Computer algebra independent integration tests

Summer 2022 edition

6-Hyperbolic-functions/6.5-Hyperbolic-secant/179-6.5.3-Hyperbolic-secant-functions

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [201]. This is test number [179].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (201)	0.00 (0)
Mathematica	95.52 (192)	4.48 (9)
Fricas	89.05 (179)	10.95 (22)
Maple	69.65 (140)	30.35 (61)
Giac	57.21 (115)	42.79 (86)
Mupad	46.77 (94)	53.23 (107)
Maxima	44.78 (90)	55.22 (111)
Sympy	5.47 (11)	94.53 (190)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

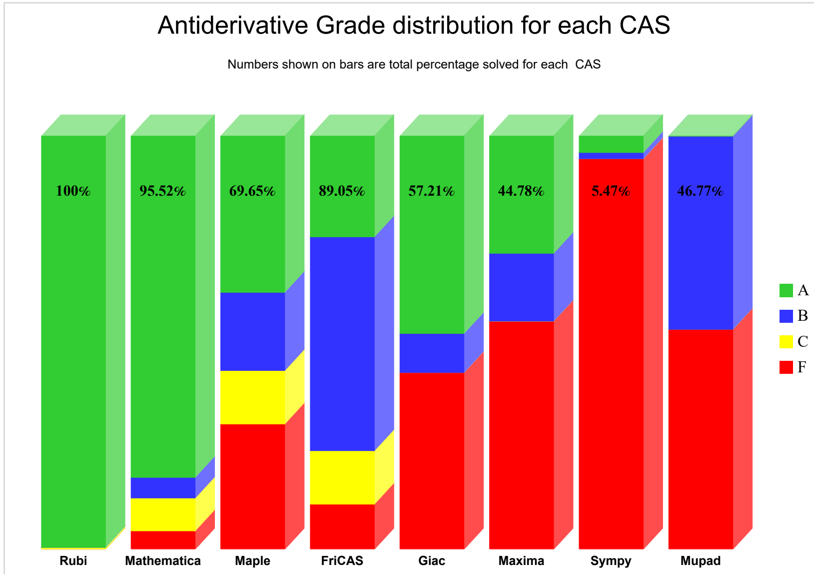
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

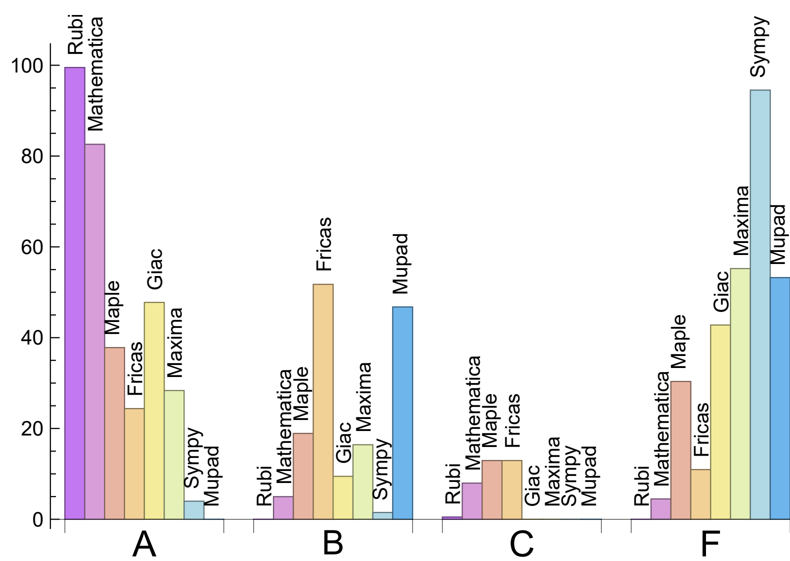
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.50	0.00	0.50	0.00
Mathematica	82.59	4.98	7.96	4.48
Giac	47.76	9.45	0.00	42.79
Maple	37.81	18.91	12.94	30.35
Maxima	28.36	16.42	0.00	55.22
Fricas	24.38	51.74	12.94	10.95
Sympy	3.98	1.49	0.00	94.53
Mupad	N/A	46.77	0.00	53.23

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	9	44.44 %	55.56 %	0.00 %
Maple	61	100.00 %	0.00 %	0.00 %
Fricas	22	81.82 %	0.00 %	18.18 %
Giac	86	68.60 %	26.74 %	4.65 %
Maxima	111	81.98 %	0.00 %	18.02 %
Sympy	190	95.26 %	2.11 %	2.63 %
Mupad	107	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

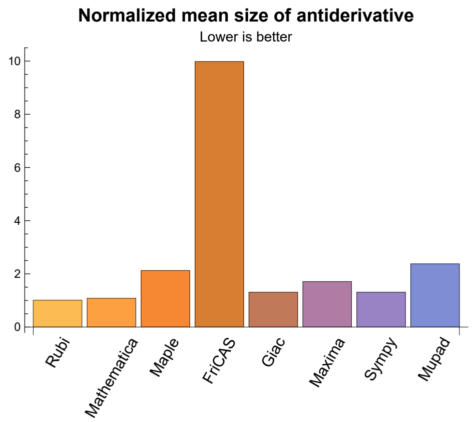
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

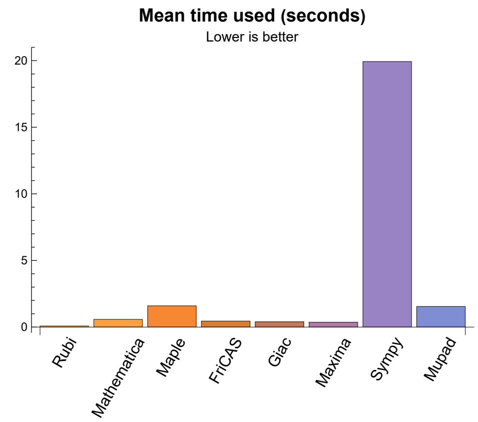
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.08	93.02	1.01	66.00	1.00
Mathematica	0.58	81.50	1.08	59.00	1.00
Maple	1.59	126.16	2.13	103.50	1.56
Maxima	0.36	88.78	1.71	62.00	1.56
Fricas	0.45	1258.22	9.98	235.00	5.33
Sympy	19.93	92.45	1.31	49.00	1.05
Giac	0.40	76.44	1.31	56.00	1.19
Mupad	1.54	154.05	2.38	75.50	2.14

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
10
8
6
4
2
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
20
15
10
5
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi $\{\}$

Mathematica $\{129, 137, 145, 146, 190\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

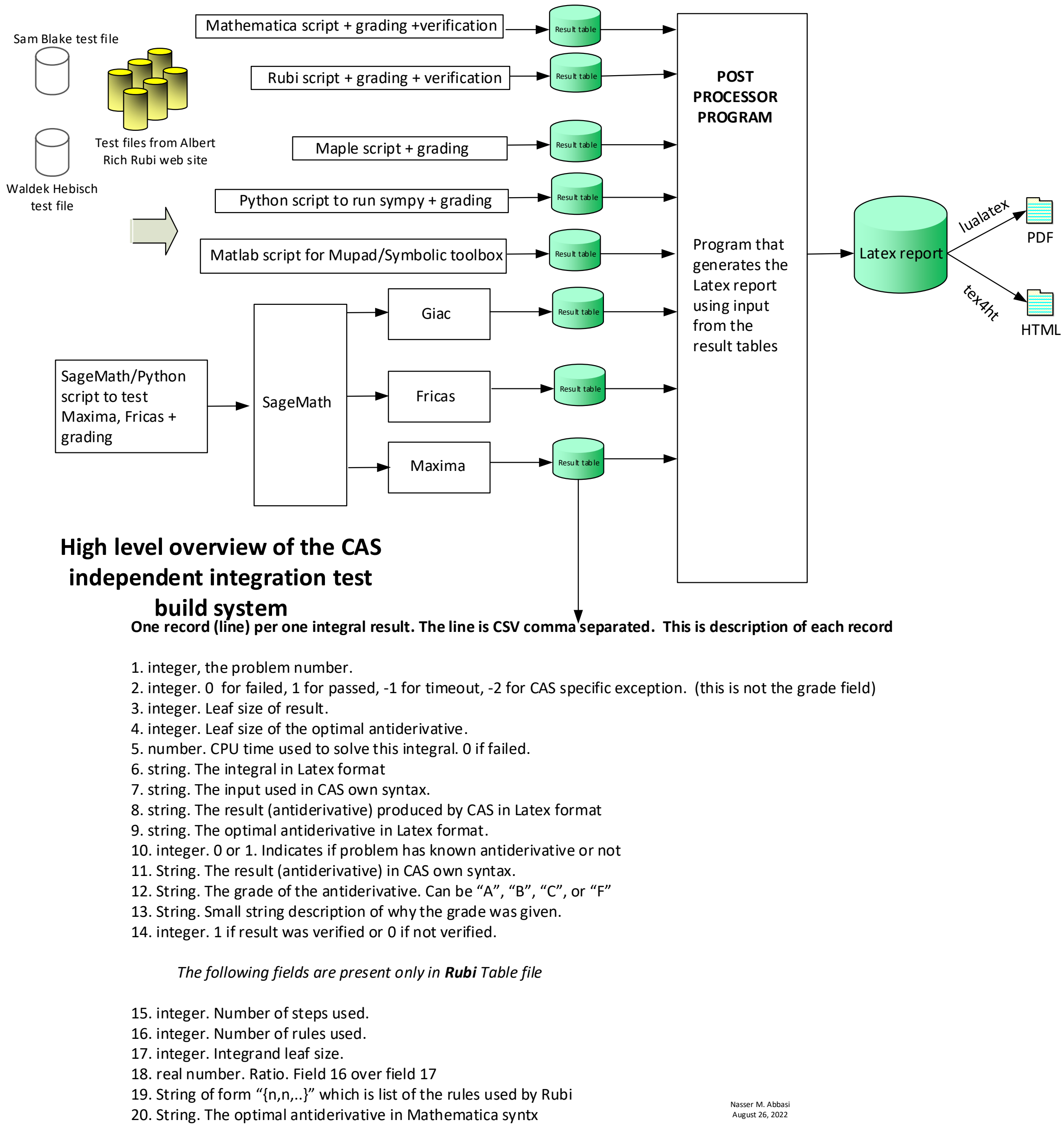
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201 }

B grade: { }

C grade: { 186 }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 133, 134, 140, 142, 143, 144, 151, 152, 153, 154, 155, 156, 157, 159, 161, 163, 164, 165, 167, 169, 171, 173, 175, 178, 179, 182, 183, 184, 186, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201 }

B grade: { 27, 82, 85, 86, 132, 135, 136, 185, 187, 188 }

C grade: { 129, 137, 145, 146, 158, 160, 162, 166, 168, 170, 172, 174, 176, 177, 180, 181 }

F grade: { 130, 131, 138, 139, 141, 147, 148, 149, 150 }

2.1.3 Maple

A grade: { 1, 2, 4, 6, 7, 8, 10, 45, 46, 47, 55, 56, 57, 58, 59, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 90, 91, 92, 93, 98, 99, 100, 101, 102, 103, 104, 108, 109, 110, 111, 112, 114, 116, 118, 119, 120, 121, 122, 123, 124, 127, 135, 144, 151, 152, 153, 154, 155, 156, 157, 159, 161, 167, 169, 171, 173, 177, 178, 191, 197, 200 }

B grade: { 9, 11, 12, 13, 14, 19, 28, 29, 30, 31, 35, 36, 37, 38, 48, 49, 50, 51, 52, 53, 54, 60, 61, 62, 95, 96, 97, 105, 106, 107, 113, 115, 117, 164, 196, 198, 199, 201 }

C grade: { 3, 5, 24, 25, 26, 27, 32, 33, 34, 87, 88, 89, 158, 160, 162, 166, 168, 170, 172, 174, 176, 186, 192, 193, 194, 195 }

F grade: { 15, 16, 17, 18, 20, 21, 22, 23, 39, 40, 41, 42, 43, 44, 78, 79, 80, 81, 82, 83, 84, 85, 86, 94, 125, 126, 128, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 141, 142, 143, 145, 146, 147, 148, 149, 150, 163, 165, 175, 179, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190 }

2.1.4 Maxima

A grade: { 1, 2, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 48, 49, 50, 51, 52, 54, 56, 64, 66, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 88, 89, 90, 107, 108, 109, 110, 111, 117, 119, 120, 122, 152, 153, 154, 155, 156, 157, 159, 171, 179, 191, 192 }

B grade: { 3, 4, 5, 6, 7, 8, 24, 25, 45, 46, 47, 53, 55, 57, 58, 59, 61, 63, 87, 103, 104, 105, 106, 112, 113, 115, 124, 151, 167, 186, 187, 188, 194 }

C grade: { }

F grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 39, 40, 41, 42, 43, 44, 60, 62, 65, 67, 78, 79, 80, 81, 82, 83, 84, 85, 86, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 114, 116, 118, 121, 123, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 158, 160, 161, 162, 163, 164, 165, 166, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 185, 189, 190, 193, 195, 196, 197, 198, 199, 200, 201 }

2.1.5 FriCAS

A grade: { 1, 27, 28, 29, 30, 31, 34, 52, 53, 54, 64, 70, 71, 72, 73, 76, 77, 90, 91, 99, 100, 107, 108, 110, 118, 119, 120, 154, 155, 156, 157, 158, 159, 161, 162, 164, 165, 167, 168, 169, 170, 173, 174, 175, 177, 179, 180, 181, 191 }

B grade: { 2, 3, 4, 5, 6, 7, 8, 24, 25, 26, 32, 33, 35, 36, 37, 38, 45, 46, 47, 48, 49, 50, 51, 55, 56, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 74, 75, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 92, 93, 95, 96, 97, 98, 101, 102, 103, 104, 105, 106, 109, 111, 112, 113, 114, 115, 116, 117, 121, 122, 123, 124, 125, 126, 127, 128, 129, 133, 134, 135, 136, 137, 142, 143, 144, 145, 146, 151, 152, 153, 163, 171, 186, 187, 188, 189, 190, 192, 193, 194, 195 }

C grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 39, 40, 41, 42, 43, 44, 196, 197, 198, 199, 200, 201 }

F grade: { 23, 94, 130, 131, 132, 138, 139, 140, 141, 147, 148, 149, 150, 160, 166, 172, 176, 178, 182, 183, 184, 185 }

2.1.6 Sympy

A grade: { 28, 29, 30, 31, 35, 36, 37, 38 }

B grade: { 108, 119, 157 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 32, 33, 34, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201 }

2.1.7 Giac

A grade: { 1, 2, 4, 6, 7, 8, 24, 25, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 87, 88, 89, 90, 91, 92, 93, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 111, 112, 114, 116, 118, 119, 120, 121, 122, 123, 151, 152, 153, 154, 155, 156, 157, 187, 188, 191, 192, 194, 195 }

B grade: { 3, 5, 26, 58, 59, 66, 79, 80, 82, 83, 85, 86, 106, 113, 115, 117, 124, 186, 193 }

C grade: { }

F grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 39, 40, 41, 42, 43, 44, 81, 84, 94, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 189, 190, 196, 197, 198, 199, 200, 201 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 28, 35, 45, 46, 47, 48, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 87, 88, 89, 90, 91, 92, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 127, 135, 144, 151, 152, 153, 159, 167, 171, 179, 186, 187, 188, 191, 192, 193, 194, 195 }

C grade: { }

F grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 49, 50, 51, 78, 79, 80, 81, 82, 83, 84, 85, 86, 93, 94, 125, 126, 128, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 141, 142, 143, 145, 146, 147, 148, 149, 150, 154, 155, 156, 157, 158, 160, 161, 162, 163, 164, 165, 166, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 185, 189, 190, 196, 197, 198, 199, 200, 201 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	19	0	12	23
N.S.	1	1.00	1.00	1.09	1.00	1.73	0.00	1.09	2.09
time (sec)	N/A	0.003	0.003	0.237	0.260	0.350	0.000	0.389	0.076

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	19	18	41	0	18	18
N.S.	1	1.00	1.00	1.90	1.80	4.10	0.00	1.80	1.80
time (sec)	N/A	0.007	0.003	0.916	0.260	0.381	0.000	0.389	0.079

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	68	65	267	0	76	81
N.S.	1	1.00	1.00	2.00	1.91	7.85	0.00	2.24	2.38
time (sec)	N/A	0.011	0.009	1.095	0.490	0.349	0.000	0.396	0.083

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	32	90	164	0	31	31
N.S.	1	1.00	1.00	1.23	3.46	6.31	0.00	1.19	1.19
time (sec)	N/A	0.009	0.006	1.085	0.270	0.347	0.000	0.394	0.062

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	47	93	112	812	0	102	189
N.S.	1	1.00	0.85	1.69	2.04	14.76	0.00	1.85	3.44
time (sec)	N/A	0.020	0.029	1.047	0.465	0.378	0.000	0.396	1.305

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	43	205	344	0	42	42
N.S.	1	1.00	1.00	1.05	5.00	8.39	0.00	1.02	1.02
time (sec)	N/A	0.011	0.008	1.133	0.263	0.344	0.000	0.386	1.354

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	19	49	116	0	18	30
N.S.	1	1.00	1.00	1.00	2.58	6.11	0.00	0.95	1.58
time (sec)	N/A	0.007	0.004	0.930	0.252	0.343	0.000	0.382	0.096

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	31	137	280	0	30	30
N.S.	1	1.00	1.00	0.89	3.91	8.00	0.00	0.86	0.86
time (sec)	N/A	0.010	0.004	1.186	0.283	0.400	0.000	0.390	1.518

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	51	217	0	190	0	0	-1
N.S.	1	1.00	0.77	3.29	0.00	2.88	0.00	0.00	-0.02
time (sec)	N/A	0.024	0.058	1.458	0.000	0.148	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	49	103	0	96	0	0	-1
N.S.	1	1.00	0.79	1.66	0.00	1.55	0.00	0.00	-0.02
time (sec)	N/A	0.023	0.035	1.675	0.000	0.088	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	135	0	24	0	0	-1
N.S.	1	1.00	1.00	3.38	0.00	0.60	0.00	0.00	-0.02
time (sec)	N/A	0.015	0.024	1.246	0.000	0.129	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	135	0	150	0	0	-1
N.S.	1	1.00	1.00	3.38	0.00	3.75	0.00	0.00	-0.02
time (sec)	N/A	0.015	0.027	1.123	0.000	0.083	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	53	174	0	223	0	0	-1
N.S.	1	1.00	0.80	2.64	0.00	3.38	0.00	0.00	-0.02
time (sec)	N/A	0.024	0.034	1.463	0.000	0.130	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	59	188	0	370	0	0	-1
N.S.	1	1.00	0.89	2.85	0.00	5.61	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.053	1.658	0.000	0.089	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	68	0	0	478	0	0	-1
N.S.	1	1.00	0.67	0.00	0.00	4.69	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.147	1.528	0.000	0.095	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	56	0	0	215	0	0	-1
N.S.	1	1.00	0.76	0.00	0.00	2.91	0.00	0.00	-0.01
time (sec)	N/A	0.027	0.052	1.330	0.000	0.139	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	52	0	0	107	0	0	-1
N.S.	1	1.00	0.74	0.00	0.00	1.53	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.031	1.306	0.000	0.090	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	0	0	27	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.64	0.00	0.00	-0.02
time (sec)	N/A	0.016	0.018	1.539	0.000	0.124	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	244	0	154	0	0	-1
N.S.	1	1.00	1.00	5.81	0.00	3.67	0.00	0.00	-0.02
time (sec)	N/A	0.016	0.025	2.360	0.000	0.084	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	63	0	0	231	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	3.04	0.00	0.00	-0.01
time (sec)	N/A	0.028	0.048	1.389	0.000	0.132	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	64	0	0	379	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	4.99	0.00	0.00	-0.01
time (sec)	N/A	0.026	0.061	1.550	0.000	0.106	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	70	0	0	483	0	0	-1
N.S.	1	1.00	0.67	0.00	0.00	4.64	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.091	1.470	0.000	0.141	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	60	0	0	0	0	0	-1
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.046	1.254	0.000	0.000	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	81	230	156	1604	0	124	-1
N.S.	1	1.00	0.90	2.56	1.73	17.82	0.00	1.38	-0.01
time (sec)	N/A	0.025	0.076	2.601	0.489	0.363	0.000	0.381	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	55	208	112	812	0	102	-1
N.S.	1	1.00	0.85	3.20	1.72	12.49	0.00	1.57	-0.02
time (sec)	N/A	0.017	0.084	2.348	0.459	0.352	0.000	0.391	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	46	183	65	267	0	76	-1
N.S.	1	1.00	1.15	4.58	1.62	6.68	0.00	1.90	-0.02
time (sec)	N/A	0.013	0.036	2.417	0.477	0.372	0.000	0.403	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	29	130	11	19	0	12	-1
N.S.	1	1.00	2.64	11.82	1.00	1.73	0.00	1.09	-0.09
time (sec)	N/A	0.009	0.013	2.489	0.264	0.374	0.000	0.391	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	97	26	10	29	23	53
N.S.	1	1.00	1.00	4.41	1.18	0.45	1.32	1.05	2.41
time (sec)	N/A	0.011	0.019	2.414	0.309	0.390	12.131	0.392	0.155

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	44	201	54	32	54	48	-1
N.S.	1	1.00	0.86	3.94	1.06	0.63	1.06	0.94	-0.02
time (sec)	N/A	0.014	0.047	2.544	0.324	0.363	12.289	0.374	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	54	305	82	66	80	70	-1
N.S.	1	1.00	0.71	4.01	1.08	0.87	1.05	0.92	-0.01
time (sec)	N/A	0.020	0.071	2.495	0.274	0.378	14.511	0.394	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	64	409	100	108	104	92	-1
N.S.	1	1.00	0.63	4.05	0.99	1.07	1.03	0.91	-0.01
time (sec)	N/A	0.025	0.110	2.466	0.265	0.364	35.117	0.387	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	42	127	72	1082	0	65	-1
N.S.	1	1.00	0.65	1.95	1.11	16.65	0.00	1.00	-0.02
time (sec)	N/A	0.023	0.025	0.989	0.518	0.396	0.000	0.399	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	29	106	39	310	0	48	-1
N.S.	1	1.00	0.63	2.30	0.85	6.74	0.00	1.04	-0.02
time (sec)	N/A	0.016	0.015	0.853	0.513	0.384	0.000	0.387	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	21	72	8	145	0	8	-1
N.S.	1	1.00	0.84	2.88	0.32	5.80	0.00	0.32	-0.04
time (sec)	N/A	0.012	0.005	1.024	0.571	0.389	0.000	0.393	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	58	17	79	12	14	33
N.S.	1	1.00	1.00	4.46	1.31	6.08	0.92	1.08	2.54
time (sec)	N/A	0.021	0.005	1.038	0.473	0.398	0.244	0.393	0.120

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	27	130	35	277	31	29	-1
N.S.	1	1.00	0.75	3.61	0.97	7.69	0.86	0.81	-0.03
time (sec)	N/A	0.015	0.016	0.865	0.466	0.366	0.378	0.387	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	36	196	53	580	49	41	-1
N.S.	1	1.00	0.65	3.56	0.96	10.55	0.89	0.75	-0.02
time (sec)	N/A	0.021	0.028	0.849	0.465	0.389	1.607	0.392	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	42	262	71	970	66	53	-1
N.S.	1	1.00	0.57	3.54	0.96	13.11	0.89	0.72	-0.01
time (sec)	N/A	0.027	0.036	1.164	0.478	0.402	12.864	0.374	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	63	0	0	1382	0	0	-1
N.S.	1	1.00	0.52	0.00	0.00	11.42	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.068	1.054	0.000	0.115	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	47	0	0	391	0	0	-1
N.S.	1	1.00	0.68	0.00	0.00	5.67	0.00	0.00	-0.01
time (sec)	N/A	0.028	0.028	0.831	0.000	0.152	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	36	0	0	60	0	0	-1
N.S.	1	1.00	0.78	0.00	0.00	1.30	0.00	0.00	-0.02
time (sec)	N/A	0.021	0.015	1.005	0.000	0.097	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	38	0	0	126	0	0	-1
N.S.	1	1.00	0.79	0.00	0.00	2.62	0.00	0.00	-0.02
time (sec)	N/A	0.021	0.031	1.001	0.000	0.131	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	47	0	0	407	0	0	-1
N.S.	1	1.00	0.61	0.00	0.00	5.29	0.00	0.00	-0.01
time (sec)	N/A	0.031	0.064	0.838	0.000	0.163	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	63	0	0	718	0	0	-1
N.S.	1	1.00	0.52	0.00	0.00	5.93	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.064	0.840	0.000	0.188	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	54	72	620	2804	0	51	498
N.S.	1	1.00	0.33	0.44	3.80	17.20	0.00	0.31	3.06
time (sec)	N/A	0.030	0.122	0.993	0.539	0.439	0.000	0.392	1.453

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	42	60	322	1475	0	39	356
N.S.	1	1.00	0.36	0.51	2.75	12.61	0.00	0.33	3.04
time (sec)	N/A	0.025	0.069	0.864	0.488	0.387	0.000	0.420	1.374

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	30	46	120	516	0	27	46
N.S.	1	1.00	0.49	0.75	1.97	8.46	0.00	0.44	0.75
time (sec)	N/A	0.017	0.040	0.862	0.492	0.368	0.000	0.398	1.344

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	29	13	81	0	13	71
N.S.	1	1.00	1.00	1.93	0.87	5.40	0.00	0.87	4.73
time (sec)	N/A	0.012	0.005	0.994	0.492	0.417	0.000	0.394	0.058

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	23	89	30	253	0	28	-1
N.S.	1	1.00	0.64	2.47	0.83	7.03	0.00	0.78	-0.03
time (sec)	N/A	0.012	0.017	1.043	0.490	0.393	0.000	0.400	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	38	230	65	1141	0	52	-1
N.S.	1	1.00	0.44	2.67	0.76	13.27	0.00	0.60	-0.01
time (sec)	N/A	0.024	0.027	0.931	0.514	0.403	0.000	0.388	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	55	362	103	2600	0	76	-1
N.S.	1	1.00	0.42	2.74	0.78	19.70	0.00	0.58	-0.01
time (sec)	N/A	0.038	0.054	0.931	0.475	0.384	0.000	0.387	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	28	105	54	36	0	42	59
N.S.	1	1.00	0.64	2.39	1.23	0.82	0.00	0.95	1.34
time (sec)	N/A	0.097	0.080	0.595	0.293	0.347	0.000	0.404	1.483

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	67	46	30	0	37	53
N.S.	1	1.00	1.00	2.91	2.00	1.30	0.00	1.61	2.30
time (sec)	N/A	0.088	0.035	0.523	0.297	0.384	0.000	0.391	1.359

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	16	65	42	14	0	28	41
N.S.	1	1.00	0.59	2.41	1.56	0.52	0.00	1.04	1.52
time (sec)	N/A	0.076	0.048	0.585	0.265	0.368	0.000	0.389	1.341

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	16	23	35	50	0	32	15
N.S.	1	1.00	0.94	1.35	2.06	2.94	0.00	1.88	0.88
time (sec)	N/A	0.053	0.013	0.589	0.267	0.360	0.000	0.388	0.068

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	44	20	48	103	0	52	51
N.S.	1	1.00	1.33	0.61	1.45	3.12	0.00	1.58	1.55
time (sec)	N/A	0.072	0.037	0.709	0.262	0.372	0.000	0.383	1.439

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	90	71	0	31	91
N.S.	1	1.00	1.09	1.00	3.91	3.09	0.00	1.35	3.96
time (sec)	N/A	0.087	0.028	0.770	0.302	0.350	0.000	0.389	1.352

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	59	38	99	630	0	90	121
N.S.	1	1.00	1.28	0.83	2.15	13.70	0.00	1.96	2.63
time (sec)	N/A	0.116	0.148	0.796	0.268	0.361	0.000	0.387	1.351

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	39	39	292	219	0	59	236
N.S.	1	1.00	1.15	1.15	8.59	6.44	0.00	1.74	6.94
time (sec)	N/A	0.101	0.044	0.810	0.265	0.352	0.000	0.392	1.376

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	219	310	0	1812	0	197	275
N.S.	1	1.00	1.66	2.35	0.00	13.73	0.00	1.49	2.08
time (sec)	N/A	0.256	0.500	0.711	0.000	0.394	0.000	0.394	2.006

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	66	209	128	490	0	87	123
N.S.	1	1.00	1.08	3.43	2.10	8.03	0.00	1.43	2.02
time (sec)	N/A	0.123	0.091	0.659	0.271	0.366	0.000	0.393	1.602

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	76	160	0	536	0	100	173
N.S.	1	1.00	0.93	1.95	0.00	6.54	0.00	1.22	2.11
time (sec)	N/A	0.131	0.128	0.683	0.000	0.368	0.000	0.388	1.670

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	19	31	46	78	0	34	20
N.S.	1	1.00	0.95	1.55	2.30	3.90	0.00	1.70	1.00
time (sec)	N/A	0.057	0.008	0.625	0.261	0.368	0.000	0.383	1.350

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	37	48	59	58	0	65	148
N.S.	1	1.00	0.70	0.91	1.11	1.09	0.00	1.23	2.79
time (sec)	N/A	0.079	0.051	0.789	0.267	0.372	0.000	0.384	1.744

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	75	77	0	452	0	64	151
N.S.	1	1.00	1.14	1.17	0.00	6.85	0.00	0.97	2.29
time (sec)	N/A	0.099	0.175	0.826	0.000	0.386	0.000	0.385	1.561

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	86	82	148	828	0	174	255
N.S.	1	1.00	1.01	0.96	1.74	9.74	0.00	2.05	3.00
time (sec)	N/A	0.163	0.233	0.875	0.291	0.446	0.000	0.390	1.831

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	156	127	0	2340	0	149	295
N.S.	1	1.00	1.41	1.14	0.00	21.08	0.00	1.34	2.66
time (sec)	N/A	0.202	0.397	0.861	0.000	0.404	0.000	0.400	1.747

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	63	110	80	139	0	86	88
N.S.	1	1.00	0.94	1.64	1.19	2.07	0.00	1.28	1.31
time (sec)	N/A	0.060	0.062	0.832	0.263	0.459	0.000	0.394	1.447

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	53	86	66	100	0	70	70
N.S.	1	1.00	0.98	1.59	1.22	1.85	0.00	1.30	1.30
time (sec)	N/A	0.056	0.051	0.795	0.261	0.424	0.000	0.382	1.358

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	45	70	56	70	0	51	52
N.S.	1	1.00	1.10	1.71	1.37	1.71	0.00	1.24	1.27
time (sec)	N/A	0.052	0.036	0.774	0.282	0.378	0.000	0.394	1.360

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	32	46	41	47	0	35	34
N.S.	1	1.00	1.23	1.77	1.58	1.81	0.00	1.35	1.31
time (sec)	N/A	0.036	0.043	0.769	0.268	0.400	0.000	0.374	1.312

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	10	9	12	14	0	11	11
N.S.	1	1.00	0.91	0.82	1.09	1.27	0.00	1.00	1.00
time (sec)	N/A	0.018	0.010	0.424	0.283	0.375	0.000	0.388	1.311

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	19	23	29	0	20	31
N.S.	1	1.00	1.10	0.95	1.15	1.45	0.00	1.00	1.55
time (sec)	N/A	0.042	0.021	0.503	0.483	0.390	0.000	0.391	1.305

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	45	33	45	127	0	36	58
N.S.	1	1.00	1.73	1.27	1.73	4.88	0.00	1.38	2.23
time (sec)	N/A	0.070	0.064	0.521	0.475	0.459	0.000	0.383	1.318

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	51	46	73	325	0	48	73
N.S.	1	1.00	1.13	1.02	1.62	7.22	0.00	1.07	1.62
time (sec)	N/A	0.054	0.059	0.671	0.479	0.382	0.000	0.385	1.345

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	58	46	33	48	0	29	24
N.S.	1	1.00	2.00	1.59	1.14	1.66	0.00	1.00	0.83
time (sec)	N/A	0.011	0.103	2.155	0.290	0.359	0.000	0.379	1.299

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	59	48	35	50	0	29	24
N.S.	1	1.00	1.97	1.60	1.17	1.67	0.00	0.97	0.80
time (sec)	N/A	0.011	0.101	1.721	0.261	0.420	0.000	0.385	1.263

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	99	0	0	924	0	151	-1
N.S.	1	1.00	1.01	0.00	0.00	9.43	0.00	1.54	-0.01
time (sec)	N/A	0.086	0.233	2.743	0.000	0.533	0.000	0.463	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	75	0	0	697	0	118	-1
N.S.	1	1.00	1.14	0.00	0.00	10.56	0.00	1.79	-0.02
time (sec)	N/A	0.028	0.137	2.525	0.000	0.489	0.000	0.453	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	60	0	0	637	0	83	-1
N.S.	1	1.00	1.62	0.00	0.00	17.22	0.00	2.24	-0.03
time (sec)	N/A	0.014	0.065	2.958	0.000	0.487	0.000	0.429	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	118	0	0	868	0	0	-1
N.S.	1	1.00	1.39	0.00	0.00	10.21	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.828	2.627	0.000	0.406	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	231	0	0	1190	0	316	-1
N.S.	1	1.00	2.03	0.00	0.00	10.44	0.00	2.77	-0.01
time (sec)	N/A	0.093	1.146	2.536	0.000	0.439	0.000	0.477	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	70	0	0	642	0	101	-1
N.S.	1	1.00	1.84	0.00	0.00	16.89	0.00	2.66	-0.03
time (sec)	N/A	0.018	1.638	2.825	0.000	0.451	0.000	0.414	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	118	0	0	871	0	0	-1
N.S.	1	1.00	1.36	0.00	0.00	10.01	0.00	0.00	-0.01
time (sec)	N/A	0.054	1.266	2.279	0.000	0.431	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	39	0	0	233	0	52	-1
N.S.	1	1.00	2.05	0.00	0.00	12.26	0.00	2.74	-0.05
time (sec)	N/A	0.013	0.027	1.214	0.000	0.396	0.000	0.385	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	51	0	0	235	0	69	-1
N.S.	1	1.00	2.43	0.00	0.00	11.19	0.00	3.29	-0.05
time (sec)	N/A	0.015	0.382	1.197	0.000	0.445	0.000	0.380	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	78	182	211	1028	0	141	233
N.S.	1	1.00	0.73	1.70	1.97	9.61	0.00	1.32	2.18
time (sec)	N/A	0.085	0.178	1.494	0.483	0.401	0.000	0.396	1.409

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	55	142	114	521	0	92	165
N.S.	1	1.00	0.75	1.95	1.56	7.14	0.00	1.26	2.26
time (sec)	N/A	0.036	0.096	1.503	0.490	0.417	0.000	0.388	1.398

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	64	41	157	0	43	70
N.S.	1	1.00	0.97	1.94	1.24	4.76	0.00	1.30	2.12
time (sec)	N/A	0.020	0.045	1.502	0.266	0.424	0.000	0.388	0.106

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	16	26	0	17	38
N.S.	1	1.00	1.00	1.06	1.00	1.62	0.00	1.06	2.38
time (sec)	N/A	0.006	0.003	0.832	0.270	0.401	0.000	0.394	1.299

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	60	83	0	270	0	56	131
N.S.	1	1.00	1.02	1.41	0.00	4.58	0.00	0.95	2.22
time (sec)	N/A	0.042	0.077	1.655	0.000	0.394	0.000	0.390	0.399

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	203	164	0	1207	0	134	296
N.S.	1	1.00	1.86	1.50	0.00	11.07	0.00	1.23	2.72
time (sec)	N/A	0.116	0.279	1.503	0.000	0.409	0.000	0.395	1.849

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	205	251	0	4125	0	261	-1
N.S.	1	1.00	1.18	1.45	0.00	23.84	0.00	1.51	-0.01
time (sec)	N/A	0.239	0.484	1.776	0.000	0.444	0.000	0.413	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	168	0	0	0	0	0	-1
N.S.	1	1.00	1.58	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.020	3.430	2.608	0.000	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	126	299	0	2402	0	182	251
N.S.	1	1.00	0.86	2.05	0.00	16.45	0.00	1.25	1.72
time (sec)	N/A	0.446	0.187	0.837	0.000	0.406	0.000	0.398	1.851

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	99	203	0	1562	0	133	209
N.S.	1	1.00	0.88	1.81	0.00	13.95	0.00	1.19	1.87
time (sec)	N/A	0.287	0.113	0.833	0.000	0.405	0.000	0.394	1.713

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	78	153	0	860	0	92	167
N.S.	1	1.00	0.92	1.80	0.00	10.12	0.00	1.08	1.96
time (sec)	N/A	0.182	0.085	0.812	0.000	0.385	0.000	0.386	1.579

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	57	94	0	430	0	62	139
N.S.	1	1.00	0.92	1.52	0.00	6.94	0.00	1.00	2.24
time (sec)	N/A	0.065	0.077	0.816	0.000	0.390	0.000	0.406	1.481

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	41	36	0	165	0	32	43
N.S.	1	1.00	0.98	0.86	0.00	3.93	0.00	0.76	1.02
time (sec)	N/A	0.038	0.018	0.422	0.000	0.382	0.000	0.391	0.116

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	51	0	219	0	45	286
N.S.	1	1.00	1.00	0.94	0.00	4.06	0.00	0.83	5.30
time (sec)	N/A	0.072	0.037	0.569	0.000	0.403	0.000	0.391	4.006

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	63	73	0	504	0	61	294
N.S.	1	1.00	0.98	1.14	0.00	7.88	0.00	0.95	4.59
time (sec)	N/A	0.107	0.072	0.603	0.000	0.411	0.000	0.398	3.881

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	82	109	0	1444	0	89	476
N.S.	1	1.00	0.94	1.25	0.00	16.60	0.00	1.02	5.47
time (sec)	N/A	0.177	0.147	0.744	0.000	0.461	0.000	0.396	5.079

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	60	75	93	686	0	69	143
N.S.	1	1.00	1.25	1.56	1.94	14.29	0.00	1.44	2.98
time (sec)	N/A	0.067	0.081	0.694	0.485	0.385	0.000	0.388	1.461

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	38	26	74	437	0	61	96
N.S.	1	1.00	1.06	0.72	2.06	12.14	0.00	1.69	2.67
time (sec)	N/A	0.042	0.050	0.663	0.475	0.400	0.000	0.383	1.429

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	41	59	51	210	0	42	67
N.S.	1	1.00	1.32	1.90	1.65	6.77	0.00	1.35	2.16
time (sec)	N/A	0.050	0.041	0.672	0.482	0.378	0.000	0.384	1.438

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	10	48	33	85	0	35	33
N.S.	1	1.00	0.71	3.43	2.36	6.07	0.00	2.50	2.36
time (sec)	N/A	0.034	0.025	0.659	0.460	0.366	0.000	0.386	1.357

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	15	32	16	14	0	14	25
N.S.	1	1.00	1.07	2.29	1.14	1.00	0.00	1.00	1.79
time (sec)	N/A	0.030	0.021	0.596	0.472	0.373	0.000	0.392	1.321

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	12	17	18	16	19	17	14
N.S.	1	1.00	1.33	1.89	2.00	1.78	2.11	1.89	1.56
time (sec)	N/A	0.018	0.007	0.579	0.272	0.355	0.069	0.403	1.305

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	44	38	52	136	0	56	65
N.S.	1	1.00	1.10	0.95	1.30	3.40	0.00	1.40	1.62
time (sec)	N/A	0.042	0.035	0.789	0.255	0.349	0.000	0.399	1.368

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	33	47	47	46	0	40	94
N.S.	1	1.00	0.87	1.24	1.24	1.21	0.00	1.05	2.47
time (sec)	N/A	0.073	0.053	1.076	0.279	0.375	0.000	0.392	1.352

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	66	56	108	773	0	94	160
N.S.	1	1.00	0.97	0.82	1.59	11.37	0.00	1.38	2.35
time (sec)	N/A	0.058	0.127	1.024	0.280	0.384	0.000	0.387	1.429

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	69	63	105	151	0	64	264
N.S.	1	1.00	1.25	1.15	1.91	2.75	0.00	1.16	4.80
time (sec)	N/A	0.081	0.072	0.830	0.275	0.387	0.000	0.395	1.535

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	132	246	332	4077	0	267	316
N.S.	1	1.00	1.09	2.03	2.74	33.69	0.00	2.21	2.61
time (sec)	N/A	0.105	0.220	0.741	0.494	0.445	0.000	0.412	1.988

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	185	266	0	4914	0	250	1001
N.S.	1	1.00	0.99	1.42	0.00	26.28	0.00	1.34	5.35
time (sec)	N/A	0.210	0.415	1.142	0.000	0.579	0.000	0.385	8.505

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	85	158	164	1280	0	152	155
N.S.	1	1.00	1.18	2.19	2.28	17.78	0.00	2.11	2.15
time (sec)	N/A	0.068	0.121	0.707	0.506	0.369	0.000	0.385	1.797

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	113	152	0	1254	0	111	700
N.S.	1	1.00	1.20	1.62	0.00	13.34	0.00	1.18	7.45
time (sec)	N/A	0.210	0.281	1.025	0.000	0.455	0.000	0.388	7.263

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	37	92	67	200	0	73	260
N.S.	1	1.00	1.06	2.63	1.91	5.71	0.00	2.09	7.43
time (sec)	N/A	0.050	0.060	0.682	0.485	0.379	0.000	0.384	1.598

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	62	84	0	193	0	52	273
N.S.	1	1.00	1.00	1.35	0.00	3.11	0.00	0.84	4.40
time (sec)	N/A	0.120	0.058	0.668	0.000	0.390	0.000	0.414	3.921

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	11	21	26	27	41	19	23
N.S.	1	1.00	0.58	1.11	1.37	1.42	2.16	1.00	1.21
time (sec)	N/A	0.023	0.014	0.545	0.271	0.371	0.247	0.399	0.109

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	44	78	67	81	0	67	271
N.S.	1	1.00	0.67	1.18	1.02	1.23	0.00	1.02	4.11
time (sec)	N/A	0.072	0.060	0.909	0.262	0.379	0.000	0.394	1.721

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	81	104	0	646	0	82	383
N.S.	1	1.00	0.71	0.91	0.00	5.67	0.00	0.72	3.36
time (sec)	N/A	0.149	0.236	0.879	0.000	0.378	0.000	0.397	1.667

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	112	113	164	1222	0	193	339
N.S.	1	1.00	0.99	1.00	1.45	10.81	0.00	1.71	3.00
time (sec)	N/A	0.135	0.214	0.944	0.269	0.422	0.000	0.397	2.218

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	166	153	0	3530	0	190	713
N.S.	1	1.00	0.80	0.74	0.00	17.05	0.00	0.92	3.44
time (sec)	N/A	0.243	0.514	0.924	0.000	0.431	0.000	0.386	1.833

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	167	162	366	5181	0	380	623
N.S.	1	1.00	0.94	0.91	2.06	29.11	0.00	2.13	3.50
time (sec)	N/A	0.225	0.680	1.265	0.302	0.490	0.000	0.392	2.746

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	160	0	0	4363	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	25.82	0.00	0.00	-0.01
time (sec)	N/A	0.138	3.551	3.495	0.000	0.832	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	108	0	0	1589	0	0	-1
N.S.	1	1.00	1.08	0.00	0.00	15.89	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.670	2.747	0.000	0.792	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	90	43	0	605	0	0	47
N.S.	1	1.00	1.76	0.84	0.00	11.86	0.00	0.00	0.92
time (sec)	N/A	0.038	0.104	1.671	0.000	0.790	0.000	0.000	1.692

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	211	0	0	8620	0	0	-1
N.S.	1	1.00	1.99	0.00	0.00	81.32	0.00	0.00	-0.01
time (sec)	N/A	0.122	1.292	2.893	0.000	0.735	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	456	0	0	16532	0	0	-1
N.S.	1	1.00	2.10	0.00	0.00	76.18	0.00	0.00	-0.00
time (sec)	N/A	0.231	18.141	3.093	0.000	1.130	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.282	180.002	2.125	0.000	0.000	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.017	5.457	2.369	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	539	0	0	0	0	0	-1
N.S.	1	1.00	2.19	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.149	16.035	2.647	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	167	0	0	2813	0	0	-1
N.S.	1	1.00	1.13	0.00	0.00	19.01	0.00	0.00	-0.01
time (sec)	N/A	0.111	3.841	3.219	0.000	0.795	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	111	0	0	925	0	0	-1
N.S.	1	1.00	1.41	0.00	0.00	11.71	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.444	2.667	0.000	0.799	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	73	26	0	558	0	0	27
N.S.	1	1.00	2.35	0.84	0.00	18.00	0.00	0.00	0.87
time (sec)	N/A	0.033	0.096	1.615	0.000	0.766	0.000	0.000	1.635

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	226	0	0	8908	0	0	-1
N.S.	1	1.00	2.13	0.00	0.00	84.04	0.00	0.00	-0.01
time (sec)	N/A	0.103	2.496	2.799	0.000	0.969	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	505	0	0	20300	0	0	-1
N.S.	1	1.00	1.93	0.00	0.00	77.48	0.00	0.00	-0.00
time (sec)	N/A	0.216	5.415	3.453	0.000	4.405	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	610	610	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.556	180.004	3.186	0.000	0.000	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.177	180.002	2.020	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	168	0	0	0	0	0	-1
N.S.	1	1.00	1.58	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.016	0.449	0.031	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.311	90.927	2.862	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	155	0	0	3745	0	0	-1
N.S.	1	1.00	1.05	0.00	0.00	25.30	0.00	0.00	-0.01
time (sec)	N/A	0.136	2.177	3.273	0.000	0.830	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	103	0	0	1107	0	0	-1
N.S.	1	1.00	1.17	0.00	0.00	12.58	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.458	2.576	0.000	0.779	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	79	46	0	917	0	0	50
N.S.	1	1.00	1.46	0.85	0.00	16.98	0.00	0.00	0.93
time (sec)	N/A	0.042	0.174	1.543	0.000	0.822	0.000	0.000	1.770

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	483	0	0	14412	0	0	-1
N.S.	1	1.00	3.40	0.00	0.00	101.49	0.00	0.00	-0.01
time (sec)	N/A	0.156	4.423	2.565	0.000	3.846	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	1084	0	0	53212	0	0	-1
N.S.	1	1.00	3.43	0.00	0.00	168.39	0.00	0.00	-0.00
time (sec)	N/A	0.310	6.787	3.313	0.000	6.470	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	907	907	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.938	180.005	3.099	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.284	180.002	2.059	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.231	72.367	2.420	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	665	665	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.680	150.719	2.837	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	84	91	386	589	0	64	405
N.S.	1	1.00	0.44	0.48	2.02	3.08	0.00	0.34	2.12
time (sec)	N/A	0.200	0.061	4.204	0.291	0.368	0.000	0.393	0.167

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	72	80	209	315	0	51	91
N.S.	1	1.00	0.51	0.57	1.48	2.23	0.00	0.36	0.65
time (sec)	N/A	0.121	0.048	4.089	0.282	0.366	0.000	0.370	1.429

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	44	69	84	120	0	38	78
N.S.	1	1.00	0.79	1.23	1.50	2.14	0.00	0.68	1.39
time (sec)	N/A	0.080	0.041	4.060	0.272	0.354	0.000	0.394	0.138

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	42	66	21	42	0	20	-1
N.S.	1	1.00	0.95	1.50	0.48	0.95	0.00	0.45	-0.02
time (sec)	N/A	0.059	0.028	4.217	0.500	0.360	0.000	0.391	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	48	106	29	66	0	33	-1
N.S.	1	1.00	0.65	1.43	0.39	0.89	0.00	0.45	-0.01
time (sec)	N/A	0.081	0.037	4.743	0.257	0.358	0.000	0.398	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	78	216	74	126	0	82	-1
N.S.	1	1.00	0.48	1.33	0.46	0.78	0.00	0.51	-0.01
time (sec)	N/A	0.107	0.046	4.927	0.266	0.355	0.000	0.397	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	106	326	112	218	532	110	-1
N.S.	1	1.00	0.42	1.30	0.45	0.87	2.13	0.44	-0.00
time (sec)	N/A	0.137	0.075	4.543	0.273	0.364	129.760	0.390	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	77	130	0	80	0	0	-1
N.S.	1	1.00	0.71	1.20	0.00	0.74	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.124	1.432	0.000	0.106	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	44	39	30	48	0	0	42
N.S.	1	1.00	1.57	1.39	1.07	1.71	0.00	0.00	1.50
time (sec)	N/A	0.029	0.032	1.210	0.507	0.371	0.000	0.000	1.471

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	65	134	0	0	0	0	-1
N.S.	1	1.00	0.32	0.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.089	0.086	1.269	0.000	0.000	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	77	97	0	90	0	0	-1
N.S.	1	1.00	1.15	1.45	0.00	1.34	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.105	1.428	0.000	0.378	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	58	114	0	70	0	0	-1
N.S.	1	1.00	0.67	1.31	0.00	0.80	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.077	1.287	0.000	0.103	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	77	0	0	100	0	0	-1
N.S.	1	1.00	1.31	0.00	0.00	1.69	0.00	0.00	-0.02
time (sec)	N/A	0.024	0.067	0.923	0.000	0.368	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	167	0	26	0	0	-1
N.S.	1	1.00	1.00	4.64	0.00	0.72	0.00	0.00	-0.03
time (sec)	N/A	0.020	0.047	2.471	0.000	0.099	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	55	0	0	57	0	0	-1
N.S.	1	1.00	1.38	0.00	0.00	1.42	0.00	0.00	-0.02
time (sec)	N/A	0.034	0.087	0.876	0.000	0.348	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	59	134	0	0	0	0	-1
N.S.	1	1.00	0.43	0.98	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.085	1.250	0.000	0.000	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	33	38	42	37	0	0	58
N.S.	1	1.00	1.43	1.65	1.83	1.61	0.00	0.00	2.52
time (sec)	N/A	0.026	0.028	1.196	0.467	0.370	0.000	0.000	1.352

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	65	117	0	66	0	0	-1
N.S.	1	1.00	0.81	1.46	0.00	0.82	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.073	1.226	0.000	0.104	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	98	121	0	109	0	0	-1
N.S.	1	1.00	0.80	0.99	0.00	0.89	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.138	1.495	0.000	0.389	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	77	138	0	88	0	0	-1
N.S.	1	1.00	0.55	0.98	0.00	0.62	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.132	1.166	0.000	0.093	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	44	47	30	56	0	0	42
N.S.	1	1.00	1.57	1.68	1.07	2.00	0.00	0.00	1.50
time (sec)	N/A	0.027	0.036	1.178	0.505	0.378	0.000	0.000	1.453

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	65	147	0	0	0	0	-1
N.S.	1	1.00	0.26	0.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.104	0.090	1.234	0.000	0.000	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	90	113	0	101	0	0	-1
N.S.	1	1.00	0.98	1.23	0.00	1.10	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.118	1.427	0.000	0.389	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	61	129	0	78	0	0	-1
N.S.	1	1.00	0.55	1.16	0.00	0.70	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.080	1.231	0.000	0.114	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	88	0	0	109	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	1.24	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.117	0.875	0.000	0.440	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	65	159	0	0	0	0	-1
N.S.	1	1.00	0.30	0.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.086	0.085	1.544	0.000	0.000	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	64	131	0	106	0	0	-1
N.S.	1	1.00	0.70	1.42	0.00	1.15	0.00	0.00	-0.01
time (sec)	N/A	0.030	0.067	1.414	0.000	0.414	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	45	127	0	0	0	0	-1
N.S.	1	1.00	0.80	2.27	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.087	2.615	0.000	0.000	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	32	0	39	28	0	0	28
N.S.	1	1.00	1.28	0.00	1.56	1.12	0.00	0.00	1.12
time (sec)	N/A	0.027	0.022	0.737	0.470	0.357	0.000	0.000	1.334

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	65	0	0	55	0	0	-1
N.S.	1	1.00	0.71	0.00	0.00	0.60	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.084	0.732	0.000	0.109	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	51	0	0	93	0	0	-1
N.S.	1	1.00	0.77	0.00	0.00	1.41	0.00	0.00	-0.02
time (sec)	N/A	0.039	0.081	0.744	0.000	0.350	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	64	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.045	0.569	0.288	0.000	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	126	0	0	0	0	0	-1
N.S.	1	1.00	1.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.049	3.710	1.507	0.000	0.000	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	101	0	0	0	0	0	-1
N.S.	1	1.00	1.44	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	1.074	1.772	0.000	0.000	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	192	0	0	0	0	0	-1
N.S.	1	1.00	2.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	9.342	1.678	0.000	0.000	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	C	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	139	29	509	96	189	0	215	66
N.S.	1	3.48	0.72	12.72	2.40	4.72	0.00	5.38	1.65
time (sec)	N/A	0.097	0.227	5.797	0.450	0.379	0.000	0.804	1.400

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	62	0	74	48	0	38	49
N.S.	1	1.00	2.48	0.00	2.96	1.92	0.00	1.52	1.96
time (sec)	N/A	0.028	0.090	2.574	0.276	0.367	0.000	0.410	1.531

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	64	0	49	49	0	37	36
N.S.	1	1.00	2.56	0.00	1.96	1.96	0.00	1.48	1.44
time (sec)	N/A	0.031	0.071	3.647	0.283	0.385	0.000	0.399	1.449

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	114	0	0	474	0	0	-1
N.S.	1	1.00	1.28	0.00	0.00	5.33	0.00	0.00	-0.01
time (sec)	N/A	0.065	3.672	1.669	0.000	0.381	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	108	0	0	538	0	0	-1
N.S.	1	1.00	1.66	0.00	0.00	8.28	0.00	0.00	-0.02
time (sec)	N/A	0.054	3.571	1.740	0.000	0.376	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	19	34	0	27	41
N.S.	1	1.00	1.00	1.05	1.00	1.79	0.00	1.42	2.16
time (sec)	N/A	0.011	0.046	2.599	0.262	0.364	0.000	0.395	1.407

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	116	28	70	0	28	24
N.S.	1	1.00	1.00	6.44	1.56	3.89	0.00	1.56	1.33
time (sec)	N/A	0.020	0.048	3.758	0.285	0.355	0.000	0.411	1.328

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	538	0	452	0	115	139
N.S.	1	1.00	1.00	9.78	0.00	8.22	0.00	2.09	2.53
time (sec)	N/A	0.027	0.043	3.999	0.000	0.369	0.000	0.389	1.403

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	222	91	272	0	47	55
N.S.	1	1.00	1.00	5.29	2.17	6.48	0.00	1.12	1.31
time (sec)	N/A	0.024	0.039	3.991	0.324	0.346	0.000	0.406	1.341

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	75	748	0	1326	0	152	314
N.S.	1	1.00	0.84	8.40	0.00	14.90	0.00	1.71	3.53
time (sec)	N/A	0.039	0.067	4.450	0.000	0.399	0.000	0.407	1.348

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	74	295	0	315	0	0	-1
N.S.	1	1.00	0.76	3.04	0.00	3.25	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.122	5.252	0.000	0.138	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	72	141	0	159	0	0	-1
N.S.	1	1.00	0.77	1.52	0.00	1.71	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.069	4.986	0.000	0.143	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	183	0	39	0	0	-1
N.S.	1	1.00	1.00	3.16	0.00	0.67	0.00	0.00	-0.02
time (sec)	N/A	0.032	0.052	4.709	0.000	0.082	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	183	0	248	0	0	-1
N.S.	1	1.00	1.00	3.16	0.00	4.28	0.00	0.00	-0.02
time (sec)	N/A	0.031	0.054	4.744	0.000	0.127	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	76	237	0	370	0	0	-1
N.S.	1	1.00	0.78	2.44	0.00	3.81	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.083	5.212	0.000	0.131	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	87	256	0	602	0	0	-1
N.S.	1	1.00	0.90	2.64	0.00	6.21	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.093	5.213	0.000	0.135	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [186] had the largest ratio of [44]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	6	0.167
2	A	2	2	1.00	8	0.250
3	A	2	2	1.00	8	0.250
4	A	2	1	1.00	8	0.125
5	A	3	2	1.00	8	0.250
6	A	2	1	1.00	8	0.125
7	A	2	1	1.00	6	0.167
8	A	2	1	1.00	6	0.167
9	A	3	3	1.00	10	0.300
10	A	3	3	1.00	10	0.300
11	A	2	2	1.00	10	0.200
12	A	2	2	1.00	10	0.200
13	A	3	3	1.00	10	0.300
14	A	3	3	1.00	10	0.300
15	A	4	3	1.00	12	0.250
16	A	3	3	1.00	12	0.250
17	A	3	3	1.00	12	0.250
18	A	2	2	1.00	12	0.167
19	A	2	2	1.00	12	0.167
20	A	3	3	1.00	12	0.250
21	A	3	3	1.00	12	0.250
22	A	4	3	1.00	12	0.250
23	A	2	2	1.00	10	0.200
24	A	5	3	1.00	12	0.250
25	A	4	3	1.00	12	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	3	3	1.00	12	0.250
27	A	2	2	1.00	12	0.167
28	A	2	2	1.00	12	0.167
29	A	3	3	1.00	12	0.250
30	A	4	3	1.00	12	0.250
31	A	5	3	1.00	12	0.250
32	A	5	4	1.00	10	0.400
33	A	4	4	1.00	10	0.400
34	A	3	3	1.00	10	0.300
35	A	2	2	1.00	10	0.200
36	A	3	3	1.00	10	0.300
37	A	4	3	1.00	10	0.300
38	A	5	3	1.00	10	0.300
39	A	7	4	1.00	10	0.400
40	A	5	4	1.00	10	0.400
41	A	4	4	1.00	10	0.400
42	A	4	4	1.00	10	0.400
43	A	5	4	1.00	10	0.400
44	A	7	4	1.00	10	0.400
45	A	3	2	1.00	10	0.200
46	A	3	2	1.00	10	0.200
47	A	3	2	1.00	10	0.200
48	A	3	3	1.00	10	0.300
49	A	3	3	1.00	10	0.300
50	A	5	3	1.00	10	0.300
51	A	7	3	1.00	10	0.300
52	A	7	7	1.00	13	0.538
53	A	6	5	1.00	13	0.385
54	A	5	5	1.00	13	0.385
55	A	5	4	1.00	11	0.364
56	A	6	6	1.00	11	0.546
57	A	6	5	1.00	13	0.385
58	A	7	7	1.00	13	0.538
59	A	7	6	1.00	13	0.462
60	A	6	5	1.00	13	0.385

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	5	4	1.00	13	0.308
62	A	5	5	1.00	13	0.385
63	A	5	4	1.00	11	0.364
64	A	4	3	1.00	11	0.273
65	A	5	5	1.00	13	0.385
66	A	6	5	1.00	13	0.385
67	A	6	5	1.00	13	0.385
68	A	7	5	1.00	13	0.385
69	A	6	5	1.00	13	0.385
70	A	5	5	1.00	13	0.385
71	A	4	4	1.00	11	0.364
72	A	1	1	1.00	11	0.091
73	A	3	3	1.00	13	0.231
74	A	4	4	1.00	13	0.308
75	A	6	6	1.00	13	0.462
76	A	2	2	1.00	12	0.167
77	A	2	2	1.00	13	0.154
78	A	5	5	1.00	14	0.357
79	A	4	4	1.00	14	0.286
80	A	2	2	1.00	14	0.143
81	A	5	4	1.00	14	0.286
82	A	6	5	1.00	14	0.357
83	A	2	2	1.00	15	0.133
84	A	5	4	1.00	15	0.267
85	A	2	2	1.00	10	0.200
86	A	2	2	1.00	10	0.200
87	A	6	5	1.00	12	0.417
88	A	5	4	1.00	12	0.333
89	A	4	4	1.00	12	0.333
90	A	2	1	1.00	10	0.100
91	A	3	3	1.00	12	0.250
92	A	5	5	1.00	12	0.417
93	A	6	6	1.00	12	0.500
94	A	1	1	1.00	14	0.071
95	A	8	6	1.00	13	0.462

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	7	6	1.00	13	0.462
97	A	6	6	1.00	13	0.462
98	A	5	5	1.00	11	0.454
99	A	3	3	1.00	11	0.273
100	A	5	5	1.00	13	0.385
101	A	6	6	1.00	13	0.462
102	A	7	7	1.00	13	0.538
103	A	5	3	1.00	13	0.231
104	A	3	2	1.00	13	0.154
105	A	4	3	1.00	13	0.231
106	A	3	2	1.00	13	0.154
107	A	3	2	1.00	13	0.154
108	A	2	2	1.00	11	0.182
109	A	3	2	1.00	11	0.182
110	A	4	3	1.00	13	0.231
111	A	3	2	1.00	13	0.154
112	A	5	3	1.00	13	0.231
113	A	3	2	1.00	13	0.154
114	A	15	8	1.00	13	0.615
115	A	3	2	1.00	13	0.154
116	A	6	6	1.00	13	0.462
117	A	3	2	1.00	13	0.154
118	A	7	7	1.00	13	0.538
119	A	4	4	1.00	11	0.364
120	A	3	2	1.00	11	0.182
121	A	9	8	1.00	13	0.615
122	A	3	2	1.00	13	0.154
123	A	15	8	1.00	13	0.615
124	A	3	2	1.00	13	0.154
125	A	5	4	1.00	23	0.174
126	A	5	4	1.00	23	0.174
127	A	4	4	1.00	21	0.190
128	A	7	5	1.00	21	0.238
129	A	13	9	1.00	23	0.391
130	A	7	7	1.00	23	0.304

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	1	1	1.00	14	0.071
132	A	5	4	1.00	23	0.174
133	A	5	4	1.00	23	0.174
134	A	5	4	1.00	23	0.174
135	A	3	3	1.00	21	0.143
136	A	7	5	1.00	21	0.238
137	A	11	6	1.00	23	0.261
138	A	11	8	1.00	23	0.348
139	A	6	6	1.00	23	0.261
140	A	1	1	1.00	14	0.071
141	A	9	8	1.00	23	0.348
142	A	5	4	1.00	23	0.174
143	A	5	4	1.00	23	0.174
144	A	4	4	1.00	21	0.190
145	A	7	4	1.00	21	0.190
146	A	11	5	1.00	23	0.217
147	A	17	11	1.00	23	0.478
148	A	7	7	1.00	23	0.304
149	A	6	6	1.00	14	0.429
150	A	14	11	1.00	23	0.478
151	A	6	5	1.00	25	0.200
152	A	6	5	1.00	25	0.200
153	A	4	4	1.00	25	0.160
154	A	4	4	1.00	25	0.160
155	A	5	4	1.00	25	0.160
156	A	6	5	1.00	25	0.200
157	A	6	5	1.00	25	0.200
158	A	6	6	1.00	15	0.400
159	A	3	3	1.00	15	0.200
160	A	8	8	1.00	15	0.533
161	A	6	6	1.00	15	0.400
162	A	5	5	1.00	13	0.385
163	A	6	6	1.00	11	0.546
164	A	3	2	1.00	15	0.133
165	A	5	5	1.00	15	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	6	6	1.00	15	0.400
167	A	3	3	1.00	15	0.200
168	A	5	5	1.00	15	0.333
169	A	8	7	1.00	15	0.467
170	A	7	6	1.00	15	0.400
171	A	3	3	1.00	15	0.200
172	A	9	8	1.00	15	0.533
173	A	7	6	1.00	15	0.400
174	A	6	5	1.00	15	0.333
175	A	7	6	1.00	15	0.400
176	A	8	7	1.00	13	0.538
177	A	7	7	1.00	11	0.636
178	A	4	3	1.00	15	0.200
179	A	3	3	1.00	15	0.200
180	A	5	5	1.00	15	0.333
181	A	6	6	1.00	15	0.400
182	A	4	4	1.00	11	0.364
183	A	4	4	1.00	13	0.308
184	A	4	4	1.00	13	0.308
185	A	4	4	1.00	13	0.308
186	C	9	4	3.48	44	0.091
187	A	3	3	1.00	15	0.200
188	A	4	4	1.00	15	0.267
189	A	3	3	1.00	20	0.150
190	A	3	3	1.00	21	0.143
191	A	2	1	1.00	15	0.067
192	A	3	2	1.00	17	0.118
193	A	3	2	1.00	17	0.118
194	A	3	1	1.00	17	0.059
195	A	4	2	1.00	17	0.118
196	A	4	3	1.00	19	0.158
197	A	4	3	1.00	19	0.158
198	A	3	2	1.00	19	0.105
199	A	3	2	1.00	19	0.105
200	A	4	3	1.00	19	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	4	3	1.00	19	0.158

Chapter 3

Listing of integrals

Local contents

3.1 $\int \text{sech}(a+bx)\,dx$

Optimal. Leaf size=11

$$\frac{\text{ArcTan}(\sinh(a+bx))}{b}$$

```
[Out] arctan(sinh(b*x+a))/b
```

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = $\{3855\}$

$$\frac{\text{ArcTan}(\sinh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x],x]
```

```
[Out] ArcTan[Sinh[a + b*x]]/b
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \text{sech}(a+bx)\,dx = \frac{\tan^{-1}(\sinh(a+bx))}{b}$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 1.00

$$\frac{\text{ArcTan}(\sinh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x],x]
```

```
[Out] ArcTan[Sinh[a + b*x]]/b
```

Maple [A]
time = 0.24, size = 12, normalized size = 1.09

method	result	size
derivativedivides	$\frac{\arctan(\sinh(bx+a))}{b}$	12
default	$\frac{\arctan(\sinh(bx+a))}{b}$	12
risch	$\frac{i\ln(e^{bx+a}+i)}{b}-\frac{i\ln(e^{bx+a}-i)}{b}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(sinh(b*x+a))/b
```

Maxima [A]
time = 0.26, size = 11, normalized size = 1.00

$$\frac{\arctan(\sinh(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a),x, algorithm="maxima")
```

```
[Out] arctan(sinh(b*x + a))/b
```

Fricas [A]
time = 0.35, size = 19, normalized size = 1.73

$$\frac{2\arctan(\cosh(bx+a)+\sinh(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a),x, algorithm="fricas")
```

```
[Out] 2*arctan(cosh(b*x + a) + sinh(b*x + a))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a),x)
```

```
[Out] Integral(sech(a + b*x), x)
```

Giac [A]
time = 0.39, size = 12, normalized size = 1.09

$$\frac{2 \arctan\left(e^{(bx+a)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a),x, algorithm="giac")
```

```
[Out] 2*arctan(e^(b*x + a))/b
```

Mupad [B]
time = 0.08, size = 23, normalized size = 2.09

$$\frac{2 \operatorname{atan}\left(\frac{e^{b x} e^{a} \sqrt{b^2}}{b}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(a + b*x),x)
```

```
[Out] (2*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2)
```

3.2 $\int \text{sech}^2(a + bx)\, dx$

Optimal. Leaf size=10

$$\frac{\tanh(a + bx)}{b}$$

```
[Out] tanh(b*x+a)/b
```

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3852, 8}

$$\frac{\tanh(a + bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]^2,x]
```

```
[Out] Tanh[a + b*x]/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned} \int \text{sech}^2(a + bx)\, dx &= \frac{i\text{Subst}\left(\int 1\, dx, x, -i\tanh(a + bx)\right)}{b} \\ &= \frac{\tanh(a + bx)}{b} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$\frac{\tanh(a + bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^2,x]
[Out] Tanh[a + b*x]/b
```

Maple [A]
time = 0.92, size = 19, normalized size = 1.90

method	result	size
risch	$-\frac{2}{b\left(e^{2bx+2a}+1\right)}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] -2/b/(exp(2*b*x+2*a)+1)
```

Maxima [A]
time = 0.26, size = 18, normalized size = 1.80

$$\frac{2}{b(e^{(-2\,bx-2\,a)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2,x, algorithm="maxima")
[Out] 2/(b*(e^(-2*b*x - 2*a) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(10) = 20$.
time = 0.38, size = 41, normalized size = 4.10

$$-\frac{2}{b\cosh\left(bx+a\right)^{2}+2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^{2}+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2,x, algorithm="fricas")
[Out] -2/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2
 + b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**2,x)
```

```
[Out] Integral(sech(a + b*x)**2, x)
```

Giac [A]
time = 0.39, size = 18, normalized size = 1.80

$$-\frac{2}{b(e^{(2\,bx+2\,a)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -2/(b*(e^(2*b*x + 2*a) + 1))
```

Mupad [B]
time = 0.08, size = 18, normalized size = 1.80

$$-\frac{2}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(a + b*x)^2,x)
```

```
[Out] -2/(b*(exp(2*a + 2*b*x) + 1))
```

3.3 $\int \operatorname{sech}^3(a+bx)\,dx$

Optimal. Leaf size=34

$$\frac{\operatorname{ArcTan}(\sinh(a+bx))}{2b}+\frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{2b}$$

```
[Out] 1/2*arctan(sinh(b*x+a))/b+1/2*sech(b*x+a)*tanh(b*x+a)/b
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = {3853, 3855}

$$\frac{\operatorname{ArcTan}(\sinh(a+bx))}{2b}+\frac{\tanh(a+bx)\operatorname{sech}(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]^3,x]
```

```
[Out] ArcTan[Sinh[a + b*x]]/(2*b) + (Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^3(a+bx)\,dx&=\frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{2b}+\frac{1}{2}\int \operatorname{sech}(a+bx)\,dx\\&=\frac{\tan^{-1}(\sinh(a+bx))}{2b}+\frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{2b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 34, normalized size = 1.00

$$\frac{\operatorname{ArcTan}(\sinh(a+bx))}{2b}+\frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^3,x]
```

```
[Out] ArcTan[Sinh[a + b*x]]/(2*b) + (Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

Maple [C] Result contains complex when optimal does not.
time = 1.10, size = 68, normalized size = 2.00

method	result	size
risch	$\frac{\mathrm{e}^{bx+a}\left(\mathrm{e}^{2bx+2a}-1\right)}{b\left(\mathrm{e}^{2bx+2a}+1\right)^{2}}+\frac{i\ln\left(\mathrm{e}^{bx+a}+i\right)}{2b}-\frac{i\ln\left(\mathrm{e}^{bx+a}-i\right)}{2b}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] exp(b*x+a)*(exp(2*b*x+2*a)-1)/b/(exp(2*b*x+2*a)+1)^2+1/2*I/b*ln(exp(b*x+a)+
I)-1/2*I/b*ln(exp(b*x+a)-I)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(30) = 60.
time = 0.49, size = 65, normalized size = 1.91

$$-\frac{\arctan\left(e^{(-bx-a)}\right)}{b}+\frac{e^{(-bx-a)}-e^{(-3\,bx-3\,a)}}{b(2\,e^{(-2\,bx-2\,a)}+e^{(-4\,bx-4\,a)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -arctan(e^(-b*x - a))/b + (e^(-b*x - a) - e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x
 - 2*a) + e^(-4*b*x - 4*a) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(30) = 60.
time = 0.35, size = 267, normalized size = 7.85

$$\frac{\cosh\left(bx+a\right)^{3}+3\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}+\sinh\left(bx+a\right)^{3}+\left(\cosh\left(bx+a\right)^{4}+4\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3}+\sinh\left(bx+a\right)^{4}+2\left(3\,\cosh\left(bx+a\right)^{2}+1\right)\sinh\left(bx+a\right)^{2}+2\,\cosh\left(bx+a\right)^{2}+4\left(\cosh\left(bx+a\right)^{3}+\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right)+1\right)\arctan\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)+\left(3\,\cosh\left(bx+a\right)^{2}-1\right)\sinh\left(bx+a\right)-\cosh\left(bx+a\right)}{b\cosh\left(bx+a\right)^{4}+4\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3}+b\sinh\left(bx+a\right)^{4}+2\,b\cosh\left(bx+a\right)^{2}+2\left(3\,b\cosh\left(bx+a\right)^{2}+b\right)\sinh\left(bx+a\right)^{2}+4\left(b\cosh\left(bx+a\right)^{3}+b\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right)+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] (cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (cos
h(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cos
h(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3
+ cosh(b*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) +
 (3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))/(b*cosh(b*x + a)^4
+ 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 + 2*b*cosh(b*x + a)
```

```
^2 + 2*(3*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 + b
*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**3,x)
```

```
[Out] Integral(sech(a + b*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(30) = 60$.
time = 0.40, size = 76, normalized size = 2.24

$$\frac{\pi + \frac{4\left(e^{(bx+a)} - e^{(-bx-a)}\right)}{\left(e^{(bx+a)} - e^{(-bx-a)}\right)^2 + 4} + 2\arctan\left(\frac{1}{2}\left(e^{(2bx+2a)} - 1\right)e^{(-bx-a)}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/4*(pi + 4*(e^(b*x + a) - e^(-b*x - a))/((e^(b*x + a) - e^(-b*x - a))^2 +
4) + 2*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)))/b
```

Mupad [B]
time = 0.08, size = 81, normalized size = 2.38

$$\frac{\operatorname{atan}\left(\frac{e^{bx}\,e^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}} - \frac{2\,e^{a+bx}}{b\,\left(2\,e^{2a+2bx} + e^{4a+4bx} + 1\right)} + \frac{e^{a+bx}}{b\,\left(e^{2a+2bx} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(a + b*x)^3,x)
```

```
[Out] atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b)/(b^2)^(1/2) - (2*exp(a + b*x))/(b*(2*
exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) + exp(a + b*x)/(b*(exp(2*a + 2*b*
x) + 1))
```

3.4 $\int \operatorname{sech}^4(a+bx)\,dx$

Optimal. Leaf size=26

$$\frac{\tanh(a+bx)}{b}-\frac{\tanh^3(a+bx)}{3b}$$

[Out] tanh(b*x+a)/b-1/3*tanh(b*x+a)^3/b

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3852}

$$\frac{\tanh(a+bx)}{b}-\frac{\tanh^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

[In] Int[Sech[a + b*x]^4,x]

[Out] Tanh[a + b*x]/b - Tanh[a + b*x]^3/(3*b)

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^4(a+bx)\,dx &= \frac{i\text{Subst}\left(\int \left(1+x^2\right)\,dx, x, -i\tanh(a+bx)\right)}{b}\\ &= \frac{\tanh(a+bx)}{b}-\frac{\tanh^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 1.00

$$\frac{\tanh(a+bx)}{b}-\frac{\tanh^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[a + b*x]^4,x]

[Out] Tanh[a + b*x]/b - Tanh[a + b*x]^3/(3*b)

Maple [A]
time = 1.08, size = 32, normalized size = 1.23

method	result	size
risch	$-\frac{4\left(3\,\mathrm{e}^{2bx+2a}+1\right)}{3b\left(\mathrm{e}^{2bx+2a}+1\right)^{3}}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -4/3*(3*exp(2*b*x+2*a)+1)/b/(exp(2*b*x+2*a)+1)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(24) = 48.
time = 0.27, size = 90, normalized size = 3.46

$$\frac{4\,e^{(-2\,bx-2\,a)}}{b(3\,e^{(-2\,bx-2\,a)}+3\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}+1)}+\frac{4}{3\,b(3\,e^{(-2\,bx-2\,a)}+3\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 4*e^(-2*b*x - 2*a)/(b*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) + e^(-6*b*x
- 6*a) + 1)) + 4/3/(b*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) + e^(-6*b*x
- 6*a) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(24) = 48.
time = 0.35, size = 164, normalized size = 6.31

$$-\frac{8\,(2\,\cosh\left(bx+a\right)+\sinh\left(bx+a\right))}{3\,(b\cosh\left(bx+a\right)^{5}+5\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{4}+b\sinh\left(bx+a\right)^{5}+3\,b\cosh\left(bx+a\right)^{3}+(10\,b\cosh\left(bx+a\right)^{2}+3\,b)\sinh\left(bx+a\right)^{3}+(10\,b\cosh\left(bx+a\right)^{3}+9\,b\cosh\left(bx+a\right))\sinh\left(bx+a\right)^{2}+4\,b\cosh\left(bx+a\right)+(5\,b\cosh\left(bx+a\right)^{4}+9\,b\cosh\left(bx+a\right)^{2}+2\,b)\sinh\left(bx+a\right))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -8/3*(2*cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^5 + 5*b*cosh(b*x +
a)*sinh(b*x + a)^4 + b*sinh(b*x + a)^5 + 3*b*cosh(b*x + a)^3 + (10*b*cosh(b
*x + a)^2 + 3*b)*sinh(b*x + a)^3 + (10*b*cosh(b*x + a)^3 + 9*b*cosh(b*x + a
))*sinh(b*x + a)^2 + 4*b*cosh(b*x + a) + (5*b*cosh(b*x + a)^4 + 9*b*cosh(b*
x + a)^2 + 2*b)*sinh(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}^{4}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**4,x)
```

```
[Out] Integral(sech(a + b*x)**4, x)
```

Giac [A]
time = 0.39, size = 31, normalized size = 1.19

$$-\frac{4\left(3\,e^{(2\,bx+2\,a)}+1\right)}{3\,b{\left(e^{(2\,bx+2\,a)}+1\right)}^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -4/3*(3*e^(2*b*x + 2*a) + 1)/(b*(e^(2*b*x + 2*a) + 1)^3)
```

Mupad [B]
time = 0.06, size = 31, normalized size = 1.19

$$-\frac{4\left(3\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)}{3\,b\,{\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(a + b*x)^4,x)
```

```
[Out] -(4*(3*exp(2*a + 2*b*x) + 1))/(3*b*(exp(2*a + 2*b*x) + 1)^3)
```

3.5 $\int \text{sech}^5(a+bx)\,dx$

Optimal. Leaf size=55

$$\frac{3\text{ArcTan}(\sinh(a+bx))}{8b}+\frac{3\text{sech}(a+bx)\tanh(a+bx)}{8b}+\frac{\text{sech}^3(a+bx)\tanh(a+bx)}{4b}$$

```
[Out] 3/8*arctan(sinh(b*x+a))/b+3/8*sech(b*x+a)*tanh(b*x+a)/b+1/4*sech(b*x+a)^3*t
anh(b*x+a)/b
```

Rubi [A]
time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = {3853, 3855}

$$\frac{3\text{ArcTan}(\sinh(a+bx))}{8b}+\frac{\tanh(a+bx)\text{sech}^3(a+bx)}{4b}+\frac{3\tanh(a+bx)\text{sech}(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]^5,x]
```

```
[Out] (3*ArcTan[Sinh[a + b*x]])/(8*b) + (3*Sech[a + b*x]*Tanh[a + b*x])/(8*b) + (
Sech[a + b*x]^3*Tanh[a + b*x])/(4*b)
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \text{sech}^5(a+bx)\,dx&=\frac{\text{sech}^3(a+bx)\tanh(a+bx)}{4b}+\frac{3}{4}\int \text{sech}^3(a+bx)\,dx\\&=\frac{3\text{sech}(a+bx)\tanh(a+bx)}{8b}+\frac{\text{sech}^3(a+bx)\tanh(a+bx)}{4b}+\frac{3}{8}\int \text{sech}(a+bx)\,dx\\&=\frac{3\tan^{-1}(\sinh(a+bx))}{8b}+\frac{3\text{sech}(a+bx)\tanh(a+bx)}{8b}+\frac{\text{sech}^3(a+bx)\tanh(a+bx)}{4b}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 47, normalized size = 0.85

$$\frac{3\text{ArcTan}(\sinh(a+bx))+3\text{sech}(a+bx)\tanh(a+bx)+2\text{sech}^3(a+bx)\tanh(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^5,x]
```

```
[Out] (3*ArcTan[Sinh[a + b*x]] + 3*Sech[a + b*x]*Tanh[a + b*x] + 2*Sech[a + b*x]^
3*Tanh[a + b*x])/(8*b)
```

Maple [C] Result contains complex when optimal does not.
time = 1.05, size = 93, normalized size = 1.69

method	result	size
risch	$\frac{e^{bx+a}\left(3\,e^{6bx+6a}+11\,e^{4bx+4a}-11\,e^{2bx+2a}-3\right)}{4b\left(e^{2bx+2a}+1\right)^4}+\frac{3i\ln\left(e^{bx+a}+i\right)}{8b}-\frac{3i\ln\left(e^{bx+a}-i\right)}{8b}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*exp(b*x+a)*(3*exp(6*b*x+6*a)+11*exp(4*b*x+4*a)-11*exp(2*b*x+2*a)-3)/b/(
exp(2*b*x+2*a)+1)^4+3/8*I/b*ln(exp(b*x+a)+I)-3/8*I/b*ln(exp(b*x+a)-I)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(49) = 98$.
time = 0.47, size = 112, normalized size = 2.04

$$-\frac{3\,\arctan\left(e^{(-bx-a)}\right)}{4\,b}+\frac{3\,e^{(-bx-a)}+11\,e^{(-3\,bx-3\,a)}-11\,e^{(-5\,bx-5\,a)}-3\,e^{(-7\,bx-7\,a)}}{4\,b(4\,e^{(-2\,bx-2\,a)}+6\,e^{(-4\,bx-4\,a)}+4\,e^{(-6\,bx-6\,a)}+e^{(-8\,bx-8\,a)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -3/4*arctan(e^(-b*x - a))/b + 1/4*(3*e^(-b*x - a) + 11*e^(-3*b*x - 3*a) - 1
1*e^(-5*b*x - 5*a) - 3*e^(-7*b*x - 7*a))/(b*(4*e^(-2*b*x - 2*a) + 6*e^(-4*b
*x - 4*a) + 4*e^(-6*b*x - 6*a) + e^(-8*b*x - 8*a) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 812 vs. $2(49) = 98$.
time = 0.38, size = 812, normalized size = 14.76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/4*(3*cosh(b*x + a)^7 + 21*cosh(b*x + a)*sinh(b*x + a)^6 + 3*sinh(b*x + a)
^7 + (63*cosh(b*x + a)^2 + 11)*sinh(b*x + a)^5 + 11*cosh(b*x + a)^5 + 5*(21
*cosh(b*x + a)^3 + 11*cosh(b*x + a))*sinh(b*x + a)^4 + (105*cosh(b*x + a)^4
 + 110*cosh(b*x + a)^2 - 11)*sinh(b*x + a)^3 - 11*cosh(b*x + a)^3 + (63*cos
h(b*x + a)^5 + 110*cosh(b*x + a)^3 - 33*cosh(b*x + a))*sinh(b*x + a)^2 + 3*
(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4*(7
*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^6 + 4*cosh(b*x + a)^6 + 8*(7*cosh(b*x +
 a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 + 30*cosh(
b*x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5
+ 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a
)^6 + 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 4*cosh(
b*x + a)^2 + 8*(cosh(b*x + a)^7 + 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 + c
osh(b*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) + (2
1*cosh(b*x + a)^6 + 55*cosh(b*x + a)^4 - 33*cosh(b*x + a)^2 - 3)*sinh(b*x +
 a) - 3*cosh(b*x + a))/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*sinh(b*x + a)
^7 + b*sinh(b*x + a)^8 + 4*b*cosh(b*x + a)^6 + 4*(7*b*cosh(b*x + a)^2 + b)*
sinh(b*x + a)^6 + 8*(7*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)
^5 + 6*b*cosh(b*x + a)^4 + 2*(35*b*cosh(b*x + a)^4 + 30*b*cosh(b*x + a)^2 +
 3*b)*sinh(b*x + a)^4 + 8*(7*b*cosh(b*x + a)^5 + 10*b*cosh(b*x + a)^3 + 3*b
*cosh(b*x + a))*sinh(b*x + a)^3 + 4*b*cosh(b*x + a)^2 + 4*(7*b*cosh(b*x + a
)^6 + 15*b*cosh(b*x + a)^4 + 9*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 8*(
b*cosh(b*x + a)^7 + 3*b*cosh(b*x + a)^5 + 3*b*cosh(b*x + a)^3 + b*cosh(b*x
+ a))*sinh(b*x + a) + b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}^5(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**5,x)
```

```
[Out] Integral(sech(a + b*x)**5, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(49) = 98.
time = 0.40, size = 102, normalized size = 1.85

$$\frac{3\,\pi+\frac{4\left(3\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3+20\,e^{(bx+a)}-20\,e^{(-bx-a)}\right)}{\left(\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+4\right)^2}+6\,\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)}-1\right)e^{(-bx-a)}\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/16*(3*pi + 4*(3*(e^(b*x + a) - e^(-b*x - a))^3 + 20*e^(b*x + a) - 20*e^(-
b*x - a))/((e^(b*x + a) - e^(-b*x - a))^2 + 4)^2 + 6*arctan(1/2*(e^(2*b*x +
 2*a) - 1)*e^(-b*x - a)))/b
```

Mupad [B]
time = 1.31, size = 189, normalized size = 3.44

$$\frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{b^2}}{b}\right)}{4\,\sqrt{b^2}}+\frac{\mathrm{e}^{a+b\,x}}{2\,b\,\left(2\,\mathrm{e}^{2\,a+2\,b\,x}+\mathrm{e}^{4\,a+4\,b\,x}+1\right)}-\frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(3\,\mathrm{e}^{2\,a+2\,b\,x}+3\,\mathrm{e}^{4\,a+4\,b\,x}+\mathrm{e}^{6\,a+6\,b\,x}+1\right)}-\frac{4\,\mathrm{e}^{3\,a+3\,b\,x}}{b\,\left(4\,\mathrm{e}^{2\,a+2\,b\,x}+6\,\mathrm{e}^{4\,a+4\,b\,x}+4\,\mathrm{e}^{6\,a+6\,b\,x}+\mathrm{e}^{8\,a+8\,b\,x}+1\right)}+\frac{3\,\mathrm{e}^{a+b\,x}}{4\,b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(a + b*x)^5,x)
```

```
[Out] (3*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(4*(b^2)^(1/2)) + exp(a + b*x)/(2
*b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - (2*exp(a + b*x))/(b*(3*ex
p(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1)) - (4*exp(3*a +
 3*b*x))/(b*(4*exp(2*a + 2*b*x) + 6*exp(4*a + 4*b*x) + 4*exp(6*a + 6*b*x) +
 exp(8*a + 8*b*x) + 1)) + (3*exp(a + b*x))/(4*b*(exp(2*a + 2*b*x) + 1))
```

3.6 $\int \operatorname{sech}^6(a+bx)\,dx$

Optimal. Leaf size=41

$$\frac{\tanh(a+bx)}{b}-\frac{2\tanh^3(a+bx)}{3b}+\frac{\tanh^5(a+bx)}{5b}$$

```
[Out] tanh(b*x+a)/b-2/3*tanh(b*x+a)^3/b+1/5*tanh(b*x+a)^5/b
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3852}

$$\frac{\tanh^5(a+bx)}{5b}-\frac{2\tanh^3(a+bx)}{3b}+\frac{\tanh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]^6,x]
```

```
[Out] Tanh[a + b*x]/b - (2*Tanh[a + b*x]^3)/(3*b) + Tanh[a + b*x]^5/(5*b)
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^6(a+bx)\,dx &= \frac{i\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx,x,-i\tanh(a+bx)\right)}{b}\\ &= \frac{\tanh(a+bx)}{b}-\frac{2\tanh^3(a+bx)}{3b}+\frac{\tanh^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 41, normalized size = 1.00

$$\frac{\tanh(a+bx)}{b}-\frac{2\tanh^3(a+bx)}{3b}+\frac{\tanh^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^6,x]
```

```
[Out] Tanh[a + b*x]/b - (2*Tanh[a + b*x]^3)/(3*b) + Tanh[a + b*x]^5/(5*b)
```

Maple [A]
time = 1.13, size = 43, normalized size = 1.05

method	result	size
risch	$-\frac{16\left(10\,{\mathrm e}^{4 b x +4 a}+5\,{\mathrm e}^{2 b x +2 a}+1\right)}{15 b \left({\mathrm e}^{2 b x +2 a}+1\right)^{5}}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^6,x,method=_RETURNVERBOSE)
```

```
[Out] -16/15*(10*exp(4*b*x+4*a)+5*exp(2*b*x+2*a)+1)/b/(exp(2*b*x+2*a)+1)^5
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(37) = 74.
time = 0.26, size = 205, normalized size = 5.00

$$\frac{16\,e^{(-2\,bx-2\,a)}}{3\,b(5\,e^{(-2\,bx-2\,a)}+10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}+5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}+1)}+\frac{32\,e^{(-4\,bx-4\,a)}}{3\,b(5\,e^{(-2\,bx-2\,a)}+10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}+5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}+1)}+\frac{16}{15\,b(5\,e^{(-2\,bx-2\,a)}+10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}+5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] 16/3*e^(-2*b*x - 2*a)/(b*(5*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4*a) + 10*e^(
-6*b*x - 6*a) + 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) + 1)) + 32/3*e^(-4*
b*x - 4*a)/(b*(5*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x - 6*
a) + 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) + 1)) + 16/15/(b*(5*e^(-2*b*x
- 2*a) + 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x - 6*a) + 5*e^(-8*b*x - 8*a) + e
^(-10*b*x - 10*a) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 344 vs. 2(37) = 74.
time = 0.34, size = 344, normalized size = 8.39

$$-\frac{16\,\left(11\,\cosh\left(bx+a\right)^{2}+18\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+11\,\sinh\left(bx+a\right)^{2}+5\right)}{15\,\left(b\cosh\left(bx+a\right)^{8}+8\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{7}+b\sinh\left(bx+a\right)^{8}+5\,b\cosh\left(bx+a\right)^{6}+\left(28\,b\cosh\left(bx+a\right)^{2}+5\,b\right)\sinh\left(bx+a\right)^{6}+2\,\left(28\,b\cosh\left(bx+a\right)^{3}+15\,b\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right)^{5}+10\,b\cosh\left(bx+a\right)^{4}+5\,\left(14\,b\cosh\left(bx+a\right)^{4}+15\,b\cosh\left(bx+a\right)^{2}+2\,b\right)\sinh\left(bx+a\right)^{4}+4\,\left(14\,b\cosh\left(bx+a\right)^{5}+25\,b\cosh\left(bx+a\right)^{3}+10\,b\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right)^{3}+11\,b\cosh\left(bx+a\right)^{2}+\left(28\,b\cosh\left(bx+a\right)^{6}+75\,b\cosh\left(bx+a\right)^{4}+60\,b\cosh\left(bx+a\right)^{2}+11\,b\right)\sinh\left(bx+a\right)^{2}+2\,\left(4\,b\cosh\left(bx+a\right)^{7}+15\,b\cosh\left(bx+a\right)^{5}+20\,b\cosh\left(bx+a\right)^{3}+9\,b\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right)+5\,b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] -16/15*(11*cosh(b*x + a)^2 + 18*cosh(b*x + a)*sinh(b*x + a) + 11*sinh(b*x +
 a)^2 + 5)/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*sinh(b*x + a)^7 + b*sinh(
b*x + a)^8 + 5*b*cosh(b*x + a)^6 + (28*b*cosh(b*x + a)^2 + 5*b)*sinh(b*x +
a)^6 + 2*(28*b*cosh(b*x + a)^3 + 15*b*cosh(b*x + a))*sinh(b*x + a)^5 + 10*b
*cosh(b*x + a)^4 + 5*(14*b*cosh(b*x + a)^4 + 15*b*cosh(b*x + a)^2 + 2*b)*si
nh(b*x + a)^4 + 4*(14*b*cosh(b*x + a)^5 + 25*b*cosh(b*x + a)^3 + 10*b*cosh(
b*x + a))*sinh(b*x + a)^3 + 11*b*cosh(b*x + a)^2 + (28*b*cosh(b*x + a)^6 +
```

```
75*b*cosh(b*x + a)^4 + 60*b*cosh(b*x + a)^2 + 11*b)*sinh(b*x + a)^2 + 2*(4*
b*cosh(b*x + a)^7 + 15*b*cosh(b*x + a)^5 + 20*b*cosh(b*x + a)^3 + 9*b*cosh(
b*x + a))*sinh(b*x + a) + 5*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}^6(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**6,x)
```

```
[Out] Integral(sech(a + b*x)**6, x)
```

Giac [A]
time = 0.39, size = 42, normalized size = 1.02

$$-\frac{16\left(10\,e^{(4\,bx+4\,a)}+5\,e^{(2\,bx+2\,a)}+1\right)}{15\,b\left(e^{(2\,bx+2\,a)}+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^6,x, algorithm="giac")
```

```
[Out] -16/15*(10*e^(4*b*x + 4*a) + 5*e^(2*b*x + 2*a) + 1)/(b*(e^(2*b*x + 2*a) + 1
)^5)
```

Mupad [B]
time = 1.35, size = 42, normalized size = 1.02

$$-\frac{16\left(5\,e^{2\,a+2\,b\,x}+10\,e^{4\,a+4\,b\,x}+1\right)}{15\,b\left(e^{2\,a+2\,b\,x}+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(a + b*x)^6,x)
```

```
[Out] -(16*(5*exp(2*a + 2*b*x) + 10*exp(4*a + 4*b*x) + 1))/(15*b*(exp(2*a + 2*b*x
) + 1)^5)
```

3.7 $\int \operatorname{sech}^4(7x)\,dx$

Optimal. Leaf size=19

$$\frac{1}{7}\tanh(7x)-\frac{1}{21}\tanh^3(7x)$$

```
[Out] 1/7*tanh(7*x)-1/21*tanh(7*x)^3
```

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {3852}

$$\frac{1}{7}\tanh(7x)-\frac{1}{21}\tanh^3(7x)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[7*x]^4,x]
```

```
[Out] Tanh[7*x]/7 - Tanh[7*x]^3/21
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^4(7x)\,dx &= \frac{1}{7}i\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,-i\tanh(7x)\right)\\ &= \frac{1}{7}\tanh(7x)-\frac{1}{21}\tanh^3(7x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 19, normalized size = 1.00

$$\frac{1}{7}\tanh(7x)-\frac{1}{21}\tanh^3(7x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[7*x]^4,x]
```

```
[Out] Tanh[7*x]/7 - Tanh[7*x]^3/21
```

Maple [A]
time = 0.93, size = 19, normalized size = 1.00

method	result	size
risch	$-\frac{4(3\,\mathrm{e}^{14x}+1)}{21(\mathrm{e}^{14x}+1)^3}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(7*x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -4/21*(3*exp(14*x)+1)/(exp(14*x)+1)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(15) = 30$.
time = 0.25, size = 49, normalized size = 2.58

$$\frac{4\,e^{(-14\,x)}}{7\left(3\,e^{(-14\,x)}+3\,e^{(-28\,x)}+e^{(-42\,x)}+1\right)}+\frac{4}{21\left(3\,e^{(-14\,x)}+3\,e^{(-28\,x)}+e^{(-42\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(7*x)^4,x, algorithm="maxima")
```

```
[Out] 4/7*e^(-14*x)/(3*e^(-14*x) + 3*e^(-28*x) + e^(-42*x) + 1) + 4/21/(3*e^(-14*
x) + 3*e^(-28*x) + e^(-42*x) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(15) = 30$.
time = 0.34, size = 116, normalized size = 6.11

$$-\frac{8\,(2\,\cosh(7\,x)+\sinh(7\,x))}{21\,(\cosh(7\,x)^5+5\,\cosh(7\,x)\sinh(7\,x)^4+\sinh(7\,x)^5+(10\,\cosh(7\,x)^2+3)\sinh(7\,x)^3+3\,\cosh(7\,x)^3+(10\,\cosh(7\,x)^3+9\,\cosh(7\,x))\sinh(7\,x)^2+(5\,\cosh(7\,x)^4+9\,\cosh(7\,x)^2+2)\sinh(7\,x)+4\,\cosh(7\,x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(7*x)^4,x, algorithm="fricas")
```

```
[Out] -8/21*(2*cosh(7*x) + sinh(7*x))/(cosh(7*x)^5 + 5*cosh(7*x)*sinh(7*x)^4 + si
nh(7*x)^5 + (10*cosh(7*x)^2 + 3)*sinh(7*x)^3 + 3*cosh(7*x)^3 + (10*cosh(7*x
)^3 + 9*cosh(7*x))*sinh(7*x)^2 + (5*cosh(7*x)^4 + 9*cosh(7*x)^2 + 2)*sinh(7
*x) + 4*cosh(7*x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}^4(7x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(7*x)**4,x)
```

```
[Out] Integral(sech(7*x)**4, x)
```

Giac [A]
time = 0.38, size = 18, normalized size = 0.95

$$-\frac{4\left(3\,e^{(14\,x)}+1\right)}{21\left(e^{(14\,x)}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(7*x)^4,x, algorithm="giac")
```

```
[Out] -4/21*(3*e^(14*x) + 1)/(e^(14*x) + 1)^3
```

Mupad [B]
time = 0.10, size = 30, normalized size = 1.58

$$-\frac{2\left(3\,\mathrm{e}^{14\,x}-3\,\mathrm{e}^{28\,x}-\mathrm{e}^{42\,x}+1\right)}{21\left(\mathrm{e}^{14\,x}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(7*x)^4,x)
```

```
[Out] -(2*(3*exp(14*x) - 3*exp(28*x) - exp(42*x) + 1))/(21*(exp(14*x) + 1)^3)
```

3.8 $\int \operatorname{sech}^6(\pi x)\,dx$

Optimal. Leaf size=35

$$\frac{\tanh(\pi x)}{\pi}-\frac{2\tanh^3(\pi x)}{3\pi}+\frac{\tanh^5(\pi x)}{5\pi}$$

```
[Out] tanh(Pi*x)/Pi-2/3*tanh(Pi*x)^3/Pi+1/5*tanh(Pi*x)^5/Pi
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3852}

$$\frac{\tanh^5(\pi x)}{5\pi}-\frac{2\tanh^3(\pi x)}{3\pi}+\frac{\tanh(\pi x)}{\pi}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[Pi*x]^6,x]
```

```
[Out] Tanh[Pi*x]/Pi - (2*Tanh[Pi*x]^3)/(3*Pi) + Tanh[Pi*x]^5/(5*Pi)
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^6(\pi x)\,dx &= \frac{i\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx, x, -i\tanh(\pi x)\right)}{\pi}\\ &= \frac{\tanh(\pi x)}{\pi}-\frac{2\tanh^3(\pi x)}{3\pi}+\frac{\tanh^5(\pi x)}{5\pi}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 35, normalized size = 1.00

$$\frac{\tanh(\pi x)}{\pi}-\frac{2\tanh^3(\pi x)}{3\pi}+\frac{\tanh^5(\pi x)}{5\pi}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[Pi*x]^6,x]
```

```
[Out] Tanh[Pi*x]/Pi - (2*Tanh[Pi*x]^3)/(3*Pi) + Tanh[Pi*x]^5/(5*Pi)
```

Maple [A]
time = 1.19, size = 31, normalized size = 0.89

method	result	size
risch	$-\frac{16\left(10\,{\rm e}^{4\pi x}+5\,{\rm e}^{2\pi x}+1\right)}{15\pi\left({\rm e}^{2\pi x}+1\right)^{5}}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(Pi*x)^6,x,method=_RETURNVERBOSE)
```

```
[Out] -16/15*(10*exp(4*Pi*x)+5*exp(2*Pi*x)+1)/Pi/(exp(2*Pi*x)+1)^5
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(31) = 62$.
time = 0.28, size = 137, normalized size = 3.91

$$\frac{16\,e^{(-2\,\pi x)}}{3\,\pi\left(5\,e^{(-2\,\pi x)}+10\,e^{(-4\,\pi x)}+10\,e^{(-6\,\pi x)}+5\,e^{(-8\,\pi x)}+e^{(-10\,\pi x)}+1\right)}+\frac{32\,e^{(-4\,\pi x)}}{3\,\pi\left(5\,e^{(-2\,\pi x)}+10\,e^{(-4\,\pi x)}+10\,e^{(-6\,\pi x)}+5\,e^{(-8\,\pi x)}+e^{(-10\,\pi x)}+1\right)}+\frac{16}{15\,\pi\left(5\,e^{(-2\,\pi x)}+10\,e^{(-4\,\pi x)}+10\,e^{(-6\,\pi x)}+5\,e^{(-8\,\pi x)}+e^{(-10\,\pi x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(pi*x)^6,x, algorithm="maxima")
```

```
[Out] 16/3*e^(-2*pi*x)/(pi*(5*e^(-2*pi*x) + 10*e^(-4*pi*x) + 10*e^(-6*pi*x) + 5*e
^(-8*pi*x) + e^(-10*pi*x) + 1)) + 32/3*e^(-4*pi*x)/(pi*(5*e^(-2*pi*x) + 10*
e^(-4*pi*x) + 10*e^(-6*pi*x) + 5*e^(-8*pi*x) + e^(-10*pi*x) + 1)) + 16/15/(
pi*(5*e^(-2*pi*x) + 10*e^(-4*pi*x) + 10*e^(-6*pi*x) + 5*e^(-8*pi*x) + e^(-1
0*pi*x) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(31) = 62$.
time = 0.40, size = 280, normalized size = 8.00

$$-\frac{16\left(11\,\cosh\left(\pi x\right)^{2}+18\,\cosh\left(\pi x\right)\sinh\left(\pi x\right)+11\,\sinh\left(\pi x\right)^{2}+5\right)}{15\left(5\,\pi+\pi\cosh\left(\pi x\right)^{8}+8\,\pi\cosh\left(\pi x\right)\sinh\left(\pi x\right)^{7}+\pi\sinh\left(\pi x\right)^{8}+5\,\pi\cosh\left(\pi x\right)^{6}+\left(5\,\pi+28\,\pi\cosh\left(\pi x\right)^{2}\right)\sinh\left(\pi x\right)^{6}+2\left(28\,\pi\cosh\left(\pi x\right)^{3}+15\,\pi\cosh\left(\pi x\right)\right)\sinh\left(\pi x\right)^{5}+10\,\pi\cosh\left(\pi x\right)^{4}+5\left(2\,\pi+14\,\pi\cosh\left(\pi x\right)^{4}+15\,\pi\cosh\left(\pi x\right)^{2}\right)\sinh\left(\pi x\right)^{4}+4\left(14\,\pi\cosh\left(\pi x\right)^{5}+25\,\pi\cosh\left(\pi x\right)^{3}+10\,\pi\cosh\left(\pi x\right)\right)\sinh\left(\pi x\right)^{3}+11\,\pi\cosh\left(\pi x\right)^{2}+\left(11\,\pi+28\,\pi\cosh\left(\pi x\right)^{6}+75\,\pi\cosh\left(\pi x\right)^{4}+60\,\pi\cosh\left(\pi x\right)^{2}\right)\sinh\left(\pi x\right)^{2}+2\left(4\,\pi\cosh\left(\pi x\right)^{7}+15\,\pi\cosh\left(\pi x\right)^{5}+20\,\pi\cosh\left(\pi x\right)^{3}+9\,\pi\cosh\left(\pi x\right)\right)\sinh\left(\pi x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(pi*x)^6,x, algorithm="fricas")
```

```
[Out] -16/15*(11*cosh(pi*x)^2 + 18*cosh(pi*x)*sinh(pi*x) + 11*sinh(pi*x)^2 + 5)/(
5*pi + pi*cosh(pi*x)^8 + 8*pi*cosh(pi*x)*sinh(pi*x)^7 + pi*sinh(pi*x)^8 + 5
*pi*cosh(pi*x)^6 + (5*pi + 28*pi*cosh(pi*x)^2)*sinh(pi*x)^6 + 2*(28*pi*cosh
(pi*x)^3 + 15*pi*cosh(pi*x))*sinh(pi*x)^5 + 10*pi*cosh(pi*x)^4 + 5*(2*pi +
14*pi*cosh(pi*x)^4 + 15*pi*cosh(pi*x)^2)*sinh(pi*x)^4 + 4*(14*pi*cosh(pi*x)
^5 + 25*pi*cosh(pi*x)^3 + 10*pi*cosh(pi*x))*sinh(pi*x)^3 + 11*pi*cosh(pi*x)
^2 + (11*pi + 28*pi*cosh(pi*x)^6 + 75*pi*cosh(pi*x)^4 + 60*pi*cosh(pi*x)^2)
```

```
*sinh(pi*x)^2 + 2*(4*pi*cosh(pi*x)^7 + 15*pi*cosh(pi*x)^5 + 20*pi*cosh(pi*x
)^3 + 9*pi*cosh(pi*x))*sinh(pi*x))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}^6(\pi x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(pi*x)**6,x)
```

```
[Out] Integral(sech(pi*x)**6, x)
```

Giac [**A**]
time = 0.39, size = 30, normalized size = 0.86

$$-\frac{16\left(10\,e^{(4\,\pi x)}+5\,e^{(2\,\pi x)}+1\right)}{15\,\pi\left(e^{(2\,\pi x)}+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(pi*x)^6,x, algorithm="giac")
```

```
[Out] -16/15*(10*e^(4*pi*x) + 5*e^(2*pi*x) + 1)/(pi*(e^(2*pi*x) + 1)^5)
```

Mupad [**B**]
time = 1.52, size = 30, normalized size = 0.86

$$-\frac{16\left(5\,\mathrm{e}^{2\,\Pi\,x}+10\,\mathrm{e}^{4\,\Pi\,x}+1\right)}{15\,\Pi\left(\mathrm{e}^{2\,\Pi\,x}+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(Pi*x)^6,x)
```

```
[Out] -(16*(5*exp(2*Pi*x) + 10*exp(4*Pi*x) + 1))/(15*Pi*(exp(2*Pi*x) + 1)^5)
```

3.9 $\int \operatorname{sech}^{\frac{5}{2}}(a+bx)\,dx$

Optimal. Leaf size=66

$$-\frac{2i\sqrt{\cosh(a+bx)}\,F\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{3b}+\frac{2\operatorname{sech}^{\frac{3}{2}}(a+bx)\sinh(a+bx)}{3b}$$

```
[Out] 2/3*sech(b*x+a)^(3/2)*sinh(b*x+a)/b-2/3*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cos
h(1/2*a+1/2*b*x)*EllipticF(I*sinh(1/2*a+1/2*b*x),2^(1/2))*cosh(b*x+a)^(1/2)
*sech(b*x+a)^(1/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3853, 3856, 2720}

$$\frac{2\sinh(a+bx)\operatorname{sech}^{\frac{3}{2}}(a+bx)}{3b}-\frac{2i\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\,F\left(\frac{1}{2}i(a+bx)\middle|2\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]^(5/2),x]
```

```
[Out] (((-2*I)/3)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(a + b*x), 2]*Sqrt[Sech[a +
 b*x]])/b + (2*Sech[a + b*x]^(3/2)*Sinh[a + b*x])/(3*b)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \operatorname{sech}^{\frac{5}{2}}(a+bx)\,dx = \frac{2\operatorname{sech}^{\frac{3}{2}}(a+bx)\sinh(a+bx)}{3b} + \frac{1}{3}\int \sqrt{\operatorname{sech}(a+bx)}\,dx$$

$$= \frac{2\operatorname{sech}^{\frac{3}{2}}(a+bx)\sinh(a+bx)}{3b} + \frac{1}{3}\left(\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\right)\int \frac{1}{\sqrt{\cosh(a+bx}}$$

$$= -\frac{2i\sqrt{\cosh(a+bx)}\,F\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{3b} + \frac{2\operatorname{sech}^{\frac{3}{2}}(a+bx)\sinh(a+bx}{3b}$$

Mathematica [A]
time = 0.06, size = 51, normalized size = 0.77

$$\frac{2\operatorname{sech}^{\frac{3}{2}}(a+bx)\left(-i\cosh^{\frac{3}{2}}(a+bx)F\left(\frac{1}{2}i(a+bx)\middle|2\right)+\sinh(a+bx)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^(5/2),x]
```

```
[Out] (2*Sech[a + b*x]^(3/2)*((-I)*Cosh[a + b*x]^(3/2)*EllipticF[(I/2)*(a + b*x),
 2] + Sinh[a + b*x]))/(3*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(82) = 164.
time = 1.46, size = 217, normalized size = 3.29

method	result
default	$\frac{2\left(2\sqrt{-\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\sqrt{-2\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\ \text{EllipticF}\left(\cosh\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+2\cosh\left(\right.\right.}{3\sqrt{2\left(\sinh^4\left(\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(2*(-sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-2*sinh(1/2*b*x+1/2*a)^2-1)^(1/2)*El
lipticF(cosh(1/2*b*x+1/2*a),2^(1/2))*sinh(1/2*b*x+1/2*a)^2+2*cosh(1/2*b*x+1
/2*a)*sinh(1/2*b*x+1/2*a)^2+(-sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-2*sinh(1/2*b*x
+1/2*a)^2-1)^(1/2)*EllipticF(cosh(1/2*b*x+1/2*a),2^(1/2)))*((2*cosh(1/2*b*x
+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)/(2*sinh(1/2*b*x+1/2*a)^4+sinh(1/2
*b*x+1/2*a)^2)^(1/2)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(3/2)/sinh(1/2*b*x+1/2*a)/
b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(b*x + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 190, normalized size = 2.88

$$\frac{2\left(\sqrt{2}\left(\cosh\left(bx+a\right)^{2}+2\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^{2}-1\right)\sqrt{\frac{\cosh\left(bx+a\right)+\sinh\left(bx+a\right)}{\cosh\left(bx+a\right)^{2}+2\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^{2}+1}}+\left(\sqrt{2}\cosh\left(bx+a\right)^{2}+2\sqrt{2}\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sqrt{2}\sinh\left(bx+a\right)^{2}+\sqrt{2}\right)\mathrm{weierstrassPInverse}\left(-4,0,\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)}{3\left(b\cosh\left(bx+a\right)^{2}+2b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^{2}+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(sqrt(2)*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x +
a)^2 - 1)*sqrt((cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b*
x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) + (sqrt(2)*cosh(b*x + a)^2 + 2
*sqrt(2)*cosh(b*x + a)*sinh(b*x + a) + sqrt(2)*sinh(b*x + a)^2 + sqrt(2))*w
eierstrassPInverse(-4, 0, cosh(b*x + a) + sinh(b*x + a)))/(b*cosh(b*x + a)^
2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 + b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}^{\frac{5}{2}}\left(a+bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**(5/2),x)
```

```
[Out] Integral(sech(a + b*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \left(\frac{1}{\cosh\left(a+b\,x\right)}\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(a + b*x))^(5/2),x)
```

```
[Out] int((1/cosh(a + b*x))^(5/2), x)
```

3.10 $\int \operatorname{sech}^{\frac{3}{2}}(a+bx)\,dx$

Optimal. Leaf size=62

$$\frac{2i\sqrt{\cosh(a+bx)}\,E\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{b}+\frac{2\sqrt{\operatorname{sech}(a+bx)}\,\sinh(a+bx)}{b}$$

```
[Out] 2*sinh(b*x+a)*sech(b*x+a)^(1/2)/b+2*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/
2*a+1/2*b*x)*EllipticE(I*sinh(1/2*a+1/2*b*x),2^(1/2))*cosh(b*x+a)^(1/2)*sec
h(b*x+a)^(1/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3853, 3856, 2719}

$$\frac{2\sinh(a+bx)\sqrt{\operatorname{sech}(a+bx)}}{b}+\frac{2i\sqrt{\cosh(a+bx)}\,\sqrt{\operatorname{sech}(a+bx)}\,E\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]^(3/2),x]
```

```
[Out] ((2*I)*Sqrt[Cosh[a + b*x]]*EllipticE[(I/2)*(a + b*x), 2]*Sqrt[Sech[a + b*x]
])/b + (2*Sqrt[Sech[a + b*x]]*Sinh[a + b*x])/b
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \operatorname{sech}^{\frac{3}{2}}(a+bx)\,dx = \frac{2\sqrt{\operatorname{sech}(a+bx)}\,\sinh(a+bx)}{b} - \int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}}\,dx$$

$$= \frac{2\sqrt{\operatorname{sech}(a+bx)}\,\sinh(a+bx)}{b} - \left(\sqrt{\cosh(a+bx)}\,\sqrt{\operatorname{sech}(a+bx)}\right)\int \sqrt{\cosh(a+bx}$$

$$= \frac{2i\sqrt{\cosh(a+bx)}\,E\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{b} + \frac{2\sqrt{\operatorname{sech}(a+bx)}\,\sinh(a+bx)}{b}$$

Mathematica [**A**]
time = 0.04, size = 49, normalized size = 0.79

$$\frac{2\sqrt{\operatorname{sech}(a+bx)}\left(i\sqrt{\cosh(a+bx)}\,E\left(\frac{1}{2}i(a+bx)\middle|2\right)+\sinh(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^(3/2),x]
```

```
[Out] (2*Sqrt[Sech[a + b*x]]*(I*Sqrt[Cosh[a + b*x]]*EllipticE[(I/2)*(a + b*x), 2]
 + Sinh[a + b*x]))/b
```

Maple [**A**]
time = 1.68, size = 103, normalized size = 1.66

method	result
default	$\frac{4\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+2\sqrt{-2\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\,\sqrt{-\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\ \mathrm{EllipticE}\left(\cosh\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right.}{\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{2\left(\cosh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\ b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(2*cosh(1/2*b*x+1/2*a)*sinh(1/2*b*x+1/2*a)^2+(-2*sinh(1/2*b*x+1/2*a)^2-1)
^(1/2)*(-sinh(1/2*b*x+1/2*a)^2)^(1/2)*EllipticE(cosh(1/2*b*x+1/2*a),2^(1/2)
))/sinh(1/2*b*x+1/2*a)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(b*x + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 96, normalized size = 1.55

$$\frac{2\left(\sqrt{2}\sqrt{\frac{\cosh\left(bx+a\right)+\sinh\left(bx+a\right)}{\cosh\left(bx+a\right)^{2}+2\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^{2}+1}}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)+\sqrt{2}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cosh\left(bx+a\right)+\sinh\left(bx+a\right)))\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(2)*sqrt((cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b
*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1))*(cosh(b*x + a) + sinh(b*x + a
)) + sqrt(2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cosh(b*x + a
) + sinh(b*x + a))))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}^{\frac{3}{2}}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**(3/2),x)
```

```
[Out] Integral(sech(a + b*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(\frac{1}{\cosh\left(a+b\,x\right)}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(a + b*x))^(3/2),x)
```

```
[Out] int((1/cosh(a + b*x))^(3/2), x)
```

3.11 $\int \sqrt{\operatorname{sech}(a+bx)}\, dx$

Optimal. Leaf size=40

$$-\frac{2i\sqrt{\cosh(a+bx)}\, F\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{b}$$

```
[Out] -2*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticF(I*sinh(1/2
*a+1/2*b*x),2^(1/2))*cosh(b*x+a)^(1/2)*sech(b*x+a)^(1/2)/b
```

Rubi [A]
time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3856, 2720}

$$-\frac{2i\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\, F\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sech[a + b*x]],x]
```

```
[Out] ((-2*I)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(a + b*x), 2]*Sqrt[Sech[a + b*x
]])/b
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\operatorname{sech}(a+bx)}\, dx &= \left(\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\right)\int \frac{1}{\sqrt{\cosh(a+bx)}}\, dx\\ &= -\frac{2i\sqrt{\cosh(a+bx)}\, F\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{b}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 40, normalized size = 1.00

$$-\frac{2i\sqrt{\cosh(a+bx)}\,F\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sech[a + b*x]],x]
```

```
[Out] ((-2*I)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(a + b*x), 2]*Sqrt[Sech[a + b*x
]])/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(62) = 124$.
time = 1.25, size = 135, normalized size = 3.38

method	result
default	$\frac{2\sqrt{\left(2\left(\cosh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\sqrt{-\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\sqrt{-2\left(\cosh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+1}}{\sqrt{2\left(\sinh^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)}\,\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{2\left(\cosh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*((2*cosh(1/2*b*x+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-sinh(1/2*b*x+
1/2*a)^2)^(1/2)*(-2*cosh(1/2*b*x+1/2*a)^2+1)^(1/2)/(2*sinh(1/2*b*x+1/2*a)^4
+sinh(1/2*b*x+1/2*a)^2)^(1/2)*EllipticF(cosh(1/2*b*x+1/2*a),2^(1/2))/sinh(1
/2*b*x+1/2*a)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sech(b*x + a)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 24, normalized size = 0.60

$$\frac{2\sqrt{2}\,\mathrm{weierstrassPInverse}(-4,0,\cosh(bx+a)+\sinh(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*weierstrassPInverse(-4, 0, cosh(b*x + a) + sinh(b*x + a))/b
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\operatorname{sech}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(sech(a + b*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sech(b*x + a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\frac{1}{\cosh(a+b\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(a + b*x))^(1/2),x)
```

```
[Out] int((1/cosh(a + b*x))^(1/2), x)
```

3.12 $\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}}\,dx$

Optimal. Leaf size=40

$$-\frac{2i\sqrt{\cosh(a+bx)}\,E\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{b}$$

```
[Out] -2*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticE(I*sinh(1/2
*a+1/2*b*x),2^(1/2))*cosh(b*x+a)^(1/2)*sech(b*x+a)^(1/2)/b
```

Rubi [A]
time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3856, 2719}

$$-\frac{2i\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\,E\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Sech[a + b*x]],x]
```

```
[Out] ((-2*I)*Sqrt[Cosh[a + b*x]]*EllipticE[(I/2)*(a + b*x), 2]*Sqrt[Sech[a + b*x
]])/b
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}}\,dx &= \left(\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\right)\int \sqrt{\cosh(a+bx)}\,dx\\ &= -\frac{2i\sqrt{\cosh(a+bx)}\,E\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{b}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 40, normalized size = 1.00

$$-\frac{2iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Sech[a + b*x]],x]
```

```
[Out] ((-2*I)*EllipticE[(I/2)*(a + b*x), 2])/(b*Sqrt[Cosh[a + b*x]]*Sqrt[Sech[a +
 b*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(62) = 124.
time = 1.12, size = 135, normalized size = 3.38

method	result
default	$-\frac{2\sqrt{\left(2\left(\cosh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,\sqrt{-\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,\sqrt{-2\left(\cosh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+1}}{\sqrt{2\left(\sinh^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)}\,\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{2\left(\cosh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-}}$
risch	$\frac{\sqrt{2}}{b\sqrt{\frac{e^{bx+a}}{e^{2bx+2a}+1}}}+\frac{\left(-\frac{2\left(e^{2bx+2a}+1\right)}{\sqrt{\left(e^{2bx+2a}+1\right)e^{bx+a}}}+\frac{i\sqrt{-i\left(e^{bx+a}+i\right)}\sqrt{2}\sqrt{i\left(e^{bx+a}-i\right)}\sqrt{ie^{bx+a}}\left(-2i\,\text{Elliptic}\right.}{\sqrt{e^{3bx+}}}\right.}{b\sqrt{\frac{e^{bx+}}{e^{2bx+2}}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sech(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*((2*cosh(1/2*b*x+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-sinh(1/2*b*x
+1/2*a)^2)^(1/2)*(-2*cosh(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticE(cosh(1/2*b*x+
1/2*a),2^(1/2))/(2*sinh(1/2*b*x+1/2*a)^4+sinh(1/2*b*x+1/2*a)^2)^(1/2)/sinh(
1/2*b*x+1/2*a)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(sech(b*x + a)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 150, normalized size = 3.75

$$-\frac{\sqrt{2}\left(\cosh\left(bx+a\right)^2+2\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^2+1\right)\sqrt{\frac{\cosh\left(bx+a\right)+\sinh\left(bx+a\right)}{\cosh\left(bx+a\right)^2+2\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^2+1}}+2\left(\sqrt{2}\cosh\left(bx+a\right)+\sqrt{2}\sinh\left(bx+a\right)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cosh\left(bx+a\right)+\sinh\left(bx+a\right)))}{b\cosh\left(bx+a\right)+b\sinh\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(2)*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^
2 + 1)*sqrt((cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b*x +
 a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) + 2*(sqrt(2)*cosh(b*x + a) + sqrt
(2)*sinh(b*x + a))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cosh(b
*x + a) + sinh(b*x + a))))/(b*cosh(b*x + a) + b*sinh(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\operatorname{sech}\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(b*x+a)**(1/2),x)
```

```
[Out] Integral(1/sqrt(sech(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(sech(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\frac{1}{\cosh\left(a+b\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cosh(a + b*x))^(1/2),x)
```

```
[Out] int(1/(1/cosh(a + b*x))^(1/2), x)
```

3.13 $\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx$

Optimal. Leaf size=66

$$-\frac{2i\sqrt{\cosh(a+bx)}\,F\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{3b}+\frac{2\sinh(a+bx)}{3b\sqrt{\operatorname{sech}(a+bx)}}$$

```
[Out] 2/3*sinh(b*x+a)/b/sech(b*x+a)^(1/2)-2/3*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cos
h(1/2*a+1/2*b*x)*EllipticF(I*sinh(1/2*a+1/2*b*x),2^(1/2))*cosh(b*x+a)^(1/2)
*sech(b*x+a)^(1/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3854, 3856, 2720}

$$\frac{2\sinh(a+bx)}{3b\sqrt{\operatorname{sech}(a+bx)}}-\frac{2i\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\,F\left(\frac{1}{2}i(a+bx)\middle|2\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]^(-3/2),x]
```

```
[Out] (((-2*I)/3)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(a + b*x), 2]*Sqrt[Sech[a +
 b*x]])/b + (2*Sinh[a + b*x])/(3*b*Sqrt[Sech[a + b*x]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx = \frac{2\sinh(a+bx)}{3b\sqrt{\operatorname{sech}(a+bx)}} + \frac{1}{3}\int \sqrt{\operatorname{sech}(a+bx)}\,dx$$
$$= \frac{2\sinh(a+bx)}{3b\sqrt{\operatorname{sech}(a+bx)}} + \frac{1}{3}\left(\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\right)\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx$$
$$= -\frac{2i\sqrt{\cosh(a+bx)}\,F\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{3b} + \frac{2\sinh(a+bx)}{3b\sqrt{\operatorname{sech}(a+bx)}}$$

Mathematica [A]
time = 0.03, size = 53, normalized size = 0.80

$$\frac{\sqrt{\operatorname{sech}(a+bx)}\left(-2i\sqrt{\cosh(a+bx)}\,F\left(\frac{1}{2}i(a+bx)\middle|2\right)+\sinh(2(a+bx))\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^(-3/2),x]
```

```
[Out] (Sqrt[Sech[a + b*x]]*((-2*I)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(a + b*x),
 2] + Sinh[2*(a + b*x)]))/(3*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(82) = 164$.
time = 1.46, size = 174, normalized size = 2.64

method	result
default	$\frac{2\sqrt{\left(2\left(\cosh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\left(4\left(\cosh^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-6\left(\cosh^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sqrt{-\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right.\right.}\right.}{3\sqrt{2\left(\sinh^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)}\,\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sech(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*((2*cosh(1/2*b*x+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(4*cosh(1/2*b
*x+1/2*a)^5-6*cosh(1/2*b*x+1/2*a)^3+(-sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-2*cosh
(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticF(cosh(1/2*b*x+1/2*a),2^(1/2))+2*cosh(1/
2*b*x+1/2*a))/(2*sinh(1/2*b*x+1/2*a)^4+sinh(1/2*b*x+1/2*a)^2)^(1/2)/sinh(1/
2*b*x+1/2*a)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(b*x + a)^(-3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 223, normalized size = 3.38

$$\frac{\sqrt{2}\left(\cosh\left(bx+a\right)^{4}+4\,\cosh\left(bx+a\right)^{3}\sinh\left(bx+a\right)+6\,\cosh\left(bx+a\right)^{2}\sinh\left(bx+a\right)^{2}+4\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3}+\sinh\left(bx+a\right)^{4}-1\right)\sqrt{\frac{\cosh\left(bx+a\right)+\sinh\left(bx+a\right)}{\cosh\left(bx+a\right)^{2}+2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^{2}+1}}+4\left(\sqrt{2}\,\cosh\left(bx+a\right)^{2}+2\,\sqrt{2}\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sqrt{2}\,\sinh\left(bx+a\right)^{2}\right)\mathrm{weierstrassPInverse}\left(-4,0,\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)}{6\left(b\cosh\left(bx+a\right)^{2}+2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(sqrt(2)*(cosh(b*x + a)^4 + 4*cosh(b*x + a)^3*sinh(b*x + a) + 6*cosh(b*
x + a)^2*sinh(b*x + a)^2 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^
4 - 1)*sqrt((cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b*x +
 a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) + 4*(sqrt(2)*cosh(b*x + a)^2 + 2*
sqrt(2)*cosh(b*x + a)*sinh(b*x + a) + sqrt(2)*sinh(b*x + a)^2)*weierstrassP
Inverse(-4, 0, cosh(b*x + a) + sinh(b*x + a)))/(b*cosh(b*x + a)^2 + 2*b*cos
h(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(b*x+a)**(3/2),x)
```

```
[Out] Integral(sech(a + b*x)**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(\frac{1}{\cosh\left(a+b\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cosh(a + b*x))^(3/2),x)
[Out] int(1/(1/cosh(a + b*x))^(3/2), x)
```

3.14 $\int \frac{1}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}\,dx$

Optimal. Leaf size=66

$$-\frac{6i\sqrt{\cosh(a+bx)}\,E\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{5b}+\frac{2\sinh(a+bx)}{5b\operatorname{sech}^{\frac{3}{2}}(a+bx)}$$

```
[Out] 2/5*sinh(b*x+a)/b/sech(b*x+a)^(3/2)-6/5*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cos
h(1/2*a+1/2*b*x)*EllipticE(I*sinh(1/2*a+1/2*b*x),2^(1/2))*cosh(b*x+a)^(1/2)
*sech(b*x+a)^(1/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3854, 3856, 2719}

$$\frac{2\sinh(a+bx)}{5b\operatorname{sech}^{\frac{3}{2}}(a+bx)}-\frac{6i\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\,E\left(\frac{1}{2}i(a+bx)\middle|2\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]^(-5/2),x]
```

```
[Out] (((-6*I)/5)*Sqrt[Cosh[a + b*x]]*EllipticE[(I/2)*(a + b*x), 2]*Sqrt[Sech[a +
 b*x]])/b + (2*Sinh[a + b*x])/(5*b*Sech[a + b*x]^(3/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}\,dx &= \frac{2\sinh(a+bx)}{5b\operatorname{sech}^{\frac{3}{2}}(a+bx)} + \frac{3}{5}\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}}\,dx \\
&= \frac{2\sinh(a+bx)}{5b\operatorname{sech}^{\frac{3}{2}}(a+bx)} + \frac{1}{5}\left(3\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\right)\int \sqrt{\cosh(a+bx)}\,dx \\
&= -\frac{6i\sqrt{\cosh(a+bx)}\,E\left(\frac{1}{2}i(a+bx)\middle|\,2\right)\sqrt{\operatorname{sech}(a+bx)}}{5b} + \frac{2\sinh(a+bx)}{5b\operatorname{sech}^{\frac{3}{2}}(a+bx)}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 59, normalized size = 0.89

$$
\frac{\sqrt{\operatorname{sech}(a+bx)}\left(-12i\sqrt{\cosh(a+bx)}\,E\left(\frac{1}{2}i(a+bx)\middle|\,2\right)+\sinh(a+bx)+\sinh(3(a+bx))\right)}{10b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^(-5/2),x]
```

```
[Out] (Sqrt[Sech[a + b*x]]*((-12*I)*Sqrt[Cosh[a + b*x]]*EllipticE[(I/2)*(a + b*x)
, 2] + Sinh[a + b*x] + Sinh[3*(a + b*x)]))/(10*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(82) = 164$.
time = 1.66, size = 188, normalized size = 2.85

method	result
default	$\frac{2\sqrt{\left(2\left(\cosh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\left(8\left(\cosh^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-16\left(\cosh^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+10\left(\cosh^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-3 \cdots\right.}{5\sqrt{2\left(\sinh^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)}\ \sinh \cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sech(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*((2*cosh(1/2*b*x+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(8*cosh(1/2*b
*x+1/2*a)^7-16*cosh(1/2*b*x+1/2*a)^5+10*cosh(1/2*b*x+1/2*a)^3-3*(-sinh(1/2*
b*x+1/2*a)^2)^(1/2)*(-2*cosh(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticE(cosh(1/2*b
*x+1/2*a),2^(1/2))-2*cosh(1/2*b*x+1/2*a))/(2*sinh(1/2*b*x+1/2*a)^4+sinh(1/2
*b*x+1/2*a)^2)^(1/2)/sinh(1/2*b*x+1/2*a)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(1/2)/
b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(b*x + a)^(-5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 370, normalized size = 5.61

$$\frac{\sqrt{2}\left(\cosh\left(bx+a\right)^{6}+6\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{5}+\sinh\left(bx+a\right)^{6}+\left(15\,\cosh\left(bx+a\right)^{2}-11\right)\sinh\left(bx+a\right)^{4}-11\,\cosh\left(bx+a\right)^{4}+4\left(5\,\cosh\left(bx+a\right)^{3}-11\,\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right)^{3}+\left(15\,\cosh\left(bx+a\right)^{4}-66\,\cosh\left(bx+a\right)^{2}-13\right)\sinh\left(bx+a\right)^{2}-13\,\cosh\left(bx+a\right)^{2}+2\left(3\,\cosh\left(bx+a\right)^{5}-22\,\cosh\left(bx+a\right)^{3}-13\,\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right)-1\right)\sqrt{\frac{\cosh\left(bx+a\right)+\sinh\left(bx+a\right)}{\cosh\left(bx+a\right)^{2}+2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^{2}+1}}-24\left(\sqrt{2}\,\cosh\left(bx+a\right)^{3}+3\,\sqrt{2}\,\cosh\left(bx+a\right)^{2}\sinh\left(bx+a\right)+3\,\sqrt{2}\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}+\sqrt{2}\,\sinh\left(bx+a\right)^{3}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)}{20\left(b\cosh\left(bx+a\right)^{3}+3\,b\cosh\left(bx+a\right)^{2}\sinh\left(bx+a\right)+3\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}+b\sinh\left(bx+a\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/20*(sqrt(2)*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x
 + a)^6 + (15*cosh(b*x + a)^2 - 11)*sinh(b*x + a)^4 - 11*cosh(b*x + a)^4 +
4*(5*cosh(b*x + a)^3 - 11*cosh(b*x + a))*sinh(b*x + a)^3 + (15*cosh(b*x + a
)^4 - 66*cosh(b*x + a)^2 - 13)*sinh(b*x + a)^2 - 13*cosh(b*x + a)^2 + 2*(3*
cosh(b*x + a)^5 - 22*cosh(b*x + a)^3 - 13*cosh(b*x + a))*sinh(b*x + a) - 1)
*sqrt((cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*si
nh(b*x + a) + sinh(b*x + a)^2 + 1)) - 24*(sqrt(2)*cosh(b*x + a)^3 + 3*sqrt(
2)*cosh(b*x + a)^2*sinh(b*x + a) + 3*sqrt(2)*cosh(b*x + a)*sinh(b*x + a)^2
+ sqrt(2)*sinh(b*x + a)^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cosh(b*x + a) + sinh(b*x + a))))/(b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)^2
*sinh(b*x + a) + 3*b*cosh(b*x + a)*sinh(b*x + a)^2 + b*sinh(b*x + a)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{sech}^{\frac{5}{2}}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(b*x+a)**(5/2),x)
```

```
[Out] Integral(sech(a + b*x)**(-5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^(-5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(\frac{1}{\cosh(a+b\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cosh(a + b*x))^(5/2),x)
[Out] int(1/(1/cosh(a + b*x))^(5/2), x)
```

3.15 $\int (b\,\mathrm{sech}(c+dx))^{7/2}\,dx$

Optimal. Leaf size=102

$$\frac{6ib^4E\left(\frac{1}{2}i(c+dx)\middle|2\right)}{5d\sqrt{\cosh(c+dx)}\sqrt{b\mathrm{sech}(c+dx)}}+\frac{6b^3\sqrt{b\mathrm{sech}(c+dx)}\sinh(c+dx)}{5d}+\frac{2b(b\mathrm{sech}(c+dx))^{5/2}\sinh(c+dx)}{5d}$$

```
[Out] 2/5*b*(b*sech(d*x+c))^(5/2)*sinh(d*x+c)/d+6/5*I*b^4*(cosh(1/2*d*x+1/2*c)^2)
^(1/2)/cosh(1/2*d*x+1/2*c)*EllipticE(I*sinh(1/2*d*x+1/2*c),2^(1/2))/d/cosh(
d*x+c)^(1/2)/(b*sech(d*x+c))^(1/2)+6/5*b^3*sinh(d*x+c)*(b*sech(d*x+c))^(1/2
)/d
```

Rubi [A]
time = 0.05, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3853, 3856, 2719}

$$\frac{6ib^4E\left(\frac{1}{2}i(c+dx)\middle|2\right)}{5d\sqrt{\cosh(c+dx)}\sqrt{b\mathrm{sech}(c+dx)}}+\frac{6b^3\sinh(c+dx)\sqrt{b\mathrm{sech}(c+dx)}}{5d}+\frac{2b\sinh(c+dx)(b\mathrm{sech}(c+dx))^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sech[c + d*x])^(7/2),x]
```

```
[Out] (((6*I)/5)*b^4*EllipticE[(I/2)*(c + d*x), 2])/(d*Sqrt[Cosh[c + d*x]]*Sqrt[b
*Sech[c + d*x]]) + (6*b^3*Sqrt[b*Sech[c + d*x]]*Sinh[c + d*x])/(5*d) + (2*b
*(b*Sech[c + d*x])^(5/2)*Sinh[c + d*x])/(5*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\int (b\text{sech}(c+dx))^{7/2}\,dx &= \frac{2b(b\text{sech}(c+dx))^{5/2}\sinh(c+dx)}{5d} + \frac{1}{5}\left(3b^2\right)\int (b\text{sech}(c+dx))^{3/2}\,dx \\
&= \frac{6b^3\sqrt{b\text{sech}(c+dx)}\,\sinh(c+dx)}{5d} + \frac{2b(b\text{sech}(c+dx))^{5/2}\sinh(c+dx)}{5d} - \frac{1}{5}\left(3b^4\right) \\
&= \frac{6b^3\sqrt{b\text{sech}(c+dx)}\,\sinh(c+dx)}{5d} + \frac{2b(b\text{sech}(c+dx))^{5/2}\sinh(c+dx)}{5d} - \frac{(3b}{5\sqrt{\cosh}} \\
&= \frac{6ib^4E\left(\frac{1}{2}i(c+dx)\middle|2\right)}{5d\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}} + \frac{6b^3\sqrt{b\text{sech}(c+dx)}\,\sinh(c+dx)}{5d} + \frac{2b(bs}{}
\end{aligned}
$$

Mathematica [A]
time = 0.15, size = 68, normalized size = 0.67

$$
\frac{b^2(b\text{sech}(c+dx))^{3/2}\left(6i\cosh^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}i(c+dx)\middle|2\right)+3\sinh(2(c+dx))+2\tanh(c+dx)\right)}{5d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sech[c + d*x])^(7/2),x]
```

```
[Out] (b^2*(b*Sech[c + d*x])^(3/2)*((6*I)*Cosh[c + d*x]^(3/2)*EllipticE[(I/2)*(c
+ d*x), 2] + 3*Sinh[2*(c + d*x)] + 2*Tanh[c + d*x]))/(5*d)
```

Maple [F]
time = 1.53, size = 0, normalized size = 0.00

$$
\int (b\,\text{sech}(dx+c))^{\frac{7}{2}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sech(d*x+c))^(7/2),x)
```

```
[Out] int((b*sech(d*x+c))^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sech(d*x + c))^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 478, normalized size = 4.69

$$\frac{2\left(3\sqrt{2}\left(b^{3}\cosh\left(dx+c\right)^{4}+4\,b^{3}\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3}+b^{3}\sinh\left(dx+c\right)^{4}+2\,b^{3}\cosh\left(dx+c\right)^{2}+b^{3}+2\left(3\,b^{3}\cosh\left(dx+c\right)^{2}+b^{3}\right)\sinh\left(dx+c\right)^{2}+4\left(b^{3}\cosh\left(dx+c\right)^{3}+b^{3}\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cosh\left(dx+c\right)+\sinh\left(dx+c\right)\right)\right)+\sqrt{2}\left(3\,b^{3}\cosh\left(dx+c\right)^{5}+15\,b^{3}\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{4}+3\,b^{3}\sinh\left(dx+c\right)^{5}+8\,b^{3}\cosh\left(dx+c\right)^{3}+b^{3}\cosh\left(dx+c\right)+2\left(15\,b^{3}\cosh\left(dx+c\right)^{2}+4\,b^{3}\right)\sinh\left(dx+c\right)^{3}+6\left(5\,b^{3}\cosh\left(dx+c\right)^{3}+4\,b^{3}\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{2}+\left(15\,b^{3}\cosh\left(dx+c\right)^{4}+24\,b^{3}\cosh\left(dx+c\right)^{2}+b^{3}\right)\sinh\left(dx+c\right)\right)\sqrt{\frac{b\cosh\left(dx+c\right)+b\sinh\left(dx+c\right)}{\cosh\left(dx+c\right)^{2}+2\,\cosh\left(dx+c\right)\sinh\left(dx+c\right)+\sinh\left(dx+c\right)^{2}+1}}\right)}{5\left(d\cosh\left(dx+c\right)^{4}+4\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3}+d\sinh\left(dx+c\right)^{4}+2\,d\cosh\left(dx+c\right)^{2}+2\left(3\,d\cosh\left(dx+c\right)^{2}+d\right)\sinh\left(dx+c\right)^{2}+4\left(d\cosh\left(dx+c\right)^{3}+d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/5*(3*sqrt(2)*(b^3*cosh(d*x + c)^4 + 4*b^3*cosh(d*x + c)*sinh(d*x + c)^3 +
 b^3*sinh(d*x + c)^4 + 2*b^3*cosh(d*x + c)^2 + b^3 + 2*(3*b^3*cosh(d*x + c)
^2 + b^3)*sinh(d*x + c)^2 + 4*(b^3*cosh(d*x + c)^3 + b^3*cosh(d*x + c))*sin
h(d*x + c))*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cosh(
d*x + c) + sinh(d*x + c))) + sqrt(2)*(3*b^3*cosh(d*x + c)^5 + 15*b^3*cosh(d
*x + c)*sinh(d*x + c)^4 + 3*b^3*sinh(d*x + c)^5 + 8*b^3*cosh(d*x + c)^3 + b
^3*cosh(d*x + c) + 2*(15*b^3*cosh(d*x + c)^2 + 4*b^3)*sinh(d*x + c)^3 + 6*(
5*b^3*cosh(d*x + c)^3 + 4*b^3*cosh(d*x + c))*sinh(d*x + c)^2 + (15*b^3*cosh
(d*x + c)^4 + 24*b^3*cosh(d*x + c)^2 + b^3)*sinh(d*x + c))*sqrt((b*cosh(d*x
 + c) + b*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
 sinh(d*x + c)^2 + 1)))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c
)^3 + d*sinh(d*x + c)^4 + 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 + d)
*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) +
d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sech(d*x + c))^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{b}{\cosh\left(c+d\,x\right)}\right)}^{7/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cosh(c + d*x))^(7/2),x)
[Out] int((b/cosh(c + d*x))^(7/2), x)
```

3.16 $\int (b\operatorname{sech}(c+dx))^{5/2}\,dx$

Optimal. Leaf size=74

$$-\frac{2ib^2\sqrt{\cosh(c+dx)}\,F\left(\frac{1}{2}i(c+dx)\middle|2\right)\sqrt{b\operatorname{sech}(c+dx)}}{3d}+\frac{2b(b\operatorname{sech}(c+dx))^{3/2}\sinh(c+dx)}{3d}$$

```
[Out] 2/3*b*(b*sech(d*x+c))^(3/2)*sinh(d*x+c)/d-2/3*I*b^2*(cosh(1/2*d*x+1/2*c)^2)
^(1/2)/cosh(1/2*d*x+1/2*c)*EllipticF(I*sinh(1/2*d*x+1/2*c),2^(1/2))*cosh(d*
x+c)^(1/2)*(b*sech(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3853, 3856, 2720}

$$\frac{2b\sinh(c+dx)(b\operatorname{sech}(c+dx))^{3/2}}{3d}-\frac{2ib^2\sqrt{\cosh(c+dx)}\,F\left(\frac{1}{2}i(c+dx)\middle|2\right)\sqrt{b\operatorname{sech}(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sech[c + d*x])^(5/2),x]
```

```
[Out] (((-2*I)/3)*b^2*Sqrt[Cosh[c + d*x]]*EllipticF[(I/2)*(c + d*x), 2]*Sqrt[b*Se
ch[c + d*x]])/d + (2*b*(b*Sech[c + d*x])^(3/2)*Sinh[c + d*x])/(3*d)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int (b\operatorname{sech}(c+dx))^{5/2}\,dx &= \frac{2b(b\operatorname{sech}(c+dx))^{3/2}\sinh(c+dx)}{3d} + \frac{1}{3}b^2\int \sqrt{b\operatorname{sech}(c+dx)}\,dx \\ &= \frac{2b(b\operatorname{sech}(c+dx))^{3/2}\sinh(c+dx)}{3d} + \frac{1}{3}\left(b^2\sqrt{\cosh(c+dx)}\sqrt{b\operatorname{sech}(c+dx)}\right)\int - \\ &= -\frac{2ib^2\sqrt{\cosh(c+dx)}\,F\left(\frac{1}{2}i(c+dx)\middle|2\right)\sqrt{b\operatorname{sech}(c+dx)}}{3d} + \frac{2b(b\operatorname{sech}(c+dx))^{3/2}}{3d}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 56, normalized size = 0.76

$$\frac{2b^2\sqrt{b\operatorname{sech}(c+dx)}\left(-i\sqrt{\cosh(c+dx)}\,F\left(\frac{1}{2}i(c+dx)\middle|2\right)+\tanh(c+dx)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sech[c + d*x])^(5/2),x]
[Out] (2*b^2*Sqrt[b*Sech[c + d*x]]*((-I)*Sqrt[Cosh[c + d*x]]*EllipticF[(I/2)*(c +
 d*x), 2] + Tanh[c + d*x]))/(3*d)
```

Maple [F]
time = 1.33, size = 0, normalized size = 0.00

$$\int \left(b\,\operatorname{sech}(dx+c)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sech(d*x+c))^(5/2),x)
[Out] int((b*sech(d*x+c))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*sech(d*x + c))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 215, normalized size = 2.91

$$\frac{2\left(\sqrt{2}\left(b^2\cosh(dx+c)^2+2\,b^2\cosh(dx+c)\sinh(dx+c)+b^2\sinh(dx+c)^2+b^2\right)\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cosh(dx+c)+\sinh(dx+c))+\sqrt{2}\left(b^2\cosh(dx+c)^2+2\,b^2\cosh(dx+c)\sinh(dx+c)+b^2\sinh(dx+c)^2-b^2\right)\sqrt{\frac{b\cosh(dx+c)+b\sinh(dx+c)}{\cosh(dx+c)^2+2\,\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2+1}}\right)}{3\left(d\cosh(dx+c)^2+2\,d\cosh(dx+c)\sinh(dx+c)+d\sinh(dx+c)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(sqrt(2)*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2
*sinh(d*x + c)^2 + b^2)*sqrt(b)*weierstrassPInverse(-4, 0, cosh(d*x + c) +
sinh(d*x + c)) + sqrt(2)*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*
x + c) + b^2*sinh(d*x + c)^2 - b^2)*sqrt((b*cosh(d*x + c) + b*sinh(d*x + c)
)/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)))
/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x + c)^2 +
 d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\mathrm{sech}\,(c+dx))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))**(5/2),x)
```

```
[Out] Integral((b*sech(c + d*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sech(d*x + c))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{b}{\cosh\left(c+d\,x\right)}\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cosh(c + d*x))^(5/2),x)
```

```
[Out] int((b/cosh(c + d*x))^(5/2), x)
```

3.17 $\int (b\,\mathrm{sech}(c+dx))^{3/2}\,dx$

Optimal. Leaf size=70

$$\frac{2ib^2E\left(\frac{1}{2}i(c+dx)\middle|2\right)}{d\sqrt{\cosh(c+dx)}\sqrt{b\,\mathrm{sech}(c+dx)}}+\frac{2b\sqrt{b\,\mathrm{sech}(c+dx)}\,\sinh(c+dx)}{d}$$

```
[Out] 2*I*b^2*(cosh(1/2*d*x+1/2*c)^2)^(1/2)/cosh(1/2*d*x+1/2*c)*EllipticE(I*sinh(
1/2*d*x+1/2*c),2^(1/2))/d/cosh(d*x+c)^(1/2)/(b*sech(d*x+c))^(1/2)+2*b*sinh(
d*x+c)*(b*sech(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3853, 3856, 2719}

$$\frac{2b\sinh(c+dx)\sqrt{b\,\mathrm{sech}(c+dx)}}{d}+\frac{2ib^2E\left(\frac{1}{2}i(c+dx)\middle|2\right)}{d\sqrt{\cosh(c+dx)}\sqrt{b\,\mathrm{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sech[c + d*x])^(3/2),x]
```

```
[Out] ((2*I)*b^2*EllipticE[(I/2)*(c + d*x), 2])/(d*Sqrt[Cosh[c + d*x]]*Sqrt[b*Sec
h[c + d*x]]) + (2*b*Sqrt[b*Sech[c + d*x]]*Sinh[c + d*x])/d
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int (b\text{sech}(c+dx))^{3/2}\,dx &= \frac{2b\sqrt{b\text{sech}(c+dx)}\,\sinh(c+dx)}{d} - b^2\int \frac{1}{\sqrt{b\text{sech}(c+dx)}}\,dx \\ &= \frac{2b\sqrt{b\text{sech}(c+dx)}\,\sinh(c+dx)}{d} - \frac{b^2\int \sqrt{\cosh(c+dx)}\,dx}{\sqrt{\cosh(c+dx)}\,\sqrt{b\text{sech}(c+dx)}} \\ &= \frac{2ib^2E\left(\frac{1}{2}i(c+dx)\middle|2\right)}{d\sqrt{\cosh(c+dx)}\,\sqrt{b\text{sech}(c+dx)}} + \frac{2b\sqrt{b\text{sech}(c+dx)}\,\sinh(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 52, normalized size = 0.74

$$\frac{2b\sqrt{b\text{sech}(c+dx)}\left(i\sqrt{\cosh(c+dx)}\,E\left(\frac{1}{2}i(c+dx)\middle|2\right)+\sinh(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sech[c + d*x])^(3/2),x]
```

```
[Out] (2*b*Sqrt[b*Sech[c + d*x]]*(I*Sqrt[Cosh[c + d*x]]*EllipticE[(I/2)*(c + d*x)
, 2] + Sinh[c + d*x]))/d
```

Maple [F]
time = 1.31, size = 0, normalized size = 0.00

$$\int \left(b\,\text{sech}(dx+c)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sech(d*x+c))^(3/2),x)
```

```
[Out] int((b*sech(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sech(d*x + c))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 107, normalized size = 1.53

$$\frac{2\left(\sqrt{2}\,b^{\frac{3}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cosh\left(dx+c\right)+\sinh\left(dx+c\right)))+\sqrt{2}\left(b\cosh\left(dx+c\right)+b\sinh\left(dx+c\right)\right)\sqrt{\frac{b\cosh\left(dx+c\right)+b\sinh\left(dx+c\right)}{\cosh\left(dx+c\right)^{2}+2\cosh\left(dx+c\right)\sinh\left(dx+c\right)+\sinh\left(dx+c\right)^{2}+1}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(2)*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cosh(d
*x + c) + sinh(d*x + c))) + sqrt(2)*(b*cosh(d*x + c) + b*sinh(d*x + c))*sqr
t((b*cosh(d*x + c) + b*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*si
nh(d*x + c) + sinh(d*x + c)^2 + 1)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\operatorname{sech}\left(c+dx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*sech(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sech(d*x + c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{b}{\cosh\left(c+d\,x\right)}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cosh(c + d*x))^(3/2),x)
```

```
[Out] int((b/cosh(c + d*x))^(3/2), x)
```

3.18 $\int \sqrt{b\text{sech}(c+dx)}\,dx$

Optimal. Leaf size=42

$$-\frac{2i\sqrt{\cosh(c+dx)}\,F\left(\frac{1}{2}i(c+dx)\middle|2\right)\sqrt{b\text{sech}(c+dx)}}{d}$$

```
[Out] -2*I*(cosh(1/2*d*x+1/2*c)^2)^(1/2)/cosh(1/2*d*x+1/2*c)*EllipticF(I*sinh(1/2
*d*x+1/2*c),2^(1/2))*cosh(d*x+c)^(1/2)*(b*sech(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3856, 2720}

$$-\frac{2i\sqrt{\cosh(c+dx)}\,F\left(\frac{1}{2}i(c+dx)\middle|2\right)\sqrt{b\text{sech}(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sech[c + d*x]],x]
```

```
[Out] ((-2*I)*Sqrt[Cosh[c + d*x]]*EllipticF[(I/2)*(c + d*x), 2]*Sqrt[b*Sech[c + d
*x]])/d
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{b\text{sech}(c+dx)}\,dx &= \left(\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}\right)\int \frac{1}{\sqrt{\cosh(c+dx)}}\,dx \\
&= -\frac{2i\sqrt{\cosh(c+dx)}\,F\left(\frac{1}{2}i(c+dx)\middle|2\right)\sqrt{b\text{sech}(c+dx)}}{d}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 42, normalized size = 1.00

$$-\frac{2i\sqrt{\cosh(c+dx)}\,F\left(\frac{1}{2}i(c+dx)\middle|2\right)\sqrt{b\text{sech}(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sech[c + d*x]],x]
[Out] ((-2*I)*Sqrt[Cosh[c + d*x]]*EllipticF[(I/2)*(c + d*x), 2]*Sqrt[b*Sech[c + d
*x]])/d
```

Maple [F]
time = 1.54, size = 0, normalized size = 0.00

$$\int\sqrt{b\,\text{sech}\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sech(d*x+c))^(1/2),x)
[Out] int((b*sech(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sech(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 27, normalized size = 0.64

$$\frac{2\sqrt{2}\sqrt{b}\,\text{weierstrassPInverse}(-4,0,\cosh\left(dx+c\right)+\sinh\left(dx+c\right))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cosh(d*x + c) + sinh(d*x + c))
/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\,\mathrm{sech}\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(b*sech(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sech(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\frac{b}{\cosh\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cosh(c + d*x))^(1/2),x)
```

```
[Out] int((b/cosh(c + d*x))^(1/2), x)
```

3.19 $\int \frac{1}{\sqrt{b\,\mathrm{sech}(c+dx)}}\,dx$

Optimal. Leaf size=42

$$-\frac{2iE\left(\frac{1}{2}i(c+dx)\middle|2\right)}{d\sqrt{\cosh(c+dx)}\sqrt{b\,\mathrm{sech}(c+dx)}}$$

```
[Out] -2*I*(cosh(1/2*d*x+1/2*c)^2)^(1/2)/cosh(1/2*d*x+1/2*c)*EllipticE(I*sinh(1/2
*d*x+1/2*c),2^(1/2))/d/cosh(d*x+c)^(1/2)/(b*sech(d*x+c))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3856, 2719}

$$-\frac{2iE\left(\frac{1}{2}i(c+dx)\middle|2\right)}{d\sqrt{\cosh(c+dx)}\sqrt{b\,\mathrm{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Sech[c + d*x]],x]
```

```
[Out] ((-2*I)*EllipticE[(I/2)*(c + d*x), 2])/(d*Sqrt[Cosh[c + d*x]]*Sqrt[b*Sech[c
 + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{b\,\mathrm{sech}(c+dx)}}\,dx &= \frac{\int \sqrt{\cosh(c+dx)}\,dx}{\sqrt{\cosh(c+dx)}\sqrt{b\,\mathrm{sech}(c+dx)}}\\ &= -\frac{2iE\left(\frac{1}{2}i(c+dx)\middle|2\right)}{d\sqrt{\cosh(c+dx)}\sqrt{b\,\mathrm{sech}(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 42, normalized size = 1.00

$$-\frac{2iE\left(\frac{1}{2}i(c+dx)\middle|2\right)}{d\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Sech[c + d*x]],x]
[Out] ((-2*I)*EllipticE[(I/2)*(c + d*x), 2])/(d*Sqrt[Cosh[c + d*x]]*Sqrt[b*Sech[c
 + d*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(64) = 128.
time = 2.36, size = 244, normalized size = 5.81

method	result
risch	$\frac{\sqrt{2}}{d\sqrt{\frac{b\,\mathrm{e}^{dx+c}}{1+\mathrm{e}^{2dx+2c}}}}+\frac{\left(-\frac{2\left(b\,\mathrm{e}^{2dx+2c}+b\right)}{b\sqrt{\mathrm{e}^{dx+c}\left(b\,\mathrm{e}^{2dx+2c}+b\right)}}+\frac{i\sqrt{-i\left(\mathrm{e}^{dx+c}+i\right)}\sqrt{2}\sqrt{i\left(\mathrm{e}^{dx+c}-i\right)}\sqrt{i\mathrm{e}^{dx+c}}\left(-2i\,\mathrm{Ellip}\right.}{\sqrt{b\,\mathrm{e}^{3d}}}\right.}{d\sqrt{\frac{b\,\mathrm{e}}{1+\mathrm{e}^{2}}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sech(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/d*2^(1/2)/(b*exp(d*x+c)/(exp(d*x+c)^2+1))^(1/2)+1/d*(-2*(b*exp(d*x+c)^2+b
)/b/(exp(d*x+c)*(b*exp(d*x+c)^2+b))^(1/2)+I*(-I*(exp(d*x+c)+I))^(1/2)*2^(1/
2)*(I*(exp(d*x+c)-I))^(1/2)*(I*exp(d*x+c))^(1/2)/(b*exp(d*x+c)^3+b*exp(d*x+
c))^(1/2)*(-2*I*EllipticE((-I*(exp(d*x+c)+I))^(1/2),1/2*2^(1/2))+I*Elliptic
F((-I*(exp(d*x+c)+I))^(1/2),1/2*2^(1/2))))*2^(1/2)/(b*exp(d*x+c)/(exp(d*x+c
)^2+1))^(1/2)*(b*exp(d*x+c)*(exp(d*x+c)^2+1))^(1/2)/(exp(d*x+c)^2+1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*sech(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 154, normalized size = 3.67

$$-\frac{2\sqrt{2}\sqrt{b}\left(\cosh(dx+c)+\sinh(dx+c)\right)\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cosh(dx+c)+\sinh(dx+c)))+\sqrt{2}\left(\cosh(dx+c)^2+2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2+1\right)\sqrt{\frac{b\cosh(dx+c)+b\sinh(dx+c)}{\cosh(dx+c)^2+2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2+1}}}{bd\cosh(dx+c)+bd\sinh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*sqrt(2)*sqrt(b)*(cosh(d*x + c) + sinh(d*x + c))*weierstrassZeta(-4, 0,
weierstrassPInverse(-4, 0, cosh(d*x + c) + sinh(d*x + c))) + sqrt(2)*(cosh(
d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*sqrt((b*c
osh(d*x + c) + b*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x
 + c) + sinh(d*x + c)^2 + 1)))/(b*d*cosh(d*x + c) + b*d*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b \operatorname{sech}(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*sech(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sech(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\frac{b}{\cosh(c + d\,x)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/cosh(c + d*x))^(1/2),x)
```

```
[Out] int(1/(b/cosh(c + d*x))^(1/2), x)
```

3.20 $\int \frac{1}{(b\,\textbf{sech}(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=76

$$-\frac{2i\sqrt{\cosh(c+dx)}\,F\left(\frac{1}{2}i(c+dx)\middle|2\right)\sqrt{b\text{sech}(c+dx)}}{3b^2d}+\frac{2\sinh(c+dx)}{3bd\sqrt{b\text{sech}(c+dx)}}$$

```
[Out] 2/3*sinh(d*x+c)/b/d/(b*sech(d*x+c))^(1/2)-2/3*I*(cosh(1/2*d*x+1/2*c)^2)^(1/
2)/cosh(1/2*d*x+1/2*c)*EllipticF(I*sinh(1/2*d*x+1/2*c),2^(1/2))*cosh(d*x+c)
^(1/2)*(b*sech(d*x+c))^(1/2)/b^2/d
```

Rubi [A]
time = 0.03, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3854, 3856, 2720}

$$\frac{2\sinh(c+dx)}{3bd\sqrt{b\text{sech}(c+dx)}}-\frac{2i\sqrt{\cosh(c+dx)}\,F\left(\frac{1}{2}i(c+dx)\middle|2\right)\sqrt{b\text{sech}(c+dx)}}{3b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sech[c + d*x])^(-3/2),x]
```

```
[Out] (((-2*I)/3)*Sqrt[Cosh[c + d*x]]*EllipticF[(I/2)*(c + d*x), 2]*Sqrt[b*Sech[c
 + d*x]])/(b^2*d) + (2*Sinh[c + d*x])/(3*b*d*Sqrt[b*Sech[c + d*x]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{(b\text{sech}(c+dx))^{3/2}}\,dx = \frac{2\sinh(c+dx)}{3bd\sqrt{b\text{sech}(c+dx)}} + \frac{\int \sqrt{b\text{sech}(c+dx)}\,dx}{3b^2}$$

$$= \frac{2\sinh(c+dx)}{3bd\sqrt{b\text{sech}(c+dx)}} + \frac{\left(\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}\right)\int \frac{1}{\sqrt{\cosh(c+dx)}}\,d}{3b^2}$$

$$= -\frac{2i\sqrt{\cosh(c+dx)}\,F\left(\frac{1}{2}i(c+dx)\middle|2\right)\sqrt{b\text{sech}(c+dx)}}{3b^2d} + \frac{2\sinh(c+dx)}{3bd\sqrt{b\text{sech}(c+dx)}}$$

Mathematica [A]
time = 0.05, size = 63, normalized size = 0.83

$$\frac{\text{sech}^2(c+dx)\left(-2i\sqrt{\cosh(c+dx)}\,F\left(\frac{1}{2}i(c+dx)\middle|2\right)+\sinh(2(c+dx))\right)}{3d(b\text{sech}(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sech[c + d*x])^(-3/2),x]
```

```
[Out] (Sech[c + d*x]^2*((-2*I)*Sqrt[Cosh[c + d*x]]*EllipticF[(I/2)*(c + d*x), 2]
+ Sinh[2*(c + d*x)]))/(3*d*(b*Sech[c + d*x])^(3/2))
```

Maple [F]
time = 1.39, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,\text{sech}\,(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sech(d*x+c))^(3/2),x)
```

```
[Out] int(1/(b*sech(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sech(d*x + c))^(-3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 231, normalized size = 3.04

$$\frac{4\sqrt{2}\left(\cosh\left(dx+c\right)^{2}+2\cosh\left(dx+c\right)\sinh\left(dx+c\right)+\sinh\left(dx+c\right)^{2}\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(-4,0,\cosh\left(dx+c\right)+\sinh\left(dx+c\right)\right)+\sqrt{2}\left(\cosh\left(dx+c\right)^{4}+4\cosh\left(dx+c\right)^{3}\sinh\left(dx+c\right)+6\cosh\left(dx+c\right)^{2}\sinh\left(dx+c\right)^{2}+4\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3}+\sinh\left(dx+c\right)^{4}-1\right)\sqrt{\frac{b\cosh\left(dx+c\right)+b\sinh\left(dx+c\right)}{\cosh\left(dx+c\right)^{2}+2\cosh\left(dx+c\right)\sinh\left(dx+c\right)+\sinh\left(dx+c\right)^{2}+1}}}{6\left(b^{2}d\cosh\left(dx+c\right)^{2}+2b^{2}d\cosh\left(dx+c\right)\sinh\left(dx+c\right)+b^{2}d\sinh\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(4*sqrt(2)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x
+ c)^2)*sqrt(b)*weierstrassPInverse(-4, 0, cosh(d*x + c) + sinh(d*x + c)) +
 sqrt(2)*(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x +
c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 -
1)*sqrt((b*cosh(d*x + c) + b*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x +
 c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)))/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d
*cosh(d*x + c)*sinh(d*x + c) + b^2*d*sinh(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \operatorname{sech}\left(c+dx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*sech(c + d*x))**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sech(d*x + c))^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(\frac{b}{\cosh\left(c+d\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/cosh(c + d*x))^(3/2),x)
```

```
[Out] int(1/(b/cosh(c + d*x))^(3/2), x)
```

3.21 $\int \frac{1}{(b\operatorname{sech}(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=76

$$-\frac{6iE\left(\frac{1}{2}i(c+dx)\middle|2\right)}{5b^2d\sqrt{\cosh(c+dx)}\sqrt{b\operatorname{sech}(c+dx)}}+\frac{2\sinh(c+dx)}{5bd(b\operatorname{sech}(c+dx))^{3/2}}$$

```
[Out] 2/5*sinh(d*x+c)/b/d/(b*sech(d*x+c))^(3/2)-6/5*I*(cosh(1/2*d*x+1/2*c)^2)^(1/
2)/cosh(1/2*d*x+1/2*c)*EllipticE(I*sinh(1/2*d*x+1/2*c),2^(1/2))/b^2/d/cosh(
d*x+c)^(1/2)/(b*sech(d*x+c))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3854, 3856, 2719}

$$\frac{2\sinh(c+dx)}{5bd(b\operatorname{sech}(c+dx))^{3/2}}-\frac{6iE\left(\frac{1}{2}i(c+dx)\middle|2\right)}{5b^2d\sqrt{\cosh(c+dx)}\sqrt{b\operatorname{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sech[c + d*x])^(-5/2),x]
```

```
[Out] (((-6*I)/5)*EllipticE[(I/2)*(c + d*x), 2])/(b^2*d*Sqrt[Cosh[c + d*x]]*Sqrt[
b*Sech[c + d*x]]) + (2*Sinh[c + d*x])/(5*b*d*(b*Sech[c + d*x])^(3/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{(b\text{sech}(c+dx))^{5/2}}\,dx = \frac{2\sinh(c+dx)}{5bd(b\text{sech}(c+dx))^{3/2}} + \frac{3\int \frac{1}{\sqrt{b\text{sech}(c+dx)}}\,dx}{5b^2}$$
$$= \frac{2\sinh(c+dx)}{5bd(b\text{sech}(c+dx))^{3/2}} + \frac{3\int \sqrt{\cosh(c+dx)}\,dx}{5b^2\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}}$$
$$= -\frac{6iE\left(\frac{1}{2}i(c+dx)\middle|2\right)}{5b^2d\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}} + \frac{2\sinh(c+dx)}{5bd(b\text{sech}(c+dx))^{3/2}}$$

Mathematica [**A**]
time = 0.06, size = 64, normalized size = 0.84

$$\frac{\sqrt{b\text{sech}(c+dx)}\left(-12i\sqrt{\cosh(c+dx)}\,E\left(\frac{1}{2}i(c+dx)\middle|2\right)+\sinh(c+dx)+\sinh(3(c+dx))\right)}{10b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sech[c + d*x])^(-5/2),x]
[Out] (Sqrt[b*Sech[c + d*x]]*((-12*I)*Sqrt[Cosh[c + d*x]]*EllipticE[(I/2)*(c + d*
x), 2] + Sinh[c + d*x] + Sinh[3*(c + d*x)]))/(10*b^3*d)
```

Maple [**F**]
time = 1.55, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,\text{sech}\,(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sech(d*x+c))^(5/2),x)
[Out] int(1/(b*sech(d*x+c))^(5/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*sech(d*x + c))^(-5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 379, normalized size = 4.99

$$-\frac{24 \sqrt{2} \left(\cosh\left(dx+c\right)^{3} + 3 \cosh\left(dx+c\right)^{2} \sinh\left(dx+c\right) + 3 \cosh\left(dx+c\right) \sinh\left(dx+c\right)^{2} + \sinh\left(dx+c\right)^{3}\right) \sqrt{b}\, \mathrm{weierstrassZeta}\left(-4, 0, \mathrm{weierstrassPInverse}\left(-4, 0, \cosh\left(dx+c\right) + \sinh\left(dx+c\right)\right)\right) - \sqrt{2} \left(\cosh\left(dx+c\right)^{6} + 6 \cosh\left(dx+c\right) \sinh\left(dx+c\right)^{5} + \sinh\left(dx+c\right)^{6} + \left(15 \cosh\left(dx+c\right)^{2} - 11\right) \sinh\left(dx+c\right)^{4} - 11 \cosh\left(dx+c\right)^{4} + 4 \left(5 \cosh\left(dx+c\right)^{3} - 11 \cosh\left(dx+c\right)\right) \sinh\left(dx+c\right)^{3} + \left(15 \cosh\left(dx+c\right)^{4} - 66 \cosh\left(dx+c\right)^{2} - 13\right) \sinh\left(dx+c\right)^{2} - 13 \cosh\left(dx+c\right)^{2} + 2 \left(3 \cosh\left(dx+c\right)^{5} - 22 \cosh\left(dx+c\right)^{3} - 13 \cosh\left(dx+c\right)\right) \sinh\left(dx+c\right) - 1\right) \sqrt{\frac{b \cosh\left(dx+c\right) + b \sinh\left(dx+c\right)}{\cosh\left(dx+c\right)^{2} + 2 \cosh\left(dx+c\right) \sinh\left(dx+c\right) + \sinh\left(dx+c\right)^{2} + 1}}}{20 \left(b^{3} d \cosh\left(dx+c\right)^{3} + 3 b^{3} d \cosh\left(dx+c\right)^{2} \sinh\left(dx+c\right) + 3 b^{3} d \cosh\left(dx+c\right) \sinh\left(dx+c\right)^{2} + b^{3} d \sinh\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/20*(24*sqrt(2)*(cosh(d*x + c)^3 + 3*cosh(d*x + c)^2*sinh(d*x + c) + 3*co
sh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3)*sqrt(b)*weierstrassZeta(-4,
0, weierstrassPInverse(-4, 0, cosh(d*x + c) + sinh(d*x + c))) - sqrt(2)*(co
sh(d*x + c)^6 + 6*cosh(d*x + c)*sinh(d*x + c)^5 + sinh(d*x + c)^6 + (15*cos
h(d*x + c)^2 - 11)*sinh(d*x + c)^4 - 11*cosh(d*x + c)^4 + 4*(5*cosh(d*x + c
)^3 - 11*cosh(d*x + c))*sinh(d*x + c)^3 + (15*cosh(d*x + c)^4 - 66*cosh(d*x
 + c)^2 - 13)*sinh(d*x + c)^2 - 13*cosh(d*x + c)^2 + 2*(3*cosh(d*x + c)^5 -
 22*cosh(d*x + c)^3 - 13*cosh(d*x + c))*sinh(d*x + c) - 1)*sqrt((b*cosh(d*x
 + c) + b*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
 sinh(d*x + c)^2 + 1)))/(b^3*d*cosh(d*x + c)^3 + 3*b^3*d*cosh(d*x + c)^2*si
nh(d*x + c) + 3*b^3*d*cosh(d*x + c)*sinh(d*x + c)^2 + b^3*d*sinh(d*x + c)^3
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \operatorname{sech}\left(c + dx\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))**(5/2),x)
```

```
[Out] Integral((b*sech(c + d*x))**(-5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sech(d*x + c))^(-5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(\frac{b}{\cosh\left(c+d\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/cosh(c + d*x))^(5/2),x)
[Out] int(1/(b/cosh(c + d*x))^(5/2), x)
```

3.22 $\int \frac{1}{(b\,\mathrm{sech}(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=104

$$-\frac{10 i \sqrt{\cosh(c+dx)}\, F\left(\frac{1}{2} i(c+dx)\middle| 2\right) \sqrt{b\mathrm{sech}(c+dx)}}{21 b^4 d}+\frac{2\sinh(c+dx)}{7bd(b\mathrm{sech}(c+dx))^{5/2}}+\frac{10\sinh(c+dx)}{21 b^3 d\sqrt{b\mathrm{sech}(c+dx)}}$$

```
[Out] 2/7*sinh(d*x+c)/b/d/(b*sech(d*x+c))^(5/2)+10/21*sinh(d*x+c)/b^3/d/(b*sech(d
*x+c))^(1/2)-10/21*I*(cosh(1/2*d*x+1/2*c)^2)^(1/2)/cosh(1/2*d*x+1/2*c)*Elli
pticF(I*sinh(1/2*d*x+1/2*c),2^(1/2))*cosh(d*x+c)^(1/2)*(b*sech(d*x+c))^(1/2
)/b^4/d
```

Rubi [A]
time = 0.04, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3854, 3856, 2720}

$$-\frac{10 i \sqrt{\cosh(c+dx)}\, F\left(\frac{1}{2} i(c+dx)\middle| 2\right) \sqrt{b\mathrm{sech}(c+dx)}}{21 b^4 d}+\frac{10\sinh(c+dx)}{21 b^3 d\sqrt{b\mathrm{sech}(c+dx)}}+\frac{2\sinh(c+dx)}{7bd(b\mathrm{sech}(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sech[c + d*x])^(-7/2),x]
```

```
[Out] (((-10*I)/21)*Sqrt[Cosh[c + d*x]]*EllipticF[(I/2)*(c + d*x), 2]*Sqrt[b*Sech
[c + d*x]])/(b^4*d) + (2*Sinh[c + d*x])/(7*b*d*(b*Sech[c + d*x])^(5/2)) + (
10*Sinh[c + d*x])/(21*b^3*d*Sqrt[b*Sech[c + d*x]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{(b\text{sech}(c+dx))^{7/2}}\,dx = \frac{2\sinh(c+dx)}{7bd(b\text{sech}(c+dx))^{5/2}} + \frac{5\int \frac{1}{(b\text{sech}(c+dx))^{3/2}}\,dx}{7b^2}$$

$$= \frac{2\sinh(c+dx)}{7bd(b\text{sech}(c+dx))^{5/2}} + \frac{10\sinh(c+dx)}{21b^3d\sqrt{b\text{sech}(c+dx)}} + \frac{5\int \sqrt{b\text{sech}(c+dx)}\,dx}{21b^4}$$

$$= \frac{2\sinh(c+dx)}{7bd(b\text{sech}(c+dx))^{5/2}} + \frac{10\sinh(c+dx)}{21b^3d\sqrt{b\text{sech}(c+dx)}} + \frac{\left(5\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+\right.}{21b}$$

$$= -\frac{10i\sqrt{\cosh(c+dx)}\,F\left(\frac{1}{2}i(c+dx)\middle|2\right)\sqrt{b\text{sech}(c+dx)}}{21b^4d} + \frac{2\sinh(c+dx)}{7bd(b\text{sech}(c+dx))^{5/2}}$$

Mathematica [A]
time = 0.09, size = 70, normalized size = 0.67

$$\frac{\sqrt{b\text{sech}(c+dx)}\left(-40i\sqrt{\cosh(c+dx)}\,F\left(\frac{1}{2}i(c+dx)\middle|2\right)+26\sinh(2(c+dx))+3\sinh(4(c+dx))\right)}{84b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sech[c + d*x])^(-7/2),x]
```

```
[Out] (Sqrt[b*Sech[c + d*x]]*((-40*I)*Sqrt[Cosh[c + d*x]]*EllipticF[(I/2)*(c + d*
x), 2] + 26*Sinh[2*(c + d*x)] + 3*Sinh[4*(c + d*x)]))/(84*b^4*d)
```

Maple [F]
time = 1.47, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,\text{sech}\,(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sech(d*x+c))^(7/2),x)
```

```
[Out] int(1/(b*sech(d*x+c))^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sech(d*x + c))^(-7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 483, normalized size = 4.64

$$\frac{80\sqrt{2}\left(\cosh(dx+c)^4+4\cosh(dx+c)^3\sinh(dx+c)+6\cosh(dx+c)^2\sinh(dx+c)^2+4\cosh(dx+c)\sinh(dx+c)^3+\sinh(dx+c)^4\right)\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cosh(dx+c)+\sinh(dx+c))+\sqrt{2}\left(3\cosh(dx+c)^8+24\cosh(dx+c)\sinh(dx+c)^7+3\sinh(dx+c)^8+2\left(42\cosh(dx+c)^2+13\right)\sinh(dx+c)^6+26\cosh(dx+c)^6+12\left(14\cosh(dx+c)^3+13\cosh(dx+c)\right)\sinh(dx+c)^5+30\left(7\cosh(dx+c)^4+13\cosh(dx+c)^2\right)\sinh(dx+c)^4+8\left(21\cosh(dx+c)^5+65\cosh(dx+c)^3\right)\sinh(dx+c)^3+2\left(42\cosh(dx+c)^6+195\cosh(dx+c)^4-13\right)\sinh(dx+c)^2-26\cosh(dx+c)^2+4\left(6\cosh(dx+c)^7+39\cosh(dx+c)^5-13\cosh(dx+c)\right)\sinh(dx+c)-3\right)\sqrt{\frac{b\cosh(dx+c)+b\sinh(dx+c)}{\cosh(dx+c)^2+2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2+1}}}{168\left(b^4d\cosh(dx+c)^4+4b^4d\cosh(dx+c)^3\sinh(dx+c)+6b^4d\cosh(dx+c)^2\sinh(dx+c)^2+4b^4d\cosh(dx+c)\sinh(dx+c)^3+b^4d\sinh(dx+c)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/168*(80*sqrt(2)*(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*co
sh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x
+ c)^4)*sqrt(b)*weierstrassPInverse(-4, 0, cosh(d*x + c) + sinh(d*x + c)) +
 sqrt(2)*(3*cosh(d*x + c)^8 + 24*cosh(d*x + c)*sinh(d*x + c)^7 + 3*sinh(d*x
 + c)^8 + 2*(42*cosh(d*x + c)^2 + 13)*sinh(d*x + c)^6 + 26*cosh(d*x + c)^6
+ 12*(14*cosh(d*x + c)^3 + 13*cosh(d*x + c))*sinh(d*x + c)^5 + 30*(7*cosh(d
*x + c)^4 + 13*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(21*cosh(d*x + c)^5 + 6
5*cosh(d*x + c)^3)*sinh(d*x + c)^3 + 2*(42*cosh(d*x + c)^6 + 195*cosh(d*x +
 c)^4 - 13)*sinh(d*x + c)^2 - 26*cosh(d*x + c)^2 + 4*(6*cosh(d*x + c)^7 + 3
9*cosh(d*x + c)^5 - 13*cosh(d*x + c))*sinh(d*x + c) - 3)*sqrt((b*cosh(d*x +
 c) + b*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + s
inh(d*x + c)^2 + 1)))/(b^4*d*cosh(d*x + c)^4 + 4*b^4*d*cosh(d*x + c)^3*sinh
(d*x + c) + 6*b^4*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b^4*d*cosh(d*x + c)
*sinh(d*x + c)^3 + b^4*d*sinh(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\operatorname{sech}\left(c+dx\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))**(7/2),x)
```

```
[Out] Integral((b*sech(c + d*x))**(-7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sech(d*x + c))^(-7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{b}{\cosh(c+d\,x)}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/cosh(c + d*x))^(7/2),x)
[Out] int(1/(b/cosh(c + d*x))^(7/2), x)
```

3.23 $\int (b\,\text{sech}(c+dx))^n\,dx$

Optimal. Leaf size=75

$$-\frac{b\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cosh^2(c+dx)\right)(b\,\text{sech}(c+dx))^{-1+n}\sinh(c+dx)}{d(1-n)\sqrt{-\sinh^2(c+dx)}}$$

```
[Out] -b*hypergeom([1/2, 1/2-1/2*n],[3/2-1/2*n],cosh(d*x+c)^2)*(b*sech(d*x+c))^(-
1+n)*sinh(d*x+c)/d/(1-n)/(-sinh(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3857, 2722}

$$-\frac{b\sinh(c+dx)(b\,\text{sech}(c+dx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cosh^2(c+dx)\right)}{d(1-n)\sqrt{-\sinh^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sech[c + d*x])^n,x]
```

```
[Out] -((b*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cosh[c + d*x]^2]*(b*Sech[
c + d*x])^(-1 + n)*Sinh[c + d*x])/(d*(1 - n)*Sqrt[-Sinh[c + d*x]^2]))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int (b\,\text{sech}(c+dx))^n\,dx &= \left(\frac{\cosh(c+dx)}{b}\right)^n (b\,\text{sech}(c+dx))^n \int \left(\frac{\cosh(c+dx)}{b}\right)^{-n} dx\\ &= -\frac{\cosh(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cosh^2(c+dx)\right)(b\,\text{sech}(c+dx))^n\sinh(c+dx)}{d(1-n)\sqrt{-\sinh^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 60, normalized size = 0.80

$$-\frac{\coth(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\mathrm{sech}^2(c+dx)\right)(b\mathrm{sech}(c+dx))^n\sqrt{\tanh^2(c+dx)}}{dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sech[c + d*x])^n,x]
```

```
[Out] -((Coth[c + d*x]*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Sech[c + d*x]^2]*(b
*Sech[c + d*x])^n*Sqrt[Tanh[c + d*x]^2])/(d*n))
```

Maple [F]
time = 1.25, size = 0, normalized size = 0.00

$$\int (b\,\mathrm{sech}(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sech(d*x+c))^n,x)
```

```
[Out] int((b*sech(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sech(d*x + c))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sech(d*x + c))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\mathrm{sech}\,(c+dx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))**n,x)
```

```
[Out] Integral((b*sech(c + d*x))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sech(d*x + c))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{b}{\cosh\left(c+d\,x\right)}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cosh(c + d*x))^n,x)
```

```
[Out] int((b/cosh(c + d*x))^n, x)
```

3.24 $\int \operatorname{sech}^2(a+bx)^{7/2}\,dx$

Optimal. Leaf size=90

$$\frac{5\text{ArcSin}(\tanh(a+bx))}{16b}+\frac{5\sqrt{\operatorname{sech}^2(a+bx)}\,\tanh(a+bx)}{16b}+\frac{5\operatorname{sech}^2(a+bx)^{3/2}\tanh(a+bx)}{24b}+\frac{\operatorname{sech}^2(a+bx)^{5/2}}{6b}$$

```
[Out] 5/16*arcsin(tanh(b*x+a))/b+5/24*(sech(b*x+a)^2)^(3/2)*tanh(b*x+a)/b+1/6*(se
ch(b*x+a)^2)^(5/2)*tanh(b*x+a)/b+5/16*(sech(b*x+a)^2)^(1/2)*tanh(b*x+a)/b
```

Rubi [A]
time = 0.02, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = {4207, 201, 222}

$$\frac{5\text{ArcSin}(\tanh(a+bx))}{16b}+\frac{\tanh(a+bx)\operatorname{sech}^2(a+bx)^{5/2}}{6b}+\frac{5\tanh(a+bx)\operatorname{sech}^2(a+bx)^{3/2}}{24b}+\frac{5\tanh(a+bx)\sqrt{\operatorname{sech}^2(a+bx)}}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[a + b*x]^2)^(7/2),x]
```

```
[Out] (5*ArcSin[Tanh[a + b*x]])/(16*b) + (5*Sqrt[Sech[a + b*x]^2]*Tanh[a + b*x])/
(16*b) + (5*(Sech[a + b*x]^2)^(3/2)*Tanh[a + b*x])/(24*b) + ((Sech[a + b*x]
^2)^(5/2)*Tanh[a + b*x])/(6*b)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}^2(a+bx)^{7/2}\,dx = \frac{\operatorname{Subst}\left(\int (1-x^2)^{5/2}\,dx, x, \tanh(a+bx)\right)}{b}$$

$$= \frac{\operatorname{sech}^2(a+bx)^{5/2}\tanh(a+bx)}{6b} + \frac{5\operatorname{Subst}\left(\int (1-x^2)^{3/2}\,dx, x, \tanh(a+bx)\right)}{6b}$$

$$= \frac{5\operatorname{sech}^2(a+bx)^{3/2}\tanh(a+bx)}{24b} + \frac{\operatorname{sech}^2(a+bx)^{5/2}\tanh(a+bx)}{6b} + \frac{5\operatorname{Subst}\left(\int \sqrt{1-\right.}{}$$

$$= \frac{5\sqrt{\operatorname{sech}^2(a+bx)}\tanh(a+bx)}{16b} + \frac{5\operatorname{sech}^2(a+bx)^{3/2}\tanh(a+bx)}{24b} + \frac{\operatorname{sech}^2(a+bx}{}$$

$$= \frac{5\sin^{-1}(\tanh(a+bx))}{16b} + \frac{5\sqrt{\operatorname{sech}^2(a+bx)}\tanh(a+bx)}{16b} + \frac{5\operatorname{sech}^2(a+bx)^{3/2}\tanh}{24b}$$

Mathematica [A]
time = 0.08, size = 81, normalized size = 0.90

$$\frac{\cosh(a+bx)\sqrt{\operatorname{sech}^2(a+bx)}\,(15\operatorname{ArcTan}(\sinh(a+bx)) + 15\operatorname{sech}(a+bx)\tanh(a+bx) + 10\operatorname{sech}^3(a+bx)\tanh(a+bx) + 8\operatorname{sech}^5(a+bx)\tanh(a+bx))}{48b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[a + b*x]^2)^(7/2),x]
```

```
[Out] (Cosh[a + b*x]*Sqrt[Sech[a + b*x]^2]*(15*ArcTan[Sinh[a + b*x]] + 15*Sech[a
+ b*x]*Tanh[a + b*x] + 10*Sech[a + b*x]^3*Tanh[a + b*x] + 8*Sech[a + b*x]^5
*Tanh[a + b*x]))/(48*b)
```

Maple [C] Result contains complex when optimal does not.
time = 2.60, size = 230, normalized size = 2.56

method	result
risch	$\frac{\sqrt{\frac{e^{2bx+2a}}{(e^{2bx+2a}+1)^2}}\,(15\,e^{10bx+10a}+85\,e^{8bx+8a}+198\,e^{6bx+6a}-198\,e^{4bx+4a}-85\,e^{2bx+2a}-15)}{24(e^{2bx+2a}+1)^5 b} + \frac{5i\ln(e^{bx}+ie^{-a})\sqrt{\frac{e^{2bx+2a}}{(e^{2bx+2a}+1)^2}}}{16b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sech(b*x+a)^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/(exp(2*b*x+2*a)+1)^5*(1/(exp(2*b*x+2*a)+1)^2*exp(2*b*x+2*a))^(1/2)*(15
*exp(10*b*x+10*a)+85*exp(8*b*x+8*a)+198*exp(6*b*x+6*a)-198*exp(4*b*x+4*a)-8
5*exp(2*b*x+2*a)-15)/b+5/16*I*ln(exp(b*x)+I*exp(-a))/b*(1/(exp(2*b*x+2*a)+1
)^2*exp(2*b*x+2*a))^(1/2)*(exp(2*b*x+2*a)+1)*exp(-b*x-a)-5/16*I*ln(exp(b*x)
```

```
-I*exp(-a))/b*(1/(exp(2*b*x+2*a)+1)^2*exp(2*b*x+2*a))^(1/2)*(exp(2*b*x+2*a)
+1)*exp(-b*x-a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(76) = 152.
time = 0.49, size = 156, normalized size = 1.73

$$-\frac{5 \arctan\left(e^{(-bx-a)}\right)}{8\,b}+\frac{15\,e^{(-bx-a)}+85\,e^{(-3\,bx-3\,a)}+198\,e^{(-5\,bx-5\,a)}-198\,e^{(-7\,bx-7\,a)}-85\,e^{(-9\,bx-9\,a)}-15\,e^{(-11\,bx-11\,a)}}{24\,b(6\,e^{(-2\,bx-2\,a)}+15\,e^{(-4\,bx-4\,a)}+20\,e^{(-6\,bx-6\,a)}+15\,e^{(-8\,bx-8\,a)}+6\,e^{(-10\,bx-10\,a)}+e^{(-12\,bx-12\,a)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(b*x+a)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -5/8*arctan(e^(-b*x - a))/b + 1/24*(15*e^(-b*x - a) + 85*e^(-3*b*x - 3*a) +
 198*e^(-5*b*x - 5*a) - 198*e^(-7*b*x - 7*a) - 85*e^(-9*b*x - 9*a) - 15*e^(
-11*b*x - 11*a))/(b*(6*e^(-2*b*x - 2*a) + 15*e^(-4*b*x - 4*a) + 20*e^(-6*b*
x - 6*a) + 15*e^(-8*b*x - 8*a) + 6*e^(-10*b*x - 10*a) + e^(-12*b*x - 12*a)
+ 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1604 vs. 2(76) = 152.
time = 0.36, size = 1604, normalized size = 17.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(b*x+a)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/24*(15*cosh(b*x + a)^11 + 165*cosh(b*x + a)*sinh(b*x + a)^10 + 15*sinh(b*
x + a)^11 + 5*(165*cosh(b*x + a)^2 + 17)*sinh(b*x + a)^9 + 85*cosh(b*x + a)
^9 + 45*(55*cosh(b*x + a)^3 + 17*cosh(b*x + a))*sinh(b*x + a)^8 + 18*(275*c
osh(b*x + a)^4 + 170*cosh(b*x + a)^2 + 11)*sinh(b*x + a)^7 + 198*cosh(b*x +
 a)^7 + 42*(165*cosh(b*x + a)^5 + 170*cosh(b*x + a)^3 + 33*cosh(b*x + a))*s
inh(b*x + a)^6 + 18*(385*cosh(b*x + a)^6 + 595*cosh(b*x + a)^4 + 231*cosh(b
*x + a)^2 - 11)*sinh(b*x + a)^5 - 198*cosh(b*x + a)^5 + 90*(55*cosh(b*x + a
)^7 + 119*cosh(b*x + a)^5 + 77*cosh(b*x + a)^3 - 11*cosh(b*x + a))*sinh(b*x
 + a)^4 + 5*(495*cosh(b*x + a)^8 + 1428*cosh(b*x + a)^6 + 1386*cosh(b*x + a
)^4 - 396*cosh(b*x + a)^2 - 17)*sinh(b*x + a)^3 - 85*cosh(b*x + a)^3 + 3*(2
75*cosh(b*x + a)^9 + 1020*cosh(b*x + a)^7 + 1386*cosh(b*x + a)^5 - 660*cosh
(b*x + a)^3 - 85*cosh(b*x + a))*sinh(b*x + a)^2 + 15*(cosh(b*x + a)^12 + 12
*cosh(b*x + a)*sinh(b*x + a)^11 + sinh(b*x + a)^12 + 6*(11*cosh(b*x + a)^2
+ 1)*sinh(b*x + a)^10 + 6*cosh(b*x + a)^10 + 20*(11*cosh(b*x + a)^3 + 3*cos
h(b*x + a))*sinh(b*x + a)^9 + 15*(33*cosh(b*x + a)^4 + 18*cosh(b*x + a)^2 +
 1)*sinh(b*x + a)^8 + 15*cosh(b*x + a)^8 + 24*(33*cosh(b*x + a)^5 + 30*cosh
(b*x + a)^3 + 5*cosh(b*x + a))*sinh(b*x + a)^7 + 4*(231*cosh(b*x + a)^6 + 3
15*cosh(b*x + a)^4 + 105*cosh(b*x + a)^2 + 5)*sinh(b*x + a)^6 + 20*cosh(b*x
```

```
 + a)^6 + 24*(33*cosh(b*x + a)^7 + 63*cosh(b*x + a)^5 + 35*cosh(b*x + a)^3
+ 5*cosh(b*x + a))*sinh(b*x + a)^5 + 15*(33*cosh(b*x + a)^8 + 84*cosh(b*x +
 a)^6 + 70*cosh(b*x + a)^4 + 20*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^4 + 15*c
osh(b*x + a)^4 + 20*(11*cosh(b*x + a)^9 + 36*cosh(b*x + a)^7 + 42*cosh(b*x
+ a)^5 + 20*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 6*(11*cosh
(b*x + a)^10 + 45*cosh(b*x + a)^8 + 70*cosh(b*x + a)^6 + 50*cosh(b*x + a)^4
 + 15*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 6*cosh(b*x + a)^2 + 12*(cosh(b
*x + a)^11 + 5*cosh(b*x + a)^9 + 10*cosh(b*x + a)^7 + 10*cosh(b*x + a)^5 +
5*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a)
+ sinh(b*x + a)) + 3*(55*cosh(b*x + a)^10 + 255*cosh(b*x + a)^8 + 462*cosh(
b*x + a)^6 - 330*cosh(b*x + a)^4 - 85*cosh(b*x + a)^2 - 5)*sinh(b*x + a) -
15*cosh(b*x + a))/(b*cosh(b*x + a)^12 + 12*b*cosh(b*x + a)*sinh(b*x + a)^11
 + b*sinh(b*x + a)^12 + 6*b*cosh(b*x + a)^10 + 6*(11*b*cosh(b*x + a)^2 + b)
*sinh(b*x + a)^10 + 20*(11*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x
+ a)^9 + 15*b*cosh(b*x + a)^8 + 15*(33*b*cosh(b*x + a)^4 + 18*b*cosh(b*x +
a)^2 + b)*sinh(b*x + a)^8 + 24*(33*b*cosh(b*x + a)^5 + 30*b*cosh(b*x + a)^3
 + 5*b*cosh(b*x + a))*sinh(b*x + a)^7 + 20*b*cosh(b*x + a)^6 + 4*(231*b*cos
h(b*x + a)^6 + 315*b*cosh(b*x + a)^4 + 105*b*cosh(b*x + a)^2 + 5*b)*sinh(b*
x + a)^6 + 24*(33*b*cosh(b*x + a)^7 + 63*b*cosh(b*x + a)^5 + 35*b*cosh(b*x
+ a)^3 + 5*b*cosh(b*x + a))*sinh(b*x + a)^5 + 15*b*cosh(b*x + a)^4 + 15*(33
*b*cosh(b*x + a)^8 + 84*b*cosh(b*x + a)^6 + 70*b*cosh(b*x + a)^4 + 20*b*cos
h(b*x + a)^2 + b)*sinh(b*x + a)^4 + 20*(11*b*cosh(b*x + a)^9 + 36*b*cosh(b*
x + a)^7 + 42*b*cosh(b*x + a)^5 + 20*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))
*sinh(b*x + a)^3 + 6*b*cosh(b*x + a)^2 + 6*(11*b*cosh(b*x + a)^10 + 45*b*co
sh(b*x + a)^8 + 70*b*cosh(b*x + a)^6 + 50*b*cosh(b*x + a)^4 + 15*b*cosh(b*x
 + a)^2 + b)*sinh(b*x + a)^2 + 12*(b*cosh(b*x + a)^11 + 5*b*cosh(b*x + a)^9
 + 10*b*cosh(b*x + a)^7 + 10*b*cosh(b*x + a)^5 + 5*b*cosh(b*x + a)^3 + b*co
sh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(b*x+a)**2)**(7/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [**A**]
time = 0.38, size = 124, normalized size = 1.38

$$\frac{15\,\pi + \frac{4\left(15\left(e^{(bx+a)}-e^{(-bx-a)}\right)^5+160\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3+528\,e^{(bx+a)}-528\,e^{(-bx-a)}\right)}{\left(\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+4\right)^3} + 30\,\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)}-1\right)e^{(-bx-a)}\right)}{96\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(b*x+a)^2)^(7/2),x, algorithm="giac")
```

```
[Out] 1/96*(15*pi + 4*(15*(e^(b*x + a) - e^(-b*x - a))^5 + 160*(e^(b*x + a) - e^(
-b*x - a))^3 + 528*e^(b*x + a) - 528*e^(-b*x - a))/((e^(b*x + a) - e^(-b*x
- a))^2 + 4)^3 + 30*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)))/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{1}{\cosh\left(a+b\,x\right)^2}\right)^{7/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(a + b*x)^2)^(7/2),x)
```

```
[Out] int((1/cosh(a + b*x)^2)^(7/2), x)
```

3.25 $\int \mathrm{sech}^2(a+bx)^{5/2}\,dx$

Optimal. Leaf size=65

$$\frac{3\mathrm{ArcSin}(\tanh(a+bx))}{8b}+\frac{3\sqrt{\mathrm{sech}^2(a+bx)}\,\tanh(a+bx)}{8b}+\frac{\mathrm{sech}^2(a+bx)^{3/2}\tanh(a+bx)}{4b}$$

```
[Out] 3/8*arcsin(tanh(b*x+a))/b+1/4*(sech(b*x+a)^2)^(3/2)*tanh(b*x+a)/b+3/8*(sech
(b*x+a)^2)^(1/2)*tanh(b*x+a)/b
```

Rubi [A]
time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4207, 201, 222}

$$\frac{3\mathrm{ArcSin}(\tanh(a+bx))}{8b}+\frac{\tanh(a+bx)\mathrm{sech}^2(a+bx)^{3/2}}{4b}+\frac{3\tanh(a+bx)\sqrt{\mathrm{sech}^2(a+bx)}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[a + b*x]^2)^(5/2),x]
```

```
[Out] (3*ArcSin[Tanh[a + b*x]])/(8*b) + (3*Sqrt[Sech[a + b*x]^2]*Tanh[a + b*x])/(
8*b) + ((Sech[a + b*x]^2)^(3/2)*Tanh[a + b*x])/(4*b)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}^2(a+bx)^{5/2}\,dx = \frac{\operatorname{Subst}\left(\int (1-x^2)^{3/2}\,dx, x, \tanh(a+bx)\right)}{b}$$
$$= \frac{\operatorname{sech}^2(a+bx)^{3/2}\tanh(a+bx)}{4b} + \frac{3\operatorname{Subst}\left(\int \sqrt{1-x^2}\,dx, x, \tanh(a+bx)\right)}{4b}$$
$$= \frac{3\sqrt{\operatorname{sech}^2(a+bx)}\,\tanh(a+bx)}{8b} + \frac{\operatorname{sech}^2(a+bx)^{3/2}\tanh(a+bx)}{4b} + \frac{3\operatorname{Subst}\left(\int \frac{}{\sqrt{1-}}\right.}{}$$
$$= \frac{3\sin^{-1}(\tanh(a+bx))}{8b} + \frac{3\sqrt{\operatorname{sech}^2(a+bx)}\,\tanh(a+bx)}{8b} + \frac{\operatorname{sech}^2(a+bx)^{3/2}\tanh(a}{4b}$$

Mathematica [A]
time = 0.08, size = 55, normalized size = 0.85

$$\frac{\operatorname{sech}^2(a+bx)^{3/2}\left(6\operatorname{ArcTan}(\sinh(a+bx))\cosh^3(a+bx)+3\sinh(2(a+bx))+4\tanh(a+bx)\right)}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[a + b*x]^2)^(5/2),x]
```

```
[Out] ((Sech[a + b*x]^2)^(3/2)*(6*ArcTan[Sinh[a + b*x]]*Cosh[a + b*x]^3 + 3*Sinh[
2*(a + b*x)] + 4*Tanh[a + b*x]))/(16*b)
```

Maple [C] Result contains complex when optimal does not.
time = 2.35, size = 208, normalized size = 3.20

method	result
risch	$\frac{\sqrt{\frac{e^{2bx+2a}}{(e^{2bx+2a}+1)^2}}\,(3\,e^{6bx+6a}+11\,e^{4bx+4a}-11\,e^{2bx+2a}-3)}{4(e^{2bx+2a}+1)^3 b} + \frac{3i\ln(e^{bx}+ie^{-a})\sqrt{\frac{e^{2bx+2a}}{(e^{2bx+2a}+1)^2}}\,(e^{2bx+2a}+1)e^{-bx-a}}{8b} - \frac{3i\ln(e^{b}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sech(b*x+a)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/(exp(2*b*x+2*a)+1)^3*(1/(exp(2*b*x+2*a)+1)^2*exp(2*b*x+2*a))^(1/2)*(3*e
xp(6*b*x+6*a)+11*exp(4*b*x+4*a)-11*exp(2*b*x+2*a)-3)/b+3/8*I*ln(exp(b*x)+I*
exp(-a))/b*(1/(exp(2*b*x+2*a)+1)^2*exp(2*b*x+2*a))^(1/2)*(exp(2*b*x+2*a)+1)
*exp(-b*x-a)-3/8*I*ln(exp(b*x)-I*exp(-a))/b*(1/(exp(2*b*x+2*a)+1)^2*exp(2*b
*x+2*a))^(1/2)*(exp(2*b*x+2*a)+1)*exp(-b*x-a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(55) = 110.

time = 0.46, size = 112, normalized size = 1.72

$$-\frac{3\arctan\left(e^{(-bx-a)}\right)}{4b}+\frac{3e^{(-bx-a)}+11e^{(-3bx-3a)}-11e^{(-5bx-5a)}-3e^{(-7bx-7a)}}{4b(4e^{(-2bx-2a)}+6e^{(-4bx-4a)}+4e^{(-6bx-6a)}+e^{(-8bx-8a)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(b*x+a)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -3/4*arctan(e^(-b*x - a))/b + 1/4*(3*e^(-b*x - a) + 11*e^(-3*b*x - 3*a) - 1
1*e^(-5*b*x - 5*a) - 3*e^(-7*b*x - 7*a))/(b*(4*e^(-2*b*x - 2*a) + 6*e^(-4*b
*x - 4*a) + 4*e^(-6*b*x - 6*a) + e^(-8*b*x - 8*a) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 812 vs. $2(55) = 110$.
time = 0.35, size = 812, normalized size = 12.49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(b*x+a)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(3*cosh(b*x + a)^7 + 21*cosh(b*x + a)*sinh(b*x + a)^6 + 3*sinh(b*x + a)
^7 + (63*cosh(b*x + a)^2 + 11)*sinh(b*x + a)^5 + 11*cosh(b*x + a)^5 + 5*(21
*cosh(b*x + a)^3 + 11*cosh(b*x + a))*sinh(b*x + a)^4 + (105*cosh(b*x + a)^4
 + 110*cosh(b*x + a)^2 - 11)*sinh(b*x + a)^3 - 11*cosh(b*x + a)^3 + (63*cos
h(b*x + a)^5 + 110*cosh(b*x + a)^3 - 33*cosh(b*x + a))*sinh(b*x + a)^2 + 3*
(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4*(7
*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^6 + 4*cosh(b*x + a)^6 + 8*(7*cosh(b*x +
 a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 + 30*cosh(
b*x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5
+ 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a
)^6 + 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 4*cosh(
b*x + a)^2 + 8*(cosh(b*x + a)^7 + 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 + c
osh(b*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) + (2
1*cosh(b*x + a)^6 + 55*cosh(b*x + a)^4 - 33*cosh(b*x + a)^2 - 3)*sinh(b*x +
 a) - 3*cosh(b*x + a))/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*sinh(b*x + a)
^7 + b*sinh(b*x + a)^8 + 4*b*cosh(b*x + a)^6 + 4*(7*b*cosh(b*x + a)^2 + b)*
sinh(b*x + a)^6 + 8*(7*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)
^5 + 6*b*cosh(b*x + a)^4 + 2*(35*b*cosh(b*x + a)^4 + 30*b*cosh(b*x + a)^2 +
 3*b)*sinh(b*x + a)^4 + 8*(7*b*cosh(b*x + a)^5 + 10*b*cosh(b*x + a)^3 + 3*b
*cosh(b*x + a))*sinh(b*x + a)^3 + 4*b*cosh(b*x + a)^2 + 4*(7*b*cosh(b*x + a
)^6 + 15*b*cosh(b*x + a)^4 + 9*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 8*(
b*cosh(b*x + a)^7 + 3*b*cosh(b*x + a)^5 + 3*b*cosh(b*x + a)^3 + b*cosh(b*x
+ a))*sinh(b*x + a) + b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\operatorname{sech}^2(a+bx)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(b*x+a)**2)**(5/2),x)
```

```
[Out] Integral((sech(a + b*x)**2)**(5/2), x)
```

Giac [A]
time = 0.39, size = 102, normalized size = 1.57

$$\frac{3\,\pi + \frac{4\left(3\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3+20\,e^{(bx+a)}-20\,e^{(-bx-a)}\right)}{\left(\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+4\right)^2} + 6\,\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)}-1\right)e^{(-bx-a)}\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(b*x+a)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/16*(3*pi + 4*(3*(e^(b*x + a) - e^(-b*x - a))^3 + 20*e^(b*x + a) - 20*e^(-
b*x - a))/((e^(b*x + a) - e^(-b*x - a))^2 + 4)^2 + 6*arctan(1/2*(e^(2*b*x +
 2*a) - 1)*e^(-b*x - a)))/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(\frac{1}{{\mathrm{cosh}\left(a+b\,x\right)}^2}\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(a + b*x)^2)^(5/2),x)
```

```
[Out] int((1/cosh(a + b*x)^2)^(5/2), x)
```

3.26 $\int \text{sech}^2(a+bx)^{3/2}\,dx$

Optimal. Leaf size=40

$$\frac{\text{ArcSin}(\tanh(a+bx))}{2b}+\frac{\sqrt{\text{sech}^2(a+bx)}\,\tanh(a+bx)}{2b}$$

```
[Out] 1/2*arcsin(tanh(b*x+a))/b+1/2*(sech(b*x+a)^2)^(1/2)*tanh(b*x+a)/b
```

Rubi [A]
time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4207, 201, 222}

$$\frac{\text{ArcSin}(\tanh(a+bx))}{2b}+\frac{\tanh(a+bx)\sqrt{\text{sech}^2(a+bx)}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[a + b*x]^2)^(3/2),x]
```

```
[Out] ArcSin[Tanh[a + b*x]]/(2*b) + (Sqrt[Sech[a + b*x]^2]*Tanh[a + b*x])/(2*b)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}^2(a+bx)^{3/2}\,dx = \frac{\operatorname{Subst}\left(\int \sqrt{1-x^2}\,dx, x, \tanh(a+bx)\right)}{b}$$
$$= \frac{\sqrt{\operatorname{sech}^2(a+bx)}\,\tanh(a+bx)}{2b} + \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \tanh(a+bx)\right)}{2b}$$
$$= \frac{\sin^{-1}(\tanh(a+bx))}{2b} + \frac{\sqrt{\operatorname{sech}^2(a+bx)}\,\tanh(a+bx)}{2b}$$

Mathematica [**A**]
time = 0.04, size = 46, normalized size = 1.15

$$\frac{\operatorname{sech}(a+bx)(\operatorname{ArcTan}(\sinh(a+bx)) + \operatorname{sech}(a+bx)\tanh(a+bx))}{2b\sqrt{\operatorname{sech}^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[a + b*x]^2)^(3/2),x]
```

```
[Out] (Sech[a + b*x]*(ArcTan[Sinh[a + b*x]] + Sech[a + b*x]*Tanh[a + b*x]))/(2*b*
Sqrt[Sech[a + b*x]^2])
```

Maple [**C**] Result contains complex when optimal does not.
time = 2.42, size = 183, normalized size = 4.58

method	result
risch	$\frac{\sqrt{\frac{e^{2bx+2a}}{\left(e^{2bx+2a}+1\right)^2}}\,\left(e^{2bx+2a}-1\right)}{\left(e^{2bx+2a}+1\right)b} + \frac{i\ln\left(e^{bx}+ie^{-a}\right)\sqrt{\frac{e^{2bx+2a}}{\left(e^{2bx+2a}+1\right)^2}}\,\left(e^{2bx+2a}+1\right)e^{-bx-a}}{2b} - \frac{i\ln\left(e^{bx}-ie^{-a}\right)\sqrt{\frac{e^{2bx+2a}}{\left(e^{2bx+2a}+1\right)^2}}\,(}{2b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sech(b*x+a)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(exp(2*b*x+2*a)+1)*(1/(exp(2*b*x+2*a)+1)^2*exp(2*b*x+2*a))^(1/2)*(exp(2*b
*x+2*a)-1)/b+1/2*I*ln(exp(b*x)+I*exp(-a))/b*(1/(exp(2*b*x+2*a)+1)^2*exp(2*b
*x+2*a))^(1/2)*(exp(2*b*x+2*a)+1)*exp(-b*x-a)-1/2*I*ln(exp(b*x)-I*exp(-a))/
b*(1/(exp(2*b*x+2*a)+1)^2*exp(2*b*x+2*a))^(1/2)*(exp(2*b*x+2*a)+1)*exp(-b*x
-a)
```

Maxima [**A**]
time = 0.48, size = 65, normalized size = 1.62

$$-\frac{\arctan\left(e^{(-bx-a)}\right)}{b} + \frac{e^{(-bx-a)} - e^{(-3\,bx-3\,a)}}{b(2\,e^{(-2\,bx-2\,a)} + e^{(-4\,bx-4\,a)} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -arctan(e^(-b*x - a))/b + (e^(-b*x - a) - e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x
 - 2*a) + e^(-4*b*x - 4*a) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(34) = 68$.
time = 0.37, size = 267, normalized size = 6.68

$$\frac{\cosh(bx+a)^3 + 3\cosh(bx+a)\sinh(bx+a)^2 + \sinh(bx+a)^3 + \left(\cosh(bx+a)^4 + 4\cosh(bx+a)\sinh(bx+a)^3 + \sinh(bx+a)^4 + 2\left(3\cosh(bx+a)^2 + 1\right)\sinh(bx+a)^2 + 2\cosh(bx+a)^2 + 4\left(\cosh(bx+a)^3 + \cosh(bx+a)\right)\sinh(bx+a) + 1\right)\arctan\left(\cosh(bx+a) + \sinh(bx+a)\right) + \left(3\cosh(bx+a)^2 - 1\right)\sinh(bx+a) - \cosh(bx+a)}{b\cosh(bx+a)^4 + 4b\cosh(bx+a)\sinh(bx+a)^3 + b\sinh(bx+a)^4 + 2b\cosh(bx+a)^2 + 2\left(3b\cosh(bx+a)^2 + b\right)\sinh(bx+a)^2 + 4\left(b\cosh(bx+a)^3 + b\cosh(bx+a)\right)\sinh(bx+a) + b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (cos
h(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cos
h(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3
+ cosh(b*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) +
 (3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))/(b*cosh(b*x + a)^4
+ 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 + 2*b*cosh(b*x + a)
^2 + 2*(3*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 + b
*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\operatorname{sech}^2\left(a+bx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(b*x+a)**2)**(3/2),x)
```

```
[Out] Integral((sech(a + b*x)**2)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(34) = 68$.
time = 0.40, size = 76, normalized size = 1.90

$$\frac{\pi + \frac{4\left(e^{(bx+a)} - e^{(-bx-a)}\right)}{\left(e^{(bx+a)} - e^{(-bx-a)}\right)^2 + 4} + 2\arctan\left(\frac{1}{2}\left(e^{(2bx+2a)} - 1\right)e^{(-bx-a)}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(b*x+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(pi + 4*(e^(b*x + a) - e^(-b*x - a))/((e^(b*x + a) - e^(-b*x - a))^2 +
4) + 2*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)))/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \left(\frac{1}{\cosh\left(a+b\,x\right)^2}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(a + b*x)^2)^(3/2),x)
[Out] int((1/cosh(a + b*x)^2)^(3/2), x)
```

3.27 $\int \sqrt{\text{sech}^2(a+bx)}\, dx$

Optimal. Leaf size=11

$$\frac{\text{ArcSin}(\tanh(a+bx))}{b}$$

```
[Out] arcsin(tanh(b*x+a))/b
```

Rubi [A]
time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4207, 222}

$$\frac{\text{ArcSin}(\tanh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sech[a + b*x]^2],x]
```

```
[Out] ArcSin[Tanh[a + b*x]]/b
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\text{sech}^2(a+bx)}\, dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\, dx, x, \tanh(a+bx)\right)}{b}\\ &= \frac{\sin^{-1}(\tanh(a+bx))}{b}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 29 vs. 2(11) = 22.
time = 0.01, size = 29, normalized size = 2.64

$$\frac{\text{ArcTan}(\sinh(a+bx))\cosh(a+bx)\sqrt{\text{sech}^2(a+bx)}}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Sech[a + b*x]^2],x]

[Out] (ArcTan[Sinh[a + b*x]]*Cosh[a + b*x]*Sqrt[Sech[a + b*x]^2])/b

Maple [C] Result contains complex when optimal does not.
time = 2.49, size = 130, normalized size = 11.82

method	result	size
risch	$\frac{i \ln\left(\mathrm{e}^{b x}+i \mathrm{e}^{-a}\right) \sqrt{\frac{\mathrm{e}^{2 b x+2 a}}{\left(\mathrm{e}^{2 b x+2 a}+1\right)^{2}}}\, \left(\mathrm{e}^{2 b x+2 a}+1\right) \mathrm{e}^{-b x-a}}{b}-\frac{i \ln\left(\mathrm{e}^{b x}-i \mathrm{e}^{-a}\right) \sqrt{\frac{\mathrm{e}^{2 b x+2 a}}{\left(\mathrm{e}^{2 b x+2 a}+1\right)^{2}}}\, \left(\mathrm{e}^{2 b x+2 a}+1\right) \mathrm{e}^{-b x-a}}{b}$	130

Verification of antiderivative is not currently implemented for this CAS.

[In] int((sech(b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] I*ln(exp(b*x)+I*exp(-a))/b*(exp(2*b*x+2*a)+1)*(1/(exp(2*b*x+2*a)+1)^2*exp(2*b*x+2*a))^(1/2)*exp(-b*x-a)-I*ln(exp(b*x)-I*exp(-a))/b*(exp(2*b*x+2*a)+1)*(1/(exp(2*b*x+2*a)+1)^2*exp(2*b*x+2*a))^(1/2)*exp(-b*x-a)

Maxima [A]
time = 0.26, size = 11, normalized size = 1.00

$$\frac{\arctan\left(\sinh\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sech(b*x+a)^2)^(1/2),x, algorithm="maxima")

[Out] arctan(sinh(b*x + a))/b

Fricas [A]
time = 0.37, size = 19, normalized size = 1.73

$$\frac{2\,\arctan\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sech(b*x+a)^2)^(1/2),x, algorithm="fricas")

[Out] 2*arctan(cosh(b*x + a) + sinh(b*x + a))/b

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\operatorname{sech}^{2}\left(a+bx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(b*x+a)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(sech(a + b*x)**2), x)
```

Giac [A]
time = 0.39, size = 12, normalized size = 1.09

$$\frac{2\arctan\left(e^{(bx+a)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(e^(b*x + a))/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.09

$$\int \sqrt{\frac{1}{\cosh\left(a+b\,x\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(a + b*x)^2)^(1/2),x)
```

```
[Out] int((1/cosh(a + b*x)^2)^(1/2), x)
```

3.28 $\int \frac{1}{\sqrt{\text{sech}^2(a+bx)}}\,dx$

Optimal. Leaf size=22

$$\frac{\tanh(a+bx)}{b\sqrt{\text{sech}^2(a+bx)}}$$

[Out] tanh(b*x+a)/b/(sech(b*x+a)^2)^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4207, 197}

$$\frac{\tanh(a+bx)}{b\sqrt{\text{sech}^2(a+bx)}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[Sech[a + b*x]^2],x]

[Out] Tanh[a + b*x]/(b*Sqrt[Sech[a + b*x]^2])

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\text{sech}^2(a+bx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^{3/2}}\,dx, x, \tanh(a+bx)\right)}{b}\\ &= \frac{\tanh(a+bx)}{b\sqrt{\text{sech}^2(a+bx)}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 1.00

$$\frac{\tanh(a+bx)}{b\sqrt{\operatorname{sech}^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Sech[a + b*x]^2],x]
[Out] Tanh[a + b*x]/(b*Sqrt[Sech[a + b*x]^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(20) = 40$.
time = 2.41, size = 97, normalized size = 4.41

method	result	size
risch	$\frac{e^{2bx+2a}}{2b(e^{2bx+2a}+1)\sqrt{\frac{e^{2bx+2a}}{(e^{2bx+2a}+1)^2}}}-\frac{1}{2b(e^{2bx+2a}+1)\sqrt{\frac{e^{2bx+2a}}{(e^{2bx+2a}+1)^2}}}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sech(b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2/b/(exp(2*b*x+2*a)+1)/(1/(exp(2*b*x+2*a)+1)^2*exp(2*b*x+2*a))^(1/2)*exp(
2*b*x+2*a)-1/2/b/(exp(2*b*x+2*a)+1)/(1/(exp(2*b*x+2*a)+1)^2*exp(2*b*x+2*a))
^(1/2)
```

Maxima [A]
time = 0.31, size = 26, normalized size = 1.18

$$\frac{e^{(bx+a)}}{2\,b}-\frac{e^{(-bx-a)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(b*x+a)^2)^(1/2),x, algorithm="maxima")
[Out] 1/2*e^(b*x + a)/b - 1/2*e^(-b*x - a)/b
```

Fricas [A]
time = 0.39, size = 10, normalized size = 0.45

$$\frac{\sinh\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

[Out] sinh(b*x + a)/b

Sympy [A]
time = 12.13, size = 29, normalized size = 1.32

$$\begin{cases} \frac{\tanh(a+bx)}{b\sqrt{\operatorname{sech}^2(a+bx)}} & \text{for } b \neq 0 \\ \frac{x}{\sqrt{\operatorname{sech}^2(a)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sech(b*x+a)**2)**(1/2),x)

[Out] Piecewise((tanh(a + b*x)/(b*sqrt(sech(a + b*x)**2)), Ne(b, 0)), (x/sqrt(sec
h(a)**2), True))

Giac [A]
time = 0.39, size = 23, normalized size = 1.05

$$\frac{e^{(bx+a)} - e^{(-bx-a)}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sech(b*x+a)^2)^(1/2),x, algorithm="giac")

[Out] 1/2*(e^(b*x + a) - e^(-b*x - a))/b

Mupad [B]
time = 0.15, size = 53, normalized size = 2.41

$$\frac{e^{-2a-2bx}\left(e^{4a+4bx}-1\right)\sqrt{\frac{4e^{2a+2bx}}{\left(e^{2a+2bx}+1\right)^2}}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1/cosh(a + b*x)^2)^(1/2),x)

[Out] (exp(- 2*a - 2*b*x)*(exp(4*a + 4*b*x) - 1)*((4*exp(2*a + 2*b*x))/(exp(2*a +
2*b*x) + 1)^2)^(1/2))/(4*b)

3.29 $\int \frac{1}{\mathrm{sech}^2(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=51

$$\frac{\tanh(a+bx)}{3b\mathrm{sech}^2(a+bx)^{3/2}}+\frac{2\tanh(a+bx)}{3b\sqrt{\mathrm{sech}^2(a+bx)}}$$

```
[Out] 1/3*tanh(b*x+a)/b/(sech(b*x+a)^2)^(3/2)+2/3*tanh(b*x+a)/b/(sech(b*x+a)^2)^(
1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4207, 198, 197}

$$\frac{2\tanh(a+bx)}{3b\sqrt{\mathrm{sech}^2(a+bx)}}+\frac{\tanh(a+bx)}{3b\mathrm{sech}^2(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[a + b*x]^2)^(-3/2),x]
```

```
[Out] Tanh[a + b*x]/(3*b*(Sech[a + b*x]^2)^(3/2)) + (2*Tanh[a + b*x])/(3*b*Sqrt[S
ech[a + b*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{3/2}}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)^{5/2}}\,dx, x, \tanh(a+bx)\right)}{b}$$

$$= \frac{\tanh(a+bx)}{3b\operatorname{sech}^2(a+bx)^{3/2}} + \frac{2\operatorname{Subst}\left(\int \frac{1}{(1-x^2)^{3/2}}\,dx, x, \tanh(a+bx)\right)}{3b}$$

$$= \frac{\tanh(a+bx)}{3b\operatorname{sech}^2(a+bx)^{3/2}} + \frac{2\tanh(a+bx)}{3b\sqrt{\operatorname{sech}^2(a+bx)}}$$

Mathematica [A]
time = 0.05, size = 44, normalized size = 0.86

$$\frac{3\operatorname{sech}^2(a+bx)\tanh(a+bx) + \tanh^3(a+bx)}{3b\operatorname{sech}^2(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[a + b*x]^2)^(-3/2),x]
```

```
[Out] (3*Sech[a + b*x]^2*Tanh[a + b*x] + Tanh[a + b*x]^3)/(3*b*(Sech[a + b*x]^2)^
(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(43) = 86$.
time = 2.54, size = 201, normalized size = 3.94

method	result
risch	$\frac{e^{4bx+4a}}{24b\left(e^{2bx+2a}+1\right)\sqrt{\frac{e^{2bx+2a}}{\left(e^{2bx+2a}+1\right)^2}}} + \frac{3\,e^{2bx+2a}}{8b\left(e^{2bx+2a}+1\right)\sqrt{\frac{e^{2bx+2a}}{\left(e^{2bx+2a}+1\right)^2}}} - \frac{3}{8b\left(e^{2bx+2a}+1\right)\sqrt{\frac{e^{2bx+2a}}{\left(e^{2bx+2a}+1\right)^2}}} - \frac{e}{24b\left(e^{2bx+2a}+\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sech(b*x+a)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/b/(exp(2*b*x+2*a)+1)/(1/(exp(2*b*x+2*a)+1)^2*exp(2*b*x+2*a))^(1/2)*exp
(4*b*x+4*a)+3/8/b/(exp(2*b*x+2*a)+1)/(1/(exp(2*b*x+2*a)+1)^2*exp(2*b*x+2*a)
)^(1/2)*exp(2*b*x+2*a)-3/8/b/(exp(2*b*x+2*a)+1)/(1/(exp(2*b*x+2*a)+1)^2*exp
(2*b*x+2*a))^(1/2)-1/24/b/(exp(2*b*x+2*a)+1)/(1/(exp(2*b*x+2*a)+1)^2*exp(2*
b*x+2*a))^(1/2)*exp(-2*b*x-2*a)
```

Maxima [A]
time = 0.32, size = 54, normalized size = 1.06

$$\frac{e^{(3\,bx+3\,a)}}{24\,b} + \frac{3\,e^{(bx+a)}}{8\,b} - \frac{3\,e^{(-bx-a)}}{8\,b} - \frac{e^{(-3\,bx-3\,a)}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/24*e^(3*b*x + 3*a)/b + 3/8*e^(b*x + a)/b - 3/8*e^(-b*x - a)/b - 1/24*e^(-
3*b*x - 3*a)/b
```

Fricas [A]
time = 0.36, size = 32, normalized size = 0.63

$$\frac{\sinh\left(bx+a\right)^{3}+3\left(\cosh\left(bx+a\right)^{2}+3\right)\sinh\left(bx+a\right)}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/12*(sinh(b*x + a)^3 + 3*(cosh(b*x + a)^2 + 3)*sinh(b*x + a))/b
```

Sympy [A]
time = 12.29, size = 54, normalized size = 1.06

$$\begin{cases} -\frac{2\tanh^{3}\left(a+bx\right)}{3b\left(\operatorname{sech}^{2}\left(a+bx\right)\right)^{\frac{3}{2}}}+\frac{\tanh\left(a+bx\right)}{b\left(\operatorname{sech}^{2}\left(a+bx\right)\right)^{\frac{3}{2}}} & \text{for}\: b\neq 0 \\ \frac{x}{\left(\operatorname{sech}^{2}\left(a\right)\right)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(b*x+a)**2)**(3/2),x)
```

```
[Out] Piecewise((-2*tanh(a + b*x)**3/(3*b*(sech(a + b*x)**2)**(3/2)) + tanh(a + b
*x)/(b*(sech(a + b*x)**2)**(3/2)), Ne(b, 0)), (x/(sech(a)**2)**(3/2), True)
)
```

Giac [A]
time = 0.37, size = 48, normalized size = 0.94

$$-\frac{\left(9\,e^{\left(2\,bx+2\,a\right)}+1\right)e^{\left(-3\,bx-3\,a\right)}-e^{\left(3\,bx+3\,a\right)}-9\,e^{\left(bx+a\right)}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(b*x+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*((9*e^(2*b*x + 2*a) + 1)*e^(-3*b*x - 3*a) - e^(3*b*x + 3*a) - 9*e^(b*
x + a))/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(\frac{1}{{\mathrm{cosh}\left(a+b\,x\right)}^2}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cosh(a + b*x)^2)^(3/2),x)
```

```
[Out] int(1/(1/cosh(a + b*x)^2)^(3/2), x)
```

3.30 $\int \frac{1}{\text{sech}^2(a+bx)^{5/2}} \, dx$

Optimal. Leaf size=76

$$\frac{\tanh(a+bx)}{5b\text{sech}^2(a+bx)^{5/2}}+\frac{4\tanh(a+bx)}{15b\text{sech}^2(a+bx)^{3/2}}+\frac{8\tanh(a+bx)}{15b\sqrt{\text{sech}^2(a+bx)}}$$

```
[Out] 1/5*tanh(b*x+a)/b/(sech(b*x+a)^2)^(5/2)+4/15*tanh(b*x+a)/b/(sech(b*x+a)^2)^
(3/2)+8/15*tanh(b*x+a)/b/(sech(b*x+a)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4207, 198, 197}

$$\frac{8\tanh(a+bx)}{15b\sqrt{\text{sech}^2(a+bx)}}+\frac{4\tanh(a+bx)}{15b\text{sech}^2(a+bx)^{3/2}}+\frac{\tanh(a+bx)}{5b\text{sech}^2(a+bx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[a + b*x]^2)^(-5/2),x]
```

```
[Out] Tanh[a + b*x]/(5*b*(Sech[a + b*x]^2)^(5/2)) + (4*Tanh[a + b*x])/(15*b*(Sech
[a + b*x]^2)^(3/2)) + (8*Tanh[a + b*x])/(15*b*Sqrt[Sech[a + b*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^{7/2}}\,dx, x, \tanh(a+bx)\right)}{b}$$
$$= \frac{\tanh(a+bx)}{5b\operatorname{sech}^2(a+bx)^{5/2}} + \frac{4\text{Subst}\left(\int \frac{1}{(1-x^2)^{5/2}}\,dx, x, \tanh(a+bx)\right)}{5b}$$
$$= \frac{\tanh(a+bx)}{5b\operatorname{sech}^2(a+bx)^{5/2}} + \frac{4\tanh(a+bx)}{15b\operatorname{sech}^2(a+bx)^{3/2}} + \frac{8\text{Subst}\left(\int \frac{1}{(1-x^2)^{3/2}}\,dx, x, \tanh(a+bx)\right)}{15b}$$
$$= \frac{\tanh(a+bx)}{5b\operatorname{sech}^2(a+bx)^{5/2}} + \frac{4\tanh(a+bx)}{15b\operatorname{sech}^2(a+bx)^{3/2}} + \frac{8\tanh(a+bx)}{15b\sqrt{\operatorname{sech}^2(a+bx)}}$$

Mathematica [**A**]
time = 0.07, size = 54, normalized size = 0.71

$$\frac{\operatorname{sech}(a+bx)(150\sinh(a+bx)+25\sinh(3(a+bx))+3\sinh(5(a+bx)))}{240b\sqrt{\operatorname{sech}^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[a + b*x]^2)^(-5/2),x]
```

```
[Out] (Sech[a + b*x]*(150*Sinh[a + b*x] + 25*Sinh[3*(a + b*x)] + 3*Sinh[5*(a + b*
x)]))/(240*b*Sqrt[Sech[a + b*x]^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(64) = 128.
time = 2.50, size = 305, normalized size = 4.01

method	result
risch	$\frac{e^{6bx+6a}}{160b\left(e^{2bx+2a}+1\right)\sqrt{\frac{e^{2bx+2a}}{\left(e^{2bx+2a}+1\right)^2}}} + \frac{5\,e^{4bx+4a}}{96b\left(e^{2bx+2a}+1\right)\sqrt{\frac{e^{2bx+2a}}{\left(e^{2bx+2a}+1\right)^2}}} + \frac{5\,e^{2bx+2a}}{16b\left(e^{2bx+2a}+1\right)\sqrt{\frac{e^{2bx+2a}}{\left(e^{2bx+2a}+1\right)^2}}} - \frac{}{16b(e^{2bx+2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sech(b*x+a)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/160/b/(exp(2*b*x+2*a)+1)/(1/(exp(2*b*x+2*a)+1)^2*exp(2*b*x+2*a))^(1/2)*ex
p(6*b*x+6*a)+5/96/b/(exp(2*b*x+2*a)+1)/(1/(exp(2*b*x+2*a)+1)^2*exp(2*b*x+2*
a))^(1/2)*exp(4*b*x+4*a)+5/16/b/(exp(2*b*x+2*a)+1)/(1/(exp(2*b*x+2*a)+1)^2*
exp(2*b*x+2*a))^(1/2)*exp(2*b*x+2*a)-5/16/b/(exp(2*b*x+2*a)+1)/(1/(exp(2*b*
x+2*a)+1)^2*exp(2*b*x+2*a))^(1/2)-5/96/b/(exp(2*b*x+2*a)+1)/(1/(exp(2*b*x+2
```

```
*a)+1)^2*exp(2*b*x+2*a))^(1/2)*exp(-2*b*x-2*a)-1/160/b/(exp(2*b*x+2*a)+1)/(
1/(exp(2*b*x+2*a)+1)^2*exp(2*b*x+2*a))^(1/2)*exp(-4*b*x-4*a)
```

Maxima [A]
time = 0.27, size = 82, normalized size = 1.08

$$\frac{e^{(5\,bx+5\,a)}}{160\,b}+\frac{5\,e^{(3\,bx+3\,a)}}{96\,b}+\frac{5\,e^{(bx+a)}}{16\,b}-\frac{5\,e^{(-bx-a)}}{16\,b}-\frac{5\,e^{(-3\,bx-3\,a)}}{96\,b}-\frac{e^{(-5\,bx-5\,a)}}{160\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(b*x+a)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/160*e^(5*b*x + 5*a)/b + 5/96*e^(3*b*x + 3*a)/b + 5/16*e^(b*x + a)/b - 5/1
6*e^(-b*x - a)/b - 5/96*e^(-3*b*x - 3*a)/b - 1/160*e^(-5*b*x - 5*a)/b
```

Fricas [A]
time = 0.38, size = 66, normalized size = 0.87

$$\frac{3\,\sinh\left(bx+a\right)^5+5\left(6\,\cosh\left(bx+a\right)^2+5\right)\sinh\left(bx+a\right)^3+15\left(\cosh\left(bx+a\right)^4+5\,\cosh\left(bx+a\right)^2+10\right)\sinh\left(bx+a\right)}{240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(b*x+a)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/240*(3*sinh(b*x + a)^5 + 5*(6*cosh(b*x + a)^2 + 5)*sinh(b*x + a)^3 + 15*(
cosh(b*x + a)^4 + 5*cosh(b*x + a)^2 + 10)*sinh(b*x + a))/b
```

Sympy [A]
time = 14.51, size = 80, normalized size = 1.05

$$\begin{cases} \frac{8\tanh^5\left(a+bx\right)}{15b\left(\operatorname{sech}^2\left(a+bx\right)\right)^{\frac{5}{2}}}-\frac{4\tanh^3\left(a+bx\right)}{3b\left(\operatorname{sech}^2\left(a+bx\right)\right)^{\frac{5}{2}}}+\frac{\tanh\left(a+bx\right)}{b\left(\operatorname{sech}^2\left(a+bx\right)\right)^{\frac{5}{2}}} & \text{for}\; b\neq 0\\ \frac{x}{\left(\operatorname{sech}^2\left(a\right)\right)^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(b*x+a)**2)**(5/2),x)
```

```
[Out] Piecewise((8*tanh(a + b*x)**5/(15*b*(sech(a + b*x)**2)**(5/2)) - 4*tanh(a +
 b*x)**3/(3*b*(sech(a + b*x)**2)**(5/2)) + tanh(a + b*x)/(b*(sech(a + b*x)*
*2)**(5/2)), Ne(b, 0)), (x/(sech(a)**2)**(5/2), True))
```

Giac [A]
time = 0.39, size = 70, normalized size = 0.92

$$-\frac{\left(150\,e^{(4\,bx+4\,a)}+25\,e^{(2\,bx+2\,a)}+3\right)e^{(-5\,bx-5\,a)}-3\,e^{(5\,bx+5\,a)}-25\,e^{(3\,bx+3\,a)}-150\,e^{(bx+a)}}{480\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(b*x+a)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/480*((150*e^(4*b*x + 4*a) + 25*e^(2*b*x + 2*a) + 3)*e^(-5*b*x - 5*a) - 3
*e^(5*b*x + 5*a) - 25*e^(3*b*x + 3*a) - 150*e^(b*x + a))/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{1}{\cosh(a+b\,x)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cosh(a + b*x)^2)^(5/2),x)
```

```
[Out] int(1/(1/cosh(a + b*x)^2)^(5/2), x)
```

3.31 $\int \frac{1}{\text{sech}^2(a+bx)^{7/2}} dx$

Optimal. Leaf size=101

$$\frac{\tanh(a+bx)}{7b\text{sech}^2(a+bx)^{7/2}}+\frac{6\tanh(a+bx)}{35b\text{sech}^2(a+bx)^{5/2}}+\frac{8\tanh(a+bx)}{35b\text{sech}^2(a+bx)^{3/2}}+\frac{16\tanh(a+bx)}{35b\sqrt{\text{sech}^2(a+bx)}}$$

```
[Out] 1/7*tanh(b*x+a)/b/(sech(b*x+a)^2)^(7/2)+6/35*tanh(b*x+a)/b/(sech(b*x+a)^2)^
(5/2)+8/35*tanh(b*x+a)/b/(sech(b*x+a)^2)^(3/2)+16/35*tanh(b*x+a)/b/(sech(b*
x+a)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4207, 198, 197}

$$\frac{16\tanh(a+bx)}{35b\sqrt{\text{sech}^2(a+bx)}}+\frac{8\tanh(a+bx)}{35b\text{sech}^2(a+bx)^{3/2}}+\frac{6\tanh(a+bx)}{35b\text{sech}^2(a+bx)^{5/2}}+\frac{\tanh(a+bx)}{7b\text{sech}^2(a+bx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[a + b*x]^2)^(-7/2),x]
```

```
[Out] Tanh[a + b*x]/(7*b*(Sech[a + b*x]^2)^(7/2)) + (6*Tanh[a + b*x])/(35*b*(Sech
[a + b*x]^2)^(5/2)) + (8*Tanh[a + b*x])/(35*b*(Sech[a + b*x]^2)^(3/2)) + (1
6*Tanh[a + b*x])/(35*b*Sqrt[Sech[a + b*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^{9/2}}\,dx, x, \tanh(a+bx)\right)}{b}$$

$$= \frac{\tanh(a+bx)}{7b\operatorname{sech}^2(a+bx)^{7/2}} + \frac{6\text{Subst}\left(\int \frac{1}{(1-x^2)^{7/2}}\,dx, x, \tanh(a+bx)\right)}{7b}$$

$$= \frac{\tanh(a+bx)}{7b\operatorname{sech}^2(a+bx)^{7/2}} + \frac{6\tanh(a+bx)}{35b\operatorname{sech}^2(a+bx)^{5/2}} + \frac{24\text{Subst}\left(\int \frac{1}{(1-x^2)^{5/2}}\,dx, x, \tanh(a+bx\right.}{35b}$$

$$= \frac{\tanh(a+bx)}{7b\operatorname{sech}^2(a+bx)^{7/2}} + \frac{6\tanh(a+bx)}{35b\operatorname{sech}^2(a+bx)^{5/2}} + \frac{8\tanh(a+bx)}{35b\operatorname{sech}^2(a+bx)^{3/2}} + \frac{16\text{Subst}\left(\int \frac{}{(1-x\right.}}{}$$

$$= \frac{\tanh(a+bx)}{7b\operatorname{sech}^2(a+bx)^{7/2}} + \frac{6\tanh(a+bx)}{35b\operatorname{sech}^2(a+bx)^{5/2}} + \frac{8\tanh(a+bx)}{35b\operatorname{sech}^2(a+bx)^{3/2}} + \frac{16\tanh(a+b}{35b\sqrt{\operatorname{sech}^2(a+}}$$

Mathematica [A]
time = 0.11, size = 64, normalized size = 0.63

$$\frac{\operatorname{sech}(a+bx)(1225\sinh(a+bx)+245\sinh(3(a+bx))+49\sinh(5(a+bx))+5\sinh(7(a+bx)))}{2240b\sqrt{\operatorname{sech}^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[a + b*x]^2)^(-7/2),x]
```

```
[Out] (Sech[a + b*x]*(1225*Sinh[a + b*x] + 245*Sinh[3*(a + b*x)] + 49*Sinh[5*(a +
 b*x)] + 5*Sinh[7*(a + b*x)]))/(2240*b*Sqrt[Sech[a + b*x]^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(85) = 170$.
time = 2.47, size = 409, normalized size = 4.05

method	result
risch	$\frac{e^{8bx+8a}}{896b(e^{2bx+2a}+1)\sqrt{\frac{e^{2bx+2a}}{(e^{2bx+2a}+1)^2}}} + \frac{7\,e^{6bx+6a}}{640b(e^{2bx+2a}+1)\sqrt{\frac{e^{2bx+2a}}{(e^{2bx+2a}+1)^2}}} + \frac{7\,e^{4bx+4a}}{128b(e^{2bx+2a}+1)\sqrt{\frac{e^{2bx+2a}}{(e^{2bx+2a}+1)^2}}} + \frac{}{128b(e^{2b}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sech(b*x+a)^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/896/b/(exp(2*b*x+2*a)+1)/(1/(exp(2*b*x+2*a)+1)^2*exp(2*b*x+2*a))^(1/2)*ex
p(8*b*x+8*a)+7/640/b/(exp(2*b*x+2*a)+1)/(1/(exp(2*b*x+2*a)+1)^2*exp(2*b*x+2
```

```
*a))^(1/2)*exp(6*b*x+6*a)+7/128/b/(exp(2*b*x+2*a)+1)/(1/(exp(2*b*x+2*a)+1)^
2*exp(2*b*x+2*a))^(1/2)*exp(4*b*x+4*a)+35/128/b/(exp(2*b*x+2*a)+1)/(1/(exp(
2*b*x+2*a)+1)^2*exp(2*b*x+2*a))^(1/2)*exp(2*b*x+2*a)-35/128/b/(exp(2*b*x+2*
a)+1)/(1/(exp(2*b*x+2*a)+1)^2*exp(2*b*x+2*a))^(1/2)-7/128/b/(exp(2*b*x+2*a)
+1)/(1/(exp(2*b*x+2*a)+1)^2*exp(2*b*x+2*a))^(1/2)*exp(-2*b*x-2*a)-7/640/b/(
exp(2*b*x+2*a)+1)/(1/(exp(2*b*x+2*a)+1)^2*exp(2*b*x+2*a))^(1/2)*exp(-4*b*x-
4*a)-1/896/b/(exp(2*b*x+2*a)+1)/(1/(exp(2*b*x+2*a)+1)^2*exp(2*b*x+2*a))^(1/
2)*exp(-6*b*x-6*a)
```

Maxima [A]
time = 0.27, size = 100, normalized size = 0.99

$$\frac{\left(49\,e^{(-2\,bx-2\,a)}+245\,e^{(-4\,bx-4\,a)}+1225\,e^{(-6\,bx-6\,a)}+5\right)e^{(7\,bx+7\,a)}}{4480\,b}-\frac{1225\,e^{(-bx-a)}+245\,e^{(-3\,bx-3\,a)}+49\,e^{(-5\,bx-5\,a)}+5\,e^{(-7\,bx-7\,a)}}{4480\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(b*x+a)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/4480*(49*e^(-2*b*x - 2*a) + 245*e^(-4*b*x - 4*a) + 1225*e^(-6*b*x - 6*a)
+ 5)*e^(7*b*x + 7*a)/b - 1/4480*(1225*e^(-b*x - a) + 245*e^(-3*b*x - 3*a) +
 49*e^(-5*b*x - 5*a) + 5*e^(-7*b*x - 7*a))/b
```

Fricas [A]
time = 0.36, size = 108, normalized size = 1.07

$$\frac{5\,\sinh\left(bx+a\right)^{7}+7\,\left(15\,\cosh\left(bx+a\right)^{2}+7\right)\sinh\left(bx+a\right)^{5}+35\,\left(5\,\cosh\left(bx+a\right)^{4}+14\,\cosh\left(bx+a\right)^{2}+7\right)\sinh\left(bx+a\right)^{3}+35\,\left(\cosh\left(bx+a\right)^{6}+7\,\cosh\left(bx+a\right)^{4}+21\,\cosh\left(bx+a\right)^{2}+35\right)\sinh\left(bx+a\right)}{2240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(b*x+a)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/2240*(5*sinh(b*x + a)^7 + 7*(15*cosh(b*x + a)^2 + 7)*sinh(b*x + a)^5 + 35
*(5*cosh(b*x + a)^4 + 14*cosh(b*x + a)^2 + 7)*sinh(b*x + a)^3 + 35*(cosh(b*
x + a)^6 + 7*cosh(b*x + a)^4 + 21*cosh(b*x + a)^2 + 35)*sinh(b*x + a))/b
```

Sympy [A]
time = 35.12, size = 104, normalized size = 1.03

$$\begin{cases} -\frac{16\tanh^{7}\left(a+bx\right)}{35b\left(\operatorname{sech}^{2}\left(a+bx\right)\right)^{\frac{7}{2}}}+\frac{8\tanh^{5}\left(a+bx\right)}{5b\left(\operatorname{sech}^{2}\left(a+bx\right)\right)^{\frac{7}{2}}}-\frac{2\tanh^{3}\left(a+bx\right)}{b\left(\operatorname{sech}^{2}\left(a+bx\right)\right)^{\frac{7}{2}}}+\frac{\tanh\left(a+bx\right)}{b\left(\operatorname{sech}^{2}\left(a+bx\right)\right)^{\frac{7}{2}}} & \text{for}\ b\neq 0 \\ \frac{x}{\left(\operatorname{sech}^{2}\left(a\right)\right)^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(b*x+a)**2)**(7/2),x)
```

```
[Out] Piecewise((-16*tanh(a + b*x)**7/(35*b*(sech(a + b*x)**2)**(7/2)) + 8*tanh(a
 + b*x)**5/(5*b*(sech(a + b*x)**2)**(7/2)) - 2*tanh(a + b*x)**3/(b*(sech(a
```

```
+ b*x)**2)**(7/2)) + tanh(a + b*x)/(b*(sech(a + b*x)**2)**(7/2)), Ne(b, 0))
, (x/(sech(a)**2)**(7/2), True))
```

Giac [A]
time = 0.39, size = 92, normalized size = 0.91

$$-\frac{\left(1225\,e^{(6\,bx+6\,a)}+245\,e^{(4\,bx+4\,a)}+49\,e^{(2\,bx+2\,a)}+5\right)e^{(-7\,bx-7\,a)}-5\,e^{(7\,bx+7\,a)}-49\,e^{(5\,bx+5\,a)}-245\,e^{(3\,bx+3\,a)}-1225\,e^{(bx+a)}}{4480\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(b*x+a)^2)^(7/2),x, algorithm="giac")
[Out] -1/4480*((1225*e^(6*b*x + 6*a) + 245*e^(4*b*x + 4*a) + 49*e^(2*b*x + 2*a) +
 5)*e^(-7*b*x - 7*a) - 5*e^(7*b*x + 7*a) - 49*e^(5*b*x + 5*a) - 245*e^(3*b*
x + 3*a) - 1225*e^(b*x + a))/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(\frac{1}{{\cosh\left(a+b\,x\right)}^2}\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cosh(a + b*x)^2)^(7/2),x)
[Out] int(1/(1/cosh(a + b*x)^2)^(7/2), x)
```

3.32 $\int \left(a\text{sech}^2(x)\right)^{5/2}\,dx$

Optimal. Leaf size=65

$$\frac{3}{8}a^{5/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a\text{sech}^2(x)}}\right)+\frac{3}{8}a^2\sqrt{a\text{sech}^2(x)}\,\tanh(x)+\frac{1}{4}a\left(a\text{sech}^2(x)\right)^{3/2}\tanh(x)$$

```
[Out] 3/8*a^(5/2)*arctan(a^(1/2)*tanh(x)/(a*sech(x)^2)^(1/2))+1/4*a*(a*sech(x)^2)
^(3/2)*tanh(x)+3/8*a^2*(a*sech(x)^2)^(1/2)*tanh(x)
```

Rubi [A]
time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4207, 201, 223, 209}

$$\frac{3}{8}a^{5/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a\text{sech}^2(x)}}\right)+\frac{3}{8}a^2\tanh(x)\sqrt{a\text{sech}^2(x)}+\frac{1}{4}a\tanh(x)\left(a\text{sech}^2(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x]^2)^(5/2),x]
```

```
[Out] (3*a^(5/2)*ArcTan[(Sqrt[a]*Tanh[x])/Sqrt[a*Sech[x]^2]])/8 + (3*a^2*Sqrt[a*S
ech[x]^2]*Tanh[x])/8 + (a*(a*Sech[x]^2)^(3/2)*Tanh[x])/4
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(a\text{sech}^2(x)\right)^{5/2} dx &= a\text{Subst}\left(\int \left(a-ax^2\right)^{3/2} dx, x, \tanh(x)\right)\\ &= \frac{1}{4}a\left(a\text{sech}^2(x)\right)^{3/2}\tanh(x) + \frac{1}{4}\left(3a^2\right)\text{Subst}\left(\int \sqrt{a-ax^2}\, dx, x, \tanh(x)\right)\\ &= \frac{3}{8}a^2\sqrt{a\text{sech}^2(x)}\tanh(x) + \frac{1}{4}a\left(a\text{sech}^2(x)\right)^{3/2}\tanh(x) + \frac{1}{8}\left(3a^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-ax^2}}\right.\\ &= \frac{3}{8}a^2\sqrt{a\text{sech}^2(x)}\tanh(x) + \frac{1}{4}a\left(a\text{sech}^2(x)\right)^{3/2}\tanh(x) + \frac{1}{8}\left(3a^3\right)\text{Subst}\left(\int \frac{1}{1+ax^2} d\right.\\ &= \frac{3}{8}a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a\text{sech}^2(x)}}\right) + \frac{3}{8}a^2\sqrt{a\text{sech}^2(x)}\tanh(x) + \frac{1}{4}a\left(a\text{sech}^2(x)\right)^{3/2}\tanh(x\end{aligned}$$

Mathematica [A]
time = 0.03, size = 42, normalized size = 0.65

$$\frac{1}{8}\cosh(x)\left(a\text{sech}^2(x)\right)^{5/2}\left(6\text{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)\cosh^4(x) + 2\sinh(x) + 3\cosh^2(x)\sinh(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x]^2)^(5/2),x]
```

```
[Out] (Cosh[x]*(a*Sech[x]^2)^(5/2)*(6*ArcTan[Tanh[x/2]]*Cosh[x]^4 + 2*Sinh[x] + 3
*Cosh[x]^2*Sinh[x]))/8
```

Maple [C] Result contains complex when optimal does not.
time = 0.99, size = 127, normalized size = 1.95

method	result
risch	$\frac{a^2\sqrt{\frac{a\,e^{2x}}{(1+e^{2x})^2}}\,(3\,e^{6x}+11\,e^{4x}-11\,e^{2x}-3)}{4(1+e^{2x})^3} + \frac{3ia^2e^{-x}(1+e^{2x})\sqrt{\frac{a\,e^{2x}}{(1+e^{2x})^2}}\,\ln(e^x+i)}{8} - \frac{3ia^2e^{-x}(1+e^{2x})\sqrt{\frac{a\,e^{2x}}{(1+e^{2x})^2}}\,\ln(e^x-i}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sech(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^2/(1+exp(2*x))^3*(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)*(3*exp(6*x)+11*exp
(4*x)-11*exp(2*x)-3)+3/8*I*a^2*exp(-x)*(1+exp(2*x))*(a*exp(2*x)/(1+exp(2*x)
)^2)^(1/2)*ln(exp(x)+I)-3/8*I*a^2*exp(-x)*(1+exp(2*x))*(a*exp(2*x)/(1+exp(2
*x))^2)^(1/2)*ln(exp(x)-I)
```

Maxima [A]
time = 0.52, size = 72, normalized size = 1.11

$$\frac{3}{4} a^{\frac{5}{2}} \arctan\left(e^{x}\right) + \frac{3 a^{\frac{5}{2}} e^{(7 x)} + 11 a^{\frac{5}{2}} e^{(5 x)} - 11 a^{\frac{5}{2}} e^{(3 x)} - 3 a^{\frac{5}{2}} e^{x}}{4 \left(e^{(8 x)} + 4 e^{(6 x)} + 6 e^{(4 x)} + 4 e^{(2 x)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 3/4*a^(5/2)*arctan(e^x) + 1/4*(3*a^(5/2)*e^(7*x) + 11*a^(5/2)*e^(5*x) - 11*
a^(5/2)*e^(3*x) - 3*a^(5/2)*e^x)/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*
x) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1082 vs. $2(49) = 98$.
time = 0.40, size = 1082, normalized size = 16.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(3*a^2*cosh(x)^7 + 3*(a^2*e^(2*x) + a^2)*sinh(x)^7 + 11*a^2*cosh(x)^5 +
 21*(a^2*cosh(x)*e^(2*x) + a^2*cosh(x))*sinh(x)^6 + (63*a^2*cosh(x)^2 + 11*
a^2 + (63*a^2*cosh(x)^2 + 11*a^2)*e^(2*x))*sinh(x)^5 - 11*a^2*cosh(x)^3 + 5
*(21*a^2*cosh(x)^3 + 11*a^2*cosh(x) + (21*a^2*cosh(x)^3 + 11*a^2*cosh(x))*e
^(2*x))*sinh(x)^4 + (105*a^2*cosh(x)^4 + 110*a^2*cosh(x)^2 - 11*a^2 + (105*
a^2*cosh(x)^4 + 110*a^2*cosh(x)^2 - 11*a^2)*e^(2*x))*sinh(x)^3 - 3*a^2*cosh
(x) + (63*a^2*cosh(x)^5 + 110*a^2*cosh(x)^3 - 33*a^2*cosh(x) + (63*a^2*cosh
(x)^5 + 110*a^2*cosh(x)^3 - 33*a^2*cosh(x))*e^(2*x))*sinh(x)^2 + 3*(a^2*cos
h(x)^8 + (a^2*e^(2*x) + a^2)*sinh(x)^8 + 4*a^2*cosh(x)^6 + 8*(a^2*cosh(x)*e
^(2*x) + a^2*cosh(x))*sinh(x)^7 + 4*(7*a^2*cosh(x)^2 + a^2 + (7*a^2*cosh(x)
^2 + a^2)*e^(2*x))*sinh(x)^6 + 6*a^2*cosh(x)^4 + 8*(7*a^2*cosh(x)^3 + 3*a^2
*cosh(x) + (7*a^2*cosh(x)^3 + 3*a^2*cosh(x))*e^(2*x))*sinh(x)^5 + 2*(35*a^2
*cosh(x)^4 + 30*a^2*cosh(x)^2 + 3*a^2 + (35*a^2*cosh(x)^4 + 30*a^2*cosh(x)^
2 + 3*a^2)*e^(2*x))*sinh(x)^4 + 4*a^2*cosh(x)^2 + 8*(7*a^2*cosh(x)^5 + 10*a
^2*cosh(x)^3 + 3*a^2*cosh(x) + (7*a^2*cosh(x)^5 + 10*a^2*cosh(x)^3 + 3*a^2*
cosh(x))*e^(2*x))*sinh(x)^3 + 4*(7*a^2*cosh(x)^6 + 15*a^2*cosh(x)^4 + 9*a^2
*cosh(x)^2 + a^2 + (7*a^2*cosh(x)^6 + 15*a^2*cosh(x)^4 + 9*a^2*cosh(x)^2 +
a^2)*e^(2*x))*sinh(x)^2 + a^2 + (a^2*cosh(x)^8 + 4*a^2*cosh(x)^6 + 6*a^2*co
sh(x)^4 + 4*a^2*cosh(x)^2 + a^2)*e^(2*x) + 8*(a^2*cosh(x)^7 + 3*a^2*cosh(x)
```

```
^5 + 3*a^2*cosh(x)^3 + a^2*cosh(x) + (a^2*cosh(x)^7 + 3*a^2*cosh(x)^5 + 3*a
^2*cosh(x)^3 + a^2*cosh(x))*e^(2*x))*sinh(x))*arctan(cosh(x) + sinh(x)) + (
3*a^2*cosh(x)^7 + 11*a^2*cosh(x)^5 - 11*a^2*cosh(x)^3 - 3*a^2*cosh(x))*e^(2
*x) + (21*a^2*cosh(x)^6 + 55*a^2*cosh(x)^4 - 33*a^2*cosh(x)^2 - 3*a^2 + (21
*a^2*cosh(x)^6 + 55*a^2*cosh(x)^4 - 33*a^2*cosh(x)^2 - 3*a^2)*e^(2*x))*sinh
(x))*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*e^x/(8*cosh(x)*e^x*sinh(x)^7 + e^x*s
inh(x)^8 + 4*(7*cosh(x)^2 + 1)*e^x*sinh(x)^6 + 8*(7*cosh(x)^3 + 3*cosh(x))*
e^x*sinh(x)^5 + 2*(35*cosh(x)^4 + 30*cosh(x)^2 + 3)*e^x*sinh(x)^4 + 8*(7*co
sh(x)^5 + 10*cosh(x)^3 + 3*cosh(x))*e^x*sinh(x)^3 + 4*(7*cosh(x)^6 + 15*cos
h(x)^4 + 9*cosh(x)^2 + 1)*e^x*sinh(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^5 + 3*co
sh(x)^3 + cosh(x))*e^x*sinh(x) + (cosh(x)^8 + 4*cosh(x)^6 + 6*cosh(x)^4 + 4
*cosh(x)^2 + 1)*e^x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\,\mathrm{sech}^2\left(x\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)**2)**(5/2),x)
```

```
[Out] Integral((a*sech(x)**2)**(5/2), x)
```

Giac [**A**]
time = 0.40, size = 65, normalized size = 1.00

$$\frac{1}{16}\left(3\,\pi - \frac{4\left(3\left(e^{(-x)} - e^{x}\right)^{3} + 20\,e^{(-x)} - 20\,e^{x}\right)}{\left(\left(e^{(-x)} - e^{x}\right)^{2} + 4\right)^{2}} + 6\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)} - 1\right)e^{(-x)}\right)\right)a^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/16*(3*pi - 4*(3*(e^(-x) - e^x)^3 + 20*e^(-x) - 20*e^x)/((e^(-x) - e^x)^2
+ 4)^2 + 6*arctan(1/2*(e^(2*x) - 1)*e^(-x)))*a^(5/2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(\frac{a}{{\mathrm{cosh}\left(x\right)}^2}\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/cosh(x)^2)^(5/2),x)
```

```
[Out] int((a/cosh(x)^2)^(5/2), x)
```

3.33 $\int \left(a\,\text{sech}^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=46

$$\frac{1}{2}a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a\text{sech}^2(x)}}\right)+\frac{1}{2}a\sqrt{a\text{sech}^2(x)}\,\tanh(x)$$

```
[Out] 1/2*a^(3/2)*arctan(a^(1/2)*tanh(x)/(a*sech(x)^2)^(1/2))+1/2*a*(a*sech(x)^2)
^(1/2)*tanh(x)
```

Rubi [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4207, 201, 223, 209}

$$\frac{1}{2}a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a\text{sech}^2(x)}}\right)+\frac{1}{2}a\tanh(x)\sqrt{a\text{sech}^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x]^2)^(3/2),x]
```

```
[Out] (a^(3/2)*ArcTan[(Sqrt[a]*Tanh[x])/Sqrt[a*Sech[x]^2]])/2 + (a*Sqrt[a*Sech[x]
^2]*Tanh[x])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(a\text{sech}^2(x)\right)^{3/2}\,dx &= a\text{Subst}\left(\int \sqrt{a-ax^2}\,dx,x,\tanh(x)\right)\\ &= \frac{1}{2}a\sqrt{a\text{sech}^2(x)}\,\tanh(x)+\frac{1}{2}a^2\text{Subst}\left(\int \frac{1}{\sqrt{a-ax^2}}\,dx,x,\tanh(x)\right)\\ &= \frac{1}{2}a\sqrt{a\text{sech}^2(x)}\,\tanh(x)+\frac{1}{2}a^2\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx,x,\frac{\tanh(x)}{\sqrt{a\text{sech}^2(x)}}\right)\\ &= \frac{1}{2}a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a\text{sech}^2(x)}}\right)+\frac{1}{2}a\sqrt{a\text{sech}^2(x)}\,\tanh(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 0.63

$$\frac{1}{2}a\sqrt{a\text{sech}^2(x)}\left(2\text{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)\cosh(x)+\tanh(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x]^2)^(3/2),x]
```

```
[Out] (a*Sqrt[a*Sech[x]^2]*(2*ArcTan[Tanh[x/2]]*Cosh[x] + Tanh[x]))/2
```

Maple [C] Result contains complex when optimal does not.
time = 0.85, size = 106, normalized size = 2.30

method	result	size
risch	$\frac{a\sqrt{\frac{a\,e^{2x}}{(1+e^{2x})^2}}\,(e^{2x}-1)}{1+e^{2x}}+\frac{ia\,e^{-x}(1+e^{2x})\sqrt{\frac{a\,e^{2x}}{(1+e^{2x})^2}}\,\ln(e^x+i)}{2}-\frac{ia\,e^{-x}(1+e^{2x})\sqrt{\frac{a\,e^{2x}}{(1+e^{2x})^2}}\,\ln(e^x-i)}{2}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sech(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a/(1+exp(2*x))*(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)*(exp(2*x)-1)+1/2*I*a*exp(-
x)*(1+exp(2*x))*(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)*ln(exp(x)+I)-1/2*I*a*exp(
-x)*(1+exp(2*x))*(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)*ln(exp(x)-I)
```

Maxima [A]
time = 0.51, size = 39, normalized size = 0.85

$$a^{\frac{3}{2}} \arctan\left(e^{x}\right) + \frac{a^{\frac{3}{2}} e^{(3\,x)} - a^{\frac{3}{2}} e^{x}}{e^{(4\,x)} + 2\,e^{(2\,x)} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] a^(3/2)*arctan(e^x) + (a^(3/2)*e^(3*x) - a^(3/2)*e^x)/(e^(4*x) + 2*e^(2*x)
+ 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(34) = 68$.
time = 0.38, size = 310, normalized size = 6.74

$$\frac{\left(a\cosh\left(x\right)^{3} + \left(ae^{(2\,x)} + a\right)\sinh\left(x\right)^{3} + 3\,\left(a\cosh\left(x\right)e^{(2\,x)} + a\cosh\left(x\right)\right)\sinh\left(x\right)^{2} + \left(a\cosh\left(x\right)^{4} + \left(ae^{(2\,x)} + a\right)\sinh\left(x\right)^{4} + 4\,\left(a\cosh\left(x\right)e^{(2\,x)} + a\cosh\left(x\right)\right)\sinh\left(x\right)^{3} + 2\,a\cosh\left(x\right)^{2} + 2\,\left(3\,a\cosh\left(x\right)^{2} + \left(3\,a\cosh\left(x\right)^{2} + a\right)e^{(2\,x)} + a\right)\sinh\left(x\right)^{2} + \left(a\cosh\left(x\right)^{4} + 2\,a\cosh\left(x\right)^{2} + a\right)e^{(2\,x)} + 4\,\left(a\cosh\left(x\right)^{3} + a\cosh\left(x\right) + \left(a\cosh\left(x\right)^{3} + a\cosh\left(x\right)\right)e^{(2\,x)}\right)\sinh\left(x\right) + a\right)\arctan\left(\cosh\left(x\right) + \sinh\left(x\right)\right) - a\cosh\left(x\right) + \left(a\cosh\left(x\right)^{3} - a\cosh\left(x\right)\right)e^{(2\,x)} + \left(3\,a\cosh\left(x\right)^{2} + \left(3\,a\cosh\left(x\right)^{2} - a\right)e^{(2\,x)} - a\right)\sinh\left(x\right)\right)\sqrt{\frac{a}{e^{(4\,x)} + 2\,e^{(2\,x)} + 1}}\,e^{x}}{4\,\cosh\left(x\right)e^{x}\sinh\left(x\right)^{3} + e^{x}\sinh\left(x\right)^{4} + 2\,\left(3\,\cosh\left(x\right)^{2} + 1\right)e^{x}\sinh\left(x\right)^{2} + 4\,\left(\cosh\left(x\right)^{3} + \cosh\left(x\right)\right)e^{x}\sinh\left(x\right) + \left(\cosh\left(x\right)^{4} + 2\,\cosh\left(x\right)^{2} + 1\right)e^{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (a*cosh(x)^3 + (a*e^(2*x) + a)*sinh(x)^3 + 3*(a*cosh(x)*e^(2*x) + a*cosh(x)
)*sinh(x)^2 + (a*cosh(x)^4 + (a*e^(2*x) + a)*sinh(x)^4 + 4*(a*cosh(x)*e^(2*
x) + a*cosh(x))*sinh(x)^3 + 2*a*cosh(x)^2 + 2*(3*a*cosh(x)^2 + (3*a*cosh(x)
^2 + a)*e^(2*x) + a)*sinh(x)^2 + (a*cosh(x)^4 + 2*a*cosh(x)^2 + a)*e^(2*x)
+ 4*(a*cosh(x)^3 + a*cosh(x) + (a*cosh(x)^3 + a*cosh(x))*e^(2*x))*sinh(x) +
 a)*arctan(cosh(x) + sinh(x)) - a*cosh(x) + (a*cosh(x)^3 - a*cosh(x))*e^(2*
x) + (3*a*cosh(x)^2 + (3*a*cosh(x)^2 - a)*e^(2*x) - a)*sinh(x))*sqrt(a/(e^(
4*x) + 2*e^(2*x) + 1))*e^x/(4*cosh(x)*e^x*sinh(x)^3 + e^x*sinh(x)^4 + 2*(3*
cosh(x)^2 + 1)*e^x*sinh(x)^2 + 4*(cosh(x)^3 + cosh(x))*e^x*sinh(x) + (cosh(
x)^4 + 2*cosh(x)^2 + 1)*e^x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \operatorname{sech}^{2}\left(x\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)**2)**(3/2),x)
```

```
[Out] Integral((a*sech(x)**2)**(3/2), x)
```

Giac [A]
time = 0.39, size = 48, normalized size = 1.04

$$\frac{1}{4} \left(\pi - \frac{4\left(e^{(-x)} - e^{x}\right)}{\left(e^{(-x)} - e^{x}\right)^{2} + 4} + 2 \arctan\left(\frac{1}{2} \left(e^{(2\,x)} - 1\right) e^{(-x)}\right)\right) a^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(pi - 4*(e^(-x) - e^x)/((e^(-x) - e^x)^2 + 4) + 2*arctan(1/2*(e^(2*x) -
 1)*e^(-x)))*a^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \left(\frac{a}{\cosh(x)^2}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/cosh(x)^2)^(3/2),x)
```

```
[Out] int((a/cosh(x)^2)^(3/2), x)
```

3.34 $\int \sqrt{a\operatorname{sech}^2(x)}\,dx$

Optimal. Leaf size=25

$$\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a\operatorname{sech}^2(x)}}\right)$$

```
[Out] arctan(a^(1/2)*tanh(x)/(a*sech(x)^2)^(1/2))*a^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4207, 223, 209}

$$\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a\operatorname{sech}^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Sech[x]^2],x]
```

```
[Out] Sqrt[a]*ArcTan[(Sqrt[a]*Tanh[x])/Sqrt[a*Sech[x]^2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sqrt{a\text{sech}^2(x)}\,dx = a\text{Subst}\left(\int \frac{1}{\sqrt{a-ax^2}}\,dx, x, \tanh(x)\right)$$
$$= a\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a\text{sech}^2(x)}}\right)$$
$$= \sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a\text{sech}^2(x)}}\right)$$

Mathematica [A]
time = 0.01, size = 21, normalized size = 0.84

$$2\text{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)\cosh(x)\sqrt{a\text{sech}^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Sech[x]^2],x]
[Out] 2*ArcTan[Tanh[x/2]]*Cosh[x]*Sqrt[a*Sech[x]^2]
```

Maple [C] Result contains complex when optimal does not.
time = 1.02, size = 72, normalized size = 2.88

method	result	size
risch	$i\sqrt{\frac{a\,e^{2x}}{(1+e^{2x})^2}}\,e^{-x}(1+e^{2x})\ln\left(e^x+i\right)-i\sqrt{\frac{a\,e^{2x}}{(1+e^{2x})^2}}\,e^{-x}(1+e^{2x})\ln\left(e^x-i\right)$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sech(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] I*(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)*exp(-x)*(1+exp(2*x))*ln(exp(x)+I)-I*(a*
exp(2*x)/(1+exp(2*x))^2)^(1/2)*exp(-x)*(1+exp(2*x))*ln(exp(x)-I)
```

Maxima [A]
time = 0.57, size = 8, normalized size = 0.32

$$2\sqrt{a}\,\arctan\left(e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^2)^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(a)*arctan(e^x)
```

Fricas [A]
time = 0.39, size = 145, normalized size = 5.80

$$\left[\sqrt{-a}\log\left(\frac{2\,a\cosh\left(x\right)e^{x}\sinh\left(x\right)+ae^{x}\sinh\left(x\right)^{2}+2\,\left(\cosh\left(x\right)e^{\left(2\,x\right)}+\left(e^{\left(2\,x\right)}+1\right)\sinh\left(x\right)+\cosh\left(x\right)\right)\sqrt{-a}\sqrt{\frac{a}{e^{\left(4\,x\right)}+2\,e^{\left(2\,x\right)}+1}}e^{x}+\left(a\cosh\left(x\right)^{2}-a\right)e^{x}}{2\,\cosh\left(x\right)e^{x}\sinh\left(x\right)+e^{x}\sinh\left(x\right)^{2}+\left(\cosh\left(x\right)^{2}+1\right)e^{x}}\right),2\,\sqrt{\frac{a}{e^{\left(4\,x\right)}+2\,e^{\left(2\,x\right)}+1}}\left(e^{\left(2\,x\right)}+1\right)\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(-a)*log((2*a*cosh(x)*e^x*sinh(x) + a*e^x*sinh(x)^2 + 2*(cosh(x)*e^(2*
x) + (e^(2*x) + 1)*sinh(x) + cosh(x))*sqrt(-a)*sqrt(a/(e^(4*x) + 2*e^(2*x)
+ 1))*e^x + (a*cosh(x)^2 - a)*e^x)/(2*cosh(x)*e^x*sinh(x) + e^x*sinh(x)^2 +
 (cosh(x)^2 + 1)*e^x)), 2*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*(e^(2*x) + 1)*a
rctan(cosh(x) + sinh(x))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\,\mathrm{sech}^{2}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a*sech(x)**2), x)
```

Giac [A]
time = 0.39, size = 8, normalized size = 0.32

$$2\,\sqrt{a}\arctan\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(a)*arctan(e^x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int\sqrt{\frac{a}{{\cosh\left(x\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/cosh(x)^2)^(1/2),x)
```

```
[Out] int((a/cosh(x)^2)^(1/2), x)
```

3.35 $\int \frac{1}{\sqrt{a\text{sech}^2(x)}}\,dx$

Optimal. Leaf size=13

$$\frac{\tanh(x)}{\sqrt{a\text{sech}^2(x)}}$$

```
[Out] tanh(x)/(a*sech(x)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4207, 197}

$$\frac{\tanh(x)}{\sqrt{a\text{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Sech[x]^2],x]
```

```
[Out] Tanh[x]/Sqrt[a*Sech[x]^2]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a\text{sech}^2(x)}}\,dx &= a\text{Subst}\left(\int \frac{1}{(a-ax^2)^{3/2}}\,dx, x, \tanh(x)\right)\\ &= \frac{\tanh(x)}{\sqrt{a\text{sech}^2(x)}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$\frac{\tanh(x)}{\sqrt{a\text{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Sech[x]^2],x]
```

```
[Out] Tanh[x]/Sqrt[a*Sech[x]^2]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(11) = 22$.
time = 1.04, size = 58, normalized size = 4.46

method	result	size
risch	$\frac{e^{2x}}{2\sqrt{\frac{a\,e^{2x}}{(1+e^{2x})^2}}\,(1+e^{2x})} - \frac{1}{2(1+e^{2x})\sqrt{\frac{a\,e^{2x}}{(1+e^{2x})^2}}}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sech(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(2*x)-1/2/(1+exp(2*x)
)/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)
```

Maxima [A]
time = 0.47, size = 17, normalized size = 1.31

$$-\frac{e^{(-x)}}{2\sqrt{a}} + \frac{e^x}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*e^(-x)/sqrt(a) + 1/2*e^x/sqrt(a)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(11) = 22$.
time = 0.40, size = 79, normalized size = 6.08

$$\frac{\left(\left(e^{(2x)}+1\right)\sinh(x)^2 + \cosh(x)^2 + \left(\cosh(x)^2-1\right)e^{(2x)} + 2\left(\cosh(x)e^{(2x)} + \cosh(x)\right)\sinh(x) - 1\right)\sqrt{\frac{a}{e^{(4x)}+2e^{(2x)}+1}}\,e^x}{2\left(a\cosh(x)e^x + ae^x\sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*((e^(2*x) + 1)*sinh(x)^2 + cosh(x)^2 + (cosh(x)^2 - 1)*e^(2*x) + 2*(cos
h(x)*e^(2*x) + cosh(x))*sinh(x) - 1)*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*e^x/
(a*cosh(x)*e^x + a*e^x*sinh(x))
```

Sympy [A]
time = 0.24, size = 12, normalized size = 0.92

$$\frac{\tanh{(x)}}{\sqrt{a\operatorname{sech}^2{(x)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)**2)**(1/2),x)
```

```
[Out] tanh(x)/sqrt(a*sech(x)**2)
```

Giac [A]
time = 0.39, size = 14, normalized size = 1.08

$$-\frac{e^{(-x)} - e^{x}}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(e^(-x) - e^x)/sqrt(a)
```

Mupad [B]
time = 0.12, size = 33, normalized size = 2.54

$$-\frac{\left(\frac{\mathrm{e}^{-2\,x}}{2}-\frac{\mathrm{e}^{2\,x}}{2}\right)\,\sqrt{\frac{1}{{\left(\frac{\mathrm{e}^{-x}}{2}+\frac{\mathrm{e}^{x}}{2}\right)}^2}}}{2\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/cosh(x)^2)^(1/2),x)
```

```
[Out] -((exp(-2*x)/2 - exp(2*x)/2)*(1/(exp(-x)/2 + exp(x)/2)^2)^(1/2))/(2*a^(1/2)
)
```

3.36 $\int \frac{1}{\left(a\,\mathrm{sech}^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=36

$$\frac{\tanh(x)}{3\left(a\mathrm{sech}^2(x)\right)^{3/2}}+\frac{2\tanh(x)}{3a\sqrt{a\mathrm{sech}^2(x)}}$$

```
[Out] 1/3*tanh(x)/(a*sech(x)^2)^(3/2)+2/3*tanh(x)/a/(a*sech(x)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4207, 198, 197}

$$\frac{2\tanh(x)}{3a\sqrt{a\mathrm{sech}^2(x)}}+\frac{\tanh(x)}{3\left(a\mathrm{sech}^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x]^2)^(-3/2),x]
```

```
[Out] Tanh[x]/(3*(a*Sech[x]^2)^(3/2)) + (2*Tanh[x])/(3*a*Sqrt[a*Sech[x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a\operatorname{sech}^2(x)\right)^{3/2}}\,dx = a\text{Subst}\left(\int \frac{1}{(a-ax^2)^{5/2}}\,dx, x, \tanh(x)\right)$$
$$= \frac{\tanh(x)}{3\left(a\operatorname{sech}^2(x)\right)^{3/2}} + \frac{2}{3}\text{Subst}\left(\int \frac{1}{(a-ax^2)^{3/2}}\,dx, x, \tanh(x)\right)$$
$$= \frac{\tanh(x)}{3\left(a\operatorname{sech}^2(x)\right)^{3/2}} + \frac{2\tanh(x)}{3a\sqrt{a\operatorname{sech}^2(x)}}$$

Mathematica [A]
time = 0.02, size = 27, normalized size = 0.75

$$\frac{\operatorname{sech}^3(x)(9\sinh(x)+\sinh(3x))}{12\left(a\operatorname{sech}^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x]^2)^(-3/2),x]
```

```
[Out] (Sech[x]^3*(9*Sinh[x] + Sinh[3*x]))/(12*(a*Sech[x]^2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(28) = 56.
time = 0.86, size = 130, normalized size = 3.61

method	result	size
risch	$\frac{e^{4x}}{24a(1+e^{2x})\sqrt{\frac{a\,e^{2x}}{(1+e^{2x})^2}}}+\frac{3\,e^{2x}}{8a(1+e^{2x})\sqrt{\frac{a\,e^{2x}}{(1+e^{2x})^2}}}-\frac{3}{8\sqrt{\frac{a\,e^{2x}}{(1+e^{2x})^2}}\,(1+e^{2x})a}-\frac{e^{-2x}}{24a(1+e^{2x})\sqrt{\frac{a\,e^{2x}}{(1+e^{2x})^2}}}$	130

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sech(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/a*exp(4*x)/(1+exp(2*x))/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)+3/8/a*exp(2*
x)/(1+exp(2*x))/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)-3/8/(a*exp(2*x)/(1+exp(2*
x))^2)^(1/2)/(1+exp(2*x))/a-1/24/a*exp(-2*x)/(1+exp(2*x))/(a*exp(2*x)/(1+ex
p(2*x))^2)^(1/2)
```

Maxima [A]
time = 0.47, size = 35, normalized size = 0.97

$$\frac{e^{(3\,x)}}{24\,a^{\frac{3}{2}}} - \frac{3\,e^{(-x)}}{8\,a^{\frac{3}{2}}} - \frac{e^{(-3\,x)}}{24\,a^{\frac{3}{2}}} + \frac{3\,e^{x}}{8\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/24*e^(3*x)/a^(3/2) - 3/8*e^(-x)/a^(3/2) - 1/24*e^(-3*x)/a^(3/2) + 3/8*e^x
/a^(3/2)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(28) = 56$.
time = 0.37, size = 277, normalized size = 7.69

$$\frac{\left(\left(e^{(2x)}+1\right)\sinh(x)^6+\cosh(x)^6+6\left(\cosh(x)e^{(2x)}+\cosh(x)\right)\sinh(x)^5+3\left(5\cosh(x)^2+\left(5\cosh(x)^2+3\right)e^{(2x)}+3\right)\sinh(x)^4+9\cosh(x)^4+4\left(5\cosh(x)^3+\left(5\cosh(x)^3+9\cosh(x)\right)e^{(2x)}+9\cosh(x)\right)\sinh(x)^3+3\left(5\cosh(x)^4+18\cosh(x)^2+\left(5\cosh(x)^4+18\cosh(x)^2-3\right)e^{(2x)}-3\right)\sinh(x)^2-9\cosh(x)^2+\left(\cosh(x)^6+9\cosh(x)^4-9\cosh(x)^2-1\right)e^{(2x)}+6\left(\cosh(x)^5+6\cosh(x)^3+\left(\cosh(x)^5+6\cosh(x)^3-3\cosh(x)\right)e^{(2x)}-3\cosh(x)\right)\sinh(x)-1\right)\sqrt{\frac{a}{e^{(4x)}+2e^{(2x)}+1}}\,e^x}{24\left(a^2\cosh(x)^3e^x+3a^2\cosh(x)^2e^x\sinh(x)+3a^2\cosh(x)e^x\sinh(x)^2+a^2e^x\sinh(x)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*((e^(2*x) + 1)*sinh(x)^6 + cosh(x)^6 + 6*(cosh(x)*e^(2*x) + cosh(x))*s
inh(x)^5 + 3*(5*cosh(x)^2 + (5*cosh(x)^2 + 3)*e^(2*x) + 3)*sinh(x)^4 + 9*co
sh(x)^4 + 4*(5*cosh(x)^3 + (5*cosh(x)^3 + 9*cosh(x))*e^(2*x) + 9*cosh(x))*s
inh(x)^3 + 3*(5*cosh(x)^4 + 18*cosh(x)^2 + (5*cosh(x)^4 + 18*cosh(x)^2 - 3)
*e^(2*x) - 3)*sinh(x)^2 - 9*cosh(x)^2 + (cosh(x)^6 + 9*cosh(x)^4 - 9*cosh(x
)^2 - 1)*e^(2*x) + 6*(cosh(x)^5 + 6*cosh(x)^3 + (cosh(x)^5 + 6*cosh(x)^3 -
3*cosh(x))*e^(2*x) - 3*cosh(x))*sinh(x) - 1)*sqrt(a/(e^(4*x) + 2*e^(2*x) +
1))*e^x/(a^2*cosh(x)^3*e^x + 3*a^2*cosh(x)^2*e^x*sinh(x) + 3*a^2*cosh(x)*e^
x*sinh(x)^2 + a^2*e^x*sinh(x)^3)
```

Sympy [**A**]
time = 0.38, size = 31, normalized size = 0.86

$$-\frac{2\tanh^3(x)}{3\left(a\operatorname{sech}^2(x)\right)^{\frac{3}{2}}}+\frac{\tanh(x)}{\left(a\operatorname{sech}^2(x)\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)**2)**(3/2),x)
```

```
[Out] -2*tanh(x)**3/(3*(a*sech(x)**2)**(3/2)) + tanh(x)/(a*sech(x)**2)**(3/2)
```

Giac [**A**]
time = 0.39, size = 29, normalized size = 0.81

$$-\frac{\left(9\,e^{(2x)}+1\right)e^{(-3x)}-e^{(3x)}-9\,e^x}{24\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^2)^(3/2),x, algorithm="giac")
```

[Out] -1/24*((9*e^(2*x) + 1)*e^(-3*x) - e^(3*x) - 9*e^x)/a^(3/2)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\left(\frac{a}{\cosh(x)^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a/cosh(x)^2)^(3/2),x)

[Out] int(1/(a/cosh(x)^2)^(3/2), x)

3.37 $\int \frac{1}{\left(a\text{sech}^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=55

$$\frac{\tanh(x)}{5\left(a\text{sech}^2(x)\right)^{5/2}}+\frac{4\tanh(x)}{15a\left(a\text{sech}^2(x)\right)^{3/2}}+\frac{8\tanh(x)}{15a^2\sqrt{a\text{sech}^2(x)}}$$

```
[Out] 1/5*tanh(x)/(a*sech(x)^2)^(5/2)+4/15*tanh(x)/a/(a*sech(x)^2)^(3/2)+8/15*tan
h(x)/a^2/(a*sech(x)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4207, 198, 197}

$$\frac{8\tanh(x)}{15a^2\sqrt{a\text{sech}^2(x)}}+\frac{4\tanh(x)}{15a\left(a\text{sech}^2(x)\right)^{3/2}}+\frac{\tanh(x)}{5\left(a\text{sech}^2(x)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x]^2)^(-5/2),x]
```

```
[Out] Tanh[x]/(5*(a*Sech[x]^2)^(5/2)) + (4*Tanh[x])/(15*a*(a*Sech[x]^2)^(3/2)) +
(8*Tanh[x])/(15*a^2*Sqrt[a*Sech[x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a\text{sech}^2(x)\right)^{5/2}}\,dx = a\text{Subst}\left(\int \frac{1}{(a-ax^2)^{7/2}}\,dx, x, \tanh(x)\right)$$
$$= \frac{\tanh(x)}{5\left(a\text{sech}^2(x)\right)^{5/2}} + \frac{4}{5}\text{Subst}\left(\int \frac{1}{(a-ax^2)^{5/2}}\,dx, x, \tanh(x)\right)$$
$$= \frac{\tanh(x)}{5\left(a\text{sech}^2(x)\right)^{5/2}} + \frac{4\tanh(x)}{15a\left(a\text{sech}^2(x)\right)^{3/2}} + \frac{8\text{Subst}\left(\int \frac{1}{(a-ax^2)^{3/2}}\,dx, x, \tanh(x)\right)}{15a}$$
$$= \frac{\tanh(x)}{5\left(a\text{sech}^2(x)\right)^{5/2}} + \frac{4\tanh(x)}{15a\left(a\text{sech}^2(x)\right)^{3/2}} + \frac{8\tanh(x)}{15a^2\sqrt{a\text{sech}^2(x)}}$$

Mathematica [A]
time = 0.03, size = 36, normalized size = 0.65

$$\frac{\cosh(x)\sqrt{a\text{sech}^2(x)}\,(150\sinh(x) + 25\sinh(3x) + 3\sinh(5x))}{240a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x]^2)^(-5/2),x]
```

```
[Out] (Cosh[x]*Sqrt[a*Sech[x]^2]*(150*Sinh[x] + 25*Sinh[3*x] + 3*Sinh[5*x]))/(240
*a^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 195 vs. 2(43) = 86.
time = 0.85, size = 196, normalized size = 3.56

method	result
risch	$\frac{e^{6x}}{160a^2(1+e^{2x})\sqrt{\frac{a\,e^{2x}}{(1+e^{2x})^2}}} + \frac{5\,e^{4x}}{96a^2(1+e^{2x})\sqrt{\frac{a\,e^{2x}}{(1+e^{2x})^2}}} + \frac{5\,e^{2x}}{16a^2(1+e^{2x})\sqrt{\frac{a\,e^{2x}}{(1+e^{2x})^2}}} - \frac{5}{16\sqrt{\frac{a\,e^{2x}}{(1+e^{2x})^2}}\,(1+e^{2x})a^2} - \frac{}{96a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sech(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/160/a^2*exp(6*x)/(1+exp(2*x))/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)+5/96/a^2*
exp(4*x)/(1+exp(2*x))/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)+5/16/a^2*exp(2*x)/(
1+exp(2*x))/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)-5/16/(a*exp(2*x)/(1+exp(2*x))
^2)^(1/2)/(1+exp(2*x))/a^2-5/96/a^2*exp(-2*x)/(1+exp(2*x))/(a*exp(2*x)/(1+e
xp(2*x))^2)^(1/2)-1/160/a^2*exp(-4*x)/(1+exp(2*x))/(a*exp(2*x)/(1+exp(2*x))
^2)^(1/2)
```

Maxima [A]
time = 0.47, size = 53, normalized size = 0.96

$$\frac{e^{(5x)}}{160\,a^{\frac{5}{2}}} + \frac{5\,e^{(3x)}}{96\,a^{\frac{5}{2}}} - \frac{5\,e^{(-x)}}{16\,a^{\frac{5}{2}}} - \frac{5\,e^{(-3x)}}{96\,a^{\frac{5}{2}}} - \frac{e^{(-5x)}}{160\,a^{\frac{5}{2}}} + \frac{5\,e^{x}}{16\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/160*e^(5*x)/a^(5/2) + 5/96*e^(3*x)/a^(5/2) - 5/16*e^(-x)/a^(5/2) - 5/96*e
^(-3*x)/a^(5/2) - 1/160*e^(-5*x)/a^(5/2) + 5/16*e^x/a^(5/2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 580 vs. $2(43) = 86$.
time = 0.39, size = 580, normalized size = 10.55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/480*(3*(e^(2*x) + 1)*sinh(x)^10 + 3*cosh(x)^10 + 30*(cosh(x)*e^(2*x) + co
sh(x))*sinh(x)^9 + 5*(27*cosh(x)^2 + (27*cosh(x)^2 + 5)*e^(2*x) + 5)*sinh(x
)^8 + 25*cosh(x)^8 + 40*(9*cosh(x)^3 + (9*cosh(x)^3 + 5*cosh(x))*e^(2*x) +
5*cosh(x))*sinh(x)^7 + 10*(63*cosh(x)^4 + 70*cosh(x)^2 + (63*cosh(x)^4 + 70
*cosh(x)^2 + 15)*e^(2*x) + 15)*sinh(x)^6 + 150*cosh(x)^6 + 4*(189*cosh(x)^5
 + 350*cosh(x)^3 + (189*cosh(x)^5 + 350*cosh(x)^3 + 225*cosh(x))*e^(2*x) +
225*cosh(x))*sinh(x)^5 + 10*(63*cosh(x)^6 + 175*cosh(x)^4 + 225*cosh(x)^2 +
 (63*cosh(x)^6 + 175*cosh(x)^4 + 225*cosh(x)^2 - 15)*e^(2*x) - 15)*sinh(x)^
4 - 150*cosh(x)^4 + 40*(9*cosh(x)^7 + 35*cosh(x)^5 + 75*cosh(x)^3 + (9*cosh
(x)^7 + 35*cosh(x)^5 + 75*cosh(x)^3 - 15*cosh(x))*e^(2*x) - 15*cosh(x))*sin
h(x)^3 + 5*(27*cosh(x)^8 + 140*cosh(x)^6 + 450*cosh(x)^4 - 180*cosh(x)^2 +
(27*cosh(x)^8 + 140*cosh(x)^6 + 450*cosh(x)^4 - 180*cosh(x)^2 - 5)*e^(2*x)
- 5)*sinh(x)^2 - 25*cosh(x)^2 + (3*cosh(x)^10 + 25*cosh(x)^8 + 150*cosh(x)^
6 - 150*cosh(x)^4 - 25*cosh(x)^2 - 3)*e^(2*x) + 10*(3*cosh(x)^9 + 20*cosh(x
)^7 + 90*cosh(x)^5 - 60*cosh(x)^3 + (3*cosh(x)^9 + 20*cosh(x)^7 + 90*cosh(x
)^5 - 60*cosh(x)^3 - 5*cosh(x))*e^(2*x) - 5*cosh(x))*sinh(x) - 3)*sqrt(a/(e
^(4*x) + 2*e^(2*x) + 1))*e^x/(a^3*cosh(x)^5*e^x + 5*a^3*cosh(x)^4*e^x*sinh(
x) + 10*a^3*cosh(x)^3*e^x*sinh(x)^2 + 10*a^3*cosh(x)^2*e^x*sinh(x)^3 + 5*a^
3*cosh(x)*e^x*sinh(x)^4 + a^3*e^x*sinh(x)^5)
```

Sympy [A]
time = 1.61, size = 49, normalized size = 0.89

$$\frac{8\tanh^{5}(x)}{15\left(a\operatorname{sech}^{2}(x)\right)^{\frac{5}{2}}} - \frac{4\tanh^{3}(x)}{3\left(a\operatorname{sech}^{2}(x)\right)^{\frac{5}{2}}} + \frac{\tanh(x)}{\left(a\operatorname{sech}^{2}(x)\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)**2)**(5/2),x)
```

```
[Out] 8*tanh(x)**5/(15*(a*sech(x)**2)**(5/2)) - 4*tanh(x)**3/(3*(a*sech(x)**2)**(
5/2)) + tanh(x)/(a*sech(x)**2)**(5/2)
```

Giac [A]
time = 0.39, size = 41, normalized size = 0.75

$$-\frac{\left(150\,e^{(4\,x)}+25\,e^{(2\,x)}+3\right)e^{(-5\,x)}-3\,e^{(5\,x)}-25\,e^{(3\,x)}-150\,e^{x}}{480\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/480*((150*e^(4*x) + 25*e^(2*x) + 3)*e^(-5*x) - 3*e^(5*x) - 25*e^(3*x) -
150*e^x)/a^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(\frac{a}{{\cosh\left(x\right)}^2}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/cosh(x)^2)^(5/2),x)
```

```
[Out] int(1/(a/cosh(x)^2)^(5/2), x)
```

3.38 $\int \frac{1}{\left(a\mathbf{sech}^2(x)\right)^{7/2}}\,dx$

Optimal. Leaf size=74

$$\frac{\tanh(x)}{7\left(a\text{sech}^2(x)\right)^{7/2}}+\frac{6\tanh(x)}{35a\left(a\text{sech}^2(x)\right)^{5/2}}+\frac{8\tanh(x)}{35a^2\left(a\text{sech}^2(x)\right)^{3/2}}+\frac{16\tanh(x)}{35a^3\sqrt{a\text{sech}^2(x)}}$$

```
[Out] 1/7*tanh(x)/(a*sech(x)^2)^(7/2)+6/35*tanh(x)/a/(a*sech(x)^2)^(5/2)+8/35*tan
h(x)/a^2/(a*sech(x)^2)^(3/2)+16/35*tanh(x)/a^3/(a*sech(x)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4207, 198, 197}

$$\frac{16\tanh(x)}{35a^3\sqrt{a\text{sech}^2(x)}}+\frac{8\tanh(x)}{35a^2\left(a\text{sech}^2(x)\right)^{3/2}}+\frac{6\tanh(x)}{35a\left(a\text{sech}^2(x)\right)^{5/2}}+\frac{\tanh(x)}{7\left(a\text{sech}^2(x)\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x]^2)^(-7/2),x]
```

```
[Out] Tanh[x]/(7*(a*Sech[x]^2)^(7/2)) + (6*Tanh[x])/(35*a*(a*Sech[x]^2)^(5/2)) +
(8*Tanh[x])/(35*a^2*(a*Sech[x]^2)^(3/2)) + (16*Tanh[x])/(35*a^3*Sqrt[a*Sech
[x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a\text{sech}^2(x)\right)^{7/2}}\,dx = a\text{Subst}\left(\int \frac{1}{(a-ax^2)^{9/2}}\,dx, x, \tanh(x)\right)$$
$$= \frac{\tanh(x)}{7\left(a\text{sech}^2(x)\right)^{7/2}} + \frac{6}{7}\text{Subst}\left(\int \frac{1}{(a-ax^2)^{7/2}}\,dx, x, \tanh(x)\right)$$
$$= \frac{\tanh(x)}{7\left(a\text{sech}^2(x)\right)^{7/2}} + \frac{6\tanh(x)}{35a\left(a\text{sech}^2(x)\right)^{5/2}} + \frac{24\text{Subst}\left(\int \frac{1}{(a-ax^2)^{5/2}}\,dx, x, \tanh(x)\right)}{35a}$$
$$= \frac{\tanh(x)}{7\left(a\text{sech}^2(x)\right)^{7/2}} + \frac{6\tanh(x)}{35a\left(a\text{sech}^2(x)\right)^{5/2}} + \frac{8\tanh(x)}{35a^2\left(a\text{sech}^2(x)\right)^{3/2}} + \frac{16\text{Subst}\left(\int \frac{1}{(a-ax^2)^{3/2}}\right.}{35a^2}$$
$$= \frac{\tanh(x)}{7\left(a\text{sech}^2(x)\right)^{7/2}} + \frac{6\tanh(x)}{35a\left(a\text{sech}^2(x)\right)^{5/2}} + \frac{8\tanh(x)}{35a^2\left(a\text{sech}^2(x)\right)^{3/2}} + \frac{16\tanh(x)}{35a^3\sqrt{a\text{sech}^2(x)}}$$

Mathematica [A]
time = 0.04, size = 42, normalized size = 0.57

$$\frac{\cosh(x)\sqrt{a\text{sech}^2(x)}\,(1225\sinh(x) + 245\sinh(3x) + 49\sinh(5x) + 5\sinh(7x))}{2240a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x]^2)^(-7/2),x]
```

```
[Out] (Cosh[x]*Sqrt[a*Sech[x]^2]*(1225*Sinh[x] + 245*Sinh[3*x] + 49*Sinh[5*x] + 5
*Sinh[7*x]))/(2240*a^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 261 vs. 2(58) = 116.
time = 1.16, size = 262, normalized size = 3.54

method	result
risch	$\frac{e^{8x}}{896a^3(1+e^{2x})\sqrt{\frac{a\,e^{2x}}{(1+e^{2x})^2}}} + \frac{7\,e^{6x}}{640a^3(1+e^{2x})\sqrt{\frac{a\,e^{2x}}{(1+e^{2x})^2}}} + \frac{7\,e^{4x}}{128a^3(1+e^{2x})\sqrt{\frac{a\,e^{2x}}{(1+e^{2x})^2}}} + \frac{35\,e^{2x}}{128a^3(1+e^{2x})\sqrt{\frac{a\,e^{2x}}{(1+e^{2x})^2}}} -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sech(x)^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/896/a^3*exp(8*x)/(1+exp(2*x))/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)+7/640/a^3
*exp(6*x)/(1+exp(2*x))/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)+7/128/a^3*exp(4*x)
```

```
/(1+exp(2*x))/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)+35/128/a^3*exp(2*x)/(1+exp(
2*x))/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)-35/128/(a*exp(2*x)/(1+exp(2*x))^2)^
(1/2)/(1+exp(2*x))/a^3-7/128/a^3*exp(-2*x)/(1+exp(2*x))/(a*exp(2*x)/(1+exp(
2*x))^2)^(1/2)-7/640/a^3*exp(-4*x)/(1+exp(2*x))/(a*exp(2*x)/(1+exp(2*x))^2)
^(1/2)-1/896/a^3*exp(-6*x)/(1+exp(2*x))/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)
```

Maxima [A]
time = 0.48, size = 71, normalized size = 0.96

$$\frac{e^{(7x)}}{896a^{\frac{7}{2}}}+\frac{7e^{(5x)}}{640a^{\frac{7}{2}}}+\frac{7e^{(3x)}}{128a^{\frac{7}{2}}}-\frac{35e^{(-x)}}{128a^{\frac{7}{2}}}-\frac{7e^{(-3x)}}{128a^{\frac{7}{2}}}-\frac{7e^{(-5x)}}{640a^{\frac{7}{2}}}-\frac{e^{(-7x)}}{896a^{\frac{7}{2}}}+\frac{35e^{x}}{128a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/896*e^(7*x)/a^(7/2) + 7/640*e^(5*x)/a^(7/2) + 7/128*e^(3*x)/a^(7/2) - 35/
128*e^(-x)/a^(7/2) - 7/128*e^(-3*x)/a^(7/2) - 7/640*e^(-5*x)/a^(7/2) - 1/89
6*e^(-7*x)/a^(7/2) + 35/128*e^x/a^(7/2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 970 vs. $2(58) = 116$.
time = 0.40, size = 970, normalized size = 13.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/4480*(5*(e^(2*x) + 1)*sinh(x)^14 + 5*cosh(x)^14 + 70*(cosh(x)*e^(2*x) + c
osh(x))*sinh(x)^13 + 7*(65*cosh(x)^2 + (65*cosh(x)^2 + 7)*e^(2*x) + 7)*sinh
(x)^12 + 49*cosh(x)^12 + 28*(65*cosh(x)^3 + (65*cosh(x)^3 + 21*cosh(x))*e^(
2*x) + 21*cosh(x))*sinh(x)^11 + 7*(715*cosh(x)^4 + 462*cosh(x)^2 + (715*cos
h(x)^4 + 462*cosh(x)^2 + 35)*e^(2*x) + 35)*sinh(x)^10 + 245*cosh(x)^10 + 70
*(143*cosh(x)^5 + 154*cosh(x)^3 + (143*cosh(x)^5 + 154*cosh(x)^3 + 35*cosh(
x))*e^(2*x) + 35*cosh(x))*sinh(x)^9 + 35*(429*cosh(x)^6 + 693*cosh(x)^4 + 3
15*cosh(x)^2 + (429*cosh(x)^6 + 693*cosh(x)^4 + 315*cosh(x)^2 + 35)*e^(2*x)
 + 35)*sinh(x)^8 + 1225*cosh(x)^8 + 8*(2145*cosh(x)^7 + 4851*cosh(x)^5 + 36
75*cosh(x)^3 + (2145*cosh(x)^7 + 4851*cosh(x)^5 + 3675*cosh(x)^3 + 1225*cos
h(x))*e^(2*x) + 1225*cosh(x))*sinh(x)^7 + 7*(2145*cosh(x)^8 + 6468*cosh(x)^
6 + 7350*cosh(x)^4 + 4900*cosh(x)^2 + (2145*cosh(x)^8 + 6468*cosh(x)^6 + 73
50*cosh(x)^4 + 4900*cosh(x)^2 - 175)*e^(2*x) - 175)*sinh(x)^6 - 1225*cosh(x
)^6 + 14*(715*cosh(x)^9 + 2772*cosh(x)^7 + 4410*cosh(x)^5 + 4900*cosh(x)^3
+ (715*cosh(x)^9 + 2772*cosh(x)^7 + 4410*cosh(x)^5 + 4900*cosh(x)^3 - 525*c
osh(x))*e^(2*x) - 525*cosh(x))*sinh(x)^5 + 35*(143*cosh(x)^10 + 693*cosh(x)
^8 + 1470*cosh(x)^6 + 2450*cosh(x)^4 - 525*cosh(x)^2 + (143*cosh(x)^10 + 69
3*cosh(x)^8 + 1470*cosh(x)^6 + 2450*cosh(x)^4 - 525*cosh(x)^2 - 7)*e^(2*x)
```

```
- 7)*sinh(x)^4 - 245*cosh(x)^4 + 140*(13*cosh(x)^11 + 77*cosh(x)^9 + 210*co
sh(x)^7 + 490*cosh(x)^5 - 175*cosh(x)^3 + (13*cosh(x)^11 + 77*cosh(x)^9 + 2
10*cosh(x)^7 + 490*cosh(x)^5 - 175*cosh(x)^3 - 7*cosh(x))*e^(2*x) - 7*cosh(
x))*sinh(x)^3 + 7*(65*cosh(x)^12 + 462*cosh(x)^10 + 1575*cosh(x)^8 + 4900*c
osh(x)^6 - 2625*cosh(x)^4 - 210*cosh(x)^2 + (65*cosh(x)^12 + 462*cosh(x)^10
 + 1575*cosh(x)^8 + 4900*cosh(x)^6 - 2625*cosh(x)^4 - 210*cosh(x)^2 - 7)*e^
(2*x) - 7)*sinh(x)^2 - 49*cosh(x)^2 + (5*cosh(x)^14 + 49*cosh(x)^12 + 245*c
osh(x)^10 + 1225*cosh(x)^8 - 1225*cosh(x)^6 - 245*cosh(x)^4 - 49*cosh(x)^2
- 5)*e^(2*x) + 14*(5*cosh(x)^13 + 42*cosh(x)^11 + 175*cosh(x)^9 + 700*cosh(
x)^7 - 525*cosh(x)^5 - 70*cosh(x)^3 + (5*cosh(x)^13 + 42*cosh(x)^11 + 175*c
osh(x)^9 + 700*cosh(x)^7 - 525*cosh(x)^5 - 70*cosh(x)^3 - 7*cosh(x))*e^(2*x
) - 7*cosh(x))*sinh(x) - 5)*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*e^x/(a^4*cosh
(x)^7*e^x + 7*a^4*cosh(x)^6*e^x*sinh(x) + 21*a^4*cosh(x)^5*e^x*sinh(x)^2 +
35*a^4*cosh(x)^4*e^x*sinh(x)^3 + 35*a^4*cosh(x)^3*e^x*sinh(x)^4 + 21*a^4*co
sh(x)^2*e^x*sinh(x)^5 + 7*a^4*cosh(x)*e^x*sinh(x)^6 + a^4*e^x*sinh(x)^7)
```

Sympy [A]
time = 12.86, size = 66, normalized size = 0.89

$$-\frac{16\tanh^7(x)}{35\left(a\operatorname{sech}^2(x)\right)^{\frac{7}{2}}}+\frac{8\tanh^5(x)}{5\left(a\operatorname{sech}^2(x)\right)^{\frac{7}{2}}}-\frac{2\tanh^3(x)}{\left(a\operatorname{sech}^2(x)\right)^{\frac{7}{2}}}+\frac{\tanh(x)}{\left(a\operatorname{sech}^2(x)\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)**2)**(7/2),x)
```

```
[Out] -16*tanh(x)**7/(35*(a*sech(x)**2)**(7/2)) + 8*tanh(x)**5/(5*(a*sech(x)**2)*
*(7/2)) - 2*tanh(x)**3/(a*sech(x)**2)**(7/2) + tanh(x)/(a*sech(x)**2)**(7/2
)
```

Giac [A]
time = 0.37, size = 53, normalized size = 0.72

$$-\frac{\left(1225\,e^{(6\,x)}+245\,e^{(4\,x)}+49\,e^{(2\,x)}+5\right)e^{(-7\,x)}-5\,e^{(7\,x)}-49\,e^{(5\,x)}-245\,e^{(3\,x)}-1225\,e^{x}}{4480\,a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/4480*((1225*e^(6*x) + 245*e^(4*x) + 49*e^(2*x) + 5)*e^(-7*x) - 5*e^(7*x)
 - 49*e^(5*x) - 245*e^(3*x) - 1225*e^x)/a^(7/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(\frac{a}{{\cosh\left(x\right)}^2}\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/cosh(x)^2)^(7/2),x)
```

```
[Out] int(1/(a/cosh(x)^2)^(7/2), x)
```

3.39 $\int \left(a\text{sech}^3(x)\right)^{5/2}\,dx$

Optimal. Leaf size=121

$$\frac{154}{195}ia^2\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)\sqrt{a\text{sech}^3(x)}+\frac{154}{195}a^2\cosh(x)\sqrt{a\text{sech}^3(x)}\,\sinh(x)+\frac{154}{585}a^2\sqrt{a\text{sech}^3(x)}\,\tanh(x)+$$

```
[Out] 154/195*I*a^2*cosh(x)^(3/2)*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*s
inh(1/2*x),2^(1/2))*(a*sech(x)^3)^(1/2)+154/195*a^2*cosh(x)*sinh(x)*(a*sech
(x)^3)^(1/2)+154/585*a^2*(a*sech(x)^3)^(1/2)*tanh(x)+22/117*a^2*sech(x)^2*(
a*sech(x)^3)^(1/2)*tanh(x)+2/13*a^2*sech(x)^4*(a*sech(x)^3)^(1/2)*tanh(x)
```

Rubi [A]
time = 0.04, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4208, 3853, 3856, 2719}

$$\frac{154}{585}a^2\tanh(x)\sqrt{a\text{sech}^3(x)}+\frac{2}{13}a^2\tanh(x)\text{sech}^4(x)\sqrt{a\text{sech}^3(x)}+\frac{22}{117}a^2\tanh(x)\text{sech}^2(x)\sqrt{a\text{sech}^3(x)}+\frac{154}{195}ia^2\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)\sqrt{a\text{sech}^3(x)}+\frac{154}{195}a^2\sinh(x)\cosh(x)\sqrt{a\text{sech}^3(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x]^3)^(5/2),x]
```

```
[Out] ((154*I)/195)*a^2*Cosh[x]^(3/2)*EllipticE[(I/2)*x, 2]*Sqrt[a*Sech[x]^3] + (
154*a^2*Cosh[x]*Sqrt[a*Sech[x]^3]*Sinh[x])/195 + (154*a^2*Sqrt[a*Sech[x]^3]
*Tanh[x])/585 + (22*a^2*Sech[x]^2*Sqrt[a*Sech[x]^3]*Tanh[x])/117 + (2*a^2*S
ech[x]^4*Sqrt[a*Sech[x]^3]*Tanh[x])/13
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\int \left(a\text{sech}^3(x)\right)^{5/2}\,dx = \frac{\left(a^2\sqrt{a\text{sech}^3(x)}\right)\int \text{sech}^{\frac{15}{2}}(x)\,dx}{\text{sech}^{\frac{3}{2}}(x)}$$

$$= \frac{2}{13}a^2\text{sech}^4(x)\sqrt{a\text{sech}^3(x)}\,\tanh(x) + \frac{\left(11a^2\sqrt{a\text{sech}^3(x)}\right)\int \text{sech}^{\frac{11}{2}}(x)\,dx}{13\text{sech}^{\frac{3}{2}}(x)}$$

$$= \frac{22}{117}a^2\text{sech}^2(x)\sqrt{a\text{sech}^3(x)}\,\tanh(x) + \frac{2}{13}a^2\text{sech}^4(x)\sqrt{a\text{sech}^3(x)}\,\tanh(x) + \frac{\left(77a^2\right.}{}$$

$$= \frac{154}{585}a^2\sqrt{a\text{sech}^3(x)}\,\tanh(x) + \frac{22}{117}a^2\text{sech}^2(x)\sqrt{a\text{sech}^3(x)}\,\tanh(x) + \frac{2}{13}a^2\text{sech}^4(x)$$

$$= \frac{154}{195}a^2\cosh(x)\sqrt{a\text{sech}^3(x)}\,\sinh(x) + \frac{154}{585}a^2\sqrt{a\text{sech}^3(x)}\,\tanh(x) + \frac{22}{117}a^2\text{sech}^2(x)$$

$$= \frac{154}{195}a^2\cosh(x)\sqrt{a\text{sech}^3(x)}\,\sinh(x) + \frac{154}{585}a^2\sqrt{a\text{sech}^3(x)}\,\tanh(x) + \frac{22}{117}a^2\text{sech}^2(x)$$

$$= \frac{154}{195}ia^2\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)\sqrt{a\text{sech}^3(x)} + \frac{154}{195}a^2\cosh(x)\sqrt{a\text{sech}^3(x)}\,\sinh(x) + \frac{15}{58}$$

Mathematica [A]
time = 0.07, size = 63, normalized size = 0.52

$$\frac{2}{585}a\text{sech}(x)\left(a\text{sech}^3(x)\right)^{3/2}\left(231i\cosh^{\frac{11}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)+55\cosh(x)\sinh(x)+77\cosh^3(x)\sinh(x)+231\cosh^5(x)\sinh(x)+45\tanh(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x]^3)^(5/2),x]
```

```
[Out] (2*a*Sech[x]*(a*Sech[x]^3)^(3/2)*((231*I)*Cosh[x]^(11/2)*EllipticE[(I/2)*x,
 2] + 55*Cosh[x]*Sinh[x] + 77*Cosh[x]^3*Sinh[x] + 231*Cosh[x]^5*Sinh[x] + 4
5*Tanh[x]))/585
```

Maple [F]
time = 1.05, size = 0, normalized size = 0.00

$$\int \left(a\,\mathrm{sech}(x)^3\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sech(x)^3)^(5/2),x)
[Out] int((a*sech(x)^3)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^3)^(5/2),x, algorithm="maxima")
[Out] integrate((a*sech(x)^3)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 1382, normalized size = 11.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^3)^(5/2),x, algorithm="fricas")
[Out] 2/585*(231*sqrt(2)*(a^2*cosh(x)^12 + 12*a^2*cosh(x)*sinh(x)^11 + a^2*sinh(x
)^12 + 6*a^2*cosh(x)^10 + 6*(11*a^2*cosh(x)^2 + a^2)*sinh(x)^10 + 15*a^2*co
sh(x)^8 + 20*(11*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^9 + 15*(33*a^2*cosh
(x)^4 + 18*a^2*cosh(x)^2 + a^2)*sinh(x)^8 + 20*a^2*cosh(x)^6 + 24*(33*a^2*c
osh(x)^5 + 30*a^2*cosh(x)^3 + 5*a^2*cosh(x))*sinh(x)^7 + 4*(231*a^2*cosh(x)
^6 + 315*a^2*cosh(x)^4 + 105*a^2*cosh(x)^2 + 5*a^2)*sinh(x)^6 + 15*a^2*cosh
(x)^4 + 24*(33*a^2*cosh(x)^7 + 63*a^2*cosh(x)^5 + 35*a^2*cosh(x)^3 + 5*a^2*
cosh(x))*sinh(x)^5 + 15*(33*a^2*cosh(x)^8 + 84*a^2*cosh(x)^6 + 70*a^2*cosh(
x)^4 + 20*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 6*a^2*cosh(x)^2 + 20*(11*a^2*cos
h(x)^9 + 36*a^2*cosh(x)^7 + 42*a^2*cosh(x)^5 + 20*a^2*cosh(x)^3 + 3*a^2*cos
h(x))*sinh(x)^3 + 6*(11*a^2*cosh(x)^10 + 45*a^2*cosh(x)^8 + 70*a^2*cosh(x)^
6 + 50*a^2*cosh(x)^4 + 15*a^2*cosh(x)^2 + a^2)*sinh(x)^2 + a^2 + 12*(a^2*co
sh(x)^11 + 5*a^2*cosh(x)^9 + 10*a^2*cosh(x)^7 + 10*a^2*cosh(x)^5 + 5*a^2*co
sh(x)^3 + a^2*cosh(x))*sinh(x))*sqrt(a)*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, cosh(x) + sinh(x))) + sqrt(2)*(231*a^2*cosh(x)^13 + 3003*a^2
*cosh(x)*sinh(x)^12 + 231*a^2*sinh(x)^13 + 1540*a^2*cosh(x)^11 + 154*(117*a
^2*cosh(x)^2 + 10*a^2)*sinh(x)^11 + 4367*a^2*cosh(x)^9 + 1694*(39*a^2*cosh(
```

```
x)^3 + 10*a^2*cosh(x))*sinh(x)^10 + 11*(15015*a^2*cosh(x)^4 + 7700*a^2*cosh
(x)^2 + 397*a^2)*sinh(x)^9 + 6808*a^2*cosh(x)^7 + 33*(9009*a^2*cosh(x)^5 +
7700*a^2*cosh(x)^3 + 1191*a^2*cosh(x))*sinh(x)^8 + 4*(99099*a^2*cosh(x)^6 +
 127050*a^2*cosh(x)^4 + 39303*a^2*cosh(x)^2 + 1702*a^2)*sinh(x)^7 + 1277*a^
2*cosh(x)^5 + 28*(14157*a^2*cosh(x)^7 + 25410*a^2*cosh(x)^5 + 13101*a^2*cos
h(x)^3 + 1702*a^2*cosh(x))*sinh(x)^6 + (297297*a^2*cosh(x)^8 + 711480*a^2*c
osh(x)^6 + 550242*a^2*cosh(x)^4 + 142968*a^2*cosh(x)^2 + 1277*a^2)*sinh(x)^
5 + 484*a^2*cosh(x)^3 + (165165*a^2*cosh(x)^9 + 508200*a^2*cosh(x)^7 + 5502
42*a^2*cosh(x)^5 + 238280*a^2*cosh(x)^3 + 6385*a^2*cosh(x))*sinh(x)^4 + 2*(
33033*a^2*cosh(x)^10 + 127050*a^2*cosh(x)^8 + 183414*a^2*cosh(x)^6 + 119140
*a^2*cosh(x)^4 + 6385*a^2*cosh(x)^2 + 242*a^2)*sinh(x)^3 + 77*a^2*cosh(x) +
 2*(9009*a^2*cosh(x)^11 + 42350*a^2*cosh(x)^9 + 78606*a^2*cosh(x)^7 + 71484
*a^2*cosh(x)^5 + 6385*a^2*cosh(x)^3 + 726*a^2*cosh(x))*sinh(x)^2 + (3003*a^
2*cosh(x)^12 + 16940*a^2*cosh(x)^10 + 39303*a^2*cosh(x)^8 + 47656*a^2*cosh(
x)^6 + 6385*a^2*cosh(x)^4 + 1452*a^2*cosh(x)^2 + 77*a^2)*sinh(x))*sqrt((a*c
osh(x) + a*sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)))/(cosh
(x)^12 + 12*cosh(x)*sinh(x)^11 + sinh(x)^12 + 6*(11*cosh(x)^2 + 1)*sinh(x)^
10 + 6*cosh(x)^10 + 20*(11*cosh(x)^3 + 3*cosh(x))*sinh(x)^9 + 15*(33*cosh(x
)^4 + 18*cosh(x)^2 + 1)*sinh(x)^8 + 15*cosh(x)^8 + 24*(33*cosh(x)^5 + 30*co
sh(x)^3 + 5*cosh(x))*sinh(x)^7 + 4*(231*cosh(x)^6 + 315*cosh(x)^4 + 105*cos
h(x)^2 + 5)*sinh(x)^6 + 20*cosh(x)^6 + 24*(33*cosh(x)^7 + 63*cosh(x)^5 + 35
*cosh(x)^3 + 5*cosh(x))*sinh(x)^5 + 15*(33*cosh(x)^8 + 84*cosh(x)^6 + 70*co
sh(x)^4 + 20*cosh(x)^2 + 1)*sinh(x)^4 + 15*cosh(x)^4 + 20*(11*cosh(x)^9 + 3
6*cosh(x)^7 + 42*cosh(x)^5 + 20*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 6*(11*co
sh(x)^10 + 45*cosh(x)^8 + 70*cosh(x)^6 + 50*cosh(x)^4 + 15*cosh(x)^2 + 1)*s
inh(x)^2 + 6*cosh(x)^2 + 12*(cosh(x)^11 + 5*cosh(x)^9 + 10*cosh(x)^7 + 10*c
osh(x)^5 + 5*cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\operatorname{sech}^3(x)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)**3)**(5/2),x)
```

```
[Out] Integral((a*sech(x)**3)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^3)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sech(x)^3)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{a}{\cosh(x)^3}\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/cosh(x)^3)^(5/2),x)
[Out] int((a/cosh(x)^3)^(5/2), x)
```

3.40 $\int \left(a\text{sech}^3(x)\right)^{3/2}\,dx$

Optimal. Leaf size=69

$$-\frac{10}{21} i a \cosh^{\frac{3}{2}}(x) F\left(\frac{ix}{2}\middle|2\right)\sqrt{a\text{sech}^3(x)}+\frac{10}{21} a\sqrt{a\text{sech}^3(x)}\,\sinh(x)+\frac{2}{7} a\text{sech}(x)\sqrt{a\text{sech}^3(x)}\,\tanh(x)$$

```
[Out] -10/21*I*a*cosh(x)^(3/2)*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh
(1/2*x),2^(1/2))*(a*sech(x)^3)^(1/2)+10/21*a*sinh(x)*(a*sech(x)^3)^(1/2)+2/
7*a*sech(x)*(a*sech(x)^3)^(1/2)*tanh(x)
```

Rubi [A]
time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4208, 3853, 3856, 2720}

$$\frac{10}{21} a\sinh(x)\sqrt{a\text{sech}^3(x)}+\frac{2}{7} a\tanh(x)\text{sech}(x)\sqrt{a\text{sech}^3(x)}-\frac{10}{21} i a \cosh^{\frac{3}{2}}(x) F\left(\frac{ix}{2}\middle|2\right)\sqrt{a\text{sech}^3(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x]^3)^(3/2),x]
```

```
[Out] ((-10*I)/21)*a*Cosh[x]^(3/2)*EllipticF[(I/2)*x, 2]*Sqrt[a*Sech[x]^3] + (10*
a*Sqrt[a*Sech[x]^3]*Sinh[x])/21 + (2*a*Sech[x]*Sqrt[a*Sech[x]^3]*Tanh[x])/7
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
```

```
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\int \left(a\text{sech}^3(x)\right)^{3/2}\,dx = \frac{\left(a\sqrt{a\text{sech}^3(x)}\right)\int \text{sech}^{\frac{9}{2}}(x)\,dx}{\text{sech}^{\frac{3}{2}}(x)}$$

$$= \frac{2}{7}a\text{sech}(x)\sqrt{a\text{sech}^3(x)}\,\tanh(x) + \frac{\left(5a\sqrt{a\text{sech}^3(x)}\right)\int \text{sech}^{\frac{5}{2}}(x)\,dx}{7\text{sech}^{\frac{3}{2}}(x)}$$

$$= \frac{10}{21}a\sqrt{a\text{sech}^3(x)}\,\sinh(x) + \frac{2}{7}a\text{sech}(x)\sqrt{a\text{sech}^3(x)}\,\tanh(x) + \frac{\left(5a\sqrt{a\text{sech}^3(x)}\right)\int \sqrt{}}{21\text{sech}^{\frac{3}{2}}(x}$$

$$= \frac{10}{21}a\sqrt{a\text{sech}^3(x)}\,\sinh(x) + \frac{2}{7}a\text{sech}(x)\sqrt{a\text{sech}^3(x)}\,\tanh(x) + \frac{1}{21}\left(5a\cosh^{\frac{3}{2}}(x)\sqrt{a\text{sec}}\right.$$

$$= -\frac{10}{21}ia\cosh^{\frac{3}{2}}(x)F\left(\frac{ix}{2}\middle|2\right)\sqrt{a\text{sech}^3(x)} + \frac{10}{21}a\sqrt{a\text{sech}^3(x)}\,\sinh(x) + \frac{2}{7}a\text{sech}(x)\sqrt{a}$$

Mathematica [**A**]
time = 0.03, size = 47, normalized size = 0.68

$$\frac{2}{21}a\text{sech}(x)\sqrt{a\text{sech}^3(x)}\left(-5i\cosh^{\frac{5}{2}}(x)F\left(\frac{ix}{2}\middle|2\right) + 5\cosh(x)\sinh(x) + 3\tanh(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x]^3)^(3/2),x]
[Out] (2*a*Sech[x]*Sqrt[a*Sech[x]^3]*((-5*I)*Cosh[x]^(5/2)*EllipticF[(I/2)*x, 2]
+ 5*Cosh[x]*Sinh[x] + 3*Tanh[x]))/21
```

Maple [**F**]
time = 0.83, size = 0, normalized size = 0.00

$$\int \left(a\,\text{sech}(x)^3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sech(x)^3)^(3/2),x)
[Out] int((a*sech(x)^3)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sech(x)^3)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 391, normalized size = 5.67

$$\frac{2\left(5\sqrt{2}\left(a\cosh(x)^6+6a\cosh(x)\sinh(x)^5+a\sinh(x)^6+3a\cosh(x)^4+3\left(5a\cosh(x)^2+a\right)\sinh(x)^4+4\left(5a\cosh(x)^3+3a\cosh(x)\right)\sinh(x)^3+3a\cosh(x)^2+3\left(5a\cosh(x)^4+6a\cosh(x)^2+a\right)\sinh(x)^2+6\left(a\cosh(x)^5+2a\cosh(x)^3+a\cosh(x)\right)\sinh(x)+a\right)\sqrt{a}\,\mathrm{weierstrassPInverse}(-4,0,\cosh(x)+\sinh(x))+\sqrt{2}\left(5a\cosh(x)^6+30a\cosh(x)\sinh(x)^5+5a\sinh(x)^6+17a\cosh(x)^4+\left(75a\cosh(x)^2+17a\right)\sinh(x)^4+4\left(25a\cosh(x)^3+17a\cosh(x)\right)\sinh(x)^3-17a\cosh(x)^2+\left(75a\cosh(x)^4+102a\cosh(x)^2-17a\right)\sinh(x)^2+2\left(15a\cosh(x)^5+34a\cosh(x)^3-17a\cosh(x)\right)\sinh(x)-5a\right)\sqrt{\frac{a\cosh(x)+a\sinh(x)}{\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2+1}}\right)}{21\left(\cosh(x)^6+6\cosh(x)\sinh(x)^5+\sinh(x)^6+3\left(5\cosh(x)^2+1\right)\sinh(x)^4+3\cosh(x)^4+4\left(5\cosh(x)^3+3\cosh(x)\right)\sinh(x)^3+3\left(5\cosh(x)^4+6\cosh(x)^2+1\right)\sinh(x)^2+3\cosh(x)^2+6\left(\cosh(x)^5+2\cosh(x)^3+\cosh(x)\right)\sinh(x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/21*(5*sqrt(2)*(a*cosh(x)^6 + 6*a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 + 3*a*co
sh(x)^4 + 3*(5*a*cosh(x)^2 + a)*sinh(x)^4 + 4*(5*a*cosh(x)^3 + 3*a*cosh(x))
*sinh(x)^3 + 3*a*cosh(x)^2 + 3*(5*a*cosh(x)^4 + 6*a*cosh(x)^2 + a)*sinh(x)^
2 + 6*(a*cosh(x)^5 + 2*a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)*sqrt(a)*weiers
trassPInverse(-4, 0, cosh(x) + sinh(x)) + sqrt(2)*(5*a*cosh(x)^6 + 30*a*cos
h(x)*sinh(x)^5 + 5*a*sinh(x)^6 + 17*a*cosh(x)^4 + (75*a*cosh(x)^2 + 17*a)*s
inh(x)^4 + 4*(25*a*cosh(x)^3 + 17*a*cosh(x))*sinh(x)^3 - 17*a*cosh(x)^2 + (
75*a*cosh(x)^4 + 102*a*cosh(x)^2 - 17*a)*sinh(x)^2 + 2*(15*a*cosh(x)^5 + 34
*a*cosh(x)^3 - 17*a*cosh(x))*sinh(x) - 5*a)*sqrt((a*cosh(x) + a*sinh(x))/(c
osh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)))/(cosh(x)^6 + 6*cosh(x)*sinh
(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 + 1)*sinh(x)^4 + 3*cosh(x)^4 + 4*(5*cosh
(x)^3 + 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4 + 6*cosh(x)^2 + 1)*sinh(x)^2
+ 3*cosh(x)^2 + 6*(cosh(x)^5 + 2*cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\,\mathrm{sech}^3\left(x\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)**3)**(3/2),x)
```

```
[Out] Integral((a*sech(x)**3)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sech(x)^3)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{a}{\cosh(x)^3}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/cosh(x)^3)^(3/2),x)
```

```
[Out] int((a/cosh(x)^3)^(3/2), x)
```

3.41 $\int \sqrt{a\operatorname{sech}^3(x)}\,dx$

Optimal. Leaf size=46

$$2i\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)\sqrt{a\operatorname{sech}^3(x)}+2\cosh(x)\sqrt{a\operatorname{sech}^3(x)}\sinh(x)$$

```
[Out] 2*I*cosh(x)^(3/2)*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*sinh(1/2*x)
,2^(1/2))*(a*sech(x)^3)^(1/2)+2*cosh(x)*sinh(x)*(a*sech(x)^3)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4208, 3853, 3856, 2719}

$$2\sinh(x)\cosh(x)\sqrt{a\operatorname{sech}^3(x)}+2i\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)\sqrt{a\operatorname{sech}^3(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Sech[x]^3],x]
```

```
[Out] (2*I)*Cosh[x]^(3/2)*EllipticE[(I/2)*x, 2]*Sqrt[a*Sech[x]^3] + 2*Cosh[x]*Sqr
t[a*Sech[x]^3]*Sinh[x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
```

```
& !IntegerQ[p]
```

Rubi steps

$$\int \sqrt{a\text{sech}^3(x)}\,dx = \frac{\sqrt{a\text{sech}^3(x)}\int \text{sech}^{\frac{3}{2}}(x)\,dx}{\text{sech}^{\frac{3}{2}}(x)}$$

$$= 2\cosh(x)\sqrt{a\text{sech}^3(x)}\,\sinh(x) - \frac{\sqrt{a\text{sech}^3(x)}\int \frac{1}{\sqrt{\text{sech}(x)}}\,dx}{\text{sech}^{\frac{3}{2}}(x)}$$

$$= 2\cosh(x)\sqrt{a\text{sech}^3(x)}\,\sinh(x) - \left(\cosh^{\frac{3}{2}}(x)\sqrt{a\text{sech}^3(x)}\right)\int \sqrt{\cosh(x)}\,dx$$

$$= 2i\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)\sqrt{a\text{sech}^3(x)} + 2\cosh(x)\sqrt{a\text{sech}^3(x)}\,\sinh(x)$$

Mathematica [**A**]
time = 0.02, size = 36, normalized size = 0.78

$$2\cosh(x)\sqrt{a\text{sech}^3(x)}\left(i\sqrt{\cosh(x)}\,E\left(\frac{ix}{2}\middle|2\right) + \sinh(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Sech[x]^3],x]
[Out] 2*Cosh[x]*Sqrt[a*Sech[x]^3]*(I*Sqrt[Cosh[x]]*EllipticE[(I/2)*x, 2] + Sinh[x
])
```

Maple [**F**]
time = 1.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\,\text{sech}\,(x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sech(x)^3)^(1/2),x)
[Out] int((a*sech(x)^3)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sech(x)^3), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 60, normalized size = 1.30

$$2\sqrt{2}\sqrt{\frac{a\cosh\left(x\right)+a\sinh\left(x\right)}{\cosh\left(x\right)^{2}+2\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}+1}}\left(\cosh\left(x\right)+\sinh\left(x\right)\right)+2\sqrt{2}\sqrt{a}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cosh\left(x\right)+\sinh\left(x\right)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*sqrt((a*cosh(x) + a*sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sin
h(x)^2 + 1))*(cosh(x) + sinh(x)) + 2*sqrt(2)*sqrt(a)*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cosh(x) + sinh(x)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\,\mathrm{sech}^{3}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)**3)**(1/2),x)
```

```
[Out] Integral(sqrt(a*sech(x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sech(x)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\frac{a}{{\cosh\left(x\right)}^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/cosh(x)^3)^(1/2),x)
```

```
[Out] int((a/cosh(x)^3)^(1/2), x)
```

3.42 $\int \frac{1}{\sqrt{a\text{sech}^3(x)}}\,dx$

Optimal. Leaf size=48

$$-\frac{2iF\left(\frac{ix}{2}\middle|2\right)}{3\cosh^{\frac{3}{2}}(x)\sqrt{a\text{sech}^3(x)}}+\frac{2\tanh(x)}{3\sqrt{a\text{sech}^3(x)}}$$

```
[Out] -2/3*I*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh(1/2*x),2^(1/2))/c
osh(x)^(3/2)/(a*sech(x)^3)^(1/2)+2/3*tanh(x)/(a*sech(x)^3)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4208, 3854, 3856, 2720}

$$\frac{2\tanh(x)}{3\sqrt{a\text{sech}^3(x)}}-\frac{2iF\left(\frac{ix}{2}\middle|2\right)}{3\cosh^{\frac{3}{2}}(x)\sqrt{a\text{sech}^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Sech[x]^3],x]
```

```
[Out] (((-2*I)/3)*EllipticF[(I/2)*x, 2])/(Cosh[x]^(3/2)*Sqrt[a*Sech[x]^3]) + (2*T
anh[x])/(3*Sqrt[a*Sech[x]^3])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{a\text{sech}^3(x)}}\,dx &= \frac{\text{sech}^{\frac{3}{2}}(x)\int \frac{1}{\text{sech}^{\frac{3}{2}}(x)}\,dx}{\sqrt{a\text{sech}^3(x)}}\\
&= \frac{2\tanh(x)}{3\sqrt{a\text{sech}^3(x)}} + \frac{\text{sech}^{\frac{3}{2}}(x)\int \sqrt{\text{sech}(x)}\,dx}{3\sqrt{a\text{sech}^3(x)}}\\
&= \frac{2\tanh(x)}{3\sqrt{a\text{sech}^3(x)}} + \frac{\int \frac{1}{\sqrt{\cosh(x)}}\,dx}{3\cosh^{\frac{3}{2}}(x)\sqrt{a\text{sech}^3(x)}}\\
&= -\frac{2iF\left(\frac{ix}{2}\middle|2\right)}{3\cosh^{\frac{3}{2}}(x)\sqrt{a\text{sech}^3(x)}} + \frac{2\tanh(x)}{3\sqrt{a\text{sech}^3(x)}}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 38, normalized size = 0.79

$$\frac{-\frac{2iF\left(\frac{ix}{2}\middle|2\right)}{\cosh^{\frac{3}{2}}(x)} + 2\tanh(x)}{3\sqrt{a\text{sech}^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Sech[x]^3],x]
[Out] (((-2*I)*EllipticF[(I/2)*x, 2])/Cosh[x]^(3/2) + 2*Tanh[x])/(3*Sqrt[a*Sech[x
]^3])
```

Maple [F]
time = 1.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\,\text{sech}\left(x\right)^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sech(x)^3)^(1/2),x)
[Out] int(1/(a*sech(x)^3)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(a*sech(x)^3), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 126, normalized size = 2.62

$$\frac{4\sqrt{2}\left(\cosh\left(x\right)^{2}+2\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-4,0,\cosh\left(x\right)+\sinh\left(x\right)\right)+\sqrt{2}\left(\cosh\left(x\right)^{4}+4\cosh\left(x\right)^{3}\sinh\left(x\right)+6\cosh\left(x\right)^{2}\sinh\left(x\right)^{2}+4\cosh\left(x\right)\sinh\left(x\right)^{3}+\sinh\left(x\right)^{4}-1\right)\sqrt{\frac{a\cosh\left(x\right)+a\sinh\left(x\right)}{\cosh\left(x\right)^{2}+2\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}+1}}}{6\left(a\cosh\left(x\right)^{2}+2a\cosh\left(x\right)\sinh\left(x\right)+a\sinh\left(x\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^3)^(1/2),x, algorithm="fricas")
[Out] 1/6*(4*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)*sqrt(a)*weierstr
assPInverse(-4, 0, cosh(x) + sinh(x)) + sqrt(2)*(cosh(x)^4 + 4*cosh(x)^3*si
nh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 - 1)*sqrt((
a*cosh(x) + a*sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)))/(a
*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\operatorname{sech}^{3}\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)**3)**(1/2),x)
[Out] Integral(1/sqrt(a*sech(x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(a*sech(x)^3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\frac{a}{\cosh\left(x\right)^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/cosh(x)^3)^(1/2),x)
[Out] int(1/(a/cosh(x)^3)^(1/2), x)
```

3.43 $\int \frac{1}{\left(a\mathbf{sech}^3(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=77

$$-\frac{14iE\left(\frac{ix}{2}\middle|2\right)}{15a\cosh^{\frac{3}{2}}(x)\sqrt{a\text{sech}^3(x)}}+\frac{14\sinh(x)}{45a\sqrt{a\text{sech}^3(x)}}+\frac{2\cosh^2(x)\sinh(x)}{9a\sqrt{a\text{sech}^3(x)}}$$

```
[Out] -14/15*I*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*sinh(1/2*x),2^(1/2))
/a/cosh(x)^(3/2)/(a*sech(x)^3)^(1/2)+14/45*sinh(x)/a/(a*sech(x)^3)^(1/2)+2/
9*cosh(x)^2*sinh(x)/a/(a*sech(x)^3)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4208, 3854, 3856, 2719}

$$\frac{14\sinh(x)}{45a\sqrt{a\text{sech}^3(x)}}-\frac{14iE\left(\frac{ix}{2}\middle|2\right)}{15a\cosh^{\frac{3}{2}}(x)\sqrt{a\text{sech}^3(x)}}+\frac{2\sinh(x)\cosh^2(x)}{9a\sqrt{a\text{sech}^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x]^3)^(-3/2),x]
```

```
[Out] (((-14*I)/15)*EllipticE[(I/2)*x, 2])/(a*Cosh[x]^(3/2)*Sqrt[a*Sech[x]^3]) +
(14*Sinh[x])/(45*a*Sqrt[a*Sech[x]^3]) + (2*Cosh[x]^2*Sinh[x])/(9*a*Sqrt[a*S
ech[x]^3])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a\text{sech}^3(x)\right)^{3/2}}\,dx &= \frac{\text{sech}^{\frac{3}{2}}(x)\int \frac{1}{\text{sech}^{\frac{9}{2}}(x)}\,dx}{a\sqrt{a\text{sech}^3(x)}}\\
&= \frac{2\cosh^2(x)\sinh(x)}{9a\sqrt{a\text{sech}^3(x)}} + \frac{\left(7\text{sech}^{\frac{3}{2}}(x)\right)\int \frac{1}{\text{sech}^{\frac{5}{2}}(x)}\,dx}{9a\sqrt{a\text{sech}^3(x)}}\\
&= \frac{14\sinh(x)}{45a\sqrt{a\text{sech}^3(x)}} + \frac{2\cosh^2(x)\sinh(x)}{9a\sqrt{a\text{sech}^3(x)}} + \frac{\left(7\text{sech}^{\frac{3}{2}}(x)\right)\int \frac{1}{\sqrt{\text{sech}(x)}}\,dx}{15a\sqrt{a\text{sech}^3(x)}}\\
&= \frac{14\sinh(x)}{45a\sqrt{a\text{sech}^3(x)}} + \frac{2\cosh^2(x)\sinh(x)}{9a\sqrt{a\text{sech}^3(x)}} + \frac{7\int \sqrt{\cosh(x)}\,dx}{15a\cosh^{\frac{3}{2}}(x)\sqrt{a\text{sech}^3(x)}}\\
&= -\frac{14iE\left(\frac{ix}{2}\middle|2\right)}{15a\cosh^{\frac{3}{2}}(x)\sqrt{a\text{sech}^3(x)}} + \frac{14\sinh(x)}{45a\sqrt{a\text{sech}^3(x)}} + \frac{2\cosh^2(x)\sinh(x)}{9a\sqrt{a\text{sech}^3(x)}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 47, normalized size = 0.61

$$\frac{-\frac{84iE\left(\frac{ix}{2}\middle|2\right)}{\cosh^{\frac{3}{2}}(x)} + 33\sinh(x) + 5\sinh(3x)}{90a\sqrt{a\text{sech}^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x]^3)^(-3/2),x]
[Out] (((-84*I)*EllipticE[(I/2)*x, 2])/Cosh[x]^(3/2) + 33*Sinh[x] + 5*Sinh[3*x])/
(90*a*Sqrt[a*Sech[x]^3])
```

Maple [F]
time = 0.84, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\,\text{sech}\left(x\right)^{3}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sech(x)^3)^(3/2),x)
```

```
[Out] int(1/(a*sech(x)^3)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sech(x)^3)^(-3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.16, size = 407, normalized size = 5.29

$$-\frac{672 \sqrt{2} \left(\cosh\left(x\right)^{5} + 5 \cosh\left(x\right)^{4} \sinh\left(x\right) + 10 \cosh\left(x\right)^{3} \sinh\left(x\right)^{2} + 10 \cosh\left(x\right)^{2} \sinh\left(x\right)^{3} + 5 \cosh\left(x\right) \sinh\left(x\right)^{4} + \sinh\left(x\right)^{5}\right) \sqrt{a} {\rm weierstrassZeta}\left(-4, 0, {\rm weierstrassPInverse}\left(-4, 0, \cosh\left(x\right) + \sinh\left(x\right)\right)\right) - \sqrt{2} \left(5 \cosh\left(x\right)^{10} + 50 \cosh\left(x\right) \sinh\left(x\right)^{9} + 5 \sinh\left(x\right)^{10} + \left(225 \cosh\left(x\right)^{2} + 43\right) \sinh\left(x\right)^{8} + 43 \cosh\left(x\right)^{8} + 8 \left(75 \cosh\left(x\right)^{3} + 43 \cosh\left(x\right)\right) \sinh\left(x\right)^{7} + 2 \left(525 \cosh\left(x\right)^{4} + 602 \cosh\left(x\right)^{2} - 149\right) \sinh\left(x\right)^{6} - 298 \cosh\left(x\right)^{6} + 4 \left(315 \cosh\left(x\right)^{5} + 602 \cosh\left(x\right)^{3} - 447 \cosh\left(x\right)\right) \sinh\left(x\right)^{5} + 2 \left(525 \cosh\left(x\right)^{6} + 1505 \cosh\left(x\right)^{4} - 2235 \cosh\left(x\right)^{2} - 187\right) \sinh\left(x\right)^{4} - 374 \cosh\left(x\right)^{4} + 8 \left(75 \cosh\left(x\right)^{7} + 301 \cosh\left(x\right)^{5} - 745 \cosh\left(x\right)^{3} - 187 \cosh\left(x\right)\right) \sinh\left(x\right)^{3} + \left(225 \cosh\left(x\right)^{8} + 1204 \cosh\left(x\right)^{6} - 4470 \cosh\left(x\right)^{4} - 2244 \cosh\left(x\right)^{2} - 43\right) \sinh\left(x\right)^{2} - 43 \cosh\left(x\right)^{2} + 2 \left(25 \cosh\left(x\right)^{9} + 172 \cosh\left(x\right)^{7} - 894 \cosh\left(x\right)^{5} - 748 \cosh\left(x\right)^{3} - 43 \cosh\left(x\right)\right) \sinh\left(x\right) - 5\right) \sqrt{\frac{a \cosh\left(x\right) + a \sinh\left(x\right)}{\cosh\left(x\right)^{2} + 2 \cosh\left(x\right) \sinh\left(x\right) + \sinh\left(x\right)^{2} + 1}}}{720 \left(a^{2} \cosh\left(x\right)^{5} + 5 a^{2} \cosh\left(x\right)^{4} \sinh\left(x\right) + 10 a^{2} \cosh\left(x\right)^{3} \sinh\left(x\right)^{2} + 10 a^{2} \cosh\left(x\right)^{2} \sinh\left(x\right)^{3} + 5 a^{2} \cosh\left(x\right) \sinh\left(x\right)^{4} + a^{2} \sinh\left(x\right)^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/720*(672*sqrt(2)*(cosh(x)^5 + 5*cosh(x)^4*sinh(x) + 10*cosh(x)^3*sinh(x)
^2 + 10*cosh(x)^2*sinh(x)^3 + 5*cosh(x)*sinh(x)^4 + sinh(x)^5)*sqrt(a)*weie
rstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cosh(x) + sinh(x))) - sqrt(2)
*(5*cosh(x)^10 + 50*cosh(x)*sinh(x)^9 + 5*sinh(x)^10 + (225*cosh(x)^2 + 43)
*sinh(x)^8 + 43*cosh(x)^8 + 8*(75*cosh(x)^3 + 43*cosh(x))*sinh(x)^7 + 2*(52
5*cosh(x)^4 + 602*cosh(x)^2 - 149)*sinh(x)^6 - 298*cosh(x)^6 + 4*(315*cosh(
x)^5 + 602*cosh(x)^3 - 447*cosh(x))*sinh(x)^5 + 2*(525*cosh(x)^6 + 1505*cos
h(x)^4 - 2235*cosh(x)^2 - 187)*sinh(x)^4 - 374*cosh(x)^4 + 8*(75*cosh(x)^7
+ 301*cosh(x)^5 - 745*cosh(x)^3 - 187*cosh(x))*sinh(x)^3 + (225*cosh(x)^8 +
 1204*cosh(x)^6 - 4470*cosh(x)^4 - 2244*cosh(x)^2 - 43)*sinh(x)^2 - 43*cosh
(x)^2 + 2*(25*cosh(x)^9 + 172*cosh(x)^7 - 894*cosh(x)^5 - 748*cosh(x)^3 - 4
3*cosh(x))*sinh(x) - 5)*sqrt((a*cosh(x) + a*sinh(x))/(cosh(x)^2 + 2*cosh(x)
*sinh(x) + sinh(x)^2 + 1)))/(a^2*cosh(x)^5 + 5*a^2*cosh(x)^4*sinh(x) + 10*a
^2*cosh(x)^3*sinh(x)^2 + 10*a^2*cosh(x)^2*sinh(x)^3 + 5*a^2*cosh(x)*sinh(x)
^4 + a^2*sinh(x)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a \operatorname{sech}^{3}\left(x\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)**3)**(3/2),x)
[Out] Integral((a*sech(x)**3)**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^3)^(3/2),x, algorithm="giac")
[Out] integrate((a*sech(x)^3)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{a}{\cosh(x)^3}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/cosh(x)^3)^(3/2),x)
[Out] int(1/(a/cosh(x)^3)^(3/2), x)
```

3.44 $\int \frac{1}{\left(a\operatorname{sech}^3(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=121

$$-\frac{26iF\left(\frac{ix}{2}\middle|2\right)}{77a^2\cosh^{\frac{3}{2}}(x)\sqrt{a\operatorname{sech}^3(x)}}+\frac{78\cosh(x)\sinh(x)}{385a^2\sqrt{a\operatorname{sech}^3(x)}}+\frac{26\cosh^3(x)\sinh(x)}{165a^2\sqrt{a\operatorname{sech}^3(x)}}+\frac{2\cosh^5(x)\sinh(x)}{15a^2\sqrt{a\operatorname{sech}^3(x)}}+\frac{26\tanh(x)}{77a^2\sqrt{a\operatorname{sech}^3(x)}}$$

```
[Out] -26/77*I*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh(1/2*x),2^(1/2))
/a^2/cosh(x)^(3/2)/(a*sech(x)^3)^(1/2)+78/385*cosh(x)*sinh(x)/a^2/(a*sech(x
)^3)^(1/2)+26/165*cosh(x)^3*sinh(x)/a^2/(a*sech(x)^3)^(1/2)+2/15*cosh(x)^5*
sinh(x)/a^2/(a*sech(x)^3)^(1/2)+26/77*tanh(x)/a^2/(a*sech(x)^3)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4208, 3854, 3856, 2720}

$$\frac{26\tanh(x)}{77a^2\sqrt{a\operatorname{sech}^3(x)}}-\frac{26iF\left(\frac{ix}{2}\middle|2\right)}{77a^2\cosh^{\frac{3}{2}}(x)\sqrt{a\operatorname{sech}^3(x)}}+\frac{2\sinh(x)\cosh^5(x)}{15a^2\sqrt{a\operatorname{sech}^3(x)}}+\frac{26\sinh(x)\cosh^3(x)}{165a^2\sqrt{a\operatorname{sech}^3(x)}}+\frac{78\sinh(x)\cosh(x)}{385a^2\sqrt{a\operatorname{sech}^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x]^3)^(-5/2),x]
```

```
[Out] (((-26*I)/77)*EllipticF[(I/2)*x, 2])/(a^2*Cosh[x]^(3/2)*Sqrt[a*Sech[x]^3])
+ (78*Cosh[x]*Sinh[x])/(385*a^2*Sqrt[a*Sech[x]^3]) + (26*Cosh[x]^3*Sinh[x])
/(165*a^2*Sqrt[a*Sech[x]^3]) + (2*Cosh[x]^5*Sinh[x])/(15*a^2*Sqrt[a*Sech[x]
^3]) + (26*Tanh[x])/(77*a^2*Sqrt[a*Sech[x]^3])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a\text{sech}^3(x)\right)^{5/2}}\,dx = \frac{\text{sech}^{\frac{3}{2}}(x)\int \frac{1}{\text{sech}^{\frac{15}{2}}(x)}\,dx}{a^2\sqrt{a\text{sech}^3(x)}}$$

$$= \frac{2\cosh^5(x)\sinh(x)}{15a^2\sqrt{a\text{sech}^3(x)}} + \frac{\left(13\text{sech}^{\frac{3}{2}}(x)\right)\int \frac{1}{\text{sech}^{\frac{11}{2}}(x)}\,dx}{15a^2\sqrt{a\text{sech}^3(x)}}$$

$$= \frac{26\cosh^3(x)\sinh(x)}{165a^2\sqrt{a\text{sech}^3(x)}} + \frac{2\cosh^5(x)\sinh(x)}{15a^2\sqrt{a\text{sech}^3(x)}} + \frac{\left(39\text{sech}^{\frac{3}{2}}(x)\right)\int \frac{1}{\text{sech}^{\frac{7}{2}}(x)}\,dx}{55a^2\sqrt{a\text{sech}^3(x)}}$$

$$= \frac{78\cosh(x)\sinh(x)}{385a^2\sqrt{a\text{sech}^3(x)}} + \frac{26\cosh^3(x)\sinh(x)}{165a^2\sqrt{a\text{sech}^3(x)}} + \frac{2\cosh^5(x)\sinh(x)}{15a^2\sqrt{a\text{sech}^3(x)}} + \frac{\left(39\text{sech}^{\frac{3}{2}}(x)\right)\int \cdots}{77a^2\sqrt{a\text{sech}\cdots}}$$

$$= \frac{78\cosh(x)\sinh(x)}{385a^2\sqrt{a\text{sech}^3(x)}} + \frac{26\cosh^3(x)\sinh(x)}{165a^2\sqrt{a\text{sech}^3(x)}} + \frac{2\cosh^5(x)\sinh(x)}{15a^2\sqrt{a\text{sech}^3(x)}} + \frac{26\tanh(x)}{77a^2\sqrt{a\text{sech}^3(x)}\cdots}$$

$$= \frac{78\cosh(x)\sinh(x)}{385a^2\sqrt{a\text{sech}^3(x)}} + \frac{26\cosh^3(x)\sinh(x)}{165a^2\sqrt{a\text{sech}^3(x)}} + \frac{2\cosh^5(x)\sinh(x)}{15a^2\sqrt{a\text{sech}^3(x)}} + \frac{26\tanh(x)}{77a^2\sqrt{a\text{sech}^3(x)}\cdots}$$

$$= -\frac{26iF\left(\frac{ix}{2}\middle|2\right)}{77a^2\cosh^{\frac{3}{2}}(x)\sqrt{a\text{sech}^3(x)}} + \frac{78\cosh(x)\sinh(x)}{385a^2\sqrt{a\text{sech}^3(x)}} + \frac{26\cosh^3(x)\sinh(x)}{165a^2\sqrt{a\text{sech}^3(x)}} + \frac{2\cosh\cdots}{15a^2\sqrt{\cdots}}$$

Mathematica [A]
time = 0.06, size = 63, normalized size = 0.52

$$\frac{\cosh(x)\sqrt{a\text{sech}^3(x)}\left(-24960i\sqrt{\cosh(x)}\,F\left(\frac{ix}{2}\middle|2\right) + 19122\sinh(2x) + 4406\sinh(4x) + 826\sinh(6x) + 77\sinh(8x)\right)}{73920a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x]^3)^(-5/2),x]
```

```
[Out] (Cosh[x]*Sqrt[a*Sech[x]^3]*((-24960*I)*Sqrt[Cosh[x]]*EllipticF[(I/2)*x, 2]
+ 19122*Sinh[2*x] + 4406*Sinh[4*x] + 826*Sinh[6*x] + 77*Sinh[8*x]))/(73920*
a^3)
```

Maple [F]
time = 0.84, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\mathrm{sech}\left(x\right)^{3}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sech(x)^3)^(5/2),x)
```

```
[Out] int(1/(a*sech(x)^3)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sech(x)^3)^(-5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.19, size = 718, normalized size = 5.93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/147840*(49920*sqrt(2)*(cosh(x)^8 + 8*cosh(x)^7*sinh(x) + 28*cosh(x)^6*sin
h(x)^2 + 56*cosh(x)^5*sinh(x)^3 + 70*cosh(x)^4*sinh(x)^4 + 56*cosh(x)^3*sin
h(x)^5 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8)*sqrt(a)*
weierstrassPInverse(-4, 0, cosh(x) + sinh(x)) + sqrt(2)*(77*cosh(x)^16 + 12
32*cosh(x)*sinh(x)^15 + 77*sinh(x)^16 + 14*(660*cosh(x)^2 + 59)*sinh(x)^14
+ 826*cosh(x)^14 + 196*(220*cosh(x)^3 + 59*cosh(x))*sinh(x)^13 + 2*(70070*c
osh(x)^4 + 37583*cosh(x)^2 + 2203)*sinh(x)^12 + 4406*cosh(x)^12 + 8*(42042*
cosh(x)^5 + 37583*cosh(x)^3 + 6609*cosh(x))*sinh(x)^11 + 2*(308308*cosh(x)^
6 + 413413*cosh(x)^4 + 145398*cosh(x)^2 + 9561)*sinh(x)^10 + 19122*cosh(x)^
```

```
10 + 4*(220220*cosh(x)^7 + 413413*cosh(x)^5 + 242330*cosh(x)^3 + 47805*cosh
(x))*sinh(x)^9 + 6*(165165*cosh(x)^8 + 413413*cosh(x)^6 + 363495*cosh(x)^4
+ 143415*cosh(x)^2)*sinh(x)^8 + 16*(55055*cosh(x)^9 + 177177*cosh(x)^7 + 21
8097*cosh(x)^5 + 143415*cosh(x)^3)*sinh(x)^7 + 2*(308308*cosh(x)^10 + 12402
39*cosh(x)^8 + 2035572*cosh(x)^6 + 2007810*cosh(x)^4 - 9561)*sinh(x)^6 - 19
122*cosh(x)^6 + 4*(84084*cosh(x)^11 + 413413*cosh(x)^9 + 872388*cosh(x)^7 +
 1204686*cosh(x)^5 - 28683*cosh(x))*sinh(x)^5 + 2*(70070*cosh(x)^12 + 41341
3*cosh(x)^10 + 1090485*cosh(x)^8 + 2007810*cosh(x)^6 - 143415*cosh(x)^2 - 2
203)*sinh(x)^4 - 4406*cosh(x)^4 + 8*(5390*cosh(x)^13 + 37583*cosh(x)^11 + 1
21165*cosh(x)^9 + 286830*cosh(x)^7 - 47805*cosh(x)^3 - 2203*cosh(x))*sinh(x
)^3 + 2*(4620*cosh(x)^14 + 37583*cosh(x)^12 + 145398*cosh(x)^10 + 430245*co
sh(x)^8 - 143415*cosh(x)^4 - 13218*cosh(x)^2 - 413)*sinh(x)^2 - 826*cosh(x)
^2 + 4*(308*cosh(x)^15 + 2891*cosh(x)^13 + 13218*cosh(x)^11 + 47805*cosh(x)
^9 - 28683*cosh(x)^5 - 4406*cosh(x)^3 - 413*cosh(x))*sinh(x) - 77)*sqrt((a*
cosh(x) + a*sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)))/(a^3
*cosh(x)^8 + 8*a^3*cosh(x)^7*sinh(x) + 28*a^3*cosh(x)^6*sinh(x)^2 + 56*a^3*
cosh(x)^5*sinh(x)^3 + 70*a^3*cosh(x)^4*sinh(x)^4 + 56*a^3*cosh(x)^3*sinh(x)
^5 + 28*a^3*cosh(x)^2*sinh(x)^6 + 8*a^3*cosh(x)*sinh(x)^7 + a^3*sinh(x)^8)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a \operatorname{sech}^{3}(x)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)**3)**(5/2),x)
[Out] Integral((a*sech(x)**3)**(-5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^3)^(5/2),x, algorithm="giac")
[Out] integrate((a*sech(x)^3)^(-5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{a}{\cosh(x)^3}\right)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/cosh(x)^3)^(5/2),x)
```

```
[Out] int(1/(a/cosh(x)^3)^(5/2), x)
```

3.45 $\int \left(a\text{sech}^4(x)\right)^{7/2} dx$

Optimal. Leaf size=163

$$a^3\cosh(x)\sqrt{a\text{sech}^4(x)}\,\sinh(x)-2a^3\sqrt{a\text{sech}^4(x)}\,\sinh^2(x)\tanh(x)+3a^3\sqrt{a\text{sech}^4(x)}\,\sinh^2(x)\tanh^3(x)-\frac{20}{7}$$

```
[Out] a^3*cosh(x)*sinh(x)*(a*sech(x)^4)^(1/2)-2*a^3*sinh(x)^2*(a*sech(x)^4)^(1/2)
*tanh(x)+3*a^3*sinh(x)^2*(a*sech(x)^4)^(1/2)*tanh(x)^3-20/7*a^3*sinh(x)^2*(
a*sech(x)^4)^(1/2)*tanh(x)^5+5/3*a^3*sinh(x)^2*(a*sech(x)^4)^(1/2)*tanh(x)^
7-6/11*a^3*sinh(x)^2*(a*sech(x)^4)^(1/2)*tanh(x)^9+1/13*a^3*sinh(x)^2*(a*se
ch(x)^4)^(1/2)*tanh(x)^11
```

Rubi [A]
time = 0.03, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4208, 3852}

$$a^3\sinh(x)\cosh(x)\sqrt{a\text{sech}^4(x)}+\frac{1}{13}a^3\sinh^2(x)\tanh^{11}(x)\sqrt{a\text{sech}^4(x)}-\frac{6}{11}a^3\sinh^2(x)\tanh^9(x)\sqrt{a\text{sech}^4(x)}+\frac{5}{3}a^3\sinh^2(x)\tanh^7(x)\sqrt{a\text{sech}^4(x)}-\frac{20}{7}a^3\sinh^2(x)\tanh^5(x)\sqrt{a\text{sech}^4(x)}+3a^3\sinh^2(x)\tanh^3(x)\sqrt{a\text{sech}^4(x)}-2a^3\sinh^2(x)\tanh(x)\sqrt{a\text{sech}^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x]^4)^(7/2),x]
```

```
[Out] a^3*Cosh[x]*Sqrt[a*Sech[x]^4]*Sinh[x] - 2*a^3*Sqrt[a*Sech[x]^4]*Sinh[x]^2*T
anh[x] + 3*a^3*Sqrt[a*Sech[x]^4]*Sinh[x]^2*Tanh[x]^3 - (20*a^3*Sqrt[a*Sech[
x]^4]*Sinh[x]^2*Tanh[x]^5)/7 + (5*a^3*Sqrt[a*Sech[x]^4]*Sinh[x]^2*Tanh[x]^7
)/3 - (6*a^3*Sqrt[a*Sech[x]^4]*Sinh[x]^2*Tanh[x]^9)/11 + (a^3*Sqrt[a*Sech[x
]^4]*Sinh[x]^2*Tanh[x]^11)/13
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\int \left(a\text{sech}^4(x)\right)^{7/2}\,dx = \left(a^3\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int \text{sech}^{14}(x)\,dx$$
$$= \left(ia^3\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\text{Subst}\left(\int \left(1+6x^2+15x^4+20x^6+15x^8+6x^{10}+x^{12}\right)\right.$$
$$= a^3\cosh(x)\sqrt{a\text{sech}^4(x)}\,\sinh(x) - 2a^3\sqrt{a\text{sech}^4(x)}\,\sinh^2(x)\tanh(x) + 3a^3\sqrt{a\text{sech}^4(x}$$

Mathematica [A]
time = 0.12, size = 54, normalized size = 0.33

$$\frac{\cosh(x)(2048+2380\cosh(2x)+1093\cosh(4x)+378\cosh(6x)+92\cosh(8x)+14\cosh(10x)+\cosh(12x))\left(a\text{sech}^4(x)\right)^{7/2}\sinh(x)}{6006}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x]^4)^(7/2),x]
```

```
[Out] (Cosh[x]*(2048 + 2380*Cosh[2*x] + 1093*Cosh[4*x] + 378*Cosh[6*x] + 92*Cosh[
8*x] + 14*Cosh[10*x] + Cosh[12*x])*(a*Sech[x]^4)^(7/2)*Sinh[x])/6006
```

Maple [A]
time = 0.99, size = 72, normalized size = 0.44

method	result	size
risch	$-\frac{2048a^3e^{-2x}\sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^4}}\,(1716\,e^{12x}+1287\,e^{10x}+715\,e^{8x}+286\,e^{6x}+78\,e^{4x}+13\,e^{2x}+1)}{3003(1+e^{2x})^{11}}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sech(x)^4)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2048/3003*a^3*exp(-2*x)/(1+exp(2*x))^11*(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*
(1716*exp(12*x)+1287*exp(10*x)+715*exp(8*x)+286*exp(6*x)+78*exp(4*x)+13*exp
(2*x)+1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 620 vs. 2(141) = 282.
time = 0.54, size = 620, normalized size = 3.80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^4)^(7/2),x, algorithm="maxima")
```

```
[Out] 2048/231*a^(7/2)*e^(-2*x)/(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-6*x) + 715*e
^(-8*x) + 1287*e^(-10*x) + 1716*e^(-12*x) + 1716*e^(-14*x) + 1287*e^(-16*x)
 + 715*e^(-18*x) + 286*e^(-20*x) + 78*e^(-22*x) + 13*e^(-24*x) + e^(-26*x)
+ 1) + 4096/77*a^(7/2)*e^(-4*x)/(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-6*x) +
 715*e^(-8*x) + 1287*e^(-10*x) + 1716*e^(-12*x) + 1716*e^(-14*x) + 1287*e^(
-16*x) + 715*e^(-18*x) + 286*e^(-20*x) + 78*e^(-22*x) + 13*e^(-24*x) + e^(-
26*x) + 1) + 4096/21*a^(7/2)*e^(-6*x)/(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-
6*x) + 715*e^(-8*x) + 1287*e^(-10*x) + 1716*e^(-12*x) + 1716*e^(-14*x) + 12
87*e^(-16*x) + 715*e^(-18*x) + 286*e^(-20*x) + 78*e^(-22*x) + 13*e^(-24*x)
+ e^(-26*x) + 1) + 10240/21*a^(7/2)*e^(-8*x)/(13*e^(-2*x) + 78*e^(-4*x) + 2
86*e^(-6*x) + 715*e^(-8*x) + 1287*e^(-10*x) + 1716*e^(-12*x) + 1716*e^(-14*
x) + 1287*e^(-16*x) + 715*e^(-18*x) + 286*e^(-20*x) + 78*e^(-22*x) + 13*e^(
-24*x) + e^(-26*x) + 1) + 6144/7*a^(7/2)*e^(-10*x)/(13*e^(-2*x) + 78*e^(-4*
x) + 286*e^(-6*x) + 715*e^(-8*x) + 1287*e^(-10*x) + 1716*e^(-12*x) + 1716*e
^(-14*x) + 1287*e^(-16*x) + 715*e^(-18*x) + 286*e^(-20*x) + 78*e^(-22*x) +
13*e^(-24*x) + e^(-26*x) + 1) + 8192/7*a^(7/2)*e^(-12*x)/(13*e^(-2*x) + 78*
e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) + 1287*e^(-10*x) + 1716*e^(-12*x) +
1716*e^(-14*x) + 1287*e^(-16*x) + 715*e^(-18*x) + 286*e^(-20*x) + 78*e^(-22
*x) + 13*e^(-24*x) + e^(-26*x) + 1) + 2048/3003*a^(7/2)/(13*e^(-2*x) + 78*e
^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) + 1287*e^(-10*x) + 1716*e^(-12*x) + 1
716*e^(-14*x) + 1287*e^(-16*x) + 715*e^(-18*x) + 286*e^(-20*x) + 78*e^(-22*
x) + 13*e^(-24*x) + e^(-26*x) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2804 vs. $2(141) = 282$.
time = 0.44, size = 2804, normalized size = 17.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^4)^(7/2),x, algorithm="fricas")
```

```
[Out] -2048/3003*(1716*a^3*cosh(x)^12 + 1287*a^3*cosh(x)^10 + 1716*(a^3*e^(4*x) +
 2*a^3*e^(2*x) + a^3)*sinh(x)^12 + 20592*(a^3*cosh(x)*e^(4*x) + 2*a^3*cosh(
x)*e^(2*x) + a^3*cosh(x))*sinh(x)^11 + 715*a^3*cosh(x)^8 + 1287*(88*a^3*cos
h(x)^2 + a^3 + (88*a^3*cosh(x)^2 + a^3)*e^(4*x) + 2*(88*a^3*cosh(x)^2 + a^3
)*e^(2*x))*sinh(x)^10 + 4290*(88*a^3*cosh(x)^3 + 3*a^3*cosh(x) + (88*a^3*co
sh(x)^3 + 3*a^3*cosh(x))*e^(4*x) + 2*(88*a^3*cosh(x)^3 + 3*a^3*cosh(x))*e^(
2*x))*sinh(x)^9 + 286*a^3*cosh(x)^6 + 715*(1188*a^3*cosh(x)^4 + 81*a^3*cosh
(x)^2 + a^3 + (1188*a^3*cosh(x)^4 + 81*a^3*cosh(x)^2 + a^3)*e^(4*x) + 2*(11
88*a^3*cosh(x)^4 + 81*a^3*cosh(x)^2 + a^3)*e^(2*x))*sinh(x)^8 + 1144*(1188*
a^3*cosh(x)^5 + 135*a^3*cosh(x)^3 + 5*a^3*cosh(x) + (1188*a^3*cosh(x)^5 + 1
35*a^3*cosh(x)^3 + 5*a^3*cosh(x))*e^(4*x) + 2*(1188*a^3*cosh(x)^5 + 135*a^3
*cosh(x)^3 + 5*a^3*cosh(x))*e^(2*x))*sinh(x)^7 + 78*a^3*cosh(x)^4 + 286*(55
44*a^3*cosh(x)^6 + 945*a^3*cosh(x)^4 + 70*a^3*cosh(x)^2 + a^3 + (5544*a^3*c
```

osh(x)^6 + 945*a^3*cosh(x)^4 + 70*a^3*cosh(x)^2 + a^3)*e^(4*x) + 2*(5544*a^
3*cosh(x)^6 + 945*a^3*cosh(x)^4 + 70*a^3*cosh(x)^2 + a^3)*e^(2*x))*sinh(x)^
6 + 572*(2376*a^3*cosh(x)^7 + 567*a^3*cosh(x)^5 + 70*a^3*cosh(x)^3 + 3*a^3*
cosh(x) + (2376*a^3*cosh(x)^7 + 567*a^3*cosh(x)^5 + 70*a^3*cosh(x)^3 + 3*a^
3*cosh(x))*e^(4*x) + 2*(2376*a^3*cosh(x)^7 + 567*a^3*cosh(x)^5 + 70*a^3*cos
h(x)^3 + 3*a^3*cosh(x))*e^(2*x))*sinh(x)^5 + 13*a^3*cosh(x)^2 + 26*(32670*a
^3*cosh(x)^8 + 10395*a^3*cosh(x)^6 + 1925*a^3*cosh(x)^4 + 165*a^3*cosh(x)^2
 + 3*a^3 + (32670*a^3*cosh(x)^8 + 10395*a^3*cosh(x)^6 + 1925*a^3*cosh(x)^4
+ 165*a^3*cosh(x)^2 + 3*a^3)*e^(4*x) + 2*(32670*a^3*cosh(x)^8 + 10395*a^3*c
osh(x)^6 + 1925*a^3*cosh(x)^4 + 165*a^3*cosh(x)^2 + 3*a^3)*e^(2*x))*sinh(x)
^4 + 104*(3630*a^3*cosh(x)^9 + 1485*a^3*cosh(x)^7 + 385*a^3*cosh(x)^5 + 55*
a^3*cosh(x)^3 + 3*a^3*cosh(x) + (3630*a^3*cosh(x)^9 + 1485*a^3*cosh(x)^7 +
385*a^3*cosh(x)^5 + 55*a^3*cosh(x)^3 + 3*a^3*cosh(x))*e^(4*x) + 2*(3630*a^3
*cosh(x)^9 + 1485*a^3*cosh(x)^7 + 385*a^3*cosh(x)^5 + 55*a^3*cosh(x)^3 + 3*
a^3*cosh(x))*e^(2*x))*sinh(x)^3 + a^3 + 13*(8712*a^3*cosh(x)^10 + 4455*a^3*
cosh(x)^8 + 1540*a^3*cosh(x)^6 + 330*a^3*cosh(x)^4 + 36*a^3*cosh(x)^2 + a^3
 + (8712*a^3*cosh(x)^10 + 4455*a^3*cosh(x)^8 + 1540*a^3*cosh(x)^6 + 330*a^3
*cosh(x)^4 + 36*a^3*cosh(x)^2 + a^3)*e^(4*x) + 2*(8712*a^3*cosh(x)^10 + 445
5*a^3*cosh(x)^8 + 1540*a^3*cosh(x)^6 + 330*a^3*cosh(x)^4 + 36*a^3*cosh(x)^2
 + a^3)*e^(2*x))*sinh(x)^2 + (1716*a^3*cosh(x)^12 + 1287*a^3*cosh(x)^10 + 7
15*a^3*cosh(x)^8 + 286*a^3*cosh(x)^6 + 78*a^3*cosh(x)^4 + 13*a^3*cosh(x)^2
+ a^3)*e^(4*x) + 2*(1716*a^3*cosh(x)^12 + 1287*a^3*cosh(x)^10 + 715*a^3*cos
h(x)^8 + 286*a^3*cosh(x)^6 + 78*a^3*cosh(x)^4 + 13*a^3*cosh(x)^2 + a^3)*e^(
2*x) + 26*(792*a^3*cosh(x)^11 + 495*a^3*cosh(x)^9 + 220*a^3*cosh(x)^7 + 66*
a^3*cosh(x)^5 + 12*a^3*cosh(x)^3 + a^3*cosh(x) + (792*a^3*cosh(x)^11 + 495*
a^3*cosh(x)^9 + 220*a^3*cosh(x)^7 + 66*a^3*cosh(x)^5 + 12*a^3*cosh(x)^3 + a
^3*cosh(x))*e^(4*x) + 2*(792*a^3*cosh(x)^11 + 495*a^3*cosh(x)^9 + 220*a^3*c
osh(x)^7 + 66*a^3*cosh(x)^5 + 12*a^3*cosh(x)^3 + a^3*cosh(x))*e^(2*x))*sinh
(x))*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)/(26*
cosh(x)*e^(2*x)*sinh(x)^25 + e^(2*x)*sinh(x)^26 + 13*(25*cosh(x)^2 + 1)*e^(
2*x)*sinh(x)^24 + 104*(25*cosh(x)^3 + 3*cosh(x))*e^(2*x)*sinh(x)^23 + 26*(5
75*cosh(x)^4 + 138*cosh(x)^2 + 3)*e^(2*x)*sinh(x)^22 + 572*(115*cosh(x)^5 +
 46*cosh(x)^3 + 3*cosh(x))*e^(2*x)*sinh(x)^21 + 286*(805*cosh(x)^6 + 483*co
sh(x)^4 + 63*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^20 + 1144*(575*cosh(x)^7 + 483*
cosh(x)^5 + 105*cosh(x)^3 + 5*cosh(x))*e^(2*x)*sinh(x)^19 + 143*(10925*cosh
(x)^8 + 12236*cosh(x)^6 + 3990*cosh(x)^4 + 380*cosh(x)^2 + 5)*e^(2*x)*sinh(
x)^18 + 286*(10925*cosh(x)^9 + 15732*cosh(x)^7 + 7182*cosh(x)^5 + 1140*cosh
(x)^3 + 45*cosh(x))*e^(2*x)*sinh(x)^17 + 143*(37145*cosh(x)^10 + 66861*cosh
(x)^8 + 40698*cosh(x)^6 + 9690*cosh(x)^4 + 765*cosh(x)^2 + 9)*e^(2*x)*sinh(
x)^16 + 208*(37145*cosh(x)^11 + 81719*cosh(x)^9 + 63954*cosh(x)^7 + 21318*c
osh(x)^5 + 2805*cosh(x)^3 + 99*cosh(x))*e^(2*x)*sinh(x)^15 + 52*(185725*cos
h(x)^12 + 490314*cosh(x)^10 + 479655*cosh(x)^8 + 213180*cosh(x)^6 + 42075*c
osh(x)^4 + 2970*cosh(x)^2 + 33)*e^(2*x)*sinh(x)^14 + 8*(1300075*cosh(x)^13
+ 4056234*cosh(x)^11 + 4849845*cosh(x)^9 + 2771340*cosh(x)^7 + 765765*cosh(
x)^5 + 90090*cosh(x)^3 + 3003*cosh(x))*e^(2*x)*sinh(x)^13 + 52*(185725*cosh

```
(x)^14 + 676039*cosh(x)^12 + 969969*cosh(x)^10 + 692835*cosh(x)^8 + 255255*
cosh(x)^6 + 45045*cosh(x)^4 + 3003*cosh(x)^2 + 33)*e^(2*x)*sinh(x)^12 + 208
*(37145*cosh(x)^15 + 156009*cosh(x)^13 + 264537*cosh(x)^11 + 230945*cosh(x)
^9 + 109395*cosh(x)^7 + 27027*cosh(x)^5 + 3003*cosh(x)^3 + 99*cosh(x))*e^(2
*x)*sinh(x)^11 + 143*(37145*cosh(x)^16 + 178296*cosh(x)^14 + 352716*cosh(x)
^12 + 369512*cosh(x)^10 + 218790*cosh(x)^8 + 72072*cosh(x)^6 + 12012*cosh(x
)^4 + 792*cosh(x)^2 + 9)*e^(2*x)*sinh(x)^10 + 286*(10925*cosh(x)^17 + 59432
*cosh(x)^15 + 135660*cosh(x)^13 + 167960*cosh(x)^11 + 121550*cosh(x)^9 + 51
480*cosh(x)^7 + 12012*cosh(x)^5 + 1320*cosh(x)^...
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)**4)**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [**A**]
time = 0.39, size = 51, normalized size = 0.31

$$-\frac{2048\,a^{\frac{7}{2}}\left(1716\,e^{(12\,x)}+1287\,e^{(10\,x)}+715\,e^{(8\,x)}+286\,e^{(6\,x)}+78\,e^{(4\,x)}+13\,e^{(2\,x)}+1\right)}{3003\,{\left(e^{(2\,x)}+1\right)}^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^4)^(7/2),x, algorithm="giac")
```

```
[Out] -2048/3003*a^(7/2)*(1716*e^(12*x) + 1287*e^(10*x) + 715*e^(8*x) + 286*e^(6*
x) + 78*e^(4*x) + 13*e^(2*x) + 1)/(e^(2*x) + 1)^13
```

Mupad [**B**]
time = 1.45, size = 498, normalized size = 3.06

$$\frac{1536\,a^3\,\sqrt{\frac{a}{{\left(\frac{e^{-x}}{2}+\frac{e^x}{2}\right)}^4}}\,\left(4\,e^{2\,x}+6\,e^{4\,x}+4\,e^{6\,x}+e^{8\,x}+1\right)}{{\left(e^{2\,x}+1\right)}^8\,\left(e^{2\,x}+2\,e^{4\,x}+e^{6\,x}\right)}-\frac{2048\,a^3\,\sqrt{\frac{a}{{\left(\frac{e^{-x}}{2}+\frac{e^x}{2}\right)}^4}}\,\left(4\,e^{2\,x}+6\,e^{4\,x}+4\,e^{6\,x}+e^{8\,x}+1\right)}{7\,{\left(e^{2\,x}+1\right)}^7\,\left(e^{2\,x}+2\,e^{4\,x}+e^{6\,x}\right)}-\frac{10240\,a^3\,\sqrt{\frac{a}{{\left(\frac{e^{-x}}{2}+\frac{e^x}{2}\right)}^4}}\,\left(4\,e^{2\,x}+6\,e^{4\,x}+4\,e^{6\,x}+e^{8\,x}+1\right)}{3\,{\left(e^{2\,x}+1\right)}^9\,\left(e^{2\,x}+2\,e^{4\,x}+e^{6\,x}\right)}+\frac{4096\,a^3\,\sqrt{\frac{a}{{\left(\frac{e^{-x}}{2}+\frac{e^x}{2}\right)}^4}}\,\left(4\,e^{2\,x}+6\,e^{4\,x}+4\,e^{6\,x}+e^{8\,x}+1\right)}{{\left(e^{2\,x}+1\right)}^{10}\,\left(e^{2\,x}+2\,e^{4\,x}+e^{6\,x}\right)}-\frac{30720\,a^3\,\sqrt{\frac{a}{{\left(\frac{e^{-x}}{2}+\frac{e^x}{2}\right)}^4}}\,\left(4\,e^{2\,x}+6\,e^{4\,x}+4\,e^{6\,x}+e^{8\,x}+1\right)}{11\,{\left(e^{2\,x}+1\right)}^{11}\,\left(e^{2\,x}+2\,e^{4\,x}+e^{6\,x}\right)}+\frac{1024\,a^3\,\sqrt{\frac{a}{{\left(\frac{e^{-x}}{2}+\frac{e^x}{2}\right)}^4}}\,\left(4\,e^{2\,x}+6\,e^{4\,x}+4\,e^{6\,x}+e^{8\,x}+1\right)}{{\left(e^{2\,x}+1\right)}^{12}\,\left(e^{2\,x}+2\,e^{4\,x}+e^{6\,x}\right)}-\frac{2048\,a^3\,\sqrt{\frac{a}{{\left(\frac{e^{-x}}{2}+\frac{e^x}{2}\right)}^4}}\,\left(4\,e^{2\,x}+6\,e^{4\,x}+4\,e^{6\,x}+e^{8\,x}+1\right)}{13\,{\left(e^{2\,x}+1\right)}^{13}\,\left(e^{2\,x}+2\,e^{4\,x}+e^{6\,x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/cosh(x)^4)^(7/2),x)
```

```
[Out] (1536*a^3*(a/(exp(-x)/2 + exp(x)/2)^4)^(1/2)*(4*exp(2*x) + 6*exp(4*x) + 4*e
xp(6*x) + exp(8*x) + 1))/((exp(2*x) + 1)^8*(exp(2*x) + 2*exp(4*x) + exp(6*x
))) - (2048*a^3*(a/(exp(-x)/2 + exp(x)/2)^4)^(1/2)*(4*exp(2*x) + 6*exp(4*x)
 + 4*exp(6*x) + exp(8*x) + 1))/(7*(exp(2*x) + 1)^7*(exp(2*x) + 2*exp(4*x) +
 exp(6*x))) - (10240*a^3*(a/(exp(-x)/2 + exp(x)/2)^4)^(1/2)*(4*exp(2*x) + 6
*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1))/(3*(exp(2*x) + 1)^9*(exp(2*x) + 2*e
```

```
xp(4*x) + exp(6*x))) + (4096*a^3*(a/(exp(-x)/2 + exp(x)/2)^4)^(1/2)*(4*exp(
2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1))/((exp(2*x) + 1)^10*(exp(2*x
) + 2*exp(4*x) + exp(6*x))) - (30720*a^3*(a/(exp(-x)/2 + exp(x)/2)^4)^(1/2)
*(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1))/(11*(exp(2*x) + 1)^
11*(exp(2*x) + 2*exp(4*x) + exp(6*x))) + (1024*a^3*(a/(exp(-x)/2 + exp(x)/2
)^4)^(1/2)*(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1))/((exp(2*x
) + 1)^12*(exp(2*x) + 2*exp(4*x) + exp(6*x))) - (2048*a^3*(a/(exp(-x)/2 + e
xp(x)/2)^4)^(1/2)*(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1))/(1
3*(exp(2*x) + 1)^13*(exp(2*x) + 2*exp(4*x) + exp(6*x)))
```

3.46 $\int \left(a\,\text{sech}^4(x)\right)^{5/2}\,dx$

Optimal. Leaf size=117

$$a^2\cosh(x)\sqrt{a\,\text{sech}^4(x)}\,\sinh(x)-\frac{4}{3}a^2\sqrt{a\,\text{sech}^4(x)}\,\sinh^2(x)\tanh(x)+\frac{6}{5}a^2\sqrt{a\,\text{sech}^4(x)}\,\sinh^2(x)\tanh^3(x)-\frac{4}{7}$$

```
[Out] a^2*cosh(x)*sinh(x)*(a*sech(x)^4)^(1/2)-4/3*a^2*sinh(x)^2*(a*sech(x)^4)^(1/
2)*tanh(x)+6/5*a^2*sinh(x)^2*(a*sech(x)^4)^(1/2)*tanh(x)^3-4/7*a^2*sinh(x)^
2*(a*sech(x)^4)^(1/2)*tanh(x)^5+1/9*a^2*sinh(x)^2*(a*sech(x)^4)^(1/2)*tanh(
x)^7
```

Rubi [A]
time = 0.02, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4208, 3852}

$$a^2\sinh(x)\cosh(x)\sqrt{a\,\text{sech}^4(x)}+\frac{1}{9}a^2\sinh^2(x)\tanh^7(x)\sqrt{a\,\text{sech}^4(x)}-\frac{4}{7}a^2\sinh^2(x)\tanh^5(x)\sqrt{a\,\text{sech}^4(x)}+\frac{6}{5}a^2\sinh^2(x)\tanh^3(x)\sqrt{a\,\text{sech}^4(x)}-\frac{4}{3}a^2\sinh^2(x)\tanh(x)\sqrt{a\,\text{sech}^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x]^4)^(5/2),x]
```

```
[Out] a^2*Cosh[x]*Sqrt[a*Sech[x]^4]*Sinh[x] - (4*a^2*Sqrt[a*Sech[x]^4]*Sinh[x]^2*
Tanh[x])/3 + (6*a^2*Sqrt[a*Sech[x]^4]*Sinh[x]^2*Tanh[x]^3)/5 - (4*a^2*Sqrt[
a*Sech[x]^4]*Sinh[x]^2*Tanh[x]^5)/7 + (a^2*Sqrt[a*Sech[x]^4]*Sinh[x]^2*Tanh
[x]^7)/9
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
& !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(a\operatorname{sech}^4(x)\right)^{5/2}\,dx &= \left(a^2\cosh^2(x)\sqrt{a\operatorname{sech}^4(x)}\right)\int \operatorname{sech}^{10}(x)\,dx\\ &= \left(ia^2\cosh^2(x)\sqrt{a\operatorname{sech}^4(x)}\right)\operatorname{Subst}\left(\int \left(1+4x^2+6x^4+4x^6+x^8\right)\,dx, x, -i\tanh(x)\right.\\ &= a^2\cosh(x)\sqrt{a\operatorname{sech}^4(x)}\,\sinh(x)-\frac{4}{3}a^2\sqrt{a\operatorname{sech}^4(x)}\,\sinh^2(x)\tanh(x)+\frac{6}{5}a^2\sqrt{a\operatorname{sech}^4(}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 42, normalized size = 0.36

$$\frac{1}{315}\cosh(x)(128+130\cosh(2x)+46\cosh(4x)+10\cosh(6x)+\cosh(8x))\left(a\operatorname{sech}^4(x)\right)^{5/2}\sinh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x]^4)^(5/2),x]
```

```
[Out] (Cosh[x]*(128 + 130*Cosh[2*x] + 46*Cosh[4*x] + 10*Cosh[6*x] + Cosh[8*x])*(a
*Sech[x]^4)^(5/2)*Sinh[x])/315
```

Maple [A]
time = 0.86, size = 60, normalized size = 0.51

method	result	size
risch	$-\frac{256a^2\mathrm{e}^{-2x}\sqrt{\frac{a\,\mathrm{e}^{4x}}{(1+\mathrm{e}^{2x})^4}}\,\left(126\,\mathrm{e}^{8x}+84\,\mathrm{e}^{6x}+36\,\mathrm{e}^{4x}+9\,\mathrm{e}^{2x}+1\right)}{315(1+\mathrm{e}^{2x})^7}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sech(x)^4)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -256/315*a^2*exp(-2*x)/(1+exp(2*x))^7*(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*(12
6*exp(8*x)+84*exp(6*x)+36*exp(4*x)+9*exp(2*x)+1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 322 vs. 2(99) = 198.
time = 0.49, size = 322, normalized size = 2.75

$$\frac{256\,a^{\frac{5}{2}}e^{(-2x)}}{35\,(9\,e^{(-2x)}+36\,e^{(-4x)}+84\,e^{(-6x)}+126\,e^{(-8x)}+126\,e^{(-10x)}+84\,e^{(-12x)}+36\,e^{(-14x)}+9\,e^{(-16x)}+e^{(-18x)}+1)}+\frac{1024\,a^{\frac{5}{2}}e^{(-4x)}}{35\,(9\,e^{(-2x)}+36\,e^{(-4x)}+84\,e^{(-6x)}+126\,e^{(-8x)}+126\,e^{(-10x)}+84\,e^{(-12x)}+36\,e^{(-14x)}+9\,e^{(-16x)}+e^{(-18x)}+1)}+\frac{1024\,a^{\frac{5}{2}}e^{(-6x)}}{15\,(9\,e^{(-2x)}+36\,e^{(-4x)}+84\,e^{(-6x)}+126\,e^{(-8x)}+126\,e^{(-10x)}+84\,e^{(-12x)}+36\,e^{(-14x)}+9\,e^{(-16x)}+e^{(-18x)}+1)}+\frac{512\,a^{\frac{5}{2}}e^{(-8x)}}{5\,(9\,e^{(-2x)}+36\,e^{(-4x)}+84\,e^{(-6x)}+126\,e^{(-8x)}+126\,e^{(-10x)}+84\,e^{(-12x)}+36\,e^{(-14x)}+9\,e^{(-16x)}+e^{(-18x)}+1)}+\frac{256\,a^{\frac{5}{2}}}{315\,(9\,e^{(-2x)}+36\,e^{(-4x)}+84\,e^{(-6x)}+126\,e^{(-8x)}+126\,e^{(-10x)}+84\,e^{(-12x)}+36\,e^{(-14x)}+9\,e^{(-16x)}+e^{(-18x)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] 256/35*a^(5/2)*e^(-2*x)/(9*e^(-2*x) + 36*e^(-4*x) + 84*e^(-6*x) + 126*e^(-8
*x) + 126*e^(-10*x) + 84*e^(-12*x) + 36*e^(-14*x) + 9*e^(-16*x) + e^(-18*x)
 + 1) + 1024/35*a^(5/2)*e^(-4*x)/(9*e^(-2*x) + 36*e^(-4*x) + 84*e^(-6*x) +
126*e^(-8*x) + 126*e^(-10*x) + 84*e^(-12*x) + 36*e^(-14*x) + 9*e^(-16*x) +
e^(-18*x) + 1) + 1024/15*a^(5/2)*e^(-6*x)/(9*e^(-2*x) + 36*e^(-4*x) + 84*e^
(-6*x) + 126*e^(-8*x) + 126*e^(-10*x) + 84*e^(-12*x) + 36*e^(-14*x) + 9*e^(
-16*x) + e^(-18*x) + 1) + 512/5*a^(5/2)*e^(-8*x)/(9*e^(-2*x) + 36*e^(-4*x)
+ 84*e^(-6*x) + 126*e^(-8*x) + 126*e^(-10*x) + 84*e^(-12*x) + 36*e^(-14*x)
+ 9*e^(-16*x) + e^(-18*x) + 1) + 256/315*a^(5/2)/(9*e^(-2*x) + 36*e^(-4*x)
+ 84*e^(-6*x) + 126*e^(-8*x) + 126*e^(-10*x) + 84*e^(-12*x) + 36*e^(-14*x)
+ 9*e^(-16*x) + e^(-18*x) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1475 vs. $2(99) = 198$.
time = 0.39, size = 1475, normalized size = 12.61

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] -256/315*(126*a^2*cosh(x)^8 + 126*(a^2*e^(4*x) + 2*a^2*e^(2*x) + a^2)*sinh(
x)^8 + 84*a^2*cosh(x)^6 + 1008*(a^2*cosh(x)*e^(4*x) + 2*a^2*cosh(x)*e^(2*x)
 + a^2*cosh(x))*sinh(x)^7 + 84*(42*a^2*cosh(x)^2 + a^2 + (42*a^2*cosh(x)^2
+ a^2)*e^(4*x) + 2*(42*a^2*cosh(x)^2 + a^2)*e^(2*x))*sinh(x)^6 + 36*a^2*cos
h(x)^4 + 504*(14*a^2*cosh(x)^3 + a^2*cosh(x) + (14*a^2*cosh(x)^3 + a^2*cosh
(x))*e^(4*x) + 2*(14*a^2*cosh(x)^3 + a^2*cosh(x))*e^(2*x))*sinh(x)^5 + 36*(
245*a^2*cosh(x)^4 + 35*a^2*cosh(x)^2 + a^2 + (245*a^2*cosh(x)^4 + 35*a^2*co
sh(x)^2 + a^2)*e^(4*x) + 2*(245*a^2*cosh(x)^4 + 35*a^2*cosh(x)^2 + a^2)*e^(
2*x))*sinh(x)^4 + 9*a^2*cosh(x)^2 + 48*(147*a^2*cosh(x)^5 + 35*a^2*cosh(x)^
3 + 3*a^2*cosh(x) + (147*a^2*cosh(x)^5 + 35*a^2*cosh(x)^3 + 3*a^2*cosh(x))*
e^(4*x) + 2*(147*a^2*cosh(x)^5 + 35*a^2*cosh(x)^3 + 3*a^2*cosh(x))*e^(2*x))
*sinh(x)^3 + 9*(392*a^2*cosh(x)^6 + 140*a^2*cosh(x)^4 + 24*a^2*cosh(x)^2 +
a^2 + (392*a^2*cosh(x)^6 + 140*a^2*cosh(x)^4 + 24*a^2*cosh(x)^2 + a^2)*e^(4
*x) + 2*(392*a^2*cosh(x)^6 + 140*a^2*cosh(x)^4 + 24*a^2*cosh(x)^2 + a^2)*e^
(2*x))*sinh(x)^2 + a^2 + (126*a^2*cosh(x)^8 + 84*a^2*cosh(x)^6 + 36*a^2*cos
h(x)^4 + 9*a^2*cosh(x)^2 + a^2)*e^(4*x) + 2*(126*a^2*cosh(x)^8 + 84*a^2*cos
h(x)^6 + 36*a^2*cosh(x)^4 + 9*a^2*cosh(x)^2 + a^2)*e^(2*x) + 18*(56*a^2*cos
h(x)^7 + 28*a^2*cosh(x)^5 + 8*a^2*cosh(x)^3 + a^2*cosh(x) + (56*a^2*cosh(x)
^7 + 28*a^2*cosh(x)^5 + 8*a^2*cosh(x)^3 + a^2*cosh(x))*e^(4*x) + 2*(56*a^2*
cosh(x)^7 + 28*a^2*cosh(x)^5 + 8*a^2*cosh(x)^3 + a^2*cosh(x))*e^(2*x))*sinh
(x))*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)/(18*
cosh(x)*e^(2*x)*sinh(x)^17 + e^(2*x)*sinh(x)^18 + 9*(17*cosh(x)^2 + 1)*e^(2
*x)*sinh(x)^16 + 48*(17*cosh(x)^3 + 3*cosh(x))*e^(2*x)*sinh(x)^15 + 36*(85*
cosh(x)^4 + 30*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^14 + 504*(17*cosh(x)^5 + 10*c
```

```
osh(x)^3 + cosh(x))*e^(2*x)*sinh(x)^13 + 84*(221*cosh(x)^6 + 195*cosh(x)^4
+ 39*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^12 + 144*(221*cosh(x)^7 + 273*cosh(x)^5
 + 91*cosh(x)^3 + 7*cosh(x))*e^(2*x)*sinh(x)^11 + 18*(2431*cosh(x)^8 + 4004
*cosh(x)^6 + 2002*cosh(x)^4 + 308*cosh(x)^2 + 7)*e^(2*x)*sinh(x)^10 + 4*(12
155*cosh(x)^9 + 25740*cosh(x)^7 + 18018*cosh(x)^5 + 4620*cosh(x)^3 + 315*co
sh(x))*e^(2*x)*sinh(x)^9 + 18*(2431*cosh(x)^10 + 6435*cosh(x)^8 + 6006*cosh
(x)^6 + 2310*cosh(x)^4 + 315*cosh(x)^2 + 7)*e^(2*x)*sinh(x)^8 + 144*(221*co
sh(x)^11 + 715*cosh(x)^9 + 858*cosh(x)^7 + 462*cosh(x)^5 + 105*cosh(x)^3 +
7*cosh(x))*e^(2*x)*sinh(x)^7 + 84*(221*cosh(x)^12 + 858*cosh(x)^10 + 1287*c
osh(x)^8 + 924*cosh(x)^6 + 315*cosh(x)^4 + 42*cosh(x)^2 + 1)*e^(2*x)*sinh(x
)^6 + 504*(17*cosh(x)^13 + 78*cosh(x)^11 + 143*cosh(x)^9 + 132*cosh(x)^7 +
63*cosh(x)^5 + 14*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x)^5 + 36*(85*cosh(x)^1
4 + 455*cosh(x)^12 + 1001*cosh(x)^10 + 1155*cosh(x)^8 + 735*cosh(x)^6 + 245
*cosh(x)^4 + 35*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^4 + 48*(17*cosh(x)^15 + 105*
cosh(x)^13 + 273*cosh(x)^11 + 385*cosh(x)^9 + 315*cosh(x)^7 + 147*cosh(x)^5
 + 35*cosh(x)^3 + 3*cosh(x))*e^(2*x)*sinh(x)^3 + 9*(17*cosh(x)^16 + 120*cos
h(x)^14 + 364*cosh(x)^12 + 616*cosh(x)^10 + 630*cosh(x)^8 + 392*cosh(x)^6 +
 140*cosh(x)^4 + 24*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^2 + 18*(cosh(x)^17 + 8*c
osh(x)^15 + 28*cosh(x)^13 + 56*cosh(x)^11 + 70*cosh(x)^9 + 56*cosh(x)^7 + 2
8*cosh(x)^5 + 8*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x) + (cosh(x)^18 + 9*cosh
(x)^16 + 36*cosh(x)^14 + 84*cosh(x)^12 + 126*cosh(x)^10 + 126*cosh(x)^8 + 8
4*cosh(x)^6 + 36*cosh(x)^4 + 9*cosh(x)^2 + 1)*e^(2*x))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\,\mathrm{sech}^4\left(x\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)**4)**(5/2),x)
```

```
[Out] Integral((a*sech(x)**4)**(5/2), x)
```

Giac [**A**]
time = 0.42, size = 39, normalized size = 0.33

$$-\frac{256\,a^{\frac{5}{2}}\left(126\,e^{(8\,x)}+84\,e^{(6\,x)}+36\,e^{(4\,x)}+9\,e^{(2\,x)}+1\right)}{315\,\left(e^{(2\,x)}+1\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] -256/315*a^(5/2)*(126*e^(8*x) + 84*e^(6*x) + 36*e^(4*x) + 9*e^(2*x) + 1)/(e
^(2*x) + 1)^9
```

Mupad [B]
time = 1.37, size = 356, normalized size = 3.04

$$\frac{256\,a^2\,\sqrt{\frac{a}{{\left(\frac{{\mathrm{e}}^{-x}}{2}+\frac{{\mathrm{e}}^x}{2}\right)}^4}}\,\left(4\,{\mathrm{e}}^{2\,x}+6\,{\mathrm{e}}^{4\,x}+4\,{\mathrm{e}}^{6\,x}+{\mathrm{e}}^{8\,x}+1\right)}{3\,{\left({\mathrm{e}}^{2\,x}+1\right)}^6\,\left({\mathrm{e}}^{2\,x}+2\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{6\,x}\right)}-\frac{128\,a^2\,\sqrt{\frac{a}{{\left(\frac{{\mathrm{e}}^{-x}}{2}+\frac{{\mathrm{e}}^x}{2}\right)}^4}}\,\left(4\,{\mathrm{e}}^{2\,x}+6\,{\mathrm{e}}^{4\,x}+4\,{\mathrm{e}}^{6\,x}+{\mathrm{e}}^{8\,x}+1\right)}{5\,{\left({\mathrm{e}}^{2\,x}+1\right)}^5\,\left({\mathrm{e}}^{2\,x}+2\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{6\,x}\right)}-\frac{768\,a^2\,\sqrt{\frac{a}{{\left(\frac{{\mathrm{e}}^{-x}}{2}+\frac{{\mathrm{e}}^x}{2}\right)}^4}}\,\left(4\,{\mathrm{e}}^{2\,x}+6\,{\mathrm{e}}^{4\,x}+4\,{\mathrm{e}}^{6\,x}+{\mathrm{e}}^{8\,x}+1\right)}{7\,{\left({\mathrm{e}}^{2\,x}+1\right)}^7\,\left({\mathrm{e}}^{2\,x}+2\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{6\,x}\right)}+\frac{64\,a^2\,\sqrt{\frac{a}{{\left(\frac{{\mathrm{e}}^{-x}}{2}+\frac{{\mathrm{e}}^x}{2}\right)}^4}}\,\left(4\,{\mathrm{e}}^{2\,x}+6\,{\mathrm{e}}^{4\,x}+4\,{\mathrm{e}}^{6\,x}+{\mathrm{e}}^{8\,x}+1\right)}{{\left({\mathrm{e}}^{2\,x}+1\right)}^8\,\left({\mathrm{e}}^{2\,x}+2\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{6\,x}\right)}-\frac{128\,a^2\,\sqrt{\frac{a}{{\left(\frac{{\mathrm{e}}^{-x}}{2}+\frac{{\mathrm{e}}^x}{2}\right)}^4}}\,\left(4\,{\mathrm{e}}^{2\,x}+6\,{\mathrm{e}}^{4\,x}+4\,{\mathrm{e}}^{6\,x}+{\mathrm{e}}^{8\,x}+1\right)}{9\,{\left({\mathrm{e}}^{2\,x}+1\right)}^9\,\left({\mathrm{e}}^{2\,x}+2\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{6\,x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/cosh(x)^4)^(5/2),x)
```

```
[Out] (256*a^2*(a/(exp(-x)/2 + exp(x)/2)^4)^(1/2)*(4*exp(2*x) + 6*exp(4*x) + 4*ex
p(6*x) + exp(8*x) + 1))/(3*(exp(2*x) + 1)^6*(exp(2*x) + 2*exp(4*x) + exp(6*
x))) - (128*a^2*(a/(exp(-x)/2 + exp(x)/2)^4)^(1/2)*(4*exp(2*x) + 6*exp(4*x)
 + 4*exp(6*x) + exp(8*x) + 1))/(5*(exp(2*x) + 1)^5*(exp(2*x) + 2*exp(4*x) +
 exp(6*x))) - (768*a^2*(a/(exp(-x)/2 + exp(x)/2)^4)^(1/2)*(4*exp(2*x) + 6*e
xp(4*x) + 4*exp(6*x) + exp(8*x) + 1))/(7*(exp(2*x) + 1)^7*(exp(2*x) + 2*exp
(4*x) + exp(6*x))) + (64*a^2*(a/(exp(-x)/2 + exp(x)/2)^4)^(1/2)*(4*exp(2*x)
 + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1))/((exp(2*x) + 1)^8*(exp(2*x) + 2
*exp(4*x) + exp(6*x))) - (128*a^2*(a/(exp(-x)/2 + exp(x)/2)^4)^(1/2)*(4*exp
(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1))/(9*(exp(2*x) + 1)^9*(exp(2
*x) + 2*exp(4*x) + exp(6*x)))
```

3.47 $\int \left(a\,\text{sech}^4(x)\right)^{3/2}\,dx$

Optimal. Leaf size=61

$$a\cosh(x)\sqrt{a\,\text{sech}^4(x)}\,\sinh(x)-\frac{2}{3}a\sqrt{a\,\text{sech}^4(x)}\,\sinh^2(x)\tanh(x)+\frac{1}{5}a\sqrt{a\,\text{sech}^4(x)}\,\sinh^2(x)\tanh^3(x)$$

```
[Out] a*cosh(x)*sinh(x)*(a*sech(x)^4)^(1/2)-2/3*a*sinh(x)^2*(a*sech(x)^4)^(1/2)*t
anh(x)+1/5*a*sinh(x)^2*(a*sech(x)^4)^(1/2)*tanh(x)^3
```

Rubi [A]
time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4208, 3852}

$$a\sinh(x)\cosh(x)\sqrt{a\,\text{sech}^4(x)}+\frac{1}{5}a\sinh^2(x)\tanh^3(x)\sqrt{a\,\text{sech}^4(x)}-\frac{2}{3}a\sinh^2(x)\tanh(x)\sqrt{a\,\text{sech}^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x]^4)^(3/2),x]
```

```
[Out] a*Cosh[x]*Sqrt[a*Sech[x]^4]*Sinh[x] - (2*a*Sqrt[a*Sech[x]^4]*Sinh[x]^2*Tanh
[x])/3 + (a*Sqrt[a*Sech[x]^4]*Sinh[x]^2*Tanh[x]^3)/5
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \left(a\,\text{sech}^4(x)\right)^{3/2}\,dx &= \left(a\cosh^2(x)\sqrt{a\,\text{sech}^4(x)}\right)\int \text{sech}^6(x)\,dx\\
&= \left(ia\cosh^2(x)\sqrt{a\,\text{sech}^4(x)}\right)\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx,x,-i\tanh(x)\right)\\
&= a\cosh(x)\sqrt{a\,\text{sech}^4(x)}\,\sinh(x)-\frac{2}{3}a\sqrt{a\,\text{sech}^4(x)}\,\sinh^2(x)\tanh(x)+\frac{1}{5}a\sqrt{a\,\text{sech}^4(x)}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 30, normalized size = 0.49

$$\frac{1}{15}\cosh(x)(8+6\cosh(2x)+\cosh(4x))\left(a\operatorname{sech}^4(x)\right)^{3/2}\sinh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x]^4)^(3/2),x]
```

```
[Out] (Cosh[x]*(8 + 6*Cosh[2*x] + Cosh[4*x])*(a*Sech[x]^4)^(3/2)*Sinh[x])/15
```

Maple [A]
time = 0.86, size = 46, normalized size = 0.75

method	result	size
risch	$-\frac{16a\,\mathrm{e}^{-2x}\sqrt{\frac{a\,\mathrm{e}^{4x}}{(1+\mathrm{e}^{2x})^4}}\,(10\,\mathrm{e}^{4x}+5\,\mathrm{e}^{2x}+1)}{15(1+\mathrm{e}^{2x})^3}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sech(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -16/15*a*exp(-2*x)/(1+exp(2*x))^3*(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*(10*exp
(4*x)+5*exp(2*x)+1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(51) = 102.
time = 0.49, size = 120, normalized size = 1.97

$$\frac{16\,a^{\frac{3}{2}}e^{(-2\,x)}}{3\,(5\,e^{(-2\,x)}+10\,e^{(-4\,x)}+10\,e^{(-6\,x)}+5\,e^{(-8\,x)}+e^{(-10\,x)}+1)}+\frac{32\,a^{\frac{3}{2}}e^{(-4\,x)}}{3\,(5\,e^{(-2\,x)}+10\,e^{(-4\,x)}+10\,e^{(-6\,x)}+5\,e^{(-8\,x)}+e^{(-10\,x)}+1)}+\frac{16\,a^{\frac{3}{2}}}{15\,(5\,e^{(-2\,x)}+10\,e^{(-4\,x)}+10\,e^{(-6\,x)}+5\,e^{(-8\,x)}+e^{(-10\,x)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] 16/3*a^(3/2)*e^(-2*x)/(5*e^(-2*x) + 10*e^(-4*x) + 10*e^(-6*x) + 5*e^(-8*x)
+ e^(-10*x) + 1) + 32/3*a^(3/2)*e^(-4*x)/(5*e^(-2*x) + 10*e^(-4*x) + 10*e^(
-6*x) + 5*e^(-8*x) + e^(-10*x) + 1) + 16/15*a^(3/2)/(5*e^(-2*x) + 10*e^(-4*
x) + 10*e^(-6*x) + 5*e^(-8*x) + e^(-10*x) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 516 vs. 2(51) = 102.
time = 0.37, size = 516, normalized size = 8.46

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] -16/15*(10*a*cosh(x)^4 + 10*(a*e^(4*x) + 2*a*e^(2*x) + a)*sinh(x)^4 + 40*(a
*cosh(x)*e^(4*x) + 2*a*cosh(x)*e^(2*x) + a*cosh(x))*sinh(x)^3 + 5*a*cosh(x)
^2 + 5*(12*a*cosh(x)^2 + (12*a*cosh(x)^2 + a)*e^(4*x) + 2*(12*a*cosh(x)^2 +
 a)*e^(2*x) + a)*sinh(x)^2 + (10*a*cosh(x)^4 + 5*a*cosh(x)^2 + a)*e^(4*x) +
 2*(10*a*cosh(x)^4 + 5*a*cosh(x)^2 + a)*e^(2*x) + 10*(4*a*cosh(x)^3 + a*cos
h(x) + (4*a*cosh(x)^3 + a*cosh(x))*e^(4*x) + 2*(4*a*cosh(x)^3 + a*cosh(x))*
e^(2*x))*sinh(x) + a)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) +
 1))*e^(2*x)/(10*cosh(x)*e^(2*x)*sinh(x)^9 + e^(2*x)*sinh(x)^10 + 5*(9*cosh
(x)^2 + 1)*e^(2*x)*sinh(x)^8 + 40*(3*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x)^7
 + 10*(21*cosh(x)^4 + 14*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^6 + 4*(63*cosh(x)^5
 + 70*cosh(x)^3 + 15*cosh(x))*e^(2*x)*sinh(x)^5 + 10*(21*cosh(x)^6 + 35*cos
h(x)^4 + 15*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^4 + 40*(3*cosh(x)^7 + 7*cosh(x)^
5 + 5*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x)^3 + 5*(9*cosh(x)^8 + 28*cosh(x)^
6 + 30*cosh(x)^4 + 12*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^2 + 10*(cosh(x)^9 + 4*
cosh(x)^7 + 6*cosh(x)^5 + 4*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x) + (cosh(x)
^10 + 5*cosh(x)^8 + 10*cosh(x)^6 + 10*cosh(x)^4 + 5*cosh(x)^2 + 1)*e^(2*x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\operatorname{sech}^4(x)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)**4)**(3/2),x)
```

```
[Out] Integral((a*sech(x)**4)**(3/2), x)
```

Giac [A]
time = 0.40, size = 27, normalized size = 0.44

$$-\frac{16\,a^{\frac{3}{2}}\left(10\,e^{(4\,x)}+5\,e^{(2\,x)}+1\right)}{15\,\left(e^{(2\,x)}+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] -16/15*a^(3/2)*(10*e^(4*x) + 5*e^(2*x) + 1)/(e^(2*x) + 1)^5
```

Mupad [B]
time = 1.34, size = 46, normalized size = 0.75

$$-\frac{4\,a\,\mathrm{e}^{-2\,x}\,\sqrt{\frac{a}{{\left(\frac{\mathrm{e}^{-x}}{2}+\frac{\mathrm{e}^{x}}{2}\right)}^4}}\,\left(5\,\mathrm{e}^{2\,x}+10\,\mathrm{e}^{4\,x}+1\right)}{15\,{\left(\mathrm{e}^{2\,x}+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/cosh(x)^4)^(3/2),x)
```

```
[Out] -(4*a*exp(-2*x)*(a/(exp(-x)/2 + exp(x)/2)^4)^(1/2)*(5*exp(2*x) + 10*exp(4*x
) + 1))/(15*(exp(2*x) + 1)^3)
```

3.48 $\int \sqrt{a\mathrm{sech}^4(x)}\,dx$

Optimal. Leaf size=15

$$\cosh(x)\sqrt{a\,\mathrm{sech}^4(x)}\,\sinh(x)$$

```
[Out] cosh(x)*sinh(x)*(a*sech(x)^4)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4208, 3852, 8}

$$\sinh(x)\cosh(x)\sqrt{a\,\mathrm{sech}^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Sech[x]^4],x]
```

```
[Out] Cosh[x]*Sqrt[a*Sech[x]^4]*Sinh[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a\,\mathrm{sech}^4(x)}\,dx &= \left(\cosh^2(x)\sqrt{a\,\mathrm{sech}^4(x)}\right)\int \mathrm{sech}^2(x)\,dx\\
&= \left(i\cosh^2(x)\sqrt{a\,\mathrm{sech}^4(x)}\right)\mathrm{Subst}\left(\int 1\,dx, x, -i\tanh(x)\right)\\
&= \cosh(x)\sqrt{a\,\mathrm{sech}^4(x)}\,\sinh(x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 1.00

$$\cosh(x)\sqrt{a\text{sech}^4(x)}\,\sinh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Sech[x]^4],x]
```

```
[Out] Cosh[x]*Sqrt[a*Sech[x]^4]*Sinh[x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(13) = 26$.
time = 0.99, size = 29, normalized size = 1.93

method	result	size
risch	$-2\sqrt{\frac{a\,\mathrm{e}^{4x}}{(1+\mathrm{e}^{2x})^4}}\;\mathrm{e}^{-2x}(1+\mathrm{e}^{2x})$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sech(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))
```

Maxima [A]
time = 0.49, size = 13, normalized size = 0.87

$$\frac{2\sqrt{a}}{e^{(-2x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(a)/(e^(-2*x) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(13) = 26$.
time = 0.42, size = 81, normalized size = 5.40

$$-\frac{2\sqrt{\frac{a}{e^{(8x)}+4e^{(6x)}+6e^{(4x)}+4e^{(2x)}+1}}\left(e^{(4x)}+2e^{(2x)}+1\right)e^{(2x)}}{2\cosh(x)e^{(2x)}\sinh(x)+e^{(2x)}\sinh(x)^2+\left(\cosh(x)^2+1\right)e^{(2x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*(e^(4*x) + 2*e
^(2*x) + 1)*e^(2*x)/(2*cosh(x)*e^(2*x)*sinh(x) + e^(2*x)*sinh(x)^2 + (cosh(
x)^2 + 1)*e^(2*x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\,\mathrm{sech}^4\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)**4)**(1/2),x)
```

```
[Out] Integral(sqrt(a*sech(x)**4), x)
```

Giac [A]
time = 0.39, size = 13, normalized size = 0.87

$$-\frac{2\,\sqrt{a}}{e^{\left(2\,x\right)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(a)/(e^(2*x) + 1)
```

Mupad [B]
time = 0.06, size = 71, normalized size = 4.73

$$-\frac{\sqrt{a}\,\sqrt{\frac{1}{{\left(\frac{{\mathrm{e}}^{-x}}{2}+\frac{{\mathrm{e}}^x}{2}\right)}^4}}\,\left(2\,{\mathrm{e}}^{2\,x}+3\,{\mathrm{e}}^{4\,x}+2\,{\mathrm{e}}^{6\,x}+\frac{{\mathrm{e}}^{8\,x}}{2}+\frac{1}{2}\right)}{\left({\mathrm{e}}^{2\,x}+1\right)\,\left({\mathrm{e}}^{2\,x}+2\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{6\,x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/cosh(x)^4)^(1/2),x)
```

```
[Out] -(a^(1/2)*(1/(exp(-x)/2 + exp(x)/2)^4)^(1/2)*(2*exp(2*x) + 3*exp(4*x) + 2*e
xp(6*x) + exp(8*x)/2 + 1/2))/((exp(2*x) + 1)*(exp(2*x) + 2*exp(4*x) + exp(6
*x)))
```

3.49 $\int \frac{1}{\sqrt{a\operatorname{sech}^4(x)}}\,dx$

Optimal. Leaf size=36

$$\frac{x\operatorname{sech}^2(x)}{2\sqrt{a\operatorname{sech}^4(x)}}+\frac{\tanh(x)}{2\sqrt{a\operatorname{sech}^4(x)}}$$

```
[Out] 1/2*x*sech(x)^2/(a*sech(x)^4)^(1/2)+1/2*tanh(x)/(a*sech(x)^4)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4208, 2715, 8}

$$\frac{x\operatorname{sech}^2(x)}{2\sqrt{a\operatorname{sech}^4(x)}}+\frac{\tanh(x)}{2\sqrt{a\operatorname{sech}^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Sech[x]^4],x]
```

```
[Out] (x*Sech[x]^2)/(2*Sqrt[a*Sech[x]^4]) + Tanh[x]/(2*Sqrt[a*Sech[x]^4])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\sqrt{a\text{sech}^4(x)}}\,dx = \frac{\text{sech}^2(x)\int \cosh^2(x)\,dx}{\sqrt{a\text{sech}^4(x)}}$$
$$= \frac{\tanh(x)}{2\sqrt{a\text{sech}^4(x)}} + \frac{\text{sech}^2(x)\int 1\,dx}{2\sqrt{a\text{sech}^4(x)}}$$
$$= \frac{x\text{sech}^2(x)}{2\sqrt{a\text{sech}^4(x)}} + \frac{\tanh(x)}{2\sqrt{a\text{sech}^4(x)}}$$

Mathematica [A]
time = 0.02, size = 23, normalized size = 0.64

$$\frac{x\text{sech}^2(x) + \tanh(x)}{2\sqrt{a\text{sech}^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Sech[x]^4],x]
```

```
[Out] (x*Sech[x]^2 + Tanh[x])/(2*Sqrt[a*Sech[x]^4])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(28) = 56$.
time = 1.04, size = 89, normalized size = 2.47

method	result	size
risch	$\frac{e^{2x}x}{2\sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^4}}\,(1+e^{2x})^2} + \frac{e^{4x}}{8\sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^4}}\,(1+e^{2x})^2} - \frac{1}{8(1+e^{2x})^2\sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^4}}}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sech(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*x+1/8/(a*exp(
4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(4*x)-1/8/(1+exp(2*x))^2/(a*ex
p(4*x)/(1+exp(2*x))^4)^(1/2)
```

Maxima [A]
time = 0.49, size = 30, normalized size = 0.83

$$-\frac{\left(\sqrt{a}\,e^{(-4\,x)} - \sqrt{a}\right)e^{(2\,x)}}{8\,a} + \frac{x}{2\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*(sqrt(a)*e^(-4*x) - sqrt(a))*e^(2*x)/a + 1/2*x/sqrt(a)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(28) = 56$.
time = 0.39, size = 253, normalized size = 7.03

$$\frac{\left(\left(e^{(4x)} + 2e^{(2x)} + 1\right)\sinh(x)^4 + \cosh(x)^4 + 4\left(\cosh(x)e^{(4x)} + 2\cosh(x)e^{(2x)} + \cosh(x)\right)\sinh(x)^3 + 4x\cosh(x)^2 + 2\left(3\cosh(x)^2 + \left(3\cosh(x)^2 + 2x\right)e^{(4x)} + 2\left(3\cosh(x)^2 + 2x\right)e^{(2x)} + 2x\right)\sinh(x)^2 + \left(\cosh(x)^4 + 4x\cosh(x)^2 - 1\right)e^{(4x)} + 2\left(\cosh(x)^4 + 4x\cosh(x)^2 - 1\right)e^{(2x)} + 4\left(\cosh(x)^3 + 2x\cosh(x) + \left(\cosh(x)^3 + 2x\cosh(x)\right)e^{(4x)} + 2\left(\cosh(x)^3 + 2x\cosh(x)\right)e^{(2x)}\right)\sinh(x) - 1\right)\sqrt{\frac{a}{e^{(8x)} + 4e^{(6x)} + 6e^{(4x)} + 4e^{(2x)} + 1}}\,e^{(2x)}}{8\left(a\cosh(x)^2 e^{(2x)} + 2a\cosh(x)e^{(2x)}\sinh(x) + ae^{(2x)}\sinh(x)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*((e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(4*x) +
 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^3 + 4*x*cosh(x)^2 + 2*(3*cosh(x)^2 +
(3*cosh(x)^2 + 2*x)*e^(4*x) + 2*(3*cosh(x)^2 + 2*x)*e^(2*x) + 2*x)*sinh(x)^
2 + (cosh(x)^4 + 4*x*cosh(x)^2 - 1)*e^(4*x) + 2*(cosh(x)^4 + 4*x*cosh(x)^2
- 1)*e^(2*x) + 4*(cosh(x)^3 + 2*x*cosh(x) + (cosh(x)^3 + 2*x*cosh(x))*e^(4*
x) + 2*(cosh(x)^3 + 2*x*cosh(x))*e^(2*x))*sinh(x) - 1)*sqrt(a/(e^(8*x) + 4*
e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)/(a*cosh(x)^2*e^(2*x) + 2*a*co
sh(x)*e^(2*x)*sinh(x) + a*e^(2*x)*sinh(x)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\,\mathrm{sech}^4(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)**4)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*sech(x)**4), x)
```

Giac [A]
time = 0.40, size = 28, normalized size = 0.78

$$-\frac{\left(2e^{(2x)} + 1\right)e^{(-2x)} - 4x - e^{(2x)}}{8\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*((2*e^(2*x) + 1)*e^(-2*x) - 4*x - e^(2*x))/sqrt(a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{\frac{a}{\cosh(x)^4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/cosh(x)^4)^(1/2),x)
[Out] int(1/(a/cosh(x)^4)^(1/2), x)
```

3.50 $\int \frac{1}{\left(a\operatorname{sech}^4(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=86

$$\frac{5x\operatorname{sech}^2(x)}{16a\sqrt{a\operatorname{sech}^4(x)}}+\frac{5\cosh(x)\sinh(x)}{24a\sqrt{a\operatorname{sech}^4(x)}}+\frac{\cosh^3(x)\sinh(x)}{6a\sqrt{a\operatorname{sech}^4(x)}}+\frac{5\tanh(x)}{16a\sqrt{a\operatorname{sech}^4(x)}}$$

```
[Out] 5/16*x*sech(x)^2/a/(a*sech(x)^4)^(1/2)+5/24*cosh(x)*sinh(x)/a/(a*sech(x)^4)
^(1/2)+1/6*cosh(x)^3*sinh(x)/a/(a*sech(x)^4)^(1/2)+5/16*tanh(x)/a/(a*sech(x
)^4)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4208, 2715, 8}

$$\frac{5x\operatorname{sech}^2(x)}{16a\sqrt{a\operatorname{sech}^4(x)}}+\frac{5\tanh(x)}{16a\sqrt{a\operatorname{sech}^4(x)}}+\frac{\sinh(x)\cosh^3(x)}{6a\sqrt{a\operatorname{sech}^4(x)}}+\frac{5\sinh(x)\cosh(x)}{24a\sqrt{a\operatorname{sech}^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x]^4)^(-3/2),x]
```

```
[Out] (5*x*Sech[x]^2)/(16*a*Sqrt[a*Sech[x]^4]) + (5*Cosh[x]*Sinh[x])/(24*a*Sqrt[a
*Sech[x]^4]) + (Cosh[x]^3*Sinh[x])/(6*a*Sqrt[a*Sech[x]^4]) + (5*Tanh[x])/(1
6*a*Sqrt[a*Sech[x]^4])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a\text{sech}^4(x)\right)^{3/2}}\,dx &= \frac{\text{sech}^2(x)\int \cosh^6(x)\,dx}{a\sqrt{a\text{sech}^4(x)}}\\
&= \frac{\cosh^3(x)\sinh(x)}{6a\sqrt{a\text{sech}^4(x)}}+\frac{\left(5\text{sech}^2(x)\right)\int \cosh^4(x)\,dx}{6a\sqrt{a\text{sech}^4(x)}}\\
&= \frac{5\cosh(x)\sinh(x)}{24a\sqrt{a\text{sech}^4(x)}}+\frac{\cosh^3(x)\sinh(x)}{6a\sqrt{a\text{sech}^4(x)}}+\frac{\left(5\text{sech}^2(x)\right)\int \cosh^2(x)\,dx}{8a\sqrt{a\text{sech}^4(x)}}\\
&= \frac{5\cosh(x)\sinh(x)}{24a\sqrt{a\text{sech}^4(x)}}+\frac{\cosh^3(x)\sinh(x)}{6a\sqrt{a\text{sech}^4(x)}}+\frac{5\tanh(x)}{16a\sqrt{a\text{sech}^4(x)}}+\frac{\left(5\text{sech}^2(x)\right)\int 1\,dx}{16a\sqrt{a\text{sech}^4(x)}}\\
&= \frac{5x\text{sech}^2(x)}{16a\sqrt{a\text{sech}^4(x)}}+\frac{5\cosh(x)\sinh(x)}{24a\sqrt{a\text{sech}^4(x)}}+\frac{\cosh^3(x)\sinh(x)}{6a\sqrt{a\text{sech}^4(x)}}+\frac{5\tanh(x)}{16a\sqrt{a\text{sech}^4(x)}}
\end{aligned}
$$

Mathematica [**A**]
time = 0.03, size = 38, normalized size = 0.44

$$\frac{\text{sech}^6(x)(60x+45\sinh(2x)+9\sinh(4x)+\sinh(6x))}{192\left(a\text{sech}^4(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x]^4)^(-3/2),x]
```

```
[Out] (Sech[x]^6*(60*x + 45*Sinh[2*x] + 9*Sinh[4*x] + Sinh[6*x]))/(192*(a*Sech[x]
^4)^(3/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(70) = 140$.
time = 0.93, size = 230, normalized size = 2.67

method	result
risch	$\frac{5\,e^{2x}x}{16a(1+e^{2x})^2\sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^4}}}+\frac{e^{8x}}{384a(1+e^{2x})^2\sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^4}}}+\frac{3\,e^{6x}}{128a(1+e^{2x})^2\sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^4}}}+\frac{15\,e^{4x}}{128a(1+e^{2x})^2\sqrt{\frac{a\,e^{4x}}{(1+e^{2x})^4}}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sech(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 5/16/a*exp(2*x)/(1+exp(2*x))^2/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*x+1/384/a*
exp(8*x)/(1+exp(2*x))^2/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)+3/128/a*exp(6*x)/
```

```
(1+exp(2*x))^2/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)+15/128/a*exp(4*x)/(1+exp(2
*x))^2/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)-15/128/(a*exp(4*x)/(1+exp(2*x))^4)
^(1/2)/(1+exp(2*x))^2/a-3/128/a*exp(-2*x)/(1+exp(2*x))^2/(a*exp(4*x)/(1+exp
(2*x))^4)^(1/2)-1/384/a*exp(-4*x)/(1+exp(2*x))^2/(a*exp(4*x)/(1+exp(2*x))^4
)^(1/2)
```

Maxima [A]
time = 0.51, size = 65, normalized size = 0.76

$$\frac{\left(9\sqrt{a}\,e^{(-2x)}+45\sqrt{a}\,e^{(-4x)}-45\sqrt{a}\,e^{(-8x)}-9\sqrt{a}\,e^{(-10x)}-\sqrt{a}\,e^{(-12x)}+\sqrt{a}\right)e^{(6x)}}{384\,a^2}+\frac{5\,x}{16\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/384*(9*sqrt(a)*e^(-2*x) + 45*sqrt(a)*e^(-4*x) - 45*sqrt(a)*e^(-8*x) - 9*s
qrt(a)*e^(-10*x) - sqrt(a)*e^(-12*x) + sqrt(a))*e^(6*x)/a^2 + 5/16*x/a^(3/2
)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1141 vs. 2(70) = 140.
time = 0.40, size = 1141, normalized size = 13.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/384*((e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^12 + cosh(x)^12 + 12*(cosh(x)*e^(4
*x) + 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^11 + 3*(22*cosh(x)^2 + (22*cosh(
x)^2 + 3)*e^(4*x) + 2*(22*cosh(x)^2 + 3)*e^(2*x) + 3)*sinh(x)^10 + 9*cosh(x
)^10 + 10*(22*cosh(x)^3 + (22*cosh(x)^3 + 9*cosh(x))*e^(4*x) + 2*(22*cosh(x
)^3 + 9*cosh(x))*e^(2*x) + 9*cosh(x))*sinh(x)^9 + 45*(11*cosh(x)^4 + 9*cosh
(x)^2 + (11*cosh(x)^4 + 9*cosh(x)^2 + 1)*e^(4*x) + 2*(11*cosh(x)^4 + 9*cosh
(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^8 + 45*cosh(x)^8 + 72*(11*cosh(x)^5 + 15*co
sh(x)^3 + (11*cosh(x)^5 + 15*cosh(x)^3 + 5*cosh(x))*e^(4*x) + 2*(11*cosh(x)
^5 + 15*cosh(x)^3 + 5*cosh(x))*e^(2*x) + 5*cosh(x))*sinh(x)^7 + 120*x*cosh(
x)^6 + 6*(154*cosh(x)^6 + 315*cosh(x)^4 + 210*cosh(x)^2 + (154*cosh(x)^6 +
315*cosh(x)^4 + 210*cosh(x)^2 + 20*x)*e^(4*x) + 2*(154*cosh(x)^6 + 315*cosh
(x)^4 + 210*cosh(x)^2 + 20*x)*e^(2*x) + 20*x)*sinh(x)^6 + 36*(22*cosh(x)^7
+ 63*cosh(x)^5 + 70*cosh(x)^3 + 20*x*cosh(x) + (22*cosh(x)^7 + 63*cosh(x)^5
 + 70*cosh(x)^3 + 20*x*cosh(x))*e^(4*x) + 2*(22*cosh(x)^7 + 63*cosh(x)^5 +
70*cosh(x)^3 + 20*x*cosh(x))*e^(2*x))*sinh(x)^5 + 45*(11*cosh(x)^8 + 42*cos
h(x)^6 + 70*cosh(x)^4 + 40*x*cosh(x)^2 + (11*cosh(x)^8 + 42*cosh(x)^6 + 70*
cosh(x)^4 + 40*x*cosh(x)^2 - 1)*e^(4*x) + 2*(11*cosh(x)^8 + 42*cosh(x)^6 +
70*cosh(x)^4 + 40*x*cosh(x)^2 - 1)*e^(2*x) - 1)*sinh(x)^4 - 45*cosh(x)^4 +
```

```
20*(11*cosh(x)^9 + 54*cosh(x)^7 + 126*cosh(x)^5 + 120*x*cosh(x)^3 + (11*cos
h(x)^9 + 54*cosh(x)^7 + 126*cosh(x)^5 + 120*x*cosh(x)^3 - 9*cosh(x))*e^(4*x
) + 2*(11*cosh(x)^9 + 54*cosh(x)^7 + 126*cosh(x)^5 + 120*x*cosh(x)^3 - 9*co
sh(x))*e^(2*x) - 9*cosh(x))*sinh(x)^3 + 3*(22*cosh(x)^10 + 135*cosh(x)^8 +
420*cosh(x)^6 + 600*x*cosh(x)^4 - 90*cosh(x)^2 + (22*cosh(x)^10 + 135*cosh(
x)^8 + 420*cosh(x)^6 + 600*x*cosh(x)^4 - 90*cosh(x)^2 - 3)*e^(4*x) + 2*(22*
cosh(x)^10 + 135*cosh(x)^8 + 420*cosh(x)^6 + 600*x*cosh(x)^4 - 90*cosh(x)^2
 - 3)*e^(2*x) - 3)*sinh(x)^2 - 9*cosh(x)^2 + (cosh(x)^12 + 9*cosh(x)^10 + 4
5*cosh(x)^8 + 120*x*cosh(x)^6 - 45*cosh(x)^4 - 9*cosh(x)^2 - 1)*e^(4*x) + 2
*(cosh(x)^12 + 9*cosh(x)^10 + 45*cosh(x)^8 + 120*x*cosh(x)^6 - 45*cosh(x)^4
 - 9*cosh(x)^2 - 1)*e^(2*x) + 6*(2*cosh(x)^11 + 15*cosh(x)^9 + 60*cosh(x)^7
 + 120*x*cosh(x)^5 - 30*cosh(x)^3 + (2*cosh(x)^11 + 15*cosh(x)^9 + 60*cosh(
x)^7 + 120*x*cosh(x)^5 - 30*cosh(x)^3 - 3*cosh(x))*e^(4*x) + 2*(2*cosh(x)^1
1 + 15*cosh(x)^9 + 60*cosh(x)^7 + 120*x*cosh(x)^5 - 30*cosh(x)^3 - 3*cosh(x
))*e^(2*x) - 3*cosh(x))*sinh(x) - 1)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x
) + 4*e^(2*x) + 1))*e^(2*x)/(a^2*cosh(x)^6*e^(2*x) + 6*a^2*cosh(x)^5*e^(2*x
)*sinh(x) + 15*a^2*cosh(x)^4*e^(2*x)*sinh(x)^2 + 20*a^2*cosh(x)^3*e^(2*x)*s
inh(x)^3 + 15*a^2*cosh(x)^2*e^(2*x)*sinh(x)^4 + 6*a^2*cosh(x)*e^(2*x)*sinh(
x)^5 + a^2*e^(2*x)*sinh(x)^6)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a \operatorname{sech}^4(x)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)**4)**(3/2),x)
[Out] Integral((a*sech(x)**4)**(-3/2), x)
```

Giac [**A**]
time = 0.39, size = 52, normalized size = 0.60

$$-\frac{\left(110\, e^{(6\,x)} + 45\, e^{(4\,x)} + 9\, e^{(2\,x)} + 1\right)e^{(-6\,x)} - 120\,x - e^{(6\,x)} - 9\, e^{(4\,x)} - 45\, e^{(2\,x)}}{384\, a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^4)^(3/2),x, algorithm="giac")
[Out] -1/384*((110*e^(6*x) + 45*e^(4*x) + 9*e^(2*x) + 1)*e^(-6*x) - 120*x - e^(6*
x) - 9*e^(4*x) - 45*e^(2*x))/a^(3/2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(\frac{a}{{\mathrm{cosh}\left(x\right)}^4}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/cosh(x)^4)^(3/2),x)
[Out] int(1/(a/cosh(x)^4)^(3/2), x)
```

3.51 $\int \frac{1}{\left(a\text{sech}^4(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=132

$$\frac{63x\text{sech}^2(x)}{256a^2\sqrt{a\text{sech}^4(x)}}+\frac{21\cosh(x)\sinh(x)}{128a^2\sqrt{a\text{sech}^4(x)}}+\frac{21\cosh^3(x)\sinh(x)}{160a^2\sqrt{a\text{sech}^4(x)}}+\frac{9\cosh^5(x)\sinh(x)}{80a^2\sqrt{a\text{sech}^4(x)}}+\frac{\cosh^7(x)\sinh(x)}{10a^2\sqrt{a\text{sech}^4(x)}}+\frac{6}{256a}$$

```
[Out] 63/256*x*sech(x)^2/a^2/(a*sech(x)^4)^(1/2)+21/128*cosh(x)*sinh(x)/a^2/(a*se
ch(x)^4)^(1/2)+21/160*cosh(x)^3*sinh(x)/a^2/(a*sech(x)^4)^(1/2)+9/80*cosh(x
)^5*sinh(x)/a^2/(a*sech(x)^4)^(1/2)+1/10*cosh(x)^7*sinh(x)/a^2/(a*sech(x)^4
)^(1/2)+63/256*tanh(x)/a^2/(a*sech(x)^4)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4208, 2715, 8}

$$\frac{63x\text{sech}^2(x)}{256a^2\sqrt{a\text{sech}^4(x)}}+\frac{63\tanh(x)}{256a^2\sqrt{a\text{sech}^4(x)}}+\frac{\sinh(x)\cosh^7(x)}{10a^2\sqrt{a\text{sech}^4(x)}}+\frac{9\sinh(x)\cosh^5(x)}{80a^2\sqrt{a\text{sech}^4(x)}}+\frac{21\sinh(x)\cosh^3(x)}{160a^2\sqrt{a\text{sech}^4(x)}}+\frac{21\sinh(x)\cosh(x)}{128a^2\sqrt{a\text{sech}^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x]^4)^(-5/2),x]
```

```
[Out] (63*x*Sech[x]^2)/(256*a^2*Sqrt[a*Sech[x]^4]) + (21*Cosh[x]*Sinh[x])/(128*a^
2*Sqrt[a*Sech[x]^4]) + (21*Cosh[x]^3*Sinh[x])/(160*a^2*Sqrt[a*Sech[x]^4]) +
 (9*Cosh[x]^5*Sinh[x])/(80*a^2*Sqrt[a*Sech[x]^4]) + (Cosh[x]^7*Sinh[x])/(10
*a^2*Sqrt[a*Sech[x]^4]) + (63*Tanh[x])/(256*a^2*Sqrt[a*Sech[x]^4])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
```

```
&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a\text{sech}^4(x)\right)^{5/2}}\,dx = \frac{\text{sech}^2(x)\int \cosh^{10}(x)\,dx}{a^2\sqrt{a\text{sech}^4(x)}}$$
$$= \frac{\cosh^7(x)\sinh(x)}{10a^2\sqrt{a\text{sech}^4(x)}} + \frac{\left(9\text{sech}^2(x)\right)\int \cosh^8(x)\,dx}{10a^2\sqrt{a\text{sech}^4(x)}}$$
$$= \frac{9\cosh^5(x)\sinh(x)}{80a^2\sqrt{a\text{sech}^4(x)}} + \frac{\cosh^7(x)\sinh(x)}{10a^2\sqrt{a\text{sech}^4(x)}} + \frac{\left(63\text{sech}^2(x)\right)\int \cosh^6(x)\,dx}{80a^2\sqrt{a\text{sech}^4(x)}}$$
$$= \frac{21\cosh^3(x)\sinh(x)}{160a^2\sqrt{a\text{sech}^4(x)}} + \frac{9\cosh^5(x)\sinh(x)}{80a^2\sqrt{a\text{sech}^4(x)}} + \frac{\cosh^7(x)\sinh(x)}{10a^2\sqrt{a\text{sech}^4(x)}} + \frac{\left(21\text{sech}^2(x)\right)\int \cos}{32a^2\sqrt{a\text{sech}^4}}$$
$$= \frac{21\cosh(x)\sinh(x)}{128a^2\sqrt{a\text{sech}^4(x)}} + \frac{21\cosh^3(x)\sinh(x)}{160a^2\sqrt{a\text{sech}^4(x)}} + \frac{9\cosh^5(x)\sinh(x)}{80a^2\sqrt{a\text{sech}^4(x)}} + \frac{\cosh^7(x)\sinh(x)}{10a^2\sqrt{a\text{sech}^4(x)}}$$
$$= \frac{21\cosh(x)\sinh(x)}{128a^2\sqrt{a\text{sech}^4(x)}} + \frac{21\cosh^3(x)\sinh(x)}{160a^2\sqrt{a\text{sech}^4(x)}} + \frac{9\cosh^5(x)\sinh(x)}{80a^2\sqrt{a\text{sech}^4(x)}} + \frac{\cosh^7(x)\sinh(x)}{10a^2\sqrt{a\text{sech}^4(x)}}$$
$$= \frac{63x\text{sech}^2(x)}{256a^2\sqrt{a\text{sech}^4(x)}} + \frac{21\cosh(x)\sinh(x)}{128a^2\sqrt{a\text{sech}^4(x)}} + \frac{21\cosh^3(x)\sinh(x)}{160a^2\sqrt{a\text{sech}^4(x)}} + \frac{9\cosh^5(x)\sinh(}{80a^2\sqrt{a\text{sech}^4(x}}$$

Mathematica [A]
time = 0.05, size = 55, normalized size = 0.42

$$\frac{\cosh^2(x)\sqrt{a\text{sech}^4(x)}\,(2520x + 2100\sinh(2x) + 600\sinh(4x) + 150\sinh(6x) + 25\sinh(8x) + 2\sinh(10x))}{10240a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x]^4)^(-5/2),x]
```

```
[Out] (Cosh[x]^2*Sqrt[a*Sech[x]^4]*(2520*x + 2100*Sinh[2*x] + 600*Sinh[4*x] + 150
*Sinh[6*x] + 25*Sinh[8*x] + 2*Sinh[10*x]))/(10240*a^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 361 vs. 2(108) = 216.
time = 0.93, size = 362, normalized size = 2.74

method	result
risch	$\frac{63\,\mathrm{e}^{2x}x}{256a^2(1+\mathrm{e}^{2x})^2\sqrt{\frac{a\,\mathrm{e}^{4x}}{(1+\mathrm{e}^{2x})^4}}}+\frac{\mathrm{e}^{12x}}{10240a^2(1+\mathrm{e}^{2x})^2\sqrt{\frac{a\,\mathrm{e}^{4x}}{(1+\mathrm{e}^{2x})^4}}}+\frac{5\,\mathrm{e}^{10x}}{4096a^2(1+\mathrm{e}^{2x})^2\sqrt{\frac{a\,\mathrm{e}^{4x}}{(1+\mathrm{e}^{2x})^4}}}+\frac{15\,\mathrm{e}^{8x}}{2048a^2(1+\mathrm{e}^{2x})^2\sqrt{\frac{a\,\mathrm{e}^{4x}}{(1+\mathrm{e}^{2x})^4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sech(x)^4)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 63/256/a^2*exp(2*x)/(1+exp(2*x))^2/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*x+1/10
240/a^2*exp(12*x)/(1+exp(2*x))^2/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)+5/4096/a
^2*exp(10*x)/(1+exp(2*x))^2/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)+15/2048/a^2*e
xp(8*x)/(1+exp(2*x))^2/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)+15/512/a^2*exp(6*x
)/(1+exp(2*x))^2/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)+105/1024/a^2*exp(4*x)/(1
+exp(2*x))^2/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)-105/1024/(a*exp(4*x)/(1+exp(
2*x))^4)^(1/2)/(1+exp(2*x))^2/a^2-15/512/a^2*exp(-2*x)/(1+exp(2*x))^2/(a*ex
p(4*x)/(1+exp(2*x))^4)^(1/2)-15/2048/a^2*exp(-4*x)/(1+exp(2*x))^2/(a*exp(4*
x)/(1+exp(2*x))^4)^(1/2)-5/4096/a^2*exp(-6*x)/(1+exp(2*x))^2/(a*exp(4*x)/(1
+exp(2*x))^4)^(1/2)-1/10240/a^2*exp(-8*x)/(1+exp(2*x))^2/(a*exp(4*x)/(1+exp
(2*x))^4)^(1/2)
```

Maxima [A]
time = 0.48, size = 103, normalized size = 0.78

$$\frac{\left(25\,\sqrt{a}\,e^{(-2\,x)}+150\,\sqrt{a}\,e^{(-4\,x)}+600\,\sqrt{a}\,e^{(-6\,x)}+2100\,\sqrt{a}\,e^{(-8\,x)}-2100\,\sqrt{a}\,e^{(-12\,x)}-600\,\sqrt{a}\,e^{(-14\,x)}-150\,\sqrt{a}\,e^{(-16\,x)}-25\,\sqrt{a}\,e^{(-18\,x)}-2\,\sqrt{a}\,e^{(-20\,x)}+2\,\sqrt{a}\right)e^{(10\,x)}}{20480\,a^3}+\frac{63\,x}{256\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/20480*(25*sqrt(a)*e^(-2*x) + 150*sqrt(a)*e^(-4*x) + 600*sqrt(a)*e^(-6*x)
+ 2100*sqrt(a)*e^(-8*x) - 2100*sqrt(a)*e^(-12*x) - 600*sqrt(a)*e^(-14*x) -
150*sqrt(a)*e^(-16*x) - 25*sqrt(a)*e^(-18*x) - 2*sqrt(a)*e^(-20*x) + 2*sqrt
(a))*e^(10*x)/a^3 + 63/256*x/a^(5/2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2600 vs. $2(108) = 216$.
time = 0.38, size = 2600, normalized size = 19.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/20480*(2*(e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^20 + 2*cosh(x)^20 + 40*(cosh(x
)*e^(4*x) + 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^19 + 5*(76*cosh(x)^2 + (76
```

*cosh(x)^2 + 5)*e^(4*x) + 2*(76*cosh(x)^2 + 5)*e^(2*x) + 5)*sinh(x)^18 + 25
cosh(x)^18 + 30(76*cosh(x)^3 + (76*cosh(x)^3 + 15*cosh(x))*e^(4*x) + 2*(7
6*cosh(x)^3 + 15*cosh(x))*e^(2*x) + 15*cosh(x))*sinh(x)^17 + 15*(646*cosh(x
)^4 + 255*cosh(x)^2 + (646*cosh(x)^4 + 255*cosh(x)^2 + 10)*e^(4*x) + 2*(646
*cosh(x)^4 + 255*cosh(x)^2 + 10)*e^(2*x) + 10)*sinh(x)^16 + 150*cosh(x)^16
+ 48*(646*cosh(x)^5 + 425*cosh(x)^3 + (646*cosh(x)^5 + 425*cosh(x)^3 + 50*c
osh(x))*e^(4*x) + 2*(646*cosh(x)^5 + 425*cosh(x)^3 + 50*cosh(x))*e^(2*x) +
50*cosh(x))*sinh(x)^15 + 60*(1292*cosh(x)^6 + 1275*cosh(x)^4 + 300*cosh(x)^
2 + (1292*cosh(x)^6 + 1275*cosh(x)^4 + 300*cosh(x)^2 + 10)*e^(4*x) + 2*(129
2*cosh(x)^6 + 1275*cosh(x)^4 + 300*cosh(x)^2 + 10)*e^(2*x) + 10)*sinh(x)^14
+ 600*cosh(x)^14 + 120*(1292*cosh(x)^7 + 1785*cosh(x)^5 + 700*cosh(x)^3 +
(1292*cosh(x)^7 + 1785*cosh(x)^5 + 700*cosh(x)^3 + 70*cosh(x))*e^(4*x) + 2*
(1292*cosh(x)^7 + 1785*cosh(x)^5 + 700*cosh(x)^3 + 70*cosh(x))*e^(2*x) + 70
*cosh(x))*sinh(x)^13 + 60*(4199*cosh(x)^8 + 7735*cosh(x)^6 + 4550*cosh(x)^4
+ 910*cosh(x)^2 + (4199*cosh(x)^8 + 7735*cosh(x)^6 + 4550*cosh(x)^4 + 910*
cosh(x)^2 + 35)*e^(4*x) + 2*(4199*cosh(x)^8 + 7735*cosh(x)^6 + 4550*cosh(x)
^4 + 910*cosh(x)^2 + 35)*e^(2*x) + 35)*sinh(x)^12 + 2100*cosh(x)^12 + 80*(4
199*cosh(x)^9 + 9945*cosh(x)^7 + 8190*cosh(x)^5 + 2730*cosh(x)^3 + (4199*co
sh(x)^9 + 9945*cosh(x)^7 + 8190*cosh(x)^5 + 2730*cosh(x)^3 + 315*cosh(x))*e
^(4*x) + 2*(4199*cosh(x)^9 + 9945*cosh(x)^7 + 8190*cosh(x)^5 + 2730*cosh(x)
^3 + 315*cosh(x))*e^(2*x) + 315*cosh(x))*sinh(x)^11 + 5040*x*cosh(x)^10 + 2
*(184756*cosh(x)^10 + 546975*cosh(x)^8 + 600600*cosh(x)^6 + 300300*cosh(x)^
4 + 69300*cosh(x)^2 + (184756*cosh(x)^10 + 546975*cosh(x)^8 + 600600*cosh(x
)^6 + 300300*cosh(x)^4 + 69300*cosh(x)^2 + 2520*x)*e^(4*x) + 2*(184756*cosh
(x)^10 + 546975*cosh(x)^8 + 600600*cosh(x)^6 + 300300*cosh(x)^4 + 69300*cos
h(x)^2 + 2520*x)*e^(2*x) + 2520*x)*sinh(x)^10 + 20*(16796*cosh(x)^11 + 6077
5*cosh(x)^9 + 85800*cosh(x)^7 + 60060*cosh(x)^5 + 23100*cosh(x)^3 + 2520*x*
cosh(x) + (16796*cosh(x)^11 + 60775*cosh(x)^9 + 85800*cosh(x)^7 + 60060*cos
h(x)^5 + 23100*cosh(x)^3 + 2520*x*cosh(x))*e^(4*x) + 2*(16796*cosh(x)^11 +
60775*cosh(x)^9 + 85800*cosh(x)^7 + 60060*cosh(x)^5 + 23100*cosh(x)^3 + 252
0*x*cosh(x))*e^(2*x))*sinh(x)^9 + 30*(8398*cosh(x)^12 + 36465*cosh(x)^10 +
64350*cosh(x)^8 + 60060*cosh(x)^6 + 34650*cosh(x)^4 + 7560*x*cosh(x)^2 + (8
398*cosh(x)^12 + 36465*cosh(x)^10 + 64350*cosh(x)^8 + 60060*cosh(x)^6 + 346
50*cosh(x)^4 + 7560*x*cosh(x)^2 - 70)*e^(4*x) + 2*(8398*cosh(x)^12 + 36465*
cosh(x)^10 + 64350*cosh(x)^8 + 60060*cosh(x)^6 + 34650*cosh(x)^4 + 7560*x*c
osh(x)^2 - 70)*e^(2*x) - 70)*sinh(x)^8 - 2100*cosh(x)^8 + 240*(646*cosh(x)^
13 + 3315*cosh(x)^11 + 7150*cosh(x)^9 + 8580*cosh(x)^7 + 6930*cosh(x)^5 + 2
520*x*cosh(x)^3 + (646*cosh(x)^13 + 3315*cosh(x)^11 + 7150*cosh(x)^9 + 8580
*cosh(x)^7 + 6930*cosh(x)^5 + 2520*x*cosh(x)^3 - 70*cosh(x))*e^(4*x) + 2*(6
46*cosh(x)^13 + 3315*cosh(x)^11 + 7150*cosh(x)^9 + 8580*cosh(x)^7 + 6930*co
sh(x)^5 + 2520*x*cosh(x)^3 - 70*cosh(x))*e^(2*x) - 70*cosh(x))*sinh(x)^7 +
60*(1292*cosh(x)^14 + 7735*cosh(x)^12 + 20020*cosh(x)^10 + 30030*cosh(x)^8
+ 32340*cosh(x)^6 + 17640*x*cosh(x)^4 - 980*cosh(x)^2 + (1292*cosh(x)^14 +
7735*cosh(x)^12 + 20020*cosh(x)^10 + 30030*cosh(x)^8 + 32340*cosh(x)^6 + 17
640*x*cosh(x)^4 - 980*cosh(x)^2 - 10)*e^(4*x) + 2*(1292*cosh(x)^14 + 7735*c

```
osh(x)^12 + 20020*cosh(x)^10 + 30030*cosh(x)^8 + 32340*cosh(x)^6 + 17640*x*
cosh(x)^4 - 980*cosh(x)^2 - 10)*e^(2*x) - 10)*sinh(x)^6 - 600*cosh(x)^6 + 2
4*(1292*cosh(x)^15 + 8925*cosh(x)^13 + 27300*cosh(x)^11 + 50050*cosh(x)^9 +
 69300*cosh(x)^7 + 52920*x*cosh(x)^5 - 4900*cosh(x)^3 + (1292*cosh(x)^15 +
8925*cosh(x)^13 + 27300*cosh(x)^11 + 50050*cosh(x)^9 + 69300*cosh(x)^7 + 52
920*x*cosh(x)^5 - 4900*cosh(x)^3 - 150*cosh(x))*e^(4*x) + 2*(1292*cosh(x)^1
5 + 8925*cosh(x)^13 + 27300*cosh(x)^11 + 50050*cosh(x)^9 + 69300*cosh(x)^7
+ 52920*x*cosh(x)^5 - 4900*cosh(x)^3 - 150*cosh(x))*e^(2*x) - 150*cosh(x))*
sinh(x)^5 + 30*(323*cosh(x)^16 + 2550*cosh(x)^14 + 9100*cosh(x)^12 + 20020*
cosh(x)^10 + 34650*cosh(x)^8 + 35280*x*cosh(x)^6 - 4900*cosh(x)^4 - 300*cos
h(x)^2 + (323*cosh(x)^16 + 2550*cosh(x)^14 + 9100*cosh(x)^12 + 20020*cosh(x
)^10 + 34650*cosh(x)^8 + 35280*x*cosh(x)^6 - 4900*cosh(x)^4 - 300*cosh(x)^2
 - 5)*e^(4*x) + 2*(323*cosh(x)^16 + 2550*cosh(x)^14 + 9100*cosh(x)^12 + 200
20*cosh(x)^10 + 34650*cosh(x)^8 + 35280*x*cosh(x)^6 - 4900*cosh(x)^4 - 300*
cosh(x)^2 - 5)*e^(2*x) - 5)*sinh(x)^4 - 150*cosh(x)^4 + 120*(19*cosh(x)^17
+ 170*cosh(x)^15 + 700*cosh(x)^13 + 1820*cosh(x)^11 + 3850*cosh(x)^9 + 5040
*x*cosh(x)^7 - 980*cosh(x)^5 - 100*cosh(x)^3 + (19*cosh(x)^17 + 170*cosh(x)
^15 + 700*cosh(x)^13 + 1820*cosh(x)^11 + 3850*cosh(x)^9 + 5040*x*cosh(x)^7
- 980*cosh(x)^5 - 100*cosh(x)^3 - 5*cosh(x))*e^(4*x) + 2*(19*cosh(x)^17 + 1
70*cosh(x)^15 + 700*cosh(x)^13 + 1820*cosh(x)^1...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\operatorname{sech}^{4}\left(x\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)**4)**(5/2),x)
```

```
[Out] Integral((a*sech(x)**4)**(-5/2), x)
```

Giac [A]
time = 0.39, size = 76, normalized size = 0.58

$$-\frac{\left(5754\,e^{(10\,x)} + 2100\,e^{(8\,x)} + 600\,e^{(6\,x)} + 150\,e^{(4\,x)} + 25\,e^{(2\,x)} + 2\right)e^{(-10\,x)} - 5040\,x - 2\,e^{(10\,x)} - 25\,e^{(8\,x)} - 150\,e^{(6\,x)} - 600\,e^{(4\,x)} - 2100\,e^{(2\,x)}}{20480\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] -1/20480*((5754*e^(10*x) + 2100*e^(8*x) + 600*e^(6*x) + 150*e^(4*x) + 25*e^
(2*x) + 2)*e^(-10*x) - 5040*x - 2*e^(10*x) - 25*e^(8*x) - 150*e^(6*x) - 600
*e^(4*x) - 2100*e^(2*x))/a^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{a}{\cosh(x)^4}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/cosh(x)^4)^(5/2),x)
```

```
[Out] int(1/(a/cosh(x)^4)^(5/2), x)
```

3.52 $\int \frac{\sinh^4(x)}{a+a\operatorname{sech}(x)} \, dx$

Optimal. Leaf size=44

$$-\frac{x}{8a}-\frac{\cosh(x)\sinh(x)}{8a}+\frac{\cosh^3(x)\sinh(x)}{4a}-\frac{\sinh^3(x)}{3a}$$

```
[Out] -1/8*x/a-1/8*cosh(x)*sinh(x)/a+1/4*cosh(x)^3*sinh(x)/a-1/3*sinh(x)^3/a
```

Rubi [A]
time = 0.10, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3957, 2918, 2644, 30, 2648, 2715, 8}

$$-\frac{x}{8a}-\frac{\sinh^3(x)}{3a}+\frac{\sinh(x)\cosh^3(x)}{4a}-\frac{\sinh(x)\cosh(x)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^4/(a + a*Sech[x]),x]
```

```
[Out] -1/8*x/a - (Cosh[x]*Sinh[x])/(8*a) + (Cosh[x]^3*Sinh[x])/(4*a) - Sinh[x]^3/
(3*a)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^4(x)}{a+a\text{sech}(x)}\,dx &= -\int \frac{\cosh(x)\sinh^4(x)}{-a-a\cosh(x)}\,dx\\
&= -\frac{\int \cosh(x)\sinh^2(x)\,dx}{a}+\frac{\int \cosh^2(x)\sinh^2(x)\,dx}{a}\\
&= \frac{\cosh^3(x)\sinh(x)}{4a}-\frac{i\text{Subst}\left(\int x^2\,dx,x,i\sinh(x)\right)}{a}-\frac{\int \cosh^2(x)\,dx}{4a}\\
&= -\frac{\cosh(x)\sinh(x)}{8a}+\frac{\cosh^3(x)\sinh(x)}{4a}-\frac{\sinh^3(x)}{3a}-\frac{\int 1\,dx}{8a}\\
&= -\frac{x}{8a}-\frac{\cosh(x)\sinh(x)}{8a}+\frac{\cosh^3(x)\sinh(x)}{4a}-\frac{\sinh^3(x)}{3a}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 28, normalized size = 0.64

$$\frac{24\sinh(x)-8\sinh(3x)+3(-4x+\sinh(4x))}{96a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^4/(a + a*Sech[x]),x]
```

```
[Out] (24*Sinh[x] - 8*Sinh[3*x] + 3*(-4*x + Sinh[4*x]))/(96*a)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(36) = 72$.
time = 0.60, size = 105, normalized size = 2.39

method	result
risch	$-\frac{x}{8a}+\frac{e^{4x}}{64a}-\frac{e^{3x}}{24a}+\frac{e^{x}}{8a}-\frac{e^{-x}}{8a}+\frac{e^{-3x}}{24a}-\frac{e^{-4x}}{64a}$
default	$\frac{-\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)+1\right)^{4}}+\frac{5}{6\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}-\frac{7}{8\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{16}{128\tanh\left(\frac{x}{2}\right)+128}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{8}+\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)-1\right)^{4}}+\frac{5}{6\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}+\frac{}{8(\tanh(}}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 16/a*(-1/64/(tanh(1/2*x)+1)^4+5/96/(tanh(1/2*x)+1)^3-7/128/(tanh(1/2*x)+1)^
2+1/128/(tanh(1/2*x)+1)-1/128*ln(tanh(1/2*x)+1)+1/64/(tanh(1/2*x)-1)^4+5/96
/(tanh(1/2*x)-1)^3+7/128/(tanh(1/2*x)-1)^2+1/128/(tanh(1/2*x)-1)+1/128*ln(t
anh(1/2*x)-1))
```

Maxima [A]
time = 0.29, size = 54, normalized size = 1.23

$$-\frac{\left(8\,e^{(-x)}-24\,e^{(-3\,x)}-3\right)e^{(4\,x)}}{192\,a}-\frac{x}{8\,a}-\frac{24\,e^{(-x)}-8\,e^{(-3\,x)}+3\,e^{(-4\,x)}}{192\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] -1/192*(8*e^(-x) - 24*e^(-3*x) - 3)*e^(4*x)/a - 1/8*x/a - 1/192*(24*e^(-x)
- 8*e^(-3*x) + 3*e^(-4*x))/a
```

Fricas [A]
time = 0.35, size = 36, normalized size = 0.82

$$\frac{(3\cosh(x)-2)\sinh(x)^{3}+3\left(\cosh(x)^{3}-2\cosh(x)^{2}+2\right)\sinh(x)-3\,x}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] 1/24*((3*cosh(x) - 2)*sinh(x)^3 + 3*(cosh(x)^3 - 2*cosh(x)^2 + 2)*sinh(x) -
 3*x)/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sinh^{4}(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**4/(a+a*sech(x)),x)
```

```
[Out] Integral(sinh(x)**4/(sech(x) + 1), x)/a
```

Giac [A]
time = 0.40, size = 42, normalized size = 0.95

$$-\frac{\left(24\,e^{(3\,x)}-8\,e^{x}+3\right)e^{(-4\,x)}+24\,x-3\,e^{(4\,x)}+8\,e^{(3\,x)}-24\,e^{x}}{192\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] -1/192*((24*e^(3*x) - 8*e^x + 3)*e^(-4*x) + 24*x - 3*e^(4*x) + 8*e^(3*x) -
24*e^x)/a
```

Mupad [B]
time = 1.48, size = 59, normalized size = 1.34

$$\frac{\mathrm{e}^{-3\,x}}{24\,a}-\frac{\mathrm{e}^{-x}}{8\,a}-\frac{\mathrm{e}^{3\,x}}{24\,a}-\frac{\mathrm{e}^{-4\,x}}{64\,a}+\frac{\mathrm{e}^{4\,x}}{64\,a}-\frac{x}{8\,a}+\frac{\mathrm{e}^{x}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(a + a/cosh(x)),x)
```

```
[Out] exp(-3*x)/(24*a) - exp(-x)/(8*a) - exp(3*x)/(24*a) - exp(-4*x)/(64*a) + exp
(4*x)/(64*a) - x/(8*a) + exp(x)/(8*a)
```

3.53 $\int \frac{\sinh^3(x)}{a+a\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=23

$$\frac{\cosh^3(x)}{3a} - \frac{\sinh^2(x)}{2a}$$

[Out] 1/3*cosh(x)^3/a-1/2*sinh(x)^2/a

Rubi [A]
time = 0.09, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3957, 2914, 2644, 30, 2645}

$$\frac{\cosh^3(x)}{3a} - \frac{\sinh^2(x)}{2a}$$

Antiderivative was successfully verified.

[In] Int[Sinh[x]^3/(a + a*Sech[x]),x]

[Out] Cosh[x]^3/(3*a) - Sinh[x]^2/(2*a)

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2914

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
```

```
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^3(x)}{a+a\text{sech}(x)}\,dx &= -\int \frac{\cosh(x)\sinh^3(x)}{-a-a\cosh(x)}\,dx\\ &= -\frac{\int \cosh(x)\sinh(x)\,dx}{a}+\frac{\int \cosh^2(x)\sinh(x)\,dx}{a}\\ &= \frac{\text{Subst}(\int x\,dx,x,i\sinh(x))}{a}+\frac{\text{Subst}\left(\int x^2\,dx,x,\cosh(x)\right)}{a}\\ &= \frac{\cosh^3(x)}{3a}-\frac{\sinh^2(x)}{2a}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 23, normalized size = 1.00

$$\frac{-7+3\cosh(x)-3\cosh(2x)+\cosh(3x)}{12a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(a + a*Sech[x]),x]
[Out] (-7 + 3*Cosh[x] - 3*Cosh[2*x] + Cosh[3*x])/(12*a)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(19) = 38.
time = 0.52, size = 67, normalized size = 2.91

method	result	size
risch	$\frac{e^{3x}}{24a}-\frac{e^{2x}}{8a}+\frac{e^{x}}{8a}+\frac{e^{-x}}{8a}-\frac{e^{-2x}}{8a}+\frac{e^{-3x}}{24a}$	54
default	$\frac{-\frac{1}{3\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}-\frac{1}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{1}{\tanh\left(\frac{x}{2}\right)-1}+\frac{1}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}-\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{8}{8\tanh\left(\frac{x}{2}\right)+8}}{a}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 8/a*(-1/24/(tanh(1/2*x)-1)^3-1/8/(tanh(1/2*x)-1)^2-1/8/(tanh(1/2*x)-1)+1/24
/(tanh(1/2*x)+1)^3-1/8/(tanh(1/2*x)+1)^2+1/8/(tanh(1/2*x)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(19) = 38$.
time = 0.30, size = 46, normalized size = 2.00

$$-\frac{\left(3\,e^{(-x)}-3\,e^{(-2\,x)}-1\right)e^{(3\,x)}}{24\,a}+\frac{3\,e^{(-x)}-3\,e^{(-2\,x)}+e^{(-3\,x)}}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] -1/24*(3*e^(-x) - 3*e^(-2*x) - 1)*e^(3*x)/a + 1/24*(3*e^(-x) - 3*e^(-2*x) +
 e^(-3*x))/a
```

Fricas [A]
time = 0.38, size = 30, normalized size = 1.30

$$\frac{\cosh\left(x\right)^{3}+3\left(\cosh\left(x\right)-1\right)\sinh\left(x\right)^{2}-3\,\cosh\left(x\right)^{2}+3\,\cosh\left(x\right)}{12\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] 1/12*(cosh(x)^3 + 3*(cosh(x) - 1)*sinh(x)^2 - 3*cosh(x)^2 + 3*cosh(x))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sinh^{3}\left(x\right)}{\operatorname{sech}\left(x\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(a+a*sech(x)),x)
```

```
[Out] Integral(sinh(x)**3/(sech(x) + 1), x)/a
```

Giac [A]
time = 0.39, size = 37, normalized size = 1.61

$$\frac{\left(3\,e^{(2\,x)}-3\,e^{x}+1\right)e^{(-3\,x)}+e^{(3\,x)}-3\,e^{(2\,x)}+3\,e^{x}}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] 1/24*((3*e^(2*x) - 3*e^x + 1)*e^(-3*x) + e^(3*x) - 3*e^(2*x) + 3*e^x)/a
```

Mupad [B]
time = 1.36, size = 53, normalized size = 2.30

$$\frac{e^{-x}}{8\,a}-\frac{e^{-2\,x}}{8\,a}-\frac{e^{2\,x}}{8\,a}+\frac{e^{-3\,x}}{24\,a}+\frac{e^{3\,x}}{24\,a}+\frac{e^{x}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a + a/cosh(x)),x)
```

```
[Out] exp(-x)/(8*a) - exp(-2*x)/(8*a) - exp(2*x)/(8*a) + exp(-3*x)/(24*a) + exp(3
*x)/(24*a) + exp(x)/(8*a)
```

3.54 $\int \frac{\sinh^2(x)}{a+a\text{sech}(x)}\,dx$

Optimal. Leaf size=27

$$\frac{x}{2a}-\frac{\sinh(x)}{a}+\frac{\cosh(x)\sinh(x)}{2a}$$

```
[Out] 1/2*x/a-sinh(x)/a+1/2*cosh(x)*sinh(x)/a
```

Rubi [A]
time = 0.08, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3957, 2918, 2717, 2715, 8}

$$\frac{x}{2a}-\frac{\sinh(x)}{a}+\frac{\sinh(x)\cosh(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(a + a*Sech[x]),x]
```

```
[Out] x/(2*a) - Sinh[x]/a + (Cosh[x]*Sinh[x])/(2*a)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^2(x)}{a+a\text{sech}(x)}\,dx &= -\int \frac{\cosh(x)\sinh^2(x)}{-a-a\cosh(x)}\,dx \\
&= -\frac{\int \cosh(x)\,dx}{a}+\frac{\int \cosh^2(x)\,dx}{a} \\
&= -\frac{\sinh(x)}{a}+\frac{\cosh(x)\sinh(x)}{2a}+\frac{\int 1\,dx}{2a} \\
&= \frac{x}{2a}-\frac{\sinh(x)}{a}+\frac{\cosh(x)\sinh(x)}{2a}
\end{aligned}$$

Mathematica [**A**]
time = 0.05, size = 16, normalized size = 0.59

$$\frac{x+(-2+\cosh(x))\sinh(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(a + a*Sech[x]),x]
```

```
[Out] (x + (-2 + Cosh[x])*Sinh[x])/(2*a)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(23) = 46$.
time = 0.58, size = 65, normalized size = 2.41

method	result	size
risch	$\frac{x}{2a}+\frac{e^{2x}}{8a}-\frac{e^{x}}{2a}+\frac{e^{-x}}{2a}-\frac{e^{-2x}}{8a}$	42
default	$\frac{\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{3}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2}-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{3}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2}}{a}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 4/a*(1/8/(tanh(1/2*x)-1)^2+3/8/(tanh(1/2*x)-1)-1/8*ln(tanh(1/2*x)-1)-1/8/(t
anh(1/2*x)+1)^2+3/8/(tanh(1/2*x)+1)+1/8*ln(tanh(1/2*x)+1))
```

Maxima [A]
time = 0.26, size = 42, normalized size = 1.56

$$-\frac{\left(4\,e^{(-x)}-1\right)e^{(2\,x)}}{8\,a}+\frac{x}{2\,a}+\frac{4\,e^{(-x)}-e^{(-2\,x)}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+a*sech(x)),x, algorithm="maxima")
[Out] -1/8*(4*e^(-x) - 1)*e^(2*x)/a + 1/2*x/a + 1/8*(4*e^(-x) - e^(-2*x))/a
```

Fricas [A]
time = 0.37, size = 14, normalized size = 0.52

$$\frac{(\cosh(x)-2)\sinh(x)+x}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+a*sech(x)),x, algorithm="fricas")
[Out] 1/2*((cosh(x) - 2)*sinh(x) + x)/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sinh^2(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(a+a*sech(x)),x)
[Out] Integral(sinh(x)**2/(sech(x) + 1), x)/a
```

Giac [A]
time = 0.39, size = 28, normalized size = 1.04

$$\frac{(4\,e^{x}-1)e^{(-2\,x)}+4\,x+e^{(2\,x)}-4\,e^{x}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+a*sech(x)),x, algorithm="giac")
[Out] 1/8*((4*e^x - 1)*e^(-2*x) + 4*x + e^(2*x) - 4*e^x)/a
```

Mupad [B]
time = 1.34, size = 41, normalized size = 1.52

$$\frac{\mathrm{e}^{-x}}{2\,a}-\frac{\mathrm{e}^{-2\,x}}{8\,a}+\frac{\mathrm{e}^{2\,x}}{8\,a}+\frac{x}{2\,a}-\frac{\mathrm{e}^{x}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a + a/cosh(x)),x)
[Out] exp(-x)/(2*a) - exp(-2*x)/(8*a) + exp(2*x)/(8*a) + x/(2*a) - exp(x)/(2*a)
```

3.55 $\int \frac{\sinh(x)}{a+a\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=17

$$\frac{\cosh(x)}{a} - \frac{\log(1+\cosh(x))}{a}$$

[Out] cosh(x)/a-ln(1+cosh(x))/a

Rubi [A]
time = 0.05, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3957, 2912, 12, 45}

$$\frac{\cosh(x)}{a} - \frac{\log(\cosh(x)+1)}{a}$$

Antiderivative was successfully verified.

[In] Int[Sinh[x]/(a + a*Sech[x]),x]

[Out] Cosh[x]/a - Log[1 + Cosh[x]]/a

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh(x)}{a+a\text{sech}(x)}\,dx &= -\int \frac{\cosh(x)\sinh(x)}{-a-a\cosh(x)}\,dx \\
&= -\frac{\text{Subst}\left(\int \frac{x}{a(-a+x)}\,dx, x, -a\cosh(x)\right)}{a} \\
&= -\frac{\text{Subst}\left(\int \frac{x}{-a+x}\,dx, x, -a\cosh(x)\right)}{a^2} \\
&= -\frac{\text{Subst}\left(\int \left(1-\frac{a}{a-x}\right)\,dx, x, -a\cosh(x)\right)}{a^2} \\
&= \frac{\cosh(x)}{a} - \frac{\log(1+\cosh(x))}{a}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 0.94

$$\frac{\cosh(x)-2\log\left(\cosh\left(\frac{x}{2}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(a + a*Sech[x]),x]
```

```
[Out] (Cosh[x] - 2*Log[Cosh[x/2]])/a
```

Maple [A]
time = 0.59, size = 23, normalized size = 1.35

method	result	size
derivativedivides	$-\frac{-\frac{1}{\text{sech}(x)}-\ln(\text{sech}(x))+\ln(1+\text{sech}(x))}{a}$	23
default	$-\frac{-\frac{1}{\text{sech}(x)}-\ln(\text{sech}(x))+\ln(1+\text{sech}(x))}{a}$	23
risch	$\frac{x}{a}+\frac{e^{x}}{2a}+\frac{e^{-x}}{2a}-\frac{2\ln(e^{x}+1)}{a}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a*(-1/sech(x)-ln(sech(x))+ln(1+sech(x)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.
time = 0.27, size = 35, normalized size = 2.06

$$-\frac{x}{a}+\frac{e^{(-x)}}{2\,a}+\frac{e^{x}}{2\,a}-\frac{2\,\log\left(e^{(-x)}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] -x/a + 1/2*e^(-x)/a + 1/2*e^x/a - 2*log(e^(-x) + 1)/a
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(17) = 34$.
time = 0.36, size = 50, normalized size = 2.94

$$\frac{2\,x\cosh\left(x\right)+\cosh\left(x\right)^{2}-4\left(\cosh\left(x\right)+\sinh\left(x\right)\right)\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)+2\left(x+\cosh\left(x\right)\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}+1}{2\left(a\cosh\left(x\right)+a\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*x*cosh(x) + cosh(x)^2 - 4*(cosh(x) + sinh(x))*log(cosh(x) + sinh(x)
+ 1) + 2*(x + cosh(x))*sinh(x) + sinh(x)^2 + 1)/(a*cosh(x) + a*sinh(x))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sinh\left(x\right)}{\operatorname{sech}\left(x\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+a*sech(x)),x)
```

```
[Out] Integral(sinh(x)/(sech(x) + 1), x)/a
```

Giac [**A**]
time = 0.39, size = 32, normalized size = 1.88

$$\frac{x}{a}+\frac{e^{(-x)}}{2\,a}+\frac{e^{x}}{2\,a}-\frac{2\,\log\left(e^{x}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] x/a + 1/2*e^(-x)/a + 1/2*e^x/a - 2*log(e^x + 1)/a
```

Mupad [**B**]
time = 0.07, size = 15, normalized size = 0.88

$$-\frac{\ln\left(\cosh(x)+1\right)-\cosh(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a + a/cosh(x)),x)
```

```
[Out] -(log(cosh(x) + 1) - cosh(x))/a
```

3.56 $\int \frac{\operatorname{csch}(x)}{a+a\operatorname{sech}(x)} \, dx$

Optimal. Leaf size=33

$$-\frac{\tanh^{-1}(\cosh(x))}{2a}-\frac{\coth(x)\operatorname{csch}(x)}{2a}+\frac{\operatorname{csch}^2(x)}{2a}$$

```
[Out] -1/2*arctanh(cosh(x))/a-1/2*coth(x)*csch(x)/a+1/2*csch(x)^2/a
```

Rubi [A]
time = 0.07, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {3957, 2785, 2686, 30, 2691, 3855}

$$\frac{\operatorname{csch}^2(x)}{2a}-\frac{\tanh^{-1}(\cosh(x))}{2a}-\frac{\coth(x)\operatorname{csch}(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]/(a + a*Sech[x]),x]
```

```
[Out] -1/2*ArcTanh[Cosh[x]]/a - (Coth[x]*Csch[x])/(2*a) + Csch[x]^2/(2*a)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
```

```
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}(x)}{a+a\operatorname{sech}(x)}\,dx &= -\int \frac{\coth(x)}{-a-a\cosh(x)}\,dx\\ &= \frac{\int \coth^2(x)\operatorname{csch}(x)\,dx}{a}-\frac{\int \coth(x)\operatorname{csch}^2(x)\,dx}{a}\\ &= -\frac{\coth(x)\operatorname{csch}(x)}{2a}+\frac{\int \operatorname{csch}(x)\,dx}{2a}-\frac{\operatorname{Subst}\left(\int x\,dx,x,-i\operatorname{csch}(x)\right)}{a}\\ &= -\frac{\tanh^{-1}(\cosh(x))}{2a}-\frac{\coth(x)\operatorname{csch}(x)}{2a}+\frac{\operatorname{csch}^2(x)}{2a}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 44, normalized size = 1.33

$$-\frac{\left(1+2\cosh^2\left(\frac{x}{2}\right)\left(\log\left(\cosh\left(\frac{x}{2}\right)\right)-\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)\right)\operatorname{sech}(x)}{2a(1+\operatorname{sech}(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]/(a + a*Sech[x]),x]
```

```
[Out] -1/2*((1 + 2*Cosh[x/2]^2*(Log[Cosh[x/2]] - Log[Sinh[x/2]]))*Sech[x])/(a*(1
+ Sech[x]))
```

Maple [A]
time = 0.71, size = 20, normalized size = 0.61

method	result	size

default	$\frac{\frac{\left(\tanh^2\left(\frac{x}{2}\right)\right)}{2}+\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2a}$	20
risch	$-\frac{e^x}{\left(e^x+1\right)^2 a}+\frac{\ln\left(e^x-1\right)}{2a}-\frac{\ln\left(e^x+1\right)}{2a}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a*(1/2*tanh(1/2*x)^2+ln(tanh(1/2*x)))
```

Maxima [A]
time = 0.26, size = 48, normalized size = 1.45

$$-\frac{e^{(-x)}}{2\,ae^{(-x)}+ae^{(-2\,x)}+a}-\frac{\log\left(e^{(-x)}+1\right)}{2\,a}+\frac{\log\left(e^{(-x)}-1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] -e^(-x)/(2*a*e^(-x) + a*e^(-2*x) + a) - 1/2*log(e^(-x) + 1)/a + 1/2*log(e^(
-x) - 1)/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(27) = 54.
time = 0.37, size = 103, normalized size = 3.12

$$-\frac{\left(\cosh\left(x\right)^2+2\left(\cosh\left(x\right)+1\right)\sinh\left(x\right)+\sinh\left(x\right)^2+2\,\cosh\left(x\right)+1\right)\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)-\left(\cosh\left(x\right)^2+2\left(\cosh\left(x\right)+1\right)\sinh\left(x\right)+\sinh\left(x\right)^2+2\,\cosh\left(x\right)+1\right)\log\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)+2\,\cosh\left(x\right)+2\,\sinh\left(x\right)}{2\left(a\cosh\left(x\right)^2+a\sinh\left(x\right)^2+2\,a\cosh\left(x\right)+2\left(a\cosh\left(x\right)+a\right)\sinh\left(x\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/2*((cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)*log
(cosh(x) + sinh(x) + 1) - (cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2
 + 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*cosh(x) + 2*sinh(x))/(a*cos
h(x)^2 + a*sinh(x)^2 + 2*a*cosh(x) + 2*(a*cosh(x) + a)*sinh(x) + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{csch}\left(x\right)}{\operatorname{sech}\left(x\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+a*sech(x)),x)
```

```
[Out] Integral(csch(x)/(sech(x) + 1), x)/a
```

Giac [A]
time = 0.38, size = 52, normalized size = 1.58

$$-\frac{\log\left(e^{(-x)}+e^{x}+2\right)}{4\,a}+\frac{\log\left(e^{(-x)}+e^{x}-2\right)}{4\,a}+\frac{e^{(-x)}+e^{x}-2}{4\,a(e^{(-x)}+e^{x}+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] -1/4*log(e^(-x) + e^x + 2)/a + 1/4*log(e^(-x) + e^x - 2)/a + 1/4*(e^(-x) +
e^x - 2)/(a*(e^(-x) + e^x + 2))
```

Mupad [B]
time = 1.44, size = 51, normalized size = 1.55

$$\frac{1}{a\,\left(\mathrm{e}^{2\,x}+2\,\mathrm{e}^{x}+1\right)}-\frac{1}{a\,\left(\mathrm{e}^{x}+1\right)}-\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{x}\,\sqrt{-a^2}}{a}\right)}{\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)*(a + a/cosh(x))),x)
```

```
[Out] 1/(a*(exp(2*x) + 2*exp(x) + 1)) - 1/(a*(exp(x) + 1)) - atan((exp(x)*(-a^2)^
(1/2))/a)/(-a^2)^(1/2)
```

3.57 $\int \frac{\operatorname{csch}^2(x)}{a+a\operatorname{sech}(x)}\,dx$

Optimal. Leaf size=23

$$-\frac{\coth^3(x)}{3a}+\frac{\operatorname{csch}^3(x)}{3a}$$

```
[Out] -1/3*coth(x)^3/a+1/3*csch(x)^3/a
```

Rubi [A]
time = 0.09, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3957, 2918, 2686, 30, 2687}

$$\frac{\operatorname{csch}^3(x)}{3a}-\frac{\coth^3(x)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^2/(a + a*Sech[x]),x]
```

```
[Out] -1/3*Coth[x]^3/a + Csch[x]^3/(3*a)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
```

```
e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^2(x)}{a+a\operatorname{sech}(x)}\,dx &= -\int \frac{\coth(x)\operatorname{csch}(x)}{-a-a\cosh(x)}\,dx\\ &= \frac{\int \coth^2(x)\operatorname{csch}^2(x)\,dx}{a}-\frac{\int \coth(x)\operatorname{csch}^3(x)\,dx}{a}\\ &= -\frac{i\operatorname{Subst}\left(\int x^2\,dx,x,i\coth(x)\right)}{a}-\frac{i\operatorname{Subst}\left(\int x^2\,dx,x,-i\operatorname{csch}(x)\right)}{a}\\ &= -\frac{\coth^3(x)}{3a}+\frac{\operatorname{csch}^3(x)}{3a}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 25, normalized size = 1.09

$$-\frac{(3+2\cosh(x)+\cosh(2x))\operatorname{csch}(x)}{6a(1+\cosh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^2/(a + a*Sech[x]),x]
[Out] -1/6*((3 + 2*Cosh[x] + Cosh[2*x])*Csch[x])/(a*(1 + Cosh[x]))
```

Maple [A]
time = 0.77, size = 23, normalized size = 1.00

method	result	size
default	$\frac{-\frac{\left(\tanh^3\left(\frac{x}{2}\right)\right)}{3}-\frac{1}{\tanh\left(\frac{x}{2}\right)}}{4a}$	23
risch	$-\frac{2\left(3\,\mathrm{e}^{2x}+2\,\mathrm{e}^{x}+1\right)}{3\left(\mathrm{e}^{x}+1\right)^{3}a\left(\mathrm{e}^{x}-1\right)}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^2/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a*(-1/3*tanh(1/2*x)^3-1/tanh(1/2*x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(19) = 38$.
time = 0.30, size = 90, normalized size = 3.91

$$-\frac{4\,e^{(-x)}}{3\,(2\,ae^{(-x)} - 2\,ae^{(-3\,x)} - ae^{(-4\,x)} + a)} - \frac{2\,e^{(-2\,x)}}{2\,ae^{(-x)} - 2\,ae^{(-3\,x)} - ae^{(-4\,x)} + a} - \frac{2}{3\,(2\,ae^{(-x)} - 2\,ae^{(-3\,x)} - ae^{(-4\,x)} + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] -4/3*e^(-x)/(2*a*e^(-x) - 2*a*e^(-3*x) - a*e^(-4*x) + a) - 2*e^(-2*x)/(2*a*
e^(-x) - 2*a*e^(-3*x) - a*e^(-4*x) + a) - 2/3/(2*a*e^(-x) - 2*a*e^(-3*x) -
a*e^(-4*x) + a)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(19) = 38$.
time = 0.35, size = 71, normalized size = 3.09

$$-\frac{4\,(2\,\cosh(x) + \sinh(x) + 1)}{3\,\left(a\cosh(x)^3 + a\sinh(x)^3 + 2\,a\cosh(x)^2 + (3\,a\cosh(x) + 2\,a)\sinh(x)^2 - a\cosh(x) + (3\,a\cosh(x)^2 + 4\,a\cosh(x) + a)\sinh(x) - 2\,a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] -4/3*(2*cosh(x) + sinh(x) + 1)/(a*cosh(x)^3 + a*sinh(x)^3 + 2*a*cosh(x)^2 +
 (3*a*cosh(x) + 2*a)*sinh(x)^2 - a*cosh(x) + (3*a*cosh(x)^2 + 4*a*cosh(x) +
 a)*sinh(x) - 2*a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{csch}^2(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2/(a+a*sech(x)),x)
```

```
[Out] Integral(csch(x)**2/(sech(x) + 1), x)/a
```

Giac [A]
time = 0.39, size = 31, normalized size = 1.35

$$-\frac{1}{2\,a(e^x - 1)} + \frac{3\,e^{(2\,x)} + 1}{6\,a(e^x + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] -1/2/(a*(e^x - 1)) + 1/6*(3*e^(2*x) + 1)/(a*(e^x + 1)^3)
```

Mupad [B]
time = 1.35, size = 91, normalized size = 3.96

$$\frac{\frac{e^{2x}}{6a}+\frac{1}{6a}-\frac{e^x}{3a}}{3e^{2x}+e^{3x}+3e^x+1}-\frac{\frac{1}{6a}-\frac{e^x}{6a}}{e^{2x}+2e^x+1}-\frac{1}{2a\,(e^x-1)}+\frac{1}{6a\,(e^x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^2*(a + a/cosh(x))),x)
```

```
[Out] (exp(2*x)/(6*a) + 1/(6*a) - exp(x)/(3*a))/(3*exp(2*x) + exp(3*x) + 3*exp(x)
 + 1) - (1/(6*a) - exp(x)/(6*a))/(exp(2*x) + 2*exp(x) + 1) - 1/(2*a*(exp(x)
 - 1)) + 1/(6*a*(exp(x) + 1))
```

3.58 $\int \frac{\operatorname{csch}^3(x)}{a+a\operatorname{sech}(x)}\,dx$

Optimal. Leaf size=46

$$\frac{\tanh^{-1}(\cosh(x))}{8a}-\frac{\coth(x)\operatorname{csch}(x)}{8a}-\frac{\coth(x)\operatorname{csch}^3(x)}{4a}+\frac{\operatorname{csch}^4(x)}{4a}$$

```
[Out] 1/8*arctanh(cosh(x))/a-1/8*coth(x)*csch(x)/a-1/4*coth(x)*csch(x)^3/a+1/4*cs
ch(x)^4/a
```

Rubi [A]
time = 0.12, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3957, 2914, 2686, 30, 2691, 3853, 3855}

$$\frac{\operatorname{csch}^4(x)}{4a}+\frac{\tanh^{-1}(\cosh(x))}{8a}-\frac{\coth(x)\operatorname{csch}^3(x)}{4a}-\frac{\coth(x)\operatorname{csch}(x)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^3/(a + a*Sech[x]),x]
```

```
[Out] ArcTanh[Cosh[x]]/(8*a) - (Coth[x]*Csch[x])/(8*a) - (Coth[x]*Csch[x]^3)/(4*a
) + Csch[x]^4/(4*a)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2914

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{csch}^3(x)}{a+a\operatorname{sech}(x)}\,dx &= -\int \frac{\coth(x)\operatorname{csch}^2(x)}{-a-a\cosh(x)}\,dx\\
&= \frac{\int \coth^2(x)\operatorname{csch}^3(x)\,dx}{a}-\frac{\int \coth(x)\operatorname{csch}^4(x)\,dx}{a}\\
&= -\frac{\coth(x)\operatorname{csch}^3(x)}{4a}+\frac{\int \operatorname{csch}^3(x)\,dx}{4a}+\frac{\operatorname{Subst}\left(\int x^3\,dx,x,-i\operatorname{csch}(x)\right)}{a}\\
&= -\frac{\coth(x)\operatorname{csch}(x)}{8a}-\frac{\coth(x)\operatorname{csch}^3(x)}{4a}+\frac{\operatorname{csch}^4(x)}{4a}-\frac{\int \operatorname{csch}(x)\,dx}{8a}\\
&= \frac{\tanh^{-1}(\cosh(x))}{8a}-\frac{\coth(x)\operatorname{csch}(x)}{8a}-\frac{\coth(x)\operatorname{csch}^3(x)}{4a}+\frac{\operatorname{csch}^4(x)}{4a}
\end{aligned}
$$

Mathematica [**A**]
time = 0.15, size = 59, normalized size = 1.28

$$
\frac{\cosh^2\left(\frac{x}{2}\right)\left(-2\operatorname{csch}^2\left(\frac{x}{2}\right)+4\log\left(\cosh\left(\frac{x}{2}\right)\right)-4\log\left(\sinh\left(\frac{x}{2}\right)\right)+\operatorname{sech}^4\left(\frac{x}{2}\right)\right)\operatorname{sech}(x)}{16(a+a\operatorname{sech}(x))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^3/(a + a*Sech[x]),x]
```

```
[Out] (Cosh[x/2]^2*(-2*Csch[x/2]^2 + 4*Log[Cosh[x/2]] - 4*Log[Sinh[x/2]] + Sech[x
/2]^4)*Sech[x])/(16*(a + a*Sech[x]))
```

Maple [A]
time = 0.80, size = 38, normalized size = 0.83

method	result	size
default	$\frac{\frac{\left(\tanh^4\left(\frac{x}{2}\right)\right)}{4}-\frac{\left(\tanh^2\left(\frac{x}{2}\right)\right)}{2}-\frac{1}{2\tanh\left(\frac{x}{2}\right)^2}-\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{8a}$	38
risch	$-\frac{e^x\left(e^{4x}+2\,e^{3x}+10\,e^{2x}+2\,e^x+1\right)}{4\left(e^x+1\right)^4 a\left(e^x-1\right)^2}+\frac{\ln\left(e^x+1\right)}{8a}-\frac{\ln\left(e^x-1\right)}{8a}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^3/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/a*(1/4*tanh(1/2*x)^4-1/2*tanh(1/2*x)^2-1/2/tanh(1/2*x)^2-ln(tanh(1/2*x)
))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(38) = 76$.
time = 0.27, size = 99, normalized size = 2.15

$$-\frac{e^{(-x)}+2\,e^{(-2\,x)}+10\,e^{(-3\,x)}+2\,e^{(-4\,x)}+e^{(-5\,x)}}{4\left(2\,ae^{(-x)}-ae^{(-2\,x)}-4\,ae^{(-3\,x)}-ae^{(-4\,x)}+2\,ae^{(-5\,x)}+ae^{(-6\,x)}+a\right)}+\frac{\log\left(e^{(-x)}+1\right)}{8\,a}-\frac{\log\left(e^{(-x)}-1\right)}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] -1/4*(e^(-x) + 2*e^(-2*x) + 10*e^(-3*x) + 2*e^(-4*x) + e^(-5*x))/(2*a*e^(-x
) - a*e^(-2*x) - 4*a*e^(-3*x) - a*e^(-4*x) + 2*a*e^(-5*x) + a*e^(-6*x) + a)
 + 1/8*log(e^(-x) + 1)/a - 1/8*log(e^(-x) - 1)/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 630 vs. $2(38) = 76$.
time = 0.36, size = 630, normalized size = 13.70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/8*(2*cosh(x)^5 + 2*(5*cosh(x) + 2)*sinh(x)^4 + 2*sinh(x)^5 + 4*cosh(x)^4
 + 4*(5*cosh(x)^2 + 4*cosh(x) + 5)*sinh(x)^3 + 20*cosh(x)^3 + 4*(5*cosh(x)^
```

```
3 + 6*cosh(x)^2 + 15*cosh(x) + 1)*sinh(x)^2 + 4*cosh(x)^2 - (cosh(x)^6 + 2*
(3*cosh(x) + 1)*sinh(x)^5 + sinh(x)^6 + 2*cosh(x)^5 + (15*cosh(x)^2 + 10*co
sh(x) - 1)*sinh(x)^4 - cosh(x)^4 + 4*(5*cosh(x)^3 + 5*cosh(x)^2 - cosh(x) -
 1)*sinh(x)^3 - 4*cosh(x)^3 + (15*cosh(x)^4 + 20*cosh(x)^3 - 6*cosh(x)^2 -
12*cosh(x) - 1)*sinh(x)^2 - cosh(x)^2 + 2*(3*cosh(x)^5 + 5*cosh(x)^4 - 2*co
sh(x)^3 - 6*cosh(x)^2 - cosh(x) + 1)*sinh(x) + 2*cosh(x) + 1)*log(cosh(x) +
 sinh(x) + 1) + (cosh(x)^6 + 2*(3*cosh(x) + 1)*sinh(x)^5 + sinh(x)^6 + 2*co
sh(x)^5 + (15*cosh(x)^2 + 10*cosh(x) - 1)*sinh(x)^4 - cosh(x)^4 + 4*(5*cosh
(x)^3 + 5*cosh(x)^2 - cosh(x) - 1)*sinh(x)^3 - 4*cosh(x)^3 + (15*cosh(x)^4
+ 20*cosh(x)^3 - 6*cosh(x)^2 - 12*cosh(x) - 1)*sinh(x)^2 - cosh(x)^2 + 2*(3
*cosh(x)^5 + 5*cosh(x)^4 - 2*cosh(x)^3 - 6*cosh(x)^2 - cosh(x) + 1)*sinh(x)
 + 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*(5*cosh(x)^4 + 8*cosh(x)^3
 + 30*cosh(x)^2 + 4*cosh(x) + 1)*sinh(x) + 2*cosh(x))/(a*cosh(x)^6 + a*sinh
(x)^6 + 2*a*cosh(x)^5 + 2*(3*a*cosh(x) + a)*sinh(x)^5 - a*cosh(x)^4 + (15*a
*cosh(x)^2 + 10*a*cosh(x) - a)*sinh(x)^4 - 4*a*cosh(x)^3 + 4*(5*a*cosh(x)^3
 + 5*a*cosh(x)^2 - a*cosh(x) - a)*sinh(x)^3 - a*cosh(x)^2 + (15*a*cosh(x)^4
 + 20*a*cosh(x)^3 - 6*a*cosh(x)^2 - 12*a*cosh(x) - a)*sinh(x)^2 + 2*a*cosh(
x) + 2*(3*a*cosh(x)^5 + 5*a*cosh(x)^4 - 2*a*cosh(x)^3 - 6*a*cosh(x)^2 - a*c
osh(x) + a)*sinh(x) + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{csch}^{3}(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**3/(a+a*sech(x)),x)
[Out] Integral(csch(x)**3/(sech(x) + 1), x)/a
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(38) = 76.
time = 0.39, size = 90, normalized size = 1.96

$$\frac{\log\left(e^{(-x)}+e^{x}+2\right)}{16\,a}-\frac{\log\left(e^{(-x)}+e^{x}-2\right)}{16\,a}+\frac{e^{(-x)}+e^{x}-6}{16\,a(e^{(-x)}+e^{x}-2)}-\frac{3\left(e^{(-x)}+e^{x}\right)^{2}+12\,e^{(-x)}+12\,e^{x}-4}{32\,a(e^{(-x)}+e^{x}+2)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+a*sech(x)),x, algorithm="giac")
[Out] 1/16*log(e^(-x) + e^x + 2)/a - 1/16*log(e^(-x) + e^x - 2)/a + 1/16*(e^(-x)
+ e^x - 6)/(a*(e^(-x) + e^x - 2)) - 1/32*(3*(e^(-x) + e^x)^2 + 12*e^(-x) +
12*e^x - 4)/(a*(e^(-x) + e^x + 2)^2)
```

Mupad [B]
time = 1.35, size = 121, normalized size = 2.63

$$\frac{1}{2\,a\,\left({\mathrm{e}}^{2\,x}+2\,{\mathrm{e}}^{x}+1\right)}-\frac{1}{4\,a\,\left({\mathrm{e}}^{2\,x}-2\,{\mathrm{e}}^{x}+1\right)}+\frac{1}{2\,a\,\left(6\,{\mathrm{e}}^{2\,x}+4\,{\mathrm{e}}^{3\,x}+{\mathrm{e}}^{4\,x}+4\,{\mathrm{e}}^{x}+1\right)}-\frac{1}{4\,a\,\left({\mathrm{e}}^{x}-1\right)}+\frac{\mathrm{atan}\left(\frac{{\mathrm{e}}^{x}\,\sqrt{-a^2}}{a}\right)}{4\,\sqrt{-a^2}}-\frac{1}{a\,\left(3\,{\mathrm{e}}^{2\,x}+{\mathrm{e}}^{3\,x}+3\,{\mathrm{e}}^{x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^3*(a + a/cosh(x))),x)
```

```
[Out] 1/(2*a*(exp(2*x) + 2*exp(x) + 1)) - 1/(4*a*(exp(2*x) - 2*exp(x) + 1)) + 1/(
2*a*(6*exp(2*x) + 4*exp(3*x) + exp(4*x) + 4*exp(x) + 1)) - 1/(4*a*(exp(x) -
 1)) + atan((exp(x)*(-a^2)^(1/2))/a)/(4*(-a^2)^(1/2)) - 1/(a*(3*exp(2*x) +
exp(3*x) + 3*exp(x) + 1))
```

3.59 $\int \frac{\operatorname{csch}^4(x)}{a+a\operatorname{sech}(x)}\,dx$

Optimal. Leaf size=34

$$\frac{\coth^3(x)}{3a}-\frac{\coth^5(x)}{5a}+\frac{\operatorname{csch}^5(x)}{5a}$$

```
[Out] 1/3*coth(x)^3/a-1/5*coth(x)^5/a+1/5*csch(x)^5/a
```

Rubi [A]
time = 0.10, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3957, 2918, 2686, 30, 2687, 14}

$$-\frac{\coth^5(x)}{5a}+\frac{\coth^3(x)}{3a}+\frac{\operatorname{csch}^5(x)}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^4/(a + a*Sech[x]),x]
```

```
[Out] Coth[x]^3/(3*a) - Coth[x]^5/(5*a) + Csch[x]^5/(5*a)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{csch}^4(x)}{a+a\operatorname{sech}(x)}\,dx &= -\int \frac{\coth(x)\operatorname{csch}^3(x)}{-a-a\cosh(x)}\,dx \\
&= \frac{\int \coth^2(x)\operatorname{csch}^4(x)\,dx}{a} - \frac{\int \coth(x)\operatorname{csch}^5(x)\,dx}{a} \\
&= \frac{i\text{Subst}\left(\int x^4\,dx, x, -i\operatorname{csch}(x)\right)}{a} + \frac{i\text{Subst}\left(\int x^2(1+x^2)\,dx, x, i\coth(x)\right)}{a} \\
&= \frac{\operatorname{csch}^5(x)}{5a} + \frac{i\text{Subst}\left(\int (x^2+x^4)\,dx, x, i\coth(x)\right)}{a} \\
&= \frac{\coth^3(x)}{3a} - \frac{\coth^5(x)}{5a} + \frac{\operatorname{csch}^5(x)}{5a}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 39, normalized size = 1.15

$$
\frac{(-15-6\cosh(x)-2\cosh(2x)+2\cosh(3x)+\cosh(4x))\operatorname{csch}^3(x)}{60a(1+\cosh(x))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^4/(a + a*Sech[x]),x]
```

```
[Out] ((-15 - 6*Cosh[x] - 2*Cosh[2*x] + 2*Cosh[3*x] + Cosh[4*x])*Csch[x]^3)/(60*a
*(1 + Cosh[x]))
```

Maple [A]
time = 0.81, size = 39, normalized size = 1.15

method	result	size
default	$\frac{-\frac{\left(\tanh^5\left(\frac{x}{2}\right)\right)}{5}+\frac{2\left(\tanh^3\left(\frac{x}{2}\right)\right)}{3}-\frac{1}{3\tanh\left(\frac{x}{2}\right)^3}+\frac{2}{\tanh\left(\frac{x}{2}\right)}}{16a}$	39
risch	$-\frac{4\left(15\,\mathrm{e}^{4x}+6\,\mathrm{e}^{3x}+2\,\mathrm{e}^{2x}-2\,\mathrm{e}^{x}-1\right)}{15\left(\mathrm{e}^{x}-1\right)^3 a\left(\mathrm{e}^{x}+1\right)^5}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^4/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/a*(-1/5*tanh(1/2*x)^5+2/3*tanh(1/2*x)^3-1/3/tanh(1/2*x)^3+2/tanh(1/2*x
))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(28) = 56$.
time = 0.26, size = 292, normalized size = 8.59

$$\frac{8\,e^{(-x)}}{15\,(2\,ae^{(-x)}-2\,ae^{(-2x)}-6\,ae^{(-3x)}+6\,ae^{(-5x)}+2\,ae^{(-6x)}-2\,ae^{(-7x)}-ae^{(-8x)}+a)}-\frac{8\,e^{(-2x)}}{15\,(2\,ae^{(-x)}-2\,ae^{(-2x)}-6\,ae^{(-3x)}+6\,ae^{(-5x)}+2\,ae^{(-6x)}-2\,ae^{(-7x)}-ae^{(-8x)}+a)}-\frac{8\,e^{(-3x)}}{5\,(2\,ae^{(-x)}-2\,ae^{(-2x)}-6\,ae^{(-3x)}+6\,ae^{(-5x)}+2\,ae^{(-6x)}-2\,ae^{(-7x)}-ae^{(-8x)}+a)}-\frac{4\,e^{(-4x)}}{2\,ae^{(-x)}-2\,ae^{(-2x)}-6\,ae^{(-3x)}+6\,ae^{(-5x)}+2\,ae^{(-6x)}-2\,ae^{(-7x)}-ae^{(-8x)}+a}+\frac{4}{15\,(2\,ae^{(-x)}-2\,ae^{(-2x)}-6\,ae^{(-3x)}+6\,ae^{(-5x)}+2\,ae^{(-6x)}-2\,ae^{(-7x)}-ae^{(-8x)}+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] 8/15*e^(-x)/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*
e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a) - 8/15*e^(-2*x)/(2*a*e^(-x) - 2*a
*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e
^(-8*x) + a) - 8/5*e^(-3*x)/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a
*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a) - 4*e^(-4*x)/(2*a
*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e
^(-7*x) - a*e^(-8*x) + a) + 4/15/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x)
+ 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(28) = 56$.
time = 0.35, size = 219, normalized size = 6.44

$$-\frac{8\,(7\,\cosh(x)^2+4\,(4\,\cosh(x)+1)\sinh(x)+7\,\sinh(x)^2+2\,\cosh(x)+1)}{15\,(a\cosh(x)^6+a\sinh(x)^6+2\,a\cosh(x)^5+2\,(3\,a\cosh(x)+a)\sinh(x)^5-2\,a\cosh(x)^4+(15\,a\cosh(x)^2+10\,a\cosh(x)-2\,a)\sinh(x)^4-6\,a\cosh(x)^3+2\,(10\,a\cosh(x)^3+10\,a\cosh(x)^2-4\,a\cosh(x)-3\,a)\sinh(x)^3-a\cosh(x)^2+(15\,a\cosh(x)^4+20\,a\cosh(x)^3-12\,a\cosh(x)^2-18\,a\cosh(x)-a)\sinh(x)^2+4\,a\cosh(x)+2\,(3\,a\cosh(x)^5+5\,a\cosh(x)^4-4\,a\cosh(x)^3-9\,a\cosh(x)^2+a\cosh(x)+4\,a)\sinh(x)+2\,a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] -8/15*(7*cosh(x)^2 + 4*(4*cosh(x) + 1)*sinh(x) + 7*sinh(x)^2 + 2*cosh(x) +
1)/(a*cosh(x)^6 + a*sinh(x)^6 + 2*a*cosh(x)^5 + 2*(3*a*cosh(x) + a)*sinh(x)
^5 - 2*a*cosh(x)^4 + (15*a*cosh(x)^2 + 10*a*cosh(x) - 2*a)*sinh(x)^4 - 6*a*
cosh(x)^3 + 2*(10*a*cosh(x)^3 + 10*a*cosh(x)^2 - 4*a*cosh(x) - 3*a)*sinh(x)
^3 - a*cosh(x)^2 + (15*a*cosh(x)^4 + 20*a*cosh(x)^3 - 12*a*cosh(x)^2 - 18*a
```

```
*cosh(x) - a)*sinh(x)^2 + 4*a*cosh(x) + 2*(3*a*cosh(x)^5 + 5*a*cosh(x)^4 -
4*a*cosh(x)^3 - 9*a*cosh(x)^2 + a*cosh(x) + 4*a)*sinh(x) + 2*a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{csch}^4(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**4/(a+a*sech(x)),x)
```

```
[Out] Integral(csch(x)**4/(sech(x) + 1), x)/a
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(28) = 56$.
time = 0.39, size = 59, normalized size = 1.74

$$\frac{3e^{(2x)} - 12e^{x} + 5}{24a(e^{x} - 1)^{3}} - \frac{15e^{(4x)} + 60e^{(3x)} + 10e^{(2x)} + 20e^{x} + 7}{120a(e^{x} + 1)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] 1/24*(3*e^(2*x) - 12*e^x + 5)/(a*(e^x - 1)^3) - 1/120*(15*e^(4*x) + 60*e^(3
*x) + 10*e^(2*x) + 20*e^x + 7)/(a*(e^x + 1)^5)
```

Mupad [B]
time = 1.38, size = 236, normalized size = 6.94

$$\frac{1}{6a\left(3e^{2x}-e^{3x}-3e^{x}+1\right)}-\frac{\frac{3e^{2x}}{40a}+\frac{e^{3x}}{40a}+\frac{1}{40a}-\frac{e^{x}}{8a}}{6e^{2x}+4e^{3x}+e^{4x}+4e^{x}+1}-\frac{\frac{e^{2x}}{40a}-\frac{1}{24a}+\frac{e^{x}}{20a}}{3e^{2x}+e^{3x}+3e^{x}+1}-\frac{\frac{e^{3x}}{10a}-\frac{e^{2x}}{4a}+\frac{e^{4x}}{40a}+\frac{1}{40a}+\frac{e^{x}}{10a}}{10e^{2x}+10e^{3x}+5e^{4x}+e^{5x}+5e^{x}+1}-\frac{1}{4a\left(e^{2x}-2e^{x}+1\right)}+\frac{1}{8a\left(e^{x}-1\right)}-\frac{1}{20a\left(e^{x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^4*(a + a/cosh(x))),x)
```

```
[Out] 1/(6*a*(3*exp(2*x) - exp(3*x) - 3*exp(x) + 1)) - ((3*exp(2*x))/(40*a) + exp
(3*x)/(40*a) + 1/(40*a) - exp(x)/(8*a))/(6*exp(2*x) + 4*exp(3*x) + exp(4*x)
 + 4*exp(x) + 1) - (exp(2*x)/(40*a) - 1/(24*a) + exp(x)/(20*a))/(3*exp(2*x)
 + exp(3*x) + 3*exp(x) + 1) - (exp(3*x)/(10*a) - exp(2*x)/(4*a) + exp(4*x)/
(40*a) + 1/(40*a) + exp(x)/(10*a))/(10*exp(2*x) + 10*exp(3*x) + 5*exp(4*x)
+ exp(5*x) + 5*exp(x) + 1) - 1/(4*a*(exp(2*x) - 2*exp(x) + 1)) + 1/(8*a*(ex
p(x) - 1)) - 1/(20*a*(exp(x) + 1))
```

3.60 $\int \frac{\sinh^4(x)}{a+b\textbf{sech}(x)}\,dx$

Optimal. Leaf size=132

$$\frac{(3a^4-12a^2b^2+8b^4)\,x}{8a^5}-\frac{2(a-b)^{3/2}b(a+b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^5}+\frac{(8b(a^2-b^2)-a(3a^2-4b^2)\cosh}{8a^4}$$

```
[Out] 1/8*(3*a^4-12*a^2*b^2+8*b^4)*x/a^5-2*(a-b)^(3/2)*b*(a+b)^(3/2)*arctan((a-b)
^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/a^5+1/8*(8*b*(a^2-b^2)-a*(3*a^2-4*b^2)*cosh
(x))*sinh(x)/a^4-1/12*(4*b-3*a*cosh(x))*sinh(x)^3/a^2
```

Rubi [A]
time = 0.26, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3957, 2944, 2814, 2738, 211}

$$-\frac{2b(a-b)^{3/2}(a+b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^5}-\frac{\sinh^3(x)(4b-3a\cosh(x))}{12a^2}+\frac{\sinh(x)\left(8b(a^2-b^2)-a(3a^2-4b^2)\cosh(x)\right)}{8a^4}+\frac{x(3a^4-12a^2b^2+8b^4)}{8a^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^4/(a + b*Sech[x]),x]
```

```
[Out] ((3*a^4 - 12*a^2*b^2 + 8*b^4)*x)/(8*a^5) - (2*(a - b)^(3/2)*b*(a + b)^(3/2)
*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/a^5 + ((8*b*(a^2 - b^2) - a*(
3*a^2 - 4*b^2)*Cosh[x])*Sinh[x])/(8*a^4) - ((4*b - 3*a*Cosh[x])*Sinh[x]^3)/
(12*a^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sinh^4(x)}{a+b\text{sech}(x)}\,dx = -\int \frac{\cosh(x)\sinh^4(x)}{-b-a\cosh(x)}\,dx$$

$$= -\frac{(4b-3a\cosh(x))\sinh^3(x)}{12a^2} + \frac{\int \frac{(-ab+(3a^2-4b^2)\cosh(x))\sinh^2(x)}{-b-a\cosh(x)}\,dx}{4a^2}$$

$$= \frac{(8b(a^2-b^2)-a(3a^2-4b^2)\cosh(x))\sinh(x)}{8a^4} - \frac{(4b-3a\cosh(x))\sinh^3(x)}{12a^2} - \frac{\int \frac{-ab(5a^2-}{}}{}$$

$$= \frac{(3a^4-12a^2b^2+8b^4)\,x}{8a^5} + \frac{(8b(a^2-b^2)-a(3a^2-4b^2)\cosh(x))\sinh(x)}{8a^4} - \frac{(4b-3a\cosh}{12}$$

$$= \frac{(3a^4-12a^2b^2+8b^4)\,x}{8a^5} + \frac{(8b(a^2-b^2)-a(3a^2-4b^2)\cosh(x))\sinh(x)}{8a^4} - \frac{(4b-3a\cosh}{12}$$

$$= \frac{(3a^4-12a^2b^2+8b^4)\,x}{8a^5} - \frac{2(a-b)^{3/2}b(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^5} + \frac{(8b(a^2-b^2}{}$$

Mathematica [A]
time = 0.50, size = 219, normalized size = 1.66

$$\frac{36a^4x-144a^2b^2x+96b^4x+\frac{192a^4b\text{ArcTan}\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-\frac{384a^2b^3\text{ArcTan}\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{192b^5\text{ArcTan}\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+24ab(5a^2-4b^2)\sinh(x)-24a^2(a^2-b^2)\sinh(2x)-8a^3b\sinh(3x)+3a^4\sinh(4x)}{96a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^4/(a + b*Sech[x]),x]
```

```
[Out] (36*a^4*x - 144*a^2*b^2*x + 96*b^4*x + (192*a^4*b*ArcTan[((-a + b)*Tanh[x/2
])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - (384*a^2*b^3*ArcTan[((-a + b)*Tanh[x
/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + (192*b^5*ArcTan[((-a + b)*Tanh[x/2
])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + 24*a*b*(5*a^2 - 4*b^2)*Sinh[x] - 24*
a^2*(a^2 - b^2)*Sinh[2*x] - 8*a^3*b*Sinh[3*x] + 3*a^4*Sinh[4*x])/(96*a^5)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(116) = 232$.
time = 0.71, size = 310, normalized size = 2.35

method	result
risch	$\frac{3x}{8a} - \frac{3x\,b^2}{2a^3} + \frac{x\,b^4}{a^5} + \frac{e^{4x}}{64a} - \frac{b\,e^{3x}}{24a^2} - \frac{e^{2x}}{8a} + \frac{e^{2x}b^2}{8a^3} + \frac{5b\,e^{x}}{8a^2} - \frac{b^3 e^{x}}{2a^4} - \frac{5b\,e^{-x}}{8a^2} + \frac{b^3 e^{-x}}{2a^4} + \frac{e^{-2x}}{8a} - \frac{e^{-2x}b^2}{8a^3} + \frac{b\,e^{-3x}}{24a^2}$
default	$\frac{1}{4a\left(\tanh\left(\frac{x}{2}\right)-1\right)^4} - \frac{-3a-2b}{6a^2\left(\tanh\left(\frac{x}{2}\right)-1\right)^3} - \frac{a^2-4ab-4b^2}{8a^3\left(\tanh\left(\frac{x}{2}\right)-1\right)^2} + \frac{\left(-3a^4+12a^2b^2-8b^4\right)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{8a^5} - \frac{3a^3+8a^2b-4a\,b^2-8b}{8a^4\left(\tanh\left(\frac{x}{2}\right)-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a/(tanh(1/2*x)-1)^4-1/6*(-3*a-2*b)/a^2/(tanh(1/2*x)-1)^3-1/8*(a^2-4*a*b
-4*b^2)/a^3/(tanh(1/2*x)-1)^2+1/8/a^5*(-3*a^4+12*a^2*b^2-8*b^4)*ln(tanh(1/2
*x)-1)-1/8*(3*a^3+8*a^2*b-4*a*b^2-8*b^3)/a^4/(tanh(1/2*x)-1)-1/4/a/(tanh(1/
2*x)+1)^4-1/6*(-3*a-2*b)/a^2/(tanh(1/2*x)+1)^3-1/8*(-a^2+4*a*b+4*b^2)/a^3/(
tanh(1/2*x)+1)^2+1/8*(3*a^4-12*a^2*b^2+8*b^4)/a^5*ln(tanh(1/2*x)+1)-1/8*(3*
a^3+8*a^2*b-4*a*b^2-8*b^3)/a^4/(tanh(1/2*x)+1)-2*b*(a^4-2*a^2*b^2+b^4)/a^5/
((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 866 vs. $2(115) = 230$.

time = 0.39, size = 1812, normalized size = 13.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [1/192*(3*a^4*cosh(x)^8 + 3*a^4*sinh(x)^8 - 8*a^3*b*cosh(x)^7 + 8*(3*a^4*co
sh(x) - a^3*b)*sinh(x)^7 - 24*(a^4 - a^2*b^2)*cosh(x)^6 + 4*(21*a^4*cosh(x)
^2 - 14*a^3*b*cosh(x) - 6*a^4 + 6*a^2*b^2)*sinh(x)^6 + 24*(3*a^4 - 12*a^2*b
^2 + 8*b^4)*x*cosh(x)^4 + 24*(5*a^3*b - 4*a*b^3)*cosh(x)^5 + 24*(7*a^4*cosh
(x)^3 - 7*a^3*b*cosh(x)^2 + 5*a^3*b - 4*a*b^3 - 6*(a^4 - a^2*b^2)*cosh(x))*
sinh(x)^5 + 8*a^3*b*cosh(x) + 2*(105*a^4*cosh(x)^4 - 140*a^3*b*cosh(x)^3 -
180*(a^4 - a^2*b^2)*cosh(x)^2 + 12*(3*a^4 - 12*a^2*b^2 + 8*b^4)*x + 60*(5*a
^3*b - 4*a*b^3)*cosh(x))*sinh(x)^4 - 3*a^4 - 24*(5*a^3*b - 4*a*b^3)*cosh(x)
^3 + 8*(21*a^4*cosh(x)^5 - 35*a^3*b*cosh(x)^4 - 15*a^3*b + 12*a*b^3 - 60*(a
^4 - a^2*b^2)*cosh(x)^3 + 12*(3*a^4 - 12*a^2*b^2 + 8*b^4)*x*cosh(x) + 30*(5
*a^3*b - 4*a*b^3)*cosh(x)^2)*sinh(x)^3 + 24*(a^4 - a^2*b^2)*cosh(x)^2 + 12*
(7*a^4*cosh(x)^6 - 14*a^3*b*cosh(x)^5 - 30*(a^4 - a^2*b^2)*cosh(x)^4 + 2*a^
4 - 2*a^2*b^2 + 12*(3*a^4 - 12*a^2*b^2 + 8*b^4)*x*cosh(x)^2 + 20*(5*a^3*b -
 4*a*b^3)*cosh(x)^3 - 6*(5*a^3*b - 4*a*b^3)*cosh(x))*sinh(x)^2 - 192*((a^2*
b - b^3)*cosh(x)^4 + 4*(a^2*b - b^3)*cosh(x)^3*sinh(x) + 6*(a^2*b - b^3)*co
sh(x)^2*sinh(x)^2 + 4*(a^2*b - b^3)*cosh(x)*sinh(x)^3 + (a^2*b - b^3)*sinh(
x)^4)*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) -
 a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-a^2 + b^2)*(a*cosh(x
) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x)
 + b)*sinh(x) + a)) + 8*(3*a^4*cosh(x)^7 - 7*a^3*b*cosh(x)^6 - 18*(a^4 - a^
2*b^2)*cosh(x)^5 + 12*(3*a^4 - 12*a^2*b^2 + 8*b^4)*x*cosh(x)^3 + 15*(5*a^3*
b - 4*a*b^3)*cosh(x)^4 + a^3*b - 9*(5*a^3*b - 4*a*b^3)*cosh(x)^2 + 6*(a^4 -
 a^2*b^2)*cosh(x))*sinh(x))/(a^5*cosh(x)^4 + 4*a^5*cosh(x)^3*sinh(x) + 6*a^
5*cosh(x)^2*sinh(x)^2 + 4*a^5*cosh(x)*sinh(x)^3 + a^5*sinh(x)^4), 1/192*(3*
a^4*cosh(x)^8 + 3*a^4*sinh(x)^8 - 8*a^3*b*cosh(x)^7 + 8*(3*a^4*cosh(x) - a^
3*b)*sinh(x)^7 - 24*(a^4 - a^2*b^2)*cosh(x)^6 + 4*(21*a^4*cosh(x)^2 - 14*a^
3*b*cosh(x) - 6*a^4 + 6*a^2*b^2)*sinh(x)^6 + 24*(3*a^4 - 12*a^2*b^2 + 8*b^4
)*x*cosh(x)^4 + 24*(5*a^3*b - 4*a*b^3)*cosh(x)^5 + 24*(7*a^4*cosh(x)^3 - 7*
a^3*b*cosh(x)^2 + 5*a^3*b - 4*a*b^3 - 6*(a^4 - a^2*b^2)*cosh(x))*sinh(x)^5
+ 8*a^3*b*cosh(x) + 2*(105*a^4*cosh(x)^4 - 140*a^3*b*cosh(x)^3 - 180*(a^4 -
 a^2*b^2)*cosh(x)^2 + 12*(3*a^4 - 12*a^2*b^2 + 8*b^4)*x + 60*(5*a^3*b - 4*a
*b^3)*cosh(x))*sinh(x)^4 - 3*a^4 - 24*(5*a^3*b - 4*a*b^3)*cosh(x)^3 + 8*(21
*a^4*cosh(x)^5 - 35*a^3*b*cosh(x)^4 - 15*a^3*b + 12*a*b^3 - 60*(a^4 - a^2*b
^2)*cosh(x)^3 + 12*(3*a^4 - 12*a^2*b^2 + 8*b^4)*x*cosh(x) + 30*(5*a^3*b - 4
*a*b^3)*cosh(x)^2)*sinh(x)^3 + 24*(a^4 - a^2*b^2)*cosh(x)^2 + 12*(7*a^4*cos
h(x)^6 - 14*a^3*b*cosh(x)^5 - 30*(a^4 - a^2*b^2)*cosh(x)^4 + 2*a^4 - 2*a^2*
b^2 + 12*(3*a^4 - 12*a^2*b^2 + 8*b^4)*x*cosh(x)^2 + 20*(5*a^3*b - 4*a*b^3)*
cosh(x)^3 - 6*(5*a^3*b - 4*a*b^3)*cosh(x))*sinh(x)^2 + 384*((a^2*b - b^3)*c
```

```
osh(x)^4 + 4*(a^2*b - b^3)*cosh(x)^3*sinh(x) + 6*(a^2*b - b^3)*cosh(x)^2*si
nh(x)^2 + 4*(a^2*b - b^3)*cosh(x)*sinh(x)^3 + (a^2*b - b^3)*sinh(x)^4)*sqrt
(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^2 - b^2)) + 8*(3*a^4
*cosh(x)^7 - 7*a^3*b*cosh(x)^6 - 18*(a^4 - a^2*b^2)*cosh(x)^5 + 12*(3*a^4 -
 12*a^2*b^2 + 8*b^4)*x*cosh(x)^3 + 15*(5*a^3*b - 4*a*b^3)*cosh(x)^4 + a^3*b
 - 9*(5*a^3*b - 4*a*b^3)*cosh(x)^2 + 6*(a^4 - a^2*b^2)*cosh(x))*sinh(x))/(a
^5*cosh(x)^4 + 4*a^5*cosh(x)^3*sinh(x) + 6*a^5*cosh(x)^2*sinh(x)^2 + 4*a^5*
cosh(x)*sinh(x)^3 + a^5*sinh(x)^4)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^4(x)}{a + b\,\mathrm{sech}\,(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**4/(a+b*sech(x)),x)
[Out] Integral(sinh(x)**4/(a + b*sech(x)), x)
```

Giac [A]

time = 0.39, size = 197, normalized size = 1.49

$$\frac{3\,a^3e^{(4\,x)} - 8\,a^2be^{(3\,x)} - 24\,a^3e^{(2\,x)} + 24\,ab^2e^{(2\,x)} + 120\,a^2be^x - 96\,b^3e^x}{192\,a^4} + \frac{(3\,a^4 - 12\,a^2b^2 + 8\,b^4)x}{8\,a^5} + \frac{(8\,a^3be^x - 3\,a^4 - 24\,(5\,a^3b - 4\,ab^3)e^{(3\,x)} + 24\,(a^4 - a^2b^2)e^{(2\,x)})e^{(-4\,x)}}{192\,a^5} - \frac{2\,(a^4b - 2\,a^2b^3 + b^5)\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*sech(x)),x, algorithm="giac")
[Out] 1/192*(3*a^3*e^(4*x) - 8*a^2*b*e^(3*x) - 24*a^3*e^(2*x) + 24*a*b^2*e^(2*x)
+ 120*a^2*b*e^x - 96*b^3*e^x)/a^4 + 1/8*(3*a^4 - 12*a^2*b^2 + 8*b^4)*x/a^5
+ 1/192*(8*a^3*b*e^x - 3*a^4 - 24*(5*a^3*b - 4*a*b^3)*e^(3*x) + 24*(a^4 - a
^2*b^2)*e^(2*x))*e^(-4*x)/a^5 - 2*(a^4*b - 2*a^2*b^3 + b^5)*arctan((a*e^x +
 b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a^5)
```

Mupad [B]

time = 2.01, size = 275, normalized size = 2.08

$$\frac{\mathrm{e}^{4\,x}}{64\,a} - \frac{\mathrm{e}^{-4\,x}}{64\,a} + \frac{x\,(3\,a^4 - 12\,a^2\,b^2 + 8\,b^4)}{8\,a^5} - \frac{\mathrm{e}^{-x}\,(5\,a^2\,b - 4\,b^3)}{8\,a^4} + \frac{\mathrm{e}^{-2\,x}\,(a^2 - b^2)}{8\,a^3} - \frac{\mathrm{e}^{2\,x}\,(a^2 - b^2)}{8\,a^3} + \frac{b\,\mathrm{e}^{-3\,x}}{24\,a^2} - \frac{b\,\mathrm{e}^{3\,x}}{24\,a^2} + \frac{\mathrm{e}^{x}\,(5\,a^2\,b - 4\,b^3)}{8\,a^4} + \frac{b\,\ln\left(\frac{2\,\mathrm{e}^x\,(a^4\,b - 2\,a^2\,b^3 + b^5)}{a^6} - \frac{2\,b\,(a+b)^{3/2}\,(a + b\,\mathrm{e}^x)\,(b-a)^{3/2}}{a^6}\right)\,(a+b)^{3/2}\,(b-a)^{3/2}}{a^5} - \frac{b\,\ln\left(\frac{2\,\mathrm{e}^x\,(a^4\,b - 2\,a^2\,b^3 + b^5)}{a^6} + \frac{2\,b\,(a+b)^{3/2}\,(a + b\,\mathrm{e}^x)\,(b-a)^{3/2}}{a^6}\right)\,(a+b)^{3/2}\,(b-a)^{3/2}}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(a + b/cosh(x)),x)
[Out] exp(4*x)/(64*a) - exp(-4*x)/(64*a) + (x*(3*a^4 + 8*b^4 - 12*a^2*b^2))/(8*a^
5) - (exp(-x)*(5*a^2*b - 4*b^3))/(8*a^4) + (exp(-2*x)*(a^2 - b^2))/(8*a^3)
- (exp(2*x)*(a^2 - b^2))/(8*a^3) + (b*exp(-3*x))/(24*a^2) - (b*exp(3*x))/(2
4*a^2) + (exp(x)*(5*a^2*b - 4*b^3))/(8*a^4) + (b*log((2*exp(x)*(a^4*b + b^5
 - 2*a^2*b^3))/a^6 - (2*b*(a + b)^(3/2)*(a + b*exp(x))*(b - a)^(3/2))/a^6)*
(a + b)^(3/2)*(b - a)^(3/2))/a^5 - (b*log((2*exp(x)*(a^4*b + b^5 - 2*a^2*b^
3))/a^6 + (2*b*(a + b)^(3/2)*(a + b*exp(x))*(b - a)^(3/2))/a^6)*(a + b)^(3/
2)*(b - a)^(3/2))/a^5
```

3.61 $\int \frac{\sinh^3(x)}{a+b\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=61

$$-\frac{(a^2-b^2)\cosh(x)}{a^3}-\frac{b\cosh^2(x)}{2a^2}+\frac{\cosh^3(x)}{3a}+\frac{b(a^2-b^2)\log(b+a\cosh(x))}{a^4}$$

```
[Out] -(a^2-b^2)*cosh(x)/a^3-1/2*b*cosh(x)^2/a^2+1/3*cosh(x)^3/a+b*(a^2-b^2)*ln(b
+a*cosh(x))/a^4
```

Rubi [A]
time = 0.12, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3957, 2916, 12, 786}

$$-\frac{b\cosh^2(x)}{2a^2}+\frac{b(a^2-b^2)\log(a\cosh(x)+b)}{a^4}-\frac{(a^2-b^2)\cosh(x)}{a^3}+\frac{\cosh^3(x)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(a + b*Sech[x]),x]
```

```
[Out] -(((a^2 - b^2)*Cosh[x])/a^3) - (b*Cosh[x]^2)/(2*a^2) + Cosh[x]^3/(3*a) + (b
*(a^2 - b^2)*Log[b + a*Cosh[x]])/a^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
```

```
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^3(x)}{a+b\mathrm{sech}(x)}\,dx &= -\int \frac{\cosh(x)\sinh^3(x)}{-b-a\cosh(x)}\,dx\\ &= \frac{\mathrm{Subst}\left(\int \frac{x(a^2-x^2)}{a(-b+x)}\,dx, x, -a\cosh(x)\right)}{a^3}\\ &= \frac{\mathrm{Subst}\left(\int \frac{x(a^2-x^2)}{-b+x}\,dx, x, -a\cosh(x)\right)}{a^4}\\ &= \frac{\mathrm{Subst}\left(\int \left(a^2\left(1-\frac{b^2}{a^2}\right)+\frac{-a^2b+b^3}{b-x}-bx-x^2\right)\,dx, x, -a\cosh(x)\right)}{a^4}\\ &= -\frac{(a^2-b^2)\cosh(x)}{a^3}-\frac{b\cosh^2(x)}{2a^2}+\frac{\cosh^3(x)}{3a}+\frac{b(a^2-b^2)\log(b+a\cosh(x))}{a^4}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 66, normalized size = 1.08

$$\frac{(-9a^3+12ab^2)\cosh(x)-3a^2b\cosh(2x)+a^3\cosh(3x)+12a^2b\log(b+a\cosh(x))-12b^3\log(b+a\cosh(x))}{12a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(a + b*Sech[x]),x]
```

```
[Out] ((-9*a^3 + 12*a*b^2)*Cosh[x] - 3*a^2*b*Cosh[2*x] + a^3*Cosh[3*x] + 12*a^2*b
*Log[b + a*Cosh[x]] - 12*b^3*Log[b + a*Cosh[x]])/(12*a^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 208 vs. 2(57) = 114.
time = 0.66, size = 209, normalized size = 3.43

method	result
risch	$-\frac{bx}{a^2}+\frac{x\,b^3}{a^4}+\frac{e^{3x}}{24a}-\frac{b\,e^{2x}}{8a^2}-\frac{3\,e^{x}}{8a}+\frac{e^{x}b^2}{2a^3}-\frac{3\,e^{-x}}{8a}+\frac{e^{-x}b^2}{2a^3}-\frac{b\,e^{-2x}}{8a^2}+\frac{e^{-3x}}{24a}+\frac{b\ln\left(e^{2x}+\frac{2b\,e^{x}}{a}+1\right)}{a^2}-\frac{b^3\ln\left(e^{2x}+\right.}{a^4}$
default	$\frac{b(a^3-a^2b-a\,b^2+b^3)\ln\left(a(\tanh^2(\frac{x}{2}))-b(\tanh^2(\frac{x}{2}))+a+b\right)}{a^4(a-b)}-\frac{-a^2+ab+2b^2}{2a^3(\tanh(\frac{x}{2})-1)}-\frac{a+b}{2a^2(\tanh(\frac{x}{2})-1)^2}-\frac{1}{3a(\tanh(\frac{x}{2})-1)^3}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

[Out] b*(a^3-a^2*b-a*b^2+b^3)/a^4/(a-b)*ln(a*tanh(1/2*x)^2-b*tanh(1/2*x)^2+a+b)-1/2*(-a^2+a*b+2*b^2)/a^3/(tanh(1/2*x)-1)-1/2*(a+b)/a^2/(tanh(1/2*x)-1)^2-1/3/a/(tanh(1/2*x)-1)^3-b*(a^2-b^2)/a^4*ln(tanh(1/2*x)-1)-1/2*(a+b)/a^2/(tanh(1/2*x)+1)^2-1/2*(a^2-a*b-2*b^2)/a^3/(tanh(1/2*x)+1)+1/3/a/(tanh(1/2*x)+1)^3-b*(a^2-b^2)/a^4*ln(tanh(1/2*x)+1)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(57) = 114$.
time = 0.27, size = 128, normalized size = 2.10

$$-\frac{\left(3\,abe^{(-x)}-a^2+3\,(3\,a^2-4\,b^2)e^{(-2\,x)}\right)e^{(3\,x)}}{24\,a^3}-\frac{3\,abe^{(-2\,x)}-a^2e^{(-3\,x)}+3\,(3\,a^2-4\,b^2)e^{(-x)}}{24\,a^3}+\frac{(a^2b-b^3)x}{a^4}+\frac{(a^2b-b^3)\log\left(2\,be^{(-x)}+ae^{(-2\,x)}+a\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(x)^3/(a+b*sech(x)),x, algorithm="maxima")

[Out] -1/24*(3*a*b*e^(-x) - a^2 + 3*(3*a^2 - 4*b^2)*e^(-2*x))*e^(3*x)/a^3 - 1/24*(3*a*b*e^(-2*x) - a^2*e^(-3*x) + 3*(3*a^2 - 4*b^2)*e^(-x))/a^3 + (a^2*b - b^3)*x/a^4 + (a^2*b - b^3)*log(2*b*e^(-x) + a*e^(-2*x) + a)/a^4

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(57) = 114$.
time = 0.37, size = 490, normalized size = 8.03

$$\frac{a^3\cosh(x)^6+a^3\sinh(x)^6-3\,a^2b\cosh(x)^5+3\,(2\,a^3\cosh(x)-a^2b)\sinh(x)^5-24\,(a^2b-b^3)x\cosh(x)^3-3\,(3\,a^3-4\,ab^2)\cosh(x)^4+3\,(5\,a^3\cosh(x)^2-5\,a^2b\cosh(x)-3\,a^3+4\,ab^2)\sinh(x)^4-3\,a^2b\cosh(x)+2\,(10\,a^3\cosh(x)^3-15\,a^2b\cosh(x)^2-12\,(a^2b-b^3)x-6\,(3\,a^3-4\,ab^2)\cosh(x))\sinh(x)^3+a^3-3\,(3\,a^3-4\,ab^2)\cosh(x)^2+3\,(5\,a^3\cosh(x)^4-10\,a^2b\cosh(x)^3-3\,a^3+4\,ab^2-24\,(a^2b-b^3)x\cosh(x)-6\,(3\,a^3-4\,ab^2)\cosh(x)^2)\sinh(x)^2+24\,((a^2b-b^3)\cosh(x)^3+3\,(a^2b-b^3)\cosh(x)^2\sinh(x)+3\,(a^2b-b^3)\cosh(x)\sinh(x)^2+(a^2b-b^3)\sinh(x)^3)\log\left(\frac{2\,(a\cosh(x)+b)}{\cosh(x)-\sinh(x)}\right)+3\,(2\,a^3\cosh(x)^5-5\,a^2b\cosh(x)^4-24\,(a^2b-b^3)x\cosh(x)^2-4\,(3\,a^3-4\,ab^2)\cosh(x)^3-a^2b-2\,(3\,a^3-4\,ab^2)\cosh(x))\sinh(x)}{24\,(a^4\cosh(x)^3+3\,a^4\cosh(x)^2\sinh(x)+3\,a^4\cosh(x)\sinh(x)^2+a^4\sinh(x)^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(x)^3/(a+b*sech(x)),x, algorithm="fricas")

[Out] 1/24*(a^3*cosh(x)^6 + a^3*sinh(x)^6 - 3*a^2*b*cosh(x)^5 + 3*(2*a^3*cosh(x) - a^2*b)*sinh(x)^5 - 24*(a^2*b - b^3)*x*cosh(x)^3 - 3*(3*a^3 - 4*a*b^2)*cosh(x)^4 + 3*(5*a^3*cosh(x)^2 - 5*a^2*b*cosh(x) - 3*a^3 + 4*a*b^2)*sinh(x)^4 - 3*a^2*b*cosh(x) + 2*(10*a^3*cosh(x)^3 - 15*a^2*b*cosh(x)^2 - 12*(a^2*b - b^3)*x - 6*(3*a^3 - 4*a*b^2)*cosh(x))*sinh(x)^3 + a^3 - 3*(3*a^3 - 4*a*b^2)*cosh(x)^2 + 3*(5*a^3*cosh(x)^4 - 10*a^2*b*cosh(x)^3 - 3*a^3 + 4*a*b^2 - 24*(a^2*b - b^3)*x*cosh(x) - 6*(3*a^3 - 4*a*b^2)*cosh(x)^2)*sinh(x)^2 + 24*((a^2*b - b^3)*cosh(x)^3 + 3*(a^2*b - b^3)*cosh(x)^2*sinh(x) + 3*(a^2*b - b^3)*cosh(x)*sinh(x)^2 + (a^2*b - b^3)*sinh(x)^3)*log(2*(a*cosh(x) + b)/(cosh(x) - sinh(x))) + 3*(2*a^3*cosh(x)^5 - 5*a^2*b*cosh(x)^4 - 24*(a^2*b - b^3)*x*cosh(x)^2 - 4*(3*a^3 - 4*a*b^2)*cosh(x)^3 - a^2*b - 2*(3*a^3 - 4*a*b^2)*cosh(x))*sinh(x))/(a^4*cosh(x)^3 + 3*a^4*cosh(x)^2*sinh(x) + 3*a^4*cosh(x)*sinh(x)^2 + a^4*sinh(x)^3)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sinh^3(x)}{a+b\operatorname{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(a+b*sech(x)),x)
```

```
[Out] Integral(sinh(x)**3/(a + b*sech(x)), x)
```

Giac [A]
time = 0.39, size = 87, normalized size = 1.43

$$\frac{a^2\left(e^{(-x)}+e^x\right)^3-3\,ab\left(e^{(-x)}+e^x\right)^2-12\,a^2\left(e^{(-x)}+e^x\right)+12\,b^2\left(e^{(-x)}+e^x\right)}{24\,a^3}+\frac{\left(a^2b-b^3\right)\log\left(\left|a\left(e^{(-x)}+e^x\right)+2\,b\right|\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] 1/24*(a^2*(e^(-x) + e^x)^3 - 3*a*b*(e^(-x) + e^x)^2 - 12*a^2*(e^(-x) + e^x)
 + 12*b^2*(e^(-x) + e^x))/a^3 + (a^2*b - b^3)*log(abs(a*(e^(-x) + e^x) + 2*
b))/a^4
```

Mupad [B]
time = 1.60, size = 123, normalized size = 2.02

$$\frac{\mathrm{e}^{-3\,x}}{24\,a}+\frac{\mathrm{e}^{3\,x}}{24\,a}-\frac{x\,\left(a^2\,b-b^3\right)}{a^4}-\frac{\mathrm{e}^x\,\left(3\,a^2-4\,b^2\right)}{8\,a^3}-\frac{b\,\mathrm{e}^{-2\,x}}{8\,a^2}-\frac{b\,\mathrm{e}^{2\,x}}{8\,a^2}+\frac{\ln\left(a+2\,b\,\mathrm{e}^x+a\,\mathrm{e}^{2\,x}\right)\,\left(a^2\,b-b^3\right)}{a^4}-\frac{\mathrm{e}^{-x}\,\left(3\,a^2-4\,b^2\right)}{8\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a + b/cosh(x)),x)
```

```
[Out] exp(-3*x)/(24*a) + exp(3*x)/(24*a) - (x*(a^2*b - b^3))/a^4 - (exp(x)*(3*a^2
 - 4*b^2))/(8*a^3) - (b*exp(-2*x))/(8*a^2) - (b*exp(2*x))/(8*a^2) + (log(a
+ 2*b*exp(x) + a*exp(2*x))*(a^2*b - b^3))/a^4 - (exp(-x)*(3*a^2 - 4*b^2))/(
8*a^3)
```

3.62 $\int \frac{\sinh^2(x)}{a+b\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=82

$$-\frac{(a^2-2b^2)\,x}{2a^3}+\frac{2\sqrt{a-b}\,b\sqrt{a+b}\,\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^3}-\frac{(2b-a\cosh(x))\sinh(x)}{2a^2}$$

```
[Out] -1/2*(a^2-2*b^2)*x/a^3-1/2*(2*b-a*cosh(x))*sinh(x)/a^2+2*b*arctan((a-b)^(1/
2)*tanh(1/2*x)/(a+b)^(1/2))*(a-b)^(1/2)*(a+b)^(1/2)/a^3
```

Rubi [A]
time = 0.13, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3957, 2944, 2814, 2738, 211}

$$\frac{2b\sqrt{a-b}\,\sqrt{a+b}\,\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^3}-\frac{\sinh(x)(2b-a\cosh(x))}{2a^2}-\frac{x(a^2-2b^2)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(a + b*Sech[x]),x]
```

```
[Out] -1/2*((a^2 - 2*b^2)*x)/a^3 + (2*Sqrt[a - b]*b*Sqrt[a + b]*ArcTan[(Sqrt[a -
b]*Tanh[x/2])/Sqrt[a + b]])/a^3 - ((2*b - a*Cosh[x])*Sinh[x])/(2*a^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^2(x)}{a+b\text{sech}(x)}\,dx &= -\int \frac{\cosh(x)\sinh^2(x)}{-b-a\cosh(x)}\,dx\\
&= -\frac{(2b-a\cosh(x))\sinh(x)}{2a^2}+\frac{\int \frac{-ab+(a^2-2b^2)\cosh(x)}{-b-a\cosh(x)}\,dx}{2a^2}\\
&= -\frac{(a^2-2b^2)\,x}{2a^3}-\frac{(2b-a\cosh(x))\sinh(x)}{2a^2}-\frac{(b(a^2-b^2))\int \frac{1}{-b-a\cosh(x)}\,dx}{a^3}\\
&= -\frac{(a^2-2b^2)\,x}{2a^3}-\frac{(2b-a\cosh(x))\sinh(x)}{2a^2}-\frac{(2b(a^2-b^2))\,\text{Subst}\left(\int \frac{1}{-a-b-(a-b)x^2}\,dx,x,
\end{aligned}
$$

$$
= -\frac{(a^2-2b^2)\,x}{2a^3}+\frac{2\sqrt{a-b}\,b\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^3}-\frac{(2b-a\cosh(x))\,\text{sinh}
$$

Mathematica [A]
time = 0.13, size = 76, normalized size = 0.93

$$
\frac{-2a^2x+4b^2x-8b\sqrt{a^2-b^2}\,\text{ArcTan}\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)-4ab\sinh(x)+a^2\sinh(2x)}{4a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(a + b*Sech[x]),x]
```

[Out] (-2*a^2*x + 4*b^2*x - 8*b*Sqrt[a^2 - b^2]*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[
a^2 - b^2]] - 4*a*b*Sinh[x] + a^2*Sinh[2*x])/(4*a^3)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(68) = 136$.
time = 0.68, size = 160, normalized size = 1.95

method	result
risch	$-\frac{x}{2a}+\frac{x\,b^2}{a^3}+\frac{e^{2x}}{8a}-\frac{b\,e^{x}}{2a^2}+\frac{b\,e^{-x}}{2a^2}-\frac{e^{-2x}}{8a}+\frac{\sqrt{-a^2+b^2}\,b\ln\left(e^{x}+\frac{b+\sqrt{-a^2+b^2}}{a}\right)}{a^3}-\frac{\sqrt{-a^2+b^2}\,b\ln\left(e^{x}-\cdots\right)}{a^3}$
default	$-\frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}-\frac{-2b-a}{2a^2\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{\left(-a^2+2b^2\right)\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2a^3}+\frac{2b\left(a^2-b^2\right)\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^3\sqrt{(a+b)(a-b)}}+\frac{\cdots}{2a\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a/(tanh(1/2*x)+1)^2-1/2*(-2*b-a)/a^2/(tanh(1/2*x)+1)+1/2/a^3*(-a^2+2*b
^2)*ln(tanh(1/2*x)+1)+2*b*(a^2-b^2)/a^3/((a+b)*(a-b))^(1/2)*arctan((a-b)*ta
nh(1/2*x)/((a+b)*(a-b))^(1/2))+1/2/a/(tanh(1/2*x)-1)^2+1/2/a^3*(a^2-2*b^2)*
ln(tanh(1/2*x)-1)-1/2*(-2*b-a)/a^2/(tanh(1/2*x)-1)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(67) = 134$.
time = 0.37, size = 536, normalized size = 6.54

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [1/8*(a^2*cosh(x)^4 + a^2*sinh(x)^4 - 4*a*b*cosh(x)^3 - 4*(a^2 - 2*b^2)*x*c
osh(x)^2 + 4*(a^2*cosh(x) - a*b)*sinh(x)^3 + 4*a*b*cosh(x) + 2*(3*a^2*cosh(
x)^2 - 6*a*b*cosh(x) - 2*(a^2 - 2*b^2)*x)*sinh(x)^2 + 8*(b*cosh(x)^2 + 2*b*
cosh(x)*sinh(x) + b*sinh(x)^2)*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*si
nh(x)^2 + 2*a*b*cosh(x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*s
qrt(-a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2
*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) + a)) - a^2 + 4*(a^2*cosh(x)^3 - 3*a
*b*cosh(x)^2 - 2*(a^2 - 2*b^2)*x*cosh(x) + a*b)*sinh(x))/(a^3*cosh(x)^2 + 2
*a^3*cosh(x)*sinh(x) + a^3*sinh(x)^2), 1/8*(a^2*cosh(x)^4 + a^2*sinh(x)^4 -
 4*a*b*cosh(x)^3 - 4*(a^2 - 2*b^2)*x*cosh(x)^2 + 4*(a^2*cosh(x) - a*b)*sinh
(x)^3 + 4*a*b*cosh(x) + 2*(3*a^2*cosh(x)^2 - 6*a*b*cosh(x) - 2*(a^2 - 2*b^2
)*x)*sinh(x)^2 - 16*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2)*sqrt(
a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^2 - b^2)) - a^2 + 4*(
a^2*cosh(x)^3 - 3*a*b*cosh(x)^2 - 2*(a^2 - 2*b^2)*x*cosh(x) + a*b)*sinh(x))
/(a^3*cosh(x)^2 + 2*a^3*cosh(x)*sinh(x) + a^3*sinh(x)^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(x)}{a + b\,\mathrm{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(a+b*sech(x)),x)
```

```
[Out] Integral(sinh(x)**2/(a + b*sech(x)), x)
```

Giac [**A**]
time = 0.39, size = 100, normalized size = 1.22

$$\frac{ae^{(2x)} - 4\,be^{x}}{8\,a^2} - \frac{(a^2 - 2\,b^2)x}{2\,a^3} + \frac{(4\,abe^{x} - a^2)e^{(-2\,x)}}{8\,a^3} + \frac{2\,(a^2 b - b^3)\arctan\left(\frac{ae^{x}+b}{\sqrt{a^2 - b^2}}\right)}{\sqrt{a^2 - b^2}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] 1/8*(a*e^(2*x) - 4*b*e^x)/a^2 - 1/2*(a^2 - 2*b^2)*x/a^3 + 1/8*(4*a*b*e^x -
a^2)*e^(-2*x)/a^3 + 2*(a^2*b - b^3)*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sq
rt(a^2 - b^2)*a^3)
```

Mupad [**B**]
time = 1.67, size = 173, normalized size = 2.11

$$\frac{\mathrm{e}^{2\,x}}{8\,a} - \frac{\mathrm{e}^{-2\,x}}{8\,a} - \frac{b\,\mathrm{e}^{x}}{2\,a^2} + \frac{b\,\mathrm{e}^{-x}}{2\,a^2} - \frac{x\,(a^2 - 2\,b^2)}{2\,a^3} + \frac{b\,\ln\left(-\frac{2\,b\,\mathrm{e}^{x}\,(a^2-b^2)}{a^4} - \frac{2\,b\,\sqrt{a+b}\,(a+b\,\mathrm{e}^{x})\,\sqrt{b-a}}{a^4}\right)\sqrt{a+b}\,\sqrt{b-a}}{a^3} - \frac{b\,\ln\left(\frac{2\,b\,\sqrt{a+b}\,(a+b\,\mathrm{e}^{x})\,\sqrt{b-a}}{a^4} - \frac{2\,b\,\mathrm{e}^{x}\,(a^2-b^2)}{a^4}\right)\sqrt{a+b}\,\sqrt{b-a}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a + b/cosh(x)),x)
```

```
[Out] exp(2*x)/(8*a) - exp(-2*x)/(8*a) - (b*exp(x))/(2*a^2) + (b*exp(-x))/(2*a^2)
 - (x*(a^2 - 2*b^2))/(2*a^3) + (b*log(- (2*b*exp(x)*(a^2 - b^2))/a^4 - (2*b
*(a + b)^(1/2)*(a + b*exp(x))*(b - a)^(1/2))/a^4)*(a + b)^(1/2)*(b - a)^(1/
2))/a^3 - (b*log((2*b*(a + b)^(1/2)*(a + b*exp(x))*(b - a)^(1/2))/a^4 - (2*
b*exp(x)*(a^2 - b^2))/a^4)*(a + b)^(1/2)*(b - a)^(1/2))/a^3
```

3.63 $\int \frac{\sinh(x)}{a+b\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=20

$$\frac{\cosh(x)}{a} - \frac{b\log(b + a\cosh(x))}{a^2}$$

```
[Out] cosh(x)/a-b*ln(b+a*cosh(x))/a^2
```

Rubi [A]
time = 0.06, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3957, 2912, 12, 45}

$$\frac{\cosh(x)}{a} - \frac{b\log(a\cosh(x) + b)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(a + b*Sech[x]),x]
```

```
[Out] Cosh[x]/a - (b*Log[b + a*Cosh[x]])/a^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sinh(x)}{a + b\text{sech}(x)}\,dx = -\int \frac{\cosh(x)\sinh(x)}{-b - a\cosh(x)}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{x}{a(-b+x)}\,dx, x, -a\cosh(x)\right)}{a}$$
$$= -\frac{\text{Subst}\left(\int \frac{x}{-b+x}\,dx, x, -a\cosh(x)\right)}{a^2}$$
$$= -\frac{\text{Subst}\left(\int \left(1 - \frac{b}{b-x}\right)\,dx, x, -a\cosh(x)\right)}{a^2}$$
$$= \frac{\cosh(x)}{a} - \frac{b\log(b + a\cosh(x))}{a^2}$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 0.95

$$\frac{a\cosh(x) - b\log(b + a\cosh(x))}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(a + b*Sech[x]),x]
[Out] (a*Cosh[x] - b*Log[b + a*Cosh[x]])/a^2
```

Maple [A]
time = 0.62, size = 31, normalized size = 1.55

method	result	size
derivativedivides	$-\frac{b\ln(a+b\,\text{sech}(x))}{a^2}+\frac{1}{a\,\text{sech}(x)}+\frac{b\ln(\text{sech}(x))}{a^2}$	31
default	$-\frac{b\ln(a+b\,\text{sech}(x))}{a^2}+\frac{1}{a\,\text{sech}(x)}+\frac{b\ln(\text{sech}(x))}{a^2}$	31
risch	$\frac{bx}{a^2}+\frac{e^x}{2a}+\frac{e^{-x}}{2a}-\frac{b\ln\left(e^{2x}+\frac{2b\,e^x}{a}+1\right)}{a^2}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a+b*sech(x)),x,method=_RETURNVERBOSE)
[Out] -b/a^2*ln(a+b*sech(x))+1/a/sech(x)+b/a^2*ln(sech(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(20) = 40.
time = 0.26, size = 46, normalized size = 2.30

$$-\frac{bx}{a^2} + \frac{e^{(-x)}}{2\,a} + \frac{e^x}{2\,a} - \frac{b\log\left(2\,be^{(-x)} + ae^{(-2\,x)} + a\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] -b*x/a^2 + 1/2*e^(-x)/a + 1/2*e^x/a - b*log(2*b*e^(-x) + a*e^(-2*x) + a)/a^
2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(20) = 40$.

time = 0.37, size = 78, normalized size = 3.90

$$\frac{2\,bx\cosh\left(x\right) + a\cosh\left(x\right)^{2} + a\sinh\left(x\right)^{2} - 2\,\left(b\cosh\left(x\right) + b\sinh\left(x\right)\right)\log\left(\frac{2\,\left(a\cosh\left(x\right)+b\right)}{\cosh\left(x\right)-\sinh\left(x\right)}\right) + 2\,\left(bx + a\cosh\left(x\right)\right)\sinh\left(x\right) + a}{2\,\left(a^{2}\cosh\left(x\right) + a^{2}\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x*cosh(x) + a*cosh(x)^2 + a*sinh(x)^2 - 2*(b*cosh(x) + b*sinh(x))*
log(2*(a*cosh(x) + b)/(cosh(x) - sinh(x))) + 2*(b*x + a*cosh(x))*sinh(x) +
a)/(a^2*cosh(x) + a^2*sinh(x))
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sinh\left(x\right)}{a + b\operatorname{sech}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sech(x)),x)
```

```
[Out] Integral(sinh(x)/(a + b*sech(x)), x)
```

Giac [A]

time = 0.38, size = 34, normalized size = 1.70

$$\frac{e^{(-x)} + e^{x}}{2\,a} - \frac{b\log\left(\left|a\left(e^{(-x)} + e^{x}\right) + 2\,b\right|\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] 1/2*(e^(-x) + e^x)/a - b*log(abs(a*(e^(-x) + e^x) + 2*b))/a^2
```

Mupad [B]

time = 1.35, size = 20, normalized size = 1.00

$$\frac{\cosh(x)}{a} - \frac{b\,\ln\left(b + a\cosh(x)\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a + b/cosh(x)),x)
```

```
[Out] cosh(x)/a - (b*log(b + a*cosh(x)))/a^2
```

3.64 $\int \frac{\operatorname{csch}(x)}{a+b\operatorname{sech}(x)}\,dx$

Optimal. Leaf size=53

$$\frac{\log(1-\cosh(x))}{2(a+b)}-\frac{\log(1+\cosh(x))}{2(a-b)}+\frac{b\log(b+a\cosh(x))}{a^2-b^2}$$

```
[Out] 1/2*ln(1-cosh(x))/(a+b)-1/2*ln(1+cosh(x))/(a-b)+b*ln(b+a*cosh(x))/(a^2-b^2)
```

Rubi [A]
time = 0.08, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3957, 2800, 815}

$$\frac{b\log(a\cosh(x)+b)}{a^2-b^2}+\frac{\log(1-\cosh(x))}{2(a+b)}-\frac{\log(\cosh(x)+1)}{2(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]/(a + b*Sech[x]),x]
```

```
[Out] Log[1 - Cosh[x]]/(2*(a + b)) - Log[1 + Cosh[x]]/(2*(a - b)) + (b*Log[b + a*
Cosh[x]])/(a^2 - b^2)
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}(x)}{a+b\operatorname{sech}(x)}\,dx &= -\int \frac{\coth(x)}{-b-a\cosh(x)}\,dx\\
&= \text{Subst}\left(\int \frac{x}{(-b+x)\,(a^2-x^2)}\,dx, x, -a\cosh(x)\right)\\
&= \text{Subst}\left(\int \left(\frac{1}{2(a-b)(a-x)}-\frac{b}{(a-b)(a+b)(b-x)}+\frac{1}{2(a+b)(a+x)}\right)\,dx, x, -a\operatorname{co}\right.\\
&= \frac{\log(1-\cosh(x))}{2(a+b)}-\frac{\log(1+\cosh(x))}{2(a-b)}+\frac{b\log(b+a\cosh(x))}{a^2-b^2}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 37, normalized size = 0.70

$$\frac{b\log(b+a\cosh(x))-b\log(\sinh(x))+a\log\left(\tanh\left(\frac{x}{2}\right)\right)}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]/(a + b*Sech[x]),x]
[Out] (b*Log[b + a*Cosh[x]] - b*Log[Sinh[x]] + a*Log[Tanh[x/2]])/(a^2 - b^2)
```

Maple [A]
time = 0.79, size = 48, normalized size = 0.91

method	result	size
default	$\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a+b}+\frac{b\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-b\left(\tanh^2\left(\frac{x}{2}\right)\right)+a+b\right)}{(a+b)(a-b)}$	48
risch	$-\frac{x}{a+b}+\frac{x}{a-b}-\frac{2bx}{a^2-b^2}+\frac{\ln(e^x-1)}{a+b}-\frac{\ln(e^x+1)}{a-b}+\frac{b\ln\left(e^{2x}+\frac{2b\,e^x}{a}+1\right)}{a^2-b^2}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)/(a+b*sech(x)),x,method=_RETURNVERBOSE)
[Out] 1/(a+b)*ln(tanh(1/2*x))+b/(a+b)/(a-b)*ln(a*tanh(1/2*x)^2-b*tanh(1/2*x)^2+a+
b)
```

Maxima [A]
time = 0.27, size = 59, normalized size = 1.11

$$\frac{b\log\left(2\,be^{(-x)}+ae^{(-2\,x)}+a\right)}{a^2-b^2}-\frac{\log\left(e^{(-x)}+1\right)}{a-b}+\frac{\log\left(e^{(-x)}-1\right)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] b*log(2*b*e^(-x) + a*e^(-2*x) + a)/(a^2 - b^2) - log(e^(-x) + 1)/(a - b) +
log(e^(-x) - 1)/(a + b)
```

Fricas [A]
time = 0.37, size = 58, normalized size = 1.09

$$\frac{b\log\left(\frac{2\,(a\cosh(x)+b)}{\cosh(x)-\sinh(x)}\right)-(a+b)\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)+(a-b)\log\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] (b*log(2*(a*cosh(x) + b)/(cosh(x) - sinh(x))) - (a + b)*log(cosh(x) + sinh(
x) + 1) + (a - b)*log(cosh(x) + sinh(x) - 1))/(a^2 - b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{csch}\left(x\right)}{a+b\operatorname{sech}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*sech(x)),x)
```

```
[Out] Integral(csch(x)/(a + b*sech(x)), x)
```

Giac [A]
time = 0.38, size = 65, normalized size = 1.23

$$\frac{ab\log\left(\left|a\left(e^{(-x)}+e^x\right)+2\,b\right|\right)}{a^3-ab^2}-\frac{\log\left(e^{(-x)}+e^x+2\right)}{2\,(a-b)}+\frac{\log\left(e^{(-x)}+e^x-2\right)}{2\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] a*b*log(abs(a*(e^(-x) + e^x) + 2*b))/(a^3 - a*b^2) - 1/2*log(e^(-x) + e^x +
 2)/(a - b) + 1/2*log(e^(-x) + e^x - 2)/(a + b)
```

Mupad [B]
time = 1.74, size = 148, normalized size = 2.79

$$\frac{\ln\left(128\,a\,b-32\,a^2-128\,b^2+32\,a^2\,\mathrm{e}^x+128\,b^2\,\mathrm{e}^x-128\,a\,b\,\mathrm{e}^x\right)}{a+b}-\frac{\ln\left(-128\,a\,b-32\,a^2-128\,b^2-32\,a^2\,\mathrm{e}^x-128\,b^2\,\mathrm{e}^x-128\,a\,b\,\mathrm{e}^x\right)}{a-b}+\frac{b\,\ln\left(16\,a\,b^2-4\,a^3\,\mathrm{e}^{2\,x}-4\,a^3+32\,b^3\,\mathrm{e}^x-8\,a^2\,b\,\mathrm{e}^x+16\,a\,b^2\,\mathrm{e}^{2\,x}\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)*(a + b/cosh(x))),x)
```

```
[Out] log(128*a*b - 32*a^2 - 128*b^2 + 32*a^2*exp(x) + 128*b^2*exp(x) - 128*a*b*e
xp(x))/(a + b) - log(- 128*a*b - 32*a^2 - 128*b^2 - 32*a^2*exp(x) - 128*b^2
*exp(x) - 128*a*b*exp(x))/(a - b) + (b*log(16*a*b^2 - 4*a^3*exp(2*x) - 4*a^
3 + 32*b^3*exp(x) - 8*a^2*b*exp(x) + 16*a*b^2*exp(2*x)))/(a^2 - b^2)
```

3.65 $\int \frac{\operatorname{csch}^2(x)}{a+b\operatorname{sech}(x)}\,dx$

Optimal. Leaf size=66

$$\frac{2ab\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}+\frac{(b-a\cosh(x))\operatorname{csch}(x)}{a^2-b^2}$$

```
[Out] 2*a*b*arctan((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a+b)^(3/2)/(a-b)^(3/2)+(
b-a*cosh(x))*csch(x)/(a^2-b^2)
```

Rubi [A]
time = 0.10, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3957, 2945, 12, 2738, 211}

$$\frac{\operatorname{csch}(x)(b-a\cosh(x))}{a^2-b^2}+\frac{2ab\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^2/(a + b*Sech[x]),x]
```

```
[Out] (2*a*b*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/((a - b)^(3/2)*(a + b)^
(3/2)) + ((b - a*Cosh[x])*Csch[x])/(a^2 - b^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^2(x)}{a+b\operatorname{sech}(x)}\,dx &= -\int \frac{\coth(x)\operatorname{csch}(x)}{-b-a\cosh(x)}\,dx \\
&= \frac{(b-a\cosh(x))\operatorname{csch}(x)}{a^2-b^2} - \frac{\int \frac{ab}{-b-a\cosh(x)}\,dx}{a^2-b^2} \\
&= \frac{(b-a\cosh(x))\operatorname{csch}(x)}{a^2-b^2} - \frac{(ab)\int \frac{1}{-b-a\cosh(x)}\,dx}{a^2-b^2} \\
&= \frac{(b-a\cosh(x))\operatorname{csch}(x)}{a^2-b^2} - \frac{(2ab)\operatorname{Subst}\left(\int \frac{1}{-a-b-(a-b)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{a^2-b^2} \\
&= \frac{2ab\tan^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}} + \frac{(b-a\cosh(x))\operatorname{csch}(x)}{a^2-b^2}
\end{aligned}$$

Mathematica [A]
time = 0.17, size = 75, normalized size = 1.14

$$\frac{1}{2}\left(-\frac{4ab\text{ArcTan}\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} - \frac{\coth\left(\frac{x}{2}\right)}{a+b} + \frac{\tanh\left(\frac{x}{2}\right)}{-a+b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^2/(a + b*Sech[x]),x]
```

```
[Out] ((-4*a*b*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) -
Coth[x/2]/(a + b) + Tanh[x/2]/(-a + b))/2
```

Maple [A]
time = 0.83, size = 77, normalized size = 1.17

method	result	size
default	$-\frac{\tanh\left(\frac{x}{2}\right)}{2(a-b)}+\frac{2ab\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}-\frac{1}{2(a+b)\tanh\left(\frac{x}{2}\right)}$	77
risch	$-\frac{2(-e^{x}b+a)}{(e^{2x}-1)(a^2-b^2)}-\frac{ba\ln\left(e^{x}+\frac{b\sqrt{-a^2+b^2}-a^2+b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)}+\frac{ba\ln\left(e^{x}+\frac{b\sqrt{-a^2+b^2}+a^2-b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)}$	165

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^2/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(a-b)*tanh(1/2*x)+2/(a+b)/(a-b)*a*b/((a+b)*(a-b))^(1/2)*arctan((a-b)*t
anh(1/2*x)/((a+b)*(a-b))^(1/2))-1/2/(a+b)/tanh(1/2*x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(58) = 116$.
time = 0.39, size = 452, normalized size = 6.85

$$\left[\frac{2a^3-2ab^2-\left(ab\cosh(x)^2+2ab\cosh(x)\sinh(x)+ab\sinh(x)^2-ab\right)\sqrt{-a^2+b^2}\log\left(\frac{a^2\cosh(x)^2+a^2\sinh(x)^2+2ab\cosh(x)-a^2+2b^2+2\left(a^2\cosh(x)+ab\right)\sinh(x)+2\sqrt{-a^2+b^2}\left(a\cosh(x)+a\sinh(x)+b\right)}{a\cosh(x)^2+a\sinh(x)^2+2b\cosh(x)+2\left(a\cosh(x)+b\right)\sinh(x)+a}\right)-2\left(a^2b-b^3\right)\cosh(x)-2\left(a^2b-b^3\right)\sinh(x)}{a^4-2a^2b^2+b^4-\left(a^4-2a^2b^2+b^4\right)\cosh(x)^2-2\left(a^4-2a^2b^2+b^4\right)\cosh(x)\sinh(x)-\left(a^4-2a^2b^2+b^4\right)\sinh(x)^2}, \frac{2\left(a^3-ab^2+\left(ab\cosh(x)^2+2ab\cosh(x)\sinh(x)+ab\sinh(x)^2-ab\right)\sqrt{a^2-b^2}\arctan\left(-\frac{a\cosh(x)+a\sinh(x)+b}{\sqrt{a^2-b^2}}\right)-\left(a^2b-b^3\right)\cosh(x)-\left(a^2b-b^3\right)\sinh(x)\right)}{a^4-2a^2b^2+b^4-\left(a^4-2a^2b^2+b^4\right)\cosh(x)^2-2\left(a^4-2a^2b^2+b^4\right)\cosh(x)\sinh(x)-\left(a^4-2a^2b^2+b^4\right)\sinh(x)^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [(2*a^3 - 2*a*b^2 - (a*b*cosh(x)^2 + 2*a*b*cosh(x)*sinh(x) + a*b*sinh(x)^2
- a*b)*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x)
- a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-a^2 + b^2)*(a*cosh(
```

```
x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x
) + b)*sinh(x) + a)) - 2*(a^2*b - b^3)*cosh(x) - 2*(a^2*b - b^3)*sinh(x))/(
a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 - 2*(a^4 - 2*a^2*
b^2 + b^4)*cosh(x)*sinh(x) - (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^2), 2*(a^3 - a
*b^2 + (a*b*cosh(x)^2 + 2*a*b*cosh(x)*sinh(x) + a*b*sinh(x)^2 - a*b)*sqrt(a
^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^2 - b^2)) - (a^2*b - b
^3)*cosh(x) - (a^2*b - b^3)*sinh(x))/(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*
b^2 + b^4)*cosh(x)^2 - 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x) - (a^4 - 2
*a^2*b^2 + b^4)*sinh(x)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(x)}{a + b\operatorname{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2/(a+b*sech(x)),x)
[Out] Integral(csch(x)**2/(a + b*sech(x)), x)
```

Giac [A]
time = 0.38, size = 64, normalized size = 0.97

$$\frac{2\,ab\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{\frac{3}{2}}} + \frac{2\,(be^x-a)}{(a^2-b^2)(e^{(2\,x)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*sech(x)),x, algorithm="giac")
[Out] 2*a*b*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(a^2 - b^2)^(3/2) + 2*(b*e^x - a)
/((a^2 - b^2)*(e^(2*x) - 1))
```

Mupad [B]
time = 1.56, size = 151, normalized size = 2.29

$$\frac{a\,b\,\ln\left(-\frac{2\,b\,\mathrm{e}^x}{a^2-b^2}-\frac{2\,b\,(a+b\,\mathrm{e}^x)}{(a+b)^{3/2}\,(b-a)^{3/2}}\right)}{(a+b)^{3/2}\,(b-a)^{3/2}} - \frac{\frac{2\,a}{a^2-b^2}-\frac{2\,b\,\mathrm{e}^x}{a^2-b^2}}{\mathrm{e}^{2\,x}-1} - \frac{a\,b\,\ln\left(\frac{2\,b\,(a+b\,\mathrm{e}^x)}{(a+b)^{3/2}\,(b-a)^{3/2}}-\frac{2\,b\,\mathrm{e}^x}{a^2-b^2}\right)}{(a+b)^{3/2}\,(b-a)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^2*(a + b/cosh(x))),x)
[Out] (a*b*log(- (2*b*exp(x))/(a^2 - b^2) - (2*b*(a + b*exp(x)))/((a + b)^(3/2)*(
b - a)^(3/2))))/((a + b)^(3/2)*(b - a)^(3/2)) - ((2*a)/(a^2 - b^2) - (2*b*e
xp(x))/(a^2 - b^2))/(exp(2*x) - 1) - (a*b*log((2*b*(a + b*exp(x)))/((a + b)
^(3/2)*(b - a)^(3/2)) - (2*b*exp(x))/(a^2 - b^2)))/((a + b)^(3/2)*(b - a)^(
3/2))
```

3.66 $\int \frac{\operatorname{csch}^3(x)}{a+b\operatorname{sech}(x)}\,dx$

Optimal. Leaf size=85

$$\frac{(b-a\cosh(x))\operatorname{csch}^2(x)}{2(a^2-b^2)}-\frac{a\log(1-\cosh(x))}{4(a+b)^2}+\frac{a\log(1+\cosh(x))}{4(a-b)^2}-\frac{a^2b\log(b+a\cosh(x))}{(a^2-b^2)^2}$$

```
[Out] 1/2*(b-a*cosh(x))*csch(x)^2/(a^2-b^2)-1/4*a*ln(1-cosh(x))/(a+b)^2+1/4*a*ln(
1+cosh(x))/(a-b)^2-a^2*b*ln(b+a*cosh(x))/(a^2-b^2)^2
```

Rubi [A]
time = 0.16, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3957, 2916, 12, 837, 815}

$$-\frac{a^2b\log(a\cosh(x)+b)}{(a^2-b^2)^2}+\frac{\operatorname{csch}^2(x)(b-a\cosh(x))}{2(a^2-b^2)}-\frac{a\log(1-\cosh(x))}{4(a+b)^2}+\frac{a\log(\cosh(x)+1)}{4(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^3/(a + b*Sech[x]),x]
```

```
[Out] ((b - a*Cosh[x])*Csch[x]^2)/(2*(a^2 - b^2)) - (a*Log[1 - Cosh[x]])/(4*(a +
b)^2) + (a*Log[1 + Cosh[x]])/(4*(a - b)^2) - (a^2*b*Log[b + a*Cosh[x]])/(a^
2 - b^2)^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
```

```
[2*m, 2*p])
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{csch}^3(x)}{a+b\operatorname{sech}(x)}\,dx &= -\int \frac{\coth(x)\operatorname{csch}^2(x)}{-b-a\cosh(x)}\,dx\\
&= -\left(a^3\operatorname{Subst}\left(\int \frac{x}{a(-b+x)\left(a^2-x^2\right)^2}\,dx,x,-a\cosh(x)\right)\right)\\
&= -\left(a^2\operatorname{Subst}\left(\int \frac{x}{(-b+x)\left(a^2-x^2\right)^2}\,dx,x,-a\cosh(x)\right)\right)\\
&= \frac{(b-a\cosh(x))\operatorname{csch}^2(x)}{2\left(a^2-b^2\right)}-\frac{\operatorname{Subst}\left(\int \frac{a^2b+a^2x}{(-b+x)(a^2-x^2)}\,dx,x,-a\cosh(x)\right)}{2\left(a^2-b^2\right)}\\
&= \frac{(b-a\cosh(x))\operatorname{csch}^2(x)}{2\left(a^2-b^2\right)}-\frac{\operatorname{Subst}\left(\int \left(\frac{a(a+b)}{2(a-b)(a-x)}-\frac{2a^2b}{(a-b)(a+b)(b-x)}+\frac{a(a-b)}{2(a+b)(a+x)}\right)\,dx,x,-\right.}{2\left(a^2-b^2\right)}\\
&= \frac{(b-a\cosh(x))\operatorname{csch}^2(x)}{2\left(a^2-b^2\right)}-\frac{a\log(1-\cosh(x))}{4(a+b)^2}+\frac{a\log(1+\cosh(x))}{4(a-b)^2}-\frac{a^2b\log(b+a\cos}{\left(a^2-b^2\right)^2}
\end{aligned}
$$

Mathematica [A]
time = 0.23, size = 86, normalized size = 1.01

$$
\frac{1}{8}\left(-\frac{\operatorname{csch}^2\left(\frac{x}{2}\right)}{a+b}-\frac{4a\left(2ab\log(b+a\cosh(x))-2ab\log(\sinh(x))+\left(a^2+b^2\right)\log\left(\tanh\left(\frac{x}{2}\right)\right)\right)}{(a-b)^2(a+b)^2}-\frac{\operatorname{sech}^2\left(\frac{x}{2}\right)}{a-b}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^3/(a + b*Sech[x]),x]
```

```
[Out] (-(Csch[x/2]^2/(a + b)) - (4*a*(2*a*b*Log[b + a*Cosh[x]] - 2*a*b*Log[Sinh[x
]] + (a^2 + b^2)*Log[Tanh[x/2]]))/((a - b)^2*(a + b)^2) - Sech[x/2]^2/(a -
b))/8
```

Maple [A]
time = 0.88, size = 82, normalized size = 0.96

method	result
default	$\frac{\tanh^2(\frac{x}{2})}{8a-8b}-\frac{1}{8(a+b)\tanh(\frac{x}{2})^2}-\frac{a\ln(\tanh(\frac{x}{2}))}{2(a+b)^2}-\frac{a^2b\ln(a(\tanh^2(\frac{x}{2}))-b(\tanh^2(\frac{x}{2}))+a+b)}{(a+b)^2(a-b)^2}$
risch	$-\frac{xa}{2(a^2-2ab+b^2)}+\frac{ax}{2a^2+4ab+2b^2}+\frac{2a^2bx}{a^4-2a^2b^2+b^4}-\frac{e^x(e^{2x}a-2e^xb+a)}{(e^{2x}-1)^2(a^2-b^2)}+\frac{a\ln(e^x+1)}{2a^2-4ab+2b^2}-\frac{a\ln(e^x-1)}{2(a^2+2ab+b^2)}-\frac{a^2b\ln(e^{2x}}{a^4-2a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^3/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*tanh(1/2*x)^2/(a-b)-1/8/(a+b)/tanh(1/2*x)^2-1/2*a/(a+b)^2*ln(tanh(1/2*x
))-a^2*b/(a+b)^2/(a-b)^2*ln(a*tanh(1/2*x)^2-b*tanh(1/2*x)^2+a+b)
```

Maxima [A]
time = 0.29, size = 148, normalized size = 1.74

$$-\frac{a^2b\log\left(2\,be^{(-x)}+ae^{(-2\,x)}+a\right)}{a^4-2\,a^2b^2+b^4}+\frac{a\log\left(e^{(-x)}+1\right)}{2\,(a^2-2\,ab+b^2)}-\frac{a\log\left(e^{(-x)}-1\right)}{2\,(a^2+2\,ab+b^2)}-\frac{ae^{(-x)}-2\,be^{(-2\,x)}+ae^{(-3\,x)}}{a^2-b^2-2\,(a^2-b^2)e^{(-2\,x)}+(a^2-b^2)e^{(-4\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] -a^2*b*log(2*b*e^(-x) + a*e^(-2*x) + a)/(a^4 - 2*a^2*b^2 + b^4) + 1/2*a*log
(e^(-x) + 1)/(a^2 - 2*a*b + b^2) - 1/2*a*log(e^(-x) - 1)/(a^2 + 2*a*b + b^2
) - (a*e^(-x) - 2*b*e^(-2*x) + a*e^(-3*x))/(a^2 - b^2 - 2*(a^2 - b^2)*e^(-2
*x) + (a^2 - b^2)*e^(-4*x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 828 vs. $2(80) = 160$.
time = 0.45, size = 828, normalized size = 9.74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^3 - a*b^2)*cosh(x)^3 + 2*(a^3 - a*b^2)*sinh(x)^3 - 4*(a^2*b - b^
3)*cosh(x)^2 - 2*(2*a^2*b - 2*b^3 - 3*(a^3 - a*b^2)*cosh(x))*sinh(x)^2 + 2*
(a^3 - a*b^2)*cosh(x) + 2*(a^2*b*cosh(x)^4 + 4*a^2*b*cosh(x)*sinh(x)^3 + a^
2*b*sinh(x)^4 - 2*a^2*b*cosh(x)^2 + a^2*b + 2*(3*a^2*b*cosh(x)^2 - a^2*b)*s
```

```
inh(x)^2 + 4*(a^2*b*cosh(x)^3 - a^2*b*cosh(x))*sinh(x))*log(2*(a*cosh(x) +
b)/(cosh(x) - sinh(x))) - ((a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 4*(a^3 + 2*a
^2*b + a*b^2)*cosh(x)*sinh(x)^3 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^4 + a^3 +
 2*a^2*b + a*b^2 - 2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2 - 2*(a^3 + 2*a^2*b +
 a*b^2 - 3*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 2*a^2*b
 + a*b^2)*cosh(x)^3 - (a^3 + 2*a^2*b + a*b^2)*cosh(x))*sinh(x))*log(cosh(x)
 + sinh(x) + 1) + ((a^3 - 2*a^2*b + a*b^2)*cosh(x)^4 + 4*(a^3 - 2*a^2*b + a
*b^2)*cosh(x)*sinh(x)^3 + (a^3 - 2*a^2*b + a*b^2)*sinh(x)^4 + a^3 - 2*a^2*b
 + a*b^2 - 2*(a^3 - 2*a^2*b + a*b^2)*cosh(x)^2 - 2*(a^3 - 2*a^2*b + a*b^2 -
 3*(a^3 - 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 - 2*a^2*b + a*b^2
)*cosh(x)^3 - (a^3 - 2*a^2*b + a*b^2)*cosh(x))*sinh(x))*log(cosh(x) + sinh(
x) - 1) + 2*(a^3 - a*b^2 + 3*(a^3 - a*b^2)*cosh(x)^2 - 4*(a^2*b - b^3)*cosh
(x))*sinh(x))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + 4*(a^4 - 2*a^2*b^2 + b^4
)*cosh(x)*sinh(x)^3 + (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^4 + a^4 - 2*a^2*b^2 +
 b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 - 2*(a^4 - 2*a^2*b^2 + b^4 - 3*(
a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^2 + 4*((a^4 - 2*a^2*b^2 + b^4)*co
sh(x)^3 - (a^4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(x)}{a + b\operatorname{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**3/(a+b*sech(x)),x)
[Out] Integral(csch(x)**3/(a + b*sech(x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(80) = 160.
time = 0.39, size = 174, normalized size = 2.05

$$-\frac{a^3 b\log\left(\left|a\left(e^{(-x)}+e^x\right)+2\,b\right|\right)}{a^5-2\,a^3b^2+ab^4}+\frac{a\log\left(e^{(-x)}+e^x+2\right)}{4\,(a^2-2\,ab+b^2)}-\frac{a\log\left(e^{(-x)}+e^x-2\right)}{4\,(a^2+2\,ab+b^2)}-\frac{a^2b\left(e^{(-x)}+e^x\right)^2+2\,a^3\left(e^{(-x)}+e^x\right)-2\,ab^2\left(e^{(-x)}+e^x\right)-8\,a^2b+4\,b^3}{2\,(a^4-2\,a^2b^2+b^4)\left(\left(e^{(-x)}+e^x\right)^2-4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*sech(x)),x, algorithm="giac")
[Out] -a^3*b*log(abs(a*(e^(-x) + e^x) + 2*b))/(a^5 - 2*a^3*b^2 + a*b^4) + 1/4*a*l
og(e^(-x) + e^x + 2)/(a^2 - 2*a*b + b^2) - 1/4*a*log(e^(-x) + e^x - 2)/(a^2
 + 2*a*b + b^2) - 1/2*(a^2*b*(e^(-x) + e^x)^2 + 2*a^3*(e^(-x) + e^x) - 2*a*
b^2*(e^(-x) + e^x) - 8*a^2*b + 4*b^3)/((a^4 - 2*a^2*b^2 + b^4)*((e^(-x) + e
^x)^2 - 4))
```

Mupad [B]
time = 1.83, size = 255, normalized size = 3.00

$$\frac{\frac{2\,(a^2\,b-b^3)}{(a^2-b^2)^2}+\frac{\mathrm{e}^x\,(a\,b^2-a^3)}{(a^2-b^2)^2}}{\mathrm{e}^{2\,x}-1}+\frac{\frac{2\,b}{a^2-b^2}-\frac{2\,a\,\mathrm{e}^x}{a^2-b^2}}{\mathrm{e}^{4\,x}-2\,\mathrm{e}^{2\,x}+1}-\frac{a\,\ln\left(\mathrm{e}^x-1\right)}{2\,a^2+4\,a\,b+2\,b^2}+\frac{a\,\ln\left(\mathrm{e}^x+1\right)}{2\,a^2-4\,a\,b+2\,b^2}-\frac{a^2\,b\,\ln\left(a^6\,\mathrm{e}^{2\,x}+a^6+a^2\,b^4-14\,a^4\,b^2+a^2\,b^4\,\mathrm{e}^{2\,x}-14\,a^4\,b^2\,\mathrm{e}^{2\,x}+2\,a\,b^5\,\mathrm{e}^x+2\,a^5\,b\,\mathrm{e}^x-28\,a^3\,b^3\,\mathrm{e}^x\right)}{a^4-2\,a^2\,b^2+b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^3*(a + b/cosh(x))),x)
```

```
[Out] ((2*(a^2*b - b^3))/(a^2 - b^2)^2 + (exp(x)*(a*b^2 - a^3))/(a^2 - b^2)^2)/(e
xp(2*x) - 1) + ((2*b)/(a^2 - b^2) - (2*a*exp(x))/(a^2 - b^2))/(exp(4*x) - 2
*exp(2*x) + 1) - (a*log(exp(x) - 1))/(4*a*b + 2*a^2 + 2*b^2) + (a*log(exp(x
) + 1))/(2*a^2 - 4*a*b + 2*b^2) - (a^2*b*log(a^6*exp(2*x) + a^6 + a^2*b^4 -
 14*a^4*b^2 + a^2*b^4*exp(2*x) - 14*a^4*b^2*exp(2*x) + 2*a*b^5*exp(x) + 2*a
^5*b*exp(x) - 28*a^3*b^3*exp(x)))/(a^4 + b^4 - 2*a^2*b^2)
```

3.67 $\int \frac{\mathbf{csch}^4(x)}{a+b\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=111

$$-\frac{2a^3b\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}}-\frac{\left(3a^2b-a\left(2a^2+b^2\right)\cosh(x)\right)\text{csch}(x)}{3\left(a^2-b^2\right)^2}+\frac{(b-a\cosh(x))\text{csch}^3(x)}{3\left(a^2-b^2\right)}$$

```
[Out] -2*a^3*b*arctan((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a+b)^(5/2)/(a-b)^(5/2
)-1/3*(3*a^2*b-a*(2*a^2+b^2)*cosh(x))*csch(x)/(a^2-b^2)^2+1/3*(b-a*cosh(x))
*csch(x)^3/(a^2-b^2)
```

Rubi [A]
time = 0.20, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3957, 2945, 12, 2738, 211}

$$-\frac{2a^3b\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}}+\frac{\text{csch}^3(x)(b-a\cosh(x))}{3\left(a^2-b^2\right)}-\frac{\text{csch}(x)\left(3a^2b-a\left(2a^2+b^2\right)\cosh(x)\right)}{3\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^4/(a + b*Sech[x]),x]
```

```
[Out] (-2*a^3*b*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/((a - b)^(5/2)*(a +
b)^(5/2)) - ((3*a^2*b - a*(2*a^2 + b^2)*Cosh[x])*Csch[x])/(3*(a^2 - b^2)^2)
 + ((b - a*Cosh[x])*Csch[x]^3)/(3*(a^2 - b^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\operatorname{csch}^4(x)}{a+b\operatorname{sech}(x)}\,dx = -\int \frac{\coth(x)\operatorname{csch}^3(x)}{-b-a\cosh(x)}\,dx$$

$$= \frac{(b-a\cosh(x))\operatorname{csch}^3(x)}{3(a^2-b^2)} - \frac{\int \frac{(ab-2a^2\cosh(x))\operatorname{csch}^2(x)}{-b-a\cosh(x)}\,dx}{3(a^2-b^2)}$$

$$= -\frac{(3a^2b-a(2a^2+b^2)\cosh(x))\operatorname{csch}(x)}{3(a^2-b^2)^2} + \frac{(b-a\cosh(x))\operatorname{csch}^3(x)}{3(a^2-b^2)} + \frac{\int \frac{3a^3b}{-b-a\cosh(x)}\,dx}{3(a^2-b^2)^2}$$

$$= -\frac{(3a^2b-a(2a^2+b^2)\cosh(x))\operatorname{csch}(x)}{3(a^2-b^2)^2} + \frac{(b-a\cosh(x))\operatorname{csch}^3(x)}{3(a^2-b^2)} + \frac{(a^3b)\int \frac{1}{-b-a\cosh(}}{(a^2-b^2)^2}$$

$$= -\frac{(3a^2b-a(2a^2+b^2)\cosh(x))\operatorname{csch}(x)}{3(a^2-b^2)^2} + \frac{(b-a\cosh(x))\operatorname{csch}^3(x)}{3(a^2-b^2)} + \frac{(2a^3b)\operatorname{Subst}\left(\int \right.}{}$$

$$= -\frac{2a^3b\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}} - \frac{(3a^2b-a(2a^2+b^2)\cosh(x))\operatorname{csch}(x)}{3(a^2-b^2)^2} + \frac{(b-a\cosh}{3(a}$$

Mathematica [A]
time = 0.40, size = 156, normalized size = 1.41

$$\frac{(b+a\cosh(x))\operatorname{sech}(x)\left(\frac{48a^3b\operatorname{ArcTan}\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} + \frac{2(4a+b)\coth\left(\frac{x}{2}\right)}{(a+b)^2} + \frac{8\operatorname{csch}^3(x)\sinh^4\left(\frac{x}{2}\right)}{a-b} - \frac{\operatorname{csch}^4\left(\frac{x}{2}\right)\sinh(x)}{2(a+b)} + \frac{8a\tanh\left(\frac{x}{2}\right)}{(a-b)^2} - \frac{2b\tanh\left(\frac{x}{2}\right)}{(a-b)^2}\right)}{24(a+b\operatorname{sech}(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^4/(a + b*Sech[x]),x]
```

```
[Out] ((b + a*Cosh[x])*Sech[x]*((48*a^3*b*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 -
b^2]])/(a^2 - b^2)^(5/2) + (2*(4*a + b)*Coth[x/2])/(a + b)^2 + (8*Csch[x]^3
*Sinh[x/2]^4)/(a - b) - (Csch[x/2]^4*Sinh[x])/(2*(a + b)) + (8*a*Tanh[x/2])
/(a - b)^2 - (2*b*Tanh[x/2])/(a - b)^2))/(24*(a + b*Sech[x]))
```

Maple [A]
time = 0.86, size = 127, normalized size = 1.14

method	result
default	$-\frac{\frac{a\left(\tanh^3\left(\frac{x}{2}\right)\right)}{3}-\frac{b\left(\tanh^3\left(\frac{x}{2}\right)\right)}{3}-3a\tanh\left(\frac{x}{2}\right)+b\tanh\left(\frac{x}{2}\right)}{8(a-b)^2}-\frac{2a^3b\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a-b)^2(a+b)^2\sqrt{(a+b)(a-b)}}-\frac{1}{24(a+b)\tanh\left(\frac{x}{2}\right)^3}-$ 8(a-
risch	$-\frac{2\left(3a^2b\,e^{5x}-3a\,b^2e^{4x}-10a^2b\,e^{3x}+4b^3e^{3x}+6a^3e^{2x}+3a^2b\,e^{x}-2a^3-a\,b^2\right)}{3(a^2-b^2)^2(e^{2x}-1)^3}-\frac{b\,a^3\ln\left(e^x+\frac{b\sqrt{-a^2+b^2}+a^2-b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,(a+b)^2(a-b)^2}+$ $\frac{b\,a^3\ln\left(e^x+\right.}{\sqrt{-a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^4/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/(a-b)^2*(1/3*a*tanh(1/2*x)^3-1/3*b*tanh(1/2*x)^3-3*a*tanh(1/2*x)+b*tan
h(1/2*x))-2/(a-b)^2/(a+b)^2*a^3*b/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2
*x)/((a+b)*(a-b))^(1/2))-1/24/(a+b)/tanh(1/2*x)^3-1/8/(a+b)^2*(-3*a-b)/tanh
(1/2*x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1129 vs. 2(98) = 196.
time = 0.40, size = 2340, normalized size = 21.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [-1/3*(6*(a^4*b - a^2*b^3)*cosh(x)^5 + 6*(a^4*b - a^2*b^3)*sinh(x)^5 - 4*a^
5 + 2*a^3*b^2 + 2*a*b^4 - 6*(a^3*b^2 - a*b^4)*cosh(x)^4 - 6*(a^3*b^2 - a*b^
4 - 5*(a^4*b - a^2*b^3)*cosh(x))*sinh(x)^4 - 4*(5*a^4*b - 7*a^2*b^3 + 2*b^5
)*cosh(x)^3 - 4*(5*a^4*b - 7*a^2*b^3 + 2*b^5 - 15*(a^4*b - a^2*b^3)*cosh(x)
^2 + 6*(a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^3 + 12*(a^5 - a^3*b^2)*cosh(x)^2
+ 12*(a^5 - a^3*b^2 + 5*(a^4*b - a^2*b^3)*cosh(x)^3 - 3*(a^3*b^2 - a*b^4)*c
osh(x)^2 - (5*a^4*b - 7*a^2*b^3 + 2*b^5)*cosh(x))*sinh(x)^2 + 3*(a^3*b*cosh
(x)^6 + 6*a^3*b*cosh(x)*sinh(x)^5 + a^3*b*sinh(x)^6 - 3*a^3*b*cosh(x)^4 + 3
*a^3*b*cosh(x)^2 + 3*(5*a^3*b*cosh(x)^2 - a^3*b)*sinh(x)^4 - a^3*b + 4*(5*a
^3*b*cosh(x)^3 - 3*a^3*b*cosh(x))*sinh(x)^3 + 3*(5*a^3*b*cosh(x)^4 - 6*a^3*
b*cosh(x)^2 + a^3*b)*sinh(x)^2 + 6*(a^3*b*cosh(x)^5 - 2*a^3*b*cosh(x)^3 + a
^3*b*cosh(x))*sinh(x))*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2
+ 2*a*b*cosh(x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-a^2
 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(
x) + 2*(a*cosh(x) + b)*sinh(x) + a)) + 6*(a^4*b - a^2*b^3)*cosh(x) + 6*(a^4
*b - a^2*b^3 + 5*(a^4*b - a^2*b^3)*cosh(x)^4 - 4*(a^3*b^2 - a*b^4)*cosh(x)^
3 - 2*(5*a^4*b - 7*a^2*b^3 + 2*b^5)*cosh(x)^2 + 4*(a^5 - a^3*b^2)*cosh(x))*
sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^6 + 6*(a^6 - 3*a^4*b^
2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^5 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6
)*sinh(x)^6 - a^6 + 3*a^4*b^2 - 3*a^2*b^4 + b^6 - 3*(a^6 - 3*a^4*b^2 + 3*a^
2*b^4 - b^6)*cosh(x)^4 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 - 5*(a^6 - 3*
a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^6 - 3*a^4*b^2 + 3
*a^2*b^4 - b^6)*cosh(x)^3 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*
sinh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2 + 3*(a^6 - 3*a^
4*b^2 + 3*a^2*b^4 - b^6 + 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 -
 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^2 + 6*((a^6 - 3*a
^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^5 - 2*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
*cosh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x)), -2/3*(3
*(a^4*b - a^2*b^3)*cosh(x)^5 + 3*(a^4*b - a^2*b^3)*sinh(x)^5 - 2*a^5 + a^3*
b^2 + a*b^4 - 3*(a^3*b^2 - a*b^4)*cosh(x)^4 - 3*(a^3*b^2 - a*b^4 - 5*(a^4*b
 - a^2*b^3)*cosh(x))*sinh(x)^4 - 2*(5*a^4*b - 7*a^2*b^3 + 2*b^5)*cosh(x)^3
- 2*(5*a^4*b - 7*a^2*b^3 + 2*b^5 - 15*(a^4*b - a^2*b^3)*cosh(x)^2 + 6*(a^3*
b^2 - a*b^4)*cosh(x))*sinh(x)^3 + 6*(a^5 - a^3*b^2)*cosh(x)^2 + 6*(a^5 - a^
3*b^2 + 5*(a^4*b - a^2*b^3)*cosh(x)^3 - 3*(a^3*b^2 - a*b^4)*cosh(x)^2 - (5*
a^4*b - 7*a^2*b^3 + 2*b^5)*cosh(x))*sinh(x)^2 - 3*(a^3*b*cosh(x)^6 + 6*a^3*
b*cosh(x)*sinh(x)^5 + a^3*b*sinh(x)^6 - 3*a^3*b*cosh(x)^4 + 3*a^3*b*cosh(x)
^2 + 3*(5*a^3*b*cosh(x)^2 - a^3*b)*sinh(x)^4 - a^3*b + 4*(5*a^3*b*cosh(x)^3
 - 3*a^3*b*cosh(x))*sinh(x)^3 + 3*(5*a^3*b*cosh(x)^4 - 6*a^3*b*cosh(x)^2 +
a^3*b)*sinh(x)^2 + 6*(a^3*b*cosh(x)^5 - 2*a^3*b*cosh(x)^3 + a^3*b*cosh(x))*
sinh(x))*sqrt(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^2 - b^2
)) + 3*(a^4*b - a^2*b^3)*cosh(x) + 3*(a^4*b - a^2*b^3 + 5*(a^4*b - a^2*b^3)
```

```
*cosh(x)^4 - 4*(a^3*b^2 - a*b^4)*cosh(x)^3 - 2*(5*a^4*b - 7*a^2*b^3 + 2*b^5
)*cosh(x)^2 + 4*(a^5 - a^3*b^2)*cosh(x))*sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^2
*b^4 - b^6)*cosh(x)^6 + 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(
x)^5 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^6 - a^6 + 3*a^4*b^2 - 3*
a^2*b^4 + b^6 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 - 3*(a^6 -
3*a^4*b^2 + 3*a^2*b^4 - b^6 - 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)
^2)*sinh(x)^4 + 4*(5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 - 3*(a^6
 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3
*a^2*b^4 - b^6)*cosh(x)^2 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 5*(a^6 -
 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 - 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 -
b^6)*cosh(x)^2)*sinh(x)^2 + 6*((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^
5 - 2*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + (a^6 - 3*a^4*b^2 + 3*
a^2*b^4 - b^6)*cosh(x))*sinh(x))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^4(x)}{a + b\operatorname{sech}(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**4/(a+b*sech(x)),x)
```

```
[Out] Integral(csch(x)**4/(a + b*sech(x)), x)
```

Giac [A]
time = 0.40, size = 149, normalized size = 1.34

$$-\frac{2\,a^3 b \arctan\left(\frac{a e^x + b}{\sqrt{a^2 - b^2}}\right)}{(a^4 - 2\,a^2 b^2 + b^4)\sqrt{a^2 - b^2}} - \frac{2\left(3\,a^2 b e^{(5\,x)} - 3\,a b^2 e^{(4\,x)} - 10\,a^2 b e^{(3\,x)} + 4\,b^3 e^{(3\,x)} + 6\,a^3 e^{(2\,x)} + 3\,a^2 b e^x - 2\,a^3 - a b^2\right)}{3\,(a^4 - 2\,a^2 b^2 + b^4)(e^{(2\,x)} - 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] -2*a^3*b*arctan((a*e^x + b)/sqrt(a^2 - b^2))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(
a^2 - b^2)) - 2/3*(3*a^2*b*e^(5*x) - 3*a*b^2*e^(4*x) - 10*a^2*b*e^(3*x) + 4
*b^3*e^(3*x) + 6*a^3*e^(2*x) + 3*a^2*b*e^x - 2*a^3 - a*b^2)/((a^4 - 2*a^2*b
^2 + b^4)*(e^(2*x) - 1)^3)
```

Mupad [B]
time = 1.75, size = 295, normalized size = 2.66

$$\frac{\frac{4\,(a\,b^2-a^3)}{(a^2-b^2)^2}+\frac{8\,\mathrm{e}^x\,(a^2\,b-b^3)}{3\,(a^2-b^2)^2}}{\mathrm{e}^{4\,x}-2\,\mathrm{e}^{2\,x}+1}-\frac{\frac{8\,a}{3\,(a^2-b^2)}-\frac{8\,b\,\mathrm{e}^x}{3\,(a^2-b^2)}}{3\,\mathrm{e}^{2\,x}-3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}-1}+\frac{\frac{2\,a\,b^2}{(a^2-b^2)^2}-\frac{2\,a^2\,b\,\mathrm{e}^x}{(a^2-b^2)^2}}{\mathrm{e}^{2\,x}-1}+\frac{a^3\,b\,\ln\left(\frac{2\,a^2\,b\,\mathrm{e}^x}{(a^2-b^2)^2}-\frac{2\,a^2\,b\,(a+b\,\mathrm{e}^x)}{(a+b)^{5/2}\,(b-a)^{5/2}}\right)}{(a+b)^{5/2}\,(b-a)^{5/2}}-\frac{a^3\,b\,\ln\left(\frac{2\,a^2\,b\,\mathrm{e}^x}{(a^2-b^2)^2}+\frac{2\,a^2\,b\,(a+b\,\mathrm{e}^x)}{(a+b)^{5/2}\,(b-a)^{5/2}}\right)}{(a+b)^{5/2}\,(b-a)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^4*(a + b/cosh(x))),x)

[Out] ((4*(a*b^2 - a^3))/(a^2 - b^2)^2 + (8*exp(x)*(a^2*b - b^3))/(3*(a^2 - b^2)^
2))/(exp(4*x) - 2*exp(2*x) + 1) - ((8*a)/(3*(a^2 - b^2)) - (8*b*exp(x))/(3*
(a^2 - b^2)))/(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1) + ((2*a*b^2)/(a^2 -
b^2)^2 - (2*a^2*b*exp(x))/(a^2 - b^2)^2)/(exp(2*x) - 1) + (a^3*b*log((2*a^2
*b*exp(x))/(a^2 - b^2)^2 - (2*a^2*b*(a + b*exp(x)))/((a + b)^(5/2)*(b - a)^
(5/2))))/((a + b)^(5/2)*(b - a)^(5/2)) - (a^3*b*log((2*a^2*b*exp(x))/(a^2 -
 b^2)^2 + (2*a^2*b*(a + b*exp(x)))/((a + b)^(5/2)*(b - a)^(5/2))))/((a + b)
^(5/2)*(b - a)^(5/2))
```

3.68 $\int \frac{\cosh^4(x)}{a+a\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=67

$$\frac{15x}{8a}-\frac{4\sinh(x)}{a}+\frac{15\cosh(x)\sinh(x)}{8a}+\frac{5\cosh^3(x)\sinh(x)}{4a}-\frac{\cosh^3(x)\sinh(x)}{a+a\,\mathrm{sech}(x)}-\frac{4\sinh^3(x)}{3a}$$

```
[Out] 15/8*x/a-4*sinh(x)/a+15/8*cosh(x)*sinh(x)/a+5/4*cosh(x)^3*sinh(x)/a-cosh(x)
^3*sinh(x)/(a+a*sech(x))-4/3*sinh(x)^3/a
```

Rubi [A]
time = 0.06, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3904, 3872, 2715, 8, 2713}

$$\frac{15x}{8a}-\frac{4\sinh^3(x)}{3a}-\frac{4\sinh(x)}{a}+\frac{5\sinh(x)\cosh^3(x)}{4a}+\frac{15\sinh(x)\cosh(x)}{8a}-\frac{\sinh(x)\cosh^3(x)}{a\,\mathrm{sech}(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^4/(a + a*Sech[x]),x]
```

```
[Out] (15*x)/(8*a) - (4*Sinh[x])/a + (15*Cosh[x]*Sinh[x])/(8*a) + (5*Cosh[x]^3*Si
nh[x])/(4*a) - (Cosh[x]^3*Sinh[x])/(a + a*Sech[x]) - (4*Sinh[x]^3)/(3*a)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^4(x)}{a+a\text{sech}(x)}\,dx &= -\frac{\cosh^3(x)\sinh(x)}{a+a\text{sech}(x)}-\frac{\int \cosh^4(x)(-5a+4a\text{sech}(x))\,dx}{a^2}\\ &= -\frac{\cosh^3(x)\sinh(x)}{a+a\text{sech}(x)}-\frac{4\int \cosh^3(x)\,dx}{a}+\frac{5\int \cosh^4(x)\,dx}{a}\\ &= \frac{5\cosh^3(x)\sinh(x)}{4a}-\frac{\cosh^3(x)\sinh(x)}{a+a\text{sech}(x)}-\frac{(4i)\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-i\sinh(x)\right)}{a}+\frac{1}{}\\ &= -\frac{4\sinh(x)}{a}+\frac{15\cosh(x)\sinh(x)}{8a}+\frac{5\cosh^3(x)\sinh(x)}{4a}-\frac{\cosh^3(x)\sinh(x)}{a+a\text{sech}(x)}-\frac{4\sinh^3(}{3a}\\ &= \frac{15x}{8a}-\frac{4\sinh(x)}{a}+\frac{15\cosh(x)\sinh(x)}{8a}+\frac{5\cosh^3(x)\sinh(x)}{4a}-\frac{\cosh^3(x)\sinh(x)}{a+a\text{sech}(x)}-\frac{4\,\text{s}}{}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 63, normalized size = 0.94

$$\frac{\text{sech}\left(\frac{x}{2}\right)\left(360x\cosh\left(\frac{x}{2}\right)-360\sinh\left(\frac{x}{2}\right)-120\sinh\left(\frac{3x}{2}\right)+40\sinh\left(\frac{5x}{2}\right)-5\sinh\left(\frac{7x}{2}\right)+3\sinh\left(\frac{9x}{2}\right)\right)}{192a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^4/(a + a*Sech[x]),x]
```

```
[Out] (Sech[x/2]*(360*x*Cosh[x/2] - 360*Sinh[x/2] - 120*Sinh[(3*x)/2] + 40*Sinh[(
5*x)/2] - 5*Sinh[(7*x)/2] + 3*Sinh[(9*x)/2]))/(192*a)
```

Maple [A]
time = 0.83, size = 110, normalized size = 1.64

method	result
risch	$\frac{15x}{8a}+\frac{e^{4x}}{64a}-\frac{e^{3x}}{24a}+\frac{e^{2x}}{4a}-\frac{7\,e^{x}}{8a}+\frac{7\,e^{-x}}{8a}-\frac{e^{-2x}}{4a}+\frac{e^{-3x}}{24a}-\frac{e^{-4x}}{64a}+\frac{2}{(e^{x}+1)a}$

default	$\frac{-\tanh\left(\frac{x}{2}\right)-\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)+1\right)^{4}}+\frac{5}{6\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}-\frac{15}{8\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{25}{8\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{15\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{8}+\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)-1\right)^{4}}+\frac{5}{6\left(\tanh\left(\frac{x}{2}\right)-1\right)}}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-tanh(1/2*x)-1/4/(tanh(1/2*x)+1)^4+5/6/(tanh(1/2*x)+1)^3-15/8/(tanh(1/
2*x)+1)^2+25/8/(tanh(1/2*x)+1)+15/8*ln(tanh(1/2*x)+1)+1/4/(tanh(1/2*x)-1)^4
+5/6/(tanh(1/2*x)-1)^3+15/8/(tanh(1/2*x)-1)^2+25/8/(tanh(1/2*x)-1)-15/8*ln(
tanh(1/2*x)-1))
```

Maxima [A]
time = 0.26, size = 80, normalized size = 1.19

$$\frac{15\,x}{8\,a}+\frac{168\,e^{(-x)}-48\,e^{(-2\,x)}+8\,e^{(-3\,x)}-3\,e^{(-4\,x)}}{192\,a}-\frac{5\,e^{(-x)}-40\,e^{(-2\,x)}+120\,e^{(-3\,x)}+552\,e^{(-4\,x)}-3}{192\,\left(ae^{(-4\,x)}+ae^{(-5\,x)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] 15/8*x/a + 1/192*(168*e^(-x) - 48*e^(-2*x) + 8*e^(-3*x) - 3*e^(-4*x))/a - 1
/192*(5*e^(-x) - 40*e^(-2*x) + 120*e^(-3*x) + 552*e^(-4*x) - 3)/(a*e^(-4*x)
 + a*e^(-5*x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(59) = 118$.
time = 0.46, size = 139, normalized size = 2.07

$$\frac{3\,\cosh\left(x\right)^{5}+\left(15\,\cosh\left(x\right)-8\right)\sinh\left(x\right)^{4}+3\,\sinh\left(x\right)^{5}-8\,\cosh\left(x\right)^{4}+\left(30\,\cosh\left(x\right)^{2}-8\,\cosh\left(x\right)+35\right)\sinh\left(x\right)^{3}+45\,\cosh\left(x\right)^{3}+\left(30\,\cosh\left(x\right)^{3}-48\,\cosh\left(x\right)^{2}+135\,\cosh\left(x\right)-160\right)\sinh\left(x\right)^{2}+24\left(15\,x-2\right)\cosh\left(x\right)-160\,\cosh\left(x\right)^{2}+\left(15\,\cosh\left(x\right)^{4}-8\,\cosh\left(x\right)^{3}+105\,\cosh\left(x\right)^{2}+360\,x-160\,\cosh\left(x\right)-288\right)\sinh\left(x\right)+360\,x+552}{192\left(a\cosh\left(x\right)+a\sinh\left(x\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] 1/192*(3*cosh(x)^5 + (15*cosh(x) - 8)*sinh(x)^4 + 3*sinh(x)^5 - 8*cosh(x)^4
 + (30*cosh(x)^2 - 8*cosh(x) + 35)*sinh(x)^3 + 45*cosh(x)^3 + (30*cosh(x)^3
 - 48*cosh(x)^2 + 135*cosh(x) - 160)*sinh(x)^2 + 24*(15*x - 2)*cosh(x) - 16
0*cosh(x)^2 + (15*cosh(x)^4 - 8*cosh(x)^3 + 105*cosh(x)^2 + 360*x - 160*cos
h(x) - 288)*sinh(x) + 360*x + 552)/(a*cosh(x) + a*sinh(x) + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cosh^{4}\left(x\right)}{\operatorname{sech}\left(x\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**4/(a+a*sech(x)),x)
```

```
[Out] Integral(cosh(x)**4/(sech(x) + 1), x)/a
```

Giac [A]
time = 0.39, size = 86, normalized size = 1.28

$$\frac{15\,x}{8\,a}+\frac{\left(552\,e^{(4\,x)}+120\,e^{(3\,x)}-40\,e^{(2\,x)}+5\,e^{x}-3\right)e^{(-4\,x)}}{192\,a(e^{x}+1)}+\frac{3\,a^{3}e^{(4\,x)}-8\,a^{3}e^{(3\,x)}+48\,a^{3}e^{(2\,x)}-168\,a^{3}e^{x}}{192\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] 15/8*x/a + 1/192*(552*e^(4*x) + 120*e^(3*x) - 40*e^(2*x) + 5*e^x - 3)*e^(-4
*x)/(a*(e^x + 1)) + 1/192*(3*a^3*e^(4*x) - 8*a^3*e^(3*x) + 48*a^3*e^(2*x) -
 168*a^3*e^x)/a^4
```

Mupad [B]
time = 1.45, size = 88, normalized size = 1.31

$$\frac{7\,\mathrm{e}^{-x}}{8\,a}-\frac{\mathrm{e}^{-2\,x}}{4\,a}+\frac{\mathrm{e}^{2\,x}}{4\,a}+\frac{\mathrm{e}^{-3\,x}}{24\,a}-\frac{\mathrm{e}^{3\,x}}{24\,a}-\frac{\mathrm{e}^{-4\,x}}{64\,a}+\frac{\mathrm{e}^{4\,x}}{64\,a}+\frac{15\,x}{8\,a}+\frac{2}{a\,\left(\mathrm{e}^{x}+1\right)}-\frac{7\,\mathrm{e}^{x}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a + a/cosh(x)),x)
```

```
[Out] (7*exp(-x))/(8*a) - exp(-2*x)/(4*a) + exp(2*x)/(4*a) + exp(-3*x)/(24*a) - e
xp(3*x)/(24*a) - exp(-4*x)/(64*a) + exp(4*x)/(64*a) + (15*x)/(8*a) + 2/(a*(
exp(x) + 1)) - (7*exp(x))/(8*a)
```

3.69 $\int \frac{\cosh^3(x)}{a+a\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=54

$$-\frac{3x}{2a}+\frac{4\sinh(x)}{a}-\frac{3\cosh(x)\sinh(x)}{2a}-\frac{\cosh^2(x)\sinh(x)}{a+a\mathrm{sech}(x)}+\frac{4\sinh^3(x)}{3a}$$

```
[Out] -3/2*x/a+4*sinh(x)/a-3/2*cosh(x)*sinh(x)/a-cosh(x)^2*sinh(x)/(a+a*sech(x))+
4/3*sinh(x)^3/a
```

Rubi [A]
time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3904, 3872, 2713, 2715, 8}

$$-\frac{3x}{2a}+\frac{4\sinh^3(x)}{3a}+\frac{4\sinh(x)}{a}-\frac{3\sinh(x)\cosh(x)}{2a}-\frac{\sinh(x)\cosh^2(x)}{a\mathrm{sech}(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(a + a*Sech[x]),x]
```

```
[Out] (-3*x)/(2*a) + (4*Sinh[x])/a - (3*Cosh[x]*Sinh[x])/(2*a) - (Cosh[x]^2*Sinh[
x])/(a + a*Sech[x]) + (4*Sinh[x]^3)/(3*a)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^3(x)}{a+a\text{sech}(x)}\,dx &= -\frac{\cosh^2(x)\sinh(x)}{a+a\text{sech}(x)}-\frac{\int \cosh^3(x)(-4a+3a\text{sech}(x))\,dx}{a^2}\\ &= -\frac{\cosh^2(x)\sinh(x)}{a+a\text{sech}(x)}-\frac{3\int \cosh^2(x)\,dx}{a}+\frac{4\int \cosh^3(x)\,dx}{a}\\ &= -\frac{3\cosh(x)\sinh(x)}{2a}-\frac{\cosh^2(x)\sinh(x)}{a+a\text{sech}(x)}+\frac{(4i)\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-i\sinh(x)\right)}{a}-\\ &= -\frac{3x}{2a}+\frac{4\sinh(x)}{a}-\frac{3\cosh(x)\sinh(x)}{2a}-\frac{\cosh^2(x)\sinh(x)}{a+a\text{sech}(x)}+\frac{4\sinh^3(x)}{3a}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 53, normalized size = 0.98

$$\frac{\text{sech}\left(\frac{x}{2}\right)\left(-36x\cosh\left(\frac{x}{2}\right)+45\sinh\left(\frac{x}{2}\right)+18\sinh\left(\frac{3x}{2}\right)-2\sinh\left(\frac{5x}{2}\right)+\sinh\left(\frac{7x}{2}\right)\right)}{24a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(a + a*Sech[x]),x]
[Out] (Sech[x/2]*(-36*x*Cosh[x/2] + 45*Sinh[x/2] + 18*Sinh[(3*x)/2] - 2*Sinh[(5*x
)/2] + Sinh[(7*x)/2]))/(24*a)
```

Maple [A]
time = 0.80, size = 86, normalized size = 1.59

method	result
risch	$-\frac{3x}{2a}+\frac{e^{3x}}{24a}-\frac{e^{2x}}{8a}+\frac{7\,e^{x}}{8a}-\frac{7\,e^{-x}}{8a}+\frac{e^{-2x}}{8a}-\frac{e^{-3x}}{24a}-\frac{2}{(e^{x}+1)a}$
default	$\frac{\tanh\left(\frac{x}{2}\right)-\frac{1}{3\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}-\frac{1}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{5}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2}-\frac{1}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}+\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}-\frac{5}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)}-}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(tanh(1/2*x)-1/3/(tanh(1/2*x)-1)^3-1/(tanh(1/2*x)-1)^2-5/2/(tanh(1/2*x)
-1)+3/2*ln(tanh(1/2*x)-1)-1/3/(tanh(1/2*x)+1)^3+1/(tanh(1/2*x)+1)^2-5/2/(ta
nh(1/2*x)+1)-3/2*ln(tanh(1/2*x)+1))
```

Maxima [A]
time = 0.26, size = 66, normalized size = 1.22

$$-\frac{3\,x}{2\,a}-\frac{21\,e^{(-x)}-3\,e^{(-2\,x)}+e^{(-3\,x)}}{24\,a}-\frac{2\,e^{(-x)}-18\,e^{(-2\,x)}-69\,e^{(-3\,x)}-1}{24\,(ae^{(-3\,x)}+ae^{(-4\,x)})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] -3/2*x/a - 1/24*(21*e^(-x) - 3*e^(-2*x) + e^(-3*x))/a - 1/24*(2*e^(-x) - 18
*e^(-2*x) - 69*e^(-3*x) - 1)/(a*e^(-3*x) + a*e^(-4*x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(48) = 96.
time = 0.42, size = 100, normalized size = 1.85

$$\frac{\cosh(x)^4+(4\,\cosh(x)-1)\sinh(x)^3+\sinh(x)^4-3\,\cosh(x)^3+(6\,\cosh(x)^2-9\,\cosh(x)+20)\sinh(x)^2-3\,(12\,x-1)\cosh(x)+20\,\cosh(x)^2+(4\,\cosh(x)^3-3\,\cosh(x)^2-36\,x+32\,\cosh(x)+39)\sinh(x)-36\,x-69}{24\,(a\cosh(x)+a\sinh(x)+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] 1/24*(cosh(x)^4 + (4*cosh(x) - 1)*sinh(x)^3 + sinh(x)^4 - 3*cosh(x)^3 + (6*
cosh(x)^2 - 9*cosh(x) + 20)*sinh(x)^2 - 3*(12*x - 1)*cosh(x) + 20*cosh(x)^2
 + (4*cosh(x)^3 - 3*cosh(x)^2 - 36*x + 32*cosh(x) + 39)*sinh(x) - 36*x - 69
)/(a*cosh(x) + a*sinh(x) + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cosh^3(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(a+a*sech(x)),x)
```

```
[Out] Integral(cosh(x)**3/(sech(x) + 1), x)/a
```

Giac [A]
time = 0.38, size = 70, normalized size = 1.30

$$-\frac{3\,x}{2\,a}-\frac{\left(69\,e^{(3\,x)}+18\,e^{(2\,x)}-2\,e^{x}+1\right)e^{(-3\,x)}}{24\,a(e^{x}+1)}+\frac{a^{2}e^{(3\,x)}-3\,a^{2}e^{(2\,x)}+21\,a^{2}e^{x}}{24\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] -3/2*x/a - 1/24*(69*e^(3*x) + 18*e^(2*x) - 2*e^x + 1)*e^(-3*x)/(a*(e^x + 1)
) + 1/24*(a^2*e^(3*x) - 3*a^2*e^(2*x) + 21*a^2*e^x)/a^3
```

Mupad [B]
time = 1.36, size = 70, normalized size = 1.30

$$\frac{\mathrm{e}^{-2\,x}}{8\,a}-\frac{7\,\mathrm{e}^{-x}}{8\,a}-\frac{\mathrm{e}^{2\,x}}{8\,a}-\frac{\mathrm{e}^{-3\,x}}{24\,a}+\frac{\mathrm{e}^{3\,x}}{24\,a}-\frac{3\,x}{2\,a}-\frac{2}{a\,\left(\mathrm{e}^{x}+1\right)}+\frac{7\,\mathrm{e}^{x}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a + a/cosh(x)),x)
```

```
[Out] exp(-2*x)/(8*a) - (7*exp(-x))/(8*a) - exp(2*x)/(8*a) - exp(-3*x)/(24*a) + e
xp(3*x)/(24*a) - (3*x)/(2*a) - 2/(a*(exp(x) + 1)) + (7*exp(x))/(8*a)
```

3.70 $\int \frac{\cosh^2(x)}{a+a\textbf{sech}(x)}\,dx$

Optimal. Leaf size=41

$$\frac{3x}{2a}-\frac{2\sinh(x)}{a}+\frac{3\cosh(x)\sinh(x)}{2a}-\frac{\cosh(x)\sinh(x)}{a+a\text{sech}(x)}$$

```
[Out] 3/2*x/a-2*sinh(x)/a+3/2*cosh(x)*sinh(x)/a-cosh(x)*sinh(x)/(a+a*sech(x))
```

Rubi [A]
time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3904, 3872, 2715, 8, 2717}

$$\frac{3x}{2a}-\frac{2\sinh(x)}{a}+\frac{3\sinh(x)\cosh(x)}{2a}-\frac{\sinh(x)\cosh(x)}{a\text{sech}(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(a + a*Sech[x]),x]
```

```
[Out] (3*x)/(2*a) - (2*Sinh[x])/a + (3*Cosh[x]*Sinh[x])/(2*a) - (Cosh[x]*Sinh[x])
/(a + a*Sech[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^2(x)}{a+a\operatorname{sech}(x)}\,dx &= -\frac{\cosh(x)\sinh(x)}{a+a\operatorname{sech}(x)}-\frac{\int \cosh^2(x)(-3a+2a\operatorname{sech}(x))\,dx}{a^2}\\
&= -\frac{\cosh(x)\sinh(x)}{a+a\operatorname{sech}(x)}-\frac{2\int \cosh(x)\,dx}{a}+\frac{3\int \cosh^2(x)\,dx}{a}\\
&= -\frac{2\sinh(x)}{a}+\frac{3\cosh(x)\sinh(x)}{2a}-\frac{\cosh(x)\sinh(x)}{a+a\operatorname{sech}(x)}+\frac{3\int 1\,dx}{2a}\\
&= \frac{3x}{2a}-\frac{2\sinh(x)}{a}+\frac{3\cosh(x)\sinh(x)}{2a}-\frac{\cosh(x)\sinh(x)}{a+a\operatorname{sech}(x)}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 45, normalized size = 1.10

$$\frac{\operatorname{sech}\left(\frac{x}{2}\right)\left(12x\cosh\left(\frac{x}{2}\right)-12\sinh\left(\frac{x}{2}\right)-3\sinh\left(\frac{3x}{2}\right)+\sinh\left(\frac{5x}{2}\right)\right)}{8a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(a + a*Sech[x]),x]
[Out] (Sech[x/2]*(12*x*Cosh[x/2] - 12*Sinh[x/2] - 3*Sinh[(3*x)/2] + Sinh[(5*x)/2]
))/(8*a)
```

Maple [A]
time = 0.77, size = 70, normalized size = 1.71

method	result	size
risch	$\frac{3x}{2a}+\frac{e^{2x}}{8a}-\frac{e^{x}}{2a}+\frac{e^{-x}}{2a}-\frac{e^{-2x}}{8a}+\frac{2}{(e^{x}+1)a}$	53
default	$\frac{-\tanh\left(\frac{x}{2}\right)-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{3}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2}+\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{3}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2}}{a}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a+a*sech(x)),x,method=_RETURNVERBOSE)
[Out] 1/a*(-tanh(1/2*x)-1/2/(tanh(1/2*x)+1)^2+3/2/(tanh(1/2*x)+1)+3/2*ln(tanh(1/2
*x)+1)+1/2/(tanh(1/2*x)-1)^2+3/2/(tanh(1/2*x)-1)-3/2*ln(tanh(1/2*x)-1))
```

Maxima [A]
time = 0.28, size = 56, normalized size = 1.37

$$\frac{3\,x}{2\,a}+\frac{4\,e^{(-x)}-e^{(-2\,x)}}{8\,a}-\frac{3\,e^{(-x)}+20\,e^{(-2\,x)}-1}{8\,\left(ae^{(-2\,x)}+ae^{(-3\,x)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] 3/2*x/a + 1/8*(4*e^(-x) - e^(-2*x))/a - 1/8*(3*e^(-x) + 20*e^(-2*x) - 1)/(a
*e^(-2*x) + a*e^(-3*x))
```

Fricas [A]
time = 0.38, size = 70, normalized size = 1.71

$$\frac{\cosh\left(x\right)^{3}+\left(3\,\cosh\left(x\right)-4\right)\sinh\left(x\right)^{2}+\sinh\left(x\right)^{3}+\left(12\,x-1\right)\cosh\left(x\right)-4\,\cosh\left(x\right)^{2}+\left(3\,\cosh\left(x\right)^{2}+12\,x-4\,\cosh\left(x\right)-7\right)\sinh\left(x\right)+12\,x+20}{8\,\left(a\cosh\left(x\right)+a\sinh\left(x\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] 1/8*(cosh(x)^3 + (3*cosh(x) - 4)*sinh(x)^2 + sinh(x)^3 + (12*x - 1)*cosh(x)
 - 4*cosh(x)^2 + (3*cosh(x)^2 + 12*x - 4*cosh(x) - 7)*sinh(x) + 12*x + 20)/
(a*cosh(x) + a*sinh(x) + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cosh^{2}\left(x\right)}{\operatorname{sech}\left(x\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(a+a*sech(x)),x)
```

```
[Out] Integral(cosh(x)**2/(sech(x) + 1), x)/a
```

Giac [A]
time = 0.39, size = 51, normalized size = 1.24

$$\frac{3\,x}{2\,a}+\frac{\left(20\,e^{(2\,x)}+3\,e^{x}-1\right)e^{(-2\,x)}}{8\,a(e^{x}+1)}+\frac{ae^{(2\,x)}-4\,ae^{x}}{8\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] 3/2*x/a + 1/8*(20*e^(2*x) + 3*e^x - 1)*e^(-2*x)/(a*(e^x + 1)) + 1/8*(a*e^(2
*x) - 4*a*e^x)/a^2
```

Mupad [B]
time = 1.36, size = 52, normalized size = 1.27

$$\frac{e^{-x}}{2a} - \frac{e^{-2x}}{8a} + \frac{e^{2x}}{8a} + \frac{3x}{2a} + \frac{2}{a(e^x + 1)} - \frac{e^x}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a + a/cosh(x)),x)
[Out] exp(-x)/(2*a) - exp(-2*x)/(8*a) + exp(2*x)/(8*a) + (3*x)/(2*a) + 2/(a*(exp(
x) + 1)) - exp(x)/(2*a)
```

3.71 $\int \frac{\cosh(x)}{a+a\text{sech}(x)}\,dx$

Optimal. Leaf size=26

$$-\frac{x}{a}+\frac{2\sinh(x)}{a}-\frac{\sinh(x)}{a+a\text{sech}(x)}$$

```
[Out] -x/a+2*sinh(x)/a-sinh(x)/(a+a*sech(x))
```

Rubi [A]
time = 0.04, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3904, 3872, 2717, 8}

$$-\frac{x}{a}+\frac{2\sinh(x)}{a}-\frac{\sinh(x)}{a\text{sech}(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/(a + a*Sech[x]),x]
```

```
[Out] -(x/a) + (2*Sinh[x])/a - Sinh[x]/(a + a*Sech[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(x)}{a+a\operatorname{sech}(x)}\,dx &= -\frac{\sinh(x)}{a+a\operatorname{sech}(x)} - \frac{\int \cosh(x)(-2a+a\operatorname{sech}(x))\,dx}{a^2}\\ &= -\frac{\sinh(x)}{a+a\operatorname{sech}(x)} - \frac{\int 1\,dx}{a} + \frac{2\int \cosh(x)\,dx}{a}\\ &= -\frac{x}{a} + \frac{2\sinh(x)}{a} - \frac{\sinh(x)}{a+a\operatorname{sech}(x)}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 32, normalized size = 1.23

$$\frac{-2x+\operatorname{sech}\left(\frac{x}{2}\right)\sinh\left(\frac{3x}{2}\right)+3\tanh\left(\frac{x}{2}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(a + a*Sech[x]),x]
```

```
[Out] (-2*x + Sech[x/2]*Sinh[(3*x)/2] + 3*Tanh[x/2])/(2*a)
```

Maple [A]
time = 0.77, size = 46, normalized size = 1.77

method	result	size
risch	$-\frac{x}{a}+\frac{e^{x}}{2a}-\frac{e^{-x}}{2a}-\frac{2}{(e^{x}+1)a}$	35
default	$\frac{\tanh\left(\frac{x}{2}\right)-\frac{1}{\tanh\left(\frac{x}{2}\right)-1}+\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\frac{1}{\tanh\left(\frac{x}{2}\right)+1}-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(tanh(1/2*x)-1/(tanh(1/2*x)-1)+ln(tanh(1/2*x)-1)-1/(tanh(1/2*x)+1)-ln(t
anh(1/2*x)+1))
```

Maxima [A]
time = 0.27, size = 41, normalized size = 1.58

$$-\frac{x}{a}+\frac{5\,e^{(-x)}+1}{2\,(ae^{(-x)}+ae^{(-2\,x)})}-\frac{e^{(-x)}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] -x/a + 1/2*(5*e^(-x) + 1)/(a*e^(-x) + a*e^(-2*x)) - 1/2*e^(-x)/a
```

Fricas [A]
time = 0.40, size = 47, normalized size = 1.81

$$-\frac{2\,x\cosh\left(x\right)-\cosh\left(x\right)^{2}+2\,\left(x-\cosh\left(x\right)-1\right)\sinh\left(x\right)-\sinh\left(x\right)^{2}+2\,x+5}{2\,\left(a\cosh\left(x\right)+a\sinh\left(x\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*x*cosh(x) - cosh(x)^2 + 2*(x - cosh(x) - 1)*sinh(x) - sinh(x)^2 + 2
*x + 5)/(a*cosh(x) + a*sinh(x) + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cosh\left(x\right)}{\operatorname{sech}\left(x\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*sech(x)),x)
```

```
[Out] Integral(cosh(x)/(sech(x) + 1), x)/a
```

Giac [A]
time = 0.37, size = 35, normalized size = 1.35

$$-\frac{x}{a}-\frac{(5\,e^{x}+1)e^{(-x)}}{2\,a(e^{x}+1)}+\frac{e^{x}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] -x/a - 1/2*(5*e^x + 1)*e^(-x)/(a*(e^x + 1)) + 1/2*e^x/a
```

Mupad [B]
time = 1.31, size = 34, normalized size = 1.31

$$\frac{\mathrm{e}^{x}}{2\,a}-\frac{x}{a}-\frac{2}{a\,\left(\mathrm{e}^{x}+1\right)}-\frac{\mathrm{e}^{-x}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a + a/cosh(x)),x)
```

```
[Out] exp(x)/(2*a) - x/a - 2/(a*(exp(x) + 1)) - exp(-x)/(2*a)
```

3.72 $\int \frac{\text{sech}(x)}{a+a\text{sech}(x)}\,dx$

Optimal. Leaf size=11

$$\frac{\tanh(x)}{a+a\text{sech}(x)}$$

```
[Out] tanh(x)/(a+a*sech(x))
```

Rubi [A]
time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3879}

$$\frac{\tanh(x)}{a\text{sech}(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]/(a + a*Sech[x]),x]
```

```
[Out] Tanh[x]/(a + a*Sech[x])
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\text{sech}(x)}{a+a\text{sech}(x)}\,dx = \frac{\tanh(x)}{a+a\text{sech}(x)}$$

Mathematica [A]
time = 0.01, size = 10, normalized size = 0.91

$$\frac{\tanh\left(\frac{x}{2}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(a + a*Sech[x]),x]
```

```
[Out] Tanh[x/2]/a
```

Maple [A]
time = 0.42, size = 9, normalized size = 0.82

method	result	size
default	$\frac{\tanh\left(\frac{x}{2}\right)}{a}$	9
risch	$-\frac{2}{\left(e^{x}+1\right)a}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*tanh(1/2*x)
```

Maxima [A]
time = 0.28, size = 12, normalized size = 1.09

$$\frac{2}{ae^{(-x)} + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] 2/(a*e^(-x) + a)
```

Fricas [A]
time = 0.37, size = 14, normalized size = 1.27

$$-\frac{2}{a\cosh\left(x\right) + a\sinh\left(x\right) + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] -2/(a*cosh(x) + a*sinh(x) + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{sech}\left(x\right)}{\operatorname{sech}\left(x\right)+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+a*sech(x)),x)
```

```
[Out] Integral(sech(x)/(sech(x) + 1), x)/a
```

Giac [A]
time = 0.39, size = 11, normalized size = 1.00

$$-\frac{2}{a(e^x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+a*sech(x)),x, algorithm="giac")
[Out] -2/(a*(e^x + 1))
```

Mupad [B]
time = 1.31, size = 11, normalized size = 1.00

$$-\frac{2}{a\,\left(\mathrm{e}^x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)*(a + a/cosh(x))),x)
[Out] -2/(a*(exp(x) + 1))
```

3.73 $\int \frac{\operatorname{sech}^2(x)}{a+a\operatorname{sech}(x)}\,dx$

Optimal. Leaf size=20

$$\frac{\operatorname{ArcTan}(\sinh(x))}{a}-\frac{\tanh(x)}{a+a\operatorname{sech}(x)}$$

```
[Out] arctan(sinh(x))/a-tanh(x)/(a+a*sech(x))
```

Rubi [A]
time = 0.04, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3874, 3855, 3879}

$$\frac{\operatorname{ArcTan}(\sinh(x))}{a}-\frac{\tanh(x)}{a\operatorname{sech}(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(a + a*Sech[x]),x]
```

```
[Out] ArcTan[Sinh[x]]/a - Tanh[x]/(a + a*Sech[x])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3874

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^2(x)}{a+a\operatorname{sech}(x)}\,dx&=\frac{\int \operatorname{sech}(x)\,dx}{a}-\int \frac{\operatorname{sech}(x)}{a+a\operatorname{sech}(x)}\,dx\\&=\frac{\tan^{-1}(\sinh(x))}{a}-\frac{\tanh(x)}{a+a\operatorname{sech}(x)}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 1.10

$$\frac{2\text{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)-\tanh\left(\frac{x}{2}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(a + a*Sech[x]),x]
[Out] (2*ArcTan[Tanh[x/2]] - Tanh[x/2])/a
```

Maple [A]
time = 0.50, size = 19, normalized size = 0.95

method	result	size
default	$\frac{-\tanh\left(\frac{x}{2}\right)+2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$	19
risch	$\frac{2}{\left(e^{x}+1\right)a}+\frac{i\ln\left(e^{x}+i\right)}{a}-\frac{i\ln\left(e^{x}-i\right)}{a}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(a+a*sech(x)),x,method=_RETURNVERBOSE)
[Out] 1/a*(-tanh(1/2*x)+2*arctan(tanh(1/2*x)))
```

Maxima [A]
time = 0.48, size = 23, normalized size = 1.15

$$-\frac{2\arctan\left(e^{(-x)}\right)}{a}-\frac{2}{ae^{(-x)}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+a*sech(x)),x, algorithm="maxima")
[Out] -2*arctan(e^(-x))/a - 2/(a*e^(-x) + a)
```

Fricas [A]
time = 0.39, size = 29, normalized size = 1.45

$$\frac{2\left(\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)+1\right)}{a\cosh\left(x\right)+a\sinh\left(x\right)+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+a*sech(x)),x, algorithm="fricas")
[Out] 2*((cosh(x) + sinh(x) + 1)*arctan(cosh(x) + sinh(x)) + 1)/(a*cosh(x) + a*si
nh(x) + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{sech}^2(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(a+a*sech(x)),x)
```

```
[Out] Integral(sech(x)**2/(sech(x) + 1), x)/a
```

Giac [A]
time = 0.39, size = 20, normalized size = 1.00

$$\frac{2\arctan(e^x)}{a} + \frac{2}{a(e^x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] 2*arctan(e^x)/a + 2/(a*(e^x + 1))
```

Mupad [B]
time = 1.30, size = 31, normalized size = 1.55

$$\frac{2}{a\,(e^x+1)} + \frac{2\,\mathrm{atan}\left(\frac{e^x\,\sqrt{a^2}}{a}\right)}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2*(a + a/cosh(x))),x)
```

```
[Out] 2/(a*(exp(x) + 1)) + (2*atan((exp(x)*(a^2)^(1/2))/a))/(a^2)^(1/2)
```

3.74 $\int \frac{\text{sech}^3(x)}{a+a\text{sech}(x)}\,dx$

Optimal. Leaf size=26

$$-\frac{\text{ArcTan}(\sinh(x))}{a}+\frac{\tanh(x)}{a}+\frac{\tanh(x)}{a+a\text{sech}(x)}$$

```
[Out] -arctan(sinh(x))/a+tanh(x)/a+tanh(x)/(a+a*sech(x))
```

Rubi [A]
time = 0.07, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3875, 3874, 3855, 3879}

$$-\frac{\text{ArcTan}(\sinh(x))}{a}+\frac{\tanh(x)}{a}+\frac{\tanh(x)}{a\text{sech}(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^3/(a + a*Sech[x]),x]
```

```
[Out] -(ArcTan[Sinh[x]]/a) + Tanh[x]/a + Tanh[x]/(a + a*Sech[x])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3874

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3875

```
Int[csc[(e_.) + (f_.)*(x_)]^3/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Simp[-Cot[e + f*x]/(b*f), x] - Dist[a/b, Int[Csc[e + f*x]^2/(a + b*
Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{\tanh(x)}{a} - \int \frac{\operatorname{sech}^2(x)}{a+a\operatorname{sech}(x)}\,dx$$
$$= \frac{\tanh(x)}{a} - \frac{\int \operatorname{sech}(x)\,dx}{a} + \int \frac{\operatorname{sech}(x)}{a+a\operatorname{sech}(x)}\,dx$$
$$= -\frac{\tan^{-1}(\sinh(x))}{a} + \frac{\tanh(x)}{a} + \frac{\tanh(x)}{a+a\operatorname{sech}(x)}$$

Mathematica [A]
time = 0.06, size = 45, normalized size = 1.73

$$\frac{2\cosh\left(\frac{x}{2}\right)\operatorname{sech}(x)\left(\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)\left(-2\text{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)+\tanh(x)\right)\right)}{a(1+\operatorname{sech}(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^3/(a + a*Sech[x]),x]
[Out] (2*Cosh[x/2]*Sech[x]*(Sinh[x/2] + Cosh[x/2]*(-2*ArcTan[Tanh[x/2]] + Tanh[x]
)))/(a*(1 + Sech[x]))
```

Maple [A]
time = 0.52, size = 33, normalized size = 1.27

method	result	size
default	$\frac{\tanh\left(\frac{x}{2}\right)+\frac{2\tanh\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)+1}-2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$	33
risch	$-\frac{2(e^{2x}+e^x+2)}{(e^x+1)a(1+e^{2x})}+\frac{i\ln(e^x-i)}{a}-\frac{i\ln(e^x+i)}{a}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^3/(a+a*sech(x)),x,method=_RETURNVERBOSE)
[Out] 1/a*(tanh(1/2*x)+2*tanh(1/2*x)/(tanh(1/2*x)^2+1)-2*arctan(tanh(1/2*x)))
```

Maxima [A]
time = 0.48, size = 45, normalized size = 1.73

$$\frac{2\left(e^{(-x)}+e^{(-2x)}+2\right)}{ae^{(-x)}+ae^{(-2x)}+ae^{(-3x)}+a}+\frac{2\arctan\left(e^{(-x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] 2*(e^(-x) + e^(-2*x) + 2)/(a*e^(-x) + a*e^(-2*x) + a*e^(-3*x) + a) + 2*arct
an(e^(-x))/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(26) = 52$.
time = 0.46, size = 127, normalized size = 4.88

$$-\frac{2\left(\left(\cosh(x)^3 + (3\cosh(x)+1)\sinh(x)^2 + \sinh(x)^3 + \cosh(x)^2 + (3\cosh(x)^2 + 2\cosh(x) + 1)\sinh(x) + \cosh(x) + 1\right)\arctan(\cosh(x)+\sinh(x)) + \cosh(x)^2 + (2\cosh(x)+1)\sinh(x) + \sinh(x)^2 + \cosh(x) + 2\right)}{a\cosh(x)^3 + a\sinh(x)^3 + a\cosh(x)^2 + (3a\cosh(x)+a)\sinh(x)^2 + a\cosh(x) + (3a\cosh(x)^2 + 2a\cosh(x) + a)\sinh(x) + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] -2*((cosh(x)^3 + (3*cosh(x) + 1)*sinh(x)^2 + sinh(x)^3 + cosh(x)^2 + (3*cos
h(x)^2 + 2*cosh(x) + 1)*sinh(x) + cosh(x) + 1)*arctan(cosh(x) + sinh(x)) +
cosh(x)^2 + (2*cosh(x) + 1)*sinh(x) + sinh(x)^2 + cosh(x) + 2)/(a*cosh(x)^3
 + a*sinh(x)^3 + a*cosh(x)^2 + (3*a*cosh(x) + a)*sinh(x)^2 + a*cosh(x) + (3
*a*cosh(x)^2 + 2*a*cosh(x) + a)*sinh(x) + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{sech}^3(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**3/(a+a*sech(x)),x)
```

```
[Out] Integral(sech(x)**3/(sech(x) + 1), x)/a
```

Giac [A]
time = 0.38, size = 36, normalized size = 1.38

$$-\frac{2\arctan(e^x)}{a} - \frac{2\left(e^{(2x)} + e^x + 2\right)}{a(e^{(3x)} + e^{(2x)} + e^x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] -2*arctan(e^x)/a - 2*(e^(2*x) + e^x + 2)/(a*(e^(3*x) + e^(2*x) + e^x + 1))
```

Mupad [B]
time = 1.32, size = 58, normalized size = 2.23

$$-\frac{\frac{2\,e^{2\,x}}{a} + \frac{4}{a} + \frac{2\,e^x}{a}}{e^{2\,x} + e^{3\,x} + e^x + 1} - \frac{2\,\mathrm{atan}\left(\frac{e^x\,\sqrt{a^2}}{a}\right)}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^3*(a + a/cosh(x))),x)
```

```
[Out] - ((2*exp(2*x))/a + 4/a + (2*exp(x))/a)/(exp(2*x) + exp(3*x) + exp(x) + 1)
- (2*atan((exp(x)*(a^2)^(1/2))/a))/(a^2)^(1/2)
```

3.75 $\int \frac{\operatorname{sech}^4(x)}{a+a\operatorname{sech}(x)}\,dx$

Optimal. Leaf size=45

$$\frac{3\text{ArcTan}(\sinh(x))}{2a}-\frac{2\tanh(x)}{a}+\frac{3\text{sech}(x)\tanh(x)}{2a}-\frac{\text{sech}^2(x)\tanh(x)}{a+a\text{sech}(x)}$$

```
[Out] 3/2*arctan(sinh(x))/a-2*tanh(x)/a+3/2*sech(x)*tanh(x)/a-sech(x)^2*tanh(x)/(
a+a*sech(x))
```

Rubi [A]
time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3903, 3872, 3852, 8, 3853, 3855}

$$\frac{3\text{ArcTan}(\sinh(x))}{2a}-\frac{2\tanh(x)}{a}-\frac{\tanh(x)\text{sech}^2(x)}{a\text{sech}(x)+a}+\frac{3\tanh(x)\text{sech}(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^4/(a + a*Sech[x]),x]
```

```
[Out] (3*ArcTan[Sinh[x]])/(2*a) - (2*Tanh[x])/a + (3*Sech[x]*Tanh[x])/(2*a) - (Se
ch[x]^2*Tanh[x])/(a + a*Sech[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3903

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(f*(a +
b*Csc[e + f*x]))), x] - Dist[d^2/(a*b), Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
- 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^4(x)}{a+a\operatorname{sech}(x)}\,dx &= -\frac{\operatorname{sech}^2(x)\tanh(x)}{a+a\operatorname{sech}(x)}-\frac{\int \operatorname{sech}^2(x)(2a-3a\operatorname{sech}(x))\,dx}{a^2}\\ &= -\frac{\operatorname{sech}^2(x)\tanh(x)}{a+a\operatorname{sech}(x)}-\frac{2\int \operatorname{sech}^2(x)\,dx}{a}+\frac{3\int \operatorname{sech}^3(x)\,dx}{a}\\ &= \frac{3\operatorname{sech}(x)\tanh(x)}{2a}-\frac{\operatorname{sech}^2(x)\tanh(x)}{a+a\operatorname{sech}(x)}-\frac{(2i)\operatorname{Subst}\left(\int 1\,dx,x,-i\tanh(x)\right)}{a}+\frac{3\int \operatorname{sech}(x}{2a}\\ &= \frac{3\tan^{-1}(\sinh(x))}{2a}-\frac{2\tanh(x)}{a}+\frac{3\operatorname{sech}(x)\tanh(x)}{2a}-\frac{\operatorname{sech}^2(x)\tanh(x)}{a+a\operatorname{sech}(x)}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 51, normalized size = 1.13

$$\frac{\cosh\left(\frac{x}{2}\right)\operatorname{sech}(x)\left(-2\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)\left(6\operatorname{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)+(-2+\operatorname{sech}(x))\tanh(x)\right)\right)}{a(1+\operatorname{sech}(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^4/(a + a*Sech[x]),x]
```

```
[Out] (Cosh[x/2]*Sech[x]*(-2*Sinh[x/2] + Cosh[x/2]*(6*ArcTan[Tanh[x/2]] + (-2 + S
ech[x])*Tanh[x])))/(a*(1 + Sech[x]))
```

Maple [A]
time = 0.67, size = 46, normalized size = 1.02

method	result	size

default	$\frac{-\tanh\left(\frac{x}{2}\right)+\frac{-3\left(\tanh^{3}\left(\frac{x}{2}\right)\right)-\tanh\left(\frac{x}{2}\right)}{\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)^{2}}+3\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$	46
risch	$\frac{3\,\mathrm{e}^{4x}+3\,\mathrm{e}^{3x}+5\,\mathrm{e}^{2x}+\mathrm{e}^{x}+4}{\left(1+\mathrm{e}^{2x}\right)^{2}a\left(\mathrm{e}^{x}+1\right)}+\frac{3i\ln\left(\mathrm{e}^{x}+i\right)}{2a}-\frac{3i\ln\left(\mathrm{e}^{x}-i\right)}{2a}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^4/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-tanh(1/2*x)+2*(-3/2*tanh(1/2*x)^3-1/2*tanh(1/2*x))/(tanh(1/2*x)^2+1)^
2+3*arctan(tanh(1/2*x)))
```

Maxima [A]
time = 0.48, size = 73, normalized size = 1.62

$$-\frac{e^{(-x)}+5\,e^{(-2\,x)}+3\,e^{(-3\,x)}+3\,e^{(-4\,x)}+4}{ae^{(-x)}+2\,ae^{(-2\,x)}+2\,ae^{(-3\,x)}+ae^{(-4\,x)}+ae^{(-5\,x)}+a}-\frac{3\,\arctan\left(e^{(-x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] -(e^(-x) + 5*e^(-2*x) + 3*e^(-3*x) + 3*e^(-4*x) + 4)/(a*e^(-x) + 2*a*e^(-2*
x) + 2*a*e^(-3*x) + a*e^(-4*x) + a*e^(-5*x) + a) - 3*arctan(e^(-x))/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 325 vs. 2(41) = 82.
time = 0.38, size = 325, normalized size = 7.22

$$\frac{3\,\cosh\left(x\right)^{4}+3\,\left(4\,\cosh\left(x\right)+1\right)\sinh\left(x\right)^{3}+3\,\sinh\left(x\right)^{4}+3\,\cosh\left(x\right)^{3}+\left(18\,\cosh\left(x\right)^{2}+9\,\cosh\left(x\right)+5\right)\sinh\left(x\right)^{2}+3\,\left(\cosh\left(x\right)^{5}+\left(5\,\cosh\left(x\right)+1\right)\sinh\left(x\right)^{4}+\sinh\left(x\right)^{5}+\cosh\left(x\right)^{4}+2\,\left(5\,\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)+1\right)\sinh\left(x\right)^{3}+2\,\cosh\left(x\right)^{3}+2\,\left(5\,\cosh\left(x\right)^{3}+3\,\cosh\left(x\right)^{2}+3\,\cosh\left(x\right)+1\right)\sinh\left(x\right)^{2}+2\,\cosh\left(x\right)^{2}+\left(5\,\cosh\left(x\right)^{4}+4\,\cosh\left(x\right)^{3}+6\,\cosh\left(x\right)^{2}+4\,\cosh\left(x\right)+1\right)\sinh\left(x\right)+\cosh\left(x\right)+1\right)\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)+5\,\cosh\left(x\right)^{2}+\left(12\,\cosh\left(x\right)^{3}+9\,\cosh\left(x\right)^{2}+10\,\cosh\left(x\right)+1\right)\sinh\left(x\right)+\cosh\left(x\right)+4}{a\cosh\left(x\right)^{5}+a\sinh\left(x\right)^{5}+a\cosh\left(x\right)^{4}+\left(5\,a\cosh\left(x\right)+a\right)\sinh\left(x\right)^{4}+2\,a\cosh\left(x\right)^{3}+2\,\left(5\,a\cosh\left(x\right)^{2}+2\,a\cosh\left(x\right)+a\right)\sinh\left(x\right)^{3}+2\,a\cosh\left(x\right)^{2}+2\,\left(5\,a\cosh\left(x\right)^{3}+3\,a\cosh\left(x\right)^{2}+3\,a\cosh\left(x\right)+a\right)\sinh\left(x\right)^{2}+a\cosh\left(x\right)+\left(5\,a\cosh\left(x\right)^{4}+4\,a\cosh\left(x\right)^{3}+6\,a\cosh\left(x\right)^{2}+4\,a\cosh\left(x\right)+a\right)\sinh\left(x\right)+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] (3*cosh(x)^4 + 3*(4*cosh(x) + 1)*sinh(x)^3 + 3*sinh(x)^4 + 3*cosh(x)^3 + (1
8*cosh(x)^2 + 9*cosh(x) + 5)*sinh(x)^2 + 3*(cosh(x)^5 + (5*cosh(x) + 1)*sin
h(x)^4 + sinh(x)^5 + cosh(x)^4 + 2*(5*cosh(x)^2 + 2*cosh(x) + 1)*sinh(x)^3
+ 2*cosh(x)^3 + 2*(5*cosh(x)^3 + 3*cosh(x)^2 + 3*cosh(x) + 1)*sinh(x)^2 + 2
*cosh(x)^2 + (5*cosh(x)^4 + 4*cosh(x)^3 + 6*cosh(x)^2 + 4*cosh(x) + 1)*sinh
(x) + cosh(x) + 1)*arctan(cosh(x) + sinh(x)) + 5*cosh(x)^2 + (12*cosh(x)^3
+ 9*cosh(x)^2 + 10*cosh(x) + 1)*sinh(x) + cosh(x) + 4)/(a*cosh(x)^5 + a*sin
h(x)^5 + a*cosh(x)^4 + (5*a*cosh(x) + a)*sinh(x)^4 + 2*a*cosh(x)^3 + 2*(5*a
*cosh(x)^2 + 2*a*cosh(x) + a)*sinh(x)^3 + 2*a*cosh(x)^2 + 2*(5*a*cosh(x)^3
+ 3*a*cosh(x)^2 + 3*a*cosh(x) + a)*sinh(x)^2 + a*cosh(x) + (5*a*cosh(x)^4 +
 4*a*cosh(x)^3 + 6*a*cosh(x)^2 + 4*a*cosh(x) + a)*sinh(x) + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{sech}^4(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**4/(a+a*sech(x)),x)
```

```
[Out] Integral(sech(x)**4/(sech(x) + 1), x)/a
```

Giac [A]
time = 0.38, size = 48, normalized size = 1.07

$$\frac{3\arctan\left(e^{x}\right)}{a} + \frac{e^{(3x)} + 2e^{(2x)} - e^{x} + 2}{a\left(e^{(2x)} + 1\right)^{2}} + \frac{2}{a\left(e^{x} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] 3*arctan(e^x)/a + (e^(3*x) + 2*e^(2*x) - e^x + 2)/(a*(e^(2*x) + 1)^2) + 2/(
a*(e^x + 1))
```

Mupad [B]
time = 1.35, size = 73, normalized size = 1.62

$$\frac{2}{a\,\left(e^{x}+1\right)} + \frac{\frac{2}{a}+\frac{e^{x}}{a}}{e^{2x}+1} + \frac{3\,\mathrm{atan}\left(\frac{e^{x}\sqrt{a^2}}{a}\right)}{\sqrt{a^2}} - \frac{2\,e^{x}}{a\,\left(2\,e^{2x}+e^{4x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^4*(a + a/cosh(x))),x)
```

```
[Out] 2/(a*(exp(x) + 1)) + (2/a + exp(x)/a)/(exp(2*x) + 1) + (3*atan((exp(x)*(a^2
)^(1/2))/a))/(a^2)^(1/2) - (2*exp(x))/(a*(2*exp(2*x) + exp(4*x) + 1))
```

3.76 $\int \frac{1}{a+a\text{sech}(c+dx)}\,dx$

Optimal. Leaf size=29

$$\frac{x}{a}-\frac{\tanh(c+dx)}{d(a+a\text{sech}(c+dx))}$$

```
[Out] x/a-tanh(d*x+c)/d/(a+a*sech(d*x+c))
```

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3862, 8}

$$\frac{x}{a}-\frac{\tanh(c+dx)}{d(a\text{sech}(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sech[c + d*x])^(-1),x]
```

```
[Out] x/a - Tanh[c + d*x]/(d*(a + a*Sech[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+a\text{sech}(c+dx)}\,dx &= -\frac{\tanh(c+dx)}{d(a+a\text{sech}(c+dx))}+\frac{\int a\,dx}{a^2}\\ &= \frac{x}{a}-\frac{\tanh(c+dx)}{d(a+a\text{sech}(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 58, normalized size = 2.00

$$\frac{\text{sech}\left(\frac{c}{2}\right)\text{sech}\left(\frac{1}{2}(c+dx)\right)\left(dx\cosh\left(\frac{dx}{2}\right)+dx\cosh\left(c+\frac{dx}{2}\right)-2\sinh\left(\frac{dx}{2}\right)\right)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sech[c + d*x])^(-1),x]
```

```
[Out] (Sech[c/2]*Sech[(c + d*x)/2]*(d*x*Cosh[(d*x)/2] + d*x*Cosh[c + (d*x)/2] - 2
*Sinh[(d*x)/2]))/(2*a*d)
```

Maple [A]
time = 2.16, size = 46, normalized size = 1.59

method	result	size
risch	$\frac{x}{a}+\frac{2}{ad\left(e^{dx+c}+1\right)}$	25
derivativedivides	$\frac{-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}$	46
default	$\frac{-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sech(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-tanh(1/2*d*x+1/2*c)-ln(tanh(1/2*d*x+1/2*c)-1)+ln(tanh(1/2*d*x+1/2*c
)+1))
```

Maxima [A]
time = 0.29, size = 33, normalized size = 1.14

$$\frac{dx+c}{ad}-\frac{2}{\left(ae^{(-dx-c)}+a\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sech(d*x+c)),x, algorithm="maxima")
```

```
[Out] (d*x + c)/(a*d) - 2/((a*e^(-d*x - c) + a)*d)
```

Fricas [A]
time = 0.36, size = 48, normalized size = 1.66

$$\frac{dx\cosh\left(dx+c\right)+dx\sinh\left(dx+c\right)+dx+2}{ad\cosh\left(dx+c\right)+ad\sinh\left(dx+c\right)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sech(d*x+c)),x, algorithm="fricas")
```

```
[Out] (d*x*cosh(d*x + c) + d*x*sinh(d*x + c) + d*x + 2)/(a*d*cosh(d*x + c) + a*d*
sinh(d*x + c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\operatorname{sech}(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sech(d*x+c)),x)
[Out] Integral(1/(sech(c + d*x) + 1), x)/a
```

Giac [A]
time = 0.38, size = 29, normalized size = 1.00

$$\frac{\frac{dx+c}{a} + \frac{2}{a\left(e^{(dx+c)}+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sech(d*x+c)),x, algorithm="giac")
[Out] ((d*x + c)/a + 2/(a*(e^(d*x + c) + 1)))/d
```

Mupad [B]
time = 1.30, size = 24, normalized size = 0.83

$$\frac{x}{a} + \frac{2}{a\,d\,\left(e^{c+d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a/cosh(c + d*x)),x)
[Out] x/a + 2/(a*d*(exp(c + d*x) + 1))
```

3.77 $\int \frac{1}{a-a\text{sech}(c+dx)} dx$

Optimal. Leaf size=30

$$\frac{x}{a} - \frac{\tanh(c+dx)}{d(a - a\text{sech}(c+dx))}$$

```
[Out] x/a-tanh(d*x+c)/d/(a-a*sech(d*x+c))
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3862, 8}

$$\frac{x}{a} - \frac{\tanh(c+dx)}{d(a - a\text{sech}(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sech[c + d*x])^(-1),x]
```

```
[Out] x/a - Tanh[c + d*x]/(d*(a - a*Sech[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{a - a\text{sech}(c+dx)} \, dx &= -\frac{\tanh(c+dx)}{d(a - a\text{sech}(c+dx))} + \frac{\int a \, dx}{a^2} \\ &= \frac{x}{a} - \frac{\tanh(c+dx)}{d(a - a\text{sech}(c+dx))} \end{aligned}$$

Mathematica [A]
time = 0.10, size = 59, normalized size = 1.97

$$\frac{\text{csch}\left(\frac{c}{2}\right) \text{csch}\left(\frac{1}{2}(c+dx)\right) \left(-dx \cosh\left(\frac{dx}{2}\right) + dx \cosh\left(c + \frac{dx}{2}\right) + 2\sinh\left(\frac{dx}{2}\right)\right)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sech[c + d*x])^(-1),x]
```

```
[Out] (Csch[c/2]*Csch[(c + d*x)/2]*(-(d*x*Cosh[(d*x)/2]) + d*x*Cosh[c + (d*x)/2]
+ 2*Sinh[(d*x)/2]))/(2*a*d)
```

Maple [A]
time = 1.72, size = 48, normalized size = 1.60

method	result	size
risch	$\frac{x}{a}-\frac{2}{ad\left(\mathrm{e}^{dx+c}-1\right)}$	25
derivativedivides	$\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{1}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}}{da}$	48
default	$\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{1}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}}{da}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sech(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(ln(tanh(1/2*d*x+1/2*c)+1)-ln(tanh(1/2*d*x+1/2*c)-1)-1/tanh(1/2*d*x+1
/2*c))
```

Maxima [A]
time = 0.26, size = 35, normalized size = 1.17

$$\frac{dx+c}{ad}+\frac{2}{\left(ae^{(-dx-c)}-a\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sech(d*x+c)),x, algorithm="maxima")
```

```
[Out] (d*x + c)/(a*d) + 2/((a*e^(-d*x - c) - a)*d)
```

Fricas [A]
time = 0.42, size = 50, normalized size = 1.67

$$\frac{dx\cosh\left(dx+c\right)+dx\sinh\left(dx+c\right)-dx-2}{ad\cosh\left(dx+c\right)+ad\sinh\left(dx+c\right)-ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sech(d*x+c)),x, algorithm="fricas")
```

```
[Out] (d*x*cosh(d*x + c) + d*x*sinh(d*x + c) - d*x - 2)/(a*d*cosh(d*x + c) + a*d*
sinh(d*x + c) - a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{\operatorname{sech}(c+dx)-1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sech(d*x+c)),x)
```

```
[Out] -Integral(1/(sech(c + d*x) - 1), x)/a
```

Giac [A]
time = 0.38, size = 29, normalized size = 0.97

$$\frac{\frac{dx+c}{a} - \frac{2}{a\left(e^{(dx+c)}-1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sech(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)/a - 2/(a*(e^(d*x + c) - 1)))/d
```

Mupad [B]
time = 1.26, size = 24, normalized size = 0.80

$$\frac{x}{a} - \frac{2}{a\,d\,\left(e^{c+d\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - a/cosh(c + d*x)),x)
```

```
[Out] x/a - 2/(a*d*(exp(c + d*x) - 1))
```

3.78 $\int (a+a\text{sech}(c+dx))^{5/2}\,dx$

Optimal. Leaf size=98

$$\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a+a\text{sech}(c+dx)}}\right)}{d}+\frac{14a^3\tanh(c+dx)}{3d\sqrt{a+a\text{sech}(c+dx)}}+\frac{2a^2\sqrt{a+a\text{sech}(c+dx)}\tanh(c+dx)}{3d}$$

```
[Out] 2*a^(5/2)*arctanh(a^(1/2)*tanh(d*x+c)/(a+a*sech(d*x+c))^(1/2))/d+14/3*a^3*t
anh(d*x+c)/d/(a+a*sech(d*x+c))^(1/2)+2/3*a^2*(a+a*sech(d*x+c))^(1/2)*tanh(d
*x+c)/d
```

Rubi [A]
time = 0.09, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3860, 4000, 3859, 209, 3877}

$$\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a\text{sech}(c+dx)+a}}\right)}{d}+\frac{14a^3\tanh(c+dx)}{3d\sqrt{a\text{sech}(c+dx)+a}}+\frac{2a^2\tanh(c+dx)\sqrt{a\text{sech}(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sech[c + d*x])^(5/2),x]
```

```
[Out] (2*a^(5/2)*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a + a*Sech[c + d*x]]])/d +
(14*a^3*Tanh[c + d*x])/(3*d*Sqrt[a + a*Sech[c + d*x]]) + (2*a^2*Sqrt[a + a*
Sech[c + d*x]]*Tanh[c + d*x])/(3*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3860

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Dist[a/(n - 1)
, Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Integ
```

```
erQ[2*n]
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4000

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dis
t[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+a\operatorname{sech}(c+dx))^{5/2}\,dx = \frac{2a^2\sqrt{a+a\operatorname{sech}(c+dx)}\,\tanh(c+dx)}{3d}+\frac{1}{3}(2a)\int\sqrt{a+a\operatorname{sech}(c+dx)}\left(\frac{3a}{2}\right.$$

$$=\frac{2a^2\sqrt{a+a\operatorname{sech}(c+dx)}\,\tanh(c+dx)}{3d}+a^2\int\sqrt{a+a\operatorname{sech}(c+dx)}\,dx+\frac{1}{3}(7$$

$$=\frac{14a^3\tanh(c+dx)}{3d\sqrt{a+a\operatorname{sech}(c+dx)}}+\frac{2a^2\sqrt{a+a\operatorname{sech}(c+dx)}\,\tanh(c+dx)}{3d}+\frac{(2ia^3)\,\mathrm{Su}}{}$$

$$=\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(c+dx)}{\sqrt{a+a\operatorname{sech}(c+dx)}}\right)}{d}+\frac{14a^3\tanh(c+dx)}{3d\sqrt{a+a\operatorname{sech}(c+dx)}}+\frac{2a^2\sqrt{a+}}{}$$

Mathematica [A]
time = 0.23, size = 99, normalized size = 1.01

$$\frac{a^2\operatorname{sech}\left(\frac{1}{2}(c+dx)\right)\operatorname{sech}(c+dx)\sqrt{a(1+\operatorname{sech}(c+dx))}\left(3\sqrt{2}\,\sinh^{-1}\left(\sqrt{2}\,\sinh\left(\frac{1}{2}(c+dx)\right)\right)\cosh^{\frac{3}{2}}(c+dx)-6\sinh\left(\frac{1}{2}(c+dx)\right)+8\sinh\left(\frac{3}{2}(c+dx)\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sech[c + d*x])^(5/2),x]
```

```
[Out] (a^2*Sech[(c + d*x)/2]*Sech[c + d*x]*Sqrt[a*(1 + Sech[c + d*x])]*(3*Sqrt[2]
*ArcSinh[Sqrt[2]*Sinh[(c + d*x)/2]]*Cosh[c + d*x]^(3/2) - 6*Sinh[(c + d*x)/
2] + 8*Sinh[(3*(c + d*x))/2]))/(3*d)
```

Maple [F]
time = 2.74, size = 0, normalized size = 0.00

$$\int (a + a\,\mathrm{sech}(dx + c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sech(d*x+c))^(5/2),x)
```

```
[Out] int((a+a*sech(d*x+c))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sech(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sech(d*x + c) + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 924 vs. 2(84) = 168.
time = 0.53, size = 924, normalized size = 9.43

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sech(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*(a^2*cosh(d*x + c)^2 + 2*a^2*cosh(d*x + c)*sinh(d*x + c) + a^2*sinh(
d*x + c)^2 + a^2)*sqrt(a)*log(-(a*cosh(d*x + c)^4 + a*sinh(d*x + c)^4 - 3*a
*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) - 3*a)*sinh(d*x + c)^3 + 5*a*cosh(d*x
 + c)^2 + (6*a*cosh(d*x + c)^2 - 9*a*cosh(d*x + c) + 5*a)*sinh(d*x + c)^2 +
 (cosh(d*x + c)^5 + (5*cosh(d*x + c) - 3)*sinh(d*x + c)^4 + sinh(d*x + c)^5
 - 3*cosh(d*x + c)^4 + (10*cosh(d*x + c)^2 - 12*cosh(d*x + c) + 5)*sinh(d*x
 + c)^3 + 5*cosh(d*x + c)^3 + (10*cosh(d*x + c)^3 - 18*cosh(d*x + c)^2 + 15
*cosh(d*x + c) - 7)*sinh(d*x + c)^2 - 7*cosh(d*x + c)^2 + (5*cosh(d*x + c)^
4 - 12*cosh(d*x + c)^3 + 15*cosh(d*x + c)^2 - 14*cosh(d*x + c) + 4)*sinh(d*
x + c) + 4*cosh(d*x + c) - 4)*sqrt(a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x
+ c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) - 4*a*cosh(d*x + c) + (4*a*cosh(
d*x + c)^3 - 9*a*cosh(d*x + c)^2 + 10*a*cosh(d*x + c) - 4*a)*sinh(d*x + c)
+ 4*a)/(cosh(d*x + c)^3 + 3*cosh(d*x + c)^2*sinh(d*x + c) + 3*cosh(d*x + c)
*sinh(d*x + c)^2 + sinh(d*x + c)^3)) + 3*(a^2*cosh(d*x + c)^2 + 2*a^2*cosh(
d*x + c)*sinh(d*x + c) + a^2*sinh(d*x + c)^2 + a^2)*sqrt(a)*log((a*cosh(d*x
 + c)^2 + a*sinh(d*x + c)^2 + (cosh(d*x + c)^3 + (3*cosh(d*x + c) + 1)*sinh
```

```
(d*x + c)^2 + sinh(d*x + c)^3 + cosh(d*x + c)^2 + (3*cosh(d*x + c)^2 + 2*co
sh(d*x + c) + 1)*sinh(d*x + c) + cosh(d*x + c) + 1)*sqrt(a)*sqrt(a/(cosh(d*
x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) + a*cosh(d
*x + c) + (2*a*cosh(d*x + c) + a)*sinh(d*x + c) + a)/(cosh(d*x + c) + sinh(
d*x + c))) + 8*(4*a^2*cosh(d*x + c)^3 + 4*a^2*sinh(d*x + c)^3 - 3*a^2*cosh(
d*x + c)^2 + 3*a^2*cosh(d*x + c) + 3*(4*a^2*cosh(d*x + c) - a^2)*sinh(d*x +
 c)^2 - 4*a^2 + 3*(4*a^2*cosh(d*x + c)^2 - 2*a^2*cosh(d*x + c) + a^2)*sinh(
d*x + c))*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*
x + c)^2 + 1)))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*si
nh(d*x + c)^2 + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a\,\mathrm{sech}\,(c+dx)+a)^{\frac{5}{2}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sech(d*x+c))**(5/2),x)
```

```
[Out] Integral((a*sech(c + d*x) + a)**(5/2), x)
```

Giac [A]
time = 0.46, size = 151, normalized size = 1.54

$$\frac{\frac{6\,a^{3}\arctan\left(-\frac{\sqrt{a}\,e^{(dx+c)}-\sqrt{ae^{(2\,dx+2\,c)}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-3\,a^{\frac{5}{2}}\log\left(\left|-\sqrt{a}\,e^{(dx+c)}+\sqrt{ae^{(2\,dx+2\,c)}+a}\right|\right)-\frac{4\,\left(4\,a^{4}-\left(3\,a^{4}e^{c}+\left(4\,a^{4}e^{(dx+3\,c)}-3\,a^{4}e^{(2\,c)}\right)e^{(dx)}\right)e^{(dx)}\right)}{\left(ae^{(2\,dx+2\,c)}+a\right)^{\frac{3}{2}}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sech(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(6*a^3*arctan(-(sqrt(a)*e^(d*x + c) - sqrt(a*e^(2*d*x + 2*c) + a))/sqrt
(-a))/sqrt(-a) - 3*a^(5/2)*log(abs(-sqrt(a)*e^(d*x + c) + sqrt(a*e^(2*d*x +
 2*c) + a))) - 4*(4*a^4 - (3*a^4*e^c + (4*a^4*e^(d*x + 3*c) - 3*a^4*e^(2*c)
)*e^(d*x))*e^(d*x))/(a*e^(2*d*x + 2*c) + a)^(3/2))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+\frac{a}{\cosh\left(c+d\,x\right)}\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cosh(c + d*x))^(5/2),x)
```

```
[Out] int((a + a/cosh(c + d*x))^(5/2), x)
```

3.79 $\int (a + a\,\textbf{sech}(c + dx))^{3/2}\,dx$

Optimal. Leaf size=66

$$\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(c+dx)}{\sqrt{a+a\,\text{sech}(c+dx)}}\right)}{d}+\frac{2a^2\tanh(c+dx)}{d\sqrt{a+a\,\text{sech}(c+dx)}}$$

```
[Out] 2*a^(3/2)*arctanh(a^(1/2)*tanh(d*x+c)/(a+a*sech(d*x+c))^(1/2))/d+2*a^2*tanh
(d*x+c)/d/(a+a*sech(d*x+c))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3860, 21, 3859, 209}

$$\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(c+dx)}{\sqrt{a\,\text{sech}(c+dx)+a}}\right)}{d}+\frac{2a^2\tanh(c+dx)}{d\sqrt{a\,\text{sech}(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sech[c + d*x])^(3/2),x]
```

```
[Out] (2*a^(3/2)*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a + a*Sech[c + d*x]]])/d +
(2*a^2*Tanh[c + d*x])/(d*Sqrt[a + a*Sech[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3860

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Dist[a/(n - 1)
, Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Integ
erQ[2*n]
```

Rubi steps

$$\begin{aligned}
\int (a+a\text{sech}(c+dx))^{3/2}\,dx &= \frac{2a^2\tanh(c+dx)}{d\sqrt{a+a\text{sech}(c+dx)}}+(2a)\int \frac{\frac{a}{2}+\frac{1}{2}a\text{sech}(c+dx)}{\sqrt{a+a\text{sech}(c+dx)}}\,dx\\
&= \frac{2a^2\tanh(c+dx)}{d\sqrt{a+a\text{sech}(c+dx)}}+a\int \sqrt{a+a\text{sech}(c+dx)}\,dx\\
&= \frac{2a^2\tanh(c+dx)}{d\sqrt{a+a\text{sech}(c+dx)}}+\frac{(2ia^2)\,\text{Subst}\left(\int \frac{1}{a+x^2}\,dx,x,-\frac{ia\tanh(c+dx)}{\sqrt{a+a\text{sech}(c+dx)}}\right)}{d}\\
&= \frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(c+dx)}{\sqrt{a+a\text{sech}(c+dx)}}\right)}{d}+\frac{2a^2\tanh(c+dx)}{d\sqrt{a+a\text{sech}(c+dx)}}
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 75, normalized size = 1.14

$$\frac{a\text{sech}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\text{sech}(c+dx))}\left(\sqrt{2}\,\sinh^{-1}\left(\sqrt{2}\,\sinh\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cosh(c+dx)}+2\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sech[c + d*x])^(3/2),x]
[Out] (a*Sech[(c + d*x)/2]*Sqrt[a*(1 + Sech[c + d*x])]*(Sqrt[2]*ArcSinh[Sqrt[2]*S
inh[(c + d*x)/2]]*Sqrt[Cosh[c + d*x]] + 2*Sinh[(c + d*x)/2]))/d
```

Maple [F]
time = 2.52, size = 0, normalized size = 0.00

$$\int (a+a\,\text{sech}(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sech(d*x+c))^(3/2),x)
[Out] int((a+a*sech(d*x+c))^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sech(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sech(d*x + c) + a)^(3/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 697 vs. 2(58) = 116.
time = 0.49, size = 697, normalized size = 10.56

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(a^(3/2)*log(-(a*cosh(d*x + c)^4 + a*sinh(d*x + c)^4 - 3*a*cosh(d*x + c
)^3 + (4*a*cosh(d*x + c) - 3*a)*sinh(d*x + c)^3 + 5*a*cosh(d*x + c)^2 + (6*
a*cosh(d*x + c)^2 - 9*a*cosh(d*x + c) + 5*a)*sinh(d*x + c)^2 + (cosh(d*x +
c)^5 + (5*cosh(d*x + c) - 3)*sinh(d*x + c)^4 + sinh(d*x + c)^5 - 3*cosh(d*x
 + c)^4 + (10*cosh(d*x + c)^2 - 12*cosh(d*x + c) + 5)*sinh(d*x + c)^3 + 5*c
osh(d*x + c)^3 + (10*cosh(d*x + c)^3 - 18*cosh(d*x + c)^2 + 15*cosh(d*x + c
) - 7)*sinh(d*x + c)^2 - 7*cosh(d*x + c)^2 + (5*cosh(d*x + c)^4 - 12*cosh(d
*x + c)^3 + 15*cosh(d*x + c)^2 - 14*cosh(d*x + c) + 4)*sinh(d*x + c) + 4*co
sh(d*x + c) - 4)*sqrt(a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x
 + c) + sinh(d*x + c)^2 + 1)) - 4*a*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 -
9*a*cosh(d*x + c)^2 + 10*a*cosh(d*x + c) - 4*a)*sinh(d*x + c) + 4*a)/(cosh(
d*x + c)^3 + 3*cosh(d*x + c)^2*sinh(d*x + c) + 3*cosh(d*x + c)*sinh(d*x + c
)^2 + sinh(d*x + c)^3)) + a^(3/2)*log((a*cosh(d*x + c)^2 + a*sinh(d*x + c)^
2 + (cosh(d*x + c)^3 + (3*cosh(d*x + c) + 1)*sinh(d*x + c)^2 + sinh(d*x + c
)^3 + cosh(d*x + c)^2 + (3*cosh(d*x + c)^2 + 2*cosh(d*x + c) + 1)*sinh(d*x
+ c) + cosh(d*x + c) + 1)*sqrt(a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)
*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) + a*cosh(d*x + c) + (2*a*cosh(d*x +
c) + a)*sinh(d*x + c) + a)/(cosh(d*x + c) + sinh(d*x + c))) + 4*(a*cosh(d*x
 + c) + a*sinh(d*x + c) - a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh
(d*x + c) + sinh(d*x + c)^2 + 1)))/d
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a\,\mathrm{sech}\,(c+dx)+a)^{\frac{3}{2}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sech(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*sech(c + d*x) + a)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(58) = 116.
time = 0.45, size = 118, normalized size = 1.79

$$\frac{\frac{2\,a^{2}\arctan\left(-\frac{\sqrt{a}\,e^{(dx+c)}-\sqrt{ae^{(2\,dx+2\,c)}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-a^{\frac{3}{2}}\log\left(\left|-\sqrt{a}\,e^{(dx+c)}+\sqrt{ae^{(2\,dx+2\,c)}+a}\right|\right)+\frac{2\,(a^{2}e^{(dx+c)}-a^{2})}{\sqrt{ae^{(2\,dx+2\,c)}+a}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sech(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] (2*a^2*arctan(-(sqrt(a)*e^(d*x + c) - sqrt(a*e^(2*d*x + 2*c) + a))/sqrt(-a)
)/sqrt(-a) - a^(3/2)*log(abs(-sqrt(a)*e^(d*x + c) + sqrt(a*e^(2*d*x + 2*c)
+ a))) + 2*(a^2*e^(d*x + c) - a^2)/sqrt(a*e^(2*d*x + 2*c) + a))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \left(a+\frac{a}{\cosh\left(c+d\,x\right)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cosh(c + d*x))^(3/2),x)
```

```
[Out] int((a + a/cosh(c + d*x))^(3/2), x)
```

3.80 $\int \sqrt{a+a\,\mathrm{sech}(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(c+dx)}{\sqrt{a+a\,\mathrm{sech}(c+dx)}}\right)}{d}$$

```
[Out] 2*arctanh(a^(1/2)*tanh(d*x+c)/(a+a*sech(d*x+c))^(1/2))*a^(1/2)/d
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3859, 209}

$$\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(c+dx)}{\sqrt{a\,\mathrm{sech}(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sech[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a + a*Sech[c + d*x]]])/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+a\,\mathrm{sech}(c+dx)}\,dx &= \frac{(2ia)\mathrm{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{ia\tanh(c+dx)}{\sqrt{a+a\,\mathrm{sech}(c+dx)}}\right)}{d}\\ &= \frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(c+dx)}{\sqrt{a+a\,\mathrm{sech}(c+dx)}}\right)}{d}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 60, normalized size = 1.62

$$\frac{\sqrt{2}\,\sinh^{-1}\left(\sqrt{2}\,\sinh\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cosh(c+dx)}\,\mathrm{sech}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\mathrm{sech}(c+dx))}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sech[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcSinh[Sqrt[2]*Sinh[(c + d*x)/2]]*Sqrt[Cosh[c + d*x]]*Sech[(c + d
*x)/2]*Sqrt[a*(1 + Sech[c + d*x])])/d
```

Maple [F]
time = 2.96, size = 0, normalized size = 0.00

$$\int \sqrt{a+a\,\mathrm{sech}\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sech(d*x+c))^(1/2),x)
```

```
[Out] int((a+a*sech(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sech(d*x + c) + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 637 vs. 2(31) = 62.
time = 0.49, size = 637, normalized size = 17.22

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a)*log(-(a*cosh(d*x + c)^4 + a*sinh(d*x + c)^4 - 3*a*cosh(d*x + c
)^3 + (4*a*cosh(d*x + c) - 3*a)*sinh(d*x + c)^3 + 5*a*cosh(d*x + c)^2 + (6*
a*cosh(d*x + c)^2 - 9*a*cosh(d*x + c) + 5*a)*sinh(d*x + c)^2 + (cosh(d*x +
c)^5 + (5*cosh(d*x + c) - 3)*sinh(d*x + c)^4 + sinh(d*x + c)^5 - 3*cosh(d*x
```

```
 + c)^4 + (10*cosh(d*x + c)^2 - 12*cosh(d*x + c) + 5)*sinh(d*x + c)^3 + 5*c
osh(d*x + c)^3 + (10*cosh(d*x + c)^3 - 18*cosh(d*x + c)^2 + 15*cosh(d*x + c
) - 7)*sinh(d*x + c)^2 - 7*cosh(d*x + c)^2 + (5*cosh(d*x + c)^4 - 12*cosh(d
*x + c)^3 + 15*cosh(d*x + c)^2 - 14*cosh(d*x + c) + 4)*sinh(d*x + c) + 4*co
sh(d*x + c) - 4)*sqrt(a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x
 + c) + sinh(d*x + c)^2 + 1)) - 4*a*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 -
9*a*cosh(d*x + c)^2 + 10*a*cosh(d*x + c) - 4*a)*sinh(d*x + c) + 4*a)/(cosh(
d*x + c)^3 + 3*cosh(d*x + c)^2*sinh(d*x + c) + 3*cosh(d*x + c)*sinh(d*x + c
)^2 + sinh(d*x + c)^3)) + sqrt(a)*log((a*cosh(d*x + c)^2 + a*sinh(d*x + c)^
2 + (cosh(d*x + c)^3 + (3*cosh(d*x + c) + 1)*sinh(d*x + c)^2 + sinh(d*x + c
)^3 + cosh(d*x + c)^2 + (3*cosh(d*x + c)^2 + 2*cosh(d*x + c) + 1)*sinh(d*x
+ c) + cosh(d*x + c) + 1)*sqrt(a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)
*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) + a*cosh(d*x + c) + (2*a*cosh(d*x +
c) + a)*sinh(d*x + c) + a)/(cosh(d*x + c) + sinh(d*x + c))))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\,\mathrm{sech}\,(c+dx)+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sech(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*sech(c + d*x) + a), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(31) = 62.
time = 0.43, size = 83, normalized size = 2.24

$$\frac{\frac{2\,a\arctan\left(-\frac{\sqrt{a}\,e^{(dx+c)}-\sqrt{ae^{(2\,dx+2\,c)}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\sqrt{a}\,\log\left(\left|-\sqrt{a}\,e^{(dx+c)}+\sqrt{ae^{(2\,dx+2\,c)}+a}\right|\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] (2*a*arctan(-(sqrt(a)*e^(d*x + c) - sqrt(a*e^(2*d*x + 2*c) + a))/sqrt(-a))/
sqrt(-a) - sqrt(a)*log(abs(-sqrt(a)*e^(d*x + c) + sqrt(a*e^(2*d*x + 2*c) +
a))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{a+\frac{a}{\cosh\left(c+d\,x\right)}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cosh(c + d*x))^(1/2),x)
[Out] int((a + a/cosh(c + d*x))^(1/2), x)
```

3.81 $\int \frac{1}{\sqrt{a+a\mathbf{sech}(c+dx)}}\,dx$

Optimal. Leaf size=85

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a+a\operatorname{sech}(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{2}\sqrt{a+a\operatorname{sech}(c+dx)}}\right)}{\sqrt{a}\,d}$$

```
[Out] 2*arctanh(a^(1/2)*tanh(d*x+c)/(a+a*sech(d*x+c))^(1/2))/d/a^(1/2)-arctanh(1/
2*a^(1/2)*tanh(d*x+c)*2^(1/2)/(a+a*sech(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3861, 3859, 209, 3880}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a\operatorname{sech}(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{2}\sqrt{a\operatorname{sech}(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + a*Sech[c + d*x]],x]
```

```
[Out] (2*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a + a*Sech[c + d*x]]])/(Sqrt[a]*d)
- (Sqrt[2]*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sech[c + d*x
]])])/(Sqrt[a]*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3861

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[1/a, In
t[Sqrt[a + b*Csc[c + d*x]], x], x] - Dist[b/a, Int[Csc[c + d*x]/Sqrt[a + b*
Csc[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+a\text{sech}(c+dx)}}\,dx &= \frac{\int \sqrt{a+a\text{sech}(c+dx)}\,dx}{a}-\int \frac{\text{sech}(c+dx)}{\sqrt{a+a\text{sech}(c+dx)}}\,dx\\ &= \frac{(2i)\text{Subst}\left(\int \frac{1}{a+x^2}\,dx,x,-\frac{ia\tanh(c+dx)}{\sqrt{a+a\text{sech}(c+dx)}}\right)}{d}-\frac{(2i)\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx,x,\right.}{}\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a+a\text{sech}(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{2}\sqrt{a+a\text{sech}(c+dx)}}\right)}{\sqrt{a}\,d}\end{aligned}$$

Mathematica [A]
time = 0.83, size = 118, normalized size = 1.39

$$\frac{\left(1+e^{c+dx}\right)\left(\sqrt{2}\sinh^{-1}\left(e^{c+dx}\right)-2\tanh^{-1}\left(\frac{-1+e^{c+dx}}{\sqrt{2}\sqrt{1+e^{2(c+dx)}}}\right)-\sqrt{2}\tanh^{-1}\left(\sqrt{1+e^{2(c+dx)}}\right)\right)}{\sqrt{2}\,d\sqrt{1+e^{2(c+dx)}}\sqrt{a(1+\text{sech}(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + a*Sech[c + d*x]],x]
[Out] ((1 + E^(c + d*x))*(Sqrt[2]*ArcSinh[E^(c + d*x)] - 2*ArcTanh[(-1 + E^(c + d
*x))/(Sqrt[2]*Sqrt[1 + E^(2*(c + d*x))])] - Sqrt[2]*ArcTanh[Sqrt[1 + E^(2*(
c + d*x))]]))/(Sqrt[2]*d*Sqrt[1 + E^(2*(c + d*x))]*Sqrt[a*(1 + Sech[c + d*x
])])
```

Maple [F]
time = 2.63, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+a\,\text{sech}\,(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sech(d*x+c))^(1/2),x)
[Out] int(1/(a+a*sech(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*sech(d*x + c) + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 868 vs. $2(70) = 140$.
time = 0.41, size = 868, normalized size = 10.21

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*sqrt(a)*log(-(3*cosh(d*x + c)^2 + 2*(3*cosh(d*x + c) - 1)*sinh
(d*x + c) + 3*sinh(d*x + c)^2 - 2*sqrt(2)*(cosh(d*x + c)^3 + (3*cosh(d*x +
c) - 1)*sinh(d*x + c)^2 + sinh(d*x + c)^3 - cosh(d*x + c)^2 + (3*cosh(d*x +
 c)^2 - 2*cosh(d*x + c) + 1)*sinh(d*x + c) + cosh(d*x + c) - 1)*sqrt(a/(cos
h(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1))/sqrt(a
) - 2*cosh(d*x + c) + 3)/(cosh(d*x + c)^2 + 2*(cosh(d*x + c) + 1)*sinh(d*x
+ c) + sinh(d*x + c)^2 + 2*cosh(d*x + c) + 1)) + sqrt(a)*log(-(a*cosh(d*x +
 c)^4 + a*sinh(d*x + c)^4 - 3*a*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) - 3*a)
*sinh(d*x + c)^3 + 5*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 - 9*a*cosh(d*
x + c) + 5*a)*sinh(d*x + c)^2 + (cosh(d*x + c)^5 + (5*cosh(d*x + c) - 3)*si
nh(d*x + c)^4 + sinh(d*x + c)^5 - 3*cosh(d*x + c)^4 + (10*cosh(d*x + c)^2 -
 12*cosh(d*x + c) + 5)*sinh(d*x + c)^3 + 5*cosh(d*x + c)^3 + (10*cosh(d*x +
 c)^3 - 18*cosh(d*x + c)^2 + 15*cosh(d*x + c) - 7)*sinh(d*x + c)^2 - 7*cosh
(d*x + c)^2 + (5*cosh(d*x + c)^4 - 12*cosh(d*x + c)^3 + 15*cosh(d*x + c)^2
- 14*cosh(d*x + c) + 4)*sinh(d*x + c) + 4*cosh(d*x + c) - 4)*sqrt(a)*sqrt(a
/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) -
 4*a*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 - 9*a*cosh(d*x + c)^2 + 10*a*cosh
(d*x + c) - 4*a)*sinh(d*x + c) + 4*a)/(cosh(d*x + c)^3 + 3*cosh(d*x + c)^2*
sinh(d*x + c) + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3)) + sqrt(
a)*log((a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + (cosh(d*x + c)^3 + (3*cosh(
d*x + c) + 1)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + cosh(d*x + c)^2 + (3*cosh
(d*x + c)^2 + 2*cosh(d*x + c) + 1)*sinh(d*x + c) + cosh(d*x + c) + 1)*sqrt(
a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^
2 + 1)) + a*cosh(d*x + c) + (2*a*cosh(d*x + c) + a)*sinh(d*x + c) + a)/(cos
h(d*x + c) + sinh(d*x + c))))/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\,\mathrm{sech}\,(c+dx)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sech(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*sech(c + d*x) + a), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+\frac{a}{\cosh\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a/cosh(c + d*x))^(1/2),x)
```

```
[Out] int(1/(a + a/cosh(c + d*x))^(1/2), x)
```

3.82 $\int \frac{1}{(a+a\,\mathbf{sech}(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=114

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(c+dx)}{\sqrt{a+a\text{sech}(c+dx)}}\right)}{a^{3/2}d}-\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(c+dx)}{\sqrt{2}\,\sqrt{a+a\text{sech}(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{\tanh(c+dx)}{2d(a+a\text{sech}(c+dx))^{3/2}}$$

```
[Out] 2*arctanh(a^(1/2)*tanh(d*x+c)/(a+a*sech(d*x+c))^(1/2))/a^(3/2)/d-5/4*arctan
h(1/2*a^(1/2)*tanh(d*x+c)*2^(1/2)/(a+a*sech(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2
)-1/2*tanh(d*x+c)/d/(a+a*sech(d*x+c))^(3/2)
```

Rubi [A]
time = 0.09, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3862, 4005, 3859, 209, 3880}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(c+dx)}{\sqrt{a\text{sech}(c+dx)+a}}\right)}{a^{3/2}d}-\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(c+dx)}{\sqrt{2}\,\sqrt{a\text{sech}(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{\tanh(c+dx)}{2d(a\text{sech}(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sech[c + d*x])^(-3/2),x]
```

```
[Out] (2*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a + a*Sech[c + d*x]]])/(a^(3/2)*d)
- (5*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sech[c + d*x]])])/
(2*Sqrt[2]*a^(3/2)*d) - Tanh[c + d*x]/(2*d*(a + a*Sech[c + d*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
```

```
gerQ[2*n]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\text{sech}(c+dx))^{3/2}}\,dx = -\frac{\tanh(c+dx)}{2d(a+a\text{sech}(c+dx))^{3/2}} - \frac{\int \frac{-2a+\frac{1}{2}a\text{sech}(c+dx)}{\sqrt{a+a\text{sech}(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\tanh(c+dx)}{2d(a+a\text{sech}(c+dx))^{3/2}} + \frac{\int \sqrt{a+a\text{sech}(c+dx)}\,dx}{a^2} - \frac{5\int \frac{\text{sech}(c+}{\sqrt{a+a\text{sech}(}}}{4a}$$

$$= -\frac{\tanh(c+dx)}{2d(a+a\text{sech}(c+dx))^{3/2}} + \frac{(2i)\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{ia\tanh(c+dx)}{\sqrt{a+a\text{sech}(c+dx)}}\right.}{ad}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a+a\text{sech}(c+dx)}}\right)}{a^{3/2}d} - \frac{5\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{2}\sqrt{a+a\text{sech}(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 231 vs. 2(114) = 228.
time = 1.15, size = 231, normalized size = 2.03

$$\frac{\cosh^3\left(\frac{1}{2}(c+dx)\right)\text{sech}^{\frac{3}{2}}(c+dx)\left(\sqrt{2}\,e^{\frac{1}{2}(-c-dx)}\sqrt{\frac{e^{c+dx}}{1+e^{2(c+dx)}}}\sqrt{1+e^{2(c+dx)}}\left(4c+4dx+4\sinh^{-1}\left(e^{c+dx}\right)-5\sqrt{2}\log\left(1+e^{c+dx}\right)-4\log\left(1+\sqrt{1+e^{2(c+dx)}}\right)+5\sqrt{2}\log\left(1-e^{c+dx}+\sqrt{2}\sqrt{1+e^{2(c+dx)}}\right)\right)-\frac{2\text{sech}\left(\frac{1}{2}(c+dx)\right)\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\text{sech}(c+dx)}}\right)}{2d(a(1+\text{sech}(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sech[c + d*x])^(-3/2),x]
```

```
[Out] (Cosh[(c + d*x)/2]^3*Sech[c + d*x]^(3/2)*(Sqrt[2]*E^((-c - d*x)/2)*Sqrt[E^(
c + d*x)/(1 + E^(2*(c + d*x)))]*Sqrt[1 + E^(2*(c + d*x))]*(4*c + 4*d*x + 4*
```

```
ArcSinh[E^(c + d*x)] - 5*Sqrt[2]*Log[1 + E^(c + d*x)] - 4*Log[1 + Sqrt[1 +
E^(2*(c + d*x))]] + 5*Sqrt[2]*Log[1 - E^(c + d*x) + Sqrt[2]*Sqrt[1 + E^(2*(
c + d*x))]]) - (2*Sech[(c + d*x)/2]*Tanh[(c + d*x)/2])/Sqrt[Sech[c + d*x]])
)/(2*d*(a*(1 + Sech[c + d*x]))^(3/2))
```

Maple [F]
time = 2.54, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+a\,\mathrm{sech}\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sech(d*x+c))^(3/2),x)
```

```
[Out] int(1/(a+a*sech(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sech(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sech(d*x + c) + a)^(-3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1190 vs. $2(93) = 186$.
time = 0.44, size = 1190, normalized size = 10.44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(5*sqrt(2)*(cosh(d*x + c)^2 + 2*(cosh(d*x + c) + 1)*sinh(d*x + c) + sin
h(d*x + c)^2 + 2*cosh(d*x + c) + 1)*sqrt(a)*log(-(3*a*cosh(d*x + c)^2 + 3*a
*sinh(d*x + c)^2 - 2*sqrt(2)*(cosh(d*x + c)^3 + (3*cosh(d*x + c) - 1)*sinh(
d*x + c)^2 + sinh(d*x + c)^3 - cosh(d*x + c)^2 + (3*cosh(d*x + c)^2 - 2*cos
h(d*x + c) + 1)*sinh(d*x + c) + cosh(d*x + c) - 1)*sqrt(a)*sqrt(a/(cosh(d*x
 + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) - 2*a*cosh(
d*x + c) + 2*(3*a*cosh(d*x + c) - a)*sinh(d*x + c) + 3*a)/(cosh(d*x + c)^2
+ 2*(cosh(d*x + c) + 1)*sinh(d*x + c) + sinh(d*x + c)^2 + 2*cosh(d*x + c) +
 1)) + 4*(cosh(d*x + c)^2 + 2*(cosh(d*x + c) + 1)*sinh(d*x + c) + sinh(d*x
+ c)^2 + 2*cosh(d*x + c) + 1)*sqrt(a)*log(-(a*cosh(d*x + c)^4 + a*sinh(d*x
+ c)^4 - 3*a*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) - 3*a)*sinh(d*x + c)^3 +
```

```
5*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 - 9*a*cosh(d*x + c) + 5*a)*sinh(
d*x + c)^2 + (cosh(d*x + c)^5 + (5*cosh(d*x + c) - 3)*sinh(d*x + c)^4 + sin
h(d*x + c)^5 - 3*cosh(d*x + c)^4 + (10*cosh(d*x + c)^2 - 12*cosh(d*x + c) +
 5)*sinh(d*x + c)^3 + 5*cosh(d*x + c)^3 + (10*cosh(d*x + c)^3 - 18*cosh(d*x
 + c)^2 + 15*cosh(d*x + c) - 7)*sinh(d*x + c)^2 - 7*cosh(d*x + c)^2 + (5*co
sh(d*x + c)^4 - 12*cosh(d*x + c)^3 + 15*cosh(d*x + c)^2 - 14*cosh(d*x + c)
+ 4)*sinh(d*x + c) + 4*cosh(d*x + c) - 4)*sqrt(a)*sqrt(a/(cosh(d*x + c)^2 +
 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) - 4*a*cosh(d*x + c)
+ (4*a*cosh(d*x + c)^3 - 9*a*cosh(d*x + c)^2 + 10*a*cosh(d*x + c) - 4*a)*si
nh(d*x + c) + 4*a)/(cosh(d*x + c)^3 + 3*cosh(d*x + c)^2*sinh(d*x + c) + 3*c
osh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3)) + 4*(cosh(d*x + c)^2 + 2*(
cosh(d*x + c) + 1)*sinh(d*x + c) + sinh(d*x + c)^2 + 2*cosh(d*x + c) + 1)*s
qrt(a)*log((a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + (cosh(d*x + c)^3 + (3*c
osh(d*x + c) + 1)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + cosh(d*x + c)^2 + (3*
cosh(d*x + c)^2 + 2*cosh(d*x + c) + 1)*sinh(d*x + c) + cosh(d*x + c) + 1)*s
qrt(a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
 c)^2 + 1)) + a*cosh(d*x + c) + (2*a*cosh(d*x + c) + a)*sinh(d*x + c) + a)/
(cosh(d*x + c) + sinh(d*x + c))) - 4*(cosh(d*x + c)^3 + (3*cosh(d*x + c) -
1)*sinh(d*x + c)^2 + sinh(d*x + c)^3 - cosh(d*x + c)^2 + (3*cosh(d*x + c)^2
 - 2*cosh(d*x + c) + 1)*sinh(d*x + c) + cosh(d*x + c) - 1)*sqrt(a/(cosh(d*x
 + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)))/(a^2*d*cos
h(d*x + c)^2 + a^2*d*sinh(d*x + c)^2 + 2*a^2*d*cosh(d*x + c) + a^2*d + 2*(a
^2*d*cosh(d*x + c) + a^2*d)*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a \operatorname{sech}(c + dx) + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sech(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*sech(c + d*x) + a)**(-3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(93) = 186.
time = 0.48, size = 316, normalized size = 2.77

$$-\frac{\frac{5\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}e^{(dx+c)}-\sqrt{ae^{(2dx+2c)}+a}+\sqrt{a}\right)}{2\sqrt{-a}}\right)}{\sqrt{-a}\,a}-\frac{4\arctan\left(-\frac{\sqrt{a}e^{(dx+c)}-\sqrt{ae^{(2dx+2c)}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{2\log\left(\left|-\sqrt{a}e^{(dx+c)}+\sqrt{ae^{(2dx+2c)}+a}\right|\right)}{a^{\frac{3}{2}}}+\frac{2\left(3\left(\sqrt{a}e^{(dx+c)}-\sqrt{ae^{(2dx+2c)}+a}\right)^{3}+\left(\sqrt{a}e^{(dx+c)}-\sqrt{ae^{(2dx+2c)}+a}\right)^{2}\sqrt{a}-\left(\sqrt{a}e^{(dx+c)}-\sqrt{ae^{(2dx+2c)}+a}\right)a+a^{\frac{3}{2}}\right)}{\left(\left(\sqrt{a}e^{(dx+c)}-\sqrt{ae^{(2dx+2c)}+a}\right)^{2}+2\left(\sqrt{a}e^{(dx+c)}-\sqrt{ae^{(2dx+2c)}+a}\right)\sqrt{a}-a\right)^{2}a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sech(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(5*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*e^(d*x + c) - sqrt(a*e^(2*d*x
+ 2*c) + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*a) - 4*arctan(-(sqrt(a)*e^(d*x +
 c) - sqrt(a*e^(2*d*x + 2*c) + a))/sqrt(-a))/(sqrt(-a)*a) + 2*log(abs(-sqrt
(a)*e^(d*x + c) + sqrt(a*e^(2*d*x + 2*c) + a)))/a^(3/2) + 2*(3*(sqrt(a)*e^(
d*x + c) - sqrt(a*e^(2*d*x + 2*c) + a))^3 + (sqrt(a)*e^(d*x + c) - sqrt(a*e
^(2*d*x + 2*c) + a))^2*sqrt(a) - (sqrt(a)*e^(d*x + c) - sqrt(a*e^(2*d*x + 2
*c) + a))*a + a^(3/2))/(((sqrt(a)*e^(d*x + c) - sqrt(a*e^(2*d*x + 2*c) + a)
)^2 + 2*(sqrt(a)*e^(d*x + c) - sqrt(a*e^(2*d*x + 2*c) + a))*sqrt(a) - a)^2*
a))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{a}{\cosh(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a/cosh(c + d*x))^(3/2),x)
```

```
[Out] int(1/(a + a/cosh(c + d*x))^(3/2), x)
```

3.83 $\int \sqrt{a - a\,\mathrm{sech}(c + dx)}\, dx$

Optimal. Leaf size=38

$$\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(c+dx)}{\sqrt{a-a\,\mathrm{sech}(c+dx)}}\right)}{d}$$

```
[Out] 2*arctanh(a^(1/2)*tanh(d*x+c)/(a-a*sech(d*x+c))^(1/2))*a^(1/2)/d
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3859, 209}

$$\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(c+dx)}{\sqrt{a-a\,\mathrm{sech}(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Sech[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a - a*Sech[c + d*x]]])/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a-a\,\mathrm{sech}(c+dx)}\,dx &= -\frac{(2ia)\mathrm{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, \frac{ia\tanh(c+dx)}{\sqrt{a-a\,\mathrm{sech}(c+dx)}}\right)}{d}\\ &= \frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(c+dx)}{\sqrt{a-a\,\mathrm{sech}(c+dx)}}\right)}{d}\end{aligned}$$

Mathematica [A]
time = 1.64, size = 70, normalized size = 1.84

$$\frac{\sqrt{1+e^{2(c+dx)}}\left(\sinh^{-1}\left(e^{c+dx}\right)+\tanh^{-1}\left(\sqrt{1+e^{2(c+dx)}}\right)\right)\sqrt{a-a\operatorname{sech}(c+dx)}}{d\left(-1+e^{c+dx}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Sech[c + d*x]],x]
```

```
[Out] (Sqrt[1 + E^(2*(c + d*x))]*(ArcSinh[E^(c + d*x)] + ArcTanh[Sqrt[1 + E^(2*(c
 + d*x))]])*Sqrt[a - a*Sech[c + d*x]])/(d*(-1 + E^(c + d*x)))
```

Maple [F]
time = 2.82, size = 0, normalized size = 0.00

$$\int \sqrt{a-a\,\mathrm{sech}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sech(d*x+c))^(1/2),x)
```

```
[Out] int((a-a*sech(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*sech(d*x + c) + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 642 vs. 2(32) = 64.
time = 0.45, size = 642, normalized size = 16.89

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a)*log((a*cosh(d*x + c)^4 + a*sinh(d*x + c)^4 + 3*a*cosh(d*x + c)
^3 + (4*a*cosh(d*x + c) + 3*a)*sinh(d*x + c)^3 + 5*a*cosh(d*x + c)^2 + (6*a
*cosh(d*x + c)^2 + 9*a*cosh(d*x + c) + 5*a)*sinh(d*x + c)^2 + (cosh(d*x + c
)^5 + (5*cosh(d*x + c) + 3)*sinh(d*x + c)^4 + sinh(d*x + c)^5 + 3*cosh(d*x
```

```
+ c)^4 + (10*cosh(d*x + c)^2 + 12*cosh(d*x + c) + 5)*sinh(d*x + c)^3 + 5*co
sh(d*x + c)^3 + (10*cosh(d*x + c)^3 + 18*cosh(d*x + c)^2 + 15*cosh(d*x + c)
 + 7)*sinh(d*x + c)^2 + 7*cosh(d*x + c)^2 + (5*cosh(d*x + c)^4 + 12*cosh(d*
x + c)^3 + 15*cosh(d*x + c)^2 + 14*cosh(d*x + c) + 4)*sinh(d*x + c) + 4*cos
h(d*x + c) + 4)*sqrt(a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x
+ c) + sinh(d*x + c)^2 + 1)) + 4*a*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 9
*a*cosh(d*x + c)^2 + 10*a*cosh(d*x + c) + 4*a)*sinh(d*x + c) + 4*a)/(cosh(d
*x + c)^3 + 3*cosh(d*x + c)^2*sinh(d*x + c) + 3*cosh(d*x + c)*sinh(d*x + c)
^2 + sinh(d*x + c)^3)) + sqrt(a)*log(-(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^
2 + (cosh(d*x + c)^3 + (3*cosh(d*x + c) - 1)*sinh(d*x + c)^2 + sinh(d*x + c
)^3 - cosh(d*x + c)^2 + (3*cosh(d*x + c)^2 - 2*cosh(d*x + c) + 1)*sinh(d*x
+ c) + cosh(d*x + c) - 1)*sqrt(a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)
*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) - a*cosh(d*x + c) + (2*a*cosh(d*x +
c) - a)*sinh(d*x + c) + a)/(cosh(d*x + c) + sinh(d*x + c))))/d
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a \operatorname{sech}(c + dx) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sech(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(-a*sech(c + d*x) + a), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(32) = 64.
time = 0.41, size = 101, normalized size = 2.66

$$-\frac{\frac{2\, a \arctan\left(-\frac{\sqrt{a}\, e^{(dx+c)} - \sqrt{a e^{(2\, dx+2\, c)} + a}}{\sqrt{-a}}\right) \operatorname{sgn}\left(e^{(dx+c)} - 1\right)}{\sqrt{-a}} + \sqrt{a} \log\left(\left|-\sqrt{a}\, e^{(dx+c)} + \sqrt{a e^{(2\, dx+2\, c)} + a}\right|\right) \operatorname{sgn}\left(e^{(dx+c)} - 1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -(2*a*arctan(-(sqrt(a)*e^(d*x + c) - sqrt(a*e^(2*d*x + 2*c) + a))/sqrt(-a))
*sgn(e^(d*x + c) - 1)/sqrt(-a) + sqrt(a)*log(abs(-sqrt(a)*e^(d*x + c) + sqr
t(a*e^(2*d*x + 2*c) + a)))*sgn(e^(d*x + c) - 1))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{a - \frac{a}{\cosh\left(c + d\, x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a/cosh(c + d*x))^(1/2),x)
[Out] int((a - a/cosh(c + d*x))^(1/2), x)
```

3.84 $\int \frac{1}{\sqrt{a - a\,\mathbf{sech}(c + dx)}}\,dx$

Optimal. Leaf size=87

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(c+dx)}{\sqrt{a-a\,\mathrm{sech}(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(c+dx)}{\sqrt{2}\,\sqrt{a-a\,\mathrm{sech}(c+dx)}}\right)}{\sqrt{a}\,d}$$

```
[Out] 2*arctanh(a^(1/2)*tanh(d*x+c)/(a-a*sech(d*x+c))^(1/2))/d/a^(1/2)-arctanh(1/
2*a^(1/2)*tanh(d*x+c)*2^(1/2)/(a-a*sech(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3861, 3859, 209, 3880}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(c+dx)}{\sqrt{a-a\,\mathrm{sech}(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(c+dx)}{\sqrt{2}\,\sqrt{a-a\,\mathrm{sech}(c+dx)}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a - a*Sech[c + d*x]],x]
```

```
[Out] (2*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a - a*Sech[c + d*x]]])/(Sqrt[a]*d)
- (Sqrt[2]*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sech[c + d*x
]])])/(Sqrt[a]*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3861

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[1/a, In
t[Sqrt[a + b*Csc[c + d*x]], x], x] - Dist[b/a, Int[Csc[c + d*x]/Sqrt[a + b*
Csc[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a-a\text{sech}(c+dx)}}\,dx = \frac{\int \sqrt{a-a\text{sech}(c+dx)}\,dx}{a} + \int \frac{\text{sech}(c+dx)}{\sqrt{a-a\text{sech}(c+dx)}}\,dx$$

$$= -\frac{(2i)\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, \frac{ia\tanh(c+dx)}{\sqrt{a-a\text{sech}(c+dx)}}\right)}{d} + \frac{(2i)\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx, \right.}{}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a-a\text{sech}(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{2}\sqrt{a-a\text{sech}(c+dx)}}\right)}{\sqrt{a}\,d}$$

Mathematica [A]
time = 1.27, size = 118, normalized size = 1.36

$$\frac{\left(-1+e^{c+dx}\right)\left(\sqrt{2}\sinh^{-1}\left(e^{c+dx}\right)-2\tanh^{-1}\left(\frac{1+e^{c+dx}}{\sqrt{2}\sqrt{1+e^{2(c+dx)}}}\right)+\sqrt{2}\tanh^{-1}\left(\sqrt{1+e^{2(c+dx)}}\right)\right)}{\sqrt{2}\,d\sqrt{1+e^{2(c+dx)}}\sqrt{a-a\text{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a - a*Sech[c + d*x]],x]
[Out] ((-1 + E^(c + d*x))*(Sqrt[2]*ArcSinh[E^(c + d*x)] - 2*ArcTanh[(1 + E^(c + d
*x))/(Sqrt[2]*Sqrt[1 + E^(2*(c + d*x))])] + Sqrt[2]*ArcTanh[Sqrt[1 + E^(2*(
c + d*x))]]))/(Sqrt[2]*d*Sqrt[1 + E^(2*(c + d*x))]*Sqrt[a - a*Sech[c + d*x]
])
```

Maple [F]
time = 2.28, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a-a\,\text{sech}\,(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sech(d*x+c))^(1/2),x)
[Out] int(1/(a-a*sech(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-a*sech(d*x + c) + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 871 vs. 2(72) = 144.
time = 0.43, size = 871, normalized size = 10.01

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*sqrt(a)*log(-(3*cosh(d*x + c)^2 + 2*(3*cosh(d*x + c) + 1)*sinh
(d*x + c) + 3*sinh(d*x + c)^2 - 2*sqrt(2)*(cosh(d*x + c)^3 + (3*cosh(d*x +
c) + 1)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + cosh(d*x + c)^2 + (3*cosh(d*x +
 c)^2 + 2*cosh(d*x + c) + 1)*sinh(d*x + c) + cosh(d*x + c) + 1)*sqrt(a/(cos
h(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1))/sqrt(a
) + 2*cosh(d*x + c) + 3)/(cosh(d*x + c)^2 + 2*(cosh(d*x + c) - 1)*sinh(d*x
+ c) + sinh(d*x + c)^2 - 2*cosh(d*x + c) + 1)) + sqrt(a)*log((a*cosh(d*x +
c)^4 + a*sinh(d*x + c)^4 + 3*a*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) + 3*a)*
sinh(d*x + c)^3 + 5*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 9*a*cosh(d*x
 + c) + 5*a)*sinh(d*x + c)^2 + (cosh(d*x + c)^5 + (5*cosh(d*x + c) + 3)*sin
h(d*x + c)^4 + sinh(d*x + c)^5 + 3*cosh(d*x + c)^4 + (10*cosh(d*x + c)^2 +
12*cosh(d*x + c) + 5)*sinh(d*x + c)^3 + 5*cosh(d*x + c)^3 + (10*cosh(d*x +
c)^3 + 18*cosh(d*x + c)^2 + 15*cosh(d*x + c) + 7)*sinh(d*x + c)^2 + 7*cosh(
d*x + c)^2 + (5*cosh(d*x + c)^4 + 12*cosh(d*x + c)^3 + 15*cosh(d*x + c)^2 +
 14*cosh(d*x + c) + 4)*sinh(d*x + c) + 4*cosh(d*x + c) + 4)*sqrt(a)*sqrt(a/
(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) +
4*a*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 9*a*cosh(d*x + c)^2 + 10*a*cosh(
d*x + c) + 4*a)*sinh(d*x + c) + 4*a)/(cosh(d*x + c)^3 + 3*cosh(d*x + c)^2*s
inh(d*x + c) + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3)) + sqrt(a
)*log(-(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + (cosh(d*x + c)^3 + (3*cosh(
d*x + c) - 1)*sinh(d*x + c)^2 + sinh(d*x + c)^3 - cosh(d*x + c)^2 + (3*cosh
(d*x + c)^2 - 2*cosh(d*x + c) + 1)*sinh(d*x + c) + cosh(d*x + c) - 1)*sqrt(
a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^
2 + 1)) - a*cosh(d*x + c) + (2*a*cosh(d*x + c) - a)*sinh(d*x + c) + a)/(cos
h(d*x + c) + sinh(d*x + c))))/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a\operatorname{sech}\left(c+dx\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sech(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(-a*sech(c + d*x) + a), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(ex
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a-\frac{a}{\cosh\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - a/cosh(c + d*x))^(1/2),x)
```

```
[Out] int(1/(a - a/cosh(c + d*x))^(1/2), x)
```

3.85 $\int \sqrt{3+3\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=19

$$2\sqrt{3}\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1+\operatorname{sech}(x)}}\right)$$

```
[Out] 2*arctanh(tanh(x)/(1+sech(x))^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3859, 209}

$$2\sqrt{3}\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{\operatorname{sech}(x)+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 + 3*Sech[x]],x]
```

```
[Out] 2*Sqrt[3]*ArcTanh[Tanh[x]/Sqrt[1 + Sech[x]]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{3+3\operatorname{sech}(x)}\,dx &= 6i\text{Subst}\left(\int \frac{1}{3+x^2}\,dx, x, -\frac{3i\tanh(x)}{\sqrt{3+3\operatorname{sech}(x)}}\right)\\ &= 2\sqrt{3}\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1+\operatorname{sech}(x)}}\right)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 39 vs. 2(19) = 38.

time = 0.03, size = 39, normalized size = 2.05

$$\sqrt{6}\,\sinh^{-1}\left(\sqrt{2}\,\sinh\left(\frac{x}{2}\right)\right)\sqrt{\cosh(x)}\,\text{sech}\left(\frac{x}{2}\right)\sqrt{1+\text{sech}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 + 3*Sech[x]],x]
```

```
[Out] Sqrt[6]*ArcSinh[Sqrt[2]*Sinh[x/2]]*Sqrt[Cosh[x]]*Sech[x/2]*Sqrt[1 + Sech[x]
]
```

Maple [F]
time = 1.21, size = 0, normalized size = 0.00

$$\int \sqrt{3+3\,\text{sech}\,(x)}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+3*sech(x))^(1/2),x)
```

```
[Out] int((3+3*sech(x))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+3*sech(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(3*sech(x) + 3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(15) = 30.
time = 0.40, size = 233, normalized size = 12.26

$$\frac{1}{2}\sqrt{3}\log\left(-\frac{\cosh(x)^4+(4\cosh(x)-3)\sinh(x)^3+\sinh(x)^4-3\cosh(x)^3+(6\cosh(x)^2-9\cosh(x)+5)\sinh(x)^2+\sqrt{2}\left(\cosh(x)^3+3(\cosh(x)-1)\sinh(x)^2+\sinh(x)^3-3\cosh(x)^2+(3\cosh(x)^2-6\cosh(x)+4)\sinh(x)+4\cosh(x)-4\right)\sqrt{\frac{\cosh(x)}{\cosh(x)-\sinh(x)}}+5\cosh(x)^2+(4\cosh(x)^3-9\cosh(x)^2+10\cosh(x)-4)\sinh(x)-4\cosh(x)+4}{\cosh(x)^3+3\cosh(x)^2\sinh(x)+3\cosh(x)\sinh(x)^2+\sinh(x)^3}\right)+\frac{1}{2}\sqrt{3}\log\left(\frac{\sqrt{2}\sqrt{\frac{\cosh(x)}{\cosh(x)-\sinh(x)}}(\cosh(x)+\sinh(x)+1)+\cosh(x)^2+(2\cosh(x)+1)\sinh(x)+\sinh(x)^2+\cosh(x)+1}{\cosh(x)+\sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+3*sech(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*log(-(cosh(x)^4 + (4*cosh(x) - 3)*sinh(x)^3 + sinh(x)^4 - 3*cos
h(x)^3 + (6*cosh(x)^2 - 9*cosh(x) + 5)*sinh(x)^2 + sqrt(2)*(cosh(x)^3 + 3*(
cosh(x) - 1)*sinh(x)^2 + sinh(x)^3 - 3*cosh(x)^2 + (3*cosh(x)^2 - 6*cosh(x)
 + 4)*sinh(x) + 4*cosh(x) - 4)*sqrt(cosh(x)/(cosh(x) - sinh(x))) + 5*cosh(x
)^2 + (4*cosh(x)^3 - 9*cosh(x)^2 + 10*cosh(x) - 4)*sinh(x) - 4*cosh(x) + 4)
/(cosh(x)^3 + 3*cosh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2 + sinh(x)^3)) + 1/2
```

```
*sqrt(3)*log((sqrt(2)*sqrt(cosh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sinh(x)
+ 1) + cosh(x)^2 + (2*cosh(x) + 1)*sinh(x) + sinh(x)^2 + cosh(x) + 1)/(cosh
(x) + sinh(x)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\sqrt{3}\int\sqrt{\operatorname{sech}(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+3*sech(x))**(1/2),x)
```

```
[Out] sqrt(3)*Integral(sqrt(sech(x) + 1), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(15) = 30$.
time = 0.39, size = 52, normalized size = 2.74

$$-\sqrt{3}\left(\log\left(\sqrt{e^{(2x)}+1}-e^{x}+1\right)+\log\left(\sqrt{e^{(2x)}+1}-e^{x}\right)-\log\left(-\sqrt{e^{(2x)}+1}+e^{x}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+3*sech(x))^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(3)*(log(sqrt(e^(2*x) + 1) - e^x + 1) + log(sqrt(e^(2*x) + 1) - e^x) -
 log(-sqrt(e^(2*x) + 1) + e^x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int\sqrt{\frac{3}{\cosh(x)}+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3/cosh(x) + 3)^(1/2),x)
```

```
[Out] int((3/cosh(x) + 3)^(1/2), x)
```

3.86 $\int \sqrt{3-3\textbf{sech}(x)}\,dx$

Optimal. Leaf size=21

$$2\sqrt{3}\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1-\text{sech}(x)}}\right)$$

```
[Out] 2*arctanh(tanh(x)/(1-sech(x))^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3859, 209}

$$2\sqrt{3}\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1-\text{sech}(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - 3*Sech[x]],x]
```

```
[Out] 2*Sqrt[3]*ArcTanh[Tanh[x]/Sqrt[1 - Sech[x]]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{3-3\text{sech}(x)}\,dx &= -\left(6i\text{Subst}\left(\int \frac{1}{3+x^2}\,dx, x, \frac{3i\tanh(x)}{\sqrt{3-3\text{sech}(x)}}\right)\right)\\ &= 2\sqrt{3}\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1-\text{sech}(x)}}\right)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 51 vs. 2(21) = 42.

time = 0.38, size = 51, normalized size = 2.43

$$\frac{\sqrt{3}\,\sqrt{1+e^{2x}}\left(\sinh^{-1}\left(e^{x}\right)+\tanh^{-1}\left(\sqrt{1+e^{2x}}\right)\right)\sqrt{1-\operatorname{sech}(x)}}{-1+e^{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - 3*Sech[x]],x]
```

```
[Out] (Sqrt[3]*Sqrt[1 + E^(2*x)]*(ArcSinh[E^x] + ArcTanh[Sqrt[1 + E^(2*x)]])*Sqrt
[1 - Sech[x]])/(-1 + E^x)
```

Maple [**F**]
time = 1.20, size = 0, normalized size = 0.00

$$\int \sqrt{3-3\,\operatorname{sech}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-3*sech(x))^(1/2),x)
```

```
[Out] int((3-3*sech(x))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*sech(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-3*sech(x) + 3), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(17) = 34$.
time = 0.45, size = 235, normalized size = 11.19

$$\frac{1}{2}\sqrt{3}\log\left(\frac{\cosh\left(x\right)^{4}+\left(4\cosh\left(x\right)+3\right)\sinh\left(x\right)^{3}+\sinh\left(x\right)^{4}+3\cosh\left(x\right)^{3}+\left(6\cosh\left(x\right)^{2}+9\cosh\left(x\right)+5\right)\sinh\left(x\right)^{2}+\sqrt{2}\left(\cosh\left(x\right)^{3}+3\left(\cosh\left(x\right)+1\right)\sinh\left(x\right)^{2}+\sinh\left(x\right)^{3}+3\cosh\left(x\right)^{2}+\left(3\cosh\left(x\right)^{2}+6\cosh\left(x\right)+4\right)\sinh\left(x\right)+4\cosh\left(x\right)+4\right)\sqrt{\frac{\cosh\left(x\right)}{\cosh\left(x\right)-\sinh\left(x\right)}}+5\cosh\left(x\right)^{2}+\left(4\cosh\left(x\right)^{3}+9\cosh\left(x\right)^{2}+10\cosh\left(x\right)+4\right)\sinh\left(x\right)+4\cosh\left(x\right)+4}{\cosh\left(x\right)^{3}+3\cosh\left(x\right)^{2}\sinh\left(x\right)+3\cosh\left(x\right)\sinh\left(x\right)^{2}+\sinh\left(x\right)^{3}}\right)+\frac{1}{2}\sqrt{3}\log\left(-\frac{\sqrt{2}\sqrt{\frac{\cosh\left(x\right)}{\cosh\left(x\right)-\sinh\left(x\right)}}\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)+\cosh\left(x\right)^{2}+\left(2\cosh\left(x\right)-1\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}-\cosh\left(x\right)+1}{\cosh\left(x\right)+\sinh\left(x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*sech(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*log((cosh(x)^4 + (4*cosh(x) + 3)*sinh(x)^3 + sinh(x)^4 + 3*cosh
(x)^3 + (6*cosh(x)^2 + 9*cosh(x) + 5)*sinh(x)^2 + sqrt(2)*(cosh(x)^3 + 3*(c
osh(x) + 1)*sinh(x)^2 + sinh(x)^3 + 3*cosh(x)^2 + (3*cosh(x)^2 + 6*cosh(x)
+ 4)*sinh(x) + 4*cosh(x) + 4)*sqrt(cosh(x)/(cosh(x) - sinh(x))) + 5*cosh(x)
^2 + (4*cosh(x)^3 + 9*cosh(x)^2 + 10*cosh(x) + 4)*sinh(x) + 4*cosh(x) + 4)/
```

```
(cosh(x)^3 + 3*cosh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2 + sinh(x)^3)) + 1/2*
sqrt(3)*log(-(sqrt(2)*sqrt(cosh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sinh(x)
- 1) + cosh(x)^2 + (2*cosh(x) - 1)*sinh(x) + sinh(x)^2 - cosh(x) + 1)/(cosh
(x) + sinh(x)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\sqrt{3}\int\sqrt{1-\operatorname{sech}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*sech(x))**(1/2),x)
[Out] sqrt(3)*Integral(sqrt(1 - sech(x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(17) = 34.
time = 0.38, size = 69, normalized size = 3.29

$$\sqrt{3}\left(\log\left(\sqrt{e^{(2x)}+1}-e^{x}+1\right)\operatorname{sgn}(e^{x}-1)-\log\left(\sqrt{e^{(2x)}+1}-e^{x}\right)\operatorname{sgn}(e^{x}-1)-\log\left(-\sqrt{e^{(2x)}+1}+e^{x}+1\right)\operatorname{sgn}(e^{x}-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*sech(x))^(1/2),x, algorithm="giac")
[Out] sqrt(3)*(log(sqrt(e^(2*x) + 1) - e^x + 1)*sgn(e^x - 1) - log(sqrt(e^(2*x) +
 1) - e^x)*sgn(e^x - 1) - log(-sqrt(e^(2*x) + 1) + e^x + 1)*sgn(e^x - 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int\sqrt{3-\frac{3}{\cosh\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3 - 3/cosh(x))^(1/2),x)
[Out] int((3 - 3/cosh(x))^(1/2), x)
```

3.87 $\int (a+b\text{sech}(c+dx))^4\,dx$

Optimal. Leaf size=107

$$a^4x+\frac{2ab(2a^2+b^2)\,\text{ArcTan}(\sinh(c+dx))}{d}+\frac{b^2(17a^2+2b^2)\tanh(c+dx)}{3d}+\frac{4ab^3\text{sech}(c+dx)\tanh(c+dx)}{3d}+\frac{b^2}{}$$

```
[Out] a^4*x+2*a*b*(2*a^2+b^2)*arctan(sinh(d*x+c))/d+1/3*b^2*(17*a^2+2*b^2)*tanh(d
*x+c)/d+4/3*a*b^3*sech(d*x+c)*tanh(d*x+c)/d+1/3*b^2*(a+b*sech(d*x+c))^2*tan
h(d*x+c)/d
```

Rubi [A]
time = 0.09, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3867, 4133, 3855, 3852, 8}

$$a^4x+\frac{2ab(2a^2+b^2)\,\text{ArcTan}(\sinh(c+dx))}{d}+\frac{b^2(17a^2+2b^2)\tanh(c+dx)}{3d}+\frac{4ab^3\tanh(c+dx)\text{sech}(c+dx)}{3d}+\frac{b^2\tanh(c+dx)(a+b\text{sech}(c+dx))^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x])^4,x]
```

```
[Out] a^4*x + (2*a*b*(2*a^2 + b^2)*ArcTan[Sinh[c + d*x]])/d + (b^2*(17*a^2 + 2*b^
2)*Tanh[c + d*x])/(3*d) + (4*a*b^3*Sech[c + d*x]*Tanh[c + d*x])/(3*d) + (b^
2*(a + b*Sech[c + d*x])^2*Tanh[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3867

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Dist[1/(n - 1)
, Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a
^2*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /;
```

```
FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 4133

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(-b)*C*Csc[e +
 f*x]*(Cot[e + f*x]/(2*f)), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A
 + C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x]
```

Rubi steps

$$\int (a+b\text{sech}(c+dx))^4\,dx = \frac{b^2(a+b\text{sech}(c+dx))^2\tanh(c+dx)}{3d} + \frac{1}{3}\int (a+b\text{sech}(c+dx))\left(3a^3+b\left(9a^2+\right.\right.$$
$$= \frac{4ab^3\text{sech}(c+dx)\tanh(c+dx)}{3d} + \frac{b^2(a+b\text{sech}(c+dx))^2\tanh(c+dx)}{3d} + \frac{1}{6}\int$$
$$= a^4x + \frac{4ab^3\text{sech}(c+dx)\tanh(c+dx)}{3d} + \frac{b^2(a+b\text{sech}(c+dx))^2\tanh(c+dx)}{3d} +$$
$$= a^4x + \frac{2ab(2a^2+b^2)\tan^{-1}(\sinh(c+dx))}{d} + \frac{4ab^3\text{sech}(c+dx)\tanh(c+dx)}{3d} +$$
$$= a^4x + \frac{2ab(2a^2+b^2)\tan^{-1}(\sinh(c+dx))}{d} + \frac{b^2(17a^2+2b^2)\tanh(c+dx)}{3d} + \frac{4ab}{}$$

Mathematica [A]
time = 0.18, size = 78, normalized size = 0.73

$$\frac{3a^4dx+6ab(2a^2+b^2)\,\text{ArcTan}(\sinh(c+dx))+3b^2(6a^2+b^2+2ab\text{sech}(c+dx))\tanh(c+dx)-b^4\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x])^4,x]
[Out] (3*a^4*d*x + 6*a*b*(2*a^2 + b^2)*ArcTan[Sinh[c + d*x]] + 3*b^2*(6*a^2 + b^2
 + 2*a*b*Sech[c + d*x])*Tanh[c + d*x] - b^4*Tanh[c + d*x]^3)/(3*d)
```

Maple [C] Result contains complex when optimal does not.
time = 1.49, size = 182, normalized size = 1.70

method	result
risch	$x\,a^4-\frac{4b^2\left(-3ab\,e^{5dx+5c}+9a^2e^{4dx+4c}+18a^2e^{2dx+2c}+3b^2e^{2dx+2c}+3ab\,e^{dx+c}+9a^2+b^2\right)}{3d\left(1+e^{2dx+2c}\right)^3}+\frac{4ia^3b\ln\left(e^{dx+c}+i\right)}{d}+\frac{2ia\,b^3\ln\left(e^{dx+}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sech(d*x+c))^4,x,method=_RETURNVERBOSE)

[Out] x*a^4-4/3*b^2*(-3*a*b*exp(5*d*x+5*c)+9*a^2*exp(4*d*x+4*c)+18*a^2*exp(2*d*x+2*c)+3*b^2*exp(2*d*x+2*c)+3*a*b*exp(d*x+c)+9*a^2+b^2)/d/(1+exp(2*d*x+2*c))^3+4*I*a^3*b/d*ln(exp(d*x+c)+I)+2*I*a*b^3/d*ln(exp(d*x+c)+I)-4*I*a^3*b/d*ln(exp(d*x+c)-I)-2*I*a*b^3/d*ln(exp(d*x+c)-I)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(101) = 202$.
time = 0.48, size = 211, normalized size = 1.97

$$a^4x - 4\,ab^3\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)} - e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)} + e^{(-4\,dx-4\,c)} + 1)}\right) + \frac{4}{3}\,b^4\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)} + 3\,e^{(-4\,dx-4\,c)} + e^{(-6\,dx-6\,c)} + 1)} + \frac{1}{d(3\,e^{(-2\,dx-2\,c)} + 3\,e^{(-4\,dx-4\,c)} + e^{(-6\,dx-6\,c)} + 1)}\right) + \frac{4\,a^3b\arctan\left(\sinh\left(dx+c\right)\right)}{d} + \frac{12\,a^2b^2}{d(e^{(-2\,dx-2\,c)} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c))^4,x, algorithm="maxima")

[Out] a^4*x - 4*a*b^3*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + 4/3*b^4*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 4*a^3*b*arctan(sinh(d*x + c))/d + 12*a^2*b^2/(d*(e^(-2*d*x - 2*c) + 1))

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1028 vs. $2(101) = 202$.
time = 0.40, size = 1028, normalized size = 9.61

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sech(d*x+c))^4,x, algorithm="fricas")

[Out] 1/3*(3*a^4*d*x*cosh(d*x + c)^6 + 3*a^4*d*x*sinh(d*x + c)^6 + 12*a*b^3*cosh(d*x + c)^5 + 3*a^4*d*x + 6*(3*a^4*d*x*cosh(d*x + c) + 2*a*b^3)*sinh(d*x + c)^5 - 12*a*b^3*cosh(d*x + c) + 9*(a^4*d*x - 4*a^2*b^2)*cosh(d*x + c)^4 + 3*(15*a^4*d*x*cosh(d*x + c)^2 + 3*a^4*d*x + 20*a*b^3*cosh(d*x + c) - 12*a^2*b^2)*sinh(d*x + c)^4 - 36*a^2*b^2 - 4*b^4 + 12*(5*a^4*d*x*cosh(d*x + c)^3 + 10*a*b^3*cosh(d*x + c)^2 + 3*(a^4*d*x - 4*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(3*a^4*d*x - 24*a^2*b^2 - 4*b^4)*cosh(d*x + c)^2 + 3*(15*a^4*d*x*cosh(d*x + c)^4 + 40*a*b^3*cosh(d*x + c)^3 + 3*a^4*d*x - 24*a^2*b^2 - 4*b^4 + 18*(a^4*d*x - 4*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 12*((2*a^3*b + a*b^3)*cosh(d*x + c)^6 + 6*(2*a^3*b + a*b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + (2*a^3*b + a*b^3)*sinh(d*x + c)^6 + 3*(2*a^3*b + a*b^3)*cosh(d*x + c)

```
^4 + 3*(2*a^3*b + a*b^3 + 5*(2*a^3*b + a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c
)^4 + 2*a^3*b + a*b^3 + 4*(5*(2*a^3*b + a*b^3)*cosh(d*x + c)^3 + 3*(2*a^3*b
 + a*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(2*a^3*b + a*b^3)*cosh(d*x + c
)^2 + 3*(5*(2*a^3*b + a*b^3)*cosh(d*x + c)^4 + 2*a^3*b + a*b^3 + 6*(2*a^3*b
 + a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 6*((2*a^3*b + a*b^3)*cosh(d*x
+ c)^5 + 2*(2*a^3*b + a*b^3)*cosh(d*x + c)^3 + (2*a^3*b + a*b^3)*cosh(d*x +
 c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) + 6*(3*a^4*d*x*co
sh(d*x + c)^5 + 10*a*b^3*cosh(d*x + c)^4 - 2*a*b^3 + 6*(a^4*d*x - 4*a^2*b^2
)*cosh(d*x + c)^3 + (3*a^4*d*x - 24*a^2*b^2 - 4*b^4)*cosh(d*x + c))*sinh(d*
x + c))/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*x
 + c)^6 + 3*d*cosh(d*x + c)^4 + 3*(5*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4
 + 4*(5*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*d*cosh(d
*x + c)^2 + 3*(5*d*cosh(d*x + c)^4 + 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)
^2 + 6*(d*cosh(d*x + c)^5 + 2*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x
 + c) + d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\,\mathrm{sech}\,(c + dx))^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))**4,x)
```

```
[Out] Integral((a + b*sech(c + d*x))**4, x)
```

Giac [**A**]
time = 0.40, size = 141, normalized size = 1.32

$$\frac{3\,(dx+c)a^4 + 12\,(2\,a^3b + ab^3)\arctan\left(e^{(dx+c)}\right) + \frac{4\,\left(3\,ab^3e^{(5\,dx+5\,c)} - 9\,a^2b^2e^{(4\,dx+4\,c)} - 18\,a^2b^2e^{(2\,dx+2\,c)} - 3\,b^4e^{(2\,dx+2\,c)} - 3\,ab^3e^{(dx+c)} - 9\,a^2b^2 - b^4\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*a^4 + 12*(2*a^3*b + a*b^3)*arctan(e^(d*x + c)) + 4*(3*a*b^
3*e^(5*d*x + 5*c) - 9*a^2*b^2*e^(4*d*x + 4*c) - 18*a^2*b^2*e^(2*d*x + 2*c)
- 3*b^4*e^(2*d*x + 2*c) - 3*a*b^3*e^(d*x + c) - 9*a^2*b^2 - b^4)/(e^(2*d*x
+ 2*c) + 1)^3)/d
```

Mupad [**B**]
time = 1.41, size = 233, normalized size = 2.18

$$a^4\,x - \frac{\frac{12\,a^2\,b^2}{d} - \frac{4\,a\,b^3\,\mathrm{e}^{c+d\,x}}{d}}{\mathrm{e}^{2\,c+2\,d\,x}+1} - \frac{\frac{4\,b^4}{d} + \frac{8\,a\,b^3\,\mathrm{e}^{c+d\,x}}{d}}{2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1} + \frac{8\,b^4}{3\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1\right)} + \frac{4\,\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(a\,b^3\,\sqrt{d^2}+2\,a^3\,b\,\sqrt{d^2}\right)}{d\,\sqrt{4\,a^6\,b^2+4\,a^4\,b^4+a^2\,b^6}}\right)\,\sqrt{4\,a^6\,b^2+4\,a^4\,b^4+a^2\,b^6}}{\sqrt{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x))^4,x)
```

```
[Out] a^4*x - ((12*a^2*b^2)/d - (4*a*b^3*exp(c + d*x))/d)/(exp(2*c + 2*d*x) + 1)
- ((4*b^4)/d + (8*a*b^3*exp(c + d*x))/d)/(2*exp(2*c + 2*d*x) + exp(4*c + 4*
d*x) + 1) + (8*b^4)/(3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c
 + 6*d*x) + 1)) + (4*atan((exp(d*x)*exp(c)*(a*b^3*(d^2)^(1/2) + 2*a^3*b*(d^
2)^(1/2)))/(d*(a^2*b^6 + 4*a^4*b^4 + 4*a^6*b^2)^(1/2)))*(a^2*b^6 + 4*a^4*b^
4 + 4*a^6*b^2)^(1/2))/(d^2)^(1/2)
```

3.88 $\int (a+b\,\mathrm{sech}(c+dx))^3\,dx$

Optimal. Leaf size=73

$$a^3x+\frac{b(6a^2+b^2)\,\mathrm{ArcTan}(\sinh(c+dx))}{2d}+\frac{5ab^2\tanh(c+dx)}{2d}+\frac{b^2(a+b\,\mathrm{sech}(c+dx))\tanh(c+dx)}{2d}$$

```
[Out] a^3*x+1/2*b*(6*a^2+b^2)*arctan(sinh(d*x+c))/d+5/2*a*b^2*tanh(d*x+c)/d+1/2*b
^2*(a+b*sech(d*x+c))*tanh(d*x+c)/d
```

Rubi [A]
time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used = {3867, 3855, 3852, 8}

$$a^3x+\frac{b(6a^2+b^2)\,\mathrm{ArcTan}(\sinh(c+dx))}{2d}+\frac{5ab^2\tanh(c+dx)}{2d}+\frac{b^2\tanh(c+dx)(a+b\,\mathrm{sech}(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x])^3,x]
```

```
[Out] a^3*x + (b*(6*a^2 + b^2)*ArcTan[Sinh[c + d*x]])/(2*d) + (5*a*b^2*Tanh[c + d
*x])/(2*d) + (b^2*(a + b*Sech[c + d*x])*Tanh[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3867

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Dist[1/(n - 1)
, Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a
^2*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rubi steps

$$\int (a+b\text{sech}(c+dx))^3\,dx = \frac{b^2(a+b\text{sech}(c+dx))\tanh(c+dx)}{2d} + \frac{1}{2}\int \left(2a^3+b\left(6a^2+b^2\right)\text{sech}(c+dx)+5a\right.$$
$$= a^3x + \frac{b^2(a+b\text{sech}(c+dx))\tanh(c+dx)}{2d} + \frac{1}{2}\left(5ab^2\right)\int \text{sech}^2(c+dx)\,dx + \frac{1}{2}(b$$
$$= a^3x + \frac{b(6a^2+b^2)\tan^{-1}(\sinh(c+dx))}{2d} + \frac{b^2(a+b\text{sech}(c+dx))\tanh(c+dx)}{2d} +$$
$$= a^3x + \frac{b(6a^2+b^2)\tan^{-1}(\sinh(c+dx))}{2d} + \frac{5ab^2\tanh(c+dx)}{2d} + \frac{b^2(a+b\text{sech}(c+$$

Mathematica [A]
time = 0.10, size = 55, normalized size = 0.75

$$\frac{2a^3dx+b(6a^2+b^2)\,\text{ArcTan}(\sinh(c+dx))+b^2(6a+b\text{sech}(c+dx))\tanh(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x])^3,x]
```

```
[Out] (2*a^3*d*x + b*(6*a^2 + b^2)*ArcTan[Sinh[c + d*x]] + b^2*(6*a + b*Sech[c +
d*x])*Tanh[c + d*x])/(2*d)
```

Maple [C] Result contains complex when optimal does not.
time = 1.50, size = 142, normalized size = 1.95

method	result
risch	$a^3x - \frac{b^2\left(-b\,e^{3dx+3c}+6a\,e^{2dx+2c}+b\,e^{dx+c}+6a\right)}{d\left(1+e^{2dx+2c}\right)^2} + \frac{3ib\ln\left(e^{dx+c}+i\right)a^2}{d} + \frac{ib^3\ln\left(e^{dx+c}+i\right)}{2d} - \frac{3ib\ln\left(e^{dx+c}-i\right)a^2}{d} - \frac{ib^3\ln\left(e^{dx+c}\right.}{2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*x-b^2*(-b*exp(3*d*x+3*c)+6*a*exp(2*d*x+2*c)+b*exp(d*x+c)+6*a)/d/(1+exp(
2*d*x+2*c))^2+3*I*b/d*ln(exp(d*x+c)+I)*a^2+1/2*I*b^3/d*ln(exp(d*x+c)+I)-3*I
*b/d*ln(exp(d*x+c)-I)*a^2-1/2*I*b^3/d*ln(exp(d*x+c)-I)
```

Maxima [A]
time = 0.49, size = 114, normalized size = 1.56

$$a^3x - b^3\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right) + \frac{3\,a^2b\arctan\left(\sinh\left(dx+c\right)\right)}{d} + \frac{6\,ab^2}{d(e^{(-2\,dx-2\,c)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] a^3*x - b^3*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*
(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + 3*a^2*b*arctan(sinh(d*x + c
))/d + 6*a*b^2/(d*(e^(-2*d*x - 2*c) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 521 vs. $2(67) = 134$.
time = 0.42, size = 521, normalized size = 7.14

$$\frac{a^{3} d x \cosh\left(d x + c\right)^{4} + a^{3} d x \sinh\left(d x + c\right)^{4} + b^{3} \cosh\left(d x + c\right)^{3} + a^{3} d x - b^{3} \cosh\left(d x + c\right) + \left(4 \, a^{3} d x \cosh\left(d x + c\right) + b^{3}\right) \sinh\left(d x + c\right)^{3} - 6 \, a b^{2} + 2 \, \left(a^{3} d x - 3 \, a b^{2}\right) \cosh\left(d x + c\right)^{2} + \left(6 \, a^{3} d x \cosh\left(d x + c\right)^{2} + 2 \, a^{3} d x + 3 \, b^{3} \cosh\left(d x + c\right) - 6 \, a b^{2}\right) \sinh\left(d x + c\right)^{2} + \left(\left(6 \, a^{2} b + b^{3}\right) \cosh\left(d x + c\right)^{4} + 4 \, \left(6 \, a^{2} b + b^{3}\right) \cosh\left(d x + c\right) \sinh\left(d x + c\right)^{3} + \left(6 \, a^{2} b + b^{3}\right) \sinh\left(d x + c\right)^{4} + 6 \, a^{2} b + b^{3} + 2 \, \left(6 \, a^{2} b + b^{3}\right) \cosh\left(d x + c\right)^{2} + 2 \, \left(6 \, a^{2} b + b^{3} + 3 \, \left(6 \, a^{2} b + b^{3}\right) \cosh\left(d x + c\right)^{2}\right) \sinh\left(d x + c\right)^{2} + 4 \, \left(\left(6 \, a^{2} b + b^{3}\right) \cosh\left(d x + c\right)^{3} + \left(6 \, a^{2} b + b^{3}\right) \cosh\left(d x + c\right)\right) \sinh\left(d x + c\right)\right) \arctan\left(\cosh\left(d x + c\right) + \sinh\left(d x + c\right)\right) + \left(4 \, a^{3} d x \cosh\left(d x + c\right)^{3} + 3 \, b^{3} \cosh\left(d x + c\right)^{2} - b^{3} + 4 \, \left(a^{3} d x - 3 \, a b^{2}\right) \cosh\left(d x + c\right)\right) \sinh\left(d x + c\right)}{d \cosh\left(d x + c\right)^{4} + 4 \, d \cosh\left(d x + c\right) \sinh\left(d x + c\right)^{3} + d \sinh\left(d x + c\right)^{4} + 2 \, d \cosh\left(d x + c\right)^{2} + 2 \, \left(3 \, d \cosh\left(d x + c\right)^{2} + d\right) \sinh\left(d x + c\right)^{2} + 4 \, \left(d \cosh\left(d x + c\right)^{3} + d \cosh\left(d x + c\right)\right) \sinh\left(d x + c\right) + d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] (a^3*d*x*cosh(d*x + c)^4 + a^3*d*x*sinh(d*x + c)^4 + b^3*cosh(d*x + c)^3 +
a^3*d*x - b^3*cosh(d*x + c) + (4*a^3*d*x*cosh(d*x + c) + b^3)*sinh(d*x + c)
^3 - 6*a*b^2 + 2*(a^3*d*x - 3*a*b^2)*cosh(d*x + c)^2 + (6*a^3*d*x*cosh(d*x
+ c)^2 + 2*a^3*d*x + 3*b^3*cosh(d*x + c) - 6*a*b^2)*sinh(d*x + c)^2 + ((6*a
^2*b + b^3)*cosh(d*x + c)^4 + 4*(6*a^2*b + b^3)*cosh(d*x + c)*sinh(d*x + c)
^3 + (6*a^2*b + b^3)*sinh(d*x + c)^4 + 6*a^2*b + b^3 + 2*(6*a^2*b + b^3)*co
sh(d*x + c)^2 + 2*(6*a^2*b + b^3 + 3*(6*a^2*b + b^3)*cosh(d*x + c)^2)*sinh(
d*x + c)^2 + 4*((6*a^2*b + b^3)*cosh(d*x + c)^3 + (6*a^2*b + b^3)*cosh(d*x
+ c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) + (4*a^3*d*x*cos
h(d*x + c)^3 + 3*b^3*cosh(d*x + c)^2 - b^3 + 4*(a^3*d*x - 3*a*b^2)*cosh(d*x
 + c))*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^
3 + d*sinh(d*x + c)^4 + 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 + d)*s
inh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \operatorname{sech}\left(c + dx\right)\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sech(c + d*x))**3, x)
```

Giac [A]
time = 0.39, size = 92, normalized size = 1.26

$$\frac{(dx + c)a^3 + \left(6\,a^2 b + b^3\right)\arctan\left(e^{(dx+c)}\right) + \frac{b^3 e^{(3\,dx+3\,c)} - 6\,ab^2 e^{(2\,dx+2\,c)} - b^3 e^{(dx+c)} - 6\,ab^2}{\left(e^{(2\,dx+2\,c)}+1\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^3,x, algorithm="giac")
```

```
[Out] ((d*x + c)*a^3 + (6*a^2*b + b^3)*arctan(e^(d*x + c)) + (b^3*e^(3*d*x + 3*c)
 - 6*a*b^2*e^(2*d*x + 2*c) - b^3*e^(d*x + c) - 6*a*b^2)/(e^(2*d*x + 2*c) +
1)^2)/d
```

Mupad [B]
time = 1.40, size = 165, normalized size = 2.26

$$a^3\,x-\frac{\frac{6\,a\,b^2}{d}-\frac{b^3\,{\mathrm{e}}^{c+d\,x}}{d}}{{\mathrm{e}}^{2\,c+2\,d\,x}+1}+\frac{\mathrm{atan}\left(\frac{{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^c\,\left(b^3\,\sqrt{d^2}+6\,a^2\,b\,\sqrt{d^2}\right)}{d\,\sqrt{36\,a^4\,b^2+12\,a^2\,b^4+b^6}}\right)\,\sqrt{36\,a^4\,b^2+12\,a^2\,b^4+b^6}}{\sqrt{d^2}}-\frac{2\,b^3\,{\mathrm{e}}^{c+d\,x}}{d\,\left(2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x))^3,x)
```

```
[Out] a^3*x - ((6*a*b^2)/d - (b^3*exp(c + d*x))/d)/(exp(2*c + 2*d*x) + 1) + (atan
((exp(d*x)*exp(c)*(b^3*(d^2)^(1/2) + 6*a^2*b*(d^2)^(1/2)))/(d*(b^6 + 12*a^2
*b^4 + 36*a^4*b^2)^(1/2)))*(b^6 + 12*a^2*b^4 + 36*a^4*b^2)^(1/2))/(d^2)^(1/
2) - (2*b^3*exp(c + d*x))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.89 $\int (a+b\,\mathrm{sech}(c+dx))^2\,dx$

Optimal. Leaf size=33

$$a^2x+\frac{2ab\mathrm{ArcTan}(\sinh(c+dx))}{d}+\frac{b^2\tanh(c+dx)}{d}$$

```
[Out] a^2*x+2*a*b*arctan(sinh(d*x+c))/d+b^2*tanh(d*x+c)/d
```

Rubi [A]
time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used = {3858, 3855, 3852, 8}

$$a^2x+\frac{2ab\mathrm{ArcTan}(\sinh(c+dx))}{d}+\frac{b^2\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x])^2,x]
```

```
[Out] a^2*x + (2*a*b*ArcTan[Sinh[c + d*x]])/d + (b^2*Tanh[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3858

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Dist[2*a*b, Int[Csc[c + d*x], x], x] + Dist[b^2, Int[Csc[c + d*x]^2, x],
x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int (a+b\text{sech}(c+dx))^2\,dx = a^2x+(2ab)\int \text{sech}(c+dx)\,dx+b^2\int \text{sech}^2(c+dx)\,dx$$
$$= a^2x+\frac{2ab\tan^{-1}(\sinh(c+dx))}{d}+\frac{(ib^2)\,\text{Subst}(\int 1\,dx,x,-i\tanh(c+dx))}{d}$$
$$= a^2x+\frac{2ab\tan^{-1}(\sinh(c+dx))}{d}+\frac{b^2\tanh(c+dx)}{d}$$

Mathematica [A]
time = 0.04, size = 32, normalized size = 0.97

$$\frac{a(adx+2b\text{ArcTan}(\sinh(c+dx)))+b^2\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x])^2,x]
```

```
[Out] (a*(a*d*x + 2*b*ArcTan[Sinh[c + d*x]]) + b^2*Tanh[c + d*x])/d
```

Maple [C] Result contains complex when optimal does not.
time = 1.50, size = 64, normalized size = 1.94

method	result	size
risch	$a^2x-\frac{2b^2}{d(1+e^{2dx+2c})}+\frac{2iba\ln(e^{dx+c}+i)}{d}-\frac{2iba\ln(e^{dx+c}-i)}{d}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*x-2*b^2/d/(1+exp(2*d*x+2*c))+2*I*b*a/d*ln(exp(d*x+c)+I)-2*I*b*a/d*ln(ex
p(d*x+c)-I)
```

Maxima [A]
time = 0.27, size = 41, normalized size = 1.24

$$a^2x+\frac{2\,ab\arctan(\sinh(dx+c))}{d}+\frac{2\,b^2}{d(e^{(-2\,dx-2\,c)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 2*a*b*arctan(sinh(d*x + c))/d + 2*b^2/(d*(e^(-2*d*x - 2*c) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(33) = 66$.
time = 0.42, size = 157, normalized size = 4.76

$$\frac{a^2dx\cosh(dx+c)^2+2\,a^2dx\cosh(dx+c)\sinh(dx+c)+a^2dx\sinh(dx+c)^2+a^2dx-2\,b^2+4\,\left(ab\cosh(dx+c)^2+2\,ab\cosh(dx+c)\sinh(dx+c)+ab\sinh(dx+c)^2+ab\right)\arctan(\cosh(dx+c)+\sinh(dx+c))}{d\cosh(dx+c)^2+2\,d\cosh(dx+c)\sinh(dx+c)+d\sinh(dx+c)^2+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (a^2*d*x*cosh(d*x + c)^2 + 2*a^2*d*x*cosh(d*x + c)*sinh(d*x + c) + a^2*d*x*
sinh(d*x + c)^2 + a^2*d*x - 2*b^2 + 4*(a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*x
 + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 + a*b)*arctan(cosh(d*x + c) + sin
h(d*x + c)))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(
d*x + c)^2 + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\operatorname{sech}(c+dx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x))**2, x)
```

Giac [A]
time = 0.39, size = 43, normalized size = 1.30

$$\frac{(dx+c)a^2+4\,ab\arctan\left(e^{(dx+c)}\right)-\frac{2\,b^2}{e^{(2\,dx+2\,c)}+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^2,x, algorithm="giac")
```

```
[Out] ((d*x + c)*a^2 + 4*a*b*arctan(e^(d*x + c)) - 2*b^2/(e^(2*d*x + 2*c) + 1))/d
```

Mupad [B]
time = 0.11, size = 70, normalized size = 2.12

$$a^2\,x-\frac{2\,b^2}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}+\frac{4\,\mathrm{atan}\left(\frac{a\,b\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{d^2}}{d\,\sqrt{a^2\,b^2}}\right)\,\sqrt{a^2\,b^2}}{\sqrt{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x))^2,x)
```

```
[Out] a^2*x - (2*b^2)/(d*(exp(2*c + 2*d*x) + 1)) + (4*atan((a*b*exp(d*x)*exp(c)*(
d^2)^(1/2))/(d*(a^2*b^2)^(1/2)))*(a^2*b^2)^(1/2))/(d^2)^(1/2)
```

3.90 $\int (a + b\text{sech}(c + dx))\, dx$

Optimal. Leaf size=16

$$ax + \frac{b\text{ArcTan}(\sinh(c + dx))}{d}$$

```
[Out] a*x+b*arctan(sinh(d*x+c))/d
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3855}

$$ax + \frac{b\text{ArcTan}(\sinh(c + dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sech[c + d*x],x]
```

```
[Out] a*x + (b*ArcTan[Sinh[c + d*x]])/d
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a + b\text{sech}(c + dx))\, dx &= ax + b\int \text{sech}(c + dx)\, dx \\ &= ax + \frac{b\tan^{-1}(\sinh(c + dx))}{d}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$ax + \frac{b\text{ArcTan}(\sinh(c + dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sech[c + d*x],x]
```

```
[Out] a*x + (b*ArcTan[Sinh[c + d*x]])/d
```

Maple [A]
time = 0.83, size = 17, normalized size = 1.06

method	result	size
default	$ax+\frac{b\arctan(\sinh(dx+c))}{d}$	17
derivativedivides	$\frac{(dx+c)a+b\arctan(\sinh(dx+c))}{d}$	22
risch	$ax+\frac{ib\ln(\mathrm{e}^{dx+c}+i)}{d}-\frac{ib\ln(\mathrm{e}^{dx+c}-i)}{d}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sech(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+b*arctan(sinh(d*x+c))/d
```

Maxima [A]
time = 0.27, size = 16, normalized size = 1.00

$$ax+\frac{b\arctan\left(\sinh\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sech(d*x+c),x, algorithm="maxima")
```

```
[Out] a*x + b*arctan(sinh(d*x + c))/d
```

Fricas [A]
time = 0.40, size = 26, normalized size = 1.62

$$\frac{adx+2\,b\arctan\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sech(d*x+c),x, algorithm="fricas")
```

```
[Out] (a*d*x + 2*b*arctan(cosh(d*x + c) + sinh(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\operatorname{sech}\left(c+dx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sech(d*x+c),x)
```

```
[Out] Integral(a + b*sech(c + d*x), x)
```

Giac [A]
time = 0.39, size = 17, normalized size = 1.06

$$ax + \frac{2 \, b \arctan \left(e^{(dx+c)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sech(d*x+c),x, algorithm="giac")
```

```
[Out] a*x + 2*b*arctan(e^(d*x + c))/d
```

Mupad [B]
time = 1.30, size = 38, normalized size = 2.38

$$a\,x + \frac{2\,\mathrm{atan}\left(\frac{b\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{d^2}}{d\,\sqrt{b^2}}\right)\,\sqrt{b^2}}{\sqrt{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b/cosh(c + d*x),x)
```

```
[Out] a*x + (2*atan((b*exp(d*x)*exp(c)*(d^2)^(1/2))/(d*(b^2)^(1/2)))*(b^2)^(1/2))
/(d^2)^(1/2)
```

3.91 $\int \frac{1}{a+b\,\mathrm{sech}(c+dx)}\,dx$

Optimal. Leaf size=59

$$\frac{x}{a}-\frac{2b\,\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\,\sqrt{a+b}\,d}$$

```
[Out] x/a-2*b*arctan((a-b)^(1/2)*tanh(1/2*d*x+1/2*c)/(a+b)^(1/2))/a/d/(a-b)^(1/2)
/(a+b)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3868, 2738, 214}

$$\frac{x}{a}-\frac{2b\,\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\,\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x])^(-1),x]
```

```
[Out] x/a - (2*b*ArcTan[(Sqrt[a - b]*Tanh[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a -
 b]*Sqrt[a + b]*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3868

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{a+b\operatorname{sech}(c+dx)}\,dx = \frac{x}{a} - \frac{\int \frac{1}{1+\frac{a\cosh(c+dx)}{b}}\,dx}{a}$$
$$= \frac{x}{a} + \frac{(2i)\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{ad}$$
$$= \frac{x}{a} - \frac{2b\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}\,d}$$

Mathematica [A]
time = 0.08, size = 60, normalized size = 1.02

$$\frac{\frac{c}{d}+x+\frac{2b\text{ArcTan}\left(\frac{(-a+b)\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x])^(-1),x]
[Out] (c/d + x + (2*b*ArcTan[((-a + b)*Tanh[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(Sqrt
[a^2 - b^2]*d))/a
```

Maple [A]
time = 1.66, size = 83, normalized size = 1.41

method	result	size
derivativedivides	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a}-\frac{2b\arctan\left(\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a\sqrt{(a+b)(a-b)}}}{d}$	83
default	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a}-\frac{2b\arctan\left(\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a\sqrt{(a+b)(a-b)}}}{d}$	83
risch	$\frac{x}{a}-\frac{b\ln\left(e^{dx+c}+\frac{b\sqrt{-a^2+b^2}+a^2-b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,da}+\frac{b\ln\left(e^{dx+c}+\frac{b\sqrt{-a^2+b^2}-a^2+b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,da}$	136

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sech(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a*ln(tanh(1/2*d*x+1/2*c)+1)-1/a*ln(tanh(1/2*d*x+1/2*c)-1)-2/a*b/((a+
b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.39, size = 270, normalized size = 4.58

$$\left[\frac{(a^2-b^2)dx-\sqrt{-a^2+b^2}\,b\log\left(\frac{a^2\cosh(dx+c)^2+a^2\sinh(dx+c)^2+2ab\cosh(dx+c)-a^2+2b^2+2(a^2\cosh(dx+c)+ab)\sinh(dx+c)+2\sqrt{-a^2+b^2}(a\cosh(dx+c)+a\sinh(dx+c)+b)}{a\cosh(dx+c)^2+a\sinh(dx+c)^2+2b\cosh(dx+c)+2(a\cosh(dx+c)+b)\sinh(dx+c)+a}\right)}{(a^3-ab^2)d}, \frac{(a^2-b^2)dx+2\sqrt{a^2-b^2}\,b\arctan\left(-\frac{a\cosh(dx+c)+a\sinh(dx+c)+b}{\sqrt{a^2-b^2}}\right)}{(a^3-ab^2)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)),x, algorithm="fricas")
```

```
[Out] [((a^2 - b^2)*d*x - sqrt(-a^2 + b^2)*b*log((a^2*cosh(d*x + c)^2 + a^2*sinh(
d*x + c)^2 + 2*a*b*cosh(d*x + c) - a^2 + 2*b^2 + 2*(a^2*cosh(d*x + c) + a*b
)*sinh(d*x + c) + 2*sqrt(-a^2 + b^2)*(a*cosh(d*x + c) + a*sinh(d*x + c) + b
))/(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*(a*cosh(d
*x + c) + b)*sinh(d*x + c) + a)))/((a^3 - a*b^2)*d), ((a^2 - b^2)*d*x + 2*s
qrt(a^2 - b^2)*b*arctan(-(a*cosh(d*x + c) + a*sinh(d*x + c) + b)/sqrt(a^2 -
 b^2)))/((a^3 - a*b^2)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\,\mathrm{sech}\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)),x)
```

```
[Out] Integral(1/(a + b*sech(c + d*x)), x)
```

Giac [A]
time = 0.39, size = 56, normalized size = 0.95

$$-\frac{\frac{2b\arctan\left(\frac{ae^{(dx+c)}+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,a}-\frac{dx+c}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)),x, algorithm="giac")
```

```
[Out] -(2*b*arctan((a*e^(d*x + c) + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a) - (d*
x + c)/a)/d
```

Mupad [B]
time = 0.40, size = 131, normalized size = 2.22

$$\frac{x}{a}+\frac{b\,\ln\left(\frac{2\,b\,{\mathrm{e}}^{c+d\,x}}{a^2}-\frac{2\,b\,\left(a+b\,{\mathrm{e}}^{c+d\,x}\right)}{a^2\,\sqrt{a+b}\,\sqrt{b-a}}\right)}{a\,d\,\sqrt{a+b}\,\sqrt{b-a}}-\frac{b\,\ln\left(\frac{2\,b\,{\mathrm{e}}^{c+d\,x}}{a^2}+\frac{2\,b\,\left(a+b\,{\mathrm{e}}^{c+d\,x}\right)}{a^2\,\sqrt{a+b}\,\sqrt{b-a}}\right)}{a\,d\,\sqrt{a+b}\,\sqrt{b-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cosh(c + d*x)),x)
```

```
[Out] x/a + (b*log((2*b*exp(c + d*x))/a^2 - (2*b*(a + b*exp(c + d*x)))/(a^2*(a +
b)^(1/2)*(b - a)^(1/2))))/(a*d*(a + b)^(1/2)*(b - a)^(1/2)) - (b*log((2*b*e
xp(c + d*x))/a^2 + (2*b*(a + b*exp(c + d*x)))/(a^2*(a + b)^(1/2)*(b - a)^(1
/2))))/(a*d*(a + b)^(1/2)*(b - a)^(1/2))
```

3.92 $\int \frac{1}{(a+b\,\mathbf{sech}(c+dx))^2}\,dx$

Optimal. Leaf size=109

$$\frac{x}{a^2}-\frac{2b(2a^2-b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d}+\frac{b^2\tanh(c+dx)}{a\,(a^2-b^2)\,d(a+b\mathrm{sech}(c+dx))}$$

```
[Out] x/a^2-2*b*(2*a^2-b^2)*arctan((a-b)^(1/2)*tanh(1/2*d*x+1/2*c)/(a+b)^(1/2))/a
^2/(a-b)^(3/2)/(a+b)^(3/2)/d+b^2*tanh(d*x+c)/a/(a^2-b^2)/d/(a+b*sech(d*x+c)
)
```

Rubi [A]
time = 0.12, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3870, 4004, 3916, 2738, 214}

$$-\frac{2b(2a^2-b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{3/2}(a+b)^{3/2}}+\frac{b^2\tanh(c+dx)}{ad\,(a^2-b^2)\,(a+b\mathrm{sech}(c+dx))}+\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x])^(-2),x]
```

```
[Out] x/a^2 - (2*b*(2*a^2 - b^2)*ArcTan[(Sqrt[a - b]*Tanh[(c + d*x)/2])/Sqrt[a +
b]])/(a^2*(a - b)^(3/2)*(a + b)^(3/2)*d) + (b^2*Tanh[c + d*x])/(a*(a^2 - b^
2)*d*(a + b*Sech[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
```

```
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+b\operatorname{sech}(c+dx))^2}\,dx &= \frac{b^2\tanh(c+dx)}{a\left(a^2-b^2\right)d(a+b\operatorname{sech}(c+dx))} - \frac{\int \frac{-a^2+b^2+ab\operatorname{sech}(c+dx)}{a+b\operatorname{sech}(c+dx)}\,dx}{a\left(a^2-b^2\right)} \\
&= \frac{x}{a^2} + \frac{b^2\tanh(c+dx)}{a\left(a^2-b^2\right)d(a+b\operatorname{sech}(c+dx))} - \frac{\left(b\left(2a^2-b^2\right)\right)\int \frac{\operatorname{sech}(c+dx)}{a+b\operatorname{sech}(c+dx)}\,dx}{a^2\left(a^2-b^2\right)} \\
&= \frac{x}{a^2} + \frac{b^2\tanh(c+dx)}{a\left(a^2-b^2\right)d(a+b\operatorname{sech}(c+dx))} - \frac{\left(2a^2-b^2\right)\int \frac{1}{1+\frac{a\cosh(c+dx)}{b}}\,dx}{a^2\left(a^2-b^2\right)} \\
&= \frac{x}{a^2} + \frac{b^2\tanh(c+dx)}{a\left(a^2-b^2\right)d(a+b\operatorname{sech}(c+dx))} + \frac{\left(2i\left(2a^2-b^2\right)\right)\operatorname{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx \right.}{a^2\left(a^2-b^2\right)d} \\
&= \frac{x}{a^2} - \frac{2b\left(2a^2-b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d} + \frac{b^2\tanh(c+dx)}{a\left(a^2-b^2\right)d(a+b\operatorname{sech}(c+
\end{aligned}$$

Mathematica [A]
time = 0.28, size = 203, normalized size = 1.86

$$\frac{a\left(\left(a^2-b^2\right)^{3/2}(c+dx)+\left(4a^2b-2b^3\right)\operatorname{ArcTan}\left(\frac{(-a+b)\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)\right)\cosh(c+dx)+b\left(\left(a^2-b^2\right)^{3/2}(c+dx)+\left(4a^2b-2b^3\right)\operatorname{ArcTan}\left(\frac{(-a+b)\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)+ab\sqrt{a^2-b^2}\sinh(c+dx)\right)}{a^2(a-b)(a+b)\sqrt{a^2-b^2}\,d(b+a\cosh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x])^(-2),x]
```

[Out] (a*((a^2 - b^2)^(3/2)*(c + d*x) + (4*a^2*b - 2*b^3)*ArcTan[((-a + b)*Tanh[(c + d*x)/2])/Sqrt[a^2 - b^2]])*Cosh[c + d*x] + b*((a^2 - b^2)^(3/2)*(c + d*x) + (4*a^2*b - 2*b^3)*ArcTan[((-a + b)*Tanh[(c + d*x)/2])/Sqrt[a^2 - b^2]] + a*b*Sqrt[a^2 - b^2]*Sinh[c + d*x]))/(a^2*(a - b)*(a + b)*Sqrt[a^2 - b^2]*d*(b + a*Cosh[c + d*x]))

Maple [A]
time = 1.50, size = 164, normalized size = 1.50

method	result
derivativedivides	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2}-\frac{2b\left(-\frac{ab\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a^2-b^2)\left(a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)}+\frac{(2a^2-b^2)\arctan\left(\right.}{(a+b)(a-b)\sqrt{}}\right.}{a^2}}{d}$
default	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2}-\frac{2b\left(-\frac{ab\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a^2-b^2)\left(a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)}+\frac{(2a^2-b^2)\arctan\left(\right.}{(a+b)(a-b)\sqrt{}}\right.}{a^2}}{d}$
risch	$\frac{x}{a^2}-\frac{2b^2(b\,e^{dx+c}+a)}{d\,a^2(a^2-b^2)(a\,e^{2dx+2c}+2b\,e^{dx+c}+a)}-\frac{2b\ln\left(e^{dx+c}+\frac{b\sqrt{-a^2+b^2}+a^2-b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)d}+\frac{b^3\ln\left(e^{dx+c}+\frac{b\sqrt{-a^2}}{\sqrt{-}}\right.}{\sqrt{-a^2+b^2}\,(a}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*sech(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(1/a^2*ln(tanh(1/2*d*x+1/2*c)+1)-1/a^2*ln(tanh(1/2*d*x+1/2*c)-1)-2*b/a^2*(-1/(a^2-b^2)*a*b*tanh(1/2*d*x+1/2*c)/(a*tanh(1/2*d*x+1/2*c)^2-b*tanh(1/2*d*x+1/2*c)^2+a+b)+(2*a^2-b^2)/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))))

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sech(d*x+c))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for more de

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 544 vs. $2(100) = 200$.
time = 0.41, size = 1207, normalized size = 11.07

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-(2*a^3*b^2 - 2*a*b^4 - (a^5 - 2*a^3*b^2 + a*b^4)*d*x*cosh(d*x + c)^2 - (a
^5 - 2*a^3*b^2 + a*b^4)*d*x*sinh(d*x + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4)*d*x
 + (2*a^3*b - a*b^3 + (2*a^3*b - a*b^3)*cosh(d*x + c)^2 + (2*a^3*b - a*b^3)
*sinh(d*x + c)^2 + 2*(2*a^2*b^2 - b^4)*cosh(d*x + c) + 2*(2*a^2*b^2 - b^4 +
 (2*a^3*b - a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 + b^2)*log((a^2*
cosh(d*x + c)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) - a^2 + 2*b^2 +
 2*(a^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(-a^2 + b^2)*(a*cosh(d*x
 + c) + a*sinh(d*x + c) + b))/(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + 2*b*
cosh(d*x + c) + 2*(a*cosh(d*x + c) + b)*sinh(d*x + c) + a)) + 2*(a^2*b^3 -
b^5 - (a^4*b - 2*a^2*b^3 + b^5)*d*x)*cosh(d*x + c) + 2*(a^2*b^3 - b^5 - (a^
5 - 2*a^3*b^2 + a*b^4)*d*x*cosh(d*x + c) - (a^4*b - 2*a^2*b^3 + b^5)*d*x)*s
inh(d*x + c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x + c)^2 + (a^7 - 2*a^5
*b^2 + a^3*b^4)*d*sinh(d*x + c)^2 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*d*cosh(
d*x + c) + (a^7 - 2*a^5*b^2 + a^3*b^4)*d + 2*((a^7 - 2*a^5*b^2 + a^3*b^4)*d
*cosh(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d)*sinh(d*x + c)), -(2*a^3*b
^2 - 2*a*b^4 - (a^5 - 2*a^3*b^2 + a*b^4)*d*x*cosh(d*x + c)^2 - (a^5 - 2*a^3
*b^2 + a*b^4)*d*x*sinh(d*x + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4)*d*x - 2*(2*a^
3*b - a*b^3 + (2*a^3*b - a*b^3)*cosh(d*x + c)^2 + (2*a^3*b - a*b^3)*sinh(d*
x + c)^2 + 2*(2*a^2*b^2 - b^4)*cosh(d*x + c) + 2*(2*a^2*b^2 - b^4 + (2*a^3*
b - a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cosh(d*
x + c) + a*sinh(d*x + c) + b)/sqrt(a^2 - b^2)) + 2*(a^2*b^3 - b^5 - (a^4*b
- 2*a^2*b^3 + b^5)*d*x)*cosh(d*x + c) + 2*(a^2*b^3 - b^5 - (a^5 - 2*a^3*b^2
 + a*b^4)*d*x*cosh(d*x + c) - (a^4*b - 2*a^2*b^3 + b^5)*d*x)*sinh(d*x + c))
/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x + c)^2 + (a^7 - 2*a^5*b^2 + a^3*b^
4)*d*sinh(d*x + c)^2 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*d*cosh(d*x + c) + (a
^7 - 2*a^5*b^2 + a^3*b^4)*d + 2*((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x + c
) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d)*sinh(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\operatorname{sech}\left(c + dx\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x))**(-2), x)
```

Giac [A]
time = 0.40, size = 134, normalized size = 1.23

$$-\frac{\frac{2\,\left(2\,a^{2}b-b^{3}\right)\arctan\left(\frac{ae^{(dx+c)}+b}{\sqrt{a^{2}-b^{2}}}\right)}{\left(a^{4}-a^{2}b^{2}\right)\sqrt{a^{2}-b^{2}}}+\frac{2\,\left(b^{3}e^{(dx+c)}+ab^{2}\right)}{\left(a^{4}-a^{2}b^{2}\right)\left(ae^{(2\,dx+2\,c)}+2\,be^{(dx+c)}+a\right)}-\frac{dx+c}{a^{2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(2*(2*a^2*b - b^3)*arctan((a*e^(d*x + c) + b)/sqrt(a^2 - b^2))/((a^4 - a^2
*b^2)*sqrt(a^2 - b^2)) + 2*(b^3*e^(d*x + c) + a*b^2)/((a^4 - a^2*b^2)*(a*e^
(2*d*x + 2*c) + 2*b*e^(d*x + c) + a)) - (d*x + c)/a^2)/d
```

Mupad [B]
time = 1.85, size = 296, normalized size = 2.72

$$\frac{\frac{2\,b^{2}}{d\,\left(a\,b^{2}-a^{3}\right)}+\frac{2\,b^{3}\,\mathrm{e}^{c+d\,x}}{a\,d\,\left(a\,b^{2}-a^{3}\right)}}{a+2\,b\,\mathrm{e}^{c+d\,x}+a\,\mathrm{e}^{2\,c+2\,d\,x}}+\frac{x}{a^{2}}+\frac{b\,\ln\left(\frac{2\,\mathrm{e}^{c+d\,x}\,\left(2\,a^{2}\,b-b^{3}\right)}{a^{3}\,\left(a^{2}-b^{2}\right)}-\frac{2\,b\,\left(2\,a^{2}-b^{2}\right)\,\left(a+b\,\mathrm{e}^{c+d\,x}\right)}{a^{3}\,{\left(a+b\right)}^{3/2}\,{\left(b-a\right)}^{3/2}}\right)\,\left(2\,a^{2}-b^{2}\right)}{a^{2}\,d\,{\left(a+b\right)}^{3/2}\,{\left(b-a\right)}^{3/2}}-\frac{b\,\ln\left(\frac{2\,\mathrm{e}^{c+d\,x}\,\left(2\,a^{2}\,b-b^{3}\right)}{a^{3}\,\left(a^{2}-b^{2}\right)}+\frac{2\,b\,\left(2\,a^{2}-b^{2}\right)\,\left(a+b\,\mathrm{e}^{c+d\,x}\right)}{a^{3}\,{\left(a+b\right)}^{3/2}\,{\left(b-a\right)}^{3/2}}\right)\,\left(2\,a^{2}-b^{2}\right)}{a^{2}\,d\,{\left(a+b\right)}^{3/2}\,{\left(b-a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cosh(c + d*x))^2,x)
```

```
[Out] ((2*b^2)/(d*(a*b^2 - a^3)) + (2*b^3*exp(c + d*x))/(a*d*(a*b^2 - a^3)))/(a +
 2*b*exp(c + d*x) + a*exp(2*c + 2*d*x)) + x/a^2 + (b*log((2*exp(c + d*x)*(2
*a^2*b - b^3))/(a^3*(a^2 - b^2)) - (2*b*(2*a^2 - b^2)*(a + b*exp(c + d*x)))
/(a^3*(a + b)^(3/2)*(b - a)^(3/2)))*(2*a^2 - b^2))/(a^2*d*(a + b)^(3/2)*(b
- a)^(3/2)) - (b*log((2*exp(c + d*x)*(2*a^2*b - b^3))/(a^3*(a^2 - b^2)) + (
2*b*(2*a^2 - b^2)*(a + b*exp(c + d*x)))/(a^3*(a + b)^(3/2)*(b - a)^(3/2)))*
(2*a^2 - b^2))/(a^2*d*(a + b)^(3/2)*(b - a)^(3/2))
```

3.93 $\int \frac{1}{(a+b\,\text{sech}(c+dx))^3}\,dx$

Optimal. Leaf size=173

$$\frac{x}{a^3}-\frac{b(6a^4-5a^2b^2+2b^4)\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{5/2}(a+b)^{5/2}d}+\frac{b^2\tanh(c+dx)}{2a\,(a^2-b^2)\,d(a+b\text{sech}(c+dx))^2}+\frac{b^2(5a^2-}{2a^2\,(a^2-b^2)^{}}$$

```
[Out] x/a^3-b*(6*a^4-5*a^2*b^2+2*b^4)*arctan((a-b)^(1/2)*tanh(1/2*d*x+1/2*c)/(a+b
)^(1/2))/a^3/(a-b)^(5/2)/(a+b)^(5/2)/d+1/2*b^2*tanh(d*x+c)/a/(a^2-b^2)/d/(a
+b*sech(d*x+c))^2+1/2*b^2*(5*a^2-2*b^2)*tanh(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*
sech(d*x+c))
```

Rubi [A]

time = 0.24, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3870, 4145, 4004, 3916, 2738, 214}

$$\frac{x}{a^3}+\frac{b^2(5a^2-2b^2)\tanh(c+dx)}{2a^2d\,(a^2-b^2)^2\,(a+b\text{sech}(c+dx))}+\frac{b^2\tanh(c+dx)}{2ad\,(a^2-b^2)\,(a+b\text{sech}(c+dx))^2}-\frac{b(6a^4-5a^2b^2+2b^4)\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x])^(-3),x]
```

```
[Out] x/a^3 - (b*(6*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTan[(Sqrt[a - b]*Tanh[(c + d*x)/2
])/Sqrt[a + b]])/(a^3*(a - b)^(5/2)*(a + b)^(5/2)*d) + (b^2*Tanh[c + d*x])/
(2*a*(a^2 - b^2)*d*(a + b*Sech[c + d*x])^2) + (b^2*(5*a^2 - 2*b^2)*Tanh[c +
 d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Sech[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
```

```
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4145

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+b\text{sech}(c+dx))^3}\,dx = \frac{b^2\tanh(c+dx)}{2a\,(a^2-b^2)\,d(a+b\text{sech}(c+dx))^2} - \frac{\int \frac{-2(a^2-b^2)+2ab\text{sech}(c+dx)-b^2\text{sech}^2(c+dx)}{(a+b\text{sech}(c+dx))^2}\,dx}{2a\,(a^2-b^2)}$$

$$= \frac{b^2\tanh(c+dx)}{2a\,(a^2-b^2)\,d(a+b\text{sech}(c+dx))^2} + \frac{b^2(5a^2-2b^2)\tanh(c+dx)}{2a^2\,(a^2-b^2)^2\,d(a+b\text{sech}(c+dx))} + \frac{\int 2\ldots}{\ldots}$$

$$= \frac{x}{a^3} + \frac{b^2\tanh(c+dx)}{2a\,(a^2-b^2)\,d(a+b\text{sech}(c+dx))^2} + \frac{b^2(5a^2-2b^2)\tanh(c+dx)}{2a^2\,(a^2-b^2)^2\,d(a+b\text{sech}(c+dx))}$$

$$= \frac{x}{a^3} + \frac{b^2\tanh(c+dx)}{2a\,(a^2-b^2)\,d(a+b\text{sech}(c+dx))^2} + \frac{b^2(5a^2-2b^2)\tanh(c+dx)}{2a^2\,(a^2-b^2)^2\,d(a+b\text{sech}(c+dx))}$$

$$= \frac{x}{a^3} + \frac{b^2\tanh(c+dx)}{2a\,(a^2-b^2)\,d(a+b\text{sech}(c+dx))^2} + \frac{b^2(5a^2-2b^2)\tanh(c+dx)}{2a^2\,(a^2-b^2)^2\,d(a+b\text{sech}(c+dx))}$$

$$= \frac{x}{a^3} - \frac{b(6a^4-5a^2b^2+2b^4)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{5/2}(a+b)^{5/2}d} + \frac{b^2\tanh(c\ldots}{2a\,(a^2-b^2)\,d(a+\ldots}$$

Mathematica [A]
time = 0.48, size = 205, normalized size = 1.18

$$\frac{(b+a\cosh(c+dx))\text{sech}^3(c+dx)\left(2(c+dx)(b+a\cosh(c+dx))^2+\frac{2b(6a^4-5a^2b^2+2b^4)\text{ArcTan}\left(\frac{(-a+b)\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)(b+a\cosh(c+dx))^2}{(a^2-b^2)^{5/2}}+\frac{ab^3\sinh(c+dx)}{(-a+b)(a+b)}+\frac{3ab^2(2a^2-b^2)(b+a\cosh(c+dx))\sinh(c+dx)}{(a-b)^2(a+b)^2}\right)}{2a^3d(a+b\text{sech}(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x])^(-3),x]
```

```
[Out] ((b + a*Cosh[c + d*x])*Sech[c + d*x]^3*(2*(c + d*x)*(b + a*Cosh[c + d*x])^2
 + (2*b*(6*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTan[((-a + b)*Tanh[(c + d*x)/2])/Sqr
t[a^2 - b^2]]*(b + a*Cosh[c + d*x])^2)/(a^2 - b^2)^(5/2) + (a*b^3*Sinh[c +
d*x])/((-a + b)*(a + b)) + (3*a*b^2*(2*a^2 - b^2)*(b + a*Cosh[c + d*x])*Sin
h[c + d*x])/((a - b)^2*(a + b)^2)))/(2*a^3*d*(a + b*Sech[c + d*x])^3)
```

Maple [A]
time = 1.78, size = 251, normalized size = 1.45

method	result

derivativedivides	$-\frac{2b\left(\frac{-\frac{\left(6a^2+ab-2b^2\right)ab\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^2+2ab+b^2\right)}-\frac{\left(6a^2-ab-2b^2\right)ab\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ab+b^2\right)}}{\left(a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^2}+\frac{\left(6a^4-5a^2b^2+2b^4\right)\arctan\left(\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-}}\right.}{2\left(a^4-2a^2b^2+b^4\right)\sqrt{(a+b)(a-b)}}\right.}{a^3}}{d}$
default	$-\frac{2b\left(\frac{-\frac{\left(6a^2+ab-2b^2\right)ab\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^2+2ab+b^2\right)}-\frac{\left(6a^2-ab-2b^2\right)ab\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ab+b^2\right)}}{\left(a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^2}+\frac{\left(6a^4-5a^2b^2+2b^4\right)\arctan\left(\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-}}\right.}{2\left(a^4-2a^2b^2+b^4\right)\sqrt{(a+b)(a-b)}}\right.}{a^3}}{d}$
risch	$\frac{x}{a^3}-\frac{b^2\left(7a^3b\,e^{3dx+3c}-4a\,b^3e^{3dx+3c}+6a^4e^{2dx+2c}+9a^2b^2e^{2dx+2c}-6b^4e^{2dx+2c}+17a^3b\,e^{dx+c}-8b^3e^{dx+c}a+6a^4-3a^2b^2\right.}{a^3\left(a^2-b^2\right)^2d\left(a\,e^{2dx+2c}+2b\,e^{dx+c}+a\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sech(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/a^3*b*((-1/2*(6*a^2+a*b-2*b^2)*a*b/(a-b)/(a^2+2*a*b+b^2)*tanh(1/2*d
*x+1/2*c)^3-1/2*(6*a^2-a*b-2*b^2)*a*b/(a+b)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/
2*c))/(a*tanh(1/2*d*x+1/2*c)^2-b*tanh(1/2*d*x+1/2*c)^2+a+b)^2+1/2*(6*a^4-5*
a^2*b^2+2*b^4)/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/
2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))-1/a^3*ln(tanh(1/2*d*x+1/2*c)-1)+1/a^3*ln
(tanh(1/2*d*x+1/2*c)+1))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2000 vs. 2(160) = 320.
time = 0.44, size = 4125, normalized size = 23.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))^3,x, algorithm="fricas")

[Out] [-1/2*(12*a^6*b^2 - 18*a^4*b^4 + 6*a^2*b^6 - 2*(a^8 - 3*a^6*b^2 + 3*a^4*b^4
 - a^2*b^6)*d*x*cosh(d*x + c)^4 - 2*(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)
*d*x*sinh(d*x + c)^4 + 2*(7*a^5*b^3 - 11*a^3*b^5 + 4*a*b^7 - 4*(a^7*b - 3*a
^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x)*cosh(d*x + c)^3 + 2*(7*a^5*b^3 - 11*a^3*b^
5 + 4*a*b^7 - 4*(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*x*cosh(d*x + c) -
 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x)*sinh(d*x + c)^3 - 2*(a^8 -
3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*x + 2*(6*a^6*b^2 + 3*a^4*b^4 - 15*a^2*b^
6 + 6*b^8 - 2*(a^8 - a^6*b^2 - 3*a^4*b^4 + 5*a^2*b^6 - 2*b^8)*d*x)*cosh(d*x
 + c)^2 + 2*(6*a^6*b^2 + 3*a^4*b^4 - 15*a^2*b^6 + 6*b^8 - 6*(a^8 - 3*a^6*b^
2 + 3*a^4*b^4 - a^2*b^6)*d*x*cosh(d*x + c)^2 - 2*(a^8 - a^6*b^2 - 3*a^4*b^4
 + 5*a^2*b^6 - 2*b^8)*d*x + 3*(7*a^5*b^3 - 11*a^3*b^5 + 4*a*b^7 - 4*(a^7*b
- 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x)*cosh(d*x + c))*sinh(d*x + c)^2 + (6*a
^6*b - 5*a^4*b^3 + 2*a^2*b^5 + (6*a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*cosh(d*x +
 c)^4 + (6*a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*sinh(d*x + c)^4 + 4*(6*a^5*b^2 -
5*a^3*b^4 + 2*a*b^6)*cosh(d*x + c)^3 + 4*(6*a^5*b^2 - 5*a^3*b^4 + 2*a*b^6 +
 (6*a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(6*a^
6*b + 7*a^4*b^3 - 8*a^2*b^5 + 4*b^7)*cosh(d*x + c)^2 + 2*(6*a^6*b + 7*a^4*b
^3 - 8*a^2*b^5 + 4*b^7 + 3*(6*a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*cosh(d*x + c)^
2 + 6*(6*a^5*b^2 - 5*a^3*b^4 + 2*a*b^6)*cosh(d*x + c))*sinh(d*x + c)^2 + 4*
(6*a^5*b^2 - 5*a^3*b^4 + 2*a*b^6)*cosh(d*x + c) + 4*(6*a^5*b^2 - 5*a^3*b^4
+ 2*a*b^6 + (6*a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*cosh(d*x + c)^3 + 3*(6*a^5*b^
2 - 5*a^3*b^4 + 2*a*b^6)*cosh(d*x + c)^2 + (6*a^6*b + 7*a^4*b^3 - 8*a^2*b^5
 + 4*b^7)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 + b^2)*log((a^2*cosh(d*x
+ c)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) - a^2 + 2*b^2 + 2*(a^2*c
osh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(-a^2 + b^2)*(a*cosh(d*x + c) + a
*sinh(d*x + c) + b))/(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + 2*b*cosh(d*x
+ c) + 2*(a*cosh(d*x + c) + b)*sinh(d*x + c) + a)) + 2*(17*a^5*b^3 - 25*a^3
*b^5 + 8*a*b^7 - 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x)*cosh(d*x +
c) + 2*(17*a^5*b^3 - 25*a^3*b^5 + 8*a*b^7 - 4*(a^8 - 3*a^6*b^2 + 3*a^4*b^4
- a^2*b^6)*d*x*cosh(d*x + c)^3 - 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*
d*x + 3*(7*a^5*b^3 - 11*a^3*b^5 + 4*a*b^7 - 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^
5 - a*b^7)*d*x)*cosh(d*x + c)^2 + 2*(6*a^6*b^2 + 3*a^4*b^4 - 15*a^2*b^6 + 6
*b^8 - 2*(a^8 - a^6*b^2 - 3*a^4*b^4 + 5*a^2*b^6 - 2*b^8)*d*x)*cosh(d*x + c)
)*sinh(d*x + c))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*cosh(d*x + c)^
4 + (a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*sinh(d*x + c)^4 + 4*(a^10*b
- 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*cosh(d*x + c)^3 + 2*(a^11 - a^9*b^2 -
3*a^7*b^4 + 5*a^5*b^6 - 2*a^3*b^8)*d*cosh(d*x + c)^2 + 4*((a^11 - 3*a^9*b^2
 + 3*a^7*b^4 - a^5*b^6)*d*cosh(d*x + c) + (a^10*b - 3*a^8*b^3 + 3*a^6*b^5 -
 a^4*b^7)*d)*sinh(d*x + c)^3 + 4*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)
*d*cosh(d*x + c) + 2*(3*(a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*cosh(d*x
 + c)^2 + 6*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*cosh(d*x + c) + (a
^11 - a^9*b^2 - 3*a^7*b^4 + 5*a^5*b^6 - 2*a^3*b^8)*d)*sinh(d*x + c)^2 + (a^
11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d + 4*((a^11 - 3*a^9*b^2 + 3*a^7*b^4
```

```
- a^5*b^6)*d*cosh(d*x + c)^3 + 3*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)
*d*cosh(d*x + c)^2 + (a^11 - a^9*b^2 - 3*a^7*b^4 + 5*a^5*b^6 - 2*a^3*b^8)*d
*cosh(d*x + c) + (a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d)*sinh(d*x + c
)), -(6*a^6*b^2 - 9*a^4*b^4 + 3*a^2*b^6 - (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^
2*b^6)*d*x*cosh(d*x + c)^4 - (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*x*si
nh(d*x + c)^4 + (7*a^5*b^3 - 11*a^3*b^5 + 4*a*b^7 - 4*(a^7*b - 3*a^5*b^3 +
3*a^3*b^5 - a*b^7)*d*x)*cosh(d*x + c)^3 + (7*a^5*b^3 - 11*a^3*b^5 + 4*a*b^7
 - 4*(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*x*cosh(d*x + c) - 4*(a^7*b -
 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x)*sinh(d*x + c)^3 - (a^8 - 3*a^6*b^2 + 3
*a^4*b^4 - a^2*b^6)*d*x + (6*a^6*b^2 + 3*a^4*b^4 - 15*a^2*b^6 + 6*b^8 - 2*(
a^8 - a^6*b^2 - 3*a^4*b^4 + 5*a^2*b^6 - 2*b^8)*d*x)*cosh(d*x + c)^2 + (6*a^
6*b^2 + 3*a^4*b^4 - 15*a^2*b^6 + 6*b^8 - 6*(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a
^2*b^6)*d*x*cosh(d*x + c)^2 - 2*(a^8 - a^6*b^2 - 3*a^4*b^4 + 5*a^2*b^6 - 2*
b^8)*d*x + 3*(7*a^5*b^3 - 11*a^3*b^5 + 4*a*b^7 - 4*(a^7*b - 3*a^5*b^3 + 3*a
^3*b^5 - a*b^7)*d*x)*cosh(d*x + c))*sinh(d*x + c)^2 - (6*a^6*b - 5*a^4*b^3
+ 2*a^2*b^5 + (6*a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*cosh(d*x + c)^4 + (6*a^6*b
- 5*a^4*b^3 + 2*a^2*b^5)*sinh(d*x + c)^4 + 4*(6*a^5*b^2 - 5*a^3*b^4 + 2*a*b
^6)*cosh(d*x + c)^3 + 4*(6*a^5*b^2 - 5*a^3*b^4 + 2*a*b^6 + (6*a^6*b - 5*a^4
*b^3 + 2*a^2*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(6*a^6*b + 7*a^4*b^3 -
 8*a^2*b^5 + 4*b^7)*cosh(d*x + c)^2 + 2*(6*a^6*b + 7*a^4*b^3 - 8*a^2*b^5 +
4*b^7 + 3*(6*a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*cosh(d*x + c)^2 + 6*(6*a^5*b^2
- 5*a^3*b^4 + 2*a*b^6)*cosh(d*x + c))*sinh(d*x + c)^2 + 4*(6*a^5*b^2 - 5*a^
3*b^4 + 2*a*b^6)*cosh(d*x + c) + 4*(6*a^5*b^2 - 5*a^3*b^4 + 2*a*b^6 + (6*a^
6*b - 5*a^4*b^3 + 2*a^2*b^5)*cosh(d*x + c)^3 + ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\operatorname{sech}\left(c + dx\right)\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sech(c + d*x))**(-3), x)
```

Giac [A]
time = 0.41, size = 261, normalized size = 1.51

$$-\frac{\frac{\left(6\,a^{4}b-5\,a^{2}b^{3}+2\,b^{5}\right)\arctan\left(\frac{ae^{(dx+c)}+b}{\sqrt{a^{2}-b^{2}}}\right)}{\left(a^{7}-2\,a^{5}b^{2}+a^{3}b^{4}\right)\sqrt{a^{2}-b^{2}}}+\frac{7\,a^{3}b^{3}e^{(3\,dx+3\,c)}-4\,ab^{5}e^{(3\,dx+3\,c)}+6\,a^{4}b^{2}e^{(2\,dx+2\,c)}+9\,a^{2}b^{4}e^{(2\,dx+2\,c)}-6\,b^{6}e^{(2\,dx+2\,c)}+17\,a^{3}b^{3}e^{(dx+c)}-8\,ab^{5}e^{(dx+c)}+6\,a^{4}b^{2}-3\,a^{2}b^{4}}{\left(a^{7}-2\,a^{5}b^{2}+a^{3}b^{4}\right)\left(ae^{(2\,dx+2\,c)}+2\,be^{(dx+c)}+a\right)^{2}}-\frac{dx+c}{a^{3}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -((6*a^4*b - 5*a^2*b^3 + 2*b^5)*arctan((a*e^(d*x + c) + b)/sqrt(a^2 - b^2))
/((a^7 - 2*a^5*b^2 + a^3*b^4)*sqrt(a^2 - b^2)) + (7*a^3*b^3*e^(3*d*x + 3*c)
 - 4*a*b^5*e^(3*d*x + 3*c) + 6*a^4*b^2*e^(2*d*x + 2*c) + 9*a^2*b^4*e^(2*d*x
 + 2*c) - 6*b^6*e^(2*d*x + 2*c) + 17*a^3*b^3*e^(d*x + c) - 8*a*b^5*e^(d*x +
 c) + 6*a^4*b^2 - 3*a^2*b^4)/((a^7 - 2*a^5*b^2 + a^3*b^4)*(a*e^(2*d*x + 2*c
) + 2*b*e^(d*x + c) + a)^2) - (d*x + c)/a^3)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{b}{\cosh(c+d\,x)}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cosh(c + d*x))^3,x)
```

```
[Out] int(1/(a + b/cosh(c + d*x))^3, x)
```

3.94 $\int \frac{1}{\sqrt{a+b\,\mathrm{sech}(c+dx)}}\,dx$

Optimal. Leaf size=106

$$\frac{2\sqrt{a+b}\,\coth(c+dx)\Pi\left(\frac{a+b}{a};\mathrm{ArcSin}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\mathrm{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\mathrm{sech}}{a-}}}{ad}$$

```
[Out] 2*coth(d*x+c)*EllipticPi((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)
/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+
c))/(a-b))^(1/2)/a/d
```

Rubi [A]
time = 0.02, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3869}

$$\frac{2\sqrt{a+b}\,\coth(c+dx)\sqrt{\frac{b(1-\mathrm{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\mathrm{sech}(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\mathrm{ArcSin}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sech[c
 + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b
)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a*d)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\,\mathrm{sech}(c+dx)}}\,dx = \frac{2\sqrt{a+b}\,\coth(c+dx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-}{}}}{ad}$$

Mathematica [A]
time = 3.43, size = 168, normalized size = 1.58

$$\frac{2b\sqrt{b+a\cosh(c+dx)}\,\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a}\,\sqrt{b+a\cosh(c+dx)}}{\sqrt{a+b}\,\sqrt{a\cosh(c+dx)}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1+\text{sech}(c+dx))}{-a+b}}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a}\,\sqrt{a+b}\,d\sqrt{a\cosh(c+dx)}\,\sqrt{-\frac{b(-1+\text{sech}(c+dx))}{a+b}}\,\sqrt{a+b\text{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*b*Sqrt[b + a*Cosh[c + d*x]]*EllipticPi[(a + b)/a, ArcSin[(Sqrt[a]*Sqrt[b
 + a*Cosh[c + d*x]])/(Sqrt[a + b]*Sqrt[a*Cosh[c + d*x]])], (a + b)/(a - b)]
*Sqrt[(b*(1 + Sech[c + d*x]))/(-a + b)]*Tanh[(c + d*x)/2])/(Sqrt[a]*Sqrt[a
+ b]*d*Sqrt[a*Cosh[c + d*x]]*Sqrt[-((b*(-1 + Sech[c + d*x]))/(a + b))]*Sqrt
[a + b*Sech[c + d*x]])
```

Maple [F]
time = 2.61, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\,\text{sech}\,(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sech(d*x+c))^(1/2),x)
```

```
[Out] int(1/(a+b*sech(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sech(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*sech(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\operatorname{sech}(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))**(1/2),x)
[Out] Integral(1/sqrt(a + b*sech(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(b*sech(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a + \frac{b}{\cosh(c + d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cosh(c + d*x))^(1/2),x)
[Out] int(1/(a + b/cosh(c + d*x))^(1/2), x)
```

3.95 $\int \frac{\cosh^4(x)}{a+b\mathbf{sech}(x)}\, dx$

Optimal. Leaf size=146

$$\frac{(3a^4+4a^2b^2+8b^4)\,x}{8a^5}-\frac{2b^5\,\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^5\sqrt{a-b}\,\sqrt{a+b}}-\frac{b(2a^2+3b^2)\sinh(x)}{3a^4}+\frac{(3a^2+4b^2)\cosh(x)\sinh(x)}{8a^3}-\frac{b}{}$$

[Out] 1/8*(3*a^4+4*a^2*b^2+8*b^4)*x/a^5-1/3*b*(2*a^2+3*b^2)*sinh(x)/a^4+1/8*(3*a^2+4*b^2)*cosh(x)*sinh(x)/a^3-1/3*b*cosh(x)^2*sinh(x)/a^2+1/4*cosh(x)^3*sinh(x)/a-2*b^5*arctan((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/a^5/(a-b)^(1/2)/(a+b)^(1/2)

Rubi [A]
time = 0.45, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$,
Rules used = {3938, 4189, 4004, 3916, 2738, 211}

$$-\frac{2b^5\,\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^5\sqrt{a-b}\,\sqrt{a+b}}-\frac{b\sinh(x)\cosh^2(x)}{3a^2}-\frac{b(2a^2+3b^2)\sinh(x)}{3a^4}+\frac{(3a^2+4b^2)\sinh(x)\cosh(x)}{8a^3}+\frac{x(3a^4+4a^2b^2+8b^4)}{8a^5}+\frac{\sinh(x)\cosh^3(x)}{4a}$$

Antiderivative was successfully verified.

[In] Int[Cosh[x]^4/(a + b*Sech[x]),x]

[Out] ((3*a^4 + 4*a^2*b^2 + 8*b^4)*x)/(8*a^5) - (2*b^5*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(a^5*Sqrt[a - b]*Sqrt[a + b]) - (b*(2*a^2 + 3*b^2)*Sinh[x])/(3*a^4) + ((3*a^2 + 4*b^2)*Cosh[x]*Sinh[x])/(8*a^3) - (b*Cosh[x]^2*Sinh[x])/(3*a^2) + (Cosh[x]^3*Sinh[x])/(4*a)

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
```

```
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3938

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n -
 a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cosh^4(x)}{a+b\text{sech}(x)}\,dx = \frac{\cosh^3(x)\sinh(x)}{4a} + \frac{\int \frac{\cosh^3(x)\left(-4b+3a\text{sech}(x)+3b\text{sech}^2(x)\right)}{a+b\text{sech}(x)}\,dx}{4a}$$

$$= -\frac{b\cosh^2(x)\sinh(x)}{3a^2} + \frac{\cosh^3(x)\sinh(x)}{4a} - \frac{\int \frac{\cosh^2(x)\left(-3(3a^2+4b^2)-ab\text{sech}(x)+8b^2\text{sech}^2(x)\right)}{a+b\text{sech}(x)}\,dx}{12a^2}$$

$$= \frac{(3a^2+4b^2)\cosh(x)\sinh(x)}{8a^3} - \frac{b\cosh^2(x)\sinh(x)}{3a^2} + \frac{\cosh^3(x)\sinh(x)}{4a} + \frac{\int \frac{\cosh(x)\left(-8b(2a^2 \cdots\right.}{\cdots}}{\cdots}$$

$$= -\frac{b(2a^2+3b^2)\sinh(x)}{3a^4} + \frac{(3a^2+4b^2)\cosh(x)\sinh(x)}{8a^3} - \frac{b\cosh^2(x)\sinh(x)}{3a^2} + \frac{\cosh^3(x)\,\text{s}\cdots}{4a\cdots}$$

$$= \frac{(3a^4+4a^2b^2+8b^4)\,x}{8a^5} - \frac{b(2a^2+3b^2)\sinh(x)}{3a^4} + \frac{(3a^2+4b^2)\cosh(x)\sinh(x)}{8a^3} - \frac{b\cosh^2(\cdots}{3\cdots}$$

$$= \frac{(3a^4+4a^2b^2+8b^4)\,x}{8a^5} - \frac{b(2a^2+3b^2)\sinh(x)}{3a^4} + \frac{(3a^2+4b^2)\cosh(x)\sinh(x)}{8a^3} - \frac{b\cosh^2(\cdots}{3\cdots}$$

$$= \frac{(3a^4+4a^2b^2+8b^4)\,x}{8a^5} - \frac{b(2a^2+3b^2)\sinh(x)}{3a^4} + \frac{(3a^2+4b^2)\cosh(x)\sinh(x)}{8a^3} - \frac{b\cosh^2(\cdots}{3\cdots}$$

$$= \frac{(3a^4+4a^2b^2+8b^4)\,x}{8a^5} - \frac{2b^5\tan^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^5\sqrt{a-b}\,\sqrt{a+b}} - \frac{b(2a^2+3b^2)\sinh(x)}{3a^4} + \frac{(3a^2+\cdots}{\cdots}$$

Mathematica [A]
time = 0.19, size = 126, normalized size = 0.86

$$\frac{12(3a^4+4a^2b^2+8b^4)\,x + \frac{192b^5\text{ArcTan}\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - 24ab(3a^2+4b^2)\sinh(x) + 24a^2(a^2+b^2)\sinh(2x) - 8a^3b\sinh(3x) + 3a^4\sinh(4x)}{96a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^4/(a + b*Sech[x]),x]
```

```
[Out] (12*(3*a^4 + 4*a^2*b^2 + 8*b^4)*x + (192*b^5*ArcTan[((-a + b)*Tanh[x/2])/Sq
rt[a^2 - b^2]])/Sqrt[a^2 - b^2] - 24*a*b*(3*a^2 + 4*b^2)*Sinh[x] + 24*a^2*(
a^2 + b^2)*Sinh[2*x] - 8*a^3*b*Sinh[3*x] + 3*a^4*Sinh[4*x])/(96*a^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(126) = 252$.
time = 0.84, size = 299, normalized size = 2.05

method	result
risch	$\frac{3x}{8a}+\frac{x\,b^2}{2a^3}+\frac{x\,b^4}{a^5}+\frac{e^{4x}}{64a}-\frac{b\,e^{3x}}{24a^2}+\frac{e^{2x}}{8a}+\frac{e^{2x}b^2}{8a^3}-\frac{3b\,e^{x}}{8a^2}-\frac{b^3e^{x}}{2a^4}+\frac{3b\,e^{-x}}{8a^2}+\frac{b^3e^{-x}}{2a^4}-\frac{e^{-2x}}{8a}-\frac{e^{-2x}b^2}{8a^3}+\frac{b\,e^{-3x}}{24a^2}-$
default	$\frac{1}{4a(\tanh(\frac{x}{2})-1)^4}-\frac{-3a-2b}{6a^2(\tanh(\frac{x}{2})-1)^3}-\frac{-7a^2-4ab-4b^2}{8a^3(\tanh(\frac{x}{2})-1)^2}+\frac{(-3a^4-4a^2b^2-8b^4)\ln(\tanh(\frac{x}{2})-1)}{8a^5}-\frac{-5a^3-8a^2b-4a\,b^2-8}{8a^4(\tanh(\frac{x}{2})-1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a/(tanh(1/2*x)-1)^4-1/6*(-3*a-2*b)/a^2/(tanh(1/2*x)-1)^3-1/8*(-7*a^2-4*
a*b-4*b^2)/a^3/(tanh(1/2*x)-1)^2+1/8/a^5*(-3*a^4-4*a^2*b^2-8*b^4)*ln(tanh(1
/2*x)-1)-1/8*(-5*a^3-8*a^2*b-4*a*b^2-8*b^3)/a^4/(tanh(1/2*x)-1)-1/4/a/(tanh
(1/2*x)+1)^4-1/6*(-3*a-2*b)/a^2/(tanh(1/2*x)+1)^3-1/8*(7*a^2+4*a*b+4*b^2)/a
^3/(tanh(1/2*x)+1)^2+1/8*(3*a^4+4*a^2*b^2+8*b^4)/a^5*ln(tanh(1/2*x)+1)-1/8*
(-5*a^3-8*a^2*b-4*a*b^2-8*b^3)/a^4/(tanh(1/2*x)+1)-2*b^5/a^5/((a+b)*(a-b))^
(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1161 vs. 2(126) = 252.
time = 0.41, size = 2402, normalized size = 16.45

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(a^6 - a^4*b^2)*cosh(x)^8 + 3*(a^6 - a^4*b^2)*sinh(x)^8 - 8*(a^5*
b - a^3*b^3)*cosh(x)^7 - 8*(a^5*b - a^3*b^3 - 3*(a^6 - a^4*b^2)*cosh(x))*si
```

nh(x)^7 + 24*(a^6 - a^2*b^4)*cosh(x)^6 + 4*(6*a^6 - 6*a^2*b^4 + 21*(a^6 - a
^4*b^2)*cosh(x)^2 - 14*(a^5*b - a^3*b^3)*cosh(x))*sinh(x)^6 - 3*a^6 + 3*a^4
b^2 + 24(3*a^6 + a^4*b^2 + 4*a^2*b^4 - 8*b^6)*x*cosh(x)^4 - 24*(3*a^5*b +
 a^3*b^3 - 4*a*b^5)*cosh(x)^5 - 24*(3*a^5*b + a^3*b^3 - 4*a*b^5 - 7*(a^6 -
a^4*b^2)*cosh(x)^3 + 7*(a^5*b - a^3*b^3)*cosh(x)^2 - 6*(a^6 - a^2*b^4)*cosh
(x))*sinh(x)^5 + 2*(105*(a^6 - a^4*b^2)*cosh(x)^4 - 140*(a^5*b - a^3*b^3)*c
osh(x)^3 + 180*(a^6 - a^2*b^4)*cosh(x)^2 + 12*(3*a^6 + a^4*b^2 + 4*a^2*b^4
 - 8*b^6)*x - 60*(3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x))*sinh(x)^4 + 24*(3*a^
5*b + a^3*b^3 - 4*a*b^5)*cosh(x)^3 + 8*(9*a^5*b + 3*a^3*b^3 - 12*a*b^5 + 21
*(a^6 - a^4*b^2)*cosh(x)^5 - 35*(a^5*b - a^3*b^3)*cosh(x)^4 + 60*(a^6 - a^2
*b^4)*cosh(x)^3 + 12*(3*a^6 + a^4*b^2 + 4*a^2*b^4 - 8*b^6)*x*cosh(x) - 30*(
3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x)^2)*sinh(x)^3 - 24*(a^6 - a^2*b^4)*cosh
(x)^2 + 12*(7*(a^6 - a^4*b^2)*cosh(x)^6 - 2*a^6 + 2*a^2*b^4 - 14*(a^5*b - a
^3*b^3)*cosh(x)^5 + 30*(a^6 - a^2*b^4)*cosh(x)^4 + 12*(3*a^6 + a^4*b^2 + 4*
a^2*b^4 - 8*b^6)*x*cosh(x)^2 - 20*(3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x)^3 +
 6*(3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x))*sinh(x)^2 - 192*(b^5*cosh(x)^4 +
4*b^5*cosh(x)^3*sinh(x) + 6*b^5*cosh(x)^2*sinh(x)^2 + 4*b^5*cosh(x)*sinh(x)
^3 + b^5*sinh(x)^4)*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2
*a*b*cosh(x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-a^2 +
b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x)
 + 2*(a*cosh(x) + b)*sinh(x) + a)) + 8*(a^5*b - a^3*b^3)*cosh(x) + 8*(3*(a^6
 - a^4*b^2)*cosh(x)^7 - 7*(a^5*b - a^3*b^3)*cosh(x)^6 + a^5*b - a^3*b^3 + 1
8*(a^6 - a^2*b^4)*cosh(x)^5 + 12*(3*a^6 + a^4*b^2 + 4*a^2*b^4 - 8*b^6)*x*co
sh(x)^3 - 15*(3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x)^4 + 9*(3*a^5*b + a^3*b^3
 - 4*a*b^5)*cosh(x)^2 - 6*(a^6 - a^2*b^4)*cosh(x))*sinh(x))/((a^7 - a^5*b^2
)*cosh(x)^4 + 4*(a^7 - a^5*b^2)*cosh(x)^3*sinh(x) + 6*(a^7 - a^5*b^2)*cosh(
x)^2*sinh(x)^2 + 4*(a^7 - a^5*b^2)*cosh(x)*sinh(x)^3 + (a^7 - a^5*b^2)*sinh
(x)^4), 1/192*(3*(a^6 - a^4*b^2)*cosh(x)^8 + 3*(a^6 - a^4*b^2)*sinh(x)^8 -
8*(a^5*b - a^3*b^3)*cosh(x)^7 - 8*(a^5*b - a^3*b^3 - 3*(a^6 - a^4*b^2)*cosh
(x))*sinh(x)^7 + 24*(a^6 - a^2*b^4)*cosh(x)^6 + 4*(6*a^6 - 6*a^2*b^4 + 21*(
a^6 - a^4*b^2)*cosh(x)^2 - 14*(a^5*b - a^3*b^3)*cosh(x))*sinh(x)^6 - 3*a^6
 + 3*a^4*b^2 + 24*(3*a^6 + a^4*b^2 + 4*a^2*b^4 - 8*b^6)*x*cosh(x)^4 - 24*(3*
a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x)^5 - 24*(3*a^5*b + a^3*b^3 - 4*a*b^5 - 7*
(a^6 - a^4*b^2)*cosh(x)^3 + 7*(a^5*b - a^3*b^3)*cosh(x)^2 - 6*(a^6 - a^2*b^
4)*cosh(x))*sinh(x)^5 + 2*(105*(a^6 - a^4*b^2)*cosh(x)^4 - 140*(a^5*b - a^3
*b^3)*cosh(x)^3 + 180*(a^6 - a^2*b^4)*cosh(x)^2 + 12*(3*a^6 + a^4*b^2 + 4*a
^2*b^4 - 8*b^6)*x - 60*(3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x))*sinh(x)^4 + 2
4*(3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x)^3 + 8*(9*a^5*b + 3*a^3*b^3 - 12*a*b
^5 + 21*(a^6 - a^4*b^2)*cosh(x)^5 - 35*(a^5*b - a^3*b^3)*cosh(x)^4 + 60*(a^
6 - a^2*b^4)*cosh(x)^3 + 12*(3*a^6 + a^4*b^2 + 4*a^2*b^4 - 8*b^6)*x*cosh(x)
 - 30*(3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x)^2)*sinh(x)^3 - 24*(a^6 - a^2*b^
4)*cosh(x)^2 + 12*(7*(a^6 - a^4*b^2)*cosh(x)^6 - 2*a^6 + 2*a^2*b^4 - 14*(a^
5*b - a^3*b^3)*cosh(x)^5 + 30*(a^6 - a^2*b^4)*cosh(x)^4 + 12*(3*a^6 + a^4*b
^2 + 4*a^2*b^4 - 8*b^6)*x*cosh(x)^2 - 20*(3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh
(x)^3 + 6*(3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x))*sinh(x)^2 + 384*(b^5*cosh(

```
x)^4 + 4*b^5*cosh(x)^3*sinh(x) + 6*b^5*cosh(x)^2*sinh(x)^2 + 4*b^5*cosh(x)*
sinh(x)^3 + b^5*sinh(x)^4)*sqrt(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) +
 b)/sqrt(a^2 - b^2)) + 8*(a^5*b - a^3*b^3)*cosh(x) + 8*(3*(a^6 - a^4*b^2)*c
osh(x)^7 - 7*(a^5*b - a^3*b^3)*cosh(x)^6 + a^5*b - a^3*b^3 + 18*(a^6 - a^2*
b^4)*cosh(x)^5 + 12*(3*a^6 + a^4*b^2 + 4*a^2*b^4 - 8*b^6)*x*cosh(x)^3 - 15*
(3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x)^4 + 9*(3*a^5*b + a^3*b^3 - 4*a*b^5)*c
osh(x)^2 - 6*(a^6 - a^2*b^4)*cosh(x))*sinh(x))/((a^7 - a^5*b^2)*cosh(x)^4 +
 4*(a^7 - a^5*b^2)*cosh(x)^3*sinh(x) + 6*(a^7 - a^5*b^2)*cosh(x)^2*sinh(x)^
2 + 4*(a^7 - a^5*b^2)*cosh(x)*sinh(x)^3 + (a^7 - a^5*b^2)*sinh(x)^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^4(x)}{a + b\operatorname{sech}(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**4/(a+b*sech(x)),x)
```

```
[Out] Integral(cosh(x)**4/(a + b*sech(x)), x)
```

Giac [A]
time = 0.40, size = 182, normalized size = 1.25

$$-\frac{2\,b^5\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,a^5}+\frac{3\,a^3e^{(4\,x)}-8\,a^2be^{(3\,x)}+24\,a^3e^{(2\,x)}+24\,ab^2e^{(2\,x)}-72\,a^2be^x-96\,b^3e^x}{192\,a^4}+\frac{(3\,a^4+4\,a^2b^2+8\,b^4)x}{8\,a^5}+\frac{\left(8\,a^3be^x-3\,a^4+24\,(3\,a^3b+4\,ab^3)e^{(3\,x)}-24\,(a^4+a^2b^2)e^{(2\,x)}\right)e^{(-4\,x)}}{192\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] -2*b^5*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a^5) + 1/192*(3
*a^3*e^(4*x) - 8*a^2*b*e^(3*x) + 24*a^3*e^(2*x) + 24*a*b^2*e^(2*x) - 72*a^2
*b*e^x - 96*b^3*e^x)/a^4 + 1/8*(3*a^4 + 4*a^2*b^2 + 8*b^4)*x/a^5 + 1/192*(8
*a^3*b*e^x - 3*a^4 + 24*(3*a^3*b + 4*a*b^3)*e^(3*x) - 24*(a^4 + a^2*b^2)*e^
(2*x))*e^(-4*x)/a^5
```

Mupad [B]
time = 1.85, size = 251, normalized size = 1.72

$$\frac{\mathrm{e}^{4\,x}}{64\,a}-\frac{\mathrm{e}^{-4\,x}}{64\,a}+\frac{x\,\left(3\,a^4+4\,a^2\,b^2+8\,b^4\right)}{8\,a^5}-\frac{\mathrm{e}^{-2\,x}\,\left(a^2+b^2\right)}{8\,a^3}+\frac{\mathrm{e}^{2\,x}\,\left(a^2+b^2\right)}{8\,a^3}+\frac{\mathrm{e}^{-x}\,\left(3\,a^2\,b+4\,b^3\right)}{8\,a^4}+\frac{b\,\mathrm{e}^{-3\,x}}{24\,a^2}-\frac{b\,\mathrm{e}^{3\,x}}{24\,a^2}-\frac{\mathrm{e}^{x}\,\left(3\,a^2\,b+4\,b^3\right)}{8\,a^4}+\frac{b^5\,\ln\left(\frac{2\,b^5\,\mathrm{e}^{x}}{a^6}-\frac{2\,b^5\,\left(a+b\,\mathrm{e}^{x}\right)}{a^6\,\sqrt{a+b}\,\sqrt{b-a}}\right)}{a^5\,\sqrt{a+b}\,\sqrt{b-a}}-\frac{b^5\,\ln\left(\frac{2\,b^5\,\mathrm{e}^{x}}{a^6}+\frac{2\,b^5\,\left(a+b\,\mathrm{e}^{x}\right)}{a^6\,\sqrt{a+b}\,\sqrt{b-a}}\right)}{a^5\,\sqrt{a+b}\,\sqrt{b-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a + b/cosh(x)),x)
```

```
[Out] exp(4*x)/(64*a) - exp(-4*x)/(64*a) + (x*(3*a^4 + 8*b^4 + 4*a^2*b^2))/(8*a^5
) - (exp(-2*x)*(a^2 + b^2))/(8*a^3) + (exp(2*x)*(a^2 + b^2))/(8*a^3) + (exp
```

```
(-x)*(3*a^2*b + 4*b^3))/(8*a^4) + (b*exp(-3*x))/(24*a^2) - (b*exp(3*x))/(24
*a^2) - (exp(x)*(3*a^2*b + 4*b^3))/(8*a^4) + (b^5*log((2*b^5*exp(x))/a^6 -
(2*b^5*(a + b*exp(x)))/(a^6*(a + b)^(1/2)*(b - a)^(1/2))))/(a^5*(a + b)^(1/
2)*(b - a)^(1/2)) - (b^5*log((2*b^5*exp(x))/a^6 + (2*b^5*(a + b*exp(x)))/(a
^6*(a + b)^(1/2)*(b - a)^(1/2))))/(a^5*(a + b)^(1/2)*(b - a)^(1/2))
```

3.96 $\int \frac{\cosh^3(x)}{a+b\,\mathrm{sech}(x)}\,dx$

Optimal. Leaf size=112

$$-\frac{b(a^2+2b^2)\,x}{2a^4}+\frac{2b^4\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\,\sqrt{a+b}}+\frac{(2a^2+3b^2)\sinh(x)}{3a^3}-\frac{b\cosh(x)\sinh(x)}{2a^2}+\frac{\cosh^2(x)\sinh(x)}{3a}$$

```
[Out] -1/2*b*(a^2+2*b^2)*x/a^4+1/3*(2*a^2+3*b^2)*sinh(x)/a^3-1/2*b*cosh(x)*sinh(x
)/a^2+1/3*cosh(x)^2*sinh(x)/a+2*b^4*arctan((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1
/2))/a^4/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3938, 4189, 4004, 3916, 2738, 211}

$$\frac{2b^4\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\,\sqrt{a+b}}-\frac{b\sinh(x)\cosh(x)}{2a^2}-\frac{bx(a^2+2b^2)}{2a^4}+\frac{(2a^2+3b^2)\sinh(x)}{3a^3}+\frac{\sinh(x)\cosh^2(x)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(a + b*Sech[x]),x]
```

```
[Out] -1/2*(b*(a^2 + 2*b^2)*x)/a^4 + (2*b^4*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a
 + b]])/(a^4*Sqrt[a - b]*Sqrt[a + b]) + ((2*a^2 + 3*b^2)*Sinh[x])/(3*a^3) -
 (b*Cosh[x]*Sinh[x])/(2*a^2) + (Cosh[x]^2*Sinh[x])/(3*a)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3938

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n -
 a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cosh^3(x)}{a+b\text{sech}(x)}\,dx = \frac{\cosh^2(x)\sinh(x)}{3a} + \frac{\int \frac{\cosh^2(x)\left(-3b+2a\text{sech}(x)+2b\text{sech}^2(x)\right)}{a+b\text{sech}(x)}\,dx}{3a}$$

$$= -\frac{b\cosh(x)\sinh(x)}{2a^2} + \frac{\cosh^2(x)\sinh(x)}{3a} - \frac{\int \frac{\cosh(x)\left(-2(2a^2+3b^2)-ab\text{sech}(x)+3b^2\text{sech}^2(x)\right)}{a+b\text{sech}(x)}\,dx}{6a^2}$$

$$= \frac{(2a^2+3b^2)\sinh(x)}{3a^3} - \frac{b\cosh(x)\sinh(x)}{2a^2} + \frac{\cosh^2(x)\sinh(x)}{3a} + \frac{\int \frac{-3b(a^2+2b^2)-3ab^2\text{sech}(x}{a+b\text{sech}(x)}}{6a^3}$$

$$= -\frac{b(a^2+2b^2)\,x}{2a^4} + \frac{(2a^2+3b^2)\sinh(x)}{3a^3} - \frac{b\cosh(x)\sinh(x)}{2a^2} + \frac{\cosh^2(x)\sinh(x)}{3a} + \frac{b^4\int}{}$$

$$= -\frac{b(a^2+2b^2)\,x}{2a^4} + \frac{(2a^2+3b^2)\sinh(x)}{3a^3} - \frac{b\cosh(x)\sinh(x)}{2a^2} + \frac{\cosh^2(x)\sinh(x)}{3a} + \frac{b^3\int}{}$$

$$= -\frac{b(a^2+2b^2)\,x}{2a^4} + \frac{(2a^2+3b^2)\sinh(x)}{3a^3} - \frac{b\cosh(x)\sinh(x)}{2a^2} + \frac{\cosh^2(x)\sinh(x)}{3a} + \frac{(2b^3}{}$$

$$= -\frac{b(a^2+2b^2)\,x}{2a^4} + \frac{2b^4\tan^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\,\sqrt{a+b}} + \frac{(2a^2+3b^2)\sinh(x)}{3a^3} - \frac{b\cosh(x)\sinh}{2a^2}$$

Mathematica [A]
time = 0.11, size = 99, normalized size = 0.88

$$\frac{-6b(a^2+2b^2)\,x - \frac{24b^4\text{ArcTan}\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + 3a(3a^2+4b^2)\sinh(x) - 3a^2b\sinh(2x) + a^3\sinh(3x)}{12a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(a + b*Sech[x]),x]
```

```
[Out] (-6*b*(a^2 + 2*b^2)*x - (24*b^4*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]
])/Sqrt[a^2 - b^2] + 3*a*(3*a^2 + 4*b^2)*Sinh[x] - 3*a^2*b*Sinh[2*x] + a^3*
Sinh[3*x])/(12*a^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(94) = 188.
time = 0.83, size = 203, normalized size = 1.81

method	result

default	$-\frac{1}{3a\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}-\frac{-a-b}{2a^{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}-\frac{2a^{2}+ab+2b^{2}}{2a^{3}\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{b\left(a^{2}+2b^{2}\right)\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2a^{4}}-\frac{1}{3a\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}-\frac{a+}{2a^{2}(\tanh}$
risch	$-\frac{bx}{2a^{2}}-\frac{x\,b^{3}}{a^{4}}+\frac{e^{3x}}{24a}-\frac{b\,e^{2x}}{8a^{2}}+\frac{3\,e^{x}}{8a}+\frac{e^{x}b^{2}}{2a^{3}}-\frac{3\,e^{-x}}{8a}-\frac{e^{-x}b^{2}}{2a^{3}}+\frac{b\,e^{-2x}}{8a^{2}}-\frac{e^{-3x}}{24a}-\frac{b^{4}\ln\left(e^{x}+\frac{b\sqrt{-a^{2}+b^{2}}-a^{2}+b^{2}}{\sqrt{-a^{2}+b^{2}}\,a}\right)}{\sqrt{-a^{2}+b^{2}}\,a^{4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a/(tanh(1/2*x)+1)^3-1/2*(-a-b)/a^2/(tanh(1/2*x)+1)^2-1/2*(2*a^2+a*b+2*
b^2)/a^3/(tanh(1/2*x)+1)-1/2*b*(a^2+2*b^2)/a^4*ln(tanh(1/2*x)+1)-1/3/a/(tan
h(1/2*x)-1)^3-1/2*(a+b)/a^2/(tanh(1/2*x)-1)^2-1/2*(2*a^2+a*b+2*b^2)/a^3/(ta
nh(1/2*x)-1)+1/2*b*(a^2+2*b^2)/a^4*ln(tanh(1/2*x)-1)+2*b^4/a^4/((a+b)*(a-b)
)^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 741 vs. $2(94) = 188$.
time = 0.40, size = 1562, normalized size = 13.95

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [1/24*((a^5 - a^3*b^2)*cosh(x)^6 + (a^5 - a^3*b^2)*sinh(x)^6 - 3*(a^4*b - a
^2*b^3)*cosh(x)^5 - 3*(a^4*b - a^2*b^3 - 2*(a^5 - a^3*b^2)*cosh(x))*sinh(x)
^5 - a^5 + a^3*b^2 - 12*(a^4*b + a^2*b^3 - 2*b^5)*x*cosh(x)^3 + 3*(3*a^5 +
a^3*b^2 - 4*a*b^4)*cosh(x)^4 + 3*(3*a^5 + a^3*b^2 - 4*a*b^4 + 5*(a^5 - a^3*
b^2)*cosh(x)^2 - 5*(a^4*b - a^2*b^3)*cosh(x))*sinh(x)^4 + 2*(10*(a^5 - a^3*
```

```
b^2)*cosh(x)^3 - 15*(a^4*b - a^2*b^3)*cosh(x)^2 - 6*(a^4*b + a^2*b^3 - 2*b^
5)*x + 6*(3*a^5 + a^3*b^2 - 4*a*b^4)*cosh(x))*sinh(x)^3 - 3*(3*a^5 + a^3*b^
2 - 4*a*b^4)*cosh(x)^2 - 3*(3*a^5 + a^3*b^2 - 4*a*b^4 - 5*(a^5 - a^3*b^2)*c
osh(x)^4 + 10*(a^4*b - a^2*b^3)*cosh(x)^3 + 12*(a^4*b + a^2*b^3 - 2*b^5)*x*
cosh(x) - 6*(3*a^5 + a^3*b^2 - 4*a*b^4)*cosh(x)^2)*sinh(x)^2 - 24*(b^4*cosh
(x)^3 + 3*b^4*cosh(x)^2*sinh(x) + 3*b^4*cosh(x)*sinh(x)^2 + b^4*sinh(x)^3)*
sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) - a^2 +
 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(-a^2 + b^2)*(a*cosh(x) + a*
sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*
sinh(x) + a)) + 3*(a^4*b - a^2*b^3)*cosh(x) + 3*(2*(a^5 - a^3*b^2)*cosh(x)^
5 + a^4*b - a^2*b^3 - 5*(a^4*b - a^2*b^3)*cosh(x)^4 - 12*(a^4*b + a^2*b^3 -
 2*b^5)*x*cosh(x)^2 + 4*(3*a^5 + a^3*b^2 - 4*a*b^4)*cosh(x)^3 - 2*(3*a^5 +
a^3*b^2 - 4*a*b^4)*cosh(x))*sinh(x))/((a^6 - a^4*b^2)*cosh(x)^3 + 3*(a^6 -
a^4*b^2)*cosh(x)^2*sinh(x) + 3*(a^6 - a^4*b^2)*cosh(x)*sinh(x)^2 + (a^6 - a
^4*b^2)*sinh(x)^3), 1/24*((a^5 - a^3*b^2)*cosh(x)^6 + (a^5 - a^3*b^2)*sinh(
x)^6 - 3*(a^4*b - a^2*b^3)*cosh(x)^5 - 3*(a^4*b - a^2*b^3 - 2*(a^5 - a^3*b^
2)*cosh(x))*sinh(x)^5 - a^5 + a^3*b^2 - 12*(a^4*b + a^2*b^3 - 2*b^5)*x*cosh
(x)^3 + 3*(3*a^5 + a^3*b^2 - 4*a*b^4)*cosh(x)^4 + 3*(3*a^5 + a^3*b^2 - 4*a*
b^4 + 5*(a^5 - a^3*b^2)*cosh(x)^2 - 5*(a^4*b - a^2*b^3)*cosh(x))*sinh(x)^4
+ 2*(10*(a^5 - a^3*b^2)*cosh(x)^3 - 15*(a^4*b - a^2*b^3)*cosh(x)^2 - 6*(a^4
*b + a^2*b^3 - 2*b^5)*x + 6*(3*a^5 + a^3*b^2 - 4*a*b^4)*cosh(x))*sinh(x)^3
- 3*(3*a^5 + a^3*b^2 - 4*a*b^4)*cosh(x)^2 - 3*(3*a^5 + a^3*b^2 - 4*a*b^4 -
5*(a^5 - a^3*b^2)*cosh(x)^4 + 10*(a^4*b - a^2*b^3)*cosh(x)^3 + 12*(a^4*b +
a^2*b^3 - 2*b^5)*x*cosh(x) - 6*(3*a^5 + a^3*b^2 - 4*a*b^4)*cosh(x)^2)*sinh(
x)^2 - 48*(b^4*cosh(x)^3 + 3*b^4*cosh(x)^2*sinh(x) + 3*b^4*cosh(x)*sinh(x)^
2 + b^4*sinh(x)^3)*sqrt(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt
(a^2 - b^2)) + 3*(a^4*b - a^2*b^3)*cosh(x) + 3*(2*(a^5 - a^3*b^2)*cosh(x)^5
 + a^4*b - a^2*b^3 - 5*(a^4*b - a^2*b^3)*cosh(x)^4 - 12*(a^4*b + a^2*b^3 -
2*b^5)*x*cosh(x)^2 + 4*(3*a^5 + a^3*b^2 - 4*a*b^4)*cosh(x)^3 - 2*(3*a^5 + a
^3*b^2 - 4*a*b^4)*cosh(x))*sinh(x))/((a^6 - a^4*b^2)*cosh(x)^3 + 3*(a^6 - a
^4*b^2)*cosh(x)^2*sinh(x) + 3*(a^6 - a^4*b^2)*cosh(x)*sinh(x)^2 + (a^6 - a^
4*b^2)*sinh(x)^3)]
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3(x)}{a + b\,\mathrm{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(a+b*sech(x)),x)
```

```
[Out] Integral(cosh(x)**3/(a + b*sech(x)), x)
```

Giac [**A**]

time = 0.39, size = 133, normalized size = 1.19

$$\frac{2\,b^4\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,a^4}+\frac{a^2e^{(3\,x)}-3\,abe^{(2\,x)}+9\,a^2e^x+12\,b^2e^x}{24\,a^3}-\frac{(a^2b+2\,b^3)x}{2\,a^4}+\frac{\left(3\,a^2be^x-a^3-3\,(3\,a^3+4\,ab^2)e^{(2\,x)}\right)e^{(-3\,x)}}{24\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] 2*b^4*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a^4) + 1/24*(a^2
*e^(3*x) - 3*a*b*e^(2*x) + 9*a^2*e^x + 12*b^2*e^x)/a^3 - 1/2*(a^2*b + 2*b^3
)*x/a^4 + 1/24*(3*a^2*b*e^x - a^3 - 3*(3*a^3 + 4*a*b^2)*e^(2*x))*e^(-3*x)/a
^4
```

Mupad [B]
time = 1.71, size = 209, normalized size = 1.87

$$\frac{e^{3\,x}}{24\,a}-\frac{e^{-3\,x}}{24\,a}-\frac{x\,(a^2\,b+2\,b^3)}{2\,a^4}+\frac{e^x\,(3\,a^2+4\,b^2)}{8\,a^3}+\frac{b\,e^{-2\,x}}{8\,a^2}-\frac{b\,e^{2\,x}}{8\,a^2}-\frac{e^{-x}\,(3\,a^2+4\,b^2)}{8\,a^3}+\frac{b^4\,\ln\left(-\frac{2\,b^4\,e^x}{a^5}-\frac{2\,b^4\,(a+b\,e^x)}{a^5\,\sqrt{a+b}\,\sqrt{b-a}}\right)}{a^4\,\sqrt{a+b}\,\sqrt{b-a}}-\frac{b^4\,\ln\left(\frac{2\,b^4\,(a+b\,e^x)}{a^5\,\sqrt{a+b}\,\sqrt{b-a}}-\frac{2\,b^4\,e^x}{a^5}\right)}{a^4\,\sqrt{a+b}\,\sqrt{b-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a + b/cosh(x)),x)
```

```
[Out] exp(3*x)/(24*a) - exp(-3*x)/(24*a) - (x*(a^2*b + 2*b^3))/(2*a^4) + (exp(x)*
(3*a^2 + 4*b^2))/(8*a^3) + (b*exp(-2*x))/(8*a^2) - (b*exp(2*x))/(8*a^2) - (
exp(-x)*(3*a^2 + 4*b^2))/(8*a^3) + (b^4*log(- (2*b^4*exp(x))/a^5 - (2*b^4*(
a + b*exp(x)))/(a^5*(a + b)^(1/2)*(b - a)^(1/2))))/(a^4*(a + b)^(1/2)*(b -
a)^(1/2)) - (b^4*log((2*b^4*(a + b*exp(x)))/(a^5*(a + b)^(1/2)*(b - a)^(1/2
)) - (2*b^4*exp(x))/a^5))/(a^4*(a + b)^(1/2)*(b - a)^(1/2))
```

3.97 $\int \frac{\cosh^2(x)}{a+b\operatorname{sech}(x)}\,dx$

Optimal. Leaf size=85

$$\frac{(a^2+2b^2)\,x}{2a^3}-\frac{2b^3\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\,\sqrt{a+b}}-\frac{b\sinh(x)}{a^2}+\frac{\cosh(x)\sinh(x)}{2a}$$

```
[Out] 1/2*(a^2+2*b^2)*x/a^3-b*sinh(x)/a^2+1/2*cosh(x)*sinh(x)/a-2*b^3*arctan((a-b
)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/a^3/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3938, 4189, 4004, 3916, 2738, 211}

$$-\frac{2b^3\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\,\sqrt{a+b}}-\frac{b\sinh(x)}{a^2}+\frac{x(a^2+2b^2)}{2a^3}+\frac{\sinh(x)\cosh(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(a + b*Sech[x]),x]
```

```
[Out] ((a^2 + 2*b^2)*x)/(2*a^3) - (2*b^3*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a +
b]])/(a^3*Sqrt[a - b]*Sqrt[a + b]) - (b*Sinh[x])/a^2 + (Cosh[x]*Sinh[x])/(2
*a)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3938

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n -
 a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cosh^2(x)}{a+b\text{sech}(x)}\,dx = \frac{\cosh(x)\sinh(x)}{2a} + \frac{\int \frac{\cosh(x)\left(-2b+a\text{sech}(x)+b\text{sech}^2(x)\right)}{a+b\text{sech}(x)}\,dx}{2a}$$

$$= -\frac{b\sinh(x)}{a^2} + \frac{\cosh(x)\sinh(x)}{2a} - \frac{\int \frac{-a^2-2b^2-ab\text{sech}(x)}{a+b\text{sech}(x)}\,dx}{2a^2}$$

$$= \frac{(a^2+2b^2)\,x}{2a^3} - \frac{b\sinh(x)}{a^2} + \frac{\cosh(x)\sinh(x)}{2a} - \frac{b^3\int \frac{\text{sech}(x)}{a+b\text{sech}(x)}\,dx}{a^3}$$

$$= \frac{(a^2+2b^2)\,x}{2a^3} - \frac{b\sinh(x)}{a^2} + \frac{\cosh(x)\sinh(x)}{2a} - \frac{b^2\int \frac{1}{1+\frac{a\cosh(x)}{b}}\,dx}{a^3}$$

$$= \frac{(a^2+2b^2)\,x}{2a^3} - \frac{b\sinh(x)}{a^2} + \frac{\cosh(x)\sinh(x)}{2a} - \frac{(2b^2)\,\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right.\right.}{a^3}$$

$$= \frac{(a^2+2b^2)\,x}{2a^3} - \frac{2b^3\tan^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\,\sqrt{a+b}} - \frac{b\sinh(x)}{a^2} + \frac{\cosh(x)\sinh(x)}{2a}$$

Mathematica [**A**]
time = 0.09, size = 78, normalized size = 0.92

$$\frac{2a^2x+4b^2x+\frac{8b^3\operatorname{ArcTan}\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-4ab\sinh(x)+a^2\sinh(2x)}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(a + b*Sech[x]),x]
```

```
[Out] (2*a^2*x + 4*b^2*x + (8*b^3*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]])/S
qrt[a^2 - b^2] - 4*a*b*Sinh[x] + a^2*Sinh[2*x])/(4*a^3)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(71) = 142.
time = 0.81, size = 153, normalized size = 1.80

method	result
default	$\frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}-\frac{-2b-a}{2a^2\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{\left(-a^2-2b^2\right)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2a^3}-\frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}-\frac{-2b-a}{2a^2\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{\left(a^2+2b^2\right)\mathrm{l}}{}$
risch	$\frac{x}{2a}+\frac{x\,b^2}{a^3}+\frac{\mathrm{e}^{2x}}{8a}-\frac{b\,\mathrm{e}^{x}}{2a^2}+\frac{b\,\mathrm{e}^{-x}}{2a^2}-\frac{\mathrm{e}^{-2x}}{8a}-\frac{b^3\ln\left(\mathrm{e}^{x}+\frac{b\sqrt{-a^2+b^2}+a^2-b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,a^3}+\frac{b^3\ln\left(\mathrm{e}^{x}+\frac{b\sqrt{-a^2+b^2}-a^2+b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a/(tanh(1/2*x)-1)^2-1/2*(-2*b-a)/a^2/(tanh(1/2*x)-1)+1/2/a^3*(-a^2-2*b^
2)*ln(tanh(1/2*x)-1)-1/2/a/(tanh(1/2*x)+1)^2-1/2*(-2*b-a)/a^2/(tanh(1/2*x)+
1)+1/2*(a^2+2*b^2)/a^3*ln(tanh(1/2*x)+1)-2*b^3/a^3/((a+b)*(a-b))^(1/2)*arct
an((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 390 vs. $2(71) = 142$.
time = 0.38, size = 860, normalized size = 10.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [1/8*((a^4 - a^2*b^2)*cosh(x)^4 + (a^4 - a^2*b^2)*sinh(x)^4 - a^4 + a^2*b^2
 + 4*(a^4 + a^2*b^2 - 2*b^4)*x*cosh(x)^2 - 4*(a^3*b - a*b^3)*cosh(x)^3 - 4*
(a^3*b - a*b^3 - (a^4 - a^2*b^2)*cosh(x))*sinh(x)^3 + 2*(3*(a^4 - a^2*b^2)*
cosh(x)^2 + 2*(a^4 + a^2*b^2 - 2*b^4)*x - 6*(a^3*b - a*b^3)*cosh(x))*sinh(x
)^2 - 8*(b^3*cosh(x)^2 + 2*b^3*cosh(x)*sinh(x) + b^3*sinh(x)^2)*sqrt(-a^2 +
 b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) - a^2 + 2*b^2 + 2*
(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b
))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) + a
)) + 4*(a^3*b - a*b^3)*cosh(x) + 4*(a^3*b - a*b^3 + (a^4 - a^2*b^2)*cosh(x)
^3 + 2*(a^4 + a^2*b^2 - 2*b^4)*x*cosh(x) - 3*(a^3*b - a*b^3)*cosh(x)^2)*sin
h(x))/((a^5 - a^3*b^2)*cosh(x)^2 + 2*(a^5 - a^3*b^2)*cosh(x)*sinh(x) + (a^5
 - a^3*b^2)*sinh(x)^2), 1/8*((a^4 - a^2*b^2)*cosh(x)^4 + (a^4 - a^2*b^2)*si
nh(x)^4 - a^4 + a^2*b^2 + 4*(a^4 + a^2*b^2 - 2*b^4)*x*cosh(x)^2 - 4*(a^3*b
- a*b^3)*cosh(x)^3 - 4*(a^3*b - a*b^3 - (a^4 - a^2*b^2)*cosh(x))*sinh(x)^3
+ 2*(3*(a^4 - a^2*b^2)*cosh(x)^2 + 2*(a^4 + a^2*b^2 - 2*b^4)*x - 6*(a^3*b -
 a*b^3)*cosh(x))*sinh(x)^2 + 16*(b^3*cosh(x)^2 + 2*b^3*cosh(x)*sinh(x) + b^
3*sinh(x)^2)*sqrt(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^2 -
 b^2)) + 4*(a^3*b - a*b^3)*cosh(x) + 4*(a^3*b - a*b^3 + (a^4 - a^2*b^2)*cos
h(x)^3 + 2*(a^4 + a^2*b^2 - 2*b^4)*x*cosh(x) - 3*(a^3*b - a*b^3)*cosh(x)^2)
*sinh(x))/((a^5 - a^3*b^2)*cosh(x)^2 + 2*(a^5 - a^3*b^2)*cosh(x)*sinh(x) +
(a^5 - a^3*b^2)*sinh(x)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(x)}{a + b\,\mathrm{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(a+b*sech(x)),x)
```

```
[Out] Integral(cosh(x)**2/(a + b*sech(x)), x)
```

Giac [A]
time = 0.39, size = 92, normalized size = 1.08

$$-\frac{2\,b^{3}\arctan\left(\frac{ae^{x}+b}{\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,a^{3}}+\frac{ae^{(2\,x)}-4\,be^{x}}{8\,a^{2}}+\frac{(a^{2}+2\,b^{2})x}{2\,a^{3}}+\frac{(4\,abe^{x}-a^{2})e^{(-2\,x)}}{8\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] -2*b^3*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a^3) + 1/8*(a*e
^(2*x) - 4*b*e^x)/a^2 + 1/2*(a^2 + 2*b^2)*x/a^3 + 1/8*(4*a*b*e^x - a^2)*e^(
-2*x)/a^3
```

Mupad [B]
time = 1.58, size = 167, normalized size = 1.96

$$\frac{e^{2\,x}}{8\,a}-\frac{e^{-2\,x}}{8\,a}-\frac{b\,e^{x}}{2\,a^{2}}+\frac{b\,e^{-x}}{2\,a^{2}}+\frac{x\,(a^{2}+2\,b^{2})}{2\,a^{3}}+\frac{b^{3}\,\ln\left(\frac{2\,b^{3}\,e^{x}}{a^{4}}-\frac{2\,b^{3}\,(a+b\,e^{x})}{a^{4}\,\sqrt{a+b}\,\sqrt{b-a}}\right)}{a^{3}\,\sqrt{a+b}\,\sqrt{b-a}}-\frac{b^{3}\,\ln\left(\frac{2\,b^{3}\,e^{x}}{a^{4}}+\frac{2\,b^{3}\,(a+b\,e^{x})}{a^{4}\,\sqrt{a+b}\,\sqrt{b-a}}\right)}{a^{3}\,\sqrt{a+b}\,\sqrt{b-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a + b/cosh(x)),x)
```

```
[Out] exp(2*x)/(8*a) - exp(-2*x)/(8*a) - (b*exp(x))/(2*a^2) + (b*exp(-x))/(2*a^2)
 + (x*(a^2 + 2*b^2))/(2*a^3) + (b^3*log((2*b^3*exp(x))/a^4 - (2*b^3*(a + b*
exp(x)))/(a^4*(a + b)^(1/2)*(b - a)^(1/2))))/(a^3*(a + b)^(1/2)*(b - a)^(1/
2)) - (b^3*log((2*b^3*exp(x))/a^4 + (2*b^3*(a + b*exp(x)))/(a^4*(a + b)^(1/
2)*(b - a)^(1/2))))/(a^3*(a + b)^(1/2)*(b - a)^(1/2))
```

3.98 $\int \frac{\cosh(x)}{a+b\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=62

$$-\frac{bx}{a^2}+\frac{2b^2\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\,\sqrt{a+b}}+\frac{\sinh(x)}{a}$$

```
[Out] -b*x/a^2+sinh(x)/a+2*b^2*arctan((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/a^2/(a
-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {3938, 12, 3868, 2738, 211}

$$\frac{2b^2\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\,\sqrt{a+b}}-\frac{bx}{a^2}+\frac{\sinh(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/(a + b*Sech[x]),x]
```

```
[Out] -((b*x)/a^2) + (2*b^2*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(a^2*Sqr
t[a - b]*Sqrt[a + b]) + Sinh[x]/a
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3868

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x
```

```
] && NeQ[a^2 - b^2, 0]
```

Rule 3938

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n -
 a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(x)}{a+b\text{sech}(x)}\,dx &= \frac{\sinh(x)}{a}-\frac{\int \frac{b}{a+b\text{sech}(x)}\,dx}{a}\\ &= \frac{\sinh(x)}{a}-\frac{b\int \frac{1}{a+b\text{sech}(x)}\,dx}{a}\\ &= -\frac{bx}{a^2}+\frac{\sinh(x)}{a}+\frac{b\int \frac{1}{1+\frac{a\cosh(x)}{b}}\,dx}{a^2}\\ &= -\frac{bx}{a^2}+\frac{\sinh(x)}{a}+\frac{(2b)\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\left(1-\frac{a}{b}\right)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2}\\ &= -\frac{bx}{a^2}+\frac{2b^2\tan^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\,\sqrt{a+b}}+\frac{\sinh(x)}{a}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 57, normalized size = 0.92

$$\frac{b\left(-x-\frac{2b\text{ArcTan}\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\right)+a\sinh(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(a + b*Sech[x]),x]
```

```
[Out] (b*(-x - (2*b*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
) + a*Sinh[x])/a^2
```

Maple [A]
time = 0.82, size = 94, normalized size = 1.52

method	result	size
default	$-\frac{1}{a\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{b\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a^{2}}+\frac{2b^{2}\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^{2}\sqrt{(a+b)(a-b)}}-\frac{1}{a\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{b\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a^{2}}$	94
risch	$-\frac{bx}{a^{2}}+\frac{e^{x}}{2a}-\frac{e^{-x}}{2a}-\frac{b^{2}\ln\left(e^{x}+\frac{b\sqrt{-a^{2}+b^{2}}-a^{2}+b^{2}}{\sqrt{-a^{2}+b^{2}}\,a}\right)}{\sqrt{-a^{2}+b^{2}}\,a^{2}}+\frac{b^{2}\ln\left(e^{x}+\frac{b\sqrt{-a^{2}+b^{2}}+a^{2}-b^{2}}{\sqrt{-a^{2}+b^{2}}\,a}\right)}{\sqrt{-a^{2}+b^{2}}\,a^{2}}$	144

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a/(tanh(1/2*x)-1)+b/a^2*ln(tanh(1/2*x)-1)+2*b^2/a^2/((a+b)*(a-b))^(1/2)*
arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))-1/a/(tanh(1/2*x)+1)-b/a^2*ln(
tanh(1/2*x)+1)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(52) = 104.
time = 0.39, size = 430, normalized size = 6.94

$$\left[-\frac{a^{3}-ab^{2}+2\,(a^{2}b-b^{3})x\cosh(x)-(a^{3}-ab^{2})\cosh(x)^{2}-(a^{3}-ab^{2})\sinh(x)^{2}+2\,(b^{2}\cosh(x)+b^{2}\sinh(x))\sqrt{-a^{2}+b^{2}}\log\left(\frac{a^{2}\cosh(x)^{2}+a^{2}\sinh(x)^{2}+2\,ab\cosh(x)-a^{2}+2\,b^{2}+2\,(a^{2}\cosh(x)+ab)\sinh(x)-2\sqrt{-a^{2}+b^{2}}\,(a\cosh(x)+a\sinh(x)+b)}{a\cosh(x)^{2}+a\sinh(x)^{2}+2\,b\cosh(x)+2\,(a\cosh(x)+b)\sinh(x)+a}\right)+2\,((a^{2}b-b^{3})x-(a^{3}-ab^{2})\cosh(x))\sinh(x)}{2\,((a^{4}-a^{2}b^{2})\cosh(x)+(a^{4}-a^{2}b^{2})\sinh(x))},\ -\frac{a^{3}-ab^{2}+2\,(a^{2}b-b^{3})x\cosh(x)-(a^{3}-ab^{2})\cosh(x)^{2}-(a^{3}-ab^{2})\sinh(x)^{2}+4\,(b^{2}\cosh(x)+b^{2}\sinh(x))\sqrt{a^{2}-b^{2}}\arctan\left(-\frac{a\cosh(x)+a\sinh(x)+b}{\sqrt{a^{2}-b^{2}}}\right)+2\,((a^{2}b-b^{3})x-(a^{3}-ab^{2})\cosh(x))\sinh(x)}{2\,((a^{4}-a^{2}b^{2})\cosh(x)+(a^{4}-a^{2}b^{2})\sinh(x))}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(a^3 - a*b^2 + 2*(a^2*b - b^3)*x*cosh(x) - (a^3 - a*b^2)*cosh(x)^2 -
(a^3 - a*b^2)*sinh(x)^2 + 2*(b^2*cosh(x) + b^2*sinh(x))*sqrt(-a^2 + b^2)*lo
g((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) - a^2 + 2*b^2 + 2*(a^2*cos
h(x) + a*b)*sinh(x) - 2*sqrt(-a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*co
sh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) + a)) + 2*(
(a^2*b - b^3)*x - (a^3 - a*b^2)*cosh(x))*sinh(x))/((a^4 - a^2*b^2)*cosh(x)
```

```
+ (a^4 - a^2*b^2)*sinh(x)), -1/2*(a^3 - a*b^2 + 2*(a^2*b - b^3)*x*cosh(x) -
 (a^3 - a*b^2)*cosh(x)^2 - (a^3 - a*b^2)*sinh(x)^2 + 4*(b^2*cosh(x) + b^2*s
inh(x))*sqrt(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^2 - b^2)
) + 2*((a^2*b - b^3)*x - (a^3 - a*b^2)*cosh(x))*sinh(x))/((a^4 - a^2*b^2)*c
osh(x) + (a^4 - a^2*b^2)*sinh(x))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(x)}{a + b\operatorname{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*sech(x)),x)
```

```
[Out] Integral(cosh(x)/(a + b*sech(x)), x)
```

Giac [A]
time = 0.41, size = 62, normalized size = 1.00

$$\frac{2\,b^2\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,a^2}-\frac{bx}{a^2}-\frac{e^{(-x)}}{2\,a}+\frac{e^x}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] 2*b^2*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a^2) - b*x/a^2 -
 1/2*e^(-x)/a + 1/2*e^x/a
```

Mupad [B]
time = 1.48, size = 139, normalized size = 2.24

$$\frac{e^x}{2\,a}-\frac{e^{-x}}{2\,a}-\frac{b\,x}{a^2}+\frac{b^2\,\ln\left(-\frac{2\,b^2\,e^x}{a^3}-\frac{2\,b^2\,(a+b\,e^x)}{a^3\,\sqrt{a+b}\,\sqrt{b-a}}\right)}{a^2\,\sqrt{a+b}\,\sqrt{b-a}}-\frac{b^2\,\ln\left(\frac{2\,b^2\,(a+b\,e^x)}{a^3\,\sqrt{a+b}\,\sqrt{b-a}}-\frac{2\,b^2\,e^x}{a^3}\right)}{a^2\,\sqrt{a+b}\,\sqrt{b-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a + b/cosh(x)),x)
```

```
[Out] exp(x)/(2*a) - exp(-x)/(2*a) - (b*x)/a^2 + (b^2*log(- (2*b^2*exp(x))/a^3 -
(2*b^2*(a + b*exp(x)))/(a^3*(a + b)^(1/2)*(b - a)^(1/2))))/(a^2*(a + b)^(1/
2)*(b - a)^(1/2)) - (b^2*log((2*b^2*(a + b*exp(x)))/(a^3*(a + b)^(1/2)*(b -
 a)^(1/2)) - (2*b^2*exp(x))/a^3))/(a^2*(a + b)^(1/2)*(b - a)^(1/2))
```

3.99 $\int \frac{\text{sech}(x)}{a+b\text{sech}(x)}\,dx$

Optimal. Leaf size=42

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}$$

```
[Out] 2*arctan((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3916, 2738, 211}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]/(a + b*Sech[x]),x]
```

```
[Out] (2*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\operatorname{sech}(x)}{a+b\operatorname{sech}(x)}\,dx = \frac{\int \frac{1}{1+\frac{a\cosh(x)}{b}}\,dx}{b}$$
$$= \frac{2\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b}$$
$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 0.98

$$-\frac{2\text{ArcTan}\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(a + b*Sech[x]),x]
[Out] (-2*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
```

Maple [A]
time = 0.42, size = 36, normalized size = 0.86

method	result	size
default	$\frac{2\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}$	36
risch	$-\frac{\ln\left(e^x+\frac{b\sqrt{-a^2+b^2}-a^2+b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}}+\frac{\ln\left(e^x+\frac{b\sqrt{-a^2+b^2}+a^2-b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}}$	109

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(a+b*sech(x)),x,method=_RETURNVERBOSE)
[Out] 2/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.38, size = 165, normalized size = 3.93

$$\left[-\frac{\sqrt{-a^2+b^2}\log\left(\frac{a^2\cosh(x)^2+a^2\sinh(x)^2+2\,ab\cosh(x)-a^2+2\,b^2+2\,(a^2\cosh(x)+ab)\sinh(x)-2\,\sqrt{-a^2+b^2}\,(a\cosh(x)+a\sinh(x)+b)}{a\cosh(x)^2+a\sinh(x)^2+2\,b\cosh(x)+2\,(a\cosh(x)+b)\sinh(x)+a}\right)}{a^2-b^2},\ -\frac{2\arctan\left(-\frac{a\cosh(x)+a\sinh(x)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [-sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) - a^2
 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(-a^2 + b^2)*(a*cosh(x) +
a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b
)*sinh(x) + a))/(a^2 - b^2), -2*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^
2 - b^2))/sqrt(a^2 - b^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{sech}(x)}{a+b\operatorname{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*sech(x)),x)
```

```
[Out] Integral(sech(x)/(a + b*sech(x)), x)
```

Giac [A]
time = 0.39, size = 32, normalized size = 0.76

$$\frac{2\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] 2*arctan((a*e^x + b)/sqrt(a^2 - b^2))/sqrt(a^2 - b^2)
```

Mupad [B]
time = 0.12, size = 43, normalized size = 1.02

$$\frac{2\,\mathrm{atan}\left(\frac{b}{\sqrt{a^2-b^2}}+\frac{a\,\mathrm{e}^x}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)*(a + b/cosh(x))),x)
```

```
[Out] (2*atan(b/(a^2 - b^2)^(1/2) + (a*exp(x))/(a^2 - b^2)^(1/2)))/(a^2 - b^2)^(1
/2)
```

3.100 $\int \frac{\text{sech}^2(x)}{a+b\text{sech}(x)}\,dx$

Optimal. Leaf size=54

$$\frac{\text{ArcTan}(\sinh(x))}{b}-\frac{2a\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b\sqrt{a+b}}$$

```
[Out] arctan(sinh(x))/b-2*a*arctan((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/b/(a-b)^(
1/2)/(a+b)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3874, 3855, 3916, 2738, 211}

$$\frac{\text{ArcTan}(\sinh(x))}{b}-\frac{2a\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\,\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(a + b*Sech[x]),x]
```

```
[Out] ArcTan[Sinh[x]]/b - (2*a*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt
[a - b]*b*Sqrt[a + b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3874

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^2(x)}{a+b\operatorname{sech}(x)}\,dx &= \frac{\int \operatorname{sech}(x)\,dx}{b} - \frac{a\int \frac{\operatorname{sech}(x)}{a+b\operatorname{sech}(x)}\,dx}{b}\\
&= \frac{\tan^{-1}(\sinh(x))}{b} - \frac{a\int \frac{1}{1+\frac{a\cosh(x)}{b}}\,dx}{b^2}\\
&= \frac{\tan^{-1}(\sinh(x))}{b} - \frac{(2a)\operatorname{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b^2}\\
&= \frac{\tan^{-1}(\sinh(x))}{b} - \frac{2a\tan^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b\sqrt{a+b}}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 54, normalized size = 1.00

$$\frac{2\left(\operatorname{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right) + \frac{a\operatorname{ArcTan}\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(a + b*Sech[x]),x]
[Out] (2*(ArcTan[Tanh[x/2]] + (a*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]])/Sq
rt[a^2 - b^2]))/b
```

Maple [A]
time = 0.57, size = 51, normalized size = 0.94

method	result	size
default	$\frac{2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{b}-\frac{2a\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b\sqrt{(a+b)(a-b)}}$	51
risch	$-\frac{a\ln\left(\mathrm{e}^{x}+\frac{b\sqrt{-a^2+b^2}+a^2-b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,b}+\frac{a\ln\left(\mathrm{e}^{x}+\frac{b\sqrt{-a^2+b^2}-a^2+b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,b}+\frac{i\ln(\mathrm{e}^{x}+i)}{b}-\frac{i\ln(\mathrm{e}^{x}-i)}{b}$	141

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b*arctan(tanh(1/2*x))-2*a/b/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/
((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.40, size = 219, normalized size = 4.06

$$\left[-\frac{\sqrt{-a^2+b^2}\,a\log\left(\frac{a^2\cosh(x)^2+a^2\sinh(x)^2+2\,ab\cosh(x)-a^2+2\,b^2+2\,(a^2\cosh(x)+ab)\sinh(x)+2\,\sqrt{-a^2+b^2}\,(a\cosh(x)+a\sinh(x)+b)}{a\cosh(x)^2+a\sinh(x)^2+2\,b\cosh(x)+2\,(a\cosh(x)+b)\sinh(x)+a}\right)-2\,(a^2-b^2)\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}{a^2b-b^3},\ \frac{2\left(\sqrt{a^2-b^2}\,a\arctan\left(-\frac{a\cosh(x)+a\sinh(x)+b}{\sqrt{a^2-b^2}}\right)+(a^2-b^2)\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)\right)}{a^2b-b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [-(sqrt(-a^2 + b^2)*a*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) -
a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-a^2 + b^2)*(a*cosh(x)
 + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x)
+ b)*sinh(x) + a)) - 2*(a^2 - b^2)*arctan(cosh(x) + sinh(x)))/(a^2*b - b^3)
, 2*(sqrt(a^2 - b^2)*a*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^2 - b^2))
 + (a^2 - b^2)*arctan(cosh(x) + sinh(x)))/(a^2*b - b^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(x)}{a + b\operatorname{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(a+b*sech(x)),x)
```

```
[Out] Integral(sech(x)**2/(a + b*sech(x)), x)
```

Giac [A]
time = 0.39, size = 45, normalized size = 0.83

$$-\frac{2\,a\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,b}+\frac{2\,\arctan\left(e^x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] -2*a*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*b) + 2*arctan(e^x
)/b
```

Mupad [B]
time = 4.01, size = 286, normalized size = 5.30

$$\frac{a\,\ln\left(64\,a\,b^4-64\,a^3\,b^2+128\,b^5\,\mathrm{e}^x-64\,a\,b^3\,\sqrt{b^2-a^2}+32\,a^3\,b\,\sqrt{b^2-a^2}+32\,a^4\,b\,\mathrm{e}^x-128\,b^4\,\mathrm{e}^x\,\sqrt{b^2-a^2}-160\,a^2\,b^3\,\mathrm{e}^x+96\,a^2\,b^2\,\mathrm{e}^x\,\sqrt{b^2-a^2}\right)}{b\,\sqrt{b^2-a^2}}-\frac{\ln\left(\mathrm{e}^x-\mathrm{i}\right)\,1\mathrm{i}-\ln\left(\mathrm{e}^x+1\mathrm{i}\right)\,1\mathrm{i}}{b}-\frac{a\,\ln\left(64\,a\,b^4-64\,a^3\,b^2+128\,b^5\,\mathrm{e}^x+64\,a\,b^3\,\sqrt{b^2-a^2}-32\,a^3\,b\,\sqrt{b^2-a^2}+32\,a^4\,b\,\mathrm{e}^x+128\,b^4\,\mathrm{e}^x\,\sqrt{b^2-a^2}-160\,a^2\,b^3\,\mathrm{e}^x-96\,a^2\,b^2\,\mathrm{e}^x\,\sqrt{b^2-a^2}\right)}{b\,\sqrt{b^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2*(a + b/cosh(x))),x)
```

```
[Out] (a*log(64*a*b^4 - 64*a^3*b^2 + 128*b^5*exp(x) - 64*a*b^3*(b^2 - a^2)^(1/2)
+ 32*a^3*b*(b^2 - a^2)^(1/2) + 32*a^4*b*exp(x) - 128*b^4*exp(x)*(b^2 - a^2)
^(1/2) - 160*a^2*b^3*exp(x) + 96*a^2*b^2*exp(x)*(b^2 - a^2)^(1/2)))/(b*(b^2
 - a^2)^(1/2)) - (log(exp(x) - 1i)*1i - log(exp(x) + 1i)*1i)/b - (a*log(64*
a*b^4 - 64*a^3*b^2 + 128*b^5*exp(x) + 64*a*b^3*(b^2 - a^2)^(1/2) - 32*a^3*b
*(b^2 - a^2)^(1/2) + 32*a^4*b*exp(x) + 128*b^4*exp(x)*(b^2 - a^2)^(1/2) - 1
60*a^2*b^3*exp(x) - 96*a^2*b^2*exp(x)*(b^2 - a^2)^(1/2)))/(b*(b^2 - a^2)^(1
/2))
```

3.101 $\int \frac{\text{sech}^3(x)}{a+b\text{sech}(x)} dx$

Optimal. Leaf size=64

$$-\frac{a\text{ArcTan}(\sinh(x))}{b^2}+\frac{2a^2\text{ArcTan}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^2\sqrt{a+b}}+\frac{\tanh(x)}{b}$$

```
[Out] -a*arctan(sinh(x))/b^2+2*a^2*arctan((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/b^
2/(a-b)^(1/2)/(a+b)^(1/2)+tanh(x)/b
```

Rubi [A]
time = 0.11, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3875, 3874, 3855, 3916, 2738, 211}

$$\frac{2a^2\text{ArcTan}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^2\sqrt{a-b}\,\sqrt{a+b}}-\frac{a\text{ArcTan}(\sinh(x))}{b^2}+\frac{\tanh(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^3/(a + b*Sech[x]),x]
```

```
[Out] -((a*ArcTan[Sinh[x]])/b^2) + (2*a^2*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a +
 b]])/(Sqrt[a - b]*b^2*Sqrt[a + b]) + Tanh[x]/b
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3874

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3875

```
Int[csc[(e_.) + (f_.)*(x_)]^3/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Simp[-Cot[e + f*x]/(b*f), x] - Dist[a/b, Int[Csc[e + f*x]^2/(a + b*
Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^3(x)}{a+b\operatorname{sech}(x)}\,dx &= \frac{\tanh(x)}{b} - \frac{a\int \frac{\operatorname{sech}^2(x)}{a+b\operatorname{sech}(x)}\,dx}{b} \\
&= \frac{\tanh(x)}{b} - \frac{a\int \operatorname{sech}(x)\,dx}{b^2} + \frac{a^2\int \frac{\operatorname{sech}(x)}{a+b\operatorname{sech}(x)}\,dx}{b^2} \\
&= -\frac{a\tan^{-1}(\sinh(x))}{b^2} + \frac{\tanh(x)}{b} + \frac{a^2\int \frac{1}{1+\frac{a\cosh(x)}{b}}\,dx}{b^3} \\
&= -\frac{a\tan^{-1}(\sinh(x))}{b^2} + \frac{\tanh(x)}{b} + \frac{(2a^2)\operatorname{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b^3} \\
&= -\frac{a\tan^{-1}(\sinh(x))}{b^2} + \frac{2a^2\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^2\sqrt{a+b}} + \frac{\tanh(x)}{b}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 63, normalized size = 0.98

$$\frac{-2a\operatorname{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right) - \frac{2a^2\operatorname{ArcTan}\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + b\tanh(x)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^3/(a + b*Sech[x]),x]
```

```
[Out] (-2*a*ArcTan[Tanh[x/2]] - (2*a^2*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2
]])/Sqrt[a^2 - b^2] + b*Tanh[x])/b^2
```

Maple [A]
time = 0.60, size = 73, normalized size = 1.14

method	result	s
default	$\frac{2a^2 \arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^2\sqrt{(a+b)(a-b)}} - \frac{2\left(-\frac{b\tanh\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)+1} + a\arctan\left(\tanh\left(\frac{x}{2}\right)\right)\right)}{b^2}$	7
risch	$-\frac{2}{b(1+e^{2x})} - \frac{a^2\ln\left(e^x + \frac{b\sqrt{-a^2+b^2}-a^2+b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,b^2} + \frac{a^2\ln\left(e^x + \frac{b\sqrt{-a^2+b^2}+a^2-b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,b^2} + \frac{ia\ln(e^x-i)}{b^2} - \frac{ia\ln(e^x+i)}{b^2}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^3/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a^2/b^2/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
-2/b^2*(-b*tanh(1/2*x)/(tanh(1/2*x)^2+1)+a*arctan(tanh(1/2*x)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(54) = 108.
time = 0.41, size = 504, normalized size = 7.88

$$\left[-\frac{2a^2b - 2b^3 + \left(a^2\cosh(x)^2 + 2a^2\cosh(x)\sinh(x) + a^2\sinh(x)^2 + a^2\right)\sqrt{-a^2+b^2}\log\left(\frac{a^2\cosh(x)^2 + a^2\sinh(x)^2 + 2ab\cosh(x) - a^2 + 2b^2 + 2\left(a^2\cosh(x) + ab\right)\sinh(x) - 2\sqrt{-a^2+b^2}\left(a\cosh(x) + a\sinh(x) + b\right)}{a\cosh(x)^2 + a\sinh(x)^2 + 2b\cosh(x) + 2\left(a\cosh(x) + b\right)\sinh(x) + a}\right) + 2\left(a^3 - ab^2 + \left(a^3 - ab^2\right)\cosh(x)^2 + 2\left(a^3 - ab^2\right)\cosh(x)\sinh(x) + \left(a^3 - ab^2\right)\sinh(x)^2\right)\arctan\left(\cosh(x) + \sinh(x)\right)}{a^2b^2 - b^4 + \left(a^2b^2 - b^4\right)\cosh(x)^2 + 2\left(a^2b^2 - b^4\right)\cosh(x)\sinh(x) + \left(a^2b^2 - b^4\right)\sinh(x)^2}, -\frac{2\left(a^2b - b^3 + \left(a^2\cosh(x)^2 + 2a^2\cosh(x)\sinh(x) + a^2\sinh(x)^2 + a^2\right)\sqrt{a^2-b^2}\arctan\left(-\frac{a\cosh(x) + a\sinh(x) + b}{\sqrt{a^2-b^2}}\right) + \left(a^3 - ab^2 + \left(a^3 - ab^2\right)\cosh(x)^2 + 2\left(a^3 - ab^2\right)\cosh(x)\sinh(x) + \left(a^3 - ab^2\right)\sinh(x)^2\right)\arctan\left(\cosh(x) + \sinh(x)\right)\right)}{a^2b^2 - b^4 + \left(a^2b^2 - b^4\right)\cosh(x)^2 + 2\left(a^2b^2 - b^4\right)\cosh(x)\sinh(x) + \left(a^2b^2 - b^4\right)\sinh(x)^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [-(2*a^2*b - 2*b^3 + (a^2*cosh(x)^2 + 2*a^2*cosh(x)*sinh(x) + a^2*sinh(x)^2
 + a^2)*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x)
 - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(-a^2 + b^2)*(a*cosh
```

```
(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(
x) + b)*sinh(x) + a)) + 2*(a^3 - a*b^2 + (a^3 - a*b^2)*cosh(x)^2 + 2*(a^3 -
 a*b^2)*cosh(x)*sinh(x) + (a^3 - a*b^2)*sinh(x)^2)*arctan(cosh(x) + sinh(x)
))/(a^2*b^2 - b^4 + (a^2*b^2 - b^4)*cosh(x)^2 + 2*(a^2*b^2 - b^4)*cosh(x)*s
inh(x) + (a^2*b^2 - b^4)*sinh(x)^2), -2*(a^2*b - b^3 + (a^2*cosh(x)^2 + 2*a
^2*cosh(x)*sinh(x) + a^2*sinh(x)^2 + a^2)*sqrt(a^2 - b^2)*arctan(-(a*cosh(x
) + a*sinh(x) + b)/sqrt(a^2 - b^2)) + (a^3 - a*b^2 + (a^3 - a*b^2)*cosh(x)^
2 + 2*(a^3 - a*b^2)*cosh(x)*sinh(x) + (a^3 - a*b^2)*sinh(x)^2)*arctan(cosh(
x) + sinh(x)))/(a^2*b^2 - b^4 + (a^2*b^2 - b^4)*cosh(x)^2 + 2*(a^2*b^2 - b^
4)*cosh(x)*sinh(x) + (a^2*b^2 - b^4)*sinh(x)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^3(x)}{a + b\operatorname{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**3/(a+b*sech(x)),x)
```

```
[Out] Integral(sech(x)**3/(a + b*sech(x)), x)
```

Giac [A]
time = 0.40, size = 61, normalized size = 0.95

$$\frac{2\,a^2 \arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,b^2} - \frac{2\,a\arctan\left(e^x\right)}{b^2} - \frac{2}{b\left(e^{(2\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] 2*a^2*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*b^2) - 2*a*arcta
n(e^x)/b^2 - 2/(b*(e^(2*x) + 1))
```

Mupad [B]
time = 3.88, size = 294, normalized size = 4.59

$$\frac{a^2\,\ln\left(64\,a^3\,b-64\,a\,b^3+32\,a^3\,\sqrt{b^2-a^2}-32\,a^4\,e^x-128\,b^4\,e^x-64\,a\,b^2\,\sqrt{b^2-a^2}-128\,b^3\,e^x\,\sqrt{b^2-a^2}+160\,a^2\,b^2\,e^x+96\,a^2\,b\,e^x\,\sqrt{b^2-a^2}\right)}{b^2\,\sqrt{b^2-a^2}}+\frac{a\,\left(\ln\left(32\,e^x-32i\right)\,1i-\ln\left(32\,e^x+32i\right)\,1i\right)}{b^2}-\frac{2}{b+b\,e^{2\,x}}-\frac{a^2\,\ln\left(64\,a\,b^3-64\,a^3\,b+32\,a^3\,\sqrt{b^2-a^2}+32\,a^4\,e^x+128\,b^4\,e^x-64\,a\,b^2\,\sqrt{b^2-a^2}-128\,b^3\,e^x\,\sqrt{b^2-a^2}-160\,a^2\,b^2\,e^x+96\,a^2\,b\,e^x\,\sqrt{b^2-a^2}\right)}{b^2\,\sqrt{b^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^3*(a + b/cosh(x))),x)
```

```
[Out] (a*(log(32*exp(x) - 32i)*1i - log(32*exp(x) + 32i)*1i))/b^2 - 2/(b + b*exp(
2*x)) + (a^2*log(64*a^3*b - 64*a*b^3 + 32*a^3*(b^2 - a^2)^(1/2) - 32*a^4*ex
p(x) - 128*b^4*exp(x) - 64*a*b^2*(b^2 - a^2)^(1/2) - 128*b^3*exp(x)*(b^2 -
```

```
a^2)^(1/2) + 160*a^2*b^2*exp(x) + 96*a^2*b*exp(x)*(b^2 - a^2)^(1/2)))/(b^2*
(b^2 - a^2)^(1/2)) - (a^2*log(64*a*b^3 - 64*a^3*b + 32*a^3*(b^2 - a^2)^(1/2
) + 32*a^4*exp(x) + 128*b^4*exp(x) - 64*a*b^2*(b^2 - a^2)^(1/2) - 128*b^3*e
xp(x)*(b^2 - a^2)^(1/2) - 160*a^2*b^2*exp(x) + 96*a^2*b*exp(x)*(b^2 - a^2)^
(1/2)))/(b^2*(b^2 - a^2)^(1/2))
```

3.102 $\int \frac{\operatorname{sech}^4(x)}{a+b\operatorname{sech}(x)}\,dx$

Optimal. Leaf size=87

$$\frac{(2a^2+b^2)\operatorname{ArcTan}(\sinh(x))}{2b^3}-\frac{2a^3\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^3\sqrt{a+b}}-\frac{a\tanh(x)}{b^2}+\frac{\operatorname{sech}(x)\tanh(x)}{2b}$$

```
[Out] 1/2*(2*a^2+b^2)*arctan(sinh(x))/b^3-2*a^3*arctan((a-b)^(1/2)*tanh(1/2*x)/(a
+b)^(1/2))/b^3/(a-b)^(1/2)/(a+b)^(1/2)-a*tanh(x)/b^2+1/2*sech(x)*tanh(x)/b
```

Rubi [A]
time = 0.18, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3936, 4167, 4083, 3855, 3916, 2738, 211}

$$-\frac{2a^3\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^3\sqrt{a-b}\sqrt{a+b}}+\frac{(2a^2+b^2)\operatorname{ArcTan}(\sinh(x))}{2b^3}-\frac{a\tanh(x)}{b^2}+\frac{\tanh(x)\operatorname{sech}(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^4/(a + b*Sech[x]),x]
```

```
[Out] ((2*a^2 + b^2)*ArcTan[Sinh[x]])/(2*b^3) - (2*a^3*ArcTan[(Sqrt[a - b]*Tanh[x
/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^3*Sqrt[a + b]) - (a*Tanh[x])/b^2 + (Sech[
x]*Tanh[x])/(2*b)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3936

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-d^3)*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 3)/(b*f*
(n - 2))), x] + Dist[d^3/(b*(n - 2)), Int[(d*Csc[e + f*x])^(n - 3)*(Simp[a*
(n - 3) + b*(n - 3)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x]/(a + b*Csc[
e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ
[n, 3]
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4167

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b
*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\text{sech}^4(x)}{a+b\text{sech}(x)}\,dx = \frac{\text{sech}(x)\tanh(x)}{2b} + \frac{\int \frac{\text{sech}(x)\left(a+b\text{sech}(x)-2a\text{sech}^2(x)\right)}{a+b\text{sech}(x)}\,dx}{2b}$$

$$= -\frac{a\tanh(x)}{b^2} + \frac{\text{sech}(x)\tanh(x)}{2b} + \frac{\int \frac{\text{sech}(x)\left(ab+(2a^2+b^2)\text{sech}(x)\right)}{a+b\text{sech}(x)}\,dx}{2b^2}$$

$$= -\frac{a\tanh(x)}{b^2} + \frac{\text{sech}(x)\tanh(x)}{2b} - \frac{a^3\int \frac{\text{sech}(x)}{a+b\text{sech}(x)}\,dx}{b^3} + \frac{(2a^2+b^2)\int \text{sech}(x)\,dx}{2b^3}$$

$$= \frac{(2a^2+b^2)\tan^{-1}(\sinh(x))}{2b^3} - \frac{a\tanh(x)}{b^2} + \frac{\text{sech}(x)\tanh(x)}{2b} - \frac{a^3\int \frac{1}{1+\frac{a\cosh(x)}{b}}\,dx}{b^4}$$

$$= \frac{(2a^2+b^2)\tan^{-1}(\sinh(x))}{2b^3} - \frac{a\tanh(x)}{b^2} + \frac{\text{sech}(x)\tanh(x)}{2b} - \frac{(2a^3)\,\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-(1-\frac{a}{b}}\right.}{b^4}$$

$$= \frac{(2a^2+b^2)\tan^{-1}(\sinh(x))}{2b^3} - \frac{2a^3\tan^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^3\sqrt{a+b}} - \frac{a\tanh(x)}{b^2} + \frac{\text{sech}(x)\tan}{2b}$$

Mathematica [A]
time = 0.15, size = 82, normalized size = 0.94

$$\frac{2(2a^2+b^2)\,\text{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right) + \frac{4a^3\text{ArcTan}\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + b(-2a+b\text{sech}(x))\tanh(x)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^4/(a + b*Sech[x]),x]
```

```
[Out] (2*(2*a^2 + b^2)*ArcTan[Tanh[x/2]] + (4*a^3*ArcTan[((-a + b)*Tanh[x/2])/Sqr
t[a^2 - b^2]])/Sqrt[a^2 - b^2] + b*(-2*a + b*Sech[x])*Tanh[x])/(2*b^3)
```

Maple [A]
time = 0.74, size = 109, normalized size = 1.25

method	result
default	$-\frac{2a^3\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^3\sqrt{(a+b)(a-b)}} + \frac{\frac{2\left(\left(-ab-\frac{1}{2}b^2\right)\left(\tanh^3\left(\frac{x}{2}\right)\right)+\left(-ab+\frac{1}{2}b^2\right)\tanh\left(\frac{x}{2}\right)\right)}{\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^2}+(2a^2+b^2)\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{b^3}$

risch	$\frac{b\,e^{3x}+2\,e^{2x}a-e^{x}b+2a}{(1+e^{2x})^2 b^2}+\frac{i\ln(e^x+i)a^2}{b^3}+\frac{i\ln(e^x+i)}{2b}-\frac{i\ln(e^x-i)a^2}{b^3}-\frac{i\ln(e^x-i)}{2b}-\frac{a^3\ln\left(e^x+\frac{b\sqrt{-a^2+b^2}+a^2-b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,b^3}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^4/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*a^3/b^3/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2)
)+2/b^3*(((-a*b-1/2*b^2)*tanh(1/2*x)^3+(-a*b+1/2*b^2)*tanh(1/2*x))/(tanh(1/
2*x)^2+1)^2+1/2*(2*a^2+b^2)*arctan(tanh(1/2*x)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 682 vs. 2(73) = 146.
time = 0.46, size = 1444, normalized size = 16.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [(2*a^3*b - 2*a*b^3 + (a^2*b^2 - b^4)*cosh(x)^3 + (a^2*b^2 - b^4)*sinh(x)^3
 + 2*(a^3*b - a*b^3)*cosh(x)^2 + (2*a^3*b - 2*a*b^3 + 3*(a^2*b^2 - b^4)*cos
h(x))*sinh(x)^2 - (a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*sinh(x)^4
+ 2*a^3*cosh(x)^2 + a^3 + 2*(3*a^3*cosh(x)^2 + a^3)*sinh(x)^2 + 4*(a^3*cosh
(x)^3 + a^3*cosh(x))*sinh(x))*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sin
h(x)^2 + 2*a*b*cosh(x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sq
rt(-a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*
b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) + a)) + ((2*a^4 - a^2*b^2 - b^4)*cosh
(x)^4 + 4*(2*a^4 - a^2*b^2 - b^4)*cosh(x)*sinh(x)^3 + (2*a^4 - a^2*b^2 - b^
4)*sinh(x)^4 + 2*a^4 - a^2*b^2 - b^4 + 2*(2*a^4 - a^2*b^2 - b^4)*cosh(x)^2
+ 2*(2*a^4 - a^2*b^2 - b^4 + 3*(2*a^4 - a^2*b^2 - b^4)*cosh(x)^2)*sinh(x)^2
 + 4*((2*a^4 - a^2*b^2 - b^4)*cosh(x)^3 + (2*a^4 - a^2*b^2 - b^4)*cosh(x))*
```

```
sinh(x))*arctan(cosh(x) + sinh(x)) - (a^2*b^2 - b^4)*cosh(x) - (a^2*b^2 - b
^4 - 3*(a^2*b^2 - b^4)*cosh(x)^2 - 4*(a^3*b - a*b^3)*cosh(x))*sinh(x))/(a^2
*b^3 - b^5 + (a^2*b^3 - b^5)*cosh(x)^4 + 4*(a^2*b^3 - b^5)*cosh(x)*sinh(x)^
3 + (a^2*b^3 - b^5)*sinh(x)^4 + 2*(a^2*b^3 - b^5)*cosh(x)^2 + 2*(a^2*b^3 -
b^5 + 3*(a^2*b^3 - b^5)*cosh(x)^2)*sinh(x)^2 + 4*((a^2*b^3 - b^5)*cosh(x)^3
 + (a^2*b^3 - b^5)*cosh(x))*sinh(x)), (2*a^3*b - 2*a*b^3 + (a^2*b^2 - b^4)*
cosh(x)^3 + (a^2*b^2 - b^4)*sinh(x)^3 + 2*(a^3*b - a*b^3)*cosh(x)^2 + (2*a^
3*b - 2*a*b^3 + 3*(a^2*b^2 - b^4)*cosh(x))*sinh(x)^2 + 2*(a^3*cosh(x)^4 + 4
*a^3*cosh(x)*sinh(x)^3 + a^3*sinh(x)^4 + 2*a^3*cosh(x)^2 + a^3 + 2*(3*a^3*c
osh(x)^2 + a^3)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + a^3*cosh(x))*sinh(x))*sqrt(a
^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^2 - b^2)) + ((2*a^4 -
a^2*b^2 - b^4)*cosh(x)^4 + 4*(2*a^4 - a^2*b^2 - b^4)*cosh(x)*sinh(x)^3 + (2
*a^4 - a^2*b^2 - b^4)*sinh(x)^4 + 2*a^4 - a^2*b^2 - b^4 + 2*(2*a^4 - a^2*b^
2 - b^4)*cosh(x)^2 + 2*(2*a^4 - a^2*b^2 - b^4 + 3*(2*a^4 - a^2*b^2 - b^4)*c
osh(x)^2)*sinh(x)^2 + 4*((2*a^4 - a^2*b^2 - b^4)*cosh(x)^3 + (2*a^4 - a^2*b
^2 - b^4)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) - (a^2*b^2 - b^4)*cos
h(x) - (a^2*b^2 - b^4 - 3*(a^2*b^2 - b^4)*cosh(x)^2 - 4*(a^3*b - a*b^3)*cos
h(x))*sinh(x))/(a^2*b^3 - b^5 + (a^2*b^3 - b^5)*cosh(x)^4 + 4*(a^2*b^3 - b^
5)*cosh(x)*sinh(x)^3 + (a^2*b^3 - b^5)*sinh(x)^4 + 2*(a^2*b^3 - b^5)*cosh(x
)^2 + 2*(a^2*b^3 - b^5 + 3*(a^2*b^3 - b^5)*cosh(x)^2)*sinh(x)^2 + 4*((a^2*b
^3 - b^5)*cosh(x)^3 + (a^2*b^3 - b^5)*cosh(x))*sinh(x))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^4(x)}{a + b\operatorname{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**4/(a+b*sech(x)),x)
[Out] Integral(sech(x)**4/(a + b*sech(x)), x)
```

Giac [A]
time = 0.40, size = 89, normalized size = 1.02

$$-\frac{2\,a^3\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,b^3}+\frac{\left(2\,a^2+b^2\right)\arctan\left(e^x\right)}{b^3}+\frac{be^{(3\,x)}+2\,ae^{(2\,x)}-be^x+2\,a}{b^2\left(e^{(2\,x)}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*sech(x)),x, algorithm="giac")
[Out] -2*a^3*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*b^3) + (2*a^2 +
 b^2)*arctan(e^x)/b^3 + (b*e^(3*x) + 2*a*e^(2*x) - b*e^x + 2*a)/(b^2*(e^(2*
x) + 1)^2)
```

Mupad [B]
time = 5.08, size = 476, normalized size = 5.47

$$\frac{e^x}{b+b\,e^{2x}}-\frac{2\,e^x}{b+2\,b\,e^{2x}+b\,e^{4x}}+\frac{2\,a}{b^2\,e^{2x}+b^2}-\frac{\ln\left(1+e^x\,1i\right)\,1i-\ln\left(e^x+1i\right)\,1i}{2\,b}-\frac{a^2\,\left(\ln\left(1+e^x\,1i\right)\,1i-\ln\left(e^x+1i\right)\,1i\right)}{b^3}-\frac{a^3\,\ln\left(16\,a\,b^5-48\,a^5\,b-24\,a^5\,\sqrt{b^2-a^2}+32\,a^3\,b^3+24\,a^6\,e^x+32\,b^6\,e^x+16\,a\,b^4\,\sqrt{b^2-a^2}+40\,a^3\,b^2\,\sqrt{b^2-a^2}+32\,b^5\,e^x\,\sqrt{b^2-a^2}+56\,a^2\,b^4\,e^x-112\,a^4\,b^2\,e^x+72\,a^2\,b^3\,e^x\,\sqrt{b^2-a^2}-72\,a^4\,b\,e^x\,\sqrt{b^2-a^2}\right)}{b^3\,\sqrt{b^2-a^2}}+\frac{a^3\,\ln\left(16\,a\,b^5-48\,a^5\,b+24\,a^5\,\sqrt{b^2-a^2}+32\,a^3\,b^3+24\,a^6\,e^x+32\,b^6\,e^x-16\,a\,b^4\,\sqrt{b^2-a^2}-40\,a^3\,b^2\,\sqrt{b^2-a^2}-32\,b^5\,e^x\,\sqrt{b^2-a^2}+56\,a^2\,b^4\,e^x-112\,a^4\,b^2\,e^x-72\,a^2\,b^3\,e^x\,\sqrt{b^2-a^2}+72\,a^4\,b\,e^x\,\sqrt{b^2-a^2}\right)}{b^3\,\sqrt{b^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^4*(a + b/cosh(x))),x)
```

```
[Out] exp(x)/(b + b*exp(2*x)) - (2*exp(x))/(b + 2*b*exp(2*x) + b*exp(4*x)) + (2*a
)/(b^2*exp(2*x) + b^2) - (log(exp(x)*1i + 1)*1i - log(exp(x) + 1i)*1i)/(2*b
) - (a^2*(log(exp(x)*1i + 1)*1i - log(exp(x) + 1i)*1i))/b^3 - (a^3*log(16*a
*b^5 - 48*a^5*b - 24*a^5*(b^2 - a^2)^(1/2) + 32*a^3*b^3 + 24*a^6*exp(x) + 3
2*b^6*exp(x) + 16*a*b^4*(b^2 - a^2)^(1/2) + 40*a^3*b^2*(b^2 - a^2)^(1/2) +
32*b^5*exp(x)*(b^2 - a^2)^(1/2) + 56*a^2*b^4*exp(x) - 112*a^4*b^2*exp(x) +
72*a^2*b^3*exp(x)*(b^2 - a^2)^(1/2) - 72*a^4*b*exp(x)*(b^2 - a^2)^(1/2)))/(
b^3*(b^2 - a^2)^(1/2)) + (a^3*log(16*a*b^5 - 48*a^5*b + 24*a^5*(b^2 - a^2)^
(1/2) + 32*a^3*b^3 + 24*a^6*exp(x) + 32*b^6*exp(x) - 16*a*b^4*(b^2 - a^2)^(
1/2) - 40*a^3*b^2*(b^2 - a^2)^(1/2) - 32*b^5*exp(x)*(b^2 - a^2)^(1/2) + 56*
a^2*b^4*exp(x) - 112*a^4*b^2*exp(x) - 72*a^2*b^3*exp(x)*(b^2 - a^2)^(1/2) +
 72*a^4*b*exp(x)*(b^2 - a^2)^(1/2)))/(b^3*(b^2 - a^2)^(1/2))
```

3.103 $\int \frac{\tanh^6(x)}{a+a\operatorname{sech}(x)}\,dx$

Optimal. Leaf size=48

$$\frac{x}{a}-\frac{3\text{ArcTan}(\sinh(x))}{8a}-\frac{(8-3\text{sech}(x))\tanh(x)}{8a}-\frac{(4-3\text{sech}(x))\tanh^3(x)}{12a}$$

```
[Out] x/a-3/8*arctan(sinh(x))/a-1/8*(8-3*sech(x))*tanh(x)/a-1/12*(4-3*sech(x))*ta
nh(x)^3/a
```

Rubi [A]
time = 0.07, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3973, 3966, 3855}

$$-\frac{3\text{ArcTan}(\sinh(x))}{8a}+\frac{x}{a}-\frac{\tanh^3(x)(4-3\text{sech}(x))}{12a}-\frac{\tanh(x)(8-3\text{sech}(x))}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^6/(a + a*Sech[x]),x]
```

```
[Out] x/a - (3*ArcTan[Sinh[x]])/(8*a) - ((8 - 3*Sech[x])*Tanh[x])/(8*a) - ((4 - 3
*Sech[x])*Tanh[x]^3)/(12*a)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\tanh^6(x)}{a+a\text{sech}(x)}\,dx = -\frac{\int(-a+a\text{sech}(x))\tanh^4(x)\,dx}{a^2}$$
$$= -\frac{(4-3\text{sech}(x))\tanh^3(x)}{12a} - \frac{\int(-4a+3a\text{sech}(x))\tanh^2(x)\,dx}{4a^2}$$
$$= -\frac{(8-3\text{sech}(x))\tanh(x)}{8a} - \frac{(4-3\text{sech}(x))\tanh^3(x)}{12a} - \frac{\int(-8a+3a\text{sech}(x))\,dx}{8a^2}$$
$$= \frac{x}{a} - \frac{(8-3\text{sech}(x))\tanh(x)}{8a} - \frac{(4-3\text{sech}(x))\tanh^3(x)}{12a} - \frac{3\int\text{sech}(x)\,dx}{8a}$$
$$= \frac{x}{a} - \frac{3\tan^{-1}(\sinh(x))}{8a} - \frac{(8-3\text{sech}(x))\tanh(x)}{8a} - \frac{(4-3\text{sech}(x))\tanh^3(x)}{12a}$$

Mathematica [A]
time = 0.08, size = 60, normalized size = 1.25

$$\frac{\cosh^2\left(\frac{x}{2}\right)\text{sech}(x)\left(6\left(4x-3\text{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)\right)+\left(-32+15\text{sech}(x)+8\text{sech}^2(x)-6\text{sech}^3(x)\right)\tanh(x)\right)}{12a(1+\text{sech}(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^6/(a + a*Sech[x]),x]
```

```
[Out] (Cosh[x/2]^2*Sech[x]*(6*(4*x - 3*ArcTan[Tanh[x/2]]) + (-32 + 15*Sech[x] + 8
*Sech[x]^2 - 6*Sech[x]^3)*Tanh[x]))/(12*a*(1 + Sech[x]))
```

Maple [A]
time = 0.69, size = 75, normalized size = 1.56

method	result	size
default	$\frac{\frac{2\left(-\frac{11\left(\tanh^7\left(\frac{x}{2}\right)\right)}{8}-\frac{137\left(\tanh^5\left(\frac{x}{2}\right)\right)}{24}-\frac{71\left(\tanh^3\left(\frac{x}{2}\right)\right)}{24}-\frac{5\tanh\left(\frac{x}{2}\right)}{8}\right)}{\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^4}-\frac{3\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{4}+\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}$	75
risch	$\frac{x}{a}+\frac{15\,e^{7x}+48\,e^{6x}-9\,e^{5x}+96\,e^{4x}+9\,e^{3x}+80\,e^{2x}-15\,e^{x}+32}{12\left(1+e^{2x}\right)^4 a}+\frac{3i\ln\left(e^{x}-i\right)}{8a}-\frac{3i\ln\left(e^{x}+i\right)}{8a}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^6/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 64/a*(1/32*(-11/8*tanh(1/2*x)^7-137/24*tanh(1/2*x)^5-71/24*tanh(1/2*x)^3-5/
8*tanh(1/2*x))/(tanh(1/2*x)^2+1)^4-3/256*arctan(tanh(1/2*x))+1/64*ln(tanh(1
/2*x)+1)-1/64*ln(tanh(1/2*x)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(42) = 84.

time = 0.49, size = 93, normalized size = 1.94

$$\frac{x}{a}+\frac{15\,e^{(-x)}-80\,e^{(-2\,x)}-9\,e^{(-3\,x)}-96\,e^{(-4\,x)}+9\,e^{(-5\,x)}-48\,e^{(-6\,x)}-15\,e^{(-7\,x)}-32}{12\,(4\,ae^{(-2\,x)}+6\,ae^{(-4\,x)}+4\,ae^{(-6\,x)}+ae^{(-8\,x)}+a)}+\frac{3\,\arctan\left(e^{(-x)}\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^6/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] x/a + 1/12*(15*e^(-x) - 80*e^(-2*x) - 9*e^(-3*x) - 96*e^(-4*x) + 9*e^(-5*x)
 - 48*e^(-6*x) - 15*e^(-7*x) - 32)/(4*a*e^(-2*x) + 6*a*e^(-4*x) + 4*a*e^(-6
*x) + a*e^(-8*x) + a) + 3/4*arctan(e^(-x))/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 686 vs. 2(42) = 84.
time = 0.39, size = 686, normalized size = 14.29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^6/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] 1/12*(12*x*cosh(x)^8 + 12*x*sinh(x)^8 + 3*(32*x*cosh(x) + 5)*sinh(x)^7 + 48
*(x + 1)*cosh(x)^6 + 15*cosh(x)^7 + 3*(112*x*cosh(x)^2 + 16*x + 35*cosh(x)
+ 16)*sinh(x)^6 + 3*(224*x*cosh(x)^3 + 96*(x + 1)*cosh(x) + 105*cosh(x)^2 -
 3)*sinh(x)^5 + 24*(3*x + 4)*cosh(x)^4 - 9*cosh(x)^5 + 3*(280*x*cosh(x)^4 +
 240*(x + 1)*cosh(x)^2 + 175*cosh(x)^3 + 24*x - 15*cosh(x) + 32)*sinh(x)^4
+ 3*(224*x*cosh(x)^5 + 320*(x + 1)*cosh(x)^3 + 175*cosh(x)^4 + 32*(3*x + 4)
*cosh(x) - 30*cosh(x)^2 + 3)*sinh(x)^3 + 16*(3*x + 5)*cosh(x)^2 + 9*cosh(x)
^3 + (336*x*cosh(x)^6 + 720*(x + 1)*cosh(x)^4 + 315*cosh(x)^5 + 144*(3*x +
4)*cosh(x)^2 - 90*cosh(x)^3 + 48*x + 27*cosh(x) + 80)*sinh(x)^2 - 9*(cosh(x
)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 + 1)*sinh(x)^6 + 4*c
osh(x)^6 + 8*(7*cosh(x)^3 + 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 + 30*cos
h(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cosh(x)^5 + 10*cosh(x)^3 + 3*cos
h(x))*sinh(x)^3 + 4*(7*cosh(x)^6 + 15*cosh(x)^4 + 9*cosh(x)^2 + 1)*sinh(x)^
2 + 4*cosh(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^5 + 3*cosh(x)^3 + cosh(x))*sinh(
x) + 1)*arctan(cosh(x) + sinh(x)) + (96*x*cosh(x)^7 + 288*(x + 1)*cosh(x)^5
 + 105*cosh(x)^6 + 96*(3*x + 4)*cosh(x)^3 - 45*cosh(x)^4 + 32*(3*x + 5)*cos
h(x) + 27*cosh(x)^2 - 15)*sinh(x) + 12*x - 15*cosh(x) + 32)/(a*cosh(x)^8 +
8*a*cosh(x)*sinh(x)^7 + a*sinh(x)^8 + 4*a*cosh(x)^6 + 4*(7*a*cosh(x)^2 + a)
*sinh(x)^6 + 8*(7*a*cosh(x)^3 + 3*a*cosh(x))*sinh(x)^5 + 6*a*cosh(x)^4 + 2*
(35*a*cosh(x)^4 + 30*a*cosh(x)^2 + 3*a)*sinh(x)^4 + 8*(7*a*cosh(x)^5 + 10*a
*cosh(x)^3 + 3*a*cosh(x))*sinh(x)^3 + 4*a*cosh(x)^2 + 4*(7*a*cosh(x)^6 + 15
*a*cosh(x)^4 + 9*a*cosh(x)^2 + a)*sinh(x)^2 + 8*(a*cosh(x)^7 + 3*a*cosh(x)^
5 + 3*a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tanh^6(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**6/(a+a*sech(x)),x)
```

```
[Out] Integral(tanh(x)**6/(sech(x) + 1), x)/a
```

Giac [A]
time = 0.39, size = 69, normalized size = 1.44

$$\frac{x}{a} - \frac{3\arctan\left(e^{x}\right)}{4a} + \frac{15e^{(7x)} + 48e^{(6x)} - 9e^{(5x)} + 96e^{(4x)} + 9e^{(3x)} + 80e^{(2x)} - 15e^{x} + 32}{12a\left(e^{(2x)} + 1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^6/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] x/a - 3/4*arctan(e^x)/a + 1/12*(15*e^(7*x) + 48*e^(6*x) - 9*e^(5*x) + 96*e^
(4*x) + 9*e^(3*x) + 80*e^(2*x) - 15*e^x + 32)/(a*(e^(2*x) + 1)^4)
```

Mupad [B]
time = 1.46, size = 143, normalized size = 2.98

$$\frac{\frac{8}{3a} + \frac{6e^x}{a}}{3e^{2x} + 3e^{4x} + e^{6x} + 1} - \frac{\frac{4}{a} + \frac{9e^x}{2a}}{2e^{2x} + e^{4x} + 1} + \frac{x}{a} + \frac{\frac{4}{a} + \frac{5e^x}{4a}}{e^{2x} + 1} - \frac{3\,\mathrm{atan}\left(\frac{e^x\sqrt{a^2}}{a}\right)}{4\sqrt{a^2}} - \frac{4e^x}{a\left(4e^{2x} + 6e^{4x} + 4e^{6x} + e^{8x} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^6/(a + a/cosh(x)),x)
```

```
[Out] (8/(3*a) + (6*exp(x))/a)/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - (4/a +
(9*exp(x))/(2*a))/(2*exp(2*x) + exp(4*x) + 1) + x/a + (4/a + (5*exp(x))/(4*
a))/(exp(2*x) + 1) - (3*atan((exp(x)*(a^2)^(1/2))/a))/(4*(a^2)^(1/2)) - (4*
exp(x))/(a*(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1))
```

3.104 $\int \frac{\tanh^5(x)}{a+a\text{sech}(x)}\,dx$

Optimal. Leaf size=36

$$\frac{\log(\cosh(x))}{a}+\frac{\text{sech}(x)}{a}+\frac{\text{sech}^2(x)}{2a}-\frac{\text{sech}^3(x)}{3a}$$

```
[Out] ln(cosh(x))/a+sech(x)/a+1/2*sech(x)^2/a-1/3*sech(x)^3/a
```

Rubi [A]
time = 0.04, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3964, 76}

$$-\frac{\text{sech}^3(x)}{3a}+\frac{\text{sech}^2(x)}{2a}+\frac{\text{sech}(x)}{a}+\frac{\log(\cosh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^5/(a + a*Sech[x]),x]
```

```
[Out] Log[Cosh[x]]/a + Sech[x]/a + Sech[x]^2/(2*a) - Sech[x]^3/(3*a)
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^5(x)}{a+a\text{sech}(x)}\,dx &= \frac{\text{Subst}\left(\int \frac{(a-ax)^2(a+ax)}{x^4}\,dx,x,\cosh(x)\right)}{a^4}\\
&= \frac{\text{Subst}\left(\int \left(\frac{a^3}{x^4}-\frac{a^3}{x^3}-\frac{a^3}{x^2}+\frac{a^3}{x}\right)\,dx,x,\cosh(x)\right)}{a^4}\\
&= \frac{\log(\cosh(x))}{a}+\frac{\text{sech}(x)}{a}+\frac{\text{sech}^2(x)}{2a}-\frac{\text{sech}^3(x)}{3a}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 38, normalized size = 1.06

$$\frac{(2+6\cosh(2x)+3\cosh(3x)\log(\cosh(x))+\cosh(x)(6+9\log(\cosh(x))))\operatorname{sech}^3(x)}{12a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^5/(a + a*Sech[x]),x]
```

```
[Out] ((2 + 6*Cosh[2*x] + 3*Cosh[3*x]*Log[Cosh[x]] + Cosh[x]*(6 + 9*Log[Cosh[x]])
)*Sech[x]^3)/(12*a)
```

Maple [A]
time = 0.66, size = 26, normalized size = 0.72

method	result	size
derivativedivides	$-\frac{\frac{\operatorname{sech}(x)^3}{3}-\frac{\operatorname{sech}(x)^2}{2}-\operatorname{sech}(x)+\ln(\operatorname{sech}(x))}{a}$	26
default	$-\frac{\frac{\operatorname{sech}(x)^3}{3}-\frac{\operatorname{sech}(x)^2}{2}-\operatorname{sech}(x)+\ln(\operatorname{sech}(x))}{a}$	26
risch	$-\frac{x}{a}+\frac{2\,e^{x}\left(3\,e^{4x}+3\,e^{3x}+2\,e^{2x}+3\,e^{x}+3\right)}{3\left(1+e^{2x}\right)^3 a}+\frac{\ln\left(1+e^{2x}\right)}{a}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a*(1/3*sech(x)^3-1/2*sech(x)^2-sech(x)+ln(sech(x)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(32) = 64.
time = 0.48, size = 74, normalized size = 2.06

$$\frac{x}{a}+\frac{2\left(3\,e^{(-x)}+3\,e^{(-2\,x)}+2\,e^{(-3\,x)}+3\,e^{(-4\,x)}+3\,e^{(-5\,x)}\right)}{3\left(3\,ae^{(-2\,x)}+3\,ae^{(-4\,x)}+ae^{(-6\,x)}+a\right)}+\frac{\log\left(e^{(-2\,x)}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] x/a + 2/3*(3*e^(-x) + 3*e^(-2*x) + 2*e^(-3*x) + 3*e^(-4*x) + 3*e^(-5*x))/(3
*a*e^(-2*x) + 3*a*e^(-4*x) + a*e^(-6*x) + a) + log(e^(-2*x) + 1)/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 437 vs. 2(32) = 64.
time = 0.40, size = 437, normalized size = 12.14

$$-\frac{3\,x\cosh(x)^6+3\,x\sinh(x)^6+6\,(3\,x\cosh(x)-1)\sinh(x)^5+3\,(3\,x-2)\cosh(x)^4-6\cosh(x)^5+3\,(15\,x\cosh(x)^2+3\,x-10\cosh(x)-2)\sinh(x)^4+4\,(15\,x\cosh(x)^3+3\,(3\,x-2)\cosh(x)-15\cosh(x)^2-1)\sinh(x)^3+3\,(3\,x-2)\cosh(x)^2-4\cosh(x)^3+3\,(15\,x\cosh(x)^4+6\,(3\,x-2)\cosh(x)^2-20\cosh(x)^3+3\,x-4\cosh(x)-2)\sinh(x)^2-3\,(\cosh(x)^6+6\cosh(x)\sinh(x)^5+\sinh(x)^6+3\,(5\cosh(x)^2+1)\sinh(x)^4+3\cosh(x)^4+4\,(5\cosh(x)^3+3\cosh(x))\sinh(x)^3+3\,(5\cosh(x)^4+6\cosh(x)^2+1)\sinh(x)^2+3\cosh(x)^2+6\,(\cosh(x)^5+2\cosh(x)^3+\cosh(x))\sinh(x)+1)\log\left(\frac{2\cosh(x)}{\cosh(x)-\sinh(x)}\right)+6\,(3\,x\cosh(x)^5+2\,(3\,x-2)\cosh(x)^3-5\cosh(x)^4+(3\,x-2)\cosh(x)-2\cosh(x)^2-1)\sinh(x)+3\,x-6\cosh(x)}{3\,(a\cosh(x)^6+6\,a\cosh(x)\sinh(x)^5+a\sinh(x)^6+3\,a\cosh(x)^4+3\,(5\,a\cosh(x)^2+a)\sinh(x)^4+4\,(5\,a\cosh(x)^3+3\,a\cosh(x))\sinh(x)^3+3\,a\cosh(x)^2+3\,(5\,a\cosh(x)^4+6\,a\cosh(x)^2+a)\sinh(x)^2+6\,(a\cosh(x)^5+2\,a\cosh(x)^3+a\cosh(x))\sinh(x)+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/3*(3*x*cosh(x)^6 + 3*x*sinh(x)^6 + 6*(3*x*cosh(x) - 1)*sinh(x)^5 + 3*(3*
x - 2)*cosh(x)^4 - 6*cosh(x)^5 + 3*(15*x*cosh(x)^2 + 3*x - 10*cosh(x) - 2)*
sinh(x)^4 + 4*(15*x*cosh(x)^3 + 3*(3*x - 2)*cosh(x) - 15*cosh(x)^2 - 1)*sin
h(x)^3 + 3*(3*x - 2)*cosh(x)^2 - 4*cosh(x)^3 + 3*(15*x*cosh(x)^4 + 6*(3*x -
 2)*cosh(x)^2 - 20*cosh(x)^3 + 3*x - 4*cosh(x) - 2)*sinh(x)^2 - 3*(cosh(x)^
6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 + 1)*sinh(x)^4 + 3*cos
h(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4 + 6*cosh(x)
^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 + 2*cosh(x)^3 + cosh(x))*sin
h(x) + 1)*log(2*cosh(x)/(cosh(x) - sinh(x))) + 6*(3*x*cosh(x)^5 + 2*(3*x -
2)*cosh(x)^3 - 5*cosh(x)^4 + (3*x - 2)*cosh(x) - 2*cosh(x)^2 - 1)*sinh(x) +
 3*x - 6*cosh(x))/(a*cosh(x)^6 + 6*a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 + 3*a*
cosh(x)^4 + 3*(5*a*cosh(x)^2 + a)*sinh(x)^4 + 4*(5*a*cosh(x)^3 + 3*a*cosh(x
))*sinh(x)^3 + 3*a*cosh(x)^2 + 3*(5*a*cosh(x)^4 + 6*a*cosh(x)^2 + a)*sinh(x
)^2 + 6*(a*cosh(x)^5 + 2*a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tanh^5(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**5/(a+a*sech(x)),x)
```

```
[Out] Integral(tanh(x)**5/(sech(x) + 1), x)/a
```

Giac [A]
time = 0.38, size = 61, normalized size = 1.69

$$\frac{\log\left(e^{(-x)}+e^{x}\right)}{a}-\frac{11\left(e^{(-x)}+e^{x}\right)^{3}-12\left(e^{(-x)}+e^{x}\right)^{2}-12\,e^{(-x)}-12\,e^{x}+16}{6\,a(e^{(-x)}+e^{x})^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] log(e^(-x) + e^x)/a - 1/6*(11*(e^(-x) + e^x)^3 - 12*(e^(-x) + e^x)^2 - 12*e
^(-x) - 12*e^x + 16)/(a*(e^(-x) + e^x)^3)
```

Mupad [B]
time = 1.43, size = 96, normalized size = 2.67

$$\frac{\ln\left(\mathrm{e}^{2\,x}+1\right)}{a}-\frac{\frac{2}{a}+\frac{8\,\mathrm{e}^{x}}{3\,a}}{2\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1}-\frac{x}{a}+\frac{\frac{2}{a}+\frac{2\,\mathrm{e}^{x}}{a}}{\mathrm{e}^{2\,x}+1}+\frac{8\,\mathrm{e}^{x}}{3\,a\,\left(3\,\mathrm{e}^{2\,x}+3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a + a/cosh(x)),x)
```

```
[Out] log(exp(2*x) + 1)/a - (2/a + (8*exp(x))/(3*a))/(2*exp(2*x) + exp(4*x) + 1)
- x/a + (2/a + (2*exp(x))/a)/(exp(2*x) + 1) + (8*exp(x))/(3*a*(3*exp(2*x) +
 3*exp(4*x) + exp(6*x) + 1))
```

3.105 $\int \frac{\tanh^4(x)}{a+a\operatorname{sech}(x)} dx$

Optimal. Leaf size=31

$$\frac{x}{a} - \frac{\text{ArcTan}(\sinh(x))}{2a} - \frac{(2-\operatorname{sech}(x))\tanh(x)}{2a}$$

```
[Out] x/a-1/2*arctan(sinh(x))/a-1/2*(2-sech(x))*tanh(x)/a
```

Rubi [A]
time = 0.05, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3973, 3966, 3855}

$$-\frac{\text{ArcTan}(\sinh(x))}{2a} + \frac{x}{a} - \frac{\tanh(x)(2-\operatorname{sech}(x))}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/(a + a*Sech[x]),x]
```

```
[Out] x/a - ArcTan[Sinh[x]]/(2*a) - ((2 - Sech[x])*Tanh[x])/(2*a)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\tanh^4(x)}{a+a\text{sech}(x)}\,dx = -\frac{\int(-a+a\text{sech}(x))\tanh^2(x)\,dx}{a^2}$$
$$= -\frac{(2-\text{sech}(x))\tanh(x)}{2a} - \frac{\int(-2a+a\text{sech}(x))\,dx}{2a^2}$$
$$= \frac{x}{a} - \frac{(2-\text{sech}(x))\tanh(x)}{2a} - \frac{\int\text{sech}(x)\,dx}{2a}$$
$$= \frac{x}{a} - \frac{\tan^{-1}(\sinh(x))}{2a} - \frac{(2-\text{sech}(x))\tanh(x)}{2a}$$

Mathematica [A]
time = 0.04, size = 41, normalized size = 1.32

$$\frac{\cosh^2\left(\frac{x}{2}\right)\text{sech}(x)\left(2\left(x-\text{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)\right)+(-2+\text{sech}(x))\tanh(x)\right)}{a(1+\text{sech}(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(a + a*Sech[x]),x]
[Out] (Cosh[x/2]^2*Sech[x]*(2*(x - ArcTan[Tanh[x/2]]) + (-2 + Sech[x])*Tanh[x]))/
(a*(1 + Sech[x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(27) = 54$.
time = 0.67, size = 59, normalized size = 1.90

method	result	size
default	$\frac{-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{2\left(-\frac{3\left(\tanh^3\left(\frac{x}{2}\right)\right)}{2}-\frac{\tanh\left(\frac{x}{2}\right)}{2}\right)}{\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^2}-\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$	59
risch	$\frac{x}{a}+\frac{e^{3x}+2\,e^{2x}-e^{x}+2}{\left(1+e^{2x}\right)^2 a}+\frac{i\ln\left(e^{x}-i\right)}{2a}-\frac{i\ln\left(e^{x}+i\right)}{2a}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a+a*sech(x)),x,method=_RETURNVERBOSE)
[Out] 16/a*(-1/16*ln(tanh(1/2*x)-1)+1/16*ln(tanh(1/2*x)+1)+1/8*(-3/2*tanh(1/2*x)^
3-1/2*tanh(1/2*x))/(tanh(1/2*x)^2+1)^2-1/16*arctan(tanh(1/2*x)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(25) = 50$.
time = 0.48, size = 51, normalized size = 1.65

$$\frac{x}{a} + \frac{e^{(-x)} - 2\,e^{(-2\,x)} - e^{(-3\,x)} - 2}{2\,ae^{(-2\,x)} + ae^{(-4\,x)} + a} + \frac{\arctan\left(e^{(-x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] x/a + (e^(-x) - 2*e^(-2*x) - e^(-3*x) - 2)/(2*a*e^(-2*x) + a*e^(-4*x) + a)
+ arctan(e^(-x))/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(25) = 50$.
time = 0.38, size = 210, normalized size = 6.77

$$\frac{x\cosh(x)^4 + x\sinh(x)^4 + (4x\cosh(x)+1)\sinh(x)^3 + 2(x+1)\cosh(x)^2 + \cosh(x)^3 + (6x\cosh(x)^2 + 2x + 3\cosh(x) + 2)\sinh(x)^2 - (\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2(3\cosh(x)^2+1)\sinh(x)^2 + 2\cosh(x)^2 + 4(\cosh(x)^3 + \cosh(x))\sinh(x) + 1)\arctan(\cosh(x)+\sinh(x)) + (4x\cosh(x)^3 + 4(x+1)\cosh(x) + 3\cosh(x)^2 - 1)\sinh(x) + x - \cosh(x) + 2}{a\cosh(x)^4 + 4a\cosh(x)\sinh(x)^3 + a\sinh(x)^4 + 2a\cosh(x)^2 + 2(3a\cosh(x)^2 + a)\sinh(x)^2 + 4(a\cosh(x)^3 + a\cosh(x))\sinh(x) + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] (x*cosh(x)^4 + x*sinh(x)^4 + (4*x*cosh(x) + 1)*sinh(x)^3 + 2*(x + 1)*cosh(x
)^2 + cosh(x)^3 + (6*x*cosh(x)^2 + 2*x + 3*cosh(x) + 2)*sinh(x)^2 - (cosh(x
)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*c
osh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)*arctan(cosh(x) + sinh(x)) +
 (4*x*cosh(x)^3 + 4*(x + 1)*cosh(x) + 3*cosh(x)^2 - 1)*sinh(x) + x - cosh(x
) + 2)/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*a*cosh(x)^2 +
 2*(3*a*cosh(x)^2 + a)*sinh(x)^2 + 4*(a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tanh^4(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(a+a*sech(x)),x)
```

```
[Out] Integral(tanh(x)**4/(sech(x) + 1), x)/a
```

Giac [A]
time = 0.38, size = 42, normalized size = 1.35

$$\frac{x}{a} - \frac{\arctan(e^x)}{a} + \frac{e^{(3x)} + 2e^{(2x)} - e^x + 2}{a(e^{(2x)}+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] x/a - arctan(e^x)/a + (e^(3*x) + 2*e^(2*x) - e^x + 2)/(a*(e^(2*x) + 1)^2)
```

Mupad [B]
time = 1.44, size = 67, normalized size = 2.16

$$\frac{x}{a}+\frac{\frac{2}{a}+\frac{e^x}{a}}{e^{2x}+1}-\frac{\operatorname{atan}\left(\frac{e^x\sqrt{a^2}}{a}\right)}{\sqrt{a^2}}-\frac{2\,e^x}{a\,\left(2\,e^{2x}+e^{4x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a + a/cosh(x)),x)
```

```
[Out] x/a + (2/a + exp(x)/a)/(exp(2*x) + 1) - atan((exp(x)*(a^2)^(1/2))/a)/(a^2)^
(1/2) - (2*exp(x))/(a*(2*exp(2*x) + exp(4*x) + 1))
```

3.106 $\int \frac{\tanh^3(x)}{a+a\text{sech}(x)}\,dx$

Optimal. Leaf size=14

$$\frac{\log(\cosh(x))}{a}+\frac{\text{sech}(x)}{a}$$

[Out] ln(cosh(x))/a+sech(x)/a

Rubi [A]

time = 0.03, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3964, 45}

$$\frac{\text{sech}(x)}{a}+\frac{\log(\cosh(x))}{a}$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]^3/(a + a*Sech[x]),x]

[Out] Log[Cosh[x]]/a + Sech[x]/a

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^3(x)}{a+a\text{sech}(x)}\,dx &= -\frac{\text{Subst}\left(\int \frac{a-ax}{x^2}\,dx,x,\cosh(x)\right)}{a^2}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{a}{x^2}-\frac{a}{x}\right)\,dx,x,\cosh(x)\right)}{a^2}\\
&= \frac{\log(\cosh(x))}{a}+\frac{\text{sech}(x)}{a}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 10, normalized size = 0.71

$$\frac{\log(\cosh(x)) + \operatorname{sech}(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(a + a*Sech[x]),x]
```

```
[Out] (Log[Cosh[x]] + Sech[x])/a
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(14) = 28$.
time = 0.66, size = 48, normalized size = 3.43

method	result	size
risch	$-\frac{x}{a}+\frac{2\,\mathrm{e}^{x}}{a\left(1+\mathrm{e}^{2x}\right)}+\frac{\ln\left(1+\mathrm{e}^{2x}\right)}{a}$	34
default	$\frac{-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\ln\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)+\frac{8}{4\left(\tanh^{2}\left(\frac{x}{2}\right)\right)+4}}{a}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 8/a*(-1/8*ln(tanh(1/2*x)-1)-1/8*ln(tanh(1/2*x)+1)+1/8*ln(tanh(1/2*x)^2+1)+1
/4/(tanh(1/2*x)^2+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(14) = 28$.
time = 0.46, size = 33, normalized size = 2.36

$$\frac{x}{a} + \frac{2\,e^{(-x)}}{ae^{(-2\,x)} + a} + \frac{\log\left(e^{(-2\,x)} + 1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] x/a + 2*e^(-x)/(a*e^(-2*x) + a) + log(e^(-2*x) + 1)/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(14) = 28$.
time = 0.37, size = 85, normalized size = 6.07

$$-\frac{x\cosh\left(x\right)^{2} + x\sinh\left(x\right)^{2} - \left(\cosh\left(x\right)^{2} + 2\,\cosh\left(x\right)\sinh\left(x\right) + \sinh\left(x\right)^{2} + 1\right)\log\left(\frac{2\,\cosh\left(x\right)}{\cosh\left(x\right)-\sinh\left(x\right)}\right) + 2\left(x\cosh\left(x\right) - 1\right)\sinh\left(x\right) + x - 2\,\cosh\left(x\right)}{a\cosh\left(x\right)^{2} + 2\,a\cosh\left(x\right)\sinh\left(x\right) + a\sinh\left(x\right)^{2} + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] -(x*cosh(x)^2 + x*sinh(x)^2 - (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 +
1)*log(2*cosh(x)/(cosh(x) - sinh(x))) + 2*(x*cosh(x) - 1)*sinh(x) + x - 2*c
osh(x))/(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tanh^3(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(a+a*sech(x)),x)
```

```
[Out] Integral(tanh(x)**3/(sech(x) + 1), x)/a
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(14) = 28.
time = 0.39, size = 35, normalized size = 2.50

$$\frac{\log\left(e^{(-x)}+e^x\right)}{a}-\frac{e^{(-x)}+e^x-2}{a(e^{(-x)}+e^x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] log(e^(-x) + e^x)/a - (e^(-x) + e^x - 2)/(a*(e^(-x) + e^x))
```

Mupad [B]
time = 1.36, size = 33, normalized size = 2.36

$$\frac{\ln\left(e^{2x}+1\right)}{a}-\frac{x}{a}+\frac{2\,e^x}{a\,\left(e^{2x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a + a/cosh(x)),x)
```

```
[Out] log(exp(2*x) + 1)/a - x/a + (2*exp(x))/(a*(exp(2*x) + 1))
```

3.107 $\int \frac{\tanh^2(x)}{a+a\text{sech}(x)} dx$

Optimal. Leaf size=14

$$\frac{x}{a}-\frac{\text{ArcTan}(\sinh(x))}{a}$$

```
[Out] x/a-arctan(sinh(x))/a
```

Rubi [A]

time = 0.03, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3973, 3855}

$$\frac{x}{a}-\frac{\text{ArcTan}(\sinh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(a + a*Sech[x]),x]
```

```
[Out] x/a - ArcTan[Sinh[x]]/a
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^2(x)}{a+a\text{sech}(x)}\,dx &= -\frac{\int (-a+a\text{sech}(x))\,dx}{a^2}\\
&= \frac{x}{a}-\frac{\int \text{sech}(x)\,dx}{a}\\
&= \frac{x}{a}-\frac{\tan^{-1}(\sinh(x))}{a}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 15, normalized size = 1.07

$$\frac{x-2\text{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(a + a*Sech[x]),x]
[Out] (x - 2*ArcTan[Tanh[x/2]])/a
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(14) = 28$.
time = 0.60, size = 32, normalized size = 2.29

method	result	size
risch	$\frac{x}{a}+\frac{i\ln(e^{x}-i)}{a}-\frac{i\ln(e^{x}+i)}{a}$	31
default	$\frac{\ln(\tanh(\frac{x}{2})+1)-2\arctan(\tanh(\frac{x}{2}))-\ln(\tanh(\frac{x}{2})-1)}{a}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a+a*sech(x)),x,method=_RETURNVERBOSE)
[Out] 4/a*(1/4*ln(tanh(1/2*x)+1)-1/2*arctan(tanh(1/2*x))-1/4*ln(tanh(1/2*x)-1))
```

Maxima [A]
time = 0.47, size = 16, normalized size = 1.14

$$\frac{x}{a}+\frac{2\arctan\left(e^{(-x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+a*sech(x)),x, algorithm="maxima")
[Out] x/a + 2*arctan(e^(-x))/a
```

Fricas [A]
time = 0.37, size = 14, normalized size = 1.00

$$\frac{x-2\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+a*sech(x)),x, algorithm="fricas")
[Out] (x - 2*arctan(cosh(x) + sinh(x)))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tanh^2(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(a+a*sech(x)),x)
```

```
[Out] Integral(tanh(x)**2/(sech(x) + 1), x)/a
```

Giac [A]
time = 0.39, size = 14, normalized size = 1.00

$$\frac{x}{a} - \frac{2\arctan(e^x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] x/a - 2*arctan(e^x)/a
```

Mupad [B]
time = 1.32, size = 25, normalized size = 1.79

$$\frac{x}{a} - \frac{2\,\mathrm{atan}\left(\frac{e^x\sqrt{a^2}}{a}\right)}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a + a/cosh(x)),x)
```

```
[Out] x/a - (2*atan((exp(x)*(a^2)^(1/2))/a))/(a^2)^(1/2)
```

3.108 $\int \frac{\tanh(x)}{a+a\operatorname{sech}(x)}\,dx$

Optimal. Leaf size=9

$$\frac{\log(1+\cosh(x))}{a}$$

[Out] ln(1+cosh(x))/a

Rubi [A]
time = 0.02, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3964, 31}

$$\frac{\log(\cosh(x)+1)}{a}$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]/(a + a*Sech[x]),x]

[Out] Log[1 + Cosh[x]]/a

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh(x)}{a+a\operatorname{sech}(x)}\,dx &= \operatorname{Subst}\left(\int \frac{1}{a+ax}\,dx, x, \cosh(x)\right)\\ &= \frac{\log(1+\cosh(x))}{a}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 1.33

$$\frac{2\log\left(\cosh\left(\frac{x}{2}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(a + a*Sech[x]),x]
[Out] (2*Log[Cosh[x/2]])/a
```

Maple [A]
time = 0.58, size = 17, normalized size = 1.89

method	result	size
derivativedivides	$-\frac{\ln(\operatorname{sech}(x))-\ln(1+\operatorname{sech}(x))}{a}$	17
default	$-\frac{\ln(\operatorname{sech}(x))-\ln(1+\operatorname{sech}(x))}{a}$	17
risch	$-\frac{x}{a}+\frac{2\ln(\mathrm{e}^{x}+1)}{a}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+a*sech(x)),x,method=_RETURNVERBOSE)
[Out] -1/a*(ln(sech(x))-ln(1+sech(x)))
```

Maxima [A]
time = 0.27, size = 18, normalized size = 2.00

$$\frac{x}{a}+\frac{2\,\log\left(e^{(-x)}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+a*sech(x)),x, algorithm="maxima")
[Out] x/a + 2*log(e^(-x) + 1)/a
```

Fricas [A]
time = 0.35, size = 16, normalized size = 1.78

$$-\frac{x-2\,\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+a*sech(x)),x, algorithm="fricas")
[Out] -(x - 2*log(cosh(x) + sinh(x) + 1))/a
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(7) = 14$.
time = 0.07, size = 19, normalized size = 2.11

$$\frac{x}{a}-\frac{\log\left(\tanh\left(x\right)+1\right)}{a}+\frac{\log\left(\operatorname{sech}\left(x\right)+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+a*sech(x)),x)
```

```
[Out] x/a - log(tanh(x) + 1)/a + log(sech(x) + 1)/a
```

Giac [A]
time = 0.40, size = 17, normalized size = 1.89

$$-\frac{x}{a} + \frac{2\,\log\left(e^{x} + 1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] -x/a + 2*log(e^x + 1)/a
```

Mupad [B]
time = 1.31, size = 14, normalized size = 1.56

$$-\frac{x - 2\,\ln\left(\mathrm{e}^{x} + 1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a + a/cosh(x)),x)
```

```
[Out] -(x - 2*log(exp(x) + 1))/a
```

3.109 $\int \frac{\coth(x)}{a+a\,\mathrm{sech}(x)}\,dx$

Optimal. Leaf size=40

$$\frac{1}{2a(1+\cosh(x))}+\frac{\log(1-\cosh(x))}{4a}+\frac{3\log(1+\cosh(x))}{4a}$$

```
[Out] 1/2/a/(1+cosh(x))+1/4*ln(1-cosh(x))/a+3/4*ln(1+cosh(x))/a
```

Rubi [A]
time = 0.04, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = $\{3964, 90\}$

$$\frac{1}{2a(\cosh(x)+1)}+\frac{\log(1-\cosh(x))}{4a}+\frac{3\log(\cosh(x)+1)}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(a + a*Sech[x]),x]
```

```
[Out] 1/(2*a*(1 + Cosh[x])) + Log[1 - Cosh[x]]/(4*a) + (3*Log[1 + Cosh[x]])/(4*a)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth(x)}{a+a\,\mathrm{sech}(x)}\,dx &= -\left(a^2\mathrm{Subst}\left(\int \frac{x^2}{(a-ax)(a+ax)^2}\,dx,x,\cosh(x)\right)\right)\\ &= -\left(a^2\mathrm{Subst}\left(\int \left(-\frac{1}{4a^3(-1+x)}+\frac{1}{2a^3(1+x)^2}-\frac{3}{4a^3(1+x)}\right)\,dx,x,\cosh(x)\right)\right)\\ &= \frac{1}{2a(1+\cosh(x))}+\frac{\log(1-\cosh(x))}{4a}+\frac{3\log(1+\cosh(x))}{4a}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 44, normalized size = 1.10

$$\frac{\left(1+2\cosh^2\left(\frac{x}{2}\right)\left(3\log\left(\cosh\left(\frac{x}{2}\right)\right)+\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)\right)\operatorname{sech}(x)}{2a(1+\operatorname{sech}(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(a + a*Sech[x]),x]
```

```
[Out] ((1 + 2*Cosh[x/2]^2*(3*Log[Cosh[x/2]] + Log[Sinh[x/2]]))*Sech[x])/(2*a*(1 +
 Sech[x]))
```

Maple [A]
time = 0.79, size = 38, normalized size = 0.95

method	result	size
default	$\frac{-\frac{\left(\tanh^2\left(\frac{x}{2}\right)\right)}{2}+\ln\left(\tanh\left(\frac{x}{2}\right)\right)-2\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-2\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2a}$	38
risch	$-\frac{x}{a}+\frac{e^x}{\left(e^x+1\right)^2 a}+\frac{\ln\left(e^x-1\right)}{2a}+\frac{3\ln\left(e^x+1\right)}{2a}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a*(-1/2*tanh(1/2*x)^2+ln(tanh(1/2*x))-2*ln(tanh(1/2*x)-1)-2*ln(tanh(1/2
*x)+1))
```

Maxima [A]
time = 0.25, size = 52, normalized size = 1.30

$$\frac{x}{a}+\frac{e^{(-x)}}{2\,ae^{(-x)}+ae^{(-2\,x)}+a}+\frac{3\,\log\left(e^{(-x)}+1\right)}{2\,a}+\frac{\log\left(e^{(-x)}-1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] x/a + e^(-x)/(2*a*e^(-x) + a*e^(-2*x) + a) + 3/2*log(e^(-x) + 1)/a + 1/2*lo
g(e^(-x) - 1)/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(34) = 68$.
time = 0.35, size = 136, normalized size = 3.40

$$-\frac{2\,x\cosh\left(x\right)^2+2\,x\sinh\left(x\right)^2+2\,(2\,x-1)\cosh\left(x\right)-3\,\left(\cosh\left(x\right)^2+2\,(\cosh\left(x\right)+1)\sinh\left(x\right)+\sinh\left(x\right)^2+2\,\cosh\left(x\right)+1\right)\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)-\left(\cosh\left(x\right)^2+2\,(\cosh\left(x\right)+1)\sinh\left(x\right)+\sinh\left(x\right)^2+2\,\cosh\left(x\right)+1\right)\log\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)+2\,(2\,x\cosh\left(x\right)+2\,x-1)\sinh\left(x\right)+2\,x}{2\,\left(a\cosh\left(x\right)^2+a\sinh\left(x\right)^2+2\,a\cosh\left(x\right)+2\,(a\cosh\left(x\right)+a)\sinh\left(x\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*x*cosh(x)^2 + 2*x*sinh(x)^2 + 2*(2*x - 1)*cosh(x) - 3*(cosh(x)^2 +
2*(cosh(x) + 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)*log(cosh(x) + sinh(x)
+ 1) - (cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)*lo
g(cosh(x) + sinh(x) - 1) + 2*(2*x*cosh(x) + 2*x - 1)*sinh(x) + 2*x)/(a*cosh
(x)^2 + a*sinh(x)^2 + 2*a*cosh(x) + 2*(a*cosh(x) + a)*sinh(x) + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\coth(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+a*sech(x)),x)
```

```
[Out] Integral(coth(x)/(sech(x) + 1), x)/a
```

Giac [A]
time = 0.40, size = 56, normalized size = 1.40

$$\frac{3\,\log\left(e^{(-x)}+e^x+2\right)}{4\,a}+\frac{\log\left(e^{(-x)}+e^x-2\right)}{4\,a}-\frac{3\,e^{(-x)}+3\,e^x+2}{4\,a(e^{(-x)}+e^x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] 3/4*log(e^(-x) + e^x + 2)/a + 1/4*log(e^(-x) + e^x - 2)/a - 1/4*(3*e^(-x) +
 3*e^x + 2)/(a*(e^(-x) + e^x + 2))
```

Mupad [B]
time = 1.37, size = 65, normalized size = 1.62

$$\frac{\ln\left(\mathrm{e}^{2\,x}-1\right)}{a}-\frac{x}{a}-\frac{1}{a+2\,a\,\mathrm{e}^x+a\,\mathrm{e}^{2\,x}}+\frac{\mathrm{atan}\left(\frac{\mathrm{e}^x\,\sqrt{-a^2}}{a}\right)}{\sqrt{-a^2}}+\frac{1}{a+a\,\mathrm{e}^x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a + a/cosh(x)),x)
```

```
[Out] log(exp(2*x) - 1)/a - x/a - 1/(a + 2*a*exp(x) + a*exp(2*x)) + atan((exp(x)*
(-a^2)^(1/2))/a)/(-a^2)^(1/2) + 1/(a + a*exp(x))
```

3.110 $\int \frac{\coth^2(x)}{a+a\operatorname{sech}(x)}\,dx$

Optimal. Leaf size=38

$$\frac{x}{a}-\frac{\coth(x)(3-2\operatorname{sech}(x))}{3a}-\frac{\coth^3(x)(1-\operatorname{sech}(x))}{3a}$$

```
[Out] x/a-1/3*coth(x)*(3-2*sech(x))/a-1/3*coth(x)^3*(1-sech(x))/a
```

Rubi [A]
time = 0.07, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.231$, Rules used = {3973, 3967, 8}

$$\frac{x}{a}-\frac{\coth^3(x)(1-\operatorname{sech}(x))}{3a}-\frac{\coth(x)(3-2\operatorname{sech}(x))}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(a + a*Sech[x]),x]
[Out] x/a - (Coth[x]*(3 - 2*Sech[x]))/(3*a) - (Coth[x]^3*(1 - Sech[x]))/(3*a)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\coth^2(x)}{a+a\operatorname{sech}(x)}\,dx = -\frac{\int \coth^4(x)(-a+a\operatorname{sech}(x))\,dx}{a^2}$$
$$= -\frac{\coth^3(x)(1-\operatorname{sech}(x))}{3a} + \frac{\int \coth^2(x)(3a-2a\operatorname{sech}(x))\,dx}{3a^2}$$
$$= -\frac{\coth(x)(3-2\operatorname{sech}(x))}{3a} - \frac{\coth^3(x)(1-\operatorname{sech}(x))}{3a} - \frac{\int -3a\,dx}{3a^2}$$
$$= \frac{x}{a} - \frac{\coth(x)(3-2\operatorname{sech}(x))}{3a} - \frac{\coth^3(x)(1-\operatorname{sech}(x))}{3a}$$

Mathematica [A]
time = 0.05, size = 33, normalized size = 0.87

$$\frac{6x-4\coth(x)-2\operatorname{csch}(x)+6x\operatorname{sech}(x)-4\tanh(x)}{6a+6a\operatorname{sech}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(a + a*Sech[x]),x]
[Out] (6*x - 4*Coth[x] - 2*Csch[x] + 6*x*Sech[x] - 4*Tanh[x])/(6*a + 6*a*Sech[x])
```

Maple [A]
time = 1.08, size = 47, normalized size = 1.24

method	result	size
risch	$\frac{x}{a}+\frac{2\,{\rm e}^{3x}-\frac{10\,{\rm e}^{x}}{3}-\frac{8}{3}}{a({\rm e}^{x}+1)^{3}({\rm e}^{x}-1)}$	36
default	$\frac{-\frac{\left(\tanh^3\left(\frac{x}{2}\right)\right)}{3}-4\tanh\left(\frac{x}{2}\right)-\frac{1}{\tanh\left(\frac{x}{2}\right)}+4\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-4\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{4a}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a+a*sech(x)),x,method=_RETURNVERBOSE)
[Out] 1/4/a*(-1/3*tanh(1/2*x)^3-4*tanh(1/2*x)-1/tanh(1/2*x)+4*ln(tanh(1/2*x)+1)-4
*ln(tanh(1/2*x)-1))
```

Maxima [A]
time = 0.28, size = 47, normalized size = 1.24

$$\frac{x}{a}-\frac{2\left(5\,e^{(-x)}-3\,e^{(-3\,x)}+4\right)}{3\left(2\,ae^{(-x)}-2\,ae^{(-3\,x)}-ae^{(-4\,x)}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] x/a - 2/3*(5*e^(-x) - 3*e^(-3*x) + 4)/(2*a*e^(-x) - 2*a*e^(-3*x) - a*e^(-4*
x) + a)
```

Fricas [A]
time = 0.38, size = 46, normalized size = 1.21

$$-\frac{2 \cosh (x)^{2}-((3 x+4) \cosh (x)+3 x+4) \sinh (x)+2 \sinh (x)^{2}+\cosh (x)}{3(a \cosh (x)+a) \sinh (x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/3*(2*cosh(x)^2 - ((3*x + 4)*cosh(x) + 3*x + 4)*sinh(x) + 2*sinh(x)^2 + c
osh(x))/((a*cosh(x) + a)*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\coth^{2}(x)}{\operatorname{sech}(x)+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(a+a*sech(x)),x)
```

```
[Out] Integral(coth(x)**2/(sech(x) + 1), x)/a
```

Giac [A]
time = 0.39, size = 40, normalized size = 1.05

$$\frac{x}{a}-\frac{1}{2 a(e^{x}-1)}+\frac{15 e^{(2 x)}+24 e^{x}+13}{6 a(e^{x}+1)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] x/a - 1/2/(a*(e^x - 1)) + 1/6*(15*e^(2*x) + 24*e^x + 13)/(a*(e^x + 1)^3)
```

Mupad [B]
time = 1.35, size = 94, normalized size = 2.47

$$\frac{\frac{5 e^{2 x}}{6 a}+\frac{5}{6 a}+\frac{e^{x}}{a}}{3 e^{2 x}+e^{3 x}+3 e^{x}+1}+\frac{\frac{1}{2 a}+\frac{5 e^{x}}{6 a}}{e^{2 x}+2 e^{x}+1}+\frac{x}{a}-\frac{1}{2 a\ (e^{x}-1)}+\frac{5}{6 a\ (e^{x}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a + a/cosh(x)),x)
```

```
[Out] ((5*exp(2*x))/(6*a) + 5/(6*a) + exp(x)/a)/(3*exp(2*x) + exp(3*x) + 3*exp(x)
 + 1) + (1/(2*a) + (5*exp(x))/(6*a))/(exp(2*x) + 2*exp(x) + 1) + x/a - 1/(2
*a*(exp(x) - 1)) + 5/(6*a*(exp(x) + 1))
```

3.111 $\int \frac{\coth^3(x)}{a+a\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=68

$$\frac{1}{8a(1-\cosh(x))}-\frac{1}{8a(1+\cosh(x))^2}+\frac{3}{4a(1+\cosh(x))}+\frac{5\log(1-\cosh(x))}{16a}+\frac{11\log(1+\cosh(x))}{16a}$$

```
[Out] 1/8/a/(1-cosh(x))-1/8/a/(1+cosh(x))^2+3/4/a/(1+cosh(x))+5/16*ln(1-cosh(x))/
a+11/16*ln(1+cosh(x))/a
```

Rubi [A]
time = 0.06, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3964, 90}

$$\frac{1}{8a(1-\cosh(x))}+\frac{3}{4a(\cosh(x)+1)}-\frac{1}{8a(\cosh(x)+1)^2}+\frac{5\log(1-\cosh(x))}{16a}+\frac{11\log(\cosh(x)+1)}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/(a + a*Sech[x]),x]
```

```
[Out] 1/(8*a*(1 - Cosh[x])) - 1/(8*a*(1 + Cosh[x])^2) + 3/(4*a*(1 + Cosh[x])) + (
5*Log[1 - Cosh[x]])/(16*a) + (11*Log[1 + Cosh[x]])/(16*a)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int \frac{\coth^3(x)}{a+a\text{sech}(x)}\,dx = a^4\text{Subst}\left(\int \frac{x^4}{(a-ax)^2(a+ax)^3}\,dx, x, \cosh(x)\right)$$
$$= a^4\text{Subst}\left(\int \left(\frac{1}{8a^5(-1+x)^2} + \frac{5}{16a^5(-1+x)} + \frac{1}{4a^5(1+x)^3} - \frac{3}{4a^5(1+x)^2} + \frac{11}{16a^5(1$$
$$= \frac{1}{8a(1-\cosh(x))} - \frac{1}{8a(1+\cosh(x))^2} + \frac{3}{4a(1+\cosh(x))} + \frac{5\log(1-\cosh(x))}{16a} + \frac{11\,\mathrm{lo}$$

Mathematica [A]
time = 0.13, size = 66, normalized size = 0.97

$$\frac{\left(12-2\coth^2\left(\frac{x}{2}\right)+4\cosh^2\left(\frac{x}{2}\right)\left(11\log\left(\cosh\left(\frac{x}{2}\right)\right)+5\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)-\text{sech}^2\left(\frac{x}{2}\right)\right)\text{sech}(x)}{16a(1+\text{sech}(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/(a + a*Sech[x]),x]
```

```
[Out] ((12 - 2*Coth[x/2]^2 + 4*Cosh[x/2]^2*(11*Log[Cosh[x/2]] + 5*Log[Sinh[x/2]])
 - Sech[x/2]^2)*Sech[x])/(16*a*(1 + Sech[x]))
```

Maple [A]
time = 1.02, size = 56, normalized size = 0.82

method	result	size
default	$\frac{-\frac{\left(\tanh^4\left(\frac{x}{2}\right)\right)}{4}-\frac{5\left(\tanh^2\left(\frac{x}{2}\right)\right)}{2}-8\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-8\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{1}{2\tanh\left(\frac{x}{2}\right)^2}+5\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{8a}$	56
risch	$-\frac{x}{a}+\frac{e^x\left(5\,e^{4x}-6\,e^{3x}-14\,e^{2x}-6\,e^x+5\right)}{4a\left(e^x+1\right)^4\left(e^x-1\right)^2}+\frac{5\ln\left(e^x-1\right)}{8a}+\frac{11\ln\left(e^x+1\right)}{8a}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/a*(-1/4*tanh(1/2*x)^4-5/2*tanh(1/2*x)^2-8*ln(tanh(1/2*x)-1)-8*ln(tanh(1
/2*x)+1)-1/2/tanh(1/2*x)^2+5*ln(tanh(1/2*x)))
```

Maxima [A]
time = 0.28, size = 108, normalized size = 1.59

$$\frac{x}{a}+\frac{5\,e^{(-x)}-6\,e^{(-2\,x)}-14\,e^{(-3\,x)}-6\,e^{(-4\,x)}+5\,e^{(-5\,x)}}{4\left(2\,ae^{(-x)}-ae^{(-2\,x)}-4\,ae^{(-3\,x)}-ae^{(-4\,x)}+2\,ae^{(-5\,x)}+ae^{(-6\,x)}+a\right)}+\frac{11\,\log\left(e^{(-x)}+1\right)}{8\,a}+\frac{5\,\log\left(e^{(-x)}-1\right)}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] x/a + 1/4*(5*e^(-x) - 6*e^(-2*x) - 14*e^(-3*x) - 6*e^(-4*x) + 5*e^(-5*x))/(
2*a*e^(-x) - a*e^(-2*x) - 4*a*e^(-3*x) - a*e^(-4*x) + 2*a*e^(-5*x) + a*e^(-
6*x) + a) + 11/8*log(e^(-x) + 1)/a + 5/8*log(e^(-x) - 1)/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 773 vs. $2(56) = 112$.
time = 0.38, size = 773, normalized size = 11.37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/8*(8*x*cosh(x)^6 + 8*x*sinh(x)^6 + 2*(8*x - 5)*cosh(x)^5 + 2*(24*x*cosh(
x) + 8*x - 5)*sinh(x)^5 - 4*(2*x - 3)*cosh(x)^4 + 2*(60*x*cosh(x)^2 + 5*(8*
x - 5)*cosh(x) - 4*x + 6)*sinh(x)^4 - 4*(8*x - 7)*cosh(x)^3 + 4*(40*x*cosh(
x)^3 + 5*(8*x - 5)*cosh(x)^2 - 4*(2*x - 3)*cosh(x) - 8*x + 7)*sinh(x)^3 - 4
*(2*x - 3)*cosh(x)^2 + 4*(30*x*cosh(x)^4 + 5*(8*x - 5)*cosh(x)^3 - 6*(2*x -
 3)*cosh(x)^2 - 3*(8*x - 7)*cosh(x) - 2*x + 3)*sinh(x)^2 + 2*(8*x - 5)*cosh
(x) - 11*(cosh(x)^6 + 2*(3*cosh(x) + 1)*sinh(x)^5 + sinh(x)^6 + 2*cosh(x)^5
 + (15*cosh(x)^2 + 10*cosh(x) - 1)*sinh(x)^4 - cosh(x)^4 + 4*(5*cosh(x)^3 +
 5*cosh(x)^2 - cosh(x) - 1)*sinh(x)^3 - 4*cosh(x)^3 + (15*cosh(x)^4 + 20*co
sh(x)^3 - 6*cosh(x)^2 - 12*cosh(x) - 1)*sinh(x)^2 - cosh(x)^2 + 2*(3*cosh(x
)^5 + 5*cosh(x)^4 - 2*cosh(x)^3 - 6*cosh(x)^2 - cosh(x) + 1)*sinh(x) + 2*co
sh(x) + 1)*log(cosh(x) + sinh(x) + 1) - 5*(cosh(x)^6 + 2*(3*cosh(x) + 1)*si
nh(x)^5 + sinh(x)^6 + 2*cosh(x)^5 + (15*cosh(x)^2 + 10*cosh(x) - 1)*sinh(x)
^4 - cosh(x)^4 + 4*(5*cosh(x)^3 + 5*cosh(x)^2 - cosh(x) - 1)*sinh(x)^3 - 4*
cosh(x)^3 + (15*cosh(x)^4 + 20*cosh(x)^3 - 6*cosh(x)^2 - 12*cosh(x) - 1)*si
nh(x)^2 - cosh(x)^2 + 2*(3*cosh(x)^5 + 5*cosh(x)^4 - 2*cosh(x)^3 - 6*cosh(x
)^2 - cosh(x) + 1)*sinh(x) + 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*
(24*x*cosh(x)^5 + 5*(8*x - 5)*cosh(x)^4 - 8*(2*x - 3)*cosh(x)^3 - 6*(8*x -
7)*cosh(x)^2 - 4*(2*x - 3)*cosh(x) + 8*x - 5)*sinh(x) + 8*x)/(a*cosh(x)^6 +
 a*sinh(x)^6 + 2*a*cosh(x)^5 + 2*(3*a*cosh(x) + a)*sinh(x)^5 - a*cosh(x)^4
+ (15*a*cosh(x)^2 + 10*a*cosh(x) - a)*sinh(x)^4 - 4*a*cosh(x)^3 + 4*(5*a*co
sh(x)^3 + 5*a*cosh(x)^2 - a*cosh(x) - a)*sinh(x)^3 - a*cosh(x)^2 + (15*a*co
sh(x)^4 + 20*a*cosh(x)^3 - 6*a*cosh(x)^2 - 12*a*cosh(x) - a)*sinh(x)^2 + 2*
a*cosh(x) + 2*(3*a*cosh(x)^5 + 5*a*cosh(x)^4 - 2*a*cosh(x)^3 - 6*a*cosh(x)^
2 - a*cosh(x) + a)*sinh(x) + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\coth^3(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(a+a*sech(x)),x)
```

```
[Out] Integral(coth(x)**3/(sech(x) + 1), x)/a
```

Giac [A]
time = 0.39, size = 94, normalized size = 1.38

$$\frac{11\,\log\left(e^{(-x)}+e^{x}+2\right)}{16\,a}+\frac{5\,\log\left(e^{(-x)}+e^{x}-2\right)}{16\,a}-\frac{5\,e^{(-x)}+5\,e^{x}-6}{16\,a(e^{(-x)}+e^{x}-2)}-\frac{33\,\left(e^{(-x)}+e^{x}\right)^{2}+84\,e^{(-x)}+84\,e^{x}+52}{32\,a(e^{(-x)}+e^{x}+2)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] 11/16*log(e^(-x) + e^x + 2)/a + 5/16*log(e^(-x) + e^x - 2)/a - 1/16*(5*e^(-
x) + 5*e^x - 6)/(a*(e^(-x) + e^x - 2)) - 1/32*(33*(e^(-x) + e^x)^2 + 84*e^(
-x) + 84*e^x + 52)/(a*(e^(-x) + e^x + 2)^2)
```

Mupad [B]
time = 1.43, size = 160, normalized size = 2.35

$$\frac{\ln\left(9\,e^{2\,x}-9\right)}{a}-\frac{x}{a}-\frac{1}{2\,\left(a+4\,a\,e^{x}+6\,a\,e^{2\,x}+4\,a\,e^{3\,x}+a\,e^{4\,x}\right)}+\frac{1}{a+3\,a\,e^{x}+3\,a\,e^{2\,x}+a\,e^{3\,x}}-\frac{1}{4\,\left(a-2\,a\,e^{x}+a\,e^{2\,x}\right)}-\frac{2}{a+2\,a\,e^{x}+a\,e^{2\,x}}+\frac{3\,\mathrm{atan}\left(\frac{e^{x}\,\sqrt{-a^2}}{a}\right)}{4\,\sqrt{-a^2}}+\frac{3}{2\,\left(a+a\,e^{x}\right)}+\frac{1}{4\,\left(a-a\,e^{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a + a/cosh(x)),x)
```

```
[Out] log(9*exp(2*x) - 9)/a - x/a - 1/(2*(a + 4*a*exp(x) + 6*a*exp(2*x) + 4*a*exp
(3*x) + a*exp(4*x))) + 1/(a + 3*a*exp(x) + 3*a*exp(2*x) + a*exp(3*x)) - 1/(
4*(a - 2*a*exp(x) + a*exp(2*x))) - 2/(a + 2*a*exp(x) + a*exp(2*x)) + (3*ata
n((exp(x)*(-a^2)^(1/2))/a))/(4*(-a^2)^(1/2)) + 3/(2*(a + a*exp(x))) + 1/(4*
(a - a*exp(x)))
```

3.112 $\int \frac{\coth^4(x)}{a+a\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=55

$$\frac{x}{a}-\frac{\coth(x)(15-8\text{sech}(x))}{15a}-\frac{\coth^3(x)(5-4\text{sech}(x))}{15a}-\frac{\coth^5(x)(1-\text{sech}(x))}{5a}$$

```
[Out] x/a-1/15*coth(x)*(15-8*sech(x))/a-1/15*coth(x)^3*(5-4*sech(x))/a-1/5*coth(x
)^5*(1-sech(x))/a
```

Rubi [A]
time = 0.08, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3973, 3967, 8}

$$\frac{x}{a}-\frac{\coth^5(x)(1-\text{sech}(x))}{5a}-\frac{\coth^3(x)(5-4\text{sech}(x))}{15a}-\frac{\coth(x)(15-8\text{sech}(x))}{15a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^4/(a + a*Sech[x]),x]
```

```
[Out] x/a - (Coth[x]*(15 - 8*Sech[x]))/(15*a) - (Coth[x]^3*(5 - 4*Sech[x]))/(15*a
) - (Coth[x]^5*(1 - Sech[x]))/(5*a)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\coth^4(x)}{a+a\operatorname{sech}(x)}\,dx &= -\frac{\int \coth^6(x)(-a+a\operatorname{sech}(x))\,dx}{a^2} \\
&= -\frac{\coth^5(x)(1-\operatorname{sech}(x))}{5a} + \frac{\int \coth^4(x)(5a-4a\operatorname{sech}(x))\,dx}{5a^2} \\
&= -\frac{\coth^3(x)(5-4\operatorname{sech}(x))}{15a} - \frac{\coth^5(x)(1-\operatorname{sech}(x))}{5a} - \frac{\int \coth^2(x)(-15a+8a\operatorname{sech}(x))}{15a^2} \\
&= -\frac{\coth(x)(15-8\operatorname{sech}(x))}{15a} - \frac{\coth^3(x)(5-4\operatorname{sech}(x))}{15a} - \frac{\coth^5(x)(1-\operatorname{sech}(x))}{5a} + \frac{\int 15}{15} \\
&= \frac{x}{a} - \frac{\coth(x)(15-8\operatorname{sech}(x))}{15a} - \frac{\coth^3(x)(5-4\operatorname{sech}(x))}{15a} - \frac{\coth^5(x)(1-\operatorname{sech}(x))}{5a}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 69, normalized size = 1.25

$$
\frac{\operatorname{csch}^3(x)\operatorname{sech}(x)(-25+8\cosh(x)+16\cosh(2x)-16\cosh(3x)-23\cosh(4x)-90x\sinh(x)-30x\sinh(2x)+30x\sinh(3x)+15x\sinh(4x))}{120a(1+\operatorname{sech}(x))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4/(a + a*Sech[x]),x]
```

```
[Out] (Csch[x]^3*Sech[x]*(-25 + 8*Cosh[x] + 16*Cosh[2*x] - 16*Cosh[3*x] - 23*Cosh
[4*x] - 90*x*Sinh[x] - 30*x*Sinh[2*x] + 30*x*Sinh[3*x] + 15*x*Sinh[4*x]))/(
120*a*(1 + Sech[x]))
```

Maple [A]
time = 0.83, size = 63, normalized size = 1.15

method	result	size
default	$\frac{-\frac{\left(\tanh^5\left(\frac{x}{2}\right)\right)}{5}-2\left(\tanh^3\left(\frac{x}{2}\right)\right)-16\tanh\left(\frac{x}{2}\right)+16\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{1}{3\tanh\left(\frac{x}{2}\right)^3}-\frac{6}{\tanh\left(\frac{x}{2}\right)}-16\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{16a}$	63
risch	$\frac{x}{a}+\frac{2\,e^{7x}-2\,e^{6x}-\frac{26\,e^{5x}}{3}-\frac{10\,e^{4x}}{3}+\frac{146\,e^{3x}}{15}+\frac{62\,e^{2x}}{15}-\frac{62\,e^{x}}{15}-\frac{46}{15}}{a(e^x-1)^3(e^x+1)^5}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/a*(-1/5*tanh(1/2*x)^5-2*tanh(1/2*x)^3-16*tanh(1/2*x)+16*ln(tanh(1/2*x)
+1)-1/3/tanh(1/2*x)^3-6/tanh(1/2*x)-16*ln(tanh(1/2*x)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(47) = 94$.

time = 0.27, size = 105, normalized size = 1.91

$$\frac{x}{a} - \frac{2\left(31\,e^{(-x)} - 31\,e^{(-2\,x)} - 73\,e^{(-3\,x)} + 25\,e^{(-4\,x)} + 65\,e^{(-5\,x)} + 15\,e^{(-6\,x)} - 15\,e^{(-7\,x)} + 23\right)}{15\,(2\,ae^{(-x)} - 2\,ae^{(-2\,x)} - 6\,ae^{(-3\,x)} + 6\,ae^{(-5\,x)} + 2\,ae^{(-6\,x)} - 2\,ae^{(-7\,x)} - ae^{(-8\,x)} + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] x/a - 2/15*(31*e^(-x) - 31*e^(-2*x) - 73*e^(-3*x) + 25*e^(-4*x) + 65*e^(-5*
x) + 15*e^(-6*x) - 15*e^(-7*x) + 23)/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3
*x) + 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(47) = 94.
time = 0.39, size = 151, normalized size = 2.75

$$-\frac{23\,\cosh(x)^4 - 2\,(2\,(15\,x + 23)\cosh(x) + 15\,x + 23)\sinh(x)^3 + 23\,\sinh(x)^4 + 16\,\cosh(x)^3 + 2\,(69\,\cosh(x)^2 + 24\,\cosh(x) - 8)\sinh(x)^2 - 16\,\cosh(x)^2 - 2\,(2\,(15\,x + 23)\cosh(x)^3 + 3\,(15\,x + 23)\cosh(x)^2 - 2\,(15\,x + 23)\cosh(x) - 45\,x - 69)\sinh(x) - 8\,\cosh(x) + 25}{30\,((2\,a\cosh(x) + a)\sinh(x)^3 + (2\,a\cosh(x)^3 + 3\,a\cosh(x)^2 - 2\,a\cosh(x) - 3\,a)\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/30*(23*cosh(x)^4 - 2*(2*(15*x + 23)*cosh(x) + 15*x + 23)*sinh(x)^3 + 23*
sinh(x)^4 + 16*cosh(x)^3 + 2*(69*cosh(x)^2 + 24*cosh(x) - 8)*sinh(x)^2 - 16
*cosh(x)^2 - 2*(2*(15*x + 23)*cosh(x)^3 + 3*(15*x + 23)*cosh(x)^2 - 2*(15*x
 + 23)*cosh(x) - 45*x - 69)*sinh(x) - 8*cosh(x) + 25)/((2*a*cosh(x) + a)*si
nh(x)^3 + (2*a*cosh(x)^3 + 3*a*cosh(x)^2 - 2*a*cosh(x) - 3*a)*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\coth^4(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4/(a+a*sech(x)),x)
```

```
[Out] Integral(coth(x)**4/(sech(x) + 1), x)/a
```

Giac [A]
time = 0.40, size = 64, normalized size = 1.16

$$\frac{x}{a} - \frac{21\,e^{(2\,x)} - 36\,e^x + 19}{24\,a(e^x - 1)^3} + \frac{115\,e^{(4\,x)} + 380\,e^{(3\,x)} + 530\,e^{(2\,x)} + 340\,e^x + 91}{40\,a(e^x + 1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] x/a - 1/24*(21*e^(2*x) - 36*e^x + 19)/(a*(e^x - 1)^3) + 1/40*(115*e^(4*x) +
 380*e^(3*x) + 530*e^(2*x) + 340*e^x + 91)/(a*(e^x + 1)^5)
```

Mupad [B]
time = 1.53, size = 264, normalized size = 4.80

$$\frac{\frac{9\,e^{2\,x}}{4\,a}+\frac{3\,e^{3\,x}}{2\,a}+\frac{23\,e^{4\,x}}{40\,a}+\frac{23}{40\,a}+\frac{3\,e^{x}}{2\,a}}{10\,e^{2\,x}+10\,e^{3\,x}+5\,e^{4\,x}+e^{5\,x}+5\,e^{x}+1}+\frac{\frac{9\,e^{2\,x}}{8\,a}+\frac{23\,e^{3\,x}}{40\,a}+\frac{3}{8\,a}+\frac{9\,e^{x}}{8\,a}}{6\,e^{2\,x}+4\,e^{3\,x}+e^{4\,x}+4\,e^{x}+1}+\frac{\frac{23\,e^{2\,x}}{40\,a}+\frac{3}{8\,a}+\frac{3\,e^{x}}{4\,a}}{3\,e^{2\,x}+e^{3\,x}+3\,e^{x}+1}+\frac{\frac{3}{8\,a}+\frac{23\,e^{x}}{40\,a}}{e^{2\,x}+2\,e^{x}+1}+\frac{1}{6\,a\,\left(3\,e^{2\,x}-e^{3\,x}-3\,e^{x}+1\right)}-\frac{1}{4\,a\,\left(e^{2\,x}-2\,e^{x}+1\right)}+\frac{x}{a}-\frac{7}{8\,a\,\left(e^{x}-1\right)}+\frac{23}{40\,a\,\left(e^{x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(a + a/cosh(x)),x)
```

```
[Out] ((9*exp(2*x))/(4*a) + (3*exp(3*x))/(2*a) + (23*exp(4*x))/(40*a) + 23/(40*a)
 + (3*exp(x))/(2*a))/(10*exp(2*x) + 10*exp(3*x) + 5*exp(4*x) + exp(5*x) + 5
*exp(x) + 1) + ((9*exp(2*x))/(8*a) + (23*exp(3*x))/(40*a) + 3/(8*a) + (9*ex
p(x))/(8*a))/(6*exp(2*x) + 4*exp(3*x) + exp(4*x) + 4*exp(x) + 1) + ((23*exp
(2*x))/(40*a) + 3/(8*a) + (3*exp(x))/(4*a))/(3*exp(2*x) + exp(3*x) + 3*exp(
x) + 1) + (3/(8*a) + (23*exp(x))/(40*a))/(exp(2*x) + 2*exp(x) + 1) + 1/(6*a
*(3*exp(2*x) - exp(3*x) - 3*exp(x) + 1)) - 1/(4*a*(exp(2*x) - 2*exp(x) + 1)
) + x/a - 7/(8*a*(exp(x) - 1)) + 23/(40*a*(exp(x) + 1))
```

3.113 $\int \frac{\tanh^7(x)}{a+b\,\mathrm{sech}(x)}\,dx$

Optimal. Leaf size=121

$$\frac{\log(\cosh(x))}{a}-\frac{(a^2-b^2)^3\log(a+b\,\mathrm{sech}(x))}{ab^6}+\frac{(a^4-3a^2b^2+3b^4)\,\mathrm{sech}(x)}{b^5}-\frac{a(a^2-3b^2)\,\mathrm{sech}^2(x)}{2b^4}+\frac{(a^2-3b^2)\,\mathrm{se}}{3b^3}$$

```
[Out] ln(cosh(x))/a-(a^2-b^2)^3*ln(a+b*sech(x))/a/b^6+(a^4-3*a^2*b^2+3*b^4)*sech(
x)/b^5-1/2*a*(a^2-3*b^2)*sech(x)^2/b^4+1/3*(a^2-3*b^2)*sech(x)^3/b^3-1/4*a*
sech(x)^4/b^2+1/5*sech(x)^5/b
```

Rubi [A]
time = 0.11, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3970, 908}

$$-\frac{(a^2-b^2)^3\log(a+b\,\mathrm{sech}(x))}{ab^6}-\frac{a(a^2-3b^2)\,\mathrm{sech}^2(x)}{2b^4}+\frac{(a^2-3b^2)\,\mathrm{sech}^3(x)}{3b^3}+\frac{(a^4-3a^2b^2+3b^4)\,\mathrm{sech}(x)}{b^5}-\frac{a\,\mathrm{sech}^4(x)}{4b^2}+\frac{\log(\cosh(x))}{a}+\frac{\mathrm{sech}^5(x)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^7/(a + b*Sech[x]),x]
```

```
[Out] Log[Cosh[x]]/a - ((a^2 - b^2)^3*Log[a + b*Sech[x]])/(a*b^6) + ((a^4 - 3*a^2
*b^2 + 3*b^4)*Sech[x])/b^5 - (a*(a^2 - 3*b^2)*Sech[x]^2)/(2*b^4) + ((a^2 -
3*b^2)*Sech[x]^3)/(3*b^3) - (a*Sech[x]^4)/(4*b^2) + Sech[x]^5/(5*b)
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\tanh^7(x)}{a+b\operatorname{sech}(x)}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{(b^2-x^2)^3}{x(a+x)}\,dx, x, b\operatorname{sech}(x)\right)}{b^6}$$

$$= -\frac{\operatorname{Subst}\left(\int \left(-a^4\left(1+\frac{3b^2(-a^2+b^2)}{a^4}\right)+\frac{b^6}{ax}+a(a^2-3b^2)\,x-(a^2-3b^2)\,x^2+ax^3-x^4+\right.\right.}{b^6}$$

$$= \frac{\log(\cosh(x))}{a} - \frac{(a^2-b^2)^3\log(a+b\operatorname{sech}(x))}{ab^6} + \frac{(a^4-3a^2b^2+3b^4)\operatorname{sech}(x)}{b^5} - \frac{a(a^2-3}{}$$

Mathematica [A]
time = 0.22, size = 132, normalized size = 1.09

$$\frac{60a(a^4-3a^2b^2+3b^4)\log(\cosh(x)) - \frac{60(a^2-b^2)^3\log(b+a\cosh(x))}{a} + 60b(a^4-3a^2b^2+3b^4)\operatorname{sech}(x) - 30ab^2(a^2-3b^2)\operatorname{sech}^2(x) + 20b^3(a^2-3b^2)\operatorname{sech}^3(x) - 15ab^4\operatorname{sech}^4(x) + 12b^5\operatorname{sech}^5(x)}{60b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^7/(a + b*Sech[x]),x]
```

```
[Out] (60*a*(a^4 - 3*a^2*b^2 + 3*b^4)*Log[Cosh[x]] - (60*(a^2 - b^2)^3*Log[b + a*
Cosh[x]])/a + 60*b*(a^4 - 3*a^2*b^2 + 3*b^4)*Sech[x] - 30*a*b^2*(a^2 - 3*b^
2)*Sech[x]^2 + 20*b^3*(a^2 - 3*b^2)*Sech[x]^3 - 15*a*b^4*Sech[x]^4 + 12*b^5
*Sech[x]^5)/(60*b^6)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(113) = 226$.
time = 0.74, size = 246, normalized size = 2.03

method	result
default	$-\frac{(a-b)^3(a^3+3a^2b+3a\,b^2+b^3)\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-b\left(\tanh^2\left(\frac{x}{2}\right)\right)+a+b\right)}{a\,b^6} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a} + \frac{\frac{32b^5}{5\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^5}+\frac{8b^3\left(a^2+3ab+3b^2\right)}{3\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^3}}{}$
risch	$-\frac{x}{a} + \frac{2\,\mathrm{e}^x\left(15a^4\mathrm{e}^{8x}-45a^2b^2\mathrm{e}^{8x}+45b^4\mathrm{e}^{8x}-15a^3b\,\mathrm{e}^{7x}+45a\,b^3\mathrm{e}^{7x}+60a^4\mathrm{e}^{6x}-160a^2b^2\mathrm{e}^{6x}+120b^4\mathrm{e}^{6x}-45a^3b\,\mathrm{e}^{5x}+105a\,b^3\mathrm{e}^{5x}+90a^4\mathrm{e}\right.}{15b^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^7/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -(a-b)^3*(a^3+3*a^2*b+3*a*b^2+b^3)/a/b^6*ln(a*tanh(1/2*x)^2-b*tanh(1/2*x)^2
+a+b)-1/a*ln(tanh(1/2*x)+1)+1/b^6*(32/5*b^5/(tanh(1/2*x)^2+1)^5+8/3*b^3*(a^
2+3*a*b+3*b^2)/(tanh(1/2*x)^2+1)^3+a*(a^4-3*a^2*b^2+3*b^4)*ln(tanh(1/2*x)^2
+1)-2*b^2*(a^3+2*a^2*b-2*b^3)/(tanh(1/2*x)^2+1)^2-4*b^4*(a+4*b)/(tanh(1/2*x
)^2+1)^4+2*b*(a^4+a^3*b-2*a^2*b^2-2*a*b^3+b^4)/(tanh(1/2*x)^2+1))-1/a*ln(ta
nh(1/2*x)-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(113) = 226$.
time = 0.49, size = 332, normalized size = 2.74

$$\frac{2\left(15\left(a^{4}-3a^{2}b^{2}+3b^{4}\right)e^{(-x)}-15\left(a^{3}b-3ab^{3}\right)e^{(-2x)}+20\left(3a^{4}-8a^{2}b^{2}+6b^{4}\right)e^{(-3x)}-15\left(3a^{3}b-7ab^{3}\right)e^{(-4x)}+2\left(45a^{4}-115a^{2}b^{2}+99b^{4}\right)e^{(-5x)}-15\left(3a^{3}b-7ab^{3}\right)e^{(-6x)}+20\left(3a^{4}-8a^{2}b^{2}+6b^{4}\right)e^{(-7x)}-15\left(a^{3}b-3ab^{3}\right)e^{(-8x)}+15\left(a^{4}-3a^{2}b^{2}+3b^{4}\right)e^{(-9x)}\right)}{15\left(5b^{5}e^{(-2x)}+10b^{5}e^{(-4x)}+10b^{5}e^{(-6x)}+5b^{5}e^{(-8x)}+b^{5}e^{(-10x)}+b^{5}\right)}+\frac{x}{a}+\frac{\left(a^{5}-3a^{3}b^{2}+3ab^{4}\right)\log\left(e^{(-2x)}+1\right)}{b^{6}}-\frac{\left(a^{6}-3a^{4}b^{2}+3a^{2}b^{4}-b^{6}\right)\log\left(2be^{(-x)}+ae^{(-2x)}+a\right)}{ab^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^7/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] 2/15*(15*(a^4 - 3*a^2*b^2 + 3*b^4)*e^(-x) - 15*(a^3*b - 3*a*b^3)*e^(-2*x) +
 20*(3*a^4 - 8*a^2*b^2 + 6*b^4)*e^(-3*x) - 15*(3*a^3*b - 7*a*b^3)*e^(-4*x)
+ 2*(45*a^4 - 115*a^2*b^2 + 99*b^4)*e^(-5*x) - 15*(3*a^3*b - 7*a*b^3)*e^(-6
*x) + 20*(3*a^4 - 8*a^2*b^2 + 6*b^4)*e^(-7*x) - 15*(a^3*b - 3*a*b^3)*e^(-8*
x) + 15*(a^4 - 3*a^2*b^2 + 3*b^4)*e^(-9*x))/(5*b^5*e^(-2*x) + 10*b^5*e^(-4*
x) + 10*b^5*e^(-6*x) + 5*b^5*e^(-8*x) + b^5*e^(-10*x) + b^5) + x/a + (a^5 -
 3*a^3*b^2 + 3*a*b^4)*log(e^(-2*x) + 1)/b^6 - (a^6 - 3*a^4*b^2 + 3*a^2*b^4
- b^6)*log(2*b*e^(-x) + a*e^(-2*x) + a)/(a*b^6)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4077 vs. $2(113) = 226$.
time = 0.44, size = 4077, normalized size = 33.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^7/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/15*(15*b^6*x*cosh(x)^10 + 15*b^6*x*sinh(x)^10 - 30*(a^5*b - 3*a^3*b^3 +
3*a*b^5)*cosh(x)^9 + 30*(5*b^6*x*cosh(x) - a^5*b + 3*a^3*b^3 - 3*a*b^5)*sin
h(x)^9 + 15*(5*b^6*x + 2*a^4*b^2 - 6*a^2*b^4)*cosh(x)^8 + 15*(45*b^6*x*cosh
(x)^2 + 5*b^6*x + 2*a^4*b^2 - 6*a^2*b^4 - 18*(a^5*b - 3*a^3*b^3 + 3*a*b^5)*
cosh(x))*sinh(x)^8 - 40*(3*a^5*b - 8*a^3*b^3 + 6*a*b^5)*cosh(x)^7 + 40*(45*
b^6*x*cosh(x)^3 - 3*a^5*b + 8*a^3*b^3 - 6*a*b^5 - 27*(a^5*b - 3*a^3*b^3 + 3
*a*b^5)*cosh(x)^2 + 3*(5*b^6*x + 2*a^4*b^2 - 6*a^2*b^4)*cosh(x))*sinh(x)^7
+ 15*b^6*x + 30*(5*b^6*x + 3*a^4*b^2 - 7*a^2*b^4)*cosh(x)^6 + 10*(315*b^6*x
*cosh(x)^4 + 15*b^6*x + 9*a^4*b^2 - 21*a^2*b^4 - 252*(a^5*b - 3*a^3*b^3 + 3
*a*b^5)*cosh(x)^3 + 42*(5*b^6*x + 2*a^4*b^2 - 6*a^2*b^4)*cosh(x)^2 - 28*(3*
a^5*b - 8*a^3*b^3 + 6*a*b^5)*cosh(x))*sinh(x)^6 - 4*(45*a^5*b - 115*a^3*b^3
 + 99*a*b^5)*cosh(x)^5 + 4*(945*b^6*x*cosh(x)^5 - 45*a^5*b + 115*a^3*b^3 -
99*a*b^5 - 945*(a^5*b - 3*a^3*b^3 + 3*a*b^5)*cosh(x)^4 + 210*(5*b^6*x + 2*a
^4*b^2 - 6*a^2*b^4)*cosh(x)^3 - 210*(3*a^5*b - 8*a^3*b^3 + 6*a*b^5)*cosh(x)
^2 + 45*(5*b^6*x + 3*a^4*b^2 - 7*a^2*b^4)*cosh(x))*sinh(x)^5 + 30*(5*b^6*x
+ 3*a^4*b^2 - 7*a^2*b^4)*cosh(x)^4 + 10*(315*b^6*x*cosh(x)^6 + 15*b^6*x + 9
*a^4*b^2 - 21*a^2*b^4 - 378*(a^5*b - 3*a^3*b^3 + 3*a*b^5)*cosh(x)^5 + 105*(
5*b^6*x + 2*a^4*b^2 - 6*a^2*b^4)*cosh(x)^4 - 140*(3*a^5*b - 8*a^3*b^3 + 6*a
```

*b^5)*cosh(x)^3 + 45*(5*b^6*x + 3*a^4*b^2 - 7*a^2*b^4)*cosh(x)^2 - 2*(45*a^5*b - 115*a^3*b^3 + 99*a*b^5)*cosh(x))*sinh(x)^4 - 40*(3*a^5*b - 8*a^3*b^3 + 6*a*b^5)*cosh(x)^3 + 40*(45*b^6*x*cosh(x)^7 - 63*(a^5*b - 3*a^3*b^3 + 3*a*b^5)*cosh(x)^6 - 3*a^5*b + 8*a^3*b^3 - 6*a*b^5 + 21*(5*b^6*x + 2*a^4*b^2 - 6*a^2*b^4)*cosh(x)^5 - 35*(3*a^5*b - 8*a^3*b^3 + 6*a*b^5)*cosh(x)^4 + 15*(5*b^6*x + 3*a^4*b^2 - 7*a^2*b^4)*cosh(x)^3 - (45*a^5*b - 115*a^3*b^3 + 99*a*b^5)*cosh(x)^2 + 3*(5*b^6*x + 3*a^4*b^2 - 7*a^2*b^4)*cosh(x))*sinh(x)^3 + 15*(5*b^6*x + 2*a^4*b^2 - 6*a^2*b^4)*cosh(x)^2 + 5*(135*b^6*x*cosh(x)^8 - 216*(a^5*b - 3*a^3*b^3 + 3*a*b^5)*cosh(x)^7 + 15*b^6*x + 84*(5*b^6*x + 2*a^4*b^2 - 6*a^2*b^4)*cosh(x)^6 + 6*a^4*b^2 - 18*a^2*b^4 - 168*(3*a^5*b - 8*a^3*b^3 + 6*a*b^5)*cosh(x)^5 + 90*(5*b^6*x + 3*a^4*b^2 - 7*a^2*b^4)*cosh(x)^4 - 8*(45*a^5*b - 115*a^3*b^3 + 99*a*b^5)*cosh(x)^3 + 36*(5*b^6*x + 3*a^4*b^2 - 7*a^2*b^4)*cosh(x)^2 - 24*(3*a^5*b - 8*a^3*b^3 + 6*a*b^5)*cosh(x))*sinh(x)^2 - 30*(a^5*b - 3*a^3*b^3 + 3*a*b^5)*cosh(x) + 15*((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^10 + 10*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^9 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^10 + 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^8 + 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 9*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^8 + 40*(3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x)^7 + 10*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^6 + 10*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 21*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 + 14*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^6 + a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 4*(63*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^5 + 70*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + 15*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x)^5 + 10*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 + 10*(21*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^6 + a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 35*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 + 15*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^4 + 40*(3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^7 + 7*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^5 + 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x)^3 + 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2 + 5*(9*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^8 + 28*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^6 + a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 30*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 + 12*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^2 + 10*((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^9 + 4*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^7 + 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^5 + 4*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x))*log(2*(a*cosh(x) + b)/(cosh(x) - sinh(x))) - 15*((a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^10 + 10*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)*sinh(x)^9 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4)*sinh(x)^10 + 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^8 + 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 + 9*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^2)*sinh(x)^8 + 40*(3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x))*sinh(x)^7 + 10*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^6 + 10*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 +

```
21*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^4 + 14*(a^6 - 3*a^4*b^2 + 3*a^2*b^
4)*cosh(x)^2)*sinh(x)^6 + a^6 - 3*a^4*b^2 + 3*a^2*b^4 + 4*(63*(a^6 - 3*a^4*
b^2 + 3*a^2*b^4)*cosh(x)^5 + 70*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^3 + 1
5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x))*sinh(x...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^7(x)}{a + b\,\mathrm{sech}\,(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**7/(a+b*sech(x)),x)
```

```
[Out] Integral(tanh(x)**7/(a + b*sech(x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(113) = 226.
time = 0.41, size = 267, normalized size = 2.21

$$\frac{(a^5 - 3\,a^3b^2 + 3\,ab^4)\log\left(e^{(-x)} + e^x\right)}{b^6} - \frac{(a^6 - 3\,a^4b^2 + 3\,a^2b^4 - b^6)\log\left(\left|a\left(e^{(-x)} + e^x\right) + 2\,b\right|\right)}{ab^6} - \frac{137\,a^5\left(e^{(-x)} + e^x\right)^5 - 411\,a^3b^2\left(e^{(-x)} + e^x\right)^5 + 411\,ab^4\left(e^{(-x)} + e^x\right)^5 - 120\,a^4b\left(e^{(-x)} + e^x\right)^4 + 360\,a^2b^3\left(e^{(-x)} + e^x\right)^4 - 360\,b^5\left(e^{(-x)} + e^x\right)^4 + 120\,a^3b^2\left(e^{(-x)} + e^x\right)^3 - 360\,ab^4\left(e^{(-x)} + e^x\right)^3 - 160\,a^2b^3\left(e^{(-x)} + e^x\right)^2 + 480\,b^5\left(e^{(-x)} + e^x\right)^2 + 240\,ab^4\left(e^{(-x)} + e^x\right) - 384\,b^5}{60\,b^6\left(e^{(-x)} + e^x\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^7/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] (a^5 - 3*a^3*b^2 + 3*a*b^4)*log(e^(-x) + e^x)/b^6 - (a^6 - 3*a^4*b^2 + 3*a^
2*b^4 - b^6)*log(abs(a*(e^(-x) + e^x) + 2*b))/(a*b^6) - 1/60*(137*a^5*(e^(-
x) + e^x)^5 - 411*a^3*b^2*(e^(-x) + e^x)^5 + 411*a*b^4*(e^(-x) + e^x)^5 - 1
20*a^4*b*(e^(-x) + e^x)^4 + 360*a^2*b^3*(e^(-x) + e^x)^4 - 360*b^5*(e^(-x)
+ e^x)^4 + 120*a^3*b^2*(e^(-x) + e^x)^3 - 360*a*b^4*(e^(-x) + e^x)^3 - 160*
a^2*b^3*(e^(-x) + e^x)^2 + 480*b^5*(e^(-x) + e^x)^2 + 240*a*b^4*(e^(-x) + e
^x) - 384*b^5)/(b^6*(e^(-x) + e^x)^5)
```

Mupad [B]
time = 1.99, size = 316, normalized size = 2.61

$$\frac{\frac{8\,a}{b^2} - \frac{8\,\mathrm{e}^x\,(5\,a^2 - 27\,b^2)}{15\,b^3}}{3\,\mathrm{e}^{2\,x} + 3\,\mathrm{e}^{4\,x} + \mathrm{e}^{6\,x} + 1} - \frac{\frac{4\,a}{b^2} + \frac{64\,\mathrm{e}^x}{5\,b}}{4\,\mathrm{e}^{2\,x} + 6\,\mathrm{e}^{4\,x} + 4\,\mathrm{e}^{6\,x} + \mathrm{e}^{8\,x} + 1} + \frac{\frac{8\,\mathrm{e}^x\,(a^2 - 3\,b^2)}{3\,b^3} + \frac{2\,(a^4 - 5\,a^2\,b^2)}{a\,b^4}}{2\,\mathrm{e}^{2\,x} + \mathrm{e}^{4\,x} + 1} - \frac{x}{a} + \frac{\frac{2\,\mathrm{e}^x\,(a^4 - 3\,a^2\,b^2 + 3\,b^4)}{b^5} - \frac{2\,(a^4 - 3\,a^2\,b^2)}{a\,b^4}}{\mathrm{e}^{2\,x} + 1} + \frac{32\,\mathrm{e}^x}{5\,b\,(5\,\mathrm{e}^{2\,x} + 10\,\mathrm{e}^{4\,x} + 10\,\mathrm{e}^{6\,x} + 5\,\mathrm{e}^{8\,x} + \mathrm{e}^{10\,x} + 1)} + \frac{\ln(\mathrm{e}^{2\,x} + 1)\,(a^5 - 3\,a^3\,b^2 + 3\,a\,b^4)}{b^6} - \frac{\ln(a + 2\,b\,\mathrm{e}^x + a\,\mathrm{e}^{2\,x})\,(a^6 - 3\,a^4\,b^2 + 3\,a^2\,b^4 - b^6)}{a\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^7/(a + b/cosh(x)),x)
```

```
[Out] ((8*a)/b^2 - (8*exp(x)*(5*a^2 - 27*b^2))/(15*b^3))/(3*exp(2*x) + 3*exp(4*x)
 + exp(6*x) + 1) - ((4*a)/b^2 + (64*exp(x))/(5*b))/(4*exp(2*x) + 6*exp(4*x)
 + 4*exp(6*x) + exp(8*x) + 1) + ((8*exp(x)*(a^2 - 3*b^2))/(3*b^3) + (2*(a^4
 - 5*a^2*b^2))/(a*b^4))/(2*exp(2*x) + exp(4*x) + 1) - x/a + ((2*exp(x)*(a^4
 + 3*b^4 - 3*a^2*b^2))/b^5 - (2*(a^4 - 3*a^2*b^2))/(a*b^4))/(exp(2*x) + 1)
+ (32*exp(x))/(5*b*(5*exp(2*x) + 10*exp(4*x) + 10*exp(6*x) + 5*exp(8*x) + e
xp(10*x) + 1)) + (log(exp(2*x) + 1)*(3*a*b^4 + a^5 - 3*a^3*b^2))/b^6 - (log
(a + 2*b*exp(x) + a*exp(2*x))*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))/(a*b^6)
```

3.114 $\int \frac{\tanh^6(x)}{a+b\,\text{sech}(x)}\,dx$

Optimal. Leaf size=187

$$\frac{x}{a}-\frac{3\text{ArcTan}(\sinh(x))}{8b}-\frac{(a^2-3b^2)\,\text{ArcTan}(\sinh(x))}{2b^3}-\frac{(a^4-3a^2b^2+3b^4)\,\text{ArcTan}(\sinh(x))}{b^5}+\frac{2(a-b)^{5/2}(a+$$

```
[Out] x/a-3/8*arctan(sinh(x))/b-1/2*(a^2-3*b^2)*arctan(sinh(x))/b^3-(a^4-3*a^2*b^
2+3*b^4)*arctan(sinh(x))/b^5+2*(a-b)^(5/2)*(a+b)^(5/2)*arctan((a-b)^(1/2)*t
anh(1/2*x)/(a+b)^(1/2))/a/b^5+a*tanh(x)/b^2+a*(a^2-3*b^2)*tanh(x)/b^4-3/8*s
ech(x)*tanh(x)/b-1/2*(a^2-3*b^2)*sech(x)*tanh(x)/b^3-1/4*sech(x)^3*tanh(x)/
b-1/3*a*tanh(x)^3/b^2
```

Rubi [A]
time = 0.21, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {3983, 2976, 2738, 211, 3855, 3852, 8, 3853}

$$-\frac{(a^2-3b^2)\,\text{ArcTan}(\sinh(x))}{2b^3}+\frac{a(a^2-3b^2)\tanh(x)}{b^4}-\frac{(a^2-3b^2)\tanh(x)\text{sech}(x)}{2b^3}-\frac{(a^4-3a^2b^2+3b^4)\,\text{ArcTan}(\sinh(x))}{b^5}+\frac{2(a-b)^{5/2}(a+b)^{5/2}\text{ArcTan}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{ab^5}-\frac{a\tanh^3(x)}{3b^2}+\frac{a\tanh(x)}{b^2}+\frac{x}{a}-\frac{3\text{ArcTan}(\sinh(x))}{8b}-\frac{\tanh(x)\text{sech}^3(x)}{4b}-\frac{3\tanh(x)\text{sech}(x)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^6/(a + b*Sech[x]),x]
```

```
[Out] x/a - (3*ArcTan[Sinh[x]])/(8*b) - ((a^2 - 3*b^2)*ArcTan[Sinh[x]])/(2*b^3) -
 ((a^4 - 3*a^2*b^2 + 3*b^4)*ArcTan[Sinh[x]])/b^5 + (2*(a - b)^(5/2)*(a + b)
^(5/2)*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(a*b^5) + (a*Tanh[x])/b
^2 + (a*(a^2 - 3*b^2)*Tanh[x])/b^4 - (3*Sech[x]*Tanh[x])/(8*b) - ((a^2 - 3*
b^2)*Sech[x]*Tanh[x])/(2*b^3) - (Sech[x]^3*Tanh[x])/(4*b) - (a*Tanh[x]^3)/(
3*b^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 2976

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3983

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
 + n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$\int \frac{\tanh^6(x)}{a+b\text{sech}(x)}\,dx = \int \frac{\sinh(x)\tanh^5(x)}{b+a\cosh(x)}\,dx$$

$$= -\int \left(-\frac{1}{a} - \frac{(a^2-b^2)^3}{ab^5(b+a\cosh(x))} + \frac{(a^4-3a^2b^2+3b^4)\,\text{sech}(x)}{b^5} + \frac{(-a^3+3ab^2)\,\text{sech}^2(x}{b^4}\right.$$

$$= \frac{x}{a} + \frac{a\int \text{sech}^4(x)\,dx}{b^2} - \frac{\int \text{sech}^5(x)\,dx}{b} + \frac{(a(a^2-3b^2))\int \text{sech}^2(x)\,dx}{b^4} - \frac{(a^2-3b^2)\int \text{se}}{b^3}$$

$$= \frac{x}{a} - \frac{(a^4-3a^2b^2+3b^4)\tan^{-1}(\sinh(x))}{b^5} - \frac{(a^2-3b^2)\,\text{sech}(x)\tanh(x)}{2b^3} - \frac{\text{sech}^3(x)\tanh}{4b}$$

$$= \frac{x}{a} - \frac{(a^2-3b^2)\tan^{-1}(\sinh(x))}{2b^3} - \frac{(a^4-3a^2b^2+3b^4)\tan^{-1}(\sinh(x))}{b^5} + \frac{2(a-b)^{5/2}(a+}{}$$

$$= \frac{x}{a} - \frac{3\tan^{-1}(\sinh(x))}{8b} - \frac{(a^2-3b^2)\tan^{-1}(\sinh(x))}{2b^3} - \frac{(a^4-3a^2b^2+3b^4)\tan^{-1}(\sinh(x}{b^5}$$

Mathematica [A]
time = 0.42, size = 185, normalized size = 0.99

$$\frac{-12(8a^4-20a^2b^2+15b^4)\,\text{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right) + \frac{48\left(b^5\sqrt{a^2-b^2}\,x-2(a^2-b^2)^3\text{ArcTan}\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)\right)}{a\sqrt{a^2-b^2}} + b(-12a^2b+15b^3+4a(9a^2-17b^2)\cosh(x)+3b(-4a^2+9b^2)\cosh(2x)+12a^3\cosh(3x)-28ab^2\cosh(3x))\,\text{sech}^3(x)\tanh(x)}{48b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^6/(a + b*Sech[x]),x]
```

```
[Out] (-12*(8*a^4 - 20*a^2*b^2 + 15*b^4)*ArcTan[Tanh[x/2]] + (48*(b^5*Sqrt[a^2 -
b^2]*x - 2*(a^2 - b^2)^3*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]]))/(a*
Sqrt[a^2 - b^2]) + b*(-12*a^2*b + 15*b^3 + 4*a*(9*a^2 - 17*b^2)*Cosh[x] + 3
*b*(-4*a^2 + 9*b^2)*Cosh[2*x] + 12*a^3*Cosh[3*x] - 28*a*b^2*Cosh[3*x])*Sech
[x]^3*Tanh[x])/(48*b^5)
```

Maple [A]
time = 1.14, size = 266, normalized size = 1.42

method	result
default	$-\frac{2\left(\frac{\left(-a^3b-\frac{1}{2}a^2b^2+2a\,b^3+\frac{7}{8}b^4\right)\left(\tanh^7\left(\frac{x}{2}\right)\right)+\left(-3a^3b-\frac{1}{2}a^2b^2+\frac{15}{8}b^4+\frac{22}{3}a\,b^3\right)\left(\tanh^5\left(\frac{x}{2}\right)\right)+\left(\frac{1}{2}a^2b^2-\frac{15}{8}b^4-3a^3b+\frac{22}{3}a\,b^3\right)\left(\tanh^3\left(\frac{x}{2}\right)\right)+\left(-\right.}{\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^4}}{b^5}$

risch	$\frac{x}{a} - \frac{12a^2b\,e^{7x} - 27b^3e^{7x} + 24a^3e^{6x} - 72a\,b^2e^{6x} + 12a^2b\,e^{5x} - 3b^3e^{5x} + 72a^3e^{4x} - 168a\,b^2e^{4x} - 12a^2b\,e^{3x} + 3b^3e^{3x} + 72a^3e^{2x} - 152a\,b^2e^{2x} - 1}{12b^4(1+e^{2x})^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^6/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/b^5*(((-a^3*b-1/2*a^2*b^2+2*a*b^3+7/8*b^4)*tanh(1/2*x)^7+(-3*a^3*b-1/2*a
^2*b^2+15/8*b^4+22/3*a*b^3)*tanh(1/2*x)^5+(1/2*a^2*b^2-15/8*b^4-3*a^3*b+22/
3*a*b^3)*tanh(1/2*x)^3+(-a^3*b+2*a*b^3+1/2*a^2*b^2-7/8*b^4)*tanh(1/2*x))/(t
anh(1/2*x)^2+1)^4+1/8*(8*a^4-20*a^2*b^2+15*b^4)*arctan(tanh(1/2*x)))+1/a*ln
(tanh(1/2*x)+1)-1/a*ln(tanh(1/2*x)-1)+2/a*(a-b)^3*(a^3+3*a^2*b+3*a*b^2+b^3)
/b^5/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^6/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2417 vs. $2(165) = 330$.
time = 0.58, size = 4914, normalized size = 26.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^6/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [1/12*(12*b^5*x*cosh(x)^8 + 12*b^5*x*sinh(x)^8 - 3*(4*a^3*b^2 - 9*a*b^4)*co
sh(x)^7 + 3*(32*b^5*x*cosh(x) - 4*a^3*b^2 + 9*a*b^4)*sinh(x)^7 + 24*(2*b^5*
x - a^4*b + 3*a^2*b^3)*cosh(x)^6 + 3*(112*b^5*x*cosh(x)^2 + 16*b^5*x - 8*a^
4*b + 24*a^2*b^3 - 7*(4*a^3*b^2 - 9*a*b^4)*cosh(x))*sinh(x)^6 + 12*b^5*x -
3*(4*a^3*b^2 - a*b^4)*cosh(x)^5 + 3*(224*b^5*x*cosh(x)^3 - 4*a^3*b^2 + a*b^
4 - 21*(4*a^3*b^2 - 9*a*b^4)*cosh(x)^2 + 48*(2*b^5*x - a^4*b + 3*a^2*b^3)*c
osh(x))*sinh(x)^5 - 24*a^4*b + 56*a^2*b^3 + 24*(3*b^5*x - 3*a^4*b + 7*a^2*b
^3)*cosh(x)^4 + 3*(280*b^5*x*cosh(x)^4 + 24*b^5*x - 24*a^4*b + 56*a^2*b^3 -
 35*(4*a^3*b^2 - 9*a*b^4)*cosh(x)^3 + 120*(2*b^5*x - a^4*b + 3*a^2*b^3)*cos
```

h(x)^2 - 5*(4*a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^4 + 3*(4*a^3*b^2 - a*b^4)*c
osh(x)^3 + 3*(224*b^5*x*cosh(x)^5 + 4*a^3*b^2 - a*b^4 - 35*(4*a^3*b^2 - 9*a
*b^4)*cosh(x)^4 + 160*(2*b^5*x - a^4*b + 3*a^2*b^3)*cosh(x)^3 - 10*(4*a^3*b
^2 - a*b^4)*cosh(x)^2 + 32*(3*b^5*x - 3*a^4*b + 7*a^2*b^3)*cosh(x))*sinh(x)
^3 + 8*(6*b^5*x - 9*a^4*b + 19*a^2*b^3)*cosh(x)^2 + (336*b^5*x*cosh(x)^6 +
48*b^5*x - 63*(4*a^3*b^2 - 9*a*b^4)*cosh(x)^5 - 72*a^4*b + 152*a^2*b^3 + 36
0*(2*b^5*x - a^4*b + 3*a^2*b^3)*cosh(x)^4 - 30*(4*a^3*b^2 - a*b^4)*cosh(x)^
3 + 144*(3*b^5*x - 3*a^4*b + 7*a^2*b^3)*cosh(x)^2 + 9*(4*a^3*b^2 - a*b^4)*c
osh(x))*sinh(x)^2 + 12*((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^8 + 8*(a^4 - 2*a^2*
b^2 + b^4)*cosh(x)*sinh(x)^7 + (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^8 + 4*(a^4 -
 2*a^2*b^2 + b^4)*cosh(x)^6 + 4*(a^4 - 2*a^2*b^2 + b^4 + 7*(a^4 - 2*a^2*b^2
 + b^4)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^3 + 3*(
a^4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh(x)^5 + 6*(a^4 - 2*a^2*b^2 + b^4)*cosh(
x)^4 + 2*(35*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + 3*a^4 - 6*a^2*b^2 + 3*b^4
+ 30*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^4 + a^4 - 2*a^2*b^2 + b^4 +
 8*(7*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^5 + 10*(a^4 - 2*a^2*b^2 + b^4)*cosh(x
)^3 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh(x)^3 + 4*(a^4 - 2*a^2*b^2 + b
^4)*cosh(x)^2 + 4*(7*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^6 + 15*(a^4 - 2*a^2*b^
2 + b^4)*cosh(x)^4 + a^4 - 2*a^2*b^2 + b^4 + 9*(a^4 - 2*a^2*b^2 + b^4)*cosh
(x)^2)*sinh(x)^2 + 8*((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^7 + 3*(a^4 - 2*a^2*b^
2 + b^4)*cosh(x)^5 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^3 + (a^4 - 2*a^2*b^2
 + b^4)*cosh(x))*sinh(x))*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)
^2 + 2*a*b*cosh(x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-
a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*co
sh(x) + 2*(a*cosh(x) + b)*sinh(x) + a)) - 3*((8*a^5 - 20*a^3*b^2 + 15*a*b^4
)*cosh(x)^8 + 8*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)*sinh(x)^7 + (8*a^5
- 20*a^3*b^2 + 15*a*b^4)*sinh(x)^8 + 4*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh
(x)^6 + 4*(8*a^5 - 20*a^3*b^2 + 15*a*b^4 + 7*(8*a^5 - 20*a^3*b^2 + 15*a*b^4
)*cosh(x)^2)*sinh(x)^6 + 8*(7*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + 3
*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^5 + 8*a^5 - 20*a^3*b^2 +
15*a*b^4 + 6*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 2*(24*a^5 - 60*a^3
*b^2 + 45*a*b^4 + 35*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 30*(8*a^5
- 20*a^3*b^2 + 15*a*b^4)*cosh(x)^2)*sinh(x)^4 + 8*(7*(8*a^5 - 20*a^3*b^2 +
15*a*b^4)*cosh(x)^5 + 10*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + 3*(8*a
^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^3 + 4*(8*a^5 - 20*a^3*b^2 + 15
*a*b^4)*cosh(x)^2 + 4*(7*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^6 + 8*a^5
- 20*a^3*b^2 + 15*a*b^4 + 15*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 9*
(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((8*a^5 - 20*a^3*b
^2 + 15*a*b^4)*cosh(x)^7 + 3*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^5 + 3*
(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + (8*a^5 - 20*a^3*b^2 + 15*a*b^4)
*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) + 3*(4*a^3*b^2 - 9*a*b^4)*cosh
(x) + (96*b^5*x*cosh(x)^7 - 21*(4*a^3*b^2 - 9*a*b^4)*cosh(x)^6 + 144*(2*b^5
*x - a^4*b + 3*a^2*b^3)*cosh(x)^5 + 12*a^3*b^2 - 27*a*b^4 - 15*(4*a^3*b^2 -
 a*b^4)*cosh(x)^4 + 96*(3*b^5*x - 3*a^4*b + 7*a^2*b^3)*cosh(x)^3 + 9*(4*a^3
*b^2 - a*b^4)*cosh(x)^2 + 16*(6*b^5*x - 9*a^4*b + 19*a^2*b^3)*cosh(x))*sinh

```
(x))/(a*b^5*cosh(x)^8 + 8*a*b^5*cosh(x)*sinh(x)^7 + a*b^5*sinh(x)^8 + 4*a*b
^5*cosh(x)^6 + 6*a*b^5*cosh(x)^4 + 4*a*b^5*cosh(x)^2 + 4*(7*a*b^5*cosh(x)^2
 + a*b^5)*sinh(x)^6 + a*b^5 + 8*(7*a*b^5*cosh(x)^3 + 3*a*b^5*cosh(x))*sinh(
x)^5 + 2*(35*a*b^5*cosh(x)^4 + 30*a*b^5*cosh(x)^2 + 3*a*b^5)*sinh(x)^4 + 8*
(7*a*b^5*cosh(x)^5 + 10*a*b^5*cosh(x)^3 + 3*a*b^5*cosh(x))*sinh(x)^3 + 4*(7
*a*b^5*cosh(x)^6 + 15*a*b^5*cosh(x)^4 + 9*a*b^5*cosh(x)^2 + a*b^5)*sinh(x)^
2 + 8*(a*b^5*cosh(x)^7 + 3*a*b^5*cosh(x)^5 + 3*a*b^5*cosh(x)^3 + a*b^5*cosh
(x))*sinh(x)), 1/12*(12*b^5*x*cosh(x)^8 + 12*b^5*x*sinh(x)^8 - 3*(4*a^3*b^2
 - 9*a*b^4)*cosh(x)^7 + 3*(32*b^5*x*cosh(x) - 4*a^3*b^2 + 9*a*b^4)*sinh(x)^
7 + 24*(2*b^5*x - a^4*b + 3*a^2*b^3)*cosh(x)^6 + 3*(112*b^5*x*cosh(x)^2 + 1
6*b^5*x - 8*a^4*b + 24*a^2*b^3 - 7*(4*a^3*b^2 - 9*a*b^4)*cosh(x))*sinh(x)^6
 + 12*b^5*x - 3*(4*a^3*b^2 - a*b^4)*cosh(x)^5 + 3*(224*b^5*x*cosh(x)^3 - 4*
a^3*b^2 + a*b^4 - 21*(4*a^3*b^2 - 9*a*b^4)*cosh...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^6(x)}{a + b\operatorname{sech}(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**6/(a+b*sech(x)),x)
```

```
[Out] Integral(tanh(x)**6/(a + b*sech(x)), x)
```

Giac [A]
time = 0.38, size = 250, normalized size = 1.34

$$\frac{x}{a} - \frac{(8\,a^4 - 20\,a^2b^2 + 15\,b^4)\arctan(e^x)}{4\,b^5} + \frac{2\,(a^6 - 3\,a^4b^2 + 3\,a^2b^4 - b^6)\arctan\left(\frac{ae^x + b}{\sqrt{a^2 - b^2}}\right)}{\sqrt{a^2 - b^2}\,ab^5} - \frac{12\,a^2be^{(7\,x)} - 27\,b^3e^{(7\,x)} + 24\,a^3e^{(6\,x)} - 72\,ab^2e^{(6\,x)} + 12\,a^2be^{(5\,x)} - 3\,b^3e^{(5\,x)} + 72\,a^3e^{(4\,x)} - 168\,ab^2e^{(4\,x)} - 12\,a^2be^{(3\,x)} + 3\,b^3e^{(3\,x)} + 72\,a^3e^{(2\,x)} - 152\,ab^2e^{(2\,x)} - 12\,a^2be^x + 27\,b^3e^x + 24\,a^3 - 56\,ab^2}{12\,b^4(e^{(2\,x)} + 1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^6/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] x/a - 1/4*(8*a^4 - 20*a^2*b^2 + 15*b^4)*arctan(e^x)/b^5 + 2*(a^6 - 3*a^4*b^
2 + 3*a^2*b^4 - b^6)*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a
*b^5) - 1/12*(12*a^2*b*e^(7*x) - 27*b^3*e^(7*x) + 24*a^3*e^(6*x) - 72*a*b^2
*e^(6*x) + 12*a^2*b*e^(5*x) - 3*b^3*e^(5*x) + 72*a^3*e^(4*x) - 168*a*b^2*e^
(4*x) - 12*a^2*b*e^(3*x) + 3*b^3*e^(3*x) + 72*a^3*e^(2*x) - 152*a*b^2*e^(2*
x) - 12*a^2*b*e^x + 27*b^3*e^x + 24*a^3 - 56*a*b^2)/(b^4*(e^(2*x) + 1)^4)
```

Mupad [B]
time = 8.50, size = 1001, normalized size = 5.35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^6/(a + b/cosh(x)),x)

[Out] ((8*a)/(3*b^2) + (6*exp(x))/b)/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - (
(exp(x)*(4*a^2 - 9*b^2))/(4*b^3) + (2*(a^4 - 3*a^2*b^2))/(a*b^4))/(exp(2*x)
 + 1) - ((4*a)/b^2 - (exp(x)*(4*a^2 - 13*b^2))/(2*b^3))/(2*exp(2*x) + exp(4
*x) + 1) + x/a + (log(exp(x) - 1i)*(a^4*8i + b^4*15i - a^2*b^2*20i))/(8*b^5
) - (log(exp(x) + 1i)*(a^4*8i + b^4*15i - a^2*b^2*20i))/(8*b^5) - (4*exp(x)
)/(b*(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1)) + (log(((-(a +
b)^5*(a - b)^5)^(1/2)*((128*a^12 + 64*b^12 - 834*a^2*b^10 + 2385*a^4*b^8 -
3160*a^6*b^6 + 2240*a^8*b^4 - 832*a^10*b^2 - 900*a*b^11*exp(x) + 192*a^11*b
*exp(x) + 3075*a^3*b^9*exp(x) - 4360*a^5*b^7*exp(x) + 3200*a^7*b^5*exp(x) -
 1216*a^9*b^3*exp(x))/(2*a^6*b^8) - ((-(a + b)^5*(a - b)^5)^(1/2)*((4*(a^2
- b^2)*(16*a*b^4 + 16*a^5 - 32*a^3*b^2 + 32*b^5*exp(x) + 28*a^4*b*exp(x) -
57*a^2*b^3*exp(x)))/(a^6*b^2) + (32*(-(a + b)^5*(a - b)^5)^(1/2)*(3*a*b^2 -
 2*a^3 + 4*b^3*exp(x) - 3*a^2*b*exp(x)))/(a^6*b^3)))/(a*b^5)))/(a*b^5) - ((
a^2 - b^2)^3*(8*a^4 + 15*b^4 - 20*a^2*b^2)*(30*a*b^4 + 16*a^5 - 40*a^3*b^2
+ 52*b^5*exp(x) + 28*a^4*b*exp(x) - 71*a^2*b^3*exp(x)))/(2*a^6*b^12))*(-(a
+ b)^5*(a - b)^5)^(1/2))/(a*b^5) - (log(- ((-(a + b)^5*(a - b)^5)^(1/2)*((1
28*a^12 + 64*b^12 - 834*a^2*b^10 + 2385*a^4*b^8 - 3160*a^6*b^6 + 2240*a^8*b
^4 - 832*a^10*b^2 - 900*a*b^11*exp(x) + 192*a^11*b*exp(x) + 3075*a^3*b^9*ex
p(x) - 4360*a^5*b^7*exp(x) + 3200*a^7*b^5*exp(x) - 1216*a^9*b^3*exp(x))/(2*
a^6*b^8) + ((-(a + b)^5*(a - b)^5)^(1/2)*((4*(a^2 - b^2)*(16*a*b^4 + 16*a^5
 - 32*a^3*b^2 + 32*b^5*exp(x) + 28*a^4*b*exp(x) - 57*a^2*b^3*exp(x)))/(a^6*
b^2) - (32*(-(a + b)^5*(a - b)^5)^(1/2)*(3*a*b^2 - 2*a^3 + 4*b^3*exp(x) - 3
*a^2*b*exp(x)))/(a^6*b^3)))/(a*b^5)))/(a*b^5) - ((a^2 - b^2)^3*(8*a^4 + 15*
b^4 - 20*a^2*b^2)*(30*a*b^4 + 16*a^5 - 40*a^3*b^2 + 52*b^5*exp(x) + 28*a^4*
b*exp(x) - 71*a^2*b^3*exp(x)))/(2*a^6*b^12))*(-(a + b)^5*(a - b)^5)^(1/2))/
(a*b^5)
```

3.115 $\int \frac{\tanh^5(x)}{a+b\,\mathrm{sech}(x)}\,dx$

Optimal. Leaf size=72

$$\frac{\log(\cosh(x))}{a}+\frac{(a^2-b^2)^2\log(a+b\,\mathrm{sech}(x))}{ab^4}-\frac{(a^2-2b^2)\,\mathrm{sech}(x)}{b^3}+\frac{a\,\mathrm{sech}^2(x)}{2b^2}-\frac{\mathrm{sech}^3(x)}{3b}$$

```
[Out] ln(cosh(x))/a+(a^2-b^2)^2*ln(a+b*sech(x))/a/b^4-(a^2-2*b^2)*sech(x)/b^3+1/2
*a*sech(x)^2/b^2-1/3*sech(x)^3/b
```

Rubi [A]
time = 0.07, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3970, 908}

$$\frac{(a^2-b^2)^2\log(a+b\,\mathrm{sech}(x))}{ab^4}-\frac{(a^2-2b^2)\,\mathrm{sech}(x)}{b^3}+\frac{a\,\mathrm{sech}^2(x)}{2b^2}+\frac{\log(\cosh(x))}{a}-\frac{\mathrm{sech}^3(x)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^5/(a + b*Sech[x]),x]
```

```
[Out] Log[Cosh[x]]/a + ((a^2 - b^2)^2*Log[a + b*Sech[x]])/(a*b^4) - ((a^2 - 2*b^2
)*Sech[x])/b^3 + (a*Sech[x]^2)/(2*b^2) - Sech[x]^3/(3*b)
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\tanh^5(x)}{a+b\text{sech}(x)}\,dx = -\frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x(a+x)}\,dx, x, b\text{sech}(x)\right)}{b^4}$$
$$= -\frac{\text{Subst}\left(\int \left(a^2\left(1-\frac{2b^2}{a^2}\right)+\frac{b^4}{ax}-ax+x^2-\frac{(a^2-b^2)^2}{a(a+x)}\right)\,dx, x, b\text{sech}(x)\right)}{b^4}$$
$$= \frac{\log(\cosh(x))}{a}+\frac{(a^2-b^2)^2\log(a+b\text{sech}(x))}{ab^4}-\frac{(a^2-2b^2)\,\text{sech}(x)}{b^3}+\frac{a\text{sech}^2(x)}{2b^2}-\frac{\text{sech}}{3}$$

Mathematica [A]
time = 0.12, size = 85, normalized size = 1.18

$$\frac{-6a^2(a^2-2b^2)\log(\cosh(x))+6(a^2-b^2)^2\log(b+a\cosh(x))-6ab(a^2-2b^2)\,\text{sech}(x)+3a^2b^2\text{sech}^2(x)-2ab^3\text{sech}^3(x)}{6ab^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^5/(a + b*Sech[x]),x]
[Out] (-6*a^2*(a^2 - 2*b^2)*Log[Cosh[x]] + 6*(a^2 - b^2)^2*Log[b + a*Cosh[x]] - 6
*a*b*(a^2 - 2*b^2)*Sech[x] + 3*a^2*b^2*Sech[x]^2 - 2*a*b^3*Sech[x]^3)/(6*a*
b^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(68) = 136.
time = 0.71, size = 158, normalized size = 2.19

method	result
default	$-\frac{\ln(\tanh(\frac{x}{2})+1)}{a}-\frac{a(a^2-2b^2)\ln(\tanh^2(\frac{x}{2})+1)+\frac{2b(a^2+ab-b^2)}{\tanh^2(\frac{x}{2})+1}+\frac{8b^3}{3(\tanh^2(\frac{x}{2})+1)^3}-\frac{2b^2(2b+a)}{(\tanh^2(\frac{x}{2})+1)^2}}{b^4}-\frac{\ln(\tanh(\frac{x}{2})-1)}{a}+\frac{(a}{}$
risch	$-\frac{x}{a}-\frac{2\,\mathrm{e}^{x}(3a^2\mathrm{e}^{4x}-6b^2\mathrm{e}^{4x}-3ab\,\mathrm{e}^{3x}+6a^2\mathrm{e}^{2x}-8b^2\mathrm{e}^{2x}-3b\,\mathrm{e}^{x}a+3a^2-6b^2)}{3b^3(1+\mathrm{e}^{2x})^3}-\frac{a^3\ln(1+\mathrm{e}^{2x})}{b^4}+\frac{2a\ln(1+\mathrm{e}^{2x})}{b^2}+\frac{a^3\ln\left(\mathrm{e}^{2x}+\frac{2b\,\mathrm{e}^{x}}{a}\right.}{b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a+b*sech(x)),x,method=_RETURNVERBOSE)
[Out] -1/a*ln(tanh(1/2*x)+1)-1/b^4*(a*(a^2-2*b^2)*ln(tanh(1/2*x)^2+1)+2*b*(a^2+a*
b-b^2)/(tanh(1/2*x)^2+1)+8/3*b^3/(tanh(1/2*x)^2+1)^3-2*b^2*(2*b+a)/(tanh(1/
2*x)^2+1)^2)-1/a*ln(tanh(1/2*x)-1)+(a-b)^2*(a^2+2*a*b+b^2)/a/b^4*ln(a*tanh(
1/2*x)^2-b*tanh(1/2*x)^2+a+b)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(68) = 136.
time = 0.51, size = 164, normalized size = 2.28

$$\frac{2\left(3\,abe^{(-2x)}+3\,abe^{(-4x)}-3\,(a^2-2\,b^2)e^{(-x)}-2\,(3\,a^2-4\,b^2)e^{(-3x)}-3\,(a^2-2\,b^2)e^{(-5x)}\right)}{3\left(3\,b^3e^{(-2x)}+3\,b^3e^{(-4x)}+b^3e^{(-6x)}+b^3\right)}+\frac{x}{a}-\frac{(a^3-2\,ab^2)\log\left(e^{(-2x)}+1\right)}{b^4}+\frac{(a^4-2\,a^2b^2+b^4)\log\left(2\,be^{(-x)}+ae^{(-2x)}+a\right)}{ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] 2/3*(3*a*b*e^(-2*x) + 3*a*b*e^(-4*x) - 3*(a^2 - 2*b^2)*e^(-x) - 2*(3*a^2 -
4*b^2)*e^(-3*x) - 3*(a^2 - 2*b^2)*e^(-5*x))/(3*b^3*e^(-2*x) + 3*b^3*e^(-4*x
) + b^3*e^(-6*x) + b^3) + x/a - (a^3 - 2*a*b^2)*log(e^(-2*x) + 1)/b^4 + (a^
4 - 2*a^2*b^2 + b^4)*log(2*b*e^(-x) + a*e^(-2*x) + a)/(a*b^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1280 vs. $2(68) = 136$.
time = 0.37, size = 1280, normalized size = 17.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^4*x*cosh(x)^6 + 3*b^4*x*sinh(x)^6 + 6*(a^3*b - 2*a*b^3)*cosh(x)^5
 + 6*(3*b^4*x*cosh(x) + a^3*b - 2*a*b^3)*sinh(x)^5 + 3*b^4*x + 3*(3*b^4*x -
 2*a^2*b^2)*cosh(x)^4 + 3*(15*b^4*x*cosh(x)^2 + 3*b^4*x - 2*a^2*b^2 + 10*(a
^3*b - 2*a*b^3)*cosh(x))*sinh(x)^4 + 4*(3*a^3*b - 4*a*b^3)*cosh(x)^3 + 4*(1
5*b^4*x*cosh(x)^3 + 3*a^3*b - 4*a*b^3 + 15*(a^3*b - 2*a*b^3)*cosh(x)^2 + 3*
(3*b^4*x - 2*a^2*b^2)*cosh(x))*sinh(x)^3 + 3*(3*b^4*x - 2*a^2*b^2)*cosh(x)^
2 + 3*(15*b^4*x*cosh(x)^4 + 3*b^4*x - 2*a^2*b^2 + 20*(a^3*b - 2*a*b^3)*cosh
(x)^3 + 6*(3*b^4*x - 2*a^2*b^2)*cosh(x)^2 + 4*(3*a^3*b - 4*a*b^3)*cosh(x))*
sinh(x)^2 + 6*(a^3*b - 2*a*b^3)*cosh(x) - 3*((a^4 - 2*a^2*b^2 + b^4)*cosh(x
)^6 + 6*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^5 + (a^4 - 2*a^2*b^2 + b^4)
*sinh(x)^6 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + 3*(a^4 - 2*a^2*b^2 + b^4
 + 5*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^4 + a^4 - 2*a^2*b^2 + b^4 +
 4*(5*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^3 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)
)*sinh(x)^3 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 + 3*(5*(a^4 - 2*a^2*b^2 +
 b^4)*cosh(x)^4 + a^4 - 2*a^2*b^2 + b^4 + 6*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)
^2)*sinh(x)^2 + 6*((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^5 + 2*(a^4 - 2*a^2*b^2 +
 b^4)*cosh(x)^3 + (a^4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh(x))*log(2*(a*cosh(x
) + b)/(cosh(x) - sinh(x))) + 3*((a^4 - 2*a^2*b^2)*cosh(x)^6 + 6*(a^4 - 2*a
^2*b^2)*cosh(x)*sinh(x)^5 + (a^4 - 2*a^2*b^2)*sinh(x)^6 + 3*(a^4 - 2*a^2*b^
2)*cosh(x)^4 + 3*(a^4 - 2*a^2*b^2 + 5*(a^4 - 2*a^2*b^2)*cosh(x)^2)*sinh(x)^
4 + a^4 - 2*a^2*b^2 + 4*(5*(a^4 - 2*a^2*b^2)*cosh(x)^3 + 3*(a^4 - 2*a^2*b^2
)*cosh(x))*sinh(x)^3 + 3*(a^4 - 2*a^2*b^2)*cosh(x)^2 + 3*(5*(a^4 - 2*a^2*b^
2)*cosh(x)^4 + a^4 - 2*a^2*b^2 + 6*(a^4 - 2*a^2*b^2)*cosh(x)^2)*sinh(x)^2 +
 6*((a^4 - 2*a^2*b^2)*cosh(x)^5 + 2*(a^4 - 2*a^2*b^2)*cosh(x)^3 + (a^4 - 2*
a^2*b^2)*cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) + 6*(3*b^4*x*
cosh(x)^5 + 5*(a^3*b - 2*a*b^3)*cosh(x)^4 + a^3*b - 2*a*b^3 + 2*(3*b^4*x -
2*a^2*b^2)*cosh(x)^3 + 2*(3*a^3*b - 4*a*b^3)*cosh(x)^2 + (3*b^4*x - 2*a^2*b
^2)*cosh(x))*sinh(x))/(a*b^4*cosh(x)^6 + 6*a*b^4*cosh(x)*sinh(x)^5 + a*b^4*
```

```
sinh(x)^6 + 3*a*b^4*cosh(x)^4 + 3*a*b^4*cosh(x)^2 + a*b^4 + 3*(5*a*b^4*cosh
(x)^2 + a*b^4)*sinh(x)^4 + 4*(5*a*b^4*cosh(x)^3 + 3*a*b^4*cosh(x))*sinh(x)^
3 + 3*(5*a*b^4*cosh(x)^4 + 6*a*b^4*cosh(x)^2 + a*b^4)*sinh(x)^2 + 6*(a*b^4*
cosh(x)^5 + 2*a*b^4*cosh(x)^3 + a*b^4*cosh(x))*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^5(x)}{a + b\operatorname{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**5/(a+b*sech(x)),x)
```

```
[Out] Integral(tanh(x)**5/(a + b*sech(x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(68) = 136.
time = 0.39, size = 152, normalized size = 2.11

$$-\frac{(a^3 - 2\,ab^2)\log\left(e^{(-x)} + e^x\right)}{b^4} + \frac{(a^4 - 2\,a^2b^2 + b^4)\log\left(\left|a\left(e^{(-x)} + e^x\right) + 2\,b\right|\right)}{ab^4} + \frac{11\,a^3\left(e^{(-x)} + e^x\right)^3 - 22\,ab^2\left(e^{(-x)} + e^x\right)^3 - 12\,a^2b\left(e^{(-x)} + e^x\right)^2 + 24\,b^3\left(e^{(-x)} + e^x\right)^2 + 12\,ab^2\left(e^{(-x)} + e^x\right) - 16\,b^3}{6\,b^4\left(e^{(-x)} + e^x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] -(a^3 - 2*a*b^2)*log(e^(-x) + e^x)/b^4 + (a^4 - 2*a^2*b^2 + b^4)*log(abs(a*
(e^(-x) + e^x) + 2*b))/(a*b^4) + 1/6*(11*a^3*(e^(-x) + e^x)^3 - 22*a*b^2*(e
^(-x) + e^x)^3 - 12*a^2*b*(e^(-x) + e^x)^2 + 24*b^3*(e^(-x) + e^x)^2 + 12*a
*b^2*(e^(-x) + e^x) - 16*b^3)/(b^4*(e^(-x) + e^x)^3)
```

Mupad [B]
time = 1.80, size = 155, normalized size = 2.15

$$\frac{\frac{2\,a}{b^2} - \frac{2\,e^x\,(a^2-2\,b^2)}{b^3}}{e^{2\,x}+1} - \frac{x}{a} - \frac{\frac{2\,a}{b^2} + \frac{8\,e^x}{3\,b}}{2\,e^{2\,x} + e^{4\,x} + 1} + \frac{\ln\left(e^{2\,x}+1\right)\,\left(2\,a\,b^2 - a^3\right)}{b^4} + \frac{8\,e^x}{3\,b\,\left(3\,e^{2\,x} + 3\,e^{4\,x} + e^{6\,x} + 1\right)} + \frac{\ln\left(a + 2\,b\,e^x + a\,e^{2\,x}\right)\,\left(a^4 - 2\,a^2\,b^2 + b^4\right)}{a\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a + b/cosh(x)),x)
```

```
[Out] ((2*a)/b^2 - (2*exp(x)*(a^2 - 2*b^2))/b^3)/(exp(2*x) + 1) - x/a - ((2*a)/b^
2 + (8*exp(x))/(3*b))/(2*exp(2*x) + exp(4*x) + 1) + (log(exp(2*x) + 1)*(2*a
*b^2 - a^3))/b^4 + (8*exp(x))/(3*b*(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1)
) + (log(a + 2*b*exp(x) + a*exp(2*x))*(a^4 + b^4 - 2*a^2*b^2))/(a*b^4)
```

3.116 $\int \frac{\tanh^4(x)}{a+b\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=94

$$\frac{x}{a}+\frac{(2a^2-3b^2)\,\mathrm{ArcTan}(\sinh(x))}{2b^3}-\frac{2(a-b)^{3/2}(a+b)^{3/2}\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{ab^3}-\frac{a\tanh(x)}{b^2}+\frac{\mathrm{sech}(x)\tanh}{2b}$$

```
[Out] x/a+1/2*(2*a^2-3*b^2)*arctan(sinh(x))/b^3-2*(a-b)^(3/2)*(a+b)^(3/2)*arctan(
(a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/a/b^3-a*tanh(x)/b^2+1/2*sech(x)*tanh(x
)/b
```

Rubi [A]
time = 0.21, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3983, 2972, 3136, 2738, 211, 3855}

$$\frac{(2a^2-3b^2)\,\mathrm{ArcTan}(\sinh(x))}{2b^3}-\frac{2(a-b)^{3/2}(a+b)^{3/2}\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{ab^3}-\frac{a\tanh(x)}{b^2}+\frac{x}{a}+\frac{\tanh(x)\mathrm{sech}(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/(a + b*Sech[x]),x]
```

```
[Out] x/a + ((2*a^2 - 3*b^2)*ArcTan[Sinh[x]])/(2*b^3) - (2*(a - b)^(3/2)*(a + b)^
(3/2)*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(a*b^3) - (a*Tanh[x])/b^
2 + (Sech[x]*Tanh[x])/(2*b)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2972

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
```

```
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((
d*Sin[e + f*x])^(n + 2)/(a^2*d^2*f*(n + 1)*(n + 2))), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3136

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[C*(x/(b*d)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)
/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f*x]), x], x] - Dist[(c^2*C - B*c*d
+ A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a,
b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && Ne
Q[c^2 - d^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3983

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
 + n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$\int \frac{\tanh^4(x)}{a+b\operatorname{sech}(x)}\,dx = \int \frac{\sinh(x)\tanh^3(x)}{b+a\cosh(x)}\,dx$$

$$= -\frac{a\tanh(x)}{b^2} + \frac{\operatorname{sech}(x)\tanh(x)}{2b} - \frac{\int \frac{(-2a^2+3b^2-ab\cosh(x)-2b^2\cosh^2(x))\operatorname{sech}(x)}{b+a\cosh(x)}\,dx}{2b^2}$$

$$= \frac{x}{a} - \frac{a\tanh(x)}{b^2} + \frac{\operatorname{sech}(x)\tanh(x)}{2b} - \frac{(a^2-b^2)^2\int \frac{1}{b+a\cosh(x)}\,dx}{ab^3} - \frac{(-2a^2+3b^2)\int \operatorname{sech}(}{2b^3}$$

$$= \frac{x}{a} + \frac{(2a^2-3b^2)\tan^{-1}(\sinh(x))}{2b^3} - \frac{a\tanh(x)}{b^2} + \frac{\operatorname{sech}(x)\tanh(x)}{2b} - \frac{\left(2(a^2-b^2)^2\right)\operatorname{Sub}}{}$$

$$= \frac{x}{a} + \frac{(2a^2-3b^2)\tan^{-1}(\sinh(x))}{2b^3} - \frac{2(a-b)^{3/2}(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{ab^3} - \frac{a}{}$$

Mathematica [A]
time = 0.28, size = 113, normalized size = 1.20

$$\frac{(b+a\cosh(x))\operatorname{sech}^2(x)\left(2\left(b^3x+a(2a^2-3b^2)\operatorname{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)+2(a^2-b^2)^{3/2}\operatorname{ArcTan}\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)\right)\cosh(x)+ab(-2a\sinh(x)+b\tanh(x))\right)}{2ab^3(a+b\operatorname{sech}(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(a + b*Sech[x]),x]
```

```
[Out] ((b + a*Cosh[x])*Sech[x]^2*(2*(b^3*x + a*(2*a^2 - 3*b^2)*ArcTan[Tanh[x/2]]
+ 2*(a^2 - b^2)^(3/2)*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]])*Cosh[x]
 + a*b*(-2*a*Sinh[x] + b*Tanh[x])))/(2*a*b^3*(a + b*Sech[x]))
```

Maple [A]
time = 1.02, size = 152, normalized size = 1.62

method	result
default	$-\frac{2(a-b)^2(a^2+2ab+b^2)\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a\,b^3\sqrt{(a+b)(a-b)}}+\frac{\frac{2\left(\left(-ab-\frac{1}{2}b^2\right)\left(\tanh^3\left(\frac{x}{2}\right)\right)+\left(-ab+\frac{1}{2}b^2\right)\tanh\left(\frac{x}{2}\right)\right)}{\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^2}+(2a^2-3b^2)\arctan(}{b^3}$
risch	$\frac{x}{a}+\frac{b\,e^{3x}+2\,e^{2x}a-e^{x}b+2a}{(1+e^{2x})^2b^2}+\frac{\sqrt{-a^2+b^2}\,a\ln\left(e^{x}-\frac{\sqrt{-a^2+b^2}-b}{a}\right)}{b^3}-\frac{\sqrt{-a^2+b^2}\,\ln\left(e^{x}-\frac{\sqrt{-a^2+b^2}-b}{a}\right)}{ba}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/a*(a-b)^2*(a^2+2*a*b+b^2)/b^3/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*
x)/((a+b)*(a-b))^(1/2))+2/b^3*(((-a*b-1/2*b^2)*tanh(1/2*x)^3+(-a*b+1/2*b^2)
*tanh(1/2*x))/(tanh(1/2*x)^2+1)^2+1/2*(2*a^2-3*b^2)*arctan(tanh(1/2*x)))-1/
a*ln(tanh(1/2*x)-1)+1/a*ln(tanh(1/2*x)+1)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 587 vs. $2(80) = 160$.

time = 0.46, size = 1254, normalized size = 13.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [(b^3*x*cosh(x)^4 + b^3*x*sinh(x)^4 + a*b^2*cosh(x)^3 + b^3*x - a*b^2*cosh(
x) + (4*b^3*x*cosh(x) + a*b^2)*sinh(x)^3 + 2*a^2*b + 2*(b^3*x + a^2*b)*cosh
(x)^2 + (6*b^3*x*cosh(x)^2 + 2*b^3*x + 3*a*b^2*cosh(x) + 2*a^2*b)*sinh(x)^2
 - ((a^2 - b^2)*cosh(x)^4 + 4*(a^2 - b^2)*cosh(x)*sinh(x)^3 + (a^2 - b^2)*s
inh(x)^4 + 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*(a^2 - b^2)*cosh(x)^2 + a^2 - b^2
)*sinh(x)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*s
inh(x))*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x)
 - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-a^2 + b^2)*(a*cosh
(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(
x) + b)*sinh(x) + a)) + ((2*a^3 - 3*a*b^2)*cosh(x)^4 + 4*(2*a^3 - 3*a*b^2)*
cosh(x)*sinh(x)^3 + (2*a^3 - 3*a*b^2)*sinh(x)^4 + 2*a^3 - 3*a*b^2 + 2*(2*a^
3 - 3*a*b^2)*cosh(x)^2 + 2*(2*a^3 - 3*a*b^2 + 3*(2*a^3 - 3*a*b^2)*cosh(x)^2
)*sinh(x)^2 + 4*((2*a^3 - 3*a*b^2)*cosh(x)^3 + (2*a^3 - 3*a*b^2)*cosh(x))*s
inh(x))*arctan(cosh(x) + sinh(x)) + (4*b^3*x*cosh(x)^3 + 3*a*b^2*cosh(x)^2
 - a*b^2 + 4*(b^3*x + a^2*b)*cosh(x))*sinh(x))/(a*b^3*cosh(x)^4 + 4*a*b^3*co
sh(x)*sinh(x)^3 + a*b^3*sinh(x)^4 + 2*a*b^3*cosh(x)^2 + a*b^3 + 2*(3*a*b^3*
cosh(x)^2 + a*b^3)*sinh(x)^2 + 4*(a*b^3*cosh(x)^3 + a*b^3*cosh(x))*sinh(x))
, (b^3*x*cosh(x)^4 + b^3*x*sinh(x)^4 + a*b^2*cosh(x)^3 + b^3*x - a*b^2*cosh
(x) + (4*b^3*x*cosh(x) + a*b^2)*sinh(x)^3 + 2*a^2*b + 2*(b^3*x + a^2*b)*cos
h(x)^2 + (6*b^3*x*cosh(x)^2 + 2*b^3*x + 3*a*b^2*cosh(x) + 2*a^2*b)*sinh(x)^
2 + 2*((a^2 - b^2)*cosh(x)^4 + 4*(a^2 - b^2)*cosh(x)*sinh(x)^3 + (a^2 - b^2
)*sinh(x)^4 + 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*(a^2 - b^2)*cosh(x)^2 + a^2 -
b^2)*sinh(x)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x)
)*sinh(x))*sqrt(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^2 - b
^2)) + ((2*a^3 - 3*a*b^2)*cosh(x)^4 + 4*(2*a^3 - 3*a*b^2)*cosh(x)*sinh(x)^3
 + (2*a^3 - 3*a*b^2)*sinh(x)^4 + 2*a^3 - 3*a*b^2 + 2*(2*a^3 - 3*a*b^2)*cosh
(x)^2 + 2*(2*a^3 - 3*a*b^2 + 3*(2*a^3 - 3*a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*(
(2*a^3 - 3*a*b^2)*cosh(x)^3 + (2*a^3 - 3*a*b^2)*cosh(x))*sinh(x))*arctan(co
sh(x) + sinh(x)) + (4*b^3*x*cosh(x)^3 + 3*a*b^2*cosh(x)^2 - a*b^2 + 4*(b^3*
x + a^2*b)*cosh(x))*sinh(x))/(a*b^3*cosh(x)^4 + 4*a*b^3*cosh(x)*sinh(x)^3 +
 a*b^3*sinh(x)^4 + 2*a*b^3*cosh(x)^2 + a*b^3 + 2*(3*a*b^3*cosh(x)^2 + a*b^3
)*sinh(x)^2 + 4*(a*b^3*cosh(x)^3 + a*b^3*cosh(x))*sinh(x))]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(x)}{a + b\operatorname{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(a+b*sech(x)),x)
[Out] Integral(tanh(x)**4/(a + b*sech(x)), x)
```

Giac [A]
time = 0.39, size = 111, normalized size = 1.18

$$\frac{x}{a}+\frac{\left(2\,a^{2}-3\,b^{2}\right)\arctan\left(e^{x}\right)}{b^{3}}-\frac{2\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\arctan\left(\frac{ae^{x}+b}{\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,ab^{3}}+\frac{be^{(3\,x)}+2\,ae^{(2\,x)}-be^{x}+2\,a}{b^{2}\left(e^{(2\,x)}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sech(x)),x, algorithm="giac")
[Out] x/a + (2*a^2 - 3*b^2)*arctan(e^x)/b^3 - 2*(a^4 - 2*a^2*b^2 + b^4)*arctan((a
*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a*b^3) + (b*e^(3*x) + 2*a*e^(2*
x) - b*e^x + 2*a)/(b^2*(e^(2*x) + 1)^2)
```

Mupad [B]
time = 7.26, size = 700, normalized size = 7.45

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a + b/cosh(x)),x)
[Out] ((2*a)/b^2 + exp(x)/b)/(exp(2*x) + 1) + x/a - (log(exp(x) - 1i)*(a^2*2i - b
^2*3i))/(2*b^3) + (log(exp(x) + 1i)*(a^2*2i - b^2*3i))/(2*b^3) - (2*exp(x))
/(b*(2*exp(2*x) + exp(4*x) + 1)) + (log((((64*a^8 + 32*b^8 - 272*a^2*b^6 +
456*a^4*b^4 - 288*a^6*b^2 - 288*a*b^7*exp(x) + 96*a^7*b*exp(x) + 600*a^3*b^
5*exp(x) - 416*a^5*b^3*exp(x))/(a^6*b^4) - (((16*(a^2 - b^2)*(4*a*b^2 - 4*a
^3 + 8*b^3*exp(x) - 7*a^2*b*exp(x)))/a^6 + (32*(-(a + b)^3*(a - b)^3)^(1/2)
*(3*a*b^2 - 2*a^3 + 4*b^3*exp(x) - 3*a^2*b*exp(x)))/(a^6*b))*(-(a + b)^3*(a
 - b)^3)^(1/2))/(a*b^3))*(-(a + b)^3*(a - b)^3)^(1/2))/(a*b^3) - (8*(a^2 -
b^2)^2*(2*a^2 - 3*b^2)*(6*a*b^2 - 4*a^3 + 10*b^3*exp(x) - 7*a^2*b*exp(x)))/
(a^6*b^6))*(-(a + b)^3*(a - b)^3)^(1/2))/(a*b^3) - (log(- (((64*a^8 + 32*b^
8 - 272*a^2*b^6 + 456*a^4*b^4 - 288*a^6*b^2 - 288*a*b^7*exp(x) + 96*a^7*b*e
xp(x) + 600*a^3*b^5*exp(x) - 416*a^5*b^3*exp(x))/(a^6*b^4) + (((16*(a^2 - b
^2)*(4*a*b^2 - 4*a^3 + 8*b^3*exp(x) - 7*a^2*b*exp(x)))/a^6 - (32*(-(a + b)^
3*(a - b)^3)^(1/2)*(3*a*b^2 - 2*a^3 + 4*b^3*exp(x) - 3*a^2*b*exp(x)))/(a^6*
b))*(-(a + b)^3*(a - b)^3)^(1/2))/(a*b^3))*(-(a + b)^3*(a - b)^3)^(1/2))/(a
*b^3) - (8*(a^2 - b^2)^2*(2*a^2 - 3*b^2)*(6*a*b^2 - 4*a^3 + 10*b^3*exp(x) -
 7*a^2*b*exp(x)))/(a^6*b^6))*(-(a + b)^3*(a - b)^3)^(1/2))/(a*b^3)
```

3.117 $\int \frac{\tanh^3(x)}{a+b\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=35

$$\frac{\log(\cosh(x))}{a}+\frac{\left(1-\frac{a^2}{b^2}\right)\log(a+b\operatorname{sech}(x))}{a}+\frac{\operatorname{sech}(x)}{b}$$

[Out] ln(cosh(x))/a+(1-a^2/b^2)*ln(a+b*sech(x))/a+sech(x)/b

Rubi [A]
time = 0.05, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3970, 908}

$$\frac{\left(1-\frac{a^2}{b^2}\right)\log(a+b\operatorname{sech}(x))}{a}+\frac{\log(\cosh(x))}{a}+\frac{\operatorname{sech}(x)}{b}$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]^3/(a + b*Sech[x]),x]

[Out] Log[Cosh[x]]/a + ((1 - a^2/b^2)*Log[a + b*Sech[x]])/a + Sech[x]/b

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\tanh^3(x)}{a+b\text{sech}(x)}\,dx = -\frac{\text{Subst}\left(\int \frac{b^2-x^2}{x(a+x)}\,dx, x, b\text{sech}(x)\right)}{b^2}$$
$$= -\frac{\text{Subst}\left(\int \left(-1+\frac{b^2}{ax}+\frac{a^2-b^2}{a(a+x)}\right)\,dx, x, b\text{sech}(x)\right)}{b^2}$$
$$= \frac{\log(\cosh(x))}{a} + \frac{\left(1-\frac{a^2}{b^2}\right)\log(a+b\text{sech}(x))}{a} + \frac{\text{sech}(x)}{b}$$

Mathematica [A]
time = 0.06, size = 37, normalized size = 1.06

$$\frac{a^2\log(\cosh(x)) + (-a^2+b^2)\log(b+a\cosh(x)) + ab\text{sech}(x)}{ab^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(a + b*Sech[x]),x]
```

```
[Out] (a^2*Log[Cosh[x]] + (-a^2 + b^2)*Log[b + a*Cosh[x]] + a*b*Sech[x])/(a*b^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(35) = 70$.
time = 0.68, size = 92, normalized size = 2.63

method	result
risch	$-\frac{x}{a}+\frac{2\,e^{x}}{b(1+e^{2x})}+\frac{a\ln(1+e^{2x})}{b^2}-\frac{a\ln\left(e^{2x}+\frac{2b\,e^{x}}{a}+1\right)}{b^2}+\frac{\ln\left(e^{2x}+\frac{2b\,e^{x}}{a}+1\right)}{a}$
default	$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}-\frac{(a-b)(a+b)\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-b\left(\tanh^2\left(\frac{x}{2}\right)\right)+a+b\right)}{a\,b^2}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}+\frac{\frac{2b}{\tanh^2\left(\frac{x}{2}\right)+1}+a\ln\left(\tanh^2\left(\frac{x}{2}\right)+1\right)}{b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a*ln(tanh(1/2*x)+1)-(a-b)*(a+b)/a/b^2*ln(a*tanh(1/2*x)^2-b*tanh(1/2*x)^2
+a+b)-1/a*ln(tanh(1/2*x)-1)+1/b^2*(2*b/(tanh(1/2*x)^2+1)+a*ln(tanh(1/2*x)^2
+1))
```

Maxima [A]
time = 0.49, size = 67, normalized size = 1.91

$$\frac{x}{a} + \frac{2\,e^{(-x)}}{be^{(-2\,x)}+b} + \frac{a\log\left(e^{(-2\,x)}+1\right)}{b^2} - \frac{(a^2-b^2)\log\left(2\,be^{(-x)}+ae^{(-2\,x)}+a\right)}{ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] x/a + 2*e^(-x)/(b*e^(-2*x) + b) + a*log(e^(-2*x) + 1)/b^2 - (a^2 - b^2)*log
(2*b*e^(-x) + a*e^(-2*x) + a)/(a*b^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(35) = 70$.
time = 0.38, size = 200, normalized size = 5.71

$$-\frac{b^2x\cosh(x)^2 + b^2x\sinh(x)^2 + b^2x - 2ab\cosh(x) + \left((a^2-b^2)\cosh(x)^2 + 2(a^2-b^2)\cosh(x)\sinh(x) + (a^2-b^2)\sinh(x)^2 + a^2 - b^2\right)\log\left(\frac{2(a\cosh(x)+b)}{\cosh(x)-\sinh(x)}\right) - \left(a^2\cosh(x)^2 + 2a^2\cosh(x)\sinh(x) + a^2\sinh(x)^2 + a^2\right)\log\left(\frac{2\cosh(x)}{\cosh(x)-\sinh(x)}\right) + 2\left(b^2x\cosh(x) - ab\right)\sinh(x)}{ab^2\cosh(x)^2 + 2ab^2\cosh(x)\sinh(x) + ab^2\sinh(x)^2 + ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] -(b^2*x*cosh(x)^2 + b^2*x*sinh(x)^2 + b^2*x - 2*a*b*cosh(x) + ((a^2 - b^2)*
cosh(x)^2 + 2*(a^2 - b^2)*cosh(x)*sinh(x) + (a^2 - b^2)*sinh(x)^2 + a^2 - b
^2)*log(2*(a*cosh(x) + b)/(cosh(x) - sinh(x))) - (a^2*cosh(x)^2 + 2*a^2*cos
h(x)*sinh(x) + a^2*sinh(x)^2 + a^2)*log(2*cosh(x)/(cosh(x) - sinh(x))) + 2*
(b^2*x*cosh(x) - a*b)*sinh(x))/(a*b^2*cosh(x)^2 + 2*a*b^2*cosh(x)*sinh(x) +
 a*b^2*sinh(x)^2 + a*b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(x)}{a + b\operatorname{sech}(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(a+b*sech(x)),x)
```

```
[Out] Integral(tanh(x)**3/(a + b*sech(x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(35) = 70$.
time = 0.38, size = 73, normalized size = 2.09

$$\frac{a\log\left(e^{(-x)} + e^x\right)}{b^2} - \frac{(a^2-b^2)\log\left(\left|a\left(e^{(-x)} + e^x\right) + 2b\right|\right)}{ab^2} - \frac{a\left(e^{(-x)} + e^x\right) - 2b}{b^2\left(e^{(-x)} + e^x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] a*log(e^(-x) + e^x)/b^2 - (a^2 - b^2)*log(abs(a*(e^(-x) + e^x) + 2*b))/(a*b
^2) - (a*(e^(-x) + e^x) - 2*b)/(b^2*(e^(-x) + e^x))
```

Mupad [B]
time = 1.60, size = 260, normalized size = 7.43

$$\frac{2\,e^x}{b+b\,e^{2\,x}}-\frac{x}{a}+\frac{\ln\left(16\,a^5\,e^{2\,x}+4\,a\,b^4+16\,a^5-16\,a^3\,b^2+8\,b^5\,e^x-16\,a^3\,b^2\,e^{2\,x}+32\,a^4\,b\,e^x+4\,a\,b^4\,e^{2\,x}-32\,a^2\,b^3\,e^x\right)}{a}-\frac{a\,\ln\left(16\,a^5\,e^{2\,x}+4\,a\,b^4+16\,a^5-16\,a^3\,b^2+8\,b^5\,e^x-16\,a^3\,b^2\,e^{2\,x}+32\,a^4\,b\,e^x+4\,a\,b^4\,e^{2\,x}-32\,a^2\,b^3\,e^x\right)}{b^2}+\frac{a\,\ln\left(16\,a^6\,e^{2\,x}-4\,b^6\,e^{2\,x}+16\,a^6-4\,b^6+20\,a^2\,b^4-32\,a^4\,b^2+20\,a^2\,b^4\,e^{2\,x}-32\,a^4\,b^2\,e^{2\,x}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a + b/cosh(x)),x)
```

```
[Out] (2*exp(x))/(b + b*exp(2*x)) - x/a + log(16*a^5*exp(2*x) + 4*a*b^4 + 16*a^5
- 16*a^3*b^2 + 8*b^5*exp(x) - 16*a^3*b^2*exp(2*x) + 32*a^4*b*exp(x) + 4*a*b
^4*exp(2*x) - 32*a^2*b^3*exp(x))/a - (a*log(16*a^5*exp(2*x) + 4*a*b^4 + 16*
a^5 - 16*a^3*b^2 + 8*b^5*exp(x) - 16*a^3*b^2*exp(2*x) + 32*a^4*b*exp(x) + 4
*a*b^4*exp(2*x) - 32*a^2*b^3*exp(x)))/b^2 + (a*log(16*a^6*exp(2*x) - 4*b^6*
exp(2*x) + 16*a^6 - 4*b^6 + 20*a^2*b^4 - 32*a^4*b^2 + 20*a^2*b^4*exp(2*x) -
 32*a^4*b^2*exp(2*x)))/b^2
```

3.118 $\int \frac{\tanh^2(x)}{a+b\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=62

$$\frac{x}{a}-\frac{\text{ArcTan}(\sinh(x))}{b}+\frac{2\sqrt{a-b}\,\sqrt{a+b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{ab}$$

```
[Out] x/a-arctan(sinh(x))/b+2*arctan((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))*(a-b)^(
1/2)*(a+b)^(1/2)/a/b
```

Rubi [A]
time = 0.12, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3979, 4136, 3855, 4004, 3916, 2738, 211}

$$\frac{2\sqrt{a-b}\,\sqrt{a+b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{ab}+\frac{x}{a}-\frac{\text{ArcTan}(\sinh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(a + b*Sech[x]),x]
```

```
[Out] x/a - ArcTan[Sinh[x]]/b + (2*Sqrt[a - b]*Sqrt[a + b]*ArcTan[(Sqrt[a - b]*Ta
nh[x/2])/Sqrt[a + b]])/(a*b)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
```

```
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3979

```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4136

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)), x_Symbol] :> Dist[C/b, Int[Csc[e + f*x], x], x] + Dist[1/b, Int[
(A*b - a*C*Csc[e + f*x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 A, C}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^2(x)}{a+b\operatorname{sech}(x)}\,dx &= -\int \frac{-1+\operatorname{sech}^2(x)}{a+b\operatorname{sech}(x)}\,dx\\
&= -\frac{\int \operatorname{sech}(x)\,dx}{b}-\frac{\int \frac{-b-a\operatorname{sech}(x)}{a+b\operatorname{sech}(x)}\,dx}{b}\\
&= \frac{x}{a}-\frac{\tan^{-1}(\sinh(x))}{b}+\left(\frac{a}{b}-\frac{b}{a}\right)\int \frac{\operatorname{sech}(x)}{a+b\operatorname{sech}(x)}\,dx\\
&= \frac{x}{a}-\frac{\tan^{-1}(\sinh(x))}{b}+\frac{\left(\frac{a}{b}-\frac{b}{a}\right)\int \frac{1}{1+\frac{a\cosh(x)}{b}}\,dx}{b}\\
&= \frac{x}{a}-\frac{\tan^{-1}(\sinh(x))}{b}+\frac{\left(2\left(\frac{a}{b}-\frac{b}{a}\right)\right)\operatorname{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\left(1-\frac{a}{b}\right)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b}\\
&= \frac{x}{a}-\frac{\tan^{-1}(\sinh(x))}{b}+\frac{2\sqrt{a-b}\,\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{ab}
\end{aligned}
$$

Mathematica [A]

time = 0.06, size = 62, normalized size = 1.00

$$\frac{bx - 2a\text{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right) - 2\sqrt{a^2-b^2}\,\text{ArcTan}\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(a + b*Sech[x]),x]
```

```
[Out] (b*x - 2*a*ArcTan[Tanh[x/2]] - 2*Sqrt[a^2 - b^2]*ArcTan[((-a + b)*Tanh[x/2]
)/Sqrt[a^2 - b^2]])/(a*b)
```

Maple [A]
time = 0.67, size = 84, normalized size = 1.35

method	result
default	$-\frac{2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{b}+\frac{2(a+b)(a-b)\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{ab\sqrt{(a+b)(a-b)}}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}$
risch	$\frac{x}{a}+\frac{\sqrt{-a^2+b^2}\,\ln\left(e^x+\frac{b+\sqrt{-a^2+b^2}}{a}\right)}{ba}-\frac{\sqrt{-a^2+b^2}\,\ln\left(e^x-\frac{\sqrt{-a^2+b^2}-b}{a}\right)}{ba}+\frac{i\ln(e^x-i)}{b}-\frac{i\ln(e^x+i}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/b*arctan(tanh(1/2*x))+2/a*(a+b)*(a-b)/b/((a+b)*(a-b))^(1/2)*arctan((a-b)
*tanh(1/2*x)/((a+b)*(a-b))^(1/2))+1/a*ln(tanh(1/2*x)+1)-1/a*ln(tanh(1/2*x)-
1)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.39, size = 193, normalized size = 3.11

$$\left[\frac{bx - 2\,a\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right) + \sqrt{-a^2+b^2}\,\log\left(\frac{a^2\cosh(x)^2+a^2\sinh(x)^2+2\,ab\cosh(x)-a^2+2\,b^2+2\,\left(a^2\cosh(x)+ab\right)\sinh(x)+2\,\sqrt{-a^2+b^2}\,\left(a\cosh(x)+a\sinh(x)+b\right)}{a\cosh(x)^2+a\sinh(x)^2+2\,b\cosh(x)+2\,\left(a\cosh(x)+b\right)\sinh(x)+a}\right)}{ab}, \frac{bx - 2\,a\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right) - 2\,\sqrt{a^2-b^2}\,\arctan\left(-\frac{a\cosh(x)+a\sinh(x)+b}{\sqrt{a^2-b^2}}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [(b*x - 2*a*arctan(cosh(x) + sinh(x)) + sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2
 + a^2*sinh(x)^2 + 2*a*b*cosh(x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh
(x) + 2*sqrt(-a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh
(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) + a)))/(a*b), (b*x - 2*a*ar
ctan(cosh(x) + sinh(x)) - 2*sqrt(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x)
+ b)/sqrt(a^2 - b^2)))/(a*b)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(x)}{a + b\,\mathrm{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(a+b*sech(x)),x)
```

```
[Out] Integral(tanh(x)**2/(a + b*sech(x)), x)
```

Giac [A]
time = 0.41, size = 52, normalized size = 0.84

$$\frac{x}{a} - \frac{2\,\arctan\left(e^{x}\right)}{b} + \frac{2\,\sqrt{a^2-b^2}\,\arctan\left(\frac{ae^{x}+b}{\sqrt{a^2-b^2}}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] x/a - 2*arctan(e^x)/b + 2*sqrt(a^2 - b^2)*arctan((a*e^x + b)/sqrt(a^2 - b^2
))/(a*b)
```

Mupad [B]
time = 3.92, size = 273, normalized size = 4.40

$$\frac{\ln\left(\mathrm{e}^{x}-\mathrm{i}\right)\,1\mathrm{i}-\ln\left(\mathrm{e}^{x}+1\mathrm{i}\right)\,1\mathrm{i}}{b}+\frac{\ln\left(2\,a\,b^3-2\,a^3\,b+a^3\,\sqrt{b^2-a^2}+a^4\,\mathrm{e}^{x}+4\,b^4\,\mathrm{e}^{x}-2\,a\,b^2\,\sqrt{b^2-a^2}-4\,b^3\,\mathrm{e}^{x}\,\sqrt{b^2-a^2}-5\,a^2\,b^2\,\mathrm{e}^{x}+3\,a^2\,b\,\mathrm{e}^{x}\,\sqrt{b^2-a^2}\right)\,\sqrt{b^2-a^2}-\ln\left(2\,a\,b^3-2\,a^3\,b-a^3\,\sqrt{b^2-a^2}+a^4\,\mathrm{e}^{x}+4\,b^4\,\mathrm{e}^{x}+2\,a\,b^2\,\sqrt{b^2-a^2}+4\,b^3\,\mathrm{e}^{x}\,\sqrt{b^2-a^2}-5\,a^2\,b^2\,\mathrm{e}^{x}-3\,a^2\,b\,\mathrm{e}^{x}\,\sqrt{b^2-a^2}\right)\,\sqrt{b^2-a^2}+b\,x}{a\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a + b/cosh(x)),x)
```

```
[Out] (log(exp(x) - 1i)*1i - log(exp(x) + 1i)*1i)/b + (log(2*a*b^3 - 2*a^3*b + a^
3*(b^2 - a^2)^(1/2) + a^4*exp(x) + 4*b^4*exp(x) - 2*a*b^2*(b^2 - a^2)^(1/2)
 - 4*b^3*exp(x)*(b^2 - a^2)^(1/2) - 5*a^2*b^2*exp(x) + 3*a^2*b*exp(x)*(b^2
- a^2)^(1/2))*(b^2 - a^2)^(1/2) - log(2*a*b^3 - 2*a^3*b - a^3*(b^2 - a^2)^(
1/2) + a^4*exp(x) + 4*b^4*exp(x) + 2*a*b^2*(b^2 - a^2)^(1/2) + 4*b^3*exp(x)
*(b^2 - a^2)^(1/2) - 5*a^2*b^2*exp(x) - 3*a^2*b*exp(x)*(b^2 - a^2)^(1/2))*(
b^2 - a^2)^(1/2) + b*x)/(a*b)
```

3.119 $\int \frac{\tanh(x)}{a+b\text{sech}(x)}\,dx$

Optimal. Leaf size=19

$$\frac{\log(\cosh(x))}{a}+\frac{\log(a+b\text{sech}(x))}{a}$$

```
[Out] ln(cosh(x))/a+ln(a+b*sech(x))/a
```

Rubi [A]
time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.364$, Rules used = {3970, 36, 29, 31}

$$\frac{\log(a+b\text{sech}(x))}{a}+\frac{\log(\cosh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(a + b*Sech[x]),x]
```

```
[Out] Log[Cosh[x]]/a + Log[a + b*Sech[x]]/a
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\tanh(x)}{a+b\text{sech}(x)}\,dx = -\text{Subst}\left(\int \frac{1}{x(a+x)}\,dx, x, b\text{sech}(x)\right)$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, b\text{sech}(x)\right)}{a} + \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\text{sech}(x)\right)}{a}$$
$$= \frac{\log(\cosh(x))}{a} + \frac{\log(a+b\text{sech}(x))}{a}$$

Mathematica [A]
time = 0.01, size = 11, normalized size = 0.58

$$\frac{\log(b+a\cosh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(a + b*Sech[x]),x]
```

```
[Out] Log[b + a*Cosh[x]]/a
```

Maple [A]
time = 0.54, size = 21, normalized size = 1.11

method	result	size
derivativedivides	$\frac{\ln(a+b\,\text{sech}(x))}{a}-\frac{\ln(\text{sech}(x))}{a}$	21
default	$\frac{\ln(a+b\,\text{sech}(x))}{a}-\frac{\ln(\text{sech}(x))}{a}$	21
risch	$-\frac{x}{a}+\frac{\ln\left(e^{2x}+\frac{2b\,e^{x}}{a}+1\right)}{a}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(a+b*sech(x))/a-1/a*ln(sech(x))
```

Maxima [A]
time = 0.27, size = 26, normalized size = 1.37

$$\frac{x}{a} + \frac{\log\left(2\,be^{(-x)} + ae^{(-2\,x)} + a\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] x/a + log(2*b*e^(-x) + a*e^(-2*x) + a)/a
```

Fricas [A]
time = 0.37, size = 27, normalized size = 1.42

$$-\frac{x-\log\left(\frac{2\,(a\cosh(x)+b)}{\cosh(x)-\sinh(x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)),x, algorithm="fricas")
[Out] -(x - log(2*(a*cosh(x) + b)/(cosh(x) - sinh(x))))/a
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(15) = 30$.
time = 0.25, size = 41, normalized size = 2.16

$$\begin{cases} \frac{\tilde{\infty}}{\operatorname{sech}(x)} & \text{for } a = 0 \wedge b = 0 \\ \frac{1}{b\operatorname{sech}(x)} & \text{for } a = 0 \\ \frac{x-\log(\tanh(x)+1)}{a} & \text{for } b = 0 \\ \frac{x}{a} + \frac{\log\left(\frac{a}{b}+\operatorname{sech}(x)\right)}{a} - \frac{\log(\tanh(x)+1)}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)),x)
[Out] Piecewise((zoo/sech(x), Eq(a, 0) & Eq(b, 0)), (1/(b*sech(x)), Eq(a, 0)), ((
x - log(tanh(x) + 1))/a, Eq(b, 0)), (x/a + log(a/b + sech(x))/a - log(tanh(
x) + 1)/a, True))
```

Giac [A]
time = 0.40, size = 19, normalized size = 1.00

$$\frac{\log\left(\left|a\left(e^{(-x)}+e^{x}\right)+2\,b\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)),x, algorithm="giac")
[Out] log(abs(a*(e^(-x) + e^x) + 2*b))/a
```

Mupad [B]
time = 0.11, size = 23, normalized size = 1.21

$$-\frac{x-\ln\left(a+2\,b\,\mathrm{e}^{x}+a\,\mathrm{e}^{2\,x}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a + b/cosh(x)),x)
[Out] -(x - log(a + 2*b*exp(x) + a*exp(2*x)))/a
```

3.120 $\int \frac{\coth(x)}{a+b\,\text{sech}(x)}\,dx$

Optimal. Leaf size=66

$$\frac{\log(\cosh(x))}{a}+\frac{\log(1-\text{sech}(x))}{2(a+b)}+\frac{\log(1+\text{sech}(x))}{2(a-b)}-\frac{b^2\log(a+b\text{sech}(x))}{a\,(a^2-b^2)}$$

```
[Out] ln(cosh(x))/a+1/2*ln(1-sech(x))/(a+b)+1/2*ln(1+sech(x))/(a-b)-b^2*ln(a+b*se
ch(x))/a/(a^2-b^2)
```

Rubi [A]
time = 0.07, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.182$, Rules used = {3970, 908}

$$-\frac{b^2\log(a+b\text{sech}(x))}{a\,(a^2-b^2)}+\frac{\log(1-\text{sech}(x))}{2(a+b)}+\frac{\log(\text{sech}(x)+1)}{2(a-b)}+\frac{\log(\cosh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(a + b*Sech[x]),x]
```

```
[Out] Log[Cosh[x]]/a + Log[1 - Sech[x]]/(2*(a + b)) + Log[1 + Sech[x]]/(2*(a - b)
) - (b^2*Log[a + b*Sech[x]])/(a*(a^2 - b^2))
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\coth(x)}{a+b\text{sech}(x)}\,dx = -\left(b^2\text{Subst}\left(\int \frac{1}{x(a+x)\,(b^2-x^2)}\,dx, x, b\text{sech}(x)\right)\right)$$
$$= -\left(b^2\text{Subst}\left(\int \left(\frac{1}{2b^2(a+b)(b-x)} + \frac{1}{ab^2x} + \frac{1}{a(a-b)(a+b)(a+x)} - \frac{1}{2(a-b)b^2(b\right.\right.\right.$$
$$= \frac{\log(\cosh(x))}{a} + \frac{\log(1-\text{sech}(x))}{2(a+b)} + \frac{\log(1+\text{sech}(x))}{2(a-b)} - \frac{b^2\log(a+b\text{sech}(x))}{a\,(a^2-b^2)}$$

Mathematica [A]
time = 0.06, size = 44, normalized size = 0.67

$$-\frac{b^2\log(b+a\cosh(x)) - a^2\log(\sinh(x)) + ab\log\left(\tanh\left(\frac{x}{2}\right)\right)}{a^3-ab^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(a + b*Sech[x]),x]
[Out] -((b^2*Log[b + a*Cosh[x]] - a^2*Log[Sinh[x]] + a*b*Log[Tanh[x/2]])/(a^3 - a
*b^2))
```

Maple [A]
time = 0.91, size = 78, normalized size = 1.18

method	result	size
default	$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}-\frac{b^2\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-b\left(\tanh^2\left(\frac{x}{2}\right)\right)+a+b\right)}{a(a+b)(a-b)}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a+b}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}$	78
risch	$\frac{x}{a}-\frac{x}{a+b}-\frac{x}{a-b}+\frac{2b^2x}{(a^2-b^2)a}+\frac{\ln(e^x-1)}{a+b}+\frac{\ln(e^x+1)}{a-b}-\frac{b^2\ln\left(e^{2x}+\frac{2b\,e^x}{a}+1\right)}{a(a^2-b^2)}$	103

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*sech(x)),x,method=_RETURNVERBOSE)
[Out] -1/a*ln(tanh(1/2*x)-1)-b^2/a/(a+b)/(a-b)*ln(a*tanh(1/2*x)^2-b*tanh(1/2*x)^2
+a+b)+1/(a+b)*ln(tanh(1/2*x))-1/a*ln(tanh(1/2*x)+1)
```

Maxima [A]
time = 0.26, size = 67, normalized size = 1.02

$$-\frac{b^2\log\left(2\,be^{(-x)}+ae^{(-2\,x)}+a\right)}{a^3-ab^2}+\frac{x}{a}+\frac{\log\left(e^{(-x)}+1\right)}{a-b}+\frac{\log\left(e^{(-x)}-1\right)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] -b^2*log(2*b*e^(-x) + a*e^(-2*x) + a)/(a^3 - a*b^2) + x/a + log(e^(-x) + 1)
/(a - b) + log(e^(-x) - 1)/(a + b)
```

Fricas [A]
time = 0.38, size = 81, normalized size = 1.23

$$-\frac{b^2 \log\left(\frac{2\,(a\cosh(x)+b)}{\cosh(x)-\sinh(x)}\right)+(a^2-b^2)x-(a^2+ab)\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)-(a^2-ab)\log\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)}{a^3-ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] -(b^2*log(2*(a*cosh(x) + b)/(cosh(x) - sinh(x))) + (a^2 - b^2)*x - (a^2 + a
*b)*log(cosh(x) + sinh(x) + 1) - (a^2 - a*b)*log(cosh(x) + sinh(x) - 1))/(a
^3 - a*b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(x\right)}{a+b\,\mathrm{sech}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)),x)
```

```
[Out] Integral(coth(x)/(a + b*sech(x)), x)
```

Giac [A]
time = 0.39, size = 67, normalized size = 1.02

$$-\frac{b^2\log\left(\left|a\left(e^{(-x)}+e^x\right)+2\,b\right|\right)}{a^3-ab^2}+\frac{\log\left(e^{(-x)}+e^x+2\right)}{2\,(a-b)}+\frac{\log\left(e^{(-x)}+e^x-2\right)}{2\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] -b^2*log(abs(a*(e^(-x) + e^x) + 2*b))/(a^3 - a*b^2) + 1/2*log(e^(-x) + e^x
+ 2)/(a - b) + 1/2*log(e^(-x) + e^x - 2)/(a + b)
```

Mupad [B]
time = 1.72, size = 271, normalized size = 4.11

$$\frac{\ln\left(64\,a\,b^4+32\,a^4\,b+32\,b^5+96\,a^2\,b^3+64\,a^3\,b^2+32\,b^5\,\mathrm{e}^x+64\,a\,b^4\,\mathrm{e}^x+32\,a^4\,b\,\mathrm{e}^x+96\,a^2\,b^3\,\mathrm{e}^x+64\,a^3\,b^2\,\mathrm{e}^x\right)}{a-b}-\frac{x}{a}+\frac{\ln\left(64\,a\,b^4-32\,a^4\,b-32\,b^5-96\,a^2\,b^3+64\,a^3\,b^2+32\,b^5\,\mathrm{e}^x-64\,a\,b^4\,\mathrm{e}^x+32\,a^4\,b\,\mathrm{e}^x+96\,a^2\,b^3\,\mathrm{e}^x-64\,a^3\,b^2\,\mathrm{e}^x\right)}{a+b}+\frac{b^2\,\ln\left(4\,a^5\,\mathrm{e}^{2\,x}+4\,a\,b^4+4\,a^5+4\,a^3\,b^2+8\,b^5\,\mathrm{e}^x+4\,a^3\,b^2\,\mathrm{e}^{2\,x}+8\,a^4\,b\,\mathrm{e}^x+4\,a\,b^4\,\mathrm{e}^{2\,x}+8\,a^2\,b^3\,\mathrm{e}^x\right)}{a\,b^2-a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a + b/cosh(x)),x)
```

```
[Out] log(64*a*b^4 + 32*a^4*b + 32*b^5 + 96*a^2*b^3 + 64*a^3*b^2 + 32*b^5*exp(x)
+ 64*a*b^4*exp(x) + 32*a^4*b*exp(x) + 96*a^2*b^3*exp(x) + 64*a^3*b^2*exp(x)
)/(a - b) - x/a + log(64*a*b^4 - 32*a^4*b - 32*b^5 - 96*a^2*b^3 + 64*a^3*b^
2 + 32*b^5*exp(x) - 64*a*b^4*exp(x) + 32*a^4*b*exp(x) + 96*a^2*b^3*exp(x) -
 64*a^3*b^2*exp(x))/(a + b) + (b^2*log(4*a^5*exp(2*x) + 4*a*b^4 + 4*a^5 + 4
*a^3*b^2 + 8*b^5*exp(x) + 4*a^3*b^2*exp(2*x) + 8*a^4*b*exp(x) + 4*a*b^4*exp
(2*x) + 8*a^2*b^3*exp(x)))/(a*b^2 - a^3)
```

3.121 $\int \frac{\coth^2(x)}{a+b\operatorname{sech}(x)}\,dx$

Optimal. Leaf size=114

$$\frac{ax}{a^2-b^2}-\frac{b^2x}{a\left(a^2-b^2\right)}+\frac{2b^3\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a(a-b)^{3/2}(a+b)^{3/2}}-\frac{a\coth(x)}{a^2-b^2}+\frac{b\operatorname{csch}(x)}{a^2-b^2}$$

```
[Out] a*x/(a^2-b^2)-b^2*x/a/(a^2-b^2)+2*b^3*arctan((a-b)^(1/2)*tanh(1/2*x)/(a+b)^
(1/2))/a/(a-b)^(3/2)/(a+b)^(3/2)-a*coth(x)/(a^2-b^2)+b*csch(x)/(a^2-b^2)
```

Rubi [A]
time = 0.15, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {3983, 2981, 2686, 8, 3554, 2814, 2738, 211}

$$-\frac{b^2x}{a\left(a^2-b^2\right)}+\frac{ax}{a^2-b^2}-\frac{a\coth(x)}{a^2-b^2}+\frac{b\operatorname{csch}(x)}{a^2-b^2}+\frac{2b^3\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(a + b*Sech[x]),x]
```

```
[Out] (a*x)/(a^2 - b^2) - (b^2*x)/(a*(a^2 - b^2)) + (2*b^3*ArcTan[(Sqrt[a - b]*Ta
nh[x/2])/Sqrt[a + b]])/(a*(a - b)^(3/2)*(a + b)^(3/2)) - (a*Coth[x])/(a^2 -
 b^2) + (b*Csch[x])/(a^2 - b^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2981

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a*(d^2/(a^2
- b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[b
*(d/(a^2 - b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[a^2*(d^2/(g^2*(a^2 - b^2))), Int[(g*Cos[e + f*x])^(p + 2)*((d*Sin[e +
 f*x])^(n - 2)/(a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3983

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
 + n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\coth^2(x)}{a+b\operatorname{sech}(x)}\,dx &= \int \frac{\cosh(x)\coth^2(x)}{b+a\cosh(x)}\,dx \\
&= \frac{a\int \coth^2(x)\,dx}{a^2-b^2} - \frac{b\int \coth(x)\operatorname{csch}(x)\,dx}{a^2-b^2} - \frac{b^2\int \frac{\cosh(x)}{b+a\cosh(x)}\,dx}{a^2-b^2} \\
&= -\frac{b^2x}{a(a^2-b^2)} - \frac{a\coth(x)}{a^2-b^2} + \frac{a\int 1\,dx}{a^2-b^2} + \frac{(ib)\text{Subst}(\int 1\,dx, x, -i\operatorname{csch}(x))}{a^2-b^2} + \frac{b^3\int \frac{1}{b+a\cosh(x}}{a(a^2-b^{}} \\
&= \frac{ax}{a^2-b^2} - \frac{b^2x}{a(a^2-b^2)} - \frac{a\coth(x)}{a^2-b^2} + \frac{b\operatorname{csch}(x)}{a^2-b^2} + \frac{(2b^3)\,\text{Subst}\left(\int \frac{1}{a+b-(-a+b)x^2}\,dx, x, \tanh\right.}{a(a^2-b^2)} \\
&= \frac{ax}{a^2-b^2} - \frac{b^2x}{a(a^2-b^2)} + \frac{2b^3\tan^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a(a-b)^{3/2}(a+b)^{3/2}} - \frac{a\coth(x)}{a^2-b^2} + \frac{b\operatorname{csch}(x)}{a^2-b^2}
\end{aligned}
$$

Mathematica [A]
time = 0.24, size = 81, normalized size = 0.71

$$
\frac{a^2x - b^2x + \frac{2b^3\operatorname{ArcTan}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - a^2\coth(x) + ab\operatorname{csch}(x)}{a^3-ab^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(a + b*Sech[x]),x]
```

```
[Out] (a^2*x - b^2*x + (2*b^3*ArcTan[((a - b)*Tanh[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a
^2 - b^2] - a^2*Coth[x] + a*b*Csch[x])/(a^3 - a*b^2)
```

Maple [A]
time = 0.88, size = 104, normalized size = 0.91

method	result	size
default	$-\frac{\tanh\left(\frac{x}{2}\right)}{2(a-b)} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a} + \frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a} + \frac{2b^3\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a-b)a(a+b)\sqrt{(a+b)(a-b)}} - \frac{1}{2(a+b)\tanh\left(\frac{x}{2}\right)}$	104
risch	$\frac{x}{a} - \frac{2(-e^x b+a)}{(e^{2x}-1)(a^2-b^2)} - \frac{b^3\ln\left(e^x + \frac{b\sqrt{-a^2+b^2}-a^2+b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)a} + \frac{b^3\ln\left(e^x + \frac{b\sqrt{-a^2+b^2}+a^2-b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)a}$	178

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(a-b)*tanh(1/2*x)-1/a*ln(tanh(1/2*x)-1)+1/a*ln(tanh(1/2*x)+1)+2/(a-b)/
a/(a+b)*b^3/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2
))-1/2/(a+b)/tanh(1/2*x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(104) = 208$.
time = 0.38, size = 646, normalized size = 5.67

$$\left[\frac{2a^4 - 2a^2b^2 - (a^4 - 2a^2b^2 + b^4)x\cosh(x)^2 - (a^4 - 2a^2b^2 + b^4)x\sinh(x)^2 - (b^3\cosh(x)^2 + 2b^3\cosh(x)\sinh(x) + b^3\sinh(x)^2 - b^3)\sqrt{-a^2+b^2}\log\left(\frac{a^2\cosh(x)^2 + a^2\sinh(x)^2 + 2ab\cosh(x) - a^2 + 2b^2 + 2(a^2\cosh(x) + ab)\sinh(x) + 2\sqrt{-a^2+b^2}(a\cosh(x) + a\sinh(x) + b)}{a\cosh(x)^2 + a\sinh(x)^2 + 2b\cosh(x) + 2(a\cosh(x) + b)\sinh(x) + a}\right) + (a^4 - 2a^2b^2 + b^4)x - 2(a^3b - ab^3)\cosh(x) - 2(a^3b - ab^3 + (a^4 - 2a^2b^2 + b^4)x\cosh(x))\sinh(x)}{a^5 - 2a^3b^2 + ab^4 - (a^5 - 2a^3b^2 + ab^4)\cosh(x)^2 - 2(a^5 - 2a^3b^2 + ab^4)\cosh(x)\sinh(x) - (a^5 - 2a^3b^2 + ab^4)\sinh(x)^2}, \frac{2a^4 - 2a^2b^2 - (a^4 - 2a^2b^2 + b^4)x\cosh(x)^2 - (a^4 - 2a^2b^2 + b^4)x\sinh(x)^2 + 2(b^3\cosh(x)^2 + 2b^3\cosh(x)\sinh(x) + b^3\sinh(x)^2 - b^3)\sqrt{a^2-b^2}\arctan\left(-\frac{a\cosh(x) + a\sinh(x) + b}{\sqrt{a^2-b^2}}\right) + (a^4 - 2a^2b^2 + b^4)x - 2(a^3b - ab^3)\cosh(x) - 2(a^3b - ab^3 + (a^4 - 2a^2b^2 + b^4)x\cosh(x))\sinh(x)}{a^5 - 2a^3b^2 + ab^4 - (a^5 - 2a^3b^2 + ab^4)\cosh(x)^2 - 2(a^5 - 2a^3b^2 + ab^4)\cosh(x)\sinh(x) - (a^5 - 2a^3b^2 + ab^4)\sinh(x)^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [(2*a^4 - 2*a^2*b^2 - (a^4 - 2*a^2*b^2 + b^4)*x*cosh(x)^2 - (a^4 - 2*a^2*b^
2 + b^4)*x*sinh(x)^2 - (b^3*cosh(x)^2 + 2*b^3*cosh(x)*sinh(x) + b^3*sinh(x)
^2 - b^3)*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(
x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-a^2 + b^2)*(a*co
sh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cos
h(x) + b)*sinh(x) + a)) + (a^4 - 2*a^2*b^2 + b^4)*x - 2*(a^3*b - a*b^3)*cos
h(x) - 2*(a^3*b - a*b^3 + (a^4 - 2*a^2*b^2 + b^4)*x*cosh(x))*sinh(x))/(a^5
- 2*a^3*b^2 + a*b^4 - (a^5 - 2*a^3*b^2 + a*b^4)*cosh(x)^2 - 2*(a^5 - 2*a^3*
b^2 + a*b^4)*cosh(x)*sinh(x) - (a^5 - 2*a^3*b^2 + a*b^4)*sinh(x)^2), (2*a^4
 - 2*a^2*b^2 - (a^4 - 2*a^2*b^2 + b^4)*x*cosh(x)^2 - (a^4 - 2*a^2*b^2 + b^4
)*x*sinh(x)^2 + 2*(b^3*cosh(x)^2 + 2*b^3*cosh(x)*sinh(x) + b^3*sinh(x)^2 -
b^3)*sqrt(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^2 - b^2)) +
 (a^4 - 2*a^2*b^2 + b^4)*x - 2*(a^3*b - a*b^3)*cosh(x) - 2*(a^3*b - a*b^3 +
 (a^4 - 2*a^2*b^2 + b^4)*x*cosh(x))*sinh(x))/(a^5 - 2*a^3*b^2 + a*b^4 - (a^
5 - 2*a^3*b^2 + a*b^4)*cosh(x)^2 - 2*(a^5 - 2*a^3*b^2 + a*b^4)*cosh(x)*sinh
(x) - (a^5 - 2*a^3*b^2 + a*b^4)*sinh(x)^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(x)}{a + b\operatorname{sech}(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(a+b*sech(x)),x)
```

```
[Out] Integral(coth(x)**2/(a + b*sech(x)), x)
```

Giac [A]
time = 0.40, size = 82, normalized size = 0.72

$$\frac{2\,b^3\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{(a^3-ab^2)\sqrt{a^2-b^2}}+\frac{x}{a}+\frac{2\,(be^x-a)}{(a^2-b^2)(e^{(2\,x)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] 2*b^3*arctan((a*e^x + b)/sqrt(a^2 - b^2))/((a^3 - a*b^2)*sqrt(a^2 - b^2)) +
 x/a + 2*(b*e^x - a)/((a^2 - b^2)*(e^(2*x) - 1))
```

Mupad [B]
time = 1.67, size = 383, normalized size = 3.36

$$\frac{x}{a}-\frac{\frac{2\,a}{a^2-b^2}-\frac{2\,b\,\mathrm{e}^x}{a^2-b^2}}{\mathrm{e}^{2\,x}-1}-\frac{2\,\mathrm{atan}\left(\left(\left(\mathrm{e}^x\left(\frac{2\,b^3}{a^3\,(a\,b^2-a^3)\,(a^2-b^2)\,\sqrt{b^6}}-\frac{2\,\left(a\,b^3\,\sqrt{b^6}-a^3\,b\,\sqrt{b^6}\right)}{a^2\,b^2\,(a\,b^2-a^3)\,\sqrt{a^2\,(a^2-b^2)^3}\,\sqrt{a^8-3\,a^6\,b^2+3\,a^4\,b^4-a^2\,b^6}}\right)+\frac{2\,\left(a^4\,\sqrt{b^6}-a^2\,b^2\,\sqrt{b^6}\right)}{a^2\,b^2\,(a\,b^2-a^3)\,\sqrt{a^2\,(a^2-b^2)^3}\,\sqrt{a^8-3\,a^6\,b^2+3\,a^4\,b^4-a^2\,b^6}}\right)\left(\frac{a^4\,\sqrt{a^8-3\,a^6\,b^2+3\,a^4\,b^4-a^2\,b^6}}{2}-\frac{a^2\,b^2\,\sqrt{a^8-3\,a^6\,b^2+3\,a^4\,b^4-a^2\,b^6}}{2}\right)\right)\sqrt{b^6}}{\sqrt{a^8-3\,a^6\,b^2+3\,a^4\,b^4-a^2\,b^6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a + b/cosh(x)),x)
```

```
[Out] x/a - ((2*a)/(a^2 - b^2) - (2*b*exp(x))/(a^2 - b^2))/(exp(2*x) - 1) - (2*at
an((exp(x)*((2*b^3)/(a^3*(a*b^2 - a^3)*(a^2 - b^2)*(b^6)^(1/2)) - (2*(a*b^3
*(b^6)^(1/2) - a^3*b*(b^6)^(1/2)))/(a^2*b^2*(a*b^2 - a^3)*(a^2*(a^2 - b^2)^
3)^(1/2)*(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)^(1/2))) + (2*(a^4*(b^6)^(1
/2) - a^2*b^2*(b^6)^(1/2)))/(a^2*b^2*(a*b^2 - a^3)*(a^2*(a^2 - b^2)^3)^(1/2
)*(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)^(1/2)))*((a^4*(a^8 - a^2*b^6 + 3*
a^4*b^4 - 3*a^6*b^2)^(1/2))/2 - (a^2*b^2*(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6
*b^2)^(1/2))/2))*(b^6)^(1/2))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)^(1/2)
```

3.122 $\int \frac{\coth^3(x)}{a+b\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=113

$$\frac{\log(\cosh(x))}{a}+\frac{(2a+3b)\log(1-\operatorname{sech}(x))}{4(a+b)^2}+\frac{(2a-3b)\log(1+\operatorname{sech}(x))}{4(a-b)^2}+\frac{b^4\log(a+b\operatorname{sech}(x))}{a\,(a^2-b^2)^2}-\frac{1}{4(a+b)(1-$$

```
[Out] ln(cosh(x))/a+1/4*(2*a+3*b)*ln(1-sech(x))/(a+b)^2+1/4*(2*a-3*b)*ln(1+sech(x
))/(a-b)^2+b^4*ln(a+b*sech(x))/a/(a^2-b^2)^2-1/4/(a+b)/(1-sech(x))-1/4/(a-b
)/(1+sech(x))
```

Rubi [A]
time = 0.13, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3970, 908}

$$\frac{b^4\log(a+b\operatorname{sech}(x))}{a\,(a^2-b^2)^2}-\frac{1}{4(a+b)(1-\operatorname{sech}(x))}-\frac{1}{4(a-b)(\operatorname{sech}(x)+1)}+\frac{(2a+3b)\log(1-\operatorname{sech}(x))}{4(a+b)^2}+\frac{(2a-3b)\log(\operatorname{sech}(x)+1)}{4(a-b)^2}+\frac{\log(\cosh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/(a + b*Sech[x]),x]
```

```
[Out] Log[Cosh[x]]/a + ((2*a + 3*b)*Log[1 - Sech[x]])/(4*(a + b)^2) + ((2*a - 3*b
)*Log[1 + Sech[x]])/(4*(a - b)^2) + (b^4*Log[a + b*Sech[x]])/(a*(a^2 - b^2)
^2) - 1/(4*(a + b)*(1 - Sech[x])) - 1/(4*(a - b)*(1 + Sech[x]))
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\coth^3(x)}{a+b\text{sech}(x)}\,dx = -\left(b^4\text{Subst}\left(\int \frac{1}{x(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\text{sech}(x)\right)\right)$$
$$= -\left(b^4\text{Subst}\left(\int \left(\frac{1}{4b^3(a+b)(b-x)^2} + \frac{2a+3b}{4b^4(a+b)^2(b-x)} + \frac{1}{ab^4x} - \frac{1}{a(a-b)^2(a+b)^2(\right.\right.\right.$$
$$= \frac{\log(\cosh(x))}{a} + \frac{(2a+3b)\log(1-\text{sech}(x))}{4(a+b)^2} + \frac{(2a-3b)\log(1+\text{sech}(x))}{4(a-b)^2} + \frac{b^4\log(a+b}{a\left(a^2-\right.}$$

Mathematica [A]
time = 0.21, size = 112, normalized size = 0.99

$$\frac{-a(a-b)^2(a+b)\text{csch}^2\left(\frac{x}{2}\right)+8b^4\log(b+a\cosh(x))+4a\left(2a\left(a^2-2b^2\right)\log(\sinh(x))+b\left(-a^2+3b^2\right)\log\left(\tanh\left(\frac{x}{2}\right)\right)\right)+a(a-b)(a+b)^2\text{sech}^2\left(\frac{x}{2}\right)}{8a(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/(a + b*Sech[x]),x]
```

```
[Out] (-(a*(a - b)^2*(a + b)*Csch[x/2]^2) + 8*b^4*Log[b + a*Cosh[x]] + 4*a*(2*a*(
a^2 - 2*b^2)*Log[Sinh[x]] + b*(-a^2 + 3*b^2)*Log[Tanh[x/2]]) + a*(a - b)*(a
 + b)^2*Sech[x/2]^2)/(8*a*(a - b)^2*(a + b)^2)
```

Maple [A]
time = 0.94, size = 113, normalized size = 1.00

method	result
default	$-\frac{\tanh^2\left(\frac{x}{2}\right)}{8(a-b)}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}+\frac{b^4\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-b\left(\tanh^2\left(\frac{x}{2}\right)\right)+a+b\right)}{(a-b)^2(a+b)^2a}-\frac{1}{8(a+b)\tanh\left(\frac{x}{2}\right)^2}+\frac{(4a+6b)}{4(}$
risch	$\frac{x}{a}-\frac{xa}{a^2+2ab+b^2}-\frac{3xb}{2(a^2+2ab+b^2)}-\frac{xa}{a^2-2ab+b^2}+\frac{3xb}{2(a^2-2ab+b^2)}-\frac{2x\,b^4}{a(a^4-2a^2b^2+b^4)}-\frac{e^x\left(-b\,e^{2x}+2a\,e^x-b\right)}{\left(e^{2x}-1\right)^2(a^2-b^2)}+\frac{a\ln(e^x-}{a^2+2ab+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*tanh(1/2*x)^2/(a-b)-1/a*ln(tanh(1/2*x)+1)-1/a*ln(tanh(1/2*x)-1)+1/(a-b
)^2*b^4/(a+b)^2/a*ln(a*tanh(1/2*x)^2-b*tanh(1/2*x)^2+a+b)-1/8/(a+b)/tanh(1/
2*x)^2+1/4/(a+b)^2*(4*a+6*b)*ln(tanh(1/2*x))
```

Maxima [A]
time = 0.27, size = 164, normalized size = 1.45

$$\frac{b^4\log\left(2\,be^{(-x)}+ae^{(-2\,x)}+a\right)}{a^5-2\,a^3b^2+ab^4}+\frac{(2\,a-3\,b)\log\left(e^{(-x)}+1\right)}{2\,(a^2-2\,ab+b^2)}+\frac{(2\,a+3\,b)\log\left(e^{(-x)}-1\right)}{2\,(a^2+2\,ab+b^2)}+\frac{be^{(-x)}-2\,ae^{(-2\,x)}+be^{(-3\,x)}}{a^2-b^2-2\,(a^2-b^2)e^{(-2\,x)}+(a^2-b^2)e^{(-4\,x)}}+\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] b^4*log(2*b*e^(-x) + a*e^(-2*x) + a)/(a^5 - 2*a^3*b^2 + a*b^4) + 1/2*(2*a -
 3*b)*log(e^(-x) + 1)/(a^2 - 2*a*b + b^2) + 1/2*(2*a + 3*b)*log(e^(-x) - 1)
/(a^2 + 2*a*b + b^2) + (b*e^(-x) - 2*a*e^(-2*x) + b*e^(-3*x))/(a^2 - b^2 -
2*(a^2 - b^2)*e^(-2*x) + (a^2 - b^2)*e^(-4*x)) + x/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1222 vs. $2(103) = 206$.
time = 0.42, size = 1222, normalized size = 10.81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^4 - 2*a^2*b^2 + b^4)*x*cosh(x)^4 + 2*(a^4 - 2*a^2*b^2 + b^4)*x*s
inh(x)^4 - 2*(a^3*b - a*b^3)*cosh(x)^3 - 2*(a^3*b - a*b^3 - 4*(a^4 - 2*a^2*
b^2 + b^4)*x*cosh(x))*sinh(x)^3 + 4*(a^4 - a^2*b^2 - (a^4 - 2*a^2*b^2 + b^4
)*x)*cosh(x)^2 + 2*(2*a^4 - 2*a^2*b^2 + 6*(a^4 - 2*a^2*b^2 + b^4)*x*cosh(x)
^2 - 2*(a^4 - 2*a^2*b^2 + b^4)*x - 3*(a^3*b - a*b^3)*cosh(x))*sinh(x)^2 + 2
*(a^4 - 2*a^2*b^2 + b^4)*x - 2*(a^3*b - a*b^3)*cosh(x) - 2*(b^4*cosh(x)^4 +
 4*b^4*cosh(x)*sinh(x)^3 + b^4*sinh(x)^4 - 2*b^4*cosh(x)^2 + b^4 + 2*(3*b^4
*cosh(x)^2 - b^4)*sinh(x)^2 + 4*(b^4*cosh(x)^3 - b^4*cosh(x))*sinh(x))*log(
2*(a*cosh(x) + b)/(cosh(x) - sinh(x))) - ((2*a^4 + a^3*b - 4*a^2*b^2 - 3*a*
b^3)*cosh(x)^4 + 4*(2*a^4 + a^3*b - 4*a^2*b^2 - 3*a*b^3)*cosh(x)*sinh(x)^3
+ (2*a^4 + a^3*b - 4*a^2*b^2 - 3*a*b^3)*sinh(x)^4 + 2*a^4 + a^3*b - 4*a^2*b
^2 - 3*a*b^3 - 2*(2*a^4 + a^3*b - 4*a^2*b^2 - 3*a*b^3)*cosh(x)^2 - 2*(2*a^4
 + a^3*b - 4*a^2*b^2 - 3*a*b^3 - 3*(2*a^4 + a^3*b - 4*a^2*b^2 - 3*a*b^3)*co
sh(x)^2)*sinh(x)^2 + 4*((2*a^4 + a^3*b - 4*a^2*b^2 - 3*a*b^3)*cosh(x)^3 - (
2*a^4 + a^3*b - 4*a^2*b^2 - 3*a*b^3)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x
) + 1) - ((2*a^4 - a^3*b - 4*a^2*b^2 + 3*a*b^3)*cosh(x)^4 + 4*(2*a^4 - a^3*
b - 4*a^2*b^2 + 3*a*b^3)*cosh(x)*sinh(x)^3 + (2*a^4 - a^3*b - 4*a^2*b^2 + 3
*a*b^3)*sinh(x)^4 + 2*a^4 - a^3*b - 4*a^2*b^2 + 3*a*b^3 - 2*(2*a^4 - a^3*b
- 4*a^2*b^2 + 3*a*b^3)*cosh(x)^2 - 2*(2*a^4 - a^3*b - 4*a^2*b^2 + 3*a*b^3 -
 3*(2*a^4 - a^3*b - 4*a^2*b^2 + 3*a*b^3)*cosh(x)^2)*sinh(x)^2 + 4*((2*a^4 -
 a^3*b - 4*a^2*b^2 + 3*a*b^3)*cosh(x)^3 - (2*a^4 - a^3*b - 4*a^2*b^2 + 3*a*
b^3)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) + 2*(4*(a^4 - 2*a^2*b^2 +
 b^4)*x*cosh(x)^3 - a^3*b + a*b^3 - 3*(a^3*b - a*b^3)*cosh(x)^2 + 4*(a^4 -
a^2*b^2 - (a^4 - 2*a^2*b^2 + b^4)*x)*cosh(x))*sinh(x))/(a^5 - 2*a^3*b^2 + a
*b^4 + (a^5 - 2*a^3*b^2 + a*b^4)*cosh(x)^4 + 4*(a^5 - 2*a^3*b^2 + a*b^4)*co
sh(x)*sinh(x)^3 + (a^5 - 2*a^3*b^2 + a*b^4)*sinh(x)^4 - 2*(a^5 - 2*a^3*b^2
+ a*b^4)*cosh(x)^2 - 2*(a^5 - 2*a^3*b^2 + a*b^4 - 3*(a^5 - 2*a^3*b^2 + a*b^
4)*cosh(x)^2)*sinh(x)^2 + 4*((a^5 - 2*a^3*b^2 + a*b^4)*cosh(x)^3 - (a^5 - 2
*a^3*b^2 + a*b^4)*cosh(x))*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(x)}{a + b\,\mathrm{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(a+b*sech(x)),x)
```

```
[Out] Integral(coth(x)**3/(a + b*sech(x)), x)
```

Giac [A]
time = 0.40, size = 193, normalized size = 1.71

$$\frac{b^4\log\left(\left|a\left(e^{(-x)}+e^x\right)+2\,b\right|\right)}{a^5-2\,a^3b^2+ab^4}+\frac{(2\,a-3\,b)\log\left(e^{(-x)}+e^x+2\right)}{4\,(a^2-2\,ab+b^2)}+\frac{(2\,a+3\,b)\log\left(e^{(-x)}+e^x-2\right)}{4\,(a^2+2\,ab+b^2)}-\frac{a^3\left(e^{(-x)}+e^x\right)^2-2\,ab^2\left(e^{(-x)}+e^x\right)^2-2\,a^2b\left(e^{(-x)}+e^x\right)+2\,b^3\left(e^{(-x)}+e^x\right)+4\,ab^2}{2\,(a^4-2\,a^2b^2+b^4)\left(\left(e^{(-x)}+e^x\right)^2-4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] b^4*log(abs(a*(e^(-x) + e^x) + 2*b))/(a^5 - 2*a^3*b^2 + a*b^4) + 1/4*(2*a -
 3*b)*log(e^(-x) + e^x + 2)/(a^2 - 2*a*b + b^2) + 1/4*(2*a + 3*b)*log(e^(-x
) + e^x - 2)/(a^2 + 2*a*b + b^2) - 1/2*(a^3*(e^(-x) + e^x)^2 - 2*a*b^2*(e^(
-x) + e^x)^2 - 2*a^2*b*(e^(-x) + e^x) + 2*b^3*(e^(-x) + e^x) + 4*a*b^2)/((a
^4 - 2*a^2*b^2 + b^4)*((e^(-x) + e^x)^2 - 4))
```

Mupad [B]
time = 2.22, size = 339, normalized size = 3.00

$$\frac{\ln\left(e^x-1\right)\,(2\,a+3\,b)}{2\,a^2+4\,a\,b+2\,b^2}-\frac{x}{a}-\frac{\frac{2\,a}{a^2-b^2}-\frac{2\,b\,e^x}{a^2-b^2}}{e^{4\,x}-2\,e^{2\,x}+1}-\frac{\frac{2\,(a^4-a^2\,b^2)}{a\,(a^2-b^2)^2}-\frac{e^x\,(a^2\,b-b^3)}{(a^2-b^2)^2}}{e^{2\,x}-1}+\frac{\ln\left(e^x+1\right)\,(2\,a-3\,b)}{2\,a^2-4\,a\,b+2\,b^2}+\frac{b^4\,\ln\left(4\,a^9\,e^{2\,x}+4\,a\,b^8+4\,a^9+7\,a^3\,b^6+14\,a^5\,b^4-17\,a^7\,b^2+8\,b^9\,e^x+7\,a^3\,b^6\,e^{2\,x}+14\,a^5\,b^4\,e^{2\,x}-17\,a^7\,b^2\,e^{2\,x}+8\,a^8\,b\,e^x+4\,a\,b^8\,e^{2\,x}+14\,a^2\,b^7\,e^x+28\,a^4\,b^5\,e^x-34\,a^6\,b^3\,e^x\right)}{a^5-2\,a^3\,b^2+a\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a + b/cosh(x)),x)
```

```
[Out] (log(exp(x) - 1)*(2*a + 3*b))/(4*a*b + 2*a^2 + 2*b^2) - x/a - ((2*a)/(a^2 -
 b^2) - (2*b*exp(x))/(a^2 - b^2))/(exp(4*x) - 2*exp(2*x) + 1) - ((2*(a^4 -
a^2*b^2))/(a*(a^2 - b^2)^2) - (exp(x)*(a^2*b - b^3))/(a^2 - b^2)^2)/(exp(2*
x) - 1) + (log(exp(x) + 1)*(2*a - 3*b))/(2*a^2 - 4*a*b + 2*b^2) + (b^4*log(
4*a^9*exp(2*x) + 4*a*b^8 + 4*a^9 + 7*a^3*b^6 + 14*a^5*b^4 - 17*a^7*b^2 + 8*
b^9*exp(x) + 7*a^3*b^6*exp(2*x) + 14*a^5*b^4*exp(2*x) - 17*a^7*b^2*exp(2*x)
 + 8*a^8*b*exp(x) + 4*a*b^8*exp(2*x) + 14*a^2*b^7*exp(x) + 28*a^4*b^5*exp(x
) - 34*a^6*b^3*exp(x)))/(a*b^4 + a^5 - 2*a^3*b^2)
```

3.123 $\int \frac{\coth^4(x)}{a+b\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=207

$$-\frac{ab^2x}{(a^2-b^2)^2}+\frac{b^4x}{a\,(a^2-b^2)^2}+\frac{ax}{a^2-b^2}-\frac{2b^5\,\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a(a-b)^{5/2}(a+b)^{5/2}}+\frac{ab^2\coth(x)}{(a^2-b^2)^2}-\frac{a\coth(x)}{a^2-b^2}-\frac{a\coth^3(x)}{3\,(a^2-b^2)}-$$

```
[Out] -a*b^2*x/(a^2-b^2)^2+b^4*x/a/(a^2-b^2)^2+a*x/(a^2-b^2)-2*b^5*arctan((a-b)^(
1/2)*tanh(1/2*x)/(a+b)^(1/2))/a/(a-b)^(5/2)/(a+b)^(5/2)+a*b^2*coth(x)/(a^2-
b^2)^2-a*coth(x)/(a^2-b^2)-1/3*a*coth(x)^3/(a^2-b^2)-b^3*csch(x)/(a^2-b^2)^
2+b*csch(x)/(a^2-b^2)+1/3*b*csch(x)^3/(a^2-b^2)
```

Rubi [A]
time = 0.24, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {3983, 2981, 2686, 3554, 8, 2814, 2738, 211}

$$-\frac{ab^2x}{(a^2-b^2)^2}+\frac{ax}{a^2-b^2}-\frac{a\coth^3(x)}{3\,(a^2-b^2)}+\frac{ab^2\coth(x)}{(a^2-b^2)^2}-\frac{a\coth(x)}{a^2-b^2}+\frac{b\,\mathrm{csch}^3(x)}{3\,(a^2-b^2)}+\frac{b\,\mathrm{csch}(x)}{a^2-b^2}+\frac{b^4x}{a\,(a^2-b^2)^2}-\frac{b^3\mathrm{csch}(x)}{(a^2-b^2)^2}-\frac{2b^5\,\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^4/(a + b*Sech[x]),x]
```

```
[Out] -((a*b^2*x)/(a^2 - b^2)^2) + (b^4*x)/(a*(a^2 - b^2)^2) + (a*x)/(a^2 - b^2)
- (2*b^5*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(a*(a - b)^(5/2)*(a +
 b)^(5/2)) + (a*b^2*Coth[x])/(a^2 - b^2)^2 - (a*Coth[x])/(a^2 - b^2) - (a*C
oth[x]^3)/(3*(a^2 - b^2)) - (b^3*Csch[x])/(a^2 - b^2)^2 + (b*Csch[x])/(a^2
- b^2) + (b*Csch[x]^3)/(3*(a^2 - b^2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2981

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a*(d^2/(a^2
- b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[b
*(d/(a^2 - b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[a^2*(d^2/(g^2*(a^2 - b^2))), Int[(g*Cos[e + f*x])^(p + 2)*((d*Sin[e +
 f*x])^(n - 2)/(a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3983

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
 + n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$\int \frac{\coth^4(x)}{a+b\text{sech}(x)}\,dx = \int \frac{\cosh(x)\coth^4(x)}{b+a\cosh(x)}\,dx$$

$$= \frac{a\int \coth^4(x)\,dx}{a^2-b^2} - \frac{b\int \coth^3(x)\text{csch}(x)\,dx}{a^2-b^2} - \frac{b^2\int \frac{\cosh(x)\coth^2(x)}{b+a\cosh(x)}\,dx}{a^2-b^2}$$

$$= -\frac{a\coth^3(x)}{3(a^2-b^2)} - \frac{(ab^2)\int \coth^2(x)\,dx}{(a^2-b^2)^2} + \frac{b^3\int \coth(x)\text{csch}(x)\,dx}{(a^2-b^2)^2} + \frac{b^4\int \frac{\cosh(x)}{b+a\cosh(x)}\,dx}{(a^2-b^2)^2} + \frac{a}{}$$

$$= \frac{b^4x}{a(a^2-b^2)^2} + \frac{ab^2\coth(x)}{(a^2-b^2)^2} - \frac{a\coth(x)}{a^2-b^2} - \frac{a\coth^3(x)}{3(a^2-b^2)} + \frac{b\text{csch}(x)}{a^2-b^2} + \frac{b\text{csch}^3(x)}{3(a^2-b^2)} - \frac{(ab}{(a}$$

$$= -\frac{ab^2x}{(a^2-b^2)^2} + \frac{b^4x}{a(a^2-b^2)^2} + \frac{ax}{a^2-b^2} + \frac{ab^2\coth(x)}{(a^2-b^2)^2} - \frac{a\coth(x)}{a^2-b^2} - \frac{a\coth^3(x)}{3(a^2-b^2)} - \frac{b^3\text{c}}{(a^2}$$

$$= -\frac{ab^2x}{(a^2-b^2)^2} + \frac{b^4x}{a(a^2-b^2)^2} + \frac{ax}{a^2-b^2} - \frac{2b^5\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a(a-b)^{5/2}(a+b)^{5/2}} + \frac{ab^2\coth(x)}{(a^2-b^2)^2}$$

Mathematica [A]

time = 0.51, size = 166, normalized size = 0.80

$$\frac{(b+a\cosh(x))\text{sech}(x)\left(\frac{24x}{a} + \frac{48b^5\text{ArcTan}\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a(a^2-b^2)^{5/2}} - \frac{2(8a+11b)\coth\left(\frac{x}{2}\right)}{(a+b)^2} + \frac{8\text{csch}^3(x)\sinh^4\left(\frac{x}{2}\right)}{a-b} - \frac{\text{csch}^4\left(\frac{x}{2}\right)\sinh(x)}{2(a+b)} - \frac{16a\tanh\left(\frac{x}{2}\right)}{(a-b)^2} + \frac{22b\tanh\left(\frac{x}{2}\right)}{(a-b)^2}\right)}{24(a+b\text{sech}(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4/(a + b*Sech[x]),x]
```

```
[Out] ((b + a*Cosh[x])*Sech[x]*((24*x)/a + (48*b^5*ArcTan[((-a + b)*Tanh[x/2])/Sq
rt[a^2 - b^2]])/(a*(a^2 - b^2)^(5/2)) - (2*(8*a + 11*b)*Coth[x/2])/(a + b)^
2 + (8*Csch[x]^3*Sinh[x/2]^4)/(a - b) - (Csch[x/2]^4*Sinh[x])/(2*(a + b)) -
 (16*a*Tanh[x/2])/(a - b)^2 + (22*b*Tanh[x/2])/(a - b)^2))/(24*(a + b*Sech[
x]))
```

Maple [A]

time = 0.92, size = 153, normalized size = 0.74

method	result
default	$-\frac{\frac{a\left(\tanh^3\left(\frac{x}{2}\right)\right)}{3} - \frac{b\left(\tanh^3\left(\frac{x}{2}\right)\right)}{3} + 5a\tanh\left(\frac{x}{2}\right) - 7b\tanh\left(\frac{x}{2}\right)}{8(a-b)^2} + \frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a} - \frac{1}{24(a+b)\tanh\left(\frac{x}{2}\right)^3} - \frac{}{8(a+}$

risch	$\frac{x}{a}-\frac{2(-3a^2b\,\mathrm{e}^{5x}+6b^3\mathrm{e}^{5x}+6a^3\mathrm{e}^{4x}-9a\,b^2\mathrm{e}^{4x}+2a^2b\,\mathrm{e}^{3x}-8b^3\mathrm{e}^{3x}-6a^3\mathrm{e}^{2x}+12a\,b^2\mathrm{e}^{2x}-3a^2b\,\mathrm{e}^{x}+6b^3\mathrm{e}^{x}+4a^3-7a\,b^2)}{3(a^4-2a^2b^2+b^4)(\mathrm{e}^{2x}-1)^3}-\frac{b^5\ln\left(\mathrm{e}^x+\frac{b\sqrt{}}{}\right.}{\sqrt{-a^2+}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)^4/(a+b*sech(x)),x,method=_RETURNVERBOSE)

[Out] -1/8/(a-b)^2*(1/3*a*tanh(1/2*x)^3-1/3*b*tanh(1/2*x)^3+5*a*tanh(1/2*x)-7*b*tanh(1/2*x))+1/a*ln(tanh(1/2*x)+1)-1/a*ln(tanh(1/2*x)-1)-1/24/(a+b)/tanh(1/2*x)^3-1/8*(5*a+7*b)/(a+b)^2/tanh(1/2*x)-2/(a-b)^2/(a+b)^2/a*b^5/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^4/(a+b*sech(x)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for more de

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1725 vs. $2(193) = 386$.
time = 0.43, size = 3530, normalized size = 17.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^4/(a+b*sech(x)),x, algorithm="fricas")

[Out] [-1/3*(3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^6 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*sinh(x)^6 - 8*a^6 + 22*a^4*b^2 - 14*a^2*b^4 + 6*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cosh(x)^5 + 6*(a^5*b - 3*a^3*b^3 + 2*a*b^5 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x))*sinh(x)^5 - 3*(4*a^6 - 10*a^4*b^2 + 6*a^2*b^4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x)^4 - 3*(4*a^6 - 10*a^4*b^2 + 6*a^2*b^4 - 15*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^2 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x - 10*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cosh(x))*sinh(x)^4 - 4*(a^5*b - 5*a^3*b^3 + 4*a*b^5)*cosh(x)^3 - 4*(a^5*b - 5*a^3*b^3 + 4*a*b^5 - 15*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^3 - 15*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cosh(x)^2 + 3*(4*a^6 - 10*a^4*b^2 + 6*a^2*b^4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x))*sinh(x)

```
^3 + 3*(4*a^6 - 12*a^4*b^2 + 8*a^2*b^4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b
^6)*x)*cosh(x)^2 + 3*(4*a^6 - 12*a^4*b^2 + 8*a^2*b^4 + 15*(a^6 - 3*a^4*b^2
+ 3*a^2*b^4 - b^6)*x*cosh(x)^4 + 20*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cosh(x)^3
 - 6*(4*a^6 - 10*a^4*b^2 + 6*a^2*b^4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6
)*x)*cosh(x)^2 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x - 4*(a^5*b - 5*a^3
*b^3 + 4*a*b^5)*cosh(x))*sinh(x)^2 - 3*(b^5*cosh(x)^6 + 6*b^5*cosh(x)*sinh(
x)^5 + b^5*sinh(x)^6 - 3*b^5*cosh(x)^4 + 3*b^5*cosh(x)^2 - b^5 + 3*(5*b^5*c
osh(x)^2 - b^5)*sinh(x)^4 + 4*(5*b^5*cosh(x)^3 - 3*b^5*cosh(x))*sinh(x)^3 +
 3*(5*b^5*cosh(x)^4 - 6*b^5*cosh(x)^2 + b^5)*sinh(x)^2 + 6*(b^5*cosh(x)^5 -
 2*b^5*cosh(x)^3 + b^5*cosh(x))*sinh(x))*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^
2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sin
h(x) + 2*sqrt(-a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sin
h(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) + a)) - 3*(a^6 - 3*a^4*b^2
 + 3*a^2*b^4 - b^6)*x + 6*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cosh(x) + 6*(3*(a^6
 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^5 + a^5*b - 3*a^3*b^3 + 2*a*b^5 +
 5*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cosh(x)^4 - 2*(4*a^6 - 10*a^4*b^2 + 6*a^2*
b^4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x)^3 - 2*(a^5*b - 5*a^3
*b^3 + 4*a*b^5)*cosh(x)^2 + (4*a^6 - 12*a^4*b^2 + 8*a^2*b^4 + 3*(a^6 - 3*a^
4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x))*sinh(x))/(a^7 - 3*a^5*b^2 + 3*a^3*b^4
- a*b^6 - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^6 - 6*(a^7 - 3*a^5*
b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)*sinh(x)^5 - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 -
 a*b^6)*sinh(x)^6 + 3*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^4 + 3*(
a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6 - 5*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^
6)*cosh(x)^2)*sinh(x)^4 - 4*(5*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x
)^3 - 3*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x))*sinh(x)^3 - 3*(a^7 -
 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^2 - 3*(a^7 - 3*a^5*b^2 + 3*a^3*b^4
- a*b^6 + 5*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^4 - 6*(a^7 - 3*a^
5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^2)*sinh(x)^2 - 6*((a^7 - 3*a^5*b^2 + 3*a
^3*b^4 - a*b^6)*cosh(x)^5 - 2*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)
^3 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x))*sinh(x)), -1/3*(3*(a^6
- 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^6 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4
 - b^6)*x*sinh(x)^6 - 8*a^6 + 22*a^4*b^2 - 14*a^2*b^4 + 6*(a^5*b - 3*a^3*b^
3 + 2*a*b^5)*cosh(x)^5 + 6*(a^5*b - 3*a^3*b^3 + 2*a*b^5 + 3*(a^6 - 3*a^4*b^
2 + 3*a^2*b^4 - b^6)*x*cosh(x))*sinh(x)^5 - 3*(4*a^6 - 10*a^4*b^2 + 6*a^2*b
^4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x)^4 - 3*(4*a^6 - 10*a^4
*b^2 + 6*a^2*b^4 - 15*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^2 + 3*(
a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x - 10*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cos
h(x))*sinh(x)^4 - 4*(a^5*b - 5*a^3*b^3 + 4*a*b^5)*cosh(x)^3 - 4*(a^5*b - 5*
a^3*b^3 + 4*a*b^5 - 15*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^3 - 15
*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cosh(x)^2 + 3*(4*a^6 - 10*a^4*b^2 + 6*a^2*b^
4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x))*sinh(x)^3 + 3*(4*a^6
- 12*a^4*b^2 + 8*a^2*b^4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x)
^2 + 3*(4*a^6 - 12*a^4*b^2 + 8*a^2*b^4 + 15*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 -
b^6)*x*cosh(x)^4 + 20*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cosh(x)^3 - 6*(4*a^6 -
10*a^4*b^2 + 6*a^2*b^4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x)^2
```

```
 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x - 4*(a^5*b - 5*a^3*b^3 + 4*a*b^5
)*cosh(x))*sinh(x)^2 + 6*(b^5*cosh(x)^6 + 6*b^5*cosh(x)*sinh(x)^5 + b^5*sin
h(x)^6 - 3*b^5*cosh(x)^4 + 3*b^5*cosh(x)^2 - b^5 + 3*(5*b^5*cosh(x)^2 - b^5
)*sinh(x)^4 + 4*(5*b^5*cosh(x)^3 - 3*b^5*cosh(x))*sinh(x)^3 + 3*(5*b^5*cosh
(x)^4 - 6*b^5*cosh(x)^2 + b^5)*sinh(x)^2 + 6*(b^5*cosh(x)^5 - 2*b^5*cosh(x)
^3 + b^5*cosh(x))*sinh(x))*sqrt(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) +
 b)/sqrt(a^2 - b^2)) - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x + 6*(a^5*b -
 3*a^3*b^3 + 2*a*b^5)*cosh(x) + 6*(3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*
cosh(x)^5 + a^5*b - 3*a^3*b^3 + 2*a*b^5 + 5*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*c
osh(x)^4 - 2*(4*a^6 - 10*a^4*b^2 + 6*a^2*b^4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b
^4 - b^6)*x)*cosh(x)^3 - 2*(a^5*b - 5*a^3*b^3 +...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^4(x)}{a + b\,\mathrm{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4/(a+b*sech(x)),x)
```

```
[Out] Integral(coth(x)**4/(a + b*sech(x)), x)
```

Giac [A]
time = 0.39, size = 190, normalized size = 0.92

$$-\frac{2\,b^5\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{(a^5-2\,a^3b^2+ab^4)\sqrt{a^2-b^2}}+\frac{x}{a}+\frac{2\,\left(3\,a^2be^{(5\,x)}-6\,b^3e^{(5\,x)}-6\,a^3e^{(4\,x)}+9\,ab^2e^{(4\,x)}-2\,a^2be^{(3\,x)}+8\,b^3e^{(3\,x)}+6\,a^3e^{(2\,x)}-12\,ab^2e^{(2\,x)}+3\,a^2be^x-6\,b^3e^x-4\,a^3+7\,ab^2\right)}{3\,(a^4-2\,a^2b^2+b^4)(e^{(2\,x)}-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] -2*b^5*arctan((a*e^x + b)/sqrt(a^2 - b^2))/((a^5 - 2*a^3*b^2 + a*b^4)*sqrt(
a^2 - b^2)) + x/a + 2/3*(3*a^2*b*e^(5*x) - 6*b^3*e^(5*x) - 6*a^3*e^(4*x) +
9*a*b^2*e^(4*x) - 2*a^2*b*e^(3*x) + 8*b^3*e^(3*x) + 6*a^3*e^(2*x) - 12*a*b^
2*e^(2*x) + 3*a^2*b*e^x - 6*b^3*e^x - 4*a^3 + 7*a*b^2)/((a^4 - 2*a^2*b^2 +
b^4)*(e^(2*x) - 1)^3)
```

Mupad [B]
time = 1.83, size = 713, normalized size = 3.44

$$\frac{x}{a}-\frac{\frac{8\,a}{3\,(a^2-b^2)}-\frac{8\,b\,e^x}{3\,(a^2-b^2)}}{3\,e^{2\,x}-3\,e^{4\,x}+e^{6\,x}-1}-\frac{\frac{2\,(2\,a^4-3\,a^2\,b^2)}{a\,{(a^2-b^2)}^2}-\frac{2\,e^x\,(a^2\,b-2\,b^3)}{{(a^2-b^2)}^2}}{e^{2\,x}-1}-\frac{\frac{4\,(a^4-a^2\,b^2)}{a\,{(a^2-b^2)}^2}-\frac{8\,e^x\,(a^2\,b-b^3)}{3\,{(a^2-b^2)}^2}}{e^{4\,x}-2\,e^{2\,x}+1}-\frac{2\,\mathrm{atan}\left(\left(e^x\left(\frac{2\,b^5}{a^3\,{(a^2-b^2)}^2\,\sqrt{b^{10}}\,(a^5-2\,a^3\,b^2+a\,b^4)}+\frac{2\,\left(a\,b^5\,\sqrt{b^{10}}-2\,a^3\,b^3\,\sqrt{b^{10}}+a^5\,b\,\sqrt{b^{10}}\right)}{a^2\,b^4\,\sqrt{a^2\,{(a^2-b^2)}^5}\,(a^5-2\,a^3\,b^2+a\,b^4)\,\sqrt{a^{12}-5\,a^{10}\,b^2+10\,a^8\,b^4-10\,a^6\,b^6+5\,a^4\,b^8-a^2\,b^{10}}}\right)+\frac{2\,\left(a^6\,\sqrt{b^{10}}+a^2\,b^4\,\sqrt{b^{10}}-2\,a^4\,b^2\,\sqrt{b^{10}}\right)}{a^2\,b^4\,\sqrt{a^2\,{(a^2-b^2)}^5}\,(a^5-2\,a^3\,b^2+a\,b^4)\,\sqrt{a^{12}-5\,a^{10}\,b^2+10\,a^8\,b^4-10\,a^6\,b^6+5\,a^4\,b^8-a^2\,b^{10}}}\right)\left(\frac{a^6\,\sqrt{a^{12}-5\,a^{10}\,b^2+10\,a^8\,b^4-10\,a^6\,b^6+5\,a^4\,b^8-a^2\,b^{10}}}{2}+\frac{a^2\,b^4\,\sqrt{a^{12}-5\,a^{10}\,b^2+10\,a^8\,b^4-10\,a^6\,b^6+5\,a^4\,b^8-a^2\,b^{10}}}{2}-a^4\,b^2\,\sqrt{a^{12}-5\,a^{10}\,b^2+10\,a^8\,b^4-10\,a^6\,b^6+5\,a^4\,b^8-a^2\,b^{10}}\right)\right)\sqrt{b^{10}}}{\sqrt{a^{12}-5\,a^{10}\,b^2+10\,a^8\,b^4-10\,a^6\,b^6+5\,a^4\,b^8-a^2\,b^{10}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(a + b/cosh(x)),x)
```

```
[Out] x/a - ((8*a)/(3*(a^2 - b^2)) - (8*b*exp(x))/(3*(a^2 - b^2)))/(3*exp(2*x) -
3*exp(4*x) + exp(6*x) - 1) - ((2*(2*a^4 - 3*a^2*b^2))/(a*(a^2 - b^2)^2) - (
2*exp(x)*(a^2*b - 2*b^3))/(a^2 - b^2)^2)/(exp(2*x) - 1) - ((4*(a^4 - a^2*b^
2))/(a*(a^2 - b^2)^2) - (8*exp(x)*(a^2*b - b^3))/(3*(a^2 - b^2)^2))/(exp(4*
x) - 2*exp(2*x) + 1) - (2*atan((exp(x)*((2*b^5)/(a^3*(a^2 - b^2)^2*(b^10)^(
1/2)*(a*b^4 + a^5 - 2*a^3*b^2)) + (2*(a*b^5*(b^10)^(1/2) - 2*a^3*b^3*(b^10)
^(1/2) + a^5*b*(b^10)^(1/2)))/(a^2*b^4*(a^2*(a^2 - b^2)^5)^(1/2)*(a*b^4 + a
^5 - 2*a^3*b^2)*(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*
a^10*b^2)^(1/2))) + (2*(a^6*(b^10)^(1/2) + a^2*b^4*(b^10)^(1/2) - 2*a^4*b^2
*(b^10)^(1/2)))/(a^2*b^4*(a^2*(a^2 - b^2)^5)^(1/2)*(a*b^4 + a^5 - 2*a^3*b^2
)*(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2)^(1/2
)))*((a^6*(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b
^2)^(1/2))/2 + (a^2*b^4*(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*
b^4 - 5*a^10*b^2)^(1/2))/2 - a^4*b^2*(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*
b^6 + 10*a^8*b^4 - 5*a^10*b^2)^(1/2)))*(b^10)^(1/2))/(a^12 - a^2*b^10 + 5*a
^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2)^(1/2)
```

3.124 $\int \frac{\coth^5(x)}{a+b\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=178

$$\frac{\log(\cosh(x))}{a}+\frac{(8a^2+21ab+15b^2)\log(1-\operatorname{sech}(x))}{16(a+b)^3}+\frac{(8a^2-21ab+15b^2)\log(1+\operatorname{sech}(x))}{16(a-b)^3}-\frac{b^6\log(a+b\operatorname{sech}}{a\,(a^2-b^2)^3}$$

```
[Out] ln(cosh(x))/a+1/16*(8*a^2+21*a*b+15*b^2)*ln(1-sech(x))/(a+b)^3+1/16*(8*a^2-
21*a*b+15*b^2)*ln(1+sech(x))/(a-b)^3-b^6*ln(a+b*sech(x))/a/(a^2-b^2)^3-1/16
/(a+b)/(1-sech(x))^2+1/16*(-5*a-7*b)/(a+b)^2/(1-sech(x))-1/16/(a-b)/(1+sech
(x))^2+1/16*(-5*a+7*b)/(a-b)^2/(1+sech(x))
```

Rubi [A]
time = 0.23, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3970, 908}

$$\frac{(8a^2+21ab+15b^2)\log(1-\operatorname{sech}(x))}{16(a+b)^3}+\frac{(8a^2-21ab+15b^2)\log(\operatorname{sech}(x)+1)}{16(a-b)^3}-\frac{b^6\log(a+b\operatorname{sech}(x))}{a\,(a^2-b^2)^3}-\frac{5a+7b}{16(a+b)^2(1-\operatorname{sech}(x))}-\frac{5a-7b}{16(a-b)^2(\operatorname{sech}(x)+1)}-\frac{1}{16(a+b)(1-\operatorname{sech}(x))^2}-\frac{1}{16(a-b)(\operatorname{sech}(x)+1)^2}+\frac{\log(\cosh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^5/(a + b*Sech[x]),x]
```

```
[Out] Log[Cosh[x]]/a + ((8*a^2 + 21*a*b + 15*b^2)*Log[1 - Sech[x]])/(16*(a + b)^3
) + ((8*a^2 - 21*a*b + 15*b^2)*Log[1 + Sech[x]])/(16*(a - b)^3) - (b^6*Log[
a + b*Sech[x]])/(a*(a^2 - b^2)^3) - 1/(16*(a + b)*(1 - Sech[x])^2) - (5*a +
 7*b)/(16*(a + b)^2*(1 - Sech[x])) - 1/(16*(a - b)*(1 + Sech[x])^2) - (5*a
- 7*b)/(16*(a - b)^2*(1 + Sech[x]))
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\coth^5(x)}{a+b\text{sech}(x)}\,dx = -\left(b^6\text{Subst}\left(\int \frac{1}{x(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\text{sech}(x)\right)\right)$$
$$= -\left(b^6\text{Subst}\left(\int \left(\frac{1}{8b^4(a+b)(b-x)^3} + \frac{5a+7b}{16b^5(a+b)^2(b-x)^2} + \frac{8a^2+21ab+15b^2}{16b^6(a+b)^3(b-x)} + \right.\right.\right.$$
$$= \frac{\log(\cosh(x))}{a} + \frac{(8a^2+21ab+15b^2)\log(1-\text{sech}(x))}{16(a+b)^3} + \frac{(8a^2-21ab+15b^2)\log(1+\text{s}}{16(a-b)^3}$$

Mathematica [**A**]
time = 0.68, size = 167, normalized size = 0.94

$$\frac{1}{64}\left(-\frac{2(7a+9b)\text{csch}^2\left(\frac{x}{2}\right)}{(a+b)^2} - \frac{\text{csch}^4\left(\frac{x}{2}\right)}{a+b} - \frac{8\left(8b^6\log(b+a\cosh(x)) + a\left(-8a(a^4-3a^2b^2+3b^4)\log(\sinh(x)) + b(3a^4-10a^2b^2+15b^4)\log\left(\tanh\left(\frac{x}{2}\right)\right)\right)\right)}{a(a-b)^3(a+b)^3} + \frac{2(7a-9b)\text{sech}^2\left(\frac{x}{2}\right)}{(a-b)^2} - \frac{\text{sech}^4\left(\frac{x}{2}\right)}{a-b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^5/(a + b*Sech[x]),x]
```

```
[Out] ((-2*(7*a + 9*b)*Csch[x/2]^2)/(a + b)^2 - Csch[x/2]^4/(a + b) - (8*(8*b^6*L
og[b + a*Cosh[x]] + a*(-8*a*(a^4 - 3*a^2*b^2 + 3*b^4)*Log[Sinh[x]] + b*(3*a
^4 - 10*a^2*b^2 + 15*b^4)*Log[Tanh[x/2]])))/(a*(a - b)^3*(a + b)^3) + (2*(7
*a - 9*b)*Sech[x/2]^2)/(a - b)^2 - Sech[x/2]^4/(a - b))/64
```

Maple [**A**]
time = 1.26, size = 162, normalized size = 0.91

method	result
default	$-\frac{\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-b\left(\tanh^2\left(\frac{x}{2}\right)\right)+6a-8b\right)^2}{64(a-b)^3} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a} - \frac{1}{64(a+b)\tanh\left(\frac{x}{2}\right)^4} - \frac{6a+8b}{32(a+b)^2\tanh\left(\frac{x}{2}\right)^2} + \frac{(16a^2+42ab+30\ldots}{16(a\ldots}$
risch	$\frac{x}{a} - \frac{x\,a^2}{a^3-3a^2b+3a\,b^2-b^3} + \frac{21xab}{8(a^3-3a^2b+3a\,b^2-b^3)} - \frac{15x\,b^2}{8(a^3-3a^2b+3a\,b^2-b^3)} - \frac{x\,a^2}{a^3+3a^2b+3a\,b^2+b^3} - \frac{21xab}{8(a^3+3a^2b+3a\,b^2+b^3)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^5/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/64*(a*tanh(1/2*x)^2-b*tanh(1/2*x)^2+6*a-8*b)^2/(a-b)^3-1/a*ln(tanh(1/2*x
)-1)-1/64/(a+b)/tanh(1/2*x)^4-1/32*(6*a+8*b)/(a+b)^2/tanh(1/2*x)^2+1/16/(a+
b)^3*(16*a^2+42*a*b+30*b^2)*ln(tanh(1/2*x))-1/a*ln(tanh(1/2*x)+1)-1/(a-b)^3
*b^6/(a+b)^3/a*ln(a*tanh(1/2*x)^2-b*tanh(1/2*x)^2+a+b)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(162) = 324$.
time = 0.30, size = 366, normalized size = 2.06

$$-\frac{b^6\log\left(2\,be^{(-x)}+ae^{(-2\,x)}+a\right)}{a^7-3\,a^5b^2+3\,a^3b^4-ab^6} + \frac{(8\,a^2-21\,ab+15\,b^2)\log\left(e^{(-x)}+1\right)}{8\,(a^3-3\,a^2b+3\,ab^2-b^3)} + \frac{(8\,a^2+21\,ab+15\,b^2)\log\left(e^{(-x)}-1\right)}{8\,(a^3+3\,a^2b+3\,ab^2+b^3)} + \frac{(5\,a^2b-9\,b^3)e^{(-x)} - 8\,(2\,a^3-3\,ab^2)e^{(-2\,x)} + (3\,a^2b+b^3)e^{(-3\,x)} + 16\,(a^3-2\,ab^2)e^{(-4\,x)} + (3\,a^2b+b^3)e^{(-5\,x)} - 8\,(2\,a^3-3\,ab^2)e^{(-6\,x)} + (5\,a^2b-9\,b^3)e^{(-7\,x)}}{4\,(a^4-2\,a^2b^2+b^4-4\,(a^4-2\,a^2b^2+b^4)e^{(-2\,x)}+6\,(a^4-2\,a^2b^2+b^4)e^{(-4\,x)}-4\,(a^4-2\,a^2b^2+b^4)e^{(-6\,x)}+(a^4-2\,a^2b^2+b^4)e^{(-8\,x)})} + \frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] -b^6*log(2*b*e^(-x) + a*e^(-2*x) + a)/(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)
 + 1/8*(8*a^2 - 21*a*b + 15*b^2)*log(e^(-x) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 -
 b^3) + 1/8*(8*a^2 + 21*a*b + 15*b^2)*log(e^(-x) - 1)/(a^3 + 3*a^2*b + 3*a*
b^2 + b^3) + 1/4*((5*a^2*b - 9*b^3)*e^(-x) - 8*(2*a^3 - 3*a*b^2)*e^(-2*x) +
 (3*a^2*b + b^3)*e^(-3*x) + 16*(a^3 - 2*a*b^2)*e^(-4*x) + (3*a^2*b + b^3)*e
^(-5*x) - 8*(2*a^3 - 3*a*b^2)*e^(-6*x) + (5*a^2*b - 9*b^3)*e^(-7*x))/(a^4 -
 2*a^2*b^2 + b^4 - 4*(a^4 - 2*a^2*b^2 + b^4)*e^(-2*x) + 6*(a^4 - 2*a^2*b^2
+ b^4)*e^(-4*x) - 4*(a^4 - 2*a^2*b^2 + b^4)*e^(-6*x) + (a^4 - 2*a^2*b^2 + b
^4)*e^(-8*x)) + x/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5181 vs. $2(162) = 324$.
time = 0.49, size = 5181, normalized size = 29.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/8*(8*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^8 + 8*(a^6 - 3*a^4*b^
2 + 3*a^2*b^4 - b^6)*x*sinh(x)^8 - 2*(5*a^5*b - 14*a^3*b^3 + 9*a*b^5)*cosh(
x)^7 - 2*(5*a^5*b - 14*a^3*b^3 + 9*a*b^5 - 32*(a^6 - 3*a^4*b^2 + 3*a^2*b^4
- b^6)*x*cosh(x))*sinh(x)^7 + 16*(2*a^6 - 5*a^4*b^2 + 3*a^2*b^4 - 2*(a^6 -
3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x)^6 + 2*(16*a^6 - 40*a^4*b^2 + 24*a^2
*b^4 + 112*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^2 - 16*(a^6 - 3*a^
4*b^2 + 3*a^2*b^4 - b^6)*x - 7*(5*a^5*b - 14*a^3*b^3 + 9*a*b^5)*cosh(x))*si
nh(x)^6 - 2*(3*a^5*b - 2*a^3*b^3 - a*b^5)*cosh(x)^5 - 2*(3*a^5*b - 2*a^3*b^
3 - a*b^5 - 224*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^3 + 21*(5*a^5
*b - 14*a^3*b^3 + 9*a*b^5)*cosh(x)^2 - 48*(2*a^6 - 5*a^4*b^2 + 3*a^2*b^4 -
2*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x))*sinh(x)^5 - 16*(2*a^6 - 6
*a^4*b^2 + 4*a^2*b^4 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x)^4 -
 2*(16*a^6 - 48*a^4*b^2 + 32*a^2*b^4 - 280*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b
^6)*x*cosh(x)^4 + 35*(5*a^5*b - 14*a^3*b^3 + 9*a*b^5)*cosh(x)^3 - 120*(2*a^
6 - 5*a^4*b^2 + 3*a^2*b^4 - 2*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x
)^2 - 24*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x + 5*(3*a^5*b - 2*a^3*b^3 - a
*b^5)*cosh(x))*sinh(x)^4 - 2*(3*a^5*b - 2*a^3*b^3 - a*b^5)*cosh(x)^3 + 2*(2
24*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^5 - 3*a^5*b + 2*a^3*b^3 +
a*b^5 - 35*(5*a^5*b - 14*a^3*b^3 + 9*a*b^5)*cosh(x)^4 + 160*(2*a^6 - 5*a^4*
b^2 + 3*a^2*b^4 - 2*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x)^3 - 10*(
3*a^5*b - 2*a^3*b^3 - a*b^5)*cosh(x)^2 - 32*(2*a^6 - 6*a^4*b^2 + 4*a^2*b^4
- 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x))*sinh(x)^3 + 16*(2*a^6 -
 5*a^4*b^2 + 3*a^2*b^4 - 2*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x)^2
```

+ 2*(112*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^6 + 16*a^6 - 40*a^4
*b^2 + 24*a^2*b^4 - 21*(5*a^5*b - 14*a^3*b^3 + 9*a*b^5)*cosh(x)^5 + 120*(2*
a^6 - 5*a^4*b^2 + 3*a^2*b^4 - 2*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh
(x)^4 - 10*(3*a^5*b - 2*a^3*b^3 - a*b^5)*cosh(x)^3 - 48*(2*a^6 - 6*a^4*b^2
+ 4*a^2*b^4 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x)^2 - 16*(a^6
- 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x - 3*(3*a^5*b - 2*a^3*b^3 - a*b^5)*cosh(x))
sinh(x)^2 + 8(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x - 2*(5*a^5*b - 14*a^3*
b^3 + 9*a*b^5)*cosh(x) + 8*(b^6*cosh(x)^8 + 8*b^6*cosh(x)*sinh(x)^7 + b^6*s
inh(x)^8 - 4*b^6*cosh(x)^6 + 6*b^6*cosh(x)^4 - 4*b^6*cosh(x)^2 + 4*(7*b^6*c
osh(x)^2 - b^6)*sinh(x)^6 + b^6 + 8*(7*b^6*cosh(x)^3 - 3*b^6*cosh(x))*sinh(
x)^5 + 2*(35*b^6*cosh(x)^4 - 30*b^6*cosh(x)^2 + 3*b^6)*sinh(x)^4 + 8*(7*b^6
*cosh(x)^5 - 10*b^6*cosh(x)^3 + 3*b^6*cosh(x))*sinh(x)^3 + 4*(7*b^6*cosh(x)
^6 - 15*b^6*cosh(x)^4 + 9*b^6*cosh(x)^2 - b^6)*sinh(x)^2 + 8*(b^6*cosh(x)^7
- 3*b^6*cosh(x)^5 + 3*b^6*cosh(x)^3 - b^6*cosh(x))*sinh(x))*log(2*(a*cosh(
x) + b)/(cosh(x) - sinh(x))) - ((8*a^6 + 3*a^5*b - 24*a^4*b^2 - 10*a^3*b^3
+ 24*a^2*b^4 + 15*a*b^5)*cosh(x)^8 + 8*(8*a^6 + 3*a^5*b - 24*a^4*b^2 - 10*a
^3*b^3 + 24*a^2*b^4 + 15*a*b^5)*cosh(x)*sinh(x)^7 + (8*a^6 + 3*a^5*b - 24*a
^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5)*sinh(x)^8 - 4*(8*a^6 + 3*a^5*b
- 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5)*cosh(x)^6 - 4*(8*a^6 +
3*a^5*b - 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5 - 7*(8*a^6 + 3*a^
5*b - 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5)*cosh(x)^2)*sinh(x)^6
+ 8*a^6 + 3*a^5*b - 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5 + 8*(7
*(8*a^6 + 3*a^5*b - 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5)*cosh(x
)^3 - 3*(8*a^6 + 3*a^5*b - 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5)
*cosh(x))*sinh(x)^5 + 6*(8*a^6 + 3*a^5*b - 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2
*b^4 + 15*a*b^5)*cosh(x)^4 + 2*(24*a^6 + 9*a^5*b - 72*a^4*b^2 - 30*a^3*b^3
+ 72*a^2*b^4 + 45*a*b^5 + 35*(8*a^6 + 3*a^5*b - 24*a^4*b^2 - 10*a^3*b^3 + 2
4*a^2*b^4 + 15*a*b^5)*cosh(x)^4 - 30*(8*a^6 + 3*a^5*b - 24*a^4*b^2 - 10*a^3
*b^3 + 24*a^2*b^4 + 15*a*b^5)*cosh(x)^2)*sinh(x)^4 + 8*(7*(8*a^6 + 3*a^5*b
- 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5)*cosh(x)^5 - 10*(8*a^6 +
3*a^5*b - 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5)*cosh(x)^3 + 3*(8
*a^6 + 3*a^5*b - 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5)*cosh(x))*
sinh(x)^3 - 4*(8*a^6 + 3*a^5*b - 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*
a*b^5)*cosh(x)^2 + 4*(7*(8*a^6 + 3*a^5*b - 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2
*b^4 + 15*a*b^5)*cosh(x)^6 - 8*a^6 - 3*a^5*b + 24*a^4*b^2 + 10*a^3*b^3 - 24
*a^2*b^4 - 15*a*b^5 - 15*(8*a^6 + 3*a^5*b - 24*a^4*b^2 - 10*a^3*b^3 + 24*a^
2*b^4 + 15*a*b^5)*cosh(x)^4 + 9*(8*a^6 + 3*a^5*b - 24*a^4*b^2 - 10*a^3*b^3
+ 24*a^2*b^4 + 15*a*b^5)*cosh(x)^2)*sinh(x)^2 + 8*((8*a^6 + 3*a^5*b - 24*a^
4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5)*cosh(x)^7 - 3*(8*a^6 + 3*a^5*b
- 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5)*cosh(x)^5 + 3*(8*a^6 + 3
*a^5*b - 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5)*cosh(x)^3 - (8*a^
6 + 3*a^5*b - 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5)*cosh(x))*sin
h(x))*log(cosh(x) + sinh(x) + 1) - ((8*a^6 - 3*a^5*b - 24*a^4*b^2 + 10*a^3*
b^3 + 24*a^2*b^4 - 15*a*b^5)*cosh(x)^8 + 8*(8*a...

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^5(x)}{a + b\operatorname{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**5/(a+b*sech(x)),x)
```

```
[Out] Integral(coth(x)**5/(a + b*sech(x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 380 vs. 2(162) = 324.
time = 0.39, size = 380, normalized size = 2.13

$$-\frac{b^6\log\left(\left|a\left(e^{(-x)}+e^x\right)+2\,b\right|\right)}{a^7-3\,a^5b^2+3\,a^3b^4-ab^6}+\frac{\left(8\,a^2-21\,ab+15\,b^2\right)\log\left(e^{(-x)}+e^x+2\right)}{16\left(a^3-3\,a^2b+3\,ab^2-b^3\right)}+\frac{\left(8\,a^2+21\,ab+15\,b^2\right)\log\left(e^{(-x)}+e^x-2\right)}{16\left(a^3+3\,a^2b+3\,ab^2+b^3\right)}-\frac{3\,a^5\left(e^{(-x)}+e^x\right)^4-9\,a^3b^2\left(e^{(-x)}+e^x\right)^4+9\,ab^4\left(e^{(-x)}+e^x\right)^4-5\,a^4b\left(e^{(-x)}+e^x\right)^3+14\,a^2b^3\left(e^{(-x)}+e^x\right)^3-9\,b^5\left(e^{(-x)}+e^x\right)^3-8\,a^5\left(e^{(-x)}+e^x\right)^2+32\,a^3b^2\left(e^{(-x)}+e^x\right)^2-48\,ab^4\left(e^{(-x)}+e^x\right)^2+12\,a^4b\left(e^{(-x)}+e^x\right)-40\,a^2b^3\left(e^{(-x)}+e^x\right)+28\,b^5\left(e^{(-x)}+e^x\right)-16\,a^3b^2+64\,ab^4}{4\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\left(\left(e^{(-x)}+e^x\right)^2-4\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] -b^6*log(abs(a*(e^(-x) + e^x) + 2*b))/(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)
 + 1/16*(8*a^2 - 21*a*b + 15*b^2)*log(e^(-x) + e^x + 2)/(a^3 - 3*a^2*b + 3*
a*b^2 - b^3) + 1/16*(8*a^2 + 21*a*b + 15*b^2)*log(e^(-x) + e^x - 2)/(a^3 +
3*a^2*b + 3*a*b^2 + b^3) - 1/4*(3*a^5*(e^(-x) + e^x)^4 - 9*a^3*b^2*(e^(-x)
+ e^x)^4 + 9*a*b^4*(e^(-x) + e^x)^4 - 5*a^4*b*(e^(-x) + e^x)^3 + 14*a^2*b^3
*(e^(-x) + e^x)^3 - 9*b^5*(e^(-x) + e^x)^3 - 8*a^5*(e^(-x) + e^x)^2 + 32*a^
3*b^2*(e^(-x) + e^x)^2 - 48*a*b^4*(e^(-x) + e^x)^2 + 12*a^4*b*(e^(-x) + e^x
) - 40*a^2*b^3*(e^(-x) + e^x) + 28*b^5*(e^(-x) + e^x) - 16*a^3*b^2 + 64*a*b
^4)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*((e^(-x) + e^x)^2 - 4)^2)
```

Mupad [B]
time = 2.75, size = 623, normalized size = 3.50

$$\frac{\ln\left(e^x-1\right)\left(8\,a^2+21\,a\,b+15\,b^2\right)}{8\,a^3+24\,a^2\,b+24\,a\,b^2+8\,b^3}-\frac{\frac{2\left(4\,a^4-5\,a^2\,b^2\right)}{a\left(a^2-b^2\right)^2}-\frac{e^x\left(9\,a^2\,b-13\,b^3\right)}{2\left(a^2-b^2\right)^2}}{e^{4\,x}-2\,e^{2\,x}+1}-\frac{\frac{2\left(2\,a^6-5\,a^4\,b^2+3\,a^2\,b^4\right)}{a\left(a^2-b^2\right)^3}-\frac{e^x\left(5\,a^4\,b-14\,a^2\,b^3+9\,b^5\right)}{4\left(a^2-b^2\right)^3}}{e^{2\,x}-1}-\frac{\frac{8\left(a^4-a^2\,b^2\right)}{a\left(a^2-b^2\right)^2}-\frac{6\,e^x\left(a^2\,b-b^3\right)}{\left(a^2-b^2\right)^2}}{3\,e^{2\,x}-3\,e^{4\,x}+e^{6\,x}-1}-\frac{x}{a}-\frac{\frac{4\,a}{a^2-b^2}-\frac{4\,b\,e^x}{a^2-b^2}}{6\,e^{4\,x}-4\,e^{2\,x}-4\,e^{6\,x}+e^{8\,x}+1}+\frac{\ln\left(e^x+1\right)\left(8\,a^2-21\,a\,b+15\,b^2\right)}{8\,a^3-24\,a^2\,b+24\,a\,b^2-8\,b^3}+\frac{b^6\,\ln\left(64\,a^{13}\,e^{2\,x}+64\,a\,b^{12}+64\,a^{13}+159\,a^3\,b^{10}+492\,a^5\,b^8-1214\,a^7\,b^6+1020\,a^9\,b^4-393\,a^{11}\,b^2+128\,b^{13}\,e^x+159\,a^3\,b^{10}\,e^{2\,x}+492\,a^5\,b^8\,e^{2\,x}-1214\,a^7\,b^6\,e^{2\,x}+1020\,a^9\,b^4\,e^{2\,x}-393\,a^{11}\,b^2\,e^{2\,x}+128\,a^{12}\,b\,e^x+64\,a\,b^{12}\,e^{2\,x}+318\,a^2\,b^{11}\,e^x+984\,a^4\,b^9\,e^x-2428\,a^6\,b^7\,e^x+2040\,a^8\,b^5\,e^x-786\,a^{10}\,b^3\,e^x\right)}{-a^7+3\,a^5\,b^2-3\,a^3\,b^4+a\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^5/(a + b/cosh(x)),x)
```

```
[Out] (log(exp(x) - 1)*(21*a*b + 8*a^2 + 15*b^2))/(24*a*b^2 + 24*a^2*b + 8*a^3 +
8*b^3) - ((2*(4*a^4 - 5*a^2*b^2))/(a*(a^2 - b^2)^2) - (exp(x)*(9*a^2*b - 13
*b^3))/(2*(a^2 - b^2)^2))/(exp(4*x) - 2*exp(2*x) + 1) - ((2*(2*a^6 + 3*a^2*
b^4 - 5*a^4*b^2))/(a*(a^2 - b^2)^3) - (exp(x)*(5*a^4*b + 9*b^5 - 14*a^2*b^3
))/(4*(a^2 - b^2)^3))/(exp(2*x) - 1) - ((8*(a^4 - a^2*b^2))/(a*(a^2 - b^2)^
2) - (6*exp(x)*(a^2*b - b^3))/(a^2 - b^2)^2)/(3*exp(2*x) - 3*exp(4*x) + exp
(6*x) - 1) - x/a - ((4*a)/(a^2 - b^2) - (4*b*exp(x))/(a^2 - b^2))/(6*exp(4*
```

```
x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1) + (log(exp(x) + 1)*(8*a^2 - 21
*a*b + 15*b^2))/(24*a*b^2 - 24*a^2*b + 8*a^3 - 8*b^3) + (b^6*log(64*a^13*ex
p(2*x) + 64*a*b^12 + 64*a^13 + 159*a^3*b^10 + 492*a^5*b^8 - 1214*a^7*b^6 +
1020*a^9*b^4 - 393*a^11*b^2 + 128*b^13*exp(x) + 159*a^3*b^10*exp(2*x) + 492
*a^5*b^8*exp(2*x) - 1214*a^7*b^6*exp(2*x) + 1020*a^9*b^4*exp(2*x) - 393*a^1
1*b^2*exp(2*x) + 128*a^12*b*exp(x) + 64*a*b^12*exp(2*x) + 318*a^2*b^11*exp(
x) + 984*a^4*b^9*exp(x) - 2428*a^6*b^7*exp(x) + 2040*a^8*b^5*exp(x) - 786*a
^10*b^3*exp(x)))/(a*b^6 - a^7 - 3*a^3*b^4 + 3*a^5*b^2)
```

3.125 $\int \sqrt{a+b\text{sech}(c+dx)}\tanh^5(c+dx)\,dx$

Optimal. Leaf size=169

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{2\sqrt{a+b\text{sech}(c+dx)}}{d}+\frac{2a(a^2-2b^2)(a+b\text{sech}(c+dx))^{3/2}}{3b^4d}-2(3a^2$$

```
[Out] 2/3*a*(a^2-2*b^2)*(a+b*sech(d*x+c))^(3/2)/b^4/d-2/5*(3*a^2-2*b^2)*(a+b*sech
(d*x+c))^(5/2)/b^4/d+6/7*a*(a+b*sech(d*x+c))^(7/2)/b^4/d-2/9*(a+b*sech(d*x+
c))^(9/2)/b^4/d+2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d-2*(a+b
*sech(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.14, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3970, 912, 1275, 213}

$$-\frac{2(3a^2-2b^2)(a+b\text{sech}(c+dx))^{5/2}}{5b^4d}+\frac{2a(a^2-2b^2)(a+b\text{sech}(c+dx))^{3/2}}{3b^4d}-\frac{2(a+b\text{sech}(c+dx))^{9/2}}{9b^4d}+\frac{6a(a+b\text{sech}(c+dx))^{7/2}}{7b^4d}-\frac{2\sqrt{a+b\text{sech}(c+dx)}}{d}+\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x]^5,x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/d - (2*Sqrt[a + b*Se
ch[c + d*x]])/d + (2*a*(a^2 - 2*b^2)*(a + b*Sech[c + d*x])^(3/2))/(3*b^4*d)
 - (2*(3*a^2 - 2*b^2)*(a + b*Sech[c + d*x])^(5/2))/(5*b^4*d) + (6*a*(a + b*
Sech[c + d*x])^(7/2))/(7*b^4*d) - (2*(a + b*Sech[c + d*x])^(9/2))/(9*b^4*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
```

```
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\text{sech}(c+dx)}\,\tanh^5(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{\sqrt{a+x}\,(b^2-x^2)^2}{x}\,dx, x, b\text{sech}(c+dx)\right)}{b^4 d}$$

$$= -\frac{2\text{Subst}\left(\int \frac{x^2(-a^2+b^2+2ax^2-x^4)^2}{-a+x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{b^4 d}$$

$$= -\frac{2\text{Subst}\left(\int \left(b^4-a(a^2-2b^2)\,x^2+(3a^2-2b^2)\,x^4-3ax^6+x^8\right.}{b^4 d}$$

$$= -\frac{2\sqrt{a+b\text{sech}(c+dx)}}{d} + \frac{2a(a^2-2b^2)\,(a+b\text{sech}(c+dx))^{3/2}}{3b^4 d}$$

$$= \frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{2\sqrt{a+b\text{sech}(c+dx)}}{d}$$

Mathematica [A]
time = 3.55, size = 160, normalized size = 0.95

$$\frac{2\sqrt{a+b\text{sech}(c+dx)}\left(-315+\frac{16a^4}{b^4}-\frac{84a^2}{b^2}+\frac{315\tanh^{-1}\left(\frac{\sqrt{b+a\cosh(c+dx)}}{\sqrt{a\cosh(c+dx)}}\right)\sqrt{a\cosh(c+dx)}}{\sqrt{b+a\cosh(c+dx)}}+\left(-\frac{8a^3}{b^3}+\frac{42a}{b}\right)\text{sech}(c+dx)+\left(126+\frac{6a^2}{b^2}\right)\text{sech}^2(c+dx)-\frac{5a\text{sech}^3(c+dx)}{b}-35\text{sech}^4(c+dx)\right)}{315d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x]^5,x]
```

```
[Out] (2*Sqrt[a + b*Sech[c + d*x]]*(-315 + (16*a^4)/b^4 - (84*a^2)/b^2 + (315*Arc
Tanh[Sqrt[b + a*Cosh[c + d*x]]/Sqrt[a*Cosh[c + d*x]]]*Sqrt[a*Cosh[c + d*x]]
)/Sqrt[b + a*Cosh[c + d*x]] + ((-8*a^3)/b^3 + (42*a)/b)*Sech[c + d*x] + (12
6 + (6*a^2)/b^2)*Sech[c + d*x]^2 - (5*a*Sech[c + d*x]^3)/b - 35*Sech[c + d*
x]^4))/(315*d)
```

Maple [F]
time = 3.50, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\,\mathrm{sech}\,(dx+c)}\,\left(\tanh^5(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^5,x)
```

```
[Out] int((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^5,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c)^5, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2052 vs. 2(145) = 290.
time = 0.83, size = 4363, normalized size = 25.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] [1/630*(315*(b^4*cosh(d*x + c)^8 + 8*b^4*cosh(d*x + c)*sinh(d*x + c)^7 + b^
4*sinh(d*x + c)^8 + 4*b^4*cosh(d*x + c)^6 + 6*b^4*cosh(d*x + c)^4 + 4*(7*b^
4*cosh(d*x + c)^2 + b^4)*sinh(d*x + c)^6 + 4*b^4*cosh(d*x + c)^2 + 8*(7*b^4
*cosh(d*x + c)^3 + 3*b^4*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*b^4*cosh(d*
x + c)^4 + 30*b^4*cosh(d*x + c)^2 + 3*b^4)*sinh(d*x + c)^4 + b^4 + 8*(7*b^4
*cosh(d*x + c)^5 + 10*b^4*cosh(d*x + c)^3 + 3*b^4*cosh(d*x + c))*sinh(d*x +
 c)^3 + 4*(7*b^4*cosh(d*x + c)^6 + 15*b^4*cosh(d*x + c)^4 + 9*b^4*cosh(d*x
+ c)^2 + b^4)*sinh(d*x + c)^2 + 8*(b^4*cosh(d*x + c)^7 + 3*b^4*cosh(d*x + c
)^5 + 3*b^4*cosh(d*x + c)^3 + b^4*cosh(d*x + c))*sinh(d*x + c))*sqrt(a)*log
(-(2*a^2*cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c)^4 + 4*a*b*cosh(d*x + c)^3 +
4*(2*a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)^3 + 4*a*b*cosh(d*x + c) + (4*a^
2 + b^2)*cosh(d*x + c)^2 + (12*a^2*cosh(d*x + c)^2 + 12*a*b*cosh(d*x + c) +
 4*a^2 + b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*cosh(d*x + c)^4 + a*sinh(d*x +
 c)^4 + b*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*a*c
osh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a)*sinh(d*x +
 c)^2 + b*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 4*a*
```

cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)*sqrt((a*cosh(d*x + c) + b)/co
sh(d*x + c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a*b*cosh(d*x + c)^2 + 2*a*b + (
4*a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x +
c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 4*((16*a^4 - 84*a^2*b^2 - 315*b^4)*c
osh(d*x + c)^8 + (16*a^4 - 84*a^2*b^2 - 315*b^4)*sinh(d*x + c)^8 - 4*(4*a^3
*b - 21*a*b^3)*cosh(d*x + c)^7 - 4*(4*a^3*b - 21*a*b^3 - 2*(16*a^4 - 84*a^2
*b^2 - 315*b^4)*cosh(d*x + c))*sinh(d*x + c)^7 + 4*(16*a^4 - 78*a^2*b^2 - 1
89*b^4)*cosh(d*x + c)^6 + 4*(16*a^4 - 78*a^2*b^2 - 189*b^4 + 7*(16*a^4 - 84
*a^2*b^2 - 315*b^4)*cosh(d*x + c)^2 - 7*(4*a^3*b - 21*a*b^3)*cosh(d*x + c))
*sinh(d*x + c)^6 - 4*(12*a^3*b - 53*a*b^3)*cosh(d*x + c)^5 - 4*(12*a^3*b -
53*a*b^3 - 14*(16*a^4 - 84*a^2*b^2 - 315*b^4)*cosh(d*x + c)^3 + 21*(4*a^3*b
 - 21*a*b^3)*cosh(d*x + c)^2 - 6*(16*a^4 - 78*a^2*b^2 - 189*b^4)*cosh(d*x +
 c))*sinh(d*x + c)^5 + 2*(48*a^4 - 228*a^2*b^2 - 721*b^4)*cosh(d*x + c)^4 +
 2*(35*(16*a^4 - 84*a^2*b^2 - 315*b^4)*cosh(d*x + c)^4 + 48*a^4 - 228*a^2*b
^2 - 721*b^4 - 70*(4*a^3*b - 21*a*b^3)*cosh(d*x + c)^3 + 30*(16*a^4 - 78*a^
2*b^2 - 189*b^4)*cosh(d*x + c)^2 - 10*(12*a^3*b - 53*a*b^3)*cosh(d*x + c))*
sinh(d*x + c)^4 + 16*a^4 - 84*a^2*b^2 - 315*b^4 - 4*(12*a^3*b - 53*a*b^3)*c
osh(d*x + c)^3 + 4*(14*(16*a^4 - 84*a^2*b^2 - 315*b^4)*cosh(d*x + c)^5 - 35
*(4*a^3*b - 21*a*b^3)*cosh(d*x + c)^4 - 12*a^3*b + 53*a*b^3 + 20*(16*a^4 -
78*a^2*b^2 - 189*b^4)*cosh(d*x + c)^3 - 10*(12*a^3*b - 53*a*b^3)*cosh(d*x +
 c)^2 + 2*(48*a^4 - 228*a^2*b^2 - 721*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 +
 4*(16*a^4 - 78*a^2*b^2 - 189*b^4)*cosh(d*x + c)^2 + 4*(7*(16*a^4 - 84*a^2*
b^2 - 315*b^4)*cosh(d*x + c)^6 - 21*(4*a^3*b - 21*a*b^3)*cosh(d*x + c)^5 +
15*(16*a^4 - 78*a^2*b^2 - 189*b^4)*cosh(d*x + c)^4 + 16*a^4 - 78*a^2*b^2 -
189*b^4 - 10*(12*a^3*b - 53*a*b^3)*cosh(d*x + c)^3 + 3*(48*a^4 - 228*a^2*b^
2 - 721*b^4)*cosh(d*x + c)^2 - 3*(12*a^3*b - 53*a*b^3)*cosh(d*x + c))*sinh(
d*x + c)^2 - 4*(4*a^3*b - 21*a*b^3)*cosh(d*x + c) + 4*(2*(16*a^4 - 84*a^2*b
^2 - 315*b^4)*cosh(d*x + c)^7 - 7*(4*a^3*b - 21*a*b^3)*cosh(d*x + c)^6 + 6*
(16*a^4 - 78*a^2*b^2 - 189*b^4)*cosh(d*x + c)^5 - 5*(12*a^3*b - 53*a*b^3)*c
osh(d*x + c)^4 - 4*a^3*b + 21*a*b^3 + 2*(48*a^4 - 228*a^2*b^2 - 721*b^4)*co
sh(d*x + c)^3 - 3*(12*a^3*b - 53*a*b^3)*cosh(d*x + c)^2 + 2*(16*a^4 - 78*a^
2*b^2 - 189*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a*cosh(d*x + c) + b)/c
osh(d*x + c)))/(b^4*d*cosh(d*x + c)^8 + 8*b^4*d*cosh(d*x + c)*sinh(d*x + c)
^7 + b^4*d*sinh(d*x + c)^8 + 4*b^4*d*cosh(d*x + c)^6 + 6*b^4*d*cosh(d*x + c
)^4 + 4*b^4*d*cosh(d*x + c)^2 + 4*(7*b^4*d*cosh(d*x + c)^2 + b^4*d)*sinh(d*
x + c)^6 + 8*(7*b^4*d*cosh(d*x + c)^3 + 3*b^4*d*cosh(d*x + c))*sinh(d*x + c
)^5 + b^4*d + 2*(35*b^4*d*cosh(d*x + c)^4 + 30*b^4*d*cosh(d*x + c)^2 + 3*b^
4*d)*sinh(d*x + c)^4 + 8*(7*b^4*d*cosh(d*x + c)^5 + 10*b^4*d*cosh(d*x + c)^
3 + 3*b^4*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*b^4*d*cosh(d*x + c)^6 + 1
5*b^4*d*cosh(d*x + c)^4 + 9*b^4*d*cosh(d*x + c)^2 + b^4*d)*sinh(d*x + c)^2
 + 8*(b^4*d*cosh(d*x + c)^7 + 3*b^4*d*cosh(d*x + c)^5 + 3*b^4*d*cosh(d*x + c
)^3 + b^4*d*cosh(d*x + c))*sinh(d*x + c)), -1/315*(315*(b^4*cosh(d*x + c)^8
 + 8*b^4*cosh(d*x + c)*sinh(d*x + c)^7 + b^4*sinh(d*x + c)^8 + 4*b^4*cosh(d
*x + c)^6 + 6*b^4*cosh(d*x + c)^4 + 4*(7*b^4*cosh(d*x + c)^2 + b^4)*sinh(d*
x + c)^6 + 4*b^4*cosh(d*x + c)^2 + 8*(7*b^4*cosh(d*x + c)^3 + 3*b^4*cosh(d*

```
x + c))*sinh(d*x + c)^5 + 2*(35*b^4*cosh(d*x + c)^4 + 30*b^4*cosh(d*x + c)^
2 + 3*b^4)*sinh(d*x + c)^4 + b^4 + 8*(7*b^4*cosh(d*x + c)^5 + 10*b^4*cosh(d
*x + c)^3 + 3*b^4*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*b^4*cosh(d*x + c)^6
 + 15*b^4*cosh(d*x + c)^4 + 9*b^4*cosh(d*x + c)^2 + b^4)*sinh(d*x + c)^2 +
8*(b^4*cosh(d*x + c)^7 + 3*b^4*cosh(d*x + c)^5 + 3*b^4*cosh(d*x + c)^3 + b^
4*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a)*arctan...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\operatorname{sech}(c + dx)}\tanh^5(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))**(1/2)*tanh(d*x+c)**5,x)
```

```
[Out] Integral(sqrt(a + b*sech(c + d*x))*tanh(c + d*x)**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^5,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c)^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh(c + d\,x)^5\,\sqrt{a + \frac{b}{\cosh(c + d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^5*(a + b/cosh(c + d*x))^(1/2),x)
```

```
[Out] int(tanh(c + d*x)^5*(a + b/cosh(c + d*x))^(1/2), x)
```

3.126 $\int \sqrt{a+b\text{sech}(c+dx)}\tanh^3(c+dx)\,dx$

Optimal. Leaf size=100

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{2\sqrt{a+b\text{sech}(c+dx)}}{d}-\frac{2a(a+b\text{sech}(c+dx))^{3/2}}{3b^2d}+\frac{2(a+b\text{sech}(c-}{5b^2d}$$

```
[Out] -2/3*a*(a+b*sech(d*x+c))^(3/2)/b^2/d+2/5*(a+b*sech(d*x+c))^(5/2)/b^2/d+2*ar
ctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d-2*(a+b*sech(d*x+c))^(1/2)/
d
```

Rubi [A]
time = 0.09, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3970, 912, 1275, 213}

$$\frac{2(a+b\text{sech}(c+dx))^{5/2}}{5b^2d}-\frac{2a(a+b\text{sech}(c+dx))^{3/2}}{3b^2d}-\frac{2\sqrt{a+b\text{sech}(c+dx)}}{d}+\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x]^3,x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/d - (2*Sqrt[a + b*Se
ch[c + d*x]])/d - (2*a*(a + b*Sech[c + d*x])^(3/2))/(3*b^2*d) + (2*(a + b*S
ech[c + d*x])^(5/2))/(5*b^2*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
```

```
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+b\text{sech}(c+dx)}\tanh^3(c+dx)\,dx &= -\frac{\text{Subst}\left(\int \frac{\sqrt{a+x}\,(b^2-x^2)}{x}\,dx, x, b\text{sech}(c+dx)\right)}{b^2 d}\\ &= -\frac{2\text{Subst}\left(\int \frac{x^2(-a^2+b^2+2ax^2-x^4)}{-a+x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{b^2 d}\\ &= -\frac{2\text{Subst}\left(\int \left(b^2+ax^2-x^4+\frac{ab^2}{-a+x^2}\right)dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{b^2 d}\\ &= -\frac{2\sqrt{a+b\text{sech}(c+dx)}}{d}-\frac{2a(a+b\text{sech}(c+dx))^{3/2}}{3b^2 d}+\frac{2(a+b\text{se}}{}\\ &= \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{2\sqrt{a+b\text{sech}(c+dx)}}{d}\end{aligned}$$

Mathematica [A]
time = 0.67, size = 108, normalized size = 1.08

$$\frac{2\sqrt{a+b\text{sech}(c+dx)}\left(-15-\frac{2a^2}{b^2}+\frac{15\tanh^{-1}\left(\frac{\sqrt{b+a\cosh(c+dx)}}{\sqrt{a\cosh(c+dx)}}\right)\sqrt{a\cosh(c+dx)}}{\sqrt{b+a\cosh(c+dx)}}+\frac{a\text{sech}(c+dx)}{b}+3\text{sech}^2(c+dx)\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x]^3,x]
```

```
[Out] (2*Sqrt[a + b*Sech[c + d*x]]*(-15 - (2*a^2)/b^2 + (15*ArcTanh[Sqrt[b + a*Co
sh[c + d*x]]/Sqrt[a*Cosh[c + d*x]]]*Sqrt[a*Cosh[c + d*x]])/Sqrt[b + a*Cosh[
c + d*x]] + (a*Sech[c + d*x])/b + 3*Sech[c + d*x]^2))/(15*d)
```

Maple [F]
time = 2.75, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\,\mathrm{sech}\,(dx+c)}\,\left(\tanh^{3}(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^3,x)
```

```
[Out] int((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c)^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 665 vs. 2(84) = 168.
time = 0.79, size = 1589, normalized size = 15.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] [1/30*(15*(b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*
sinh(d*x + c)^4 + 2*b^2*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + b^2)*s
inh(d*x + c)^2 + b^2 + 4*(b^2*cosh(d*x + c)^3 + b^2*cosh(d*x + c))*sinh(d*x
 + c))*sqrt(a)*log(-(2*a^2*cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c)^4 + 4*a*b*
cosh(d*x + c)^3 + 4*(2*a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)^3 + 4*a*b*cos
h(d*x + c) + (4*a^2 + b^2)*cosh(d*x + c)^2 + (12*a^2*cosh(d*x + c)^2 + 12*a
*b*cosh(d*x + c) + 4*a^2 + b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*cosh(d*x + c
)^4 + a*sinh(d*x + c)^4 + b*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) + b)*sinh(
d*x + c)^3 + 2*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*cosh(d*x + c)
 + 2*a)*sinh(d*x + c)^2 + b*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 3*b*cosh
(d*x + c)^2 + 4*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)*sqrt((a*cos
h(d*x + c) + b)/cosh(d*x + c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a*b*cosh(d*x
+ c)^2 + 2*a*b + (4*a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)
^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 4*(2*a*b*cosh(d*x
+ c)^3 - (2*a^2 + 15*b^2)*cosh(d*x + c)^4 - (2*a^2 + 15*b^2)*sinh(d*x + c)^
4 + 2*(a*b - 2*(2*a^2 + 15*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*a*b*cosh
```

```
(d*x + c) - 2*(2*a^2 + 9*b^2)*cosh(d*x + c)^2 + 2*(3*a*b*cosh(d*x + c) - 3*
(2*a^2 + 15*b^2)*cosh(d*x + c)^2 - 2*a^2 - 9*b^2)*sinh(d*x + c)^2 - 2*a^2 -
 15*b^2 + 2*(3*a*b*cosh(d*x + c)^2 - 2*(2*a^2 + 15*b^2)*cosh(d*x + c)^3 + a
*b - 2*(2*a^2 + 9*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a*cosh(d*x + c)
+ b)/cosh(d*x + c)))/(b^2*d*cosh(d*x + c)^4 + 4*b^2*d*cosh(d*x + c)*sinh(d*
x + c)^3 + b^2*d*sinh(d*x + c)^4 + 2*b^2*d*cosh(d*x + c)^2 + b^2*d + 2*(3*b
^2*d*cosh(d*x + c)^2 + b^2*d)*sinh(d*x + c)^2 + 4*(b^2*d*cosh(d*x + c)^3 +
b^2*d*cosh(d*x + c))*sinh(d*x + c)), -1/15*(15*(b^2*cosh(d*x + c)^4 + 4*b^2
*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*b^2*cosh(d*x + c)^
2 + 2*(3*b^2*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^2 + b^2 + 4*(b^2*cosh(d*x
 + c)^3 + b^2*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a)*arctan((a*cosh(d*x + c
)^2 + a*sinh(d*x + c)^2 + b*cosh(d*x + c) + (2*a*cosh(d*x + c) + b)*sinh(d*
x + c) + a)*sqrt(-a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c))/(a^2*cosh(d*
x + c)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + a^2 + 2*(a^2*cosh(d*
x + c) + a*b)*sinh(d*x + c))) - 2*(2*a*b*cosh(d*x + c)^3 - (2*a^2 + 15*b^2)
*cosh(d*x + c)^4 - (2*a^2 + 15*b^2)*sinh(d*x + c)^4 + 2*(a*b - 2*(2*a^2 + 1
5*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*a*b*cosh(d*x + c) - 2*(2*a^2 + 9*
b^2)*cosh(d*x + c)^2 + 2*(3*a*b*cosh(d*x + c) - 3*(2*a^2 + 15*b^2)*cosh(d*x
 + c)^2 - 2*a^2 - 9*b^2)*sinh(d*x + c)^2 - 2*a^2 - 15*b^2 + 2*(3*a*b*cosh(d
*x + c)^2 - 2*(2*a^2 + 15*b^2)*cosh(d*x + c)^3 + a*b - 2*(2*a^2 + 9*b^2)*co
sh(d*x + c))*sinh(d*x + c))*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)))/(b^2
*d*cosh(d*x + c)^4 + 4*b^2*d*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*d*sinh(d*x
 + c)^4 + 2*b^2*d*cosh(d*x + c)^2 + b^2*d + 2*(3*b^2*d*cosh(d*x + c)^2 + b^
2*d)*sinh(d*x + c)^2 + 4*(b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c))*sinh
(d*x + c))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\operatorname{sech}(c + dx)} \tanh^3(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))**(1/2)*tanh(d*x+c)**3,x)
```

```
[Out] Integral(sqrt(a + b*sech(c + d*x))*tanh(c + d*x)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh(c+d\,x)^3\,\sqrt{a+\frac{b}{\cosh\left(c+d\,x\right)}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^3*(a + b/cosh(c + d*x))^(1/2),x)
[Out] int(tanh(c + d*x)^3*(a + b/cosh(c + d*x))^(1/2), x)
```

3.127 $\int \sqrt{a+b\text{sech}(c+dx)} \tanh(c+dx)\,dx$

Optimal. Leaf size=51

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{2\sqrt{a+b\text{sech}(c+dx)}}{d}$$

```
[Out] 2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d-2*(a+b*sech(d*x+c))^(1
/2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3970, 52, 65, 213}

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{2\sqrt{a+b\text{sech}(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/d - (2*Sqrt[a + b*Se
ch[c + d*x]])/d
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\text{sech}(c+dx)}\tanh(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{\sqrt{a+x}}{x}\,dx, x, b\text{sech}(c+dx)\right)}{d}$$

$$= -\frac{2\sqrt{a+b\text{sech}(c+dx)}}{d} - \frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+x}}\,dx, x, b\text{sech}(c+$$

$$= -\frac{2\sqrt{a+b\text{sech}(c+dx)}}{d} - \frac{(2a)\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\text{sec}$$

$$= \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{2\sqrt{a+b\text{sech}(c+dx)}}{d}$$

Mathematica [A]
time = 0.10, size = 90, normalized size = 1.76

$$-\frac{2\left(-\tanh^{-1}\left(\frac{\sqrt{b+a\cosh(c+dx)}}{\sqrt{a\cosh(c+dx)}}\right)\sqrt{a\cosh(c+dx)}+\sqrt{b+a\cosh(c+dx)}\right)\sqrt{a+b\text{sech}(c+dx)}}{d\sqrt{b+a\cosh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x],x]
```

```
[Out] (-2*(-(ArcTanh[Sqrt[b + a*Cosh[c + d*x]]/Sqrt[a*Cosh[c + d*x]]]*Sqrt[a*Cosh
[c + d*x]]) + Sqrt[b + a*Cosh[c + d*x]])*Sqrt[a + b*Sech[c + d*x]])/(d*Sqrt
[b + a*Cosh[c + d*x]])
```

Maple [A]
time = 1.67, size = 43, normalized size = 0.84

method	result	size

derivativedivides	$-\frac{2\sqrt{a+b\,\mathrm{sech}\left(dx+c\right)}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{sech}\left(dx+c\right)}}{\sqrt{a}}\right)}{d}$	43
default	$-\frac{2\sqrt{a+b\,\mathrm{sech}\left(dx+c\right)}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{sech}\left(dx+c\right)}}{\sqrt{a}}\right)}{d}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(2*(a+b*sech(d*x+c))^(1/2)-2*a^(1/2)*arctanh((a+b*sech(d*x+c))^(1/2)/a
^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(43) = 86$.
time = 0.79, size = 605, normalized size = 11.86

$$\left[\frac{\sqrt{a}\log\left(-\frac{2a^2\cosh(dx+c)^4+2a^2\sinh(dx+c)^4+4ab\cosh(dx+c)^3+4\left(2a^2\cosh(dx+c)+ab\right)\sinh(dx+c)^3+4ab\cosh(dx+c)+\left(4a^2+b^2\right)\cosh(dx+c)^2+\left(12a^2\cosh(dx+c)^2+12ab\cosh(dx+c)+4a^2+b^2\right)\sinh(dx+c)^2+2a^2+2\left(a\cosh(dx+c)^4+a\sinh(dx+c)^4+b\cosh(dx+c)^3+\left(4a\cosh(dx+c)+b\right)\sinh(dx+c)^3+2a\cosh(dx+c)^2+\left(6a\cosh(dx+c)^2+3b\cosh(dx+c)+2a\right)\sinh(dx+c)^2+b\cosh(dx+c)+\left(4a\cosh(dx+c)^3+3b\cosh(dx+c)^2+4a\cosh(dx+c)+b\right)\sinh(dx+c)+a\right)\sqrt{a}\sqrt{\frac{a\cosh(dx+c)+b}{\cosh(dx+c)}}+2\left(4a^2\cosh(dx+c)^3+6ab\cosh(dx+c)^2+2ab+\left(4a^2+b^2\right)\cosh(dx+c)\right)\sinh(dx+c)}{\cosh(dx+c)^2+2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2}\right)-4\sqrt{\frac{a\cosh(dx+c)+b}{\cosh(dx+c)}}}{2d},\ -\frac{\sqrt{-a}\arctan\left(\frac{\left(a\cosh(dx+c)^2+a\sinh(dx+c)^2+b\cosh(dx+c)+\left(2a\cosh(dx+c)+b\right)\sinh(dx+c)+a\right)\sqrt{-a}\sqrt{\frac{a\cosh(dx+c)+b}{\cosh(dx+c)}}}{a^2\cosh(dx+c)^2+a^2\sinh(dx+c)^2+2ab\cosh(dx+c)+a^2+2\left(a^2\cosh(dx+c)+ab\right)\sinh(dx+c)}\right)+2\sqrt{\frac{a\cosh(dx+c)+b}{\cosh(dx+c)}}}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*log(-(2*a^2*cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c)^4 + 4*a*b*c
osh(d*x + c)^3 + 4*(2*a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)^3 + 4*a*b*cosh
(d*x + c) + (4*a^2 + b^2)*cosh(d*x + c)^2 + (12*a^2*cosh(d*x + c)^2 + 12*a*
b*cosh(d*x + c) + 4*a^2 + b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*cosh(d*x + c)
^4 + a*sinh(d*x + c)^4 + b*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) + b)*sinh(d
*x + c)^3 + 2*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*cosh(d*x + c)
+ 2*a)*sinh(d*x + c)^2 + b*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 3*b*cosh(
d*x + c)^2 + 4*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)*sqrt((a*cosh
(d*x + c) + b)/cosh(d*x + c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a*b*cosh(d*x +
 c)^2 + 2*a*b + (4*a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^
2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 4*sqrt((a*cosh(d*x
+ c) + b)/cosh(d*x + c)))/d, -(sqrt(-a)*arctan((a*cosh(d*x + c)^2 + a*sinh(
```

```
d*x + c)^2 + b*cosh(d*x + c) + (2*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*s
qrt(-a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c))/(a^2*cosh(d*x + c)^2 + a^
2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + a^2 + 2*(a^2*cosh(d*x + c) + a*b)
*sinh(d*x + c))) + 2*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)))/d]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\operatorname{sech}\left(c + dx\right)}\tanh\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))**(1/2)*tanh(d*x+c),x)
```

```
[Out] Integral(sqrt(a + b*sech(c + d*x))*tanh(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c), x)
```

Mupad [B]
time = 1.69, size = 47, normalized size = 0.92

$$\frac{2\sqrt{a}\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{\cosh\left(c+d\,x\right)}}}{\sqrt{a}}\right)}{d}-\frac{2\sqrt{a+\frac{b}{\cosh\left(c+d\,x\right)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)*(a + b/cosh(c + d*x))^(1/2),x)
```

```
[Out] (2*a^(1/2)*atanh((a + b/cosh(c + d*x))^(1/2)/a^(1/2)))/d - (2*(a + b/cosh(c
 + d*x))^(1/2))/d
```

3.128 $\int \coth(c+dx)\sqrt{a+b\,\mathrm{sech}(c+dx)}\,dx$

Optimal. Leaf size=106

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a-b}}\right)}{d}-\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right)}{d}$$

```
[Out] 2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d-arctanh((a+b*sech(d*x+
c))^(1/2)/(a-b)^(1/2))*(a-b)^(1/2)/d-arctanh((a+b*sech(d*x+c))^(1/2)/(a+b)^
(1/2))*(a+b)^(1/2)/d
```

Rubi [A]
time = 0.12, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3970, 912, 1301, 212, 213}

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a-b}}\right)}{d}-\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]*Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/d - (Sqrt[a - b]*Arc
Tanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a - b]])/d - (Sqrt[a + b]*ArcTanh[Sqrt[
a + b*Sech[c + d*x]]/Sqrt[a + b]])/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
```

```
, p] && FractionQ[m]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \coth(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx = -\frac{b^2\text{Subst}\left(\int \frac{\sqrt{a+x}}{x(b^2-x^2)}\,dx, x, b\text{sech}(c+dx)\right)}{d}$$

$$= -\frac{(2b^2)\,\text{Subst}\left(\int \frac{x^2}{(-a+x^2)(-a^2+b^2+2ax^2-x^4)}\,dx, x, \sqrt{a+b\text{sech}(c+dx}\right.}{d}$$

$$= -\frac{(2b^2)\,\text{Subst}\left(\int \left(-\frac{a}{b^2(a-x^2)}+\frac{a+b}{2b^2(a+b-x^2)}+\frac{-a+b}{2b^2(-a+b+x^2)}\right)dx, x,\right.}{d}$$

$$= \frac{(2a)\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{d} + \frac{(a-b)\text{Subst}\left(\right.}{}$$

$$= \frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a}}{}\right.}{d}$$

Mathematica [A]
time = 1.29, size = 211, normalized size = 1.99

$$\frac{\left(-\sqrt{-a+b}\,\text{ArcTan}\left(\frac{a+a\cosh(c+dx)-\sqrt{a\cosh(c+dx)}\sqrt{b+a\cosh(c+dx)}}{\sqrt{a}\sqrt{-a+b}}\right)+2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{b+a\cosh(c+dx)}}{\sqrt{a\cosh(c+dx)}}\right)-\sqrt{a+b}\,\tanh^{-1}\left(\frac{a-a\cosh(c+dx)+\sqrt{a\cosh(c+dx)}\sqrt{b+a\cosh(c+dx)}}{\sqrt{a}\sqrt{a+b}}\right)\right)\sqrt{a\cosh(c+dx)}\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}\,d\sqrt{b+a\cosh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]*Sqrt[a + b*Sech[c + d*x]],x]
[Out] ((-(Sqrt[-a + b]*ArcTan[(a + a*Cosh[c + d*x] - Sqrt[a*Cosh[c + d*x]]*Sqrt[b
 + a*Cosh[c + d*x]])/(Sqrt[a]*Sqrt[-a + b])]) + 2*Sqrt[a]*ArcTanh[Sqrt[b +
```

```
a*Cosh[c + d*x]]/Sqrt[a*Cosh[c + d*x]]] - Sqrt[a + b]*ArcTanh[(a - a*Cosh[c
 + d*x] + Sqrt[a*Cosh[c + d*x]]*Sqrt[b + a*Cosh[c + d*x]])/(Sqrt[a]*Sqrt[a
+ b])])*Sqrt[a*Cosh[c + d*x]]*Sqrt[a + b*Sech[c + d*x]])/(Sqrt[a]*d*Sqrt[b
+ a*Cosh[c + d*x]])
```

Maple [F]
time = 2.89, size = 0, normalized size = 0.00

$$\int \coth (dx+c) \sqrt{a+b\,\mathrm{sech}\,(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)*(a+b*sech(d*x+c))^(1/2),x)
[Out] int(coth(d*x+c)*(a+b*sech(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sech(d*x + c) + a)*coth(d*x + c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 410 vs. 2(88) = 176.
time = 0.73, size = 8620, normalized size = 81.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
[Out] [1/4*(sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cosh(d*x + c)^4 + (8*a^2 - 8*
a*b + b^2)*sinh(d*x + c)^4 + 4*(4*a*b - 3*b^2)*cosh(d*x + c)^3 + 4*(4*a*b -
 3*b^2 + (8*a^2 - 8*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(8*a^2 -
8*a*b + 3*b^2)*cosh(d*x + c)^2 + 2*(3*(8*a^2 - 8*a*b + b^2)*cosh(d*x + c)^2
 + 8*a^2 - 8*a*b + 3*b^2 + 6*(4*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2
 + 8*a^2 - 8*a*b + b^2 - 4*((2*a - b)*cosh(d*x + c)^4 + (2*a - b)*sinh(d*x
+ c)^4 + 2*b*cosh(d*x + c)^3 + 2*(2*(2*a - b)*cosh(d*x + c) + b)*sinh(d*x +
 c)^3 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*(2*a - b)*cosh(d*x + c)^2 + 3*b*
cosh(d*x + c) + 2*a - b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*(2*(2*a -
b)*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 2*(2*a - b)*cosh(d*x + c) + b)*s
inh(d*x + c) + 2*a - b)*sqrt(a - b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c
```

```
)) + 4*(4*a*b - 3*b^2)*cosh(d*x + c) + 4*((8*a^2 - 8*a*b + b^2)*cosh(d*x +
c)^3 + 3*(4*a*b - 3*b^2)*cosh(d*x + c)^2 + 4*a*b - 3*b^2 + (8*a^2 - 8*a*b +
 3*b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^4 + 4*(cosh(d*x + c) +
 1)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 4*cosh(d*x + c)^3 + 6*(cosh(d*x + c
)^2 + 2*cosh(d*x + c) + 1)*sinh(d*x + c)^2 + 6*cosh(d*x + c)^2 + 4*(cosh(d*
x + c)^3 + 3*cosh(d*x + c)^2 + 3*cosh(d*x + c) + 1)*sinh(d*x + c) + 4*cosh(
d*x + c) + 1)) + sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cosh(d*x + c)^4 +
(8*a^2 + 8*a*b + b^2)*sinh(d*x + c)^4 + 4*(4*a*b + 3*b^2)*cosh(d*x + c)^3 +
 4*(4*a*b + 3*b^2 + (8*a^2 + 8*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 +
2*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 2*(3*(8*a^2 + 8*a*b + b^2)*cosh
(d*x + c)^2 + 8*a^2 + 8*a*b + 3*b^2 + 6*(4*a*b + 3*b^2)*cosh(d*x + c))*sinh
(d*x + c)^2 + 8*a^2 + 8*a*b + b^2 - 4*((2*a + b)*cosh(d*x + c)^4 + (2*a + b
)*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^3 + 2*(2*(2*a + b)*cosh(d*x + c) + b)
*sinh(d*x + c)^3 + 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*(2*a + b)*cosh(d*x +
c)^2 + 3*b*cosh(d*x + c) + 2*a + b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2
*(2*(2*a + b)*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 2*(2*a + b)*cosh(d*x
+ c) + b)*sinh(d*x + c) + 2*a + b)*sqrt(a + b)*sqrt((a*cosh(d*x + c) + b)/c
osh(d*x + c)) + 4*(4*a*b + 3*b^2)*cosh(d*x + c) + 4*((8*a^2 + 8*a*b + b^2)*
cosh(d*x + c)^3 + 3*(4*a*b + 3*b^2)*cosh(d*x + c)^2 + 4*a*b + 3*b^2 + (8*a^
2 + 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^4 + 4*(cosh
(d*x + c) - 1)*sinh(d*x + c)^3 + sinh(d*x + c)^4 - 4*cosh(d*x + c)^3 + 6*(c
osh(d*x + c)^2 - 2*cosh(d*x + c) + 1)*sinh(d*x + c)^2 + 6*cosh(d*x + c)^2 +
 4*(cosh(d*x + c)^3 - 3*cosh(d*x + c)^2 + 3*cosh(d*x + c) - 1)*sinh(d*x + c
) - 4*cosh(d*x + c) + 1)) + 2*sqrt(a)*log(-(2*a^2*cosh(d*x + c)^4 + 2*a^2*s
inh(d*x + c)^4 + 4*a*b*cosh(d*x + c)^3 + 4*(2*a^2*cosh(d*x + c) + a*b)*sinh
(d*x + c)^3 + 4*a*b*cosh(d*x + c) + (4*a^2 + b^2)*cosh(d*x + c)^2 + (12*a^2
*cosh(d*x + c)^2 + 12*a*b*cosh(d*x + c) + 4*a^2 + b^2)*sinh(d*x + c)^2 + 2*
a^2 + 2*(a*cosh(d*x + c)^4 + a*sinh(d*x + c)^4 + b*cosh(d*x + c)^3 + (4*a*c
osh(d*x + c) + b)*sinh(d*x + c)^3 + 2*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c
)^2 + 3*b*cosh(d*x + c) + 2*a)*sinh(d*x + c)^2 + b*cosh(d*x + c) + (4*a*cos
h(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 4*a*cosh(d*x + c) + b)*sinh(d*x + c) +
 a)*sqrt(a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 2*(4*a^2*cosh(d*x +
 c)^3 + 6*a*b*cosh(d*x + c)^2 + 2*a*b + (4*a^2 + b^2)*cosh(d*x + c))*sinh(d
*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2
)))/d, -1/4*(4*sqrt(-a)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x
+ c) + sinh(d*x + c)^2 + 1)*sqrt(-a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x +
c))/(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + b*cosh(d*x + c) + (2*a*cosh(d*
x + c) + b)*sinh(d*x + c) + a)) - sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*c
osh(d*x + c)^4 + (8*a^2 - 8*a*b + b^2)*sinh(d*x + c)^4 + 4*(4*a*b - 3*b^2)*
cosh(d*x + c)^3 + 4*(4*a*b - 3*b^2 + (8*a^2 - 8*a*b + b^2)*cosh(d*x + c))*s
inh(d*x + c)^3 + 2*(8*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 2*(3*(8*a^2 -
8*a*b + b^2)*cosh(d*x + c)^2 + 8*a^2 - 8*a*b + 3*b^2 + 6*(4*a*b - 3*b^2)*co
sh(d*x + c))*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 - 4*((2*a - b)*cosh(d*x
+ c)^4 + (2*a - b)*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^3 + 2*(2*(2*a - b)*c
osh(d*x + c) + b)*sinh(d*x + c)^3 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*(2*a
```

```
 - b)*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a - b)*sinh(d*x + c)^2 + 2*b*
cosh(d*x + c) + 2*(2*(2*a - b)*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 2*(2
*a - b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a - b)*sqrt(a - b)*sqrt((a*cos
h(d*x + c) + b)/cosh(d*x + c)) + 4*(4*a*b - 3*b^2)*cosh(d*x + c) + 4*((8*a^
2 - 8*a*b + b^2)*cosh(d*x + c)^3 + 3*(4*a*b - 3*b^2)*cosh(d*x + c)^2 + 4*a*
b - 3*b^2 + (8*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x
 + c)^4 + 4*(cosh(d*x + c) + 1)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 4*cosh(
d*x + c)^3 + 6*(cosh(d*x + c)^2 + 2*cosh(d*x + c) + 1)*sinh(d*x + c)^2 + 6*
cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)^2 + 3*cosh(d*x + c)
+ 1)*sinh(d*x + c) + 4*cosh(d*x + c) + 1)) - sqrt(a + b)*log(-((8*a^2 + 8*a
*b + b^2)*cosh(d*x + c)^4 + (8*a^2 + 8*a*b + b^...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\operatorname{sech}\left(c + dx\right)}\coth\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*sech(c + d*x))*coth(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sech(d*x + c) + a)*coth(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \coth(c + d\,x)\,\sqrt{a + \frac{b}{\cosh\left(c + d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)*(a + b/cosh(c + d*x))^(1/2),x)
[Out] int(coth(c + d*x)*(a + b/cosh(c + d*x))^(1/2), x)
```

3.129 $\int \coth^3(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx$

Optimal. Leaf size=217

$$\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{a\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,d}+\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+d}}{\sqrt{a-b}}\right.}{4\sqrt{a-b}\,d}$$

```
[Out] 2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d-a*arctanh((a+b*sech(d*
x+c))^(1/2)/(a-b)^(1/2))/d/(a-b)^(1/2)+3/4*b*arctanh((a+b*sech(d*x+c))^(1/2
)/(a-b)^(1/2))/d/(a-b)^(1/2)-a*arctanh((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2))
/d/(a+b)^(1/2)-3/4*b*arctanh((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2))/d/(a+b)^(
1/2)-1/2*coth(d*x+c)^2*(a+b*sech(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.23, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3970, 912, 1329, 1192, 12, 1107, 212, 1184, 213}

$$-\frac{\coth^2(c+dx)\sqrt{a+b\text{sech}(c+dx)}}{2d}+\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{4d\sqrt{a-b}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{d\sqrt{a-b}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{d\sqrt{a+b}}-\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{4d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^3*Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/d - (a*ArcTanh[Sqrt[
a + b*Sech[c + d*x]]/Sqrt[a - b]])/(Sqrt[a - b]*d) + (3*b*ArcTanh[Sqrt[a +
b*Sech[c + d*x]]/Sqrt[a - b]])/(4*Sqrt[a - b]*d) - (a*ArcTanh[Sqrt[a + b*Se
ch[c + d*x]]/Sqrt[a + b]])/(Sqrt[a + b]*d) - (3*b*ArcTanh[Sqrt[a + b*Sech[c
 + d*x]]/Sqrt[a + b]])/(4*Sqrt[a + b]*d) - (Coth[c + d*x]^2*Sqrt[a + b*Sech
[c + d*x]])/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1184

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1329

```
Int[(((f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[f^2/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^(
m - 2)*(a*e + c*d*x^2)*(a + b*x^2 + c*x^4)^p, x], x] - Dist[d*e*(f^2/(c*d^2
 - b*d*e + a*e^2)), Int[(f*x)^(m - 2)*((a + b*x^2 + c*x^4)^(p + 1)/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
p, -1] && GtQ[m, 0]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \coth^3(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx = -\frac{b^4\text{Subst}\left(\int \frac{\sqrt{a+x}}{x(b^2-x^2)^2}\,dx, x, b\text{sech}(c+dx)\right)}{d}$$

$$= -\frac{(2b^4)\,\text{Subst}\left(\int \frac{x^2}{(-a+x^2)(-a^2+b^2+2ax^2-x^4)^2}\,dx, x, \sqrt{a+b\text{sech}(c+$$

$$= -\frac{(2b^2)\,\text{Subst}\left(\int \frac{-a^2+b^2+ax^2}{(-a^2+b^2+2ax^2-x^4)^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{d}$$

$$= \frac{b^2\sqrt{a+b\text{sech}(c+dx)}}{2d\left(a^2-b^2-2a(a+b\text{sech}(c+dx))+(a+b\text{sech}(c+dx))^2\right)} -$$

$$= \frac{b^2\sqrt{a+b\text{sech}(c+dx)}}{2d\left(a^2-b^2-2a(a+b\text{sech}(c+dx))+(a+b\text{sech}(c+dx))^2\right)} -$$

$$= \frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{a\tanh^{-1}\left(\frac{\sqrt{a+b\text{sec}}}{\sqrt{a}}\right.}{\sqrt{a-b}\,d}$$

$$= \frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{a\tanh^{-1}\left(\frac{\sqrt{a+b\text{sec}}}{\sqrt{a}}\right.}{\sqrt{a-b}\,d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 18.14, size = 456, normalized size = 2.10

$$\frac{\left(\frac{-\sqrt{a}\sqrt{-a+b}\,(2a+3b)\tanh^{-1}\left(\frac{a-a\cosh(c+dx)+\sqrt{a\cosh(c+dx)}\sqrt{b+a\cosh(c+dx)}}{\sqrt{a}\sqrt{a+b}}\right)\cosh(c+dx)+2i\sqrt{a}\sqrt{-a+b}\tanh^{-1}\left(\frac{a-a\cosh(c+dx)-i\sqrt{-a\cosh(c+dx)}\sqrt{b+a\cosh(c+dx)}}{\sqrt{a}\sqrt{a+b}}\right)\sqrt{-a^2\cosh^2(c+dx)}+\sqrt{a+b}\left(\sqrt{a}\,(2a-3b)\text{ArcTan}\left(\frac{a+a\cosh(c+dx)-\sqrt{a\cosh(c+dx)}\sqrt{b+a\cosh(c+dx)}}{\sqrt{a}\sqrt{-a+b}}\right)\cosh(c+dx)+2\left(4\sqrt{-a+b}\,\text{ArcTan}\left(\frac{\sqrt{b+a\cosh(c+dx)}}{\sqrt{-a\cosh(c+dx)}}\right)-i\sqrt{a}\,\text{ArcTan}\left(\frac{a+a\cosh(c+dx)+i\sqrt{-a\cosh(c+dx)}\sqrt{b+a\cosh(c+dx)}}{\sqrt{a}\sqrt{-a+b}}\right)\right)\sqrt{-a^2\cosh^2(c+dx)}\right)}{\sqrt{-a+b}\sqrt{a+b}\sqrt{a\cosh(c+dx)}\sqrt{b+a\cosh(c+dx)}}-2\coth^2(c+dx)\right)\sqrt{a+b\text{sech}(c+dx)}}{4d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[c + d*x]^3*Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (((-(Sqrt[a]*Sqrt[-a + b]*(2*a + 3*b)*ArcTanh[(a - a*Cosh[c + d*x] + Sqrt[a
*Cosh[c + d*x]]*Sqrt[b + a*Cosh[c + d*x]])/(Sqrt[a]*Sqrt[a + b])]*Cosh[c +
d*x]) + (2*I)*Sqrt[a]*Sqrt[-a + b]*ArcTanh[(a - a*Cosh[c + d*x] - I*Sqrt[-(
a*Cosh[c + d*x])]*Sqrt[b + a*Cosh[c + d*x]])/(Sqrt[a]*Sqrt[a + b])]*Sqrt[-(
a^2*Cosh[c + d*x]^2)] + Sqrt[a + b]*(Sqrt[a]*(2*a - 3*b)*ArcTan[(a + a*Cosh
```

```
[c + d*x] - Sqrt[a*Cosh[c + d*x]]*Sqrt[b + a*Cosh[c + d*x]])/(Sqrt[a]*Sqrt[
-a + b])]*Cosh[c + d*x] + 2*(4*Sqrt[-a + b]*ArcTan[Sqrt[b + a*Cosh[c + d*x]
]/Sqrt[-(a*Cosh[c + d*x])]] - I*Sqrt[a]*ArcTan[(a + a*Cosh[c + d*x] + I*Sqr
t[-(a*Cosh[c + d*x])]*Sqrt[b + a*Cosh[c + d*x]])/(Sqrt[a]*Sqrt[-a + b])])*S
qrt[-(a^2*Cosh[c + d*x]^2)]))/(Sqrt[-a + b]*Sqrt[a + b]*Sqrt[a*Cosh[c + d*x
]]*Sqrt[b + a*Cosh[c + d*x]]) - 2*Coth[c + d*x]^2)*Sqrt[a + b*Sech[c + d*x]
])/(4*d)
```

Maple [F]
time = 3.09, size = 0, normalized size = 0.00

$$\int \left(\coth^3\left(dx+c\right)\right)\sqrt{a+b\,\mathrm{sech}\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^3*(a+b*sech(d*x+c))^(1/2),x)
```

```
[Out] int(coth(d*x+c)^3*(a+b*sech(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(d*x + c) + a)*coth(d*x + c)^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1399 vs. $2(179) = 358$.
time = 1.13, size = 16532, normalized size = 76.18

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(((4*a^2 + a*b - 3*b^2)*cosh(d*x + c)^4 + 4*(4*a^2 + a*b - 3*b^2)*co
sh(d*x + c)*sinh(d*x + c)^3 + (4*a^2 + a*b - 3*b^2)*sinh(d*x + c)^4 - 2*(4*
a^2 + a*b - 3*b^2)*cosh(d*x + c)^2 + 2*(3*(4*a^2 + a*b - 3*b^2)*cosh(d*x +
c)^2 - 4*a^2 - a*b + 3*b^2)*sinh(d*x + c)^2 + 4*a^2 + a*b - 3*b^2 + 4*((4*a
^2 + a*b - 3*b^2)*cosh(d*x + c)^3 - (4*a^2 + a*b - 3*b^2)*cosh(d*x + c))*si
nh(d*x + c))*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cosh(d*x + c)^4 + (8*a
^2 - 8*a*b + b^2)*sinh(d*x + c)^4 + 4*(4*a*b - 3*b^2)*cosh(d*x + c)^3 + 4*(
4*a*b - 3*b^2 + (8*a^2 - 8*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(8
```

*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 2*(3*(8*a^2 - 8*a*b + b^2)*cosh(d*x + c)^2 + 8*a^2 - 8*a*b + 3*b^2 + 6*(4*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*((2*a - b)*cosh(d*x + c)^4 + (2*a - b)*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^3 + 2*(2*(2*a - b)*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*(2*a - b)*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a - b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*(2*(2*a - b)*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 2*(2*a - b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a - b)*sqrt(a - b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 4*(4*a*b - 3*b^2)*cosh(d*x + c) + 4*((8*a^2 - 8*a*b + b^2)*cosh(d*x + c)^3 + 3*(4*a*b - 3*b^2)*cosh(d*x + c)^2 + 4*a*b - 3*b^2 + (8*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^4 + 4*(cosh(d*x + c) + 1)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 4*cosh(d*x + c)^3 + 6*(cosh(d*x + c)^2 + 2*cosh(d*x + c) + 1)*sinh(d*x + c)^2 + 6*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)^2 + 3*cosh(d*x + c) + 1)*sinh(d*x + c) + 4*cosh(d*x + c) + 1)) - ((4*a^2 - a*b - 3*b^2)*cosh(d*x + c)^4 + 4*(4*a^2 - a*b - 3*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (4*a^2 - a*b - 3*b^2)*sinh(d*x + c)^4 - 2*(4*a^2 - a*b - 3*b^2)*cosh(d*x + c)^2 + 2*(3*(4*a^2 - a*b - 3*b^2)*cosh(d*x + c)^2 - 4*a^2 + a*b + 3*b^2)*sinh(d*x + c)^2 + 4*a^2 - a*b - 3*b^2 + 4*((4*a^2 - a*b - 3*b^2)*cosh(d*x + c)^3 - (4*a^2 - a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cosh(d*x + c)^4 + (8*a^2 + 8*a*b + b^2)*sinh(d*x + c)^4 + 4*(4*a*b + 3*b^2)*cosh(d*x + c)^3 + 4*(4*a*b + 3*b^2 + (8*a^2 + 8*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 2*(3*(8*a^2 + 8*a*b + b^2)*cosh(d*x + c)^2 + 8*a^2 + 8*a*b + 3*b^2 + 6*(4*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 8*a^2 + 8*a*b + b^2 - 4*((2*a + b)*cosh(d*x + c)^4 + (2*a + b)*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^3 + 2*(2*(2*a + b)*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*(2*a + b)*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a + b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*(2*(2*a + b)*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 2*(2*a + b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a + b)*sqrt(a + b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 4*(4*a*b + 3*b^2)*cosh(d*x + c) + 4*((8*a^2 + 8*a*b + b^2)*cosh(d*x + c)^3 + 3*(4*a*b + 3*b^2)*cosh(d*x + c)^2 + 4*a*b + 3*b^2 + (8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^4 + 4*(cosh(d*x + c) - 1)*sinh(d*x + c)^3 + sinh(d*x + c)^4 - 4*cosh(d*x + c)^3 + 6*(cosh(d*x + c)^2 - 2*cosh(d*x + c) + 1)*sinh(d*x + c)^2 + 6*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - 3*cosh(d*x + c)^2 + 3*cosh(d*x + c) - 1)*sinh(d*x + c) - 4*cosh(d*x + c) + 1)) - 8*((a^2 - b^2)*cosh(d*x + c)^4 + 4*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 - b^2)*sinh(d*x + c)^4 - 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 - b^2)*cosh(d*x + c)^2 - a^2 + b^2)*sinh(d*x + c)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(d*x + c)^3 - (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a)*log(-(2*a^2*cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c)^4 + 4*a*b*cosh(d*x + c)^3 + 4*(2*a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)^3 + 4*a*b*cosh(d*x + c) + (4*a^2 + b^2)*cosh(d*x + c)^2 + (12*a^2*cosh(d*x + c)^2 + 12*a*b*cosh(d*x + c) + 4*a^2 + b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*cosh(d*x + c)^4 + a*sinh(d*x + c)^4 + b*cosh(d*x + c)^3 + (4*a

```
*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*a*cosh(d*x + c)^2 + (6*a*cosh(d*x +
 c)^2 + 3*b*cosh(d*x + c) + 2*a)*sinh(d*x + c)^2 + b*cosh(d*x + c) + (4*a*c
osh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 4*a*cosh(d*x + c) + b)*sinh(d*x + c)
 + a)*sqrt(a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 2*(4*a^2*cosh(d*x
 + c)^3 + 6*a*b*cosh(d*x + c)^2 + 2*a*b + (4*a^2 + b^2)*cosh(d*x + c))*sinh
(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2)) + 8*((a^2 - b^2)*cosh(d*x + c)^4 + 4*(a^2 - b^2)*cosh(d*x + c)*sinh(d*
x + c)^3 + (a^2 - b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*
(3*(a^2 - b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - b^2 + 4
*((a^2 - b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*s
qrt((a*cosh(d*x + c) + b)/cosh(d*x + c)))/((a^2 - b^2)*d*cosh(d*x + c)^4 +
4*(a^2 - b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 - b^2)*d*sinh(d*x + c)
^4 - 2*(a^2 - b^2)*d*cosh(d*x + c)^2 + 2*(3*(a^2 - b^2)*d*cosh(d*x + c)^2 -
 (a^2 - b^2)*d)*sinh(d*x + c)^2 + (a^2 - b^2)*d...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\operatorname{sech}\left(c + dx\right)}\,\coth^{3}\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3*(a+b*sech(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sech(c + d*x))*coth(c + d*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sech(d*x + c) + a)*coth(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{coth}(c+d\,x)^3\,\sqrt{a+\frac{b}{\cosh\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^3*(a + b/cosh(c + d*x))^(1/2),x)
```

```
[Out] int(coth(c + d*x)^3*(a + b/cosh(c + d*x))^(1/2), x)
```

3.130 $\int \sqrt{a+b\,\mathrm{sech}(c+dx)}\,\tanh^2(c+dx)\,dx$

Optimal. Leaf size=344

$$-\frac{2a(a-b)\sqrt{a+b}\,\coth(c+dx)E\left(\mathrm{ArcSin}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\mathrm{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+}{}}}{3b^2d}$$

```
[Out] -2/3*a*(a-b)*coth(d*x+c)*EllipticE((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+
b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*
x+c))/(a-b))^(1/2)/b^2/d-2/3*(a+2*b)*coth(d*x+c)*EllipticF((a+b*sech(d*x+c)
)^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+c))/(a+
b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/b/d+2*coth(d*x+c)*EllipticPi((a+
b*sech(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(
b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/d-2/3*(a+b*
sech(d*x+c))^(1/2)*tanh(d*x+c)/d
```

Rubi [A]
time = 0.28, antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3979, 4142, 4143, 4006, 3869, 3917, 4089}

$$-\frac{2a(a-b)\sqrt{a+b}\,\coth(c+dx)\sqrt{\frac{b(1-\mathrm{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\mathrm{sech}(c+dx)+1)}{a-b}}\,E\left(\mathrm{ArcSin}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3b^2d}-\frac{2\sqrt{a+b}\,(a+2b)\coth(c+dx)\sqrt{\frac{b(1-\mathrm{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\mathrm{sech}(c+dx)+1)}{a-b}}\,F\left(\mathrm{ArcSin}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3bd}+\frac{2\sqrt{a+b}\,\coth(c+dx)\sqrt{\frac{b(1-\mathrm{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\mathrm{sech}(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\mathrm{ArcSin}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d}-\frac{2\tanh(c+dx)\sqrt{a+b\,\mathrm{sech}(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x]^2,x]
```

```
[Out] (-2*a*(a - b)*Sqrt[a + b]*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c
+ d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)
]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(3*b^2*d) - (2*Sqrt[a + b]*(a +
 2*b)*Coth[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]]
, (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec
h[c + d*x]))/(a - b))])/(3*b*d) + (2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(
a + b)/a, ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*S
qrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b)
)])/d - (2*Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x])/(3*d)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3979

```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4142

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_.), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^
m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[
a*A*(m + 1) + (A*b*(m + 1) + b*C*m)*Csc[e + f*x] + a*C*m*Csc[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m,
0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\text{sech}(c+dx)}\tanh^2(c+dx)\,dx = -\int \sqrt{a+b\text{sech}(c+dx)}\left(-1+\text{sech}^2(c+dx)\right)\,dx$$

$$= -\frac{2\sqrt{a+b\text{sech}(c+dx)}\tanh(c+dx)}{3d} - \frac{2}{3}\int \frac{-\frac{3a}{2} - b\text{sech}(c+}{\sqrt{a+b}}$$

$$= -\frac{2\sqrt{a+b\text{sech}(c+dx)}\tanh(c+dx)}{3d} - \frac{2}{3}\int \frac{-\frac{3a}{2} + \left(-\frac{a}{2} - b\right)}{\sqrt{a+b\text{sech}(}}$$

$$= -\frac{2a(a-b)\sqrt{a+b}\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx}}{\sqrt{a+b}}\right.\right.}{3b^2}$$

$$= -\frac{2a(a-b)\sqrt{a+b}\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx}}{\sqrt{a+b}}\right.\right.}{3b^2}$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x]^2,x]
[Out] $Aborted
```

Maple [F]
time = 2.12, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\,\text{sech}\,(dx+c)}\,\left(\tanh^2(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^2,x)
[Out] int((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^2,x, algorithm="maxima")
[Out] integrate(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^2,x, algorithm="fricas")
[Out] integral(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\operatorname{sech}(c + dx)}\tanh^2(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))**(1/2)*tanh(d*x+c)**2,x)
[Out] Integral(sqrt(a + b*sech(c + d*x))*tanh(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^2,x, algorithm="giac")
[Out] integrate(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \tanh(c + d\,x)^2\,\sqrt{a + \frac{b}{\cosh(c + d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^2*(a + b/cosh(c + d*x))^(1/2),x)
[Out] int(tanh(c + d*x)^2*(a + b/cosh(c + d*x))^(1/2), x)
```

3.131 $\int \sqrt{a+b\,\mathrm{sech}(c+dx)}\,dx$

Optimal. Leaf size=125

$$\frac{2\coth(c+dx)\Pi\left(\frac{a}{a+b};\mathrm{ArcSin}\left(\frac{\sqrt{a+b}}{\sqrt{a+b\,\mathrm{sech}(c+dx)}}\right)|\frac{a-b}{a+b}\right)\sqrt{-\frac{b(1-\mathrm{sech}(c+dx))}{a+b\,\mathrm{sech}(c+dx)}}\sqrt{\frac{b(1+\mathrm{sech}(c+dx}{a+b\,\mathrm{sech}(c+dx}}}{\sqrt{a+b}\,d}$$

```
[Out] 2*coth(d*x+c)*EllipticPi((a+b)^(1/2)/(a+b*sech(d*x+c))^(1/2),a/(a+b),((a-b)
/(a+b))^(1/2))*(a+b*sech(d*x+c))*(-b*(1-sech(d*x+c))/(a+b*sech(d*x+c)))^(1/
2)*(b*(1+sech(d*x+c))/(a+b*sech(d*x+c)))^(1/2)/d/(a+b)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$,
Rules used = $\{3865\}$

$$\frac{2\coth(c+dx)\sqrt{-\frac{b(1-\mathrm{sech}(c+dx))}{a+b\,\mathrm{sech}(c+dx)}}\sqrt{\frac{b(\mathrm{sech}(c+dx)+1)}{a+b\,\mathrm{sech}(c+dx)}}(a+b\,\mathrm{sech}(c+dx))\Pi\left(\frac{a}{a+b};\mathrm{ArcSin}\left(\frac{\sqrt{a+b}}{\sqrt{a+b\,\mathrm{sech}(c+dx)}}\right)|\frac{a-b}{a+b}\right)}{d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*Coth[c + d*x]*EllipticPi[a/(a + b), ArcSin[Sqrt[a + b]/Sqrt[a + b*Sech[c
 + d*x]]], (a - b)/(a + b)]*Sqrt[-((b*(1 - Sech[c + d*x]))/(a + b*Sech[c +
d*x]))]*Sqrt[(b*(1 + Sech[c + d*x]))/(a + b*Sech[c + d*x])]*(a + b*Sech[c +
 d*x]))/(Sqrt[a + b]*d)
```

Rule 3865

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*((a + b
*Csc[c + d*x])/(d*Rt[a + b, 2]*Cot[c + d*x]))*Sqrt[b*((1 + Csc[c + d*x])/(a
 + b*Csc[c + d*x]))]*Sqrt[(-b)*((1 - Csc[c + d*x])/(a + b*Csc[c + d*x]))]*E
llipticPi[a/(a + b), ArcSin[Rt[a + b, 2]/Sqrt[a + b*Csc[c + d*x]]], (a - b)
/(a + b)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\,\mathrm{sech}(c+dx)}\,dx = \frac{2\coth(c+dx)\Pi\left(\frac{a}{a+b};\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b\,\mathrm{sech}(c+dx)}}\right)|\frac{a-b}{a+b}\right)\sqrt{-\frac{b(1-\mathrm{sech}(c}{a+b\,\mathrm{sech}(c}}}{\sqrt{a+b}\,d}$$

Mathematica [F]
time = 5.46, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\text{sech}(c + dx)}\, dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sech[c + d*x]],x]
[Out] Integrate[Sqrt[a + b*Sech[c + d*x]], x]
```

Maple [F]
time = 2.37, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\,\text{sech}\,(dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c))^(1/2),x)
[Out] int((a+b*sech(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sech(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sech(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\,\text{sech}\,(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sech(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sech(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+\frac{b}{\cosh\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x))^(1/2),x)
```

```
[Out] int((a + b/cosh(c + d*x))^(1/2), x)
```

3.132 $\int \coth^2(c+dx)\sqrt{a+b\,\mathrm{sech}(c+dx)}\,dx$

Optimal. Leaf size=246

$$\frac{\sqrt{a+b}\,\coth(c+dx)F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\,\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx}{a-b}}}{d}$$

```
[Out] coth(d*x+c)*EllipticF((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/
2))*(a+b)^(1/2)*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^
(1/2)/d+2*coth(d*x+c)*EllipticPi((a+b)^(1/2)/(a+b*sech(d*x+c))^(1/2),a/(a+b
),((a-b)/(a+b))^(1/2))*(a+b*sech(d*x+c))*(-b*(1-sech(d*x+c))/(a+b*sech(d*x+
c)))^(1/2)*(b*(1+sech(d*x+c))/(a+b*sech(d*x+c)))^(1/2)/d/(a+b)^(1/2)-coth(d
*x+c)*(a+b*sech(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.15, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3981, 3865, 3960, 3917}

$$\frac{\sqrt{a+b}\,\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\,\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d}+\frac{2\coth(c+dx)\sqrt{-\frac{b(1-\text{sech}(c+dx))}{a+b\,\text{sech}(c+dx)}}\sqrt{\frac{b(\text{sech}(c+dx)+1)}{a+b\,\text{sech}(c+dx)}}\,(a+b\,\text{sech}(c+dx))\Pi\left(\frac{a}{a+b};\text{ArcSin}\left(\frac{\sqrt{a+b}}{\sqrt{a+b\,\text{sech}(c+dx)}}\right)|\frac{a-b}{a+b}\right)}{d\sqrt{a+b}}-\frac{\coth(c+dx)\sqrt{a+b\,\text{sech}(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^2*Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (Sqrt[a + b]*Coth[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(
1 + Sech[c + d*x]))/(a - b))])/d - (Coth[c + d*x]*Sqrt[a + b*Sech[c + d*x]]
)/d + (2*Coth[c + d*x]*EllipticPi[a/(a + b), ArcSin[Sqrt[a + b]/Sqrt[a + b*
Sech[c + d*x]]], (a - b)/(a + b)]*Sqrt[-((b*(1 - Sech[c + d*x]))/(a + b*Sec
h[c + d*x]))]*Sqrt[(b*(1 + Sech[c + d*x]))/(a + b*Sech[c + d*x])]*(a + b*Se
ch[c + d*x]))/(Sqrt[a + b]*d)
```

Rule 3865

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*((a + b
*Csc[c + d*x])/(d*Rt[a + b, 2]*Cot[c + d*x]))*Sqrt[b*((1 + Csc[c + d*x])/(a
 + b*Csc[c + d*x]))]*Sqrt[(-b)*((1 - Csc[c + d*x])/(a + b*Csc[c + d*x]))]*E
llipticPi[a/(a + b), ArcSin[Rt[a + b, 2]/Sqrt[a + b*Csc[c + d*x]]], (a - b)
/(a + b)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
```

```
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[Tan[e + f*x]*((a + b*Csc[e + f*x])^m/f), x] + Dist[b*m, I
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
m}, x]
```

Rule 3981

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Sec[c + d
*x]^2)^(-m/2), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] &&
ILtQ[m/2, 0] && IntegerQ[n - 1/2] && EqQ[m, -2]
```

Rubi steps

$$\int \coth^2(c+dx)\sqrt{a+b\operatorname{sech}(c+dx)}\,dx = -\int \left(-\sqrt{a+b\operatorname{sech}(c+dx)} - \operatorname{csch}^2(c+dx)\sqrt{a+b\operatorname{sech}(c+d$$

$$= \int \sqrt{a+b\operatorname{sech}(c+dx)}\,dx + \int \operatorname{csch}^2(c+dx)\sqrt{a+b\operatorname{sech}(c+d$$

$$= -\frac{\coth(c+dx)\sqrt{a+b\operatorname{sech}(c+dx)}}{d} + \frac{2\coth(c+dx)\Pi\left(\frac{a}{a+b};\operatorname{si}\right.}{}$$

$$= \frac{\sqrt{a+b}\,\coth(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{}}{d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 539 vs. 2(246) = 492.
time = 16.04, size = 539, normalized size = 2.19

$$-\frac{\coth(c+dx)\sqrt{a+b\operatorname{sech}(c+dx)}}{d}+\frac{\sqrt{a+b\operatorname{sech}(c+dx)}\left(\frac{2\sqrt{b}\,(a-a\cosh(c+dx))^{3/2}\sqrt{\frac{(a+b)(a+a\cosh(c+dx))}{(a-b)(a-a\cosh(c+dx))}}\,F\left(\operatorname{ArcSin}\left(\frac{\sqrt{a}\sqrt{b+a\cosh(c+dx)}}{\sqrt{b}\sqrt{a-a\cosh(c+dx)}}\right)|-\frac{2b}{a-b}\right)\sinh(c+dx)}{a^{3/2}\sqrt{-1+\cosh(c+dx)}\sqrt{1+\cosh(c+dx)}\sqrt{-\frac{a(a+b)\cosh(c+dx)}{b(a-a\cosh(c+dx))}}\left(-\frac{a-a\cosh(c+dx)}{a}\right)^{3/2}\sqrt{\frac{a+a\cosh(c+dx)}{a}}\sqrt{\operatorname{sech}(c+dx)}}-\frac{4b(a-a\cosh(c+dx))\Pi\left(\frac{a+b}{a};\operatorname{ArcSin}\left(\frac{\sqrt{a}\sqrt{b+a\cosh(c+dx)}}{\sqrt{a+b}\sqrt{a\cosh(c+dx)}}\right)|\frac{a+b}{a-b}\right)\sqrt{-\frac{b(a+a\cosh(c+dx))\operatorname{sech}(c+dx)}{a(a-b)}}\sinh(c+dx)}{\sqrt{a}\sqrt{a+b}\sqrt{-1+\cosh(c+dx)}\sqrt{a\cosh(c+dx)}\sqrt{1+\cosh(c+dx)}\sqrt{-\frac{a-a\cosh(c+dx)}{a}}\sqrt{\frac{a+a\cosh(c+dx)}{a}}\sqrt{\operatorname{sech}(c+dx)}\sqrt{-\frac{b(a-a\cosh(c+dx))\operatorname{sech}(c+dx)}{a(a+b)}}}\right)}{2d\sqrt{b+a\cosh(c+dx)}\sqrt{\operatorname{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^2*Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] -((Coth[c + d*x]*Sqrt[a + b*Sech[c + d*x]])/d) + (Sqrt[a + b*Sech[c + d*x]]
*((2*Sqrt[b]*(a - a*Cosh[c + d*x])^(3/2)*Sqrt[((a + b)*(a + a*Cosh[c + d*x]
))/((a - b)*(a - a*Cosh[c + d*x]))]*EllipticF[ArcSin[(Sqrt[a]*Sqrt[b + a*Co
sh[c + d*x]])/(Sqrt[b]*Sqrt[a - a*Cosh[c + d*x]])], (-2*b)/(a - b)]*Sinh[c
+ d*x])/(a^(3/2)*Sqrt[-1 + Cosh[c + d*x]]*Sqrt[1 + Cosh[c + d*x]]*Sqrt[-((a
*(a + b)*Cosh[c + d*x])/(b*(a - a*Cosh[c + d*x])))]*(-((a - a*Cosh[c + d*x]
)/a))^(3/2)*Sqrt[(a + a*Cosh[c + d*x])/a]*Sqrt[Sech[c + d*x]]) - (4*b*(a -
a*Cosh[c + d*x])*EllipticPi[(a + b)/a, ArcSin[(Sqrt[a]*Sqrt[b + a*Cosh[c +
d*x]])/(Sqrt[a + b]*Sqrt[a*Cosh[c + d*x]])], (a + b)/(a - b)]*Sqrt[-((b*(a
+ a*Cosh[c + d*x])*Sech[c + d*x])/(a*(a - b)))]*Sinh[c + d*x])/(Sqrt[a]*Sqr
t[a + b]*Sqrt[-1 + Cosh[c + d*x]]*Sqrt[a*Cosh[c + d*x]]*Sqrt[1 + Cosh[c + d
*x]]*Sqrt[-((a - a*Cosh[c + d*x])/a)]*Sqrt[(a + a*Cosh[c + d*x])/a]*Sqrt[Se
ch[c + d*x]]*Sqrt[-((b*(a - a*Cosh[c + d*x])*Sech[c + d*x])/(a*(a + b)))]))
)/(2*d*Sqrt[b + a*Cosh[c + d*x]]*Sqrt[Sech[c + d*x]])
```

Maple [**F**]
time = 2.65, size = 0, normalized size = 0.00

$$\int \left(\coth^2\left(dx+c\right)\right)\sqrt{a+b\,\mathrm{sech}\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2*(a+b*sech(d*x+c))^(1/2),x)
```

```
[Out] int(coth(d*x+c)^2*(a+b*sech(d*x+c))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(d*x + c) + a)*coth(d*x + c)^2, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sech(d*x + c) + a)*coth(d*x + c)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\operatorname{sech}(c + dx)}\, \coth^2(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2*(a+b*sech(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sech(c + d*x))*coth(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sech(d*x + c) + a)*coth(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \coth(c + d\,x)^2\, \sqrt{a + \frac{b}{\cosh(c + d\,x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^2*(a + b/cosh(c + d*x))^(1/2),x)
```

```
[Out] int(coth(c + d*x)^2*(a + b/cosh(c + d*x))^(1/2), x)
```

3.133 $\int \frac{\tanh^5(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx$

Optimal. Leaf size=148

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}+\frac{2a(a^2-2b^2)\sqrt{a+b\text{sech}(c+dx)}}{b^4d}-\frac{2(3a^2-2b^2)(a+b\text{sech}(c+dx))^{3/2}}{3b^4d}$$

```
[Out] -2/3*(3*a^2-2*b^2)*(a+b*sech(d*x+c))^(3/2)/b^4/d+6/5*a*(a+b*sech(d*x+c))^(5
/2)/b^4/d-2/7*(a+b*sech(d*x+c))^(7/2)/b^4/d+2*arctanh((a+b*sech(d*x+c))^(1/
2)/a^(1/2))/d/a^(1/2)+2*a*(a^2-2*b^2)*(a+b*sech(d*x+c))^(1/2)/b^4/d
```

Rubi [A]
time = 0.11, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3970, 912, 1167, 213}

$$-\frac{2(3a^2-2b^2)(a+b\text{sech}(c+dx))^{3/2}}{3b^4d}+\frac{2a(a^2-2b^2)\sqrt{a+b\text{sech}(c+dx)}}{b^4d}-\frac{2(a+b\text{sech}(c+dx))^{7/2}}{7b^4d}+\frac{6a(a+b\text{sech}(c+dx))^{5/2}}{5b^4d}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^5/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) + (2*a*(a^2 - 2*
b^2)*Sqrt[a + b*Sech[c + d*x]])/(b^4*d) - (2*(3*a^2 - 2*b^2)*(a + b*Sech[c
+ d*x])^(3/2))/(3*b^4*d) + (6*a*(a + b*Sech[c + d*x])^(5/2))/(5*b^4*d) - (2
*(a + b*Sech[c + d*x])^(7/2))/(7*b^4*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\tanh^5(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = -\frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x\sqrt{a+x}}\,dx, x, b\text{sech}(c+dx)\right)}{b^4 d}$$

$$= -\frac{2\text{Subst}\left(\int \frac{(-a^2+b^2+2ax^2-x^4)^2}{-a+x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{b^4 d}$$

$$= -\frac{2\text{Subst}\left(\int \left(-a^3+2ab^2+(3a^2-2b^2)x^2-3ax^4+x^6+\frac{b^4}{-a+x^2}\right)dx, x, \sqrt{a-\cdots}\right.}{b^4 d}$$

$$= \frac{2a(a^2-2b^2)\sqrt{a+b\text{sech}(c+dx)}}{b^4 d} - \frac{2(3a^2-2b^2)(a+b\text{sech}(c+dx))^{3/2}}{3b^4 d} + \cdots$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d} + \frac{2a(a^2-2b^2)\sqrt{a+b\text{sech}(c+dx)}}{b^4 d} - \cdots$$

Mathematica [A]

time = 3.84, size = 167, normalized size = 1.13

$$\frac{2\left(48a^4-140a^2b^2+\frac{105b^4\tanh^{-1}\left(\frac{\sqrt{b+a\cosh(c+dx)}}{\sqrt{a\cosh(c+dx)}}\right)\sqrt{b+a\cosh(c+dx)}}{\sqrt{a\cosh(c+dx)}}+(24a^3b-70ab^3)\text{sech}(c+dx)+(-6a^2b^2+70b^4)\text{sech}^2(c+dx)+3ab^3\text{sech}^3(c+dx)-15b^4\text{sech}^4(c+dx)\right)}{105b^4d\sqrt{a+b\text{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^5/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*(48*a^4 - 140*a^2*b^2 + (105*b^4*ArcTanh[Sqrt[b + a*Cosh[c + d*x]]/Sqrt[
a*Cosh[c + d*x]]]*Sqrt[b + a*Cosh[c + d*x]])/Sqrt[a*Cosh[c + d*x]] + (24*a^
3*b - 70*a*b^3)*Sech[c + d*x] + (-6*a^2*b^2 + 70*b^4)*Sech[c + d*x]^2 + 3*a
```

```
*b^3*Sech[c + d*x]^3 - 15*b^4*Sech[c + d*x]^4))/(105*b^4*d*Sqrt[a + b*Sech[
c + d*x]])
```

Maple [F]
time = 3.22, size = 0, normalized size = 0.00

$$\int \frac{\tanh^5(dx+c)}{\sqrt{a+b\,\mathrm{sech}\,(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(1/2),x)
[Out] int(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(tanh(d*x + c)^5/sqrt(b*sech(d*x + c) + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1277 vs. 2(128) = 256.
time = 0.79, size = 2813, normalized size = 19.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
[Out] [1/210*(105*(b^4*cosh(d*x + c)^6 + 6*b^4*cosh(d*x + c)*sinh(d*x + c)^5 + b^
4*sinh(d*x + c)^6 + 3*b^4*cosh(d*x + c)^4 + 3*b^4*cosh(d*x + c)^2 + 3*(5*b^
4*cosh(d*x + c)^2 + b^4)*sinh(d*x + c)^4 + b^4 + 4*(5*b^4*cosh(d*x + c)^3 +
 3*b^4*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*b^4*cosh(d*x + c)^4 + 6*b^4*co
sh(d*x + c)^2 + b^4)*sinh(d*x + c)^2 + 6*(b^4*cosh(d*x + c)^5 + 2*b^4*cosh(
d*x + c)^3 + b^4*cosh(d*x + c))*sinh(d*x + c))*sqrt(a)*log(-(2*a^2*cosh(d*x
 + c)^4 + 2*a^2*sinh(d*x + c)^4 + 4*a*b*cosh(d*x + c)^3 + 4*(2*a^2*cosh(d*x
 + c) + a*b)*sinh(d*x + c)^3 + 4*a*b*cosh(d*x + c) + (4*a^2 + b^2)*cosh(d*x
 + c)^2 + (12*a^2*cosh(d*x + c)^2 + 12*a*b*cosh(d*x + c) + 4*a^2 + b^2)*sin
h(d*x + c)^2 + 2*a^2 + 2*(a*cosh(d*x + c)^4 + a*sinh(d*x + c)^4 + b*cosh(d*
x + c)^3 + (4*a*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*a*cosh(d*x + c)^2 +
(6*a*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a)*sinh(d*x + c)^2 + b*cosh(d*
x + c) + (4*a*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 4*a*cosh(d*x + c) + b
```

)*sinh(d*x + c) + a)*sqrt(a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 2*
(4*a^2*cosh(d*x + c)^3 + 6*a*b*cosh(d*x + c)^2 + 2*a*b + (4*a^2 + b^2)*cosh
(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c)
+ sinh(d*x + c)^2)) + 16*((12*a^4 - 35*a^2*b^2)*cosh(d*x + c)^6 + (12*a^4 -
 35*a^2*b^2)*sinh(d*x + c)^6 - (12*a^3*b - 35*a*b^3)*cosh(d*x + c)^5 - (12*
a^3*b - 35*a*b^3 - 6*(12*a^4 - 35*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 +
 3*(12*a^4 - 29*a^2*b^2)*cosh(d*x + c)^4 + (36*a^4 - 87*a^2*b^2 + 15*(12*a^
4 - 35*a^2*b^2)*cosh(d*x + c)^2 - 5*(12*a^3*b - 35*a*b^3)*cosh(d*x + c))*si
nh(d*x + c)^4 + 12*a^4 - 35*a^2*b^2 - 8*(3*a^3*b - 5*a*b^3)*cosh(d*x + c)^3
 - 2*(12*a^3*b - 20*a*b^3 - 10*(12*a^4 - 35*a^2*b^2)*cosh(d*x + c)^3 + 5*(1
2*a^3*b - 35*a*b^3)*cosh(d*x + c)^2 - 6*(12*a^4 - 29*a^2*b^2)*cosh(d*x + c)
)*sinh(d*x + c)^3 + 3*(12*a^4 - 29*a^2*b^2)*cosh(d*x + c)^2 + (15*(12*a^4 -
 35*a^2*b^2)*cosh(d*x + c)^4 + 36*a^4 - 87*a^2*b^2 - 10*(12*a^3*b - 35*a*b^
3)*cosh(d*x + c)^3 + 18*(12*a^4 - 29*a^2*b^2)*cosh(d*x + c)^2 - 24*(3*a^3*b
 - 5*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - (12*a^3*b - 35*a*b^3)*cosh(d*x
 + c) + (6*(12*a^4 - 35*a^2*b^2)*cosh(d*x + c)^5 - 5*(12*a^3*b - 35*a*b^3)*
cosh(d*x + c)^4 - 12*a^3*b + 35*a*b^3 + 12*(12*a^4 - 29*a^2*b^2)*cosh(d*x +
 c)^3 - 24*(3*a^3*b - 5*a*b^3)*cosh(d*x + c)^2 + 6*(12*a^4 - 29*a^2*b^2)*co
sh(d*x + c))*sinh(d*x + c))*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)))/(a*b
^4*d*cosh(d*x + c)^6 + 6*a*b^4*d*cosh(d*x + c)*sinh(d*x + c)^5 + a*b^4*d*si
nh(d*x + c)^6 + 3*a*b^4*d*cosh(d*x + c)^4 + 3*a*b^4*d*cosh(d*x + c)^2 + a*b
^4*d + 3*(5*a*b^4*d*cosh(d*x + c)^2 + a*b^4*d)*sinh(d*x + c)^4 + 4*(5*a*b^4
*d*cosh(d*x + c)^3 + 3*a*b^4*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*a*b^4*
d*cosh(d*x + c)^4 + 6*a*b^4*d*cosh(d*x + c)^2 + a*b^4*d)*sinh(d*x + c)^2 +
6*(a*b^4*d*cosh(d*x + c)^5 + 2*a*b^4*d*cosh(d*x + c)^3 + a*b^4*d*cosh(d*x +
 c))*sinh(d*x + c)), -1/105*(105*(b^4*cosh(d*x + c)^6 + 6*b^4*cosh(d*x + c)
*sinh(d*x + c)^5 + b^4*sinh(d*x + c)^6 + 3*b^4*cosh(d*x + c)^4 + 3*b^4*cosh
(d*x + c)^2 + 3*(5*b^4*cosh(d*x + c)^2 + b^4)*sinh(d*x + c)^4 + b^4 + 4*(5*
b^4*cosh(d*x + c)^3 + 3*b^4*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*b^4*cosh(
d*x + c)^4 + 6*b^4*cosh(d*x + c)^2 + b^4)*sinh(d*x + c)^2 + 6*(b^4*cosh(d*x
 + c)^5 + 2*b^4*cosh(d*x + c)^3 + b^4*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a
)*arctan((a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + b*cosh(d*x + c) + (2*a*co
sh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(-a)*sqrt((a*cosh(d*x + c) + b)/cos
h(d*x + c))/(a^2*cosh(d*x + c)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c
) + a^2 + 2*(a^2*cosh(d*x + c) + a*b)*sinh(d*x + c))) - 8*((12*a^4 - 35*a^2
*b^2)*cosh(d*x + c)^6 + (12*a^4 - 35*a^2*b^2)*sinh(d*x + c)^6 - (12*a^3*b -
 35*a*b^3)*cosh(d*x + c)^5 - (12*a^3*b - 35*a*b^3 - 6*(12*a^4 - 35*a^2*b^2)
*cosh(d*x + c))*sinh(d*x + c)^5 + 3*(12*a^4 - 29*a^2*b^2)*cosh(d*x + c)^4 +
 (36*a^4 - 87*a^2*b^2 + 15*(12*a^4 - 35*a^2*b^2)*cosh(d*x + c)^2 - 5*(12*a^
3*b - 35*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 12*a^4 - 35*a^2*b^2 - 8*(3
*a^3*b - 5*a*b^3)*cosh(d*x + c)^3 - 2*(12*a^3*b - 20*a*b^3 - 10*(12*a^4 - 3
5*a^2*b^2)*cosh(d*x + c)^3 + 5*(12*a^3*b - 35*a*b^3)*cosh(d*x + c)^2 - 6*(1
2*a^4 - 29*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(12*a^4 - 29*a^2*b^2
)*cosh(d*x + c)^2 + (15*(12*a^4 - 35*a^2*b^2)*cosh(d*x + c)^4 + 36*a^4 - 87
*a^2*b^2 - 10*(12*a^3*b - 35*a*b^3)*cosh(d*x + c)^3 + 18*(12*a^4 - 29*a^2*b

```
^2)*cosh(d*x + c)^2 - 24*(3*a^3*b - 5*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^2
 - (12*a^3*b - 35*a*b^3)*cosh(d*x + c) + (6*(12*a^4 - 35*a^2*b^2)*cosh(d*x
+ c)^5 - 5*(12*a^3*b - 35*a*b^3)*cosh(d*x + c)^4 - 12*a^3*b + 35*a*b^3 + 12
*(12*a^4 - 29*a^2*b^2)*cosh(d*x + c)^3 - 24*(3*a^3*b - 5*a*b^3)*cosh(d*x +
c)^2 + 6*(12*a^4 - 29*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a*cosh(d
*x + c) + b)/cosh(d*x + c)))/(a*b^4*d*cosh(d*x + c)^6 + 6*a*b^4*d*cosh(d*x
+ c)*sinh(d*x + c)^5 + a*b^4*d*sinh(d*x + c)^6 + 3*a*b^4*d*cosh(d*x + c)^4
+ 3*a*b^4*d*cosh(d*x + c)^2 + a*b^4*d + 3*(5*a*b^4*d*cosh(d*x + c)^2 + a*b^
4*d)*sinh(d*x + c)^4 + 4*(5*a*b^4*d*cosh(d*x + ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^5(c+dx)}{\sqrt{a+b\,\mathrm{sech}\,(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**5/(a+b*sech(d*x+c))**(1/2),x)
```

```
[Out] Integral(tanh(c + d*x)**5/sqrt(a + b*sech(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(d*x + c)^5/sqrt(b*sech(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(c+d\,x)^5}{\sqrt{a+\frac{b}{\cosh\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^5/(a + b/cosh(c + d*x))^(1/2),x)
```

```
[Out] int(tanh(c + d*x)^5/(a + b/cosh(c + d*x))^(1/2), x)
```

3.134 $\int \frac{\tanh^3(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx$

Optimal. Leaf size=79

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}-\frac{2a\sqrt{a+b\text{sech}(c+dx)}}{b^2d}+\frac{2(a+b\text{sech}(c+dx))^{3/2}}{3b^2d}$$

```
[Out] 2/3*(a+b*sech(d*x+c))^(3/2)/b^2/d+2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2)
)/d/a^(1/2)-2*a*(a+b*sech(d*x+c))^(1/2)/b^2/d
```

Rubi [A]
time = 0.08, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.174$, Rules used = {3970, 912, 1167, 213}

$$\frac{2(a+b\text{sech}(c+dx))^{3/2}}{3b^2d}-\frac{2a\sqrt{a+b\text{sech}(c+dx)}}{b^2d}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^3/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) - (2*a*Sqrt[a +
b*Sech[c + d*x]])/(b^2*d) + (2*(a + b*Sech[c + d*x])^(3/2))/(3*b^2*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
```

```
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^3(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx &= -\frac{\text{Subst}\left(\int \frac{b^2-x^2}{x\sqrt{a+x}}\,dx, x, b\text{sech}(c+dx)\right)}{b^2d} \\
&= -\frac{2\text{Subst}\left(\int \frac{-a^2+b^2+2ax^2-x^4}{-a+x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{b^2d} \\
&= -\frac{2\text{Subst}\left(\int \left(a-x^2+\frac{b^2}{-a+x^2}\right)dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{b^2d} \\
&= -\frac{2a\sqrt{a+b\text{sech}(c+dx)}}{b^2d}+\frac{2(a+b\text{sech}(c+dx))^{3/2}}{3b^2d}-\frac{2\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x,\right.}{} \\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}-\frac{2a\sqrt{a+b\text{sech}(c+dx)}}{b^2d}+\frac{2(a+b\text{sech}(c}{3b^2d}
\end{aligned}$$

Mathematica [A]
time = 0.44, size = 111, normalized size = 1.41

$$\frac{2\left(-2a^2+\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{b+a\cosh(c+dx)}}{\sqrt{a\cosh(c+dx)}}\right)\sqrt{b+a\cosh(c+dx)}}{\sqrt{a\cosh(c+dx)}}-ab\text{sech}(c+dx)+b^2\text{sech}^2(c+dx)\right)}{3b^2d\sqrt{a+b\text{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^3/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*(-2*a^2 + (3*b^2*ArcTanh[Sqrt[b + a*Cosh[c + d*x]]/Sqrt[a*Cosh[c + d*x]]
]*Sqrt[b + a*Cosh[c + d*x]])/Sqrt[a*Cosh[c + d*x]] - a*b*Sech[c + d*x] + b^
2*Sech[c + d*x]^2))/(3*b^2*d*Sqrt[a + b*Sech[c + d*x]])
```

Maple [F]
time = 2.67, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(dx+c)}{\sqrt{a+b\,\mathrm{sech}\,(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x)
[Out] int(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(tanh(d*x + c)^3/sqrt(b*sech(d*x + c) + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(67) = 134$.
time = 0.80, size = 925, normalized size = 11.71

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
[Out] [1/6*(3*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh
(d*x + c)^2 + b^2)*sqrt(a)*log(-(2*a^2*cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c
)^4 + 4*a*b*cosh(d*x + c)^3 + 4*(2*a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)^3
 + 4*a*b*cosh(d*x + c) + (4*a^2 + b^2)*cosh(d*x + c)^2 + (12*a^2*cosh(d*x +
 c)^2 + 12*a*b*cosh(d*x + c) + 4*a^2 + b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*
cosh(d*x + c)^4 + a*sinh(d*x + c)^4 + b*cosh(d*x + c)^3 + (4*a*cosh(d*x + c
) + b)*sinh(d*x + c)^3 + 2*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*c
osh(d*x + c) + 2*a)*sinh(d*x + c)^2 + b*cosh(d*x + c) + (4*a*cosh(d*x + c)^
3 + 3*b*cosh(d*x + c)^2 + 4*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)
*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a
*b*cosh(d*x + c)^2 + 2*a*b + (4*a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))/(c
osh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 8*(a^2
*cosh(d*x + c)^2 + a^2*sinh(d*x + c)^2 - a*b*cosh(d*x + c) + a^2 + (2*a^2*c
osh(d*x + c) - a*b)*sinh(d*x + c))*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)
))/(a*b^2*d*cosh(d*x + c)^2 + 2*a*b^2*d*cosh(d*x + c)*sinh(d*x + c) + a*b^2
```

```
*d*sinh(d*x + c)^2 + a*b^2*d), -1/3*(3*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*
x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 + b^2)*sqrt(-a)*arctan((a*cosh(d
*x + c)^2 + a*sinh(d*x + c)^2 + b*cosh(d*x + c) + (2*a*cosh(d*x + c) + b)*s
inh(d*x + c) + a)*sqrt(-a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c))/(a^2*c
osh(d*x + c)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + a^2 + 2*(a^2*c
osh(d*x + c) + a*b)*sinh(d*x + c))) + 4*(a^2*cosh(d*x + c)^2 + a^2*sinh(d*x
 + c)^2 - a*b*cosh(d*x + c) + a^2 + (2*a^2*cosh(d*x + c) - a*b)*sinh(d*x +
c))*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)))/(a*b^2*d*cosh(d*x + c)^2 + 2
*a*b^2*d*cosh(d*x + c)*sinh(d*x + c) + a*b^2*d*sinh(d*x + c)^2 + a*b^2*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(c+dx)}{\sqrt{a+b\,\mathrm{sech}\,(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**3/(a+b*sech(d*x+c))**(1/2),x)
[Out] Integral(tanh(c + d*x)**3/sqrt(a + b*sech(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(tanh(d*x + c)^3/sqrt(b*sech(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(c+d\,x)^3}{\sqrt{a+\frac{b}{\cosh\,(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^3/(a + b/cosh(c + d*x))^(1/2),x)
[Out] int(tanh(c + d*x)^3/(a + b/cosh(c + d*x))^(1/2), x)
```

3.135 $\int \frac{\tanh(c+dx)}{\sqrt{a+b\,\mathrm{sech}(c+dx)}}\,dx$

Optimal. Leaf size=31

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

```
[Out] 2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3970, 65, 213}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\tanh(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+x}}\,dx, x, b\text{sech}(c+dx)\right)}{d}$$
$$= -\frac{2\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{d}$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 73 vs. 2(31) = 62.
time = 0.10, size = 73, normalized size = 2.35

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b+a\cosh(c+dx)}}{\sqrt{a\cosh(c+dx)}}\right)\sqrt{b+a\cosh(c+dx)}}{d\sqrt{a\cosh(c+dx)}\sqrt{a+b\text{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sqrt[b + a*Cosh[c + d*x]]/Sqrt[a*Cosh[c + d*x]]]*Sqrt[b + a*Cosh
[c + d*x]])/(d*Sqrt[a*Cosh[c + d*x]]*Sqrt[a + b*Sech[c + d*x]])
```

Maple [A]
time = 1.62, size = 26, normalized size = 0.84

method	result	size
derivativedivides	$\frac{2\,\text{arctanh}\left(\frac{\sqrt{a+b\,\text{sech}\,(dx+c)}}{\sqrt{a}}\right)}{d\sqrt{a}}$	26
default	$\frac{2\,\text{arctanh}\left(\frac{\sqrt{a+b\,\text{sech}\,(dx+c)}}{\sqrt{a}}\right)}{d\sqrt{a}}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)/(a+b*sech(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(d*x + c)/sqrt(b*sech(d*x + c) + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(25) = 50.
time = 0.77, size = 558, normalized size = 18.00

$$\left[\frac{\log\left(-\frac{2\,a^{2}\cosh(dx+c)^{4}+2\,a^{2}\sinh(dx+c)^{4}+4\,ab\cosh(dx+c)^{3}+4\,(2\,a^{2}\cosh(dx+c)+ab)\sinh(dx+c)^{3}+4\,ab\cosh(dx+c)+(4\,a^{2}+b^{2})\cosh(dx+c)^{2}+\left(12\,a^{2}\cosh(dx+c)^{2}+12\,ab\cosh(dx+c)+4\,a^{2}+b^{2}\right)\sinh(dx+c)^{2}+2\,a^{2}+2\left(a\cosh(dx+c)^{4}+a\sinh(dx+c)^{4}+b\cosh(dx+c)^{3}+(4\,a\cosh(dx+c)+b)\sinh(dx+c)^{3}+2\,a\cosh(dx+c)^{2}+\left(6\,a\cosh(dx+c)^{2}+3\,b\cosh(dx+c)+2\,a\right)\sinh(dx+c)^{2}+b\cosh(dx+c)+\left(4\,a\cosh(dx+c)^{3}+3\,b\cosh(dx+c)^{2}+4\,a\cosh(dx+c)+b\right)\sinh(dx+c)+a\right)\sqrt{a}\sqrt{\frac{a\cosh(dx+c)+b}{\cosh(dx+c)}}+2\left(4\,a^{2}\cosh(dx+c)^{3}+6\,ab\cosh(dx+c)^{2}+2\,ab+(4\,a^{2}+b^{2})\cosh(dx+c)\right)\sinh(dx+c)}{\cosh(dx+c)^{2}+2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^{2}}\right)}{2\sqrt{a}\,d},\ -\frac{\sqrt{-a}\arctan\left(\frac{\left(a\cosh(dx+c)^{2}+a\sinh(dx+c)^{2}+b\cosh(dx+c)+(2\,a\cosh(dx+c)+b)\sinh(dx+c)+a\right)\sqrt{-a}\sqrt{\frac{a\cosh(dx+c)+b}{\cosh(dx+c)}}}{a^{2}\cosh(dx+c)^{2}+a^{2}\sinh(dx+c)^{2}+2\,ab\cosh(dx+c)+a^{2}+2\,(a^{2}\cosh(dx+c)+ab)\sinh(dx+c)}\right)}{ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(2*a^2*cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c)^4 + 4*a*b*cosh(d*x +
 c)^3 + 4*(2*a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)^3 + 4*a*b*cosh(d*x + c)
 + (4*a^2 + b^2)*cosh(d*x + c)^2 + (12*a^2*cosh(d*x + c)^2 + 12*a*b*cosh(d*
x + c) + 4*a^2 + b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*cosh(d*x + c)^4 + a*si
nh(d*x + c)^4 + b*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) + b)*sinh(d*x + c)^3
 + 2*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a)*si
nh(d*x + c)^2 + b*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^
2 + 4*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)*sqrt((a*cosh(d*x + c)
 + b)/cosh(d*x + c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a*b*cosh(d*x + c)^2 + 2
*a*b + (4*a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cos
h(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2))/(sqrt(a)*d), -sqrt(-a)*arctan(
(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + b*cosh(d*x + c) + (2*a*cosh(d*x +
c) + b)*sinh(d*x + c) + a)*sqrt(-a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c
))/(a^2*cosh(d*x + c)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + a^2 +
 2*(a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)))/(a*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(c+dx)}{\sqrt{a+b\,\mathrm{sech}\,(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c))**(1/2),x)
```

```
[Out] Integral(tanh(c + d*x)/sqrt(a + b*sech(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(tanh(d*x + c)/sqrt(b*sech(d*x + c) + a), x)
```

Mupad [B]
time = 1.64, size = 27, normalized size = 0.87

$$\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{\cosh\left(c+d\,x\right)}}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)/(a + b/cosh(c + d*x))^(1/2),x)
[Out] (2*atanh((a + b/cosh(c + d*x))^(1/2)/a^(1/2)))/(a^(1/2)*d)
```

3.136 $\int \frac{\coth(c+dx)}{\sqrt{a+b\,\mathrm{sech}(c+dx)}}\,dx$

Optimal. Leaf size=106

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}\,d}$$

```
[Out] 2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)-arctanh((a+b*sech(d*x+
c))^(1/2)/(a-b)^(1/2))/d/(a-b)^(1/2)-arctanh((a+b*sech(d*x+c))^(1/2)/(a+b)^
(1/2))/d/(a+b)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3970, 912, 1184, 212, 213}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a-b}}\right)}{d\sqrt{a-b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right)}{d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) - ArcTanh[Sqrt[a
 + b*Sech[c + d*x]]/Sqrt[a - b]]/(Sqrt[a - b]*d) - ArcTanh[Sqrt[a + b*Sech[
c + d*x]]/Sqrt[a + b]]/(Sqrt[a + b]*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
```

```
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1184

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx &= -\frac{b^2\text{Subst}\left(\int \frac{1}{x\sqrt{a+x}\,(b^2-x^2)}\,dx,x,b\operatorname{sech}(c+dx)\right)}{d}\\ &= -\frac{(2b^2)\,\text{Subst}\left(\int \frac{1}{(-a+x^2)(-a^2+b^2+2ax^2-x^4)}\,dx,x,\sqrt{a+b\operatorname{sech}(c+dx)}\right)}{d}\\ &= -\frac{(2b^2)\,\text{Subst}\left(\int \left(-\frac{1}{b^2(a-x^2)}+\frac{1}{2b^2(a+b-x^2)}-\frac{1}{2b^2(-a+b+x^2)}\right)dx,x,\sqrt{a+b\operatorname{sech}(c\cdots}\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx,x,\sqrt{a+b\operatorname{sech}(c+dx)}\right)}{d}+\frac{\text{Subst}\left(\int \frac{1}{-a+b+x^2}\,dx,x,\sqrt{a\cdots}\right)}{d}\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,d}-\frac{\tanh\cdots}{\cdots}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 226 vs. 2(106) = 212.

time = 2.50, size = 226, normalized size = 2.13

$$\frac{\sqrt{b+a\cosh(c+dx)}\left(-\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\sqrt{\cosh(c+dx)}}{\sqrt{b+a\cosh(c+dx)}}\right)}{\sqrt{a-b}}+\frac{-\frac{\tanh^{-1}\left(\frac{\sqrt{-a-b}\sqrt{\cosh(c+dx)}}{\sqrt{-b-a\cosh(c+dx)}}\right)\sqrt{-b-a\cosh(c+dx)}}{\sqrt{-a-b}}+\frac{2\sqrt{b}\sinh^{-1}\left(\frac{\sqrt{a}\sqrt{\cosh(c+dx)}}{\sqrt{b}}\right)\sqrt{1+\frac{a\cosh(c+dx)}{b}}}{\sqrt{a}}}{\sqrt{b+a\cosh(c+dx)}}\right)}{d\sqrt{\cosh(c+dx)}\sqrt{a+b\operatorname{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (Sqrt[b + a*Cosh[c + d*x]]*(-(ArcTanh[(Sqrt[a - b]*Sqrt[Cosh[c + d*x]])/Sqr
t[b + a*Cosh[c + d*x]]]/Sqrt[a - b]) + (-((ArcTanh[(Sqrt[-a - b]*Sqrt[Cosh[
c + d*x]])/Sqrt[-b - a*Cosh[c + d*x]]]*Sqrt[-b - a*Cosh[c + d*x]])/Sqrt[-a
- b]) + (2*Sqrt[b]*ArcSinh[(Sqrt[a]*Sqrt[Cosh[c + d*x]])/Sqrt[b]]*Sqrt[1 +
(a*Cosh[c + d*x])/b])/Sqrt[a])/Sqrt[b + a*Cosh[c + d*x]]))/(d*Sqrt[Cosh[c +
 d*x]]*Sqrt[a + b*Sech[c + d*x]])
```

Maple [F]
time = 2.80, size = 0, normalized size = 0.00

$$\int \frac{\coth \left(dx+c\right)}{\sqrt{a+b\,\mathrm{sech}\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)/(a+b*sech(d*x+c))^(1/2),x)
```

```
[Out] int(coth(d*x+c)/(a+b*sech(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(d*x + c)/sqrt(b*sech(d*x + c) + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 446 vs. $2(88) = 176$.
time = 0.97, size = 8908, normalized size = 84.04

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a^2 + a*b)*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cosh(d*x + c)^4 +
 (8*a^2 - 8*a*b + b^2)*sinh(d*x + c)^4 + 4*(4*a*b - 3*b^2)*cosh(d*x + c)^3
+ 4*(4*a*b - 3*b^2 + (8*a^2 - 8*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 +
 2*(8*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 2*(3*(8*a^2 - 8*a*b + b^2)*cos
h(d*x + c)^2 + 8*a^2 - 8*a*b + 3*b^2 + 6*(4*a*b - 3*b^2)*cosh(d*x + c))*sin
h(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 - 4*((2*a - b)*cosh(d*x + c)^4 + (2*a -
```

b)*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^3 + 2*(2*(2*a - b)*cosh(d*x + c) + b
)*sinh(d*x + c)^3 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*(2*a - b)*cosh(d*x +
c)^2 + 3*b*cosh(d*x + c) + 2*a - b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) +
2*(2*(2*a - b)*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 2*(2*a - b)*cosh(d*x
+ c) + b)*sinh(d*x + c) + 2*a - b)*sqrt(a - b)*sqrt((a*cosh(d*x + c) + b)/
cosh(d*x + c)) + 4*(4*a*b - 3*b^2)*cosh(d*x + c) + 4*((8*a^2 - 8*a*b + b^2)
*cosh(d*x + c)^3 + 3*(4*a*b - 3*b^2)*cosh(d*x + c)^2 + 4*a*b - 3*b^2 + (8*a
^2 - 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^4 + 4*(cos
h(d*x + c) + 1)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 4*cosh(d*x + c)^3 + 6*(
cosh(d*x + c)^2 + 2*cosh(d*x + c) + 1)*sinh(d*x + c)^2 + 6*cosh(d*x + c)^2
+ 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)^2 + 3*cosh(d*x + c) + 1)*sinh(d*x +
c) + 4*cosh(d*x + c) + 1)) + (a^2 - a*b)*sqrt(a + b)*log(-((8*a^2 + 8*a*b +
b^2)*cosh(d*x + c)^4 + (8*a^2 + 8*a*b + b^2)*sinh(d*x + c)^4 + 4*(4*a*b +
3*b^2)*cosh(d*x + c)^3 + 4*(4*a*b + 3*b^2 + (8*a^2 + 8*a*b + b^2)*cosh(d*x
+ c))*sinh(d*x + c)^3 + 2*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 2*(3*(8
*a^2 + 8*a*b + b^2)*cosh(d*x + c)^2 + 8*a^2 + 8*a*b + 3*b^2 + 6*(4*a*b + 3*
b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 8*a^2 + 8*a*b + b^2 - 4*((2*a + b)*co
sh(d*x + c)^4 + (2*a + b)*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^3 + 2*(2*(2*a
+ b)*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*(2*a + b)*cosh(d*x + c)^2 + 2*
(3*(2*a + b)*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a + b)*sinh(d*x + c)^2
+ 2*b*cosh(d*x + c) + 2*(2*(2*a + b)*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2
+ 2*(2*a + b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a + b)*sqrt(a + b)*sqrt
((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 4*(4*a*b + 3*b^2)*cosh(d*x + c) + 4
*((8*a^2 + 8*a*b + b^2)*cosh(d*x + c)^3 + 3*(4*a*b + 3*b^2)*cosh(d*x + c)^2
+ 4*a*b + 3*b^2 + (8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c))/(c
osh(d*x + c)^4 + 4*(cosh(d*x + c) - 1)*sinh(d*x + c)^3 + sinh(d*x + c)^4 -
4*cosh(d*x + c)^3 + 6*(cosh(d*x + c)^2 - 2*cosh(d*x + c) + 1)*sinh(d*x + c)
^2 + 6*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - 3*cosh(d*x + c)^2 + 3*cosh(d*
x + c) - 1)*sinh(d*x + c) - 4*cosh(d*x + c) + 1)) + 2*(a^2 - b^2)*sqrt(a)*l
og(-(2*a^2*cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c)^4 + 4*a*b*cosh(d*x + c)^3
+ 4*(2*a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)^3 + 4*a*b*cosh(d*x + c) + (4*
a^2 + b^2)*cosh(d*x + c)^2 + (12*a^2*cosh(d*x + c)^2 + 12*a*b*cosh(d*x + c)
+ 4*a^2 + b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*cosh(d*x + c)^4 + a*sinh(d*x
+ c)^4 + b*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*a
*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a)*sinh(d*x
+ c)^2 + b*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 4*
a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)*sqrt((a*cosh(d*x + c) + b)/
cosh(d*x + c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a*b*cosh(d*x + c)^2 + 2*a*b +
(4*a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x
+ c)*sinh(d*x + c) + sinh(d*x + c)^2)))/((a^3 - a*b^2)*d), 1/4*(2*(a^2 - a*
b)*sqrt(-a - b)*arctan(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
sinh(d*x + c)^2 + 1)*sqrt(-a - b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)
)/((2*a + b)*cosh(d*x + c)^2 + (2*a + b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c
) + 2*((2*a + b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a + b)) + (a^2 + a*b)
*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cosh(d*x + c)^4 + (8*a^2 - 8*a*b +

```
 b^2)*sinh(d*x + c)^4 + 4*(4*a*b - 3*b^2)*cosh(d*x + c)^3 + 4*(4*a*b - 3*b^
2 + (8*a^2 - 8*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(8*a^2 - 8*a*b
 + 3*b^2)*cosh(d*x + c)^2 + 2*(3*(8*a^2 - 8*a*b + b^2)*cosh(d*x + c)^2 + 8*
a^2 - 8*a*b + 3*b^2 + 6*(4*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 8*
a^2 - 8*a*b + b^2 - 4*((2*a - b)*cosh(d*x + c)^4 + (2*a - b)*sinh(d*x + c)^
4 + 2*b*cosh(d*x + c)^3 + 2*(2*(2*a - b)*cosh(d*x + c) + b)*sinh(d*x + c)^3
 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*(2*a - b)*cosh(d*x + c)^2 + 3*b*cosh(
d*x + c) + 2*a - b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*(2*(2*a - b)*co
sh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 2*(2*a - b)*cosh(d*x + c) + b)*sinh(d
*x + c) + 2*a - b)*sqrt(a - b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) +
4*(4*a*b - 3*b^2)*cosh(d*x + c) + 4*((8*a^2 - 8*a*b + b^2)*cosh(d*x + c)^3
+ 3*(4*a*b - 3*b^2)*cosh(d*x + c)^2 + 4*a*b - 3*b^2 + (8*a^2 - 8*a*b + 3*b^
2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^4 + 4*(cosh(d*x + c) + 1)*s
inh(d*x + c)^3 + sinh(d*x + c)^4 + 4*cosh(d*x + c)^3 + 6*(cosh(d*x + c)^2 +
 2*cosh(d*x + c) + 1)*sinh(d*x + c)^2 + 6*cosh(d*x + c)^2 + 4*(cosh(d*x + c
)^3 + 3*cosh(d*x + c)^2 + 3*cosh(d*x + c) + 1)*...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c))**(1/2),x)
[Out] Integral(coth(c + d*x)/sqrt(a + b*sech(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(coth(d*x + c)/sqrt(b*sech(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(c+d\,x)}{\sqrt{a+\frac{b}{\cosh(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)/(a + b/cosh(c + d*x))^(1/2),x)
[Out] int(coth(c + d*x)/(a + b/cosh(c + d*x))^(1/2), x)
```

3.137 $\int \frac{\coth^3(c+dx)}{\sqrt{a+b\,\mathrm{sech}(c+dx)}}\,dx$

Optimal. Leaf size=262

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,d}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a-b}}\right)}{4(a-b)^{3/2}d}-\cdots b$$

```
[Out] 1/4*b*arctanh((a+b*sech(d*x+c))^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)/d-1/4*b*arct
anh((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)/d+2*arctanh((a+b*sech(
d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)-arctanh((a+b*sech(d*x+c))^(1/2)/(a-b)^(1/2
))/d/(a-b)^(1/2)-arctanh((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2))/d/(a+b)^(1/2)
-1/4*(a+b*sech(d*x+c))^(1/2)/(a+b)/d/(1-sech(d*x+c))-1/4*(a+b*sech(d*x+c))^
(1/2)/(a-b)/d/(1+sech(d*x+c))
```

Rubi [A]

time = 0.22, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3970, 912, 1252, 212, 205, 213}

$$-\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{4d(a+b)(1-\mathrm{sech}(c+dx))}-\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{4d(a-b)(\mathrm{sech}(c+dx)+1)}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a-b}}\right)}{4d(a-b)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a-b}}\right)}{d\sqrt{a-b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right)}{d\sqrt{a+b}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right)}{4d(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^3/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) - ArcTanh[Sqrt[a
 + b*Sech[c + d*x]]/Sqrt[a - b]]/(Sqrt[a - b]*d) + (b*ArcTanh[Sqrt[a + b*Se
ch[c + d*x]]/Sqrt[a - b]])/(4*(a - b)^(3/2)*d) - (b*ArcTanh[Sqrt[a + b*Sech
[c + d*x]]/Sqrt[a + b]])/(4*(a + b)^(3/2)*d) - ArcTanh[Sqrt[a + b*Sech[c +
d*x]]/Sqrt[a + b]]/(Sqrt[a + b]*d) - Sqrt[a + b*Sech[c + d*x]]/(4*(a + b)*d
*(1 - Sech[c + d*x])) - Sqrt[a + b*Sech[c + d*x]]/(4*(a - b)*d*(1 + Sech[c
+ d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1252

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p]
 && IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\coth^3(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx = -\frac{b^4\text{Subst}\left(\int \frac{1}{x\sqrt{a+x}\,(b^2-x^2)^2}\,dx, x, b\operatorname{sech}(c+dx)\right)}{d}$$

$$= -\frac{(2b^4)\,\text{Subst}\left(\int \frac{1}{(-a+x^2)(-a^2+b^2+2ax^2-x^4)^2}\,dx, x, \sqrt{a+b\operatorname{sech}(c+dx)}\right)}{d}$$

$$= -\frac{(2b^4)\,\text{Subst}\left(\int \left(-\frac{1}{b^4(a-x^2)} + \frac{1}{4b^3(a+b-x^2)^2} + \frac{1}{2b^4(a+b-x^2)} - \frac{1}{4b^3(-a+b+x^2)^2} - \frac{}{2b^4(-a}\right.\right.}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \sqrt{a+b\operatorname{sech}(c+dx)}\right)}{d} + \frac{\text{Subst}\left(\int \frac{1}{-a+b+x^2}\,dx, x, \sqrt{a}\right.}{d}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,d} - \text{tanh}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,d} + b\,\text{tan}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.42, size = 505, normalized size = 1.93

$$\frac{\frac{-2a^2\coth^2(c+dx)+2b^2\operatorname{csch}^2(c+dx)}{a^2-b^2} + \frac{\sqrt{b+a\cosh(c+dx)}\left(a^{3/2}\sqrt{-a+b}\,(2a^2+ab-3b^2)\tanh^{-1}\left(\frac{a-a\cosh(c+dx)+\sqrt{a\cosh(c+dx)}\sqrt{b+a\cosh(c+dx)}}{\sqrt{a}\sqrt{a+b}}\right)-(a+b)\left(-2i\sqrt{a}(-a+b)^{3/2}\tanh^{-1}\left(\frac{a-a\cosh(c+dx)-i\sqrt{-a\cosh(c+dx)}\sqrt{b+a\cosh(c+dx)}}{\sqrt{a}\sqrt{a+b}}\right)\sqrt{-a^2\cosh^2(c+dx)}+\sqrt{a+b}\left(a^{3/2}(2a-3b)\text{ArcTan}\left(\frac{a+a\cosh(c+dx)-\sqrt{a\cosh(c+dx)}\sqrt{b+a\cosh(c+dx)}}{\sqrt{a}\sqrt{-a+b}}\right)\cosh(c+dx)+2(a-b)\left(4\sqrt{-a+b}\,\text{ArcTan}\left(\frac{\sqrt{b+a\cosh(c+dx)}}{\sqrt{-a\cosh(c+dx)}}\right)-i\sqrt{a}\,\text{ArcTan}\left(\frac{a+a\cosh(c+dx)+i\sqrt{-a\cosh(c+dx)}\sqrt{b+a\cosh(c+dx)}}{\sqrt{a}\sqrt{-a+b}}\right)\right)\sqrt{-a^2\cosh^2(c+dx)}\right)\right)\operatorname{sech}(c+dx)\right)}{a(-a+b)^{3/2}(a+b)^{3/2}\sqrt{a\cosh(c+dx)}}}{4d\sqrt{a+b\operatorname{sech}(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[c + d*x]^3/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] ((-2*a^2*Coth[c + d*x]^2 + 2*b^2*Csch[c + d*x]^2)/(a^2 - b^2) + (Sqrt[b + a
*Cosh[c + d*x]]*(a^(3/2)*Sqrt[-a + b]*(2*a^2 + a*b - 3*b^2)*ArcTanh[(a - a*
Cosh[c + d*x] + Sqrt[a*Cosh[c + d*x]]*Sqrt[b + a*Cosh[c + d*x]])/(Sqrt[a]*S
qrt[a + b])] - (a + b)*((-2*I)*Sqrt[a]*(-a + b)^(3/2)*ArcTanh[(a - a*Cosh[c
 + d*x] - I*Sqrt[-(a*Cosh[c + d*x])]*Sqrt[b + a*Cosh[c + d*x]])/(Sqrt[a]*Sq
rt[a + b])]*Sqrt[-(a^2*Cosh[c + d*x]^2)] + Sqrt[a + b]*(a^(3/2)*(2*a - 3*b)
*ArcTan[(a + a*Cosh[c + d*x] - Sqrt[a*Cosh[c + d*x]]*Sqrt[b + a*Cosh[c + d*
x]])/(Sqrt[a]*Sqrt[-a + b])]*Cosh[c + d*x] + 2*(a - b)*(4*Sqrt[-a + b]*ArcT
an[Sqrt[b + a*Cosh[c + d*x]]/Sqrt[-(a*Cosh[c + d*x])]] - I*Sqrt[a]*ArcTan[(
a + a*Cosh[c + d*x] + I*Sqrt[-(a*Cosh[c + d*x])]*Sqrt[b + a*Cosh[c + d*x]])
/(Sqrt[a]*Sqrt[-a + b])])*Sqrt[-(a^2*Cosh[c + d*x]^2)]))*Sech[c + d*x]))/(a
*(-a + b)^(3/2)*(a + b)^(3/2)*Sqrt[a*Cosh[c + d*x]]))/(4*d*Sqrt[a + b*Sech[
c + d*x]])
```

Maple [F]
time = 3.45, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(dx+c)}{\sqrt{a+b\,\mathrm{sech}\,(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x)

[Out] int(coth(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(coth(d*x + c)^3/sqrt(b*sech(d*x + c) + a), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1870 vs. $2(218) = 436$.
time = 4.40, size = 20300, normalized size = 77.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")

```
[Out] [1/16*(((4*a^4 + 3*a^3*b - 6*a^2*b^2 - 5*a*b^3)*cosh(d*x + c)^4 + 4*(4*a^4
+ 3*a^3*b - 6*a^2*b^2 - 5*a*b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + (4*a^4 + 3
*a^3*b - 6*a^2*b^2 - 5*a*b^3)*sinh(d*x + c)^4 + 4*a^4 + 3*a^3*b - 6*a^2*b^2
 - 5*a*b^3 - 2*(4*a^4 + 3*a^3*b - 6*a^2*b^2 - 5*a*b^3)*cosh(d*x + c)^2 - 2*
(4*a^4 + 3*a^3*b - 6*a^2*b^2 - 5*a*b^3 - 3*(4*a^4 + 3*a^3*b - 6*a^2*b^2 - 5
*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((4*a^4 + 3*a^3*b - 6*a^2*b^2
- 5*a*b^3)*cosh(d*x + c)^3 - (4*a^4 + 3*a^3*b - 6*a^2*b^2 - 5*a*b^3)*cosh(d
*x + c))*sinh(d*x + c))*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cosh(d*x +
c)^4 + (8*a^2 - 8*a*b + b^2)*sinh(d*x + c)^4 + 4*(4*a*b - 3*b^2)*cosh(d*x +
 c)^3 + 4*(4*a*b - 3*b^2 + (8*a^2 - 8*a*b + b^2)*cosh(d*x + c))*sinh(d*x +
c)^3 + 2*(8*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 2*(3*(8*a^2 - 8*a*b + b^
2)*cosh(d*x + c)^2 + 8*a^2 - 8*a*b + 3*b^2 + 6*(4*a*b - 3*b^2)*cosh(d*x + c
))*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 - 4*((2*a - b)*cosh(d*x + c)^4 + (
2*a - b)*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^3 + 2*(2*(2*a - b)*cosh(d*x +
c) + b)*sinh(d*x + c)^3 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*(2*a - b)*cosh
```

(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a - b)*sinh(d*x + c)^2 + 2*b*cosh(d*x +
c) + 2*(2*(2*a - b)*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 2*(2*a - b)*co
sh(d*x + c) + b)*sinh(d*x + c) + 2*a - b)*sqrt(a - b)*sqrt((a*cosh(d*x + c)
+ b)/cosh(d*x + c)) + 4*(4*a*b - 3*b^2)*cosh(d*x + c) + 4*((8*a^2 - 8*a*b
+ b^2)*cosh(d*x + c)^3 + 3*(4*a*b - 3*b^2)*cosh(d*x + c)^2 + 4*a*b - 3*b^2
+ (8*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^4 +
4*(cosh(d*x + c) + 1)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 4*cosh(d*x + c)^3
+ 6*(cosh(d*x + c)^2 + 2*cosh(d*x + c) + 1)*sinh(d*x + c)^2 + 6*cosh(d*x +
c)^2 + 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)^2 + 3*cosh(d*x + c) + 1)*sinh(
d*x + c) + 4*cosh(d*x + c) + 1)) + ((4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3)
*cosh(d*x + c)^4 + 4*(4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)*
sinh(d*x + c)^3 + (4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3)*sinh(d*x + c)^4 +
4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3 - 2*(4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5
*a*b^3)*cosh(d*x + c)^2 - 2*(4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3 - 3*(4*a
^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((
4*a^4 - 3*a^3*b - 6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c)^3 - (4*a^4 - 3*a^3*b -
6*a^2*b^2 + 5*a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a + b)*log(-((8*a^
2 + 8*a*b + b^2)*cosh(d*x + c)^4 + (8*a^2 + 8*a*b + b^2)*sinh(d*x + c)^4 +
4*(4*a*b + 3*b^2)*cosh(d*x + c)^3 + 4*(4*a*b + 3*b^2 + (8*a^2 + 8*a*b + b^2
)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^
2 + 2*(3*(8*a^2 + 8*a*b + b^2)*cosh(d*x + c)^2 + 8*a^2 + 8*a*b + 3*b^2 + 6*
(4*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 8*a^2 + 8*a*b + b^2 - 4*((
2*a + b)*cosh(d*x + c)^4 + (2*a + b)*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^3
+ 2*(2*(2*a + b)*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*(2*a + b)*cosh(d*x
+ c)^2 + 2*(3*(2*a + b)*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a + b)*sinh
(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*(2*(2*a + b)*cosh(d*x + c)^3 + 3*b*cosh
(d*x + c)^2 + 2*(2*a + b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a + b)*sqrt(
a + b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 4*(4*a*b + 3*b^2)*cosh(d
x + c) + 4((8*a^2 + 8*a*b + b^2)*cosh(d*x + c)^3 + 3*(4*a*b + 3*b^2)*cosh
(d*x + c)^2 + 4*a*b + 3*b^2 + (8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d
*x + c))/(cosh(d*x + c)^4 + 4*(cosh(d*x + c) - 1)*sinh(d*x + c)^3 + sinh(d*
x + c)^4 - 4*cosh(d*x + c)^3 + 6*(cosh(d*x + c)^2 - 2*cosh(d*x + c) + 1)*si
nh(d*x + c)^2 + 6*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - 3*cosh(d*x + c)^2
+ 3*cosh(d*x + c) - 1)*sinh(d*x + c) - 4*cosh(d*x + c) + 1)) + 8*((a^4 - 2*
a^2*b^2 + b^4)*cosh(d*x + c)^4 + 4*(a^4 - 2*a^2*b^2 + b^4)*cosh(d*x + c)*si
nh(d*x + c)^3 + (a^4 - 2*a^2*b^2 + b^4)*sinh(d*x + c)^4 + a^4 - 2*a^2*b^2 +
b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(d*x + c)^2 - 2*(a^4 - 2*a^2*b^2 + b^4
- 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^4 - 2
*a^2*b^2 + b^4)*cosh(d*x + c)^3 - (a^4 - 2*a^2*b^2 + b^4)*cosh(d*x + c))*si
nh(d*x + c))*sqrt(a)*log(-(2*a^2*cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c)^4 +
4*a*b*cosh(d*x + c)^3 + 4*(2*a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)^3 + 4*a
*b*cosh(d*x + c) + (4*a^2 + b^2)*cosh(d*x + c)^2 + (12*a^2*cosh(d*x + c)^2
+ 12*a*b*cosh(d*x + c) + 4*a^2 + b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*cosh(d
*x + c)^4 + a*sinh(d*x + c)^4 + b*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) + b)
*sinh(d*x + c)^3 + 2*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*cosh(d*

```
x + c) + 2*a)*sinh(d*x + c)^2 + b*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 3*
b*cosh(d*x + c)^2 + 4*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)*sqrt(
(a*cosh(d*x + c) + b)/cosh(d*x + c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a*b*cos
h(d*x + c)^2 + 2*a*b + (4*a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*
x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 8*((a^4 - a^
2*b^2)*cosh(d*x + c)^4 + (a^4 - a^2*b^2)*sinh(d*x + c)^4 + a^4 - a^2*b^2 -
2*(a^3*b - a*b^3)*cosh(d*x + c)^3 - 2*(a^3*b - ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3/(a+b*sech(d*x+c))**(1/2),x)
[Out] Integral(coth(c + d*x)**3/sqrt(a + b*sech(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(coth(d*x + c)^3/sqrt(b*sech(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\coth(c+d\,x)^3}{\sqrt{a+\frac{b}{\cosh(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^3/(a + b/cosh(c + d*x))^(1/2),x)
[Out] int(coth(c + d*x)^3/(a + b/cosh(c + d*x))^(1/2), x)
```

3.138 $\int \frac{\tanh^4(c+dx)}{\sqrt{a+b\,\text{sech}(c+dx)}}\,dx$

Optimal. Leaf size=610

$$-\frac{4(a-b)\sqrt{a+b}\,\coth(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{s}}{}}}{b^2d}$$

```
[Out] -4*(a-b)*coth(d*x+c)*EllipticE((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(
a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c)
)/(a-b))^(1/2)/b^2/d+2/15*(a-b)*(8*a^2+9*b^2)*coth(d*x+c)*EllipticE((a+b*se
ch(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*
x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/b^4/d-4*coth(d*x+c)*Ell
ipticF((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)
*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/b/d+2/15*
(8*a^2-2*a*b+9*b^2)*coth(d*x+c)*EllipticF((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/
2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+
sech(d*x+c))/(a-b))^(1/2)/b^3/d+2*coth(d*x+c)*EllipticPi((a+b*sech(d*x+c))^
(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+c
))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/a/d-8/15*a*(a+b*sech(d*x+c
))^(1/2)*tanh(d*x+c)/b^2/d+2/5*sech(d*x+c)*(a+b*sech(d*x+c))^(1/2)*tanh(d*x
+c)/b/d
```

Rubi [A]
time = 0.56, antiderivative size = 610, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3980, 3869, 3922, 3917, 4089, 3945, 4167, 4090}

$$\frac{2(a-b)\sqrt{a+b}\,(8a^2+9b^2)\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15b^4d}+\frac{2\sqrt{a+b}\,(8a^2-2ab+9b^2)\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15b^3d}-\frac{4(a-b)\sqrt{a+b}\,\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}-\frac{4\sqrt{a+b}\,\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bd}+\frac{2\sqrt{a+b}\,\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad}-\frac{8a\tanh(c+dx)\sqrt{a+b\text{sech}(c+dx)}}{15b^2d}+\frac{2\tanh(c+dx)\text{sech}(c+dx)\sqrt{a+b\text{sech}(c+dx)}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^4/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (-4*(a - b)*Sqrt[a + b]*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*
Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(b^2*d) + (2*(a - b)*Sqrt[a + b]*
(8*a^2 + 9*b^2)*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sq
rt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((
b*(1 + Sech[c + d*x]))/(a - b))])/(15*b^4*d) - (4*Sqrt[a + b]*Coth[c + d*x]
*EllipticF[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b
))])/(b*d) + (2*Sqrt[a + b]*(8*a^2 - 2*a*b + 9*b^2)*Coth[c + d*x]*EllipticF
[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(15*b
```

```
^3*d) + (2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a +
b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x])
)/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a*d) - (8*a*Sqrt[a +
b*Sech[c + d*x]]*Tanh[c + d*x])/(15*b^2*d) + (2*Sech[c + d*x]*Sqrt[a + b*Se
ch[c + d*x]]*Tanh[c + d*x])/(5*b*d)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3922

```
Int[csc[(e_.) + (f_.)*(x_)]^2/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x
_Symbol] :> -Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x] + Int[Csc[e + f*
x]*((1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, e, f},
 x] && NeQ[a^2 - b^2, 0]
```

Rule 3945

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d^2*Cos[e + f*x]*(d*Csc[e + f*x])^(n - 2)*(S
qrt[a + b*Csc[e + f*x]]/(b*f*(2*n - 3))), x] + Dist[d^3/(b*(2*n - 3)), Int[
((d*Csc[e + f*x])^(n - 3)/Sqrt[a + b*Csc[e + f*x]])*Simp[2*a*(n - 3) + b*(2
*n - 5)*Csc[e + f*x] - 2*a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 3980

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Csc[c + d
*x]^2)^(m/2), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] && I
GtQ[m/2, 0] && IntegerQ[n - 1/2]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4167

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b
*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^4(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \left(\frac{1}{\sqrt{a+b\text{sech}(c+dx)}} - \frac{2\text{sech}^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}} + \frac{\text{sech}^4(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\right)$$

$$= -\left(2\int \frac{\text{sech}^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx\right) + \int \frac{1}{\sqrt{a+b\text{sech}(c+dx)}}\,dx + \int \frac{\text{se}}{\sqrt{a+}}$$

$$= \frac{2\sqrt{a+b}\,\coth(c+dx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-}{}}}{ad}$$

$$= -\frac{4(a-b)\sqrt{a+b}\,\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(}{}}}{b^2d}$$

$$= -\frac{4(a-b)\sqrt{a+b}\,\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(}{}}}{b^2d}$$

$$= -\frac{4(a-b)\sqrt{a+b}\,\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(}{}}}{b^2d}$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[Tanh[c + d*x]^4/Sqrt[a + b*Sech[c + d*x]],x]
[Out] $Aborted
```

Maple [F]
time = 3.19, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(dx+c)}{\sqrt{a+b\,\text{sech}\,(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(1/2),x)
[Out] int(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(tanh(d*x + c)^4/sqrt(b*sech(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(tanh(d*x + c)^4/sqrt(b*sech(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{4}(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**4/(a+b*sech(d*x+c))**(1/2),x)
[Out] Integral(tanh(c + d*x)**4/sqrt(a + b*sech(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(tanh(d*x + c)^4/sqrt(b*sech(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tanh(c+d\,x)^4}{\sqrt{a+\frac{b}{\cosh(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^4/(a + b/cosh(c + d*x))^(1/2),x)
[Out] int(tanh(c + d*x)^4/(a + b/cosh(c + d*x))^(1/2), x)
```

3.139 $\int \frac{\tanh^2(c+dx)}{\sqrt{a+b\,\mathbf{sech}(c+dx)}}\,dx$

Optimal. Leaf size=310

$$-\frac{2(a-b)\sqrt{a+b}\coth(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+}{}}}{b^2d}$$

```
[Out] -2*(a-b)*coth(d*x+c)*EllipticE((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(
a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c)
)/(a-b))^(1/2)/b^2/d-2*coth(d*x+c)*EllipticF((a+b*sech(d*x+c))^(1/2)/(a+b)^
(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*
(1+sech(d*x+c))/(a-b))^(1/2)/b/d+2*coth(d*x+c)*EllipticPi((a+b*sech(d*x+c))
^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+
c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/a/d
```

Rubi [A]
time = 0.18, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3979, 4144, 4006, 3869, 3917, 4089}

$$-\frac{2(a-b)\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}-\frac{2\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bd}+\frac{2\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^2/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*
Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(b^2*d) - (2*Sqrt[a + b]*Coth[c +
 d*x]*EllipticF[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
 b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(
a - b))])/(b*d) + (2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(a + b)/a, ArcSin
[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech
[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a*d)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3979

```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4144

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*
(b_.) + (a_)], x_Symbol] :> Int[(A - C*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x
]], x] + Dist[C, Int[Csc[e + f*x]*((1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*
x]]), x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\tanh^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = -\int \frac{-1+\text{sech}^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx$$

$$= -\int \frac{-1-\text{sech}(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx - \int \frac{\text{sech}(c+dx)(1+\text{sech}(c+dx))}{\sqrt{a+b\text{sech}(c+dx)}}\,dx$$

$$= -\frac{2(a-b)\sqrt{a+b}\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(}{}}}{b^2d}$$

$$= -\frac{2(a-b)\sqrt{a+b}\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(}{}}}{b^2d}$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[Tanh[c + d*x]^2/Sqrt[a + b*Sech[c + d*x]],x]
[Out] $Aborted
```

Maple [F]
time = 2.02, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(dx+c)}{\sqrt{a+b\,\text{sech}\,(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x)
[Out] int(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(tanh(d*x + c)^2/sqrt(b*sech(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(tanh(d*x + c)^2/sqrt(b*sech(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**2/(a+b*sech(d*x+c))**(1/2),x)
[Out] Integral(tanh(c + d*x)**2/sqrt(a + b*sech(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(tanh(d*x + c)^2/sqrt(b*sech(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tanh(c+d\,x)^2}{\sqrt{a+\frac{b}{\cosh(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^2/(a + b/cosh(c + d*x))^(1/2),x)
[Out] int(tanh(c + d*x)^2/(a + b/cosh(c + d*x))^(1/2), x)
```

3.140 $\int \frac{1}{\sqrt{a+b\text{sech}(c+dx)}}\,dx$

Optimal. Leaf size=106

$$\frac{2\sqrt{a+b}\coth(c+dx)\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{ad}$$

[Out] 2*coth(d*x+c)*EllipticPi((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/a/d

Rubi [A]
time = 0.02, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3869}

$$\frac{2\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sech[c
 + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b
)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a*d)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \frac{2\sqrt{a+b}\coth(c+dx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-}{}}}{ad}$$

Mathematica [A]
time = 0.45, size = 168, normalized size = 1.58

$$\frac{2b\sqrt{b+a\cosh(c+dx)}\,\Pi\left(\frac{a+b}{a};\mathrm{ArcSin}\left(\frac{\sqrt{a}\,\sqrt{b+a\cosh(c+dx)}}{\sqrt{a+b}\,\sqrt{a\cosh(c+dx)}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1+\mathrm{sech}(c+dx))}{-a+b}}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a}\,\sqrt{a+b}\,d\sqrt{a\cosh(c+dx)}\sqrt{-\frac{b(-1+\mathrm{sech}(c+dx))}{a+b}}\sqrt{a+b\mathrm{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*b*Sqrt[b + a*Cosh[c + d*x]]*EllipticPi[(a + b)/a, ArcSin[(Sqrt[a]*Sqrt[b
 + a*Cosh[c + d*x]])/(Sqrt[a + b]*Sqrt[a*Cosh[c + d*x]])], (a + b)/(a - b)]
*Sqrt[(b*(1 + Sech[c + d*x]))/(-a + b)]*Tanh[(c + d*x)/2])/(Sqrt[a]*Sqrt[a
+ b]*d*Sqrt[a*Cosh[c + d*x]]*Sqrt[-((b*(-1 + Sech[c + d*x]))/(a + b))]*Sqrt
[a + b*Sech[c + d*x]])
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\,\mathrm{sech}\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sech(d*x+c))^(1/2),x)
```

```
[Out] int(1/(a+b*sech(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sech(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*sech(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\operatorname{sech}\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))**(1/2),x)
[Out] Integral(1/sqrt(a + b*sech(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(b*sech(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+\frac{b}{\cosh\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cosh(c + d*x))^(1/2),x)
[Out] int(1/(a + b/cosh(c + d*x))^(1/2), x)
```

3.141 $\int \frac{\coth^2(c+dx)}{\sqrt{a+b\,\mathrm{sech}(c+dx)}}\,dx$

Optimal. Leaf size=362

$$\frac{\coth(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\,\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{\sqrt{a+b}\,d}-\text{co}$$

```
[Out] coth(d*x+c)*EllipticE((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/
2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/d/(a+b
)^(1/2)-coth(d*x+c)*EllipticF((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a
-b))^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2
)/d/(a+b)^(1/2)+2*coth(d*x+c)*EllipticPi((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2
),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+c))/(a+b))^(1/2)*
(-b*(1+sech(d*x+c))/(a-b))^(1/2)/a/d-coth(d*x+c)/d/(a+b*sech(d*x+c))^(1/2)-
b^2*tanh(d*x+c)/(a^2-b^2)/d/(a+b*sech(d*x+c))^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3981, 3869, 3960, 3918, 21, 3914, 3917, 4089}

$$-\frac{b^2\tanh(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\,\text{sech}(c+dx)}}-\frac{\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\,\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d\sqrt{a+b}}+\frac{\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\,\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d\sqrt{a+b}}+\frac{2\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\,\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad}-\frac{\coth(c+dx)}{d\sqrt{a+b\,\text{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^2/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a
+ b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c +
 d*x]))/(a - b))])/(Sqrt[a + b]*d) - (Coth[c + d*x]*EllipticF[ArcSin[Sqrt[a
 + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*
x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(Sqrt[a + b]*d) + (
2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sech[c
+ d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)
]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a*d) - Coth[c + d*x]/(d*Sqrt[a
 + b*Sech[c + d*x]]) - (b^2*Tanh[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sech[c
 + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3914

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[a - b, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[b, Int[Csc[e + f*x]*((1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x],
x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3918

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*C
sc[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[Tan[e + f*x]*((a + b*Csc[e + f*x])^m/f), x] + Dist[b*m, I
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
m}, x]
```

Rule 3981

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Sec[c + d
*x]^2)^(-m/2), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] &&
ILtQ[m/2, 0] && IntegerQ[n - 1/2] && EqQ[m, -2]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\coth^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = -\int\left(-\frac{1}{\sqrt{a+b\text{sech}(c+dx)}}-\frac{\text{csch}^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\right)dx$$

$$= \int\frac{1}{\sqrt{a+b\text{sech}(c+dx)}}\,dx+\int\frac{\text{csch}^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx$$

$$= \frac{2\sqrt{a+b}\,\coth(c+dx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{se}}{}}}{ad}$$

$$= \frac{2\sqrt{a+b}\,\coth(c+dx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{se}}{}}}{ad}$$

$$= \frac{2\sqrt{a+b}\,\coth(c+dx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{se}}{}}}{ad}$$

$$= \frac{2\sqrt{a+b}\,\coth(c+dx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{se}}{}}}{ad}$$

$$= \frac{\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}}{\sqrt{a+b}\,d}$$

Mathematica [F]
time = 90.93, size = 0, normalized size = 0.00

$$\int\frac{\coth^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Coth[c + d*x]^2/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] Integrate[Coth[c + d*x]^2/Sqrt[a + b*Sech[c + d*x]], x]
```

Maple [F]
time = 2.86, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(dx+c)}{\sqrt{a+b\,\mathrm{sech}\,(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x)
[Out] int(coth(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(coth(d*x + c)^2/sqrt(b*sech(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(coth(d*x + c)^2/sqrt(b*sech(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(c+dx)}{\sqrt{a+b\,\mathrm{sech}\,(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2/(a+b*sech(d*x+c))**(1/2),x)
[Out] Integral(coth(c + d*x)**2/sqrt(a + b*sech(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(coth(d*x + c)^2/sqrt(b*sech(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\coth(c+d\,x)^2}{\sqrt{a+\frac{b}{\cosh\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^2/(a + b/cosh(c + d*x))^(1/2),x)
[Out] int(coth(c + d*x)^2/(a + b/cosh(c + d*x))^(1/2), x)
```

3.142 $\int \frac{\tanh^5(c+dx)}{(a+b\,\text{sech}(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=148

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{2(a^2-b^2)^2}{ab^4d\sqrt{a+b\text{sech}(c+dx)}}-\frac{2(3a^2-2b^2)\sqrt{a+b\text{sech}(c+dx)}}{b^4d}+\frac{2a(a+$$

```
[Out] 2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d+2*a*(a+b*sech(d*x+c))^
(3/2)/b^4/d-2/5*(a+b*sech(d*x+c))^(5/2)/b^4/d-2*(a^2-b^2)^2/a/b^4/d/(a+b*se
ch(d*x+c))^(1/2)-2*(3*a^2-2*b^2)*(a+b*sech(d*x+c))^(1/2)/b^4/d
```

Rubi [A]
time = 0.14, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3970, 912, 1275, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{2(3a^2-2b^2)\sqrt{a+b\text{sech}(c+dx)}}{b^4d}-\frac{2(a^2-b^2)^2}{ab^4d\sqrt{a+b\text{sech}(c+dx)}}-\frac{2(a+b\text{sech}(c+dx))^{5/2}}{5b^4d}+\frac{2a(a+b\text{sech}(c+dx))^{3/2}}{b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^5/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) - (2*(a^2 - b^2)
^2)/(a*b^4*d*Sqrt[a + b*Sech[c + d*x]]) - (2*(3*a^2 - 2*b^2)*Sqrt[a + b*Sec
h[c + d*x]])/(b^4*d) + (2*a*(a + b*Sech[c + d*x])^(3/2))/(b^4*d) - (2*(a +
b*Sech[c + d*x])^(5/2))/(5*b^4*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\tanh^5(c+dx)}{(a+b\operatorname{sech}(c+dx))^{3/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x(a+x)^{3/2}}\,dx, x, b\operatorname{sech}(c+dx)\right)}{b^4 d}$$

$$= -\frac{2\text{Subst}\left(\int \frac{(-a^2+b^2+2ax^2-x^4)^2}{x^2(-a+x^2)}\,dx, x, \sqrt{a+b\operatorname{sech}(c+dx)}\right)}{b^4 d}$$

$$= -\frac{2\text{Subst}\left(\int \left(3a^2\left(1-\frac{2b^2}{3a^2}\right)-\frac{(a^2-b^2)^2}{ax^2}-3ax^2+x^4-\frac{b^4}{a(a-x^2)}\right)dx, x, \sqrt{a+bs\right.}{b^4 d}$$

$$= -\frac{2(a^2-b^2)^2}{ab^4 d\sqrt{a+b\operatorname{sech}(c+dx)}} - \frac{2(3a^2-2b^2)\sqrt{a+b\operatorname{sech}(c+dx)}}{b^4 d} + \frac{2a(a+b}{}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{2(a^2-b^2)^2}{ab^4 d\sqrt{a+b\operatorname{sech}(c+dx)}} - \frac{2(3a^2-2b}{}$$

Mathematica [A]
time = 2.18, size = 155, normalized size = 1.05

$$-\frac{2\left(16a^4-20a^2b^2+5b^4-\frac{5b^4\tanh^{-1}\left(\frac{\sqrt{b+a\cosh(c+dx)}}{\sqrt{a\cosh(c+dx)}}\right)\sqrt{b+a\cosh(c+dx)}}{\sqrt{a\cosh(c+dx)}}+2ab(4a^2-5b^2)\operatorname{sech}(c+dx)-2a^2b^2\operatorname{sech}^2(c+dx)+ab^3\operatorname{sech}^3(c+dx)\right)}{5ab^4d\sqrt{a+b\operatorname{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^5/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] (-2*(16*a^4 - 20*a^2*b^2 + 5*b^4 - (5*b^4*ArcTanh[Sqrt[b + a*Cosh[c + d*x]]
/Sqrt[a*Cosh[c + d*x]]]*Sqrt[b + a*Cosh[c + d*x]])/Sqrt[a*Cosh[c + d*x]] +
```

```
2*a*b*(4*a^2 - 5*b^2)*Sech[c + d*x] - 2*a^2*b^2*Sech[c + d*x]^2 + a*b^3*Sec
h[c + d*x]^3))/(5*a*b^4*d*Sqrt[a + b*Sech[c + d*x]])
```

Maple [F]
time = 3.27, size = 0, normalized size = 0.00

$$\int \frac{\tanh^5(dx+c)}{(a+b\,\mathrm{sech}\,(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(3/2),x)
```

```
[Out] int(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(d*x + c)^5/(b*sech(d*x + c) + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1743 vs. $2(132) = 264$.
time = 0.83, size = 3745, normalized size = 25.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/10*(5*(a*b^4*cosh(d*x + c)^6 + a*b^4*sinh(d*x + c)^6 + 2*b^5*cosh(d*x +
c)^5 + 3*a*b^4*cosh(d*x + c)^4 + 4*b^5*cosh(d*x + c)^3 + 3*a*b^4*cosh(d*x +
 c)^2 + 2*b^5*cosh(d*x + c) + 2*(3*a*b^4*cosh(d*x + c) + b^5)*sinh(d*x + c)
^5 + a*b^4 + (15*a*b^4*cosh(d*x + c)^2 + 10*b^5*cosh(d*x + c) + 3*a*b^4)*si
nh(d*x + c)^4 + 4*(5*a*b^4*cosh(d*x + c)^3 + 5*b^5*cosh(d*x + c)^2 + 3*a*b^
4*cosh(d*x + c) + b^5)*sinh(d*x + c)^3 + (15*a*b^4*cosh(d*x + c)^4 + 20*b^5
*cosh(d*x + c)^3 + 18*a*b^4*cosh(d*x + c)^2 + 12*b^5*cosh(d*x + c) + 3*a*b^
4)*sinh(d*x + c)^2 + 2*(3*a*b^4*cosh(d*x + c)^5 + 5*b^5*cosh(d*x + c)^4 + 6
*a*b^4*cosh(d*x + c)^3 + 6*b^5*cosh(d*x + c)^2 + 3*a*b^4*cosh(d*x + c) + b^
5)*sinh(d*x + c))*sqrt(a)*log(-(2*a^2*cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c)
^4 + 4*a*b*cosh(d*x + c)^3 + 4*(2*a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)^3
+ 4*a*b*cosh(d*x + c) + (4*a^2 + b^2)*cosh(d*x + c)^2 + (12*a^2*cosh(d*x +
c)^2 + 12*a*b*cosh(d*x + c) + 4*a^2 + b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*c
```

osh(d*x + c)^4 + a*sinh(d*x + c)^4 + b*cosh(d*x + c)^3 + (4*a*cosh(d*x + c)
 + b)*sinh(d*x + c)^3 + 2*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*co
sh(d*x + c) + 2*a)*sinh(d*x + c)^2 + b*cosh(d*x + c) + (4*a*cosh(d*x + c)^3
 + 3*b*cosh(d*x + c)^2 + 4*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)*
sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a*
b*cosh(d*x + c)^2 + 2*a*b + (4*a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))/(co
sh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 4*((16*
a^5 - 20*a^3*b^2 + 5*a*b^4)*cosh(d*x + c)^6 + (16*a^5 - 20*a^3*b^2 + 5*a*b^
4)*sinh(d*x + c)^6 + 4*(4*a^4*b - 5*a^2*b^3)*cosh(d*x + c)^5 + 2*(8*a^4*b -
 10*a^2*b^3 + 3*(16*a^5 - 20*a^3*b^2 + 5*a*b^4)*cosh(d*x + c))*sinh(d*x + c
)^5 + 16*a^5 - 20*a^3*b^2 + 5*a*b^4 + (48*a^5 - 68*a^3*b^2 + 15*a*b^4)*cosh
(d*x + c)^4 + (48*a^5 - 68*a^3*b^2 + 15*a*b^4 + 15*(16*a^5 - 20*a^3*b^2 + 5
*a*b^4)*cosh(d*x + c)^2 + 20*(4*a^4*b - 5*a^2*b^3)*cosh(d*x + c))*sinh(d*x
+ c)^4 + 32*(a^4*b - a^2*b^3)*cosh(d*x + c)^3 + 4*(8*a^4*b - 8*a^2*b^3 + 5*
(16*a^5 - 20*a^3*b^2 + 5*a*b^4)*cosh(d*x + c)^3 + 10*(4*a^4*b - 5*a^2*b^3)*
cosh(d*x + c)^2 + (48*a^5 - 68*a^3*b^2 + 15*a*b^4)*cosh(d*x + c))*sinh(d*x
+ c)^3 + (48*a^5 - 68*a^3*b^2 + 15*a*b^4)*cosh(d*x + c)^2 + (48*a^5 - 68*a^
3*b^2 + 15*a*b^4 + 15*(16*a^5 - 20*a^3*b^2 + 5*a*b^4)*cosh(d*x + c)^4 + 40*
(4*a^4*b - 5*a^2*b^3)*cosh(d*x + c)^3 + 6*(48*a^5 - 68*a^3*b^2 + 15*a*b^4)*
cosh(d*x + c)^2 + 96*(a^4*b - a^2*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 4*(
4*a^4*b - 5*a^2*b^3)*cosh(d*x + c) + 2*(3*(16*a^5 - 20*a^3*b^2 + 5*a*b^4)*c
osh(d*x + c)^5 + 8*a^4*b - 10*a^2*b^3 + 10*(4*a^4*b - 5*a^2*b^3)*cosh(d*x +
 c)^4 + 2*(48*a^5 - 68*a^3*b^2 + 15*a*b^4)*cosh(d*x + c)^3 + 48*(a^4*b - a^
2*b^3)*cosh(d*x + c)^2 + (48*a^5 - 68*a^3*b^2 + 15*a*b^4)*cosh(d*x + c))*si
nh(d*x + c))*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)))/(a^3*b^4*d*cosh(d*x
 + c)^6 + a^3*b^4*d*sinh(d*x + c)^6 + 2*a^2*b^5*d*cosh(d*x + c)^5 + 3*a^3*b
^4*d*cosh(d*x + c)^4 + 4*a^2*b^5*d*cosh(d*x + c)^3 + 3*a^3*b^4*d*cosh(d*x +
 c)^2 + 2*a^2*b^5*d*cosh(d*x + c) + a^3*b^4*d + 2*(3*a^3*b^4*d*cosh(d*x + c
) + a^2*b^5*d)*sinh(d*x + c)^5 + (15*a^3*b^4*d*cosh(d*x + c)^2 + 10*a^2*b^5
*d*cosh(d*x + c) + 3*a^3*b^4*d)*sinh(d*x + c)^4 + 4*(5*a^3*b^4*d*cosh(d*x +
 c)^3 + 5*a^2*b^5*d*cosh(d*x + c)^2 + 3*a^3*b^4*d*cosh(d*x + c) + a^2*b^5*d
)*sinh(d*x + c)^3 + (15*a^3*b^4*d*cosh(d*x + c)^4 + 20*a^2*b^5*d*cosh(d*x +
 c)^3 + 18*a^3*b^4*d*cosh(d*x + c)^2 + 12*a^2*b^5*d*cosh(d*x + c) + 3*a^3*b
^4*d)*sinh(d*x + c)^2 + 2*(3*a^3*b^4*d*cosh(d*x + c)^5 + 5*a^2*b^5*d*cosh(d
*x + c)^4 + 6*a^3*b^4*d*cosh(d*x + c)^3 + 6*a^2*b^5*d*cosh(d*x + c)^2 + 3*a
^3*b^4*d*cosh(d*x + c) + a^2*b^5*d)*sinh(d*x + c)), -1/5*(5*(a*b^4*cosh(d*x
 + c)^6 + a*b^4*sinh(d*x + c)^6 + 2*b^5*cosh(d*x + c)^5 + 3*a*b^4*cosh(d*x
+ c)^4 + 4*b^5*cosh(d*x + c)^3 + 3*a*b^4*cosh(d*x + c)^2 + 2*b^5*cosh(d*x +
 c) + 2*(3*a*b^4*cosh(d*x + c) + b^5)*sinh(d*x + c)^5 + a*b^4 + (15*a*b^4*c
osh(d*x + c)^2 + 10*b^5*cosh(d*x + c) + 3*a*b^4)*sinh(d*x + c)^4 + 4*(5*a*b
^4*cosh(d*x + c)^3 + 5*b^5*cosh(d*x + c)^2 + 3*a*b^4*cosh(d*x + c) + b^5)*s
inh(d*x + c)^3 + (15*a*b^4*cosh(d*x + c)^4 + 20*b^5*cosh(d*x + c)^3 + 18*a*
b^4*cosh(d*x + c)^2 + 12*b^5*cosh(d*x + c) + 3*a*b^4)*sinh(d*x + c)^2 + 2*(
3*a*b^4*cosh(d*x + c)^5 + 5*b^5*cosh(d*x + c)^4 + 6*a*b^4*cosh(d*x + c)^3 +
 6*b^5*cosh(d*x + c)^2 + 3*a*b^4*cosh(d*x + c) + b^5)*sinh(d*x + c))*sqrt(-

```
a)*arctan((a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + b*cosh(d*x + c) + (2*a*c
osh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(-a)*sqrt((a*cosh(d*x + c) + b)/co
sh(d*x + c))/(a^2*cosh(d*x + c)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x +
c) + a^2 + 2*(a^2*cosh(d*x + c) + a*b)*sinh(d*x + c))) + 2*((16*a^5 - 20*a^
3*b^2 + 5*a*b^4)*cosh(d*x + c)^6 + (16*a^5 - 20*a^3*b^2 + 5*a*b^4)*sinh(d*x
 + c)^6 + 4*(4*a^4*b - 5*a^2*b^3)*cosh(d*x + c)^5 + 2*(8*a^4*b - 10*a^2*b^3
 + 3*(16*a^5 - 20*a^3*b^2 + 5*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 16*a^
5 - 20*a^3*b^2 + 5*a*b^4 + (48*a^5 - 68*a^3*b^2 + 15*a*b^4)*cosh(d*x + c)^4
 + (48*a^5 - 68*a^3*b^2 + 15*a*b^4 + 15*(16*a^5...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^5(c+dx)}{(a+b\operatorname{sech}(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**5/(a+b*sech(d*x+c))**(3/2),x)
[Out] Integral(tanh(c + d*x)**5/(a + b*sech(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(tanh(d*x + c)^5/(b*sech(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(c+d\,x)^5}{\left(a+\frac{b}{\cosh(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^5/(a + b/cosh(c + d*x))^(3/2),x)
[Out] int(tanh(c + d*x)^5/(a + b/cosh(c + d*x))^(3/2), x)
```

3.143 $\int \frac{\tanh^3(c+dx)}{(a+b\,\text{sech}(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=88

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\,\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2(a^2-b^2)}{ab^2d\sqrt{a+b\,\text{sech}(c+dx)}}+\frac{2\sqrt{a+b\,\text{sech}(c+dx)}}{b^2d}$$

```
[Out] 2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d+2*(a^2-b^2)/a/b^2/d/(a
+b*sech(d*x+c))^(1/2)+2*(a+b*sech(d*x+c))^(1/2)/b^2/d
```

Rubi [A]
time = 0.09, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3970, 912, 1275, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\,\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2(a^2-b^2)}{ab^2d\sqrt{a+b\,\text{sech}(c+dx)}}+\frac{2\sqrt{a+b\,\text{sech}(c+dx)}}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^3/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + (2*(a^2 - b^2)
)/(a*b^2*d*Sqrt[a + b*Sech[c + d*x]]) + (2*Sqrt[a + b*Sech[c + d*x]])/(b^2*
d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
```

```
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^3(c+dx)}{(a+b\operatorname{sech}(c+dx))^{3/2}}\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{b^2-x^2}{x(a+x)^{3/2}}\,dx, x, b\operatorname{sech}(c+dx)\right)}{b^2d}\\ &= -\frac{2\operatorname{Subst}\left(\int \frac{-a^2+b^2+2ax^2-x^4}{x^2(-a+x^2)}\,dx, x, \sqrt{a+b\operatorname{sech}(c+dx)}\right)}{b^2d}\\ &= -\frac{2\operatorname{Subst}\left(\int \left(-1+\frac{a^2-b^2}{ax^2}-\frac{b^2}{a(a-x^2)}\right)dx, x, \sqrt{a+b\operatorname{sech}(c+dx)}\right)}{b^2d}\\ &= \frac{2(a^2-b^2)}{ab^2d\sqrt{a+b\operatorname{sech}(c+dx)}}+\frac{2\sqrt{a+b\operatorname{sech}(c+dx)}}{b^2d}+\frac{2\operatorname{Subst}\left(\int \frac{1}{a-x^2}\,dx, x,\right.}{}\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2(a^2-b^2)}{ab^2d\sqrt{a+b\operatorname{sech}(c+dx)}}+\frac{2\sqrt{a+b}}{}\end{aligned}$$

Mathematica [A]
time = 0.46, size = 103, normalized size = 1.17

$$\frac{2\left(2a^2-b^2+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{b+a\cosh(c+dx)}}{\sqrt{a\cosh(c+dx)}}\right)\sqrt{b+a\cosh(c+dx)}}{\sqrt{a\cosh(c+dx)}}+ab\operatorname{sech}(c+dx)\right)}{ab^2d\sqrt{a+b\operatorname{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^3/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] (2*(2*a^2 - b^2 + (b^2*ArcTanh[Sqrt[b + a*Cosh[c + d*x]]/Sqrt[a*Cosh[c + d*
x]]]*Sqrt[b + a*Cosh[c + d*x]])/Sqrt[a*Cosh[c + d*x]] + a*b*Sech[c + d*x]))
/(a*b^2*d*Sqrt[a + b*Sech[c + d*x]])
```

Maple [F]
time = 2.58, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(dx+c)}{(a+b\,\mathrm{sech}(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x)
```

```
[Out] int(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(d*x + c)^3/(b*sech(d*x + c) + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 424 vs. 2(78) = 156.
time = 0.78, size = 1107, normalized size = 12.58

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*b^2*cosh(d*x + c)^2 + a*b^2*sinh(d*x + c)^2 + 2*b^3*cosh(d*x + c)
+ a*b^2 + 2*(a*b^2*cosh(d*x + c) + b^3)*sinh(d*x + c))*sqrt(a)*log(-(2*a^2*
cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c)^4 + 4*a*b*cosh(d*x + c)^3 + 4*(2*a^2*
cosh(d*x + c) + a*b)*sinh(d*x + c)^3 + 4*a*b*cosh(d*x + c) + (4*a^2 + b^2)*
cosh(d*x + c)^2 + (12*a^2*cosh(d*x + c)^2 + 12*a*b*cosh(d*x + c) + 4*a^2 +
b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*cosh(d*x + c)^4 + a*sinh(d*x + c)^4 + b
*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*a*cosh(d*x +
 c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a)*sinh(d*x + c)^2 + b
*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 4*a*cosh(d*x
+ c) + b)*sinh(d*x + c) + a)*sqrt(a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x +
c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a*b*cosh(d*x + c)^2 + 2*a*b + (4*a^2 + b
^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d
*x + c) + sinh(d*x + c)^2)) + 4*(2*a^2*b*cosh(d*x + c) + 2*a^3 - a*b^2 + (2
*a^3 - a*b^2)*cosh(d*x + c)^2 + (2*a^3 - a*b^2)*sinh(d*x + c)^2 + 2*(a^2*b
+ (2*a^3 - a*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a*cosh(d*x + c) + b)/
```

```
cosh(d*x + c)))/(a^3*b^2*d*cosh(d*x + c)^2 + a^3*b^2*d*sinh(d*x + c)^2 + 2*
a^2*b^3*d*cosh(d*x + c) + a^3*b^2*d + 2*(a^3*b^2*d*cosh(d*x + c) + a^2*b^3*
d)*sinh(d*x + c)), -((a*b^2*cosh(d*x + c)^2 + a*b^2*sinh(d*x + c)^2 + 2*b^3
*cosh(d*x + c) + a*b^2 + 2*(a*b^2*cosh(d*x + c) + b^3)*sinh(d*x + c))*sqrt(
-a)*arctan((a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + b*cosh(d*x + c) + (2*a*
cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(-a)*sqrt((a*cosh(d*x + c) + b)/c
osh(d*x + c))/(a^2*cosh(d*x + c)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x +
 c) + a^2 + 2*(a^2*cosh(d*x + c) + a*b)*sinh(d*x + c))) - 2*(2*a^2*b*cosh(d
*x + c) + 2*a^3 - a*b^2 + (2*a^3 - a*b^2)*cosh(d*x + c)^2 + (2*a^3 - a*b^2)
*sinh(d*x + c)^2 + 2*(a^2*b + (2*a^3 - a*b^2)*cosh(d*x + c))*sinh(d*x + c))
*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)))/(a^3*b^2*d*cosh(d*x + c)^2 + a^
3*b^2*d*sinh(d*x + c)^2 + 2*a^2*b^3*d*cosh(d*x + c) + a^3*b^2*d + 2*(a^3*b^
2*d*cosh(d*x + c) + a^2*b^3*d)*sinh(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(c+dx)}{(a+b\operatorname{sech}(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**3/(a+b*sech(d*x+c))**(3/2),x)
[Out] Integral(tanh(c + d*x)**3/(a + b*sech(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(tanh(d*x + c)^3/(b*sech(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(c+d\,x)^3}{\left(a+\frac{b}{\cosh(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^3/(a + b/cosh(c + d*x))^(3/2),x)
[Out] int(tanh(c + d*x)^3/(a + b/cosh(c + d*x))^(3/2), x)
```

3.144 $\int \frac{\tanh(c+dx)}{(a+b\,\mathrm{sech}(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=54

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\mathrm{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{2}{ad\sqrt{a+b\mathrm{sech}(c+dx)}}$$

```
[Out] 2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d-2/a/d/(a+b*sech(d*x+c)
)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3970, 53, 65, 213}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\mathrm{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{2}{ad\sqrt{a+b\mathrm{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) - 2/(a*d*Sqrt[a
+ b*Sech[c + d*x]])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh(c+dx)}{(a+b\operatorname{sech}(c+dx))^{3/2}}\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{1}{x(a+x)^{3/2}}\,dx, x, b\operatorname{sech}(c+dx)\right)}{d}\\
&= -\frac{2}{ad\sqrt{a+b\operatorname{sech}(c+dx)}} - \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+x}}\,dx, x, b\operatorname{sech}(c+dx)\right)}{ad}\\
&= -\frac{2}{ad\sqrt{a+b\operatorname{sech}(c+dx)}} - \frac{2\operatorname{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\operatorname{sech}(c+dx)}\right)}{ad}\\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{2}{ad\sqrt{a+b\operatorname{sech}(c+dx)}}
\end{aligned}$$

Mathematica [A]
time = 0.17, size = 79, normalized size = 1.46

$$\frac{2\left(-1+\frac{\tanh^{-1}\left(\frac{\sqrt{b+a\cosh(c+dx)}}{\sqrt{a\cosh(c+dx)}}\right)\sqrt{b+a\cosh(c+dx)}}{\sqrt{a\cosh(c+dx)}}\right)}{ad\sqrt{a+b\operatorname{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] (2*(-1 + (ArcTanh[Sqrt[b + a*Cosh[c + d*x]]/Sqrt[a*Cosh[c + d*x]]]*Sqrt[b +
 a*Cosh[c + d*x]])/Sqrt[a*Cosh[c + d*x]]))/(a*d*Sqrt[a + b*Sech[c + d*x]])
```

Maple [A]
time = 1.54, size = 46, normalized size = 0.85

method	result	size
derivativedivides	$-\frac{\frac{2}{a\sqrt{a+b\,\mathrm{sech}\left(dx+c\right)}}-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{sech}\left(dx+c\right)}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}}{d}$	46
default	$-\frac{\frac{2}{a\sqrt{a+b\,\mathrm{sech}\left(dx+c\right)}}-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{sech}\left(dx+c\right)}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}}{d}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)/(a+b*sech(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(2/a/(a+b*sech(d*x+c))^(1/2)-2/a^(3/2)*arctanh((a+b*sech(d*x+c))^(1/2)
/a^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(d*x + c)/(b*sech(d*x + c) + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(46) = 92$.
time = 0.82, size = 917, normalized size = 16.98

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*(a*cos
h(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)*log(-(2*a^2*cosh(d*x + c)^4 + 2*
a^2*sinh(d*x + c)^4 + 4*a*b*cosh(d*x + c)^3 + 4*(2*a^2*cosh(d*x + c) + a*b)
*sinh(d*x + c)^3 + 4*a*b*cosh(d*x + c) + (4*a^2 + b^2)*cosh(d*x + c)^2 + (1
2*a^2*cosh(d*x + c)^2 + 12*a*b*cosh(d*x + c) + 4*a^2 + b^2)*sinh(d*x + c)^2
 + 2*a^2 + 2*(a*cosh(d*x + c)^4 + a*sinh(d*x + c)^4 + b*cosh(d*x + c)^3 + (
4*a*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*a*cosh(d*x + c)^2 + (6*a*cosh(d*
```

```
x + c)^2 + 3*b*cosh(d*x + c) + 2*a)*sinh(d*x + c)^2 + b*cosh(d*x + c) + (4*
a*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 4*a*cosh(d*x + c) + b)*sinh(d*x +
 c) + a)*sqrt(a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 2*(4*a^2*cosh(
d*x + c)^3 + 6*a*b*cosh(d*x + c)^2 + 2*a*b + (4*a^2 + b^2)*cosh(d*x + c))*s
inh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
 c)^2)) - 4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d
*x + c)^2 + a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)))/(a^3*d*cosh(d*x +
 c)^2 + a^3*d*sinh(d*x + c)^2 + 2*a^2*b*d*cosh(d*x + c) + a^3*d + 2*(a^3*d*
cosh(d*x + c) + a^2*b*d)*sinh(d*x + c)), -((a*cosh(d*x + c)^2 + a*sinh(d*x
+ c)^2 + 2*b*cosh(d*x + c) + 2*(a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqr
t(-a)*arctan((a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + b*cosh(d*x + c) + (2*
a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(-a)*sqrt((a*cosh(d*x + c) + b)
/cosh(d*x + c))/(a^2*cosh(d*x + c)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x
 + c) + a^2 + 2*(a^2*cosh(d*x + c) + a*b)*sinh(d*x + c))) + 2*(a*cosh(d*x +
 c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a)*sqrt((a*co
sh(d*x + c) + b)/cosh(d*x + c)))/(a^3*d*cosh(d*x + c)^2 + a^3*d*sinh(d*x +
c)^2 + 2*a^2*b*d*cosh(d*x + c) + a^3*d + 2*(a^3*d*cosh(d*x + c) + a^2*b*d)*
sinh(d*x + c))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh{(c+dx)}}{(a+b\operatorname{sech}{(c+dx)})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c))**(3/2),x)
[Out] Integral(tanh(c + d*x)/(a + b*sech(c + d*x))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(tanh(d*x + c)/(b*sech(d*x + c) + a)^(3/2), x)
```

Mupad [**B**]

time = 1.77, size = 50, normalized size = 0.93

$$\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{\cosh\left(c+d\,x\right)}}}{\sqrt{a}}\right)}{a^{3/2}\,d}-\frac{2}{a\,d\,\sqrt{a+\frac{b}{\cosh\left(c+d\,x\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)/(a + b/cosh(c + d*x))^(3/2),x)
```

```
[Out] (2*atanh((a + b/cosh(c + d*x))^(1/2)/a^(1/2)))/(a^(3/2)*d) - 2/(a*d*(a + b/
cosh(c + d*x))^(1/2))
```

3.145 $\int \frac{\coth(c+dx)}{(a+b\,\mathrm{sech}(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=142

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}d}+\frac{}{a}$$

```
[Out] 2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d-arctanh((a+b*sech(d*x+
c))^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)/d-arctanh((a+b*sech(d*x+c))^(1/2)/(a+b)^
(1/2))/(a+b)^(3/2)/d+2*b^2/a/(a^2-b^2)/d/(a+b*sech(d*x+c))^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3970, 912, 1301, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2b^2}{ad\left(a^2-b^2\right)\sqrt{a+b\,\mathrm{sech}(c+dx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a-b}}\right)}{d(a-b)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right)}{d(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) - ArcTanh[Sqrt[a
 + b*Sech[c + d*x]]/Sqrt[a - b]]/((a - b)^(3/2)*d) - ArcTanh[Sqrt[a + b*Sec
h[c + d*x]]/Sqrt[a + b]]/((a + b)^(3/2)*d) + (2*b^2)/(a*(a^2 - b^2)*d*Sqrt[
a + b*Sech[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\coth(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = -\frac{b^2\text{Subst}\left(\int \frac{1}{x(a+x)^{3/2}(b^2-x^2)}\,dx, x, b\text{sech}(c+dx)\right)}{d}$$

$$= -\frac{(2b^2)\,\text{Subst}\left(\int \frac{1}{x^2(-a+x^2)(-a^2+b^2+2ax^2-x^4)}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{d}$$

$$= -\frac{(2b^2)\,\text{Subst}\left(\int \left(\frac{1}{a(a^2-b^2)x^2} - \frac{1}{ab^2(a-x^2)} + \frac{1}{2(a-b)b^2(a-b-x^2)} + \frac{1}{2b^2(a+b)(a+b-x^2)}\right) d}{d}$$

$$= \frac{2b^2}{a\,(a^2-b^2)\,d\sqrt{a+b\text{sech}(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx}}{ad}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}d} - \frac{\tan}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.42, size = 483, normalized size = 3.40

$$\frac{\text{sech}(c+dx)\left(\frac{4ab^2(b+a\cosh(c+dx))}{a^2-b^2} + \frac{(b+a\cosh(c+dx))^{3/2}\left(a^{3/2}(-a+b)^{5/2}\tanh^{-1}\left(\frac{a-a\cosh(c+dx)+\sqrt{a\cosh(c+dx)}\sqrt{b+a\cosh(c+dx)}}{\sqrt{a}\sqrt{a+b}}\right) - \frac{1}{2}(a+b)\left(-2i\sqrt{a}(-a+b)^{3/2}\tanh^{-1}\left(\frac{a-a\cosh(c+dx)-i\sqrt{-a\cosh(c+dx)}\sqrt{b+a\cosh(c+dx)}}{\sqrt{a}\sqrt{a+b}}\right)\sqrt{-a^2\cosh^2(c+dx)} + 2\sqrt{a+b}\left(a^{3/2}(a+b)\text{ArcTan}\left(\frac{a+a\cosh(c+dx)-\sqrt{a\cosh(c+dx)}\sqrt{b+a\cosh(c+dx)}}{\sqrt{a}\sqrt{-a+b}}\right)\cosh(c+dx) + (a-b)\left(4\sqrt{-a+b}\,\text{ArcTan}\left(\frac{\sqrt{b+a\cosh(c+dx)}}{\sqrt{-a\cosh(c+dx)}}\right) - i\sqrt{a}\,\text{ArcTan}\left(\frac{a+a\cosh(c+dx)+i\sqrt{-a\cosh(c+dx)}\sqrt{b+a\cosh(c+dx)}}{\sqrt{a}\sqrt{-a+b}}\right)\right)\sqrt{-a^2\cosh^2(c+dx)}\right)\text{sech}(c+dx)\right)\right)}{(-a+b)^{3/2}(a+b)^{3/2}\sqrt{a\cosh(c+dx)}}\right)}{2a^2d(a+b\text{sech}(c+dx))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[c + d*x]/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] (Sech[c + d*x]*((4*a*b^2*(b + a*Cosh[c + d*x]))/(a^2 - b^2) + ((b + a*Cosh[
c + d*x])^(3/2)*(a^(3/2)*(-a + b)^(5/2)*ArcTanh[(a - a*Cosh[c + d*x] + Sqrt
[a*Cosh[c + d*x]]*Sqrt[b + a*Cosh[c + d*x]])/(Sqrt[a]*Sqrt[a + b])] - ((a +
 b)*((-2*I)*Sqrt[a]*(-a + b)^(3/2)*ArcTanh[(a - a*Cosh[c + d*x] - I*Sqrt[-(
```

```
a*Cosh[c + d*x])]*Sqrt[b + a*Cosh[c + d*x]])/(Sqrt[a]*Sqrt[a + b])]*Sqrt[-(
a^2*Cosh[c + d*x]^2)] + 2*Sqrt[a + b]*(a^(3/2)*(a + b)*ArcTan[(a + a*Cosh[c
 + d*x] - Sqrt[a*Cosh[c + d*x]]*Sqrt[b + a*Cosh[c + d*x]])/(Sqrt[a]*Sqrt[-a
 + b])]*Cosh[c + d*x] + (a - b)*(4*Sqrt[-a + b]*ArcTan[Sqrt[b + a*Cosh[c +
d*x]]/Sqrt[-(a*Cosh[c + d*x])]] - I*Sqrt[a]*ArcTan[(a + a*Cosh[c + d*x] + I
*Sqrt[-(a*Cosh[c + d*x])]*Sqrt[b + a*Cosh[c + d*x]])/(Sqrt[a]*Sqrt[-a + b])
])*Sqrt[-(a^2*Cosh[c + d*x]^2)]))*Sech[c + d*x])/2))/((-a + b)^(3/2)*(a + b
)^(3/2)*Sqrt[a*Cosh[c + d*x]])))/(2*a^2*d*(a + b*Sech[c + d*x])^(3/2))
```

Maple [F]
time = 2.56, size = 0, normalized size = 0.00

$$\int \frac{\coth(dx+c)}{(a+b\,\mathrm{sech}(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)/(a+b*sech(d*x+c))^(3/2),x)
[Out] int(coth(d*x+c)/(a+b*sech(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(coth(d*x + c)/(b*sech(d*x + c) + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1134 vs. $2(122) = 244$.
time = 3.85, size = 14412, normalized size = 101.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")
[Out] [-1/4*((a^5 + 2*a^4*b + a^3*b^2 + (a^5 + 2*a^4*b + a^3*b^2)*cosh(d*x + c)^2
 + (a^5 + 2*a^4*b + a^3*b^2)*sinh(d*x + c)^2 + 2*(a^4*b + 2*a^3*b^2 + a^2*b
^3)*cosh(d*x + c) + 2*(a^4*b + 2*a^3*b^2 + a^2*b^3 + (a^5 + 2*a^4*b + a^3*b
^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*c
osh(d*x + c)^4 + (8*a^2 - 8*a*b + b^2)*sinh(d*x + c)^4 + 4*(4*a*b - 3*b^2)*
cosh(d*x + c)^3 + 4*(4*a*b - 3*b^2 + (8*a^2 - 8*a*b + b^2)*cosh(d*x + c))*s
inh(d*x + c)^3 + 2*(8*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 2*(3*(8*a^2 -
```

8*a*b + b^2)*cosh(d*x + c)^2 + 8*a^2 - 8*a*b + 3*b^2 + 6*(4*a*b - 3*b^2)*co
sh(d*x + c))*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*((2*a - b)*cosh(d*x
+ c)^4 + (2*a - b)*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^3 + 2*(2*(2*a - b)*c
osh(d*x + c) + b)*sinh(d*x + c)^3 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*(2*a
 - b)*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a - b)*sinh(d*x + c)^2 + 2*b*
cosh(d*x + c) + 2*(2*(2*a - b)*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 2*(2
*a - b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a - b)*sqrt(a - b)*sqrt((a*cos
h(d*x + c) + b)/cosh(d*x + c)) + 4*(4*a*b - 3*b^2)*cosh(d*x + c) + 4*((8*a^
2 - 8*a*b + b^2)*cosh(d*x + c)^3 + 3*(4*a*b - 3*b^2)*cosh(d*x + c)^2 + 4*a*
b - 3*b^2 + (8*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x
 + c)^4 + 4*(cosh(d*x + c) + 1)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 4*cosh(
d*x + c)^3 + 6*(cosh(d*x + c)^2 + 2*cosh(d*x + c) + 1)*sinh(d*x + c)^2 + 6*
cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)^2 + 3*cosh(d*x + c)
+ 1)*sinh(d*x + c) + 4*cosh(d*x + c) + 1)) - (a^5 - 2*a^4*b + a^3*b^2 + (a^
5 - 2*a^4*b + a^3*b^2)*cosh(d*x + c)^2 + (a^5 - 2*a^4*b + a^3*b^2)*sinh(d*x
 + c)^2 + 2*(a^4*b - 2*a^3*b^2 + a^2*b^3)*cosh(d*x + c) + 2*(a^4*b - 2*a^3*
b^2 + a^2*b^3 + (a^5 - 2*a^4*b + a^3*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqr
t(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cosh(d*x + c)^4 + (8*a^2 + 8*a*b + b^2
)*sinh(d*x + c)^4 + 4*(4*a*b + 3*b^2)*cosh(d*x + c)^3 + 4*(4*a*b + 3*b^2 +
(8*a^2 + 8*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(8*a^2 + 8*a*b + 3
*b^2)*cosh(d*x + c)^2 + 2*(3*(8*a^2 + 8*a*b + b^2)*cosh(d*x + c)^2 + 8*a^2
+ 8*a*b + 3*b^2 + 6*(4*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 8*a^2
+ 8*a*b + b^2 - 4*((2*a + b)*cosh(d*x + c)^4 + (2*a + b)*sinh(d*x + c)^4 +
2*b*cosh(d*x + c)^3 + 2*(2*(2*a + b)*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2
*(2*a + b)*cosh(d*x + c)^2 + 2*(3*(2*a + b)*cosh(d*x + c)^2 + 3*b*cosh(d*x
+ c) + 2*a + b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*(2*(2*a + b)*cosh(d
*x + c)^3 + 3*b*cosh(d*x + c)^2 + 2*(2*a + b)*cosh(d*x + c) + b)*sinh(d*x +
 c) + 2*a + b)*sqrt(a + b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 4*(4
*a*b + 3*b^2)*cosh(d*x + c) + 4*((8*a^2 + 8*a*b + b^2)*cosh(d*x + c)^3 + 3*
(4*a*b + 3*b^2)*cosh(d*x + c)^2 + 4*a*b + 3*b^2 + (8*a^2 + 8*a*b + 3*b^2)*c
osh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^4 + 4*(cosh(d*x + c) - 1)*sinh(
d*x + c)^3 + sinh(d*x + c)^4 - 4*cosh(d*x + c)^3 + 6*(cosh(d*x + c)^2 - 2*c
osh(d*x + c) + 1)*sinh(d*x + c)^2 + 6*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3
- 3*cosh(d*x + c)^2 + 3*cosh(d*x + c) - 1)*sinh(d*x + c) - 4*cosh(d*x + c)
+ 1)) - 2*(a^5 - 2*a^3*b^2 + a*b^4 + (a^5 - 2*a^3*b^2 + a*b^4)*cosh(d*x + c
)^2 + (a^5 - 2*a^3*b^2 + a*b^4)*sinh(d*x + c)^2 + 2*(a^4*b - 2*a^2*b^3 + b^
5)*cosh(d*x + c) + 2*(a^4*b - 2*a^2*b^3 + b^5 + (a^5 - 2*a^3*b^2 + a*b^4)*c
osh(d*x + c))*sinh(d*x + c))*sqrt(a)*log(-(2*a^2*cosh(d*x + c)^4 + 2*a^2*si
nh(d*x + c)^4 + 4*a*b*cosh(d*x + c)^3 + 4*(2*a^2*cosh(d*x + c) + a*b)*sinh(
d*x + c)^3 + 4*a*b*cosh(d*x + c) + (4*a^2 + b^2)*cosh(d*x + c)^2 + (12*a^2*
cosh(d*x + c)^2 + 12*a*b*cosh(d*x + c) + 4*a^2 + b^2)*sinh(d*x + c)^2 + 2*a
^2 + 2*(a*cosh(d*x + c)^4 + a*sinh(d*x + c)^4 + b*cosh(d*x + c)^3 + (4*a*co
sh(d*x + c) + b)*sinh(d*x + c)^3 + 2*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)
^2 + 3*b*cosh(d*x + c) + 2*a)*sinh(d*x + c)^2 + b*cosh(d*x + c) + (4*a*cosh
(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 4*a*cosh(d*x + c) + b)*sinh(d*x + c) +

```
a)*sqrt(a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 2*(4*a^2*cosh(d*x +
c)^3 + 6*a*b*cosh(d*x + c)^2 + 2*a*b + (4*a^2 + b^2)*cosh(d*x + c))*sinh(d*
x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)
) - 8*(a^3*b^2 - a*b^4 + (a^3*b^2 - a*b^4)*cosh(d*x + c)^2 + 2*(a^3*b^2 - a
*b^4)*cosh(d*x + c)*sinh(d*x + c) + (a^3*b^2 - a*b^4)*sinh(d*x + c)^2)*sqrt
((a*cosh(d*x + c) + b)/cosh(d*x + c)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cosh(
d*x + c)^2 + (a^7 - 2*a^5*b^2 + a^3*b^4)*d*sinh(d*x + c)^2 + 2*(a^6*b - 2*a
^4*b^3 + a^2*b^5)*d*cosh(d*x + c) + (a^7 - 2*a^5*b^2 + a^3*b^4)*d + 2*((a^7
 - 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d)*
sinh(d*x + c)), 1/4*(2*(a^5 - 2*a^4*b + a^3*b^2 + (a^5 - 2*a^4*b + a^3*b^2)
*cosh(d*x + c)^2 + (a^5 - 2*a^4*b + a^3*b^2)*sinh(d*x + c)^2 + 2*(a^4*b - 2
*a^3*b^2 + a^2*b^3)*cosh(d*x + c) + 2*(a^4*b - 2*a^3*b^2 + a^2*b^3 + (a^5 -
 2*a^4*b + a^3*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a - b)*arctan(2*(co
sh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*sqrt(-
a - b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(c+dx)}{(a+b\,\mathrm{sech}\,(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c))**(3/2),x)
[Out] Integral(coth(c + d*x)/(a + b*sech(c + d*x))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(coth(d*x + c)/(b*sech(d*x + c) + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(c+d\,x)}{{\left(a+\frac{b}{\cosh(c+d\,x)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)/(a + b/cosh(c + d*x))^(3/2),x)
[Out] int(coth(c + d*x)/(a + b/cosh(c + d*x))^(3/2), x)
```

3.146 $\int \frac{\coth^3(c+dx)}{(a+b\,\mathbf{sech}(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=316

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{2(a-b)^{5/2}d}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+}{\sqrt{a-b}}\right.}{4(a-b)^{5/2}d}$$

```
[Out] 2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d-1/2*(2*a-3*b)*arctanh(
(a+b*sech(d*x+c))^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)/d+1/4*b*arctanh((a+b*sech(
d*x+c))^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)/d-1/4*b*arctanh((a+b*sech(d*x+c))^(1
/2)/(a+b)^(1/2))/(a+b)^(5/2)/d-1/2*(2*a+3*b)*arctanh((a+b*sech(d*x+c))^(1/2
)/(a+b)^(1/2))/(a+b)^(5/2)/d-2*b^4/a/(a^2-b^2)^2/d/(a+b*sech(d*x+c))^(1/2)-
1/4*(a+b*sech(d*x+c))^(1/2)/(a+b)^2/d/(1-sech(d*x+c))-1/4*(a+b*sech(d*x+c))
^(1/2)/(a-b)^2/d/(1+sech(d*x+c))
```

Rubi [A]

time = 0.31, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3970, 912, 1349, 212, 205}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{2b^4}{ad\left(a^2-b^2\right)^2\sqrt{a+b\text{sech}(c+dx)}}-\frac{\sqrt{a+b\text{sech}(c+dx)}}{4d(a+b)^2(1-\text{sech}(c+dx))}-\frac{\sqrt{a+b\text{sech}(c+dx)}}{4d(a-b)^2(\text{sech}(c+dx)+1)}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{4d(a-b)^{5/2}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{4d(a+b)^{5/2}}-\frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{2d(a-b)^{5/2}}-\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{2d(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^3/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) - ((2*a - 3*b)*A
rcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a - b]])/(2*(a - b)^(5/2)*d) + (b*Arc
Tanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a - b]])/(4*(a - b)^(5/2)*d) - (b*ArcTa
nh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]])/(4*(a + b)^(5/2)*d) - ((2*a + 3*
b)*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]])/(2*(a + b)^(5/2)*d) - (2
*b^4)/(a*(a^2 - b^2)^2*d*Sqrt[a + b*Sech[c + d*x]]) - Sqrt[a + b*Sech[c + d
*x]]/(4*(a + b)^2*d*(1 - Sech[c + d*x])) - Sqrt[a + b*Sech[c + d*x]]/(4*(a
- b)^2*d*(1 + Sech[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\coth^3(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = -\frac{b^4\text{Subst}\left(\int \frac{1}{x(a+x)^{3/2}(b^2-x^2)^2}\,dx, x, b\text{sech}(c+dx)\right)}{d}$$

$$= -\frac{(2b^4)\,\text{Subst}\left(\int \frac{1}{x^2(-a+x^2)(-a^2+b^2+2ax^2-x^4)^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{d}$$

$$= -\frac{(2b^4)\,\text{Subst}\left(\int \left(-\frac{1}{a(a-b)^2(a+b)^2x^2} - \frac{1}{ab^4(a-x^2)} - \frac{1}{4(a-b)b^3(a-b-x^2)^2} + \frac{2a-3b}{4(a-b)^2b^4(a-b}\right.\right.}{d}$$

$$= -\frac{2b^4}{a\left(a^2-b^2\right)^2 d\sqrt{a+b\text{sech}(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{a+b\text{sech}(c+\right.}{ad}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right.}{2(a-b)^{5/2}d}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right.}{2(a-b)^{5/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.79, size = 1084, normalized size = 3.43

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[c + d*x]^3/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] ((b + a*Cosh[c + d*x])^(3/2)*(-(((-(a^3*b) + 7*a*b^3)*(Sqrt[a + b]*ArcTan[(
a + a*Cosh[c + d*x] - Sqrt[a*Cosh[c + d*x]]*Sqrt[b + a*Cosh[c + d*x]])/(Sqr
t[a]*Sqrt[-a + b])] + Sqrt[-a + b]*ArcTanh[(a + b + Sqrt[b + a*Cosh[c + d*x
]]*(Sqrt[a*Cosh[c + d*x]] - Sqrt[b + a*Cosh[c + d*x]]))/(Sqrt[a]*Sqrt[a + b
])])*Sqrt[(-a + a*Cosh[c + d*x])/(a + a*Cosh[c + d*x])]*(a + a*Cosh[c + d*x
]))/(Sqrt[a]*Sqrt[-a + b]*Sqrt[a + b]*Sqrt[-1 + Cosh[c + d*x]]*Sqrt[a*Cosh[
c + d*x]]*Sqrt[1 + Cosh[c + d*x]]*Sqrt[Sech[c + d*x]])) + ((2*a^4 - 6*a^2*b
^2 - 2*b^4)*(Sqrt[a + b]*ArcTan[(a + a*Cosh[c + d*x] - Sqrt[a*Cosh[c + d*x]
]*Sqrt[b + a*Cosh[c + d*x]])/(Sqrt[a]*Sqrt[-a + b])] - Sqrt[-a + b]*ArcTanh
[(a + b + Sqrt[b + a*Cosh[c + d*x]]*(Sqrt[a*Cosh[c + d*x]] - Sqrt[b + a*Cos
h[c + d*x]]))/(Sqrt[a]*Sqrt[a + b])])*Sqrt[a*Cosh[c + d*x]]*Sqrt[(-a + a*Co
sh[c + d*x])/(a + a*Cosh[c + d*x])]*(a + a*Cosh[c + d*x])*Sqrt[Sech[c + d*x
]])/(a^(3/2)*Sqrt[-a + b]*Sqrt[a + b]*Sqrt[-1 + Cosh[c + d*x]]*Sqrt[1 + Cos
h[c + d*x]]) - (I*(2*a^4 - 4*a^2*b^2 + 2*b^4)*(-(Sqrt[a + b]*((4*I)*Sqrt[-a
 + b]*ArcTan[Sqrt[b + a*Cosh[c + d*x]]/Sqrt[-(a*Cosh[c + d*x])]] + Sqrt[a]*
```

```
ArcTan[(a + a*Cosh[c + d*x] + I*Sqrt[-(a*Cosh[c + d*x])]*Sqrt[b + a*Cosh[c
+ d*x]])/(Sqrt[a]*Sqrt[-a + b])])) + Sqrt[a]*Sqrt[-a + b]*ArcTanh[(a - a*Co
sh[c + d*x] - I*Sqrt[-(a*Cosh[c + d*x])]*Sqrt[b + a*Cosh[c + d*x]])/(Sqrt[a
]*Sqrt[a + b])])*Sqrt[-(a*Cosh[c + d*x])]*Sqrt[(-a + a*Cosh[c + d*x])/(a +
a*Cosh[c + d*x])]*(a + a*Cosh[c + d*x])*Cosh[2*(c + d*x)]*Sqrt[Sech[c + d*x
]])/(Sqrt[-a + b]*Sqrt[a + b]*Sqrt[-1 + Cosh[c + d*x]]*Sqrt[1 + Cosh[c + d*
x]]*(a^2 - 2*b^2 + 4*b*(b + a*Cosh[c + d*x]) - 2*(b + a*Cosh[c + d*x])^2)))
*Sech[c + d*x]^(3/2))/(4*a*(a - b)^2*(a + b)^2*d*(a + b*Sech[c + d*x])^(3/2
)) + ((b + a*Cosh[c + d*x])^2*(-1/2*(a^4 + a^2*b^2 + 4*b^4)/(a^2*(-a^2 + b^
2)^2) + (2*b^5)/(a^2*(a^2 - b^2)^2*(b + a*Cosh[c + d*x])) + ((-a^2 - b^2 +
2*a*b*Cosh[c + d*x])*Csch[c + d*x]^2)/(2*(-a^2 + b^2)^2))*Sech[c + d*x]^2)/
(d*(a + b*Sech[c + d*x])^(3/2))
```

Maple [F]
time = 3.31, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(dx+c)}{(a+b\,\mathrm{sech}\,(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x)
[Out] int(coth(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(coth(d*x + c)^3/(b*sech(d*x + c) + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5984 vs. $2(266) = 532$.
time = 6.47, size = 53212, normalized size = 168.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")
[Out] [-1/16*((4*a^7 + 5*a^6*b - 9*a^5*b^2 - 17*a^4*b^3 - 7*a^3*b^4 + (4*a^7 + 5*
a^6*b - 9*a^5*b^2 - 17*a^4*b^3 - 7*a^3*b^4)*cosh(d*x + c)^6 + (4*a^7 + 5*a^
6*b - 9*a^5*b^2 - 17*a^4*b^3 - 7*a^3*b^4)*sinh(d*x + c)^6 + 2*(4*a^6*b + 5*
```

a^5*b^2 - 9*a^4*b^3 - 17*a^3*b^4 - 7*a^2*b^5)*cosh(d*x + c)^5 + 2*(4*a^6*b
+ 5*a^5*b^2 - 9*a^4*b^3 - 17*a^3*b^4 - 7*a^2*b^5 + 3*(4*a^7 + 5*a^6*b - 9*a
^5*b^2 - 17*a^4*b^3 - 7*a^3*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 - (4*a^7 +
5*a^6*b - 9*a^5*b^2 - 17*a^4*b^3 - 7*a^3*b^4)*cosh(d*x + c)^4 - (4*a^7 + 5*
a^6*b - 9*a^5*b^2 - 17*a^4*b^3 - 7*a^3*b^4 - 15*(4*a^7 + 5*a^6*b - 9*a^5*b^
2 - 17*a^4*b^3 - 7*a^3*b^4)*cosh(d*x + c)^2 - 10*(4*a^6*b + 5*a^5*b^2 - 9*a
^4*b^3 - 17*a^3*b^4 - 7*a^2*b^5)*cosh(d*x + c))*sinh(d*x + c)^4 - 4*(4*a^6*
b + 5*a^5*b^2 - 9*a^4*b^3 - 17*a^3*b^4 - 7*a^2*b^5)*cosh(d*x + c)^3 - 4*(4*
a^6*b + 5*a^5*b^2 - 9*a^4*b^3 - 17*a^3*b^4 - 7*a^2*b^5 - 5*(4*a^7 + 5*a^6*b
 - 9*a^5*b^2 - 17*a^4*b^3 - 7*a^3*b^4)*cosh(d*x + c)^3 - 5*(4*a^6*b + 5*a^5
*b^2 - 9*a^4*b^3 - 17*a^3*b^4 - 7*a^2*b^5)*cosh(d*x + c)^2 + (4*a^7 + 5*a^6
*b - 9*a^5*b^2 - 17*a^4*b^3 - 7*a^3*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 - (
4*a^7 + 5*a^6*b - 9*a^5*b^2 - 17*a^4*b^3 - 7*a^3*b^4)*cosh(d*x + c)^2 - (4*
a^7 + 5*a^6*b - 9*a^5*b^2 - 17*a^4*b^3 - 7*a^3*b^4 - 15*(4*a^7 + 5*a^6*b -
9*a^5*b^2 - 17*a^4*b^3 - 7*a^3*b^4)*cosh(d*x + c)^4 - 20*(4*a^6*b + 5*a^5*b
^2 - 9*a^4*b^3 - 17*a^3*b^4 - 7*a^2*b^5)*cosh(d*x + c)^3 + 6*(4*a^7 + 5*a^6
*b - 9*a^5*b^2 - 17*a^4*b^3 - 7*a^3*b^4)*cosh(d*x + c)^2 + 12*(4*a^6*b + 5*
a^5*b^2 - 9*a^4*b^3 - 17*a^3*b^4 - 7*a^2*b^5)*cosh(d*x + c))*sinh(d*x + c)^
2 + 2*(4*a^6*b + 5*a^5*b^2 - 9*a^4*b^3 - 17*a^3*b^4 - 7*a^2*b^5)*cosh(d*x +
 c) + 2*(4*a^6*b + 5*a^5*b^2 - 9*a^4*b^3 - 17*a^3*b^4 - 7*a^2*b^5 + 3*(4*a^
7 + 5*a^6*b - 9*a^5*b^2 - 17*a^4*b^3 - 7*a^3*b^4)*cosh(d*x + c)^5 + 5*(4*a^
6*b + 5*a^5*b^2 - 9*a^4*b^3 - 17*a^3*b^4 - 7*a^2*b^5)*cosh(d*x + c)^4 - 2*(
4*a^7 + 5*a^6*b - 9*a^5*b^2 - 17*a^4*b^3 - 7*a^3*b^4)*cosh(d*x + c)^3 - 6*(
4*a^6*b + 5*a^5*b^2 - 9*a^4*b^3 - 17*a^3*b^4 - 7*a^2*b^5)*cosh(d*x + c)^2 -
 (4*a^7 + 5*a^6*b - 9*a^5*b^2 - 17*a^4*b^3 - 7*a^3*b^4)*cosh(d*x + c))*sinh
(d*x + c))*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cosh(d*x + c)^4 + (8*a^2
 - 8*a*b + b^2)*sinh(d*x + c)^4 + 4*(4*a*b - 3*b^2)*cosh(d*x + c)^3 + 4*(4*
a*b - 3*b^2 + (8*a^2 - 8*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(8*a
^2 - 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 2*(3*(8*a^2 - 8*a*b + b^2)*cosh(d*x +
 c)^2 + 8*a^2 - 8*a*b + 3*b^2 + 6*(4*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x +
 c)^2 + 8*a^2 - 8*a*b + b^2 + 4*((2*a - b)*cosh(d*x + c)^4 + (2*a - b)*sinh
(d*x + c)^4 + 2*b*cosh(d*x + c)^3 + 2*(2*(2*a - b)*cosh(d*x + c) + b)*sinh(
d*x + c)^3 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*(2*a - b)*cosh(d*x + c)^2 +
 3*b*cosh(d*x + c) + 2*a - b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*(2*(2
*a - b)*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 2*(2*a - b)*cosh(d*x + c) +
 b)*sinh(d*x + c) + 2*a - b)*sqrt(a - b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*
x + c)) + 4*(4*a*b - 3*b^2)*cosh(d*x + c) + 4*((8*a^2 - 8*a*b + b^2)*cosh(d
x + c)^3 + 3(4*a*b - 3*b^2)*cosh(d*x + c)^2 + 4*a*b - 3*b^2 + (8*a^2 - 8*
a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^4 + 4*(cosh(d*x +
 c) + 1)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 4*cosh(d*x + c)^3 + 6*(cosh(d*
x + c)^2 + 2*cosh(d*x + c) + 1)*sinh(d*x + c)^2 + 6*cosh(d*x + c)^2 + 4*(co
sh(d*x + c)^3 + 3*cosh(d*x + c)^2 + 3*cosh(d*x + c) + 1)*sinh(d*x + c) + 4*
cosh(d*x + c) + 1)) - (4*a^7 - 5*a^6*b - 9*a^5*b^2 + 17*a^4*b^3 - 7*a^3*b^4
 + (4*a^7 - 5*a^6*b - 9*a^5*b^2 + 17*a^4*b^3 - 7*a^3*b^4)*cosh(d*x + c)^6 +
 (4*a^7 - 5*a^6*b - 9*a^5*b^2 + 17*a^4*b^3 - 7*a^3*b^4)*sinh(d*x + c)^6 + 2

```
*(4*a^6*b - 5*a^5*b^2 - 9*a^4*b^3 + 17*a^3*b^4 - 7*a^2*b^5)*cosh(d*x + c)^5
 + 2*(4*a^6*b - 5*a^5*b^2 - 9*a^4*b^3 + 17*a^3*b^4 - 7*a^2*b^5 + 3*(4*a^7 -
 5*a^6*b - 9*a^5*b^2 + 17*a^4*b^3 - 7*a^3*b^4)*cosh(d*x + c))*sinh(d*x + c)
^5 - (4*a^7 - 5*a^6*b - 9*a^5*b^2 + 17*a^4*b^3 - 7*a^3*b^4)*cosh(d*x + c)^4
 - (4*a^7 - 5*a^6*b - 9*a^5*b^2 + 17*a^4*b^3 - 7*a^3*b^4 - 15*(4*a^7 - 5*a^
6*b - 9*a^5*b^2 + 17*a^4*b^3 - 7*a^3*b^4)*cosh(d*x + c)^2 - 10*(4*a^6*b - 5
*a^5*b^2 - 9*a^4*b^3 + 17*a^3*b^4 - 7*a^2*b^5)*cosh(d*x + c))*sinh(d*x + c)
^4 - 4*(4*a^6*b - 5*a^5*b^2 - 9*a^4*b^3 + 17*a^3*b^4 - 7*a^2*b^5)*cosh(d*x
+ c)^3 - 4*(4*a^6*b - 5*a^5*b^2 - 9*a^4*b^3 + 17*a^3*b^4 - 7*a^2*b^5 - 5*(4
*a^7 - 5*a^6*b - 9*a^5*b^2 + 17*a^4*b^3 - 7*a^3*b^4)*cosh(d*x + c)^3 - 5*(4
*a^6*b - 5*a^5*b^2 - 9*a^4*b^3 + 17*a^3*b^4 - 7*a^2*b^5)*cosh(d*x + c)^2 +
(4*a^7 - 5*a^6*b - 9*a^5*b^2 + 17*a^4*b^3 - 7*a^3*b^4)*cosh(d*x + c))*sinh(
d*x + c)^3 - (4*a^7 - 5*a^6*b - 9*a^5*b^2 + 17*a^4*b^3 - 7*a^3*b^4)*cosh(d*
x + c)^2 - (4*a^7 - 5*a^6*b - 9*a^5*b^2 + 17*a^4*b^3 - 7*a^3*b^4 - 15*(4*a^
7 - 5*a^6*b - 9*a^5*b^2 + 17*a^4*b^3 - 7*a^3*b^4)*cosh(d*x + c)^4 - 20*(4*a
^6*b - 5*a^5*b^2 - 9*a^4*b^3 + 17*a^3*b^4 - 7*a^2*b^5)*cosh(d*x + c)^3 + 6*
(4*a^7 - 5*a^6*b - 9*a^5*b^2 + 17*a^4*b^3 - 7*a^3*b^4)*cosh(d*x + c)^2 + 12
*(4*a^6*b - 5*a^5*b^2 - 9*a^4*b^3 + 17*a^3*b^4 - 7*a^2*b^5)*cosh(d*x + c))*
sinh(d*x + c)^2 + 2*(4*a^6*b - 5*a^5*b^2 - 9*a^...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(c+dx)}{(a+b\operatorname{sech}(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3/(a+b*sech(d*x+c))**(3/2),x)
[Out] Integral(coth(c + d*x)**3/(a + b*sech(c + d*x))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(coth(d*x + c)^3/(b*sech(d*x + c) + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\coth(c+d\,x)^3}{\left(a+\frac{b}{\cosh(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^3/(a + b/cosh(c + d*x))^(3/2),x)
[Out] int(coth(c + d*x)^3/(a + b/cosh(c + d*x))^(3/2), x)
```

3.147 $\int \frac{\tanh^4(c+dx)}{(a+b\,\mathbf{sech}(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=907

$$-\frac{2\coth(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{a\sqrt{a+b}\,d}$$

[Out] -2*coth(d*x+c)*EllipticE((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/a/d/(a+b)^(1/2)+4*a*coth(d*x+c)*EllipticE((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/b^2/d/(a+b)^(1/2)-2/3*a*(8*a^2-5*b^2)*coth(d*x+c)*EllipticE((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/b^4/d/(a+b)^(1/2)+2*coth(d*x+c)*EllipticF((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/a/d/(a+b)^(1/2)+4*coth(d*x+c)*EllipticF((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/b/d/(a+b)^(1/2)-2/3*(2*a+b)*(4*a+b)*coth(d*x+c)*EllipticF((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/b^3/d/(a+b)^(1/2)+2*coth(d*x+c)*EllipticPi((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/a^2/d-4*a*tanh(d*x+c)/(a^2-b^2)/d/(a+b*sech(d*x+c))^(1/2)+2*b^2*tanh(d*x+c)/a/(a^2-b^2)/d/(a+b*sech(d*x+c))^(1/2)-2*a^2*sech(d*x+c)*tanh(d*x+c)/b/(a^2-b^2)/d/(a+b*sech(d*x+c))^(1/2)+2/3*(4*a^2-b^2)*(a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)/b^2/(a^2-b^2)/d

Rubi [A]
time = 0.94, antiderivative size = 907, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3980, 3870, 4143, 4006, 3869, 3917, 4089, 3921, 4090, 3930, 4167}

Antiderivative was successfully verified.

[In] Int[Tanh[c + d*x]^4/(a + b*Sech[c + d*x])^(3/2),x]

[Out] (-2*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a*Sqrt[a + b]*d) + (4*a*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(b^2*Sqrt[

```
a + b]*d) - (2*a*(8*a^2 - 5*b^2)*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*
Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/
(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(3*b^4*Sqrt[a + b]*d) +
(2*Coth[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (
a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c
 + d*x]))/(a - b))])/(a*Sqrt[a + b]*d) + (4*Coth[c + d*x]*EllipticF[ArcSin[
Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[
c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(b*Sqrt[a + b
]*d) - (2*(2*a + b)*(4*a + b)*Coth[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec
h[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a
+ b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(3*b^3*Sqrt[a + b]*d) + (2*
Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sech[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*
Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a^2*d) - (4*a*Tanh[c + d*x])/((a
^2 - b^2)*d*Sqrt[a + b*Sech[c + d*x]]) + (2*b^2*Tanh[c + d*x])/(a*(a^2 - b^
2)*d*Sqrt[a + b*Sech[c + d*x]]) - (2*a^2*Sech[c + d*x]*Tanh[c + d*x])/(b*(a
^2 - b^2)*d*Sqrt[a + b*Sech[c + d*x]]) + (2*(4*a^2 - b^2)*Sqrt[a + b*Sech[c
 + d*x]]*Tanh[c + d*x])/(3*b^2*(a^2 - b^2)*d)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
```

```
x_Symbol] :> Simp[a*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(
a^2 - b^2))), x] - Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(b*(m + 1) - a*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{
a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3930

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[d^3/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^
(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m
 + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n,
 2]))
```

Rule 3980

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Csc[c + d
*x]^2)^(m/2), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] && I
GtQ[m/2, 0] && IntegerQ[n - 1/2]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

```
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4167

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b
*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^4(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \left(\frac{1}{(a+b\text{sech}(c+dx))^{3/2}} - \frac{2\text{sech}^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}} + \frac{\text{sech}^4(c+dx)}{(a+b\text{sech}(c+dx))^{3/}}\right.$$

$$= -\left(2\int \frac{\text{sech}^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx\right) + \int \frac{1}{(a+b\text{sech}(c+dx))^{3/2}}\,dx + \int \frac{}{(a+}$$

$$= -\frac{4a\tanh(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\text{sech}(c+dx)}} + \frac{2b^2\tanh(c+dx)}{a\,(a^2-b^2)\,d\sqrt{a+b\text{sech}(c+dx)}} - \frac{2}{b\,(a}$$

$$= -\frac{4a\tanh(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\text{sech}(c+dx)}} + \frac{2b^2\tanh(c+dx)}{a\,(a^2-b^2)\,d\sqrt{a+b\text{sech}(c+dx)}} - \frac{2}{b\,(a}$$

$$= -\frac{2\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx}{a+b}}}{a\sqrt{a+b}\,d}$$

$$= -\frac{2\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx}{a+b}}}{a\sqrt{a+b}\,d}$$

Mathematica [**F**]

time = 180.01, size = 0, normalized size = 0.00

$$\text{\$Aborted}$$

Verification is not applicable to the result.

```
[In] Integrate[Tanh[c + d*x]^4/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] $Aborted
```

Maple [F]
time = 3.10, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{4}\left(dx+c\right)}{\left(a+b\,\mathrm{sech}\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(3/2),x)
```

```
[Out] int(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(d*x + c)^4/(b*sech(d*x + c) + a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c)^4/(b^2*sech(d*x + c)^2 + 2
*a*b*sech(d*x + c) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{4}\left(c+dx\right)}{\left(a+b\,\mathrm{sech}\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**4/(a+b*sech(d*x+c))**(3/2),x)
```

```
[Out] Integral(tanh(c + d*x)**4/(a + b*sech(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(d*x + c)^4/(b*sech(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tanh(c+d\,x)^4}{\left(a+\frac{b}{\cosh(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^4/(a + b/cosh(c + d*x))^(3/2),x)
```

```
[Out] int(tanh(c + d*x)^4/(a + b/cosh(c + d*x))^(3/2), x)
```

3.148 $\int \frac{\tanh^2(c+dx)}{(a+b\,\mathbf{sech}(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=344

$$\frac{2(a-b)\sqrt{a+b}\,\coth(c+dx)E\left(\mathrm{ArcSin}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\mathrm{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\mathrm{se}\cdots}{\cdots}}}{ab^2d}$$

```
[Out] 2*(a-b)*coth(d*x+c)*EllipticE((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a
-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))
/(a-b))^(1/2)/a/b^2/d+2*coth(d*x+c)*EllipticF((a+b*sech(d*x+c))^(1/2)/(a+b)
^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b
*(1+sech(d*x+c))/(a-b))^(1/2)/a/b/d+2*coth(d*x+c)*EllipticPi((a+b*sech(d*x+
c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d
*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/a^2/d-2*tanh(d*x+c)/a/
d/(a+b*sech(d*x+c))^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3979, 4146, 4144, 4006, 3869, 3917, 4089}

$$\frac{2\sqrt{a+b}\,\coth(c+dx)\sqrt{\frac{b(1-\mathrm{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\mathrm{sech}(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\mathrm{ArcSin}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^2d}+\frac{2(a-b)\sqrt{a+b}\,\coth(c+dx)\sqrt{\frac{b(1-\mathrm{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\mathrm{sech}(c+dx)+1)}{a-b}}\,E\left(\mathrm{ArcSin}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ab^2d}+\frac{2\sqrt{a+b}\,\coth(c+dx)\sqrt{\frac{b(1-\mathrm{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\mathrm{sech}(c+dx)+1)}{a-b}}\,F\left(\mathrm{ArcSin}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{abd}-\frac{2\tanh(c+dx)}{ad\sqrt{a+b\,\mathrm{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^2/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c + d
*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a*b^2*d) + (2*Sqrt[a + b]*Coth[c
+ d*x]*EllipticF[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a
- b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/
(a - b))])/(a*b*d) + (2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(a + b)/a, Arc
Sin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - S
ech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a^2*d) -
 (2*Tanh[c + d*x])/(a*d*Sqrt[a + b*Sech[c + d*x]])
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3979

```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4144

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*
(b_.) + (a_)], x_Symbol] :> Int[(A - C*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x
]], x] + Dist[C, Int[Csc[e + f*x]*((1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*
x]]), x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4146

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 + a^2*C)*Cot[e + f*x]*((a + b*Csc[
e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 -
 b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*b*(
A + C)*(m + 1)*Csc[e + f*x] + (A*b^2 + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = -\int \frac{-1+\text{sech}^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx$$

$$= -\frac{2\tanh(c+dx)}{ad\sqrt{a+b\text{sech}(c+dx)}} + \frac{2\int \frac{\frac{1}{2}(a^2-b^2)+\frac{1}{2}(a^2-b^2)\text{sech}^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx}{a(a^2-b^2)}$$

$$= -\frac{2\tanh(c+dx)}{ad\sqrt{a+b\text{sech}(c+dx)}} + \frac{\int \frac{\text{sech}(c+dx)(1+\text{sech}(c+dx))}{\sqrt{a+b\text{sech}(c+dx)}}\,dx}{a} + \frac{2\int \frac{\frac{1}{2}(a^2-b^2)-\frac{1}{2}(a}{\sqrt{a+b\text{se}}}}{a(a^2}$$

$$= \frac{2(a-b)\sqrt{a+b}\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(}{}}}{ab^2d}$$

$$= \frac{2(a-b)\sqrt{a+b}\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(}{}}}{ab^2d}$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[Tanh[c + d*x]^2/(a + b*Sech[c + d*x])^(3/2),x]
[Out] $Aborted
```

Maple [F]
time = 2.06, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(dx+c)}{(a+b\,\text{sech}(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x)
[Out] int(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(d*x + c)^2/(b*sech(d*x + c) + a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c)^2/(b^2*sech(d*x + c)^2 + 2
*a*b*sech(d*x + c) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(c + dx)}{(a + b\operatorname{sech}(c + dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**2/(a+b*sech(d*x+c))**(3/2),x)
```

```
[Out] Integral(tanh(c + d*x)**2/(a + b*sech(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(d*x + c)^2/(b*sech(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tanh(c + d\,x)^2}{\left(a + \frac{b}{\cosh(c + d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^2/(a + b/cosh(c + d*x))^(3/2),x)
```

```
[Out] int(tanh(c + d*x)^2/(a + b/cosh(c + d*x))^(3/2), x)
```

3.149 $\int \frac{1}{(a+b\mathbf{sech}(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=347

$$-\frac{2\coth(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{a\sqrt{a+b}\,d}$$

```
[Out] -2*coth(d*x+c)*EllipticE((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^
(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/a/d
/(a+b)^(1/2)+2*coth(d*x+c)*EllipticF((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((
a+b)/(a-b))^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b
))^(1/2)/a/d/(a+b)^(1/2)+2*coth(d*x+c)*EllipticPi((a+b*sech(d*x+c))^(1/2)/(
a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+c))/(a+b
))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/a^2/d+2*b^2*tanh(d*x+c)/a/(a^2-b^
2)/d/(a+b*sech(d*x+c))^(1/2)
```

Rubi [A]

time = 0.23, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3870, 4143, 4006, 3869, 3917, 4089}

$$\frac{2\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^2d}+\frac{2b^2\tanh(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\text{sech}(c+dx)}}+\frac{2\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad\sqrt{a+b}}-\frac{2\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x])^(-3/2),x]
```

```
[Out] (-2*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]],
(a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[
c + d*x]))/(a - b))])/(a*Sqrt[a + b]*d) + (2*Coth[c + d*x]*EllipticF[ArcSin
[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech
[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a*Sqrt[a +
b]*d) + (2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a +
b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x])
)/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a^2*d) + (2*b^2*Tanh[
c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sech[c + d*x]])
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \frac{2b^2\tanh(c+dx)}{a(a^2-b^2)\,d\sqrt{a+b\text{sech}(c+dx)}} - \frac{2\int \frac{\frac{1}{2}(-a^2+b^2)+\frac{1}{2}ab\text{sech}(c+dx)+\frac{1}{2}b^2\text{sech}^2(c+\cdots}{\sqrt{a+b\text{sech}(c+dx)}}}{a(a^2-b^2)}$$

$$= \frac{2b^2\tanh(c+dx)}{a(a^2-b^2)\,d\sqrt{a+b\text{sech}(c+dx)}} - \frac{2\int \frac{\frac{1}{2}(-a^2+b^2)+\left(\frac{ab}{2}-\frac{b^2}{2}\right)\text{sech}(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx}{a(a^2-b^2)} - \cdots b^2\cdots$$

$$= -\frac{2\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+\cdots}{a+b}}\cdots}{a\sqrt{a+b}\,d}$$

$$= -\frac{2\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+\cdots}{a+b}}\cdots}{a\sqrt{a+b}\,d}$$

Mathematica [F]
time = 72.37, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\text{sech}(c+dx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sech[c + d*x])^(-3/2),x]
```

```
[Out] Integrate[(a + b*Sech[c + d*x])^(-3/2), x]
```

Maple [F]
time = 2.42, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\,\text{sech}\,(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sech(d*x+c))^(3/2),x)
```

```
[Out] int(1/(a+b*sech(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sech(d*x + c) + a)^(-3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sech(d*x + c) + a)/(b^2*sech(d*x + c)^2 + 2*a*b*sech(d*x +
c) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{sech}\left(c + dx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*sech(c + d*x))**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sech(d*x + c) + a)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+\frac{b}{\mathrm{cosh}\left(c+d\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cosh(c + d*x))^(3/2),x)
```

```
[Out] int(1/(a + b/cosh(c + d*x))^(3/2), x)
```

3.150 $\int \frac{\coth^2(c+dx)}{(a+b\,\mathbf{sech}(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=665

$$\frac{4a\coth(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{(a-b)(a+b)^{3/2}d}$$

```
[Out] -coth(d*x+c)/d/(a+b*sech(d*x+c))^(3/2)+4*a*coth(d*x+c)*EllipticE((a+b*sech(
d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1
/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/(a-b)/(a+b)^(3/2)/d-(3*a-b)*coth(d*x+c
)*EllipticF((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-
sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/(a-b)/(a+b)^(3/2
)/d-2*coth(d*x+c)*EllipticE((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b
))^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/
a/d/(a+b)^(1/2)+2*coth(d*x+c)*EllipticF((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2)
,((a+b)/(a-b))^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(
a-b))^(1/2)/a/d/(a+b)^(1/2)+2*coth(d*x+c)*EllipticPi((a+b*sech(d*x+c))^(1/2
)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+c))/(
a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/a^2/d-b^2*tanh(d*x+c)/(a^2-b^2
)/d/(a+b*sech(d*x+c))^(3/2)-4*a*b^2*tanh(d*x+c)/(a^2-b^2)^2/d/(a+b*sech(d*x
+c))^(1/2)+2*b^2*tanh(d*x+c)/a/(a^2-b^2)/d/(a+b*sech(d*x+c))^(1/2)
```

Rubi [A]
time = 0.68, antiderivative size = 665, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3981, 3870, 4143, 4006, 3869, 3917, 4089, 3960, 3918, 4088, 4090}

$$\frac{2\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^2d}+\frac{2b^2\tanh(c+dx)}{ad(a^2-b^2)\sqrt{a+b\text{sech}(c+dx)}}-\frac{4ab^2\tanh(c+dx)}{d(a^2-b^2)^2\sqrt{a+b\text{sech}(c+dx)}}-\frac{b^2\tanh(c+dx)}{d(a^2-b^2)(a+b\text{sech}(c+dx))^{3/2}}+\frac{2\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad\sqrt{a+b}}-\frac{(3a-b)\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d(a-b)(a+b)^{3/2}}-\frac{2\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad\sqrt{a+b}}+\frac{4a\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d(a-b)(a+b)^{3/2}}-\frac{\coth(c+dx)}{d(a+b\text{sech}(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^2/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] (4*a*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]],
 (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech
[c + d*x]))/(a - b))])/((a - b)*(a + b)^(3/2)*d) - (2*Coth[c + d*x]*Ellipti
cE[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*
(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a*
Sqrt[a + b]*d) - ((3*a - b)*Coth[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sech[
c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a +
b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/((a - b)*(a + b)^(3/2)*d) + (
2*Coth[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a
 + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c
+ d*x]))/(a - b))])/(a*Sqrt[a + b]*d) + (2*Sqrt[a + b]*Coth[c + d*x]*Ellipt
```

```
icPi[(a + b)/a, ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
 b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(
a - b))]])/(a^2*d) - Coth[c + d*x]/(d*(a + b*Sech[c + d*x])^(3/2)) - (b^2*Ta
nh[c + d*x])/((a^2 - b^2)*d*(a + b*Sech[c + d*x])^(3/2)) - (4*a*b^2*Tanh[c
+ d*x])/((a^2 - b^2)^2*d*Sqrt[a + b*Sech[c + d*x]]) + (2*b^2*Tanh[c + d*x])
/(a*(a^2 - b^2)*d*Sqrt[a + b*Sech[c + d*x]])
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3918

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*C
sc[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[Tan[e + f*x]*((a + b*Csc[e + f*x])^m/f), x] + Dist[b*m, I
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
m}, x]
```

Rule 3981

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Sec[c + d
*x]^2)^(-m/2), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] &&
ILtQ[m/2, 0] && IntegerQ[n - 1/2] && EqQ[m, -2]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4088

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1
/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[
(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
```

```
+ Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\coth^2(c+dx)}{(a+b\mathrm{sech}(c+dx))^{3/2}}\,dx = -\int \left(-\frac{1}{(a+b\mathrm{sech}(c+dx))^{3/2}} - \frac{\mathrm{csch}^2(c+dx)}{(a+b\mathrm{sech}(c+dx))^{3/2}}\right) dx$$

$$= \int \frac{1}{(a+b\mathrm{sech}(c+dx))^{3/2}}\,dx + \int \frac{\mathrm{csch}^2(c+dx)}{(a+b\mathrm{sech}(c+dx))^{3/2}}\,dx$$

$$= -\frac{\coth(c+dx)}{d(a+b\mathrm{sech}(c+dx))^{3/2}} + \frac{2b^2\tanh(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\mathrm{sech}(c+dx)}} + \frac{1}{2}(3b)\int \frac{}{(a}$$

$$= -\frac{\coth(c+dx)}{d(a+b\mathrm{sech}(c+dx))^{3/2}} - \frac{b^2\tanh(c+dx)}{\left(a^2-b^2\right)d(a+b\mathrm{sech}(c+dx))^{3/2}} + \frac{2b^2}{a\left(a^2-b^2\right)d}$$

$$= -\frac{2\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\mathrm{sech}(c+dx}{a+b}}}{a\sqrt{a+b}\,d}$$

$$= -\frac{2\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\mathrm{sech}(c+dx}{a+b}}}{a\sqrt{a+b}\,d}$$

$$= \frac{4a\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\mathrm{sech}(c+dx}{a+b}}}{(a-b)(a+b)^{3/2}d}$$

Mathematica [F]
time = 150.72, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(c+dx)}{(a+b\mathrm{sech}(c+dx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Coth[c + d*x]^2/(a + b*Sech[c + d*x])^(3/2),x]
[Out] Integrate[Coth[c + d*x]^2/(a + b*Sech[c + d*x])^(3/2), x]
```

Maple [F]
time = 2.84, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(dx+c)}{(a+b\,\mathrm{sech}(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x)
```

```
[Out] int(coth(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(d*x + c)^2/(b*sech(d*x + c) + a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sech(d*x + c) + a)*coth(d*x + c)^2/(b^2*sech(d*x + c)^2 + 2
*a*b*sech(d*x + c) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(c+dx)}{(a+b\operatorname{sech}(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2/(a+b*sech(d*x+c))**(3/2),x)
```

```
[Out] Integral(coth(c + d*x)**2/(a + b*sech(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(coth(d*x + c)^2/(b*sech(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\coth(c+d\,x)^2}{\left(a+\frac{b}{\cosh(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^2/(a + b/cosh(c + d*x))^(3/2),x)
[Out] int(coth(c + d*x)^2/(a + b/cosh(c + d*x))^(3/2), x)
```

3.151 $\int e^{c(a+bx)}\mathbf{sech}^2(ac+bcx)^{7/2}\,dx$

Optimal. Leaf size=191

$$\frac{32\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{3bc\left(1+e^{2c(a+bx)}\right)^6}-\frac{192\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{5bc\left(1+e^{2c(a+bx)}\right)^5}+\frac{48\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac}{bc\left(1+e^{2c(a+bx)}\right)^4}$$

```
[Out] 32/3*cosh(b*c*x+a*c)*(sech(b*c*x+a*c)^2)^(1/2)/b/c/(1+exp(2*c*(b*x+a)))^6-1
92/5*cosh(b*c*x+a*c)*(sech(b*c*x+a*c)^2)^(1/2)/b/c/(1+exp(2*c*(b*x+a)))^5+4
8*cosh(b*c*x+a*c)*(sech(b*c*x+a*c)^2)^(1/2)/b/c/(1+exp(2*c*(b*x+a)))^4-64/3
*cosh(b*c*x+a*c)*(sech(b*c*x+a*c)^2)^(1/2)/b/c/(1+exp(2*c*(b*x+a)))^3
```

Rubi [A]
time = 0.20, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6852, 2320, 12, 272, 45}

$$-\frac{64\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{3bc\left(e^{2c(a+bx)}+1\right)^3}+\frac{48\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{bc\left(e^{2c(a+bx)}+1\right)^4}-\frac{192\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{5bc\left(e^{2c(a+bx)}+1\right)^5}+\frac{32\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{3bc\left(e^{2c(a+bx)}+1\right)^6}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*(Sech[a*c + b*c*x]^2)^(7/2),x]
```

```
[Out] (32*Cosh[a*c + b*c*x]*Sqrt[Sech[a*c + b*c*x]^2])/(3*b*c*(1 + E^(2*c*(a + b*
x)))^6) - (192*Cosh[a*c + b*c*x]*Sqrt[Sech[a*c + b*c*x]^2])/(5*b*c*(1 + E^(
2*c*(a + b*x)))^5) + (48*Cosh[a*c + b*c*x]*Sqrt[Sech[a*c + b*c*x]^2])/(b*c*
(1 + E^(2*c*(a + b*x)))^4) - (64*Cosh[a*c + b*c*x]*Sqrt[Sech[a*c + b*c*x]^2
])/(3*b*c*(1 + E^(2*c*(a + b*x)))^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int e^{c(a+bx)}\operatorname{sech}^2(ac+bcx)^{7/2}\,dx = \left(\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\int e^{c(a+bx)}\operatorname{sech}^7(ac+bcx)\,dx$$

$$= \frac{\left(\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \frac{128x^7}{(1+x^2)^7}\,dx, x, e^{c(a+bx)}\right)}{bc}$$

$$= \frac{\left(128\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \frac{x^7}{(1+x^2)^7}\,dx, x, e^{c(a+bx)}\right)}{bc}$$

$$= \frac{\left(64\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \frac{x^3}{(1+x)^7}\,dx, x, e^{2c(a+bx)}\right)}{bc}$$

$$= \frac{\left(64\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \left(-\frac{1}{(1+x)^7}+\frac{3}{(1+x)^6}-\frac{3}{(1+x}\right.\right.}{bc}$$

$$= \frac{32\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{3bc\left(1+e^{2c(a+bx)}\right)^6} - \frac{192\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bc}}{5bc\left(1+e^{2c(a+bx)}\right)^5}$$

Mathematica [A]
time = 0.06, size = 84, normalized size = 0.44

$$-\frac{16\left(1+6e^{2c(a+bx)}+15e^{4c(a+bx)}+20e^{6c(a+bx)}\right)\cosh(c(a+bx))\sqrt{\operatorname{sech}^2(c(a+bx))}}{15bc\left(1+e^{2c(a+bx)}\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*(Sech[a*c + b*c*x]^2)^(7/2),x]
```

```
[Out] (-16*(1 + 6*E^(2*c*(a + b*x)) + 15*E^(4*c*(a + b*x)) + 20*E^(6*c*(a + b*x))
)*Cosh[c*(a + b*x)]*Sqrt[Sech[c*(a + b*x)]^2])/(15*b*c*(1 + E^(2*c*(a + b*x
)))^6)
```

Maple [A]
time = 4.20, size = 91, normalized size = 0.48

method	result	size
risch	$-\frac{16\left(20\,e^{6c(bx+a)}+15\,e^{4c(bx+a)}+6\,e^{2c(bx+a)}+1\right)\sqrt{\frac{e^{2c(bx+a)}}{\left(1+e^{2c(bx+a)}\right)^{2}}}\;e^{-c(bx+a)}}{15cb\left(1+e^{2c(bx+a)}\right)^{5}}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -16/15/c/b*(20*exp(6*c*(b*x+a))+15*exp(4*c*(b*x+a))+6*exp(2*c*(b*x+a))+1)*(
1/(1+exp(2*c*(b*x+a)))^2*exp(2*c*(b*x+a)))^(1/2)/(1+exp(2*c*(b*x+a)))^5*exp
(-c*(b*x+a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 386 vs. 2(173) = 346.
time = 0.29, size = 386, normalized size = 2.02

$$-\frac{64\,e^{(6\,bcx+6\,ac)}}{3\,bc(e^{(12\,bcx+12\,ac)}+6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}+20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}+1)}-\frac{16\,e^{(4\,bcx+4\,ac)}}{bc(e^{(12\,bcx+12\,ac)}+6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}+20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}+1)}-\frac{32\,e^{(2\,bcx+2\,ac)}}{5\,bc(e^{(12\,bcx+12\,ac)}+6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}+20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}+1)}-\frac{16}{15\,bc(e^{(12\,bcx+12\,ac)}+6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}+20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -64/3*e^(6*b*c*x + 6*a*c)/(b*c*(e^(12*b*c*x + 12*a*c) + 6*e^(10*b*c*x + 10*
a*c) + 15*e^(8*b*c*x + 8*a*c) + 20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*
a*c) + 6*e^(2*b*c*x + 2*a*c) + 1)) - 16*e^(4*b*c*x + 4*a*c)/(b*c*(e^(12*b*c
*x + 12*a*c) + 6*e^(10*b*c*x + 10*a*c) + 15*e^(8*b*c*x + 8*a*c) + 20*e^(6*b
*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*a*c) + 1)) - 32/5
*e^(2*b*c*x + 2*a*c)/(b*c*(e^(12*b*c*x + 12*a*c) + 6*e^(10*b*c*x + 10*a*c)
+ 15*e^(8*b*c*x + 8*a*c) + 20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*a*c)
+ 6*e^(2*b*c*x + 2*a*c) + 1)) - 16/15/(b*c*(e^(12*b*c*x + 12*a*c) + 6*e^(10
*b*c*x + 10*a*c) + 15*e^(8*b*c*x + 8*a*c) + 20*e^(6*b*c*x + 6*a*c) + 15*e^(
4*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*a*c) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 589 vs. 2(173) = 346.
time = 0.37, size = 589, normalized size = 3.08

$$-\frac{16\left(21\,\cosh\left(bcx+ac\right)^{3}+63\,\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^{2}+19\,\sinh\left(bcx+ac\right)^{3}+3\left(19\,\cosh\left(bcx+ac\right)^{2}+3\right)\sinh\left(bcx+ac\right)+21\,\cosh\left(bcx+ac\right)\right)}{15\left(bc\cosh\left(bcx+ac\right)^{9}+9\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^{8}+bc\sinh\left(bcx+ac\right)^{9}+6\,bc\cosh\left(bcx+ac\right)^{7}+6\left(6\,bc\cosh\left(bcx+ac\right)^{2}+bc\right)\sinh\left(bcx+ac\right)^{7}+15\,bc\cosh\left(bcx+ac\right)^{5}+42\left(2\,bc\cosh\left(bcx+ac\right)^{3}+bc\cosh\left(bcx+ac\right)\right)\sinh\left(bcx+ac\right)^{6}+3\left(42\,bc\cosh\left(bcx+ac\right)^{4}+42\,bc\cosh\left(bcx+ac\right)^{2}+5\,bc\right)\sinh\left(bcx+ac\right)^{5}+21\,bc\cosh\left(bcx+ac\right)^{3}+3\left(42\,bc\cosh\left(bcx+ac\right)^{5}+70\,bc\cosh\left(bcx+ac\right)^{3}+25\,bc\cosh\left(bcx+ac\right)\right)\sinh\left(bcx+ac\right)^{4}+\left(84\,bc\cosh\left(bcx+ac\right)^{6}+210\,bc\cosh\left(bcx+ac\right)^{4}+150\,bc\cosh\left(bcx+ac\right)^{2}+19\,bc\right)\sinh\left(bcx+ac\right)^{3}+21\,bc\cosh\left(bcx+ac\right)+3\left(12\,bc\cosh\left(bcx+ac\right)^{7}+42\,bc\cosh\left(bcx+ac\right)^{5}+50\,bc\cosh\left(bcx+ac\right)^{3}+21\,bc\cosh\left(bcx+ac\right)\right)\sinh\left(bcx+ac\right)^{2}+3\left(3\,bc\cosh\left(bcx+ac\right)^{8}+14\,bc\cosh\left(bcx+ac\right)^{6}+25\,bc\cosh\left(bcx+ac\right)^{4}+19\,bc\cosh\left(bcx+ac\right)^{2}+3\,bc\right)\sinh\left(bcx+ac\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -16/15*(21*cosh(b*c*x + a*c)^3 + 63*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 +
 19*sinh(b*c*x + a*c)^3 + 3*(19*cosh(b*c*x + a*c)^2 + 3)*sinh(b*c*x + a*c)
+ 21*cosh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^9 + 9*b*c*cosh(b*c*x + a*c)*
sinh(b*c*x + a*c)^8 + b*c*sinh(b*c*x + a*c)^9 + 6*b*c*cosh(b*c*x + a*c)^7 +
 6*(6*b*c*cosh(b*c*x + a*c)^2 + b*c)*sinh(b*c*x + a*c)^7 + 15*b*c*cosh(b*c*
x + a*c)^5 + 42*(2*b*c*cosh(b*c*x + a*c)^3 + b*c*cosh(b*c*x + a*c))*sinh(b*
c*x + a*c)^6 + 3*(42*b*c*cosh(b*c*x + a*c)^4 + 42*b*c*cosh(b*c*x + a*c)^2 +
 5*b*c)*sinh(b*c*x + a*c)^5 + 21*b*c*cosh(b*c*x + a*c)^3 + 3*(42*b*c*cosh(b
*c*x + a*c)^5 + 70*b*c*cosh(b*c*x + a*c)^3 + 25*b*c*cosh(b*c*x + a*c))*sinh
(b*c*x + a*c)^4 + (84*b*c*cosh(b*c*x + a*c)^6 + 210*b*c*cosh(b*c*x + a*c)^4
 + 150*b*c*cosh(b*c*x + a*c)^2 + 19*b*c)*sinh(b*c*x + a*c)^3 + 21*b*c*cosh(
b*c*x + a*c) + 3*(12*b*c*cosh(b*c*x + a*c)^7 + 42*b*c*cosh(b*c*x + a*c)^5 +
 50*b*c*cosh(b*c*x + a*c)^3 + 21*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^2
 + 3*(3*b*c*cosh(b*c*x + a*c)^8 + 14*b*c*cosh(b*c*x + a*c)^6 + 25*b*c*cosh(
b*c*x + a*c)^4 + 19*b*c*cosh(b*c*x + a*c)^2 + 3*b*c)*sinh(b*c*x + a*c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)**2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.39, size = 64, normalized size = 0.34

$$-\frac{16\left(20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}+1\right)}{15\,bc(e^{(2\,bcx+2\,ac)}+1)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(7/2),x, algorithm="giac")
```

```
[Out] -16/15*(20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*
a*c) + 1)/(b*c*(e^(2*b*c*x + 2*a*c) + 1)^6)
```

Mupad [B]
time = 0.17, size = 405, normalized size = 2.12

$$\frac{24\,\sqrt{\frac{1}{{\left(\frac{{\mathrm{e}}^{a\,c+b\,c\,x}}{2}+\frac{{\mathrm{e}}^{-a\,c-b\,c\,x}}{2}\right)}^2}}\,\left(2\,{\mathrm{e}}^{2\,a\,c+2\,b\,c\,x}+{\mathrm{e}}^{4\,a\,c+4\,b\,c\,x}+1\right)}{b\,c\,\left({\mathrm{e}}^{a\,c+b\,c\,x}+{\mathrm{e}}^{3\,a\,c+3\,b\,c\,x}\right)\,{\left({\mathrm{e}}^{2\,a\,c+2\,b\,c\,x}+1\right)}^4}-\frac{32\,\sqrt{\frac{1}{{\left(\frac{{\mathrm{e}}^{a\,c+b\,c\,x}}{2}+\frac{{\mathrm{e}}^{-a\,c-b\,c\,x}}{2}\right)}^2}}\,\left(2\,{\mathrm{e}}^{2\,a\,c+2\,b\,c\,x}+{\mathrm{e}}^{4\,a\,c+4\,b\,c\,x}+1\right)}{3\,b\,c\,\left({\mathrm{e}}^{a\,c+b\,c\,x}+{\mathrm{e}}^{3\,a\,c+3\,b\,c\,x}\right)\,{\left({\mathrm{e}}^{2\,a\,c+2\,b\,c\,x}+1\right)}^3}-\frac{96\,\sqrt{\frac{1}{{\left(\frac{{\mathrm{e}}^{a\,c+b\,c\,x}}{2}+\frac{{\mathrm{e}}^{-a\,c-b\,c\,x}}{2}\right)}^2}}\,\left(2\,{\mathrm{e}}^{2\,a\,c+2\,b\,c\,x}+{\mathrm{e}}^{4\,a\,c+4\,b\,c\,x}+1\right)}{5\,b\,c\,\left({\mathrm{e}}^{a\,c+b\,c\,x}+{\mathrm{e}}^{3\,a\,c+3\,b\,c\,x}\right)\,{\left({\mathrm{e}}^{2\,a\,c+2\,b\,c\,x}+1\right)}^5}+\frac{16\,\sqrt{\frac{1}{{\left(\frac{{\mathrm{e}}^{a\,c+b\,c\,x}}{2}+\frac{{\mathrm{e}}^{-a\,c-b\,c\,x}}{2}\right)}^2}}\,\left(2\,{\mathrm{e}}^{2\,a\,c+2\,b\,c\,x}+{\mathrm{e}}^{4\,a\,c+4\,b\,c\,x}+1\right)}{3\,b\,c\,\left({\mathrm{e}}^{a\,c+b\,c\,x}+{\mathrm{e}}^{3\,a\,c+3\,b\,c\,x}\right)\,{\left({\mathrm{e}}^{2\,a\,c+2\,b\,c\,x}+1\right)}^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*(1/cosh(a*c + b*c*x)^2)^(7/2),x)
```

```
[Out] (24*(1/(exp(a*c + b*c*x)/2 + exp(- a*c - b*c*x)/2)^2)^(1/2)*(2*exp(2*a*c +
2*b*c*x) + exp(4*a*c + 4*b*c*x) + 1))/(b*c*(exp(a*c + b*c*x) + exp(3*a*c +
3*b*c*x))*(exp(2*a*c + 2*b*c*x) + 1)^4) - (32*(1/(exp(a*c + b*c*x)/2 + exp(
- a*c - b*c*x)/2)^2)^(1/2)*(2*exp(2*a*c + 2*b*c*x) + exp(4*a*c + 4*b*c*x) +
 1))/(3*b*c*(exp(a*c + b*c*x) + exp(3*a*c + 3*b*c*x))*(exp(2*a*c + 2*b*c*x)
 + 1)^3) - (96*(1/(exp(a*c + b*c*x)/2 + exp(- a*c - b*c*x)/2)^2)^(1/2)*(2*e
xp(2*a*c + 2*b*c*x) + exp(4*a*c + 4*b*c*x) + 1))/(5*b*c*(exp(a*c + b*c*x) +
 exp(3*a*c + 3*b*c*x))*(exp(2*a*c + 2*b*c*x) + 1)^5) + (16*(1/(exp(a*c + b*
c*x)/2 + exp(- a*c - b*c*x)/2)^2)^(1/2)*(2*exp(2*a*c + 2*b*c*x) + exp(4*a*c
 + 4*b*c*x) + 1))/(3*b*c*(exp(a*c + b*c*x) + exp(3*a*c + 3*b*c*x))*(exp(2*a
*c + 2*b*c*x) + 1)^6)
```

3.152 $\int e^{c(a+bx)}\text{sech}^2(ac+bcx)^{5/2}\,dx$

Optimal. Leaf size=141

$$-\frac{4\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}}{bc\left(1+e^{2c(a+bx)}\right)^4}+\frac{32\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}}{3bc\left(1+e^{2c(a+bx)}\right)^3}-\frac{8\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}}{bc\left(1+e^{2c(a+bx)}\right)^2}$$

```
[Out] -4*cosh(b*c*x+a*c)*(sech(b*c*x+a*c)^2)^(1/2)/b/c/(1+exp(2*c*(b*x+a)))^4+32/
3*cosh(b*c*x+a*c)*(sech(b*c*x+a*c)^2)^(1/2)/b/c/(1+exp(2*c*(b*x+a)))^3-8*co
sh(b*c*x+a*c)*(sech(b*c*x+a*c)^2)^(1/2)/b/c/(1+exp(2*c*(b*x+a)))^2
```

Rubi [A]
time = 0.12, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6852, 2320, 12, 272, 45}

$$-\frac{8\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}}{bc\left(e^{2c(a+bx)}+1\right)^2}+\frac{32\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}}{3bc\left(e^{2c(a+bx)}+1\right)^3}-\frac{4\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}}{bc\left(e^{2c(a+bx)}+1\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*(Sech[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] (-4*Cosh[a*c + b*c*x]*Sqrt[Sech[a*c + b*c*x]^2])/(b*c*(1 + E^(2*c*(a + b*x)
))^4) + (32*Cosh[a*c + b*c*x]*Sqrt[Sech[a*c + b*c*x]^2])/(3*b*c*(1 + E^(2*c
*(a + b*x)))^3) - (8*Cosh[a*c + b*c*x]*Sqrt[Sech[a*c + b*c*x]^2])/(b*c*(1 +
 E^(2*c*(a + b*x)))^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int e^{c(a+bx)}\operatorname{sech}^2(ac+bcx)^{5/2}\,dx = \left(\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\int e^{c(a+bx)}\operatorname{sech}^5(ac+bcx)\,dx$$
$$= \frac{\left(\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \frac{32x^5}{(1+x^2)^5}\,dx, x, e^{c(a+bx)}\right)}{bc}$$
$$= \frac{\left(32\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \frac{x^5}{(1+x^2)^5}\,dx, x, e^{c(a+bx)}\right)}{bc}$$
$$= \frac{\left(16\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \frac{x^2}{(1+x)^5}\,dx, x, e^{2c(a+bx)}\right)}{bc}$$
$$= \frac{\left(16\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \left(\frac{1}{(1+x)^5} - \frac{2}{(1+x)^4} + \frac{1}{(1+x}\right.\right.}{bc}$$
$$= -\frac{4\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{bc\left(1+e^{2c(a+bx)}\right)^4} + \frac{32\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+b}}{3bc\left(1+e^{2c(a+bx)}\right)^3}$$

Mathematica [A]
time = 0.05, size = 72, normalized size = 0.51

$$-\frac{4\left(1+4e^{2c(a+bx)}+6e^{4c(a+bx)}\right)\cosh(c(a+bx))\sqrt{\operatorname{sech}^2(c(a+bx))}}{3bc\left(1+e^{2c(a+bx)}\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*(Sech[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] (-4*(1 + 4*E^(2*c*(a + b*x)) + 6*E^(4*c*(a + b*x)))*Cosh[c*(a + b*x)]*Sqrt[
Sech[c*(a + b*x)]^2])/(3*b*c*(1 + E^(2*c*(a + b*x)))^4)
```

Maple [A]
time = 4.09, size = 80, normalized size = 0.57

method	result	size
risch	$-\frac{4\left(6\,{e}^{4c(bx+a)}+4\,{e}^{2c(bx+a)}+1\right)\sqrt{\frac{{e}^{2c(bx+a)}}{\left(1+{e}^{2c(bx+a)}\right)^{2}}}\,{e}^{-c(bx+a)}}{3cb\left(1+{e}^{2c(bx+a)}\right)^{3}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/3/c/b*(6*exp(4*c*(b*x+a))+4*exp(2*c*(b*x+a))+1)*(1/(1+exp(2*c*(b*x+a)))^
2*exp(2*c*(b*x+a)))^(1/2)/(1+exp(2*c*(b*x+a)))^3*exp(-c*(b*x+a))
```

Maxima [A]
time = 0.28, size = 209, normalized size = 1.48

$$-\frac{8\,e^{(4\,bcx+4\,ac)}}{bc\left(e^{(8\,bcx+8\,ac)}+4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}+4\,e^{(2\,bcx+2\,ac)}+1\right)}-\frac{16\,e^{(2\,bcx+2\,ac)}}{3\,bc\left(e^{(8\,bcx+8\,ac)}+4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}+4\,e^{(2\,bcx+2\,ac)}+1\right)}-\frac{4}{3\,bc\left(e^{(8\,bcx+8\,ac)}+4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}+4\,e^{(2\,bcx+2\,ac)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -8*e^(4*b*c*x + 4*a*c)/(b*c*(e^(8*b*c*x + 8*a*c) + 4*e^(6*b*c*x + 6*a*c) +
6*e^(4*b*c*x + 4*a*c) + 4*e^(2*b*c*x + 2*a*c) + 1)) - 16/3*e^(2*b*c*x + 2*a
*c)/(b*c*(e^(8*b*c*x + 8*a*c) + 4*e^(6*b*c*x + 6*a*c) + 6*e^(4*b*c*x + 4*a*
c) + 4*e^(2*b*c*x + 2*a*c) + 1)) - 4/3/(b*c*(e^(8*b*c*x + 8*a*c) + 4*e^(6*b
*c*x + 6*a*c) + 6*e^(4*b*c*x + 4*a*c) + 4*e^(2*b*c*x + 2*a*c) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 315 vs. 2(130) = 260.
time = 0.37, size = 315, normalized size = 2.23

$$-\frac{4\left(7\cosh\left(bcx+ac\right)^{2}+10\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)+7\sinh\left(bcx+ac\right)^{2}+4\right)}{3\left(bc\cosh\left(bcx+ac\right)^{6}+6\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^{5}+bc\sinh\left(bcx+ac\right)^{6}+4\,bc\cosh\left(bcx+ac\right)^{4}+\left(15\,bc\cosh\left(bcx+ac\right)^{2}+4\,bc\right)\sinh\left(bcx+ac\right)^{4}+7\,bc\cosh\left(bcx+ac\right)^{2}+4\left(5\,bc\cosh\left(bcx+ac\right)^{3}+4\,bc\cosh\left(bcx+ac\right)\right)\sinh\left(bcx+ac\right)^{3}+\left(15\,bc\cosh\left(bcx+ac\right)^{4}+24\,bc\cosh\left(bcx+ac\right)^{2}+7\,bc\right)\sinh\left(bcx+ac\right)^{2}+4\,bc+2\left(3\,bc\cosh\left(bcx+ac\right)^{5}+8\,bc\cosh\left(bcx+ac\right)^{3}+5\,bc\cosh\left(bcx+ac\right)\right)\sinh\left(bcx+ac\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -4/3*(7*cosh(b*c*x + a*c)^2 + 10*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + 7*si
nh(b*c*x + a*c)^2 + 4)/(b*c*cosh(b*c*x + a*c)^6 + 6*b*c*cosh(b*c*x + a*c)*s
inh(b*c*x + a*c)^5 + b*c*sinh(b*c*x + a*c)^6 + 4*b*c*cosh(b*c*x + a*c)^4 +
(15*b*c*cosh(b*c*x + a*c)^2 + 4*b*c)*sinh(b*c*x + a*c)^4 + 7*b*c*cosh(b*c*x
 + a*c)^2 + 4*(5*b*c*cosh(b*c*x + a*c)^3 + 4*b*c*cosh(b*c*x + a*c))*sinh(b*
```

```
c*x + a*c)^3 + (15*b*c*cosh(b*c*x + a*c)^4 + 24*b*c*cosh(b*c*x + a*c)^2 + 7
*b*c)*sinh(b*c*x + a*c)^2 + 4*b*c + 2*(3*b*c*cosh(b*c*x + a*c)^5 + 8*b*c*co
sh(b*c*x + a*c)^3 + 5*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)**2)**(5/2),x)
[Out] Timed out
```

Giac [A]
time = 0.37, size = 51, normalized size = 0.36

$$-\frac{4\left(6\,e^{(4\,bcx+4\,ac)}+4\,e^{(2\,bcx+2\,ac)}+1\right)}{3\,bc{\left(e^{(2\,bcx+2\,ac)}+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
[Out] -4/3*(6*e^(4*b*c*x + 4*a*c) + 4*e^(2*b*c*x + 2*a*c) + 1)/(b*c*(e^(2*b*c*x +
 2*a*c) + 1)^4)
```

Mupad [B]
time = 1.43, size = 91, normalized size = 0.65

$$-\frac{2\,\mathrm{e}^{-a\,c-b\,c\,x}\,\sqrt{\frac{1}{{\left(\frac{\mathrm{e}^{a\,c+b\,c\,x}}{2}+\frac{\mathrm{e}^{-a\,c-b\,c\,x}}{2}\right)}^2}}\,\left(4\,\mathrm{e}^{2\,a\,c+2\,b\,c\,x}+6\,\mathrm{e}^{4\,a\,c+4\,b\,c\,x}+1\right)}{3\,b\,c\,{\left(\mathrm{e}^{2\,a\,c+2\,b\,c\,x}+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*(1/cosh(a*c + b*c*x)^2)^(5/2),x)
[Out] -(2*exp(- a*c - b*c*x)*(1/(exp(a*c + b*c*x)/2 + exp(- a*c - b*c*x)/2)^2)^(1
/2)*(4*exp(2*a*c + 2*b*c*x) + 6*exp(4*a*c + 4*b*c*x) + 1))/(3*b*c*(exp(2*a*
c + 2*b*c*x) + 1)^3)
```

3.153 $\int e^{c(a+bx)}\textbf{sech}^2(ac+bcx)^{3/2}\,dx$

Optimal. Leaf size=56

$$\frac{2e^{4c(a+bx)}\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{bc\left(1+e^{2c(a+bx)}\right)^2}$$

```
[Out] 2*exp(4*c*(b*x+a))*cosh(b*c*x+a*c)*(sech(b*c*x+a*c)^2)^(1/2)/b/c/(1+exp(2*c
*(b*x+a)))^2
```

Rubi [A]

time = 0.08, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6852, 2320, 12, 270}

$$\frac{2e^{4c(a+bx)}\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{bc\left(e^{2c(a+bx)}+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*(Sech[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] (2*E^(4*c*(a + b*x))*Cosh[a*c + b*c*x]*Sqrt[Sech[a*c + b*c*x]^2])/(b*c*(1 +
 E^(2*c*(a + b*x)))^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int e^{c(a+bx)}\operatorname{sech}^2(ac+bcx)^{3/2}\,dx &= \left(\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\int e^{c(a+bx)}\operatorname{sech}^3(ac+bcx)\,dx\\
&= \frac{\left(\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\operatorname{Subst}\left(\int\frac{8x^3}{(1+x^2)^3}\,dx,x,e^{c(a+bx)}\right)}{bc}\\
&= \frac{\left(8\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\operatorname{Subst}\left(\int\frac{x^3}{(1+x^2)^3}\,dx,x,e^{c(a+bx)}\right)}{bc}\\
&= \frac{2e^{4c(a+bx)}\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{bc\left(1+e^{2c(a+bx)}\right)^2}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 44, normalized size = 0.79

$$\frac{e^{3c(a+bx)}\sqrt{\operatorname{sech}^2(c(a+bx))}}{bc+bce^{2c(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*(Sech[a*c + b*c*x]^2)^(3/2),x]
[Out] (E^(3*c*(a + b*x))*Sqrt[Sech[c*(a + b*x)]^2])/(b*c + b*c*E^(2*c*(a + b*x)))
```

Maple [A]
time = 4.06, size = 69, normalized size = 1.23

method	result	size
risch	$-\frac{2\left(2\,\mathrm{e}^{2c(bx+a)}+1\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(1+\mathrm{e}^{2c(bx+a)}\right)^2}}\;\mathrm{e}^{-c(bx+a)}}{cb\left(1+\mathrm{e}^{2c(bx+a)}\right)}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/c/b*(2*exp(2*c*(b*x+a))+1)*(1/(1+exp(2*c*(b*x+a)))^2*exp(2*c*(b*x+a)))^(
1/2)/(1+exp(2*c*(b*x+a)))*exp(-c*(b*x+a))
```

Maxima [A]
time = 0.27, size = 84, normalized size = 1.50

$$-\frac{4\,e^{(2\,bcx+2\,ac)}}{bc(e^{(4\,bcx+4\,ac)}+2\,e^{(2\,bcx+2\,ac)}+1)}-\frac{2}{bc(e^{(4\,bcx+4\,ac)}+2\,e^{(2\,bcx+2\,ac)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -4*e^(2*b*c*x + 2*a*c)/(b*c*(e^(4*b*c*x + 4*a*c) + 2*e^(2*b*c*x + 2*a*c) +
1)) - 2/(b*c*(e^(4*b*c*x + 4*a*c) + 2*e^(2*b*c*x + 2*a*c) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(52) = 104$.
time = 0.35, size = 120, normalized size = 2.14

$$-\frac{2\,(3\,\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right))}{bc\cosh\left(bcx+ac\right)^{3}+3\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^{2}+bc\sinh\left(bcx+ac\right)^{3}+3\,bc\cosh\left(bcx+ac\right)+\left(3\,bc\cosh\left(bcx+ac\right)^{2}+bc\right)\sinh\left(bcx+ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(3*cosh(b*c*x + a*c) + sinh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^3 + 3*b
*c*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 + b*c*sinh(b*c*x + a*c)^3 + 3*b*c*
cosh(b*c*x + a*c) + (3*b*c*cosh(b*c*x + a*c)^2 + b*c)*sinh(b*c*x + a*c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.39, size = 38, normalized size = 0.68

$$-\frac{2\left(2\,e^{(2\,bcx+2\,ac)}+1\right)}{bc(e^{(2\,bcx+2\,ac)}+1)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

[Out] -2*(2*e^(2*b*c*x + 2*a*c) + 1)/(b*c*(e^(2*b*c*x + 2*a*c) + 1)^2)

Mupad [B]
time = 0.14, size = 78, normalized size = 1.39

$$-\frac{e^{-a\,c-b\,c\,x}\,\left(2\,e^{2\,a\,c+2\,b\,c\,x}+1\right)\,\sqrt{\frac{1}{{\left(\frac{e^{a\,c+b\,c\,x}}{2}+\frac{e^{-a\,c-b\,c\,x}}{2}\right)}^2}}}{b\,c\,\left(e^{2\,a\,c+2\,b\,c\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(c*(a + b*x))*(1/cosh(a*c + b*c*x)^2)^(3/2),x)

[Out] -(exp(- a*c - b*c*x)*(2*exp(2*a*c + 2*b*c*x) + 1)*(1/(exp(a*c + b*c*x)/2 + exp(- a*c - b*c*x)/2)^2)^(1/2))/(b*c*(exp(2*a*c + 2*b*c*x) + 1))

3.154 $\int e^{c(a+bx)} \sqrt{\operatorname{sech}^2(ac+bcx)}\, dx$

Optimal. Leaf size=44

$$\frac{\cosh(ac+bcx)\log\left(1+e^{2c(a+bx)}\right)\sqrt{\operatorname{sech}^2(ac+bcx)}}{bc}$$

[Out] cosh(b*c*x+a*c)*ln(1+exp(2*c*(b*x+a)))*(sech(b*c*x+a*c)^2)^(1/2)/b/c

Rubi [A]
time = 0.06, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6852, 2320, 12, 266}

$$\frac{\log\left(e^{2c(a+bx)}+1\right)\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{bc}$$

Antiderivative was successfully verified.

[In] Int[E^(c*(a + b*x))*Sqrt[Sech[a*c + b*c*x]^2],x]

[Out] (Cosh[a*c + b*c*x]*Log[1 + E^(2*c*(a + b*x))]*Sqrt[Sech[a*c + b*c*x]^2])/(b*c)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
```

```
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int e^{c(a+bx)}\sqrt{\operatorname{sech}^2(ac+bcx)}\,dx &= \left(\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\int e^{c(a+bx)}\operatorname{sech}(ac+bcx)\,dx\\
&= \frac{\left(\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \frac{2x}{1+x^2}\,dx, x, e^{c(a+bx)}\right)}{bc}\\
&= \frac{\left(2\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, e^{c(a+bx)}\right)}{bc}\\
&= \frac{\cosh(ac+bcx)\log\left(1+e^{2c(a+bx)}\right)\sqrt{\operatorname{sech}^2(ac+bcx)}}{bc}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 42, normalized size = 0.95

$$\frac{\cosh(c(a+bx))\log\left(1+e^{2c(a+bx)}\right)\sqrt{\operatorname{sech}^2(c(a+bx))}}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*Sqrt[Sech[a*c + b*c*x]^2],x]
```

```
[Out] (Cosh[c*(a + b*x)]*Log[1 + E^(2*c*(a + b*x))]*Sqrt[Sech[c*(a + b*x)]^2])/(b
*c)
```

Maple [A]
time = 4.22, size = 66, normalized size = 1.50

method	result	size
risch	$\frac{\ln\left(e^{2bcx}+e^{-2ac}\right)\left(1+e^{2c(bx+a)}\right)\sqrt{\frac{e^{2c(bx+a)}}{\left(1+e^{2c(bx+a)}\right)^2}}\;e^{-c(bx+a)}}{cb}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ln(exp(2*b*c*x)+exp(-2*a*c))/c/b*(1+exp(2*c*(b*x+a)))*(1/(1+exp(2*c*(b*x+a)
))^2*exp(2*c*(b*x+a)))^(1/2)*exp(-c*(b*x+a))
```

Maxima [A]
time = 0.50, size = 21, normalized size = 0.48

$$\frac{\log\left(e^{(2\,bcx+2\,ac)}+1\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
[Out] log(e^(2*b*c*x + 2*a*c) + 1)/(b*c)
```

Fricas [A]
time = 0.36, size = 42, normalized size = 0.95

$$\frac{\log\left(\frac{2\,\cosh(bcx+ac)}{\cosh(bcx+ac)-\sinh(bcx+ac)}\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
[Out] log(2*cosh(b*c*x + a*c)/(cosh(b*c*x + a*c) - sinh(b*c*x + a*c)))/(b*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int\sqrt{\operatorname{sech}^2\left(ac+bcx\right)}\,e^{bcx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)**2)**(1/2),x)
[Out] exp(a*c)*Integral(sqrt(sech(a*c + b*c*x)**2)*exp(b*c*x), x)
```

Giac [A]
time = 0.39, size = 20, normalized size = 0.45

$$\frac{\log\left(e^{(2\,bcx)}+e^{(-2\,ac)}\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
[Out] log(e^(2*b*c*x) + e^(-2*a*c))/(b*c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{e}^{c\,(a+b\,x)}\,\sqrt{\frac{1}{{\cosh\left(a\,c+b\,c\,x\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*(1/cosh(a*c + b*c*x)^2)^(1/2),x)
[Out] int(exp(c*(a + b*x))*(1/cosh(a*c + b*c*x)^2)^(1/2), x)
```

3.155 $\int \frac{e^{c(a+bx)}}{\sqrt{\operatorname{sech}^2(ac+bcx)}}\,dx$

Optimal. Leaf size=74

$$\frac{e^{2c(a+bx)}\operatorname{sech}(ac+bcx)}{4bc\sqrt{\operatorname{sech}^2(ac+bcx)}}+\frac{x\operatorname{sech}(ac+bcx)}{2\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

```
[Out] 1/4*exp(2*c*(b*x+a))*sech(b*c*x+a*c)/b/c/(sech(b*c*x+a*c)^2)^(1/2)+1/2*x*se
ch(b*c*x+a*c)/(sech(b*c*x+a*c)^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6852, 2320, 12, 14}

$$\frac{e^{2c(a+bx)}\operatorname{sech}(ac+bcx)}{4bc\sqrt{\operatorname{sech}^2(ac+bcx)}}+\frac{x\operatorname{sech}(ac+bcx)}{2\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/Sqrt[Sech[a*c + b*c*x]^2],x]
```

```
[Out] (E^(2*c*(a + b*x))*Sech[a*c + b*c*x])/(4*b*c*Sqrt[Sech[a*c + b*c*x]^2]) + (
x*Sech[a*c + b*c*x])/(2*Sqrt[Sech[a*c + b*c*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{c(a+bx)}}{\sqrt{\operatorname{sech}^2(ac+bcx)}}\,dx &= \frac{\operatorname{sech}(ac+bcx)\int e^{c(a+bx)}\cosh(ac+bcx)\,dx}{\sqrt{\operatorname{sech}^2(ac+bcx)}}\\
&= \frac{\operatorname{sech}(ac+bcx)\operatorname{Subst}\left(\int \frac{1+x^2}{2x}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\operatorname{sech}^2(ac+bcx)}}\\
&= \frac{\operatorname{sech}(ac+bcx)\operatorname{Subst}\left(\int \frac{1+x^2}{x}\,dx, x, e^{c(a+bx)}\right)}{2bc\sqrt{\operatorname{sech}^2(ac+bcx)}}\\
&= \frac{\operatorname{sech}(ac+bcx)\operatorname{Subst}\left(\int \left(\frac{1}{x}+x\right)\,dx, x, e^{c(a+bx)}\right)}{2bc\sqrt{\operatorname{sech}^2(ac+bcx)}}\\
&= \frac{e^{2c(a+bx)}\operatorname{sech}(ac+bcx)}{4bc\sqrt{\operatorname{sech}^2(ac+bcx)}} + \frac{x\operatorname{sech}(ac+bcx)}{2\sqrt{\operatorname{sech}^2(ac+bcx)}}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 48, normalized size = 0.65

$$
\frac{\left(e^{2c(a+bx)}+2bcx\right)\operatorname{sech}(c(a+bx))}{4bc\sqrt{\operatorname{sech}^2(c(a+bx))}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/Sqrt[Sech[a*c + b*c*x]^2],x]
[Out] ((E^(2*c*(a + b*x)) + 2*b*c*x)*Sech[c*(a + b*x)])/(4*b*c*Sqrt[Sech[c*(a + b
*x)]^2])
```

Maple [A]
time = 4.74, size = 106, normalized size = 1.43

method	result	size

risch	$\frac{x\,\mathrm{e}^{c(bx+a)}}{2\left(1+\mathrm{e}^{2c(bx+a)}\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(1+\mathrm{e}^{2c(bx+a)}\right)^{2}}}}+\frac{\mathrm{e}^{3c(bx+a)}}{4cb\left(1+\mathrm{e}^{2c(bx+a)}\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(1+\mathrm{e}^{2c(bx+a)}\right)^{2}}}}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x/(1+exp(2*c*(b*x+a)))/(1/(1+exp(2*c*(b*x+a)))^2*exp(2*c*(b*x+a)))^(1/2
)*exp(c*(b*x+a))+1/4/c/b/(1+exp(2*c*(b*x+a)))/(1/(1+exp(2*c*(b*x+a)))^2*exp
(2*c*(b*x+a)))^(1/2)*exp(3*c*(b*x+a))
```

Maxima [A]
time = 0.26, size = 29, normalized size = 0.39

$$\frac{1}{2}\,x+\frac{a}{2\,b}+\frac{e^{(2\,bcx+2\,ac)}}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/2*a/b + 1/4*e^(2*b*c*x + 2*a*c)/(b*c)
```

Fricas [A]
time = 0.36, size = 66, normalized size = 0.89

$$\frac{(2\,bcx+1)\cosh\left(bcx+ac\right)-(2\,bcx-1)\sinh\left(bcx+ac\right)}{4\,(bc\cosh\left(bcx+ac\right)-bc\sinh\left(bcx+ac\right))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*((2*b*c*x + 1)*cosh(b*c*x + a*c) - (2*b*c*x - 1)*sinh(b*c*x + a*c))/(b*
c*cosh(b*c*x + a*c) - b*c*sinh(b*c*x + a*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int\frac{e^{bcx}}{\sqrt{\operatorname{sech}^{2}\left(ac+bcx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)**2)**(1/2),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)/sqrt(sech(a*c + b*c*x)**2), x)
```

Giac [**A**]
time = 0.40, size = 33, normalized size = 0.45

$$\frac{\left(2\,bcxe^{(-ac)} + e^{(2\,bcx+ac)}\right)e^{(ac)}}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
[Out] 1/4*(2*b*c*x*e^(-a*c) + e^(2*b*c*x + a*c))*e^(a*c)/(b*c)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{e}^{c\,(a+b\,x)}}{\sqrt{\frac{1}{\cosh\left(a\,c+b\,c\,x\right)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))/(1/cosh(a*c + b*c*x)^2)^(1/2),x)
[Out] int(exp(c*(a + b*x))/(1/cosh(a*c + b*c*x)^2)^(1/2), x)
```

3.156 $\int \frac{e^{c(a+bx)}}{\mathbf{sech}^2(ac+bcx)^{3/2}}\,dx$

Optimal. Leaf size=162

$$-\frac{e^{-2c(a+bx)}\text{sech}(ac+bcx)}{16bc\sqrt{\text{sech}^2(ac+bcx)}}+\frac{3e^{2c(a+bx)}\text{sech}(ac+bcx)}{16bc\sqrt{\text{sech}^2(ac+bcx)}}+\frac{e^{4c(a+bx)}\text{sech}(ac+bcx)}{32bc\sqrt{\text{sech}^2(ac+bcx)}}+\frac{3x\text{sech}(ac+bcx)}{8\sqrt{\text{sech}^2(ac+bcx)}}$$

```
[Out] -1/16*sech(b*c*x+a*c)/b/c/exp(2*c*(b*x+a))/(sech(b*c*x+a*c)^2)^(1/2)+3/16*e
xp(2*c*(b*x+a))*sech(b*c*x+a*c)/b/c/(sech(b*c*x+a*c)^2)^(1/2)+1/32*exp(4*c*
(b*x+a))*sech(b*c*x+a*c)/b/c/(sech(b*c*x+a*c)^2)^(1/2)+3/8*x*sech(b*c*x+a*c
)/(sech(b*c*x+a*c)^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6852, 2320, 12, 272, 45}

$$-\frac{e^{-2c(a+bx)}\text{sech}(ac+bcx)}{16bc\sqrt{\text{sech}^2(ac+bcx)}}+\frac{3e^{2c(a+bx)}\text{sech}(ac+bcx)}{16bc\sqrt{\text{sech}^2(ac+bcx)}}+\frac{e^{4c(a+bx)}\text{sech}(ac+bcx)}{32bc\sqrt{\text{sech}^2(ac+bcx)}}+\frac{3x\text{sech}(ac+bcx)}{8\sqrt{\text{sech}^2(ac+bcx)}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/(Sech[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] -1/16*Sech[a*c + b*c*x]/(b*c*E^(2*c*(a + b*x))*Sqrt[Sech[a*c + b*c*x]^2]) +
 (3*E^(2*c*(a + b*x))*Sech[a*c + b*c*x])/(16*b*c*Sqrt[Sech[a*c + b*c*x]^2])
 + (E^(4*c*(a + b*x))*Sech[a*c + b*c*x])/(32*b*c*Sqrt[Sech[a*c + b*c*x]^2])
 + (3*x*Sech[a*c + b*c*x])/(8*Sqrt[Sech[a*c + b*c*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{c(a+bx)}}{\operatorname{sech}^2(ac+bcx)^{3/2}}\,dx &= \frac{\operatorname{sech}(ac+bcx)\int e^{c(a+bx)}\cosh^3(ac+bcx)\,dx}{\sqrt{\operatorname{sech}^2(ac+bcx)}}\\
&= \frac{\operatorname{sech}(ac+bcx)\operatorname{Subst}\left(\int \frac{(1+x^2)^3}{8x^3}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\operatorname{sech}^2(ac+bcx)}}\\
&= \frac{\operatorname{sech}(ac+bcx)\operatorname{Subst}\left(\int \frac{(1+x^2)^3}{x^3}\,dx, x, e^{c(a+bx)}\right)}{8bc\sqrt{\operatorname{sech}^2(ac+bcx)}}\\
&= \frac{\operatorname{sech}(ac+bcx)\operatorname{Subst}\left(\int \frac{(1+x)^3}{x^2}\,dx, x, e^{2c(a+bx)}\right)}{16bc\sqrt{\operatorname{sech}^2(ac+bcx)}}\\
&= \frac{\operatorname{sech}(ac+bcx)\operatorname{Subst}\left(\int \left(3+\frac{1}{x^2}+\frac{3}{x}+x\right)\,dx, x, e^{2c(a+bx)}\right)}{16bc\sqrt{\operatorname{sech}^2(ac+bcx)}}\\
&= -\frac{e^{-2c(a+bx)}\operatorname{sech}(ac+bcx)}{16bc\sqrt{\operatorname{sech}^2(ac+bcx)}}+\frac{3e^{2c(a+bx)}\operatorname{sech}(ac+bcx)}{16bc\sqrt{\operatorname{sech}^2(ac+bcx)}}+\frac{e^{4c(a+bx)}\operatorname{sech}(ac+bcx)}{32bc\sqrt{\operatorname{sech}^2(ac+bcx)}}
\end{aligned}
$$

Mathematica [A]

time = 0.05, size = 78, normalized size = 0.48

$$\frac{\left(-e^{-2c(a+bx)}+3e^{2c(a+bx)}+\frac{1}{2}e^{4c(a+bx)}+6bcx\right)\operatorname{sech}^3(c(a+bx))}{16bc\operatorname{sech}^2(c(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[E^(c*(a + b*x))/(Sech[a*c + b*c*x]^2)^(3/2),x]

[Out] ((-E^(-2*c*(a + b*x)) + 3*E^(2*c*(a + b*x)) + E^(4*c*(a + b*x))/2 + 6*b*c*x)*Sech[c*(a + b*x)]^3)/(16*b*c*(Sech[c*(a + b*x)]^2)^(3/2))

Maple [A]
time = 4.93, size = 216, normalized size = 1.33

method	result
risch	$\frac{3x\,e^{c(bx+a)}}{8\left(1+e^{2c(bx+a)}\right)\sqrt{\frac{e^{2c(bx+a)}}{\left(1+e^{2c(bx+a)}\right)^2}}}+\frac{e^{5c(bx+a)}}{32cb\left(1+e^{2c(bx+a)}\right)\sqrt{\frac{e^{2c(bx+a)}}{\left(1+e^{2c(bx+a)}\right)^2}}}+\frac{3\,e^{3c(bx+a)}}{16cb\left(1+e^{2c(bx+a)}\right)\sqrt{\frac{e^{2c(bx+a)}}{\left(1+e^{2c(bx+a)}\right)^2}}}-$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] 3/8*x/(1+exp(2*c*(b*x+a)))/(1/(1+exp(2*c*(b*x+a)))^2*exp(2*c*(b*x+a)))^(1/2)*exp(c*(b*x+a))+1/32/c/b/(1+exp(2*c*(b*x+a)))/(1/(1+exp(2*c*(b*x+a)))^2*exp(2*c*(b*x+a)))^(1/2)*exp(5*c*(b*x+a))+3/16/c/b/(1+exp(2*c*(b*x+a)))/(1/(1+exp(2*c*(b*x+a)))^2*exp(2*c*(b*x+a)))^(1/2)*exp(3*c*(b*x+a))-1/16/c/b/(1+exp(2*c*(b*x+a)))/(1/(1+exp(2*c*(b*x+a)))^2*exp(2*c*(b*x+a)))^(1/2)*exp(-c*(b*x+a))

Maxima [A]
time = 0.27, size = 74, normalized size = 0.46

$$\frac{3\,(bcx+ac)}{8\,bc}+\frac{e^{(4\,bcx+4\,ac)}}{32\,bc}+\frac{3\,e^{(2\,bcx+2\,ac)}}{16\,bc}-\frac{e^{(-2\,bcx-2\,ac)}}{16\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")

[Out] 3/8*(b*c*x + a*c)/(b*c) + 1/32*e^(4*b*c*x + 4*a*c)/(b*c) + 3/16*e^(2*b*c*x + 2*a*c)/(b*c) - 1/16*e^(-2*b*c*x - 2*a*c)/(b*c)

Fricas [A]
time = 0.35, size = 126, normalized size = 0.78

$$-\frac{\cosh\left(bcx+ac\right)^3+3\,\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^2-3\,\sinh\left(bcx+ac\right)^3-6\,(2\,bcx+1)\cosh\left(bcx+ac\right)+3\,\left(4\,bcx-3\,\cosh\left(bcx+ac\right)^2-2\right)\sinh\left(bcx+ac\right)}{32\,(bc\cosh\left(bcx+ac\right)-bc\sinh\left(bcx+ac\right))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")

[Out] -1/32*(cosh(b*c*x + a*c)^3 + 3*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 - 3*sinh(b*c*x + a*c)^3 - 6*(2*b*c*x + 1)*cosh(b*c*x + a*c) + 3*(4*b*c*x - 3*cosh

```
(b*c*x + a*c)^2 - 2)*sinh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c) - b*c*sinh(b
*c*x + a*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int\frac{e^{bcx}}{\left(\operatorname{sech}^2\left(ac+bcx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)**2)**(3/2),x)
[Out] exp(a*c)*Integral(exp(b*c*x)/(sech(a*c + b*c*x)**2)**(3/2), x)
```

Giac [A]
time = 0.40, size = 82, normalized size = 0.51

$$\frac{\left(12\,bcxe^{(-ac)}-2\,\left(3\,e^{(2\,bcx+2\,ac)}+1\right)e^{(-2\,bcx-3\,ac)}+\left(e^{(4\,bcx+9\,ac)}+6\,e^{(2\,bcx+7\,ac)}\right)e^{(-6\,ac)}\right)e^{(ac)}}{32\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
[Out] 1/32*(12*b*c*x*e^(-a*c) - 2*(3*e^(2*b*c*x + 2*a*c) + 1)*e^(-2*b*c*x - 3*a*c
) + (e^(4*b*c*x + 9*a*c) + 6*e^(2*b*c*x + 7*a*c))*e^(-6*a*c))*e^(a*c)/(b*c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\mathrm{e}^{c\,(a+b\,x)}}{{\left(\frac{1}{{\mathrm{cosh}\left(a\,c+b\,c\,x\right)}^2}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))/(1/cosh(a*c + b*c*x)^2)^(3/2),x)
[Out] int(exp(c*(a + b*x))/(1/cosh(a*c + b*c*x)^2)^(3/2), x)
```

3.157 $\int \frac{e^{c(a+bx)}}{\operatorname{sech}^2(ac+bcx)^{5/2}}\,dx$

Optimal. Leaf size=250

$$-\frac{e^{-4c(a+bx)}\operatorname{sech}(ac+bcx)}{128bc\sqrt{\operatorname{sech}^2(ac+bcx)}}-\frac{5e^{-2c(a+bx)}\operatorname{sech}(ac+bcx)}{64bc\sqrt{\operatorname{sech}^2(ac+bcx)}}+\frac{5e^{2c(a+bx)}\operatorname{sech}(ac+bcx)}{32bc\sqrt{\operatorname{sech}^2(ac+bcx)}}+\frac{5e^{4c(a+bx)}\operatorname{sech}(ac+bcx)}{128bc\sqrt{\operatorname{sech}^2(ac+bcx)}}+$$

```
[Out] -1/128*sech(b*c*x+a*c)/b/c/exp(4*c*(b*x+a))/(sech(b*c*x+a*c)^2)^(1/2)-5/64*
sech(b*c*x+a*c)/b/c/exp(2*c*(b*x+a))/(sech(b*c*x+a*c)^2)^(1/2)+5/32*exp(2*c
*(b*x+a))*sech(b*c*x+a*c)/b/c/(sech(b*c*x+a*c)^2)^(1/2)+5/128*exp(4*c*(b*x+
a))*sech(b*c*x+a*c)/b/c/(sech(b*c*x+a*c)^2)^(1/2)+1/192*exp(6*c*(b*x+a))*se
ch(b*c*x+a*c)/b/c/(sech(b*c*x+a*c)^2)^(1/2)+5/16*x*sech(b*c*x+a*c)/(sech(b*
c*x+a*c)^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6852, 2320, 12, 272, 45}

$$-\frac{e^{-4c(a+bx)}\operatorname{sech}(ac+bcx)}{128bc\sqrt{\operatorname{sech}^2(ac+bcx)}}-\frac{5e^{-2c(a+bx)}\operatorname{sech}(ac+bcx)}{64bc\sqrt{\operatorname{sech}^2(ac+bcx)}}+\frac{5e^{2c(a+bx)}\operatorname{sech}(ac+bcx)}{32bc\sqrt{\operatorname{sech}^2(ac+bcx)}}+\frac{5e^{4c(a+bx)}\operatorname{sech}(ac+bcx)}{128bc\sqrt{\operatorname{sech}^2(ac+bcx)}}+\frac{e^{6c(a+bx)}\operatorname{sech}(ac+bcx)}{192bc\sqrt{\operatorname{sech}^2(ac+bcx)}}+\frac{5x\operatorname{sech}(ac+bcx)}{16\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/(Sech[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] -1/128*Sech[a*c + b*c*x]/(b*c*E^(4*c*(a + b*x))*Sqrt[Sech[a*c + b*c*x]^2])
- (5*Sech[a*c + b*c*x])/(64*b*c*E^(2*c*(a + b*x))*Sqrt[Sech[a*c + b*c*x]^2]
) + (5*E^(2*c*(a + b*x))*Sech[a*c + b*c*x])/(32*b*c*Sqrt[Sech[a*c + b*c*x]^
2]) + (5*E^(4*c*(a + b*x))*Sech[a*c + b*c*x])/(128*b*c*Sqrt[Sech[a*c + b*c*
x]^2]) + (E^(6*c*(a + b*x))*Sech[a*c + b*c*x])/(192*b*c*Sqrt[Sech[a*c + b*c
*x]^2]) + (5*x*Sech[a*c + b*c*x])/(16*Sqrt[Sech[a*c + b*c*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{c(a+bx)}}{\operatorname{sech}^2(ac+bcx)^{5/2}}\,dx &= \frac{\operatorname{sech}(ac+bcx)\int e^{c(a+bx)}\cosh^5(ac+bcx)\,dx}{\sqrt{\operatorname{sech}^2(ac+bcx)}}\\
&= \frac{\operatorname{sech}(ac+bcx)\text{Subst}\left(\int \frac{(1+x^2)^5}{32x^5}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\operatorname{sech}^2(ac+bcx)}}\\
&= \frac{\operatorname{sech}(ac+bcx)\text{Subst}\left(\int \frac{(1+x^2)^5}{x^5}\,dx, x, e^{c(a+bx)}\right)}{32bc\sqrt{\operatorname{sech}^2(ac+bcx)}}\\
&= \frac{\operatorname{sech}(ac+bcx)\text{Subst}\left(\int \frac{(1+x)^5}{x^3}\,dx, x, e^{2c(a+bx)}\right)}{64bc\sqrt{\operatorname{sech}^2(ac+bcx)}}\\
&= \frac{\operatorname{sech}(ac+bcx)\text{Subst}\left(\int \left(10+\frac{1}{x^3}+\frac{5}{x^2}+\frac{10}{x}+5x+x^2\right)\,dx, x, e^{2c(a+bx)}\right)}{64bc\sqrt{\operatorname{sech}^2(ac+bcx)}}\\
&= -\frac{e^{-4c(a+bx)}\operatorname{sech}(ac+bcx)}{128bc\sqrt{\operatorname{sech}^2(ac+bcx)}} - \frac{5e^{-2c(a+bx)}\operatorname{sech}(ac+bcx)}{64bc\sqrt{\operatorname{sech}^2(ac+bcx)}} + \frac{5e^{2c(a+bx)}\operatorname{sech}(ac+bc}{32bc\sqrt{\operatorname{sech}^2(ac+bc}}
\end{aligned}$$

Mathematica [A]

time = 0.07, size = 106, normalized size = 0.42

$$\frac{\left(-\frac{1}{2}e^{-4c(a+bx)}-5e^{-2c(a+bx)}+10e^{2c(a+bx)}+\frac{5}{2}e^{4c(a+bx)}+\frac{1}{3}e^{6c(a+bx)}+20bcx\right)\mathrm{sech}^5(c(a+bx))}{64bc\,\mathrm{sech}^2(c(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/(Sech[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] ((-1/2*1/E^(4*c*(a + b*x)) - 5/E^(2*c*(a + b*x)) + 10*E^(2*c*(a + b*x)) + (
5*E^(4*c*(a + b*x)))/2 + E^(6*c*(a + b*x))/3 + 20*b*c*x)*Sech[c*(a + b*x)]^
5)/(64*b*c*(Sech[c*(a + b*x)]^2)^(5/2))
```

Maple [A]
time = 4.54, size = 326, normalized size = 1.30

method	result
risch	$\frac{5x\,\mathrm{e}^{c(bx+a)}}{16\left(1+\mathrm{e}^{2c(bx+a)}\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(1+\mathrm{e}^{2c(bx+a)}\right)^2}}}+\frac{\mathrm{e}^{7c(bx+a)}}{192cb\left(1+\mathrm{e}^{2c(bx+a)}\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(1+\mathrm{e}^{2c(bx+a)}\right)^2}}}+\frac{5\,\mathrm{e}^{5c(bx+a)}}{128cb\left(1+\mathrm{e}^{2c(bx+a)}\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(1+\mathrm{e}^{2c(bx+a)}\right)^2}}}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 5/16*x/(1+exp(2*c*(b*x+a)))/(1/(1+exp(2*c*(b*x+a)))^2*exp(2*c*(b*x+a)))^(1/
2)*exp(c*(b*x+a))+1/192/c/b/(1+exp(2*c*(b*x+a)))/(1/(1+exp(2*c*(b*x+a)))^2*
exp(2*c*(b*x+a)))^(1/2)*exp(7*c*(b*x+a))+5/128/c/b/(1+exp(2*c*(b*x+a)))/(1/
(1+exp(2*c*(b*x+a)))^2*exp(2*c*(b*x+a)))^(1/2)*exp(5*c*(b*x+a))+5/32/c/b/(1
+exp(2*c*(b*x+a)))/(1/(1+exp(2*c*(b*x+a)))^2*exp(2*c*(b*x+a)))^(1/2)*exp(3*
c*(b*x+a))-5/64/c/b/(1+exp(2*c*(b*x+a)))/(1/(1+exp(2*c*(b*x+a)))^2*exp(2*c*
(b*x+a)))^(1/2)*exp(-c*(b*x+a))-1/128/c/b/(1+exp(2*c*(b*x+a)))/(1/(1+exp(2*
c*(b*x+a)))^2*exp(2*c*(b*x+a)))^(1/2)*exp(-3*c*(b*x+a))
```

Maxima [A]
time = 0.27, size = 112, normalized size = 0.45

$$\frac{5\,(bcx+ac)}{16\,bc}+\frac{e^{(6\,bcx+6\,ac)}}{192\,bc}+\frac{5\,e^{(4\,bcx+4\,ac)}}{128\,bc}+\frac{5\,e^{(2\,bcx+2\,ac)}}{32\,bc}-\frac{5\,e^{(-2\,bcx-2\,ac)}}{64\,bc}-\frac{e^{(-4\,bcx-4\,ac)}}{128\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 5/16*(b*c*x + a*c)/(b*c) + 1/192*e^(6*b*c*x + 6*a*c)/(b*c) + 5/128*e^(4*b*c
*x + 4*a*c)/(b*c) + 5/32*e^(2*b*c*x + 2*a*c)/(b*c) - 5/64*e^(-2*b*c*x - 2*a
*c)/(b*c) - 1/128*e^(-4*b*c*x - 4*a*c)/(b*c)
```

Fricas [A]
time = 0.36, size = 218, normalized size = 0.87

$$-\frac{\cosh\left(bcx+ac\right)^{5}+5\,\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^{4}-5\,\sinh\left(bcx+ac\right)^{5}-5\,\left(10\,\cosh\left(bcx+ac\right)^{2}+9\right)\sinh\left(bcx+ac\right)^{3}+15\,\cosh\left(bcx+ac\right)^{3}+5\,\left(2\,\cosh\left(bcx+ac\right)^{3}+9\,\cosh\left(bcx+ac\right)\right)\sinh\left(bcx+ac\right)^{2}-60\,\left(2\,bcx+1\right)\cosh\left(bcx+ac\right)-5\,\left(5\,\cosh\left(bcx+ac\right)^{4}-24\,bcx+27\,\cosh\left(bcx+ac\right)^{2}+12\right)\sinh\left(bcx+ac\right)}{384\,\left(bc\cosh\left(bcx+ac\right)-bc\sinh\left(bcx+ac\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/384*(cosh(b*c*x + a*c)^5 + 5*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^4 - 5*s
inh(b*c*x + a*c)^5 - 5*(10*cosh(b*c*x + a*c)^2 + 9)*sinh(b*c*x + a*c)^3 + 1
5*cosh(b*c*x + a*c)^3 + 5*(2*cosh(b*c*x + a*c)^3 + 9*cosh(b*c*x + a*c))*sin
h(b*c*x + a*c)^2 - 60*(2*b*c*x + 1)*cosh(b*c*x + a*c) - 5*(5*cosh(b*c*x + a
*c)^4 - 24*b*c*x + 27*cosh(b*c*x + a*c)^2 + 12)*sinh(b*c*x + a*c))/(b*c*cos
h(b*c*x + a*c) - b*c*sinh(b*c*x + a*c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 532 vs. $2(238) = 476$.
time = 129.76, size = 532, normalized size = 2.13

$$\begin{cases} x & \text{for}\: c = 0 \wedge \left(b = 0 \vee c = 0\right) \\ \frac{x e^{ac}}{\left(\operatorname{sech}^{2}{\left(ac \right)}\right)^{\frac{5}{2}}} & \text{for}\: b = 0 \\ - \frac{5 x e^{ac} e^{bcx} \tanh^{5}{\left(ac + bcx \right)}}{16 \left(\operatorname{sech}^{2}{\left(ac + bcx \right)}\right)^{\frac{5}{2}}} + \frac{5 x e^{ac} e^{bcx} \tanh^{4}{\left(ac + bcx \right)}}{16 \left(\operatorname{sech}^{2}{\left(ac + bcx \right)}\right)^{\frac{5}{2}}} + \frac{5 x e^{ac} e^{bcx} \tanh^{3}{\left(ac + bcx \right)}}{8 \left(\operatorname{sech}^{2}{\left(ac + bcx \right)}\right)^{\frac{5}{2}}} - \frac{5 x e^{ac} e^{bcx} \tanh^{2}{\left(ac + bcx \right)}}{8 \left(\operatorname{sech}^{2}{\left(ac + bcx \right)}\right)^{\frac{5}{2}}} - \frac{5 x e^{ac} e^{bcx} \tanh{\left(ac + bcx \right)}}{16 \left(\operatorname{sech}^{2}{\left(ac + bcx \right)}\right)^{\frac{5}{2}}} + \frac{5 x e^{ac} e^{bcx}}{16 \left(\operatorname{sech}^{2}{\left(ac + bcx \right)}\right)^{\frac{5}{2}}} + \frac{8 e^{ac} e^{bcx} \tanh^{5}{\left(ac + bcx \right)}}{15 bc \left(\operatorname{sech}^{2}{\left(ac + bcx \right)}\right)^{\frac{5}{2}}} - \frac{53 e^{ac} e^{bcx} \tanh^{4}{\left(ac + bcx \right)}}{240 bc \left(\operatorname{sech}^{2}{\left(ac + bcx \right)}\right)^{\frac{5}{2}}} - \frac{331 e^{ac} e^{bcx} \tanh^{3}{\left(ac + bcx \right)}}{240 bc \left(\operatorname{sech}^{2}{\left(ac + bcx \right)}\right)^{\frac{5}{2}}} + \frac{131 e^{ac} e^{bcx} \tanh^{2}{\left(ac + bcx \right)}}{240 bc \left(\operatorname{sech}^{2}{\left(ac + bcx \right)}\right)^{\frac{5}{2}}} + \frac{253 e^{ac} e^{bcx} \tanh{\left(ac + bcx \right)}}{240 bc \left(\operatorname{sech}^{2}{\left(ac + bcx \right)}\right)^{\frac{5}{2}}} - \frac{11 e^{ac} e^{bcx}}{30 bc \left(\operatorname{sech}^{2}{\left(ac + bcx \right)}\right)^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)**2)**(5/2),x)
```

```
[Out] Piecewise((x, Eq(c, 0) & (Eq(b, 0) | Eq(c, 0))), (x*exp(a*c)/(sech(a*c)**2)
**(5/2), Eq(b, 0)), (-5*x*exp(a*c)*exp(b*c*x)*tanh(a*c + b*c*x)**5/(16*(sec
h(a*c + b*c*x)**2)**(5/2)) + 5*x*exp(a*c)*exp(b*c*x)*tanh(a*c + b*c*x)**4/(
16*(sech(a*c + b*c*x)**2)**(5/2)) + 5*x*exp(a*c)*exp(b*c*x)*tanh(a*c + b*c*
x)**3/(8*(sech(a*c + b*c*x)**2)**(5/2)) - 5*x*exp(a*c)*exp(b*c*x)*tanh(a*c
+ b*c*x)**2/(8*(sech(a*c + b*c*x)**2)**(5/2)) - 5*x*exp(a*c)*exp(b*c*x)*tan
h(a*c + b*c*x)/(16*(sech(a*c + b*c*x)**2)**(5/2)) + 5*x*exp(a*c)*exp(b*c*x)
/(16*(sech(a*c + b*c*x)**2)**(5/2)) + 8*exp(a*c)*exp(b*c*x)*tanh(a*c + b*c*
x)**5/(15*b*c*(sech(a*c + b*c*x)**2)**(5/2)) - 53*exp(a*c)*exp(b*c*x)*tanh(
a*c + b*c*x)**4/(240*b*c*(sech(a*c + b*c*x)**2)**(5/2)) - 331*exp(a*c)*exp(
b*c*x)*tanh(a*c + b*c*x)**3/(240*b*c*(sech(a*c + b*c*x)**2)**(5/2)) + 131*e
xp(a*c)*exp(b*c*x)*tanh(a*c + b*c*x)**2/(240*b*c*(sech(a*c + b*c*x)**2)**(5
/2)) + 253*exp(a*c)*exp(b*c*x)*tanh(a*c + b*c*x)/(240*b*c*(sech(a*c + b*c*x
)**2)**(5/2)) - 11*exp(a*c)*exp(b*c*x)/(30*b*c*(sech(a*c + b*c*x)**2)**(5/2
)), True))
```

Giac [A]
time = 0.39, size = 110, normalized size = 0.44

$$\frac{\left(120\,bcxe^{\left(-ac\right)}-3\,\left(30\,e^{\left(4\,bcx+4\,ac\right)}+10\,e^{\left(2\,bcx+2\,ac\right)}+1\right)e^{\left(-4\,bcx-5\,ac\right)}+\left(2\,e^{\left(6\,bcx+20\,ac\right)}+15\,e^{\left(4\,bcx+18\,ac\right)}+60\,e^{\left(2\,bcx+16\,ac\right)}\right)e^{\left(-15\,ac\right)}\right)e^{\left(ac\right)}}{384\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/384*(120*b*c*x*e^(-a*c) - 3*(30*e^(4*b*c*x + 4*a*c) + 10*e^(2*b*c*x + 2*a
*c) + 1)*e^(-4*b*c*x - 5*a*c) + (2*e^(6*b*c*x + 20*a*c) + 15*e^(4*b*c*x + 1
8*a*c) + 60*e^(2*b*c*x + 16*a*c))*e^(-15*a*c))*e^(a*c)/(b*c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e^{c\,(a+b\,x)}}{\left(\frac{1}{\cosh(a\,c+b\,c\,x)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))/(1/cosh(a*c + b*c*x)^2)^(5/2),x)
```

```
[Out] int(exp(c*(a + b*x))/(1/cosh(a*c + b*c*x)^2)^(5/2), x)
```

3.158 $\int \frac{x^5}{\sqrt{\mathbf{sech}(2\log(cx))}}\,dx$

Optimal. Leaf size=108

$$\frac{2x^2}{21c^4\sqrt{\operatorname{sech}(2\log(cx))}}+\frac{x^6}{7\sqrt{\operatorname{sech}(2\log(cx))}}+\frac{\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)F\left(2\cot^{-1}(cx)\middle|\frac{1}{2}\right)}{21c^5\left(c^4+\frac{1}{x^4}\right)x\sqrt{\operatorname{sech}(2\log(cx))}}$$

```
[Out] 2/21*x^2/c^4/sech(2*ln(c*x))^(1/2)+1/7*x^6/sech(2*ln(c*x))^(1/2)+1/21*(c^2+
1/x^2)*(cos(2*arccot(c*x))^2)^(1/2)/cos(2*arccot(c*x))*EllipticF(sin(2*arcc
ot(c*x)),1/2*2^(1/2))*((c^4+1/x^4)/(c^2+1/x^2)^2)^(1/2)/c^5/(c^4+1/x^4)/x/s
ech(2*ln(c*x))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5670, 5668, 342, 283, 331, 226}

$$\frac{2x^2}{21c^4\sqrt{\operatorname{sech}(2\log(cx))}}+\frac{\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)F\left(2\cot^{-1}(cx)\middle|\frac{1}{2}\right)}{21c^5x\left(c^4+\frac{1}{x^4}\right)\sqrt{\operatorname{sech}(2\log(cx))}}+\frac{x^6}{7\sqrt{\operatorname{sech}(2\log(cx))}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[Sech[2*Log[c*x]]],x]
```

```
[Out] (2*x^2)/(21*c^4*Sqrt[Sech[2*Log[c*x]]]) + x^6/(7*Sqrt[Sech[2*Log[c*x]]]) +
(Sqrt[(c^4 + x^(-4))/(c^2 + x^(-2))^2]*(c^2 + x^(-2))*EllipticF[2*ArcCot[c*
x], 1/2])/(21*c^5*(c^4 + x^(-4))*x*Sqrt[Sech[2*Log[c*x]]])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5668

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5670

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{x^5}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^5}{\sqrt{\operatorname{sech}(2\log(x))}}\,dx, x, cx\right)}{c^6}$$

$$= \frac{\operatorname{Subst}\left(\int \sqrt{1+\frac{1}{x^4}}\,x^6\,dx, x, cx\right)}{c^7\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}}$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{\sqrt{1+x^4}}{x^8}\,dx, x, \frac{1}{cx}\right)}{c^7\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}}$$

$$= \frac{x^6}{7\sqrt{\operatorname{sech}(2\log(cx))}} - \frac{2\operatorname{Subst}\left(\int \frac{1}{x^4\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)}{7c^7\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}}$$

$$= \frac{2x^2}{21c^4\sqrt{\operatorname{sech}(2\log(cx))}} + \frac{x^6}{7\sqrt{\operatorname{sech}(2\log(cx))}} + \frac{2\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx, x, \frac{1}{c\ldots}\right.}{21c^7\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log\ldots}}$$

$$= \frac{2x^2}{21c^4\sqrt{\operatorname{sech}(2\log(cx))}} + \frac{x^6}{7\sqrt{\operatorname{sech}(2\log(cx))}} + \frac{\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)F\left(2\operatorname{co}\ldots\right.}{21c^5\left(c^4+\frac{1}{x^4}\right)x\sqrt{\operatorname{sech}(2\operatorname{lo}\ldots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.12, size = 77, normalized size = 0.71

$$\frac{\sqrt{1+c^4x^4}\sqrt{\frac{c^2x^2}{2+2c^4x^4}}\left((1+c^4x^4)^{3/2} - {}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-c^4x^4\right)\right)}{7c^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[Sech[2*Log[c*x]]],x]
```

```
[Out] (Sqrt[1 + c^4*x^4]*Sqrt[(c^2*x^2)/(2 + 2*c^4*x^4)]*((1 + c^4*x^4)^(3/2) - H
ypergeometric2F1[-1/2, 1/4, 5/4, -(c^4*x^4)]))/(7*c^6)
```

Maple [C] Result contains complex when optimal does not.
time = 1.43, size = 130, normalized size = 1.20

method	result	size
risch	$\frac{x^2(3c^4x^4+2)\sqrt{2}}{42c^4\sqrt{\frac{c^2x^2}{c^4x^4+1}}} - \frac{\sqrt{-ic^2x^2+1}\,\sqrt{ic^2x^2+1}\,\text{EllipticF}\left(x\sqrt{ic^2}, i\right)\sqrt{2}\,x}{21c^4\sqrt{ic^2}\,(c^4x^4+1)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}$	130

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/sech(2*ln(c*x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/42*x^2*(3*c^4*x^4+2)/c^4*2^(1/2)/(c^2*x^2/(c^4*x^4+1))^(1/2)-1/21/c^4/(I*
c^2)^(1/2)*(1-I*c^2*x^2)^(1/2)*(1+I*c^2*x^2)^(1/2)/(c^4*x^4+1)*EllipticF(x*
(I*c^2)^(1/2),I)*2^(1/2)*x/(c^2*x^2/(c^4*x^4+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/sech(2*log(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^5/sqrt(sech(2*log(c*x))), x)
```

Fricas [A]
time = 0.11, size = 80, normalized size = 0.74

$$-\frac{2\sqrt{2}\sqrt{c^4}\,c\left(-\frac{1}{c^4}\right)^{\frac{3}{4}}\text{ellipticF}\left(\frac{\left(-\frac{1}{c^4}\right)^{\frac{1}{4}}}{x}, -1\right) - \sqrt{2}\left(3\,c^8x^8+5\,c^4x^4+2\right)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{42\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/sech(2*log(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/42*(2*sqrt(2)*sqrt(c^4)*c*(-1/c^4)^(3/4)*ellipticF((-1/c^4)^(1/4)/x, -1)
 - sqrt(2)*(3*c^8*x^8 + 5*c^4*x^4 + 2)*sqrt(c^2*x^2/(c^4*x^4 + 1)))/c^6
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/sech(2*ln(c*x))**(1/2),x)
```

```
[Out] Integral(x**5/sqrt(sech(2*log(c*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/sech(2*log(c*x))^(1/2),x, algorithm="giac")
[Out] integrate(x^5/sqrt(sech(2*log(c*x))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\sqrt{\frac{1}{\cosh\left(2\,\ln\left(c\,x\right)\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(1/cosh(2*log(c*x)))^(1/2),x)
[Out] int(x^5/(1/cosh(2*log(c*x)))^(1/2), x)
```

3.159 $\int \frac{x^4}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx$

Optimal. Leaf size=28

$$\frac{\left(c^4+\frac{1}{x^4}\right)x^5}{6c^4\sqrt{\operatorname{sech}(2\log(cx))}}$$

```
[Out] 1/6*(c^4+1/x^4)*x^5/c^4/sech(2*ln(c*x))^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5670, 5668, 270}

$$\frac{x^5\left(c^4+\frac{1}{x^4}\right)}{6c^4\sqrt{\operatorname{sech}(2\log(cx))}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[Sech[2*Log[c*x]]],x]
```

```
[Out] ((c^4 + x^(-4))*x^5)/(6*c^4*Sqrt[Sech[2*Log[c*x]]])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5668

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5670

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{x^4}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\sqrt{\operatorname{sech}(2\log(x))}}\,dx, x, cx\right)}{c^5}$$

$$= \frac{\text{Subst}\left(\int \sqrt{1+\frac{1}{x^4}}\,x^5\,dx, x, cx\right)}{c^6\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}}$$

$$= \frac{\left(c^4+\frac{1}{x^4}\right)x^5}{6c^4\sqrt{\operatorname{sech}(2\log(cx))}}$$

Mathematica [A]
time = 0.03, size = 44, normalized size = 1.57

$$\frac{(1+c^4x^4)^2\sqrt{\frac{c^2x^2}{2+2c^4x^4}}}{6c^6x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[Sech[2*Log[c*x]]],x]
[Out] ((1 + c^4*x^4)^2*Sqrt[(c^2*x^2)/(2 + 2*c^4*x^4)])/(6*c^6*x)
```

Maple [A]
time = 1.21, size = 39, normalized size = 1.39

method	result	size
risch	$\frac{\sqrt{2}\,x(c^4x^4+1)}{12\sqrt{\frac{c^2x^2}{c^4x^4+1}}\,c^4}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/sech(2*ln(c*x))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/12*2^(1/2)*x/(c^2*x^2/(c^4*x^4+1))^(1/2)*(c^4*x^4+1)/c^4
```

Maxima [A]
time = 0.51, size = 30, normalized size = 1.07

$$\frac{\left(\sqrt{2}\,c^4x^4+\sqrt{2}\right)\sqrt{c^4x^4+1}}{12\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/sech(2*log(c*x))^(1/2),x, algorithm="maxima")
[Out] 1/12*(sqrt(2)*c^4*x^4 + sqrt(2))*sqrt(c^4*x^4 + 1)/c^5
```

Fricas [A]
time = 0.37, size = 48, normalized size = 1.71

$$\frac{\sqrt{2}\,(c^8x^8+2\,c^4x^4+1)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{12\,c^6x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/sech(2*log(c*x))^(1/2),x, algorithm="fricas")
[Out] 1/12*sqrt(2)*(c^8*x^8 + 2*c^4*x^4 + 1)*sqrt(c^2*x^2/(c^4*x^4 + 1))/(c^6*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/sech(2*ln(c*x))**(1/2),x)
[Out] Integral(x**4/sqrt(sech(2*log(c*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/sech(2*log(c*x))^(1/2),x, algorithm="giac")
[Out] integrate(x^4/sqrt(sech(2*log(c*x))), x)
```

Mupad [B]
time = 1.47, size = 42, normalized size = 1.50

$$\frac{{\left(c^4\,x^4+1\right)}^2\,\sqrt{\frac{2\,c^2\,x^2}{c^4\,x^4+1}}}{12\,c^6\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(1/cosh(2*log(c*x)))^(1/2),x)
[Out] ((c^4*x^4 + 1)^2*((2*c^2*x^2)/(c^4*x^4 + 1))^(1/2))/(12*c^6*x)
```

3.160 $\int \frac{x^3}{\sqrt{\mathrm{sech}(2\log(cx))}}\,dx$

Optimal. Leaf size=203

$$\frac{2}{5c^4\sqrt{\operatorname{sech}(2\log(cx))}}-\frac{2}{5c^4\left(c^2+\frac{1}{x^2}\right)x^2\sqrt{\operatorname{sech}(2\log(cx))}}+\frac{x^4}{5\sqrt{\operatorname{sech}(2\log(cx))}}+\frac{2\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)}{5c^3\left(c^4+\frac{1}{x^4}\right)x\sqrt{\operatorname{sech}}}$$

```
[Out] 2/5/c^4/sech(2*ln(c*x))^(1/2)-2/5/c^4/(c^2+1/x^2)/x^2/sech(2*ln(c*x))^(1/2)
+1/5*x^4/sech(2*ln(c*x))^(1/2)+2/5*(c^2+1/x^2)*(cos(2*arccot(c*x))^2)^(1/2)
/cos(2*arccot(c*x))*EllipticE(sin(2*arccot(c*x)),1/2*2^(1/2))*((c^4+1/x^4)/
(c^2+1/x^2)^2)^(1/2)/c^3/(c^4+1/x^4)/x/sech(2*ln(c*x))^(1/2)-1/5*(c^2+1/x^2
)*(cos(2*arccot(c*x))^2)^(1/2)/cos(2*arccot(c*x))*EllipticF(sin(2*arccot(c*
x)),1/2*2^(1/2))*((c^4+1/x^4)/(c^2+1/x^2)^2)^(1/2)/c^3/(c^4+1/x^4)/x/sech(2
*ln(c*x))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {5670, 5668, 342, 283, 331, 311, 226, 1210}

$$\frac{2}{5c^4\sqrt{\operatorname{sech}(2\log(cx))}}-\frac{2}{5c^4x^2\left(c^2+\frac{1}{x^2}\right)\sqrt{\operatorname{sech}(2\log(cx))}}-\frac{\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)F\left(2\cot^{-1}(cx)\middle|\frac{1}{2}\right)}{5c^3x\left(c^4+\frac{1}{x^4}\right)\sqrt{\operatorname{sech}(2\log(cx))}}+\frac{2\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)E\left(2\cot^{-1}(cx)\middle|\frac{1}{2}\right)}{5c^3x\left(c^4+\frac{1}{x^4}\right)\sqrt{\operatorname{sech}(2\log(cx))}}+\frac{x^4}{5\sqrt{\operatorname{sech}(2\log(cx))}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[Sech[2*Log[c*x]]],x]
```

```
[Out] 2/(5*c^4*Sqrt[Sech[2*Log[c*x]]]) - 2/(5*c^4*(c^2 + x^(-2))*x^2*Sqrt[Sech[2*
Log[c*x]]]) + x^4/(5*Sqrt[Sech[2*Log[c*x]]]) + (2*Sqrt[(c^4 + x^(-4))/(c^2
+ x^(-2))^2]*(c^2 + x^(-2))*EllipticE[2*ArcCot[c*x], 1/2])/(5*c^3*(c^4 + x^
(-4))*x*Sqrt[Sech[2*Log[c*x]]]) - (Sqrt[(c^4 + x^(-4))/(c^2 + x^(-2))^2]*(c
^2 + x^(-2))*EllipticF[2*ArcCot[c*x], 1/2])/(5*c^3*(c^4 + x^(-4))*x*Sqrt[Se
ch[2*Log[c*x]]])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
```

```
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 5668

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5670

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{x^3}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^3}{\sqrt{\operatorname{sech}(2\log(x))}}\,dx, x, cx\right)}{c^4}$$

$$= \frac{\operatorname{Subst}\left(\int \sqrt{1+\frac{1}{x^4}}\,x^4\,dx, x, cx\right)}{c^5\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}}$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{\sqrt{1+x^4}}{x^6}\,dx, x, \frac{1}{cx}\right)}{c^5\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}}$$

$$= \frac{x^4}{5\sqrt{\operatorname{sech}(2\log(cx))}} - \frac{2\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)}{5c^5\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}}$$

$$= \frac{2}{5c^4\sqrt{\operatorname{sech}(2\log(cx))}} + \frac{x^4}{5\sqrt{\operatorname{sech}(2\log(cx))}} - \frac{2\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)}{5c^5\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}}$$

$$= \frac{2}{5c^4\sqrt{\operatorname{sech}(2\log(cx))}} + \frac{x^4}{5\sqrt{\operatorname{sech}(2\log(cx))}} - \frac{2\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)}{5c^5\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}}$$

$$= \frac{2}{5c^4\sqrt{\operatorname{sech}(2\log(cx))}} - \frac{2}{5c^4\left(c^2+\frac{1}{x^2}\right)x^2\sqrt{\operatorname{sech}(2\log(cx))}} + \frac{x^4}{5\sqrt{\operatorname{sech}(2\log(cx))}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.09, size = 65, normalized size = 0.32

$$\frac{\left(\frac{c^2x^2}{1+c^4x^4}\right)^{3/2}\left(1+c^4x^4\right)^{3/2}\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-c^4x^4\right)}{3\sqrt{2}\,c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[Sech[2*Log[c*x]]],x]
```

```
[Out] (((c^2*x^2)/(1 + c^4*x^4))^(3/2)*(1 + c^4*x^4)^(3/2)*Hypergeometric2F1[-1/2
, 3/4, 7/4, -(c^4*x^4)])/(3*Sqrt[2]*c^4)
```

Maple [C] Result contains complex when optimal does not.
time = 1.27, size = 134, normalized size = 0.66

method	result	size
risch	$\frac{x^{4}\sqrt{2}}{10\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}+1}}}+\frac{i\sqrt{-ic^{2}x^{2}+1}\,\sqrt{ic^{2}x^{2}+1}\,\left(\mathrm{EllipticF}\left(x\sqrt{ic^{2}},i\right)-\mathrm{EllipticE}\left(x\sqrt{ic^{2}},i\right)\right)\sqrt{2}\,x}{5\sqrt{ic^{2}}\,(c^{4}x^{4}+1)c^{2}\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}+1}}}$	134

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/sech(2*ln(c*x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*x^4*2^(1/2)/(c^2*x^2/(c^4*x^4+1))^(1/2)+1/5*I/(I*c^2)^(1/2)*(1-I*c^2*x
^2)^(1/2)*(1+I*c^2*x^2)^(1/2)/(c^4*x^4+1)/c^2*(EllipticF(x*(I*c^2)^(1/2),I)
-EllipticE(x*(I*c^2)^(1/2),I))*2^(1/2)*x/(c^2*x^2/(c^4*x^4+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/sech(2*log(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt(sech(2*log(c*x))), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/sech(2*log(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/sech(2*ln(c*x))**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(sech(2*log(c*x))), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/sech(2*log(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\sqrt{\frac{1}{\cosh\left(2\,\ln\left(c\,x\right)\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(1/cosh(2*log(c*x)))^(1/2),x)
```

```
[Out] int(x^3/(1/cosh(2*log(c*x)))^(1/2), x)
```

3.161 $\int \frac{x^2}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx$

Optimal. Leaf size=67

$$\frac{x^3}{4\sqrt{\operatorname{sech}(2\log(cx))}}+\frac{\tanh^{-1}\left(\sqrt{1+\frac{1}{c^4x^4}}\right)}{4c^4\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}}$$

```
[Out] 1/4*x^3/sech(2*ln(c*x))^(1/2)+1/4*arctanh((1+1/c^4/x^4)^(1/2))/c^4/x/(1+1/c
^4/x^4)^(1/2)/sech(2*ln(c*x))^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5670, 5668, 272, 43, 65, 213}

$$\frac{\tanh^{-1}\left(\sqrt{\frac{1}{c^4x^4}+1}\right)}{4c^4x\sqrt{\frac{1}{c^4x^4}+1}\,\sqrt{\operatorname{sech}(2\log(cx))}}+\frac{x^3}{4\sqrt{\operatorname{sech}(2\log(cx))}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[Sech[2*Log[c*x]]],x]
```

```
[Out] x^3/(4*Sqrt[Sech[2*Log[c*x]]]) + ArcTanh[Sqrt[1 + 1/(c^4*x^4)]]/(4*c^4*Sqrt
[1 + 1/(c^4*x^4)]*x*Sqrt[Sech[2*Log[c*x]]])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5668

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5670

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{x^2}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\sqrt{\operatorname{sech}(2\log(x))}}\,dx, x, cx\right)}{c^3}$$

$$= \frac{\text{Subst}\left(\int \sqrt{1+\frac{1}{x^4}}\,x^3\,dx, x, cx\right)}{c^4\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}}$$

$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{1+x}}{x^2}\,dx, x, \frac{1}{c^4x^4}\right)}{4c^4\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}}$$

$$= \frac{x^3}{4\sqrt{\operatorname{sech}(2\log(cx))}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, \frac{1}{c^4x^4}\right)}{8c^4\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}}$$

$$= \frac{x^3}{4\sqrt{\operatorname{sech}(2\log(cx))}} - \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+\frac{1}{c^4x^4}}\right)}{4c^4\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}}$$

$$= \frac{x^3}{4\sqrt{\operatorname{sech}(2\log(cx))}} + \frac{\tanh^{-1}\left(\sqrt{1+\frac{1}{c^4x^4}}\right)}{4c^4\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}}$$

Mathematica [A]
time = 0.11, size = 77, normalized size = 1.15

$$\frac{x\left(c^2x^2\sqrt{1+c^4x^4}+\sinh^{-1}\left(c^2x^2\right)\right)}{4\sqrt{2}\,c^2\sqrt{\frac{c^2x^2}{1+c^4x^4}}\,\sqrt{1+c^4x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[Sech[2*Log[c*x]]],x]
[Out] (x*(c^2*x^2*Sqrt[1 + c^4*x^4] + ArcSinh[c^2*x^2]))/(4*Sqrt[2]*c^2*Sqrt[(c^2
*x^2)/(1 + c^4*x^4)]*Sqrt[1 + c^4*x^4])
```

Maple [A]
time = 1.43, size = 97, normalized size = 1.45

method	result	size
risch	$\frac{x^{3}\sqrt{2}}{8\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}+1}}}+\frac{\ln\left(\frac{c^{4}x^{2}}{\sqrt{c^{4}}}+\sqrt{c^{4}x^{4}+1}\right)\sqrt{2}\,x}{8\sqrt{c^{4}}\,\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}+1}}\,\sqrt{c^{4}x^{4}+1}}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/sech(2*ln(c*x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*x^3*2^(1/2)/(c^2*x^2/(c^4*x^4+1))^(1/2)+1/8*ln(c^4*x^2/(c^4)^(1/2)+(c^4
*x^4+1)^(1/2))/(c^4)^(1/2)*2^(1/2)*x/(c^2*x^2/(c^4*x^4+1))^(1/2)/(c^4*x^4+1
)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/sech(2*log(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(sech(2*log(c*x))), x)
```

Fricas [A]
time = 0.38, size = 90, normalized size = 1.34

$$\frac{2\sqrt{2}\left(c^{5}x^{5}+cx\right)\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}+1}}+\sqrt{2}\log\left(-2\,c^{4}x^{4}-2\left(c^{5}x^{5}+cx\right)\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}+1}}-1\right)}{16\,c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/sech(2*log(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(2*sqrt(2)*(c^5*x^5 + c*x)*sqrt(c^2*x^2/(c^4*x^4 + 1)) + sqrt(2)*log(-
2*c^4*x^4 - 2*(c^5*x^5 + c*x)*sqrt(c^2*x^2/(c^4*x^4 + 1)) - 1))/c^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{2}}{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/sech(2*ln(c*x))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(sech(2*log(c*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/sech(2*log(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(sech(2*log(c*x))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{\frac{1}{\cosh\left(2\,\ln\left(c\,x\right)\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1/cosh(2*log(c*x)))^(1/2),x)
```

```
[Out] int(x^2/(1/cosh(2*log(c*x)))^(1/2), x)
```

3.162 $\int \frac{x}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx$

Optimal. Leaf size=87

$$\frac{x^2}{3\sqrt{\operatorname{sech}(2\log(cx))}}-\frac{\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)F\left(2\cot^{-1}(cx)\middle|\frac{1}{2}\right)}{3c\left(c^4+\frac{1}{x^4}\right)x\sqrt{\operatorname{sech}(2\log(cx))}}$$

```
[Out] 1/3*x^2/sech(2*ln(c*x))^(1/2)-1/3*(c^2+1/x^2)*(cos(2*arccot(c*x))^2)^(1/2)/
cos(2*arccot(c*x))*EllipticF(sin(2*arccot(c*x)),1/2*2^(1/2))*((c^4+1/x^4)/(
c^2+1/x^2)^2)^(1/2)/c/(c^4+1/x^4)/x/sech(2*ln(c*x))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5670, 5668, 342, 283, 226}

$$\frac{x^2}{3\sqrt{\operatorname{sech}(2\log(cx))}}-\frac{\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)F\left(2\cot^{-1}(cx)\middle|\frac{1}{2}\right)}{3cx\left(c^4+\frac{1}{x^4}\right)\sqrt{\operatorname{sech}(2\log(cx))}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[Sech[2*Log[c*x]]],x]
```

```
[Out] x^2/(3*Sqrt[Sech[2*Log[c*x]]]) - (Sqrt[(c^4 + x^(-4))/(c^2 + x^(-2))^2]*(c^
2 + x^(-2))*EllipticF[2*ArcCot[c*x], 1/2])/(3*c*(c^4 + x^(-4))*x*Sqrt[Sech[
2*Log[c*x]]])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5668

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5670

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{\sqrt{\operatorname{sech}(2\log(x))}}\,dx, x, cx\right)}{c^2}\\
&= \frac{\text{Subst}\left(\int \sqrt{1+\frac{1}{x^4}}\,x^2\,dx, x, cx\right)}{c^3\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}}\\
&= -\frac{\text{Subst}\left(\int \frac{\sqrt{1+x^4}}{x^4}\,dx, x, \frac{1}{cx}\right)}{c^3\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}}\\
&= \frac{x^2}{3\sqrt{\operatorname{sech}(2\log(cx))}} - \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)}{3c^3\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}}\\
&= \frac{x^2}{3\sqrt{\operatorname{sech}(2\log(cx))}} - \frac{\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)F\left(2\cot^{-1}(cx)\middle|\frac{1}{2}\right)}{3c\left(c^4+\frac{1}{x^4}\right)x\sqrt{\operatorname{sech}(2\log(cx))}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.08, size = 58, normalized size = 0.67

$$\frac{\sqrt{1+c^4x^4}\sqrt{\frac{c^2x^2}{2+2c^4x^4}}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-c^4x^4\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[Sech[2*Log[c*x]]],x]
```

```
[Out] (Sqrt[1 + c^4*x^4]*Sqrt[(c^2*x^2)/(2 + 2*c^4*x^4)]*Hypergeometric2F1[-1/2,
1/4, 5/4, -(c^4*x^4)])/c^2
```

Maple [C] Result contains complex when optimal does not.
time = 1.29, size = 114, normalized size = 1.31

method	result	size
risch	$\frac{x^2\sqrt{2}}{6\sqrt{\frac{c^2x^2}{c^4x^4+1}}}+\frac{\sqrt{-ic^2x^2+1}\,\sqrt{ic^2x^2+1}\,\mathrm{EllipticF}\left(x\sqrt{ic^2},i\right)\sqrt{2}\,x}{3\sqrt{ic^2}\,(c^4x^4+1)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}$	114

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/sech(2*ln(c*x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*x^2*2^(1/2)/(c^2*x^2/(c^4*x^4+1))^(1/2)+1/3/(I*c^2)^(1/2)*(1-I*c^2*x^2)
^(1/2)*(1+I*c^2*x^2)^(1/2)/(c^4*x^4+1)*EllipticF(x*(I*c^2)^(1/2),I)*2^(1/2)
*x/(c^2*x^2/(c^4*x^4+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sech(2*log(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(sech(2*log(c*x))), x)
```

Fricas [A]
time = 0.10, size = 70, normalized size = 0.80

$$\frac{2\sqrt{2}\sqrt{c^4}\,c\left(-\frac{1}{c^4}\right)^{\frac{3}{4}}\mathrm{ellipticF}\left(\frac{\left(-\frac{1}{c^4}\right)^{\frac{1}{4}}}{x},-1\right)+\sqrt{2}\,(c^4x^4+1)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{6\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sech(2*log(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(2)*sqrt(c^4)*c*(-1/c^4)^(3/4)*ellipticF((-1/c^4)^(1/4)/x, -1) +
 sqrt(2)*(c^4*x^4 + 1)*sqrt(c^2*x^2/(c^4*x^4 + 1)))/c^2
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sech(2*ln(c*x))**(1/2),x)
```

```
[Out] Integral(x/sqrt(sech(2*log(c*x))), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sech(2*log(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{\frac{1}{\cosh\left(2\,\ln\left(c\,x\right)\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1/cosh(2*log(c*x)))^(1/2),x)
```

```
[Out] int(x/(1/cosh(2*log(c*x)))^(1/2), x)
```

3.163 $\int \frac{1}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx$

Optimal. Leaf size=59

$$\frac{x}{2\sqrt{\operatorname{sech}(2\log(cx))}}-\frac{\operatorname{csch}^{-1}(c^2x^2)}{2c^2\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}}$$

```
[Out] 1/2*x/sech(2*ln(c*x))^(1/2)-1/2*arccsch(c^2*x^2)/c^2/x/(1+1/c^4/x^4)^(1/2)/
sech(2*ln(c*x))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {5664, 5662, 342, 281, 283, 221}

$$\frac{x}{2\sqrt{\operatorname{sech}(2\log(cx))}}-\frac{\operatorname{csch}^{-1}(c^2x^2)}{2c^2x\sqrt{\frac{1}{c^4x^4}+1}\,\sqrt{\operatorname{sech}(2\log(cx))}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Sech[2*Log[c*x]]],x]
```

```
[Out] x/(2*Sqrt[Sech[2*Log[c*x]]]) - ArcCsch[c^2*x^2]/(2*c^2*Sqrt[1 + 1/(c^4*x^4)
]*x*Sqrt[Sech[2*Log[c*x]]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5662

```
Int[Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Dist[Sech[d*(a
+ b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*d*p)), Int[1/(x^(b*
d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; FreeQ[{a, b, d, p}, x] &&
 !IntegerQ[p]
```

Rule 5664

```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\operatorname{sech}(2\log(x))}}\,dx, x, cx\right)}{c} \\
&= \frac{\text{Subst}\left(\int \sqrt{1+\frac{1}{x^4}}\,x\,dx, x, cx\right)}{c^2\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}} \\
&= -\frac{\text{Subst}\left(\int \frac{\sqrt{1+x^4}}{x^3}\,dx, x, \frac{1}{cx}\right)}{c^2\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}} \\
&= -\frac{\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{x^2}\,dx, x, \frac{1}{c^2x^2}\right)}{2c^2\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}} \\
&= \frac{x}{2\sqrt{\operatorname{sech}(2\log(cx))}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \frac{1}{c^2x^2}\right)}{2c^2\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}} \\
&= \frac{x}{2\sqrt{\operatorname{sech}(2\log(cx))}} - \frac{\operatorname{csch}^{-1}(c^2x^2)}{2c^2\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}}
\end{aligned}
$$

Mathematica [**A**]
time = 0.07, size = 77, normalized size = 1.31

$$
\frac{x\left(2\sqrt{1+c^4x^4}-2\tanh^{-1}\left(\sqrt{1+c^4x^4}\right)\right)}{4\sqrt{2}\sqrt{\frac{c^2x^2}{1+c^4x^4}}\sqrt{1+c^4x^4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Sech[2*Log[c*x]]],x]
[Out] (x*(2*Sqrt[1 + c^4*x^4] - 2*ArcTanh[Sqrt[1 + c^4*x^4]]))/(4*Sqrt[2]*Sqrt[(c
^2*x^2)/(1 + c^4*x^4)]*Sqrt[1 + c^4*x^4])
```

Maple [**F**]
time = 0.92, size = 0, normalized size = 0.00

$$
\int \frac{1}{\sqrt{\operatorname{sech}\left(2\ln\left(cx\right)\right)}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sech(2*ln(c*x))^(1/2),x)
```

```
[Out] int(1/sech(2*ln(c*x))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(2*log(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(sech(2*log(c*x))), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(49) = 98$.
time = 0.37, size = 100, normalized size = 1.69

$$\frac{\sqrt{2}\,cx\log\left(\frac{c^5x^5+2\,cx-2\,(c^4x^4+1)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{cx^5}\right)+2\,\sqrt{2}\,(c^4x^4+1)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{8\,c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(2*log(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(2)*c*x*log((c^5*x^5 + 2*c*x - 2*(c^4*x^4 + 1)*sqrt(c^2*x^2/(c^4*x
^4 + 1)))/(c*x^5)) + 2*sqrt(2)*(c^4*x^4 + 1)*sqrt(c^2*x^2/(c^4*x^4 + 1)))/(
c^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(2*ln(c*x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(sech(2*log(c*x))), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(2*log(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\frac{1}{\cosh\left(2\,\ln\left(c\,x\right)\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cosh(2*log(c*x)))^(1/2),x)
```

```
[Out] int(1/(1/cosh(2*log(c*x)))^(1/2), x)
```

3.164 $\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x}\,dx$

Optimal. Leaf size=36

$$-i\sqrt{\cosh(2\log(cx))}\,F(i\log(cx)|2)\sqrt{\operatorname{sech}(2\log(cx))}$$

```
[Out] -I*((1/2*c*x+1/2/c/x)^2)^(1/2)/(1/2*c*x+1/2/c/x)*EllipticF(I*(1/2*c*x-1/2/c
/x),2^(1/2))*cosh(2*ln(c*x))^(1/2)*sech(2*ln(c*x))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3856, 2720}

$$-i\sqrt{\operatorname{sech}(2\log(cx))}\,\sqrt{\cosh(2\log(cx))}\,F(i\log(cx)|2)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sech[2*Log[c*x]]]/x,x]
```

```
[Out] (-I)*Sqrt[Cosh[2*Log[c*x]]]*EllipticF[I*Log[c*x], 2]*Sqrt[Sech[2*Log[c*x]]]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x}\,dx &= \operatorname{Subst}\left(\int \sqrt{\operatorname{sech}(2x)}\,dx, x, \log(cx)\right)\\
&= \left(\sqrt{\cosh(2\log(cx))}\,\sqrt{\operatorname{sech}(2\log(cx))}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{\cosh(2x)}}\,dx, x, \log(cx)\right)\\
&= -i\sqrt{\cosh(2\log(cx))}\,F(i\log(cx)|2)\sqrt{\operatorname{sech}(2\log(cx))}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 36, normalized size = 1.00

$$-i\sqrt{\cosh(2\log(cx))}\,F(i\log(cx)|2)\sqrt{\operatorname{sech}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sech[2*Log[c*x]]]/x,x]
```

```
[Out] (-I)*Sqrt[Cosh[2*Log[c*x]]]*EllipticF[I*Log[c*x], 2]*Sqrt[Sech[2*Log[c*x]]]
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(73) = 146$.
time = 2.47, size = 167, normalized size = 4.64

method	result
derivativedivides	$\frac{\sqrt{\left(2\left(\frac{cx}{2}+\frac{1}{2cx}\right)^2-1\right)\left(\frac{cx}{2}-\frac{1}{2cx}\right)^2}\sqrt{-\left(\frac{cx}{2}-\frac{1}{2cx}\right)^2}\sqrt{-2\left(\frac{cx}{2}+\frac{1}{2cx}\right)^2+1}\ \mathrm{EllipticF}\left(\frac{cx}{2}+\right.}{\sqrt{2\left(\frac{cx}{2}-\frac{1}{2cx}\right)^4+\left(\frac{cx}{2}-\frac{1}{2cx}\right)^2}\left(\frac{cx}{2}-\frac{1}{2cx}\right)\sqrt{2\left(\frac{cx}{2}+\frac{1}{2cx}\right)^2-1}}$
default	$\frac{\sqrt{\left(2\left(\frac{cx}{2}+\frac{1}{2cx}\right)^2-1\right)\left(\frac{cx}{2}-\frac{1}{2cx}\right)^2}\sqrt{-\left(\frac{cx}{2}-\frac{1}{2cx}\right)^2}\sqrt{-2\left(\frac{cx}{2}+\frac{1}{2cx}\right)^2+1}\ \mathrm{EllipticF}\left(\frac{cx}{2}+\right.}{\sqrt{2\left(\frac{cx}{2}-\frac{1}{2cx}\right)^4+\left(\frac{cx}{2}-\frac{1}{2cx}\right)^2}\left(\frac{cx}{2}-\frac{1}{2cx}\right)\sqrt{2\left(\frac{cx}{2}+\frac{1}{2cx}\right)^2-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(2*ln(c*x))^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] ((2*(1/2*c*x+1/2/c/x)^2-1)*(1/2*c*x-1/2/c/x)^2)^(1/2)*(-(1/2*c*x-1/2/c/x)^2
)^(1/2)*(-2*(1/2*c*x+1/2/c/x)^2+1)^(1/2)/(2*(1/2*c*x-1/2/c/x)^4+(1/2*c*x-1/
2/c/x)^2)^(1/2)*EllipticF(1/2*c*x+1/2/c/x,2^(1/2))/(1/2*c*x-1/2/c/x)/(2*(1/
2*c*x+1/2/c/x)^2-1)^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sech(2*log(c*x)))/x, x)
```

Fricas [**A**]
time = 0.10, size = 26, normalized size = 0.72

$$-\frac{\sqrt{2}\left(-c^4\right)^{\frac{3}{4}}\mathrm{ellipticF}\left(\left(-c^4\right)^{\frac{1}{4}}x,-1\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] -sqrt(2)*(-c^4)^(3/4)*ellipticF((-c^4)^(1/4)*x, -1)/c^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*ln(c*x))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(sech(2*log(c*x)))/x, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(1/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{\frac{1}{\cosh(2\ln(c\,x))}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(2*log(c*x)))^(1/2)/x,x)
```

```
[Out] int((1/cosh(2*log(c*x)))^(1/2)/x, x)
```

3.165 $\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^2}\,dx$

Optimal. Leaf size=40

$$-\frac{1}{2}c^2\sqrt{1+\frac{1}{c^4x^4}}\,x\operatorname{csch}^{-1}\left(c^2x^2\right)\sqrt{\operatorname{sech}(2\log(cx))}$$

```
[Out] -1/2*c^2*x*arccsch(c^2*x^2)*(1+1/c^4/x^4)^(1/2)*sech(2*ln(c*x))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5670, 5668, 342, 281, 221}

$$-\frac{1}{2}c^2x\sqrt{\frac{1}{c^4x^4}+1}\operatorname{csch}^{-1}\left(c^2x^2\right)\sqrt{\operatorname{sech}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sech[2*Log[c*x]]]/x^2,x]
```

```
[Out] -1/2*(c^2*Sqrt[1 + 1/(c^4*x^4)]*x*ArcCsch[c^2*x^2]*Sqrt[Sech[2*Log[c*x]]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5668

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5670

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^2}\,dx &= c\operatorname{Subst}\left(\int \frac{\sqrt{\operatorname{sech}(2\log(x))}}{x^2}\,dx, x, cx\right)\\
&= \left(c^2\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{1}{x^4}}\,x^3}\,dx, x, cx\right)\\
&= -\left(\left(c^2\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}\right)\operatorname{Subst}\left(\int \frac{x}{\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)\right)\\
&= -\left(\frac{1}{2}\left(c^2\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \frac{1}{c^2x^2}\right)\right)\\
&= -\frac{1}{2}c^2\sqrt{1+\frac{1}{c^4x^4}}\,x\operatorname{csch}^{-1}\left(c^2x^2\right)\sqrt{\operatorname{sech}(2\log(cx))}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 55, normalized size = 1.38

$$-\frac{\sqrt{1+c^4x^4}\sqrt{\frac{c^2x^2}{2+2c^4x^4}}\tanh^{-1}\left(\sqrt{1+c^4x^4}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sech[2*Log[c*x]]]/x^2,x]
```

```
[Out] -((Sqrt[1 + c^4*x^4]*Sqrt[(c^2*x^2)/(2 + 2*c^4*x^4)]*ArcTanh[Sqrt[1 + c^4*x
^4]])/x)
```

Maple [F]
time = 0.88, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{sech}\left(2\ln\left(cx\right)\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(2*ln(c*x))^(1/2)/x^2,x)
```

```
[Out] int(sech(2*ln(c*x))^(1/2)/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sech(2*log(c*x)))/x^2, x)
```

Fricas [A]
time = 0.35, size = 57, normalized size = 1.42

$$\frac{1}{4}\sqrt{2}\,c\log\left(\frac{c^5x^5+2\,cx-2\,(c^4x^4+1)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{cx^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*c*log((c^5*x^5 + 2*c*x - 2*(c^4*x^4 + 1)*sqrt(c^2*x^2/(c^4*x^4
+ 1)))/(c*x^5))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*ln(c*x))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(sech(2*log(c*x)))/x**2, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\frac{1}{\cosh \left(2 \ln \left(c\, x\right)\right)}}}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(2*log(c*x)))^(1/2)/x^2,x)
```

```
[Out] int((1/cosh(2*log(c*x)))^(1/2)/x^2, x)
```

3.166 $\int \frac{\sqrt{\mathrm{sech}(2\log(cx))}}{x^3}\,dx$

Optimal. Leaf size=137

$$-\frac{\left(c^4+\frac{1}{x^4}\right)\sqrt{\mathrm{sech}(2\log(cx))}}{c^2+\frac{1}{x^2}}+c\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)xE\left(2\cot^{-1}(cx)|\frac{1}{2}\right)\sqrt{\mathrm{sech}(2\log(cx))}-\frac{1}{2}c\sqrt{\frac{}{(c}}$$

```
[Out] -(c^4+1/x^4)*sech(2*ln(c*x))^(1/2)/(c^2+1/x^2)+c*(c^2+1/x^2)*x*(cos(2*arcco
t(c*x))^2)^(1/2)/cos(2*arccot(c*x))*EllipticE(sin(2*arccot(c*x)),1/2*2^(1/2
))*((c^4+1/x^4)/(c^2+1/x^2)^2)^(1/2)*sech(2*ln(c*x))^(1/2)-1/2*c*(c^2+1/x^2
)*x*(cos(2*arccot(c*x))^2)^(1/2)/cos(2*arccot(c*x))*EllipticF(sin(2*arccot(
c*x)),1/2*2^(1/2))*((c^4+1/x^4)/(c^2+1/x^2)^2)^(1/2)*sech(2*ln(c*x))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5670, 5668, 342, 311, 226, 1210}

$$-\frac{\left(c^4+\frac{1}{x^4}\right)\sqrt{\mathrm{sech}(2\log(cx))}}{c^2+\frac{1}{x^2}}-\frac{1}{2}cx\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\sqrt{\mathrm{sech}(2\log(cx))}F\left(2\cot^{-1}(cx)|\frac{1}{2}\right)+cx\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\sqrt{\mathrm{sech}(2\log(cx))}E\left(2\cot^{-1}(cx)|\frac{1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sech[2*Log[c*x]]]/x^3,x]
```

```
[Out] -(((c^4 + x^(-4))*Sqrt[Sech[2*Log[c*x]]])/(c^2 + x^(-2))) + c*Sqrt[(c^4 + x
^(-4))/(c^2 + x^(-2))^2]*(c^2 + x^(-2))*x*EllipticE[2*ArcCot[c*x], 1/2]*Sqr
t[Sech[2*Log[c*x]]] - (c*Sqrt[(c^4 + x^(-4))/(c^2 + x^(-2))^2]*(c^2 + x^(-2
))*x*EllipticF[2*ArcCot[c*x], 1/2]*Sqrt[Sech[2*Log[c*x]]])/2
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
```

```
egerQ[m]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 5668

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5670

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^3}\,dx &= c^2\text{Subst}\left(\int \frac{\sqrt{\operatorname{sech}(2\log(x))}}{x^3}\,dx, x, cx\right)\\
&= \left(c^3\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{1}{x^4}}\,x^4}\,dx, x, cx\right)\\
&= -\left(\left(c^3\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)\right)\\
&= -\left(\left(c^3\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)\right) + \left(c \cdots\right.\\
&= -\frac{\left(c^4+\frac{1}{x^4}\right)\sqrt{\operatorname{sech}(2\log(cx))}}{c^2+\frac{1}{x^2}} + c\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)xE\left(2\cot^{-1}(cx)|\frac{1}{2}\right)
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.08, size = 59, normalized size = 0.43

$$-\frac{c^2\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-c^4x^4\right)}{\sqrt{1+c^4x^4}\sqrt{\frac{c^2x^2}{2+2c^4x^4}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sech[2*Log[c*x]]]/x^3,x]
```

```
[Out] -((c^2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(c^4*x^4)])/(Sqrt[1 + c^4*x^4]*Sq
rt[(c^2*x^2)/(2 + 2*c^4*x^4)]))
```

Maple [C] Result contains complex when optimal does not.
time = 1.25, size = 134, normalized size = 0.98

method	result
risch	$-\frac{(c^4x^4+1)\sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{x^2}+\frac{ic^2\sqrt{-ic^2x^2+1}\sqrt{ic^2x^2+1}\left(\text{EllipticF}\left(x\sqrt{ic^2},i\right)-\text{EllipticE}\left(x\sqrt{ic^2},i\right)\right)\sqrt{2}}{\sqrt{ic^2}\,x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(2*ln(c*x))^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -(c^4*x^4+1)/x^2*2^(1/2)*(c^2*x^2/(c^4*x^4+1))^(1/2)+I*c^2/(I*c^2)^(1/2)*(1
-I*c^2*x^2)^(1/2)*(1+I*c^2*x^2)^(1/2)*(EllipticF(x*(I*c^2)^(1/2),I)-Ellipti
cE(x*(I*c^2)^(1/2),I))*2^(1/2)*(c^2*x^2/(c^4*x^4+1))^(1/2)/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sech(2*log(c*x)))/x^3, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*ln(c*x))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(sech(2*log(c*x)))/x**3, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1}{\cosh\left(2\,\ln\left(c\,x\right)\right)}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(2*log(c*x)))^(1/2)/x^3,x)
```

```
[Out] int((1/cosh(2*log(c*x)))^(1/2)/x^3, x)
```

3.167 $\int \frac{\sqrt{\mathrm{sech}(2\log(cx))}}{x^4}\,dx$

Optimal. Leaf size=23

$$-\frac{1}{2}\left(c^4+\frac{1}{x^4}\right)x\sqrt{\mathrm{sech}(2\log(cx))}$$

```
[Out] -1/2*(c^4+1/x^4)*x*sech(2*ln(c*x))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5670, 5668, 267}

$$-\frac{1}{2}x\left(c^4+\frac{1}{x^4}\right)\sqrt{\mathrm{sech}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sech[2*Log[c*x]]]/x^4,x]
```

```
[Out] -1/2*((c^4 + x^(-4))*x*Sqrt[Sech[2*Log[c*x]]])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5668

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5670

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^4}\,dx = c^3\text{Subst}\left(\int \frac{\sqrt{\operatorname{sech}(2\log(x))}}{x^4}\,dx, x, cx\right)$$

$$= \left(c^4\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{1}{x^4}}\,x^5}\,dx, x, cx\right)$$

$$= -\frac{1}{2}\left(c^4+\frac{1}{x^4}\right)x\sqrt{\operatorname{sech}(2\log(cx))}$$

Mathematica [A]
time = 0.03, size = 33, normalized size = 1.43

$$-\frac{c^2}{2x\sqrt{\frac{c^2x^2}{2+2c^4x^4}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sech[2*Log[c*x]]]/x^4,x]
```

```
[Out] -1/2*c^2/(x*Sqrt[(c^2*x^2)/(2 + 2*c^4*x^4)])
```

Maple [A]
time = 1.20, size = 38, normalized size = 1.65

method	result	size
risch	$-\frac{\sqrt{2}\,\sqrt{\frac{c^2x^2}{c^4x^4+1}}\,(c^4x^4+1)}{2x^3}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(2*ln(c*x))^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*2^(1/2)*(c^2*x^2/(c^4*x^4+1))^(1/2)/x^3*(c^4*x^4+1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(19) = 38.
time = 0.47, size = 42, normalized size = 1.83

$$-\frac{1}{2}c^3\left(\frac{\sqrt{2}}{\sqrt{\frac{1}{c^4x^4}+1}}+\frac{\sqrt{2}}{c^4x^4\sqrt{\frac{1}{c^4x^4}+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^4,x, algorithm="maxima")
[Out] -1/2*c^3*(sqrt(2)/sqrt(1/(c^4*x^4) + 1) + sqrt(2)/(c^4*x^4*sqrt(1/(c^4*x^4)
 + 1)))
```

Fricas [A]
time = 0.37, size = 37, normalized size = 1.61

$$-\frac{\sqrt{2}\,(c^4x^4+1)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^4,x, algorithm="fricas")
[Out] -1/2*sqrt(2)*(c^4*x^4 + 1)*sqrt(c^2*x^2/(c^4*x^4 + 1))/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*ln(c*x))**(1/2)/x**4,x)
[Out] Integral(sqrt(sech(2*log(c*x)))/x**4, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^4,x, algorithm="giac")
[Out] Timed out
```

Mupad [B]
time = 1.35, size = 58, normalized size = 2.52

$$-\frac{\sqrt{\frac{2\,c^2\,x^2}{c^4\,x^4+1}}}{2\,x^3}-\frac{c^4\,x\,\sqrt{\frac{2\,c^2\,x^2}{c^4\,x^4+1}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(2*log(c*x)))^(1/2)/x^4,x)
[Out] - ((2*c^2*x^2)/(c^4*x^4 + 1))^(1/2)/(2*x^3) - (c^4*x*((2*c^2*x^2)/(c^4*x^4
+ 1))^(1/2))/2
```

3.168 $\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^5}\,dx$

Optimal. Leaf size=80

$$-\frac{1}{3}\left(c^4+\frac{1}{x^4}\right)\sqrt{\operatorname{sech}(2\log(cx))}+\frac{1}{6}c^3\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)xF\left(2\cot^{-1}(cx)|\frac{1}{2}\right)\sqrt{\operatorname{sech}(2\log(cx))}$$

```
[Out] -1/3*(c^4+1/x^4)*sech(2*ln(c*x))^(1/2)+1/6*c^3*(c^2+1/x^2)*x*(cos(2*arccot(
c*x))^2)^(1/2)/cos(2*arccot(c*x))*EllipticF(sin(2*arccot(c*x)),1/2*2^(1/2))
*((c^4+1/x^4)/(c^2+1/x^2)^2)^(1/2)*sech(2*ln(c*x))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5670, 5668, 342, 327, 226}

$$\frac{1}{6}c^3x\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\sqrt{\operatorname{sech}(2\log(cx))}\,F\left(2\cot^{-1}(cx)|\frac{1}{2}\right)-\frac{1}{3}\left(c^4+\frac{1}{x^4}\right)\sqrt{\operatorname{sech}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sech[2*Log[c*x]]]/x^5,x]
```

```
[Out] -1/3*((c^4 + x^(-4))*Sqrt[Sech[2*Log[c*x]]]) + (c^3*Sqrt[(c^4 + x^(-4))/(c^
2 + x^(-2))^2]*(c^2 + x^(-2))*x*EllipticF[2*ArcCot[c*x], 1/2]*Sqrt[Sech[2*L
og[c*x]]])/6
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
```

```
egerQ[m]
```

Rule 5668

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5670

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^5}\,dx &= c^4\text{Subst}\left(\int \frac{\sqrt{\operatorname{sech}(2\log(x))}}{x^5}\,dx, x, cx\right)\\ &= \left(c^5\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{1}{x^4}}\,x^6}\,dx, x, cx\right)\\ &= -\left(\left(c^5\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)\right)\\ &= -\frac{1}{3}\left(c^4+\frac{1}{x^4}\right)\sqrt{\operatorname{sech}(2\log(cx))}+\frac{1}{3}\left(c^5\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}\right)\text{Sub}\\ &= -\frac{1}{3}\left(c^4+\frac{1}{x^4}\right)\sqrt{\operatorname{sech}(2\log(cx))}+\frac{1}{6}c^3\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)xF\left(2\cot^{-1}(c\right.\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.07, size = 65, normalized size = 0.81

$$-\frac{\sqrt{2}\sqrt{\frac{c^2x^2}{1+c^4x^4}}\sqrt{1+c^4x^4}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};-c^4x^4\right)}{3x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sech[2*Log[c*x]]]/x^5,x]
```

```
[Out] -1/3*(Sqrt[2]*Sqrt[(c^2*x^2)/(1 + c^4*x^4)]*Sqrt[1 + c^4*x^4]*Hypergeometri
c2F1[-3/4, 1/2, 1/4, -(c^4*x^4)])/x^4
```

Maple [C] Result contains complex when optimal does not.
time = 1.23, size = 117, normalized size = 1.46

method	result	size
risch	$-\frac{(c^4x^4+1)\sqrt{2}\,\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{3x^4}-\frac{c^4\sqrt{-ic^2x^2+1}\,\sqrt{ic^2x^2+1}\,\mathrm{EllipticF}\left(x\sqrt{ic^2}\,,i\right)\sqrt{2}\,\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{3\sqrt{ic^2}\,x}$	117

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(2*ln(c*x))^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(c^4*x^4+1)/x^4*2^(1/2)*(c^2*x^2/(c^4*x^4+1))^(1/2)-1/3*c^4/(I*c^2)^(1
/2)*(1-I*c^2*x^2)^(1/2)*(1+I*c^2*x^2)^(1/2)*EllipticF(x*(I*c^2)^(1/2),I)*2^
(1/2)*(c^2*x^2/(c^4*x^4+1))^(1/2)/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sech(2*log(c*x)))/x^5, x)
```

Fricas [A]
time = 0.10, size = 66, normalized size = 0.82

$$\frac{\sqrt{2}\,(-c^4)^{\frac{3}{4}}\,cx^4\mathrm{ellipticF}\left((-c^4)^{\frac{1}{4}}\,x,-1\right)-\sqrt{2}\,(c^4x^4+1)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{3\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-c^4)^(3/4)*c*x^4*ellipticF((-c^4)^(1/4)*x, -1) - sqrt(2)*(c^
4*x^4 + 1)*sqrt(c^2*x^2/(c^4*x^4 + 1)))/x^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\mathrm{sech}\,(2\log{(cx)})}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*ln(c*x))**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(sech(2*log(c*x)))/x**5, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1}{\cosh\left(2\,\ln\left(c\,x\right)\right)}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(2*log(c*x)))^(1/2)/x^5,x)
```

```
[Out] int((1/cosh(2*log(c*x)))^(1/2)/x^5, x)
```

3.169 $\int \frac{x^8}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=122

$$\frac{x}{32c^4\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x^5}{16\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x^9}{12\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}-\frac{\tanh^{-1}\left(\sqrt{1+\ldots}\right.}{32c^{12}\left(1+\frac{1}{c^4x^4}\right)^{3/2}x^3\mathrm{sec}\ldots}$$

```
[Out] 1/32*x/c^4/(c^4+1/x^4)/sech(2*ln(c*x))^(3/2)+1/16*x^5/(c^4+1/x^4)/sech(2*ln
(c*x))^(3/2)+1/12*x^9/sech(2*ln(c*x))^(3/2)-1/32*arctanh((1+1/c^4/x^4)^(1/2
))/c^12/(1+1/c^4/x^4)^(3/2)/x^3/sech(2*ln(c*x))^(3/2)
```

Rubi [A]
time = 0.05, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {5670, 5668, 272, 43, 44, 65, 213}

$$\frac{x}{32c^4\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x^5}{16\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}-\frac{\tanh^{-1}\left(\sqrt{\frac{1}{c^4x^4}+1}\right)}{32c^{12}x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x^9}{12\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] x/(32*c^4*(c^4 + x^(-4))*Sech[2*Log[c*x]]^(3/2)) + x^5/(16*(c^4 + x^(-4))*S
ech[2*Log[c*x]]^(3/2)) + x^9/(12*Sech[2*Log[c*x]]^(3/2)) - ArcTanh[Sqrt[1 +
 1/(c^4*x^4)]]/(32*c^12*(1 + 1/(c^4*x^4))^(3/2)*x^3*Sech[2*Log[c*x]]^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5668

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5670

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{x^8}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^8}{\operatorname{sech}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^9}$$

$$= \frac{\operatorname{Subst}\left(\int \left(1+\frac{1}{x^4}\right)^{3/2} x^{11}\,dx, x, cx\right)}{c^{12}\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{(1+x)^{3/2}}{x^4}\,dx, x, \frac{1}{c^4x^4}\right)}{4c^{12}\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x^9}{12\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{\operatorname{Subst}\left(\int \frac{\sqrt{1+x}}{x^3}\,dx, x, \frac{1}{c^4x^4}\right)}{8c^{12}\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x^5}{16\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^9}{12\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{1+x}}\,dx, x,\right.}{32c^{12}\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2}$$

$$= \frac{x}{32c^4\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^5}{16\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^9}{12\operatorname{sech}^{\frac{3}{2}}(2\log(cx}$$

$$= \frac{x}{32c^4\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^5}{16\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^9}{12\operatorname{sech}^{\frac{3}{2}}(2\log(cx}$$

$$= \frac{x}{32c^4\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^5}{16\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^9}{12\operatorname{sech}^{\frac{3}{2}}(2\log(cx}$$

Mathematica [A]
time = 0.14, size = 98, normalized size = 0.80

$$\frac{c^3x^3\sqrt{1+c^4x^4}\,(3+14c^4x^4+8c^8x^8)-3cx\sinh^{-1}(c^2x^2)}{192\sqrt{2}\,c^9\sqrt{\frac{c^2x^2}{1+c^4x^4}}\sqrt{1+c^4x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/Sech[2*Log[c*x]]^(3/2),x]
[Out] (c^3*x^3*Sqrt[1 + c^4*x^4]*(3 + 14*c^4*x^4 + 8*c^8*x^8) - 3*c*x*ArcSinh[c^2
*x^2])/(192*Sqrt[2]*c^9*Sqrt[(c^2*x^2)/(1 + c^4*x^4)]*Sqrt[1 + c^4*x^4])
```

Maple [A]
time = 1.50, size = 121, normalized size = 0.99

method	result	size
risch	$\frac{x^{3}\left(8 c^{8} x^{8}+14 c^{4} x^{4}+3\right) \sqrt{2}}{384 c^{6} \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}+1}}}-\frac{\ln \left(\frac{c^{4} x^{2}}{\sqrt{c^{4}}}+\sqrt{c^{4} x^{4}+1}\right) \sqrt{2}\, x}{128 c^{6} \sqrt{c^{4}}\, \sqrt{c^{4} x^{4}+1}\, \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}+1}}}$	121

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/sech(2*ln(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/384*x^3*(8*c^8*x^8+14*c^4*x^4+3)/c^6*2^(1/2)/(c^2*x^2/(c^4*x^4+1))^(1/2)-
1/128/c^6*ln(c^4*x^2/(c^4)^(1/2)+(c^4*x^4+1)^(1/2))/(c^4)^(1/2)*2^(1/2)*x/(
c^4*x^4+1)^(1/2)/(c^2*x^2/(c^4*x^4+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/sech(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^8/sech(2*log(c*x))^(3/2), x)
```

Fricas [A]
time = 0.39, size = 109, normalized size = 0.89

$$\frac{2 \sqrt{2} \left(8 \, c^{13} x^{13} + 22 \, c^{9} x^{9} + 17 \, c^{5} x^{5} + 3 \, c x\right) \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4} + 1}} + 3 \sqrt{2} \log \left(-2 \, c^{4} x^{4} + 2 \left(c^{5} x^{5} + c x\right) \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4} + 1}} - 1\right)}{768 \, c^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/sech(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/768*(2*sqrt(2)*(8*c^13*x^13 + 22*c^9*x^9 + 17*c^5*x^5 + 3*c*x)*sqrt(c^2*x
^2/(c^4*x^4 + 1)) + 3*sqrt(2)*log(-2*c^4*x^4 + 2*(c^5*x^5 + c*x)*sqrt(c^2*x
^2/(c^4*x^4 + 1)) - 1))/c^9
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/sech(2*ln(c*x))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/sech(2*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^8/sech(2*log(c*x))^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^8}{\left(\frac{1}{\cosh(2\,\ln(c\,x))}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(1/cosh(2*log(c*x)))^(3/2),x)
```

```
[Out] int(x^8/(1/cosh(2*log(c*x)))^(3/2), x)
```

3.170 $\int \frac{x^7}{\mathrm{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=141

$$\frac{4}{77c^4\left(c^4+\frac{1}{x^4}\right)\mathrm{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{6x^4}{77\left(c^4+\frac{1}{x^4}\right)\mathrm{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x^8}{11\mathrm{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{2\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\cdots\right.}{77c^5\left(c^4+\frac{1}{x^4}\right)^2x^3\cdots}$$

```
[Out] 4/77/c^4/(c^4+1/x^4)/sech(2*ln(c*x))^(3/2)+6/77*x^4/(c^4+1/x^4)/sech(2*ln(c
*x))^(3/2)+1/11*x^8/sech(2*ln(c*x))^(3/2)+2/77*(c^2+1/x^2)*(cos(2*arccot(c*
x))^2)^(1/2)/cos(2*arccot(c*x))*EllipticF(sin(2*arccot(c*x)),1/2*2^(1/2))*(
(c^4+1/x^4)/(c^2+1/x^2)^2)^(1/2)/c^5/(c^4+1/x^4)^2/x^3/sech(2*ln(c*x))^(3/2
)
```

Rubi [A]
time = 0.07, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5670, 5668, 342, 283, 331, 226}

$$\frac{6x^4}{77\left(c^4+\frac{1}{x^4}\right)\mathrm{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{4}{77c^4\left(c^4+\frac{1}{x^4}\right)\mathrm{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{2\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)F\left(2\cot^{-1}(cx)\middle|\frac{1}{2}\right)}{77c^5x^3\left(c^4+\frac{1}{x^4}\right)^2\mathrm{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x^8}{11\mathrm{sech}^{\frac{3}{2}}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] 4/(77*c^4*(c^4 + x^(-4))*Sech[2*Log[c*x]]^(3/2)) + (6*x^4)/(77*(c^4 + x^(-4
))*Sech[2*Log[c*x]]^(3/2)) + x^8/(11*Sech[2*Log[c*x]]^(3/2)) + (2*Sqrt[(c^4
 + x^(-4))/(c^2 + x^(-2))^2]*(c^2 + x^(-2))*EllipticF[2*ArcCot[c*x], 1/2])/
(77*c^5*(c^4 + x^(-4))^2*x^3*Sech[2*Log[c*x]]^(3/2))
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
```

```
nomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5668

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5670

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{x^7}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^7}{\operatorname{sech}^{\frac{3}{2}}(2\log(x))}\,dx,x,cx\right)}{c^8}$$

$$= \frac{\operatorname{Subst}\left(\int \left(1+\frac{1}{x^4}\right)^{3/2} x^{10}\,dx,x,cx\right)}{c^{11}\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{(1+x^4)^{3/2}}{x^{12}}\,dx,x,\frac{1}{cx}\right)}{c^{11}\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x^8}{11\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{6\operatorname{Subst}\left(\int \frac{\sqrt{1+x^4}}{x^8}\,dx,x,\frac{1}{cx}\right)}{11c^{11}\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{6x^4}{77\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^8}{11\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{12\operatorname{Subst}\left(\int \frac{1}{x^4\sqrt{1+x^4}}\,dx \ldots\right.}{77c^{11}\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(\ldots}$$

$$= \frac{4}{77c^4\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{6x^4}{77\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^8}{11\operatorname{sech}^{\frac{3}{2}}(2\log(\ldots}$$

$$= \frac{4}{77c^4\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{6x^4}{77\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^8}{11\operatorname{sech}^{\frac{3}{2}}(2\log(\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.13, size = 77, normalized size = 0.55

$$\frac{\sqrt{1+c^4x^4}\sqrt{\frac{c^2x^2}{2+2c^4x^4}}\left((1+c^4x^4)^{5/2} - {}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-c^4x^4\right)\right)}{22c^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] (Sqrt[1 + c^4*x^4]*Sqrt[(c^2*x^2)/(2 + 2*c^4*x^4)]*((1 + c^4*x^4)^(5/2) - H
ypergeometric2F1[-3/2, 1/4, 5/4, -(c^4*x^4)]))/(22*c^8)
```

Maple [C] Result contains complex when optimal does not.
time = 1.17, size = 138, normalized size = 0.98

method	result	size
risch	$\frac{x^{2}\left(7 c^{8} x^{8}+13 c^{4} x^{4}+4\right) \sqrt{2}}{308 c^{6} \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}+1}}}-\frac{\sqrt{-i c^{2} x^{2}+1}\, \sqrt{i c^{2} x^{2}+1}\, \mathrm{EllipticF}\left(x \sqrt{i c^{2}}, i\right) \sqrt{2}\, x}{77 c^{6} \sqrt{i c^{2}}\, \left(c^{4} x^{4}+1\right) \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}+1}}}$	138

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/sech(2*ln(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/308*x^2*(7*c^8*x^8+13*c^4*x^4+4)/c^6*2^(1/2)/(c^2*x^2/(c^4*x^4+1))^(1/2)-
1/77/c^6/(I*c^2)^(1/2)*(1-I*c^2*x^2)^(1/2)*(1+I*c^2*x^2)^(1/2)/(c^4*x^4+1)*
EllipticF(x*(I*c^2)^(1/2),I)*2^(1/2)*x/(c^2*x^2/(c^4*x^4+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/sech(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^7/sech(2*log(c*x))^(3/2), x)
```

Fricas [A]
time = 0.09, size = 88, normalized size = 0.62

$$-\frac{4 \sqrt{2}\, \sqrt{c^{4}}\, c\left(-\frac{1}{c^{4}}\right)^{\frac{3}{4}} \mathrm{ellipticF}\left(\frac{\left(-\frac{1}{c^{4}}\right)^{\frac{1}{4}}}{x},-1\right)-\sqrt{2}\, \left(7 \, c^{12} x^{12}+20 \, c^{8} x^{8}+17 \, c^{4} x^{4}+4\right) \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}+1}}}{308 \, c^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/sech(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/308*(4*sqrt(2)*sqrt(c^4)*c*(-1/c^4)^(3/4)*ellipticF((-1/c^4)^(1/4)/x, -1
) - sqrt(2)*(7*c^12*x^12 + 20*c^8*x^8 + 17*c^4*x^4 + 4)*sqrt(c^2*x^2/(c^4*x
^4 + 1)))/c^8
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{7}}{\operatorname{sech}^{\frac{3}{2}}\left(2 \log \left(c x\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/sech(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(x**7/sech(2*log(c*x))**(3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/sech(2*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7}{\left(\frac{1}{\cosh(2\,\ln(c\,x))}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(1/cosh(2*log(c*x)))^(3/2),x)
```

```
[Out] int(x^7/(1/cosh(2*log(c*x)))^(3/2), x)
```

3.171 $\int \frac{x^6}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=28

$$\frac{\left(c^4+\frac{1}{x^4}\right)x^7}{10c^4\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] 1/10*(c^4+1/x^4)*x^7/c^4/sech(2*ln(c*x))^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5670, 5668, 270}

$$\frac{x^7\left(c^4+\frac{1}{x^4}\right)}{10c^4\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] ((c^4 + x^(-4))*x^7)/(10*c^4*Sech[2*Log[c*x]]^(3/2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5668

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5670

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{x^6}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^6}{\operatorname{sech}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^7}$$
$$= \frac{\operatorname{Subst}\left(\int \left(1+\frac{1}{x^4}\right)^{3/2} x^9\,dx, x, cx\right)}{c^{10}\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$
$$= \frac{\left(c^4+\frac{1}{x^4}\right)x^7}{10c^4\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

Mathematica [A]
time = 0.04, size = 44, normalized size = 1.57

$$\frac{(1+c^4x^4)^3\sqrt{\frac{c^2x^2}{2+2c^4x^4}}}{20c^8x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/Sech[2*Log[c*x]]^(3/2),x]
[Out] ((1 + c^4*x^4)^3*Sqrt[(c^2*x^2)/(2 + 2*c^4*x^4)])/(20*c^8*x)
```

Maple [A]
time = 1.18, size = 47, normalized size = 1.68

method	result	size
risch	$\frac{\sqrt{2}\,x\left(c^8x^8+2c^4x^4+1\right)}{40c^6\sqrt{\frac{c^2x^2}{c^4x^4+1}}}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/sech(2*ln(c*x))^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/40*2^(1/2)/c^6*x/(c^2*x^2/(c^4*x^4+1))^(1/2)*(c^8*x^8+2*c^4*x^4+1)
```

Maxima [A]
time = 0.51, size = 30, normalized size = 1.07

$$\frac{\left(\sqrt{2}\,c^4x^4+\sqrt{2}\right)(c^4x^4+1)^{\frac{3}{2}}}{40\,c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/sech(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/40*(sqrt(2)*c^4*x^4 + sqrt(2))*(c^4*x^4 + 1)^(3/2)/c^7
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(24) = 48$.
time = 0.38, size = 56, normalized size = 2.00

$$\frac{\sqrt{2}\,(c^{12}x^{12}+3\,c^8x^8+3\,c^4x^4+1)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{40\,c^8x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/sech(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/40*sqrt(2)*(c^12*x^12 + 3*c^8*x^8 + 3*c^4*x^4 + 1)*sqrt(c^2*x^2/(c^4*x^4
+ 1))/(c^8*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/sech(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(x**6/sech(2*log(c*x))**(3/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/sech(2*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 1.45, size = 42, normalized size = 1.50

$$\frac{(c^4\,x^4+1)^3\,\sqrt{\frac{2\,c^2\,x^2}{c^4\,x^4+1}}}{40\,c^8\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(1/cosh(2*log(c*x)))^(3/2),x)
```

```
[Out] ((c^4*x^4 + 1)^3*((2*c^2*x^2)/(c^4*x^4 + 1))^(1/2))/(40*c^8*x)
```

3.172 $\int \frac{x^5}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=251

$$-\frac{4}{15c^4\left(c^4+\frac{1}{x^4}\right)\left(c^2+\frac{1}{x^2}\right)x^4\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{4}{15c^4\left(c^4+\frac{1}{x^4}\right)x^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{2x^2}{15\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(c$$

[Out] -4/15/c^4/(c^4+1/x^4)/(c^2+1/x^2)/x^4/sech(2*ln(c*x))^(3/2)+4/15/c^4/(c^4+1/x^4)/x^2/sech(2*ln(c*x))^(3/2)+2/15*x^2/(c^4+1/x^4)/sech(2*ln(c*x))^(3/2)+1/9*x^6/sech(2*ln(c*x))^(3/2)+4/15*(c^2+1/x^2)*(cos(2*arccot(c*x))^2)^(1/2)/cos(2*arccot(c*x))*EllipticE(sin(2*arccot(c*x)),1/2*2^(1/2))*((c^4+1/x^4)/(c^2+1/x^2)^2)^(1/2)/c^3/(c^4+1/x^4)^2/x^3/sech(2*ln(c*x))^(3/2)-2/15*(c^2+1/x^2)*(cos(2*arccot(c*x))^2)^(1/2)/cos(2*arccot(c*x))*EllipticF(sin(2*arccot(c*x)),1/2*2^(1/2))*((c^4+1/x^4)/(c^2+1/x^2)^2)^(1/2)/c^3/(c^4+1/x^4)^2/x^3/sech(2*ln(c*x))^(3/2)

Rubi [A]
time = 0.10, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {5670, 5668, 342, 283, 331, 311, 226, 1210}

$$\frac{2x^2}{15\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{4}{15c^4x^2\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}-\frac{4}{15c^4x^4\left(c^4+\frac{1}{x^4}\right)\left(c^2+\frac{1}{x^2}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}-\frac{2\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)F\left(2\cot^{-1}(cx)\middle|\frac{1}{2}\right)}{15c^3x^3\left(c^4+\frac{1}{x^4}\right)^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{4\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)E\left(2\cot^{-1}(cx)\middle|\frac{1}{2}\right)}{15c^3x^3\left(c^4+\frac{1}{x^4}\right)^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x^6}{9\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

Antiderivative was successfully verified.

[In] Int[x^5/Sech[2*Log[c*x]]^(3/2),x]

[Out] -4/(15*c^4*(c^4 + x^(-4))*(c^2 + x^(-2))*x^4*Sech[2*Log[c*x]]^(3/2)) + 4/(15*c^4*(c^4 + x^(-4))*x^2*Sech[2*Log[c*x]]^(3/2)) + (2*x^2)/(15*(c^4 + x^(-4))*Sech[2*Log[c*x]]^(3/2)) + x^6/(9*Sech[2*Log[c*x]]^(3/2)) + (4*Sqrt[(c^4 + x^(-4))/(c^2 + x^(-2))^2]*(c^2 + x^(-2))*EllipticE[2*ArcCot[c*x], 1/2])/(15*c^3*(c^4 + x^(-4))^2*x^3*Sech[2*Log[c*x]]^(3/2)) - (2*Sqrt[(c^4 + x^(-4))/(c^2 + x^(-2))^2]*(c^2 + x^(-2))*EllipticF[2*ArcCot[c*x], 1/2])/(15*c^3*(c^4 + x^(-4))^2*x^3*Sech[2*Log[c*x]]^(3/2))

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 5668

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5670

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
```

```
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{x^5}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{\operatorname{sech}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^6}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^4}\right)^{3/2} x^8\,dx, x, cx\right)}{c^9\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$
$$= -\frac{\text{Subst}\left(\int \frac{(1+x^4)^{3/2}}{x^{10}}\,dx, x, \frac{1}{cx}\right)}{c^9\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$
$$= \frac{x^6}{9\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{2\text{Subst}\left(\int \frac{\sqrt{1+x^4}}{x^6}\,dx, x, \frac{1}{cx}\right)}{3c^9\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$
$$= \frac{2x^2}{15\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^6}{9\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{4\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x^4}}\,dx,\right.}{15c^9\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{sech}^{\frac{3}{2}}(2}$$
$$= \frac{4}{15c^4\left(c^4+\frac{1}{x^4}\right)x^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{2x^2}{15\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^6}{9\operatorname{sech}^{\frac{3}{2}}(2\log(}$$
$$= \frac{4}{15c^4\left(c^4+\frac{1}{x^4}\right)x^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{2x^2}{15\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^6}{9\operatorname{sech}^{\frac{3}{2}}(2\log(}$$
$$= -\frac{4}{15c^4\left(c^4+\frac{1}{x^4}\right)\left(c^2+\frac{1}{x^2}\right)x^4\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{4}{15c^4\left(c^4+\frac{1}{x^4}\right)x^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.09, size = 65, normalized size = 0.26

$$\frac{\left(\frac{c^2x^2}{1+c^4x^4}\right)^{3/2}(1+c^4x^4)^{3/2}\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-c^4x^4\right)}{6\sqrt{2}\,c^6}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/Sech[2*Log[c*x]]^(3/2),x]

[Out] (((c^2*x^2)/(1 + c^4*x^4))^(3/2)*(1 + c^4*x^4)^(3/2)*Hypergeometric2F1[-3/2, 3/4, 7/4, -(c^4*x^4)])/(6*Sqrt[2]*c^6)

Maple [C] Result contains complex when optimal does not.
time = 1.23, size = 147, normalized size = 0.59

method	result	size
risch	$\frac{x^{4}\left(5c^{4}x^{4}+11\right)\sqrt{2}}{180c^{2}\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}+1}}}+\frac{i\sqrt{-ic^{2}x^{2}+1}\,\sqrt{ic^{2}x^{2}+1}\,\left(\mathrm{EllipticF}\left(x\sqrt{ic^{2}},i\right)-\mathrm{EllipticE}\left(x\sqrt{ic^{2}},i\right)\right)\sqrt{2}\,x}{15\sqrt{ic^{2}}\,\left(c^{4}x^{4}+1\right)c^{4}\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}+1}}}$	147

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/sech(2*ln(c*x))^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/180*x^4*(5*c^4*x^4+11)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4+1))^(1/2)+1/15*I/(I*c^2)^(1/2)*(1-I*c^2*x^2)^(1/2)*(1+I*c^2*x^2)^(1/2)/(c^4*x^4+1)/c^4*(EllipticF(x*(I*c^2)^(1/2),I)-EllipticE(x*(I*c^2)^(1/2),I))*2^(1/2)*x/(c^2*x^2/(c^4*x^4+1))^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/sech(2*log(c*x))^(3/2),x, algorithm="maxima")

[Out] integrate(x^5/sech(2*log(c*x))^(3/2), x)

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/sech(2*log(c*x))^(3/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly 1 arguments (2 given)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5}}{\operatorname{sech}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/sech(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(x**5/sech(2*log(c*x))**(3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/sech(2*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{\left(\frac{1}{\cosh(2\,\ln(c\,x))}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(1/cosh(2*log(c*x)))^(3/2),x)
```

```
[Out] int(x^5/(1/cosh(2*log(c*x)))^(3/2), x)
```

3.173 $\int \frac{x^4}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=92

$$\frac{3x}{16\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x^5}{8\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{3\tanh^{-1}\left(\sqrt{1+\frac{1}{c^4x^4}}\right)}{16c^8\left(1+\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] 3/16*x/(c^4+1/x^4)/sech(2*ln(c*x))^(3/2)+1/8*x^5/sech(2*ln(c*x))^(3/2)+3/16
*arctanh((1+1/c^4/x^4)^(1/2))/c^8/(1+1/c^4/x^4)^(3/2)/x^3/sech(2*ln(c*x))^(
3/2)
```

Rubi [A]
time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5670, 5668, 272, 43, 65, 213}

$$\frac{3x}{16\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{3\tanh^{-1}\left(\sqrt{\frac{1}{c^4x^4}+1}\right)}{16c^8x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x^5}{8\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] (3*x)/(16*(c^4 + x^(-4))*Sech[2*Log[c*x]]^(3/2)) + x^5/(8*Sech[2*Log[c*x]]^
(3/2)) + (3*ArcTanh[Sqrt[1 + 1/(c^4*x^4)]])/(16*c^8*(1 + 1/(c^4*x^4))^(3/2)
*x^3*Sech[2*Log[c*x]]^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5668

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5670

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{x^4}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^4}{\operatorname{sech}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^5}$$

$$= \frac{\operatorname{Subst}\left(\int \left(1+\frac{1}{x^4}\right)^{3/2} x^7\,dx, x, cx\right)}{c^8\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{(1+x)^{3/2}}{x^3}\,dx, x, \frac{1}{c^4x^4}\right)}{4c^8\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x^5}{8\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{3\operatorname{Subst}\left(\int \frac{\sqrt{1+x}}{x^2}\,dx, x, \frac{1}{c^4x^4}\right)}{16c^8\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{3x}{16\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^5}{8\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{3\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, \frac{}{c}\right.}{32c^8\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\operatorname{lo}}$$

$$= \frac{3x}{16\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^5}{8\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{3\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+}\right.}{16c^8\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\operatorname{lo}}$$

$$= \frac{3x}{16\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^5}{8\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{3\tanh^{-1}\left(\sqrt{1+\frac{1}{c^4x^4}}\right)}{16c^8\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\operatorname{lo}}$$

<u>**Mathematica**</u> [**A**]
time = 0.12, size = 90, normalized size = 0.98

$$\frac{c^3x^3\sqrt{1+c^4x^4}\,(5+2c^4x^4)+3cx\sinh^{-1}(c^2x^2)}{32\sqrt{2}\,c^5\sqrt{\frac{c^2x^2}{1+c^4x^4}}\,\sqrt{1+c^4x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] (c^3*x^3*Sqrt[1 + c^4*x^4]*(5 + 2*c^4*x^4) + 3*c*x*ArcSinh[c^2*x^2])/(32*Sq
rt[2]*c^5*Sqrt[(c^2*x^2)/(1 + c^4*x^4)]*Sqrt[1 + c^4*x^4])
```

<u>**Maple**</u> [**A**]
time = 1.43, size = 113, normalized size = 1.23

method	result	size
risch	$\frac{x^{3}(2c^{4}x^{4}+5)\sqrt{2}}{64c^{2}\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}+1}}}+\frac{3\ln\left(\frac{c^{4}x^{2}}{\sqrt{c^{4}}}+\sqrt{c^{4}x^{4}+1}\right)\sqrt{2}\,x}{64\sqrt{c^{4}}\,c^{2}\sqrt{c^{4}x^{4}+1}\,\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}+1}}}$	113

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/sech(2*ln(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/64*x^3*(2*c^4*x^4+5)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4+1))^(1/2)+3/64*ln(c^4*
x^2/(c^4)^(1/2)+(c^4*x^4+1)^(1/2))/(c^4)^(1/2)*2^(1/2)/c^2*x/(c^4*x^4+1)^(1
/2)/(c^2*x^2/(c^4*x^4+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/sech(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sech(2*log(c*x))^(3/2), x)
```

Fricas [A]
time = 0.39, size = 101, normalized size = 1.10

$$\frac{2\sqrt{2}\left(2c^{9}x^{9}+7c^{5}x^{5}+5cx\right)\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}+1}}+3\sqrt{2}\log\left(-2c^{4}x^{4}-2\left(c^{5}x^{5}+cx\right)\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}+1}}-1\right)}{128c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/sech(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/128*(2*sqrt(2)*(2*c^9*x^9 + 7*c^5*x^5 + 5*c*x)*sqrt(c^2*x^2/(c^4*x^4 + 1)
) + 3*sqrt(2)*log(-2*c^4*x^4 - 2*(c^5*x^5 + c*x)*sqrt(c^2*x^2/(c^4*x^4 + 1)
) - 1))/c^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{4}}{\operatorname{sech}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/sech(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(x**4/sech(2*log(c*x))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/sech(2*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sech(2*log(c*x))^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{{\left(\frac{1}{\cosh\left(2\,\ln\left(c\,x\right)\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(1/cosh(2*log(c*x)))^(3/2),x)
```

```
[Out] int(x^4/(1/cosh(2*log(c*x)))^(3/2), x)
```

3.174 $\int \frac{x^3}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=111

$$\frac{2}{7\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x^4}{7\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}-\frac{2\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)F\left(2\cot^{-1}(cx)|\frac{1}{2}\right)}{7c\left(c^4+\frac{1}{x^4}\right)^2x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] 2/7/(c^4+1/x^4)/sech(2*ln(c*x))^(3/2)+1/7*x^4/sech(2*ln(c*x))^(3/2)-2/7*(c^
2+1/x^2)*(cos(2*arccot(c*x))^2)^(1/2)/cos(2*arccot(c*x))*EllipticF(sin(2*ar
ccot(c*x)),1/2*2^(1/2))*((c^4+1/x^4)/(c^2+1/x^2)^2)^(1/2)/c/(c^4+1/x^4)^2/x
^3/sech(2*ln(c*x))^(3/2)
```

Rubi [A]
time = 0.06, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5670, 5668, 342, 283, 226}

$$\frac{2}{7\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}-\frac{2\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)F\left(2\cot^{-1}(cx)|\frac{1}{2}\right)}{7cx^3\left(c^4+\frac{1}{x^4}\right)^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x^4}{7\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] 2/(7*(c^4 + x^(-4))*Sech[2*Log[c*x]]^(3/2)) + x^4/(7*Sech[2*Log[c*x]]^(3/2)
) - (2*Sqrt[(c^4 + x^(-4))/(c^2 + x^(-2))^2]*(c^2 + x^(-2))*EllipticF[2*Arc
Cot[c*x], 1/2])/(7*c*(c^4 + x^(-4))^2*x^3*Sech[2*Log[c*x]]^(3/2))
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5668

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5670

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{x^3}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^3}{\operatorname{sech}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^4}$$

$$= \frac{\operatorname{Subst}\left(\int \left(1+\frac{1}{x^4}\right)^{3/2} x^6\,dx, x, cx\right)}{c^7\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{(1+x^4)^{3/2}}{x^8}\,dx, x, \frac{1}{cx}\right)}{c^7\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x^4}{7\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{6\operatorname{Subst}\left(\int \frac{\sqrt{1+x^4}}{x^4}\,dx, x, \frac{1}{cx}\right)}{7c^7\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{2}{7\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^4}{7\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{4\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right.}{7c^7\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(}$$

$$= \frac{2}{7\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^4}{7\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{2\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)F\left(2\,\mathrm{c}\right.}{7c\left(c^4+\frac{1}{x^4}\right)^2 x^3\operatorname{sech}^{\frac{3}{2}}(2\log}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.08, size = 61, normalized size = 0.55

$$\frac{\sqrt{1+c^4x^4}\sqrt{\frac{c^2x^2}{2+2c^4x^4}}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-c^4x^4\right)}{2c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] (Sqrt[1 + c^4*x^4]*Sqrt[(c^2*x^2)/(2 + 2*c^4*x^4)]*Hypergeometric2F1[-3/2,
1/4, 5/4, -(c^4*x^4)])/(2*c^4)
```

Maple [C] Result contains complex when optimal does not.
time = 1.23, size = 129, normalized size = 1.16

method	result	size
risch	$\frac{x^2(c^4x^4+3)\sqrt{2}}{28c^2\sqrt{\frac{c^2x^2}{c^4x^4+1}}}+\frac{\sqrt{-ic^2x^2+1}\,\sqrt{ic^2x^2+1}\,\mathrm{EllipticF}\left(x\sqrt{ic^2},i\right)\sqrt{2}\,x}{7\sqrt{ic^2}\,(c^4x^4+1)c^2\sqrt{\frac{c^2x^2}{c^4x^4+1}}}$	129

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/sech(2*ln(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/28*x^2*(c^4*x^4+3)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4+1))^(1/2)+1/7/(I*c^2)^(1
/2)*(1-I*c^2*x^2)^(1/2)*(1+I*c^2*x^2)^(1/2)/(c^4*x^4+1)*EllipticF(x*(I*c^2)
^(1/2),I)*2^(1/2)/c^2*x/(c^2*x^2/(c^4*x^4+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/sech(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sech(2*log(c*x))^(3/2), x)
```

Fricas [A]
time = 0.11, size = 78, normalized size = 0.70

$$\frac{4\sqrt{2}\sqrt{c^4}\,c\left(-\frac{1}{c^4}\right)^{\frac{3}{4}}\mathrm{ellipticF}\left(\frac{\left(-\frac{1}{c^4}\right)^{\frac{1}{4}}}{x},-1\right)+\sqrt{2}\left(c^8x^8+4\,c^4x^4+3\right)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{28\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/sech(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/28*(4*sqrt(2)*sqrt(c^4)*c*(-1/c^4)^(3/4)*ellipticF((-1/c^4)^(1/4)/x, -1)
+ sqrt(2)*(c^8*x^8 + 4*c^4*x^4 + 3)*sqrt(c^2*x^2/(c^4*x^4 + 1)))/c^4
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/sech(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(x**3/sech(2*log(c*x))**(3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/sech(2*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{{\left(\frac{1}{\cosh(2\,\ln(c\,x))}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(1/cosh(2*log(c*x)))^(3/2),x)
```

```
[Out] int(x^3/(1/cosh(2*log(c*x)))^(3/2), x)
```

3.175 $\int \frac{x^2}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=88

$$\frac{1}{2\left(c^4+\frac{1}{x^4}\right)x\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x^3}{6\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}-\frac{\operatorname{csch}^{-1}(c^2x^2)}{2c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] 1/2/(c^4+1/x^4)/x/sech(2*ln(c*x))^(3/2)+1/6*x^3/sech(2*ln(c*x))^(3/2)-1/2*a
rccsch(c^2*x^2)/c^6/(1+1/c^4/x^4)^(3/2)/x^3/sech(2*ln(c*x))^(3/2)
```

Rubi [A]
time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5670, 5668, 342, 281, 283, 221}

$$\frac{1}{2x\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}-\frac{\operatorname{csch}^{-1}(c^2x^2)}{2c^6x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x^3}{6\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] 1/(2*(c^4 + x^(-4))*x*Sech[2*Log[c*x]]^(3/2)) + x^3/(6*Sech[2*Log[c*x]]^(3/
2)) - ArcCsch[c^2*x^2]/(2*c^6*(1 + 1/(c^4*x^4))^(3/2)*x^3*Sech[2*Log[c*x]]^
(3/2))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5668

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5670

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{x^2}{\operatorname{sech}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^3} \\
&= \frac{\operatorname{Subst}\left(\int \left(1+\frac{1}{x^4}\right)^{3/2} x^5\,dx, x, cx\right)}{c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{sech}^{\frac{3}{2}}(2\log(cx))} \\
&= -\frac{\operatorname{Subst}\left(\int \frac{(1+x^4)^{3/2}}{x^7}\,dx, x, \frac{1}{cx}\right)}{c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{sech}^{\frac{3}{2}}(2\log(cx))} \\
&= -\frac{\operatorname{Subst}\left(\int \frac{(1+x^2)^{3/2}}{x^4}\,dx, x, \frac{1}{c^2x^2}\right)}{2c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{sech}^{\frac{3}{2}}(2\log(cx))} \\
&= \frac{x^3}{6\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{\operatorname{Subst}\left(\int \frac{\sqrt{1+x^2}}{x^2}\,dx, x, \frac{1}{c^2x^2}\right)}{2c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{sech}^{\frac{3}{2}}(2\log(cx))} \\
&= \frac{1}{2\left(c^4+\frac{1}{x^4}\right) x \operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^3}{6\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \frac{1}{c^2x^2}\right)}{2c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{sech}^{\frac{3}{2}}(2\log(cx))} \\
&= \frac{1}{2\left(c^4+\frac{1}{x^4}\right) x \operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^3}{6\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{\operatorname{csch}^{-1}(c^2x^2)}{2c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{sech}^{\frac{3}{2}}(2\log(cx))}
\end{aligned}
$$

Mathematica [A]
time = 0.12, size = 88, normalized size = 1.00

$$\frac{x\left(\sqrt{1+c^4x^4}\,(4+c^4x^4)-3\tanh^{-1}\left(\sqrt{1+c^4x^4}\right)\right)}{12\sqrt{2}\,c^2\sqrt{\frac{c^2x^2}{1+c^4x^4}}\,\sqrt{1+c^4x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] (x*(Sqrt[1 + c^4*x^4]*(4 + c^4*x^4) - 3*ArcTanh[Sqrt[1 + c^4*x^4]]))/(12*Sq
rt[2]*c^2*Sqrt[(c^2*x^2)/(1 + c^4*x^4)]*Sqrt[1 + c^4*x^4])
```

Maple [F]
time = 0.88, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{sech}\left(2\ln\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/sech(2*ln(c*x))^(3/2),x)
```

```
[Out] int(x^2/sech(2*ln(c*x))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/sech(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sech(2*log(c*x))^(3/2), x)
```

Fricas [A]
time = 0.44, size = 109, normalized size = 1.24

$$\frac{3\sqrt{2}\,cx\log\left(\frac{c^5x^5+2\,cx-2\,(c^4x^4+1)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{cx^5}\right)+2\sqrt{2}\,(c^8x^8+5\,c^4x^4+4)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{48\,c^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/sech(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/48*(3*sqrt(2)*c*x*log((c^5*x^5 + 2*c*x - 2*(c^4*x^4 + 1)*sqrt(c^2*x^2/(c^
4*x^4 + 1)))/(c*x^5)) + 2*sqrt(2)*(c^8*x^8 + 5*c^4*x^4 + 4)*sqrt(c^2*x^2/(c
^4*x^4 + 1)))/(c^4*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/sech(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(x**2/sech(2*log(c*x))**(3/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/sech(2*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(\frac{1}{\cosh(2\ln(c\,x))}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1/cosh(2*log(c*x)))^(3/2),x)
```

```
[Out] int(x^2/(1/cosh(2*log(c*x)))^(3/2), x)
```

3.176 $\int \frac{x}{\text{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=214

$$-\frac{12}{5\left(c^4+\frac{1}{x^4}\right)\left(c^2+\frac{1}{x^2}\right)x^4\text{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{6}{5\left(c^4+\frac{1}{x^4}\right)x^2\text{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x^2}{5\text{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{12c\sqrt{\frac{c^4+\cdots}{(c^2+\cdots}}}{5\left(c^4\cdots\right.}$$

```
[Out] -12/5/(c^4+1/x^4)/(c^2+1/x^2)/x^4/sech(2*ln(c*x))^(3/2)+6/5/(c^4+1/x^4)/x^2
/sech(2*ln(c*x))^(3/2)+1/5*x^2/sech(2*ln(c*x))^(3/2)+12/5*c*(c^2+1/x^2)*(co
s(2*arccot(c*x))^2)^(1/2)/cos(2*arccot(c*x))*EllipticE(sin(2*arccot(c*x)),1
/2*2^(1/2))*((c^4+1/x^4)/(c^2+1/x^2)^2)^(1/2)/(c^4+1/x^4)^2/x^3/sech(2*ln(c
*x))^(3/2)-6/5*c*(c^2+1/x^2)*(cos(2*arccot(c*x))^2)^(1/2)/cos(2*arccot(c*x)
)*EllipticF(sin(2*arccot(c*x)),1/2*2^(1/2))*((c^4+1/x^4)/(c^2+1/x^2)^2)^(1/
2)/(c^4+1/x^4)^2/x^3/sech(2*ln(c*x))^(3/2)
```

Rubi [A]
time = 0.09, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {5670, 5668, 342, 283, 311, 226, 1210}

$$\frac{6}{5x^2\left(c^4+\frac{1}{x^4}\right)\text{sech}^{\frac{3}{2}}(2\log(cx))}-\frac{12}{5x^4\left(c^4+\frac{1}{x^4}\right)\left(c^2+\frac{1}{x^2}\right)\text{sech}^{\frac{3}{2}}(2\log(cx))}-\frac{6c\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)F\left(2\cot^{-1}(cx)|\frac{1}{2}\right)}{5x^3\left(c^4+\frac{1}{x^4}\right)^2\text{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{12c\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)E\left(2\cot^{-1}(cx)|\frac{1}{2}\right)}{5x^3\left(c^4+\frac{1}{x^4}\right)^2\text{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x^2}{5\text{sech}^{\frac{3}{2}}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] -12/(5*(c^4 + x^(-4))*(c^2 + x^(-2))*x^4*Sech[2*Log[c*x]]^(3/2)) + 6/(5*(c^
4 + x^(-4))*x^2*Sech[2*Log[c*x]]^(3/2)) + x^2/(5*Sech[2*Log[c*x]]^(3/2)) +
(12*c*Sqrt[(c^4 + x^(-4))/(c^2 + x^(-2))^2]*(c^2 + x^(-2))*EllipticE[2*ArcC
ot[c*x], 1/2])/(5*(c^4 + x^(-4))^2*x^3*Sech[2*Log[c*x]]^(3/2)) - (6*c*Sqrt[
(c^4 + x^(-4))/(c^2 + x^(-2))^2]*(c^2 + x^(-2))*EllipticF[2*ArcCot[c*x], 1/
2])/(5*(c^4 + x^(-4))^2*x^3*Sech[2*Log[c*x]]^(3/2))
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
```

```
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 5668

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5670

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{x}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x}{\operatorname{sech}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^2}$$

$$= \frac{\operatorname{Subst}\left(\int \left(1+\frac{1}{x^4}\right)^{3/2} x^4\,dx, x, cx\right)}{c^5\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{(1+x^4)^{3/2}}{x^6}\,dx, x, \frac{1}{cx}\right)}{c^5\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x^2}{5\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{6\operatorname{Subst}\left(\int \frac{\sqrt{1+x^4}}{x^2}\,dx, x, \frac{1}{cx}\right)}{5c^5\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{6}{5\left(c^4+\frac{1}{x^4}\right)x^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^2}{5\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{12\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{1+x^4}}\,dx,\right.}{5c^5\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2}$$

$$= \frac{6}{5\left(c^4+\frac{1}{x^4}\right)x^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^2}{5\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{12\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx,\right.}{5c^5\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2}$$

$$= -\frac{12}{5\left(c^4+\frac{1}{x^4}\right)\left(c^2+\frac{1}{x^2}\right)x^4\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{6}{5\left(c^4+\frac{1}{x^4}\right)x^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{}{5\mathrm{se}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.09, size = 65, normalized size = 0.30

$$-\frac{{}_2F_1\left(-\frac{3}{2},-\frac{1}{4};\frac{3}{4};-c^4x^4\right)}{2\sqrt{2}\,c^2\sqrt{\frac{c^2x^2}{1+c^4x^4}}\,\sqrt{1+c^4x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] -1/2*Hypergeometric2F1[-3/2, -1/4, 3/4, -(c^4*x^4)]/(Sqrt[2]*c^2*Sqrt[(c^2*
x^2)/(1 + c^4*x^4)]*Sqrt[1 + c^4*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 1.54, size = 159, normalized size = 0.74

method	result	s
risch	$\frac{\left(c^{8} x^{8}-4 c^{4} x^{4}-5\right) \sqrt{2}}{20\left(c^{4} x^{4}+1\right) c^{2} \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}+1}}}+\frac{3 i \sqrt{-i c^{2} x^{2}+1}\, \sqrt{i c^{2} x^{2}+1}\, \left(\operatorname{EllipticF}\left(x \sqrt{i c^{2}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{i c^{2}}, i\right)\right) \sqrt{2}\, x}{5 \sqrt{i c^{2}}\, \left(c^{4} x^{4}+1\right) \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}+1}}}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/sech(2*ln(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/20*(c^8*x^8-4*c^4*x^4-5)/(c^4*x^4+1)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4+1))^(1
/2)+3/5*I/(I*c^2)^(1/2)*(1-I*c^2*x^2)^(1/2)*(1+I*c^2*x^2)^(1/2)/(c^4*x^4+1)
*(EllipticF(x*(I*c^2)^(1/2),I)-EllipticE(x*(I*c^2)^(1/2),I))*2^(1/2)*x/(c^2
*x^2/(c^4*x^4+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sech(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sech(2*log(c*x))^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sech(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(x/sech(2*log(c*x))^(3/2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{sech}^{\frac{3}{2}}\left(2 \log\left(cx\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sech(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(x/sech(2*log(c*x))**(3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sech(2*log(c*x))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\left(\frac{1}{\cosh(2\,\ln(c\,x))}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1/cosh(2*log(c*x)))^(3/2),x)
[Out] int(x/(1/cosh(2*log(c*x)))^(3/2), x)
```

3.177 $\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=92

$$-\frac{3}{4\left(c^4+\frac{1}{x^4}\right)x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x}{4\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{3\tanh^{-1}\left(\sqrt{1+\frac{1}{c^4x^4}}\right)}{4c^4\left(1+\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] -3/4/(c^4+1/x^4)/x^3/sech(2*ln(c*x))^(3/2)+1/4*x/sech(2*ln(c*x))^(3/2)+3/4*
arctanh((1+1/c^4/x^4)^(1/2))/c^4/(1+1/c^4/x^4)^(3/2)/x^3/sech(2*ln(c*x))^(3
/2)
```

Rubi [A]
time = 0.03, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {5664, 5662, 272, 43, 52, 65, 213}

$$-\frac{3}{4x^3\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{3\tanh^{-1}\left(\sqrt{\frac{1}{c^4x^4}+1}\right)}{4c^4x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x}{4\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[2*Log[c*x]]^(-3/2),x]
```

```
[Out] -3/(4*(c^4 + x^(-4))*x^3*Sech[2*Log[c*x]]^(3/2)) + x/(4*Sech[2*Log[c*x]]^(3
/2)) + (3*ArcTanh[Sqrt[1 + 1/(c^4*x^4)]])/(4*c^4*(1 + 1/(c^4*x^4))^(3/2)*x^
3*Sech[2*Log[c*x]]^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5662

```
Int[Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Dist[Sech[d*(a
+ b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*d*p)), Int[1/(x^(b*
d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; FreeQ[{a, b, d, p}, x] &&
 !IntegerQ[p]
```

Rule 5664

```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c}$$

$$= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^4}\right)^{3/2} x^3\,dx, x, cx\right)}{c^4\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{\text{Subst}\left(\int \frac{(1+x)^{3/2}}{x^2}\,dx, x, \frac{1}{c^4x^4}\right)}{4c^4\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x}{4\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{3\text{Subst}\left(\int \frac{\sqrt{1+x}}{x}\,dx, x, \frac{1}{c^4x^4}\right)}{8c^4\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{3}{4\left(c^4+\frac{1}{x^4}\right) x^3 \operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x}{4\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{3\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x\right.}{8c^4\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{sech}^{\frac{3}{2}}(2}$$

$$= -\frac{3}{4\left(c^4+\frac{1}{x^4}\right) x^3 \operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x}{4\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{3\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1\right.}{4c^4\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{sech}^{\frac{3}{2}}(}$$

$$= -\frac{3}{4\left(c^4+\frac{1}{x^4}\right) x^3 \operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x}{4\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{3\tanh^{-1}\left(\sqrt{1+\frac{1}{c^4x^4}}\right.}{4c^4\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{sech}^{\frac{3}{2}}(2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.07, size = 64, normalized size = 0.70

$$-\frac{\sqrt{1+c^4x^4}\sqrt{\frac{c^2x^2}{2+2c^4x^4}}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-c^4x^4\right)}{4c^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[2*Log[c*x]]^(-3/2),x]
```

```
[Out] -1/4*(Sqrt[1 + c^4*x^4]*Sqrt[(c^2*x^2)/(2 + 2*c^4*x^4)]*Hypergeometric2F1[-
3/2, -1/2, 1/2, -(c^4*x^4)])/(c^4*x^3)
```

Maple [A]
time = 1.41, size = 131, normalized size = 1.42

method	result	size
risch	$\frac{(c^{8}x^{8}-c^{4}x^{4}-2)\sqrt{2}}{16x(c^{4}x^{4}+1)c^{2}\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}+1}}}+\frac{3c^{2}\ln\left(\frac{c^{4}x^{2}}{\sqrt{c^{4}}}+\sqrt{c^{4}x^{4}+1}\right)\sqrt{2}\,x}{16\sqrt{c^{4}}\,\sqrt{c^{4}x^{4}+1}\,\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}+1}}}$	131

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sech(2*ln(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*(c^8*x^8-c^4*x^4-2)/x/(c^4*x^4+1)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4+1))^(1
/2)+3/16*c^2*ln(c^4*x^2/(c^4)^(1/2)+(c^4*x^4+1)^(1/2))/(c^4)^(1/2)*2^(1/2)*
x/(c^4*x^4+1)^(1/2)/(c^2*x^2/(c^4*x^4+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(2*log(c*x))^(-3/2), x)
```

Fricas [A]
time = 0.41, size = 106, normalized size = 1.15

$$\frac{3\sqrt{2}\,c^{3}x^{3}\log\left(-2\,c^{4}x^{4}-2\,(c^{5}x^{5}+cx)\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}+1}}-1\right)+2\sqrt{2}\,(c^{8}x^{8}-c^{4}x^{4}-2)\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}+1}}}{32\,c^{4}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/32*(3*sqrt(2)*c^3*x^3*log(-2*c^4*x^4 - 2*(c^5*x^5 + c*x)*sqrt(c^2*x^2/(c^
4*x^4 + 1)) - 1) + 2*sqrt(2)*(c^8*x^8 - c^4*x^4 - 2)*sqrt(c^2*x^2/(c^4*x^4
+ 1)))/(c^4*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\operatorname{sech}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(sech(2*log(c*x))**(-3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(2*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(\frac{1}{\cosh\left(2\,\ln\left(c\,x\right)\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cosh(2*log(c*x)))^(3/2),x)
```

```
[Out] int(1/(1/cosh(2*log(c*x)))^(3/2), x)
```

3.178 $\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x}\,dx$

Optimal. Leaf size=56

$$i\sqrt{\cosh(2\log(cx))}\,E(i\log(cx)|2)\sqrt{\operatorname{sech}(2\log(cx))}+\sqrt{\operatorname{sech}(2\log(cx))}\sinh(2\log(cx))$$

```
[Out] sinh(2*ln(c*x))*sech(2*ln(c*x))^(1/2)+I*((1/2*c*x+1/2/c/x)^2)^(1/2)/(1/2*c*
x+1/2/c/x)*EllipticE(I*(1/2*c*x-1/2/c/x),2^(1/2))*cosh(2*ln(c*x))^(1/2)*sec
h(2*ln(c*x))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3853, 3856, 2719}

$$\sinh(2\log(cx))\sqrt{\operatorname{sech}(2\log(cx))}+i\sqrt{\operatorname{sech}(2\log(cx))}\sqrt{\cosh(2\log(cx))}\,E(i\log(cx)|2)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[2*Log[c*x]]^(3/2)/x,x]
```

```
[Out] I*Sqrt[Cosh[2*Log[c*x]]]*EllipticE[I*Log[c*x], 2]*Sqrt[Sech[2*Log[c*x]]] +
Sqrt[Sech[2*Log[c*x]]]*Sinh[2*Log[c*x]]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x}\,dx = \operatorname{Subst}\left(\int \operatorname{sech}^{\frac{3}{2}}(2x)\,dx, x, \log(cx)\right)$$
$$= \sqrt{\operatorname{sech}(2\log(cx))}\sinh(2\log(cx)) - \operatorname{Subst}\left(\int \frac{1}{\sqrt{\operatorname{sech}(2x)}}\,dx, x, \log(cx)\right)$$
$$= \sqrt{\operatorname{sech}(2\log(cx))}\sinh(2\log(cx)) - \left(\sqrt{\cosh(2\log(cx))}\sqrt{\operatorname{sech}(2\log(cx))}\right)\operatorname{Subst}($$
$$= i\sqrt{\cosh(2\log(cx))}E(i\log(cx)|2)\sqrt{\operatorname{sech}(2\log(cx))} + \sqrt{\operatorname{sech}(2\log(cx))}\sinh(2\log$$

Mathematica [A]
time = 0.09, size = 45, normalized size = 0.80

$$\frac{\frac{iE(i\log(cx)|2)}{\sqrt{\cosh(2\log(cx))}} + \tanh(2\log(cx))}{\sqrt{\operatorname{sech}(2\log(cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[2*Log[c*x]]^(3/2)/x,x]
```

```
[Out] ((I*EllipticE[I*Log[c*x], 2])/Sqrt[Cosh[2*Log[c*x]]] + Tanh[2*Log[c*x]])/Sq
rt[Sech[2*Log[c*x]]]
```

Maple [A]
time = 2.62, size = 127, normalized size = 2.27

method	result	size
derivativedivides	$\frac{2\left(\frac{cx}{2}+\frac{1}{2cx}\right)\left(\frac{cx}{2}-\frac{1}{2cx}\right)^{2}+\sqrt{-2\left(\frac{cx}{2}-\frac{1}{2cx}\right)^{2}-1}\ \operatorname{EllipticE}\left(\frac{cx}{2}+\frac{1}{2cx},\sqrt{2}\right)\sqrt{-\left(\frac{cx}{2}-\frac{1}{2cx}\right)^{2}}}{\left(\frac{cx}{2}-\frac{1}{2cx}\right)\sqrt{2\left(\frac{cx}{2}+\frac{1}{2cx}\right)^{2}-1}}$	127
default	$\frac{2\left(\frac{cx}{2}+\frac{1}{2cx}\right)\left(\frac{cx}{2}-\frac{1}{2cx}\right)^{2}+\sqrt{-2\left(\frac{cx}{2}-\frac{1}{2cx}\right)^{2}-1}\ \operatorname{EllipticE}\left(\frac{cx}{2}+\frac{1}{2cx},\sqrt{2}\right)\sqrt{-\left(\frac{cx}{2}-\frac{1}{2cx}\right)^{2}}}{\left(\frac{cx}{2}-\frac{1}{2cx}\right)\sqrt{2\left(\frac{cx}{2}+\frac{1}{2cx}\right)^{2}-1}}$	127

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(2*ln(c*x))^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] (2*(1/2*c*x+1/2/c/x)*(1/2*c*x-1/2/c/x)^2+(-2*(1/2*c*x-1/2/c/x)^2-1)^(1/2)*E
llipticE(1/2*c*x+1/2/c/x,2^(1/2))*(-(1/2*c*x-1/2/c/x)^2)^(1/2))/(1/2*c*x-1/
2/c/x)/(2*(1/2*c*x+1/2/c/x)^2-1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(3/2)/x,x, algorithm="maxima")
[Out] integrate(sech(2*log(c*x))^(3/2)/x, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(3/2)/x,x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*ln(c*x))**(3/2)/x,x)
[Out] Integral(sech(2*log(c*x))**(3/2)/x, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(3/2)/x,x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(\frac{1}{\cosh\left(2\,\ln\left(c\,x\right)\right)}\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(2*log(c*x)))^(3/2)/x,x)
[Out] int((1/cosh(2*log(c*x)))^(3/2)/x, x)
```

3.179 $\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx$

Optimal. Leaf size=25

$$\frac{1}{2}\left(c^4+\frac{1}{x^4}\right)x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))$$

```
[Out] 1/2*(c^4+1/x^4)*x^3*sech(2*ln(c*x))^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5670, 5668, 267}

$$\frac{1}{2}x^3\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))$$

Antiderivative was successfully verified.

```
[In] Int[Sech[2*Log[c*x]]^(3/2)/x^2,x]
```

```
[Out] ((c^4 + x^(-4))*x^3*Sech[2*Log[c*x]]^(3/2))/2
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5668

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5670

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx = c\mathrm{Subst}\left(\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(x))}{x^2}\,dx, x, cx\right)$$

$$= \left(c^4\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))\right)\mathrm{Subst}\left(\int \frac{1}{\left(1+\frac{1}{x^4}\right)^{3/2}x^5}\,dx, x, cx\right)$$

$$= \frac{1}{2}\left(c^4+\frac{1}{x^4}\right)x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))$$

Mathematica [A]
time = 0.02, size = 32, normalized size = 1.28

$$\sqrt{2}\,c^2x\sqrt{\frac{c^2x^2}{1+c^4x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[2*Log[c*x]]^(3/2)/x^2,x]
```

```
[Out] Sqrt[2]*c^2*x*Sqrt[(c^2*x^2)/(1 + c^4*x^4)]
```

Maple [F]
time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(2\ln(cx))^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(2*ln(c*x))^(3/2)/x^2,x)
```

```
[Out] int(sech(2*ln(c*x))^(3/2)/x^2,x)
```

Maxima [A]
time = 0.47, size = 39, normalized size = 1.56

$$c\left(\frac{\sqrt{2}}{\left(\frac{1}{c^4x^4}+1\right)^{\frac{3}{2}}}+\frac{\sqrt{2}}{c^4x^4\left(\frac{1}{c^4x^4}+1\right)^{\frac{3}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] c*(sqrt(2)/(1/(c^4*x^4) + 1)^(3/2) + sqrt(2)/(c^4*x^4*(1/(c^4*x^4) + 1)^(3/
2)))
```

Fricas [A]
time = 0.36, size = 28, normalized size = 1.12

$$\sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4+1}}\,c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] sqrt(2)*sqrt(c^2*x^2/(c^4*x^4 + 1))*c^2*x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*ln(c*x))**(3/2)/x**2,x)
```

```
[Out] Integral(sech(2*log(c*x))**(3/2)/x**2, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 1.33, size = 28, normalized size = 1.12

$$c^2\,x\,\sqrt{\frac{2\,c^2\,x^2}{c^4\,x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(2*log(c*x)))^(3/2)/x^2,x)
```

```
[Out] c^2*x*((2*c^2*x^2)/(c^4*x^4 + 1))^(1/2)
```

3.180 $\int \frac{\mathrm{sech}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx$

Optimal. Leaf size=92

$$\frac{1}{2}\left(c^4+\frac{1}{x^4}\right)x^2\mathrm{sech}^{\frac{3}{2}}(2\log(cx))-\frac{\left(c^4+\frac{1}{x^4}\right)\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)x^3F\left(2\cot^{-1}(cx)|\frac{1}{2}\right)\mathrm{sech}^{\frac{3}{2}}(2\log(cx))}{4c}$$

```
[Out] 1/2*(c^4+1/x^4)*x^2*sech(2*ln(c*x))^(3/2)-1/4*(c^4+1/x^4)*(c^2+1/x^2)*x^3*(
cos(2*arccot(c*x))^2)^(1/2)/cos(2*arccot(c*x))*EllipticF(sin(2*arccot(c*x))
,1/2*2^(1/2))*sech(2*ln(c*x))^(3/2)*((c^4+1/x^4)/(c^2+1/x^2)^2)^(1/2)/c
```

Rubi [A]
time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5670, 5668, 342, 294, 226}

$$\frac{1}{2}x^2\left(c^4+\frac{1}{x^4}\right)\mathrm{sech}^{\frac{3}{2}}(2\log(cx))-\frac{x^3\left(c^4+\frac{1}{x^4}\right)\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\mathrm{sech}^{\frac{3}{2}}(2\log(cx))F\left(2\cot^{-1}(cx)|\frac{1}{2}\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[2*Log[c*x]]^(3/2)/x^3,x]
```

```
[Out] ((c^4 + x^(-4))*x^2*Sech[2*Log[c*x]]^(3/2))/2 - ((c^4 + x^(-4))*Sqrt[(c^4 +
 x^(-4))/(c^2 + x^(-2))^2]*(c^2 + x^(-2))*x^3*EllipticF[2*ArcCot[c*x], 1/2]
*Sech[2*Log[c*x]]^(3/2))/(4*c)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5668

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5670

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx &= c^2\text{Subst}\left(\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(x))}{x^3}\,dx,x,cx\right)\\
&= \left(c^5\left(1+\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{1}{x^4}\right)^{3/2}x^6}\,dx,x,cx\right)\\
&= -\left(\left(c^5\left(1+\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int \frac{x^4}{(1+x^4)^{3/2}}\,dx,x,\frac{1}{cx}\right)\right)\\
&= \frac{1}{2}\left(c^4+\frac{1}{x^4}\right)x^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))-\frac{1}{2}\left(c^5\left(1+\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\Big(\\
&= \frac{1}{2}\left(c^4+\frac{1}{x^4}\right)x^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))-\frac{\left(c^4+\frac{1}{x^4}\right)\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)x^3F\left(2\cot^{-1}(c\right.}{4c}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.08, size = 65, normalized size = 0.71

$$\sqrt{2}\,c^2\sqrt{\frac{c^2x^2}{1+c^4x^4}}\left(1+\sqrt{1+c^4x^4}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-c^4x^4\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[2*Log[c*x]]^(3/2)/x^3,x]
```

```
[Out] Sqrt[2]*c^2*Sqrt[(c^2*x^2)/(1 + c^4*x^4)]*(1 + Sqrt[1 + c^4*x^4]*Hypergeome
tric2F1[1/4, 1/2, 5/4, -(c^4*x^4)])
```

Maple [**F**]
time = 0.73, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{sech}(2\ln(cx))^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(2*ln(c*x))^(3/2)/x^3,x)
```

```
[Out] int(sech(2*ln(c*x))^(3/2)/x^3,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sech(2*log(c*x))^(3/2)/x^3, x)
```

Fricas [**A**]
time = 0.11, size = 55, normalized size = 0.60

$$\frac{\sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4+1}}\,c^3 - \sqrt{2}\,(-c^4)^{\frac{3}{4}}\,\mathrm{ellipticF}\left((-c^4)^{\frac{1}{4}}\,x, -1\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] (sqrt(2)*sqrt(c^2*x^2/(c^4*x^4 + 1))*c^3 - sqrt(2)*(-c^4)^(3/4)*ellipticF((
-c^4)^(1/4)*x, -1))/c
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{sech}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*ln(c*x))**(3/2)/x**3,x)
```

```
[Out] Integral(sech(2*log(c*x))**(3/2)/x**3, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cosh(2\,\ln(c\,x))}\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(2*log(c*x)))^(3/2)/x^3,x)
```

```
[Out] int((1/cosh(2*log(c*x)))^(3/2)/x^3, x)
```

3.181 $\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx$

Optimal. Leaf size=66

$$\frac{1}{2}\left(c^4+\frac{1}{x^4}\right)x\operatorname{sech}^{\frac{3}{2}}(2\log(cx))-\frac{1}{2}c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{csch}^{-1}\left(c^2x^2\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))$$

```
[Out] 1/2*(c^4+1/x^4)*x*sech(2*ln(c*x))^(3/2)-1/2*c^6*(1+1/c^4/x^4)^(3/2)*x^3*arc
csch(c^2*x^2)*sech(2*ln(c*x))^(3/2)
```

Rubi [A]
time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5670, 5668, 342, 281, 294, 221}

$$\frac{1}{2}x\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))-\frac{1}{2}c^6x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{csch}^{-1}\left(c^2x^2\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))$$

Antiderivative was successfully verified.

```
[In] Int[Sech[2*Log[c*x]]^(3/2)/x^4,x]
```

```
[Out] ((c^4 + x^(-4))*x*Sech[2*Log[c*x]]^(3/2))/2 - (c^6*(1 + 1/(c^4*x^4))^(3/2)*
x^3*ArcCsch[c^2*x^2]*Sech[2*Log[c*x]]^(3/2))/2
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5668

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5670

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx &= c^3\text{Subst}\left(\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(x))}{x^4}\,dx, x, cx\right) \\
&= \left(c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{1}{x^4}\right)^{3/2}x^7}\,dx, x, cx\right) \\
&= -\left(\left(c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int \frac{x^5}{(1+x^4)^{3/2}}\,dx, x, \frac{1}{cx}\right)\right) \\
&= -\left(\frac{1}{2}\left(c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int \frac{x^2}{(1+x^2)^{3/2}}\,dx, x, \frac{1}{c^2x^2}\right)\right) \\
&= \frac{1}{2}\left(c^4+\frac{1}{x^4}\right)x\operatorname{sech}^{\frac{3}{2}}(2\log(cx)) - \frac{1}{2}\left(c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\Bigg(\\
&= \frac{1}{2}\left(c^4+\frac{1}{x^4}\right)x\operatorname{sech}^{\frac{3}{2}}(2\log(cx)) - \frac{1}{2}c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{-1}(c^2x^2)\operatorname{sech}^{\frac{3}{2}}(2\log(cx
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.08, size = 51, normalized size = 0.77

$$
\frac{\sqrt{2}\,c^2\sqrt{\frac{c^2x^2}{1+c^4x^4}}\;{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1+c^4x^4\right)}{x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[2*Log[c*x]]^(3/2)/x^4,x]
```

```
[Out] (Sqrt[2]*c^2*Sqrt[(c^2*x^2)/(1 + c^4*x^4)]*Hypergeometric2F1[-1/2, 1, 1/2,
1 + c^4*x^4])/x
```

Maple [F]
time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(2\ln(cx))^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(2*ln(c*x))^(3/2)/x^4,x)
```

```
[Out] int(sech(2*ln(c*x))^(3/2)/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sech(2*log(c*x))^(3/2)/x^4, x)
```

Fricas [A]
time = 0.35, size = 93, normalized size = 1.41

$$\frac{\sqrt{2}\,c^3x\log\left(\frac{c^5x^5+2\,cx-2\,(c^4x^4+1)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{cx^5}\right)+2\,\sqrt{2}\,\sqrt{\frac{c^2x^2}{c^4x^4+1}}\,c^2}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*c^3*x*log((c^5*x^5 + 2*c*x - 2*(c^4*x^4 + 1)*sqrt(c^2*x^2/(c^4
*x^4 + 1)))/(c*x^5)) + 2*sqrt(2)*sqrt(c^2*x^2/(c^4*x^4 + 1))*c^2)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*ln(c*x))**(3/2)/x**4,x)
```

```
[Out] Integral(sech(2*log(c*x))**(3/2)/x**4, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(\frac{1}{\cosh(2\,\ln(c\,x))}\right)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(2*log(c*x)))^(3/2)/x^4,x)
```

```
[Out] int((1/cosh(2*log(c*x)))^(3/2)/x^4, x)
```

3.182 $\int \operatorname{sech}(a+b\log(cx^n))\,dx$

Optimal. Leaf size=63

$$\frac{2e^a x(cx^n)^b\ {}_2F_1\left(1,\frac{b+\frac{1}{n}}{2b};\frac{1}{2}\left(3+\frac{1}{bn}\right);-e^{2a}(cx^n)^{2b}\right)}{1+bn}$$

```
[Out] 2*exp(a)*x*(c*x^n)^b*hypergeom([1, 1/2*(b+1/n)/b],[3/2+1/2/b/n],-exp(2*a)*(
c*x^n)^(2*b))/(b*n+1)
```

Rubi [A]
time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5664, 5666, 269, 371}

$$\frac{2e^a x(cx^n)^b\ {}_2F_1\left(1,\frac{b+\frac{1}{n}}{2b};\frac{1}{2}\left(3+\frac{1}{bn}\right);-e^{2a}(cx^n)^{2b}\right)}{bn+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*Log[c*x^n]],x]
```

```
[Out] (2*E^a*x*(c*x^n)^b*Hypergeometric2F1[1, (b + n^(-1))/(2*b), (3 + 1/(b*n))/2
, -(E^(2*a)*(c*x^n)^(2*b))])/(1 + b*n)
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5664

```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5666

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*
```

```
d)))^p)), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}(a+b\log(cx^n))\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{sech}(a+b\log(x))\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(2e^{-a}x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1-b+\frac{1}{n}}}{1+e^{-2a}x^{-2b}}\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(2e^{-a}x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1+b+\frac{1}{n}}}{e^{-2a}+x^{2b}}\,dx,x,cx^n\right)}{n}$$
$$= \frac{2e^a x(cx^n)^b\ {}_2F_1\left(1,\frac{b+\frac{1}{n}}{2b};\frac{1}{2}\left(3+\frac{1}{bn}\right);-e^{2a}(cx^n)^{2b}\right)}{1+bn}$$

Mathematica [A]
time = 0.57, size = 64, normalized size = 1.02

$$\frac{2e^a x(cx^n)^b\ {}_2F_1\left(1,\frac{1}{2}\left(1+\frac{1}{bn}\right);\frac{1}{2}\left(3+\frac{1}{bn}\right);-e^{2a}(cx^n)^{2b}\right)}{1+bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*Log[c*x^n]],x]
[Out] (2*E^a*x*(c*x^n)^b*Hypergeometric2F1[1, (1 + 1/(b*n))/2, (3 + 1/(b*n))/2, -
(E^(2*a)*(c*x^n)^(2*b))])/(1 + b*n)
```

Maple [F]
time = 0.29, size = 0, normalized size = 0.00

$$\int \operatorname{sech}(a+b\ln(c\,x^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(a+b*ln(c*x^n)),x)
[Out] int(sech(a+b*ln(c*x^n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] integrate(sech(b*log(c*x^n) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] integral(sech(b*log(c*x^n) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}\left(a + b\log\left(cx^{n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*ln(c*x**n)),x)
```

```
[Out] Integral(sech(a + b*log(c*x**n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] integrate(sech(b*log(c*x^n) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cosh\left(a + b\,\ln\left(c\,x^{n}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(a + b*log(c*x^n)),x)
```

```
[Out] int(1/cosh(a + b*log(c*x^n)), x)
```

3.183 $\int \operatorname{sech}^2(a+b\log(cx^n))\,dx$

Optimal. Leaf size=69

$$\frac{4e^{2a}x(cx^n)^{2b}\,{}_2F_1\left(2,\frac{1}{2}\left(2+\frac{1}{bn}\right);\frac{1}{2}\left(4+\frac{1}{bn}\right);-e^{2a}(cx^n)^{2b}\right)}{1+2bn}$$

```
[Out] 4*exp(2*a)*x*(c*x^n)^(2*b)*hypergeom([2, 1+1/2/b/n],[2+1/2/b/n],-exp(2*a)*(
c*x^n)^(2*b))/(2*b*n+1)
```

Rubi [A]
time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5664, 5666, 269, 371}

$$\frac{4e^{2a}x(cx^n)^{2b}\,{}_2F_1\left(2,\frac{1}{2}\left(2+\frac{1}{bn}\right);\frac{1}{2}\left(4+\frac{1}{bn}\right);-e^{2a}(cx^n)^{2b}\right)}{2bn+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (4*E^(2*a)*x*(c*x^n)^(2*b)*Hypergeometric2F1[2, (2 + 1/(b*n))/2, (4 + 1/(b*
n))/2, -(E^(2*a)*(c*x^n)^(2*b))])/(1 + 2*b*n)
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5664

```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5666

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*
```

```
d)))^p)), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}^2(a+b\log(cx^n))\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{sech}^2(a+b\log(x))\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(4e^{-2a}x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1-2b+\frac{1}{n}}}{(1+e^{-2a}x^{-2b})^2}\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(4e^{-2a}x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1+2b+\frac{1}{n}}}{(e^{-2a}+x^{2b})^2}\,dx,x,cx^n\right)}{n}$$
$$= \frac{4e^{2a}x(cx^n)^{2b}\,{}_2F_1\left(2,\frac{1}{2}\left(2+\frac{1}{bn}\right);\frac{1}{2}\left(4+\frac{1}{bn}\right);-e^{2a}(cx^n)^{2b}\right)}{1+2bn}$$

Mathematica [A]
time = 3.71, size = 126, normalized size = 1.83

$$\frac{x\left(-\frac{e^{2a}(cx^n)^{2b}\,{}_2F_1\left(1,1+\frac{1}{2bn};2+\frac{1}{2bn};-e^{2a}(cx^n)^{2b}\right)}{1+2bn}+{}_2F_1\left(1,\frac{1}{2bn};1+\frac{1}{2bn};-e^{2a}(cx^n)^{2b}\right)+\tanh(a+b\log(cx^n))\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (x*(-((E^(2*a)*(c*x^n)^(2*b)*Hypergeometric2F1[1, 1 + 1/(2*b*n), 2 + 1/(2*b
*n), -(E^(2*a)*(c*x^n)^(2*b))])/(1 + 2*b*n)) + Hypergeometric2F1[1, 1/(2*b*
n), 1 + 1/(2*b*n), -(E^(2*a)*(c*x^n)^(2*b))] + Tanh[a + b*Log[c*x^n]]))/(b*
n)
```

Maple [F]
time = 1.51, size = 0, normalized size = 0.00

$$\int \operatorname{sech}(a+b\ln(c\,x^n))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(a+b*ln(c*x^n))^2,x)
```

```
[Out] int(sech(a+b*ln(c*x^n))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] -2*x/(b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n) + 4*integrate(1/2/(b*c^(2*b
)*n*e^(2*b*log(x^n) + 2*a) + b*n), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] integral(sech(b*log(c*x^n) + a)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}^2\left(a + b\log\left(cx^n\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*ln(c*x**n))**2,x)
```

```
[Out] Integral(sech(a + b*log(c*x**n))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] integrate(sech(b*log(c*x^n) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(a + b\,\ln\left(c\,x^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(a + b*log(c*x^n))^2,x)
```

```
[Out] int(1/cosh(a + b*log(c*x^n))^2, x)
```

3.184 $\int \operatorname{sech}^3(a + b\log(cx^n))\,dx$

Optimal. Leaf size=70

$$\frac{8e^{3a}x(cx^n)^{3b}\ {}_2F_1\left(3,\frac{3b+\frac{1}{n}}{2b};\frac{1}{2}\left(5+\frac{1}{bn}\right);-e^{2a}(cx^n)^{2b}\right)}{1+3bn}$$

```
[Out] 8*exp(3*a)*x*(c*x^n)^(3*b)*hypergeom([3, 1/2*(3*b+1/n)/b],[5/2+1/2/b/n],-ex
p(2*a)*(c*x^n)^(2*b))/(3*b*n+1)
```

Rubi [A]
time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5664, 5666, 269, 371}

$$\frac{8e^{3a}x(cx^n)^{3b}\ {}_2F_1\left(3,\frac{3b+\frac{1}{n}}{2b};\frac{1}{2}\left(5+\frac{1}{bn}\right);-e^{2a}(cx^n)^{2b}\right)}{3bn+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (8*E^(3*a)*x*(c*x^n)^(3*b)*Hypergeometric2F1[3, (3*b + n^(-1))/(2*b), (5 +
1/(b*n))/2, -(E^(2*a)*(c*x^n)^(2*b))])/(1 + 3*b*n)
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5664

```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5666

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*
```

```
d)))^p)), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}^3(a+b\log(cx^n))\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{sech}^3(a+b\log(x))\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(8e^{-3a}x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1-3b+\frac{1}{n}}}{(1+e^{-2a}x^{-2b})^3}\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(8e^{-3a}x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1+3b+\frac{1}{n}}}{(e^{-2a}+x^{2b})^3}\,dx,x,cx^n\right)}{n}$$
$$= \frac{8e^{3a}x(cx^n)^{3b}\,{}_2F_1\left(3,\frac{3b+\frac{1}{n}}{2b};\frac{1}{2}\left(5+\frac{1}{bn}\right);-e^{2a}(cx^n)^{2b}\right)}{1+3bn}$$

Mathematica [A]
time = 1.07, size = 101, normalized size = 1.44

$$\frac{x\left(2e^a(-1+bn)(cx^n)^b\,{}_2F_1\left(1,\frac{1}{2}\left(1+\frac{1}{bn}\right);\frac{1}{2}\left(3+\frac{1}{bn}\right);-e^{2a}(cx^n)^{2b}\right)+\operatorname{sech}(a+b\log(cx^n))\left(1+bn\tanh(a+b\log(cx^n))\right)\right)}{2b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*Log[c*x^n]]^3,x]
[Out] (x*(2*E^a*(-1 + b*n)*(c*x^n)^b*Hypergeometric2F1[1, (1 + 1/(b*n))/2, (3 + 1
/(b*n))/2, -(E^(2*a)*(c*x^n)^(2*b))] + Sech[a + b*Log[c*x^n]]*(1 + b*n*Tanh
[a + b*Log[c*x^n]])))/(2*b^2*n^2)
```

Maple [F]
time = 1.77, size = 0, normalized size = 0.00

$$\int \operatorname{sech}(a+b\ln(c\,x^n))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(a+b*ln(c*x^n))^3,x)
[Out] int(sech(a+b*ln(c*x^n))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] 8*(b^2*c^b*n^2 - c^b)*integrate(1/8*e^(b*log(x^n) + a)/(b^2*c^(2*b)*n^2*e^(
2*b*log(x^n) + 2*a) + b^2*n^2), x) + ((b*c^(3*b)*n + c^(3*b))*x*e^(3*b*log(
x^n) + 3*a) - (b*c^b*n - c^b)*x*e^(b*log(x^n) + a))/(b^2*c^(4*b)*n^2*e^(4*b
*log(x^n) + 4*a) + 2*b^2*c^(2*b)*n^2*e^(2*b*log(x^n) + 2*a) + b^2*n^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] integral(sech(b*log(c*x^n) + a)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}^3\left(a + b\log\left(cx^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*ln(c*x**n))**3,x)
```

```
[Out] Integral(sech(a + b*log(c*x**n))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] integrate(sech(b*log(c*x^n) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\cosh\left(a+b\,\ln\left(c\,x^n\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(a + b*log(c*x^n))^3,x)
```

```
[Out] int(1/cosh(a + b*log(c*x^n))^3, x)
```

3.185 $\int \operatorname{sech}^4(a + b\log(cx^n))\,dx$

Optimal. Leaf size=69

$$\frac{16e^{4a}x(cx^n)^{4b}\,{}_2F_1\left(4,\frac{1}{2}\left(4+\frac{1}{bn}\right);\frac{1}{2}\left(6+\frac{1}{bn}\right);-e^{2a}(cx^n)^{2b}\right)}{1+4bn}$$

```
[Out] 16*exp(4*a)*x*(c*x^n)^(4*b)*hypergeom([4, 2+1/2/b/n],[3+1/2/b/n],-exp(2*a)*
(c*x^n)^(2*b))/(4*b*n+1)
```

Rubi [A]
time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5664, 5666, 269, 371}

$$\frac{16e^{4a}x(cx^n)^{4b}\,{}_2F_1\left(4,\frac{1}{2}\left(4+\frac{1}{bn}\right);\frac{1}{2}\left(6+\frac{1}{bn}\right);-e^{2a}(cx^n)^{2b}\right)}{4bn+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (16*E^(4*a)*x*(c*x^n)^(4*b)*Hypergeometric2F1[4, (4 + 1/(b*n))/2, (6 + 1/(b
*n))/2, -(E^(2*a)*(c*x^n)^(2*b))])/(1 + 4*b*n)
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5664

```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5666

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*
```

```
d)))^p)), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{sech}^4(a+b\log(cx^n))\,dx &= \frac{\left(x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{sech}^4(a+b\log(x))\,dx,x,cx^n\right)}{n}\\
&= \frac{\left(16e^{-4a}x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1-4b+\frac{1}{n}}}{(1+e^{-2a}x^{-2b})^4}\,dx,x,cx^n\right)}{n}\\
&= \frac{\left(16e^{-4a}x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1+4b+\frac{1}{n}}}{(e^{-2a}+x^{2b})^4}\,dx,x,cx^n\right)}{n}\\
&= \frac{16e^{4a}x(cx^n)^{4b}\,{}_2F_1\left(4,\frac{1}{2}\left(4+\frac{1}{bn}\right);\frac{1}{2}\left(6+\frac{1}{bn}\right);-e^{2a}(cx^n)^{2b}\right)}{1+4bn}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 192 vs. 2(69) = 138.
time = 9.34, size = 192, normalized size = 2.78

$$\frac{x\left(-2e^{2a}(-1+2bn)(cx^n)^{2b}\,{}_2F_1\left(1,1+\frac{1}{2bn};2+\frac{1}{2bn};-e^{2a}(cx^n)^{2b}\right)+(-2+8b^2n^2)\,{}_2F_1\left(1,\frac{1}{2bn};1+\frac{1}{2bn};-e^{2a}(cx^n)^{2b}\right)+\operatorname{sech}^2(a+b\log(cx^n))\,(2bn+(-1+8b^2n^2+(-1+4b^2n^2)\cosh(2(a+b\log(cx^n))))\tanh(a+b\log(cx^n)))\right)}{12b^3n^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (x*(-2*E^(2*a)*(-1 + 2*b*n)*(c*x^n)^(2*b)*Hypergeometric2F1[1, 1 + 1/(2*b*n
), 2 + 1/(2*b*n), -(E^(2*a)*(c*x^n)^(2*b))] + (-2 + 8*b^2*n^2)*Hypergeometr
ic2F1[1, 1/(2*b*n), 1 + 1/(2*b*n), -(E^(2*a)*(c*x^n)^(2*b))] + Sech[a + b*L
og[c*x^n]]^2*(2*b*n + (-1 + 8*b^2*n^2 + (-1 + 4*b^2*n^2)*Cosh[2*(a + b*Log[
c*x^n])])*Tanh[a + b*Log[c*x^n]])))/(12*b^3*n^3)
```

Maple [F]
time = 1.68, size = 0, normalized size = 0.00

$$\int \operatorname{sech}(a+b\ln(c\,x^n))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(a+b*ln(c*x^n))^4,x)
```

```
[Out] int(sech(a+b*ln(c*x^n))^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^4,x, algorithm="maxima")
```

```
[Out] 16*(4*b^2*n^2 - 1)*integrate(1/48/(b^3*c^(2*b)*n^3*e^(2*b*log(x^n) + 2*a) +
 b^3*n^3), x) + 1/3*((2*b*c^(4*b)*n + c^(4*b))*x*e^(4*b*log(x^n) + 4*a) - 2
*(6*b^2*c^(2*b)*n^2 - b*c^(2*b)*n - c^(2*b))*x*e^(2*b*log(x^n) + 2*a) - (4*
b^2*n^2 - 1)*x)/(b^3*c^(6*b)*n^3*e^(6*b*log(x^n) + 6*a) + 3*b^3*c^(4*b)*n^3
*e^(4*b*log(x^n) + 4*a) + 3*b^3*c^(2*b)*n^3*e^(2*b*log(x^n) + 2*a) + b^3*n^
3)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

```
[Out] integral(sech(b*log(c*x^n) + a)^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}^4\left(a + b\log\left(cx^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*ln(c*x**n))**4,x)
```

```
[Out] Integral(sech(a + b*log(c*x**n))**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^4,x, algorithm="giac")
```

```
[Out] integrate(sech(b*log(c*x^n) + a)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(a+b\,\ln\left(c\,x^n\right)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(a + b*log(c*x^n))^4,x)
[Out] int(1/cosh(a + b*log(c*x^n))^4, x)
```

3.186 $\int \left(\left(1-b^2n^2\right)\textbf{sech}(a+b\log\left(cx^n\right))+2b^2n^2\textbf{sech}^3(a+\right.$

Optimal. Leaf size=40

$$x\text{sech}(a+b\log\left(cx^n\right))+bnx\text{sech}(a+b\log\left(cx^n\right))\tanh\left(a+b\log\left(cx^n\right)\right)$$

```
[Out] x*sech(a+b*ln(c*x^n))+b*n*x*sech(a+b*ln(c*x^n))*tanh(a+b*ln(c*x^n))
```

Rubi [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.10, antiderivative size = 139, normalized size of antiderivative = 3.48, number of steps used = 9, number of rules used = 4, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$,
Rules used = {5664, 5666, 269, 371}

$$\frac{16e^{3a}b^2n^2x(cx^n)^{3b}\ {}_2F_1\left(3,\frac{3b+\frac{1}{n}}{2b};\frac{1}{2}\left(5+\frac{1}{bn}\right);-e^{2a}(cx^n)^{2b}\right)}{3bn+1}+2e^ax(1-bn)\left(cx^n\right)^b\ {}_2F_1\left(1,\frac{b+\frac{1}{n}}{2b};\frac{1}{2}\left(3+\frac{1}{bn}\right);-e^{2a}(cx^n)^{2b}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - b^2*n^2)*Sech[a + b*Log[c*x^n]] + 2*b^2*n^2*Sech[a + b*Log[c*x^n]]
^3,x]
```

```
[Out] 2*E^a*(1 - b*n)*x*(c*x^n)^b*Hypergeometric2F1[1, (b + n^(-1))/(2*b), (3 + 1
/(b*n))/2, -(E^(2*a)*(c*x^n)^(2*b))] + (16*b^2*E^(3*a)*n^2*x*(c*x^n)^(3*b)*
Hypergeometric2F1[3, (3*b + n^(-1))/(2*b), (5 + 1/(b*n))/2, -(E^(2*a)*(c*x^
n)^(2*b))])/(1 + 3*b*n)
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5664

```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5666

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*
```

```
d)))^p)), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\int \left(\left(1-b^2n^2\right)\operatorname{sech}(a+b\log(cx^n))+2b^2n^2\operatorname{sech}^3(a+b\log(cx^n))\right)dx=\left(2b^2n^2\right)\int \operatorname{sech}^3(a+b\log(cx^n))\,d$$

$$=\left(2b^2nx(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\right.$$

$$=\left(16b^2e^{-3a}nx(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int\right.$$

$$=\left(16b^2e^{-3a}nx(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int\right.$$

$$=2e^a(1-bn)x(cx^n)^b\ {}_2F_1\left(1,\frac{b+\frac{1}{n}}{2b};\right.$$

Mathematica [A]
time = 0.23, size = 29, normalized size = 0.72

$$x\operatorname{sech}(a+b\log(cx^n))\left(1+bn\tanh\left(a+b\log(cx^n)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - b^2*n^2)*Sech[a + b*Log[c*x^n]] + 2*b^2*n^2*Sech[a + b*Log[c
*x^n]]^3,x]
[Out] x*Sech[a + b*Log[c*x^n]]*(1 + b*n*Tanh[a + b*Log[c*x^n]])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 5.80, size = 509, normalized size = 12.72

method	result
risch	$\dfrac{2c^b(x^n)^b x\left(nb(x^n)^{2b}c^{2b}e^{3a}e^{-\frac{3ib\pi\operatorname{csgn}(icx^n)^3}{2}}e^{\frac{3ib\operatorname{csgn}(icx^n)^2\operatorname{csgn}(ic)\pi}{2}}e^{\frac{3ib\operatorname{csgn}(icx^n)^2\operatorname{csgn}(ix^n)\pi}{2}}e^{-\frac{3ib\operatorname{csgn}(icx^n)\operatorname{csgn}(ic)\operatorname{csgn}(ix^n)\pi}{2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^2*n^2+1)*sech(a+b*ln(c*x^n))+2*b^2*n^2*sech(a+b*ln(c*x^n))^3,x,meth
od=_RETURNVERBOSE)
```

```
[Out] 2*c^b*(x^n)^b*x/((((x^n)^b)^2*(c^b)^2*exp(2*a)*exp(-I*b*csgn(I*c*x^n)^3*Pi)*
exp(I*b*csgn(I*c*x^n)^2*csgn(I*c)*Pi)*exp(I*b*csgn(I*c*x^n)^2*csgn(I*x^n)*P
i)*exp(-I*b*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)*Pi)+1)^2*(n*b*((x^n)^b)^2*(
c^b)^2*exp(3*a)*exp(-3/2*I*b*csgn(I*c*x^n)^3*Pi)*exp(3/2*I*b*csgn(I*c*x^n)^
2*csgn(I*c)*Pi)*exp(3/2*I*b*csgn(I*c*x^n)^2*csgn(I*x^n)*Pi)*exp(-3/2*I*b*cs
gn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)*Pi)-exp(a)*exp(-1/2*I*b*csgn(I*c*x^n)^3*P
i)*exp(1/2*I*b*csgn(I*c*x^n)^2*csgn(I*c)*Pi)*exp(1/2*I*b*csgn(I*c*x^n)^2*cs
gn(I*x^n)*Pi)*exp(-1/2*I*b*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)*Pi)*b*n+((x^
n)^b)^2*(c^b)^2*exp(3*a)*exp(-3/2*I*b*csgn(I*c*x^n)^3*Pi)*exp(3/2*I*b*csgn(
I*c*x^n)^2*csgn(I*c)*Pi)*exp(3/2*I*b*csgn(I*c*x^n)^2*csgn(I*x^n)*Pi)*exp(-3
/2*I*b*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)*Pi)+exp(a)*exp(-1/2*I*b*csgn(I*c
*x^n)^3*Pi)*exp(1/2*I*b*csgn(I*c*x^n)^2*csgn(I*c)*Pi)*exp(1/2*I*b*csgn(I*c*
x^n)^2*csgn(I*x^n)*Pi)*exp(-1/2*I*b*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)*Pi)
)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(40) = 80$.
time = 0.45, size = 96, normalized size = 2.40

$$\frac{2\left(\left(bc^{3\,b}n + c^{3\,b}\right)xe^{\left(3\,b\log\left(x^{n}\right) + 3\,a\right)} - \left(bc^{b}n - c^{b}\right)xe^{\left(b\log\left(x^{n}\right) + a\right)}\right)}{c^{4\,b}e^{\left(4\,b\log\left(x^{n}\right) + 4\,a\right)} + 2\,c^{2\,b}e^{\left(2\,b\log\left(x^{n}\right) + 2\,a\right)} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*n^2+1)*sech(a+b*log(c*x^n))+2*b^2*n^2*sech(a+b*log(c*x^n))^
3,x, algorithm="maxima")
```

```
[Out] 2*((b*c^(3*b)*n + c^(3*b))*x*e^(3*b*log(x^n) + 3*a) - (b*c^b*n - c^b)*x*e^(
b*log(x^n) + a))/(c^(4*b)*e^(4*b*log(x^n) + 4*a) + 2*c^(2*b)*e^(2*b*log(x^n
) + 2*a) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(40) = 80$.
time = 0.38, size = 189, normalized size = 4.72

$$\frac{2\left((bn+1)x\cosh\left(bn\log\left(x\right) + b\log\left(c\right) + a\right)^{2} + 2\,(bn+1)x\cosh\left(bn\log\left(x\right) + b\log\left(c\right) + a\right)\sinh\left(bn\log\left(x\right) + b\log\left(c\right) + a\right) + (bn+1)x\sinh\left(bn\log\left(x\right) + b\log\left(c\right) + a\right)^{2} - (bn-1)x\right)}{\cosh\left(bn\log\left(x\right) + b\log\left(c\right) + a\right)^{3} + 3\,\cosh\left(bn\log\left(x\right) + b\log\left(c\right) + a\right)\sinh\left(bn\log\left(x\right) + b\log\left(c\right) + a\right)^{2} + \sinh\left(bn\log\left(x\right) + b\log\left(c\right) + a\right)^{3} + \left(3\,\cosh\left(bn\log\left(x\right) + b\log\left(c\right) + a\right)^{2} + 1\right)\sinh\left(bn\log\left(x\right) + b\log\left(c\right) + a\right) + 3\,\cosh\left(bn\log\left(x\right) + b\log\left(c\right) + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*n^2+1)*sech(a+b*log(c*x^n))+2*b^2*n^2*sech(a+b*log(c*x^n))^
3,x, algorithm="fricas")
```

```
[Out] 2*((b*n + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*(b*n + 1)*x*cosh(b*n*l
og(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + (b*n + 1)*x*sinh(b*
n*log(x) + b*log(c) + a)^2 - (b*n - 1)*x)/(cosh(b*n*log(x) + b*log(c) + a)^
3 + 3*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^2 + s
inh(b*n*log(x) + b*log(c) + a)^3 + (3*cosh(b*n*log(x) + b*log(c) + a)^2 + 1
)*sinh(b*n*log(x) + b*log(c) + a) + 3*cosh(b*n*log(x) + b*log(c) + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(2b^2n^2 \operatorname{sech}^2\left(a+b\log\left(cx^n\right)\right)-b^2n^2+1\right)\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**2*n**2+1)*sech(a+b*ln(c*x**n))+2*b**2*n**2*sech(a+b*ln(c*x**
n))**3,x)
```

```
[Out] Integral((2*b**2*n**2*sech(a + b*log(c*x**n))**2 - b**2*n**2 + 1)*sech(a +
b*log(c*x**n)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(40) = 80$.
time = 0.80, size = 215, normalized size = 5.38

$$\frac{2\,bc^{3\,b}nxx^{3\,bn}e^{(3\,a)}}{c^{4\,b}x^{4\,bn}e^{(4\,a)}+2\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+1}-\frac{2\,bc^{b}nxx^{bn}e^{a}}{c^{4\,b}x^{4\,bn}e^{(4\,a)}+2\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+1}+\frac{2\,c^{3\,b}xx^{3\,bn}e^{(3\,a)}}{c^{4\,b}x^{4\,bn}e^{(4\,a)}+2\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+1}+\frac{2\,c^{b}xx^{bn}e^{a}}{c^{4\,b}x^{4\,bn}e^{(4\,a)}+2\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*n^2+1)*sech(a+b*log(c*x^n))+2*b^2*n^2*sech(a+b*log(c*x^n))^
3,x, algorithm="giac")
```

```
[Out] 2*b*c^(3*b)*n*x*x^(3*b*n)*e^(3*a)/(c^(4*b)*x^(4*b*n)*e^(4*a) + 2*c^(2*b)*x^
(2*b*n)*e^(2*a) + 1) - 2*b*c^b*n*x*x^(b*n)*e^a/(c^(4*b)*x^(4*b*n)*e^(4*a) +
 2*c^(2*b)*x^(2*b*n)*e^(2*a) + 1) + 2*c^(3*b)*x*x^(3*b*n)*e^(3*a)/(c^(4*b)*
x^(4*b*n)*e^(4*a) + 2*c^(2*b)*x^(2*b*n)*e^(2*a) + 1) + 2*c^b*x*x^(b*n)*e^a/
(c^(4*b)*x^(4*b*n)*e^(4*a) + 2*c^(2*b)*x^(2*b*n)*e^(2*a) + 1)
```

Mupad [B]
time = 1.40, size = 66, normalized size = 1.65

$$\frac{2\,x\,\mathrm{e}^{a}\,{\left(c\,x^n\right)}^b\,\left(\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}-b\,n+b\,n\,\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}+1\right)}{{\left(\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*b^2*n^2)/cosh(a + b*log(c*x^n))^3 - (b^2*n^2 - 1)/cosh(a + b*log(c*x
^n)),x)
```

```
[Out] (2*x*exp(a)*(c*x^n)^b*(exp(2*a)*(c*x^n)^(2*b) - b*n + b*n*exp(2*a)*(c*x^n)^
(2*b) + 1))/(exp(2*a)*(c*x^n)^(2*b) + 1)^2
```

3.187 $\int \operatorname{sech}^3\left(a+2 \log \left(c \sqrt{x}\right)\right) \, dx$

Optimal. Leaf size=25

$$\frac{2 c^6 e^{-a}}{\left(c^4+\frac{e^{-2 a}}{x^2}\right)^2}$$

[Out] 2*c^6/exp(a)/(c^4+1/exp(2*a)/x^2)^2

Rubi [A]
time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5664, 5666, 267}

$$\frac{2 e^{-a} c^6}{\left(\frac{e^{-2 a}}{x^2}+c^4\right)^2}$$

Antiderivative was successfully verified.

[In] Int[Sech[a + 2*Log[c*Sqrt[x]]]^3,x]

[Out] (2*c^6)/(E^a*(c^4 + 1/(E^(2*a)*x^2))^2)

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5664

```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5666

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*
d)))^p)), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}^3\left(a+2\log\left(c\sqrt{x}\right)\right)\,dx = \frac{2\text{Subst}\left(\int x\operatorname{sech}^3(a+2\log(x))\,dx,x,c\sqrt{x}\right)}{c^2}$$
$$= \frac{(16e^{-3a})\,\text{Subst}\left(\int \frac{1}{\left(1+\frac{e^{-2a}}{x^4}\right)^3x^5}\,dx,x,c\sqrt{x}\right)}{c^2}$$
$$= \frac{2c^6e^{-a}}{\left(c^4+\frac{e^{-2a}}{x^2}\right)^2}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 62 vs. 2(25) = 50.
time = 0.09, size = 62, normalized size = 2.48

$$-\frac{2(\cosh(a)-\sinh(a))\left(2c^4x^2+\cosh^2(a)-2\cosh(a)\sinh(a)+\sinh^2(a)\right)}{c^2\left((1+c^4x^2)\cosh(a)+(-1+c^4x^2)\sinh(a)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + 2*Log[c*Sqrt[x]]]^3,x]
[Out] (-2*(Cosh[a] - Sinh[a])*(2*c^4*x^2 + Cosh[a]^2 - 2*Cosh[a]*Sinh[a] + Sinh[a
]^2))/(c^2*((1 + c^4*x^2)*Cosh[a] + (-1 + c^4*x^2)*Sinh[a])^2)
```

Maple [F]
time = 2.57, size = 0, normalized size = 0.00

$$\int \operatorname{sech}\left(a+2\ln\left(c\sqrt{x}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(a+2*ln(c*x^(1/2)))^3,x)
[Out] int(sech(a+2*ln(c*x^(1/2)))^3,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(23) = 46.
time = 0.28, size = 74, normalized size = 2.96

$$-\frac{2\left(\frac{2\,c^4x^2e^{(2\,a)}}{c^8x^4e^{(5\,a)}+2\,c^4x^2e^{(3\,a)}+e^a}+\frac{1}{c^8x^4e^{(5\,a)}+2\,c^4x^2e^{(3\,a)}+e^a}\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+2*log(c*x^(1/2)))^3,x, algorithm="maxima")
```

```
[Out] -2*(2*c^4*x^2*e^(2*a)/(c^8*x^4*e^(5*a) + 2*c^4*x^2*e^(3*a) + e^a) + 1/(c^8*
x^4*e^(5*a) + 2*c^4*x^2*e^(3*a) + e^a))/c^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(23) = 46$.
time = 0.37, size = 48, normalized size = 1.92

$$-\frac{2\left(2\,c^4x^2e^{(2\,a)}+1\right)}{c^{10}x^4e^{(5\,a)}+2\,c^6x^2e^{(3\,a)}+c^2e^a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+2*log(c*x^(1/2)))^3,x, algorithm="fricas")
```

```
[Out] -2*(2*c^4*x^2*e^(2*a) + 1)/(c^10*x^4*e^(5*a) + 2*c^6*x^2*e^(3*a) + c^2*e^a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}^3\left(a+2\log\left(c\sqrt{x}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+2*ln(c*x**(1/2)))**3,x)
```

```
[Out] Integral(sech(a + 2*log(c*sqrt(x)))**3, x)
```

Giac [A]
time = 0.41, size = 38, normalized size = 1.52

$$-\frac{2\left(2\,c^4x^2e^{(2\,a)}+1\right)e^{(-a)}}{\left(c^4x^2e^{(2\,a)}+1\right)^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+2*log(c*x^(1/2)))^3,x, algorithm="giac")
```

```
[Out] -2*(2*c^4*x^2*e^(2*a) + 1)*e^(-a)/((c^4*x^2*e^(2*a) + 1)^2*c^2)
```

Mupad [B]
time = 1.53, size = 49, normalized size = 1.96

$$-\frac{\frac{2\,\mathrm{e}^{-a}}{c^2}+4\,c^2\,x^2\,\mathrm{e}^a}{\mathrm{e}^{4\,a}\,c^8\,x^4+2\,\mathrm{e}^{2\,a}\,c^4\,x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(a + 2*log(c*x^(1/2)))^3,x)
```

```
[Out] -((2*exp(-a))/c^2 + 4*c^2*x^2*exp(a))/(2*c^4*x^2*exp(2*a) + c^8*x^4*exp(4*a
) + 1)
```

3.188 $\int \operatorname{sech}^3\left(a+2 \log \left(\frac{c}{\sqrt{x}}\right)\right) \, dx$

Optimal. Leaf size=25

$$\frac{2c^2e^{-3a}}{\left(e^{-2a}+\frac{c^4}{x^2}\right)^2}$$

```
[Out] 2*c^2/exp(3*a)/(exp(-2*a)+c^4/x^2)^2
```

Rubi [A]
time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {5664, 5666, 269, 267}

$$\frac{2e^{-3a}c^2}{\left(e^{-2a}+\frac{c^4}{x^2}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + 2*Log[c/Sqrt[x]]]^3,x]
```

```
[Out] (2*c^2)/(E^(3*a)*(E^(-2*a) + c^4/x^2)^2)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 5664

```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5666

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*
d)))^p)), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)dx = -\left((2c^2)\operatorname{Subst}\left(\int \frac{\operatorname{sech}^3(a+2\log(x))}{x^3}\,dx, x, \frac{c}{\sqrt{x}}\right)\right)$$
$$= -\left((16c^2e^{-3a})\operatorname{Subst}\left(\int \frac{1}{\left(1+\frac{e^{-2a}}{x^4}\right)^3 x^9}\,dx, x, \frac{c}{\sqrt{x}}\right)\right)$$
$$= -\left((16c^2e^{-3a})\operatorname{Subst}\left(\int \frac{x^3}{(e^{-2a}+x^4)^3}\,dx, x, \frac{c}{\sqrt{x}}\right)\right)$$
$$= \frac{2c^2e^{-3a}}{\left(e^{-2a}+\frac{c^4}{x^2}\right)^2}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 64 vs. 2(25) = 50.
time = 0.07, size = 64, normalized size = 2.56

$$-\frac{2c^6((c^4+2x^2)\cosh(a)+(c^4-2x^2)\sinh(a))(\cosh(2a)+\sinh(2a))}{((c^4+x^2)\cosh(a)+(c^4-x^2)\sinh(a))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + 2*Log[c/Sqrt[x]]]^3,x]
[Out] (-2*c^6*((c^4 + 2*x^2)*Cosh[a] + (c^4 - 2*x^2)*Sinh[a])*(Cosh[2*a] + Sinh[2
*a]))/((c^4 + x^2)*Cosh[a] + (c^4 - x^2)*Sinh[a])^2
```

Maple [F]
time = 3.65, size = 0, normalized size = 0.00

$$\int \operatorname{sech}\left(a+2\ln\left(\frac{c}{\sqrt{x}}\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(a+2*ln(c/x^(1/2)))^3,x)
[Out] int(sech(a+2*ln(c/x^(1/2)))^3,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(23) = 46.
time = 0.28, size = 49, normalized size = 1.96

$$-\frac{2\left(c^{10}e^{(5a)}+2c^6x^2e^{(3a)}\right)}{c^8e^{(4a)}+2c^4x^2e^{(2a)}+x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+2*log(c/x^(1/2)))^3,x, algorithm="maxima")
```

```
[Out] -2*(c^10*e^(5*a) + 2*c^6*x^2*e^(3*a))/(c^8*e^(4*a) + 2*c^4*x^2*e^(2*a) + x^
4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(23) = 46$.
time = 0.39, size = 49, normalized size = 1.96

$$-\frac{2\left(c^{10}e^{(5a)} + 2c^{6}x^{2}e^{(3a)}\right)}{c^{8}e^{(4a)} + 2c^{4}x^{2}e^{(2a)} + x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+2*log(c/x^(1/2)))^3,x, algorithm="fricas")
```

```
[Out] -2*(c^10*e^(5*a) + 2*c^6*x^2*e^(3*a))/(c^8*e^(4*a) + 2*c^4*x^2*e^(2*a) + x^
4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}^{3}\left(a + 2\log\left(\frac{c}{\sqrt{x}}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+2*ln(c/x**(1/2)))**3,x)
```

```
[Out] Integral(sech(a + 2*log(c/sqrt(x)))**3, x)
```

Giac [A]
time = 0.40, size = 37, normalized size = 1.48

$$-\frac{2\left(c^{10}e^{(5a)} + 2c^{6}x^{2}e^{(3a)}\right)}{\left(c^{4}e^{(2a)} + x^{2}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+2*log(c/x^(1/2)))^3,x, algorithm="giac")
```

```
[Out] -2*(c^10*e^(5*a) + 2*c^6*x^2*e^(3*a))/(c^4*e^(2*a) + x^2)^2
```

Mupad [B]
time = 1.45, size = 36, normalized size = 1.44

$$\frac{2\,c^{2}\,x^{4}\,\mathrm{e}^{a}}{\mathrm{e}^{4\,a}\,c^{8} + 2\,\mathrm{e}^{2\,a}\,c^{4}\,x^{2} + x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(a + 2*log(c/x^(1/2)))^3,x)
```

```
[Out] (2*c^2*x^4*exp(a))/(c^8*exp(4*a) + x^4 + 2*c^4*x^2*exp(2*a))
```

3.189 $\int \operatorname{sech}^p\left(a+\frac{\log(cx^n)}{n(-2+p)}\right)\,dx$

Optimal. Leaf size=89

$$\frac{e^{2a}(2-p)x(cx^n)^{-\frac{2}{n(2-p)}}\left(1+e^{-2a}(cx^n)^{\frac{2}{n(2-p)}}\right)\operatorname{sech}^p\left(a-\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

```
[Out] 1/2*exp(2*a)*(2-p)*x*(1+(c*x^n)^(2/n/(2-p))/exp(2*a))*sech(a-ln(c*x^n)/n/(2
-p))^p/(1-p)/((c*x^n)^(2/n/(2-p)))
```

Rubi [A]
time = 0.06, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5664, 5668, 267}

$$\frac{e^{2a}(2-p)x(cx^n)^{-\frac{2}{n(2-p)}}\left(e^{-2a}(cx^n)^{\frac{2}{n(2-p)}}+1\right)\operatorname{sech}^p\left(a-\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + Log[c*x^n]/(n*(-2 + p))]^p,x]
```

```
[Out] (E^(2*a)*(2 - p)*x*(1 + (c*x^n)^(2/(n*(2 - p)))/E^(2*a))*Sech[a - Log[c*x^n
]/(n*(2 - p))]^p)/(2*(1 - p)*(c*x^n)^(2/(n*(2 - p))))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5664

```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5668

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}^p\left(a+\frac{\log\left(cx^n\right)}{n(-2+p)}\right)\,dx=\frac{\left(x\left(cx^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{sech}^p\left(a+\frac{\log(x)}{n(-2+p)}\right)\,dx,x,cx^n\right)}{n}$$

$$=\frac{\left(x\left(cx^n\right)^{-\frac{1}{n}+\frac{p}{n(-2+p)}}\left(1+e^{-2a}\left(cx^n\right)^{-\frac{2}{n(-2+p)}}\right)^p\operatorname{sech}^p\left(a+\frac{\log(cx^n)}{n(-2+p)}\right)\right)\operatorname{Subst}\left(\right.}{n}$$

$$=\frac{e^{2a}(2-p)x\left(cx^n\right)^{-\frac{2}{n(2-p)}}\left(1+e^{-2a}\left(cx^n\right)^{\frac{2}{n(2-p)}}\right)\operatorname{sech}^p\left(a-\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

Mathematica [A]
time = 3.67, size = 114, normalized size = 1.28

$$-\frac{2^{-1+p}(-2+p)x\left(\frac{e^a\left(cx^n\right)^{\frac{1}{2n-np}}}{e^{2a}+\left(cx^n\right)^{-\frac{2}{n(-2+p)}}}\right)^p\left(-1+e^{2a}\left(cx^n\right)^{\frac{2}{n(-2+p)}}\left(-1+\left(1+e^{-2a}\left(cx^n\right)^{-\frac{2}{n(-2+p)}}\right)^p\right)\right)}{-1+p}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + Log[c*x^n]/(n*(-2 + p))]^p,x]
```

```
[Out] -((2^(-1 + p)*(-2 + p)*x*((E^a*(c*x^n)^(2*n - n*p)^(-1))/(E^(2*a) + (c*x^n)
^(-2/(n*(-2 + p)))))^p*(-1 + E^(2*a)*(c*x^n)^(2/(n*(-2 + p)))*(-1 + (1 + 1/
(E^(2*a)*(c*x^n)^(2/(n*(-2 + p)))))^p)))/(-1 + p))
```

Maple [F]
time = 1.67, size = 0, normalized size = 0.00

$$\int \operatorname{sech}\left(a+\frac{\ln\left(c\,x^n\right)}{n\left(-2+p\right)}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(a+ln(c*x^n)/n/(-2+p))^p,x)
```

```
[Out] int(sech(a+ln(c*x^n)/n/(-2+p))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+log(c*x^n)/n/(-2+p))^p,x, algorithm="maxima")
```

```
[Out] integrate(sech(a + log(c*x^n)/(n*(p - 2)))^p, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(76) = 152$.
time = 0.38, size = 474, normalized size = 5.33

$$\frac{(p-2)x\cosh\left(p\log\left(\frac{2\left(\cosh\left(\frac{anp-2\,an+n\log(x)+\log(c)}{np-2\,n}\right)+\sinh\left(\frac{anp-2\,an+n\log(x)+\log(c)}{np-2\,n}\right)\right)}{\cosh\left(\frac{anp-2\,an+n\log(x)+\log(c)}{np-2\,n}\right)^{2}+2\cosh\left(\frac{anp-2\,an+n\log(x)+\log(c)}{np-2\,n}\right)\sinh\left(\frac{anp-2\,an+n\log(x)+\log(c)}{np-2\,n}\right)+\sinh\left(\frac{anp-2\,an+n\log(x)+\log(c)}{np-2\,n}\right)^{2}+1}\right)\right)\cosh\left(\frac{anp-2\,an+n\log(x)+\log(c)}{np-2\,n}\right)+(p-2)x\cosh\left(\frac{anp-2\,an+n\log(x)+\log(c)}{np-2\,n}\right)\sinh\left(p\log\left(\frac{2\left(\cosh\left(\frac{anp-2\,an+n\log(x)+\log(c)}{np-2\,n}\right)+\sinh\left(\frac{anp-2\,an+n\log(x)+\log(c)}{np-2\,n}\right)\right)}{\cosh\left(\frac{anp-2\,an+n\log(x)+\log(c)}{np-2\,n}\right)^{2}+2\cosh\left(\frac{anp-2\,an+n\log(x)+\log(c)}{np-2\,n}\right)\sinh\left(\frac{anp-2\,an+n\log(x)+\log(c)}{np-2\,n}\right)+\sinh\left(\frac{anp-2\,an+n\log(x)+\log(c)}{np-2\,n}\right)^{2}+1}\right)\right)}{(p-1)\cosh\left(\frac{anp-2\,an+n\log(x)+\log(c)}{np-2\,n}\right)-(p-1)\sinh\left(\frac{anp-2\,an+n\log(x)+\log(c)}{np-2\,n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+log(c*x^n)/n/(-2+p))^p,x, algorithm="fricas")
```

```
[Out] ((p - 2)*x*cosh(p*log(2*(cosh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*
n)) + sinh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n)))/(cosh((a*n*p -
 2*a*n + n*log(x) + log(c))/(n*p - 2*n))^2 + 2*cosh((a*n*p - 2*a*n + n*log(
x) + log(c))/(n*p - 2*n))*sinh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2
*n)) + sinh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n))^2 + 1)))*cosh(
(a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n)) + (p - 2)*x*cosh((a*n*p -
2*a*n + n*log(x) + log(c))/(n*p - 2*n))*sinh(p*log(2*(cosh((a*n*p - 2*a*n +
 n*log(x) + log(c))/(n*p - 2*n)) + sinh((a*n*p - 2*a*n + n*log(x) + log(c))
/(n*p - 2*n)))/(cosh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n))^2 + 2
*cosh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n))*sinh((a*n*p - 2*a*n
+ n*log(x) + log(c))/(n*p - 2*n)) + sinh((a*n*p - 2*a*n + n*log(x) + log(c)
)/(n*p - 2*n))^2 + 1))))/((p - 1)*cosh((a*n*p - 2*a*n + n*log(x) + log(c))/
(n*p - 2*n)) - (p - 1)*sinh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n)
))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}^{p}\left(a + \frac{\log\left(cx^{n}\right)}{n\left(p-2\right)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+ln(c*x**n)/n/(-2+p))**p,x)
```

```
[Out] Integral(sech(a + log(c*x**n)/(n*(p - 2)))**p, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+log(c*x^n)/n/(-2+p))^p,x, algorithm="giac")
```

```
[Out] integrate(sech(a + log(c*x^n)/(n*(p - 2)))^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{1}{\cosh\left(a+\frac{\ln(c\,x^n)}{n\,(p-2)}\right)}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(a + log(c*x^n)/(n*(p - 2))))^p,x)
[Out] int((1/cosh(a + log(c*x^n)/(n*(p - 2))))^p, x)
```

3.190 $\int \operatorname{sech}^p\left(a-\frac{\log(cx^n)}{n(-2+p)}\right)\,dx$

Optimal. Leaf size=65

$$\frac{(2-p)x\left(1+e^{-2a}(cx^n)^{-\frac{2}{n(2-p)}}\right)\operatorname{sech}^p\left(a+\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

```
[Out] 1/2*(2-p)*x*(1+1/exp(2*a)/((c*x^n)^(2/n/(2-p))))*sech(a+ln(c*x^n)/n/(2-p))^
p/(1-p)
```

Rubi [A]
time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5664, 5668, 270}

$$\frac{(2-p)x\left(e^{-2a}(cx^n)^{-\frac{2}{n(2-p)}}+1\right)\operatorname{sech}^p\left(a+\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a - Log[c*x^n]/(n*(-2 + p))]^p,x]
```

```
[Out] ((2 - p)*x*(1 + 1/(E^(2*a)*(c*x^n)^(2/(n*(2 - p)))))*Sech[a + Log[c*x^n]/(n
*(2 - p))]^p)/(2*(1 - p))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5664

```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5668

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}^p\left(a-\frac{\log(cx^n)}{n(-2+p)}\right)dx = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{sech}^p\left(a-\frac{\log(x)}{n(-2+p)}\right)dx,x,cx^n\right)}{n}$$

$$= \frac{\left(x(cx^n)^{-\frac{1}{n}-\frac{p}{n(-2+p)}}\left(1+e^{-2a}(cx^n)^{\frac{2}{n(-2+p)}}\right)^p\operatorname{sech}^p\left(a-\frac{\log(cx^n)}{n(-2+p)}\right)\right)\text{Subst}\left(\int\right.}{n}$$

$$= \frac{(2-p)x\left(1+e^{-2a}(cx^n)^{-\frac{2}{n(2-p)}}\right)\operatorname{sech}^p\left(a+\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

Mathematica [A]
time = 3.57, size = 108, normalized size = 1.66

$$\frac{2^{-1+p}e^{-a}(-2+p)x(cx^n)^{\frac{1}{n(-2+p)}}\left(\frac{e^{\frac{a(2+p)}{-2+p}}(cx^n)^{\frac{1}{n(-2+p)}}}{e^{\frac{2ap}{-2+p}}+e^{\frac{4a}{-2+p}}(cx^n)^{\frac{2}{n(-2+p)}}}\right)^{-1+p}}{-1+p}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[a - Log[c*x^n]/(n*(-2 + p))]^p,x]
[Out] (2^(-1 + p)*(-2 + p)*x*(c*x^n)^(1/(n*(-2 + p)))*((E^((a*(2 + p))/(-2 + p))*
(c*x^n)^(1/(n*(-2 + p))))/(E^((2*a*p)/(-2 + p)) + E^((4*a)/(-2 + p))*(c*x^n
)^(2/(n*(-2 + p)))))^(-1 + p))/(E^a*(-1 + p))
```

Maple [F]
time = 1.74, size = 0, normalized size = 0.00

$$\int \operatorname{sech}\left(a-\frac{\ln(c\,x^n)}{n\,(-2+p)}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(a-ln(c*x^n)/n/(-2+p))^p,x)
[Out] int(sech(a-ln(c*x^n)/n/(-2+p))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a-log(c*x^n)/n/(-2+p))^p,x, algorithm="maxima")
```

[Out] integrate(sech(-a + log(c*x^n)/(n*(p - 2)))^p, x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 538 vs. $2(55) = 110$.
time = 0.38, size = 538, normalized size = 8.28

$$\frac{(p-2)x\cosh\left(p\log\left(\frac{2\left(\cosh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)+\sinh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)\right)}{\cosh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)^2+2\cosh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)\sinh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)+\sinh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)^2+1}\right)\right)\cosh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)+(p-2)x\cosh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)\sinh\left(p\log\left(\frac{2\left(\cosh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)+\sinh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)\right)}{\cosh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)^2+2\cosh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)\sinh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)+\sinh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)^2+1}\right)\right)}{(p-1)\cosh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)-(p-1)\sinh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(a-log(c*x^n)/n/(-2+p))^p,x, algorithm="fricas")

[Out] ((p - 2)*x*cosh(p*log(2*(cosh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)) + sinh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)))/(cosh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n))^2 + 2*cosh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n))*sinh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)) + sinh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n))^2 + 1)))*cosh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)) + (p - 2)*x*cosh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n))*sinh(p*log(2*(cosh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)) + sinh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)))/(cosh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n))^2 + 2*cosh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n))*sinh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)) + sinh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n))^2 + 1))))/((p - 1)*cosh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)) - (p - 1)*sinh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)))

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}^{p}\left(a - \frac{\log\left(cx^{n}\right)}{n\left(p - 2\right)}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(a-ln(c*x**n)/n/(-2+p))**p,x)

[Out] Integral(sech(a - log(c*x**n)/(n*(p - 2)))**p, x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(a-log(c*x^n)/n/(-2+p))^p,x, algorithm="giac")

[Out] integrate(sech(a - log(c*x^n)/(n*(p - 2)))^p, x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \left(\frac{1}{\cosh\left(a-\frac{\ln(c\,x^n)}{n\,(p-2)}\right)}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(a - log(c*x^n)/(n*(p - 2))))^p,x)
[Out] int((1/cosh(a - log(c*x^n)/(n*(p - 2))))^p, x)
```

3.191 $\int \frac{\operatorname{sech}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=19

$$\frac{\operatorname{ArcTan}(\sinh(a+b\log(cx^n)))}{bn}$$

```
[Out] arctan(sinh(a+b*ln(c*x^n)))/b/n
```

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3855}

$$\frac{\operatorname{ArcTan}(\sinh(a+b\log(cx^n)))}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*Log[c*x^n]]/x,x]
```

```
[Out] ArcTan[Sinh[a + b*Log[c*x^n]]]/(b*n)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}(a+b\log(cx^n))}{x}\,dx &= \frac{\operatorname{Subst}\left(\int \operatorname{sech}(a+bx)\,dx, x, \log(cx^n)\right)}{n}\\ &= \frac{\tan^{-1}(\sinh(a+b\log(cx^n)))}{bn}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 19, normalized size = 1.00

$$\frac{\operatorname{ArcTan}(\sinh(a+b\log(cx^n)))}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*Log[c*x^n]]/x,x]
```

```
[Out] ArcTan[Sinh[a + b*Log[c*x^n]]]/(b*n)
```

Maple [A]
time = 2.60, size = 20, normalized size = 1.05

method	result
derivativedivides	$\frac{\arctan(\sinh(a+b\ln(c\,x^{n})))}{bn}$
default	$\frac{\arctan(\sinh(a+b\ln(c\,x^{n})))}{bn}$
risch	$\frac{i\ln\left(c^{b}(x^{n})^{b}\mathrm{e}^{a}\mathrm{e}^{-\frac{ib\pi \mathrm{csgn}(ic\,x^{n})^{3}}{2}}\mathrm{e}^{\frac{ib\mathrm{csgn}(ic\,x^{n})^{2}\mathrm{csgn}(ic)\pi}{2}}\mathrm{e}^{\frac{ib\mathrm{csgn}(ic\,x^{n})^{2}\mathrm{csgn}(ix^{n})\pi}{2}}\mathrm{e}^{-\frac{ib\,\mathrm{csgn}(ic\,x^{n})\mathrm{csgn}(ic)\mathrm{csgn}(ix^{n})\pi}{2}}+i\right)}{bn}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(a+b*ln(c*x^n))/x,x,method=_RETURNVERBOSE)
```

```
[Out] arctan(sinh(a+b*ln(c*x^n)))/b/n
```

Maxima [A]
time = 0.26, size = 19, normalized size = 1.00

$$\frac{\arctan\left(\sinh\left(b\log\left(cx^{n}\right)+a\right)\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))/x,x, algorithm="maxima")
```

```
[Out] arctan(sinh(b*log(c*x^n) + a))/(b*n)
```

Fricas [A]
time = 0.36, size = 34, normalized size = 1.79

$$\frac{2\arctan\left(\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))/x,x, algorithm="fricas")
```

```
[Out] 2*arctan(cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a))
/(b*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{sech}\left(a+b\log\left(cx^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*ln(c*x**n))/x,x)
```

```
[Out] Integral(sech(a + b*log(c*x**n))/x, x)
```

Giac [A]
time = 0.40, size = 27, normalized size = 1.42

$$\frac{2 \arctan\left(\frac{c^{2\,b} x^{bn} e^{a}}{c^{b}}\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))/x,x, algorithm="giac")
```

```
[Out] 2*arctan(c^(2*b)*x^(b*n)*e^a/c^b)/(b*n)
```

Mupad [B]
time = 1.41, size = 41, normalized size = 2.16

$$-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{-a}\,\sqrt{b^2\,n^2}}{b\,n\,{\left(c\,x^n\right)}^b}\right)}{\sqrt{b^2\,n^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*cosh(a + b*log(c*x^n))),x)
```

```
[Out] -(2*atan((exp(-a)*(b^2*n^2)^(1/2))/(b*n*(c*x^n)^b)))/(b^2*n^2)^(1/2)
```

3.192 $\int \frac{\text{sech}^2(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=18

$$\frac{\tanh\left(a+b\log\left(cx^n\right)\right)}{bn}$$

```
[Out] tanh(a+b*ln(c*x^n))/b/n
```

Rubi [A]
time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3852, 8}

$$\frac{\tanh\left(a+b\log\left(cx^n\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*Log[c*x^n]]^2/x,x]
```

```
[Out] Tanh[a + b*Log[c*x^n]]/(b*n)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\text{sech}^2(a+b\log(cx^n))}{x}\,dx &= \frac{\text{Subst}\left(\int \text{sech}^2(a+bx)\,dx, x, \log(cx^n)\right)}{n}\\ &= \frac{i\text{Subst}\left(\int 1\,dx, x, -i\tanh\left(a+b\log\left(cx^n\right)\right)\right)}{bn}\\ &= \frac{\tanh\left(a+b\log\left(cx^n\right)\right)}{bn}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 18, normalized size = 1.00

$$\frac{\tanh\left(a+b\log\left(cx^n\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*Log[c*x^n]]^2/x,x]
```

```
[Out] Tanh[a + b*Log[c*x^n]]/(b*n)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 3.76, size = 116, normalized size = 6.44

method	result	size
risch	$-\frac{2}{bn\left(\left(x^{n}\right)^{2b} c^{2b} \mathrm{e}^{2a} \mathrm{e}^{-ib\pi \operatorname{csgn}(ic\,x^{n})^{3}} \mathrm{e}^{ib \operatorname{csgn}(ic\,x^{n})^{2} \operatorname{csgn}(ic)\pi} \mathrm{e}^{ib \operatorname{csgn}(ic\,x^{n})^{2} \operatorname{csgn}(ix^{n})\pi} \mathrm{e}^{-ib \operatorname{csgn}(ic\,x^{n}) \operatorname{csgn}(ic) \operatorname{csgn}(ix^{n})\pi}+1\right)}$	116

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(a+b*ln(c*x^n))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] -2/b/n/(((x^n)^b)^2*(c^b)^2*exp(2*a)*exp(-I*b*csgn(I*c*x^n)^3*Pi)*exp(I*b*c
sgn(I*c*x^n)^2*csgn(I*c)*Pi)*exp(I*b*csgn(I*c*x^n)^2*csgn(I*x^n)*Pi)*exp(-I
*b*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)*Pi)+1)
```

Maxima [A]
time = 0.29, size = 28, normalized size = 1.56

$$-\frac{2}{bc^{2\,b}ne^{\left(2\,b\log\left(x^{n}\right)+2\,a\right)}+bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^2/x,x, algorithm="maxima")
```

```
[Out] -2/(b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(18) = 36.
time = 0.36, size = 70, normalized size = 3.89

$$-\frac{2}{bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2\,bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+bn\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^2/x,x, algorithm="fricas")
```

```
[Out] -2/(b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*b*n*cosh(b*n*log(x) + b*log(c
) + a)*sinh(b*n*log(x) + b*log(c) + a) + b*n*sinh(b*n*log(x) + b*log(c) + a
)^2 + b*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^{2}\left(a+b\log\left(cx^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*ln(c*x**n))**2/x,x)
```

```
[Out] Integral(sech(a + b*log(c*x**n))**2/x, x)
```

Giac [A]
time = 0.41, size = 28, normalized size = 1.56

$$-\frac{2}{(c^{2\,b}x^{2\,bn}e^{(2\,a)}+1)bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^2/x,x, algorithm="giac")
```

```
[Out] -2/((c^(2*b)*x^(2*b*n)*e^(2*a) + 1)*b*n)
```

Mupad [B]
time = 1.33, size = 24, normalized size = 1.33

$$-\frac{2}{b\,n+b\,n\,\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*cosh(a + b*log(c*x^n))^2),x)
```

```
[Out] -2/(b*n + b*n*exp(2*a)*(c*x^n)^(2*b))
```

3.193 $\int \frac{\operatorname{sech}^3(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=55

$$\frac{\operatorname{ArcTan}(\sinh(a+b\log(cx^n)))}{2bn}+\frac{\operatorname{sech}(a+b\log(cx^n))\tanh(a+b\log(cx^n))}{2bn}$$

```
[Out] 1/2*arctan(sinh(a+b*ln(c*x^n)))/b/n+1/2*sech(a+b*ln(c*x^n))*tanh(a+b*ln(c*x
^n))/b/n
```

Rubi [A]
time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.118$, Rules used = {3853, 3855}

$$\frac{\operatorname{ArcTan}(\sinh(a+b\log(cx^n)))}{2bn}+\frac{\tanh(a+b\log(cx^n))\operatorname{sech}(a+b\log(cx^n))}{2bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] ArcTan[Sinh[a + b*Log[c*x^n]]]/(2*b*n) + (Sech[a + b*Log[c*x^n]]*Tanh[a + b
*Log[c*x^n]])/(2*b*n)
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^3(a+b\log(cx^n))}{x}\,dx &= \frac{\operatorname{Subst}\left(\int \operatorname{sech}^3(a+bx)\,dx,x,\log(cx^n)\right)}{n}\\ &= \frac{\operatorname{sech}(a+b\log(cx^n))\tanh(a+b\log(cx^n))}{2bn}+\frac{\operatorname{Subst}\left(\int \operatorname{sech}(a+bx)\,dx,x,\log(\right.}{2n}\\ &= \frac{\tan^{-1}(\sinh(a+b\log(cx^n)))}{2bn}+\frac{\operatorname{sech}(a+b\log(cx^n))\tanh(a+b\log(cx^n))}{2bn}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 55, normalized size = 1.00

$$\frac{\text{ArcTan}(\sinh(a+b\log(cx^n)))}{2bn}+\frac{\text{sech}(a+b\log(cx^n))\tanh(a+b\log(cx^n))}{2bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] ArcTan[Sinh[a + b*Log[c*x^n]]]/(2*b*n) + (Sech[a + b*Log[c*x^n]]*Tanh[a + b
*Log[c*x^n]])/(2*b*n)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 4.00, size = 538, normalized size = 9.78

method	result
risch	$\frac{c^b(x^n)^b\left((x^n)^{2b}c^{2b}e^{3a}e^{-\frac{3ib\pi\text{csgn}(icx^n)^3}{2}}e^{\frac{3ib\text{csgn}(icx^n)^2\text{csgn}(ic)\pi}{2}}e^{\frac{3ib\text{csgn}(icx^n)^2\text{csgn}(ix^n)\pi}{2}}e^{-\frac{3ib\,\text{csgn}(icx^n)\text{csgn}(ic)\text{csgn}(ix^n)\pi}{2}}-e^a e^{\cdots}\right.}{bn\left((x^n)^{2b}c^{2b}e^{2a}e^{-ib\pi\text{csgn}(icx^n)^3}e^{ib\text{csgn}(icx^n)^2\text{csgn}(ic)\pi}e^{ib\text{csgn}(icx^n)^2\text{csgn}(\cdots}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(a+b*ln(c*x^n))^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] c^b*(x^n)^b/b/n/(((x^n)^b)^2*(c^b)^2*exp(2*a)*exp(-I*b*csgn(I*c*x^n)^3*Pi)*
exp(I*b*csgn(I*c*x^n)^2*csgn(I*c)*Pi)*exp(I*b*csgn(I*c*x^n)^2*csgn(I*x^n)*P
i)*exp(-I*b*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)*Pi)+1)^2*(((x^n)^b)^2*(c^b)
^2*exp(3*a)*exp(-3/2*I*b*csgn(I*c*x^n)^3*Pi)*exp(3/2*I*b*csgn(I*c*x^n)^2*cs
gn(I*c)*Pi)*exp(3/2*I*b*csgn(I*c*x^n)^2*csgn(I*x^n)*Pi)*exp(-3/2*I*b*csgn(I
*c*x^n)*csgn(I*c)*csgn(I*x^n)*Pi)-exp(a)*exp(-1/2*I*b*csgn(I*c*x^n)^3*Pi)*e
xp(1/2*I*b*csgn(I*c*x^n)^2*csgn(I*c)*Pi)*exp(1/2*I*b*csgn(I*c*x^n)^2*csgn(I
*x^n)*Pi)*exp(-1/2*I*b*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)*Pi))+1/2*I/b/n*l
n(c^b*(x^n)^b*exp(a)*exp(-1/2*I*b*csgn(I*c*x^n)^3*Pi)*exp(1/2*I*b*csgn(I*c*
x^n)^2*csgn(I*c)*Pi)*exp(1/2*I*b*csgn(I*c*x^n)^2*csgn(I*x^n)*Pi)*exp(-1/2*I
*b*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)*Pi)+I)-1/2*I/b/n*ln(c^b*(x^n)^b*exp(
a)*exp(-1/2*I*b*csgn(I*c*x^n)^3*Pi)*exp(1/2*I*b*csgn(I*c*x^n)^2*csgn(I*c)*P
i)*exp(1/2*I*b*csgn(I*c*x^n)^2*csgn(I*x^n)*Pi)*exp(-1/2*I*b*csgn(I*c*x^n)*c
sgn(I*c)*csgn(I*x^n)*Pi)-I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^3/x,x, algorithm="maxima")
```

```
[Out] 8*c^b*integrate(1/8*e^(b*log(x^n) + a)/(c^(2*b)*x*e^(2*b*log(x^n) + 2*a) +
x), x) + (c^(3*b)*e^(3*b*log(x^n) + 3*a) - c^b*e^(b*log(x^n) + a))/(b*c^(4*
b)*n*e^(4*b*log(x^n) + 4*a) + 2*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 452 vs. $2(51) = 102$.
time = 0.37, size = 452, normalized size = 8.22

$$\frac{\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}+3\,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}+\left(\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{4}+4\,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{4}+2\,\left(3\,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+1\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2\,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+4\,\left(\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}+\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+1\right)\arctan\left(\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)+\left(3\,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}-1\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{4}+4\,bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}+bn\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{4}+2\,bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2\,\left(3\,bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+bn\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+bn+4\,\left(bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}+bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^3/x,x, algorithm="fricas")
```

```
[Out] (cosh(b*n*log(x) + b*log(c) + a)^3 + 3*cosh(b*n*log(x) + b*log(c) + a)*sinh
(b*n*log(x) + b*log(c) + a)^2 + sinh(b*n*log(x) + b*log(c) + a)^3 + (cosh(b
*n*log(x) + b*log(c) + a)^4 + 4*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*lo
g(x) + b*log(c) + a)^3 + sinh(b*n*log(x) + b*log(c) + a)^4 + 2*(3*cosh(b*n*
log(x) + b*log(c) + a)^2 + 1)*sinh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*
n*log(x) + b*log(c) + a)^2 + 4*(cosh(b*n*log(x) + b*log(c) + a)^3 + cosh(b*
n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a) + 1)*arctan(cosh(
b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)) + (3*cosh(b*n
*log(x) + b*log(c) + a)^2 - 1)*sinh(b*n*log(x) + b*log(c) + a) - cosh(b*n*l
og(x) + b*log(c) + a))/(b*n*cosh(b*n*log(x) + b*log(c) + a)^4 + 4*b*n*cosh(
b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3 + b*n*sinh(b*n
*log(x) + b*log(c) + a)^4 + 2*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*(3*
b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + b*n)*sinh(b*n*log(x) + b*log(c) + a
)^2 + b*n + 4*(b*n*cosh(b*n*log(x) + b*log(c) + a)^3 + b*n*cosh(b*n*log(x)
+ b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^{3}\left(a + b\log\left(cx^{n}\right)\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*ln(c*x**n))**3/x,x)
```

```
[Out] Integral(sech(a + b*log(c*x**n))**3/x, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(51) = 102$.
time = 0.39, size = 115, normalized size = 2.09

$$c^{3\,b} \left(\frac{\arctan\left(\frac{c^{2\,b} x^{bn} e^{a}}{c^{b}}\right) e^{\left(-3\,a\right)}}{bc^{2\,b} c^{b} n} + \frac{{\left(c^{2\,b} x^{3\,bn} e^{\left(2\,a\right)} - x^{bn}\right)} e^{\left(-2\,a\right)}}{{\left(c^{2\,b} x^{2\,bn} e^{\left(2\,a\right)} + 1\right)}^{2} bc^{2\,b} n}\right)} e^{\left(3\,a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^3/x,x, algorithm="giac")
```

```
[Out] c^(3*b)*(arctan(c^(2*b)*x^(b*n)*e^a/c^b)*e^(-3*a)/(b*c^(2*b)*c^b*n) + (c^(2
*b)*x^(3*b*n)*e^(2*a) - x^(b*n))*e^(-2*a)/((c^(2*b)*x^(2*b*n)*e^(2*a) + 1)^
2*b*c^(2*b)*n))*e^(3*a)
```

Mupad [B]
time = 1.40, size = 139, normalized size = 2.53

$$\frac{2\,\mathrm{e}^{-a}}{{\left(c\,x^n\right)}^b\,\left(b\,n+\frac{2\,b\,n\,\mathrm{e}^{-2\,a}}{{\left(c\,x^n\right)}^{2\,b}}+\frac{b\,n\,\mathrm{e}^{-4\,a}}{{\left(c\,x^n\right)}^{4\,b}}\right)}-\frac{\mathrm{e}^{-a}}{{\left(c\,x^n\right)}^b\,\left(b\,n+\frac{b\,n\,\mathrm{e}^{-2\,a}}{{\left(c\,x^n\right)}^{2\,b}}\right)}-\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{-a}\,\sqrt{b^2\,n^2}}{b\,n\,{\left(c\,x^n\right)}^b}\right)}{\sqrt{b^2\,n^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*cosh(a + b*log(c*x^n))^3),x)
```

```
[Out] (2*exp(-a))/((c*x^n)^b*(b*n + (2*b*n*exp(-2*a))/(c*x^n)^(2*b) + (b*n*exp(-4
*a))/(c*x^n)^(4*b))) - exp(-a)/((c*x^n)^b*(b*n + (b*n*exp(-2*a))/(c*x^n)^(2
*b))) - atan((exp(-a)*(b^2*n^2)^(1/2))/(b*n*(c*x^n)^b))/(b^2*n^2)^(1/2)
```

3.194 $\int \frac{\operatorname{sech}^4(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=42

$$\frac{\tanh\left(a+b\log\left(cx^n\right)\right)}{bn}-\frac{\tanh^3\left(a+b\log\left(cx^n\right)\right)}{3bn}$$

```
[Out] tanh(a+b*ln(c*x^n))/b/n-1/3*tanh(a+b*ln(c*x^n))^3/b/n
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3852}

$$\frac{\tanh\left(a+b\log\left(cx^n\right)\right)}{bn}-\frac{\tanh^3\left(a+b\log\left(cx^n\right)\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] Tanh[a + b*Log[c*x^n]]/(b*n) - Tanh[a + b*Log[c*x^n]]^3/(3*b*n)
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^4(a+b\log\left(cx^n\right))}{x}\,dx&=\frac{\operatorname{Subst}\left(\int \operatorname{sech}^4(a+bx)\,dx,x,\log\left(cx^n\right)\right)}{n}\\&=\frac{i\operatorname{Subst}\left(\int\left(1+x^2\right)\,dx,x,-i\tanh\left(a+b\log\left(cx^n\right)\right)\right)}{bn}\\&=\frac{\tanh\left(a+b\log\left(cx^n\right)\right)}{bn}-\frac{\tanh^3\left(a+b\log\left(cx^n\right)\right)}{3bn}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 42, normalized size = 1.00

$$\frac{\tanh\left(a+b\log\left(cx^n\right)\right)}{bn}-\frac{\tanh^3\left(a+b\log\left(cx^n\right)\right)}{3bn}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[a + b*Log[c*x^n]]^4/x,x]

[Out] Tanh[a + b*Log[c*x^n]]/(b*n) - Tanh[a + b*Log[c*x^n]]^3/(3*b*n)

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 3.99, size = 222, normalized size = 5.29

method	result	size
risch	$-\frac{4\left(3(x^n)^{2b}c^{2b}e^{2a}e^{-ib\pi\mathrm{csgn}(icx^n)^3}e^{ib\mathrm{csgn}(icx^n)^2\mathrm{csgn}(ic)\pi}e^{ib\mathrm{csgn}(icx^n)^2\mathrm{csgn}(ix^n)\pi}e^{-ib\,\mathrm{csgn}(icx^n)\mathrm{csgn}(ic)\mathrm{csgn}(ix^n)\pi}+1\right)}{3bn\left((x^n)^{2b}c^{2b}e^{2a}e^{-ib\pi\mathrm{csgn}(icx^n)^3}e^{ib\mathrm{csgn}(icx^n)^2\mathrm{csgn}(ic)\pi}e^{ib\mathrm{csgn}(icx^n)^2\mathrm{csgn}(ix^n)\pi}e^{-ib\,\mathrm{csgn}(icx^n)\mathrm{csgn}(ic)\mathrm{csgn}(ix^n)\pi}+1\right)^3}$	222

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(a+b*ln(c*x^n))^4/x,x,method=_RETURNVERBOSE)

[Out] -4/3*(3*((x^n)^b)^2*(c^b)^2*exp(2*a)*exp(-I*b*csgn(I*c*x^n)^3*Pi)*exp(I*b*csgn(I*c*x^n)^2*csgn(I*c)*Pi)*exp(I*b*csgn(I*c*x^n)^2*csgn(I*x^n)*Pi)*exp(-I*b*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)*Pi)+1)/b/n/(((x^n)^b)^2*(c^b)^2*exp(2*a)*exp(-I*b*csgn(I*c*x^n)^3*Pi)*exp(I*b*csgn(I*c*x^n)^2*csgn(I*c)*Pi)*exp(I*b*csgn(I*c*x^n)^2*csgn(I*x^n)*Pi)*exp(-I*b*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)*Pi)+1)^3

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(40) = 80.
time = 0.32, size = 91, normalized size = 2.17

$$-\frac{4\left(3\,c^{2\,b}e^{\left(2\,b\log\left(x^n\right)+2\,a\right)}+1\right)}{3\left(bc^{6\,b}ne^{\left(6\,b\log\left(x^n\right)+6\,a\right)}+3\,bc^{4\,b}ne^{\left(4\,b\log\left(x^n\right)+4\,a\right)}+3\,bc^{2\,b}ne^{\left(2\,b\log\left(x^n\right)+2\,a\right)}+bn\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(a+b*log(c*x^n))^4/x,x, algorithm="maxima")

[Out] -4/3*(3*c^(2*b)*e^(2*b*log(x^n) + 2*a) + 1)/(b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a) + 3*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 3*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(40) = 80.
time = 0.35, size = 272, normalized size = 6.48

$$-\frac{8\left(2\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)}{3\left(bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^5+5\,bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^4+bn\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^5+3\,bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^3+\left(10\,bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2+3\,bn\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^3+4\,bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\left(10\,bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^3+9\,bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2+\left(5\,bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^4+9\,bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2+2\,bn\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(a+b*log(c*x^n))^4/x,x, algorithm="fricas")

[Out] -8/3*(2*cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a))/(b*n*cosh(b*n*log(x) + b*log(c) + a)^5 + 5*b*n*cosh(b*n*log(x) + b*log(c) +

```
 a)*sinh(b*n*log(x) + b*log(c) + a)^4 + b*n*sinh(b*n*log(x) + b*log(c) + a)
^5 + 3*b*n*cosh(b*n*log(x) + b*log(c) + a)^3 + (10*b*n*cosh(b*n*log(x) + b*
log(c) + a)^2 + 3*b*n)*sinh(b*n*log(x) + b*log(c) + a)^3 + 4*b*n*cosh(b*n*l
og(x) + b*log(c) + a) + (10*b*n*cosh(b*n*log(x) + b*log(c) + a)^3 + 9*b*n*c
osh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^2 + (5*b*n*
cosh(b*n*log(x) + b*log(c) + a)^4 + 9*b*n*cosh(b*n*log(x) + b*log(c) + a)^2
 + 2*b*n)*sinh(b*n*log(x) + b*log(c) + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^4\left(a+b\log\left(cx^n\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*ln(c*x**n))**4/x,x)
```

```
[Out] Integral(sech(a + b*log(c*x**n))**4/x, x)
```

Giac [A]
time = 0.41, size = 47, normalized size = 1.12

$$-\frac{4\left(3\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+1\right)}{3\left(c^{2\,b}x^{2\,bn}e^{(2\,a)}+1\right)^3bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^4/x,x, algorithm="giac")
```

```
[Out] -4/3*(3*c^(2*b)*x^(2*b*n)*e^(2*a) + 1)/((c^(2*b)*x^(2*b*n)*e^(2*a) + 1)^3*b
*n)
```

Mupad [B]
time = 1.34, size = 55, normalized size = 1.31

$$\frac{4\,\mathrm{e}^{4\,a}\,\left(c\,x^n\right)^{4\,b}\,\left(\mathrm{e}^{2\,a}\,\left(c\,x^n\right)^{2\,b}+3\right)}{3\,b\,n\,\left(\mathrm{e}^{2\,a}\,\left(c\,x^n\right)^{2\,b}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*cosh(a + b*log(c*x^n))^4),x)
```

```
[Out] (4*exp(4*a)*(c*x^n)^(4*b)*(exp(2*a)*(c*x^n)^(2*b) + 3))/(3*b*n*(exp(2*a)*(c
*x^n)^(2*b) + 1)^3)
```

3.195 $\int \frac{\text{sech}^5(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=89

$$\frac{3\text{ArcTan}(\sinh(a+b\log(cx^n)))}{8bn}+\frac{3\text{sech}(a+b\log(cx^n))\tanh(a+b\log(cx^n))}{8bn}+\frac{\text{sech}^3(a+b\log(cx^n))\tanh(}{4bn}$$

```
[Out] 3/8*arctan(sinh(a+b*ln(c*x^n)))/b/n+3/8*sech(a+b*ln(c*x^n))*tanh(a+b*ln(c*x
^n))/b/n+1/4*sech(a+b*ln(c*x^n))^3*tanh(a+b*ln(c*x^n))/b/n
```

Rubi [A]

time = 0.04, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3853, 3855}

$$\frac{3\text{ArcTan}(\sinh(a+b\log(cx^n)))}{8bn}+\frac{\tanh(a+b\log(cx^n))\,\text{sech}^3(a+b\log(cx^n))}{4bn}+\frac{3\tanh(a+b\log(cx^n))\,\text{sech}(a+b\log(cx^n))}{8bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*Log[c*x^n]]^5/x,x]
```

```
[Out] (3*ArcTan[Sinh[a + b*Log[c*x^n]]])/(8*b*n) + (3*Sech[a + b*Log[c*x^n]]*Tanh
[a + b*Log[c*x^n]])/(8*b*n) + (Sech[a + b*Log[c*x^n]]^3*Tanh[a + b*Log[c*x^
n]])/(4*b*n)
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\text{sech}^5(a+b\log(cx^n))}{x}\,dx &= \frac{\text{Subst}\left(\int \text{sech}^5(a+bx)\,dx, x, \log(cx^n)\right)}{n}\\ &= \frac{\text{sech}^3(a+b\log(cx^n))\tanh(a+b\log(cx^n))}{4bn}+\frac{3\text{Subst}\left(\int \text{sech}^3(a+bx)\,dx, x,\right.}{4n}\\ &= \frac{3\text{sech}(a+b\log(cx^n))\tanh(a+b\log(cx^n))}{8bn}+\frac{\text{sech}^3(a+b\log(cx^n))\tanh(a}{4bn}\\ &= \frac{3\tan^{-1}(\sinh(a+b\log(cx^n)))}{8bn}+\frac{3\text{sech}(a+b\log(cx^n))\tanh(a+b\log(cx^n))}{8bn}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 75, normalized size = 0.84

$$\frac{3\text{ArcTan}(\sinh(a+b\log(cx^n)))+3\text{sech}(a+b\log(cx^n))\tanh(a+b\log(cx^n))+2\text{sech}^3(a+b\log(cx^n))\tanh(a+b\log(cx^n))}{8bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*Log[c*x^n]]^5/x,x]
```

```
[Out] (3*ArcTan[Sinh[a + b*Log[c*x^n]]] + 3*Sech[a + b*Log[c*x^n]]*Tanh[a + b*Log
[c*x^n]] + 2*Sech[a + b*Log[c*x^n]]^3*Tanh[a + b*Log[c*x^n]])/(8*b*n)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 4.45, size = 748, normalized size = 8.40

method	result
risch	$c^b(x^n)^b\left(3(x^n)^{6b}c^{6b}e^{7a}e^{-\frac{7ib\pi\text{csgn}(icx^n)^3}{2}}e^{\frac{7ib\text{csgn}(icx^n)^2\text{csgn}(ic)\pi}{2}}e^{\frac{7ib\text{csgn}(icx^n)^2\text{csgn}(ix^n)\pi}{2}}e^{-\frac{7ib\,\text{csgn}(icx^n)\text{csgn}(ic)\text{csgn}(ix^n)\pi}{2}}+11(x^{\cdots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(a+b*ln(c*x^n))^5/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*c^b*(x^n)^b/b/n/(((x^n)^b)^2*(c^b)^2*exp(2*a)*exp(-I*b*csgn(I*c*x^n)^3*
Pi)*exp(I*b*csgn(I*c*x^n)^2*csgn(I*c)*Pi)*exp(I*b*csgn(I*c*x^n)^2*csgn(I*x^
n)*Pi)*exp(-I*b*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)*Pi)+1)^4*(3*((x^n)^b)^6
*(c^b)^6*exp(7*a)*exp(-7/2*I*b*csgn(I*c*x^n)^3*Pi)*exp(7/2*I*b*csgn(I*c*x^n
)^2*csgn(I*c)*Pi)*exp(7/2*I*b*csgn(I*c*x^n)^2*csgn(I*x^n)*Pi)*exp(-7/2*I*b*
csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)*Pi)+11*((x^n)^b)^4*(c^b)^4*exp(5*a)*exp
(-5/2*I*b*csgn(I*c*x^n)^3*Pi)*exp(5/2*I*b*csgn(I*c*x^n)^2*csgn(I*c)*Pi)*exp
(5/2*I*b*csgn(I*c*x^n)^2*csgn(I*x^n)*Pi)*exp(-5/2*I*b*csgn(I*c*x^n)*csgn(I*
c)*csgn(I*x^n)*Pi)-11*((x^n)^b)^2*(c^b)^2*exp(3*a)*exp(-3/2*I*b*csgn(I*c*x^
n)^3*Pi)*exp(3/2*I*b*csgn(I*c*x^n)^2*csgn(I*c)*Pi)*exp(3/2*I*b*csgn(I*c*x^n
)^2*csgn(I*x^n)*Pi)*exp(-3/2*I*b*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)*Pi)-3*
exp(a)*exp(-1/2*I*b*csgn(I*c*x^n)^3*Pi)*exp(1/2*I*b*csgn(I*c*x^n)^2*csgn(I*
c)*Pi)*exp(1/2*I*b*csgn(I*c*x^n)^2*csgn(I*x^n)*Pi)*exp(-1/2*I*b*csgn(I*c*x^
n)*csgn(I*c)*csgn(I*x^n)*Pi))+3/8*I/b/n*ln(c^b*(x^n)^b*exp(a)*exp(-1/2*I*b*
csgn(I*c*x^n)^3*Pi)*exp(1/2*I*b*csgn(I*c*x^n)^2*csgn(I*c)*Pi)*exp(1/2*I*b*c
sgn(I*c*x^n)^2*csgn(I*x^n)*Pi)*exp(-1/2*I*b*csgn(I*c*x^n)*csgn(I*c)*csgn(I*
x^n)*Pi)+I)-3/8*I/b/n*ln(c^b*(x^n)^b*exp(a)*exp(-1/2*I*b*csgn(I*c*x^n)^3*Pi
)*exp(1/2*I*b*csgn(I*c*x^n)^2*csgn(I*c)*Pi)*exp(1/2*I*b*csgn(I*c*x^n)^2*csg
n(I*x^n)*Pi)*exp(-1/2*I*b*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)*Pi)-I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^5/x,x, algorithm="maxima")
```

```
[Out] 96*c^b*integrate(1/128*e^(b*log(x^n) + a)/(c^(2*b)*x*e^(2*b*log(x^n) + 2*a)
 + x), x) + 1/4*(3*c^(7*b)*e^(7*b*log(x^n) + 7*a) + 11*c^(5*b)*e^(5*b*log(x
^n) + 5*a) - 11*c^(3*b)*e^(3*b*log(x^n) + 3*a) - 3*c^b*e^(b*log(x^n) + a))/
(b*c^(8*b)*n*e^(8*b*log(x^n) + 8*a) + 4*b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a)
+ 6*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 4*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*
a) + b*n)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1326 vs. $2(83) = 166$.
time = 0.40, size = 1326, normalized size = 14.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^5/x,x, algorithm="fricas")
```

```
[Out] 1/4*(3*cosh(b*n*log(x) + b*log(c) + a)^7 + 21*cosh(b*n*log(x) + b*log(c) +
a)*sinh(b*n*log(x) + b*log(c) + a)^6 + 3*sinh(b*n*log(x) + b*log(c) + a)^7
+ (63*cosh(b*n*log(x) + b*log(c) + a)^2 + 11)*sinh(b*n*log(x) + b*log(c) +
a)^5 + 11*cosh(b*n*log(x) + b*log(c) + a)^5 + 5*(21*cosh(b*n*log(x) + b*log
(c) + a)^3 + 11*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c)
 + a)^4 + (105*cosh(b*n*log(x) + b*log(c) + a)^4 + 110*cosh(b*n*log(x) + b*
log(c) + a)^2 - 11)*sinh(b*n*log(x) + b*log(c) + a)^3 - 11*cosh(b*n*log(x)
+ b*log(c) + a)^3 + (63*cosh(b*n*log(x) + b*log(c) + a)^5 + 110*cosh(b*n*lo
g(x) + b*log(c) + a)^3 - 33*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x
) + b*log(c) + a)^2 + 3*(cosh(b*n*log(x) + b*log(c) + a)^8 + 8*cosh(b*n*log
(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^7 + sinh(b*n*log(x) + b
*log(c) + a)^8 + 4*(7*cosh(b*n*log(x) + b*log(c) + a)^2 + 1)*sinh(b*n*log(x
) + b*log(c) + a)^6 + 4*cosh(b*n*log(x) + b*log(c) + a)^6 + 8*(7*cosh(b*n*l
og(x) + b*log(c) + a)^3 + 3*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x
) + b*log(c) + a)^5 + 2*(35*cosh(b*n*log(x) + b*log(c) + a)^4 + 30*cosh(b*n
*log(x) + b*log(c) + a)^2 + 3)*sinh(b*n*log(x) + b*log(c) + a)^4 + 6*cosh(b
*n*log(x) + b*log(c) + a)^4 + 8*(7*cosh(b*n*log(x) + b*log(c) + a)^5 + 10*c
osh(b*n*log(x) + b*log(c) + a)^3 + 3*cosh(b*n*log(x) + b*log(c) + a))*sinh(
b*n*log(x) + b*log(c) + a)^3 + 4*(7*cosh(b*n*log(x) + b*log(c) + a)^6 + 15*
cosh(b*n*log(x) + b*log(c) + a)^4 + 9*cosh(b*n*log(x) + b*log(c) + a)^2 + 1
)*sinh(b*n*log(x) + b*log(c) + a)^2 + 4*cosh(b*n*log(x) + b*log(c) + a)^2 +
 8*(cosh(b*n*log(x) + b*log(c) + a)^7 + 3*cosh(b*n*log(x) + b*log(c) + a)^5
 + 3*cosh(b*n*log(x) + b*log(c) + a)^3 + cosh(b*n*log(x) + b*log(c) + a))*s
inh(b*n*log(x) + b*log(c) + a) + 1)*arctan(cosh(b*n*log(x) + b*log(c) + a)
+ sinh(b*n*log(x) + b*log(c) + a)) + (21*cosh(b*n*log(x) + b*log(c) + a)^6
+ 55*cosh(b*n*log(x) + b*log(c) + a)^4 - 33*cosh(b*n*log(x) + b*log(c) + a)
```

```
^2 - 3)*sinh(b*n*log(x) + b*log(c) + a) - 3*cosh(b*n*log(x) + b*log(c) + a)
)/(b*n*cosh(b*n*log(x) + b*log(c) + a)^8 + 8*b*n*cosh(b*n*log(x) + b*log(c)
 + a)*sinh(b*n*log(x) + b*log(c) + a)^7 + b*n*sinh(b*n*log(x) + b*log(c) +
a)^8 + 4*b*n*cosh(b*n*log(x) + b*log(c) + a)^6 + 4*(7*b*n*cosh(b*n*log(x) +
 b*log(c) + a)^2 + b*n)*sinh(b*n*log(x) + b*log(c) + a)^6 + 6*b*n*cosh(b*n*
log(x) + b*log(c) + a)^4 + 8*(7*b*n*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*b
*n*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^5 + 2*(
35*b*n*cosh(b*n*log(x) + b*log(c) + a)^4 + 30*b*n*cosh(b*n*log(x) + b*log(c
) + a)^2 + 3*b*n)*sinh(b*n*log(x) + b*log(c) + a)^4 + 4*b*n*cosh(b*n*log(x)
 + b*log(c) + a)^2 + 8*(7*b*n*cosh(b*n*log(x) + b*log(c) + a)^5 + 10*b*n*co
sh(b*n*log(x) + b*log(c) + a)^3 + 3*b*n*cosh(b*n*log(x) + b*log(c) + a))*si
nh(b*n*log(x) + b*log(c) + a)^3 + 4*(7*b*n*cosh(b*n*log(x) + b*log(c) + a)^
6 + 15*b*n*cosh(b*n*log(x) + b*log(c) + a)^4 + 9*b*n*cosh(b*n*log(x) + b*lo
g(c) + a)^2 + b*n)*sinh(b*n*log(x) + b*log(c) + a)^2 + b*n + 8*(b*n*cosh(b*
n*log(x) + b*log(c) + a)^7 + 3*b*n*cosh(b*n*log(x) + b*log(c) + a)^5 + 3*b*
n*cosh(b*n*log(x) + b*log(c) + a)^3 + b*n*cosh(b*n*log(x) + b*log(c) + a))*
sinh(b*n*log(x) + b*log(c) + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^5\left(a+b\log\left(cx^n\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*ln(c*x**n))**5/x,x)
```

```
[Out] Integral(sech(a + b*log(c*x**n))**5/x, x)
```

Giac [A]
time = 0.41, size = 152, normalized size = 1.71

$$\frac{1}{4}\,c^{5\,b}\left(\frac{3\,\arctan\left(\frac{c^{2\,b}x^{bn}e^{a}}{c^{b}}\right)e^{(-5\,a)}}{bc^{4\,b}c^{b}n}+\frac{\left(3\,c^{6\,b}x^{7\,bn}e^{(6\,a)}+11\,c^{4\,b}x^{5\,bn}e^{(4\,a)}-11\,c^{2\,b}x^{3\,bn}e^{(2\,a)}-3\,x^{bn}\right)e^{(-4\,a)}}{\left(c^{2\,b}x^{2\,bn}e^{(2\,a)}+1\right)^{4}bc^{4\,b}n}\right)e^{(5\,a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^5/x,x, algorithm="giac")
```

```
[Out] 1/4*c^(5*b)*(3*arctan(c^(2*b)*x^(b*n)*e^a/c^b)*e^(-5*a)/(b*c^(4*b)*c^b*n) +
 (3*c^(6*b)*x^(7*b*n)*e^(6*a) + 11*c^(4*b)*x^(5*b*n)*e^(4*a) - 11*c^(2*b)*x
^(3*b*n)*e^(2*a) - 3*x^(b*n))*e^(-4*a)/((c^(2*b)*x^(2*b*n)*e^(2*a) + 1)^4*b
*c^(4*b)*n))*e^(5*a)
```

Mupad [B]
time = 1.35, size = 314, normalized size = 3.53

$$\frac{2\,\mathrm{e}^{-a}}{{\left(c\,x^n\right)}^b\,\left(b\,n+\frac{3\,b\,n\,\mathrm{e}^{-2\,a}}{{\left(c\,x^n\right)}^{2\,b}}+\frac{3\,b\,n\,\mathrm{e}^{-4\,a}}{{\left(c\,x^n\right)}^{4\,b}}+\frac{b\,n\,\mathrm{e}^{-6\,a}}{{\left(c\,x^n\right)}^{6\,b}}\right)}-\frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^{-a}\,\sqrt{b^2\,n^2}}{b\,n\,{\left(c\,x^n\right)}^b}\right)}{4\,\sqrt{b^2\,n^2}}-\frac{3\,\mathrm{e}^{-a}}{4\,{\left(c\,x^n\right)}^b\,\left(b\,n+\frac{b\,n\,\mathrm{e}^{-2\,a}}{{\left(c\,x^n\right)}^{2\,b}}\right)}+\frac{4\,\mathrm{e}^{-3\,a}}{{\left(c\,x^n\right)}^{3\,b}\,\left(b\,n+\frac{4\,b\,n\,\mathrm{e}^{-2\,a}}{{\left(c\,x^n\right)}^{2\,b}}+\frac{6\,b\,n\,\mathrm{e}^{-4\,a}}{{\left(c\,x^n\right)}^{4\,b}}+\frac{4\,b\,n\,\mathrm{e}^{-6\,a}}{{\left(c\,x^n\right)}^{6\,b}}+\frac{b\,n\,\mathrm{e}^{-8\,a}}{{\left(c\,x^n\right)}^{8\,b}}\right)}-\frac{\mathrm{e}^{-a}}{2\,{\left(c\,x^n\right)}^b\,\left(b\,n+\frac{2\,b\,n\,\mathrm{e}^{-2\,a}}{{\left(c\,x^n\right)}^{2\,b}}+\frac{b\,n\,\mathrm{e}^{-4\,a}}{{\left(c\,x^n\right)}^{4\,b}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*cosh(a + b*log(c*x^n))^5),x)
```

```
[Out] (2*exp(-a))/((c*x^n)^b*(b*n + (3*b*n*exp(-2*a))/(c*x^n)^(2*b) + (3*b*n*exp(
-4*a))/(c*x^n)^(4*b) + (b*n*exp(-6*a))/(c*x^n)^(6*b))) - (3*atan((exp(-a)*(
b^2*n^2)^(1/2))/(b*n*(c*x^n)^b)))/(4*(b^2*n^2)^(1/2)) - (3*exp(-a))/(4*(c*x
^n)^b*(b*n + (b*n*exp(-2*a))/(c*x^n)^(2*b))) + (4*exp(-3*a))/((c*x^n)^(3*b)
*(b*n + (4*b*n*exp(-2*a))/(c*x^n)^(2*b) + (6*b*n*exp(-4*a))/(c*x^n)^(4*b) +
 (4*b*n*exp(-6*a))/(c*x^n)^(6*b) + (b*n*exp(-8*a))/(c*x^n)^(8*b))) - exp(-a
)/(2*(c*x^n)^b*(b*n + (2*b*n*exp(-2*a))/(c*x^n)^(2*b) + (b*n*exp(-4*a))/(c*
x^n)^(4*b)))
```

3.196 $\int \frac{\operatorname{sech}^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=97

$$-\frac{2i\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}\,F\left(\frac{1}{2}i(a+b\log\left(cx^n\right))\middle|\,2\right)\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}}{3bn}+\frac{2\operatorname{sech}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)\sinh(}{3bn}$$

```
[Out] 2/3*sech(a+b*ln(c*x^n))^(3/2)*sinh(a+b*ln(c*x^n))/b/n-2/3*I*(cosh(1/2*a+1/2
*b*ln(c*x^n))^2)^(1/2)/cosh(1/2*a+1/2*b*ln(c*x^n))*EllipticF(I*sinh(1/2*a+1
/2*b*ln(c*x^n)),2^(1/2))*cosh(a+b*ln(c*x^n))^(1/2)*sech(a+b*ln(c*x^n))^(1/2
)/b/n
```

Rubi [A]
time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3853, 3856, 2720}

$$\frac{2\sinh\left(a+b\log\left(cx^n\right)\right)\operatorname{sech}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{3bn}-\frac{2i\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}\,F\left(\frac{1}{2}i(a+b\log\left(cx^n\right))\middle|\,2\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] (((-2*I)/3)*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticF[(I/2)*(a + b*Log[c*x^n])
, 2]*Sqrt[Sech[a + b*Log[c*x^n]]])/(b*n) + (2*Sech[a + b*Log[c*x^n]]^(3/2)*
Sinh[a + b*Log[c*x^n]])/(3*b*n)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\operatorname{sech}^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{\operatorname{Subst}\left(\int \operatorname{sech}^{\frac{5}{2}}(a+bx)\,dx, x, \log(cx^n)\right)}{n}$$

$$= \frac{2\operatorname{sech}^{\frac{3}{2}}(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{3bn} + \frac{\operatorname{Subst}\left(\int \sqrt{\operatorname{sech}(a+bx)}\,dx\right.}{3n}$$

$$= \frac{2\operatorname{sech}^{\frac{3}{2}}(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{3bn} + \frac{\left(\sqrt{\cosh(a+b\log(cx^n))}\sqrt{}\right.}{}$$

$$= -\frac{2i\sqrt{\cosh(a+b\log(cx^n))}\,F\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)\sqrt{\operatorname{sech}(a+b\log(cx^n)}}{3bn}$$

Mathematica [A]
time = 0.12, size = 74, normalized size = 0.76

$$\frac{2\operatorname{sech}^{\frac{3}{2}}(a+b\log(cx^n))\left(-i\cosh^{\frac{3}{2}}(a+b\log(cx^n))\,F\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)+\sinh(a+b\log(cx^n))\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] (2*Sech[a + b*Log[c*x^n]]^(3/2)*((-I)*Cosh[a + b*Log[c*x^n]]^(3/2)*Elliptic
F[(I/2)*(a + b*Log[c*x^n]), 2] + Sinh[a + b*Log[c*x^n]]))/(3*b*n)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(123) = 246$.
time = 5.25, size = 295, normalized size = 3.04

method	result
derivativedivides	$2\left(2\sqrt{-\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)}\sqrt{-2\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)-1}\ \operatorname{EllipticF}\left(\cosh\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right.\right.\right.$
default	$2\left(2\sqrt{-\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)}\sqrt{-2\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)-1}\ \operatorname{EllipticF}\left(\cosh\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(a+b*ln(c*x^n))^(5/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/n*(2*(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*sinh(1/2*a+1/2*b*ln(c*x
^n))^2-1)^(1/2)*EllipticF(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))*sinh(1/2*a+1
/2*b*ln(c*x^n))^2+2*cosh(1/2*a+1/2*b*ln(c*x^n))*sinh(1/2*a+1/2*b*ln(c*x^n))
^2+(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*sinh(1/2*a+1/2*b*ln(c*x^n))^2
-1)^(1/2)*EllipticF(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2)))*((2*cosh(1/2*a+1/
2*b*ln(c*x^n))^2-1)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/(2*sinh(1/2*a+1/2*
b*ln(c*x^n))^4+sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/(2*cosh(1/2*a+1/2*b*ln(
c*x^n))^2-1)^(3/2)/sinh(1/2*a+1/2*b*ln(c*x^n))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^(5/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sech(b*log(c*x^n) + a)^(5/2)/x, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 315, normalized size = 3.25

$$\frac{2\left(\sqrt{2}\left(\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}-1\right)\sqrt{\frac{\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+1}}+\left(\sqrt{2}\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2\sqrt{2}\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sqrt{2}\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+\sqrt{2}\right){\rm weierstrassPInverse}\left(-4,0,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)\right)}{3\left(bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+bn\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+bn\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")
```

```
[Out] 2/3*(sqrt(2)*(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log
(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^
2 - 1)*sqrt((cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) +
 a))/(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)
*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 + 1))
+ (sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*sqrt(2)*cosh(b*n*log(x) +
b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sqrt(2)*sinh(b*n*log(x) + b
*log(c) + a)^2 + sqrt(2))*weierstrassPInverse(-4, 0, cosh(b*n*log(x) + b*lo
g(c) + a) + sinh(b*n*log(x) + b*log(c) + a)))/(b*n*cosh(b*n*log(x) + b*log(
c) + a)^2 + 2*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c
) + a) + b*n*sinh(b*n*log(x) + b*log(c) + a)^2 + b*n)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*ln(c*x**n))**(5/2)/x,x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^(5/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cosh(a+b\,\ln(c\,x^n))}\right)^{5/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(a + b*log(c*x^n)))^(5/2)/x,x)
```

```
[Out] int((1/cosh(a + b*log(c*x^n)))^(5/2)/x, x)
```

3.197 $\int \frac{\operatorname{sech}^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=93

$$\frac{2i\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}\,E\left(\frac{1}{2}i(a+b\log\left(cx^n\right))\middle|2\right)\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}}{bn}+\frac{2\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}\,\sinh}{bn}$$

[Out] 2*sinh(a+b*ln(c*x^n))*sech(a+b*ln(c*x^n))^(1/2)/b/n+2*I*(cosh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/cosh(1/2*a+1/2*b*ln(c*x^n))*EllipticE(I*sinh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))*cosh(a+b*ln(c*x^n))^(1/2)*sech(a+b*ln(c*x^n))^(1/2)/b/n

Rubi [A]
time = 0.04, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3853, 3856, 2719}

$$\frac{2\sinh\left(a+b\log\left(cx^n\right)\right)\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}}{bn}+\frac{2i\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}\,E\left(\frac{1}{2}i(a+b\log\left(cx^n\right))\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

[In] Int[Sech[a + b*Log[c*x^n]]^(3/2)/x,x]

[Out] ((2*I)*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2]*Sqrt[Sech[a + b*Log[c*x^n]]])/(b*n) + (2*Sqrt[Sech[a + b*Log[c*x^n]]]*Sinh[a + b*Log[c*x^n]])/(b*n)

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{\operatorname{Subst}\left(\int \operatorname{sech}^{\frac{3}{2}}(a+bx)\,dx, x, \log(cx^n)\right)}{n}$$

$$= \frac{2\sqrt{\operatorname{sech}(a+b\log(cx^n))}\sinh(a+b\log(cx^n))}{bn} - \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}}\right.}{n}$$

$$= \frac{2\sqrt{\operatorname{sech}(a+b\log(cx^n))}\sinh(a+b\log(cx^n))}{bn} - \frac{\left(\sqrt{\cosh(a+b\log(cx^n))}\right.}{}$$

$$= \frac{2i\sqrt{\cosh(a+b\log(cx^n))}\,E\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)\sqrt{\operatorname{sech}(a+b\log(cx^n))}}{bn}$$

Mathematica [A]
time = 0.07, size = 72, normalized size = 0.77

$$\frac{2\sqrt{\operatorname{sech}(a+b\log(cx^n))}\left(i\sqrt{\cosh(a+b\log(cx^n))}\,E\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)+\sinh(a+b\log(cx^n))\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] (2*Sqrt[Sech[a + b*Log[c*x^n]]]*(I*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticE[(
I/2)*(a + b*Log[c*x^n]), 2] + Sinh[a + b*Log[c*x^n]]))/(b*n)
```

Maple [A]
time = 4.99, size = 141, normalized size = 1.52

method	result
derivativedivides	$\frac{4\cosh\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)+2\sqrt{-\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)}\sqrt{-2\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(\right.\right.}}}{n\sinh\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\sqrt{2\left(\cosh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)-1}}$
default	$\frac{4\cosh\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)+2\sqrt{-\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)}\sqrt{-2\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(\right.\right.}}}{n\sinh\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\sqrt{2\left(\cosh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(a+b*ln(c*x^n))^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/n*(2*cosh(1/2*a+1/2*b*ln(c*x^n))*sinh(1/2*a+1/2*b*ln(c*x^n))^2+(-sinh(1/2
*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*sinh(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)*Ell
```

```
ipticE(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2)))/sinh(1/2*a+1/2*b*ln(c*x^n))/(2
*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sech(b*log(c*x^n) + a)^(3/2)/x, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 159, normalized size = 1.71

$$\frac{2 \left(\sqrt{2} \sqrt{\frac{\cosh \left(bn \log \left(x\right) + b \log \left(c\right) + a\right) + \sinh \left(bn \log \left(x\right) + b \log \left(c\right) + a\right)}{\cosh \left(bn \log \left(x\right) + b \log \left(c\right) + a\right)^{2} + 2 \cosh \left(bn \log \left(x\right) + b \log \left(c\right) + a\right) \sinh \left(bn \log \left(x\right) + b \log \left(c\right) + a\right) + \sinh \left(bn \log \left(x\right) + b \log \left(c\right) + a\right)^{2} + 1}} \left(\cosh \left(bn \log \left(x\right) + b \log \left(c\right) + a\right) + \sinh \left(bn \log \left(x\right) + b \log \left(c\right) + a\right)\right) + \sqrt{2}\, \mathrm{weierstrassZeta}\left(-4, 0, \mathrm{weierstrassPInverse}\left(-4, 0, \cosh \left(bn \log \left(x\right) + b \log \left(c\right) + a\right) + \sinh \left(bn \log \left(x\right) + b \log \left(c\right) + a\right)\right)\right)\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 2*(sqrt(2)*sqrt((cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(
c) + a))/(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c)
+ a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 +
1))*(cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)) + s
qrt(2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cosh(b*n*log(x) +
b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a))))/(b*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}{\left(a + b \log{\left(c x^{n} \right)} \right)}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*ln(c*x**n))**(3/2)/x,x)
```

```
[Out] Integral(sech(a + b*log(c*x**n))**(3/2)/x, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^(3/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cosh(a+b\,\ln(c\,x^n))}\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(a + b*log(c*x^n)))^(3/2)/x,x)
[Out] int((1/cosh(a + b*log(c*x^n)))^(3/2)/x, x)
```

3.198 $\int \frac{\sqrt{\operatorname{sech}(a+b\log(cx^n))}}{x}\,dx$

Optimal. Leaf size=58

$$-\frac{2i\sqrt{\cosh(a+b\log(cx^n))}\,F\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)\sqrt{\operatorname{sech}(a+b\log(cx^n))}}{bn}$$

```
[Out] -2*I*(cosh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/cosh(1/2*a+1/2*b*ln(c*x^n))*Elli
pticF(I*sinh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))*cosh(a+b*ln(c*x^n))^(1/2)*sech
(a+b*ln(c*x^n))^(1/2)/b/n
```

Rubi [A]
time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3856, 2720}

$$-\frac{2i\sqrt{\operatorname{sech}(a+b\log(cx^n))}\sqrt{\cosh(a+b\log(cx^n))}\,F\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sech[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] ((-2*I)*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticF[(I/2)*(a + b*Log[c*x^n]), 2]
*Sqrt[Sech[a + b*Log[c*x^n]]])/(b*n)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\sqrt{\operatorname{sech}(a+b\log(cx^n))}}{x}\,dx = \frac{\operatorname{Subst}\left(\int \sqrt{\operatorname{sech}(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$

$$= \frac{\left(\sqrt{\cosh(a+b\log(cx^n))}\sqrt{\operatorname{sech}(a+b\log(cx^n))}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{\cosh(a+b}\right.}{n}$$

$$= -\frac{2i\sqrt{\cosh(a+b\log(cx^n))}\,F\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)\sqrt{\operatorname{sech}(a+b\log(cx^n}}{bn}$$

Mathematica [**A**]
time = 0.05, size = 58, normalized size = 1.00

$$-\frac{2i\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}\,F\left(\frac{1}{2}i(a+b\log\left(cx^n\right))\middle|\,2\right)\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sech[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] ((-2*I)*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticF[(I/2)*(a + b*Log[c*x^n]), 2]
*Sqrt[Sech[a + b*Log[c*x^n]]])/(b*n)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(90) = 180.
time = 4.71, size = 183, normalized size = 3.16

method	result
derivativedivides	$\frac{2\sqrt{\left(2\left(\cosh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)}\sqrt{-\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right.}}{n\sqrt{2\left(\sinh^4\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)+\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)}\sinh\left(\frac{a}{2}+\frac{b\,l}{}\right.}$
default	$\frac{2\sqrt{\left(2\left(\cosh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)}\sqrt{-\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right.}}{n\sqrt{2\left(\sinh^4\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)+\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)}\sinh\left(\frac{a}{2}+\frac{b\,l}{}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(a+b*ln(c*x^n))^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/n*((2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/
2)*(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*cosh(1/2*a+1/2*b*ln(c*x^n))^2
+1)^(1/2)/(2*sinh(1/2*a+1/2*b*ln(c*x^n))^4+sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(
1/2)*EllipticF(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/sinh(1/2*a+1/2*b*ln(c*x
^n))/(2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sech(b*log(c*x^n) + a))/x, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 39, normalized size = 0.67

$$\frac{2\sqrt{2}\,\mathrm{weierstrassPInverse}(-4,0,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right))}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*weierstrassPInverse(-4, 0, cosh(b*n*log(x) + b*log(c) + a) + sinh
(b*n*log(x) + b*log(c) + a))/(b*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\operatorname{sech}\left(a+b\log\left(cx^{n}\right)\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*ln(c*x**n))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(sech(a + b*log(c*x**n)))/x, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^(1/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\sqrt{\frac{1}{\cosh\left(a+b\,\ln\left(c\,x^n\right)\right)}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(a + b*log(c*x^n)))^(1/2)/x,x)
```

```
[Out] int((1/cosh(a + b*log(c*x^n)))^(1/2)/x, x)
```

3.199 $\int \frac{1}{x\sqrt{\text{sech}(a+b\log(cx^n))}}\,dx$

Optimal. Leaf size=58

$$-\frac{2i\sqrt{\cosh(a+b\log(cx^n))}\,E\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)\sqrt{\text{sech}(a+b\log(cx^n))}}{bn}$$

```
[Out] -2*I*(cosh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/cosh(1/2*a+1/2*b*ln(c*x^n))*Elli
pticE(I*sinh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))*cosh(a+b*ln(c*x^n))^(1/2)*sech
(a+b*ln(c*x^n))^(1/2)/b/n
```

Rubi [A]
time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3856, 2719}

$$-\frac{2i\sqrt{\text{sech}(a+b\log(cx^n))}\sqrt{\cosh(a+b\log(cx^n))}\,E\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[Sech[a + b*Log[c*x^n]]]),x]
```

```
[Out] ((-2*I)*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2]
*Sqrt[Sech[a + b*Log[c*x^n]]])/(b*n)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{\text{sech}(a+b\log(cx^n))}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\text{sech}(a+bx)}}\,dx, x, \log(cx^n)\right)}{n}\\
&= \frac{\left(\sqrt{\cosh(a+b\log(cx^n))}\sqrt{\text{sech}(a+b\log(cx^n))}\right)\text{Subst}\left(\int \sqrt{\cosh(a+\right.}{n}\\
&= -\frac{2i\sqrt{\cosh(a+b\log(cx^n))}\,E\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)\sqrt{\text{sech}(a+b\log(c}{bn}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 58, normalized size = 1.00

$$-\frac{2iE\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{bn\sqrt{\cosh(a+b\log(cx^n))}\sqrt{\operatorname{sech}(a+b\log(cx^n))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[Sech[a + b*Log[c*x^n]]]),x]
```

```
[Out] ((-2*I)*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2])/(b*n*Sqrt[Cosh[a + b*Log[c*
x^n]]]*Sqrt[Sech[a + b*Log[c*x^n]]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(90) = 180.
time = 4.74, size = 183, normalized size = 3.16

method	result
derivativedivides	$-\frac{2\sqrt{\left(2\left(\cosh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)}\sqrt{-\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right.}}{n\sqrt{2\left(\sinh^4\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)+\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)}\sinh\left(\frac{a}{2}+\frac{b}{}\right.}$
default	$-\frac{2\sqrt{\left(2\left(\cosh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)}\sqrt{-\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right.}}{n\sqrt{2\left(\sinh^4\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)+\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)}\sinh\left(\frac{a}{2}+\frac{b}{}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/sech(a+b*ln(c*x^n))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/n*((2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1
/2)*(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*cosh(1/2*a+1/2*b*ln(c*x^n))^
2+1)^(1/2)*EllipticE(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/(2*sinh(1/2*a+1/2
*b*ln(c*x^n))^4+sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/sinh(1/2*a+1/2*b*ln(c*
x^n))/(2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sech(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*sqrt(sech(b*log(c*x^n) + a))), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 248, normalized size = 4.28

$$-\frac{\sqrt{2}\left(\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+1\right)\sqrt{\frac{\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+1}}+2\left(\sqrt{2}\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sqrt{2}\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)\right)}{bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+bn\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sech(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
[Out] -(sqrt(2)*(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c)
 + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 +
 1)*sqrt((cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)
)/(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*si
nh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 + 1)) + 2
*(sqrt(2)*cosh(b*n*log(x) + b*log(c) + a) + sqrt(2)*sinh(b*n*log(x) + b*log
(c) + a))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cosh(b*n*log(x)
 + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a))))/(b*n*cosh(b*n*log(x)
+ b*log(c) + a) + b*n*sinh(b*n*log(x) + b*log(c) + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\operatorname{sech}\left(a+b\log\left(cx^{n}\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sech(a+b*ln(c*x**n))**(1/2),x)
[Out] Integral(1/(x*sqrt(sech(a + b*log(c*x**n)))), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sech(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\sqrt{\frac{1}{\cosh\left(a+b\,\ln\left(c\,x^{n}\right)\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1/cosh(a + b*log(c*x^n)))^(1/2)),x)
[Out] int(1/(x*(1/cosh(a + b*log(c*x^n)))^(1/2)), x)
```

3.200 $\int \frac{1}{x\text{sech}^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=97

$$-\frac{2i\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}\,F\left(\frac{1}{2}i(a+b\log\left(cx^n\right))\middle|2\right)\sqrt{\text{sech}\left(a+b\log\left(cx^n\right)\right)}}{3bn}+\frac{2\sinh\left(a+b\log\left(cx^n\right)\right)}{3bn\sqrt{\text{sech}\left(a+b\log\left(cx^n\right)\right)}}$$

```
[Out] 2/3*sinh(a+b*ln(c*x^n))/b/n/sech(a+b*ln(c*x^n))^(1/2)-2/3*I*(cosh(1/2*a+1/2
*b*ln(c*x^n))^2)^(1/2)/cosh(1/2*a+1/2*b*ln(c*x^n))*EllipticF(I*sinh(1/2*a+1
/2*b*ln(c*x^n)),2^(1/2))*cosh(a+b*ln(c*x^n))^(1/2)*sech(a+b*ln(c*x^n))^(1/2
)/b/n
```

Rubi [A]
time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3854, 3856, 2720}

$$\frac{2\sinh\left(a+b\log\left(cx^n\right)\right)}{3bn\sqrt{\text{sech}\left(a+b\log\left(cx^n\right)\right)}}-\frac{2i\sqrt{\text{sech}\left(a+b\log\left(cx^n\right)\right)}\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}\,F\left(\frac{1}{2}i(a+b\log\left(cx^n\right))\middle|2\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sech[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] (((-2*I)/3)*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticF[(I/2)*(a + b*Log[c*x^n])
, 2]*Sqrt[Sech[a + b*Log[c*x^n]]])/(b*n) + (2*Sinh[a + b*Log[c*x^n]])/(3*b*
n*Sqrt[Sech[a + b*Log[c*x^n]]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{x\text{sech}^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\text{sech}^{\frac{3}{2}}(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$

$$= \frac{2\sinh(a+b\log(cx^n))}{3bn\sqrt{\text{sech}(a+b\log(cx^n))}} + \frac{\text{Subst}\left(\int \sqrt{\text{sech}(a+bx)}\,dx, x, \log(cx^n)\right)}{3n}$$

$$= \frac{2\sinh(a+b\log(cx^n))}{3bn\sqrt{\text{sech}(a+b\log(cx^n))}} + \frac{\left(\sqrt{\cosh(a+b\log(cx^n))}\sqrt{\text{sech}(a+b\log$$

$$= -\frac{2i\sqrt{\cosh(a+b\log(cx^n))}\,F\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)\sqrt{\text{sech}(a+b\log(cx}{3bn}$$

Mathematica [A]
time = 0.08, size = 76, normalized size = 0.78

$$\frac{\sqrt{\text{sech}(a+b\log(cx^n))}\left(-2i\sqrt{\cosh(a+b\log(cx^n))}\,F\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)+\sinh(2(a+b\log(cx^n)))\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sech[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] (Sqrt[Sech[a + b*Log[c*x^n]]]*((-2*I)*Sqrt[Cosh[a + b*Log[c*x^n]]]*Elliptic
F[(I/2)*(a + b*Log[c*x^n]), 2] + Sinh[2*(a + b*Log[c*x^n])]))/(3*b*n)
```

Maple [A]
time = 5.21, size = 237, normalized size = 2.44

method	result
derivativedivides	$\frac{2\sqrt{\left(2\left(\cosh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)}\left(4\left(\cosh^5\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)-6\left(\cosh^3\right.\right.}{3n\sqrt{2\left(\sinh^4\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)+}}$
default	$\frac{2\sqrt{\left(2\left(\cosh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)}\left(4\left(\cosh^5\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)-6\left(\cosh^3\right.\right.}{3n\sqrt{2\left(\sinh^4\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/sech(a+b*ln(c*x^n))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/n*((2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(
1/2)*(4*cosh(1/2*a+1/2*b*ln(c*x^n))^5-6*cosh(1/2*a+1/2*b*ln(c*x^n))^3+(-sin
h(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*cosh(1/2*a+1/2*b*ln(c*x^n))^2+1)^(1/2
)*EllipticF(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))+2*cosh(1/2*a+1/2*b*ln(c*x^
n)))/(2*sinh(1/2*a+1/2*b*ln(c*x^n))^4+sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/
sinh(1/2*a+1/2*b*ln(c*x^n))/(2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sech(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*sech(b*log(c*x^n) + a)^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 370, normalized size = 3.81

$$\frac{\sqrt{2}\left(\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{4}+4\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+6\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+4\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{4}-1\right)\sqrt{\frac{\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+1}}+4\left(\sqrt{2}\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2\sqrt{2}\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sqrt{2}\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}\right)\mathrm{weierstrassPInverse}\left(-4,0,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)}{6\left(bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+bn\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sech(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(sqrt(2)*(cosh(b*n*log(x) + b*log(c) + a)^4 + 4*cosh(b*n*log(x) + b*log
(c) + a)^3*sinh(b*n*log(x) + b*log(c) + a) + 6*cosh(b*n*log(x) + b*log(c) +
 a)^2*sinh(b*n*log(x) + b*log(c) + a)^2 + 4*cosh(b*n*log(x) + b*log(c) + a)
*sinh(b*n*log(x) + b*log(c) + a)^3 + sinh(b*n*log(x) + b*log(c) + a)^4 - 1)
*sqrt((cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a))/(
cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(
b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 + 1)) + 4*(s
qrt(2)*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*sqrt(2)*cosh(b*n*log(x) + b*lo
g(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sqrt(2)*sinh(b*n*log(x) + b*log
(c) + a)^2)*weierstrassPInverse(-4, 0, cosh(b*n*log(x) + b*log(c) + a) + si
nh(b*n*log(x) + b*log(c) + a)))/(b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*
b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + b*n*s
inh(b*n*log(x) + b*log(c) + a)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\,\mathrm{sech}^{\frac{3}{2}}\left(a+b\log\left(cx^{n}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sech(a+b*ln(c*x**n))**(3/2),x)
[Out] Integral(1/(x*sech(a + b*log(c*x**n))**(3/2)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sech(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,{\left(\frac{1}{\mathrm{cosh}\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1/cosh(a + b*log(c*x^n)))^(3/2)),x)
[Out] int(1/(x*(1/cosh(a + b*log(c*x^n)))^(3/2)), x)
```

3.201 $\int \frac{1}{x\mathbf{sech}^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=97

$$-\frac{6i\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}\,E\left(\frac{1}{2}i(a+b\log\left(cx^n\right))\middle|2\right)\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}}{5bn}+\frac{2\sinh\left(a+b\log\left(cx^n\right)\right)}{5bn\operatorname{sech}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}$$

```
[Out] 2/5*sinh(a+b*ln(c*x^n))/b/n/sech(a+b*ln(c*x^n))^(3/2)-6/5*I*(cosh(1/2*a+1/2
*b*ln(c*x^n))^2)^(1/2)/cosh(1/2*a+1/2*b*ln(c*x^n))*EllipticE(I*sinh(1/2*a+1
/2*b*ln(c*x^n)),2^(1/2))*cosh(a+b*ln(c*x^n))^(1/2)*sech(a+b*ln(c*x^n))^(1/2
)/b/n
```

Rubi [A]
time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3854, 3856, 2719}

$$\frac{2\sinh\left(a+b\log\left(cx^n\right)\right)}{5bn\operatorname{sech}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}-\frac{6i\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}\,E\left(\frac{1}{2}i(a+b\log\left(cx^n\right))\middle|2\right)}{5bn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sech[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] (((-6*I)/5)*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticE[(I/2)*(a + b*Log[c*x^n])
, 2]*Sqrt[Sech[a + b*Log[c*x^n]]])/(b*n) + (2*Sinh[a + b*Log[c*x^n]])/(5*b*
n*Sech[a + b*Log[c*x^n]]^(3/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{x\text{sech}^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\text{sech}^{\frac{5}{2}}(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$
$$= \frac{2\sinh(a+b\log(cx^n))}{5bn\text{sech}^{\frac{3}{2}}(a+b\log(cx^n))} + \frac{3\text{Subst}\left(\int \frac{1}{\sqrt{\text{sech}(a+bx)}}\,dx, x, \log(cx^n)\right)}{5n}$$
$$= \frac{2\sinh(a+b\log(cx^n))}{5bn\text{sech}^{\frac{3}{2}}(a+b\log(cx^n))} + \frac{\left(3\sqrt{\cosh(a+b\log(cx^n))}\sqrt{\text{sech}(a+b\log(\right.}{}$$
$$= -\frac{6i\sqrt{\cosh(a+b\log(cx^n))}\,E\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)\sqrt{\text{sech}(a+b\log(cx}{5bn}$$

Mathematica [A]
time = 0.09, size = 87, normalized size = 0.90

$$\frac{\sqrt{\text{sech}(a+b\log(cx^n))}\left(-12i\sqrt{\cosh(a+b\log(cx^n))}\,E\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)+\sinh(a+b\log(cx^n))+\sinh(3(a+b\log(cx^n)))\right)}{10bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sech[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] (Sqrt[Sech[a + b*Log[c*x^n]]]*((-12*I)*Sqrt[Cosh[a + b*Log[c*x^n]]]*Ellipti
cE[(I/2)*(a + b*Log[c*x^n]), 2] + Sinh[a + b*Log[c*x^n]] + Sinh[3*(a + b*Lo
g[c*x^n])]))/(10*b*n)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(123) = 246$.
time = 5.21, size = 256, normalized size = 2.64

method	result
derivativedivides	$\frac{2\sqrt{\left(2\left(\cosh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)}\left(8\left(\cosh^7\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)-16\left(\cosh\right.\right.}{5n\sqrt{2\left(\sinh^4\left(\frac{a}{2}+\frac{b}{}\right.\right.}}$

default	$\frac{2\sqrt{\left(2\left(\cosh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)}\left(8\left(\cosh^7\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)-16\left(\cosh^5\right.\right.}{5n\sqrt{2\left(\sinh^4\left(\frac{a}{2}+\frac{b\ln}{}\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/sech(a+b*ln(c*x^n))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5/n*((2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(
1/2)*(8*cosh(1/2*a+1/2*b*ln(c*x^n))^7-16*cosh(1/2*a+1/2*b*ln(c*x^n))^5+10*c
osh(1/2*a+1/2*b*ln(c*x^n))^3-3*(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*c
osh(1/2*a+1/2*b*ln(c*x^n))^2+1)^(1/2)*EllipticE(cosh(1/2*a+1/2*b*ln(c*x^n))
,2^(1/2))-2*cosh(1/2*a+1/2*b*ln(c*x^n)))/(2*sinh(1/2*a+1/2*b*ln(c*x^n))^4+s
inh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/sinh(1/2*a+1/2*b*ln(c*x^n))/(2*cosh(1/2
*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sech(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*sech(b*log(c*x^n) + a)^(5/2)), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 602, normalized size = 6.21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sech(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/20*(sqrt(2)*(cosh(b*n*log(x) + b*log(c) + a)^6 + 6*cosh(b*n*log(x) + b*lo
g(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^5 + sinh(b*n*log(x) + b*log(c) +
a)^6 + (15*cosh(b*n*log(x) + b*log(c) + a)^2 - 11)*sinh(b*n*log(x) + b*log(
c) + a)^4 - 11*cosh(b*n*log(x) + b*log(c) + a)^4 + 4*(5*cosh(b*n*log(x) + b
*log(c) + a)^3 - 11*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*lo
g(c) + a)^3 + (15*cosh(b*n*log(x) + b*log(c) + a)^4 - 66*cosh(b*n*log(x) +
b*log(c) + a)^2 - 13)*sinh(b*n*log(x) + b*log(c) + a)^2 - 13*cosh(b*n*log(x
) + b*log(c) + a)^2 + 2*(3*cosh(b*n*log(x) + b*log(c) + a)^5 - 22*cosh(b*n*
log(x) + b*log(c) + a)^3 - 13*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log
(x) + b*log(c) + a) - 1)*sqrt((cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*l
```

```
og(x) + b*log(c) + a))/(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(
x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*lo
g(c) + a)^2 + 1)) - 24*(sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*sqrt(
2)*cosh(b*n*log(x) + b*log(c) + a)^2*sinh(b*n*log(x) + b*log(c) + a) + 3*sq
rt(2)*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^2 + s
qrt(2)*sinh(b*n*log(x) + b*log(c) + a)^3)*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(
c) + a))))/(b*n*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*b*n*cosh(b*n*log(x) +
 b*log(c) + a)^2*sinh(b*n*log(x) + b*log(c) + a) + 3*b*n*cosh(b*n*log(x) +
b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^2 + b*n*sinh(b*n*log(x) + b*l
og(c) + a)^3)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sech(a+b*ln(c*x**n))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5009 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sech(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(\frac{1}{\cosh(a+b\,\ln(c\,x^n))}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1/cosh(a + b*log(c*x^n)))^(5/2)),x)
[Out] int(1/(x*(1/cosh(a + b*log(c*x^n)))^(5/2)), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
   #debug=False
   m = func.name() in ['exp','log','ln',
           'sin','cos','tan','cot','sec','csc',
           'arcsin','arccos','arctan','arccot','arcsec','arccsc',
           'sinh','cosh','tanh','coth','sech','csch',
           'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
       'arctan2','floor','abs'
       ]
   if debug:
      if m:
         print ("func ", func , " is elementary_function")
      else:
         print ("func ", func , " is NOT elementary_function")


   return m

def is_special_function(func):
   #debug=False
   if debug:
      print ("type(func)=", type(func))

   m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

   if debug:
      print ("m=",m)
      if m:
         print ("func ", func ," is special_function")
      else:
         print ("func ", func ," is NOT special_function")


   return m


def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])   #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```