.
time = 0.41, size = 807, normalized size = 7.54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] 1/24*((a^5 + a^3*b^2)*cosh(x)^6 + (a^5 + a^3*b^2)*sinh(x)^6 - 3*(a^4*b + a^
2*b^3)*cosh(x)^5 - 3*(a^4*b + a^2*b^3 - 2*(a^5 + a^3*b^2)*cosh(x))*sinh(x)^
5 + a^5 + a^3*b^2 + 12*(a^4*b - a^2*b^3 - 2*b^5)*x*cosh(x)^3 - 3*(3*a^5 - a
^3*b^2 - 4*a*b^4)*cosh(x)^4 - 3*(3*a^5 - a^3*b^2 - 4*a*b^4 - 5*(a^5 + a^3*b
```

```
^2)*cosh(x)^2 + 5*(a^4*b + a^2*b^3)*cosh(x))*sinh(x)^4 + 2*(10*(a^5 + a^3*b
^2)*cosh(x)^3 - 15*(a^4*b + a^2*b^3)*cosh(x)^2 + 6*(a^4*b - a^2*b^3 - 2*b^5
)*x - 6*(3*a^5 - a^3*b^2 - 4*a*b^4)*cosh(x))*sinh(x)^3 - 3*(3*a^5 - a^3*b^2
 - 4*a*b^4)*cosh(x)^2 - 3*(3*a^5 - a^3*b^2 - 4*a*b^4 - 5*(a^5 + a^3*b^2)*co
sh(x)^4 + 10*(a^4*b + a^2*b^3)*cosh(x)^3 - 12*(a^4*b - a^2*b^3 - 2*b^5)*x*c
osh(x) + 6*(3*a^5 - a^3*b^2 - 4*a*b^4)*cosh(x)^2)*sinh(x)^2 + 24*(b^4*cosh(
x)^3 + 3*b^4*cosh(x)^2*sinh(x) + 3*b^4*cosh(x)*sinh(x)^2 + b^4*sinh(x)^3)*s
qrt(a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) + a^2 + 2
*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(a*cosh(x) + a*sin
h(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sin
h(x) - a)) + 3*(a^4*b + a^2*b^3)*cosh(x) + 3*(2*(a^5 + a^3*b^2)*cosh(x)^5 +
 a^4*b + a^2*b^3 - 5*(a^4*b + a^2*b^3)*cosh(x)^4 + 12*(a^4*b - a^2*b^3 - 2*
b^5)*x*cosh(x)^2 - 4*(3*a^5 - a^3*b^2 - 4*a*b^4)*cosh(x)^3 - 2*(3*a^5 - a^3
*b^2 - 4*a*b^4)*cosh(x))*sinh(x))/((a^6 + a^4*b^2)*cosh(x)^3 + 3*(a^6 + a^4
*b^2)*cosh(x)^2*sinh(x) + 3*(a^6 + a^4*b^2)*cosh(x)*sinh(x)^2 + (a^6 + a^4*
b^2)*sinh(x)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3(x)}{a + b \operatorname{csch}(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(a+b*csch(x)),x)
[Out] Integral(sinh(x)**3/(a + b*csch(x)), x)
```

Giac [A]
time = 0.40, size = 155, normalized size = 1.45

$$\frac{b^4 \log\left(\frac{\left|2\, ae^x + 2\, b - 2\, \sqrt{a^2 + b^2}\right|}{\left|2\, ae^x + 2\, b + 2\, \sqrt{a^2 + b^2}\right|}\right)}{\sqrt{a^2 + b^2}\, a^4} + \frac{a^2 e^{(3\,x)} - 3\, abe^{(2\,x)} - 9\, a^2 e^x + 12\, b^2 e^x}{24\, a^3} + \frac{(a^2 b - 2\, b^3) x}{2\, a^4} + \frac{\left(3\, a^2 b e^x + a^3 - 3\, (3\, a^3 - 4\, ab^2) e^{(2\,x)}\right) e^{(-3\,x)}}{24\, a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*csch(x)),x, algorithm="giac")
[Out] b^4*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*a^4) + 1/24*(a^2*e^(3*x) - 3*a*b*e^(2*x) - 9*a
^2*e^x + 12*b^2*e^x)/a^3 + 1/2*(a^2*b - 2*b^3)*x/a^4 + 1/24*(3*a^2*b*e^x +
a^3 - 3*(3*a^3 - 4*a*b^2)*e^(2*x))*e^(-3*x)/a^4
```

Mupad [B]
time = 1.83, size = 199, normalized size = 1.86

$$\frac{\mathrm{e}^{-3\,x}}{24\,a} + \frac{\mathrm{e}^{3\,x}}{24\,a} + \frac{x\,(a^2\,b - 2\,b^3)}{2\,a^4} - \frac{\mathrm{e}^x\,(3\,a^2 - 4\,b^2)}{8\,a^3} + \frac{b\,\mathrm{e}^{-2\,x}}{8\,a^2} - \frac{b\,\mathrm{e}^{2\,x}}{8\,a^2} - \frac{\mathrm{e}^{-x}\,(3\,a^2 - 4\,b^2)}{8\,a^3} - \frac{b^4\,\ln\left(-\frac{2\,b^4\,\mathrm{e}^x}{a^5} - \frac{2\,b^4\,(a - b\,\mathrm{e}^x)}{a^5\,\sqrt{a^2 + b^2}}\right)}{a^4\,\sqrt{a^2 + b^2}} + \frac{b^4\,\ln\left(\frac{2\,b^4\,(a - b\,\mathrm{e}^x)}{a^5\,\sqrt{a^2 + b^2}} - \frac{2\,b^4\,\mathrm{e}^x}{a^5}\right)}{a^4\,\sqrt{a^2 + b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a + b/sinh(x)),x)
```

```
[Out] exp(-3*x)/(24*a) + exp(3*x)/(24*a) + (x*(a^2*b - 2*b^3))/(2*a^4) - (exp(x)*
(3*a^2 - 4*b^2))/(8*a^3) + (b*exp(-2*x))/(8*a^2) - (b*exp(2*x))/(8*a^2) - (
exp(-x)*(3*a^2 - 4*b^2))/(8*a^3) - (b^4*log(- (2*b^4*exp(x))/a^5 - (2*b^4*(
a - b*exp(x)))/(a^5*(a^2 + b^2)^(1/2))))/(a^4*(a^2 + b^2)^(1/2)) + (b^4*log
((2*b^4*(a - b*exp(x)))/(a^5*(a^2 + b^2)^(1/2)) - (2*b^4*exp(x))/a^5))/(a^4
*(a^2 + b^2)^(1/2))
```

3.78 $\int \frac{\sinh^2(x)}{a+b\text{csch}(x)} dx$

Optimal. Leaf size=80

$$-\frac{(a^2-2b^2)\,x}{2a^3}+\frac{2b^3\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}}-\frac{b\cosh(x)}{a^2}+\frac{\cosh(x)\sinh(x)}{2a}$$

```
[Out] -1/2*(a^2-2*b^2)*x/a^3-b*cosh(x)/a^2+1/2*cosh(x)*sinh(x)/a+2*b^3*arctanh((a
-b*tanh(1/2*x))/(a^2+b^2)^(1/2))/a^3/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3938, 4189, 4004, 3916, 2739, 632, 212}

$$-\frac{b\cosh(x)}{a^2}-\frac{x(a^2-2b^2)}{2a^3}+\frac{2b^3\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}}+\frac{\sinh(x)\cosh(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(a + b*Csch[x]),x]
```

```
[Out] -1/2*((a^2 - 2*b^2)*x)/a^3 + (2*b^3*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^
2]])/(a^3*Sqrt[a^2 + b^2]) - (b*Cosh[x])/a^2 + (Cosh[x]*Sinh[x])/(2*a)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3938

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n -
 a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\sinh^2(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{\cosh(x)\sinh(x)}{2a} - \frac{i\int \frac{\left(-2ib-ia\operatorname{csch}(x)-ib\operatorname{csch}^2(x)\right)\sinh(x)}{a+b\operatorname{csch}(x)}\,dx}{2a}$$

$$= -\frac{b\cosh(x)}{a^2} + \frac{\cosh(x)\sinh(x)}{2a} + \frac{\int \frac{-a^2+2b^2-ab\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}\,dx}{2a^2}$$

$$= -\frac{(a^2-2b^2)\,x}{2a^3} - \frac{b\cosh(x)}{a^2} + \frac{\cosh(x)\sinh(x)}{2a} - \frac{b^3\int \frac{\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}\,dx}{a^3}$$

$$= -\frac{(a^2-2b^2)\,x}{2a^3} - \frac{b\cosh(x)}{a^2} + \frac{\cosh(x)\sinh(x)}{2a} - \frac{b^2\int \frac{1}{1+\frac{a\sinh(x)}{b}}\,dx}{a^3}$$

$$= -\frac{(a^2-2b^2)\,x}{2a^3} - \frac{b\cosh(x)}{a^2} + \frac{\cosh(x)\sinh(x)}{2a} - \frac{(2b^2)\operatorname{Subst}\left(\int \frac{1}{1+\frac{2ax}{b}-x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right.}{a^3}$$

$$= -\frac{(a^2-2b^2)\,x}{2a^3} - \frac{b\cosh(x)}{a^2} + \frac{\cosh(x)\sinh(x)}{2a} + \frac{(4b^2)\operatorname{Subst}\left(\int \frac{1}{4\left(1+\frac{a^2}{b^2}\right)-x^2}\,dx, x, \frac{2a}{b} - 2\right.}{a^3}$$

$$= -\frac{(a^2-2b^2)\,x}{2a^3} + \frac{2b^3\tanh^{-1}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}} - \frac{b\cosh(x)}{a^2} + \frac{\cosh(x)\sinh(x)}{2a}$$

Mathematica [A]
time = 0.09, size = 82, normalized size = 1.02

$$\frac{-2a^2x+4b^2x-\frac{8b^3\operatorname{ArcTan}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-4ab\cosh(x)+a^2\sinh(2x)}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(a + b*Csch[x]),x]
```

```
[Out] (-2*a^2*x + 4*b^2*x - (8*b^3*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sq
rt[-a^2 - b^2] - 4*a*b*Cosh[x] + a^2*Sinh[2*x])/(4*a^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(70) = 140.
time = 0.83, size = 152, normalized size = 1.90

method	result

default	$\frac{2b^3 \operatorname{arctanh}\left(\frac{-2b\tanh\left(\frac{x}{2}\right)+2a}{2\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}} + \frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)-1\right)^2} - \frac{-2b-a}{2a^2\left(\tanh\left(\frac{x}{2}\right)-1\right)} + \frac{\left(a^2-2b^2\right)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2a^3} - \frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)+1\right)^2} -$
risch	$-\frac{x}{2a} + \frac{x\,b^2}{a^3} + \frac{e^{2x}}{8a} - \frac{b\,e^{x}}{2a^2} - \frac{b\,e^{-x}}{2a^2} - \frac{e^{-2x}}{8a} + \frac{b^3\ln\left(e^{x}+\frac{b\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,a}\right)}{\sqrt{a^2+b^2}\,a^3} - \frac{b^3\ln\left(e^{x}+\frac{b\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,a}\right)}{\sqrt{a^2+b^2}\,a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^3/a^3/(a^2+b^2)^(1/2)*arctanh(1/2*(-2*b*tanh(1/2*x)+2*a)/(a^2+b^2)^(1/2
))+1/2/a/(tanh(1/2*x)-1)^2-1/2*(-2*b-a)/a^2/(tanh(1/2*x)-1)+1/2*(a^2-2*b^2)
/a^3*ln(tanh(1/2*x)-1)-1/2/a/(tanh(1/2*x)+1)^2-1/2*(2*b-a)/a^2/(tanh(1/2*x)
+1)+1/2/a^3*(-a^2+2*b^2)*ln(tanh(1/2*x)+1)
```

Maxima [A]
time = 0.48, size = 116, normalized size = 1.45

$$-\frac{b^3\log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,a^3} - \frac{\left(4\,be^{(-x)}-a\right)e^{(2\,x)}}{8\,a^2} - \frac{4\,be^{(-x)}+ae^{(-2\,x)}}{8\,a^2} - \frac{\left(a^2-2\,b^2\right)x}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -b^3*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))
/(sqrt(a^2 + b^2)*a^3) - 1/8*(4*b*e^(-x) - a)*e^(2*x)/a^2 - 1/8*(4*b*e^(-x)
 + a*e^(-2*x))/a^2 - 1/2*(a^2 - 2*b^2)*x/a^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 456 vs. 2(72) = 144.
time = 0.44, size = 456, normalized size = 5.70

$$\frac{\left(a^4+a^2b^2\right)\cosh\left(x\right)^4+\left(a^4+a^2b^2\right)\sinh\left(x\right)^4-a^4-a^2b^2-4\left(a^4-a^2b^2-2\,b^4\right)x\cosh\left(x\right)^2-4\left(a^3b+ab^3\right)\cosh\left(x\right)^3-4\left(a^3b+ab^3-\left(a^4+a^2b^2\right)\cosh\left(x\right)\right)\sinh\left(x\right)^3+2\left(3\left(a^4+a^2b^2\right)\cosh\left(x\right)^2-2\left(a^4-a^2b^2-2\,b^4\right)x-6\left(a^3b+ab^3\right)\cosh\left(x\right)\right)\sinh\left(x\right)^2+8\left(b^3\cosh\left(x\right)^2+2\,b^3\cosh\left(x\right)\sinh\left(x\right)+b^3\sinh\left(x\right)^2\right)\sqrt{a^2+b^2}\log\left(\frac{a^2\cosh(x)^2+a^2\sinh(x)^2+2\,ab\cosh(x)+a^2+2\,b^2+2\left(a^2\cosh(x)+ab\right)\sinh(x)+2\sqrt{a^2+b^2}\left(a\cosh(x)+a\sinh(x)+b\right)}{a\cosh(x)^2+a\sinh(x)^2+2\,b\cosh(x)+2\left(a\cosh(x)+b\right)\sinh(x)-a}\right)-4\left(a^3b+ab^3\right)\cosh\left(x\right)-4\left(a^3b+ab^3-\left(a^4+a^2b^2\right)\cosh\left(x\right)^3+2\left(a^4-a^2b^2-2\,b^4\right)x\cosh\left(x\right)+3\left(a^3b+ab^3\right)\cosh\left(x\right)^2\right)\sinh\left(x\right)}{8\left(\left(a^5+a^3b^2\right)\cosh\left(x\right)^2+2\left(a^5+a^3b^2\right)\cosh\left(x\right)\sinh\left(x\right)+\left(a^5+a^3b^2\right)\sinh\left(x\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] 1/8*((a^4 + a^2*b^2)*cosh(x)^4 + (a^4 + a^2*b^2)*sinh(x)^4 - a^4 - a^2*b^2
- 4*(a^4 - a^2*b^2 - 2*b^4)*x*cosh(x)^2 - 4*(a^3*b + a*b^3)*cosh(x)^3 - 4*(
a^3*b + a*b^3 - (a^4 + a^2*b^2)*cosh(x))*sinh(x)^3 + 2*(3*(a^4 + a^2*b^2)*c
osh(x)^2 - 2*(a^4 - a^2*b^2 - 2*b^4)*x - 6*(a^3*b + a*b^3)*cosh(x))*sinh(x)
^2 + 8*(b^3*cosh(x)^2 + 2*b^3*cosh(x)*sinh(x) + b^3*sinh(x)^2)*sqrt(a^2 + b
^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) + a^2 + 2*b^2 + 2*(a
```

```
^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/
(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) - a))
- 4*(a^3*b + a*b^3)*cosh(x) - 4*(a^3*b + a*b^3 - (a^4 + a^2*b^2)*cosh(x)^3
+ 2*(a^4 - a^2*b^2 - 2*b^4)*x*cosh(x) + 3*(a^3*b + a*b^3)*cosh(x)^2)*sinh(x
))/((a^5 + a^3*b^2)*cosh(x)^2 + 2*(a^5 + a^3*b^2)*cosh(x)*sinh(x) + (a^5 +
a^3*b^2)*sinh(x)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(x)}{a + b\,\mathrm{csch}\,(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(a+b*csch(x)),x)
```

```
[Out] Integral(sinh(x)**2/(a + b*csch(x)), x)
```

Giac [A]
time = 0.39, size = 115, normalized size = 1.44

$$-\frac{b^3\log\left(\frac{\left|2\,ae^x+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^x+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,a^3}+\frac{ae^{(2\,x)}-4\,be^x}{8\,a^2}-\frac{(a^2-2\,b^2)x}{2\,a^3}-\frac{(4\,abe^x+a^2)e^{(-2\,x)}}{8\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -b^3*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(
a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3) + 1/8*(a*e^(2*x) - 4*b*e^x)/a^2 - 1/2*(a
^2 - 2*b^2)*x/a^3 - 1/8*(4*a*b*e^x + a^2)*e^(-2*x)/a^3
```

Mupad [B]
time = 1.67, size = 157, normalized size = 1.96

$$\frac{\mathrm{e}^{2\,x}}{8\,a}-\frac{\mathrm{e}^{-2\,x}}{8\,a}-\frac{b\,\mathrm{e}^x}{2\,a^2}-\frac{b\,\mathrm{e}^{-x}}{2\,a^2}-\frac{x\,(a^2-2\,b^2)}{2\,a^3}-\frac{b^3\,\ln\left(\frac{2\,b^3\,\mathrm{e}^x}{a^4}-\frac{2\,b^3\,(a-b\,\mathrm{e}^x)}{a^4\,\sqrt{a^2+b^2}}\right)}{a^3\,\sqrt{a^2+b^2}}+\frac{b^3\,\ln\left(\frac{2\,b^3\,\mathrm{e}^x}{a^4}+\frac{2\,b^3\,(a-b\,\mathrm{e}^x)}{a^4\,\sqrt{a^2+b^2}}\right)}{a^3\,\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a + b/sinh(x)),x)
```

```
[Out] exp(2*x)/(8*a) - exp(-2*x)/(8*a) - (b*exp(x))/(2*a^2) - (b*exp(-x))/(2*a^2)
 - (x*(a^2 - 2*b^2))/(2*a^3) - (b^3*log((2*b^3*exp(x))/a^4 - (2*b^3*(a - b*
exp(x)))/(a^4*(a^2 + b^2)^(1/2))))/(a^3*(a^2 + b^2)^(1/2)) + (b^3*log((2*b^
3*exp(x))/a^4 + (2*b^3*(a - b*exp(x)))/(a^4*(a^2 + b^2)^(1/2))))/(a^3*(a^2
+ b^2)^(1/2))
```

3.79 $\int \frac{\sinh(x)}{a+b\,\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=57

$$-\frac{bx}{a^2}-\frac{2b^2\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}+\frac{\cosh(x)}{a}$$

```
[Out] -b*x/a^2+cosh(x)/a-2*b^2*arctanh((a-b*tanh(1/2*x))/(a^2+b^2)^(1/2))/a^2/(a^
2+b^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {3938, 12, 3868, 2739, 632, 212}

$$-\frac{2b^2\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}-\frac{bx}{a^2}+\frac{\cosh(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(a + b*Csch[x]),x]
```

```
[Out] -((b*x)/a^2) - (2*b^2*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2*Sqrt
[a^2 + b^2]) + Cosh[x]/a
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3868

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rule 3938

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n -
 a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh(x)}{a+b\text{csch}(x)}\,dx &= \frac{\cosh(x)}{a}-\frac{\int \frac{b}{a+b\text{csch}(x)}\,dx}{a}\\
&= \frac{\cosh(x)}{a}-\frac{b\int \frac{1}{a+b\text{csch}(x)}\,dx}{a}\\
&= -\frac{bx}{a^2}+\frac{\cosh(x)}{a}+\frac{b\int \frac{1}{1+\frac{a\sinh(x)}{b}}\,dx}{a^2}\\
&= -\frac{bx}{a^2}+\frac{\cosh(x)}{a}+\frac{(2b)\text{Subst}\left(\int \frac{1}{1+\frac{2ax}{b}-x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2}\\
&= -\frac{bx}{a^2}+\frac{\cosh(x)}{a}-\frac{(4b)\text{Subst}\left(\int \frac{1}{4\left(1+\frac{a^2}{b^2}\right)-x^2}\,dx,x,\frac{2a}{b}-2\tanh\left(\frac{x}{2}\right)\right)}{a^2}\\
&= -\frac{bx}{a^2}-\frac{2b^2\tanh^{-1}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}+\frac{\cosh(x)}{a}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 61, normalized size = 1.07

$$\frac{b\left(-x+\frac{2b\text{ArcTan}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}\right)+a\cosh(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(a + b*Csch[x]),x]
```

```
[Out] (b*(-x + (2*b*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2])
 + a*Cosh[x])/a^2
```

Maple [A]
time = 0.85, size = 92, normalized size = 1.61

method	result	size
default	$-\frac{1}{a\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{b\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a^{2}}-\frac{2b^{2}\operatorname{arctanh}\left(\frac{-2b\tanh\left(\frac{x}{2}\right)+2a}{2\sqrt{a^{2}+b^{2}}}\right)}{a^{2}\sqrt{a^{2}+b^{2}}}+\frac{1}{a\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{b\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a^{2}}$	92
risch	$-\frac{bx}{a^{2}}+\frac{e^{x}}{2a}+\frac{e^{-x}}{2a}+\frac{b^{2}\ln\left(e^{x}+\frac{b\sqrt{a^{2}+b^{2}}-a^{2}-b^{2}}{\sqrt{a^{2}+b^{2}}\,a}\right)}{\sqrt{a^{2}+b^{2}}\,a^{2}}-\frac{b^{2}\ln\left(e^{x}+\frac{b\sqrt{a^{2}+b^{2}}+a^{2}+b^{2}}{\sqrt{a^{2}+b^{2}}\,a}\right)}{\sqrt{a^{2}+b^{2}}\,a^{2}}$	132

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a/(tanh(1/2*x)-1)+b/a^2*ln(tanh(1/2*x)-1)-2*b^2/a^2/(a^2+b^2)^(1/2)*arct
anh(1/2*(-2*b*tanh(1/2*x)+2*a)/(a^2+b^2)^(1/2))+1/a/(tanh(1/2*x)+1)-b/a^2*l
n(tanh(1/2*x)+1)
```

Maxima [A]
time = 0.47, size = 84, normalized size = 1.47

$$\frac{b^{2}\log\left(\frac{ae^{(-x)}-b-\sqrt{a^{2}+b^{2}}}{ae^{(-x)}-b+\sqrt{a^{2}+b^{2}}}\right)}{\sqrt{a^{2}+b^{2}}\,a^{2}}-\frac{bx}{a^{2}}+\frac{e^{(-x)}}{2\,a}+\frac{e^{x}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] b^2*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))/
(sqrt(a^2 + b^2)*a^2) - b*x/a^2 + 1/2*e^(-x)/a + 1/2*e^x/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(53) = 106.
time = 0.36, size = 238, normalized size = 4.18

$$\frac{a^{3}+ab^{2}-2\,(a^{2}b+b^{3})x\cosh\left(x\right)+(a^{3}+ab^{2})\cosh\left(x\right)^{2}+(a^{3}+ab^{2})\sinh\left(x\right)^{2}+2\,(b^{2}\cosh\left(x\right)+b^{2}\sinh\left(x\right))\sqrt{a^{2}+b^{2}}\log\left(\frac{a^{2}\cosh(x)^{2}+a^{2}\sinh(x)^{2}+2\,ab\cosh(x)+a^{2}+2\,b^{2}+2\,(a^{2}\cosh(x)+ab)\sinh(x)-2\,\sqrt{a^{2}+b^{2}}\,(a\cosh(x)+a\sinh(x)+b)}{a\cosh(x)^{2}+a\sinh(x)^{2}+2\,b\cosh(x)+2\,(a\cosh(x)+b)\sinh(x)-a}\right)-2\,((a^{2}b+b^{3})x-(a^{3}+ab^{2})\cosh\left(x\right))\sinh\left(x\right)}{2\,((a^{4}+a^{2}b^{2})\cosh\left(x\right)+(a^{4}+a^{2}b^{2})\sinh\left(x\right))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*csch(x)),x, algorithm="fricas")
```

[Out] 1/2*(a^3 + a*b^2 - 2*(a^2*b + b^3)*x*cosh(x) + (a^3 + a*b^2)*cosh(x)^2 + (a^3 + a*b^2)*sinh(x)^2 + 2*(b^2*cosh(x) + b^2*sinh(x))*sqrt(a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) + a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) - a)) - 2*((a^2*b + b^3)*x - (a^3 + a*b^2)*cosh(x))*sinh(x))/((a^4 + a^2*b^2)*cosh(x) + (a^4 + a^2*b^2)*sinh(x))

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(x)}{a + b\operatorname{csch}(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(x)/(a+b*csch(x)),x)

[Out] Integral(sinh(x)/(a + b*csch(x)), x)

Giac [A]
time = 0.39, size = 86, normalized size = 1.51

$$\frac{b^2 \log\left(\frac{\left|2\, a e^x + 2\, b - 2\, \sqrt{a^2 + b^2}\right|}{\left|2\, a e^x + 2\, b + 2\, \sqrt{a^2 + b^2}\right|}\right)}{\sqrt{a^2 + b^2}\, a^2} - \frac{bx}{a^2} + \frac{e^{(-x)}}{2\, a} + \frac{e^x}{2\, a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(x)/(a+b*csch(x)),x, algorithm="giac")

[Out] b^2*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^2) - b*x/a^2 + 1/2*e^(-x)/a + 1/2*e^x/a

Mupad [B]
time = 1.58, size = 129, normalized size = 2.26

$$\frac{e^{-x}}{2\, a} + \frac{e^x}{2\, a} - \frac{b\, x}{a^2} - \frac{b^2\, \ln\left(-\frac{2\, b^2\, e^x}{a^3} - \frac{2\, b^2\, (a - b\, e^x)}{a^3\, \sqrt{a^2 + b^2}}\right)}{a^2\, \sqrt{a^2 + b^2}} + \frac{b^2\, \ln\left(\frac{2\, b^2\, (a - b\, e^x)}{a^3\, \sqrt{a^2 + b^2}} - \frac{2\, b^2\, e^x}{a^3}\right)}{a^2\, \sqrt{a^2 + b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(x)/(a + b/sinh(x)),x)

[Out] exp(-x)/(2*a) + exp(x)/(2*a) - (b*x)/a^2 - (b^2*log(- (2*b^2*exp(x))/a^3 - (2*b^2*(a - b*exp(x)))/(a^3*(a^2 + b^2)^(1/2))))/(a^2*(a^2 + b^2)^(1/2)) + (b^2*log((2*b^2*(a - b*exp(x)))/(a^3*(a^2 + b^2)^(1/2)) - (2*b^2*exp(x))/a^3))/(a^2*(a^2 + b^2)^(1/2))

3.80 $\int \frac{\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=37

$$-\frac{2\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

```
[Out] -2*arctanh((a-b*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3916, 2739, 632, 212}

$$-\frac{2\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]/(a + b*Csch[x]),x]
```

```
[Out] (-2*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
```

```
}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{\int \frac{1}{1+\frac{a\sinh(x)}{b}}\,dx}{b}$$
$$= \frac{2\text{Subst}\left(\int \frac{1}{1+\frac{2ax}{b}-x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b}$$
$$= -\frac{4\text{Subst}\left(\int \frac{1}{4\left(1+\frac{a^2}{b^2}\right)-x^2}\,dx, x, \frac{2a}{b}-2\tanh\left(\frac{x}{2}\right)\right)}{b}$$
$$= -\frac{2\tanh^{-1}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Mathematica [A]
time = 0.02, size = 45, normalized size = 1.22

$$\frac{2\text{ArcTan}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]/(a + b*Csch[x]),x]
[Out] (2*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2]
```

Maple [A]
time = 0.43, size = 35, normalized size = 0.95

method	result	size
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{-2b\tanh\left(\frac{x}{2}\right)+2a}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$	35
risch	$\frac{\ln\left(\mathrm{e}^x+\frac{b\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,a}\right)}{\sqrt{a^2+b^2}}-\frac{\ln\left(\mathrm{e}^x+\frac{b\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,a}\right)}{\sqrt{a^2+b^2}}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(a^2+b^2)^(1/2)*arctanh(1/2*(-2*b*tanh(1/2*x)+2*a)/(a^2+b^2)^(1/2))
```

Maxima [A]
time = 0.49, size = 54, normalized size = 1.46

$$\frac{\log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))/sqrt
(a^2 + b^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(33) = 66$.
time = 0.37, size = 111, normalized size = 3.00

$$\frac{\log\left(\frac{a^2\cosh(x)^2+a^2\sinh(x)^2+2\,ab\cosh(x)+a^2+2\,b^2+2\,(a^2\cosh(x)+ab)\sinh(x)-2\,\sqrt{a^2+b^2}\,(a\cosh(x)+a\sinh(x)+b)}{a\cosh(x)^2+a\sinh(x)^2+2\,b\cosh(x)+2\,(a\cosh(x)+b)\sinh(x)-a}\right)}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) + a^2 + 2*b^2 + 2*(a^2*c
osh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*c
osh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) - a))/sqrt
(a^2 + b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*csch(x)),x)
```

```
[Out] Integral(csch(x)/(a + b*csch(x)), x)
```

Giac [A]
time = 0.39, size = 56, normalized size = 1.51

$$\frac{\log\left(\frac{\left|2\,ae^x+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^x+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(a^2 +
 b^2)))/sqrt(a^2 + b^2)
```

Mupad [B]
time = 1.52, size = 49, normalized size = 1.32

$$\frac{2\,\mathrm{atan}\left(\frac{b}{\sqrt{-a^2-b^2}}+\frac{a\,\mathrm{e}^x}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)*(a + b/sinh(x))),x)
```

```
[Out] (2*atan(b/(- a^2 - b^2)^(1/2) + (a*exp(x))/(- a^2 - b^2)^(1/2)))/(- a^2 - b
^2)^(1/2)
```

3.81 $\int \frac{\operatorname{csch}^2(x)}{a+b\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=50

$$-\frac{\tanh^{-1}(\cosh(x))}{b}+\frac{2a\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}$$

```
[Out] -arctanh(cosh(x))/b+2*a*arctanh((a-b*tanh(1/2*x))/(a^2+b^2)^(1/2))/b/(a^2+b
^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3874, 3855, 3916, 2739, 632, 212}

$$\frac{2a\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}-\frac{\tanh^{-1}(\cosh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^2/(a + b*Csch[x]),x]
```

```
[Out] -(ArcTanh[Cosh[x]]/b) + (2*a*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/(b
*Sqrt[a^2 + b^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3874

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^2(x)}{a+b\operatorname{csch}(x)}\,dx &= \frac{\int \operatorname{csch}(x)\,dx}{b}-\frac{a\int \frac{\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}\,dx}{b}\\
&= -\frac{\tanh^{-1}(\cosh(x))}{b}-\frac{a\int \frac{1}{1+\frac{a\sinh(x)}{b}}\,dx}{b^2}\\
&= -\frac{\tanh^{-1}(\cosh(x))}{b}-\frac{(2a)\operatorname{Subst}\left(\int \frac{1}{1+\frac{2ax}{b}-x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b^2}\\
&= -\frac{\tanh^{-1}(\cosh(x))}{b}+\frac{(4a)\operatorname{Subst}\left(\int \frac{1}{4\left(1+\frac{a^2}{b^2}\right)-x^2}\,dx,x,\frac{2a}{b}-2\tanh\left(\frac{x}{2}\right)\right)}{b^2}\\
&= -\frac{\tanh^{-1}(\cosh(x))}{b}+\frac{2a\tanh^{-1}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 58, normalized size = 1.16

$$\frac{-\frac{2a\operatorname{ArcTan}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+\log\left(\tanh\left(\frac{x}{2}\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^2/(a + b*Csch[x]),x]
```

```
[Out] ((-2*a*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + Log[T
anh[x/2]])/b
```

Maple [A]
time = 0.55, size = 49, normalized size = 0.98

method	result	size
default	$\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{b}+\frac{2a\,\mathrm{arctanh}\left(\frac{-2b\tanh\left(\frac{x}{2}\right)+2a}{2\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}$	49
risch	$-\frac{\ln(\mathrm{e}^x+1)}{b}+\frac{a\ln\left(\mathrm{e}^x+\frac{b\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,a}\right)}{\sqrt{a^2+b^2}\,b}-\frac{a\ln\left(\mathrm{e}^x+\frac{b\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,a}\right)}{\sqrt{a^2+b^2}\,b}+\frac{\ln(\mathrm{e}^x-1)}{b}$	124

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^2/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*ln(tanh(1/2*x))+2*a/b/(a^2+b^2)^(1/2)*arctanh(1/2*(-2*b*tanh(1/2*x)+2*a
)/(a^2+b^2)^(1/2))
```

Maxima [A]
time = 0.47, size = 83, normalized size = 1.66

$$-\frac{a\log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,b}-\frac{\log\left(e^{(-x)}+1\right)}{b}+\frac{\log\left(e^{(-x)}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -a*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))/(
sqrt(a^2 + b^2)*b) - log(e^(-x) + 1)/b + log(e^(-x) - 1)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(46) = 92.
time = 0.38, size = 156, normalized size = 3.12

$$\frac{\sqrt{a^2+b^2}\,a\log\left(\frac{a^2\cosh(x)^2+a^2\sinh(x)^2+2\,ab\cosh(x)+a^2+2\,b^2+2\,(a^2\cosh(x)+ab)\sinh(x)+2\,\sqrt{a^2+b^2}\,(a\cosh(x)+a\sinh(x)+b)}{a\cosh(x)^2+a\sinh(x)^2+2\,b\cosh(x)+2\,(a\cosh(x)+b)\sinh(x)-a}\right)-(a^2+b^2)\log\left(\cosh(x)+\sinh(x)+1\right)+(a^2+b^2)\log\left(\cosh(x)+\sinh(x)-1\right)}{a^2b+b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] (sqrt(a^2 + b^2)*a*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) + a^2
 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(a*cosh(x) + a
```

```
*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)
*sinh(x) - a)) - (a^2 + b^2)*log(cosh(x) + sinh(x) + 1) + (a^2 + b^2)*log(c
osh(x) + sinh(x) - 1))/(a^2*b + b^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(x)}{a + b\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2/(a+b*csch(x)),x)
```

```
[Out] Integral(csch(x)**2/(a + b*csch(x)), x)
```

Giac [**A**]
time = 0.38, size = 82, normalized size = 1.64

$$-\frac{a\log\left(\frac{\left|2\,ae^{x}+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^{x}+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b} - \frac{\log\left(e^{x}+1\right)}{b} + \frac{\log\left(\left|e^{x}-1\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -a*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(a^
2 + b^2)))/(sqrt(a^2 + b^2)*b) - log(e^x + 1)/b + log(abs(e^x - 1))/b
```

Mupad [**B**]
time = 1.68, size = 287, normalized size = 5.74

$$\frac{\ln\left(32\,b-32\,b\,e^{x}\right)}{b}-\frac{\ln\left(32\,b+32\,b\,e^{x}\right)}{b}+\frac{a\,\ln\left(128\,b^5\,e^{x}-64\,a^3\,b^2-64\,a\,b^4-128\,b^4\,e^{x}\,\sqrt{a^2+b^2}+32\,a^4\,b\,e^{x}+160\,a^2\,b^3\,e^{x}+64\,a\,b^3\,\sqrt{a^2+b^2}+32\,a^3\,b\,\sqrt{a^2+b^2}-96\,a^2\,b^2\,e^{x}\,\sqrt{a^2+b^2}\right)\,\sqrt{a^2+b^2}}{a^2\,b+b^3}-\frac{a\,\ln\left(64\,a\,b^4+64\,a^3\,b^2-128\,b^5\,e^{x}-128\,b^4\,e^{x}\,\sqrt{a^2+b^2}-32\,a^4\,b\,e^{x}-160\,a^2\,b^3\,e^{x}+64\,a\,b^3\,\sqrt{a^2+b^2}+32\,a^3\,b\,\sqrt{a^2+b^2}-96\,a^2\,b^2\,e^{x}\,\sqrt{a^2+b^2}\right)\,\sqrt{a^2+b^2}}{a^2\,b+b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^2*(a + b/sinh(x))),x)
```

```
[Out] log(32*b - 32*b*exp(x))/b - log(32*b + 32*b*exp(x))/b + (a*log(128*b^5*exp(
x) - 64*a^3*b^2 - 64*a*b^4 - 128*b^4*exp(x)*(a^2 + b^2)^(1/2) + 32*a^4*b*ex
p(x) + 160*a^2*b^3*exp(x) + 64*a*b^3*(a^2 + b^2)^(1/2) + 32*a^3*b*(a^2 + b^
2)^(1/2) - 96*a^2*b^2*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/(a^2*b +
 b^3) - (a*log(64*a*b^4 + 64*a^3*b^2 - 128*b^5*exp(x) - 128*b^4*exp(x)*(a^2
 + b^2)^(1/2) - 32*a^4*b*exp(x) - 160*a^2*b^3*exp(x) + 64*a*b^3*(a^2 + b^2)
^(1/2) + 32*a^3*b*(a^2 + b^2)^(1/2) - 96*a^2*b^2*exp(x)*(a^2 + b^2)^(1/2))*
(a^2 + b^2)^(1/2))/(a^2*b + b^3)
```

3.82 $\int \frac{\operatorname{csch}^3(x)}{a+b\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=59

$$\frac{a\tanh^{-1}(\cosh(x))}{b^2}-\frac{2a^2\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}-\frac{\coth(x)}{b}$$

```
[Out] a*arctanh(cosh(x))/b^2-coth(x)/b-2*a^2*arctanh((a-b*tanh(1/2*x))/(a^2+b^2)^
(1/2))/b^2/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3875, 3874, 3855, 3916, 2739, 632, 212}

$$-\frac{2a^2\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}+\frac{a\tanh^{-1}(\cosh(x))}{b^2}-\frac{\coth(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^3/(a + b*Csch[x]),x]
```

```
[Out] (a*ArcTanh[Cosh[x]])/b^2 - (2*a^2*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]
])/(b^2*Sqrt[a^2 + b^2]) - Coth[x]/b
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3874

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3875

```
Int[csc[(e_.) + (f_.)*(x_)]^3/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Simp[-Cot[e + f*x]/(b*f), x] - Dist[a/b, Int[Csc[e + f*x]^2/(a + b*
Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{csch}^3(x)}{a+b\operatorname{csch}(x)}\,dx &= -\frac{\coth(x)}{b}-\frac{a\int \frac{\operatorname{csch}^2(x)}{a+b\operatorname{csch}(x)}\,dx}{b}\\
&= -\frac{\coth(x)}{b}-\frac{a\int \operatorname{csch}(x)\,dx}{b^2}+\frac{a^2\int \frac{\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}\,dx}{b^2}\\
&= \frac{a\tanh^{-1}(\cosh(x))}{b^2}-\frac{\coth(x)}{b}+\frac{a^2\int \frac{1}{1+\frac{a\sinh(x)}{b}}\,dx}{b^3}\\
&= \frac{a\tanh^{-1}(\cosh(x))}{b^2}-\frac{\coth(x)}{b}+\frac{(2a^2)\operatorname{Subst}\left(\int \frac{1}{1+\frac{2ax}{b}-x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b^3}\\
&= \frac{a\tanh^{-1}(\cosh(x))}{b^2}-\frac{\coth(x)}{b}-\frac{(4a^2)\operatorname{Subst}\left(\int \frac{1}{4\left(1+\frac{a^2}{b^2}\right)-x^2}\,dx,x,\frac{2a}{b}-2\tanh\left(\frac{x}{2}\right)\right)}{b^3}\\
&= \frac{a\tanh^{-1}(\cosh(x))}{b^2}-\frac{2a^2\tanh^{-1}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}-\frac{\coth(x)}{b}
\end{aligned}
$$

Mathematica [A]
time = 0.20, size = 71, normalized size = 1.20

$$\frac{\frac{4a^2\text{ArcTan}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-2b\coth(x)-2a\log\left(\tanh\left(\frac{x}{2}\right)\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^3/(a + b*Csch[x]),x]
```

```
[Out] ((4*a^2*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - 2*b*
Coth[x] - 2*a*Log[Tanh[x/2]])/(2*b^2)
```

Maple [A]
time = 0.56, size = 73, normalized size = 1.24

method	result	size
default	$-\frac{\tanh\left(\frac{x}{2}\right)}{2b}-\frac{1}{2b\tanh\left(\frac{x}{2}\right)}-\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{b^2}-\frac{2a^2\,\mathrm{arctanh}\left(\frac{-2b\tanh\left(\frac{x}{2}\right)+2a}{2\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}$	73
risch	$-\frac{2}{b(\mathrm{e}^{2x}-1)}+\frac{a^2\ln\left(\mathrm{e}^x+\frac{b\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,a}\right)}{\sqrt{a^2+b^2}\,b^2}-\frac{a^2\ln\left(\mathrm{e}^x+\frac{b\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,a}\right)}{\sqrt{a^2+b^2}\,b^2}+\frac{a\ln(\mathrm{e}^x+1)}{b^2}-\frac{a\ln(\mathrm{e}^x-1)}{b^2}$	143

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^3/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b*tanh(1/2*x)-1/2/b/tanh(1/2*x)-a/b^2*ln(tanh(1/2*x))-2*a^2/b^2/(a^2+b
^2)^(1/2)*arctanh(1/2*(-2*b*tanh(1/2*x)+2*a)/(a^2+b^2)^(1/2))
```

Maxima [A]
time = 0.47, size = 100, normalized size = 1.69

$$\frac{a^2\log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,b^2}+\frac{a\log\left(e^{(-x)}+1\right)}{b^2}-\frac{a\log\left(e^{(-x)}-1\right)}{b^2}+\frac{2}{be^{(-2\,x)}-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] a^2*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))/
(sqrt(a^2 + b^2)*b^2) + a*log(e^(-x) + 1)/b^2 - a*log(e^(-x) - 1)/b^2 + 2/(
b*e^(-2*x) - b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(55) = 110$.
time = 0.38, size = 345, normalized size = 5.85

$$\frac{2\,a^{2}b+2\,b^{3}-\left(a^{2}\cosh\left(x\right)^{2}+2\,a^{2}\cosh\left(x\right)\sinh\left(x\right)+a^{2}\sinh\left(x\right)^{2}-a^{2}\right)\sqrt{a^{2}+b^{2}}\log\left(\frac{a^{2}\cosh(x)^{2}+a^{2}\sinh(x)^{2}+2\,ab\cosh(x)+a^{2}+2\,b^{2}+2\,\left(a^{2}\cosh(x)+ab\right)\sinh(x)-2\,\sqrt{a^{2}+b^{2}}\left(a\cosh(x)+a\sinh(x)+b\right)}{a\cosh(x)^{2}+a\sinh(x)^{2}+2\,b\cosh(x)+2\,\left(a\cosh(x)+b\right)\sinh(x)-a}\right)+\left(a^{3}+ab^{2}-\left(a^{3}+ab^{2}\right)\cosh\left(x\right)^{2}-2\,\left(a^{3}+ab^{2}\right)\cosh\left(x\right)\sinh\left(x\right)-\left(a^{3}+ab^{2}\right)\sinh\left(x\right)^{2}\right)\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)-\left(a^{3}+ab^{2}-\left(a^{3}+ab^{2}\right)\cosh\left(x\right)^{2}-2\,\left(a^{3}+ab^{2}\right)\cosh\left(x\right)\sinh\left(x\right)-\left(a^{3}+ab^{2}\right)\sinh\left(x\right)^{2}\right)\log\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)}{a^{2}b^{2}+b^{4}-\left(a^{2}b^{2}+b^{4}\right)\cosh\left(x\right)^{2}-2\,\left(a^{2}b^{2}+b^{4}\right)\cosh\left(x\right)\sinh\left(x\right)-\left(a^{2}b^{2}+b^{4}\right)\sinh\left(x\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] (2*a^2*b + 2*b^3 - (a^2*cosh(x)^2 + 2*a^2*cosh(x)*sinh(x) + a^2*sinh(x)^2 -
 a^2)*sqrt(a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) +
a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(a*cosh(x)
+ a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) +
 b)*sinh(x) - a)) + (a^3 + a*b^2 - (a^3 + a*b^2)*cosh(x)^2 - 2*(a^3 + a*b^2
)*cosh(x)*sinh(x) - (a^3 + a*b^2)*sinh(x)^2)*log(cosh(x) + sinh(x) + 1) - (
a^3 + a*b^2 - (a^3 + a*b^2)*cosh(x)^2 - 2*(a^3 + a*b^2)*cosh(x)*sinh(x) - (
a^3 + a*b^2)*sinh(x)^2)*log(cosh(x) + sinh(x) - 1))/(a^2*b^2 + b^4 - (a^2*b
^2 + b^4)*cosh(x)^2 - 2*(a^2*b^2 + b^4)*cosh(x)*sinh(x) - (a^2*b^2 + b^4)*s
inh(x)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^{3}\left(x\right)}{a+b\operatorname{csch}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**3/(a+b*csch(x)),x)
```

```
[Out] Integral(csch(x)**3/(a + b*csch(x)), x)
```

Giac [A]
time = 0.39, size = 98, normalized size = 1.66

$$\frac{a^{2}\log\left(\frac{\left|2\,ae^{x}+2\,b-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,ae^{x}+2\,b+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{\sqrt{a^{2}+b^{2}}\,b^{2}}+\frac{a\log\left(e^{x}+1\right)}{b^{2}}-\frac{a\log\left(\left|e^{x}-1\right|\right)}{b^{2}}-\frac{2}{b(e^{(2\,x)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] a^2*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*b^2) + a*log(e^x + 1)/b^2 - a*log(abs(e^x - 1)
)/b^2 - 2/(b*(e^(2*x) - 1))
```

Mupad [B]
time = 1.73, size = 292, normalized size = 4.95

$$\frac{2}{b-b\,e^{2\,x}}-\frac{a\,\ln\left(32\,e^x-32\right)}{b^2}+\frac{a\,\ln\left(32\,e^x+32\right)}{b^2}+\frac{a^2\,\ln\left(32\,a^4\,e^x-64\,a\,b^3-64\,a^3\,b-32\,a^3\,\sqrt{a^2+b^2}+128\,b^4\,e^x+128\,b^3\,e^x\,\sqrt{a^2+b^2}+160\,a^2\,b^2\,e^x-64\,a\,b^2\,\sqrt{a^2+b^2}+96\,a^2\,b\,e^x\,\sqrt{a^2+b^2}\right)\,\sqrt{a^2+b^2}}{a^2\,b^2+b^4}-\frac{a^2\,\ln\left(32\,a^3\,\sqrt{a^2+b^2}-64\,a\,b^3-64\,a^3\,b+32\,a^4\,e^x+128\,b^4\,e^x-128\,b^3\,e^x\,\sqrt{a^2+b^2}+160\,a^2\,b^2\,e^x+64\,a\,b^2\,\sqrt{a^2+b^2}-96\,a^2\,b\,e^x\,\sqrt{a^2+b^2}\right)\,\sqrt{a^2+b^2}}{a^2\,b^2+b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^3*(a + b/sinh(x))),x)
```

```
[Out] 2/(b - b*exp(2*x)) - (a*log(32*exp(x) - 32))/b^2 + (a*log(32*exp(x) + 32))/
b^2 + (a^2*log(32*a^4*exp(x) - 64*a*b^3 - 64*a^3*b - 32*a^3*(a^2 + b^2)^(1/
2) + 128*b^4*exp(x) + 128*b^3*exp(x)*(a^2 + b^2)^(1/2) + 160*a^2*b^2*exp(x)
 - 64*a*b^2*(a^2 + b^2)^(1/2) + 96*a^2*b*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b
^2)^(1/2))/(b^4 + a^2*b^2) - (a^2*log(32*a^3*(a^2 + b^2)^(1/2) - 64*a*b^3 -
 64*a^3*b + 32*a^4*exp(x) + 128*b^4*exp(x) - 128*b^3*exp(x)*(a^2 + b^2)^(1/
2) + 160*a^2*b^2*exp(x) + 64*a*b^2*(a^2 + b^2)^(1/2) - 96*a^2*b*exp(x)*(a^2
 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/(b^4 + a^2*b^2)
```

3.83 $\int \frac{\operatorname{csch}^4(x)}{a+b\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=83

$$-\frac{(2a^2-b^2)\tanh^{-1}(\cosh(x))}{2b^3}+\frac{2a^3\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}}+\frac{a\coth(x)}{b^2}-\frac{\coth(x)\operatorname{csch}(x)}{2b}$$

```
[Out] -1/2*(2*a^2-b^2)*arctanh(cosh(x))/b^3+a*coth(x)/b^2-1/2*coth(x)*csch(x)/b+2
*a^3*arctanh((a-b*tanh(1/2*x))/(a^2+b^2)^(1/2))/b^3/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {3936, 4167, 4083, 3855, 3916, 2739, 632, 212}

$$-\frac{(2a^2-b^2)\tanh^{-1}(\cosh(x))}{2b^3}+\frac{2a^3\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}}+\frac{a\coth(x)}{b^2}-\frac{\coth(x)\operatorname{csch}(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^4/(a + b*Csch[x]),x]
```

```
[Out] -1/2*((2*a^2 - b^2)*ArcTanh[Cosh[x]])/b^3 + (2*a^3*ArcTanh[(a - b*Tanh[x/2]
)/Sqrt[a^2 + b^2]])/(b^3*Sqrt[a^2 + b^2]) + (a*Coth[x])/b^2 - (Coth[x]*Csch
[x])/(2*b)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3936

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-d^3)*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 3)/(b*f*
(n - 2))), x] + Dist[d^3/(b*(n - 2)), Int[(d*Csc[e + f*x])^(n - 3)*(Simp[a*
(n - 3) + b*(n - 3)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x]/(a + b*Csc[
e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ
[n, 3]
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4167

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b
*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\operatorname{csch}^4(x)}{a+b\operatorname{csch}(x)}\,dx = -\frac{\coth(x)\operatorname{csch}(x)}{2b} - \frac{\int \frac{\operatorname{csch}(x)\left(a+b\operatorname{csch}(x)+2a\operatorname{csch}^2(x)\right)}{a+b\operatorname{csch}(x)}\,dx}{2b}$$

$$= \frac{a\coth(x)}{b^2} - \frac{\coth(x)\operatorname{csch}(x)}{2b} + \frac{i\int \frac{\operatorname{csch}(x)\left(iab-i(2a^2-b^2)\operatorname{csch}(x)\right)}{a+b\operatorname{csch}(x)}\,dx}{2b^2}$$

$$= \frac{a\coth(x)}{b^2} - \frac{\coth(x)\operatorname{csch}(x)}{2b} - \frac{a^3\int \frac{\operatorname{csch}(x)}{a+b\operatorname{csch}(x)}\,dx}{b^3} + \frac{(2a^2-b^2)\int \operatorname{csch}(x)\,dx}{2b^3}$$

$$= -\frac{(2a^2-b^2)\tanh^{-1}(\cosh(x))}{2b^3} + \frac{a\coth(x)}{b^2} - \frac{\coth(x)\operatorname{csch}(x)}{2b} - \frac{a^3\int \frac{1}{1+\frac{a\sinh(x)}{b}}\,dx}{b^4}$$

$$= -\frac{(2a^2-b^2)\tanh^{-1}(\cosh(x))}{2b^3} + \frac{a\coth(x)}{b^2} - \frac{\coth(x)\operatorname{csch}(x)}{2b} - \frac{(2a^3)\operatorname{Subst}\left(\int \frac{1}{1+\frac{2ax}{b}-x^2}\right.}{b^4}$$

$$= -\frac{(2a^2-b^2)\tanh^{-1}(\cosh(x))}{2b^3} + \frac{a\coth(x)}{b^2} - \frac{\coth(x)\operatorname{csch}(x)}{2b} + \frac{(4a^3)\operatorname{Subst}\left(\int \frac{1}{4\left(1+\frac{a^2}{b^2}\right)-}\right.}{}$$

$$= -\frac{(2a^2-b^2)\tanh^{-1}(\cosh(x))}{2b^3} + \frac{2a^3\tanh^{-1}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}} + \frac{a\coth(x)}{b^2} - \frac{\coth(x)\operatorname{csch}}{2b}$$

Mathematica [A]
time = 0.33, size = 124, normalized size = 1.49

$$-\frac{\frac{16a^3\operatorname{ArcTan}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} - 4ab\coth\left(\frac{x}{2}\right) + b^2\operatorname{csch}^2\left(\frac{x}{2}\right) - 8a^2\log\left(\tanh\left(\frac{x}{2}\right)\right) + 4b^2\log\left(\tanh\left(\frac{x}{2}\right)\right) + b^2\operatorname{sech}^2\left(\frac{x}{2}\right) - 4ab\tanh\left(\frac{x}{2}\right)}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^4/(a + b*Csch[x]),x]
```

```
[Out] -1/8*((16*a^3*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2]
- 4*a*b*Coth[x/2] + b^2*Csch[x/2]^2 - 8*a^2*Log[Tanh[x/2]] + 4*b^2*Log[Tanh
[x/2]] + b^2*Sech[x/2]^2 - 4*a*b*Tanh[x/2])/b^3
```

Maple [A]
time = 0.68, size = 108, normalized size = 1.30

method	result
default	$\frac{\frac{b\left(\tanh^2\left(\frac{x}{2}\right)\right)}{2}+2a\tanh\left(\frac{x}{2}\right)}{4b^2} - \frac{1}{8b\tanh\left(\frac{x}{2}\right)^2} + \frac{(4a^2-2b^2)\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{4b^3} + \frac{a}{2b^2\tanh\left(\frac{x}{2}\right)} + \frac{2a^3\operatorname{arctanh}\left(\frac{-2b\tanh\left(\frac{x}{2}\right)+2a}{2\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}}$

risch	$\frac{-b\,e^{3x}+2\,e^{2x}a-e^{x}b-2a}{\left(e^{2x}-1\right)^{2}b^{2}}+\frac{a^{3}\ln\left(e^{x}+\frac{b\sqrt{a^{2}+b^{2}}+a^{2}+b^{2}}{\sqrt{a^{2}+b^{2}}\,a}\right)}{\sqrt{a^{2}+b^{2}}\,b^{3}}-\frac{a^{3}\ln\left(e^{x}+\frac{b\sqrt{a^{2}+b^{2}}-a^{2}-b^{2}}{\sqrt{a^{2}+b^{2}}\,a}\right)}{\sqrt{a^{2}+b^{2}}\,b^{3}}+\frac{\ln(e^{x}-1)a^{2}}{b^{3}}-\frac{\ln(e^{x}-}{2b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^4/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b^2*(1/2*b*tanh(1/2*x)^2+2*a*tanh(1/2*x))-1/8/b/tanh(1/2*x)^2+1/4/b^3*(
4*a^2-2*b^2)*ln(tanh(1/2*x))+1/2*a/b^2/tanh(1/2*x)+2/b^3*a^3/(a^2+b^2)^(1/2
)*arctanh(1/2*(-2*b*tanh(1/2*x)+2*a)/(a^2+b^2)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(75) = 150$.
time = 0.47, size = 158, normalized size = 1.90

$$-\frac{a^{3}\log\left(\frac{ae^{(-x)}-b-\sqrt{a^{2}+b^{2}}}{ae^{(-x)}-b+\sqrt{a^{2}+b^{2}}}\right)}{\sqrt{a^{2}+b^{2}}\,b^{3}}+\frac{be^{(-x)}+2\,ae^{(-2\,x)}+be^{(-3\,x)}-2\,a}{2\,b^{2}e^{(-2\,x)}-b^{2}e^{(-4\,x)}-b^{2}}-\frac{(2\,a^{2}-b^{2})\log\left(e^{(-x)}+1\right)}{2\,b^{3}}+\frac{(2\,a^{2}-b^{2})\log\left(e^{(-x)}-1\right)}{2\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -a^3*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))
/(sqrt(a^2 + b^2)*b^3) + (b*e^(-x) + 2*a*e^(-2*x) + b*e^(-3*x) - 2*a)/(2*b^
2*e^(-2*x) - b^2*e^(-4*x) - b^2) - 1/2*(2*a^2 - b^2)*log(e^(-x) + 1)/b^3 +
1/2*(2*a^2 - b^2)*log(e^(-x) - 1)/b^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 947 vs. $2(75) = 150$.
time = 0.42, size = 947, normalized size = 11.41

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(4*a^3*b + 4*a*b^3 + 2*(a^2*b^2 + b^4)*cosh(x)^3 + 2*(a^2*b^2 + b^4)*s
inh(x)^3 - 4*(a^3*b + a*b^3)*cosh(x)^2 - 2*(2*a^3*b + 2*a*b^3 - 3*(a^2*b^2
+ b^4)*cosh(x))*sinh(x)^2 - 2*(a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^
3*sinh(x)^4 - 2*a^3*cosh(x)^2 + a^3 + 2*(3*a^3*cosh(x)^2 - a^3)*sinh(x)^2 +
 4*(a^3*cosh(x)^3 - a^3*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((a^2*cosh(x)^
2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) + a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sin
h(x) + 2*sqrt(a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh
(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) - a)) + 2*(a^2*b^2 + b^4)*c
osh(x) + ((2*a^4 + a^2*b^2 - b^4)*cosh(x)^4 + 4*(2*a^4 + a^2*b^2 - b^4)*cos
h(x)*sinh(x)^3 + (2*a^4 + a^2*b^2 - b^4)*sinh(x)^4 + 2*a^4 + a^2*b^2 - b^4
```

```
- 2*(2*a^4 + a^2*b^2 - b^4)*cosh(x)^2 - 2*(2*a^4 + a^2*b^2 - b^4 - 3*(2*a^4
 + a^2*b^2 - b^4)*cosh(x)^2)*sinh(x)^2 + 4*((2*a^4 + a^2*b^2 - b^4)*cosh(x)
^3 - (2*a^4 + a^2*b^2 - b^4)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) -
 ((2*a^4 + a^2*b^2 - b^4)*cosh(x)^4 + 4*(2*a^4 + a^2*b^2 - b^4)*cosh(x)*sin
h(x)^3 + (2*a^4 + a^2*b^2 - b^4)*sinh(x)^4 + 2*a^4 + a^2*b^2 - b^4 - 2*(2*a
^4 + a^2*b^2 - b^4)*cosh(x)^2 - 2*(2*a^4 + a^2*b^2 - b^4 - 3*(2*a^4 + a^2*b
^2 - b^4)*cosh(x)^2)*sinh(x)^2 + 4*((2*a^4 + a^2*b^2 - b^4)*cosh(x)^3 - (2*
a^4 + a^2*b^2 - b^4)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) + 2*(a^2*
b^2 + b^4 + 3*(a^2*b^2 + b^4)*cosh(x)^2 - 4*(a^3*b + a*b^3)*cosh(x))*sinh(x
))/(a^2*b^3 + b^5 + (a^2*b^3 + b^5)*cosh(x)^4 + 4*(a^2*b^3 + b^5)*cosh(x)*s
inh(x)^3 + (a^2*b^3 + b^5)*sinh(x)^4 - 2*(a^2*b^3 + b^5)*cosh(x)^2 - 2*(a^2
*b^3 + b^5 - 3*(a^2*b^3 + b^5)*cosh(x)^2)*sinh(x)^2 + 4*((a^2*b^3 + b^5)*co
sh(x)^3 - (a^2*b^3 + b^5)*cosh(x))*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^4(x)}{a + b\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**4/(a+b*csch(x)),x)
```

```
[Out] Integral(csch(x)**4/(a + b*csch(x)), x)
```

Giac [A]
time = 0.39, size = 141, normalized size = 1.70

$$-\frac{a^3\log\left(\frac{\left|2\,ae^x+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^x+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b^3}-\frac{(2\,a^2-b^2)\log\left(e^x+1\right)}{2\,b^3}+\frac{(2\,a^2-b^2)\log\left(|e^x-1|\right)}{2\,b^3}-\frac{be^{(3\,x)}-2\,ae^{(2\,x)}+be^x+2\,a}{b^2\left(e^{(2\,x)}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -a^3*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(
a^2 + b^2)))/(sqrt(a^2 + b^2)*b^3) - 1/2*(2*a^2 - b^2)*log(e^x + 1)/b^3 + 1
/2*(2*a^2 - b^2)*log(abs(e^x - 1))/b^3 - (b*e^(3*x) - 2*a*e^(2*x) + b*e^x +
 2*a)/(b^2*(e^(2*x) - 1)^2)
```

Mupad [B]
time = 2.09, size = 617, normalized size = 7.43

$$\frac{e^x}{b-b\,e^{2\,x}}-\frac{2\,e^x}{b-2\,b\,e^{2\,x}+b\,e^{4\,x}}-\frac{\ln\left(24\,a^4+4\,b^4-20\,a^2\,b^2-24\,a^4\,e^x-4\,b^4\,e^x+20\,a^2\,b^2\,e^x\right)}{2\,b}+\frac{\ln\left(24\,a^4+4\,b^4-20\,a^2\,b^2+24\,a^4\,e^x+4\,b^4\,e^x-20\,a^2\,b^2\,e^x\right)}{2\,b}+\frac{2\,a}{b^2\,e^{2\,x}-b^2}+\frac{a^2\,\ln\left(24\,a^4+4\,b^4-20\,a^2\,b^2-24\,a^4\,e^x-4\,b^4\,e^x+20\,a^2\,b^2\,e^x\right)}{b^3}-\frac{a^2\,\ln\left(24\,a^4+4\,b^4-20\,a^2\,b^2+24\,a^4\,e^x+4\,b^4\,e^x-20\,a^2\,b^2\,e^x\right)}{b^3}-\frac{a^3\,\ln\left(16\,a\,b^5-24\,a^5\,\sqrt{a^2+b^2}-48\,a^5\,b-32\,a^3\,b^3+24\,a^6\,e^x-32\,b^6\,e^x-40\,a^3\,b^2\,\sqrt{a^2+b^2}-32\,b^5\,e^x\,\sqrt{a^2+b^2}+56\,a^2\,b^4\,e^x+112\,a^4\,b^2\,e^x+16\,a\,b^4\,\sqrt{a^2+b^2}+72\,a^4\,b\,e^x\,\sqrt{a^2+b^2}+72\,a^2\,b^3\,e^x\,\sqrt{a^2+b^2}\right)\,\sqrt{a^2+b^2}}{a^2\,b^3+b^5}+\frac{a^3\,\ln\left(24\,a^5\,\sqrt{a^2+b^2}+16\,a\,b^5-48\,a^5\,b-32\,a^3\,b^3+24\,a^6\,e^x-32\,b^6\,e^x+40\,a^3\,b^2\,\sqrt{a^2+b^2}+32\,b^5\,e^x\,\sqrt{a^2+b^2}+56\,a^2\,b^4\,e^x+112\,a^4\,b^2\,e^x-16\,a\,b^4\,\sqrt{a^2+b^2}-72\,a^4\,b\,e^x\,\sqrt{a^2+b^2}-72\,a^2\,b^3\,e^x\,\sqrt{a^2+b^2}\right)\,\sqrt{a^2+b^2}}{a^2\,b^3+b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^4*(a + b/sinh(x))),x)
```

```
[Out] exp(x)/(b - b*exp(2*x)) - (2*exp(x))/(b - 2*b*exp(2*x) + b*exp(4*x)) - log(
24*a^4 + 4*b^4 - 20*a^2*b^2 - 24*a^4*exp(x) - 4*b^4*exp(x) + 20*a^2*b^2*exp
(x))/(2*b) + log(24*a^4 + 4*b^4 - 20*a^2*b^2 + 24*a^4*exp(x) + 4*b^4*exp(x)
 - 20*a^2*b^2*exp(x))/(2*b) + (2*a)/(b^2*exp(2*x) - b^2) + (a^2*log(24*a^4
+ 4*b^4 - 20*a^2*b^2 - 24*a^4*exp(x) - 4*b^4*exp(x) + 20*a^2*b^2*exp(x)))/b
^3 - (a^2*log(24*a^4 + 4*b^4 - 20*a^2*b^2 + 24*a^4*exp(x) + 4*b^4*exp(x) -
20*a^2*b^2*exp(x)))/b^3 - (a^3*log(16*a*b^5 - 24*a^5*(a^2 + b^2)^(1/2) - 48
*a^5*b - 32*a^3*b^3 + 24*a^6*exp(x) - 32*b^6*exp(x) - 40*a^3*b^2*(a^2 + b^2
)^(1/2) - 32*b^5*exp(x)*(a^2 + b^2)^(1/2) + 56*a^2*b^4*exp(x) + 112*a^4*b^2
*exp(x) + 16*a*b^4*(a^2 + b^2)^(1/2) + 72*a^4*b*exp(x)*(a^2 + b^2)^(1/2) +
72*a^2*b^3*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/(b^5 + a^2*b^3) + (
a^3*log(24*a^5*(a^2 + b^2)^(1/2) + 16*a*b^5 - 48*a^5*b - 32*a^3*b^3 + 24*a^
6*exp(x) - 32*b^6*exp(x) + 40*a^3*b^2*(a^2 + b^2)^(1/2) + 32*b^5*exp(x)*(a^
2 + b^2)^(1/2) + 56*a^2*b^4*exp(x) + 112*a^4*b^2*exp(x) - 16*a*b^4*(a^2 + b
^2)^(1/2) - 72*a^4*b*exp(x)*(a^2 + b^2)^(1/2) - 72*a^2*b^3*exp(x)*(a^2 + b^
2)^(1/2))*(a^2 + b^2)^(1/2))/(b^5 + a^2*b^3)
```

3.84 $\int \frac{\cosh^4(x)}{i+\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=38

$$\frac{ix}{8}+\frac{\cosh^3(x)}{3}+\frac{1}{8}i\cosh(x)\sinh(x)-\frac{1}{4}i\cosh^3(x)\sinh(x)$$

```
[Out] 1/8*I*x+1/3*cosh(x)^3+1/8*I*cosh(x)*sinh(x)-1/4*I*cosh(x)^3*sinh(x)
```

Rubi [A]
time = 0.09, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3957, 2918, 2645, 30, 2648, 2715, 8}

$$\frac{ix}{8}+\frac{\cosh^3(x)}{3}-\frac{1}{4}i\sinh(x)\cosh^3(x)+\frac{1}{8}i\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^4/(I + Csch[x]),x]
```

```
[Out] (I/8)*x + Cosh[x]^3/3 + (I/8)*Cosh[x]*Sinh[x] - (I/4)*Cosh[x]^3*Sinh[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^4(x)}{i+\operatorname{csch}(x)}\,dx &= i\int \frac{\cosh^4(x)\sinh(x)}{i-\sinh(x)}\,dx\\
&= -\left(i\int \cosh^2(x)\sinh^2(x)\,dx\right)+\int \cosh^2(x)\sinh(x)\,dx\\
&= -\frac{1}{4}i\cosh^3(x)\sinh(x)+\frac{1}{4}i\int \cosh^2(x)\,dx+\operatorname{Subst}\left(\int x^2\,dx,x,\cosh(x)\right)\\
&= \frac{\cosh^3(x)}{3}+\frac{1}{8}i\cosh(x)\sinh(x)-\frac{1}{4}i\cosh^3(x)\sinh(x)+\frac{1}{8}i\int 1\,dx\\
&= \frac{ix}{8}+\frac{\cosh^3(x)}{3}+\frac{1}{8}i\cosh(x)\sinh(x)-\frac{1}{4}i\cosh^3(x)\sinh(x)
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 32, normalized size = 0.84

$$\frac{ix}{8}+\frac{\cosh(x)}{4}+\frac{1}{12}\cosh(3x)-\frac{1}{32}i\sinh(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^4/(I + Csch[x]),x]
```

```
[Out] (I/8)*x + Cosh[x]/4 + Cosh[3*x]/12 - (I/32)*Sinh[4*x]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(27) = 54.
time = 0.82, size = 116, normalized size = 3.05

method	result
risch	$\frac{ix}{8}-\frac{i\mathrm{e}^{4x}}{64}+\frac{\mathrm{e}^{3x}}{24}+\frac{\mathrm{e}^{x}}{8}+\frac{\mathrm{e}^{-x}}{8}+\frac{\mathrm{e}^{-3x}}{24}+\frac{i\mathrm{e}^{-4x}}{64}$
default	$-\frac{i\ln(\tanh(\frac{x}{2})-1)}{8}-\frac{i}{4(\tanh(\frac{x}{2})-1)^4}+\frac{-\frac{1}{3}-\frac{i}{2}}{(\tanh(\frac{x}{2})-1)^3}+\frac{-\frac{1}{2}-\frac{3i}{8}}{(\tanh(\frac{x}{2})-1)^2}+\frac{-\frac{1}{2}-\frac{i}{8}}{\tanh(\frac{x}{2})-1}+\frac{i}{4(\tanh(\frac{x}{2})+1)^4}+\frac{i\ln(\tanh(\frac{x}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(I+csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*I*ln(tanh(1/2*x)-1)-1/4*I/(tanh(1/2*x)-1)^4-(1/3+1/2*I)/(tanh(1/2*x)-1
)^3-(1/2+3/8*I)/(tanh(1/2*x)-1)^2-(1/2+1/8*I)/(tanh(1/2*x)-1)+1/4*I/(tanh(1
/2*x)+1)^4+1/8*I*ln(tanh(1/2*x)+1)+(1/3-1/2*I)/(tanh(1/2*x)+1)^3+(1/2-1/8*I
)/(tanh(1/2*x)+1)+(-1/2+3/8*I)/(tanh(1/2*x)+1)^2
```

Maxima [A]
time = 0.26, size = 42, normalized size = 1.11

$$\frac{1}{192}\left(8e^{(-x)}+24e^{(-3x)}-3i\right)e^{(4x)}+\frac{1}{8}ix+\frac{1}{8}e^{(-x)}+\frac{1}{24}e^{(-3x)}+\frac{1}{64}ie^{(-4x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] 1/192*(8*e^(-x) + 24*e^(-3*x) - 3*I)*e^(4*x) + 1/8*I*x + 1/8*e^(-x) + 1/24*
e^(-3*x) + 1/64*I*e^(-4*x)
```

Fricas [A]
time = 0.37, size = 43, normalized size = 1.13

$$\frac{1}{192}\left(24i\,xe^{(4x)}-3i\,e^{(8x)}+8e^{(7x)}+24e^{(5x)}+24e^{(3x)}+8e^{x}+3i\right)e^{(-4x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] 1/192*(24*I*x*e^(4*x) - 3*I*e^(8*x) + 8*e^(7*x) + 24*e^(5*x) + 24*e^(3*x) +
 8*e^x + 3*I)*e^(-4*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cosh^4(x)}{\operatorname{csch}(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**4/(I+csch(x)),x)
```

```
[Out] Integral(cosh(x)**4/(csch(x) + I), x)
```

Giac [A]
time = 0.40, size = 38, normalized size = 1.00

$$\frac{1}{192}\left(24\,e^{(3\,x)}+8\,e^{x}+3i\right)e^{(-4\,x)}+\frac{1}{8}i\,x-\frac{1}{64}i\,e^{(4\,x)}+\frac{1}{24}\,e^{(3\,x)}+\frac{1}{8}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(I+csch(x)),x, algorithm="giac")
```

```
[Out] 1/192*(24*e^(3*x) + 8*e^x + 3*I)*e^(-4*x) + 1/8*I*x - 1/64*I*e^(4*x) + 1/24
*e^(3*x) + 1/8*e^x
```

Mupad [B]
time = 1.55, size = 41, normalized size = 1.08

$$\frac{\mathrm{e}^{-x}}{8}+\frac{\mathrm{e}^{-3\,x}}{24}+\frac{\mathrm{e}^{3\,x}}{24}+\frac{\mathrm{e}^{x}}{8}+\frac{x\,1\mathrm{i}}{8}+\frac{\mathrm{e}^{-4\,x}\,1\mathrm{i}}{64}-\frac{\mathrm{e}^{4\,x}\,1\mathrm{i}}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(1/sinh(x) + 1i),x)
```

```
[Out] (x*1i)/8 + exp(-x)/8 + exp(-3*x)/24 + exp(3*x)/24 + (exp(-4*x)*1i)/64 - (ex
p(4*x)*1i)/64 + exp(x)/8
```

3.85 $\int \frac{\cosh^3(x)}{i+\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=19

$$\frac{\sinh^2(x)}{2}-\frac{1}{3}i\sinh^3(x)$$

```
[Out] 1/2*sinh(x)^2-1/3*I*sinh(x)^3
```

Rubi [A]
time = 0.07, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3957, 2914, 2644, 30}

$$\frac{\sinh^2(x)}{2}-\frac{1}{3}i\sinh^3(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(I + Csch[x]),x]
```

```
[Out] Sinh[x]^2/2 - (I/3)*Sinh[x]^3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2914

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
```

```
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^3(x)}{i+\operatorname{csch}(x)}\,dx &= i\int \frac{\cosh^3(x)\sinh(x)}{i-\sinh(x)}\,dx\\
&= -\left(i\int \cosh(x)\sinh^2(x)\,dx\right)+\int \cosh(x)\sinh(x)\,dx\\
&= -\text{Subst}\left(\int x\,dx,x,i\sinh(x)\right)+\text{Subst}\left(\int x^2\,dx,x,i\sinh(x)\right)\\
&= \frac{\sinh^2(x)}{2}-\frac{1}{3}i\sinh^3(x)
\end{aligned}
$$

Mathematica [**A**]
time = 0.01, size = 19, normalized size = 1.00

$$\frac{\sinh^2(x)}{2}-\frac{1}{3}i\sinh^3(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(I + Csch[x]),x]
[Out] Sinh[x]^2/2 - (I/3)*Sinh[x]^3
```

Maple [**A**]
time = 0.50, size = 15, normalized size = 0.79

method	result	size
derivativedivides	$-\frac{i}{3\operatorname{csch}(x)^3}+\frac{1}{2\operatorname{csch}(x)^2}$	15
default	$-\frac{i}{3\operatorname{csch}(x)^3}+\frac{1}{2\operatorname{csch}(x)^2}$	15
risch	$-\frac{i\mathrm{e}^{3x}}{24}+\frac{\mathrm{e}^{2x}}{8}+\frac{i\mathrm{e}^{x}}{8}-\frac{i\mathrm{e}^{-x}}{8}+\frac{\mathrm{e}^{-2x}}{8}+\frac{i\mathrm{e}^{-3x}}{24}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(I+csch(x)),x,method=_RETURNVERBOSE)
[Out] -1/3*I/csch(x)^3+1/2/csch(x)^2
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(13) = 26.
time = 0.26, size = 39, normalized size = 2.05

$$\frac{1}{24}\left(3\,e^{(-x)}+3i\,e^{(-2\,x)}-i\right)e^{(3\,x)}-\frac{1}{8}i\,e^{(-x)}+\frac{1}{8}\,e^{(-2\,x)}+\frac{1}{24}i\,e^{(-3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] 1/24*(3*e^(-x) + 3*I*e^(-2*x) - I)*e^(3*x) - 1/8*I*e^(-x) + 1/8*e^(-2*x) +
1/24*I*e^(-3*x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(13) = 26$.
time = 0.42, size = 36, normalized size = 1.89

$$\frac{1}{24} \left(-i\, e^{(6\, x)} + 3\, e^{(5\, x)} + 3 i\, e^{(4\, x)} - 3 i\, e^{(2\, x)} + 3\, e^{x} + i\right) e^{(-3\, x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] 1/24*(-I*e^(6*x) + 3*e^(5*x) + 3*I*e^(4*x) - 3*I*e^(2*x) + 3*e^x + I)*e^(-3
*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3(x)}{\operatorname{csch}(x) + i}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(I+csch(x)),x)
```

```
[Out] Integral(cosh(x)**3/(csch(x) + I), x)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(13) = 26$.
time = 0.38, size = 35, normalized size = 1.84

$$-\frac{1}{24} \left(3 i\, e^{(2\, x)} - 3\, e^{x} - i\right) e^{(-3\, x)} - \frac{1}{24} i\, e^{(3\, x)} + \frac{1}{8}\, e^{(2\, x)} + \frac{1}{8} i\, e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(I+csch(x)),x, algorithm="giac")
```

```
[Out] -1/24*(3*I*e^(2*x) - 3*e^x - I)*e^(-3*x) - 1/24*I*e^(3*x) + 1/8*e^(2*x) + 1
/8*I*e^x
```

Mupad [B]
time = 0.14, size = 39, normalized size = 2.05

$$\frac{\mathrm{e}^{-2\,x}}{8} - \frac{\mathrm{e}^{-x}\,1\mathrm{i}}{8} + \frac{\mathrm{e}^{2\,x}}{8} + \frac{\mathrm{e}^{-3\,x}\,1\mathrm{i}}{24} - \frac{\mathrm{e}^{3\,x}\,1\mathrm{i}}{24} + \frac{\mathrm{e}^{x}\,1\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(1/sinh(x) + 1i),x)
```

```
[Out] exp(-2*x)/8 - (exp(-x)*1i)/8 + exp(2*x)/8 + (exp(-3*x)*1i)/24 - (exp(3*x)*1
i)/24 + (exp(x)*1i)/8
```

3.86 $\int \frac{\cosh^2(x)}{i+\mathbf{csch}(x)} dx$

Optimal. Leaf size=20

$$\frac{ix}{2} + \cosh(x) - \frac{1}{2} i \cosh(x) \sinh(x)$$

```
[Out] 1/2*I*x+cosh(x)-1/2*I*cosh(x)*sinh(x)
```

Rubi [A]
time = 0.07, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3957, 2918, 2718, 2715, 8}

$$\frac{ix}{2} + \cosh(x) - \frac{1}{2} i \sinh(x) \cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(I + Csch[x]),x]
```

```
[Out] (I/2)*x + Cosh[x] - (I/2)*Cosh[x]*Sinh[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^2(x)}{i+\operatorname{csch}(x)}\,dx &= i\int \frac{\cosh^2(x)\sinh(x)}{i-\sinh(x)}\,dx \\
&= -\left(i\int \sinh^2(x)\,dx\right)+\int \sinh(x)\,dx \\
&= \cosh(x)-\frac{1}{2}i\cosh(x)\sinh(x)+\frac{1}{2}i\int 1\,dx \\
&= \frac{ix}{2}+\cosh(x)-\frac{1}{2}i\cosh(x)\sinh(x)
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 20, normalized size = 1.00

$$\frac{ix}{2}+\cosh(x)-\frac{1}{4}i\sinh(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(I + Csch[x]),x]
[Out] (I/2)*x + Cosh[x] - (I/4)*Sinh[2*x]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(14) = 28.
time = 0.77, size = 68, normalized size = 3.40

method	result	size
risch	$\frac{ix}{2}-\frac{i\,{e}^{2x}}{8}+\frac{{e}^{x}}{2}+\frac{{e}^{-x}}{2}+\frac{i\,{e}^{-2x}}{8}$	30
default	$-\frac{i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2}-\frac{i}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{-1-\frac{i}{2}}{\tanh\left(\frac{x}{2}\right)-1}+\frac{i}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2}+\frac{1-\frac{i}{2}}{\tanh\left(\frac{x}{2}\right)+1}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(I+csch(x)),x,method=_RETURNVERBOSE)
[Out] -1/2*I*ln(tanh(1/2*x)-1)-1/2*I/(tanh(1/2*x)-1)^2-(1+1/2*I)/(tanh(1/2*x)-1)+
1/2*I/(tanh(1/2*x)+1)^2+1/2*I*ln(tanh(1/2*x)+1)+(1-1/2*I)/(tanh(1/2*x)+1)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(12) = 24$.
time = 0.26, size = 30, normalized size = 1.50

$$\frac{1}{8}\left(4\,e^{(-x)} - i\right)e^{(2\,x)} + \frac{1}{2}i\,x + \frac{1}{2}\,e^{(-x)} + \frac{1}{8}i\,e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] 1/8*(4*e^(-x) - I)*e^(2*x) + 1/2*I*x + 1/2*e^(-x) + 1/8*I*e^(-2*x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(12) = 24$.
time = 0.37, size = 31, normalized size = 1.55

$$\frac{1}{8}\left(4i\,xe^{(2\,x)} - i\,e^{(4\,x)} + 4\,e^{(3\,x)} + 4\,e^{x} + i\right)e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] 1/8*(4*I*x*e^(2*x) - I*e^(4*x) + 4*e^(3*x) + 4*e^x + I)*e^(-2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(x)}{\operatorname{csch}(x) + i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(I+csch(x)),x)
```

```
[Out] Integral(cosh(x)**2/(csch(x) + I), x)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.
time = 0.38, size = 26, normalized size = 1.30

$$\frac{1}{8}\left(4\,e^{x} + i\right)e^{(-2\,x)} + \frac{1}{2}i\,x - \frac{1}{8}i\,e^{(2\,x)} + \frac{1}{2}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(I+csch(x)),x, algorithm="giac")
```

```
[Out] 1/8*(4*e^x + I)*e^(-2*x) + 1/2*I*x - 1/8*I*e^(2*x) + 1/2*e^x
```

Mupad [B]
time = 1.46, size = 29, normalized size = 1.45

$$\frac{e^{-x}}{2}+\frac{e^{x}}{2}+\frac{x\,1i}{2}+\frac{e^{-2\,x}\,1i}{8}-\frac{e^{2\,x}\,1i}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(1/sinh(x) + 1i),x)
[Out] (x*1i)/2 + exp(-x)/2 + (exp(-2*x)*1i)/8 - (exp(2*x)*1i)/8 + exp(x)/2
```

3.87 $\int \frac{\cosh(x)}{i+\mathbf{csch}(x)} \, dx$

Optimal. Leaf size=16

$$\log(i - \sinh(x)) - i \sinh(x)$$

[Out] ln(I-sinh(x))-I*sinh(x)

Rubi [A]

time = 0.04, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3957, 2912, 45}

$$\log(-\sinh(x) + i) - i \sinh(x)$$

Antiderivative was successfully verified.

[In] Int[Cosh[x]/(I + Csch[x]),x]

[Out] Log[I - Sinh[x]] - I*Sinh[x]

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\cosh(x)}{i+\operatorname{csch}(x)}\,dx = i\int \frac{\cosh(x)\sinh(x)}{i-\sinh(x)}\,dx$$
$$= i\text{Subst}\left(\int \frac{x}{i+x}\,dx, x, -\sinh(x)\right)$$
$$= i\text{Subst}\left(\int \left(1-\frac{i}{i+x}\right)\,dx, x, -\sinh(x)\right)$$
$$= \log(i-\sinh(x)) - i\sinh(x)$$

Mathematica [A]
time = 0.01, size = 21, normalized size = 1.31

$$-2i\text{ArcTan}\left(\coth\left(\frac{x}{2}\right)\right) + \log(\cosh(x)) - i\sinh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(I + Csch[x]),x]
[Out] (-2*I)*ArcTan[Coth[x/2]] + Log[Cosh[x]] - I*Sinh[x]
```

Maple [A]
time = 0.99, size = 20, normalized size = 1.25

method	result	size
derivativedivides	$\ln(i+\operatorname{csch}(x)) - \ln(\operatorname{csch}(x)) - \frac{i}{\operatorname{csch}(x)}$	20
default	$\ln(i+\operatorname{csch}(x)) - \ln(\operatorname{csch}(x)) - \frac{i}{\operatorname{csch}(x)}$	20
risch	$-x - \frac{i e^{x}}{2} + \frac{i e^{-x}}{2} + 2\ln(e^{x} - i)$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(I+csch(x)),x,method=_RETURNVERBOSE)
[Out] ln(I+csch(x))-ln(csch(x))-I/csch(x)
```

Maxima [A]
time = 0.27, size = 21, normalized size = 1.31

$$x + \frac{1}{2}i\,e^{(-x)} - \frac{1}{2}i\,e^{x} + 2\,\log\left(e^{(-x)} + i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] x + 1/2*I*e^(-x) - 1/2*I*e^x + 2*log(e^(-x) + I)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(12) = 24$.
time = 0.37, size = 28, normalized size = 1.75

$$-\frac{1}{2}\left(2\,xe^{x}-4\,e^{x}\log\left(e^{x}-i\right)+i\,e^{(2\,x)}-i\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*x*e^x - 4*e^x*log(e^x - I) + I*e^(2*x) - I)*e^(-x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cosh\left(x\right)}{\operatorname{csch}\left(x\right)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(I+csch(x)),x)
```

```
[Out] Integral(cosh(x)/(csch(x) + I), x)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(12) = 24$.
time = 0.39, size = 25, normalized size = 1.56

$$\frac{1}{2}i\,e^{(-x)}-\frac{1}{2}i\,e^{x}-\log\left(-i\,e^{x}\right)+2\,\log\left(e^{x}-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(I+csch(x)),x, algorithm="giac")
```

```
[Out] 1/2*I*e^(-x) - 1/2*I*e^x - log(-I*e^x) + 2*log(e^x - I)
```

Mupad [B]
time = 0.09, size = 12, normalized size = 0.75

$$\ln\left(\sinh(x)-\mathrm{i}\right)-\sinh(x)\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(1/sinh(x) + 1i),x)
```

```
[Out] log(sinh(x) - 1i) - sinh(x)*1i
```

3.88 $\int \frac{\text{sech}(x)}{i+\text{csch}(x)} \, dx$

Optimal. Leaf size=28

$$-\frac{1}{2}i\text{ArcTan}(\sinh(x)) - \frac{\text{sech}^2(x)}{2} + \frac{1}{2}i\text{sech}(x)\tanh(x)$$

```
[Out] -1/2*I*arctan(sinh(x))-1/2*sech(x)^2+1/2*I*sech(x)*tanh(x)
```

Rubi [A]
time = 0.06, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {3957, 2785, 2686, 30, 2691, 3855}

$$-\frac{1}{2}i\text{ArcTan}(\sinh(x)) - \frac{1}{2}\text{sech}^2(x) + \frac{1}{2}i\tanh(x)\text{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]/(I + Csch[x]),x]
```

```
[Out] (-1/2*I)*ArcTan[Sinh[x]] - Sech[x]^2/2 + (I/2)*Sech[x]*Tanh[x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
```

```
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}(x)}{i+\operatorname{csch}(x)}\,dx &= i\int \frac{\tanh(x)}{i-\sinh(x)}\,dx\\ &= -\left(i\int \operatorname{sech}(x)\tanh^2(x)\,dx\right)+\int \operatorname{sech}^2(x)\tanh(x)\,dx\\ &= \frac{1}{2}i\operatorname{sech}(x)\tanh(x)-\frac{1}{2}i\int \operatorname{sech}(x)\,dx-\operatorname{Subst}\left(\int x\,dx,x,\operatorname{sech}(x)\right)\\ &= -\frac{1}{2}i\tan^{-1}(\sinh(x))-\frac{\operatorname{sech}^2(x)}{2}+\frac{1}{2}i\operatorname{sech}(x)\tanh(x)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 20, normalized size = 0.71

$$-\frac{1}{2}i\left(\operatorname{ArcTan}(\sinh(x))+\frac{1}{i-\sinh(x)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(I + Csch[x]),x]
[Out] (-1/2*I)*(ArcTan[Sinh[x]] + (I - Sinh[x])^(-1))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(20) = 40.
time = 1.27, size = 43, normalized size = 1.54

method	result	size

risch	$\frac{i e^{x}}{\left(e^{x}-i\right)^{2}}-\frac{\ln\left(e^{x}-i\right)}{2}+\frac{\ln\left(e^{x}+i\right)}{2}$	30
default	$-\frac{i}{-i+\tanh\left(\frac{x}{2}\right)}+\frac{1}{\left(-i+\tanh\left(\frac{x}{2}\right)\right)^{2}}-\frac{\ln\left(-i+\tanh\left(\frac{x}{2}\right)\right)}{2}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)}{2}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(I+csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -I/(-I+tanh(1/2*x))+1/(-I+tanh(1/2*x))^2-1/2*ln(-I+tanh(1/2*x))+1/2*ln(tanh
(1/2*x)+I)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(18) = 36$.
time = 0.26, size = 41, normalized size = 1.46

$$-\frac{2i\,e^{(-x)}}{4i\,e^{(-x)}+2\,e^{(-2\,x)}-2}-\frac{1}{2}\,\log\left(e^{(-x)}+i\right)+\frac{1}{2}\,\log\left(e^{(-x)}-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -2*I*e^(-x)/(4*I*e^(-x) + 2*e^(-2*x) - 2) - 1/2*log(e^(-x) + I) + 1/2*log(e
^(-x) - I)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(18) = 36$.
time = 0.37, size = 52, normalized size = 1.86

$$\frac{\left(e^{(2\,x)}-2i\,e^{x}-1\right)\log\left(e^{x}+i\right)-\left(e^{(2\,x)}-2i\,e^{x}-1\right)\log\left(e^{x}-i\right)+2i\,e^{x}}{2\left(e^{(2\,x)}-2i\,e^{x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] 1/2*((e^(2*x) - 2*I*e^x - 1)*log(e^x + I) - (e^(2*x) - 2*I*e^x - 1)*log(e^x
 - I) + 2*I*e^x)/(e^(2*x) - 2*I*e^x - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{sech}\left(x\right)}{\operatorname{csch}\left(x\right)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+csch(x)),x)
```

```
[Out] Integral(sech(x)/(csch(x) + I), x)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(18) = 36$.
time = 0.39, size = 53, normalized size = 1.89

$$\frac{e^{(-x)} - e^x - 2i}{4\left(e^{(-x)} - e^x + 2i\right)} + \frac{1}{4}\log\left(-i\,e^{(-x)} + i\,e^x - 2\right) - \frac{1}{4}\log\left(-e^{(-x)} + e^x - 2i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+csch(x)),x, algorithm="giac")
```

```
[Out] 1/4*(e^(-x) - e^x - 2*I)/(e^(-x) - e^x + 2*I) + 1/4*log(-I*e^(-x) + I*e^x -
 2) - 1/4*log(-e^(-x) + e^x - 2*I)
```

Mupad [B]
time = 0.23, size = 46, normalized size = 1.64

$$\frac{\ln\left(-1 + e^x\,1i\right)}{2} - \frac{\ln\left(1 + e^x\,1i\right)}{2} + \frac{1}{1 - e^{2\,x} + e^x\,2i} + \frac{1i}{e^x - i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)*(1/sinh(x) + 1i)),x)
```

```
[Out] log(exp(x)*1i - 1)/2 - log(exp(x)*1i + 1)/2 + 1/(exp(x)*2i - exp(2*x) + 1)
+ 1i/(exp(x) - 1i)
```

3.89 $\int \frac{\operatorname{sech}^2(x)}{i+\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=19

$$-\frac{1}{3}\operatorname{sech}^3(x)-\frac{1}{3}i\tanh^3(x)$$

```
[Out] -1/3*sech(x)^3-1/3*I*tanh(x)^3
```

Rubi [A]
time = 0.08, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.385$, Rules used = {3957, 2918, 2686, 30, 2687}

$$-\frac{1}{3}\operatorname{sech}^3(x)-\frac{1}{3}i\tanh^3(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(I + Csch[x]),x]
```

```
[Out] -1/3*Sech[x]^3 - (I/3)*Tanh[x]^3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
```

```
e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^2(x)}{i+\operatorname{csch}(x)}\,dx &= i\int \frac{\operatorname{sech}(x)\tanh(x)}{i-\sinh(x)}\,dx\\ &= -\left(i\int \operatorname{sech}^2(x)\tanh^2(x)\,dx\right)+\int \operatorname{sech}^3(x)\tanh(x)\,dx\\ &= -\operatorname{Subst}\left(\int x^2\,dx, x, \operatorname{sech}(x)\right)+\operatorname{Subst}\left(\int x^2\,dx, x, i\tanh(x)\right)\\ &= -\frac{1}{3}\operatorname{sech}^3(x)-\frac{1}{3}i\tanh^3(x)\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 64 vs. 2(19) = 38.
time = 0.04, size = 64, normalized size = 3.37

$$\frac{-3+\cosh(x)+\cosh(2x)-2i\sinh(x)+i\cosh(x)\sinh(x)}{6\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(I + Csch[x]),x]
```

```
[Out] (-3 + Cosh[x] + Cosh[2*x] - (2*I)*Sinh[x] + I*Cosh[x]*Sinh[x])/(6*(Cosh[x/2
] - I*Sinh[x/2])*(Cosh[x/2] + I*Sinh[x/2])^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(14) = 28.
time = 1.09, size = 49, normalized size = 2.58

method	result	size
risch	$\frac{2i\left(-2i\,{\rm e}^{x}+3\,{\rm e}^{2x}-1\right)}{3\left({\rm e}^{x}+i\right)\left({\rm e}^{x}-i\right)^{3}}$	31
default	$\frac{i}{-2i+2\tanh\left(\frac{x}{2}\right)}-\frac{2i}{3\left(-i+\tanh\left(\frac{x}{2}\right)\right)^{3}}-\frac{1}{\left(-i+\tanh\left(\frac{x}{2}\right)\right)^{2}}-\frac{i}{2\left(\tanh\left(\frac{x}{2}\right)+i\right)}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(I+csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I/(-I+tanh(1/2*x))-2/3*I/(-I+tanh(1/2*x))^3-1/(-I+tanh(1/2*x))^2-1/2*I/
(tanh(1/2*x)+I)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(13) = 26$.
time = 0.26, size = 81, normalized size = 4.26

$$\frac{8\,e^{(-x)}}{12i\,e^{(-x)} + 12i\,e^{(-3\,x)} + 6\,e^{(-4\,x)} - 6} - \frac{12i\,e^{(-2\,x)}}{12i\,e^{(-x)} + 12i\,e^{(-3\,x)} + 6\,e^{(-4\,x)} - 6} + \frac{4i}{12i\,e^{(-x)} + 12i\,e^{(-3\,x)} + 6\,e^{(-4\,x)} - 6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] 8*e^(-x)/(12*I*e^(-x) + 12*I*e^(-3*x) + 6*e^(-4*x) - 6) - 12*I*e^(-2*x)/(12
*I*e^(-x) + 12*I*e^(-3*x) + 6*e^(-4*x) - 6) + 4*I/(12*I*e^(-x) + 12*I*e^(-3
*x) + 6*e^(-4*x) - 6)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(13) = 26$.
time = 0.36, size = 32, normalized size = 1.68

$$-\frac{2\,\left(-3i\,e^{(2\,x)} - 2\,e^{x} + i\right)}{3\,\left(e^{(4\,x)} - 2i\,e^{(3\,x)} - 2i\,e^{x} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] -2/3*(-3*I*e^(2*x) - 2*e^x + I)/(e^(4*x) - 2*I*e^(3*x) - 2*I*e^x - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(x)}{\operatorname{csch}(x) + i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(I+csch(x)),x)
```

```
[Out] Integral(sech(x)**2/(csch(x) + I), x)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(13) = 26$.
time = 0.40, size = 27, normalized size = 1.42

$$-\frac{i}{2\,(i\,e^x-1)}+\frac{3\,e^{(2\,x)}-1}{6\,(e^x-i)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(I+csch(x)),x, algorithm="giac")
[Out] -1/2*I/(I*e^x - 1) + 1/6*(3*e^(2*x) - 1)/(e^x - I)^3
```

Mupad [B]
time = 1.57, size = 31, normalized size = 1.63

$$-\frac{\frac{2}{3}-2\,\mathrm{e}^{2\,x}+\frac{\mathrm{e}^x\,4\mathrm{i}}{3}}{(\mathrm{e}^x+1\mathrm{i})\,(1+\mathrm{e}^x\,1\mathrm{i})^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2*(1/sinh(x) + 1i)),x)
[Out] -((exp(x)*4i)/3 - 2*exp(2*x) + 2/3)/((exp(x) + 1i)*(exp(x)*1i + 1)^3)
```

3.90 $\int \frac{\operatorname{sech}^3(x)}{i+\operatorname{csch}(x)} \, dx$

Optimal. Leaf size=40

$$-\frac{1}{8}i\text{ArcTan}(\sinh(x)) - \frac{\operatorname{sech}^4(x)}{4} - \frac{1}{8}i\operatorname{sech}(x)\tanh(x) + \frac{1}{4}i\operatorname{sech}^3(x)\tanh(x)$$

```
[Out] -1/8*I*arctan(sinh(x))-1/4*sech(x)^4-1/8*I*sech(x)*tanh(x)+1/4*I*sech(x)^3*
tanh(x)
```

Rubi [A]
time = 0.09, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3957, 2914, 2686, 30, 2691, 3853, 3855}

$$-\frac{1}{8}i\text{ArcTan}(\sinh(x)) - \frac{1}{4}\operatorname{sech}^4(x) + \frac{1}{4}i\tanh(x)\operatorname{sech}^3(x) - \frac{1}{8}i\tanh(x)\operatorname{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^3/(I + Csch[x]),x]
```

```
[Out] (-1/8*I)*ArcTan[Sinh[x]] - Sech[x]^4/4 - (I/8)*Sech[x]*Tanh[x] + (I/4)*Sech
[x]^3*Tanh[x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2914

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{sech}^3(x)}{i+\operatorname{csch}(x)}\,dx &= i\int \frac{\operatorname{sech}^2(x)\tanh(x)}{i-\sinh(x)}\,dx \\
&= -\left(i\int \operatorname{sech}^3(x)\tanh^2(x)\,dx\right)+\int \operatorname{sech}^4(x)\tanh(x)\,dx \\
&= \frac{1}{4}i\operatorname{sech}^3(x)\tanh(x)-\frac{1}{4}i\int \operatorname{sech}^3(x)\,dx-\operatorname{Subst}\left(\int x^3\,dx,x,\operatorname{sech}(x)\right) \\
&= -\frac{1}{4}\operatorname{sech}^4(x)-\frac{1}{8}i\operatorname{sech}(x)\tanh(x)+\frac{1}{4}i\operatorname{sech}^3(x)\tanh(x)-\frac{1}{8}i\int \operatorname{sech}(x)\,dx \\
&= -\frac{1}{8}i\tan^{-1}(\sinh(x))-\frac{\operatorname{sech}^4(x)}{4}-\frac{1}{8}i\operatorname{sech}(x)\tanh(x)+\frac{1}{4}i\operatorname{sech}^3(x)\tanh(x)
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 60, normalized size = 1.50

$$
\frac{1}{4}i\operatorname{ArcTan}\left(\coth\left(\frac{x}{2}\right)\right)-\frac{1}{8\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)^2}-\frac{1}{8\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^3/(I + Csch[x]),x]
```

```
[Out] (I/4)*ArcTan[Coth[x/2]] - 1/(8*(Cosh[x/2] - I*Sinh[x/2])^2) - 1/(8*(Cosh[x/
2] + I*Sinh[x/2])^4)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(29) = 58$.
time = 1.13, size = 89, normalized size = 2.22

method	result
risch	$-\frac{i e^{x} \left(-2 i e^{3 x}+e^{4 x}+2 i e^{x}-10 e^{2 x}+1\right)}{4 \left(e^{x}-i\right)^{4} \left(e^{x}+i\right)^{2}}+\frac{\ln \left(e^{x}+i\right)}{8}-\frac{\ln \left(e^{x}-i\right)}{8}$
default	$\frac{i}{\left(-i+\tanh \left(\frac{x}{2}\right)\right)^{3}}-\frac{i}{2 \left(-i+\tanh \left(\frac{x}{2}\right)\right)}-\frac{1}{2 \left(-i+\tanh \left(\frac{x}{2}\right)\right)^{4}}+\frac{1}{\left(-i+\tanh \left(\frac{x}{2}\right)\right)^{2}}-\frac{\ln \left(-i+\tanh \left(\frac{x}{2}\right)\right)}{8}+\frac{i}{4 \tanh \left(\frac{x}{2}\right)+4 i}+\frac{}{4(\tan}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^3/(I+csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] I/(-I+tanh(1/2*x))^3-1/2*I/(-I+tanh(1/2*x))-1/2/(-I+tanh(1/2*x))^4+1/(-I+ta
nh(1/2*x))^2-1/8*ln(-I+tanh(1/2*x))+1/4*I/(tanh(1/2*x)+I)+1/4/(tanh(1/2*x)+
I)^2+1/8*ln(tanh(1/2*x)+I)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(26) = 52$.
time = 0.40, size = 142, normalized size = 3.55

$$\frac{\left(e^{(6x)} - 2i\,e^{(5x)} + e^{(4x)} - 4i\,e^{(3x)} - e^{(2x)} - 2i\,e^{x} - 1\right)\log\left(e^{x} + i\right) - \left(e^{(6x)} - 2i\,e^{(5x)} + e^{(4x)} - 4i\,e^{(3x)} - e^{(2x)} - 2i\,e^{x} - 1\right)\log\left(e^{x} - i\right) - 2i\,e^{(5x)} - 4\,e^{(4x)} + 20i\,e^{(3x)} + 4\,e^{(2x)} - 2i\,e^{x}}{8\left(e^{(6x)} - 2i\,e^{(5x)} + e^{(4x)} - 4i\,e^{(3x)} - e^{(2x)} - 2i\,e^{x} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] 1/8*((e^(6*x) - 2*I*e^(5*x) + e^(4*x) - 4*I*e^(3*x) - e^(2*x) - 2*I*e^x - 1
)*log(e^x + I) - (e^(6*x) - 2*I*e^(5*x) + e^(4*x) - 4*I*e^(3*x) - e^(2*x) -
 2*I*e^x - 1)*log(e^x - I) - 2*I*e^(5*x) - 4*e^(4*x) + 20*I*e^(3*x) + 4*e^(
```

```
2*x) - 2*I*e^x)/(e^(6*x) - 2*I*e^(5*x) + e^(4*x) - 4*I*e^(3*x) - e^(2*x) -
2*I*e^x - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^3(x)}{\operatorname{csch}(x) + i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**3/(I+csch(x)),x)
```

```
[Out] Integral(sech(x)**3/(csch(x) + I), x)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(26) = 52.
time = 0.38, size = 94, normalized size = 2.35

$$-\frac{-i\,e^{(-x)} + i\,e^{x} - 6}{16\,(-i\,e^{(-x)} + i\,e^{x} - 2)} + \frac{3\,(e^{(-x)} - e^{x})^2 + 12i\,e^{(-x)} - 12i\,e^{x} + 4}{32\,(e^{(-x)} - e^{x} + 2i)^2} + \frac{1}{16}\log\left(-e^{(-x)} + e^{x} + 2i\right) - \frac{1}{16}\log\left(-e^{(-x)} + e^{x} - 2i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(I+csch(x)),x, algorithm="giac")
```

```
[Out] -1/16*(-I*e^(-x) + I*e^x - 6)/(-I*e^(-x) + I*e^x - 2) + 1/32*(3*(e^(-x) - e
^x)^2 + 12*I*e^(-x) - 12*I*e^x + 4)/(e^(-x) - e^x + 2*I)^2 + 1/16*log(-e^(-
x) + e^x + 2*I) - 1/16*log(-e^(-x) + e^x - 2*I)
```

Mupad [B]
time = 1.95, size = 122, normalized size = 3.05

$$\frac{\ln\left(-\frac{1}{4} + \frac{e^{x}\,1i}{4}\right)}{8} - \frac{\ln\left(\frac{1}{4} + \frac{e^{x}\,1i}{4}\right)}{8} - \frac{1i}{e^{2x}\,3i - e^{3x} + 3\,e^{x} - i} - \frac{1}{4\,(e^{2x} - 1 + e^{x}\,2i)} - \frac{1}{2\,(e^{4x} - 6\,e^{2x} + 1 - e^{3x}\,4i + e^{x}\,4i)} - \frac{1}{2\,(1 - e^{2x} + e^{x}\,2i)} - \frac{1i}{4\,(e^{x} + 1i)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^3*(1/sinh(x) + 1i)),x)
```

```
[Out] log((exp(x)*1i)/4 - 1/4)/8 - log((exp(x)*1i)/4 + 1/4)/8 - 1i/(exp(2*x)*3i -
 exp(3*x) + 3*exp(x) - 1i) - 1/(4*(exp(2*x) + exp(x)*2i - 1)) - 1/(2*(exp(4
*x) - exp(3*x)*4i - 6*exp(2*x) + exp(x)*4i + 1)) - 1/(2*(exp(x)*2i - exp(2*
x) + 1)) - 1i/(4*(exp(x) + 1i))
```

3.91 $\int \frac{\operatorname{sech}^4(x)}{i+\operatorname{csch}(x)} dx$

Optimal. Leaf size=29

$$-\frac{1}{5}\operatorname{sech}^5(x) - \frac{1}{3}i\tanh^3(x) + \frac{1}{5}i\tanh^5(x)$$

```
[Out] -1/5*sech(x)^5-1/3*I*tanh(x)^3+1/5*I*tanh(x)^5
```

Rubi [A]
time = 0.09, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3957, 2918, 2686, 30, 2687, 14}

$$\frac{1}{5}i\tanh^5(x) - \frac{1}{3}i\tanh^3(x) - \frac{1}{5}\operatorname{sech}^5(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^4/(I + Csch[x]),x]
```

```
[Out] -1/5*Sech[x]^5 - (I/3)*Tanh[x]^3 + (I/5)*Tanh[x]^5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^4(x)}{i+\operatorname{csch}(x)}\,dx &= i\int \frac{\operatorname{sech}^3(x)\tanh(x)}{i-\sinh(x)}\,dx\\ &= -\left(i\int \operatorname{sech}^4(x)\tanh^2(x)\,dx\right)+\int \operatorname{sech}^5(x)\tanh(x)\,dx\\ &= -\operatorname{Subst}\left(\int x^4\,dx,x,\operatorname{sech}(x)\right)+\operatorname{Subst}\left(\int x^2\left(1+x^2\right)\,dx,x,i\tanh(x)\right)\\ &= -\frac{1}{5}\operatorname{sech}^5(x)+\operatorname{Subst}\left(\int \left(x^2+x^4\right)\,dx,x,i\tanh(x)\right)\\ &= -\frac{1}{5}\operatorname{sech}^5(x)-\frac{1}{3}i\tanh^3(x)+\frac{1}{5}i\tanh^5(x)\end{aligned}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 96 vs. $2(29) = 58$.
time = 0.08, size = 96, normalized size = 3.31

$$\frac{-240+54\cosh(x)+32\cosh(2x)+18\cosh(3x)+16\cosh(4x)-96i\sinh(x)+18i\sinh(2x)-32i\sinh(3x)+9i\sinh(4x)}{960\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)^3\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^4/(I + Csch[x]),x]
```

```
[Out] (-240 + 54*Cosh[x] + 32*Cosh[2*x] + 18*Cosh[3*x] + 16*Cosh[4*x] - (96*I)*Si
nh[x] + (18*I)*Sinh[2*x] - (32*I)*Sinh[3*x] + (9*I)*Sinh[4*x])/(960*(Cosh[x
/2] - I*Sinh[x/2])^3*(Cosh[x/2] + I*Sinh[x/2])^5)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(21) = 42$.
time = 1.12, size = 93, normalized size = 3.21

method	result
risch	$\frac{4i\left(-6i\,\mathrm{e}^{3x}+15\,\mathrm{e}^{4x}-2i\,\mathrm{e}^{x}-2\,\mathrm{e}^{2x}-1\right)}{15\left(\mathrm{e}^{x}+i\right)^{3}\left(\mathrm{e}^{x}-i\right)^{5}}$
default	$\frac{i}{6\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}-\frac{3i}{8\left(\tanh\left(\frac{x}{2}\right)+i\right)}-\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}-\frac{4i}{3\left(-i+\tanh\left(\frac{x}{2}\right)\right)^{3}}+\frac{3i}{8\left(-i+\tanh\left(\frac{x}{2}\right)\right)}+\frac{2i}{5\left(-i+\tanh\left(\frac{x}{2}\right)\right)^{5}}+\frac{}{(-i+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^4/(I+csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*I/(tanh(1/2*x)+I)^3-3/8*I/(tanh(1/2*x)+I)-1/4/(tanh(1/2*x)+I)^2-4/3*I/(
-I+tanh(1/2*x))^3+3/8*I/(-I+tanh(1/2*x))+2/5*I/(-I+tanh(1/2*x))^5+1/(-I+tan
h(1/2*x))^4-1/(-I+tanh(1/2*x))^2
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(19) = 38.
time = 0.36, size = 68, normalized size = 2.34

$$-\frac{4\left(-15i\,e^{(4x)}-6\,e^{(3x)}+2i\,e^{(2x)}-2\,e^{x}+i\right)}{15\left(e^{(8x)}-2i\,e^{(7x)}+2\,e^{(6x)}-6i\,e^{(5x)}-6i\,e^{(3x)}-2\,e^{(2x)}-2i\,e^{x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] -4/15*(-15*I*e^(4*x) - 6*e^(3*x) + 2*I*e^(2*x) - 2*e^x + I)/(e^(8*x) - 2*I*
e^(7*x) + 2*e^(6*x) - 6*I*e^(5*x) - 6*I*e^(3*x) - 2*e^(2*x) - 2*I*e^x - 1)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{sech}^{4}(x)}{\operatorname{csch}(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**4/(I+csch(x)),x)
```

```
[Out] Integral(sech(x)**4/(csch(x) + I), x)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(19) = 38$.
time = 0.40, size = 55, normalized size = 1.90

$$-\frac{-3i\,e^{(2\,x)}+12\,e^{x}+5i}{24\,(i\,e^{x}-1)^{3}}+\frac{15\,e^{(4\,x)}-60i\,e^{(3\,x)}-10\,e^{(2\,x)}+20i\,e^{x}+7}{120\,(e^{x}-i)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(I+csch(x)),x, algorithm="giac")
```

```
[Out] -1/24*(-3*I*e^(2*x) + 12*e^x + 5*I)/(I*e^x - 1)^3 + 1/120*(15*e^(4*x) - 60*
I*e^(3*x) - 10*e^(2*x) + 20*I*e^x + 7)/(e^x - I)^5
```

Mupad [B]
time = 1.92, size = 207, normalized size = 7.14

$$-\frac{1\mathrm{i}}{4\,\left(\mathrm{e}^{2\,x}-1+\mathrm{e}^{x}\,2\mathrm{i}\right)}+\frac{1}{20\,\left(\mathrm{e}^{x}-\mathrm{i}\right)}-\frac{1}{8\,\left(\mathrm{e}^{x}+1\mathrm{i}\right)}+\frac{\frac{\mathrm{e}^{3\,x}}{40}-\frac{\mathrm{e}^{2\,x}\,3\mathrm{i}}{40}+\frac{\mathrm{e}^{x}}{8}+\frac{1}{40}\mathrm{i}}{\mathrm{e}^{4\,x}-6\,\mathrm{e}^{2\,x}+1-\mathrm{e}^{3\,x}\,4\mathrm{i}+\mathrm{e}^{x}\,4\mathrm{i}}-\frac{\frac{\mathrm{e}^{2\,x}}{40}+\frac{1}{24}-\frac{\mathrm{e}^{x}\,1\mathrm{i}}{20}}{\mathrm{e}^{2\,x}\,3\mathrm{i}-\mathrm{e}^{3\,x}+3\,\mathrm{e}^{x}-\mathrm{i}}-\frac{1}{6\,\left(\mathrm{e}^{2\,x}\,3\mathrm{i}+\mathrm{e}^{3\,x}-3\,\mathrm{e}^{x}-\mathrm{i}\right)}+\frac{\frac{\mathrm{e}^{2\,x}}{4}+\frac{\mathrm{e}^{4\,x}}{40}+\frac{1}{40}-\frac{\mathrm{e}^{3\,x}\,1\mathrm{i}}{10}+\frac{\mathrm{e}^{x}\,1\mathrm{i}}{10}}{\mathrm{e}^{2\,x}\,10\mathrm{i}-10\,\mathrm{e}^{3\,x}-\mathrm{e}^{4\,x}\,5\mathrm{i}+\mathrm{e}^{5\,x}+5\,\mathrm{e}^{x}-\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^4*(1/sinh(x) + 1i)),x)
```

```
[Out] 1/(20*(exp(x) - 1i)) - 1i/(4*(exp(2*x) + exp(x)*2i - 1)) - 1/(8*(exp(x) + 1
i)) + (exp(3*x)/40 - (exp(2*x)*3i)/40 + exp(x)/8 + 1i/40)/(exp(4*x) - exp(3
*x)*4i - 6*exp(2*x) + exp(x)*4i + 1) - (exp(2*x)/40 - (exp(x)*1i)/20 + 1/24
)/(exp(2*x)*3i - exp(3*x) + 3*exp(x) - 1i) - 1/(6*(exp(2*x)*3i + exp(3*x) -
 3*exp(x) - 1i)) + (exp(2*x)/4 - (exp(3*x)*1i)/10 + exp(4*x)/40 + (exp(x)*1
i)/10 + 1/40)/(exp(2*x)*10i - 10*exp(3*x) - exp(4*x)*5i + exp(5*x) + 5*exp(
x) - 1i)
```

3.92 $\int \frac{\cosh^5(x)}{a+b\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=102

$$-\frac{b(a^2+b^2)^2\log(b+a\sinh(x))}{a^6}+\frac{(a^2+b^2)^2\sinh(x)}{a^5}-\frac{b(2a^2+b^2)\sinh^2(x)}{2a^4}+\frac{(2a^2+b^2)\sinh^3(x)}{3a^3}-\frac{b\sinh^4(x)}{4a^2}$$

```
[Out] -b*(a^2+b^2)^2*ln(b+a*sinh(x))/a^6+(a^2+b^2)^2*sinh(x)/a^5-1/2*b*(2*a^2+b^2
)*sinh(x)^2/a^4+1/3*(2*a^2+b^2)*sinh(x)^3/a^3-1/4*b*sinh(x)^4/a^2+1/5*sinh(
x)^5/a
```

Rubi [A]
time = 0.14, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3957, 2916, 12, 786}

$$-\frac{b\sinh^4(x)}{4a^2}-\frac{b(a^2+b^2)^2\log(a\sinh(x)+b)}{a^6}+\frac{(a^2+b^2)^2\sinh(x)}{a^5}-\frac{b(2a^2+b^2)\sinh^2(x)}{2a^4}+\frac{(2a^2+b^2)\sinh^3(x)}{3a^3}+\frac{\sinh^5(x)}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^5/(a + b*Csch[x]),x]
```

```
[Out] -((b*(a^2 + b^2)^2*Log[b + a*Sinh[x]])/a^6) + ((a^2 + b^2)^2*Sinh[x])/a^5 -
 (b*(2*a^2 + b^2)*Sinh[x]^2)/(2*a^4) + ((2*a^2 + b^2)*Sinh[x]^3)/(3*a^3) -
(b*Sinh[x]^4)/(4*a^2) + Sinh[x]^5/(5*a)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^5(x)}{a+b\text{csch}(x)}\,dx &= i\int \frac{\cosh^5(x)\sinh(x)}{ib+ia\sinh(x)}\,dx\\
&= -\frac{i\text{Subst}\left(\int \frac{x(a^2-x^2)^2}{a(ib+x)}\,dx,x,ia\sinh(x)\right)}{a^5}\\
&= -\frac{i\text{Subst}\left(\int \frac{x(a^2-x^2)^2}{ib+x}\,dx,x,ia\sinh(x)\right)}{a^6}\\
&= -\frac{i\text{Subst}\left(\int \left((a^2+b^2)^2-\frac{b(a^2+b^2)^2}{b-ix}+ib(2a^2+b^2)\,x-(2a^2+b^2)\,x^2-ibx^3+x^4\right)\,dx,x,\right.}{a^6}\\
&= -\frac{b(a^2+b^2)^2\log(b+a\sinh(x))}{a^6}+\frac{(a^2+b^2)^2\sinh(x)}{a^5}-\frac{b(2a^2+b^2)\sinh^2(x)}{2a^4}+\frac{(2a^2+b}{}
\end{aligned}$$

Mathematica [A]
time = 0.27, size = 124, normalized size = 1.22

$$\frac{-15a^4b\cosh(2x)+15a(3a^4+8a^2b^2+4b^4)\sinh(x)-30a^2b(a^2+b^2)\sinh^2(x)+20a^3(a^2+b^2)\sinh^3(x)-15a^4b\sinh^4(x)+12a^5\sinh^5(x)+5\left(-12b(a^2+b^2)^2\log(b+a\sinh(x))+a^5\sinh(3x)\right)}{60a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^5/(a + b*Csch[x]),x]
```

```
[Out] (-15*a^4*b*Cosh[2*x] + 15*a*(3*a^4 + 8*a^2*b^2 + 4*b^4)*Sinh[x] - 30*a^2*b*
(a^2 + b^2)*Sinh[x]^2 + 20*a^3*(a^2 + b^2)*Sinh[x]^3 - 15*a^4*b*Sinh[x]^4 +
 12*a^5*Sinh[x]^5 + 5*(-12*b*(a^2 + b^2)^2*Log[b + a*Sinh[x]] + a^5*Sinh[3*
x]))/(60*a^6)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 377 vs. 2(94) = 188.
time = 0.68, size = 378, normalized size = 3.71

method	result
risch	$\frac{bx}{a^2}+\frac{2b^3x}{a^4}+\frac{b^5x}{a^6}+\frac{e^{5x}}{160a}-\frac{b\,e^{4x}}{64a^2}+\frac{5\,e^{3x}}{96a}+\frac{e^{3x}b^2}{24a^3}-\frac{3b\,e^{2x}}{16a^2}-\frac{b^3e^{2x}}{8a^4}+\frac{5\,e^x}{16a}+\frac{7\,e^xb^2}{8a^3}+\frac{e^xb^4}{2a^5}-\frac{5\,e^{-x}}{16a}-\frac{7\,e^{-x}b^2}{8a^3}-$
default	$-\frac{1}{5a\left(\tanh\left(\frac{x}{2}\right)-1\right)^5}-\frac{2a+b}{4a^2\left(\tanh\left(\frac{x}{2}\right)-1\right)^4}-\frac{11a^2+6ab+4b^2}{12a^3\left(\tanh\left(\frac{x}{2}\right)-1\right)^3}-\frac{7a^3+9a^2b+4a\,b^2+4b^3}{8a^4\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}-\frac{8a^4+7a^3b+16a^2b^2+4a\,b^3+8b^4}{8a^5\left(\tanh\left(\frac{x}{2}\right)-1\right)}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^5/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/a/(tanh(1/2*x)-1)^5-1/4*(2*a+b)/a^2/(tanh(1/2*x)-1)^4-1/12*(11*a^2+6*a
*b+4*b^2)/a^3/(tanh(1/2*x)-1)^3-1/8*(7*a^3+9*a^2*b+4*a*b^2+4*b^3)/a^4/(tanh
(1/2*x)-1)^2-1/8*(8*a^4+7*a^3*b+16*a^2*b^2+4*a*b^3+8*b^4)/a^5/(tanh(1/2*x)-
1)+b*(a^4+2*a^2*b^2+b^4)/a^6*ln(tanh(1/2*x)-1)-1/5/a/(tanh(1/2*x)+1)^5-1/4*
(-2*a+b)/a^2/(tanh(1/2*x)+1)^4-1/12*(11*a^2-6*a*b+4*b^2)/a^3/(tanh(1/2*x)+1
)^3-1/8*(-7*a^3+9*a^2*b-4*a*b^2+4*b^3)/a^4/(tanh(1/2*x)+1)^2-1/8*(8*a^4-7*a
^3*b+16*a^2*b^2-4*a*b^3+8*b^4)/a^5/(tanh(1/2*x)+1)+b*(a^4+2*a^2*b^2+b^4)/a^
6*ln(tanh(1/2*x)+1)+2*b/a^6*(-1/2*a^4-a^2*b^2-1/2*b^4)*ln(-b*tanh(1/2*x)^2+
2*a*tanh(1/2*x)+b)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(94) = 188$.
time = 0.26, size = 242, normalized size = 2.37

$$-\frac{\left(15\,a^3be^{(-x)} - 6\,a^4 - 10\,(5\,a^4 + 4\,a^2b^2)e^{(-2\,x)} + 60\,(3\,a^3b + 2\,ab^3)e^{(-3\,x)} - 60\,(5\,a^4 + 14\,a^2b^2 + 8\,b^4)e^{(-4\,x)}\right)e^{(5\,x)}}{960\,a^5} - \frac{15\,a^3be^{(-4\,x)} + 6\,a^4e^{(-5\,x)} + 60\,(5\,a^4 + 14\,a^2b^2 + 8\,b^4)e^{(-x)} + 60\,(3\,a^3b + 2\,ab^3)e^{(-2\,x)} + 10\,(5\,a^4 + 4\,a^2b^2)e^{(-3\,x)}}{960\,a^5} - \frac{(a^4b + 2\,a^2b^3 + b^5)x}{a^6} - \frac{(a^4b + 2\,a^2b^3 + b^5)\log\left(-2\,be^{(-x)} + ae^{(-2\,x)} - a\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -1/960*(15*a^3*b*e^(-x) - 6*a^4 - 10*(5*a^4 + 4*a^2*b^2)*e^(-2*x) + 60*(3*a
^3*b + 2*a*b^3)*e^(-3*x) - 60*(5*a^4 + 14*a^2*b^2 + 8*b^4)*e^(-4*x))*e^(5*x
)/a^5 - 1/960*(15*a^3*b*e^(-4*x) + 6*a^4*e^(-5*x) + 60*(5*a^4 + 14*a^2*b^2
+ 8*b^4)*e^(-x) + 60*(3*a^3*b + 2*a*b^3)*e^(-2*x) + 10*(5*a^4 + 4*a^2*b^2)*
e^(-3*x))/a^5 - (a^4*b + 2*a^2*b^3 + b^5)*x/a^6 - (a^4*b + 2*a^2*b^3 + b^5)
*log(-2*b*e^(-x) + a*e^(-2*x) - a)/a^6
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1398 vs. $2(94) = 188$.
time = 0.43, size = 1398, normalized size = 13.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] 1/960*(6*a^5*cosh(x)^10 + 6*a^5*sinh(x)^10 - 15*a^4*b*cosh(x)^9 + 15*(4*a^5
*cosh(x) - a^4*b)*sinh(x)^9 + 10*(5*a^5 + 4*a^3*b^2)*cosh(x)^8 + 5*(54*a^5*
cosh(x)^2 - 27*a^4*b*cosh(x) + 10*a^5 + 8*a^3*b^2)*sinh(x)^8 - 60*(3*a^4*b
+ 2*a^2*b^3)*cosh(x)^7 + 20*(36*a^5*cosh(x)^3 - 27*a^4*b*cosh(x)^2 - 9*a^4*
b - 6*a^2*b^3 + 4*(5*a^5 + 4*a^3*b^2)*cosh(x))*sinh(x)^7 + 960*(a^4*b + 2*a
```

```
^2*b^3 + b^5)*x*cosh(x)^5 + 60*(5*a^5 + 14*a^3*b^2 + 8*a*b^4)*cosh(x)^6 + 2
0*(63*a^5*cosh(x)^4 - 63*a^4*b*cosh(x)^3 + 15*a^5 + 42*a^3*b^2 + 24*a*b^4 +
 14*(5*a^5 + 4*a^3*b^2)*cosh(x)^2 - 21*(3*a^4*b + 2*a^2*b^3)*cosh(x))*sinh(
x)^6 - 15*a^4*b*cosh(x) + 2*(756*a^5*cosh(x)^5 - 945*a^4*b*cosh(x)^4 + 280*
(5*a^5 + 4*a^3*b^2)*cosh(x)^3 - 630*(3*a^4*b + 2*a^2*b^3)*cosh(x)^2 + 480*(
a^4*b + 2*a^2*b^3 + b^5)*x + 180*(5*a^5 + 14*a^3*b^2 + 8*a*b^4)*cosh(x))*si
nh(x)^5 - 6*a^5 - 60*(5*a^5 + 14*a^3*b^2 + 8*a*b^4)*cosh(x)^4 + 10*(126*a^5
*cosh(x)^6 - 189*a^4*b*cosh(x)^5 - 30*a^5 - 84*a^3*b^2 - 48*a*b^4 + 70*(5*a
^5 + 4*a^3*b^2)*cosh(x)^4 - 210*(3*a^4*b + 2*a^2*b^3)*cosh(x)^3 + 480*(a^4*
b + 2*a^2*b^3 + b^5)*x*cosh(x) + 90*(5*a^5 + 14*a^3*b^2 + 8*a*b^4)*cosh(x)^
2)*sinh(x)^4 - 60*(3*a^4*b + 2*a^2*b^3)*cosh(x)^3 + 20*(36*a^5*cosh(x)^7 -
63*a^4*b*cosh(x)^6 + 28*(5*a^5 + 4*a^3*b^2)*cosh(x)^5 - 9*a^4*b - 6*a^2*b^3
 - 105*(3*a^4*b + 2*a^2*b^3)*cosh(x)^4 + 480*(a^4*b + 2*a^2*b^3 + b^5)*x*co
sh(x)^2 + 60*(5*a^5 + 14*a^3*b^2 + 8*a*b^4)*cosh(x)^3 - 12*(5*a^5 + 14*a^3*
b^2 + 8*a*b^4)*cosh(x))*sinh(x)^3 - 10*(5*a^5 + 4*a^3*b^2)*cosh(x)^2 + 10*(
27*a^5*cosh(x)^8 - 54*a^4*b*cosh(x)^7 + 28*(5*a^5 + 4*a^3*b^2)*cosh(x)^6 -
126*(3*a^4*b + 2*a^2*b^3)*cosh(x)^5 - 5*a^5 - 4*a^3*b^2 + 960*(a^4*b + 2*a^
2*b^3 + b^5)*x*cosh(x)^3 + 90*(5*a^5 + 14*a^3*b^2 + 8*a*b^4)*cosh(x)^4 - 36
*(5*a^5 + 14*a^3*b^2 + 8*a*b^4)*cosh(x)^2 - 18*(3*a^4*b + 2*a^2*b^3)*cosh(x
))*sinh(x)^2 - 960*((a^4*b + 2*a^2*b^3 + b^5)*cosh(x)^5 + 5*(a^4*b + 2*a^2*
b^3 + b^5)*cosh(x)^4*sinh(x) + 10*(a^4*b + 2*a^2*b^3 + b^5)*cosh(x)^3*sinh(
x)^2 + 10*(a^4*b + 2*a^2*b^3 + b^5)*cosh(x)^2*sinh(x)^3 + 5*(a^4*b + 2*a^2*
b^3 + b^5)*cosh(x)*sinh(x)^4 + (a^4*b + 2*a^2*b^3 + b^5)*sinh(x)^5)*log(2*(
a*sinh(x) + b)/(cosh(x) - sinh(x))) + 5*(12*a^5*cosh(x)^9 - 27*a^4*b*cosh(x
)^8 + 16*(5*a^5 + 4*a^3*b^2)*cosh(x)^7 - 84*(3*a^4*b + 2*a^2*b^3)*cosh(x)^6
 + 960*(a^4*b + 2*a^2*b^3 + b^5)*x*cosh(x)^4 + 72*(5*a^5 + 14*a^3*b^2 + 8*a
*b^4)*cosh(x)^5 - 3*a^4*b - 48*(5*a^5 + 14*a^3*b^2 + 8*a*b^4)*cosh(x)^3 - 3
6*(3*a^4*b + 2*a^2*b^3)*cosh(x)^2 - 4*(5*a^5 + 4*a^3*b^2)*cosh(x))*sinh(x))
/(a^6*cosh(x)^5 + 5*a^6*cosh(x)^4*sinh(x) + 10*a^6*cosh(x)^3*sinh(x)^2 + 10
*a^6*cosh(x)^2*sinh(x)^3 + 5*a^6*cosh(x)*sinh(x)^4 + a^6*sinh(x)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^5(x)}{a + b\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**5/(a+b*csch(x)),x)
```

```
[Out] Integral(cosh(x)**5/(a + b*csch(x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(94) = 188.
time = 0.41, size = 194, normalized size = 1.90

$$-\frac{6\,a^4\left(e^{(-x)}-e^x\right)^5+15\,a^3b\left(e^{(-x)}-e^x\right)^4+80\,a^4\left(e^{(-x)}-e^x\right)^3+40\,a^2b^2\left(e^{(-x)}-e^x\right)^3+240\,a^3b\left(e^{(-x)}-e^x\right)^2+120\,ab^3\left(e^{(-x)}-e^x\right)^2+480\,a^4\left(e^{(-x)}-e^x\right)+960\,a^2b^2\left(e^{(-x)}-e^x\right)+480\,b^4\left(e^{(-x)}-e^x\right)}{960\,a^5}-\frac{\left(a^4b+2\,a^2b^3+b^5\right)\log\left(\left|-a\left(e^{(-x)}-e^x\right)+2\,b\right|\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -1/960*(6*a^4*(e^(-x) - e^x)^5 + 15*a^3*b*(e^(-x) - e^x)^4 + 80*a^4*(e^(-x)
 - e^x)^3 + 40*a^2*b^2*(e^(-x) - e^x)^3 + 240*a^3*b*(e^(-x) - e^x)^2 + 120*
a*b^3*(e^(-x) - e^x)^2 + 480*a^4*(e^(-x) - e^x) + 960*a^2*b^2*(e^(-x) - e^x
) + 480*b^4*(e^(-x) - e^x))/a^5 - (a^4*b + 2*a^2*b^3 + b^5)*log(abs(-a*(e^(
-x) - e^x) + 2*b))/a^6
```

Mupad [B]
time = 2.07, size = 228, normalized size = 2.24

$$\frac{e^{5x}}{160a}-\frac{e^{-5x}}{160a}-\frac{e^{-2x}\left(3a^2b+2b^3\right)}{16a^4}-\frac{e^{2x}\left(3a^2b+2b^3\right)}{16a^4}+\frac{e^{x}\left(5a^4+14a^2b^2+8b^4\right)}{16a^5}-\frac{b\,e^{-4x}}{64a^2}-\frac{b\,e^{4x}}{64a^2}-\frac{\ln\left(2b\,e^{x}-a+a\,e^{2x}\right)\left(a^4b+2a^2b^3+b^5\right)}{a^6}-\frac{e^{-x}\left(5a^4+14a^2b^2+8b^4\right)}{16a^5}-\frac{e^{-3x}\left(5a^2+4b^2\right)}{96a^3}+\frac{e^{3x}\left(5a^2+4b^2\right)}{96a^3}+\frac{b\,x\left(a^2+b^2\right)^2}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^5/(a + b/sinh(x)),x)
```

```
[Out] exp(5*x)/(160*a) - exp(-5*x)/(160*a) - (exp(-2*x)*(3*a^2*b + 2*b^3))/(16*a^
4) - (exp(2*x)*(3*a^2*b + 2*b^3))/(16*a^4) + (exp(x)*(5*a^4 + 8*b^4 + 14*a^
2*b^2))/(16*a^5) - (b*exp(-4*x))/(64*a^2) - (b*exp(4*x))/(64*a^2) - (log(2*
b*exp(x) - a + a*exp(2*x))*(a^4*b + b^5 + 2*a^2*b^3))/a^6 - (exp(-x)*(5*a^4
 + 8*b^4 + 14*a^2*b^2))/(16*a^5) - (exp(-3*x)*(5*a^2 + 4*b^2))/(96*a^3) + (
exp(3*x)*(5*a^2 + 4*b^2))/(96*a^3) + (b*x*(a^2 + b^2)^2)/a^6
```

3.93 $\int \frac{\cosh^4(x)}{a+b\,\text{csch}(x)}\,dx$

Optimal. Leaf size=125

$$\frac{(3a^4+12a^2b^2+8b^4)\,x}{8a^5}+\frac{2b(a^2+b^2)^{3/2}\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^5}-\frac{\cosh^3(x)(4b-3a\sinh(x))}{12a^2}-\frac{\cosh(x)\,(8b(a^2+b$$

```
[Out] 1/8*(3*a^4+12*a^2*b^2+8*b^4)*x/a^5+2*b*(a^2+b^2)^(3/2)*arctanh((a-b*tanh(1/
2*x))/(a^2+b^2)^(1/2))/a^5-1/12*cosh(x)^3*(4*b-3*a*sinh(x))/a^2-1/8*cosh(x)
*(8*b*(a^2+b^2)-a*(3*a^2+4*b^2)*sinh(x))/a^4
```

Rubi [A]
time = 0.27, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3957, 2944, 2814, 2739, 632, 210}

$$-\frac{\cosh^3(x)(4b-3a\sinh(x))}{12a^2}+\frac{2b(a^2+b^2)^{3/2}\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^5}-\frac{\cosh(x)\,(8b(a^2+b^2)-a(3a^2+4b^2)\sinh(x))}{8a^4}+\frac{x(3a^4+12a^2b^2+8b^4)}{8a^5}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^4/(a + b*Csch[x]),x]
```

```
[Out] ((3*a^4 + 12*a^2*b^2 + 8*b^4)*x)/(8*a^5) + (2*b*(a^2 + b^2)^(3/2)*ArcTanh[(
a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/a^5 - (Cosh[x]^3*(4*b - 3*a*Sinh[x]))/(1
2*a^2) - (Cosh[x]*(8*b*(a^2 + b^2) - a*(3*a^2 + 4*b^2)*Sinh[x]))/(8*a^4)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^4(x)}{a+b\text{csch}(x)}\,dx &= i\int \frac{\cosh^4(x)\sinh(x)}{ib+ia\sinh(x)}\,dx \\
&= -\frac{\cosh^3(x)(4b-3a\sinh(x))}{12a^2}+\frac{\int \frac{\cosh^2(x)\left(-iab+i\left(3a^2+4b^2\right)\sinh(x)\right)}{ib+ia\sinh(x)}\,dx}{4a^2} \\
&= -\frac{\cosh^3(x)(4b-3a\sinh(x))}{12a^2}-\frac{\cosh(x)\left(8b\left(a^2+b^2\right)-a\left(3a^2+4b^2\right)\sinh(x)\right)}{8a^4}+\frac{\int \frac{-iab(}{}}{} \\
&= \frac{\left(3a^4+12a^2b^2+8b^4\right)x}{8a^5}-\frac{\cosh^3(x)(4b-3a\sinh(x))}{12a^2}-\frac{\cosh(x)\left(8b\left(a^2+b^2\right)-a\left(3a^2+\right.\right.}{8a^4} \\
&= \frac{\left(3a^4+12a^2b^2+8b^4\right)x}{8a^5}-\frac{\cosh^3(x)(4b-3a\sinh(x))}{12a^2}-\frac{\cosh(x)\left(8b\left(a^2+b^2\right)-a\left(3a^2+\right.\right.}{8a^4} \\
&= \frac{\left(3a^4+12a^2b^2+8b^4\right)x}{8a^5}-\frac{\cosh^3(x)(4b-3a\sinh(x))}{12a^2}-\frac{\cosh(x)\left(8b\left(a^2+b^2\right)-a\left(3a^2+\right.\right.}{8a^4} \\
&= \frac{\left(3a^4+12a^2b^2+8b^4\right)x}{8a^5}+\frac{2b\left(a^2+b^2\right)^{3/2}\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^5}-\frac{\cosh^3(x)(4b-3a\sin}{12a^2}
\end{aligned}$$

Mathematica [A]
time = 0.82, size = 180, normalized size = 1.44

$$\frac{-24ab(5a^2+4b^2)\cosh(x)-8a^3b\cosh(3x)+3\left(12a^4x+48a^2b^2x+32b^4x+64a^2b\sqrt{-a^2-b^2}\,\text{ArcTan}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)+64b^3\sqrt{-a^2-b^2}\,\text{ArcTan}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)+8a^2(a^2+b^2)\sinh(2x)+a^4\sinh(4x)\right)}{96a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^4/(a + b*Csch[x]),x]
```

```
[Out] (-24*a*b*(5*a^2 + 4*b^2)*Cosh[x] - 8*a^3*b*Cosh[3*x] + 3*(12*a^4*x + 48*a^2
*b^2*x + 32*b^4*x + 64*a^2*b*Sqrt[-a^2 - b^2]*ArcTan[(a - b*Tanh[x/2])/Sqrt
[-a^2 - b^2]] + 64*b^3*Sqrt[-a^2 - b^2]*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2
- b^2]] + 8*a^2*(a^2 + b^2)*Sinh[2*x] + a^4*Sinh[4*x]))/(96*a^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(113) = 226.
time = 0.64, size = 311, normalized size = 2.49

method	result
risch	$\frac{3x}{8a}+\frac{3x\,b^2}{2a^3}+\frac{x\,b^4}{a^5}+\frac{e^{4x}}{64a}-\frac{b\,e^{3x}}{24a^2}+\frac{e^{2x}}{8a}+\frac{e^{2x}b^2}{8a^3}-\frac{5b\,e^{x}}{8a^2}-\frac{b^3e^{x}}{2a^4}-\frac{5b\,e^{-x}}{8a^2}-\frac{b^3e^{-x}}{2a^4}-\frac{e^{-2x}}{8a}-\frac{e^{-2x}b^2}{8a^3}-\frac{b\,e^{-3x}}{24a^2}-$
default	$-\frac{1}{4a\left(\tanh\left(\frac{x}{2}\right)+1\right)^4}-\frac{-3a+2b}{6a^2\left(\tanh\left(\frac{x}{2}\right)+1\right)^3}-\frac{7a^2-4ab+4b^2}{8a^3\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}+\frac{\left(3a^4+12a^2b^2+8b^4\right)\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{8a^5}-\frac{-5a^3+12a^2b-4a\,b^2+}{8a^4\left(\tanh\left(\frac{x}{2}\right)+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a/(tanh(1/2*x)+1)^4-1/6*(-3*a+2*b)/a^2/(tanh(1/2*x)+1)^3-1/8*(7*a^2-4*
a*b+4*b^2)/a^3/(tanh(1/2*x)+1)^2+1/8*(3*a^4+12*a^2*b^2+8*b^4)/a^5*ln(tanh(1
/2*x)+1)-1/8*(-5*a^3+12*a^2*b-4*a*b^2+8*b^3)/a^4/(tanh(1/2*x)+1)+1/4/a/(tan
h(1/2*x)-1)^4-1/6*(-3*a-2*b)/a^2/(tanh(1/2*x)-1)^3-1/8*(-7*a^2-4*a*b-4*b^2)
/a^3/(tanh(1/2*x)-1)^2+1/8/a^5*(-3*a^4-12*a^2*b^2-8*b^4)*ln(tanh(1/2*x)-1)-
1/8*(-5*a^3-12*a^2*b-4*a*b^2-8*b^3)/a^4/(tanh(1/2*x)-1)+2*b*(a^4+2*a^2*b^2+
b^4)/a^5/(a^2+b^2)^(1/2)*arctanh(1/2*(-2*b*tanh(1/2*x)+2*a)/(a^2+b^2)^(1/2)
)
```

Maxima [A]
time = 0.49, size = 217, normalized size = 1.74

$$-\frac{\left(8\,a^2be^{(-x)}-3\,a^3-24\,(a^3+ab^2)e^{(-2\,x)}+24\,(5\,a^2b+4\,b^3)e^{(-3\,x)}\right)e^{(4\,x)}}{192\,a^4}-\frac{8\,a^2be^{(-3\,x)}+3\,a^3e^{(-4\,x)}+24\,(5\,a^2b+4\,b^3)e^{(-x)}+24\,(a^3+ab^2)e^{(-2\,x)}}{192\,a^4}+\frac{(3\,a^4+12\,a^2b^2+8\,b^4)x}{8\,a^5}-\frac{(a^4b+2\,a^2b^3+b^5)\log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -1/192*(8*a^2*b*e^(-x) - 3*a^3 - 24*(a^3 + a*b^2)*e^(-2*x) + 24*(5*a^2*b +
4*b^3)*e^(-3*x))*e^(4*x)/a^4 - 1/192*(8*a^2*b*e^(-3*x) + 3*a^3*e^(-4*x) + 2
4*(5*a^2*b + 4*b^3)*e^(-x) + 24*(a^3 + a*b^2)*e^(-2*x))/a^4 + 1/8*(3*a^4 +
12*a^2*b^2 + 8*b^4)*x/a^5 - (a^4*b + 2*a^2*b^3 + b^5)*log((a*e^(-x) - b - s
qrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 924 vs. $2(114) = 228$.
time = 0.42, size = 924, normalized size = 7.39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] 1/192*(3*a^4*cosh(x)^8 + 3*a^4*sinh(x)^8 - 8*a^3*b*cosh(x)^7 + 8*(3*a^4*cos
h(x) - a^3*b)*sinh(x)^7 + 24*(a^4 + a^2*b^2)*cosh(x)^6 + 4*(21*a^4*cosh(x)^
2 - 14*a^3*b*cosh(x) + 6*a^4 + 6*a^2*b^2)*sinh(x)^6 + 24*(3*a^4 + 12*a^2*b^
2 + 8*b^4)*x*cosh(x)^4 - 24*(5*a^3*b + 4*a*b^3)*cosh(x)^5 + 24*(7*a^4*cosh(
x)^3 - 7*a^3*b*cosh(x)^2 - 5*a^3*b - 4*a*b^3 + 6*(a^4 + a^2*b^2)*cosh(x))*s
inh(x)^5 - 8*a^3*b*cosh(x) + 2*(105*a^4*cosh(x)^4 - 140*a^3*b*cosh(x)^3 + 1
80*(a^4 + a^2*b^2)*cosh(x)^2 + 12*(3*a^4 + 12*a^2*b^2 + 8*b^4)*x - 60*(5*a^
3*b + 4*a*b^3)*cosh(x))*sinh(x)^4 - 3*a^4 - 24*(5*a^3*b + 4*a*b^3)*cosh(x)^
3 + 8*(21*a^4*cosh(x)^5 - 35*a^3*b*cosh(x)^4 - 15*a^3*b - 12*a*b^3 + 60*(a^
4 + a^2*b^2)*cosh(x)^3 + 12*(3*a^4 + 12*a^2*b^2 + 8*b^4)*x*cosh(x) - 30*(5*
a^3*b + 4*a*b^3)*cosh(x)^2)*sinh(x)^3 - 24*(a^4 + a^2*b^2)*cosh(x)^2 + 12*(
7*a^4*cosh(x)^6 - 14*a^3*b*cosh(x)^5 + 30*(a^4 + a^2*b^2)*cosh(x)^4 - 2*a^4
 - 2*a^2*b^2 + 12*(3*a^4 + 12*a^2*b^2 + 8*b^4)*x*cosh(x)^2 - 20*(5*a^3*b +
4*a*b^3)*cosh(x)^3 - 6*(5*a^3*b + 4*a*b^3)*cosh(x))*sinh(x)^2 + 192*((a^2*b
 + b^3)*cosh(x)^4 + 4*(a^2*b + b^3)*cosh(x)^3*sinh(x) + 6*(a^2*b + b^3)*cos
h(x)^2*sinh(x)^2 + 4*(a^2*b + b^3)*cosh(x)*sinh(x)^3 + (a^2*b + b^3)*sinh(x
)^4)*sqrt(a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) + a
^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(a*cosh(x) +
 a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) +
b)*sinh(x) - a)) + 8*(3*a^4*cosh(x)^7 - 7*a^3*b*cosh(x)^6 + 18*(a^4 + a^2*b
^2)*cosh(x)^5 + 12*(3*a^4 + 12*a^2*b^2 + 8*b^4)*x*cosh(x)^3 - 15*(5*a^3*b +
 4*a*b^3)*cosh(x)^4 - a^3*b - 9*(5*a^3*b + 4*a*b^3)*cosh(x)^2 - 6*(a^4 + a^
2*b^2)*cosh(x))*sinh(x))/(a^5*cosh(x)^4 + 4*a^5*cosh(x)^3*sinh(x) + 6*a^5*c
osh(x)^2*sinh(x)^2 + 4*a^5*cosh(x)*sinh(x)^3 + a^5*sinh(x)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^4(x)}{a + b\,\text{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**4/(a+b*csch(x)),x)
```

```
[Out] Integral(cosh(x)**4/(a + b*csch(x)), x)
```

Giac [A]
time = 0.40, size = 221, normalized size = 1.77

$$\frac{3\,a^3e^{(4\,x)} - 8\,a^2be^{(3\,x)} + 24\,a^3e^{(2\,x)} + 24\,ab^2e^{(2\,x)} - 120\,a^2be^x - 96\,b^3e^x}{192\,a^4} + \frac{(3\,a^4 + 12\,a^2b^2 + 8\,b^4)x}{8\,a^5} - \frac{(8\,a^3be^x + 3\,a^4 + 24\,(5\,a^3b + 4\,ab^3)e^{(3\,x)} + 24\,(a^4 + a^2b^2)e^{(2\,x)})e^{(-4\,x)}}{192\,a^5} - \frac{(a^4b + 2\,a^2b^3 + b^5)\log\left(\frac{|2\,ae^x + 2\,b - 2\,\sqrt{a^2 + b^2}|}{|2\,ae^x + 2\,b + 2\,\sqrt{a^2 + b^2}|}\right)}{\sqrt{a^2 + b^2}\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] 1/192*(3*a^3*e^(4*x) - 8*a^2*b*e^(3*x) + 24*a^3*e^(2*x) + 24*a*b^2*e^(2*x)
- 120*a^2*b*e^x - 96*b^3*e^x)/a^4 + 1/8*(3*a^4 + 12*a^2*b^2 + 8*b^4)*x/a^5
- 1/192*(8*a^3*b*e^x + 3*a^4 + 24*(5*a^3*b + 4*a*b^3)*e^(3*x) + 24*(a^4 + a
^2*b^2)*e^(2*x))*e^(-4*x)/a^5 - (a^4*b + 2*a^2*b^3 + b^5)*log(abs(2*a*e^x +
 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2
 + b^2)*a^5)
```

Mupad [B]
time = 2.02, size = 247, normalized size = 1.98

$$\frac{e^{4\,x}}{64\,a} - \frac{e^{-4\,x}}{64\,a} + \frac{x\,(3\,a^4 + 12\,a^2\,b^2 + 8\,b^4)}{8\,a^5} - \frac{e^{-2\,x}\,(a^2 + b^2)}{8\,a^3} + \frac{e^{2\,x}\,(a^2 + b^2)}{8\,a^3} - \frac{e^{-x}\,(5\,a^2\,b + 4\,b^3)}{8\,a^4} - \frac{b\,e^{-3\,x}}{24\,a^2} - \frac{b\,e^{3\,x}}{24\,a^2} - \frac{e^x\,(5\,a^2\,b + 4\,b^3)}{8\,a^4} - \frac{b\,\ln\left(\frac{2\,b\,e^x\,(a^2+b^2)^2}{a^6} - \frac{2\,b\,(a-b\,e^x)\,(a^2+b^2)^{3/2}}{a^6}\right)\,(a^2 + b^2)^{3/2}}{a^5} + \frac{b\,\ln\left(\frac{2\,b\,(a-b\,e^x)\,(a^2+b^2)^{3/2}}{a^6} + \frac{2\,b\,e^x\,(a^2+b^2)^2}{a^6}\right)\,(a^2 + b^2)^{3/2}}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a + b/sinh(x)),x)
```

```
[Out] exp(4*x)/(64*a) - exp(-4*x)/(64*a) + (x*(3*a^4 + 8*b^4 + 12*a^2*b^2))/(8*a^
5) - (exp(-2*x)*(a^2 + b^2))/(8*a^3) + (exp(2*x)*(a^2 + b^2))/(8*a^3) - (ex
p(-x)*(5*a^2*b + 4*b^3))/(8*a^4) - (b*exp(-3*x))/(24*a^2) - (b*exp(3*x))/(2
4*a^2) - (exp(x)*(5*a^2*b + 4*b^3))/(8*a^4) - (b*log((2*b*exp(x)*(a^2 + b^2
)^2)/a^6 - (2*b*(a - b*exp(x))*(a^2 + b^2)^(3/2))/a^6)*(a^2 + b^2)^(3/2))/a
^5 + (b*log((2*b*(a - b*exp(x))*(a^2 + b^2)^(3/2))/a^6 + (2*b*exp(x)*(a^2 +
 b^2)^2)/a^6)*(a^2 + b^2)^(3/2))/a^5
```

3.94 $\int \frac{\cosh^3(x)}{a+b\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=57

$$-\frac{b(a^2+b^2)\log(b+a\sinh(x))}{a^4}+\frac{(a^2+b^2)\sinh(x)}{a^3}-\frac{b\sinh^2(x)}{2a^2}+\frac{\sinh^3(x)}{3a}$$

```
[Out] -b*(a^2+b^2)*ln(b+a*sinh(x))/a^4+(a^2+b^2)*sinh(x)/a^3-1/2*b*sinh(x)^2/a^2+
1/3*sinh(x)^3/a
```

Rubi [A]

time = 0.11, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3957, 2916, 12, 786}

$$-\frac{b\sinh^2(x)}{2a^2}-\frac{b(a^2+b^2)\log(a\sinh(x)+b)}{a^4}+\frac{(a^2+b^2)\sinh(x)}{a^3}+\frac{\sinh^3(x)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(a + b*Csch[x]),x]
```

```
[Out] -((b*(a^2 + b^2)*Log[b + a*Sinh[x]])/a^4) + ((a^2 + b^2)*Sinh[x])/a^3 - (b*
Sinh[x]^2)/(2*a^2) + Sinh[x]^3/(3*a)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
```

```
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^3(x)}{a+b\text{csch}(x)}\,dx &= i\int \frac{\cosh^3(x)\sinh(x)}{ib+ia\sinh(x)}\,dx \\
&= -\frac{i\text{Subst}\left(\int \frac{x(a^2-x^2)}{a(ib+x)}\,dx, x, ia\sinh(x)\right)}{a^3} \\
&= -\frac{i\text{Subst}\left(\int \frac{x(a^2-x^2)}{ib+x}\,dx, x, ia\sinh(x)\right)}{a^4} \\
&= -\frac{i\text{Subst}\left(\int \left(a^2\left(1+\frac{b^2}{a^2}\right)-\frac{b(a^2+b^2)}{b-ix}+ibx-x^2\right)\,dx, x, ia\sinh(x)\right)}{a^4} \\
&= -\frac{b(a^2+b^2)\log(b+a\sinh(x))}{a^4}+\frac{(a^2+b^2)\sinh(x)}{a^3}-\frac{b\sinh^2(x)}{2a^2}+\frac{\sinh^3(x)}{3a}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 67, normalized size = 1.18

$$\frac{-3a^2b\cosh(2x)-12a^2b\log(b+a\sinh(x))-12b^3\log(b+a\sinh(x))+3a(3a^2+4b^2)\sinh(x)+a^3\sinh(3x)}{12a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(a + b*Csch[x]),x]
```

```
[Out] (-3*a^2*b*Cosh[2*x] - 12*a^2*b*Log[b + a*Sinh[x]] - 12*b^3*Log[b + a*Sinh[x
]] + 3*a*(3*a^2 + 4*b^2)*Sinh[x] + a^3*Sinh[3*x])/(12*a^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(53) = 106.
time = 0.68, size = 191, normalized size = 3.35

method	result
risch	$\frac{bx}{a^2}+\frac{b^3x}{a^4}+\frac{e^{3x}}{24a}-\frac{b\,e^{2x}}{8a^2}+\frac{3\,e^{x}}{8a}+\frac{e^{x}b^2}{2a^3}-\frac{3\,e^{-x}}{8a}-\frac{e^{-x}b^2}{2a^3}-\frac{b\,e^{-2x}}{8a^2}-\frac{e^{-3x}}{24a}-\frac{b\ln\left(e^{2x}+\frac{2b\,e^{x}}{a}-1\right)}{a^2}-\frac{b^3\ln\left(e^{2x}+\frac{2b\,e^{x}}{a}\right.}{a^4}$
default	$\frac{2b\left(-\frac{a^2}{2}-\frac{b^2}{2}\right)\ln\left(-b\left(\tanh^2\left(\frac{x}{2}\right)\right)+2a\tanh\left(\frac{x}{2}\right)+b\right)}{a^4}-\frac{1}{3a\left(\tanh\left(\frac{x}{2}\right)-1\right)^3}-\frac{a+b}{2a^2\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}-\frac{2a^2+ab+2b^2}{2a^3\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{b(a^2+b^2)\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/a^4*b*(-1/2*a^2-1/2*b^2)*ln(-b*tanh(1/2*x)^2+2*a*tanh(1/2*x)+b)-1/3/a/(ta
nh(1/2*x)-1)^3-1/2*(a+b)/a^2/(tanh(1/2*x)-1)^2-1/2*(2*a^2+a*b+2*b^2)/a^3/(t
anh(1/2*x)-1)+1/a^4*b*(a^2+b^2)*ln(tanh(1/2*x)-1)-1/3/a/(tanh(1/2*x)+1)^3-1
/2*(-a+b)/a^2/(tanh(1/2*x)+1)^2-1/2*(2*a^2-a*b+2*b^2)/a^3/(tanh(1/2*x)+1)+1
/a^4*b*(a^2+b^2)*ln(tanh(1/2*x)+1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(53) = 106$.
time = 0.27, size = 127, normalized size = 2.23

$$-\frac{\left(3\,abe^{(-x)} - a^2 - 3\,(3\,a^2 + 4\,b^2)e^{(-2\,x)}\right)e^{(3\,x)}}{24\,a^3} - \frac{3\,abe^{(-2\,x)} + a^2e^{(-3\,x)} + 3\,(3\,a^2 + 4\,b^2)e^{(-x)}}{24\,a^3} - \frac{(a^2b + b^3)x}{a^4} - \frac{(a^2b + b^3)\log\left(-2\,be^{(-x)} + ae^{(-2\,x)} - a\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -1/24*(3*a*b*e^(-x) - a^2 - 3*(3*a^2 + 4*b^2)*e^(-2*x))*e^(3*x)/a^3 - 1/24*
(3*a*b*e^(-2*x) + a^2*e^(-3*x) + 3*(3*a^2 + 4*b^2)*e^(-x))/a^3 - (a^2*b + b
^3)*x/a^4 - (a^2*b + b^3)*log(-2*b*e^(-x) + a*e^(-2*x) - a)/a^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 476 vs. $2(53) = 106$.
time = 0.38, size = 476, normalized size = 8.35

$$\frac{a^3\cosh(x)^6 + a^3\sinh(x)^6 - 3\,a^2b\cosh(x)^5 + 3\,(2\,a^3\cosh(x) - a^2b)\sinh(x)^5 + 24\,(a^2b + b^3)x\cosh(x)^3 + 3\,(3\,a^3 + 4\,ab^2)\cosh(x)^4 + 3\,\left(5\,a^3\cosh(x)^2 - 5\,a^2b\cosh(x) + 3\,a^3 + 4\,ab^2\right)\sinh(x)^4 - 3\,a^2b\cosh(x) + 2\,\left(10\,a^3\cosh(x)^3 - 15\,a^2b\cosh(x)^2 + 12\,(a^2b + b^3)x + 6\,(3\,a^3 + 4\,ab^2)\cosh(x)\right)\sinh(x)^3 - a^3 - 3\,(3\,a^3 + 4\,ab^2)\cosh(x)^2 + 3\,\left(5\,a^3\cosh(x)^4 - 10\,a^2b\cosh(x)^3 - 3\,a^3 - 4\,ab^2 + 24\,(a^2b + b^3)x\cosh(x) + 6\,(3\,a^3 + 4\,ab^2)\cosh(x)^2\right)\sinh(x)^2 - 24\,\left((a^2b + b^3)\cosh(x)^3 + 3\,(a^2b + b^3)\cosh(x)^2\sinh(x) + 3\,(a^2b + b^3)\cosh(x)\sinh(x)^2 + (a^2b + b^3)\sinh(x)^3\right)\log\left(\frac{2\,(a\sinh(x) + b)}{\cosh(x) - \sinh(x)}\right) + 3\,\left(2\,a^3\cosh(x)^5 - 5\,a^2b\cosh(x)^4 + 24\,(a^2b + b^3)x\cosh(x)^2 + 4\,(3\,a^3 + 4\,ab^2)\cosh(x)^3 - a^2b - 2\,(3\,a^3 + 4\,ab^2)\cosh(x)\right)\sinh(x)}{24\,\left(a^4\cosh(x)^3 + 3\,a^4\cosh(x)^2\sinh(x) + 3\,a^4\cosh(x)\sinh(x)^2 + a^4\sinh(x)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] 1/24*(a^3*cosh(x)^6 + a^3*sinh(x)^6 - 3*a^2*b*cosh(x)^5 + 3*(2*a^3*cosh(x)
- a^2*b)*sinh(x)^5 + 24*(a^2*b + b^3)*x*cosh(x)^3 + 3*(3*a^3 + 4*a*b^2)*cos
h(x)^4 + 3*(5*a^3*cosh(x)^2 - 5*a^2*b*cosh(x) + 3*a^3 + 4*a*b^2)*sinh(x)^4
- 3*a^2*b*cosh(x) + 2*(10*a^3*cosh(x)^3 - 15*a^2*b*cosh(x)^2 + 12*(a^2*b +
b^3)*x + 6*(3*a^3 + 4*a*b^2)*cosh(x))*sinh(x)^3 - a^3 - 3*(3*a^3 + 4*a*b^2)
*cosh(x)^2 + 3*(5*a^3*cosh(x)^4 - 10*a^2*b*cosh(x)^3 - 3*a^3 - 4*a*b^2 + 24
*(a^2*b + b^3)*x*cosh(x) + 6*(3*a^3 + 4*a*b^2)*cosh(x)^2)*sinh(x)^2 - 24*((
a^2*b + b^3)*cosh(x)^3 + 3*(a^2*b + b^3)*cosh(x)^2*sinh(x) + 3*(a^2*b + b^3
)*cosh(x)*sinh(x)^2 + (a^2*b + b^3)*sinh(x)^3)*log(2*(a*sinh(x) + b)/(cosh(
x) - sinh(x))) + 3*(2*a^3*cosh(x)^5 - 5*a^2*b*cosh(x)^4 + 24*(a^2*b + b^3)*
x*cosh(x)^2 + 4*(3*a^3 + 4*a*b^2)*cosh(x)^3 - a^2*b - 2*(3*a^3 + 4*a*b^2)*c
osh(x))*sinh(x))/(a^4*cosh(x)^3 + 3*a^4*cosh(x)^2*sinh(x) + 3*a^4*cosh(x)*s
inh(x)^2 + a^4*sinh(x)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3(x)}{a + b\operatorname{csch}(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(a+b*csch(x)),x)
```

```
[Out] Integral(cosh(x)**3/(a + b*csch(x)), x)
```

Giac [A]
time = 0.39, size = 97, normalized size = 1.70

$$-\frac{a^2\left(e^{(-x)}-e^x\right)^3+3\,ab\left(e^{(-x)}-e^x\right)^2+12\,a^2\left(e^{(-x)}-e^x\right)+12\,b^2\left(e^{(-x)}-e^x\right)}{24\,a^3}-\frac{\left(a^2b+b^3\right)\log\left(\left|-a\left(e^{(-x)}-e^x\right)+2\,b\right|\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -1/24*(a^2*(e^(-x) - e^x)^3 + 3*a*b*(e^(-x) - e^x)^2 + 12*a^2*(e^(-x) - e^x
) + 12*b^2*(e^(-x) - e^x))/a^3 - (a^2*b + b^3)*log(abs(-a*(e^(-x) - e^x) +
2*b))/a^4
```

Mupad [B]
time = 1.68, size = 121, normalized size = 2.12

$$\frac{e^{3\,x}}{24\,a}-\frac{e^{-3\,x}}{24\,a}+\frac{x\,\left(a^2\,b+b^3\right)}{a^4}+\frac{e^x\,\left(3\,a^2+4\,b^2\right)}{8\,a^3}-\frac{b\,e^{-2\,x}}{8\,a^2}-\frac{b\,e^{2\,x}}{8\,a^2}-\frac{\ln\left(2\,b\,e^x-a+a\,e^{2\,x}\right)\,\left(a^2\,b+b^3\right)}{a^4}-\frac{e^{-x}\,\left(3\,a^2+4\,b^2\right)}{8\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a + b/sinh(x)),x)
```

```
[Out] exp(3*x)/(24*a) - exp(-3*x)/(24*a) + (x*(a^2*b + b^3))/a^4 + (exp(x)*(3*a^2
 + 4*b^2))/(8*a^3) - (b*exp(-2*x))/(8*a^2) - (b*exp(2*x))/(8*a^2) - (log(2*
b*exp(x) - a + a*exp(2*x))*(a^2*b + b^3))/a^4 - (exp(-x)*(3*a^2 + 4*b^2))/(
8*a^3)
```

3.95 $\int \frac{\cosh^2(x)}{a+b\text{csch}(x)}\,dx$

Optimal. Leaf size=77

$$\frac{(a^2+2b^2)\,x}{2a^3}+\frac{2b\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3}-\frac{\cosh(x)(2b-a\sinh(x))}{2a^2}$$

[Out] 1/2*(a^2+2*b^2)*x/a^3-1/2*cosh(x)*(2*b-a*sinh(x))/a^2+2*b*arctanh((a-b*tanh(1/2*x))/(a^2+b^2)^(1/2))*(a^2+b^2)^(1/2)/a^3

Rubi [A]
time = 0.14, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3957, 2944, 2814, 2739, 632, 210}

$$-\frac{\cosh(x)(2b-a\sinh(x))}{2a^2}+\frac{x(a^2+2b^2)}{2a^3}+\frac{2b\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3}$$

Antiderivative was successfully verified.

[In] Int[Cosh[x]^2/(a + b*Csch[x]),x]

[Out] ((a^2 + 2*b^2)*x)/(2*a^3) + (2*b*Sqrt[a^2 + b^2]*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/a^3 - (Cosh[x]*(2*b - a*Sinh[x]))/(2*a^2)

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^2(x)}{a+b\operatorname{csch}(x)}\,dx &= i\int \frac{\cosh^2(x)\sinh(x)}{ib+ia\sinh(x)}\,dx\\
&= -\frac{\cosh(x)(2b-a\sinh(x))}{2a^2}+\frac{\int \frac{-iab+i(a^2+2b^2)\sinh(x)}{ib+ia\sinh(x)}\,dx}{2a^2}\\
&= \frac{(a^2+2b^2)\,x}{2a^3}-\frac{\cosh(x)(2b-a\sinh(x))}{2a^2}-\frac{(ib(a^2+b^2))\int \frac{1}{ib+ia\sinh(x)}\,dx}{a^3}\\
&= \frac{(a^2+2b^2)\,x}{2a^3}-\frac{\cosh(x)(2b-a\sinh(x))}{2a^2}-\frac{(2ib(a^2+b^2))\operatorname{Subst}\left(\int \frac{1}{ib+2iax-ibx^2}\,dx,x,\tanh\right.}{a^3}\\
&= \frac{(a^2+2b^2)\,x}{2a^3}-\frac{\cosh(x)(2b-a\sinh(x))}{2a^2}+\frac{(4ib(a^2+b^2))\operatorname{Subst}\left(\int \frac{1}{-4(a^2+b^2)-x^2}\,dx,x,2ia\right.}{a^3}\\
&= \frac{(a^2+2b^2)\,x}{2a^3}+\frac{2b\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3}-\frac{\cosh(x)(2b-a\sinh(x))}{2a^2}
\end{aligned}
$$

Mathematica [**A**]

time = 0.15, size = 80, normalized size = 1.04

$$\frac{2a^2x+4b^2x+8b\sqrt{-a^2-b^2}\,\mathrm{ArcTan}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)-4ab\cosh(x)+a^2\sinh(2x)}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(a + b*Csch[x]),x]
```

```
[Out] (2*a^2*x + 4*b^2*x + 8*b*Sqrt[-a^2 - b^2]*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^
2 - b^2]] - 4*a*b*Cosh[x] + a^2*Sinh[2*x])/(4*a^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(67) = 134$.
time = 0.70, size = 150, normalized size = 1.95

method	result
risch	$\frac{x}{2a}+\frac{x\,b^2}{a^3}+\frac{\mathrm{e}^{2x}}{8a}-\frac{b\,\mathrm{e}^{x}}{2a^2}-\frac{b\,\mathrm{e}^{-x}}{2a^2}-\frac{\mathrm{e}^{-2x}}{8a}+\frac{\sqrt{a^2+b^2}\,b\ln\left(\mathrm{e}^{x}+\frac{b+\sqrt{a^2+b^2}}{a}\right)}{a^3}-\frac{\sqrt{a^2+b^2}\,b\ln\left(\mathrm{e}^{x}-\frac{-b+\sqrt{a^2}}{a}\right.}{a^3}$
default	$-\frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}-\frac{2b-a}{2a^2\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{(a^2+2b^2)\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2a^3}+\frac{2b\sqrt{a^2+b^2}\,\operatorname{arctanh}\left(\frac{-2b\tanh\left(\frac{x}{2}\right)+2a}{2\sqrt{a^2+b^2}}\right)}{a^3}+\frac{}{2a(\mathrm{ta}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a/(tanh(1/2*x)+1)^2-1/2*(2*b-a)/a^2/(tanh(1/2*x)+1)+1/2*(a^2+2*b^2)/a^
3*ln(tanh(1/2*x)+1)+2*b*(a^2+b^2)^(1/2)/a^3*arctanh(1/2*(-2*b*tanh(1/2*x)+2
*a)/(a^2+b^2)^(1/2))+1/2/a/(tanh(1/2*x)-1)^2-1/2*(-2*b-a)/a^2/(tanh(1/2*x)-
1)+1/2/a^3*(-a^2-2*b^2)*ln(tanh(1/2*x)-1)
```

Maxima [A]
time = 0.47, size = 122, normalized size = 1.58

$$-\frac{\left(4\,be^{(-x)}-a\right)e^{(2\,x)}}{8\,a^2}-\frac{4\,be^{(-x)}+ae^{(-2\,x)}}{8\,a^2}+\frac{(a^2+2\,b^2)x}{2\,a^3}-\frac{(a^2b+b^3)\log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*b*e^(-x) - a)*e^(2*x)/a^2 - 1/8*(4*b*e^(-x) + a*e^(-2*x))/a^2 + 1/2
*(a^2 + 2*b^2)*x/a^3 - (a^2*b + b^3)*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(
a*e^(-x) - b + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(68) = 136$.
time = 0.37, size = 304, normalized size = 3.95

$$\frac{a^2\cosh(x)^4 + a^2\sinh(x)^4 - 4\,ab\cosh(x)^3 + 4\,(a^2 + 2\,b^2)x\cosh(x)^2 + 4\,(a^2\cosh(x) - ab)\sinh(x)^3 - 4\,ab\cosh(x) + 2\,\left(3\,a^2\cosh(x)^2 - 6\,ab\cosh(x) + 2\,(a^2 + 2\,b^2)x\right)\sinh(x)^2 + 8\,\left(b\cosh(x)^2 + 2\,b\cosh(x)\sinh(x) + b\sinh(x)^2\right)\sqrt{a^2 + b^2}\log\left(\frac{a^2\cosh(x)^2 + a^2\sinh(x)^2 + 2\,ab\cosh(x) + a^2 + 2\,b^2 + 2\,(a^2\cosh(x) + ab)\sinh(x) + 2\,\sqrt{a^2 + b^2}\,(a\cosh(x) + a\sinh(x) + b)}{a\cosh(x)^2 + a\sinh(x)^2 + 2\,b\cosh(x) + 2\,(a\cosh(x) + b)\sinh(x) - a}\right) - a^2 + 4\,\left(a^2\cosh(x)^3 - 3\,ab\cosh(x)^2 + 2\,(a^2 + 2\,b^2)x\cosh(x) - ab\right)\sinh(x)}{8\,\left(a^3\cosh(x)^2 + 2\,a^3\cosh(x)\sinh(x) + a^3\sinh(x)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] 1/8*(a^2*cosh(x)^4 + a^2*sinh(x)^4 - 4*a*b*cosh(x)^3 + 4*(a^2 + 2*b^2)*x*co
sh(x)^2 + 4*(a^2*cosh(x) - a*b)*sinh(x)^3 - 4*a*b*cosh(x) + 2*(3*a^2*cosh(x
)^2 - 6*a*b*cosh(x) + 2*(a^2 + 2*b^2)*x)*sinh(x)^2 + 8*(b*cosh(x)^2 + 2*b*c
osh(x)*sinh(x) + b*sinh(x)^2)*sqrt(a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh
(x)^2 + 2*a*b*cosh(x) + a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqr
t(a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*
cosh(x) + 2*(a*cosh(x) + b)*sinh(x) - a)) - a^2 + 4*(a^2*cosh(x)^3 - 3*a*b*
cosh(x)^2 + 2*(a^2 + 2*b^2)*x*cosh(x) - a*b)*sinh(x))/(a^3*cosh(x)^2 + 2*a^
3*cosh(x)*sinh(x) + a^3*sinh(x)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(x)}{a + b\,\mathrm{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(a+b*csch(x)),x)
```

```
[Out] Integral(cosh(x)**2/(a + b*csch(x)), x)
```

Giac [A]
time = 0.39, size = 121, normalized size = 1.57

$$\frac{ae^{(2\,x)} - 4\,be^x}{8\,a^2} + \frac{(a^2 + 2\,b^2)x}{2\,a^3} - \frac{(4\,abe^x + a^2)e^{(-2\,x)}}{8\,a^3} - \frac{(a^2b + b^3)\log\left(\frac{\left|2\,ae^x + 2\,b - 2\,\sqrt{a^2 + b^2}\right|}{\left|2\,ae^x + 2\,b + 2\,\sqrt{a^2 + b^2}\right|}\right)}{\sqrt{a^2 + b^2}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] 1/8*(a*e^(2*x) - 4*b*e^x)/a^2 + 1/2*(a^2 + 2*b^2)*x/a^3 - 1/8*(4*a*b*e^x +
a^2)*e^(-2*x)/a^3 - (a^2*b + b^3)*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2)
)/abs(2*a*e^x + 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3)
```

Mupad [B]
time = 1.69, size = 159, normalized size = 2.06

$$\frac{e^{2x}}{8a}-\frac{e^{-2x}}{8a}-\frac{b\,e^{x}}{2a^2}-\frac{b\,e^{-x}}{2a^2}+\frac{x\,(a^2+2\,b^2)}{2\,a^3}-\frac{b\ln\left(\frac{2\,b\,e^{x}\,(a^2+b^2)}{a^4}-\frac{2\,b\,(a-b\,e^{x})\,\sqrt{a^2+b^2}}{a^4}\right)\sqrt{a^2+b^2}}{a^3}+\frac{b\ln\left(\frac{2\,b\,(a-b\,e^{x})\,\sqrt{a^2+b^2}}{a^4}+\frac{2\,b\,e^{x}\,(a^2+b^2)}{a^4}\right)\sqrt{a^2+b^2}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a + b/sinh(x)),x)
```

```
[Out] exp(2*x)/(8*a) - exp(-2*x)/(8*a) - (b*exp(x))/(2*a^2) - (b*exp(-x))/(2*a^2)
 + (x*(a^2 + 2*b^2))/(2*a^3) - (b*log((2*b*exp(x)*(a^2 + b^2))/a^4 - (2*b*(
a - b*exp(x))*(a^2 + b^2)^(1/2))/a^4)*(a^2 + b^2)^(1/2))/a^3 + (b*log((2*b*
(a - b*exp(x))*(a^2 + b^2)^(1/2))/a^4 + (2*b*exp(x)*(a^2 + b^2))/a^4)*(a^2
+ b^2)^(1/2))/a^3
```

3.96 $\int \frac{\cosh(x)}{a+b\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=20

$$-\frac{b\log(b+a\sinh(x))}{a^2}+\frac{\sinh(x)}{a}$$

[Out] -b*ln(b+a*sinh(x))/a^2+sinh(x)/a

Rubi [A]
time = 0.05, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3957, 2912, 12, 45}

$$\frac{\sinh(x)}{a}-\frac{b\log(a\sinh(x)+b)}{a^2}$$

Antiderivative was successfully verified.

[In] Int[Cosh[x]/(a + b*Csch[x]),x]

[Out] -((b*Log[b + a*Sinh[x]])/a^2) + Sinh[x]/a

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\cosh(x)}{a+b\text{csch}(x)}\,dx = i\int \frac{\cosh(x)\sinh(x)}{ib+ia\sinh(x)}\,dx$$
$$= -\frac{i\text{Subst}\left(\int \frac{x}{a(ib+x)}\,dx, x, ia\sinh(x)\right)}{a}$$
$$= -\frac{i\text{Subst}\left(\int \frac{x}{ib+x}\,dx, x, ia\sinh(x)\right)}{a^2}$$
$$= -\frac{i\text{Subst}\left(\int \left(1-\frac{b}{b-ix}\right)\,dx, x, ia\sinh(x)\right)}{a^2}$$
$$= -\frac{b\log(b+a\sinh(x))}{a^2}+\frac{\sinh(x)}{a}$$

Mathematica [**A**]
time = 0.01, size = 19, normalized size = 0.95

$$\frac{-b\log(b+a\sinh(x))+a\sinh(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(a + b*Csch[x]),x]
[Out] (-(b*Log[b + a*Sinh[x]]) + a*Sinh[x])/a^2
```

Maple [**A**]
time = 0.65, size = 31, normalized size = 1.55

method	result	size
derivativedivides	$\frac{1}{a\,\text{csch}(x)}+\frac{b\ln(\text{csch}(x))}{a^2}-\frac{b\ln(a+b\,\text{csch}(x))}{a^2}$	31
default	$\frac{1}{a\,\text{csch}(x)}+\frac{b\ln(\text{csch}(x))}{a^2}-\frac{b\ln(a+b\,\text{csch}(x))}{a^2}$	31
risch	$\frac{bx}{a^2}+\frac{e^x}{2a}-\frac{e^{-x}}{2a}-\frac{b\ln\left(e^{2x}+\frac{2b\,e^x}{a}-1\right)}{a^2}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a+b*csch(x)),x,method=_RETURNVERBOSE)
[Out] 1/a/csch(x)+b/a^2*ln(csch(x))-b/a^2*ln(a+b*csch(x))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(20) = 40.
time = 0.26, size = 48, normalized size = 2.40

$$-\frac{bx}{a^2}-\frac{e^{(-x)}}{2\,a}+\frac{e^x}{2\,a}-\frac{b\log\left(-2\,be^{(-x)}+ae^{(-2\,x)}-a\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -b*x/a^2 - 1/2*e^(-x)/a + 1/2*e^x/a - b*log(-2*b*e^(-x) + a*e^(-2*x) - a)/a
^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(20) = 40$.
time = 0.36, size = 80, normalized size = 4.00

$$\frac{2\,bx\cosh\left(x\right)+a\cosh\left(x\right)^{2}+a\sinh\left(x\right)^{2}-2\,\left(b\cosh\left(x\right)+b\sinh\left(x\right)\right)\log\left(\frac{2\,\left(a\sinh\left(x\right)+b\right)}{\cosh\left(x\right)-\sinh\left(x\right)}\right)+2\,\left(bx+a\cosh\left(x\right)\right)\sinh\left(x\right)-a}{2\,\left(a^{2}\cosh\left(x\right)+a^{2}\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x*cosh(x) + a*cosh(x)^2 + a*sinh(x)^2 - 2*(b*cosh(x) + b*sinh(x))*
log(2*(a*sinh(x) + b)/(cosh(x) - sinh(x))) + 2*(b*x + a*cosh(x))*sinh(x) -
a)/(a^2*cosh(x) + a^2*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cosh\left(x\right)}{a+b\operatorname{csch}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*csch(x)),x)
```

```
[Out] Integral(cosh(x)/(a + b*csch(x)), x)
```

Giac [A]
time = 0.40, size = 39, normalized size = 1.95

$$-\frac{e^{(-x)}-e^{x}}{2\,a}-\frac{b\log\left(\left|-a\left(e^{(-x)}-e^{x}\right)+2\,b\right|\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -1/2*(e^(-x) - e^x)/a - b*log(abs(-a*(e^(-x) - e^x) + 2*b))/a^2
```

Mupad [B]
time = 0.08, size = 20, normalized size = 1.00

$$\frac{\sinh(x)}{a}-\frac{b\,\ln\left(b+a\sinh(x)\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a + b/sinh(x)),x)
```

```
[Out] sinh(x)/a - (b*log(b + a*sinh(x)))/a^2
```

3.97 $\int \frac{\operatorname{sech}(x)}{a+b\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=64

$$\frac{\log(i-\sinh(x))}{2(ia+b)}-\frac{\log(i+\sinh(x))}{2(ia-b)}-\frac{b\log(b+a\sinh(x))}{a^2+b^2}$$

```
[Out] 1/2*ln(I-sinh(x))/(I*a+b)-1/2*ln(I+sinh(x))/(I*a-b)-b*ln(b+a*sinh(x))/(a^2+
b^2)
```

Rubi [A]
time = 0.08, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3957, 2800, 815}

$$-\frac{b\log(a\sinh(x)+b)}{a^2+b^2}+\frac{\log(-\sinh(x)+i)}{2(b+ia)}-\frac{\log(\sinh(x)+i)}{2(-b+ia)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]/(a + b*Csch[x]),x]
```

```
[Out] Log[I - Sinh[x]]/(2*(I*a + b)) - Log[I + Sinh[x]]/(2*(I*a - b)) - (b*Log[b
+ a*Sinh[x]])/(a^2 + b^2)
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{sech}(x)}{a+b\text{csch}(x)}\,dx &= i\int \frac{\tanh(x)}{ib+ia\sinh(x)}\,dx \\
&= -\left(i\text{Subst}\left(\int \frac{x}{(ib+x)\left(a^2-x^2\right)}\,dx, x, ia\sinh(x)\right)\right) \\
&= -\left(i\text{Subst}\left(\int \left(\frac{1}{2(a+ib)(a-x)} - \frac{b}{\left(a^2+b^2\right)(b-ix)} + \frac{1}{2(a-ib)(a+x)}\right)\,dx, x, ia\sin\right.\right. \\
&= \frac{\log(i-\sinh(x))}{2(ia+b)} - \frac{\log(i+\sinh(x))}{2(ia-b)} - \frac{b\log(b+a\sinh(x))}{a^2+b^2}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 36, normalized size = 0.56

$$\frac{2a\text{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right) + b\log(\cosh(x)) - b\log(b+a\sinh(x))}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(a + b*Csch[x]),x]
```

```
[Out] (2*a*ArcTan[Tanh[x/2]] + b*Log[Cosh[x]] - b*Log[b + a*Sinh[x]])/(a^2 + b^2)
```

Maple [A]
time = 0.76, size = 72, normalized size = 1.12

method	result	size
default	$\frac{2b\ln\left(\tanh^2\left(\frac{x}{2}\right)+1\right)+4a\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{2a^2+2b^2} - \frac{2b\ln\left(-b\left(\tanh^2\left(\frac{x}{2}\right)\right)+2a\tanh\left(\frac{x}{2}\right)+b\right)}{2a^2+2b^2}$	72
risch	$\frac{i\ln(e^x+i)a}{a^2+b^2} + \frac{\ln(e^x+i)b}{a^2+b^2} - \frac{i\ln(e^x-i)a}{a^2+b^2} + \frac{\ln(e^x-i)b}{a^2+b^2} - \frac{b\ln\left(e^{2x}+\frac{2b\,e^x}{a}-1\right)}{a^2+b^2}$	101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 4/(2*a^2+2*b^2)*(1/2*b*ln(tanh(1/2*x)^2+1)+a*arctan(tanh(1/2*x)))-2*b/(2*a^
2+2*b^2)*ln(-b*tanh(1/2*x)^2+2*a*tanh(1/2*x)+b)
```

Maxima [A]
time = 0.49, size = 66, normalized size = 1.03

$$-\frac{2\,a\arctan\left(e^{(-x)}\right)}{a^2+b^2} - \frac{b\log\left(-2\,be^{(-x)}+ae^{(-2\,x)}-a\right)}{a^2+b^2} + \frac{b\log\left(e^{(-2\,x)}+1\right)}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -2*a*arctan(e^(-x))/(a^2 + b^2) - b*log(-2*b*e^(-x) + a*e^(-2*x) - a)/(a^2
+ b^2) + b*log(e^(-2*x) + 1)/(a^2 + b^2)
```

Fricas [A]
time = 0.39, size = 57, normalized size = 0.89

$$\frac{2\,a\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)-b\log\left(\frac{2\,\left(a\sinh(x)+b\right)}{\cosh(x)-\sinh(x)}\right)+b\log\left(\frac{2\,\cosh(x)}{\cosh(x)-\sinh(x)}\right)}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] (2*a*arctan(cosh(x) + sinh(x)) - b*log(2*(a*sinh(x) + b)/(cosh(x) - sinh(x)
)) + b*log(2*cosh(x)/(cosh(x) - sinh(x))))/(a^2 + b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{sech}\left(x\right)}{a+b\operatorname{csch}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*csch(x)),x)
```

```
[Out] Integral(sech(x)/(a + b*csch(x)), x)
```

Giac [A]
time = 0.39, size = 89, normalized size = 1.39

$$-\frac{ab\log\left(\left|-a\left(e^{(-x)}-e^x\right)+2\,b\right|\right)}{a^3+ab^2}+\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\,\left(e^{(2\,x)}-1\right)e^{(-x)}\right)\right)a}{2\,\left(a^2+b^2\right)}+\frac{b\log\left(\left(e^{(-x)}-e^x\right)^2+4\right)}{2\,\left(a^2+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -a*b*log(abs(-a*(e^(-x) - e^x) + 2*b))/(a^3 + a*b^2) + 1/2*(pi + 2*arctan(1
/2*(e^(2*x) - 1)*e^(-x)))*a/(a^2 + b^2) + 1/2*b*log((e^(-x) - e^x)^2 + 4)/(
a^2 + b^2)
```

Mupad [B]
time = 2.36, size = 93, normalized size = 1.45

$$\frac{\ln\left(1+\mathrm{e}^x\,1\mathrm{i}\right)}{b+a\,1\mathrm{i}}-\frac{b\,\ln\left(a^3\,\mathrm{e}^{2\,x}-4\,a\,b^2-a^3+8\,b^3\,\mathrm{e}^x+2\,a^2\,b\,\mathrm{e}^x+4\,a\,b^2\,\mathrm{e}^{2\,x}\right)}{a^2+b^2}+\frac{\ln\left(\mathrm{e}^x+1\mathrm{i}\right)\,1\mathrm{i}}{a+b\,1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)*(a + b/sinh(x))),x)
```

```
[Out] log(exp(x)*1i + 1)/(a*1i + b) + (log(exp(x) + 1i)*1i)/(a + b*1i) - (b*log(a
^3*exp(2*x) - 4*a*b^2 - a^3 + 8*b^3*exp(x) + 2*a^2*b*exp(x) + 4*a*b^2*exp(2
*x)))/(a^2 + b^2)
```

3.98 $\int \frac{\text{sech}^2(x)}{a+b\text{csch}(x)}\,dx$

Optimal. Leaf size=60

$$\frac{2ab\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{\text{sech}(x)(b-a\sinh(x))}{a^2+b^2}$$

```
[Out] 2*a*b*arctanh((a-b*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)-sech(x)*(b
-a*sinh(x))/(a^2+b^2)
```

Rubi [A]
time = 0.10, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3957, 2945, 12, 2739, 632, 210}

$$\frac{2ab\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{\text{sech}(x)(b-a\sinh(x))}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(a + b*Csch[x]),x]
```

```
[Out] (2*a*b*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2) - (Sec
h[x]*(b - a*Sinh[x]))/(a^2 + b^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^2(x)}{a+b\operatorname{csch}(x)}\,dx &= i\int \frac{\operatorname{sech}(x)\tanh(x)}{ib+ia\sinh(x)}\,dx \\
&= -\frac{\operatorname{sech}(x)(b-a\sinh(x))}{a^2+b^2}-\frac{i\int \frac{ab}{ib+ia\sinh(x)}\,dx}{a^2+b^2} \\
&= -\frac{\operatorname{sech}(x)(b-a\sinh(x))}{a^2+b^2}-\frac{(iab)\int \frac{1}{ib+ia\sinh(x)}\,dx}{a^2+b^2} \\
&= -\frac{\operatorname{sech}(x)(b-a\sinh(x))}{a^2+b^2}-\frac{(2iab)\operatorname{Subst}\left(\int \frac{1}{ib+2iax-ibx^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2} \\
&= -\frac{\operatorname{sech}(x)(b-a\sinh(x))}{a^2+b^2}+\frac{(4iab)\operatorname{Subst}\left(\int \frac{1}{-4(a^2+b^2)-x^2}\,dx,x,2ia-2ib\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2} \\
&= \frac{2ab\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{\operatorname{sech}(x)(b-a\sinh(x))}{a^2+b^2}
\end{aligned}$$

Mathematica [A]

time = 0.13, size = 67, normalized size = 1.12

$$\frac{-b\operatorname{sech}(x)+a\left(-\frac{2b\operatorname{ArcTan}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+\tanh(x)\right)}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(a + b*Csch[x]),x]
```

```
[Out] (-(b*Sech[x]) + a*((-2*b*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-
a^2 - b^2] + Tanh[x]))/(a^2 + b^2)
```

Maple [A]
time = 0.82, size = 81, normalized size = 1.35

method	result	size
default	$\frac{4ab\,\mathrm{arctanh}\left(\frac{-2b\tanh\left(\frac{x}{2}\right)+2a}{2\sqrt{a^2+b^2}}\right)}{(2a^2+2b^2)\sqrt{a^2+b^2}}-\frac{2\left(b-a\tanh\left(\frac{x}{2}\right)\right)}{(a^2+b^2)\left(\tanh^2\left(\frac{x}{2}\right)+1\right)}$	81
risch	$-\frac{2(\mathrm{e}^x b+a)}{(1+\mathrm{e}^{2x})(a^2+b^2)}+\frac{ba\ln\left(\mathrm{e}^x+\frac{\left(a^2+b^2\right)^{\frac{3}{2}}b+a^4+2a^2b^2+b^4}{a\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}}-\frac{ba\ln\left(\mathrm{e}^x+\frac{\left(a^2+b^2\right)^{\frac{3}{2}}b-a^4-2a^2b^2-b^4}{a\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}}$	142

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 4*a*b/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(-2*b*tanh(1/2*x)+2*a)/(a^2
+b^2)^(1/2))-2/(a^2+b^2)*(b-a*tanh(1/2*x))/(tanh(1/2*x)^2+1)
```

Maxima [A]
time = 0.47, size = 91, normalized size = 1.52

$$-\frac{ab\log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}}-\frac{2\left(be^{(-x)}-a\right)}{a^2+b^2+(a^2+b^2)e^{(-2x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -a*b*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))
/(a^2 + b^2)^(3/2) - 2*(b*e^(-x) - a)/(a^2 + b^2 + (a^2 + b^2)*e^(-2*x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(56) = 112$.
time = 0.41, size = 256, normalized size = 4.27

$$-\frac{2\,a^{3}+2\,ab^{2}-\left(ab\cosh\left(x\right)^{2}+2\,ab\cosh\left(x\right)\sinh\left(x\right)+ab\sinh\left(x\right)^{2}+ab\right)\sqrt{a^{2}+b^{2}}\log\left(\frac{a^{2}\cosh(x)^{2}+a^{2}\sinh(x)^{2}+2\,ab\cosh(x)+a^{2}+2\,b^{2}+2\,(a^{2}\cosh(x)+ab)\sinh(x)+2\,\sqrt{a^{2}+b^{2}}\,(a\cosh(x)+a\sinh(x)+b)}{a\cosh(x)^{2}+a\sinh(x)^{2}+2\,b\cosh(x)+2\,(a\cosh(x)+b)\sinh(x)-a}\right)+2\,(a^{2}b+b^{3})\cosh\left(x\right)+2\,(a^{2}b+b^{3})\sinh\left(x\right)}{a^{4}+2\,a^{2}b^{2}+b^{4}+(a^{4}+2\,a^{2}b^{2}+b^{4})\cosh\left(x\right)^{2}+2\,(a^{4}+2\,a^{2}b^{2}+b^{4})\cosh\left(x\right)\sinh\left(x\right)+(a^{4}+2\,a^{2}b^{2}+b^{4})\sinh\left(x\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] -(2*a^3 + 2*a*b^2 - (a*b*cosh(x)^2 + 2*a*b*cosh(x)*sinh(x) + a*b*sinh(x)^2
+ a*b)*sqrt(a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) +
 a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(a*cosh(x)
 + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x)
+ b)*sinh(x) - a)) + 2*(a^2*b + b^3)*cosh(x) + 2*(a^2*b + b^3)*sinh(x))/(a^
4 + 2*a^2*b^2 + b^4 + (a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2 + 2*(a^4 + 2*a^2*b^
2 + b^4)*cosh(x)*sinh(x) + (a^4 + 2*a^2*b^2 + b^4)*sinh(x)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{sech}^{2}\left(x\right)}{a+b\operatorname{csch}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(a+b*csch(x)),x)
```

```
[Out] Integral(sech(x)**2/(a + b*csch(x)), x)
```

Giac [A]
time = 0.40, size = 85, normalized size = 1.42

$$-\frac{ab\log\left(\frac{\left|2\,ae^{x}+2\,b-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,ae^{x}+2\,b+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}-\frac{2\,\left(be^{x}+a\right)}{(a^{2}+b^{2})(e^{(2\,x)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -a*b*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(
a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(b*e^x + a)/((a^2 + b^2)*(e^(2*x) + 1))
```

Mupad [B]
time = 1.66, size = 133, normalized size = 2.22

$$\frac{a\,b\,\ln\left(\frac{2\,b\,(a-b\,\mathrm{e}^{x})}{(a^{2}+b^{2})^{3/2}}+\frac{2\,b\,\mathrm{e}^{x}}{a^{2}+b^{2}}\right)}{\left(a^{2}+b^{2}\right)^{3/2}}-\frac{a\,b\,\ln\left(\frac{2\,b\,\mathrm{e}^{x}}{a^{2}+b^{2}}-\frac{2\,b\,(a-b\,\mathrm{e}^{x})}{(a^{2}+b^{2})^{3/2}}\right)}{\left(a^{2}+b^{2}\right)^{3/2}}-\frac{\frac{2\,a}{a^{2}+b^{2}}+\frac{2\,b\,\mathrm{e}^{x}}{a^{2}+b^{2}}}{\mathrm{e}^{2\,x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2*(a + b/sinh(x))),x)
```

```
[Out] (a*b*log((2*b*(a - b*exp(x)))/(a^2 + b^2)^(3/2) + (2*b*exp(x))/(a^2 + b^2))
)/(a^2 + b^2)^(3/2) - (a*b*log((2*b*exp(x))/(a^2 + b^2) - (2*b*(a - b*exp(x
)))/(a^2 + b^2)^(3/2)))/(a^2 + b^2)^(3/2) - ((2*a)/(a^2 + b^2) + (2*b*exp(x
))/(a^2 + b^2))/(exp(2*x) + 1)
```

3.99 $\int \frac{\operatorname{sech}^3(x)}{a+b\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=95

$$-\frac{ia\log(i-\sinh(x))}{4(a-ib)^2}+\frac{ia\log(i+\sinh(x))}{4(a+ib)^2}-\frac{a^2b\log(b+a\sinh(x))}{(a^2+b^2)^2}-\frac{\operatorname{sech}^2(x)(b-a\sinh(x))}{2\,(a^2+b^2)}$$

```
[Out] -1/4*I*a*ln(I-sinh(x))/(a-I*b)^2+1/4*I*a*ln(I+sinh(x))/(a+I*b)^2-a^2*b*ln(b
+a*sinh(x))/(a^2+b^2)^2-1/2*sech(x)^2*(b-a*sinh(x))/(a^2+b^2)
```

Rubi [A]
time = 0.16, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3957, 2916, 12, 837, 815}

$$-\frac{a^2b\log(a\sinh(x)+b)}{(a^2+b^2)^2}-\frac{\operatorname{sech}^2(x)(b-a\sinh(x))}{2\,(a^2+b^2)}-\frac{ia\log(-\sinh(x)+i)}{4(a-ib)^2}+\frac{ia\log(\sinh(x)+i)}{4(a+ib)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^3/(a + b*Csch[x]),x]
```

```
[Out] ((-1/4*I)*a*Log[I - Sinh[x]])/(a - I*b)^2 + ((I/4)*a*Log[I + Sinh[x]])/(a +
 I*b)^2 - (a^2*b*Log[b + a*Sinh[x]])/(a^2 + b^2)^2 - (Sech[x]^2*(b - a*Sinh
[x]))/(2*(a^2 + b^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
```

```
[2*m, 2*p])
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^3(x)}{a+b\operatorname{csch}(x)}\,dx &= i\int \frac{\operatorname{sech}^2(x)\tanh(x)}{ib+ia\sinh(x)}\,dx\\
&= -\left(\left(ia^3\right)\operatorname{Subst}\left(\int \frac{x}{a(ib+x)\left(a^2-x^2\right)^2}\,dx, x, ia\sinh(x)\right)\right)\\
&= -\left(\left(ia^2\right)\operatorname{Subst}\left(\int \frac{x}{(ib+x)\left(a^2-x^2\right)^2}\,dx, x, ia\sinh(x)\right)\right)\\
&= -\frac{\operatorname{sech}^2(x)(b-a\sinh(x))}{2\left(a^2+b^2\right)} - \frac{i\operatorname{Subst}\left(\int \frac{-ia^2b+a^2x}{(ib+x)(a^2-x^2)}\,dx, x, ia\sinh(x)\right)}{2\left(a^2+b^2\right)}\\
&= -\frac{\operatorname{sech}^2(x)(b-a\sinh(x))}{2\left(a^2+b^2\right)} - \frac{i\operatorname{Subst}\left(\int \left(\frac{a(a-ib)}{2(a+ib)(a-x)} - \frac{2a^2b}{(a^2+b^2)(b-ix)} + \frac{a(a+ib)}{2(a-ib)(a+x)}\right)\,dx, x\right.}{2\left(a^2+b^2\right)}\\
&= -\frac{ia\log(i-\sinh(x))}{4(a-ib)^2} + \frac{ia\log(i+\sinh(x))}{4(a+ib)^2} - \frac{a^2b\log(b+a\sinh(x))}{\left(a^2+b^2\right)^2} - \frac{\operatorname{sech}^2(x)(b-a}{2\left(a^2+b\right.}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 78, normalized size = 0.82

$$\frac{2a\left(\left(a^2-b^2\right)\operatorname{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)+ab(\log(\cosh(x))-\log(b+a\sinh(x)))\right)-b\left(a^2+b^2\right)\operatorname{sech}^2(x)+a\left(a^2+b^2\right)\operatorname{sech}(x)\tanh(x)}{2\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^3/(a + b*Csch[x]),x]
```

[Out] (2*a*((a^2 - b^2)*ArcTan[Tanh[x/2]] + a*b*(Log[Cosh[x]] - Log[b + a*Sinh[x]
])) - b*(a^2 + b^2)*Sech[x]^2 + a*(a^2 + b^2)*Sech[x]*Tanh[x])/(2*(a^2 + b^
2)^2)

Maple [A]
time = 0.93, size = 143, normalized size = 1.51

method	result
default	$\frac{\frac{2\left(\left(-\frac{1}{2}a^3-\frac{1}{2}a\,b^2\right)\left(\tanh^3\left(\frac{x}{2}\right)\right)+\left(a^2b+b^3\right)\left(\tanh^2\left(\frac{x}{2}\right)\right)+\left(\frac{1}{2}a^3+\frac{1}{2}a\,b^2\right)\tanh\left(\frac{x}{2}\right)\right)}{\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^2}+a\left(ab\ln\left(\tanh^2\left(\frac{x}{2}\right)+1\right)+\left(a^2-b^2\right)\arctan\left(\tanh\left(\frac{x}{2}\right)\right)\right)}{\left(a^2+b^2\right)^2}-$
risch	$\frac{e^x\left(e^{2x}a-2\,e^x b-a\right)}{\left(1+e^{2x}\right)^2\left(a^2+b^2\right)}-\frac{ia^3\ln\left(e^x-i\right)}{2\left(a^4+2a^2b^2+b^4\right)}+\frac{ia\ln\left(e^x-i\right)b^2}{2a^4+4a^2b^2+2b^4}+\frac{a^2\ln\left(e^x-i\right)b}{a^4+2a^2b^2+b^4}+\frac{ia^3\ln\left(e^x+i\right)}{2a^4+4a^2b^2+2b^4}-\frac{ia\ln\left(e^x+i\right)b^2}{2\left(a^4+2a^2b^2+b^4\right)}+$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(x)^3/(a+b*csch(x)),x,method=_RETURNVERBOSE)

[Out] 2/(a^2+b^2)^2*(((-1/2*a^3-1/2*a*b^2)*tanh(1/2*x)^3+(a^2*b+b^3)*tanh(1/2*x)^
2+(1/2*a^3+1/2*a*b^2)*tanh(1/2*x))/(tanh(1/2*x)^2+1)^2+1/2*a*(a*b*ln(tanh(1
/2*x)^2+1)+(a^2-b^2)*arctan(tanh(1/2*x))))-a^2*b/(a^2+b^2)^2*ln(-b*tanh(1/2
*x)^2+2*a*tanh(1/2*x)+b)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 161 vs. 2(78) = 156.
time = 0.49, size = 161, normalized size = 1.69

$$-\frac{a^2b\log\left(-2\,be^{(-x)}+ae^{(-2\,x)}-a\right)}{a^4+2\,a^2b^2+b^4}+\frac{a^2b\log\left(e^{(-2\,x)}+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{\left(a^3-ab^2\right)\arctan\left(e^{(-x)}\right)}{a^4+2\,a^2b^2+b^4}+\frac{ae^{(-x)}-2\,be^{(-2\,x)}-ae^{(-3\,x)}}{a^2+b^2+2\left(a^2+b^2\right)e^{(-2\,x)}+\left(a^2+b^2\right)e^{(-4\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(x)^3/(a+b*csch(x)),x, algorithm="maxima")

[Out] -a^2*b*log(-2*b*e^(-x) + a*e^(-2*x) - a)/(a^4 + 2*a^2*b^2 + b^4) + a^2*b*lo
g(e^(-2*x) + 1)/(a^4 + 2*a^2*b^2 + b^4) - (a^3 - a*b^2)*arctan(e^(-x))/(a^4
 + 2*a^2*b^2 + b^4) + (a*e^(-x) - 2*b*e^(-2*x) - a*e^(-3*x))/(a^2 + b^2 + 2
*(a^2 + b^2)*e^(-2*x) + (a^2 + b^2)*e^(-4*x))

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 675 vs. 2(78) = 156.
time = 0.48, size = 675, normalized size = 7.11

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(x)^3/(a+b*csch(x)),x, algorithm="fricas")

```
[Out] ((a^3 + a*b^2)*cosh(x)^3 + (a^3 + a*b^2)*sinh(x)^3 - 2*(a^2*b + b^3)*cosh(x
)^2 - (2*a^2*b + 2*b^3 - 3*(a^3 + a*b^2)*cosh(x))*sinh(x)^2 + ((a^3 - a*b^2
)*cosh(x)^4 + 4*(a^3 - a*b^2)*cosh(x)*sinh(x)^3 + (a^3 - a*b^2)*sinh(x)^4 +
 a^3 - a*b^2 + 2*(a^3 - a*b^2)*cosh(x)^2 + 2*(a^3 - a*b^2 + 3*(a^3 - a*b^2)
*cosh(x)^2)*sinh(x)^2 + 4*((a^3 - a*b^2)*cosh(x)^3 + (a^3 - a*b^2)*cosh(x))
*sinh(x))*arctan(cosh(x) + sinh(x)) - (a^3 + a*b^2)*cosh(x) - (a^2*b*cosh(x
)^4 + 4*a^2*b*cosh(x)*sinh(x)^3 + a^2*b*sinh(x)^4 + 2*a^2*b*cosh(x)^2 + a^2
*b + 2*(3*a^2*b*cosh(x)^2 + a^2*b)*sinh(x)^2 + 4*(a^2*b*cosh(x)^3 + a^2*b*c
osh(x))*sinh(x))*log(2*(a*sinh(x) + b)/(cosh(x) - sinh(x))) + (a^2*b*cosh(x
)^4 + 4*a^2*b*cosh(x)*sinh(x)^3 + a^2*b*sinh(x)^4 + 2*a^2*b*cosh(x)^2 + a^2
*b + 2*(3*a^2*b*cosh(x)^2 + a^2*b)*sinh(x)^2 + 4*(a^2*b*cosh(x)^3 + a^2*b*c
osh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) - (a^3 + a*b^2 - 3*(a^3
 + a*b^2)*cosh(x)^2 + 4*(a^2*b + b^3)*cosh(x))*sinh(x))/((a^4 + 2*a^2*b^2 +
 b^4)*cosh(x)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^3 + (a^4 + 2*a^
2*b^2 + b^4)*sinh(x)^4 + a^4 + 2*a^2*b^2 + b^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*
cosh(x)^2 + 2*(a^4 + 2*a^2*b^2 + b^4 + 3*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2)
*sinh(x)^2 + 4*((a^4 + 2*a^2*b^2 + b^4)*cosh(x)^3 + (a^4 + 2*a^2*b^2 + b^4)
*cosh(x))*sinh(x))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^3(x)}{a+b\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**3/(a+b*csch(x)),x)
```

```
[Out] Integral(sech(x)**3/(a + b*csch(x)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(78) = 156.
time = 0.40, size = 218, normalized size = 2.29

$$-\frac{a^3 b\log\left(\left|-a\left(e^{(-x)}-e^x\right)+2\,b\right|\right)}{a^5+2\,a^3b^2+ab^4}+\frac{a^2b\log\left(\left(e^{(-x)}-e^x\right)^2+4\right)}{2\,(a^4+2\,a^2b^2+b^4)}+\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)}-1\right)e^{(-x)}\right)\right)(a^3-ab^2)}{4\,(a^4+2\,a^2b^2+b^4)}-\frac{a^2b\left(e^{(-x)}-e^x\right)^2+2\,a^3\left(e^{(-x)}-e^x\right)+2\,ab^2\left(e^{(-x)}-e^x\right)+8\,a^2b+4\,b^3}{2\,(a^4+2\,a^2b^2+b^4)\left(\left(e^{(-x)}-e^x\right)^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -a^3*b*log(abs(-a*(e^(-x) - e^x) + 2*b))/(a^5 + 2*a^3*b^2 + a*b^4) + 1/2*a^
2*b*log((e^(-x) - e^x)^2 + 4)/(a^4 + 2*a^2*b^2 + b^4) + 1/4*(pi + 2*arctan(
1/2*(e^(2*x) - 1)*e^(-x)))*(a^3 - a*b^2)/(a^4 + 2*a^2*b^2 + b^4) - 1/2*(a^2
*b*(e^(-x) - e^x)^2 + 2*a^3*(e^(-x) - e^x) + 2*a*b^2*(e^(-x) - e^x) + 8*a^2
*b + 4*b^3)/((a^4 + 2*a^2*b^2 + b^4)*((e^(-x) - e^x)^2 + 4))
```

Mupad [B]
time = 2.99, size = 256, normalized size = 2.69

$$\frac{\frac{2\,b}{a^2+b^2}-\frac{2\,a\,{\mathrm{e}}^x}{a^2+b^2}}{2\,{\mathrm{e}}^{2\,x}+{\mathrm{e}}^{4\,x}+1}-\frac{\frac{2\,\left(a^2\,b+b^3\right)}{{\left(a^2+b^2\right)}^2}-\frac{{\mathrm{e}}^x\,\left(a^3+a\,b^2\right)}{{\left(a^2+b^2\right)}^2}}{{\mathrm{e}}^{2\,x}+1}+\frac{a\,\ln\left({\mathrm{e}}^x+1{}\mathrm{i}\right)}{2\,\left(-a^2\,1{}\mathrm{i}+2\,a\,b+b^2\,1{}\mathrm{i}\right)}-\frac{a^2\,b\,\ln\left(a^6\,{\mathrm{e}}^{2\,x}-a^6-a^2\,b^4-14\,a^4\,b^2+a^2\,b^4\,{\mathrm{e}}^{2\,x}+14\,a^4\,b^2\,{\mathrm{e}}^{2\,x}+2\,a\,b^5\,{\mathrm{e}}^x+2\,a^5\,b\,{\mathrm{e}}^x+28\,a^3\,b^3\,{\mathrm{e}}^x\right)}{a^4+2\,a^2\,b^2+b^4}+\frac{a\,\ln\left(1+{\mathrm{e}}^x\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{2\,\left(-a^2+a\,b\,2{}\mathrm{i}+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^3*(a + b/sinh(x))),x)
```

```
[Out] ((2*b)/(a^2 + b^2) - (2*a*exp(x))/(a^2 + b^2))/(2*exp(2*x) + exp(4*x) + 1)
- ((2*(a^2*b + b^3))/(a^2 + b^2)^2 - (exp(x)*(a*b^2 + a^3))/(a^2 + b^2)^2)/
(exp(2*x) + 1) + (a*log(exp(x)*1i + 1)*1i)/(2*(a*b*2i - a^2 + b^2)) + (a*lo
g(exp(x) + 1i))/(2*(2*a*b - a^2*1i + b^2*1i)) - (a^2*b*log(a^6*exp(2*x) - a
^6 - a^2*b^4 - 14*a^4*b^2 + a^2*b^4*exp(2*x) + 14*a^4*b^2*exp(2*x) + 2*a*b^
5*exp(x) + 2*a^5*b*exp(x) + 28*a^3*b^3*exp(x)))/(a^4 + b^4 + 2*a^2*b^2)
```

3.100 $\int \frac{\operatorname{sech}^4(x)}{a+b\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=104

$$\frac{2a^3b\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}-\frac{\operatorname{sech}^3(x)(b-a\sinh(x))}{3\,(a^2+b^2)}-\frac{\operatorname{sech}(x)\left(3a^2b-a(2a^2-b^2)\sinh(x)\right)}{3\,(a^2+b^2)^2}$$

```
[Out] 2*a^3*b*arctanh((a-b*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)-1/3*sech
(x)^3*(b-a*sinh(x))/(a^2+b^2)-1/3*sech(x)*(3*a^2*b-a*(2*a^2-b^2)*sinh(x))/(
a^2+b^2)^2
```

Rubi [A]
time = 0.19, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3957, 2945, 12, 2739, 632, 210}

$$-\frac{\operatorname{sech}^3(x)(b-a\sinh(x))}{3\,(a^2+b^2)}-\frac{\operatorname{sech}(x)\left(3a^2b-a(2a^2-b^2)\sinh(x)\right)}{3\,(a^2+b^2)^2}+\frac{2a^3b\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^4/(a + b*Csch[x]),x]
```

```
[Out] (2*a^3*b*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) - (S
ech[x]^3*(b - a*Sinh[x]))/(3*(a^2 + b^2)) - (Sech[x]*(3*a^2*b - a*(2*a^2 -
b^2)*Sinh[x]))/(3*(a^2 + b^2)^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\operatorname{sech}^4(x)}{a+b\operatorname{csch}(x)}\,dx = i\int \frac{\operatorname{sech}^3(x)\tanh(x)}{ib+ia\sinh(x)}\,dx$$

$$= -\frac{\operatorname{sech}^3(x)(b-a\sinh(x))}{3(a^2+b^2)} + \frac{\int \frac{\operatorname{sech}^2(x)(-iab+2ia^2\sinh(x))}{ib+ia\sinh(x)}\,dx}{3(a^2+b^2)}$$

$$= -\frac{\operatorname{sech}^3(x)(b-a\sinh(x))}{3(a^2+b^2)} - \frac{\operatorname{sech}(x)\left(3a^2b-a(2a^2-b^2)\sinh(x)\right)}{3(a^2+b^2)^2} + \frac{\int -\frac{3ia^3b}{ib+ia\sinh(x)}\,dx}{3(a^2+b^2)^2}$$

$$= -\frac{\operatorname{sech}^3(x)(b-a\sinh(x))}{3(a^2+b^2)} - \frac{\operatorname{sech}(x)\left(3a^2b-a(2a^2-b^2)\sinh(x)\right)}{3(a^2+b^2)^2} - \frac{(ia^3b)\int \frac{1}{ib+ia\sinh(}}{(a^2+b^2)^2}$$

$$= -\frac{\operatorname{sech}^3(x)(b-a\sinh(x))}{3(a^2+b^2)} - \frac{\operatorname{sech}(x)\left(3a^2b-a(2a^2-b^2)\sinh(x)\right)}{3(a^2+b^2)^2} - \frac{(2ia^3b)\operatorname{Subst}(\int}{}$$

$$= -\frac{\operatorname{sech}^3(x)(b-a\sinh(x))}{3(a^2+b^2)} - \frac{\operatorname{sech}(x)\left(3a^2b-a(2a^2-b^2)\sinh(x)\right)}{3(a^2+b^2)^2} + \frac{(4ia^3b)\operatorname{Subst}\left(\int\right.}{}$$

$$= \frac{2a^3b\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} - \frac{\operatorname{sech}^3(x)(b-a\sinh(x))}{3(a^2+b^2)} - \frac{\operatorname{sech}(x)\left(3a^2b-a(2a^2-b^2)\operatorname{s}\right.}{3(a^2+b^2)^2}$$

Mathematica [A]
time = 0.45, size = 114, normalized size = 1.10

$$-\frac{\frac{6a^3b\operatorname{ArcTan}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + 3a^2b\operatorname{sech}(x) + b(a^2+b^2)\operatorname{sech}^3(x) + (-2a^3+ab^2)\tanh(x) - a(a^2+b^2)\operatorname{sech}^2(x)\tanh(x)}{3(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^4/(a + b*Csch[x]),x]
[Out] -1/3*((6*a^3*b*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2]
 + 3*a^2*b*Sech[x] + b*(a^2 + b^2)*Sech[x]^3 + (-2*a^3 + a*b^2)*Tanh[x] - a
*(a^2 + b^2)*Sech[x]^2*Tanh[x])/(a^2 + b^2)^2
```

Maple [A]
time = 0.87, size = 170, normalized size = 1.63

method	result
default	$-\frac{2\left(-a^3\left(\tanh^5\left(\frac{x}{2}\right)\right)+(2a^2b+b^3)\left(\tanh^4\left(\frac{x}{2}\right)\right)+\left(-\frac{2}{3}a^3+\frac{4}{3}a\,b^2\right)\left(\tanh^3\left(\frac{x}{2}\right)\right)+2a^2b\left(\tanh^2\left(\frac{x}{2}\right)\right)-a^3\tanh\left(\frac{x}{2}\right)+\frac{4a^2b}{3}+\frac{b^3}{3}\right)}{(a^4+2a^2b^2+b^4)\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^3}+\frac{4a^3b\,\mathrm{arc}}{(2a^4+4}$

risch	$-\frac{2(3a^2b\,e^{5x}-3a\,b^2e^{4x}+10a^2b\,e^{3x}+4b^3e^{3x}+6a^3e^{2x}+3a^2b\,e^{x}+2a^3-a\,b^2)}{3(1+e^{2x})^3(a^2+b^2)^2}+\frac{b\,a^3\ln\left(e^{x}+\frac{(a^2+b^2)^{\frac{5}{2}}b+a^6+3a^4b^2+3a^2b^4+b^6}{a(a^2+b^2)^{\frac{5}{2}}}\right)}{(a^2+b^2)^{\frac{5}{2}}}-\frac{b\,a^3}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^4/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(a^4+2*a^2*b^2+b^4)*(-a^3*tanh(1/2*x)^5+(2*a^2*b+b^3)*tanh(1/2*x)^4+(-2/
3*a^3+4/3*a*b^2)*tanh(1/2*x)^3+2*a^2*b*tanh(1/2*x)^2-a^3*tanh(1/2*x)+4/3*a^
2*b+1/3*b^3)/(tanh(1/2*x)^2+1)^3+4*a^3*b/(2*a^4+4*a^2*b^2+2*b^4)/(a^2+b^2)^
(1/2)*arctanh(1/2*(-2*b*tanh(1/2*x)+2*a)/(a^2+b^2)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(97) = 194$.
time = 0.48, size = 226, normalized size = 2.17

$$-\frac{a^3b\log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}}-\frac{2\left(3\,a^2be^{(-x)}-6\,a^3e^{(-2\,x)}+3\,ab^2e^{(-4\,x)}+3\,a^2be^{(-5\,x)}-2\,a^3+ab^2+2\,(5\,a^2b+2\,b^3)e^{(-3\,x)}\right)}{3\,(a^4+2\,a^2b^2+b^4+3\,(a^4+2\,a^2b^2+b^4)e^{(-2\,x)}+3\,(a^4+2\,a^2b^2+b^4)e^{(-4\,x)}+(a^4+2\,a^2b^2+b^4)e^{(-6\,x)})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -a^3*b*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)
))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) - 2/3*(3*a^2*b*e^(-x) - 6*a^3*
e^(-2*x) + 3*a*b^2*e^(-4*x) + 3*a^2*b*e^(-5*x) - 2*a^3 + a*b^2 + 2*(5*a^2*b
 + 2*b^3)*e^(-3*x))/(a^4 + 2*a^2*b^2 + b^4 + 3*(a^4 + 2*a^2*b^2 + b^4)*e^(-
2*x) + 3*(a^4 + 2*a^2*b^2 + b^4)*e^(-4*x) + (a^4 + 2*a^2*b^2 + b^4)*e^(-6*x
))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1155 vs. $2(97) = 194$.
time = 0.39, size = 1155, normalized size = 11.11

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] -1/3*(6*(a^4*b + a^2*b^3)*cosh(x)^5 + 6*(a^4*b + a^2*b^3)*sinh(x)^5 + 4*a^5
 + 2*a^3*b^2 - 2*a*b^4 - 6*(a^3*b^2 + a*b^4)*cosh(x)^4 - 6*(a^3*b^2 + a*b^4
 - 5*(a^4*b + a^2*b^3)*cosh(x))*sinh(x)^4 + 4*(5*a^4*b + 7*a^2*b^3 + 2*b^5)
*cosh(x)^3 + 4*(5*a^4*b + 7*a^2*b^3 + 2*b^5 + 15*(a^4*b + a^2*b^3)*cosh(x)^
2 - 6*(a^3*b^2 + a*b^4)*cosh(x))*sinh(x)^3 + 12*(a^5 + a^3*b^2)*cosh(x)^2 +
 12*(a^5 + a^3*b^2 + 5*(a^4*b + a^2*b^3)*cosh(x)^3 - 3*(a^3*b^2 + a*b^4)*co
sh(x)^2 + (5*a^4*b + 7*a^2*b^3 + 2*b^5)*cosh(x))*sinh(x)^2 - 3*(a^3*b*cosh(
```

```
x)^6 + 6*a^3*b*cosh(x)*sinh(x)^5 + a^3*b*sinh(x)^6 + 3*a^3*b*cosh(x)^4 + 3*
a^3*b*cosh(x)^2 + 3*(5*a^3*b*cosh(x)^2 + a^3*b)*sinh(x)^4 + a^3*b + 4*(5*a^
3*b*cosh(x)^3 + 3*a^3*b*cosh(x))*sinh(x)^3 + 3*(5*a^3*b*cosh(x)^4 + 6*a^3*b
*cosh(x)^2 + a^3*b)*sinh(x)^2 + 6*(a^3*b*cosh(x)^5 + 2*a^3*b*cosh(x)^3 + a^
3*b*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 +
2*a*b*cosh(x) + a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 +
b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x)
+ 2*(a*cosh(x) + b)*sinh(x) - a)) + 6*(a^4*b + a^2*b^3)*cosh(x) + 6*(a^4*b
+ a^2*b^3 + 5*(a^4*b + a^2*b^3)*cosh(x)^4 - 4*(a^3*b^2 + a*b^4)*cosh(x)^3 +
 2*(5*a^4*b + 7*a^2*b^3 + 2*b^5)*cosh(x)^2 + 4*(a^5 + a^3*b^2)*cosh(x))*sin
h(x))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^6 + 6*(a^6 + 3*a^4*b^2 +
 3*a^2*b^4 + b^6)*cosh(x)*sinh(x)^5 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*s
inh(x)^6 + a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b
^4 + b^6)*cosh(x)^4 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 5*(a^6 + 3*a^4
*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^6 + 3*a^4*b^2 + 3*a^
2*b^4 + b^6)*cosh(x)^3 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*sin
h(x)^3 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2 + 3*(a^6 + 3*a^4*b
^2 + 3*a^2*b^4 + b^6 + 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^4 + 6*
(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(x)^2 + 6*((a^6 + 3*a^4*
b^2 + 3*a^2*b^4 + b^6)*cosh(x)^5 + 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*co
sh(x)^3 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^4(x)}{a + b\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**4/(a+b*csch(x)),x)
```

```
[Out] Integral(sech(x)**4/(a + b*csch(x)), x)
```

Giac [A]
time = 0.40, size = 174, normalized size = 1.67

$$-\frac{a^3 b \log\left(\frac{\left|2\,a e^x + 2\,b - 2\,\sqrt{a^2 + b^2}\right|}{\left|2\,a e^x + 2\,b + 2\,\sqrt{a^2 + b^2}\right|}\right)}{(a^4 + 2\,a^2b^2 + b^4)\sqrt{a^2 + b^2}} - \frac{2\left(3\,a^2be^{(5\,x)} - 3\,ab^2e^{(4\,x)} + 10\,a^2be^{(3\,x)} + 4\,b^3e^{(3\,x)} + 6\,a^3e^{(2\,x)} + 3\,a^2be^x + 2\,a^3 - ab^2\right)}{3\,(a^4 + 2\,a^2b^2 + b^4)(e^{(2\,x)} + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -a^3*b*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqr
t(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) - 2/3*(3*a^2*b*e^(
```

```
5*x) - 3*a*b^2*e^(4*x) + 10*a^2*b*e^(3*x) + 4*b^3*e^(3*x) + 6*a^3*e^(2*x) +
 3*a^2*b*e^x + 2*a^3 - a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*(e^(2*x) + 1)^3)
```

Mupad [B]
time = 1.79, size = 269, normalized size = 2.59

$$\frac{\frac{2\,a\,b^2}{(a^2+b^2)^2}-\frac{2\,a^2\,b\,e^x}{(a^2+b^2)^2}}{e^{2\,x}+1}-\frac{\frac{4\,(a^3+a\,b^2)}{(a^2+b^2)^2}+\frac{8\,e^x\,(a^2\,b+b^3)}{3\,(a^2+b^2)^2}}{2\,e^{2\,x}+e^{4\,x}+1}+\frac{\frac{8\,a}{3\,(a^2+b^2)}+\frac{8\,b\,e^x}{3\,(a^2+b^2)}}{3\,e^{2\,x}+3\,e^{4\,x}+e^{6\,x}+1}-\frac{a^3\,b\,\ln\left(\frac{2\,a^2\,b\,e^x}{(a^2+b^2)^2}-\frac{2\,a^2\,b\,(a-b\,e^x)}{(a^2+b^2)^{5/2}}\right)}{(a^2+b^2)^{5/2}}+\frac{a^3\,b\,\ln\left(\frac{2\,a^2\,b\,(a-b\,e^x)}{(a^2+b^2)^{5/2}}+\frac{2\,a^2\,b\,e^x}{(a^2+b^2)^2}\right)}{(a^2+b^2)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^4*(a + b/sinh(x))),x)
```

```
[Out] ((2*a*b^2)/(a^2 + b^2)^2 - (2*a^2*b*exp(x))/(a^2 + b^2)^2)/(exp(2*x) + 1) -
 ((4*(a*b^2 + a^3))/(a^2 + b^2)^2 + (8*exp(x)*(a^2*b + b^3))/(3*(a^2 + b^2)
^2))/(2*exp(2*x) + exp(4*x) + 1) + ((8*a)/(3*(a^2 + b^2)) + (8*b*exp(x))/(3
*(a^2 + b^2)))/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - (a^3*b*log((2*a^2
*b*exp(x))/(a^2 + b^2)^2 - (2*a^2*b*(a - b*exp(x)))/(a^2 + b^2)^(5/2)))/(a^
2 + b^2)^(5/2) + (a^3*b*log((2*a^2*b*(a - b*exp(x)))/(a^2 + b^2)^(5/2) + (2
*a^2*b*exp(x))/(a^2 + b^2)^2))/(a^2 + b^2)^(5/2)
```

3.101 $\int \frac{\mathbf{sech}^5(x)}{a+b\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=149

$$-\frac{a(3ia+b)\log(i-\sinh(x))}{16(a-ib)^3}+\frac{a(3a+ib)\log(i+\sinh(x))}{16(ia-b)^3}-\frac{a^4b\log(b+a\sinh(x))}{(a^2+b^2)^3}-\frac{\operatorname{sech}^4(x)(b-a\sinh(x))}{4(a^2+b^2)}$$

```
[Out] -1/16*a*(3*I*a+b)*ln(I-sinh(x))/(a-I*b)^3+1/16*a*(3*a+I*b)*ln(I+sinh(x))/(I
*a-b)^3-a^4*b*ln(b+a*sinh(x))/(a^2+b^2)^3-1/4*sech(x)^4*(b-a*sinh(x))/(a^2+
b^2)-1/8*sech(x)^2*(4*a^2*b-a*(3*a^2-b^2)*sinh(x))/(a^2+b^2)^2
```

Rubi [A]
time = 0.26, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3957, 2916, 12, 837, 815}

$$-\frac{\operatorname{sech}^4(x)(b-a\sinh(x))}{4(a^2+b^2)}-\frac{\operatorname{sech}^2(x)\left(4a^2b-a(3a^2-b^2)\sinh(x)\right)}{8(a^2+b^2)^2}-\frac{a^4b\log(a\sinh(x)+b)}{(a^2+b^2)^3}-\frac{a(b+3ia)\log(-\sinh(x)+i)}{16(a-ib)^3}+\frac{a(3a+ib)\log(\sinh(x)+i)}{16(-b+ia)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^5/(a + b*Csch[x]),x]
```

```
[Out] -1/16*(a*((3*I)*a + b)*Log[I - Sinh[x]])/(a - I*b)^3 + (a*(3*a + I*b)*Log[I
 + Sinh[x]])/(16*(I*a - b)^3) - (a^4*b*Log[b + a*Sinh[x]])/(a^2 + b^2)^3 -
(Sech[x]^4*(b - a*Sinh[x]))/(4*(a^2 + b^2)) - (Sech[x]^2*(4*a^2*b - a*(3*a^
2 - b^2)*Sinh[x]))/(8*(a^2 + b^2)^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
```

```
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^5(x)}{a+b\operatorname{csch}(x)}\,dx &= i\int \frac{\operatorname{sech}^4(x)\tanh(x)}{ib+ia\sinh(x)}\,dx\\
&= -\left(\left(ia^5\right)\operatorname{Subst}\left(\int \frac{x}{a(ib+x)\left(a^2-x^2\right)^3}\,dx,x,ia\sinh(x)\right)\right)\\
&= -\left(\left(ia^4\right)\operatorname{Subst}\left(\int \frac{x}{(ib+x)\left(a^2-x^2\right)^3}\,dx,x,ia\sinh(x)\right)\right)\\
&= -\frac{\operatorname{sech}^4(x)(b-a\sinh(x))}{4\left(a^2+b^2\right)}-\frac{\left(ia^2\right)\operatorname{Subst}\left(\int \frac{-ia^2b+3a^2x}{(ib+x)\left(a^2-x^2\right)^2}\,dx,x,ia\sinh(x)\right)}{4\left(a^2+b^2\right)}\\
&= -\frac{\operatorname{sech}^4(x)(b-a\sinh(x))}{4\left(a^2+b^2\right)}-\frac{\operatorname{sech}^2(x)\left(4a^2b-a\left(3a^2-b^2\right)\sinh(x)\right)}{8\left(a^2+b^2\right)^2}-\frac{i\operatorname{Subst}\left(\int \frac{-ia^2b(5a^2}{(i}\right.}{}\\
&= -\frac{\operatorname{sech}^4(x)(b-a\sinh(x))}{4\left(a^2+b^2\right)}-\frac{\operatorname{sech}^2(x)\left(4a^2b-a\left(3a^2-b^2\right)\sinh(x)\right)}{8\left(a^2+b^2\right)^2}-\frac{i\operatorname{Subst}\left(\int \left(\frac{a(a-ib)}{2(a+i}\right.\right.}{}\\
&= -\frac{a(3ia+b)\log(i-\sinh(x))}{16(a-ib)^3}+\frac{a(3a+ib)\log(i+\sinh(x))}{16(ia-b)^3}-\frac{a^4b\log(b+a\sinh(x))}{\left(a^2+b^2\right)^3}-
\end{aligned}$$

Mathematica [A]

time = 0.20, size = 138, normalized size = 0.93

$$\frac{2a\left(3a^4-6a^2b^2-b^4\right)\operatorname{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)+8a^4b(\log(\cosh(x))-\log(b+a\sinh(x)))-4a^2b\left(a^2+b^2\right)\operatorname{sech}^2(x)-2b\left(a^2+b^2\right)^2\operatorname{sech}^4(x)+a\left(3a^4+2a^2b^2-b^4\right)\operatorname{sech}(x)\tanh(x)+2a\left(a^2+b^2\right)^2\operatorname{sech}^3(x)\tanh(x)}{8\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^5/(a + b*Csch[x]),x]
```

```
[Out] (2*a*(3*a^4 - 6*a^2*b^2 - b^4)*ArcTan[Tanh[x/2]] + 8*a^4*b*(Log[Cosh[x]] -
Log[b + a*Sinh[x]]) - 4*a^2*b*(a^2 + b^2)*Sech[x]^2 - 2*b*(a^2 + b^2)^2*Sec
h[x]^4 + a*(3*a^4 + 2*a^2*b^2 - b^4)*Sech[x]*Tanh[x] + 2*a*(a^2 + b^2)^2*Se
ch[x]^3*Tanh[x])/(8*(a^2 + b^2)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(135) = 270$.
time = 1.08, size = 316, normalized size = 2.12

method	result
default	$-\frac{a^4 b \ln\left(-b\left(\tanh^2\left(\frac{x}{2}\right)\right)+2a\tanh\left(\frac{x}{2}\right)+b\right)}{(a^4+2a^2b^2+b^4)(a^2+b^2)}+\frac{2\left(\left(-\frac{5}{8}a^5-\frac{3}{4}a^3b^2-\frac{1}{8}a\,b^4\right)\left(\tanh^7\left(\frac{x}{2}\right)\right)+\left(2a^4b+3a^2b^3+b^5\right)\left(\tanh^6\left(\frac{x}{2}\right)\right)+\left(\frac{3}{8}a^5+\frac{5}{4}a^3b^2+\frac{7}{8}a\right.}{}$
risch	$\frac{\left(3a^3\mathrm{e}^{6x}-a\,b^2\mathrm{e}^{6x}-8a^2b\,\mathrm{e}^{5x}+11a^3\mathrm{e}^{4x}+7a\,b^2\mathrm{e}^{4x}-32a^2b\,\mathrm{e}^{3x}-16b^3\mathrm{e}^{3x}-11a^3\mathrm{e}^{2x}-7a\,b^2\mathrm{e}^{2x}-8a^2b\,\mathrm{e}^{x}-3a^3+a\,b^2\right)\mathrm{e}^{x}}{4(a^4+2a^2b^2+b^4)(1+\mathrm{e}^{2x})^4}-\frac{3ia^5\ln(\mathrm{e}^x}{8(a^6+3a^4b^2+3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^5/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -a^4*b/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*ln(-b*tanh(1/2*x)^2+2*a*tanh(1/2*x)+b)
+2/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*(((-5/8*a^5-3/4*a^3*b^2-1/8*a*b^4)*tanh(1/
2*x)^7+(2*a^4*b+3*a^2*b^3+b^5)*tanh(1/2*x)^6+(3/8*a^5+5/4*a^3*b^2+7/8*a*b^4
)*tanh(1/2*x)^5+(2*a^4*b+2*a^2*b^3)*tanh(1/2*x)^4+(-3/8*a^5-5/4*a^3*b^2-7/8
*a*b^4)*tanh(1/2*x)^3+(2*a^4*b+3*a^2*b^3+b^5)*tanh(1/2*x)^2+(5/8*a^5+3/4*a^
3*b^2+1/8*a*b^4)*tanh(1/2*x))/(tanh(1/2*x)^2+1)^4+1/8*a*(4*a^3*b*ln(tanh(1/
2*x)^2+1)+(3*a^4-6*a^2*b^2-b^4)*arctan(tanh(1/2*x))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(130) = 260$.
time = 0.48, size = 348, normalized size = 2.34

$$-\frac{a^4 b \log\left(-2\,b e^{(-x)}+a e^{(-2\,x)}-a\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{a^4 b \log\left(e^{(-2\,x)}+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{(3\,a^5-6\,a^3b^2-ab^4)\arctan\left(e^{(-x)}\right)}{4\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)}-\frac{8\,a^2be^{(-2\,x)}+8\,a^2be^{(-6\,x)}-(3\,a^3-ab^2)e^{(-x)}-(11\,a^3+7\,ab^2)e^{(-3\,x)}+16\,(2\,a^2b+b^3)e^{(-4\,x)}+(11\,a^3+7\,ab^2)e^{(-5\,x)}+(3\,a^3-ab^2)e^{(-7\,x)}}{4\,(a^4+2\,a^2b^2+b^4+4\,(a^4+2\,a^2b^2+b^4)e^{(-2\,x)}+6\,(a^4+2\,a^2b^2+b^4)e^{(-4\,x)}+4\,(a^4+2\,a^2b^2+b^4)e^{(-6\,x)}+(a^4+2\,a^2b^2+b^4)e^{(-8\,x)})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^5/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -a^4*b*log(-2*b*e^(-x) + a*e^(-2*x) - a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
) + a^4*b*log(e^(-2*x) + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 1/4*(3*a^
5 - 6*a^3*b^2 - a*b^4)*arctan(e^(-x))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) -
 1/4*(8*a^2*b*e^(-2*x) + 8*a^2*b*e^(-6*x) - (3*a^3 - a*b^2)*e^(-x) - (11*a^
3 + 7*a*b^2)*e^(-3*x) + 16*(2*a^2*b + b^3)*e^(-4*x) + (11*a^3 + 7*a*b^2)*e^
(-5*x) + (3*a^3 - a*b^2)*e^(-7*x))/(a^4 + 2*a^2*b^2 + b^4 + 4*(a^4 + 2*a^2*
```

```
b^2 + b^4)*e^(-2*x) + 6*(a^4 + 2*a^2*b^2 + b^4)*e^(-4*x) + 4*(a^4 + 2*a^2*b
^2 + b^4)*e^(-6*x) + (a^4 + 2*a^2*b^2 + b^4)*e^(-8*x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2778 vs. $2(130) = 260$.
time = 0.46, size = 2778, normalized size = 18.64

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^5/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] 1/4*((3*a^5 + 2*a^3*b^2 - a*b^4)*cosh(x)^7 + (3*a^5 + 2*a^3*b^2 - a*b^4)*si
nh(x)^7 - 8*(a^4*b + a^2*b^3)*cosh(x)^6 - (8*a^4*b + 8*a^2*b^3 - 7*(3*a^5 +
 2*a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^6 + (11*a^5 + 18*a^3*b^2 + 7*a*b^4)*co
sh(x)^5 + (11*a^5 + 18*a^3*b^2 + 7*a*b^4 + 21*(3*a^5 + 2*a^3*b^2 - a*b^4)*c
osh(x)^2 - 48*(a^4*b + a^2*b^3)*cosh(x))*sinh(x)^5 - 16*(2*a^4*b + 3*a^2*b^
3 + b^5)*cosh(x)^4 - (32*a^4*b + 48*a^2*b^3 + 16*b^5 - 35*(3*a^5 + 2*a^3*b^
2 - a*b^4)*cosh(x)^3 + 120*(a^4*b + a^2*b^3)*cosh(x)^2 - 5*(11*a^5 + 18*a^3
*b^2 + 7*a*b^4)*cosh(x))*sinh(x)^4 - (11*a^5 + 18*a^3*b^2 + 7*a*b^4)*cosh(x
)^3 - (11*a^5 + 18*a^3*b^2 + 7*a*b^4 - 35*(3*a^5 + 2*a^3*b^2 - a*b^4)*cosh(
x)^4 + 160*(a^4*b + a^2*b^3)*cosh(x)^3 - 10*(11*a^5 + 18*a^3*b^2 + 7*a*b^4)
*cosh(x)^2 + 64*(2*a^4*b + 3*a^2*b^3 + b^5)*cosh(x))*sinh(x)^3 - 8*(a^4*b +
 a^2*b^3)*cosh(x)^2 + (21*(3*a^5 + 2*a^3*b^2 - a*b^4)*cosh(x)^5 - 8*a^4*b -
 8*a^2*b^3 - 120*(a^4*b + a^2*b^3)*cosh(x)^4 + 10*(11*a^5 + 18*a^3*b^2 + 7*
a*b^4)*cosh(x)^3 - 96*(2*a^4*b + 3*a^2*b^3 + b^5)*cosh(x)^2 - 3*(11*a^5 + 1
8*a^3*b^2 + 7*a*b^4)*cosh(x))*sinh(x)^2 + ((3*a^5 - 6*a^3*b^2 - a*b^4)*cosh
(x)^8 + 8*(3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x)*sinh(x)^7 + (3*a^5 - 6*a^3*b^
2 - a*b^4)*sinh(x)^8 + 4*(3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x)^6 + 4*(3*a^5 -
 6*a^3*b^2 - a*b^4 + 7*(3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x)^2)*sinh(x)^6 + 8
*(7*(3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x)^3 + 3*(3*a^5 - 6*a^3*b^2 - a*b^4)*c
osh(x))*sinh(x)^5 + 3*a^5 - 6*a^3*b^2 - a*b^4 + 6*(3*a^5 - 6*a^3*b^2 - a*b^
4)*cosh(x)^4 + 2*(9*a^5 - 18*a^3*b^2 - 3*a*b^4 + 35*(3*a^5 - 6*a^3*b^2 - a*
b^4)*cosh(x)^4 + 30*(3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x)^2)*sinh(x)^4 + 8*(7
*(3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x)^5 + 10*(3*a^5 - 6*a^3*b^2 - a*b^4)*cos
h(x)^3 + 3*(3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^3 + 4*(3*a^5 - 6*a^
3*b^2 - a*b^4)*cosh(x)^2 + 4*(7*(3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x)^6 + 3*a
^5 - 6*a^3*b^2 - a*b^4 + 15*(3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x)^4 + 9*(3*a^
5 - 6*a^3*b^2 - a*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((3*a^5 - 6*a^3*b^2 - a*b^4
)*cosh(x)^7 + 3*(3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x)^5 + 3*(3*a^5 - 6*a^3*b^
2 - a*b^4)*cosh(x)^3 + (3*a^5 - 6*a^3*b^2 - a*b^4)*cosh(x))*sinh(x))*arctan
(cosh(x) + sinh(x)) - (3*a^5 + 2*a^3*b^2 - a*b^4)*cosh(x) - 4*(a^4*b*cosh(x
)^8 + 8*a^4*b*cosh(x)*sinh(x)^7 + a^4*b*sinh(x)^8 + 4*a^4*b*cosh(x)^6 + 6*a
^4*b*cosh(x)^4 + 4*a^4*b*cosh(x)^2 + 4*(7*a^4*b*cosh(x)^2 + a^4*b)*sinh(x)^
6 + 8*(7*a^4*b*cosh(x)^3 + 3*a^4*b*cosh(x))*sinh(x)^5 + a^4*b + 2*(35*a^4*b
```

```
*cosh(x)^4 + 30*a^4*b*cosh(x)^2 + 3*a^4*b)*sinh(x)^4 + 8*(7*a^4*b*cosh(x)^5
 + 10*a^4*b*cosh(x)^3 + 3*a^4*b*cosh(x))*sinh(x)^3 + 4*(7*a^4*b*cosh(x)^6 +
 15*a^4*b*cosh(x)^4 + 9*a^4*b*cosh(x)^2 + a^4*b)*sinh(x)^2 + 8*(a^4*b*cosh(
x)^7 + 3*a^4*b*cosh(x)^5 + 3*a^4*b*cosh(x)^3 + a^4*b*cosh(x))*sinh(x))*log(
2*(a*sinh(x) + b)/(cosh(x) - sinh(x))) + 4*(a^4*b*cosh(x)^8 + 8*a^4*b*cosh(
x)*sinh(x)^7 + a^4*b*sinh(x)^8 + 4*a^4*b*cosh(x)^6 + 6*a^4*b*cosh(x)^4 + 4*
a^4*b*cosh(x)^2 + 4*(7*a^4*b*cosh(x)^2 + a^4*b)*sinh(x)^6 + 8*(7*a^4*b*cosh
(x)^3 + 3*a^4*b*cosh(x))*sinh(x)^5 + a^4*b + 2*(35*a^4*b*cosh(x)^4 + 30*a^4
*b*cosh(x)^2 + 3*a^4*b)*sinh(x)^4 + 8*(7*a^4*b*cosh(x)^5 + 10*a^4*b*cosh(x)
^3 + 3*a^4*b*cosh(x))*sinh(x)^3 + 4*(7*a^4*b*cosh(x)^6 + 15*a^4*b*cosh(x)^4
 + 9*a^4*b*cosh(x)^2 + a^4*b)*sinh(x)^2 + 8*(a^4*b*cosh(x)^7 + 3*a^4*b*cosh
(x)^5 + 3*a^4*b*cosh(x)^3 + a^4*b*cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x)
- sinh(x))) + (7*(3*a^5 + 2*a^3*b^2 - a*b^4)*cosh(x)^6 - 48*(a^4*b + a^2*b^
3)*cosh(x)^5 - 3*a^5 - 2*a^3*b^2 + a*b^4 + 5*(11*a^5 + 18*a^3*b^2 + 7*a*b^4
)*cosh(x)^4 - 64*(2*a^4*b + 3*a^2*b^3 + b^5)*cosh(x)^3 - 3*(11*a^5 + 18*a^3
*b^2 + 7*a*b^4)*cosh(x)^2 - 16*(a^4*b + a^2*b^3)*cosh(x))*sinh(x))/((a^6 +
3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^8 + 8*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b
^6)*cosh(x)*sinh(x)^7 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sinh(x)^8 + 4*(
a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^6 + 4*(a^6 + 3*a^4*b^2 + 3*a^2*b
^4 + b^6 + 7*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(x)^6 + a^6
 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 8*(7*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c
osh(x)^3 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*sinh(x)^5 + 6*(a^
6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^4 + 2*(3*a^6 + 9*a^4*b^2 + 9*a^2*b
^4 + 3*b^6 + 35*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^4 + 30*(a^6 + 3
*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^6 + 3*a^4*b^2 +
3*a^2*b^4 + b^6)*cosh(x)^5 + 10*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)
^3 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*sinh(x)^3 + 4*(a^6 + 3*
a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2 + 4*(7*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
b^6)*cosh(x)^6 + a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 15*(a^6 + 3*a^4*b^2 +
3*a^2*b^4 + b^6)*cosh(x)^4 + 9*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^
2)*sinh(x)^2 + 8*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^7 + 3*(a^6 +
3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^5 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b
^6)*cosh(x)^3 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*sinh(x))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^5(x)}{a + b\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**5/(a+b*csch(x)),x)
```

```
[Out] Integral(sech(x)**5/(a + b*csch(x)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 374 vs. 2(130) = 260.

time = 0.40, size = 374, normalized size = 2.51

$$-\frac{a^{5}b\log\left(\left|-a\left(e^{(-x)}-e^{x}\right)+2\,b\right|\right)}{a^{7}+3\,a^{5}b^{2}+3\,a^{3}b^{4}+ab^{6}}+\frac{a^{4}b\log\left(\left(e^{(-x)}-e^{x}\right)^{2}+4\right)}{2\left(a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}\right)}+\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)}-1\right)e^{(-x)}\right)\right)\left(3\,a^{5}-6\,a^{3}b^{2}-ab^{4}\right)}{16\left(a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}\right)}-\frac{3\,a^{4}b\left(e^{(-x)}-e^{x}\right)^{4}+3\,a^{5}\left(e^{(-x)}-e^{x}\right)^{3}+2\,a^{3}b^{2}\left(e^{(-x)}-e^{x}\right)^{3}-ab^{4}\left(e^{(-x)}-e^{x}\right)^{3}+32\,a^{4}b\left(e^{(-x)}-e^{x}\right)^{2}+8\,a^{2}b^{3}\left(e^{(-x)}-e^{x}\right)^{2}+20\,a^{5}\left(e^{(-x)}-e^{x}\right)+24\,a^{3}b^{2}\left(e^{(-x)}-e^{x}\right)+4\,ab^{4}\left(e^{(-x)}-e^{x}\right)+96\,a^{4}b+64\,a^{2}b^{3}+16\,b^{5}}{4\left(a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}\right)\left(\left(e^{(-x)}-e^{x}\right)^{2}+4\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^5/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -a^5*b*log(abs(-a*(e^(-x) - e^x) + 2*b))/(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b
^6) + 1/2*a^4*b*log((e^(-x) - e^x)^2 + 4)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^
6) + 1/16*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))*(3*a^5 - 6*a^3*b^2 - a*
b^4)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 1/4*(3*a^4*b*(e^(-x) - e^x)^4 +
3*a^5*(e^(-x) - e^x)^3 + 2*a^3*b^2*(e^(-x) - e^x)^3 - a*b^4*(e^(-x) - e^x)^
3 + 32*a^4*b*(e^(-x) - e^x)^2 + 8*a^2*b^3*(e^(-x) - e^x)^2 + 20*a^5*(e^(-x)
 - e^x) + 24*a^3*b^2*(e^(-x) - e^x) + 4*a*b^4*(e^(-x) - e^x) + 96*a^4*b + 6
4*a^2*b^3 + 16*b^5)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*((e^(-x) - e^x)^2
+ 4)^2)
```

Mupad [B]
time = 5.51, size = 513, normalized size = 3.44

$$\frac{\frac{8\left(a^{2}\,b+b^{3}\right)}{\left(a^{2}+b^{2}\right)^{2}}-\frac{6\,e^{x}\left(a^{3}+a\,b^{2}\right)}{\left(a^{2}+b^{2}\right)^{2}}}{3\,e^{2\,x}+3\,e^{4\,x}+e^{6\,x}+1}-\frac{\frac{2\left(a^{2}\,b+2\,b^{3}\right)}{\left(a^{2}+b^{2}\right)^{2}}-\frac{e^{x}\left(a^{3}+5\,a\,b^{2}\right)}{2\left(a^{2}+b^{2}\right)^{2}}}{2\,e^{2\,x}+e^{4\,x}+1}-\frac{\frac{4\,b}{a^{2}+b^{2}}-\frac{4\,a\,e^{x}}{a^{2}+b^{2}}}{4\,e^{2\,x}+6\,e^{4\,x}+4\,e^{6\,x}+e^{8\,x}+1}-\frac{\frac{2\left(a^{4}\,b+a^{2}\,b^{3}\right)}{\left(a^{2}+b^{2}\right)^{3}}-\frac{e^{x}\left(3\,a^{5}+2\,a^{3}\,b^{2}-a\,b^{4}\right)}{4\left(a^{2}+b^{2}\right)^{3}}}{e^{2\,x}+1}+\frac{\ln\left(e^{x}+1\mathrm{i}\right)\,\left(a\,b-a^{2}\,3\mathrm{i}\right)}{8\left(-a^{3}-a^{2}\,b\,3\mathrm{i}+3\,a\,b^{2}+b^{3}\,1\mathrm{i}\right)}+\frac{\ln\left(1+e^{x}\,1\mathrm{i}\right)\,\left(-3\,a^{2}+a\,b\,1\mathrm{i}\right)}{8\left(-a^{3}\,1\mathrm{i}-3\,a^{2}\,b+a\,b^{2}\,3\mathrm{i}+b^{3}\right)}-\frac{a^{4}\,b\,\ln\left(9\,a^{10}\,e^{2\,x}-9\,a^{10}-a^{2}\,b^{8}-12\,a^{4}\,b^{6}-30\,a^{6}\,b^{4}-220\,a^{8}\,b^{2}+a^{2}\,b^{8}\,e^{2\,x}+12\,a^{4}\,b^{6}\,e^{2\,x}+30\,a^{6}\,b^{4}\,e^{2\,x}+220\,a^{8}\,b^{2}\,e^{2\,x}+2\,a\,b^{9}\,e^{x}+18\,a^{9}\,b\,e^{x}+24\,a^{3}\,b^{7}\,e^{x}+60\,a^{5}\,b^{5}\,e^{x}+440\,a^{7}\,b^{3}\,e^{x}\right)}{a^{6}+3\,a^{4}\,b^{2}+3\,a^{2}\,b^{4}+b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^5*(a + b/sinh(x))),x)
```

```
[Out] ((8*(a^2*b + b^3))/(a^2 + b^2)^2 - (6*exp(x)*(a*b^2 + a^3))/(a^2 + b^2)^2)/
(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - ((2*(a^2*b + 2*b^3))/(a^2 + b^2)
^2 - (exp(x)*(5*a*b^2 + a^3))/(2*(a^2 + b^2)^2))/(2*exp(2*x) + exp(4*x) + 1
) - ((4*b)/(a^2 + b^2) - (4*a*exp(x))/(a^2 + b^2))/(4*exp(2*x) + 6*exp(4*x)
 + 4*exp(6*x) + exp(8*x) + 1) - ((2*(a^4*b + a^2*b^3))/(a^2 + b^2)^3 - (exp
(x)*(3*a^5 - a*b^4 + 2*a^3*b^2))/(4*(a^2 + b^2)^3))/(exp(2*x) + 1) + (log(e
xp(x) + 1i)*(a*b - a^2*3i))/(8*(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)) + (log(
exp(x)*1i + 1)*(a*b*1i - 3*a^2))/(8*(a*b^2*3i - 3*a^2*b - a^3*1i + b^3)) -
(a^4*b*log(9*a^10*exp(2*x) - 9*a^10 - a^2*b^8 - 12*a^4*b^6 - 30*a^6*b^4 - 2
20*a^8*b^2 + a^2*b^8*exp(2*x) + 12*a^4*b^6*exp(2*x) + 30*a^6*b^4*exp(2*x) +
 220*a^8*b^2*exp(2*x) + 2*a*b^9*exp(x) + 18*a^9*b*exp(x) + 24*a^3*b^7*exp(x
) + 60*a^5*b^5*exp(x) + 440*a^7*b^3*exp(x)))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4
*b^2)
```

3.102 $\int \frac{\tanh^5(x)}{i+\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=109

$$-\frac{21}{32}i\log(i-\sinh(x))-\frac{11}{32}i\log(i+\sinh(x))+\frac{i}{32(1-i\sinh(x))^2}-\frac{i}{4(1-i\sinh(x))}-\frac{i}{24(1+i\sinh(x))^3}+\frac{\ }{32(1}$$

```
[Out] -21/32*I*ln(I-sinh(x))-11/32*I*ln(I+sinh(x))+1/32*I/(1-I*sinh(x))^2-1/4*I/(
1-I*sinh(x))-1/24*I/(1+I*sinh(x))^3+9/32*I/(1+I*sinh(x))^2-15/16*I/(1+I*sin
h(x))
```

Rubi [A]
time = 0.06, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3964, 90}

$$-\frac{i}{4(1-i\sinh(x))}-\frac{15i}{16(1+i\sinh(x))}+\frac{i}{32(1-i\sinh(x))^2}+\frac{9i}{32(1+i\sinh(x))^2}-\frac{i}{24(1+i\sinh(x))^3}-\frac{21}{32}i\log(-\sinh(x)+i)-\frac{11}{32}i\log(\sinh(x)+i)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^5/(I + Csch[x]),x]
```

```
[Out] ((-21*I)/32)*Log[I - Sinh[x]] - ((11*I)/32)*Log[I + Sinh[x]] + (I/32)/(1 -
I*Sinh[x])^2 - (I/4)/(1 - I*Sinh[x]) - (I/24)/(1 + I*Sinh[x])^3 + ((9*I)/32
)/(1 + I*Sinh[x])^2 - ((15*I)/16)/(1 + I*Sinh[x])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int \frac{\tanh^5(x)}{i+\operatorname{csch}(x)}\,dx = \text{Subst}\left(\int \frac{x^6}{(i-ix)^3(i+ix)^4}\,dx, x, i\sinh(x)\right)$$
$$= \text{Subst}\left(\int \left(-\frac{i}{16(-1+x)^3} - \frac{i}{4(-1+x)^2} - \frac{11i}{32(-1+x)} + \frac{i}{8(1+x)^4} - \frac{9i}{16(1+x)^3} + \frac{}{16}\right.\right.$$
$$= -\frac{21}{32}i\log(i-\sinh(x)) - \frac{11}{32}i\log(i+\sinh(x)) + \frac{i}{32(1-i\sinh(x))^2} - \frac{i}{4(1-i\sinh(x))} -$$

Mathematica [A]
time = 0.14, size = 75, normalized size = 0.69

$$\frac{1}{96}\left(-63i\log(i-\sinh(x)) - 33i\log(i+\sinh(x)) - \frac{2(44+29i\sinh(x)+79\sinh^2(x)+39i\sinh^3(x)+33\sinh^4(x))}{(-i+\sinh(x))^3(i+\sinh(x))^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^5/(I + Csch[x]),x]
```

```
[Out] ((-63*I)*Log[I - Sinh[x]] - (33*I)*Log[I + Sinh[x]] - (2*(44 + (29*I)*Sinh[
x] + 79*Sinh[x]^2 + (39*I)*Sinh[x]^3 + 33*Sinh[x]^4))/((-I + Sinh[x])^3*(I
+ Sinh[x])^2))/96
```

Maple [A]
time = 1.56, size = 155, normalized size = 1.42

method	result
risch	$ix - \frac{2ie^{4x}+184\,e^{3x}-78ie^{2x}+270\,e^{5x}+33\,e^{x}-2ie^{6x}+184\,e^{7x}+78ie^{8x}+33\,e^{9x}}{24(e^{x}-i)^6(e^{x}+i)^4} - \frac{21i\ln(e^{x}-i)}{16} - \frac{11i\ln(e^{x}+i)}{16}$
default	$-\frac{21i\ln\left(-i+\tanh\left(\frac{x}{2}\right)\right)}{16} + \frac{3i}{8\left(-i+\tanh\left(\frac{x}{2}\right)\right)^2} + \frac{i}{3\left(-i+\tanh\left(\frac{x}{2}\right)\right)^6} - \frac{3i}{8\left(-i+\tanh\left(\frac{x}{2}\right)\right)^4} + \frac{1}{\left(-i+\tanh\left(\frac{x}{2}\right)\right)^5} + \frac{11}{12\left(-i+\tanh\left(\frac{x}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(I+csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -21/16*I*ln(-I+tanh(1/2*x))+3/8*I/(-I+tanh(1/2*x))^2+1/3*I/(-I+tanh(1/2*x))
^6-3/8*I/(-I+tanh(1/2*x))^4+1/(-I+tanh(1/2*x))^5+11/12/(-I+tanh(1/2*x))^3+1
/(-I+tanh(1/2*x))-11/16*I*ln(tanh(1/2*x)+I)+1/8*I/(tanh(1/2*x)+I)^4+1/4*I/(
tanh(1/2*x)+I)^2-1/4/(tanh(1/2*x)+I)^3-3/8/(tanh(1/2*x)+I)+I*ln(tanh(1/2*x)
+1)+I*ln(tanh(1/2*x)-1)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(67) = 134.
time = 0.33, size = 144, normalized size = 1.32

$$-ix + \frac{33\,e^{(-x)} + 78i\,e^{(-2x)} + 184\,e^{(-3x)} - 2i\,e^{(-4x)} + 270\,e^{(-5x)} + 2i\,e^{(-6x)} + 184\,e^{(-7x)} - 78i\,e^{(-8x)} + 33\,e^{(-9x)}}{48i\,e^{(-x)} - 72\,e^{(-2x)} + 192i\,e^{(-3x)} - 48\,e^{(-4x)} + 288i\,e^{(-5x)} + 48\,e^{(-6x)} + 192i\,e^{(-7x)} + 72\,e^{(-8x)} + 48i\,e^{(-9x)} + 24\,e^{(-10x)} - 24} - \frac{11}{16}i\log\left(e^{(-x)} - i\right) - \frac{21}{16}i\log\left(i\,e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -I*x + (33*e^(-x) + 78*I*e^(-2*x) + 184*e^(-3*x) - 2*I*e^(-4*x) + 270*e^(-5
*x) + 2*I*e^(-6*x) + 184*e^(-7*x) - 78*I*e^(-8*x) + 33*e^(-9*x))/(48*I*e^(-
x) - 72*e^(-2*x) + 192*I*e^(-3*x) - 48*e^(-4*x) + 288*I*e^(-5*x) + 48*e^(-6
*x) + 192*I*e^(-7*x) + 72*e^(-8*x) + 48*I*e^(-9*x) + 24*e^(-10*x) - 24) - 1
1/16*I*log(e^(-x) - I) - 21/16*I*log(I*e^(-x) - 1)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(67) = 134$.
time = 0.38, size = 304, normalized size = 2.79

$$\frac{48i\,xe^{(10\,x)} + 6\,(16\,x - 11)e^{(9\,x)} - 12\,(-12i\,x + 13i)e^{(8\,x)} + 16\,(24\,x - 23)e^{(7\,x)} - 4\,(-24i\,x - i)e^{(6\,x)} + 36\,(16\,x - 15)e^{(5\,x)} - 4\,(24i\,x + i)e^{(4\,x)} + 16\,(24\,x - 23)e^{(3\,x)} - 12\,(12i\,x - 13i)e^{(2\,x)} + 6\,(16\,x - 11)e^{x} - 33\,\left(i\,e^{(10\,x)} + 2\,e^{(9\,x)} + 3i\,e^{(8\,x)} + 8\,e^{(7\,x)} + 2i\,e^{(6\,x)} + 12\,e^{(5\,x)} - 2i\,e^{(4\,x)} + 8\,e^{(3\,x)} - 3i\,e^{(2\,x)} + 2\,e^{x} - i\right)\log\left(e^{x} + i\right) - 63\,\left(i\,e^{(10\,x)} + 2\,e^{(9\,x)} + 3i\,e^{(8\,x)} + 8\,e^{(7\,x)} + 2i\,e^{(6\,x)} + 12\,e^{(5\,x)} - 2i\,e^{(4\,x)} + 8\,e^{(3\,x)} - 3i\,e^{(2\,x)} + 2\,e^{x} - i\right)\log\left(e^{x} - i\right) - 48i\,x}{48\,\left(e^{(10\,x)} - 2i\,e^{(9\,x)} + 3\,e^{(8\,x)} - 8i\,e^{(7\,x)} + 2\,e^{(6\,x)} - 12i\,e^{(5\,x)} - 2\,e^{(4\,x)} - 8i\,e^{(3\,x)} - 3\,e^{(2\,x)} - 2i\,e^{x} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] 1/48*(48*I*x*e^(10*x) + 6*(16*x - 11)*e^(9*x) - 12*(-12*I*x + 13*I)*e^(8*x)
 + 16*(24*x - 23)*e^(7*x) - 4*(-24*I*x - I)*e^(6*x) + 36*(16*x - 15)*e^(5*x
) - 4*(24*I*x + I)*e^(4*x) + 16*(24*x - 23)*e^(3*x) - 12*(12*I*x - 13*I)*e^
(2*x) + 6*(16*x - 11)*e^x - 33*(I*e^(10*x) + 2*e^(9*x) + 3*I*e^(8*x) + 8*e^
(7*x) + 2*I*e^(6*x) + 12*e^(5*x) - 2*I*e^(4*x) + 8*e^(3*x) - 3*I*e^(2*x) +
2*e^x - I)*log(e^x + I) - 63*(I*e^(10*x) + 2*e^(9*x) + 3*I*e^(8*x) + 8*e^(7
*x) + 2*I*e^(6*x) + 12*e^(5*x) - 2*I*e^(4*x) + 8*e^(3*x) - 3*I*e^(2*x) + 2*
e^x - I)*log(e^x - I) - 48*I*x)/(e^(10*x) - 2*I*e^(9*x) + 3*e^(8*x) - 8*I*e
^(7*x) + 2*e^(6*x) - 12*I*e^(5*x) - 2*e^(4*x) - 8*I*e^(3*x) - 3*e^(2*x) - 2
*I*e^x - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{5}\left(x\right)}{\operatorname{csch}\left(x\right) + i}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**5/(I+csch(x)),x)
```

```
[Out] Integral(tanh(x)**5/(csch(x) + I), x)
```

Giac [A]
time = 0.40, size = 120, normalized size = 1.10

$$-\frac{33i\,\left(e^{(-x)} - e^{x}\right)^{2} + 100\,e^{(-x)} - 100\,e^{x} - 76i}{64\,\left(-i\,e^{(-x)} + i\,e^{x} - 2\right)^{2}} - \frac{-231i\,\left(e^{(-x)} - e^{x}\right)^{3} + 1026\,\left(e^{(-x)} - e^{x}\right)^{2} + 1548i\,e^{(-x)} - 1548i\,e^{x} - 776}{192\,\left(e^{(-x)} - e^{x} + 2i\right)^{3}} - \frac{11}{32}i\,\log\left(-e^{(-x)} + e^{x} + 2i\right) - \frac{21}{32}i\,\log\left(-e^{(-x)} + e^{x} - 2i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(I+csch(x)),x, algorithm="giac")
```

```
[Out] -1/64*(33*I*(e^(-x) - e^x)^2 + 100*e^(-x) - 100*e^x - 76*I)/(-I*e^(-x) + I*
e^x - 2)^2 - 1/192*(-231*I*(e^(-x) - e^x)^3 + 1026*(e^(-x) - e^x)^2 + 1548*
I*e^(-x) - 1548*I*e^x - 776)/(e^(-x) - e^x + 2*I)^3 - 11/32*I*log(-e^(-x) +
 e^x + 2*I) - 21/32*I*log(-e^(-x) + e^x - 2*I)
```

Mupad [B]
time = 4.11, size = 274, normalized size = 2.51

$$x\,1\mathrm{i}-\ln\left(\left(\frac{5\,\mathrm{e}^x}{8}-\frac{5}{8}\mathrm{i}\right)\,\left(\frac{5\,\mathrm{e}^x}{8}+\frac{5}{8}\mathrm{i}\right)\right)\,1\mathrm{i}+\frac{5\,\mathrm{atan}\left(\mathrm{e}^x\right)}{8}+\frac{1\mathrm{i}}{3\,\left(15\,\mathrm{e}^{4\,x}-15\,\mathrm{e}^{2\,x}-\mathrm{e}^{6\,x}+1-\mathrm{e}^{3\,x}\,20\mathrm{i}+\mathrm{e}^{5\,x}\,6\mathrm{i}+\mathrm{e}^x\,6\mathrm{i}\right)}-\frac{1}{\mathrm{e}^{2\,x}\,10\mathrm{i}-10\,\mathrm{e}^{3\,x}-\mathrm{e}^{4\,x}\,5\mathrm{i}+\mathrm{e}^{5\,x}+5\,\mathrm{e}^x-\mathrm{i}}-\frac{31}{12\,\left(\mathrm{e}^{2\,x}\,3\mathrm{i}-\mathrm{e}^{3\,x}+3\,\mathrm{e}^x-\mathrm{i}\right)}-\frac{5\mathrm{i}}{8\,\left(\mathrm{e}^{2\,x}-1+\mathrm{e}^x\,2\mathrm{i}\right)}+\frac{17\mathrm{i}}{8\,\left(\mathrm{e}^{4\,x}-6\,\mathrm{e}^{2\,x}+1-\mathrm{e}^{3\,x}\,4\mathrm{i}+\mathrm{e}^x\,4\mathrm{i}\right)}+\frac{1\mathrm{i}}{8\,\left(\mathrm{e}^{4\,x}-6\,\mathrm{e}^{2\,x}+1+\mathrm{e}^{3\,x}\,4\mathrm{i}-\mathrm{e}^x\,4\mathrm{i}\right)}+\frac{3\mathrm{i}}{1-\mathrm{e}^{2\,x}+\mathrm{e}^x\,2\mathrm{i}}-\frac{15}{8\,\left(\mathrm{e}^x-\mathrm{i}\right)}+\frac{1}{2\,\left(\mathrm{e}^x+1\mathrm{i}\right)}-\frac{1}{4\,\left(\mathrm{e}^{2\,x}\,3\mathrm{i}+\mathrm{e}^{3\,x}-3\,\mathrm{e}^x-\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(1/sinh(x) + 1i),x)
```

```
[Out] x*1i - log(((5*exp(x))/8 - 5i/8)*((5*exp(x))/8 + 5i/8))*1i + (5*atan(exp(x)
))/8 + 1i/(3*(15*exp(4*x) - exp(3*x)*20i - 15*exp(2*x) + exp(5*x)*6i - exp(
6*x) + exp(x)*6i + 1)) - 1/(exp(2*x)*10i - 10*exp(3*x) - exp(4*x)*5i + exp(
5*x) + 5*exp(x) - 1i) - 31/(12*(exp(2*x)*3i - exp(3*x) + 3*exp(x) - 1i)) -
5i/(8*(exp(2*x) + exp(x)*2i - 1)) + 17i/(8*(exp(4*x) - exp(3*x)*4i - 6*exp(
2*x) + exp(x)*4i + 1)) + 1i/(8*(exp(3*x)*4i - 6*exp(2*x) + exp(4*x) - exp(x
)*4i + 1)) + 3i/(exp(x)*2i - exp(2*x) + 1) - 15/(8*(exp(x) - 1i)) + 1/(2*(e
xp(x) + 1i)) - 1/(4*(exp(2*x)*3i + exp(3*x) - 3*exp(x) - 1i))
```

3.103 $\int \frac{\tanh^4(x)}{i+\operatorname{csch}(x)} \, dx$

Optimal. Leaf size=52

$$-ix + \frac{1}{15}(15i - 8\operatorname{csch}(x))\tanh(x) + \frac{1}{15}(5i - 4\operatorname{csch}(x))\tanh^3(x) + \frac{1}{5}(i - \operatorname{csch}(x))\tanh^5(x)$$

```
[Out] -I*x+1/15*(15*I-8*csch(x))*tanh(x)+1/15*(5*I-4*csch(x))*tanh(x)^3+1/5*(I-cs
ch(x))*tanh(x)^5
```

Rubi [A]

time = 0.08, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3973, 3967, 8}

$$-ix + \frac{1}{5}\tanh^5(x)(-\operatorname{csch}(x) + i) + \frac{1}{15}\tanh^3(x)(-4\operatorname{csch}(x) + 5i) + \frac{1}{15}\tanh(x)(-8\operatorname{csch}(x) + 15i)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/(I + Csch[x]),x]
```

```
[Out] (-I)*x + ((15*I - 8*Csch[x])*Tanh[x])/15 + ((5*I - 4*Csch[x])*Tanh[x]^3)/15
 + ((I - Csch[x])*Tanh[x]^5)/5
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\tanh^4(x)}{i+\operatorname{csch}(x)}\,dx = \int (-i+\operatorname{csch}(x))\tanh^6(x)\,dx$$
$$= \frac{1}{5}(i-\operatorname{csch}(x))\tanh^5(x) - \frac{1}{5}\int (5i-4\operatorname{csch}(x))\tanh^4(x)\,dx$$
$$= \frac{1}{15}(5i-4\operatorname{csch}(x))\tanh^3(x) + \frac{1}{5}(i-\operatorname{csch}(x))\tanh^5(x) + \frac{1}{15}\int (-15i+8\operatorname{csch}(x))\tanh^2($$
$$= \frac{1}{15}(15i-8\operatorname{csch}(x))\tanh(x) + \frac{1}{15}(5i-4\operatorname{csch}(x))\tanh^3(x) + \frac{1}{5}(i-\operatorname{csch}(x))\tanh^5(x) -$$
$$= -ix + \frac{1}{15}(15i-8\operatorname{csch}(x))\tanh(x) + \frac{1}{15}(5i-4\operatorname{csch}(x))\tanh^3(x) + \frac{1}{5}(i-\operatorname{csch}(x))\tanh^5$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 126 vs. $2(52) = 104$.
time = 0.09, size = 126, normalized size = 2.42

$$\frac{-200+6(89-120ix)\cosh(x)-128\cosh(2x)+178\cosh(3x)-240ix\cosh(3x)-184\cosh(4x)+64i\sinh(x)+178i\sinh(2x)+240x\sinh(2x)+128i\sinh(3x)+89i\sinh(4x)+120x\sinh(4x)}{960\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)^3\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(I + Csch[x]),x]
```

```
[Out] (-200 + 6*(89 - (120*I)*x)*Cosh[x] - 128*Cosh[2*x] + 178*Cosh[3*x] - (240*I
)*x*Cosh[3*x] - 184*Cosh[4*x] + (64*I)*Sinh[x] + (178*I)*Sinh[2*x] + 240*x*
Sinh[2*x] + (128*I)*Sinh[3*x] + (89*I)*Sinh[4*x] + 120*x*Sinh[4*x])/(960*(C
osh[x/2] - I*Sinh[x/2])^3*(Cosh[x/2] + I*Sinh[x/2])^5)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(42) = 84$.
time = 1.49, size = 99, normalized size = 1.90

method	result
risch	$-ix - \frac{2(-31ie^{2x}+73\,e^{3x}+31\,e^{x}-25ie^{4x}+65\,e^{5x}-23i+15ie^{6x}+15\,e^{7x})}{15(e^{x}+i)^3(e^{x}-i)^5}$
default	$\frac{11i}{8(-i+\tanh(\frac{x}{2}))} + \frac{2i}{5(-i+\tanh(\frac{x}{2}))^5} + \frac{1}{(-i+\tanh(\frac{x}{2}))^4} + \frac{1}{(-i+\tanh(\frac{x}{2}))^2} - i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right) + i\ln\left(\tanh\left(\frac{x}{2}\right)\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(I+csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 11/8*I/(-I+tanh(1/2*x))+2/5*I/(-I+tanh(1/2*x))^5+1/(-I+tanh(1/2*x))^4+1/(-I
+tanh(1/2*x))^2-I*ln(tanh(1/2*x)+1)+I*ln(tanh(1/2*x)-1)+5/8*I/(tanh(1/2*x)+
I)+1/6*I/(tanh(1/2*x)+I)^3-1/4/(tanh(1/2*x)+I)^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(36) = 72$.
time = 0.27, size = 96, normalized size = 1.85

$$-ix - \frac{2\left(31\,e^{(-x)} + 31i\,e^{(-2\,x)} + 73\,e^{(-3\,x)} + 25i\,e^{(-4\,x)} + 65\,e^{(-5\,x)} - 15i\,e^{(-6\,x)} + 15\,e^{(-7\,x)} + 23i\right)}{30i\,e^{(-x)} - 30\,e^{(-2\,x)} + 90i\,e^{(-3\,x)} + 90i\,e^{(-5\,x)} + 30\,e^{(-6\,x)} + 30i\,e^{(-7\,x)} + 15\,e^{(-8\,x)} - 15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -I*x - 2*(31*e^(-x) + 31*I*e^(-2*x) + 73*e^(-3*x) + 25*I*e^(-4*x) + 65*e^(-
5*x) - 15*I*e^(-6*x) + 15*e^(-7*x) + 23*I)/(30*I*e^(-x) - 30*e^(-2*x) + 90*
I*e^(-3*x) + 90*I*e^(-5*x) + 30*e^(-6*x) + 30*I*e^(-7*x) + 15*e^(-8*x) - 15
)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(36) = 72$.
time = 0.39, size = 124, normalized size = 2.38

$$\frac{-15i\,xe^{(8\,x)} - 30\,(x+1)e^{(7\,x)} - 30\,(i\,x+i)e^{(6\,x)} - 10\,(9\,x+13)e^{(5\,x)} - 2\,(45\,x+73)e^{(3\,x)} - 2\,(-15i\,x - 31i)e^{(2\,x)} - 2\,(15\,x+31)e^{x} + 15i\,x + 50i\,e^{(4\,x)} + 46i}{15\left(e^{(8\,x)} - 2i\,e^{(7\,x)} + 2\,e^{(6\,x)} - 6i\,e^{(5\,x)} - 6i\,e^{(3\,x)} - 2\,e^{(2\,x)} - 2i\,e^{x} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] 1/15*(-15*I*x*e^(8*x) - 30*(x + 1)*e^(7*x) - 30*(I*x + I)*e^(6*x) - 10*(9*x
 + 13)*e^(5*x) - 2*(45*x + 73)*e^(3*x) - 2*(-15*I*x - 31*I)*e^(2*x) - 2*(15
*x + 31)*e^x + 15*I*x + 50*I*e^(4*x) + 46*I)/(e^(8*x) - 2*I*e^(7*x) + 2*e^(
6*x) - 6*I*e^(5*x) - 6*I*e^(3*x) - 2*e^(2*x) - 2*I*e^x - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(x)}{\operatorname{csch}(x) + i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(I+csch(x)),x)
```

```
[Out] Integral(tanh(x)**4/(csch(x) + I), x)
```

Giac [A]
time = 0.39, size = 62, normalized size = 1.19

$$-\frac{21i\,e^{(2\,x)} - 36\,e^{x} - 19i}{24\,(i\,e^{x} - 1)^3} - \frac{115\,e^{(4\,x)} - 380i\,e^{(3\,x)} - 530\,e^{(2\,x)} + 340i\,e^{x} + 91}{40\,(e^{x} - i)^5} - i\,\log\left(i\,e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(I+csch(x)),x, algorithm="giac")
```

```
[Out] -1/24*(21*I*e^(2*x) - 36*e^x - 19*I)/(I*e^x - 1)^3 - 1/40*(115*e^(4*x) - 38
0*I*e^(3*x) - 530*e^(2*x) + 340*I*e^x + 91)/(e^x - I)^5 - I*log(I*e^x)
```

Mupad [B]
time = 2.22, size = 237, normalized size = 4.56

$$-x\,1\mathrm{i}-\frac{1\mathrm{i}}{4\,\left({\mathrm{e}}^{2\,x}-1+{\mathrm{e}}^x\,2\mathrm{i}\right)}+\frac{\frac{23\,{\mathrm{e}}^x}{40}-\frac{3}{8}\mathrm{i}}{1-{\mathrm{e}}^{2\,x}+{\mathrm{e}}^x\,2\mathrm{i}}-\frac{23}{40\,\left({\mathrm{e}}^x-\mathrm{i}\right)}+\frac{7}{8\,\left({\mathrm{e}}^x+1\mathrm{i}\right)}+\frac{\frac{{\mathrm{e}}^{2\,x}\,9\mathrm{i}}{8}-\frac{23\,{\mathrm{e}}^{3\,x}}{40}+\frac{9\,{\mathrm{e}}^x}{8}-\frac{3}{8}\mathrm{i}}{{\mathrm{e}}^{4\,x}-6\,{\mathrm{e}}^{2\,x}+1-{\mathrm{e}}^{3\,x}\,4\mathrm{i}+{\mathrm{e}}^x\,4\mathrm{i}}-\frac{\frac{3}{8}-\frac{23\,{\mathrm{e}}^{2\,x}}{40}+\frac{{\mathrm{e}}^x\,3\mathrm{i}}{4}}{{\mathrm{e}}^{2\,x}\,3\mathrm{i}-{\mathrm{e}}^{3\,x}+3\,{\mathrm{e}}^x-\mathrm{i}}-\frac{1}{6\,\left({\mathrm{e}}^{2\,x}\,3\mathrm{i}+{\mathrm{e}}^{3\,x}-3\,{\mathrm{e}}^x-\mathrm{i}\right)}-\frac{\frac{23\,{\mathrm{e}}^{4\,x}}{40}-\frac{9\,{\mathrm{e}}^{2\,x}}{4}+\frac{23}{40}-\frac{{\mathrm{e}}^{3\,x}\,3\mathrm{i}}{2}+\frac{{\mathrm{e}}^x\,3\mathrm{i}}{2}}{{\mathrm{e}}^{2\,x}\,10\mathrm{i}-10\,{\mathrm{e}}^{3\,x}-{\mathrm{e}}^{4\,x}\,5\mathrm{i}+{\mathrm{e}}^{5\,x}+5\,{\mathrm{e}}^x-\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(1/sinh(x) + 1i),x)
```

```
[Out] ((23*exp(x))/40 - 3i/8)/(exp(x)*2i - exp(2*x) + 1) - 1i/(4*(exp(2*x) + exp(
x)*2i - 1)) - x*1i - 23/(40*(exp(x) - 1i)) + 7/(8*(exp(x) + 1i)) + ((exp(2*
x)*9i)/8 - (23*exp(3*x))/40 + (9*exp(x))/8 - 3i/8)/(exp(4*x) - exp(3*x)*4i
- 6*exp(2*x) + exp(x)*4i + 1) - ((exp(x)*3i)/4 - (23*exp(2*x))/40 + 3/8)/(e
xp(2*x)*3i - exp(3*x) + 3*exp(x) - 1i) - 1/(6*(exp(2*x)*3i + exp(3*x) - 3*e
xp(x) - 1i)) - ((23*exp(4*x))/40 - (exp(3*x)*3i)/2 - (9*exp(2*x))/4 + (exp(
x)*3i)/2 + 23/40)/(exp(2*x)*10i - 10*exp(3*x) - exp(4*x)*5i + exp(5*x) + 5*
exp(x) - 1i)
```

3.104 $\int \frac{\tanh^3(x)}{i+\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=77

$$-\frac{11}{16}i\log(i-\sinh(x))-\frac{5}{16}i\log(i+\sinh(x))-\frac{i}{8(1-i\sinh(x))}+\frac{i}{8(1+i\sinh(x))^2}-\frac{3i}{4(1+i\sinh(x))}$$

```
[Out] -11/16*I*ln(I-sinh(x))-5/16*I*ln(I+sinh(x))-1/8*I/(1-I*sinh(x))+1/8*I/(1+I*
sinh(x))^2-3/4*I/(1+I*sinh(x))
```

Rubi [A]
time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3964, 90}

$$-\frac{i}{8(1-i\sinh(x))}-\frac{3i}{4(1+i\sinh(x))}+\frac{i}{8(1+i\sinh(x))^2}-\frac{11}{16}i\log(-\sinh(x)+i)-\frac{5}{16}i\log(\sinh(x)+i)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/(I + Csch[x]),x]
```

```
[Out] ((-11*I)/16)*Log[I - Sinh[x]] - ((5*I)/16)*Log[I + Sinh[x]] - (I/8)/(1 - I*
Sinh[x]) + (I/8)/(1 + I*Sinh[x])^2 - ((3*I)/4)/(1 + I*Sinh[x])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int \frac{\tanh^3(x)}{i+\operatorname{csch}(x)}\,dx = \text{Subst}\left(\int \frac{x^4}{(i-ix)^2(i+ix)^3}\,dx, x, i\sinh(x)\right)$$
$$= \text{Subst}\left(\int \left(-\frac{i}{8(-1+x)^2} - \frac{5i}{16(-1+x)} - \frac{i}{4(1+x)^3} + \frac{3i}{4(1+x)^2} - \frac{11i}{16(1+x)}\right) dx, x,$$
$$= -\frac{11}{16} i\log(i-\sinh(x)) - \frac{5}{16} i\log(i+\sinh(x)) - \frac{i}{8(1-i\sinh(x))} + \frac{i}{8(1+i\sinh(x))^2} -$$

Mathematica [A]
time = 0.09, size = 61, normalized size = 0.79

$$\frac{1}{16}\left(-11i\log(i-\sinh(x)) - 5i\log(i+\sinh(x)) - \frac{2(6+3i\sinh(x)+5\sinh^2(x))}{(-i+\sinh(x))^2(i+\sinh(x))}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(I + Csch[x]),x]
```

```
[Out] ((-11*I)*Log[I - Sinh[x]] - (5*I)*Log[I + Sinh[x]] - (2*(6 + (3*I)*Sinh[x]
+ 5*Sinh[x]^2))/((-I + Sinh[x])^2*(I + Sinh[x])))/16
```

Maple [A]
time = 1.54, size = 109, normalized size = 1.42

method	result
risch	$ix - \frac{-6ie^{2x}+5\,e^{x}+6ie^{4x}+14\,e^{3x}+5\,e^{5x}}{4(e^{x}+i)^2(e^{x}-i)^4} - \frac{5i\ln(e^{x}+i)}{8} - \frac{11i\ln(e^{x}-i)}{8}$
default	$i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right) - \frac{11i\ln(-i+\tanh(\frac{x}{2}))}{8} + \frac{i}{2(-i+\tanh(\frac{x}{2}))^4} + \frac{i}{2(-i+\tanh(\frac{x}{2}))^2} + \frac{1}{(-i+\tanh(\frac{x}{2}))^3} + \frac{1}{-i+\tanh(\frac{x}{2})}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(I+csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] I*ln(tanh(1/2*x)-1)-11/8*I*ln(-I+tanh(1/2*x))+1/2*I/(-I+tanh(1/2*x))^4+1/2*
I/(-I+tanh(1/2*x))^2+1/(-I+tanh(1/2*x))^3+1/(-I+tanh(1/2*x))+I*ln(tanh(1/2*
x)+1)-5/8*I*ln(tanh(1/2*x)+I)+1/4*I/(tanh(1/2*x)+I)^2-1/4/(tanh(1/2*x)+I)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(47) = 94.
time = 0.29, size = 96, normalized size = 1.25

$$-ix + \frac{5\,e^{(-x)} + 6i\,e^{(-2\,x)} + 14\,e^{(-3\,x)} - 6i\,e^{(-4\,x)} + 5\,e^{(-5\,x)}}{8i\,e^{(-x)} - 4\,e^{(-2\,x)} + 16i\,e^{(-3\,x)} + 4\,e^{(-4\,x)} + 8i\,e^{(-5\,x)} + 4\,e^{(-6\,x)} - 4} - \frac{5}{8}i\log\left(e^{(-x)} - i\right) - \frac{11}{8}i\log\left(i\,e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -I*x + (5*e^(-x) + 6*I*e^(-2*x) + 14*e^(-3*x) - 6*I*e^(-4*x) + 5*e^(-5*x))/
(8*I*e^(-x) - 4*e^(-2*x) + 16*I*e^(-3*x) + 4*e^(-4*x) + 8*I*e^(-5*x) + 4*e^
(-6*x) - 4) - 5/8*I*log(e^(-x) - I) - 11/8*I*log(I*e^(-x) - 1)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(47) = 94$.
time = 0.43, size = 186, normalized size = 2.42

$$\frac{8i\,xe^{(6\,x)} + 2\,(8\,x - 5)e^{(5\,x)} - 4\,(-2i\,x + 3i)e^{(4\,x)} + 4\,(8\,x - 7)e^{(3\,x)} - 4\,(2i\,x - 3i)e^{(2\,x)} + 2\,(8\,x - 5)e^{x} - 5\,\left(i\,e^{(6\,x)} + 2\,e^{(5\,x)} + i\,e^{(4\,x)} + 4\,e^{(3\,x)} - i\,e^{(2\,x)} + 2\,e^{x} - i\right)\log\left(e^{x} + i\right) - 11\,\left(i\,e^{(6\,x)} + 2\,e^{(5\,x)} + i\,e^{(4\,x)} + 4\,e^{(3\,x)} - i\,e^{(2\,x)} + 2\,e^{x} - i\right)\log\left(e^{x} - i\right) - 8i\,x}{8\,\left(e^{(6\,x)} - 2i\,e^{(5\,x)} + e^{(4\,x)} - 4i\,e^{(3\,x)} - e^{(2\,x)} - 2i\,e^{x} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] 1/8*(8*I*x*e^(6*x) + 2*(8*x - 5)*e^(5*x) - 4*(-2*I*x + 3*I)*e^(4*x) + 4*(8*
x - 7)*e^(3*x) - 4*(2*I*x - 3*I)*e^(2*x) + 2*(8*x - 5)*e^x - 5*(I*e^(6*x) +
 2*e^(5*x) + I*e^(4*x) + 4*e^(3*x) - I*e^(2*x) + 2*e^x - I)*log(e^x + I) -
11*(I*e^(6*x) + 2*e^(5*x) + I*e^(4*x) + 4*e^(3*x) - I*e^(2*x) + 2*e^x - I)*
log(e^x - I) - 8*I*x)/(e^(6*x) - 2*I*e^(5*x) + e^(4*x) - 4*I*e^(3*x) - e^(2
*x) - 2*I*e^x - 1)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{3}(x)}{\operatorname{csch}(x) + i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(I+csch(x)),x)
```

```
[Out] Integral(tanh(x)**3/(csch(x) + I), x)
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(47) = 94$.
time = 0.39, size = 98, normalized size = 1.27

$$\frac{5\,e^{(-x)} - 5\,e^{x} - 6i}{16\,\left(-i\,e^{(-x)} + i\,e^{x} - 2\right)} + \frac{33i\,\left(e^{(-x)} - e^{x}\right)^{2} - 84\,e^{(-x)} + 84\,e^{x} - 52i}{32\,\left(e^{(-x)} - e^{x} + 2i\right)^{2}} - \frac{5}{16}i\,\log\left(i\,e^{(-x)} - i\,e^{x} + 2\right) - \frac{11}{16}i\,\log\left(i\,e^{(-x)} - i\,e^{x} - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(I+csch(x)),x, algorithm="giac")
```

```
[Out] 1/16*(5*e^(-x) - 5*e^x - 6*I)/(-I*e^(-x) + I*e^x - 2) + 1/32*(33*I*(e^(-x)
- e^x)^2 - 84*e^(-x) + 84*e^x - 52*I)/(e^(-x) - e^x + 2*I)^2 - 5/16*I*log(I
*e^(-x) - I*e^x + 2) - 11/16*I*log(I*e^(-x) - I*e^x - 2)
```

Mupad [B]
time = 0.58, size = 140, normalized size = 1.82

$$x\,1\mathrm{i}-\ln\left(\left(\frac{3\,\mathrm{e}^x}{4}-\frac{3}{4}\mathrm{i}\right)\,\left(\frac{3\,\mathrm{e}^x}{4}+\frac{3}{4}\mathrm{i}\right)\right)\,1\mathrm{i}+\frac{3\,\mathrm{atan}\left(\mathrm{e}^x\right)}{4}-\frac{1}{\mathrm{e}^{2\,x}\,3\mathrm{i}-\mathrm{e}^{3\,x}+3\,\mathrm{e}^x-\mathrm{i}}-\frac{1\mathrm{i}}{4\,\left(\mathrm{e}^{2\,x}-1+\mathrm{e}^x\,2\mathrm{i}\right)}+\frac{1\mathrm{i}}{2\,\left(\mathrm{e}^{4\,x}-6\,\mathrm{e}^{2\,x}+1-\mathrm{e}^{3\,x}\,4\mathrm{i}+\mathrm{e}^x\,4\mathrm{i}\right)}+\frac{2\mathrm{i}}{1-\mathrm{e}^{2\,x}+\mathrm{e}^x\,2\mathrm{i}}-\frac{3}{2\,\left(\mathrm{e}^x-\mathrm{i}\right)}+\frac{1}{4\,\left(\mathrm{e}^x+1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(1/sinh(x) + 1i),x)
```

```
[Out] x*1i - log(((3*exp(x))/4 - 3i/4)*((3*exp(x))/4 + 3i/4))*1i + (3*atan(exp(x)
))/4 - 1/(exp(2*x)*3i - exp(3*x) + 3*exp(x) - 1i) - 1i/(4*(exp(2*x) + exp(x
)*2i - 1)) + 1i/(2*(exp(4*x) - exp(3*x)*4i - 6*exp(2*x) + exp(x)*4i + 1)) +
 2i/(exp(x)*2i - exp(2*x) + 1) - 3/(2*(exp(x) - 1i)) + 1/(4*(exp(x) + 1i))
```

3.105 $\int \frac{\tanh^2(x)}{i+\text{csch}(x)} \, dx$

Optimal. Leaf size=36

$$-ix + \frac{1}{3}(3i - 2\text{csch}(x))\tanh(x) + \frac{1}{3}(i - \text{csch}(x))\tanh^3(x)$$

```
[Out] -I*x+1/3*(3*I-2*csch(x))*tanh(x)+1/3*(I-csch(x))*tanh(x)^3
```

Rubi [A]
time = 0.06, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3973, 3967, 8}

$$-ix + \frac{1}{3}\tanh^3(x)(-\text{csch}(x) + i) + \frac{1}{3}\tanh(x)(-2\text{csch}(x) + 3i)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(I + Csch[x]),x]
```

```
[Out] (-I)*x + ((3*I - 2*Csch[x])*Tanh[x])/3 + ((I - Csch[x])*Tanh[x]^3)/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\tanh^2(x)}{i+\text{csch}(x)}\,dx = \int (-i+\text{csch}(x))\tanh^4(x)\,dx$$
$$= \frac{1}{3}(i-\text{csch}(x))\tanh^3(x) - \frac{1}{3}\int (3i-2\text{csch}(x))\tanh^2(x)\,dx$$
$$= \frac{1}{3}(3i-2\text{csch}(x))\tanh(x) + \frac{1}{3}(i-\text{csch}(x))\tanh^3(x) + \frac{1}{3}\int -3i\,dx$$
$$= -ix + \frac{1}{3}(3i-2\text{csch}(x))\tanh(x) + \frac{1}{3}(i-\text{csch}(x))\tanh^3(x)$$

Mathematica [A]
time = 0.06, size = 71, normalized size = 1.97

$$\frac{-4\cosh(2x)+2i\sinh(x)+(5i+6x)\cosh(x)(-i+\sinh(x))}{6\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(I + Csch[x]),x]
```

```
[Out] (-4*Cosh[2*x] + (2*I)*Sinh[x] + (5*I + 6*x)*Cosh[x]*(-I + Sinh[x]))/(6*(Cos
h[x/2] - I*Sinh[x/2])*(Cosh[x/2] + I*Sinh[x/2])^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(29) = 58.
time = 1.36, size = 67, normalized size = 1.86

method	result
risch	$-ix-\frac{2(5\,\mathrm{e}^{x}-4i+3\,\mathrm{e}^{3x})}{3(\mathrm{e}^{x}+i)(\mathrm{e}^{x}-i)^3}$
default	$\frac{i}{2\tanh\left(\frac{x}{2}\right)+2i}+\frac{3i}{2\left(-i+\tanh\left(\frac{x}{2}\right)\right)}+\frac{2i}{3\left(-i+\tanh\left(\frac{x}{2}\right)\right)^3}+\frac{1}{\left(-i+\tanh\left(\frac{x}{2}\right)\right)^2}-i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+i\ln\left(\tanh\left(\frac{x}{2}\right)-\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(I+csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I/(tanh(1/2*x)+I)+3/2*I/(-I+tanh(1/2*x))+2/3*I/(-I+tanh(1/2*x))^3+1/(-I
+tanh(1/2*x))^2-I*ln(tanh(1/2*x)+1)+I*ln(tanh(1/2*x)-1)
```

Maxima [A]
time = 0.27, size = 42, normalized size = 1.17

$$-i\,x-\frac{2\left(5\,e^{(-x)}+3\,e^{(-3\,x)}+4i\right)}{6i\,e^{(-x)}+6i\,e^{(-3\,x)}+3\,e^{(-4\,x)}-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -I*x - 2*(5*e^(-x) + 3*e^(-3*x) + 4*I)/(6*I*e^(-x) + 6*I*e^(-3*x) + 3*e^(-4
*x) - 3)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(24) = 48$.
time = 0.44, size = 50, normalized size = 1.39

$$\frac{-3i\,xe^{(4\,x)} - 6\,(x+1)e^{(3\,x)} - 2\,(3\,x+5)e^{x} + 3i\,x + 8i}{3\,(e^{(4\,x)} - 2i\,e^{(3\,x)} - 2i\,e^{x} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] 1/3*(-3*I*x*e^(4*x) - 6*(x + 1)*e^(3*x) - 2*(3*x + 5)*e^x + 3*I*x + 8*I)/(e
^(4*x) - 2*I*e^(3*x) - 2*I*e^x - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(x)}{\operatorname{csch}(x) + i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(I+csch(x)),x)
```

```
[Out] Integral(tanh(x)**2/(csch(x) + I), x)
```

Giac [A]
time = 0.39, size = 38, normalized size = 1.06

$$\frac{i}{2\,(i\,e^{x} - 1)} - \frac{15\,e^{(2\,x)} - 24i\,e^{x} - 13}{6\,(e^{x} - i)^{3}} - i\,\log\left(i\,e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(I+csch(x)),x, algorithm="giac")
```

```
[Out] 1/2*I/(I*e^x - 1) - 1/6*(15*e^(2*x) - 24*I*e^x - 13)/(e^x - I)^3 - I*log(I*
e^x)
```

Mupad [B]
time = 1.63, size = 85, normalized size = 2.36

$$-x\,1\mathrm{i} + \frac{\frac{5\,\mathrm{e}^{x}}{6} - \frac{1}{2}\mathrm{i}}{1 - \mathrm{e}^{2\,x} + \mathrm{e}^{x}\,2\mathrm{i}} - \frac{5}{6\,\left(\mathrm{e}^{x} - \mathrm{i}\right)} + \frac{1}{2\,\left(\mathrm{e}^{x} + 1\mathrm{i}\right)} - \frac{\frac{5}{6} - \frac{5\,\mathrm{e}^{2\,x}}{6} + \mathrm{e}^{x}\,1\mathrm{i}}{\mathrm{e}^{2\,x}\,3\mathrm{i} - \mathrm{e}^{3\,x} + 3\,\mathrm{e}^{x} - \mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(1/sinh(x) + 1i),x)
```

```
[Out] ((5*exp(x))/6 - 1i/2)/(exp(x)*2i - exp(2*x) + 1) - x*1i - 5/(6*(exp(x) - 1i
)) + 1/(2*(exp(x) + 1i)) - (exp(x)*1i - (5*exp(2*x))/6 + 5/6)/(exp(2*x)*3i
- exp(3*x) + 3*exp(x) - 1i)
```

3.106 $\int \frac{\tanh(x)}{i+\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=45

$$-\frac{3}{4}i\log(i-\sinh(x))-\frac{1}{4}i\log(i+\sinh(x))-\frac{i}{2(1+i\sinh(x))}$$

```
[Out] -3/4*I*ln(I-sinh(x))-1/4*I*ln(I+sinh(x))-1/2*I/(1+I*sinh(x))
```

Rubi [A]
time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.182$, Rules used = {3964, 90}

$$-\frac{i}{2(1+i\sinh(x))}-\frac{3}{4}i\log(-\sinh(x)+i)-\frac{1}{4}i\log(\sinh(x)+i)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(I + Csch[x]),x]
```

```
[Out] ((-3*I)/4)*Log[I - Sinh[x]] - (I/4)*Log[I + Sinh[x]] - (I/2)/(1 + I*Sinh[x]
)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh(x)}{i+\operatorname{csch}(x)}\,dx &= \text{Subst}\left(\int \frac{x^2}{(i-ix)(i+ix)^2}\,dx, x, i\sinh(x)\right)\\
&= \text{Subst}\left(\int \left(-\frac{i}{4(-1+x)}+\frac{i}{2(1+x)^2}-\frac{3i}{4(1+x)}\right)dx, x, i\sinh(x)\right)\\
&= -\frac{3}{4}i\log(i-\sinh(x))-\frac{1}{4}i\log(i+\sinh(x))-\frac{i}{2(1+i\sinh(x))}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 39, normalized size = 0.87

$$\frac{1}{4}\left(-3i\log(i-\sinh(x))-i\log(i+\sinh(x))-\frac{2}{-i+\sinh(x)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(I + Csch[x]),x]
[Out] ((-3*I)*Log[I - Sinh[x]] - I*Log[I + Sinh[x]] - 2/(-I + Sinh[x]))/4
```

Maple [A]
time = 1.41, size = 65, normalized size = 1.44

method	result
risch	$ix-\frac{e^{x}}{(e^{x}-i)^{2}}-\frac{3i\ln(e^{x}-i)}{2}-\frac{i\ln(e^{x}+i)}{2}$
default	$-\frac{3i\ln\left(-i+\tanh\left(\frac{x}{2}\right)\right)}{2}+\frac{i}{\left(-i+\tanh\left(\frac{x}{2}\right)\right)^{2}}+\frac{1}{-i+\tanh\left(\frac{x}{2}\right)}-\frac{i\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)}{2}+i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+i\ln\left(\tanh\left(\frac{x}{2}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(I+csch(x)),x,method=_RETURNVERBOSE)
[Out] -3/2*I*ln(-I+tanh(1/2*x))+I/(-I+tanh(1/2*x))^2+1/(-I+tanh(1/2*x))-1/2*I*ln(
tanh(1/2*x)+I)+I*ln(tanh(1/2*x)-1)+I*ln(tanh(1/2*x)+1)
```

Maxima [A]
time = 0.26, size = 45, normalized size = 1.00

$$-i\,x+\frac{e^{(-x)}}{2i\,e^{(-x)}+e^{(-2\,x)}-1}-\frac{1}{2}i\,\log\left(i\,e^{(-x)}+1\right)-\frac{3}{2}i\,\log\left(i\,e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(I+csch(x)),x, algorithm="maxima")
[Out] -I*x + e^(-x)/(2*I*e^(-x) + e^(-2*x) - 1) - 1/2*I*log(I*e^(-x) + 1) - 3/2*I
*log(I*e^(-x) - 1)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(27) = 54.
time = 0.41, size = 71, normalized size = 1.58

$$\frac{2i\,xe^{(2\,x)}+2\,(2\,x-1)e^{x}+\left(-i\,e^{(2\,x)}-2\,e^{x}+i\right)\log\left(e^{x}+i\right)-3\left(i\,e^{(2\,x)}+2\,e^{x}-i\right)\log\left(e^{x}-i\right)-2i\,x}{2\left(e^{(2\,x)}-2i\,e^{x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*I*x*e^(2*x) + 2*(2*x - 1)*e^x + (-I*e^(2*x) - 2*e^x + I)*log(e^x + I
) - 3*(I*e^(2*x) + 2*e^x - I)*log(e^x - I) - 2*I*x)/(e^(2*x) - 2*I*e^x - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{\operatorname{csch}(x) + i}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(I+csch(x)),x)
```

```
[Out] Integral(tanh(x)/(csch(x) + I), x)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(27) = 54.
time = 0.40, size = 55, normalized size = 1.22

$$\frac{3i\,e^{(-x)} - 3i\,e^{x} - 2}{4\left(e^{(-x)} - e^{x} + 2i\right)} - \frac{1}{4}i\,\log\left(-i\,e^{(-x)} + i\,e^{x} - 2\right) - \frac{3}{4}i\,\log\left(-e^{(-x)} + e^{x} - 2i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(I+csch(x)),x, algorithm="giac")
```

```
[Out] 1/4*(3*I*e^(-x) - 3*I*e^x - 2)/(e^(-x) - e^x + 2*I) - 1/4*I*log(-I*e^(-x) +
 I*e^x - 2) - 3/4*I*log(-e^(-x) + e^x - 2*I)
```

Mupad [B]
time = 0.23, size = 50, normalized size = 1.11

$$x\,1\mathrm{i} + \mathrm{atan}(\mathrm{e}^{x}) - \ln\left(\left(\mathrm{e}^{x} - \mathrm{i}\right)\,\left(\mathrm{e}^{x} + 1\mathrm{i}\right)\right)\,1\mathrm{i} + \frac{1\mathrm{i}}{1 - \mathrm{e}^{2\,x} + \mathrm{e}^{x}\,2\mathrm{i}} - \frac{1}{\mathrm{e}^{x} - \mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(1/sinh(x) + 1i),x)
```

```
[Out] x*1i + atan(exp(x)) - log((exp(x) - 1i)*(exp(x) + 1i))*1i + 1i/(exp(x)*2i -
 exp(2*x) + 1) - 1/(exp(x) - 1i)
```

3.107 $\int \frac{\coth(x)}{i+\operatorname{csch}(x)} dx$

Optimal. Leaf size=13

$$-i\log(i-\sinh(x))$$

```
[Out] -I*ln(I-sinh(x))
```

Rubi [A]
time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3964, 31}

$$-i\log(-\sinh(x)+i)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(I + Csch[x]),x]
```

```
[Out] (-I)*Log[I - Sinh[x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth(x)}{i+\operatorname{csch}(x)} dx &= \operatorname{Subst}\left(\int \frac{1}{i+ix} dx, x, i\sinh(x)\right)\\ &= -i\log(i-\sinh(x))\end{aligned}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$-i\log(i-\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(I + Csch[x]),x]
```

```
[Out] (-I)*Log[I - Sinh[x]]
```

Maple [A]
time = 0.93, size = 17, normalized size = 1.31

method	result	size
risch	$ix-2i\ln\left(\mathrm{e}^{x}-i\right)$	15
derivativedivides	$-i\ln\left(i+\operatorname{csch}(x)\right)+i\ln\left(\operatorname{csch}(x)\right)$	17
default	$-i\ln\left(i+\operatorname{csch}(x)\right)+i\ln\left(\operatorname{csch}(x)\right)$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(I+csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -I*ln(I+csch(x))+I*ln(csch(x))
```

Maxima [A]
time = 0.26, size = 15, normalized size = 1.15

$$-i\,x-2i\,\log\left(i\,e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -I*x - 2*I*log(I*e^(-x) - 1)
```

Fricas [A]
time = 0.38, size = 11, normalized size = 0.85

$$i\,x-2i\,\log\left(e^{x}-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] I*x - 2*I*log(e^x - I)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\coth\left(x\right)}{\operatorname{csch}\left(x\right)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(I+csch(x)),x)
```

```
[Out] Integral(coth(x)/(csch(x) + I), x)
```

Giac [**A**]
time = 0.40, size = 13, normalized size = 1.00

$$ix - 2i\log\left(ie^{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(I+csch(x)),x, algorithm="giac")
```

```
[Out] I*x - 2*I*log(I*e^x + 1)
```

Mupad [**B**]
time = 1.46, size = 14, normalized size = 1.08

$$x\,1\mathrm{i} - \ln\left(\mathrm{e}^{x} - \mathrm{i}\right)\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(1/sinh(x) + 1i),x)
```

```
[Out] x*1i - log(exp(x) - 1i)*2i
```

3.108 $\int \frac{\coth^2(x)}{i+\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=11

$$-ix-\tanh^{-1}(\cosh(x))$$

```
[Out] -I*x-arctanh(cosh(x))
```

Rubi [A]
time = 0.03, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3973, 3855}

$$-\tanh^{-1}(\cosh(x))-ix$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(I + Csch[x]),x]
```

```
[Out] (-I)*x - ArcTanh[Cosh[x]]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^2(x)}{i+\operatorname{csch}(x)}\,dx &= \int (-i+\operatorname{csch}(x))\,dx\\ &= -ix+\int \operatorname{csch}(x)\,dx\\ &= -ix-\tanh^{-1}(\cosh(x))\end{aligned}$$

Mathematica [A]
time = 0.02, size = 13, normalized size = 1.18

$$-ix+\log\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(I + Csch[x]),x]
```

```
[Out] (-I)*x + Log[Tanh[x/2]]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(10) = 20.
time = 0.80, size = 27, normalized size = 2.45

method	result	size
risch	$-ix+\ln\left(\mathrm{e}^{x}-1\right)-\ln\left(\mathrm{e}^{x}+1\right)$	18
default	$i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\ln\left(\tanh\left(\frac{x}{2}\right)\right)$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(I+csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] I*ln(tanh(1/2*x)-1)-I*ln(tanh(1/2*x)+1)+ln(tanh(1/2*x))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(9) = 18.
time = 0.26, size = 20, normalized size = 1.82

$$-i\,x-\log\left(e^{(-x)}+1\right)+\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -I*x - log(e^(-x) + 1) + log(e^(-x) - 1)
```

Fricas [A]
time = 0.46, size = 16, normalized size = 1.45

$$-i\,x-\log\left(e^{x}+1\right)+\log\left(e^{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] -I*x - log(e^x + 1) + log(e^x - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\coth^{2}\left(x\right)}{\operatorname{csch}\left(x\right)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(I+csch(x)),x)
[Out] Integral(coth(x)**2/(csch(x) + I), x)
```

Giac [**A**]
time = 0.39, size = 17, normalized size = 1.55

$$-i\,x - \log\left(e^{x} + 1\right) + \log\left(\left|e^{x} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(I+csch(x)),x, algorithm="giac")
[Out] -I*x - log(e^x + 1) + log(abs(e^x - 1))
```

Mupad [**B**]
time = 0.18, size = 21, normalized size = 1.91

$$\ln\left(2 - 2\,\mathrm{e}^{x}\right) - \ln\left(-2\,\mathrm{e}^{x} - 2\right) - x\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(1/sinh(x) + 1i),x)
[Out] log(2 - 2*exp(x)) - log(- 2*exp(x) - 2) - x*1i
```

3.109 $\int \frac{\coth^3(x)}{i+\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=12

$$-\operatorname{csch}(x) - i\log(\sinh(x))$$

```
[Out] -csch(x)-I*ln(sinh(x))
```

Rubi [A]

time = 0.03, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3964, 45}

$$-\operatorname{csch}(x) - i\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/(I + Csch[x]),x]
```

```
[Out] -Csch[x] - I*Log[Sinh[x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^3(x)}{i+\operatorname{csch}(x)}\,dx &= \operatorname{Subst}\left(\int \frac{i-ix}{x^2}\,dx, x, i\sinh(x)\right)\\
&= \operatorname{Subst}\left(\int \left(\frac{i}{x^2}-\frac{i}{x}\right)\,dx, x, i\sinh(x)\right)\\
&= -\operatorname{csch}(x) - i\log(\sinh(x))
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 1.00

$$-\mathrm{csch}(x) - i\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/(I + Csch[x]),x]
```

```
[Out] -Csch[x] - I*Log[Sinh[x]]
```

Maple [A]
time = 0.85, size = 12, normalized size = 1.00

method	result	size
derivativedivides	$-\mathrm{csch}(x)+i\ln(\mathrm{csch}(x))$	12
default	$-\mathrm{csch}(x)+i\ln(\mathrm{csch}(x))$	12
risch	$ix-\frac{2\,\mathrm{e}^{x}}{\mathrm{e}^{2x}-1}-i\ln\left(\mathrm{e}^{2x}-1\right)$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(I+csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -csch(x)+I*ln(csch(x))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(10) = 20.
time = 0.26, size = 36, normalized size = 3.00

$$-i\,x+\frac{2\,e^{(-x)}}{e^{(-2\,x)}-1}-i\,\log\left(e^{(-x)}+1\right)-i\,\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -I*x + 2*e^(-x)/(e^(-2*x) - 1) - I*log(e^(-x) + 1) - I*log(e^(-x) - 1)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(10) = 20.
time = 0.46, size = 40, normalized size = 3.33

$$\frac{i\,xe^{(2\,x)}+\left(-i\,e^{(2\,x)}+i\right)\log\left(e^{(2\,x)}-1\right)-i\,x-2\,e^{x}}{e^{(2\,x)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(I+csch(x)),x, algorithm="fricas")
```

[Out] (I*x*e^(2*x) + (-I*e^(2*x) + I)*log(e^(2*x) - 1) - I*x - 2*e^x)/(e^(2*x) - 1)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(x)}{\operatorname{csch}(x) + i}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)**3/(I+csch(x)),x)

[Out] Integral(coth(x)**3/(csch(x) + I), x)

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(10) = 20.
time = 0.39, size = 39, normalized size = 3.25

$$-\frac{-i\,e^{(-x)} + i\,e^{x} - 2}{e^{(-x)} - e^{x}} - i\,\log\left(\left|-e^{(-x)} + e^{x}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^3/(I+csch(x)),x, algorithm="giac")

[Out] -(-I*e^(-x) + I*e^x - 2)/(e^(-x) - e^x) - I*log(abs(-e^(-x) + e^x))

Mupad [B]
time = 1.57, size = 27, normalized size = 2.25

$$-\frac{2\,e^{x}}{e^{2\,x} - 1} + x\,1i - \ln\left(e^{2\,x} - 1\right)\,1i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)^3/(1/sinh(x) + 1i),x)

[Out] x*1i - log(exp(2*x) - 1)*1i - (2*exp(x))/(exp(2*x) - 1)

3.110 $\int \frac{\coth^4(x)}{i+\operatorname{csch}(x)} dx$

Optimal. Leaf size=27

$$-ix-\frac{1}{2}\tanh^{-1}(\cosh(x))+\frac{1}{2}\coth(x)(2i-\operatorname{csch}(x))$$

```
[Out] -I*x-1/2*arctanh(cosh(x))+1/2*coth(x)*(2*I-csch(x))
```

Rubi [A]
time = 0.04, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3973, 3966, 3855}

$$-ix-\frac{1}{2}\tanh^{-1}(\cosh(x))+\frac{1}{2}\coth(x)(-\operatorname{csch}(x)+2i)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^4/(I + Csch[x]),x]
```

```
[Out] (-I)*x - ArcTanh[Cosh[x]]/2 + (Coth[x]*(2*I - Csch[x]))/2
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\coth^4(x)}{i+\operatorname{csch}(x)}\,dx &= \int \coth^2(x)(-i+\operatorname{csch}(x))\,dx \\
&= \frac{1}{2}\coth(x)(2i-\operatorname{csch}(x)) + \frac{1}{2}\int (-2i+\operatorname{csch}(x))\,dx \\
&= -ix + \frac{1}{2}\coth(x)(2i-\operatorname{csch}(x)) + \frac{1}{2}\int \operatorname{csch}(x)\,dx \\
&= -ix - \frac{1}{2}\tanh^{-1}(\cosh(x)) + \frac{1}{2}\coth(x)(2i-\operatorname{csch}(x))
\end{aligned}
$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 65 vs. $2(27) = 54$.
time = 0.03, size = 65, normalized size = 2.41

$$
-ix + \frac{1}{2}i\coth\left(\frac{x}{2}\right) - \frac{1}{8}\operatorname{csch}^2\left(\frac{x}{2}\right) + \frac{1}{2}\log\left(\tanh\left(\frac{x}{2}\right)\right) - \frac{1}{8}\operatorname{sech}^2\left(\frac{x}{2}\right) + \frac{1}{2}i\tanh\left(\frac{x}{2}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4/(I + Csch[x]),x]
```

```
[Out] (-I)*x + (I/2)*Coth[x/2] - Csch[x/2]^2/8 + Log[Tanh[x/2]]/2 - Sech[x/2]^2/8
 + (I/2)*Tanh[x/2]
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(21) = 42$.
time = 1.08, size = 61, normalized size = 2.26

method	result
risch	$-ix - \frac{-2i e^{2x}+e^{3x}+2i+e^{x}}{\left(e^{2x}-1\right)^2} - \frac{\ln(e^x+1)}{2} + \frac{\ln(e^x-1)}{2}$
default	$\frac{i\tanh\left(\frac{x}{2}\right)}{2} + \frac{\left(\tanh^2\left(\frac{x}{2}\right)\right)}{8} - \frac{1}{8\tanh\left(\frac{x}{2}\right)^2} + \frac{i}{2\tanh\left(\frac{x}{2}\right)} + \frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2} + i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right) - i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(I+csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*tanh(1/2*x)+1/8*tanh(1/2*x)^2-1/8/tanh(1/2*x)^2+1/2*I/tanh(1/2*x)+1/2
*ln(tanh(1/2*x))+I*ln(tanh(1/2*x)-1)-I*ln(tanh(1/2*x)+1)
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(17) = 34$.
time = 0.26, size = 55, normalized size = 2.04

$$
-i\,x + \frac{e^{(-x)} + 2i\,e^{(-2\,x)} + e^{(-3\,x)} - 2i}{2\,e^{(-2\,x)} - e^{(-4\,x)} - 1} - \frac{1}{2}\,\log\left(e^{(-x)} + 1\right) + \frac{1}{2}\,\log\left(e^{(-x)} - 1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -I*x + (e^(-x) + 2*I*e^(-2*x) + e^(-3*x) - 2*I)/(2*e^(-2*x) - e^(-4*x) - 1)
 - 1/2*log(e^(-x) + 1) + 1/2*log(e^(-x) - 1)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(17) = 34$.
time = 0.40, size = 86, normalized size = 3.19

$$\frac{-2i\,xe^{(4\,x)} - 4\,(-i\,x - i)e^{(2\,x)} - \left(e^{(4\,x)} - 2\,e^{(2\,x)} + 1\right)\log\left(e^{x} + 1\right) + \left(e^{(4\,x)} - 2\,e^{(2\,x)} + 1\right)\log\left(e^{x} - 1\right) - 2i\,x - 2\,e^{(3\,x)} - 2\,e^{x} - 4i}{2\left(e^{(4\,x)} - 2\,e^{(2\,x)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(-2*I*x*e^(4*x) - 4*(-I*x - I)*e^(2*x) - (e^(4*x) - 2*e^(2*x) + 1)*log(
e^x + 1) + (e^(4*x) - 2*e^(2*x) + 1)*log(e^x - 1) - 2*I*x - 2*e^(3*x) - 2*e
^x - 4*I)/(e^(4*x) - 2*e^(2*x) + 1)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^{4}(x)}{\operatorname{csch}(x) + i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4/(I+csch(x)),x)
```

```
[Out] Integral(coth(x)**4/(csch(x) + I), x)
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(17) = 34$.
time = 0.39, size = 43, normalized size = 1.59

$$-i\,x - \frac{e^{(3\,x)} - 2i\,e^{(2\,x)} + e^{x} + 2i}{\left(e^{(2\,x)} - 1\right)^{2}} - \frac{1}{2}\,\log\left(e^{x} + 1\right) + \frac{1}{2}\,\log\left(|e^{x} - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(I+csch(x)),x, algorithm="giac")
```

```
[Out] -I*x - (e^(3*x) - 2*I*e^(2*x) + e^x + 2*I)/(e^(2*x) - 1)^2 - 1/2*log(e^x +
1) + 1/2*log(abs(e^x - 1))
```

Mupad [**B**]
time = 1.64, size = 56, normalized size = 2.07

$$\frac{\ln\left(1-{\mathrm{e}}^{x}\right)}{2}-\frac{\ln\left(-{\mathrm{e}}^{x}-1\right)}{2}-\frac{{\mathrm{e}}^{x}-2{}\mathrm{i}}{{\mathrm{e}}^{2\,x}-1}-\frac{2\,{\mathrm{e}}^{x}}{{\mathrm{e}}^{4\,x}-2\,{\mathrm{e}}^{2\,x}+1}-x\,1{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(1/sinh(x) + 1i),x)
```

```
[Out] log(1 - exp(x))/2 - log(- exp(x) - 1)/2 - x*1i - (exp(x) - 2i)/(exp(2*x) -
1) - (2*exp(x))/(exp(4*x) - 2*exp(2*x) + 1)
```

3.111 $\int \frac{\coth^5(x)}{i+\mathbf{csch}(x)} dx$

Optimal. Leaf size=30

$$-\text{csch}(x)+\frac{1}{2}i\text{csch}^2(x)-\frac{\text{csch}^3(x)}{3}-i\log(\sinh(x))$$

```
[Out] -csch(x)+1/2*I*csch(x)^2-1/3*csch(x)^3-I*ln(sinh(x))
```

Rubi [A]
time = 0.04, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3964, 76}

$$-\frac{1}{3}\text{csch}^3(x)+\frac{1}{2}i\text{csch}^2(x)-\text{csch}(x)-i\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^5/(I + Csch[x]),x]
```

```
[Out] -Csch[x] + (I/2)*Csch[x]^2 - Csch[x]^3/3 - I*Log[Sinh[x]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^5(x)}{i+\text{csch}(x)}\,dx &= \text{Subst}\left(\int \frac{(i-ix)^2(i+ix)}{x^4}\,dx, x, i\sinh(x)\right)\\ &= \text{Subst}\left(\int \left(-\frac{i}{x^4}+\frac{i}{x^3}+\frac{i}{x^2}-\frac{i}{x}\right)\,dx, x, i\sinh(x)\right)\\ &= -\text{csch}(x)+\frac{1}{2}i\text{csch}^2(x)-\frac{\text{csch}^3(x)}{3}-i\log(\sinh(x))\end{aligned}$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 1.00

$$-\operatorname{csch}(x)+\frac{1}{2}i\operatorname{csch}^2(x)-\frac{\operatorname{csch}^3(x)}{3}-i\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^5/(I + Csch[x]),x]
[Out] -Csch[x] + (I/2)*Csch[x]^2 - Csch[x]^3/3 - I*Log[Sinh[x]]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(24) = 48.
time = 1.43, size = 78, normalized size = 2.60

method	result
risch	$ix-\frac{2\,e^{x}\left(-3ie^{3x}+3\,e^{4x}+3ie^{x}-2\,e^{2x}+3\right)}{3\left(e^{2x}-1\right)^{3}}-i\ln\left(e^{2x}-1\right)$
default	$\frac{3\tanh\left(\frac{x}{2}\right)}{8}+\frac{\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{24}+\frac{i\left(\tanh^{2}\left(\frac{x}{2}\right)\right)}{8}-\frac{1}{24\tanh\left(\frac{x}{2}\right)^{3}}-i\ln\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{i}{8\tanh\left(\frac{x}{2}\right)^{2}}-\frac{3}{8\tanh\left(\frac{x}{2}\right)}+i\ln\left(\tanh\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^5/(I+csch(x)),x,method=_RETURNVERBOSE)
[Out] 3/8*tanh(1/2*x)+1/24*tanh(1/2*x)^3+1/8*I*tanh(1/2*x)^2-1/24/tanh(1/2*x)^3-I
*ln(tanh(1/2*x))+1/8*I/tanh(1/2*x)^2-3/8/tanh(1/2*x)+I*ln(tanh(1/2*x)-1)+I*
ln(tanh(1/2*x)+1)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(22) = 44.
time = 0.26, size = 75, normalized size = 2.50

$$-i\,x+\frac{2\left(3\,e^{(-x)}-3i\,e^{(-2\,x)}-2\,e^{(-3\,x)}+3i\,e^{(-4\,x)}+3\,e^{(-5\,x)}\right)}{3\left(3\,e^{(-2\,x)}-3\,e^{(-4\,x)}+e^{(-6\,x)}-1\right)}-i\,\log\left(e^{(-x)}+1\right)-i\,\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(I+csch(x)),x, algorithm="maxima")
[Out] -I*x + 2/3*(3*e^(-x) - 3*I*e^(-2*x) - 2*e^(-3*x) + 3*I*e^(-4*x) + 3*e^(-5*x
))/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1) - I*log(e^(-x) + 1) - I*log(e^(
-x) - 1)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(22) = 44.
time = 0.41, size = 100, normalized size = 3.33

$$\frac{3i\,xe^{(6\,x)}-3\,(3i\,x-2i)e^{(4\,x)}-3\,(-3i\,x+2i)e^{(2\,x)}-3\left(i\,e^{(6\,x)}-3i\,e^{(4\,x)}+3i\,e^{(2\,x)}-i\right)\log\left(e^{(2\,x)}-1\right)-3i\,x-6\,e^{(5\,x)}+4\,e^{(3\,x)}-6\,e^{x}}{3\left(e^{(6\,x)}-3\,e^{(4\,x)}+3\,e^{(2\,x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] 1/3*(3*I*x*e^(6*x) - 3*(3*I*x - 2*I)*e^(4*x) - 3*(-3*I*x + 2*I)*e^(2*x) - 3
*(I*e^(6*x) - 3*I*e^(4*x) + 3*I*e^(2*x) - I)*log(e^(2*x) - 1) - 3*I*x - 6*e
^(5*x) + 4*e^(3*x) - 6*e^x)/(e^(6*x) - 3*e^(4*x) + 3*e^(2*x) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^5(x)}{\operatorname{csch}(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**5/(I+csch(x)),x)
```

```
[Out] Integral(coth(x)**5/(csch(x) + I), x)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(22) = 44.
time = 0.39, size = 65, normalized size = 2.17

$$\frac{11i\left(e^{(-x)}-e^{x}\right)^{3}+12\left(e^{(-x)}-e^{x}\right)^{2}+12i\,e^{(-x)}-12i\,e^{x}+16}{6\left(e^{(-x)}-e^{x}\right)^{3}}-i\,\log\left(\left|-e^{(-x)}+e^{x}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(I+csch(x)),x, algorithm="giac")
```

```
[Out] 1/6*(11*I*(e^(-x) - e^x)^3 + 12*(e^(-x) - e^x)^2 + 12*I*e^(-x) - 12*I*e^x +
 16)/(e^(-x) - e^x)^3 - I*log(abs(-e^(-x) + e^x))
```

Mupad [B]
time = 1.68, size = 81, normalized size = 2.70

$$x\,1\mathrm{i}-\ln\left(\mathrm{e}^{2\,x}-1\right)\,1\mathrm{i}-\frac{8\,\mathrm{e}^{x}}{3\,\left(3\,\mathrm{e}^{2\,x}-3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}-1\right)}-\frac{\frac{8\,\mathrm{e}^{x}}{3}-2\mathrm{i}}{\mathrm{e}^{4\,x}-2\,\mathrm{e}^{2\,x}+1}-\frac{2\,\mathrm{e}^{x}-2\mathrm{i}}{\mathrm{e}^{2\,x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^5/(1/sinh(x) + 1i),x)
```

```
[Out] x*1i - log(exp(2*x) - 1)*1i - (8*exp(x))/(3*(3*exp(2*x) - 3*exp(4*x) + exp(
6*x) - 1)) - ((8*exp(x))/3 - 2i)/(exp(4*x) - 2*exp(2*x) + 1) - (2*exp(x) -
2i)/(exp(2*x) - 1)
```

3.112 $\int \frac{\coth^6(x)}{i+\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=43

$$-ix-\frac{3}{8}\tanh^{-1}(\cosh(x))+\frac{1}{12}\coth^3(x)(4i-3\mathrm{csch}(x))+\frac{1}{8}\coth(x)(8i-3\mathrm{csch}(x))$$

```
[Out] -I*x-3/8*arctanh(cosh(x))+1/12*coth(x)^3*(4*I-3*csch(x))+1/8*coth(x)*(8*I-3
*csch(x))
```

Rubi [A]
time = 0.05, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3973, 3966, 3855}

$$-ix-\frac{3}{8}\tanh^{-1}(\cosh(x))+\frac{1}{12}\coth^3(x)(-3\mathrm{csch}(x)+4i)+\frac{1}{8}\coth(x)(-3\mathrm{csch}(x)+8i)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^6/(I + Csch[x]),x]
```

```
[Out] (-I)*x - (3*ArcTanh[Cosh[x]])/8 + (Coth[x]^3*(4*I - 3*Csch[x]))/12 + (Coth[
x]*(8*I - 3*Csch[x]))/8
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\coth^6(x)}{i+\operatorname{csch}(x)}\,dx &= \int \coth^4(x)(-i+\operatorname{csch}(x))\,dx \\
&= \frac{1}{12}\coth^3(x)(4i-3\operatorname{csch}(x)) + \frac{1}{4}\int \coth^2(x)(-4i+3\operatorname{csch}(x))\,dx \\
&= \frac{1}{12}\coth^3(x)(4i-3\operatorname{csch}(x)) + \frac{1}{8}\coth(x)(8i-3\operatorname{csch}(x)) + \frac{1}{8}\int (-8i+3\operatorname{csch}(x))\,dx \\
&= -ix + \frac{1}{12}\coth^3(x)(4i-3\operatorname{csch}(x)) + \frac{1}{8}\coth(x)(8i-3\operatorname{csch}(x)) + \frac{3}{8}\int \operatorname{csch}(x)\,dx \\
&= -ix - \frac{3}{8}\tanh^{-1}(\cosh(x)) + \frac{1}{12}\coth^3(x)(4i-3\operatorname{csch}(x)) + \frac{1}{8}\coth(x)(8i-3\operatorname{csch}(x))
\end{aligned}
$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 129 vs. $2(43) = 86$.
time = 0.03, size = 129, normalized size = 3.00

$$
-ix + \frac{2}{3}i\coth\left(\frac{x}{2}\right) - \frac{5}{32}\operatorname{csch}^2\left(\frac{x}{2}\right) + \frac{1}{24}i\coth\left(\frac{x}{2}\right)\operatorname{csch}^2\left(\frac{x}{2}\right) - \frac{1}{64}\operatorname{csch}^4\left(\frac{x}{2}\right) + \frac{3}{8}\log\left(\tanh\left(\frac{x}{2}\right)\right) - \frac{5}{32}\operatorname{sech}^2\left(\frac{x}{2}\right) + \frac{1}{64}\operatorname{sech}^4\left(\frac{x}{2}\right) + \frac{2}{3}i\tanh\left(\frac{x}{2}\right) - \frac{1}{24}i\operatorname{sech}^2\left(\frac{x}{2}\right)\tanh\left(\frac{x}{2}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^6/(I + Csch[x]),x]
```

```
[Out] (-I)*x + ((2*I)/3)*Coth[x/2] - (5*Csch[x/2]^2)/32 + (I/24)*Coth[x/2]*Csch[x
/2]^2 - Csch[x/2]^4/64 + (3*Log[Tanh[x/2]])/8 - (5*Sech[x/2]^2)/32 + Sech[x
/2]^4/64 + ((2*I)/3)*Tanh[x/2] - (I/24)*Sech[x/2]^2*Tanh[x/2]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(34) = 68$.
time = 1.39, size = 95, normalized size = 2.21

method	result
risch	$-ix - \frac{-48i e^{6x}+15 e^{7x}+96i e^{4x}+9 e^{5x}-80i e^{2x}+9 e^{3x}+32i+15 e^{x}}{12(e^{2x}-1)^4} + \frac{3\ln(e^x-1)}{8} - \frac{3\ln(e^x+1)}{8}$
default	$\frac{5i\tanh\left(\frac{x}{2}\right)}{8} + \frac{\left(\tanh^4\left(\frac{x}{2}\right)\right)}{64} + \frac{i\left(\tanh^3\left(\frac{x}{2}\right)\right)}{24} + \frac{\left(\tanh^2\left(\frac{x}{2}\right)\right)}{8} + i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right) - \frac{1}{64\tanh\left(\frac{x}{2}\right)^4} + \frac{5i}{8\tanh\left(\frac{x}{2}\right)} + \frac{}{24\mathrm{t}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^6/(I+csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 5/8*I*tanh(1/2*x)+1/64*tanh(1/2*x)^4+1/24*I*tanh(1/2*x)^3+1/8*tanh(1/2*x)^2
+I*ln(tanh(1/2*x)-1)-1/64/tanh(1/2*x)^4+5/8*I/tanh(1/2*x)+1/24*I/tanh(1/2*x
)^3-1/8/tanh(1/2*x)^2+3/8*ln(tanh(1/2*x))-I*ln(tanh(1/2*x)+1)
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(31) = 62$.
time = 0.26, size = 96, normalized size = 2.23

$$-i\,x+\frac{15\,e^{(-x)}+80i\,e^{(-2\,x)}+9\,e^{(-3\,x)}-96i\,e^{(-4\,x)}+9\,e^{(-5\,x)}+48i\,e^{(-6\,x)}+15\,e^{(-7\,x)}-32i}{12\,(4\,e^{(-2\,x)}-6\,e^{(-4\,x)}+4\,e^{(-6\,x)}-e^{(-8\,x)}-1)}-\frac{3}{8}\,\log\left(e^{(-x)}+1\right)+\frac{3}{8}\,\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^6/(I+csch(x)),x, algorithm="maxima")
```

```
[Out] -I*x + 1/12*(15*e^(-x) + 80*I*e^(-2*x) + 9*e^(-3*x) - 96*I*e^(-4*x) + 9*e^(
-5*x) + 48*I*e^(-6*x) + 15*e^(-7*x) - 32*I)/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^
(-6*x) - e^(-8*x) - 1) - 3/8*log(e^(-x) + 1) + 3/8*log(e^(-x) - 1)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(31) = 62$.
time = 0.45, size = 157, normalized size = 3.65

$$\frac{-24i\,xe^{(8\,x)}-96\,(-i\,x-i)e^{(6\,x)}-48\,(3i\,x+4i)e^{(4\,x)}-32\,(-3i\,x-5i)e^{(2\,x)}-9\,\left(e^{(8\,x)}-4\,e^{(6\,x)}+6\,e^{(4\,x)}-4\,e^{(2\,x)}+1\right)\log\left(e^{x}+1\right)+9\,\left(e^{(8\,x)}-4\,e^{(6\,x)}+6\,e^{(4\,x)}-4\,e^{(2\,x)}+1\right)\log\left(e^{x}-1\right)-24i\,x-30\,e^{(7\,x)}-18\,e^{(5\,x)}-18\,e^{(3\,x)}-30\,e^{x}-64i}{24\,\left(e^{(8\,x)}-4\,e^{(6\,x)}+6\,e^{(4\,x)}-4\,e^{(2\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^6/(I+csch(x)),x, algorithm="fricas")
```

```
[Out] 1/24*(-24*I*x*e^(8*x) - 96*(-I*x - I)*e^(6*x) - 48*(3*I*x + 4*I)*e^(4*x) -
32*(-3*I*x - 5*I)*e^(2*x) - 9*(e^(8*x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*x)
+ 1)*log(e^x + 1) + 9*(e^(8*x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*x) + 1)*log
(e^x - 1) - 24*I*x - 30*e^(7*x) - 18*e^(5*x) - 18*e^(3*x) - 30*e^x - 64*I)/
(e^(8*x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*x) + 1)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^{6}(x)}{\operatorname{csch}(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**6/(I+csch(x)),x)
```

```
[Out] Integral(coth(x)**6/(csch(x) + I), x)
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(31) = 62$.
time = 0.39, size = 71, normalized size = 1.65

$$-i\,x-\frac{15\,e^{(7\,x)}-48i\,e^{(6\,x)}+9\,e^{(5\,x)}+96i\,e^{(4\,x)}+9\,e^{(3\,x)}-80i\,e^{(2\,x)}+15\,e^{x}+32i}{12\,\left(e^{(2\,x)}-1\right)^{4}}-\frac{3}{8}\,\log\left(e^{x}+1\right)+\frac{3}{8}\,\log\left(|e^{x}-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^6/(I+csch(x)),x, algorithm="giac")
```

```
[Out] -I*x - 1/12*(15*e^(7*x) - 48*I*e^(6*x) + 9*e^(5*x) + 96*I*e^(4*x) + 9*e^(3*
x) - 80*I*e^(2*x) + 15*e^x + 32*I)/(e^(2*x) - 1)^4 - 3/8*log(e^x + 1) + 3/8
*log(abs(e^x - 1))
```

Mupad [B]
time = 1.86, size = 106, normalized size = 2.47

$$\frac{3\,\ln\left(\frac{3}{4}-\frac{3\,e^x}{4}\right)}{8}-x\,1i-\frac{3\,\ln\left(\frac{3\,e^x}{4}+\frac{3}{4}\right)}{8}-\frac{5\,e^x}{4\,\left(e^{2\,x}-1\right)}-\frac{9\,e^x}{2\,{\left(e^{2\,x}-1\right)}^2}-\frac{6\,e^x}{{\left(e^{2\,x}-1\right)}^3}-\frac{4\,e^x}{{\left(e^{2\,x}-1\right)}^4}+\frac{4i}{e^{2\,x}-1}+\frac{4i}{{\left(e^{2\,x}-1\right)}^2}+\frac{8i}{3\,{\left(e^{2\,x}-1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^6/(1/sinh(x) + 1i),x)
```

```
[Out] (3*log(3/4 - (3*exp(x))/4))/8 - x*1i - (3*log((3*exp(x))/4 + 3/4))/8 - (5*e
xp(x))/(4*(exp(2*x) - 1)) - (9*exp(x))/(2*(exp(2*x) - 1)^2) - (6*exp(x))/(e
xp(2*x) - 1)^3 - (4*exp(x))/(exp(2*x) - 1)^4 + 4i/(exp(2*x) - 1) + 4i/(exp(
2*x) - 1)^2 + 8i/(3*(exp(2*x) - 1)^3)
```

3.113 $\int \frac{\tanh^5(x)}{a+b\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=194

$$-\frac{b^5\text{ArcTan}(\sinh(x))}{(a^2+b^2)^3}-\frac{b^3\text{ArcTan}(\sinh(x))}{2(a^2+b^2)^2}-\frac{3b\text{ArcTan}(\sinh(x))}{8(a^2+b^2)}+\frac{b^6\log(a+b\operatorname{csch}(x))}{a(a^2+b^2)^3}+\frac{\log(\sinh(x))}{a}-\frac{a(a^4+\cdots}{\cdots}$$

```
[Out] -b^5*arctan(sinh(x))/(a^2+b^2)^3-1/2*b^3*arctan(sinh(x))/(a^2+b^2)^2-3/8*b*
arctan(sinh(x))/(a^2+b^2)+b^6*ln(a+b*csch(x))/a/(a^2+b^2)^3+ln(sinh(x))/a-a
*(a^4+3*a^2*b^2+3*b^4)*ln(tanh(x))/(a^2+b^2)^3+3/8*b*sech(x)*tanh(x)/(a^2+b
^2)-1/2*(a*(a^2+2*b^2)-b^3*csch(x))*tanh(x)^2/(a^2+b^2)^2-1/4*(a-b*csch(x))
*tanh(x)^4/(a^2+b^2)
```

Rubi [A]
time = 0.20, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3970, 908, 653, 205, 209, 649, 266}

$$-\frac{3b\text{ArcTan}(\sinh(x))}{8(a^2+b^2)}-\frac{b^5\text{ArcTan}(\sinh(x))}{(a^2+b^2)^3}-\frac{b^3\text{ArcTan}(\sinh(x))}{2(a^2+b^2)^2}-\frac{\tanh^4(x)(a-b\operatorname{csch}(x))}{4(a^2+b^2)}+\frac{3b\tanh(x)\operatorname{sech}(x)}{8(a^2+b^2)}+\frac{b^6\log(a+b\operatorname{csch}(x))}{a(a^2+b^2)^3}-\frac{\tanh^2(x)\left(a(a^2+2b^2)-b^3\operatorname{csch}(x)\right)}{2(a^2+b^2)^2}-\frac{a(a^4+3a^2b^2+3b^4)\log(\tanh(x))}{(a^2+b^2)^3}+\frac{\log(\sinh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^5/(a + b*Csch[x]),x]
```

```
[Out] -((b^5*ArcTan[Sinh[x]])/(a^2 + b^2)^3) - (b^3*ArcTan[Sinh[x]])/(2*(a^2 + b^
2)^2) - (3*b*ArcTan[Sinh[x]])/(8*(a^2 + b^2)) + (b^6*Log[a + b*Csch[x]])/(a
*(a^2 + b^2)^3) + Log[Sinh[x]]/a - (a*(a^4 + 3*a^2*b^2 + 3*b^4)*Log[Tanh[x]
])/(a^2 + b^2)^3 + (3*b*Sech[x]*Tanh[x])/(8*(a^2 + b^2)) - ((a*(a^2 + 2*b^2
) - b^3*Csch[x])*Tanh[x]^2)/(2*(a^2 + b^2)^2) - ((a - b*Csch[x])*Tanh[x]^4)
/(4*(a^2 + b^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\tanh^5(x)}{a+b\operatorname{csch}(x)}\,dx = b^6\text{Subst}\left(\int \frac{1}{x(a+x)\left(-b^2-x^2\right)^3}\,dx, x, b\operatorname{csch}(x)\right)$$

$$= b^6\text{Subst}\left(\int \left(-\frac{1}{ab^6x}+\frac{1}{a\left(a^2+b^2\right)^3(a+x)}+\frac{b^2+ax}{b^2\left(a^2+b^2\right)\left(b^2+x^2\right)^3}+\frac{b^4+a(a^2+2}{b^4\left(a^2+b^2\right)^2(b^2}\right.\right.$$

$$= \frac{b^6\log(a+b\operatorname{csch}(x))}{a\left(a^2+b^2\right)^3}+\frac{\log(\sinh(x))}{a}+\frac{\text{Subst}\left(\int \frac{b^6+a\left(a^4+3a^2b^2+3b^4\right)x}{b^2+x^2}\,dx, x, b\operatorname{csch}(x)\right)}{\left(a^2+b^2\right)^3}+$$

$$= \frac{b^6\log(a+b\operatorname{csch}(x))}{a\left(a^2+b^2\right)^3}+\frac{\log(\sinh(x))}{a}-\frac{\left(a(a^2+2b^2)-b^3\operatorname{csch}(x)\right)\tanh^2(x)}{2\left(a^2+b^2\right)^2}-\frac{(a-b\operatorname{csc}}{4(}$$

$$= -\frac{b^5\tan^{-1}(\sinh(x))}{\left(a^2+b^2\right)^3}-\frac{b^3\tan^{-1}(\sinh(x))}{2\left(a^2+b^2\right)^2}+\frac{b^6\log(a+b\operatorname{csch}(x))}{a\left(a^2+b^2\right)^3}+\frac{\log(\sinh(x))}{a}-\frac{a(a^4}{}$$

$$= -\frac{b^5\tan^{-1}(\sinh(x))}{\left(a^2+b^2\right)^3}-\frac{b^3\tan^{-1}(\sinh(x))}{2\left(a^2+b^2\right)^2}-\frac{3b\tan^{-1}(\sinh(x))}{8\left(a^2+b^2\right)}+\frac{b^6\log(a+b\operatorname{csch}(x))}{a\left(a^2+b^2\right)^3}+$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.35, size = 253, normalized size = 1.30

$$\frac{ab(5a^4+14a^2b^2+9b^4)\,\text{ArcTan}(\sinh(x))+4a(a^5+ia^4b+3a^3b^2+3ia^2b^3+3ab^4+3ib^5)\log(i-\sinh(x))+4a(a^5-ia^4b+3a^3b^2-3ia^2b^3+3ab^4-3ib^5)\log(i+\sinh(x))+8b^6\log(b+a\sinh(x))+4a^2(2a^4+5a^2b^2+3b^4)\operatorname{sech}^2(x)-2a^2\left(a^2+b^2\right)^2\operatorname{sech}^4(x)+ab(5a^4+14a^2b^2+9b^4)\operatorname{sech}(x)\tanh(x)-2ab\left(a^2+b^2\right)^2\operatorname{sech}^3(x)\tanh(x)}{8a\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^5/(a + b*Csch[x]),x]
```

```
[Out] (a*b*(5*a^4 + 14*a^2*b^2 + 9*b^4)*ArcTan[Sinh[x]] + 4*a*(a^5 + I*a^4*b + 3*
a^3*b^2 + (3*I)*a^2*b^3 + 3*a*b^4 + (3*I)*b^5)*Log[I - Sinh[x]] + 4*a*(a^5
- I*a^4*b + 3*a^3*b^2 - (3*I)*a^2*b^3 + 3*a*b^4 - (3*I)*b^5)*Log[I + Sinh[x
]] + 8*b^6*Log[b + a*Sinh[x]] + 4*a^2*(2*a^4 + 5*a^2*b^2 + 3*b^4)*Sech[x]^2
 - 2*a^2*(a^2 + b^2)^2*Sech[x]^4 + a*b*(5*a^4 + 14*a^2*b^2 + 9*b^4)*Sech[x]
*Tanh[x] - 2*a*b*(a^2 + b^2)^2*Sech[x]^3*Tanh[x])/(8*a*(a^2 + b^2)^3)
```

Maple [A]
time = 1.66, size = 364, normalized size = 1.88

method	result
default	$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}+\frac{b^6\ln\left(-b\left(\tanh^2\left(\frac{x}{2}\right)\right)+2a\tanh\left(\frac{x}{2}\right)+b\right)}{\left(a^4+2a^2b^2+b^4\right)\left(a^2+b^2\right)a}+\frac{2\left(\left(-\frac{3}{8}a^4b-\frac{5}{4}a^2b^3-\frac{7}{8}b^5\right)\left(\tanh^7\left(\frac{x}{2}\right)\right)+\left(-a^5-3a^3b^2-2a\,b^4\right)\left(\tanh^6\left(\frac{x}{2}\right)\right)}{}$
risch	$\frac{x}{a}-\frac{2x\,a^5}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{6x\,a^3b^2}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{6xa\,b^4}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{2x\,b^6}{a\left(a^6+3a^4b^2+3a^2b^4+b^6\right)}+\frac{\left(5a^2b\,e^{6x}+9b^3e^{6x}+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a*ln(tanh(1/2*x)-1)+b^6/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/a*ln(-b*tanh(1/2*x
)^2+2*a*tanh(1/2*x)+b)+2/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*(((-3/8*a^4*b-5/4*a^
2*b^3-7/8*b^5)*tanh(1/2*x)^7+(-a^5-3*a^3*b^2-2*a*b^4)*tanh(1/2*x)^6+(-13/4*
a^2*b^3-15/8*b^5-11/8*a^4*b)*tanh(1/2*x)^5+(-4*a^5-10*a^3*b^2-6*a*b^4)*tanh
(1/2*x)^4+(13/4*a^2*b^3+15/8*b^5+11/8*a^4*b)*tanh(1/2*x)^3+(-a^5-3*a^3*b^2-
2*a*b^4)*tanh(1/2*x)^2+(3/8*a^4*b+5/4*a^2*b^3+7/8*b^5)*tanh(1/2*x))/(tanh(1
/2*x)^2+1)^4+1/16*(8*a^5+24*a^3*b^2+24*a*b^4)*ln(tanh(1/2*x)^2+1)+1/8*(-3*a
^4*b-10*a^2*b^3-15*b^5)*arctan(tanh(1/2*x)))-1/a*ln(tanh(1/2*x)+1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(185) = 370$.
time = 0.49, size = 383, normalized size = 1.97

$$\frac{b^{6} \log\left(-2\, b e^{(-x)} + a e^{(-2\, x)} - a\right)}{a^{7} + 3\, a^{5} b^{2} + 3\, a^{3} b^{4} + a b^{6}} + \frac{(3\, a^{4} b + 10\, a^{2} b^{3} + 15\, b^{5}) \arctan\left(e^{(-x)}\right)}{4\, (a^{6} + 3\, a^{4} b^{2} + 3\, a^{2} b^{4} + b^{6})} + \frac{(a^{5} + 3\, a^{3} b^{2} + 3\, a b^{4}) \log\left(e^{(-2\, x)} + 1\right)}{a^{6} + 3\, a^{4} b^{2} + 3\, a^{2} b^{4} + b^{6}} + \frac{(5\, a^{2} b + 9\, b^{3}) e^{(-x)} + 8\, (2\, a^{3} + 3\, a b^{2}) e^{(-2\, x)} - (3\, a^{2} b - b^{3}) e^{(-3\, x)} + 16\, (a^{3} + 2\, a b^{2}) e^{(-4\, x)} + (3\, a^{2} b - b^{3}) e^{(-5\, x)} + 8\, (2\, a^{3} + 3\, a b^{2}) e^{(-6\, x)} - (5\, a^{2} b + 9\, b^{3}) e^{(-7\, x)}}{4\, (a^{4} + 2\, a^{2} b^{2} + b^{4} + 4\, (a^{4} + 2\, a^{2} b^{2} + b^{4}) e^{(-2\, x)} + 6\, (a^{4} + 2\, a^{2} b^{2} + b^{4}) e^{(-4\, x)} + 4\, (a^{4} + 2\, a^{2} b^{2} + b^{4}) e^{(-6\, x)} + (a^{4} + 2\, a^{2} b^{2} + b^{4}) e^{(-8\, x)})} + \frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] b^6*log(-2*b*e^(-x) + a*e^(-2*x) - a)/(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)
 + 1/4*(3*a^4*b + 10*a^2*b^3 + 15*b^5)*arctan(e^(-x))/(a^6 + 3*a^4*b^2 + 3*
a^2*b^4 + b^6) + (a^5 + 3*a^3*b^2 + 3*a*b^4)*log(e^(-2*x) + 1)/(a^6 + 3*a^4
*b^2 + 3*a^2*b^4 + b^6) + 1/4*((5*a^2*b + 9*b^3)*e^(-x) + 8*(2*a^3 + 3*a*b^
2)*e^(-2*x) - (3*a^2*b - b^3)*e^(-3*x) + 16*(a^3 + 2*a*b^2)*e^(-4*x) + (3*a
^2*b - b^3)*e^(-5*x) + 8*(2*a^3 + 3*a*b^2)*e^(-6*x) - (5*a^2*b + 9*b^3)*e^(
-7*x))/(a^4 + 2*a^2*b^2 + b^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*e^(-2*x) + 6*(a^4
 + 2*a^2*b^2 + b^4)*e^(-4*x) + 4*(a^4 + 2*a^2*b^2 + b^4)*e^(-6*x) + (a^4 +
2*a^2*b^2 + b^4)*e^(-8*x)) + x/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4025 vs. $2(185) = 370$.
time = 0.46, size = 4025, normalized size = 20.75

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] -1/4*(4*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^8 + 4*(a^6 + 3*a^4*b^
2 + 3*a^2*b^4 + b^6)*x*sinh(x)^8 - (5*a^5*b + 14*a^3*b^3 + 9*a*b^5)*cosh(x)
^7 - (5*a^5*b + 14*a^3*b^3 + 9*a*b^5 - 32*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^
6)*x*cosh(x))*sinh(x)^7 - 8*(2*a^6 + 5*a^4*b^2 + 3*a^2*b^4 - 2*(a^6 + 3*a^4
*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x)^6 - (16*a^6 + 40*a^4*b^2 + 24*a^2*b^4 -
112*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^2 - 16*(a^6 + 3*a^4*b^2 +
 3*a^2*b^4 + b^6)*x + 7*(5*a^5*b + 14*a^3*b^3 + 9*a*b^5)*cosh(x))*sinh(x)^6
```

+ (3*a^5*b + 2*a^3*b^3 - a*b^5)*cosh(x)^5 + (3*a^5*b + 2*a^3*b^3 - a*b^5 + 224*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^3 - 21*(5*a^5*b + 14*a^3 *b^3 + 9*a*b^5)*cosh(x)^2 - 48*(2*a^6 + 5*a^4*b^2 + 3*a^2*b^4 - 2*(a^6 + 3* a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x))*sinh(x)^5 - 8*(2*a^6 + 6*a^4*b^2 + 4 *a^2*b^4 - 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x)^4 - (16*a^6 + 4 8*a^4*b^2 + 32*a^2*b^4 - 280*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^ 4 + 35*(5*a^5*b + 14*a^3*b^3 + 9*a*b^5)*cosh(x)^3 + 120*(2*a^6 + 5*a^4*b^2 + 3*a^2*b^4 - 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x)^2 - 24*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x - 5*(3*a^5*b + 2*a^3*b^3 - a*b^5)*cosh(x)) *sinh(x)^4 - (3*a^5*b + 2*a^3*b^3 - a*b^5)*cosh(x)^3 + (224*(a^6 + 3*a^4*b^ 2 + 3*a^2*b^4 + b^6)*x*cosh(x)^5 - 3*a^5*b - 2*a^3*b^3 + a*b^5 - 35*(5*a^5* b + 14*a^3*b^3 + 9*a*b^5)*cosh(x)^4 - 160*(2*a^6 + 5*a^4*b^2 + 3*a^2*b^4 - 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x)^3 + 10*(3*a^5*b + 2*a^3*b^ 3 - a*b^5)*cosh(x)^2 - 32*(2*a^6 + 6*a^4*b^2 + 4*a^2*b^4 - 3*(a^6 + 3*a^4*b ^2 + 3*a^2*b^4 + b^6)*x)*cosh(x))*sinh(x)^3 - 8*(2*a^6 + 5*a^4*b^2 + 3*a^2* b^4 - 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x)^2 + (112*(a^6 + 3*a^ 4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^6 - 16*a^6 - 40*a^4*b^2 - 24*a^2*b^4 - 2 1*(5*a^5*b + 14*a^3*b^3 + 9*a*b^5)*cosh(x)^5 - 120*(2*a^6 + 5*a^4*b^2 + 3*a ^2*b^4 - 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x)^4 + 10*(3*a^5*b + 2*a^3*b^3 - a*b^5)*cosh(x)^3 - 48*(2*a^6 + 6*a^4*b^2 + 4*a^2*b^4 - 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x)^2 + 16*(a^6 + 3*a^4*b^2 + 3*a^2*b ^4 + b^6)*x - 3*(3*a^5*b + 2*a^3*b^3 - a*b^5)*cosh(x))*sinh(x)^2 + 4*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x + ((3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x)^8 + 8*(3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x)*sinh(x)^7 + (3*a^5*b + 10 *a^3*b^3 + 15*a*b^5)*sinh(x)^8 + 4*(3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x)^6 + 4*(3*a^5*b + 10*a^3*b^3 + 15*a*b^5 + 7*(3*a^5*b + 10*a^3*b^3 + 15*a*b ^5)*cosh(x)^2)*sinh(x)^6 + 3*a^5*b + 10*a^3*b^3 + 15*a*b^5 + 8*(7*(3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x)^3 + 3*(3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*co sh(x))*sinh(x)^5 + 6*(3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x)^4 + 2*(9*a^5 *b + 30*a^3*b^3 + 45*a*b^5 + 35*(3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x)^4 + 30*(3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x)^2)*sinh(x)^4 + 8*(7*(3*a^5* b + 10*a^3*b^3 + 15*a*b^5)*cosh(x)^5 + 10*(3*a^5*b + 10*a^3*b^3 + 15*a*b^5) *cosh(x)^3 + 3*(3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x))*sinh(x)^3 + 4*(3* a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x)^2 + 4*(7*(3*a^5*b + 10*a^3*b^3 + 15* a*b^5)*cosh(x)^6 + 3*a^5*b + 10*a^3*b^3 + 15*a*b^5 + 15*(3*a^5*b + 10*a^3*b ^3 + 15*a*b^5)*cosh(x)^4 + 9*(3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x)^2)*s inh(x)^2 + 8*((3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x)^7 + 3*(3*a^5*b + 10 *a^3*b^3 + 15*a*b^5)*cosh(x)^5 + 3*(3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x)^3 + (3*a^5*b + 10*a^3*b^3 + 15*a*b^5)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) + (5*a^5*b + 14*a^3*b^3 + 9*a*b^5)*cosh(x) - 4*(b^6*cosh(x)^8 + 8* b^6*cosh(x)*sinh(x)^7 + b^6*sinh(x)^8 + 4*b^6*cosh(x)^6 + 6*b^6*cosh(x)^4 + 4*b^6*cosh(x)^2 + 4*(7*b^6*cosh(x)^2 + b^6)*sinh(x)^6 + b^6 + 8*(7*b^6*cos h(x)^3 + 3*b^6*cosh(x))*sinh(x)^5 + 2*(35*b^6*cosh(x)^4 + 30*b^6*cosh(x)^2 + 3*b^6)*sinh(x)^4 + 8*(7*b^6*cosh(x)^5 + 10*b^6*cosh(x)^3 + 3*b^6*cosh(x)) *sinh(x)^3 + 4*(7*b^6*cosh(x)^6 + 15*b^6*cosh(x)^4 + 9*b^6*cosh(x)^2 + b^6)

```
*sinh(x)^2 + 8*(b^6*cosh(x)^7 + 3*b^6*cosh(x)^5 + 3*b^6*cosh(x)^3 + b^6*cos
h(x))*sinh(x))*log(2*(a*sinh(x) + b)/(cosh(x) - sinh(x))) - 4*((a^6 + 3*a^4
*b^2 + 3*a^2*b^4)*cosh(x)^8 + 8*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)*sinh(
x)^7 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4)*sinh(x)^8 + 4*(a^6 + 3*a^4*b^2 + 3*a^2
*b^4)*cosh(x)^6 + 4*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + 7*(a^6 + 3*a^4*b^2 + 3*a
^2*b^4)*cosh(x)^2)*sinh(x)^6 + a^6 + 3*a^4*b^2 + 3*a^2*b^4 + 8*(7*(a^6 + 3*
a^4*b^2 + 3*a^2*b^4)*cosh(x)^3 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x))*s
inh(x)^5 + 6*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^4 + 2*(3*a^6 + 9*a^4*b^2
 + 9*a^2*b^4 + 35*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^4 + 30*(a^6 + 3*a^4
*b^2 + 3*a^2*b^4)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)
*cosh(x)^5 + 10*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^3 + 3*(a^6 + 3*a^4*b^
2 + 3*a^2*b^4)*cosh(x))*sinh(x)^3 + 4*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)
^2 + 4*(7*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^6 + a^6 + 3*a^4*b^2 + 3*a^2
*b^4 + 15*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^4 + 9*(a^6 + 3*a^4*b^2 + 3*
a^2*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((a^6 + 3*a^4...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^5(x)}{a + b\,\mathrm{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**5/(a+b*csch(x)),x)
```

```
[Out] Integral(tanh(x)**5/(a + b*csch(x)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 432 vs. 2(185) = 370.
time = 0.39, size = 432, normalized size = 2.23

$$\frac{b^6\log\left(\left|-a\left(e^{(-x)}-e^x\right)+2\,b\right|\right)}{a^7+3\,a^5b^2+3\,a^3b^4+ab^6}-\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2x)}-1\right)e^{(-x)}\right)\right)\left(3\,a^4b+10\,a^2b^3+15\,b^5\right)}{16\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)}+\frac{\left(a^5+3\,a^3b^2+3\,ab^4\right)\log\left(\left(e^{(-x)}-e^x\right)^2+4\right)}{2\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)}-\frac{3\,a^5\left(e^{(-x)}-e^x\right)^4+9\,a^3b^2\left(e^{(-x)}-e^x\right)^4+9\,ab^4\left(e^{(-x)}-e^x\right)^4+5\,a^4b\left(e^{(-x)}-e^x\right)^3+14\,a^2b^3\left(e^{(-x)}-e^x\right)^3+9\,b^5\left(e^{(-x)}-e^x\right)^3+8\,a^5\left(e^{(-x)}-e^x\right)^2+32\,a^3b^2\left(e^{(-x)}-e^x\right)^2+48\,ab^4\left(e^{(-x)}-e^x\right)^2+12\,a^4b\left(e^{(-x)}-e^x\right)+40\,a^2b^3\left(e^{(-x)}-e^x\right)+28\,b^5\left(e^{(-x)}-e^x\right)+16\,a^3b^2+64\,ab^4}{4\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\left(\left(e^{(-x)}-e^x\right)^2+4\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] b^6*log(abs(-a*(e^(-x) - e^x) + 2*b))/(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)
 - 1/16*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))*(3*a^4*b + 10*a^2*b^3 + 1
5*b^5)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/2*(a^5 + 3*a^3*b^2 + 3*a*b^4
)*log((e^(-x) - e^x)^2 + 4)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 1/4*(3*a^
5*(e^(-x) - e^x)^4 + 9*a^3*b^2*(e^(-x) - e^x)^4 + 9*a*b^4*(e^(-x) - e^x)^4
+ 5*a^4*b*(e^(-x) - e^x)^3 + 14*a^2*b^3*(e^(-x) - e^x)^3 + 9*b^5*(e^(-x) -
e^x)^3 + 8*a^5*(e^(-x) - e^x)^2 + 32*a^3*b^2*(e^(-x) - e^x)^2 + 48*a*b^4*(e
^(-x) - e^x)^2 + 12*a^4*b*(e^(-x) - e^x) + 40*a^2*b^3*(e^(-x) - e^x) + 28*b
^5*(e^(-x) - e^x) + 16*a^3*b^2 + 64*a*b^4)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
b^6)*((e^(-x) - e^x)^2 + 4)^2)
```

Mupad [B]
time = 7.04, size = 611, normalized size = 3.15

$$\frac{\frac{6\,\mathrm{e}^x\,\left(a^2\,b+b^3\right)}{{\left(a^2+b^2\right)}^2}+\frac{8\,\left(a^4+a^2\,b^2\right)}{a\,{\left(a^2+b^2\right)}^2}}{3\,\mathrm{e}^{2\,x}+3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}+1}-\frac{\frac{4\,a}{a^2+b^2}+\frac{4\,b\,\mathrm{e}^x}{a^2+b^2}}{4\,\mathrm{e}^{2\,x}+6\,\mathrm{e}^{4\,x}+4\,\mathrm{e}^{6\,x}+\mathrm{e}^{8\,x}+1}-\frac{x}{a}-\frac{\frac{\mathrm{e}^x\,\left(9\,a^2\,b+13\,b^3\right)}{2\,{\left(a^2+b^2\right)}^2}+\frac{2\,\left(4\,a^4+5\,a^2\,b^2\right)}{a\,{\left(a^2+b^2\right)}^2}}{2\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1}+\frac{\frac{\mathrm{e}^x\,\left(5\,a^4\,b+14\,a^2\,b^3+9\,b^5\right)}{4\,{\left(a^2+b^2\right)}^3}+\frac{2\,\left(2\,a^6+5\,a^4\,b^2+3\,a^2\,b^4\right)}{a\,{\left(a^2+b^2\right)}^3}}{\mathrm{e}^{2\,x}+1}+\frac{\ln\left(1+\mathrm{e}^x\,1\mathrm{i}\right)\,\left(-8\,a^2+a\,b\,21\mathrm{i}+15\,b^2\right)}{8\,\left(-a^3+a^2\,b\,3\mathrm{i}+3\,a\,b^2-b^3\,1\mathrm{i}\right)}+\frac{b^6\,\ln\left(64\,a^{13}\,\mathrm{e}^{2\,x}-64\,a\,b^{12}-64\,a^{13}+159\,a^3\,b^{10}-492\,a^5\,b^8-1214\,a^7\,b^6-1020\,a^9\,b^4-393\,a^{11}\,b^2+128\,b^{13}\,\mathrm{e}^x-159\,a^3\,b^{10}\,\mathrm{e}^{2\,x}+492\,a^5\,b^8\,\mathrm{e}^{2\,x}+1214\,a^7\,b^6\,\mathrm{e}^{2\,x}+1020\,a^9\,b^4\,\mathrm{e}^{2\,x}+393\,a^{11}\,b^2\,\mathrm{e}^{2\,x}+128\,a^{12}\,b\,\mathrm{e}^x+64\,a\,b^{12}\,\mathrm{e}^{2\,x}-318\,a^2\,b^{11}\,\mathrm{e}^x+984\,a^4\,b^9\,\mathrm{e}^x+2428\,a^6\,b^7\,\mathrm{e}^x+2040\,a^8\,b^5\,\mathrm{e}^x+786\,a^{10}\,b^3\,\mathrm{e}^x\right)}{a^7+3\,a^5\,b^2+3\,a^3\,b^4+a\,b^6}+\frac{\ln\left(\mathrm{e}^x+1\mathrm{i}\right)\,\left(-a^2\,8\mathrm{i}+21\,a\,b+b^2\,15\mathrm{i}\right)}{8\,\left(-a^3\,1\mathrm{i}+3\,a^2\,b+a\,b^2\,3\mathrm{i}-b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a + b/sinh(x)),x)
```

```
[Out] ((6*exp(x)*(a^2*b + b^3))/(a^2 + b^2)^2 + (8*(a^4 + a^2*b^2))/(a*(a^2 + b^2
)^2))/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - ((4*a)/(a^2 + b^2) + (4*b*
exp(x))/(a^2 + b^2))/(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1)
- x/a - ((exp(x)*(9*a^2*b + 13*b^3))/(2*(a^2 + b^2)^2) + (2*(4*a^4 + 5*a^2*
b^2))/(a*(a^2 + b^2)^2))/(2*exp(2*x) + exp(4*x) + 1) + ((exp(x)*(5*a^4*b +
9*b^5 + 14*a^2*b^3))/(4*(a^2 + b^2)^3) + (2*(2*a^6 + 3*a^2*b^4 + 5*a^4*b^2)
)/(a*(a^2 + b^2)^3))/(exp(2*x) + 1) + (log(exp(x)*1i + 1)*(a*b*21i - 8*a^2
+ 15*b^2))/(8*(3*a*b^2 + a^2*b*3i - a^3 - b^3*1i)) + (b^6*log(64*a^13*exp(2
*x) - 64*a*b^12 - 64*a^13 + 159*a^3*b^10 - 492*a^5*b^8 - 1214*a^7*b^6 - 102
0*a^9*b^4 - 393*a^11*b^2 + 128*b^13*exp(x) - 159*a^3*b^10*exp(2*x) + 492*a^
5*b^8*exp(2*x) + 1214*a^7*b^6*exp(2*x) + 1020*a^9*b^4*exp(2*x) + 393*a^11*b
^2*exp(2*x) + 128*a^12*b*exp(x) + 64*a*b^12*exp(2*x) - 318*a^2*b^11*exp(x)
+ 984*a^4*b^9*exp(x) + 2428*a^6*b^7*exp(x) + 2040*a^8*b^5*exp(x) + 786*a^10
*b^3*exp(x)))/(a*b^6 + a^7 + 3*a^3*b^4 + 3*a^5*b^2) + (log(exp(x) + 1i)*(21
*a*b - a^2*8i + b^2*15i))/(8*(a*b^2*3i + 3*a^2*b - a^3*1i - b^3))
```

3.114 $\int \frac{\tanh^4(x)}{a+b\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=183

$$\frac{ab^2x}{(a^2+b^2)^2}+\frac{b^4x}{a\,(a^2+b^2)^2}+\frac{ax}{a^2+b^2}+\frac{2b^5\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)^{5/2}}+\frac{b^3\text{sech}(x)}{(a^2+b^2)^2}+\frac{b\text{sech}(x)}{a^2+b^2}-\frac{b\text{sech}^3(x)}{3\,(a^2+b^2)}-\frac{ab^2\tanh(}{(a^2+b^2}$$

[Out] a*b^2*x/(a^2+b^2)^2+b^4*x/a/(a^2+b^2)^2+a*x/(a^2+b^2)+2*b^5*arctanh((a-b*tanh(1/2*x))/(a^2+b^2)^(1/2))/a/(a^2+b^2)^(5/2)+b^3*sech(x)/(a^2+b^2)^2+b*sech(x)/(a^2+b^2)-1/3*b*sech(x)^3/(a^2+b^2)-a*b^2*tanh(x)/(a^2+b^2)^2-a*tanh(x)/(a^2+b^2)-1/3*a*tanh(x)^3/(a^2+b^2)

Rubi [A]
time = 0.29, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {3983, 2981, 2686, 3554, 8, 2814, 2739, 632, 210}

$$\frac{ab^2x}{(a^2+b^2)^2}+\frac{ax}{a^2+b^2}-\frac{a\tanh^3(x)}{3\,(a^2+b^2)}-\frac{ab^2\tanh(x)}{(a^2+b^2)^2}-\frac{a\tanh(x)}{a^2+b^2}-\frac{b\text{sech}^3(x)}{3\,(a^2+b^2)}+\frac{b\text{sech}(x)}{a^2+b^2}+\frac{2b^5\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)^{5/2}}+\frac{b^4x}{a\,(a^2+b^2)^2}+\frac{b^3\text{sech}(x)}{(a^2+b^2)^2}$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]^4/(a + b*Csch[x]),x]

[Out] (a*b^2*x)/(a^2 + b^2)^2 + (b^4*x)/(a*(a^2 + b^2)^2) + (a*x)/(a^2 + b^2) + (2*b^5*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a*(a^2 + b^2)^(5/2)) + (b^3*Sech[x])/(a^2 + b^2)^2 + (b*Sech[x])/(a^2 + b^2) - (b*Sech[x]^3)/(3*(a^2 + b^2)) - (a*b^2*Tanh[x])/(a^2 + b^2)^2 - (a*Tanh[x])/(a^2 + b^2) - (a*Tanh[x]^3)/(3*(a^2 + b^2))

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2981

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a*(d^2/(a^2
- b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[b
*(d/(a^2 - b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[a^2*(d^2/(g^2*(a^2 - b^2))), Int[(g*Cos[e + f*x])^(p + 2)*((d*Sin[e +
 f*x])^(n - 2)/(a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3983

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
 + n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$\int \frac{\tanh^4(x)}{a+b\text{csch}(x)}\,dx = i\int \frac{\sinh(x)\tanh^4(x)}{ib+ia\sinh(x)}\,dx$$
$$= \frac{a\int \tanh^4(x)\,dx}{a^2+b^2} - \frac{b\int \text{sech}(x)\tanh^3(x)\,dx}{a^2+b^2} + \frac{(ib^2)\int \frac{\sinh(x)\tanh^2(x)}{ib+ia\sinh(x)}\,dx}{a^2+b^2}$$
$$= -\frac{a\tanh^3(x)}{3(a^2+b^2)} + \frac{(ab^2)\int \tanh^2(x)\,dx}{(a^2+b^2)^2} - \frac{b^3\int \text{sech}(x)\tanh(x)\,dx}{(a^2+b^2)^2} + \frac{(ib^4)\int \frac{\sinh(x)}{ib+ia\sinh(x)}\,dx}{(a^2+b^2)^2}$$
$$= \frac{b^4x}{a(a^2+b^2)^2} + \frac{b\text{sech}(x)}{a^2+b^2} - \frac{b\text{sech}^3(x)}{3(a^2+b^2)} - \frac{ab^2\tanh(x)}{(a^2+b^2)^2} - \frac{a\tanh(x)}{a^2+b^2} - \frac{a\tanh^3(x)}{3(a^2+b^2)} + \frac{(ab}{(a}$$
$$= \frac{ab^2x}{(a^2+b^2)^2} + \frac{b^4x}{a(a^2+b^2)^2} + \frac{ax}{a^2+b^2} + \frac{b^3\text{sech}(x)}{(a^2+b^2)^2} + \frac{b\text{sech}(x)}{a^2+b^2} - \frac{b\text{sech}^3(x)}{3(a^2+b^2)} - \frac{ab^2\tanh}{(a^2+b}$$
$$= \frac{ab^2x}{(a^2+b^2)^2} + \frac{b^4x}{a(a^2+b^2)^2} + \frac{ax}{a^2+b^2} + \frac{b^3\text{sech}(x)}{(a^2+b^2)^2} + \frac{b\text{sech}(x)}{a^2+b^2} - \frac{b\text{sech}^3(x)}{3(a^2+b^2)} - \frac{ab^2\tanh}{(a^2+b}$$
$$= \frac{ab^2x}{(a^2+b^2)^2} + \frac{b^4x}{a(a^2+b^2)^2} + \frac{ax}{a^2+b^2} + \frac{2b^5\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)^{5/2}} + \frac{b^3\text{sech}(x)}{(a^2+b^2)^2} + \frac{b\text{sech}}{a^2+}$$

Mathematica [A]
time = 0.52, size = 141, normalized size = 0.77

$$\frac{1}{3}\left(\frac{3\left(x-\frac{2b^5\text{ArcTan}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{(-a^2-b^2)^{5/2}}\right)}{a} + \frac{3b(a^2+2b^2)\,\text{sech}(x)}{(a^2+b^2)^2} - \frac{b\text{sech}^3(x)}{a^2+b^2} - \frac{a(4a^2+7b^2)\tanh(x)}{(a^2+b^2)^2} + \frac{a\text{sech}^2(x)\tanh(x)}{a^2+b^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(a + b*Csch[x]),x]
```

```
[Out] ((3*(x - (2*b^5*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/(-a^2 - b^2)^(5
/2)))/a + (3*b*(a^2 + 2*b^2)*Sech[x])/(a^2 + b^2)^2 - (b*Sech[x]^3)/(a^2 +
b^2) - (a*(4*a^2 + 7*b^2)*Tanh[x])/(a^2 + b^2)^2 + (a*Sech[x]^2*Tanh[x])/(a
^2 + b^2))/3
```

Maple [A]
time = 1.12, size = 207, normalized size = 1.13

method	result

default	$\frac{2\left(-a^{3}-2a\,b^{2}\right)\left(\tanh^{5}\left(\frac{x}{2}\right)\right)+2b^{3}\left(\tanh^{4}\left(\frac{x}{2}\right)\right)+2\left(-\frac{10}{3}a^{3}-\frac{16}{3}a\,b^{2}\right)\left(\tanh^{3}\left(\frac{x}{2}\right)\right)+2\left(2a^{2}b+4b^{3}\right)\left(\tanh^{2}\left(\frac{x}{2}\right)\right)+2\left(-a^{3}-2a\,b^{2}\right)\tanh\left(\frac{x}{2}\right)+\frac{4a^{2}}{3}}{\left(a^{4}+2a^{2}b^{2}+b^{4}\right)\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)^{3}}$
risch	$\frac{x}{a}+\frac{2a^{2}b\,e^{5x}+4b^{3}e^{5x}+4a^{3}e^{4x}+6a\,b^{2}e^{4x}+\frac{4a^{2}b\,e^{3x}}{3}+\frac{16b^{3}e^{3x}}{3}+4a^{3}e^{2x}+8a\,b^{2}e^{2x}+2a^{2}b\,e^{x}+4b^{3}e^{x}+\frac{8a^{3}}{3}+\frac{14a\,b^{2}}{3}}{\left(a^{4}+2a^{2}b^{2}+b^{4}\right)\left(1+e^{2x}\right)^{3}}+\frac{b^{5}\ln\left(e^{x}+\frac{\left(a^{2}+b^{2}\right.}{}\right.}{(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(a^4+2*a^2*b^2+b^4)*((-a^3-2*a*b^2)*tanh(1/2*x)^5+b^3*tanh(1/2*x)^4+(-10/
3*a^3-16/3*a*b^2)*tanh(1/2*x)^3+(2*a^2*b+4*b^3)*tanh(1/2*x)^2+(-a^3-2*a*b^2
)*tanh(1/2*x)+2/3*a^2*b+5/3*b^3)/(tanh(1/2*x)^2+1)^3+2/a*b^5/(a^4+2*a^2*b^2
+b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(-2*b*tanh(1/2*x)+2*a)/(a^2+b^2)^(1/2))+1
/a*ln(tanh(1/2*x)+1)-1/a*ln(tanh(1/2*x)-1)
```

Maxima [A]
time = 0.49, size = 261, normalized size = 1.43

$$-\frac{b^{5}\log\left(\frac{ae^{(-x)}-b-\sqrt{a^{2}+b^{2}}}{ae^{(-x)}-b+\sqrt{a^{2}+b^{2}}}\right)}{\left(a^{5}+2\,a^{3}b^{2}+ab^{4}\right)\sqrt{a^{2}+b^{2}}}-\frac{2\,\left(4\,a^{3}+7\,ab^{2}-3\,\left(a^{2}b+2\,b^{3}\right)e^{(-x)}+6\,\left(a^{3}+2\,ab^{2}\right)e^{(-2\,x)}-2\,\left(a^{2}b+4\,b^{3}\right)e^{(-3\,x)}+3\,\left(2\,a^{3}+3\,ab^{2}\right)e^{(-4\,x)}-3\,\left(a^{2}b+2\,b^{3}\right)e^{(-5\,x)}\right)}{3\,\left(a^{4}+2\,a^{2}b^{2}+b^{4}+3\,\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)e^{(-2\,x)}+3\,\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)e^{(-4\,x)}+\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)e^{(-6\,x)}\right)}+\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -b^5*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))
/((a^5 + 2*a^3*b^2 + a*b^4)*sqrt(a^2 + b^2)) - 2/3*(4*a^3 + 7*a*b^2 - 3*(a^
2*b + 2*b^3)*e^(-x) + 6*(a^3 + 2*a*b^2)*e^(-2*x) - 2*(a^2*b + 4*b^3)*e^(-3*
x) + 3*(2*a^3 + 3*a*b^2)*e^(-4*x) - 3*(a^2*b + 2*b^3)*e^(-5*x))/(a^4 + 2*a^
2*b^2 + b^4 + 3*(a^4 + 2*a^2*b^2 + b^4)*e^(-2*x) + 3*(a^4 + 2*a^2*b^2 + b^4
)*e^(-4*x) + (a^4 + 2*a^2*b^2 + b^4)*e^(-6*x)) + x/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1746 vs. $2(175) = 350$.
time = 0.40, size = 1746, normalized size = 9.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^6 + 3*(a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6)*x*sinh(x)^6 + 8*a^6 + 22*a^4*b^2 + 14*a^2*b^4 + 6*(a^5*
b + 3*a^3*b^3 + 2*a*b^5)*cosh(x)^5 + 6*(a^5*b + 3*a^3*b^3 + 2*a*b^5 + 3*(a^
```

```
6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x))*sinh(x)^5 + 3*(4*a^6 + 10*a^4*b
^2 + 6*a^2*b^4 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x)^4 + 3*(4*
a^6 + 10*a^4*b^2 + 6*a^2*b^4 + 15*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cos
h(x)^2 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x + 10*(a^5*b + 3*a^3*b^3 +
2*a*b^5)*cosh(x))*sinh(x)^4 + 4*(a^5*b + 5*a^3*b^3 + 4*a*b^5)*cosh(x)^3 + 4
*(a^5*b + 5*a^3*b^3 + 4*a*b^5 + 15*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*co
sh(x)^3 + 15*(a^5*b + 3*a^3*b^3 + 2*a*b^5)*cosh(x)^2 + 3*(4*a^6 + 10*a^4*b^
2 + 6*a^2*b^4 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x))*sinh(x)^3
 + 3*(4*a^6 + 12*a^4*b^2 + 8*a^2*b^4 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
)*x)*cosh(x)^2 + 3*(4*a^6 + 12*a^4*b^2 + 8*a^2*b^4 + 15*(a^6 + 3*a^4*b^2 +
3*a^2*b^4 + b^6)*x*cosh(x)^4 + 20*(a^5*b + 3*a^3*b^3 + 2*a*b^5)*cosh(x)^3 +
 6*(4*a^6 + 10*a^4*b^2 + 6*a^2*b^4 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*
x)*cosh(x)^2 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x + 4*(a^5*b + 5*a^3*b
^3 + 4*a*b^5)*cosh(x))*sinh(x)^2 + 3*(b^5*cosh(x)^6 + 6*b^5*cosh(x)*sinh(x)
^5 + b^5*sinh(x)^6 + 3*b^5*cosh(x)^4 + 3*b^5*cosh(x)^2 + b^5 + 3*(5*b^5*cos
h(x)^2 + b^5)*sinh(x)^4 + 4*(5*b^5*cosh(x)^3 + 3*b^5*cosh(x))*sinh(x)^3 + 3
*(5*b^5*cosh(x)^4 + 6*b^5*cosh(x)^2 + b^5)*sinh(x)^2 + 6*(b^5*cosh(x)^5 + 2
*b^5*cosh(x)^3 + b^5*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((a^2*cosh(x)^2 +
 a^2*sinh(x)^2 + 2*a*b*cosh(x) + a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x
) + 2*sqrt(a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)
^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) - a)) + 3*(a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6)*x + 6*(a^5*b + 3*a^3*b^3 + 2*a*b^5)*cosh(x) + 6*(3*(a^6 + 3
*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^5 + a^5*b + 3*a^3*b^3 + 2*a*b^5 + 5*(
a^5*b + 3*a^3*b^3 + 2*a*b^5)*cosh(x)^4 + 2*(4*a^6 + 10*a^4*b^2 + 6*a^2*b^4
+ 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x)*cosh(x)^3 + 2*(a^5*b + 5*a^3*b^3
 + 4*a*b^5)*cosh(x)^2 + (4*a^6 + 12*a^4*b^2 + 8*a^2*b^4 + 3*(a^6 + 3*a^4*b^
2 + 3*a^2*b^4 + b^6)*x)*cosh(x))*sinh(x))/(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*
b^6 + (a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^6 + 6*(a^7 + 3*a^5*b^2
+ 3*a^3*b^4 + a*b^6)*cosh(x)*sinh(x)^5 + (a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b
^6)*sinh(x)^6 + 3*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^4 + 3*(a^7
+ 3*a^5*b^2 + 3*a^3*b^4 + a*b^6 + 5*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*c
osh(x)^2)*sinh(x)^4 + 4*(5*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^3
+ 3*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x))*sinh(x)^3 + 3*(a^7 + 3*a
^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^2 + 3*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*
b^6 + 5*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^4 + 6*(a^7 + 3*a^5*b^
2 + 3*a^3*b^4 + a*b^6)*cosh(x)^2)*sinh(x)^2 + 6*((a^7 + 3*a^5*b^2 + 3*a^3*b
^4 + a*b^6)*cosh(x)^5 + 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^3 +
 (a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x))*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(x)}{a + b\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(a+b*csch(x)),x)
```

```
[Out] Integral(tanh(x)**4/(a + b*csch(x)), x)
```

Giac [A]
time = 0.40, size = 215, normalized size = 1.17

$$-\frac{b^5\log\left(\frac{\left|2\,ae^x+2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,ae^x+2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^5+2\,a^3b^2+ab^4\right)\sqrt{a^2+b^2}}+\frac{x}{a}+\frac{2\left(3\,a^2be^{(5\,x)}+6\,b^3e^{(5\,x)}+6\,a^3e^{(4\,x)}+9\,ab^2e^{(4\,x)}+2\,a^2be^{(3\,x)}+8\,b^3e^{(3\,x)}+6\,a^3e^{(2\,x)}+12\,ab^2e^{(2\,x)}+3\,a^2be^x+6\,b^3e^x+4\,a^3+7\,ab^2\right)}{3\left(a^4+2\,a^2b^2+b^4\right)\left(e^{(2\,x)}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -b^5*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(
a^2 + b^2)))/((a^5 + 2*a^3*b^2 + a*b^4)*sqrt(a^2 + b^2)) + x/a + 2/3*(3*a^2
*b*e^(5*x) + 6*b^3*e^(5*x) + 6*a^3*e^(4*x) + 9*a*b^2*e^(4*x) + 2*a^2*b*e^(3
*x) + 8*b^3*e^(3*x) + 6*a^3*e^(2*x) + 12*a*b^2*e^(2*x) + 3*a^2*b*e^x + 6*b^
3*e^x + 4*a^3 + 7*a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*(e^(2*x) + 1)^3)
```

Mupad [B]
time = 3.43, size = 707, normalized size = 3.86

$$\frac{x}{a}+\frac{\frac{8\,a}{3\left(a^2+b^2\right)}+\frac{8\,b\,e^x}{3\left(a^2+b^2\right)}}{3\,e^{2\,x}+3\,e^{4\,x}+e^{6\,x}+1}-\frac{\frac{8\,e^x\left(a^2b+b^3\right)}{3\left(a^2+b^2\right)^2}+\frac{4\left(a^4+a^2b^2\right)}{a\left(a^2+b^2\right)^2}}{2\,e^{2\,x}+e^{4\,x}+1}+\frac{\frac{2\,e^x\left(a^2b+2\,b^3\right)}{\left(a^2+b^2\right)^2}+\frac{2\left(2\,a^4+3\,a^2b^2\right)}{a\left(a^2+b^2\right)^2}}{e^{2\,x}+1}+\frac{2\,\mathrm{atan}\left(\left(\left(e^x\left(\frac{2\,b^5}{a^3\sqrt{b^{10}}\left(a^2+b^2\right)^2\left(a^5+2\,a^3b^2+a\,b^4\right)}+\frac{2\left(2\,a^3b^3\sqrt{b^{10}}+a\,b^5\sqrt{b^{10}}+a^5b\sqrt{b^{10}}\right)}{a^2b^4\sqrt{-a^2\left(a^2+b^2\right)^5}\left(a^5+2\,a^3b^2+a\,b^4\right)\sqrt{-a^{12}-5\,a^{10}b^2-10\,a^8b^4-10\,a^6b^6-5\,a^4b^8-a^2b^{10}}}\right)-\frac{2\left(a^6\sqrt{b^{10}}+a^2b^4\sqrt{b^{10}}+2\,a^4b^2\sqrt{b^{10}}\right)}{a^2b^4\sqrt{-a^2\left(a^2+b^2\right)^5}\left(a^5+2\,a^3b^2+a\,b^4\right)\sqrt{-a^{12}-5\,a^{10}b^2-10\,a^8b^4-10\,a^6b^6-5\,a^4b^8-a^2b^{10}}}\right)\left(\frac{a^6\sqrt{-a^{12}-5\,a^{10}b^2-10\,a^8b^4-10\,a^6b^6-5\,a^4b^8-a^2b^{10}}}{2}+\frac{a^2b^4\sqrt{-a^{12}-5\,a^{10}b^2-10\,a^8b^4-10\,a^6b^6-5\,a^4b^8-a^2b^{10}}}{2}+a^4b^2\sqrt{-a^{12}-5\,a^{10}b^2-10\,a^8b^4-10\,a^6b^6-5\,a^4b^8-a^2b^{10}}\right)\right)\sqrt{b^{10}}}{\sqrt{-a^{12}-5\,a^{10}b^2-10\,a^8b^4-10\,a^6b^6-5\,a^4b^8-a^2b^{10}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a + b/sinh(x)),x)
```

```
[Out] x/a + ((8*a)/(3*(a^2 + b^2)) + (8*b*exp(x))/(3*(a^2 + b^2)))/(3*exp(2*x) +
3*exp(4*x) + exp(6*x) + 1) - ((8*exp(x)*(a^2*b + b^3))/(3*(a^2 + b^2)^2) +
(4*(a^4 + a^2*b^2))/(a*(a^2 + b^2)^2))/(2*exp(2*x) + exp(4*x) + 1) + ((2*ex
p(x)*(a^2*b + 2*b^3))/(a^2 + b^2)^2 + (2*(2*a^4 + 3*a^2*b^2))/(a*(a^2 + b^2
)^2))/(exp(2*x) + 1) + (2*atan((exp(x)*((2*b^5)/(a^3*(b^10)^(1/2)*(a^2 + b^
2)^2*(a*b^4 + a^5 + 2*a^3*b^2)) + (2*(2*a^3*b^3*(b^10)^(1/2) + a*b^5*(b^10)
^(1/2) + a^5*b*(b^10)^(1/2)))/(a^2*b^4*(-a^2*(a^2 + b^2)^5)^(1/2)*(a*b^4 +
a^5 + 2*a^3*b^2)*(- a^12 - a^2*b^10 - 5*a^4*b^8 - 10*a^6*b^6 - 10*a^8*b^4 -
 5*a^10*b^2)^(1/2))) - (2*(a^6*(b^10)^(1/2) + a^2*b^4*(b^10)^(1/2) + 2*a^4*
b^2*(b^10)^(1/2)))/(a^2*b^4*(-a^2*(a^2 + b^2)^5)^(1/2)*(a*b^4 + a^5 + 2*a^3
*b^2)*(- a^12 - a^2*b^10 - 5*a^4*b^8 - 10*a^6*b^6 - 10*a^8*b^4 - 5*a^10*b^2
)^(1/2)))*((a^6*(- a^12 - a^2*b^10 - 5*a^4*b^8 - 10*a^6*b^6 - 10*a^8*b^4 -
5*a^10*b^2)^(1/2))/2 + (a^2*b^4*(- a^12 - a^2*b^10 - 5*a^4*b^8 - 10*a^6*b^6
 - 10*a^8*b^4 - 5*a^10*b^2)^(1/2))/2 + a^4*b^2*(- a^12 - a^2*b^10 - 5*a^4*b
^8 - 10*a^6*b^6 - 10*a^8*b^4 - 5*a^10*b^2)^(1/2)))*(b^10)^(1/2))/(- a^12 -
a^2*b^10 - 5*a^4*b^8 - 10*a^6*b^6 - 10*a^8*b^4 - 5*a^10*b^2)^(1/2)
```

3.115 $\int \frac{\tanh^3(x)}{a+b\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=113

$$-\frac{b^3\text{ArcTan}(\sinh(x))}{(a^2+b^2)^2}-\frac{b\text{ArcTan}(\sinh(x))}{2(a^2+b^2)}+\frac{b^4\log(a+b\operatorname{csch}(x))}{a(a^2+b^2)^2}+\frac{\log(\sinh(x))}{a}-\frac{a(a^2+2b^2)\log(\tanh(x))}{(a^2+b^2)^2}-$$

```
[Out] -b^3*arctan(sinh(x))/(a^2+b^2)^2-1/2*b*arctan(sinh(x))/(a^2+b^2)+b^4*ln(a+b
*csch(x))/a/(a^2+b^2)^2+ln(sinh(x))/a-a*(a^2+2*b^2)*ln(tanh(x))/(a^2+b^2)^2
-1/2*(a-b*csch(x))*tanh(x)^2/(a^2+b^2)
```

Rubi [A]
time = 0.12, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3970, 908, 653, 209, 649, 266}

$$-\frac{b\text{ArcTan}(\sinh(x))}{2(a^2+b^2)}-\frac{b^3\text{ArcTan}(\sinh(x))}{(a^2+b^2)^2}-\frac{a(a^2+2b^2)\log(\tanh(x))}{(a^2+b^2)^2}-\frac{\tanh^2(x)(a-b\operatorname{csch}(x))}{2(a^2+b^2)}+\frac{b^4\log(a+b\operatorname{csch}(x))}{a(a^2+b^2)^2}+\frac{\log(\sinh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/(a + b*Csch[x]),x]
```

```
[Out] -((b^3*ArcTan[Sinh[x]])/(a^2 + b^2)^2) - (b*ArcTan[Sinh[x]])/(2*(a^2 + b^2)
) + (b^4*Log[a + b*Csch[x]])/(a*(a^2 + b^2)^2) + Log[Sinh[x]]/a - (a*(a^2 +
 2*b^2)*Log[Tanh[x]])/(a^2 + b^2)^2 - ((a - b*Csch[x])*Tanh[x]^2)/(2*(a^2 +
 b^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^3(x)}{a+b\operatorname{csch}(x)}\,dx &= -\left(b^4\text{Subst}\left(\int \frac{1}{x(a+x)\left(-b^2-x^2\right)^2}\,dx,x,b\operatorname{csch}(x)\right)\right)\\
&= -\left(b^4\text{Subst}\left(\int \left(\frac{1}{ab^4x}-\frac{1}{a\left(a^2+b^2\right)^2(a+x)}+\frac{-b^2-ax}{b^2\left(a^2+b^2\right)\left(b^2+x^2\right)^2}+\frac{-b^4-a(a^2\cdots}{b^4\left(a^2+b^2\right)^2(\cdots}\right.\right.\right.\\
&= \frac{b^4\log(a+b\operatorname{csch}(x))}{a\left(a^2+b^2\right)^2}+\frac{\log(\sinh(x))}{a}-\frac{\text{Subst}\left(\int \frac{-b^4-a\left(a^2+2b^2\right)x}{b^2+x^2}\,dx,x,b\operatorname{csch}(x)\right)}{\left(a^2+b^2\right)^2}-\frac{b^2\text{Sul}\cdots}{\cdots}\\
&= \frac{b^4\log(a+b\operatorname{csch}(x))}{a\left(a^2+b^2\right)^2}+\frac{\log(\sinh(x))}{a}-\frac{(a-b\operatorname{csch}(x))\tanh^2(x)}{2\left(a^2+b^2\right)}+\frac{b^4\text{Subst}\left(\int \frac{1}{b^2+x^2}\,dx,\cdots\right.}{\left(a^2+b^2\right)^2\cdots}\\
&= -\frac{b^3\tan^{-1}(\sinh(x))}{\left(a^2+b^2\right)^2}-\frac{b\tan^{-1}(\sinh(x))}{2\left(a^2+b^2\right)}+\frac{b^4\log(a+b\operatorname{csch}(x))}{a\left(a^2+b^2\right)^2}+\frac{\log(\sinh(x))}{a}-\frac{a(a^2\cdots}{\cdots}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.14, size = 191, normalized size = 1.69

$$\frac{ab(a^2+b^2)\operatorname{ArcTan}(\sinh(x))+a^4\log(i-\sinh(x))+ia^3b\log(i-\sinh(x))+2a^2b^2\log(i-\sinh(x))+2iab^3\log(i-\sinh(x))+a^4\log(i+\sinh(x))-ia^3b\log(i+\sinh(x))+2a^2b^2\log(i+\sinh(x))-2iab^3\log(i+\sinh(x))+2b^4\log(b+a\sinh(x))+a^2(a^2+b^2)\operatorname{sech}^2(x)+ab(a^2+b^2)\operatorname{sech}(x)\tanh(x)}{2a\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(a + b*Csch[x]),x]
```

```
[Out] (a*b*(a^2 + b^2)*ArcTan[Sinh[x]] + a^4*Log[I - Sinh[x]] + I*a^3*b*Log[I - S
inh[x]] + 2*a^2*b^2*Log[I - Sinh[x]] + (2*I)*a*b^3*Log[I - Sinh[x]] + a^4*L
og[I + Sinh[x]] - I*a^3*b*Log[I + Sinh[x]] + 2*a^2*b^2*Log[I + Sinh[x]] - (
2*I)*a*b^3*Log[I + Sinh[x]] + 2*b^4*Log[b + a*Sinh[x]] + a^2*(a^2 + b^2)*Se
ch[x]^2 + a*b*(a^2 + b^2)*Sech[x]*Tanh[x])/(2*a*(a^2 + b^2)^2)
```

Maple [A]
time = 1.04, size = 182, normalized size = 1.61

method	result
default	$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}+\frac{\frac{2\left(\left(-\frac{1}{2}a^{2}b-\frac{1}{2}b^{3}\right)\left(\tanh^{3}\left(\frac{x}{2}\right)\right)+\left(-a^{3}-a\,b^{2}\right)\left(\tanh^{2}\left(\frac{x}{2}\right)\right)+\left(\frac{1}{2}a^{2}b+\frac{1}{2}b^{3}\right)\tanh\left(\frac{x}{2}\right)\right)}{\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{\left(2a^{3}+4a\,b^{2}\right)\ln\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)}{2}+}{\left(a^{2}+b^{2}\right)^{2}}$
risch	$\frac{x}{a}-\frac{2x\,a^{3}}{a^{4}+2a^{2}b^{2}+b^{4}}-\frac{4xa\,b^{2}}{a^{4}+2a^{2}b^{2}+b^{4}}-\frac{2x\,b^{4}}{a(a^{4}+2a^{2}b^{2}+b^{4})}+\frac{e^{x}\left(b\,e^{2x}+2a\,e^{x}-b\right)}{(1+e^{2x})^{2}(a^{2}+b^{2})}+\frac{i\ln(e^{x}-i)a^{2}b}{2a^{4}+4a^{2}b^{2}+2b^{4}}+\frac{3i\ln(e^{x}-i)b^{3}}{2(a^{4}+2a^{2}b^{2}+b^{4})}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a*ln(tanh(1/2*x)-1)+2/(a^2+b^2)^2*(((-1/2*a^2*b-1/2*b^3)*tanh(1/2*x)^3+(
-a^3-a*b^2)*tanh(1/2*x)^2+(1/2*a^2*b+1/2*b^3)*tanh(1/2*x))/(tanh(1/2*x)^2+1
)^2+1/4*(2*a^3+4*a*b^2)*ln(tanh(1/2*x)^2+1)+1/2*(-a^2*b-3*b^3)*arctan(tanh(
1/2*x)))+b^4/a/(a^2+b^2)^2*ln(-b*tanh(1/2*x)^2+2*a*tanh(1/2*x)+b)-1/a*ln(ta
nh(1/2*x)+1)
```

Maxima [A]
time = 0.48, size = 172, normalized size = 1.52

$$\frac{b^4 \log\left(-2\,be^{(-x)} + ae^{(-2\,x)} - a\right)}{a^5 + 2\,a^3b^2 + ab^4} + \frac{\left(a^2b + 3\,b^3\right)\arctan\left(e^{(-x)}\right)}{a^4 + 2\,a^2b^2 + b^4} + \frac{\left(a^3 + 2\,ab^2\right)\log\left(e^{(-2\,x)} + 1\right)}{a^4 + 2\,a^2b^2 + b^4} + \frac{be^{(-x)} + 2\,ae^{(-2\,x)} - be^{(-3\,x)}}{a^2 + b^2 + 2\,(a^2 + b^2)e^{(-2\,x)} + (a^2 + b^2)e^{(-4\,x)}} + \frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] b^4*log(-2*b*e^(-x) + a*e^(-2*x) - a)/(a^5 + 2*a^3*b^2 + a*b^4) + (a^2*b +
3*b^3)*arctan(e^(-x))/(a^4 + 2*a^2*b^2 + b^4) + (a^3 + 2*a*b^2)*log(e^(-2*x
) + 1)/(a^4 + 2*a^2*b^2 + b^4) + (b*e^(-x) + 2*a*e^(-2*x) - b*e^(-3*x))/(a^
2 + b^2 + 2*(a^2 + b^2)*e^(-2*x) + (a^2 + b^2)*e^(-4*x)) + x/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 965 vs. 2(110) = 220.
time = 0.57, size = 965, normalized size = 8.54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] -((a^4 + 2*a^2*b^2 + b^4)*x*cosh(x)^4 + (a^4 + 2*a^2*b^2 + b^4)*x*sinh(x)^4
 - (a^3*b + a*b^3)*cosh(x)^3 - (a^3*b + a*b^3 - 4*(a^4 + 2*a^2*b^2 + b^4)*x
*cosh(x))*sinh(x)^3 - 2*(a^4 + a^2*b^2 - (a^4 + 2*a^2*b^2 + b^4)*x)*cosh(x)
^2 - (2*a^4 + 2*a^2*b^2 - 6*(a^4 + 2*a^2*b^2 + b^4)*x*cosh(x)^2 - 2*(a^4 +
2*a^2*b^2 + b^4)*x + 3*(a^3*b + a*b^3)*cosh(x))*sinh(x)^2 + (a^4 + 2*a^2*b^
2 + b^4)*x + ((a^3*b + 3*a*b^3)*cosh(x)^4 + 4*(a^3*b + 3*a*b^3)*cosh(x)*sin
h(x)^3 + (a^3*b + 3*a*b^3)*sinh(x)^4 + a^3*b + 3*a*b^3 + 2*(a^3*b + 3*a*b^3
)*cosh(x)^2 + 2*(a^3*b + 3*a*b^3 + 3*(a^3*b + 3*a*b^3)*cosh(x)^2)*sinh(x)^2
 + 4*((a^3*b + 3*a*b^3)*cosh(x)^3 + (a^3*b + 3*a*b^3)*cosh(x))*sinh(x))*arc
tan(cosh(x) + sinh(x)) + (a^3*b + a*b^3)*cosh(x) - (b^4*cosh(x)^4 + 4*b^4*c
osh(x)*sinh(x)^3 + b^4*sinh(x)^4 + 2*b^4*cosh(x)^2 + b^4 + 2*(3*b^4*cosh(x)
^2 + b^4)*sinh(x)^2 + 4*(b^4*cosh(x)^3 + b^4*cosh(x))*sinh(x))*log(2*(a*sin
h(x) + b)/(cosh(x) - sinh(x))) - ((a^4 + 2*a^2*b^2)*cosh(x)^4 + 4*(a^4 + 2*
a^2*b^2)*cosh(x)*sinh(x)^3 + (a^4 + 2*a^2*b^2)*sinh(x)^4 + a^4 + 2*a^2*b^2
+ 2*(a^4 + 2*a^2*b^2)*cosh(x)^2 + 2*(a^4 + 2*a^2*b^2 + 3*(a^4 + 2*a^2*b^2)*
cosh(x)^2)*sinh(x)^2 + 4*((a^4 + 2*a^2*b^2)*cosh(x)^3 + (a^4 + 2*a^2*b^2)*c
osh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) + (4*(a^4 + 2*a^2*b^2 +
 b^4)*x*cosh(x)^3 + a^3*b + a*b^3 - 3*(a^3*b + a*b^3)*cosh(x)^2 - 4*(a^4 +
a^2*b^2 - (a^4 + 2*a^2*b^2 + b^4)*x)*cosh(x))*sinh(x))/(a^5 + 2*a^3*b^2 + a
*b^4 + (a^5 + 2*a^3*b^2 + a*b^4)*cosh(x)^4 + 4*(a^5 + 2*a^3*b^2 + a*b^4)*co
sh(x)*sinh(x)^3 + (a^5 + 2*a^3*b^2 + a*b^4)*sinh(x)^4 + 2*(a^5 + 2*a^3*b^2
+ a*b^4)*cosh(x)^2 + 2*(a^5 + 2*a^3*b^2 + a*b^4 + 3*(a^5 + 2*a^3*b^2 + a*b^
4)*cosh(x)^2)*sinh(x)^2 + 4*((a^5 + 2*a^3*b^2 + a*b^4)*cosh(x)^3 + (a^5 + 2
*a^3*b^2 + a*b^4)*cosh(x))*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(x)}{a + b \operatorname{csch}(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(a+b*csch(x)),x)
```

```
[Out] Integral(tanh(x)**3/(a + b*csch(x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 234 vs. 2(110) = 220.
time = 0.39, size = 234, normalized size = 2.07

$$\frac{b^4 \log\left(\left|-a\left(e^{(-x)} - e^x\right) + 2\,b\right|\right)}{a^5 + 2\,a^3b^2 + ab^4} - \frac{\left(\pi + 2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)} - 1\right)e^{(-x)}\right)\right)\left(a^2b + 3\,b^3\right)}{4\left(a^4 + 2\,a^2b^2 + b^4\right)} + \frac{\left(a^3 + 2\,ab^2\right)\log\left(\left(e^{(-x)} - e^x\right)^2 + 4\right)}{2\left(a^4 + 2\,a^2b^2 + b^4\right)} - \frac{a^3\left(e^{(-x)} - e^x\right)^2 + 2\,ab^2\left(e^{(-x)} - e^x\right)^2 + 2\,a^2b\left(e^{(-x)} - e^x\right) + 2\,b^3\left(e^{(-x)} - e^x\right) + 4\,ab^2}{2\left(a^4 + 2\,a^2b^2 + b^4\right)\left(\left(e^{(-x)} - e^x\right)^2 + 4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] b^4*log(abs(-a*(e^(-x) - e^x) + 2*b))/(a^5 + 2*a^3*b^2 + a*b^4) - 1/4*(pi +
 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))*(a^2*b + 3*b^3)/(a^4 + 2*a^2*b^2 + b^4
) + 1/2*(a^3 + 2*a*b^2)*log((e^(-x) - e^x)^2 + 4)/(a^4 + 2*a^2*b^2 + b^4) -
 1/2*(a^3*(e^(-x) - e^x)^2 + 2*a*b^2*(e^(-x) - e^x)^2 + 2*a^2*b*(e^(-x) - e
^x) + 2*b^3*(e^(-x) - e^x) + 4*a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*((e^(-x) - e
^x)^2 + 4))
```

Mupad [B]
time = 4.01, size = 335, normalized size = 2.96

$$\frac{\frac{e^x\,\left(a^2\,b+b^3\right)}{{\left(a^2+b^2\right)}^2}+\frac{2\,\left(a^4+a^2\,b^2\right)}{a\,{\left(a^2+b^2\right)}^2}}{e^{2\,x}+1}-\frac{\frac{2\,a}{a^2+b^2}+\frac{2\,b\,e^x}{a^2+b^2}}{2\,e^{2\,x}+e^{4\,x}+1}-\frac{x}{a}+\frac{b^4\,\ln\left(4\,a^9\,e^{2\,x}-4\,a\,b^8-4\,a^9+7\,a^3\,b^6-14\,a^5\,b^4-17\,a^7\,b^2+8\,b^9\,e^x-7\,a^3\,b^6\,e^{2\,x}+14\,a^5\,b^4\,e^{2\,x}+17\,a^7\,b^2\,e^{2\,x}+8\,a^8\,b\,e^x+4\,a\,b^8\,e^{2\,x}-14\,a^2\,b^7\,e^x+28\,a^4\,b^5\,e^x+34\,a^6\,b^3\,e^x\right)}{a^5+2\,a^3\,b^2+a\,b^4}+\frac{\ln\left(1+e^x\,1{}\mathrm{i}\right)\,\left(3\,b+a\,2{}\mathrm{i}\right)}{2\,\left(a^2\,1{}\mathrm{i}+2\,a\,b-b^2\,1{}\mathrm{i}\right)}+\frac{\ln\left(e^x+1{}\mathrm{i}\right)\,\left(2\,a+b\,3{}\mathrm{i}\right)}{2\,\left(a^2+a\,b\,2{}\mathrm{i}-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a + b/sinh(x)),x)
```

```
[Out] ((exp(x)*(a^2*b + b^3))/(a^2 + b^2)^2 + (2*(a^4 + a^2*b^2))/(a*(a^2 + b^2)^
2))/(exp(2*x) + 1) - ((2*a)/(a^2 + b^2) + (2*b*exp(x))/(a^2 + b^2))/(2*exp(
2*x) + exp(4*x) + 1) - x/a + (b^4*log(4*a^9*exp(2*x) - 4*a*b^8 - 4*a^9 + 7*
a^3*b^6 - 14*a^5*b^4 - 17*a^7*b^2 + 8*b^9*exp(x) - 7*a^3*b^6*exp(2*x) + 14*
a^5*b^4*exp(2*x) + 17*a^7*b^2*exp(2*x) + 8*a^8*b*exp(x) + 4*a*b^8*exp(2*x)
- 14*a^2*b^7*exp(x) + 28*a^4*b^5*exp(x) + 34*a^6*b^3*exp(x)))/(a*b^4 + a^5
+ 2*a^3*b^2) + (log(exp(x)*1i + 1)*(a*2i + 3*b))/(2*(2*a*b + a^2*1i - b^2*1
i)) + (log(exp(x) + 1i)*(2*a + b*3i))/(2*(a*b*2i + a^2 - b^2))
```

3.116 $\int \frac{\tanh^2(x)}{a+b\,\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=100

$$\frac{ax}{a^2+b^2}+\frac{b^2x}{a\left(a^2+b^2\right)}+\frac{2b^3\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\left(a^2+b^2\right)^{3/2}}+\frac{b\mathrm{sech}(x)}{a^2+b^2}-\frac{a\tanh(x)}{a^2+b^2}$$

```
[Out] a*x/(a^2+b^2)+b^2*x/a/(a^2+b^2)+2*b^3*arctanh((a-b*tanh(1/2*x))/(a^2+b^2)^(
1/2))/a/(a^2+b^2)^(3/2)+b*sech(x)/(a^2+b^2)-a*tanh(x)/(a^2+b^2)
```

Rubi [A]
time = 0.15, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {3983, 2981, 2686, 8, 3554, 2814, 2739, 632, 210}

$$\frac{b^2x}{a\left(a^2+b^2\right)}+\frac{ax}{a^2+b^2}-\frac{a\tanh(x)}{a^2+b^2}+\frac{b\mathrm{sech}(x)}{a^2+b^2}+\frac{2b^3\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\left(a^2+b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(a + b*Csch[x]),x]
```

```
[Out] (a*x)/(a^2 + b^2) + (b^2*x)/(a*(a^2 + b^2)) + (2*b^3*ArcTanh[(a - b*Tanh[x/
2])/Sqrt[a^2 + b^2]])/(a*(a^2 + b^2)^(3/2)) + (b*Sech[x])/(a^2 + b^2) - (a*
Tanh[x])/(a^2 + b^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2981

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a*(d^2/(a^2
- b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[b
*(d/(a^2 - b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[a^2*(d^2/(g^2*(a^2 - b^2))), Int[(g*Cos[e + f*x])^(p + 2)*((d*Sin[e +
 f*x])^(n - 2)/(a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3983

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
 + n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$\int \frac{\tanh^2(x)}{a+b\text{csch}(x)}\,dx = i\int \frac{\sinh(x)\tanh^2(x)}{ib+ia\sinh(x)}\,dx$$
$$= \frac{a\int \tanh^2(x)\,dx}{a^2+b^2} - \frac{b\int \text{sech}(x)\tanh(x)\,dx}{a^2+b^2} + \frac{(ib^2)\int \frac{\sinh(x)}{ib+ia\sinh(x)}\,dx}{a^2+b^2}$$
$$= \frac{b^2x}{a(a^2+b^2)} - \frac{a\tanh(x)}{a^2+b^2} + \frac{a\int 1\,dx}{a^2+b^2} + \frac{b\text{Subst}(\int 1\,dx, x, \text{sech}(x))}{a^2+b^2} - \frac{(ib^3)\int \frac{1}{ib+ia\sinh(x)}\,dx}{a(a^2+b^2)}$$
$$= \frac{ax}{a^2+b^2} + \frac{b^2x}{a(a^2+b^2)} + \frac{b\text{sech}(x)}{a^2+b^2} - \frac{a\tanh(x)}{a^2+b^2} - \frac{(2ib^3)\,\text{Subst}\left(\int \frac{1}{ib+2iax-ibx^2}\,dx, x, \tanh\right.}{a(a^2+b^2)}$$
$$= \frac{ax}{a^2+b^2} + \frac{b^2x}{a(a^2+b^2)} + \frac{b\text{sech}(x)}{a^2+b^2} - \frac{a\tanh(x)}{a^2+b^2} + \frac{(4ib^3)\,\text{Subst}\left(\int \frac{1}{-4(a^2+b^2)-x^2}\,dx, x, 2ia\right.}{a(a^2+b^2)}$$
$$= \frac{ax}{a^2+b^2} + \frac{b^2x}{a(a^2+b^2)} + \frac{2b^3\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)^{3/2}} + \frac{b\text{sech}(x)}{a^2+b^2} - \frac{a\tanh(x)}{a^2+b^2}$$

Mathematica [A]
time = 0.24, size = 82, normalized size = 0.82

$$\frac{x+\frac{2b^3\text{ArcTan}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{(-a^2-b^2)^{3/2}}}{a} + \frac{b\text{sech}(x)}{a^2+b^2} - \frac{a\tanh(x)}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(a + b*Csch[x]),x]
```

```
[Out] (x + (2*b^3*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]])/(-a^2 - b^2)^(3/2))
/a + (b*Sech[x])/(a^2 + b^2) - (a*Tanh[x])/(a^2 + b^2)
```

Maple [A]
time = 0.90, size = 95, normalized size = 0.95

method	result	size
default	$\frac{2b-2a\tanh\left(\frac{x}{2}\right)}{(a^2+b^2)\left(\tanh^2\left(\frac{x}{2}\right)+1\right)} + \frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a} + \frac{2b^3\,\text{arctanh}\left(\frac{-2b\tanh\left(\frac{x}{2}\right)+2a}{2\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)^{\frac{3}{2}}}$	95
risch	$\frac{x}{a} + \frac{2\,e^{x}b+2a}{(1+e^{2x})(a^2+b^2)} + \frac{b^3\ln\left(e^{x}+\frac{(a^2+b^2)^{\frac{3}{2}}b+a^4+2a^2b^2+b^4}{a(a^2+b^2)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}a} - \frac{b^3\ln\left(e^{x}+\frac{(a^2+b^2)^{\frac{3}{2}}b-a^4-2a^2b^2-b^4}{a(a^2+b^2)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}a}$	155

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(a^2+b^2)*(b-a*tanh(1/2*x))/(tanh(1/2*x)^2+1)+1/a*ln(tanh(1/2*x)+1)-1/a*l
n(tanh(1/2*x)-1)+2/a*b^3/(a^2+b^2)^(3/2)*arctanh(1/2*(-2*b*tanh(1/2*x)+2*a)
/(a^2+b^2)^(1/2))
```

Maxima [A]
time = 0.47, size = 108, normalized size = 1.08

$$-\frac{b^3\log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{(a^3+ab^2)\sqrt{a^2+b^2}}+\frac{2\left(be^{(-x)}-a\right)}{a^2+b^2+(a^2+b^2)e^{(-2x)}}+\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -b^3*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))
/((a^3 + a*b^2)*sqrt(a^2 + b^2)) + 2*(b*e^(-x) - a)/(a^2 + b^2 + (a^2 + b^2
)*e^(-2*x)) + x/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 349 vs. 2(96) = 192.
time = 0.38, size = 349, normalized size = 3.49

$$\frac{2\,a^4+2\,a^2b^2+(a^4+2\,a^2b^2+b^4)x\cosh\left(x\right)^2+(a^4+2\,a^2b^2+b^4)x\sinh\left(x\right)^2+\left(b^3\cosh\left(x\right)^2+2\,b^3\cosh\left(x\right)\sinh\left(x\right)+b^3\sinh\left(x\right)^2+b^3\right)\sqrt{a^2+b^2}\log\left(\frac{a^2\cosh(x)^2+a^2\sinh(x)^2+2\,ab\cosh(x)+a^2+2\,b^2+2\,(a^2\cosh(x)+ab)\sinh(x)+2\,\sqrt{a^2+b^2}\,(a\cosh(x)+a\sinh(x)+b)}{a\cosh(x)^2+a\sinh(x)^2+2\,b\cosh(x)+2\,(a\cosh(x)+b)\sinh(x)-a}\right)+(a^4+2\,a^2b^2+b^4)x+2\,(a^3b+ab^3)\cosh\left(x\right)+2\,(a^3b+ab^3+(a^4+2\,a^2b^2+b^4)x\cosh\left(x\right))\sinh\left(x\right)}{a^5+2\,a^3b^2+ab^4+(a^5+2\,a^3b^2+ab^4)\cosh\left(x\right)^2+2\,(a^5+2\,a^3b^2+ab^4)\cosh\left(x\right)\sinh\left(x\right)+(a^5+2\,a^3b^2+ab^4)\sinh\left(x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] (2*a^4 + 2*a^2*b^2 + (a^4 + 2*a^2*b^2 + b^4)*x*cosh(x)^2 + (a^4 + 2*a^2*b^2
 + b^4)*x*sinh(x)^2 + (b^3*cosh(x)^2 + 2*b^3*cosh(x)*sinh(x) + b^3*sinh(x)^
2 + b^3)*sqrt(a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x)
 + a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(a*cosh(
x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x
) + b)*sinh(x) - a)) + (a^4 + 2*a^2*b^2 + b^4)*x + 2*(a^3*b + a*b^3)*cosh(x
) + 2*(a^3*b + a*b^3 + (a^4 + 2*a^2*b^2 + b^4)*x*cosh(x))*sinh(x))/(a^5 + 2
*a^3*b^2 + a*b^4 + (a^5 + 2*a^3*b^2 + a*b^4)*cosh(x)^2 + 2*(a^5 + 2*a^3*b^2
 + a*b^4)*cosh(x)*sinh(x) + (a^5 + 2*a^3*b^2 + a*b^4)*sinh(x)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tanh^2\left(x\right)}{a+b\,\mathrm{csch}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(a+b*csch(x)),x)
```

```
[Out] Integral(tanh(x)**2/(a + b*csch(x)), x)
```

Giac [A]
time = 0.40, size = 102, normalized size = 1.02

$$-\frac{b^3\log\left(\frac{|2\,ae^x+2\,b-2\,\sqrt{a^2+b^2}|}{|2\,ae^x+2\,b+2\,\sqrt{a^2+b^2}|}\right)}{(a^3+ab^2)\sqrt{a^2+b^2}}+\frac{x}{a}+\frac{2\,(be^x+a)}{(a^2+b^2)(e^{(2\,x)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -b^3*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(
a^2 + b^2)))/((a^3 + a*b^2)*sqrt(a^2 + b^2)) + x/a + 2*(b*e^x + a)/((a^2 +
b^2)*(e^(2*x) + 1))
```

Mupad [B]
time = 2.47, size = 376, normalized size = 3.76

$$\frac{x}{a}+\frac{\frac{2\,a}{a^2+b^2}+\frac{2\,b\,\mathrm{e}^x}{a^2+b^2}}{\mathrm{e}^{2\,x}+1}+\frac{2\,\mathrm{atan}\left(\left(\frac{a^4\,\sqrt{-a^8-3\,a^6\,b^2-3\,a^4\,b^4-a^2\,b^6}}{2}+\frac{a^2\,b^2\,\sqrt{-a^8-3\,a^6\,b^2-3\,a^4\,b^4-a^2\,b^6}}{2}\right)\left(\mathrm{e}^x\left(\frac{2\,b^3}{a^3\,(a^3+a\,b^2)\,\sqrt{b^6}\,(a^2+b^2)}+\frac{2\,\left(a\,b^3\,\sqrt{b^6}+a^3\,b\,\sqrt{b^6}\right)}{a^2\,b^2\,\sqrt{-a^2\,{(a^2+b^2)}^3}\,(a^3+a\,b^2)\,\sqrt{-a^8-3\,a^6\,b^2-3\,a^4\,b^4-a^2\,b^6}}\right)-\frac{2\,\left(a^4\,\sqrt{b^6}+a^2\,b^2\,\sqrt{b^6}\right)}{a^2\,b^2\,\sqrt{-a^2\,{(a^2+b^2)}^3}\,(a^3+a\,b^2)\,\sqrt{-a^8-3\,a^6\,b^2-3\,a^4\,b^4-a^2\,b^6}}\right)\right)\sqrt{b^6}}{\sqrt{-a^8-3\,a^6\,b^2-3\,a^4\,b^4-a^2\,b^6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a + b/sinh(x)),x)
```

```
[Out] x/a + ((2*a)/(a^2 + b^2) + (2*b*exp(x))/(a^2 + b^2))/(exp(2*x) + 1) + (2*at
an(((a^4*(- a^8 - a^2*b^6 - 3*a^4*b^4 - 3*a^6*b^2)^(1/2))/2 + (a^2*b^2*(- a
^8 - a^2*b^6 - 3*a^4*b^4 - 3*a^6*b^2)^(1/2))/2)*(exp(x)*((2*b^3)/(a^3*(a*b^
2 + a^3)*(b^6)^(1/2)*(a^2 + b^2)) + (2*(a*b^3*(b^6)^(1/2) + a^3*b*(b^6)^(1/
2)))/(a^2*b^2*(-a^2*(a^2 + b^2)^3)^(1/2)*(a*b^2 + a^3)*(- a^8 - a^2*b^6 - 3
*a^4*b^4 - 3*a^6*b^2)^(1/2))) - (2*(a^4*(b^6)^(1/2) + a^2*b^2*(b^6)^(1/2)))
/(a^2*b^2*(-a^2*(a^2 + b^2)^3)^(1/2)*(a*b^2 + a^3)*(- a^8 - a^2*b^6 - 3*a^4
*b^4 - 3*a^6*b^2)^(1/2))))*(b^6)^(1/2))/(- a^8 - a^2*b^6 - 3*a^4*b^4 - 3*a^
6*b^2)^(1/2)
```

3.117 $\int \frac{\tanh(x)}{a+b\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=61

$$-\frac{b\text{ArcTan}(\sinh(x))}{a^2+b^2}+\frac{b^2\log(a+b\text{csch}(x))}{a\,(a^2+b^2)}+\frac{\log(\sinh(x))}{a}-\frac{a\log(\tanh(x))}{a^2+b^2}$$

```
[Out] -b*arctan(sinh(x))/(a^2+b^2)+b^2*ln(a+b*csch(x))/a/(a^2+b^2)+ln(sinh(x))/a-
a*ln(tanh(x))/(a^2+b^2)
```

Rubi [A]
time = 0.07, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {3970, 908, 649, 209, 266}

$$-\frac{b\text{ArcTan}(\sinh(x))}{a^2+b^2}-\frac{a\log(\tanh(x))}{a^2+b^2}+\frac{b^2\log(a+b\text{csch}(x))}{a\,(a^2+b^2)}+\frac{\log(\sinh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(a + b*Csch[x]),x]
```

```
[Out] -((b*ArcTan[Sinh[x]])/(a^2 + b^2)) + (b^2*Log[a + b*Csch[x]])/(a*(a^2 + b^2
)) + Log[Sinh[x]]/a - (a*Log[Tanh[x]])/(a^2 + b^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
```

```
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\tanh(x)}{a+b\text{csch}(x)}\,dx = b^2\text{Subst}\left(\int \frac{1}{x(a+x)\left(-b^2-x^2\right)}\,dx, x, b\text{csch}(x)\right)$$

$$= b^2\text{Subst}\left(\int \left(-\frac{1}{ab^2x}+\frac{1}{a\left(a^2+b^2\right)(a+x)}+\frac{b^2+ax}{b^2\left(a^2+b^2\right)\left(b^2+x^2\right)}\right) dx, x, b\text{csch}(x)\right)$$

$$= \frac{b^2\log(a+b\text{csch}(x))}{a\left(a^2+b^2\right)}+\frac{\log(\sinh(x))}{a}+\frac{\text{Subst}\left(\int \frac{b^2+ax}{b^2+x^2}\,dx, x, b\text{csch}(x)\right)}{a^2+b^2}$$

$$= \frac{b^2\log(a+b\text{csch}(x))}{a\left(a^2+b^2\right)}+\frac{\log(\sinh(x))}{a}+\frac{a\text{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x, b\text{csch}(x)\right)}{a^2+b^2}+\frac{b^2\text{Subst}\left(\int \frac{}{b^2}\right.}{}$$

$$= -\frac{b\tan^{-1}(\sinh(x))}{a^2+b^2}+\frac{b^2\log(a+b\text{csch}(x))}{a\left(a^2+b^2\right)}+\frac{\log(\sinh(x))}{a}-\frac{a\log(\tanh(x))}{a^2+b^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.04, size = 63, normalized size = 1.03

$$\frac{a(a+ib)\log(i-\sinh(x))+a(a-ib)\log(i+\sinh(x))+2b^2\log(b+a\sinh(x))}{2a\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(a + b*Csch[x]),x]
```

```
[Out] (a*(a + I*b)*Log[I - Sinh[x]] + a*(a - I*b)*Log[I + Sinh[x]] + 2*b^2*Log[b
+ a*Sinh[x]])/(2*a*(a^2 + b^2))
```

Maple [A]
time = 0.92, size = 97, normalized size = 1.59

method	result	si

default	$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}+\frac{b^{2}\ln\left(-b\left(\tanh^{2}\left(\frac{x}{2}\right)\right)+2a\tanh\left(\frac{x}{2}\right)+b\right)}{a\left(a^{2}+b^{2}\right)}+\frac{4a\ln\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)-8b\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{4a^{2}+4b^{2}}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}$
risch	$\frac{x}{a}-\frac{2ax}{a^{2}+b^{2}}-\frac{2b^{2}x}{\left(a^{2}+b^{2}\right)a}+\frac{i\ln\left(\mathrm{e}^{x}-i\right)b}{a^{2}+b^{2}}+\frac{\ln\left(\mathrm{e}^{x}-i\right)a}{a^{2}+b^{2}}-\frac{i\ln\left(\mathrm{e}^{x}+i\right)b}{a^{2}+b^{2}}+\frac{\ln\left(\mathrm{e}^{x}+i\right)a}{a^{2}+b^{2}}+\frac{b^{2}\ln\left(\mathrm{e}^{2x}+\frac{2b\,\mathrm{e}^{x}}{a}-1\right)}{a\left(a^{2}+b^{2}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a*ln(tanh(1/2*x)-1)+b^2/a/(a^2+b^2)*ln(-b*tanh(1/2*x)^2+2*a*tanh(1/2*x)+
b)+8/(4*a^2+4*b^2)*(1/2*a*ln(tanh(1/2*x)^2+1)-b*arctan(tanh(1/2*x)))-1/a*ln
(tanh(1/2*x)+1)
```

Maxima [A]
time = 0.46, size = 74, normalized size = 1.21

$$\frac{b^{2}\log\left(-2\,be^{(-x)}+ae^{(-2\,x)}-a\right)}{a^{3}+ab^{2}}+\frac{2\,b\arctan\left(e^{(-x)}\right)}{a^{2}+b^{2}}+\frac{a\log\left(e^{(-2\,x)}+1\right)}{a^{2}+b^{2}}+\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] b^2*log(-2*b*e^(-x) + a*e^(-2*x) - a)/(a^3 + a*b^2) + 2*b*arctan(e^(-x))/(a
^2 + b^2) + a*log(e^(-2*x) + 1)/(a^2 + b^2) + x/a
```

Fricas [A]
time = 0.46, size = 75, normalized size = 1.23

$$-\frac{2\,ab\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)-b^{2}\log\left(\frac{2\,\left(a\sinh\left(x\right)+b\right)}{\cosh\left(x\right)-\sinh\left(x\right)}\right)-a^{2}\log\left(\frac{2\,\cosh\left(x\right)}{\cosh\left(x\right)-\sinh\left(x\right)}\right)+\left(a^{2}+b^{2}\right)x}{a^{3}+ab^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] -(2*a*b*arctan(cosh(x) + sinh(x)) - b^2*log(2*(a*sinh(x) + b)/(cosh(x) - si
nh(x))) - a^2*log(2*cosh(x)/(cosh(x) - sinh(x))) + (a^2 + b^2)*x)/(a^3 + a*
b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tanh\left(x\right)}{a+b\operatorname{csch}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*csch(x)),x)
```

```
[Out] Integral(tanh(x)/(a + b*csch(x)), x)
```

Giac [A]
time = 0.39, size = 89, normalized size = 1.46

$$\frac{b^2 \log\left(\left|-a\left(e^{(-x)} - e^x\right) + 2\,b\right|\right)}{a^3 + ab^2} - \frac{\left(\pi + 2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)} - 1\right)e^{(-x)}\right)\right)b}{2\left(a^2 + b^2\right)} + \frac{a \log\left(\left(e^{(-x)} - e^x\right)^2 + 4\right)}{2\left(a^2 + b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] b^2*log(abs(-a*(e^(-x) - e^x) + 2*b))/(a^3 + a*b^2) - 1/2*(pi + 2*arctan(1/
2*(e^(2*x) - 1)*e^(-x)))*b/(a^2 + b^2) + 1/2*a*log((e^(-x) - e^x)^2 + 4)/(a
^2 + b^2)
```

Mupad [B]
time = 2.53, size = 132, normalized size = 2.16

$$\frac{\ln\left(1 + e^x\,1i\right)}{a - b\,1i} - \frac{x}{a} + \frac{b^2\,\ln\left(a^5\,e^{2\,x} - a\,b^4 - a^5 + a^3\,b^2 + 2\,b^5\,e^x - a^3\,b^2\,e^{2\,x} + 2\,a^4\,b\,e^x + a\,b^4\,e^{2\,x} - 2\,a^2\,b^3\,e^x\right)}{a^3 + a\,b^2} + \frac{\ln\left(e^x + 1i\right)\,1i}{-b + a\,1i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a + b/sinh(x)),x)
```

```
[Out] log(exp(x)*1i + 1)/(a - b*1i) + (log(exp(x) + 1i)*1i)/(a*1i - b) - x/a + (b
^2*log(a^5*exp(2*x) - a*b^4 - a^5 + a^3*b^2 + 2*b^5*exp(x) - a^3*b^2*exp(2*
x) + 2*a^4*b*exp(x) + a*b^4*exp(2*x) - 2*a^2*b^3*exp(x)))/(a*b^2 + a^3)
```

3.118 $\int \frac{\coth(x)}{a+b\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=19

$$\frac{\log(a+b\operatorname{csch}(x))}{a}+\frac{\log(\sinh(x))}{a}$$

```
[Out] ln(a+b*csch(x))/a+ln(sinh(x))/a
```

Rubi [A]
time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3970, 36, 29, 31}

$$\frac{\log(a+b\operatorname{csch}(x))}{a}+\frac{\log(\sinh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(a + b*Csch[x]),x]
```

```
[Out] Log[a + b*Csch[x]]/a + Log[Sinh[x]]/a
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\coth(x)}{a+b\operatorname{csch}(x)}\,dx = -\text{Subst}\left(\int \frac{1}{x(a+x)}\,dx, x, b\operatorname{csch}(x)\right)$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, b\operatorname{csch}(x)\right)}{a} + \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\operatorname{csch}(x)\right)}{a}$$
$$= \frac{\log(a+b\operatorname{csch}(x))}{a} + \frac{\log(\sinh(x))}{a}$$

Mathematica [A]
time = 0.01, size = 11, normalized size = 0.58

$$\frac{\log(b+a\sinh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(a + b*Csch[x]),x]
[Out] Log[b + a*Sinh[x]]/a
```

Maple [A]
time = 0.62, size = 21, normalized size = 1.11

method	result	size
derivativedivides	$\frac{\ln(a+b\operatorname{csch}(x))}{a} - \frac{\ln(\operatorname{csch}(x))}{a}$	21
default	$\frac{\ln(a+b\operatorname{csch}(x))}{a} - \frac{\ln(\operatorname{csch}(x))}{a}$	21
risch	$-\frac{x}{a} + \frac{\ln\left(e^{2x}+\frac{2b\,e^{x}}{a}-1\right)}{a}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*csch(x)),x,method=_RETURNVERBOSE)
[Out] ln(a+b*csch(x))/a-1/a*ln(csch(x))
```

Maxima [A]
time = 0.29, size = 28, normalized size = 1.47

$$\frac{x}{a} + \frac{\log\left(-2\,be^{(-x)} + ae^{(-2\,x)} - a\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*csch(x)),x, algorithm="maxima")
[Out] x/a + log(-2*b*e^(-x) + a*e^(-2*x) - a)/a
```

Fricas [A]
time = 0.36, size = 27, normalized size = 1.42

$$-\frac{x-\log\left(\frac{2\,(a\sinh(x)+b)}{\cosh(x)-\sinh(x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*csch(x)),x, algorithm="fricas")
[Out] -(x - log(2*(a*sinh(x) + b)/(cosh(x) - sinh(x))))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\coth\left(x\right)}{a+b\operatorname{csch}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*csch(x)),x)
[Out] Integral(coth(x)/(a + b*csch(x)), x)
```

Giac [A]
time = 0.40, size = 22, normalized size = 1.16

$$\frac{\log\left(\left|-a\left(e^{(-x)}-e^{x}\right)+2\,b\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*csch(x)),x, algorithm="giac")
[Out] log(abs(-a*(e^(-x) - e^x) + 2*b))/a
```

Mupad [B]
time = 0.10, size = 25, normalized size = 1.32

$$-\frac{x-\ln\left(2\,b\,\mathrm{e}^{x}-a+a\,\mathrm{e}^{2\,x}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a + b/sinh(x)),x)
[Out] -(x - log(2*b*exp(x) - a + a*exp(2*x)))/a
```

3.119 $\int \frac{\coth^2(x)}{a+b\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=57

$$\frac{x}{a}-\frac{\tanh^{-1}(\cosh(x))}{b}+\frac{2\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{ab}$$

[Out] x/a-arctanh(cosh(x))/b+2*arctanh((a-b*tanh(1/2*x))/(a^2+b^2)^(1/2))*(a^2+b^2)^(1/2)/a/b

Rubi [A]
time = 0.12, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {3979, 4136, 3855, 4004, 3916, 2739, 632, 212}

$$\frac{2\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{ab}+\frac{x}{a}-\frac{\tanh^{-1}(\cosh(x))}{b}$$

Antiderivative was successfully verified.

[In] Int[Coth[x]^2/(a + b*Csch[x]),x]

[Out] x/a - ArcTanh[Cosh[x]]/b + (2*Sqrt[a^2 + b^2]*ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a*b)

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3979

```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4136

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)), x_Symbol] :> Dist[C/b, Int[Csc[e + f*x], x], x] + Dist[1/b, Int[
(A*b - a*C*Csc[e + f*x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 A, C}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\coth^2(x)}{a+b\text{csch}(x)}\,dx &= -\int \frac{-1-\text{csch}^2(x)}{a+b\text{csch}(x)}\,dx \\
&= \frac{i\int \frac{-ib+ia\text{csch}(x)}{a+b\text{csch}(x)}\,dx}{b} + \frac{\int \text{csch}(x)\,dx}{b} \\
&= \frac{x}{a} - \frac{\tanh^{-1}(\cosh(x))}{b} - \frac{(a^2+b^2)\int \frac{\text{csch}(x)}{a+b\text{csch}(x)}\,dx}{ab} \\
&= \frac{x}{a} - \frac{\tanh^{-1}(\cosh(x))}{b} - \left(\frac{1}{a}+\frac{a}{b^2}\right)\int \frac{1}{1+\frac{a\sinh(x)}{b}}\,dx \\
&= \frac{x}{a} - \frac{\tanh^{-1}(\cosh(x))}{b} - \left(2\left(\frac{1}{a}+\frac{a}{b^2}\right)\right)\text{Subst}\left(\int \frac{1}{1+\frac{2ax}{b}-x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right) \\
&= \frac{x}{a} - \frac{\tanh^{-1}(\cosh(x))}{b} + \left(4\left(\frac{1}{a}+\frac{a}{b^2}\right)\right)\text{Subst}\left(\int \frac{1}{4\left(1+\frac{a^2}{b^2}\right)-x^2}\,dx, x, \frac{2a}{b}-2\tanh\left(\right.\right. \\
&= \frac{x}{a} - \frac{\tanh^{-1}(\cosh(x))}{b} + \frac{2\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b\left(\frac{a}{b}-\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{a^2+b^2}}\right)}{ab}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 65, normalized size = 1.14

$$
\frac{bx+2\sqrt{-a^2-b^2}\,\text{ArcTan}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)+a\log\left(\tanh\left(\frac{x}{2}\right)\right)}{ab}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(a + b*Csch[x]),x]
```

```
[Out] (b*x + 2*Sqrt[-a^2 - b^2]*ArcTan[(a - b*Tanh[x/2])/Sqrt[-a^2 - b^2]] + a*Lo
g[Tanh[x/2]])/(a*b)
```

Maple [A]
time = 0.71, size = 84, normalized size = 1.47

method	result	size
default	$\frac{(2a^2+2b^2)\,\text{arctanh}\left(\frac{-2b\tanh\left(\frac{x}{2}\right)+2a}{2\sqrt{a^2+b^2}}\right)}{ab\sqrt{a^2+b^2}}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{b}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}$	84
risch	$\frac{x}{a}+\frac{\sqrt{a^2+b^2}\,\ln\left(e^x+\frac{b+\sqrt{a^2+b^2}}{a}\right)}{ba}-\frac{\sqrt{a^2+b^2}\,\ln\left(e^x-\frac{-b+\sqrt{a^2+b^2}}{a}\right)}{ba}+\frac{\ln(e^x-1)}{b}-\frac{\ln(e^x+1)}{b}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] (2*a^2+2*b^2)/a/b/(a^2+b^2)^(1/2)*arctanh(1/2*(-2*b*tanh(1/2*x)+2*a)/(a^2+b
^2)^(1/2))+1/b*ln(tanh(1/2*x))-1/a*ln(tanh(1/2*x)-1)+1/a*ln(tanh(1/2*x)+1)
```

Maxima [A]
time = 0.47, size = 90, normalized size = 1.58

$$\frac{x}{a} - \frac{\log\left(e^{(-x)} + 1\right)}{b} + \frac{\log\left(e^{(-x)} - 1\right)}{b} - \frac{\sqrt{a^2 + b^2}\log\left(\frac{ae^{(-x)} - b - \sqrt{a^2 + b^2}}{ae^{(-x)} - b + \sqrt{a^2 + b^2}}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] x/a - log(e^(-x) + 1)/b + log(e^(-x) - 1)/b - sqrt(a^2 + b^2)*log((a*e^(-x)
 - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))/(a*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(53) = 106.
time = 0.52, size = 141, normalized size = 2.47

$$\frac{bx - a\log\left(\cosh\left(x\right) + \sinh\left(x\right) + 1\right) + a\log\left(\cosh\left(x\right) + \sinh\left(x\right) - 1\right) + \sqrt{a^2 + b^2}\log\left(\frac{a^2\cosh(x)^2 + a^2\sinh(x)^2 + 2ab\cosh(x) + a^2 + 2b^2 + 2\left(a^2\cosh(x) + ab\right)\sinh(x) + 2\sqrt{a^2 + b^2}\left(a\cosh(x) + a\sinh(x) + b\right)}{a\cosh(x)^2 + a\sinh(x)^2 + 2b\cosh(x) + 2\left(a\cosh(x) + b\right)\sinh(x) - a}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] (b*x - a*log(cosh(x) + sinh(x) + 1) + a*log(cosh(x) + sinh(x) - 1) + sqrt(a
^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) + a^2 + 2*b^2
+ 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(a*cosh(x) + a*sinh(x)
+ b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x)
- a)))/(a*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2\left(x\right)}{a + b\operatorname{csch}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(a+b*csch(x)),x)
```

```
[Out] Integral(coth(x)**2/(a + b*csch(x)), x)
```

Giac [A]
time = 0.41, size = 89, normalized size = 1.56

$$\frac{x}{a} - \frac{\log\left(e^{x}+1\right)}{b} + \frac{\log\left(\left|e^{x}-1\right|\right)}{b} - \frac{\sqrt{a^{2}+b^{2}}\log\left(\frac{\left|2\,ae^{x}+2\,b-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,ae^{x}+2\,b+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] x/a - log(e^x + 1)/b + log(abs(e^x - 1))/b - sqrt(a^2 + b^2)*log(abs(2*a*e^
x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(a^2 + b^2)))/(a*b)
```

Mupad [B]
time = 0.34, size = 316, normalized size = 5.54

$$\frac{x}{a}+\frac{\ln\left(32\,a^2\,b+32\,b^3-32\,b^3\,\mathrm{e}^x-32\,a^2\,b\,\mathrm{e}^x\right)}{b}-\frac{\ln\left(32\,a^2\,b+32\,b^3+32\,b^3\,\mathrm{e}^x+32\,a^2\,b\,\mathrm{e}^x\right)}{b}+\frac{\ln\left(128\,b^5\,\mathrm{e}^x-64\,a^3\,b^2-64\,a\,b^4-128\,b^4\,\mathrm{e}^x\,\sqrt{a^2+b^2}+32\,a^4\,b\,\mathrm{e}^x+160\,a^2\,b^3\,\mathrm{e}^x+64\,a\,b^3\,\sqrt{a^2+b^2}+32\,a^3\,b\,\sqrt{a^2+b^2}-96\,a^2\,b^2\,\mathrm{e}^x\,\sqrt{a^2+b^2}\right)\,\sqrt{a^2+b^2}}{a\,b}-\frac{\ln\left(128\,b^5\,\mathrm{e}^x-64\,a^3\,b^2-64\,a\,b^4+128\,b^4\,\mathrm{e}^x\,\sqrt{a^2+b^2}+32\,a^4\,b\,\mathrm{e}^x+160\,a^2\,b^3\,\mathrm{e}^x-64\,a\,b^3\,\sqrt{a^2+b^2}-32\,a^3\,b\,\sqrt{a^2+b^2}+96\,a^2\,b^2\,\mathrm{e}^x\,\sqrt{a^2+b^2}\right)\,\sqrt{a^2+b^2}}{a\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a + b/sinh(x)),x)
```

```
[Out] x/a + log(32*a^2*b + 32*b^3 - 32*b^3*exp(x) - 32*a^2*b*exp(x))/b - log(32*a
^2*b + 32*b^3 + 32*b^3*exp(x) + 32*a^2*b*exp(x))/b + (log(128*b^5*exp(x) -
64*a^3*b^2 - 64*a*b^4 - 128*b^4*exp(x)*(a^2 + b^2)^(1/2) + 32*a^4*b*exp(x)
+ 160*a^2*b^3*exp(x) + 64*a*b^3*(a^2 + b^2)^(1/2) + 32*a^3*b*(a^2 + b^2)^(1
/2) - 96*a^2*b^2*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/(a*b) - (log(
128*b^5*exp(x) - 64*a^3*b^2 - 64*a*b^4 + 128*b^4*exp(x)*(a^2 + b^2)^(1/2) +
 32*a^4*b*exp(x) + 160*a^2*b^3*exp(x) - 64*a*b^3*(a^2 + b^2)^(1/2) - 32*a^3
*b*(a^2 + b^2)^(1/2) + 96*a^2*b^2*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/
2))/(a*b)
```

3.120 $\int \frac{\coth^3(x)}{a+b\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=32

$$-\frac{\operatorname{csch}(x)}{b}+\left(\frac{1}{a}+\frac{a}{b^2}\right)\log(a+b\operatorname{csch}(x))+\frac{\log(\sinh(x))}{a}$$

```
[Out] -csch(x)/b+(1/a+a/b^2)*ln(a+b*csch(x))+ln(sinh(x))/a
```

Rubi [A]
time = 0.05, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3970, 908}

$$\left(\frac{a}{b^2}+\frac{1}{a}\right)\log(a+b\operatorname{csch}(x))+\frac{\log(\sinh(x))}{a}-\frac{\operatorname{csch}(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/(a + b*Csch[x]),x]
```

```
[Out] -(Csch[x]/b) + (a^(-1) + a/b^2)*Log[a + b*Csch[x]] + Log[Sinh[x]]/a
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\coth^3(x)}{a+b\text{csch}(x)}\,dx &= \frac{\text{Subst}\left(\int \frac{-b^2-x^2}{x(a+x)}\,dx, x, b\text{csch}(x)\right)}{b^2} \\
&= \frac{\text{Subst}\left(\int \left(-1-\frac{b^2}{ax}+\frac{a^2+b^2}{a(a+x)}\right)\,dx, x, b\text{csch}(x)\right)}{b^2} \\
&= -\frac{\text{csch}(x)}{b} + \left(\frac{1}{a}+\frac{a}{b^2}\right)\log(a+b\text{csch}(x)) + \frac{\log(\sinh(x))}{a}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 37, normalized size = 1.16

$$\frac{-ab\text{csch}(x) - a^2\log(\sinh(x)) + (a^2+b^2)\log(b+a\sinh(x))}{ab^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/(a + b*Csch[x]),x]
```

```
[Out] (-(a*b*Csch[x]) - a^2*Log[Sinh[x]] + (a^2 + b^2)*Log[b + a*Sinh[x]])/(a*b^2
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(32) = 64$.
time = 0.73, size = 95, normalized size = 2.97

method	result
risch	$-\frac{x}{a}-\frac{2\,\mathrm{e}^{x}}{b(\mathrm{e}^{2x}-1)}+\frac{a\ln\left(\mathrm{e}^{2x}+\frac{2b\,\mathrm{e}^{x}}{a}-1\right)}{b^2}+\frac{\ln\left(\mathrm{e}^{2x}+\frac{2b\,\mathrm{e}^{x}}{a}-1\right)}{a}-\frac{a\ln(\mathrm{e}^{2x}-1)}{b^2}$
default	$\frac{\tanh\left(\frac{x}{2}\right)}{2b}-\frac{1}{2b\tanh\left(\frac{x}{2}\right)}-\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{b^2}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}+\frac{(2a^2+2b^2)\ln\left(-b\left(\tanh^2\left(\frac{x}{2}\right)\right)+2a\tanh\left(\frac{x}{2}\right)+\right.}{2a\,b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b*tanh(1/2*x)-1/2/b/tanh(1/2*x)-a/b^2*ln(tanh(1/2*x))-1/a*ln(tanh(1/2*x
)-1)-1/a*ln(tanh(1/2*x)+1)+1/2/a/b^2*(2*a^2+2*b^2)*ln(-b*tanh(1/2*x)^2+2*a*
tanh(1/2*x)+b)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(32) = 64$.
time = 0.26, size = 82, normalized size = 2.56

$$\frac{x}{a}+\frac{2\,e^{(-x)}}{be^{(-2\,x)}-b}-\frac{a\log\left(e^{(-x)}+1\right)}{b^2}-\frac{a\log\left(e^{(-x)}-1\right)}{b^2}+\frac{(a^2+b^2)\log\left(-2\,be^{(-x)}+ae^{(-2\,x)}-a\right)}{ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] x/a + 2*e^(-x)/(b*e^(-2*x) - b) - a*log(e^(-x) + 1)/b^2 - a*log(e^(-x) - 1)
/b^2 + (a^2 + b^2)*log(-2*b*e^(-x) + a*e^(-2*x) - a)/(a*b^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(32) = 64$.
time = 0.38, size = 199, normalized size = 6.22

$$-\frac{b^2x\cosh(x)^2 + b^2x\sinh(x)^2 - b^2x + 2\,ab\cosh(x) - \left((a^2+b^2)\cosh(x)^2 + 2\,(a^2+b^2)\cosh(x)\sinh(x) + (a^2+b^2)\sinh(x)^2 - a^2 - b^2\right)\log\left(\frac{2\,(a\sinh(x)+b)}{\cosh(x)-\sinh(x)}\right) + \left(a^2\cosh(x)^2 + 2\,a^2\cosh(x)\sinh(x) + a^2\sinh(x)^2 - a^2\right)\log\left(\frac{2\sinh(x)}{\cosh(x)-\sinh(x)}\right) + 2\,(b^2x\cosh(x) + ab)\sinh(x)}{ab^2\cosh(x)^2 + 2\,ab^2\cosh(x)\sinh(x) + ab^2\sinh(x)^2 - ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] -(b^2*x*cosh(x)^2 + b^2*x*sinh(x)^2 - b^2*x + 2*a*b*cosh(x) - ((a^2 + b^2)*
cosh(x)^2 + 2*(a^2 + b^2)*cosh(x)*sinh(x) + (a^2 + b^2)*sinh(x)^2 - a^2 - b
^2)*log(2*(a*sinh(x) + b)/(cosh(x) - sinh(x))) + (a^2*cosh(x)^2 + 2*a^2*cos
h(x)*sinh(x) + a^2*sinh(x)^2 - a^2)*log(2*sinh(x)/(cosh(x) - sinh(x))) + 2*
(b^2*x*cosh(x) + a*b)*sinh(x))/(a*b^2*cosh(x)^2 + 2*a*b^2*cosh(x)*sinh(x) +
 a*b^2*sinh(x)^2 - a*b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(x)}{a + b\,\mathrm{csch}\,(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(a+b*csch(x)),x)
```

```
[Out] Integral(coth(x)**3/(a + b*csch(x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(32) = 64$.
time = 0.41, size = 80, normalized size = 2.50

$$-\frac{a\log\left(\left|-e^{(-x)} + e^x\right|\right)}{b^2} + \frac{(a^2+b^2)\log\left(\left|-a\left(e^{(-x)} - e^x\right) + 2\,b\right|\right)}{ab^2} + \frac{a\left(e^{(-x)} - e^x\right) + 2\,b}{b^2\left(e^{(-x)} - e^x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -a*log(abs(-e^(-x) + e^x))/b^2 + (a^2 + b^2)*log(abs(-a*(e^(-x) - e^x) + 2*
b))/(a*b^2) + (a*(e^(-x) - e^x) + 2*b)/(b^2*(e^(-x) - e^x))
```

Mupad [B]
time = 1.84, size = 261, normalized size = 8.16

$$\frac{2\,{\mathrm{e}}^x}{b-b\,{\mathrm{e}}^{2\,x}}-\frac{x}{a}+\frac{\ln\left(16\,a^5\,{\mathrm{e}}^{2\,x}-4\,a\,b^4-16\,a^5-16\,a^3\,b^2+8\,b^5\,{\mathrm{e}}^x+16\,a^3\,b^2\,{\mathrm{e}}^{2\,x}+32\,a^4\,b\,{\mathrm{e}}^x+4\,a\,b^4\,{\mathrm{e}}^{2\,x}+32\,a^2\,b^3\,{\mathrm{e}}^x\right)}{a}+\frac{a\,\ln\left(16\,a^5\,{\mathrm{e}}^{2\,x}-4\,a\,b^4-16\,a^5-16\,a^3\,b^2+8\,b^5\,{\mathrm{e}}^x+16\,a^3\,b^2\,{\mathrm{e}}^{2\,x}+32\,a^4\,b\,{\mathrm{e}}^x+4\,a\,b^4\,{\mathrm{e}}^{2\,x}+32\,a^2\,b^3\,{\mathrm{e}}^x\right)}{b^2}-\frac{a\,\ln\left(16\,a^6\,{\mathrm{e}}^{2\,x}+4\,b^6\,{\mathrm{e}}^{2\,x}-16\,a^6-4\,b^6-20\,a^2\,b^4-32\,a^4\,b^2+20\,a^2\,b^4\,{\mathrm{e}}^{2\,x}+32\,a^4\,b^2\,{\mathrm{e}}^{2\,x}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a + b/sinh(x)),x)
```

```
[Out] (2*exp(x))/(b - b*exp(2*x)) - x/a + log(16*a^5*exp(2*x) - 4*a*b^4 - 16*a^5
- 16*a^3*b^2 + 8*b^5*exp(x) + 16*a^3*b^2*exp(2*x) + 32*a^4*b*exp(x) + 4*a*b
^4*exp(2*x) + 32*a^2*b^3*exp(x))/a + (a*log(16*a^5*exp(2*x) - 4*a*b^4 - 16*
a^5 - 16*a^3*b^2 + 8*b^5*exp(x) + 16*a^3*b^2*exp(2*x) + 32*a^4*b*exp(x) + 4
*a*b^4*exp(2*x) + 32*a^2*b^3*exp(x)))/b^2 - (a*log(16*a^6*exp(2*x) + 4*b^6*
exp(2*x) - 16*a^6 - 4*b^6 - 20*a^2*b^4 - 32*a^4*b^2 + 20*a^2*b^4*exp(2*x) +
 32*a^4*b^2*exp(2*x)))/b^2
```

3.121 $\int \frac{\coth^4(x)}{a+b\text{csch}(x)}\,dx$

Optimal. Leaf size=88

$$\frac{x}{a}-\frac{(2a^2+3b^2)\tanh^{-1}(\cosh(x))}{2b^3}+\frac{2(a^2+b^2)^{3/2}\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{ab^3}+\frac{a\coth(x)}{b^2}-\frac{\coth(x)\text{csch}(x)}{2b}$$

```
[Out] x/a-1/2*(2*a^2+3*b^2)*arctanh(cosh(x))/b^3+2*(a^2+b^2)^(3/2)*arctanh((a-b*t
anh(1/2*x))/(a^2+b^2)^(1/2))/a/b^3+a*coth(x)/b^2-1/2*coth(x)*csch(x)/b
```

Rubi [A]
time = 0.23, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3983, 2972, 3136, 2739, 632, 210, 3855}

$$\frac{2(a^2+b^2)^{3/2}\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{ab^3}-\frac{(2a^2+3b^2)\tanh^{-1}(\cosh(x))}{2b^3}+\frac{a\coth(x)}{b^2}+\frac{x}{a}-\frac{\coth(x)\text{csch}(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^4/(a + b*Csch[x]),x]
```

```
[Out] x/a - ((2*a^2 + 3*b^2)*ArcTanh[Cosh[x]])/(2*b^3) + (2*(a^2 + b^2)^(3/2)*Arc
Tanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a*b^3) + (a*Coth[x])/b^2 - (Coth[
x]*Csch[x])/(2*b)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2972

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((
d*Sin[e + f*x])^(n + 2)/(a^2*d^2*f*(n + 1)*(n + 2))), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3136

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[C*(x/(b*d)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)
/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f*x]), x], x] - Dist[(c^2*C - B*c*d
+ A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a,
b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && Ne
Q[c^2 - d^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3983

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
 + n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$\int \frac{\coth^4(x)}{a+b\text{csch}(x)}\,dx = i\int \frac{\cosh(x)\coth^3(x)}{ib+ia\sinh(x)}\,dx$$

$$= \frac{a\coth(x)}{b^2} - \frac{\coth(x)\text{csch}(x)}{2b} - \frac{i\int \frac{\text{csch}(x)(-2a^2-3b^2+ab\sinh(x)-2b^2\sinh^2(x))}{ib+ia\sinh(x)}\,dx}{2b^2}$$

$$= \frac{x}{a} + \frac{a\coth(x)}{b^2} - \frac{\coth(x)\text{csch}(x)}{2b} - \frac{\left(i(a^2+b^2)^2\right)\int \frac{1}{ib+ia\sinh(x)}\,dx}{ab^3} + \frac{(2a^2+3b^2)\int \text{csc}}{2b^3}$$

$$= \frac{x}{a} - \frac{(2a^2+3b^2)\tanh^{-1}(\cosh(x))}{2b^3} + \frac{a\coth(x)}{b^2} - \frac{\coth(x)\text{csch}(x)}{2b} - \frac{\left(2i(a^2+b^2)^2\right)\text{Su}}{}$$

$$= \frac{x}{a} - \frac{(2a^2+3b^2)\tanh^{-1}(\cosh(x))}{2b^3} + \frac{a\coth(x)}{b^2} - \frac{\coth(x)\text{csch}(x)}{2b} + \frac{\left(4i(a^2+b^2)^2\right)\text{Su}}{}$$

$$= \frac{x}{a} - \frac{(2a^2+3b^2)\tanh^{-1}(\cosh(x))}{2b^3} + \frac{2(a^2+b^2)^{3/2}\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{ab^3} + \frac{a\coth(x)}{b^2} -$$

Mathematica [A]
time = 0.41, size = 151, normalized size = 1.72

$$\frac{\text{csch}(x)(b+a\sinh(x))\left(8b^3x-16(-a^2-b^2)^{3/2}\text{ArcTan}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)+4a^2b\coth\left(\frac{x}{2}\right)-ab^2\text{csch}^2\left(\frac{x}{2}\right)+4a(2a^2+3b^2)\log\left(\tanh\left(\frac{x}{2}\right)\right)-ab^2\text{sech}^2\left(\frac{x}{2}\right)+4a^2b\tanh\left(\frac{x}{2}\right)\right)}{8ab^3(a+b\text{csch}(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4/(a + b*Csch[x]),x]
```

```
[Out] (Csch[x]*(b + a*Sinh[x])*(8*b^3*x - 16*(-a^2 - b^2)^(3/2)*ArcTan[(a - b*Tan
h[x/2])/Sqrt[-a^2 - b^2]] + 4*a^2*b*Coth[x/2] - a*b^2*Csch[x/2]^2 + 4*a*(2*
a^2 + 3*b^2)*Log[Tanh[x/2]] - a*b^2*Sech[x/2]^2 + 4*a^2*b*Tanh[x/2]))/(8*a*
b^3*(a + b*Csch[x]))
```

Maple [A]
time = 0.91, size = 150, normalized size = 1.70

method	result
default	$\frac{\frac{b\left(\tanh^2\left(\frac{x}{2}\right)\right)}{2}+2a\tanh\left(\frac{x}{2}\right)}{4b^2} - \frac{1}{8b\tanh\left(\frac{x}{2}\right)^2} + \frac{(4a^2+6b^2)\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{4b^3} + \frac{a}{2b^2\tanh\left(\frac{x}{2}\right)} + \frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a} + \frac{(8a^4+16a^2b^2+8b}{4a}$
risch	$\frac{x}{a} + \frac{-b\,e^{3x}+2\,e^{2x}a-e^{x}b-2a}{(e^{2x}-1)^2b^2} - \frac{\ln(e^{x}+1)a^2}{b^3} - \frac{3\ln(e^{x}+1)}{2b} + \frac{\ln(e^{x}-1)a^2}{b^3} + \frac{3\ln(e^{x}-1)}{2b} + \frac{(a^2+b^2)^{\frac{3}{2}}\ln\left(e^{x}+\frac{(a^2+b^2)^{\frac{3}{2}}+a^2b}{(a^2+b^2)a}\right.}{b^3a}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)^4/(a+b*csch(x)),x,method=_RETURNVERBOSE)

[Out] 1/4/b^2*(1/2*b*tanh(1/2*x)^2+2*a*tanh(1/2*x))-1/8/b/tanh(1/2*x)^2+1/4/b^3*(4*a^2+6*b^2)*ln(tanh(1/2*x))+1/2*a/b^2/tanh(1/2*x)+1/a*ln(tanh(1/2*x)+1)+1/4*(8*a^4+16*a^2*b^2+8*b^4)/a/b^3/(a^2+b^2)^(1/2)*arctanh(1/2*(-2*b*tanh(1/2*x)+2*a)/(a^2+b^2)^(1/2))-1/a*ln(tanh(1/2*x)-1)

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(80) = 160$.
time = 0.51, size = 178, normalized size = 2.02

$$\frac{be^{(-x)} + 2\,ae^{(-2\,x)} + be^{(-3\,x)} - 2\,a}{2\,b^2e^{(-2\,x)} - b^2e^{(-4\,x)} - b^2} + \frac{x}{a} - \frac{(2\,a^2 + 3\,b^2)\log\left(e^{(-x)} + 1\right)}{2\,b^3} + \frac{(2\,a^2 + 3\,b^2)\log\left(e^{(-x)} - 1\right)}{2\,b^3} - \frac{(a^4 + 2\,a^2b^2 + b^4)\log\left(\frac{ae^{(-x)} - b - \sqrt{a^2 + b^2}}{ae^{(-x)} - b + \sqrt{a^2 + b^2}}\right)}{\sqrt{a^2 + b^2}\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^4/(a+b*csch(x)),x, algorithm="maxima")

[Out] (b*e^(-x) + 2*a*e^(-2*x) + b*e^(-3*x) - 2*a)/(2*b^2*e^(-2*x) - b^2*e^(-4*x) - b^2) + x/a - 1/2*(2*a^2 + 3*b^2)*log(e^(-x) + 1)/b^3 + 1/2*(2*a^2 + 3*b^2)*log(e^(-x) - 1)/b^3 - (a^4 + 2*a^2*b^2 + b^4)*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a*b^3)

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 831 vs. $2(80) = 160$.
time = 0.44, size = 831, normalized size = 9.44

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^4/(a+b*csch(x)),x, algorithm="fricas")

[Out] 1/2*(2*b^3*x*cosh(x)^4 + 2*b^3*x*sinh(x)^4 - 2*a*b^2*cosh(x)^3 + 2*b^3*x - 2*a*b^2*cosh(x) + 2*(4*b^3*x*cosh(x) - a*b^2)*sinh(x)^3 - 4*a^2*b - 4*(b^3*x - a^2*b)*cosh(x)^2 + 2*(6*b^3*x*cosh(x)^2 - 2*b^3*x - 3*a*b^2*cosh(x) + 2*a^2*b)*sinh(x)^2 + 2*((a^2 + b^2)*cosh(x)^4 + 4*(a^2 + b^2)*cosh(x)*sinh(x)^3 + (a^2 + b^2)*sinh(x)^4 - 2*(a^2 + b^2)*cosh(x)^2 + 2*(3*(a^2 + b^2)*cosh(x)^2 - a^2 - b^2)*sinh(x)^2 + a^2 + b^2 + 4*((a^2 + b^2)*cosh(x)^3 - (a^2 + b^2)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) + a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) - a)) - ((2*a^3 + 3*a*b^2)*cosh(x)^4 + 4*(2*a^3 + 3*a*b^2)*cosh(x)*sinh(x)^3 + (2*a^3 + 3*a*b^2)*sinh(x)^4 + 2*a^3 +

```
3*a*b^2 - 2*(2*a^3 + 3*a*b^2)*cosh(x)^2 - 2*(2*a^3 + 3*a*b^2 - 3*(2*a^3 + 3
*a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((2*a^3 + 3*a*b^2)*cosh(x)^3 - (2*a^3 + 3*
a*b^2)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) + ((2*a^3 + 3*a*b^2)*co
sh(x)^4 + 4*(2*a^3 + 3*a*b^2)*cosh(x)*sinh(x)^3 + (2*a^3 + 3*a*b^2)*sinh(x)
^4 + 2*a^3 + 3*a*b^2 - 2*(2*a^3 + 3*a*b^2)*cosh(x)^2 - 2*(2*a^3 + 3*a*b^2 -
 3*(2*a^3 + 3*a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((2*a^3 + 3*a*b^2)*cosh(x)^3
- (2*a^3 + 3*a*b^2)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) + 2*(4*b^3
*x*cosh(x)^3 - 3*a*b^2*cosh(x)^2 - a*b^2 - 4*(b^3*x - a^2*b)*cosh(x))*sinh(
x))/(a*b^3*cosh(x)^4 + 4*a*b^3*cosh(x)*sinh(x)^3 + a*b^3*sinh(x)^4 - 2*a*b^
3*cosh(x)^2 + a*b^3 + 2*(3*a*b^3*cosh(x)^2 - a*b^3)*sinh(x)^2 + 4*(a*b^3*co
sh(x)^3 - a*b^3*cosh(x))*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^4(x)}{a + b\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4/(a+b*csch(x)),x)
```

```
[Out] Integral(coth(x)**4/(a + b*csch(x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 161 vs. 2(80) = 160.
time = 0.40, size = 161, normalized size = 1.83

$$\frac{x}{a} - \frac{(2a^2 + 3b^2)\log(e^x + 1)}{2b^3} + \frac{(2a^2 + 3b^2)\log(|e^x - 1|)}{2b^3} - \frac{(a^4 + 2a^2b^2 + b^4)\log\left(\frac{|2ae^x + 2b - 2\sqrt{a^2 + b^2}|}{|2ae^x + 2b + 2\sqrt{a^2 + b^2}|}\right)}{\sqrt{a^2 + b^2}\,ab^3} - \frac{be^{(3x)} - 2ae^{(2x)} + be^x + 2a}{b^2(e^{(2x)} - 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] x/a - 1/2*(2*a^2 + 3*b^2)*log(e^x + 1)/b^3 + 1/2*(2*a^2 + 3*b^2)*log(abs(e^
x - 1))/b^3 - (a^4 + 2*a^2*b^2 + b^4)*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 +
b^2))/abs(2*a*e^x + 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a*b^3) - (b*
e^(3*x) - 2*a*e^(2*x) + b*e^x + 2*a)/(b^2*(e^(2*x) - 1)^2)
```

Mupad [B]
time = 2.62, size = 378, normalized size = 4.30

$$\frac{\frac{2a}{b^2} - \frac{e^x}{b}}{e^{2x} - 1} + \frac{x}{a} + \frac{\ln(e^x - 1)\,(2a^2 + 3b^2)}{2b^3} - \frac{\ln(e^x + 1)\,(2a^2 + 3b^2)}{2b^3} - \frac{2e^x}{b\,(e^{4x} - 2e^{2x} + 1)} + \frac{\ln\left(a^3\sqrt{(a^2 + b^2)^3} - 2a^5 b - 2ab^5 - 4a^3b^3 + a^6 e^x + 4b^6 e^x + 2ab^2\sqrt{(a^2 + b^2)^3} - 4b^3 e^x\sqrt{(a^2 + b^2)^3} + 9a^2b^4 e^x + 6a^4b^2 e^x - 3a^2 b e^x\sqrt{(a^2 + b^2)^3}\right)\sqrt{(a^2 + b^2)^3}}{ab^3} - \frac{\ln\left(a^6 e^x - 2a^5 b - a^3\sqrt{(a^2 + b^2)^3} - 4a^3b^3 - 2ab^5 + 4b^6 e^x - 2ab^2\sqrt{(a^2 + b^2)^3} + 4b^3 e^x\sqrt{(a^2 + b^2)^3} + 9a^2b^4 e^x + 6a^4b^2 e^x + 3a^2 b e^x\sqrt{(a^2 + b^2)^3}\right)\sqrt{(a^2 + b^2)^3}}{ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(a + b/sinh(x)),x)
```

```
[Out] ((2*a)/b^2 - exp(x)/b)/(exp(2*x) - 1) + x/a + (log(exp(x) - 1)*(2*a^2 + 3*b
^2))/(2*b^3) - (log(exp(x) + 1)*(2*a^2 + 3*b^2))/(2*b^3) - (2*exp(x))/(b*(e
xp(4*x) - 2*exp(2*x) + 1)) + (log(a^3*((a^2 + b^2)^3)^(1/2) - 2*a^5*b - 2*a
*b^5 - 4*a^3*b^3 + a^6*exp(x) + 4*b^6*exp(x) + 2*a*b^2*((a^2 + b^2)^3)^(1/2
) - 4*b^3*exp(x)*((a^2 + b^2)^3)^(1/2) + 9*a^2*b^4*exp(x) + 6*a^4*b^2*exp(x
) - 3*a^2*b*exp(x)*((a^2 + b^2)^3)^(1/2))*((a^2 + b^2)^3)^(1/2))/(a*b^3) -
(log(a^6*exp(x) - 2*a^5*b - a^3*((a^2 + b^2)^3)^(1/2) - 4*a^3*b^3 - 2*a*b^5
 + 4*b^6*exp(x) - 2*a*b^2*((a^2 + b^2)^3)^(1/2) + 4*b^3*exp(x)*((a^2 + b^2)
^3)^(1/2) + 9*a^2*b^4*exp(x) + 6*a^4*b^2*exp(x) + 3*a^2*b*exp(x)*((a^2 + b^
2)^3)^(1/2))*((a^2 + b^2)^3)^(1/2))/(a*b^3)
```

3.122 $\int \frac{\coth^5(x)}{a+b\operatorname{csch}(x)}\,dx$

Optimal. Leaf size=70

$$-\frac{(a^2+2b^2)\operatorname{csch}(x)}{b^3}+\frac{a\operatorname{csch}^2(x)}{2b^2}-\frac{\operatorname{csch}^3(x)}{3b}+\frac{(a^2+b^2)^2\log(a+b\operatorname{csch}(x))}{ab^4}+\frac{\log(\sinh(x))}{a}$$

```
[Out] -(a^2+2*b^2)*csch(x)/b^3+1/2*a*csch(x)^2/b^2-1/3*csch(x)^3/b+(a^2+b^2)^2*ln
(a+b*csch(x))/a/b^4+ln(sinh(x))/a
```

Rubi [A]
time = 0.07, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3970, 908}

$$\frac{(a^2+b^2)^2\log(a+b\operatorname{csch}(x))}{ab^4}-\frac{(a^2+2b^2)\operatorname{csch}(x)}{b^3}+\frac{a\operatorname{csch}^2(x)}{2b^2}+\frac{\log(\sinh(x))}{a}-\frac{\operatorname{csch}^3(x)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^5/(a + b*Csch[x]),x]
```

```
[Out] -(((a^2 + 2*b^2)*Csch[x])/b^3) + (a*Csch[x]^2)/(2*b^2) - Csch[x]^3/(3*b) +
((a^2 + b^2)^2*Log[a + b*Csch[x]])/(a*b^4) + Log[Sinh[x]]/a
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\coth^5(x)}{a+b\text{csch}(x)}\,dx = -\frac{\text{Subst}\left(\int \frac{(-b^2-x^2)^2}{x(a+x)}\,dx, x, b\text{csch}(x)\right)}{b^4}$$

$$= -\frac{\text{Subst}\left(\int \left(a^2\left(1+\frac{2b^2}{a^2}\right)+\frac{b^4}{ax}-ax+x^2-\frac{(a^2+b^2)^2}{a(a+x)}\right)\,dx, x, b\text{csch}(x)\right)}{b^4}$$

$$= -\frac{(a^2+2b^2)\,\text{csch}(x)}{b^3}+\frac{a\text{csch}^2(x)}{2b^2}-\frac{\text{csch}^3(x)}{3b}+\frac{(a^2+b^2)^2\log(a+b\text{csch}(x))}{ab^4}+\frac{\log(\sinh}{a}$$

Mathematica [A]
time = 0.09, size = 83, normalized size = 1.19

$$\frac{-6ab(a^2+2b^2)\,\text{csch}(x)+3a^2b^2\text{csch}^2(x)-2ab^3\text{csch}^3(x)-6a^2(a^2+2b^2)\log(\sinh(x))+6(a^2+b^2)^2\log(b+a\sinh(x))}{6ab^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^5/(a + b*Csch[x]),x]
```

```
[Out] (-6*a*b*(a^2 + 2*b^2)*Csch[x] + 3*a^2*b^2*Csch[x]^2 - 2*a*b^3*Csch[x]^3 - 6
*a^2*(a^2 + 2*b^2)*Log[Sinh[x]] + 6*(a^2 + b^2)^2*Log[b + a*Sinh[x]])/(6*a*
b^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(66) = 132$.
time = 0.75, size = 181, normalized size = 2.59

method	result
risch	$-\frac{x}{a}-\frac{2\,\mathrm{e}^{x}\left(3a^{2}\mathrm{e}^{4x}+6b^{2}\mathrm{e}^{4x}-3ab\,\mathrm{e}^{3x}-6a^{2}\mathrm{e}^{2x}-8b^{2}\mathrm{e}^{2x}+3b\,\mathrm{e}^{x}a+3a^{2}+6b^{2}\right)}{3b^{3}\left(\mathrm{e}^{2x}-1\right)^{3}}-\frac{a^{3}\ln\left(\mathrm{e}^{2x}-1\right)}{b^{4}}-\frac{2a\ln\left(\mathrm{e}^{2x}-1\right)}{b^{2}}+\frac{a^{3}\ln\left(\mathrm{e}^{2x}+\frac{2b\,\mathrm{e}^{x}}{a}-\right.}{b^{4}}$
default	$\frac{\frac{\left(\tanh^{3}\left(\frac{x}{2}\right)\right)b^{2}}{3}+b\left(\tanh^{2}\left(\frac{x}{2}\right)\right)a+4a^{2}\tanh\left(\frac{x}{2}\right)+7b^{2}\tanh\left(\frac{x}{2}\right)}{8b^{3}}+\frac{\left(8a^{4}+16a^{2}b^{2}+8b^{4}\right)\ln\left(-b\left(\tanh^{2}\left(\frac{x}{2}\right)\right)+2a\tanh\left(\frac{x}{2}\right)+b\right)}{8a\,b^{4}}-\frac{\ln(\tanh(\frac{x}{2}}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^5/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/b^3*(1/3*tanh(1/2*x)^3*b^2+b*tanh(1/2*x)^2*a+4*a^2*tanh(1/2*x)+7*b^2*ta
nh(1/2*x))+1/8/a/b^4*(8*a^4+16*a^2*b^2+8*b^4)*ln(-b*tanh(1/2*x)^2+2*a*tanh(
1/2*x)+b)-1/a*ln(tanh(1/2*x)+1)-1/24/b/tanh(1/2*x)^3-1/8*(4*a^2+7*b^2)/b^3/
tanh(1/2*x)+1/8*a/b^2/tanh(1/2*x)^2-1/b^4*a*(a^2+2*b^2)*ln(tanh(1/2*x))-1/a
*ln(tanh(1/2*x)-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(66) = 132$.

time = 0.27, size = 190, normalized size = 2.71

$$-\frac{2\left(3\,abe^{(-2\,x)}-3\,abe^{(-4\,x)}-3\,(a^2+2\,b^2)e^{(-x)}+2\,(3\,a^2+4\,b^2)e^{(-3\,x)}-3\,(a^2+2\,b^2)e^{(-5\,x)}\right)}{3\,(3\,b^3e^{(-2\,x)}-3\,b^3e^{(-4\,x)}+b^3e^{(-6\,x)}-b^3)}+\frac{x}{a}-\frac{(a^3+2\,ab^2)\log\left(e^{(-x)}+1\right)}{b^4}-\frac{(a^3+2\,ab^2)\log\left(e^{(-x)}-1\right)}{b^4}+\frac{(a^4+2\,a^2b^2+b^4)\log\left(-2\,be^{(-x)}+ae^{(-2\,x)}-a\right)}{ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -2/3*(3*a*b*e^(-2*x) - 3*a*b*e^(-4*x) - 3*(a^2 + 2*b^2)*e^(-x) + 2*(3*a^2 +
 4*b^2)*e^(-3*x) - 3*(a^2 + 2*b^2)*e^(-5*x))/(3*b^3*e^(-2*x) - 3*b^3*e^(-4*
x) + b^3*e^(-6*x) - b^3) + x/a - (a^3 + 2*a*b^2)*log(e^(-x) + 1)/b^4 - (a^3
 + 2*a*b^2)*log(e^(-x) - 1)/b^4 + (a^4 + 2*a^2*b^2 + b^4)*log(-2*b*e^(-x) +
 a*e^(-2*x) - a)/(a*b^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1288 vs. $2(66) = 132$.
time = 0.38, size = 1288, normalized size = 18.40

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^4*x*cosh(x)^6 + 3*b^4*x*sinh(x)^6 + 6*(a^3*b + 2*a*b^3)*cosh(x)^5
 + 6*(3*b^4*x*cosh(x) + a^3*b + 2*a*b^3)*sinh(x)^5 - 3*b^4*x - 3*(3*b^4*x +
 2*a^2*b^2)*cosh(x)^4 + 3*(15*b^4*x*cosh(x)^2 - 3*b^4*x - 2*a^2*b^2 + 10*(a
^3*b + 2*a*b^3)*cosh(x))*sinh(x)^4 - 4*(3*a^3*b + 4*a*b^3)*cosh(x)^3 + 4*(1
5*b^4*x*cosh(x)^3 - 3*a^3*b - 4*a*b^3 + 15*(a^3*b + 2*a*b^3)*cosh(x)^2 - 3*
(3*b^4*x + 2*a^2*b^2)*cosh(x))*sinh(x)^3 + 3*(3*b^4*x + 2*a^2*b^2)*cosh(x)^
2 + 3*(15*b^4*x*cosh(x)^4 + 3*b^4*x + 2*a^2*b^2 + 20*(a^3*b + 2*a*b^3)*cosh
(x)^3 - 6*(3*b^4*x + 2*a^2*b^2)*cosh(x)^2 - 4*(3*a^3*b + 4*a*b^3)*cosh(x))*
sinh(x)^2 + 6*(a^3*b + 2*a*b^3)*cosh(x) - 3*((a^4 + 2*a^2*b^2 + b^4)*cosh(x
)^6 + 6*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^5 + (a^4 + 2*a^2*b^2 + b^4)
*sinh(x)^6 - 3*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^4 - 3*(a^4 + 2*a^2*b^2 + b^4
 - 5*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^4 - a^4 - 2*a^2*b^2 - b^4 +
 4*(5*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^3 - 3*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)
)*sinh(x)^3 + 3*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2 + 3*(5*(a^4 + 2*a^2*b^2 +
 b^4)*cosh(x)^4 + a^4 + 2*a^2*b^2 + b^4 - 6*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)
^2)*sinh(x)^2 + 6*((a^4 + 2*a^2*b^2 + b^4)*cosh(x)^5 - 2*(a^4 + 2*a^2*b^2 +
 b^4)*cosh(x)^3 + (a^4 + 2*a^2*b^2 + b^4)*cosh(x))*sinh(x))*log(2*(a*sinh(x
) + b)/(cosh(x) - sinh(x))) + 3*((a^4 + 2*a^2*b^2)*cosh(x)^6 + 6*(a^4 + 2*a
^2*b^2)*cosh(x)*sinh(x)^5 + (a^4 + 2*a^2*b^2)*sinh(x)^6 - 3*(a^4 + 2*a^2*b^
2)*cosh(x)^4 - 3*(a^4 + 2*a^2*b^2 - 5*(a^4 + 2*a^2*b^2)*cosh(x)^2)*sinh(x)^
4 - a^4 - 2*a^2*b^2 + 4*(5*(a^4 + 2*a^2*b^2)*cosh(x)^3 - 3*(a^4 + 2*a^2*b^2
)*cosh(x))*sinh(x)^3 + 3*(a^4 + 2*a^2*b^2)*cosh(x)^2 + 3*(5*(a^4 + 2*a^2*b^
2)*cosh(x)^4 + a^4 + 2*a^2*b^2 - 6*(a^4 + 2*a^2*b^2)*cosh(x)^2)*sinh(x)^2 +
```

```
6*((a^4 + 2*a^2*b^2)*cosh(x)^5 - 2*(a^4 + 2*a^2*b^2)*cosh(x)^3 + (a^4 + 2*
a^2*b^2)*cosh(x))*sinh(x))*log(2*sinh(x)/(cosh(x) - sinh(x))) + 6*(3*b^4*x*
cosh(x)^5 + 5*(a^3*b + 2*a*b^3)*cosh(x)^4 + a^3*b + 2*a*b^3 - 2*(3*b^4*x +
2*a^2*b^2)*cosh(x)^3 - 2*(3*a^3*b + 4*a*b^3)*cosh(x)^2 + (3*b^4*x + 2*a^2*b
^2)*cosh(x))*sinh(x))/(a*b^4*cosh(x)^6 + 6*a*b^4*cosh(x)*sinh(x)^5 + a*b^4*
sinh(x)^6 - 3*a*b^4*cosh(x)^4 + 3*a*b^4*cosh(x)^2 - a*b^4 + 3*(5*a*b^4*cosh
(x)^2 - a*b^4)*sinh(x)^4 + 4*(5*a*b^4*cosh(x)^3 - 3*a*b^4*cosh(x))*sinh(x)^
3 + 3*(5*a*b^4*cosh(x)^4 - 6*a*b^4*cosh(x)^2 + a*b^4)*sinh(x)^2 + 6*(a*b^4*
cosh(x)^5 - 2*a*b^4*cosh(x)^3 + a*b^4*cosh(x))*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^5(x)}{a + b\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**5/(a+b*csch(x)),x)
```

```
[Out] Integral(coth(x)**5/(a + b*csch(x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(66) = 132.
time = 0.40, size = 170, normalized size = 2.43

$$-\frac{(a^3 + 2\,ab^2)\log\left(\left|-e^{(-x)} + e^x\right|\right)}{b^4} + \frac{(a^4 + 2\,a^2b^2 + b^4)\log\left(\left|-a\left(e^{(-x)} - e^x\right) + 2\,b\right|\right)}{ab^4} + \frac{11\,a^3\left(e^{(-x)} - e^x\right)^3 + 22\,ab^2\left(e^{(-x)} - e^x\right)^3 + 12\,a^2b\left(e^{(-x)} - e^x\right)^2 + 24\,b^3\left(e^{(-x)} - e^x\right)^2 + 12\,ab^2\left(e^{(-x)} - e^x\right) + 16\,b^3}{6\,b^4\left(e^{(-x)} - e^x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -(a^3 + 2*a*b^2)*log(abs(-e^(-x) + e^x))/b^4 + (a^4 + 2*a^2*b^2 + b^4)*log(
abs(-a*(e^(-x) - e^x) + 2*b))/(a*b^4) + 1/6*(11*a^3*(e^(-x) - e^x)^3 + 22*a
*b^2*(e^(-x) - e^x)^3 + 12*a^2*b*(e^(-x) - e^x)^2 + 24*b^3*(e^(-x) - e^x)^2
 + 12*a*b^2*(e^(-x) - e^x) + 16*b^3)/(b^4*(e^(-x) - e^x)^3)
```

Mupad [B]
time = 1.97, size = 155, normalized size = 2.21

$$\frac{\frac{2\,a}{b^2} - \frac{2\,e^x\,(a^2+2\,b^2)}{b^3}}{e^{2\,x} - 1} - \frac{x}{a} + \frac{\frac{2\,a}{b^2} - \frac{8\,e^x}{3\,b}}{e^{4\,x} - 2\,e^{2\,x} + 1} - \frac{8\,e^x}{3\,b\,\left(3\,e^{2\,x} - 3\,e^{4\,x} + e^{6\,x} - 1\right)} - \frac{\ln\left(e^{2\,x} - 1\right)\,\left(a^3 + 2\,a\,b^2\right)}{b^4} + \frac{\ln\left(2\,b\,e^x - a + a\,e^{2\,x}\right)\,\left(a^4 + 2\,a^2\,b^2 + b^4\right)}{a\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^5/(a + b/sinh(x)),x)
```

```
[Out] ((2*a)/b^2 - (2*exp(x)*(a^2 + 2*b^2))/b^3)/(exp(2*x) - 1) - x/a + ((2*a)/b^
2 - (8*exp(x))/(3*b))/(exp(4*x) - 2*exp(2*x) + 1) - (8*exp(x))/(3*b*(3*exp(
2*x) - 3*exp(4*x) + exp(6*x) - 1)) - (log(exp(2*x) - 1)*(2*a*b^2 + a^3))/b^
4 + (log(2*b*exp(x) - a + a*exp(2*x))*(a^4 + b^4 + 2*a^2*b^2))/(a*b^4)
```

3.123 $\int \frac{\coth^6(x)}{a+b\,\text{csch}(x)}\,dx$

Optimal. Leaf size=183

$$\frac{x}{a}-\frac{3\tanh^{-1}(\cosh(x))}{8b}+\frac{(a^2+3b^2)\tanh^{-1}(\cosh(x))}{2b^3}-\frac{(a^4+3a^2b^2+3b^4)\tanh^{-1}(\cosh(x))}{b^5}+\frac{2(a^2+b^2)^{5/2}\tan}{a}$$

```
[Out] x/a-3/8*arctanh(cosh(x))/b+1/2*(a^2+3*b^2)*arctanh(cosh(x))/b^3-(a^4+3*a^2*
b^2+3*b^4)*arctanh(cosh(x))/b^5+2*(a^2+b^2)^(5/2)*arctanh((a-b*tanh(1/2*x))
/(a^2+b^2)^(1/2))/a/b^5-a*coth(x)/b^2+a*(a^2+3*b^2)*coth(x)/b^4+1/3*a*coth(
x)^3/b^2+3/8*coth(x)*csch(x)/b-1/2*(a^2+3*b^2)*coth(x)*csch(x)/b^3-1/4*coth
(x)*csch(x)^3/b
```

Rubi [A]
time = 0.25, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {3983, 2976, 3855, 3852, 8, 3853, 2739, 632, 210}

$$\frac{2(a^2+b^2)^{5/2}\tanh^{-1}\left(\frac{a-b\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{ab^5}+\frac{a(a^2+3b^2)\coth(x)}{b^4}+\frac{(a^2+3b^2)\tanh^{-1}(\cosh(x))}{2b^3}-\frac{(a^2+3b^2)\coth(x)\text{csch}(x)}{2b^3}-\frac{(a^4+3a^2b^2+3b^4)\tanh^{-1}(\cosh(x))}{b^5}+\frac{a\coth^3(x)}{3b^2}-\frac{a\coth(x)}{b^2}+\frac{x}{a}-\frac{3\tanh^{-1}(\cosh(x))}{8b}-\frac{\coth(x)\text{csch}^3(x)}{4b}+\frac{3\coth(x)\text{csch}(x)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^6/(a + b*Csch[x]),x]
```

```
[Out] x/a - (3*ArcTanh[Cosh[x]])/(8*b) + ((a^2 + 3*b^2)*ArcTanh[Cosh[x]])/(2*b^3)
 - ((a^4 + 3*a^2*b^2 + 3*b^4)*ArcTanh[Cosh[x]])/b^5 + (2*(a^2 + b^2)^(5/2)*
ArcTanh[(a - b*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a*b^5) - (a*Coth[x])/b^2 + (a*
(a^2 + 3*b^2)*Coth[x])/b^4 + (a*Coth[x]^3)/(3*b^2) + (3*Coth[x]*Csch[x])/(8
*b) - ((a^2 + 3*b^2)*Coth[x]*Csch[x])/(2*b^3) - (Coth[x]*Csch[x]^3)/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2976

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3983

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
 + n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rule steps

$$\int \frac{\coth^6(x)}{a+b\text{csch}(x)}\,dx = i\int \frac{\cosh(x)\coth^5(x)}{ib+ia\sinh(x)}\,dx$$
$$= -\int \left(-\frac{1}{a} - \frac{(a^4+3a^2b^2+3b^4)\,\text{csch}(x)}{b^5} - \frac{a(-a^2-3b^2)\,\text{csch}^2(x)}{b^4} - \frac{(a^2+3b^2)\,\text{csch}^3(\right.$$
$$= \frac{x}{a} - \frac{a\int \text{csch}^4(x)\,dx}{b^2} + \frac{\int \text{csch}^5(x)\,dx}{b} - \frac{\left(i(a^2+b^2)^3\right)\int \frac{1}{ib+ia\sinh(x)}\,dx}{ab^5} - \frac{(a(a^2+3b^2)}{}$$
$$= \frac{x}{a} - \frac{(a^4+3a^2b^2+3b^4)\tanh^{-1}(\cosh(x))}{b^5} - \frac{(a^2+3b^2)\coth(x)\text{csch}(x)}{2b^3} - \frac{\coth(x)\text{csch}^3}{4b}$$
$$= \frac{x}{a} + \frac{(a^2+3b^2)\tanh^{-1}(\cosh(x))}{2b^3} - \frac{(a^4+3a^2b^2+3b^4)\tanh^{-1}(\cosh(x))}{b^5} - \frac{a\coth(x)}{b^2} +$$
$$= \frac{x}{a} - \frac{3\tanh^{-1}(\cosh(x))}{8b} + \frac{(a^2+3b^2)\tanh^{-1}(\cosh(x))}{2b^3} - \frac{(a^4+3a^2b^2+3b^4)\tanh^{-1}(\text{co}}{b^5}$$

Mathematica [A]
time = 1.08, size = 269, normalized size = 1.47

$$\frac{\text{csch}(x)(b+a\sinh(x))\left(192b^5x+384(-a^2-b^2)^{5/2}\text{ArcTan}\left(\frac{a-b\tanh(\frac{x}{2})}{\sqrt{-a^2-b^2}}\right)+32a^2b(3a^2+7b^2)\coth\left(\frac{x}{2}\right)-6ab^2(4a^2+9b^2)\text{csch}^2\left(\frac{x}{2}\right)-3ab^4\text{csch}^4\left(\frac{x}{2}\right)+24a(8a^4+20a^2b^2+15b^4)\log\left(\tanh\left(\frac{x}{2}\right)\right)-6ab^2(4a^2+9b^2)\text{sech}^2\left(\frac{x}{2}\right)+3ab^4\text{sech}^4\left(\frac{x}{2}\right)-64a^2b^3\text{csch}^3(x)\sinh^4\left(\frac{x}{2}\right)+4a^2b^3\text{csch}^4\left(\frac{x}{2}\right)\sinh(x)+32a^2b(3a^2+7b^2)\tanh\left(\frac{x}{2}\right)\right)}{192ab^5(a+b\text{csch}(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^6/(a + b*Csch[x]),x]
```

```
[Out] (Csch[x]*(b + a*Sinh[x])*(192*b^5*x + 384*(-a^2 - b^2)^(5/2)*ArcTan[(a - b*
Tanh[x/2])/Sqrt[-a^2 - b^2]] + 32*a^2*b*(3*a^2 + 7*b^2)*Coth[x/2] - 6*a*b^2
*(4*a^2 + 9*b^2)*Csch[x/2]^2 - 3*a*b^4*Csch[x/2]^4 + 24*a*(8*a^4 + 20*a^2*b
^2 + 15*b^4)*Log[Tanh[x/2]] - 6*a*b^2*(4*a^2 + 9*b^2)*Sech[x/2]^2 + 3*a*b^4
*Sech[x/2]^4 - 64*a^2*b^3*Csch[x]^3*Sinh[x/2]^4 + 4*a^2*b^3*Csch[x/2]^4*Sin
h[x] + 32*a^2*b*(3*a^2 + 7*b^2)*Tanh[x/2]))/(192*a*b^5*(a + b*Csch[x]))
```

Maple [A]
time = 0.97, size = 260, normalized size = 1.42

method	result
default	$\frac{\frac{b^3\left(\tanh^4\left(\frac{x}{2}\right)\right)}{4}+\frac{2a\left(\tanh^3\left(\frac{x}{2}\right)\right)b^2}{3}+2a^2b\left(\tanh^2\left(\frac{x}{2}\right)\right)+4b^3\left(\tanh^2\left(\frac{x}{2}\right)\right)+8a^3\tanh\left(\frac{x}{2}\right)+18a\,b^2\tanh\left(\frac{x}{2}\right)}{16b^4} - \frac{1}{64b\tanh\left(\frac{x}{2}\right)^4} - \frac{4a^2+8}{32b^3\tanh}$

risch	$\frac{x}{a}+\frac{-12a^2b\,e^{7x}-27b^3e^{7x}+24a^3e^{6x}+72a\,b^2e^{6x}+12a^2b\,e^{5x}+3b^3e^{5x}-72a^3e^{4x}-168a\,b^2e^{4x}+12a^2b\,e^{3x}+3b^3e^{3x}+72a^3e^{2x}+152a\,b^2e^{2x}-}{12b^4(e^{2x}-1)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^6/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/b^4*(1/4*b^3*tanh(1/2*x)^4+2/3*a*tanh(1/2*x)^3*b^2+2*a^2*b*tanh(1/2*x)
^2+4*b^3*tanh(1/2*x)^2+8*a^3*tanh(1/2*x)+18*a*b^2*tanh(1/2*x))-1/64/b/tanh(
1/2*x)^4-1/32*(4*a^2+8*b^2)/b^3/tanh(1/2*x)^2+1/16/b^5*(16*a^4+40*a^2*b^2+3
0*b^4)*ln(tanh(1/2*x))+1/24*a/b^2/tanh(1/2*x)^3+1/8*a*(4*a^2+9*b^2)/b^4/tan
h(1/2*x)+1/16*(32*a^6+96*a^4*b^2+96*a^2*b^4+32*b^6)/a/b^5/(a^2+b^2)^(1/2)*a
rctanh(1/2*(-2*b*tanh(1/2*x)+2*a)/(a^2+b^2)^(1/2))+1/a*ln(tanh(1/2*x)+1)-1/
a*ln(tanh(1/2*x)-1)
```

Maxima [A]
time = 0.48, size = 330, normalized size = 1.80

$$-\frac{24\,a^3+56\,ab^2-3\,(4\,a^2b+9\,b^3)e^{(-x)}-8\,(9\,a^3+19\,ab^2)e^{(-2\,x)}+3\,(4\,a^2b+b^3)e^{(-3\,x)}+24\,(3\,a^3+7\,ab^2)e^{(-4\,x)}+3\,(4\,a^2b+b^3)e^{(-5\,x)}-24\,(a^3+3\,ab^2)e^{(-6\,x)}-3\,(4\,a^2b+9\,b^3)e^{(-7\,x)}}{12\,(4\,b^4e^{(-2\,x)}-6\,b^4e^{(-4\,x)}+4\,b^4e^{(-6\,x)}-b^4e^{(-8\,x)}-b^4)}+\frac{x}{a}-\frac{(8\,a^4+20\,a^2b^2+15\,b^4)\log\left(e^{(-x)}+1\right)}{8\,b^5}+\frac{(8\,a^4+20\,a^2b^2+15\,b^4)\log\left(e^{(-x)}-1\right)}{8\,b^5}-\frac{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,ab^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^6/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] -1/12*(24*a^3 + 56*a*b^2 - 3*(4*a^2*b + 9*b^3)*e^(-x) - 8*(9*a^3 + 19*a*b^2
)*e^(-2*x) + 3*(4*a^2*b + b^3)*e^(-3*x) + 24*(3*a^3 + 7*a*b^2)*e^(-4*x) + 3
*(4*a^2*b + b^3)*e^(-5*x) - 24*(a^3 + 3*a*b^2)*e^(-6*x) - 3*(4*a^2*b + 9*b^
3)*e^(-7*x))/(4*b^4*e^(-2*x) - 6*b^4*e^(-4*x) + 4*b^4*e^(-6*x) - b^4*e^(-8*
x) - b^4) + x/a - 1/8*(8*a^4 + 20*a^2*b^2 + 15*b^4)*log(e^(-x) + 1)/b^5 + 1
/8*(8*a^4 + 20*a^2*b^2 + 15*b^4)*log(e^(-x) - 1)/b^5 - (a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6)*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*a*b^5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3160 vs. 2(167) = 334.
time = 0.63, size = 3160, normalized size = 17.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^6/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] 1/24*(24*b^5*x*cosh(x)^8 + 24*b^5*x*sinh(x)^8 - 6*(4*a^3*b^2 + 9*a*b^4)*cos
h(x)^7 + 6*(32*b^5*x*cosh(x) - 4*a^3*b^2 - 9*a*b^4)*sinh(x)^7 - 48*(2*b^5*x
 - a^4*b - 3*a^2*b^3)*cosh(x)^6 + 6*(112*b^5*x*cosh(x)^2 - 16*b^5*x + 8*a^4
*b + 24*a^2*b^3 - 7*(4*a^3*b^2 + 9*a*b^4)*cosh(x))*sinh(x)^6 + 24*b^5*x + 6
```

```
*(4*a^3*b^2 + a*b^4)*cosh(x)^5 + 6*(224*b^5*x*cosh(x)^3 + 4*a^3*b^2 + a*b^4
 - 21*(4*a^3*b^2 + 9*a*b^4)*cosh(x)^2 - 48*(2*b^5*x - a^4*b - 3*a^2*b^3)*co
sh(x))*sinh(x)^5 - 48*a^4*b - 112*a^2*b^3 + 48*(3*b^5*x - 3*a^4*b - 7*a^2*b
^3)*cosh(x)^4 + 6*(280*b^5*x*cosh(x)^4 + 24*b^5*x - 24*a^4*b - 56*a^2*b^3 -
 35*(4*a^3*b^2 + 9*a*b^4)*cosh(x)^3 - 120*(2*b^5*x - a^4*b - 3*a^2*b^3)*cos
h(x)^2 + 5*(4*a^3*b^2 + a*b^4)*cosh(x))*sinh(x)^4 + 6*(4*a^3*b^2 + a*b^4)*c
osh(x)^3 + 6*(224*b^5*x*cosh(x)^5 + 4*a^3*b^2 + a*b^4 - 35*(4*a^3*b^2 + 9*a
*b^4)*cosh(x)^4 - 160*(2*b^5*x - a^4*b - 3*a^2*b^3)*cosh(x)^3 + 10*(4*a^3*b
^2 + a*b^4)*cosh(x)^2 + 32*(3*b^5*x - 3*a^4*b - 7*a^2*b^3)*cosh(x))*sinh(x)
^3 - 16*(6*b^5*x - 9*a^4*b - 19*a^2*b^3)*cosh(x)^2 + 2*(336*b^5*x*cosh(x)^6
 - 48*b^5*x - 63*(4*a^3*b^2 + 9*a*b^4)*cosh(x)^5 + 72*a^4*b + 152*a^2*b^3 -
 360*(2*b^5*x - a^4*b - 3*a^2*b^3)*cosh(x)^4 + 30*(4*a^3*b^2 + a*b^4)*cosh(
x)^3 + 144*(3*b^5*x - 3*a^4*b - 7*a^2*b^3)*cosh(x)^2 + 9*(4*a^3*b^2 + a*b^4
)*cosh(x))*sinh(x)^2 + 24*((a^4 + 2*a^2*b^2 + b^4)*cosh(x)^8 + 8*(a^4 + 2*a
^2*b^2 + b^4)*cosh(x)*sinh(x)^7 + (a^4 + 2*a^2*b^2 + b^4)*sinh(x)^8 - 4*(a^
4 + 2*a^2*b^2 + b^4)*cosh(x)^6 - 4*(a^4 + 2*a^2*b^2 + b^4 - 7*(a^4 + 2*a^2*
b^2 + b^4)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^3 -
3*(a^4 + 2*a^2*b^2 + b^4)*cosh(x))*sinh(x)^5 + 6*(a^4 + 2*a^2*b^2 + b^4)*co
sh(x)^4 + 2*(35*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^4 + 3*a^4 + 6*a^2*b^2 + 3*b
^4 - 30*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^4 + a^4 + 2*a^2*b^2 + b^
4 + 8*(7*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^5 - 10*(a^4 + 2*a^2*b^2 + b^4)*cos
h(x)^3 + 3*(a^4 + 2*a^2*b^2 + b^4)*cosh(x))*sinh(x)^3 - 4*(a^4 + 2*a^2*b^2
+ b^4)*cosh(x)^2 + 4*(7*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^6 - 15*(a^4 + 2*a^2
*b^2 + b^4)*cosh(x)^4 - a^4 - 2*a^2*b^2 - b^4 + 9*(a^4 + 2*a^2*b^2 + b^4)*c
osh(x)^2)*sinh(x)^2 + 8*((a^4 + 2*a^2*b^2 + b^4)*cosh(x)^7 - 3*(a^4 + 2*a^2
*b^2 + b^4)*cosh(x)^5 + 3*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^3 - (a^4 + 2*a^2*
b^2 + b^4)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(
x)^2 + 2*a*b*cosh(x) + a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt
(a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*c
osh(x) + 2*(a*cosh(x) + b)*sinh(x) - a)) - 6*(4*a^3*b^2 + 9*a*b^4)*cosh(x)
- 3*((8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^8 + 8*(8*a^5 + 20*a^3*b^2 + 15
*a*b^4)*cosh(x)*sinh(x)^7 + (8*a^5 + 20*a^3*b^2 + 15*a*b^4)*sinh(x)^8 - 4*(
8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^6 - 4*(8*a^5 + 20*a^3*b^2 + 15*a*b^4
 - 7*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^2)*sinh(x)^6 + 8*(7*(8*a^5 + 2
0*a^3*b^2 + 15*a*b^4)*cosh(x)^3 - 3*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)
)*sinh(x)^5 + 8*a^5 + 20*a^3*b^2 + 15*a*b^4 + 6*(8*a^5 + 20*a^3*b^2 + 15*a*
b^4)*cosh(x)^4 + 2*(24*a^5 + 60*a^3*b^2 + 45*a*b^4 + 35*(8*a^5 + 20*a^3*b^2
 + 15*a*b^4)*cosh(x)^4 - 30*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^2)*sinh
(x)^4 + 8*(7*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^5 - 10*(8*a^5 + 20*a^3
*b^2 + 15*a*b^4)*cosh(x)^3 + 3*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x))*sin
h(x)^3 - 4*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^2 + 4*(7*(8*a^5 + 20*a^3
*b^2 + 15*a*b^4)*cosh(x)^6 - 8*a^5 - 20*a^3*b^2 - 15*a*b^4 - 15*(8*a^5 + 20
*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 9*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^
2)*sinh(x)^2 + 8*((8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^7 - 3*(8*a^5 + 20
*a^3*b^2 + 15*a*b^4)*cosh(x)^5 + 3*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^
```

```
3 - (8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x)
 + 1) + 3*((8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^8 + 8*(8*a^5 + 20*a^3*b^
2 + 15*a*b^4)*cosh(x)*sinh(x)^7 + (8*a^5 + 20*a^3*b^2 + 15*a*b^4)*sinh(x)^8
 - 4*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^6 - 4*(8*a^5 + 20*a^3*b^2 + 15
*a*b^4 - 7*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^2)*sinh(x)^6 + 8*(7*(8*a
^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^3 - 3*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*c
osh(x))*sinh(x)^5 + 8*a^5 + 20*a^3*b^2 + 15*a*b^4 + 6*(8*a^5 + 20*a^3*b^2 +
 15*a*b^4)*cosh(x)^4 + 2*(24*a^5 + 60*a^3*b^2 + 45*a*b^4 + 35*(8*a^5 + 20*a
^3*b^2 + 15*a*b^4)*cosh(x)^4 - 30*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^2
)*sinh(x)^4 + 8*(7*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^5 - 10*(8*a^5 +
20*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + 3*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x
))*sinh(x)^3 - 4*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^2 + 4*(7*(8*a^5 +
20*a^3*b^2 + 15*a*b^4)*cosh(x)^6 - 8*a^5 - 20*a^3*b^2 - 15*a*b^4 - 15*(8*a^
5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 9*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*co
sh(x)^2)*sinh(x)^2 + 8*((8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^7 - 3*(8*a^
5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x)^5 + 3*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*co
sh(x)^3 - (8*a^5 + 20*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x))*log(cosh(x) + s
inh(x) - 1) + 2*(96*b^5*x*cosh(x)^7 - 21*(4*a^3...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^6(x)}{a + b\,\mathrm{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**6/(a+b*csch(x)),x)
```

```
[Out] Integral(coth(x)**6/(a + b*csch(x)), x)
```

Giac [A]
time = 0.41, size = 305, normalized size = 1.67

$$\frac{x}{a} - \frac{(8\,a^4 + 20\,a^2b^2 + 15\,b^4)\log(e^x + 1)}{8\,b^5} + \frac{(8\,a^4 + 20\,a^2b^2 + 15\,b^4)\log(|e^x - 1|)}{8\,b^5} - \frac{(a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6)\log\left(\frac{|2\,ae^x + 2\,b - 2\,\sqrt{a^2 + b^2}|}{|2\,ae^x + 2\,b + 2\,\sqrt{a^2 + b^2}|}\right)}{\sqrt{a^2 + b^2}\,ab^5} - \frac{12\,a^2be^{(7\,x)} + 27\,b^3e^{(7\,x)} - 24\,a^3e^{(6\,x)} - 72\,ab^2e^{(6\,x)} - 12\,a^2be^{(5\,x)} - 3\,b^3e^{(5\,x)} + 72\,a^3e^{(4\,x)} + 168\,ab^2e^{(4\,x)} - 12\,a^2be^{(3\,x)} - 3\,b^3e^{(3\,x)} - 72\,a^3e^{(2\,x)} - 152\,ab^2e^{(2\,x)} + 12\,a^2be^x + 27\,b^3e^x + 24\,a^3 + 56\,ab^2}{12\,b^4(e^{(2\,x)} - 1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^6/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] x/a - 1/8*(8*a^4 + 20*a^2*b^2 + 15*b^4)*log(e^x + 1)/b^5 + 1/8*(8*a^4 + 20*
a^2*b^2 + 15*b^4)*log(abs(e^x - 1))/b^5 - (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^
6)*log(abs(2*a*e^x + 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*e^x + 2*b + 2*sqrt(a^
2 + b^2)))/(sqrt(a^2 + b^2)*a*b^5) - 1/12*(12*a^2*b*e^(7*x) + 27*b^3*e^(7*x
) - 24*a^3*e^(6*x) - 72*a*b^2*e^(6*x) - 12*a^2*b*e^(5*x) - 3*b^3*e^(5*x) +
72*a^3*e^(4*x) + 168*a*b^2*e^(4*x) - 12*a^2*b*e^(3*x) - 3*b^3*e^(3*x) - 72*
a^3*e^(2*x) - 152*a*b^2*e^(2*x) + 12*a^2*b*e^x + 27*b^3*e^x + 24*a^3 + 56*a
*b^2)/(b^4*(e^(2*x) - 1)^4)
```

Mupad [B]
time = 2.90, size = 543, normalized size = 2.97

$$\frac{\frac{8a}{3b^2}-\frac{6e^x}{b}}{3e^{2x}-3e^{4x}+e^{6x}-1}-\frac{\frac{e^x(4a^2+9b^2)}{4b^3}-\frac{2(a^4+3a^2b^2)}{ab^4}}{e^{2x}-1}+\frac{\frac{4a}{b^2}-\frac{e^x(4a^2+13b^2)}{2b^3}}{e^{4x}-2e^{2x}+1}+\frac{x}{a}+\frac{\ln(e^x-1)\,(8a^4+20a^2b^2+15b^4)}{8b^5}-\frac{\ln(e^x+1)\,(8a^4+20a^2b^2+15b^4)}{8b^5}-\frac{4e^x}{b\,(6e^{4x}-4e^{2x}-4e^{6x}+e^{8x}+1)}+\frac{\ln\left(a^3\sqrt{(a^2+b^2)^5}-2a^7b-2ab^7-6a^3b^5-6a^5b^3+a^8e^x+4b^8e^x+2ab^2\sqrt{(a^2+b^2)^5}-4b^3e^x\sqrt{(a^2+b^2)^5}+13a^2b^6e^x+15a^4b^4e^x+7a^6b^2e^x-3a^2be^x\sqrt{(a^2+b^2)^5}\right)\sqrt{(a^2+b^2)^5}}{ab^5}-\frac{\ln\left(a^8e^x-2a^7b-a^3\sqrt{(a^2+b^2)^5}-6a^3b^5-6a^5b^3-2ab^7+4b^8e^x-2ab^2\sqrt{(a^2+b^2)^5}+4b^3e^x\sqrt{(a^2+b^2)^5}+13a^2b^6e^x+15a^4b^4e^x+7a^6b^2e^x+3a^2be^x\sqrt{(a^2+b^2)^5}\right)\sqrt{(a^2+b^2)^5}}{ab^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^6/(a + b/sinh(x)),x)
```

```
[Out] ((8*a)/(3*b^2) - (6*exp(x))/b)/(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1) - (
(exp(x)*(4*a^2 + 9*b^2))/(4*b^3) - (2*(a^4 + 3*a^2*b^2))/(a*b^4))/(exp(2*x)
 - 1) + ((4*a)/b^2 - (exp(x)*(4*a^2 + 13*b^2))/(2*b^3))/(exp(4*x) - 2*exp(2
*x) + 1) + x/a + (log(exp(x) - 1)*(8*a^4 + 15*b^4 + 20*a^2*b^2))/(8*b^5) -
(log(exp(x) + 1)*(8*a^4 + 15*b^4 + 20*a^2*b^2))/(8*b^5) - (4*exp(x))/(b*(6*
exp(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1)) + (log(a^3*((a^2 + b^2)
^5)^(1/2) - 2*a^7*b - 2*a*b^7 - 6*a^3*b^5 - 6*a^5*b^3 + a^8*exp(x) + 4*b^8*
exp(x) + 2*a*b^2*((a^2 + b^2)^5)^(1/2) - 4*b^3*exp(x)*((a^2 + b^2)^5)^(1/2)
 + 13*a^2*b^6*exp(x) + 15*a^4*b^4*exp(x) + 7*a^6*b^2*exp(x) - 3*a^2*b*exp(x
)*((a^2 + b^2)^5)^(1/2))*((a^2 + b^2)^5)^(1/2))/(a*b^5) - (log(a^8*exp(x) -
 2*a^7*b - a^3*((a^2 + b^2)^5)^(1/2) - 6*a^3*b^5 - 6*a^5*b^3 - 2*a*b^7 + 4*
b^8*exp(x) - 2*a*b^2*((a^2 + b^2)^5)^(1/2) + 4*b^3*exp(x)*((a^2 + b^2)^5)^(
1/2) + 13*a^2*b^6*exp(x) + 15*a^4*b^4*exp(x) + 7*a^6*b^2*exp(x) + 3*a^2*b*e
xp(x)*((a^2 + b^2)^5)^(1/2))*((a^2 + b^2)^5)^(1/2))/(a*b^5)
```

3.124 $\int \frac{\coth^7(x)}{a+b\mathbf{csch}(x)}\,dx$

Optimal. Leaf size=119

$$-\frac{(a^4+3a^2b^2+3b^4)\operatorname{csch}(x)}{b^5}+\frac{a(a^2+3b^2)\operatorname{csch}^2(x)}{2b^4}-\frac{(a^2+3b^2)\operatorname{csch}^3(x)}{3b^3}+\frac{a\operatorname{csch}^4(x)}{4b^2}-\frac{\operatorname{csch}^5(x)}{5b}+\frac{(a^2+b^2)^3\operatorname{lo}}{}$$

```
[Out] -(a^4+3*a^2*b^2+3*b^4)*csch(x)/b^5+1/2*a*(a^2+3*b^2)*csch(x)^2/b^4-1/3*(a^2
+3*b^2)*csch(x)^3/b^3+1/4*a*csch(x)^4/b^2-1/5*csch(x)^5/b+(a^2+b^2)^3*ln(a+
b*csch(x))/a/b^6+ln(sinh(x))/a
```

Rubi [A]
time = 0.10, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3970, 908}

$$\frac{(a^2+b^2)^3\log(a+b\operatorname{csch}(x))}{ab^6}+\frac{a(a^2+3b^2)\operatorname{csch}^2(x)}{2b^4}-\frac{(a^2+3b^2)\operatorname{csch}^3(x)}{3b^3}-\frac{(a^4+3a^2b^2+3b^4)\operatorname{csch}(x)}{b^5}+\frac{a\operatorname{csch}^4(x)}{4b^2}+\frac{\log(\sinh(x))}{a}-\frac{\operatorname{csch}^5(x)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^7/(a + b*Csch[x]),x]
```

```
[Out] -(((a^4 + 3*a^2*b^2 + 3*b^4)*Csch[x])/b^5) + (a*(a^2 + 3*b^2)*Csch[x]^2)/(2
*b^4) - ((a^2 + 3*b^2)*Csch[x]^3)/(3*b^3) + (a*Csch[x]^4)/(4*b^2) - Csch[x]
^5/(5*b) + ((a^2 + b^2)^3*Log[a + b*Csch[x]])/(a*b^6) + Log[Sinh[x]]/a
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\coth^7(x)}{a+b\operatorname{csch}(x)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{(-b^2-x^2)^3}{x(a+x)}\,dx, x, b\operatorname{csch}(x)\right)}{b^6}$$

$$= \frac{\operatorname{Subst}\left(\int \left(-a^4\left(1+\frac{3b^2(a^2+b^2)}{a^4}\right)-\frac{b^6}{ax}+a(a^2+3b^2)\,x-(a^2+3b^2)\,x^2+ax^3-x^4+\frac{(a^2}{a(}\right.\right.}{b^6}$$

$$= -\frac{(a^4+3a^2b^2+3b^4)\operatorname{csch}(x)}{b^5}+\frac{a(a^2+3b^2)\operatorname{csch}^2(x)}{2b^4}-\frac{(a^2+3b^2)\operatorname{csch}^3(x)}{3b^3}+\frac{a\operatorname{csch}^4(}{4b^2}$$

Mathematica [A]
time = 0.18, size = 130, normalized size = 1.09

$$\frac{-60b(a^4+3a^2b^2+3b^4)\operatorname{csch}(x)+30ab^2(a^2+3b^2)\operatorname{csch}^2(x)-20b^3(a^2+3b^2)\operatorname{csch}^3(x)+15ab^4\operatorname{csch}^4(x)-12b^5\operatorname{csch}^5(x)-60a(a^4+3a^2b^2+3b^4)\log(\sinh(x))+\frac{60(a^2+b^2)^3\log(b+a\sinh(x))}{a}}{60b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^7/(a + b*Csch[x]),x]
```

```
[Out] (-60*b*(a^4 + 3*a^2*b^2 + 3*b^4)*Csch[x] + 30*a*b^2*(a^2 + 3*b^2)*Csch[x]^2
 - 20*b^3*(a^2 + 3*b^2)*Csch[x]^3 + 15*a*b^4*Csch[x]^4 - 12*b^5*Csch[x]^5 -
 60*a*(a^4 + 3*a^2*b^2 + 3*b^4)*Log[Sinh[x]] + (60*(a^2 + b^2)^3*Log[b + a*
Sinh[x]])/a)/(60*b^6)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(111) = 222$.
time = 0.78, size = 314, normalized size = 2.64

method	result
default	$\frac{\frac{b^4\left(\tanh^5\left(\frac{x}{2}\right)\right)}{5}+\frac{a\left(\tanh^4\left(\frac{x}{2}\right)\right)b^3}{2}+\frac{4a^2b^2\left(\tanh^3\left(\frac{x}{2}\right)\right)}{3}+3\left(\tanh^3\left(\frac{x}{2}\right)\right)b^4+4a^3b\left(\tanh^2\left(\frac{x}{2}\right)\right)+10b^3\left(\tanh^2\left(\frac{x}{2}\right)\right)a+16a^4\tanh\left(\frac{x}{2}\right)+44a^2b^2\,\mathrm{t}}{32b^5}$
risch	$-\frac{x}{a}-\frac{2\,\mathrm{e}^{x}\left(15a^4\mathrm{e}^{8x}+45a^2b^2\mathrm{e}^{8x}+45b^4\mathrm{e}^{8x}-15a^3b\,\mathrm{e}^{7x}-45a\,b^3\mathrm{e}^{7x}-60a^4\mathrm{e}^{6x}-160a^2b^2\mathrm{e}^{6x}-120b^4\mathrm{e}^{6x}+45a^3b\,\mathrm{e}^{5x}+105a\,b^3\mathrm{e}^{5x}+90a^4\mathrm{e}\right.}{15b^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^7/(a+b*csch(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/b^5*(1/5*b^4*tanh(1/2*x)^5+1/2*a*tanh(1/2*x)^4*b^3+4/3*a^2*b^2*tanh(1/
2*x)^3+3*tanh(1/2*x)^3*b^4+4*a^3*b*tanh(1/2*x)^2+10*b^3*tanh(1/2*x)^2*a+16*
a^4*tanh(1/2*x)+44*a^2*b^2*tanh(1/2*x)+38*b^4*tanh(1/2*x))-1/a*ln(tanh(1/2*
x)+1)+1/32/b^6/a*(32*a^6+96*a^4*b^2+96*a^2*b^4+32*b^6)*ln(-b*tanh(1/2*x)^2+
2*a*tanh(1/2*x)+b)-1/160/b/tanh(1/2*x)^5-1/96/b^3*(4*a^2+9*b^2)/tanh(1/2*x)
^3-1/32*(16*a^4+44*a^2*b^2+38*b^4)/b^5/tanh(1/2*x)+1/64*a/b^2/tanh(1/2*x)^4
```

```
+1/16*a/b^4*(2*a^2+5*b^2)/tanh(1/2*x)^2-1/b^6*a*(a^4+3*a^2*b^2+3*b^4)*ln(ta
nh(1/2*x))-1/a*ln(tanh(1/2*x)-1)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(111) = 222$.
time = 0.28, size = 364, normalized size = 3.06

$$\frac{2\left(15\left(a^{4}+3a^{2}b^{2}+3b^{4}\right)e^{(-x)}-15\left(a^{3}b+3ab^{3}\right)e^{(-2x)}-20\left(3a^{4}+8a^{2}b^{2}+6b^{4}\right)e^{(-3x)}+15\left(3a^{3}b+7ab^{3}\right)e^{(-4x)}+2\left(45a^{4}+115a^{2}b^{2}+99b^{4}\right)e^{(-5x)}-15\left(3a^{3}b+7ab^{3}\right)e^{(-6x)}-20\left(3a^{4}+8a^{2}b^{2}+6b^{4}\right)e^{(-7x)}+15\left(a^{3}b+3ab^{3}\right)e^{(-8x)}+15\left(a^{4}+3a^{2}b^{2}+3b^{4}\right)e^{(-9x)}\right)}{15\left(5b^{5}e^{(-2x)}-10b^{5}e^{(-4x)}+10b^{5}e^{(-6x)}-5b^{5}e^{(-8x)}+b^{5}e^{(-10x)}-b^{5}\right)}+\frac{x}{a}-\frac{\left(a^{5}+3a^{3}b^{2}+3ab^{4}\right)\log\left(e^{(-x)}+1\right)}{b^{6}}-\frac{\left(a^{5}+3a^{3}b^{2}+3ab^{4}\right)\log\left(e^{(-x)}-1\right)}{b^{6}}+\frac{\left(a^{6}+3a^{4}b^{2}+3a^{2}b^{4}+b^{6}\right)\log\left(-2be^{(-x)}+ae^{(-2x)}-a\right)}{ab^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^7/(a+b*csch(x)),x, algorithm="maxima")
```

```
[Out] 2/15*(15*(a^4 + 3*a^2*b^2 + 3*b^4)*e^(-x) - 15*(a^3*b + 3*a*b^3)*e^(-2*x) -
 20*(3*a^4 + 8*a^2*b^2 + 6*b^4)*e^(-3*x) + 15*(3*a^3*b + 7*a*b^3)*e^(-4*x)
+ 2*(45*a^4 + 115*a^2*b^2 + 99*b^4)*e^(-5*x) - 15*(3*a^3*b + 7*a*b^3)*e^(-6
*x) - 20*(3*a^4 + 8*a^2*b^2 + 6*b^4)*e^(-7*x) + 15*(a^3*b + 3*a*b^3)*e^(-8*
x) + 15*(a^4 + 3*a^2*b^2 + 3*b^4)*e^(-9*x))/(5*b^5*e^(-2*x) - 10*b^5*e^(-4*
x) + 10*b^5*e^(-6*x) - 5*b^5*e^(-8*x) + b^5*e^(-10*x) - b^5) + x/a - (a^5 +
 3*a^3*b^2 + 3*a*b^4)*log(e^(-x) + 1)/b^6 - (a^5 + 3*a^3*b^2 + 3*a*b^4)*log
(e^(-x) - 1)/b^6 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*log(-2*b*e^(-x) + a*
e^(-2*x) - a)/(a*b^6)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4024 vs. $2(111) = 222$.
time = 0.52, size = 4024, normalized size = 33.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^7/(a+b*csch(x)),x, algorithm="fricas")
```

```
[Out] -1/15*(15*b^6*x*cosh(x)^10 + 15*b^6*x*sinh(x)^10 + 30*(a^5*b + 3*a^3*b^3 +
3*a*b^5)*cosh(x)^9 + 30*(5*b^6*x*cosh(x) + a^5*b + 3*a^3*b^3 + 3*a*b^5)*sin
h(x)^9 - 15*(5*b^6*x + 2*a^4*b^2 + 6*a^2*b^4)*cosh(x)^8 + 15*(45*b^6*x*cosh
(x)^2 - 5*b^6*x - 2*a^4*b^2 - 6*a^2*b^4 + 18*(a^5*b + 3*a^3*b^3 + 3*a*b^5)*
cosh(x))*sinh(x)^8 - 40*(3*a^5*b + 8*a^3*b^3 + 6*a*b^5)*cosh(x)^7 + 40*(45*
b^6*x*cosh(x)^3 - 3*a^5*b - 8*a^3*b^3 - 6*a*b^5 + 27*(a^5*b + 3*a^3*b^3 + 3
*a*b^5)*cosh(x)^2 - 3*(5*b^6*x + 2*a^4*b^2 + 6*a^2*b^4)*cosh(x))*sinh(x)^7
- 15*b^6*x + 30*(5*b^6*x + 3*a^4*b^2 + 7*a^2*b^4)*cosh(x)^6 + 10*(315*b^6*x
*cosh(x)^4 + 15*b^6*x + 9*a^4*b^2 + 21*a^2*b^4 + 252*(a^5*b + 3*a^3*b^3 + 3
*a*b^5)*cosh(x)^3 - 42*(5*b^6*x + 2*a^4*b^2 + 6*a^2*b^4)*cosh(x)^2 - 28*(3*
a^5*b + 8*a^3*b^3 + 6*a*b^5)*cosh(x))*sinh(x)^6 + 4*(45*a^5*b + 115*a^3*b^3
 + 99*a*b^5)*cosh(x)^5 + 4*(945*b^6*x*cosh(x)^5 + 45*a^5*b + 115*a^3*b^3 +
99*a*b^5 + 945*(a^5*b + 3*a^3*b^3 + 3*a*b^5)*cosh(x)^4 - 210*(5*b^6*x + 2*a
^4*b^2 + 6*a^2*b^4)*cosh(x)^3 - 210*(3*a^5*b + 8*a^3*b^3 + 6*a*b^5)*cosh(x)
^2 + 45*(5*b^6*x + 3*a^4*b^2 + 7*a^2*b^4)*cosh(x))*sinh(x)^5 - 30*(5*b^6*x
```

```
+ 3*a^4*b^2 + 7*a^2*b^4)*cosh(x)^4 + 10*(315*b^6*x*cosh(x)^6 - 15*b^6*x - 9
*a^4*b^2 - 21*a^2*b^4 + 378*(a^5*b + 3*a^3*b^3 + 3*a*b^5)*cosh(x)^5 - 105*(
5*b^6*x + 2*a^4*b^2 + 6*a^2*b^4)*cosh(x)^4 - 140*(3*a^5*b + 8*a^3*b^3 + 6*a
*b^5)*cosh(x)^3 + 45*(5*b^6*x + 3*a^4*b^2 + 7*a^2*b^4)*cosh(x)^2 + 2*(45*a^
5*b + 115*a^3*b^3 + 99*a*b^5)*cosh(x))*sinh(x)^4 - 40*(3*a^5*b + 8*a^3*b^3
+ 6*a*b^5)*cosh(x)^3 + 40*(45*b^6*x*cosh(x)^7 + 63*(a^5*b + 3*a^3*b^3 + 3*a
*b^5)*cosh(x)^6 - 3*a^5*b - 8*a^3*b^3 - 6*a*b^5 - 21*(5*b^6*x + 2*a^4*b^2 +
 6*a^2*b^4)*cosh(x)^5 - 35*(3*a^5*b + 8*a^3*b^3 + 6*a*b^5)*cosh(x)^4 + 15*(
5*b^6*x + 3*a^4*b^2 + 7*a^2*b^4)*cosh(x)^3 + (45*a^5*b + 115*a^3*b^3 + 99*a
*b^5)*cosh(x)^2 - 3*(5*b^6*x + 3*a^4*b^2 + 7*a^2*b^4)*cosh(x))*sinh(x)^3 +
15*(5*b^6*x + 2*a^4*b^2 + 6*a^2*b^4)*cosh(x)^2 + 5*(135*b^6*x*cosh(x)^8 + 2
16*(a^5*b + 3*a^3*b^3 + 3*a*b^5)*cosh(x)^7 + 15*b^6*x - 84*(5*b^6*x + 2*a^4
*b^2 + 6*a^2*b^4)*cosh(x)^6 + 6*a^4*b^2 + 18*a^2*b^4 - 168*(3*a^5*b + 8*a^3
*b^3 + 6*a*b^5)*cosh(x)^5 + 90*(5*b^6*x + 3*a^4*b^2 + 7*a^2*b^4)*cosh(x)^4
+ 8*(45*a^5*b + 115*a^3*b^3 + 99*a*b^5)*cosh(x)^3 - 36*(5*b^6*x + 3*a^4*b^2
 + 7*a^2*b^4)*cosh(x)^2 - 24*(3*a^5*b + 8*a^3*b^3 + 6*a*b^5)*cosh(x))*sinh(
x)^2 + 30*(a^5*b + 3*a^3*b^3 + 3*a*b^5)*cosh(x) - 15*((a^6 + 3*a^4*b^2 + 3*
a^2*b^4 + b^6)*cosh(x)^10 + 10*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)*
sinh(x)^9 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sinh(x)^10 - 5*(a^6 + 3*a^4
*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^8 - 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 -
9*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(x)^8 + 40*(3*(a^6 + 3
*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^3 - (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
*cosh(x))*sinh(x)^7 + 10*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^6 + 10
*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 21*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
)*cosh(x)^4 - 14*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(x)^6 -
 a^6 - 3*a^4*b^2 - 3*a^2*b^4 - b^6 + 4*(63*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b
^6)*cosh(x)^5 - 70*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^3 + 15*(a^6
+ 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*sinh(x)^5 - 10*(a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6)*cosh(x)^4 + 10*(21*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh
(x)^6 - a^6 - 3*a^4*b^2 - 3*a^2*b^4 - b^6 - 35*(a^6 + 3*a^4*b^2 + 3*a^2*b^4
 + b^6)*cosh(x)^4 + 15*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(
x)^4 + 40*(3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^7 - 7*(a^6 + 3*a^4
*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^5 + 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c
osh(x)^3 - (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*sinh(x)^3 + 5*(a^6
+ 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2 + 5*(9*(a^6 + 3*a^4*b^2 + 3*a^2*b^
4 + b^6)*cosh(x)^8 - 28*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^6 + a^6
 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 30*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cos
h(x)^4 - 12*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(x)^2 + 10*(
(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^9 - 4*(a^6 + 3*a^4*b^2 + 3*a^2*
b^4 + b^6)*cosh(x)^7 + 6*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^5 - 4*
(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^3 + (a^6 + 3*a^4*b^2 + 3*a^2*b^
4 + b^6)*cosh(x))*sinh(x))*log(2*(a*sinh(x) + b)/(cosh(x) - sinh(x))) + 15*
((a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^10 + 10*(a^6 + 3*a^4*b^2 + 3*a^2*b^4
)*cosh(x)*sinh(x)^9 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4)*sinh(x)^10 - 5*(a^6 + 3
*a^4*b^2 + 3*a^2*b^4)*cosh(x)^8 - 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 - 9*(a^6 +
```

```
 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^2)*sinh(x)^8 + 40*(3*(a^6 + 3*a^4*b^2 + 3*a
^2*b^4)*cosh(x)^3 - (a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x))*sinh(x)^7 + 10*(
a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^6 + 10*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
21*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^4 - 14*(a^6 + 3*a^4*b^2 + 3*a^2*b^
4)*cosh(x)^2)*sinh(x)^6 - a^6 - 3*a^4*b^2 - 3*a^2*b^4 + 4*(63*(a^6 + 3*a^4*
b^2 + 3*a^2*b^4)*cosh(x)^5 - 70*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^3 + 1
5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4)*cosh(x))*sinh(x...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^7(x)}{a + b\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**7/(a+b*csch(x)),x)
```

```
[Out] Integral(coth(x)**7/(a + b*csch(x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 295 vs. 2(111) = 222.
time = 0.40, size = 295, normalized size = 2.48

$$-\frac{(a^5 + 3a^3b^2 + 3ab^4)\log(|-e^{(-x)} + e^x|)}{b^6} + \frac{(a^6 + 3a^4b^2 + 3a^2b^4 + b^6)\log(|-a(e^{(-x)} - e^x) + 2b|)}{ab^6} + \frac{137a^5(e^{(-x)} - e^x)^5 + 411a^3b^2(e^{(-x)} - e^x)^5 + 411ab^4(e^{(-x)} - e^x)^5 + 120a^4b(e^{(-x)} - e^x)^4 + 360a^2b^3(e^{(-x)} - e^x)^4 + 360b^5(e^{(-x)} - e^x)^4 + 120a^3b^2(e^{(-x)} - e^x)^3 + 360ab^4(e^{(-x)} - e^x)^3 + 160a^2b^3(e^{(-x)} - e^x)^2 + 480b^5(e^{(-x)} - e^x)^2 + 240ab^4(e^{(-x)} - e^x) + 384b^5}{60b^6(e^{(-x)} - e^x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^7/(a+b*csch(x)),x, algorithm="giac")
```

```
[Out] -(a^5 + 3*a^3*b^2 + 3*a*b^4)*log(abs(-e^(-x) + e^x))/b^6 + (a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6)*log(abs(-a*(e^(-x) - e^x) + 2*b))/(a*b^6) + 1/60*(137*a
^5*(e^(-x) - e^x)^5 + 411*a^3*b^2*(e^(-x) - e^x)^5 + 411*a*b^4*(e^(-x) - e^
x)^5 + 120*a^4*b*(e^(-x) - e^x)^4 + 360*a^2*b^3*(e^(-x) - e^x)^4 + 360*b^5*
(e^(-x) - e^x)^4 + 120*a^3*b^2*(e^(-x) - e^x)^3 + 360*a*b^4*(e^(-x) - e^x)^
3 + 160*a^2*b^3*(e^(-x) - e^x)^2 + 480*b^5*(e^(-x) - e^x)^2 + 240*a*b^4*(e^
(-x) - e^x) + 384*b^5)/(b^6*(e^(-x) - e^x)^5)
```

Mupad [B]
time = 2.18, size = 317, normalized size = 2.66

$$\frac{\frac{4a}{b^2} - \frac{64e^x}{5b}}{6e^{4x} - 4e^{2x} - 4e^{6x} + e^{8x} + 1} + \frac{\frac{8a}{b^2} - \frac{8e^x(5a^2+27b^2)}{15b^3}}{3e^{2x} - 3e^{4x} + e^{6x} - 1} - \frac{\frac{8e^x(a^2+3b^2)}{3b^3} - \frac{2(a^4+5a^2b^2)}{ab^4}}{e^{4x} - 2e^{2x} + 1} - \frac{x}{a} - \frac{\frac{2e^x(a^4+3a^2b^2+3b^4)}{b^5} - \frac{2(a^4+3a^2b^2)}{ab^4}}{e^{2x} - 1} - \frac{32e^x}{5b(5e^{2x} - 10e^{4x} + 10e^{6x} - 5e^{8x} + e^{10x} - 1)} - \frac{\ln(e^{2x} - 1)(a^5 + 3a^3b^2 + 3ab^4)}{b^6} + \frac{\ln(2be^x - a + ae^{2x})(a^6 + 3a^4b^2 + 3a^2b^4 + b^6)}{ab^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^7/(a + b/sinh(x)),x)
```

```
[Out] ((4*a)/b^2 - (64*exp(x))/(5*b))/(6*exp(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp
(8*x) + 1) + ((8*a)/b^2 - (8*exp(x)*(5*a^2 + 27*b^2))/(15*b^3))/(3*exp(2*x)
```

```
 - 3*exp(4*x) + exp(6*x) - 1) - ((8*exp(x)*(a^2 + 3*b^2))/(3*b^3) - (2*(a^4
 + 5*a^2*b^2))/(a*b^4))/(exp(4*x) - 2*exp(2*x) + 1) - x/a - ((2*exp(x)*(a^4
 + 3*b^4 + 3*a^2*b^2))/b^5 - (2*(a^4 + 3*a^2*b^2))/(a*b^4))/(exp(2*x) - 1)
- (32*exp(x))/(5*b*(5*exp(2*x) - 10*exp(4*x) + 10*exp(6*x) - 5*exp(8*x) + e
xp(10*x) - 1)) - (log(exp(2*x) - 1)*(3*a*b^4 + a^5 + 3*a^3*b^2))/b^6 + (log
(2*b*exp(x) - a + a*exp(2*x))*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))/(a*b^6)
```

3.125 $\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{7/2}\,dx$

Optimal. Leaf size=199

$$-\frac{32\sqrt{\operatorname{csch}^2(ac+bcx)}\,\sinh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^6}+\frac{192\sqrt{\operatorname{csch}^2(ac+bcx)}\,\sinh(ac+bcx)}{5bc\left(1-e^{2c(a+bx)}\right)^5}-\frac{48\sqrt{\operatorname{csch}^2(ac+bcx)}\,\sinh(a}{bc\left(1-e^{2c(a+bx)}\right)^4}$$

```
[Out] -32/3*sinh(b*c*x+a*c)*(csch(b*c*x+a*c)^2)^(1/2)/b/c/(1-exp(2*c*(b*x+a)))^6+
192/5*sinh(b*c*x+a*c)*(csch(b*c*x+a*c)^2)^(1/2)/b/c/(1-exp(2*c*(b*x+a)))^5-
48*sinh(b*c*x+a*c)*(csch(b*c*x+a*c)^2)^(1/2)/b/c/(1-exp(2*c*(b*x+a)))^4+64/
3*sinh(b*c*x+a*c)*(csch(b*c*x+a*c)^2)^(1/2)/b/c/(1-exp(2*c*(b*x+a)))^3
```

Rubi [A]

time = 0.21, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6852, 2320, 12, 272, 45}

$$\frac{64\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}}{3bc\left(1-e^{2c(a+bx)}\right)^3}-\frac{48\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}}{bc\left(1-e^{2c(a+bx)}\right)^4}+\frac{192\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}}{5bc\left(1-e^{2c(a+bx)}\right)^5}-\frac{32\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}}{3bc\left(1-e^{2c(a+bx)}\right)^6}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*(Csch[a*c + b*c*x]^2)^(7/2),x]
```

```
[Out] (-32*Sqrt[Csch[a*c + b*c*x]^2]*Sinh[a*c + b*c*x])/(3*b*c*(1 - E^(2*c*(a + b
*x)))^6) + (192*Sqrt[Csch[a*c + b*c*x]^2]*Sinh[a*c + b*c*x])/(5*b*c*(1 - E^
(2*c*(a + b*x)))^5) - (48*Sqrt[Csch[a*c + b*c*x]^2]*Sinh[a*c + b*c*x])/(b*c
*(1 - E^(2*c*(a + b*x)))^4) + (64*Sqrt[Csch[a*c + b*c*x]^2]*Sinh[a*c + b*c*
x])/(3*b*c*(1 - E^(2*c*(a + b*x)))^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int e^{c(a+bx)}\text{csch}^2(ac+bcx)^{7/2}\,dx = \left(\sqrt{\text{csch}^2(ac+bcx)}\,\sinh(ac+bcx)\right)\int e^{c(a+bx)}\text{csch}^7(ac+bcx)\,dx$$

$$= \frac{\left(\sqrt{\text{csch}^2(ac+bcx)}\,\sinh(ac+bcx)\right)\text{Subst}\left(\int \frac{128x^7}{(-1+x^2)^7}\,dx, x, e^{c(a+bx)}\right)}{bc}$$

$$= \frac{\left(128\sqrt{\text{csch}^2(ac+bcx)}\,\sinh(ac+bcx)\right)\text{Subst}\left(\int \frac{x^7}{(-1+x^2)^7}\,dx, x, e^{c(a+bx)}\right)}{bc}$$

$$= \frac{\left(64\sqrt{\text{csch}^2(ac+bcx)}\,\sinh(ac+bcx)\right)\text{Subst}\left(\int \frac{x^3}{(-1+x)^7}\,dx, x, e^{2c(a+bx)}\right)}{bc}$$

$$= \frac{\left(64\sqrt{\text{csch}^2(ac+bcx)}\,\sinh(ac+bcx)\right)\text{Subst}\left(\int \left(\frac{1}{(-1+x)^7} + \frac{3}{(-1+x)^6} + \cdots\right.\right.}{bc}$$

$$= -\frac{32\sqrt{\text{csch}^2(ac+bcx)}\,\sinh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^6} + \frac{192\sqrt{\text{csch}^2(ac+bcx)}\,\sinh(ac+\cdots}{5bc\left(1-e^{2c(a+bx)}\right)^5}$$

Mathematica [A]
time = 0.06, size = 84, normalized size = 0.42

$$-\frac{16\left(-1+6e^{2c(a+bx)}-15e^{4c(a+bx)}+20e^{6c(a+bx)}\right)\sqrt{\text{csch}^2(c(a+bx))}\,\sinh(c(a+bx))}{15bc\left(-1+e^{2c(a+bx)}\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*(Csch[a*c + b*c*x]^2)^(7/2),x]
```

```
[Out] (-16*(-1 + 6*E^(2*c*(a + b*x)) - 15*E^(4*c*(a + b*x)) + 20*E^(6*c*(a + b*x)
))*Sqrt[Csch[c*(a + b*x)]^2]*Sinh[c*(a + b*x)])/(15*b*c*(-1 + E^(2*c*(a + b
*x)))^6)
```

Maple [A]
time = 3.61, size = 91, normalized size = 0.46

method	result	size
risch	$-\frac{16\left(20\,{\mathrm e}^{6c(bx+a)}-15\,{\mathrm e}^{4c(bx+a)}+6\,{\mathrm e}^{2c(bx+a)}-1\right)\sqrt{\frac{{\mathrm e}^{2c(bx+a)}}{\left({\mathrm e}^{2c(bx+a)}-1\right)^{2}}}\;{\mathrm e}^{-c(bx+a)}}{15cb\left({\mathrm e}^{2c(bx+a)}-1\right)^{5}}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -16/15/c/b*(20*exp(6*c*(b*x+a))-15*exp(4*c*(b*x+a))+6*exp(2*c*(b*x+a))-1)*(
1/(exp(2*c*(b*x+a))-1)^2*exp(2*c*(b*x+a)))^(1/2)/(exp(2*c*(b*x+a))-1)^5*exp
(-c*(b*x+a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(173) = 346$.
time = 0.48, size = 386, normalized size = 1.94

$$-\frac{64\,e^{(6\,bcx+6\,ac)}}{3\,bc\left(e^{(12\,bcx+12\,ac)}-6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}-20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}-6\,e^{(2\,bcx+2\,ac)}+1\right)}+\frac{16\,e^{(4\,bcx+4\,ac)}}{bc\left(e^{(12\,bcx+12\,ac)}-6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}-20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}-6\,e^{(2\,bcx+2\,ac)}+1\right)}-\frac{32\,e^{(2\,bcx+2\,ac)}}{5\,bc\left(e^{(12\,bcx+12\,ac)}-6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}-20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}-6\,e^{(2\,bcx+2\,ac)}+1\right)}+\frac{16}{15\,bc\left(e^{(12\,bcx+12\,ac)}-6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}-20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}-6\,e^{(2\,bcx+2\,ac)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -64/3*e^(6*b*c*x + 6*a*c)/(b*c*(e^(12*b*c*x + 12*a*c) - 6*e^(10*b*c*x + 10*
a*c) + 15*e^(8*b*c*x + 8*a*c) - 20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*
a*c) - 6*e^(2*b*c*x + 2*a*c) + 1)) + 16*e^(4*b*c*x + 4*a*c)/(b*c*(e^(12*b*c
*x + 12*a*c) - 6*e^(10*b*c*x + 10*a*c) + 15*e^(8*b*c*x + 8*a*c) - 20*e^(6*b
*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*a*c) - 6*e^(2*b*c*x + 2*a*c) + 1)) - 32/5
*e^(2*b*c*x + 2*a*c)/(b*c*(e^(12*b*c*x + 12*a*c) - 6*e^(10*b*c*x + 10*a*c)
+ 15*e^(8*b*c*x + 8*a*c) - 20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*a*c)
- 6*e^(2*b*c*x + 2*a*c) + 1)) + 16/15/(b*c*(e^(12*b*c*x + 12*a*c) - 6*e^(10
*b*c*x + 10*a*c) + 15*e^(8*b*c*x + 8*a*c) - 20*e^(6*b*c*x + 6*a*c) + 15*e^(
4*b*c*x + 4*a*c) - 6*e^(2*b*c*x + 2*a*c) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 592 vs. $2(173) = 346$.
time = 0.46, size = 592, normalized size = 2.97

$$-\frac{16\left(19\cosh\left(bcx+ac\right)^{3}+57\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^{2}+21\sinh\left(bcx+ac\right)^{3}+21\left(3\cosh\left(bcx+ac\right)^{2}-1\right)\sinh\left(bcx+ac\right)-9\cosh\left(bcx+ac\right)\right)}{15\left(bc\cosh\left(bcx+ac\right)^{9}+9\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^{8}+bc\sinh\left(bcx+ac\right)^{9}-6\,bc\cosh\left(bcx+ac\right)^{7}+6\left(6\,bc\cosh\left(bcx+ac\right)^{2}-bc\right)\sinh\left(bcx+ac\right)^{7}+15\,bc\cosh\left(bcx+ac\right)^{5}+42\left(2\,bc\cosh\left(bcx+ac\right)^{3}-bc\cosh\left(bcx+ac\right)\right)\sinh\left(bcx+ac\right)^{6}+3\left(42\,bc\cosh\left(bcx+ac\right)^{4}-42\,bc\cosh\left(bcx+ac\right)^{2}+5\,bc\right)\sinh\left(bcx+ac\right)^{5}-19\,bc\cosh\left(bcx+ac\right)^{3}+3\left(42\,bc\cosh\left(bcx+ac\right)^{5}-70\,bc\cosh\left(bcx+ac\right)^{3}+25\,bc\cosh\left(bcx+ac\right)\right)\sinh\left(bcx+ac\right)^{4}+3\left(28\,bc\cosh\left(bcx+ac\right)^{6}-70\,bc\cosh\left(bcx+ac\right)^{4}+50\,bc\cosh\left(bcx+ac\right)^{2}-7\,bc\right)\sinh\left(bcx+ac\right)^{3}+9\,bc\cosh\left(bcx+ac\right)+3\left(12\,bc\cosh\left(bcx+ac\right)^{7}-42\,bc\cosh\left(bcx+ac\right)^{5}+50\,bc\cosh\left(bcx+ac\right)^{3}-19\,bc\cosh\left(bcx+ac\right)\right)\sinh\left(bcx+ac\right)^{2}+3\left(3\,bc\cosh\left(bcx+ac\right)^{8}-14\,bc\cosh\left(bcx+ac\right)^{6}+25\,bc\cosh\left(bcx+ac\right)^{4}-21\,bc\cosh\left(bcx+ac\right)^{2}+7\,bc\right)\sinh\left(bcx+ac\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -16/15*(19*cosh(b*c*x + a*c)^3 + 57*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 +
 21*sinh(b*c*x + a*c)^3 + 21*(3*cosh(b*c*x + a*c)^2 - 1)*sinh(b*c*x + a*c)
- 9*cosh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^9 + 9*b*c*cosh(b*c*x + a*c)*s
inh(b*c*x + a*c)^8 + b*c*sinh(b*c*x + a*c)^9 - 6*b*c*cosh(b*c*x + a*c)^7 +
6*(6*b*c*cosh(b*c*x + a*c)^2 - b*c)*sinh(b*c*x + a*c)^7 + 15*b*c*cosh(b*c*x
 + a*c)^5 + 42*(2*b*c*cosh(b*c*x + a*c)^3 - b*c*cosh(b*c*x + a*c))*sinh(b*c
*x + a*c)^6 + 3*(42*b*c*cosh(b*c*x + a*c)^4 - 42*b*c*cosh(b*c*x + a*c)^2 +
5*b*c)*sinh(b*c*x + a*c)^5 - 19*b*c*cosh(b*c*x + a*c)^3 + 3*(42*b*c*cosh(b*
c*x + a*c)^5 - 70*b*c*cosh(b*c*x + a*c)^3 + 25*b*c*cosh(b*c*x + a*c))*sinh(
b*c*x + a*c)^4 + 3*(28*b*c*cosh(b*c*x + a*c)^6 - 70*b*c*cosh(b*c*x + a*c)^4
 + 50*b*c*cosh(b*c*x + a*c)^2 - 7*b*c)*sinh(b*c*x + a*c)^3 + 9*b*c*cosh(b*c
*x + a*c) + 3*(12*b*c*cosh(b*c*x + a*c)^7 - 42*b*c*cosh(b*c*x + a*c)^5 + 50
*b*c*cosh(b*c*x + a*c)^3 - 19*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^2 +
3*(3*b*c*cosh(b*c*x + a*c)^8 - 14*b*c*cosh(b*c*x + a*c)^6 + 25*b*c*cosh(b*c
*x + a*c)^4 - 21*b*c*cosh(b*c*x + a*c)^2 + 7*b*c)*sinh(b*c*x + a*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{ac} \int \left(\operatorname{csch}^2(ac + bcx)\right)^{\frac{7}{2}} e^{bcx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)**2)**(7/2),x)
```

```
[Out] exp(a*c)*Integral((csch(a*c + b*c*x)**2)**(7/2)*exp(b*c*x), x)
```

Giac [A]
time = 0.41, size = 90, normalized size = 0.45

$$-\frac{16 \left(20\, e^{(6\, bcx + 6\, ac)} - 15\, e^{(4\, bcx + 4\, ac)} + 6\, e^{(2\, bcx + 2\, ac)} - 1\right)}{15\, bc\left(e^{(2\, bcx + 2\, ac)} - 1\right)^6 \operatorname{sgn}\left(e^{(bcx + ac)} - e^{(-bcx - ac)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(7/2),x, algorithm="giac")
```

```
[Out] -16/15*(20*e^(6*b*c*x + 6*a*c) - 15*e^(4*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*
a*c) - 1)/(b*c*(e^(2*b*c*x + 2*a*c) - 1)^6*sgn(e^(b*c*x + a*c) - e^(-b*c*x
- a*c)))
```

Mupad [B]
time = 1.60, size = 413, normalized size = 2.08

$$\frac{32\sqrt{\frac{1}{\left(\frac{e^{ac+bcx}}{2}-\frac{e^{-ac-bcx}}{2}\right)^2}}\left(e^{4ac+4bcx}-2\,e^{2ac+2bcx}+1\right)}{3\,b\,c\left(e^{ac+bcx}-e^{3ac+3bcx}\right)\left(e^{2ac+2bcx}-1\right)^3}+\frac{24\sqrt{\frac{1}{\left(\frac{e^{ac+bcx}}{2}-\frac{e^{-ac-bcx}}{2}\right)^2}}\left(e^{4ac+4bcx}-2\,e^{2ac+2bcx}+1\right)}{b\,c\left(e^{ac+bcx}-e^{3ac+3bcx}\right)\left(e^{2ac+2bcx}-1\right)^4}+\frac{96\sqrt{\frac{1}{\left(\frac{e^{ac+bcx}}{2}-\frac{e^{-ac-bcx}}{2}\right)^2}}\left(e^{4ac+4bcx}-2\,e^{2ac+2bcx}+1\right)}{5\,b\,c\left(e^{ac+bcx}-e^{3ac+3bcx}\right)\left(e^{2ac+2bcx}-1\right)^5}+\frac{16\sqrt{\frac{1}{\left(\frac{e^{ac+bcx}}{2}-\frac{e^{-ac-bcx}}{2}\right)^2}}\left(e^{4ac+4bcx}-2\,e^{2ac+2bcx}+1\right)}{3\,b\,c\left(e^{ac+bcx}-e^{3ac+3bcx}\right)\left(e^{2ac+2bcx}-1\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*(1/sinh(a*c + b*c*x)^2)^(7/2),x)
```

```
[Out] (32*(1/(exp(a*c + b*c*x)/2 - exp(- a*c - b*c*x)/2)^2)^(1/2)*(exp(4*a*c + 4*
b*c*x) - 2*exp(2*a*c + 2*b*c*x) + 1))/(3*b*c*(exp(a*c + b*c*x) - exp(3*a*c
+ 3*b*c*x))*(exp(2*a*c + 2*b*c*x) - 1)^3) + (24*(1/(exp(a*c + b*c*x)/2 - ex
p(- a*c - b*c*x)/2)^2)^(1/2)*(exp(4*a*c + 4*b*c*x) - 2*exp(2*a*c + 2*b*c*x)
 + 1))/(b*c*(exp(a*c + b*c*x) - exp(3*a*c + 3*b*c*x))*(exp(2*a*c + 2*b*c*x)
 - 1)^4) + (96*(1/(exp(a*c + b*c*x)/2 - exp(- a*c - b*c*x)/2)^2)^(1/2)*(exp
(4*a*c + 4*b*c*x) - 2*exp(2*a*c + 2*b*c*x) + 1))/(5*b*c*(exp(a*c + b*c*x) -
 exp(3*a*c + 3*b*c*x))*(exp(2*a*c + 2*b*c*x) - 1)^5) + (16*(1/(exp(a*c + b*
c*x)/2 - exp(- a*c - b*c*x)/2)^2)^(1/2)*(exp(4*a*c + 4*b*c*x) - 2*exp(2*a*c
 + 2*b*c*x) + 1))/(3*b*c*(exp(a*c + b*c*x) - exp(3*a*c + 3*b*c*x))*(exp(2*a
*c + 2*b*c*x) - 1)^6)
```

3.126 $\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{5/2}\,dx$

Optimal. Leaf size=147

$$-\frac{4\sqrt{\operatorname{csch}^2(ac+bcx)}\,\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^4}+\frac{32\sqrt{\operatorname{csch}^2(ac+bcx)}\,\sinh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^3}-\frac{8\sqrt{\operatorname{csch}^2(ac+bcx)}\,\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2}$$

```
[Out] -4*sinh(b*c*x+a*c)*(csch(b*c*x+a*c)^2)^(1/2)/b/c/(1-exp(2*c*(b*x+a)))^4+32/
3*sinh(b*c*x+a*c)*(csch(b*c*x+a*c)^2)^(1/2)/b/c/(1-exp(2*c*(b*x+a)))^3-8*si
nh(b*c*x+a*c)*(csch(b*c*x+a*c)^2)^(1/2)/b/c/(1-exp(2*c*(b*x+a)))^2
```

Rubi [A]
time = 0.12, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6852, 2320, 12, 272, 45}

$$-\frac{8\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}}{bc\left(1-e^{2c(a+bx)}\right)^2}+\frac{32\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}}{3bc\left(1-e^{2c(a+bx)}\right)^3}-\frac{4\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}}{bc\left(1-e^{2c(a+bx)}\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*(Csch[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] (-4*Sqrt[Csch[a*c + b*c*x]^2]*Sinh[a*c + b*c*x])/(b*c*(1 - E^(2*c*(a + b*x)
))^4) + (32*Sqrt[Csch[a*c + b*c*x]^2]*Sinh[a*c + b*c*x])/(3*b*c*(1 - E^(2*c
*(a + b*x)))^3) - (8*Sqrt[Csch[a*c + b*c*x]^2]*Sinh[a*c + b*c*x])/(b*c*(1 -
 E^(2*c*(a + b*x)))^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{5/2}\,dx = \left(\sqrt{\operatorname{csch}^2(ac+bcx)}\,\sinh(ac+bcx)\right)\int e^{c(a+bx)}\operatorname{csch}^5(ac+bcx)\,dx$$

$$= \frac{\left(\sqrt{\operatorname{csch}^2(ac+bcx)}\,\sinh(ac+bcx)\right)\operatorname{Subst}\left(\int \frac{32x^5}{(-1+x^2)^5}\,dx, x, e^{c(a+bx)}\right)}{bc}$$

$$= \frac{\left(32\sqrt{\operatorname{csch}^2(ac+bcx)}\,\sinh(ac+bcx)\right)\operatorname{Subst}\left(\int \frac{x^5}{(-1+x^2)^5}\,dx, x, e^{c(a+bx)}\right)}{bc}$$

$$= \frac{\left(16\sqrt{\operatorname{csch}^2(ac+bcx)}\,\sinh(ac+bcx)\right)\operatorname{Subst}\left(\int \frac{x^2}{(-1+x)^5}\,dx, x, e^{2c(a+bx)}\right)}{bc}$$

$$= \frac{\left(16\sqrt{\operatorname{csch}^2(ac+bcx)}\,\sinh(ac+bcx)\right)\operatorname{Subst}\left(\int \left(\frac{1}{(-1+x)^5} + \frac{2}{(-1+x)^4} + \frac{}{(-1}\right.\right.}{bc}$$

$$= -\frac{4\sqrt{\operatorname{csch}^2(ac+bcx)}\,\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^4} + \frac{32\sqrt{\operatorname{csch}^2(ac+bcx)}\,\sinh(ac+bc}{3bc\left(1-e^{2c(a+bx)}\right)^3}$$

Mathematica [A]
time = 0.05, size = 72, normalized size = 0.49

$$-\frac{4\left(1-4e^{2c(a+bx)}+6e^{4c(a+bx)}\right)\sqrt{\operatorname{csch}^2(c(a+bx))}\,\sinh(c(a+bx))}{3bc\left(-1+e^{2c(a+bx)}\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*(Csch[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] (-4*(1 - 4*E^(2*c*(a + b*x)) + 6*E^(4*c*(a + b*x)))*Sqrt[Csch[c*(a + b*x)]^
2]*Sinh[c*(a + b*x)])/(3*b*c*(-1 + E^(2*c*(a + b*x)))^4)
```

Maple [A]
time = 3.56, size = 80, normalized size = 0.54

method	result	size
risch	$-\frac{4\left(6\,{\mathrm e}^{4c(bx+a)}-4\,{\mathrm e}^{2c(bx+a)}+1\right)\sqrt{\frac{{\mathrm e}^{2c(bx+a)}}{\left({\mathrm e}^{2c(bx+a)}-1\right)^{2}}}\;{\mathrm e}^{-c(bx+a)}}{3cb\left({\mathrm e}^{2c(bx+a)}-1\right)^{3}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/3/c/b*(6*exp(4*c*(b*x+a))-4*exp(2*c*(b*x+a))+1)*(1/(exp(2*c*(b*x+a))-1)^
2*exp(2*c*(b*x+a)))^(1/2)/(exp(2*c*(b*x+a))-1)^3*exp(-c*(b*x+a))
```

Maxima [A]
time = 0.48, size = 209, normalized size = 1.42

$$-\frac{8\,e^{(4\,bcx+4\,ac)}}{bc\left(e^{(8\,bcx+8\,ac)}-4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}-4\,e^{(2\,bcx+2\,ac)}+1\right)}+\frac{16\,e^{(2\,bcx+2\,ac)}}{3\,bc\left(e^{(8\,bcx+8\,ac)}-4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}-4\,e^{(2\,bcx+2\,ac)}+1\right)}-\frac{4}{3\,bc\left(e^{(8\,bcx+8\,ac)}-4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}-4\,e^{(2\,bcx+2\,ac)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -8*e^(4*b*c*x + 4*a*c)/(b*c*(e^(8*b*c*x + 8*a*c) - 4*e^(6*b*c*x + 6*a*c) +
6*e^(4*b*c*x + 4*a*c) - 4*e^(2*b*c*x + 2*a*c) + 1)) + 16/3*e^(2*b*c*x + 2*a
*c)/(b*c*(e^(8*b*c*x + 8*a*c) - 4*e^(6*b*c*x + 6*a*c) + 6*e^(4*b*c*x + 4*a*
c) - 4*e^(2*b*c*x + 2*a*c) + 1)) - 4/3/(b*c*(e^(8*b*c*x + 8*a*c) - 4*e^(6*b
*c*x + 6*a*c) + 6*e^(4*b*c*x + 4*a*c) - 4*e^(2*b*c*x + 2*a*c) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 315 vs. 2(130) = 260.
time = 0.39, size = 315, normalized size = 2.14

$$-\frac{4\left(7\,\cosh\left(bcx+ac\right)^{2}+10\,\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)+7\,\sinh\left(bcx+ac\right)^{2}-4\right)}{3\left(bc\cosh\left(bcx+ac\right)^{6}+6\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^{5}+bc\sinh\left(bcx+ac\right)^{6}-4\,bc\cosh\left(bcx+ac\right)^{4}+\left(15\,bc\cosh\left(bcx+ac\right)^{2}-4\,bc\right)\sinh\left(bcx+ac\right)^{4}+7\,bc\cosh\left(bcx+ac\right)^{2}+4\left(5\,bc\cosh\left(bcx+ac\right)^{3}-4\,bc\cosh\left(bcx+ac\right)\right)\sinh\left(bcx+ac\right)^{3}+\left(15\,bc\cosh\left(bcx+ac\right)^{4}-24\,bc\cosh\left(bcx+ac\right)^{2}+7\,bc\right)\sinh\left(bcx+ac\right)^{2}-4\,bc+2\left(3\,bc\cosh\left(bcx+ac\right)^{5}-8\,bc\cosh\left(bcx+ac\right)^{3}+5\,bc\cosh\left(bcx+ac\right)\right)\sinh\left(bcx+ac\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -4/3*(7*cosh(b*c*x + a*c)^2 + 10*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + 7*si
nh(b*c*x + a*c)^2 - 4)/(b*c*cosh(b*c*x + a*c)^6 + 6*b*c*cosh(b*c*x + a*c)*s
inh(b*c*x + a*c)^5 + b*c*sinh(b*c*x + a*c)^6 - 4*b*c*cosh(b*c*x + a*c)^4 +
(15*b*c*cosh(b*c*x + a*c)^2 - 4*b*c)*sinh(b*c*x + a*c)^4 + 7*b*c*cosh(b*c*x
 + a*c)^2 + 4*(5*b*c*cosh(b*c*x + a*c)^3 - 4*b*c*cosh(b*c*x + a*c))*sinh(b*
```

```
c*x + a*c)^3 + (15*b*c*cosh(b*c*x + a*c)^4 - 24*b*c*cosh(b*c*x + a*c)^2 + 7
*b*c)*sinh(b*c*x + a*c)^2 - 4*b*c + 2*(3*b*c*cosh(b*c*x + a*c)^5 - 8*b*c*co
sh(b*c*x + a*c)^3 + 5*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int \left(\operatorname{csch}^2\left(ac+bcx\right)\right)^{\frac{5}{2}} e^{bcx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)**2)**(5/2),x)
```

```
[Out] exp(a*c)*Integral((csch(a*c + b*c*x)**2)**(5/2)*exp(b*c*x), x)
```

Giac [A]
time = 0.41, size = 77, normalized size = 0.52

$$-\frac{4\left(6\,e^{(4\,bcx+4\,ac)}-4\,e^{(2\,bcx+2\,ac)}+1\right)}{3\,bc(e^{(2\,bcx+2\,ac)}-1)^4\operatorname{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -4/3*(6*e^(4*b*c*x + 4*a*c) - 4*e^(2*b*c*x + 2*a*c) + 1)/(b*c*(e^(2*b*c*x +
 2*a*c) - 1)^4*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)))
```

Mupad [B]
time = 1.53, size = 91, normalized size = 0.62

$$-\frac{2\,\mathrm{e}^{-a\,c-b\,c\,x}\,\sqrt{\frac{1}{{\left(\frac{\mathrm{e}^{a\,c+b\,c\,x}}{2}-\frac{\mathrm{e}^{-a\,c-b\,c\,x}}{2}\right)}^2}}\,\left(6\,\mathrm{e}^{4\,a\,c+4\,b\,c\,x}-4\,\mathrm{e}^{2\,a\,c+2\,b\,c\,x}+1\right)}{3\,b\,c\,{\left(\mathrm{e}^{2\,a\,c+2\,b\,c\,x}-1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*(1/sinh(a*c + b*c*x)^2)^(5/2),x)
```

```
[Out] -(2*exp(- a*c - b*c*x)*(1/(exp(a*c + b*c*x)/2 - exp(- a*c - b*c*x)/2)^2)^(1
/2)*(6*exp(4*a*c + 4*b*c*x) - 4*exp(2*a*c + 2*b*c*x) + 1))/(3*b*c*(exp(2*a*
c + 2*b*c*x) - 1)^3)
```

3.127 $\int e^{c(a+bx)}\mathbf{csch}^2(ac+bcx)^{3/2}\,dx$

Optimal. Leaf size=58

$$-\frac{2e^{4c(a+bx)}\sqrt{\operatorname{csch}^2(ac+bcx)}\,\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2}$$

[Out] -2*exp(4*c*(b*x+a))*sinh(b*c*x+a*c)*(csch(b*c*x+a*c)^2)^(1/2)/b/c/(1-exp(2*c*(b*x+a)))^2

Rubi [A]
time = 0.08, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6852, 2320, 12, 270}

$$-\frac{2e^{4c(a+bx)}\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}}{bc\left(1-e^{2c(a+bx)}\right)^2}$$

Antiderivative was successfully verified.

[In] Int[E^(c*(a + b*x))*(Csch[a*c + b*c*x]^2)^(3/2),x]

[Out] (-2*E^(4*c*(a + b*x))*Sqrt[Csch[a*c + b*c*x]^2]*Sinh[a*c + b*c*x])/(b*c*(1 - E^(2*c*(a + b*x)))^2)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int e^{c(a+bx)}\operatorname{csch}^2(ac+bcx)^{3/2}\,dx &= \left(\sqrt{\operatorname{csch}^2(ac+bcx)}\,\sinh(ac+bcx)\right)\int e^{c(a+bx)}\operatorname{csch}^3(ac+bcx)\,dx\\
&= \frac{\left(\sqrt{\operatorname{csch}^2(ac+bcx)}\,\sinh(ac+bcx)\right)\operatorname{Subst}\left(\int \frac{8x^3}{(-1+x^2)^3}\,dx, x, e^{c(a+bx)}\right)}{bc}\\
&= \frac{\left(8\sqrt{\operatorname{csch}^2(ac+bcx)}\,\sinh(ac+bcx)\right)\operatorname{Subst}\left(\int \frac{x^3}{(-1+x^2)^3}\,dx, x, e^{c(a+bx)}\right)}{bc}\\
&= -\frac{2e^{4c(a+bx)}\sqrt{\operatorname{csch}^2(ac+bcx)}\,\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 56, normalized size = 0.97

$$-\frac{2e^{4c(a+bx)}\operatorname{csch}^2(c(a+bx))^{3/2}\sinh^3(c(a+bx))}{bc\left(-1+e^{2c(a+bx)}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*(Csch[a*c + b*c*x]^2)^(3/2),x]
[Out] (-2*E^(4*c*(a + b*x))*(Csch[c*(a + b*x)]^2)^(3/2)*Sinh[c*(a + b*x)]^3)/(b*c
*(-1 + E^(2*c*(a + b*x)))^2)
```

Maple [A]
time = 3.51, size = 69, normalized size = 1.19

method	result	size
risch	$-\frac{2\left(2\,e^{2c(bx+a)}-1\right)\sqrt{\frac{e^{2c(bx+a)}}{\left(e^{2c(bx+a)}-1\right)^2}}\;e^{-c(bx+a)}}{cb\left(e^{2c(bx+a)}-1\right)}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/c/b*(2*exp(2*c*(b*x+a))-1)*(1/(exp(2*c*(b*x+a))-1)^2*exp(2*c*(b*x+a)))^(
1/2)/(exp(2*c*(b*x+a))-1)*exp(-c*(b*x+a))
```

Maxima [A]
time = 0.48, size = 84, normalized size = 1.45

$$-\frac{4\,e^{(2\,bcx+2\,ac)}}{bc(e^{(4\,bcx+4\,ac)}-2\,e^{(2\,bcx+2\,ac)}+1)}+\frac{2}{bc(e^{(4\,bcx+4\,ac)}-2\,e^{(2\,bcx+2\,ac)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -4*e^(2*b*c*x + 2*a*c)/(b*c*(e^(4*b*c*x + 4*a*c) - 2*e^(2*b*c*x + 2*a*c) +
1)) + 2/(b*c*(e^(4*b*c*x + 4*a*c) - 2*e^(2*b*c*x + 2*a*c) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(52) = 104$.
time = 0.39, size = 121, normalized size = 2.09

$$-\frac{2\left(\cosh\left(bcx+ac\right)+3\,\sinh\left(bcx+ac\right)\right)}{bc\cosh\left(bcx+ac\right)^{3}+3\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^{2}+bc\sinh\left(bcx+ac\right)^{3}-bc\cosh\left(bcx+ac\right)+3\left(bc\cosh\left(bcx+ac\right)^{2}-bc\right)\sinh\left(bcx+ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(cosh(b*c*x + a*c) + 3*sinh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^3 + 3*b
*c*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 + b*c*sinh(b*c*x + a*c)^3 - b*c*co
sh(b*c*x + a*c) + 3*(b*c*cosh(b*c*x + a*c)^2 - b*c)*sinh(b*c*x + a*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int\left(\operatorname{csch}^{2}\left(ac+bcx\right)\right)^{\frac{3}{2}}e^{bcx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)**2)**(3/2),x)
```

```
[Out] exp(a*c)*Integral((csch(a*c + b*c*x)**2)**(3/2)*exp(b*c*x), x)
```

Giac [A]
time = 0.40, size = 64, normalized size = 1.10

$$-\frac{2\left(2\,e^{(2\,bcx+2\,ac)}-1\right)}{bc(e^{(2\,bcx+2\,ac)}-1)^{2}\operatorname{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(2*e^(2*b*c*x + 2*a*c) - 1)/(b*c*(e^(2*b*c*x + 2*a*c) - 1)^2*sgn(e^(b*c*
x + a*c) - e^(-b*c*x - a*c)))
```

Mupad [B]
time = 1.55, size = 78, normalized size = 1.34

$$-\frac{e^{-a\,c-b\,c\,x}\,\left(2\,e^{2\,a\,c+2\,b\,c\,x}-1\right)\,\sqrt{\frac{1}{{\left(\frac{e^{a\,c+b\,c\,x}}{2}-\frac{e^{-a\,c-b\,c\,x}}{2}\right)}^2}}}{b\,c\,\left(e^{2\,a\,c+2\,b\,c\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*(1/sinh(a*c + b*c*x)^2)^(3/2),x)
```

```
[Out] -(exp(- a*c - b*c*x)*(2*exp(2*a*c + 2*b*c*x) - 1)*(1/(exp(a*c + b*c*x)/2 -
exp(- a*c - b*c*x)/2)^2)^(1/2))/(b*c*(exp(2*a*c + 2*b*c*x) - 1))
```

3.128 $\int e^{c(a+bx)} \sqrt{\operatorname{csch}^2(ac+bcx)}\, dx$

Optimal. Leaf size=46

$$\frac{\sqrt{\operatorname{csch}^2(ac+bcx)}\,\log\left(1-e^{2c(a+bx)}\right)\sinh(ac+bcx)}{bc}$$

```
[Out] ln(1-exp(2*c*(b*x+a)))*sinh(b*c*x+a*c)*(csch(b*c*x+a*c)^2)^(1/2)/b/c
```

Rubi [A]
time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6852, 2320, 12, 266}

$$\frac{\log\left(1-e^{2c(a+bx)}\right)\sinh(ac+bcx)\sqrt{\operatorname{csch}^2(ac+bcx)}}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*Sqrt[Csch[a*c + b*c*x]^2],x]
```

```
[Out] (Sqrt[Csch[a*c + b*c*x]^2]*Log[1 - E^(2*c*(a + b*x))]*Sinh[a*c + b*c*x])/(b
*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
```

```
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int e^{c(a+bx)}\sqrt{\operatorname{csch}^2(ac+bcx)}\,dx = \left(\sqrt{\operatorname{csch}^2(ac+bcx)}\,\sinh(ac+bcx)\right)\int e^{c(a+bx)}\operatorname{csch}(ac+bcx)\,dx$$
$$= \frac{\left(\sqrt{\operatorname{csch}^2(ac+bcx)}\,\sinh(ac+bcx)\right)\operatorname{Subst}\left(\int \frac{2x}{-1+x^2}\,dx, x, e^{c(a+bx)}\right)}{bc}$$
$$= \frac{\left(2\sqrt{\operatorname{csch}^2(ac+bcx)}\,\sinh(ac+bcx)\right)\operatorname{Subst}\left(\int \frac{x}{-1+x^2}\,dx, x, e^{c(a+bx)}\right)}{bc}$$
$$= \frac{\sqrt{\operatorname{csch}^2(ac+bcx)}\,\log\left(1-e^{2c(a+bx)}\right)\sinh(ac+bcx)}{bc}$$

Mathematica [A]
time = 0.03, size = 44, normalized size = 0.96

$$\frac{\sqrt{\operatorname{csch}^2(c(a+bx))}\,\log\left(1-e^{2c(a+bx)}\right)\sinh(c(a+bx))}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*Sqrt[Csch[a*c + b*c*x]^2],x]
[Out] (Sqrt[Csch[c*(a + b*x)]^2]*Log[1 - E^(2*c*(a + b*x))]*Sinh[c*(a + b*x)])/(b
*c)
```

Maple [A]
time = 3.77, size = 68, normalized size = 1.48

method	result	size
risch	$\frac{\ln\left(e^{2bcx}-e^{-2ac}\right)\left(e^{2c(bx+a)}-1\right)\sqrt{\frac{e^{2c(bx+a)}}{\left(e^{2c(bx+a)}-1\right)^2}}\,e^{-c(bx+a)}}{cb}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] ln(exp(2*b*c*x)-exp(-2*a*c))/c/b*(exp(2*c*(b*x+a))-1)*(1/(exp(2*c*(b*x+a))-
1)^2*exp(2*c*(b*x+a)))^(1/2)*exp(-c*(b*x+a))
```

Maxima [A]
time = 0.47, size = 39, normalized size = 0.85

$$\frac{\log\left(e^{(bcx+ac)}+1\right)}{bc}+\frac{\log\left(e^{(bcx+ac)}-1\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(e^(b*c*x + a*c) + 1)/(b*c) + log(e^(b*c*x + a*c) - 1)/(b*c)
```

Fricas [A]
time = 0.38, size = 42, normalized size = 0.91

$$\frac{\log\left(\frac{2\sinh(bcx+ac)}{\cosh(bcx+ac)-\sinh(bcx+ac)}\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] log(2*sinh(b*c*x + a*c)/(cosh(b*c*x + a*c) - sinh(b*c*x + a*c)))/(b*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int\sqrt{\operatorname{csch}^2(ac+bcx)}\,e^{bcx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)**2)**(1/2),x)
```

```
[Out] exp(a*c)*Integral(sqrt(csch(a*c + b*c*x)**2)*exp(b*c*x), x)
```

Giac [A]
time = 0.39, size = 48, normalized size = 1.04

$$\frac{\log\left(\left|e^{(2\,bcx+2\,ac)}-1\right|\right)}{bc\operatorname{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(csch(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] log(abs(e^(2*b*c*x + 2*a*c) - 1))/(b*c*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*
c)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int e^{c(a+bx)}\sqrt{\frac{1}{\sinh(ac+bcx)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*(1/sinh(a*c + b*c*x)^2)^(1/2),x)
[Out] int(exp(c*(a + b*x))*(1/sinh(a*c + b*c*x)^2)^(1/2), x)
```

3.129 $\int \frac{e^{c(a+bx)}}{\sqrt{\operatorname{csch}^2(ac+bcx)}}\,dx$

Optimal. Leaf size=74

$$\frac{e^{2c(a+bx)}\operatorname{csch}(ac+bcx)}{4bc\sqrt{\operatorname{csch}^2(ac+bcx)}}-\frac{x\operatorname{csch}(ac+bcx)}{2\sqrt{\operatorname{csch}^2(ac+bcx)}}$$

```
[Out] 1/4*exp(2*c*(b*x+a))*csch(b*c*x+a*c)/b/c/(csch(b*c*x+a*c)^2)^(1/2)-1/2*x*cs
ch(b*c*x+a*c)/(csch(b*c*x+a*c)^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6852, 2320, 12, 14}

$$\frac{e^{2c(a+bx)}\operatorname{csch}(ac+bcx)}{4bc\sqrt{\operatorname{csch}^2(ac+bcx)}}-\frac{x\operatorname{csch}(ac+bcx)}{2\sqrt{\operatorname{csch}^2(ac+bcx)}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/Sqrt[Csch[a*c + b*c*x]^2],x]
```

```
[Out] (E^(2*c*(a + b*x))*Csch[a*c + b*c*x])/(4*b*c*Sqrt[Csch[a*c + b*c*x]^2]) - (
x*Csch[a*c + b*c*x])/(2*Sqrt[Csch[a*c + b*c*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{c(a+bx)}}{\sqrt{\operatorname{csch}^2(ac+bcx)}}\,dx &= \frac{\operatorname{csch}(ac+bcx)\int e^{c(a+bx)}\sinh(ac+bcx)\,dx}{\sqrt{\operatorname{csch}^2(ac+bcx)}}\\ &= \frac{\operatorname{csch}(ac+bcx)\operatorname{Subst}\left(\int \frac{-1+x^2}{2x}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\operatorname{csch}^2(ac+bcx)}}\\ &= \frac{\operatorname{csch}(ac+bcx)\operatorname{Subst}\left(\int \frac{-1+x^2}{x}\,dx, x, e^{c(a+bx)}\right)}{2bc\sqrt{\operatorname{csch}^2(ac+bcx)}}\\ &= \frac{\operatorname{csch}(ac+bcx)\operatorname{Subst}\left(\int \left(-\frac{1}{x}+x\right)\,dx, x, e^{c(a+bx)}\right)}{2bc\sqrt{\operatorname{csch}^2(ac+bcx)}}\\ &= \frac{e^{2c(a+bx)}\operatorname{csch}(ac+bcx)}{4bc\sqrt{\operatorname{csch}^2(ac+bcx)}} - \frac{x\operatorname{csch}(ac+bcx)}{2\sqrt{\operatorname{csch}^2(ac+bcx)}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 48, normalized size = 0.65

$$\frac{\left(e^{2c(a+bx)}-2bcx\right)\operatorname{csch}(c(a+bx))}{4bc\sqrt{\operatorname{csch}^2(c(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/Sqrt[Csch[a*c + b*c*x]^2],x]
```

```
[Out] ((E^(2*c*(a + b*x)) - 2*b*c*x)*Csch[c*(a + b*x)])/(4*b*c*Sqrt[Csch[c*(a + b
*x)]^2])
```

Maple [A]
time = 3.27, size = 106, normalized size = 1.43

method	result	size

risch	$-\frac{x\,\mathrm{e}^{c(bx+a)}}{2\left(\mathrm{e}^{2c(bx+a)}-1\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(\mathrm{e}^{2c(bx+a)}-1\right)^{2}}}}+\frac{\mathrm{e}^{3c(bx+a)}}{4cb\left(\mathrm{e}^{2c(bx+a)}-1\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(\mathrm{e}^{2c(bx+a)}-1\right)^{2}}}}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x/(exp(2*c*(b*x+a))-1)/(1/(exp(2*c*(b*x+a))-1)^2*exp(2*c*(b*x+a)))^(1/
2)*exp(c*(b*x+a))+1/4/c/b/(exp(2*c*(b*x+a))-1)/(1/(exp(2*c*(b*x+a))-1)^2*ex
p(2*c*(b*x+a)))^(1/2)*exp(3*c*(b*x+a))
```

Maxima [A]
time = 0.48, size = 36, normalized size = 0.49

$$-\frac{bcx+ac}{2\,bc}+\frac{e^{(2\,bcx+2\,ac)}}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(b*c*x + a*c)/(b*c) + 1/4*e^(2*b*c*x + 2*a*c)/(b*c)
```

Fricas [A]
time = 0.48, size = 66, normalized size = 0.89

$$-\frac{(2\,bcx-1)\cosh\left(bcx+ac\right)-(2\,bcx+1)\sinh\left(bcx+ac\right)}{4\left(bc\cosh\left(bcx+ac\right)-bc\sinh\left(bcx+ac\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*((2*b*c*x - 1)*cosh(b*c*x + a*c) - (2*b*c*x + 1)*sinh(b*c*x + a*c))/(b
*c*cosh(b*c*x + a*c) - b*c*sinh(b*c*x + a*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int\frac{e^{bcx}}{\sqrt{\operatorname{csch}^{2}\left(ac+bcx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)**2)**(1/2),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)/sqrt(csch(a*c + b*c*x)**2), x)
```

Giac [A]
time = 0.40, size = 71, normalized size = 0.96

$$-\frac{1}{2}\,x\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)+\frac{e^{(2\,bcx+2\,ac)}\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
[Out] -1/2*x*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) + 1/4*e^(2*b*c*x + 2*a*c)*sg
n(e^(b*c*x + a*c) - e^(-b*c*x - a*c))/(b*c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{e}^{c\,(a+b\,x)}}{\sqrt{\frac{1}{{\mathrm{sinh}\left(a\,c+b\,c\,x\right)}^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))/(1/sinh(a*c + b*c*x)^2)^(1/2),x)
[Out] int(exp(c*(a + b*x))/(1/sinh(a*c + b*c*x)^2)^(1/2), x)
```

3.130 $\int \frac{e^{c(a+bx)}}{\operatorname{csch}^2(ac+bcx)^{3/2}}\,dx$

Optimal. Leaf size=162

$$\frac{e^{-2c(a+bx)}\operatorname{csch}(ac+bcx)}{16bc\sqrt{\operatorname{csch}^2(ac+bcx)}}-\frac{3e^{2c(a+bx)}\operatorname{csch}(ac+bcx)}{16bc\sqrt{\operatorname{csch}^2(ac+bcx)}}+\frac{e^{4c(a+bx)}\operatorname{csch}(ac+bcx)}{32bc\sqrt{\operatorname{csch}^2(ac+bcx)}}+\frac{3x\operatorname{csch}(ac+bcx)}{8\sqrt{\operatorname{csch}^2(ac+bcx)}}$$

```
[Out] 1/16*csch(b*c*x+a*c)/b/c/exp(2*c*(b*x+a))/(csch(b*c*x+a*c)^2)^(1/2)-3/16*ex
p(2*c*(b*x+a))*csch(b*c*x+a*c)/b/c/(csch(b*c*x+a*c)^2)^(1/2)+1/32*exp(4*c*(
b*x+a))*csch(b*c*x+a*c)/b/c/(csch(b*c*x+a*c)^2)^(1/2)+3/8*x*csch(b*c*x+a*c)
/(csch(b*c*x+a*c)^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6852, 2320, 12, 272, 45}

$$\frac{e^{-2c(a+bx)}\operatorname{csch}(ac+bcx)}{16bc\sqrt{\operatorname{csch}^2(ac+bcx)}}-\frac{3e^{2c(a+bx)}\operatorname{csch}(ac+bcx)}{16bc\sqrt{\operatorname{csch}^2(ac+bcx)}}+\frac{e^{4c(a+bx)}\operatorname{csch}(ac+bcx)}{32bc\sqrt{\operatorname{csch}^2(ac+bcx)}}+\frac{3x\operatorname{csch}(ac+bcx)}{8\sqrt{\operatorname{csch}^2(ac+bcx)}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/(Csch[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] Csch[a*c + b*c*x]/(16*b*c*E^(2*c*(a + b*x))*Sqrt[Csch[a*c + b*c*x]^2]) - (3
*E^(2*c*(a + b*x))*Csch[a*c + b*c*x])/(16*b*c*Sqrt[Csch[a*c + b*c*x]^2]) +
(E^(4*c*(a + b*x))*Csch[a*c + b*c*x])/(32*b*c*Sqrt[Csch[a*c + b*c*x]^2]) +
(3*x*Csch[a*c + b*c*x])/(8*Sqrt[Csch[a*c + b*c*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{c(a+bx)}}{\operatorname{csch}^2(ac+bcx)^{3/2}}\,dx &= \frac{\operatorname{csch}(ac+bcx)\int e^{c(a+bx)}\sinh^3(ac+bcx)\,dx}{\sqrt{\operatorname{csch}^2(ac+bcx)}}\\
&= \frac{\operatorname{csch}(ac+bcx)\operatorname{Subst}\left(\int \frac{(-1+x^2)^3}{8x^3}\,dx,x,e^{c(a+bx)}\right)}{bc\sqrt{\operatorname{csch}^2(ac+bcx)}}\\
&= \frac{\operatorname{csch}(ac+bcx)\operatorname{Subst}\left(\int \frac{(-1+x^2)^3}{x^3}\,dx,x,e^{c(a+bx)}\right)}{8bc\sqrt{\operatorname{csch}^2(ac+bcx)}}\\
&= \frac{\operatorname{csch}(ac+bcx)\operatorname{Subst}\left(\int \frac{(-1+x)^3}{x^2}\,dx,x,e^{2c(a+bx)}\right)}{16bc\sqrt{\operatorname{csch}^2(ac+bcx)}}\\
&= \frac{\operatorname{csch}(ac+bcx)\operatorname{Subst}\left(\int \left(-3-\frac{1}{x^2}+\frac{3}{x}+x\right)\,dx,x,e^{2c(a+bx)}\right)}{16bc\sqrt{\operatorname{csch}^2(ac+bcx)}}\\
&= \frac{e^{-2c(a+bx)}\operatorname{csch}(ac+bcx)}{16bc\sqrt{\operatorname{csch}^2(ac+bcx)}}-\frac{3e^{2c(a+bx)}\operatorname{csch}(ac+bcx)}{16bc\sqrt{\operatorname{csch}^2(ac+bcx)}}+\frac{e^{4c(a+bx)}\operatorname{csch}(ac+bcx)}{32bc\sqrt{\operatorname{csch}^2(ac+bcx)}}+
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 76, normalized size = 0.47

$$\frac{\left(e^{-2c(a+bx)}-3e^{2c(a+bx)}+\frac{1}{2}e^{4c(a+bx)}+6bcx\right)\operatorname{csch}^3(c(a+bx))}{16bc\operatorname{csch}^2(c(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[E^(c*(a + b*x))/(Csch[a*c + b*c*x]^2)^(3/2),x]

[Out] ((E^(-2*c*(a + b*x)) - 3*E^(2*c*(a + b*x)) + E^(4*c*(a + b*x))/2 + 6*b*c*x)*Csch[c*(a + b*x)]^3)/(16*b*c*(Csch[c*(a + b*x)]^2)^(3/2))

Maple [A]
time = 3.25, size = 216, normalized size = 1.33

method	result
risch	$\frac{3x\,e^{c(bx+a)}}{8\left(e^{2c(bx+a)}-1\right)\sqrt{\frac{e^{2c(bx+a)}}{\left(e^{2c(bx+a)}-1\right)^{2}}}}+\frac{e^{5c(bx+a)}}{32cb\left(e^{2c(bx+a)}-1\right)\sqrt{\frac{e^{2c(bx+a)}}{\left(e^{2c(bx+a)}-1\right)^{2}}}}-\frac{3\,e^{3c(bx+a)}}{16cb\left(e^{2c(bx+a)}-1\right)\sqrt{\frac{e^{2c(bx+a)}}{\left(e^{2c(bx+a)}-1\right)^{2}}}}+$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] 3/8*x/(exp(2*c*(b*x+a))-1)/(1/(exp(2*c*(b*x+a))-1)^2*exp(2*c*(b*x+a)))^(1/2)*exp(c*(b*x+a))+1/32/c/b/(exp(2*c*(b*x+a))-1)/(1/(exp(2*c*(b*x+a))-1)^2*exp(2*c*(b*x+a)))^(1/2)*exp(5*c*(b*x+a))-3/16/c/b/(exp(2*c*(b*x+a))-1)/(1/(exp(2*c*(b*x+a))-1)^2*exp(2*c*(b*x+a)))^(1/2)*exp(3*c*(b*x+a))+1/16/c/b/(exp(2*c*(b*x+a))-1)/(1/(exp(2*c*(b*x+a))-1)^2*exp(2*c*(b*x+a)))^(1/2)*exp(-c*(b*x+a))

Maxima [A]
time = 0.48, size = 62, normalized size = 0.38

$$\frac{\left(e^{(6\,bcx+6\,ac)}-6\,e^{(4\,bcx+4\,ac)}+2\right)e^{(-2\,bcx-2\,ac)}}{32\,bc}+\frac{3\,(bcx+ac)}{8\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")

[Out] 1/32*(e^(6*b*c*x + 6*a*c) - 6*e^(4*b*c*x + 4*a*c) + 2)*e^(-2*b*c*x - 2*a*c)/(b*c) + 3/8*(b*c*x + a*c)/(b*c)

Fricas [A]
time = 0.40, size = 126, normalized size = 0.78

$$\frac{3\,\cosh\left(bcx+ac\right)^{3}+9\,\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^{2}-\sinh\left(bcx+ac\right)^{3}+6\,(2\,bcx-1)\cosh\left(bcx+ac\right)-3\,\left(4\,bcx+\cosh\left(bcx+ac\right)^{2}+2\right)\sinh\left(bcx+ac\right)}{32\,(bc\cosh\left(bcx+ac\right)-bc\sinh\left(bcx+ac\right))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")

[Out] 1/32*(3*cosh(b*c*x + a*c)^3 + 9*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 - sinh(b*c*x + a*c)^3 + 6*(2*b*c*x - 1)*cosh(b*c*x + a*c) - 3*(4*b*c*x + cosh(b*

```
c*x + a*c)^2 + 2)*sinh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c) - b*c*sinh(b*c*
x + a*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int\frac{e^{bcx}}{\left(\operatorname{csch}^{2}\left(ac+bcx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)**2)**(3/2),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)/(csch(a*c + b*c*x)**2)**(3/2), x)
```

Giac [A]
time = 0.41, size = 204, normalized size = 1.26

$$\frac{\left(12\,bcxe^{(-ac)}\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)-2\,\left(3\,e^{(2\,bcx+2\,ac)}\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)-\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)\right)e^{(-2\,bcx-3\,ac)}+\left(e^{(4\,bcx+9\,ac)}\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)-6\,e^{(2\,bcx+7\,ac)}\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)\right)e^{(-6\,ac)}\right)e^{(ac)}}{32\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/32*(12*b*c*x*e^(-a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - 2*(3*e^(2
*b*c*x + 2*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - sgn(e^(b*c*x + a*
c) - e^(-b*c*x - a*c)))*e^(-2*b*c*x - 3*a*c) + (e^(4*b*c*x + 9*a*c)*sgn(e^(
b*c*x + a*c) - e^(-b*c*x - a*c)) - 6*e^(2*b*c*x + 7*a*c)*sgn(e^(b*c*x + a*c
) - e^(-b*c*x - a*c)))*e^(-6*a*c))*e^(a*c)/(b*c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\mathrm{e}^{c\,\left(a+b\,x\right)}}{{\left(\frac{1}{{\mathrm{sinh}\left(a\,c+b\,c\,x\right)}^2}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))/(1/sinh(a*c + b*c*x)^2)^(3/2),x)
```

```
[Out] int(exp(c*(a + b*x))/(1/sinh(a*c + b*c*x)^2)^(3/2), x)
```

3.131 $\int \frac{e^{c(a+bx)}}{\operatorname{csch}^2(ac+bcx)^{5/2}}\,dx$

Optimal. Leaf size=250

$$\frac{e^{-4c(a+bx)}\operatorname{csch}(ac+bcx)}{128bc\sqrt{\operatorname{csch}^2(ac+bcx)}}-\frac{5e^{-2c(a+bx)}\operatorname{csch}(ac+bcx)}{64bc\sqrt{\operatorname{csch}^2(ac+bcx)}}+\frac{5e^{2c(a+bx)}\operatorname{csch}(ac+bcx)}{32bc\sqrt{\operatorname{csch}^2(ac+bcx)}}-\frac{5e^{4c(a+bx)}\operatorname{csch}(ac+bcx)}{128bc\sqrt{\operatorname{csch}^2(ac+bcx)}}+$$

```
[Out] 1/128*csch(b*c*x+a*c)/b/c/exp(4*c*(b*x+a))/(csch(b*c*x+a*c)^2)^(1/2)-5/64*c
sch(b*c*x+a*c)/b/c/exp(2*c*(b*x+a))/(csch(b*c*x+a*c)^2)^(1/2)+5/32*exp(2*c*
(b*x+a))*csch(b*c*x+a*c)/b/c/(csch(b*c*x+a*c)^2)^(1/2)-5/128*exp(4*c*(b*x+a
))*csch(b*c*x+a*c)/b/c/(csch(b*c*x+a*c)^2)^(1/2)+1/192*exp(6*c*(b*x+a))*csc
h(b*c*x+a*c)/b/c/(csch(b*c*x+a*c)^2)^(1/2)-5/16*x*csch(b*c*x+a*c)/(csch(b*c
*x+a*c)^2)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6852, 2320, 12, 272, 45}

$$\frac{e^{-4c(a+bx)}\operatorname{csch}(ac+bcx)}{128bc\sqrt{\operatorname{csch}^2(ac+bcx)}}-\frac{5e^{-2c(a+bx)}\operatorname{csch}(ac+bcx)}{64bc\sqrt{\operatorname{csch}^2(ac+bcx)}}+\frac{5e^{2c(a+bx)}\operatorname{csch}(ac+bcx)}{32bc\sqrt{\operatorname{csch}^2(ac+bcx)}}-\frac{5e^{4c(a+bx)}\operatorname{csch}(ac+bcx)}{128bc\sqrt{\operatorname{csch}^2(ac+bcx)}}+\frac{e^{6c(a+bx)}\operatorname{csch}(ac+bcx)}{192bc\sqrt{\operatorname{csch}^2(ac+bcx)}}-\frac{5x\operatorname{csch}(ac+bcx)}{16\sqrt{\operatorname{csch}^2(ac+bcx)}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/(Csch[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] Csch[a*c + b*c*x]/(128*b*c*E^(4*c*(a + b*x))*Sqrt[Csch[a*c + b*c*x]^2]) - (
5*Csch[a*c + b*c*x])/(64*b*c*E^(2*c*(a + b*x))*Sqrt[Csch[a*c + b*c*x]^2]) +
 (5*E^(2*c*(a + b*x))*Csch[a*c + b*c*x])/(32*b*c*Sqrt[Csch[a*c + b*c*x]^2])
 - (5*E^(4*c*(a + b*x))*Csch[a*c + b*c*x])/(128*b*c*Sqrt[Csch[a*c + b*c*x]^
2]) + (E^(6*c*(a + b*x))*Csch[a*c + b*c*x])/(192*b*c*Sqrt[Csch[a*c + b*c*x]
^2]) - (5*x*Csch[a*c + b*c*x])/(16*Sqrt[Csch[a*c + b*c*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{c(a+bx)}}{\operatorname{csch}^2(ac+bcx)^{5/2}}\,dx &= \frac{\operatorname{csch}(ac+bcx)\int e^{c(a+bx)}\sinh^5(ac+bcx)\,dx}{\sqrt{\operatorname{csch}^2(ac+bcx)}}\\
&= \frac{\operatorname{csch}(ac+bcx)\operatorname{Subst}\left(\int \frac{(-1+x^2)^5}{32x^5}\,dx,x,e^{c(a+bx)}\right)}{bc\sqrt{\operatorname{csch}^2(ac+bcx)}}\\
&= \frac{\operatorname{csch}(ac+bcx)\operatorname{Subst}\left(\int \frac{(-1+x^2)^5}{x^5}\,dx,x,e^{c(a+bx)}\right)}{32bc\sqrt{\operatorname{csch}^2(ac+bcx)}}\\
&= \frac{\operatorname{csch}(ac+bcx)\operatorname{Subst}\left(\int \frac{(-1+x)^5}{x^3}\,dx,x,e^{2c(a+bx)}\right)}{64bc\sqrt{\operatorname{csch}^2(ac+bcx)}}\\
&= \frac{\operatorname{csch}(ac+bcx)\operatorname{Subst}\left(\int \left(10-\frac{1}{x^3}+\frac{5}{x^2}-\frac{10}{x}-5x+x^2\right)\,dx,x,e^{2c(a+bx)}\right)}{64bc\sqrt{\operatorname{csch}^2(ac+bcx)}}\\
&= \frac{e^{-4c(a+bx)}\operatorname{csch}(ac+bcx)}{128bc\sqrt{\operatorname{csch}^2(ac+bcx)}}-\frac{5e^{-2c(a+bx)}\operatorname{csch}(ac+bcx)}{64bc\sqrt{\operatorname{csch}^2(ac+bcx)}}+\frac{5e^{2c(a+bx)}\operatorname{csch}(ac+bcx)}{32bc\sqrt{\operatorname{csch}^2(ac+bcx)}}
\end{aligned}
$$

Mathematica [A]

time = 0.07, size = 106, normalized size = 0.42

$$\frac{\left(\frac{1}{2}e^{-4c(a+bx)} - 5e^{-2c(a+bx)} + 10e^{2c(a+bx)} - \frac{5}{2}e^{4c(a+bx)} + \frac{1}{3}e^{6c(a+bx)} - 20bcx\right)\operatorname{csch}^5(c(a+bx))}{64bc\operatorname{csch}^2(c(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/(Csch[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] ((1/(2*E^(4*c*(a + b*x))) - 5/E^(2*c*(a + b*x)) + 10*E^(2*c*(a + b*x)) - (5
*E^(4*c*(a + b*x)))/2 + E^(6*c*(a + b*x))/3 - 20*b*c*x)*Csch[c*(a + b*x)]^5
)/(64*b*c*(Csch[c*(a + b*x)]^2)^(5/2))
```

Maple [A]
time = 3.17, size = 326, normalized size = 1.30

method	result
risch	$-\frac{5x\,\mathrm{e}^{c(bx+a)}}{16\left(\mathrm{e}^{2c(bx+a)}-1\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(\mathrm{e}^{2c(bx+a)}-1\right)^2}}}+\frac{\mathrm{e}^{7c(bx+a)}}{192cb\left(\mathrm{e}^{2c(bx+a)}-1\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(\mathrm{e}^{2c(bx+a)}-1\right)^2}}}-\frac{5\,\mathrm{e}^{5c(bx+a)}}{128cb\left(\mathrm{e}^{2c(bx+a)}-1\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(\mathrm{e}^{2c(bx+a)}-1\right)^2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -5/16*x/(exp(2*c*(b*x+a))-1)/(1/(exp(2*c*(b*x+a))-1)^2*exp(2*c*(b*x+a)))^(1
/2)*exp(c*(b*x+a))+1/192/c/b/(exp(2*c*(b*x+a))-1)/(1/(exp(2*c*(b*x+a))-1)^2
*exp(2*c*(b*x+a)))^(1/2)*exp(7*c*(b*x+a))-5/128/c/b/(exp(2*c*(b*x+a))-1)/(1
/(exp(2*c*(b*x+a))-1)^2*exp(2*c*(b*x+a)))^(1/2)*exp(5*c*(b*x+a))+5/32/c/b/(
exp(2*c*(b*x+a))-1)/(1/(exp(2*c*(b*x+a))-1)^2*exp(2*c*(b*x+a)))^(1/2)*exp(3
*c*(b*x+a))-5/64/c/b/(exp(2*c*(b*x+a))-1)/(1/(exp(2*c*(b*x+a))-1)^2*exp(2*c
*(b*x+a)))^(1/2)*exp(-c*(b*x+a))+1/128/c/b/(exp(2*c*(b*x+a))-1)/(1/(exp(2*c
*(b*x+a))-1)^2*exp(2*c*(b*x+a)))^(1/2)*exp(-3*c*(b*x+a))
```

Maxima [A]
time = 0.48, size = 90, normalized size = 0.36

$$\frac{\left(2\,e^{(10\,bcx+10\,ac)} - 15\,e^{(8\,bcx+8\,ac)} + 60\,e^{(6\,bcx+6\,ac)} - 30\,e^{(2\,bcx+2\,ac)} + 3\right)e^{(-4\,bcx-4\,ac)}}{384\,bc} - \frac{5\,(bcx+ac)}{16\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/384*(2*e^(10*b*c*x + 10*a*c) - 15*e^(8*b*c*x + 8*a*c) + 60*e^(6*b*c*x + 6
*a*c) - 30*e^(2*b*c*x + 2*a*c) + 3)*e^(-4*b*c*x - 4*a*c)/(b*c) - 5/16*(b*c*
x + a*c)/(b*c)
```

Fricas [A]
time = 0.42, size = 218, normalized size = 0.87

$$\frac{5 \cosh\left(bcx+ac\right)^{5}+25 \cosh\left(bcx+ac\right) \sinh\left(bcx+ac\right)^{4}-\sinh\left(bcx+ac\right)^{5}-5 \left(2 \cosh\left(bcx+ac\right)^{2}-3\right) \sinh\left(bcx+ac\right)^{3}-45 \cosh\left(bcx+ac\right)^{3}+5 \left(10 \cosh\left(bcx+ac\right)^{3}-27 \cosh\left(bcx+ac\right)\right) \sinh\left(bcx+ac\right)^{2}-60 \left(2 bcx-1\right) \cosh\left(bcx+ac\right)-5 \left(\cosh\left(bcx+ac\right)^{4}-24 bcx-9 \cosh\left(bcx+ac\right)^{2}-12\right) \sinh\left(bcx+ac\right)}{384 \left(bc \cosh\left(bcx+ac\right)-bc \sinh\left(bcx+ac\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/384*(5*cosh(b*c*x + a*c)^5 + 25*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^4 - s
inh(b*c*x + a*c)^5 - 5*(2*cosh(b*c*x + a*c)^2 - 3)*sinh(b*c*x + a*c)^3 - 45
*cosh(b*c*x + a*c)^3 + 5*(10*cosh(b*c*x + a*c)^3 - 27*cosh(b*c*x + a*c))*si
nh(b*c*x + a*c)^2 - 60*(2*b*c*x - 1)*cosh(b*c*x + a*c) - 5*(cosh(b*c*x + a*
c)^4 - 24*b*c*x - 9*cosh(b*c*x + a*c)^2 - 12)*sinh(b*c*x + a*c))/(b*c*cosh(
b*c*x + a*c) - b*c*sinh(b*c*x + a*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{ac} \int \frac{e^{bcx}}{\left(\operatorname{csch}^{2}\left(ac+bcx\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)**2)**(5/2),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)/(csch(a*c + b*c*x)**2)**(5/2), x)
```

Giac [A]
time = 0.41, size = 278, normalized size = 1.11

$$-\frac{\left(120 bcxe^{(-ac)}\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)-3 \left(30 e^{(4 bcx+4 ac)}\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)-10 e^{(2 bcx+2 ac)}\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)+\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)\right)e^{(-4 bcx-5 ac)}-\left(2 e^{(6 bcx+20 ac)}\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)-15 e^{(4 bcx+18 ac)}\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)+60 e^{(2 bcx+16 ac)}\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)\right)e^{(-15 ac)}\right)e^{(ac)}}{384 bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(csch(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/384*(120*b*c*x*e^(-a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - 3*(30*
e^(4*b*c*x + 4*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - 10*e^(2*b*c*x
 + 2*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) + sgn(e^(b*c*x + a*c) - e
^(-b*c*x - a*c)))*e^(-4*b*c*x - 5*a*c) - (2*e^(6*b*c*x + 20*a*c)*sgn(e^(b*c
*x + a*c) - e^(-b*c*x - a*c)) - 15*e^(4*b*c*x + 18*a*c)*sgn(e^(b*c*x + a*c)
 - e^(-b*c*x - a*c)) + 60*e^(2*b*c*x + 16*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*
c*x - a*c)))*e^(-15*a*c))*e^(a*c)/(b*c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{e}^{c\,\left(a+b\,x\right)}}{{\left(\frac{1}{{\mathrm{sinh}\left(a\,c+b\,c\,x\right)}^2}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))/(1/sinh(a*c + b*c*x)^2)^(5/2),x)
[Out] int(exp(c*(a + b*x))/(1/sinh(a*c + b*c*x)^2)^(5/2), x)
```

3.132 $\int \frac{x^5}{\sqrt{\text{csch}(2\log(cx))}}\,dx$

Optimal. Leaf size=81

$$-\frac{2x^2}{21c^4\sqrt{\text{csch}(2\log(cx))}}+\frac{x^6}{7\sqrt{\text{csch}(2\log(cx))}}+\frac{2F(\csc^{-1}(cx)|-1)}{21c^7\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\text{csch}(2\log(cx))}}$$

```
[Out] -2/21*x^2/c^4/csch(2*ln(c*x))^(1/2)+1/7*x^6/csch(2*ln(c*x))^(1/2)+2/21*Elli
pticF(1/c/x,I)/c^7/x/(1-1/c^4/x^4)^(1/2)/csch(2*ln(c*x))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5671, 5669, 342, 283, 331, 227}

$$-\frac{2x^2}{21c^4\sqrt{\text{csch}(2\log(cx))}}+\frac{2F(\csc^{-1}(cx)|-1)}{21c^7x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\text{csch}(2\log(cx))}}+\frac{x^6}{7\sqrt{\text{csch}(2\log(cx))}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[Csch[2*Log[c*x]]],x]
```

```
[Out] (-2*x^2)/(21*c^4*Sqrt[Csch[2*Log[c*x]]]) + x^6/(7*Sqrt[Csch[2*Log[c*x]]]) +
 (2*EllipticF[ArcCsc[c*x], -1])/(21*c^7*Sqrt[1 - 1/(c^4*x^4)]*x*Sqrt[Csch[2
*Log[c*x]]])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5669

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5671

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{x^5}{\sqrt{\operatorname{csch}(2\log(x))}}\,dx, x, cx\right)}{c^6} \\
&= \frac{\operatorname{Subst}\left(\int \sqrt{1-\frac{1}{x^4}}\,x^6\,dx, x, cx\right)}{c^7\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}} \\
&= -\frac{\operatorname{Subst}\left(\int \frac{\sqrt{1-x^4}}{x^8}\,dx, x, \frac{1}{cx}\right)}{c^7\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}} \\
&= \frac{x^6}{7\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{2\operatorname{Subst}\left(\int \frac{1}{x^4\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)}{7c^7\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}} \\
&= -\frac{2x^2}{21c^4\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{x^6}{7\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{2\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^4}}\,dx, x, \right.}{21c^7\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log}} \\
&= -\frac{2x^2}{21c^4\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{x^6}{7\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{2F(\csc^{-1}(cx)|-1)}{21c^7\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log}}
\end{aligned}
$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.12, size = 80, normalized size = 0.99

$$
\frac{x^2\left(-(1-c^4x^4)^{3/2} + {}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};c^4x^4\right)\right)}{7c^4\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{-1+c^4x^4}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[Csch[2*Log[c*x]]],x]
```

```
[Out] (x^2*(-(1 - c^4*x^4)^(3/2) + Hypergeometric2F1[-1/2, 1/4, 5/4, c^4*x^4]))/(
7*c^4*Sqrt[2 - 2*c^4*x^4]*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)])
```

Maple [**A**]
time = 1.16, size = 125, normalized size = 1.54

method	result	size
risch	$\frac{x^{2}\left(3 c^{4} x^{4}-2\right) \sqrt{2}}{42 c^{4} \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}-1}}}-\frac{\sqrt{c^{2} x^{2}+1}\, \sqrt{-c^{2} x^{2}+1}\, \operatorname{EllipticF}\left(x \sqrt{-c^{2}}, i\right) \sqrt{2}\, x}{21 c^{4} \sqrt{-c^{2}}\, \left(c^{4} x^{4}-1\right) \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}-1}}}$	125

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/csch(2*ln(c*x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/42*x^2*(3*c^4*x^4-2)/c^4*2^(1/2)/(c^2*x^2/(c^4*x^4-1))^(1/2)-1/21/c^4/(-c
^2)^(1/2)*(c^2*x^2+1)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^4*x^4-1)*EllipticF(x*(-c^
2)^(1/2),I)*2^(1/2)*x/(c^2*x^2/(c^4*x^4-1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/csch(2*log(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^5/sqrt(csch(2*log(c*x))), x)
```

Fricas [A]
time = 0.10, size = 71, normalized size = 0.88

$$\frac{\sqrt{2}\,\left(3\,c^{10}x^{8}-5\,c^{6}x^{4}+2\,c^{2}\right)\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}-1}}+2\,\sqrt{2}\,\sqrt{c^{4}}\,\operatorname{ellipticF}\left(\frac{1}{cx},-1\right)}{42\,c^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/csch(2*log(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/42*(sqrt(2)*(3*c^10*x^8 - 5*c^6*x^4 + 2*c^2)*sqrt(c^2*x^2/(c^4*x^4 - 1))
+ 2*sqrt(2)*sqrt(c^4)*ellipticF(1/(c*x), -1))/c^8
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5}}{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/csch(2*ln(c*x))**(1/2),x)
```

```
[Out] Integral(x**5/sqrt(csch(2*log(c*x))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/csch(2*log(c*x))^(1/2),x, algorithm="giac")
[Out] integrate(x^5/sqrt(csch(2*log(c*x))), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\sqrt{\frac{1}{\sinh\left(2\,\ln\left(c\,x\right)\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(1/sinh(2*log(c*x)))^(1/2),x)
[Out] int(x^5/(1/sinh(2*log(c*x)))^(1/2), x)
```

3.133 $\int \frac{x^4}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx$

Optimal. Leaf size=30

$$\frac{\left(c^4-\frac{1}{x^4}\right)x^5}{6c^4\sqrt{\operatorname{csch}(2\log(cx))}}$$

```
[Out] 1/6*(c^4-1/x^4)*x^5/c^4/csch(2*ln(c*x))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5671, 5669, 270}

$$\frac{x^5\left(c^4-\frac{1}{x^4}\right)}{6c^4\sqrt{\operatorname{csch}(2\log(cx))}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[Csch[2*Log[c*x]]],x]
```

```
[Out] ((c^4 - x^(-4))*x^5)/(6*c^4*Sqrt[Csch[2*Log[c*x]]])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5669

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5671

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{x^4}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^4}{\sqrt{\operatorname{csch}(2\log(x))}}\,dx, x, cx\right)}{c^5}$$
$$= \frac{\operatorname{Subst}\left(\int \sqrt{1-\frac{1}{x^4}}\, x^5\,dx, x, cx\right)}{c^6\sqrt{1-\frac{1}{c^4x^4}}\, x\sqrt{\operatorname{csch}(2\log(cx))}}$$
$$= \frac{\left(c^4-\frac{1}{x^4}\right)x^5}{6c^4\sqrt{\operatorname{csch}(2\log(cx))}}$$

Mathematica [A]
time = 0.03, size = 44, normalized size = 1.47

$$\frac{(-1+c^4x^4)^2\sqrt{\frac{c^2x^2}{-2+2c^4x^4}}}{6c^6x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[Csch[2*Log[c*x]]],x]
[Out] ((-1 + c^4*x^4)^2*Sqrt[(c^2*x^2)/(-2 + 2*c^4*x^4)])/(6*c^6*x)
```

Maple [A]
time = 0.90, size = 39, normalized size = 1.30

method	result	size
risch	$\frac{\sqrt{2}\,x(c^4x^4-1)}{12\sqrt{\frac{c^2x^2}{c^4x^4-1}}\;c^4}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/csch(2*ln(c*x))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/12*2^(1/2)*x/(c^2*x^2/(c^4*x^4-1))^(1/2)*(c^4*x^4-1)/c^4
```

Maxima [A]
time = 0.52, size = 46, normalized size = 1.53

$$\frac{\left(\sqrt{2}\,c^4x^4-\sqrt{2}\right)\sqrt{c^2x^2+1}\,\sqrt{cx+1}\,\sqrt{cx-1}}{12\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/csch(2*log(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(sqrt(2)*c^4*x^4 - sqrt(2))*sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(c*x -
 1)/c^5
```

Fricas [A]
time = 0.38, size = 48, normalized size = 1.60

$$\frac{\sqrt{2}\,(c^8x^8-2\,c^4x^4+1)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{12\,c^6x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/csch(2*log(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(2)*(c^8*x^8 - 2*c^4*x^4 + 1)*sqrt(c^2*x^2/(c^4*x^4 - 1))/(c^6*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^4}{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/csch(2*ln(c*x))**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(csch(2*log(c*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/csch(2*log(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(csch(2*log(c*x))), x)
```

Mupad [B]
time = 2.15, size = 42, normalized size = 1.40

$$\frac{{\left(c^4\,x^4-1\right)}^2\,\sqrt{\frac{2\,c^2\,x^2}{c^4\,x^4-1}}}{12\,c^6\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(1/sinh(2*log(c*x)))^(1/2),x)
```

```
[Out] ((c^4*x^4 - 1)^2*((2*c^2*x^2)/(c^4*x^4 - 1))^(1/2))/(12*c^6*x)
```

3.134 $\int \frac{x^3}{\sqrt{\text{csch}(2\log(cx))}}\,dx$

Optimal. Leaf size=119

$$-\frac{2}{5c^4\sqrt{\text{csch}(2\log(cx))}}+\frac{x^4}{5\sqrt{\text{csch}(2\log(cx))}}-\frac{2E(\csc^{-1}(cx)|-1)}{5c^5\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\text{csch}(2\log(cx))}}+\frac{2F(\csc^{-1}(cx)|-1)}{5c^5\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\text{csch}(2\log(cx))}}$$

```
[Out] -2/5/c^4/csch(2*ln(c*x))^(1/2)+1/5*x^4/csch(2*ln(c*x))^(1/2)-2/5*EllipticE(
1/c/x,I)/c^5/x/(1-1/c^4/x^4)^(1/2)/csch(2*ln(c*x))^(1/2)+2/5*EllipticF(1/c/
x,I)/c^5/x/(1-1/c^4/x^4)^(1/2)/csch(2*ln(c*x))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5671, 5669, 342, 283, 331, 313, 227, 1195, 435}

$$-\frac{2}{5c^4\sqrt{\text{csch}(2\log(cx))}}+\frac{2F(\csc^{-1}(cx)|-1)}{5c^5x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\text{csch}(2\log(cx))}}-\frac{2E(\csc^{-1}(cx)|-1)}{5c^5x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\text{csch}(2\log(cx))}}+\frac{x^4}{5\sqrt{\text{csch}(2\log(cx))}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[Csch[2*Log[c*x]]],x]
```

```
[Out] -2/(5*c^4*Sqrt[Csch[2*Log[c*x]]]) + x^4/(5*Sqrt[Csch[2*Log[c*x]]]) - (2*Ell
ipticE[ArcCsc[c*x], -1])/(5*c^5*Sqrt[1 - 1/(c^4*x^4)]*x*Sqrt[Csch[2*Log[c*x
]]]) + (2*EllipticF[ArcCsc[c*x], -1])/(5*c^5*Sqrt[1 - 1/(c^4*x^4)]*x*Sqrt[C
sch[2*Log[c*x]]])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
```

```
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[(d + e*x^2)/(Sqrt[q + c*x^2]*Sqrt[q - c
*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 5669

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5671

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{x^3}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^3}{\sqrt{\operatorname{csch}(2\log(x))}}\,dx, x, cx\right)}{c^4}$$

$$= \frac{\operatorname{Subst}\left(\int \sqrt{1-\frac{1}{x^4}}\,x^4\,dx, x, cx\right)}{c^5\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}}$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{\sqrt{1-x^4}}{x^6}\,dx, x, \frac{1}{cx}\right)}{c^5\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}}$$

$$= \frac{x^4}{5\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{2\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)}{5c^5\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}}$$

$$= -\frac{2}{5c^4\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{x^4}{5\sqrt{\operatorname{csch}(2\log(cx))}} - \frac{2\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right.}{5c^5\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(c}$$

$$= -\frac{2}{5c^4\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{x^4}{5\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{2\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right.}{5c^5\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(c}$$

$$= -\frac{2}{5c^4\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{x^4}{5\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{2F(\csc^{-1}(cx)|-1)}{5c^5\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(c}$$

$$= -\frac{2}{5c^4\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{x^4}{5\sqrt{\operatorname{csch}(2\log(cx))}} - \frac{2E(\csc^{-1}(cx)|-1)}{5c^5\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(c}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.08, size = 60, normalized size = 0.50

$$\frac{x^4\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};c^4x^4\right)}{3\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{-1+c^4x^4}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[Csch[2*Log[c*x]]],x]
```

```
[Out] (x^4*Hypergeometric2F1[-1/2, 3/4, 7/4, c^4*x^4])/(3*Sqrt[2 - 2*c^4*x^4]*Sqr
t[(c^2*x^2)/(-1 + c^4*x^4)])
```

Maple [A]
time = 0.99, size = 127, normalized size = 1.07

method	result	size
risch	$\frac{x^{4} \sqrt{2}}{10 \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}-1}}}-\frac{\sqrt{c^{2} x^{2}+1}\, \sqrt{-c^{2} x^{2}+1}\, \left(\operatorname{EllipticF}\left(x \sqrt{-c^{2}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{-c^{2}}, i\right)\right) \sqrt{2}\, x}{5 \sqrt{-c^{2}}\, \left(c^{4} x^{4}-1\right) c^{2} \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}-1}}}$	127

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/csch(2*ln(c*x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*x^4*2^(1/2)/(c^2*x^2/(c^4*x^4-1))^(1/2)-1/5/(-c^2)^(1/2)*(c^2*x^2+1)^(
1/2)*(-c^2*x^2+1)^(1/2)/(c^4*x^4-1)/c^2*(EllipticF(x*(-c^2)^(1/2),I)-Ellipt
icE(x*(-c^2)^(1/2),I))*2^(1/2)*x/(c^2*x^2/(c^4*x^4-1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/csch(2*log(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt(csch(2*log(c*x))), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/csch(2*log(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\sqrt{\operatorname{csch}\left(2 \log \left(cx\right)\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/csch(2*ln(c*x))**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(csch(2*log(c*x))), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/csch(2*log(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\sqrt{\frac{1}{\sinh\left(2\,\ln\left(c\,x\right)\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(1/sinh(2*log(c*x)))^(1/2),x)
```

```
[Out] int(x^3/(1/sinh(2*log(c*x)))^(1/2), x)
```

3.135 $\int \frac{x^2}{\sqrt{\mathrm{csch}(2\log(cx))}}\,dx$

Optimal. Leaf size=69

$$\frac{x^3}{4\sqrt{\mathrm{csch}(2\log(cx))}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)}{4c^4\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\mathrm{csch}(2\log(cx))}}$$

```
[Out] 1/4*x^3/csch(2*ln(c*x))^(1/2)-1/4*arctanh((1-1/c^4/x^4)^(1/2))/c^4/x/(1-1/c
^4/x^4)^(1/2)/csch(2*ln(c*x))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5671, 5669, 272, 43, 65, 212}

$$\frac{x^3}{4\sqrt{\mathrm{csch}(2\log(cx))}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)}{4c^4x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\mathrm{csch}(2\log(cx))}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[Csch[2*Log[c*x]]],x]
```

```
[Out] x^3/(4*Sqrt[Csch[2*Log[c*x]]]) - ArcTanh[Sqrt[1 - 1/(c^4*x^4)]]/(4*c^4*Sqrt
[1 - 1/(c^4*x^4)]*x*Sqrt[Csch[2*Log[c*x]]])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5669

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5671

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{x^2}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{\operatorname{csch}(2\log(x))}}\,dx, x, cx\right)}{c^3}$$

$$= \frac{\operatorname{Subst}\left(\int \sqrt{1-\frac{1}{x^4}}\,x^3\,dx, x, cx\right)}{c^4\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}}$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{\sqrt{1-x}}{x^2}\,dx, x, \frac{1}{c^4x^4}\right)}{4c^4\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}}$$

$$= \frac{x^3}{4\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx, x, \frac{1}{c^4x^4}\right)}{8c^4\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}}$$

$$= \frac{x^3}{4\sqrt{\operatorname{csch}(2\log(cx))}} - \frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-\frac{1}{c^4x^4}}\right)}{4c^4\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}}$$

$$= \frac{x^3}{4\sqrt{\operatorname{csch}(2\log(cx))}} - \frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)}{4c^4\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}}$$

Mathematica [A]
time = 0.10, size = 74, normalized size = 1.07

$$\frac{x\left(c^2x^2\sqrt{1-c^4x^4}+\operatorname{ArcSin}(c^2x^2)\right)}{4c^2\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{-1+c^4x^4}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[Csch[2*Log[c*x]]],x]
```

```
[Out] (x*(c^2*x^2*Sqrt[1 - c^4*x^4] + ArcSin[c^2*x^2]))/(4*c^2*Sqrt[2 - 2*c^4*x^4
]*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)])
```

Maple [A]
time = 1.19, size = 97, normalized size = 1.41

method	result	size
risch	$\frac{x^{3}\sqrt{2}}{8\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}-1}}}-\frac{\ln\left(\frac{c^{4}x^{2}}{\sqrt{c^{4}}}+\sqrt{c^{4}x^{4}-1}\right)\sqrt{2}\,x}{8\sqrt{c^{4}}\,\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}-1}}\,\sqrt{c^{4}x^{4}-1}}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/csch(2*ln(c*x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*x^3*2^(1/2)/(c^2*x^2/(c^4*x^4-1))^(1/2)-1/8*ln(c^4*x^2/(c^4)^(1/2)+(c^4
*x^4-1)^(1/2))/(c^4)^(1/2)*2^(1/2)*x/(c^2*x^2/(c^4*x^4-1))^(1/2)/(c^4*x^4-1
)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/csch(2*log(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(csch(2*log(c*x))), x)
```

Fricas [A]
time = 0.39, size = 92, normalized size = 1.33

$$\frac{2\,\sqrt{2}\,(c^{5}x^{5}-cx)\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}-1}}+\sqrt{2}\,\log\left(2\,c^{4}x^{4}-2\,(c^{5}x^{5}-cx)\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}-1}}-1\right)}{16\,c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/csch(2*log(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(2*sqrt(2)*(c^5*x^5 - c*x)*sqrt(c^2*x^2/(c^4*x^4 - 1)) + sqrt(2)*log(2
*c^4*x^4 - 2*(c^5*x^5 - c*x)*sqrt(c^2*x^2/(c^4*x^4 - 1)) - 1))/c^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{2}}{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/csch(2*ln(c*x))**(1/2),x)
[Out] Integral(x**2/sqrt(csch(2*log(c*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/csch(2*log(c*x))^(1/2),x, algorithm="giac")
[Out] integrate(x^2/sqrt(csch(2*log(c*x))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{\frac{1}{\sinh\left(2\,\ln\left(c\,x\right)\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1/sinh(2*log(c*x)))^(1/2),x)
[Out] int(x^2/(1/sinh(2*log(c*x)))^(1/2), x)
```

3.136 $\int \frac{x}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx$

Optimal. Leaf size=60

$$\frac{x^2}{3\sqrt{\operatorname{csch}(2\log(cx))}}+\frac{2F\left(\csc^{-1}(cx)|-1\right)}{3c^3\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}}$$

```
[Out] 1/3*x^2/csch(2*ln(c*x))^(1/2)+2/3*EllipticF(1/c/x,I)/c^3/x/(1-1/c^4/x^4)^(1
/2)/csch(2*ln(c*x))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5671, 5669, 342, 283, 227}

$$\frac{2F\left(\csc^{-1}(cx)|-1\right)}{3c^3x\sqrt{1-\frac{1}{c^4x^4}}\sqrt{\operatorname{csch}(2\log(cx))}}+\frac{x^2}{3\sqrt{\operatorname{csch}(2\log(cx))}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[Csch[2*Log[c*x]]],x]
```

```
[Out] x^2/(3*Sqrt[Csch[2*Log[c*x]]]) + (2*EllipticF[ArcCsc[c*x], -1])/(3*c^3*Sqrt
[1 - 1/(c^4*x^4)]*x*Sqrt[Csch[2*Log[c*x]]])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5669

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5671

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{x}{\sqrt{\operatorname{csch}(2\log(x))}}\,dx, x, cx\right)}{c^2}\\
&= \frac{\operatorname{Subst}\left(\int \sqrt{1-\frac{1}{x^4}}\,x^2\,dx, x, cx\right)}{c^3\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}}\\
&= -\frac{\operatorname{Subst}\left(\int \frac{\sqrt{1-x^4}}{x^4}\,dx, x, \frac{1}{cx}\right)}{c^3\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}}\\
&= \frac{x^2}{3\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{2\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)}{3c^3\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}}\\
&= \frac{x^2}{3\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{2F\left(\csc^{-1}(cx)\middle|-1\right)}{3c^3\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.08, size = 57, normalized size = 0.95

$$\frac{x^2\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};c^4x^4\right)}{\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{-1+c^4x^4}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[Csch[2*Log[c*x]]],x]
```

```
[Out] (x^2*Hypergeometric2F1[-1/2, 1/4, 5/4, c^4*x^4])/(Sqrt[2 - 2*c^4*x^4]*Sqrt[
(c^2*x^2)/(-1 + c^4*x^4)])
```

Maple [A]
time = 0.96, size = 109, normalized size = 1.82

method	result	size
risch	$\frac{x^{2} \sqrt{2}}{6 \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}-1}}}-\frac{\sqrt{c^{2} x^{2}+1}\, \sqrt{-c^{2} x^{2}+1}\, \mathrm{EllipticF}\left(x \sqrt{-c^{2}}, i\right) \sqrt{2}\, x}{3 \sqrt{-c^{2}}\, \left(c^{4} x^{4}-1\right) \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}-1}}}$	109

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/csch(2*ln(c*x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*x^2*2^(1/2)/(c^2*x^2/(c^4*x^4-1))^(1/2)-1/3/(-c^2)^(1/2)*(c^2*x^2+1)^(1
/2)*(-c^2*x^2+1)^(1/2)/(c^4*x^4-1)*EllipticF(x*(-c^2)^(1/2),I)*2^(1/2)*x/(c
^2*x^2/(c^4*x^4-1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(2*log(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(csch(2*log(c*x))), x)
```

Fricas [A]
time = 0.10, size = 62, normalized size = 1.03

$$\frac{\sqrt{2}\left(c^{6} x^{4}-c^{2}\right) \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}-1}}+2 \sqrt{2} \sqrt{c^{4}}\, \mathrm{ellipticF}\left(\frac{1}{c x},-1\right)}{6\, c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(2*log(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(sqrt(2)*(c^6*x^4 - c^2)*sqrt(c^2*x^2/(c^4*x^4 - 1)) + 2*sqrt(2)*sqrt(c
^4)*ellipticF(1/(c*x), -1))/c^4
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(2*ln(c*x))**(1/2),x)
[Out] Integral(x/sqrt(csch(2*log(c*x))), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(2*log(c*x))^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\sqrt{\frac{1}{\sinh\left(2\,\ln\left(c\,x\right)\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1/sinh(2*log(c*x)))^(1/2),x)
[Out] int(x/(1/sinh(2*log(c*x)))^(1/2), x)
```

3.137 $\int \frac{1}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx$

Optimal. Leaf size=60

$$\frac{x}{2\sqrt{\operatorname{csch}(2\log(cx))}}+\frac{\csc^{-1}(c^2x^2)}{2c^2\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}}$$

```
[Out] 1/2*x/csch(2*ln(c*x))^(1/2)+1/2*arccsc(c^2*x^2)/c^2/x/(1-1/c^4/x^4)^(1/2)/c
sch(2*ln(c*x))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {5665, 5663, 342, 281, 283, 222}

$$\frac{\csc^{-1}(c^2x^2)}{2c^2x\sqrt{1-\frac{1}{c^4x^4}}\,\sqrt{\operatorname{csch}(2\log(cx))}}+\frac{x}{2\sqrt{\operatorname{csch}(2\log(cx))}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Csch[2*Log[c*x]]],x]
```

```
[Out] x/(2*Sqrt[Csch[2*Log[c*x]]]) + ArcCsc[c^2*x^2]/(2*c^2*Sqrt[1 - 1/(c^4*x^4)]
*x*Sqrt[Csch[2*Log[c*x]]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5663

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Dist[Csch[d*(a
+ b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*d*p)), Int[1/(x^(b*
d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; FreeQ[{a, b, d, p}, x] &&
 !IntegerQ[p]
```

Rule 5665

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{1}{\sqrt{\operatorname{csch}(2\log(cx))}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\operatorname{csch}(2\log(x))}}\,dx, x, cx\right)}{c}$$

$$= \frac{\text{Subst}\left(\int \sqrt{1-\frac{1}{x^4}}\,x\,dx, x, cx\right)}{c^2\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}}$$

$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{1-x^4}}{x^3}\,dx, x, \frac{1}{cx}\right)}{c^2\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}}$$

$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{x^2}\,dx, x, \frac{1}{c^2x^2}\right)}{2c^2\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}}$$

$$= \frac{x}{2\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \frac{1}{c^2x^2}\right)}{2c^2\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}}$$

$$= \frac{x}{2\sqrt{\operatorname{csch}(2\log(cx))}} + \frac{\csc^{-1}\left(c^2x^2\right)}{2c^2\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}}$$

Mathematica [**A**]
time = 0.07, size = 77, normalized size = 1.28

$$\frac{x\left(2\sqrt{-1+c^4x^4} - 2\text{ArcTan}\left(\sqrt{-1+c^4x^4}\right)\right)}{4\sqrt{2}\sqrt{\frac{c^2x^2}{-1+c^4x^4}}\sqrt{-1+c^4x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Csch[2*Log[c*x]]],x]
```

```
[Out] (x*(2*Sqrt[-1 + c^4*x^4] - 2*ArcTan[Sqrt[-1 + c^4*x^4]]))/(4*Sqrt[2]*Sqrt[(
c^2*x^2)/(-1 + c^4*x^4)]*Sqrt[-1 + c^4*x^4])
```

Maple [**F**]
time = 0.76, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\operatorname{csch}\left(2\ln\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/csch(2*ln(c*x))^(1/2),x)
[Out] int(1/csch(2*ln(c*x))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(2*log(c*x))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(csch(2*log(c*x))), x)
```

Fricas [A]
time = 0.48, size = 86, normalized size = 1.43

$$-\frac{\sqrt{2}\,cx\arctan\left(\frac{(c^4x^4-1)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{cx}\right)-\sqrt{2}\,(c^4x^4-1)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{4\,c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(2*log(c*x))^(1/2),x, algorithm="fricas")
[Out] -1/4*(sqrt(2)*c*x*arctan((c^4*x^4 - 1)*sqrt(c^2*x^2/(c^4*x^4 - 1))/(c*x)) -
 sqrt(2)*(c^4*x^4 - 1)*sqrt(c^2*x^2/(c^4*x^4 - 1)))/(c^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(2*ln(c*x))**(1/2),x)
[Out] Integral(1/sqrt(csch(2*log(c*x))), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(2*log(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\frac{1}{\sinh\left(2\,\ln\left(c\,x\right)\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/sinh(2*log(c*x)))^(1/2),x)
```

```
[Out] int(1/(1/sinh(2*log(c*x)))^(1/2), x)
```

3.138 $\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x}\,dx$

Optimal. Leaf size=46

$$i\sqrt{\operatorname{csch}(2\log(cx))}\,F\left(\frac{\pi}{4}-i\log(cx)\middle|2\right)\sqrt{i\sinh(2\log(cx))}$$

```
[Out] I*(sin(1/4*Pi+I*ln(c*x))^2)^(1/2)/sin(1/4*Pi+I*ln(c*x))*EllipticF(cos(1/4*P
i+I*ln(c*x)),2^(1/2))*csch(2*ln(c*x))^(1/2)*(I*sinh(2*ln(c*x)))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3856, 2720}

$$i\sqrt{i\sinh(2\log(cx))}\sqrt{\operatorname{csch}(2\log(cx))}\,F\left(\frac{\pi}{4}-i\log(cx)\middle|2\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Csch[2*Log[c*x]]]/x,x]
```

```
[Out] I*Sqrt[Csch[2*Log[c*x]]]*EllipticF[Pi/4 - I*Log[c*x], 2]*Sqrt[I*Sinh[2*Log[
c*x]]]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x}\,dx &= \operatorname{Subst}\left(\int \sqrt{\operatorname{csch}(2x)}\,dx, x, \log(cx)\right)\\
&= \left(\sqrt{\operatorname{csch}(2\log(cx))}\sqrt{i\sinh(2\log(cx))}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{i\sinh(2x)}}\,dx, x, \log(cx)\right)\\
&= i\sqrt{\operatorname{csch}(2\log(cx))}\,F\left(\frac{\pi}{4}-i\log(cx)\middle|2\right)\sqrt{i\sinh(2\log(cx))}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 43, normalized size = 0.93

$$\operatorname{csch}^{\frac{3}{2}}(2\log(cx))F\left(\frac{\pi}{4}-i\log(cx)\middle|2\right)(i\sinh(2\log(cx)))^{3/2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Csch[2*Log[c*x]]]/x,x]

[Out] Csch[2*Log[c*x]]^(3/2)*EllipticF[Pi/4 - I*Log[c*x], 2]*(I*Sinh[2*Log[c*x]])^(3/2)

Maple [A]
time = 2.91, size = 90, normalized size = 1.96

method	result
derivativedivides	$\frac{i\sqrt{-i(\sinh(2\ln(cx))+i)}\,\sqrt{2}\,\sqrt{-i(-\sinh(2\ln(cx))+i)}\,\sqrt{i\sinh(2\ln(cx))}\,\mathrm{EllipticF}(}{2\cosh(2\ln(cx))\sqrt{\sinh(2\ln(cx))}}$
default	$\frac{i\sqrt{-i(\sinh(2\ln(cx))+i)}\,\sqrt{2}\,\sqrt{-i(-\sinh(2\ln(cx))+i)}\,\sqrt{i\sinh(2\ln(cx))}\,\mathrm{EllipticF}(}{2\cosh(2\ln(cx))\sqrt{\sinh(2\ln(cx))}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(2*ln(c*x))^(1/2)/x,x,method=_RETURNVERBOSE)

[Out] 1/2*I*(-I*(sinh(2*ln(c*x))+I))^(1/2)*2^(1/2)*(-I*(-sinh(2*ln(c*x))+I))^(1/2)*(I*sinh(2*ln(c*x)))^(1/2)*EllipticF((-I*(sinh(2*ln(c*x))+I))^(1/2),1/2*2^(1/2))/cosh(2*ln(c*x))/sinh(2*ln(c*x))^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(2*log(c*x))^(1/2)/x,x, algorithm="maxima")

[Out] integrate(sqrt(csch(2*log(c*x)))/x, x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 0
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*ln(c*x))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(csch(2*log(c*x)))/x, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(1/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\frac{1}{\sinh\left(2\,\ln\left(c\,x\right)\right)}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sinh(2*log(c*x)))^(1/2)/x,x)
```

```
[Out] int((1/sinh(2*log(c*x)))^(1/2)/x, x)
```

3.139 $\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^2}\,dx$

Optimal. Leaf size=41

$$-\frac{1}{2}c^2\sqrt{1-\frac{1}{c^4x^4}}\,x\csc^{-1}\left(c^2x^2\right)\sqrt{\operatorname{csch}(2\log(cx))}$$

[Out] -1/2*c^2*x*arccsc(c^2*x^2)*(1-1/c^4/x^4)^(1/2)*csch(2*ln(c*x))^(1/2)

Rubi [A]
time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5671, 5669, 342, 281, 222}

$$-\frac{1}{2}c^2x\sqrt{1-\frac{1}{c^4x^4}}\csc^{-1}\left(c^2x^2\right)\sqrt{\operatorname{csch}(2\log(cx))}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Csch[2*Log[c*x]]]/x^2,x]

[Out] -1/2*(c^2*Sqrt[1 - 1/(c^4*x^4)]*x*ArcCsc[c^2*x^2]*Sqrt[Csch[2*Log[c*x]]])

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5669

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5671

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^2}\,dx &= c\text{Subst}\left(\int \frac{\sqrt{\operatorname{csch}(2\log(x))}}{x^2}\,dx, x, cx\right)\\
&= \left(c^2\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{1}{x^4}}\,x^3}\,dx, x, cx\right)\\
&= -\left(\left(c^2\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}\right)\text{Subst}\left(\int \frac{x}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)\right)\\
&= -\left(\frac{1}{2}\left(c^2\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \frac{1}{c^2x^2}\right)\right)\\
&= -\frac{1}{2}c^2\sqrt{1-\frac{1}{c^4x^4}}\,x\csc^{-1}\left(c^2x^2\right)\sqrt{\operatorname{csch}(2\log(cx))}
\end{aligned}$$

Mathematica [**A**]
time = 0.08, size = 54, normalized size = 1.32

$$\frac{\sqrt{-1+c^4x^4}\sqrt{\frac{c^2x^2}{-2+2c^4x^4}}\,\text{ArcTan}\left(\sqrt{-1+c^4x^4}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Csch[2*Log[c*x]]]/x^2,x]
```

```
[Out] (Sqrt[-1 + c^4*x^4]*Sqrt[(c^2*x^2)/(-2 + 2*c^4*x^4)]*ArcTan[Sqrt[-1 + c^4*x
^4]])/x
```

Maple [**F**]
time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{csch}\left(2\ln\left(cx\right)\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(2*ln(c*x))^(1/2)/x^2,x)
```

```
[Out] int(csch(2*ln(c*x))^(1/2)/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(csch(2*log(c*x)))/x^2, x)
```

Fricas [A]
time = 0.43, size = 43, normalized size = 1.05

$$\frac{1}{2}\sqrt{2}\,c\arctan\left(\frac{(c^4x^4-1)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{cx}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*c*arctan((c^4*x^4 - 1)*sqrt(c^2*x^2/(c^4*x^4 - 1))/(c*x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*ln(c*x))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(csch(2*log(c*x)))/x**2, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\frac{1}{\sinh\left(2\,\ln\left(c\,x\right)\right)}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sinh(2*log(c*x)))^(1/2)/x^2,x)
[Out] int((1/sinh(2*log(c*x)))^(1/2)/x^2, x)
```

3.140 $\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^3}\,dx$

Optimal. Leaf size=74

$$-c^3\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}\,E\left(\csc^{-1}(cx)\middle|-1\right)+c^3\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}\,F\left(\csc^{-1}(cx)\middle|-1\right)$$

```
[Out] -c^3*x*EllipticE(1/c/x,I)*(1-1/c^4/x^4)^(1/2)*csch(2*ln(c*x))^(1/2)+c^3*x*E
llipticF(1/c/x,I)*(1-1/c^4/x^4)^(1/2)*csch(2*ln(c*x))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {5671, 5669, 342, 313, 227, 1195, 435}

$$c^3x\sqrt{1-\frac{1}{c^4x^4}}\,F\left(\csc^{-1}(cx)\middle|-1\right)\sqrt{\operatorname{csch}(2\log(cx))}-c^3x\sqrt{1-\frac{1}{c^4x^4}}\,E\left(\csc^{-1}(cx)\middle|-1\right)\sqrt{\operatorname{csch}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Csch[2*Log[c*x]]]/x^3,x]
```

```
[Out] -(c^3*Sqrt[1 - 1/(c^4*x^4)]*x*Sqrt[Csch[2*Log[c*x]]]*EllipticE[ArcCsc[c*x],
 -1]) + c^3*Sqrt[1 - 1/(c^4*x^4)]*x*Sqrt[Csch[2*Log[c*x]]]*EllipticF[ArcCsc
[c*x], -1]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
```

```
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[(d + e*x^2)/(Sqrt[q + c*x^2]*Sqrt[q - c
*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 5669

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5671

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^3}\,dx &= c^2\text{Subst}\left(\int \frac{\sqrt{\operatorname{csch}(2\log(x))}}{x^3}\,dx, x, cx\right)\\
&= \left(c^3\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{1}{x^4}}\,x^4}\,dx, x, cx\right)\\
&= -\left(\left(c^3\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)\right)\\
&= \left(c^3\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right) - \Bigg(c^3\sqrt{1\\
&= c^3\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}\,F\left(\csc^{-1}(cx)\middle|-1\right) - \Bigg(c^3\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}\\
&= -c^3\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}\,E\left(\csc^{-1}(cx)\middle|-1\right) + c^3\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 0.07, size = 58, normalized size = 0.78

$$-\frac{\sqrt{2-2c^4x^4}\,\sqrt{\frac{c^2x^2}{-1+c^4x^4}}\;{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};c^4x^4\right)}{x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Csch[2*Log[c*x]]]/x^3,x]
```

```
[Out] -((Sqrt[2 - 2*c^4*x^4]*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)]*Hypergeometric2F1[-1/
4, 1/2, 3/4, c^4*x^4])/x^2)
```

Maple [A]
time = 0.89, size = 126, normalized size = 1.70

method	result
risch	$\frac{(c^4x^4-1)\sqrt{2}\,\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{x^2}-\frac{c^2\sqrt{c^2x^2+1}\,\sqrt{-c^2x^2+1}\,\left(\text{EllipticF}\left(x\sqrt{-c^2},i\right)-\text{EllipticE}\left(x\sqrt{-c^2},i\right)\right)\sqrt{2}\,\sqrt{}}{\sqrt{-c^2}\,x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(2*ln(c*x))^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] (c^4*x^4-1)/x^2*2^(1/2)*(c^2*x^2/(c^4*x^4-1))^(1/2)-c^2/(-c^2)^(1/2)*(c^2*x
^2+1)^(1/2)*(-c^2*x^2+1)^(1/2)*(EllipticF(x*(-c^2)^(1/2),I)-EllipticE(x*(-c
^2)^(1/2),I))*2^(1/2)*(c^2*x^2/(c^4*x^4-1))^(1/2)/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(csch(2*log(c*x)))/x^3, x)
```

Fricas [A]
time = 0.11, size = 36, normalized size = 0.49

$$\frac{\sqrt{2}\,(c^4x^4-1)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] sqrt(2)*(c^4*x^4 - 1)*sqrt(c^2*x^2/(c^4*x^4 - 1))/x^2
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*ln(c*x))**(1/2)/x**3,x)
[Out] Integral(sqrt(csch(2*log(c*x)))/x**3, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^3,x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1}{\sinh\left(2\,\ln\left(c\,x\right)\right)}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sinh(2*log(c*x)))^(1/2)/x^3,x)
[Out] int((1/sinh(2*log(c*x)))^(1/2)/x^3, x)
```

3.141 $\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^4} dx$

Optimal. Leaf size=25

$$\frac{1}{2}\left(c^4-\frac{1}{x^4}\right)x\sqrt{\operatorname{csch}(2\log(cx))}$$

```
[Out] 1/2*(c^4-1/x^4)*x*csch(2*ln(c*x))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5671, 5669, 267}

$$\frac{1}{2}x\left(c^4-\frac{1}{x^4}\right)\sqrt{\operatorname{csch}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Csch[2*Log[c*x]]]/x^4,x]
```

```
[Out] ((c^4 - x^(-4))*x*Sqrt[Csch[2*Log[c*x]]])/2
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5669

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5671

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^4}\,dx = c^3\text{Subst}\left(\int \frac{\sqrt{\operatorname{csch}(2\log(x))}}{x^4}\,dx, x, cx\right)$$
$$= \left(c^4\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{1}{x^4}}\,x^5}\,dx, x, cx\right)$$
$$= \frac{1}{2}\left(c^4-\frac{1}{x^4}\right)x\sqrt{\operatorname{csch}(2\log(cx))}$$

Mathematica [A]
time = 0.03, size = 33, normalized size = 1.32

$$\frac{c^2}{2x\sqrt{\frac{c^2x^2}{-2+2c^4x^4}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Csch[2*Log[c*x]]]/x^4,x]
```

```
[Out] c^2/(2*x*Sqrt[(c^2*x^2)/(-2 + 2*c^4*x^4)])
```

Maple [A]
time = 0.82, size = 38, normalized size = 1.52

method	result	size
risch	$\frac{\sqrt{2}\,\sqrt{\frac{c^2x^2}{c^4x^4-1}}\,\left(c^4x^4-1\right)}{2x^3}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(2*ln(c*x))^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*2^(1/2)*(c^2*x^2/(c^4*x^4-1))^(1/2)/x^3*(c^4*x^4-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(21) = 42$.
time = 0.48, size = 89, normalized size = 3.56

$$\frac{1}{2}c^3\left(\frac{\sqrt{2}}{\sqrt{\frac{1}{cx}+1}\,\sqrt{-\frac{1}{cx}+1}\,\sqrt{\frac{1}{c^2x^2}+1}} - \frac{\sqrt{2}}{c^4x^4\sqrt{\frac{1}{cx}+1}\,\sqrt{-\frac{1}{cx}+1}\,\sqrt{\frac{1}{c^2x^2}+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/2*c^3*(sqrt(2)/(sqrt(1/(c*x) + 1)*sqrt(-1/(c*x) + 1)*sqrt(1/(c^2*x^2) + 1
)) - sqrt(2)/(c^4*x^4*sqrt(1/(c*x) + 1)*sqrt(-1/(c*x) + 1)*sqrt(1/(c^2*x^2)
 + 1)))
```

Fricas [A]
time = 0.39, size = 37, normalized size = 1.48

$$\frac{\sqrt{2}\,(c^4x^4-1)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*(c^4*x^4 - 1)*sqrt(c^2*x^2/(c^4*x^4 - 1))/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*ln(c*x))**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(csch(2*log(c*x)))/x**4, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 1.47, size = 58, normalized size = 2.32

$$\frac{c^4\,x\,\sqrt{\frac{2\,c^2\,x^2}{c^4\,x^4-1}}}{2}-\frac{\sqrt{\frac{2\,c^2\,x^2}{c^4\,x^4-1}}}{2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sinh(2*log(c*x)))^(1/2)/x^4,x)
```

```
[Out] (c^4*x*((2*c^2*x^2)/(c^4*x^4 - 1))^(1/2))/2 - ((2*c^2*x^2)/(c^4*x^4 - 1))^(
1/2)/(2*x^3)
```

3.142 $\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^5}\,dx$

Optimal. Leaf size=64

$$\frac{1}{3}\left(c^4-\frac{1}{x^4}\right)\sqrt{\operatorname{csch}(2\log(cx))}-\frac{1}{3}c^5\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}\,F\left(\csc^{-1}(cx)\middle|-1\right)$$

```
[Out] 1/3*(c^4-1/x^4)*csch(2*ln(c*x))^(1/2)-1/3*c^5*x*EllipticF(1/c/x,I)*(1-1/c^4
/x^4)^(1/2)*csch(2*ln(c*x))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5671, 5669, 342, 327, 227}

$$\frac{1}{3}\left(c^4-\frac{1}{x^4}\right)\sqrt{\operatorname{csch}(2\log(cx))}-\frac{1}{3}c^5x\sqrt{1-\frac{1}{c^4x^4}}\,F\left(\csc^{-1}(cx)\middle|-1\right)\sqrt{\operatorname{csch}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Csch[2*Log[c*x]]]/x^5,x]
```

```
[Out] ((c^4 - x^(-4))*Sqrt[Csch[2*Log[c*x]]])/3 - (c^5*Sqrt[1 - 1/(c^4*x^4)]*x*Sq
rt[Csch[2*Log[c*x]]]*EllipticF[ArcCsc[c*x], -1])/3
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5669

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5671

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt{\operatorname{csch}(2\log(cx))}}{x^5}\,dx = c^4\text{Subst}\left(\int \frac{\sqrt{\operatorname{csch}(2\log(x))}}{x^5}\,dx, x, cx\right)$$
$$= \left(c^5\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{1}{x^4}}\,x^6}\,dx, x, cx\right)$$
$$= -\left(\left(c^5\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{1-x^4}}\,dx, x, \frac{1}{cx}\right)\right)$$
$$= \frac{1}{3}\left(c^4-\frac{1}{x^4}\right)\sqrt{\operatorname{csch}(2\log(cx))} - \frac{1}{3}\left(c^5\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}\right)\text{Subst}\Big($$
$$= \frac{1}{3}\left(c^4-\frac{1}{x^4}\right)\sqrt{\operatorname{csch}(2\log(cx))} - \frac{1}{3}c^5\sqrt{1-\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{csch}(2\log(cx))}\,F\big(\csc^{-1}(c$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.07, size = 60, normalized size = 0.94

$$-\frac{\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{-1+c^4x^4}}\;{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};c^4x^4\right)}{3x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Csch[2*Log[c*x]]]/x^5,x]
```

```
[Out] -1/3*(Sqrt[2 - 2*c^4*x^4]*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)]*Hypergeometric2F1[
-3/4, 1/2, 1/4, c^4*x^4])/x^4
```

Maple [A]
time = 0.89, size = 112, normalized size = 1.75

method	result	size
risch	$\frac{\left(c^{4} x^{4}-1\right) \sqrt{2}\, \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}-1}}}{3 x^{4}}+\frac{c^{4} \sqrt{c^{2} x^{2}+1}\, \sqrt{-c^{2} x^{2}+1}\, \operatorname{EllipticF}\left(x \sqrt{-c^{2}}, i\right) \sqrt{2}\, \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}-1}}}{3 \sqrt{-c^{2}}\, x}$	112

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(2*ln(c*x))^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(c^4*x^4-1)/x^4*2^(1/2)*(c^2*x^2/(c^4*x^4-1))^(1/2)+1/3*c^4/(-c^2)^(1/2
)*(c^2*x^2+1)^(1/2)*(-c^2*x^2+1)^(1/2)*EllipticF(x*(-c^2)^(1/2),I)*2^(1/2)*
(c^2*x^2/(c^4*x^4-1))^(1/2)/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(csch(2*log(c*x)))/x^5, x)
```

Fricas [A]
time = 0.08, size = 37, normalized size = 0.58

$$\frac{\sqrt{2}\,\left(c^{4} x^{4}-1\right)\sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}-1}}}{3\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(2)*(c^4*x^4 - 1)*sqrt(c^2*x^2/(c^4*x^4 - 1))/x^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{csch}\left(2\log\left(cx\right)\right)}}{x^{5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*ln(c*x))**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(csch(2*log(c*x)))/x**5, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(1/2)/x^5,x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\frac{1}{\sinh\left(2\,\ln\left(c\,x\right)\right)}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sinh(2*log(c*x)))^(1/2)/x^5,x)
[Out] int((1/sinh(2*log(c*x)))^(1/2)/x^5, x)
```

3.143 $\int \frac{x^8}{\mathrm{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=128

$$\frac{x}{32c^4\left(c^4-\frac{1}{x^4}\right)\mathrm{csch}^{\frac{3}{2}}(2\log(cx))}-\frac{x^5}{16\left(c^4-\frac{1}{x^4}\right)\mathrm{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x^9}{12\mathrm{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{\tanh^{-1}\left(\sqrt{1-\right.}{32c^{12}\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\mathrm{c}}$$

```
[Out] 1/32*x/c^4/(c^4-1/x^4)/csch(2*ln(c*x))^(3/2)-1/16*x^5/(c^4-1/x^4)/csch(2*ln
(c*x))^(3/2)+1/12*x^9/csch(2*ln(c*x))^(3/2)+1/32*arctanh((1-1/c^4/x^4)^(1/2
))/c^12/(1-1/c^4/x^4)^(3/2)/x^3/csch(2*ln(c*x))^(3/2)
```

Rubi [A]
time = 0.06, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {5671, 5669, 272, 43, 44, 65, 212}

$$\frac{x}{32c^4\left(c^4-\frac{1}{x^4}\right)\mathrm{csch}^{\frac{3}{2}}(2\log(cx))}-\frac{x^5}{16\left(c^4-\frac{1}{x^4}\right)\mathrm{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)}{32c^{12}x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\mathrm{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x^9}{12\mathrm{csch}^{\frac{3}{2}}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] x/(32*c^4*(c^4 - x^(-4))*Csch[2*Log[c*x]]^(3/2)) - x^5/(16*(c^4 - x^(-4))*C
sch[2*Log[c*x]]^(3/2)) + x^9/(12*Csch[2*Log[c*x]]^(3/2)) + ArcTanh[Sqrt[1 -
 1/(c^4*x^4)]]/(32*c^12*(1 - 1/(c^4*x^4))^(3/2)*x^3*Csch[2*Log[c*x]]^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5669

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5671

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{x^8}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\text{Subst}\left(\int \frac{x^8}{\operatorname{csch}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^9}$$

$$= \frac{\text{Subst}\left(\int \left(1-\frac{1}{x^4}\right)^{3/2} x^{11}\,dx, x, cx\right)}{c^{12}\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{\text{Subst}\left(\int \frac{(1-x)^{3/2}}{x^4}\,dx, x, \frac{1}{c^4x^4}\right)}{4c^{12}\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x^9}{12\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{\text{Subst}\left(\int \frac{\sqrt{1-x}}{x^3}\,dx, x, \frac{1}{c^4x^4}\right)}{8c^{12}\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{x^5}{16\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^9}{12\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x^2}\,dx\right.}{32c^{12}\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}}$$

$$= \frac{x}{32c^4\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{x^5}{16\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^9}{12\operatorname{csch}^{\frac{3}{2}}(2\log(c}$$

$$= \frac{x}{32c^4\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{x^5}{16\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^9}{12\operatorname{csch}^{\frac{3}{2}}(2\log(c}$$

$$= \frac{x}{32c^4\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{x^5}{16\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^9}{12\operatorname{csch}^{\frac{3}{2}}(2\log(c}$$

__Mathematica__ [A]
time = 0.14, size = 95, normalized size = 0.74

$$\frac{c^3x^3\sqrt{1-c^4x^4}\,(3-14c^4x^4+8c^8x^8)-3cx\text{ArcSin}(c^2x^2)}{192c^9\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{-1+c^4x^4}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/Csch[2*Log[c*x]]^(3/2),x]
[Out] (c^3*x^3*Sqrt[1 - c^4*x^4]*(3 - 14*c^4*x^4 + 8*c^8*x^8) - 3*c*x*ArcSin[c^2*
x^2])/(192*c^9*Sqrt[2 - 2*c^4*x^4]*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)])
```

Maple [A]
time = 1.19, size = 121, normalized size = 0.95

method	result	size
risch	$\frac{x^{3}\left(8c^{8}x^{8}-14c^{4}x^{4}+3\right)\sqrt{2}}{384c^{6}\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}-1}}}+\frac{\ln\left(\frac{c^{4}x^{2}}{\sqrt{c^{4}}}+\sqrt{c^{4}x^{4}-1}\right)\sqrt{2}\,x}{128c^{6}\sqrt{c^{4}}\,\sqrt{c^{4}x^{4}-1}\,\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}-1}}}$	121

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/csch(2*ln(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/384*x^3*(8*c^8*x^8-14*c^4*x^4+3)/c^6*2^(1/2)/(c^2*x^2/(c^4*x^4-1))^(1/2)+
1/128/c^6*ln(c^4*x^2/(c^4)^(1/2)+(c^4*x^4-1)^(1/2))/(c^4)^(1/2)*2^(1/2)*x/(
c^4*x^4-1)^(1/2)/(c^2*x^2/(c^4*x^4-1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/csch(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^8/csch(2*log(c*x))^(3/2), x)
```

Fricas [A]
time = 0.39, size = 110, normalized size = 0.86

$$\frac{2\sqrt{2}\left(8\,c^{13}x^{13}-22\,c^{9}x^{9}+17\,c^{5}x^{5}-3\,cx\right)\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}-1}}+3\sqrt{2}\,\log\left(2\,c^{4}x^{4}+2\left(c^{5}x^{5}-cx\right)\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}-1}}-1\right)}{768\,c^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/csch(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/768*(2*sqrt(2)*(8*c^13*x^13 - 22*c^9*x^9 + 17*c^5*x^5 - 3*c*x)*sqrt(c^2*x
^2/(c^4*x^4 - 1)) + 3*sqrt(2)*log(2*c^4*x^4 + 2*(c^5*x^5 - c*x)*sqrt(c^2*x^
2/(c^4*x^4 - 1)) - 1))/c^9
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{8}}{\operatorname{csch}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/csch(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(x**8/csch(2*log(c*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/csch(2*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^8/csch(2*log(c*x))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^8}{{\left(\frac{1}{\mathrm{sinh}\left(2\,\ln\left(c\,x\right)\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(1/sinh(2*log(c*x)))^(3/2),x)
```

```
[Out] int(x^8/(1/sinh(2*log(c*x)))^(3/2), x)
```

3.144 $\int \frac{x^7}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=118

$$\frac{4}{77c^4\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}-\frac{6x^4}{77\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x^8}{11\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}-\frac{4F(\csc^{-1}(cx)|}{77c^{11}\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{csc}}$$

```
[Out] 4/77/c^4/(c^4-1/x^4)/csch(2*ln(c*x))^(3/2)-6/77*x^4/(c^4-1/x^4)/csch(2*ln(c
*x))^(3/2)+1/11*x^8/csch(2*ln(c*x))^(3/2)-4/77*EllipticF(1/c/x,I)/c^11/(1-1
/c^4/x^4)^(3/2)/x^3/csch(2*ln(c*x))^(3/2)
```

Rubi [A]
time = 0.06, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5671, 5669, 342, 283, 331, 227}

$$-\frac{6x^4}{77\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{4}{77c^4\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}-\frac{4F(\csc^{-1}(cx)|-1)}{77c^{11}x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x^8}{11\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] 4/(77*c^4*(c^4 - x^(-4))*Csch[2*Log[c*x]]^(3/2)) - (6*x^4)/(77*(c^4 - x^(-4
))*Csch[2*Log[c*x]]^(3/2)) + x^8/(11*Csch[2*Log[c*x]]^(3/2)) - (4*EllipticF
[ArcCsc[c*x], -1])/(77*c^11*(1 - 1/(c^4*x^4))^(3/2)*x^3*Csch[2*Log[c*x]]^(3
/2))
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5669

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5671

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{x^7}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^7}{\operatorname{csch}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^8}$$

$$= \frac{\operatorname{Subst}\left(\int \left(1-\frac{1}{x^4}\right)^{3/2} x^{10}\,dx, x, cx\right)}{c^{11}\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{(1-x^4)^{3/2}}{x^{12}}\,dx, x, \frac{1}{cx}\right)}{c^{11}\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x^8}{11\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{6\operatorname{Subst}\left(\int \frac{\sqrt{1-x^4}}{x^8}\,dx, x, \frac{1}{cx}\right)}{11c^{11}\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{6x^4}{77\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^8}{11\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{12\operatorname{Subst}\left(\int \frac{1}{x^4\sqrt{1-x^4}}\,d\right.}{77c^{11}\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}}$$

$$= \frac{4}{77c^4\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{6x^4}{77\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^8}{11\operatorname{csch}^{\frac{3}{2}}(2\log(cx}$$

$$= \frac{4}{77c^4\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{6x^4}{77\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^8}{11\operatorname{csch}^{\frac{3}{2}}(2\log(cx}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.12, size = 80, normalized size = 0.68

$$\frac{x^2\left(\left(1-c^4x^4\right)^{5/2} - {}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};c^4x^4\right)\right)}{22c^6\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{-1+c^4x^4}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] (x^2*((1 - c^4*x^4)^(5/2) - Hypergeometric2F1[-3/2, 1/4, 5/4, c^4*x^4]))/(2
2*c^6*Sqrt[2 - 2*c^4*x^4]*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)])
```

Maple [A]
time = 0.97, size = 133, normalized size = 1.13

method	result	size
risch	$\frac{x^{2}\left(7 c^{8} x^{8}-13 c^{4} x^{4}+4\right) \sqrt{2}}{308 c^{6} \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}-1}}}+\frac{\sqrt{c^{2} x^{2}+1}\, \sqrt{-c^{2} x^{2}+1}\, \operatorname{EllipticF}\left(x \sqrt{-c^{2}}, i\right) \sqrt{2}\, x}{77 c^{6} \sqrt{-c^{2}}\, \left(c^{4} x^{4}-1\right) \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}-1}}}$	133

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/csch(2*ln(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/308*x^2*(7*c^8*x^8-13*c^4*x^4+4)/c^6*2^(1/2)/(c^2*x^2/(c^4*x^4-1))^(1/2)+
1/77/c^6/(-c^2)^(1/2)*(c^2*x^2+1)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^4*x^4-1)*Elli
pticF(x*(-c^2)^(1/2),I)*2^(1/2)*x/(c^2*x^2/(c^4*x^4-1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/csch(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^7/csch(2*log(c*x))^(3/2), x)
```

Fricas [A]
time = 0.10, size = 79, normalized size = 0.67

$$\frac{\sqrt{2}\left(7 c^{14} x^{12}-20 c^{10} x^{8}+17 c^{6} x^{4}-4 c^{2}\right) \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}-1}}-4 \sqrt{2} \sqrt{c^{4}} \operatorname{ellipticF}\left(\frac{1}{c x},-1\right)}{308 c^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/csch(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/308*(sqrt(2)*(7*c^14*x^12 - 20*c^10*x^8 + 17*c^6*x^4 - 4*c^2)*sqrt(c^2*x^
2/(c^4*x^4 - 1)) - 4*sqrt(2)*sqrt(c^4)*ellipticF(1/(c*x), -1))/c^10
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{7}}{\operatorname{csch}^{\frac{3}{2}}\left(2 \log\left(cx\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/csch(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(x**7/csch(2*log(c*x))**(3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/csch(2*log(c*x))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7}{\left(\frac{1}{\sinh(2\,\ln(c\,x))}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(1/sinh(2*log(c*x)))^(3/2),x)
[Out] int(x^7/(1/sinh(2*log(c*x)))^(3/2), x)
```

3.145 $\int \frac{x^6}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=30

$$\frac{\left(c^4-\frac{1}{x^4}\right)x^7}{10c^4\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

[Out] 1/10*(c^4-1/x^4)*x^7/c^4/csch(2*ln(c*x))^(3/2)

Rubi [A]
time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5671, 5669, 270}

$$\frac{x^7\left(c^4-\frac{1}{x^4}\right)}{10c^4\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

Antiderivative was successfully verified.

[In] Int[x^6/Csch[2*Log[c*x]]^(3/2),x]

[Out] ((c^4 - x^(-4))*x^7)/(10*c^4*Csch[2*Log[c*x]]^(3/2))

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5669

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5671

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{x^6}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^6}{\operatorname{csch}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^7}$$
$$= \frac{\operatorname{Subst}\left(\int \left(1-\frac{1}{x^4}\right)^{3/2} x^9\,dx, x, cx\right)}{c^{10}\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$
$$= \frac{\left(c^4-\frac{1}{x^4}\right)x^7}{10c^4\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

Mathematica [A]
time = 0.03, size = 44, normalized size = 1.47

$$\frac{(-1+c^4x^4)^3\sqrt{\frac{c^2x^2}{-2+2c^4x^4}}}{20c^8x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/Csch[2*Log[c*x]]^(3/2),x]
[Out] ((-1 + c^4*x^4)^3*Sqrt[(c^2*x^2)/(-2 + 2*c^4*x^4)])/(20*c^8*x)
```

Maple [A]
time = 0.89, size = 47, normalized size = 1.57

method	result	size
risch	$\frac{\sqrt{2}\, x\left(c^8x^8-2c^4x^4+1\right)}{40c^6\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/csch(2*ln(c*x))^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/40*2^(1/2)/c^6*x/(c^2*x^2/(c^4*x^4-1))^(1/2)*(c^8*x^8-2*c^4*x^4+1)
```

Maxima [A]
time = 0.51, size = 46, normalized size = 1.53

$$\frac{\left(\sqrt{2}\,c^4x^4-\sqrt{2}\right)(c^2x^2+1)^{\frac{3}{2}}(cx+1)^{\frac{3}{2}}(cx-1)^{\frac{3}{2}}}{40\,c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/csch(2*log(c*x))^(3/2),x, algorithm="maxima")

[Out] 1/40*(sqrt(2)*c^4*x^4 - sqrt(2))*(c^2*x^2 + 1)^(3/2)*(c*x + 1)^(3/2)*(c*x - 1)^(3/2)/c^7

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(26) = 52$.
time = 0.45, size = 56, normalized size = 1.87

$$\frac{\sqrt{2}\,(c^{12}x^{12} - 3\,c^8x^8 + 3\,c^4x^4 - 1)\sqrt{\frac{c^2x^2}{c^4x^4 - 1}}}{40\,c^8x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/csch(2*log(c*x))^(3/2),x, algorithm="fricas")

[Out] 1/40*sqrt(2)*(c^12*x^12 - 3*c^8*x^8 + 3*c^4*x^4 - 1)*sqrt(c^2*x^2/(c^4*x^4 - 1))/(c^8*x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\operatorname{csch}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6/csch(2*ln(c*x))**(3/2),x)

[Out] Integral(x**6/csch(2*log(c*x))**(3/2), x)

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/csch(2*log(c*x))^(3/2),x, algorithm="giac")

[Out] Timed out

Mupad [B]
time = 1.56, size = 42, normalized size = 1.40

$$\frac{{\left(c^4\,x^4-1\right)}^3\,\sqrt{\frac{2\,c^2\,x^2}{c^4\,x^4-1}}}{40\,c^8\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(1/sinh(2*log(c*x)))^(3/2),x)

[Out] ((c^4*x^4 - 1)^3*((2*c^2*x^2)/(c^4*x^4 - 1))^(1/2))/(40*c^8*x)

3.146 $\int \frac{x^5}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=162

$$\frac{4}{15c^4\left(c^4-\frac{1}{x^4}\right)x^2\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}-\frac{2x^2}{15\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x^6}{9\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{4E(\csc^{-1}(cx)}{15c^9\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\text{cs}}$$

```
[Out] 4/15/c^4/(c^4-1/x^4)/x^2/csch(2*ln(c*x))^(3/2)-2/15*x^2/(c^4-1/x^4)/csch(2*
ln(c*x))^(3/2)+1/9*x^6/csch(2*ln(c*x))^(3/2)+4/15*EllipticE(1/c/x,I)/c^9/(1
-1/c^4/x^4)^(3/2)/x^3/csch(2*ln(c*x))^(3/2)-4/15*EllipticF(1/c/x,I)/c^9/(1-
1/c^4/x^4)^(3/2)/x^3/csch(2*ln(c*x))^(3/2)
```

Rubi [A]
time = 0.07, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5671, 5669, 342, 283, 331, 313, 227, 1195, 435}

$$-\frac{2x^2}{15\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{4}{15c^4x^2\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}-\frac{4F(\csc^{-1}(cx)|-1)}{15c^9x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{4E(\csc^{-1}(cx)|-1)}{15c^9x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x^6}{9\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] 4/(15*c^4*(c^4 - x^(-4))*x^2*Csch[2*Log[c*x]]^(3/2)) - (2*x^2)/(15*(c^4 - x
^(-4))*Csch[2*Log[c*x]]^(3/2)) + x^6/(9*Csch[2*Log[c*x]]^(3/2)) + (4*Ellipt
icE[ArcCsc[c*x], -1])/(15*c^9*(1 - 1/(c^4*x^4))^(3/2)*x^3*Csch[2*Log[c*x]]^
(3/2)) - (4*EllipticF[ArcCsc[c*x], -1])/(15*c^9*(1 - 1/(c^4*x^4))^(3/2)*x^3
*Csch[2*Log[c*x]]^(3/2))
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
```

```
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[(d + e*x^2)/(Sqrt[q + c*x^2]*Sqrt[q - c
*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 5669

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5671

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{x^5}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{\operatorname{csch}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^6}$$

$$= \frac{\text{Subst}\left(\int \left(1-\frac{1}{x^4}\right)^{3/2} x^8\,dx, x, cx\right)}{c^9\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{\text{Subst}\left(\int \frac{(1-x^4)^{3/2}}{x^{10}}\,dx, x, \frac{1}{cx}\right)}{c^9\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x^6}{9\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{2\text{Subst}\left(\int \frac{\sqrt{1-x^4}}{x^6}\,dx, x, \frac{1}{cx}\right)}{3c^9\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{2x^2}{15\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^6}{9\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{4\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-x^4}}\,dx, \right.}{15c^9\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2}$$

$$= \frac{4}{15c^4\left(c^4-\frac{1}{x^4}\right)x^2\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{2x^2}{15\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^6}{9\operatorname{csch}^{\frac{3}{2}}(2\log(c}$$

$$= \frac{4}{15c^4\left(c^4-\frac{1}{x^4}\right)x^2\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{2x^2}{15\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^6}{9\operatorname{csch}^{\frac{3}{2}}(2\log(c}$$

$$= \frac{4}{15c^4\left(c^4-\frac{1}{x^4}\right)x^2\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{2x^2}{15\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^6}{9\operatorname{csch}^{\frac{3}{2}}(2\log(c}$$

$$= \frac{4}{15c^4\left(c^4-\frac{1}{x^4}\right)x^2\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{2x^2}{15\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^6}{9\operatorname{csch}^{\frac{3}{2}}(2\log(c}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.09, size = 63, normalized size = 0.39

$$-\frac{x^4\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};c^4x^4\right)}{6c^2\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{-1+c^4x^4}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] -1/6*(x^4*Hypergeometric2F1[-3/2, 3/4, 7/4, c^4*x^4])/(c^2*Sqrt[2 - 2*c^4*x
^4]*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)])
```

Maple [A]
time = 0.99, size = 140, normalized size = 0.86

method	result	size
risch	$\frac{x^4(5c^4x^4-11)\sqrt{2}}{180c^2\sqrt{\frac{c^2x^2}{c^4x^4-1}}}+\frac{\sqrt{c^2x^2+1}\,\sqrt{-c^2x^2+1}\,\left(\mathrm{EllipticF}\left(x\sqrt{-c^2}\,,i\right)-\mathrm{EllipticE}\left(x\sqrt{-c^2}\,,i\right)\right)\sqrt{2}\,x}{15\sqrt{-c^2}\,(c^4x^4-1)c^4\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$	140

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/csch(2*ln(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/180*x^4*(5*c^4*x^4-11)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4-1))^(1/2)+1/15/(-c^2
)^(1/2)*(c^2*x^2+1)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^4*x^4-1)/c^4*(EllipticF(x*(
-c^2)^(1/2),I)-EllipticE(x*(-c^2)^(1/2),I))*2^(1/2)*x/(c^2*x^2/(c^4*x^4-1))
^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/csch(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^5/csch(2*log(c*x))^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/csch(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/csch(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(x**5/csch(2*log(c*x))**(3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/csch(2*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\left(\frac{1}{\sinh(2\,\ln(c\,x))}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(1/sinh(2*log(c*x)))^(3/2),x)
```

```
[Out] int(x^5/(1/sinh(2*log(c*x)))^(3/2), x)
```

3.147 $\int \frac{x^4}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=96

$$-\frac{3x}{16\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x^5}{8\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)}{16c^8\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] -3/16*x/(c^4-1/x^4)/csch(2*ln(c*x))^(3/2)+1/8*x^5/csch(2*ln(c*x))^(3/2)+3/1
6*arctanh((1-1/c^4/x^4)^(1/2))/c^8/(1-1/c^4/x^4)^(3/2)/x^3/csch(2*ln(c*x))^
(3/2)
```

Rubi [A]
time = 0.05, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5671, 5669, 272, 43, 65, 212}

$$-\frac{3x}{16\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)}{16c^8x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x^5}{8\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] (-3*x)/(16*(c^4 - x^(-4))*Csch[2*Log[c*x]]^(3/2)) + x^5/(8*Csch[2*Log[c*x]]
^(3/2)) + (3*ArcTanh[Sqrt[1 - 1/(c^4*x^4)]])/(16*c^8*(1 - 1/(c^4*x^4))^(3/2
)*x^3*Csch[2*Log[c*x]]^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5669

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5671

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{x^4}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\operatorname{csch}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^5}$$

$$= \frac{\text{Subst}\left(\int \left(1-\frac{1}{x^4}\right)^{3/2} x^7\,dx, x, cx\right)}{c^8\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{\text{Subst}\left(\int \frac{(1-x)^{3/2}}{x^3}\,dx, x, \frac{1}{c^4x^4}\right)}{4c^8\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x^5}{8\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{3\text{Subst}\left(\int \frac{\sqrt{1-x}}{x^2}\,dx, x, \frac{1}{c^4x^4}\right)}{16c^8\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{3x}{16\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^5}{8\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{3\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx,\right.}{32c^8\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(}$$

$$= -\frac{3x}{16\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^5}{8\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{3\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1\right.}{16c^8\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(}$$

$$= -\frac{3x}{16\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^5}{8\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{c^4x}}\right.}{16c^8\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(}$$

Mathematica [A]
time = 0.12, size = 87, normalized size = 0.91

$$\frac{c^3x^3\sqrt{1-c^4x^4}\,(-5+2c^4x^4) - 3cx\operatorname{ArcSin}(c^2x^2)}{32c^5\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{-1+c^4x^4}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] (c^3*x^3*Sqrt[1 - c^4*x^4]*(-5 + 2*c^4*x^4) - 3*c*x*ArcSin[c^2*x^2])/(32*c^
5*Sqrt[2 - 2*c^4*x^4]*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)])
```

Maple [A]
time = 1.15, size = 113, normalized size = 1.18

method	result	size
risch	$\frac{x^{3} \left(2 c^{4} x^{4}-5\right) \sqrt{2}}{64 c^{2} \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}-1}}}+\frac{3 \ln \left(\frac{c^{4} x^{2}}{\sqrt{c^{4}}}+\sqrt{c^{4} x^{4}-1}\right) \sqrt{2}\, x}{64 \sqrt{c^{4}}\, c^{2} \sqrt{c^{4} x^{4}-1}\, \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}-1}}}$	113

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/csch(2*ln(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/64*x^3*(2*c^4*x^4-5)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4-1))^(1/2)+3/64*ln(c^4*
x^2/(c^4)^(1/2)+(c^4*x^4-1)^(1/2))/(c^4)^(1/2)*2^(1/2)/c^2*x/(c^4*x^4-1)^(1
/2)/(c^2*x^2/(c^4*x^4-1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/csch(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/csch(2*log(c*x))^(3/2), x)
```

Fricas [A]
time = 0.42, size = 102, normalized size = 1.06

$$\frac{2 \sqrt{2} \left(2 c^{9} x^{9}-7 c^{5} x^{5}+5 c x\right) \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}-1}}+3 \sqrt{2} \log \left(2 c^{4} x^{4}+2 \left(c^{5} x^{5}-c x\right) \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}-1}}-1\right)}{128 c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/csch(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/128*(2*sqrt(2)*(2*c^9*x^9 - 7*c^5*x^5 + 5*c*x)*sqrt(c^2*x^2/(c^4*x^4 - 1)
) + 3*sqrt(2)*log(2*c^4*x^4 + 2*(c^5*x^5 - c*x)*sqrt(c^2*x^2/(c^4*x^4 - 1))
 - 1))/c^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}}{\operatorname{csch}^{\frac{3}{2}}\left(2 \log \left(c x\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/csch(2*ln(c*x))**(3/2),x)
[Out] Integral(x**4/csch(2*log(c*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/csch(2*log(c*x))^(3/2),x, algorithm="giac")
[Out] integrate(x^4/csch(2*log(c*x))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\left(\frac{1}{\sinh(2\,\ln(c\,x))}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(1/sinh(2*log(c*x)))^(3/2),x)
[Out] int(x^4/(1/sinh(2*log(c*x)))^(3/2), x)
```

3.148 $\int \frac{x^3}{\mathbf{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=86

$$-\frac{2}{7\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x^4}{7\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}-\frac{4F\left(\csc^{-1}(cx)|-1\right)}{7c^7\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] -2/7/(c^4-1/x^4)/csch(2*ln(c*x))^(3/2)+1/7*x^4/csch(2*ln(c*x))^(3/2)-4/7*El
lipticF(1/c/x,I)/c^7/(1-1/c^4/x^4)^(3/2)/x^3/csch(2*ln(c*x))^(3/2)
```

Rubi [A]
time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5671, 5669, 342, 283, 227}

$$-\frac{2}{7\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}-\frac{4F\left(\csc^{-1}(cx)|-1\right)}{7c^7x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x^4}{7\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] -2/(7*(c^4 - x^(-4))*Csch[2*Log[c*x]]^(3/2)) + x^4/(7*Csch[2*Log[c*x]]^(3/2
)) - (4*EllipticF[ArcCsc[c*x], -1])/(7*c^7*(1 - 1/(c^4*x^4))^(3/2)*x^3*Csch
[2*Log[c*x]]^(3/2))
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5669

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5671

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{x^3}{\operatorname{csch}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^4} \\
&= \frac{\operatorname{Subst}\left(\int \left(1-\frac{1}{x^4}\right)^{3/2} x^6\,dx, x, cx\right)}{c^7\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{csch}^{\frac{3}{2}}(2\log(cx))} \\
&= -\frac{\operatorname{Subst}\left(\int \frac{\left(1-x^4\right)^{3/2}}{x^8}\,dx, x, \frac{1}{cx}\right)}{c^7\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{csch}^{\frac{3}{2}}(2\log(cx))} \\
&= \frac{x^4}{7\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{6\operatorname{Subst}\left(\int \frac{\sqrt{1-x^4}}{x^4}\,dx, x, \frac{1}{cx}\right)}{7c^7\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{csch}^{\frac{3}{2}}(2\log(cx))} \\
&= -\frac{2}{7\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^4}{7\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{4\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^4}}\,dx, x\right.}{7c^7\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{csch}^{\frac{3}{2}}(2\mathrm{l}} \\
&= -\frac{2}{7\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^4}{7\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{4F(\csc^{-1}(cx)|-1)}{7c^7\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{csch}^{\frac{3}{2}}(2\mathrm{l}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.09, size = 65, normalized size = 0.76

$$
\frac{\sqrt{1-c^4x^4}\sqrt{\frac{c^2x^2}{-1+c^4x^4}}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};c^4x^4\right)}{2\sqrt{2}\,c^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] (Sqrt[1 - c^4*x^4]*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)]*Hypergeometric2F1[-3/2, 1
/4, 5/4, c^4*x^4])/(2*Sqrt[2]*c^4)
```

Maple [A]
time = 0.97, size = 124, normalized size = 1.44

method	result	size
risch	$\frac{x^{2}\left(c^{4} x^{4}-3\right) \sqrt{2}}{28 c^{2} \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}-1}}}+\frac{\sqrt{c^{2} x^{2}+1}\, \sqrt{-c^{2} x^{2}+1}\, \mathrm{EllipticF}\left(x \sqrt{-c^{2}}, i\right) \sqrt{2}\, x}{7 \sqrt{-c^{2}}\, \left(c^{4} x^{4}-1\right) c^{2} \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}-1}}}$	124

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/csch(2*ln(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/28*x^2*(c^4*x^4-3)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4-1))^(1/2)+1/7/(-c^2)^(1/
2)*(c^2*x^2+1)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^4*x^4-1)*EllipticF(x*(-c^2)^(1/2
),I)*2^(1/2)/c^2*x/(c^2*x^2/(c^4*x^4-1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/csch(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/csch(2*log(c*x))^(3/2), x)
```

Fricas [A]
time = 0.11, size = 70, normalized size = 0.81

$$\frac{\sqrt{2}\,\left(c^{10} x^{8}-4\,c^{6} x^{4}+3\,c^{2}\right) \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}-1}}-4\,\sqrt{2}\,\sqrt{c^{4}}\,\mathrm{ellipticF}\left(\frac{1}{cx},-1\right)}{28\,c^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/csch(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/28*(sqrt(2)*(c^10*x^8 - 4*c^6*x^4 + 3*c^2)*sqrt(c^2*x^2/(c^4*x^4 - 1)) -
4*sqrt(2)*sqrt(c^4)*ellipticF(1/(c*x), -1))/c^6
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/csch(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(x**3/csch(2*log(c*x))**(3/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/csch(2*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\left(\frac{1}{\sinh(2\ln(c\,x))}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(1/sinh(2*log(c*x)))^(3/2),x)
```

```
[Out] int(x^3/(1/sinh(2*log(c*x)))^(3/2), x)
```

3.149 $\int \frac{x^2}{\mathbf{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=91

$$-\frac{1}{2\left(c^4-\frac{1}{x^4}\right)x\text{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x^3}{6\text{csch}^{\frac{3}{2}}(2\log(cx))}-\frac{\csc^{-1}\left(c^2x^2\right)}{2c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\text{csch}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] -1/2/(c^4-1/x^4)/x/csch(2*ln(c*x))^(3/2)+1/6*x^3/csch(2*ln(c*x))^(3/2)-1/2*
arccsc(c^2*x^2)/c^6/(1-1/c^4/x^4)^(3/2)/x^3/csch(2*ln(c*x))^(3/2)
```

Rubi [A]
time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5671, 5669, 342, 281, 283, 222}

$$-\frac{1}{2x\left(c^4-\frac{1}{x^4}\right)\text{csch}^{\frac{3}{2}}(2\log(cx))}-\frac{\csc^{-1}\left(c^2x^2\right)}{2c^6x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\text{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x^3}{6\text{csch}^{\frac{3}{2}}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] -1/2*1/((c^4 - x^(-4))*x*Csch[2*Log[c*x]]^(3/2)) + x^3/(6*Csch[2*Log[c*x]]^
(3/2)) - ArcCsc[c^2*x^2]/(2*c^6*(1 - 1/(c^4*x^4))^(3/2)*x^3*Csch[2*Log[c*x]
]^(3/2))
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5669

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5671

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^3}\\
&= \frac{\operatorname{Subst}\left(\int \left(1-\frac{1}{x^4}\right)^{3/2} x^5\,dx, x, cx\right)}{c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\\
&= -\frac{\operatorname{Subst}\left(\int \frac{\left(1-x^4\right)^{3/2}}{x^7}\,dx, x, \frac{1}{cx}\right)}{c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\\
&= -\frac{\operatorname{Subst}\left(\int \frac{\left(1-x^2\right)^{3/2}}{x^4}\,dx, x, \frac{1}{c^2x^2}\right)}{2c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\\
&= \frac{x^3}{6\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{\operatorname{Subst}\left(\int \frac{\sqrt{1-x^2}}{x^2}\,dx, x, \frac{1}{c^2x^2}\right)}{2c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\\
&= -\frac{1}{2\left(c^4-\frac{1}{x^4}\right)x\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^3}{6\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x\right.}{2c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2}\\
&= -\frac{1}{2\left(c^4-\frac{1}{x^4}\right)x\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^3}{6\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{\csc^{-1}\left(c^2x^2\right)}{2c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2}
\end{aligned}
$$

Mathematica [A]
time = 0.11, size = 88, normalized size = 0.97

$$\frac{x\left((-4+c^4x^4)\sqrt{-1+c^4x^4}+3\text{ArcTan}\left(\sqrt{-1+c^4x^4}\right)\right)}{12\sqrt{2}\,c^2\sqrt{\frac{c^2x^2}{-1+c^4x^4}}\sqrt{-1+c^4x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] (x*((-4 + c^4*x^4)*Sqrt[-1 + c^4*x^4] + 3*ArcTan[Sqrt[-1 + c^4*x^4]]))/(12*
Sqrt[2]*c^2*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)]*Sqrt[-1 + c^4*x^4])
```

Maple [F]
time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{csch}(2\ln(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/csch(2*ln(c*x))^(3/2),x)
```

```
[Out] int(x^2/csch(2*ln(c*x))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/csch(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/csch(2*log(c*x))^(3/2), x)
```

Fricas [A]
time = 0.45, size = 94, normalized size = 1.03

$$\frac{3\sqrt{2}\,cx\arctan\left(\frac{(c^4x^4-1)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{cx}\right)+\sqrt{2}\,(c^8x^8-5\,c^4x^4+4)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{24\,c^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/csch(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*(3*sqrt(2)*c*x*arctan((c^4*x^4 - 1)*sqrt(c^2*x^2/(c^4*x^4 - 1))/(c*x))
 + sqrt(2)*(c^8*x^8 - 5*c^4*x^4 + 4)*sqrt(c^2*x^2/(c^4*x^4 - 1)))/(c^4*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/csch(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(x**2/csch(2*log(c*x))**(3/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/csch(2*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(\frac{1}{\sinh(2\,\ln(c\,x))}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1/sinh(2*log(c*x)))^(3/2),x)
```

```
[Out] int(x^2/(1/sinh(2*log(c*x)))^(3/2), x)
```

3.150 $\int \frac{x}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=130

$$-\frac{6}{5\left(c^4-\frac{1}{x^4}\right)x^2\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x^2}{5\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}-\frac{12E(\csc^{-1}(cx)|-1)}{5c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{12F(\csc^{-}}{5c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2}}$$

```
[Out] -6/5/(c^4-1/x^4)/x^2/csch(2*ln(c*x))^(3/2)+1/5*x^2/csch(2*ln(c*x))^(3/2)-12
/5*EllipticE(1/c/x,I)/c^5/(1-1/c^4/x^4)^(3/2)/x^3/csch(2*ln(c*x))^(3/2)+12/
5*EllipticF(1/c/x,I)/c^5/(1-1/c^4/x^4)^(3/2)/x^3/csch(2*ln(c*x))^(3/2)
```

Rubi [A]
time = 0.05, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {5671, 5669, 342, 283, 313, 227, 1195, 435}

$$-\frac{6}{5x^2\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{12F(\csc^{-1}(cx)|-1)}{5c^5x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}-\frac{12E(\csc^{-1}(cx)|-1)}{5c^5x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x^2}{5\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[x/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] -6/(5*(c^4 - x^(-4))*x^2*Csch[2*Log[c*x]]^(3/2)) + x^2/(5*Csch[2*Log[c*x]]^
(3/2)) - (12*EllipticE[ArcCsc[c*x], -1])/(5*c^5*(1 - 1/(c^4*x^4))^(3/2)*x^3
*Csch[2*Log[c*x]]^(3/2)) + (12*EllipticF[ArcCsc[c*x], -1])/(5*c^5*(1 - 1/(c
^4*x^4))^(3/2)*x^3*Csch[2*Log[c*x]]^(3/2))
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
```

```
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[(d + e*x^2)/(Sqrt[q + c*x^2]*Sqrt[q - c
*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 5669

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5671

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{x}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x}{\operatorname{csch}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^2}$$

$$= \frac{\operatorname{Subst}\left(\int \left(1-\frac{1}{x^4}\right)^{3/2} x^4\,dx, x, cx\right)}{c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{(1-x^4)^{3/2}}{x^6}\,dx, x, \frac{1}{cx}\right)}{c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x^2}{5\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{6\operatorname{Subst}\left(\int \frac{\sqrt{1-x^4}}{x^2}\,dx, x, \frac{1}{cx}\right)}{5c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{6}{5\left(c^4-\frac{1}{x^4}\right)x^2\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^2}{5\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{12\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{1-x^4}}\,dx, \right.}{5c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2}$$

$$= -\frac{6}{5\left(c^4-\frac{1}{x^4}\right)x^2\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^2}{5\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{12\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^4}}\,dx, \right.}{5c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2}$$

$$= -\frac{6}{5\left(c^4-\frac{1}{x^4}\right)x^2\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^2}{5\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{12F(\csc^{-1}(cx)|-1}{5c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2}$$

$$= -\frac{6}{5\left(c^4-\frac{1}{x^4}\right)x^2\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x^2}{5\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{12E(\csc^{-1}(cx)|-1}{5c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.08, size = 60, normalized size = 0.46

$$\frac{{}_2F_1\left(-\frac{3}{2},-\frac{1}{4};\frac{3}{4};c^4x^4\right)}{2c^2\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{-1+c^4x^4}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Csch[2*Log[c*x]]^(3/2),x]
```

```
[Out] Hypergeometric2F1[-3/2, -1/4, 3/4, c^4*x^4]/(2*c^2*Sqrt[2 - 2*c^4*x^4]*Sqrt
[(c^2*x^2)/(-1 + c^4*x^4)])
```

Maple [A]
time = 0.97, size = 152, normalized size = 1.17

method	result
risch	$\frac{(c^8x^8+4c^4x^4-5)\sqrt{2}}{20(c^4x^4-1)c^2\sqrt{\frac{c^2x^2}{c^4x^4-1}}}-\frac{3\sqrt{c^2x^2+1}\,\sqrt{-c^2x^2+1}\,\left(\mathrm{EllipticF}\left(x\sqrt{-c^2}\,,i\right)-\mathrm{EllipticE}\left(x\sqrt{-c^2}\,,i\right)\right)\sqrt{2}\,x}{5\sqrt{-c^2}\,(c^4x^4-1)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/csch(2*ln(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/20*(c^8*x^8+4*c^4*x^4-5)/(c^4*x^4-1)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4-1))^(1
/2)-3/5/(-c^2)^(1/2)*(c^2*x^2+1)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^4*x^4-1)*(Elli
pticF(x*(-c^2)^(1/2),I)-EllipticE(x*(-c^2)^(1/2),I))*2^(1/2)*x/(c^2*x^2/(c^
4*x^4-1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/csch(2*log(c*x))^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(x/csch(2*log(c*x))^(3/2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(x/csch(2*log(c*x))**(3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(2*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\left(\frac{1}{\sinh(2\,\ln(c\,x))}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1/sinh(2*log(c*x)))^(3/2),x)
```

```
[Out] int(x/(1/sinh(2*log(c*x)))^(3/2), x)
```

3.151 $\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=96

$$\frac{3}{4\left(c^4-\frac{1}{x^4}\right)x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x}{4\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}-\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)}{4c^4\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] 3/4/(c^4-1/x^4)/x^3/csch(2*ln(c*x))^(3/2)+1/4*x/csch(2*ln(c*x))^(3/2)-3/4*a
rctanh((1-1/c^4/x^4)^(1/2))/c^4/(1-1/c^4/x^4)^(3/2)/x^3/csch(2*ln(c*x))^(3/
2)
```

Rubi [A]
time = 0.03, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {5665, 5663, 272, 43, 52, 65, 212}

$$\frac{3}{4x^3\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}-\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{c^4x^4}}\right)}{4c^4x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}+\frac{x}{4\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[2*Log[c*x]]^(-3/2),x]
```

```
[Out] 3/(4*(c^4 - x^(-4))*x^3*Csch[2*Log[c*x]]^(3/2)) + x/(4*Csch[2*Log[c*x]]^(3/
2)) - (3*ArcTanh[Sqrt[1 - 1/(c^4*x^4)]])/(4*c^4*(1 - 1/(c^4*x^4))^(3/2)*x^3
*Csch[2*Log[c*x]]^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5663

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Dist[Csch[d*(a
+ b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*d*p)), Int[1/(x^(b*
d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; FreeQ[{a, b, d, p}, x] &&
 !IntegerQ[p]
```

Rule 5665

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c}$$

$$= \frac{\text{Subst}\left(\int \left(1-\frac{1}{x^4}\right)^{3/2} x^3\,dx, x, cx\right)}{c^4\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{\text{Subst}\left(\int \frac{(1-x)^{3/2}}{x^2}\,dx, x, \frac{1}{c^4x^4}\right)}{4c^4\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x}{4\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{3\text{Subst}\left(\int \frac{\sqrt{1-x}}{x}\,dx, x, \frac{1}{c^4x^4}\right)}{8c^4\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{3}{4\left(c^4-\frac{1}{x^4}\right) x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x}{4\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{3\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx, x\right.}{8c^4\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2}$$

$$= \frac{3}{4\left(c^4-\frac{1}{x^4}\right) x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x}{4\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{3\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-}\right.}{4c^4\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2}$$

$$= \frac{3}{4\left(c^4-\frac{1}{x^4}\right) x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} + \frac{x}{4\operatorname{csch}^{\frac{3}{2}}(2\log(cx))} - \frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{c^4x^4}}\right.}{4c^4\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.07, size = 63, normalized size = 0.66

$$\frac{{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};c^4x^4\right)}{4c^2x\sqrt{2-2c^4x^4}\sqrt{\frac{c^2x^2}{-1+c^4x^4}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[2*Log[c*x]]^(-3/2),x]
```

```
[Out] Hypergeometric2F1[-3/2, -1/2, 1/2, c^4*x^4]/(4*c^2*x*Sqrt[2 - 2*c^4*x^4]*Sq
rt[(c^2*x^2)/(-1 + c^4*x^4)])
```

Maple [A]
time = 1.09, size = 130, normalized size = 1.35

method	result	size
risch	$\frac{(c^8x^8+c^4x^4-2)\sqrt{2}}{16x(c^4x^4-1)c^2\sqrt{\frac{c^2x^2}{c^4x^4-1}}}-\frac{3c^2\ln\left(\frac{c^4x^2}{\sqrt{c^4}}+\sqrt{c^4x^4-1}\right)\sqrt{2}\,x}{16\sqrt{c^4}\,\sqrt{c^4x^4-1}\,\sqrt{\frac{c^2x^2}{c^4x^4-1}}}$	130

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/csch(2*ln(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*(c^8*x^8+c^4*x^4-2)/x/(c^4*x^4-1)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4-1))^(1
/2)-3/16*c^2*ln(c^4*x^2/(c^4)^(1/2)+(c^4*x^4-1)^(1/2))/(c^4)^(1/2)*2^(1/2)*
x/(c^4*x^4-1)^(1/2)/(c^2*x^2/(c^4*x^4-1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csch(2*log(c*x))^(-3/2), x)
```

Fricas [A]
time = 0.40, size = 106, normalized size = 1.10

$$\frac{3\sqrt{2}\,c^3x^3\log\left(2\,c^4x^4-2\,(c^5x^5-cx)\sqrt{\frac{c^2x^2}{c^4x^4-1}}-1\right)+2\sqrt{2}\,(c^8x^8+c^4x^4-2)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{32\,c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/32*(3*sqrt(2)*c^3*x^3*log(2*c^4*x^4 - 2*(c^5*x^5 - c*x)*sqrt(c^2*x^2/(c^4
*x^4 - 1)) - 1) + 2*sqrt(2)*(c^8*x^8 + c^4*x^4 - 2)*sqrt(c^2*x^2/(c^4*x^4 -
 1)))/(c^4*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\operatorname{csch}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(2*ln(c*x))**(3/2),x)
[Out] Integral(csch(2*log(c*x))**(-3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csch(2*log(c*x))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(\frac{1}{\sinh\left(2\,\ln\left(c\,x\right)\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/sinh(2*log(c*x)))^(3/2),x)
[Out] int(1/(1/sinh(2*log(c*x)))^(3/2), x)
```

3.152 $\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x}\,dx$

Optimal. Leaf size=67

$$-\cosh(2\log(cx))\sqrt{\operatorname{csch}(2\log(cx))}+\frac{iE\left(\frac{\pi}{4}-i\log(cx)\middle|2\right)}{\sqrt{\operatorname{csch}(2\log(cx))}\sqrt{i\sinh(2\log(cx))}}$$

```
[Out] -cosh(2*ln(c*x))*csch(2*ln(c*x))^(1/2)+I*(sin(1/4*Pi+I*ln(c*x))^2)^(1/2)/si
n(1/4*Pi+I*ln(c*x))*EllipticE(cos(1/4*Pi+I*ln(c*x)),2^(1/2))/csch(2*ln(c*x)
)^(1/2)/(I*sinh(2*ln(c*x)))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3853, 3856, 2719}

$$-\cosh(2\log(cx))\sqrt{\operatorname{csch}(2\log(cx))}+\frac{iE\left(\frac{\pi}{4}-i\log(cx)\middle|2\right)}{\sqrt{i\sinh(2\log(cx))}\sqrt{\operatorname{csch}(2\log(cx))}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[2*Log[c*x]]^(3/2)/x,x]
```

```
[Out] -(Cosh[2*Log[c*x]]*Sqrt[Csch[2*Log[c*x]]]) + (I*EllipticE[Pi/4 - I*Log[c*x]
, 2])/(Sqrt[Csch[2*Log[c*x]]]*Sqrt[I*Sinh[2*Log[c*x]]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x}\,dx = \operatorname{Subst}\left(\int \operatorname{csch}^{\frac{3}{2}}(2x)\,dx, x, \log(cx)\right)$$
$$= -\cosh(2\log(cx))\sqrt{\operatorname{csch}(2\log(cx))} + \operatorname{Subst}\left(\int \frac{1}{\sqrt{\operatorname{csch}(2x)}}\,dx, x, \log(cx)\right)$$
$$= -\cosh(2\log(cx))\sqrt{\operatorname{csch}(2\log(cx))} + \frac{\operatorname{Subst}\left(\int \sqrt{i\sinh(2x)}\,dx, x, \log(cx)\right)}{\sqrt{\operatorname{csch}(2\log(cx))}\sqrt{i\sinh(2\log(cx))}}$$
$$= -\cosh(2\log(cx))\sqrt{\operatorname{csch}(2\log(cx))} + \frac{iE\left(\frac{\pi}{4} - i\log(cx)\middle|2\right)}{\sqrt{\operatorname{csch}(2\log(cx))}\sqrt{i\sinh(2\log(cx))}}$$

Mathematica [A]
time = 0.07, size = 54, normalized size = 0.81

$$\sqrt{\operatorname{csch}(2\log(cx))}\left(-\cosh(2\log(cx)) + E\left(\frac{\pi}{4} - i\log(cx)\middle|2\right)\sqrt{i\sinh(2\log(cx))}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[2*Log[c*x]]^(3/2)/x,x]
```

```
[Out] Sqrt[Csch[2*Log[c*x]]]*(-Cosh[2*Log[c*x]] + EllipticE[Pi/4 - I*Log[c*x], 2]
*Sqrt[I*Sinh[2*Log[c*x]]])
```

Maple [A]
time = 2.56, size = 163, normalized size = 2.43

method	result
derivativedivides	$\frac{2\sqrt{1-i\sinh(2\ln(cx))}\sqrt{2}\sqrt{1+i\sinh(2\ln(cx))}\sqrt{i\sinh(2\ln(cx))}\operatorname{EllipticE}\left(\sqrt{1-\right.}{}$
default	$\frac{2\sqrt{1-i\sinh(2\ln(cx))}\sqrt{2}\sqrt{1+i\sinh(2\ln(cx))}\sqrt{i\sinh(2\ln(cx))}\operatorname{EllipticE}\left(\sqrt{1-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(2*ln(c*x))^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(2*(1-I*sinh(2*ln(c*x)))^(1/2)*2^(1/2)*(1+I*sinh(2*ln(c*x)))^(1/2)*(I*s
inh(2*ln(c*x)))^(1/2)*EllipticE((1-I*sinh(2*ln(c*x)))^(1/2),1/2*2^(1/2))-(1
-I*sinh(2*ln(c*x)))^(1/2)*2^(1/2)*(1+I*sinh(2*ln(c*x)))^(1/2)*(I*sinh(2*ln(
c*x)))^(1/2)*EllipticF((1-I*sinh(2*ln(c*x)))^(1/2),1/2*2^(1/2))-2*cosh(2*ln
(c*x))^2)/cosh(2*ln(c*x))/sinh(2*ln(c*x))^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(3/2)/x,x, algorithm="maxima")
[Out] integrate(csch(2*log(c*x))^(3/2)/x, x)
```

Fricas [**A**]
time = 0.10, size = 31, normalized size = 0.46

$$-\sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4-1}}\,c^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(3/2)/x,x, algorithm="fricas")
[Out] -sqrt(2)*sqrt(c^2*x^2/(c^4*x^4 - 1))*c^2*x^2
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*ln(c*x))**(3/2)/x,x)
[Out] Integral(csch(2*log(c*x))**(3/2)/x, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(3/2)/x,x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(\frac{1}{\sinh(2\,\ln(c\,x))}\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sinh(2*log(c*x)))^(3/2)/x,x)
[Out] int((1/sinh(2*log(c*x)))^(3/2)/x, x)
```

3.153 $\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^2} dx$

Optimal. Leaf size=27

$$-\frac{1}{2}\left(c^4-\frac{1}{x^4}\right)x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))$$

```
[Out] -1/2*(c^4-1/x^4)*x^3*csch(2*ln(c*x))^(3/2)
```

Rubi [A]

time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5671, 5669, 267}

$$-\frac{1}{2}x^3\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))$$

Antiderivative was successfully verified.

```
[In] Int[Csch[2*Log[c*x]]^(3/2)/x^2,x]
```

```
[Out] -1/2*((c^4 - x^(-4))*x^3*Csch[2*Log[c*x]]^(3/2))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5669

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5671

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx = c\operatorname{Subst}\left(\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(x))}{x^2}\,dx, x, cx\right)$$
$$= \left(c^4\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))\right)\operatorname{Subst}\left(\int \frac{1}{\left(1-\frac{1}{x^4}\right)^{3/2}x^5}\,dx, x, cx\right)$$
$$= -\frac{1}{2}\left(c^4-\frac{1}{x^4}\right)x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))$$

Mathematica [**A**]
time = 0.02, size = 33, normalized size = 1.22

$$-\sqrt{2}\,c^2x\sqrt{\frac{c^2x^2}{-1+c^4x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[2*Log[c*x]]^(3/2)/x^2,x]
[Out] -(Sqrt[2]*c^2*x*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)])
```

Maple [**F**]
time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(2\ln(cx))^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(2*ln(c*x))^(3/2)/x^2,x)
[Out] int(csch(2*ln(c*x))^(3/2)/x^2,x)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(23) = 46.
time = 0.48, size = 87, normalized size = 3.22

$$-c\left(\frac{\sqrt{2}}{\left(\frac{1}{cx}+1\right)^{\frac{3}{2}}\left(-\frac{1}{cx}+1\right)^{\frac{3}{2}}\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}}-\frac{\sqrt{2}}{c^4x^4\left(\frac{1}{cx}+1\right)^{\frac{3}{2}}\left(-\frac{1}{cx}+1\right)^{\frac{3}{2}}\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] -c*(sqrt(2)/((1/(c*x) + 1)^(3/2)*(-1/(c*x) + 1)^(3/2)*(1/(c^2*x^2) + 1)^(3/
2)) - sqrt(2)/(c^4*x^4*(1/(c*x) + 1)^(3/2)*(-1/(c*x) + 1)^(3/2)*(1/(c^2*x^2
) + 1)^(3/2)))
```

Fricas [A]
time = 0.37, size = 29, normalized size = 1.07

$$-\sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4-1}}\,c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(3/2)/x^2,x, algorithm="fricas")
[Out] -sqrt(2)*sqrt(c^2*x^2/(c^4*x^4 - 1))*c^2*x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*ln(c*x))**(3/2)/x**2,x)
[Out] Integral(csch(2*log(c*x))**(3/2)/x**2, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(3/2)/x^2,x, algorithm="giac")
[Out] Timed out
```

Mupad [B]
time = 1.47, size = 29, normalized size = 1.07

$$-c^2\,x\,\sqrt{\frac{2\,c^2\,x^2}{c^4\,x^4-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sinh(2*log(c*x)))^(3/2)/x^2,x)
[Out] -c^2*x*((2*c^2*x^2)/(c^4*x^4 - 1))^(1/2)
```

3.154 $\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx$

Optimal. Leaf size=69

$$-\frac{1}{2}\left(c^4-\frac{1}{x^4}\right)x^2\operatorname{csch}^{\frac{3}{2}}(2\log(cx))+\frac{1}{2}c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))F\left(\csc^{-1}(cx)\middle|-1\right)$$

```
[Out] -1/2*(c^4-1/x^4)*x^2*csch(2*ln(c*x))^(3/2)+1/2*c^5*(1-1/c^4/x^4)^(3/2)*x^3*
csch(2*ln(c*x))^(3/2)*EllipticF(1/c/x,I)
```

Rubi [A]
time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5671, 5669, 342, 294, 227}

$$\frac{1}{2}c^5x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}F\left(\csc^{-1}(cx)\middle|-1\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))-\frac{1}{2}x^2\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))$$

Antiderivative was successfully verified.

```
[In] Int[Csch[2*Log[c*x]]^(3/2)/x^3,x]
```

```
[Out] -1/2*((c^4 - x^(-4))*x^2*Csch[2*Log[c*x]]^(3/2)) + (c^5*(1 - 1/(c^4*x^4))^(
3/2)*x^3*Csch[2*Log[c*x]]^(3/2)*EllipticF[ArcCsc[c*x], -1])/2
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5669

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5671

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx &= c^2\text{Subst}\left(\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(x))}{x^3}\,dx, x, cx\right)\\
&= \left(c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{1}{x^4}\right)^{3/2} x^6}\,dx, x, cx\right)\\
&= -\left(\left(c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int \frac{x^4}{\left(1-x^4\right)^{3/2}}\,dx, x, \frac{1}{cx}\right)\right)\\
&= -\frac{1}{2}\left(c^4-\frac{1}{x^4}\right)x^2\operatorname{csch}^{\frac{3}{2}}(2\log(cx))+\frac{1}{2}\left(c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))\right)\text{Sub}\\
&= -\frac{1}{2}\left(c^4-\frac{1}{x^4}\right)x^2\operatorname{csch}^{\frac{3}{2}}(2\log(cx))+\frac{1}{2}c^5\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))F\left(\csc^{-}\right.
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.08, size = 66, normalized size = 0.96

$$-\sqrt{2}\,c^2\sqrt{\frac{c^2x^2}{-1+c^4x^4}}\left(1+\sqrt{1-c^4x^4}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};c^4x^4\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[2*Log[c*x]]^(3/2)/x^3,x]
[Out] -(Sqrt[2]*c^2*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)]*(1 + Sqrt[1 - c^4*x^4]*Hyperge
ometric2F1[1/4, 1/2, 5/4, c^4*x^4]))
```

Maple [F]
time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(2\ln(cx))^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(2*ln(c*x))^(3/2)/x^3,x)
```

```
[Out] int(csch(2*ln(c*x))^(3/2)/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(csch(2*log(c*x))^(3/2)/x^3, x)
```

Fricas [A]
time = 0.12, size = 28, normalized size = 0.41

$$-\sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4-1}}\,c^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] -sqrt(2)*sqrt(c^2*x^2/(c^4*x^4 - 1))*c^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*ln(c*x))**(3/2)/x**3,x)
```

```
[Out] Integral(csch(2*log(c*x))**(3/2)/x**3, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\sinh(2\ln(c\,x))}\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sinh(2*log(c*x)))^(3/2)/x^3,x)
```

```
[Out] int((1/sinh(2*log(c*x)))^(3/2)/x^3, x)
```

3.155 $\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx$

Optimal. Leaf size=69

$$-\frac{1}{2}\left(c^4-\frac{1}{x^4}\right)x\operatorname{csch}^{\frac{3}{2}}(2\log(cx))+\frac{1}{2}c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2}x^3\csc^{-1}\left(c^2x^2\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))$$

```
[Out] -1/2*(c^4-1/x^4)*x*csch(2*ln(c*x))^(3/2)+1/2*c^6*(1-1/c^4/x^4)^(3/2)*x^3*ar
ccsc(c^2*x^2)*csch(2*ln(c*x))^(3/2)
```

Rubi [A]
time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5671, 5669, 342, 281, 294, 222}

$$\frac{1}{2}c^6x^3\left(1-\frac{1}{c^4x^4}\right)^{3/2}\csc^{-1}\left(c^2x^2\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))-\frac{1}{2}x\left(c^4-\frac{1}{x^4}\right)\operatorname{csch}^{\frac{3}{2}}(2\log(cx))$$

Antiderivative was successfully verified.

```
[In] Int[Csch[2*Log[c*x]]^(3/2)/x^4,x]
```

```
[Out] -1/2*((c^4 - x^(-4))*x*Csch[2*Log[c*x]]^(3/2)) + (c^6*(1 - 1/(c^4*x^4))^(3/
2)*x^3*ArcCsc[c^2*x^2]*Csch[2*Log[c*x]]^(3/2))/2
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5669

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5671

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Csch[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx = c^3\text{Subst}\left(\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(x))}{x^4}\,dx, x, cx\right)$$
$$= \left(c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{1}{x^4}\right)^{3/2}x^7}\,dx, x, cx\right)$$
$$= -\left(\left(c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int \frac{x^5}{(1-x^4)^{3/2}}\,dx, x, \frac{1}{cx}\right)\right)$$
$$= -\left(\frac{1}{2}\left(c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int \frac{x^2}{(1-x^2)^{3/2}}\,dx, x, \frac{1}{c^2x^2}\right)\right)$$
$$= -\frac{1}{2}\left(c^4-\frac{1}{x^4}\right)x\operatorname{csch}^{\frac{3}{2}}(2\log(cx)) + \frac{1}{2}\left(c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{\frac{3}{2}}(2\log(cx))\right)\text{Subs}$$
$$= -\frac{1}{2}\left(c^4-\frac{1}{x^4}\right)x\operatorname{csch}^{\frac{3}{2}}(2\log(cx)) + \frac{1}{2}c^6\left(1-\frac{1}{c^4x^4}\right)^{3/2} x^3\csc^{-1}\left(c^2x^2\right)\operatorname{csch}^{\frac{3}{2}}(2\log($$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.07, size = 53, normalized size = 0.77

$$-\frac{\sqrt{2}\,c^2\sqrt{\frac{c^2x^2}{-1+c^4x^4}}\;{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1-c^4x^4\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[2*Log[c*x]]^(3/2)/x^4,x]
```

```
[Out] -((Sqrt[2]*c^2*Sqrt[(c^2*x^2)/(-1 + c^4*x^4)]*Hypergeometric2F1[-1/2, 1, 1/
2, 1 - c^4*x^4])/x)
```

Maple [**F**]
time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(2\ln(cx))^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(2*ln(c*x))^(3/2)/x^4,x)
```

```
[Out] int(csch(2*ln(c*x))^(3/2)/x^4,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(csch(2*log(c*x))^(3/2)/x^4, x)
```

Fricas [**A**]
time = 0.40, size = 78, normalized size = 1.13

$$-\frac{\sqrt{2}\,c^3x\arctan\left(\frac{(c^4x^4-1)\sqrt{\frac{c^2x^2}{c^4x^4-1}}}{cx}\right)+\sqrt{2}\,\sqrt{\frac{c^2x^2}{c^4x^4-1}}\,c^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] -(sqrt(2)*c^3*x*arctan((c^4*x^4 - 1)*sqrt(c^2*x^2/(c^4*x^4 - 1))/(c*x)) + s
qrt(2)*sqrt(c^2*x^2/(c^4*x^4 - 1))*c^2)/x
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*ln(c*x))**(3/2)/x**4,x)
```

```
[Out] Integral(csch(2*log(c*x))**(3/2)/x**4, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(2*log(c*x))^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\sinh(2\,\ln(c\,x))}\right)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sinh(2*log(c*x)))^(3/2)/x^4,x)
```

```
[Out] int((1/sinh(2*log(c*x)))^(3/2)/x^4, x)
```

3.156 $\int \operatorname{csch}(a+b\log(cx^n))\,dx$

Optimal. Leaf size=62

$$-\frac{2e^a x (cx^n)^b\ {}_2F_1\left(1,\frac{b+\frac{1}{n}}{2b};\frac{1}{2}\left(3+\frac{1}{bn}\right);e^{2a}(cx^n)^{2b}\right)}{1+bn}$$

```
[Out] -2*exp(a)*x*(c*x^n)^b*hypergeom([1, 1/2*(b+1/n)/b],[3/2+1/2/b/n],exp(2*a)*(
c*x^n)^(2*b))/(b*n+1)
```

Rubi [A]
time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5665, 5667, 269, 371}

$$-\frac{2e^a x (cx^n)^b\ {}_2F_1\left(1,\frac{b+\frac{1}{n}}{2b};\frac{1}{2}\left(3+\frac{1}{bn}\right);e^{2a}(cx^n)^{2b}\right)}{bn+1}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*Log[c*x^n]],x]
```

```
[Out] (-2*E^a*x*(c*x^n)^b*Hypergeometric2F1[1, (b + n^(-1))/(2*b), (3 + 1/(b*n))/
2, E^(2*a)*(c*x^n)^(2*b)])/(1 + b*n)
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5665

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5667

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*
```

```
d)))^p)), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}(a+b\log(cx^n))\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{csch}(a+b\log(x))\,dx, x, cx^n\right)}{n}$$
$$= \frac{\left(2e^{-a}x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1-b+\frac{1}{n}}}{1-e^{-2a}x^{-2b}}\,dx, x, cx^n\right)}{n}$$
$$= \frac{\left(2e^{-a}x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1+b+\frac{1}{n}}}{-e^{-2a}+x^{2b}}\,dx, x, cx^n\right)}{n}$$
$$= -\frac{2e^a x(cx^n)^b\,{}_2F_1\left(1, \frac{b+\frac{1}{n}}{2b}; \frac{1}{2}\left(3+\frac{1}{bn}\right); e^{2a}(cx^n)^{2b}\right)}{1+bn}$$

Mathematica [A]
time = 0.79, size = 62, normalized size = 1.00

$$-\frac{2e^a x(cx^n)^b\,{}_2F_1\left(1, \frac{1}{2}\left(1+\frac{1}{bn}\right); \frac{1}{2}\left(3+\frac{1}{bn}\right); e^{2(a+b\log(cx^n))}\right)}{1+bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*Log[c*x^n]],x]
[Out] (-2*E^a*x*(c*x^n)^b*Hypergeometric2F1[1, (1 + 1/(b*n))/2, (3 + 1/(b*n))/2,
E^(2*(a + b*Log[c*x^n]))])/(1 + b*n)
```

Maple [F]
time = 1.36, size = 0, normalized size = 0.00

$$\int \operatorname{csch}(a+b\ln(c\,x^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a+b*ln(c*x^n)),x)
[Out] int(csch(a+b*ln(c*x^n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n)),x, algorithm="maxima")
[Out] integrate(csch(b*log(c*x^n) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] integral(csch(b*log(c*x^n) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}\left(a + b\log\left(cx^{n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*ln(c*x**n)),x)
[Out] Integral(csch(a + b*log(c*x**n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n)),x, algorithm="giac")
[Out] integrate(csch(b*log(c*x^n) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sinh\left(a + b\,\ln\left(c\,x^{n}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(a + b*log(c*x^n)),x)
[Out] int(1/sinh(a + b*log(c*x^n)), x)
```

3.157 $\int \operatorname{csch}^2(a+b\log(cx^n))\,dx$

Optimal. Leaf size=68

$$\frac{4e^{2a}x(cx^n)^{2b}\ {}_2F_1\left(2,\frac{1}{2}\left(2+\frac{1}{bn}\right);\frac{1}{2}\left(4+\frac{1}{bn}\right);e^{2a}(cx^n)^{2b}\right)}{1+2bn}$$

```
[Out] 4*exp(2*a)*x*(c*x^n)^(2*b)*hypergeom([2, 1+1/2/b/n],[2+1/2/b/n],exp(2*a)*(c
*x^n)^(2*b))/(2*b*n+1)
```

Rubi [A]
time = 0.07, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5665, 5667, 269, 371}

$$\frac{4e^{2a}x(cx^n)^{2b}\ {}_2F_1\left(2,\frac{1}{2}\left(2+\frac{1}{bn}\right);\frac{1}{2}\left(4+\frac{1}{bn}\right);e^{2a}(cx^n)^{2b}\right)}{2bn+1}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (4*E^(2*a)*x*(c*x^n)^(2*b)*Hypergeometric2F1[2, (2 + 1/(b*n))/2, (4 + 1/(b*
n))/2, E^(2*a)*(c*x^n)^(2*b)])/(1 + 2*b*n)
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5665

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5667

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*
```

```
d)))^p)), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^2(a+b\log(cx^n))\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{csch}^2(a+b\log(x))\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(4e^{-2a}x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1-2b+\frac{1}{n}}}{(1-e^{-2a}x^{-2b})^2}\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(4e^{-2a}x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1+2b+\frac{1}{n}}}{(-e^{-2a}+x^{2b})^2}\,dx,x,cx^n\right)}{n}$$
$$= \frac{4e^{2a}x(cx^n)^{2b}\,{}_2F_1\left(2,\frac{1}{2}\left(2+\frac{1}{bn}\right);\frac{1}{2}\left(4+\frac{1}{bn}\right);e^{2a}(cx^n)^{2b}\right)}{1+2bn}$$

Mathematica [A]
time = 2.89, size = 126, normalized size = 1.85

$$\frac{x\left(-\coth(a+b\log(cx^n))-\frac{e^{2a}(cx^n)^{2b}\,{}_2F_1\left(1,1+\frac{1}{2bn};2+\frac{1}{2bn};e^{2(a+b\log(cx^n))}\right)}{1+2bn}-{}_2F_1\left(1,\frac{1}{2bn};1+\frac{1}{2bn};e^{2(a+b\log(cx^n))}\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (x*(-Coth[a + b*Log[c*x^n]] - (E^(2*a)*(c*x^n)^(2*b)*Hypergeometric2F1[1, 1
 + 1/(2*b*n), 2 + 1/(2*b*n), E^(2*(a + b*Log[c*x^n]))])/(1 + 2*b*n) - Hyper
geometric2F1[1, 1/(2*b*n), 1 + 1/(2*b*n), E^(2*(a + b*Log[c*x^n]))]))/(b*n)
```

Maple [F]
time = 1.28, size = 0, normalized size = 0.00

$$\int \operatorname{csch}(a+b\ln(c\,x^n))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a+b*ln(c*x^n))^2,x)
```

```
[Out] int(csch(a+b*ln(c*x^n))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] -2*x/(b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) - b*n) - 4*integrate(1/4/(b*c^b*n*
e^(b*log(x^n) + a) + b*n), x) + 4*integrate(1/4/(b*c^b*n*e^(b*log(x^n) + a)
 - b*n), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] integral(csch(b*log(c*x^n) + a)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^2\left(a+b\log\left(cx^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*ln(c*x**n))**2,x)
```

```
[Out] Integral(csch(a + b*log(c*x**n))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] integrate(csch(b*log(c*x^n) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\operatorname{sinh}\left(a+b\,\ln\left(c\,x^n\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(a + b*log(c*x^n))^2,x)
```

```
[Out] int(1/sinh(a + b*log(c*x^n))^2, x)
```

3.158 $\int \operatorname{csch}^3(a+b\log(cx^n))\,dx$

Optimal. Leaf size=69

$$-\frac{8e^{3a}x(cx^n)^{3b}\ {}_2F_1\left(3,\frac{3b+\frac{1}{n}}{2b};\frac{1}{2}\left(5+\frac{1}{bn}\right);e^{2a}(cx^n)^{2b}\right)}{1+3bn}$$

```
[Out] -8*exp(3*a)*x*(c*x^n)^(3*b)*hypergeom([3, 1/2*(3*b+1/n)/b],[5/2+1/2/b/n],ex
p(2*a)*(c*x^n)^(2*b))/(3*b*n+1)
```

Rubi [A]
time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5665, 5667, 269, 371}

$$-\frac{8e^{3a}x(cx^n)^{3b}\ {}_2F_1\left(3,\frac{3b+\frac{1}{n}}{2b};\frac{1}{2}\left(5+\frac{1}{bn}\right);e^{2a}(cx^n)^{2b}\right)}{3bn+1}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (-8*E^(3*a)*x*(c*x^n)^(3*b)*Hypergeometric2F1[3, (3*b + n^(-1))/(2*b), (5 +
 1/(b*n))/2, E^(2*a)*(c*x^n)^(2*b)])/(1 + 3*b*n)
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5665

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5667

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*
```

```
d)))^p)), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^3(a+b\log(cx^n))\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{csch}^3(a+b\log(x))\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(8e^{-3a}x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1-3b+\frac{1}{n}}}{(1-e^{-2a}x^{-2b})^3}\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(8e^{-3a}x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1+3b+\frac{1}{n}}}{(-e^{-2a}+x^{2b})^3}\,dx,x,cx^n\right)}{n}$$
$$= -\frac{8e^{3a}x(cx^n)^{3b}\,{}_2F_1\left(3,\frac{3b+\frac{1}{n}}{2b};\frac{1}{2}\left(5+\frac{1}{bn}\right);e^{2a}(cx^n)^{2b}\right)}{1+3bn}$$

Mathematica [A]
time = 4.07, size = 101, normalized size = 1.46

$$\frac{-4x(1+bn\coth(a+b\log(cx^n)))\operatorname{csch}(a+b\log(cx^n))+8e^a(-1+bn)x(cx^n)^b\,{}_2F_1\left(1,\frac{1}{2}\left(1+\frac{1}{bn}\right);\frac{1}{2}\left(3+\frac{1}{bn}\right);e^{2(a+b\log(cx^n))}\right)}{8b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*Log[c*x^n]]^3,x]
[Out] (-4*x*(1 + b*n*Coth[a + b*Log[c*x^n]])*Csch[a + b*Log[c*x^n]] + 8*E^a*(-1 +
 b*n)*x*(c*x^n)^b*Hypergeometric2F1[1, (1 + 1/(b*n))/2, (3 + 1/(b*n))/2, E^
(2*(a + b*Log[c*x^n]))])/(8*b^2*n^2)
```

Maple [F]
time = 1.92, size = 0, normalized size = 0.00

$$\int \operatorname{csch}(a+b\ln(c\,x^n))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a+b*ln(c*x^n))^3,x)
[Out] int(csch(a+b*ln(c*x^n))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] -8*(b^2*n^2 - 1)*integrate(1/16/(b^2*c^b*n^2*e^(b*log(x^n) + a) + b^2*n^2),
 x) - 8*(b^2*n^2 - 1)*integrate(1/16/(b^2*c^b*n^2*e^(b*log(x^n) + a) - b^2*
n^2), x) - ((b*c^(3*b)*n + c^(3*b))*x*e^(3*b*log(x^n) + 3*a) + (b*c^b*n - c
^b)*x*e^(b*log(x^n) + a))/(b^2*c^(4*b)*n^2*e^(4*b*log(x^n) + 4*a) - 2*b^2*c
^(2*b)*n^2*e^(2*b*log(x^n) + 2*a) + b^2*n^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] integral(csch(b*log(c*x^n) + a)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^{3}\left(a + b \log \left(cx^{n}\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*ln(c*x**n))**3,x)
```

```
[Out] Integral(csch(a + b*log(c*x**n))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] integrate(csch(b*log(c*x^n) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\mathrm{sinh}\left(a+b\,\ln\left(c\,x^n\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(a + b*log(c*x^n))^3,x)
```

```
[Out] int(1/sinh(a + b*log(c*x^n))^3, x)
```

3.159 $\int \operatorname{csch}^4(a+b\log(cx^n))\,dx$

Optimal. Leaf size=68

$$\frac{16e^{4a}x(cx^n)^{4b}\,{}_2F_1\left(4,\frac{1}{2}\left(4+\frac{1}{bn}\right);\frac{1}{2}\left(6+\frac{1}{bn}\right);e^{2a}(cx^n)^{2b}\right)}{1+4bn}$$

```
[Out] 16*exp(4*a)*x*(c*x^n)^(4*b)*hypergeom([4, 2+1/2/b/n],[3+1/2/b/n],exp(2*a)*(
c*x^n)^(2*b))/(4*b*n+1)
```

Rubi [A]
time = 0.07, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5665, 5667, 269, 371}

$$\frac{16e^{4a}x(cx^n)^{4b}\,{}_2F_1\left(4,\frac{1}{2}\left(4+\frac{1}{bn}\right);\frac{1}{2}\left(6+\frac{1}{bn}\right);e^{2a}(cx^n)^{2b}\right)}{4bn+1}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (16*E^(4*a)*x*(c*x^n)^(4*b)*Hypergeometric2F1[4, (4 + 1/(b*n))/2, (6 + 1/(b
*n))/2, E^(2*a)*(c*x^n)^(2*b)])/(1 + 4*b*n)
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5665

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5667

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*
```

```
d)))^p)), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^4(a+b\log(cx^n))\,dx &= \frac{\left(x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{csch}^4(a+b\log(x))\,dx,x,cx^n\right)}{n}\\
&= \frac{\left(16e^{-4a}x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1-4b+\frac{1}{n}}}{(1-e^{-2a}x^{-2b})^4}\,dx,x,cx^n\right)}{n}\\
&= \frac{\left(16e^{-4a}x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1+4b+\frac{1}{n}}}{(-e^{-2a}+x^{2b})^4}\,dx,x,cx^n\right)}{n}\\
&= \frac{16e^{4a}x(cx^n)^{4b}\,{}_2F_1\left(4,\frac{1}{2}\left(4+\frac{1}{bn}\right);\frac{1}{2}\left(6+\frac{1}{bn}\right);e^{2a}(cx^n)^{2b}\right)}{1+4bn}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 200 vs. 2(68) = 136.
time = 5.94, size = 200, normalized size = 2.94

$$\frac{x\left(4e^{2a}(-1+2bn)(cx^n)^{2b}\,{}_2F_1\left(1,1+\frac{1}{2bn};2+\frac{1}{2bn};e^{2(a+b\log(cx^n))}\right)+4(-1+4b^2n^2)\,{}_2F_1\left(1,\frac{1}{2bn};1+\frac{1}{2bn};e^{2(a+b\log(cx^n))}\right)+\operatorname{csch}^3(a+b\log(cx^n))\left((1-12b^2n^2)\cosh(a+b\log(cx^n))+(-1+4b^2n^2)\cosh(3(a+b\log(cx^n)))-4bn\sinh(a+b\log(cx^n))\right)\right)}{24b^3n^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (x*(4*E^(2*a)*(-1 + 2*b*n)*(c*x^n)^(2*b)*Hypergeometric2F1[1, 1 + 1/(2*b*n)
, 2 + 1/(2*b*n), E^(2*(a + b*Log[c*x^n]))] + 4*(-1 + 4*b^2*n^2)*Hypergeomet
ric2F1[1, 1/(2*b*n), 1 + 1/(2*b*n), E^(2*(a + b*Log[c*x^n]))] + Csch[a + b*
Log[c*x^n]]^3*((1 - 12*b^2*n^2)*Cosh[a + b*Log[c*x^n]] + (-1 + 4*b^2*n^2)*C
osh[3*(a + b*Log[c*x^n])] - 4*b*n*Sinh[a + b*Log[c*x^n]])))/(24*b^3*n^3)
```

Maple [F]
time = 1.62, size = 0, normalized size = 0.00

$$\int \operatorname{csch}(a+b\ln(c\,x^n))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a+b*ln(c*x^n))^4,x)
```

```
[Out] int(csch(a+b*ln(c*x^n))^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^4,x, algorithm="maxima")
```

```
[Out] 16*(4*b^2*n^2 - 1)*integrate(1/96/(b^3*c^b*n^3*e^(b*log(x^n) + a) + b^3*n^3
), x) - 16*(4*b^2*n^2 - 1)*integrate(1/96/(b^3*c^b*n^3*e^(b*log(x^n) + a) -
 b^3*n^3), x) - 1/3*((2*b*c^(4*b)*n + c^(4*b))*x*e^(4*b*log(x^n) + 4*a) + 2
*(6*b^2*c^(2*b)*n^2 - b*c^(2*b)*n - c^(2*b))*x*e^(2*b*log(x^n) + 2*a) - (4*
b^2*n^2 - 1)*x)/(b^3*c^(6*b)*n^3*e^(6*b*log(x^n) + 6*a) - 3*b^3*c^(4*b)*n^3
*e^(4*b*log(x^n) + 4*a) + 3*b^3*c^(2*b)*n^3*e^(2*b*log(x^n) + 2*a) - b^3*n^
3)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

```
[Out] integral(csch(b*log(c*x^n) + a)^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^{4}\left(a + b\log\left(cx^{n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*ln(c*x**n))**4,x)
```

```
[Out] Integral(csch(a + b*log(c*x**n))**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^4,x, algorithm="giac")
```

```
[Out] integrate(csch(b*log(c*x^n) + a)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sinh\left(a+b\,\ln\left(c\,x^n\right)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(a + b*log(c*x^n))^4,x)
[Out] int(1/sinh(a + b*log(c*x^n))^4, x)
```

3.160 $\int \left(-\left(\left(1-b^2n^2\right) \operatorname{csch}(a+b\log\left(cx^n\right))\right)+2b^2n^2\operatorname{csch}$

Optimal. Leaf size=42

$$-x\operatorname{csch}(a+b\log\left(cx^n\right))-bnx\coth\left(a+b\log\left(cx^n\right)\right)\operatorname{csch}(a+b\log\left(cx^n\right))$$

```
[Out] -x*csch(a+b*ln(c*x^n))-b*n*x*coth(a+b*ln(c*x^n))*csch(a+b*ln(c*x^n))
```

Rubi [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.11, antiderivative size = 137, normalized size of antiderivative = 3.26, number of steps used = 9, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.089$, Rules used = {5665, 5667, 269, 371}

$$2e^a x(1-bn)\left(cx^n\right)^b {}_2F_1\left(1,\frac{b+\frac{1}{n}}{2b};\frac{1}{2}\left(3+\frac{1}{bn}\right);e^{2a}\left(cx^n\right)^{2b}\right)-\frac{16e^{3a}b^2n^2x\left(cx^n\right)^{3b} {}_2F_1\left(3,\frac{3b+\frac{1}{n}}{2b};\frac{1}{2}\left(5+\frac{1}{bn}\right);e^{2a}\left(cx^n\right)^{2b}\right)}{3bn+1}$$

Antiderivative was successfully verified.

```
[In] Int[-((1 - b^2*n^2)*Csch[a + b*Log[c*x^n]]) + 2*b^2*n^2*Csch[a + b*Log[c*x^
n]]^3,x]
```

```
[Out] 2*E^a*(1 - b*n)*x*(c*x^n)^b*Hypergeometric2F1[1, (b + n^(-1))/(2*b), (3 + 1
/(b*n))/2, E^(2*a)*(c*x^n)^(2*b)] - (16*b^2*E^(3*a)*n^2*x*(c*x^n)^(3*b)*Hyp
ergeometric2F1[3, (3*b + n^(-1))/(2*b), (5 + 1/(b*n))/2, E^(2*a)*(c*x^n)^(2
*b)])/(1 + 3*b*n)
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5665

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5667

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*
```

```
d)))^p)), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\int \left(-\left(1-b^2n^2\right)\operatorname{csch}(a+b\log(cx^n))+2b^2n^2\operatorname{csch}^3(a+b\log(cx^n))\right)\,dx = \left(2b^2n^2\right)\int \operatorname{csch}^3(a+b\log(cx^n))\,d$$

$$= \left(2b^2nx(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+}\right.$$

$$= \left(16b^2e^{-3a}nx(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int\right.$$

$$= \left(16b^2e^{-3a}nx(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int\right.$$

$$= 2e^a(1-bn)x(cx^n)^b\ {}_2F_1\left(1,\frac{b+\frac{1}{n}}{2b};\right.$$

Mathematica [A]
time = 0.31, size = 30, normalized size = 0.71

$$-x(1+bn\coth(a+b\log(cx^n)))\operatorname{csch}(a+b\log(cx^n))$$

Antiderivative was successfully verified.

```
[In] Integrate[-((1 - b^2*n^2)*Csch[a + b*Log[c*x^n]]) + 2*b^2*n^2*Csch[a + b*Lo
g[c*x^n]]^3,x]
[Out] -(x*(1 + b*n*Coth[a + b*Log[c*x^n]])*Csch[a + b*Log[c*x^n]])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 7.54, size = 509, normalized size = 12.12

method	result
risch	$-\frac{2c^b(x^n)^b x\left(nb(x^n)^{2b}c^{2b}e^{3a}e^{-\frac{3ib\pi\operatorname{csgn}(icx^n)^3}{2}}e^{\frac{3ib\operatorname{csgn}(icx^n)^2\operatorname{csgn}(ic)\pi}{2}}e^{\frac{3ib\operatorname{csgn}(icx^n)^2\operatorname{csgn}(ix^n)\pi}{2}}e^{-\frac{3ib\operatorname{csgn}(icx^n)\operatorname{csgn}(ic)\operatorname{csgn}(ix^n)\pi}{2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(-b^2*n^2+1)*csch(a+b*ln(c*x^n))+2*b^2*n^2*csch(a+b*ln(c*x^n))^3,x,met
hod=_RETURNVERBOSE)
```

```
[Out] -2*c^b*(x^n)^b*x/(((x^n)^b)^2*(c^b)^2*exp(2*a)*exp(-I*b*csgn(I*c*x^n)^3*Pi)
*exp(I*b*csgn(I*c*x^n)^2*csgn(I*c)*Pi)*exp(I*b*csgn(I*c*x^n)^2*csgn(I*x^n)*
Pi)*exp(-I*b*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)*Pi)-1)^2*(n*b*((x^n)^b)^2*
(c^b)^2*exp(3*a)*exp(-3/2*I*b*csgn(I*c*x^n)^3*Pi)*exp(3/2*I*b*csgn(I*c*x^n)
^2*csgn(I*c)*Pi)*exp(3/2*I*b*csgn(I*c*x^n)^2*csgn(I*x^n)*Pi)*exp(-3/2*I*b*c
sgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)*Pi)+exp(a)*exp(-1/2*I*b*csgn(I*c*x^n)^3*
Pi)*exp(1/2*I*b*csgn(I*c*x^n)^2*csgn(I*c)*Pi)*exp(1/2*I*b*csgn(I*c*x^n)^2*c
sgn(I*x^n)*Pi)*exp(-1/2*I*b*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)*Pi)*b*n+((x
^n)^b)^2*(c^b)^2*exp(3*a)*exp(-3/2*I*b*csgn(I*c*x^n)^3*Pi)*exp(3/2*I*b*csgn
(I*c*x^n)^2*csgn(I*c)*Pi)*exp(3/2*I*b*csgn(I*c*x^n)^2*csgn(I*x^n)*Pi)*exp(-
3/2*I*b*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)*Pi)-exp(a)*exp(-1/2*I*b*csgn(I*
c*x^n)^3*Pi)*exp(1/2*I*b*csgn(I*c*x^n)^2*csgn(I*c)*Pi)*exp(1/2*I*b*csgn(I*c
*x^n)^2*csgn(I*x^n)*Pi)*exp(-1/2*I*b*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)*Pi
))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(42) = 84$.
time = 0.49, size = 95, normalized size = 2.26

$$-\frac{2\left(\left(bc^{3\,b}n+c^{3\,b}\right)xe^{\left(3\,b\log\left(x^{n}\right)+3\,a\right)}+\left(bc^{b}n-c^{b}\right)xe^{\left(b\log\left(x^{n}\right)+a\right)}\right)}{c^{4\,b}e^{\left(4\,b\log\left(x^{n}\right)+4\,a\right)}-2\,c^{2\,b}e^{\left(2\,b\log\left(x^{n}\right)+2\,a\right)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(-b^2*n^2+1)*csch(a+b*log(c*x^n))+2*b^2*n^2*csch(a+b*log(c*x^n))
^3,x, algorithm="maxima")
```

```
[Out] -2*((b*c^(3*b)*n + c^(3*b))*x*e^(3*b*log(x^n) + 3*a) + (b*c^b*n - c^b)*x*e^
(b*log(x^n) + a))/(c^(4*b)*e^(4*b*log(x^n) + 4*a) - 2*c^(2*b)*e^(2*b*log(x^
n) + 2*a) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(42) = 84$.
time = 0.38, size = 187, normalized size = 4.45

$$-\frac{2\left((bn+1)x\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2\,(bn+1)x\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+(bn+1)x\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+(bn-1)x\right)}{\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}+3\,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}+3\left(\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}-1\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(-b^2*n^2+1)*csch(a+b*log(c*x^n))+2*b^2*n^2*csch(a+b*log(c*x^n))
^3,x, algorithm="fricas")
```

```
[Out] -2*((b*n + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*(b*n + 1)*x*cosh(b*n*
log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + (b*n + 1)*x*sinh(b
*n*log(x) + b*log(c) + a)^2 + (b*n - 1)*x)/(cosh(b*n*log(x) + b*log(c) + a)
^3 + 3*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^2 +
sinh(b*n*log(x) + b*log(c) + a)^3 + 3*(cosh(b*n*log(x) + b*log(c) + a)^2 -
1)*sinh(b*n*log(x) + b*log(c) + a) - cosh(b*n*log(x) + b*log(c) + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(2b^2n^2\operatorname{csch}^2\left(a+b\log\left(cx^n\right)\right)+b^2n^2-1\right)\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(-b**2*n**2+1)*csch(a+b*ln(c*x**n))+2*b**2*n**2*csch(a+b*ln(c*x*
*n))**3,x)
```

```
[Out] Integral((2*b**2*n**2*csch(a + b*log(c*x**n))**2 + b**2*n**2 - 1)*csch(a +
b*log(c*x**n)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(42) = 84.
time = 0.59, size = 215, normalized size = 5.12

$$-\frac{2\,bc^{3\,b}nxx^{3\,bn}e^{(3\,a)}}{c^{4\,b}x^{4\,bn}e^{(4\,a)}-2\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+1}-\frac{2\,bc^{b}nxx^{bn}e^{a}}{c^{4\,b}x^{4\,bn}e^{(4\,a)}-2\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+1}-\frac{2\,c^{3\,b}xx^{3\,bn}e^{(3\,a)}}{c^{4\,b}x^{4\,bn}e^{(4\,a)}-2\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+1}+\frac{2\,c^{b}xx^{bn}e^{a}}{c^{4\,b}x^{4\,bn}e^{(4\,a)}-2\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(-b^2*n^2+1)*csch(a+b*log(c*x^n))+2*b^2*n^2*csch(a+b*log(c*x^n))
^3,x, algorithm="giac")
```

```
[Out] -2*b*c^(3*b)*n*x*x^(3*b*n)*e^(3*a)/(c^(4*b)*x^(4*b*n)*e^(4*a) - 2*c^(2*b)*x
^(2*b*n)*e^(2*a) + 1) - 2*b*c^b*n*x*x^(b*n)*e^a/(c^(4*b)*x^(4*b*n)*e^(4*a)
- 2*c^(2*b)*x^(2*b*n)*e^(2*a) + 1) - 2*c^(3*b)*x*x^(3*b*n)*e^(3*a)/(c^(4*b)
*x^(4*b*n)*e^(4*a) - 2*c^(2*b)*x^(2*b*n)*e^(2*a) + 1) + 2*c^b*x*x^(b*n)*e^a
/(c^(4*b)*x^(4*b*n)*e^(4*a) - 2*c^(2*b)*x^(2*b*n)*e^(2*a) + 1)
```

Mupad [B]
time = 1.52, size = 65, normalized size = 1.55

$$-\frac{2\,x\,\mathrm{e}^{a}\,{\left(c\,x^n\right)}^b\,\left(b\,n+\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}+b\,n\,\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}-1\right)}{{\left(\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}-1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*n^2 - 1)/sinh(a + b*log(c*x^n)) + (2*b^2*n^2)/sinh(a + b*log(c*x^n
))^3,x)
```

```
[Out] -(2*x*exp(a)*(c*x^n)^b*(b*n + exp(2*a)*(c*x^n)^(2*b) + b*n*exp(2*a)*(c*x^n)
^(2*b) - 1))/(exp(2*a)*(c*x^n)^(2*b) - 1)^2
```

3.161 $\int \operatorname{csch}^3\left(a+2\log\left(c\sqrt{x}\right)\right)\,dx$

Optimal. Leaf size=26

$$-\frac{2c^6e^{-a}}{\left(c^4-\frac{e^{-2a}}{x^2}\right)^2}$$

```
[Out] -2*c^6/exp(a)/(c^4-1/exp(2*a)/x^2)^2
```

Rubi [A]

time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5665, 5667, 267}

$$-\frac{2e^{-a}c^6}{\left(c^4-\frac{e^{-2a}}{x^2}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + 2*Log[c*Sqrt[x]]]^3,x]
```

```
[Out] (-2*c^6)/(E^a*(c^4 - 1/(E^(2*a)*x^2))^2)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5665

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5667

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*
d)))^p)), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^3\left(a+2\log\left(c\sqrt{x}\right)\right)\,dx = \frac{2\text{Subst}\left(\int x\operatorname{csch}^3(a+2\log(x))\,dx,x,c\sqrt{x}\right)}{c^2}$$
$$= \frac{(16e^{-3a})\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{e^{-2a}}{x^4}\right)^3x^5}\,dx,x,c\sqrt{x}\right)}{c^2}$$
$$= -\frac{2c^6e^{-a}}{\left(c^4-\frac{e^{-2a}}{x^2}\right)^2}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 62 vs. 2(26) = 52.
time = 0.09, size = 62, normalized size = 2.38

$$\frac{2(\cosh(a)-\sinh(a))\left(-2c^4x^2+\cosh^2(a)-2\cosh(a)\sinh(a)+\sinh^2(a)\right)}{c^2\left((-1+c^4x^2)\cosh(a)+(1+c^4x^2)\sinh(a)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + 2*Log[c*Sqrt[x]]]^3,x]
```

```
[Out] (2*(Cosh[a] - Sinh[a])*(-2*c^4*x^2 + Cosh[a]^2 - 2*Cosh[a]*Sinh[a] + Sinh[a
]^2))/(c^2*((-1 + c^4*x^2)*Cosh[a] + (1 + c^4*x^2)*Sinh[a])^2)
```

Maple [**F**]
time = 2.72, size = 0, normalized size = 0.00

$$\int \operatorname{csch}\left(a+2\ln\left(c\sqrt{x}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a+2*ln(c*x^(1/2)))^3,x)
```

```
[Out] int(csch(a+2*ln(c*x^(1/2)))^3,x)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(24) = 48.
time = 0.28, size = 76, normalized size = 2.92

$$-\frac{2\left(\frac{2\,c^4x^2e^{(2\,a)}}{c^8x^4e^{(5\,a)}-2\,c^4x^2e^{(3\,a)}+e^a}-\frac{1}{c^8x^4e^{(5\,a)}-2\,c^4x^2e^{(3\,a)}+e^a}\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+2*log(c*x^(1/2)))^3,x, algorithm="maxima")
```

```
[Out] -2*(2*c^4*x^2*e^(2*a)/(c^8*x^4*e^(5*a) - 2*c^4*x^2*e^(3*a) + e^a) - 1/(c^8*
x^4*e^(5*a) - 2*c^4*x^2*e^(3*a) + e^a))/c^2
```

Fricas [A]
time = 0.39, size = 48, normalized size = 1.85

$$-\frac{2\left(2\,c^4x^2e^{(2\,a)}-1\right)}{c^{10}x^4e^{(5\,a)}-2\,c^6x^2e^{(3\,a)}+c^2e^a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+2*log(c*x^(1/2)))^3,x, algorithm="fricas")
[Out] -2*(2*c^4*x^2*e^(2*a) - 1)/(c^10*x^4*e^(5*a) - 2*c^6*x^2*e^(3*a) + c^2*e^a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^3\left(a+2\log\left(c\sqrt{x}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+2*ln(c*x**(1/2)))**3,x)
[Out] Integral(csch(a + 2*log(c*sqrt(x)))**3, x)
```

Giac [A]
time = 0.40, size = 38, normalized size = 1.46

$$-\frac{2\left(2\,c^4x^2e^{(2\,a)}-1\right)e^{(-a)}}{\left(c^4x^2e^{(2\,a)}-1\right)^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+2*log(c*x^(1/2)))^3,x, algorithm="giac")
[Out] -2*(2*c^4*x^2*e^(2*a) - 1)*e^(-a)/((c^4*x^2*e^(2*a) - 1)^2*c^2)
```

Mupad [B]
time = 1.65, size = 48, normalized size = 1.85

$$\frac{\frac{2\,\mathrm{e}^{-a}}{c^2}-4\,c^2\,x^2\,\mathrm{e}^a}{\mathrm{e}^{4\,a}\,c^8\,x^4-2\,\mathrm{e}^{2\,a}\,c^4\,x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(a + 2*log(c*x^(1/2)))^3,x)
[Out] ((2*exp(-a))/c^2 - 4*c^2*x^2*exp(a))/(c^8*x^4*exp(4*a) - 2*c^4*x^2*exp(2*a)
 + 1)
```

3.162 $\int \operatorname{csch}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)\,dx$

Optimal. Leaf size=26

$$\frac{2c^2e^{-3a}}{\left(e^{-2a}-\frac{c^4}{x^2}\right)^2}$$

[Out] 2*c^2/exp(3*a)/(exp(-2*a)-c^4/x^2)^2

Rubi [A]
time = 0.04, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {5665, 5667, 269, 267}

$$\frac{2e^{-3a}c^2}{\left(e^{-2a}-\frac{c^4}{x^2}\right)^2}$$

Antiderivative was successfully verified.

[In] Int[Csch[a + 2*Log[c/Sqrt[x]]]^3,x]

[Out] (2*c^2)/(E^(3*a)*(E^(-2*a) - c^4/x^2)^2)

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 5665

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5667

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*
d)))^p)), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)\,dx &= -\left(\left(2c^2\right)\operatorname{Subst}\left(\int \frac{\operatorname{csch}^3(a+2\log(x))}{x^3}\,dx, x, \frac{c}{\sqrt{x}}\right)\right)\\ &= -\left(\left(16c^2e^{-3a}\right)\operatorname{Subst}\left(\int \frac{1}{\left(1-\frac{e^{-2a}}{x^4}\right)^3 x^9}\,dx, x, \frac{c}{\sqrt{x}}\right)\right)\\ &= -\left(\left(16c^2e^{-3a}\right)\operatorname{Subst}\left(\int \frac{x^3}{\left(-e^{-2a}+x^4\right)^3}\,dx, x, \frac{c}{\sqrt{x}}\right)\right)\\ &= \frac{2c^2e^{-3a}}{\left(e^{-2a}-\frac{c^4}{x^2}\right)^2}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 65 vs. 2(26) = 52.
time = 0.07, size = 65, normalized size = 2.50

$$-\frac{2c^6\left(\left(c^4-2x^2\right)\cosh(a)+\left(c^4+2x^2\right)\sinh(a)\right)\left(\cosh(2a)+\sinh(2a)\right)}{\left(\left(-c^4+x^2\right)\cosh(a)-\left(c^4+x^2\right)\sinh(a)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + 2*Log[c/Sqrt[x]]]^3,x]
[Out] (-2*c^6*((c^4 - 2*x^2)*Cosh[a] + (c^4 + 2*x^2)*Sinh[a])*(Cosh[2*a] + Sinh[2
*a]))/((-c^4 + x^2)*Cosh[a] - (c^4 + x^2)*Sinh[a])^2
```

Maple [F]
time = 3.84, size = 0, normalized size = 0.00

$$\int \operatorname{csch}\left(a+2\ln\left(\frac{c}{\sqrt{x}}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a+2*ln(c/x^(1/2)))^3,x)
[Out] int(csch(a+2*ln(c/x^(1/2)))^3,x)
```

Maxima [A]
time = 0.28, size = 49, normalized size = 1.88

$$-\frac{2\left(c^{10}e^{(5a)}-2c^6x^2e^{(3a)}\right)}{c^8e^{(4a)}-2c^4x^2e^{(2a)}+x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+2*log(c/x^(1/2)))^3,x, algorithm="maxima")
```

```
[Out] -2*(c^10*e^(5*a) - 2*c^6*x^2*e^(3*a))/(c^8*e^(4*a) - 2*c^4*x^2*e^(2*a) + x^
4)
```

Fricas [A]
time = 0.39, size = 49, normalized size = 1.88

$$-\frac{2\left(c^{10}e^{(5\,a)}-2\,c^{6}x^{2}e^{(3\,a)}\right)}{c^{8}e^{(4\,a)}-2\,c^{4}x^{2}e^{(2\,a)}+x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+2*log(c/x^(1/2)))^3,x, algorithm="fricas")
```

```
[Out] -2*(c^10*e^(5*a) - 2*c^6*x^2*e^(3*a))/(c^8*e^(4*a) - 2*c^4*x^2*e^(2*a) + x^
4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^{3}\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+2*ln(c/x**(1/2)))**3,x)
```

```
[Out] Integral(csch(a + 2*log(c/sqrt(x)))**3, x)
```

Giac [A]
time = 0.39, size = 39, normalized size = 1.50

$$-\frac{2\left(c^{10}e^{(5\,a)}-2\,c^{6}x^{2}e^{(3\,a)}\right)}{\left(c^{4}e^{(2\,a)}-x^{2}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+2*log(c/x^(1/2)))^3,x, algorithm="giac")
```

```
[Out] -2*(c^10*e^(5*a) - 2*c^6*x^2*e^(3*a))/(c^4*e^(2*a) - x^2)^2
```

Mupad [B]
time = 1.59, size = 36, normalized size = 1.38

$$\frac{2\,c^{2}\,x^{4}\,\mathrm{e}^{a}}{\mathrm{e}^{4\,a}\,c^{8}-2\,\mathrm{e}^{2\,a}\,c^{4}\,x^{2}+x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(a + 2*log(c/x^(1/2)))^3,x)
```

```
[Out] (2*c^2*x^4*exp(a))/(c^8*exp(4*a) + x^4 - 2*c^4*x^2*exp(2*a))
```

3.163 $\int \operatorname{csch}^p\left(a+\frac{\log(cx^n)}{n(-2+p)}\right)\,dx$

Optimal. Leaf size=90

$$-\frac{e^{2a}(2-p)x(cx^n)^{-\frac{2}{n(2-p)}}\left(1-e^{-2a}(cx^n)^{\frac{2}{n(2-p)}}\right)\operatorname{csch}^p\left(a-\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

```
[Out] -1/2*exp(2*a)*(2-p)*x*(1-(c*x^n)^(2/n/(2-p))/exp(2*a))*csch(a-ln(c*x^n)/n/(
2-p))^p/(1-p)/((c*x^n)^(2/n/(2-p)))
```

Rubi [A]
time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {5665, 5669, 267}

$$-\frac{e^{2a}(2-p)x(cx^n)^{-\frac{2}{n(2-p)}}\left(1-e^{-2a}(cx^n)^{\frac{2}{n(2-p)}}\right)\operatorname{csch}^p\left(a-\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + Log[c*x^n]/(n*(-2 + p))]^p,x]
```

```
[Out] -1/2*(E^(2*a)*(2 - p)*x*(1 - (c*x^n)^(2/(n*(2 - p)))/E^(2*a))*Csch[a - Log[
c*x^n]/(n*(2 - p))]^p)/((1 - p)*(c*x^n)^(2/(n*(2 - p))))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5665

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5669

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^p\left(a+\frac{\log(cx^n)}{n(-2+p)}\right)\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{csch}^p\left(a+\frac{\log(x)}{n(-2+p)}\right)\,dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x(cx^n)^{-\frac{1}{n}+\frac{p}{n(-2+p)}}\left(1-e^{-2a}(cx^n)^{-\frac{2}{n(-2+p)}}\right)^p\operatorname{csch}^p\left(a+\frac{\log(cx^n)}{n(-2+p)}\right)\right)\operatorname{Subst}\left(\int\right.}{n}$$

$$= -\frac{e^{2a}(2-p)x(cx^n)^{-\frac{2}{n(2-p)}}\left(1-e^{-2a}(cx^n)^{\frac{2}{n(2-p)}}\right)\operatorname{csch}^p\left(a-\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

Mathematica [A]
time = 3.57, size = 115, normalized size = 1.28

$$\frac{2^{-1+p}(-2+p)x\left(\frac{e^a(cx^n)^{\frac{1}{n(-2+p)}}}{-1+e^{2a}(cx^n)^{\frac{2}{n(-2+p)}}}\right)^p\left(1+e^{2a}(cx^n)^{\frac{2}{n(-2+p)}}\left(-1+\left(1-e^{-2a}(cx^n)^{-\frac{2}{n(-2+p)}}\right)^p\right)\right)}{-1+p}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + Log[c*x^n]/(n*(-2 + p))]^p,x]
```

```
[Out] (2^(-1 + p)*(-2 + p)*x*((E^a*(c*x^n)^(1/(n*(-2 + p))))/(-1 + E^(2*a)*(c*x^n
)^(2/(n*(-2 + p)))))^p*(1 + E^(2*a)*(c*x^n)^(2/(n*(-2 + p)))*(-1 + (1 - 1/(
E^(2*a)*(c*x^n)^(2/(n*(-2 + p)))))^p)))/(-1 + p)
```

Maple [F]
time = 2.20, size = 0, normalized size = 0.00

$$\int \operatorname{csch}\left(a+\frac{\ln(c\,x^n)}{n\,(-2+p)}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a+ln(c*x^n)/n/(-2+p))^p,x)
```

```
[Out] int(csch(a+ln(c*x^n)/n/(-2+p))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+log(c*x^n)/n/(-2+p))^p,x, algorithm="maxima")
```

```
[Out] integrate(csch(a + log(c*x^n)/(n*(p - 2)))^p, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 475 vs. $2(76) = 152$.
time = 0.48, size = 475, normalized size = 5.28

$$-\frac{(p-2)x\cosh\left(p\log\left(\frac{2\left(\cosh\left(\frac{anp-2an+n\log(x)+\log(c)}{np-2n}\right)+\sinh\left(\frac{anp-2an+n\log(x)+\log(c)}{np-2n}\right)\right)}{\cosh\left(\frac{anp-2an+n\log(x)+\log(c)}{np-2n}\right)^2+2\cosh\left(\frac{anp-2an+n\log(x)+\log(c)}{np-2n}\right)\sinh\left(\frac{anp-2an+n\log(x)+\log(c)}{np-2n}\right)+\sinh\left(\frac{anp-2an+n\log(x)+\log(c)}{np-2n}\right)^2-1}\right)\right)\sinh\left(\frac{anp-2an+n\log(x)+\log(c)}{np-2n}\right)+(p-2)x\sinh\left(p\log\left(\frac{2\left(\cosh\left(\frac{anp-2an+n\log(x)+\log(c)}{np-2n}\right)+\sinh\left(\frac{anp-2an+n\log(x)+\log(c)}{np-2n}\right)\right)}{\cosh\left(\frac{anp-2an+n\log(x)+\log(c)}{np-2n}\right)^2+2\cosh\left(\frac{anp-2an+n\log(x)+\log(c)}{np-2n}\right)\sinh\left(\frac{anp-2an+n\log(x)+\log(c)}{np-2n}\right)+\sinh\left(\frac{anp-2an+n\log(x)+\log(c)}{np-2n}\right)^2-1}\right)\right)\sinh\left(\frac{anp-2an+n\log(x)+\log(c)}{np-2n}\right)}{(p-1)\cosh\left(\frac{anp-2an+n\log(x)+\log(c)}{np-2n}\right)-(p-1)\sinh\left(\frac{anp-2an+n\log(x)+\log(c)}{np-2n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+log(c*x^n)/n/(-2+p))^p,x, algorithm="fricas")
```

```
[Out] -((p - 2)*x*cosh(p*log(2*(cosh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2
*n)) + sinh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n)))/(cosh((a*n*p
- 2*a*n + n*log(x) + log(c))/(n*p - 2*n))^2 + 2*cosh((a*n*p - 2*a*n + n*log
(x) + log(c))/(n*p - 2*n))*sinh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p -
2*n)) + sinh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n))^2 - 1)))*sinh
((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n)) + (p - 2)*x*sinh(p*log(2*
(cosh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n)) + sinh((a*n*p - 2*a*
n + n*log(x) + log(c))/(n*p - 2*n)))/(cosh((a*n*p - 2*a*n + n*log(x) + log(
c))/(n*p - 2*n))^2 + 2*cosh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n)
)*sinh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n)) + sinh((a*n*p - 2*a
*n + n*log(x) + log(c))/(n*p - 2*n))^2 - 1)))*sinh((a*n*p - 2*a*n + n*log(x
) + log(c))/(n*p - 2*n)))/((p - 1)*cosh((a*n*p - 2*a*n + n*log(x) + log(c))
/(n*p - 2*n)) - (p - 1)*sinh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n
)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^p\left(a+\frac{\log(cx^n)}{n(p-2)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+ln(c*x**n)/n/(-2+p))**p,x)
```

```
[Out] Integral(csch(a + log(c*x**n)/(n*(p - 2)))**p, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+log(c*x^n)/n/(-2+p))^p,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{1}{\sinh\left(a+\frac{\ln(c\,x^n)}{n\,(p-2)}\right)}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sinh(a + log(c*x^n)/(n*(p - 2))))^p,x)
[Out] int((1/sinh(a + log(c*x^n)/(n*(p - 2))))^p, x)
```

3.164 $\int \operatorname{csch}^p\left(a-\frac{\log(cx^n)}{n(-2+p)}\right)\,dx$

Optimal. Leaf size=66

$$\frac{(2-p)x\left(1-e^{-2a}(cx^n)^{-\frac{2}{n(2-p)}}\right)\operatorname{csch}^p\left(a+\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

```
[Out] 1/2*(2-p)*x*(1-1/exp(2*a)/((c*x^n)^(2/n/(2-p))))*csch(a+ln(c*x^n)/n/(2-p))^
p/(1-p)
```

Rubi [A]
time = 0.07, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5665, 5669, 270}

$$\frac{(2-p)x\left(1-e^{-2a}(cx^n)^{-\frac{2}{n(2-p)}}\right)\operatorname{csch}^p\left(a+\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a - Log[c*x^n]/(n*(-2 + p))]^p,x]
```

```
[Out] ((2 - p)*x*(1 - 1/(E^(2*a)*(c*x^n)^(2/(n*(2 - p)))))*Csch[a + Log[c*x^n]/(n
*(2 - p))]^p)/(2*(1 - p))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5665

```
Int[Csch[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Csch[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5669

```
Int[Csch[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Dist[Csch[d*(a + b*Log[x])]^p*((1 - 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^p\left(a-\frac{\log\left(cx^n\right)}{n(-2+p)}\right)\,dx=\frac{\left(x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{csch}^p\left(a-\frac{\log(x)}{n(-2+p)}\right)\,dx,x,cx^n\right)}{n}$$
$$=\frac{\left(x(cx^n)^{-\frac{1}{n}-\frac{p}{n(-2+p)}}\left(1-e^{-2a}(cx^n)^{\frac{2}{n(-2+p)}}\right)^p\operatorname{csch}^p\left(a-\frac{\log(cx^n)}{n(-2+p)}\right)\right)\operatorname{Subst}\left(\int x\right.}{n}$$
$$=\frac{(2-p)x\left(1-e^{-2a}(cx^n)^{-\frac{2}{n(2-p)}}\right)\operatorname{csch}^p\left(a+\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 140 vs. 2(66) = 132.
time = 3.35, size = 140, normalized size = 2.12

$$\frac{2^{-1+p}e^{-\frac{2ap}{-2+p}}(-2+p)x\left(e^{\frac{2ap}{-2+p}}-e^{\frac{4a}{-2+p}}(cx^n)^{\frac{2}{n(-2+p)}}\right)\left(-\frac{e^{\frac{a(2+p)}{-2+p}}(cx^n)^{\frac{1}{n(-2+p)}}}{-e^{\frac{2ap}{-2+p}}+e^{\frac{4a}{-2+p}}(cx^n)^{\frac{2}{n(-2+p)}}}\right)^p}{-1+p}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csch[a - Log[c*x^n]/(n*(-2 + p))]^p,x]
```

```
[Out] (2^(-1 + p)*(-2 + p)*x*(E^((2*a*p)/(-2 + p)) - E^((4*a)/(-2 + p))*(c*x^n)^(
2/(n*(-2 + p))))*(-((E^((a*(2 + p))/(-2 + p))*(c*x^n)^(1/(n*(-2 + p))))/(-E
^((2*a*p)/(-2 + p)) + E^((4*a)/(-2 + p))*(c*x^n)^(2/(n*(-2 + p))))))^p)/(E^
((2*a*p)/(-2 + p))*(-1 + p))
```

Maple [F]
time = 2.13, size = 0, normalized size = 0.00

$$\int \operatorname{csch}\left(a-\frac{\ln\left(c\,x^n\right)}{n\left(-2+p\right)}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a-ln(c*x^n)/n/(-2+p))^p,x)
```

```
[Out] int(csch(a-ln(c*x^n)/n/(-2+p))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a-log(c*x^n)/n/(-2+p))^p,x, algorithm="maxima")
```

```
[Out] integrate((-csch(-a + log(c*x^n)/(n*(p - 2))))^p, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 539 vs. $2(55) = 110$.
time = 0.38, size = 539, normalized size = 8.17

$$-\frac{(p-2)x\cosh\left(p\log\left(-\frac{2\left(\cosh\left(-\frac{anp-2an-n\log(x)-\log(c)}{np-2n}\right)+\sinh\left(-\frac{anp-2an-n\log(x)-\log(c)}{np-2n}\right)\right)}{\cosh\left(-\frac{anp-2an-n\log(x)-\log(c)}{np-2n}\right)^2+2\cosh\left(-\frac{anp-2an-n\log(x)-\log(c)}{np-2n}\right)\sinh\left(-\frac{anp-2an-n\log(x)-\log(c)}{np-2n}\right)+\sinh\left(-\frac{anp-2an-n\log(x)-\log(c)}{np-2n}\right)^2-1}\right)\right)\sinh\left(-\frac{anp-2an-n\log(x)-\log(c)}{np-2n}\right)+(p-2)x\sinh\left(p\log\left(-\frac{2\left(\cosh\left(-\frac{anp-2an-n\log(x)-\log(c)}{np-2n}\right)+\sinh\left(-\frac{anp-2an-n\log(x)-\log(c)}{np-2n}\right)\right)}{\cosh\left(-\frac{anp-2an-n\log(x)-\log(c)}{np-2n}\right)^2+2\cosh\left(-\frac{anp-2an-n\log(x)-\log(c)}{np-2n}\right)\sinh\left(-\frac{anp-2an-n\log(x)-\log(c)}{np-2n}\right)+\sinh\left(-\frac{anp-2an-n\log(x)-\log(c)}{np-2n}\right)^2-1}\right)\right)\sinh\left(-\frac{anp-2an-n\log(x)-\log(c)}{np-2n}\right)}{(p-1)\cosh\left(-\frac{anp-2an-n\log(x)-\log(c)}{np-2n}\right)-(p-1)\sinh\left(-\frac{anp-2an-n\log(x)-\log(c)}{np-2n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a-log(c*x^n)/n/(-2+p))^p,x, algorithm="fricas")
```

```
[Out] -((p - 2)*x*cosh(p*log(-2*(cosh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p -
 2*n)) + sinh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)))/(cosh(-(a*
n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n))^2 + 2*cosh(-(a*n*p - 2*a*n -
n*log(x) - log(c))/(n*p - 2*n))*sinh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(
n*p - 2*n)) + sinh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n))^2 - 1)
))*sinh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)) + (p - 2)*x*sinh(
p*log(-2*(cosh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)) + sinh(-(a
*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)))/(cosh(-(a*n*p - 2*a*n - n*l
og(x) - log(c))/(n*p - 2*n))^2 + 2*cosh(-(a*n*p - 2*a*n - n*log(x) - log(c)
)/(n*p - 2*n))*sinh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)) + sin
h(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n))^2 - 1)))*sinh(-(a*n*p -
 2*a*n - n*log(x) - log(c))/(n*p - 2*n)))/((p - 1)*cosh(-(a*n*p - 2*a*n - n
*log(x) - log(c))/(n*p - 2*n)) - (p - 1)*sinh(-(a*n*p - 2*a*n - n*log(x) -
log(c))/(n*p - 2*n)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^{p}\left(a - \frac{\log\left(cx^{n}\right)}{n\left(p-2\right)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a-ln(c*x**n)/n/(-2+p))**p,x)
```

```
[Out] Integral(csch(a - log(c*x**n)/(n*(p - 2)))**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a-log(c*x^n)/n/(-2+p))^p,x, algorithm="giac")
```

```
[Out] integrate(csch(a - log(c*x^n)/(n*(p - 2)))^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \left(\frac{1}{\sinh\left(a-\frac{\ln(c\,x^n)}{n\,(p-2)}\right)}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sinh(a - log(c*x^n)/(n*(p - 2))))^p,x)
[Out] int((1/sinh(a - log(c*x^n)/(n*(p - 2))))^p, x)
```

3.165 $\int \frac{\operatorname{csch}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=20

$$-\frac{\tanh^{-1}\left(\cosh\left(a+b\log\left(cx^n\right)\right)\right)}{bn}$$

```
[Out] -arctanh(cosh(a+b*ln(c*x^n)))/b/n
```

Rubi [A]
time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3855}

$$-\frac{\tanh^{-1}\left(\cosh\left(a+b\log\left(cx^n\right)\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*Log[c*x^n]]/x,x]
```

```
[Out] -(ArcTanh[Cosh[a + b*Log[c*x^n]]]/(b*n))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}(a+b\log(cx^n))}{x}\,dx &= \frac{\operatorname{Subst}\left(\int \operatorname{csch}(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n}\\ &= -\frac{\tanh^{-1}\left(\cosh\left(a+b\log\left(cx^n\right)\right)\right)}{bn}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 54 vs. 2(20) = 40.
time = 0.05, size = 54, normalized size = 2.70

$$-\frac{\log\left(\cosh\left(\frac{a}{2}+\frac{1}{2}b\log\left(cx^n\right)\right)\right)}{bn}+\frac{\log\left(\sinh\left(\frac{a}{2}+\frac{1}{2}b\log\left(cx^n\right)\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*Log[c*x^n]]/x,x]
```

[Out] -(Log[Cosh[a/2 + (b*Log[c*x^n])/2]]/(b*n)) + Log[Sinh[a/2 + (b*Log[c*x^n])/2]]/(b*n)

Maple [A]
time = 3.02, size = 23, normalized size = 1.15

method	result
derivativedivides	$\frac{\ln\left(\tanh\left(\frac{a}{2}+\frac{b\ln\left(c\,x^{n}\right)}{2}\right)\right)}{nb}$
default	$\frac{\ln\left(\tanh\left(\frac{a}{2}+\frac{b\ln\left(c\,x^{n}\right)}{2}\right)\right)}{nb}$
risch	$\frac{\ln\left(c^{b}\left(x^{n}\right)^{b}\mathrm{e}^{a}\mathrm{e}^{-\frac{ib\pi\operatorname{csgn}\left(ic\,x^{n}\right)^{3}}{2}}\mathrm{e}^{\frac{ib\operatorname{csgn}\left(ic\,x^{n}\right)^{2}\operatorname{csgn}\left(ic\right)\pi}{2}}\mathrm{e}^{\frac{ib\operatorname{csgn}\left(ic\,x^{n}\right)^{2}\operatorname{csgn}\left(ix^{n}\right)\pi}{2}}\mathrm{e}^{-\frac{ib\operatorname{csgn}\left(ic\,x^{n}\right)\operatorname{csgn}\left(ic\right)\operatorname{csgn}\left(ix^{n}\right)\pi}{2}}-1\right)}{bn} -$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(a+b*ln(c*x^n))/x,x,method=_RETURNVERBOSE)

[Out] 1/n/b*ln(tanh(1/2*a+1/2*b*ln(c*x^n)))

Maxima [A]
time = 0.26, size = 22, normalized size = 1.10

$$\frac{\log\left(\tanh\left(\frac{1}{2}\,b\log\left(cx^{n}\right)+\frac{1}{2}\,a\right)\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(a+b*log(c*x^n))/x,x, algorithm="maxima")

[Out] log(tanh(1/2*b*log(c*x^n) + 1/2*a))/(b*n)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(20) = 40$.
time = 0.42, size = 65, normalized size = 3.25

$$-\frac{\log\left(\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+1\right)-\log\left(\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-1\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(a+b*log(c*x^n))/x,x, algorithm="fricas")

[Out] -(log(cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a) + 1) - log(cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a) - 1))/(b*n)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{csch}\left(a+b\log\left(cx^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*ln(c*x**n))/x,x)
```

```
[Out] Integral(csch(a + b*log(c*x**n))/x, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 145 vs. 2(20) = 40.

time = 0.42, size = 145, normalized size = 7.25

$$-c^{b}\left(\frac{c^{b}e^{(-a)}\log\left(\sqrt{2\,x^{bn}|c|^{b}\cos\left(-\frac{1}{2}\,\pi b\mathrm{sgn}\left(c\right)+\frac{1}{2}\,\pi b\right)e^{a}+x^{2\,bn}|c|^{2\,b}e^{(2\,a)}+1}\right)}{bc^{2\,b}n}-\frac{c^{b}e^{(-a)}\log\left(\sqrt{-2\,x^{bn}|c|^{b}\cos\left(-\frac{1}{2}\,\pi b\mathrm{sgn}\left(c\right)+\frac{1}{2}\,\pi b\right)e^{a}+x^{2\,bn}|c|^{2\,b}e^{(2\,a)}+1}\right)}{bc^{2\,b}n}\right)e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))/x,x, algorithm="giac")
```

```
[Out] -c^b*(c^b*e^(-a)*log(sqrt(2*x^(b*n)*abs(c)^b*cos(-1/2*pi*b*sgn(c) + 1/2*pi*
b)*e^a + x^(2*b*n)*abs(c)^(2*b)*e^(2*a) + 1))/(b*c^(2*b)*n) - c^b*e^(-a)*lo
g(sqrt(-2*x^(b*n)*abs(c)^b*cos(-1/2*pi*b*sgn(c) + 1/2*pi*b)*e^a + x^(2*b*n)
*abs(c)^(2*b)*e^(2*a) + 1))/(b*c^(2*b)*n))*e^a
```

Mupad [B]

time = 1.62, size = 43, normalized size = 2.15

$$-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{-a}\,\sqrt{-b^2\,n^2}}{b\,n\,{\left(c\,x^n\right)}^b}\right)}{\sqrt{-b^2\,n^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*sinh(a + b*log(c*x^n))),x)
```

```
[Out] -(2*atan((exp(-a)*(-b^2*n^2)^(1/2))/(b*n*(c*x^n)^b)))/(-b^2*n^2)^(1/2)
```

3.166 $\int \frac{\operatorname{csch}^2(a+b\log(cx^n))}{x} dx$

Optimal. Leaf size=19

$$-\frac{\coth(a+b\log(cx^n))}{bn}$$

```
[Out] -coth(a+b*ln(c*x^n))/b/n
```

Rubi [A]
time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3852, 8}

$$-\frac{\coth(a+b\log(cx^n))}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*Log[c*x^n]]^2/x,x]
```

```
[Out] -(Coth[a + b*Log[c*x^n]]/(b*n))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^2(a+b\log(cx^n))}{x} dx &= \frac{\operatorname{Subst}\left(\int \operatorname{csch}^2(a+bx) dx, x, \log(cx^n)\right)}{n}\\ &= -\frac{i\operatorname{Subst}\left(\int 1 dx, x, -i\coth(a+b\log(cx^n))\right)}{bn}\\ &= -\frac{\coth(a+b\log(cx^n))}{bn}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 19, normalized size = 1.00

$$-\frac{\coth(a+b\log(cx^n))}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*Log[c*x^n]]^2/x,x]
```

```
[Out] -(Coth[a + b*Log[c*x^n]]/(b*n))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 3.66, size = 116, normalized size = 6.11

method	result	size
risch	$-\frac{2}{bn\left(\left(x^{n}\right)^{2b} c^{2b} \mathrm{e}^{2a} \mathrm{e}^{-ib\pi \operatorname{csgn}\left(ic\,x^{n}\right)^{3}} \mathrm{e}^{ib \operatorname{csgn}\left(ic\,x^{n}\right)^{2} \operatorname{csgn}\left(ic\right)\pi} \mathrm{e}^{ib \operatorname{csgn}\left(ic\,x^{n}\right)^{2} \operatorname{csgn}\left(ix^{n}\right)\pi} \mathrm{e}^{-ib \operatorname{csgn}\left(ic\,x^{n}\right) \operatorname{csgn}\left(ic\right) \operatorname{csgn}\left(ix^{n}\right)\pi}-1\right)}$	116

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a+b*ln(c*x^n))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] -2/b/n/(((x^n)^b)^2*(c^b)^2*exp(2*a)*exp(-I*b*csgn(I*c*x^n)^3*Pi)*exp(I*b*c
sgn(I*c*x^n)^2*csgn(I*c)*Pi)*exp(I*b*csgn(I*c*x^n)^2*csgn(I*x^n)*Pi)*exp(-I
*b*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)*Pi)-1)
```

Maxima [A]
time = 0.29, size = 29, normalized size = 1.53

$$-\frac{2}{bc^{2\,b}ne^{\left(2\,b\log\left(x^{n}\right)+2\,a\right)}-bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^2/x,x, algorithm="maxima")
```

```
[Out] -2/(b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) - b*n)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(19) = 38.
time = 0.35, size = 71, normalized size = 3.74

$$-\frac{2}{bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2\,bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+bn\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}-bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^2/x,x, algorithm="fricas")
```

```
[Out] -2/(b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*b*n*cosh(b*n*log(x) + b*log(c
) + a)*sinh(b*n*log(x) + b*log(c) + a) + b*n*sinh(b*n*log(x) + b*log(c) + a
)^2 - b*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{csch}^{2}\left(a+b\log\left(cx^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*ln(c*x**n))**2/x,x)
```

```
[Out] Integral(csch(a + b*log(c*x**n))**2/x, x)
```

Giac [A]
time = 0.39, size = 28, normalized size = 1.47

$$-\frac{2}{(c^{2\,b}x^{2\,bn}e^{(2\,a)}-1)bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^2/x,x, algorithm="giac")
```

```
[Out] -2/((c^(2*b)*x^(2*b*n)*e^(2*a) - 1)*b*n)
```

Mupad [B]
time = 1.44, size = 25, normalized size = 1.32

$$\frac{2}{b\,n-b\,n\,\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*sinh(a + b*log(c*x^n))^2),x)
```

```
[Out] 2/(b*n - b*n*exp(2*a)*(c*x^n)^(2*b))
```

3.167 $\int \frac{\text{csch}^3(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=55

$$\frac{\tanh^{-1}(\cosh(a+b\log(cx^n)))}{2bn}-\frac{\coth(a+b\log(cx^n))\,\text{csch}(a+b\log(cx^n))}{2bn}$$

```
[Out] 1/2*arctanh(cosh(a+b*ln(c*x^n)))/b/n-1/2*coth(a+b*ln(c*x^n))*csch(a+b*ln(c*
x^n))/b/n
```

Rubi [A]
time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3853, 3855}

$$\frac{\tanh^{-1}(\cosh(a+b\log(cx^n)))}{2bn}-\frac{\coth(a+b\log(cx^n))\,\text{csch}(a+b\log(cx^n))}{2bn}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] ArcTanh[Cosh[a + b*Log[c*x^n]]]/(2*b*n) - (Coth[a + b*Log[c*x^n]]*Csch[a +
b*Log[c*x^n]])/(2*b*n)
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\text{csch}^3(a+b\log(cx^n))}{x}\,dx &= \frac{\text{Subst}\left(\int \text{csch}^3(a+bx)\,dx,x,\log(cx^n)\right)}{n}\\ &= -\frac{\coth(a+b\log(cx^n))\,\text{csch}(a+b\log(cx^n))}{2bn}-\frac{\text{Subst}\left(\int \text{csch}(a+bx)\,dx,x,\text{lo}\right.}{2n}\\ &= \frac{\tanh^{-1}(\cosh(a+b\log(cx^n)))}{2bn}-\frac{\coth(a+b\log(cx^n))\,\text{csch}(a+b\log(cx^n))}{2bn}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 81, normalized size = 1.47

$$-\frac{\operatorname{csch}^2\left(\frac{1}{2}(a+b\log(cx^n))\right)}{8bn}-\frac{\log\left(\tanh\left(\frac{1}{2}(a+b\log(cx^n))\right)\right)}{2bn}-\frac{\operatorname{sech}^2\left(\frac{1}{2}(a+b\log(cx^n))\right)}{8bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] -1/8*Csch[(a + b*Log[c*x^n])/2]^2/(b*n) - Log[Tanh[(a + b*Log[c*x^n])/2]]/(
2*b*n) - Sech[(a + b*Log[c*x^n])/2]^2/(8*b*n)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 4.17, size = 534, normalized size = 9.71

method	result
risch	$-\frac{c^b (x^n)^b \big((x^n)^{2b} c^{2b} e^{3a} e^{-\frac{3ib\pi \operatorname{csgn}(icx^n)^3}{2}} e^{\frac{3ib \operatorname{csgn}(icx^n)^2 \operatorname{csgn}(ic)\pi}{2}} e^{\frac{3ib \operatorname{csgn}(icx^n)^2 \operatorname{csgn}(ix^n)\pi}{2}} e^{-\frac{3ib \operatorname{csgn}(icx^n) \operatorname{csgn}(ic) \operatorname{csgn}(ix^n)\pi}{2}} + e^a e}{bn \big((x^n)^{2b} c^{2b} e^{2a} e^{-ib\pi \operatorname{csgn}(icx^n)^3} e^{ib \operatorname{csgn}(icx^n)^2 \operatorname{csgn}(ic)\pi} e^{ib \operatorname{csgn}(icx^n)^2 \operatorname{csgn}(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a+b*ln(c*x^n))^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] -c^b*(x^n)^b/b/n/(((x^n)^b)^2*(c^b)^2*exp(2*a)*exp(-I*b*csgn(I*c*x^n)^3*Pi)
*exp(I*b*csgn(I*c*x^n)^2*csgn(I*c)*Pi)*exp(I*b*csgn(I*c*x^n)^2*csgn(I*x^n)*
Pi)*exp(-I*b*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)*Pi)-1)^2*(((x^n)^b)^2*(c^b
)^2*exp(3*a)*exp(-3/2*I*b*csgn(I*c*x^n)^3*Pi)*exp(3/2*I*b*csgn(I*c*x^n)^2*c
sgn(I*c)*Pi)*exp(3/2*I*b*csgn(I*c*x^n)^2*csgn(I*x^n)*Pi)*exp(-3/2*I*b*csgn(
I*c*x^n)*csgn(I*c)*csgn(I*x^n)*Pi)+exp(a)*exp(-1/2*I*b*csgn(I*c*x^n)^3*Pi)*
exp(1/2*I*b*csgn(I*c*x^n)^2*csgn(I*c)*Pi)*exp(1/2*I*b*csgn(I*c*x^n)^2*csgn(
I*x^n)*Pi)*exp(-1/2*I*b*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)*Pi))-1/2/b/n*ln
(c^b*(x^n)^b*exp(a)*exp(-1/2*I*b*csgn(I*c*x^n)^3*Pi)*exp(1/2*I*b*csgn(I*c*x
^n)^2*csgn(I*c)*Pi)*exp(1/2*I*b*csgn(I*c*x^n)^2*csgn(I*x^n)*Pi)*exp(-1/2*I*
b*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)*Pi)-1)+1/2/b/n*ln(c^b*(x^n)^b*exp(a)*
exp(-1/2*I*b*csgn(I*c*x^n)^3*Pi)*exp(1/2*I*b*csgn(I*c*x^n)^2*csgn(I*c)*Pi)*
exp(1/2*I*b*csgn(I*c*x^n)^2*csgn(I*x^n)*Pi)*exp(-1/2*I*b*csgn(I*c*x^n)*csgn
(I*c)*csgn(I*x^n)*Pi)+1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 150 vs. 2(51) = 102.
time = 0.28, size = 150, normalized size = 2.73

$$-\frac{c^{3b}e^{(3b\log(x^n)+3a)}+c^b e^{(b\log(x^n)+a)}}{bc^{4b}ne^{(4b\log(x^n)+4a)}-2bc^{2b}ne^{(2b\log(x^n)+2a)}+bn}+\frac{\log\left(\frac{\left(c^b e^{(b\log(x^n)+a)}+1\right)e^{(-a)}}{c^b}\right)}{2bn}-\frac{\log\left(\frac{\left(c^b e^{(b\log(x^n)+a)}-1\right)e^{(-a)}}{c^b}\right)}{2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^3/x,x, algorithm="maxima")
```

```
[Out] -(c^(3*b)*e^(3*b*log(x^n) + 3*a) + c^b*e^(b*log(x^n) + a))/(b*c^(4*b)*n*e^(
4*b*log(x^n) + 4*a) - 2*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n) + 1/2*log
((c^b*e^(b*log(x^n) + a) + 1)*e^(-a)/c^b)/(b*n) - 1/2*log((c^b*e^(b*log(x^n
) + a) - 1)*e^(-a)/c^b)/(b*n)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 643 vs. $2(51) = 102$.
time = 0.37, size = 643, normalized size = 11.69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^3/x,x, algorithm="fricas")
```

```
[Out] -1/2*(2*cosh(b*n*log(x) + b*log(c) + a)^3 + 6*cosh(b*n*log(x) + b*log(c) +
a)*sinh(b*n*log(x) + b*log(c) + a)^2 + 2*sinh(b*n*log(x) + b*log(c) + a)^3
- (cosh(b*n*log(x) + b*log(c) + a)^4 + 4*cosh(b*n*log(x) + b*log(c) + a)*si
nh(b*n*log(x) + b*log(c) + a)^3 + sinh(b*n*log(x) + b*log(c) + a)^4 + 2*(3*
cosh(b*n*log(x) + b*log(c) + a)^2 - 1)*sinh(b*n*log(x) + b*log(c) + a)^2 -
2*cosh(b*n*log(x) + b*log(c) + a)^2 + 4*(cosh(b*n*log(x) + b*log(c) + a)^3
- cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a) + 1)*log
(cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a) + 1) + (
cosh(b*n*log(x) + b*log(c) + a)^4 + 4*cosh(b*n*log(x) + b*log(c) + a)*sinh(
b*n*log(x) + b*log(c) + a)^3 + sinh(b*n*log(x) + b*log(c) + a)^4 + 2*(3*cos
h(b*n*log(x) + b*log(c) + a)^2 - 1)*sinh(b*n*log(x) + b*log(c) + a)^2 - 2*c
osh(b*n*log(x) + b*log(c) + a)^2 + 4*(cosh(b*n*log(x) + b*log(c) + a)^3 - c
osh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a) + 1)*log(co
sh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a) - 1) + 2*(3
*cosh(b*n*log(x) + b*log(c) + a)^2 + 1)*sinh(b*n*log(x) + b*log(c) + a) + 2
*cosh(b*n*log(x) + b*log(c) + a))/(b*n*cosh(b*n*log(x) + b*log(c) + a)^4 +
4*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3 + b
*n*sinh(b*n*log(x) + b*log(c) + a)^4 - 2*b*n*cosh(b*n*log(x) + b*log(c) + a
)^2 + 2*(3*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 - b*n)*sinh(b*n*log(x) + b
*log(c) + a)^2 + b*n + 4*(b*n*cosh(b*n*log(x) + b*log(c) + a)^3 - b*n*cosh(
b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(a + b\log(cx^n))}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*ln(c*x**n))**3/x,x)
```

```
[Out] Integral(csch(a + b*log(c*x**n))**3/x, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(51) = 102.
time = 0.41, size = 210, normalized size = 3.82

$$\frac{1}{2} c^{3b} \left(\frac{c^b e^{(-3a)} \log \left(\sqrt{2 x^{bn} |c|^b \cos \left(-\frac{1}{2} \pi b \mathrm{sgn}\left(c\right) + \frac{1}{2} \pi b \right) e^a + x^{2bn} |c|^{2b} e^{(2a)} + 1} \right)}{b c^{4b} n} - \frac{c^b e^{(-3a)} \log \left(\sqrt{-2 x^{bn} |c|^b \cos \left(-\frac{1}{2} \pi b \mathrm{sgn}\left(c\right) + \frac{1}{2} \pi b \right) e^a + x^{2bn} |c|^{2b} e^{(2a)} + 1} \right)}{b c^{4b} n} - \frac{2 \left(c^{2b} x^{3bn} e^{(2a)} + x^{bn} \right) e^{(-2a)}}{\left(c^{2b} x^{2bn} e^{(2a)} - 1 \right)^2 b c^{2b} n} \right) e^{(3a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^3/x,x, algorithm="giac")
```

```
[Out] 1/2*c^(3*b)*(c^b*e^(-3*a)*log(sqrt(2*x^(b*n)*abs(c)^b*cos(-1/2*pi*b*sgn(c)
+ 1/2*pi*b)*e^a + x^(2*b*n)*abs(c)^(2*b)*e^(2*a) + 1))/(b*c^(4*b)*n) - c^b*
e^(-3*a)*log(sqrt(-2*x^(b*n)*abs(c)^b*cos(-1/2*pi*b*sgn(c) + 1/2*pi*b)*e^a
+ x^(2*b*n)*abs(c)^(2*b)*e^(2*a) + 1))/(b*c^(4*b)*n) - 2*(c^(2*b)*x^(3*b*n)
*e^(2*a) + x^(b*n))*e^(-2*a)/((c^(2*b)*x^(2*b*n)*e^(2*a) - 1)^2*b*c^(2*b)*n
))*e^(3*a)
```

Mupad [B]
time = 1.49, size = 140, normalized size = 2.55

$$\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{-a}\,\sqrt{-b^2\,n^2}}{b\,n\,{\left(c\,x^n\right)}^b}\right)}{\sqrt{-b^2\,n^2}}+\frac{\mathrm{e}^{-a}}{{\left(c\,x^n\right)}^b\,\left(b\,n-\frac{b\,n\,\mathrm{e}^{-2\,a}}{{\left(c\,x^n\right)}^{2\,b}}\right)}-\frac{2\,\mathrm{e}^{-a}}{{\left(c\,x^n\right)}^b\,\left(b\,n-\frac{2\,b\,n\,\mathrm{e}^{-2\,a}}{{\left(c\,x^n\right)}^{2\,b}}+\frac{b\,n\,\mathrm{e}^{-4\,a}}{{\left(c\,x^n\right)}^{4\,b}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*sinh(a + b*log(c*x^n))^3),x)
```

```
[Out] atan((exp(-a)*(-b^2*n^2)^(1/2))/(b*n*(c*x^n)^b))/(-b^2*n^2)^(1/2) + exp(-a)
/((c*x^n)^b*(b*n - (b*n*exp(-2*a))/(c*x^n)^(2*b))) - (2*exp(-a))/((c*x^n)^b
*(b*n - (2*b*n*exp(-2*a))/(c*x^n)^(2*b) + (b*n*exp(-4*a))/(c*x^n)^(4*b)))
```

3.168 $\int \frac{\operatorname{csch}^4(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=42

$$\frac{\coth\left(a+b\log\left(cx^n\right)\right)}{bn}-\frac{\coth^3\left(a+b\log\left(cx^n\right)\right)}{3bn}$$

```
[Out] coth(a+b*ln(c*x^n))/b/n-1/3*coth(a+b*ln(c*x^n))^3/b/n
```

Rubi [A]
time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3852}

$$\frac{\coth\left(a+b\log\left(cx^n\right)\right)}{bn}-\frac{\coth^3\left(a+b\log\left(cx^n\right)\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] Coth[a + b*Log[c*x^n]]/(b*n) - Coth[a + b*Log[c*x^n]]^3/(3*b*n)
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^4(a+b\log(cx^n))}{x}\,dx &= \frac{\operatorname{Subst}\left(\int \operatorname{csch}^4(a+bx)\,dx,x,\log\left(cx^n\right)\right)}{n}\\ &= \frac{i\operatorname{Subst}\left(\int \left(1+x^2\right)\,dx,x,-i\coth\left(a+b\log\left(cx^n\right)\right)\right)}{bn}\\ &= \frac{\coth\left(a+b\log\left(cx^n\right)\right)}{bn}-\frac{\coth^3\left(a+b\log\left(cx^n\right)\right)}{3bn}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 56, normalized size = 1.33

$$\frac{2\coth\left(a+b\log\left(cx^n\right)\right)}{3bn}-\frac{\coth\left(a+b\log\left(cx^n\right)\right)\operatorname{csch}^2(a+b\log\left(cx^n\right))}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] (2*Coth[a + b*Log[c*x^n]])/(3*b*n) - (Coth[a + b*Log[c*x^n]]*Csch[a + b*Log
[c*x^n]]^2)/(3*b*n)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 3.98, size = 222, normalized size = 5.29

method	result	size
risch	$-\frac{4\left(3\left(x^{n}\right)^{2b} c^{2b} \mathrm{e}^{2a} \mathrm{e}^{-ib\pi \mathrm{csgn}\left(ic\,x^{n}\right)^{3}} \mathrm{e}^{ib\mathrm{csgn}\left(ic\,x^{n}\right)^{2} \mathrm{csgn}\left(ic\right)\pi} \mathrm{e}^{ib\mathrm{csgn}\left(ic\,x^{n}\right)^{2} \mathrm{csgn}\left(ix^{n}\right)\pi} \mathrm{e}^{-ib\,\mathrm{csgn}\left(ic\,x^{n}\right)\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(ix^{n}\right)\pi}-1\right)}{3bn\left(\left(x^{n}\right)^{2b} c^{2b} \mathrm{e}^{2a} \mathrm{e}^{-ib\pi \mathrm{csgn}\left(ic\,x^{n}\right)^{3}} \mathrm{e}^{ib\mathrm{csgn}\left(ic\,x^{n}\right)^{2} \mathrm{csgn}\left(ic\right)\pi} \mathrm{e}^{ib\mathrm{csgn}\left(ic\,x^{n}\right)^{2} \mathrm{csgn}\left(ix^{n}\right)\pi} \mathrm{e}^{-ib\,\mathrm{csgn}\left(ic\,x^{n}\right)\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(ix^{n}\right)\pi}-1\right)^{3}}$	222

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a+b*ln(c*x^n))^4/x,x,method=_RETURNVERBOSE)
```

```
[Out] -4/3*(3*((x^n)^b)^2*(c^b)^2*exp(2*a)*exp(-I*b*csgn(I*c*x^n)^3*Pi)*exp(I*b*c
sgn(I*c*x^n)^2*csgn(I*c)*Pi)*exp(I*b*csgn(I*c*x^n)^2*csgn(I*x^n)*Pi)*exp(-I
*b*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)*Pi)-1)/b/n/(((x^n)^b)^2*(c^b)^2*exp(
2*a)*exp(-I*b*csgn(I*c*x^n)^3*Pi)*exp(I*b*csgn(I*c*x^n)^2*csgn(I*c)*Pi)*exp
(I*b*csgn(I*c*x^n)^2*csgn(I*x^n)*Pi)*exp(-I*b*csgn(I*c*x^n)*csgn(I*c)*csgn(
I*x^n)*Pi)-1)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(40) = 80.
time = 0.28, size = 92, normalized size = 2.19

$$-\frac{4\left(3\,c^{2\,b}e^{\left(2\,b\log\left(x^{n}\right)+2\,a\right)}-1\right)}{3\left(bc^{6\,b}ne^{\left(6\,b\log\left(x^{n}\right)+6\,a\right)}-3\,bc^{4\,b}ne^{\left(4\,b\log\left(x^{n}\right)+4\,a\right)}+3\,bc^{2\,b}ne^{\left(2\,b\log\left(x^{n}\right)+2\,a\right)}-bn\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^4/x,x, algorithm="maxima")
```

```
[Out] -4/3*(3*c^(2*b)*e^(2*b*log(x^n) + 2*a) - 1)/(b*c^(6*b)*n*e^(6*b*log(x^n) +
6*a) - 3*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 3*b*c^(2*b)*n*e^(2*b*log(x^n)
 + 2*a) - b*n)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(40) = 80.
time = 0.36, size = 272, normalized size = 6.48

$$-\frac{8\left(\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+2\,\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)}{3\left(bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{5}+5\,bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{4}+bn\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{5}-3\,bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}+\left(10\,bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}-3\,bn\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}+2\,bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\left(10\,bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}-9\,bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+\left(5\,bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{4}-9\,bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+4\,bn\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^4/x,x, algorithm="fricas")
```

```
[Out] -8/3*(cosh(b*n*log(x) + b*log(c) + a) + 2*sinh(b*n*log(x) + b*log(c) + a))/
(b*n*cosh(b*n*log(x) + b*log(c) + a)^5 + 5*b*n*cosh(b*n*log(x) + b*log(c) +
 a)*sinh(b*n*log(x) + b*log(c) + a)^4 + b*n*sinh(b*n*log(x) + b*log(c) + a)
^5 - 3*b*n*cosh(b*n*log(x) + b*log(c) + a)^3 + (10*b*n*cosh(b*n*log(x) + b*
log(c) + a)^2 - 3*b*n)*sinh(b*n*log(x) + b*log(c) + a)^3 + 2*b*n*cosh(b*n*l
og(x) + b*log(c) + a) + (10*b*n*cosh(b*n*log(x) + b*log(c) + a)^3 - 9*b*n*c
osh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^2 + (5*b*n*
cosh(b*n*log(x) + b*log(c) + a)^4 - 9*b*n*cosh(b*n*log(x) + b*log(c) + a)^2
 + 4*b*n)*sinh(b*n*log(x) + b*log(c) + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^4\left(a + b\log\left(cx^n\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*ln(c*x**n))**4/x,x)
```

```
[Out] Integral(csch(a + b*log(c*x**n))**4/x, x)
```

Giac [A]
time = 0.39, size = 47, normalized size = 1.12

$$-\frac{4\left(3\,c^{2\,b}x^{2\,bn}e^{(2\,a)}-1\right)}{3\left(c^{2\,b}x^{2\,bn}e^{(2\,a)}-1\right)^3bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^4/x,x, algorithm="giac")
```

```
[Out] -4/3*(3*c^(2*b)*x^(2*b*n)*e^(2*a) - 1)/((c^(2*b)*x^(2*b*n)*e^(2*a) - 1)^3*b
*n)
```

Mupad [B]
time = 1.45, size = 55, normalized size = 1.31

$$\frac{4\,\mathrm{e}^{4\,a}\,{\left(c\,x^n\right)}^{4\,b}\,\left(\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}-3\right)}{3\,b\,n\,{\left(\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}-1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*sinh(a + b*log(c*x^n))^4),x)
```

```
[Out] (4*exp(4*a)*(c*x^n)^(4*b)*(exp(2*a)*(c*x^n)^(2*b) - 3))/(3*b*n*(exp(2*a)*(c
*x^n)^(2*b) - 1)^3)
```

3.169 $\int \frac{\operatorname{csch}^5(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=89

$$-\frac{3\tanh^{-1}(\cosh(a+b\log(cx^n)))}{8bn}+\frac{3\coth(a+b\log(cx^n))\operatorname{csch}(a+b\log(cx^n))}{8bn}-\frac{\coth(a+b\log(cx^n))\operatorname{csch}^3}{4bn}$$

```
[Out] -3/8*arctanh(cosh(a+b*ln(c*x^n)))/b/n+3/8*coth(a+b*ln(c*x^n))*csch(a+b*ln(c
*x^n))/b/n-1/4*coth(a+b*ln(c*x^n))*csch(a+b*ln(c*x^n))^3/b/n
```

Rubi [A]

time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3853, 3855}

$$-\frac{3\tanh^{-1}(\cosh(a+b\log(cx^n)))}{8bn}-\frac{\coth(a+b\log(cx^n))\operatorname{csch}^3(a+b\log(cx^n))}{4bn}+\frac{3\coth(a+b\log(cx^n))\operatorname{csch}(a+b\log(cx^n))}{8bn}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*Log[c*x^n]]^5/x,x]
```

```
[Out] (-3*ArcTanh[Cosh[a + b*Log[c*x^n]]])/(8*b*n) + (3*Coth[a + b*Log[c*x^n]]*Cs
ch[a + b*Log[c*x^n]])/(8*b*n) - (Coth[a + b*Log[c*x^n]]*Csch[a + b*Log[c*x^
n]]^3)/(4*b*n)
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^5(a+b\log(cx^n))}{x}\,dx &= \frac{\operatorname{Subst}\left(\int \operatorname{csch}^5(a+bx)\,dx,x,\log(cx^n)\right)}{n}\\
&= -\frac{\coth(a+b\log(cx^n))\operatorname{csch}^3(a+b\log(cx^n))}{4bn}-\frac{3\operatorname{Subst}\left(\int \operatorname{csch}^3(a+bx)\,dx,x,\right.}{4n}\\
&= \frac{3\coth(a+b\log(cx^n))\operatorname{csch}(a+b\log(cx^n))}{8bn}-\frac{\coth(a+b\log(cx^n))\operatorname{csch}^3(a+}{4bn}\\
&= -\frac{3\tanh^{-1}(\cosh(a+b\log(cx^n)))}{8bn}+\frac{3\coth(a+b\log(cx^n))\operatorname{csch}(a+b\log(cx}{8bn}
\end{aligned}$$

Mathematica [**A**]
time = 0.05, size = 135, normalized size = 1.52

$$\frac{3\text{csch}^2\left(\frac{1}{2}(a+b\log\left(cx^n\right))\right)}{32bn}-\frac{\text{csch}^4\left(\frac{1}{2}(a+b\log\left(cx^n\right))\right)}{64bn}+\frac{3\log\left(\tanh\left(\frac{1}{2}(a+b\log\left(cx^n\right))\right)\right)}{8bn}+\frac{3\text{sech}^2\left(\frac{1}{2}(a+b\log\left(cx^n\right))\right)}{32bn}+\frac{\text{sech}^4\left(\frac{1}{2}(a+b\log\left(cx^n\right))\right)}{64bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*Log[c*x^n]]^5/x,x]
```

```
[Out] (3*Csch[(a + b*Log[c*x^n])/2]^2)/(32*b*n) - Csch[(a + b*Log[c*x^n])/2]^4/(6
4*b*n) + (3*Log[Tanh[(a + b*Log[c*x^n])/2]])/(8*b*n) + (3*Sech[(a + b*Log[c
*x^n])/2]^2)/(32*b*n) + Sech[(a + b*Log[c*x^n])/2]^4/(64*b*n)
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 4.31, size = 744, normalized size = 8.36

method	result
risch	$\frac{c^b (x^n)^b \left(3(x^n)^{6b} c^{6b} e^{7a} e^{-\frac{7ib\pi \text{csgn}(ic\,x^n)^3}{2}} e^{\frac{7ib\text{csgn}(ic\,x^n)^2 \text{csgn}(ic)\pi}{2}} e^{\frac{7ib\text{csgn}(ic\,x^n)^2 \text{csgn}(ix^n)\pi}{2}} e^{-\frac{7ib\,\text{csgn}(ic\,x^n)\text{csgn}(ic)\text{csgn}(ix^n)\pi}{2}} -11(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a+b*ln(c*x^n))^5/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*c^b*(x^n)^b/b/n/(((x^n)^b)^2*(c^b)^2*exp(2*a)*exp(-I*b*csgn(I*c*x^n)^3*
Pi)*exp(I*b*csgn(I*c*x^n)^2*csgn(I*c)*Pi)*exp(I*b*csgn(I*c*x^n)^2*csgn(I*x^
n)*Pi)*exp(-I*b*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)*Pi)-1)^4*(3*((x^n)^b)^6
*(c^b)^6*exp(7*a)*exp(-7/2*I*b*csgn(I*c*x^n)^3*Pi)*exp(7/2*I*b*csgn(I*c*x^n
)^2*csgn(I*c)*Pi)*exp(7/2*I*b*csgn(I*c*x^n)^2*csgn(I*x^n)*Pi)*exp(-7/2*I*b*
csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)*Pi)-11*((x^n)^b)^4*(c^b)^4*exp(5*a)*exp
(-5/2*I*b*csgn(I*c*x^n)^3*Pi)*exp(5/2*I*b*csgn(I*c*x^n)^2*csgn(I*c)*Pi)*exp
(5/2*I*b*csgn(I*c*x^n)^2*csgn(I*x^n)*Pi)*exp(-5/2*I*b*csgn(I*c*x^n)*csgn(I*
c)*csgn(I*x^n)*Pi)-11*((x^n)^b)^2*(c^b)^2*exp(3*a)*exp(-3/2*I*b*csgn(I*c*x^
n)^3*Pi)*exp(3/2*I*b*csgn(I*c*x^n)^2*csgn(I*c)*Pi)*exp(3/2*I*b*csgn(I*c*x^n
)^2*csgn(I*x^n)*Pi)*exp(-3/2*I*b*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)*Pi)+3*
exp(a)*exp(-1/2*I*b*csgn(I*c*x^n)^3*Pi)*exp(1/2*I*b*csgn(I*c*x^n)^2*csgn(I*
c)*Pi)*exp(1/2*I*b*csgn(I*c*x^n)^2*csgn(I*x^n)*Pi)*exp(-1/2*I*b*csgn(I*c*x^
n)*csgn(I*c)*csgn(I*x^n)*Pi))+3/8/b/n*ln(c^b*(x^n)^b*exp(a)*exp(-1/2*I*b*cs
gn(I*c*x^n)^3*Pi)*exp(1/2*I*b*csgn(I*c*x^n)^2*csgn(I*c)*Pi)*exp(1/2*I*b*csg
n(I*c*x^n)^2*csgn(I*x^n)*Pi)*exp(-1/2*I*b*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^
n)*Pi)-1)-3/8/b/n*ln(c^b*(x^n)^b*exp(a)*exp(-1/2*I*b*csgn(I*c*x^n)^3*Pi)*ex
p(1/2*I*b*csgn(I*c*x^n)^2*csgn(I*c)*Pi)*exp(1/2*I*b*csgn(I*c*x^n)^2*csgn(I*
x^n)*Pi)*exp(-1/2*I*b*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)*Pi)+1)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(83) = 166$.

time = 0.29, size = 232, normalized size = 2.61

$$\frac{3\,c^{7\,b}e^{(7\,b\log(x^n)+7\,a)} - 11\,c^{5\,b}e^{(5\,b\log(x^n)+5\,a)} - 11\,c^{3\,b}e^{(3\,b\log(x^n)+3\,a)} + 3\,c^{b}e^{(b\log(x^n)+a)}}{4\,(bc^{8\,b}ne^{(8\,b\log(x^n)+8\,a)} - 4\,bc^{6\,b}ne^{(6\,b\log(x^n)+6\,a)} + 6\,bc^{4\,b}ne^{(4\,b\log(x^n)+4\,a)} - 4\,bc^{2\,b}ne^{(2\,b\log(x^n)+2\,a)} + bn)} - \frac{3\,\log\left(\frac{\left(c^{b}e^{(b\log(x^n)+a)}+1\right)e^{(-a)}}{c^{b}}\right)}{8\,bn} + \frac{3\,\log\left(\frac{\left(c^{b}e^{(b\log(x^n)+a)}-1\right)e^{(-a)}}{c^{b}}\right)}{8\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^5/x,x, algorithm="maxima")
```

```
[Out] 1/4*(3*c^(7*b)*e^(7*b*log(x^n) + 7*a) - 11*c^(5*b)*e^(5*b*log(x^n) + 5*a) -
 11*c^(3*b)*e^(3*b*log(x^n) + 3*a) + 3*c^b*e^(b*log(x^n) + a))/(b*c^(8*b)*n
*e^(8*b*log(x^n) + 8*a) - 4*b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a) + 6*b*c^(4*b
)*n*e^(4*b*log(x^n) + 4*a) - 4*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n) -
3/8*log((c^b*e^(b*log(x^n) + a) + 1)*e^(-a)/c^b)/(b*n) + 3/8*log((c^b*e^(b*
log(x^n) + a) - 1)*e^(-a)/c^b)/(b*n)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1806 vs. 2(83) = 166.
time = 0.45, size = 1806, normalized size = 20.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^5/x,x, algorithm="fricas")
```

```
[Out] 1/8*(6*cosh(b*n*log(x) + b*log(c) + a)^7 + 42*cosh(b*n*log(x) + b*log(c) +
a)*sinh(b*n*log(x) + b*log(c) + a)^6 + 6*sinh(b*n*log(x) + b*log(c) + a)^7
+ 2*(63*cosh(b*n*log(x) + b*log(c) + a)^2 - 11)*sinh(b*n*log(x) + b*log(c)
+ a)^5 - 22*cosh(b*n*log(x) + b*log(c) + a)^5 + 10*(21*cosh(b*n*log(x) + b*
log(c) + a)^3 - 11*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log
(c) + a)^4 + 2*(105*cosh(b*n*log(x) + b*log(c) + a)^4 - 110*cosh(b*n*log(x)
 + b*log(c) + a)^2 - 11)*sinh(b*n*log(x) + b*log(c) + a)^3 - 22*cosh(b*n*lo
g(x) + b*log(c) + a)^3 + 2*(63*cosh(b*n*log(x) + b*log(c) + a)^5 - 110*cosh
(b*n*log(x) + b*log(c) + a)^3 - 33*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*
n*log(x) + b*log(c) + a)^2 - 3*(cosh(b*n*log(x) + b*log(c) + a)^8 + 8*cosh(
b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^7 + sinh(b*n*log
(x) + b*log(c) + a)^8 + 4*(7*cosh(b*n*log(x) + b*log(c) + a)^2 - 1)*sinh(b*
n*log(x) + b*log(c) + a)^6 - 4*cosh(b*n*log(x) + b*log(c) + a)^6 + 8*(7*cos
h(b*n*log(x) + b*log(c) + a)^3 - 3*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*
n*log(x) + b*log(c) + a)^5 + 2*(35*cosh(b*n*log(x) + b*log(c) + a)^4 - 30*c
osh(b*n*log(x) + b*log(c) + a)^2 + 3)*sinh(b*n*log(x) + b*log(c) + a)^4 + 6
*cosh(b*n*log(x) + b*log(c) + a)^4 + 8*(7*cosh(b*n*log(x) + b*log(c) + a)^5
 - 10*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*cosh(b*n*log(x) + b*log(c) + a)
)*sinh(b*n*log(x) + b*log(c) + a)^3 + 4*(7*cosh(b*n*log(x) + b*log(c) + a)^
6 - 15*cosh(b*n*log(x) + b*log(c) + a)^4 + 9*cosh(b*n*log(x) + b*log(c) + a
)^2 - 1)*sinh(b*n*log(x) + b*log(c) + a)^2 - 4*cosh(b*n*log(x) + b*log(c) +
```

```
a)^2 + 8*(cosh(b*n*log(x) + b*log(c) + a)^7 - 3*cosh(b*n*log(x) + b*log(c)
 + a)^5 + 3*cosh(b*n*log(x) + b*log(c) + a)^3 - cosh(b*n*log(x) + b*log(c)
+ a))*sinh(b*n*log(x) + b*log(c) + a) + 1)*log(cosh(b*n*log(x) + b*log(c) +
 a) + sinh(b*n*log(x) + b*log(c) + a) + 1) + 3*(cosh(b*n*log(x) + b*log(c)
+ a)^8 + 8*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^
7 + sinh(b*n*log(x) + b*log(c) + a)^8 + 4*(7*cosh(b*n*log(x) + b*log(c) + a
)^2 - 1)*sinh(b*n*log(x) + b*log(c) + a)^6 - 4*cosh(b*n*log(x) + b*log(c) +
 a)^6 + 8*(7*cosh(b*n*log(x) + b*log(c) + a)^3 - 3*cosh(b*n*log(x) + b*log(
c) + a))*sinh(b*n*log(x) + b*log(c) + a)^5 + 2*(35*cosh(b*n*log(x) + b*log(
c) + a)^4 - 30*cosh(b*n*log(x) + b*log(c) + a)^2 + 3)*sinh(b*n*log(x) + b*l
og(c) + a)^4 + 6*cosh(b*n*log(x) + b*log(c) + a)^4 + 8*(7*cosh(b*n*log(x) +
 b*log(c) + a)^5 - 10*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*cosh(b*n*log(x)
 + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^3 + 4*(7*cosh(b*n*log(x)
+ b*log(c) + a)^6 - 15*cosh(b*n*log(x) + b*log(c) + a)^4 + 9*cosh(b*n*log(x
) + b*log(c) + a)^2 - 1)*sinh(b*n*log(x) + b*log(c) + a)^2 - 4*cosh(b*n*log
(x) + b*log(c) + a)^2 + 8*(cosh(b*n*log(x) + b*log(c) + a)^7 - 3*cosh(b*n*l
og(x) + b*log(c) + a)^5 + 3*cosh(b*n*log(x) + b*log(c) + a)^3 - cosh(b*n*lo
g(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a) + 1)*log(cosh(b*n*log
(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a) - 1) + 2*(21*cosh(b*n
*log(x) + b*log(c) + a)^6 - 55*cosh(b*n*log(x) + b*log(c) + a)^4 - 33*cosh(
b*n*log(x) + b*log(c) + a)^2 + 3)*sinh(b*n*log(x) + b*log(c) + a) + 6*cosh(
b*n*log(x) + b*log(c) + a))/(b*n*cosh(b*n*log(x) + b*log(c) + a)^8 + 8*b*n*
cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^7 + b*n*sin
h(b*n*log(x) + b*log(c) + a)^8 - 4*b*n*cosh(b*n*log(x) + b*log(c) + a)^6 +
4*(7*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 - b*n)*sinh(b*n*log(x) + b*log(c
) + a)^6 + 6*b*n*cosh(b*n*log(x) + b*log(c) + a)^4 + 8*(7*b*n*cosh(b*n*log(
x) + b*log(c) + a)^3 - 3*b*n*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(
x) + b*log(c) + a)^5 + 2*(35*b*n*cosh(b*n*log(x) + b*log(c) + a)^4 - 30*b*n
*cosh(b*n*log(x) + b*log(c) + a)^2 + 3*b*n)*sinh(b*n*log(x) + b*log(c) + a)
^4 - 4*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 8*(7*b*n*cosh(b*n*log(x) + b
*log(c) + a)^5 - 10*b*n*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*b*n*cosh(b*n*
log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^3 + 4*(7*b*n*cosh(b
*n*log(x) + b*log(c) + a)^6 - 15*b*n*cosh(b*n*log(x) + b*log(c) + a)^4 + 9*
b*n*cosh(b*n*log(x) + b*log(c) + a)^2 - b*n)*sinh(b*n*log(x) + b*log(c) + a
)^2 + b*n + 8*(b*n*cosh(b*n*log(x) + b*log(c) + a)^7 - 3*b*n*cosh(b*n*log(x
) + b*log(c) + a)^5 + 3*b*n*cosh(b*n*log(x) + b*log(c) + a)^3 - b*n*cosh(b*
n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a))
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^5\left(a + b\log\left(cx^n\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*ln(c*x**n))**5/x,x)
```

```
[Out] Integral(csch(a + b*log(c*x**n))**5/x, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 248 vs. 2(83) = 166.
time = 0.42, size = 248, normalized size = 2.79

$$-\frac{1}{8}\,c^{5\,b}\left(\frac{3\,c^{b}e^{(-5\,a)}\log\left(\sqrt{2\,x^{bn}|c|^{b}\cos\left(-\frac{1}{2}\,\pi b\mathrm{sgn}\left(c\right)+\frac{1}{2}\,\pi b\right)e^{a}+x^{2\,bn}|c|^{2\,b}e^{(2\,a)}+1}\right)}{bc^{6\,b}n}-\frac{3\,c^{b}e^{(-5\,a)}\log\left(\sqrt{-2\,x^{bn}|c|^{b}\cos\left(-\frac{1}{2}\,\pi b\mathrm{sgn}\left(c\right)+\frac{1}{2}\,\pi b\right)e^{a}+x^{2\,bn}|c|^{2\,b}e^{(2\,a)}+1}\right)}{bc^{6\,b}n}-\frac{2\,\left(3\,c^{6\,b}x^{7\,bn}e^{(6\,a)}-11\,c^{4\,b}x^{5\,bn}e^{(4\,a)}-11\,c^{2\,b}x^{3\,bn}e^{(2\,a)}+3\,x^{bn}\right)e^{(-4\,a)}}{\left(c^{2\,b}x^{2\,bn}e^{(2\,a)}-1\right)^{4}bc^{4\,b}n}\right)e^{(5\,a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^5/x,x, algorithm="giac")
```

```
[Out] -1/8*c^(5*b)*(3*c^b*e^(-5*a)*log(sqrt(2*x^(b*n)*abs(c)^b*cos(-1/2*pi*b*sgn(
c) + 1/2*pi*b)*e^a + x^(2*b*n)*abs(c)^(2*b)*e^(2*a) + 1))/(b*c^(6*b)*n) - 3
*c^b*e^(-5*a)*log(sqrt(-2*x^(b*n)*abs(c)^b*cos(-1/2*pi*b*sgn(c) + 1/2*pi*b)
*e^a + x^(2*b*n)*abs(c)^(2*b)*e^(2*a) + 1))/(b*c^(6*b)*n) - 2*(3*c^(6*b)*x^
(7*b*n)*e^(6*a) - 11*c^(4*b)*x^(5*b*n)*e^(4*a) - 11*c^(2*b)*x^(3*b*n)*e^(2*
a) + 3*x^(b*n))*e^(-4*a)/((c^(2*b)*x^(2*b*n)*e^(2*a) - 1)^4*b*c^(4*b)*n))*e
^(5*a)
```

Mupad [B]
time = 1.48, size = 318, normalized size = 3.57

$$\frac{2\,\mathrm{e}^{-a}}{{\left(c\,x^n\right)}^b\,\left(b\,n-\frac{3\,b\,n\,\mathrm{e}^{-2\,a}}{{\left(c\,x^n\right)}^{2\,b}}+\frac{3\,b\,n\,\mathrm{e}^{-4\,a}}{{\left(c\,x^n\right)}^{4\,b}}-\frac{b\,n\,\mathrm{e}^{-6\,a}}{{\left(c\,x^n\right)}^{6\,b}}\right)}-\frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^{-a}\,\sqrt{-b^2\,n^2}}{b\,n\,{\left(c\,x^n\right)}^b}\right)}{4\,\sqrt{-b^2\,n^2}}-\frac{3\,\mathrm{e}^{-a}}{4\,{\left(c\,x^n\right)}^b\,\left(b\,n-\frac{b\,n\,\mathrm{e}^{-2\,a}}{{\left(c\,x^n\right)}^{2\,b}}\right)}-\frac{4\,\mathrm{e}^{-3\,a}}{{\left(c\,x^n\right)}^{3\,b}\,\left(b\,n-\frac{4\,b\,n\,\mathrm{e}^{-2\,a}}{{\left(c\,x^n\right)}^{2\,b}}+\frac{6\,b\,n\,\mathrm{e}^{-4\,a}}{{\left(c\,x^n\right)}^{4\,b}}-\frac{4\,b\,n\,\mathrm{e}^{-6\,a}}{{\left(c\,x^n\right)}^{6\,b}}+\frac{b\,n\,\mathrm{e}^{-8\,a}}{{\left(c\,x^n\right)}^{8\,b}}\right)}-\frac{\mathrm{e}^{-a}}{2\,{\left(c\,x^n\right)}^b\,\left(b\,n-\frac{2\,b\,n\,\mathrm{e}^{-2\,a}}{{\left(c\,x^n\right)}^{2\,b}}+\frac{b\,n\,\mathrm{e}^{-4\,a}}{{\left(c\,x^n\right)}^{4\,b}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*sinh(a + b*log(c*x^n))^5),x)
```

```
[Out] (2*exp(-a))/((c*x^n)^b*(b*n - (3*b*n*exp(-2*a))/(c*x^n)^(2*b) + (3*b*n*exp(
-4*a))/(c*x^n)^(4*b) - (b*n*exp(-6*a))/(c*x^n)^(6*b))) - (3*atan((exp(-a)*(
-b^2*n^2)^(1/2))/(b*n*(c*x^n)^b)))/(4*(-b^2*n^2)^(1/2)) - (3*exp(-a))/(4*(c
*x^n)^b*(b*n - (b*n*exp(-2*a))/(c*x^n)^(2*b))) - (4*exp(-3*a))/((c*x^n)^(3*
b)*(b*n - (4*b*n*exp(-2*a))/(c*x^n)^(2*b) + (6*b*n*exp(-4*a))/(c*x^n)^(4*b)
 - (4*b*n*exp(-6*a))/(c*x^n)^(6*b) + (b*n*exp(-8*a))/(c*x^n)^(8*b))) - exp(
-a)/(2*(c*x^n)^b*(b*n - (2*b*n*exp(-2*a))/(c*x^n)^(2*b) + (b*n*exp(-4*a))/(
c*x^n)^(4*b)))
```

3.170 $\int \frac{\operatorname{csch}^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=111

$$-\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)\operatorname{csch}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{3bn}+\frac{2i\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}\,F\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log\left(cx^n\right)\right)\middle|\,2\right)\ \cdots}{3bn}$$

```
[Out] -2/3*cosh(a+b*ln(c*x^n))*csch(a+b*ln(c*x^n))^(3/2)/b/n-2/3*I*(sin(1/2*I*a+1
/4*Pi+1/2*I*b*ln(c*x^n))^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n))*Ell
ipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n)),2^(1/2))*csch(a+b*ln(c*x^n))^(
1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)/b/n
```

Rubi [A]

time = 0.06, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3853, 3856, 2720}

$$-\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)\operatorname{csch}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{3bn}+\frac{2i\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}\,F\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|\,2\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] (-2*Cosh[a + b*Log[c*x^n]]*Csch[a + b*Log[c*x^n]]^(3/2))/(3*b*n) + (((2*I)/
3)*Sqrt[Csch[a + b*Log[c*x^n]]]*EllipticF[(I*a - Pi/2 + I*b*Log[c*x^n])/2,
2]*Sqrt[I*Sinh[a + b*Log[c*x^n]]])/(b*n)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule steps

$$\int \frac{\operatorname{csch}^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{\text{Subst}\left(\int \operatorname{csch}^{\frac{5}{2}}(a+bx)\,dx, x, \log(cx^n)\right)}{n}$$

$$= -\frac{2\cosh(a+b\log(cx^n))\operatorname{csch}^{\frac{3}{2}}(a+b\log(cx^n))}{3bn} - \frac{\text{Subst}\left(\int \sqrt{\operatorname{csch}(a+bx)}\,dx\right.}{3n}$$

$$= -\frac{2\cosh(a+b\log(cx^n))\operatorname{csch}^{\frac{3}{2}}(a+b\log(cx^n))}{3bn} - \frac{\left(\sqrt{\operatorname{csch}(a+b\log(cx^n))}\,\sqrt{\right.}{}$$

$$= -\frac{2\cosh(a+b\log(cx^n))\operatorname{csch}^{\frac{3}{2}}(a+b\log(cx^n))}{3bn} + \frac{2i\sqrt{\operatorname{csch}(a+b\log(cx^n))}\,F}{}$$

Mathematica [A]
time = 0.15, size = 84, normalized size = 0.76

$$-\frac{2\sqrt{\operatorname{csch}(a+b\log(cx^n))}\left(\coth(a+b\log(cx^n)) + iF\left(\frac{1}{4}(-2ia+\pi-2ib\log(cx^n))\middle|2\right)\sqrt{i\sinh(a+b\log(cx^n))}\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] (-2*Sqrt[Csch[a + b*Log[c*x^n]]]*(Coth[a + b*Log[c*x^n]] + I*EllipticF[((-2
*I)*a + Pi - (2*I)*b*Log[c*x^n])/4, 2]*Sqrt[I*Sinh[a + b*Log[c*x^n]]]))/(3*
b*n)
```

Maple [A]
time = 5.41, size = 144, normalized size = 1.30

method	result
derivativedivides	$-\frac{i\sqrt{1-i\sinh(a+b\ln(c\,x^n))}\,\sqrt{2}\,\sqrt{1+i\sinh(a+b\ln(c\,x^n))}\,\sqrt{i\sinh(a+b\ln(c\,x^n)}}{3n\sinh(a+b\ln(c\,x^n))^{\frac{3}{2}}\cosh}$
default	$-\frac{i\sqrt{1-i\sinh(a+b\ln(c\,x^n))}\,\sqrt{2}\,\sqrt{1+i\sinh(a+b\ln(c\,x^n))}\,\sqrt{i\sinh(a+b\ln(c\,x^n)}}{3n\sinh(a+b\ln(c\,x^n))^{\frac{3}{2}}\cosh}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a+b*ln(c*x^n))^(5/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/n/sinh(a+b*ln(c*x^n))^(3/2)*(I*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)
*(1+I*sinh(a+b*ln(c*x^n)))^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((1
-I*sinh(a+b*ln(c*x^n)))^(1/2),1/2*2^(1/2))*sinh(a+b*ln(c*x^n))+2*cosh(a+b*l
n(c*x^n))^2)/cosh(a+b*ln(c*x^n))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^(5/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(csch(b*log(c*x^n) + a)^(5/2)/x, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 318, normalized size = 2.86

$$-\frac{2\left(\sqrt{2}\left(\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2+2\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2+1\right)\sqrt{\frac{\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2+2\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2-1}}+\left(\sqrt{2}\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2+2\sqrt{2}\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sqrt{2}\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2-\sqrt{2}\right)\mathrm{weierstrassPInverse}\left(4,0,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)\right)}{3\left(bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2+2bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+bn\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2-bn\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")
```

```
[Out] -2/3*(sqrt(2)*(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*lo
g(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)
^2 + 1)*sqrt((cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c)
+ a))/(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a
)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1))
 + (sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*sqrt(2)*cosh(b*n*log(x) +
 b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sqrt(2)*sinh(b*n*log(x) +
b*log(c) + a)^2 - sqrt(2))*weierstrassPInverse(4, 0, cosh(b*n*log(x) + b*lo
g(c) + a) + sinh(b*n*log(x) + b*log(c) + a)))/(b*n*cosh(b*n*log(x) + b*log(
c) + a)^2 + 2*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c
) + a) + b*n*sinh(b*n*log(x) + b*log(c) + a)^2 - b*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^{\frac{5}{2}}\left(a + b\log\left(cx^{n}\right)\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*ln(c*x**n))**(5/2)/x,x)
```

```
[Out] Integral(csch(a + b*log(c*x**n))**(5/2)/x, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^(5/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\sinh(a+b\,\ln(c\,x^n))}\right)^{5/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sinh(a + b*log(c*x^n)))^(5/2)/x,x)
```

```
[Out] int((1/sinh(a + b*log(c*x^n)))^(5/2)/x, x)
```

3.171 $\int \frac{\operatorname{csch}^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=107

$$-\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}}{bn}-\frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log\left(cx^n\right)\right)\middle|2\right)}{bn\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

```
[Out] -2*cosh(a+b*ln(c*x^n))*csch(a+b*ln(c*x^n))^(1/2)/b/n+2*I*(sin(1/2*I*a+1/4*P
i+1/2*I*b*ln(c*x^n))^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n))*Ellipti
cE(cos(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n)),2^(1/2))/b/n/csch(a+b*ln(c*x^n))^(
1/2)/(I*sinh(a+b*ln(c*x^n)))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3853, 3856, 2719}

$$-\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}}{bn}-\frac{2iE\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|2\right)}{bn\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] (-2*Cosh[a + b*Log[c*x^n]]*Sqrt[Csch[a + b*Log[c*x^n]]])/(b*n) - ((2*I)*Ell
ipticE[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2])/(b*n*Sqrt[Csch[a + b*Log[c*x^n]
]]*Sqrt[I*Sinh[a + b*Log[c*x^n]]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{\text{Subst}\left(\int \operatorname{csch}^{\frac{3}{2}}(a+bx)\,dx, x, \log(cx^n)\right)}{n}$$

$$= -\frac{2\cosh(a+b\log(cx^n))\sqrt{\operatorname{csch}(a+b\log(cx^n))}}{bn} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\operatorname{csch}(a+bx)}}\right.}{n}$$

$$= -\frac{2\cosh(a+b\log(cx^n))\sqrt{\operatorname{csch}(a+b\log(cx^n))}}{bn} + \frac{\text{Subst}\left(\int \sqrt{i\sinh(a+\right.}{n\sqrt{\operatorname{csch}(a+b\log(cx^n))}}$$

$$= -\frac{2\cosh(a+b\log(cx^n))\sqrt{\operatorname{csch}(a+b\log(cx^n))}}{bn} - \frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}\right.\right.}{bn\sqrt{\operatorname{csch}(a+b\log(cx^n)}}$$

Mathematica [A]
time = 0.08, size = 80, normalized size = 0.75

$$-\frac{2\sqrt{\operatorname{csch}(a+b\log(cx^n))}\left(\cosh(a+b\log(cx^n)) - E\left(\frac{1}{4}(-2ia+\pi-2ib\log(cx^n))\middle|2\right)\sqrt{i\sinh(a+b\log(cx^n))}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] (-2*Sqrt[Csch[a + b*Log[c*x^n]]]*(Cosh[a + b*Log[c*x^n]] - EllipticE[((-2*I
)*a + Pi - (2*I)*b*Log[c*x^n])/4, 2]*Sqrt[I*Sinh[a + b*Log[c*x^n]]]))/(b*n)
```

Maple [A]
time = 5.32, size = 212, normalized size = 1.98

method	result
derivativedivides	$\frac{2\sqrt{1-i\sinh(a+b\ln(c\,x^n))}\,\sqrt{2}\,\sqrt{1+i\sinh(a+b\ln(c\,x^n))}\,\sqrt{i\sinh(a+b\ln(c\,x^n))}}{}$
default	$\frac{2\sqrt{1-i\sinh(a+b\ln(c\,x^n))}\,\sqrt{2}\,\sqrt{1+i\sinh(a+b\ln(c\,x^n))}\,\sqrt{i\sinh(a+b\ln(c\,x^n))}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a+b*ln(c*x^n))^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(2*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(1+I*sinh(a+b*ln(c*x^n)))^(1
/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticE((1-I*sinh(a+b*ln(c*x^n)))^(1/2)
```

```
,1/2*2^(1/2))-(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(1+I*sinh(a+b*ln(c*x^
n)))^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((1-I*sinh(a+b*ln(c*x^n))
)^(1/2),1/2*2^(1/2))-2*cosh(a+b*ln(c*x^n))^2)/cosh(a+b*ln(c*x^n))/sinh(a+b*
ln(c*x^n))^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(csch(b*log(c*x^n) + a)^(3/2)/x, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 159, normalized size = 1.49

$$-\frac{2\left(\sqrt{2}\sqrt{\frac{\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}-1}}\left(\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)+\sqrt{2}\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)))\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] -2*(sqrt(2)*sqrt((cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log
(c) + a))/(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c)
 + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 -
 1))*(cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)) +
sqrt(2)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cosh(b*n*log(x) + b
*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a))))/(b*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^{\frac{3}{2}}\left(a+b\log\left(cx^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*ln(c*x**n))**(3/2)/x,x)
```

```
[Out] Integral(csch(a + b*log(c*x**n))**(3/2)/x, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^(3/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\sinh(a+b\ \ln(c\,x^n))}\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sinh(a + b*log(c*x^n)))^(3/2)/x,x)
```

```
[Out] int((1/sinh(a + b*log(c*x^n)))^(3/2)/x, x)
```

3.172 $\int \frac{\sqrt{\operatorname{csch}(a+b\log(cx^n))}}{x} \, dx$

Optimal. Leaf size=72

$$-\frac{2i\sqrt{\operatorname{csch}(a+b\log(cx^n))}\,F\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log(cx^n)\right)\middle|2\right)\sqrt{i\sinh(a+b\log(cx^n))}}{bn}$$

```
[Out] 2*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n))^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*
I*b*ln(c*x^n))*EllipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n)),2^(1/2))*csc
h(a+b*ln(c*x^n))^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)/b/n
```

Rubi [A]
time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3856, 2720}

$$-\frac{2i\sqrt{i\sinh(a+b\log(cx^n))}\sqrt{\operatorname{csch}(a+b\log(cx^n))}\,F\left(\frac{1}{2}\left(ia+ib\log(cx^n)-\frac{\pi}{2}\right)\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Csch[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] ((-2*I)*Sqrt[Csch[a + b*Log[c*x^n]]]*EllipticF[(I*a - Pi/2 + I*b*Log[c*x^n]
)/2, 2]*Sqrt[I*Sinh[a + b*Log[c*x^n]]])/(b*n)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\sqrt{\operatorname{csch}(a+b\log(cx^n))}}{x} \, dx = \frac{\operatorname{Subst}\left(\int \sqrt{\operatorname{csch}(a+bx)} \, dx, x, \log(cx^n)\right)}{n}$$

$$= \frac{\left(\sqrt{\operatorname{csch}(a+b\log(cx^n))}\sqrt{i\sinh(a+b\log(cx^n))}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{i\sinh(a}}\right.}{n}$$

$$= -\frac{2i\sqrt{\operatorname{csch}(a+b\log(cx^n))}\,F\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log(cx^n)\right)\middle|2\right)\sqrt{i\sinh(a+b}}{bn}$$

Mathematica [A]
time = 0.07, size = 66, normalized size = 0.92

$$\frac{2\operatorname{csch}^{\frac{3}{2}}(a+b\log(cx^n))\,F\left(\frac{1}{4}(-2ia+\pi-2ib\log(cx^n))\middle|\,2\right)(i\sinh(a+b\log(cx^n)))^{3/2}}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Csch[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] (2*Csch[a + b*Log[c*x^n]]^(3/2)*EllipticF[((-2*I)*a + Pi - (2*I)*b*Log[c*x^
n])/4, 2]*(I*Sinh[a + b*Log[c*x^n]])^(3/2))/(b*n)
```

Maple [A]
time = 4.68, size = 120, normalized size = 1.67

method	result
derivativedivides	$\frac{i\sqrt{-i(\sinh(a+b\ln(cx^n))+i)}\,\sqrt{2}\,\sqrt{-i(-\sinh(a+b\ln(cx^n))+i)}\,\sqrt{i\sinh(a+b\ln}}{n\cosh(a+b\ln(cx^n))\sqrt{\sinh(a+b\ln(cx^n}}$
default	$\frac{i\sqrt{-i(\sinh(a+b\ln(cx^n))+i)}\,\sqrt{2}\,\sqrt{-i(-\sinh(a+b\ln(cx^n))+i)}\,\sqrt{i\sinh(a+b\ln}}{n\cosh(a+b\ln(cx^n))\sqrt{\sinh(a+b\ln(cx^n}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(a+b*ln(c*x^n))^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] I/n*(-I*(sinh(a+b*ln(c*x^n))+I))^(1/2)*2^(1/2)*(-I*(-sinh(a+b*ln(c*x^n))+I)
)^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((-I*(sinh(a+b*ln(c*x^n))+I)
)^(1/2),1/2*2^(1/2))/cosh(a+b*ln(c*x^n))/sinh(a+b*ln(c*x^n))^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(csch(b*log(c*x^n) + a))/x, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 39, normalized size = 0.54

$$\frac{2\sqrt{2}\,\text{weierstrassPInverse}(4,0,\cosh(bn\log(x)+b\log(c)+a)+\sinh(bn\log(x)+b\log(c)+a))}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*weierstrassPInverse(4, 0, cosh(b*n*log(x) + b*log(c) + a) + sinh(
b*n*log(x) + b*log(c) + a))/(b*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{csch}\left(a+b\log\left(cx^{n}\right)\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*ln(c*x**n))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(csch(a + b*log(c*x**n)))/x, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(a+b*log(c*x^n))^(1/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1}{\sinh\left(a+b\,\ln\left(c\,x^n\right)\right)}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sinh(a + b*log(c*x^n)))^(1/2)/x,x)
```

```
[Out] int((1/sinh(a + b*log(c*x^n)))^(1/2)/x, x)
```

3.173 $\int \frac{1}{x\sqrt{\operatorname{csch}(a+b\log(cx^n))}}\,dx$

Optimal. Leaf size=72

$$-\frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log\left(cx^n\right)\right)\middle|2\right)}{bn\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

```
[Out] 2*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n))^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*
I*b*ln(c*x^n))*EllipticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n)),2^(1/2))/b/n
/csch(a+b*ln(c*x^n))^(1/2)/(I*sinh(a+b*ln(c*x^n)))^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3856, 2719}

$$-\frac{2iE\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|2\right)}{bn\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\sqrt{\operatorname{csch}\left(a+b\log\left(cx^n\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[Csch[a + b*Log[c*x^n]]]),x]
```

```
[Out] ((-2*I)*EllipticE[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2])/(b*n*Sqrt[Csch[a + b
*Log[c*x^n]]]*Sqrt[I*Sinh[a + b*Log[c*x^n]]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{x\sqrt{\operatorname{csch}(a+b\log(cx^n))}}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{\operatorname{csch}(a+bx)}}\,dx, x, \log(cx^n)\right)}{n}$$

$$= \frac{\operatorname{Subst}\left(\int \sqrt{i\sinh(a+bx)}\,dx, x, \log(cx^n)\right)}{n\sqrt{\operatorname{csch}(a+b\log(cx^n))}\sqrt{i\sinh(a+b\log(cx^n))}}$$

$$= -\frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log(cx^n)\right)\middle|2\right)}{bn\sqrt{\operatorname{csch}(a+b\log(cx^n))}\sqrt{i\sinh(a+b\log(cx^n))}}$$

Mathematica [**A**]
time = 0.06, size = 68, normalized size = 0.94

$$\frac{2\sqrt{\operatorname{csch}(a+b\log(cx^n))}\,E\left(\frac{1}{2}\left(\frac{\pi}{2}-i(a+b\log(cx^n))\right)\middle|2\right)\sqrt{i\sinh(a+b\log(cx^n))}}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[Csch[a + b*Log[c*x^n]]]),x]
```

```
[Out] (2*Sqrt[Csch[a + b*Log[c*x^n]]]*EllipticE[(Pi/2 - I*(a + b*Log[c*x^n]))/2,
2]*Sqrt[I*Sinh[a + b*Log[c*x^n]]])/(b*n)
```

Maple [**A**]
time = 4.98, size = 146, normalized size = 2.03

method	result
derivativedivides	$\frac{\sqrt{-i(\sinh(a+b\ln(c\,x^n))+i)}\,\sqrt{2}\,\sqrt{-i(-\sinh(a+b\ln(c\,x^n))+i)}\,\sqrt{i\sinh(a+b}}{n\cosh(a+b\ln(c\,x}$
default	$\frac{\sqrt{-i(\sinh(a+b\ln(c\,x^n))+i)}\,\sqrt{2}\,\sqrt{-i(-\sinh(a+b\ln(c\,x^n))+i)}\,\sqrt{i\sinh(a+b}}{n\cosh(a+b\ln(c\,x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/csch(a+b*ln(c*x^n))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(-I*(sinh(a+b*ln(c*x^n))+I))^(1/2)*2^(1/2)*(-I*(-sinh(a+b*ln(c*x^n))+I)
)^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*(2*EllipticE((1-I*sinh(a+b*ln(c*x^n))
)^(1/2),1/2*2^(1/2))-EllipticF((1-I*sinh(a+b*ln(c*x^n)))^(1/2),1/2*2^(1/2))
)/cosh(a+b*ln(c*x^n))/sinh(a+b*ln(c*x^n))^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/csch(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*sqrt(csch(b*log(c*x^n) + a))), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 248, normalized size = 3.44

$$-\frac{\sqrt{2}\left(\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2\,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}-1\right)\sqrt{\frac{\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2\,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}-1}}+2\left(\sqrt{2}\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sqrt{2}\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right){\rm weierstrassZeta}\left(4,0,{\rm weierstrassPInverse}\left(4,0,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)\right)}{bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+bn\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/csch(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(2)*(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c)
 + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 -
 1)*sqrt((cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)
)/(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*si
nh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)) + 2
*(sqrt(2)*cosh(b*n*log(x) + b*log(c) + a) + sqrt(2)*sinh(b*n*log(x) + b*log
(c) + a))*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cosh(b*n*log(x) +
 b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a))))/(b*n*cosh(b*n*log(x) +
b*log(c) + a) + b*n*sinh(b*n*log(x) + b*log(c) + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\operatorname{csch}\left(a + b\log\left(cx^{n}\right)\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/csch(a+b*ln(c*x**n))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(csch(a + b*log(c*x**n)))), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/csch(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\sqrt{\frac{1}{\sinh\left(a+b\ln\left(c\,x^n\right)\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1/sinh(a + b*log(c*x^n)))^(1/2)),x)
[Out] int(1/(x*(1/sinh(a + b*log(c*x^n)))^(1/2)), x)
```

3.174 $\int \frac{1}{x\text{csch}^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=111

$$\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)}{3bn\sqrt{\text{csch}\left(a+b\log\left(cx^n\right)\right)}}+\frac{2i\sqrt{\text{csch}\left(a+b\log\left(cx^n\right)\right)}\,F\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log\left(cx^n\right)\right)\middle|2\right)\sqrt{i\sinh\left(a+b\log\left(cx\right.\right.}}{3bn}$$

```
[Out] 2/3*cosh(a+b*ln(c*x^n))/b/n/csch(a+b*ln(c*x^n))^(1/2)-2/3*I*(sin(1/2*I*a+1/
4*Pi+1/2*I*b*ln(c*x^n))^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n))*Elli
pticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n)),2^(1/2))*csch(a+b*ln(c*x^n))^(1
/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)/b/n
```

Rubi [A]
time = 0.05, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3854, 3856, 2720}

$$\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)}{3bn\sqrt{\text{csch}\left(a+b\log\left(cx^n\right)\right)}}+\frac{2i\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\,\sqrt{\text{csch}\left(a+b\log\left(cx^n\right)\right)}\,F\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|2\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Csch[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] (2*Cosh[a + b*Log[c*x^n]])/(3*b*n*Sqrt[Csch[a + b*Log[c*x^n]]]) + (((2*I)/3
)*Sqrt[Csch[a + b*Log[c*x^n]]]*EllipticF[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2
]*Sqrt[I*Sinh[a + b*Log[c*x^n]]])/(b*n)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{x\text{csch}^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\text{csch}^{\frac{3}{2}}(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$
$$= \frac{2\cosh(a+b\log(cx^n))}{3bn\sqrt{\text{csch}(a+b\log(cx^n))}} - \frac{\text{Subst}\left(\int \sqrt{\text{csch}(a+bx)}\,dx, x, \log(cx^n)\right)}{3n}$$
$$= \frac{2\cosh(a+b\log(cx^n))}{3bn\sqrt{\text{csch}(a+b\log(cx^n))}} - \frac{\left(\sqrt{\text{csch}(a+b\log(cx^n))}\sqrt{i\sinh(a+b\log}\right.}{}$$
$$= \frac{2\cosh(a+b\log(cx^n))}{3bn\sqrt{\text{csch}(a+b\log(cx^n))}} + \frac{2i\sqrt{\text{csch}(a+b\log(cx^n))}\,F\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\right.\right.}{3bn}$$

Mathematica [A]
time = 0.09, size = 86, normalized size = 0.77

$$\frac{\sqrt{\text{csch}(a+b\log(cx^n))}\left(-2iF\left(\frac{1}{4}(-2ia+\pi-2ib\log(cx^n))\middle|2\right)\sqrt{i\sinh(a+b\log(cx^n))}+\sinh(2(a+b\log(cx^n)))\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Csch[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] (Sqrt[Csch[a + b*Log[c*x^n]]]*((-2*I)*EllipticF[((-2*I)*a + Pi - (2*I)*b*Lo
g[c*x^n])/4, 2]*Sqrt[I*Sinh[a + b*Log[c*x^n]]] + Sinh[2*(a + b*Log[c*x^n])]
))/(3*b*n)
```

Maple [A]
time = 5.16, size = 143, normalized size = 1.29

method	result
derivativedivides	$-\frac{\frac{i\sqrt{1-i\sinh(a+b\ln(cx^n))}\sqrt{2}\sqrt{1+i\sinh(a+b\ln(cx^n))}\sqrt{i\sinh(a+b\ln(cx^n}}{3}}{n\cosh(a+b\ln(cx^n))\sqrt{\sinh(a-}}$
default	$-\frac{\frac{i\sqrt{1-i\sinh(a+b\ln(cx^n))}\sqrt{2}\sqrt{1+i\sinh(a+b\ln(cx^n))}\sqrt{i\sinh(a+b\ln(cx^n}}{3}}{n\cosh(a+b\ln(cx^n))\sqrt{\sinh(a-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/csch(a+b*ln(c*x^n))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(-1/3*I*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(1+I*sinh(a+b*ln(c*x^n)
))^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((1-I*sinh(a+b*ln(c*x^n)))^
(1/2),1/2*2^(1/2))+2/3*cosh(a+b*ln(c*x^n))^2*sinh(a+b*ln(c*x^n)))/cosh(a+b*
ln(c*x^n))/sinh(a+b*ln(c*x^n))^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/csch(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*csch(b*log(c*x^n) + a)^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 370, normalized size = 3.33

$$\frac{\sqrt{2}\left(\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{4}+4\,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+6\,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+4\,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{4}-1\right)\sqrt{\frac{\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2\,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}-1}}-4\left(\sqrt{2}\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2\,\sqrt{2}\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sqrt{2}\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}\right){\rm weierstrassPInverse}\left(4,0,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)}{6\left(bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2\,bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+bn\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/csch(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(sqrt(2)*(cosh(b*n*log(x) + b*log(c) + a)^4 + 4*cosh(b*n*log(x) + b*log
(c) + a)^3*sinh(b*n*log(x) + b*log(c) + a) + 6*cosh(b*n*log(x) + b*log(c) +
 a)^2*sinh(b*n*log(x) + b*log(c) + a)^2 + 4*cosh(b*n*log(x) + b*log(c) + a)
*sinh(b*n*log(x) + b*log(c) + a)^3 + sinh(b*n*log(x) + b*log(c) + a)^4 - 1)
*sqrt((cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a))/(
cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(
b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)) - 4*(s
qrt(2)*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*sqrt(2)*cosh(b*n*log(x) + b*lo
g(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sqrt(2)*sinh(b*n*log(x) + b*log
(c) + a)^2)*weierstrassPInverse(4, 0, cosh(b*n*log(x) + b*log(c) + a) + sin
h(b*n*log(x) + b*log(c) + a)))/(b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*b
*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + b*n*si
nh(b*n*log(x) + b*log(c) + a)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{csch}^{\frac{3}{2}}\left(a + b \log\left(c x^{n}\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/csch(a+b*ln(c*x**n))**(3/2),x)
```

```
[Out] Integral(1/(x*csch(a + b*log(c*x**n))**(3/2)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/csch(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(\frac{1}{\sinh(a+b\,\ln(c\,x^n))}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1/sinh(a + b*log(c*x^n)))^(3/2)),x)
```

```
[Out] int(1/(x*(1/sinh(a + b*log(c*x^n)))^(3/2)), x)
```

3.175 $\int \frac{1}{x\mathbf{csch}^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=111

$$\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)}{5bn\text{csch}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}+\frac{6iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log\left(cx^n\right)\right)\middle|2\right)}{5bn\sqrt{\text{csch}\left(a+b\log\left(cx^n\right)\right)}\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

```
[Out] 2/5*cosh(a+b*ln(c*x^n))/b/n/csch(a+b*ln(c*x^n))^(3/2)-6/5*I*(sin(1/2*I*a+1/
4*Pi+1/2*I*b*ln(c*x^n))^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n))*Elli
pticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n)),2^(1/2))/b/n/csch(a+b*ln(c*x^n)
)^(1/2)/(I*sinh(a+b*ln(c*x^n)))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3854, 3856, 2719}

$$\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)}{5bn\text{csch}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}+\frac{6iE\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|2\right)}{5bn\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\sqrt{\text{csch}\left(a+b\log\left(cx^n\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Csch[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] (2*Cosh[a + b*Log[c*x^n]])/(5*b*n*Csch[a + b*Log[c*x^n]]^(3/2)) + (((6*I)/5
)*EllipticE[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2])/(b*n*Sqrt[Csch[a + b*Log[c
*x^n]]]*Sqrt[I*Sinh[a + b*Log[c*x^n]]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{x\text{csch}^{\frac{5}{2}}(a+b\log(cx^n))}dx = \frac{\text{Subst}\left(\int \frac{1}{\text{csch}^{\frac{5}{2}}(a+bx)}dx, x, \log(cx^n)\right)}{n}$$

$$= \frac{2\cosh(a+b\log(cx^n))}{5bn\text{csch}^{\frac{3}{2}}(a+b\log(cx^n))} - \frac{3\text{Subst}\left(\int \frac{1}{\sqrt{\text{csch}(a+bx)}}dx, x, \log(cx^n)\right)}{5n}$$

$$= \frac{2\cosh(a+b\log(cx^n))}{5bn\text{csch}^{\frac{3}{2}}(a+b\log(cx^n))} - \frac{3\text{Subst}\left(\int \sqrt{i\sinh(a+bx)}\,dx, x, \log(cx^n\right.}{5n\sqrt{\text{csch}(a+b\log(cx^n))}\sqrt{i\sinh(a+b\log}}$$

$$= \frac{2\cosh(a+b\log(cx^n))}{5bn\text{csch}^{\frac{3}{2}}(a+b\log(cx^n))} + \frac{6iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log(cx^n)\right)\middle|2\right)}{5bn\sqrt{\text{csch}(a+b\log(cx^n))}\sqrt{i\sinh(a+b\log}}$$

Mathematica [A]
time = 0.13, size = 95, normalized size = 0.86

$$\frac{2\left(\cosh(a+b\log(cx^n)) - 3\text{csch}^2(a+b\log(cx^n))\,E\left(\frac{1}{4}(-2ia+\pi-2ib\log(cx^n))\middle|2\right)\sqrt{i\sinh(a+b\log(cx^n))}\right)}{5bn\text{csch}^{\frac{3}{2}}(a+b\log(cx^n))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Csch[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] (2*(Cosh[a + b*Log[c*x^n]] - 3*Csch[a + b*Log[c*x^n]]^2*EllipticE[((-2*I)*a
 + Pi - (2*I)*b*Log[c*x^n])/4, 2]*Sqrt[I*Sinh[a + b*Log[c*x^n]]]))/(5*b*n*C
sch[a + b*Log[c*x^n]]^(3/2))
```

Maple [A]
time = 5.47, size = 227, normalized size = 2.05

method	result
derivativedivides	$-\frac{\frac{6\sqrt{1-i\sinh(a+b\ln(c\,x^n))}\sqrt{2}\sqrt{1+i\sinh(a+b\ln(c\,x^n))}\sqrt{i\sinh(a+b\ln(c\,x^n}}{5}}{}$
default	$-\frac{\frac{6\sqrt{1-i\sinh(a+b\ln(c\,x^n))}\sqrt{2}\sqrt{1+i\sinh(a+b\ln(c\,x^n))}\sqrt{i\sinh(a+b\ln(c\,x^n}}{5}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/csch(a+b*ln(c*x^n))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(-6/5*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(1+I*sinh(a+b*ln(c*x^n)))
^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticE((1-I*sinh(a+b*ln(c*x^n)))^(1
/2),1/2*2^(1/2))+3/5*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(1+I*sinh(a+b*
ln(c*x^n)))^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((1-I*sinh(a+b*ln(
c*x^n)))^(1/2),1/2*2^(1/2))+2/5*cosh(a+b*ln(c*x^n))^4-2/5*cosh(a+b*ln(c*x^n
))^2)/cosh(a+b*ln(c*x^n))/sinh(a+b*ln(c*x^n))^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/csch(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*csch(b*log(c*x^n) + a)^(5/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 602, normalized size = 5.42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/csch(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/20*(sqrt(2)*(cosh(b*n*log(x) + b*log(c) + a)^6 + 6*cosh(b*n*log(x) + b*lo
g(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^5 + sinh(b*n*log(x) + b*log(c) +
a)^6 + (15*cosh(b*n*log(x) + b*log(c) + a)^2 + 11)*sinh(b*n*log(x) + b*log(
c) + a)^4 + 11*cosh(b*n*log(x) + b*log(c) + a)^4 + 4*(5*cosh(b*n*log(x) + b
*log(c) + a)^3 + 11*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*lo
g(c) + a)^3 + (15*cosh(b*n*log(x) + b*log(c) + a)^4 + 66*cosh(b*n*log(x) +
b*log(c) + a)^2 - 13)*sinh(b*n*log(x) + b*log(c) + a)^2 - 13*cosh(b*n*log(x
) + b*log(c) + a)^2 + 2*(3*cosh(b*n*log(x) + b*log(c) + a)^5 + 22*cosh(b*n*
log(x) + b*log(c) + a)^3 - 13*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log
(x) + b*log(c) + a) + 1)*sqrt((cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*l
og(x) + b*log(c) + a))/(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(
x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*lo
g(c) + a)^2 - 1)) + 24*(sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*sqrt(
2)*cosh(b*n*log(x) + b*log(c) + a)^2*sinh(b*n*log(x) + b*log(c) + a) + 3*sq
rt(2)*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^2 + s
qrt(2)*sinh(b*n*log(x) + b*log(c) + a)^3)*weierstrassZeta(4, 0, weierstrass
PInverse(4, 0, cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c)
 + a))))/(b*n*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*b*n*cosh(b*n*log(x) + b
*log(c) + a)^2*sinh(b*n*log(x) + b*log(c) + a) + 3*b*n*cosh(b*n*log(x) + b*
```

```
log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^2 + b*n*sinh(b*n*log(x) + b*log
(c) + a)^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\,\mathrm{csch}^{\frac{5}{2}}\left(a+b\log\left(cx^{n}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/csch(a+b*ln(c*x**n))**(5/2),x)
[Out] Integral(1/(x*csch(a + b*log(c*x**n))**(5/2)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/csch(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(\frac{1}{\sinh(a+b\,\ln(c\,x^n))}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1/sinh(a + b*log(c*x^n)))^(5/2)),x)
[Out] int(1/(x*(1/sinh(a + b*log(c*x^n)))^(5/2)), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1]))  #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
   #debug=False
   m = func.name() in ['exp','log','ln',
         'sin','cos','tan','cot','sec','csc',
         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
         'sinh','cosh','tanh','coth','sech','csch',
         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
      'arctan2','floor','abs'
      ]
   if debug:
      if m:
         print ("func ", func , " is elementary_function")
      else:
         print ("func ", func , " is NOT elementary_function")


   return m

def is_special_function(func):
   #debug=False
   if debug:
      print ("type(func)=", type(func))

   m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
         'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
         'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
         'elliptic_pi','exp_integral_e','log_integral']

   if debug:
      print ("m=",m)
      if m:
         print ("func ", func ," is special_function")
      else:
         print ("func ", func ," is NOT special_function")


   return m


def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])   #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)  #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(5,m1)  #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(6,m1)  #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```