Computer algebra independent integration tests

Summer 2022 edition

6-Hyperbolic-functions/6.4-Hyperbolic-cotangent/175-6.4.2-Hyperbolic-cotangent-functions

Nasser M. Abbasi

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [224]. This is test number [175].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (224)	0.00 (0)
Mathematica	100.00 (224)	0.00 (0)
Fricas	79.91 (179)	20.09 (45)
Maple	73.21 (164)	26.79 (60)
Giac	61.16 (137)	38.84 (87)
Mupad	58.48 (131)	41.52 (93)
Maxima	46.88 (105)	53.12 (119)
Sympy	14.73 (33)	85.27 (191)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

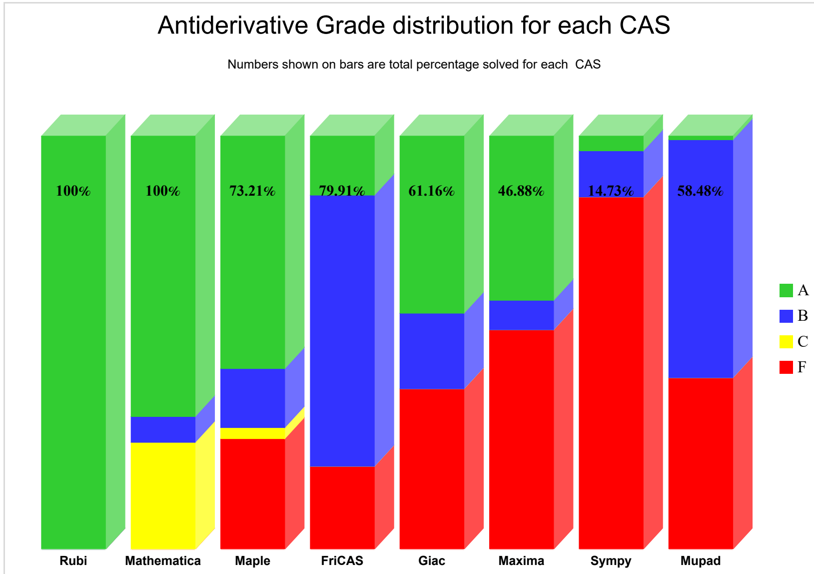
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

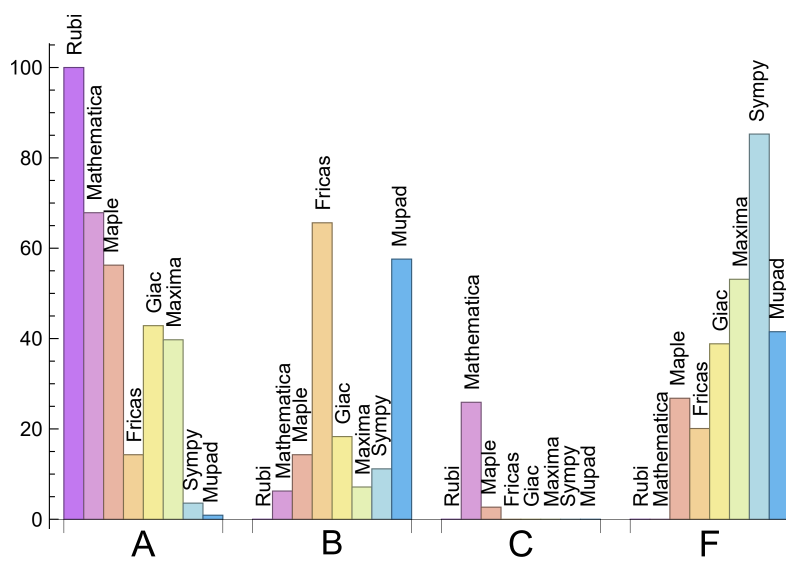
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	67.86	6.25	25.89	0.00
Maple	56.25	14.29	2.68	26.79
Giac	42.86	18.30	0.00	38.84
Maxima	39.73	7.14	0.00	53.12
Fricas	14.29	65.62	0.00	20.09
Sympy	3.57	11.16	0.00	85.27
Mupad	N/A	57.59	0.00	41.52

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	60	100.00 %	0.00 %	0.00 %
Fricas	45	77.78 %	0.00 %	22.22 %
Giac	87	82.76 %	10.34 %	6.90 %
Maxima	119	93.28 %	0.00 %	6.72 %
Sympy	191	90.58 %	7.85 %	1.57 %
Mupad	93	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

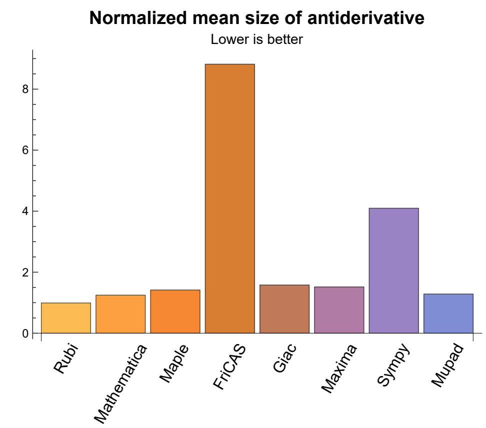
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

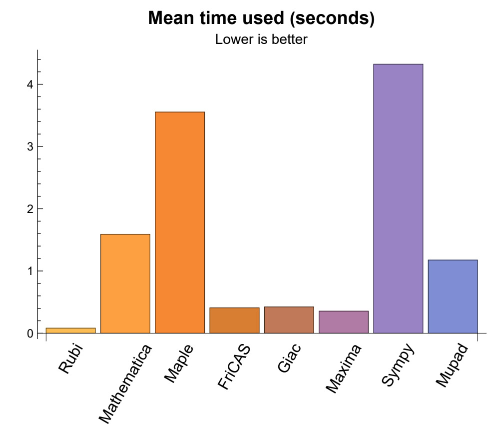
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.08	85.42	0.99	60.00	1.00
Mathematica	1.59	85.42	1.25	60.00	1.03
Maple	3.55	87.71	1.42	68.00	1.16
Maxima	0.35	82.30	1.52	47.00	1.12
Fricas	0.41	1009.93	8.82	287.00	4.94
Sympy	4.32	221.91	4.10	136.00	3.79
Giac	0.42	96.26	1.58	66.00	1.27
Mupad	1.18	72.42	1.28	42.00	0.92

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
2
4
6
8
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
1
2
3
4
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{220, 224\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {168, 169, 172, 173, 174, 175, 176, 197, 198}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `sleliev re` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```python
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

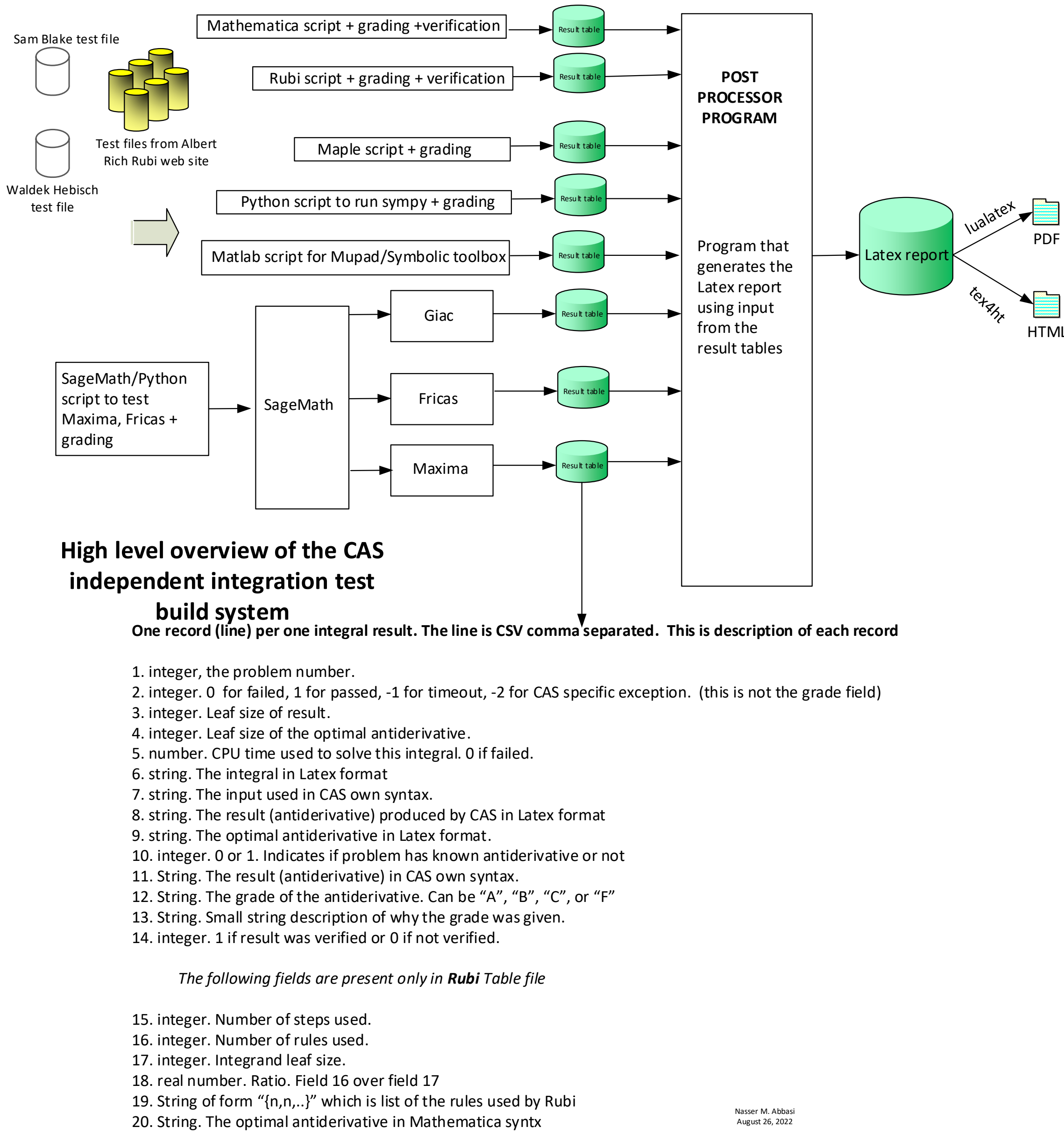
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 10, 12, 15, 16, 17, 18, 19, 20, 21, 22, 24, 28, 29, 30, 35, 36, 37, 38, 39, 42, 43, 45, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 64, 65, 66, 67, 68, 69, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 153, 155, 157, 158, 165, 166, 167, 169, 170, 171, 172, 181, 184, 185, 186, 187, 189, 190, 191, 193, 194, 195, 196, 198, 199, 200, 201, 202, 205, 206, 208, 209, 211, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224 }

B grade: { 151, 168, 174, 175, 176, 177, 178, 179, 180, 182, 183, 197, 207, 210 }

C grade: { 6, 7, 8, 9, 11, 13, 14, 23, 25, 26, 27, 31, 32, 33, 34, 40, 41, 44, 46, 47, 48, 49, 61, 62, 63, 70, 71, 72, 73, 74, 75, 76, 87, 88, 113, 132, 133, 134, 135, 136, 137, 138, 139, 152, 154, 156, 159, 160, 161, 162, 163, 164, 173, 188, 192, 203, 204, 212 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 18, 19, 20, 21, 29, 30, 31, 32, 40, 41, 42, 43, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 93, 94, 97, 98, 99, 100, 101, 102, 103, 109, 114, 115, 116, 117, 118, 120, 121, 122, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 153, 157, 158, 159, 160, 161, 164, 181, 191, 192, 193, 199, 200, 201, 202, 203, 204, 205, 206, 207, 211, 212, 217, 218, 219, 220, 221, 222, 223, 224 }

B grade: { 33, 34, 35, 36, 37, 38, 89, 90, 91, 92, 95, 96, 104, 105, 106, 107, 108, 110, 111, 112, 119, 123, 124, 125, 126, 140, 152, 154, 155, 162, 188, 213 }

C grade: { 156, 163, 210, 214, 215, 216 }

F grade: { 15, 16, 17, 22, 23, 24, 25, 26, 27, 28, 39, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 113, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 185, 186, 187, 189, 190, 194, 195, 196, 197, 198, 208, 209 }

2.1.4 Maxima

A grade: { 18, 19, 20, 21, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 64, 65, 66, 67, 68, 69, 80, 81, 82, 85, 86, 89, 90, 91, 92, 93, 94, 97, 99, 102, 105, 106, 107, 108, 109, 110, 114, 116, 119, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 181, 188, 211, 212, 213, 214, 215, 216, 220, 224 }

B grade: { 61, 62, 63, 77, 78, 79, 83, 84, 95, 96, 104, 111, 112, 191, 192, 193 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 39, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 70, 71, 72, 73, 74, 75, 76, 87, 88, 98, 100, 101, 103, 113, 115, 117, 118, 120, 132, 133, 134, 135, 136, 137, 138, 139, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 185, 186, 187, 189, 190, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 217, 218, 219, 221, 222, 223 }

2.1.5 FriCAS

A grade: { 9, 10, 46, 81, 85, 86, 91, 92, 93, 101, 107, 108, 118, 122, 143, 144, 145, 146, 151, 152, 153, 155, 156, 158, 163, 164, 213, 214, 218, 220, 222, 224 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 11, 12, 13, 14, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 47, 48, 49, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 82, 83, 84, 87, 88, 89, 90, 94, 95, 96, 97, 98, 99, 100, 102, 103, 104, 105, 106, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 147, 148, 149, 150, 154, 157, 159, 160, 161, 162, 181, 188, 191, 192, 193, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 215, 216, 217, 219, 221, 223 }

C grade: { }

F grade: { 15, 16, 17, 28, 39, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 185, 186, 187, 189, 190, 194, 195, 196, 197, 198 }

2.1.6 Sympy

A grade: { 61, 62, 63, 64, 85, 86, 220, 224 }

B grade: { 33, 34, 65, 66, 67, 68, 69, 77, 78, 79, 80, 81, 127, 128, 129, 130, 131, 144, 145, 146, 147, 148, 149, 155, 162 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 70, 71, 72, 73, 74, 75, 76, 82, 83, 84, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 150, 151, 152, 153, 154, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 221, 222, 223 }

2.1.7 Giac

A grade: { 18, 19, 21, 40, 41, 42, 43, 61, 62, 63, 64, 65, 66, 67, 68, 69, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 89, 90, 91, 92, 93, 94, 96, 97, 98, 99, 100, 101, 103, 105, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 120, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 151, 152, 153, 154, 156, 157, 158, 159, 160, 161, 162, 163, 164, 188, 192, 211, 212, 213, 214, 215, 216, 220, 224 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 11, 12, 20, 29, 30, 31, 32, 70, 71, 72, 73, 74, 75, 76, 95, 102, 104, 113, 119, 121, 132, 133, 134, 135, 136, 137, 138, 139, 150, 155, 181, 191, 193 }

C grade: { }

F grade: { 9, 10, 13, 14, 15, 16, 17, 22, 23, 24, 25, 26, 27, 28, 33, 34, 35, 36, 37, 38, 39, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 87, 88, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 185, 186, 187, 189, 190, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 217, 218, 219, 221, 222, 223 }

2.1.8 Mupad

A grade: { 220, 224 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 20, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 181, 188, 191, 192, 193, 199, 200, 201, 202, 203, 204 }

C grade: { }

F grade: $\{$ 15, 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 113, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 185, 186, 187, 189, 190, 194, 195, 196, 197, 198, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 221, 222, 223 $\}$

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	83	74	0	1574	0	379	83
N.S.	1	1.00	0.86	0.76	0.00	16.23	0.00	3.91	0.86
time (sec)	N/A	0.052	0.165	2.188	0.000	0.372	0.000	0.528	1.580

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	68	60	0	988	0	224	62
N.S.	1	1.00	0.87	0.77	0.00	12.67	0.00	2.87	0.79
time (sec)	N/A	0.037	0.152	2.138	0.000	0.418	0.000	0.531	1.372

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	61	58	0	637	0	168	61
N.S.	1	1.00	0.81	0.77	0.00	8.49	0.00	2.24	0.81
time (sec)	N/A	0.039	0.064	2.170	0.000	0.531	0.000	0.464	1.275

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	51	48	0	594	0	101	41
N.S.	1	1.00	0.88	0.83	0.00	10.24	0.00	1.74	0.71
time (sec)	N/A	0.024	0.029	2.329	0.000	0.421	0.000	0.451	1.215

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	49	48	0	598	0	102	38
N.S.	1	1.00	0.86	0.84	0.00	10.49	0.00	1.79	0.67
time (sec)	N/A	0.024	0.025	2.272	0.000	0.518	0.000	0.487	1.294

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	36	62	0	923	0	174	64
N.S.	1	1.00	0.46	0.79	0.00	11.83	0.00	2.23	0.82
time (sec)	N/A	0.038	0.053	2.097	0.000	0.592	0.000	0.479	1.390

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	38	63	0	1428	0	213	63
N.S.	1	1.00	0.48	0.80	0.00	18.08	0.00	2.70	0.80
time (sec)	N/A	0.037	0.047	2.112	0.000	0.508	0.000	0.508	1.491

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	38	77	0	2132	0	333	80
N.S.	1	1.00	0.38	0.77	0.00	21.32	0.00	3.33	0.80
time (sec)	N/A	0.052	0.069	2.043	0.000	0.511	0.000	0.554	1.596

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	36	199	0	292	0	0	249
N.S.	1	1.00	0.15	0.84	0.00	1.24	0.00	0.00	1.06
time (sec)	N/A	0.212	0.022	1.536	0.000	0.373	0.000	0.000	1.881

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	149	181	0	310	0	0	233
N.S.	1	1.00	0.68	0.83	0.00	1.42	0.00	0.00	1.07
time (sec)	N/A	0.205	0.125	1.551	0.000	0.488	0.000	0.000	1.499

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	38	109	0	291	0	217	146
N.S.	1	1.00	0.29	0.83	0.00	2.20	0.00	1.64	1.11
time (sec)	N/A	0.076	0.030	1.391	0.000	0.379	0.000	0.558	1.516

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	98	109	0	1598	0	216	147
N.S.	1	1.00	0.74	0.83	0.00	12.11	0.00	1.64	1.11
time (sec)	N/A	0.072	0.085	1.231	0.000	0.551	0.000	0.489	1.652

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	36	193	0	356	0	0	197
N.S.	1	1.00	0.17	0.89	0.00	1.63	0.00	0.00	0.90
time (sec)	N/A	0.169	0.019	1.445	0.000	0.569	0.000	0.000	1.386

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	36	207	0	3348	0	0	165
N.S.	1	1.00	0.15	0.87	0.00	14.07	0.00	0.00	0.69
time (sec)	N/A	0.228	0.041	1.434	0.000	0.567	0.000	0.000	1.419

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	45	0	0	0	0	0	-1
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.016	0.046	1.830	0.000	0.000	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	51	0	0	0	0	0	-1
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.020	0.029	1.707	0.000	0.000	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	47	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.030	0.036	2.257	0.000	0.000	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	56	53	97	823	0	90	-1
N.S.	1	1.00	0.92	0.87	1.59	13.49	0.00	1.48	-0.02
time (sec)	N/A	0.026	0.090	1.411	0.495	0.513	0.000	0.424	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	39	45	54	125	0	54	-1
N.S.	1	1.00	1.26	1.45	1.74	4.03	0.00	1.74	-0.03
time (sec)	N/A	0.014	0.033	1.642	0.488	0.425	0.000	0.399	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	56	34	128	0	60	30
N.S.	1	1.00	1.00	1.81	1.10	4.13	0.00	1.94	0.97
time (sec)	N/A	0.016	0.044	1.572	0.499	0.434	0.000	0.425	1.264

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	48	79	84	817	0	104	-1
N.S.	1	1.00	0.74	1.22	1.29	12.57	0.00	1.60	-0.02
time (sec)	N/A	0.025	0.101	1.407	0.495	0.458	0.000	0.423	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	166	0	0	1994	0	0	-1
N.S.	1	1.00	0.56	0.00	0.00	6.71	0.00	0.00	-0.00
time (sec)	N/A	0.163	0.207	1.754	0.000	0.392	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	43	0	0	2037	0	0	-1
N.S.	1	1.00	0.15	0.00	0.00	7.05	0.00	0.00	-0.00
time (sec)	N/A	0.131	0.033	1.727	0.000	0.423	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	151	0	0	1618	0	0	-1
N.S.	1	1.00	0.57	0.00	0.00	6.13	0.00	0.00	-0.00
time (sec)	N/A	0.164	0.071	1.757	0.000	0.396	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	41	0	0	8338	0	0	-1
N.S.	1	1.00	0.16	0.00	0.00	31.58	0.00	0.00	-0.00
time (sec)	N/A	0.122	0.040	1.877	0.000	0.534	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	41	0	0	2066	0	0	-1
N.S.	1	1.00	0.14	0.00	0.00	7.15	0.00	0.00	-0.00
time (sec)	N/A	0.159	0.044	1.829	0.000	0.415	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	43	0	0	14359	0	0	-1
N.S.	1	1.00	0.14	0.00	0.00	46.47	0.00	0.00	-0.00
time (sec)	N/A	0.137	0.104	1.825	0.000	0.536	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	53	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.030	0.033	2.265	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	82	107	0	2152	0	788	-1
N.S.	1	1.00	0.61	0.80	0.00	16.06	0.00	5.88	-0.01
time (sec)	N/A	0.045	0.351	2.289	0.000	0.419	0.000	0.619	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	63	86	0	633	0	269	-1
N.S.	1	1.00	0.61	0.83	0.00	6.09	0.00	2.59	-0.01
time (sec)	N/A	0.035	0.067	2.573	0.000	0.368	0.000	0.495	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	41	92	0	907	0	279	-1
N.S.	1	1.00	0.39	0.88	0.00	8.64	0.00	2.66	-0.01
time (sec)	N/A	0.033	0.023	2.419	0.000	0.399	0.000	0.574	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	43	106	0	3022	0	521	-1
N.S.	1	1.00	0.30	0.75	0.00	21.43	0.00	3.70	-0.01
time (sec)	N/A	0.044	0.047	2.228	0.000	0.437	0.000	0.594	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	43	145	87	1046	162	0	-1
N.S.	1	1.00	0.58	1.96	1.18	14.14	2.19	0.00	-0.01
time (sec)	N/A	0.026	0.046	2.654	0.488	0.376	106.751	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	41	119	34	392	136	0	-1
N.S.	1	1.00	0.82	2.38	0.68	7.84	2.72	0.00	-0.02
time (sec)	N/A	0.018	0.024	2.547	0.492	0.360	9.640	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	39	192	51	148	0	0	-1
N.S.	1	1.00	1.26	6.19	1.65	4.77	0.00	0.00	-0.03
time (sec)	N/A	0.014	0.022	2.618	0.490	0.357	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	192	32	187	0	0	-1
N.S.	1	1.00	1.00	6.19	1.03	6.03	0.00	0.00	-0.03
time (sec)	N/A	0.014	0.019	2.597	0.502	0.385	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	40	119	37	287	0	0	-1
N.S.	1	1.00	0.80	2.38	0.74	5.74	0.00	0.00	-0.02
time (sec)	N/A	0.017	0.044	2.589	0.481	0.354	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	51	149	89	1579	0	0	-1
N.S.	1	1.00	0.64	1.86	1.11	19.74	0.00	0.00	-0.01
time (sec)	N/A	0.026	0.064	2.609	0.487	0.409	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	51	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.040	2.414	0.000	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	43	77	137	3421	0	77	-1
N.S.	1	1.00	0.39	0.70	1.25	31.10	0.00	0.70	-0.01
time (sec)	N/A	0.033	0.047	1.591	0.501	0.377	0.000	0.445	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	41	55	34	415	0	27	-1
N.S.	1	1.00	0.82	1.10	0.68	8.30	0.00	0.54	-0.02
time (sec)	N/A	0.017	0.022	1.605	0.495	0.346	0.000	0.446	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	40	59	36	422	0	32	-1
N.S.	1	1.00	0.80	1.18	0.72	8.44	0.00	0.64	-0.02
time (sec)	N/A	0.017	0.042	1.579	0.488	0.365	0.000	0.424	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	68	84	155	3473	0	99	-1
N.S.	1	1.00	0.58	0.71	1.31	29.43	0.00	0.84	-0.01
time (sec)	N/A	0.036	0.164	1.437	0.520	0.389	0.000	0.471	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	68	0	0	2864	0	0	-1
N.S.	1	1.00	0.19	0.00	0.00	8.11	0.00	0.00	-0.00
time (sec)	N/A	0.147	0.095	1.992	0.000	0.427	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	166	0	0	618	0	0	-1
N.S.	1	1.00	0.57	0.00	0.00	2.12	0.00	0.00	-0.00
time (sec)	N/A	0.157	0.226	1.961	0.000	0.362	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	43	0	0	288	0	0	-1
N.S.	1	1.00	0.15	0.00	0.00	1.00	0.00	0.00	-0.00
time (sec)	N/A	0.124	0.022	1.971	0.000	0.410	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	41	0	0	3316	0	0	-1
N.S.	1	1.00	0.14	0.00	0.00	11.47	0.00	0.00	-0.00
time (sec)	N/A	0.156	0.030	1.956	0.000	0.425	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	43	0	0	1159	0	0	-1
N.S.	1	1.00	0.15	0.00	0.00	3.98	0.00	0.00	-0.00
time (sec)	N/A	0.135	0.030	1.912	0.000	0.376	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	43	0	0	15579	0	0	-1
N.S.	1	1.00	0.12	0.00	0.00	42.22	0.00	0.00	-0.00
time (sec)	N/A	0.178	0.030	2.026	0.000	0.636	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	55	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.031	0.036	180.000	0.000	0.000	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	58	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.034	0.053	2.319	0.000	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.032	0.029	2.554	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	58	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.036	0.039	3.013	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	58	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.038	0.064	2.670	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	60	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.035	0.052	1.956	0.000	0.000	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.034	0.031	1.782	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.036	0.031	1.773	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	58	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.033	0.034	2.191	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.035	0.031	2.181	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	60	0	0	0	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.048	2.200	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	94	31	140	448	48	41	88
N.S.	1	1.00	2.29	0.76	3.41	10.93	1.17	1.00	2.15
time (sec)	N/A	0.031	0.171	0.200	0.276	0.340	0.890	0.406	1.126

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	84	25	95	273	37	35	60
N.S.	1	1.00	2.71	0.81	3.06	8.81	1.19	1.13	1.94
time (sec)	N/A	0.023	0.130	0.190	0.302	0.361	0.571	0.442	1.144

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	61	19	55	142	31	29	36
N.S.	1	1.00	2.65	0.83	2.39	6.17	1.35	1.26	1.57
time (sec)	N/A	0.015	0.109	0.192	0.268	0.337	0.395	0.406	0.041

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	13	19	53	22	21	20
N.S.	1	1.00	1.00	1.00	1.46	4.08	1.69	1.62	1.54
time (sec)	N/A	0.009	0.006	0.192	0.266	0.362	0.227	0.414	1.143

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	24	10	26	27	10	14
N.S.	1	1.00	1.12	1.50	0.62	1.62	1.69	0.62	0.88
time (sec)	N/A	0.006	0.023	0.262	0.271	0.341	0.265	0.414	0.051

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	30	32	16	52	88	18	16
N.S.	1	1.00	1.15	1.23	0.62	2.00	3.38	0.69	0.62
time (sec)	N/A	0.013	0.044	0.261	0.275	0.353	0.495	0.408	0.046

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	44	40	22	86	182	24	22
N.S.	1	1.00	1.22	1.11	0.61	2.39	5.06	0.67	0.61
time (sec)	N/A	0.019	0.064	0.259	0.265	0.353	0.634	0.415	0.056

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	53	48	28	121	299	30	28
N.S.	1	1.00	1.15	1.04	0.61	2.63	6.50	0.65	0.61
time (sec)	N/A	0.028	0.089	0.256	0.274	0.357	0.889	0.417	1.145

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	62	56	34	159	444	36	34
N.S.	1	1.00	1.11	1.00	0.61	2.84	7.93	0.64	0.61
time (sec)	N/A	0.035	0.102	0.267	0.282	0.349	1.214	0.413	1.149

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	101	43	0	438	0	160	44
N.S.	1	1.00	1.77	0.75	0.00	7.68	0.00	2.81	0.77
time (sec)	N/A	0.032	0.187	0.680	0.000	0.372	0.000	0.420	1.265

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	92	35	0	259	0	112	54
N.S.	1	1.00	2.04	0.78	0.00	5.76	0.00	2.49	1.20
time (sec)	N/A	0.025	0.111	0.642	0.000	0.351	0.000	0.424	1.195

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	69	27	0	131	0	63	26
N.S.	1	1.00	2.09	0.82	0.00	3.97	0.00	1.91	0.79
time (sec)	N/A	0.017	0.073	0.645	0.000	0.339	0.000	0.415	1.185

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	45	17	0	50	0	37	16
N.S.	1	1.00	2.14	0.81	0.00	2.38	0.00	1.76	0.76
time (sec)	N/A	0.010	0.055	0.772	0.000	0.388	0.000	0.406	1.204

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	51	27	0	85	0	66	26
N.S.	1	1.00	1.59	0.84	0.00	2.66	0.00	2.06	0.81
time (sec)	N/A	0.016	0.211	0.713	0.000	0.374	0.000	0.421	1.243

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	86	35	0	168	0	113	32
N.S.	1	1.00	1.76	0.71	0.00	3.43	0.00	2.31	0.65
time (sec)	N/A	0.024	0.218	0.668	0.000	0.393	0.000	0.426	1.200

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	94	43	0	266	0	161	40
N.S.	1	1.00	1.54	0.70	0.00	4.36	0.00	2.64	0.66
time (sec)	N/A	0.033	0.527	0.658	0.000	0.380	0.000	0.438	1.200

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	141	182	348	2748	641	226	244
N.S.	1	1.00	0.99	1.28	2.45	19.35	4.51	1.59	1.72
time (sec)	N/A	0.151	0.543	0.322	0.281	0.409	2.362	0.425	1.282

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	109	134	219	1396	488	153	158
N.S.	1	1.00	1.08	1.33	2.17	13.82	4.83	1.51	1.56
time (sec)	N/A	0.091	0.609	0.363	0.275	0.379	1.445	0.437	1.238

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	86	93	136	654	369	99	97
N.S.	1	1.00	1.25	1.35	1.97	9.48	5.35	1.43	1.41
time (sec)	N/A	0.046	0.277	0.372	0.279	0.376	0.965	0.437	0.109

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	65	61	49	205	236	57	51
N.S.	1	1.00	1.71	1.61	1.29	5.39	6.21	1.50	1.34
time (sec)	N/A	0.017	0.090	0.300	0.268	0.353	0.610	0.421	0.104

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	64	71	52	62	236	62	55
N.S.	1	1.00	1.28	1.42	1.04	1.24	4.72	1.24	1.10
time (sec)	N/A	0.039	0.057	0.739	0.279	0.365	1.002	0.430	1.225

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	100	93	124	426	0	130	104
N.S.	1	1.00	1.18	1.09	1.46	5.01	0.00	1.53	1.22
time (sec)	N/A	0.068	1.075	0.680	0.282	0.408	0.000	0.420	1.300

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	134	130	322	1431	0	203	195
N.S.	1	1.00	1.04	1.01	2.50	11.09	0.00	1.57	1.51
time (sec)	N/A	0.128	2.436	0.794	0.285	0.397	0.000	0.430	1.399

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	214	163	522	3698	0	303	310
N.S.	1	1.00	1.27	0.96	3.09	21.88	0.00	1.79	1.83
time (sec)	N/A	0.193	6.155	0.931	0.325	0.407	0.000	0.429	1.386

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	55	42	28	49	42	29	22
N.S.	1	1.00	1.77	1.35	0.90	1.58	1.35	0.94	0.71
time (sec)	N/A	0.031	0.032	0.754	0.263	0.366	0.424	0.409	0.045

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	55	42	29	48	42	24	22
N.S.	1	1.00	1.77	1.35	0.94	1.55	1.35	0.77	0.71
time (sec)	N/A	0.029	0.031	0.750	0.263	0.385	0.421	0.425	0.044

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	128	70	0	2231	0	0	151
N.S.	1	1.00	1.73	0.95	0.00	30.15	0.00	0.00	2.04
time (sec)	N/A	0.051	2.833	2.331	0.000	0.464	0.000	0.000	1.408

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	129	70	0	2307	0	0	242
N.S.	1	1.00	1.74	0.95	0.00	31.18	0.00	0.00	3.27
time (sec)	N/A	0.044	6.000	1.787	0.000	0.448	0.000	0.000	1.471

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	42	110	36	93	0	42	34
N.S.	1	1.00	0.70	1.83	0.60	1.55	0.00	0.70	0.57
time (sec)	N/A	0.046	0.075	0.543	0.275	0.355	0.000	0.413	1.377

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	36	80	33	60	0	31	29
N.S.	1	1.00	1.24	2.76	1.14	2.07	0.00	1.07	1.00
time (sec)	N/A	0.032	0.057	0.505	0.290	0.356	0.000	0.425	1.267

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	30	70	22	50	0	30	22
N.S.	1	1.00	0.79	1.84	0.58	1.32	0.00	0.79	0.58
time (sec)	N/A	0.036	0.038	0.542	0.260	0.360	0.000	0.405	1.245

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	40	17	25	0	19	17
N.S.	1	1.00	1.11	2.11	0.89	1.32	0.00	1.00	0.89
time (sec)	N/A	0.024	0.034	0.510	0.256	0.350	0.000	0.418	1.242

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	7	11	6	9	0	6	6
N.S.	1	1.00	0.70	1.10	0.60	0.90	0.00	0.60	0.60
time (sec)	N/A	0.014	0.005	0.296	0.265	0.407	0.000	0.408	1.197

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	18	0	12	11
N.S.	1	1.00	1.00	1.14	1.00	2.57	0.00	1.71	1.57
time (sec)	N/A	0.022	0.003	0.500	0.260	0.425	0.000	0.409	1.176

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	14	23	31	77	0	26	29
N.S.	1	1.00	1.75	2.88	3.88	9.62	0.00	3.25	3.62
time (sec)	N/A	0.026	0.030	0.556	0.272	0.385	0.000	0.407	0.079

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	32	41	55	0	10	16
N.S.	1	1.00	1.00	2.91	3.73	5.00	0.00	0.91	1.45
time (sec)	N/A	0.023	0.020	0.614	0.258	0.345	0.000	0.416	1.176

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	156	263	166	1279	0	229	143
N.S.	1	1.00	1.01	1.70	1.07	8.25	0.00	1.48	0.92
time (sec)	N/A	0.180	0.206	0.740	0.289	0.380	0.000	0.415	1.629

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	171	175	0	1859	0	163	172
N.S.	1	1.00	1.28	1.31	0.00	13.87	0.00	1.22	1.28
time (sec)	N/A	0.177	0.552	0.705	0.000	0.398	0.000	0.408	1.865

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	75	155	83	331	0	114	85
N.S.	1	1.00	0.82	1.68	0.90	3.60	0.00	1.24	0.92
time (sec)	N/A	0.103	0.114	0.716	0.274	0.368	0.000	0.431	1.469

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	80	93	0	431	0	72	156
N.S.	1	1.00	1.10	1.27	0.00	5.90	0.00	0.99	2.14
time (sec)	N/A	0.081	0.295	0.606	0.000	0.377	0.000	0.394	1.496

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	46	39	0	147	0	35	35
N.S.	1	1.00	1.21	1.03	0.00	3.87	0.00	0.92	0.92
time (sec)	N/A	0.025	0.027	0.430	0.000	0.361	0.000	0.396	0.172

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	20	13	12	43	0	46	51
N.S.	1	1.00	1.67	1.08	1.00	3.58	0.00	3.83	4.25
time (sec)	N/A	0.030	0.045	0.571	0.264	0.363	0.000	0.421	0.156

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	65	85	0	384	0	85	230
N.S.	1	1.00	1.14	1.49	0.00	6.74	0.00	1.49	4.04
time (sec)	N/A	0.075	0.078	0.769	0.000	0.406	0.000	0.414	1.457

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	50	102	110	434	0	106	88
N.S.	1	1.00	1.25	2.55	2.75	10.85	0.00	2.65	2.20
time (sec)	N/A	0.048	0.123	0.714	0.258	0.386	0.000	0.430	1.442

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	42	118	36	92	0	42	34
N.S.	1	1.00	0.70	1.97	0.60	1.53	0.00	0.70	0.57
time (sec)	N/A	0.048	0.066	0.418	0.260	0.387	0.000	0.421	1.422

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	34	82	27	56	0	25	23
N.S.	1	1.00	1.36	3.28	1.08	2.24	0.00	1.00	0.92
time (sec)	N/A	0.121	0.044	0.378	0.266	0.361	0.000	0.405	1.315

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	24	78	22	51	0	30	22
N.S.	1	1.00	0.63	2.05	0.58	1.34	0.00	0.79	0.58
time (sec)	N/A	0.041	0.038	0.404	0.260	0.363	0.000	0.397	0.107

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	42	11	23	0	11	11
N.S.	1	1.00	1.12	2.47	0.65	1.35	0.00	0.65	0.65
time (sec)	N/A	0.077	0.012	0.373	0.266	0.360	0.000	0.412	1.186

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	16	19	12	23	0	10	10
N.S.	1	1.00	1.60	1.90	1.20	2.30	0.00	1.00	1.00
time (sec)	N/A	0.077	0.019	0.522	0.496	0.333	0.000	0.396	1.281

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	9	36	18	78	0	27	21
N.S.	1	1.00	0.60	2.40	1.20	5.20	0.00	1.80	1.40
time (sec)	N/A	0.026	0.024	0.578	0.484	0.350	0.000	0.410	1.189

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	45	33	140	0	25	22
N.S.	1	1.00	1.00	2.25	1.65	7.00	0.00	1.25	1.10
time (sec)	N/A	0.115	0.030	0.591	0.478	0.349	0.000	0.407	1.255

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	38	75	84	0	18	18
N.S.	1	1.00	1.00	2.24	4.41	4.94	0.00	1.06	1.06
time (sec)	N/A	0.032	0.039	0.593	0.264	0.363	0.000	0.412	0.074

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	133	0	0	231	0	122	-1
N.S.	1	1.00	6.33	0.00	0.00	11.00	0.00	5.81	-0.05
time (sec)	N/A	0.031	13.800	2.089	0.000	0.385	0.000	0.412	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	144	249	154	1229	0	216	135
N.S.	1	1.00	0.98	1.69	1.05	8.36	0.00	1.47	0.92
time (sec)	N/A	0.253	0.402	0.773	0.283	0.363	0.000	0.409	1.743

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	167	176	0	1873	0	164	262
N.S.	1	1.00	1.24	1.30	0.00	13.87	0.00	1.21	1.94
time (sec)	N/A	0.182	1.009	0.730	0.000	0.405	0.000	0.413	2.044

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	73	146	80	334	0	104	82
N.S.	1	1.00	0.86	1.72	0.94	3.93	0.00	1.22	0.96
time (sec)	N/A	0.115	0.168	0.728	0.281	0.360	0.000	0.430	1.398

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	79	92	0	431	0	71	158
N.S.	1	1.00	1.10	1.28	0.00	5.99	0.00	0.99	2.19
time (sec)	N/A	0.082	0.206	0.704	0.000	0.395	0.000	0.415	1.529

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	60	54	0	200	0	48	164
N.S.	1	1.00	1.20	1.08	0.00	4.00	0.00	0.96	3.28
time (sec)	N/A	0.097	0.078	0.796	0.000	0.425	0.000	0.404	3.223

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	27	61	46	117	0	76	323
N.S.	1	1.00	0.93	2.10	1.59	4.03	0.00	2.62	11.14
time (sec)	N/A	0.038	0.068	0.783	0.500	0.348	0.000	0.401	1.584

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	85	121	0	856	0	102	166
N.S.	1	1.00	1.02	1.46	0.00	10.31	0.00	1.23	2.00
time (sec)	N/A	0.166	0.133	0.931	0.000	0.376	0.000	0.399	3.938

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	68	151	133	909	0	201	123
N.S.	1	1.00	0.86	1.91	1.68	11.51	0.00	2.54	1.56
time (sec)	N/A	0.070	0.214	0.830	0.476	0.366	0.000	0.419	1.450

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	38	41	38	41	0	26	65
N.S.	1	1.00	1.23	1.32	1.23	1.32	0.00	0.84	2.10
time (sec)	N/A	0.073	0.038	1.276	0.496	0.351	0.000	0.414	0.513

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	40	96	55	571	0	47	69
N.S.	1	1.00	0.93	2.23	1.28	13.28	0.00	1.09	1.60
time (sec)	N/A	0.083	0.070	0.572	0.480	0.360	0.000	0.405	1.303

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	33	80	43	354	0	39	35
N.S.	1	1.00	0.89	2.16	1.16	9.57	0.00	1.05	0.95
time (sec)	N/A	0.071	0.041	0.579	0.488	0.359	0.000	0.407	1.206

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	27	65	29	186	0	35	29
N.S.	1	1.00	0.93	2.24	1.00	6.41	0.00	1.21	1.00
time (sec)	N/A	0.056	0.035	0.491	0.485	0.353	0.000	0.402	1.216

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	23	47	17	73	0	17	17
N.S.	1	1.00	1.21	2.47	0.89	3.84	0.00	0.89	0.89
time (sec)	N/A	0.031	0.022	0.510	0.498	0.363	0.000	0.417	1.170

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	24	10	26	27	10	14
N.S.	1	1.00	1.12	1.50	0.62	1.62	1.69	0.62	0.88
time (sec)	N/A	0.006	0.020	0.236	0.258	0.367	0.266	0.416	0.002

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	24	10	26	27	10	12
N.S.	1	1.00	1.12	1.50	0.62	1.62	1.69	0.62	0.75
time (sec)	N/A	0.015	0.014	0.273	0.273	0.332	0.275	0.412	0.049

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	23	24	24	73	92	18	21
N.S.	1	1.00	1.21	1.26	1.26	3.84	4.84	0.95	1.11
time (sec)	N/A	0.026	0.022	0.257	0.275	0.359	0.362	0.409	0.059

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	27	28	38	196	160	36	21
N.S.	1	1.00	0.87	0.90	1.23	6.32	5.16	1.16	0.68
time (sec)	N/A	0.036	0.032	0.352	0.261	0.376	0.561	0.421	1.164

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	33	32	54	357	197	40	29
N.S.	1	1.00	0.89	0.86	1.46	9.65	5.32	1.08	0.78
time (sec)	N/A	0.050	0.045	0.424	0.269	0.387	0.761	0.403	0.066

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	90	35	0	259	0	135	34
N.S.	1	1.00	2.00	0.78	0.00	5.76	0.00	3.00	0.76
time (sec)	N/A	0.038	0.097	0.599	0.000	0.345	0.000	0.413	1.238

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	53	26	0	131	0	71	25
N.S.	1	1.00	1.66	0.81	0.00	4.09	0.00	2.22	0.78
time (sec)	N/A	0.026	0.080	0.683	0.000	0.362	0.000	0.411	1.197

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	97	25	0	85	0	64	24
N.S.	1	1.00	3.23	0.83	0.00	2.83	0.00	2.13	0.80
time (sec)	N/A	0.027	0.136	0.747	0.000	0.350	0.000	0.408	1.230

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	84	35	0	166	0	89	32
N.S.	1	1.00	1.71	0.71	0.00	3.39	0.00	1.82	0.65
time (sec)	N/A	0.035	0.233	0.688	0.000	0.364	0.000	0.404	1.218

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	70	35	0	436	0	197	34
N.S.	1	1.00	1.56	0.78	0.00	9.69	0.00	4.38	0.76
time (sec)	N/A	0.048	0.176	0.676	0.000	0.378	0.000	0.423	1.248

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	61	26	0	242	0	133	25
N.S.	1	1.00	1.79	0.76	0.00	7.12	0.00	3.91	0.74
time (sec)	N/A	0.034	0.123	0.786	0.000	0.362	0.000	0.434	1.195

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	81	35	0	189	0	88	36
N.S.	1	1.00	1.93	0.83	0.00	4.50	0.00	2.10	0.86
time (sec)	N/A	0.042	0.254	0.770	0.000	0.358	0.000	0.416	1.256

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	86	35	0	166	0	113	31
N.S.	1	1.00	1.76	0.71	0.00	3.39	0.00	2.31	0.63
time (sec)	N/A	0.058	0.241	0.756	0.000	0.356	0.000	0.422	1.232

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	105	190	146	1294	0	141	163
N.S.	1	1.00	1.08	1.96	1.51	13.34	0.00	1.45	1.68
time (sec)	N/A	0.356	0.246	0.681	0.509	0.398	0.000	0.414	1.612

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	88	138	94	637	0	97	111
N.S.	1	1.00	1.16	1.82	1.24	8.38	0.00	1.28	1.46
time (sec)	N/A	0.223	0.229	0.658	0.509	0.397	0.000	0.410	1.511

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	64	111	67	264	0	74	73
N.S.	1	1.00	1.07	1.85	1.12	4.40	0.00	1.23	1.22
time (sec)	N/A	0.128	0.105	0.639	0.479	0.373	0.000	0.418	1.450

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	46	88	50	73	0	57	58
N.S.	1	1.00	0.90	1.73	0.98	1.43	0.00	1.12	1.14
time (sec)	N/A	0.058	0.069	0.638	0.472	0.383	0.000	0.408	0.324

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	29	55	37	42	148	43	42
N.S.	1	1.00	0.74	1.41	0.95	1.08	3.79	1.10	1.08
time (sec)	N/A	0.031	0.045	0.277	0.277	0.357	0.496	0.414	0.084

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	29	55	36	43	134	43	42
N.S.	1	1.00	0.74	1.41	0.92	1.10	3.44	1.10	1.08
time (sec)	N/A	0.042	0.045	0.289	0.274	0.379	0.505	0.405	0.064

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	49	60	63	76	372	59	57
N.S.	1	1.00	0.78	0.95	1.00	1.21	5.90	0.94	0.90
time (sec)	N/A	0.066	0.062	0.292	0.278	0.400	0.865	0.413	1.480

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	64	67	82	271	636	76	74
N.S.	1	1.00	1.00	1.05	1.28	4.23	9.94	1.19	1.16
time (sec)	N/A	0.092	0.098	0.435	0.276	0.380	1.317	0.429	1.509

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	88	76	119	648	882	100	110
N.S.	1	1.00	1.16	1.00	1.57	8.53	11.61	1.32	1.45
time (sec)	N/A	0.158	0.152	0.389	0.288	0.376	1.818	0.410	1.627

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	108	95	169	1299	1013	143	164
N.S.	1	1.00	1.15	1.01	1.80	13.82	10.78	1.52	1.74
time (sec)	N/A	0.266	0.225	0.380	0.294	0.416	3.021	0.436	1.631

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	49	73	68	184	0	169	68
N.S.	1	1.00	0.91	1.35	1.26	3.41	0.00	3.13	1.26
time (sec)	N/A	0.062	0.136	1.303	0.412	0.418	0.000	0.423	1.282

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	64	24	36	28	0	24	23
N.S.	1	1.00	2.13	0.80	1.20	0.93	0.00	0.80	0.77
time (sec)	N/A	0.028	0.019	0.438	0.272	0.367	0.000	0.410	1.251

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	64	83	48	62	0	54	39
N.S.	1	1.00	1.42	1.84	1.07	1.38	0.00	1.20	0.87
time (sec)	N/A	0.030	0.168	0.714	0.514	0.359	0.000	0.402	1.213

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	26	37	36	33	0	37	25
N.S.	1	1.00	1.13	1.61	1.57	1.43	0.00	1.61	1.09
time (sec)	N/A	0.020	0.140	0.569	0.283	0.354	0.000	0.414	1.226

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	58	71	45	58	0	51	36
N.S.	1	1.00	1.45	1.78	1.12	1.45	0.00	1.28	0.90
time (sec)	N/A	0.014	0.132	0.708	0.489	0.359	0.000	0.418	1.186

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	21	26	10	18	27	21	18
N.S.	1	1.00	1.75	2.17	0.83	1.50	2.25	1.75	1.50
time (sec)	N/A	0.010	0.023	0.792	0.269	0.367	0.531	0.393	1.212

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	62	88	47	54	0	52	37
N.S.	1	1.00	1.51	2.15	1.15	1.32	0.00	1.27	0.90
time (sec)	N/A	0.024	0.123	0.704	0.473	0.391	0.000	0.408	1.205

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	27	35	30	38	0	33	25
N.S.	1	1.00	1.29	1.67	1.43	1.81	0.00	1.57	1.19
time (sec)	N/A	0.021	0.113	0.509	0.264	0.338	0.000	0.412	1.214

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	86	41	53	61	0	40	40
N.S.	1	1.00	1.83	0.87	1.13	1.30	0.00	0.85	0.85
time (sec)	N/A	0.046	0.078	0.422	0.273	0.362	0.000	0.411	1.247

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	154	100	66	104	0	72	60
N.S.	1	1.00	2.26	1.47	0.97	1.53	0.00	1.06	0.88
time (sec)	N/A	0.050	2.115	0.589	0.478	0.356	0.000	0.421	1.237

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	163	54	53	74	0	54	42
N.S.	1	1.00	3.98	1.32	1.29	1.80	0.00	1.32	1.02
time (sec)	N/A	0.033	2.140	0.428	0.264	0.350	0.000	0.419	1.226

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	153	86	60	97	0	66	54
N.S.	1	1.00	2.55	1.43	1.00	1.62	0.00	1.10	0.90
time (sec)	N/A	0.029	1.625	0.600	0.479	0.358	0.000	0.399	1.205

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	28	35	19	28	78	21	28
N.S.	1	1.00	2.00	2.50	1.36	2.00	5.57	1.50	2.00
time (sec)	N/A	0.018	0.041	0.793	0.273	0.334	2.653	0.423	1.188

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	153	104	69	97	0	77	60
N.S.	1	1.00	1.78	1.21	0.80	1.13	0.00	0.90	0.70
time (sec)	N/A	0.043	1.926	0.608	0.479	0.347	0.000	0.401	1.212

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	155	55	50	82	0	57	48
N.S.	1	1.00	2.58	0.92	0.83	1.37	0.00	0.95	0.80
time (sec)	N/A	0.038	2.119	0.383	0.261	0.360	0.000	0.411	1.230

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	46	0	0	0	0	0	-1
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.026	0.070	0.464	0.000	0.000	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	77	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.124	0.447	0.000	0.000	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	108	0	0	0	0	0	-1
N.S.	1	1.00	0.61	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	0.152	0.596	0.000	0.000	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	259	0	0	0	0	0	-1
N.S.	1	1.00	3.28	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.030	1.361	0.846	0.000	0.000	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	126	0	0	0	0	0	-1
N.S.	1	1.00	1.27	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	2.148	0.451	0.000	0.000	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	83	0	0	0	0	0	-1
N.S.	1	1.00	1.60	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.017	0.379	0.848	0.000	0.000	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	125	0	0	0	0	0	-1
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.488	0.836	0.000	0.000	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	142	0	0	0	0	0	-1
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.505	0.847	0.000	0.000	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	176	0	0	0	0	0	-1
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.109	2.036	0.851	0.000	0.000	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	171	0	0	0	0	0	-1
N.S.	1	1.00	2.80	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.022	1.110	0.842	0.000	0.000	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	171	0	0	0	0	0	-1
N.S.	1	1.00	2.80	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.025	1.253	0.848	0.000	0.000	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	171	0	0	0	0	0	-1
N.S.	1	1.00	2.80	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.025	1.320	0.858	0.000	0.000	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	198	0	0	0	0	0	-1
N.S.	1	1.00	3.41	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.047	4.872	0.906	0.000	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	207	0	0	0	0	0	-1
N.S.	1	1.00	3.34	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.044	3.512	0.944	0.000	0.000	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	193	0	0	0	0	0	-1
N.S.	1	1.00	3.57	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.039	4.748	0.960	0.000	0.000	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	198	0	0	0	0	0	-1
N.S.	1	1.00	3.81	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.037	6.162	0.659	0.000	0.000	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	40	48	24	76	0	74	34
N.S.	1	1.00	1.60	1.92	0.96	3.04	0.00	2.96	1.36
time (sec)	N/A	0.015	0.046	2.287	0.264	0.374	0.000	0.487	1.193

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	197	0	0	0	0	0	-1
N.S.	1	1.00	3.40	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.043	2.607	0.559	0.000	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	191	0	0	0	0	0	-1
N.S.	1	1.00	3.47	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.043	2.594	0.545	0.000	0.000	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	155	0	0	0	0	0	-1
N.S.	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	4.925	0.548	0.000	0.000	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	165	0	0	0	0	0	-1
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.122	3.094	0.552	0.000	0.000	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	151	0	0	0	0	0	-1
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.112	4.500	0.569	0.000	0.000	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	160	0	0	0	0	0	-1
N.S.	1	1.00	1.27	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.108	5.695	0.543	0.000	0.000	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	49	63	37	72	0	37	34
N.S.	1	1.00	1.75	2.25	1.32	2.57	0.00	1.32	1.21
time (sec)	N/A	0.021	0.082	2.317	0.324	0.347	0.000	0.479	1.189

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	158	0	0	0	0	0	-1
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	2.463	0.572	0.000	0.000	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	156	0	0	0	0	0	-1
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.120	2.432	0.424	0.000	0.000	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	52	56	330	572	0	127	95
N.S.	1	1.00	1.21	1.30	7.67	13.30	0.00	2.95	2.21
time (sec)	N/A	0.030	0.157	2.381	0.330	0.350	0.000	0.468	1.225

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	44	69	499	171	0	67	163
N.S.	1	1.00	0.98	1.53	11.09	3.80	0.00	1.49	3.62
time (sec)	N/A	0.030	0.085	2.357	0.352	0.360	0.000	0.498	1.208

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	67	71	855	1576	0	161	229
N.S.	1	1.00	1.02	1.08	12.95	23.88	0.00	2.44	3.47
time (sec)	N/A	0.043	0.202	2.474	0.399	0.363	0.000	0.490	1.199

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	158	0	0	0	0	0	-1
N.S.	1	1.00	1.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	9.101	0.574	0.000	0.000	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	312	0	0	0	0	0	-1
N.S.	1	1.00	1.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.147	9.901	0.432	0.000	0.000	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	600	0	0	0	0	0	-1
N.S.	1	1.00	1.96	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.327	13.208	1.396	0.000	0.000	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	387	0	0	0	0	0	-1
N.S.	1	1.00	3.37	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.086	2.458	1.181	0.000	0.000	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	174	0	0	0	0	0	-1
N.S.	1	1.00	1.29	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.106	3.470	1.055	0.000	0.000	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	64	76	0	626	0	0	65
N.S.	1	1.00	0.88	1.04	0.00	8.58	0.00	0.00	0.89
time (sec)	N/A	0.039	0.208	5.122	0.000	0.378	0.000	0.000	2.241

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	57	74	0	334	0	0	51
N.S.	1	1.00	0.81	1.06	0.00	4.77	0.00	0.00	0.73
time (sec)	N/A	0.035	0.108	4.647	0.000	0.374	0.000	0.000	1.860

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	61	0	305	0	0	39
N.S.	1	1.00	1.00	1.27	0.00	6.35	0.00	0.00	0.81
time (sec)	N/A	0.027	0.057	4.793	0.000	0.383	0.000	0.000	1.505

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	37	0	303	0	0	36
N.S.	1	1.00	1.00	0.79	0.00	6.45	0.00	0.00	0.77
time (sec)	N/A	0.027	0.083	4.688	0.000	0.373	0.000	0.000	1.644

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	44	76	0	625	0	0	65
N.S.	1	1.00	0.62	1.07	0.00	8.80	0.00	0.00	0.92
time (sec)	N/A	0.038	0.107	4.709	0.000	0.359	0.000	0.000	1.742

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	46	74	0	1104	0	0	64
N.S.	1	1.00	0.64	1.03	0.00	15.33	0.00	0.00	0.89
time (sec)	N/A	0.037	0.142	4.701	0.000	0.434	0.000	0.000	2.391

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	266	149	0	8951	0	0	-1
N.S.	1	1.00	1.97	1.10	0.00	66.30	0.00	0.00	-0.01
time (sec)	N/A	0.262	7.090	1.288	0.000	1.249	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	199	90	0	6695	0	0	-1
N.S.	1	1.00	1.90	0.86	0.00	63.76	0.00	0.00	-0.01
time (sec)	N/A	0.146	98.244	1.201	0.000	1.009	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	141	52	0	1752	0	0	-1
N.S.	1	1.00	2.43	0.90	0.00	30.21	0.00	0.00	-0.02
time (sec)	N/A	0.083	53.723	1.184	0.000	0.780	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	203	0	0	6705	0	0	-1
N.S.	1	1.00	1.92	0.00	0.00	63.25	0.00	0.00	-0.01
time (sec)	N/A	0.167	6.288	2.552	0.000	0.971	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	278	0	0	9148	0	0	-1
N.S.	1	1.00	1.52	0.00	0.00	49.99	0.00	0.00	-0.01
time (sec)	N/A	0.238	8.386	2.528	0.000	1.181	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	304	850	0	7964	0	0	-1
N.S.	1	1.00	2.30	6.44	0.00	60.33	0.00	0.00	-0.01
time (sec)	N/A	0.162	5.837	1.010	0.000	1.444	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	164	320	167	1617	0	181	-1
N.S.	1	1.00	0.51	1.00	0.52	5.07	0.00	0.57	-0.00
time (sec)	N/A	0.636	10.193	5.644	0.496	0.373	0.000	0.421	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	334	298	112	613	0	155	-1
N.S.	1	1.00	1.70	1.51	0.57	3.11	0.00	0.79	-0.01
time (sec)	N/A	0.207	2.750	5.130	0.486	0.378	0.000	0.406	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	51	213	56	70	0	94	-1
N.S.	1	1.00	0.61	2.57	0.67	0.84	0.00	1.13	-0.01
time (sec)	N/A	0.102	0.045	5.889	0.480	0.353	0.000	0.409	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	51	218	35	53	0	60	-1
N.S.	1	1.00	0.61	2.63	0.42	0.64	0.00	0.72	-0.01
time (sec)	N/A	0.143	0.091	5.757	0.477	0.353	0.000	0.410	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	104	301	90	458	0	130	-1
N.S.	1	1.00	0.54	1.56	0.47	2.37	0.00	0.67	-0.01
time (sec)	N/A	0.599	0.213	5.248	0.484	0.353	0.000	0.413	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	133	324	145	1226	0	185	-1
N.S.	1	1.00	0.43	1.04	0.47	3.94	0.00	0.59	-0.00
time (sec)	N/A	1.217	0.328	5.322	0.480	0.356	0.000	0.411	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	124	118	0	296	0	0	-1
N.S.	1	1.00	0.79	0.75	0.00	1.89	0.00	0.00	-0.01
time (sec)	N/A	0.275	0.190	4.293	0.000	0.419	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	88	88	0	155	0	0	-1
N.S.	1	1.00	0.77	0.77	0.00	1.35	0.00	0.00	-0.01
time (sec)	N/A	0.193	0.134	3.458	0.000	0.352	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	59	58	0	149	0	0	-1
N.S.	1	1.00	0.77	0.75	0.00	1.94	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.100	3.317	0.000	0.386	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.055	1.943	180.000	0.000	0.000	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	124	118	0	298	0	0	-1
N.S.	1	1.00	0.79	0.75	0.00	1.90	0.00	0.00	-0.01
time (sec)	N/A	0.269	0.198	3.467	0.000	0.380	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	88	88	0	155	0	0	-1
N.S.	1	1.00	0.77	0.77	0.00	1.35	0.00	0.00	-0.01
time (sec)	N/A	0.189	0.133	2.430	0.000	0.424	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	62	58	0	151	0	0	-1
N.S.	1	1.00	0.81	0.75	0.00	1.96	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.082	3.010	0.000	0.358	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	4.147	180.000	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [211] had the largest ratio of [25]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	6	1.00	12	0.500
2	A	6	6	1.00	12	0.500
3	A	6	6	1.00	12	0.500
4	A	5	5	1.00	12	0.417
5	A	5	5	1.00	12	0.417
6	A	6	6	1.00	12	0.500
7	A	6	6	1.00	12	0.500
8	A	7	6	1.00	12	0.500
9	A	13	9	1.00	12	0.750
10	A	12	8	1.00	12	0.667
11	A	9	9	1.00	12	0.750
12	A	9	9	1.00	12	0.750
13	A	12	8	1.00	12	0.667
14	A	13	9	1.00	12	0.750
15	A	2	2	1.00	8	0.250
16	A	2	2	1.00	10	0.200
17	A	3	3	1.00	12	0.250
18	A	3	3	1.00	14	0.214
19	A	2	2	1.00	14	0.143
20	A	2	2	1.00	14	0.143
21	A	3	3	1.00	14	0.214
22	A	14	10	1.00	14	0.714
23	A	14	10	1.00	14	0.714
24	A	13	9	1.00	14	0.643
25	A	13	9	1.00	14	0.643

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	14	10	1.00	14	0.714
27	A	14	10	1.00	14	0.714
28	A	3	3	1.00	12	0.250
29	A	8	7	1.00	14	0.500
30	A	7	7	1.00	14	0.500
31	A	7	7	1.00	14	0.500
32	A	8	7	1.00	14	0.500
33	A	4	3	1.00	14	0.214
34	A	3	3	1.00	14	0.214
35	A	2	2	1.00	14	0.143
36	A	2	2	1.00	14	0.143
37	A	3	3	1.00	14	0.214
38	A	4	3	1.00	14	0.214
39	A	3	3	1.00	12	0.250
40	A	5	3	1.00	14	0.214
41	A	3	3	1.00	14	0.214
42	A	3	3	1.00	14	0.214
43	A	5	3	1.00	14	0.214
44	A	16	10	1.00	14	0.714
45	A	14	10	1.00	14	0.714
46	A	14	10	1.00	14	0.714
47	A	14	10	1.00	14	0.714
48	A	14	10	1.00	14	0.714
49	A	16	10	1.00	14	0.714
50	A	3	3	1.00	12	0.250
51	A	3	3	1.00	14	0.214
52	A	3	3	1.00	14	0.214
53	A	3	3	1.00	14	0.214
54	A	3	3	1.00	14	0.214
55	A	3	3	1.00	14	0.214
56	A	3	3	1.00	14	0.214
57	A	3	3	1.00	14	0.214
58	A	3	3	1.00	14	0.214
59	A	3	3	1.00	14	0.214
60	A	3	3	1.00	14	0.214

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	5	3	1.00	6	0.500
62	A	4	3	1.00	6	0.500
63	A	3	3	1.00	6	0.500
64	A	2	2	1.00	6	0.333
65	A	2	2	1.00	6	0.333
66	A	3	2	1.00	6	0.333
67	A	4	2	1.00	6	0.333
68	A	5	2	1.00	6	0.333
69	A	6	2	1.00	6	0.333
70	A	5	3	1.00	8	0.375
71	A	4	3	1.00	8	0.375
72	A	3	3	1.00	8	0.375
73	A	2	2	1.00	8	0.250
74	A	3	3	1.00	8	0.375
75	A	4	3	1.00	8	0.375
76	A	5	3	1.00	8	0.375
77	A	5	4	1.00	12	0.333
78	A	4	4	1.00	12	0.333
79	A	3	3	1.00	12	0.250
80	A	2	2	1.00	12	0.167
81	A	2	2	1.00	12	0.167
82	A	3	3	1.00	12	0.250
83	A	4	4	1.00	12	0.333
84	A	5	4	1.00	12	0.333
85	A	2	2	1.00	12	0.167
86	A	2	2	1.00	12	0.167
87	A	5	4	1.00	14	0.286
88	A	5	4	1.00	14	0.286
89	A	4	3	1.00	11	0.273
90	A	3	2	1.00	11	0.182
91	A	4	3	1.00	11	0.273
92	A	2	2	1.00	9	0.222
93	A	1	1	1.00	9	0.111
94	A	2	2	1.00	11	0.182
95	A	2	2	1.00	11	0.182

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	2	1	1.00	11	0.091
97	A	5	4	1.00	13	0.308
98	A	9	6	1.00	13	0.462
99	A	4	3	1.00	13	0.231
100	A	5	5	1.00	11	0.454
101	A	2	2	1.00	11	0.182
102	A	2	2	1.00	13	0.154
103	A	5	5	1.00	13	0.385
104	A	3	2	1.00	13	0.154
105	A	5	4	1.00	11	0.364
106	A	9	7	1.00	11	0.636
107	A	5	4	1.00	11	0.364
108	A	8	6	1.00	9	0.667
109	A	8	7	1.00	9	0.778
110	A	3	2	1.00	11	0.182
111	A	8	7	1.00	11	0.636
112	A	4	3	1.00	11	0.273
113	A	4	4	1.00	13	0.308
114	A	5	3	1.00	13	0.231
115	A	10	9	1.00	13	0.692
116	A	4	3	1.00	13	0.231
117	A	6	6	1.00	11	0.546
118	A	6	5	1.00	11	0.454
119	A	3	2	1.00	13	0.154
120	A	9	7	1.00	13	0.538
121	A	3	2	1.00	13	0.154
122	A	6	5	1.00	13	0.385
123	A	6	4	1.00	11	0.364
124	A	5	4	1.00	11	0.364
125	A	4	4	1.00	11	0.364
126	A	4	4	1.00	9	0.444
127	A	2	2	1.00	6	0.333
128	A	2	2	1.00	9	0.222
129	A	3	2	1.00	11	0.182
130	A	3	3	1.00	11	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
131	A	4	4	1.00	11	0.364
132	A	4	4	1.00	11	0.364
133	A	3	3	1.00	11	0.273
134	A	3	3	1.00	11	0.273
135	A	4	4	1.00	11	0.364
136	A	4	4	1.00	13	0.308
137	A	3	3	1.00	13	0.231
138	A	4	4	1.00	13	0.308
139	A	4	4	1.00	13	0.308
140	A	6	6	1.00	13	0.462
141	A	5	5	1.00	13	0.385
142	A	4	4	1.00	13	0.308
143	A	3	3	1.00	11	0.273
144	A	2	2	1.00	8	0.250
145	A	2	2	1.00	11	0.182
146	A	4	4	1.00	13	0.308
147	A	5	5	1.00	13	0.385
148	A	6	6	1.00	13	0.462
149	A	7	7	1.00	13	0.538
150	A	3	3	1.00	14	0.214
151	A	4	3	1.00	11	0.273
152	A	5	5	1.00	11	0.454
153	A	4	4	1.00	9	0.444
154	A	5	5	1.00	7	0.714
155	A	2	1	1.00	11	0.091
156	A	5	5	1.00	11	0.454
157	A	4	4	1.00	11	0.364
158	A	4	3	1.00	13	0.231
159	A	6	6	1.00	13	0.462
160	A	5	5	1.00	11	0.454
161	A	7	6	1.00	9	0.667
162	A	3	2	1.00	13	0.154
163	A	6	6	1.00	13	0.462
164	A	5	5	1.00	13	0.385
165	A	3	3	1.00	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	4	4	1.00	15	0.267
167	A	5	5	1.00	15	0.333
168	A	3	3	1.00	9	0.333
169	A	3	3	1.00	15	0.200
170	A	2	2	1.00	11	0.182
171	A	4	4	1.00	11	0.364
172	A	5	5	1.00	11	0.454
173	A	5	5	1.00	11	0.454
174	A	3	3	1.00	7	0.429
175	A	3	3	1.00	9	0.333
176	A	3	3	1.00	9	0.333
177	A	4	4	1.00	17	0.235
178	A	4	4	1.00	17	0.235
179	A	4	4	1.00	15	0.267
180	A	4	4	1.00	13	0.308
181	A	2	1	1.00	17	0.059
182	A	4	4	1.00	17	0.235
183	A	4	4	1.00	17	0.235
184	A	5	5	1.00	19	0.263
185	A	5	5	1.00	19	0.263
186	A	5	5	1.00	17	0.294
187	A	5	5	1.00	15	0.333
188	A	3	2	1.00	19	0.105
189	A	5	5	1.00	19	0.263
190	A	5	5	1.00	19	0.263
191	A	3	2	1.00	17	0.118
192	A	4	2	1.00	17	0.118
193	A	4	2	1.00	17	0.118
194	A	4	4	1.00	19	0.210
195	A	5	5	1.00	21	0.238
196	A	6	6	1.00	21	0.286
197	A	4	4	1.00	15	0.267
198	A	4	4	1.00	21	0.190
199	A	7	6	1.00	19	0.316
200	A	7	6	1.00	19	0.316

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	6	5	1.00	19	0.263
202	A	6	5	1.00	19	0.263
203	A	7	6	1.00	19	0.316
204	A	7	6	1.00	19	0.316
205	A	8	7	1.00	23	0.304
206	A	7	6	1.00	23	0.261
207	A	4	4	1.00	21	0.190
208	A	8	5	1.00	21	0.238
209	A	11	6	1.00	23	0.261
210	A	8	7	1.00	21	0.333
211	A	9	7	1.00	25	0.280
212	A	8	7	1.00	25	0.280
213	A	4	4	1.00	25	0.160
214	A	4	4	1.00	25	0.160
215	A	8	7	1.00	25	0.280
216	A	9	7	1.00	25	0.280
217	A	19	5	1.00	9	0.556
218	A	13	5	1.00	9	0.556
219	A	9	4	1.00	7	0.571
220	A	0	0	0.00	0	0.000
221	A	19	5	1.00	9	0.556
222	A	13	5	1.00	9	0.556
223	A	9	4	1.00	7	0.571
224	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

Local contents

3.1 $\int (b\coth(c+dx))^{7/2}\,dx$

Optimal. Leaf size=97

$$\frac{b^{7/2}\text{ArcTan}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}+\frac{b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{2b^3\sqrt{b\coth(c+dx)}}{d}-\frac{2b(b\coth(c+d}{5d}$$

```
[Out] b^(7/2)*arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/d+b^(7/2)*arctanh((b*coth(d*x
+c))^(1/2)/b^(1/2))/d-2/5*b*(b*coth(d*x+c))^(5/2)/d-2*b^3*(b*coth(d*x+c))^(
1/2)/d
```

Rubi [A]
time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3554, 3557, 335, 218, 212, 209}

$$\frac{b^{7/2}\text{ArcTan}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}+\frac{b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{2b^3\sqrt{b\coth(c+dx)}}{d}-\frac{2b(b\coth(c+dx))^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x])^(7/2),x]
```

```
[Out] (b^(7/2)*ArcTan[Sqrt[b*Coth[c + d*x]]/Sqrt[b]])/d + (b^(7/2)*ArcTanh[Sqrt[b
*Coth[c + d*x]]/Sqrt[b]])/d - (2*b^3*Sqrt[b*Coth[c + d*x]])/d - (2*b*(b*Cot
h[c + d*x])^(5/2))/(5*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int (b\coth(c+dx))^{7/2}\,dx &= -\frac{2b(b\coth(c+dx))^{5/2}}{5d}+b^2\int (b\coth(c+dx))^{3/2}\,dx\\ &= -\frac{2b^3\sqrt{b\coth(c+dx)}}{d}-\frac{2b(b\coth(c+dx))^{5/2}}{5d}+b^4\int \frac{1}{\sqrt{b\coth(c+dx)}}\,dx\\ &= -\frac{2b^3\sqrt{b\coth(c+dx)}}{d}-\frac{2b(b\coth(c+dx))^{5/2}}{5d}-\frac{b^5\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(-b^2+x^2)}\,dx,x,\right.}{d}\\ &= -\frac{2b^3\sqrt{b\coth(c+dx)}}{d}-\frac{2b(b\coth(c+dx))^{5/2}}{5d}-\frac{(2b^5)\,\text{Subst}\left(\int \frac{1}{-b^2+x^4}\,dx,x,\sqrt{\right.}}{d}\\ &= -\frac{2b^3\sqrt{b\coth(c+dx)}}{d}-\frac{2b(b\coth(c+dx))^{5/2}}{5d}+\frac{b^4\text{Subst}\left(\int \frac{1}{b-x^2}\,dx,x,\sqrt{b\coth}\right.}{d}\\ &= \frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}+\frac{b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{2b^3\sqrt{b\,\text{c}}}{}\end{aligned}$$

Mathematica [A]
time = 0.17, size = 83, normalized size = 0.86

$$\frac{b^3\sqrt{b\coth(c+dx)}\left(5\text{ArcTan}\left(\sqrt{\coth(c+dx)}\right)+5\tanh^{-1}\left(\sqrt{\coth(c+dx)}\right)-10\sqrt{\coth(c+dx)}-2\coth^{\frac{5}{2}}(c+dx)\right)}{5d\sqrt{\coth(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x])^(7/2),x]
```

```
[Out] (b^3*Sqrt[b*Coth[c + d*x]]*(5*ArcTan[Sqrt[Coth[c + d*x]]] + 5*ArcTanh[Sqrt[
Coth[c + d*x]]] - 10*Sqrt[Coth[c + d*x]] - 2*Coth[c + d*x]^(5/2)))/(5*d*Sqr
t[Coth[c + d*x]])
```

Maple [A]
time = 2.19, size = 74, normalized size = 0.76

method	result
derivativedivides	$-\frac{2b\left(\frac{(b\coth(dx+c))^{\frac{5}{2}}}{5}+b^2\sqrt{b\coth(dx+c)}-\frac{b^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{2}-\frac{b^{\frac{5}{2}}\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{2}\right)}{d}$
default	$-\frac{2b\left(\frac{(b\coth(dx+c))^{\frac{5}{2}}}{5}+b^2\sqrt{b\coth(dx+c)}-\frac{b^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{2}-\frac{b^{\frac{5}{2}}\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{2}\right)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*b*(1/5*(b*coth(d*x+c))^(5/2)+b^2*(b*coth(d*x+c))^(1/2)-1/2*b^(5/2)*arc
tanh((b*coth(d*x+c))^(1/2)/b^(1/2))-1/2*b^(5/2)*arctan((b*coth(d*x+c))^(1/2
)/b^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c))^(7/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 762 vs. $2(79) = 158$.
time = 0.37, size = 1574, normalized size = 16.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] [-1/20*(10*(b^3*cosh(d*x + c)^4 + 4*b^3*cosh(d*x + c)*sinh(d*x + c)^3 + b^3
*sinh(d*x + c)^4 - 2*b^3*cosh(d*x + c)^2 + b^3 + 2*(3*b^3*cosh(d*x + c)^2 -
 b^3)*sinh(d*x + c)^2 + 4*(b^3*cosh(d*x + c)^3 - b^3*cosh(d*x + c))*sinh(d*
x + c))*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x +
 c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - 5*(b^3*
cosh(d*x + c)^4 + 4*b^3*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*sinh(d*x + c)^4
 - 2*b^3*cosh(d*x + c)^2 + b^3 + 2*(3*b^3*cosh(d*x + c)^2 - b^3)*sinh(d*x +
 c)^2 + 4*(b^3*cosh(d*x + c)^3 - b^3*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b)
*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d*x
 + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x +
c)^4 - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2
 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d*x + c)^4
+ 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*c
osh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 16*(3*b^3*cosh(d*x + c)^
4 + 12*b^3*cosh(d*x + c)*sinh(d*x + c)^3 + 3*b^3*sinh(d*x + c)^4 - 4*b^3*co
sh(d*x + c)^2 + 3*b^3 + 2*(9*b^3*cosh(d*x + c)^2 - 2*b^3)*sinh(d*x + c)^2 +
 4*(3*b^3*cosh(d*x + c)^3 - 2*b^3*cosh(d*x + c))*sinh(d*x + c))*sqrt(b*cosh
(d*x + c)/sinh(d*x + c)))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x +
 c)^3 + d*sinh(d*x + c)^4 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 -
d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c)
+ d), 1/20*(10*(b^3*cosh(d*x + c)^4 + 4*b^3*cosh(d*x + c)*sinh(d*x + c)^3 +
 b^3*sinh(d*x + c)^4 - 2*b^3*cosh(d*x + c)^2 + b^3 + 2*(3*b^3*cosh(d*x + c)
^2 - b^3)*sinh(d*x + c)^2 + 4*(b^3*cosh(d*x + c)^3 - b^3*cosh(d*x + c))*sin
h(d*x + c))*sqrt(b)*arctan(sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*c
osh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b))
+ 5*(b^3*cosh(d*x + c)^4 + 4*b^3*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*sinh(d
*x + c)^4 - 2*b^3*cosh(d*x + c)^2 + b^3 + 2*(3*b^3*cosh(d*x + c)^2 - b^3)*s
inh(d*x + c)^2 + 4*(b^3*cosh(d*x + c)^3 - b^3*cosh(d*x + c))*sinh(d*x + c))
*sqrt(b)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x + c) + 12*b
*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c)^3 + 2*b*
sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + si
nh(d*x + c)^4 + (6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c)^2 +
 2*(2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sqrt(b*cosh(d
*x + c)/sinh(d*x + c)) - b) - 16*(3*b^3*cosh(d*x + c)^4 + 12*b^3*cosh(d*x +
 c)*sinh(d*x + c)^3 + 3*b^3*sinh(d*x + c)^4 - 4*b^3*cosh(d*x + c)^2 + 3*b^3
 + 2*(9*b^3*cosh(d*x + c)^2 - 2*b^3)*sinh(d*x + c)^2 + 4*(3*b^3*cosh(d*x +
c)^3 - 2*b^3*cosh(d*x + c))*sinh(d*x + c))*sqrt(b*cosh(d*x + c)/sinh(d*x +
c)))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x +
c)^4 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 +
4*(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)]
```

Sympy [**F(-1)**] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 379 vs. 2(79) = 158.
time = 0.53, size = 379, normalized size = 3.91

$$-\frac{10 b^{\frac{7}{2}} \arctan\left(-\frac{\sqrt{b} e^{(2dx+2c)} - \sqrt{be^{(4dx+4c)} - b}}{\sqrt{b}}\right) \operatorname{sgn}\left(e^{(2dx+2c)} - 1\right) + 5 b^{\frac{7}{2}} \log\left(\left|-\sqrt{b} e^{(2dx+2c)} + \sqrt{be^{(4dx+4c)} - b}\right|\right) \operatorname{sgn}\left(e^{(2dx+2c)} - 1\right) - \frac{16\left(5\left(\sqrt{b}e^{(2dx+2c)} - \sqrt{be^{(4dx+4c)}-b}\right)^4 b^4 \operatorname{sgn}\left(e^{(2dx+2c)}-1\right) - 10\left(\sqrt{b}e^{(2dx+2c)} - \sqrt{be^{(4dx+4c)}-b}\right)^3 b^{\frac{9}{2}} \operatorname{sgn}\left(e^{(2dx+2c)}-1\right) + 20\left(\sqrt{b}e^{(2dx+2c)} - \sqrt{be^{(4dx+4c)}-b}\right)^2 b^5 \operatorname{sgn}\left(e^{(2dx+2c)}-1\right) - 10\left(\sqrt{b}e^{(2dx+2c)} - \sqrt{be^{(4dx+4c)}-b}\right) b^{\frac{11}{2}} \operatorname{sgn}\left(e^{(2dx+2c)}-1\right) + 3 b^6 \operatorname{sgn}\left(e^{(2dx+2c)}-1\right)\right)}{\left(\sqrt{b}e^{(2dx+2c)} - \sqrt{be^{(4dx+4c)}-b} - \sqrt{b}\right)^5}}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] -1/10*(10*b^(7/2)*arctan(-(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c)
 - b))/sqrt(b))*sgn(e^(2*d*x + 2*c) - 1) + 5*b^(7/2)*log(abs(-sqrt(b)*e^(2*
d*x + 2*c) + sqrt(b*e^(4*d*x + 4*c) - b)))*sgn(e^(2*d*x + 2*c) - 1) - 16*(5
*(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))^4*b^4*sgn(e^(2*d*x
 + 2*c) - 1) - 10*(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))^3
*b^(9/2)*sgn(e^(2*d*x + 2*c) - 1) + 20*(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^
(4*d*x + 4*c) - b))^2*b^5*sgn(e^(2*d*x + 2*c) - 1) - 10*(sqrt(b)*e^(2*d*x +
 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))*b^(11/2)*sgn(e^(2*d*x + 2*c) - 1) + 3*
b^6*sgn(e^(2*d*x + 2*c) - 1))/(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x +
4*c) - b) - sqrt(b))^5)/d
```

Mupad [B]
time = 1.58, size = 83, normalized size = 0.86

$$\frac{b^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{b\coth\left(c+d\,x\right)}}{\sqrt{b}}\right)}{d}-\frac{2\,b^3\,\sqrt{b\coth\left(c+d\,x\right)}}{d}-\frac{2\,b\,{\left(b\coth(c+d\,x)\right)}^{5/2}}{5\,d}-\frac{b^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{b\coth\left(c+d\,x\right)}\,1{}\mathrm{i}}{\sqrt{b}}\right)\,1{}\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x))^(7/2),x)
```

```
[Out] (b^(7/2)*atan((b*coth(c + d*x))^(1/2)/b^(1/2)))/d - (2*b^3*(b*coth(c + d*x)
)^(1/2))/d - (2*b*(b*coth(c + d*x))^(5/2))/(5*d) - (b^(7/2)*atan(((b*coth(c
 + d*x))^(1/2)*1i)/b^(1/2))*1i)/d
```

3.2 $\int (b\coth(c+dx))^{5/2}\,dx$

Optimal. Leaf size=78

$$-\frac{b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}+\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{2b(b\coth(c+dx))^{3/2}}{3d}$$

```
[Out] -b^(5/2)*arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/d+b^(5/2)*arctanh((b*coth(d*
x+c))^(1/2)/b^(1/2))/d-2/3*b*(b*coth(d*x+c))^(3/2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.500$, Rules used = {3554, 3557, 335, 304, 209, 212}

$$-\frac{b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}+\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{2b(b\coth(c+dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x])^(5/2),x]
```

```
[Out] -((b^(5/2)*ArcTan[Sqrt[b*Coth[c + d*x]]/Sqrt[b]])/d) + (b^(5/2)*ArcTanh[Sqr
t[b*Coth[c + d*x]]/Sqrt[b]])/d - (2*b*(b*Coth[c + d*x])^(3/2))/(3*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int (b\coth(c+dx))^{5/2}\,dx &= -\frac{2b(b\coth(c+dx))^{3/2}}{3d}+b^2\int \sqrt{b\coth(c+dx)}\,dx\\ &= -\frac{2b(b\coth(c+dx))^{3/2}}{3d}-\frac{b^3\text{Subst}\left(\int \frac{\sqrt{x}}{-b^2+x^2}\,dx,x,b\coth(c+dx)\right)}{d}\\ &= -\frac{2b(b\coth(c+dx))^{3/2}}{3d}-\frac{(2b^3)\,\text{Subst}\left(\int \frac{x^2}{-b^2+x^4}\,dx,x,\sqrt{b\coth(c+dx)}\right)}{d}\\ &= -\frac{2b(b\coth(c+dx))^{3/2}}{3d}+\frac{b^3\text{Subst}\left(\int \frac{1}{b-x^2}\,dx,x,\sqrt{b\coth(c+dx)}\right)}{d}-\frac{b^3\text{Subst}\left(\int\right.}{}\\ &= -\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}+\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{2b(b\coth}{}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 68, normalized size = 0.87

$$-\frac{(b\coth(c+dx))^{5/2}\left(3\text{ArcTan}\left(\sqrt{\coth(c+dx)}\right)-3\tanh^{-1}\left(\sqrt{\coth(c+dx)}\right)+2\coth^{\frac{3}{2}}(c+dx)\right)}{3d\coth^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x])^(5/2),x]
```

```
[Out] -1/3*((b*Coth[c + d*x])^(5/2)*(3*ArcTan[Sqrt[Coth[c + d*x]]] - 3*ArcTanh[Sq
rt[Coth[c + d*x]]] + 2*Coth[c + d*x]^(3/2)))/(d*Coth[c + d*x]^(5/2))
```

Maple [A]
time = 2.14, size = 60, normalized size = 0.77

method	result	size
derivativedivides	$-\frac{2b\left(\frac{(b\coth(dx+c))^{\frac{3}{2}}}{3}+\frac{b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{2}-\frac{b^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{2}\right)}{d}$	60
default	$-\frac{2b\left(\frac{(b\coth(dx+c))^{\frac{3}{2}}}{3}+\frac{b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{2}-\frac{b^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{2}\right)}{d}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*b*(1/3*(b*coth(d*x+c))^(3/2)+1/2*b^(3/2)*arctan((b*coth(d*x+c))^(1/2)/
b^(1/2))-1/2*b^(3/2)*arctanh((b*coth(d*x+c))^(1/2)/b^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c))^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 469 vs. 2(62) = 124.
time = 0.42, size = 988, normalized size = 12.67

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(6*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*si
nh(d*x + c)^2 - b^2)*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sin
h(d*x + c) + sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/
```

```
(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 +
b)) - 3*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh
(d*x + c)^2 - b^2)*sqrt(-b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*s
inh(d*x + c) + 6*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh
(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh
(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c
)) - 2*b)/(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x +
 c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4))
 + 8*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*
x + c)^2 + b^2)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)))/(d*cosh(d*x + c)^2 + 2
*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x + c)^2 - d), -1/12*(6*(b^2*cosh
(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 - b^2
)*sqrt(b)*arctan(sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x +
c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - 3*(b^2*c
osh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 -
b^2)*sqrt(b)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x + c) +
12*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c)^3 +
2*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3
+ sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c)
^2 + 2*(2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sqrt(b*co
sh(d*x + c)/sinh(d*x + c)) - b) + 8*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x +
 c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 + b^2)*sqrt(b*cosh(d*x + c)/sinh(d*
x + c)))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x
+ c)^2 - d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\coth(c+dx))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))**(5/2),x)
```

```
[Out] Integral((b*coth(c + d*x))**(5/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(62) = 124.
time = 0.53, size = 224, normalized size = 2.87

$$\frac{6\,b^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{b}\,e^{(2\,dx+2\,c)}-\sqrt{be^{(4\,dx+4\,c)}-b}}{\sqrt{b}}\right)\mathrm{sgn}\left(e^{(2\,dx+2\,c)}-1\right)-3\,b^{\frac{5}{2}}\log\left(\left|-\sqrt{b}\,e^{(2\,dx+2\,c)}+\sqrt{be^{(4\,dx+4\,c)}-b}\right|\right)\mathrm{sgn}\left(e^{(2\,dx+2\,c)}-1\right)+\frac{8\left(3\left(\sqrt{b}\,e^{(2\,dx+2\,c)}-\sqrt{be^{(4\,dx+4\,c)}-b}\right)^{2}b^{3}\mathrm{sgn}\left(e^{(2\,dx+2\,c)}-1\right)+b^{4}\mathrm{sgn}\left(e^{(2\,dx+2\,c)}-1\right)\right)}{\left(\sqrt{b}\,e^{(2\,dx+2\,c)}-\sqrt{be^{(4\,dx+4\,c)}-b}-\sqrt{b}\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*(6*b^(5/2)*arctan(-(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) -
b))/sqrt(b))*sgn(e^(2*d*x + 2*c) - 1) - 3*b^(5/2)*log(abs(-sqrt(b)*e^(2*d*x
 + 2*c) + sqrt(b*e^(4*d*x + 4*c) - b)))*sgn(e^(2*d*x + 2*c) - 1) + 8*(3*(sq
rt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))^2*b^3*sgn(e^(2*d*x + 2
*c) - 1) + b^4*sgn(e^(2*d*x + 2*c) - 1))/(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*
e^(4*d*x + 4*c) - b) - sqrt(b))^3)/d
```

Mupad [B]
time = 1.37, size = 62, normalized size = 0.79

$$\frac{b^{5/2}\,\mathrm{atanh}\left(\frac{\sqrt{b\coth\left(c+d\,x\right)}}{\sqrt{b}}\right)}{d}-\frac{b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b\coth\left(c+d\,x\right)}}{\sqrt{b}}\right)}{d}-\frac{2\,b\,{\left(b\coth(c+d\,x)\right)}^{3/2}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x))^(5/2),x)
```

```
[Out] (b^(5/2)*atanh((b*coth(c + d*x))^(1/2)/b^(1/2)))/d - (b^(5/2)*atan((b*coth(
c + d*x))^(1/2)/b^(1/2)))/d - (2*b*(b*coth(c + d*x))^(3/2))/(3*d)
```

3.3 $\int (b\coth(c+dx))^{3/2}\,dx$

Optimal. Leaf size=75

$$\frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}+\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{2b\sqrt{b\coth(c+dx)}}{d}$$

```
[Out] b^(3/2)*arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/d+b^(3/2)*arctanh((b*coth(d*x
+c))^(1/2)/b^(1/2))/d-2*b*(b*coth(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3554, 3557, 335, 218, 212, 209}

$$\frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}+\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{2b\sqrt{b\coth(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x])^(3/2),x]
```

```
[Out] (b^(3/2)*ArcTan[Sqrt[b*Coth[c + d*x]]/Sqrt[b]])/d + (b^(3/2)*ArcTanh[Sqrt[b
*Coth[c + d*x]]/Sqrt[b]])/d - (2*b*Sqrt[b*Coth[c + d*x]])/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int (b\coth(c+dx))^{3/2}\,dx &= -\frac{2b\sqrt{b\coth(c+dx)}}{d}+b^2\int \frac{1}{\sqrt{b\coth(c+dx)}}\,dx\\ &= -\frac{2b\sqrt{b\coth(c+dx)}}{d}-\frac{b^3\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(-b^2+x^2)}\,dx,x,b\coth(c+dx)\right)}{d}\\ &= -\frac{2b\sqrt{b\coth(c+dx)}}{d}-\frac{(2b^3)\,\text{Subst}\left(\int \frac{1}{-b^2+x^4}\,dx,x,\sqrt{b\coth(c+dx)}\right)}{d}\\ &= -\frac{2b\sqrt{b\coth(c+dx)}}{d}+\frac{b^2\text{Subst}\left(\int \frac{1}{b-x^2}\,dx,x,\sqrt{b\coth(c+dx)}\right)}{d}+\frac{b^2\text{Subst}\left(\int\right.}{}\\ &= \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}+\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{2b\sqrt{b\,\text{co}}}{}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 61, normalized size = 0.81

$$\frac{\left(\text{ArcTan}\left(\sqrt{\coth(c+dx)}\right)+\tanh^{-1}\left(\sqrt{\coth(c+dx)}\right)-2\sqrt{\coth(c+dx)}\right)(b\coth(c+dx))^{3/2}}{d\coth^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x])^(3/2),x]
```

```
[Out] ((ArcTan[Sqrt[Coth[c + d*x]]] + ArcTanh[Sqrt[Coth[c + d*x]]] - 2*Sqrt[Coth[
c + d*x]])*(b*Coth[c + d*x])^(3/2))/(d*Coth[c + d*x]^(3/2))
```

Maple [A]
time = 2.17, size = 58, normalized size = 0.77

method	result
derivativedivides	$-\frac{2b\left(\sqrt{b\coth\left(dx+c\right)}-\frac{\sqrt{b}\,\arctan\left(\frac{\sqrt{b\coth\left(dx+c\right)}}{\sqrt{b}}\right)}{2}-\frac{\sqrt{b}\,\operatorname{arctanh}\left(\frac{\sqrt{b\coth\left(dx+c\right)}}{\sqrt{b}}\right)}{2}\right)}{d}$
default	$-\frac{2b\left(\sqrt{b\coth\left(dx+c\right)}-\frac{\sqrt{b}\,\arctan\left(\frac{\sqrt{b\coth\left(dx+c\right)}}{\sqrt{b}}\right)}{2}-\frac{\sqrt{b}\,\operatorname{arctanh}\left(\frac{\sqrt{b\coth\left(dx+c\right)}}{\sqrt{b}}\right)}{2}\right)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*b*((b*coth(d*x+c))^(1/2)-1/2*b^(1/2)*arctan((b*coth(d*x+c))^(1/2)/b^(1
/2))-1/2*b^(1/2)*arctanh((b*coth(d*x+c))^(1/2)/b^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c))^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(61) = 122$.
time = 0.53, size = 637, normalized size = 8.49

$$\left[-\frac{2\sqrt{-b}\,b\arctan\left(\frac{\left(\cosh(dx+c)^2+2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2\right)\sqrt{-b}\sqrt{\frac{b\cosh(dx+c)}{\sinh(dx+c)}}}{b\cosh(dx+c)^2+2b\cosh(dx+c)\sinh(dx+c)+b\sinh(dx+c)^2+b}\right)-\sqrt{-b}\,b\log\left(-\frac{b\cosh(dx+c)^4+4b\cosh(dx+c)^3\sinh(dx+c)+6b\cosh(dx+c)^2\sinh(dx+c)^2+4b\cosh(dx+c)\sinh(dx+c)^3+b\sinh(dx+c)^4-2\left(\cosh(dx+c)^2+2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2-1\right)\sqrt{-b}\sqrt{\frac{b\cosh(dx+c)}{\sinh(dx+c)}}-2b}{\cosh(dx+c)^4+4\cosh(dx+c)^3\sinh(dx+c)+6\cosh(dx+c)^2\sinh(dx+c)^2+4\cosh(dx+c)\sinh(dx+c)^3+\sinh(dx+c)^4}\right)+8b\sqrt{\frac{b\cosh(dx+c)}{\sinh(dx+c)}}}{4d},\ \frac{2b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{b}\sqrt{\frac{b\cosh(dx+c)}{\sinh(dx+c)}}}{b\cosh(dx+c)^2+2b\cosh(dx+c)\sinh(dx+c)+b\sinh(dx+c)^2+b}\right)+b^{\frac{3}{2}}\log\left(2b\cosh(dx+c)^4+8b\cosh(dx+c)^3\sinh(dx+c)+12b\cosh(dx+c)^2\sinh(dx+c)^2+8b\cosh(dx+c)\sinh(dx+c)^3+2b\sinh(dx+c)^4+2\left(\cosh(dx+c)^4+4\cosh(dx+c)\sinh(dx+c)^3+\sinh(dx+c)^4+\left(6\cosh(dx+c)^2-1\right)\sinh(dx+c)^2-\cosh(dx+c)^2+2\left(2\cosh(dx+c)^3-\cosh(dx+c)\right)\sinh(dx+c)\right)\sqrt{b}\sqrt{\frac{b\cosh(dx+c)}{\sinh(dx+c)}}-b\right)-8b\sqrt{\frac{b\cosh(dx+c)}{\sinh(dx+c)}}}{4d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*sqrt(-b)*b*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c)
 + sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*
```

```
x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - sqrt
(-b)*b*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*co
sh(d*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(
d*x + c)^4 - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x
+ c)^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d*x +
 c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2
 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 8*b*sqrt(b*cosh(d*
x + c)/sinh(d*x + c)))/d, 1/4*(2*b^(3/2)*arctan(sqrt(b)*sqrt(b*cosh(d*x + c
)/sinh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*s
inh(d*x + c)^2 + b)) + b^(3/2)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^
3*sinh(d*x + c) + 12*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*
sinh(d*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c
)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)
^2 - cosh(d*x + c)^2 + 2*(2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c))
*sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - b) - 8*b*sqrt(b*cosh(d*x + c
)/sinh(d*x + c)))/d]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\coth(c+dx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))**(3/2),x)
[Out] Integral((b*coth(c + d*x))**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(61) = 122.
time = 0.46, size = 168, normalized size = 2.24

$$-\frac{\left(2\sqrt{b}\arctan\left(-\frac{\sqrt{b}e^{(2dx+2c)}-\sqrt{be^{(4dx+4c)}-b}}{\sqrt{b}}\right)\mathrm{sgn}\left(e^{(2dx+2c)}-1\right)+\sqrt{b}\log\left(\left|-\sqrt{b}e^{(2dx+2c)}+\sqrt{be^{(4dx+4c)}-b}\right|\right)\mathrm{sgn}\left(e^{(2dx+2c)}-1\right)-\frac{8b\,\mathrm{sgn}\left(e^{(2dx+2c)}-1\right)}{\sqrt{b}e^{(2dx+2c)}-\sqrt{be^{(4dx+4c)}-b}-\sqrt{b}}\right)b}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(3/2),x, algorithm="giac")
[Out] -1/2*(2*sqrt(b)*arctan(-(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) -
 b))/sqrt(b))*sgn(e^(2*d*x + 2*c) - 1) + sqrt(b)*log(abs(-sqrt(b)*e^(2*d*x
+ 2*c) + sqrt(b*e^(4*d*x + 4*c) - b)))*sgn(e^(2*d*x + 2*c) - 1) - 8*b*sgn(e
^(2*d*x + 2*c) - 1)/(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b)
- sqrt(b)))*b/d
```

Mupad [B]
time = 1.27, size = 61, normalized size = 0.81

$$\frac{b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b\coth(c+d\,x)}}{\sqrt{b}}\right)}{d}-\frac{2\,b\,\sqrt{b\coth(c+d\,x)}}{d}+\frac{b^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{b\coth(c+d\,x)}}{\sqrt{b}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x))^(3/2),x)
```

```
[Out] (b^(3/2)*atan((b*coth(c + d*x))^(1/2)/b^(1/2)))/d - (2*b*(b*coth(c + d*x))^
(1/2))/d + (b^(3/2)*atanh((b*coth(c + d*x))^(1/2)/b^(1/2)))/d
```

3.4 $\int \sqrt{b\coth(c+dx)}\,dx$

Optimal. Leaf size=58

$$-\frac{\sqrt{b}\,\mathrm{ArcTan}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}$$

```
[Out] -arctan((b*coth(d*x+c))^(1/2)/b^(1/2))*b^(1/2)/d+arctanh((b*coth(d*x+c))^(1
/2)/b^(1/2))*b^(1/2)/d
```

Rubi [A]
time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3557, 335, 304, 209, 212}

$$\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{\sqrt{b}\,\mathrm{ArcTan}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Coth[c + d*x]],x]
```

```
[Out] -((Sqrt[b]*ArcTan[Sqrt[b*Coth[c + d*x]]/Sqrt[b]])/d) + (Sqrt[b]*ArcTanh[Sqr
t[b*Coth[c + d*x]]/Sqrt[b]])/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \sqrt{b\coth(c+dx)}\,dx &= -\frac{b\text{Subst}\left(\int \frac{\sqrt{x}}{-b^2+x^2}\,dx, x, b\coth(c+dx)\right)}{d}\\ &= -\frac{(2b)\text{Subst}\left(\int \frac{x^2}{-b^2+x^4}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{d}\\ &= \frac{b\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{d} - \frac{b\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{d}\\ &= -\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d} + \frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}\end{aligned}$$

Mathematica [**A**]
time = 0.03, size = 51, normalized size = 0.88

$$\frac{\left(-\text{ArcTan}\left(\sqrt{\coth(c+dx)}\right)+\tanh^{-1}\left(\sqrt{\coth(c+dx)}\right)\right)\sqrt{b\coth(c+dx)}}{d\sqrt{\coth(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Coth[c + d*x]],x]
```

```
[Out] ((-ArcTan[Sqrt[Coth[c + d*x]]] + ArcTanh[Sqrt[Coth[c + d*x]]])*Sqrt[b*Coth[
c + d*x]])/(d*Sqrt[Coth[c + d*x]])
```

Maple [**A**]
time = 2.33, size = 48, normalized size = 0.83

method	result	size

derivativedivides	$-\frac{2b\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{2\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{2\sqrt{b}}\right)}{d}$	48
default	$-\frac{2b\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{2\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{2\sqrt{b}}\right)}{d}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*b*(-1/2/b^(1/2)*arctanh((b*coth(d*x+c))^(1/2)/b^(1/2))+1/2/b^(1/2)*arc
tan((b*coth(d*x+c))^(1/2)/b^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*coth(d*x + c)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(46) = 92.
time = 0.42, size = 594, normalized size = 10.24

$$\left[-\frac{2\sqrt{-b}\arctan\left(\frac{\left(\cosh(dx+c)^2+2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2\right)\sqrt{-b}\sqrt{\frac{b\cosh(dx+c)}{\sinh(dx+c)}}}{b\cosh(dx+c)^2+2b\cosh(dx+c)\sinh(dx+c)+b\sinh(dx+c)^2+b}\right)-\sqrt{-b}\log\left(-\frac{b\cosh(dx+c)^4+4b\cosh(dx+c)^3\sinh(dx+c)+6b\cosh(dx+c)^2\sinh(dx+c)^2+4b\cosh(dx+c)\sinh(dx+c)^3+b\sinh(dx+c)^4+2\left(\cosh(dx+c)^2+2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2-1\right)\sqrt{-b}\sqrt{\frac{b\cosh(dx+c)}{\sinh(dx+c)}}-2b}{\cosh(dx+c)^4+4\cosh(dx+c)^3\sinh(dx+c)+6\cosh(dx+c)^2\sinh(dx+c)^2+4\cosh(dx+c)\sinh(dx+c)^3+\sinh(dx+c)^4}\right)}{4d}, -\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{\frac{b\cosh(dx+c)}{\sinh(dx+c)}}}{b\cosh(dx+c)^2+2b\cosh(dx+c)\sinh(dx+c)+b\sinh(dx+c)^2+b}\right)-\sqrt{b}\log\left(2b\cosh(dx+c)^4+8b\cosh(dx+c)^3\sinh(dx+c)+12b\cosh(dx+c)^2\sinh(dx+c)^2+8b\cosh(dx+c)\sinh(dx+c)^3+2b\sinh(dx+c)^4+2\left(\cosh(dx+c)^4+4\cosh(dx+c)\sinh(dx+c)^3+\sinh(dx+c)^4+\left(6\cosh(dx+c)^2-1\right)\sinh(dx+c)^2-\cosh(dx+c)^2+2\left(2\cosh(dx+c)^3-\cosh(dx+c)\right)\sinh(dx+c)\right)\sqrt{b}\sqrt{\frac{b\cosh(dx+c)}{\sinh(dx+c)}}-b\right)}{4d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
 sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x
+ c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - sqrt(-
b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d
*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x
+ c)^4 + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d*x + c)^
4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4
```

```
*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)))/d, -1/4*(2*sqrt(b)*arct
an(sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cos
h(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - sqrt(b)*log(2*b*cosh(d
*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x + c) + 12*b*cosh(d*x + c)^2*sinh(d
*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*(co
sh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + (6*cosh
(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c)^2 + 2*(2*cosh(d*x + c)^3 -
 cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))
- b))/d]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\coth(c+dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(b*coth(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(46) = 92.
time = 0.45, size = 101, normalized size = 1.74

$$\frac{\left(2\sqrt{b}\arctan\left(-\frac{\sqrt{b}e^{(2dx+2c)}-\sqrt{be^{(4dx+4c)}-b}}{\sqrt{b}}\right)-\sqrt{b}\log\left(\left|-\sqrt{b}e^{(2dx+2c)}+\sqrt{be^{(4dx+4c)}-b}\right|\right)\right)\mathrm{sgn}\left(e^{(2dx+2c)}-1\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*sqrt(b)*arctan(-(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) -
b))/sqrt(b)) - sqrt(b)*log(abs(-sqrt(b)*e^(2*d*x + 2*c) + sqrt(b*e^(4*d*x +
 4*c) - b))))*sgn(e^(2*d*x + 2*c) - 1)/d
```

Mupad [B]
time = 1.22, size = 41, normalized size = 0.71

$$-\frac{\sqrt{b}\left(\mathrm{atan}\left(\frac{\sqrt{b\coth(c+d\,x)}}{\sqrt{b}}\right)-\mathrm{atanh}\left(\frac{\sqrt{b\coth(c+d\,x)}}{\sqrt{b}}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x))^(1/2),x)
```

```
[Out] -(b^(1/2)*(atan((b*coth(c + d*x))^(1/2)/b^(1/2)) - atanh((b*coth(c + d*x))^
(1/2)/b^(1/2))))/d
```

3.5 $\int \frac{1}{\sqrt{b\coth(c+dx)}}\,dx$

Optimal. Leaf size=57

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}\,d}+\frac{\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}\,d}$$

```
[Out] arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/d/b^(1/2)+arctanh((b*coth(d*x+c))^(1/
2)/b^(1/2))/d/b^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3557, 335, 218, 212, 209}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}\,d}+\frac{\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Coth[c + d*x]],x]
```

```
[Out] ArcTan[Sqrt[b*Coth[c + d*x]]/Sqrt[b]]/(Sqrt[b]*d) + ArcTanh[Sqrt[b*Coth[c +
 d*x]]/Sqrt[b]]/(Sqrt[b]*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{b\coth(c+dx)}}\,dx &= -\frac{b\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(-b^2+x^2)}\,dx, x, b\coth(c+dx)\right)}{d}\\ &= -\frac{(2b)\text{Subst}\left(\int \frac{1}{-b^2+x^4}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{d} + \frac{\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{d}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}\,d} + \frac{\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}\,d}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 49, normalized size = 0.86

$$\frac{\left(\text{ArcTan}\left(\sqrt{\coth(c+dx)}\right)+\tanh^{-1}\left(\sqrt{\coth(c+dx)}\right)\right)\sqrt{\coth(c+dx)}}{d\sqrt{b\coth(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Coth[c + d*x]],x]
```

```
[Out] ((ArcTan[Sqrt[Coth[c + d*x]]] + ArcTanh[Sqrt[Coth[c + d*x]]])*Sqrt[Coth[c +
 d*x]])/(d*Sqrt[b*Coth[c + d*x]])
```

Maple [A]
time = 2.27, size = 48, normalized size = 0.84

method	result	size
derivativedivides	$-\frac{2b\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{2b^{\frac{3}{2}}}-\frac{\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{2b^{\frac{3}{2}}}\right)}{d}$	48
default	$-\frac{2b\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{2b^{\frac{3}{2}}}-\frac{\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{2b^{\frac{3}{2}}}\right)}{d}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*b*(-1/2/b^(3/2)*arctanh((b*coth(d*x+c))^(1/2)/b^(1/2))-1/2/b^(3/2)*arc
tan((b*coth(d*x+c))^(1/2)/b^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*coth(d*x + c)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(45) = 90$.
time = 0.52, size = 598, normalized size = 10.49

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
 sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x
+ c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) + sqrt(-
b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d
*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x
+ c)^4 + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d*x + c)^
```

```
4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4
*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)))/(b*d), 1/4*(2*sqrt(b)*a
rctan(sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*
cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) + sqrt(b)*log(2*b*cos
h(d*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x + c) + 12*b*cosh(d*x + c)^2*sin
h(d*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*
(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + (6*c
osh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c)^2 + 2*(2*cosh(d*x + c)^
3 - cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c
)) - b))/(b*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\coth(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*coth(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(45) = 90.
time = 0.49, size = 102, normalized size = 1.79

$$-\frac{\frac{2\arctan\left(-\frac{\sqrt{b}\,e^{(2\,dx+2\,c)}-\sqrt{be^{(4\,dx+4\,c)}-b}}{\sqrt{b}}\right)}{\sqrt{b}}+\frac{\log\left(\left|-\sqrt{b}\,e^{(2\,dx+2\,c)}+\sqrt{be^{(4\,dx+4\,c)}-b}\right|\right)}{\sqrt{b}}}{2\,d\mathrm{sgn}\left(e^{(2\,dx+2\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(2*arctan(-(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))/sqr
t(b))/sqrt(b) + log(abs(-sqrt(b)*e^(2*d*x + 2*c) + sqrt(b*e^(4*d*x + 4*c) -
 b)))/sqrt(b))/(d*sgn(e^(2*d*x + 2*c) - 1))
```

Mupad [B]
time = 1.29, size = 38, normalized size = 0.67

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b\coth\left(c+d\,x\right)}}{\sqrt{b}}\right)+\mathrm{atanh}\left(\frac{\sqrt{b\coth\left(c+d\,x\right)}}{\sqrt{b}}\right)}{\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x))^(1/2),x)
```

```
[Out] (atan((b*coth(c + d*x))^(1/2)/b^(1/2)) + atanh((b*coth(c + d*x))^(1/2)/b^(1
/2)))/(b^(1/2)*d)
```

3.6 $\int \frac{1}{(b\coth(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=78

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}-\frac{2}{bd\sqrt{b\coth(c+dx)}}$$

```
[Out] -arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/b^(3/2)/d+arctanh((b*coth(d*x+c))^(1
/2)/b^(1/2))/b^(3/2)/d-2/b/d/(b*coth(d*x+c))^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3555, 3557, 335, 304, 209, 212}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}-\frac{2}{bd\sqrt{b\coth(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x])^(-3/2),x]
```

```
[Out] -(ArcTan[Sqrt[b*Coth[c + d*x]]/Sqrt[b]]/(b^(3/2)*d)) + ArcTanh[Sqrt[b*Coth[
c + d*x]]/Sqrt[b]]/(b^(3/2)*d) - 2/(b*d*Sqrt[b*Coth[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(b\coth(c+dx))^{3/2}}\,dx &= -\frac{2}{bd\sqrt{b\coth(c+dx)}} + \frac{\int \sqrt{b\coth(c+dx)}\,dx}{b^2} \\
&= -\frac{2}{bd\sqrt{b\coth(c+dx)}} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{-b^2+x^2}\,dx, x, b\coth(c+dx)\right)}{bd} \\
&= -\frac{2}{bd\sqrt{b\coth(c+dx)}} - \frac{2\text{Subst}\left(\int \frac{x^2}{-b^2+x^4}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{bd} \\
&= -\frac{2}{bd\sqrt{b\coth(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{bd} - \frac{\text{Subst}\left(\int \frac{1}{b+x^2}\right.}{} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} + \frac{\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} - \frac{2}{bd\sqrt{b\coth(c+}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.05, size = 36, normalized size = 0.46

$$-\frac{2\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};\coth^2(c+dx)\right)}{bd\sqrt{b\coth(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Coth[c + d*x])^(-3/2),x]

[Out] (-2*Hypergeometric2F1[-1/4, 1, 3/4, Coth[c + d*x]^2])/(b*d*Sqrt[b*Coth[c + d*x]])

Maple [A]
time = 2.10, size = 62, normalized size = 0.79

method	result	size
derivativedivides	$-\frac{2b\left(\frac{1}{b^{2}\sqrt{b\coth\left(dx+c\right)}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\coth\left(dx+c\right)}}{\sqrt{b}}\right)}{2b^{\frac{5}{2}}}+\frac{\arctan\left(\frac{\sqrt{b\coth\left(dx+c\right)}}{\sqrt{b}}\right)}{2b^{\frac{5}{2}}}\right)}{d}$	62
default	$-\frac{2b\left(\frac{1}{b^{2}\sqrt{b\coth\left(dx+c\right)}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\coth\left(dx+c\right)}}{\sqrt{b}}\right)}{2b^{\frac{5}{2}}}+\frac{\arctan\left(\frac{\sqrt{b\coth\left(dx+c\right)}}{\sqrt{b}}\right)}{2b^{\frac{5}{2}}}\right)}{d}$	62

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*coth(d*x+c))^(3/2),x,method=_RETURNVERBOSE)

[Out] -2/d*b*(1/b^2/(b*coth(d*x+c))^(1/2)-1/2/b^(5/2)*arctanh((b*coth(d*x+c))^(1/2)/b^(1/2))+1/2/b^(5/2)*arctan((b*coth(d*x+c))^(1/2)/b^(1/2)))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((b*coth(d*x + c))^(-3/2), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(64) = 128$.
time = 0.59, size = 923, normalized size = 11.83

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c))^(3/2),x, algorithm="fricas")

```
[Out] [-1/4*(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2
 + 1)*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + si
nh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x + c
)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) + (cosh(d*x
 + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*sqrt(-b)*log
(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d*x + c
)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4
 - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1
)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d*x + c)^4 + 4*
cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(
d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 8*(cosh(d*x + c)^2 + 2*cosh(
d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(b*cosh(d*x + c)/sinh(d*x
 + c)))/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x + c)*sinh(d*x + c) + b^2*
d*sinh(d*x + c)^2 + b^2*d), -1/4*(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh
(d*x + c) + sinh(d*x + c)^2 + 1)*sqrt(b)*arctan(sqrt(b)*sqrt(b*cosh(d*x + c
)/sinh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*s
inh(d*x + c)^2 + b)) - (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + s
inh(d*x + c)^2 + 1)*sqrt(b)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^3*s
inh(d*x + c) + 12*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*sin
h(d*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*s
inh(d*x + c)^3 + sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2
- cosh(d*x + c)^2 + 2*(2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c))*sq
rt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - b) + 8*(cosh(d*x + c)^2 + 2*cos
h(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(b*cosh(d*x + c)/sinh(d
*x + c)))/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x + c)*sinh(d*x + c) + b^
2*d*sinh(d*x + c)^2 + b^2*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\coth{(c+dx)})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*coth(c + d*x))**(-3/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(64) = 128.
time = 0.48, size = 174, normalized size = 2.23

$$\frac{\frac{2\arctan\left(-\frac{\sqrt{b}e^{(2dx+2c)}-\sqrt{be^{(4dx+4c)}-b}}{\sqrt{b}}\right)}{\sqrt{b}\,\mathrm{sgn}\left(e^{(2dx+2c)}-1\right)}-\frac{\log\left(\left|-\sqrt{b}e^{(2dx+2c)}+\sqrt{be^{(4dx+4c)}-b}\right|\right)}{\sqrt{b}\,\mathrm{sgn}\left(e^{(2dx+2c)}-1\right)}-\frac{8}{\left(\sqrt{b}e^{(2dx+2c)}-\sqrt{be^{(4dx+4c)}-b}+\sqrt{b}\right)\mathrm{sgn}\left(e^{(2dx+2c)}-1\right)}}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*arctan(-(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))/sqrt
(b))/(sqrt(b)*sgn(e^(2*d*x + 2*c) - 1)) - log(abs(-sqrt(b)*e^(2*d*x + 2*c)
+ sqrt(b*e^(4*d*x + 4*c) - b)))/(sqrt(b)*sgn(e^(2*d*x + 2*c) - 1)) - 8/((sq
rt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b) + sqrt(b))*sgn(e^(2*d*x
 + 2*c) - 1)))/(b*d)
```

Mupad [B]
time = 1.39, size = 64, normalized size = 0.82

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{b\coth\left(c+d\,x\right)}}{\sqrt{b}}\right)}{b^{3/2}\,d}-\frac{\operatorname{atan}\left(\frac{\sqrt{b\coth\left(c+d\,x\right)}}{\sqrt{b}}\right)}{b^{3/2}\,d}-\frac{2}{b\,d\,\sqrt{b\coth\left(c+d\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x))^(3/2),x)
```

```
[Out] atanh((b*coth(c + d*x))^(1/2)/b^(1/2))/(b^(3/2)*d) - atan((b*coth(c + d*x))
^(1/2)/b^(1/2))/(b^(3/2)*d) - 2/(b*d*(b*coth(c + d*x))^(1/2))
```

3.7 $\int \frac{1}{(b\coth(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=79

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}d}-\frac{2}{3bd(b\coth(c+dx))^{3/2}}$$

```
[Out] arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/b^(5/2)/d+arctanh((b*coth(d*x+c))^(1/
2)/b^(1/2))/b^(5/2)/d-2/3/b/d/(b*coth(d*x+c))^(3/2)
```

Rubi [A]
time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3555, 3557, 335, 218, 212, 209}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}d}-\frac{2}{3bd(b\coth(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x])^(-5/2),x]
```

```
[Out] ArcTan[Sqrt[b*Coth[c + d*x]]/Sqrt[b]]/(b^(5/2)*d) + ArcTanh[Sqrt[b*Coth[c +
 d*x]]/Sqrt[b]]/(b^(5/2)*d) - 2/(3*b*d*(b*Coth[c + d*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(b\coth(c+dx))^{5/2}}\,dx &= -\frac{2}{3bd(b\coth(c+dx))^{3/2}}+\frac{\int \frac{1}{\sqrt{b\coth(c+dx)}}\,dx}{b^2}\\
&= -\frac{2}{3bd(b\coth(c+dx))^{3/2}}-\frac{\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}\,(-b^2+x^2)}\,dx,x,b\coth(c+dx)\right)}{bd}\\
&= -\frac{2}{3bd(b\coth(c+dx))^{3/2}}-\frac{2\mathrm{Subst}\left(\int \frac{1}{-b^2+x^4}\,dx,x,\sqrt{b\coth(c+dx)}\right)}{bd}\\
&= -\frac{2}{3bd(b\coth(c+dx))^{3/2}}+\frac{\mathrm{Subst}\left(\int \frac{1}{b-x^2}\,dx,x,\sqrt{b\coth(c+dx)}\right)}{b^2d}+\frac{\mathrm{Subst}\left(\int\right.}{}\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}d}-\frac{2}{3bd(b\coth(c+}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.05, size = 38, normalized size = 0.48

$$-\frac{2\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};\coth^2(c+dx)\right)}{3bd(b\coth(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x])^(-5/2),x]
[Out] (-2*Hypergeometric2F1[-3/4, 1, 1/4, Coth[c + d*x]^2])/(3*b*d*(b*Coth[c + d*x])^(3/2))
```

Maple [A]
time = 2.11, size = 63, normalized size = 0.80

method	result	size
derivativedivides	$-\frac{2b\left(-\frac{\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{2b^{\frac{7}{2}}}+\frac{1}{3b^{2}(b\coth(dx+c))^{\frac{3}{2}}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{2b^{\frac{7}{2}}}\right)}{d}$	63
default	$-\frac{2b\left(-\frac{\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{2b^{\frac{7}{2}}}+\frac{1}{3b^{2}(b\coth(dx+c))^{\frac{3}{2}}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{2b^{\frac{7}{2}}}\right)}{d}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
[Out] -2/d*b*(-1/2/b^(7/2)*arctan((b*coth(d*x+c))^(1/2)/b^(1/2))+1/3/b^2/(b*coth(d*x+c))^(3/2)-1/2/b^(7/2)*arctanh((b*coth(d*x+c))^(1/2)/b^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c))^(-5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 689 vs. $2(63) = 126$.
time = 0.51, size = 1428, normalized size = 18.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(5/2),x, algorithm="fricas")
[Out] [-1/12*(6*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(co
```

```
sh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(-b)*arctan((cosh(d*x
 + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*
cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*
x + c) + b*sinh(d*x + c)^2 + b)) + 3*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sin
h(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2
+ 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1
)*sqrt(-b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*
b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*s
inh(d*x + c)^4 + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(
d*x + c)^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d
*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x +
c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 8*(cosh(d*x +
c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x +
c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d
*x + c))*sinh(d*x + c) + 1)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)))/(b^3*d*cos
h(d*x + c)^4 + 4*b^3*d*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*d*sinh(d*x + c)^
4 + 2*b^3*d*cosh(d*x + c)^2 + b^3*d + 2*(3*b^3*d*cosh(d*x + c)^2 + b^3*d)*s
inh(d*x + c)^2 + 4*(b^3*d*cosh(d*x + c)^3 + b^3*d*cosh(d*x + c))*sinh(d*x +
 c)), 1/12*(6*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x
 + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4
*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(b)*arctan(sqrt(b
)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c
)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) + 3*(cosh(d*x + c)^4 + 4*cosh(d*x
 + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*
x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x
 + c) + 1)*sqrt(b)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x +
 c) + 12*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c
)^3 + 2*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x +
 c)^3 + sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*
x + c)^2 + 2*(2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sqr
t(b*cosh(d*x + c)/sinh(d*x + c)) - b) - 8*(cosh(d*x + c)^4 + 4*cosh(d*x + c
)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x +
c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c
) + 1)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)))/(b^3*d*cosh(d*x + c)^4 + 4*b^3*
d*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*d*sinh(d*x + c)^4 + 2*b^3*d*cosh(d*x
+ c)^2 + b^3*d + 2*(3*b^3*d*cosh(d*x + c)^2 + b^3*d)*sinh(d*x + c)^2 + 4*(b
^3*d*cosh(d*x + c)^3 + b^3*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\coth(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))**(5/2),x)
```

```
[Out] Integral((b*coth(c + d*x))**(-5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 213 vs. 2(63) = 126.
time = 0.51, size = 213, normalized size = 2.70

$$-\frac{\frac{6\arctan\left(-\frac{\sqrt{b}\,e^{(2\,dx+2\,c)}-\sqrt{be^{(4\,dx+4\,c)}-b}}{\sqrt{b}}\right)}{b^{\frac{5}{2}}\operatorname{sgn}\left(e^{(2\,dx+2\,c)}-1\right)}+\frac{3\log\left(\left|-\sqrt{b}\,e^{(2\,dx+2\,c)}+\sqrt{be^{(4\,dx+4\,c)}-b}\right|\right)}{b^{\frac{5}{2}}\operatorname{sgn}\left(e^{(2\,dx+2\,c)}-1\right)}+\frac{8\left(3\left(\sqrt{b}\,e^{(2\,dx+2\,c)}-\sqrt{be^{(4\,dx+4\,c)}-b}\right)^{2}+b\right)}{\left(\sqrt{b}\,e^{(2\,dx+2\,c)}-\sqrt{be^{(4\,dx+4\,c)}-b}+\sqrt{b}\right)^{3}b^{2}\operatorname{sgn}\left(e^{(2\,dx+2\,c)}-1\right)}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/6*(6*arctan(-(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))/sqr
t(b))/(b^(5/2)*sgn(e^(2*d*x + 2*c) - 1)) + 3*log(abs(-sqrt(b)*e^(2*d*x + 2*
c) + sqrt(b*e^(4*d*x + 4*c) - b)))/(b^(5/2)*sgn(e^(2*d*x + 2*c) - 1)) + 8*(
3*(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))^2 + b)/((sqrt(b)*
e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b) + sqrt(b))^3*b^2*sgn(e^(2*d*x
 + 2*c) - 1)))/d
```

Mupad [B]
time = 1.49, size = 63, normalized size = 0.80

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b\coth\left(c+d\,x\right)}}{\sqrt{b}}\right)}{b^{5/2}\,d}-\frac{2}{3\,b\,d\,{\left(b\coth\left(c+d\,x\right)\right)}^{3/2}}+\frac{\operatorname{atanh}\left(\frac{\sqrt{b\coth\left(c+d\,x\right)}}{\sqrt{b}}\right)}{b^{5/2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x))^(5/2),x)
```

```
[Out] atan((b*coth(c + d*x))^(1/2)/b^(1/2))/(b^(5/2)*d) - 2/(3*b*d*(b*coth(c + d*
x))^(3/2)) + atanh((b*coth(c + d*x))^(1/2)/b^(1/2))/(b^(5/2)*d)
```

3.8 $\int \frac{1}{(b\coth(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=100

$$-\frac{\mathrm{ArcTan}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{7/2}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{7/2}d}-\frac{2}{5bd(b\coth(c+dx))^{5/2}}-\frac{2}{b^3d\sqrt{b\coth(c+dx)}}$$

```
[Out] -arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/b^(7/2)/d+arctanh((b*coth(d*x+c))^(1
/2)/b^(1/2))/b^(7/2)/d-2/5/b/d/(b*coth(d*x+c))^(5/2)-2/b^3/d/(b*coth(d*x+c)
)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3555, 3557, 335, 304, 209, 212}

$$-\frac{\mathrm{ArcTan}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{7/2}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{7/2}d}-\frac{2}{b^3d\sqrt{b\coth(c+dx)}}-\frac{2}{5bd(b\coth(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x])^(-7/2),x]
```

```
[Out] -(ArcTan[Sqrt[b*Coth[c + d*x]]/Sqrt[b]]/(b^(7/2)*d)) + ArcTanh[Sqrt[b*Coth[
c + d*x]]/Sqrt[b]]/(b^(7/2)*d) - 2/(5*b*d*(b*Coth[c + d*x])^(5/2)) - 2/(b^3
*d*Sqrt[b*Coth[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(b\coth(c+dx))^{7/2}}\,dx &= -\frac{2}{5bd(b\coth(c+dx))^{5/2}}+\frac{\int \frac{1}{(b\coth(c+dx))^{3/2}}\,dx}{b^2}\\
&= -\frac{2}{5bd(b\coth(c+dx))^{5/2}}-\frac{2}{b^3d\sqrt{b\coth(c+dx)}}+\frac{\int \sqrt{b\coth(c+dx)}\,dx}{b^4}\\
&= -\frac{2}{5bd(b\coth(c+dx))^{5/2}}-\frac{2}{b^3d\sqrt{b\coth(c+dx)}}-\frac{\text{Subst}\left(\int \frac{\sqrt{x}}{-b^2+x^2}\,dx,x,b\coth(\right.}{b^3d}\\
&= -\frac{2}{5bd(b\coth(c+dx))^{5/2}}-\frac{2}{b^3d\sqrt{b\coth(c+dx)}}-\frac{2\text{Subst}\left(\int \frac{x^2}{-b^2+x^4}\,dx,x,\sqrt{b\,\text{co}}\right.}{b^3d}\\
&= -\frac{2}{5bd(b\coth(c+dx))^{5/2}}-\frac{2}{b^3d\sqrt{b\coth(c+dx)}}+\frac{\text{Subst}\left(\int \frac{1}{b-x^2}\,dx,x,\sqrt{b\coth(}\right.}{b^3d}\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{7/2}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{7/2}d}-\frac{2}{5bd(b\coth(c+\right.}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.07, size = 38, normalized size = 0.38

$$-\frac{2\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};\coth^2(c+dx)\right)}{5bd(b\coth(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x])^(-7/2),x]
```

```
[Out] (-2*Hypergeometric2F1[-5/4, 1, -1/4, Coth[c + d*x]^2])/(5*b*d*(b*Coth[c + d
*x])^(5/2))
```

Maple [A]
time = 2.04, size = 77, normalized size = 0.77

method	result
derivativedivides	$-\frac{2b\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{2b^{\frac{9}{2}}}+\frac{\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{2b^{\frac{9}{2}}}+\frac{1}{b^4\sqrt{b\coth(dx+c)}}+\frac{}{5b^2(b\,c}\right.}{d}$
default	$-\frac{2b\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{2b^{\frac{9}{2}}}+\frac{\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{2b^{\frac{9}{2}}}+\frac{1}{b^4\sqrt{b\coth(dx+c)}}+\frac{}{5b^2(b\,c}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*b*(-1/2/b^(9/2)*arctanh((b*coth(d*x+c))^(1/2)/b^(1/2))+1/2/b^(9/2)*arc
tan((b*coth(d*x+c))^(1/2)/b^(1/2))+1/b^4/(b*coth(d*x+c))^(1/2)+1/5/b^2/(b*c
oth(d*x+c))^(5/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c))^(-7/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1041 vs. $2(82) = 164$.
time = 0.51, size = 2132, normalized size = 21.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c))^(7/2),x, algorithm="fricas")

[Out] [-1/20*(10*(cosh(d*x + c)^6 + 6*cosh(d*x + c)*sinh(d*x + c)^5 + sinh(d*x + c)^6 + 3*(5*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^4 + 3*cosh(d*x + c)^4 + 4*(5*cosh(d*x + c)^3 + 3*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*cosh(d*x + c)^4 + 6*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 3*cosh(d*x + c)^2 + 6*(cosh(d*x + c)^5 + 2*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) + 5*(cosh(d*x + c)^6 + 6*cosh(d*x + c)*sinh(d*x + c)^5 + sinh(d*x + c)^6 + 3*(5*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^4 + 3*cosh(d*x + c)^4 + 4*(5*cosh(d*x + c)^3 + 3*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*cosh(d*x + c)^4 + 6*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 3*cosh(d*x + c)^2 + 6*(cosh(d*x + c)^5 + 2*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(-b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 16*(3*cosh(d*x + c)^6 + 18*cosh(d*x + c)*sinh(d*x + c)^5 + 3*sinh(d*x + c)^6 + (45*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^4 + cosh(d*x + c)^4 + 4*(15*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c)^3 + (45*cosh(d*x + c)^4 + 6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c)^2 + 2*(9*cosh(d*x + c)^5 + 2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) - 3)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)))/(b^4*d*cosh(d*x + c)^6 + 6*b^4*d*cosh(d*x + c)*sinh(d*x + c)^5 + b^4*d*sinh(d*x + c)^6 + 3*b^4*d*cosh(d*x + c)^4 + 3*b^4*d*cosh(d*x + c)^2 + b^4*d + 3*(5*b^4*d*cosh(d*x + c)^2 + b^4*d)*sinh(d*x + c)^4 + 4*(5*b^4*d*cosh(d*x + c)^3 + 3*b^4*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*b^4*d*cosh(d*x + c)^4 + 6*b^4*d*cosh(d*x + c)^2 + b^4*d)*sinh(d*x + c)^2 + 6*(b^4*d*cosh(d*x + c)^5 + 2*b^4*d*cosh(d*x + c)^3 + b^4*d*cosh(d*x + c))*sinh(d*x + c)), -1/20*(10*(cosh(d*x + c)^6 + 6*cosh(d*x + c)*sinh(d*x + c)^5 + sinh(d*x + c)^6 + 3*(5*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^4 + 3*cosh(d*x + c)^4 + 4*(5*cosh(d*x + c)^3 + 3*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*cosh(d*x + c)^4 + 6*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 3*cosh(d*x + c)^2 + 6*(cosh(d*x + c)^5 + 2*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(b)*arctan(sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - 5*(cosh(d*x + c)^6 + 6*cosh(d*x + c)*sinh(d*x + c)^5 + sinh(d*x + c)^6 + 3*(5*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^4 + 3*cosh(d*x + c)^4 + 4*(5*cosh(d*x + c)^3 + 3*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*cosh(d*x + c)^4 + 6*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 3*cosh(d*x + c)^2 + 6*(cosh(d*x + c)^5 + 2*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(b)*log(2*b*cosh(d*

```
x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x + c) + 12*b*cosh(d*x + c)^2*sinh(d*
x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*(cos
h(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + (6*cosh(
d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c)^2 + 2*(2*cosh(d*x + c)^3 -
cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) -
 b) + 16*(3*cosh(d*x + c)^6 + 18*cosh(d*x + c)*sinh(d*x + c)^5 + 3*sinh(d*x
 + c)^6 + (45*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^4 + cosh(d*x + c)^4 + 4*(1
5*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c)^3 + (45*cosh(d*x + c)^4 +
6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c)^2 + 2*(9*cosh(d*x +
c)^5 + 2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) - 3)*sqrt(b*cosh(d*
x + c)/sinh(d*x + c)))/(b^4*d*cosh(d*x + c)^6 + 6*b^4*d*cosh(d*x + c)*sinh(
d*x + c)^5 + b^4*d*sinh(d*x + c)^6 + 3*b^4*d*cosh(d*x + c)^4 + 3*b^4*d*cosh
(d*x + c)^2 + b^4*d + 3*(5*b^4*d*cosh(d*x + c)^2 + b^4*d)*sinh(d*x + c)^4 +
 4*(5*b^4*d*cosh(d*x + c)^3 + 3*b^4*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5
*b^4*d*cosh(d*x + c)^4 + 6*b^4*d*cosh(d*x + c)^2 + b^4*d)*sinh(d*x + c)^2 +
 6*(b^4*d*cosh(d*x + c)^5 + 2*b^4*d*cosh(d*x + c)^3 + b^4*d*cosh(d*x + c))*
sinh(d*x + c))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\coth(c+dx))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))**(7/2),x)
```

```
[Out] Integral((b*coth(c + d*x))**(-7/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(82) = 164.
time = 0.55, size = 333, normalized size = 3.33

$$\frac{\frac{10\arctan\left(-\frac{\sqrt{b}e^{(2dx+2c)}-\sqrt{be^{(4dx+4c)}-b}}{\sqrt{b}}\right)}{b^{\frac{7}{2}}\mathrm{sgn}\left(e^{(2dx+2c)}-1\right)}-\frac{5\log\left(\left|-\sqrt{b}e^{(2dx+2c)}+\sqrt{be^{(4dx+4c)}-b}\right|\right)}{b^{\frac{7}{2}}\mathrm{sgn}\left(e^{(2dx+2c)}-1\right)}-\frac{16\left(5\left(\sqrt{b}e^{(2dx+2c)}-\sqrt{be^{(4dx+4c)}-b}\right)^4+10\left(\sqrt{b}e^{(2dx+2c)}-\sqrt{be^{(4dx+4c)}-b}\right)^3\sqrt{b}+20\left(\sqrt{b}e^{(2dx+2c)}-\sqrt{be^{(4dx+4c)}-b}\right)^2b+10\left(\sqrt{b}e^{(2dx+2c)}-\sqrt{be^{(4dx+4c)}-b}\right)b^{\frac{3}{2}}+3b^2\right)}{\left(\sqrt{b}e^{(2dx+2c)}-\sqrt{be^{(4dx+4c)}-b}+\sqrt{b}\right)^5b^3\mathrm{sgn}\left(e^{(2dx+2c)}-1\right)}}{10d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] 1/10*(10*arctan(-(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))/sq
rt(b))/(b^(7/2)*sgn(e^(2*d*x + 2*c) - 1)) - 5*log(abs(-sqrt(b)*e^(2*d*x + 2
*c) + sqrt(b*e^(4*d*x + 4*c) - b)))/(b^(7/2)*sgn(e^(2*d*x + 2*c) - 1)) - 16
*(5*(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))^4 + 10*(sqrt(b)
*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))^3*sqrt(b) + 20*(sqrt(b)*e^(
2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))^2*b + 10*(sqrt(b)*e^(2*d*x + 2*
c) - sqrt(b*e^(4*d*x + 4*c) - b))*b^(3/2) + 3*b^2)/((sqrt(b)*e^(2*d*x + 2*c
```

```
) - sqrt(b*e^(4*d*x + 4*c) - b) + sqrt(b))^5*b^3*sgn(e^(2*d*x + 2*c) - 1)))
/d
```

Mupad [B]
time = 1.60, size = 80, normalized size = 0.80

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{b\coth\left(c+d\,x\right)}}{\sqrt{b}}\right)}{b^{7/2}\,d}-\frac{\operatorname{atan}\left(\frac{\sqrt{b\coth\left(c+d\,x\right)}}{\sqrt{b}}\right)}{b^{7/2}\,d}-\frac{\frac{2}{5\,b}+\frac{2\coth(c+d\,x)^2}{b}}{d\,\left(b\coth\left(c+d\,x\right)\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x))^(7/2),x)
```

```
[Out] atanh((b*coth(c + d*x))^(1/2)/b^(1/2))/(b^(7/2)*d) - atan((b*coth(c + d*x))
^(1/2)/b^(1/2))/(b^(7/2)*d) - (2/(5*b) + (2*coth(c + d*x)^2)/b)/(d*(b*coth(
c + d*x))^(5/2))
```

3.9 $\int (b\coth(c+dx))^{4/3}\,dx$

Optimal. Leaf size=236

$$-\frac{\sqrt{3}\,b^{4/3}\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2d}+\frac{\sqrt{3}\,b^{4/3}\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2d}+\frac{b^{4/3}\tanh^{-1}\left(\frac{\sqrt[3]{b\,c}}{}\right.}{d}$$

```
[Out] b^(4/3)*arctanh((b*coth(d*x+c))^(1/3)/b^(1/3))/d-3*b*(b*coth(d*x+c))^(1/3)/
d-1/4*b^(4/3)*ln(b^(2/3)-b^(1/3)*(b*coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3
))/d+1/4*b^(4/3)*ln(b^(2/3)+b^(1/3)*(b*coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(
2/3))/d-1/2*b^(4/3)*arctan(1/3*(1-2*(b*coth(d*x+c))^(1/3)/b^(1/3))*3^(1/2))
*3^(1/2)/d+1/2*b^(4/3)*arctan(1/3*(1+2*(b*coth(d*x+c))^(1/3)/b^(1/3))*3^(1/
2))*3^(1/2)/d
```

Rubi [A]
time = 0.21, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3554, 3557, 335, 216, 648, 632, 210, 642, 212}

$$-\frac{\sqrt{3}\,b^{4/3}\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2d}+\frac{\sqrt{3}\,b^{4/3}\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}+1}{\sqrt{3}}\right)}{2d}-\frac{b^{4/3}\log\left(b^{2/3}-\sqrt[3]{b}\,\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4d}+\frac{b^{4/3}\log\left(b^{2/3}+\sqrt[3]{b}\,\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4d}+\frac{b^{4/3}\tanh^{-1}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{d}-\frac{3b\sqrt[3]{b\coth(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x])^(4/3),x]
```

```
[Out] -1/2*(Sqrt[3]*b^(4/3)*ArcTan[(1 - (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))/Sqrt
[3]])/d + (Sqrt[3]*b^(4/3)*ArcTan[(1 + (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))
/Sqrt[3]])/(2*d) + (b^(4/3)*ArcTanh[(b*Coth[c + d*x])^(1/3)/b^(1/3)])/d - (
3*b*(b*Coth[c + d*x])^(1/3))/d - (b^(4/3)*Log[b^(2/3) - b^(1/3)*(b*Coth[c +
 d*x])^(1/3) + (b*Coth[c + d*x])^(2/3)])/(4*d) + (b^(4/3)*Log[b^(2/3) + b^(
1/3)*(b*Coth[c + d*x])^(1/3) + (b*Coth[c + d*x])^(2/3)])/(4*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int (b\coth(c+dx))^{4/3}\,dx = -\frac{3b\sqrt[3]{b\coth(c+dx)}}{d} + b^2\int \frac{1}{(b\coth(c+dx))^{2/3}}\,dx$$

$$= -\frac{3b\sqrt[3]{b\coth(c+dx)}}{d} - \frac{b^3\text{Subst}\left(\int \frac{1}{x^{2/3}(-b^2+x^2)}\,dx, x, b\coth(c+dx)\right)}{d}$$

$$= -\frac{3b\sqrt[3]{b\coth(c+dx)}}{d} - \frac{(3b^3)\,\text{Subst}\left(\int \frac{1}{-b^2+x^6}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{d}$$

$$= -\frac{3b\sqrt[3]{b\coth(c+dx)}}{d} + \frac{b^{4/3}\text{Subst}\left(\int \frac{\sqrt[3]{b}-\frac{x}{2}}{b^{2/3}-\sqrt[3]{b}\,x+x^2}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{d} +$$

$$= \frac{b^{4/3}\tanh^{-1}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{d} - \frac{3b\sqrt[3]{b\coth(c+dx)}}{d} - \frac{b^{4/3}\text{Subst}\left(\int \frac{-\sqrt[3]{b}}{b^{2/3}-\sqrt[3]{b}}\right.}{}$$

$$= \frac{b^{4/3}\tanh^{-1}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{d} - \frac{3b\sqrt[3]{b\coth(c+dx)}}{d} - \frac{b^{4/3}\log\left(b^{2/3}-\sqrt[3]{b}\right.}{}$$

$$= -\frac{\sqrt{3}\,b^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2d} + \frac{\sqrt{3}\,b^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b\coth(c+d}}{\sqrt[3]{b}}}{\sqrt{3}}\right.}{2d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.02, size = 36, normalized size = 0.15

$$\frac{3b\sqrt[3]{b\coth(c+dx)}\left(-1+{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};\coth^2(c+dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x])^(4/3),x]
```

```
[Out] (3*b*(b*Coth[c + d*x])^(1/3)*(-1 + Hypergeometric2F1[1/6, 1, 7/6, Coth[c + 
d*x]^2]))/d
```

Maple [A]
time = 1.54, size = 199, normalized size = 0.84

method	result

derivativedivides	$-\frac{3b\left((b\coth(dx+c))^{\frac{1}{3}}+\frac{\left(\frac{\ln\left((b\coth(dx+c))^{\frac{1}{3}}-b^{\frac{1}{3}}\right)}{3b^{\frac{2}{3}}}-\frac{\ln\left(b^{\frac{2}{3}}+b^{\frac{1}{3}}(b\coth(dx+c))^{\frac{1}{3}}+(b\coth(dx+c))^{\frac{2}{3}}\right)}{6b^{\frac{2}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\left(1+\frac{2(b}{}\right.}{}\right.}{3b}\right.}{2}\right.}{}$
default	$-\frac{3b\left((b\coth(dx+c))^{\frac{1}{3}}+\frac{\left(\frac{\ln\left((b\coth(dx+c))^{\frac{1}{3}}-b^{\frac{1}{3}}\right)}{3b^{\frac{2}{3}}}-\frac{\ln\left(b^{\frac{2}{3}}+b^{\frac{1}{3}}(b\coth(dx+c))^{\frac{1}{3}}+(b\coth(dx+c))^{\frac{2}{3}}\right)}{6b^{\frac{2}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\left(1+\frac{2(b}{}\right.}{}\right.}{3b}\right.}{2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c))^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3/d*b*((b*coth(d*x+c))^(1/3)+1/2*(1/3/b^(2/3)*ln((b*coth(d*x+c))^(1/3)-b^(
1/3))-1/6/b^(2/3)*ln(b^(2/3)+b^(1/3)*(b*coth(d*x+c))^(1/3)+(b*coth(d*x+c))^
(2/3))-1/3/b^(2/3)*3^(1/2)*arctan(1/3*(1+2*(b*coth(d*x+c))^(1/3)/b^(1/3))*3
^(1/2)))*b-1/2*(1/3/b^(2/3)*ln((b*coth(d*x+c))^(1/3)+b^(1/3))-1/6/b^(2/3)*l
n(b^(2/3)-b^(1/3)*(b*coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3))+1/3/b^(2/3)*
3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*coth(d*x+c))^(1/3)/b^(1/3)-1)))*b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c))^(4/3), x)
```

Fricas [A]
time = 0.37, size = 292, normalized size = 1.24

$$-\frac{2\sqrt{3}(-b)^{\frac{1}{3}}b\arctan\left(\frac{\sqrt{3}b+2\sqrt{3}(-b)^{\frac{2}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}}{3b}\right)-2\sqrt{3}b^{\frac{4}{3}}\arctan\left(-\frac{\sqrt{3}b-2\sqrt{3}b^{\frac{2}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}}{3b}\right)+(-b)^{\frac{1}{3}}b\log\left((-b)^{\frac{2}{3}}-(-b)^{\frac{1}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}+\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{2}{3}}\right)+b^{\frac{4}{3}}\log\left(b^{\frac{2}{3}}-b^{\frac{1}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}+\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{2}{3}}\right)-2(-b)^{\frac{1}{3}}b\log\left((-b)^{\frac{1}{3}}+\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}\right)-2b^{\frac{4}{3}}\log\left(b^{\frac{1}{3}}+\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}\right)+12b\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*(-b)^(1/3)*b*arctan(1/3*(sqrt(3)*b + 2*sqrt(3)*(-b)^(2/3)*(
b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/b) - 2*sqrt(3)*b^(4/3)*arctan(-1/3*(s
qrt(3)*b - 2*sqrt(3)*b^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/b) + (-
b)^(1/3)*b*log((-b)^(2/3) - (-b)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3
) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + b^(4/3)*log(b^(2/3) - b^(1/3)*
(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/
3)) - 2*(-b)^(1/3)*b*log((-b)^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3)
) - 2*b^(4/3)*log(b^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) + 12*b*(
b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\coth(c+dx))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))**(4/3),x)
```

```
[Out] Integral((b*coth(c + d*x))**(4/3), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Minimal poly. in rootof must be fract
ion free Error: Bad Argument ValueMinimal poly. in rootof must be fraction
free E
```

Mupad [B]
time = 1.88, size = 249, normalized size = 1.06

$$-\frac{3b(b\coth(c+dx))^{1/3}}{d}-\frac{b^{4/3}\,\mathrm{atan}\left(\frac{(b\coth(c+dx))^{1/3}\,1\mathrm{i}}{b^{1/3}}\right)\,1\mathrm{i}}{d}-\frac{b^{4/3}\ln\left(\frac{486b^{37/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{d^4}-\frac{486b^{12}(b\coth(c+dx))^{1/3}}{d^4}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2d}-\frac{b^{4/3}\ln\left(\frac{486b^{37/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{d^4}-\frac{486b^{12}(b\coth(c+dx))^{1/3}}{d^4}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2d}+\frac{b^{4/3}\ln\left(\frac{972b^{37/3}\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{d^4}+\frac{486b^{12}(b\coth(c+dx))^{1/3}}{d^4}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{d}+\frac{b^{4/3}\ln\left(\frac{972b^{37/3}\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{d^4}+\frac{486b^{12}(b\coth(c+dx))^{1/3}}{d^4}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x))^(4/3),x)
```

```
[Out] (b^(4/3)*log((972*b^(37/3)*((3^(1/2)*1i)/4 - 1/4))/d^4 + (486*b^12*(b*coth(
c + d*x))^(1/3))/d^4)*((3^(1/2)*1i)/4 - 1/4))/d - (b^(4/3)*atan(((b*coth(c
+ d*x))^(1/3)*1i)/b^(1/3))*1i)/d - (b^(4/3)*log((486*b^(37/3)*((3^(1/2)*1i)
/2 - 1/2))/d^4 - (486*b^12*(b*coth(c + d*x))^(1/3))/d^4)*((3^(1/2)*1i)/2 -
1/2))/(2*d) - (b^(4/3)*log((486*b^(37/3)*((3^(1/2)*1i)/2 + 1/2))/d^4 - (486
*b^12*(b*coth(c + d*x))^(1/3))/d^4)*((3^(1/2)*1i)/2 + 1/2))/(2*d) - (3*b*(b
*coth(c + d*x))^(1/3))/d + (b^(4/3)*log((972*b^(37/3)*((3^(1/2)*1i)/4 + 1/4
))/d^4 + (486*b^12*(b*coth(c + d*x))^(1/3))/d^4)*((3^(1/2)*1i)/4 + 1/4))/d
```

3.10 $\int (b\coth(c+dx))^{2/3}\,dx$

Optimal. Leaf size=218

$$\frac{\sqrt{3}\,b^{2/3}\mathrm{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2d}-\frac{\sqrt{3}\,b^{2/3}\mathrm{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2d}+\frac{b^{2/3}\tanh^{-1}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\cdots}\right.}{d}$$

```
[Out] b^(2/3)*arctanh((b*coth(d*x+c))^(1/3)/b^(1/3))/d-1/4*b^(2/3)*ln(b^(2/3)-b^(
1/3)*(b*coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3))/d+1/4*b^(2/3)*ln(b^(2/3)+
b^(1/3)*(b*coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3))/d+1/2*b^(2/3)*arctan(1
/3*(1-2*(b*coth(d*x+c))^(1/3)/b^(1/3))*3^(1/2))*3^(1/2)/d-1/2*b^(2/3)*arcta
n(1/3*(1+2*(b*coth(d*x+c))^(1/3)/b^(1/3))*3^(1/2))*3^(1/2)/d
```

Rubi [A]
time = 0.20, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3557, 335, 302, 648, 632, 210, 642, 212}

$$\frac{\sqrt{3}\,b^{2/3}\mathrm{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2d}-\frac{\sqrt{3}\,b^{2/3}\mathrm{ArcTan}\left(\frac{\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}+1}{\sqrt{3}}\right)}{2d}-\frac{b^{2/3}\log\left(b^{2/3}-\sqrt[3]{b}\,\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4d}+\frac{b^{2/3}\log\left(b^{2/3}+\sqrt[3]{b}\,\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4d}+\frac{b^{2/3}\tanh^{-1}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x])^(2/3),x]
```

```
[Out] (Sqrt[3]*b^(2/3)*ArcTan[(1 - (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))/Sqrt[3]])
/(2*d) - (Sqrt[3]*b^(2/3)*ArcTan[(1 + (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))/
Sqrt[3]])/(2*d) + (b^(2/3)*ArcTanh[(b*Coth[c + d*x])^(1/3)/b^(1/3)])/d - (b
^(2/3)*Log[b^(2/3) - b^(1/3)*(b*Coth[c + d*x])^(1/3) + (b*Coth[c + d*x])^(2
/3)])/(4*d) + (b^(2/3)*Log[b^(2/3) + b^(1/3)*(b*Coth[c + d*x])^(1/3) + (b*C
oth[c + d*x])^(2/3)])/(4*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int (b\coth(c+dx))^{2/3}\,dx = -\frac{b\text{Subst}\left(\int \frac{x^{2/3}}{-b^2+x^2}\,dx, x, b\coth(c+dx)\right)}{d}$$

$$= -\frac{(3b)\text{Subst}\left(\int \frac{x^4}{-b^2+x^6}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{d}$$

$$= \frac{b^{2/3}\text{Subst}\left(\int \frac{-\frac{\sqrt[3]{b}}{2}-\frac{x}{2}}{b^{2/3}-\sqrt[3]{b}\,x+x^2}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{d} + \frac{b^{2/3}\text{Subst}\left(\int \frac{-\frac{\sqrt[3]{b}}{2}+\frac{x}{2}}{b^{2/3}+\sqrt[3]{b}\,x+\cdots}\right.}{}$$

$$= \frac{b^{2/3}\tanh^{-1}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{d} - \frac{b^{2/3}\text{Subst}\left(\int \frac{-\sqrt[3]{b}+2x}{b^{2/3}-\sqrt[3]{b}\,x+x^2}\,dx, x, \sqrt[3]{b\coth(c\cdots}\right.}{4d}$$

$$= \frac{b^{2/3}\tanh^{-1}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{d} - \frac{b^{2/3}\log\left(b^{2/3}-\sqrt[3]{b}\,\sqrt[3]{b\coth(c+dx)}+(b\,\mathrm{c}\cdots\right.}{4d}$$

$$= \frac{\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2d} - \frac{\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right.}{2d}$$

Mathematica [A]
time = 0.12, size = 149, normalized size = 0.68

$$\frac{(b\coth(c+dx))^{2/3}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)-2\sqrt{3}\,\text{ArcTan}\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)+4\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)-\log\left(1-\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)+\log\left(1+\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)\right)}{4d\coth^{\frac{2}{3}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x])^(2/3),x]
```

```
[Out] ((b*Coth[c + d*x])^(2/3)*(2*Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt
[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]] + 4*ArcTanh[Co
th[c + d*x]^(1/3)] - Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)] + L
og[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)]))/(4*d*Coth[c + d*x]^(2/3
))
```

Maple [A]
time = 1.55, size = 181, normalized size = 0.83

method	result

derivativedivides	$-\frac{3b\left(\frac{\ln\left((b\coth(dx+c))^{\frac{1}{3}}-b^{\frac{1}{3}}\right)}{6b^{\frac{1}{3}}}-\frac{\ln\left(b^{\frac{2}{3}}+b^{\frac{1}{3}}(b\coth(dx+c))^{\frac{1}{3}}+(b\coth(dx+c))^{\frac{2}{3}}\right)}{12b^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\left(1+\frac{2(b\coth(dx+c))^{\frac{1}{3}}}{b^{\frac{1}{3}}}\right)\sqrt{3}}{3}\right)}{6b^{\frac{1}{3}}}\right.}{}$
default	$-\frac{3b\left(\frac{\ln\left((b\coth(dx+c))^{\frac{1}{3}}-b^{\frac{1}{3}}\right)}{6b^{\frac{1}{3}}}-\frac{\ln\left(b^{\frac{2}{3}}+b^{\frac{1}{3}}(b\coth(dx+c))^{\frac{1}{3}}+(b\coth(dx+c))^{\frac{2}{3}}\right)}{12b^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\left(1+\frac{2(b\coth(dx+c))^{\frac{1}{3}}}{b^{\frac{1}{3}}}\right)\sqrt{3}}{3}\right)}{6b^{\frac{1}{3}}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c))^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3/d*b*(1/6/b^(1/3)*ln((b*coth(d*x+c))^(1/3)-b^(1/3))-1/12/b^(1/3)*ln(b^(2/
3)+b^(1/3)*(b*coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3))+1/6*3^(1/2)/b^(1/3)
*arctan(1/3*(1+2*(b*coth(d*x+c))^(1/3)/b^(1/3))*3^(1/2))-1/6/b^(1/3)*ln((b*
coth(d*x+c))^(1/3)+b^(1/3))+1/12/b^(1/3)*ln(b^(2/3)-b^(1/3)*(b*coth(d*x+c))
^(1/3)+(b*coth(d*x+c))^(2/3))+1/6*3^(1/2)/b^(1/3)*arctan(1/3*3^(1/2)*(2*(b*
coth(d*x+c))^(1/3)/b^(1/3)-1)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c))^(2/3), x)
```

Fricas [A]
time = 0.49, size = 310, normalized size = 1.42

$$-\frac{2\sqrt{3}\left(-b^{2}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}b-2\sqrt{3}\left(-b^{2}\right)^{\frac{1}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}}{3b}\right)+2\sqrt{3}\left(b^{2}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}b-2\sqrt{3}\left(b^{2}\right)^{\frac{1}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}}{3b}\right)+\left(-b^{2}\right)^{\frac{1}{3}}\log\left(b\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{2}{3}}-\left(-b^{2}\right)^{\frac{1}{3}}b+\left(-b^{2}\right)^{\frac{2}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}\right)+\left(b^{2}\right)^{\frac{1}{3}}\log\left(b\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{2}{3}}+\left(b^{2}\right)^{\frac{1}{3}}b-\left(b^{2}\right)^{\frac{2}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}\right)-2\left(-b^{2}\right)^{\frac{1}{3}}\log\left(b\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}-\left(-b^{2}\right)^{\frac{2}{3}}\right)-2\left(b^{2}\right)^{\frac{1}{3}}\log\left(b\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}+\left(b^{2}\right)^{\frac{2}{3}}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*(-b^2)^(1/3)*arctan(-1/3*(sqrt(3)*b - 2*sqrt(3)*(-b^2)^(1/3
)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/b) + 2*sqrt(3)*(b^2)^(1/3)*arctan(
```

```
-1/3*(sqrt(3)*b - 2*sqrt(3)*(b^2)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/
3))/b) + (-b^2)^(1/3)*log(b*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3) - (-b^2)^
(1/3)*b + (-b^2)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) + (b^2)^(1/3)
*log(b*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3) + (b^2)^(1/3)*b - (b^2)^(2/3)*
(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) - 2*(-b^2)^(1/3)*log(b*(b*cosh(d*x +
 c)/sinh(d*x + c))^(1/3) - (-b^2)^(2/3)) - 2*(b^2)^(1/3)*log(b*(b*cosh(d*x
+ c)/sinh(d*x + c))^(1/3) + (b^2)^(2/3)))/d
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (b\coth(c+dx))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))**(2/3),x)
```

```
[Out] Integral((b*coth(c + d*x))**(2/3), x)
```

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Minimal poly. in rootof must be fract
ion free Error: Bad Argument ValueMinimal poly. in rootof must be fraction
free E
```

Mupad [B]

time = 1.50, size = 233, normalized size = 1.07

$$-\frac{b^{2/3}\,\mathrm{atan}\left(\frac{(b\coth(c+d\,x))^{1/3}\,1\mathrm{i}}{b^{1/3}}\right)\,1\mathrm{i}}{d}-\frac{b^{2/3}\,\ln\left(\frac{972\,b^9}{d^3}-\frac{972\,b^{26/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(b\coth(c+d\,x))^{1/3}}{d^3}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,d}-\frac{b^{2/3}\,\ln\left(\frac{972\,b^9}{d^3}-\frac{972\,b^{26/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(b\coth(c+d\,x))^{1/3}}{d^3}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,d}+\frac{b^{2/3}\,\ln\left(\frac{972\,b^9}{d^3}+\frac{1944\,b^{26/3}\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)\,(b\coth(c+d\,x))^{1/3}}{d^3}\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{d}+\frac{b^{2/3}\,\ln\left(\frac{972\,b^9}{d^3}+\frac{1944\,b^{26/3}\,\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)\,(b\coth(c+d\,x))^{1/3}}{d^3}\right)\,\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x))^(2/3),x)
```

```
[Out] (b^(2/3)*log((972*b^9)/d^3 + (1944*b^(26/3)*((3^(1/2)*1i)/4 - 1/4)*(b*coth(
c + d*x))^(1/3))/d^3)*((3^(1/2)*1i)/4 - 1/4))/d - (b^(2/3)*log((972*b^9)/d^
3 - (972*b^(26/3)*((3^(1/2)*1i)/2 - 1/2)*(b*coth(c + d*x))^(1/3))/d^3)*((3^
(1/2)*1i)/2 - 1/2))/(2*d) - (b^(2/3)*log((972*b^9)/d^3 - (972*b^(26/3)*((3^
(1/2)*1i)/2 + 1/2)*(b*coth(c + d*x))^(1/3))/d^3)*((3^(1/2)*1i)/2 + 1/2))/(2
*d) - (b^(2/3)*atan(((b*coth(c + d*x))^(1/3)*1i)/b^(1/3))*1i)/d + (b^(2/3)*
log((972*b^9)/d^3 + (1944*b^(26/3)*((3^(1/2)*1i)/4 + 1/4)*(b*coth(c + d*x))
^(1/3))/d^3)*((3^(1/2)*1i)/4 + 1/4))/d
```

3.11 $\int \sqrt[3]{b\coth(c+dx)}\, dx$

Optimal. Leaf size=132

$$-\frac{\sqrt{3}\sqrt[3]{b}\,\text{ArcTan}\left(\frac{b^{2/3}+2(b\coth(c+dx))^{2/3}}{\sqrt{3}\,b^{2/3}}\right)}{2d}-\frac{\sqrt[3]{b}\,\log\left(b^{2/3}-(b\coth(c+dx))^{2/3}\right)}{2d}+\frac{\sqrt[3]{b}\,\log\left(b^{4/3}+b^{2/3}(b\coth(c\right.}{}$$

```
[Out] -1/2*b^(1/3)*ln(b^(2/3)-(b*coth(d*x+c))^(2/3))/d+1/4*b^(1/3)*ln(b^(4/3)+b^(
2/3)*(b*coth(d*x+c))^(2/3)+(b*coth(d*x+c))^(4/3))/d-1/2*b^(1/3)*arctan(1/3*
(b^(2/3)+2*(b*coth(d*x+c))^(2/3))/b^(2/3)*3^(1/2))*3^(1/2)/d
```

Rubi [A]
time = 0.08, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3557, 335, 281, 298, 31, 648, 631, 210, 642}

$$-\frac{\sqrt{3}\sqrt[3]{b}\,\text{ArcTan}\left(\frac{b^{2/3}+2(b\coth(c+dx))^{2/3}}{\sqrt{3}\,b^{2/3}}\right)}{2d}-\frac{\sqrt[3]{b}\,\log\left(b^{2/3}-(b\coth(c+dx))^{2/3}\right)}{2d}+\frac{\sqrt[3]{b}\,\log\left(b^{2/3}(b\coth(c+dx))^{2/3}+b^{4/3}+(b\coth(c+dx))^{4/3}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x])^(1/3),x]
```

```
[Out] -1/2*(Sqrt[3]*b^(1/3)*ArcTan[(b^(2/3) + 2*(b*Coth[c + d*x])^(2/3))/(Sqrt[3]
*b^(2/3))])/d - (b^(1/3)*Log[b^(2/3) - (b*Coth[c + d*x])^(2/3)])/(2*d) + (b
^(1/3)*Log[b^(4/3) + b^(2/3)*(b*Coth[c + d*x])^(2/3) + (b*Coth[c + d*x])^(4
/3)])/(4*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \sqrt[3]{b\coth(c+dx)}\,dx = -\frac{b\text{Subst}\left(\int \frac{\sqrt[3]{x}}{-b^2+x^2}\,dx, x, b\coth(c+dx)\right)}{d}$$
$$= -\frac{(3b)\text{Subst}\left(\int \frac{x^3}{-b^2+x^6}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{d}$$
$$= -\frac{(3b)\text{Subst}\left(\int \frac{x}{-b^2+x^3}\,dx, x, (b\coth(c+dx))^{2/3}\right)}{2d}$$
$$= -\frac{\sqrt[3]{b}\,\text{Subst}\left(\int \frac{1}{-b^{2/3}+x}\,dx, x, (b\coth(c+dx))^{2/3}\right)}{2d} + \frac{\sqrt[3]{b}\,\text{Subst}\left(\int \frac{-b^{2/3}+x}{b^{4/3}+b^{2/3}x+x^2}\,dx, x\right.}{2d}$$
$$= -\frac{\sqrt[3]{b}\,\log\left(b^{2/3}-(b\coth(c+dx))^{2/3}\right)}{2d} + \frac{\sqrt[3]{b}\,\text{Subst}\left(\int \frac{b^{2/3}+2x}{b^{4/3}+b^{2/3}x+x^2}\,dx, x, (b\coth(c\right.}{4d}$$
$$= -\frac{\sqrt[3]{b}\,\log\left(b^{2/3}-(b\coth(c+dx))^{2/3}\right)}{2d} + \frac{\sqrt[3]{b}\,\log\left(b^{4/3}+b^{2/3}(b\coth(c+dx))^{2/3}+(\right.}{4d}$$
$$= -\frac{\sqrt{3}\,\sqrt[3]{b}\,\tan^{-1}\left(\frac{1+\frac{2(b\coth(c+dx))^{2/3}}{b^{2/3}}}{\sqrt{3}}\right)}{2d} - \frac{\sqrt[3]{b}\,\log\left(b^{2/3}-(b\coth(c+dx))^{2/3}\right)}{2d} + \frac{\sqrt[3]{b}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 38, normalized size = 0.29

$$\frac{3(b\coth(c+dx))^{4/3}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\coth^2(c+dx)\right)}{4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x])^(1/3),x]
```

```
[Out] (3*(b*Coth[c + d*x])^(4/3)*Hypergeometric2F1[2/3, 1, 5/3, Coth[c + d*x]^2])
/(4*b*d)
```

Maple [A]
time = 1.39, size = 109, normalized size = 0.83

method	result

derivativedivides	$-\frac{3b\left(\frac{\ln\left((b\coth(dx+c))^{\frac{2}{3}}-\left(b^{2}\right)^{\frac{1}{3}}\right)}{6\left(b^{2}\right)^{\frac{1}{3}}}-\frac{\ln\left((b\coth(dx+c))^{\frac{4}{3}}+\left(b^{2}\right)^{\frac{1}{3}}(b\coth(dx+c))^{\frac{2}{3}}+\left(b^{2}\right)^{\frac{2}{3}}\right)}{12\left(b^{2}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(b\coth(}{(b^{2}}\right.}{3}\right.}{6\left(b^{2}\right)^{\frac{1}{3}}}\right.}{d}$
default	$-\frac{3b\left(\frac{\ln\left((b\coth(dx+c))^{\frac{2}{3}}-\left(b^{2}\right)^{\frac{1}{3}}\right)}{6\left(b^{2}\right)^{\frac{1}{3}}}-\frac{\ln\left((b\coth(dx+c))^{\frac{4}{3}}+\left(b^{2}\right)^{\frac{1}{3}}(b\coth(dx+c))^{\frac{2}{3}}+\left(b^{2}\right)^{\frac{2}{3}}\right)}{12\left(b^{2}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(b\coth(}{(b^{2}}\right.}{3}\right.}{6\left(b^{2}\right)^{\frac{1}{3}}}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c))^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3/d*b*(1/6/(b^2)^(1/3)*ln((b*coth(d*x+c))^(2/3)-(b^2)^(1/3))-1/12/(b^2)^(1
/3)*ln((b*coth(d*x+c))^(4/3)+(b^2)^(1/3)*(b*coth(d*x+c))^(2/3)+(b^2)^(2/3))
+1/6*3^(1/2)/(b^2)^(1/3)*arctan(1/3*3^(1/2)*(2/(b^2)^(1/3)*(b*coth(d*x+c))^
(2/3)+1)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c))^(1/3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 291 vs. 2(99) = 198.
time = 0.38, size = 291, normalized size = 2.20

$$-\frac{2\sqrt{3}(-b)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}b-2\sqrt{3}(-b)^{\frac{1}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{2}{3}}}{3b}\right)-2(-b)^{\frac{1}{3}}\log\left(-(-b)^{\frac{2}{3}}+\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{2}{3}}\right)+(-b)^{\frac{1}{3}}\log\left(\frac{\left(\cosh(dx+c)^{2}+2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^{2}-1\right)(-b)^{\frac{2}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{2}{3}}-\left(b\cosh(dx+c)^{2}+2b\cosh(dx+c)\sinh(dx+c)+b\sinh(dx+c)^{2}-b\right)(-b)^{\frac{1}{3}}+\left(b\cosh(dx+c)^{2}+2b\cosh(dx+c)\sinh(dx+c)+b\sinh(dx+c)^{2}+b\right)\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}}{\cosh(dx+c)^{2}+2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^{2}-1}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*(-b)^(1/3)*arctan(-1/3*(sqrt(3)*b - 2*sqrt(3)*(-b)^(1/3)*(b
*cosh(d*x + c)/sinh(d*x + c))^(2/3))/b) - 2*(-b)^(1/3)*log(-(-b)^(2/3) + (b
*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + (-b)^(1/3)*log(((cosh(d*x + c)^2 + 2
*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3)*(b*cosh(d*x
+ c)/sinh(d*x + c))^(2/3) - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x
 + c) + b*sinh(d*x + c)^2 - b)*(-b)^(1/3) + (b*cosh(d*x + c)^2 + 2*b*cosh(d
*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*(b*cosh(d*x + c)/sinh(d*x +
c))^(1/3))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2 - 1)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{b \coth (c+dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))**(1/3),x)
```

```
[Out] Integral((b*coth(c + d*x))**(1/3), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(99) = 198.
time = 0.56, size = 217, normalized size = 1.64

$$-\frac{b\left(\frac{2\sqrt{3}\,|b|^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(\frac{be^{(2dx+2c)}+b}{e^{(2dx+2c)}-1}\right)^{\frac{2}{3}}+|b|^{\frac{2}{3}}\right)}{3\,|b|^{\frac{2}{3}}}\right)}{b^2}-\frac{|b|^{\frac{4}{3}}\log\left(\left(\frac{be^{(2dx+2c)}+b}{e^{(2dx+2c)}-1}\right)^{\frac{2}{3}}|b|^{\frac{2}{3}}+|b|^{\frac{4}{3}}+\frac{\left(be^{(2dx+2c)}+b\right)\left(\frac{be^{(2dx+2c)}+b}{e^{(2dx+2c)}-1}\right)^{\frac{1}{3}}}{e^{(2dx+2c)}-1}\right)}{b^2}+\frac{2\,|b|^{\frac{4}{3}}\log\left(\left|\left(\frac{be^{(2dx+2c)}+b}{e^{(2dx+2c)}-1}\right)^{\frac{2}{3}}-|b|^{\frac{2}{3}}\right|\right)}{b^2}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] -1/4*b*(2*sqrt(3)*abs(b)^(4/3)*arctan(1/3*sqrt(3)*(2*((b*e^(2*d*x + 2*c) +
b)/(e^(2*d*x + 2*c) - 1))^(2/3) + abs(b)^(2/3))/abs(b)^(2/3))/b^2 - abs(b)^
(4/3)*log(((b*e^(2*d*x + 2*c) + b)/(e^(2*d*x + 2*c) - 1))^(2/3)*abs(b)^(2/3
) + abs(b)^(4/3) + (b*e^(2*d*x + 2*c) + b)*((b*e^(2*d*x + 2*c) + b)/(e^(2*d
*x + 2*c) - 1))^(1/3)/(e^(2*d*x + 2*c) - 1))/b^2 + 2*abs(b)^(4/3)*log(abs((
(b*e^(2*d*x + 2*c) + b)/(e^(2*d*x + 2*c) - 1))^(2/3) - abs(b)^(2/3)))/b^2)/
d
```

Mupad [B]
time = 1.52, size = 146, normalized size = 1.11

$$\frac{(-b)^{1/3}\ln\left(81\,(-b)^{16/3}\,(b\coth(c+d\,x))^{2/3}-81\,b^6\right)}{2\,d}-\frac{(-b)^{1/3}\ln\left(-\frac{81\,b^6}{d^4}-\frac{81\,(-b)^{16/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(b\coth(c+d\,x))^{2/3}}{d^4}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,d}+\frac{(-b)^{1/3}\ln\left(-\frac{81\,b^6}{d^4}+\frac{162\,(-b)^{16/3}\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)(b\coth(c+d\,x))^{2/3}}{d^4}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x))^(1/3),x)
```

```
[Out] ((-b)^(1/3)*log(81*(-b)^(16/3)*(b*coth(c + d*x))^(2/3) - 81*b^6))/(2*d) - (
(-b)^(1/3)*log(- (81*b^6)/d^4 - (81*(-b)^(16/3)*((3^(1/2)*1i)/2 + 1/2)*(b*c
oth(c + d*x))^(2/3))/d^4)*((3^(1/2)*1i)/2 + 1/2))/(2*d) + ((-b)^(1/3)*log((
162*(-b)^(16/3)*((3^(1/2)*1i)/4 - 1/4)*(b*coth(c + d*x))^(2/3))/d^4 - (81*b
^6)/d^4)*((3^(1/2)*1i)/4 - 1/4))/d
```

3.12 $\int \frac{1}{\sqrt[3]{b\coth(c+dx)}}\,dx$

Optimal. Leaf size=132

$$\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{b^{2/3}+2(b\coth(c+dx))^{2/3}}{\sqrt{3}\,b^{2/3}}\right)}{2\sqrt[3]{b}\,d}-\frac{\log\left(b^{2/3}-(b\coth(c+dx))^{2/3}\right)}{2\sqrt[3]{b}\,d}+\frac{\log\left(b^{4/3}+b^{2/3}(b\coth(c+dx))^{2/3}+(b\right.}{4\sqrt[3]{b}\,d}$$

```
[Out] -1/2*ln(b^(2/3)-(b*coth(d*x+c))^(2/3))/b^(1/3)/d+1/4*ln(b^(4/3)+b^(2/3)*(b*
coth(d*x+c))^(2/3)+(b*coth(d*x+c))^(4/3))/b^(1/3)/d+1/2*arctan(1/3*(b^(2/3)
+2*(b*coth(d*x+c))^(2/3))/b^(2/3)*3^(1/2))*3^(1/2)/b^(1/3)/d
```

Rubi [A]
time = 0.07, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3557, 335, 281, 206, 31, 648, 631, 210, 642}

$$\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{b^{2/3}+2(b\coth(c+dx))^{2/3}}{\sqrt{3}\,b^{2/3}}\right)}{2\sqrt[3]{b}\,d}-\frac{\log\left(b^{2/3}-(b\coth(c+dx))^{2/3}\right)}{2\sqrt[3]{b}\,d}+\frac{\log\left(b^{2/3}(b\coth(c+dx))^{2/3}+b^{4/3}+(b\coth(c+dx))^{4/3}\right)}{4\sqrt[3]{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x])^(-1/3),x]
```

```
[Out] (Sqrt[3]*ArcTan[(b^(2/3) + 2*(b*Coth[c + d*x])^(2/3))/(Sqrt[3]*b^(2/3))])/(
2*b^(1/3)*d) - Log[b^(2/3) - (b*Coth[c + d*x])^(2/3)]/(2*b^(1/3)*d) + Log[b
^(4/3) + b^(2/3)*(b*Coth[c + d*x])^(2/3) + (b*Coth[c + d*x])^(4/3)]/(4*b^(1
/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{b\coth(c+dx)}}\,dx = -\frac{b\text{Subst}\left(\int \frac{1}{\sqrt[3]{x}\,(-b^2+x^2)}\,dx, x, b\coth(c+dx)\right)}{d}$$

$$= -\frac{(3b)\text{Subst}\left(\int \frac{x}{-b^2+x^6}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{d}$$

$$= -\frac{(3b)\text{Subst}\left(\int \frac{1}{-b^2+x^3}\,dx, x, (b\coth(c+dx))^{2/3}\right)}{2d}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{-b^{2/3}+x}\,dx, x, (b\coth(c+dx))^{2/3}\right)}{2\sqrt[3]{b}\,d} - \frac{\text{Subst}\left(\int \frac{-2b^{2/3}-x}{b^{4/3}+b^{2/3}x+x^2}\,dx, x, (b\cot\right.}{2\sqrt[3]{b}\,d}$$

$$= -\frac{\log\left(b^{2/3}-(b\coth(c+dx))^{2/3}\right)}{2\sqrt[3]{b}\,d} + \frac{\text{Subst}\left(\int \frac{b^{2/3}+2x}{b^{4/3}+b^{2/3}x+x^2}\,dx, x, (b\coth(c+dx))^{2/}\right.}{4\sqrt[3]{b}\,d}$$

$$= -\frac{\log\left(b^{2/3}-(b\coth(c+dx))^{2/3}\right)}{2\sqrt[3]{b}\,d} + \frac{\log\left(b^{4/3}+b^{2/3}(b\coth(c+dx))^{2/3}+(b\coth(c\right.}{4\sqrt[3]{b}\,d}$$

$$= \frac{\sqrt{3}\,\tan^{-1}\left(\frac{1+\frac{2(b\coth(c+dx))^{2/3}}{b^{2/3}}}{\sqrt{3}}\right)}{2\sqrt[3]{b}\,d} - \frac{\log\left(b^{2/3}-(b\coth(c+dx))^{2/3}\right)}{2\sqrt[3]{b}\,d} + \frac{\log\left(b^{4/3}+b^{2/\right.}{}$$

Mathematica [A]
time = 0.08, size = 98, normalized size = 0.74

$$\frac{\sqrt[3]{\coth(c+dx)}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{1+2\coth^{\frac{2}{3}}(c+dx)}{\sqrt{3}}\right)-2\log\left(1-\coth^{\frac{2}{3}}(c+dx)\right)+\log\left(1+\coth^{\frac{2}{3}}(c+dx)+\coth^{\frac{4}{3}}(c+dx)\right)\right)}{4d\sqrt[3]{b\coth(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x])^(-1/3),x]
```

```
[Out] (Coth[c + d*x]^(1/3)*(2*Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(2/3))/Sqrt[3]]
 - 2*Log[1 - Coth[c + d*x]^(2/3)] + Log[1 + Coth[c + d*x]^(2/3) + Coth[c +
d*x]^(4/3)]))/(4*d*(b*Coth[c + d*x])^(1/3))
```

Maple [A]
time = 1.23, size = 109, normalized size = 0.83

method	result

derivativedivides	$-\frac{3b\left(\frac{\ln\left((b\coth(dx+c))^{\frac{2}{3}}-(b^2)^{\frac{1}{3}}\right)}{6(b^2)^{\frac{2}{3}}}-\frac{\ln\left((b\coth(dx+c))^{\frac{4}{3}}+(b^2)^{\frac{1}{3}}(b\coth(dx+c))^{\frac{2}{3}}+(b^2)^{\frac{2}{3}}\right)}{12(b^2)^{\frac{2}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(b\coth(}{(b^2}\right.}{3}\right.}{6(b^2)^{\frac{2}{3}}}\right.}{d}$
default	$-\frac{3b\left(\frac{\ln\left((b\coth(dx+c))^{\frac{2}{3}}-(b^2)^{\frac{1}{3}}\right)}{6(b^2)^{\frac{2}{3}}}-\frac{\ln\left((b\coth(dx+c))^{\frac{4}{3}}+(b^2)^{\frac{1}{3}}(b\coth(dx+c))^{\frac{2}{3}}+(b^2)^{\frac{2}{3}}\right)}{12(b^2)^{\frac{2}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(b\coth(}{(b^2}\right.}{3}\right.}{6(b^2)^{\frac{2}{3}}}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c))^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3/d*b*(1/6/(b^2)^(2/3)*ln((b*coth(d*x+c))^(2/3)-(b^2)^(1/3))-1/12/(b^2)^(2
/3)*ln((b*coth(d*x+c))^(4/3)+(b^2)^(1/3)*(b*coth(d*x+c))^(2/3)+(b^2)^(2/3))
-1/6/(b^2)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(b^2)^(1/3)*(b*coth(d*x+c))^
(2/3)+1)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c))^(-1/3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 742 vs. 2(99) = 198.
time = 0.55, size = 1598, normalized size = 12.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(1/3),x, algorithm="fricas")

[Out] [1/4*(sqrt(3)*b*sqrt((-b)^(1/3)/b)*log((3*b*cosh(d*x + c)^4 + 12*b*cosh(d*x
 + c)*sinh(d*x + c)^3 + 3*b*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^2 + 2*(9*b*
cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 3*(cosh(d*x + c)^4 + 4*cosh(d*x + c)
*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c
)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c)
 + 1)*(-b)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3) - sqrt(3)*((cosh(d*x
 + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x
 + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cos
h(d*x + c))*sinh(d*x + c) + 1)*(-b)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(
2/3) - (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x
+ c)^4 - 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^2
+ 4*(b*cosh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c) + b)*(-b)^(1/3) - 2
*(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d*x + c)
^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4
- b)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))*sqrt((-b)^(1/3)/b) + 4*(3*b*cos
h(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) + 3*b)/(cosh(d*x + c)^2 + 2*c
osh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 2*(-b)^(2/3)*log(-(-b)^(2/
3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + (-b)^(2/3)*log(((cosh(d*x + c
)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3)*(b*co
sh(d*x + c)/sinh(d*x + c))^(2/3) - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*s
inh(d*x + c) + b*sinh(d*x + c)^2 - b)*(-b)^(1/3) + (b*cosh(d*x + c)^2 + 2*b
*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*(b*cosh(d*x + c)/sinh
(d*x + c))^(1/3))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d
*x + c)^2 - 1)))/(b*d), 1/4*(2*sqrt(3)*b*sqrt(-(-b)^(1/3)/b)*arctan((2*sqrt
(3)*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 +
2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x
 + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*(-b)^(2/3)*(b*cosh(d*x + c)/sin
h(d*x + c))^(2/3)*sqrt(-(-b)^(1/3)/b) + sqrt(3)*(b*cosh(d*x + c)^4 + 4*b*co
sh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*b*cosh(d*x + c)^2 + 2*(
3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - b*cosh(d*
x + c))*sinh(d*x + c) + b)*(-b)^(1/3)*sqrt(-(-b)^(1/3)/b) - 4*sqrt(3)*(b*co
sh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d*x + c)^2*sin
h(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - b)*(
b*cosh(d*x + c)/sinh(d*x + c))^(1/3)*sqrt(-(-b)^(1/3)/b))/(9*b*cosh(d*x + c
)^4 + 36*b*cosh(d*x + c)*sinh(d*x + c)^3 + 9*b*sinh(d*x + c)^4 + 14*b*cosh(
d*x + c)^2 + 2*(27*b*cosh(d*x + c)^2 + 7*b)*sinh(d*x + c)^2 + 4*(9*b*cosh(d
*x + c)^3 + 7*b*cosh(d*x + c))*sinh(d*x + c) + 9*b)) - 2*(-b)^(2/3)*log(-(-
b)^(2/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + (-b)^(2/3)*log(((cosh(d
*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3)
*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3) - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x
+ c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*(-b)^(1/3) + (b*cosh(d*x + c)^2
 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*(b*cosh(d*x + c
)/sinh(d*x + c))^(1/3))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
```

```
sinh(d*x + c)^2 - 1)))/(b*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{b\coth(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))**(1/3),x)
```

```
[Out] Integral((b*coth(c + d*x))**(-1/3), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(99) = 198.
time = 0.49, size = 216, normalized size = 1.64

$$\frac{b\left(\frac{2\sqrt{3}\,|b|^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(\frac{be^{(2dx+2c)}+b}{e^{(2dx+2c)}-1}\right)^{\frac{2}{3}}+|b|^{\frac{2}{3}}\right)}{3|b|^{\frac{2}{3}}}\right)}{b^2}+\frac{|b|^{\frac{2}{3}}\log\left(\left(\frac{be^{(2dx+2c)}+b}{e^{(2dx+2c)}-1}\right)^{\frac{2}{3}}|b|^{\frac{2}{3}}+|b|^{\frac{4}{3}}+\frac{\left(be^{(2dx+2c)}+b\right)\left(\frac{be^{(2dx+2c)}+b}{e^{(2dx+2c)}-1}\right)^{\frac{1}{3}}}{e^{(2dx+2c)}-1}\right)}{b^2}-\frac{2|b|^{\frac{2}{3}}\log\left(\left|\left(\frac{be^{(2dx+2c)}+b}{e^{(2dx+2c)}-1}\right)^{\frac{2}{3}}-|b|^{\frac{2}{3}}\right|\right)}{b^2}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] 1/4*b*(2*sqrt(3)*abs(b)^(2/3)*arctan(1/3*sqrt(3)*(2*((b*e^(2*d*x + 2*c) + b
)/(e^(2*d*x + 2*c) - 1))^(2/3) + abs(b)^(2/3))/abs(b)^(2/3))/b^2 + abs(b)^(
2/3)*log(((b*e^(2*d*x + 2*c) + b)/(e^(2*d*x + 2*c) - 1))^(2/3)*abs(b)^(2/3)
 + abs(b)^(4/3) + (b*e^(2*d*x + 2*c) + b)*((b*e^(2*d*x + 2*c) + b)/(e^(2*d*
x + 2*c) - 1))^(1/3)/(e^(2*d*x + 2*c) - 1))/b^2 - 2*abs(b)^(2/3)*log(abs(((
b*e^(2*d*x + 2*c) + b)/(e^(2*d*x + 2*c) - 1))^(2/3) - abs(b)^(2/3)))/b^2)/d
```

Mupad [**B**]
time = 1.65, size = 147, normalized size = 1.11

$$\frac{\ln\left(162\,(-b)^{11/3}+162\,b^3\,(b\coth(c+d\,x))^{2/3}\right)}{2\,(-b)^{1/3}\,d}+\frac{\ln\left(\frac{81\,(-b)^{11/3}\,\left(-1+\sqrt{3}\,1i\right)}{d^3}+\frac{162\,b^3\,(b\coth(c+d\,x))^{2/3}}{d^3}\right)\left(-1+\sqrt{3}\,1i\right)}{4\,(-b)^{1/3}\,d}-\frac{\ln\left(\frac{81\,(-b)^{11/3}\,\left(1+\sqrt{3}\,1i\right)}{d^3}-\frac{162\,b^3\,(b\coth(c+d\,x))^{2/3}}{d^3}\right)\left(1+\sqrt{3}\,1i\right)}{4\,(-b)^{1/3}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x))^(1/3),x)
```

```
[Out] log(162*(-b)^(11/3) + 162*b^3*(b*coth(c + d*x))^(2/3))/(2*(-b)^(1/3)*d) + (
log((81*(-b)^(11/3)*(3^(1/2)*1i - 1))/d^3 + (162*b^3*(b*coth(c + d*x))^(2/3
))/d^3)*(3^(1/2)*1i - 1))/(4*(-b)^(1/3)*d) - (log((81*(-b)^(11/3)*(3^(1/2)*
1i + 1))/d^3 - (162*b^3*(b*coth(c + d*x))^(2/3))/d^3)*(3^(1/2)*1i + 1))/(4*
(-b)^(1/3)*d)
```

3.13 $\int \frac{1}{(b\coth(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=218

$$-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2b^{2/3}d}+\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2b^{2/3}d}+\frac{\tanh^{-1}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{b^{2/3}d}$$

```
[Out] arctanh((b*coth(d*x+c))^(1/3)/b^(1/3))/b^(2/3)/d-1/4*ln(b^(2/3)-b^(1/3)*(b*
coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3))/b^(2/3)/d+1/4*ln(b^(2/3)+b^(1/3)*
(b*coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3))/b^(2/3)/d-1/2*arctan(1/3*(1-2*
(b*coth(d*x+c))^(1/3)/b^(1/3))*3^(1/2))*3^(1/2)/b^(2/3)/d+1/2*arctan(1/3*(1
+2*(b*coth(d*x+c))^(1/3)/b^(1/3))*3^(1/2))*3^(1/2)/b^(2/3)/d
```

Rubi [A]
time = 0.17, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3557, 335, 216, 648, 632, 210, 642, 212}

$$-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2b^{2/3}d}+\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}+1}{\sqrt{3}}\right)}{2b^{2/3}d}-\frac{\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4b^{2/3}d}+\frac{\log\left(b^{2/3}+\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4b^{2/3}d}+\frac{\tanh^{-1}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{b^{2/3}d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x])^(-2/3),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(1 - (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))/Sqrt[3]])/(b
^(2/3)*d) + (Sqrt[3]*ArcTan[(1 + (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))/Sqrt[
3]])/(2*b^(2/3)*d) + ArcTanh[(b*Coth[c + d*x])^(1/3)/b^(1/3)]/(b^(2/3)*d) -
 Log[b^(2/3) - b^(1/3)*(b*Coth[c + d*x])^(1/3) + (b*Coth[c + d*x])^(2/3)]/(
4*b^(2/3)*d) + Log[b^(2/3) + b^(1/3)*(b*Coth[c + d*x])^(1/3) + (b*Coth[c +
d*x])^(2/3)]/(4*b^(2/3)*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{(b\coth(c+dx))^{2/3}}\,dx = -\frac{b\text{Subst}\left(\int \frac{1}{x^{2/3}(-b^2+x^2)}\,dx, x, b\coth(c+dx)\right)}{d}$$

$$= -\frac{(3b)\text{Subst}\left(\int \frac{1}{-b^2+x^6}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt[3]{b}-\frac{x}{2}}{b^{2/3}-\sqrt[3]{b}\,x+x^2}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{b^{2/3}d} + \frac{\text{Subst}\left(\int \frac{\sqrt[3]{b}+\frac{x}{2}}{b^{2/3}+\sqrt[3]{b}\,x+x^2}\,dx, x,\right.}{b^{2/3}d}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{b^{2/3}d} - \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{b}+2x}{b^{2/3}-\sqrt[3]{b}\,x+x^2}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{4b^{2/3}d}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{b^{2/3}d} - \frac{\log\left(b^{2/3}-\sqrt[3]{b}\,\sqrt[3]{b\coth(c+dx)}+(b\coth(c+\right.}{4b^{2/3}d}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2b^{2/3}d} + \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2b^{2/3}d} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.02, size = 36, normalized size = 0.17

$$\frac{3\sqrt[3]{b\coth(c+dx)}\;{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};\coth^2(c+dx)\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x])^(-2/3),x]
```

```
[Out] (3*(b*Coth[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1, 7/6, Coth[c + d*x]^2])
/(b*d)
```

Maple [A]
time = 1.44, size = 193, normalized size = 0.89

method	result

derivativedivides	$-\frac{3b\left(-\frac{\frac{\ln\left((b\coth(dx+c))^{\frac{1}{3}}+b^{\frac{1}{3}}\right)}{3b^{\frac{2}{3}}}-\frac{\ln\left(b^{\frac{2}{3}}-b^{\frac{1}{3}}(b\coth(dx+c))^{\frac{1}{3}}+(b\coth(dx+c))^{\frac{2}{3}}\right)}{6b^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(b\coth(dx+c))^{\frac{1}{3}}}{b^{\frac{1}{3}}}\cdots\right)}{3}\right)}{3b^{\frac{2}{3}}}}{2b}\cdots\right)}{\cdots}$
default	$-\frac{3b\left(-\frac{\frac{\ln\left((b\coth(dx+c))^{\frac{1}{3}}+b^{\frac{1}{3}}\right)}{3b^{\frac{2}{3}}}-\frac{\ln\left(b^{\frac{2}{3}}-b^{\frac{1}{3}}(b\coth(dx+c))^{\frac{1}{3}}+(b\coth(dx+c))^{\frac{2}{3}}\right)}{6b^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(b\coth(dx+c))^{\frac{1}{3}}}{b^{\frac{1}{3}}}\cdots\right)}{3}\right)}{3b^{\frac{2}{3}}}}{2b}\cdots\right)}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c))^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3/d*b*(-1/2*(1/3/b^(2/3)*ln((b*coth(d*x+c))^(1/3)+b^(1/3))-1/6/b^(2/3)*ln(
b^(2/3)-b^(1/3)*(b*coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3))+1/3/b^(2/3)*3^
(1/2)*arctan(1/3*3^(1/2)*(2*(b*coth(d*x+c))^(1/3)/b^(1/3)-1)))/b+1/2*(1/3/b
^(2/3)*ln((b*coth(d*x+c))^(1/3)-b^(1/3))-1/6/b^(2/3)*ln(b^(2/3)+b^(1/3)*(b*
coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3))-1/3/b^(2/3)*3^(1/2)*arctan(1/3*(1
+2*(b*coth(d*x+c))^(1/3)/b^(1/3))*3^(1/2)))/b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c))^(-2/3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(166) = 332$.
time = 0.57, size = 356, normalized size = 1.63

$$\frac{2\sqrt{3}\,b\sqrt{-(-b^2)^{\frac{1}{3}}}\arctan\left(-\frac{\sqrt{3}(-b^2)^{\frac{1}{3}}b\sqrt{-(-b^2)^{\frac{1}{3}}}-2\sqrt{3}(-b^2)^{\frac{2}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}\sqrt{-(-b^2)^{\frac{1}{3}}}}{3b^2}\right)+2\sqrt{3}(b^2)^{\frac{1}{6}}b\arctan\left(-\frac{\sqrt{3}(b^2)^{\frac{1}{6}}\left((b^2)^{\frac{1}{3}}b-2(b^2)^{\frac{2}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}\right)}{3b^2}\right)+(-b^2)^{\frac{2}{3}}\log\left(b\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{2}{3}}-(-b^2)^{\frac{1}{3}}b+(-b^2)^{\frac{2}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}\right)-(b^2)^{\frac{2}{3}}\log\left(b\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{2}{3}}+(b^2)^{\frac{1}{3}}b-(b^2)^{\frac{2}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}\right)-2(-b^2)^{\frac{2}{3}}\log\left(b\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}-(-b^2)^{\frac{2}{3}}\right)+2(b^2)^{\frac{2}{3}}\log\left(b\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}+(b^2)^{\frac{2}{3}}\right)}{4b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] 1/4*(2*sqrt(3)*b*sqrt(-(-b^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-b^2)^(1/3)*b*sq
rt(-(-b^2)^(1/3)) - 2*sqrt(3)*(-b^2)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^
(1/3)*sqrt(-(-b^2)^(1/3)))/b^2) + 2*sqrt(3)*(b^2)^(1/6)*b*arctan(-1/3*sqrt(
3)*(b^2)^(1/6)*((b^2)^(1/3)*b - 2*(b^2)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c
))^(1/3))/b^2) + (-b^2)^(2/3)*log(b*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3) -
 (-b^2)^(1/3)*b + (-b^2)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) - (b^
2)^(2/3)*log(b*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3) + (b^2)^(1/3)*b - (b^2
)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) - 2*(-b^2)^(2/3)*log(b*(b*co
sh(d*x + c)/sinh(d*x + c))^(1/3) - (-b^2)^(2/3)) + 2*(b^2)^(2/3)*log(b*(b*c
osh(d*x + c)/sinh(d*x + c))^(1/3) + (b^2)^(2/3)))/(b^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\coth\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))**(2/3),x)
```

```
[Out] Integral((b*coth(c + d*x))**(-2/3), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Minimal poly. in rootof must be fract
ion free Error: Bad Argument ValueMinimal poly. in rootof must be fraction
free E
```

Mupad [B]
time = 1.39, size = 197, normalized size = 0.90

$$\frac{\mathrm{atanh}\left(\frac{(b\coth(c+d\,x))^{1/3}}{b^{1/3}}\right)}{b^{2/3}\,d}-\frac{\mathrm{atan}\left(\frac{b^{10/3}\,(b\coth(c+d\,x))^{1/3}\,243\mathrm{i}}{-243\,b^{11/3}+\sqrt{3}\,b^{11/3}\,243\mathrm{i}}-\frac{243\,\sqrt{3}\,b^{10/3}\,(b\coth(c+d\,x))^{1/3}}{-243\,b^{11/3}+\sqrt{3}\,b^{11/3}\,243\mathrm{i}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,b^{2/3}\,d}-\frac{\mathrm{atan}\left(\frac{b^{10/3}\,(b\coth(c+d\,x))^{1/3}\,243\mathrm{i}}{243\,b^{11/3}+\sqrt{3}\,b^{11/3}\,243\mathrm{i}}+\frac{243\,\sqrt{3}\,b^{10/3}\,(b\coth(c+d\,x))^{1/3}}{243\,b^{11/3}+\sqrt{3}\,b^{11/3}\,243\mathrm{i}}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,b^{2/3}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x))^(2/3),x)
```

```
[Out] atanh((b*coth(c + d*x))^(1/3)/b^(1/3))/(b^(2/3)*d) - (atan((b^(10/3)*(b*cot
h(c + d*x))^(1/3)*243i)/(3^(1/2)*b^(11/3)*243i - 243*b^(11/3)) - (243*3^(1/
```

```
2)*b^(10/3)*(b*coth(c + d*x))^(1/3))/(3^(1/2)*b^(11/3)*243i - 243*b^(11/3))
)*(3^(1/2)*1i + 1)*1i)/(2*b^(2/3)*d) - (atan((b^(10/3)*(b*coth(c + d*x))^(1
/3)*243i)/(3^(1/2)*b^(11/3)*243i + 243*b^(11/3)) + (243*3^(1/2)*b^(10/3)*(b
*coth(c + d*x))^(1/3))/(3^(1/2)*b^(11/3)*243i + 243*b^(11/3)))*(3^(1/2)*1i
- 1)*1i)/(2*b^(2/3)*d)
```

3.14 $\int \frac{1}{(b\coth(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=238

$$\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2b^{4/3}d}-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{1+\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2b^{4/3}d}+\frac{\tanh^{-1}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{b^{4/3}d}$$

```
[Out] arctanh((b*coth(d*x+c))^(1/3)/b^(1/3))/b^(4/3)/d-3/b/d/(b*coth(d*x+c))^(1/3
)-1/4*ln(b^(2/3)-b^(1/3)*(b*coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3))/b^(4/
3)/d+1/4*ln(b^(2/3)+b^(1/3)*(b*coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3))/b^
(4/3)/d+1/2*arctan(1/3*(1-2*(b*coth(d*x+c))^(1/3)/b^(1/3))*3^(1/2))*3^(1/2)
/b^(4/3)/d-1/2*arctan(1/3*(1+2*(b*coth(d*x+c))^(1/3)/b^(1/3))*3^(1/2))*3^(1
/2)/b^(4/3)/d
```

Rubi [A]
time = 0.23, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3555, 3557, 335, 302, 648, 632, 210, 642, 212}

$$\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2b^{4/3}d}-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}+1}{\sqrt{3}}\right)}{2b^{4/3}d}-\frac{\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4b^{4/3}d}+\frac{\log\left(b^{2/3}+\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4b^{4/3}d}+\frac{\tanh^{-1}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{b^{4/3}d}-\frac{3}{bd\sqrt[3]{b\coth(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x])^(-4/3),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 - (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))/Sqrt[3]])/(2*b^(4
/3)*d) - (Sqrt[3]*ArcTan[(1 + (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))/Sqrt[3]]
)/(2*b^(4/3)*d) + ArcTanh[(b*Coth[c + d*x])^(1/3)/b^(1/3)]/(b^(4/3)*d) - 3/
(b*d*(b*Coth[c + d*x])^(1/3)) - Log[b^(2/3) - b^(1/3)*(b*Coth[c + d*x])^(1/
3) + (b*Coth[c + d*x])^(2/3)]/(4*b^(4/3)*d) + Log[b^(2/3) + b^(1/3)*(b*Coth
[c + d*x])^(1/3) + (b*Coth[c + d*x])^(2/3)]/(4*b^(4/3)*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(b\coth(c+dx))^{4/3}}\,dx &= -\frac{3}{bd\sqrt[3]{b\coth(c+dx)}} + \frac{\int (b\coth(c+dx))^{2/3}\,dx}{b^2} \\
&= -\frac{3}{bd\sqrt[3]{b\coth(c+dx)}} - \frac{\text{Subst}\left(\int \frac{x^{2/3}}{-b^2+x^2}\,dx, x, b\coth(c+dx)\right)}{bd} \\
&= -\frac{3}{bd\sqrt[3]{b\coth(c+dx)}} - \frac{3\text{Subst}\left(\int \frac{x^4}{-b^2+x^6}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{bd} \\
&= -\frac{3}{bd\sqrt[3]{b\coth(c+dx)}} + \frac{\text{Subst}\left(\int \frac{-\frac{\sqrt[3]{b}}{2}-\frac{x}{2}}{b^{2/3}-\sqrt[3]{b}\,x+x^2}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{b^{4/3}d} + \text{Sub} \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{b^{4/3}d} - \frac{3}{bd\sqrt[3]{b\coth(c+dx)}} - \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{b}+2x}{b^{2/3}-\sqrt[3]{b}\,x+x^2}\,dx\right.}{4b^{4/}} \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{b^{4/3}d} - \frac{3}{bd\sqrt[3]{b\coth(c+dx)}} - \frac{\log\left(b^{2/3}-\sqrt[3]{b}\,\sqrt[3]{b\coth(\right.}{} \\
&= \frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2b^{4/3}d} - \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2b^{4/3}d} + \text{ta}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.04, size = 36, normalized size = 0.15

$$-\frac{3\,{}_2F_1\left(-\frac{1}{6},1;\frac{5}{6};\coth^2(c+dx)\right)}{bd\sqrt[3]{b\coth(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x])^(-4/3),x]
```

[Out] (-3*Hypergeometric2F1[-1/6, 1, 5/6, Coth[c + d*x]^2])/(b*d*(b*Coth[c + d*x]
)^(1/3))

Maple [A]
time = 1.43, size = 207, normalized size = 0.87

method	result
derivativedivides	$-\frac{3b\left(\frac{\frac{\ln\left((b\coth(dx+c))^{\frac{1}{3}}-b^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}}-\frac{\ln\left(b^{\frac{2}{3}}+b^{\frac{1}{3}}(b\coth(dx+c))^{\frac{1}{3}}+(b\coth(dx+c))^{\frac{2}{3}}\right)}{6b^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\left(1+\frac{2(b\coth(dx+c))^{\frac{1}{3}}}{b^{\frac{1}{3}}}\right)\sqrt{3}}{3}\right)}{3b^{\frac{1}{3}}}}{2b^{2}}\right.}{}$
default	$-\frac{3b\left(\frac{\frac{\ln\left((b\coth(dx+c))^{\frac{1}{3}}-b^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}}-\frac{\ln\left(b^{\frac{2}{3}}+b^{\frac{1}{3}}(b\coth(dx+c))^{\frac{1}{3}}+(b\coth(dx+c))^{\frac{2}{3}}\right)}{6b^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\left(1+\frac{2(b\coth(dx+c))^{\frac{1}{3}}}{b^{\frac{1}{3}}}\right)\sqrt{3}}{3}\right)}{3b^{\frac{1}{3}}}}{2b^{2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*coth(d*x+c))^(4/3),x,method=_RETURNVERBOSE)

[Out] -3/d*b*(1/2*(1/3/b^(1/3)*ln((b*coth(d*x+c))^(1/3)-b^(1/3))-1/6/b^(1/3)*ln(b
^(2/3)+b^(1/3)*(b*coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3))+1/3*3^(1/2)/b^(
1/3)*arctan(1/3*(1+2*(b*coth(d*x+c))^(1/3)/b^(1/3))*3^(1/2)))/b^2+1/b^2/(b*
coth(d*x+c))^(1/3)+1/2*(-1/3/b^(1/3)*ln((b*coth(d*x+c))^(1/3)+b^(1/3))+1/6/
b^(1/3)*ln(b^(2/3)-b^(1/3)*(b*coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3))+1/3
*3^(1/2)/b^(1/3)*arctan(1/3*3^(1/2)*(2*(b*coth(d*x+c))^(1/3)/b^(1/3)-1)))/b
^2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*coth(d*x+c))^(4/3),x, algorithm="maxima")

[Out] integrate((b*coth(d*x + c))^(-4/3), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 593 vs. $2(184) = 368$.
time = 0.57, size = 3348, normalized size = 14.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh
(d*x + c)^2 + b)*sqrt((-b)^(1/3)/b)*log(3*b*cosh(d*x + c)^2 + 6*b*cosh(d*x
+ c)*sinh(d*x + c) + 3*b*sinh(d*x + c)^2 - 3*(cosh(d*x + c)^2 + 2*cosh(d*x
+ c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3)*(b*cosh(d*x + c)/sinh(
d*x + c))^(1/3) - sqrt(3)*(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x +
c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3)
+ (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2
- b)*(-b)^(1/3) - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*
sinh(d*x + c)^2 - b)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))*sqrt((-b)^(1/3)
/b) + b) + sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b
*sinh(d*x + c)^2 + b)*sqrt(-1/b^(2/3))*log(-(2*sqrt(3)*(cosh(d*x + c)^2 + 2
*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*b^(2/3)*(b*cosh(d*x + c
)/sinh(d*x + c))^(2/3)*sqrt(-1/b^(2/3)) - b*cosh(d*x + c)^2 - 2*b*cosh(d*x
+ c)*sinh(d*x + c) - b*sinh(d*x + c)^2 - sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*c
osh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*b^(1/3)*sqrt(-1/b^(2/3)
) + (sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(
d*x + c)^2 - b)*sqrt(-1/b^(2/3)) + 3*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sin
h(d*x + c) + sinh(d*x + c)^2 - 1)*b^(2/3))*(b*cosh(d*x + c)/sinh(d*x + c))^
(1/3) - 3*b)/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
c)^2)) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2
 + 1)*(-b)^(2/3)*log((-b)^(2/3) - (-b)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c)
)^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) - (cosh(d*x + c)^2 + 2*cos
h(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*b^(2/3)*log(b^(2/3) - b^(1/
3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^
(2/3)) - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2 + 1)*(-b)^(2/3)*log((-b)^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3))
+ 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)
*b^(2/3)*log(b^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) - 12*(cosh(d*
x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(b*cosh(d*x
 + c)/sinh(d*x + c))^(2/3))/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x + c)*
sinh(d*x + c) + b^2*d*sinh(d*x + c)^2 + b^2*d), -1/4*(2*sqrt(3)*(b*cosh(d*x
 + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*sqrt(-(-
b)^(1/3)/b)*arctan(-1/3*sqrt(3)*(-b)^(1/3)*sqrt(-(-b)^(1/3)/b) + 2/3*sqrt(3
)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)*sqrt(-(-b)^(1/3)/b)) - sqrt(3)*(b*c
osh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*s
qrt(-1/b^(2/3))*log(-(2*sqrt(3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x
```

```
 + c) + sinh(d*x + c)^2 - 1)*b^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3)*
sqrt(-1/b^(2/3)) - b*cosh(d*x + c)^2 - 2*b*cosh(d*x + c)*sinh(d*x + c) - b*
sinh(d*x + c)^2 - sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x +
 c) + b*sinh(d*x + c)^2 - b)*b^(1/3)*sqrt(-1/b^(2/3)) + (sqrt(3)*(b*cosh(d*
x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*sqrt(-1
/b^(2/3)) + 3*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
 c)^2 - 1)*b^(2/3))*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) - 3*b)/(cosh(d*x
+ c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - (cosh(d*x + c)
^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(2/3)*log((-
b)^(2/3) - (-b)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x +
 c)/sinh(d*x + c))^(2/3)) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c
) + sinh(d*x + c)^2 + 1)*b^(2/3)*log(b^(2/3) - b^(1/3)*(b*cosh(d*x + c)/sin
h(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + 2*(cosh(d*x +
c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(2/3)*log(
(-b)^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) - 2*(cosh(d*x + c)^2 +
2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*b^(2/3)*log(b^(1/3) +
(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) + 12*(cosh(d*x + c)^2 + 2*cosh(d*x +
 c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(b*cosh(d*x + c)/sinh(d*x + c))^(2
/3))/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x + c)*sinh(d*x + c) + b^2*d*s
inh(d*x + c)^2 + b^2*d), 1/4*(sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c
)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*sqrt((-b)^(1/3)/b)*log(3*b*cosh(d*
x + c)^2 + 6*b*cosh(d*x + c)*sinh(d*x + c) + 3*b*sinh(d*x + c)^2 - 3*(cosh(
d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3
)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) - sqrt(3)*(2*(cosh(d*x + c)^2 + 2*c
osh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3)*(b*cosh(d*x +
c)/sinh(d*x + c))^(2/3) + (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x +
 c) + b*sinh(d*x + c)^2 - b)*(-b)^(1/3) - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x
 + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*(b*cosh(d*x + c)/sinh(d*x + c)
)^(1/3))*sqrt((-b)^(1/3)/b) + b) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(
d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(2/3)*log((-b)^(2/3) - (-b)^(1/3)*(b*c
osh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\coth(c+dx))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))**(4/3),x)
```

```
[Out] Integral((b*coth(c + d*x))**(-4/3), x)
```

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Minimal poly. in rootof must be fract
ion free Error: Bad Argument ValueMinimal poly. in rootof must be fraction
free E
```

Mupad [B]
time = 1.42, size = 165, normalized size = 0.69

$$-\frac{3}{b\,d\,{\left(b\,\mathrm{coth}\left(c+d\,x\right)\right)}^{1/3}}-\frac{\mathrm{atan}\left(\frac{{\left(b\,\mathrm{coth}\left(c+d\,x\right)\right)}^{1/3}\,1{}\mathrm{i}}{b^{1/3}}\right)\,1{}\mathrm{i}}{b^{4/3}\,d}-\frac{\mathrm{atan}\left(\frac{b^9\,d^4\,{\left(b\,\mathrm{coth}\left(c+d\,x\right)\right)}^{1/3}\,486{}\mathrm{i}}{243\,b^{28/3}\,d^4-\sqrt{3}\,b^{28/3}\,d^4\,243{}\mathrm{i}}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{2\,b^{4/3}\,d}+\frac{\mathrm{atan}\left(\frac{b^9\,d^4\,{\left(b\,\mathrm{coth}\left(c+d\,x\right)\right)}^{1/3}\,486{}\mathrm{i}}{243\,b^{28/3}\,d^4+\sqrt{3}\,b^{28/3}\,d^4\,243{}\mathrm{i}}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{2\,b^{4/3}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x))^(4/3),x)
```

```
[Out] (atan((b^9*d^4*(b*coth(c + d*x))^(1/3)*486i)/(243*b^(28/3)*d^4 + 3^(1/2)*b^
(28/3)*d^4*243i))*(3^(1/2)*1i - 1)*1i)/(2*b^(4/3)*d) - (atan(((b*coth(c + d
*x))^(1/3)*1i)/b^(1/3))*1i)/(b^(4/3)*d) - (atan((b^9*d^4*(b*coth(c + d*x))^
(1/3)*486i)/(243*b^(28/3)*d^4 - 3^(1/2)*b^(28/3)*d^4*243i))*(3^(1/2)*1i + 1
)*1i)/(2*b^(4/3)*d) - 3/(b*d*(b*coth(c + d*x))^(1/3))
```

3.15 $\int \coth^n(a+bx)\,dx$

Optimal. Leaf size=43

$$\frac{\coth^{1+n}(a+bx)\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};\coth^2(a+bx)\right)}{b(1+n)}$$

```
[Out] coth(b*x+a)^(1+n)*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],coth(b*x+a)^2)/b/(1+
n)
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3557, 371}

$$\frac{\coth^{n+1}(a+bx)\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\coth^2(a+bx)\right)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*x]^n,x]
```

```
[Out] (Coth[a + b*x]^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Coth[a +
b*x]^2])/(b*(1 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \coth^n(a+bx)\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{x^n}{-1+x^2}\,dx,x,\coth(a+bx)\right)}{b}\\ &= \frac{\coth^{1+n}(a+bx)\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};\coth^2(a+bx)\right)}{b(1+n)}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 45, normalized size = 1.05

$$\frac{\coth^{1+n}(a+bx)\,{}_2F_1\left(1,\frac{1+n}{2};1+\frac{1+n}{2};\coth^2(a+bx)\right)}{b(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*x]^n,x]
```

```
[Out] (Coth[a + b*x]^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, 1 + (1 + n)/2, Coth[
a + b*x]^2])/(b*(1 + n))
```

Maple [F]
time = 1.83, size = 0, normalized size = 0.00

$$\int \coth^{n}\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(b*x+a)^n,x)
```

```
[Out] int(coth(b*x+a)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] integrate(coth(b*x + a)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] integral(coth(b*x + a)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \coth^{n}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)**n,x)
```

```
[Out] Integral(coth(a + b*x)**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^n,x, algorithm="giac")
```

```
[Out] integrate(coth(b*x + a)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \coth(a+b\,x)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + b*x)^n,x)
```

```
[Out] int(coth(a + b*x)^n, x)
```

3.16 $\int (b\coth(c+dx))^n\,dx$

Optimal. Leaf size=48

$$\frac{(b\coth(c+dx))^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};\coth^2(c+dx)\right)}{bd(1+n)}$$

```
[Out] (b*coth(d*x+c))^(1+n)*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],coth(d*x+c)^2)/b
/d/(1+n)
```

Rubi [A]
time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3557, 371}

$$\frac{(b\coth(c+dx))^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\coth^2(c+dx)\right)}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x])^n,x]
```

```
[Out] ((b*Coth[c + d*x])^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Coth[
c + d*x]^2])/(b*d*(1 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int (b\coth(c+dx))^n\,dx &= -\frac{b\text{Subst}\left(\int \frac{x^n}{-b^2+x^2}\,dx,x,b\coth(c+dx)\right)}{d}\\ &= \frac{(b\coth(c+dx))^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};\coth^2(c+dx)\right)}{bd(1+n)}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 51, normalized size = 1.06

$$\frac{\coth(c+dx)(b\coth(c+dx))^n\,{}_2F_1\left(1,\frac{1+n}{2};1+\frac{1+n}{2};\coth^2(c+dx)\right)}{d(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x])^n,x]
[Out] (Coth[c + d*x]*(b*Coth[c + d*x])^n*Hypergeometric2F1[1, (1 + n)/2, 1 + (1 +
 n)/2, Coth[c + d*x]^2])/(d*(1 + n))
```

Maple [F]
time = 1.71, size = 0, normalized size = 0.00

$$\int (b\coth(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c))^n,x)
[Out] int((b*coth(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*coth(d*x + c))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\coth(c+dx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))**n,x)
```

```
[Out] Integral((b*coth(c + d*x))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int (b\coth(c+d\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x))^n,x)
```

```
[Out] int((b*coth(c + d*x))^n, x)
```

3.17 $\int \left(b \coth^2(c+dx)\right)^n \, dx$

Optimal. Leaf size=57

$$\frac{\coth(c+dx)\left(b\coth^2(c+dx)\right)^n {}_2F_1\left(1,\frac{1}{2}(1+2n);\frac{1}{2}(3+2n);\coth^2(c+dx)\right)}{d(1+2n)}$$

```
[Out] coth(d*x+c)*(b*coth(d*x+c)^2)^n*hypergeom([1, 1/2+n],[3/2+n],coth(d*x+c)^2)
/d/(1+2*n)
```

Rubi [A]
time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3739, 3557, 371}

$$\frac{\coth(c+dx)\left(b\coth^2(c+dx)\right)^n {}_2F_1\left(1,\frac{1}{2}(2n+1);\frac{1}{2}(2n+3);\coth^2(c+dx)\right)}{d(2n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^2)^n,x]
```

```
[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^2)^n*Hypergeometric2F1[1, (1 + 2*n)/2, (3 +
 2*n)/2, Coth[c + d*x]^2])/(d*(1 + 2*n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(b\coth^2(c+dx)\right)^n\,dx = \left(\coth^{-2n}(c+dx)\left(b\coth^2(c+dx)\right)^n\right)\int \coth^{2n}(c+dx)\,dx$$
$$= -\frac{\left(\coth^{-2n}(c+dx)\left(b\coth^2(c+dx)\right)^n\right)\text{Subst}\left(\int \frac{x^{2n}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d}$$
$$= \frac{\coth(c+dx)\left(b\coth^2(c+dx)\right)^n {}_2F_1\left(1,\frac{1}{2}(1+2n);\frac{1}{2}(3+2n);\coth^2(c+dx)\right)}{d(1+2n)}$$

Mathematica [A]
time = 0.04, size = 47, normalized size = 0.82

$$\frac{\coth(c+dx)\left(b\coth^2(c+dx)\right)^n {}_2F_1\left(1,\frac{1}{2}+n;\frac{3}{2}+n;\coth^2(c+dx)\right)}{d+2dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^2)^n,x]
```

```
[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^2)^n*Hypergeometric2F1[1, 1/2 + n, 3/2 + n,
 Coth[c + d*x]^2])/(d + 2*d*n)
```

Maple [F]
time = 2.26, size = 0, normalized size = 0.00

$$\int \left(b\left(\coth^2\left(dx+c\right)\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^2)^n,x)
```

```
[Out] int((b*coth(d*x+c)^2)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^2)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^n,x, algorithm="fricas")
```

```
[Out] integral((b*coth(d*x + c)^2)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\coth^2\left(c+dx\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**2)**n,x)
```

```
[Out] Integral((b*coth(c + d*x)**2)**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^n,x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^2)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(b\,{\mathrm{coth}\left(c+d\,x\right)}^2\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^2)^n,x)
```

```
[Out] int((b*coth(c + d*x)^2)^n, x)
```

3.18 $\int \left(b\coth^2(c+dx)\right)^{3/2}\,dx$

Optimal. Leaf size=61

$$-\frac{b\coth(c+dx)\sqrt{b\coth^2(c+dx)}}{2d}+\frac{b\sqrt{b\coth^2(c+dx)}\,\log(\sinh(c+dx))\tanh(c+dx)}{d}$$

```
[Out] -1/2*b*coth(d*x+c)*(b*coth(d*x+c)^2)^(1/2)/d+b*ln(sinh(d*x+c))*(b*coth(d*x+
c)^2)^(1/2)*tanh(d*x+c)/d
```

Rubi [A]
time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3739, 3554, 3556}

$$\frac{b\tanh(c+dx)\sqrt{b\coth^2(c+dx)}\,\log(\sinh(c+dx))}{d}-\frac{b\coth(c+dx)\sqrt{b\coth^2(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^2)^(3/2),x]
```

```
[Out] -1/2*(b*Coth[c + d*x]*Sqrt[b*Coth[c + d*x]^2])/d + (b*Sqrt[b*Coth[c + d*x]^
2]*Log[Sinh[c + d*x]]*Tanh[c + d*x])/d
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(b\coth^2(c+dx)\right)^{3/2}\,dx = \left(b\sqrt{b\coth^2(c+dx)}\,\tanh(c+dx)\right)\int \coth^3(c+dx)\,dx$$
$$= -\frac{b\coth(c+dx)\sqrt{b\coth^2(c+dx)}}{2d} + \left(b\sqrt{b\coth^2(c+dx)}\,\tanh(c+dx)\right)\int$$
$$= -\frac{b\coth(c+dx)\sqrt{b\coth^2(c+dx)}}{2d} + \frac{b\sqrt{b\coth^2(c+dx)}\,\log(\sinh(c+dx))\,\mathrm{ta}}{d}$$

Mathematica [A]
time = 0.09, size = 56, normalized size = 0.92

$$-\frac{\left(b\coth^2(c+dx)\right)^{3/2}\left(\coth^2(c+dx)-2\log(\cosh(c+dx))-2\log(\tanh(c+dx))\right)\tanh^3(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^2)^(3/2),x]
```

```
[Out] -1/2*((b*Coth[c + d*x]^2)^(3/2)*(Coth[c + d*x]^2 - 2*Log[Cosh[c + d*x]] - 2
*Log[Tanh[c + d*x]])*Tanh[c + d*x]^3)/d
```

Maple [A]
time = 1.41, size = 53, normalized size = 0.87

method	result
derivativedivides	$-\frac{\left(b\left(\coth^2(dx+c)\right)\right)^{\frac{3}{2}}\left(\coth^2(dx+c)+\ln(\coth(dx+c)-1)+\ln(\coth(dx+c)+1)\right)}{2d\coth(dx+c)^3}$
default	$-\frac{\left(b\left(\coth^2(dx+c)\right)\right)^{\frac{3}{2}}\left(\coth^2(dx+c)+\ln(\coth(dx+c)-1)+\ln(\coth(dx+c)+1)\right)}{2d\coth(dx+c)^3}$
risch	$\frac{b\left(\mathrm{e}^{2dx+2c}-1\right)\sqrt{\frac{b\left(1+\mathrm{e}^{2dx+2c}\right)^2}{\left(\mathrm{e}^{2dx+2c}-1\right)^2}}\,x}{1+\mathrm{e}^{2dx+2c}} - \frac{2b\left(\mathrm{e}^{2dx+2c}-1\right)\sqrt{\frac{b\left(1+\mathrm{e}^{2dx+2c}\right)^2}{\left(\mathrm{e}^{2dx+2c}-1\right)^2}}\,(dx+c)}{\left(1+\mathrm{e}^{2dx+2c}\right)d} - \frac{2b\sqrt{\frac{b\left(1+\mathrm{e}^{2dx+2c}\right)^2}{\left(\mathrm{e}^{2dx+2c}-1\right)^2}}\,\mathrm{e}^{2dx+2}}{\left(1+\mathrm{e}^{2dx+2c}\right)\left(\mathrm{e}^{2dx+2c}-1\right)d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(b*coth(d*x+c)^2)^(3/2)*(coth(d*x+c)^2+ln(coth(d*x+c)-1)+ln(coth(d*x
+c)+1))/coth(d*x+c)^3
```

Maxima [A]
time = 0.50, size = 97, normalized size = 1.59

$$-\frac{(dx+c)b^{\frac{3}{2}}}{d} - \frac{b^{\frac{3}{2}}\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{b^{\frac{3}{2}}\log\left(e^{(-dx-c)}-1\right)}{d} - \frac{2\,b^{\frac{3}{2}}e^{(-2\,dx-2\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -(d*x + c)*b^(3/2)/d - b^(3/2)*log(e^(-d*x - c) + 1)/d - b^(3/2)*log(e^(-d*
x - c) - 1)/d - 2*b^(3/2)*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d
*x - 4*c) - 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 823 vs. $2(55) = 110$.
time = 0.51, size = 823, normalized size = 13.49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (b*d*x*cosh(d*x + c)^4 - (b*d*x*e^(2*d*x + 2*c) - b*d*x)*sinh(d*x + c)^4 -
4*(b*d*x*cosh(d*x + c)*e^(2*d*x + 2*c) - b*d*x*cosh(d*x + c))*sinh(d*x + c)
^3 + b*d*x - 2*(b*d*x - b)*cosh(d*x + c)^2 + 2*(3*b*d*x*cosh(d*x + c)^2 - b
*d*x - (3*b*d*x*cosh(d*x + c)^2 - b*d*x + b)*e^(2*d*x + 2*c) + b)*sinh(d*x
+ c)^2 - (b*d*x*cosh(d*x + c)^4 + b*d*x - 2*(b*d*x - b)*cosh(d*x + c)^2)*e^
(2*d*x + 2*c) - (b*cosh(d*x + c)^4 - (b*e^(2*d*x + 2*c) - b)*sinh(d*x + c)^
4 - 4*(b*cosh(d*x + c)*e^(2*d*x + 2*c) - b*cosh(d*x + c))*sinh(d*x + c)^3 -
 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - (3*b*cosh(d*x + c)^2 - b)*e
^(2*d*x + 2*c) - b)*sinh(d*x + c)^2 - (b*cosh(d*x + c)^4 - 2*b*cosh(d*x + c
)^2 + b)*e^(2*d*x + 2*c) + 4*(b*cosh(d*x + c)^3 - b*cosh(d*x + c) - (b*cosh
(d*x + c)^3 - b*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c) + b)*log(2*si
nh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(b*d*x*cosh(d*x + c)^3 - (
b*d*x - b)*cosh(d*x + c) - (b*d*x*cosh(d*x + c)^3 - (b*d*x - b)*cosh(d*x +
c))*e^(2*d*x + 2*c))*sinh(d*x + c))*sqrt((b*e^(4*d*x + 4*c) + 2*b*e^(2*d*x
+ 2*c) + b)/(e^(4*d*x + 4*c) - 2*e^(2*d*x + 2*c) + 1))/(d*cosh(d*x + c)^4 +
 (d*e^(2*d*x + 2*c) + d)*sinh(d*x + c)^4 + 4*(d*cosh(d*x + c)*e^(2*d*x + 2*
c) + d*cosh(d*x + c))*sinh(d*x + c)^3 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d
*x + c)^2 + (3*d*cosh(d*x + c)^2 - d)*e^(2*d*x + 2*c) - d)*sinh(d*x + c)^2
+ (d*cosh(d*x + c)^4 - 2*d*cosh(d*x + c)^2 + d)*e^(2*d*x + 2*c) + 4*(d*cosh
(d*x + c)^3 - d*cosh(d*x + c) + (d*cosh(d*x + c)^3 - d*cosh(d*x + c))*e^(2*
d*x + 2*c))*sinh(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\coth^2\left(c+dx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**2)**(3/2),x)
```

```
[Out] Integral((b*coth(c + d*x)**2)**(3/2), x)
```

Giac [A]
time = 0.42, size = 90, normalized size = 1.48

$$-\frac{\left((dx+c)\operatorname{sgn}\left(e^{(4\,dx+4\,c)}-1\right)-\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)\operatorname{sgn}\left(e^{(4\,dx+4\,c)}-1\right)+\frac{2\,e^{(2\,dx+2\,c)}\operatorname{sgn}\left(e^{(4\,dx+4\,c)}-1\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^{2}}\right)b^{\frac{3}{2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -((d*x + c)*sgn(e^(4*d*x + 4*c) - 1) - log(abs(e^(2*d*x + 2*c) - 1))*sgn(e^
(4*d*x + 4*c) - 1) + 2*e^(2*d*x + 2*c)*sgn(e^(4*d*x + 4*c) - 1)/(e^(2*d*x +
 2*c) - 1)^2)*b^(3/2)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(b\,{\mathrm{coth}\left(c+d\,x\right)}^2\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^2)^(3/2),x)
```

```
[Out] int((b*coth(c + d*x)^2)^(3/2), x)
```

3.19 $\int \sqrt{b \coth^2(c+dx)}\, dx$

Optimal. Leaf size=31

$$\frac{\sqrt{b\coth^2(c+dx)}\,\log(\sinh(c+dx))\tanh(c+dx)}{d}$$

```
[Out] ln(sinh(d*x+c))*(b*coth(d*x+c)^2)^(1/2)*tanh(d*x+c)/d
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3739, 3556}

$$\frac{\tanh(c+dx)\sqrt{b\coth^2(c+dx)}\,\log(\sinh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Coth[c + d*x]^2],x]
```

```
[Out] (Sqrt[b*Coth[c + d*x]^2]*Log[Sinh[c + d*x]]*Tanh[c + d*x])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \sqrt{b\coth^2(c+dx)}\,dx &= \left(\sqrt{b\coth^2(c+dx)}\,\tanh(c+dx)\right)\int \coth(c+dx)\,dx\\ &= \frac{\sqrt{b\coth^2(c+dx)}\,\log(\sinh(c+dx))\tanh(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 39, normalized size = 1.26

$$\frac{\sqrt{b\coth^2(c+dx)}\,(\log(\cosh(c+dx))+\log(\tanh(c+dx)))\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Coth[c + d*x]^2],x]
```

```
[Out] (Sqrt[b*Coth[c + d*x]^2]*(Log[Cosh[c + d*x]] + Log[Tanh[c + d*x]])*Tanh[c +
 d*x])/d
```

Maple [A]
time = 1.64, size = 45, normalized size = 1.45

method	result
derivativedivides	$-\frac{\sqrt{b\left(\coth^2\left(dx+c\right)\right)}\,(\ln(\coth(dx+c)-1)+\ln(\coth(dx+c)+1))}{2d\coth(dx+c)}$
default	$-\frac{\sqrt{b\left(\coth^2\left(dx+c\right)\right)}\,(\ln(\coth(dx+c)-1)+\ln(\coth(dx+c)+1))}{2d\coth(dx+c)}$
risch	$\frac{\sqrt{\frac{b\left(1+e^{2dx+2c}\right)^2}{\left(e^{2dx+2c}-1\right)^2}}\,\left(e^{2dx+2c}-1\right)x}{1+e^{2dx+2c}}-\frac{2\sqrt{\frac{b\left(1+e^{2dx+2c}\right)^2}{\left(e^{2dx+2c}-1\right)^2}}\,\left(e^{2dx+2c}-1\right)(dx+c)}{\left(1+e^{2dx+2c}\right)d}+\frac{\sqrt{\frac{b\left(1+e^{2dx+2c}\right)^2}{\left(e^{2dx+2c}-1\right)^2}}\,\left(e^{2dx+2c}-\right.}{\left(1+e^{2dx+2c}\right)d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(b*coth(d*x+c)^2)^(1/2)*(ln(coth(d*x+c)-1)+ln(coth(d*x+c)+1))/coth(d
*x+c)
```

Maxima [A]
time = 0.49, size = 54, normalized size = 1.74

$$-\frac{(dx+c)\sqrt{b}}{d}-\frac{\sqrt{b}\,\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\sqrt{b}\,\log\left(e^{(-dx-c)}-1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(d*x + c)*sqrt(b)/d - sqrt(b)*log(e^(-d*x - c) + 1)/d - sqrt(b)*log(e^(-d*
x - c) - 1)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(29) = 58.

time = 0.43, size = 125, normalized size = 4.03

$$-\frac{\left(dxe^{(2\,dx+2\,c)}-dx-\left(e^{(2\,dx+2\,c)}-1\right)\log\left(\frac{2\,\sinh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)\right)\sqrt{\frac{be^{(4\,dx+4\,c)}+2\,be^{(2\,dx+2\,c)}+b}{e^{(4\,dx+4\,c)}-2\,e^{(2\,dx+2\,c)}+1}}}{de^{(2\,dx+2\,c)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(d*x*e^(2*d*x + 2*c) - d*x - (e^(2*d*x + 2*c) - 1)*log(2*sinh(d*x + c)/(co
sh(d*x + c) - sinh(d*x + c))))*sqrt((b*e^(4*d*x + 4*c) + 2*b*e^(2*d*x + 2*c
) + b)/(e^(4*d*x + 4*c) - 2*e^(2*d*x + 2*c) + 1))/(d*e^(2*d*x + 2*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\coth^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(b*coth(c + d*x)**2), x)
```

Giac [A]
time = 0.40, size = 54, normalized size = 1.74

$$-\frac{\left((dx+c)\mathrm{sgn}\left(e^{(4\,dx+4\,c)}-1\right)-\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)\mathrm{sgn}\left(e^{(4\,dx+4\,c)}-1\right)\right)\sqrt{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -((d*x + c)*sgn(e^(4*d*x + 4*c) - 1) - log(abs(e^(2*d*x + 2*c) - 1))*sgn(e^
(4*d*x + 4*c) - 1))*sqrt(b)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int\sqrt{b\,\mathrm{coth}\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^2)^(1/2),x)
```

```
[Out] int((b*coth(c + d*x)^2)^(1/2), x)
```

3.20 $\int \frac{1}{\sqrt{b\coth^2(c+dx)}}\,dx$

Optimal. Leaf size=31

$$\frac{\coth(c+dx)\log(\cosh(c+dx))}{d\sqrt{b\coth^2(c+dx)}}$$

```
[Out] coth(d*x+c)*ln(cosh(d*x+c))/d/(b*coth(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3739, 3556}

$$\frac{\coth(c+dx)\log(\cosh(c+dx))}{d\sqrt{b\coth^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Coth[c + d*x]^2],x]
```

```
[Out] (Coth[c + d*x]*Log[Cosh[c + d*x]])/(d*Sqrt[b*Coth[c + d*x]^2])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{b\coth^2(c+dx)}}\,dx &= \frac{\coth(c+dx)\int \tanh(c+dx)\,dx}{\sqrt{b\coth^2(c+dx)}}\\
&= \frac{\coth(c+dx)\log(\cosh(c+dx))}{d\sqrt{b\coth^2(c+dx)}}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 31, normalized size = 1.00

$$\frac{\coth(c+dx)\log(\cosh(c+dx))}{d\sqrt{b\coth^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Coth[c + d*x]^2],x]
[Out] (Coth[c + d*x]*Log[Cosh[c + d*x]])/(d*Sqrt[b*Coth[c + d*x]^2])
```

Maple [A]
time = 1.57, size = 56, normalized size = 1.81

method	result	siz
derivativedivides	$\frac{\coth(dx+c)(2\ln(\coth(dx+c))-\ln(\coth(dx+c)+1)-\ln(\coth(dx+c)-1))}{2d\sqrt{b\left(\coth^2\left(dx+c\right)\right)}}$	56
default	$\frac{\coth(dx+c)(2\ln(\coth(dx+c))-\ln(\coth(dx+c)+1)-\ln(\coth(dx+c)-1))}{2d\sqrt{b\left(\coth^2\left(dx+c\right)\right)}}$	56
risch	$\frac{(1+e^{2dx+2c})x}{\sqrt{\frac{b(1+e^{2dx+2c})^2}{(e^{2dx+2c}-1)^2}}\,(e^{2dx+2c}-1)}-\frac{2(1+e^{2dx+2c})(dx+c)}{\sqrt{\frac{b(1+e^{2dx+2c})^2}{(e^{2dx+2c}-1)^2}}\,(e^{2dx+2c}-1)d}+\frac{(1+e^{2dx+2c})\ln(1+e^{2dx+2c})}{\sqrt{\frac{b(1+e^{2dx+2c})^2}{(e^{2dx+2c}-1)^2}}\,(e^{2dx+2c}-1)d}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2/d*coth(d*x+c)*(2*ln(coth(d*x+c))-ln(coth(d*x+c)+1)-ln(coth(d*x+c)-1))/(
b*coth(d*x+c)^2)^(1/2)
```

Maxima [A]
time = 0.50, size = 34, normalized size = 1.10

$$-\frac{dx+c}{\sqrt{b}\,d}-\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^2)^(1/2),x, algorithm="maxima")
[Out] -(d*x + c)/(sqrt(b)*d) - log(e^(-2*d*x - 2*c) + 1)/(sqrt(b)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(29) = 58$.
time = 0.43, size = 128, normalized size = 4.13

$$-\frac{\left(dxe^{(2\,dx+2\,c)}-dx-\left(e^{(2\,dx+2\,c)}-1\right)\log\left(\frac{2\,\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)\right)\sqrt{\frac{be^{(4\,dx+4\,c)}+2\,be^{(2\,dx+2\,c)}+b}{e^{(4\,dx+4\,c)}-2\,e^{(2\,dx+2\,c)}+1}}}{bde^{(2\,dx+2\,c)}+bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(d*x*e^(2*d*x + 2*c) - d*x - (e^(2*d*x + 2*c) - 1)*log(2*cosh(d*x + c)/(co
sh(d*x + c) - sinh(d*x + c))))*sqrt((b*e^(4*d*x + 4*c) + 2*b*e^(2*d*x + 2*c
) + b)/(e^(4*d*x + 4*c) - 2*e^(2*d*x + 2*c) + 1))/(b*d*e^(2*d*x + 2*c) + b*
d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b \coth^2{(c + dx)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*coth(c + d*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(29) = 58.
time = 0.42, size = 60, normalized size = 1.94

$$-\frac{\frac{dx+c}{\sqrt{b}\,\mathrm{sgn}\left(e^{(4\,dx+4\,c)}-1\right)} - \frac{\log\left(e^{(2\,dx+2\,c)}+1\right)}{\sqrt{b}\,\mathrm{sgn}\left(e^{(4\,dx+4\,c)}-1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -((d*x + c)/(sqrt(b)*sgn(e^(4*d*x + 4*c) - 1)) - log(e^(2*d*x + 2*c) + 1)/(
sqrt(b)*sgn(e^(4*d*x + 4*c) - 1)))/d
```

Mupad [B]
time = 1.26, size = 30, normalized size = 0.97

$$\frac{\mathrm{atanh}\left(\frac{\sqrt{b}\,\mathrm{coth}(c+d\,x)}{\sqrt{b\,{\mathrm{coth}\left(c+d\,x\right)}^2}}\right)}{\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x)^2)^(1/2),x)
```

```
[Out] atanh((b^(1/2)*coth(c + d*x))/(b*coth(c + d*x)^2)^(1/2))/(b^(1/2)*d)
```

3.21 $\int \frac{1}{\left(b\coth^2(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=65

$$\frac{\coth(c+dx)\log(\cosh(c+dx))}{bd\sqrt{b\coth^2(c+dx)}}-\frac{\tanh(c+dx)}{2bd\sqrt{b\coth^2(c+dx)}}$$

```
[Out] coth(d*x+c)*ln(cosh(d*x+c))/b/d/(b*coth(d*x+c)^2)^(1/2)-1/2*tanh(d*x+c)/b/d
/(b*coth(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3739, 3554, 3556}

$$\frac{\coth(c+dx)\log(\cosh(c+dx))}{bd\sqrt{b\coth^2(c+dx)}}-\frac{\tanh(c+dx)}{2bd\sqrt{b\coth^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^2)^(-3/2),x]
```

```
[Out] (Coth[c + d*x]*Log[Cosh[c + d*x]])/(b*d*Sqrt[b*Coth[c + d*x]^2]) - Tanh[c +
 d*x]/(2*b*d*Sqrt[b*Coth[c + d*x]^2])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{3/2}}\,dx = \frac{\coth(c+dx)\int \tanh^3(c+dx)\,dx}{b\sqrt{b\coth^2(c+dx)}}$$
$$= -\frac{\tanh(c+dx)}{2bd\sqrt{b\coth^2(c+dx)}} + \frac{\coth(c+dx)\int \tanh(c+dx)\,dx}{b\sqrt{b\coth^2(c+dx)}}$$
$$= \frac{\coth(c+dx)\log(\cosh(c+dx))}{bd\sqrt{b\coth^2(c+dx)}} - \frac{\tanh(c+dx)}{2bd\sqrt{b\coth^2(c+dx)}}$$

Mathematica [**A**]
time = 0.10, size = 48, normalized size = 0.74

$$\frac{2\coth(c+dx)\log(\cosh(c+dx)) - \tanh(c+dx)}{2bd\sqrt{b\coth^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^2)^(-3/2),x]
```

```
[Out] (2*Coth[c + d*x]*Log[Cosh[c + d*x]] - Tanh[c + d*x])/(2*b*d*Sqrt[b*Coth[c + 
 d*x]^2])
```

Maple [**A**]
time = 1.41, size = 79, normalized size = 1.22

method	result
derivativedivides	$-\frac{\coth(dx+c)\left(\ln(\coth(dx+c)+1)\left(\coth^2(dx+c)\right)+\ln(\coth(dx+c)-1)\left(\coth^2(dx+c)\right)-2\ln(\coth(dx+c))\left(\coth^2(dx+c)\right)+}{2d\left(b\left(\coth^2(dx+c)\right)\right)^{\frac{3}{2}}}$
default	$-\frac{\coth(dx+c)\left(\ln(\coth(dx+c)+1)\left(\coth^2(dx+c)\right)+\ln(\coth(dx+c)-1)\left(\coth^2(dx+c)\right)-2\ln(\coth(dx+c))\left(\coth^2(dx+c)\right)+}{2d\left(b\left(\coth^2(dx+c)\right)\right)^{\frac{3}{2}}}$
risch	$\frac{\left(1+e^{2dx+2c}\right)x}{b\left(e^{2dx+2c}-1\right)\sqrt{\frac{b\left(1+e^{2dx+2c}\right)^2}{\left(e^{2dx+2c}-1\right)^2}}} - \frac{2\left(1+e^{2dx+2c}\right)(dx+c)}{b\left(e^{2dx+2c}-1\right)\sqrt{\frac{b\left(1+e^{2dx+2c}\right)^2}{\left(e^{2dx+2c}-1\right)^2}}\,d} + \frac{2\,e^{2dx+2c}}{b\left(1+e^{2dx+2c}\right)\left(e^{2dx+2c}-1\right)\sqrt{\frac{b(1+e^{2c}}{(e^{2dx+}}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*coth(d*x+c)*(ln(coth(d*x+c)+1)*coth(d*x+c)^2+ln(coth(d*x+c)-1)*coth(
d*x+c)^2-2*ln(coth(d*x+c))*coth(d*x+c)^2+1)/(b*coth(d*x+c)^2)^(3/2)
```

Maxima [A]
time = 0.49, size = 84, normalized size = 1.29

$$-\frac{2\sqrt{b}e^{(-2\,dx-2\,c)}}{(2\,b^2e^{(-2\,dx-2\,c)}+b^2e^{(-4\,dx-4\,c)}+b^2)d}-\frac{dx+c}{b^{\frac{3}{2}}d}-\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{b^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(b)*e^(-2*d*x - 2*c)/((2*b^2*e^(-2*d*x - 2*c) + b^2*e^(-4*d*x - 4*c)
 + b^2)*d) - (d*x + c)/(b^(3/2)*d) - log(e^(-2*d*x - 2*c) + 1)/(b^(3/2)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 817 vs. $2(59) = 118$.
time = 0.46, size = 817, normalized size = 12.57

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (d*x*cosh(d*x + c)^4 - (d*x*e^(2*d*x + 2*c) - d*x)*sinh(d*x + c)^4 - 4*(d*x
*cosh(d*x + c)*e^(2*d*x + 2*c) - d*x*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(d*
x - 1)*cosh(d*x + c)^2 + 2*(3*d*x*cosh(d*x + c)^2 + d*x - (3*d*x*cosh(d*x +
 c)^2 + d*x - 1)*e^(2*d*x + 2*c) - 1)*sinh(d*x + c)^2 + d*x - (d*x*cosh(d*x
 + c)^4 + 2*(d*x - 1)*cosh(d*x + c)^2 + d*x)*e^(2*d*x + 2*c) + ((e^(2*d*x +
 2*c) - 1)*sinh(d*x + c)^4 - cosh(d*x + c)^4 + 4*(cosh(d*x + c)*e^(2*d*x +
2*c) - cosh(d*x + c))*sinh(d*x + c)^3 - 2*(3*cosh(d*x + c)^2 - (3*cosh(d*x
+ c)^2 + 1)*e^(2*d*x + 2*c) + 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + (cos
h(d*x + c)^4 + 2*cosh(d*x + c)^2 + 1)*e^(2*d*x + 2*c) - 4*(cosh(d*x + c)^3
- (cosh(d*x + c)^3 + cosh(d*x + c))*e^(2*d*x + 2*c) + cosh(d*x + c))*sinh(d
*x + c) - 1)*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(d*x*
cosh(d*x + c)^3 + (d*x - 1)*cosh(d*x + c) - (d*x*cosh(d*x + c)^3 + (d*x - 1
)*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c))*sqrt((b*e^(4*d*x + 4*c) +
2*b*e^(2*d*x + 2*c) + b)/(e^(4*d*x + 4*c) - 2*e^(2*d*x + 2*c) + 1))/(b^2*d*
cosh(d*x + c)^4 + 2*b^2*d*cosh(d*x + c)^2 + (b^2*d*e^(2*d*x + 2*c) + b^2*d)
*sinh(d*x + c)^4 + 4*(b^2*d*cosh(d*x + c)*e^(2*d*x + 2*c) + b^2*d*cosh(d*x
+ c))*sinh(d*x + c)^3 + b^2*d + 2*(3*b^2*d*cosh(d*x + c)^2 + b^2*d + (3*b^2
*d*cosh(d*x + c)^2 + b^2*d)*e^(2*d*x + 2*c))*sinh(d*x + c)^2 + (b^2*d*cosh(
d*x + c)^4 + 2*b^2*d*cosh(d*x + c)^2 + b^2*d)*e^(2*d*x + 2*c) + 4*(b^2*d*co
sh(d*x + c)^3 + b^2*d*cosh(d*x + c) + (b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d
*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(b\coth^2\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**2)**(3/2),x)
```

```
[Out] Integral((b*coth(c + d*x)**2)**(-3/2), x)
```

Giac [A]
time = 0.42, size = 104, normalized size = 1.60

$$-\frac{\frac{dx+c}{\sqrt{b}\,\mathrm{sgn}\left(e^{(4\,dx+4\,c)}-1\right)}-\frac{\log\left(e^{(2\,dx+2\,c)}+1\right)}{\sqrt{b}\,\mathrm{sgn}\left(e^{(4\,dx+4\,c)}-1\right)}-\frac{2\,e^{(2\,dx+2\,c)}}{\sqrt{b}\,\left(e^{(2\,dx+2\,c)}+1\right)^{2}\mathrm{sgn}\left(e^{(4\,dx+4\,c)}-1\right)}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -((d*x + c)/(sqrt(b)*sgn(e^(4*d*x + 4*c) - 1)) - log(e^(2*d*x + 2*c) + 1)/(
sqrt(b)*sgn(e^(4*d*x + 4*c) - 1)) - 2*e^(2*d*x + 2*c)/(sqrt(b)*(e^(2*d*x +
2*c) + 1)^2*sgn(e^(4*d*x + 4*c) - 1)))/(b*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(b\,{\mathrm{coth}\left(c+d\,x\right)}^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x)^2)^(3/2),x)
```

```
[Out] int(1/(b*coth(c + d*x)^2)^(3/2), x)
```

3.22 $\int \left(b \coth^2(c+dx)\right)^{4/3} dx$

Optimal. Leaf size=297

$$\frac{\sqrt{3}\, b \text{ArcTan}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^2(c+dx)}}{2d\coth^{\frac{2}{3}}(c+dx)}-\frac{\sqrt{3}\, b \text{ArcTan}\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^2(c+a}}{2d\coth^{\frac{2}{3}}(c+dx)}$$

```
[Out] b*arctanh(coth(d*x+c)^(1/3))*(b*coth(d*x+c)^2)^(1/3)/d/coth(d*x+c)^(2/3)-3/
5*b*coth(d*x+c)*(b*coth(d*x+c)^2)^(1/3)/d-1/4*b*(b*coth(d*x+c)^2)^(1/3)*ln(
1-coth(d*x+c)^(1/3)+coth(d*x+c)^(2/3))/d/coth(d*x+c)^(2/3)+1/4*b*(b*coth(d*
x+c)^2)^(1/3)*ln(1+coth(d*x+c)^(1/3)+coth(d*x+c)^(2/3))/d/coth(d*x+c)^(2/3)
+1/2*b*arctan(1/3*(1-2*coth(d*x+c)^(1/3))*3^(1/2))*(b*coth(d*x+c)^2)^(1/3)*
3^(1/2)/d/coth(d*x+c)^(2/3)-1/2*b*arctan(1/3*(1+2*coth(d*x+c)^(1/3))*3^(1/2
))*(b*coth(d*x+c)^2)^(1/3)*3^(1/2)/d/coth(d*x+c)^(2/3)
```

Rubi [A]
time = 0.16, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3739, 3554, 3557, 335, 302, 648, 632, 210, 642, 212}

$$\frac{\sqrt{3}\, b \text{ArcTan}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^2(c+dx)}}{2d\coth^{\frac{2}{3}}(c+dx)}-\frac{\sqrt{3}\, b \text{ArcTan}\left(\frac{2\sqrt[3]{\coth(c+dx)}+1}{\sqrt{3}}\right)\sqrt[3]{b\coth^2(c+dx)}}{2d\coth^{\frac{2}{3}}(c+dx)}-\frac{3b\coth(c+dx)\sqrt[3]{b\coth^2(c+dx)}}{5d}-\frac{b\sqrt[3]{b\coth^2(c+dx)}\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\coth^{\frac{2}{3}}(c+dx)}+\frac{b\sqrt[3]{b\coth^2(c+dx)}\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}+1\right)}{4d\coth^{\frac{2}{3}}(c+dx)}+\frac{b\sqrt[3]{b\coth^2(c+dx)}\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)}{d\coth^{\frac{2}{3}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^2)^(4/3),x]
```

```
[Out] (Sqrt[3]*b*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^2)^
(1/3))/(2*d*Coth[c + d*x]^(2/3)) - (Sqrt[3]*b*ArcTan[(1 + 2*Coth[c + d*x]^(
1/3))/Sqrt[3]]*(b*Coth[c + d*x]^2)^(1/3))/(2*d*Coth[c + d*x]^(2/3)) + (b*Ar
cTanh[Coth[c + d*x]^(1/3)]*(b*Coth[c + d*x]^2)^(1/3))/(d*Coth[c + d*x]^(2/3
)) - (3*b*Coth[c + d*x]*(b*Coth[c + d*x]^2)^(1/3))/(5*d) - (b*(b*Coth[c + d
*x]^2)^(1/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*Coth[
c + d*x]^(2/3)) + (b*(b*Coth[c + d*x]^2)^(1/3)*Log[1 + Coth[c + d*x]^(1/3)
+ Coth[c + d*x]^(2/3)])/(4*d*Coth[c + d*x]^(2/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(b\coth^2(c+dx)\right)^{4/3}\,dx = \frac{\left(b\sqrt[3]{b\coth^2(c+dx)}\right)\int \coth^{\frac{8}{3}}(c+dx)\,dx}{\coth^{\frac{2}{3}}(c+dx)}$$

$$= -\frac{3b\coth(c+dx)\sqrt[3]{b\coth^2(c+dx)}}{5d} + \frac{\left(b\sqrt[3]{b\coth^2(c+dx)}\right)\int \coth^{\frac{2}{3}}(c+dx)}{\coth^{\frac{2}{3}}(c+dx)}$$

$$= -\frac{3b\coth(c+dx)\sqrt[3]{b\coth^2(c+dx)}}{5d} - \frac{\left(b\sqrt[3]{b\coth^2(c+dx)}\right)\text{Subst}\left(\int \frac{x^{2/3}}{-1+x^2}\,d\right.}{d\coth^{\frac{2}{3}}(c+dx)}$$

$$= -\frac{3b\coth(c+dx)\sqrt[3]{b\coth^2(c+dx)}}{5d} - \frac{\left(3b\sqrt[3]{b\coth^2(c+dx)}\right)\text{Subst}\left(\int \frac{x^4}{-1+x^6}\right.}{d\coth^{\frac{2}{3}}(c+dx)}$$

$$= -\frac{3b\coth(c+dx)\sqrt[3]{b\coth^2(c+dx)}}{5d} + \frac{\left(b\sqrt[3]{b\coth^2(c+dx)}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx\right.}{d\coth^{\frac{2}{3}}(c+dx)}$$

$$= \frac{b\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^2(c+dx)}}{d\coth^{\frac{2}{3}}(c+dx)} - \frac{3b\coth(c+dx)\sqrt[3]{b\coth^2(c+}}{5d}$$

$$= \frac{b\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^2(c+dx)}}{d\coth^{\frac{2}{3}}(c+dx)} - \frac{3b\coth(c+dx)\sqrt[3]{b\coth^2(c+}}{5d}$$

$$= \frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^2(c+dx)}}{2d\coth^{\frac{2}{3}}(c+dx)} - \frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1+2\sqrt[3]{\cot}}{\sqrt{}}\right.}{2d\,c}$$

Mathematica [A]
time = 0.21, size = 166, normalized size = 0.56

$$-\frac{\left(b\coth^2(c+dx)\right)^{4/3}\left(-20\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)+12\coth^{\frac{5}{3}}(c+dx)-5\left(2\sqrt{3}\,\mathrm{ArcTan}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)-2\sqrt{3}\,\mathrm{ArcTan}\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)-\log\left(1-\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)+\log\left(1+\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)\right)\right)}{20d\coth^{\frac{8}{3}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^2)^(4/3),x]
```

```
[Out] -1/20*((b*Coth[c + d*x]^2)^(4/3)*(-20*ArcTanh[Coth[c + d*x]^(1/3)] + 12*Cot
h[c + d*x]^(5/3) - 5*(2*Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]
 - 2*Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]] - Log[1 - Coth[c +
 d*x]^(1/3) + Coth[c + d*x]^(2/3)] + Log[1 + Coth[c + d*x]^(1/3) + Coth[c +
 d*x]^(2/3)])))/(d*Coth[c + d*x]^(8/3))
```

Maple [F]
time = 1.75, size = 0, normalized size = 0.00

$$\int \left(b\left(\coth^2\left(dx+c\right)\right)\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^2)^(4/3),x)
```

```
[Out] int((b*coth(d*x+c)^2)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^2)^(4/3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1994 vs. $2(245) = 490$.
time = 0.39, size = 1994, normalized size = 6.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(4/3),x, algorithm="fricas")
```

```
[Out] -1/20*(10*(sqrt(3)*b*cosh(d*x + c)^2 + 2*sqrt(3)*b*cosh(d*x + c)*sinh(d*x +
 c) + sqrt(3)*b*sinh(d*x + c)^2 - sqrt(3)*b)*(-b)^(1/3)*arctan(1/3*(sqrt(3)
```

```
*b*cosh(d*x + c)^2 + 2*sqrt(3)*b*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*b*si
nh(d*x + c)^2 + 2*(sqrt(3)*cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c)*sinh(d
*x + c) + sqrt(3)*sinh(d*x + c)^2 - sqrt(3))*(-b)^(2/3)*((b*cosh(d*x + c)^2
 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) +
sqrt(3)*b)/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*
x + c)^2 + b)) - 10*(sqrt(3)*b*cosh(d*x + c)^2 + 2*sqrt(3)*b*cosh(d*x + c)*
sinh(d*x + c) + sqrt(3)*b*sinh(d*x + c)^2 - sqrt(3)*b)*b^(1/3)*arctan(-1/3*
(sqrt(3)*b*cosh(d*x + c)^2 + 2*sqrt(3)*b*cosh(d*x + c)*sinh(d*x + c) + sqrt
(3)*b*sinh(d*x + c)^2 - 2*(sqrt(3)*cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c
)*sinh(d*x + c) + sqrt(3)*sinh(d*x + c)^2 - sqrt(3))*b^(2/3)*((b*cosh(d*x +
 c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/
3) + sqrt(3)*b)/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*si
nh(d*x + c)^2 + b)) + 5*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c
) + b*sinh(d*x + c)^2 - b)*(-b)^(1/3)*log(((cosh(d*x + c)^4 + 4*cosh(d*x +
c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x +
 c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x +
c) + 1)*(-b)^(2/3) - (cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6
*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d
*x + c)^4 - 1)*(-b)^(1/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cos
h(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) + (cosh(d*x + c)^4 + 4*cosh(d*x
+ c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x
 + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x
+ c) + 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + s
inh(d*x + c)^2 - 1))^(2/3))/(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c
)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(
d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)) + 5*(b
*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)
*b^(1/3)*log(((cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x
 + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4
*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*b^(2/3) - (cosh(d*x +
 c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2
 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 - 1)*b^(1/3)*((b*cosh(
d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1)
)^(1/3) + (cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c
)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(co
sh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*((b*cosh(d*x + c)^2 + b*s
inh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3))/(cosh(d
*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d
*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + c
osh(d*x + c))*sinh(d*x + c) + 1)) - 10*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x +
c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*(-b)^(1/3)*log(((cosh(d*x + c)^2
+ 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(1/3) + (cosh(d
*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*((b*cosh(d
*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))
^(1/3))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2
```

```
+ 1)) - 10*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*
x + c)^2 - b)*b^(1/3)*log(((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c)
 + sinh(d*x + c)^2 + 1)*b^(1/3) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d
*x + c) + sinh(d*x + c)^2 - 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)
/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3))/(cosh(d*x + c)^2 + 2*cosh(
d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) + 12*(b*cosh(d*x + c)^2 + 2*
b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*((b*cosh(d*x + c)^2
+ b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3))/(d
*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x + c)^2 - d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\coth^2{(c+dx)}\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**2)**(4/3),x)
[Out] Integral((b*coth(c + d*x)**2)**(4/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(4/3),x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^2)^(4/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(b\,{\mathrm{coth}\left(c+d\,x\right)}^2\right)}^{4/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^2)^(4/3),x)
[Out] int((b*coth(c + d*x)^2)^(4/3), x)
```

3.23 $\int \left(b\coth^2(c+dx)\right)^{2/3}\,dx$

Optimal. Leaf size=289

$$-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\left(b\coth^2(c+dx)\right)^{2/3}}{2d\coth^{\frac{4}{3}}(c+dx)}+\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\left(b\coth^2(c+d\right.}{2d\coth^{\frac{4}{3}}(c+dx)}$$

```
[Out] arctanh(coth(d*x+c)^(1/3))*(b*coth(d*x+c)^2)^(2/3)/d/coth(d*x+c)^(4/3)-1/4*
(b*coth(d*x+c)^2)^(2/3)*ln(1-coth(d*x+c)^(1/3)+coth(d*x+c)^(2/3))/d/coth(d*
x+c)^(4/3)+1/4*(b*coth(d*x+c)^2)^(2/3)*ln(1+coth(d*x+c)^(1/3)+coth(d*x+c)^(
2/3))/d/coth(d*x+c)^(4/3)-1/2*arctan(1/3*(1-2*coth(d*x+c)^(1/3))*3^(1/2))*(
b*coth(d*x+c)^2)^(2/3)*3^(1/2)/d/coth(d*x+c)^(4/3)+1/2*arctan(1/3*(1+2*coth
(d*x+c)^(1/3))*3^(1/2))*(b*coth(d*x+c)^2)^(2/3)*3^(1/2)/d/coth(d*x+c)^(4/3)
-3*(b*coth(d*x+c)^2)^(2/3)*tanh(d*x+c)/d
```

Rubi [A]

time = 0.13, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3739, 3554, 3557, 335, 216, 648, 632, 210, 642, 212}

$$-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\left(b\coth^2(c+dx)\right)^{2/3}}{2d\coth^{\frac{4}{3}}(c+dx)}+\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{2\sqrt[3]{\coth(c+dx)}+1}{\sqrt{3}}\right)\left(b\coth^2(c+dx)\right)^{2/3}}{2d\coth^{\frac{4}{3}}(c+dx)}-\frac{3\tanh(c+dx)\left(b\coth^2(c+dx)\right)^{2/3}}{d}-\frac{\left(b\coth^2(c+dx)\right)^{2/3}\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\coth^{\frac{4}{3}}(c+dx)}+\frac{\left(b\coth^2(c+dx)\right)^{2/3}\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}+1\right)}{4d\coth^{\frac{4}{3}}(c+dx)}+\frac{\left(b\coth^2(c+dx)\right)^{2/3}\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)}{d\coth^{\frac{4}{3}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^2)^(2/3),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^
2)^(2/3))/(d*Coth[c + d*x]^(4/3)) + (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1
/3))/Sqrt[3]]*(b*Coth[c + d*x]^2)^(2/3))/(2*d*Coth[c + d*x]^(4/3)) + (ArcTa
nh[Coth[c + d*x]^(1/3)]*(b*Coth[c + d*x]^2)^(2/3))/(d*Coth[c + d*x]^(4/3))
 - ((b*Coth[c + d*x]^2)^(2/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2
/3)])/(4*d*Coth[c + d*x]^(4/3)) + ((b*Coth[c + d*x]^2)^(2/3)*Log[1 + Coth[c
 + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*Coth[c + d*x]^(4/3)) - (3*(b*Cot
h[c + d*x]^2)^(2/3)*Tanh[c + d*x])/d
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(b\coth^2(c+dx)\right)^{2/3}dx = \frac{\left(b\coth^2(c+dx)\right)^{2/3}\int \coth^{\frac{4}{3}}(c+dx)\,dx}{\coth^{\frac{4}{3}}(c+dx)}$$

$$= -\frac{3\left(b\coth^2(c+dx)\right)^{2/3}\tanh(c+dx)}{d} + \frac{\left(b\coth^2(c+dx)\right)^{2/3}\int \frac{1}{\coth^{\frac{2}{3}}(c+dx)}\,dx}{\coth^{\frac{4}{3}}(c+dx)}$$

$$= -\frac{3\left(b\coth^2(c+dx)\right)^{2/3}\tanh(c+dx)}{d} - \frac{\left(b\coth^2(c+dx)\right)^{2/3}\text{Subst}\left(\int \frac{1}{x^{2/3}(-1+x^2}\right.}{d\coth^{\frac{4}{3}}(c+dx)}$$

$$= -\frac{3\left(b\coth^2(c+dx)\right)^{2/3}\tanh(c+dx)}{d} - \frac{\left(3\left(b\coth^2(c+dx)\right)^{2/3}\right)\text{Subst}\left(\int \frac{1}{-1+x^6}\right.}{d\coth^{\frac{4}{3}}(c+dx)}$$

$$= -\frac{3\left(b\coth^2(c+dx)\right)^{2/3}\tanh(c+dx)}{d} + \frac{\left(b\coth^2(c+dx)\right)^{2/3}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x\right.}{d\coth^{\frac{4}{3}}(c+dx)}$$

$$= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\left(b\coth^2(c+dx)\right)^{2/3}}{d\coth^{\frac{4}{3}}(c+dx)} - \frac{3\left(b\coth^2(c+dx)\right)^{2/3}\tanh(c}{d}$$

$$= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\left(b\coth^2(c+dx)\right)^{2/3}}{d\coth^{\frac{4}{3}}(c+dx)} - \frac{\left(b\coth^2(c+dx)\right)^{2/3}\log\left(1-\right.}{4d\,\text{cot}}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\left(b\coth^2(c+dx)\right)^{2/3}}{2d\coth^{\frac{4}{3}}(c+dx)} + \frac{\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{\text{co}}}{}\right.}{2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.03, size = 43, normalized size = 0.15

$$\frac{3\left(b\coth^2(c+dx)\right)^{2/3}\left(-1+{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};\coth^2(c+dx)\right)\right)\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^2)^(2/3),x]
```

```
[Out] (3*(b*Coth[c + d*x]^2)^(2/3)*(-1 + Hypergeometric2F1[1/6, 1, 7/6, Coth[c +
d*x]^2])*Tanh[c + d*x])/d
```

Maple [F]
time = 1.73, size = 0, normalized size = 0.00

$$\int \left(b\left(\coth^2\left(dx+c\right)\right)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^2)^(2/3),x)
```

```
[Out] int((b*coth(d*x+c)^2)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^2)^(2/3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2037 vs. 2(239) = 478.
time = 0.42, size = 2037, normalized size = 7.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*(sqrt(3)*cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c)*sinh(d*x + c) +
sqrt(3)*sinh(d*x + c)^2 + sqrt(3))*(-b^2)^(1/3)*arctan(-1/3*(sqrt(3)*b*cosh
(d*x + c)^2 + 2*sqrt(3)*b*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*b*sinh(d*x
+ c)^2 - 2*(sqrt(3)*cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c)*sinh(d*x + c)
 + sqrt(3)*sinh(d*x + c)^2 - sqrt(3))*(-b^2)^(1/3)*((b*cosh(d*x + c)^2 + b*
```

```
sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) + sqrt(
3)*b)/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c
)^2 + b)) + 2*(sqrt(3)*cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c)*sinh(d*x +
 c) + sqrt(3)*sinh(d*x + c)^2 + sqrt(3))*(b^2)^(1/3)*arctan(-1/3*(sqrt(3)*b
*cosh(d*x + c)^2 + 2*sqrt(3)*b*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*b*sinh
(d*x + c)^2 - 2*(sqrt(3)*cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c)*sinh(d*x
 + c) + sqrt(3)*sinh(d*x + c)^2 - sqrt(3))*(b^2)^(1/3)*((b*cosh(d*x + c)^2
+ b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) + s
qrt(3)*b)/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x
 + c)^2 + b)) + (-b^2)^(1/3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x +
c) + sinh(d*x + c)^2 + 1)*log(((cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*
x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^
3 + sinh(d*x + c)^4 - 1)*(-b^2)^(2/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)
^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) + (b*cosh(d*x + c)^4
 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*b*cosh(d*x + c
)^2 + 2*(3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 -
b*cosh(d*x + c))*sinh(d*x + c) + b)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2
 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3) - (b*cosh(d*x + c)^4 +
 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^
2 + 2*(3*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + b*
cosh(d*x + c))*sinh(d*x + c) + b)*(-b^2)^(1/3))/(cosh(d*x + c)^4 + 4*cosh(d
*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(
d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d
*x + c) + 1)) + (b^2)^(1/3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c
) + sinh(d*x + c)^2 + 1)*log(-((cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*
x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^
3 + sinh(d*x + c)^4 - 1)*(b^2)^(2/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^
2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) - (b*cosh(d*x + c)^4
+ 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*b*cosh(d*x + c)
^2 + 2*(3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - b
*cosh(d*x + c))*sinh(d*x + c) + b)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2
+ b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3) - (b*cosh(d*x + c)^4 +
4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^2
 + 2*(3*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + b*c
osh(d*x + c))*sinh(d*x + c) + b)*(b^2)^(1/3))/(cosh(d*x + c)^4 + 4*cosh(d*x
 + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*
x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x
 + c) + 1)) - 2*(-b^2)^(1/3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x +
c) + sinh(d*x + c)^2 + 1)*log(-((-b^2)^(2/3)*(cosh(d*x + c)^2 + 2*cosh(d*x
+ c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1) - (b*cosh(d*x + c)^2 + 2*b*cosh(d
*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*((b*cosh(d*x + c)^2 + b*sinh
(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3))/(cosh(d*x
+ c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) - 2*(b^2)^(1
/3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)
*log(((b^2)^(2/3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d
```

```
*x + c)^2 + 1) + (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*s
inh(d*x + c)^2 - b)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x
+ c)^2 + sinh(d*x + c)^2 - 1))^(1/3))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*si
nh(d*x + c) + sinh(d*x + c)^2 + 1)) + 12*(cosh(d*x + c)^2 + 2*cosh(d*x + c)
*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)
^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3))/(d*cosh(d*x + c)^2
+ 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x + c)^2 + d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\coth^2\left(c+dx\right)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**2)**(2/3),x)
[Out] Integral((b*coth(c + d*x)**2)**(2/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(2/3),x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^2)^(2/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(b\,{\mathrm{coth}\left(c+d\,x\right)}^2\right)}^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^2)^(2/3),x)
[Out] int((b*coth(c + d*x)^2)^(2/3), x)
```

3.24 $\int \sqrt[3]{b\coth^2(c+dx)}\,dx$

Optimal. Leaf size=264

$$\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^2(c+dx)}}{2d\coth^{\frac{2}{3}}(c+dx)}-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^2(c+dx}}{2d\coth^{\frac{2}{3}}(c+dx)}$$

```
[Out] arctanh(coth(d*x+c)^(1/3))*(b*coth(d*x+c)^2)^(1/3)/d/coth(d*x+c)^(2/3)-1/4*
(b*coth(d*x+c)^2)^(1/3)*ln(1-coth(d*x+c)^(1/3)+coth(d*x+c)^(2/3))/d/coth(d*
x+c)^(2/3)+1/4*(b*coth(d*x+c)^2)^(1/3)*ln(1+coth(d*x+c)^(1/3)+coth(d*x+c)^(
2/3))/d/coth(d*x+c)^(2/3)+1/2*arctan(1/3*(1-2*coth(d*x+c)^(1/3))*3^(1/2))*(
b*coth(d*x+c)^2)^(1/3)*3^(1/2)/d/coth(d*x+c)^(2/3)-1/2*arctan(1/3*(1+2*coth
(d*x+c)^(1/3))*3^(1/2))*(b*coth(d*x+c)^2)^(1/3)*3^(1/2)/d/coth(d*x+c)^(2/3)
```

Rubi [A]
time = 0.16, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {3739, 3557, 335, 302, 648, 632, 210, 642, 212}

$$\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^2(c+dx)}}{2d\coth^{\frac{2}{3}}(c+dx)}-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{2\sqrt[3]{\coth(c+dx)}+1}{\sqrt{3}}\right)\sqrt[3]{b\coth^2(c+dx)}}{2d\coth^{\frac{2}{3}}(c+dx)}-\frac{\sqrt[3]{b\coth^2(c+dx)}\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\coth^{\frac{2}{3}}(c+dx)}+\frac{\sqrt[3]{b\coth^2(c+dx)}\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}+1\right)}{4d\coth^{\frac{2}{3}}(c+dx)}+\frac{\sqrt[3]{b\coth^2(c+dx)}\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)}{d\coth^{\frac{2}{3}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^2)^(1/3),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^2)^(1
/3))/(2*d*Coth[c + d*x]^(2/3)) - (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3)
)/Sqrt[3]]*(b*Coth[c + d*x]^2)^(1/3))/(2*d*Coth[c + d*x]^(2/3)) + (ArcTanh[
Coth[c + d*x]^(1/3)]*(b*Coth[c + d*x]^2)^(1/3))/(d*Coth[c + d*x]^(2/3)) - (
(b*Coth[c + d*x]^2)^(1/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)
])/(4*d*Coth[c + d*x]^(2/3)) + ((b*Coth[c + d*x]^2)^(1/3)*Log[1 + Coth[c +
d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*Coth[c + d*x]^(2/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
```

```
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \sqrt[3]{b\coth^2(c+dx)}\,dx &= \frac{\sqrt[3]{b\coth^2(c+dx)}\int \coth^{\frac{2}{3}}(c+dx)\,dx}{\coth^{\frac{2}{3}}(c+dx)} \\
&= -\frac{\sqrt[3]{b\coth^2(c+dx)}\,\text{Subst}\left(\int \frac{x^{2/3}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d\coth^{\frac{2}{3}}(c+dx)} \\
&= -\frac{\left(3\sqrt[3]{b\coth^2(c+dx)}\right)\text{Subst}\left(\int \frac{x^4}{-1+x^6}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d\coth^{\frac{2}{3}}(c+dx)} \\
&= \frac{\sqrt[3]{b\coth^2(c+dx)}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d\coth^{\frac{2}{3}}(c+dx)} + \frac{\sqrt[3]{b\coth^2(c+dx)}\,\text{Sul}}{} \\
&= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^2(c+dx)}}{d\coth^{\frac{2}{3}}(c+dx)} - \frac{\sqrt[3]{b\coth^2(c+dx)}\,\text{Subst}\left(\int \frac{-1+2}{1-x+}\right.}{4d\coth^{\frac{2}{3}}(c-} \\
&= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^2(c+dx)}}{d\coth^{\frac{2}{3}}(c+dx)} - \frac{\sqrt[3]{b\coth^2(c+dx)}\,\log\left(1-\sqrt[3]{\text{co}}\right.}{4d\coth^{\frac{2}{3}}(} \\
&= \frac{\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^2(c+dx)}}{2d\coth^{\frac{2}{3}}(c+dx)} - \frac{\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{\coth(c}}{\sqrt{3}}\right.}{2d\coth}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 151, normalized size = 0.57

$$\frac{\sqrt[3]{b\coth^2(c+dx)}\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)-2\sqrt{3}\,\text{ArcTan}\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)+4\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)-\log\left(1-\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)+\log\left(1+\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)\right)}{4d\coth^{\frac{2}{3}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^2)^(1/3),x]
```

```
[Out] ((b*Coth[c + d*x]^2)^(1/3)*(2*Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sq
rt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]] + 4*ArcTanh[
Coth[c + d*x]^(1/3)] - Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)] +
```

```
Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)]))/(4*d*Coth[c + d*x]^(2
/3))
```

Maple [F]
time = 1.76, size = 0, normalized size = 0.00

$$\int \left(b\left(\coth^{2}\left(dx+c\right)\right)\right)^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^2)^(1/3),x)
```

```
[Out] int((b*coth(d*x+c)^2)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^2)^(1/3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1618 vs. 2(216) = 432.
time = 0.40, size = 1618, normalized size = 6.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*(-b)^(1/3)*arctan(1/3*(sqrt(3)*b*cosh(d*x + c)^2 + 2*sqrt(3
)*b*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*b*sinh(d*x + c)^2 + 2*(sqrt(3)*co
sh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*sinh(d*x +
c)^2 - sqrt(3))*(-b)^(2/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(co
sh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) + sqrt(3)*b)/(b*cosh(d*x + c)^2
 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - 2*sqrt(3)*b^
(1/3)*arctan(-1/3*(sqrt(3)*b*cosh(d*x + c)^2 + 2*sqrt(3)*b*cosh(d*x + c)*si
nh(d*x + c) + sqrt(3)*b*sinh(d*x + c)^2 - 2*(sqrt(3)*cosh(d*x + c)^2 + 2*sq
rt(3)*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*sinh(d*x + c)^2 - sqrt(3))*b^(2
/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*
x + c)^2 - 1))^(1/3) + sqrt(3)*b)/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*si
nh(d*x + c) + b*sinh(d*x + c)^2 + b)) + (-b)^(1/3)*log(((cosh(d*x + c)^4 +
4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 +
```

```
1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c)
)*sinh(d*x + c) + 1)*(-b)^(2/3) - (cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh
(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x +
c)^3 + sinh(d*x + c)^4 - 1)*(-b)^(1/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c
)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) + (cosh(d*x + c)^4
+ 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2
- 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x +
c))*sinh(d*x + c) + 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d
*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3))/(cosh(d*x + c)^4 + 4*cosh(d*x + c)
*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c
)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c)
 + 1)) + b^(1/3)*log(((cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 +
sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x +
c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*b^(2/3) - (co
sh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*
x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 - 1)*b^(1/3)*(
(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c
)^2 - 1))^(1/3) + (cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh
(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2
 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*((b*cosh(d*x + c)
^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3))
/(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(
3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x +
c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)) - 2*(-b)^(1/3)*log(((cosh(d*x + c
)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(1/3) + (co
sh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*((b*co
sh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 -
 1))^(1/3))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c
)^2 + 1)) - 2*b^(1/3)*log(((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c)
 + sinh(d*x + c)^2 + 1)*b^(1/3) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d
*x + c) + sinh(d*x + c)^2 - 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)
/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3))/(cosh(d*x + c)^2 + 2*cosh(
d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{b\coth^2\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**2)**(1/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**2)**(1/3), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(1/3),x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^2)^(1/3), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(b\,{\mathrm{coth}\left(c+d\,x\right)}^2\right)}^{1/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^2)^(1/3),x)
[Out] int((b*coth(c + d*x)^2)^(1/3), x)
```

3.25 $\int \frac{1}{\sqrt[3]{b\coth^2(c+dx)}}\,dx$

Optimal. Leaf size=264

$$-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{2}{3}}(c+dx)}{2d\sqrt[3]{b\coth^2(c+dx)}}+\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{2}{3}}(c+dx)}{2d\sqrt[3]{b\coth^2(c+dx)}}+\text{tan}$$

```
[Out] arctanh(coth(d*x+c)^(1/3))*coth(d*x+c)^(2/3)/d/(b*coth(d*x+c)^2)^(1/3)-1/4*
coth(d*x+c)^(2/3)*ln(1-coth(d*x+c)^(1/3)+coth(d*x+c)^(2/3))/d/(b*coth(d*x+c
)^2)^(1/3)+1/4*coth(d*x+c)^(2/3)*ln(1+coth(d*x+c)^(1/3)+coth(d*x+c)^(2/3))/
d/(b*coth(d*x+c)^2)^(1/3)-1/2*arctan(1/3*(1-2*coth(d*x+c)^(1/3))*3^(1/2))*c
oth(d*x+c)^(2/3)*3^(1/2)/d/(b*coth(d*x+c)^2)^(1/3)+1/2*arctan(1/3*(1+2*coth
(d*x+c)^(1/3))*3^(1/2))*coth(d*x+c)^(2/3)*3^(1/2)/d/(b*coth(d*x+c)^2)^(1/3)
```

Rubi [A]
time = 0.12, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {3739, 3557, 335, 216, 648, 632, 210, 642, 212}

$$-\frac{\sqrt{3}\coth^{\frac{2}{3}}(c+dx)\text{ArcTan}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)}{2d\sqrt[3]{b\coth^2(c+dx)}}+\frac{\sqrt{3}\coth^{\frac{2}{3}}(c+dx)\text{ArcTan}\left(\frac{2\sqrt[3]{\coth(c+dx)}+1}{\sqrt{3}}\right)}{2d\sqrt[3]{b\coth^2(c+dx)}}-\frac{\coth^{\frac{2}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\sqrt[3]{b\coth^2(c+dx)}}+\frac{\coth^{\frac{2}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}+1\right)}{4d\sqrt[3]{b\coth^2(c+dx)}}+\frac{\coth^{\frac{2}{3}}(c+dx)\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)}{d\sqrt[3]{b\coth^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^2)^(-1/3),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c + d*x]^(2/
3))/(d*(b*Coth[c + d*x]^2)^(1/3)) + (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1
/3))/Sqrt[3]]*Coth[c + d*x]^(2/3))/(2*d*(b*Coth[c + d*x]^2)^(1/3)) + (ArcTa
nh[Coth[c + d*x]^(1/3)]*Coth[c + d*x]^(2/3))/(d*(b*Coth[c + d*x]^2)^(1/3))
- (Coth[c + d*x]^(2/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/
(4*d*(b*Coth[c + d*x]^2)^(1/3)) + (Coth[c + d*x]^(2/3)*Log[1 + Coth[c + d*x
]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*(b*Coth[c + d*x]^2)^(1/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
```

```
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{b\coth^2(c+dx)}}\,dx = \frac{\coth^{\frac{2}{3}}(c+dx)\int \frac{1}{\coth^{\frac{2}{3}}(c+dx)}\,dx}{\sqrt[3]{b\coth^2(c+dx)}}$$

$$= -\frac{\coth^{\frac{2}{3}}(c+dx)\text{Subst}\left(\int \frac{1}{x^{2/3}(-1+x^2)}\,dx, x, \coth(c+dx)\right)}{d\sqrt[3]{b\coth^2(c+dx)}}$$

$$= -\frac{\left(3\coth^{\frac{2}{3}}(c+dx)\right)\text{Subst}\left(\int \frac{1}{-1+x^6}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d\sqrt[3]{b\coth^2(c+dx)}}$$

$$= \frac{\coth^{\frac{2}{3}}(c+dx)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d\sqrt[3]{b\coth^2(c+dx)}} + \frac{\coth^{\frac{2}{3}}(c+dx)\text{Subst}\left(\int \frac{1}{1-\cdots}\right.}{d\sqrt[3]{b\cot\cdots}}$$

$$= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{2}{3}}(c+dx)}{d\sqrt[3]{b\coth^2(c+dx)}} - \frac{\coth^{\frac{2}{3}}(c+dx)\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \cdots\right.}{4d\sqrt[3]{b\coth^2(c+dx)}}$$

$$= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{2}{3}}(c+dx)}{d\sqrt[3]{b\coth^2(c+dx)}} - \frac{\coth^{\frac{2}{3}}(c+dx)\log\left(1-\sqrt[3]{\coth(c+d\cdots}\right.}{4d\sqrt[3]{b\coth^2(c+d\cdots}}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{2}{3}}(c+dx)}{2d\sqrt[3]{b\coth^2(c+dx)}} + \frac{\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{\coth(c+\cdots}}{\sqrt{3}}\right.}{2d\sqrt[3]{b\coth^2(c\cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.04, size = 41, normalized size = 0.16

$$\frac{3\coth(c+dx)\,{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};\coth^2(c+dx)\right)}{d\sqrt[3]{b\coth^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^2)^(-1/3),x]
```

```
[Out] (3*Coth[c + d*x]*Hypergeometric2F1[1/6, 1, 7/6, Coth[c + d*x]^2])/(d*(b*Cot
h[c + d*x]^2)^(1/3))
```

Maple [F]
time = 1.88, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \left(\coth^{2}\left(dx+c\right)\right)\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^2)^(1/3),x)
[Out] int(1/(b*coth(d*x+c)^2)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^2)^(1/3),x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c)^2)^(-1/3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1594 vs. $2(216) = 432$.
time = 0.53, size = 8338, normalized size = 31.58

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^2)^(1/3),x, algorithm="fricas")
[Out] [1/4*(sqrt(3)*b*sqrt(-1/b^(2/3))*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)
*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*sqrt(3)*(cosh(d*x + c)^4 + 4*cosh(
d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh
(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(
d*x + c) + 1)*b^(2/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*
x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3)*sqrt(-1/b^(2/3)) - 2*b*cosh(d*x + c)
^2 + 2*(3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^2 + sqrt(3)*(b*cosh(d*x + c)
^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*b*cosh(d*x +
 c)^2 + 2*(3*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3
+ b*cosh(d*x + c))*sinh(d*x + c) + b)*b^(1/3)*sqrt(-1/b^(2/3)) + 4*(b*cosh(
d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c) - (sqrt(3)*(b*cosh(d*x + c)^4 +
 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d*x + c)^2*sinh(d*x + c)^2 +
4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - b)*sqrt(-1/b^(2/3))
```

```
 + 3*(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2
*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 - 1)*b
^(2/3))*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sin
h(d*x + c)^2 - 1))^(1/3) - 3*b)/(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x
 + c)^3 + sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + cosh(
d*x + c)^2 + 2*(2*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c))) + sqrt(3
)*b*sqrt((-b)^(1/3)/b)*log(-(3*b*cosh(d*x + c)^4 + 12*b*cosh(d*x + c)*sinh(
d*x + c)^3 + 3*b*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^2 + 2*(9*b*cosh(d*x +
c)^2 + b)*sinh(d*x + c)^2 - 3*(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x
 + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3
 + sinh(d*x + c)^4 - 1)*(-b)^(2/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2
+ b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) - sqrt(3)*(2*(cosh(d*x
+ c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x
+ c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh
(d*x + c))*sinh(d*x + c) + 1)*(-b)^(2/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x +
 c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3) + (b*cosh(d*x + c
)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*b*cosh(d*x
+ c)^2 + 2*(3*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3
 + b*cosh(d*x + c))*sinh(d*x + c) + b)*(-b)^(1/3) - (b*cosh(d*x + c)^4 + 4*
b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b
*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - b)*((b*cosh(d*x + c)^2
 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3))*s
qrt((-b)^(1/3)/b) + 4*(3*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c)
 - b)/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 +
1)) + (-b)^(2/3)*log(((cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 +
sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x +
c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*(-b)^(2/3) -
(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh
(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 - 1)*(-b)^(
1/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d
*x + c)^2 - 1))^(1/3) + (cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3
+ sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x
+ c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*((b*cosh(d*
x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^
(2/3))/(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4
 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(
d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)) - b^(2/3)*log(((cosh(d*x +
c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x +
c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d
*x + c))*sinh(d*x + c) + 1)*b^(2/3) - (cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*
sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*
x + c)^3 + sinh(d*x + c)^4 - 1)*b^(1/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x +
c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) + (cosh(d*x + c)^4
 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2
 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x +
```

```
c))*sinh(d*x + c) + 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(
d*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3))/(cosh(d*x + c)^4 + 4*cosh(d*x + c
)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x +
c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c
) + 1)) - 2*(-b)^(2/3)*log(((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c
) + sinh(d*x + c)^2 + 1)*(-b)^(1/3) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*si
nh(d*x + c) + sinh(d*x + c)^2 - 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2
+ b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3))/(cosh(d*x + c)^2 + 2*c
osh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + ...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{b\coth^2\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**2)**(1/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**2)**(-1/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^2)^(-1/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,{\mathrm{coth}\left(c+d\,x\right)}^2\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x)^2)^(1/3),x)
```

```
[Out] int(1/(b*coth(c + d*x)^2)^(1/3), x)
```

3.26 $\int \frac{1}{\left(b\coth^2(c+dx)\right)^{2/3}}\,dx$

Optimal. Leaf size=289

$$-\frac{3\coth(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}}+\frac{\sqrt{3}\,\mathrm{ArcTan}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{4}{3}}(c+dx)}{2d\left(b\coth^2(c+dx)\right)^{2/3}}-\frac{\sqrt{3}\,\mathrm{ArcTan}\left(\frac{1+2\sqrt[3]{\coth(c+dx}}{\sqrt{3}}\right.}{2d\left(b\coth^2(c+dx\right.}$$

```
[Out] -3*coth(d*x+c)/d/(b*coth(d*x+c)^2)^(2/3)+arctanh(coth(d*x+c)^(1/3))*coth(d*
x+c)^(4/3)/d/(b*coth(d*x+c)^2)^(2/3)-1/4*coth(d*x+c)^(4/3)*ln(1-coth(d*x+c)
^(1/3)+coth(d*x+c)^(2/3))/d/(b*coth(d*x+c)^2)^(2/3)+1/4*coth(d*x+c)^(4/3)*l
n(1+coth(d*x+c)^(1/3)+coth(d*x+c)^(2/3))/d/(b*coth(d*x+c)^2)^(2/3)+1/2*arct
an(1/3*(1-2*coth(d*x+c)^(1/3))*3^(1/2))*coth(d*x+c)^(4/3)*3^(1/2)/d/(b*coth
(d*x+c)^2)^(2/3)-1/2*arctan(1/3*(1+2*coth(d*x+c)^(1/3))*3^(1/2))*coth(d*x+c
)^(4/3)*3^(1/2)/d/(b*coth(d*x+c)^2)^(2/3)
```

Rubi [A]
time = 0.16, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3739, 3555, 3557, 335, 302, 648, 632, 210, 642, 212}

$$\frac{\sqrt{3}\coth^{\frac{4}{3}}(c+dx)\mathrm{ArcTan}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)}{2d\left(b\coth^2(c+dx)\right)^{2/3}}-\frac{\sqrt{3}\coth^{\frac{4}{3}}(c+dx)\mathrm{ArcTan}\left(\frac{2\sqrt[3]{\coth(c+dx)}+1}{\sqrt{3}}\right)}{2d\left(b\coth^2(c+dx)\right)^{2/3}}-\frac{3\coth(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}}-\frac{\coth^{\frac{4}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\left(b\coth^2(c+dx)\right)^{2/3}}+\frac{\coth^{\frac{4}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}+1\right)}{4d\left(b\coth^2(c+dx)\right)^{2/3}}+\frac{\coth^{\frac{4}{3}}(c+dx)\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)}{d\left(b\coth^2(c+dx)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^2)^(-2/3),x]
```

```
[Out] (-3*Coth[c + d*x])/(d*(b*Coth[c + d*x]^2)^(2/3)) + (Sqrt[3]*ArcTan[(1 - 2*C
oth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c + d*x]^(4/3))/(2*d*(b*Coth[c + d*x]^2)^
(2/3)) - (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c + d*x]
^(4/3))/(2*d*(b*Coth[c + d*x]^2)^(2/3)) + (ArcTanh[Coth[c + d*x]^(1/3)]*Cot
h[c + d*x]^(4/3))/(d*(b*Coth[c + d*x]^2)^(2/3)) - (Coth[c + d*x]^(4/3)*Log[
1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*(b*Coth[c + d*x]^2)^(2
/3)) + (Coth[c + d*x]^(4/3)*Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/
3)])/(4*d*(b*Coth[c + d*x]^2)^(2/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{2/3}}\,dx = \frac{\coth^{\frac{4}{3}}(c+dx)\int \frac{1}{\coth^{\frac{4}{3}}(c+dx)}\,dx}{\left(b\coth^2(c+dx)\right)^{2/3}}$$

$$= -\frac{3\coth(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}} + \frac{\coth^{\frac{4}{3}}(c+dx)\int \coth^{\frac{2}{3}}(c+dx)\,dx}{\left(b\coth^2(c+dx)\right)^{2/3}}$$

$$= -\frac{3\coth(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}} - \frac{\coth^{\frac{4}{3}}(c+dx)\text{Subst}\left(\int \frac{x^{2/3}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d\left(b\coth^2(c+dx)\right)^{2/3}}$$

$$= -\frac{3\coth(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}} - \frac{\left(3\coth^{\frac{4}{3}}(c+dx)\right)\text{Subst}\left(\int \frac{x^4}{-1+x^6}\,dx, x, \sqrt[3]{\coth(c+\right.}{d\left(b\coth^2(c+dx)\right)^{2/3}}$$

$$= -\frac{3\coth(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}} + \frac{\coth^{\frac{4}{3}}(c+dx)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d\left(b\coth^2(c+dx)\right)^{2/3}}$$

$$= -\frac{3\coth(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}} + \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{4}{3}}(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}} - \frac{\coth^{\frac{4}{3}}(c}{}$$

$$= -\frac{3\coth(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}} + \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{4}{3}}(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}} - \frac{\coth^{\frac{4}{3}}(c}{}$$

$$= -\frac{3\coth(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}} + \frac{\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{4}{3}}(c+dx)}{2d\left(b\coth^2(c+dx)\right)^{2/3}} - $$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.04, size = 41, normalized size = 0.14

$$-\frac{3\coth(c+dx)\,{}_2F_1\left(-\frac{1}{6},1;\frac{5}{6};\coth^2(c+dx)\right)}{d\left(b\coth^2(c+dx)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^2)^(-2/3),x]
```

```
[Out] (-3*Coth[c + d*x]*Hypergeometric2F1[-1/6, 1, 5/6, Coth[c + d*x]^2])/(d*(b*C
oth[c + d*x]^2)^(2/3))
```

Maple [F]
time = 1.83, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\left(\coth^2\left(dx+c\right)\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^2)^(2/3),x)
```

```
[Out] int(1/(b*coth(d*x+c)^2)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^2)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^2)^(-2/3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2066 vs. 2(239) = 478.
time = 0.42, size = 2066, normalized size = 7.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^2)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/4*(2*sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sin
h(d*x + c)^2 + b)*sqrt(-(-b^2)^(1/3))*arctan(1/3*(2*sqrt(3)*(-b^2)^(2/3)*(c
osh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(
-(-b^2)^(1/3))*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^
```

```
2 + sinh(d*x + c)^2 - 1))^(1/3) - sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x
 + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*(-b^2)^(1/3)*sqrt(-(-b^2)^(1/3
)))/(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*x
 + c)^2 + b^2)) + 2*sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x
 + c) + b*sinh(d*x + c)^2 + b)*(b^2)^(1/6)*arctan(1/3*sqrt(3)*(b^2)^(1/6)*(
2*(b^2)^(2/3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
 c)^2 - 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 +
sinh(d*x + c)^2 - 1))^(1/3) - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d
*x + c) + b*sinh(d*x + c)^2 + b)*(b^2)^(1/3))/(b^2*cosh(d*x + c)^2 + 2*b^2*
cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 + b^2)) + (-b^2)^(2/3)*(c
osh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*log((
(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh
(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 - 1)*(-b^2)
^(2/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh
(d*x + c)^2 - 1))^(1/3) + (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x +
 c)^3 + b*sinh(d*x + c)^4 - 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 -
b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c)
+ b)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d
*x + c)^2 - 1))^(2/3) - (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c
)^3 + b*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + b)
*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) +
b)*(-b^2)^(1/3))/(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(
d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2
+ 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)) - (b^2)^(2/3)*(co
sh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*log(-(
(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh
(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 - 1)*(b^2)^
(2/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(
d*x + c)^2 - 1))^(1/3) - (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x +
c)^3 + b*sinh(d*x + c)^4 - 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - b
)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c) +
 b)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*
x + c)^2 - 1))^(2/3) - (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)
^3 + b*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + b)*
sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) + b
)*(b^2)^(1/3))/(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*
x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 +
4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)) - 2*(-b^2)^(2/3)*(c
osh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*log(-
((-b^2)^(2/3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
 c)^2 + 1) - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(
d*x + c)^2 - b)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)
^2 + sinh(d*x + c)^2 - 1))^(1/3))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d
*x + c) + sinh(d*x + c)^2 + 1)) + 2*(b^2)^(2/3)*(cosh(d*x + c)^2 + 2*cosh(d
*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*log(((b^2)^(2/3)*(cosh(d*x + c
```

```
)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1) + (b*cosh(d*x +
c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*((b*cosh(d*
x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^
(1/3))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 +
 1)) - 12*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x
 + c)^2 - b)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2
+ sinh(d*x + c)^2 - 1))^(1/3))/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x +
c)*sinh(d*x + c) + b^2*d*sinh(d*x + c)^2 + b^2*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\coth^2\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**2)**(2/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**2)**(-2/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^2)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^2)^(-2/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,{\mathrm{coth}\left(c+d\,x\right)}^2\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x)^2)^(2/3),x)
```

```
[Out] int(1/(b*coth(c + d*x)^2)^(2/3), x)
```

3.27 $\int \frac{1}{\left(b\coth^2(c+dx)\right)^{4/3}}\,dx$

Optimal. Leaf size=309

$$-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{2}{3}}(c+dx)}{2bd\sqrt[3]{b\coth^2(c+dx)}}+\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{2}{3}}(c+dx)}{2bd\sqrt[3]{b\coth^2(c+dx)}}+\frac{\tan}{}$$

[Out] arctanh(coth(d*x+c)^(1/3))*coth(d*x+c)^(2/3)/b/d/(b*coth(d*x+c)^2)^(1/3)-1/4*coth(d*x+c)^(2/3)*ln(1-coth(d*x+c)^(1/3)+coth(d*x+c)^(2/3))/b/d/(b*coth(d*x+c)^2)^(1/3)+1/4*coth(d*x+c)^(2/3)*ln(1+coth(d*x+c)^(1/3)+coth(d*x+c)^(2/3))/b/d/(b*coth(d*x+c)^2)^(1/3)-1/2*arctan(1/3*(1-2*coth(d*x+c)^(1/3))*3^(1/2))*coth(d*x+c)^(2/3)*3^(1/2)/b/d/(b*coth(d*x+c)^2)^(1/3)+1/2*arctan(1/3*(1+2*coth(d*x+c)^(1/3))*3^(1/2))*coth(d*x+c)^(2/3)*3^(1/2)/b/d/(b*coth(d*x+c)^2)^(1/3)-3/5*tanh(d*x+c)/b/d/(b*coth(d*x+c)^2)^(1/3)

Rubi [A]

time = 0.14, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3739, 3555, 3557, 335, 216, 648, 632, 210, 642, 212}

$$-\frac{\sqrt{3}\coth^{\frac{2}{3}}(c+dx)\text{ArcTan}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)}{2bd\sqrt[3]{b\coth^2(c+dx)}}+\frac{\sqrt{3}\coth^{\frac{2}{3}}(c+dx)\text{ArcTan}\left(\frac{2\sqrt[3]{\coth(c+dx)}+1}{\sqrt{3}}\right)}{2bd\sqrt[3]{b\coth^2(c+dx)}}-\frac{3\tanh(c+dx)}{5bd\sqrt[3]{b\coth^2(c+dx)}}-\frac{\coth^{\frac{2}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4bd\sqrt[3]{b\coth^2(c+dx)}}+\frac{\coth^{\frac{2}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}+1\right)}{4bd\sqrt[3]{b\coth^2(c+dx)}}+\frac{\coth^{\frac{2}{3}}(c+dx)\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)}{bd\sqrt[3]{b\coth^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b*Coth[c + d*x]^2)^(-4/3),x]

[Out] -1/2*(Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c + d*x]^(2/3))/(b*d*(b*Coth[c + d*x]^2)^(1/3)) + (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c + d*x]^(2/3))/(2*b*d*(b*Coth[c + d*x]^2)^(1/3)) + (ArcTanh[Coth[c + d*x]^(1/3)]*Coth[c + d*x]^(2/3))/(b*d*(b*Coth[c + d*x]^2)^(1/3)) - (Coth[c + d*x]^(2/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*b*d*(b*Coth[c + d*x]^2)^(1/3)) + (Coth[c + d*x]^(2/3)*Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*b*d*(b*Coth[c + d*x]^2)^(1/3)) - (3*Tanh[c + d*x])/(5*b*d*(b*Coth[c + d*x]^2)^(1/3))

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{4/3}}\,dx = \frac{\coth^{\frac{2}{3}}(c+dx)\int \frac{1}{\coth^{\frac{8}{3}}(c+dx)}\,dx}{b\sqrt[3]{b\coth^2(c+dx)}}$$

$$= -\frac{3\tanh(c+dx)}{5bd\sqrt[3]{b\coth^2(c+dx)}} + \frac{\coth^{\frac{2}{3}}(c+dx)\int \frac{1}{\coth^{\frac{2}{3}}(c+dx)}\,dx}{b\sqrt[3]{b\coth^2(c+dx)}}$$

$$= -\frac{3\tanh(c+dx)}{5bd\sqrt[3]{b\coth^2(c+dx)}} - \frac{\coth^{\frac{2}{3}}(c+dx)\text{Subst}\left(\int \frac{1}{x^{2/3}(-1+x^2)}\,dx, x, \coth(c+dx\right.}{bd\sqrt[3]{b\coth^2(c+dx)}}$$

$$= -\frac{3\tanh(c+dx)}{5bd\sqrt[3]{b\coth^2(c+dx)}} - \frac{\left(3\coth^{\frac{2}{3}}(c+dx)\right)\text{Subst}\left(\int \frac{1}{-1+x^6}\,dx, x, \sqrt[3]{\coth(c+}\right.}{bd\sqrt[3]{b\coth^2(c+dx)}}$$

$$= -\frac{3\tanh(c+dx)}{5bd\sqrt[3]{b\coth^2(c+dx)}} + \frac{\coth^{\frac{2}{3}}(c+dx)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{bd\sqrt[3]{b\coth^2(c+dx)}}$$

$$= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{2}{3}}(c+dx)}{bd\sqrt[3]{b\coth^2(c+dx)}} - \frac{3\tanh(c+dx)}{5bd\sqrt[3]{b\coth^2(c+dx)}} - \frac{\coth^{\frac{2}{3}}(c+}{}$$

$$= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{2}{3}}(c+dx)}{bd\sqrt[3]{b\coth^2(c+dx)}} - \frac{\coth^{\frac{2}{3}}(c+dx)\log\left(1-\sqrt[3]{\coth(c+}\right.}{4bd\sqrt[3]{b\coth^2(c+}}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{2}{3}}(c+dx)}{2bd\sqrt[3]{b\coth^2(c+dx)}} + \frac{\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{\coth(c}}{\sqrt{3}}\right.}{2bd\sqrt[3]{b\coth^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order

3 in optimal.
time = 0.10, size = 43, normalized size = 0.14

$$-\frac{3\coth(c+dx)\,{}_2F_1\left(-\frac{5}{6},1;\frac{1}{6};\coth^2(c+dx)\right)}{5d\left(b\coth^2(c+dx)\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^2)^(-4/3),x]
```

```
[Out] (-3*Coth[c + d*x]*Hypergeometric2F1[-5/6, 1, 1/6, Coth[c + d*x]^2])/(5*d*(b
*Coth[c + d*x]^2)^(4/3))
```

Maple [F]
time = 1.82, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\left(\coth^2\left(dx+c\right)\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^2)^(4/3),x)
```

```
[Out] int(1/(b*coth(d*x+c)^2)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^2)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^2)^(-4/3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3099 vs. 2(257) = 514.
time = 0.54, size = 14359, normalized size = 46.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^2)^(4/3),x, algorithm="fricas")
```

```
[Out] [1/20*(5*sqrt(3)*(b*cosh(d*x + c)^6 + 6*b*cosh(d*x + c)*sinh(d*x + c)^5 + b
*sinh(d*x + c)^6 + 3*b*cosh(d*x + c)^4 + 3*(5*b*cosh(d*x + c)^2 + b)*sinh(d
*x + c)^4 + 4*(5*b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c))*sinh(d*x + c)^3 + 3
```

```
*b*cosh(d*x + c)^2 + 3*(5*b*cosh(d*x + c)^4 + 6*b*cosh(d*x + c)^2 + b)*sinh
(d*x + c)^2 + 6*(b*cosh(d*x + c)^5 + 2*b*cosh(d*x + c)^3 + b*cosh(d*x + c))
*sinh(d*x + c) + b)*sqrt(-1/b^(2/3))*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x
+ c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*sqrt(3)*(cosh(d*x + c)^4 + 4*c
osh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*
sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*s
inh(d*x + c) + 1)*b^(2/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cos
h(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3)*sqrt(-1/b^(2/3)) - 2*b*cosh(d*x
+ c)^2 + 2*(3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^2 + sqrt(3)*(b*cosh(d*x
+ c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*b*cosh(d
*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c
)^3 + b*cosh(d*x + c))*sinh(d*x + c) + b)*b^(1/3)*sqrt(-1/b^(2/3)) + 4*(b*c
osh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c) - (sqrt(3)*(b*cosh(d*x + c)
^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d*x + c)^2*sinh(d*x + c)^
2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - b)*sqrt(-1/b^(2
/3)) + 3*(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x +
c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 -
1)*b^(2/3))*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 +
 sinh(d*x + c)^2 - 1))^(1/3) - 3*b)/(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh
(d*x + c)^3 + sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + c
osh(d*x + c)^2 + 2*(2*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c))) + 5*
sqrt(3)*(b*cosh(d*x + c)^6 + 6*b*cosh(d*x + c)*sinh(d*x + c)^5 + b*sinh(d*x
 + c)^6 + 3*b*cosh(d*x + c)^4 + 3*(5*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^4
 + 4*(5*b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c))*sinh(d*x + c)^3 + 3*b*cosh(d
*x + c)^2 + 3*(5*b*cosh(d*x + c)^4 + 6*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)
^2 + 6*(b*cosh(d*x + c)^5 + 2*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x
 + c) + b)*sqrt((-b)^(1/3)/b)*log(-(3*b*cosh(d*x + c)^4 + 12*b*cosh(d*x + c
)*sinh(d*x + c)^3 + 3*b*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^2 + 2*(9*b*cosh
(d*x + c)^2 + b)*sinh(d*x + c)^2 - 3*(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*s
inh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x
 + c)^3 + sinh(d*x + c)^4 - 1)*(-b)^(2/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x
+ c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) - sqrt(3)*(2*(co
sh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*co
sh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3
 - cosh(d*x + c))*sinh(d*x + c) + 1)*(-b)^(2/3)*((b*cosh(d*x + c)^2 + b*sin
h(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3) + (b*cosh(
d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*b*co
sh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x
 + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) + b)*(-b)^(1/3) - (b*cosh(d*x + c)
^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d*x + c)^2*sinh(d*x + c)^
2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - b)*((b*cosh(d*x
 + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(
1/3))*sqrt((-b)^(1/3)/b) + 4*(3*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d
*x + c) - b)/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
c)^2 + 1)) + 5*(cosh(d*x + c)^6 + 6*cosh(d*x + c)*sinh(d*x + c)^5 + sinh(d*
```

```
x + c)^6 + 3*(5*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^4 + 3*cosh(d*x + c)^4 +
4*(5*cosh(d*x + c)^3 + 3*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*cosh(d*x + c
)^4 + 6*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 3*cosh(d*x + c)^2 + 6*(cosh(
d*x + c)^5 + 2*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*(-b)^(2/
3)*log(((cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^
4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh
(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*(-b)^(2/3) - (cosh(d*x + c)
^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 +
4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 - 1)*(-b)^(1/3)*((b*cosh(
d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1)
)^(1/3) + (cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c
)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(co
sh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*((b*cosh(d*x + c)^2 + b*s
inh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3))/(cosh(d
*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d
*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + c
osh(d*x + c))*sinh(d*x + c) + 1)) - 5*(cosh(d*x + c)^6 + 6*cosh(d*x + c)*si
nh(d*x + c)^5 + sinh(d*x + c)^6 + 3*(5*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^4
 + 3*cosh(d*x + c)^4 + 4*(5*cosh(d*x + c)^3 + 3...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\coth^2\left(c+dx\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**2)**(4/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**2)**(-4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^2)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^2)^(-4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,{\mathrm{coth}\left(c+d\,x\right)}^2\right)}^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x)^2)^(4/3),x)
```

```
[Out] int(1/(b*coth(c + d*x)^2)^(4/3), x)
```

3.28 $\int \left(b\coth^3(c+dx)\right)^n\,dx$

Optimal. Leaf size=55

$$\frac{\coth(c+dx)\left(b\coth^3(c+dx)\right)^n\,{}_2F_1\left(1,\frac{1}{2}(1+3n);\frac{3(1+n)}{2};\coth^2(c+dx)\right)}{d(1+3n)}$$

```
[Out] coth(d*x+c)*(b*coth(d*x+c)^3)^n*hypergeom([1, 1/2+3/2*n],[3/2+3/2*n],coth(d
*x+c)^2)/d/(1+3*n)
```

Rubi [A]
time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3739, 3557, 371}

$$\frac{\coth(c+dx)\left(b\coth^3(c+dx)\right)^n\,{}_2F_1\left(1,\frac{1}{2}(3n+1);\frac{3(n+1)}{2};\coth^2(c+dx)\right)}{d(3n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^3)^n,x]
```

```
[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^3)^n*Hypergeometric2F1[1, (1 + 3*n)/2, (3*(
1 + n))/2, Coth[c + d*x]^2])/(d*(1 + 3*n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(b\coth^3(c+dx)\right)^n\,dx = \left(\coth^{-3n}(c+dx)\left(b\coth^3(c+dx)\right)^n\right)\int \coth^{3n}(c+dx)\,dx$$
$$= -\frac{\left(\coth^{-3n}(c+dx)\left(b\coth^3(c+dx)\right)^n\right)\text{Subst}\left(\int \frac{x^{3n}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d}$$
$$= \frac{\coth(c+dx)\left(b\coth^3(c+dx)\right)^n {}_2F_1\left(1, \frac{1}{2}(1+3n); \frac{3(1+n)}{2}; \coth^2(c+dx)\right)}{d(1+3n)}$$

Mathematica [A]
time = 0.03, size = 53, normalized size = 0.96

$$\frac{\coth(c+dx)\left(b\coth^3(c+dx)\right)^n {}_2F_1\left(1, \frac{1}{2}(1+3n); \frac{3(1+n)}{2}; \coth^2(c+dx)\right)}{d+3dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^3)^n,x]
[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^3)^n*Hypergeometric2F1[1, (1 + 3*n)/2, (3*(
1 + n))/2, Coth[c + d*x]^2])/(d + 3*d*n)
```

Maple [F]
time = 2.26, size = 0, normalized size = 0.00

$$\int \left(b\left(\coth^3(dx+c)\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^3)^n,x)
[Out] int((b*coth(d*x+c)^3)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^n,x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c)^3)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^n,x, algorithm="fricas")
```

```
[Out] integral((b*coth(d*x + c)^3)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\coth^{3}\left(c + dx\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**3)**n,x)
```

```
[Out] Integral((b*coth(c + d*x)**3)**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^n,x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^3)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(b\,{\mathrm{coth}\left(c+d\,x\right)}^3\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^3)^n,x)
```

```
[Out] int((b*coth(c + d*x)^3)^n, x)
```

3.29 $\int \left(b\coth^3(c+dx)\right)^{3/2}\,dx$

Optimal. Leaf size=134

$$-\frac{2b\sqrt{b\coth^3(c+dx)}}{3d}-\frac{b\text{ArcTan}\left(\sqrt{\coth(c+dx)}\right)\sqrt{b\coth^3(c+dx)}}{d\coth^{\frac{3}{2}}(c+dx)}+\frac{b\tanh^{-1}\left(\sqrt{\coth(c+dx)}\right)\sqrt{b\,\mathrm{co}}}{d\coth^{\frac{3}{2}}(c+dx)}$$

```
[Out] -2/3*b*(b*coth(d*x+c)^3)^(1/2)/d-b*arctan(coth(d*x+c)^(1/2))*(b*coth(d*x+c)
^3)^(1/2)/d/coth(d*x+c)^(3/2)+b*arctanh(coth(d*x+c)^(1/2))*(b*coth(d*x+c)^3
)^(1/2)/d/coth(d*x+c)^(3/2)-2/7*b*coth(d*x+c)^2*(b*coth(d*x+c)^3)^(1/2)/d
```

Rubi [A]
time = 0.05, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3739, 3554, 3557, 335, 304, 209, 212}

$$-\frac{b\text{ArcTan}\left(\sqrt{\coth(c+dx)}\right)\sqrt{b\coth^3(c+dx)}}{d\coth^{\frac{3}{2}}(c+dx)}-\frac{2b\sqrt{b\coth^3(c+dx)}}{3d}-\frac{2b\coth^2(c+dx)\sqrt{b\coth^3(c+dx)}}{7d}+\frac{b\sqrt{b\coth^3(c+dx)}\tanh^{-1}\left(\sqrt{\coth(c+dx)}\right)}{d\coth^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^3)^(3/2),x]
```

```
[Out] (-2*b*Sqrt[b*Coth[c + d*x]^3])/(3*d) - (b*ArcTan[Sqrt[Coth[c + d*x]]]*Sqrt[
b*Coth[c + d*x]^3])/(d*Coth[c + d*x]^(3/2)) + (b*ArcTanh[Sqrt[Coth[c + d*x]
]]*Sqrt[b*Coth[c + d*x]^3])/(d*Coth[c + d*x]^(3/2)) - (2*b*Coth[c + d*x]^2*
Sqrt[b*Coth[c + d*x]^3])/(7*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(b\coth^3(c+dx)\right)^{3/2}\,dx = \frac{\left(b\sqrt{b\coth^3(c+dx)}\right)\int \coth^{\frac{9}{2}}(c+dx)\,dx}{\coth^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2b\coth^2(c+dx)\sqrt{b\coth^3(c+dx)}}{7d} + \frac{\left(b\sqrt{b\coth^3(c+dx)}\right)\int \coth^{\frac{5}{2}}(c+dx$$

$$= -\frac{2b\sqrt{b\coth^3(c+dx)}}{3d} - \frac{2b\coth^2(c+dx)\sqrt{b\coth^3(c+dx)}}{7d} + \left(b\sqrt{b\coth^3(c\right.$$

$$= -\frac{2b\sqrt{b\coth^3(c+dx)}}{3d} - \frac{2b\coth^2(c+dx)\sqrt{b\coth^3(c+dx)}}{7d} - \left(b\sqrt{b\coth^3(c\right.$$

$$= -\frac{2b\sqrt{b\coth^3(c+dx)}}{3d} - \frac{2b\coth^2(c+dx)\sqrt{b\coth^3(c+dx)}}{7d} - \left(2b\sqrt{b\coth^3(c\right.$$

$$= -\frac{2b\sqrt{b\coth^3(c+dx)}}{3d} - \frac{2b\coth^2(c+dx)\sqrt{b\coth^3(c+dx)}}{7d} + \left(b\sqrt{b\coth^3(c\right.$$

$$= -\frac{2b\sqrt{b\coth^3(c+dx)}}{3d} - \frac{b\tan^{-1}\left(\sqrt{\coth(c+dx)}\right)\sqrt{b\coth^3(c+dx)}}{d\coth^{\frac{3}{2}}(c+dx)} + b\tan$$

Mathematica [A]
time = 0.35, size = 82, normalized size = 0.61

$$-\frac{\left(b\coth^3(c+dx)\right)^{3/2}\left(21\text{ArcTan}\left(\sqrt{\coth(c+dx)}\right) - 21\tanh^{-1}\left(\sqrt{\coth(c+dx)}\right) + 14\coth^{\frac{3}{2}}(c+dx) + 6\coth^{\frac{7}{2}}(c+dx)\right)}{21d\coth^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^3)^(3/2),x]
```

```
[Out] -1/21*((b*Coth[c + d*x]^3)^(3/2)*(21*ArcTan[Sqrt[Coth[c + d*x]]] - 21*ArcTa
nh[Sqrt[Coth[c + d*x]]] + 14*Coth[c + d*x]^(3/2) + 6*Coth[c + d*x]^(7/2)))/
(d*Coth[c + d*x]^(9/2))
```

Maple [A]
time = 2.29, size = 107, normalized size = 0.80

method	result
derivativedivides	$-\frac{\left(b\left(\coth^{3}(dx+c)\right)\right)^{\frac{3}{2}}\left(-21b^{\frac{7}{2}}\operatorname{arctanh}\left(\frac{\sqrt{b\coth\left(dx+c\right)}}{\sqrt{b}}\right)+21b^{\frac{7}{2}}\arctan\left(\frac{\sqrt{b\coth\left(dx+c\right)}}{\sqrt{b}}\right)+6(b\mathrm{c}}{21d\coth(dx+c)^{3}\left(b\coth(dx+c)\right)^{\frac{3}{2}}b^{2}}$
default	$-\frac{\left(b\left(\coth^{3}(dx+c)\right)\right)^{\frac{3}{2}}\left(-21b^{\frac{7}{2}}\operatorname{arctanh}\left(\frac{\sqrt{b\coth\left(dx+c\right)}}{\sqrt{b}}\right)+21b^{\frac{7}{2}}\arctan\left(\frac{\sqrt{b\coth\left(dx+c\right)}}{\sqrt{b}}\right)+6(b\mathrm{c}}{21d\coth(dx+c)^{3}\left(b\coth(dx+c)\right)^{\frac{3}{2}}b^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/21/d*(b*coth(d*x+c)^3)^(3/2)*(-21*b^(7/2)*arctanh((b*coth(d*x+c))^(1/2)/
b^(1/2))+21*b^(7/2)*arctan((b*coth(d*x+c))^(1/2)/b^(1/2))+6*(b*coth(d*x+c))
^(7/2)+14*b^2*(b*coth(d*x+c))^(3/2))/coth(d*x+c)^3/(b*coth(d*x+c))^(3/2)/b^
2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^3)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1051 vs. $2(114) = 228$.
time = 0.42, size = 2152, normalized size = 16.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/84*(42*(b*cosh(d*x + c)^6 + 6*b*cosh(d*x + c)*sinh(d*x + c)^5 + b*sinh(
d*x + c)^6 - 3*b*cosh(d*x + c)^4 + 3*(5*b*cosh(d*x + c)^2 - b)*sinh(d*x + c
)^4 + 4*(5*b*cosh(d*x + c)^3 - 3*b*cosh(d*x + c))*sinh(d*x + c)^3 + 3*b*cos
h(d*x + c)^2 + 3*(5*b*cosh(d*x + c)^4 - 6*b*cosh(d*x + c)^2 + b)*sinh(d*x +
 c)^2 + 6*(b*cosh(d*x + c)^5 - 2*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(
d*x + c) - b)*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x +
 c) + sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh
(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - 2
```

```
1*(b*cosh(d*x + c)^6 + 6*b*cosh(d*x + c)*sinh(d*x + c)^5 + b*sinh(d*x + c)^
6 - 3*b*cosh(d*x + c)^4 + 3*(5*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^4 + 4*(
5*b*cosh(d*x + c)^3 - 3*b*cosh(d*x + c))*sinh(d*x + c)^3 + 3*b*cosh(d*x + c
)^2 + 3*(5*b*cosh(d*x + c)^4 - 6*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 6
*(b*cosh(d*x + c)^5 - 2*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c)
- b)*sqrt(-b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) +
 6*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 +
b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + si
nh(d*x + c)^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cos
h(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x
 + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 16*(5*b*cos
h(d*x + c)^6 + 30*b*cosh(d*x + c)*sinh(d*x + c)^5 + 5*b*sinh(d*x + c)^6 + b
*cosh(d*x + c)^4 + (75*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^4 + 4*(25*b*cos
h(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c)^3 + b*cosh(d*x + c)^2 + (75*b
*cosh(d*x + c)^4 + 6*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 2*(15*b*cosh(
d*x + c)^5 + 2*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) + 5*b)*sq
rt(b*cosh(d*x + c)/sinh(d*x + c)))/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*s
inh(d*x + c)^5 + d*sinh(d*x + c)^6 - 3*d*cosh(d*x + c)^4 + 3*(5*d*cosh(d*x
+ c)^2 - d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*s
inh(d*x + c)^3 + 3*d*cosh(d*x + c)^2 + 3*(5*d*cosh(d*x + c)^4 - 6*d*cosh(d*
x + c)^2 + d)*sinh(d*x + c)^2 + 6*(d*cosh(d*x + c)^5 - 2*d*cosh(d*x + c)^3
+ d*cosh(d*x + c))*sinh(d*x + c) - d), -1/84*(42*(b*cosh(d*x + c)^6 + 6*b*c
osh(d*x + c)*sinh(d*x + c)^5 + b*sinh(d*x + c)^6 - 3*b*cosh(d*x + c)^4 + 3*
(5*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^4 + 4*(5*b*cosh(d*x + c)^3 - 3*b*co
sh(d*x + c))*sinh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 3*(5*b*cosh(d*x + c)^4
 - 6*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 6*(b*cosh(d*x + c)^5 - 2*b*co
sh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) - b)*sqrt(b)*arctan(sqrt(b)*
sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*
sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - 21*(b*cosh(d*x + c)^6 + 6*b*cosh(
d*x + c)*sinh(d*x + c)^5 + b*sinh(d*x + c)^6 - 3*b*cosh(d*x + c)^4 + 3*(5*b
*cosh(d*x + c)^2 - b)*sinh(d*x + c)^4 + 4*(5*b*cosh(d*x + c)^3 - 3*b*cosh(d
*x + c))*sinh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 3*(5*b*cosh(d*x + c)^4 - 6
*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 6*(b*cosh(d*x + c)^5 - 2*b*cosh(d
*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) - b)*sqrt(b)*log(2*b*cosh(d*x +
c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x + c) + 12*b*cosh(d*x + c)^2*sinh(d*x +
c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*(cosh(d*
x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + (6*cosh(d*x
+ c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c)^2 + 2*(2*cosh(d*x + c)^3 - cosh
(d*x + c))*sinh(d*x + c))*sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - b)
+ 16*(5*b*cosh(d*x + c)^6 + 30*b*cosh(d*x + c)*sinh(d*x + c)^5 + 5*b*sinh(d
*x + c)^6 + b*cosh(d*x + c)^4 + (75*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^4
+ 4*(25*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c)^3 + b*cosh(d*x +
 c)^2 + (75*b*cosh(d*x + c)^4 + 6*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 +
2*(15*b*cosh(d*x + c)^5 + 2*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x +
 c) + 5*b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)))/(d*cosh(d*x + c)^6 + 6*d*co
```

```
sh(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*x + c)^6 - 3*d*cosh(d*x + c)^4 + 3*(
5*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 - 3*d*cos
h(d*x + c))*sinh(d*x + c)^3 + 3*d*cosh(d*x + c)^2 + 3*(5*d*cosh(d*x + c)^4
- 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 6*(d*cosh(d*x + c)^5 - 2*d*cos
h(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) - d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\coth^3\left(c+dx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**3)**(3/2),x)
```

```
[Out] Integral((b*coth(c + d*x)**3)**(3/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 788 vs. 2(114) = 228.
time = 0.62, size = 788, normalized size = 5.88

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/42*(42*sqrt(b)*arctan(-(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c)
- b))/sqrt(b))*sgn(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c)
- 1)*sgn(e^(4*d*x + 4*c) - 1) - 21*sqrt(b)*log(abs(-sqrt(b)*e^(2*d*x + 2*c)
 + sqrt(b*e^(4*d*x + 4*c) - b)))*sgn(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) +
3*e^(2*d*x + 2*c) - 1)*sgn(e^(4*d*x + 4*c) - 1) + 16*(21*(sqrt(b)*e^(2*d*x
+ 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))^6*b*sgn(e^(6*d*x + 6*c) - 3*e^(4*d*x
+ 4*c) + 3*e^(2*d*x + 2*c) - 1)*sgn(e^(4*d*x + 4*c) - 1) - 42*(sqrt(b)*e^(2
*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))^5*b^(3/2)*sgn(e^(6*d*x + 6*c) -
3*e^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c) - 1)*sgn(e^(4*d*x + 4*c) - 1) + 119*(
sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))^4*b^2*sgn(e^(6*d*x +
 6*c) - 3*e^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c) - 1)*sgn(e^(4*d*x + 4*c) - 1)
 - 56*(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))^3*b^(5/2)*sgn
(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c) - 1)*sgn(e^(4*d*x
+ 4*c) - 1) + 63*(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))^2*
b^3*sgn(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c) - 1)*sgn(e^
(4*d*x + 4*c) - 1) - 14*(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) -
 b))*b^(7/2)*sgn(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c) -
1)*sgn(e^(4*d*x + 4*c) - 1) + 5*b^4*sgn(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c)
 + 3*e^(2*d*x + 2*c) - 1)*sgn(e^(4*d*x + 4*c) - 1))/(sqrt(b)*e^(2*d*x + 2*c
) - sqrt(b*e^(4*d*x + 4*c) - b) - sqrt(b))^7)*b/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\coth(c+d\,x)^3\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^3)^(3/2),x)
[Out] int((b*coth(c + d*x)^3)^(3/2), x)
```

3.30 $\int \sqrt{b \coth^3(c+dx)}\, dx$

Optimal. Leaf size=104

$$\frac{\operatorname{ArcTan}\left(\sqrt{\coth(c+dx)}\right)\sqrt{b\coth^3(c+dx)}}{d\coth^{\frac{3}{2}}(c+dx)}+\frac{\tanh^{-1}\left(\sqrt{\coth(c+dx)}\right)\sqrt{b\coth^3(c+dx)}}{d\coth^{\frac{3}{2}}(c+dx)}-\frac{2\sqrt{b\coth^3(}$$

```
[Out] arctan(coth(d*x+c)^(1/2))*(b*coth(d*x+c)^3)^(1/2)/d/coth(d*x+c)^(3/2)+arcta
nh(coth(d*x+c)^(1/2))*(b*coth(d*x+c)^3)^(1/2)/d/coth(d*x+c)^(3/2)-2*(b*coth
(d*x+c)^3)^(1/2)*tanh(d*x+c)/d
```

Rubi [A]
time = 0.04, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3739, 3554, 3557, 335, 218, 212, 209}

$$\frac{\operatorname{ArcTan}\left(\sqrt{\coth(c+dx)}\right)\sqrt{b\coth^3(c+dx)}}{d\coth^{\frac{3}{2}}(c+dx)}-\frac{2\tanh(c+dx)\sqrt{b\coth^3(c+dx)}}{d}+\frac{\sqrt{b\coth^3(c+dx)}\tanh^{-1}\left(\sqrt{\coth(c+dx)}\right)}{d\coth^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Coth[c + d*x]^3],x]
```

```
[Out] (ArcTan[Sqrt[Coth[c + d*x]]]*Sqrt[b*Coth[c + d*x]^3])/(d*Coth[c + d*x]^(3/2
)) + (ArcTanh[Sqrt[Coth[c + d*x]]]*Sqrt[b*Coth[c + d*x]^3])/(d*Coth[c + d*x
]^(3/2)) - (2*Sqrt[b*Coth[c + d*x]^3]*Tanh[c + d*x])/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \sqrt{b\coth^3(c+dx)}\,dx = \frac{\sqrt{b\coth^3(c+dx)}\int \coth^{\frac{3}{2}}(c+dx)\,dx}{\coth^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2\sqrt{b\coth^3(c+dx)}\tanh(c+dx)}{d} + \frac{\sqrt{b\coth^3(c+dx)}\int \frac{1}{\sqrt{\coth(c+dx)}}\,dx}{\coth^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2\sqrt{b\coth^3(c+dx)}\tanh(c+dx)}{d} - \frac{\sqrt{b\coth^3(c+dx)}\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(-1+x^2)}\,d\right.}{d\coth^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2\sqrt{b\coth^3(c+dx)}\tanh(c+dx)}{d} - \frac{\left(2\sqrt{b\coth^3(c+dx)}\right)\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx\right.}{d\coth^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2\sqrt{b\coth^3(c+dx)}\tanh(c+dx)}{d} + \frac{\sqrt{b\coth^3(c+dx)}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\right.}{d\coth^{\frac{3}{2}}(c+dx)}$$

$$= \frac{\tan^{-1}\left(\sqrt{\coth(c+dx)}\right)\sqrt{b\coth^3(c+dx)}}{d\coth^{\frac{3}{2}}(c+dx)} + \frac{\tanh^{-1}\left(\sqrt{\coth(c+dx)}\right)\sqrt{b\,c}}{d\coth^{\frac{3}{2}}(c+dx)}$$

Mathematica [A]
time = 0.07, size = 63, normalized size = 0.61

$$\frac{\left(\text{ArcTan}\left(\sqrt{\coth(c+dx)}\right)+\tanh^{-1}\left(\sqrt{\coth(c+dx)}\right)-2\sqrt{\coth(c+dx)}\right)\sqrt{b\coth^3(c+dx)}}{d\coth^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Coth[c + d*x]^3],x]
```

```
[Out] ((ArcTan[Sqrt[Coth[c + d*x]]] + ArcTanh[Sqrt[Coth[c + d*x]]] - 2*Sqrt[Coth[
c + d*x]])*Sqrt[b*Coth[c + d*x]^3])/(d*Coth[c + d*x]^(3/2))
```

Maple [A]
time = 2.57, size = 86, normalized size = 0.83

method	result
derivativedivides	$\frac{\sqrt{b\left(\coth^3(dx+c)\right)}\left(-2\sqrt{b\coth(dx+c)}+\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)+\sqrt{b}\,\text{arctan}\right.}{d\coth(dx+c)\sqrt{b\coth(dx+c)}}$

default	$\frac{\sqrt{b\left(\coth^{3}\left(dx+c\right)\right)}\left(-2\sqrt{b\coth\left(dx+c\right)}+\sqrt{b}\,\operatorname{arctanh}\left(\frac{\sqrt{b\coth\left(dx+c\right)}}{\sqrt{b}}\right)+\sqrt{b}\,\arctan\left(\right.\right.}{d\coth\left(dx+c\right)\sqrt{b\coth\left(dx+c\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*coth(d*x+c)^3)^(1/2)/coth(d*x+c)/(b*coth(d*x+c))^(1/2)*(-2*(b*coth(d
*x+c))^(1/2)+b^(1/2)*arctanh((b*coth(d*x+c))^(1/2)/b^(1/2))+b^(1/2)*arctan(
(b*coth(d*x+c))^(1/2)/b^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*coth(d*x + c)^3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 291 vs. 2(90) = 180.
time = 0.37, size = 633, normalized size = 6.09

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
 sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x
+ c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - sqrt(-
b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d
*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x
+ c)^4 - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d*x + c)^
4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4
*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 8*sqrt(b*cosh(d*x + c)
/sinh(d*x + c)))/d, 1/4*(2*sqrt(b)*arctan(sqrt(b)*sqrt(b*cosh(d*x + c)/sinh
(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*
x + c)^2 + b)) + sqrt(b)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh
(d*x + c) + 12*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d
*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh
```

```
(d*x + c)^3 + sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - c
osh(d*x + c)^2 + 2*(2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c))*sqrt(
b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - b) - 8*sqrt(b*cosh(d*x + c)/sinh(d
*x + c)))/d]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\coth^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**3)**(1/2),x)
```

```
[Out] Integral(sqrt(b*coth(c + d*x)**3), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(90) = 180.
time = 0.49, size = 269, normalized size = 2.59

$$-\frac{2\sqrt{b}\arctan\left(-\frac{\sqrt{b}e^{(2dx+2c)}-\sqrt{be^{(4dx+4c)}-b}}{\sqrt{b}}\right)\operatorname{sgn}\left(e^{(6dx+6c)}-3e^{(4dx+4c)}+3e^{(2dx+2c)}-1\right)\operatorname{sgn}\left(e^{(4dx+4c)}-1\right)+\sqrt{b}\log\left(\left|-\sqrt{b}e^{(2dx+2c)}+\sqrt{be^{(4dx+4c)}-b}\right|\right)\operatorname{sgn}\left(e^{(6dx+6c)}-3e^{(4dx+4c)}+3e^{(2dx+2c)}-1\right)\operatorname{sgn}\left(e^{(4dx+4c)}-1\right)-\frac{8b\operatorname{sgn}\left(e^{(6dx+6c)}-3e^{(4dx+4c)}+3e^{(2dx+2c)}-1\right)\operatorname{sgn}\left(e^{(4dx+4c)}-1\right)}{\sqrt{b}e^{(2dx+2c)}-\sqrt{be^{(4dx+4c)}-b}-\sqrt{b}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(2*sqrt(b)*arctan(-(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) -
 b))/sqrt(b))*sgn(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c) -
 1)*sgn(e^(4*d*x + 4*c) - 1) + sqrt(b)*log(abs(-sqrt(b)*e^(2*d*x + 2*c) + s
qrt(b*e^(4*d*x + 4*c) - b)))*sgn(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*e^
(2*d*x + 2*c) - 1)*sgn(e^(4*d*x + 4*c) - 1) - 8*b*sgn(e^(6*d*x + 6*c) - 3*e
^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c) - 1)*sgn(e^(4*d*x + 4*c) - 1)/(sqrt(b)*e
^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b) - sqrt(b)))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{b\,{\mathrm{coth}\left(c+d\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^3)^(1/2),x)
```

```
[Out] int((b*coth(c + d*x)^3)^(1/2), x)
```

3.31 $\int \frac{1}{\sqrt{b\coth^3(c+dx)}}\,dx$

Optimal. Leaf size=105

$$-\frac{2\coth(c+dx)}{d\sqrt{b\coth^3(c+dx)}}-\frac{\text{ArcTan}\left(\sqrt{\coth(c+dx)}\right)\coth^{\frac{3}{2}}(c+dx)}{d\sqrt{b\coth^3(c+dx)}}+\frac{\tanh^{-1}\left(\sqrt{\coth(c+dx)}\right)\coth^{\frac{3}{2}}(c+dx)}{d\sqrt{b\coth^3(c+dx)}}$$

```
[Out] -2*coth(d*x+c)/d/(b*coth(d*x+c)^3)^(1/2)-arctan(coth(d*x+c)^(1/2))*coth(d*x
+c)^(3/2)/d/(b*coth(d*x+c)^3)^(1/2)+arctanh(coth(d*x+c)^(1/2))*coth(d*x+c)^
(3/2)/d/(b*coth(d*x+c)^3)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3739, 3555, 3557, 335, 304, 209, 212}

$$-\frac{\coth^{\frac{3}{2}}(c+dx)\text{ArcTan}\left(\sqrt{\coth(c+dx)}\right)}{d\sqrt{b\coth^3(c+dx)}}-\frac{2\coth(c+dx)}{d\sqrt{b\coth^3(c+dx)}}+\frac{\coth^{\frac{3}{2}}(c+dx)\tanh^{-1}\left(\sqrt{\coth(c+dx)}\right)}{d\sqrt{b\coth^3(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Coth[c + d*x]^3],x]
```

```
[Out] (-2*Coth[c + d*x])/(d*Sqrt[b*Coth[c + d*x]^3]) - (ArcTan[Sqrt[Coth[c + d*x]
]]*Coth[c + d*x]^(3/2))/(d*Sqrt[b*Coth[c + d*x]^3]) + (ArcTanh[Sqrt[Coth[c
+ d*x]]]*Coth[c + d*x]^(3/2))/(d*Sqrt[b*Coth[c + d*x]^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
```

```
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\sqrt{b\coth^3(c+dx)}}\,dx = \frac{\coth^{\frac{3}{2}}(c+dx)\int \frac{1}{\coth^{\frac{3}{2}}(c+dx)}\,dx}{\sqrt{b\coth^3(c+dx)}}$$

$$= -\frac{2\coth(c+dx)}{d\sqrt{b\coth^3(c+dx)}} + \frac{\coth^{\frac{3}{2}}(c+dx)\int \sqrt{\coth(c+dx)}\,dx}{\sqrt{b\coth^3(c+dx)}}$$

$$= -\frac{2\coth(c+dx)}{d\sqrt{b\coth^3(c+dx)}} - \frac{\coth^{\frac{3}{2}}(c+dx)\mathrm{Subst}\left(\int \frac{\sqrt{x}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d\sqrt{b\coth^3(c+dx)}}$$

$$= -\frac{2\coth(c+dx)}{d\sqrt{b\coth^3(c+dx)}} - \frac{\left(2\coth^{\frac{3}{2}}(c+dx)\right)\mathrm{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt{\coth(c+dx)}\right.}{d\sqrt{b\coth^3(c+dx)}}$$

$$= -\frac{2\coth(c+dx)}{d\sqrt{b\coth^3(c+dx)}} + \frac{\coth^{\frac{3}{2}}(c+dx)\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\coth(c+dx)}\right)}{d\sqrt{b\coth^3(c+dx)}} - $$

$$= -\frac{2\coth(c+dx)}{d\sqrt{b\coth^3(c+dx)}} - \frac{\tan^{-1}\left(\sqrt{\coth(c+dx)}\right)\coth^{\frac{3}{2}}(c+dx)}{d\sqrt{b\coth^3(c+dx)}} + \frac{\tanh^{-1}\left(\sqrt{c\right.}}{d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.02, size = 41, normalized size = 0.39

$$-\frac{2\coth(c+dx)\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};\coth^2(c+dx)\right)}{d\sqrt{b\coth^3(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Coth[c + d*x]^3],x]
```

```
[Out] (-2*Coth[c + d*x]*Hypergeometric2F1[-1/4, 1, 3/4, Coth[c + d*x]^2])/(d*Sqrt
[b*Coth[c + d*x]^3])
```

Maple [A]
time = 2.42, size = 92, normalized size = 0.88

method	result

derivativedivides	$-\frac{\coth(dx+c)\left(2b^{\frac{5}{2}}+\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)b^{2}\sqrt{b\coth(dx+c)}-\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c}}{\sqrt{b}}\right.\right.}{d\sqrt{b\left(\coth^{3}(dx+c)\right)}\,b^{\frac{5}{2}}}$
default	$-\frac{\coth(dx+c)\left(2b^{\frac{5}{2}}+\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)b^{2}\sqrt{b\coth(dx+c)}-\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c}}{\sqrt{b}}\right.\right.}{d\sqrt{b\left(\coth^{3}(dx+c)\right)}\,b^{\frac{5}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*coth(d*x+c)*(2*b^(5/2)+arctan((b*coth(d*x+c))^(1/2)/b^(1/2))*b^2*(b*co
th(d*x+c))^(1/2)-arctanh((b*coth(d*x+c))^(1/2)/b^(1/2))*b^2*(b*coth(d*x+c))
^(1/2))/(b*coth(d*x+c)^3)^(1/2)/b^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*coth(d*x + c)^3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 429 vs. 2(91) = 182.
time = 0.40, size = 907, normalized size = 8.64

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2
 + 1)*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + si
nh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x + c
)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) + (cosh(d*x
 + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*sqrt(-b)*log
(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d*x + c
)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4
 - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1
)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d*x + c)^4 + 4*
```

```
cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(
d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 8*(cosh(d*x + c)^2 + 2*cosh(
d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(b*cosh(d*x + c)/sinh(d*x
 + c)))/(b*d*cosh(d*x + c)^2 + 2*b*d*cosh(d*x + c)*sinh(d*x + c) + b*d*sinh
(d*x + c)^2 + b*d), -1/4*(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c
) + sinh(d*x + c)^2 + 1)*sqrt(b)*arctan(sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d
*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x
+ c)^2 + b)) - (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x
+ c)^2 + 1)*sqrt(b)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x
+ c) + 12*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x +
c)^3 + 2*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x
+ c)^3 + sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d
*x + c)^2 + 2*(2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sq
rt(b*cosh(d*x + c)/sinh(d*x + c)) - b) + 8*(cosh(d*x + c)^2 + 2*cosh(d*x +
c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))
)/(b*d*cosh(d*x + c)^2 + 2*b*d*cosh(d*x + c)*sinh(d*x + c) + b*d*sinh(d*x +
 c)^2 + b*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\coth^3\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*coth(c + d*x)**3), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 279 vs. 2(91) = 182.
time = 0.57, size = 279, normalized size = 2.66

$$\frac{\frac{2\arctan\left(-\frac{\sqrt{b}\,e^{(2\,dx+2\,c)}-\sqrt{be^{(4\,dx+4\,c)}-b}}{\sqrt{b}}\right)}{\sqrt{b}\,\mathrm{sgn}\left(e^{(6\,dx+6\,c)}-3\,e^{(4\,dx+4\,c)}+3\,e^{(2\,dx+2\,c)}-1\right)\mathrm{sgn}\left(e^{(4\,dx+4\,c)}-1\right)}-\frac{\log\left(\left|-\sqrt{b}\,e^{(2\,dx+2\,c)}+\sqrt{be^{(4\,dx+4\,c)}-b}\right|\right)}{\sqrt{b}\,\mathrm{sgn}\left(e^{(6\,dx+6\,c)}-3\,e^{(4\,dx+4\,c)}+3\,e^{(2\,dx+2\,c)}-1\right)\mathrm{sgn}\left(e^{(4\,dx+4\,c)}-1\right)}-\frac{8}{\left(\sqrt{b}\,e^{(2\,dx+2\,c)}-\sqrt{be^{(4\,dx+4\,c)}-b}+\sqrt{b}\right)\mathrm{sgn}\left(e^{(6\,dx+6\,c)}-3\,e^{(4\,dx+4\,c)}+3\,e^{(2\,dx+2\,c)}-1\right)\mathrm{sgn}\left(e^{(4\,dx+4\,c)}-1\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*arctan(-(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))/sqrt
(b))/(sqrt(b)*sgn(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c) -
 1)*sgn(e^(4*d*x + 4*c) - 1)) - log(abs(-sqrt(b)*e^(2*d*x + 2*c) + sqrt(b*e
^(4*d*x + 4*c) - b)))/(sqrt(b)*sgn(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*
e^(2*d*x + 2*c) - 1)*sgn(e^(4*d*x + 4*c) - 1)) - 8/((sqrt(b)*e^(2*d*x + 2*c
) - sqrt(b*e^(4*d*x + 4*c) - b) + sqrt(b))*sgn(e^(6*d*x + 6*c) - 3*e^(4*d*x
 + 4*c) + 3*e^(2*d*x + 2*c) - 1)*sgn(e^(4*d*x + 4*c) - 1)))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{b\coth\left(c+d\,x\right)^3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x)^3)^(1/2),x)
[Out] int(1/(b*coth(c + d*x)^3)^(1/2), x)
```

3.32 $\int \frac{1}{\left(b\coth^3(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=141

$$-\frac{2}{3bd\sqrt{b\coth^3(c+dx)}}+\frac{\mathrm{ArcTan}\left(\sqrt{\coth(c+dx)}\right)\coth^{\frac{3}{2}}(c+dx)}{bd\sqrt{b\coth^3(c+dx)}}+\frac{\tanh^{-1}\left(\sqrt{\coth(c+dx)}\right)\coth^{\frac{3}{2}}(c+d$$

```
[Out] -2/3/b/d/(b*coth(d*x+c)^3)^(1/2)+arctan(coth(d*x+c)^(1/2))*coth(d*x+c)^(3/2
)/b/d/(b*coth(d*x+c)^3)^(1/2)+arctanh(coth(d*x+c)^(1/2))*coth(d*x+c)^(3/2)/
b/d/(b*coth(d*x+c)^3)^(1/2)-2/7*tanh(d*x+c)^2/b/d/(b*coth(d*x+c)^3)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3739, 3555, 3557, 335, 218, 212, 209}

$$\frac{\coth^{\frac{3}{2}}(c+dx)\mathrm{ArcTan}\left(\sqrt{\coth(c+dx)}\right)}{bd\sqrt{b\coth^3(c+dx)}}-\frac{2}{3bd\sqrt{b\coth^3(c+dx)}}-\frac{2\tanh^2(c+dx)}{7bd\sqrt{b\coth^3(c+dx)}}+\frac{\coth^{\frac{3}{2}}(c+dx)\tanh^{-1}\left(\sqrt{\coth(c+dx)}\right)}{bd\sqrt{b\coth^3(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^3)^(-3/2),x]
```

```
[Out] -2/(3*b*d*Sqrt[b*Coth[c + d*x]^3]) + (ArcTan[Sqrt[Coth[c + d*x]]]*Coth[c +
d*x]^(3/2))/(b*d*Sqrt[b*Coth[c + d*x]^3]) + (ArcTanh[Sqrt[Coth[c + d*x]]]*C
oth[c + d*x]^(3/2))/(b*d*Sqrt[b*Coth[c + d*x]^3]) - (2*Tanh[c + d*x]^2)/(7*
b*d*Sqrt[b*Coth[c + d*x]^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
```

```
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(b\coth^3(c+dx)\right)^{3/2}}\,dx &= \frac{\coth^{\frac{3}{2}}(c+dx)\int \frac{1}{\coth^{\frac{9}{2}}(c+dx)}\,dx}{b\sqrt{b\coth^3(c+dx)}} \\
&= -\frac{2\tanh^2(c+dx)}{7bd\sqrt{b\coth^3(c+dx)}} + \frac{\coth^{\frac{3}{2}}(c+dx)\int \frac{1}{\coth^{\frac{5}{2}}(c+dx)}\,dx}{b\sqrt{b\coth^3(c+dx)}} \\
&= -\frac{2}{3bd\sqrt{b\coth^3(c+dx)}} - \frac{2\tanh^2(c+dx)}{7bd\sqrt{b\coth^3(c+dx)}} + \frac{\coth^{\frac{3}{2}}(c+dx)\int \frac{1}{\sqrt{\coth(c+}}}{b\sqrt{b\coth^3(c+dx)}} \\
&= -\frac{2}{3bd\sqrt{b\coth^3(c+dx)}} - \frac{2\tanh^2(c+dx)}{7bd\sqrt{b\coth^3(c+dx)}} - \frac{\coth^{\frac{3}{2}}(c+dx)\text{Subst}\left(\int \frac{}{\sqrt{}}\right.}{bd\sqrt{b\cot}} \\
&= -\frac{2}{3bd\sqrt{b\coth^3(c+dx)}} - \frac{2\tanh^2(c+dx)}{7bd\sqrt{b\coth^3(c+dx)}} - \frac{\left(2\coth^{\frac{3}{2}}(c+dx)\right)\text{Subst}\left(\right.}{bd\sqrt{b\,c}} \\
&= -\frac{2}{3bd\sqrt{b\coth^3(c+dx)}} - \frac{2\tanh^2(c+dx)}{7bd\sqrt{b\coth^3(c+dx)}} + \frac{\coth^{\frac{3}{2}}(c+dx)\text{Subst}\left(\int \frac{1}{1-x}\right.}{bd\sqrt{b\coth^3}} \\
&= -\frac{2}{3bd\sqrt{b\coth^3(c+dx)}} + \frac{\tan^{-1}\left(\sqrt{\coth(c+dx)}\right)\coth^{\frac{3}{2}}(c+dx)}{bd\sqrt{b\coth^3(c+dx)}} + \frac{\tanh^{-1}\left(\right.}{}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.05, size = 43, normalized size = 0.30

$$
-\frac{2\coth(c+dx)\,{}_2F_1\left(-\frac{7}{4},1;-\frac{3}{4};\coth^2(c+dx)\right)}{7d\left(b\coth^3(c+dx)\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^3)^(-3/2),x]
```

```
[Out] (-2*Coth[c + d*x]*Hypergeometric2F1[-7/4, 1, -3/4, Coth[c + d*x]^2])/(7*d*(
b*Coth[c + d*x]^3)^(3/2))
```

Maple [A]
time = 2.23, size = 106, normalized size = 0.75

method	result
derivativedivides	$-\frac{\coth(dx+c)\left(14b^{\frac{15}{2}}(\coth^2(dx+c))+6b^{\frac{15}{2}}-21\,\mathrm{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)b^4(b\coth(dx+c))^{\frac{7}{2}}-21\arctan\left(\right.\right.}{21d\,b^{\frac{15}{2}}(b(\coth^3(dx+c)))^{\frac{3}{2}}}$
default	$-\frac{\coth(dx+c)\left(14b^{\frac{15}{2}}(\coth^2(dx+c))+6b^{\frac{15}{2}}-21\,\mathrm{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)b^4(b\coth(dx+c))^{\frac{7}{2}}-21\arctan\left(\right.\right.}{21d\,b^{\frac{15}{2}}(b(\coth^3(dx+c)))^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/21/d*coth(d*x+c)/b^(15/2)*(14*b^(15/2)*coth(d*x+c)^2+6*b^(15/2)-21*arcta
nh((b*coth(d*x+c))^(1/2)/b^(1/2))*b^4*(b*coth(d*x+c))^(7/2)-21*arctan((b*co
th(d*x+c))^(1/2)/b^(1/2))*b^4*(b*coth(d*x+c))^(7/2))/(b*coth(d*x+c)^3)^(3/2
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^3)^(-3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1486 vs. $2(121) = 242$.
time = 0.44, size = 3022, normalized size = 21.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/84*(42*(cosh(d*x + c)^8 + 8*cosh(d*x + c)*sinh(d*x + c)^7 + sinh(d*x +
c)^8 + 4*(7*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^6 + 4*cosh(d*x + c)^6 + 8*(7
*cosh(d*x + c)^3 + 3*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*cosh(d*x + c)^4
 + 30*cosh(d*x + c)^2 + 3)*sinh(d*x + c)^4 + 6*cosh(d*x + c)^4 + 8*(7*cosh(
d*x + c)^5 + 10*cosh(d*x + c)^3 + 3*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*c
osh(d*x + c)^6 + 15*cosh(d*x + c)^4 + 9*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^
2 + 4*cosh(d*x + c)^2 + 8*(cosh(d*x + c)^7 + 3*cosh(d*x + c)^5 + 3*cosh(d*x
 + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(-b)*arctan((cosh(d*x + c)^
```

2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) + 21*(cosh(d*x + c)^8 + 8*cosh(d*x + c)*sinh(d*x + c)^7 + sinh(d*x + c)^8 + 4*(7*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^6 + 4*cosh(d*x + c)^6 + 8*(7*cosh(d*x + c)^3 + 3*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*cosh(d*x + c)^4 + 30*cosh(d*x + c)^2 + 3)*sinh(d*x + c)^4 + 6*cosh(d*x + c)^4 + 8*(7*cosh(d*x + c)^5 + 10*cosh(d*x + c)^3 + 3*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*cosh(d*x + c)^6 + 15*cosh(d*x + c)^4 + 9*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 4*cosh(d*x + c)^2 + 8*(cosh(d*x + c)^7 + 3*cosh(d*x + c)^5 + 3*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(-b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 16*(5*cosh(d*x + c)^8 + 40*cosh(d*x + c)*sinh(d*x + c)^7 + 5*sinh(d*x + c)^8 + 2*(70*cosh(d*x + c)^2 - 3)*sinh(d*x + c)^6 - 6*cosh(d*x + c)^6 + 4*(70*cosh(d*x + c)^3 - 9*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(175*cosh(d*x + c)^4 - 45*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^4 + 2*cosh(d*x + c)^4 + 8*(35*cosh(d*x + c)^5 - 15*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c)^3 + 2*(70*cosh(d*x + c)^6 - 45*cosh(d*x + c)^4 + 6*cosh(d*x + c)^2 - 3)*sinh(d*x + c)^2 - 6*cosh(d*x + c)^2 + 4*(10*cosh(d*x + c)^7 - 9*cosh(d*x + c)^5 + 2*cosh(d*x + c)^3 - 3*cosh(d*x + c))*sinh(d*x + c) + 5)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)))/(b^2*d*cosh(d*x + c)^8 + 8*b^2*d*cosh(d*x + c)*sinh(d*x + c)^7 + b^2*d*sinh(d*x + c)^8 + 4*b^2*d*cosh(d*x + c)^6 + 6*b^2*d*cosh(d*x + c)^4 + 4*(7*b^2*d*cosh(d*x + c)^2 + b^2*d)*sinh(d*x + c)^6 + 8*(7*b^2*d*cosh(d*x + c)^3 + 3*b^2*d*cosh(d*x + c))*sinh(d*x + c)^5 + 4*b^2*d*cosh(d*x + c)^2 + 2*(35*b^2*d*cosh(d*x + c)^4 + 30*b^2*d*cosh(d*x + c)^2 + 3*b^2*d)*sinh(d*x + c)^4 + 8*(7*b^2*d*cosh(d*x + c)^5 + 10*b^2*d*cosh(d*x + c)^3 + 3*b^2*d*cosh(d*x + c))*sinh(d*x + c)^3 + b^2*d + 4*(7*b^2*d*cosh(d*x + c)^6 + 15*b^2*d*cosh(d*x + c)^4 + 9*b^2*d*cosh(d*x + c)^2 + b^2*d)*sinh(d*x + c)^2 + 8*(b^2*d*cosh(d*x + c)^7 + 3*b^2*d*cosh(d*x + c)^5 + 3*b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c))*sinh(d*x + c)), 1/84*(42*(cosh(d*x + c)^8 + 8*cosh(d*x + c)*sinh(d*x + c)^7 + sinh(d*x + c)^8 + 4*(7*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^6 + 4*cosh(d*x + c)^6 + 8*(7*cosh(d*x + c)^3 + 3*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*cosh(d*x + c)^4 + 30*cosh(d*x + c)^2 + 3)*sinh(d*x + c)^4 + 6*cosh(d*x + c)^4 + 8*(7*cosh(d*x + c)^5 + 10*cosh(d*x + c)^3 + 3*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*cosh(d*x + c)^6 + 15*cosh(d*x + c)^4 + 9*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 4*cosh(d*x + c)^2 + 8*(cosh(d*x + c)^7 + 3*cosh(d*x + c)^5 + 3*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(b)*arctan(sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) + 21*(cosh(d*x + c)^8 + 8*cosh(d*x + c)*sinh(d*x + c)^7 + sinh(d*x + c)^8 + 4*(7*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^6 + 4*cosh(d*x + c)^6 + 8*(7*cosh(d*x + c)^3 + 3*cosh(d*x + c))*si

```
nh(d*x + c)^5 + 2*(35*cosh(d*x + c)^4 + 30*cosh(d*x + c)^2 + 3)*sinh(d*x +
c)^4 + 6*cosh(d*x + c)^4 + 8*(7*cosh(d*x + c)^5 + 10*cosh(d*x + c)^3 + 3*co
sh(d*x + c))*sinh(d*x + c)^3 + 4*(7*cosh(d*x + c)^6 + 15*cosh(d*x + c)^4 +
9*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 4*cosh(d*x + c)^2 + 8*(cosh(d*x +
c)^7 + 3*cosh(d*x + c)^5 + 3*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c)
 + 1)*sqrt(b)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x + c) +
 12*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c)^3 +
 2*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3
 + sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c
)^2 + 2*(2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sqrt(b*c
osh(d*x + c)/sinh(d*x + c)) - b) - 16*(5*cosh(d*x + c)^8 + 40*cosh(d*x + c)
*sinh(d*x + c)^7 + 5*sinh(d*x + c)^8 + 2*(70*cosh(d*x + c)^2 - 3)*sinh(d*x
+ c)^6 - 6*cosh(d*x + c)^6 + 4*(70*cosh(d*x + c)^3 - 9*cosh(d*x + c))*sinh(
d*x + c)^5 + 2*(175*cosh(d*x + c)^4 - 45*cosh(d...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\coth^3\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**3)**(3/2),x)
```

```
[Out] Integral((b*coth(c + d*x)**3)**(-3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 521 vs. 2(121) = 242.
time = 0.59, size = 521, normalized size = 3.70

$$-\frac{\frac{42\arctan\left(-\frac{\sqrt{b}e^{(2dx+2c)}-\sqrt{be^{(4dx+4c)}-b}}{\sqrt{b}}\right)}{\sqrt{b}\,\mathrm{sgn}\left(e^{(6dx+6c)}-3e^{(4dx+4c)}+3e^{(2dx+2c)}-1\right)\mathrm{sgn}\left(e^{(4dx+4c)}-1\right)}+\frac{21\log\left(\left|-\sqrt{b}e^{(2dx+2c)}+\sqrt{be^{(4dx+4c)}-b}\right|\right)}{\sqrt{b}\,\mathrm{sgn}\left(e^{(6dx+6c)}-3e^{(4dx+4c)}+3e^{(2dx+2c)}-1\right)\mathrm{sgn}\left(e^{(4dx+4c)}-1\right)}+\frac{16\left(21\left(\sqrt{b}e^{(2dx+2c)}-\sqrt{be^{(4dx+4c)}-b}\right)^6+42\left(\sqrt{b}e^{(2dx+2c)}-\sqrt{be^{(4dx+4c)}-b}\right)^5\sqrt{b}+119\left(\sqrt{b}e^{(2dx+2c)}-\sqrt{be^{(4dx+4c)}-b}\right)^4b+56\left(\sqrt{b}e^{(2dx+2c)}-\sqrt{be^{(4dx+4c)}-b}\right)^3b^{\frac{3}{2}}+63\left(\sqrt{b}e^{(2dx+2c)}-\sqrt{be^{(4dx+4c)}-b}\right)^2b^2+14\left(\sqrt{b}e^{(2dx+2c)}-\sqrt{be^{(4dx+4c)}-b}\right)b^{\frac{5}{2}}+5b^3\right)}{\left(\sqrt{b}e^{(2dx+2c)}-\sqrt{be^{(4dx+4c)}-b}+\sqrt{b}\right)^7\mathrm{sgn}\left(e^{(6dx+6c)}-3e^{(4dx+4c)}+3e^{(2dx+2c)}-1\right)\mathrm{sgn}\left(e^{(4dx+4c)}-1\right)}}{42\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(3/2),x, algorithm="giac")
```

```
[Out] -1/42*(42*arctan(-(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))/s
qrt(b))/(sqrt(b)*sgn(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c
) - 1)*sgn(e^(4*d*x + 4*c) - 1)) + 21*log(abs(-sqrt(b)*e^(2*d*x + 2*c) + sq
rt(b*e^(4*d*x + 4*c) - b)))/(sqrt(b)*sgn(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c
) + 3*e^(2*d*x + 2*c) - 1)*sgn(e^(4*d*x + 4*c) - 1)) + 16*(21*(sqrt(b)*e^(2
*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))^6 + 42*(sqrt(b)*e^(2*d*x + 2*c)
- sqrt(b*e^(4*d*x + 4*c) - b))^5*sqrt(b) + 119*(sqrt(b)*e^(2*d*x + 2*c) - s
qrt(b*e^(4*d*x + 4*c) - b))^4*b + 56*(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4
*d*x + 4*c) - b))^3*b^(3/2) + 63*(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x
 + 4*c) - b))^2*b^2 + 14*(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c)
```

```
- b))*b^(5/2) + 5*b^3)/((sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) -
 b) + sqrt(b))^7*sgn(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c
) - 1)*sgn(e^(4*d*x + 4*c) - 1)))/(b*d)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\coth\left(c+d\,x\right)^3\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x)^3)^(3/2),x)
```

```
[Out] int(1/(b*coth(c + d*x)^3)^(3/2), x)
```

3.33 $\int \left(b\coth^3(c+dx)\right)^{4/3} dx$

Optimal. Leaf size=74

$$-\frac{b\sqrt[3]{b\coth^3(c+dx)}}{d}-\frac{b\coth^2(c+dx)\sqrt[3]{b\coth^3(c+dx)}}{3d}+bx\sqrt[3]{b\coth^3(c+dx)}\tanh(c+dx)$$

```
[Out] -b*(b*coth(d*x+c)^3)^(1/3)/d-1/3*b*coth(d*x+c)^2*(b*coth(d*x+c)^3)^(1/3)/d+
b*x*(b*coth(d*x+c)^3)^(1/3)*tanh(d*x+c)
```

Rubi [A]
time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3739, 3554, 8}

$$-\frac{b\sqrt[3]{b\coth^3(c+dx)}}{d}-\frac{b\coth^2(c+dx)\sqrt[3]{b\coth^3(c+dx)}}{3d}+bx\tanh(c+dx)\sqrt[3]{b\coth^3(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^3)^(4/3),x]
```

```
[Out] -((b*(b*Coth[c + d*x]^3)^(1/3))/d) - (b*Coth[c + d*x]^2*(b*Coth[c + d*x]^3)
^(1/3))/(3*d) + b*x*(b*Coth[c + d*x]^3)^(1/3)*Tanh[c + d*x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(b\coth^3(c+dx)\right)^{4/3} dx = \left(b\sqrt[3]{b\coth^3(c+dx)}\,\tanh(c+dx)\right)\int \coth^4(c+dx)\,dx$$
$$= -\frac{b\coth^2(c+dx)\sqrt[3]{b\coth^3(c+dx)}}{3d} + \left(b\sqrt[3]{b\coth^3(c+dx)}\,\tanh(c+dx)\right)\int c$$
$$= -\frac{b\sqrt[3]{b\coth^3(c+dx)}}{d} - \frac{b\coth^2(c+dx)\sqrt[3]{b\coth^3(c+dx)}}{3d} + \left(b\sqrt[3]{b\coth^3(c+c}\right.$$
$$= -\frac{b\sqrt[3]{b\coth^3(c+dx)}}{d} - \frac{b\coth^2(c+dx)\sqrt[3]{b\coth^3(c+dx)}}{3d} + bx\sqrt[3]{b\coth^3(c+d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.05, size = 43, normalized size = 0.58

$$-\frac{\left(b\coth^3(c+dx)\right)^{4/3} {}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\tanh^2(c+dx)\right)\tanh(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^3)^(4/3),x]
```

```
[Out] -1/3*((b*Coth[c + d*x]^3)^(4/3)*Hypergeometric2F1[-3/2, 1, -1/2, Tanh[c + d
*x]^2]*Tanh[c + d*x])/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(66) = 132.
time = 2.65, size = 145, normalized size = 1.96

method	result	size
risch	$\frac{b\left(e^{2dx+2c}-1\right)\left(\frac{b\left(1+e^{2dx+2c}\right)^3}{\left(e^{2dx+2c}-1\right)^3}\right)^{\frac{1}{3}} x}{1+e^{2dx+2c}} - \frac{4b\left(\frac{b\left(1+e^{2dx+2c}\right)^3}{\left(e^{2dx+2c}-1\right)^3}\right)^{\frac{1}{3}}\left(3\,e^{4dx+4c}-3\,e^{2dx+2c}+2\right)}{3\left(1+e^{2dx+2c}\right)\left(e^{2dx+2c}-1\right)^2 d}$	145

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^3)^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] b/(1+exp(2*d*x+2*c))*(exp(2*d*x+2*c)-1)*(b*(1+exp(2*d*x+2*c))^3/(exp(2*d*x+
2*c)-1)^3)^(1/3)*x-4/3*b/(1+exp(2*d*x+2*c))/(exp(2*d*x+2*c)-1)^2*(b*(1+exp(
2*d*x+2*c))^3/(exp(2*d*x+2*c)-1)^3)^(1/3)*(3*exp(4*d*x+4*c)-3*exp(2*d*x+2*c
)+2)/d
```

Maxima [A]
time = 0.49, size = 87, normalized size = 1.18

$$\frac{(dx+c)b^{\frac{4}{3}}}{d}-\frac{4\left(3\,b^{\frac{4}{3}}e^{(-2\,dx-2\,c)}-3\,b^{\frac{4}{3}}e^{(-4\,dx-4\,c)}-2\,b^{\frac{4}{3}}\right)}{3\,d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(4/3),x, algorithm="maxima")
```

```
[Out] (d*x + c)*b^(4/3)/d - 4/3*(3*b^(4/3)*e^(-2*d*x - 2*c) - 3*b^(4/3)*e^(-4*d*x
 - 4*c) - 2*b^(4/3))/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*
x - 6*c) - 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1046 vs. 2(66) = 132.
time = 0.38, size = 1046, normalized size = 14.14

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(4/3),x, algorithm="fricas")
```

```
[Out] -1/3*(3*b*d*x*cosh(d*x + c)^6 - 3*(b*d*x*e^(2*d*x + 2*c) - b*d*x)*sinh(d*x
+ c)^6 - 18*(b*d*x*cosh(d*x + c)*e^(2*d*x + 2*c) - b*d*x*cosh(d*x + c))*sin
h(d*x + c)^5 - 3*(3*b*d*x + 4*b)*cosh(d*x + c)^4 + 3*(15*b*d*x*cosh(d*x + c
)^2 - 3*b*d*x - (15*b*d*x*cosh(d*x + c)^2 - 3*b*d*x - 4*b)*e^(2*d*x + 2*c)
- 4*b)*sinh(d*x + c)^4 + 12*(5*b*d*x*cosh(d*x + c)^3 - (3*b*d*x + 4*b)*cosh
(d*x + c) - (5*b*d*x*cosh(d*x + c)^3 - (3*b*d*x + 4*b)*cosh(d*x + c))*e^(2*
d*x + 2*c))*sinh(d*x + c)^3 - 3*b*d*x + 3*(3*b*d*x + 4*b)*cosh(d*x + c)^2 +
 3*(15*b*d*x*cosh(d*x + c)^4 + 3*b*d*x - 6*(3*b*d*x + 4*b)*cosh(d*x + c)^2
- (15*b*d*x*cosh(d*x + c)^4 + 3*b*d*x - 6*(3*b*d*x + 4*b)*cosh(d*x + c)^2 +
 4*b)*e^(2*d*x + 2*c) + 4*b)*sinh(d*x + c)^2 - (3*b*d*x*cosh(d*x + c)^6 - 3
*(3*b*d*x + 4*b)*cosh(d*x + c)^4 - 3*b*d*x + 3*(3*b*d*x + 4*b)*cosh(d*x + c
)^2 - 8*b)*e^(2*d*x + 2*c) + 6*(3*b*d*x*cosh(d*x + c)^5 - 2*(3*b*d*x + 4*b)
*cosh(d*x + c)^3 + (3*b*d*x + 4*b)*cosh(d*x + c) - (3*b*d*x*cosh(d*x + c)^5
 - 2*(3*b*d*x + 4*b)*cosh(d*x + c)^3 + (3*b*d*x + 4*b)*cosh(d*x + c))*e^(2*
d*x + 2*c))*sinh(d*x + c) - 8*b)*((b*e^(6*d*x + 6*c) + 3*b*e^(4*d*x + 4*c)
+ 3*b*e^(2*d*x + 2*c) + b)/(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*e^(2*d*
x + 2*c) - 1))^(1/3)/(d*cosh(d*x + c)^6 + (d*e^(2*d*x + 2*c) + d)*sinh(d*x
+ c)^6 + 6*(d*cosh(d*x + c)*e^(2*d*x + 2*c) + d*cosh(d*x + c))*sinh(d*x + c
)^5 - 3*d*cosh(d*x + c)^4 + 3*(5*d*cosh(d*x + c)^2 + (5*d*cosh(d*x + c)^2 -
 d)*e^(2*d*x + 2*c) - d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 - 3*d*cos
h(d*x + c) + (5*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*e^(2*d*x + 2*c))*sin
h(d*x + c)^3 + 3*d*cosh(d*x + c)^2 + 3*(5*d*cosh(d*x + c)^4 - 6*d*cosh(d*x
+ c)^2 + (5*d*cosh(d*x + c)^4 - 6*d*cosh(d*x + c)^2 + d)*e^(2*d*x + 2*c) +
```

```
d)*sinh(d*x + c)^2 + (d*cosh(d*x + c)^6 - 3*d*cosh(d*x + c)^4 + 3*d*cosh(d*
x + c)^2 - d)*e^(2*d*x + 2*c) + 6*(d*cosh(d*x + c)^5 - 2*d*cosh(d*x + c)^3
+ d*cosh(d*x + c) + (d*cosh(d*x + c)^5 - 2*d*cosh(d*x + c)^3 + d*cosh(d*x +
 c))*e^(2*d*x + 2*c))*sinh(d*x + c) - d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(66) = 132$.
time = 106.75, size = 162, normalized size = 2.19

$$\begin{cases} x\left(b\coth^3(c)\right)^{\frac{4}{3}} & \text{for } d = 0 \\ -\frac{\left(b\coth^3\left(dx+\log\left(-e^{-dx}\right)\right)\right)^{\frac{4}{3}}\log\left(-e^{-dx}\right)}{d} & \text{for } c = \log\left(-e^{-dx}\right) \\ -\frac{\left(b\coth^3\left(dx+\log\left(e^{-dx}\right)\right)\right)^{\frac{4}{3}}\log\left(e^{-dx}\right)}{d} & \text{for } c = \log\left(e^{-dx}\right) \\ x\left(\frac{b}{\tanh^3(c+dx)}\right)^{\frac{4}{3}}\tanh^4(c+dx) - \frac{\left(\frac{b}{\tanh^3(c+dx)}\right)^{\frac{4}{3}}\tanh^3(c+dx)}{d} - \frac{\left(\frac{b}{\tanh^3(c+dx)}\right)^{\frac{4}{3}}\tanh(c+dx)}{3d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**3)**(4/3),x)
```

```
[Out] Piecewise((x*(b*coth(c)**3)**(4/3), Eq(d, 0)), (-(b*coth(d*x + log(-exp(-d*
x)))**3)**(4/3)*log(-exp(-d*x))/d, Eq(c, log(-exp(-d*x)))), (-(b*coth(d*x +
 log(exp(-d*x)))**3)**(4/3)*log(exp(-d*x))/d, Eq(c, log(exp(-d*x)))), (x*(b
/tanh(c + d*x)**3)**(4/3)*tanh(c + d*x)**4 - (b/tanh(c + d*x)**3)**(4/3)*ta
nh(c + d*x)**3/d - (b/tanh(c + d*x)**3)**(4/3)*tanh(c + d*x)/(3*d), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^3)^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\coth(c+d\,x)^3\right)^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^3)^(4/3),x)
```

```
[Out] int((b*coth(c + d*x)^3)^(4/3), x)
```

3.34 $\int \left(b\coth^3(c+dx)\right)^{2/3}\, dx$

Optimal. Leaf size=50

$$-\frac{\left(b\coth^3(c+dx)\right)^{2/3}\tanh(c+dx)}{d}+x\left(b\coth^3(c+dx)\right)^{2/3}\tanh^2(c+dx)$$

```
[Out] -(b*coth(d*x+c)^3)^(2/3)*tanh(d*x+c)/d+x*(b*coth(d*x+c)^3)^(2/3)*tanh(d*x+c
)^2
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3739, 3554, 8}

$$x\tanh^2(c+dx)\left(b\coth^3(c+dx)\right)^{2/3}-\frac{\tanh(c+dx)\left(b\coth^3(c+dx)\right)^{2/3}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^3)^(2/3),x]
```

```
[Out] -(((b*Coth[c + d*x]^3)^(2/3)*Tanh[c + d*x])/d) + x*(b*Coth[c + d*x]^3)^(2/3
)*Tanh[c + d*x]^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(b\coth^3(c+dx)\right)^{2/3}\,dx = \left(\left(b\coth^3(c+dx)\right)^{2/3}\tanh^2(c+dx)\right)\int \coth^2(c+dx)\,dx$$
$$= -\frac{\left(b\coth^3(c+dx)\right)^{2/3}\tanh(c+dx)}{d} + \left(\left(b\coth^3(c+dx)\right)^{2/3}\tanh^2(c+dx)\right)\int$$
$$= -\frac{\left(b\coth^3(c+dx)\right)^{2/3}\tanh(c+dx)}{d} + x\left(b\coth^3(c+dx)\right)^{2/3}\tanh^2(c+dx)$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.02, size = 41, normalized size = 0.82

$$-\frac{\left(b\coth^3(c+dx)\right)^{2/3}\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\tanh^2(c+dx)\right)\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^3)^(2/3),x]
```

```
[Out] -(((b*Coth[c + d*x]^3)^(2/3)*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[c + d*x]^
2]*Tanh[c + d*x])/d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(46) = 92.
time = 2.55, size = 119, normalized size = 2.38

method	result	size
risch	$\frac{\left(\frac{b\left(1+e^{2dx+2c}\right)^3}{\left(e^{2dx+2c}-1\right)^3}\right)^{\frac{2}{3}}\left(e^{2dx+2c}-1\right)^2 x}{\left(1+e^{2dx+2c}\right)^2}-\frac{2\left(\frac{b\left(1+e^{2dx+2c}\right)^3}{\left(e^{2dx+2c}-1\right)^3}\right)^{\frac{2}{3}}\left(e^{2dx+2c}-1\right)}{\left(1+e^{2dx+2c}\right)^2 d}$	119

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^3)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] (b*(1+exp(2*d*x+2*c))^3/(exp(2*d*x+2*c)-1)^3)^(2/3)/(1+exp(2*d*x+2*c))^2*(e
xp(2*d*x+2*c)-1)^2*x-2*(b*(1+exp(2*d*x+2*c))^3/(exp(2*d*x+2*c)-1)^3)^(2/3)/
(1+exp(2*d*x+2*c))^2*(exp(2*d*x+2*c)-1)/d
```

Maxima [A]
time = 0.49, size = 34, normalized size = 0.68

$$\frac{(dx+c)b^{\frac{2}{3}}}{d} + \frac{2\,b^{\frac{2}{3}}}{d\left(e^{(-2\,dx-2\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(2/3),x, algorithm="maxima")
```

```
[Out] (d*x + c)*b^(2/3)/d + 2*b^(2/3)/(d*(e^(-2*d*x - 2*c) - 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 392 vs. $2(46) = 92$.
time = 0.36, size = 392, normalized size = 7.84

$$\frac{\left(dx\cosh\left(dx+c\right)^2+\left(dxe^{(4\,dx+4\,c)}-2\,dxe^{(2\,dx+2\,c)}+dx\right)\sinh\left(dx+c\right)^2-dx+\left(dx\cosh\left(dx+c\right)^2-dx-2\right)e^{(4\,dx+4\,c)}-2\left(dx\cosh\left(dx+c\right)^2-dx-2\right)e^{(2\,dx+2\,c)}+2\left(dx\cosh\left(dx+c\right)e^{(4\,dx+4\,c)}-2\,dx\cosh\left(dx+c\right)e^{(2\,dx+2\,c)}+dx\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)-2\right)\left(\frac{be^{(6\,dx+6\,c)}+3\,be^{(4\,dx+4\,c)}+3\,be^{(2\,dx+2\,c)}+b}{e^{(6\,dx+6\,c)}-3\,e^{(4\,dx+4\,c)}+3\,e^{(2\,dx+2\,c)}-1}\right)^{\frac{2}{3}}}{d\cosh\left(dx+c\right)^2+\left(de^{(4\,dx+4\,c)}+2\,de^{(2\,dx+2\,c)}+d\right)\sinh\left(dx+c\right)^2+\left(d\cosh\left(dx+c\right)^2-d\right)e^{(4\,dx+4\,c)}+2\left(d\cosh\left(dx+c\right)^2-d\right)e^{(2\,dx+2\,c)}+2\left(d\cosh\left(dx+c\right)e^{(4\,dx+4\,c)}+2\,d\cosh\left(dx+c\right)e^{(2\,dx+2\,c)}+d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)-d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(2/3),x, algorithm="fricas")
```

```
[Out] (d*x*cosh(d*x + c)^2 + (d*x*e^(4*d*x + 4*c) - 2*d*x*e^(2*d*x + 2*c) + d*x)*
sinh(d*x + c)^2 - d*x + (d*x*cosh(d*x + c)^2 - d*x - 2)*e^(4*d*x + 4*c) - 2
*(d*x*cosh(d*x + c)^2 - d*x - 2)*e^(2*d*x + 2*c) + 2*(d*x*cosh(d*x + c)*e^(
4*d*x + 4*c) - 2*d*x*cosh(d*x + c)*e^(2*d*x + 2*c) + d*x*cosh(d*x + c))*sin
h(d*x + c) - 2)*((b*e^(6*d*x + 6*c) + 3*b*e^(4*d*x + 4*c) + 3*b*e^(2*d*x +
2*c) + b)/(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c) - 1))^(2
/3)/(d*cosh(d*x + c)^2 + (d*e^(4*d*x + 4*c) + 2*d*e^(2*d*x + 2*c) + d)*sinh
(d*x + c)^2 + (d*cosh(d*x + c)^2 - d)*e^(4*d*x + 4*c) + 2*(d*cosh(d*x + c)^
2 - d)*e^(2*d*x + 2*c) + 2*(d*cosh(d*x + c)*e^(4*d*x + 4*c) + 2*d*cosh(d*x
+ c)*e^(2*d*x + 2*c) + d*cosh(d*x + c))*sinh(d*x + c) - d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(44) = 88$.
time = 9.64, size = 136, normalized size = 2.72

$$\begin{cases} x\left(b\coth^3\left(c\right)\right)^{\frac{2}{3}} & \text{for}\: d=0 \\ -\frac{\left(b\coth^3\left(dx+\log\left(-e^{-dx}\right)\right)\right)^{\frac{2}{3}}\log\left(-e^{-dx}\right)}{d} & \text{for}\: c=\log\left(-e^{-dx}\right) \\ -\frac{\left(b\coth^3\left(dx+\log\left(e^{-dx}\right)\right)\right)^{\frac{2}{3}}\log\left(e^{-dx}\right)}{d} & \text{for}\: c=\log\left(e^{-dx}\right) \\ x\left(\frac{b}{\tanh^3\left(c+dx\right)}\right)^{\frac{2}{3}}\tanh^2\left(c+dx\right)-\frac{\left(\frac{b}{\tanh^3\left(c+dx\right)}\right)^{\frac{2}{3}}\tanh\left(c+dx\right)}{d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**3)**(2/3),x)
```

```
[Out] Piecewise((x*(b*coth(c)**3)**(2/3), Eq(d, 0)), (-(b*coth(d*x + log(-exp(-d*
x)))**3)**(2/3)*log(-exp(-d*x))/d, Eq(c, log(-exp(-d*x)))), (-(b*coth(d*x +
 log(exp(-d*x)))**3)**(2/3)*log(exp(-d*x))/d, Eq(c, log(exp(-d*x)))), (x*(b
/tanh(c + d*x)**3)**(2/3)*tanh(c + d*x)**2 - (b/tanh(c + d*x)**3)**(2/3)*ta
nh(c + d*x)/d, True))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(2/3),x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^3)^(2/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \left(b\,\mathrm{coth}(c+d\,x)^3\right)^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^3)^(2/3),x)
[Out] int((b*coth(c + d*x)^3)^(2/3), x)
```

3.35 $\int \sqrt[3]{b \coth^3(c+dx)}\, dx$

Optimal. Leaf size=31

$$\frac{\sqrt[3]{b \coth^3(c+dx)}\, \log(\sinh(c+dx)) \tanh(c+dx)}{d}$$

[Out] (b*coth(d*x+c)^3)^(1/3)*ln(sinh(d*x+c))*tanh(d*x+c)/d

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3739, 3556}

$$\frac{\tanh(c+dx) \sqrt[3]{b \coth^3(c+dx)}\, \log(\sinh(c+dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[(b*Coth[c + d*x]^3)^(1/3),x]

[Out] ((b*Coth[c + d*x]^3)^(1/3)*Log[Sinh[c + d*x]]*Tanh[c + d*x])/d

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{b \coth^3(c+dx)}\, dx &= \left(\sqrt[3]{b \coth^3(c+dx)}\, \tanh(c+dx)\right) \int \coth(c+dx)\, dx\\ &= \frac{\sqrt[3]{b \coth^3(c+dx)}\, \log(\sinh(c+dx)) \tanh(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 1.26

$$\frac{\sqrt[3]{b\coth^3(c+dx)}\,(\log(\cosh(c+dx))+\log(\tanh(c+dx)))\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^3)^(1/3),x]
[Out] ((b*Coth[c + d*x]^3)^(1/3)*(Log[Cosh[c + d*x]] + Log[Tanh[c + d*x]])*Tanh[c
 + d*x])/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 191 vs. 2(29) = 58.
time = 2.62, size = 192, normalized size = 6.19

method	result
risch	$\frac{\left(\frac{b\left(1+e^{2dx+2c}\right)^3}{\left(e^{2dx+2c}-1\right)^3}\right)^{\frac{1}{3}}\left(e^{2dx+2c}-1\right)x}{1+e^{2dx+2c}}-\frac{2\left(\frac{b\left(1+e^{2dx+2c}\right)^3}{\left(e^{2dx+2c}-1\right)^3}\right)^{\frac{1}{3}}\left(e^{2dx+2c}-1\right)(dx+c)}{\left(1+e^{2dx+2c}\right)d}+\frac{\left(\frac{b\left(1+e^{2dx+2c}\right)^3}{\left(e^{2dx+2c}-1\right)^3}\right)^{\frac{1}{3}}\left(e^{2dx+2c}-1\right)\ln\left(e^{2dx+2c}-\right.}{\left(1+e^{2dx+2c}\right)d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^3)^(1/3),x,method=_RETURNVERBOSE)
[Out] (b*(1+exp(2*d*x+2*c))^3/(exp(2*d*x+2*c)-1)^3)^(1/3)/(1+exp(2*d*x+2*c))*(exp
(2*d*x+2*c)-1)*x-2*(b*(1+exp(2*d*x+2*c))^3/(exp(2*d*x+2*c)-1)^3)^(1/3)/(1+e
xp(2*d*x+2*c))*(exp(2*d*x+2*c)-1)/d*(d*x+c)+(b*(1+exp(2*d*x+2*c))^3/(exp(2*
d*x+2*c)-1)^3)^(1/3)/(1+exp(2*d*x+2*c))*(exp(2*d*x+2*c)-1)/d*ln(exp(2*d*x+2
*c)-1)
```

Maxima [A]
time = 0.49, size = 51, normalized size = 1.65

$$\frac{(dx+c)b^{\frac{1}{3}}}{d}+\frac{b^{\frac{1}{3}}\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{b^{\frac{1}{3}}\log\left(e^{(-dx-c)}-1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(1/3),x, algorithm="maxima")
[Out] (d*x + c)*b^(1/3)/d + b^(1/3)*log(e^(-d*x - c) + 1)/d + b^(1/3)*log(e^(-d*x
 - c) - 1)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(29) = 58.
time = 0.36, size = 148, normalized size = 4.77

$$-\frac{\left(dxe^{(2\,dx+2\,c)}-dx-\left(e^{(2\,dx+2\,c)}-1\right)\log\left(\frac{2\,\sinh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)\right)\left(\frac{be^{(6\,dx+6\,c)}+3\,be^{(4\,dx+4\,c)}+3\,be^{(2\,dx+2\,c)}+b}{e^{(6\,dx+6\,c)}-3\,e^{(4\,dx+4\,c)}+3\,e^{(2\,dx+2\,c)}-1}\right)^{\frac{1}{3}}}{de^{(2\,dx+2\,c)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(1/3),x, algorithm="fricas")
```

```
[Out] -(d*x*e^(2*d*x + 2*c) - d*x - (e^(2*d*x + 2*c) - 1)*log(2*sinh(d*x + c)/(co
sh(d*x + c) - sinh(d*x + c))))*((b*e^(6*d*x + 6*c) + 3*b*e^(4*d*x + 4*c) +
3*b*e^(2*d*x + 2*c) + b)/(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*e^(2*d*x
+ 2*c) - 1))^(1/3)/(d*e^(2*d*x + 2*c) + d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{b\coth^3\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**3)**(1/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**3)**(1/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^3)^(1/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int {\left(b\,{\mathrm{coth}\left(c+d\,x\right)}^3\right)}^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^3)^(1/3),x)
```

```
[Out] int((b*coth(c + d*x)^3)^(1/3), x)
```

3.36 $\int \frac{1}{\sqrt[3]{b \coth^3(c+dx)}} \, dx$

Optimal. Leaf size=31

$$\frac{\coth(c+dx)\log(\cosh(c+dx))}{d\sqrt[3]{b\coth^3(c+dx)}}$$

```
[Out] coth(d*x+c)*ln(cosh(d*x+c))/d/(b*coth(d*x+c)^3)^(1/3)
```

Rubi [A]

time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3739, 3556}

$$\frac{\coth(c+dx)\log(\cosh(c+dx))}{d\sqrt[3]{b\coth^3(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^3)^(-1/3),x]
```

```
[Out] (Coth[c + d*x]*Log[Cosh[c + d*x]])/(d*(b*Coth[c + d*x]^3)^(1/3))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt[3]{b\coth^3(c+dx)}} \, dx &= \frac{\coth(c+dx) \int \tanh(c+dx) \, dx}{\sqrt[3]{b\coth^3(c+dx)}} \\ &= \frac{\coth(c+dx)\log(\cosh(c+dx))}{d\sqrt[3]{b\coth^3(c+dx)}} \end{aligned}$$

Mathematica [**A**]
time = 0.02, size = 31, normalized size = 1.00

$$\frac{\coth(c+dx)\log(\cosh(c+dx))}{d\sqrt[3]{b\coth^3(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^3)^(-1/3),x]
```

```
[Out] (Coth[c + d*x]*Log[Cosh[c + d*x]])/(d*(b*Coth[c + d*x]^3)^(1/3))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 191 vs. 2(29) = 58.
time = 2.60, size = 192, normalized size = 6.19

method	result	size
risch	$\frac{\left(1+\mathrm{e}^{2dx+2c}\right)x}{\left(\frac{b\left(1+\mathrm{e}^{2dx+2c}\right)^{3}}{\left(\mathrm{e}^{2dx+2c}-1\right)^{3}}\right)^{\frac{1}{3}}\left(\mathrm{e}^{2dx+2c}-1\right)}-\frac{2\left(1+\mathrm{e}^{2dx+2c}\right)\left(dx+c\right)}{\left(\frac{b\left(1+\mathrm{e}^{2dx+2c}\right)^{3}}{\left(\mathrm{e}^{2dx+2c}-1\right)^{3}}\right)^{\frac{1}{3}}\left(\mathrm{e}^{2dx+2c}-1\right)d}+\frac{\left(1+\mathrm{e}^{2dx+2c}\right)\ln\left(1+\mathrm{e}^{2dx+2c}\right)}{\left(\frac{b\left(1+\mathrm{e}^{2dx+2c}\right)^{3}}{\left(\mathrm{e}^{2dx+2c}-1\right)^{3}}\right)^{\frac{1}{3}}\left(\mathrm{e}^{2dx+2c}-1\right)d}$	192

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^3)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(b*(1+exp(2*d*x+2*c))^3/(exp(2*d*x+2*c)-1)^3)^(1/3)/(exp(2*d*x+2*c)-1)*(1
+exp(2*d*x+2*c))*x-2/(b*(1+exp(2*d*x+2*c))^3/(exp(2*d*x+2*c)-1)^3)^(1/3)/(e
xp(2*d*x+2*c)-1)*(1+exp(2*d*x+2*c))/d*(d*x+c)+1/(b*(1+exp(2*d*x+2*c))^3/(ex
p(2*d*x+2*c)-1)^3)^(1/3)/(exp(2*d*x+2*c)-1)*(1+exp(2*d*x+2*c))/d*ln(1+exp(2
*d*x+2*c))
```

Maxima [**A**]
time = 0.50, size = 32, normalized size = 1.03

$$\frac{dx+c}{b^{\frac{1}{3}}d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{b^{\frac{1}{3}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(1/3),x, algorithm="maxima")
```

```
[Out] (d*x + c)/(b^(1/3)*d) + log(e^(-2*d*x - 2*c) + 1)/(b^(1/3)*d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(29) = 58.
time = 0.38, size = 187, normalized size = 6.03

$$-\frac{\left(dxe^{(4\,dx+4\,c)}-2\,dxe^{(2\,dx+2\,c)}+dx-\left(e^{(4\,dx+4\,c)}-2\,e^{(2\,dx+2\,c)}+1\right)\log\left(\frac{2\,\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)\right)\left(\frac{be^{(6\,dx+6\,c)}+3\,be^{(4\,dx+4\,c)}+3\,be^{(2\,dx+2\,c)}+b}{e^{(6\,dx+6\,c)}-3\,e^{(4\,dx+4\,c)}+3\,e^{(2\,dx+2\,c)}-1}\right)^{\frac{2}{3}}}{bde^{(4\,dx+4\,c)}+2\,bde^{(2\,dx+2\,c)}+bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(1/3),x, algorithm="fricas")
```

```
[Out] -(d*x*e^(4*d*x + 4*c) - 2*d*x*e^(2*d*x + 2*c) + d*x - (e^(4*d*x + 4*c) - 2*
e^(2*d*x + 2*c) + 1)*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))*
((b*e^(6*d*x + 6*c) + 3*b*e^(4*d*x + 4*c) + 3*b*e^(2*d*x + 2*c) + b)/(e^(6*
d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c) - 1))^(2/3)/(b*d*e^(4*d*
x + 4*c) + 2*b*d*e^(2*d*x + 2*c) + b*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{b \coth^3(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**3)**(1/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**3)**(-1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^3)^(-1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\left(b \coth(c + d\,x)^3\right)^{1/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x)^3)^(1/3),x)
```

```
[Out] int(1/(b*coth(c + d*x)^3)^(1/3), x)
```

3.37 $\int \frac{1}{\left(b\coth^3(c+dx)\right)^{2/3}}\,dx$

Optimal. Leaf size=50

$$-\frac{\coth(c+dx)}{d\left(b\coth^3(c+dx)\right)^{2/3}}+\frac{x\coth^2(c+dx)}{\left(b\coth^3(c+dx)\right)^{2/3}}$$

```
[Out] -coth(d*x+c)/d/(b*coth(d*x+c)^3)^(2/3)+x*coth(d*x+c)^2/(b*coth(d*x+c)^3)^(2
/3)
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3739, 3554, 8}

$$\frac{x\coth^2(c+dx)}{\left(b\coth^3(c+dx)\right)^{2/3}}-\frac{\coth(c+dx)}{d\left(b\coth^3(c+dx)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^3)^(-2/3),x]
```

```
[Out] -(Coth[c + d*x]/(d*(b*Coth[c + d*x]^3)^(2/3))) + (x*Coth[c + d*x]^2)/(b*Cot
h[c + d*x]^3)^(2/3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{2/3}}\,dx = \frac{\coth^2(c+dx)\int \tanh^2(c+dx)\,dx}{\left(b\coth^3(c+dx)\right)^{2/3}}$$
$$= -\frac{\coth(c+dx)}{d\left(b\coth^3(c+dx)\right)^{2/3}} + \frac{\coth^2(c+dx)\int 1\,dx}{\left(b\coth^3(c+dx)\right)^{2/3}}$$
$$= -\frac{\coth(c+dx)}{d\left(b\coth^3(c+dx)\right)^{2/3}} + \frac{x\coth^2(c+dx)}{\left(b\coth^3(c+dx)\right)^{2/3}}$$

Mathematica [A]
time = 0.04, size = 40, normalized size = 0.80

$$\frac{\coth(c+dx)\left(-1+\tanh^{-1}(\tanh(c+dx))\coth(c+dx)\right)}{d\left(b\coth^3(c+dx)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^3)^(-2/3),x]
```

```
[Out] (Coth[c + d*x]*(-1 + ArcTanh[Tanh[c + d*x]]*Coth[c + d*x]))/(d*(b*Coth[c + d*x]^3)^(2/3))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(46) = 92$.
time = 2.59, size = 119, normalized size = 2.38

method	result	size
risch	$\frac{\left(1+e^{2dx+2c}\right)^2 x}{\left(\frac{b\left(1+e^{2dx+2c}\right)^3}{\left(e^{2dx+2c}-1\right)^3}\right)^{\frac{2}{3}}\left(e^{2dx+2c}-1\right)^2}+\frac{2+2\,e^{2dx+2c}}{\left(\frac{b\left(1+e^{2dx+2c}\right)^3}{\left(e^{2dx+2c}-1\right)^3}\right)^{\frac{2}{3}}\left(e^{2dx+2c}-1\right)^2 d}$	119

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^3)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(b*(1+exp(2*d*x+2*c))^3/(exp(2*d*x+2*c)-1)^3)^(2/3)/(exp(2*d*x+2*c)-1)^2*
(1+exp(2*d*x+2*c))^2*x+2/(b*(1+exp(2*d*x+2*c))^3/(exp(2*d*x+2*c)-1)^3)^(2/3
)/(exp(2*d*x+2*c)-1)^2*(1+exp(2*d*x+2*c))/d
```

Maxima [A]
time = 0.48, size = 37, normalized size = 0.74

$$\frac{dx+c}{b^{\frac{2}{3}}d} - \frac{2}{\left(b^{\frac{2}{3}}e^{(-2\,dx-2\,c)} + b^{\frac{2}{3}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(2/3),x, algorithm="maxima")
```

```
[Out] (d*x + c)/(b^(2/3)*d) - 2/((b^(2/3)*e^(-2*d*x - 2*c) + b^(2/3))*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(46) = 92$.
time = 0.35, size = 287, normalized size = 5.74

$$-\frac{\left(dx\cosh\left(dx+c\right)^{2}-\left(dxe^{\left(2\,dx+2\,c\right)}-dx\right)\sinh\left(dx+c\right)^{2}+dx-\left(dx\cosh\left(dx+c\right)^{2}+dx+2\right)e^{\left(2\,dx+2\,c\right)}-2\left(dx\cosh\left(dx+c\right)e^{\left(2\,dx+2\,c\right)}-dx\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)+2\right)\left(\frac{be^{\left(6\,dx+6\,c\right)}+3\,be^{\left(4\,dx+4\,c\right)}+3\,be^{\left(2\,dx+2\,c\right)}+b}{e^{\left(6\,dx+6\,c\right)}-3\,e^{\left(4\,dx+4\,c\right)}+3\,e^{\left(2\,dx+2\,c\right)}-1}\right)^{\frac{1}{3}}}{bd\cosh\left(dx+c\right)^{2}+\left(bde^{\left(2\,dx+2\,c\right)}+bd\right)\sinh\left(dx+c\right)^{2}+bd+\left(bd\cosh\left(dx+c\right)^{2}+bd\right)e^{\left(2\,dx+2\,c\right)}+2\left(bd\cosh\left(dx+c\right)e^{\left(2\,dx+2\,c\right)}+bd\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(2/3),x, algorithm="fricas")
```

```
[Out] -(d*x*cosh(d*x + c)^2 - (d*x*e^(2*d*x + 2*c) - d*x)*sinh(d*x + c)^2 + d*x -
 (d*x*cosh(d*x + c)^2 + d*x + 2)*e^(2*d*x + 2*c) - 2*(d*x*cosh(d*x + c)*e^(
2*d*x + 2*c) - d*x*cosh(d*x + c))*sinh(d*x + c) + 2)*((b*e^(6*d*x + 6*c) +
3*b*e^(4*d*x + 4*c) + 3*b*e^(2*d*x + 2*c) + b)/(e^(6*d*x + 6*c) - 3*e^(4*d*
x + 4*c) + 3*e^(2*d*x + 2*c) - 1))^(1/3)/(b*d*cosh(d*x + c)^2 + (b*d*e^(2*d
*x + 2*c) + b*d)*sinh(d*x + c)^2 + b*d + (b*d*cosh(d*x + c)^2 + b*d)*e^(2*d
*x + 2*c) + 2*(b*d*cosh(d*x + c)*e^(2*d*x + 2*c) + b*d*cosh(d*x + c))*sinh(
d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\coth^{3}\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**3)**(2/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**3)**(-2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^3)^(-2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(b\coth\left(c+d\,x\right)^3\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x)^3)^(2/3),x)
[Out] int(1/(b*coth(c + d*x)^3)^(2/3), x)
```

3.38 $\int \frac{1}{\left(b\coth^3(c+dx)\right)^{4/3}}\,dx$

Optimal. Leaf size=80

$$-\frac{1}{bd\sqrt[3]{b\coth^3(c+dx)}}+\frac{x\coth(c+dx)}{b\sqrt[3]{b\coth^3(c+dx)}}-\frac{\tanh^2(c+dx)}{3bd\sqrt[3]{b\coth^3(c+dx)}}$$

```
[Out] -1/b/d/(b*coth(d*x+c)^3)^(1/3)+x*coth(d*x+c)/b/(b*coth(d*x+c)^3)^(1/3)-1/3*
tanh(d*x+c)^2/b/d/(b*coth(d*x+c)^3)^(1/3)
```

Rubi [A]
time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3739, 3554, 8}

$$\frac{x\coth(c+dx)}{b\sqrt[3]{b\coth^3(c+dx)}}-\frac{1}{bd\sqrt[3]{b\coth^3(c+dx)}}-\frac{\tanh^2(c+dx)}{3bd\sqrt[3]{b\coth^3(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^3)^(-4/3),x]
```

```
[Out] -(1/(b*d*(b*Coth[c + d*x]^3)^(1/3))) + (x*Coth[c + d*x])/(b*(b*Coth[c + d*x
]^3)^(1/3)) - Tanh[c + d*x]^2/(3*b*d*(b*Coth[c + d*x]^3)^(1/3))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{4/3}}\,dx = \frac{\coth(c+dx)\int \tanh^4(c+dx)\,dx}{b\sqrt[3]{b\coth^3(c+dx)}}$$
$$= -\frac{\tanh^2(c+dx)}{3bd\sqrt[3]{b\coth^3(c+dx)}} + \frac{\coth(c+dx)\int \tanh^2(c+dx)\,dx}{b\sqrt[3]{b\coth^3(c+dx)}}$$
$$= -\frac{1}{bd\sqrt[3]{b\coth^3(c+dx)}} - \frac{\tanh^2(c+dx)}{3bd\sqrt[3]{b\coth^3(c+dx)}} + \frac{\coth(c+dx)\int 1\,dx}{b\sqrt[3]{b\coth^3(c+dx)}}$$
$$= -\frac{1}{bd\sqrt[3]{b\coth^3(c+dx)}} + \frac{x\coth(c+dx)}{b\sqrt[3]{b\coth^3(c+dx)}} - \frac{\tanh^2(c+dx)}{3bd\sqrt[3]{b\coth^3(c+dx)}}$$

Mathematica [A]
time = 0.06, size = 51, normalized size = 0.64

$$\frac{-3+3\tanh^{-1}(\tanh(c+dx))\coth(c+dx)-\tanh^2(c+dx)}{3bd\sqrt[3]{b\coth^3(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^3)^(-4/3),x]
```

```
[Out] (-3 + 3*ArcTanh[Tanh[c + d*x]]*Coth[c + d*x] - Tanh[c + d*x]^2)/(3*b*d*(b*C
oth[c + d*x]^3)^(1/3))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(72) = 144.
time = 2.61, size = 149, normalized size = 1.86

method	result	size
risch	$\frac{\left(1+e^{2dx+2c}\right)x}{b\left(e^{2dx+2c}-1\right)\left(\frac{b\left(1+e^{2dx+2c}\right)^3}{\left(e^{2dx+2c}-1\right)^3}\right)^{\frac{1}{3}}}+\frac{4\,e^{4dx+4c}+4\,e^{2dx+2c}+\frac{8}{3}}{b\left(1+e^{2dx+2c}\right)^2\left(e^{2dx+2c}-1\right)\left(\frac{b\left(1+e^{2dx+2c}\right)^3}{\left(e^{2dx+2c}-1\right)^3}\right)^{\frac{1}{3}}d}$	149

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^3)^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1+exp(2*d*x+2*c))/(exp(2*d*x+2*c)-1)/(b*(1+exp(2*d*x+2*c))^3/(exp(2*d*
x+2*c)-1)^3)^(1/3)*x+4/3/b/(1+exp(2*d*x+2*c))^2/(exp(2*d*x+2*c)-1)/(b*(1+ex
p(2*d*x+2*c))^3/(exp(2*d*x+2*c)-1)^3)^(1/3)*(3*exp(4*d*x+4*c)+3*exp(2*d*x+2
*c)+2)/d
```

Maxima [A]
time = 0.49, size = 89, normalized size = 1.11

$$-\frac{4\left(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+2\right)}{3\left(3\,b^{\frac{4}{3}}e^{(-2\,dx-2\,c)}+3\,b^{\frac{4}{3}}e^{(-4\,dx-4\,c)}+b^{\frac{4}{3}}e^{(-6\,dx-6\,c)}+b^{\frac{4}{3}}\right)d}+\frac{dx+c}{b^{\frac{4}{3}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(4/3),x, algorithm="maxima")
```

```
[Out] -4/3*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/((3*b^(4/3)*e^(-2*d*x -
2*c) + 3*b^(4/3)*e^(-4*d*x - 4*c) + b^(4/3)*e^(-6*d*x - 6*c) + b^(4/3))*d)
+ (d*x + c)/(b^(4/3)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1579 vs. 2(72) = 144.
time = 0.41, size = 1579, normalized size = 19.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(4/3),x, algorithm="fricas")
```

```
[Out] 1/3*(3*d*x*cosh(d*x + c)^6 + 3*(d*x*e^(4*d*x + 4*c) - 2*d*x*e^(2*d*x + 2*c)
 + d*x)*sinh(d*x + c)^6 + 18*(d*x*cosh(d*x + c)*e^(4*d*x + 4*c) - 2*d*x*cos
h(d*x + c)*e^(2*d*x + 2*c) + d*x*cosh(d*x + c))*sinh(d*x + c)^5 + 3*(3*d*x
+ 4)*cosh(d*x + c)^4 + 3*(15*d*x*cosh(d*x + c)^2 + 3*d*x + (15*d*x*cosh(d*x
 + c)^2 + 3*d*x + 4)*e^(4*d*x + 4*c) - 2*(15*d*x*cosh(d*x + c)^2 + 3*d*x +
4)*e^(2*d*x + 2*c) + 4)*sinh(d*x + c)^4 + 12*(5*d*x*cosh(d*x + c)^3 + (3*d*
x + 4)*cosh(d*x + c) + (5*d*x*cosh(d*x + c)^3 + (3*d*x + 4)*cosh(d*x + c))*
e^(4*d*x + 4*c) - 2*(5*d*x*cosh(d*x + c)^3 + (3*d*x + 4)*cosh(d*x + c))*e^(
2*d*x + 2*c))*sinh(d*x + c)^3 + 3*(3*d*x + 4)*cosh(d*x + c)^2 + 3*(15*d*x*c
osh(d*x + c)^4 + 6*(3*d*x + 4)*cosh(d*x + c)^2 + 3*d*x + (15*d*x*cosh(d*x +
 c)^4 + 6*(3*d*x + 4)*cosh(d*x + c)^2 + 3*d*x + 4)*e^(4*d*x + 4*c) - 2*(15*
d*x*cosh(d*x + c)^4 + 6*(3*d*x + 4)*cosh(d*x + c)^2 + 3*d*x + 4)*e^(2*d*x +
 2*c) + 4)*sinh(d*x + c)^2 + 3*d*x + (3*d*x*cosh(d*x + c)^6 + 3*(3*d*x + 4)
*cosh(d*x + c)^4 + 3*(3*d*x + 4)*cosh(d*x + c)^2 + 3*d*x + 8)*e^(4*d*x + 4*
c) - 2*(3*d*x*cosh(d*x + c)^6 + 3*(3*d*x + 4)*cosh(d*x + c)^4 + 3*(3*d*x +
4)*cosh(d*x + c)^2 + 3*d*x + 8)*e^(2*d*x + 2*c) + 6*(3*d*x*cosh(d*x + c)^5
+ 2*(3*d*x + 4)*cosh(d*x + c)^3 + (3*d*x + 4)*cosh(d*x + c) + (3*d*x*cosh(d
*x + c)^5 + 2*(3*d*x + 4)*cosh(d*x + c)^3 + (3*d*x + 4)*cosh(d*x + c))*e^(4
*d*x + 4*c) - 2*(3*d*x*cosh(d*x + c)^5 + 2*(3*d*x + 4)*cosh(d*x + c)^3 + (3
*d*x + 4)*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c) + 8)*((b*e^(6*d*x +
 6*c) + 3*b*e^(4*d*x + 4*c) + 3*b*e^(2*d*x + 2*c) + b)/(e^(6*d*x + 6*c) - 3
*e^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c) - 1))^(2/3)/(b^2*d*cosh(d*x + c)^6 + 3
```

```
*b^2*d*cosh(d*x + c)^4 + (b^2*d*e^(4*d*x + 4*c) + 2*b^2*d*e^(2*d*x + 2*c) +
 b^2*d)*sinh(d*x + c)^6 + 6*(b^2*d*cosh(d*x + c)*e^(4*d*x + 4*c) + 2*b^2*d*
cosh(d*x + c)*e^(2*d*x + 2*c) + b^2*d*cosh(d*x + c))*sinh(d*x + c)^5 + 3*b^
2*d*cosh(d*x + c)^2 + 3*(5*b^2*d*cosh(d*x + c)^2 + b^2*d + (5*b^2*d*cosh(d*
x + c)^2 + b^2*d)*e^(4*d*x + 4*c) + 2*(5*b^2*d*cosh(d*x + c)^2 + b^2*d)*e^(
2*d*x + 2*c))*sinh(d*x + c)^4 + 4*(5*b^2*d*cosh(d*x + c)^3 + 3*b^2*d*cosh(d
*x + c) + (5*b^2*d*cosh(d*x + c)^3 + 3*b^2*d*cosh(d*x + c))*e^(4*d*x + 4*c)
 + 2*(5*b^2*d*cosh(d*x + c)^3 + 3*b^2*d*cosh(d*x + c))*e^(2*d*x + 2*c))*sin
h(d*x + c)^3 + b^2*d + 3*(5*b^2*d*cosh(d*x + c)^4 + 6*b^2*d*cosh(d*x + c)^2
 + b^2*d + (5*b^2*d*cosh(d*x + c)^4 + 6*b^2*d*cosh(d*x + c)^2 + b^2*d)*e^(4
*d*x + 4*c) + 2*(5*b^2*d*cosh(d*x + c)^4 + 6*b^2*d*cosh(d*x + c)^2 + b^2*d)
*e^(2*d*x + 2*c))*sinh(d*x + c)^2 + (b^2*d*cosh(d*x + c)^6 + 3*b^2*d*cosh(d
*x + c)^4 + 3*b^2*d*cosh(d*x + c)^2 + b^2*d)*e^(4*d*x + 4*c) + 2*(b^2*d*cos
h(d*x + c)^6 + 3*b^2*d*cosh(d*x + c)^4 + 3*b^2*d*cosh(d*x + c)^2 + b^2*d)*e
^(2*d*x + 2*c) + 6*(b^2*d*cosh(d*x + c)^5 + 2*b^2*d*cosh(d*x + c)^3 + b^2*d
*cosh(d*x + c) + (b^2*d*cosh(d*x + c)^5 + 2*b^2*d*cosh(d*x + c)^3 + b^2*d*c
osh(d*x + c))*e^(4*d*x + 4*c) + 2*(b^2*d*cosh(d*x + c)^5 + 2*b^2*d*cosh(d*x
 + c)^3 + b^2*d*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\coth^3\left(c+dx\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**3)**(4/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**3)**(-4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^3)^(-4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,{\mathrm{coth}\left(c+d\,x\right)}^3\right)}^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x)^3)^(4/3),x)
[Out] int(1/(b*coth(c + d*x)^3)^(4/3), x)
```

3.39 $\int \left(b\coth^4(c+dx)\right)^n\,dx$

Optimal. Leaf size=57

$$\frac{\coth(c+dx)\left(b\coth^4(c+dx)\right)^n\,{}_2F_1\left(1,\frac{1}{2}(1+4n);\frac{1}{2}(3+4n);\coth^2(c+dx)\right)}{d(1+4n)}$$

```
[Out] coth(d*x+c)*(b*coth(d*x+c)^4)^n*hypergeom([1, 1/2+2*n],[3/2+2*n],coth(d*x+c
)^2)/d/(1+4*n)
```

Rubi [A]
time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3739, 3557, 371}

$$\frac{\coth(c+dx)\left(b\coth^4(c+dx)\right)^n\,{}_2F_1\left(1,\frac{1}{2}(4n+1);\frac{1}{2}(4n+3);\coth^2(c+dx)\right)}{d(4n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^4)^n,x]
```

```
[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^4)^n*Hypergeometric2F1[1, (1 + 4*n)/2, (3 +
 4*n)/2, Coth[c + d*x]^2])/(d*(1 + 4*n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\int \left(b\coth^4(c+dx)\right)^n\,dx &= \left(\coth^{-4n}(c+dx)\left(b\coth^4(c+dx)\right)^n\right)\int \coth^{4n}(c+dx)\,dx\\
&= -\frac{\left(\coth^{-4n}(c+dx)\left(b\coth^4(c+dx)\right)^n\right)\text{Subst}\left(\int \frac{x^{4n}}{-1+x^2}\,dx,x,\coth(c+dx)\right)}{d}\\
&= \frac{\coth(c+dx)\left(b\coth^4(c+dx)\right)^n\,{}_2F_1\left(1,\frac{1}{2}(1+4n);\frac{1}{2}(3+4n);\coth^2(c+dx)\right)}{d(1+4n)}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 51, normalized size = 0.89

$$
\frac{\coth(c+dx)\left(b\coth^4(c+dx)\right)^n\,{}_2F_1\left(1,\frac{1}{2}+2n;\frac{3}{2}+2n;\coth^2(c+dx)\right)}{d+4dn}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^4)^n,x]
```

```
[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^4)^n*Hypergeometric2F1[1, 1/2 + 2*n, 3/2 +
2*n, Coth[c + d*x]^2])/(d + 4*d*n)
```

Maple [F]
time = 2.41, size = 0, normalized size = 0.00

$$
\int \left(b\left(\coth^4\left(dx+c\right)\right)\right)^n\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^4)^n,x)
```

```
[Out] int((b*coth(d*x+c)^4)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^4)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^n,x, algorithm="fricas")
[Out] integral((b*coth(d*x + c)^4)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\coth^4\left(c+dx\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**4)**n,x)
[Out] Integral((b*coth(c + d*x)**4)**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^n,x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^4)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \left(b\,\mathrm{coth}(c+d\,x)^4\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^4)^n,x)
[Out] int((b*coth(c + d*x)^4)^n, x)
```

3.40 $\int \left(b\coth^4(c+dx)\right)^{3/2}\,dx$

Optimal. Leaf size=110

$$-\frac{b\coth(c+dx)\sqrt{b\coth^4(c+dx)}}{3d}-\frac{b\coth^3(c+dx)\sqrt{b\coth^4(c+dx)}}{5d}-\frac{b\sqrt{b\coth^4(c+dx)}\,\tanh(c+dx)}{d}-$$

```
[Out] -1/3*b*coth(d*x+c)*(b*coth(d*x+c)^4)^(1/2)/d-1/5*b*coth(d*x+c)^3*(b*coth(d*
x+c)^4)^(1/2)/d-b*(b*coth(d*x+c)^4)^(1/2)*tanh(d*x+c)/d+b*x*(b*coth(d*x+c)^
4)^(1/2)*tanh(d*x+c)^2
```

Rubi [A]
time = 0.03, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3739, 3554, 8}

$$-\frac{b\coth(c+dx)\sqrt{b\coth^4(c+dx)}}{3d}-\frac{b\coth^3(c+dx)\sqrt{b\coth^4(c+dx)}}{5d}+bx\tanh^2(c+dx)\sqrt{b\coth^4(c+dx)}-\frac{b\tanh(c+dx)\sqrt{b\coth^4(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^4)^(3/2),x]
```

```
[Out] -1/3*(b*Coth[c + d*x]*Sqrt[b*Coth[c + d*x]^4])/d - (b*Coth[c + d*x]^3*Sqrt[
b*Coth[c + d*x]^4])/(5*d) - (b*Sqrt[b*Coth[c + d*x]^4]*Tanh[c + d*x])/d + b
*x*Sqrt[b*Coth[c + d*x]^4]*Tanh[c + d*x]^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(b\coth^4(c+dx)\right)^{3/2} dx = \left(b\sqrt{b\coth^4(c+dx)}\,\tanh^2(c+dx)\right)\int \coth^6(c+dx)\,dx$$
$$= -\frac{b\coth^3(c+dx)\sqrt{b\coth^4(c+dx)}}{5d} + \left(b\sqrt{b\coth^4(c+dx)}\,\tanh^2(c+dx)\right)\int$$
$$= -\frac{b\coth(c+dx)\sqrt{b\coth^4(c+dx)}}{3d} - \frac{b\coth^3(c+dx)\sqrt{b\coth^4(c+dx)}}{5d} + \left(b\sqrt{\right.}$$
$$= -\frac{b\coth(c+dx)\sqrt{b\coth^4(c+dx)}}{3d} - \frac{b\coth^3(c+dx)\sqrt{b\coth^4(c+dx)}}{5d} - \frac{b\sqrt{b}}{}$$
$$= -\frac{b\coth(c+dx)\sqrt{b\coth^4(c+dx)}}{3d} - \frac{b\coth^3(c+dx)\sqrt{b\coth^4(c+dx)}}{5d} - \frac{b\sqrt{b}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.05, size = 43, normalized size = 0.39

$$-\frac{\left(b\coth^4(c+dx)\right)^{3/2}\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};\tanh^2(c+dx)\right)\tanh(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^4)^(3/2),x]
```

```
[Out] -1/5*((b*Coth[c + d*x]^4)^(3/2)*Hypergeometric2F1[-5/2, 1, -3/2, Tanh[c + d
*x]^2]*Tanh[c + d*x])/d
```

Maple [A]
time = 1.59, size = 77, normalized size = 0.70

method	result
derivativedivides	$-\frac{\left(b\left(\coth^4(dx+c)\right)\right)^{\frac{3}{2}}\left(6\left(\coth^5(dx+c)\right)+10\left(\coth^3(dx+c)\right)+15\ln(\coth(dx+c)-1)-15\ln(\coth(dx+c)+1)+30\coth(dx+c)\right.}{30d\coth(dx+c)^6}$
default	$-\frac{\left(b\left(\coth^4(dx+c)\right)\right)^{\frac{3}{2}}\left(6\left(\coth^5(dx+c)\right)+10\left(\coth^3(dx+c)\right)+15\ln(\coth(dx+c)-1)-15\ln(\coth(dx+c)+1)+30\coth(dx+c)\right.}{30d\coth(dx+c)^6}$
risch	$\frac{b\left(e^{2dx+2c}-1\right)^2\sqrt{\frac{b\left(1+e^{2dx+2c}\right)^4}{\left(e^{2dx+2c}-1\right)^4}}\,x}{\left(1+e^{2dx+2c}\right)^2} - \frac{2b\sqrt{\frac{b\left(1+e^{2dx+2c}\right)^4}{\left(e^{2dx+2c}-1\right)^4}}\left(45\,e^{8dx+8c}-90\,e^{6dx+6c}+140\,e^{4dx+4c}-70\,e^{2dx+2c}+23\right)}{15\left(1+e^{2dx+2c}\right)^2\left(e^{2dx+2c}-1\right)^3d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^4)^(3/2),x,method=_RETURNVERBOSE)
```

[Out] -1/30/d*(b*coth(d*x+c)^4)^(3/2)*(6*coth(d*x+c)^5+10*coth(d*x+c)^3+15*ln(coth(d*x+c)-1)-15*ln(coth(d*x+c)+1)+30*coth(d*x+c))/coth(d*x+c)^6

Maxima [A]
time = 0.50, size = 137, normalized size = 1.25

$$\frac{(dx+c)b^{\frac{3}{2}}}{d} - \frac{2\left(70\,b^{\frac{3}{2}}e^{(-2\,dx-2\,c)} - 140\,b^{\frac{3}{2}}e^{(-4\,dx-4\,c)} + 90\,b^{\frac{3}{2}}e^{(-6\,dx-6\,c)} - 45\,b^{\frac{3}{2}}e^{(-8\,dx-8\,c)} - 23\,b^{\frac{3}{2}}\right)}{15\,d(5\,e^{(-2\,dx-2\,c)} - 10\,e^{(-4\,dx-4\,c)} + 10\,e^{(-6\,dx-6\,c)} - 5\,e^{(-8\,dx-8\,c)} + e^{(-10\,dx-10\,c)} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^4)^(3/2),x, algorithm="maxima")

[Out] (d*x + c)*b^(3/2)/d - 2/15*(70*b^(3/2)*e^(-2*d*x - 2*c) - 140*b^(3/2)*e^(-4*d*x - 4*c) + 90*b^(3/2)*e^(-6*d*x - 6*c) - 45*b^(3/2)*e^(-8*d*x - 8*c) - 23*b^(3/2))/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1))

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3421 vs. 2(98) = 196.
time = 0.38, size = 3421, normalized size = 31.10

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*coth(d*x+c)^4)^(3/2),x, algorithm="fricas")

[Out] 1/15*(15*b*d*x*cosh(d*x + c)^10 + 15*(b*d*x*e^(4*d*x + 4*c) - 2*b*d*x*e^(2*d*x + 2*c) + b*d*x)*sinh(d*x + c)^10 + 150*(b*d*x*cosh(d*x + c)*e^(4*d*x + 4*c) - 2*b*d*x*cosh(d*x + c)*e^(2*d*x + 2*c) + b*d*x*cosh(d*x + c))*sinh(d*x + c)^9 - 15*(5*b*d*x + 6*b)*cosh(d*x + c)^8 + 15*(45*b*d*x*cosh(d*x + c)^2 - 5*b*d*x + (45*b*d*x*cosh(d*x + c)^2 - 5*b*d*x - 6*b)*e^(4*d*x + 4*c) - 2*(45*b*d*x*cosh(d*x + c)^2 - 5*b*d*x - 6*b)*e^(2*d*x + 2*c) - 6*b)*sinh(d*x + c)^8 + 120*(15*b*d*x*cosh(d*x + c)^3 - (5*b*d*x + 6*b)*cosh(d*x + c) + (15*b*d*x*cosh(d*x + c)^3 - (5*b*d*x + 6*b)*cosh(d*x + c))*e^(4*d*x + 4*c) - 2*(15*b*d*x*cosh(d*x + c)^3 - (5*b*d*x + 6*b)*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c)^7 + 30*(5*b*d*x + 6*b)*cosh(d*x + c)^6 + 30*(105*b*d*x*cosh(d*x + c)^4 + 5*b*d*x - 14*(5*b*d*x + 6*b)*cosh(d*x + c)^2 + (105*b*d*x*cosh(d*x + c)^4 + 5*b*d*x - 14*(5*b*d*x + 6*b)*cosh(d*x + c)^2 + 6*b)*e^(4*d*x + 4*c) - 2*(105*b*d*x*cosh(d*x + c)^4 + 5*b*d*x - 14*(5*b*d*x + 6*b)*cosh(d*x + c)^2 + 6*b)*e^(2*d*x + 2*c) + 6*b)*sinh(d*x + c)^6 + 60*(63*b*d*x*cosh(d*x + c)^5 - 14*(5*b*d*x + 6*b)*cosh(d*x + c)^3 + 3*(5*b*d*x + 6*b)*cosh(d*x + c) + (63*b*d*x*cosh(d*x + c)^5 - 14*(5*b*d*x + 6*b)*cosh(d*x + c)^3 + 3*(5*b*d*x + 6*b)*cosh(d*x + c))*e^(4*d*x + 4*c) - 2*(63*b*d*x*cosh(d*x + c)^5 - 14*(5*b*d*x + 6*b)*cosh(d*x + c)^3 + 3*(5*b*d*x + 6*b)*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c)^5 - 10*(15*b*d*x + 28*b)*cosh(d*x + c)^

```
4 + 10*(315*b*d*x*cosh(d*x + c)^6 - 105*(5*b*d*x + 6*b)*cosh(d*x + c)^4 - 1
5*b*d*x + 45*(5*b*d*x + 6*b)*cosh(d*x + c)^2 + (315*b*d*x*cosh(d*x + c)^6 -
 105*(5*b*d*x + 6*b)*cosh(d*x + c)^4 - 15*b*d*x + 45*(5*b*d*x + 6*b)*cosh(d
*x + c)^2 - 28*b)*e^(4*d*x + 4*c) - 2*(315*b*d*x*cosh(d*x + c)^6 - 105*(5*b
*d*x + 6*b)*cosh(d*x + c)^4 - 15*b*d*x + 45*(5*b*d*x + 6*b)*cosh(d*x + c)^2
 - 28*b)*e^(2*d*x + 2*c) - 28*b)*sinh(d*x + c)^4 + 40*(45*b*d*x*cosh(d*x +
c)^7 - 21*(5*b*d*x + 6*b)*cosh(d*x + c)^5 + 15*(5*b*d*x + 6*b)*cosh(d*x + c
)^3 - (15*b*d*x + 28*b)*cosh(d*x + c) + (45*b*d*x*cosh(d*x + c)^7 - 21*(5*b
*d*x + 6*b)*cosh(d*x + c)^5 + 15*(5*b*d*x + 6*b)*cosh(d*x + c)^3 - (15*b*d*
x + 28*b)*cosh(d*x + c))*e^(4*d*x + 4*c) - 2*(45*b*d*x*cosh(d*x + c)^7 - 21
*(5*b*d*x + 6*b)*cosh(d*x + c)^5 + 15*(5*b*d*x + 6*b)*cosh(d*x + c)^3 - (15
*b*d*x + 28*b)*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c)^3 - 15*b*d*x +
 5*(15*b*d*x + 28*b)*cosh(d*x + c)^2 + 5*(135*b*d*x*cosh(d*x + c)^8 - 84*(5
*b*d*x + 6*b)*cosh(d*x + c)^6 + 90*(5*b*d*x + 6*b)*cosh(d*x + c)^4 + 15*b*d
*x - 12*(15*b*d*x + 28*b)*cosh(d*x + c)^2 + (135*b*d*x*cosh(d*x + c)^8 - 84
*(5*b*d*x + 6*b)*cosh(d*x + c)^6 + 90*(5*b*d*x + 6*b)*cosh(d*x + c)^4 + 15*
b*d*x - 12*(15*b*d*x + 28*b)*cosh(d*x + c)^2 + 28*b)*e^(4*d*x + 4*c) - 2*(1
35*b*d*x*cosh(d*x + c)^8 - 84*(5*b*d*x + 6*b)*cosh(d*x + c)^6 + 90*(5*b*d*x
 + 6*b)*cosh(d*x + c)^4 + 15*b*d*x - 12*(15*b*d*x + 28*b)*cosh(d*x + c)^2 +
 28*b)*e^(2*d*x + 2*c) + 28*b)*sinh(d*x + c)^2 + (15*b*d*x*cosh(d*x + c)^10
 - 15*(5*b*d*x + 6*b)*cosh(d*x + c)^8 + 30*(5*b*d*x + 6*b)*cosh(d*x + c)^6
- 10*(15*b*d*x + 28*b)*cosh(d*x + c)^4 - 15*b*d*x + 5*(15*b*d*x + 28*b)*cos
h(d*x + c)^2 - 46*b)*e^(4*d*x + 4*c) - 2*(15*b*d*x*cosh(d*x + c)^10 - 15*(5
*b*d*x + 6*b)*cosh(d*x + c)^8 + 30*(5*b*d*x + 6*b)*cosh(d*x + c)^6 - 10*(15
*b*d*x + 28*b)*cosh(d*x + c)^4 - 15*b*d*x + 5*(15*b*d*x + 28*b)*cosh(d*x +
c)^2 - 46*b)*e^(2*d*x + 2*c) + 10*(15*b*d*x*cosh(d*x + c)^9 - 12*(5*b*d*x +
 6*b)*cosh(d*x + c)^7 + 18*(5*b*d*x + 6*b)*cosh(d*x + c)^5 - 4*(15*b*d*x +
28*b)*cosh(d*x + c)^3 + (15*b*d*x + 28*b)*cosh(d*x + c) + (15*b*d*x*cosh(d*
x + c)^9 - 12*(5*b*d*x + 6*b)*cosh(d*x + c)^7 + 18*(5*b*d*x + 6*b)*cosh(d*x
 + c)^5 - 4*(15*b*d*x + 28*b)*cosh(d*x + c)^3 + (15*b*d*x + 28*b)*cosh(d*x
+ c))*e^(4*d*x + 4*c) - 2*(15*b*d*x*cosh(d*x + c)^9 - 12*(5*b*d*x + 6*b)*co
sh(d*x + c)^7 + 18*(5*b*d*x + 6*b)*cosh(d*x + c)^5 - 4*(15*b*d*x + 28*b)*co
sh(d*x + c)^3 + (15*b*d*x + 28*b)*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x
+ c) - 46*b)*sqrt((b*e^(8*d*x + 8*c) + 4*b*e^(6*d*x + 6*c) + 6*b*e^(4*d*x +
 4*c) + 4*b*e^(2*d*x + 2*c) + b)/(e^(8*d*x + 8*c) - 4*e^(6*d*x + 6*c) + 6*e
^(4*d*x + 4*c) - 4*e^(2*d*x + 2*c) + 1))/(d*cosh(d*x + c)^10 + (d*e^(4*d*x
+ 4*c) + 2*d*e^(2*d*x + 2*c) + d)*sinh(d*x + c)^10 + 10*(d*cosh(d*x + c)*e^
(4*d*x + 4*c) + 2*d*cosh(d*x + c)*e^(2*d*x + 2*c) + d*cosh(d*x + c))*sinh(d
*x + c)^9 - 5*d*cosh(d*x + c)^8 + 5*(9*d*cosh(d*x + c)^2 + (9*d*cosh(d*x +
c)^2 - d)*e^(4*d*x + 4*c) + 2*(9*d*cosh(d*x + c)^2 - d)*e^(2*d*x + 2*c) - d
)*sinh(d*x + c)^8 + 40*(3*d*cosh(d*x + c)^3 - d*cosh(d*x + c) + (3*d*cosh(d
*x + c)^3 - d*cosh(d*x + c))*e^(4*d*x + 4*c) + 2*(3*d*cosh(d*x + c)^3 - d*c
osh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c)^7 + 10*d*cosh(d*x + c)^6 + 10*
(21*d*cosh(d*x + c)^4 - 14*d*cosh(d*x + c)^2 + (21*d*cosh(d*x + c)^4 - 14*d
*cosh(d*x + c)^2 + d)*e^(4*d*x + 4*c) + 2*(21*d*cosh(d*x + c)^4 - 14*d*cosh
```

```
(d*x + c)^2 + d)*e^(2*d*x + 2*c) + d)*sinh(d*x + c)^6 + 4*(63*d*cosh(d*x +
c)^5 - 70*d*cosh(d*x + c)^3 + 15*d*cosh(d*x + c) + (63*d*cosh(d*x + c)^5 -
70*d*cosh(d*x + c)^3 + 15*d*cosh(d*x + c))*e^(4...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\coth^4\left(c+dx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**4)**(3/2),x)
```

```
[Out] Integral((b*coth(c + d*x)**4)**(3/2), x)
```

Giac [A]
time = 0.44, size = 77, normalized size = 0.70

$$\frac{\left(15\,dx+15\,c-\frac{2\,\left(45\,e^{\left(8\,dx+8\,c\right)}-90\,e^{\left(6\,dx+6\,c\right)}+140\,e^{\left(4\,dx+4\,c\right)}-70\,e^{\left(2\,dx+2\,c\right)}+23\right)}{\left(e^{\left(2\,dx+2\,c\right)}-1\right)^{5}}\right)b^{\frac{3}{2}}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^(3/2),x, algorithm="giac")
```

```
[Out] 1/15*(15*d*x + 15*c - 2*(45*e^(8*d*x + 8*c) - 90*e^(6*d*x + 6*c) + 140*e^(4
*d*x + 4*c) - 70*e^(2*d*x + 2*c) + 23)/(e^(2*d*x + 2*c) - 1)^5)*b^(3/2)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,{\mathrm{coth}\left(c+d\,x\right)}^4\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^4)^(3/2),x)
```

```
[Out] int((b*coth(c + d*x)^4)^(3/2), x)
```

3.41 $\int \sqrt{b \coth^4(c+dx)}\, dx$

Optimal. Leaf size=50

$$-\frac{\sqrt{b \coth^4(c+dx)}\ \tanh(c+dx)}{d}+x\sqrt{b \coth^4(c+dx)}\ \tanh^2(c+dx)$$

```
[Out] -(b*coth(d*x+c)^4)^(1/2)*tanh(d*x+c)/d+x*(b*coth(d*x+c)^4)^(1/2)*tanh(d*x+c
)^2
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3739, 3554, 8}

$$x \tanh^2(c+dx)\sqrt{b \coth^4(c+dx)}-\frac{\tanh(c+dx)\sqrt{b \coth^4(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Coth[c + d*x]^4],x]
```

```
[Out] -((Sqrt[b*Coth[c + d*x]^4]*Tanh[c + d*x])/d) + x*Sqrt[b*Coth[c + d*x]^4]*Ta
nh[c + d*x]^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \sqrt{b\coth^4(c+dx)}\,dx = \left(\sqrt{b\coth^4(c+dx)}\,\tanh^2(c+dx)\right)\int \coth^2(c+dx)\,dx$$
$$= -\frac{\sqrt{b\coth^4(c+dx)}\,\tanh(c+dx)}{d} + \left(\sqrt{b\coth^4(c+dx)}\,\tanh^2(c+dx)\right)\int 1\,d$$
$$= -\frac{\sqrt{b\coth^4(c+dx)}\,\tanh(c+dx)}{d} + x\sqrt{b\coth^4(c+dx)}\,\tanh^2(c+dx)$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.02, size = 41, normalized size = 0.82

$$-\frac{\sqrt{b\coth^4(c+dx)}\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\tanh^2(c+dx)\right)\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Coth[c + d*x]^4],x]
```

```
[Out] -((Sqrt[b*Coth[c + d*x]^4]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[c + d*x]^2]
*Tanh[c + d*x])/d)
```

Maple [A]
time = 1.60, size = 55, normalized size = 1.10

method	result	size
derivativedivides	$-\frac{\sqrt{b\left(\coth^4\left(dx+c\right)\right)}\,(2\coth(dx+c)+\ln(\coth(dx+c)-1)-\ln(\coth(dx+c)+1))}{2d\coth(dx+c)^2}$	55
default	$-\frac{\sqrt{b\left(\coth^4\left(dx+c\right)\right)}\,(2\coth(dx+c)+\ln(\coth(dx+c)-1)-\ln(\coth(dx+c)+1))}{2d\coth(dx+c)^2}$	55
risch	$\frac{\sqrt{\frac{b(1+e^{2dx+2c})^4}{(e^{2dx+2c}-1)^4}}\,(e^{2dx+2c}-1)^2x}{(1+e^{2dx+2c})^2} - \frac{2\sqrt{\frac{b(1+e^{2dx+2c})^4}{(e^{2dx+2c}-1)^4}}\,(e^{2dx+2c}-1)}{(1+e^{2dx+2c})^2d}$	119

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(b*coth(d*x+c)^4)^(1/2)*(2*coth(d*x+c)+ln(coth(d*x+c)-1)-ln(coth(d*x
+c)+1))/coth(d*x+c)^2
```

Maxima [A]
time = 0.49, size = 34, normalized size = 0.68

$$\frac{(dx+c)\sqrt{b}}{d}+\frac{2\sqrt{b}}{d(e^{(-2\,dx-2\,c)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] (d*x + c)*sqrt(b)/d + 2*sqrt(b)/(d*(e^(-2*d*x - 2*c) - 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(46) = 92$.
time = 0.35, size = 415, normalized size = 8.30

$$\frac{\left(dx\cosh\left(dx+c\right)^{2}+\left(dxe^{(4\,dx+4\,c)}-2\,dxe^{(2\,dx+2\,c)}+dx\right)\sinh\left(dx+c\right)^{2}-dx+\left(dx\cosh\left(dx+c\right)^{2}-dx-2\right)e^{(4\,dx+4\,c)}-2\left(dx\cosh\left(dx+c\right)^{2}-dx-2\right)e^{(2\,dx+2\,c)}+2\left(dx\cosh\left(dx+c\right)e^{(4\,dx+4\,c)}-2\,dx\cosh\left(dx+c\right)e^{(2\,dx+2\,c)}+dx\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)-2\right)\sqrt{\frac{be^{(8\,dx+8\,c)}+4\,be^{(6\,dx+6\,c)}+6\,be^{(4\,dx+4\,c)}+4\,be^{(2\,dx+2\,c)}+b}{e^{(8\,dx+8\,c)}-4\,e^{(6\,dx+6\,c)}+6\,e^{(4\,dx+4\,c)}-4\,e^{(2\,dx+2\,c)}+1}}}{d\cosh\left(dx+c\right)^{2}+\left(de^{(4\,dx+4\,c)}+2\,de^{(2\,dx+2\,c)}+d\right)\sinh\left(dx+c\right)^{2}+\left(d\cosh\left(dx+c\right)^{2}-d\right)e^{(4\,dx+4\,c)}+2\left(d\cosh\left(dx+c\right)^{2}-d\right)e^{(2\,dx+2\,c)}+2\left(d\cosh\left(dx+c\right)e^{(4\,dx+4\,c)}+2\,d\cosh\left(dx+c\right)e^{(2\,dx+2\,c)}+d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)-d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] (d*x*cosh(d*x + c)^2 + (d*x*e^(4*d*x + 4*c) - 2*d*x*e^(2*d*x + 2*c) + d*x)*
sinh(d*x + c)^2 - d*x + (d*x*cosh(d*x + c)^2 - d*x - 2)*e^(4*d*x + 4*c) - 2
*(d*x*cosh(d*x + c)^2 - d*x - 2)*e^(2*d*x + 2*c) + 2*(d*x*cosh(d*x + c)*e^(
4*d*x + 4*c) - 2*d*x*cosh(d*x + c)*e^(2*d*x + 2*c) + d*x*cosh(d*x + c))*sin
h(d*x + c) - 2)*sqrt((b*e^(8*d*x + 8*c) + 4*b*e^(6*d*x + 6*c) + 6*b*e^(4*d*
x + 4*c) + 4*b*e^(2*d*x + 2*c) + b)/(e^(8*d*x + 8*c) - 4*e^(6*d*x + 6*c) +
6*e^(4*d*x + 4*c) - 4*e^(2*d*x + 2*c) + 1))/(d*cosh(d*x + c)^2 + (d*e^(4*d*
x + 4*c) + 2*d*e^(2*d*x + 2*c) + d)*sinh(d*x + c)^2 + (d*cosh(d*x + c)^2 -
d)*e^(4*d*x + 4*c) + 2*(d*cosh(d*x + c)^2 - d)*e^(2*d*x + 2*c) + 2*(d*cosh(
d*x + c)*e^(4*d*x + 4*c) + 2*d*cosh(d*x + c)*e^(2*d*x + 2*c) + d*cosh(d*x +
 c))*sinh(d*x + c) - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\coth^{4}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**4)**(1/2),x)
```

```
[Out] Integral(sqrt(b*coth(c + d*x)**4), x)
```

Giac [A]
time = 0.45, size = 27, normalized size = 0.54

$$\frac{\left(dx+c-\frac{2}{e^{(2\,dx+2\,c)}-1}\right)\sqrt{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] (d*x + c - 2/(e^(2*d*x + 2*c) - 1))*sqrt(b)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{b \coth(c+d\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^4)^(1/2),x)
```

```
[Out] int((b*coth(c + d*x)^4)^(1/2), x)
```

3.42 $\int \frac{1}{\sqrt{b\coth^4(c+dx)}}\,dx$

Optimal. Leaf size=50

$$-\frac{\coth(c+dx)}{d\sqrt{b\coth^4(c+dx)}}+\frac{x\coth^2(c+dx)}{\sqrt{b\coth^4(c+dx)}}$$

```
[Out] -coth(d*x+c)/d/(b*coth(d*x+c)^4)^(1/2)+x*coth(d*x+c)^2/(b*coth(d*x+c)^4)^(1
/2)
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3739, 3554, 8}

$$\frac{x\coth^2(c+dx)}{\sqrt{b\coth^4(c+dx)}}-\frac{\coth(c+dx)}{d\sqrt{b\coth^4(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Coth[c + d*x]^4],x]
```

```
[Out] -(Coth[c + d*x]/(d*Sqrt[b*Coth[c + d*x]^4])) + (x*Coth[c + d*x]^2)/Sqrt[b*C
oth[c + d*x]^4]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\sqrt{b\coth^4(c+dx)}}\,dx = \frac{\coth^2(c+dx)\int \tanh^2(c+dx)\,dx}{\sqrt{b\coth^4(c+dx)}}$$
$$= -\frac{\coth(c+dx)}{d\sqrt{b\coth^4(c+dx)}} + \frac{\coth^2(c+dx)\int 1\,dx}{\sqrt{b\coth^4(c+dx)}}$$
$$= -\frac{\coth(c+dx)}{d\sqrt{b\coth^4(c+dx)}} + \frac{x\coth^2(c+dx)}{\sqrt{b\coth^4(c+dx)}}$$

Mathematica [A]
time = 0.04, size = 40, normalized size = 0.80

$$\frac{\coth(c+dx)\left(-1+\tanh^{-1}(\tanh(c+dx))\coth(c+dx)\right)}{d\sqrt{b\coth^4(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Coth[c + d*x]^4],x]
```

```
[Out] (Coth[c + d*x]*(-1 + ArcTanh[Tanh[c + d*x]]*Coth[c + d*x]))/(d*Sqrt[b*Coth[
c + d*x]^4])
```

Maple [A]
time = 1.58, size = 59, normalized size = 1.18

method	result	size
derivativedivides	$-\frac{\coth(dx+c)(\ln(\coth(dx+c)-1)\coth(dx+c)-\ln(\coth(dx+c)+1)\coth(dx+c)+2)}{2d\sqrt{b\left(\coth^4(dx+c)\right)}}$	59
default	$-\frac{\coth(dx+c)(\ln(\coth(dx+c)-1)\coth(dx+c)-\ln(\coth(dx+c)+1)\coth(dx+c)+2)}{2d\sqrt{b\left(\coth^4(dx+c)\right)}}$	59
risch	$\frac{\left(1+e^{2dx+2c}\right)^2 x}{\sqrt{\frac{b\left(1+e^{2dx+2c}\right)^4}{\left(e^{2dx+2c}-1\right)^4}}\left(e^{2dx+2c}-1\right)^2}+\frac{2+2\,e^{2dx+2c}}{\sqrt{\frac{b\left(1+e^{2dx+2c}\right)^4}{\left(e^{2dx+2c}-1\right)^4}}\left(e^{2dx+2c}-1\right)^2 d}$	119

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*coth(d*x+c)*(ln(coth(d*x+c)-1)*coth(d*x+c)-ln(coth(d*x+c)+1)*coth(d*
x+c)+2)/(b*coth(d*x+c)^4)^(1/2)
```

Maxima [A]
time = 0.49, size = 36, normalized size = 0.72

$$\frac{dx+c}{\sqrt{b}\,d}-\frac{2\,\sqrt{b}}{\left(be^{(-2\,dx-2\,c)}+b\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] (d*x + c)/(sqrt(b)*d) - 2*sqrt(b)/((b*e^(-2*d*x - 2*c) + b)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(46) = 92$.
time = 0.37, size = 422, normalized size = 8.44

$$\frac{\left(dx\cosh\left(dx+c\right)^{2}+\left(dxe^{(4\,dx+4\,c)}-2\,dxe^{(2\,dx+2\,c)}+dx\right)\sinh\left(dx+c\right)^{2}+dx+\left(dx\cosh\left(dx+c\right)^{2}+dx+2\right)e^{(4\,dx+4\,c)}-2\,\left(dx\cosh\left(dx+c\right)^{2}+dx+2\right)e^{(2\,dx+2\,c)}+2\,\left(dx\cosh\left(dx+c\right)e^{(4\,dx+4\,c)}-2\,dx\cosh\left(dx+c\right)e^{(2\,dx+2\,c)}+dx\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)+2\right)\sqrt{\frac{be^{(8\,dx+8\,c)}+4\,be^{(6\,dx+6\,c)}+6\,be^{(4\,dx+4\,c)}+4\,be^{(2\,dx+2\,c)}+b}{e^{(8\,dx+8\,c)}-4\,e^{(6\,dx+6\,c)}+6\,e^{(4\,dx+4\,c)}-4\,e^{(2\,dx+2\,c)}+1}}}{bd\cosh\left(dx+c\right)^{2}+\left(bde^{(4\,dx+4\,c)}+2\,bde^{(2\,dx+2\,c)}+bd\right)\sinh\left(dx+c\right)^{2}+bd+\left(bd\cosh\left(dx+c\right)^{2}+bd\right)e^{(4\,dx+4\,c)}+2\,\left(bd\cosh\left(dx+c\right)^{2}+bd\right)e^{(2\,dx+2\,c)}+2\,\left(bd\cosh\left(dx+c\right)e^{(4\,dx+4\,c)}+2\,bd\cosh\left(dx+c\right)e^{(2\,dx+2\,c)}+bd\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] (d*x*cosh(d*x + c)^2 + (d*x*e^(4*d*x + 4*c) - 2*d*x*e^(2*d*x + 2*c) + d*x)*
sinh(d*x + c)^2 + d*x + (d*x*cosh(d*x + c)^2 + d*x + 2)*e^(4*d*x + 4*c) - 2
*(d*x*cosh(d*x + c)^2 + d*x + 2)*e^(2*d*x + 2*c) + 2*(d*x*cosh(d*x + c)*e^(
4*d*x + 4*c) - 2*d*x*cosh(d*x + c)*e^(2*d*x + 2*c) + d*x*cosh(d*x + c))*sin
h(d*x + c) + 2)*sqrt((b*e^(8*d*x + 8*c) + 4*b*e^(6*d*x + 6*c) + 6*b*e^(4*d*
x + 4*c) + 4*b*e^(2*d*x + 2*c) + b)/(e^(8*d*x + 8*c) - 4*e^(6*d*x + 6*c) +
6*e^(4*d*x + 4*c) - 4*e^(2*d*x + 2*c) + 1))/(b*d*cosh(d*x + c)^2 + (b*d*e^(
4*d*x + 4*c) + 2*b*d*e^(2*d*x + 2*c) + b*d)*sinh(d*x + c)^2 + b*d + (b*d*co
sh(d*x + c)^2 + b*d)*e^(4*d*x + 4*c) + 2*(b*d*cosh(d*x + c)^2 + b*d)*e^(2*d
*x + 2*c) + 2*(b*d*cosh(d*x + c)*e^(4*d*x + 4*c) + 2*b*d*cosh(d*x + c)*e^(2
*d*x + 2*c) + b*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{b\coth^{4}\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**4)**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*coth(c + d*x)**4), x)
```

Giac [A]
time = 0.42, size = 32, normalized size = 0.64

$$\frac{\frac{dx+c}{\sqrt{b}}+\frac{2}{\sqrt{b}\,\left(e^{(2\,dx+2\,c)}+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] ((d*x + c)/sqrt(b) + 2/(sqrt(b)*(e^(2*d*x + 2*c) + 1)))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{b\coth\left(c+d\,x\right)^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x)^4)^(1/2),x)
```

```
[Out] int(1/(b*coth(c + d*x)^4)^(1/2), x)
```

3.43 $\int \frac{1}{\left(b \coth^4(c+dx)\right)^{3/2}} \, dx$

Optimal. Leaf size=118

$$-\frac{\coth(c+dx)}{bd\sqrt{b\coth^4(c+dx)}}+\frac{x\coth^2(c+dx)}{b\sqrt{b\coth^4(c+dx)}}-\frac{\tanh(c+dx)}{3bd\sqrt{b\coth^4(c+dx)}}-\frac{\tanh^3(c+dx)}{5bd\sqrt{b\coth^4(c+dx)}}$$

```
[Out] -coth(d*x+c)/b/d/(b*coth(d*x+c)^4)^(1/2)+x*coth(d*x+c)^2/b/(b*coth(d*x+c)^4
)^(1/2)-1/3*tanh(d*x+c)/b/d/(b*coth(d*x+c)^4)^(1/2)-1/5*tanh(d*x+c)^3/b/d/(
b*coth(d*x+c)^4)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3739, 3554, 8}

$$-\frac{\coth(c+dx)}{bd\sqrt{b\coth^4(c+dx)}}+\frac{x\coth^2(c+dx)}{b\sqrt{b\coth^4(c+dx)}}-\frac{\tanh^3(c+dx)}{5bd\sqrt{b\coth^4(c+dx)}}-\frac{\tanh(c+dx)}{3bd\sqrt{b\coth^4(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^4)^(-3/2),x]
```

```
[Out] -(Coth[c + d*x]/(b*d*Sqrt[b*Coth[c + d*x]^4])) + (x*Coth[c + d*x]^2)/(b*Sqr
t[b*Coth[c + d*x]^4]) - Tanh[c + d*x]/(3*b*d*Sqrt[b*Coth[c + d*x]^4]) - Tan
h[c + d*x]^3/(5*b*d*Sqrt[b*Coth[c + d*x]^4])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{3/2}}\,dx = \frac{\coth^2(c+dx)\int \tanh^6(c+dx)\,dx}{b\sqrt{b\coth^4(c+dx)}}$$
$$= -\frac{\tanh^3(c+dx)}{5bd\sqrt{b\coth^4(c+dx)}} + \frac{\coth^2(c+dx)\int \tanh^4(c+dx)\,dx}{b\sqrt{b\coth^4(c+dx)}}$$
$$= -\frac{\tanh(c+dx)}{3bd\sqrt{b\coth^4(c+dx)}} - \frac{\tanh^3(c+dx)}{5bd\sqrt{b\coth^4(c+dx)}} + \frac{\coth^2(c+dx)\int \tanh^2(c-}{b\sqrt{b\coth^4(c+dx)}}$$
$$= -\frac{\coth(c+dx)}{bd\sqrt{b\coth^4(c+dx)}} - \frac{\tanh(c+dx)}{3bd\sqrt{b\coth^4(c+dx)}} - \frac{\tanh^3(c+dx)}{5bd\sqrt{b\coth^4(c+dx)}} + \frac{c}{b}$$
$$= -\frac{\coth(c+dx)}{bd\sqrt{b\coth^4(c+dx)}} + \frac{x\coth^2(c+dx)}{b\sqrt{b\coth^4(c+dx)}} - \frac{\tanh(c+dx)}{3bd\sqrt{b\coth^4(c+dx)}} - \frac{}{5bd}$$

Mathematica [A]
time = 0.16, size = 68, normalized size = 0.58

$$\frac{-15\coth(c+dx)+15\tanh^{-1}(\tanh(c+dx))\coth^2(c+dx)-5\tanh(c+dx)-3\tanh^3(c+dx)}{15bd\sqrt{b\coth^4(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^4)^(-3/2),x]
[Out] (-15*Coth[c + d*x] + 15*ArcTanh[Tanh[c + d*x]]*Coth[c + d*x]^2 - 5*Tanh[c +
 d*x] - 3*Tanh[c + d*x]^3)/(15*b*d*Sqrt[b*Coth[c + d*x]^4])
```

Maple [A]
time = 1.44, size = 84, normalized size = 0.71

method	result
derivativedivides	$\frac{\coth(dx+c)\left(15\ln(\coth(dx+c)+1)\left(\coth^5(dx+c)\right)-15\ln(\coth(dx+c)-1)\left(\coth^5(dx+c)\right)-30\left(\coth^4(dx+c)\right)-10\left(\coth^2(d\right.\right.}{30d\left(b\left(\coth^4(dx+c)\right)\right)^{\frac{3}{2}}}$
default	$\frac{\coth(dx+c)\left(15\ln(\coth(dx+c)+1)\left(\coth^5(dx+c)\right)-15\ln(\coth(dx+c)-1)\left(\coth^5(dx+c)\right)-30\left(\coth^4(dx+c)\right)-10\left(\coth^2(d\right.\right.}{30d\left(b\left(\coth^4(dx+c)\right)\right)^{\frac{3}{2}}}$
risch	$\frac{\left(1+e^{2dx+2c}\right)^2 x}{b\left(e^{2dx+2c}-1\right)^2\sqrt{\frac{b\left(1+e^{2dx+2c}\right)^4}{\left(e^{2dx+2c}-1\right)^4}}} + \frac{6\,e^{8dx+8c}+12\,e^{6dx+6c}+\frac{56\,e^{4dx+4c}}{3}+\frac{28\,e^{2dx+2c}}{3}+\frac{46}{15}}{b\left(1+e^{2dx+2c}\right)^3\left(e^{2dx+2c}-1\right)^2\sqrt{\frac{b\left(1+e^{2dx+2c}\right)^4}{\left(e^{2dx+2c}-1\right)^4}}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/30/d*coth(d*x+c)*(15*ln(coth(d*x+c)+1)*coth(d*x+c)^5-15*ln(coth(d*x+c)-1)
*coth(d*x+c)^5-30*coth(d*x+c)^4-10*coth(d*x+c)^2-6)/(b*coth(d*x+c)^4)^(3/2)
```

Maxima [A]
time = 0.52, size = 155, normalized size = 1.31

$$-\frac{2 \left(70 \sqrt{b} e^{(-2 dx-2 c)}+140 \sqrt{b} e^{(-4 dx-4 c)}+90 \sqrt{b} e^{(-6 dx-6 c)}+45 \sqrt{b} e^{(-8 dx-8 c)}+23 \sqrt{b}\right)}{15 \left(5 b^{2} e^{(-2 dx-2 c)}+10 b^{2} e^{(-4 dx-4 c)}+10 b^{2} e^{(-6 dx-6 c)}+5 b^{2} e^{(-8 dx-8 c)}+b^{2} e^{(-10 dx-10 c)}+b^{2}\right) d}+\frac{dx+c}{b^{\frac{3}{2}} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/15*(70*sqrt(b)*e^(-2*d*x - 2*c) + 140*sqrt(b)*e^(-4*d*x - 4*c) + 90*sqrt
(b)*e^(-6*d*x - 6*c) + 45*sqrt(b)*e^(-8*d*x - 8*c) + 23*sqrt(b))/((5*b^2*e^
(-2*d*x - 2*c) + 10*b^2*e^(-4*d*x - 4*c) + 10*b^2*e^(-6*d*x - 6*c) + 5*b^2*
e^(-8*d*x - 8*c) + b^2*e^(-10*d*x - 10*c) + b^2)*d) + (d*x + c)/(b^(3/2)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3473 vs. $2(106) = 212$.
time = 0.39, size = 3473, normalized size = 29.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(15*d*x*cosh(d*x + c)^10 + 15*(d*x*e^(4*d*x + 4*c) - 2*d*x*e^(2*d*x +
2*c) + d*x)*sinh(d*x + c)^10 + 150*(d*x*cosh(d*x + c)*e^(4*d*x + 4*c) - 2*d
*x*cosh(d*x + c)*e^(2*d*x + 2*c) + d*x*cosh(d*x + c))*sinh(d*x + c)^9 + 15*
(5*d*x + 6)*cosh(d*x + c)^8 + 15*(45*d*x*cosh(d*x + c)^2 + 5*d*x + (45*d*x*
cosh(d*x + c)^2 + 5*d*x + 6)*e^(4*d*x + 4*c) - 2*(45*d*x*cosh(d*x + c)^2 +
5*d*x + 6)*e^(2*d*x + 2*c) + 6)*sinh(d*x + c)^8 + 120*(15*d*x*cosh(d*x + c)
^3 + (5*d*x + 6)*cosh(d*x + c) + (15*d*x*cosh(d*x + c)^3 + (5*d*x + 6)*cosh
(d*x + c))*e^(4*d*x + 4*c) - 2*(15*d*x*cosh(d*x + c)^3 + (5*d*x + 6)*cosh(d
*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c)^7 + 30*(5*d*x + 6)*cosh(d*x + c)^6
+ 30*(105*d*x*cosh(d*x + c)^4 + 14*(5*d*x + 6)*cosh(d*x + c)^2 + 5*d*x + (1
05*d*x*cosh(d*x + c)^4 + 14*(5*d*x + 6)*cosh(d*x + c)^2 + 5*d*x + 6)*e^(4*d
*x + 4*c) - 2*(105*d*x*cosh(d*x + c)^4 + 14*(5*d*x + 6)*cosh(d*x + c)^2 + 5
*d*x + 6)*e^(2*d*x + 2*c) + 6)*sinh(d*x + c)^6 + 60*(63*d*x*cosh(d*x + c)^5
 + 14*(5*d*x + 6)*cosh(d*x + c)^3 + 3*(5*d*x + 6)*cosh(d*x + c) + (63*d*x*c
```

osh(d*x + c)^5 + 14*(5*d*x + 6)*cosh(d*x + c)^3 + 3*(5*d*x + 6)*cosh(d*x + c))*e^(4*d*x + 4*c) - 2*(63*d*x*cosh(d*x + c)^5 + 14*(5*d*x + 6)*cosh(d*x + c)^3 + 3*(5*d*x + 6)*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c)^5 + 10*(15*d*x + 28)*cosh(d*x + c)^4 + 10*(315*d*x*cosh(d*x + c)^6 + 105*(5*d*x + 6)*cosh(d*x + c)^4 + 45*(5*d*x + 6)*cosh(d*x + c)^2 + 15*d*x + (315*d*x*cosh(d*x + c)^6 + 105*(5*d*x + 6)*cosh(d*x + c)^4 + 45*(5*d*x + 6)*cosh(d*x + c)^2 + 15*d*x + 28)*e^(4*d*x + 4*c) - 2*(315*d*x*cosh(d*x + c)^6 + 105*(5*d*x + 6)*cosh(d*x + c)^4 + 45*(5*d*x + 6)*cosh(d*x + c)^2 + 15*d*x + 28)*e^(2*d*x + 2*c) + 28)*sinh(d*x + c)^4 + 40*(45*d*x*cosh(d*x + c)^7 + 21*(5*d*x + 6)*cosh(d*x + c)^5 + 15*(5*d*x + 6)*cosh(d*x + c)^3 + (15*d*x + 28)*cosh(d*x + c) + (45*d*x*cosh(d*x + c)^7 + 21*(5*d*x + 6)*cosh(d*x + c)^5 + 15*(5*d*x + 6)*cosh(d*x + c)^3 + (15*d*x + 28)*cosh(d*x + c))*e^(4*d*x + 4*c) - 2*(45*d*x*cosh(d*x + c)^7 + 21*(5*d*x + 6)*cosh(d*x + c)^5 + 15*(5*d*x + 6)*cosh(d*x + c)^3 + (15*d*x + 28)*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c)^3 + 5*(15*d*x + 28)*cosh(d*x + c)^2 + 5*(135*d*x*cosh(d*x + c)^8 + 84*(5*d*x + 6)*cosh(d*x + c)^6 + 90*(5*d*x + 6)*cosh(d*x + c)^4 + 12*(15*d*x + 28)*cosh(d*x + c)^2 + 15*d*x + (135*d*x*cosh(d*x + c)^8 + 84*(5*d*x + 6)*cosh(d*x + c)^6 + 90*(5*d*x + 6)*cosh(d*x + c)^4 + 12*(15*d*x + 28)*cosh(d*x + c)^2 + 15*d*x + 28)*e^(4*d*x + 4*c) - 2*(135*d*x*cosh(d*x + c)^8 + 84*(5*d*x + 6)*cosh(d*x + c)^6 + 90*(5*d*x + 6)*cosh(d*x + c)^4 + 12*(15*d*x + 28)*cosh(d*x + c)^2 + 15*d*x + 28)*e^(2*d*x + 2*c) + 28)*sinh(d*x + c)^2 + 15*d*x + (15*d*x*cosh(d*x + c)^10 + 15*(5*d*x + 6)*cosh(d*x + c)^8 + 30*(5*d*x + 6)*cosh(d*x + c)^6 + 10*(15*d*x + 28)*cosh(d*x + c)^4 + 5*(15*d*x + 28)*cosh(d*x + c)^2 + 15*d*x + 46)*e^(4*d*x + 4*c) - 2*(15*d*x*cosh(d*x + c)^10 + 15*(5*d*x + 6)*cosh(d*x + c)^8 + 30*(5*d*x + 6)*cosh(d*x + c)^6 + 10*(15*d*x + 28)*cosh(d*x + c)^4 + 5*(15*d*x + 28)*cosh(d*x + c)^2 + 15*d*x + 46)*e^(2*d*x + 2*c) + 10*(15*d*x*cosh(d*x + c)^9 + 12*(5*d*x + 6)*cosh(d*x + c)^7 + 18*(5*d*x + 6)*cosh(d*x + c)^5 + 4*(15*d*x + 28)*cosh(d*x + c)^3 + (15*d*x + 28)*cosh(d*x + c) + (15*d*x*cosh(d*x + c)^9 + 12*(5*d*x + 6)*cosh(d*x + c)^7 + 18*(5*d*x + 6)*cosh(d*x + c)^5 + 4*(15*d*x + 28)*cosh(d*x + c)^3 + (15*d*x + 28)*cosh(d*x + c))*e^(4*d*x + 4*c) - 2*(15*d*x*cosh(d*x + c)^9 + 12*(5*d*x + 6)*cosh(d*x + c)^7 + 18*(5*d*x + 6)*cosh(d*x + c)^5 + 4*(15*d*x + 28)*cosh(d*x + c)^3 + (15*d*x + 28)*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c) + 46)*sqrt((b*e^(8*d*x + 8*c) + 4*b*e^(6*d*x + 6*c) + 6*b*e^(4*d*x + 4*c) + 4*b*e^(2*d*x + 2*c) + b)/(e^(8*d*x + 8*c) - 4*e^(6*d*x + 6*c) + 6*e^(4*d*x + 4*c) - 4*e^(2*d*x + 2*c) + 1))/(b^2*d*cosh(d*x + c)^10 + 5*b^2*d*cosh(d*x + c)^8 + (b^2*d*e^(4*d*x + 4*c) + 2*b^2*d*e^(2*d*x + 2*c) + b^2*d)*sinh(d*x + c)^10 + 10*(b^2*d*cosh(d*x + c)*e^(4*d*x + 4*c) + 2*b^2*d*cosh(d*x + c)*e^(2*d*x + 2*c) + b^2*d*cosh(d*x + c))*sinh(d*x + c)^9 + 10*b^2*d*cosh(d*x + c)^6 + 5*(9*b^2*d*cosh(d*x + c)^2 + b^2*d + (9*b^2*d*cosh(d*x + c)^2 + b^2*d)*e^(4*d*x + 4*c) + 2*(9*b^2*d*cosh(d*x + c)^2 + b^2*d)*e^(2*d*x + 2*c))*sinh(d*x + c)^8 + 40*(3*b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c) + (3*b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c))*e^(4*d*x + 4*c) + 2*(3*b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c)^7 + 10*b^2*d*cosh(d*x + c)^4 + 10*(21*b^2*d*cosh(d*x + c)^4 + 14

```
*b^2*d*cosh(d*x + c)^2 + b^2*d + (21*b^2*d*cosh(d*x + c)^4 + 14*b^2*d*cosh(
d*x + c)^2 + b^2*d)*e^(4*d*x + 4*c) + 2*(21*b^2*d*cosh(d*x + c)^4 + 14*b^2*
d*cosh(d*x + c)^2 + b^2*d)*e^(2*d*x + 2*c))*sinh(d*x + c)^6 + 4*(63*b^2*d*c
osh(d*x + c)^5 + 70*b^2*d*cosh(d*x + c)^3 + 15*b^2*d*cosh(d*x + c) + (63*b^
2*d*cosh(d*x + c)^5 + 70*b^2*d*cosh(d*x + c)^3 + 15*b^2*d*cosh(d*x + c))*e^
(4*d*x + 4*c) + 2*(63*b^2*d*cosh(d*x + c)^5 + 70*b^2*d*cosh(d*x + c)^3 + 15
*b^2*d*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c)^5 + 5*b^2*d*cosh(d*x +
 c)^2 + 10*(21*b^2*d*cosh(d*x + c)^6 + 35*b^2*d...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**4)**(3/2),x)
[Out] Integral((b*coth(c + d*x)**4)**(-3/2), x)
```

Giac [A]
time = 0.47, size = 99, normalized size = 0.84

$$\frac{\frac{15\,(dx+c)}{\sqrt{b}} + \frac{2\left(45\sqrt{b}\,e^{(8\,dx+8\,c)}+90\sqrt{b}\,e^{(6\,dx+6\,c)}+140\sqrt{b}\,e^{(4\,dx+4\,c)}+70\sqrt{b}\,e^{(2\,dx+2\,c)}+23\sqrt{b}\right)}{b\left(e^{(2\,dx+2\,c)}+1\right)^5}}{15\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(3/2),x, algorithm="giac")
[Out] 1/15*(15*(d*x + c)/sqrt(b) + 2*(45*sqrt(b)*e^(8*d*x + 8*c) + 90*sqrt(b)*e^(
6*d*x + 6*c) + 140*sqrt(b)*e^(4*d*x + 4*c) + 70*sqrt(b)*e^(2*d*x + 2*c) + 2
3*sqrt(b))/(b*(e^(2*d*x + 2*c) + 1)^5))/(b*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\coth\left(c+d\,x\right)^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x)^4)^(3/2),x)
[Out] int(1/(b*coth(c + d*x)^4)^(3/2), x)
```

3.44 $\int \left(b\coth^4(c+dx)\right)^{4/3}\,dx$

Optimal. Leaf size=353

$$-\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^4(c+dx)}}{2d\coth^{\frac{4}{3}}(c+dx)}+\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^4(c$$

```
[Out] b*arctanh(coth(d*x+c)^(1/3))*(b*coth(d*x+c)^4)^(1/3)/d/coth(d*x+c)^(4/3)-3/
7*b*coth(d*x+c)*(b*coth(d*x+c)^4)^(1/3)/d-3/13*b*coth(d*x+c)^3*(b*coth(d*x+
c)^4)^(1/3)/d-1/4*b*(b*coth(d*x+c)^4)^(1/3)*ln(1-coth(d*x+c)^(1/3)+coth(d*x
+c)^(2/3))/d/coth(d*x+c)^(4/3)+1/4*b*(b*coth(d*x+c)^4)^(1/3)*ln(1+coth(d*x+
c)^(1/3)+coth(d*x+c)^(2/3))/d/coth(d*x+c)^(4/3)-1/2*b*arctan(1/3*(1-2*coth(
d*x+c)^(1/3))*3^(1/2))*(b*coth(d*x+c)^4)^(1/3)*3^(1/2)/d/coth(d*x+c)^(4/3)+
1/2*b*arctan(1/3*(1+2*coth(d*x+c)^(1/3))*3^(1/2))*(b*coth(d*x+c)^4)^(1/3)*3
^(1/2)/d/coth(d*x+c)^(4/3)-3*b*(b*coth(d*x+c)^4)^(1/3)*tanh(d*x+c)/d
```

Rubi [A]
time = 0.15, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3739, 3554, 3557, 335, 216, 648, 632, 210, 642, 212}

$$-\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^4(c+dx)}}{2d\coth^{\frac{4}{3}}(c+dx)}+\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{2\sqrt[3]{\coth(c+dx)}+1}{\sqrt{3}}\right)\sqrt[3]{b\coth^4(c+dx)}}{2d\coth^{\frac{4}{3}}(c+dx)}-\frac{3b\coth(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{7d}-\frac{3b\coth^3(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{13d}-\frac{3b\tanh(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{d}-\frac{b\sqrt[3]{b\coth^4(c+dx)}\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\coth^{\frac{4}{3}}(c+dx)}+\frac{b\sqrt[3]{b\coth^4(c+dx)}\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}+1\right)}{4d\coth^{\frac{4}{3}}(c+dx)}+\frac{b\sqrt[3]{b\coth^4(c+dx)}\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)}{d\coth^{\frac{4}{3}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^4)^(4/3),x]
```

```
[Out] -1/2*(Sqrt[3]*b*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x
]^4)^(1/3))/(d*Coth[c + d*x]^(4/3)) + (Sqrt[3]*b*ArcTan[(1 + 2*Coth[c + d*x
]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^4)^(1/3))/(2*d*Coth[c + d*x]^(4/3)) + (b
*ArcTanh[Coth[c + d*x]^(1/3)]*(b*Coth[c + d*x]^4)^(1/3))/(d*Coth[c + d*x]^(
4/3)) - (3*b*Coth[c + d*x]*(b*Coth[c + d*x]^4)^(1/3))/(7*d) - (3*b*Coth[c +
 d*x]^3*(b*Coth[c + d*x]^4)^(1/3))/(13*d) - (b*(b*Coth[c + d*x]^4)^(1/3)*Lo
g[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*Coth[c + d*x]^(4/3))
 + (b*(b*Coth[c + d*x]^4)^(1/3)*Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]
^(2/3)])/(4*d*Coth[c + d*x]^(4/3)) - (3*b*(b*Coth[c + d*x]^4)^(1/3)*Tanh[c
+ d*x])/d
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(b\coth^4(c+dx)\right)^{4/3} dx = \frac{\left(b\sqrt[3]{b\coth^4(c+dx)}\right)\int \coth^{\frac{16}{3}}(c+dx)\,dx}{\coth^{\frac{4}{3}}(c+dx)}$$

$$= -\frac{3b\coth^3(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{13d} + \frac{\left(b\sqrt[3]{b\coth^4(c+dx)}\right)\int \coth^{\frac{10}{3}}(c+dx$$

$$= -\frac{3b\coth(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{7d} - \frac{3b\coth^3(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{13d} + \Big($$

$$= -\frac{3b\coth(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{7d} - \frac{3b\coth^3(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{13d} - 3b$$

$$= -\frac{3b\coth(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{7d} - \frac{3b\coth^3(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{13d} - 3b$$

$$= -\frac{3b\coth(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{7d} - \frac{3b\coth^3(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{13d} - 3b$$

$$= -\frac{3b\coth(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{7d} - \frac{3b\coth^3(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{13d} - 3b$$

$$= \frac{b\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^4(c+dx)}}{d\coth^{\frac{4}{3}}(c+dx)} - \frac{3b\coth(c+dx)\sqrt[3]{b\coth^4(c+}}{7d}$$

$$= \frac{b\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^4(c+dx)}}{d\coth^{\frac{4}{3}}(c+dx)} - \frac{3b\coth(c+dx)\sqrt[3]{b\coth^4(c+}}{7d}$$

$$= -\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^4(c+dx)}}{2d\coth^{\frac{4}{3}}(c+dx)} + \frac{\sqrt{3}\,b\tan^{-1}\Big(\frac{1+2\sqrt[3]{c}}{}}{2d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.09, size = 68, normalized size = 0.19

$$-\frac{3b\sqrt[3]{b\coth^4(c+dx)}\left(91+13\coth^2(c+dx)+7\coth^4(c+dx)-91\,{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};\coth^2(c+dx)\right)\right)\tanh(c+dx)}{91d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^4)^(4/3),x]
```

```
[Out] (-3*b*(b*Coth[c + d*x]^4)^(1/3)*(91 + 13*Coth[c + d*x]^2 + 7*Coth[c + d*x]^
4 - 91*Hypergeometric2F1[1/6, 1, 7/6, Coth[c + d*x]^2])*Tanh[c + d*x])/(91*
d)
```

Maple [F]
time = 1.99, size = 0, normalized size = 0.00

$$\int \left(b\left(\coth^{4}\left(dx+c\right)\right)\right)^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^4)^(4/3),x)
```

```
[Out] int((b*coth(d*x+c)^4)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^4)^(4/3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2864 vs. $2(295) = 590$.
time = 0.43, size = 2864, normalized size = 8.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^(4/3),x, algorithm="fricas")
```

```
[Out] -1/364*(182*(sqrt(3)*b*cosh(d*x + c)^8 + 8*sqrt(3)*b*cosh(d*x + c)*sinh(d*x
 + c)^7 + sqrt(3)*b*sinh(d*x + c)^8 - 4*sqrt(3)*b*cosh(d*x + c)^6 + 4*(7*sq
rt(3)*b*cosh(d*x + c)^2 - sqrt(3)*b)*sinh(d*x + c)^6 + 8*(7*sqrt(3)*b*cosh(
d*x + c)^3 - 3*sqrt(3)*b*cosh(d*x + c))*sinh(d*x + c)^5 + 6*sqrt(3)*b*cosh(
d*x + c)^4 + 2*(35*sqrt(3)*b*cosh(d*x + c)^4 - 30*sqrt(3)*b*cosh(d*x + c)^2
 + 3*sqrt(3)*b)*sinh(d*x + c)^4 + 8*(7*sqrt(3)*b*cosh(d*x + c)^5 - 10*sqrt(
3)*b*cosh(d*x + c)^3 + 3*sqrt(3)*b*cosh(d*x + c))*sinh(d*x + c)^3 - 4*sqrt(
3)*b*cosh(d*x + c)^2 + 4*(7*sqrt(3)*b*cosh(d*x + c)^6 - 15*sqrt(3)*b*cosh(d
*x + c)^4 + 9*sqrt(3)*b*cosh(d*x + c)^2 - sqrt(3)*b)*sinh(d*x + c)^2 + 8*(s
```

```
qrt(3)*b*cosh(d*x + c)^7 - 3*sqrt(3)*b*cosh(d*x + c)^5 + 3*sqrt(3)*b*cosh(d
*x + c)^3 - sqrt(3)*b*cosh(d*x + c))*sinh(d*x + c) + sqrt(3)*b)*(-b)^(1/3)*
arctan(1/3*(sqrt(3)*b + 2*sqrt(3)*(-b)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c)
)^(1/3))/b) - 182*(sqrt(3)*b*cosh(d*x + c)^8 + 8*sqrt(3)*b*cosh(d*x + c)*si
nh(d*x + c)^7 + sqrt(3)*b*sinh(d*x + c)^8 - 4*sqrt(3)*b*cosh(d*x + c)^6 + 4
*(7*sqrt(3)*b*cosh(d*x + c)^2 - sqrt(3)*b)*sinh(d*x + c)^6 + 8*(7*sqrt(3)*b
*cosh(d*x + c)^3 - 3*sqrt(3)*b*cosh(d*x + c))*sinh(d*x + c)^5 + 6*sqrt(3)*b
*cosh(d*x + c)^4 + 2*(35*sqrt(3)*b*cosh(d*x + c)^4 - 30*sqrt(3)*b*cosh(d*x
+ c)^2 + 3*sqrt(3)*b)*sinh(d*x + c)^4 + 8*(7*sqrt(3)*b*cosh(d*x + c)^5 - 10
*sqrt(3)*b*cosh(d*x + c)^3 + 3*sqrt(3)*b*cosh(d*x + c))*sinh(d*x + c)^3 - 4
*sqrt(3)*b*cosh(d*x + c)^2 + 4*(7*sqrt(3)*b*cosh(d*x + c)^6 - 15*sqrt(3)*b*
cosh(d*x + c)^4 + 9*sqrt(3)*b*cosh(d*x + c)^2 - sqrt(3)*b)*sinh(d*x + c)^2
+ 8*(sqrt(3)*b*cosh(d*x + c)^7 - 3*sqrt(3)*b*cosh(d*x + c)^5 + 3*sqrt(3)*b*
cosh(d*x + c)^3 - sqrt(3)*b*cosh(d*x + c))*sinh(d*x + c) + sqrt(3)*b)*b^(1/
3)*arctan(-1/3*(sqrt(3)*b - 2*sqrt(3)*b^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c
))^(1/3))/b) + 91*(b*cosh(d*x + c)^8 + 8*b*cosh(d*x + c)*sinh(d*x + c)^7 +
b*sinh(d*x + c)^8 - 4*b*cosh(d*x + c)^6 + 4*(7*b*cosh(d*x + c)^2 - b)*sinh(
d*x + c)^6 + 8*(7*b*cosh(d*x + c)^3 - 3*b*cosh(d*x + c))*sinh(d*x + c)^5 +
6*b*cosh(d*x + c)^4 + 2*(35*b*cosh(d*x + c)^4 - 30*b*cosh(d*x + c)^2 + 3*b)
*sinh(d*x + c)^4 + 8*(7*b*cosh(d*x + c)^5 - 10*b*cosh(d*x + c)^3 + 3*b*cosh
(d*x + c))*sinh(d*x + c)^3 - 4*b*cosh(d*x + c)^2 + 4*(7*b*cosh(d*x + c)^6 -
 15*b*cosh(d*x + c)^4 + 9*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^2 + 8*(b*cos
h(d*x + c)^7 - 3*b*cosh(d*x + c)^5 + 3*b*cosh(d*x + c)^3 - b*cosh(d*x + c))
*sinh(d*x + c) + b)*(-b)^(1/3)*log((-b)^(2/3) - (-b)^(1/3)*(b*cosh(d*x + c)
/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + 91*(b*cosh
(d*x + c)^8 + 8*b*cosh(d*x + c)*sinh(d*x + c)^7 + b*sinh(d*x + c)^8 - 4*b*c
osh(d*x + c)^6 + 4*(7*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^6 + 8*(7*b*cosh(
d*x + c)^3 - 3*b*cosh(d*x + c))*sinh(d*x + c)^5 + 6*b*cosh(d*x + c)^4 + 2*(
35*b*cosh(d*x + c)^4 - 30*b*cosh(d*x + c)^2 + 3*b)*sinh(d*x + c)^4 + 8*(7*b
*cosh(d*x + c)^5 - 10*b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c))*sinh(d*x + c)^
3 - 4*b*cosh(d*x + c)^2 + 4*(7*b*cosh(d*x + c)^6 - 15*b*cosh(d*x + c)^4 + 9
*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^2 + 8*(b*cosh(d*x + c)^7 - 3*b*cosh(d
*x + c)^5 + 3*b*cosh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c) + b)*b^(1/
3)*log(b^(2/3) - b^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*
x + c)/sinh(d*x + c))^(2/3)) - 182*(b*cosh(d*x + c)^8 + 8*b*cosh(d*x + c)*s
inh(d*x + c)^7 + b*sinh(d*x + c)^8 - 4*b*cosh(d*x + c)^6 + 4*(7*b*cosh(d*x
+ c)^2 - b)*sinh(d*x + c)^6 + 8*(7*b*cosh(d*x + c)^3 - 3*b*cosh(d*x + c))*s
inh(d*x + c)^5 + 6*b*cosh(d*x + c)^4 + 2*(35*b*cosh(d*x + c)^4 - 30*b*cosh(
d*x + c)^2 + 3*b)*sinh(d*x + c)^4 + 8*(7*b*cosh(d*x + c)^5 - 10*b*cosh(d*x
+ c)^3 + 3*b*cosh(d*x + c))*sinh(d*x + c)^3 - 4*b*cosh(d*x + c)^2 + 4*(7*b*
cosh(d*x + c)^6 - 15*b*cosh(d*x + c)^4 + 9*b*cosh(d*x + c)^2 - b)*sinh(d*x
+ c)^2 + 8*(b*cosh(d*x + c)^7 - 3*b*cosh(d*x + c)^5 + 3*b*cosh(d*x + c)^3 -
 b*cosh(d*x + c))*sinh(d*x + c) + b)*(-b)^(1/3)*log((-b)^(1/3) + (b*cosh(d*
x + c)/sinh(d*x + c))^(1/3)) - 182*(b*cosh(d*x + c)^8 + 8*b*cosh(d*x + c)*s
inh(d*x + c)^7 + b*sinh(d*x + c)^8 - 4*b*cosh(d*x + c)^6 + 4*(7*b*cosh(d*x
```

```
+ c)^2 - b)*sinh(d*x + c)^6 + 8*(7*b*cosh(d*x + c)^3 - 3*b*cosh(d*x + c))*s
inh(d*x + c)^5 + 6*b*cosh(d*x + c)^4 + 2*(35*b*cosh(d*x + c)^4 - 30*b*cosh(
d*x + c)^2 + 3*b)*sinh(d*x + c)^4 + 8*(7*b*cosh(d*x + c)^5 - 10*b*cosh(d*x
+ c)^3 + 3*b*cosh(d*x + c))*sinh(d*x + c)^3 - 4*b*cosh(d*x + c)^2 + 4*(7*b*
cosh(d*x + c)^6 - 15*b*cosh(d*x + c)^4 + 9*b*cosh(d*x + c)^2 - b)*sinh(d*x
+ c)^2 + 8*(b*cosh(d*x + c)^7 - 3*b*cosh(d*x + c)^5 + 3*b*cosh(d*x + c)^3 -
 b*cosh(d*x + c))*sinh(d*x + c) + b)*b^(1/3)*log(b^(1/3) + (b*cosh(d*x + c)
/sinh(d*x + c))^(1/3)) + 12*(111*b*cosh(d*x + c)^8 + 888*b*cosh(d*x + c)*si
nh(d*x + c)^7 + 111*b*sinh(d*x + c)^8 - 336*b*cosh(d*x + c)^6 + 84*(37*b*co
sh(d*x + c)^2 - 4*b)*sinh(d*x + c)^6 + 168*(37*b*cosh(d*x + c)^3 - 12*b*cos
h(d*x + c))*sinh(d*x + c)^5 + 562*b*cosh(d*x + c)^4 + 2*(3885*b*cosh(d*x +
c)^4 - 2520*b*cosh(d*x + c)^2 + 281*b)*sinh(d*x + c)^4 + 8*(777*b*cosh(d*x
+ c)^5 - 840*b*cosh(d*x + c)^3 + 281*b*cosh(d*x...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\coth^4\left(c+dx\right)\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**4)**(4/3),x)
[Out] Integral((b*coth(c + d*x)**4)**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^(4/3),x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^4)^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(b\,{\mathrm{coth}\left(c+d\,x\right)}^4\right)}^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^4)^(4/3),x)
[Out] int((b*coth(c + d*x)^4)^(4/3), x)
```

3.45 $\int \left(b\coth^4(c+dx)\right)^{2/3}\,dx$

Optimal. Leaf size=291

$$\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\left(b\coth^4(c+dx)\right)^{2/3}}{2d\coth^{\frac{8}{3}}(c+dx)}-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\left(b\coth^4(c+dx\right.}{2d\coth^{\frac{8}{3}}(c+dx)}$$

```
[Out] arctanh(coth(d*x+c)^(1/3))*(b*coth(d*x+c)^4)^(2/3)/d/coth(d*x+c)^(8/3)-1/4*
(b*coth(d*x+c)^4)^(2/3)*ln(1-coth(d*x+c)^(1/3)+coth(d*x+c)^(2/3))/d/coth(d*
x+c)^(8/3)+1/4*(b*coth(d*x+c)^4)^(2/3)*ln(1+coth(d*x+c)^(1/3)+coth(d*x+c)^(
2/3))/d/coth(d*x+c)^(8/3)+1/2*arctan(1/3*(1-2*coth(d*x+c)^(1/3))*3^(1/2))*(
b*coth(d*x+c)^4)^(2/3)*3^(1/2)/d/coth(d*x+c)^(8/3)-1/2*arctan(1/3*(1+2*coth
(d*x+c)^(1/3))*3^(1/2))*(b*coth(d*x+c)^4)^(2/3)*3^(1/2)/d/coth(d*x+c)^(8/3)
-3/5*(b*coth(d*x+c)^4)^(2/3)*tanh(d*x+c)/d
```

Rubi [A]
time = 0.16, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3739, 3554, 3557, 335, 302, 648, 632, 210, 642, 212}

$$\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\left(b\coth^4(c+dx)\right)^{2/3}}{2d\coth^{\frac{8}{3}}(c+dx)}-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{2\sqrt[3]{\coth(c+dx)}+1}{\sqrt{3}}\right)\left(b\coth^4(c+dx)\right)^{2/3}}{2d\coth^{\frac{8}{3}}(c+dx)}-\frac{3\tanh(c+dx)\left(b\coth^4(c+dx)\right)^{2/3}}{5d}-\frac{\left(b\coth^4(c+dx)\right)^{2/3}\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\coth^{\frac{8}{3}}(c+dx)}+\frac{\left(b\coth^4(c+dx)\right)^{2/3}\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}+1\right)}{4d\coth^{\frac{8}{3}}(c+dx)}+\frac{\left(b\coth^4(c+dx)\right)^{2/3}\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)}{d\coth^{\frac{8}{3}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^4)^(2/3),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^4)^(2
/3))/(2*d*Coth[c + d*x]^(8/3)) - (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3)
)/Sqrt[3]]*(b*Coth[c + d*x]^4)^(2/3))/(2*d*Coth[c + d*x]^(8/3)) + (ArcTanh[
Coth[c + d*x]^(1/3)]*(b*Coth[c + d*x]^4)^(2/3))/(d*Coth[c + d*x]^(8/3)) - (
(b*Coth[c + d*x]^4)^(2/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)
])/(4*d*Coth[c + d*x]^(8/3)) + ((b*Coth[c + d*x]^4)^(2/3)*Log[1 + Coth[c +
d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*Coth[c + d*x]^(8/3)) - (3*(b*Coth[c
 + d*x]^4)^(2/3)*Tanh[c + d*x])/(5*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(b\coth^4(c+dx)\right)^{2/3}\,dx = \frac{\left(b\coth^4(c+dx)\right)^{2/3}\int \coth^{\frac{8}{3}}(c+dx)\,dx}{\coth^{\frac{8}{3}}(c+dx)}$$

$$= -\frac{3\left(b\coth^4(c+dx)\right)^{2/3}\tanh(c+dx)}{5d} + \frac{\left(b\coth^4(c+dx)\right)^{2/3}\int \coth^{\frac{2}{3}}(c+dx)\,dx}{\coth^{\frac{8}{3}}(c+dx)}$$

$$= -\frac{3\left(b\coth^4(c+dx)\right)^{2/3}\tanh(c+dx)}{5d} - \frac{\left(b\coth^4(c+dx)\right)^{2/3}\,\mathrm{Subst}\left(\int \frac{x^{2/3}}{-1+x^2}\,dx,\right.}{d\coth^{\frac{8}{3}}(c+dx)}$$

$$= -\frac{3\left(b\coth^4(c+dx)\right)^{2/3}\tanh(c+dx)}{5d} - \frac{\left(3\left(b\coth^4(c+dx)\right)^{2/3}\right)\mathrm{Subst}\left(\int \frac{x^4}{-1+x^6}\right.}{d\coth^{\frac{8}{3}}(c+dx)}$$

$$= -\frac{3\left(b\coth^4(c+dx)\right)^{2/3}\tanh(c+dx)}{5d} + \frac{\left(b\coth^4(c+dx)\right)^{2/3}\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x\right.}{d\coth^{\frac{8}{3}}(c+dx)}$$

$$= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\left(b\coth^4(c+dx)\right)^{2/3}}{d\coth^{\frac{8}{3}}(c+dx)} - \frac{3\left(b\coth^4(c+dx)\right)^{2/3}\tanh(c}{5d}$$

$$= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\left(b\coth^4(c+dx)\right)^{2/3}}{d\coth^{\frac{8}{3}}(c+dx)} - \frac{\left(b\coth^4(c+dx)\right)^{2/3}\log\left(1-\right.}{4d\,\mathrm{cot}}$$

$$= \frac{\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\left(b\coth^4(c+dx)\right)^{2/3}}{2d\coth^{\frac{8}{3}}(c+dx)} - \frac{\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{\mathrm{cot}}}{\sqrt{}}\right.}{2d}$$

Mathematica [A]

time = 0.23, size = 166, normalized size = 0.57

$$\frac{\left(b\coth^4(c+dx)\right)^{2/3}\left(20\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)-12\coth^{\frac{5}{3}}(c+dx)+5\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)-2\sqrt{3}\,\text{ArcTan}\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)-\log\left(1-\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)+\log\left(1+\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)\right)\right)}{20d\coth^{\frac{8}{3}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^4)^(2/3),x]
```

```
[Out] ((b*Coth[c + d*x]^4)^(2/3)*(20*ArcTanh[Coth[c + d*x]^(1/3)] - 12*Coth[c + d
*x]^(5/3) + 5*(2*Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]] - 2*Sq
rt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]] - Log[1 - Coth[c + d*x]^(
1/3) + Coth[c + d*x]^(2/3)] + Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(
2/3)])))/(20*d*Coth[c + d*x]^(8/3))
```

Maple [F]
time = 1.96, size = 0, normalized size = 0.00

$$\int \left(b\left(\coth^4(dx+c)\right)\right)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^4)^(2/3),x)
```

```
[Out] int((b*coth(d*x+c)^4)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^4)^(2/3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 618 vs. 2(239) = 478.
time = 0.36, size = 618, normalized size = 2.12

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/20*(10*(sqrt(3)*cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c)*sinh(d*x + c)
+ sqrt(3)*sinh(d*x + c)^2 - sqrt(3))*(-b^2)^(1/3)*arctan(-1/3*(sqrt(3)*b -
2*sqrt(3)*(-b^2)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/b) + 10*(sqrt
```

```
(3)*cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*sinh(
d*x + c)^2 - sqrt(3))*(b^2)^(1/3)*arctan(-1/3*(sqrt(3)*b - 2*sqrt(3)*(b^2)^
(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/b) + 5*(-b^2)^(1/3)*(cosh(d*x
+ c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*log(b*(b*cosh
(d*x + c)/sinh(d*x + c))^(2/3) - (-b^2)^(1/3)*b + (-b^2)^(2/3)*(b*cosh(d*x
+ c)/sinh(d*x + c))^(1/3)) + 5*(b^2)^(1/3)*(cosh(d*x + c)^2 + 2*cosh(d*x +
c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*log(b*(b*cosh(d*x + c)/sinh(d*x + c
))^(2/3) + (b^2)^(1/3)*b - (b^2)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3
)) - 10*(-b^2)^(1/3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sin
h(d*x + c)^2 - 1)*log(b*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) - (-b^2)^(2/3
)) - 10*(b^2)^(1/3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh
(d*x + c)^2 - 1)*log(b*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b^2)^(2/3))
 + 12*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 +
1)*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x
 + c)*sinh(d*x + c) + d*sinh(d*x + c)^2 - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\coth^4\left(c+dx\right)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**4)**(2/3),x)
[Out] Integral((b*coth(c + d*x)**4)**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^(2/3),x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^4)^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(b\,{\mathrm{coth}\left(c+d\,x\right)}^4\right)}^{2/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^4)^(2/3),x)
[Out] int((b*coth(c + d*x)^4)^(2/3), x)
```

3.46 $\int \sqrt[3]{b\coth^4(c+dx)}\,dx$

Optimal. Leaf size=289

$$-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^4(c+dx)}}{2d\coth^{\frac{4}{3}}(c+dx)}+\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^4(c+}$$

```
[Out] arctanh(coth(d*x+c)^(1/3))*(b*coth(d*x+c)^4)^(1/3)/d/coth(d*x+c)^(4/3)-1/4*
(b*coth(d*x+c)^4)^(1/3)*ln(1-coth(d*x+c)^(1/3)+coth(d*x+c)^(2/3))/d/coth(d*
x+c)^(4/3)+1/4*(b*coth(d*x+c)^4)^(1/3)*ln(1+coth(d*x+c)^(1/3)+coth(d*x+c)^(
2/3))/d/coth(d*x+c)^(4/3)-1/2*arctan(1/3*(1-2*coth(d*x+c)^(1/3))*3^(1/2))*(
b*coth(d*x+c)^4)^(1/3)*3^(1/2)/d/coth(d*x+c)^(4/3)+1/2*arctan(1/3*(1+2*coth
(d*x+c)^(1/3))*3^(1/2))*(b*coth(d*x+c)^4)^(1/3)*3^(1/2)/d/coth(d*x+c)^(4/3)
-3*(b*coth(d*x+c)^4)^(1/3)*tanh(d*x+c)/d
```

Rubi [A]
time = 0.12, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3739, 3554, 3557, 335, 216, 648, 632, 210, 642, 212}

$$-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^4(c+dx)}}{2d\coth^{\frac{4}{3}}(c+dx)}+\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{2\sqrt[3]{\coth(c+dx)}+1}{\sqrt{3}}\right)\sqrt[3]{b\coth^4(c+dx)}}{2d\coth^{\frac{4}{3}}(c+dx)}-\frac{3\tanh(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{d}-\frac{\sqrt[3]{b\coth^4(c+dx)}\,\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\coth^{\frac{4}{3}}(c+dx)}+\frac{\sqrt[3]{b\coth^4(c+dx)}\,\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}+1\right)}{4d\coth^{\frac{4}{3}}(c+dx)}+\frac{\sqrt[3]{b\coth^4(c+dx)}\,\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)}{d\coth^{\frac{4}{3}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^4)^(1/3),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^
4)^(1/3))/(d*Coth[c + d*x]^(4/3)) + (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1
/3))/Sqrt[3]]*(b*Coth[c + d*x]^4)^(1/3))/(2*d*Coth[c + d*x]^(4/3)) + (ArcTa
nh[Coth[c + d*x]^(1/3)]*(b*Coth[c + d*x]^4)^(1/3))/(d*Coth[c + d*x]^(4/3))
- ((b*Coth[c + d*x]^4)^(1/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2
/3)])/(4*d*Coth[c + d*x]^(4/3)) + ((b*Coth[c + d*x]^4)^(1/3)*Log[1 + Coth[c
 + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*Coth[c + d*x]^(4/3)) - (3*(b*Cot
h[c + d*x]^4)^(1/3)*Tanh[c + d*x])/d
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \sqrt[3]{b\coth^4(c+dx)}\,dx = \frac{\sqrt[3]{b\coth^4(c+dx)}\int \coth^{\frac{4}{3}}(c+dx)\,dx}{\coth^{\frac{4}{3}}(c+dx)}$$

$$= -\frac{3\sqrt[3]{b\coth^4(c+dx)}\tanh(c+dx)}{d} + \frac{\sqrt[3]{b\coth^4(c+dx)}\int \frac{1}{\coth^{\frac{2}{3}}(c+dx)}\,dx}{\coth^{\frac{4}{3}}(c+dx)}$$

$$= -\frac{3\sqrt[3]{b\coth^4(c+dx)}\tanh(c+dx)}{d} - \frac{\sqrt[3]{b\coth^4(c+dx)}\,\mathrm{Subst}\left(\int \frac{1}{x^{2/3}(-1+x^2)}\,dx\right.}{d\coth^{\frac{4}{3}}(c+dx)}$$

$$= -\frac{3\sqrt[3]{b\coth^4(c+dx)}\tanh(c+dx)}{d} - \frac{\left(3\sqrt[3]{b\coth^4(c+dx)}\right)\mathrm{Subst}\left(\int \frac{1}{-1+x^6}\,dx\right.}{d\coth^{\frac{4}{3}}(c+dx)}$$

$$= -\frac{3\sqrt[3]{b\coth^4(c+dx)}\tanh(c+dx)}{d} + \frac{\sqrt[3]{b\coth^4(c+dx)}\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[3]{\right.}{d\coth^{\frac{4}{3}}(c+dx)}$$

$$= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^4(c+dx)}}{d\coth^{\frac{4}{3}}(c+dx)} - \frac{3\sqrt[3]{b\coth^4(c+dx)}\tanh(c+d}{d}$$

$$= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^4(c+dx)}}{d\coth^{\frac{4}{3}}(c+dx)} - \frac{\sqrt[3]{b\coth^4(c+dx)}\log\left(1-\sqrt[3]{}\right.}{4d\coth^{\frac{4}{3}}}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^4(c+dx)}}{2d\coth^{\frac{4}{3}}(c+dx)} + \frac{\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{\coth}}{\sqrt{}}\right.}{2d\,c}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.02, size = 43, normalized size = 0.15

$$\frac{3\sqrt[3]{b\coth^4(c+dx)}\left(-1+{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};\coth^2(c+dx)\right)\right)\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^4)^(1/3),x]
```

```
[Out] (3*(b*Coth[c + d*x]^4)^(1/3)*(-1 + Hypergeometric2F1[1/6, 1, 7/6, Coth[c +
d*x]^2])*Tanh[c + d*x])/d
```

Maple [F]
time = 1.97, size = 0, normalized size = 0.00

$$\int \left(b\left(\coth^4\left(dx+c\right)\right)\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^4)^(1/3),x)
```

```
[Out] int((b*coth(d*x+c)^4)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^4)^(1/3), x)
```

Fricas [A]
time = 0.41, size = 288, normalized size = 1.00

$$-\frac{2\sqrt{3}\,(-b)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,b+2\sqrt{3}\,(-b)^{\frac{2}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}}{3b}\right)-2\sqrt{3}\,b^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}\,b-2\sqrt{3}\,b^{\frac{2}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}}{3b}\right)+(-b)^{\frac{1}{3}}\log\left((-b)^{\frac{2}{3}}-(-b)^{\frac{1}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}+\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{2}{3}}\right)+b^{\frac{1}{3}}\log\left(b^{\frac{2}{3}}-b^{\frac{1}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}+\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{2}{3}}\right)-2\,(-b)^{\frac{1}{3}}\log\left((-b)^{\frac{1}{3}}+\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}\right)-2\,b^{\frac{1}{3}}\log\left(b^{\frac{1}{3}}+\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}\right)+12\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*(-b)^(1/3)*arctan(1/3*(sqrt(3)*b + 2*sqrt(3)*(-b)^(2/3)*(b*
cosh(d*x + c)/sinh(d*x + c))^(1/3))/b) - 2*sqrt(3)*b^(1/3)*arctan(-1/3*(sqr
t(3)*b - 2*sqrt(3)*b^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/b) + (-b)
^(1/3)*log((-b)^(2/3) - (-b)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) +
(b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + b^(1/3)*log(b^(2/3) - b^(1/3)*(b*c
osh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3))
```

```
- 2*(-b)^(1/3)*log((-b)^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) - 2*
b^(1/3)*log(b^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) + 12*(b*cosh(d
*x + c)/sinh(d*x + c))^(1/3))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{b\coth^4(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**4)**(1/3),x)
[Out] Integral((b*coth(c + d*x)**4)**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^(1/3),x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^4)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(b\,\mathrm{coth}(c+d\,x)^4\right)^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^4)^(1/3),x)
[Out] int((b*coth(c + d*x)^4)^(1/3), x)
```

3.47 $\int \frac{1}{\sqrt[3]{b\coth^4(c+dx)}}\,dx$

Optimal. Leaf size=289

$$-\frac{3\coth(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}}+\frac{\sqrt{3}\,\mathrm{ArcTan}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{4}{3}}(c+dx)}{2d\sqrt[3]{b\coth^4(c+dx)}}-\frac{\sqrt{3}\,\mathrm{ArcTan}\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right.}{2d\sqrt[3]{b\coth^4(c+dx}}$$

```
[Out] -3*coth(d*x+c)/d/(b*coth(d*x+c)^4)^(1/3)+arctanh(coth(d*x+c)^(1/3))*coth(d*
x+c)^(4/3)/d/(b*coth(d*x+c)^4)^(1/3)-1/4*coth(d*x+c)^(4/3)*ln(1-coth(d*x+c)
^(1/3)+coth(d*x+c)^(2/3))/d/(b*coth(d*x+c)^4)^(1/3)+1/4*coth(d*x+c)^(4/3)*l
n(1+coth(d*x+c)^(1/3)+coth(d*x+c)^(2/3))/d/(b*coth(d*x+c)^4)^(1/3)+1/2*arct
an(1/3*(1-2*coth(d*x+c)^(1/3))*3^(1/2))*coth(d*x+c)^(4/3)*3^(1/2)/d/(b*coth
(d*x+c)^4)^(1/3)-1/2*arctan(1/3*(1+2*coth(d*x+c)^(1/3))*3^(1/2))*coth(d*x+c
)^(4/3)*3^(1/2)/d/(b*coth(d*x+c)^4)^(1/3)
```

Rubi [A]
time = 0.16, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3739, 3555, 3557, 335, 302, 648, 632, 210, 642, 212}

$$\frac{\sqrt{3}\coth^{\frac{4}{3}}(c+dx)\mathrm{ArcTan}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)}{2d\sqrt[3]{b\coth^4(c+dx)}}-\frac{\sqrt{3}\coth^{\frac{4}{3}}(c+dx)\mathrm{ArcTan}\left(\frac{2\sqrt[3]{\coth(c+dx)}+1}{\sqrt{3}}\right)}{2d\sqrt[3]{b\coth^4(c+dx)}}-\frac{3\coth(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}}-\frac{\coth^{\frac{4}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\sqrt[3]{b\coth^4(c+dx)}}+\frac{\coth^{\frac{4}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}+1\right)}{4d\sqrt[3]{b\coth^4(c+dx)}}+\frac{\coth^{\frac{4}{3}}(c+dx)\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)}{d\sqrt[3]{b\coth^4(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^4)^(-1/3),x]
```

```
[Out] (-3*Coth[c + d*x])/(d*(b*Coth[c + d*x]^4)^(1/3)) + (Sqrt[3]*ArcTan[(1 - 2*C
oth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c + d*x]^(4/3))/(2*d*(b*Coth[c + d*x]^4)^
(1/3)) - (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c + d*x]
^(4/3))/(2*d*(b*Coth[c + d*x]^4)^(1/3)) + (ArcTanh[Coth[c + d*x]^(1/3)]*Cot
h[c + d*x]^(4/3))/(d*(b*Coth[c + d*x]^4)^(1/3)) - (Coth[c + d*x]^(4/3)*Log[
1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*(b*Coth[c + d*x]^4)^(1
/3)) + (Coth[c + d*x]^(4/3)*Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/
3)])/(4*d*(b*Coth[c + d*x]^4)^(1/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{b\coth^4(c+dx)}}\,dx = \frac{\coth^{\frac{4}{3}}(c+dx)\int \frac{1}{\coth^{\frac{4}{3}}(c+dx)}\,dx}{\sqrt[3]{b\coth^4(c+dx)}}$$

$$= -\frac{3\coth(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}} + \frac{\coth^{\frac{4}{3}}(c+dx)\int \coth^{\frac{2}{3}}(c+dx)\,dx}{\sqrt[3]{b\coth^4(c+dx)}}$$

$$= -\frac{3\coth(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}} - \frac{\coth^{\frac{4}{3}}(c+dx)\text{Subst}\left(\int \frac{x^{2/3}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d\sqrt[3]{b\coth^4(c+dx)}}$$

$$= -\frac{3\coth(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}} - \frac{\left(3\coth^{\frac{4}{3}}(c+dx)\right)\text{Subst}\left(\int \frac{x^4}{-1+x^6}\,dx, x, \sqrt[3]{\coth(c+dx)}\right.}{d\sqrt[3]{b\coth^4(c+dx)}}$$

$$= -\frac{3\coth(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}} + \frac{\coth^{\frac{4}{3}}(c+dx)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d\sqrt[3]{b\coth^4(c+dx)}} + \cdots$$

$$= -\frac{3\coth(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}} + \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{4}{3}}(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}} - \frac{\coth^{\frac{4}{3}}(c+\cdots}{\cdots}$$

$$= -\frac{3\coth(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}} + \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{4}{3}}(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}} - \frac{\coth^{\frac{4}{3}}(c+\cdots}{\cdots}$$

$$= -\frac{3\coth(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}} + \frac{\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{4}{3}}(c+dx)}{2d\sqrt[3]{b\coth^4(c+dx)}} - \frac{\sqrt{3}\cdots}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 41, normalized size = 0.14

$$-\frac{3\coth(c+dx)\,{}_2F_1\left(-\frac{1}{6},1;\frac{5}{6};\coth^2(c+dx)\right)}{d\sqrt[3]{b\coth^4(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^4)^(-1/3),x]
```

```
[Out] (-3*Coth[c + d*x]*Hypergeometric2F1[-1/6, 1, 5/6, Coth[c + d*x]^2])/(d*(b*C
oth[c + d*x]^4)^(1/3))
```

Maple [F]
time = 1.96, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\left(\coth^4(dx+c)\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^4)^(1/3),x)
```

```
[Out] int(1/(b*coth(d*x+c)^4)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^4)^(-1/3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 585 vs. 2(239) = 478.
time = 0.43, size = 3316, normalized size = 11.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh
(d*x + c)^2 + b)*sqrt((-b)^(1/3)/b)*log(3*b*cosh(d*x + c)^2 + 6*b*cosh(d*x
```

```
+ c)*sinh(d*x + c) + 3*b*sinh(d*x + c)^2 - 3*(cosh(d*x + c)^2 + 2*cosh(d*x
+ c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3)*(b*cosh(d*x + c)/sinh(
d*x + c))^(1/3) - sqrt(3)*(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x +
c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3)
+ (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2
- b)*(-b)^(1/3) - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*
sinh(d*x + c)^2 - b)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))*sqrt((-b)^(1/3)
/b) + b) + sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b
*sinh(d*x + c)^2 + b)*sqrt(-1/b^(2/3))*log(-(2*sqrt(3)*(cosh(d*x + c)^2 + 2
*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*b^(2/3)*(b*cosh(d*x + c
)/sinh(d*x + c))^(2/3)*sqrt(-1/b^(2/3)) - b*cosh(d*x + c)^2 - 2*b*cosh(d*x
+ c)*sinh(d*x + c) - b*sinh(d*x + c)^2 - sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*c
osh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*b^(1/3)*sqrt(-1/b^(2/3)
) + (sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(
d*x + c)^2 - b)*sqrt(-1/b^(2/3)) + 3*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sin
h(d*x + c) + sinh(d*x + c)^2 - 1)*b^(2/3))*(b*cosh(d*x + c)/sinh(d*x + c))^
(1/3) - 3*b)/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
c)^2)) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2
 + 1)*(-b)^(2/3)*log((-b)^(2/3) - (-b)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c)
)^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) - (cosh(d*x + c)^2 + 2*cos
h(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*b^(2/3)*log(b^(2/3) - b^(1/
3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^
(2/3)) - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2 + 1)*(-b)^(2/3)*log((-b)^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3))
+ 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)
*b^(2/3)*log(b^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) - 12*(cosh(d*
x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(b*cosh(d*x
 + c)/sinh(d*x + c))^(2/3))/(b*d*cosh(d*x + c)^2 + 2*b*d*cosh(d*x + c)*sinh
(d*x + c) + b*d*sinh(d*x + c)^2 + b*d), -1/4*(2*sqrt(3)*(b*cosh(d*x + c)^2
+ 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*sqrt(-(-b)^(1/3)
/b)*arctan(-1/3*sqrt(3)*(-b)^(1/3)*sqrt(-(-b)^(1/3)/b) + 2/3*sqrt(3)*(b*cos
h(d*x + c)/sinh(d*x + c))^(1/3)*sqrt(-(-b)^(1/3)/b)) - sqrt(3)*(b*cosh(d*x
+ c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*sqrt(-1/b
^(2/3))*log(-(2*sqrt(3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
sinh(d*x + c)^2 - 1)*b^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3)*sqrt(-1/
b^(2/3)) - b*cosh(d*x + c)^2 - 2*b*cosh(d*x + c)*sinh(d*x + c) - b*sinh(d*x
 + c)^2 - sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*
sinh(d*x + c)^2 - b)*b^(1/3)*sqrt(-1/b^(2/3)) + (sqrt(3)*(b*cosh(d*x + c)^2
 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*sqrt(-1/b^(2/3)
) + 3*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 -
1)*b^(2/3))*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) - 3*b)/(cosh(d*x + c)^2 +
 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - (cosh(d*x + c)^2 + 2*c
osh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(2/3)*log((-b)^(2/3)
 - (-b)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh
(d*x + c))^(2/3)) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh
```

```
(d*x + c)^2 + 1)*b^(2/3)*log(b^(2/3) - b^(1/3)*(b*cosh(d*x + c)/sinh(d*x +
c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + 2*(cosh(d*x + c)^2 + 2
*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(2/3)*log((-b)^(1/
3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) - 2*(cosh(d*x + c)^2 + 2*cosh(d
*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*b^(2/3)*log(b^(1/3) + (b*cosh(
d*x + c)/sinh(d*x + c))^(1/3)) + 12*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh
(d*x + c) + sinh(d*x + c)^2 - 1)*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3))/(b*
d*cosh(d*x + c)^2 + 2*b*d*cosh(d*x + c)*sinh(d*x + c) + b*d*sinh(d*x + c)^2
 + b*d), 1/4*(sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c)
+ b*sinh(d*x + c)^2 + b)*sqrt((-b)^(1/3)/b)*log(3*b*cosh(d*x + c)^2 + 6*b*c
osh(d*x + c)*sinh(d*x + c) + 3*b*sinh(d*x + c)^2 - 3*(cosh(d*x + c)^2 + 2*c
osh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3)*(b*cosh(d*x +
c)/sinh(d*x + c))^(1/3) - sqrt(3)*(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sin
h(d*x + c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c)
)^(2/3) + (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x
 + c)^2 - b)*(-b)^(1/3) - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x +
 c) + b*sinh(d*x + c)^2 - b)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))*sqrt((-
b)^(1/3)/b) + b) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(
d*x + c)^2 + 1)*(-b)^(2/3)*log((-b)^(2/3) - (-b)^(1/3)*(b*cosh(d*x + c)/sin
h(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x +...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{b\coth^4(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**4)**(1/3),x)
[Out] Integral((b*coth(c + d*x)**4)**(-1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(1/3),x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^4)^(-1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\coth\left(c+d\,x\right)^4\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x)^4)^(1/3),x)
[Out] int(1/(b*coth(c + d*x)^4)^(1/3), x)
```

3.48 $\int \frac{1}{\left(b\coth^4(c+dx)\right)^{2/3}}\,dx$

Optimal. Leaf size=291

$$-\frac{3\coth(c+dx)}{5d\left(b\coth^4(c+dx)\right)^{2/3}}-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{8}{3}}(c+dx)}{2d\left(b\coth^4(c+dx)\right)^{2/3}}+\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)}{2d\left(b\coth^4(c+\right.}$$

```
[Out] -3/5*coth(d*x+c)/d/(b*coth(d*x+c)^4)^(2/3)+arctanh(coth(d*x+c)^(1/3))*coth(
d*x+c)^(8/3)/d/(b*coth(d*x+c)^4)^(2/3)-1/4*coth(d*x+c)^(8/3)*ln(1-coth(d*x+
c)^(1/3)+coth(d*x+c)^(2/3))/d/(b*coth(d*x+c)^4)^(2/3)+1/4*coth(d*x+c)^(8/3)
*ln(1+coth(d*x+c)^(1/3)+coth(d*x+c)^(2/3))/d/(b*coth(d*x+c)^4)^(2/3)-1/2*ar
ctan(1/3*(1-2*coth(d*x+c)^(1/3))*3^(1/2))*coth(d*x+c)^(8/3)*3^(1/2)/d/(b*co
th(d*x+c)^4)^(2/3)+1/2*arctan(1/3*(1+2*coth(d*x+c)^(1/3))*3^(1/2))*coth(d*x
+c)^(8/3)*3^(1/2)/d/(b*coth(d*x+c)^4)^(2/3)
```

Rubi [A]
time = 0.13, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3739, 3555, 3557, 335, 216, 648, 632, 210, 642, 212}

$$-\frac{\sqrt{3}\coth^{\frac{8}{3}}(c+dx)\text{ArcTan}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)}{2d\left(b\coth^4(c+dx)\right)^{2/3}}+\frac{\sqrt{3}\coth^{\frac{8}{3}}(c+dx)\text{ArcTan}\left(\frac{2\sqrt[3]{\coth(c+dx)}+1}{\sqrt{3}}\right)}{2d\left(b\coth^4(c+dx)\right)^{2/3}}-\frac{3\coth(c+dx)}{5d\left(b\coth^4(c+dx)\right)^{2/3}}-\frac{\coth^{\frac{8}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\left(b\coth^4(c+dx)\right)^{2/3}}+\frac{\coth^{\frac{8}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}+1\right)}{4d\left(b\coth^4(c+dx)\right)^{2/3}}+\frac{\coth^{\frac{8}{3}}(c+dx)\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)}{d\left(b\coth^4(c+dx)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^4)^(-2/3),x]
```

```
[Out] (-3*Coth[c + d*x])/(5*d*(b*Coth[c + d*x]^4)^(2/3)) - (Sqrt[3]*ArcTan[(1 - 2
*Coth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c + d*x]^(8/3))/(2*d*(b*Coth[c + d*x]^4
)^(2/3)) + (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c + d*
x]^(8/3))/(2*d*(b*Coth[c + d*x]^4)^(2/3)) + (ArcTanh[Coth[c + d*x]^(1/3)]*C
oth[c + d*x]^(8/3))/(d*(b*Coth[c + d*x]^4)^(2/3)) - (Coth[c + d*x]^(8/3)*Lo
g[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*(b*Coth[c + d*x]^4)^
(2/3)) + (Coth[c + d*x]^(8/3)*Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(
2/3)])/(4*d*(b*Coth[c + d*x]^4)^(2/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{2/3}}\,dx = \frac{\coth^{\frac{8}{3}}(c+dx)\int \frac{1}{\coth^{\frac{8}{3}}(c+dx)}\,dx}{\left(b\coth^4(c+dx)\right)^{2/3}}$$

$$= -\frac{3\coth(c+dx)}{5d\left(b\coth^4(c+dx)\right)^{2/3}} + \frac{\coth^{\frac{8}{3}}(c+dx)\int \frac{1}{\coth^{\frac{2}{3}}(c+dx)}\,dx}{\left(b\coth^4(c+dx)\right)^{2/3}}$$

$$= -\frac{3\coth(c+dx)}{5d\left(b\coth^4(c+dx)\right)^{2/3}} - \frac{\coth^{\frac{8}{3}}(c+dx)\text{Subst}\left(\int \frac{1}{x^{2/3}(-1+x^2)}\,dx, x, \coth(c+\right.}{d\left(b\coth^4(c+dx)\right)^{2/3}}$$

$$= -\frac{3\coth(c+dx)}{5d\left(b\coth^4(c+dx)\right)^{2/3}} - \frac{\left(3\coth^{\frac{8}{3}}(c+dx)\right)\text{Subst}\left(\int \frac{1}{-1+x^6}\,dx, x, \sqrt[3]{\coth(c}\right.}{d\left(b\coth^4(c+dx)\right)^{2/3}}$$

$$= -\frac{3\coth(c+dx)}{5d\left(b\coth^4(c+dx)\right)^{2/3}} + \frac{\coth^{\frac{8}{3}}(c+dx)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right.}{d\left(b\coth^4(c+dx)\right)^{2/3}}$$

$$= -\frac{3\coth(c+dx)}{5d\left(b\coth^4(c+dx)\right)^{2/3}} + \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{8}{3}}(c+dx)}{d\left(b\coth^4(c+dx)\right)^{2/3}} - \frac{\coth^{\frac{8}{3}}}{}$$

$$= -\frac{3\coth(c+dx)}{5d\left(b\coth^4(c+dx)\right)^{2/3}} + \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{8}{3}}(c+dx)}{d\left(b\coth^4(c+dx)\right)^{2/3}} - \frac{\coth^{\frac{8}{3}}}{}$$

$$= -\frac{3\coth(c+dx)}{5d\left(b\coth^4(c+dx)\right)^{2/3}} - \frac{\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{8}{3}}(c+dx)}{2d\left(b\coth^4(c+dx)\right)^{2/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.03, size = 43, normalized size = 0.15

$$-\frac{3\coth(c+dx)\,{}_2F_1\left(-\frac{5}{6},1;\frac{1}{6};\coth^2(c+dx)\right)}{5d\left(b\coth^4(c+dx)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^4)^(-2/3),x]
```

```
[Out] (-3*Coth[c + d*x]*Hypergeometric2F1[-5/6, 1, 1/6, Coth[c + d*x]^2])/(5*d*(b
*Coth[c + d*x]^4)^(2/3))
```

Maple [F]
time = 1.91, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\left(\coth^4\left(dx+c\right)\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^4)^(2/3),x)
```

```
[Out] int(1/(b*coth(d*x+c)^4)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^4)^(-2/3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1159 vs. 2(239) = 478.
time = 0.38, size = 1159, normalized size = 3.98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/20*(10*sqrt(3)*(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b
*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + b)*sinh(d
*x + c)^2 + 4*(b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) + b)*sqrt
(-(-b^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-b^2)^(1/3)*b*sqrt(-(-b^2)^(1/3)) - 2
```

```
*sqrt(3)*(-b^2)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)*sqrt(-(-b^2)^(1
/3)))/b^2) + 10*sqrt(3)*(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c
)^3 + b*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + b)
*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) +
b)*(b^2)^(1/6)*arctan(-1/3*sqrt(3)*(b^2)^(1/6)*((b^2)^(1/3)*b - 2*(b^2)^(2/
3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/b^2) + 5*(cosh(d*x + c)^4 + 4*cos
h(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*si
nh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sin
h(d*x + c) + 1)*(-b^2)^(2/3)*log(b*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3) -
(-b^2)^(1/3)*b + (-b^2)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) - 5*(c
osh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*c
osh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^
3 + cosh(d*x + c))*sinh(d*x + c) + 1)*(b^2)^(2/3)*log(b*(b*cosh(d*x + c)/si
nh(d*x + c))^(2/3) + (b^2)^(1/3)*b - (b^2)^(2/3)*(b*cosh(d*x + c)/sinh(d*x
+ c))^(1/3)) - 10*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh
(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2
 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*(-b^2)^(2/3)*log(
b*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) - (-b^2)^(2/3)) + 10*(cosh(d*x + c)
^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)
^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x
 + c))*sinh(d*x + c) + 1)*(b^2)^(2/3)*log(b*(b*cosh(d*x + c)/sinh(d*x + c))
^(1/3) + (b^2)^(2/3)) - 12*(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x
+ c)^3 + b*sinh(d*x + c)^4 - 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 -
 b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c)
 + b)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/(b^2*d*cosh(d*x + c)^4 + 4*b^2
*d*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*d*sinh(d*x + c)^4 + 2*b^2*d*cosh(d*x
 + c)^2 + b^2*d + 2*(3*b^2*d*cosh(d*x + c)^2 + b^2*d)*sinh(d*x + c)^2 + 4*(
b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\coth^4\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**4)**(2/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**4)**(-2/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^4)^(-2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\coth\left(c+d\,x\right)^4\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x)^4)^(2/3),x)
```

```
[Out] int(1/(b*coth(c + d*x)^4)^(2/3), x)
```

3.49 $\int \frac{1}{\left(b\coth^4(c+dx)\right)^{4/3}}\,dx$

Optimal. Leaf size=369

$$-\frac{3\coth(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}}+\frac{\sqrt{3}\,\mathrm{ArcTan}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{4}{3}}(c+dx)}{2bd\sqrt[3]{b\coth^4(c+dx)}}-\frac{\sqrt{3}\,\mathrm{ArcTan}\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right.}{2bd\sqrt[3]{b\coth^4(c+}}$$

```
[Out] -3*coth(d*x+c)/b/d/(b*coth(d*x+c)^4)^(1/3)+arctanh(coth(d*x+c)^(1/3))*coth(
d*x+c)^(4/3)/b/d/(b*coth(d*x+c)^4)^(1/3)-1/4*coth(d*x+c)^(4/3)*ln(1-coth(d*
x+c)^(1/3)+coth(d*x+c)^(2/3))/b/d/(b*coth(d*x+c)^4)^(1/3)+1/4*coth(d*x+c)^(
4/3)*ln(1+coth(d*x+c)^(1/3)+coth(d*x+c)^(2/3))/b/d/(b*coth(d*x+c)^4)^(1/3)+
1/2*arctan(1/3*(1-2*coth(d*x+c)^(1/3))*3^(1/2))*coth(d*x+c)^(4/3)*3^(1/2)/b
/d/(b*coth(d*x+c)^4)^(1/3)-1/2*arctan(1/3*(1+2*coth(d*x+c)^(1/3))*3^(1/2))*
coth(d*x+c)^(4/3)*3^(1/2)/b/d/(b*coth(d*x+c)^4)^(1/3)-3/7*tanh(d*x+c)/b/d/(
b*coth(d*x+c)^4)^(1/3)-3/13*tanh(d*x+c)^3/b/d/(b*coth(d*x+c)^4)^(1/3)
```

Rubi [A]

time = 0.18, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3739, 3555, 3557, 335, 302, 648, 632, 210, 642, 212}

$$\frac{\sqrt{3}\coth^{\frac{4}{3}}(c+dx)\mathrm{ArcTan}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)}{2bd\sqrt[3]{b\coth^4(c+dx)}}-\frac{\sqrt{3}\coth^{\frac{4}{3}}(c+dx)\mathrm{ArcTan}\left(\frac{2\sqrt[3]{\coth(c+dx)}+1}{\sqrt{3}}\right)}{2bd\sqrt[3]{b\coth^4(c+dx)}}-\frac{3\coth(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}}-\frac{3\tanh^3(c+dx)}{13bd\sqrt[3]{b\coth^4(c+dx)}}-\frac{3\tanh(c+dx)}{7bd\sqrt[3]{b\coth^4(c+dx)}}-\frac{\coth^{\frac{4}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4bd\sqrt[3]{b\coth^4(c+dx)}}+\frac{\coth^{\frac{4}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}+1\right)}{4bd\sqrt[3]{b\coth^4(c+dx)}}+\frac{\coth^{\frac{4}{3}}(c+dx)\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)}{bd\sqrt[3]{b\coth^4(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^4)^(-4/3),x]
```

```
[Out] (-3*Coth[c + d*x])/(b*d*(b*Coth[c + d*x]^4)^(1/3)) + (Sqrt[3]*ArcTan[(1 - 2
*Coth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c + d*x]^(4/3))/(2*b*d*(b*Coth[c + d*x]
^4)^(1/3)) - (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c +
d*x]^(4/3))/(2*b*d*(b*Coth[c + d*x]^4)^(1/3)) + (ArcTanh[Coth[c + d*x]^(1/3
)]*Coth[c + d*x]^(4/3))/(b*d*(b*Coth[c + d*x]^4)^(1/3)) - (Coth[c + d*x]^(4
/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*b*d*(b*Coth[c +
d*x]^4)^(1/3)) + (Coth[c + d*x]^(4/3)*Log[1 + Coth[c + d*x]^(1/3) + Coth[c
+ d*x]^(2/3)])/(4*b*d*(b*Coth[c + d*x]^4)^(1/3)) - (3*Tanh[c + d*x])/(7*b*d
*(b*Coth[c + d*x]^4)^(1/3)) - (3*Tanh[c + d*x]^3)/(13*b*d*(b*Coth[c + d*x]^
4)^(1/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{4/3}}\,dx = \frac{\coth^{\frac{4}{3}}(c+dx)\int \frac{1}{\coth^{\frac{16}{3}}(c+dx)}\,dx}{b\sqrt[3]{b\coth^4(c+dx)}}$$

$$= -\frac{3\tanh^3(c+dx)}{13bd\sqrt[3]{b\coth^4(c+dx)}} + \frac{\coth^{\frac{4}{3}}(c+dx)\int \frac{1}{\coth^{\frac{10}{3}}(c+dx)}\,dx}{b\sqrt[3]{b\coth^4(c+dx)}}$$

$$= -\frac{3\tanh(c+dx)}{7bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{3\tanh^3(c+dx)}{13bd\sqrt[3]{b\coth^4(c+dx)}} + \frac{\coth^{\frac{4}{3}}(c+dx)\int \frac{1}{\coth^{\frac{4}{3}}(c+dx)}}{b\sqrt[3]{b\coth^4(c+dx)}}$$

$$= -\frac{3\coth(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{3\tanh(c+dx)}{7bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{3\tanh^3(c+dx)}{13bd\sqrt[3]{b\coth^4(c+dx)}} + $$

$$= -\frac{3\coth(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{3\tanh(c+dx)}{7bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{3\tanh^3(c+dx)}{13bd\sqrt[3]{b\coth^4(c+dx)}} - $$

$$= -\frac{3\coth(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{3\tanh(c+dx)}{7bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{3\tanh^3(c+dx)}{13bd\sqrt[3]{b\coth^4(c+dx)}} - $$

$$= -\frac{3\coth(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{3\tanh(c+dx)}{7bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{3\tanh^3(c+dx)}{13bd\sqrt[3]{b\coth^4(c+dx)}} + $$

$$= -\frac{3\coth(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}} + \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{4}{3}}(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{3\tanh}{7bd\sqrt[3]{b\coth}}$$

$$= -\frac{3\coth(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}} + \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{4}{3}}(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}} - \coth^{\frac{4}{3}}(c$$

$$= -\frac{3\coth(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}} + \frac{\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{4}{3}}(c+dx)}{2bd\sqrt[3]{b\coth^4(c+dx)}} - $$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 43, normalized size = 0.12

$$-\frac{3\coth(c+dx)\,{}_2F_1\left(-\frac{13}{6},1;-\frac{7}{6};\coth^2(c+dx)\right)}{13d\left(b\coth^4(c+dx)\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^4)^(-4/3),x]
```

```
[Out] (-3*Coth[c + d*x]*Hypergeometric2F1[-13/6, 1, -7/6, Coth[c + d*x]^2])/(13*d
*(b*Coth[c + d*x]^4)^(4/3))
```

Maple [F]
time = 2.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \left(\coth^{4}\left(dx+c\right)\right)\right)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^4)^(4/3),x)
```

```
[Out] int(1/(b*coth(d*x+c)^4)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^4)^(-4/3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3650 vs. $2(311) = 622$.
time = 0.64, size = 15579, normalized size = 42.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(4/3),x, algorithm="fricas")
```

```
[Out] [1/364*(91*sqrt(3)*(b*cosh(d*x + c)^10 + 10*b*cosh(d*x + c)*sinh(d*x + c)^9
 + b*sinh(d*x + c)^10 + 5*b*cosh(d*x + c)^8 + 5*(9*b*cosh(d*x + c)^2 + b)*s
inh(d*x + c)^8 + 40*(3*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c)^7
 + 10*b*cosh(d*x + c)^6 + 10*(21*b*cosh(d*x + c)^4 + 14*b*cosh(d*x + c)^2 +
 b)*sinh(d*x + c)^6 + 4*(63*b*cosh(d*x + c)^5 + 70*b*cosh(d*x + c)^3 + 15*b
*cosh(d*x + c))*sinh(d*x + c)^5 + 10*b*cosh(d*x + c)^4 + 10*(21*b*cosh(d*x
+ c)^6 + 35*b*cosh(d*x + c)^4 + 15*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^4 +
 40*(3*b*cosh(d*x + c)^7 + 7*b*cosh(d*x + c)^5 + 5*b*cosh(d*x + c)^3 + b*co
sh(d*x + c))*sinh(d*x + c)^3 + 5*b*cosh(d*x + c)^2 + 5*(9*b*cosh(d*x + c)^8
```

```
 + 28*b*cosh(d*x + c)^6 + 30*b*cosh(d*x + c)^4 + 12*b*cosh(d*x + c)^2 + b)*
sinh(d*x + c)^2 + 10*(b*cosh(d*x + c)^9 + 4*b*cosh(d*x + c)^7 + 6*b*cosh(d*
x + c)^5 + 4*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) + b)*sqrt((
-b)^(1/3)/b)*log(3*b*cosh(d*x + c)^2 + 6*b*cosh(d*x + c)*sinh(d*x + c) + 3*
b*sinh(d*x + c)^2 - 3*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + si
nh(d*x + c)^2 - 1)*(-b)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) - sqrt(
3)*(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 -
1)*(-b)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3) + (b*cosh(d*x + c)^2 +
2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*(-b)^(1/3) - (b*co
sh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*(b
*cosh(d*x + c)/sinh(d*x + c))^(1/3))*sqrt((-b)^(1/3)/b) + b) + 91*sqrt(3)*(
b*cosh(d*x + c)^10 + 10*b*cosh(d*x + c)*sinh(d*x + c)^9 + b*sinh(d*x + c)^1
0 + 5*b*cosh(d*x + c)^8 + 5*(9*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^8 + 40*
(3*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c)^7 + 10*b*cosh(d*x + c
)^6 + 10*(21*b*cosh(d*x + c)^4 + 14*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^6
+ 4*(63*b*cosh(d*x + c)^5 + 70*b*cosh(d*x + c)^3 + 15*b*cosh(d*x + c))*sinh
(d*x + c)^5 + 10*b*cosh(d*x + c)^4 + 10*(21*b*cosh(d*x + c)^6 + 35*b*cosh(d
*x + c)^4 + 15*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^4 + 40*(3*b*cosh(d*x +
c)^7 + 7*b*cosh(d*x + c)^5 + 5*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*
x + c)^3 + 5*b*cosh(d*x + c)^2 + 5*(9*b*cosh(d*x + c)^8 + 28*b*cosh(d*x + c
)^6 + 30*b*cosh(d*x + c)^4 + 12*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 10
*(b*cosh(d*x + c)^9 + 4*b*cosh(d*x + c)^7 + 6*b*cosh(d*x + c)^5 + 4*b*cosh(
d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) + b)*sqrt(-1/b^(2/3))*log(-(2*s
qrt(3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 -
 1)*b^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3)*sqrt(-1/b^(2/3)) - b*cosh
(d*x + c)^2 - 2*b*cosh(d*x + c)*sinh(d*x + c) - b*sinh(d*x + c)^2 - sqrt(3)
*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 -
 b)*b^(1/3)*sqrt(-1/b^(2/3)) + (sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x +
 c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*sqrt(-1/b^(2/3)) + 3*(cosh(d*x +
 c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*b^(2/3))*(b*co
sh(d*x + c)/sinh(d*x + c))^(1/3) - 3*b)/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*
sinh(d*x + c) + sinh(d*x + c)^2)) + 91*(cosh(d*x + c)^10 + 10*cosh(d*x + c)
*sinh(d*x + c)^9 + sinh(d*x + c)^10 + 5*(9*cosh(d*x + c)^2 + 1)*sinh(d*x +
c)^8 + 5*cosh(d*x + c)^8 + 40*(3*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x
+ c)^7 + 10*(21*cosh(d*x + c)^4 + 14*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^6 +
 10*cosh(d*x + c)^6 + 4*(63*cosh(d*x + c)^5 + 70*cosh(d*x + c)^3 + 15*cosh(
d*x + c))*sinh(d*x + c)^5 + 10*(21*cosh(d*x + c)^6 + 35*cosh(d*x + c)^4 + 1
5*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^4 + 10*cosh(d*x + c)^4 + 40*(3*cosh(d*
x + c)^7 + 7*cosh(d*x + c)^5 + 5*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x
+ c)^3 + 5*(9*cosh(d*x + c)^8 + 28*cosh(d*x + c)^6 + 30*cosh(d*x + c)^4 + 1
2*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 5*cosh(d*x + c)^2 + 10*(cosh(d*x +
 c)^9 + 4*cosh(d*x + c)^7 + 6*cosh(d*x + c)^5 + 4*cosh(d*x + c)^3 + cosh(d*
x + c))*sinh(d*x + c) + 1)*(-b)^(2/3)*log((-b)^(2/3) - (-b)^(1/3)*(b*cosh(d
*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) - 91*
(cosh(d*x + c)^10 + 10*cosh(d*x + c)*sinh(d*x + c)^9 + sinh(d*x + c)^10 + 5
```

```
*(9*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^8 + 5*cosh(d*x + c)^8 + 40*(3*cosh(d
*x + c)^3 + cosh(d*x + c))*sinh(d*x + c)^7 + 10*(21*cosh(d*x + c)^4 + 14*co
sh(d*x + c)^2 + 1)*sinh(d*x + c)^6 + 10*cosh(d*x + c)^6 + 4*(63*cosh(d*x +
c)^5 + 70*cosh(d*x + c)^3 + 15*cosh(d*x + c))*sinh(d*x + c)^5 + 10*(21*cosh
(d*x + c)^6 + 35*cosh(d*x + c)^4 + 15*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^4
+ 10*cosh(d*x + c)^4 + 40*(3*cosh(d*x + c)^7 + 7*cosh(d*x + c)^5 + 5*cosh(d
*x + c)^3 + cosh(d*x + c))*sinh(d*x + c)^3 + 5*(9*cosh(d*x + c)^8 + 28*cosh
(d*x + c)^6 + 30*cosh(d*x + c)^4 + 12*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2
+ 5*cosh(d*x + c)^2 + 10*(cosh(d*x + c)^9 + 4*cosh(d*x + c)^7 + 6*cosh(d*x
+ c)^5 + 4*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*b^(2/3)*log(
b^(2/3) - b^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/
sinh(d*x + c))^(2/3)) - 182*(cosh(d*x + c)^10 + 10*cosh(d*x + c)*sinh(d*x +
 c)^9 + sinh(d*x + c)^10 + 5*(9*cosh(d*x + c)^2...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\coth^4\left(c+dx\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**4)**(4/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**4)**(-4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^4)^(-4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,{\mathrm{coth}\left(c+d\,x\right)}^4\right)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x)^4)^(4/3),x)
```

```
[Out] int(1/(b*coth(c + d*x)^4)^(4/3), x)
```

3.50 $\int (b\coth^m(c+dx))^n\,dx$

Optimal. Leaf size=57

$$\frac{\coth(c+dx)\,(b\coth^m(c+dx))^n\;{}_2F_1\left(1,\frac{1}{2}(1+mn);\frac{1}{2}(3+mn);\coth^2(c+dx)\right)}{d(1+mn)}$$

```
[Out] coth(d*x+c)*(b*coth(d*x+c)^m)^n*hypergeom([1, 1/2*m*n+1/2],[1/2*m*n+3/2],co
th(d*x+c)^2)/d/(m*n+1)
```

Rubi [A]
time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3740, 3557, 371}

$$\frac{\coth(c+dx)\,(b\coth^m(c+dx))^n\;{}_2F_1\left(1,\frac{1}{2}(mn+1);\frac{1}{2}(mn+3);\coth^2(c+dx)\right)}{d(mn+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^m)^n,x]
```

```
[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^m)^n*Hypergeometric2F1[1, (1 + m*n)/2, (3 +
 m*n)/2, Coth[c + d*x]^2])/(d*(1 + m*n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\int (b\coth^m(c+dx))^n\,dx &= \left(\coth^{-mn}(c+dx)\,(b\coth^m(c+dx))^n\right)\int \coth^{mn}(c+dx)\,dx\\
&= -\frac{\left(\coth^{-mn}(c+dx)\,(b\coth^m(c+dx))^n\right)\mathrm{Subst}\left(\int \frac{x^{mn}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d}\\
&= \frac{\coth(c+dx)\,(b\coth^m(c+dx))^n\ {}_2F_1\left(1,\frac{1}{2}(1+mn);\frac{1}{2}(3+mn);\coth^2(c+dx)\right)}{d(1+mn)}
\end{aligned}
$$

Mathematica [**A**]
time = 0.04, size = 55, normalized size = 0.96

$$
\frac{\coth(c+dx)\,(b\coth^m(c+dx))^n\ {}_2F_1\left(1,\frac{1}{2}(1+mn);\frac{1}{2}(3+mn);\coth^2(c+dx)\right)}{d+dmn}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^m)^n,x]
```

```
[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^m)^n*Hypergeometric2F1[1, (1 + m*n)/2, (3 +
 m*n)/2, Coth[c + d*x]^2])/(d + d*m*n)
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$
\int \left(b\left(\coth^{m}\left(dx+c\right)\right)\right)^{n}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^m)^n,x)
```

```
[Out] int((b*coth(d*x+c)^m)^n,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^m)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^n,x, algorithm="fricas")
[Out] integral((b*coth(d*x + c)^m)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\coth^{m}\left(c+dx\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**m)**n,x)
[Out] Integral((b*coth(c + d*x)**m)**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^n,x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^m)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(b\,{\mathrm{coth}\left(c+d\,x\right)}^m\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^m)^n,x)
[Out] int((b*coth(c + d*x)^m)^n, x)
```

3.51 $\int (b\coth^m(c+dx))^{3/2}\,dx$

Optimal. Leaf size=63

$$\frac{2b\coth^{1+m}(c+dx)\sqrt{b\coth^m(c+dx)}\ {}_2F_1\left(1,\frac{1}{4}(2+3m);\frac{3(2+m)}{4};\coth^2(c+dx)\right)}{d(2+3m)}$$

```
[Out] 2*b*coth(d*x+c)^(1+m)*hypergeom([1, 1/2+3/4*m],[3/2+3/4*m],coth(d*x+c)^2)*(
b*coth(d*x+c)^m)^(1/2)/d/(2+3*m)
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3740, 3557, 371}

$$\frac{2b\coth^{m+1}(c+dx)\sqrt{b\coth^m(c+dx)}\ {}_2F_1\left(1,\frac{1}{4}(3m+2);\frac{3(m+2)}{4};\coth^2(c+dx)\right)}{d(3m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^m)^(3/2),x]
```

```
[Out] (2*b*Coth[c + d*x]^(1 + m)*Sqrt[b*Coth[c + d*x]^m]*Hypergeometric2F1[1, (2
+ 3*m)/4, (3*(2 + m))/4, Coth[c + d*x]^2])/(d*(2 + 3*m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int (b\coth^m(c+dx))^{3/2}\,dx &= \left(b\coth^{-\frac{m}{2}}(c+dx)\sqrt{b\coth^m(c+dx)}\right)\int \coth^{\frac{3m}{2}}(c+dx)\,dx\\ &= -\frac{\left(b\coth^{-\frac{m}{2}}(c+dx)\sqrt{b\coth^m(c+dx)}\right)\text{Subst}\left(\int \frac{x^{3m/2}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d}\\ &= \frac{2b\coth^{1+m}(c+dx)\sqrt{b\coth^m(c+dx)}\ {}_2F_1\left(1, \frac{1}{4}(2+3m); \frac{3(2+m)}{4}; \coth^2(c+dx)\right)}{d(2+3m)}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 58, normalized size = 0.92

$$\frac{2\coth(c+dx)\,(b\coth^m(c+dx))^{3/2}\ {}_2F_1\left(1, \frac{1}{4}(2+3m); \frac{3(2+m)}{4}; \coth^2(c+dx)\right)}{d(2+3m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^m)^(3/2),x]
```

```
[Out] (2*Coth[c + d*x]*(b*Coth[c + d*x]^m)^(3/2)*Hypergeometric2F1[1, (2 + 3*m)/4
, (3*(2 + m))/4, Coth[c + d*x]^2])/(d*(2 + 3*m))
```

Maple [F]
time = 2.32, size = 0, normalized size = 0.00

$$\int \left(b\left(\coth^m(dx+c)\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^m)^(3/2),x)
```

```
[Out] int((b*coth(d*x+c)^m)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^m)^(3/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\coth^{m}\left(c+dx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**m)**(3/2),x)
[Out] Integral((b*coth(c + d*x)**m)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(3/2),x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^m)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(b\,{\mathrm{coth}\left(c+d\,x\right)}^m\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^m)^(3/2),x)
[Out] int((b*coth(c + d*x)^m)^(3/2), x)
```

3.52 $\int \sqrt{b\coth^m(c+dx)}\,dx$

Optimal. Leaf size=54

$$\frac{2\coth(c+dx)\sqrt{b\coth^m(c+dx)}\;{}_2F_1\left(1,\frac{2+m}{4};\frac{6+m}{4};\coth^2(c+dx)\right)}{d(2+m)}$$

```
[Out] 2*coth(d*x+c)*hypergeom([1, 1/2+1/4*m],[3/2+1/4*m],coth(d*x+c)^2)*(b*coth(d
*x+c)^m)^(1/2)/d/(2+m)
```

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3740, 3557, 371}

$$\frac{2\coth(c+dx)\sqrt{b\coth^m(c+dx)}\;{}_2F_1\left(1,\frac{m+2}{4};\frac{m+6}{4};\coth^2(c+dx)\right)}{d(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Coth[c + d*x]^m],x]
```

```
[Out] (2*Coth[c + d*x]*Sqrt[b*Coth[c + d*x]^m]*Hypergeometric2F1[1, (2 + m)/4, (6
 + m)/4, Coth[c + d*x]^2])/(d*(2 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{b\coth^m(c+dx)}\,dx &= \left(\coth^{-\frac{m}{2}}(c+dx)\sqrt{b\coth^m(c+dx)}\right)\int \coth^{\frac{m}{2}}(c+dx)\,dx\\
&= -\frac{\left(\coth^{-\frac{m}{2}}(c+dx)\sqrt{b\coth^m(c+dx)}\right)\text{Subst}\left(\int \frac{x^{m/2}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d}\\
&= \frac{2\coth(c+dx)\sqrt{b\coth^m(c+dx)}\;{}_2F_1\left(1, \frac{2+m}{4}; \frac{6+m}{4}; \coth^2(c+dx)\right)}{d(2+m)}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 54, normalized size = 1.00

$$
\frac{2\coth(c+dx)\sqrt{b\coth^m(c+dx)}\;{}_2F_1\left(1, \frac{2+m}{4}; \frac{6+m}{4}; \coth^2(c+dx)\right)}{d(2+m)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Coth[c + d*x]^m],x]
```

```
[Out] (2*Coth[c + d*x]*Sqrt[b*Coth[c + d*x]^m]*Hypergeometric2F1[1, (2 + m)/4, (6
 + m)/4, Coth[c + d*x]^2])/(d*(2 + m))
```

Maple [F]
time = 2.55, size = 0, normalized size = 0.00

$$
\int \sqrt{b\left(\coth^m\left(dx+c\right)\right)}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^m)^(1/2),x)
```

```
[Out] int((b*coth(d*x+c)^m)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*coth(d*x + c)^m), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\coth^{m}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**m)**(1/2),x)
```

```
[Out] Integral(sqrt(b*coth(c + d*x)**m), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*coth(d*x + c)^m), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{b\,{\mathrm{coth}\left(c+d\,x\right)}^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^m)^(1/2),x)
```

```
[Out] int((b*coth(c + d*x)^m)^(1/2), x)
```

3.53 $\int \frac{1}{\sqrt{b\coth^m(c+dx)}}\,dx$

Optimal. Leaf size=60

$$\frac{2\coth(c+dx)\,{}_2F_1\left(1,\frac{2-m}{4};\frac{6-m}{4};\coth^2(c+dx)\right)}{d(2-m)\sqrt{b\coth^m(c+dx)}}$$

```
[Out] 2*coth(d*x+c)*hypergeom([1, 1/2-1/4*m],[3/2-1/4*m],coth(d*x+c)^2)/d/(2-m)/(
b*coth(d*x+c)^m)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3740, 3557, 371}

$$\frac{2\coth(c+dx)\,{}_2F_1\left(1,\frac{2-m}{4};\frac{6-m}{4};\coth^2(c+dx)\right)}{d(2-m)\sqrt{b\coth^m(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Coth[c + d*x]^m],x]
```

```
[Out] (2*Coth[c + d*x]*Hypergeometric2F1[1, (2 - m)/4, (6 - m)/4, Coth[c + d*x]^2
])/(d*(2 - m)*Sqrt[b*Coth[c + d*x]^m])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{b\coth^m(c+dx)}}\,dx &= \frac{\coth^{\frac{m}{2}}(c+dx)\int \coth^{-\frac{m}{2}}(c+dx)\,dx}{\sqrt{b\coth^m(c+dx)}}\\
&= -\frac{\coth^{\frac{m}{2}}(c+dx)\text{Subst}\left(\int \frac{x^{-m/2}}{-1+x^2}\,dx,x,\coth(c+dx)\right)}{d\sqrt{b\coth^m(c+dx)}}\\
&= \frac{2\coth(c+dx)\,{}_2F_1\left(1,\frac{2-m}{4};\frac{6-m}{4};\coth^2(c+dx)\right)}{d(2-m)\sqrt{b\coth^m(c+dx)}}
\end{aligned}
$$

Mathematica [**A**]
time = 0.04, size = 58, normalized size = 0.97

$$
-\frac{2\coth(c+dx)\,{}_2F_1\left(1,\frac{2-m}{4};\frac{6-m}{4};\coth^2(c+dx)\right)}{d(-2+m)\sqrt{b\coth^m(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Coth[c + d*x]^m],x]
```

```
[Out] (-2*Coth[c + d*x]*Hypergeometric2F1[1, (2 - m)/4, (6 - m)/4, Coth[c + d*x]^
2])/(d*(-2 + m)*Sqrt[b*Coth[c + d*x]^m])
```

Maple [**F**]
time = 3.01, size = 0, normalized size = 0.00

$$
\int \frac{1}{\sqrt{b\left(\coth^m(dx+c)\right)}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^m)^(1/2),x)
```

```
[Out] int(1/(b*coth(d*x+c)^m)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*coth(d*x + c)^m), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b \coth^{m}{\left(c + dx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**m)**(1/2),x)
[Out] Integral(1/sqrt(b*coth(c + d*x)**m), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(b*coth(d*x + c)^m), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{b\,{\mathrm{coth}\left(c+d\,x\right)}^m}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x)^m)^(1/2),x)
[Out] int(1/(b*coth(c + d*x)^m)^(1/2), x)
```

3.54 $\int \frac{1}{(b\coth^m(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=69

$$\frac{2\coth^{1-m}(c+dx)\,{}_2F_1\left(1,\frac{1}{4}(2-3m);\frac{3(2-m)}{4};\coth^2(c+dx)\right)}{bd(2-3m)\sqrt{b\coth^m(c+dx)}}$$

```
[Out] 2*coth(d*x+c)^(1-m)*hypergeom([1, 1/2-3/4*m],[3/2-3/4*m],coth(d*x+c)^2)/b/d
/(2-3*m)/(b*coth(d*x+c)^m)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3740, 3557, 371}

$$\frac{2\coth^{1-m}(c+dx)\,{}_2F_1\left(1,\frac{1}{4}(2-3m);\frac{3(2-m)}{4};\coth^2(c+dx)\right)}{bd(2-3m)\sqrt{b\coth^m(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^m)^(-3/2),x]
```

```
[Out] (2*Coth[c + d*x]^(1 - m)*Hypergeometric2F1[1, (2 - 3*m)/4, (3*(2 - m))/4, C
oth[c + d*x]^2])/(b*d*(2 - 3*m)*Sqrt[b*Coth[c + d*x]^m])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
```

```
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(b\coth^m(c+dx))^{3/2}}\,dx &= \frac{\coth^{\frac{m}{2}}(c+dx)\int \coth^{-\frac{3m}{2}}(c+dx)\,dx}{b\sqrt{b\coth^m(c+dx)}}\\
&= -\frac{\coth^{\frac{m}{2}}(c+dx)\operatorname{Subst}\left(\int \frac{x^{-3m/2}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{bd\sqrt{b\coth^m(c+dx)}}\\
&= \frac{2\coth^{1-m}(c+dx)\,{}_2F_1\left(1, \frac{1}{4}(2-3m); \frac{3(2-m)}{4}; \coth^2(c+dx)\right)}{bd(2-3m)\sqrt{b\coth^m(c+dx)}}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 58, normalized size = 0.84

$$
-\frac{2\coth(c+dx)\,{}_2F_1\left(1, \frac{1}{4}(2-3m); -\frac{3}{4}(-2+m); \coth^2(c+dx)\right)}{d(-2+3m)\,(b\coth^m(c+dx))^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^m)^(-3/2),x]
```

```
[Out] (-2*Coth[c + d*x]*Hypergeometric2F1[1, (2 - 3*m)/4, (-3*(-2 + m))/4, Coth[c
 + d*x]^2])/(d*(-2 + 3*m)*(b*Coth[c + d*x]^m)^(3/2))
```

Maple [F]
time = 2.67, size = 0, normalized size = 0.00

$$
\int \frac{1}{(b(\coth^m(dx+c)))^{\frac{3}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^m)^(3/2),x)
```

```
[Out] int(1/(b*coth(d*x+c)^m)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^m)^(-3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\coth^m(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**m)**(3/2),x)
```

```
[Out] Integral((b*coth(c + d*x)**m)**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^m)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\,\coth\left(c+d\,x\right)^m\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x)^m)^(3/2),x)
```

```
[Out] int(1/(b*coth(c + d*x)^m)^(3/2), x)
```

3.55 $\int (b\coth^m(c+dx))^{4/3}\,dx$

Optimal. Leaf size=65

$$\frac{3b\coth^{1+m}(c+dx)\sqrt[3]{b\coth^m(c+dx)}\;{}_2F_1\left(1,\frac{1}{6}(3+4m);\frac{1}{6}(9+4m);\coth^2(c+dx)\right)}{d(3+4m)}$$

```
[Out] 3*b*coth(d*x+c)^(1+m)*(b*coth(d*x+c)^m)^(1/3)*hypergeom([1, 1/2+2/3*m],[3/2
+2/3*m],coth(d*x+c)^2)/d/(3+4*m)
```

Rubi [A]
time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3740, 3557, 371}

$$\frac{3b\coth^{m+1}(c+dx)\sqrt[3]{b\coth^m(c+dx)}\;{}_2F_1\left(1,\frac{1}{6}(4m+3);\frac{1}{6}(4m+9);\coth^2(c+dx)\right)}{d(4m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^m)^(4/3),x]
```

```
[Out] (3*b*Coth[c + d*x]^(1 + m)*(b*Coth[c + d*x]^m)^(1/3)*Hypergeometric2F1[1, (
3 + 4*m)/6, (9 + 4*m)/6, Coth[c + d*x]^2])/(d*(3 + 4*m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int (b\coth^m(c+dx))^{4/3}\,dx = \left(b\coth^{-\frac{m}{3}}(c+dx)\sqrt[3]{b\coth^m(c+dx)}\right)\int \coth^{\frac{4m}{3}}(c+dx)\,dx$$
$$= -\frac{\left(b\coth^{-\frac{m}{3}}(c+dx)\sqrt[3]{b\coth^m(c+dx)}\right)\text{Subst}\left(\int \frac{x^{4m/3}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d}$$
$$= \frac{3b\coth^{1+m}(c+dx)\sqrt[3]{b\coth^m(c+dx)}\;{}_2F_1\left(1, \frac{1}{6}(3+4m); \frac{1}{6}(9+4m); \coth^2(c+\right.}{d(3+4m)}$$

Mathematica [**A**]
time = 0.05, size = 60, normalized size = 0.92

$$\frac{3\coth(c+dx)\,(b\coth^m(c+dx))^{4/3}\;{}_2F_1\left(1, \frac{1}{6}(3+4m); \frac{1}{6}(9+4m); \coth^2(c+dx)\right)}{d(3+4m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^m)^(4/3),x]
```

```
[Out] (3*Coth[c + d*x]*(b*Coth[c + d*x]^m)^(4/3)*Hypergeometric2F1[1, (3 + 4*m)/6
, (9 + 4*m)/6, Coth[c + d*x]^2])/(d*(3 + 4*m))
```

Maple [**F**]
time = 1.96, size = 0, normalized size = 0.00

$$\int \left(b\left(\coth^m\left(dx+c\right)\right)\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^m)^(4/3),x)
```

```
[Out] int((b*coth(d*x+c)^m)^(4/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^m)^(4/3), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(4/3),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**m)**(4/3),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(4/3),x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^m)^(4/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(b\,{\mathrm{coth}\left(c+d\,x\right)}^m\right)}^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^m)^(4/3),x)
[Out] int((b*coth(c + d*x)^m)^(4/3), x)
```

3.56 $\int (b \coth^m(c+dx))^{2/3}\, dx$

Optimal. Leaf size=60

$$\frac{3 \coth(c+dx) \left(b \coth^m(c+dx)\right)^{2/3} {}_2F_1\left(1, \frac{1}{6}(3+2m); \frac{1}{6}(9+2m); \coth^2(c+dx)\right)}{d(3+2m)}$$

```
[Out] 3*coth(d*x+c)*(b*coth(d*x+c)^m)^(2/3)*hypergeom([1, 1/2+1/3*m],[3/2+1/3*m],
coth(d*x+c)^2)/d/(3+2*m)
```

Rubi [A]
time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3740, 3557, 371}

$$\frac{3 \coth(c+dx) \left(b \coth^m(c+dx)\right)^{2/3} {}_2F_1\left(1, \frac{1}{6}(2m+3); \frac{1}{6}(2m+9); \coth^2(c+dx)\right)}{d(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^m)^(2/3),x]
```

```
[Out] (3*Coth[c + d*x]*(b*Coth[c + d*x]^m)^(2/3)*Hypergeometric2F1[1, (3 + 2*m)/6
, (9 + 2*m)/6, Coth[c + d*x]^2])/(d*(3 + 2*m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\int (b\coth^m(c+dx))^{2/3}\,dx &= \left(\coth^{-\frac{2m}{3}}(c+dx)\,(b\coth^m(c+dx))^{2/3}\right)\int \coth^{\frac{2m}{3}}(c+dx)\,dx\\
&= -\frac{\left(\coth^{-\frac{2m}{3}}(c+dx)\,(b\coth^m(c+dx))^{2/3}\right)\text{Subst}\left(\int \frac{x^{2m/3}}{-1+x^2}\,dx,x,\coth(c+dx)\right)}{d}\\
&= \frac{3\coth(c+dx)\,(b\coth^m(c+dx))^{2/3}\,{}_2F_1\left(1,\frac{1}{6}(3+2m);\frac{1}{6}(9+2m);\coth^2(c+dx)\right)}{d(3+2m)}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 60, normalized size = 1.00

$$
\frac{3\coth(c+dx)\,(b\coth^m(c+dx))^{2/3}\,{}_2F_1\left(1,\frac{1}{6}(3+2m);\frac{1}{6}(9+2m);\coth^2(c+dx)\right)}{d(3+2m)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^m)^(2/3),x]
[Out] (3*Coth[c + d*x]*(b*Coth[c + d*x]^m)^(2/3)*Hypergeometric2F1[1, (3 + 2*m)/6
, (9 + 2*m)/6, Coth[c + d*x]^2])/(d*(3 + 2*m))
```

Maple [F]
time = 1.78, size = 0, normalized size = 0.00

$$
\int \left(b\left(\coth^m(dx+c)\right)\right)^{\frac{2}{3}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^m)^(2/3),x)
[Out] int((b*coth(d*x+c)^m)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(2/3),x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c)^m)^(2/3), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(2/3),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\coth^m (c + dx))^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**m)**(2/3),x)
[Out] Integral((b*coth(c + d*x)**m)**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(2/3),x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^m)^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(b\,{\mathrm{coth}\left(c+d\,x\right)}^m\right)}^{2/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^m)^(2/3),x)
[Out] int((b*coth(c + d*x)^m)^(2/3), x)
```

3.57 $\int \sqrt[3]{b \coth^m(c+dx)}\, dx$

Optimal. Leaf size=54

$$\frac{3\coth(c+dx)\sqrt[3]{b\coth^m(c+dx)}\ {}_2F_1\left(1,\frac{3+m}{6};\frac{9+m}{6};\coth^2(c+dx)\right)}{d(3+m)}$$

```
[Out] 3*coth(d*x+c)*(b*coth(d*x+c)^m)^(1/3)*hypergeom([1, 1/2+1/6*m],[3/2+1/6*m],
coth(d*x+c)^2)/d/(3+m)
```

Rubi [A]
time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3740, 3557, 371}

$$\frac{3\coth(c+dx)\sqrt[3]{b\coth^m(c+dx)}\ {}_2F_1\left(1,\frac{m+3}{6};\frac{m+9}{6};\coth^2(c+dx)\right)}{d(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^m)^(1/3),x]
```

```
[Out] (3*Coth[c + d*x]*(b*Coth[c + d*x]^m)^(1/3)*Hypergeometric2F1[1, (3 + m)/6,
(9 + m)/6, Coth[c + d*x]^2])/(d*(3 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\int \sqrt[3]{b \coth^m(c+dx)}\,dx &= \left(\coth^{-\frac{m}{3}}(c+dx)\sqrt[3]{b \coth^m(c+dx)}\right)\int \coth^{\frac{m}{3}}(c+dx)\,dx\\
&= -\frac{\left(\coth^{-\frac{m}{3}}(c+dx)\sqrt[3]{b \coth^m(c+dx)}\right)\operatorname{Subst}\left(\int \frac{x^{m/3}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d}\\
&= \frac{3\coth(c+dx)\sqrt[3]{b \coth^m(c+dx)}\,{}_2F_1\left(1,\frac{3+m}{6};\frac{9+m}{6};\coth^2(c+dx)\right)}{d(3+m)}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 54, normalized size = 1.00

$$\frac{3\coth(c+dx)\sqrt[3]{b \coth^m(c+dx)}\,{}_2F_1\left(1,\frac{3+m}{6};\frac{9+m}{6};\coth^2(c+dx)\right)}{d(3+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^m)^(1/3),x]
[Out] (3*Coth[c + d*x]*(b*Coth[c + d*x]^m)^(1/3)*Hypergeometric2F1[1, (3 + m)/6,
(9 + m)/6, Coth[c + d*x]^2])/(d*(3 + m))
```

Maple [F]
time = 1.77, size = 0, normalized size = 0.00

$$\int (b(\coth^m(dx+c)))^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^m)^(1/3),x)
[Out] int((b*coth(d*x+c)^m)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(1/3),x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c)^m)^(1/3), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(1/3),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{b \coth^{m} (c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**m)**(1/3),x)
[Out] Integral((b*coth(c + d*x)**m)**(1/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(1/3),x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^m)^(1/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(b\,{\mathrm{coth}\left(c+d\,x\right)}^m\right)}^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^m)^(1/3),x)
[Out] int((b*coth(c + d*x)^m)^(1/3), x)
```

3.58 $\int \frac{1}{\sqrt[3]{b\coth^m(c+dx)}}\,dx$

Optimal. Leaf size=60

$$\frac{3\coth(c+dx)\,{}_2F_1\left(1,\frac{3-m}{6};\frac{9-m}{6};\coth^2(c+dx)\right)}{d(3-m)\sqrt[3]{b\coth^m(c+dx)}}$$

```
[Out] 3*coth(d*x+c)*hypergeom([1, 1/2-1/6*m],[3/2-1/6*m],coth(d*x+c)^2)/d/(3-m)/(
b*coth(d*x+c)^m)^(1/3)
```

Rubi [A]
time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3740, 3557, 371}

$$\frac{3\coth(c+dx)\,{}_2F_1\left(1,\frac{3-m}{6};\frac{9-m}{6};\coth^2(c+dx)\right)}{d(3-m)\sqrt[3]{b\coth^m(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^m)^(-1/3),x]
```

```
[Out] (3*Coth[c + d*x]*Hypergeometric2F1[1, (3 - m)/6, (9 - m)/6, Coth[c + d*x]^2
])/(d*(3 - m)*(b*Coth[c + d*x]^m)^(1/3))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt[3]{b\coth^m(c+dx)}}\,dx &= \frac{\coth^{\frac{m}{3}}(c+dx)\int \coth^{-\frac{m}{3}}(c+dx)\,dx}{\sqrt[3]{b\coth^m(c+dx)}}\\
&= -\frac{\coth^{\frac{m}{3}}(c+dx)\text{Subst}\left(\int \frac{x^{-m/3}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d\sqrt[3]{b\coth^m(c+dx)}}\\
&= \frac{3\coth(c+dx)\,{}_2F_1\left(1,\frac{3-m}{6};\frac{9-m}{6};\coth^2(c+dx)\right)}{d(3-m)\sqrt[3]{b\coth^m(c+dx)}}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 58, normalized size = 0.97

$$
-\frac{3\coth(c+dx)\,{}_2F_1\left(1,\frac{3-m}{6};\frac{9-m}{6};\coth^2(c+dx)\right)}{d(-3+m)\sqrt[3]{b\coth^m(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^m)^(-1/3),x]
```

```
[Out] (-3*Coth[c + d*x]*Hypergeometric2F1[1, (3 - m)/6, (9 - m)/6, Coth[c + d*x]^
2])/(d*(-3 + m)*(b*Coth[c + d*x]^m)^(1/3))
```

Maple [F]
time = 2.19, size = 0, normalized size = 0.00

$$
\int \frac{1}{\left(b\left(\coth^m\left(dx+c\right)\right)\right)^{\frac{1}{3}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^m)^(1/3),x)
```

```
[Out] int(1/(b*coth(d*x+c)^m)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^m)^(-1/3), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(1/3),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{b\coth^{m}(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**m)**(1/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**m)**(-1/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^m)^(-1/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(b\coth\left(c+d\,x\right)^{m}\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x)^m)^(1/3),x)
```

```
[Out] int(1/(b*coth(c + d*x)^m)^(1/3), x)
```

3.59 $\int \frac{1}{(b\coth^m(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=60

$$\frac{3\coth(c+dx)\,{}_2F_1\left(1,\frac{1}{6}(3-2m);\frac{1}{6}(9-2m);\coth^2(c+dx)\right)}{d(3-2m)\left(b\coth^m(c+dx)\right)^{2/3}}$$

```
[Out] 3*coth(d*x+c)*hypergeom([1, 1/2-1/3*m],[3/2-1/3*m],coth(d*x+c)^2)/d/(3-2*m)
/(b*coth(d*x+c)^m)^(2/3)
```

Rubi [A]
time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3740, 3557, 371}

$$\frac{3\coth(c+dx)\,{}_2F_1\left(1,\frac{1}{6}(3-2m);\frac{1}{6}(9-2m);\coth^2(c+dx)\right)}{d(3-2m)\left(b\coth^m(c+dx)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^m)^(-2/3),x]
```

```
[Out] (3*Coth[c + d*x]*Hypergeometric2F1[1, (3 - 2*m)/6, (9 - 2*m)/6, Coth[c + d*
x]^2])/(d*(3 - 2*m)*(b*Coth[c + d*x]^m)^(2/3))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{(b\coth^m(c+dx))^{2/3}}\,dx = \frac{\coth^{\frac{2m}{3}}(c+dx)\int \coth^{-\frac{2m}{3}}(c+dx)\,dx}{(b\coth^m(c+dx))^{2/3}}$$
$$= -\frac{\coth^{\frac{2m}{3}}(c+dx)\text{Subst}\left(\int \frac{x^{-2m/3}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d\,(b\coth^m(c+dx))^{2/3}}$$
$$= \frac{3\coth(c+dx)\,{}_2F_1\left(1,\frac{1}{6}(3-2m);\frac{1}{6}(9-2m);\coth^2(c+dx)\right)}{d(3-2m)\,(b\coth^m(c+dx))^{2/3}}$$

Mathematica [**A**]
time = 0.03, size = 60, normalized size = 1.00

$$-\frac{3\coth(c+dx)\,{}_2F_1\left(1,\frac{1}{6}(3-2m);\frac{1}{6}(9-2m);\coth^2(c+dx)\right)}{d(-3+2m)\,(b\coth^m(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^m)^(-2/3),x]
```

```
[Out] (-3*Coth[c + d*x]*Hypergeometric2F1[1, (3 - 2*m)/6, (9 - 2*m)/6, Coth[c + d
*x]^2])/(d*(-3 + 2*m)*(b*Coth[c + d*x]^m)^(2/3))
```

Maple [**F**]
time = 2.18, size = 0, normalized size = 0.00

$$\int \frac{1}{(b(\coth^m(dx+c)))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^m)^(2/3),x)
```

```
[Out] int(1/(b*coth(d*x+c)^m)^(2/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^m)^(-2/3), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(2/3),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\coth^m(c+dx))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**m)**(2/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**m)**(-2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^m)^(-2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(b\coth(c+d\,x)^m)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x)^m)^(2/3),x)
```

```
[Out] int(1/(b*coth(c + d*x)^m)^(2/3), x)
```

3.60 $\int \frac{1}{(b\coth^m(c+dx))^{4/3}} \, dx$

Optimal. Leaf size=69

$$\frac{3\coth^{1-m}(c+dx)\,{}_2F_1\left(1,\frac{1}{6}(3-4m);\frac{1}{6}(9-4m);\coth^2(c+dx)\right)}{bd(3-4m)\sqrt[3]{b\coth^m(c+dx)}}$$

```
[Out] 3*coth(d*x+c)^(1-m)*hypergeom([1, 1/2-2/3*m],[3/2-2/3*m],coth(d*x+c)^2)/b/d
/(3-4*m)/(b*coth(d*x+c)^m)^(1/3)
```

Rubi [A]
time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3740, 3557, 371}

$$\frac{3\coth^{1-m}(c+dx)\,{}_2F_1\left(1,\frac{1}{6}(3-4m);\frac{1}{6}(9-4m);\coth^2(c+dx)\right)}{bd(3-4m)\sqrt[3]{b\coth^m(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^m)^(-4/3),x]
```

```
[Out] (3*Coth[c + d*x]^(1 - m)*Hypergeometric2F1[1, (3 - 4*m)/6, (9 - 4*m)/6, Cot
h[c + d*x]^2])/(b*d*(3 - 4*m)*(b*Coth[c + d*x]^m)^(1/3))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{(b\coth^m(c+dx))^{4/3}}\,dx = \frac{\coth^{\frac{m}{3}}(c+dx)\int \coth^{-\frac{4m}{3}}(c+dx)\,dx}{b\sqrt[3]{b\coth^m(c+dx)}}$$
$$= -\frac{\coth^{\frac{m}{3}}(c+dx)\text{Subst}\left(\int \frac{x^{-4m/3}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{bd\sqrt[3]{b\coth^m(c+dx)}}$$
$$= \frac{3\coth^{1-m}(c+dx)\,{}_2F_1\left(1, \frac{1}{6}(3-4m); \frac{1}{6}(9-4m); \coth^2(c+dx)\right)}{bd(3-4m)\sqrt[3]{b\coth^m(c+dx)}}$$

Mathematica [A]
time = 0.05, size = 60, normalized size = 0.87

$$-\frac{3\coth(c+dx)\,{}_2F_1\left(1, \frac{1}{6}(3-4m); \frac{1}{6}(9-4m); \coth^2(c+dx)\right)}{d(-3+4m)\left(b\coth^m(c+dx)\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^m)^(-4/3),x]
[Out] (-3*Coth[c + d*x]*Hypergeometric2F1[1, (3 - 4*m)/6, (9 - 4*m)/6, Coth[c + d
*x]^2])/(d*(-3 + 4*m)*(b*Coth[c + d*x]^m)^(4/3))
```

Maple [F]
time = 2.20, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\left(\coth^m(dx+c)\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^m)^(4/3),x)
[Out] int(1/(b*coth(d*x+c)^m)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(4/3),x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c)^m)^(-4/3), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(4/3),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\coth^{m}\left(c+dx\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**m)**(4/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**m)**(-4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^m)^(-4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,{\mathrm{coth}\left(c+d\,x\right)}^m\right)}^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x)^m)^(4/3),x)
```

```
[Out] int(1/(b*coth(c + d*x)^m)^(4/3), x)
```

3.61 $\int (1+\coth(x))^5\,dx$

Optimal. Leaf size=41

$$16x - 8\coth(x) - 2(1+\coth(x))^2 - \frac{2}{3}(1+\coth(x))^3 - \frac{1}{4}(1+\coth(x))^4 + 16\log(\sinh(x))$$

```
[Out] 16*x-8*coth(x)-2*(1+coth(x))^2-2/3*(1+coth(x))^3-1/4*(1+coth(x))^4+16*ln(si
nh(x))
```

Rubi [A]
time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3559, 3558, 3556}

$$16x - \frac{1}{4}(\coth(x)+1)^4 - \frac{2}{3}(\coth(x)+1)^3 - 2(\coth(x)+1)^2 - 8\coth(x) + 16\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Coth[x])^5,x]
```

```
[Out] 16*x - 8*Coth[x] - 2*(1 + Coth[x])^2 - (2*(1 + Coth[x])^3)/3 - (1 + Coth[x]
)^4/4 + 16*Log[Sinh[x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3558

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[b^2*(Tan[c + d*x]/d),
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rubi steps

$$\begin{aligned}\int (1+\coth(x))^5\,dx &= -\frac{1}{4}(1+\coth(x))^4 + 2\int (1+\coth(x))^4\,dx \\ &= -\frac{2}{3}(1+\coth(x))^3 - \frac{1}{4}(1+\coth(x))^4 + 4\int (1+\coth(x))^3\,dx \\ &= -2(1+\coth(x))^2 - \frac{2}{3}(1+\coth(x))^3 - \frac{1}{4}(1+\coth(x))^4 + 8\int (1+\coth(x))^2\,dx \\ &= 16x - 8\coth(x) - 2(1+\coth(x))^2 - \frac{2}{3}(1+\coth(x))^3 - \frac{1}{4}(1+\coth(x))^4 + 16\int \coth \\ &= 16x - 8\coth(x) - 2(1+\coth(x))^2 - \frac{2}{3}(1+\coth(x))^3 - \frac{1}{4}(1+\coth(x))^4 + 16\log(\sinh\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.17, size = 94, normalized size = 2.29

$$\frac{(1+\coth(x))^5\sinh(x)\left(-3\cosh^4(x)-20\cosh^3(x)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\tanh^2(x)\right)\sinh(x)-66\cosh^2(x)\sinh^2(x)-120\cosh(x)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\tanh^2(x)\right)\sinh^3(x)+12(x+16\log(\cosh(x))+16\log(\tanh(x)))\sinh^4(x)\right)}{12(\cosh(x)+\sinh(x))^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Coth[x])^5,x]
```

```
[Out] ((1 + Coth[x])^5*Sinh[x]*(-3*Cosh[x]^4 - 20*Cosh[x]^3*Hypergeometric2F1[-3/
2, 1, -1/2, Tanh[x]^2]*Sinh[x] - 66*Cosh[x]^2*Sinh[x]^2 - 120*Cosh[x]*Hyper
geometric2F1[-1/2, 1, 1/2, Tanh[x]^2]*Sinh[x]^3 + 12*(x + 16*Log[Cosh[x]] +
 16*Log[Tanh[x]])*Sinh[x]^4))/(12*(Cosh[x] + Sinh[x])^5)
```

Maple [A]
time = 0.20, size = 31, normalized size = 0.76

method	result	size
derivativedivides	$-\frac{(\coth^4(x))}{4}-\frac{5(\coth^3(x))}{3}-\frac{11(\coth^2(x))}{2}-15\coth(x)-16\ln(\coth(x)-1)$	31
default	$-\frac{(\coth^4(x))}{4}-\frac{5(\coth^3(x))}{3}-\frac{11(\coth^2(x))}{2}-15\coth(x)-16\ln(\coth(x)-1)$	31
risch	$-\frac{4(48\,e^{6x}-108\,e^{4x}+88\,e^{2x}-25)}{3(e^{2x}-1)^4}+16\ln(e^{2x}-1)$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+coth(x))^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*coth(x)^4-5/3*coth(x)^3-11/2*coth(x)^2-15*coth(x)-16*ln(coth(x)-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(37) = 74.
time = 0.28, size = 140, normalized size = 3.41

$$27\,x-\frac{20\left(3\,e^{(-2\,x)}-3\,e^{(-4\,x)}-2\right)}{3\left(3\,e^{(-2\,x)}-3\,e^{(-4\,x)}+e^{(-6\,x)}-1\right)}+\frac{4\left(e^{(-2\,x)}-e^{(-4\,x)}+e^{(-6\,x)}\right)}{4\,e^{(-2\,x)}-6\,e^{(-4\,x)}+4\,e^{(-6\,x)}-e^{(-8\,x)}-1}+\frac{20\,e^{(-2\,x)}}{2\,e^{(-2\,x)}-e^{(-4\,x)}-1}+\frac{20}{e^{(-2\,x)}-1}+11\log\left(e^{(-x)}+1\right)+11\log\left(e^{(-x)}-1\right)+5\log\left(\sinh\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^5,x, algorithm="maxima")
```

```
[Out] 27*x - 20/3*(3*e^(-2*x) - 3*e^(-4*x) - 2)/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*
x) - 1) + 4*(e^(-2*x) - e^(-4*x) + e^(-6*x))/(4*e^(-2*x) - 6*e^(-4*x) + 4*e
^(-6*x) - e^(-8*x) - 1) + 20*e^(-2*x)/(2*e^(-2*x) - e^(-4*x) - 1) + 20/(e^(
-2*x) - 1) + 11*log(e^(-x) + 1) + 11*log(e^(-x) - 1) + 5*log(sinh(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 448 vs. $2(37) = 74$.
time = 0.34, size = 448, normalized size = 10.93

$$-\frac{4\left(48\cosh(x)^6 + 288\cosh(x)\sinh(x)^5 + 48\sinh(x)^6 + 36\left(20\cosh(x)^2 - 3\right)\sinh(x)^4 - 108\cosh(x)^4 + 48\left(20\cosh(x)^3 - 9\cosh(x)\right)\sinh(x)^3 + 8\left(90\cosh(x)^4 - 81\cosh(x)^2 + 11\right)\sinh(x)^2 + 88\cosh(x)^2 - 12\left(\cosh(x)^8 + 8\cosh(x)\sinh(x)^7 + \sinh(x)^8 + 4\left(7\cosh(x)^2 - 1\right)\sinh(x)^6 - 4\cosh(x)^6 + 8\left(7\cosh(x)^3 - 3\cosh(x)\right)\sinh(x)^5 + 2\left(35\cosh(x)^4 - 30\cosh(x)^2 + 3\right)\sinh(x)^4 + 6\cosh(x)^4 + 8\left(7\cosh(x)^5 - 10\cosh(x)^3 + 3\cosh(x)\right)\sinh(x)^3 + 4\left(7\cosh(x)^6 - 15\cosh(x)^4 + 9\cosh(x)^2 - 1\right)\sinh(x)^2 - 4\cosh(x)^2 + 8\left(\cosh(x)^7 - 3\cosh(x)^5 + 3\cosh(x)^3 - \cosh(x)\right)\sinh(x) + 1\right)\log\left(\frac{2\sinh(x)}{\cosh(x) - \sinh(x)}\right) + 16\left(18\cosh(x)^5 - 27\cosh(x)^3 + 11\cosh(x)\right)\sinh(x) - 25\right)}{3\left(\cosh(x)^8 + 8\cosh(x)\sinh(x)^7 + \sinh(x)^8 + 4\left(7\cosh(x)^2 - 1\right)\sinh(x)^6 - 4\cosh(x)^6 + 8\left(7\cosh(x)^3 - 3\cosh(x)\right)\sinh(x)^5 + 2\left(35\cosh(x)^4 - 30\cosh(x)^2 + 3\right)\sinh(x)^4 + 6\cosh(x)^4 + 8\left(7\cosh(x)^5 - 10\cosh(x)^3 + 3\cosh(x)\right)\sinh(x)^3 + 4\left(7\cosh(x)^6 - 15\cosh(x)^4 + 9\cosh(x)^2 - 1\right)\sinh(x)^2 - 4\cosh(x)^2 + 8\left(\cosh(x)^7 - 3\cosh(x)^5 + 3\cosh(x)^3 - \cosh(x)\right)\sinh(x) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^5,x, algorithm="fricas")
```

```
[Out] -4/3*(48*cosh(x)^6 + 288*cosh(x)*sinh(x)^5 + 48*sinh(x)^6 + 36*(20*cosh(x)^
2 - 3)*sinh(x)^4 - 108*cosh(x)^4 + 48*(20*cosh(x)^3 - 9*cosh(x))*sinh(x)^3
+ 8*(90*cosh(x)^4 - 81*cosh(x)^2 + 11)*sinh(x)^2 + 88*cosh(x)^2 - 12*(cosh(
x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 - 1)*sinh(x)^6 - 4*
cosh(x)^6 + 8*(7*cosh(x)^3 - 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 - 30*co
sh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cosh(x)^5 - 10*cosh(x)^3 + 3*co
sh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 - 15*cosh(x)^4 + 9*cosh(x)^2 - 1)*sinh(x)
^2 - 4*cosh(x)^2 + 8*(cosh(x)^7 - 3*cosh(x)^5 + 3*cosh(x)^3 - cosh(x))*sinh
(x) + 1)*log(2*sinh(x)/(cosh(x) - sinh(x))) + 16*(18*cosh(x)^5 - 27*cosh(x)
^3 + 11*cosh(x))*sinh(x) - 25)/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8
 + 4*(7*cosh(x)^2 - 1)*sinh(x)^6 - 4*cosh(x)^6 + 8*(7*cosh(x)^3 - 3*cosh(x)
)*sinh(x)^5 + 2*(35*cosh(x)^4 - 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 +
 8*(7*cosh(x)^5 - 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 - 15
*cosh(x)^4 + 9*cosh(x)^2 - 1)*sinh(x)^2 - 4*cosh(x)^2 + 8*(cosh(x)^7 - 3*co
sh(x)^5 + 3*cosh(x)^3 - cosh(x))*sinh(x) + 1)
```

Sympy [A]
time = 0.89, size = 48, normalized size = 1.17

$$32x - 16\log\left(\tanh(x) + 1\right) + 16\log\left(\tanh(x)\right) - \frac{15}{\tanh(x)} - \frac{11}{2\tanh^2(x)} - \frac{5}{3\tanh^3(x)} - \frac{1}{4\tanh^4(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))**5,x)
```

```
[Out] 32*x - 16*log(tanh(x) + 1) + 16*log(tanh(x)) - 15/tanh(x) - 11/(2*tanh(x)**
2) - 5/(3*tanh(x)**3) - 1/(4*tanh(x)**4)
```

Giac [A]
time = 0.41, size = 41, normalized size = 1.00

$$-\frac{4\left(48\,e^{(6\,x)}-108\,e^{(4\,x)}+88\,e^{(2\,x)}-25\right)}{3\left(e^{(2\,x)}-1\right)^{4}}+16\,\log\left(\left|e^{(2\,x)}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^5,x, algorithm="giac")
```

```
[Out] -4/3*(48*e^(6*x) - 108*e^(4*x) + 88*e^(2*x) - 25)/(e^(2*x) - 1)^4 + 16*log(
abs(e^(2*x) - 1))
```

Mupad [B]
time = 1.13, size = 88, normalized size = 2.15

$$16\,\ln\left(\mathrm{e}^{2\,x}-1\right)-\frac{64}{3\,\left(3\,\mathrm{e}^{2\,x}-3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}-1\right)}-\frac{48}{\mathrm{e}^{4\,x}-2\,\mathrm{e}^{2\,x}+1}-\frac{4}{6\,\mathrm{e}^{4\,x}-4\,\mathrm{e}^{2\,x}-4\,\mathrm{e}^{6\,x}+\mathrm{e}^{8\,x}+1}-\frac{64}{\mathrm{e}^{2\,x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(x) + 1)^5,x)
```

```
[Out] 16*log(exp(2*x) - 1) - 64/(3*(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1)) - 48
/(exp(4*x) - 2*exp(2*x) + 1) - 4/(6*exp(4*x) - 4*exp(2*x) - 4*exp(6*x) + ex
p(8*x) + 1) - 64/(exp(2*x) - 1)
```

3.62 $\int (1+\coth(x))^4\,dx$

Optimal. Leaf size=31

$$8x-4\coth(x)-(1+\coth(x))^2-\frac{1}{3}(1+\coth(x))^3+8\log(\sinh(x))$$

```
[Out] 8*x-4*coth(x)-(1+coth(x))^2-1/3*(1+coth(x))^3+8*ln(sinh(x))
```

Rubi [A]
time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3559, 3558, 3556}

$$8x-\frac{1}{3}(\coth(x)+1)^3-(\coth(x)+1)^2-4\coth(x)+8\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Coth[x])^4,x]
```

```
[Out] 8*x - 4*Coth[x] - (1 + Coth[x])^2 - (1 + Coth[x])^3/3 + 8*Log[Sinh[x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3558

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[b^2*(Tan[c + d*x]/d),
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rubi steps

$$\begin{aligned}\int (1+\coth(x))^4\,dx &= -\frac{1}{3}(1+\coth(x))^3+2\int (1+\coth(x))^3\,dx\\ &= -(1+\coth(x))^2-\frac{1}{3}(1+\coth(x))^3+4\int (1+\coth(x))^2\,dx\\ &= 8x-4\coth(x)-(1+\coth(x))^2-\frac{1}{3}(1+\coth(x))^3+8\int \coth(x)\,dx\\ &= 8x-4\coth(x)-(1+\coth(x))^2-\frac{1}{3}(1+\coth(x))^3+8\log(\sinh(x))\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.13, size = 84, normalized size = 2.71

$$\frac{(1+\coth(x))^4\sinh(x)\left(-\cosh^3(x)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\tanh^2(x)\right)+3\sinh(x)\left(-2\cosh^2(x)-6\cosh(x)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\tanh^2(x)\right)\sinh(x)+(x+8\log(\cosh(x))+8\log(\tanh(x)))\sinh^2(x)\right)\right)}{3(\cosh(x)+\sinh(x))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Coth[x])^4,x]
```

```
[Out] ((1 + Coth[x])^4*Sinh[x]*(-(Cosh[x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Tanh
[x]^2]) + 3*Sinh[x]*(-2*Cosh[x]^2 - 6*Cosh[x]*Hypergeometric2F1[-1/2, 1, 1/
2, Tanh[x]^2]*Sinh[x] + (x + 8*Log[Cosh[x]] + 8*Log[Tanh[x]])*Sinh[x]^2)))/
(3*(Cosh[x] + Sinh[x])^4)
```

Maple [A]
time = 0.19, size = 25, normalized size = 0.81

method	result	size
derivativedivides	$-\frac{\left(\coth^3(x)\right)}{3}-2\left(\coth^2(x)\right)-7\coth(x)-8\ln(\coth(x)-1)$	25
default	$-\frac{\left(\coth^3(x)\right)}{3}-2\left(\coth^2(x)\right)-7\coth(x)-8\ln(\coth(x)-1)$	25
risch	$-\frac{4\left(18\,e^{4x}-27\,e^{2x}+11\right)}{3\left(e^{2x}-1\right)^3}+8\ln\left(e^{2x}-1\right)$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+coth(x))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*coth(x)^3-2*coth(x)^2-7*coth(x)-8*ln(coth(x)-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(29) = 58.
time = 0.30, size = 95, normalized size = 3.06

$$12\,x-\frac{4\left(3\,e^{(-2\,x)}-3\,e^{(-4\,x)}-2\right)}{3\left(3\,e^{(-2\,x)}-3\,e^{(-4\,x)}+e^{(-6\,x)}-1\right)}+\frac{8\,e^{(-2\,x)}}{2\,e^{(-2\,x)}-e^{(-4\,x)}-1}+\frac{12}{e^{(-2\,x)}-1}+4\,\log\left(e^{(-x)}+1\right)+4\,\log\left(e^{(-x)}-1\right)+4\,\log\left(\sinh(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^4,x, algorithm="maxima")
```

```
[Out] 12*x - 4/3*(3*e^(-2*x) - 3*e^(-4*x) - 2)/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x
) - 1) + 8*e^(-2*x)/(2*e^(-2*x) - e^(-4*x) - 1) + 12/(e^(-2*x) - 1) + 4*log
(e^(-x) + 1) + 4*log(e^(-x) - 1) + 4*log(sinh(x))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(29) = 58$.
time = 0.36, size = 273, normalized size = 8.81

$$-\frac{4\left(18\cosh\left(x\right)^{4}+72\cosh\left(x\right)\sinh\left(x\right)^{3}+18\sinh\left(x\right)^{4}+27\left(4\cosh\left(x\right)^{2}-1\right)\sinh\left(x\right)^{2}-27\cosh\left(x\right)^{2}-6\left(\cosh\left(x\right)^{6}+6\cosh\left(x\right)\sinh\left(x\right)^{5}+\sinh\left(x\right)^{6}+3\left(5\cosh\left(x\right)^{2}-1\right)\sinh\left(x\right)^{4}-3\cosh\left(x\right)^{4}+4\left(5\cosh\left(x\right)^{3}-3\cosh\left(x\right)\right)\sinh\left(x\right)^{3}+3\left(5\cosh\left(x\right)^{4}-6\cosh\left(x\right)^{2}+1\right)\sinh\left(x\right)^{2}+3\cosh\left(x\right)^{2}+6\left(\cosh\left(x\right)^{5}-2\cosh\left(x\right)^{3}+\cosh\left(x\right)\right)\sinh\left(x\right)-1\right)\log\left(\frac{2\sinh(x)}{\cosh(x)-\sinh(x)}\right)+18\left(4\cosh\left(x\right)^{3}-3\cosh\left(x\right)\right)\sinh\left(x\right)+11\right)}{3\left(\cosh\left(x\right)^{6}+6\cosh\left(x\right)\sinh\left(x\right)^{5}+\sinh\left(x\right)^{6}+3\left(5\cosh\left(x\right)^{2}-1\right)\sinh\left(x\right)^{4}-3\cosh\left(x\right)^{4}+4\left(5\cosh\left(x\right)^{3}-3\cosh\left(x\right)\right)\sinh\left(x\right)^{3}+3\left(5\cosh\left(x\right)^{4}-6\cosh\left(x\right)^{2}+1\right)\sinh\left(x\right)^{2}+3\cosh\left(x\right)^{2}+6\left(\cosh\left(x\right)^{5}-2\cosh\left(x\right)^{3}+\cosh\left(x\right)\right)\sinh\left(x\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^4,x, algorithm="fricas")
```

```
[Out] -4/3*(18*cosh(x)^4 + 72*cosh(x)*sinh(x)^3 + 18*sinh(x)^4 + 27*(4*cosh(x)^2
- 1)*sinh(x)^2 - 27*cosh(x)^2 - 6*(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x
)^6 + 3*(5*cosh(x)^2 - 1)*sinh(x)^4 - 3*cosh(x)^4 + 4*(5*cosh(x)^3 - 3*cosh
(x))*sinh(x)^3 + 3*(5*cosh(x)^4 - 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2
+ 6*(cosh(x)^5 - 2*cosh(x)^3 + cosh(x))*sinh(x) - 1)*log(2*sinh(x)/(cosh(x)
 - sinh(x))) + 18*(4*cosh(x)^3 - 3*cosh(x))*sinh(x) + 11)/(cosh(x)^6 + 6*co
sh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 - 1)*sinh(x)^4 - 3*cosh(x)^4 +
 4*(5*cosh(x)^3 - 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4 - 6*cosh(x)^2 + 1)*
sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 - 2*cosh(x)^3 + cosh(x))*sinh(x) - 1
)
```

Sympy [**A**]
time = 0.57, size = 37, normalized size = 1.19

$$16x - 8\log\left(\tanh\left(x\right)+1\right) + 8\log\left(\tanh\left(x\right)\right) - \frac{7}{\tanh\left(x\right)} - \frac{2}{\tanh^{2}\left(x\right)} - \frac{1}{3\tanh^{3}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))**4,x)
```

```
[Out] 16*x - 8*log(tanh(x) + 1) + 8*log(tanh(x)) - 7/tanh(x) - 2/tanh(x)**2 - 1/(
3*tanh(x)**3)
```

Giac [**A**]
time = 0.44, size = 35, normalized size = 1.13

$$-\frac{4\left(18\,e^{(4\,x)} - 27\,e^{(2\,x)} + 11\right)}{3\left(e^{(2\,x)} - 1\right)^{3}} + 8\,\log\left(\left|e^{(2\,x)} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^4,x, algorithm="giac")
```

```
[Out] -4/3*(18*e^(4*x) - 27*e^(2*x) + 11)/(e^(2*x) - 1)^3 + 8*log(abs(e^(2*x) - 1
))
```

Mupad [B]
time = 1.14, size = 60, normalized size = 1.94

$$8\ln\left(e^{2x}-1\right)-\frac{8}{3\left(3e^{2x}-3e^{4x}+e^{6x}-1\right)}-\frac{12}{e^{4x}-2e^{2x}+1}-\frac{24}{e^{2x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(x) + 1)^4,x)
```

```
[Out] 8*log(exp(2*x) - 1) - 8/(3*(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1)) - 12/(
exp(4*x) - 2*exp(2*x) + 1) - 24/(exp(2*x) - 1)
```

3.63 $\int (1+\coth(x))^3 \, dx$

Optimal. Leaf size=23

$$4x - 2\coth(x) - \frac{1}{2}(1+\coth(x))^2 + 4\log(\sinh(x))$$

```
[Out] 4*x-2*coth(x)-1/2*(1+coth(x))^2+4*ln(sinh(x))
```

Rubi [A]

time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3559, 3558, 3556}

$$4x - \frac{1}{2}(\coth(x)+1)^2 - 2\coth(x) + 4\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Coth[x])^3,x]
```

```
[Out] 4*x - 2*Coth[x] - (1 + Coth[x])^2/2 + 4*Log[Sinh[x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3558

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[b^2*(Tan[c + d*x]/d),
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rubi steps

$$\begin{aligned}
\int (1+\coth(x))^3 \, dx &= -\frac{1}{2}(1+\coth(x))^2 + 2\int (1+\coth(x))^2 \, dx \\
&= 4x - 2\coth(x) - \frac{1}{2}(1+\coth(x))^2 + 4\int \coth(x) \, dx \\
&= 4x - 2\coth(x) - \frac{1}{2}(1+\coth(x))^2 + 4\log(\sinh(x))
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.11, size = 61, normalized size = 2.65

$$\frac{1}{4}\operatorname{csch}^2(x)\left(-1-2x-8\log(\cosh(x))-8\log(\tanh(x))+\cosh(2x)(-1+2x+8\log(\cosh(x))+8\log(\tanh(x)))-6\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\tanh^2(x)\right)\sinh(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Coth[x])^3,x]
```

```
[Out] (Csch[x]^2*(-1 - 2*x - 8*Log[Cosh[x]] - 8*Log[Tanh[x]] + Cosh[2*x]*(-1 + 2*
x + 8*Log[Cosh[x]] + 8*Log[Tanh[x]]) - 6*Hypergeometric2F1[-1/2, 1, 1/2, Ta
nh[x]^2]*Sinh[2*x]))/4
```

Maple [A]
time = 0.19, size = 19, normalized size = 0.83

method	result	size
derivativedivides	$-\frac{(\coth^2(x))}{2}-3\coth(x)-4\ln(\coth(x)-1)$	19
default	$-\frac{(\coth^2(x))}{2}-3\coth(x)-4\ln(\coth(x)-1)$	19
risch	$-\frac{2(4\,e^{2x}-3)}{(e^{2x}-1)^2}+4\ln(e^{2x}-1)$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+coth(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*coth(x)^2-3*coth(x)-4*ln(coth(x)-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(21) = 42.
time = 0.27, size = 55, normalized size = 2.39

$$5\,x+\frac{2\,e^{(-2\,x)}}{2\,e^{(-2\,x)}-e^{(-4\,x)}-1}+\frac{6}{e^{(-2\,x)}-1}+\log\left(e^{(-x)}+1\right)+\log\left(e^{(-x)}-1\right)+3\,\log\left(\sinh\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^3,x, algorithm="maxima")
```

```
[Out] 5*x + 2*e^(-2*x)/(2*e^(-2*x) - e^(-4*x) - 1) + 6/(e^(-2*x) - 1) + log(e^(-x
) + 1) + log(e^(-x) - 1) + 3*log(sinh(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(21) = 42.
time = 0.34, size = 142, normalized size = 6.17

$$-\frac{2\left(4\,\cosh\left(x\right)^2-2\left(\cosh\left(x\right)^4+4\,\cosh\left(x\right)\sinh\left(x\right)^3+\sinh\left(x\right)^4+2\left(3\,\cosh\left(x\right)^2-1\right)\sinh\left(x\right)^2-2\,\cosh\left(x\right)^2+4\left(\cosh\left(x\right)^3-\cosh\left(x\right)\right)\sinh\left(x\right)+1\right)\log\left(\frac{2\,\sinh(x)}{\cosh(x)-\sinh(x)}\right)+8\,\cosh\left(x\right)\sinh\left(x\right)+4\,\sinh\left(x\right)^2-3\right)}{\cosh\left(x\right)^4+4\,\cosh\left(x\right)\sinh\left(x\right)^3+\sinh\left(x\right)^4+2\left(3\,\cosh\left(x\right)^2-1\right)\sinh\left(x\right)^2-2\,\cosh\left(x\right)^2+4\left(\cosh\left(x\right)^3-\cosh\left(x\right)\right)\sinh\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^3,x, algorithm="fricas")
```

```
[Out] -2*(4*cosh(x)^2 - 2*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cos
h(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)*
log(2*sinh(x)/(cosh(x) - sinh(x))) + 8*cosh(x)*sinh(x) + 4*sinh(x)^2 - 3)/(
cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2
 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)
```

Sympy [**A**]
time = 0.39, size = 31, normalized size = 1.35

$$8x - 4\log\left(\tanh\left(x\right) + 1\right) + 4\log\left(\tanh\left(x\right)\right) - \frac{3}{\tanh\left(x\right)} - \frac{1}{2\tanh^{2}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))**3,x)
```

```
[Out] 8*x - 4*log(tanh(x) + 1) + 4*log(tanh(x)) - 3/tanh(x) - 1/(2*tanh(x)**2)
```

Giac [**A**]
time = 0.41, size = 29, normalized size = 1.26

$$-\frac{2\left(4\,e^{(2\,x)} - 3\right)}{\left(e^{(2\,x)} - 1\right)^{2}} + 4\,\log\left(\left|e^{(2\,x)} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^3,x, algorithm="giac")
```

```
[Out] -2*(4*e^(2*x) - 3)/(e^(2*x) - 1)^2 + 4*log(abs(e^(2*x) - 1))
```

Mupad [**B**]
time = 0.04, size = 36, normalized size = 1.57

$$4\,\ln\left(\mathrm{e}^{2\,x} - 1\right) - \frac{2}{\mathrm{e}^{4\,x} - 2\,\mathrm{e}^{2\,x} + 1} - \frac{8}{\mathrm{e}^{2\,x} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(x) + 1)^3,x)
```

```
[Out] 4*log(exp(2*x) - 1) - 2/(exp(4*x) - 2*exp(2*x) + 1) - 8/(exp(2*x) - 1)
```

3.64 $\int (1+\coth(x))^2 \, dx$

Optimal. Leaf size=13

$$2x-\coth(x)+2\log(\sinh(x))$$

[Out] 2*x-coth(x)+2*ln(sinh(x))

Rubi [A]
time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3558, 3556}

$$2x-\coth(x)+2\log(\sinh(x))$$

Antiderivative was successfully verified.

[In] Int[(1 + Coth[x])^2,x]

[Out] 2*x - Coth[x] + 2*Log[Sinh[x]]

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3558

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[b^2*(Tan[c + d*x]/d),
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (1+\coth(x))^2 \, dx &= 2x-\coth(x)+2\int \coth(x) \, dx\\ &= 2x-\coth(x)+2\log(\sinh(x))\end{aligned}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$2x-\coth(x)+2\log(\sinh(x))$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Coth[x])^2,x]

```
[Out] 2*x - Coth[x] + 2*Log[Sinh[x]]
```

Maple [A]
time = 0.19, size = 13, normalized size = 1.00

method	result	size
derivativedivides	$-\coth(x)-2\ln(\coth(x)-1)$	13
default	$-\coth(x)-2\ln(\coth(x)-1)$	13
risch	$-\frac{2}{e^{2x}-1}+2\ln(e^{2x}-1)$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+coth(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -coth(x)-2*ln(coth(x)-1)
```

Maxima [A]
time = 0.27, size = 19, normalized size = 1.46

$$2x+\frac{2}{e^{(-2x)}-1}+2\log(\sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^2,x, algorithm="maxima")
```

```
[Out] 2*x + 2/(e^(-2*x) - 1) + 2*log(sinh(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(13) = 26$.
time = 0.36, size = 53, normalized size = 4.08

$$\frac{2\left(\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2-1\right)\log\left(\frac{2\sinh(x)}{\cosh(x)-\sinh(x)}\right)-1\right)}{\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^2,x, algorithm="fricas")
```

```
[Out] 2*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*log(2*sinh(x)/(cosh(x) -
 sinh(x))) - 1)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)
```

Sympy [A]
time = 0.23, size = 22, normalized size = 1.69

$$4x-2\log(\tanh(x)+1)+2\log(\tanh(x))-\frac{1}{\tanh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))**2,x)
```

```
[Out] 4*x - 2*log(tanh(x) + 1) + 2*log(tanh(x)) - 1/tanh(x)
```

Giac [**A**]
time = 0.41, size = 21, normalized size = 1.62

$$-\frac{2}{e^{(2x)}-1}+2\log\left(\left|e^{(2x)}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^2,x, algorithm="giac")
```

```
[Out] -2/(e^(2*x) - 1) + 2*log(abs(e^(2*x) - 1))
```

Mupad [**B**]
time = 1.14, size = 20, normalized size = 1.54

$$2\ln\left(\mathrm{e}^{2x}-1\right)-\frac{2}{\mathrm{e}^{2x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(x) + 1)^2,x)
```

```
[Out] 2*log(exp(2*x) - 1) - 2/(exp(2*x) - 1)
```

3.65 $\int \frac{1}{1+\coth(x)}\,dx$

Optimal. Leaf size=16

$$\frac{x}{2}-\frac{1}{2(1+\coth(x))}$$

```
[Out] 1/2*x-1/2/(1+coth(x))
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3560, 8}

$$\frac{x}{2}-\frac{1}{2(\coth(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Coth[x])^(-1),x]
```

```
[Out] x/2 - 1/(2*(1 + Coth[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+\coth(x)}\,dx &= -\frac{1}{2(1+\coth(x))}+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}-\frac{1}{2(1+\coth(x))}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 18, normalized size = 1.12

$$\frac{1}{4}(2x+\cosh(2x)-\sinh(2x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Coth[x])^(-1),x]
```

```
[Out] (2*x + Cosh[2*x] - Sinh[2*x])/4
```

Maple [A]
time = 0.26, size = 24, normalized size = 1.50

method	result	size
risch	$\frac{x}{2}+\frac{\mathrm{e}^{-2x}}{4}$	11
derivativedivides	$-\frac{\ln(\coth(x)-1)}{4}-\frac{1}{2(1+\coth(x))}+\frac{\ln(1+\coth(x))}{4}$	24
default	$-\frac{\ln(\coth(x)-1)}{4}-\frac{1}{2(1+\coth(x))}+\frac{\ln(1+\coth(x))}{4}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*ln(coth(x)-1)-1/2/(1+coth(x))+1/4*ln(1+coth(x))
```

Maxima [A]
time = 0.27, size = 10, normalized size = 0.62

$$\frac{1}{2}\,x+\frac{1}{4}\,e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/4*e^(-2*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.
time = 0.34, size = 26, normalized size = 1.62

$$\frac{(2\,x+1)\cosh\left(x\right)+(2\,x-1)\sinh\left(x\right)}{4\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] 1/4*((2*x + 1)*cosh(x) + (2*x - 1)*sinh(x))/(cosh(x) + sinh(x))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(10) = 20$.
time = 0.27, size = 27, normalized size = 1.69

$$\frac{x\tanh\left(x\right)}{2\tanh\left(x\right)+2}+\frac{x}{2\tanh\left(x\right)+2}+\frac{1}{2\tanh\left(x\right)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x)),x)
```

```
[Out] x*tanh(x)/(2*tanh(x) + 2) + x/(2*tanh(x) + 2) + 1/(2*tanh(x) + 2)
```

Giac [A]
time = 0.41, size = 10, normalized size = 0.62

$$\frac{1}{2}\,x+\frac{1}{4}\,e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x)),x, algorithm="giac")
```

```
[Out] 1/2*x + 1/4*e^(-2*x)
```

Mupad [B]
time = 0.05, size = 14, normalized size = 0.88

$$\frac{x}{2}-\frac{1}{2\,\left(\mathrm{coth}\left(x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x) + 1),x)
```

```
[Out] x/2 - 1/(2*(coth(x) + 1))
```

3.66 $\int \frac{1}{(1+\coth(x))^2} \, dx$

Optimal. Leaf size=26

$$\frac{x}{4} - \frac{1}{4(1+\coth(x))^2} - \frac{1}{4(1+\coth(x))}$$

```
[Out] 1/4*x-1/4/(1+coth(x))^2-1/4/(1+coth(x))
```

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = $\{3560, 8\}$

$$\frac{x}{4} - \frac{1}{4(\coth(x)+1)} - \frac{1}{4(\coth(x)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Coth[x])^(-2),x]
```

```
[Out] x/4 - 1/(4*(1 + Coth[x])^2) - 1/(4*(1 + Coth[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1+\coth(x))^2} \, dx &= -\frac{1}{4(1+\coth(x))^2} + \frac{1}{2} \int \frac{1}{1+\coth(x)} \, dx \\
&= -\frac{1}{4(1+\coth(x))^2} - \frac{1}{4(1+\coth(x))} + \frac{\int 1 \, dx}{4} \\
&= \frac{x}{4} - \frac{1}{4(1+\coth(x))^2} - \frac{1}{4(1+\coth(x))}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 30, normalized size = 1.15

$$\frac{1}{16}(4x + 4\cosh(2x) - \cosh(4x) - 4\sinh(2x) + \sinh(4x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Coth[x])^(-2),x]
```

```
[Out] (4*x + 4*Cosh[2*x] - Cosh[4*x] - 4*Sinh[2*x] + Sinh[4*x])/16
```

Maple [A]
time = 0.26, size = 32, normalized size = 1.23

method	result	size
risch	$\frac{x}{4}+\frac{{\mathrm e}^{-2 x}}{4}-\frac{{\mathrm e}^{-4 x}}{16}$	17
derivativedivides	$-\frac{\ln \left(\coth \left(x \right)-1\right)}{8}-\frac{1}{4 \left(1+\coth \left(x \right)\right)^{2}}-\frac{1}{4 \left(1+\coth \left(x \right)\right)}+\frac{\ln \left(1+\coth \left(x \right)\right)}{8}$	32
default	$-\frac{\ln \left(\coth \left(x \right)-1\right)}{8}-\frac{1}{4 \left(1+\coth \left(x \right)\right)^{2}}-\frac{1}{4 \left(1+\coth \left(x \right)\right)}+\frac{\ln \left(1+\coth \left(x \right)\right)}{8}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+coth(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*ln(coth(x)-1)-1/4/(1+coth(x))^2-1/4/(1+coth(x))+1/8*ln(1+coth(x))
```

Maxima [A]
time = 0.28, size = 16, normalized size = 0.62

$$\frac{1}{4}\,x+\frac{1}{4}\,e^{\left(-2\,x\right)}-\frac{1}{16}\,e^{\left(-4\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*x + 1/4*e^(-2*x) - 1/16*e^(-4*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(20) = 40$.
time = 0.35, size = 52, normalized size = 2.00

$$\frac{\left(4\,x-1\right)\cosh\left(x\right)^{2}+2\,\left(4\,x+1\right)\cosh\left(x\right)\sinh\left(x\right)+\left(4\,x-1\right)\sinh\left(x\right)^{2}+4}{16\,\left(\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^2,x, algorithm="fricas")
```

```
[Out] 1/16*((4*x - 1)*cosh(x)^2 + 2*(4*x + 1)*cosh(x)*sinh(x) + (4*x - 1)*sinh(x)
^2 + 4)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(20) = 40$.
time = 0.50, size = 88, normalized size = 3.38

$$\frac{x\tanh^{2}\left(x\right)}{4\tanh^{2}\left(x\right)+8\tanh\left(x\right)+4}+\frac{2x\tanh\left(x\right)}{4\tanh^{2}\left(x\right)+8\tanh\left(x\right)+4}+\frac{x}{4\tanh^{2}\left(x\right)+8\tanh\left(x\right)+4}+\frac{3\tanh\left(x\right)}{4\tanh^{2}\left(x\right)+8\tanh\left(x\right)+4}+\frac{2}{4\tanh^{2}\left(x\right)+8\tanh\left(x\right)+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))**2,x)
```

```
[Out] x*tanh(x)**2/(4*tanh(x)**2 + 8*tanh(x) + 4) + 2*x*tanh(x)/(4*tanh(x)**2 + 8
*tanh(x) + 4) + x/(4*tanh(x)**2 + 8*tanh(x) + 4) + 3*tanh(x)/(4*tanh(x)**2
+ 8*tanh(x) + 4) + 2/(4*tanh(x)**2 + 8*tanh(x) + 4)
```

Giac [A]
time = 0.41, size = 18, normalized size = 0.69

$$\frac{1}{16}\left(4\,e^{(2\,x)}-1\right)e^{(-4\,x)}+\frac{1}{4}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^2,x, algorithm="giac")
```

```
[Out] 1/16*(4*e^(2*x) - 1)*e^(-4*x) + 1/4*x
```

Mupad [B]
time = 0.05, size = 16, normalized size = 0.62

$$\frac{x}{4}+\frac{\mathrm{e}^{-2\,x}}{4}-\frac{\mathrm{e}^{-4\,x}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x) + 1)^2,x)
```

```
[Out] x/4 + exp(-2*x)/4 - exp(-4*x)/16
```

3.67 $\int \frac{1}{(1+\coth(x))^3}\,dx$

Optimal. Leaf size=36

$$\frac{x}{8}-\frac{1}{6(1+\coth(x))^3}-\frac{1}{8(1+\coth(x))^2}-\frac{1}{8(1+\coth(x))}$$

```
[Out] 1/8*x-1/6/(1+coth(x))^3-1/8/(1+coth(x))^2-1/8/(1+coth(x))
```

Rubi [A]
time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3560, 8}

$$\frac{x}{8}-\frac{1}{8(\coth(x)+1)}-\frac{1}{8(\coth(x)+1)^2}-\frac{1}{6(\coth(x)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Coth[x])^(-3),x]
```

```
[Out] x/8 - 1/(6*(1 + Coth[x])^3) - 1/(8*(1 + Coth[x])^2) - 1/(8*(1 + Coth[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a + 
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1+\coth(x))^3}\,dx &= -\frac{1}{6(1+\coth(x))^3}+\frac{1}{2}\int \frac{1}{(1+\coth(x))^2}\,dx\\
&= -\frac{1}{6(1+\coth(x))^3}-\frac{1}{8(1+\coth(x))^2}+\frac{1}{4}\int \frac{1}{1+\coth(x)}\,dx\\
&= -\frac{1}{6(1+\coth(x))^3}-\frac{1}{8(1+\coth(x))^2}-\frac{1}{8(1+\coth(x))}+\frac{\int 1\,dx}{8}\\
&= \frac{x}{8}-\frac{1}{6(1+\coth(x))^3}-\frac{1}{8(1+\coth(x))^2}-\frac{1}{8(1+\coth(x))}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 44, normalized size = 1.22

$$\frac{1}{96}(12x + 18\cosh(2x) - 9\cosh(4x) + 2\cosh(6x) - 18\sinh(2x) + 9\sinh(4x) - 2\sinh(6x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Coth[x])^(-3),x]
```

```
[Out] (12*x + 18*Cosh[2*x] - 9*Cosh[4*x] + 2*Cosh[6*x] - 18*Sinh[2*x] + 9*Sinh[4*
x] - 2*Sinh[6*x])/96
```

Maple [A]
time = 0.26, size = 40, normalized size = 1.11

method	result	size
risch	$\frac{x}{8}+\frac{3\,{\mathrm e}^{-2x}}{16}-\frac{3\,{\mathrm e}^{-4x}}{32}+\frac{{\mathrm e}^{-6x}}{48}$	23
derivativedivides	$-\frac{\ln(\coth(x)-1)}{16}-\frac{1}{6(1+\coth(x))^{3}}-\frac{1}{8(1+\coth(x))^{2}}-\frac{1}{8(1+\coth(x))}+\frac{\ln(1+\coth(x))}{16}$	40
default	$-\frac{\ln(\coth(x)-1)}{16}-\frac{1}{6(1+\coth(x))^{3}}-\frac{1}{8(1+\coth(x))^{2}}-\frac{1}{8(1+\coth(x))}+\frac{\ln(1+\coth(x))}{16}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+coth(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*ln(coth(x)-1)-1/6/(1+coth(x))^3-1/8/(1+coth(x))^2-1/8/(1+coth(x))+1/1
6*ln(1+coth(x))
```

Maxima [A]
time = 0.26, size = 22, normalized size = 0.61

$$\frac{1}{8}\,x + \frac{3}{16}\,e^{(-2\,x)} - \frac{3}{32}\,e^{(-4\,x)} + \frac{1}{48}\,e^{(-6\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^3,x, algorithm="maxima")
```

```
[Out] 1/8*x + 3/16*e^(-2*x) - 3/32*e^(-4*x) + 1/48*e^(-6*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(28) = 56.
time = 0.35, size = 86, normalized size = 2.39

$$\frac{2\,(6\,x+1)\cosh\left(x\right)^{3}+6\,(6\,x+1)\cosh\left(x\right)\sinh\left(x\right)^{2}+2\,(6\,x-1)\sinh\left(x\right)^{3}+3\,\left(2\,(6\,x-1)\cosh\left(x\right)^{2}+9\right)\sinh\left(x\right)+9\,\cosh\left(x\right)}{96\,\left(\cosh\left(x\right)^{3}+3\,\cosh\left(x\right)^{2}\sinh\left(x\right)+3\,\cosh\left(x\right)\sinh\left(x\right)^{2}+\sinh\left(x\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^3,x, algorithm="fricas")
```

```
[Out] 1/96*(2*(6*x + 1)*cosh(x)^3 + 6*(6*x + 1)*cosh(x)*sinh(x)^2 + 2*(6*x - 1)*s
inh(x)^3 + 3*(2*(6*x - 1)*cosh(x)^2 + 9)*sinh(x) + 9*cosh(x))/(cosh(x)^3 +
3*cosh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2 + sinh(x)^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(31) = 62$.
time = 0.63, size = 182, normalized size = 5.06

$$\frac{3x\tanh^{3}(x)}{24\tanh^{3}(x)+72\tanh^{2}(x)+72\tanh(x)+24}+\frac{9x\tanh^{2}(x)}{24\tanh^{3}(x)+72\tanh^{2}(x)+72\tanh(x)+24}+\frac{9x\tanh(x)}{24\tanh^{3}(x)+72\tanh^{2}(x)+72\tanh(x)+24}+\frac{3x}{24\tanh^{3}(x)+72\tanh^{2}(x)+72\tanh(x)+24}+\frac{21\tanh^{2}(x)}{24\tanh^{3}(x)+72\tanh^{2}(x)+72\tanh(x)+24}+\frac{27\tanh(x)}{24\tanh^{3}(x)+72\tanh^{2}(x)+72\tanh(x)+24}+\frac{10}{24\tanh^{3}(x)+72\tanh^{2}(x)+72\tanh(x)+24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))**3,x)
```

```
[Out] 3*x*tanh(x)**3/(24*tanh(x)**3 + 72*tanh(x)**2 + 72*tanh(x) + 24) + 9*x*tanh
(x)**2/(24*tanh(x)**3 + 72*tanh(x)**2 + 72*tanh(x) + 24) + 9*x*tanh(x)/(24*
tanh(x)**3 + 72*tanh(x)**2 + 72*tanh(x) + 24) + 3*x/(24*tanh(x)**3 + 72*tan
h(x)**2 + 72*tanh(x) + 24) + 21*tanh(x)**2/(24*tanh(x)**3 + 72*tanh(x)**2 +
 72*tanh(x) + 24) + 27*tanh(x)/(24*tanh(x)**3 + 72*tanh(x)**2 + 72*tanh(x)
+ 24) + 10/(24*tanh(x)**3 + 72*tanh(x)**2 + 72*tanh(x) + 24)
```

Giac [A]
time = 0.42, size = 24, normalized size = 0.67

$$\frac{1}{96}\left(18\,e^{(4\,x)}-9\,e^{(2\,x)}+2\right)e^{(-6\,x)}+\frac{1}{8}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^3,x, algorithm="giac")
```

```
[Out] 1/96*(18*e^(4*x) - 9*e^(2*x) + 2)*e^(-6*x) + 1/8*x
```

Mupad [B]
time = 0.06, size = 22, normalized size = 0.61

$$\frac{x}{8}+\frac{3\,\mathrm{e}^{-2\,x}}{16}-\frac{3\,\mathrm{e}^{-4\,x}}{32}+\frac{\mathrm{e}^{-6\,x}}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x) + 1)^3,x)
```

```
[Out] x/8 + (3*exp(-2*x))/16 - (3*exp(-4*x))/32 + exp(-6*x)/48
```

3.68 $\int \frac{1}{(1+\coth(x))^4} \, dx$

Optimal. Leaf size=46

$$\frac{x}{16} - \frac{1}{8(1+\coth(x))^4} - \frac{1}{12(1+\coth(x))^3} - \frac{1}{16(1+\coth(x))^2} - \frac{1}{16(1+\coth(x))}$$

```
[Out] 1/16*x-1/8/(1+coth(x))^4-1/12/(1+coth(x))^3-1/16/(1+coth(x))^2-1/16/(1+coth
(x))
```

Rubi [A]
time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3560, 8}

$$\frac{x}{16} - \frac{1}{16(\coth(x)+1)} - \frac{1}{16(\coth(x)+1)^2} - \frac{1}{12(\coth(x)+1)^3} - \frac{1}{8(\coth(x)+1)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Coth[x])^(-4),x]
```

```
[Out] x/16 - 1/(8*(1 + Coth[x])^4) - 1/(12*(1 + Coth[x])^3) - 1/(16*(1 + Coth[x])
^2) - 1/(16*(1 + Coth[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1+\coth(x))^4} \, dx &= -\frac{1}{8(1+\coth(x))^4} + \frac{1}{2} \int \frac{1}{(1+\coth(x))^3} \, dx \\
&= -\frac{1}{8(1+\coth(x))^4} - \frac{1}{12(1+\coth(x))^3} + \frac{1}{4} \int \frac{1}{(1+\coth(x))^2} \, dx \\
&= -\frac{1}{8(1+\coth(x))^4} - \frac{1}{12(1+\coth(x))^3} - \frac{1}{16(1+\coth(x))^2} + \frac{1}{8} \int \frac{1}{1+\coth(x)} \, dx \\
&= -\frac{1}{8(1+\coth(x))^4} - \frac{1}{12(1+\coth(x))^3} - \frac{1}{16(1+\coth(x))^2} - \frac{1}{16(1+\coth(x))} + \frac{\int 1 \, dx}{16} \\
&= \frac{x}{16} - \frac{1}{8(1+\coth(x))^4} - \frac{1}{12(1+\coth(x))^3} - \frac{1}{16(1+\coth(x))^2} - \frac{1}{16(1+\coth(x))}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 53, normalized size = 1.15

$$\frac{1}{384}(\cosh(4x)-\sinh(4x))(-36+64\cosh(2x)+3(-1+8x)\cosh(4x)+32\sinh(2x)+3\sinh(4x)+24x\sinh(4x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Coth[x])^(-4),x]
```

```
[Out] ((Cosh[4*x] - Sinh[4*x])*(-36 + 64*Cosh[2*x] + 3*(-1 + 8*x)*Cosh[4*x] + 32*
Sinh[2*x] + 3*Sinh[4*x] + 24*x*Sinh[4*x]))/384
```

Maple [A]
time = 0.26, size = 48, normalized size = 1.04

method	result
risch	$\frac{x}{16}+\frac{e^{-2x}}{8}-\frac{3\,e^{-4x}}{32}+\frac{e^{-6x}}{24}-\frac{e^{-8x}}{128}$
derivativedivides	$-\frac{1}{8(1+\coth(x))^4}-\frac{1}{12(1+\coth(x))^3}-\frac{1}{16(1+\coth(x))^2}-\frac{1}{16(1+\coth(x))}+\frac{\ln(1+\coth(x))}{32}-\frac{\ln(\coth(x)-1)}{32}$
default	$-\frac{1}{8(1+\coth(x))^4}-\frac{1}{12(1+\coth(x))^3}-\frac{1}{16(1+\coth(x))^2}-\frac{1}{16(1+\coth(x))}+\frac{\ln(1+\coth(x))}{32}-\frac{\ln(\coth(x)-1)}{32}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+coth(x))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/(1+coth(x))^4-1/12/(1+coth(x))^3-1/16/(1+coth(x))^2-1/16/(1+coth(x))+1
/32*ln(1+coth(x))-1/32*ln(coth(x)-1)
```

Maxima [A]
time = 0.27, size = 28, normalized size = 0.61

$$\frac{1}{16}\,x+\frac{1}{8}\,e^{(-2\,x)}-\frac{3}{32}\,e^{(-4\,x)}+\frac{1}{24}\,e^{(-6\,x)}-\frac{1}{128}\,e^{(-8\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^4,x, algorithm="maxima")
```

```
[Out] 1/16*x + 1/8*e^(-2*x) - 3/32*e^(-4*x) + 1/24*e^(-6*x) - 1/128*e^(-8*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(36) = 72$.
time = 0.36, size = 121, normalized size = 2.63

$$\frac{3\,(8\,x-1)\cosh(x)^4+12\,(8\,x+1)\cosh(x)\sinh(x)^3+3\,(8\,x-1)\sinh(x)^4+2\,(9\,(8\,x-1)\cosh(x)^2+32)\sinh(x)^2+64\,\cosh(x)^2+4\,(3\,(8\,x+1)\cosh(x)^3+16\,\cosh(x))\sinh(x)-36}{384\,(\cosh(x)^4+4\,\cosh(x)^3\sinh(x)+6\,\cosh(x)^2\sinh(x)^2+4\,\cosh(x)\sinh(x)^3+\sinh(x)^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^4,x, algorithm="fricas")
```

```
[Out] 1/384*(3*(8*x - 1)*cosh(x)^4 + 12*(8*x + 1)*cosh(x)*sinh(x)^3 + 3*(8*x - 1)
*sinh(x)^4 + 2*(9*(8*x - 1)*cosh(x)^2 + 32)*sinh(x)^2 + 64*cosh(x)^2 + 4*(3
*(8*x + 1)*cosh(x)^3 + 16*cosh(x))*sinh(x) - 36)/(cosh(x)^4 + 4*cosh(x)^3*s
inh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(41) = 82$.
time = 0.89, size = 299, normalized size = 6.50

$$\frac{3x\tanh^4(x)}{48\tanh^4(x)+192\tanh^3(x)+288\tanh^2(x)+192\tanh(x)+48}+\frac{12x\tanh^3(x)}{48\tanh^4(x)+192\tanh^3(x)+288\tanh^2(x)+192\tanh(x)+48}+\frac{18x\tanh^2(x)}{48\tanh^4(x)+192\tanh^3(x)+288\tanh^2(x)+192\tanh(x)+48}+\frac{12x\tanh(x)}{48\tanh^4(x)+192\tanh^3(x)+288\tanh^2(x)+192\tanh(x)+48}+\frac{3x}{48\tanh^4(x)+192\tanh^3(x)+288\tanh^2(x)+192\tanh(x)+48}+\frac{45\tanh^3(x)}{48\tanh^4(x)+192\tanh^3(x)+288\tanh^2(x)+192\tanh(x)+48}+\frac{84\tanh^2(x)}{48\tanh^4(x)+192\tanh^3(x)+288\tanh^2(x)+192\tanh(x)+48}+\frac{61\tanh(x)}{48\tanh^4(x)+192\tanh^3(x)+288\tanh^2(x)+192\tanh(x)+48}+\frac{16}{48\tanh^4(x)+192\tanh^3(x)+288\tanh^2(x)+192\tanh(x)+48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))**4,x)
```

```
[Out] 3*x*tanh(x)**4/(48*tanh(x)**4 + 192*tanh(x)**3 + 288*tanh(x)**2 + 192*tanh(
x) + 48) + 12*x*tanh(x)**3/(48*tanh(x)**4 + 192*tanh(x)**3 + 288*tanh(x)**2
 + 192*tanh(x) + 48) + 18*x*tanh(x)**2/(48*tanh(x)**4 + 192*tanh(x)**3 + 28
8*tanh(x)**2 + 192*tanh(x) + 48) + 12*x*tanh(x)/(48*tanh(x)**4 + 192*tanh(x
)**3 + 288*tanh(x)**2 + 192*tanh(x) + 48) + 3*x/(48*tanh(x)**4 + 192*tanh(x
)**3 + 288*tanh(x)**2 + 192*tanh(x) + 48) + 45*tanh(x)**3/(48*tanh(x)**4 +
192*tanh(x)**3 + 288*tanh(x)**2 + 192*tanh(x) + 48) + 84*tanh(x)**2/(48*tan
h(x)**4 + 192*tanh(x)**3 + 288*tanh(x)**2 + 192*tanh(x) + 48) + 61*tanh(x)/
(48*tanh(x)**4 + 192*tanh(x)**3 + 288*tanh(x)**2 + 192*tanh(x) + 48) + 16/(
48*tanh(x)**4 + 192*tanh(x)**3 + 288*tanh(x)**2 + 192*tanh(x) + 48)
```

Giac [A]
time = 0.42, size = 30, normalized size = 0.65

$$\frac{1}{384}\left(48\,e^{(6\,x)}-36\,e^{(4\,x)}+16\,e^{(2\,x)}-3\right)e^{(-8\,x)}+\frac{1}{16}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^4,x, algorithm="giac")
```

```
[Out] 1/384*(48*e^(6*x) - 36*e^(4*x) + 16*e^(2*x) - 3)*e^(-8*x) + 1/16*x
```

Mupad [B]
time = 1.14, size = 28, normalized size = 0.61

$$\frac{x}{16}+\frac{\mathrm{e}^{-2\,x}}{8}-\frac{3\,\mathrm{e}^{-4\,x}}{32}+\frac{\mathrm{e}^{-6\,x}}{24}-\frac{\mathrm{e}^{-8\,x}}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x) + 1)^4,x)
```

```
[Out] x/16 + exp(-2*x)/8 - (3*exp(-4*x))/32 + exp(-6*x)/24 - exp(-8*x)/128
```

3.69 $\int \frac{1}{(1+\coth(x))^5} dx$

Optimal. Leaf size=56

$$\frac{x}{32}-\frac{1}{10(1+\coth(x))^5}-\frac{1}{16(1+\coth(x))^4}-\frac{1}{24(1+\coth(x))^3}-\frac{1}{32(1+\coth(x))^2}-\frac{1}{32(1+\coth(x))}$$

```
[Out] 1/32*x-1/10/(1+coth(x))^5-1/16/(1+coth(x))^4-1/24/(1+coth(x))^3-1/32/(1+cot
h(x))^2-1/32/(1+coth(x))
```

Rubi [A]
time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3560, 8}

$$\frac{x}{32}-\frac{1}{32(\coth(x)+1)}-\frac{1}{32(\coth(x)+1)^2}-\frac{1}{24(\coth(x)+1)^3}-\frac{1}{16(\coth(x)+1)^4}-\frac{1}{10(\coth(x)+1)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Coth[x])^(-5),x]
```

```
[Out] x/32 - 1/(10*(1 + Coth[x])^5) - 1/(16*(1 + Coth[x])^4) - 1/(24*(1 + Coth[x]
)^3) - 1/(32*(1 + Coth[x])^2) - 1/(32*(1 + Coth[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(1+\coth(x))^5}\,dx = -\frac{1}{10(1+\coth(x))^5} + \frac{1}{2}\int \frac{1}{(1+\coth(x))^4}\,dx$$
$$= -\frac{1}{10(1+\coth(x))^5} - \frac{1}{16(1+\coth(x))^4} + \frac{1}{4}\int \frac{1}{(1+\coth(x))^3}\,dx$$
$$= -\frac{1}{10(1+\coth(x))^5} - \frac{1}{16(1+\coth(x))^4} - \frac{1}{24(1+\coth(x))^3} + \frac{1}{8}\int \frac{1}{(1+\coth(x))^2}\,dx$$
$$= -\frac{1}{10(1+\coth(x))^5} - \frac{1}{16(1+\coth(x))^4} - \frac{1}{24(1+\coth(x))^3} - \frac{1}{32(1+\coth(x))^2} + \frac{1}{16}$$
$$= -\frac{1}{10(1+\coth(x))^5} - \frac{1}{16(1+\coth(x))^4} - \frac{1}{24(1+\coth(x))^3} - \frac{1}{32(1+\coth(x))^2} - \frac{}{32}$$
$$= \frac{x}{32} - \frac{1}{10(1+\coth(x))^5} - \frac{1}{16(1+\coth(x))^4} - \frac{1}{24(1+\coth(x))^3} - \frac{1}{32(1+\coth(x))^2}$$

Mathematica [A]
time = 0.10, size = 62, normalized size = 1.11

$$\frac{(\cosh(5x)-\sinh(5x))(-100\cosh(x)+225\cosh(3x)+12\cosh(5x)+120x\cosh(5x)-500\sinh(x)+375\sinh(3x)-12\sinh(5x)+120x\sinh(5x))}{3840}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Coth[x])^(-5),x]
```

```
[Out] ((Cosh[5*x] - Sinh[5*x])*(-100*Cosh[x] + 225*Cosh[3*x] + 12*Cosh[5*x] + 120
*x*Cosh[5*x] - 500*Sinh[x] + 375*Sinh[3*x] - 12*Sinh[5*x] + 120*x*Sinh[5*x]
))/3840
```

Maple [A]
time = 0.27, size = 56, normalized size = 1.00

method	result
risch	$\frac{x}{32}+\frac{5\,e^{-2x}}{64}-\frac{5\,e^{-4x}}{64}+\frac{5\,e^{-6x}}{96}-\frac{5\,e^{-8x}}{256}+\frac{e^{-10x}}{320}$
derivativedivides	$-\frac{1}{10(1+\coth(x))^5}-\frac{1}{16(1+\coth(x))^4}-\frac{1}{24(1+\coth(x))^3}-\frac{1}{32(1+\coth(x))^2}-\frac{1}{32(1+\coth(x))}+\frac{\ln(1+\coth(x))}{64}$
default	$-\frac{1}{10(1+\coth(x))^5}-\frac{1}{16(1+\coth(x))^4}-\frac{1}{24(1+\coth(x))^3}-\frac{1}{32(1+\coth(x))^2}-\frac{1}{32(1+\coth(x))}+\frac{\ln(1+\coth(x))}{64}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+coth(x))^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/10/(1+coth(x))^5-1/16/(1+coth(x))^4-1/24/(1+coth(x))^3-1/32/(1+coth(x))^
2-1/32/(1+coth(x))+1/64*ln(1+coth(x))-1/64*ln(coth(x)-1)
```

Maxima [A]
time = 0.28, size = 34, normalized size = 0.61

$$\frac{1}{32}\,x+\frac{5}{64}\,e^{(-2\,x)}-\frac{5}{64}\,e^{(-4\,x)}+\frac{5}{96}\,e^{(-6\,x)}-\frac{5}{256}\,e^{(-8\,x)}+\frac{1}{320}\,e^{(-10\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^5,x, algorithm="maxima")
```

```
[Out] 1/32*x + 5/64*e^(-2*x) - 5/64*e^(-4*x) + 5/96*e^(-6*x) - 5/256*e^(-8*x) + 1
/320*e^(-10*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(44) = 88$.
time = 0.35, size = 159, normalized size = 2.84

$$\frac{12\,(10\,x+1)\cosh\left(x\right)^{5}+60\,(10\,x+1)\cosh\left(x\right)\sinh\left(x\right)^{4}+12\,(10\,x-1)\sinh\left(x\right)^{5}+15\,\left(8\,(10\,x-1)\cosh\left(x\right)^{2}+25\right)\sinh\left(x\right)^{3}+225\,\cosh\left(x\right)^{3}+15\,\left(8\,(10\,x+1)\cosh\left(x\right)^{3}+45\,\cosh\left(x\right)\right)\sinh\left(x\right)^{2}+5\,\left(12\,(10\,x-1)\cosh\left(x\right)^{4}+225\,\cosh\left(x\right)^{2}-100\right)\sinh\left(x\right)-100\,\cosh\left(x\right)}{3840\,\left(\cosh\left(x\right)^{5}+5\,\cosh\left(x\right)^{4}\sinh\left(x\right)+10\,\cosh\left(x\right)^{3}\sinh\left(x\right)^{2}+10\,\cosh\left(x\right)^{2}\sinh\left(x\right)^{3}+5\,\cosh\left(x\right)\sinh\left(x\right)^{4}+\sinh\left(x\right)^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^5,x, algorithm="fricas")
```

```
[Out] 1/3840*(12*(10*x + 1)*cosh(x)^5 + 60*(10*x + 1)*cosh(x)*sinh(x)^4 + 12*(10*
x - 1)*sinh(x)^5 + 15*(8*(10*x - 1)*cosh(x)^2 + 25)*sinh(x)^3 + 225*cosh(x)
^3 + 15*(8*(10*x + 1)*cosh(x)^3 + 45*cosh(x))*sinh(x)^2 + 5*(12*(10*x - 1)*
cosh(x)^4 + 225*cosh(x)^2 - 100)*sinh(x) - 100*cosh(x))/(cosh(x)^5 + 5*cosh
(x)^4*sinh(x) + 10*cosh(x)^3*sinh(x)^2 + 10*cosh(x)^2*sinh(x)^3 + 5*cosh(x)
*sinh(x)^4 + sinh(x)^5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 444 vs. $2(51) = 102$.
time = 1.21, size = 444, normalized size = 7.93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))**5,x)
```

```
[Out] 15*x*tanh(x)**5/(480*tanh(x)**5 + 2400*tanh(x)**4 + 4800*tanh(x)**3 + 4800*
tanh(x)**2 + 2400*tanh(x) + 480) + 75*x*tanh(x)**4/(480*tanh(x)**5 + 2400*t
anh(x)**4 + 4800*tanh(x)**3 + 4800*tanh(x)**2 + 2400*tanh(x) + 480) + 150*x
*tanh(x)**3/(480*tanh(x)**5 + 2400*tanh(x)**4 + 4800*tanh(x)**3 + 4800*tanh
(x)**2 + 2400*tanh(x) + 480) + 150*x*tanh(x)**2/(480*tanh(x)**5 + 2400*tanh
(x)**4 + 4800*tanh(x)**3 + 4800*tanh(x)**2 + 2400*tanh(x) + 480) + 75*x*tan
h(x)/(480*tanh(x)**5 + 2400*tanh(x)**4 + 4800*tanh(x)**3 + 4800*tanh(x)**2
+ 2400*tanh(x) + 480) + 15*x/(480*tanh(x)**5 + 2400*tanh(x)**4 + 4800*tanh(
x)**3 + 4800*tanh(x)**2 + 2400*tanh(x) + 480) + 465*tanh(x)**4/(480*tanh(x)
```

```
**5 + 2400*tanh(x)**4 + 4800*tanh(x)**3 + 4800*tanh(x)**2 + 2400*tanh(x) +
480) + 1125*tanh(x)**3/(480*tanh(x)**5 + 2400*tanh(x)**4 + 4800*tanh(x)**3
+ 4800*tanh(x)**2 + 2400*tanh(x) + 480) + 1205*tanh(x)**2/(480*tanh(x)**5 +
 2400*tanh(x)**4 + 4800*tanh(x)**3 + 4800*tanh(x)**2 + 2400*tanh(x) + 480)
+ 625*tanh(x)/(480*tanh(x)**5 + 2400*tanh(x)**4 + 4800*tanh(x)**3 + 4800*ta
nh(x)**2 + 2400*tanh(x) + 480) + 128/(480*tanh(x)**5 + 2400*tanh(x)**4 + 48
00*tanh(x)**3 + 4800*tanh(x)**2 + 2400*tanh(x) + 480)
```

Giac [A]
time = 0.41, size = 36, normalized size = 0.64

$$\frac{1}{3840}\left(300\,e^{(8\,x)}-300\,e^{(6\,x)}+200\,e^{(4\,x)}-75\,e^{(2\,x)}+12\right)e^{(-10\,x)}+\frac{1}{32}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^5,x, algorithm="giac")
```

```
[Out] 1/3840*(300*e^(8*x) - 300*e^(6*x) + 200*e^(4*x) - 75*e^(2*x) + 12)*e^(-10*x
) + 1/32*x
```

Mupad [B]
time = 1.15, size = 34, normalized size = 0.61

$$\frac{x}{32}+\frac{5\,\mathrm{e}^{-2\,x}}{64}-\frac{5\,\mathrm{e}^{-4\,x}}{64}+\frac{5\,\mathrm{e}^{-6\,x}}{96}-\frac{5\,\mathrm{e}^{-8\,x}}{256}+\frac{\mathrm{e}^{-10\,x}}{320}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x) + 1)^5,x)
```

```
[Out] x/32 + (5*exp(-2*x))/64 - (5*exp(-4*x))/64 + (5*exp(-6*x))/96 - (5*exp(-8*x
))/256 + exp(-10*x)/320
```

3.70 $\int (1+\coth(x))^{7/2}\,dx$

Optimal. Leaf size=57

$$8\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)-8\sqrt{1+\coth(x)}-\frac{4}{3}(1+\coth(x))^{3/2}-\frac{2}{5}(1+\coth(x))^{5/2}$$

```
[Out] -4/3*(1+coth(x))^(3/2)-2/5*(1+coth(x))^(5/2)+8*arctanh(1/2*(1+coth(x))^(1/2
)*2^(1/2))*2^(1/2)-8*(1+coth(x))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3559, 3561, 212}

$$-\frac{2}{5}(\coth(x)+1)^{5/2}-\frac{4}{3}(\coth(x)+1)^{3/2}-8\sqrt{\coth(x)+1}+8\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Coth[x])^(7/2),x]
```

```
[Out] 8*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 8*Sqrt[1 + Coth[x]] - (4*(1
+ Coth[x])^(3/2))/3 - (2*(1 + Coth[x])^(5/2))/5
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (1+\coth(x))^{7/2}\,dx = -\frac{2}{5}(1+\coth(x))^{5/2} + 2\int (1+\coth(x))^{5/2}\,dx$$
$$= -\frac{4}{3}(1+\coth(x))^{3/2} - \frac{2}{5}(1+\coth(x))^{5/2} + 4\int (1+\coth(x))^{3/2}\,dx$$
$$= -8\sqrt{1+\coth(x)} - \frac{4}{3}(1+\coth(x))^{3/2} - \frac{2}{5}(1+\coth(x))^{5/2} + 8\int \sqrt{1+\coth(x)}\,d$$
$$= -8\sqrt{1+\coth(x)} - \frac{4}{3}(1+\coth(x))^{3/2} - \frac{2}{5}(1+\coth(x))^{5/2} + 16\text{Subst}\left(\int \frac{1}{2-x^2}\,d\right.$$
$$= 8\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right) - 8\sqrt{1+\coth(x)} - \frac{4}{3}(1+\coth(x))^{3/2} - \frac{2}{5}(1+\text{co}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.19, size = 101, normalized size = 1.77

$$-\frac{2(1+\coth(x))^{7/2}\left(4\left((-15+15i)\text{ArcTan}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i(1+\coth(x))}\right)+19\sqrt{i(1+\coth(x))}\right)\sinh^3(x)+\sqrt{i(1+\coth(x))}\sinh(x)(3+8\sinh(2x))\right)}{15\sqrt{i(1+\coth(x))}(\cosh(x)+\sinh(x))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Coth[x])^(7/2),x]
```

```
[Out] (-2*(1 + Coth[x])^(7/2)*(4*((-15 + 15*I)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Cot
h[x])]] + 19*Sqrt[I*(1 + Coth[x])])*Sinh[x]^3 + Sqrt[I*(1 + Coth[x])]*Sinh[
x]*(3 + 8*Sinh[2*x])))/(15*Sqrt[I*(1 + Coth[x])]*(Cosh[x] + Sinh[x])^3)
```

Maple [A]
time = 0.68, size = 43, normalized size = 0.75

method	result
derivativedivides	$-\frac{4(1+\coth(x))^{\frac{3}{2}}}{3}-\frac{2(1+\coth(x))^{\frac{5}{2}}}{5}+8\,\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}-8\sqrt{1+\coth(x)}$
default	$-\frac{4(1+\coth(x))^{\frac{3}{2}}}{3}-\frac{2(1+\coth(x))^{\frac{5}{2}}}{5}+8\,\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}-8\sqrt{1+\coth(x)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+coth(x))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/3*(1+coth(x))^(3/2)-2/5*(1+coth(x))^(5/2)+8*arctanh(1/2*(1+coth(x))^(1/2
)*2^(1/2))*2^(1/2)-8*(1+coth(x))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((coth(x) + 1)^(7/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 438 vs. $2(42) = 84$.
time = 0.37, size = 438, normalized size = 7.68

$$-\frac{4 \left(2 \sqrt{2} \left(23 \sqrt{2} \cosh \left(x\right)^{5}+115 \sqrt{2} \cosh \left(x\right) \sinh \left(x\right)^{4}+23 \sqrt{2} \sinh \left(x\right)^{5}+5 \left(46 \sqrt{2} \cosh \left(x\right)^{2}-7 \sqrt{2}\right) \sinh \left(x\right)^{3}-35 \sqrt{2} \cosh \left(x\right)^{3}+5 \left(46 \sqrt{2} \cosh \left(x\right)^{3}-21 \sqrt{2} \cosh \left(x\right)\right) \sinh \left(x\right)^{2}+5 \left(23 \sqrt{2} \cosh \left(x\right)^{4}-21 \sqrt{2} \cosh \left(x\right)^{2}+3 \sqrt{2}\right) \sinh \left(x\right)+15 \sqrt{2} \cosh \left(x\right)\right) \sqrt{\frac{\sinh \left(x\right)}{\cosh \left(x\right)-\sinh \left(x\right)}}-15 \left(\sqrt{2} \cosh \left(x\right)^{6}+6 \sqrt{2} \cosh \left(x\right) \sinh \left(x\right)^{5}+\sqrt{2} \sinh \left(x\right)^{6}+3 \left(5 \sqrt{2} \cosh \left(x\right)^{2}-\sqrt{2}\right) \sinh \left(x\right)^{4}-3 \sqrt{2} \cosh \left(x\right)^{4}+4 \left(5 \sqrt{2} \cosh \left(x\right)^{3}-3 \sqrt{2} \cosh \left(x\right)\right) \sinh \left(x\right)^{3}+3 \left(5 \sqrt{2} \cosh \left(x\right)^{4}-6 \sqrt{2} \cosh \left(x\right)^{2}+\sqrt{2}\right) \sinh \left(x\right)^{2}+3 \sqrt{2} \cosh \left(x\right)^{2}+6 \left(\sqrt{2} \cosh \left(x\right)^{5}-2 \sqrt{2} \cosh \left(x\right)^{3}+\sqrt{2} \cosh \left(x\right)\right) \sinh \left(x\right)-\sqrt{2}\right) \log \left(2 \sqrt{2} \sqrt{\frac{\sinh \left(x\right)}{\cosh \left(x\right)-\sinh \left(x\right)}} \left(\cosh \left(x\right)+\sinh \left(x\right)\right)+2 \cosh \left(x\right)^{2}+4 \cosh \left(x\right) \sinh \left(x\right)+2 \sinh \left(x\right)^{2}-1\right)\right)}{15 \left(\cosh \left(x\right)^{6}+6 \cosh \left(x\right) \sinh \left(x\right)^{5}+\sinh \left(x\right)^{6}+3 \left(5 \cosh \left(x\right)^{2}-1\right) \sinh \left(x\right)^{4}-3 \cosh \left(x\right)^{4}+4 \left(5 \cosh \left(x\right)^{3}-3 \cosh \left(x\right)\right) \sinh \left(x\right)^{3}+3 \left(5 \cosh \left(x\right)^{4}-6 \cosh \left(x\right)^{2}+1\right) \sinh \left(x\right)^{2}+3 \cosh \left(x\right)^{2}+6 \left(\cosh \left(x\right)^{5}-2 \cosh \left(x\right)^{3}+\cosh \left(x\right)\right) \sinh \left(x\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^(7/2),x, algorithm="fricas")
```

```
[Out] -4/15*(2*sqrt(2)*(23*sqrt(2)*cosh(x)^5 + 115*sqrt(2)*cosh(x)*sinh(x)^4 + 23
*sqrt(2)*sinh(x)^5 + 5*(46*sqrt(2)*cosh(x)^2 - 7*sqrt(2))*sinh(x)^3 - 35*sq
rt(2)*cosh(x)^3 + 5*(46*sqrt(2)*cosh(x)^3 - 21*sqrt(2)*cosh(x))*sinh(x)^2 +
 5*(23*sqrt(2)*cosh(x)^4 - 21*sqrt(2)*cosh(x)^2 + 3*sqrt(2))*sinh(x) + 15*s
qrt(2)*cosh(x))*sqrt(sinh(x)/(cosh(x) - sinh(x))) - 15*(sqrt(2)*cosh(x)^6 +
 6*sqrt(2)*cosh(x)*sinh(x)^5 + sqrt(2)*sinh(x)^6 + 3*(5*sqrt(2)*cosh(x)^2 -
 sqrt(2))*sinh(x)^4 - 3*sqrt(2)*cosh(x)^4 + 4*(5*sqrt(2)*cosh(x)^3 - 3*sqrt
(2)*cosh(x))*sinh(x)^3 + 3*(5*sqrt(2)*cosh(x)^4 - 6*sqrt(2)*cosh(x)^2 + sqr
t(2))*sinh(x)^2 + 3*sqrt(2)*cosh(x)^2 + 6*(sqrt(2)*cosh(x)^5 - 2*sqrt(2)*co
sh(x)^3 + sqrt(2)*cosh(x))*sinh(x) - sqrt(2))*log(2*sqrt(2)*sqrt(sinh(x)/(c
osh(x) - sinh(x)))*(cosh(x) + sinh(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) +
2*sinh(x)^2 - 1))/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(
x)^2 - 1)*sinh(x)^4 - 3*cosh(x)^4 + 4*(5*cosh(x)^3 - 3*cosh(x))*sinh(x)^3 +
 3*(5*cosh(x)^4 - 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 -
 2*cosh(x)^3 + cosh(x))*sinh(x) - 1)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))**(7/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(42) = 84$.
time = 0.42, size = 160, normalized size = 2.81

$$-\frac{4}{15} \sqrt{2} \left(\frac{2 \left(45 \left(\sqrt{e^{\left(4 x\right)}-e^{\left(2 x\right)}}-e^{\left(2 x\right)}\right)^{4}+135 \left(\sqrt{e^{\left(4 x\right)}-e^{\left(2 x\right)}}-e^{\left(2 x\right)}\right)^{3}+170 \left(\sqrt{e^{\left(4 x\right)}-e^{\left(2 x\right)}}-e^{\left(2 x\right)}\right)^{2}+100 \sqrt{e^{\left(4 x\right)}-e^{\left(2 x\right)}}-100 e^{\left(2 x\right)}+23\right)}{\left(\sqrt{e^{\left(4 x\right)}-e^{\left(2 x\right)}}-e^{\left(2 x\right)}+1\right)^{5}}+15 \log \left(\left|2 \sqrt{e^{\left(4 x\right)}-e^{\left(2 x\right)}}-2 e^{\left(2 x\right)}+1\right|\right)\right) \mathrm{sgn}\left(e^{\left(2 x\right)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^(7/2),x, algorithm="giac")
```

```
[Out] -4/15*sqrt(2)*(2*(45*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^4 + 135*(sqrt(e^(4
*x) - e^(2*x)) - e^(2*x))^3 + 170*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^2 + 1
00*sqrt(e^(4*x) - e^(2*x)) - 100*e^(2*x) + 23)/(sqrt(e^(4*x) - e^(2*x)) - e
^(2*x) + 1)^5 + 15*log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1)))*sgn
(e^(2*x) - 1)
```

Mupad [B]
time = 1.27, size = 44, normalized size = 0.77

$$-8\sqrt{\coth(x)+1}-\frac{4(\coth(x)+1)^{3/2}}{3}-\frac{2(\coth(x)+1)^{5/2}}{5}-\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{\coth(x)+1}\,1\mathrm{i}}{2}\right)8\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(x) + 1)^(7/2),x)
```

```
[Out] - 2^(1/2)*atan((2^(1/2)*(coth(x) + 1)^(1/2)*1i)/2)*8i - 8*(coth(x) + 1)^(1/
2) - (4*(coth(x) + 1)^(3/2))/3 - (2*(coth(x) + 1)^(5/2))/5
```

3.71 $\int (1+\coth(x))^{5/2}\,dx$

Optimal. Leaf size=45

$$4\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)-4\sqrt{1+\coth(x)}-\frac{2}{3}(1+\coth(x))^{3/2}$$

```
[Out] -2/3*(1+coth(x))^(3/2)+4*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-4*(
1+coth(x))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3559, 3561, 212}

$$-\frac{2}{3}(\coth(x)+1)^{3/2}-4\sqrt{\coth(x)+1}+4\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Coth[x])^(5/2),x]
```

```
[Out] 4*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 4*Sqrt[1 + Coth[x]] - (2*(1
+ Coth[x])^(3/2))/3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (1+\coth(x))^{5/2}\,dx = -\frac{2}{3}(1+\coth(x))^{3/2} + 2\int (1+\coth(x))^{3/2}\,dx$$
$$= -4\sqrt{1+\coth(x)} - \frac{2}{3}(1+\coth(x))^{3/2} + 4\int \sqrt{1+\coth(x)}\,dx$$
$$= -4\sqrt{1+\coth(x)} - \frac{2}{3}(1+\coth(x))^{3/2} + 8\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+\coth(x)}\right)$$
$$= 4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right) - 4\sqrt{1+\coth(x)} - \frac{2}{3}(1+\coth(x))^{3/2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.11, size = 92, normalized size = 2.04

$$-\frac{2(1+\coth(x))^{5/2}\sinh(x)\left(\cosh(x)\sqrt{i(1+\coth(x))} + \left((-6+6i)\text{ArcTan}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i(1+\coth(x))}\right) + 7\sqrt{i(1+\coth(x))}\right)\sinh(x)\right)}{3\sqrt{i(1+\coth(x))}\,(\cosh(x)+\sinh(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Coth[x])^(5/2),x]
```

```
[Out] (-2*(1 + Coth[x])^(5/2)*Sinh[x]*(Cosh[x]*Sqrt[I*(1 + Coth[x])] + ((-6 + 6*I
)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Coth[x])]] + 7*Sqrt[I*(1 + Coth[x])])*Sinh
[x]))/(3*Sqrt[I*(1 + Coth[x])]*(Cosh[x] + Sinh[x])^2)
```

Maple [A]
time = 0.64, size = 35, normalized size = 0.78

method	result	size
derivativedivides	$-\frac{2(1+\coth(x))^{\frac{3}{2}}}{3} + 4\,\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2} - 4\sqrt{1+\coth(x)}$	35
default	$-\frac{2(1+\coth(x))^{\frac{3}{2}}}{3} + 4\,\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2} - 4\sqrt{1+\coth(x)}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+coth(x))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(1+coth(x))^(3/2)+4*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-4*(
1+coth(x))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((coth(x) + 1)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 259 vs. 2(34) = 68.
time = 0.35, size = 259, normalized size = 5.76

$$-\frac{2\left(2\sqrt{2}\left(4\sqrt{2}\cosh\left(x\right)^{3}+12\sqrt{2}\cosh\left(x\right)\sinh\left(x\right)^{2}+4\sqrt{2}\sinh\left(x\right)^{3}+3\left(4\sqrt{2}\cosh\left(x\right)^{2}-\sqrt{2}\right)\sinh\left(x\right)-3\sqrt{2}\cosh\left(x\right)\right)\sqrt{\frac{\sinh\left(x\right)}{\cosh\left(x\right)-\sinh\left(x\right)}}-3\left(\sqrt{2}\cosh\left(x\right)^{4}+4\sqrt{2}\cosh\left(x\right)\sinh\left(x\right)^{3}+\sqrt{2}\sinh\left(x\right)^{4}+2\left(3\sqrt{2}\cosh\left(x\right)^{2}-\sqrt{2}\right)\sinh\left(x\right)^{2}-2\sqrt{2}\cosh\left(x\right)^{2}+4\left(\sqrt{2}\cosh\left(x\right)^{3}-\sqrt{2}\cosh\left(x\right)\right)\sinh\left(x\right)+\sqrt{2}\right)\log\left(2\sqrt{2}\sqrt{\frac{\sinh\left(x\right)}{\cosh\left(x\right)-\sinh\left(x\right)}}\left(\cosh\left(x\right)+\sinh\left(x\right)\right)+2\cosh\left(x\right)^{2}+4\cosh\left(x\right)\sinh\left(x\right)+2\sinh\left(x\right)^{2}-1\right)\right)}{3\left(\cosh\left(x\right)^{4}+4\cosh\left(x\right)\sinh\left(x\right)^{3}+\sinh\left(x\right)^{4}+2\left(3\cosh\left(x\right)^{2}-1\right)\sinh\left(x\right)^{2}-2\cosh\left(x\right)^{2}+4\left(\cosh\left(x\right)^{3}-\cosh\left(x\right)\right)\sinh\left(x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(2*sqrt(2)*(4*sqrt(2)*cosh(x)^3 + 12*sqrt(2)*cosh(x)*sinh(x)^2 + 4*sqr
t(2)*sinh(x)^3 + 3*(4*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x) - 3*sqrt(2)*cosh
(x))*sqrt(sinh(x)/(cosh(x) - sinh(x))) - 3*(sqrt(2)*cosh(x)^4 + 4*sqrt(2)*c
osh(x)*sinh(x)^3 + sqrt(2)*sinh(x)^4 + 2*(3*sqrt(2)*cosh(x)^2 - sqrt(2))*si
nh(x)^2 - 2*sqrt(2)*cosh(x)^2 + 4*(sqrt(2)*cosh(x)^3 - sqrt(2)*cosh(x))*sin
h(x) + sqrt(2))*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x) - sinh(x)))*(cosh(x) +
sinh(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - 1))/(cosh(x)^4 +
 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x
)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\coth\left(x\right)+1\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))**(5/2),x)
```

```
[Out] Integral((coth(x) + 1)**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(34) = 68.
time = 0.42, size = 112, normalized size = 2.49

$$-\frac{2}{3}\sqrt{2}\left(\frac{2\left(6\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^{2}+9\sqrt{e^{(4x)}-e^{(2x)}}-9e^{(2x)}+4\right)}{\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}+1\right)^{3}}+3\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x)}+1\right|\right)\right)\operatorname{sgn}\left(e^{(2x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(2)*(2*(6*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^2 + 9*sqrt(e^(4*x) -
 e^(2*x)) - 9*e^(2*x) + 4)/(sqrt(e^(4*x) - e^(2*x)) - e^(2*x) + 1)^3 + 3*lo
g(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1)))*sgn(e^(2*x) - 1)
```

Mupad [B]
time = 1.19, size = 54, normalized size = 1.20

$$\sqrt{8}\ \ln\left(-2\sqrt{8}\ \sqrt{\coth\left(x\right)+1}-8\right)-\frac{2\left(\coth(x)+1\right)^{3/2}}{3}-2\sqrt{2}\ \ln\left(4\sqrt{2}\ \sqrt{\coth\left(x\right)+1}-8\right)-4\sqrt{\coth\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(x) + 1)^(5/2),x)
```

```
[Out] 8^(1/2)*log(- 2*8^(1/2)*(coth(x) + 1)^(1/2) - 8) - (2*(coth(x) + 1)^(3/2))/
3 - 2*2^(1/2)*log(4*2^(1/2)*(coth(x) + 1)^(1/2) - 8) - 4*(coth(x) + 1)^(1/2
)
```

3.72 $\int (1+\coth(x))^{3/2}\,dx$

Optimal. Leaf size=33

$$2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)-2\sqrt{1+\coth(x)}$$

```
[Out] 2*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-2*(1+coth(x))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}=0.375$, Rules used = {3559, 3561, 212}

$$2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)-2\sqrt{\coth(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Coth[x])^(3/2),x]
```

```
[Out] 2*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 2*Sqrt[1 + Coth[x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int (1+\coth(x))^{3/2}\,dx &= -2\sqrt{1+\coth(x)}+2\int \sqrt{1+\coth(x)}\,dx\\
&= -2\sqrt{1+\coth(x)}+4\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+\coth(x)}\right)\\
&= 2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)-2\sqrt{1+\coth(x)}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.07, size = 69, normalized size = 2.09

$$
-\frac{2(1+\coth(x))^{3/2}\left((-1+i)\text{ArcTan}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i(1+\coth(x))}\right)+\sqrt{i(1+\coth(x))}\right)\sinh(x)}{\sqrt{i(1+\coth(x))}\,(\cosh(x)+\sinh(x))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Coth[x])^(3/2),x]
```

```
[Out] (-2*(1 + Coth[x])^(3/2)*((-1 + I)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Coth[x])]]
 + Sqrt[I*(1 + Coth[x])])*Sinh[x])/(Sqrt[I*(1 + Coth[x])]*(Cosh[x] + Sinh[x
]))
```

Maple [A]
time = 0.64, size = 27, normalized size = 0.82

method	result	size
derivativedivides	$2\,\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}-2\sqrt{1+\coth(x)}$	27
default	$2\,\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}-2\sqrt{1+\coth(x)}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+coth(x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-2*(1+coth(x))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((coth(x) + 1)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(26) = 52$.
time = 0.34, size = 131, normalized size = 3.97

$$-\frac{2\sqrt{2}\left(\sqrt{2}\cosh\left(x\right)+\sqrt{2}\sinh\left(x\right)\right)\sqrt{\frac{\sinh\left(x\right)}{\cosh\left(x\right)-\sinh\left(x\right)}}-\left(\sqrt{2}\cosh\left(x\right)^{2}+2\sqrt{2}\cosh\left(x\right)\sinh\left(x\right)+\sqrt{2}\sinh\left(x\right)^{2}-\sqrt{2}\right)\log\left(2\sqrt{2}\sqrt{\frac{\sinh\left(x\right)}{\cosh\left(x\right)-\sinh\left(x\right)}}\left(\cosh\left(x\right)+\sinh\left(x\right)\right)+2\cosh\left(x\right)^{2}+4\cosh\left(x\right)\sinh\left(x\right)+2\sinh\left(x\right)^{2}-1\right)}{\cosh\left(x\right)^{2}+2\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^(3/2),x, algorithm="fricas")
```

```
[Out] -(2*sqrt(2)*(sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(sinh(x)/(cosh(x) - sin
h(x))) - (sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2
 - sqrt(2))*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sinh
(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - 1))/(cosh(x)^2 + 2*c
osh(x)*sinh(x) + sinh(x)^2 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\coth\left(x\right)+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))**(3/2),x)
```

```
[Out] Integral((coth(x) + 1)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(26) = 52$.
time = 0.42, size = 63, normalized size = 1.91

$$-\sqrt{2}\left(\frac{2}{\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}+1}+\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x)}+1\right|\right)\right)\mathrm{sgn}\left(e^{(2x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^(3/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*(2/(sqrt(e^(4*x) - e^(2*x)) - e^(2*x) + 1) + log(abs(2*sqrt(e^(4*x
) - e^(2*x)) - 2*e^(2*x) + 1)))*sgn(e^(2*x) - 1)
```

Mupad [B]
time = 1.19, size = 26, normalized size = 0.79

$$2\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{\mathrm{coth}\left(x\right)+1}}{2}\right)-2\sqrt{\mathrm{coth}\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(x) + 1)^(3/2),x)
[Out] 2*2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2) - 2*(coth(x) + 1)^(1/2)
```

3.73 $\int \sqrt{1+\coth(x)}\,dx$

Optimal. Leaf size=21

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)$$

```
[Out] arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3561, 212}

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Coth[x]],x]
[Out] Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1+\coth(x)}\,dx &= 2\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+\coth(x)}\right)\\ &= \sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.06, size = 45, normalized size = 2.14

$$\frac{(1+i)\text{ArcTan}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i(1+\coth(x))}\right)(1+\coth(x))^{3/2}}{(i(1+\coth(x)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Coth[x]],x]
```

```
[Out] ((1 + I)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Coth[x])]]*(1 + Coth[x])^(3/2))/(I*
(1 + Coth[x]))^(3/2)
```

Maple [A]
time = 0.77, size = 17, normalized size = 0.81

method	result	size
derivativedivides	$\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}$	17
default	$\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+coth(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(coth(x) + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(16) = 32$.
time = 0.39, size = 50, normalized size = 2.38

$$\frac{1}{2}\sqrt{2}\log\left(2\sqrt{2}\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}}\left(\cosh(x)+\sinh(x)\right)+2\cosh(x)^2+4\cosh(x)\sinh(x)+2\sinh(x)^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sinh
(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\coth(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))**(1/2),x)
```

```
[Out] Integral(sqrt(coth(x) + 1), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(16) = 32$.
time = 0.41, size = 37, normalized size = 1.76

$$-\frac{1}{2}\sqrt{2}\log\left(\left|2\sqrt{e^{(4x)} - e^{(2x)}} - 2e^{(2x)} + 1\right|\right)\operatorname{sgn}\left(e^{(2x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1))*sgn(e^(2*x
) - 1)
```

Mupad [B]
time = 1.20, size = 16, normalized size = 0.76

$$\sqrt{2}\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{\coth(x) + 1}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(x) + 1)^(1/2),x)
```

```
[Out] 2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2)
```

3.74 $\int \frac{1}{\sqrt{1+\coth(x)}}\,dx$

Optimal. Leaf size=32

$$\frac{\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{\sqrt{1+\coth(x)}}$$

```
[Out] 1/2*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-1/(1+coth(x))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3560, 3561, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{\sqrt{\coth(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + Coth[x]],x]
```

```
[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/Sqrt[2] - 1/Sqrt[1 + Coth[x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+\coth(x)}}\,dx = -\frac{1}{\sqrt{1+\coth(x)}} + \frac{1}{2}\int \sqrt{1+\coth(x)}\,dx$$
$$= -\frac{1}{\sqrt{1+\coth(x)}} + \text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+\coth(x)}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{1}{\sqrt{1+\coth(x)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.21, size = 51, normalized size = 1.59

$$\frac{-2-(1+i)\text{ArcTan}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i(1+\coth(x))}\right)\sqrt{i(1+\coth(x))}}{2\sqrt{1+\coth(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + Coth[x]],x]
```

```
[Out] (-2 - (1 + I)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Coth[x])]]*Sqrt[I*(1 + Coth[x]
)])/(2*Sqrt[1 + Coth[x]])
```

Maple [A]
time = 0.71, size = 27, normalized size = 0.84

method	result	size
derivativedivides	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{2} - \frac{1}{\sqrt{1+\coth(x)}}$	27
default	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{2} - \frac{1}{\sqrt{1+\coth(x)}}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+coth(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-1/(1+coth(x))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(coth(x) + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(26) = 52$.
time = 0.37, size = 85, normalized size = 2.66

$$\frac{\left(\sqrt{2}\cosh\left(x\right)+\sqrt{2}\sinh\left(x\right)\right)\log\left(2\sqrt{2}\sqrt{\frac{\sinh\left(x\right)}{\cosh\left(x\right)-\sinh\left(x\right)}}\left(\cosh\left(x\right)+\sinh\left(x\right)\right)+2\cosh\left(x\right)^{2}+4\cosh\left(x\right)\sinh\left(x\right)+2\sinh\left(x\right)^{2}-1\right)-4\sqrt{\frac{\sinh\left(x\right)}{\cosh\left(x\right)-\sinh\left(x\right)}}}{4\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*((sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x
) - sinh(x)))*(cosh(x) + sinh(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sin
h(x)^2 - 1) - 4*sqrt(sinh(x)/(cosh(x) - sinh(x))))/(cosh(x) + sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{\coth\left(x\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(coth(x) + 1), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(26) = 52$.
time = 0.42, size = 66, normalized size = 2.06

$$\frac{\sqrt{2}\left(\frac{2}{\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}}-\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x)}+1\right|\right)\right)}{4\,\mathrm{sgn}\left(e^{(2x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(2/(sqrt(e^(4*x) - e^(2*x)) - e^(2*x)) - log(abs(2*sqrt(e^(4*x)
 - e^(2*x)) - 2*e^(2*x) + 1)))/sgn(e^(2*x) - 1)
```

Mupad [B]
time = 1.24, size = 26, normalized size = 0.81

$$\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\sqrt{\coth\left(x\right)+1}}{2}\right)}{2}-\frac{1}{\sqrt{\coth\left(x\right)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x) + 1)^(1/2),x)
[Out] (2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2))/2 - 1/(coth(x) + 1)^(1/2)
```

3.75 $\int \frac{1}{(1+\coth(x))^{3/2}}\,dx$

Optimal. Leaf size=49

$$\frac{\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{1}{3(1+\coth(x))^{3/2}}-\frac{1}{2\sqrt{1+\coth(x)}}$$

```
[Out] -1/3/(1+coth(x))^(3/2)+1/4*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-1
/2/(1+coth(x))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3560, 3561, 212}

$$-\frac{1}{2\sqrt{\coth(x)+1}}-\frac{1}{3(\coth(x)+1)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Coth[x])^(-3/2),x]
```

```
[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/(2*Sqrt[2]) - 1/(3*(1 + Coth[x])^(3/2))
- 1/(2*Sqrt[1 + Coth[x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+\coth(x))^{3/2}}\,dx &= -\frac{1}{3(1+\coth(x))^{3/2}}+\frac{1}{2}\int \frac{1}{\sqrt{1+\coth(x)}}\,dx\\ &= -\frac{1}{3(1+\coth(x))^{3/2}}-\frac{1}{2\sqrt{1+\coth(x)}}+\frac{1}{4}\int \sqrt{1+\coth(x)}\,dx\\ &= -\frac{1}{3(1+\coth(x))^{3/2}}-\frac{1}{2\sqrt{1+\coth(x)}}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1+\coth(x)}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{1}{3(1+\coth(x))^{3/2}}-\frac{1}{2\sqrt{1+\coth(x)}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.22, size = 86, normalized size = 1.76

$$\left(\frac{1}{4}+\frac{i}{4}\right)\sqrt{1+\coth(x)}\left(-\frac{i\text{ArcTan}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i(1+\coth(x))}\right)}{\sqrt{i(1+\coth(x))}}+\left(\frac{1}{6}-\frac{i}{6}\right)(-4+5\cosh(2x)-\cosh(4x)-5\sinh(2x)+\sinh(4x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Coth[x])^(-3/2),x]
```

```
[Out] (1/4 + I/4)*Sqrt[1 + Coth[x]]*(((-I)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Coth[x]
)]])/Sqrt[I*(1 + Coth[x])] + (1/6 - I/6)*(-4 + 5*Cosh[2*x] - Cosh[4*x] - 5*
Sinh[2*x] + Sinh[4*x]))
```

Maple [A]
time = 0.67, size = 35, normalized size = 0.71

method	result	size
derivativedivides	$-\frac{1}{3(1+\coth(x))^{\frac{3}{2}}}+\frac{\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{4}-\frac{1}{2\sqrt{1+\coth(x)}}$	35
default	$-\frac{1}{3(1+\coth(x))^{\frac{3}{2}}}+\frac{\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{4}-\frac{1}{2\sqrt{1+\coth(x)}}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+coth(x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/(1+coth(x))^(3/2)+1/4*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-1
/2/(1+coth(x))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((coth(x) + 1)^(-3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(34) = 68.
time = 0.39, size = 168, normalized size = 3.43

$$-\frac{2\sqrt{2}\left(4\sqrt{2}\cosh\left(x\right)^{2}+8\sqrt{2}\cosh\left(x\right)\sinh\left(x\right)+4\sqrt{2}\sinh\left(x\right)^{2}-\sqrt{2}\right)\sqrt{\frac{\sinh\left(x\right)}{\cosh\left(x\right)-\sinh\left(x\right)}}-3\left(\sqrt{2}\cosh\left(x\right)^{3}+3\sqrt{2}\cosh\left(x\right)^{2}\sinh\left(x\right)+3\sqrt{2}\cosh\left(x\right)\sinh\left(x\right)^{2}+\sqrt{2}\sinh\left(x\right)^{3}\right)\log\left(2\sqrt{2}\sqrt{\frac{\sinh\left(x\right)}{\cosh\left(x\right)-\sinh\left(x\right)}}\left(\cosh\left(x\right)+\sinh\left(x\right)\right)+2\cosh\left(x\right)^{2}+4\cosh\left(x\right)\sinh\left(x\right)+2\sinh\left(x\right)^{2}-1\right)}{24\left(\cosh\left(x\right)^{3}+3\cosh\left(x\right)^{2}\sinh\left(x\right)+3\cosh\left(x\right)\sinh\left(x\right)^{2}+\sinh\left(x\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/24*(2*sqrt(2)*(4*sqrt(2)*cosh(x)^2 + 8*sqrt(2)*cosh(x)*sinh(x) + 4*sqrt(
2)*sinh(x)^2 - sqrt(2))*sqrt(sinh(x)/(cosh(x) - sinh(x))) - 3*(sqrt(2)*cosh
(x)^3 + 3*sqrt(2)*cosh(x)^2*sinh(x) + 3*sqrt(2)*cosh(x)*sinh(x)^2 + sqrt(2)
*sinh(x)^3)*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sinh
(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - 1))/(cosh(x)^3 + 3*c
osh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2 + sinh(x)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\coth\left(x\right)+1\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))**(3/2),x)
```

```
[Out] Integral((coth(x) + 1)**(-3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(34) = 68.
time = 0.43, size = 113, normalized size = 2.31

$$\frac{\sqrt{2}\left(\frac{2\left(6\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^{2}+3\sqrt{e^{(4x)}-e^{(2x)}}-3e^{(2x)}+1\right)}{\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^{3}}-3\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x)}+1\right|\right)\right)}{24\,\mathrm{sgn}\left(e^{(2x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^(3/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(2)*(2*(6*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^2 + 3*sqrt(e^(4*x) -
 e^(2*x)) - 3*e^(2*x) + 1)/(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^3 - 3*log(ab
s(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1)))/sgn(e^(2*x) - 1)
```

Mupad [B]
time = 1.20, size = 32, normalized size = 0.65

$$\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{\coth\left(x\right)+1}}{2}\right)}{4}-\frac{\frac{\coth(x)}{2}+\frac{5}{6}}{{\left(\coth\left(x\right)+1\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x) + 1)^(3/2),x)
```

```
[Out] (2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2))/4 - (coth(x)/2 + 5/6)/(cot
h(x) + 1)^(3/2)
```

3.76 $\int \frac{1}{(1+\coth(x))^{5/2}} dx$

Optimal. Leaf size=61

$$\frac{\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{4\sqrt{2}}-\frac{1}{5(1+\coth(x))^{5/2}}-\frac{1}{6(1+\coth(x))^{3/2}}-\frac{1}{4\sqrt{1+\coth(x)}}$$

```
[Out] -1/5/(1+coth(x))^(5/2)-1/6/(1+coth(x))^(3/2)+1/8*arctanh(1/2*(1+coth(x))^(1
/2)*2^(1/2))*2^(1/2)-1/4/(1+coth(x))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3560, 3561, 212}

$$-\frac{1}{4\sqrt{\coth(x)+1}}-\frac{1}{6(\coth(x)+1)^{3/2}}-\frac{1}{5(\coth(x)+1)^{5/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Coth[x])^(-5/2),x]
```

```
[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/(4*Sqrt[2]) - 1/(5*(1 + Coth[x])^(5/2))
- 1/(6*(1 + Coth[x])^(3/2)) - 1/(4*Sqrt[1 + Coth[x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(1+\coth(x))^{5/2}}\,dx &= -\frac{1}{5(1+\coth(x))^{5/2}}+\frac{1}{2}\int \frac{1}{(1+\coth(x))^{3/2}}\,dx \\
&= -\frac{1}{5(1+\coth(x))^{5/2}}-\frac{1}{6(1+\coth(x))^{3/2}}+\frac{1}{4}\int \frac{1}{\sqrt{1+\coth(x)}}\,dx \\
&= -\frac{1}{5(1+\coth(x))^{5/2}}-\frac{1}{6(1+\coth(x))^{3/2}}-\frac{1}{4\sqrt{1+\coth(x)}}+\frac{1}{8}\int \sqrt{1+\coth(x
\\
&= -\frac{1}{5(1+\coth(x))^{5/2}}-\frac{1}{6(1+\coth(x))^{3/2}}-\frac{1}{4\sqrt{1+\coth(x)}}+\frac{1}{4}\text{Subst}\left(\int \frac{1}{2-x^2
\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{4\sqrt{2}}-\frac{1}{5(1+\coth(x))^{5/2}}-\frac{1}{6(1+\coth(x))^{3/2}}-\frac{1}{4\sqrt{1+co
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.53, size = 94, normalized size = 1.54

$$
\frac{\left(\frac{1}{8}+\frac{i}{8}\right)\text{ArcTan}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i(1+\coth(x))}\right)(1+\coth(x))^{3/2}}{(i(1+\coth(x)))^{3/2}}-\frac{1}{60}\sqrt{1+\coth(x)}\,(\cosh(3x)-\sinh(3x))(-10\cosh(x)+10\cosh(3x)-24\sinh(x)+13\sinh(3x))
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Coth[x])^(-5/2),x]
```

```
[Out] ((1/8 + I/8)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Coth[x])]]*(1 + Coth[x])^(3/2))
/(I*(1 + Coth[x]))^(3/2) - (Sqrt[1 + Coth[x]]*(Cosh[3*x] - Sinh[3*x])*(-10*
Cosh[x] + 10*Cosh[3*x] - 24*Sinh[x] + 13*Sinh[3*x]))/60
```

Maple [A]
time = 0.66, size = 43, normalized size = 0.70

method	result
derivativedivides	$-\frac{1}{5(1+\coth(x))^{\frac{5}{2}}}-\frac{1}{6(1+\coth(x))^{\frac{3}{2}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{8}-\frac{1}{4\sqrt{1+\coth(x)}}$
default	$-\frac{1}{5(1+\coth(x))^{\frac{5}{2}}}-\frac{1}{6(1+\coth(x))^{\frac{3}{2}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{8}-\frac{1}{4\sqrt{1+\coth(x)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+coth(x))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/(1+coth(x))^(5/2)-1/6/(1+coth(x))^(3/2)+1/8*arctanh(1/2*(1+coth(x))^(1
/2)*2^(1/2))*2^(1/2)-1/4/(1+coth(x))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((coth(x) + 1)^(-5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(42) = 84.
time = 0.38, size = 266, normalized size = 4.36

$$-\frac{2\sqrt{2}\left(23\sqrt{2}\cosh(x)^4+92\sqrt{2}\cosh(x)\sinh(x)^3+23\sqrt{2}\sinh(x)^4+\left(138\sqrt{2}\cosh(x)^2-11\sqrt{2}\right)\sinh(x)^2-11\sqrt{2}\cosh(x)^2+2\left(46\sqrt{2}\cosh(x)^3-11\sqrt{2}\cosh(x)\right)\sinh(x)+3\sqrt{2}\right)\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}}-15\left(\sqrt{2}\cosh(x)^5+5\sqrt{2}\cosh(x)^4\sinh(x)+10\sqrt{2}\cosh(x)^3\sinh(x)^2+10\sqrt{2}\cosh(x)^2\sinh(x)^3+5\sqrt{2}\cosh(x)\sinh(x)^4+\sqrt{2}\sinh(x)^5\right)\log\left(2\sqrt{2}\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}}(\cosh(x)+\sinh(x))+2\cosh(x)^2+4\cosh(x)\sinh(x)+2\sinh(x)^2-1\right)}{240\left(\cosh(x)^5+5\cosh(x)^4\sinh(x)+10\cosh(x)^3\sinh(x)^2+10\cosh(x)^2\sinh(x)^3+5\cosh(x)\sinh(x)^4+\sinh(x)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/240*(2*sqrt(2)*(23*sqrt(2)*cosh(x)^4 + 92*sqrt(2)*cosh(x)*sinh(x)^3 + 23
*sqrt(2)*sinh(x)^4 + (138*sqrt(2)*cosh(x)^2 - 11*sqrt(2))*sinh(x)^2 - 11*sq
rt(2)*cosh(x)^2 + 2*(46*sqrt(2)*cosh(x)^3 - 11*sqrt(2)*cosh(x))*sinh(x) + 3
*sqrt(2))*sqrt(sinh(x)/(cosh(x) - sinh(x))) - 15*(sqrt(2)*cosh(x)^5 + 5*sqr
t(2)*cosh(x)^4*sinh(x) + 10*sqrt(2)*cosh(x)^3*sinh(x)^2 + 10*sqrt(2)*cosh(x
)^2*sinh(x)^3 + 5*sqrt(2)*cosh(x)*sinh(x)^4 + sqrt(2)*sinh(x)^5)*log(2*sqrt
(2)*sqrt(sinh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sinh(x)) + 2*cosh(x)^2 + 4
*cosh(x)*sinh(x) + 2*sinh(x)^2 - 1))/(cosh(x)^5 + 5*cosh(x)^4*sinh(x) + 10*
cosh(x)^3*sinh(x)^2 + 10*cosh(x)^2*sinh(x)^3 + 5*cosh(x)*sinh(x)^4 + sinh(x
)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\coth(x)+1)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))**(5/2),x)
```

```
[Out] Integral((coth(x) + 1)**(-5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 161 vs. 2(42) = 84.

time = 0.44, size = 161, normalized size = 2.64

$$\frac{\sqrt{2}\left(\frac{2\left(45\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^4+45\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^3+35\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^2+15\sqrt{e^{(4x)}-e^{(2x)}}-15e^{(2x)}+3\right)}{\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^5}-15\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x)}+1\right|\right)\right)}{240\,\mathrm{sgn}\left(e^{(2x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^(5/2),x, algorithm="giac")
```

```
[Out] 1/240*sqrt(2)*(2*(45*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^4 + 45*(sqrt(e^(4*
x) - e^(2*x)) - e^(2*x))^3 + 35*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^2 + 15*
sqrt(e^(4*x) - e^(2*x)) - 15*e^(2*x) + 3)/(sqrt(e^(4*x) - e^(2*x)) - e^(2*x
))^5 - 15*log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1)))/sgn(e^(2*x)
- 1)
```

Mupad [B]
time = 1.20, size = 40, normalized size = 0.66

$$\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{\coth(x)+1}}{2}\right)}{8}-\frac{\frac{\coth(x)}{6}+\frac{(\coth(x)+1)^2}{4}+\frac{11}{30}}{(\coth(x)+1)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x) + 1)^(5/2),x)
```

```
[Out] (2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2))/8 - (coth(x)/6 + (coth(x)
+ 1)^2/4 + 11/30)/(coth(x) + 1)^(5/2)
```

3.77 $\int (a+b\coth(c+dx))^5\,dx$

Optimal. Leaf size=142

$$a(a^4+10a^2b^2+5b^4)\,x-\frac{4ab^2(a^2+b^2)\coth(c+dx)}{d}-\frac{b(3a^2+b^2)\,(a+b\coth(c+dx))^2}{2d}-\frac{2ab(a+b\coth(c+d}{3d}$$

```
[Out] a*(a^4+10*a^2*b^2+5*b^4)*x-4*a*b^2*(a^2+b^2)*coth(d*x+c)/d-1/2*b*(3*a^2+b^2
)*(a+b*coth(d*x+c))^2/d-2/3*a*b*(a+b*coth(d*x+c))^3/d-1/4*b*(a+b*coth(d*x+c
))^4/d+b*(5*a^4+10*a^2*b^2+b^4)*ln(sinh(d*x+c))/d
```

Rubi [A]

time = 0.15, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3563, 3609, 3606, 3556}

$$-\frac{b(3a^2+b^2)\,(a+b\coth(c+dx))^2}{2d}-\frac{4ab^2(a^2+b^2)\coth(c+dx)}{d}+\frac{b(5a^4+10a^2b^2+b^4)\log(\sinh(c+dx))}{d}+ax(a^4+10a^2b^2+5b^4)-\frac{b(a+b\coth(c+dx))^4}{4d}-\frac{2ab(a+b\coth(c+dx))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Coth[c + d*x])^5,x]
```

```
[Out] a*(a^4 + 10*a^2*b^2 + 5*b^4)*x - (4*a*b^2*(a^2 + b^2)*Coth[c + d*x])/d - (b
*(3*a^2 + b^2)*(a + b*Coth[c + d*x])^2)/(2*d) - (2*a*b*(a + b*Coth[c + d*x]
)^3)/(3*d) - (b*(a + b*Coth[c + d*x])^4)/(4*d) + (b*(5*a^4 + 10*a^2*b^2 + b
^4)*Log[Sinh[c + d*x]])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3563

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (a+b\coth(c+dx))^5\,dx &= -\frac{b(a+b\coth(c+dx))^4}{4d}+\int (a+b\coth(c+dx))^3\left(a^2+b^2+2ab\coth(c+dx\right.\\ &= -\frac{2ab(a+b\coth(c+dx))^3}{3d}-\frac{b(a+b\coth(c+dx))^4}{4d}+\int (a+b\coth(c+dx))\\ &= -\frac{b(3a^2+b^2)\left(a+b\coth(c+dx)\right)^2}{2d}-\frac{2ab(a+b\coth(c+dx))^3}{3d}-\frac{b(a+b\coth}{4d}\\ &= a\left(a^4+10a^2b^2+5b^4\right)x-\frac{4ab^2(a^2+b^2)\coth(c+dx)}{d}-\frac{b(3a^2+b^2)\left(a+b\coth\right.}{2d}\\ &= a\left(a^4+10a^2b^2+5b^4\right)x-\frac{4ab^2(a^2+b^2)\coth(c+dx)}{d}-\frac{b(3a^2+b^2)\left(a+b\coth\right.}{2d}\end{aligned}$$

Mathematica [A]
time = 0.54, size = 141, normalized size = 0.99

$$-\frac{60ab^2(2a^2+b^2)\coth(c+dx)+6b^3(10a^2+b^2)\coth^2(c+dx)+20ab^4\coth^3(c+dx)+3b^5\coth^4(c+dx)+6(a+b)^5\log(1-\tanh(c+dx))-12b(5a^4+10a^2b^2+b^4)\log(\tanh(c+dx))-6(a-b)^5\log(1+\tanh(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Coth[c + d*x])^5,x]
```

```
[Out] -1/12*(60*a*b^2*(2*a^2 + b^2)*Coth[c + d*x] + 6*b^3*(10*a^2 + b^2)*Coth[c +
 d*x]^2 + 20*a*b^4*Coth[c + d*x]^3 + 3*b^5*Coth[c + d*x]^4 + 6*(a + b)^5*Lo
g[1 - Tanh[c + d*x]] - 12*b*(5*a^4 + 10*a^2*b^2 + b^4)*Log[Tanh[c + d*x]] -
 6*(a - b)^5*Log[1 + Tanh[c + d*x]])/d
```

Maple [A]
time = 0.32, size = 182, normalized size = 1.28

method	result
derivativedivides	$\frac{-10a^3b^2\coth(dx+c)-5a\,b^4\coth(dx+c)-5a^2b^3\left(\coth^2(dx+c)\right)-\frac{5a\,b^4\left(\coth^3(dx+c)\right)}{3}-\frac{b^5\left(\coth^2(dx+c)\right)}{2}-\frac{b^5\left(\coth^4(dx+c)\right)}{4}}{d}$
default	$\frac{-10a^3b^2\coth(dx+c)-5a\,b^4\coth(dx+c)-5a^2b^3\left(\coth^2(dx+c)\right)-\frac{5a\,b^4\left(\coth^3(dx+c)\right)}{3}-\frac{b^5\left(\coth^2(dx+c)\right)}{2}-\frac{b^5\left(\coth^4(dx+c)\right)}{4}}{d}$

risch	$a^5x - 5a^4xb + 10a^3b^2x - 10a^2b^3x + 5a\,b^4x - b^5x - \frac{10b\,a^4c}{d} - \frac{20b^3a^2c}{d} - \frac{2b^5c}{d} - \frac{4b^2(15a^3e^{6dx+6}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*coth(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-10*a^3*b^2*coth(d*x+c)-5*a*b^4*coth(d*x+c)-5*a^2*b^3*coth(d*x+c)^2-5/
3*a*b^4*coth(d*x+c)^3-1/2*b^5*coth(d*x+c)^2-1/4*b^5*coth(d*x+c)^4-1/2*(a^5+
5*a^4*b+10*a^3*b^2+10*a^2*b^3+5*a*b^4+b^5)*ln(coth(d*x+c)-1)+1/2*(a^5-5*a^4
*b+10*a^3*b^2-10*a^2*b^3+5*a*b^4-b^5)*ln(coth(d*x+c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(136) = 272$.
time = 0.28, size = 348, normalized size = 2.45

$$\frac{5}{3}ab^4\left(3x + \frac{3c}{d} - \frac{4\left(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} - 2\right)}{d(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1)}\right) + b^5\left(x + \frac{c}{d} + \frac{\log\left(e^{(-dx-c)}+1\right)}{d} + \frac{\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{4\left(e^{(-2dx-2c)} - e^{(-4dx-4c)} + e^{(-6dx-6c)}\right)}{d(4e^{(-2dx-2c)} - 6e^{(-4dx-4c)} + 4e^{(-6dx-6c)} - e^{(-8dx-8c)} - 1)}\right) + 10a^2b^3\left(x + \frac{c}{d} + \frac{\log\left(e^{(-dx-c)}+1\right)}{d} + \frac{\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2e^{(-2dx-2c)}}{d(2e^{(-2dx-2c)} - e^{(-4dx-4c)} - 1)}\right) + 10a^3b^2\left(x + \frac{c}{d} + \frac{2}{d(e^{(-2dx-2c)} - 1)}\right) + a^5x + \frac{5a^4b\log\left(\sinh\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 5/3*a*b^4*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) - 2)/(d
*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + b^5*(
x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 4*(e^(-2*d*x
- 2*c) - e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) - 6*e^
(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) + 10*a^2*b^3*
(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*d*x
- 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) + 10*a^3*b^2*(x + c
/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + a^5*x + 5*a^4*b*log(sinh(d*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2748 vs. $2(136) = 272$.
time = 0.41, size = 2748, normalized size = 19.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x*cosh(d
*x + c)^8 + 24*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*
x*cosh(d*x + c)*sinh(d*x + c)^7 + 3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^
3 + 5*a*b^4 - b^5)*d*x*sinh(d*x + c)^8 - 12*(5*a^3*b^2 + 5*a^2*b^3 + 5*a*b^
4 + b^5 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*co
sh(d*x + c)^6 - 12*(5*a^3*b^2 + 5*a^2*b^3 + 5*a*b^4 + b^5 - 7*(a^5 - 5*a^4*
b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x*cosh(d*x + c)^2 + (a^5 - 5
```

*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*sinh(d*x + c)^6 + 24
(7(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x*cosh(d*x
+ c)^3 - 3*(5*a^3*b^2 + 5*a^2*b^3 + 5*a*b^4 + b^5 + (a^5 - 5*a^4*b + 10*a^3
*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 60
*a^3*b^2 + 40*a*b^4 + 6*(30*a^3*b^2 + 20*a^2*b^3 + 20*a*b^4 + 2*b^5 + 3*(a^
5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*cosh(d*x + c)^4
 + 6*(35*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x*cosh
(d*x + c)^4 + 30*a^3*b^2 + 20*a^2*b^3 + 20*a*b^4 + 2*b^5 + 3*(a^5 - 5*a^4*b
 + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x - 30*(5*a^3*b^2 + 5*a^2*b^3
 + 5*a*b^4 + b^5 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5
)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 24*(7*(a^5 - 5*a^4*b + 10*a^3*b^2
 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x*cosh(d*x + c)^5 - 10*(5*a^3*b^2 + 5*a^2*
b^3 + 5*a*b^4 + b^5 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 -
b^5)*d*x)*cosh(d*x + c)^3 + (30*a^3*b^2 + 20*a^2*b^3 + 20*a*b^4 + 2*b^5 + 3
*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*cosh(d*x +
c))*sinh(d*x + c)^3 + 3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4
- b^5)*d*x - 4*(45*a^3*b^2 + 15*a^2*b^3 + 25*a*b^4 + 3*b^5 + 3*(a^5 - 5*a^4
*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*cosh(d*x + c)^2 + 4*(21*
(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x*cosh(d*x + c)
^6 - 45*a^3*b^2 - 15*a^2*b^3 - 25*a*b^4 - 3*b^5 - 45*(5*a^3*b^2 + 5*a^2*b^3
 + 5*a*b^4 + b^5 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5
)*d*x)*cosh(d*x + c)^4 - 3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b
^4 - b^5)*d*x + 9*(30*a^3*b^2 + 20*a^2*b^3 + 20*a*b^4 + 2*b^5 + 3*(a^5 - 5*
a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*cosh(d*x + c)^2)*sinh
(d*x + c)^2 + 3*((5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^8 + 8*(5*a^4*b
+ 10*a^2*b^3 + b^5)*cosh(d*x + c)*sinh(d*x + c)^7 + (5*a^4*b + 10*a^2*b^3 +
 b^5)*sinh(d*x + c)^8 - 4*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^6 - 4*
(5*a^4*b + 10*a^2*b^3 + b^5 - 7*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^
2)*sinh(d*x + c)^6 + 8*(7*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^3 - 3*
(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c))*sinh(d*x + c)^5 + 5*a^4*b + 10*
a^2*b^3 + b^5 + 6*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^4 + 2*(15*a^4*
b + 30*a^2*b^3 + 3*b^5 + 35*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^4 -
30*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(5*
a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^5 - 10*(5*a^4*b + 10*a^2*b^3 + b^5)
*cosh(d*x + c)^3 + 3*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c))*sinh(d*x +
 c)^3 - 4*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^2 + 4*(7*(5*a^4*b + 10
*a^2*b^3 + b^5)*cosh(d*x + c)^6 - 5*a^4*b - 10*a^2*b^3 - b^5 - 15*(5*a^4*b
+ 10*a^2*b^3 + b^5)*cosh(d*x + c)^4 + 9*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d
*x + c)^2)*sinh(d*x + c)^2 + 8*((5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^
7 - 3*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^5 + 3*(5*a^4*b + 10*a^2*b^
3 + b^5)*cosh(d*x + c)^3 - (5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c))*sinh
(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 8*(3*(a^5
 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x*cosh(d*x + c)^7 -
 9*(5*a^3*b^2 + 5*a^2*b^3 + 5*a*b^4 + b^5 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 1
0*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*cosh(d*x + c)^5 + 3*(30*a^3*b^2 + 20*a^2*b^

```
3 + 20*a*b^4 + 2*b^5 + 3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4
 - b^5)*d*x)*cosh(d*x + c)^3 - (45*a^3*b^2 + 15*a^2*b^3 + 25*a*b^4 + 3*b^5
+ 3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*cosh(d*x
 + c))*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^
7 + d*sinh(d*x + c)^8 - 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 - d)*s
inh(d*x + c)^6 + 8*(7*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^
5 + 6*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 - 30*d*cosh(d*x + c)^2 +
3*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 - 10*d*cosh(d*x + c)^3 + 3*d*
cosh(d*x + c))*sinh(d*x + c)^3 - 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)
^6 - 15*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 8*(d
*cosh(d*x + c)^7 - 3*d*cosh(d*x + c)^5 + 3*d*cosh(d*x + c)^3 - d*cosh(d*x +
 c))*sinh(d*x + c) + d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 641 vs. $2(133) = 266$.
time = 2.36, size = 641, normalized size = 4.51

$$\begin{cases} -\frac{a^5\log(-e^{-dx})}{d} - \frac{5a^4b\log(-e^{-dx})\coth(dx+\log(-e^{-dx}))}{d} - \frac{10a^3b^2\log(-e^{-dx})\coth^2(dx+\log(-e^{-dx}))}{d} - \frac{10a^2b^3\log(-e^{-dx})\coth^3(dx+\log(-e^{-dx}))}{d} - \frac{5ab^4\log(-e^{-dx})\coth^4(dx+\log(-e^{-dx}))}{d} - \frac{b^5\log(-e^{-dx})\coth^5(dx+\log(-e^{-dx}))}{d} & \text{for } c = \log(-e^{-dx}) \\ -\frac{a^5\log(e^{-dx})}{d} - \frac{5a^4b\log(e^{-dx})\coth(dx+\log(e^{-dx}))}{d} - \frac{10a^3b^2\log(e^{-dx})\coth^2(dx+\log(e^{-dx}))}{d} - \frac{10a^2b^3\log(e^{-dx})\coth^3(dx+\log(e^{-dx}))}{d} - \frac{5ab^4\log(e^{-dx})\coth^4(dx+\log(e^{-dx}))}{d} - \frac{b^5\log(e^{-dx})\coth^5(dx+\log(e^{-dx}))}{d} & \text{for } c = \log(e^{-dx}) \\ x(a+b\coth(c))^5 & \text{for } d = 0 \\ a^5x + 5a^4bx - \frac{5a^4b\log(\tanh(c+dx)+1)}{d} + \frac{5a^4b\log(\tanh(c+dx))}{d} + 10a^3b^2x - \frac{10a^3b^2}{d\tanh(c+dx)} + 10a^2b^3x - \frac{10a^2b^3\log(\tanh(c+dx)+1)}{d} + \frac{10a^2b^3\log(\tanh(c+dx))}{d} - \frac{5a^2b^3}{d\tanh^2(c+dx)} + 5ab^4x - \frac{5ab^4}{d\tanh(c+dx)} - \frac{5ab^4}{3d\tanh^3(c+dx)} + b^5x - \frac{b^5\log(\tanh(c+dx)+1)}{d} + \frac{b^5\log(\tanh(c+dx))}{d} - \frac{b^5}{2d\tanh^2(c+dx)} - \frac{b^5}{4d\tanh^4(c+dx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))**5,x)
```

```
[Out] Piecewise((-a**5*log(-exp(-d*x))/d - 5*a**4*b*log(-exp(-d*x))*coth(d*x + lo
g(-exp(-d*x)))/d - 10*a**3*b**2*log(-exp(-d*x))*coth(d*x + log(-exp(-d*x)))
**2/d - 10*a**2*b**3*log(-exp(-d*x))*coth(d*x + log(-exp(-d*x)))**3/d - 5*a
*b**4*log(-exp(-d*x))*coth(d*x + log(-exp(-d*x)))**4/d - b**5*log(-exp(-d*x
))*coth(d*x + log(-exp(-d*x)))**5/d, Eq(c, log(-exp(-d*x)))), (-a**5*log(ex
p(-d*x))/d - 5*a**4*b*log(exp(-d*x))*coth(d*x + log(exp(-d*x)))/d - 10*a**3
*b**2*log(exp(-d*x))*coth(d*x + log(exp(-d*x)))**2/d - 10*a**2*b**3*log(exp
(-d*x))*coth(d*x + log(exp(-d*x)))**3/d - 5*a*b**4*log(exp(-d*x))*coth(d*x
+ log(exp(-d*x)))**4/d - b**5*log(exp(-d*x))*coth(d*x + log(exp(-d*x)))**5/
d, Eq(c, log(exp(-d*x)))), (x*(a + b*coth(c))**5, Eq(d, 0)), (a**5*x + 5*a*
*4*b*x - 5*a**4*b*log(tanh(c + d*x) + 1)/d + 5*a**4*b*log(tanh(c + d*x))/d
+ 10*a**3*b**2*x - 10*a**3*b**2/(d*tanh(c + d*x)) + 10*a**2*b**3*x - 10*a**
2*b**3*log(tanh(c + d*x) + 1)/d + 10*a**2*b**3*log(tanh(c + d*x))/d - 5*a**
2*b**3/(d*tanh(c + d*x)**2) + 5*a*b**4*x - 5*a*b**4/(d*tanh(c + d*x)) - 5*a
*b**4/(3*d*tanh(c + d*x)**3) + b**5*x - b**5*log(tanh(c + d*x) + 1)/d + b**
5*log(tanh(c + d*x))/d - b**5/(2*d*tanh(c + d*x)**2) - b**5/(4*d*tanh(c + d
*x)**4), True))
```

Giac [A]
time = 0.42, size = 226, normalized size = 1.59

$$\frac{3\,(a^5 - 5\,a^4b + 10\,a^3b^2 - 10\,a^2b^3 + 5\,ab^4 - b^5)(dx + c) + 3\,(5\,a^4b + 10\,a^2b^3 + b^5)\log\left(\left|e^{(2\,dx+2\,c)} - 1\right|\right) + \frac{4\,\left(15\,a^3b^2+10\,ab^4-3\,(5\,a^3b^2+5\,a^2b^3+5\,ab^4+b^5)e^{(6\,dx+6\,c)}+3\,(15\,a^3b^2+10\,a^2b^3+10\,ab^4+b^5)e^{(4\,dx+4\,c)}-(45\,a^3b^2+15\,a^2b^3+25\,ab^4+3\,b^5)e^{(2\,dx+2\,c)}\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^4}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/3*(3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*(d*x + c)
+ 3*(5*a^4*b + 10*a^2*b^3 + b^5)*log(abs(e^(2*d*x + 2*c) - 1)) + 4*(15*a^3*
b^2 + 10*a*b^4 - 3*(5*a^3*b^2 + 5*a^2*b^3 + 5*a*b^4 + b^5)*e^(6*d*x + 6*c)
+ 3*(15*a^3*b^2 + 10*a^2*b^3 + 10*a*b^4 + b^5)*e^(4*d*x + 4*c) - (45*a^3*b^
2 + 15*a^2*b^3 + 25*a*b^4 + 3*b^5)*e^(2*d*x + 2*c))/(e^(2*d*x + 2*c) - 1)^4
)/d
```

Mupad [B]
time = 1.28, size = 244, normalized size = 1.72

$$x\,{\left(a-b\right)}^5-\frac{4\,\left(5\,a^3\,b^2+5\,a^2\,b^3+5\,a\,b^4+b^5\right)}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}+\frac{\ln\left({\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}-1\right)\,\left(5\,a^4\,b+10\,a^2\,b^3+b^5\right)}{d}-\frac{4\,\left(5\,a^2\,b^3+5\,a\,b^4+2\,b^5\right)}{d\,\left({\mathrm{e}}^{4\,c+4\,d\,x}-2\,{\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}-\frac{4\,b^5}{d\,\left(6\,{\mathrm{e}}^{4\,c+4\,d\,x}-4\,{\mathrm{e}}^{2\,c+2\,d\,x}-4\,{\mathrm{e}}^{6\,c+6\,d\,x}+{\mathrm{e}}^{8\,c+8\,d\,x}+1\right)}-\frac{8\,\left(3\,b^5+5\,a\,b^4\right)}{3\,d\,\left(3\,{\mathrm{e}}^{2\,c+2\,d\,x}-3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*coth(c + d*x))^5,x)
```

```
[Out] x*(a - b)^5 - (4*(5*a*b^4 + b^5 + 5*a^2*b^3 + 5*a^3*b^2))/(d*(exp(2*c + 2*d
*x) - 1)) + (log(exp(2*c)*exp(2*d*x) - 1)*(5*a^4*b + b^5 + 10*a^2*b^3))/d -
 (4*(5*a*b^4 + 2*b^5 + 5*a^2*b^3))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x
) + 1)) - (4*b^5)/(d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c +
 6*d*x) + exp(8*c + 8*d*x) + 1)) - (8*(5*a*b^4 + 3*b^5))/(3*d*(3*exp(2*c +
2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1))
```

3.78 $\int (a+b\coth(c+dx))^4\,dx$

Optimal. Leaf size=101

$$\left(a^4+6a^2b^2+b^4\right)x-\frac{b^2(3a^2+b^2)\coth(c+dx)}{d}-\frac{ab(a+b\coth(c+dx))^2}{d}-\frac{b(a+b\coth(c+dx))^3}{3d}+\frac{4ab(a^2+b^2)\log(\sinh(c+dx))}{d}$$

```
[Out] (a^4+6*a^2*b^2+b^4)*x-b^2*(3*a^2+b^2)*coth(d*x+c)/d-a*b*(a+b*coth(d*x+c))^2
/d-1/3*b*(a+b*coth(d*x+c))^3/d+4*a*b*(a^2+b^2)*ln(sinh(d*x+c))/d
```

Rubi [A]
time = 0.09, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3563, 3609, 3606, 3556}

$$-\frac{b^2(3a^2+b^2)\coth(c+dx)}{d}+\frac{4ab(a^2+b^2)\log(\sinh(c+dx))}{d}+x\left(a^4+6a^2b^2+b^4\right)-\frac{b(a+b\coth(c+dx))^3}{3d}-\frac{ab(a+b\coth(c+dx))^2}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Coth[c + d*x])^4,x]
```

```
[Out] (a^4 + 6*a^2*b^2 + b^4)*x - (b^2*(3*a^2 + b^2)*Coth[c + d*x])/d - (a*b*(a +
 b*Coth[c + d*x])^2)/d - (b*(a + b*Coth[c + d*x])^3)/(3*d) + (4*a*b*(a^2 +
b^2)*Log[Sinh[c + d*x]])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3563

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
```

```
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (a+b\coth(c+dx))^4\,dx &= -\frac{b(a+b\coth(c+dx))^3}{3d}+\int (a+b\coth(c+dx))^2\left(a^2+b^2+2ab\coth(c+dx\right.\\ &= -\frac{ab(a+b\coth(c+dx))^2}{d}-\frac{b(a+b\coth(c+dx))^3}{3d}+\int (a+b\coth(c+dx))\,(\\ &= \left(a^4+6a^2b^2+b^4\right)x-\frac{b^2(3a^2+b^2)\coth(c+dx)}{d}-\frac{ab(a+b\coth(c+dx))^2}{d}-\\ &= \left(a^4+6a^2b^2+b^4\right)x-\frac{b^2(3a^2+b^2)\coth(c+dx)}{d}-\frac{ab(a+b\coth(c+dx))^2}{d}-\end{aligned}$$

Mathematica [A]
time = 0.61, size = 109, normalized size = 1.08

$$-\frac{6b^2(6a^2+b^2)\coth(c+dx)+12ab^3\coth^2(c+dx)+2b^4\coth^3(c+dx)+3(a+b)^4\log(1-\tanh(c+dx))-24ab(a^2+b^2)\log(\tanh(c+dx))-3(a-b)^4\log(1+\tanh(c+dx))}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Coth[c + d*x])^4,x]
```

```
[Out] -1/6*(6*b^2*(6*a^2 + b^2)*Coth[c + d*x] + 12*a*b^3*Coth[c + d*x]^2 + 2*b^4*
Coth[c + d*x]^3 + 3*(a + b)^4*Log[1 - Tanh[c + d*x]] - 24*a*b*(a^2 + b^2)*L
og[Tanh[c + d*x]] - 3*(a - b)^4*Log[1 + Tanh[c + d*x]])/d
```

Maple [A]
time = 0.36, size = 134, normalized size = 1.33

method	result
derivativedivides	$\frac{-\frac{b^4\left(\coth^3(dx+c)\right)}{3}-2a\,b^3\left(\coth^2(dx+c)\right)-6a^2b^2\coth(dx+c)-b^4\coth(dx+c)-\frac{\left(a^4+4a^3b+6a^2b^2+4a\,b^3+b^4\right)\ln(\coth(dx+c)-}{2}}{d}$
default	$\frac{-\frac{b^4\left(\coth^3(dx+c)\right)}{3}-2a\,b^3\left(\coth^2(dx+c)\right)-6a^2b^2\coth(dx+c)-b^4\coth(dx+c)-\frac{\left(a^4+4a^3b+6a^2b^2+4a\,b^3+b^4\right)\ln(\coth(dx+c)-}{2}}{d}$
risch	$x\,a^4-4a^3bx+6a^2b^2x-4a\,b^3x+b^4x-\frac{8a^3bc}{d}-\frac{8a\,b^3c}{d}-\frac{4b^2\left(9a^2\mathrm{e}^{4dx+4c}+6ab\,\mathrm{e}^{4dx+4c}+3b^2\mathrm{e}^{4dx+4c}\right.}{3d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*coth(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*b^4*coth(d*x+c)^3-2*a*b^3*coth(d*x+c)^2-6*a^2*b^2*coth(d*x+c)-b^4
*coth(d*x+c)-1/2*(a^4+4*a^3*b+6*a^2*b^2+4*a*b^3+b^4)*ln(coth(d*x+c)-1)+1/2*
(a^4-4*a^3*b+6*a^2*b^2-4*a*b^3+b^4)*ln(coth(d*x+c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(99) = 198$.
time = 0.27, size = 219, normalized size = 2.17

$$\frac{1}{3}\,b^4\left(3\,x+\frac{3\,c}{d}-\frac{4\,\left(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}-2\right)}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}\right)+4\,ab^3\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)+6\,a^2b^2\left(x+\frac{c}{d}+\frac{2}{d(e^{(-2\,dx-2\,c)}-1)}\right)+a^4x+\frac{4\,a^3b\log\left(\sinh\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3*b^4*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) - 2)/(d*(
3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + 4*a*b^3
*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*d*x
 - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) + 6*a^2*b^2*(x + c
/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + a^4*x + 4*a^3*b*log(sinh(d*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1396 vs. $2(99) = 198$.
time = 0.38, size = 1396, normalized size = 13.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x*cosh(d*x + c)^6 + 18
*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x*cosh(d*x + c)*sinh(d*x + c
)^5 + 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x*sinh(d*x + c)^6 - 3
*(12*a^2*b^2 + 8*a*b^3 + 4*b^4 + 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b
^4)*d*x)*cosh(d*x + c)^4 + 3*(15*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4
)*d*x*cosh(d*x + c)^2 - 12*a^2*b^2 - 8*a*b^3 - 4*b^4 - 3*(a^4 - 4*a^3*b + 6
*a^2*b^2 - 4*a*b^3 + b^4)*d*x)*sinh(d*x + c)^4 - 36*a^2*b^2 - 8*b^4 + 12*(5
*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x*cosh(d*x + c)^3 - (12*a^2*
b^2 + 8*a*b^3 + 4*b^4 + 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x)*
cosh(d*x + c))*sinh(d*x + c)^3 - 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b
^4)*d*x + 3*(24*a^2*b^2 + 8*a*b^3 + 4*b^4 + 3*(a^4 - 4*a^3*b + 6*a^2*b^2 -
4*a*b^3 + b^4)*d*x)*cosh(d*x + c)^2 + 3*(15*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*
a*b^3 + b^4)*d*x*cosh(d*x + c)^4 + 24*a^2*b^2 + 8*a*b^3 + 4*b^4 + 3*(a^4 -
4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x - 6*(12*a^2*b^2 + 8*a*b^3 + 4*b^4
+ 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x)*cosh(d*x + c)^2)*sinh(
d*x + c)^2 + 12*((a^3*b + a*b^3)*cosh(d*x + c)^6 + 6*(a^3*b + a*b^3)*cosh(d
*x + c)*sinh(d*x + c)^5 + (a^3*b + a*b^3)*sinh(d*x + c)^6 - 3*(a^3*b + a*b^
3)*cosh(d*x + c)^4 - 3*(a^3*b + a*b^3 - 5*(a^3*b + a*b^3)*cosh(d*x + c)^2)*
```

```
sinh(d*x + c)^4 - a^3*b - a*b^3 + 4*(5*(a^3*b + a*b^3)*cosh(d*x + c)^3 - 3*
(a^3*b + a*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a^3*b + a*b^3)*cosh(d*x
 + c)^2 + 3*(5*(a^3*b + a*b^3)*cosh(d*x + c)^4 + a^3*b + a*b^3 - 6*(a^3*b +
 a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 6*((a^3*b + a*b^3)*cosh(d*x + c)
^5 - 2*(a^3*b + a*b^3)*cosh(d*x + c)^3 + (a^3*b + a*b^3)*cosh(d*x + c))*sin
h(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 6*(3*(a^
4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x*cosh(d*x + c)^5 - 2*(12*a^2*b^
2 + 8*a*b^3 + 4*b^4 + 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x)*co
sh(d*x + c)^3 + (24*a^2*b^2 + 8*a*b^3 + 4*b^4 + 3*(a^4 - 4*a^3*b + 6*a^2*b^
2 - 4*a*b^3 + b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^6 +
6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*x + c)^6 - 3*d*cosh(d*x + c)^4
 + 3*(5*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 - 3
*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*d*cosh(d*x + c)^2 + 3*(5*d*cosh(d*x +
 c)^4 - 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 6*(d*cosh(d*x + c)^5 - 2
*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) - d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(92) = 184$.
time = 1.45, size = 488, normalized size = 4.83

$$\begin{cases} x(a+b\coth(c))^4 & \text{for } d=0 \\ -\frac{a^4\log(-e^{-dx})}{d} - \frac{4a^3b\log(-e^{-dx})\coth(dx+\log(-e^{-dx}))}{d} - \frac{6a^2b^2\log(-e^{-dx})\coth^2(dx+\log(-e^{-dx}))}{d} - \frac{4ab^3\log(-e^{-dx})\coth^3(dx+\log(-e^{-dx}))}{d} - \frac{b^4\log(-e^{-dx})\coth^4(dx+\log(-e^{-dx}))}{d} & \text{for } c=\log(-e^{-dx}) \\ -\frac{a^4\log(e^{-dx})}{d} - \frac{4a^3b\log(e^{-dx})\coth(dx+\log(e^{-dx}))}{d} - \frac{6a^2b^2\log(e^{-dx})\coth^2(dx+\log(e^{-dx}))}{d} - \frac{4ab^3\log(e^{-dx})\coth^3(dx+\log(e^{-dx}))}{d} - \frac{b^4\log(e^{-dx})\coth^4(dx+\log(e^{-dx}))}{d} & \text{for } c=\log(e^{-dx}) \\ a^4x+4a^3bx-\frac{4a^3b\log(\tanh(c+dx)+1)}{d}+\frac{4a^3b\log(\tanh(c+dx))}{d}+6a^2b^2x-\frac{6a^2b^2}{d\tanh(c+dx)}+4ab^3x-\frac{4ab^3\log(\tanh(c+dx)+1)}{d}+\frac{4ab^3\log(\tanh(c+dx))}{d}-\frac{2ab^3}{d\tanh^2(c+dx)}+b^4x-\frac{b^4}{d\tanh(c+dx)}-\frac{b^4}{3d\tanh^3(c+dx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))**4,x)
```

```
[Out] Piecewise((x*(a + b*coth(c))**4, Eq(d, 0)), (-a**4*log(-exp(-d*x))/d - 4*a*
*3*b*log(-exp(-d*x))*coth(d*x + log(-exp(-d*x)))/d - 6*a**2*b**2*log(-exp(-
d*x))*coth(d*x + log(-exp(-d*x)))**2/d - 4*a*b**3*log(-exp(-d*x))*coth(d*x
+ log(-exp(-d*x)))**3/d - b**4*log(-exp(-d*x))*coth(d*x + log(-exp(-d*x)))*
*4/d, Eq(c, log(-exp(-d*x)))), (-a**4*log(exp(-d*x))/d - 4*a**3*b*log(exp(-
d*x))*coth(d*x + log(exp(-d*x)))/d - 6*a**2*b**2*log(exp(-d*x))*coth(d*x +
log(exp(-d*x)))**2/d - 4*a*b**3*log(exp(-d*x))*coth(d*x + log(exp(-d*x)))**
3/d - b**4*log(exp(-d*x))*coth(d*x + log(exp(-d*x)))**4/d, Eq(c, log(exp(-d
*x)))), (a**4*x + 4*a**3*b*x - 4*a**3*b*log(tanh(c + d*x) + 1)/d + 4*a**3*b
*log(tanh(c + d*x))/d + 6*a**2*b**2*x - 6*a**2*b**2/(d*tanh(c + d*x)) + 4*a
*b**3*x - 4*a*b**3*log(tanh(c + d*x) + 1)/d + 4*a*b**3*log(tanh(c + d*x))/d
 - 2*a*b**3/(d*tanh(c + d*x)**2) + b**4*x - b**4/(d*tanh(c + d*x)) - b**4/(
3*d*tanh(c + d*x)**3), True))
```

Giac [A]
time = 0.44, size = 153, normalized size = 1.51

$$\frac{3\left(a^4-4a^3b+6a^2b^2-4ab^3+b^4\right)(dx+c)+12\left(a^3b+ab^3\right)\log\left(\left|e^{(2dx+2c)}-1\right|\right)-\frac{4\left(9a^2b^2+2b^4+3\left(3a^2b^2+2ab^3+b^4\right)e^{(4dx+4c)}-3\left(6a^2b^2+2ab^3+b^4\right)e^{(2dx+2c)}\right)}{\left(e^{(2dx+2c)}-1\right)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*(d*x + c) + 12*(a^3*b +
a*b^3)*log(abs(e^(2*d*x + 2*c) - 1)) - 4*(9*a^2*b^2 + 2*b^4 + 3*(3*a^2*b^2
+ 2*a*b^3 + b^4)*e^(4*d*x + 4*c) - 3*(6*a^2*b^2 + 2*a*b^3 + b^4)*e^(2*d*x +
 2*c))/(e^(2*d*x + 2*c) - 1)^3)/d
```

Mupad [B]
time = 1.24, size = 158, normalized size = 1.56

$$x\,{\left(a-b\right)}^4-\frac{4\,\left(3\,a^2\,b^2+2\,a\,b^3+b^4\right)}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}-\frac{4\,\left(b^4+2\,a\,b^3\right)}{d\,\left({\mathrm{e}}^{4\,c+4\,d\,x}-2\,{\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}+\frac{\ln\left({\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}-1\right)\,\left(4\,a^3\,b+4\,a\,b^3\right)}{d}-\frac{8\,b^4}{3\,d\,\left(3\,{\mathrm{e}}^{2\,c+2\,d\,x}-3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*coth(c + d*x))^4,x)
```

```
[Out] x*(a - b)^4 - (4*(2*a*b^3 + b^4 + 3*a^2*b^2))/(d*(exp(2*c + 2*d*x) - 1)) -
(4*(2*a*b^3 + b^4))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) + (log(
exp(2*c)*exp(2*d*x) - 1)*(4*a*b^3 + 4*a^3*b))/d - (8*b^4)/(3*d*(3*exp(2*c +
 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1))
```

3.79 $\int (a+b\coth(c+dx))^3\,dx$

Optimal. Leaf size=69

$$a\left(a^2+3b^2\right)x-\frac{2ab^2\coth(c+dx)}{d}-\frac{b(a+b\coth(c+dx))^2}{2d}+\frac{b(3a^2+b^2)\log(\sinh(c+dx))}{d}$$

```
[Out] a*(a^2+3*b^2)*x-2*a*b^2*coth(d*x+c)/d-1/2*b*(a+b*coth(d*x+c))^2/d+b*(3*a^2+
b^2)*ln(sinh(d*x+c))/d
```

Rubi [A]
time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3563, 3606, 3556}

$$\frac{b(3a^2+b^2)\log(\sinh(c+dx))}{d}+ax\left(a^2+3b^2\right)-\frac{2ab^2\coth(c+dx)}{d}-\frac{b(a+b\coth(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Coth[c + d*x])^3,x]
```

```
[Out] a*(a^2 + 3*b^2)*x - (2*a*b^2*Coth[c + d*x])/d - (b*(a + b*Coth[c + d*x])^2)
/(2*d) + (b*(3*a^2 + b^2)*Log[Sinh[c + d*x]])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3563

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rubi steps

$$\int (a+b\coth(c+dx))^3\,dx = -\frac{b(a+b\coth(c+dx))^2}{2d} + \int (a+b\coth(c+dx))\left(a^2+b^2+2ab\coth(c+dx)\right)$$
$$= a\left(a^2+3b^2\right)x - \frac{2ab^2\coth(c+dx)}{d} - \frac{b(a+b\coth(c+dx))^2}{2d} + \left(b\left(3a^2+b^2\right)\right)\int$$
$$= a\left(a^2+3b^2\right)x - \frac{2ab^2\coth(c+dx)}{d} - \frac{b(a+b\coth(c+dx))^2}{2d} + \frac{b(3a^2+b^2)\log(s}{d}$$

Mathematica [A]
time = 0.28, size = 86, normalized size = 1.25

$$-\frac{6ab^2\coth(c+dx)+b^3\coth^2(c+dx)+(a+b)^3\log(1-\tanh(c+dx))-2b(3a^2+b^2)\log(\tanh(c+dx))-(a-b)^3\log(1+\tanh(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Coth[c + d*x])^3,x]
```

```
[Out] -1/2*(6*a*b^2*Coth[c + d*x] + b^3*Coth[c + d*x]^2 + (a + b)^3*Log[1 - Tanh[
c + d*x]] - 2*b*(3*a^2 + b^2)*Log[Tanh[c + d*x]] - (a - b)^3*Log[1 + Tanh[c
 + d*x]])/d
```

Maple [A]
time = 0.37, size = 93, normalized size = 1.35

method	result
derivativedivides	$\frac{-\frac{b^3\left(\coth^2(dx+c)\right)}{2}-3a\,b^2\coth(dx+c)-\frac{\left(a^3+3a^2b+3a\,b^2+b^3\right)\ln(\coth(dx+c)-1)}{2}+\frac{\left(a^3-3a^2b+3a\,b^2-b^3\right)\ln(\coth(dx+c)+1)}{2}}{d}$
default	$\frac{-\frac{b^3\left(\coth^2(dx+c)\right)}{2}-3a\,b^2\coth(dx+c)-\frac{\left(a^3+3a^2b+3a\,b^2+b^3\right)\ln(\coth(dx+c)-1)}{2}+\frac{\left(a^3-3a^2b+3a\,b^2-b^3\right)\ln(\coth(dx+c)+1)}{2}}{d}$
risch	$a^3x-3a^2bx+3a\,b^2x-b^3x-\frac{6a^2bc}{d}-\frac{2b^3c}{d}-\frac{2b^2\left(3a\,\mathrm{e}^{2dx+2c}+b\,\mathrm{e}^{2dx+2c}-3a\right)}{d\left(\mathrm{e}^{2dx+2c}-1\right)^2}+\frac{3b\ln\left(\mathrm{e}^{2dx+2c}-1\right)a^2}{d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*coth(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*b^3*coth(d*x+c)^2-3*a*b^2*coth(d*x+c)-1/2*(a^3+3*a^2*b+3*a*b^2+b^
3)*ln(coth(d*x+c)-1)+1/2*(a^3-3*a^2*b+3*a*b^2-b^3)*ln(coth(d*x+c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(67) = 134.
time = 0.28, size = 136, normalized size = 1.97

$$b^3\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)+3\,ab^2\left(x+\frac{c}{d}+\frac{2}{d(e^{(-2\,dx-2\,c)}-1)}\right)+a^3x+\frac{3\,a^2b\log\left(\sinh\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] b^3*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*
d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) + 3*a*b^2*(x +
c/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + a^3*x + 3*a^2*b*log(sinh(d*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 654 vs. $2(67) = 134$.
time = 0.38, size = 654, normalized size = 9.48

$$\frac{(a^3 - 3a^2b + 3ab^2 - b^3)dx\cosh(dx+c)^4 + 4(a^3 - 3a^2b + 3ab^2 - b^3)dx\cosh(dx+c)\sinh(dx+c)^3 + (a^3 - 3a^2b + 3ab^2 - b^3)dx\sinh(dx+c)^4 + 6ab^2 + (a^3 - 3a^2b + 3ab^2 - b^3)dx - 2(3ab^2 + b^3 + (a^3 - 3a^2b + 3ab^2 - b^3)dx)\cosh(dx+c)^2 + 2(3(a^3 - 3a^2b + 3ab^2 - b^3)dx\cosh(dx+c)^2 - 3ab^2 - b^3 - (a^3 - 3a^2b + 3ab^2 - b^3)dx)\sinh(dx+c)^2 + ((3a^2b + b^3)\cosh(dx+c)^4 + 4(3a^2b + b^3)\cosh(dx+c)\sinh(dx+c)^3 + (3a^2b + b^3)\sinh(dx+c)^4 + 3a^2b + b^3 - 2(3a^2b + b^3)\cosh(dx+c)^2 - 2(3a^2b + b^3 - 3(3a^2b + b^3)\cosh(dx+c)^2)\sinh(dx+c)^2 + 4((3a^2b + b^3)\cosh(dx+c)^3 - (3a^2b + b^3)\cosh(dx+c))\sinh(dx+c))\log\left(\frac{2\sinh(dx+c)}{\cosh(dx+c) - \sinh(dx+c)}\right) + 4((a^3 - 3a^2b + 3ab^2 - b^3)dx\cosh(dx+c)^3 - (3ab^2 + b^3 + (a^3 - 3a^2b + 3ab^2 - b^3)dx)\cosh(dx+c))\sinh(dx+c)}{d\cosh(dx+c)^4 + 4d\cosh(dx+c)\sinh(dx+c)^3 + d\sinh(dx+c)^4 - 2d\cosh(dx+c)^2 + 2(3d\cosh(dx+c)^2 - d)\sinh(dx+c)^2 + 4(d\cosh(dx+c)^3 - d\cosh(dx+c))\sinh(dx+c) + d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] ((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x*cosh(d*x + c)^4 + 4*(a^3 - 3*a^2*b + 3
*a*b^2 - b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 - 3*a^2*b + 3*a*b^2
- b^3)*d*x*sinh(d*x + c)^4 + 6*a*b^2 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x
- 2*(3*a*b^2 + b^3 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x)*cosh(d*x + c)^2 +
 2*(3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x*cosh(d*x + c)^2 - 3*a*b^2 - b^3 -
 (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x)*sinh(d*x + c)^2 + ((3*a^2*b + b^3)*co
sh(d*x + c)^4 + 4*(3*a^2*b + b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^2*b
+ b^3)*sinh(d*x + c)^4 + 3*a^2*b + b^3 - 2*(3*a^2*b + b^3)*cosh(d*x + c)^2
- 2*(3*a^2*b + b^3 - 3*(3*a^2*b + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4
*((3*a^2*b + b^3)*cosh(d*x + c)^3 - (3*a^2*b + b^3)*cosh(d*x + c))*sinh(d*x
 + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*((a^3 - 3*a
^2*b + 3*a*b^2 - b^3)*d*x*cosh(d*x + c)^3 - (3*a*b^2 + b^3 + (a^3 - 3*a^2*b
 + 3*a*b^2 - b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4
*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 - 2*d*cosh(d*x + c)^2
+ 2*(3*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 - d*co
sh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(63) = 126$.
time = 0.97, size = 369, normalized size = 5.35

$$\begin{cases} x(a + b\coth(c))^3 & \text{for}\ d = 0 \\ -\frac{a^3\log(-e^{-dx})}{d} - \frac{3a^2b\log(-e^{-dx})\coth(dx + \log(-e^{-dx}))}{d} - \frac{3ab^2\log(-e^{-dx})\coth^2(dx + \log(-e^{-dx}))}{d} - \frac{b^3\log(-e^{-dx})\coth^3(dx + \log(-e^{-dx}))}{d} & \text{for}\ c = \log(-e^{-dx}) \\ -\frac{a^3\log(e^{-dx})}{d} - \frac{3a^2b\log(e^{-dx})\coth(dx + \log(e^{-dx}))}{d} - \frac{3ab^2\log(e^{-dx})\coth^2(dx + \log(e^{-dx}))}{d} - \frac{b^3\log(e^{-dx})\coth^3(dx + \log(e^{-dx}))}{d} & \text{for}\ c = \log(e^{-dx}) \\ a^3x + 3a^2bx - \frac{3a^2b\log(\tanh(c+dx)+1)}{d} + \frac{3a^2b\log(\tanh(c+dx))}{d} + 3ab^2x - \frac{3ab^2}{d\tanh(c+dx)} + b^3x - \frac{b^3\log(\tanh(c+dx)+1)}{d} + \frac{b^3\log(\tanh(c+dx))}{d} - \frac{b^3}{2d\tanh^2(c+dx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))**3,x)
```

```
[Out] Piecewise((x*(a + b*coth(c))**3, Eq(d, 0)), (-a**3*log(-exp(-d*x))/d - 3*a*
*2*b*log(-exp(-d*x))*coth(d*x + log(-exp(-d*x)))/d - 3*a*b**2*log(-exp(-d*x
```

```
))*coth(d*x + log(-exp(-d*x)))**2/d - b**3*log(-exp(-d*x))*coth(d*x + log(-
exp(-d*x)))**3/d, Eq(c, log(-exp(-d*x)))), (-a**3*log(exp(-d*x))/d - 3*a**2
*b*log(exp(-d*x))*coth(d*x + log(exp(-d*x)))/d - 3*a*b**2*log(exp(-d*x))*co
th(d*x + log(exp(-d*x)))**2/d - b**3*log(exp(-d*x))*coth(d*x + log(exp(-d*x
)))**3/d, Eq(c, log(exp(-d*x)))), (a**3*x + 3*a**2*b*x - 3*a**2*b*log(tanh(
c + d*x) + 1)/d + 3*a**2*b*log(tanh(c + d*x))/d + 3*a*b**2*x - 3*a*b**2/(d*
tanh(c + d*x)) + b**3*x - b**3*log(tanh(c + d*x) + 1)/d + b**3*log(tanh(c +
 d*x))/d - b**3/(2*d*tanh(c + d*x)**2), True))
```

Giac [A]
time = 0.44, size = 99, normalized size = 1.43

$$\frac{(a^3-3\,a^2b+3\,ab^2-b^3)(dx+c)+(3\,a^2b+b^3)\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)+\frac{2\,(3\,ab^2-(3\,ab^2+b^3)e^{(2\,dx+2\,c)})}{\left(e^{(2\,dx+2\,c)}-1\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^3,x, algorithm="giac")
[Out] ((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(d*x + c) + (3*a^2*b + b^3)*log(abs(e^(2*d
*x + 2*c) - 1)) + 2*(3*a*b^2 - (3*a*b^2 + b^3)*e^(2*d*x + 2*c))/(e^(2*d*x +
 2*c) - 1)^2)/d
```

Mupad [B]
time = 0.11, size = 97, normalized size = 1.41

$$x\,(a-b)^3-\frac{2\,(b^3+3\,a\,b^2)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}-\frac{2\,b^3}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}+\frac{\ln\left(\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}-1\right)\,\left(3\,a^2\,b+b^3\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*coth(c + d*x))^3,x)
[Out] x*(a - b)^3 - (2*(3*a*b^2 + b^3))/(d*(exp(2*c + 2*d*x) - 1)) - (2*b^3)/(d*(
exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) + (log(exp(2*c)*exp(2*d*x) - 1)
*(3*a^2*b + b^3))/d
```

3.80 $\int (a+b\coth(c+dx))^2\,dx$

Optimal. Leaf size=38

$$\left(a^2+b^2\right)x-\frac{b^2\coth(c+dx)}{d}+\frac{2ab\log(\sinh(c+dx))}{d}$$

```
[Out] (a^2+b^2)*x-b^2*coth(d*x+c)/d+2*a*b*ln(sinh(d*x+c))/d
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3558, 3556}

$$x\left(a^2+b^2\right)+\frac{2ab\log(\sinh(c+dx))}{d}-\frac{b^2\coth(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Coth[c + d*x])^2,x]
```

```
[Out] (a^2 + b^2)*x - (b^2*Coth[c + d*x])/d + (2*a*b*Log[Sinh[c + d*x]])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3558

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[b^2*(Tan[c + d*x]/d),
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\coth(c+dx))^2\,dx &= \left(a^2+b^2\right)x-\frac{b^2\coth(c+dx)}{d}+(2ab)\int \coth(c+dx)\,dx\\ &= \left(a^2+b^2\right)x-\frac{b^2\coth(c+dx)}{d}+\frac{2ab\log(\sinh(c+dx))}{d}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 65, normalized size = 1.71

$$\frac{-2b^2\coth(c+dx)-(a+b)^2\log(1-\tanh(c+dx))+4ab\log(\tanh(c+dx))+(a-b)^2\log(1+\tanh(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Coth[c + d*x])^2,x]
```

```
[Out] (-2*b^2*Coth[c + d*x] - (a + b)^2*Log[1 - Tanh[c + d*x]] + 4*a*b*Log[Tanh[c
 + d*x]] + (a - b)^2*Log[1 + Tanh[c + d*x]])/(2*d)
```

Maple [A]
time = 0.30, size = 61, normalized size = 1.61

method	result	size
derivativedivides	$\frac{-b^{2} \coth \left(d x+c\right)-\frac{\left(a^{2}+2 a b+b^{2}\right) \ln \left(\coth \left(d x+c\right)-1\right)}{2}+\frac{\left(a^{2}-2 a b+b^{2}\right) \ln \left(\coth \left(d x+c\right)+1\right)}{2}}{d}$	61
default	$\frac{-b^{2} \coth \left(d x+c\right)-\frac{\left(a^{2}+2 a b+b^{2}\right) \ln \left(\coth \left(d x+c\right)-1\right)}{2}+\frac{\left(a^{2}-2 a b+b^{2}\right) \ln \left(\coth \left(d x+c\right)+1\right)}{2}}{d}$	61
risch	$a^{2} x-2 a b x+b^{2} x-\frac{4 a b c}{d}-\frac{2 b^{2}}{d \left(\mathrm{e}^{2 d x+2 c}-1\right)}+\frac{2 a b \ln \left(\mathrm{e}^{2 d x+2 c}-1\right)}{d}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*coth(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-b^2*coth(d*x+c)-1/2*(a^2+2*a*b+b^2)*ln(coth(d*x+c)-1)+1/2*(a^2-2*a*b+
b^2)*ln(coth(d*x+c)+1))
```

Maxima [A]
time = 0.27, size = 49, normalized size = 1.29

$$b^{2} {\left(x + \frac{c}{d} + \frac{2}{d {\left(e^{\left(-2 \, d x - 2 \, c\right)} - 1\right)}}\right)} + a^{2} x + \frac{2 \, a b \log\left(\sinh\left(d x + c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] b^2*(x + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + a^2*x + 2*a*b*log(sinh(d*x +
 c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(38) = 76$.
time = 0.35, size = 205, normalized size = 5.39

$$\frac{{\left(a^{2} - 2 \, a b + b^{2}\right)} d x \cosh\left(d x + c\right)^{2} + 2 \, {\left(a^{2} - 2 \, a b + b^{2}\right)} d x \cosh\left(d x + c\right) \sinh\left(d x + c\right) + {\left(a^{2} - 2 \, a b + b^{2}\right)} d x \sinh\left(d x + c\right)^{2} - {\left(a^{2} - 2 \, a b + b^{2}\right)} d x - 2 \, b^{2} + 2 \, {\left(a b \cosh\left(d x + c\right)^{2} + 2 \, a b \cosh\left(d x + c\right) \sinh\left(d x + c\right) + a b \sinh\left(d x + c\right)^{2} - a b\right)} \log\left(\frac{2 \, \sinh\left(d x + c\right)}{\cosh\left(d x + c\right) - \sinh\left(d x + c\right)}\right)}{d \cosh\left(d x + c\right)^{2} + 2 \, d \cosh\left(d x + c\right) \sinh\left(d x + c\right) + d \sinh\left(d x + c\right)^{2} - d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] ((a^2 - 2*a*b + b^2)*d*x*cosh(d*x + c)^2 + 2*(a^2 - 2*a*b + b^2)*d*x*cosh(d
*x + c)*sinh(d*x + c) + (a^2 - 2*a*b + b^2)*d*x*sinh(d*x + c)^2 - (a^2 - 2*
```

```
a*b + b^2)*d*x - 2*b^2 + 2*(a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*x + c)*sinh(
d*x + c) + a*b*sinh(d*x + c)^2 - a*b)*log(2*sinh(d*x + c)/(cosh(d*x + c) -
sinh(d*x + c))))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*s
inh(d*x + c)^2 - d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(34) = 68$.

time = 0.61, size = 236, normalized size = 6.21

$$\begin{cases} x(a+b\coth(c))^2 & \text{for } d=0 \\ -\frac{a^2\log(-e^{-dx})}{d} - \frac{2ab\log(-e^{-dx})\coth(dx+\log(-e^{-dx}))}{d} - \frac{b^2\log(-e^{-dx})\coth^2(dx+\log(-e^{-dx}))}{d} & \text{for } c=\log(-e^{-dx}) \\ -\frac{a^2\log(e^{-dx})}{d} - \frac{2ab\log(e^{-dx})\coth(dx+\log(e^{-dx}))}{d} - \frac{b^2\log(e^{-dx})\coth^2(dx+\log(e^{-dx}))}{d} & \text{for } c=\log(e^{-dx}) \\ a^2x + 2abx - \frac{2ab\log(\tanh(c+dx)+1)}{d} + \frac{2ab\log(\tanh(c+dx))}{d} + b^2x - \frac{b^2}{d\tanh(c+dx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))**2,x)
```

```
[Out] Piecewise((x*(a + b*coth(c))**2, Eq(d, 0)), (-a**2*log(-exp(-d*x))/d - 2*a*
b*log(-exp(-d*x))*coth(d*x + log(-exp(-d*x)))/d - b**2*log(-exp(-d*x))*coth
(d*x + log(-exp(-d*x)))**2/d, Eq(c, log(-exp(-d*x)))), (-a**2*log(exp(-d*x)
)/d - 2*a*b*log(exp(-d*x))*coth(d*x + log(exp(-d*x)))/d - b**2*log(exp(-d*x
))*coth(d*x + log(exp(-d*x)))**2/d, Eq(c, log(exp(-d*x)))), (a**2*x + 2*a*b
*x - 2*a*b*log(tanh(c + d*x) + 1)/d + 2*a*b*log(tanh(c + d*x))/d + b**2*x -
 b**2/(d*tanh(c + d*x)), True))
```

Giac [A]

time = 0.42, size = 57, normalized size = 1.50

$$\frac{2\,ab\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right) + (a^2-2\,ab+b^2)(dx+c) - \frac{2\,b^2}{e^{(2\,dx+2\,c)}-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*a*b*log(abs(e^(2*d*x + 2*c) - 1)) + (a^2 - 2*a*b + b^2)*(d*x + c) - 2*b^
2/(e^(2*d*x + 2*c) - 1))/d
```

Mupad [B]

time = 0.10, size = 51, normalized size = 1.34

$$x\,(a-b)^2 - \frac{2\,b^2}{d\,\left(e^{2\,c+2\,d\,x}-1\right)} + \frac{2\,a\,b\,\ln\left(e^{2\,c}\,e^{2\,d\,x}-1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*coth(c + d*x))^2,x)
```

```
[Out] x*(a - b)^2 - (2*b^2)/(d*(exp(2*c + 2*d*x) - 1)) + (2*a*b*log(exp(2*c)*exp(
2*d*x) - 1))/d
```

3.81 $\int \frac{1}{a+b\coth(c+dx)}\,dx$

Optimal. Leaf size=50

$$\frac{ax}{a^2-b^2}-\frac{b\log(b\cosh(c+dx)+a\sinh(c+dx))}{(a^2-b^2)\,d}$$

```
[Out] a*x/(a^2-b^2)-b*ln(b*cosh(d*x+c)+a*sinh(d*x+c))/(a^2-b^2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3565, 3611}

$$\frac{ax}{a^2-b^2}-\frac{b\log(a\sinh(c+dx)+b\cosh(c+dx))}{d\,(a^2-b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Coth[c + d*x])^(-1),x]
```

```
[Out] (a*x)/(a^2 - b^2) - (b*Log[b*Cosh[c + d*x] + a*Sinh[c + d*x]])/((a^2 - b^2)
*d)
```

Rule 3565

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[a*(x/(a^
2 + b^2)), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\coth(c+dx)}\,dx &= \frac{ax}{a^2-b^2}-\frac{(ib)\int \frac{-ib-ia\coth(c+dx)}{a+b\coth(c+dx)}\,dx}{a^2-b^2}\\ &= \frac{ax}{a^2-b^2}-\frac{b\log(b\cosh(c+dx)+a\sinh(c+dx))}{(a^2-b^2)\,d}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 64, normalized size = 1.28

$$\frac{(-a+b)\log(1-\coth(c+dx))+(a+b)\log(1+\coth(c+dx))-2b\log(a+b\coth(c+dx))}{2(a-b)(a+b)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Coth[c + d*x])^(-1),x]
```

```
[Out] ((-a + b)*Log[1 - Coth[c + d*x]] + (a + b)*Log[1 + Coth[c + d*x]] - 2*b*Log
[a + b*Coth[c + d*x]])/(2*(a - b)*(a + b)*d)
```

Maple [A]
time = 0.74, size = 71, normalized size = 1.42

method	result	size
derivativedivides	$\frac{-\frac{\ln(\coth(dx+c)-1)}{2b+2a}-\frac{b\ln(a+b\coth(dx+c))}{(a-b)(a+b)}+\frac{\ln(\coth(dx+c)+1)}{2a-2b}}{d}$	71
default	$\frac{-\frac{\ln(\coth(dx+c)-1)}{2b+2a}-\frac{b\ln(a+b\coth(dx+c))}{(a-b)(a+b)}+\frac{\ln(\coth(dx+c)+1)}{2a-2b}}{d}$	71
risch	$\frac{x}{a+b}+\frac{2bx}{a^2-b^2}+\frac{2bc}{d(a^2-b^2)}-\frac{b\ln\left(\mathrm{e}^{2dx+2c}-\frac{a-b}{a+b}\right)}{d(a^2-b^2)}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*coth(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/(2*b+2*a)*ln(coth(d*x+c)-1)-b/(a-b)/(a+b)*ln(a+b*coth(d*x+c))+1/(2*
a-2*b)*ln(coth(d*x+c)+1))
```

Maxima [A]
time = 0.28, size = 52, normalized size = 1.04

$$-\frac{b\log\left(-(a-b)e^{(-2\,dx-2\,c)}+a+b\right)}{(a^2-b^2)d}+\frac{dx+c}{(a+b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c)),x, algorithm="maxima")
```

```
[Out] -b*log(-(a - b)*e^(-2*d*x - 2*c) + a + b)/((a^2 - b^2)*d) + (d*x + c)/((a +
 b)*d)
```

Fricas [A]
time = 0.36, size = 62, normalized size = 1.24

$$\frac{(a+b)dx-b\log\left(\frac{2\,(b\cosh(dx+c)+a\sinh(dx+c))}{\cosh(dx+c)-\sinh(dx+c)}\right)}{(a^2-b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((a + b)*d*x - b*log(2*(b*cosh(d*x + c) + a*sinh(d*x + c))/(cosh(d*x + c) -
 sinh(d*x + c))))/((a^2 - b^2)*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(37) = 74$.

time = 1.00, size = 236, normalized size = 4.72

$$\begin{cases} \frac{\tilde{\infty}x}{\coth(c)} & \text{for } a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{x - \frac{\log(\tanh(c+dx)+1)}{d}}{b} & \text{for } a = 0 \\ -\frac{dx\tanh(c+dx)}{2bd\tanh(c+dx)-2bd} + \frac{dx}{2bd\tanh(c+dx)-2bd} - \frac{1}{2bd\tanh(c+dx)-2bd} & \text{for } a = -b \\ \frac{dx\tanh(c+dx)}{2bd\tanh(c+dx)+2bd} + \frac{dx}{2bd\tanh(c+dx)+2bd} + \frac{1}{2bd\tanh(c+dx)+2bd} & \text{for } a = b \\ \frac{x}{a+b\coth(c)} & \text{for } d = 0 \\ \frac{adx}{a^2d-b^2d} - \frac{bdx}{a^2d-b^2d} + \frac{b\log(\tanh(c+dx)+1)}{a^2d-b^2d} - \frac{b\log\left(\tanh(c+dx)+\frac{b}{a}\right)}{a^2d-b^2d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x/coth(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((x - log(tanh(c
 + d*x) + 1)/d)/b, Eq(a, 0)), (-d*x*tanh(c + d*x)/(2*b*d*tanh(c + d*x) - 2*
b*d) + d*x/(2*b*d*tanh(c + d*x) - 2*b*d) - 1/(2*b*d*tanh(c + d*x) - 2*b*d),
 Eq(a, -b)), (d*x*tanh(c + d*x)/(2*b*d*tanh(c + d*x) + 2*b*d) + d*x/(2*b*d*
tanh(c + d*x) + 2*b*d) + 1/(2*b*d*tanh(c + d*x) + 2*b*d), Eq(a, b)), (x/(a
+ b*coth(c)), Eq(d, 0)), (a*d*x/(a**2*d - b**2*d) - b*d*x/(a**2*d - b**2*d)
 + b*log(tanh(c + d*x) + 1)/(a**2*d - b**2*d) - b*log(tanh(c + d*x) + b/a)/
(a**2*d - b**2*d), True))
```

Giac [A]
time = 0.43, size = 62, normalized size = 1.24

$$-\frac{\frac{b\log\left(\left|ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}-a+b\right|\right)}{a^2-b^2} - \frac{dx+c}{a-b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c)),x, algorithm="giac")
```

```
[Out] -(b*log(abs(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) - a + b))/(a^2 - b^2) - (
d*x + c)/(a - b))/d
```

Mupad [B]
time = 1.22, size = 55, normalized size = 1.10

$$\frac{x}{a-b}-\frac{b\,\ln\left(b-a+a\,{\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}+b\,{\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}\right)}{a^2\,d-b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*coth(c + d*x)),x)
```

```
[Out] x/(a - b) - (b*log(b - a + a*exp(2*c)*exp(2*d*x) + b*exp(2*c)*exp(2*d*x)))/
(a^2*d - b^2*d)
```

3.82 $\int \frac{1}{(a+b\coth(c+dx))^2}\,dx$

Optimal. Leaf size=85

$$\frac{(a^2+b^2)\,x}{(a^2-b^2)^2}+\frac{b}{(a^2-b^2)\,d(a+b\coth(c+dx))}-\frac{2ab\log(b\cosh(c+dx)+a\sinh(c+dx))}{(a^2-b^2)^2\,d}$$

```
[Out] (a^2+b^2)*x/(a^2-b^2)^2+b/(a^2-b^2)/d/(a+b*coth(d*x+c))-2*a*b*ln(b*cosh(d*x
+c)+a*sinh(d*x+c))/(a^2-b^2)^2/d
```

Rubi [A]
time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3564, 3612, 3611}

$$\frac{b}{d\,(a^2-b^2)\,(a+b\coth(c+dx))}-\frac{2ab\log(a\sinh(c+dx)+b\cosh(c+dx))}{d\,(a^2-b^2)^2}+\frac{x(a^2+b^2)}{(a^2-b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Coth[c + d*x])^(-2),x]
```

```
[Out] ((a^2 + b^2)*x)/(a^2 - b^2)^2 + b/((a^2 - b^2)*d*(a + b*Coth[c + d*x])) - (
2*a*b*Log[b*Cosh[c + d*x] + a*Sinh[c + d*x]])/((a^2 - b^2)^2*d)
```

Rule 3564

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\coth(c+dx))^2}\,dx = \frac{b}{(a^2-b^2)\,d(a+b\coth(c+dx))} + \frac{\int \frac{a-b\coth(c+dx)}{a+b\coth(c+dx)}\,dx}{a^2-b^2}$$
$$= \frac{(a^2+b^2)\,x}{(a^2-b^2)^2} + \frac{b}{(a^2-b^2)\,d(a+b\coth(c+dx))} - \frac{(2iab)\int \frac{-ib-ia\coth(c+dx)}{a+b\coth(c+dx)}\,dx}{(a^2-b^2)^2}$$
$$= \frac{(a^2+b^2)\,x}{(a^2-b^2)^2} + \frac{b}{(a^2-b^2)\,d(a+b\coth(c+dx))} - \frac{2ab\log(b\cosh(c+dx)+a\sin}{(a^2-b^2)^2\,d}$$

Mathematica [A]
time = 1.08, size = 100, normalized size = 1.18

$$\frac{-\frac{\log(1-\tanh(c+dx))}{(a+b)^2} + \frac{\log(1+\tanh(c+dx))}{(a-b)^2} + \frac{2b\left(-2a^2\log(b+a\tanh(c+dx)) + \frac{-a^2b+b^3}{b+a\tanh(c+dx)}\right)}{a(a^2-b^2)^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Coth[c + d*x])^(-2),x]
[Out] (-(Log[1 - Tanh[c + d*x]]/(a + b)^2) + Log[1 + Tanh[c + d*x]]/(a - b)^2 + (
2*b*(-2*a^2*Log[b + a*Tanh[c + d*x]] + (-(a^2*b) + b^3)/(b + a*Tanh[c + d*x
]])))/(a*(a^2 - b^2)^2))/(2*d)
```

Maple [A]
time = 0.68, size = 93, normalized size = 1.09

method	result
derivativedivides	$\frac{\frac{\ln(\coth(dx+c)+1)}{2(a-b)^2} + \frac{b}{(a-b)(a+b)(a+b\coth(dx+c))} - \frac{2ab\ln(a+b\coth(dx+c))}{(a+b)^2(a-b)^2} - \frac{\ln(\coth(dx+c)-1)}{2(a+b)^2}}{d}$
default	$\frac{\frac{\ln(\coth(dx+c)+1)}{2(a-b)^2} + \frac{b}{(a-b)(a+b)(a+b\coth(dx+c))} - \frac{2ab\ln(a+b\coth(dx+c))}{(a+b)^2(a-b)^2} - \frac{\ln(\coth(dx+c)-1)}{2(a+b)^2}}{d}$
risch	$\frac{x}{a^2+2ab+b^2} + \frac{4abx}{a^4-2a^2b^2+b^4} + \frac{4abc}{d(a^4-2a^2b^2+b^4)} - \frac{2b^2}{(a-b)d(a^2+2ab+b^2)\left(a\,\mathrm{e}^{2dx+2c}+b\,\mathrm{e}^{2dx+2c}-a+b\right)} - \frac{2ab\ln\left(\mathrm{e}\right.}{d(a^4-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*coth(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d*(1/2/(a-b)^2*ln(coth(d*x+c)+1)+b/(a-b)/(a+b)/(a+b*coth(d*x+c))-2*a*b/(a
+b)^2/(a-b)^2*ln(a+b*coth(d*x+c))-1/2/(a+b)^2*ln(coth(d*x+c)-1))
```

Maxima [A]
time = 0.28, size = 124, normalized size = 1.46

$$-\frac{2\,ab\log\left(-(a-b)e^{(-2\,dx-2\,c)}+a+b\right)}{(a^4-2\,a^2b^2+b^4)d} - \frac{2\,b^2}{(a^4-2\,a^2b^2+b^4-(a^4-2\,a^3b+2\,ab^3-b^4)e^{(-2\,dx-2\,c)})d} + \frac{dx+c}{(a^2+2\,ab+b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -2*a*b*log(-(a - b)*e^(-2*d*x - 2*c) + a + b)/((a^4 - 2*a^2*b^2 + b^4)*d) -
 2*b^2/((a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^3*b + 2*a*b^3 - b^4)*e^(-2*d*x
- 2*c))*d) + (d*x + c)/((a^2 + 2*a*b + b^2)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 426 vs. $2(85) = 170$.
time = 0.41, size = 426, normalized size = 5.01

$$\frac{(a^3+3a^2b+3ab^2+b^3)dx\cosh(dx+c)^2+2(a^3+3a^2b+3ab^2+b^3)dx\cosh(dx+c)\sinh(dx+c)+(a^3+3a^2b+3ab^2+b^3)dx\sinh(dx+c)^2-2ab^2+2b^3-(a^3+a^2b-ab^2-b^3)dx+2\left(a^2b-ab^2-(a^2b+ab^2)\cosh(dx+c)^2-2(a^2b+ab^2)\cosh(dx+c)\sinh(dx+c)-(a^2b+ab^2)\sinh(dx+c)^2\right)\log\left(\frac{2(b\cosh(dx+c)+a\sinh(dx+c))}{\cosh(dx+c)-\sinh(dx+c)}\right)}{(a^5+a^4b-2a^3b^2-2a^2b^3+ab^4+b^5)d\cosh(dx+c)^2+2(a^5+a^4b-2a^3b^2-2a^2b^3+ab^4+b^5)d\cosh(dx+c)\sinh(dx+c)+(a^5+a^4b-2a^3b^2-2a^2b^3+ab^4+b^5)d\sinh(dx+c)^2-(a^5-a^4b-2a^3b^2+2a^2b^3+ab^4-b^5)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] ((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^2 + 2*(a^3 + 3*a^2*b + 3
*a*b^2 + b^3)*d*x*cosh(d*x + c)*sinh(d*x + c) + (a^3 + 3*a^2*b + 3*a*b^2 +
b^3)*d*x*sinh(d*x + c)^2 - 2*a*b^2 + 2*b^3 - (a^3 + a^2*b - a*b^2 - b^3)*d*
x + 2*(a^2*b - a*b^2 - (a^2*b + a*b^2)*cosh(d*x + c)^2 - 2*(a^2*b + a*b^2)*
cosh(d*x + c)*sinh(d*x + c) - (a^2*b + a*b^2)*sinh(d*x + c)^2)*log(2*(b*cos
h(d*x + c) + a*sinh(d*x + c))/(cosh(d*x + c) - sinh(d*x + c))))/((a^5 + a^4
*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*d*cosh(d*x + c)^2 + 2*(a^5 + a^4*
b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*d*cosh(d*x + c)*sinh(d*x + c) + (a
^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*d*sinh(d*x + c)^2 - (a^5
- a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))**2,x)
```

```
[Out] Piecewise((zoo*x/coth(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((x - tanh(c
+ d*x)/d)/b**2, Eq(a, 0)), (d*x*tanh(c + d*x)**2/(4*b**2*d*tanh(c + d*x)**2
 - 8*b**2*d*tanh(c + d*x) + 4*b**2*d) - 2*d*x*tanh(c + d*x)/(4*b**2*d*tanh(
c + d*x)**2 - 8*b**2*d*tanh(c + d*x) + 4*b**2*d) + d*x/(4*b**2*d*tanh(c + d
*x)**2 - 8*b**2*d*tanh(c + d*x) + 4*b**2*d) + 3*tanh(c + d*x)/(4*b**2*d*tan
h(c + d*x)**2 - 8*b**2*d*tanh(c + d*x) + 4*b**2*d) - 2/(4*b**2*d*tanh(c + d
*x)**2 - 8*b**2*d*tanh(c + d*x) + 4*b**2*d), Eq(a, -b)), (d*x*tanh(c + d*x)
**2/(4*b**2*d*tanh(c + d*x)**2 + 8*b**2*d*tanh(c + d*x) + 4*b**2*d) + 2*d*x
*tanh(c + d*x)/(4*b**2*d*tanh(c + d*x)**2 + 8*b**2*d*tanh(c + d*x) + 4*b**2
*d) + d*x/(4*b**2*d*tanh(c + d*x)**2 + 8*b**2*d*tanh(c + d*x) + 4*b**2*d) +
```

```
 3*tanh(c + d*x)/(4*b**2*d*tanh(c + d*x)**2 + 8*b**2*d*tanh(c + d*x) + 4*b*
*2*d) + 2/(4*b**2*d*tanh(c + d*x)**2 + 8*b**2*d*tanh(c + d*x) + 4*b**2*d),
Eq(a, b)), ((Integral(-2*exp(2*c)*exp(2*d*x)/(exp(4*c)*exp(4*d*x)*coth(c +
d*x)**2 - 2*exp(4*c)*exp(4*d*x)*coth(c + d*x) + exp(4*c)*exp(4*d*x) - 2*exp
(2*c)*exp(2*d*x)*coth(c + d*x)**2 + 2*exp(2*c)*exp(2*d*x) + coth(c + d*x)**
2 + 2*coth(c + d*x) + 1), x) + Integral(exp(4*c)*exp(4*d*x)/(exp(4*c)*exp(4
*d*x)*coth(c + d*x)**2 - 2*exp(4*c)*exp(4*d*x)*coth(c + d*x) + exp(4*c)*exp
(4*d*x) - 2*exp(2*c)*exp(2*d*x)*coth(c + d*x)**2 + 2*exp(2*c)*exp(2*d*x) +
coth(c + d*x)**2 + 2*coth(c + d*x) + 1), x) + Integral(1/(exp(4*c)*exp(4*d*
x)*coth(c + d*x)**2 - 2*exp(4*c)*exp(4*d*x)*coth(c + d*x) + exp(4*c)*exp(4*
d*x) - 2*exp(2*c)*exp(2*d*x)*coth(c + d*x)**2 + 2*exp(2*c)*exp(2*d*x) + cot
h(c + d*x)**2 + 2*coth(c + d*x) + 1), x))/b**2, Eq(a, (-b*exp(2*c)*exp(2*d*
x) - b)/(exp(2*c)*exp(2*d*x) - 1))), (x/(a + b*coth(c))**2, Eq(d, 0)), (a**
4*d*x*tanh(c + d*x)/(a**6*d*tanh(c + d*x) + a**5*b*d - 2*a**4*b**2*d*tanh(c
 + d*x) - 2*a**3*b**3*d + a**2*b**4*d*tanh(c + d*x) + a*b**5*d) - 2*a**3*b*
d*x*tanh(c + d*x)/(a**6*d*tanh(c + d*x) + a**5*b*d - 2*a**4*b**2*d*tanh(c +
 d*x) - 2*a**3*b**3*d + a**2*b**4*d*tanh(c + d*x) + a*b**5*d) + a**3*b*d*x/
(a**6*d*tanh(c + d*x) + a**5*b*d - 2*a**4*b**2*d*tanh(c + d*x) - 2*a**3*b**
3*d + a**2*b**4*d*tanh(c + d*x) + a*b**5*d) + 2*a**3*b*log(tanh(c + d*x) +
1)*tanh(c + d*x)/(a**6*d*tanh(c + d*x) + a**5*b*d - 2*a**4*b**2*d*tanh(c +
d*x) - 2*a**3*b**3*d + a**2*b**4*d*tanh(c + d*x) + a*b**5*d) - 2*a**3*b*log
(tanh(c + d*x) + b/a)*tanh(c + d*x)/(a**6*d*tanh(c + d*x) + a**5*b*d - 2*a*
*4*b**2*d*tanh(c + d*x) - 2*a**3*b**3*d + a**2*b**4*d*tanh(c + d*x) + a*b**
5*d) + a**2*b**2*d*x*tanh(c + d*x)/(a**6*d*tanh(c + d*x) + a**5*b*d - 2*a**
4*b**2*d*tanh(c + d*x) - 2*a**3*b**3*d + a**2*b**4*d*tanh(c + d*x) + a*b**5
*d) - 2*a**2*b**2*d*x/(a**6*d*tanh(c + d*x) + a**5*b*d - 2*a**4*b**2*d*tanh
(c + d*x) - 2*a**3*b**3*d + a**2*b**4*d*tanh(c + d*x) + a*b**5*d) + 2*a**2*
b**2*log(tanh(c + d*x) + 1)/(a**6*d*tanh(c + d*x) + a**5*b*d - 2*a**4*b**2*
d*tanh(c + d*x) - 2*a**3*b**3*d + a**2*b**4*d*tanh(c + d*x) + a*b**5*d) - 2
*a**2*b**2*log(tanh(c + d*x) + b/a)/(a**6*d*tanh(c + d*x) + a**5*b*d - 2*a*
*4*b**2*d*tanh(c + d*x) - 2*a**3*b**3*d + a**2*b**4*d*tanh(c + d*x) + a*b**
5*d) - a**2*b**2/(a**6*d*tanh(c + d*x) + a**5*b*d - 2*a**4*b**2*d*tanh(c +
d*x) - 2*a**3*b**3*d + a**2*b**4*d*tanh(c + d*x) + a*b**5*d) + a*b**3*d*x/(
a**6*d*tanh(c + d*x) + a**5*b*d - 2*a**4*b**2*d*tanh(c + d*x) - 2*a**3*b**3
*d + a**2*b**4*d*tanh(c + d*x) + a*b**5*d) + b**4/(a**6*d*tanh(c + d*x) + a
**5*b*d - 2*a**4*b**2*d*tanh(c + d*x) - 2*a**3*b**3*d + a**2*b**4*d*tanh(c
+ d*x) + a*b**5*d), True))
```

Giac [A]

time = 0.42, size = 130, normalized size = 1.53

$$-\frac{\frac{2\,ab\log\left(\left|ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}-a+b\right|\right)}{a^4-2\,a^2b^2+b^4}-\frac{dx+c}{a^2-2\,ab+b^2}+\frac{2\,(ab^2-b^3)}{\left(ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}-a+b\right)(a+b)^2(a-b)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(2*a*b*log(abs(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) - a + b))/(a^4 - 2*a^
2*b^2 + b^4) - (d*x + c)/(a^2 - 2*a*b + b^2) + 2*(a*b^2 - b^3)/((a*e^(2*d*x
 + 2*c) + b*e^(2*d*x + 2*c) - a + b)*(a + b)^2*(a - b)^2))/d
```

Mupad [B]
time = 1.30, size = 104, normalized size = 1.22

$$\frac{x}{(a-b)^2}-\frac{2\,a\,b\,\ln\left(b-a+a\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+b\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\right)}{d\,a^4-2\,d\,a^2\,b^2+d\,b^4}-\frac{2\,b^2}{d\,(a+b)^2\,(a-b)\,\left(b-a+\mathrm{e}^{2\,c+2\,d\,x}\,(a+b)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*coth(c + d*x))^2,x)
```

```
[Out] x/(a - b)^2 - (2*a*b*log(b - a + a*exp(2*c)*exp(2*d*x) + b*exp(2*c)*exp(2*d
*x)))/(a^4*d + b^4*d - 2*a^2*b^2*d) - (2*b^2)/(d*(a + b)^2*(a - b)*(b - a +
 exp(2*c + 2*d*x)*(a + b)))
```

3.83 $\int \frac{1}{(a+b\coth(c+dx))^3}\,dx$

Optimal. Leaf size=129

$$\frac{a(a^2+3b^2)\,x}{(a^2-b^2)^3}+\frac{b}{2\,(a^2-b^2)\,d(a+b\coth(c+dx))^2}+\frac{2ab}{(a^2-b^2)^2\,d(a+b\coth(c+dx))}-\frac{b(3a^2+b^2)\log(b\cosh($$

```
[Out] a*(a^2+3*b^2)*x/(a^2-b^2)^3+1/2*b/(a^2-b^2)/d/(a+b*coth(d*x+c))^2+2*a*b/(a^
2-b^2)^2/d/(a+b*coth(d*x+c))-b*(3*a^2+b^2)*ln(b*cosh(d*x+c)+a*sinh(d*x+c))/
(a^2-b^2)^3/d
```

Rubi [A]
time = 0.13, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3564, 3610, 3612, 3611}

$$\frac{2ab}{d\,(a^2-b^2)^2\,(a+b\coth(c+dx))}+\frac{b}{2d\,(a^2-b^2)\,(a+b\coth(c+dx))^2}-\frac{b(3a^2+b^2)\log(a\sinh(c+dx)+b\cosh(c+dx))}{d\,(a^2-b^2)^3}+\frac{ax(a^2+3b^2)}{(a^2-b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Coth[c + d*x])^(-3),x]
```

```
[Out] (a*(a^2 + 3*b^2)*x)/(a^2 - b^2)^3 + b/(2*(a^2 - b^2)*d*(a + b*Coth[c + d*x]
)^2) + (2*a*b)/((a^2 - b^2)^2*d*(a + b*Coth[c + d*x])) - (b*(3*a^2 + b^2)*L
og[b*Cosh[c + d*x] + a*Sinh[c + d*x]])/((a^2 - b^2)^3*d)
```

Rule 3564

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
```

```
NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\coth(c+dx))^3}\,dx = \frac{b}{2(a^2-b^2)\,d(a+b\coth(c+dx))^2} + \frac{\int \frac{a-b\coth(c+dx)}{(a+b\coth(c+dx))^2}\,dx}{a^2-b^2}$$

$$= \frac{b}{2(a^2-b^2)\,d(a+b\coth(c+dx))^2} + \frac{2ab}{(a^2-b^2)^2\,d(a+b\coth(c+dx))} + \frac{\int \frac{a^2+b^2}{a-}}{}$$

$$= \frac{a(a^2+3b^2)\,x}{(a^2-b^2)^3} + \frac{b}{2(a^2-b^2)\,d(a+b\coth(c+dx))^2} + \frac{2ab}{(a^2-b^2)^2\,d(a+b\coth(c}$$

$$= \frac{a(a^2+3b^2)\,x}{(a^2-b^2)^3} + \frac{b}{2(a^2-b^2)\,d(a+b\coth(c+dx))^2} + \frac{2ab}{(a^2-b^2)^2\,d(a+b\coth(c}$$

Mathematica [A]
time = 2.44, size = 134, normalized size = 1.04

$$-\frac{\frac{\log(1-\tanh(c+dx))}{(a+b)^3} - \frac{\log(1+\tanh(c+dx))}{(a-b)^3} + \frac{b\left(2(3a^2+b^2)\log(b+a\tanh(c+dx)) + \frac{b(-a^2+b^2)\left(-5a^2b+b^3+(-6a^3+2ab^2)\tanh(c+dx)\right)}{a^2(b+a\tanh(c+dx))^2}\right)}{(a^2-b^2)^3}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Coth[c + d*x])^(-3),x]
```

```
[Out] -1/2*(Log[1 - Tanh[c + d*x]]/(a + b)^3 - Log[1 + Tanh[c + d*x]]/(a - b)^3 +
 (b*(2*(3*a^2 + b^2)*Log[b + a*Tanh[c + d*x]] + (b*(-a^2 + b^2)*(-5*a^2*b +
 b^3 + (-6*a^3 + 2*a*b^2)*Tanh[c + d*x]))/(a^2*(b + a*Tanh[c + d*x])^2)))/(
a^2 - b^2)^3)/d
```

Maple [A]
time = 0.79, size = 130, normalized size = 1.01

method	result

derivativedivides	$\frac{\frac{b}{2(a-b)(a+b)(a+b\coth(dx+c))^2}+\frac{2ab}{(a+b)^2(a-b)^2(a+b\coth(dx+c))}-\frac{b\left(3a^2+b^2\right)\ln(a+b\coth(dx+c))}{(a+b)^3(a-b)^3}+\frac{\ln(\coth(dx+c)+1)}{2(a-b)^3}-\frac{\ln(\coth(d}{2(a-}}{d}$
default	$\frac{\frac{b}{2(a-b)(a+b)(a+b\coth(dx+c))^2}+\frac{2ab}{(a+b)^2(a-b)^2(a+b\coth(dx+c))}-\frac{b\left(3a^2+b^2\right)\ln(a+b\coth(dx+c))}{(a+b)^3(a-b)^3}+\frac{\ln(\coth(dx+c)+1)}{2(a-b)^3}-\frac{\ln(\coth(d}{2(a-}}{d}$
risch	$\frac{x}{a^3+3a^2b+3a\,b^2+b^3}+\frac{6b\,a^2x}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{2b^3x}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{6b\,a^2c}{d(a^6-3a^4b^2+3a^2b^4-b^6)}+\frac{2b^3}{d(a^6-3a^4b^2-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*coth(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*b/(a-b)/(a+b)/(a+b*coth(d*x+c))^2+2*a*b/(a+b)^2/(a-b)^2/(a+b*coth(
d*x+c))-b*(3*a^2+b^2)/(a+b)^3/(a-b)^3*ln(a+b*coth(d*x+c))+1/2/(a-b)^3*ln(co
th(d*x+c)+1)-1/2/(a+b)^3*ln(coth(d*x+c)-1))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(127) = 254$.
time = 0.29, size = 322, normalized size = 2.50

$$-\frac{(3\,a^2b+b^3)\log\left(-(a-b)e^{(-2\,dx-2\,c)}+a+b\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)d}-\frac{2\left(3\,a^2b^2+3\,ab^3-(3\,a^2b^2-2\,ab^3-b^4)e^{(-2\,dx-2\,c)}\right)}{(a^7+a^6b-3\,a^5b^2-3\,a^4b^3+3\,a^3b^4+3\,a^2b^5-ab^6-b^7-2\,(a^7-a^6b-3\,a^5b^2+3\,a^4b^3+3\,a^3b^4-3\,a^2b^5-ab^6+b^7)e^{(-2\,dx-2\,c)}+(a^7-3\,a^6b+a^5b^2+5\,a^4b^3-5\,a^3b^4-a^2b^5+3\,ab^6-b^7)e^{(-4\,dx-4\,c)})d}+\frac{dx+c}{(a^3+3\,a^2b+3\,ab^2+b^3)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -(3*a^2*b + b^3)*log(-(a - b)*e^(-2*d*x - 2*c) + a + b)/((a^6 - 3*a^4*b^2 +
 3*a^2*b^4 - b^6)*d) - 2*(3*a^2*b^2 + 3*a*b^3 - (3*a^2*b^2 - 2*a*b^3 - b^4)
*e^(-2*d*x - 2*c))/((a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^
2*b^5 - a*b^6 - b^7 - 2*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 -
3*a^2*b^5 - a*b^6 + b^7)*e^(-2*d*x - 2*c) + (a^7 - 3*a^6*b + a^5*b^2 + 5*a^
4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*e^(-4*d*x - 4*c))*d) + (d*x +
c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1431 vs. $2(127) = 254$.
time = 0.40, size = 1431, normalized size = 11.09

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] ((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x*cosh(d*x + c
)^4 + 4*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x*cosh(
d*x + c)*sinh(d*x + c)^3 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b
```

```
^4 + b^5)*d*x*sinh(d*x + c)^4 + 6*a^3*b^2 - 12*a^2*b^3 + 6*a*b^4 + (a^5 + a
^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*d*x - 2*(3*a^3*b^2 - a^2*b^3 -
3*a*b^4 + b^5 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*x
)*cosh(d*x + c)^2 - 2*(3*a^3*b^2 - a^2*b^3 - 3*a*b^4 + b^5 - 3*(a^5 + 5*a^4
*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x*cosh(d*x + c)^2 + (a^5 +
3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*x)*sinh(d*x + c)^2 - (3*
a^4*b - 6*a^3*b^2 + 4*a^2*b^3 - 2*a*b^4 + b^5 + (3*a^4*b + 6*a^3*b^2 + 4*a^
2*b^3 + 2*a*b^4 + b^5)*cosh(d*x + c)^4 + 4*(3*a^4*b + 6*a^3*b^2 + 4*a^2*b^3
 + 2*a*b^4 + b^5)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^4*b + 6*a^3*b^2 + 4*
a^2*b^3 + 2*a*b^4 + b^5)*sinh(d*x + c)^4 - 2*(3*a^4*b - 2*a^2*b^3 - b^5)*co
sh(d*x + c)^2 - 2*(3*a^4*b - 2*a^2*b^3 - b^5 - 3*(3*a^4*b + 6*a^3*b^2 + 4*a
^2*b^3 + 2*a*b^4 + b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((3*a^4*b + 6*
a^3*b^2 + 4*a^2*b^3 + 2*a*b^4 + b^5)*cosh(d*x + c)^3 - (3*a^4*b - 2*a^2*b^3
 - b^5)*cosh(d*x + c))*sinh(d*x + c))*log(2*(b*cosh(d*x + c) + a*sinh(d*x +
 c))/(cosh(d*x + c) - sinh(d*x + c))) + 4*((a^5 + 5*a^4*b + 10*a^3*b^2 + 10
*a^2*b^3 + 5*a*b^4 + b^5)*d*x*cosh(d*x + c)^3 - (3*a^3*b^2 - a^2*b^3 - 3*a*
b^4 + b^5 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*x)*co
sh(d*x + c))*sinh(d*x + c))/((a^8 + 2*a^7*b - 2*a^6*b^2 - 6*a^5*b^3 + 6*a^3
*b^5 + 2*a^2*b^6 - 2*a*b^7 - b^8)*d*cosh(d*x + c)^4 + 4*(a^8 + 2*a^7*b - 2*
a^6*b^2 - 6*a^5*b^3 + 6*a^3*b^5 + 2*a^2*b^6 - 2*a*b^7 - b^8)*d*cosh(d*x + c
)*sinh(d*x + c)^3 + (a^8 + 2*a^7*b - 2*a^6*b^2 - 6*a^5*b^3 + 6*a^3*b^5 + 2*
a^2*b^6 - 2*a*b^7 - b^8)*d*sinh(d*x + c)^4 - 2*(a^8 - 4*a^6*b^2 + 6*a^4*b^4
 - 4*a^2*b^6 + b^8)*d*cosh(d*x + c)^2 + 2*(3*(a^8 + 2*a^7*b - 2*a^6*b^2 - 6
*a^5*b^3 + 6*a^3*b^5 + 2*a^2*b^6 - 2*a*b^7 - b^8)*d*cosh(d*x + c)^2 - (a^8
- 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d)*sinh(d*x + c)^2 + (a^8 - 2*a^
7*b - 2*a^6*b^2 + 6*a^5*b^3 - 6*a^3*b^5 + 2*a^2*b^6 + 2*a*b^7 - b^8)*d + 4*
((a^8 + 2*a^7*b - 2*a^6*b^2 - 6*a^5*b^3 + 6*a^3*b^5 + 2*a^2*b^6 - 2*a*b^7 -
 b^8)*d*cosh(d*x + c)^3 - (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d
*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.43, size = 203, normalized size = 1.57

$$-\frac{\frac{(3\,a^2b+b^3)\log\left(\left|ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}-a+b\right|\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}-\frac{dx+c}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{2\left((3\,a^2b^2-4\,ab^3+b^4)e^{(2\,dx+2\,c)}-\frac{3\,(a^3b^2-2\,a^2b^3+ab^4)}{a+b}\right)}{\left(ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}-a+b\right)^2(a+b)^2(a-b)^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -((3*a^2*b + b^3)*log(abs(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) - a + b))/(
a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) - (d*x + c)/(a^3 - 3*a^2*b + 3*a*b^2 - b
^3) + 2*((3*a^2*b^2 - 4*a*b^3 + b^4)*e^(2*d*x + 2*c) - 3*(a^3*b^2 - 2*a^2*b
^3 + a*b^4)/(a + b))/((a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) - a + b)^2*(a
+ b)^2*(a - b)^3))/d
```

Mupad [B]
time = 1.40, size = 195, normalized size = 1.51

$$\frac{x}{(a-b)^3}-\frac{\ln\left(b-a+a\,{\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}+b\,{\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}\right)\,\left(3\,a^2\,b+b^3\right)}{d\,a^6-3\,d\,a^4\,b^2+3\,d\,a^2\,b^4-d\,b^6}+\frac{2\,b^3}{d\,{(a+b)}^3\,\left(a-b\right)\,\left({\mathrm{e}}^{4\,c+4\,d\,x}\,{(a+b)}^2+{(a-b)}^2-2\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a+b\right)\,\left(a-b\right)\right)}-\frac{2\,\left(3\,a\,b^2-b^3\right)}{d\,{(a+b)}^3\,{(a-b)}^2\,\left(b-a+{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a+b\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*coth(c + d*x))^3,x)
```

```
[Out] x/(a - b)^3 - (log(b - a + a*exp(2*c)*exp(2*d*x) + b*exp(2*c)*exp(2*d*x))*(
3*a^2*b + b^3))/(a^6*d - b^6*d + 3*a^2*b^4*d - 3*a^4*b^2*d) + (2*b^3)/(d*(a
 + b)^3*(a - b)*(exp(4*c + 4*d*x)*(a + b)^2 + (a - b)^2 - 2*exp(2*c + 2*d*x
)*(a + b)*(a - b))) - (2*(3*a*b^2 - b^3))/(d*(a + b)^3*(a - b)^2*(b - a + e
xp(2*c + 2*d*x)*(a + b)))
```

3.84 $\int \frac{1}{(a+b\coth(c+dx))^4}\,dx$

Optimal. Leaf size=169

$$\frac{(a^4+6a^2b^2+b^4)\,x}{(a^2-b^2)^4}+\frac{b}{3\,(a^2-b^2)\,d(a+b\coth(c+dx))^3}+\frac{ab}{(a^2-b^2)^2\,d(a+b\coth(c+dx))^2}+\frac{b(3a^2}{(a^2-b^2)^3\,d(a+}$$

```
[Out] (a^4+6*a^2*b^2+b^4)*x/(a^2-b^2)^4+1/3*b/(a^2-b^2)/d/(a+b*coth(d*x+c))^3+a*b
/(a^2-b^2)^2/d/(a+b*coth(d*x+c))^2+b*(3*a^2+b^2)/(a^2-b^2)^3/d/(a+b*coth(d*
x+c))-4*a*b*(a^2+b^2)*ln(b*cosh(d*x+c)+a*sinh(d*x+c))/(a^2-b^2)^4/d
```

Rubi [A]
time = 0.19, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3564, 3610, 3612, 3611}

$$\frac{b(3a^2+b^2)}{d\,(a^2-b^2)^3\,(a+b\coth(c+dx))}+\frac{ab}{d\,(a^2-b^2)^2\,(a+b\coth(c+dx))^2}+\frac{b}{3d\,(a^2-b^2)\,(a+b\coth(c+dx))^3}-\frac{4ab(a^2+b^2)\log(a\sinh(c+dx)+b\cosh(c+dx))}{d\,(a^2-b^2)^4}+\frac{x(a^4+6a^2b^2+b^4)}{(a^2-b^2)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Coth[c + d*x])^(-4),x]
```

```
[Out] ((a^4 + 6*a^2*b^2 + b^4)*x)/(a^2 - b^2)^4 + b/(3*(a^2 - b^2)*d*(a + b*Coth[
c + d*x])^3) + (a*b)/((a^2 - b^2)^2*d*(a + b*Coth[c + d*x])^2) + (b*(3*a^2
+ b^2))/((a^2 - b^2)^3*d*(a + b*Coth[c + d*x])) - (4*a*b*(a^2 + b^2)*Log[b*
Cosh[c + d*x] + a*Sinh[c + d*x]])/((a^2 - b^2)^4*d)
```

Rule 3564

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
```

```
NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\coth(c+dx))^4}\,dx = \frac{b}{3\,(a^2-b^2)\,d(a+b\coth(c+dx))^3} + \frac{\int \frac{a-b\coth(c+dx)}{(a+b\coth(c+dx))^3}\,dx}{a^2-b^2}$$

$$= \frac{b}{3\,(a^2-b^2)\,d(a+b\coth(c+dx))^3} + \frac{ab}{(a^2-b^2)^2\,d(a+b\coth(c+dx))^2} + \frac{\int \frac{a^2}{}}{}$$

$$= \frac{b}{3\,(a^2-b^2)\,d(a+b\coth(c+dx))^3} + \frac{ab}{(a^2-b^2)^2\,d(a+b\coth(c+dx))^2} + \frac{}{(a^2}$$

$$= \frac{(a^4+6a^2b^2+b^4)\,x}{(a^2-b^2)^4} + \frac{b}{3\,(a^2-b^2)\,d(a+b\coth(c+dx))^3} + \frac{ab}{(a^2-b^2)^2\,d(a+b}$$

$$= \frac{(a^4+6a^2b^2+b^4)\,x}{(a^2-b^2)^4} + \frac{b}{3\,(a^2-b^2)\,d(a+b\coth(c+dx))^3} + \frac{ab}{(a^2-b^2)^2\,d(a+b}$$

Mathematica [A]
time = 6.15, size = 214, normalized size = 1.27

$$-\frac{\log(1-\tanh(c+dx))}{2(a+b)^4d}+\frac{\log(1+\tanh(c+dx))}{2(a-b)^4d}-\frac{4ab(a^2+b^2)\log(b+a\tanh(c+dx))}{(a^2-b^2)^4\,d}-\frac{b^4}{3a^3\,(a^2-b^2)\,d(b+a\tanh(c+dx))^3}+\frac{b^3(2a^2-b^2)}{a^3\,(a^2-b^2)^2\,d(b+a\tanh(c+dx))^2}-\frac{b^2(6a^4-3a^2b^2+b^4)}{a^3\,(a^2-b^2)^3\,d(b+a\tanh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Coth[c + d*x])^(-4),x]
```

```
[Out] -1/2*Log[1 - Tanh[c + d*x]]/((a + b)^4*d) + Log[1 + Tanh[c + d*x]]/(2*(a -
b)^4*d) - (4*a*b*(a^2 + b^2)*Log[b + a*Tanh[c + d*x]])/((a^2 - b^2)^4*d) -
b^4/(3*a^3*(a^2 - b^2)*d*(b + a*Tanh[c + d*x])^3) + (b^3*(2*a^2 - b^2))/(a^
3*(a^2 - b^2)^2*d*(b + a*Tanh[c + d*x])^2) - (b^2*(6*a^4 - 3*a^2*b^2 + b^4)
)/(a^3*(a^2 - b^2)^3*d*(b + a*Tanh[c + d*x]))
```

Maple [A]
time = 0.93, size = 163, normalized size = 0.96

method	result
derivativedivides	$\frac{\frac{b}{3(a-b)(a+b)(a+b\coth(dx+c))^3}+\frac{ab}{(a+b)^2(a-b)^2(a+b\coth(dx+c))^2}+\frac{b\left(3a^2+b^2\right)}{(a+b)^3(a-b)^3(a+b\coth(dx+c))}-\frac{4ba\left(a^2+b^2\right)\ln(a+b\coth(dx+c}{(a+b)^4(a-b)^4}}{d}$
default	$\frac{\frac{b}{3(a-b)(a+b)(a+b\coth(dx+c))^3}+\frac{ab}{(a+b)^2(a-b)^2(a+b\coth(dx+c))^2}+\frac{b\left(3a^2+b^2\right)}{(a+b)^3(a-b)^3(a+b\coth(dx+c))}-\frac{4ba\left(a^2+b^2\right)\ln(a+b\coth(dx+c}{(a+b)^4(a-b)^4}}{d}$
risch	$\frac{x}{a^4+4a^3b+6a^2b^2+4a\,b^3+b^4}+\frac{8b\,a^3x}{a^8-4a^6b^2+6b^4a^4-4a^2b^6+b^8}+\frac{8b^3ax}{a^8-4a^6b^2+6b^4a^4-4a^2b^6+b^8}+\frac{8b\,a^3c}{d(a^8-4a^6b^2+6b^4a^4-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*coth(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*b/(a-b)/(a+b)/(a+b*coth(d*x+c))^3+a*b/(a+b)^2/(a-b)^2/(a+b*coth(d*
x+c))^2+b*(3*a^2+b^2)/(a+b)^3/(a-b)^3/(a+b*coth(d*x+c))-4*b*a*(a^2+b^2)/(a+
b)^4/(a-b)^4*ln(a+b*coth(d*x+c))+1/2/(a-b)^4*ln(coth(d*x+c)+1)-1/2/(a+b)^4*
ln(coth(d*x+c)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 522 vs. $2(167) = 334$.
time = 0.33, size = 522, normalized size = 3.09

$$-\frac{4\,(a^3b+ab^3)\log\left(-(a-b)e^{(-2\,dx-2\,c)}+a+b\right)}{(a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8)d}-\frac{4\,\left(9\,a^4b^2+18\,a^3b^3+11\,a^2b^4+4\,ab^5+2\,b^6-3\,(6\,a^4b^2+2\,a^3b^3-5\,a^2b^4-2\,ab^5-b^6)e^{(-2\,dx-2\,c)}+3\,(3\,a^4b^2-4\,a^3b^3+b^6)e^{(-4\,dx-4\,c)}\right)}{3\,\left(a^{10}+2\,a^9b-3\,a^8b^2-8\,a^7b^3+2\,a^6b^4+12\,a^5b^5+2\,a^4b^6-8\,a^3b^7-3\,a^2b^8+2\,ab^9+b^{10}-3\,(a^{10}-5\,a^8b^2+10\,a^6b^4-10\,a^4b^6+5\,a^2b^8-b^{10})e^{(-2\,dx-2\,c)}+3\,(a^{10}-2\,a^9b-3\,a^8b^2+8\,a^7b^3+2\,a^6b^4-12\,a^5b^5+2\,a^4b^6+8\,a^3b^7-3\,a^2b^8-2\,ab^9+b^{10})e^{(-4\,dx-4\,c)}-(a^{10}-4\,a^9b+3\,a^8b^2+8\,a^7b^3-14\,a^6b^4+14\,a^4b^6-8\,a^3b^7-3\,a^2b^8+4\,ab^9-b^{10})e^{(-6\,dx-6\,c)}\right)d}+\frac{dx+c}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -4*(a^3*b + a*b^3)*log(-(a - b)*e^(-2*d*x - 2*c) + a + b)/((a^8 - 4*a^6*b^2
 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d) - 4/3*(9*a^4*b^2 + 18*a^3*b^3 + 11*a^2*b
^4 + 4*a*b^5 + 2*b^6 - 3*(6*a^4*b^2 + 2*a^3*b^3 - 5*a^2*b^4 - 2*a*b^5 - b^6
)*e^(-2*d*x - 2*c) + 3*(3*a^4*b^2 - 4*a^3*b^3 + b^6)*e^(-4*d*x - 4*c))/((a^
10 + 2*a^9*b - 3*a^8*b^2 - 8*a^7*b^3 + 2*a^6*b^4 + 12*a^5*b^5 + 2*a^4*b^6 -
 8*a^3*b^7 - 3*a^2*b^8 + 2*a*b^9 + b^10 - 3*(a^10 - 5*a^8*b^2 + 10*a^6*b^4
- 10*a^4*b^6 + 5*a^2*b^8 - b^10)*e^(-2*d*x - 2*c) + 3*(a^10 - 2*a^9*b - 3*a
^8*b^2 + 8*a^7*b^3 + 2*a^6*b^4 - 12*a^5*b^5 + 2*a^4*b^6 + 8*a^3*b^7 - 3*a^2
*b^8 - 2*a*b^9 + b^10)*e^(-4*d*x - 4*c) - (a^10 - 4*a^9*b + 3*a^8*b^2 + 8*a
^7*b^3 - 14*a^6*b^4 + 14*a^4*b^6 - 8*a^3*b^7 - 3*a^2*b^8 + 4*a*b^9 - b^10)*
e^(-6*d*x - 6*c))*d) + (d*x + c)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^
4)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3698 vs. $2(167) = 334$.
time = 0.41, size = 3698, normalized size = 21.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*coth(d*x+c))^4,x, algorithm="fricas")

[Out] 1/3*(3*(a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a^4*b^3 + 35*a^3*b^4 + 21*a^2*b^5 + 7*a*b^6 + b^7)*d*x*cosh(d*x + c)^6 + 18*(a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a^4*b^3 + 35*a^3*b^4 + 21*a^2*b^5 + 7*a*b^6 + b^7)*d*x*cosh(d*x + c)*sinh(d*x + c)^5 + 3*(a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a^4*b^3 + 35*a^3*b^4 + 21*a^2*b^5 + 7*a*b^6 + b^7)*d*x*sinh(d*x + c)^6 - 36*a^5*b^2 + 108*a^4*b^3 - 116*a^3*b^4 + 60*a^2*b^5 - 24*a*b^6 + 8*b^7 - 3*(12*a^5*b^2 + 4*a^4*b^3 - 16*a^3*b^4 + 4*a*b^6 - 4*b^7 + 3*(a^7 + 5*a^6*b + 9*a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - 9*a^2*b^5 - 5*a*b^6 - b^7)*d*x)*cosh(d*x + c)^4 - 3*(12*a^5*b^2 + 4*a^4*b^3 - 16*a^3*b^4 + 4*a*b^6 - 4*b^7 - 15*(a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a^4*b^3 + 35*a^3*b^4 + 21*a^2*b^5 + 7*a*b^6 + b^7)*d*x*cosh(d*x + c)^2 + 3*(a^7 + 5*a^6*b + 9*a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - 9*a^2*b^5 - 5*a*b^6 - b^7)*d*x)*sinh(d*x + c)^4 + 12*(5*(a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a^4*b^3 + 35*a^3*b^4 + 21*a^2*b^5 + 7*a*b^6 + b^7)*d*x*cosh(d*x + c)^3 - (12*a^5*b^2 + 4*a^4*b^3 - 16*a^3*b^4 + 4*a*b^6 - 4*b^7 + 3*(a^7 + 5*a^6*b + 9*a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - 9*a^2*b^5 - 5*a*b^6 - b^7)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 - 3*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7)*d*x + 3*(24*a^5*b^2 - 32*a^4*b^3 - 12*a^3*b^4 + 28*a^2*b^5 - 12*a*b^6 + 4*b^7 + 3*(a^7 + 3*a^6*b + a^5*b^2 - 5*a^4*b^3 - 5*a^3*b^4 + a^2*b^5 + 3*a*b^6 + b^7)*d*x)*cosh(d*x + c)^2 + 3*(24*a^5*b^2 - 32*a^4*b^3 - 12*a^3*b^4 + 28*a^2*b^5 - 12*a*b^6 + 4*b^7 + 15*(a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a^4*b^3 + 35*a^3*b^4 + 21*a^2*b^5 + 7*a*b^6 + b^7)*d*x*cosh(d*x + c)^4 + 3*(a^7 + 3*a^6*b + a^5*b^2 - 5*a^4*b^3 - 5*a^3*b^4 + a^2*b^5 + 3*a*b^6 + b^7)*d*x - 6*(12*a^5*b^2 + 4*a^4*b^3 - 16*a^3*b^4 + 4*a*b^6 - 4*b^7 + 3*(a^7 + 5*a^6*b + 9*a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - 9*a^2*b^5 - 5*a*b^6 - b^7)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 12*(a^6*b - 3*a^5*b^2 + 4*a^4*b^3 - 4*a^3*b^4 + 3*a^2*b^5 - a*b^6 - (a^6*b + 3*a^5*b^2 + 4*a^4*b^3 + 4*a^3*b^4 + 3*a^2*b^5 + a*b^6)*cosh(d*x + c)^6 - 6*(a^6*b + 3*a^5*b^2 + 4*a^4*b^3 + 4*a^3*b^4 + 3*a^2*b^5 + a*b^6)*cosh(d*x + c)*sinh(d*x + c)^5 - (a^6*b + 3*a^5*b^2 + 4*a^4*b^3 + 4*a^3*b^4 + 3*a^2*b^5 + a*b^6)*sinh(d*x + c)^6 + 3*(a^6*b + a^5*b^2 - a^2*b^5 - a*b^6)*cosh(d*x + c)^4 + 3*(a^6*b + a^5*b^2 - a^2*b^5 - a*b^6 - 5*(a^6*b + 3*a^5*b^2 + 4*a^4*b^3 + 4*a^3*b^4 + 3*a^2*b^5 + a*b^6)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 4*(5*(a^6*b + 3*a^5*b^2 + 4*a^4*b^3 + 4*a^3*b^4 + 3*a^2*b^5 + a*b^6)*cosh(d*x + c)^3 - 3*(a^6*b + a^5*b^2 - a^2*b^5 - a*b^6)*cosh(d*x + c))*sinh(d*x + c)^3 - 3*(a^6*b - a^5*b^2 - a^2*b^5 + a*b^6)*cosh(d*x + c)^2 - 3*(a^6*b - a^5*b^2 - a^2*b^5 + a*b^6 + 5*(a^6*b + 3*a^5*b^2 + 4*a^4*b^3 + 4*a^3*b^4 + 3*a^2*b^5 + a*b^6)*cosh(d*x + c)^4 - 6*(a^6*b + a^5*b^2 - a^2*b^5 - a*b^6)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 6*((a^6*b + 3*a^5*b^2 + 4*a^4*b^3 + 4*a^3*b^4 + 3*a^2*b^5 + a*b^6)*cosh(d*x + c)^5 - 2*(a^6*b + a^5*b^2 - a^2*b^5 - a*b^6)*cosh(d*x + c)^3 + (a^6*b - a^5*b^2 - a^2*b^5 + a*b^6)*cosh(d*x + c))*sinh(d*x + c))*log(2*(b*cosh(d*x + c) + a*sinh(d*x + c))/(cosh(d*x + c) - sinh(d*x + c))) + 6*(3*

```
(a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a^4*b^3 + 35*a^3*b^4 + 21*a^2*b^5 + 7*a*b^
6 + b^7)*d*x*cosh(d*x + c)^5 - 2*(12*a^5*b^2 + 4*a^4*b^3 - 16*a^3*b^4 + 4*a
*b^6 - 4*b^7 + 3*(a^7 + 5*a^6*b + 9*a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - 9*a^2
*b^5 - 5*a*b^6 - b^7)*d*x)*cosh(d*x + c)^3 + (24*a^5*b^2 - 32*a^4*b^3 - 12*
a^3*b^4 + 28*a^2*b^5 - 12*a*b^6 + 4*b^7 + 3*(a^7 + 3*a^6*b + a^5*b^2 - 5*a^
4*b^3 - 5*a^3*b^4 + a^2*b^5 + 3*a*b^6 + b^7)*d*x)*cosh(d*x + c))*sinh(d*x +
 c))/((a^11 + 3*a^10*b - a^9*b^2 - 11*a^8*b^3 - 6*a^7*b^4 + 14*a^6*b^5 + 14
*a^5*b^6 - 6*a^4*b^7 - 11*a^3*b^8 - a^2*b^9 + 3*a*b^10 + b^11)*d*cosh(d*x +
 c)^6 + 6*(a^11 + 3*a^10*b - a^9*b^2 - 11*a^8*b^3 - 6*a^7*b^4 + 14*a^6*b^5
+ 14*a^5*b^6 - 6*a^4*b^7 - 11*a^3*b^8 - a^2*b^9 + 3*a*b^10 + b^11)*d*cosh(d
*x + c)*sinh(d*x + c)^5 + (a^11 + 3*a^10*b - a^9*b^2 - 11*a^8*b^3 - 6*a^7*b
^4 + 14*a^6*b^5 + 14*a^5*b^6 - 6*a^4*b^7 - 11*a^3*b^8 - a^2*b^9 + 3*a*b^10
+ b^11)*d*sinh(d*x + c)^6 - 3*(a^11 + a^10*b - 5*a^9*b^2 - 5*a^8*b^3 + 10*a
^7*b^4 + 10*a^6*b^5 - 10*a^5*b^6 - 10*a^4*b^7 + 5*a^3*b^8 + 5*a^2*b^9 - a*b
^10 - b^11)*d*cosh(d*x + c)^4 + 3*(5*(a^11 + 3*a^10*b - a^9*b^2 - 11*a^8*b^
3 - 6*a^7*b^4 + 14*a^6*b^5 + 14*a^5*b^6 - 6*a^4*b^7 - 11*a^3*b^8 - a^2*b^9
+ 3*a*b^10 + b^11)*d*cosh(d*x + c)^2 - (a^11 + a^10*b - 5*a^9*b^2 - 5*a^8*b
^3 + 10*a^7*b^4 + 10*a^6*b^5 - 10*a^5*b^6 - 10*a^4*b^7 + 5*a^3*b^8 + 5*a^2*
b^9 - a*b^10 - b^11)*d)*sinh(d*x + c)^4 + 3*(a^11 - a^10*b - 5*a^9*b^2 + 5*
a^8*b^3 + 10*a^7*b^4 - 10*a^6*b^5 - 10*a^5*b^6 + 10*a^4*b^7 + 5*a^3*b^8 - 5
*a^2*b^9 - a*b^10 + b^11)*d*cosh(d*x + c)^2 + 4*(5*(a^11 + 3*a^10*b - a^9*b
^2 - 11*a^8*b^3 - 6*a^7*b^4 + 14*a^6*b^5 + 14*a^5*b^6 - 6*a^4*b^7 - 11*a^3*
b^8 - a^2*b^9 + 3*a*b^10 + b^11)*d*cosh(d*x + c)^3 - 3*(a^11 + a^10*b - 5*a
^9*b^2 - 5*a^8*b^3 + 10*a^7*b^4 + 10*a^6*b^5 - 10*a^5*b^6 - 10*a^4*b^7 + 5*
a^3*b^8 + 5*a^2*b^9 - a*b^10 - b^11)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(
5*(a^11 + 3*a^10*b - a^9*b^2 - 11*a^8*b^3 - 6*a...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.43, size = 303, normalized size = 1.79

$$-\frac{\frac{12\,\left(a^{3} b+a b^{3}\right) \log\left(\left|a e^{\left(2\,d x+2\,c\right)}+b e^{\left(2\,d x+2\,c\right)}-a+b\right|\right)}{a^{8}-4\,a^{6} b^{2}+6\,a^{4} b^{4}-4\,a^{2} b^{6}+b^{8}}-\frac{3\,\left(d x+c\right)}{a^{4}-4\,a^{3} b+6\,a^{2} b^{2}-4\,a b^{3}+b^{4}}+\frac{4\,\left(3\,\left(3\,a^{4} b^{2}-2\,a^{3} b^{3}-2\,a^{2} b^{4}+2\,a b^{5}-b^{6}\right) e^{\left(4\,d x+4\,c\right)}-3\,\left(6\,a^{4} b^{2}-14\,a^{3} b^{3}+11\,a^{2} b^{4}-4\,a b^{5}+b^{6}\right) e^{\left(2\,d x+2\,c\right)}+\frac{9\,a^{5} b^{2}-27\,a^{4} b^{3}+29\,a^{3} b^{4}-15\,a^{2} b^{5}+6\,a b^{6}-2\,b^{7}}{a+b}\right)}{{\left(a e^{\left(2\,d x+2\,c\right)}+b e^{\left(2\,d x+2\,c\right)}-a+b\right)}^{3} {\left(a+b\right)}^{3} {\left(a-b\right)}^{4}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(12*(a^3*b + a*b^3)*log(abs(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) - a
+ b))/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) - 3*(d*x + c)/(a^4 -
4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) + 4*(3*(3*a^4*b^2 - 2*a^3*b^3 - 2*a^2*
b^4 + 2*a*b^5 - b^6)*e^(4*d*x + 4*c) - 3*(6*a^4*b^2 - 14*a^3*b^3 + 11*a^2*b
^4 - 4*a*b^5 + b^6)*e^(2*d*x + 2*c) + (9*a^5*b^2 - 27*a^4*b^3 + 29*a^3*b^4
- 15*a^2*b^5 + 6*a*b^6 - 2*b^7)/(a + b))/((a*e^(2*d*x + 2*c) + b*e^(2*d*x +
 2*c) - a + b)^3*(a + b)^3*(a - b)^4))/d
```

Mupad [B]
time = 1.39, size = 310, normalized size = 1.83

$$\frac{x}{(a-b)^4}-\frac{\ln\left(b-a+a\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+b\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\right)\,\left(4\,a^3\,b+4\,a\,b^3\right)}{d\,a^8-4\,d\,a^6\,b^2+6\,d\,a^4\,b^4-4\,d\,a^2\,b^6+d\,b^8}-\frac{4\,\left(3\,a^2\,b^2-2\,a\,b^3+b^4\right)}{d\,(a+b)^4\,(a-b)^3\,\left(b-a+\mathrm{e}^{2\,c+2\,d\,x}\,(a+b)\right)}-\frac{8\,b^4}{3\,d\,(a+b)^4\,(a-b)\,\left(\mathrm{e}^{6\,c+6\,d\,x}\,(a+b)^3-(a-b)^3+3\,\mathrm{e}^{2\,c+2\,d\,x}\,(a+b)\,(a-b)^2-3\,\mathrm{e}^{4\,c+4\,d\,x}\,(a+b)^2\,(a-b)\right)}+\frac{4\,\left(2\,a\,b^3-b^4\right)}{d\,(a+b)^4\,(a-b)^2\,\left(\mathrm{e}^{4\,c+4\,d\,x}\,(a+b)^2+(a-b)^2-2\,\mathrm{e}^{2\,c+2\,d\,x}\,(a+b)\,(a-b)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*coth(c + d*x))^4,x)
```

```
[Out] x/(a - b)^4 - (log(b - a + a*exp(2*c)*exp(2*d*x) + b*exp(2*c)*exp(2*d*x))*(
4*a*b^3 + 4*a^3*b))/(a^8*d + b^8*d - 4*a^2*b^6*d + 6*a^4*b^4*d - 4*a^6*b^2*
d) - (4*(b^4 - 2*a*b^3 + 3*a^2*b^2))/(d*(a + b)^4*(a - b)^3*(b - a + exp(2*
c + 2*d*x)*(a + b))) - (8*b^4)/(3*d*(a + b)^4*(a - b)*(exp(6*c + 6*d*x)*(a
+ b)^3 - (a - b)^3 + 3*exp(2*c + 2*d*x)*(a + b)*(a - b)^2 - 3*exp(4*c + 4*d
*x)*(a + b)^2*(a - b))) + (4*(2*a*b^3 - b^4))/(d*(a + b)^4*(a - b)^2*(exp(4
*c + 4*d*x)*(a + b)^2 + (a - b)^2 - 2*exp(2*c + 2*d*x)*(a + b)*(a - b)))
```

3.85 $\int \frac{1}{4+6\coth(c+dx)}\,dx$

Optimal. Leaf size=31

$$-\frac{x}{5}+\frac{3\log(3\cosh(c+dx)+2\sinh(c+dx))}{10d}$$

```
[Out] -1/5*x+3/10*ln(3*cosh(d*x+c)+2*sinh(d*x+c))/d
```

Rubi [A]
time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3565, 3611}

$$\frac{3\log(2\sinh(c+dx)+3\cosh(c+dx))}{10d}-\frac{x}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 6*Coth[c + d*x])^(-1),x]
```

```
[Out] -1/5*x + (3*Log[3*Cosh[c + d*x] + 2*Sinh[c + d*x]])/(10*d)
```

Rule 3565

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[a*(x/(a^
2 + b^2)), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{4+6\coth(c+dx)}\,dx&=-\frac{x}{5}+\frac{3}{10}i\int \frac{-6i-4i\coth(c+dx)}{4+6\coth(c+dx)}\,dx\\&=-\frac{x}{5}+\frac{3\log(3\cosh(c+dx)+2\sinh(c+dx))}{10d}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 55, normalized size = 1.77

$$-\frac{\log(6-6\coth(c+dx))}{20d}+\frac{3\log(4+6\coth(c+dx))}{10d}-\frac{\log(6+6\coth(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 6*Coth[c + d*x])^(-1),x]
```

```
[Out] -1/20*Log[6 - 6*Coth[c + d*x]]/d + (3*Log[4 + 6*Coth[c + d*x]])/(10*d) - Lo
g[6 + 6*Coth[c + d*x]]/(4*d)
```

Maple [A]
time = 0.75, size = 42, normalized size = 1.35

method	result	size
risch	$-\frac{x}{2}-\frac{3c}{5d}+\frac{3\ln\left(\mathrm{e}^{2dx+2c}+\frac{1}{5}\right)}{10d}$	28
derivativedivides	$\frac{-\frac{\ln(\coth(dx+c)+1)}{2}-\frac{\ln(\coth(dx+c)-1)}{10}+\frac{3\ln(2+3\coth(dx+c))}{5}}{2d}$	42
default	$\frac{-\frac{\ln(\coth(dx+c)+1)}{2}-\frac{\ln(\coth(dx+c)-1)}{10}+\frac{3\ln(2+3\coth(dx+c))}{5}}{2d}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4+6*coth(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(-1/2*ln(coth(d*x+c)+1)-1/10*ln(coth(d*x+c)-1)+3/5*ln(2+3*coth(d*x+c)
))
```

Maxima [A]
time = 0.26, size = 28, normalized size = 0.90

$$\frac{dx+c}{10\,d}+\frac{3\,\log\left(e^{(-2\,dx-2\,c)}+5\right)}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+6*coth(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/10*(d*x + c)/d + 3/10*log(e^(-2*d*x - 2*c) + 5)/d
```

Fricas [A]
time = 0.37, size = 49, normalized size = 1.58

$$-\frac{5\,dx-3\,\log\left(\frac{2\,(3\,\cosh(dx+c)+2\,\sinh(dx+c))}{\cosh(dx+c)-\sinh(dx+c)}\right)}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+6*coth(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/10*(5*d*x - 3*log(2*(3*cosh(d*x + c) + 2*sinh(d*x + c))/(cosh(d*x + c) -
 sinh(d*x + c))))/d
```

Sympy [A]
time = 0.42, size = 42, normalized size = 1.35

$$\begin{cases} \frac{x}{10} - \frac{3\log(\tanh(c+dx)+1)}{10d} + \frac{3\log(2\tanh(c+dx)+3)}{10d} & \text{for } d \neq 0 \\ \frac{x}{6\coth(c)+4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+6*coth(d*x+c)),x)
```

```
[Out] Piecewise((x/10 - 3*log(tanh(c + d*x) + 1)/(10*d) + 3*log(2*tanh(c + d*x) +
 3)/(10*d), Ne(d, 0)), (x/(6*coth(c) + 4), True))
```

Giac [A]
time = 0.41, size = 29, normalized size = 0.94

$$-\frac{5\,dx + 5\,c - 3\,\log\left(5\,e^{(2\,dx+2\,c)} + 1\right)}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+6*coth(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/10*(5*d*x + 5*c - 3*log(5*e^(2*d*x + 2*c) + 1))/d
```

Mupad [B]
time = 0.04, size = 22, normalized size = 0.71

$$\frac{3\,\ln\left(\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x} + \frac{1}{5}\right)}{10\,d} - \frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(6*coth(c + d*x) + 4),x)
```

```
[Out] (3*log(exp(2*c)*exp(2*d*x) + 1/5))/(10*d) - x/2
```

3.86 $\int \frac{1}{4-6\coth(c+dx)}\,dx$

Optimal. Leaf size=31

$$-\frac{x}{5}-\frac{3\log(3\cosh(c+dx)-2\sinh(c+dx))}{10d}$$

```
[Out] -1/5*x-3/10*ln(3*cosh(d*x+c)-2*sinh(d*x+c))/d
```

Rubi [A]
time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3565, 3611}

$$-\frac{3\log(3\cosh(c+dx)-2\sinh(c+dx))}{10d}-\frac{x}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(4 - 6*Coth[c + d*x])^(-1),x]
```

```
[Out] -1/5*x - (3*Log[3*Cosh[c + d*x] - 2*Sinh[c + d*x]])/(10*d)
```

Rule 3565

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[a*(x/(a^
2 + b^2)), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{4-6\coth(c+dx)}\,dx &= -\frac{x}{5}-\frac{3}{10}i\int \frac{6i-4i\coth(c+dx)}{4-6\coth(c+dx)}\,dx\\ &= -\frac{x}{5}-\frac{3\log(3\cosh(c+dx)-2\sinh(c+dx))}{10d}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 55, normalized size = 1.77

$$-\frac{3\log(4-6\coth(c+dx))}{10d}+\frac{\log(6-6\coth(c+dx))}{4d}+\frac{\log(6+6\coth(c+dx))}{20d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 - 6*Coth[c + d*x])^(-1),x]
```

```
[Out] (-3*Log[4 - 6*Coth[c + d*x]])/(10*d) + Log[6 - 6*Coth[c + d*x]]/(4*d) + Log
[6 + 6*Coth[c + d*x]]/(20*d)
```

Maple [A]
time = 0.75, size = 42, normalized size = 1.35

method	result	size
risch	$\frac{x}{10}+\frac{3c}{5d}-\frac{3\ln\left(\mathrm{e}^{2dx+2c}+5\right)}{10d}$	28
derivativedivides	$\frac{\frac{\ln(\coth(dx+c)-1)}{2}-\frac{3\ln(-2+3\coth(dx+c))}{5}+\frac{\ln(\coth(dx+c)+1)}{10}}{2d}$	42
default	$\frac{\frac{\ln(\coth(dx+c)-1)}{2}-\frac{3\ln(-2+3\coth(dx+c))}{5}+\frac{\ln(\coth(dx+c)+1)}{10}}{2d}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4-6*coth(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(1/2*ln(coth(d*x+c)-1)-3/5*ln(-2+3*coth(d*x+c))+1/10*ln(coth(d*x+c)+1
))
```

Maxima [A]
time = 0.26, size = 29, normalized size = 0.94

$$-\frac{1}{2}\,x-\frac{c}{2\,d}-\frac{3\,\log\left(5\,e^{(-2\,dx-2\,c)}+1\right)}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-6*coth(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*x - 1/2*c/d - 3/10*log(5*e^(-2*d*x - 2*c) + 1)/d
```

Fricas [A]
time = 0.38, size = 48, normalized size = 1.55

$$\frac{dx-3\,\log\left(\frac{2\,(3\,\cosh(dx+c)-2\,\sinh(dx+c))}{\cosh(dx+c)-\sinh(dx+c)}\right)}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-6*coth(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/10*(d*x - 3*log(2*(3*cosh(d*x + c) - 2*sinh(d*x + c))/(cosh(d*x + c) - si
nh(d*x + c))))/d
```

Sympy [A]
time = 0.42, size = 42, normalized size = 1.35

$$\begin{cases} -\frac{x}{2} + \frac{3\log\left(\tanh\left(c+dx\right)+1\right)}{10d} - \frac{3\log\left(2\tanh\left(c+dx\right)-3\right)}{10d} & \text{for } d \neq 0 \\ \frac{x}{4-6\coth\left(c\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-6*coth(d*x+c)),x)
```

```
[Out] Piecewise((-x/2 + 3*log(tanh(c + d*x) + 1)/(10*d) - 3*log(2*tanh(c + d*x) -
 3)/(10*d), Ne(d, 0)), (x/(4 - 6*coth(c)), True))
```

Giac [A]
time = 0.43, size = 24, normalized size = 0.77

$$\frac{dx + c - 3\,\log\left(e^{(2\,dx+2\,c)} + 5\right)}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-6*coth(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/10*(d*x + c - 3*log(e^(2*d*x + 2*c) + 5))/d
```

Mupad [B]
time = 0.04, size = 22, normalized size = 0.71

$$\frac{x}{10} - \frac{3\,\ln\left(e^{2\,c}\,e^{2\,d\,x} + 5\right)}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(6*coth(c + d*x) - 4),x)
```

```
[Out] x/10 - (3*log(exp(2*c)*exp(2*d*x) + 5))/(10*d)
```

3.87 $\int \sqrt{a+b\coth(c+dx)}\,dx$

Optimal. Leaf size=74

$$-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a-b}}\right)}{d}+\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a+b}}\right)}{d}$$

```
[Out] -arctanh((a+b*coth(d*x+c))^(1/2)/(a-b)^(1/2))*(a-b)^(1/2)/d+arctanh((a+b*co
th(d*x+c))^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)/d
```

Rubi [A]
time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3566, 714, 1144, 213}

$$\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a+b}}\right)}{d}-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a-b}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Coth[c + d*x]],x]
```

```
[Out] -((Sqrt[a - b]*ArcTanh[Sqrt[a + b*Coth[c + d*x]]/Sqrt[a - b]])/d) + (Sqrt[a
 + b]*ArcTanh[Sqrt[a + b*Coth[c + d*x]]/Sqrt[a + b]])/d
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 714

```
Int[Sqrt[(d_) + (e_.)*(x_)]/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[2*e, S
ubst[Int[x^2/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x]
 /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\coth(c+dx)}\,dx = -\frac{b\text{Subst}\left(\int \frac{\sqrt{a+x}}{-b^2+x^2}\,dx, x, b\coth(c+dx)\right)}{d}$$
$$= -\frac{(2b)\text{Subst}\left(\int \frac{x^2}{a^2-b^2-2ax^2+x^4}\,dx, x, \sqrt{a+b\coth(c+dx)}\right)}{d}$$
$$= \frac{(a-b)\text{Subst}\left(\int \frac{1}{-a+b+x^2}\,dx, x, \sqrt{a+b\coth(c+dx)}\right)}{d} - \frac{(a+b)\text{Subst}\left(\int \frac{}{-a\cdots}\right.}{}$$
$$= -\frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a-b}}\right)}{d} + \frac{\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\coth\cdots}}{\sqrt{a-\cdots}}\right.}{d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.83, size = 128, normalized size = 1.73

$$\frac{\left(-\sqrt{i(a-b)}\,\tanh^{-1}\left(\frac{\sqrt{i(a+b\coth(c+dx))}}{\sqrt{i(a-b)}}\right)+\sqrt{i(a+b)}\,\tanh^{-1}\left(\frac{\sqrt{i(a+b\coth(c+dx))}}{\sqrt{i(a+b)}}\right)\right)\sqrt{a+b\coth(c+dx)}}{d\sqrt{i(a+b\coth(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Coth[c + d*x]],x]
```

```
[Out] ((-(Sqrt[I*(a - b)]*ArcTanh[Sqrt[I*(a + b*Coth[c + d*x])]/Sqrt[I*(a - b)]])
 + Sqrt[I*(a + b)]*ArcTanh[Sqrt[I*(a + b*Coth[c + d*x])]/Sqrt[I*(a + b)]])*
Sqrt[a + b*Coth[c + d*x]])/(d*Sqrt[I*(a + b*Coth[c + d*x])])
```

Maple [A]
time = 2.33, size = 70, normalized size = 0.95

method	result
derivativedivides	$-\frac{2b\left(-\frac{\sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b\coth(dx+c)}}{\sqrt{a+b}}\right)}{2b}+\frac{\sqrt{-a+b}\,\arctan\left(\frac{\sqrt{a+b\coth(dx+c)}}{\sqrt{-a+b}}\right)}{2b}\right.}{d}$

default	$-\frac{2b\left(-\frac{\sqrt{a+b}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\coth(dx+c)}}{\sqrt{a+b}}\right)}{2b}+\frac{\sqrt{-a+b}\,\arctan\left(\frac{\sqrt{a+b\coth(dx+c)}}{\sqrt{-a+b}}\right)}{2b}\right)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*coth(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*b*(-1/2*(a+b)^(1/2)/b*arctanh((a+b*coth(d*x+c))^(1/2)/(a+b)^(1/2))+1/2
*(-a+b)^(1/2)/b*arctan((a+b*coth(d*x+c))^(1/2)/(-a+b)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*coth(d*x + c) + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(62) = 124$.
time = 0.46, size = 2231, normalized size = 30.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a + b)*log(2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 8*(a^2 + 2*a*
b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a^2 + 2*a*b + b^2)*sinh(d*x + c
)^4 - 4*(a^2 + a*b)*cosh(d*x + c)^2 + 4*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c
)^2 - a^2 - a*b)*sinh(d*x + c)^2 + 2*a^2 - b^2 + 2*((a + b)*cosh(d*x + c)^4
 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 - (2*a
 + b)*cosh(d*x + c)^2 + (6*(a + b)*cosh(d*x + c)^2 - 2*a - b)*sinh(d*x + c)
^2 + 2*(2*(a + b)*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d*x + c)
+ a)*sqrt(a + b)*sqrt((b*cosh(d*x + c) + a*sinh(d*x + c))/sinh(d*x + c)) +
8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - (a^2 + a*b)*cosh(d*x + c))*sinh(d*
x + c)) + sqrt(a - b)*log(((2*a^2 - b^2)*cosh(d*x + c)^4 + 4*(2*a^2 - b^2)*
cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2 - b^2)*sinh(d*x + c)^4 - 4*(a^2 - a*
b)*cosh(d*x + c)^2 + 2*(3*(2*a^2 - b^2)*cosh(d*x + c)^2 - 2*a^2 + 2*a*b)*si
nh(d*x + c)^2 + 2*a^2 - 4*a*b + 2*b^2 - 2*(a*cosh(d*x + c)^4 + 4*a*cosh(d*x
 + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - (2*a - b)*cosh(d*x + c)^2 + (6*
```

```
a*cosh(d*x + c)^2 - 2*a + b)*sinh(d*x + c)^2 + 2*(2*a*cosh(d*x + c)^3 - (2*
a - b)*cosh(d*x + c))*sinh(d*x + c) + a - b)*sqrt(a - b)*sqrt((b*cosh(d*x +
 c) + a*sinh(d*x + c))/sinh(d*x + c)) + 4*((2*a^2 - b^2)*cosh(d*x + c)^3 -
2*(a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^4 + 4*cosh(d*x +
 c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*s
inh(d*x + c)^3 + sinh(d*x + c)^4)))/d, -1/4*(2*sqrt(-a - b)*arctan(((a + b)
*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x
 + c)^2 - a)*sqrt(-a - b)*sqrt((b*cosh(d*x + c) + a*sinh(d*x + c))/sinh(d*x
 + c))/((a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 2*(a^2 + 2*a*b + b^2)*cosh(d*
x + c)*sinh(d*x + c) + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^2 - a^2 + b^2)) -
sqrt(a - b)*log(((2*a^2 - b^2)*cosh(d*x + c)^4 + 4*(2*a^2 - b^2)*cosh(d*x +
 c)*sinh(d*x + c)^3 + (2*a^2 - b^2)*sinh(d*x + c)^4 - 4*(a^2 - a*b)*cosh(d*
x + c)^2 + 2*(3*(2*a^2 - b^2)*cosh(d*x + c)^2 - 2*a^2 + 2*a*b)*sinh(d*x + c
)^2 + 2*a^2 - 4*a*b + 2*b^2 - 2*(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh
(d*x + c)^3 + a*sinh(d*x + c)^4 - (2*a - b)*cosh(d*x + c)^2 + (6*a*cosh(d*x
 + c)^2 - 2*a + b)*sinh(d*x + c)^2 + 2*(2*a*cosh(d*x + c)^3 - (2*a - b)*cos
h(d*x + c))*sinh(d*x + c) + a - b)*sqrt(a - b)*sqrt((b*cosh(d*x + c) + a*si
nh(d*x + c))/sinh(d*x + c)) + 4*((2*a^2 - b^2)*cosh(d*x + c)^3 - 2*(a^2 - a
*b)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh
(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x +
c)^3 + sinh(d*x + c)^4)))/d, -1/4*(2*sqrt(-a + b)*arctan(-(a*cosh(d*x + c)^
2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 - a + b)*sqrt(-a +
b)*sqrt((b*cosh(d*x + c) + a*sinh(d*x + c))/sinh(d*x + c))/((a^2 - b^2)*cos
h(d*x + c)^2 + 2*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*sinh
(d*x + c)^2 - a^2 + 2*a*b - b^2)) - sqrt(a + b)*log(2*(a^2 + 2*a*b + b^2)*c
osh(d*x + c)^4 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a
^2 + 2*a*b + b^2)*sinh(d*x + c)^4 - 4*(a^2 + a*b)*cosh(d*x + c)^2 + 4*(3*(a
^2 + 2*a*b + b^2)*cosh(d*x + c)^2 - a^2 - a*b)*sinh(d*x + c)^2 + 2*a^2 - b^
2 + 2*((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 +
(a + b)*sinh(d*x + c)^4 - (2*a + b)*cosh(d*x + c)^2 + (6*(a + b)*cosh(d*x +
 c)^2 - 2*a - b)*sinh(d*x + c)^2 + 2*(2*(a + b)*cosh(d*x + c)^3 - (2*a + b)
*cosh(d*x + c))*sinh(d*x + c) + a)*sqrt(a + b)*sqrt((b*cosh(d*x + c) + a*si
nh(d*x + c))/sinh(d*x + c)) + 8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - (a^2
 + a*b)*cosh(d*x + c))*sinh(d*x + c)))/d, -1/2*(sqrt(-a + b)*arctan(-(a*cos
h(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 - a + b)
*sqrt(-a + b)*sqrt((b*cosh(d*x + c) + a*sinh(d*x + c))/sinh(d*x + c))/((a^2
 - b^2)*cosh(d*x + c)^2 + 2*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2
- b^2)*sinh(d*x + c)^2 - a^2 + 2*a*b - b^2)) + sqrt(-a - b)*arctan(((a + b)
*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x
 + c)^2 - a)*sqrt(-a - b)*sqrt((b*cosh(d*x + c) + a*sinh(d*x + c))/sinh(d*x
 + c))/((a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 2*(a^2 + 2*a*b + b^2)*cosh(d*
x + c)*sinh(d*x + c) + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^2 - a^2 + b^2)))/d
]
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \coth(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*coth(c + d*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(ex
```

Mupad [B]
time = 1.41, size = 151, normalized size = 2.04

$$\frac{\mathrm{atan}\left(\frac{b^2\sqrt{a-b}\,\sqrt{a+b\coth(c+d\,x)}\,1\mathrm{i}+a\,b\sqrt{a-b}\,\sqrt{a+b\coth(c+d\,x)}\,1\mathrm{i}}{a^2\,b-b^3}\right)\sqrt{a-b}\,1\mathrm{i}}{d}+\frac{\mathrm{atan}\left(\frac{b^2\sqrt{a+b}\,\sqrt{a+b\coth(c+d\,x)}\,1\mathrm{i}-a\,b\sqrt{a+b}\,\sqrt{a+b\coth(c+d\,x)}\,1\mathrm{i}}{a^2\,b-b^3}\right)\sqrt{a+b}\,1\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*coth(c + d*x))^(1/2),x)
```

```
[Out] (atan((b^2*(a - b)^(1/2)*(a + b*coth(c + d*x))^(1/2)*1i + a*b*(a - b)^(1/2)
*(a + b*coth(c + d*x))^(1/2)*1i)/(a^2*b - b^3))*(a - b)^(1/2)*1i)/d + (atan
((b^2*(a + b)^(1/2)*(a + b*coth(c + d*x))^(1/2)*1i - a*b*(a + b)^(1/2)*(a +
 b*coth(c + d*x))^(1/2)*1i)/(a^2*b - b^3))*(a + b)^(1/2)*1i)/d
```

3.88 $\int \frac{1}{\sqrt{a+b\coth(c+dx)}}\,dx$

Optimal. Leaf size=74

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,d}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}\,d}$$

```
[Out] -arctanh((a+b*coth(d*x+c))^(1/2)/(a-b)^(1/2))/d/(a-b)^(1/2)+arctanh((a+b*co
th(d*x+c))^(1/2)/(a+b)^(1/2))/d/(a+b)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3566, 722, 1107, 213}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a+b}}\right)}{d\sqrt{a+b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a-b}}\right)}{d\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Coth[c + d*x]],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Coth[c + d*x]]/Sqrt[a - b]]/(Sqrt[a - b]*d)) + ArcTanh
[Sqrt[a + b*Coth[c + d*x]]/Sqrt[a + b]]/(Sqrt[a + b]*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 722

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2*
e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]],
x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\coth(c+dx)}}\,dx = -\frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a+x}\,(-b^2+x^2)}\,dx, x, b\coth(c+dx)\right)}{d}$$

$$= -\frac{(2b)\text{Subst}\left(\int \frac{1}{a^2-b^2-2ax^2+x^4}\,dx, x, \sqrt{a+b\coth(c+dx)}\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{-a-b+x^2}\,dx, x, \sqrt{a+b\coth(c+dx)}\right)}{d} + \frac{\text{Subst}\left(\int \frac{1}{-a+b+x^2}\,dx, x, \sqrt{}\right.}{d}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,d} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}\,d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.00, size = 129, normalized size = 1.74

$$-\frac{\left(\frac{\tanh^{-1}\left(\frac{\sqrt{i(a+b\coth(c+dx))}}{\sqrt{i(a-b)}}\right)}{\sqrt{i(a-b)}} - \frac{\tanh^{-1}\left(\frac{\sqrt{i(a+b\coth(c+dx))}}{\sqrt{i(a+b)}}\right)}{\sqrt{i(a+b)}}\right)\sqrt{i(a+b\coth(c+dx))}}{d\sqrt{a+b\coth(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Coth[c + d*x]],x]
```

```
[Out] -(((ArcTanh[Sqrt[I*(a + b*Coth[c + d*x])]/Sqrt[I*(a - b)]]/Sqrt[I*(a - b)]
 - ArcTanh[Sqrt[I*(a + b*Coth[c + d*x])]/Sqrt[I*(a + b)]]/Sqrt[I*(a + b)])*S
qrt[I*(a + b*Coth[c + d*x])])/(d*Sqrt[a + b*Coth[c + d*x]]))
```

Maple [A]
time = 1.79, size = 70, normalized size = 0.95

method	result	size

derivativedivides	$-\frac{2b\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\coth(dx+c)}}{\sqrt{a+b}}\right)}{2b\sqrt{a+b}}-\frac{\arctan\left(\frac{\sqrt{a+b\coth(dx+c)}}{\sqrt{-a+b}}\right)}{2b\sqrt{-a+b}}\right)}{d}$	70
default	$-\frac{2b\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\coth(dx+c)}}{\sqrt{a+b}}\right)}{2b\sqrt{a+b}}-\frac{\arctan\left(\frac{\sqrt{a+b\coth(dx+c)}}{\sqrt{-a+b}}\right)}{2b\sqrt{-a+b}}\right)}{d}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*coth(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*b*(-1/2/b/(a+b)^(1/2)*arctanh((a+b*coth(d*x+c))^(1/2)/(a+b)^(1/2))-1/2
/b/(-a+b)^(1/2)*arctan((a+b*coth(d*x+c))^(1/2)/(-a+b)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*coth(d*x + c) + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(62) = 124$.
time = 0.45, size = 2307, normalized size = 31.18

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a + b)*(a - b)*log(2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 8*(a^
2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a^2 + 2*a*b + b^2)*sinh
(d*x + c)^4 - 4*(a^2 + a*b)*cosh(d*x + c)^2 + 4*(3*(a^2 + 2*a*b + b^2)*cosh
(d*x + c)^2 - a^2 - a*b)*sinh(d*x + c)^2 + 2*a^2 - b^2 + 2*((a + b)*cosh(d*
x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^
4 - (2*a + b)*cosh(d*x + c)^2 + (6*(a + b)*cosh(d*x + c)^2 - 2*a - b)*sinh(
d*x + c)^2 + 2*(2*(a + b)*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d
*x + c) + a)*sqrt(a + b)*sqrt((b*cosh(d*x + c) + a*sinh(d*x + c))/sinh(d*x
```

```
+ c)) + 8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - (a^2 + a*b)*cosh(d*x + c))
*sinh(d*x + c)) + (a + b)*sqrt(a - b)*log(((2*a^2 - b^2)*cosh(d*x + c)^4 +
4*(2*a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2 - b^2)*sinh(d*x + c)
^4 - 4*(a^2 - a*b)*cosh(d*x + c)^2 + 2*(3*(2*a^2 - b^2)*cosh(d*x + c)^2 - 2
*a^2 + 2*a*b)*sinh(d*x + c)^2 + 2*a^2 - 4*a*b + 2*b^2 - 2*(a*cosh(d*x + c)^
4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - (2*a - b)*cosh(
d*x + c)^2 + (6*a*cosh(d*x + c)^2 - 2*a + b)*sinh(d*x + c)^2 + 2*(2*a*cosh(
d*x + c)^3 - (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + a - b)*sqrt(a - b)*sq
rt((b*cosh(d*x + c) + a*sinh(d*x + c))/sinh(d*x + c)) + 4*((2*a^2 - b^2)*co
sh(d*x + c)^3 - 2*(a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^
4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4
*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)))/((a^2 - b^2)*d), -1/4*(
2*(a - b)*sqrt(-a - b)*arctan(((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x
 + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 - a)*sqrt(-a - b)*sqrt((b*cos
h(d*x + c) + a*sinh(d*x + c))/sinh(d*x + c))/((a^2 + 2*a*b + b^2)*cosh(d*x
+ c)^2 + 2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + 2*a*b +
 b^2)*sinh(d*x + c)^2 - a^2 + b^2)) - (a + b)*sqrt(a - b)*log(((2*a^2 - b^2
)*cosh(d*x + c)^4 + 4*(2*a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2
- b^2)*sinh(d*x + c)^4 - 4*(a^2 - a*b)*cosh(d*x + c)^2 + 2*(3*(2*a^2 - b^2)
*cosh(d*x + c)^2 - 2*a^2 + 2*a*b)*sinh(d*x + c)^2 + 2*a^2 - 4*a*b + 2*b^2 -
 2*(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)
^4 - (2*a - b)*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 - 2*a + b)*sinh(d*x +
 c)^2 + 2*(2*a*cosh(d*x + c)^3 - (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + a
 - b)*sqrt(a - b)*sqrt((b*cosh(d*x + c) + a*sinh(d*x + c))/sinh(d*x + c)) +
 4*((2*a^2 - b^2)*cosh(d*x + c)^3 - 2*(a^2 - a*b)*cosh(d*x + c))*sinh(d*x +
 c))/(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2
*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)))/((a
^2 - b^2)*d), -1/4*(2*(a + b)*sqrt(-a + b)*arctan(-(a*cosh(d*x + c)^2 + 2*a
*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 - a + b)*sqrt(-a + b)*sqrt
((b*cosh(d*x + c) + a*sinh(d*x + c))/sinh(d*x + c))/((a^2 - b^2)*cosh(d*x +
 c)^2 + 2*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*sinh(d*x +
c)^2 - a^2 + 2*a*b - b^2)) - sqrt(a + b)*(a - b)*log(2*(a^2 + 2*a*b + b^2)*
cosh(d*x + c)^4 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(
a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 - 4*(a^2 + a*b)*cosh(d*x + c)^2 + 4*(3*(
a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 - a^2 - a*b)*sinh(d*x + c)^2 + 2*a^2 - b
^2 + 2*((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 +
 (a + b)*sinh(d*x + c)^4 - (2*a + b)*cosh(d*x + c)^2 + (6*(a + b)*cosh(d*x
+ c)^2 - 2*a - b)*sinh(d*x + c)^2 + 2*(2*(a + b)*cosh(d*x + c)^3 - (2*a + b
)*cosh(d*x + c))*sinh(d*x + c) + a)*sqrt(a + b)*sqrt((b*cosh(d*x + c) + a*s
inh(d*x + c))/sinh(d*x + c)) + 8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - (a^
2 + a*b)*cosh(d*x + c))*sinh(d*x + c)))/((a^2 - b^2)*d), -1/2*((a + b)*sqrt
(-a + b)*arctan(-(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*s
inh(d*x + c)^2 - a + b)*sqrt(-a + b)*sqrt((b*cosh(d*x + c) + a*sinh(d*x + c
))/sinh(d*x + c))/((a^2 - b^2)*cosh(d*x + c)^2 + 2*(a^2 - b^2)*cosh(d*x + c
)*sinh(d*x + c) + (a^2 - b^2)*sinh(d*x + c)^2 - a^2 + 2*a*b - b^2)) + (a -
```

```
b)*sqrt(-a - b)*arctan(((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*s
inh(d*x + c) + (a + b)*sinh(d*x + c)^2 - a)*sqrt(-a - b)*sqrt((b*cosh(d*x +
 c) + a*sinh(d*x + c))/sinh(d*x + c))/((a^2 + 2*a*b + b^2)*cosh(d*x + c)^2
+ 2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + 2*a*b + b^2)*s
inh(d*x + c)^2 - a^2 + b^2)))/((a^2 - b^2)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b \coth{\left(c + d x \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*coth(c + d*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(ex
```

Mupad [B]
time = 1.47, size = 242, normalized size = 3.27

$$\frac{\operatorname{atanh}\left(\frac{16\,a\,b^2\,\sqrt{a+b\coth\left(c+d\,x\right)}}{\left(\frac{16\,b^4\,d^3}{a\,d^3-b\,d^3}-\frac{16\,a\,b^3\,d^3}{a\,d^3-b\,d^3}\right)\,\sqrt{a-b}}+\frac{\left(a\,d^3-b\,d^3\right)\,\sqrt{a+b\coth\left(c+d\,x\right)}}{b\,d^3\,\sqrt{a-b}}\right)}{d\,\sqrt{a-b}}-\frac{\operatorname{atanh}\left(\frac{16\,a\,b^2\,\sqrt{a+b\coth\left(c+d\,x\right)}}{\left(\frac{16\,b^4\,d^3}{a\,d^3+b\,d^3}+\frac{16\,a\,b^3\,d^3}{a\,d^3+b\,d^3}\right)\,\sqrt{a+b}}-\frac{\left(a\,d^3+b\,d^3\right)\,\sqrt{a+b\coth\left(c+d\,x\right)}}{b\,d^3\,\sqrt{a+b}}\right)}{d\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*coth(c + d*x))^(1/2),x)
```

```
[Out] atanh((16*a*b^2*(a + b*coth(c + d*x))^(1/2))/(((16*b^4*d^3)/(a*d^3 - b*d^3)
 - (16*a*b^3*d^3)/(a*d^3 - b*d^3))*(a - b)^(1/2)) + ((a*d^3 - b*d^3)*(a + b
*coth(c + d*x))^(1/2))/(b*d^3*(a - b)^(1/2)))/(d*(a - b)^(1/2)) - atanh((16
*a*b^2*(a + b*coth(c + d*x))^(1/2))/(((16*b^4*d^3)/(a*d^3 + b*d^3) + (16*a*
b^3*d^3)/(a*d^3 + b*d^3))*(a + b)^(1/2)) - ((a*d^3 + b*d^3)*(a + b*coth(c +
 d*x))^(1/2))/(b*d^3*(a + b)^(1/2)))/(d*(a + b)^(1/2))
```

3.89 $\int \frac{\sinh^4(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=60

$$\frac{5x}{16}+\frac{1}{32(1-\coth(x))^2}+\frac{1}{8(1-\coth(x))}-\frac{1}{24(1+\coth(x))^3}-\frac{3}{32(1+\coth(x))^2}-\frac{3}{16(1+\coth(x))}$$

```
[Out] 5/16*x+1/32/(1-coth(x))^2+1/8/(1-coth(x))-1/24/(1+coth(x))^3-3/32/(1+coth(x
))^2-3/16/(1+coth(x))
```

Rubi [A]

time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3568, 46, 213}

$$\frac{5x}{16}+\frac{1}{8(1-\coth(x))}-\frac{3}{16(\coth(x)+1)}+\frac{1}{32(1-\coth(x))^2}-\frac{3}{32(\coth(x)+1)^2}-\frac{1}{24(\coth(x)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^4/(1 + Coth[x]),x]
```

```
[Out] (5*x)/16 + 1/(32*(1 - Coth[x])^2) + 1/(8*(1 - Coth[x])) - 1/(24*(1 + Coth[x
])^3) - 3/(32*(1 + Coth[x])^2) - 3/(16*(1 + Coth[x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sinh^4(x)}{1+\coth(x)}\,dx = \text{Subst}\left(\int \frac{1}{(1-x)^3(1+x)^4}\,dx, x, \coth(x)\right)$$
$$= \text{Subst}\left(\int \left(-\frac{1}{16(-1+x)^3}+\frac{1}{8(-1+x)^2}+\frac{1}{8(1+x)^4}+\frac{3}{16(1+x)^3}+\frac{3}{16(1+x)^2}-\right.\right.$$
$$= \frac{1}{32(1-\coth(x))^2}+\frac{1}{8(1-\coth(x))}-\frac{1}{24(1+\coth(x))^3}-\frac{3}{32(1+\coth(x))^2}-\frac{}{16(1+}$$
$$= \frac{5x}{16}+\frac{1}{32(1-\coth(x))^2}+\frac{1}{8(1-\coth(x))}-\frac{1}{24(1+\coth(x))^3}-\frac{3}{32(1+\coth(x))^2}-$$

Mathematica [A]
time = 0.07, size = 42, normalized size = 0.70

$$\frac{1}{192}(60x+15\cosh(2x)-6\cosh(4x)+\cosh(6x)-45\sinh(2x)+9\sinh(4x)-\sinh(6x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^4/(1 + Coth[x]),x]
```

```
[Out] (60*x + 15*Cosh[2*x] - 6*Cosh[4*x] + Cosh[6*x] - 45*Sinh[2*x] + 9*Sinh[4*x]
 - Sinh[6*x])/192
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(48) = 96$.
time = 0.54, size = 110, normalized size = 1.83

method	result
risch	$\frac{5x}{16}+\frac{e^{4x}}{128}-\frac{5\,e^{2x}}{64}+\frac{5\,e^{-2x}}{32}-\frac{5\,e^{-4x}}{128}+\frac{e^{-6x}}{192}$
default	$\frac{1}{8(\tanh(\frac{x}{2})-1)^4}+\frac{1}{4(\tanh(\frac{x}{2})-1)^3}-\frac{1}{8(\tanh(\frac{x}{2})-1)^2}-\frac{1}{4(\tanh(\frac{x}{2})-1)}-\frac{5\ln(\tanh(\frac{x}{2})-1)}{16}+\frac{1}{3(\tanh(\frac{x}{2})+1)^6}-\frac{}{(\tanh(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(1+coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/(tanh(1/2*x)-1)^4+1/4/(tanh(1/2*x)-1)^3-1/8/(tanh(1/2*x)-1)^2-1/4/(tanh
(1/2*x)-1)-5/16*ln(tanh(1/2*x)-1)+1/3/(tanh(1/2*x)+1)^6-1/(tanh(1/2*x)+1)^5
+5/8/(tanh(1/2*x)+1)^4+5/12/(tanh(1/2*x)+1)^3-3/8/(tanh(1/2*x)+1)+5/16*ln(t
anh(1/2*x)+1)
```

Maxima [A]
time = 0.27, size = 36, normalized size = 0.60

$$-\frac{1}{128}\left(10\,e^{(-2\,x)}-1\right)e^{(4\,x)}+\frac{5}{16}\,x+\frac{5}{32}\,e^{(-2\,x)}-\frac{5}{128}\,e^{(-4\,x)}+\frac{1}{192}\,e^{(-6\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] -1/128*(10*e^(-2*x) - 1)*e^(4*x) + 5/16*x + 5/32*e^(-2*x) - 5/128*e^(-4*x)
+ 1/192*e^(-6*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(44) = 88$.
time = 0.35, size = 93, normalized size = 1.55

$$\frac{5 \cosh(x)^5 + 25 \cosh(x)\sinh(x)^4 + \sinh(x)^5 + 5\left(2 \cosh(x)^2 - 3\right)\sinh(x)^3 - 45 \cosh(x)^3 + 5\left(10 \cosh(x)^3 - 27 \cosh(x)\right)\sinh(x)^2 + 60\,(2\,x + 1)\cosh(x) + 5\left(\cosh(x)^4 - 9 \cosh(x)^2 + 24\,x - 12\right)\sinh(x)}{384\left(\cosh(x) + \sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] 1/384*(5*cosh(x)^5 + 25*cosh(x)*sinh(x)^4 + sinh(x)^5 + 5*(2*cosh(x)^2 - 3)
*sinh(x)^3 - 45*cosh(x)^3 + 5*(10*cosh(x)^3 - 27*cosh(x))*sinh(x)^2 + 60*(2
*x + 1)*cosh(x) + 5*(cosh(x)^4 - 9*cosh(x)^2 + 24*x - 12)*sinh(x))/(cosh(x)
 + sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^4(x)}{\coth(x) + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**4/(1+coth(x)),x)
```

```
[Out] Integral(sinh(x)**4/(coth(x) + 1), x)
```

Giac [A]
time = 0.41, size = 42, normalized size = 0.70

$$-\frac{1}{384}\left(110\,e^{(6\,x)} - 60\,e^{(4\,x)} + 15\,e^{(2\,x)} - 2\right)e^{(-6\,x)} + \frac{5}{16}\,x + \frac{1}{128}\,e^{(4\,x)} - \frac{5}{64}\,e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(1+coth(x)),x, algorithm="giac")
```

```
[Out] -1/384*(110*e^(6*x) - 60*e^(4*x) + 15*e^(2*x) - 2)*e^(-6*x) + 5/16*x + 1/12
8*e^(4*x) - 5/64*e^(2*x)
```

Mupad [B]
time = 1.38, size = 34, normalized size = 0.57

$$\frac{5\,x}{16} + \frac{5\,\mathrm{e}^{-2\,x}}{32} - \frac{5\,\mathrm{e}^{2\,x}}{64} - \frac{5\,\mathrm{e}^{-4\,x}}{128} + \frac{\mathrm{e}^{4\,x}}{128} + \frac{\mathrm{e}^{-6\,x}}{192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(coth(x) + 1),x)
```

```
[Out] (5*x)/16 + (5*exp(-2*x))/32 - (5*exp(2*x))/64 - (5*exp(-4*x))/128 + exp(4*x
)/128 + exp(-6*x)/192
```

3.90 $\int \frac{\sinh^3(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=29

$$-\frac{4\cosh(x)}{5}+\frac{4\cosh^3(x)}{15}-\frac{\sinh^3(x)}{5(1+\coth(x))}$$

```
[Out] -4/5*cosh(x)+4/15*cosh(x)^3-1/5*sinh(x)^3/(1+coth(x))
```

Rubi [A]
time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3583, 2713}

$$\frac{4\cosh^3(x)}{15}-\frac{4\cosh(x)}{5}-\frac{\sinh^3(x)}{5(\coth(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(1 + Coth[x]),x]
```

```
[Out] (-4*Cosh[x])/5 + (4*Cosh[x]^3)/15 - Sinh[x]^3/(5*(1 + Coth[x]))
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^3(x)}{1+\coth(x)}\,dx &= -\frac{\sinh^3(x)}{5(1+\coth(x))}+\frac{4}{5}\int \sinh^3(x)\,dx \\
&= -\frac{\sinh^3(x)}{5(1+\coth(x))}-\frac{4}{5}\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,\cosh(x)\right) \\
&= -\frac{4\cosh(x)}{5}+\frac{4\cosh^3(x)}{15}-\frac{\sinh^3(x)}{5(1+\coth(x))}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 36, normalized size = 1.24

$$\frac{\operatorname{csch}(x)(-45-20\cosh(2x)+\cosh(4x)-40\sinh(2x)+4\sinh(4x))}{120(1+\coth(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(1 + Coth[x]),x]
```

```
[Out] (Csch[x]*(-45 - 20*Cosh[2*x] + Cosh[4*x] - 40*Sinh[2*x] + 4*Sinh[4*x]))/(12
0*(1 + Coth[x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(23) = 46.
time = 0.50, size = 80, normalized size = 2.76

method	result
risch	$\frac{e^{3x}}{48}-\frac{e^{x}}{4}-\frac{3\,e^{-x}}{8}+\frac{e^{-3x}}{12}-\frac{e^{-5x}}{80}$
default	$-\frac{1}{6\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}-\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{3}{8\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{2}{5\left(\tanh\left(\frac{x}{2}\right)+1\right)^{5}}+\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{4}}-\frac{1}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}-$ 2(tanh

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(1+coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/(tanh(1/2*x)-1)^3-1/4/(tanh(1/2*x)-1)^2+3/8/(tanh(1/2*x)-1)-2/5/(tanh(
1/2*x)+1)^5+1/(tanh(1/2*x)+1)^4-1/3/(tanh(1/2*x)+1)^3-1/2/(tanh(1/2*x)+1)^2
-3/8/(tanh(1/2*x)+1)
```

Maxima [A]
time = 0.29, size = 33, normalized size = 1.14

$$-\frac{1}{48}\left(12\,e^{(-2\,x)}-1\right)e^{(3\,x)}-\frac{3}{8}\,e^{(-x)}+\frac{1}{12}\,e^{(-3\,x)}-\frac{1}{80}\,e^{(-5\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] -1/48*(12*e^(-2*x) - 1)*e^(3*x) - 3/8*e^(-x) + 1/12*e^(-3*x) - 1/80*e^(-5*x
)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(23) = 46.
time = 0.36, size = 60, normalized size = 2.07

$$\frac{\cosh(x)^{4}+16\,\cosh(x)\sinh(x)^{3}+\sinh(x)^{4}+2\left(3\,\cosh(x)^{2}-10\right)\sinh(x)^{2}-20\,\cosh(x)^{2}+16\left(\cosh(x)^{3}-5\,\cosh(x)\right)\sinh(x)-45}{120\left(\cosh(x)+\sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] 1/120*(cosh(x)^4 + 16*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 10)*
sinh(x)^2 - 20*cosh(x)^2 + 16*(cosh(x)^3 - 5*cosh(x))*sinh(x) - 45)/(cosh(x
) + sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3(x)}{\coth(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(1+coth(x)),x)
```

```
[Out] Integral(sinh(x)**3/(coth(x) + 1), x)
```

Giac [A]
time = 0.43, size = 31, normalized size = 1.07

$$-\frac{1}{240}\left(90\,e^{(4\,x)}-20\,e^{(2\,x)}+3\right)e^{(-5\,x)}+\frac{1}{48}\,e^{(3\,x)}-\frac{1}{4}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1+coth(x)),x, algorithm="giac")
```

```
[Out] -1/240*(90*e^(4*x) - 20*e^(2*x) + 3)*e^(-5*x) + 1/48*e^(3*x) - 1/4*e^x
```

Mupad [B]
time = 1.27, size = 29, normalized size = 1.00

$$\frac{\mathrm{e}^{-3\,x}}{12}-\frac{3\,\mathrm{e}^{-x}}{8}+\frac{\mathrm{e}^{3\,x}}{48}-\frac{\mathrm{e}^{-5\,x}}{80}-\frac{\mathrm{e}^{x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(coth(x) + 1),x)
```

```
[Out] exp(-3*x)/12 - (3*exp(-x))/8 + exp(3*x)/48 - exp(-5*x)/80 - exp(x)/4
```

3.91 $\int \frac{\sinh^2(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=38

$$-\frac{3x}{8}-\frac{1}{8(1-\coth(x))}+\frac{1}{8(1+\coth(x))^2}+\frac{1}{4(1+\coth(x))}$$

```
[Out] -3/8*x-1/8/(1-coth(x))+1/8/(1+coth(x))^2+1/4/(1+coth(x))
```

Rubi [A]
time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3568, 46, 213}

$$-\frac{3x}{8}-\frac{1}{8(1-\coth(x))}+\frac{1}{4(\coth(x)+1)}+\frac{1}{8(\coth(x)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(1 + Coth[x]),x]
```

```
[Out] (-3*x)/8 - 1/(8*(1 - Coth[x])) + 1/(8*(1 + Coth[x])^2) + 1/(4*(1 + Coth[x])
)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sinh^2(x)}{1+\coth(x)}\,dx = -\text{Subst}\left(\int \frac{1}{(1-x)^2(1+x)^3}\,dx, x, \coth(x)\right)$$
$$= -\text{Subst}\left(\int \left(\frac{1}{8(-1+x)^2}+\frac{1}{4(1+x)^3}+\frac{1}{4(1+x)^2}-\frac{3}{8\,(-1+x^2)}\right)dx, x, \coth(x)\right)$$
$$= -\frac{1}{8(1-\coth(x))}+\frac{1}{8(1+\coth(x))^2}+\frac{1}{4(1+\coth(x))}+\frac{3}{8}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \text{cot}\right.$$
$$= -\frac{3x}{8}-\frac{1}{8(1-\coth(x))}+\frac{1}{8(1+\coth(x))^2}+\frac{1}{4(1+\coth(x))}$$

Mathematica [A]
time = 0.04, size = 30, normalized size = 0.79

$$\frac{1}{32}(-12x-4\cosh(2x)+\cosh(4x)+8\sinh(2x)-\sinh(4x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(1 + Coth[x]),x]
```

```
[Out] (-12*x - 4*Cosh[2*x] + Cosh[4*x] + 8*Sinh[2*x] - Sinh[4*x])/32
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(30) = 60.
time = 0.54, size = 70, normalized size = 1.84

method	result
risch	$-\frac{3x}{8}+\frac{e^{2x}}{16}-\frac{3\,e^{-2x}}{16}+\frac{e^{-4x}}{32}$
default	$\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}+\frac{1}{4\tanh\left(\frac{x}{2}\right)-4}+\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{8}+\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)^4}-\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^3}+\frac{1}{2\tanh\left(\frac{x}{2}\right)+2}-\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)+\right.}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(1+coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/(tanh(1/2*x)-1)^2+1/4/(tanh(1/2*x)-1)+3/8*ln(tanh(1/2*x)-1)+1/2/(tanh(1
/2*x)+1)^4-1/(tanh(1/2*x)+1)^3+1/2/(tanh(1/2*x)+1)-3/8*ln(tanh(1/2*x)+1)
```

Maxima [A]
time = 0.26, size = 22, normalized size = 0.58

$$-\frac{3}{8}\,x+\frac{1}{16}\,e^{(2\,x)}-\frac{3}{16}\,e^{(-2\,x)}+\frac{1}{32}\,e^{(-4\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] -3/8*x + 1/16*e^(2*x) - 3/16*e^(-2*x) + 1/32*e^(-4*x)
```

Fricas [A]
time = 0.36, size = 50, normalized size = 1.32

$$\frac{3\cosh(x)^3 + 9\cosh(x)\sinh(x)^2 + \sinh(x)^3 - 6(2x+1)\cosh(x) + 3\left(\cosh(x)^2 - 4x + 2\right)\sinh(x)}{32(\cosh(x) + \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] 1/32*(3*cosh(x)^3 + 9*cosh(x)*sinh(x)^2 + sinh(x)^3 - 6*(2*x + 1)*cosh(x) +
 3*(cosh(x)^2 - 4*x + 2)*sinh(x))/(cosh(x) + sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(x)}{\coth(x) + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(1+coth(x)),x)
```

```
[Out] Integral(sinh(x)**2/(coth(x) + 1), x)
```

Giac [A]
time = 0.41, size = 30, normalized size = 0.79

$$\frac{1}{32}\left(9\,e^{(4\,x)} - 6\,e^{(2\,x)} + 1\right)e^{(-4\,x)} - \frac{3}{8}\,x + \frac{1}{16}\,e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1+coth(x)),x, algorithm="giac")
```

```
[Out] 1/32*(9*e^(4*x) - 6*e^(2*x) + 1)*e^(-4*x) - 3/8*x + 1/16*e^(2*x)
```

Mupad [B]
time = 1.24, size = 22, normalized size = 0.58

$$\frac{e^{2\,x}}{16} - \frac{3\,e^{-2\,x}}{16} - \frac{3\,x}{8} + \frac{e^{-4\,x}}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(coth(x) + 1),x)
```

```
[Out] exp(2*x)/16 - (3*exp(-2*x))/16 - (3*x)/8 + exp(-4*x)/32
```

3.92 $\int \frac{\sinh(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=19

$$\frac{2\cosh(x)}{3}-\frac{\sinh(x)}{3(1+\coth(x))}$$

```
[Out] 2/3*cosh(x)-1/3*sinh(x)/(1+coth(x))
```

Rubi [A]
time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3583, 2718}

$$\frac{2\cosh(x)}{3}-\frac{\sinh(x)}{3(\coth(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(1 + Coth[x]),x]
```

```
[Out] (2*Cosh[x])/3 - Sinh[x]/(3*(1 + Coth[x]))
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(x)}{1+\coth(x)}\,dx &= -\frac{\sinh(x)}{3(1+\coth(x))}+\frac{2}{3}\int \sinh(x)\,dx\\ &= \frac{2\cosh(x)}{3}-\frac{\sinh(x)}{3(1+\coth(x))}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 21, normalized size = 1.11

$$\frac{1}{12}\left(9\cosh(x)-\cosh(3x)+4\sinh^3(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(1 + Coth[x]),x]
```

```
[Out] (9*Cosh[x] - Cosh[3*x] + 4*Sinh[x]^3)/12
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(15) = 30$.
time = 0.51, size = 40, normalized size = 2.11

method	result	size
risch	$\frac{e^x}{4}+\frac{e^{-x}}{2}-\frac{e^{-3x}}{12}$	18
default	$-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{2}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^3}+\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}+\frac{1}{2\tanh\left(\frac{x}{2}\right)+2}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(1+coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(tanh(1/2*x)-1)-2/3/(tanh(1/2*x)+1)^3+1/(tanh(1/2*x)+1)^2+1/2/(tanh(1/
2*x)+1)
```

Maxima [A]
time = 0.26, size = 17, normalized size = 0.89

$$\frac{1}{2}\,e^{(-x)}-\frac{1}{12}\,e^{(-3\,x)}+\frac{1}{4}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 1/2*e^(-x) - 1/12*e^(-3*x) + 1/4*e^x
```

Fricas [A]
time = 0.35, size = 25, normalized size = 1.32

$$\frac{\cosh\left(x\right)^{2}+4\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}+3}{6\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] 1/6*(cosh(x)^2 + 4*cosh(x)*sinh(x) + sinh(x)^2 + 3)/(cosh(x) + sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(x)}{\coth(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+coth(x)),x)
[Out] Integral(sinh(x)/(coth(x) + 1), x)
```

Giac [A]
time = 0.42, size = 19, normalized size = 1.00

$$\frac{1}{12}\left(6\,e^{(2\,x)}-1\right)e^{(-3\,x)}+\frac{1}{4}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+coth(x)),x, algorithm="giac")
[Out] 1/12*(6*e^(2*x) - 1)*e^(-3*x) + 1/4*e^x
```

Mupad [B]
time = 1.24, size = 17, normalized size = 0.89

$$\frac{\mathrm{e}^{-x}}{2}-\frac{\mathrm{e}^{-3\,x}}{12}+\frac{\mathrm{e}^{x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(coth(x) + 1),x)
[Out] exp(-x)/2 - exp(-3*x)/12 + exp(x)/4
```

3.93 $\int \frac{\operatorname{csch}(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=10

$$-\frac{\operatorname{csch}(x)}{1+\coth(x)}$$

```
[Out] -csch(x)/(1+coth(x))
```

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3569}

$$-\frac{\operatorname{csch}(x)}{\coth(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]/(1 + Coth[x]),x]
```

```
[Out] -(Csch[x]/(1 + Coth[x]))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int \frac{\operatorname{csch}(x)}{1+\coth(x)}\,dx = -\frac{\operatorname{csch}(x)}{1+\coth(x)}$$

Mathematica [A]
time = 0.01, size = 7, normalized size = 0.70

$$-\cosh(x)+\sinh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]/(1 + Coth[x]),x]
```

```
[Out] -Cosh[x] + Sinh[x]
```

Maple [A]
time = 0.30, size = 11, normalized size = 1.10

method	result	size
risch	$-\mathrm{e}^{-x}$	7
gosper	$-\frac{\operatorname{csch}(x)}{1+\coth(x)}$	11
default	$-\frac{2}{\tanh\left(\frac{x}{2}\right)+1}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)/(1+coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(tanh(1/2*x)+1)
```

Maxima [A]
time = 0.26, size = 6, normalized size = 0.60

$$-e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] -e^(-x)
```

Fricas [A]
time = 0.41, size = 9, normalized size = 0.90

$$-\frac{1}{\cosh\left(x\right)+\sinh\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] -1/(cosh(x) + sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}\left(x\right)}{\coth\left(x\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(1+coth(x)),x)
```

```
[Out] Integral(csch(x)/(coth(x) + 1), x)
```

Giac [A]
time = 0.41, size = 6, normalized size = 0.60

$$-e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(1+coth(x)),x, algorithm="giac")
```

```
[Out] -e^(-x)
```

Mupad [B]
time = 1.20, size = 6, normalized size = 0.60

$$-\mathrm{e}^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)*(coth(x) + 1)),x)
```

```
[Out] -exp(-x)
```

3.94 $\int \frac{\operatorname{csch}^2(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=7

$$-\log(1+\coth(x))$$

```
[Out] -ln(1+coth(x))
```

Rubi [A]
time = 0.02, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3568, 31}

$$-\log(\coth(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^2/(1 + Coth[x]),x]
```

```
[Out] -Log[1 + Coth[x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^2(x)}{1+\coth(x)}\,dx &= -\operatorname{Subst}\left(\int \frac{1}{1+x}\,dx, x, \coth(x)\right)\\ &= -\log(1+\coth(x))\end{aligned}$$

Mathematica [A]
time = 0.00, size = 7, normalized size = 1.00

$$-x+\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^2/(1 + Coth[x]),x]
[Out] -x + Log[Sinh[x]]
```

Maple [A]
time = 0.50, size = 8, normalized size = 1.14

method	result	size
derivativedivides	$-\ln\left(1+\coth\left(x\right)\right)$	8
default	$-\ln\left(1+\coth\left(x\right)\right)$	8
risch	$-2x+\ln\left(\mathrm{e}^{2x}-1\right)$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^2/(1+coth(x)),x,method=_RETURNVERBOSE)
[Out] -ln(1+coth(x))
```

Maxima [A]
time = 0.26, size = 7, normalized size = 1.00

$$-\log\left(\coth\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(1+coth(x)),x, algorithm="maxima")
[Out] -log(coth(x) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 18 vs. 2(7) = 14.
time = 0.42, size = 18, normalized size = 2.57

$$-2\,x+\log\left(\frac{2\,\sinh\left(x\right)}{\cosh\left(x\right)-\sinh\left(x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(1+coth(x)),x, algorithm="fricas")
[Out] -2*x + log(2*sinh(x)/(cosh(x) - sinh(x)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{csch}^{2}\left(x\right)}{\coth\left(x\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2/(1+coth(x)),x)
```

```
[Out] Integral(csch(x)**2/(coth(x) + 1), x)
```

Giac [A]
time = 0.41, size = 12, normalized size = 1.71

$$-2x + \log\left(\left|e^{(2x)} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(1+coth(x)),x, algorithm="giac")
[Out] -2*x + log(abs(e^(2*x) - 1))
```

Mupad [B]
time = 1.18, size = 11, normalized size = 1.57

$$\ln\left(e^{2x} - 1\right) - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^2*(coth(x) + 1)),x)
[Out] log(exp(2*x) - 1) - 2*x
```

3.95 $\int \frac{\operatorname{csch}^3(x)}{1+\coth(x)} \, dx$

Optimal. Leaf size=8

$$\tanh^{-1}(\cosh(x))-\operatorname{csch}(x)$$

```
[Out] arctanh(cosh(x))-csch(x)
```

Rubi [A]
time = 0.03, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3582, 3855}

$$\tanh^{-1}(\cosh(x))-\operatorname{csch}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^3/(1 + Coth[x]),x]
```

```
[Out] ArcTanh[Cosh[x]] - Csch[x]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\operatorname{csch}^3(x)}{1+\coth(x)} \, dx &= -\operatorname{csch}(x)-\int \operatorname{csch}(x) \, dx \\ &= \tanh^{-1}(\cosh(x))-\operatorname{csch}(x) \end{aligned}$$

Mathematica [A]
time = 0.03, size = 14, normalized size = 1.75

$$-\operatorname{csch}(x)-\log\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^3/(1 + Coth[x]),x]
```

```
[Out] -Csch[x] - Log[Tanh[x/2]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(8) = 16$.
time = 0.56, size = 23, normalized size = 2.88

method	result	size
default	$\frac{\tanh\left(\frac{x}{2}\right)}{2}-\frac{1}{2\tanh\left(\frac{x}{2}\right)}-\ln\left(\tanh\left(\frac{x}{2}\right)\right)$	23
risch	$-\frac{2\,\mathrm{e}^{x}}{\mathrm{e}^{2x}-1}-\ln\left(\mathrm{e}^{x}-1\right)+\ln\left(\mathrm{e}^{x}+1\right)$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^3/(1+coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*tanh(1/2*x)-1/2/tanh(1/2*x)-ln(tanh(1/2*x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(8) = 16$.
time = 0.27, size = 31, normalized size = 3.88

$$\frac{2\,e^{(-x)}}{e^{(-2\,x)}-1}+\log\left(e^{(-x)}+1\right)-\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 2*e^(-x)/(e^(-2*x) - 1) + log(e^(-x) + 1) - log(e^(-x) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(8) = 16$.
time = 0.39, size = 77, normalized size = 9.62

$$\frac{\left(\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}-1\right)\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)-\left(\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}-1\right)\log\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)-2\,\cosh\left(x\right)-2\,\sinh\left(x\right)}{\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] ((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*log(cosh(x) + sinh(x) + 1)
 - (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*log(cosh(x) + sinh(x) -
1) - 2*cosh(x) - 2*sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{csch}^{3}\left(x\right)}{\coth\left(x\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**3/(1+coth(x)),x)
```

```
[Out] Integral(csch(x)**3/(coth(x) + 1), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(8) = 16.
time = 0.41, size = 26, normalized size = 3.25

$$-\frac{2\,e^x}{e^{(2\,x)}-1}+\log\left(e^x+1\right)-\log\left(\left|e^x-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(1+coth(x)),x, algorithm="giac")
```

```
[Out] -2*e^x/(e^(2*x) - 1) + log(e^x + 1) - log(abs(e^x - 1))
```

Mupad [B]
time = 0.08, size = 29, normalized size = 3.62

$$\ln\left(2\,\mathrm{e}^x+2\right)-\ln\left(2\,\mathrm{e}^x-2\right)-\frac{2\,\mathrm{e}^x}{\mathrm{e}^{2\,x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^3*(coth(x) + 1)),x)
```

```
[Out] log(2*exp(x) + 2) - log(2*exp(x) - 2) - (2*exp(x))/(exp(2*x) - 1)
```

3.96 $\int \frac{\operatorname{csch}^4(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=11

$$\coth(x)-\frac{\coth^2(x)}{2}$$

[Out] coth(x)-1/2*coth(x)^2

Rubi [A]
time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3568}

$$\coth(x)-\frac{\coth^2(x)}{2}$$

Antiderivative was successfully verified.

[In] Int[Csch[x]^4/(1 + Coth[x]),x]

[Out] Coth[x] - Coth[x]^2/2

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^4(x)}{1+\coth(x)}\,dx &= \operatorname{Subst}\left(\int (1-x)\,dx,x,\coth(x)\right)\\ &= \coth(x)-\frac{\coth^2(x)}{2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 11, normalized size = 1.00

$$\coth(x)-\frac{\operatorname{csch}^2(x)}{2}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[x]^4/(1 + Coth[x]),x]

```
[Out] Coth[x] - Csch[x]^2/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(9) = 18$.
time = 0.61, size = 32, normalized size = 2.91

method	result	size
risch	$-\frac{2}{(e^{2x}-1)^2}$	11
default	$-\frac{(\tanh^2(\frac{x}{2}))}{8}+\frac{\tanh(\frac{x}{2})}{2}-\frac{1}{8\tanh(\frac{x}{2})^2}+\frac{1}{2\tanh(\frac{x}{2})}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^4/(1+coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*tanh(1/2*x)^2+1/2*tanh(1/2*x)-1/8/tanh(1/2*x)^2+1/2/tanh(1/2*x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(9) = 18$.
time = 0.26, size = 41, normalized size = 3.73

$$\frac{4\,e^{(-2\,x)}}{2\,e^{(-2\,x)}-e^{(-4\,x)}-1}-\frac{2}{2\,e^{(-2\,x)}-e^{(-4\,x)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 4*e^(-2*x)/(2*e^(-2*x) - e^(-4*x) - 1) - 2/(2*e^(-2*x) - e^(-4*x) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(9) = 18$.
time = 0.35, size = 55, normalized size = 5.00

$$-\frac{2}{\cosh(x)^4+4\,\cosh(x)\sinh(x)^3+\sinh(x)^4+2\,(3\,\cosh(x)^2-1)\sinh(x)^2-2\,\cosh(x)^2+4\,(\cosh(x)^3-\cosh(x))\sinh(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] -2/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(
x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{csch}^4(x)}{\coth(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**4/(1+coth(x)),x)
```

```
[Out] Integral(csch(x)**4/(coth(x) + 1), x)
```

Giac [**A**]
time = 0.42, size = 10, normalized size = 0.91

$$-\frac{2}{\left(e^{(2x)}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(1+coth(x)),x, algorithm="giac")
```

```
[Out] -2/(e^(2*x) - 1)^2
```

Mupad [**B**]
time = 1.18, size = 16, normalized size = 1.45

$$-\frac{2}{e^{4x}-2e^{2x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^4*(coth(x) + 1)),x)
```

```
[Out] -2/(exp(4*x) - 2*exp(2*x) + 1)
```

3.97 $\int \frac{\sinh^4(x)}{a+b\coth(x)} \, dx$

Optimal. Leaf size=155

$$-\frac{(3a^2+9ab+8b^2)\log(1-\coth(x))}{16(a+b)^3}+\frac{(3a^2-9ab+8b^2)\log(1+\coth(x))}{16(a-b)^3}-\frac{b^5\log(a+b\coth(x))}{(a^2-b^2)^3}-\frac{\left(4b^3-a\right.}{}$$

```
[Out] -1/16*(3*a^2+9*a*b+8*b^2)*ln(1-coth(x))/(a+b)^3+1/16*(3*a^2-9*a*b+8*b^2)*ln
(1+coth(x))/(a-b)^3-b^5*ln(a+b*coth(x))/(a^2-b^2)^3-1/8*(4*b^3-a*(7-3*a^2/b
^2)*b^2*coth(x))*sinh(x)^2/(a^2-b^2)^2-1/4*(b-a*coth(x))*sinh(x)^4/(a^2-b^2
)
```

Rubi [A]
time = 0.18, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3587, 755, 837, 815}

$$-\frac{(3a^2+9ab+8b^2)\log(1-\coth(x))}{16(a+b)^3}+\frac{(3a^2-9ab+8b^2)\log(\coth(x)+1)}{16(a-b)^3}-\frac{\sinh^4(x)(b-a\coth(x))}{4(a^2-b^2)}-\frac{b^5\log(a+b\coth(x))}{(a^2-b^2)^3}-\frac{\sinh^2(x)\left(4b^3-ab^2\left(7-\frac{3a^2}{b^2}\right)\coth(x)\right)}{8(a^2-b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^4/(a + b*Coth[x]),x]
```

```
[Out] -1/16*((3*a^2 + 9*a*b + 8*b^2)*Log[1 - Coth[x]])/(a + b)^3 + ((3*a^2 - 9*a*
b + 8*b^2)*Log[1 + Coth[x]])/(16*(a - b)^3) - (b^5*Log[a + b*Coth[x]])/(a^2
 - b^2)^3 - ((4*b^3 - a*(7 - (3*a^2)/b^2)*b^2*Coth[x])*Sinh[x]^2)/(8*(a^2 -
 b^2)^2) - ((b - a*Coth[x])*Sinh[x]^4)/(4*(a^2 - b^2))
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
```

```
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sinh^4(x)}{a+b\coth(x)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(a+x)\left(1-\frac{x^2}{b^2}\right)^3}\,dx, x, b\coth(x)\right)}{b}$$

$$= -\frac{(b-a\coth(x))\sinh^4(x)}{4\left(a^2-b^2\right)} + \frac{b\text{Subst}\left(\int \frac{-4+\frac{3a^2}{b^2}+\frac{3ax}{b^2}}{(a+x)\left(1-\frac{x^2}{b^2}\right)^2}\,dx, x, b\coth(x)\right)}{4\left(a^2-b^2\right)}$$

$$= -\frac{\left(4b^3-a\left(7-\frac{3a^2}{b^2}\right)b^2\coth(x)\right)\sinh^2(x)}{8\left(a^2-b^2\right)^2} - \frac{(b-a\coth(x))\sinh^4(x)}{4\left(a^2-b^2\right)} - \frac{b^5\text{Subst}\left(\int \frac{-3}{\ldots}\right.}{\ldots}$$

$$= -\frac{\left(4b^3-a\left(7-\frac{3a^2}{b^2}\right)b^2\coth(x)\right)\sinh^2(x)}{8\left(a^2-b^2\right)^2} - \frac{(b-a\coth(x))\sinh^4(x)}{4\left(a^2-b^2\right)} - \frac{b^5\text{Subst}\left(\int \left(-\ldots\right.\right.}{\ldots}$$

$$= -\frac{\left(3a^2+9ab+8b^2\right)\log(1-\coth(x))}{16(a+b)^3} + \frac{\left(3a^2-9ab+8b^2\right)\log(1+\coth(x))}{16(a-b)^3} - \frac{b^5\log(a \ldots}{(a^2 \ldots}$$

Mathematica [A]

time = 0.21, size = 156, normalized size = 1.01

$$\frac{12a^5x-40a^3b^2x+60ab^4x+4b\left(a^4-4a^2b^2+3b^4\right)\cosh(2x)-b\left(a^2-b^2\right)^2\cosh(4x)-32b^5\log(b\cosh(x)+a\sinh(x))-8a^5\sinh(2x)+24a^3b^2\sinh(2x)-16ab^4\sinh(2x)+a^5\sinh(4x)-2a^3b^2\sinh(4x)+ab^4\sinh(4x)}{32(a-b)^3(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^4/(a + b*Coth[x]),x]
```

[Out] (12*a^5*x - 40*a^3*b^2*x + 60*a*b^4*x + 4*b*(a^4 - 4*a^2*b^2 + 3*b^4)*Cosh[
2*x] - b*(a^2 - b^2)^2*Cosh[4*x] - 32*b^5*Log[b*Cosh[x] + a*Sinh[x]] - 8*a^
5*Sinh[2*x] + 24*a^3*b^2*Sinh[2*x] - 16*a*b^4*Sinh[2*x] + a^5*Sinh[4*x] - 2
*a^3*b^2*Sinh[4*x] + a*b^4*Sinh[4*x])/(32*(a - b)^3*(a + b)^3)

Maple [A]
time = 0.74, size = 263, normalized size = 1.70

method	result
risch	$\frac{3a^2x}{8(a+b)^3}+\frac{9axb}{8(a+b)^3}+\frac{x\,b^2}{(a+b)^3}+\frac{e^{4x}}{64a+64b}-\frac{e^{2x}a}{8(a+b)^2}-\frac{3\,e^{2x}b}{16(a+b)^2}+\frac{e^{-2x}a}{8(a-b)^2}-\frac{3\,e^{-2x}b}{16(a-b)^2}-\frac{e^{-4x}}{64(a-b)}+\frac{2b^5x}{a^6-3a^4b^2+3a}$
default	$\frac{16}{(64a+64b)\left(\tanh\left(\frac{x}{2}\right)-1\right)^4}+\frac{64}{(128a+128b)\left(\tanh\left(\frac{x}{2}\right)-1\right)^3}-\frac{a+3b}{8(a+b)^2\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}-\frac{3a+5b}{8(a+b)^2\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{(-3a^2-9ab-}{8}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(x)^4/(a+b*coth(x)),x,method=_RETURNVERBOSE)

[Out] 16/(64*a+64*b)/(tanh(1/2*x)-1)^4+64/(128*a+128*b)/(tanh(1/2*x)-1)^3-1/8*(a+
3*b)/(a+b)^2/(tanh(1/2*x)-1)^2-1/8*(3*a+5*b)/(a+b)^2/(tanh(1/2*x)-1)+1/8/(a
+b)^3*(-3*a^2-9*a*b-8*b^2)*ln(tanh(1/2*x)-1)-16/(64*a-64*b)/(tanh(1/2*x)+1)
^4+64/(128*a-128*b)/(tanh(1/2*x)+1)^3-1/8*(-a+3*b)/(a-b)^2/(tanh(1/2*x)+1)^
2-1/8*(3*a-5*b)/(a-b)^2/(tanh(1/2*x)+1)+1/8*(3*a^2-9*a*b+8*b^2)/(a-b)^3*ln(
tanh(1/2*x)+1)-b^5/(a-b)^3/(a+b)^3*ln(b*tanh(1/2*x)^2+2*a*tanh(1/2*x)+b)

Maxima [A]
time = 0.29, size = 166, normalized size = 1.07

$$-\frac{b^5\log\left(-(a-b)e^{(-2x)}+a+b\right)}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{(3a^2+9ab+8b^2)x}{8(a^3+3a^2b+3ab^2+b^3)}-\frac{\left(4(2a+3b)e^{(-2x)}-a-b\right)e^{(4x)}}{64(a^2+2ab+b^2)}+\frac{4(2a-3b)e^{(-2x)}-(a-b)e^{(-4x)}}{64(a^2-2ab+b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(x)^4/(a+b*coth(x)),x, algorithm="maxima")

[Out] -b^5*log(-(a - b)*e^(-2*x) + a + b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) + 1
/8*(3*a^2 + 9*a*b + 8*b^2)*x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 1/64*(4*(2*a
 + 3*b)*e^(-2*x) - a - b)*e^(4*x)/(a^2 + 2*a*b + b^2) + 1/64*(4*(2*a - 3*b)
*e^(-2*x) - (a - b)*e^(-4*x))/(a^2 - 2*a*b + b^2)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1279 vs. 2(147) = 294.
time = 0.38, size = 1279, normalized size = 8.25

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(x)^4/(a+b*coth(x)),x, algorithm="fricas")

```
[Out] 1/64*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^8 + 8*(a^
5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^7 + (a^5 -
 a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^8 - 4*(2*a^5 - a^4*b
- 6*a^3*b^2 + 4*a^2*b^3 + 4*a*b^4 - 3*b^5)*cosh(x)^6 - 4*(2*a^5 - a^4*b - 6
*a^3*b^2 + 4*a^2*b^3 + 4*a*b^4 - 3*b^5 - 7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2
*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x)^6 + 8*(3*a^5 - 10*a^3*b^2 + 15*a*b^4
 + 8*b^5)*x*cosh(x)^4 + 8*(7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 -
 b^5)*cosh(x)^3 - 3*(2*a^5 - a^4*b - 6*a^3*b^2 + 4*a^2*b^3 + 4*a*b^4 - 3*b^
5)*cosh(x))*sinh(x)^5 - a^5 - a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - a*b^4 - b^5 +
 2*(35*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 - 30*(
2*a^5 - a^4*b - 6*a^3*b^2 + 4*a^2*b^3 + 4*a*b^4 - 3*b^5)*cosh(x)^2 + 4*(3*a
^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*x)*sinh(x)^4 + 8*(7*(a^5 - a^4*b - 2*a^
3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 - 10*(2*a^5 - a^4*b - 6*a^3*b^2
+ 4*a^2*b^3 + 4*a*b^4 - 3*b^5)*cosh(x)^3 + 4*(3*a^5 - 10*a^3*b^2 + 15*a*b^4
 + 8*b^5)*x*cosh(x))*sinh(x)^3 + 4*(2*a^5 + a^4*b - 6*a^3*b^2 - 4*a^2*b^3 +
 4*a*b^4 + 3*b^5)*cosh(x)^2 + 4*(7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a
*b^4 - b^5)*cosh(x)^6 + 2*a^5 + a^4*b - 6*a^3*b^2 - 4*a^2*b^3 + 4*a*b^4 + 3
*b^5 - 15*(2*a^5 - a^4*b - 6*a^3*b^2 + 4*a^2*b^3 + 4*a*b^4 - 3*b^5)*cosh(x)
^4 + 12*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*x*cosh(x)^2)*sinh(x)^2 - 64
*(b^5*cosh(x)^4 + 4*b^5*cosh(x)^3*sinh(x) + 6*b^5*cosh(x)^2*sinh(x)^2 + 4*b
^5*cosh(x)*sinh(x)^3 + b^5*sinh(x)^4)*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x
) - sinh(x))) + 8*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh
(x)^7 - 3*(2*a^5 - a^4*b - 6*a^3*b^2 + 4*a^2*b^3 + 4*a*b^4 - 3*b^5)*cosh(x)
^5 + 4*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*x*cosh(x)^3 + (2*a^5 + a^4*b
 - 6*a^3*b^2 - 4*a^2*b^3 + 4*a*b^4 + 3*b^5)*cosh(x))*sinh(x))/((a^6 - 3*a^4
*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 + 4*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*c
osh(x)^3*sinh(x) + 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2*sinh(x)^
2 + 4*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^3 + (a^6 - 3*a^4*
b^2 + 3*a^2*b^4 - b^6)*sinh(x)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^4(x)}{a + b\coth(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**4/(a+b*coth(x)),x)
```

```
[Out] Integral(sinh(x)**4/(a + b*coth(x)), x)
```

Giac [A]
time = 0.41, size = 229, normalized size = 1.48

$$-\frac{b^5\log\left(\left|-ae^{(2x)} - be^{(2x)} + a - b\right|\right)}{a^6 - 3a^4b^2 + 3a^2b^4 - b^6} + \frac{(3a^2 - 9ab + 8b^2)x}{8(a^3 - 3a^2b + 3ab^2 - b^3)} - \frac{\left(18a^2e^{(4x)} - 54abe^{(4x)} + 48b^2e^{(4x)} - 8a^2e^{(2x)} + 20abe^{(2x)} - 12b^2e^{(2x)} + a^2 - 2ab + b^2\right)e^{(-4x)}}{64(a^3 - 3a^2b + 3ab^2 - b^3)} + \frac{ae^{(4x)} + be^{(4x)} - 8ae^{(2x)} - 12be^{(2x)}}{64(a^2 + 2ab + b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -b^5*log(abs(-a*e^(2*x) - b*e^(2*x) + a - b))/(a^6 - 3*a^4*b^2 + 3*a^2*b^4
- b^6) + 1/8*(3*a^2 - 9*a*b + 8*b^2)*x/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - 1/
64*(18*a^2*e^(4*x) - 54*a*b*e^(4*x) + 48*b^2*e^(4*x) - 8*a^2*e^(2*x) + 20*a
*b*e^(2*x) - 12*b^2*e^(2*x) + a^2 - 2*a*b + b^2)*e^(-4*x)/(a^3 - 3*a^2*b +
3*a*b^2 - b^3) + 1/64*(a*e^(4*x) + b*e^(4*x) - 8*a*e^(2*x) - 12*b*e^(2*x))/
(a^2 + 2*a*b + b^2)
```

Mupad [B]
time = 1.63, size = 143, normalized size = 0.92

$$\frac{e^{4x}}{64a+64b}-\frac{e^{-4x}}{64a-64b}+\frac{e^{-2x}(2a-3b)}{16(a-b)^2}-\frac{b^5\ln(b-a+ae^{2x}+be^{2x})}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{x(3a^2-9ab+8b^2)}{8(a-b)^3}-\frac{e^{2x}(2a+3b)}{16(a+b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(a + b*coth(x)),x)
```

```
[Out] exp(4*x)/(64*a + 64*b) - exp(-4*x)/(64*a - 64*b) + (exp(-2*x)*(2*a - 3*b))/
(16*(a - b)^2) - (b^5*log(b - a + a*exp(2*x) + b*exp(2*x)))/(a^6 - b^6 + 3*
a^2*b^4 - 3*a^4*b^2) + (x*(3*a^2 - 9*a*b + 8*b^2))/(8*(a - b)^3) - (exp(2*x
)*(2*a + 3*b))/(16*(a + b)^2)
```

3.98 $\int \frac{\sinh^3(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=134

$$-\frac{b^4\tanh^{-1}\left(\frac{(b+a\coth(x))\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}+\frac{ab^2\cosh(x)}{(a^2-b^2)^2}-\frac{a\cosh(x)}{a^2-b^2}+\frac{a\cosh^3(x)}{3(a^2-b^2)}-\frac{b^3\sinh(x)}{(a^2-b^2)^2}-\frac{b\sinh^3(x)}{3(a^2-b^2)}$$

```
[Out] -b^4*arctanh((b+a*coth(x))*sinh(x)/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)+a*b^2*c
osh(x)/(a^2-b^2)^2-a*cosh(x)/(a^2-b^2)+1/3*a*cosh(x)^3/(a^2-b^2)-b^3*sinh(x
)/(a^2-b^2)^2-1/3*b*sinh(x)^3/(a^2-b^2)
```

Rubi [A]
time = 0.18, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3592, 3567, 2713, 2718, 3590, 212}

$$-\frac{b\sinh^3(x)}{3(a^2-b^2)}+\frac{a\cosh^3(x)}{3(a^2-b^2)}+\frac{ab^2\cosh(x)}{(a^2-b^2)^2}-\frac{a\cosh(x)}{a^2-b^2}-\frac{b^4\tanh^{-1}\left(\frac{\sinh(x)(a\coth(x)+b)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}-\frac{b^3\sinh(x)}{(a^2-b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(a + b*Coth[x]),x]
```

```
[Out] -((b^4*ArcTanh[((b + a*Coth[x])*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2
)) + (a*b^2*Cosh[x])/(a^2 - b^2)^2 - (a*Cosh[x])/(a^2 - b^2) + (a*Cosh[x]^3
)/(3*(a^2 - b^2)) - (b^3*Sinh[x])/(a^2 - b^2)^2 - (b*Sinh[x]^3)/(3*(a^2 - b
^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3590

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> Dist[-f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*
x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3592

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(d*Sec[e + f*x])^m*(a - b*Tan[e
+ f*x]), x], x] + Dist[b^2/(d^2*(a^2 + b^2)), Int[(d*Sec[e + f*x])^(m + 2)/
(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 + b^2,
 0] && ILtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^3(x)}{a+b\coth(x)}\,dx &= \frac{\int (a-b\coth(x))\sinh^3(x)\,dx}{a^2-b^2}+\frac{b^2\int \frac{\sinh(x)}{a+b\coth(x)}\,dx}{a^2-b^2}\\
&= -\frac{b\sinh^3(x)}{3\left(a^2-b^2\right)}+\frac{b^2\int (a-b\coth(x))\sinh(x)\,dx}{\left(a^2-b^2\right)^2}+\frac{b^4\int \frac{\operatorname{csch}(x)}{a+b\coth(x)}\,dx}{\left(a^2-b^2\right)^2}+\frac{a\int \sinh^3(x)\,dx}{a^2-b^2}\\
&= -\frac{b^3\sinh(x)}{\left(a^2-b^2\right)^2}-\frac{b\sinh^3(x)}{3\left(a^2-b^2\right)}+\frac{\left(ab^2\right)\int \sinh(x)\,dx}{\left(a^2-b^2\right)^2}-\frac{b^4\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx,x,i(-ib-a\right.}{\left(a^2-b^2\right)^2}\\
&= -\frac{b^4\tanh^{-1}\left(\frac{(b+a\coth(x))\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}+\frac{ab^2\cosh(x)}{\left(a^2-b^2\right)^2}-\frac{a\cosh(x)}{a^2-b^2}+\frac{a\cosh^3(x)}{3\left(a^2-b^2\right)}-\frac{b^3\sinh(x}{\left(a^2-b^2\right.}
\end{aligned}
$$

Mathematica [A]
time = 0.55, size = 171, normalized size = 1.28

$$
\frac{24b^4\sqrt{a+b}\,\text{ArcTan}\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a+b}\,\sqrt{a+b}}\right)-3a\sqrt{-a+b}\left(3a^3+3a^2b-7ab^2-7b^3\right)\cosh(x)-a(-a+b)^{3/2}(a+b)^2\cosh(3x)+3b\sqrt{-a+b}\left(a^3+a^2b-5ab^2-5b^3\right)\sinh(x)+b(-a+b)^{3/2}(a+b)^2\sinh(3x)}{12(-a+b)^{5/2}(a+b)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(a + b*Coth[x]),x]
```

[Out] (24*b^4*Sqrt[a + b]*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a + b])] - 3*a*Sqrt[-a + b]*(3*a^3 + 3*a^2*b - 7*a*b^2 - 7*b^3)*Cosh[x] - a*(-a + b)^(3/2)*(a + b)^2*Cosh[3*x] + 3*b*Sqrt[-a + b]*(a^3 + a^2*b - 5*a*b^2 - 5*b^3)*Sinh[x] + b*(-a + b)^(3/2)*(a + b)^2*Sinh[3*x])/(12*(-a + b)^(5/2)*(a + b)^3)

Maple [A]
time = 0.70, size = 175, normalized size = 1.31

method	result
risch	$\frac{e^{3x}}{24a+24b}-\frac{3\,e^{x}a}{8(a+b)^2}-\frac{5\,e^{x}b}{8(a+b)^2}-\frac{3\,e^{-x}a}{8(a-b)^2}+\frac{5\,e^{-x}b}{8(a-b)^2}+\frac{e^{-3x}}{24a-24b}+\frac{b^4\ln\left(e^{x}-\frac{a-b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(a+b)^2(a-b)^2}-\frac{b^4\ln\left(e^{x}+\frac{a-b}{\sqrt{a^2-\ldots}}\right)}{\sqrt{a^2-b^2}\,(a+b)^2(\ldots}$
default	$\frac{2b^4\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)b+2a}{2\sqrt{-a^2+b^2}}\right)}{(a-b)^2(a+b)^2\sqrt{-a^2+b^2}}-\frac{16}{(32a-32b)\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}+\frac{32}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^3(32a-32b)}-\frac{-2b+a}{2(a-b)^2\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{\ldots}{3(\tanh(\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(x)^3/(a+b*coth(x)),x,method=_RETURNVERBOSE)

[Out] 2*b^4/(a-b)^2/(a+b)^2/(-a^2+b^2)^(1/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*a)/(-a^2+b^2)^(1/2))-16/(32*a-32*b)/(tanh(1/2*x)+1)^2+32/3/(tanh(1/2*x)+1)^3/(32*a-32*b)-1/2*(-2*b+a)/(a-b)^2/(tanh(1/2*x)+1)-32/3/(tanh(1/2*x)-1)^3/(32*a+32*b)-16/(32*a+32*b)/(tanh(1/2*x)-1)^2-1/2/(a+b)^2*(-2*b-a)/(tanh(1/2*x)-1)

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(x)^3/(a+b*coth(x)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more de

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 902 vs. 2(126) = 252.
time = 0.40, size = 1859, normalized size = 13.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] [1/24*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a
^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5
- a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 + a^5 + a^4*b - 2*
a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 - 3*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b
^3 + 7*a*b^4 - 5*b^5)*cosh(x)^4 - 3*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3
 + 7*a*b^4 - 5*b^5 - 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*
cosh(x)^2)*sinh(x)^4 + 4*(5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 -
b^5)*cosh(x)^3 - 3*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^
5)*cosh(x))*sinh(x)^3 - 3*(3*a^5 + a^4*b - 10*a^3*b^2 - 6*a^2*b^3 + 7*a*b^4
 + 5*b^5)*cosh(x)^2 - 3*(3*a^5 + a^4*b - 10*a^3*b^2 - 6*a^2*b^3 + 7*a*b^4 +
 5*b^5 - 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 +
6*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^5)*cosh(x)^2)*sin
h(x)^2 + 24*(b^4*cosh(x)^3 + 3*b^4*cosh(x)^2*sinh(x) + 3*b^4*cosh(x)*sinh(x
)^2 + b^4*sinh(x)^3)*sqrt(a^2 - b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cos
h(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(a^2 - b^2)*(cosh(x) + sinh(x)) +
a - b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 -
 a + b)) + 6*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5
 - 2*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^5)*cosh(x)^3 -
 (3*a^5 + a^4*b - 10*a^3*b^2 - 6*a^2*b^3 + 7*a*b^4 + 5*b^5)*cosh(x))*sinh(x
))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*
a^2*b^4 - b^6)*cosh(x)^2*sinh(x) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*co
sh(x)*sinh(x)^2 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^3), 1/24*((a^
5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a^5 - a^4*b
 - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5 - a^4*b -
2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 + a^5 + a^4*b - 2*a^3*b^2 -
2*a^2*b^3 + a*b^4 + b^5 - 3*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b
^4 - 5*b^5)*cosh(x)^4 - 3*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4
 - 5*b^5 - 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)
*sinh(x)^4 + 4*(5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(
x)^3 - 3*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^5)*cosh(x)
)*sinh(x)^3 - 3*(3*a^5 + a^4*b - 10*a^3*b^2 - 6*a^2*b^3 + 7*a*b^4 + 5*b^5)*
cosh(x)^2 - 3*(3*a^5 + a^4*b - 10*a^3*b^2 - 6*a^2*b^3 + 7*a*b^4 + 5*b^5 - 5
*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 6*(3*a^5 -
 a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^5)*cosh(x)^2)*sinh(x)^2 + 4
8*(b^4*cosh(x)^3 + 3*b^4*cosh(x)^2*sinh(x) + 3*b^4*cosh(x)*sinh(x)^2 + b^4*
sinh(x)^3)*sqrt(-a^2 + b^2)*arctan(sqrt(-a^2 + b^2)/((a + b)*cosh(x) + (a +
 b)*sinh(x))) + 6*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh
(x)^5 - 2*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^5)*cosh(x
)^3 - (3*a^5 + a^4*b - 10*a^3*b^2 - 6*a^2*b^3 + 7*a*b^4 + 5*b^5)*cosh(x))*s
inh(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b^2
 + 3*a^2*b^4 - b^6)*cosh(x)^2*sinh(x) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^
6)*cosh(x)*sinh(x)^2 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^3)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3(x)}{a + b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(a+b*coth(x)),x)
```

```
[Out] Integral(sinh(x)**3/(a + b*coth(x)), x)
```

Giac [A]
time = 0.41, size = 163, normalized size = 1.22

$$-\frac{2\,b^4\arctan\left(-\frac{ae^x+be^x}{\sqrt{-a^2+b^2}}\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{-a^2+b^2}}-\frac{\left(9\,ae^{(2\,x)}-15\,be^{(2\,x)}-a+b\right)e^{(-3\,x)}}{24\,\left(a^2-2\,ab+b^2\right)}+\frac{a^2e^{(3\,x)}+2\,abe^{(3\,x)}+b^2e^{(3\,x)}-9\,a^2e^x-24\,abe^x-15\,b^2e^x}{24\,\left(a^3+3\,a^2b+3\,ab^2+b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -2*b^4*arctan(-(a*e^x + b*e^x)/sqrt(-a^2 + b^2))/((a^4 - 2*a^2*b^2 + b^4)*s
qrt(-a^2 + b^2)) - 1/24*(9*a*e^(2*x) - 15*b*e^(2*x) - a + b)*e^(-3*x)/(a^2
- 2*a*b + b^2) + 1/24*(a^2*e^(3*x) + 2*a*b*e^(3*x) + b^2*e^(3*x) - 9*a^2*e^
x - 24*a*b*e^x - 15*b^2*e^x)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
```

Mupad [B]
time = 1.86, size = 172, normalized size = 1.28

$$\frac{\mathrm{e}^{-3\,x}}{24\,a-24\,b}+\frac{\mathrm{e}^{3\,x}}{24\,a+24\,b}-\frac{\mathrm{e}^{-x}\,\left(3\,a-5\,b\right)}{8\,{\left(a-b\right)}^2}-\frac{\mathrm{e}^{x}\,\left(3\,a+5\,b\right)}{8\,{\left(a+b\right)}^2}-\frac{b^4\,\ln\left(2\,a^3\,b-2\,a\,b^3+a^4-b^4+\mathrm{e}^{x}\,{\left(a+b\right)}^{7/2}\,\sqrt{a-b}\right)}{{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{5/2}}+\frac{b^4\,\ln\left(2\,a\,b^3-2\,a^3\,b-a^4+b^4+\mathrm{e}^{x}\,{\left(a+b\right)}^{7/2}\,\sqrt{a-b}\right)}{{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a + b*coth(x)),x)
```

```
[Out] exp(-3*x)/(24*a - 24*b) + exp(3*x)/(24*a + 24*b) - (exp(-x)*(3*a - 5*b))/(8
*(a - b)^2) - (exp(x)*(3*a + 5*b))/(8*(a + b)^2) - (b^4*log(2*a^3*b - 2*a*b
^3 + a^4 - b^4 + exp(x)*(a + b)^(7/2)*(a - b)^(1/2)))/((a + b)^(5/2)*(a - b
)^(5/2)) + (b^4*log(2*a*b^3 - 2*a^3*b - a^4 + b^4 + exp(x)*(a + b)^(7/2)*(a
 - b)^(1/2)))/((a + b)^(5/2)*(a - b)^(5/2))
```

3.99 $\int \frac{\sinh^2(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=92

$$\frac{(a+2b)\log(1-\coth(x))}{4(a+b)^2}-\frac{(a-2b)\log(1+\coth(x))}{4(a-b)^2}-\frac{b^3\log(a+b\coth(x))}{(a^2-b^2)^2}-\frac{(b-a\coth(x))\sinh^2(x)}{2\,(a^2-b^2)}$$

```
[Out] 1/4*(a+2*b)*ln(1-coth(x))/(a+b)^2-1/4*(a-2*b)*ln(1+coth(x))/(a-b)^2-b^3*ln(
a+b*coth(x))/(a^2-b^2)^2-1/2*(b-a*coth(x))*sinh(x)^2/(a^2-b^2)
```

Rubi [A]
time = 0.10, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3587, 755, 815}

$$-\frac{\sinh^2(x)(b-a\coth(x))}{2\,(a^2-b^2)}-\frac{b^3\log(a+b\coth(x))}{(a^2-b^2)^2}+\frac{(a+2b)\log(1-\coth(x))}{4(a+b)^2}-\frac{(a-2b)\log(\coth(x)+1)}{4(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(a + b*Coth[x]),x]
```

```
[Out] ((a + 2*b)*Log[1 - Coth[x]])/(4*(a + b)^2) - ((a - 2*b)*Log[1 + Coth[x]])/(
4*(a - b)^2) - (b^3*Log[a + b*Coth[x]])/(a^2 - b^2)^2 - ((b - a*Coth[x])*Si
nh[x]^2)/(2*(a^2 - b^2))
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sinh^2(x)}{a+b\coth(x)}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(a+x)\left(1-\frac{x^2}{b^2}\right)^2}\,dx, x, b\coth(x)\right)}{b}$$

$$= -\frac{(b-a\coth(x))\sinh^2(x)}{2(a^2-b^2)} - \frac{b\text{Subst}\left(\int \frac{-2+\frac{a^2}{b^2}+\frac{ax}{b^2}}{(a+x)\left(1-\frac{x^2}{b^2}\right)}\,dx, x, b\coth(x)\right)}{2(a^2-b^2)}$$

$$= -\frac{(b-a\coth(x))\sinh^2(x)}{2(a^2-b^2)} - \frac{b\text{Subst}\left(\int \left(\frac{(a-b)(a+2b)}{2b(a+b)(b-x)} + \frac{2b^2}{(a-b)(a+b)(a+x)} + \frac{(a-2b)(a+b)}{2(a-b)b(b+x)}\right) dx}{2(a^2-b^2)}$$

$$= \frac{(a+2b)\log(1-\coth(x))}{4(a+b)^2} - \frac{(a-2b)\log(1+\coth(x))}{4(a-b)^2} - \frac{b^3\log(a+b\coth(x))}{(a^2-b^2)^2} - \frac{(b-}{}$$

Mathematica [A]
time = 0.11, size = 75, normalized size = 0.82

$$\frac{-2a^3x+6ab^2x+(-a^2b+b^3)\cosh(2x)-4b^3\log(b\cosh(x)+a\sinh(x))+a(a^2-b^2)\sinh(2x)}{4(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(a + b*Coth[x]),x]
[Out] (-2*a^3*x + 6*a*b^2*x + (-(a^2*b) + b^3)*Cosh[2*x] - 4*b^3*Log[b*Cosh[x] +
a*Sinh[x]] + a*(a^2 - b^2)*Sinh[2*x])/(4*(a - b)^2*(a + b)^2)
```

Maple [A]
time = 0.72, size = 155, normalized size = 1.68

method	result
risch	$-\frac{xb}{(a+b)^2} - \frac{ax}{2(a+b)^2} + \frac{e^{2x}}{8a+8b} - \frac{e^{-2x}}{8(a-b)} + \frac{2b^3x}{a^4-2a^2b^2+b^4} - \frac{b^3\ln\left(e^{2x}-\frac{a-b}{a+b}\right)}{a^4-2a^2b^2+b^4}$
default	$-\frac{b^3\ln\left(b\left(\tanh^2\left(\frac{x}{2}\right)\right)+2a\tanh\left(\frac{x}{2}\right)+b\right)}{(a-b)^2(a+b)^2} + \frac{8}{(16a+16b)\left(\tanh\left(\frac{x}{2}\right)-1\right)^2} + \frac{16}{(32a+32b)\left(\tanh\left(\frac{x}{2}\right)-1\right)} + \frac{(2b+a)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2(a+b)^2} - \frac{}{(16}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a+b*coth(x)),x,method=_RETURNVERBOSE)
[Out] -b^3/(a-b)^2/(a+b)^2*ln(b*tanh(1/2*x)^2+2*a*tanh(1/2*x)+b)+8/(16*a+16*b)/(t
anh(1/2*x)-1)^2+16/(32*a+32*b)/(tanh(1/2*x)-1)+1/2*(2*b+a)/(a+b)^2*ln(tanh(
1/2*x)-1)-8/(16*a-16*b)/(tanh(1/2*x)+1)^2+16/(32*a-32*b)/(tanh(1/2*x)+1)+1/
2/(a-b)^2*(2*b-a)*ln(tanh(1/2*x)+1)
```

Maxima [A]
time = 0.27, size = 83, normalized size = 0.90

$$-\frac{b^3 \log\left(-(a-b)e^{(-2\,x)}+a+b\right)}{a^4-2\,a^2b^2+b^4}-\frac{(a+2\,b)x}{2\,(a^2+2\,ab+b^2)}+\frac{e^{(2\,x)}}{8\,(a+b)}-\frac{e^{(-2\,x)}}{8\,(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] -b^3*log(-(a - b)*e^(-2*x) + a + b)/(a^4 - 2*a^2*b^2 + b^4) - 1/2*(a + 2*b)
*x/(a^2 + 2*a*b + b^2) + 1/8*e^(2*x)/(a + b) - 1/8*e^(-2*x)/(a - b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 331 vs. 2(87) = 174.
time = 0.37, size = 331, normalized size = 3.60

$$\frac{(a^3-a^2b-ab^2+b^3)\cosh(x)^4+4\,(a^3-a^2b-ab^2+b^3)\cosh(x)\sinh(x)^3+(a^3-a^2b-ab^2+b^3)\sinh(x)^4-4\,(a^3-3\,ab^2-2\,b^3)x\cosh(x)^2-a^3-a^2b+ab^2+b^3+2\,\left(3\,(a^3-a^2b-ab^2+b^3)\cosh(x)^2-2\,(a^3-3\,ab^2-2\,b^3)x\right)\sinh(x)^2-8\,\left(b^3\cosh(x)^2+2\,b^3\cosh(x)\sinh(x)+b^3\sinh(x)^2\right)\log\left(\frac{2\,(b\cosh(x)+a\sinh(x))}{\cosh(x)-\sinh(x)}\right)+4\,\left((a^3-a^2b-ab^2+b^3)\cosh(x)^3-2\,(a^3-3\,ab^2-2\,b^3)x\cosh(x)\right)\sinh(x)}{8\,\left((a^4-2\,a^2b^2+b^4)\cosh(x)^2+2\,(a^4-2\,a^2b^2+b^4)\cosh(x)\sinh(x)+(a^4-2\,a^2b^2+b^4)\sinh(x)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] 1/8*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 - a^2*b - a*b^2 + b^3)*
cosh(x)*sinh(x)^3 + (a^3 - a^2*b - a*b^2 + b^3)*sinh(x)^4 - 4*(a^3 - 3*a*b^
2 - 2*b^3)*x*cosh(x)^2 - a^3 - a^2*b + a*b^2 + b^3 + 2*(3*(a^3 - a^2*b - a*
b^2 + b^3)*cosh(x)^2 - 2*(a^3 - 3*a*b^2 - 2*b^3)*x)*sinh(x)^2 - 8*(b^3*cosh
(x)^2 + 2*b^3*cosh(x)*sinh(x) + b^3*sinh(x)^2)*log(2*(b*cosh(x) + a*sinh(x)
)/(cosh(x) - sinh(x))) + 4*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^3 - 2*(a^3
- 3*a*b^2 - 2*b^3)*x*cosh(x))*sinh(x))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 +
 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x) + (a^4 - 2*a^2*b^2 + b^4)*sinh(x
)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(x)}{a+b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(a+b*coth(x)),x)
```

```
[Out] Integral(sinh(x)**2/(a + b*coth(x)), x)
```

Giac [A]
time = 0.43, size = 114, normalized size = 1.24

$$-\frac{b^3 \log\left(\left|-ae^{(2\,x)}-be^{(2\,x)}+a-b\right|\right)}{a^4-2\,a^2b^2+b^4}-\frac{(a-2\,b)x}{2\,(a^2-2\,ab+b^2)}+\frac{\left(2\,ae^{(2\,x)}-4\,be^{(2\,x)}-a+b\right)e^{(-2\,x)}}{8\,(a^2-2\,ab+b^2)}+\frac{e^{(2\,x)}}{8\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -b^3*log(abs(-a*e^(2*x) - b*e^(2*x) + a - b))/(a^4 - 2*a^2*b^2 + b^4) - 1/2
*(a - 2*b)*x/(a^2 - 2*a*b + b^2) + 1/8*(2*a*e^(2*x) - 4*b*e^(2*x) - a + b)*
e^(-2*x)/(a^2 - 2*a*b + b^2) + 1/8*e^(2*x)/(a + b)
```

Mupad [B]
time = 1.47, size = 85, normalized size = 0.92

$$\frac{e^{2x}}{8a+8b}-\frac{e^{-2x}}{8a-8b}-\frac{b^3\ln\left(b-a+a\,e^{2x}+b\,e^{2x}\right)}{a^4-2a^2b^2+b^4}-\frac{x\,(a-2b)}{2\,(a-b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a + b*coth(x)),x)
```

```
[Out] exp(2*x)/(8*a + 8*b) - exp(-2*x)/(8*a - 8*b) - (b^3*log(b - a + a*exp(2*x)
+ b*exp(2*x)))/(a^4 + b^4 - 2*a^2*b^2) - (x*(a - 2*b))/(2*(a - b)^2)
```

3.100 $\int \frac{\sinh(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=73

$$-\frac{b^2\tanh^{-1}\left(\frac{(b+a\coth(x))\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}+\frac{a\cosh(x)}{a^2-b^2}-\frac{b\sinh(x)}{a^2-b^2}$$

```
[Out] -b^2*arctanh((b+a*coth(x))*sinh(x)/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)+a*cosh(
x)/(a^2-b^2)-b*sinh(x)/(a^2-b^2)
```

Rubi [A]
time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {3592, 3567, 2718, 3590, 212}

$$-\frac{b\sinh(x)}{a^2-b^2}+\frac{a\cosh(x)}{a^2-b^2}-\frac{b^2\tanh^{-1}\left(\frac{\sinh(x)(a\coth(x)+b)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(a + b*Coth[x]),x]
```

```
[Out] -((b^2*ArcTanh[((b + a*Coth[x])*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2
)) + (a*Cosh[x])/(a^2 - b^2) - (b*Sinh[x])/(a^2 - b^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3590

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> Dist[-f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*
x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3592

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(d*Sec[e + f*x])^m*(a - b*Tan[e
+ f*x]), x], x] + Dist[b^2/(d^2*(a^2 + b^2)), Int[(d*Sec[e + f*x])^(m + 2)/
(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 + b^2,
 0] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(x)}{a+b\coth(x)}\,dx &= \frac{\int (a-b\coth(x))\sinh(x)\,dx}{a^2-b^2}+\frac{b^2\int \frac{\operatorname{csch}(x)}{a+b\coth(x)}\,dx}{a^2-b^2}\\ &= -\frac{b\sinh(x)}{a^2-b^2}+\frac{a\int \sinh(x)\,dx}{a^2-b^2}-\frac{b^2\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx,x,i(-ib-ia\coth(x))\sinh(x)\right)}{a^2-b^2}\\ &= -\frac{b^2\tanh^{-1}\left(\frac{(b+a\coth(x))\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}+\frac{a\cosh(x)}{a^2-b^2}-\frac{b\sinh(x)}{a^2-b^2}\end{aligned}$$

Mathematica [A]
time = 0.29, size = 80, normalized size = 1.10

$$\frac{a\cosh(x)}{a^2-b^2}+b\left(-\frac{2b\text{ArcTan}\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a+b}\,\sqrt{a+b}}\right)}{(-a+b)^{3/2}(a+b)^{3/2}}+\frac{\sinh(x)}{-a^2+b^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(a + b*Coth[x]),x]
```

```
[Out] (a*Cosh[x])/(a^2 - b^2) + b*((-2*b*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*S
qrt[a + b])])/((-a + b)^(3/2)*(a + b)^(3/2)) + Sinh[x]/(-a^2 + b^2))
```

Maple [A]
time = 0.61, size = 93, normalized size = 1.27

method	result	size

default	$\frac{8}{(8a-8b)\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{2b^2\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)b+2a}{2\sqrt{-a^2+b^2}}\right)}{(a+b)(a-b)\sqrt{-a^2+b^2}}-\frac{8}{(8a+8b)\left(\tanh\left(\frac{x}{2}\right)-1\right)}$	93
risch	$\frac{e^x}{2b+2a}+\frac{e^{-x}}{2a-2b}+\frac{b^2\ln\left(e^x-\frac{a-b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)}-\frac{b^2\ln\left(e^x+\frac{a-b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)}$	122

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 8/(8*a-8*b)/(tanh(1/2*x)+1)+2*b^2/(a+b)/(a-b)/(-a^2+b^2)^(1/2)*arctan(1/2*(
2*tanh(1/2*x)*b+2*a)/(-a^2+b^2)^(1/2))-8/(8*a+8*b)/(tanh(1/2*x)-1)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(69) = 138$.
time = 0.38, size = 431, normalized size = 5.90

$$\left[\frac{a^3+a^2b-ab^2-b^3+(a^3-a^2b-ab^2+b^3)\cosh(x)^2+2(a^3-a^2b-ab^2+b^3)\cosh(x)\sinh(x)+(a^3-a^2b-ab^2+b^3)\sinh(x)^2-2(b^2\cosh(x)+b^2\sinh(x))\sqrt{a^2-b^2}\log\left(\frac{(a+b)\cosh(x)^2+2(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2+2\sqrt{a^2-b^2}(\cosh(x)+\sinh(x))+a-b}{(a+b)\cosh(x)^2+2(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2-a+b}\right)}{2((a^4-2a^2b^2+b^4)\cosh(x)+(a^4-2a^2b^2+b^4)\sinh(x))}, \frac{a^3+a^2b-ab^2-b^3+(a^3-a^2b-ab^2+b^3)\cosh(x)^2+2(a^3-a^2b-ab^2+b^3)\cosh(x)\sinh(x)+(a^3-a^2b-ab^2+b^3)\sinh(x)^2+4(b^2\cosh(x)+b^2\sinh(x))\sqrt{-a^2+b^2}\arctan\left(\frac{\sqrt{-a^2+b^2}}{(a+b)\cosh(x)+(a+b)\sinh(x)}\right)}{2((a^4-2a^2b^2+b^4)\cosh(x)+(a^4-2a^2b^2+b^4)\sinh(x))}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] [1/2*(a^3 + a^2*b - a*b^2 - b^3 + (a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^2 + 2
*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) + (a^3 - a^2*b - a*b^2 + b^3)*
sinh(x)^2 - 2*(b^2*cosh(x) + b^2*sinh(x))*sqrt(a^2 - b^2)*log(((a + b)*cosh
(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + 2*sqrt(a^2 - b^2)*(
cosh(x) + sinh(x)) + a - b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x)
+ (a + b)*sinh(x)^2 - a + b)))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 - 2*
a^2*b^2 + b^4)*sinh(x)), 1/2*(a^3 + a^2*b - a*b^2 - b^3 + (a^3 - a^2*b - a*
b^2 + b^3)*cosh(x)^2 + 2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) + (a^3
 - a^2*b - a*b^2 + b^3)*sinh(x)^2 + 4*(b^2*cosh(x) + b^2*sinh(x))*sqrt(-a^2
```

```
+ b^2)*arctan(sqrt(-a^2 + b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))))/((a^4
- 2*a^2*b^2 + b^4)*cosh(x) + (a^4 - 2*a^2*b^2 + b^4)*sinh(x))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(x)}{a + b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*coth(x)),x)
[Out] Integral(sinh(x)/(a + b*coth(x)), x)
```

Giac [A]
time = 0.39, size = 72, normalized size = 0.99

$$\frac{2\,b^2 \arctan\left(\frac{ae^x + be^x}{\sqrt{-a^2 + b^2}}\right)}{(a^2 - b^2)\sqrt{-a^2 + b^2}} + \frac{e^{(-x)}}{2\,(a - b)} + \frac{e^x}{2\,(a + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*coth(x)),x, algorithm="giac")
[Out] 2*b^2*arctan((a*e^x + b*e^x)/sqrt(-a^2 + b^2))/((a^2 - b^2)*sqrt(-a^2 + b^2
)) + 1/2*e^(-x)/(a - b) + 1/2*e^x/(a + b)
```

Mupad [B]
time = 1.50, size = 156, normalized size = 2.14

$$\frac{e^x}{2\,a + 2\,b} + \frac{e^{-x}}{2\,a - 2\,b} - \frac{b^2\,\ln\left(\frac{2\,b^2\,e^x}{-a^3 - a^2\,b + a\,b^2 + b^3} - \frac{2\,b^2}{(a+b)^{5/2}\,\sqrt{a - b}}\right)}{(a + b)^{3/2}\,(a - b)^{3/2}} + \frac{b^2\,\ln\left(\frac{2\,b^2}{(a+b)^{5/2}\,\sqrt{a - b}} + \frac{2\,b^2\,e^x}{-a^3 - a^2\,b + a\,b^2 + b^3}\right)}{(a + b)^{3/2}\,(a - b)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a + b*coth(x)),x)
[Out] exp(x)/(2*a + 2*b) + exp(-x)/(2*a - 2*b) - (b^2*log((2*b^2*exp(x))/(a*b^2 -
 a^2*b - a^3 + b^3) - (2*b^2)/((a + b)^(5/2)*(a - b)^(1/2))))/((a + b)^(3/2
)*(a - b)^(3/2)) + (b^2*log((2*b^2)/((a + b)^(5/2)*(a - b)^(1/2)) + (2*b^2*
exp(x))/(a*b^2 - a^2*b - a^3 + b^3)))/((a + b)^(3/2)*(a - b)^(3/2))
```

3.101 $\int \frac{\operatorname{csch}(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=38

$$-\frac{\tanh^{-1}\left(\frac{(b+a\coth(x))\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

```
[Out] -arctanh((b+a*coth(x))*sinh(x)/(a^2-b^2)^(1/2))/(a^2-b^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3590, 212}

$$-\frac{\tanh^{-1}\left(\frac{\sinh(x)(a\coth(x)+b)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]/(a + b*Coth[x]),x]
```

```
[Out] -(ArcTanh[((b + a*Coth[x])*Sinh[x])/Sqrt[a^2 - b^2]]/Sqrt[a^2 - b^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3590

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> Dist[-f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*
x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}(x)}{a+b\coth(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx, x, i(-ib-ia\coth(x))\sinh(x)\right)\\ &= -\frac{i\tan^{-1}\left(\frac{(-ib-ia\coth(x))\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 46, normalized size = 1.21

$$\frac{2\text{ArcTan}\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a+b}\sqrt{a+b}}\right)}{\sqrt{-a+b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]/(a + b*Coth[x]),x]
[Out] (2*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a + b])])/(Sqrt[-a + b]*Sqrt
[a + b])
```

Maple [A]
time = 0.43, size = 39, normalized size = 1.03

method	result	size
default	$\frac{2\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)b+2a}{2\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$	39
risch	$\frac{\ln\left(e^x-\frac{a-b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-\frac{\ln\left(e^x+\frac{a-b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)/(a+b*coth(x)),x,method=_RETURNVERBOSE)
[Out] 2/(-a^2+b^2)^(1/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*a)/(-a^2+b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*coth(x)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.36, size = 147, normalized size = 3.87

$$\left[\frac{\log\left(\frac{(a+b)\cosh(x)^2+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2-2\,\sqrt{a^2-b^2}\,(\cosh(x)+\sinh(x))+a-b}{(a+b)\cosh(x)^2+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2-a+b}\right)}{\sqrt{a^2-b^2}},\ \frac{2\,\sqrt{-a^2+b^2}\,\arctan\left(\frac{\sqrt{-a^2+b^2}}{(a+b)\cosh(x)+(a+b)\sinh(x)}\right)}{a^2-b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] [log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2
*sqrt(a^2 - b^2)*(cosh(x) + sinh(x)) + a - b)/((a + b)*cosh(x)^2 + 2*(a + b
)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a + b))/sqrt(a^2 - b^2), 2*sqrt(-a^
2 + b^2)*arctan(sqrt(-a^2 + b^2)/((a + b)*cosh(x) + (a + b)*sinh(x)))/(a^2
- b^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(x)}{a+b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*coth(x)),x)
```

```
[Out] Integral(csch(x)/(a + b*coth(x)), x)
```

Giac [**A**]
time = 0.40, size = 35, normalized size = 0.92

$$\frac{2\arctan\left(\frac{ae^x+be^x}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] 2*arctan((a*e^x + b*e^x)/sqrt(-a^2 + b^2))/sqrt(-a^2 + b^2)
```

Mupad [**B**]
time = 0.17, size = 35, normalized size = 0.92

$$-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^x\,\sqrt{b^2-a^2}}{a-b}\right)}{\sqrt{b^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)*(a + b*coth(x))),x)
```

```
[Out] -(2*atan((exp(x)*(b^2 - a^2)^(1/2))/(a - b)))/(b^2 - a^2)^(1/2)
```

3.102 $\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=12

$$-\frac{\log(a+b\coth(x))}{b}$$

[Out] -ln(a+b*coth(x))/b

Rubi [A]
time = 0.03, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3587, 31}

$$-\frac{\log(a+b\coth(x))}{b}$$

Antiderivative was successfully verified.

[In] Int[Csch[x]^2/(a + b*Coth[x]),x]

[Out] -(Log[a + b*Coth[x]]/b)

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\coth(x)\right)}{b}\\ &= -\frac{\log(a+b\coth(x))}{b}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 20, normalized size = 1.67

$$\frac{\log(\sinh(x))-\log(b\cosh(x)+a\sinh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^2/(a + b*Coth[x]),x]
[Out] (Log[Sinh[x]] - Log[b*Cosh[x] + a*Sinh[x]])/b
```

Maple [A]
time = 0.57, size = 13, normalized size = 1.08

method	result	size
derivativedivides	$-\frac{\ln(a+b\coth(x))}{b}$	13
default	$-\frac{\ln(a+b\coth(x))}{b}$	13
risch	$-\frac{\ln\left(e^{2x}-\frac{a-b}{a+b}\right)}{b}+\frac{\ln(e^{2x}-1)}{b}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^2/(a+b*coth(x)),x,method=_RETURNVERBOSE)
[Out] -ln(a+b*coth(x))/b
```

Maxima [A]
time = 0.26, size = 12, normalized size = 1.00

$$-\frac{\log\left(b\coth\left(x\right)+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*coth(x)),x, algorithm="maxima")
[Out] -log(b*coth(x) + a)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(12) = 24$.
time = 0.36, size = 43, normalized size = 3.58

$$-\frac{\log\left(\frac{2\,(b\cosh(x)+a\sinh(x))}{\cosh(x)-\sinh(x)}\right)-\log\left(\frac{2\,\sinh(x)}{\cosh(x)-\sinh(x)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*coth(x)),x, algorithm="fricas")
[Out] -(log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) - log(2*sinh(x)/(cosh(
x) - sinh(x))))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{csch}^2\left(x\right)}{a+b\coth\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2/(a+b*coth(x)),x)
```

```
[Out] Integral(csch(x)**2/(a + b*coth(x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(12) = 24.
time = 0.42, size = 46, normalized size = 3.83

$$-\frac{(a+b)\log\left(\left|ae^{(2x)}+be^{(2x)}-a+b\right|\right)}{ab+b^2}+\frac{\log\left(\left|e^{(2x)}-1\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -(a + b)*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a*b + b^2) + log(abs(e^(2
*x) - 1))/b
```

Mupad [B]
time = 0.16, size = 51, normalized size = 4.25

$$-\frac{2\,\mathrm{atan}\left(\frac{a\,\mathrm{e}^{2\,x}\,\sqrt{-b^2}-a\,\sqrt{-b^2}+b\,\mathrm{e}^{2\,x}\,\sqrt{-b^2}}{b^2}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^2*(a + b*coth(x))),x)
```

```
[Out] -(2*atan((a*exp(2*x)*(-b^2)^(1/2) - a*(-b^2)^(1/2) + b*exp(2*x)*(-b^2)^(1/2
))/b^2))/(-b^2)^(1/2)
```

3.103 $\int \frac{\operatorname{csch}^3(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=57

$$\frac{a\tanh^{-1}(\cosh(x))}{b^2}-\frac{\sqrt{a^2-b^2}\tanh^{-1}\left(\frac{(b+a\coth(x))\sinh(x)}{\sqrt{a^2-b^2}}\right)}{b^2}-\frac{\operatorname{csch}(x)}{b}$$

```
[Out] a*arctanh(cosh(x))/b^2-csch(x)/b-arctanh((b+a*coth(x))*sinh(x)/(a^2-b^2)^(1
/2))*(a^2-b^2)^(1/2)/b^2
```

Rubi [A]
time = 0.08, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.385$, Rules used = {3591, 3567, 3855, 3590, 212}

$$-\frac{\sqrt{a^2-b^2}\tanh^{-1}\left(\frac{\sinh(x)(a\coth(x)+b)}{\sqrt{a^2-b^2}}\right)}{b^2}+\frac{a\tanh^{-1}(\cosh(x))}{b^2}-\frac{\operatorname{csch}(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^3/(a + b*Coth[x]),x]
```

```
[Out] (a*ArcTanh[Cosh[x]])/b^2 - (Sqrt[a^2 - b^2]*ArcTanh[((b + a*Coth[x])*Sinh[x
])/Sqrt[a^2 - b^2]])/b^2 - Csch[x]/b
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3590

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> Dist[-f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*
x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3591

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[-d^2/b^2, Int[(d*Sec[e + f*x])^(m - 2)*(a - b*Tan[e
 + f*x]), x], x] + Dist[d^2*((a^2 + b^2)/b^2), Int[(d*Sec[e + f*x])^(m - 2)
/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 + b^2
, 0] && IGtQ[m, 1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^3(x)}{a+b\coth(x)}\,dx &= -\frac{\int (a-b\coth(x))\operatorname{csch}(x)\,dx}{b^2}+\frac{(a^2-b^2)\int \frac{\operatorname{csch}(x)}{a+b\coth(x)}\,dx}{b^2}\\ &= -\frac{\operatorname{csch}(x)}{b}-\frac{a\int \operatorname{csch}(x)\,dx}{b^2}-\frac{(a^2-b^2)\operatorname{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx,x,i(-ib-ia\coth(x))\sinh\right.}{b^2}\\ &= \frac{a\tanh^{-1}(\cosh(x))}{b^2}-\frac{\sqrt{a^2-b^2}\,\tanh^{-1}\left(\frac{(b+a\coth(x))\sinh(x)}{\sqrt{a^2-b^2}}\right)}{b^2}-\frac{\operatorname{csch}(x)}{b}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 65, normalized size = 1.14

$$-\frac{2\sqrt{-a+b}\,\sqrt{a+b}\,\operatorname{ArcTan}\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a+b}\,\sqrt{a+b}}\right)+b\operatorname{csch}(x)+a\log\left(\tanh\left(\frac{x}{2}\right)\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^3/(a + b*Coth[x]),x]
[Out] -((2*Sqrt[-a + b]*Sqrt[a + b]*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a
 + b])] + b*Csch[x] + a*Log[Tanh[x/2]])/b^2)
```

Maple [A]
time = 0.77, size = 85, normalized size = 1.49

method	result
default	$\frac{\tanh\left(\frac{x}{2}\right)}{2b}-\frac{1}{2b\tanh\left(\frac{x}{2}\right)}-\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{b^2}+\frac{(4a^2-4b^2)\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)b+2a}{2\sqrt{-a^2+b^2}}\right)}{2b^2\sqrt{-a^2+b^2}}$

risch	$-\frac{2\,\mathrm{e}^{x}}{b(\mathrm{e}^{2x}-1)}+\frac{a\ln(\mathrm{e}^{x}+1)}{b^{2}}-\frac{a\ln(\mathrm{e}^{x}-1)}{b^{2}}+\frac{\sqrt{a^{2}-b^{2}}\,\ln\left(\mathrm{e}^{x}-\frac{\sqrt{a^{2}-b^{2}}}{a+b}\right)}{b^{2}}-\frac{\sqrt{a^{2}-b^{2}}\,\ln\left(\mathrm{e}^{x}+\frac{\sqrt{a^{2}-b^{2}}}{a+b}\right)}{b^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^3/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b*tanh(1/2*x)-1/2/b/tanh(1/2*x)-a/b^2*ln(tanh(1/2*x))+1/2/b^2*(4*a^2-4*
b^2)/(-a^2+b^2)^(1/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*a)/(-a^2+b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(53) = 106$.
time = 0.41, size = 384, normalized size = 6.74

$$\left[\frac{\sqrt{a^{2}-b^{2}}\left(\cosh(x)^{2}+2\cosh(x)\sinh(x)+\sinh(x)^{2}-1\right)\log\left(\frac{(a+b)\cosh(x)^{2}+2(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^{2}-2\sqrt{a^{2}-b^{2}}(\cosh(x)+\sinh(x))+a-b}{(a+b)\cosh(x)^{2}+2(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^{2}-a+b}\right)-2b\cosh(x)+\left(a\cosh(x)^{2}+2a\cosh(x)\sinh(x)+a\sinh(x)^{2}-a\right)\log(\cosh(x)+\sinh(x)+1)-\left(a\cosh(x)^{2}+2a\cosh(x)\sinh(x)+a\sinh(x)^{2}-a\right)\log(\cosh(x)+\sinh(x)-1)-2b\sinh(x)}{b^{2}\cosh(x)^{2}+2b^{2}\cosh(x)\sinh(x)+b^{2}\sinh(x)^{2}-b^{2}}, \frac{2\sqrt{-a^{2}+b^{2}}\left(\cosh(x)^{2}+2\cosh(x)\sinh(x)+\sinh(x)^{2}-1\right)\arctan\left(\frac{\sqrt{-a^{2}+b^{2}}}{(a+b)\cosh(x)+(a+b)\sinh(x)}\right)-2b\cosh(x)+\left(a\cosh(x)^{2}+2a\cosh(x)\sinh(x)+a\sinh(x)^{2}-a\right)\log(\cosh(x)+\sinh(x)+1)-\left(a\cosh(x)^{2}+2a\cosh(x)\sinh(x)+a\sinh(x)^{2}-a\right)\log(\cosh(x)+\sinh(x)-1)-2b\sinh(x)}{b^{2}\cosh(x)^{2}+2b^{2}\cosh(x)\sinh(x)+b^{2}\sinh(x)^{2}-b^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] [(sqrt(a^2 - b^2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*log(((a +
 b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(a^2
- b^2)*(cosh(x) + sinh(x)) + a - b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*
sinh(x) + (a + b)*sinh(x)^2 - a + b)) - 2*b*cosh(x) + (a*cosh(x)^2 + 2*a*co
sh(x)*sinh(x) + a*sinh(x)^2 - a)*log(cosh(x) + sinh(x) + 1) - (a*cosh(x)^2
+ 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*log(cosh(x) + sinh(x) - 1) - 2*b*s
inh(x))/(b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2 - b^2), (2*s
qrt(-a^2 + b^2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*arctan(sqrt
(-a^2 + b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) - 2*b*cosh(x) + (a*cosh(x
)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*log(cosh(x) + sinh(x) + 1) - (
a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*log(cosh(x) + sinh(x)
- 1) - 2*b*sinh(x))/(b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2
- b^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(x)}{a + b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**3/(a+b*coth(x)),x)
```

```
[Out] Integral(csch(x)**3/(a + b*coth(x)), x)
```

Giac [A]
time = 0.41, size = 85, normalized size = 1.49

$$\frac{a\log\left(e^{x}+1\right)}{b^{2}} - \frac{a\log\left(\left|e^{x}-1\right|\right)}{b^{2}} + \frac{2\left(a^{2}-b^{2}\right)\arctan\left(\frac{ae^{x}+be^{x}}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}\,b^{2}} - \frac{2\,e^{x}}{b(e^{(2\,x)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] a*log(e^x + 1)/b^2 - a*log(abs(e^x - 1))/b^2 + 2*(a^2 - b^2)*arctan((a*e^x
+ b*e^x)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b^2) - 2*e^x/(b*(e^(2*x) - 1))
```

Mupad [B]
time = 1.46, size = 230, normalized size = 4.04

$$\frac{2\,e^{x}}{b-b\,e^{2\,x}} - \frac{a\,\ln\left(32\,a\,b^{2}-64\,a^{2}\,b+32\,a^{3}-32\,a^{3}\,e^{x}-32\,a\,b^{2}\,e^{x}+64\,a^{2}\,b\,e^{x}\right)}{b^{2}} + \frac{a\,\ln\left(32\,a\,b^{2}-64\,a^{2}\,b+32\,a^{3}+32\,a^{3}\,e^{x}+32\,a\,b^{2}\,e^{x}-64\,a^{2}\,b\,e^{x}\right)}{b^{2}} + \frac{\ln\left(32\,a\,\sqrt{a^{2}-b^{2}}-32\,b\,\sqrt{a^{2}-b^{2}}-32\,a^{2}\,e^{x}+32\,b^{2}\,e^{x}\right)\,\sqrt{a^{2}-b^{2}}}{b^{2}} - \frac{\ln\left(32\,a\,\sqrt{a^{2}-b^{2}}-32\,b\,\sqrt{a^{2}-b^{2}}+32\,a^{2}\,e^{x}-32\,b^{2}\,e^{x}\right)\,\sqrt{a^{2}-b^{2}}}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^3*(a + b*coth(x))),x)
```

```
[Out] (2*exp(x))/(b - b*exp(2*x)) - (a*log(32*a*b^2 - 64*a^2*b + 32*a^3 - 32*a^3*
exp(x) - 32*a*b^2*exp(x) + 64*a^2*b*exp(x)))/b^2 + (a*log(32*a*b^2 - 64*a^2
*b + 32*a^3 + 32*a^3*exp(x) + 32*a*b^2*exp(x) - 64*a^2*b*exp(x)))/b^2 + (lo
g(32*a*(a^2 - b^2)^(1/2) - 32*b*(a^2 - b^2)^(1/2) - 32*a^2*exp(x) + 32*b^2*
exp(x))*(a^2 - b^2)^(1/2))/b^2 - (log(32*a*(a^2 - b^2)^(1/2) - 32*b*(a^2 -
b^2)^(1/2) + 32*a^2*exp(x) - 32*b^2*exp(x))*(a^2 - b^2)^(1/2))/b^2
```

3.104 $\int \frac{\operatorname{csch}^4(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=40

$$\frac{a\coth(x)}{b^2}-\frac{\coth^2(x)}{2b}-\frac{(a^2-b^2)\log(a+b\coth(x))}{b^3}$$

```
[Out] a*coth(x)/b^2-1/2*coth(x)^2/b-(a^2-b^2)*ln(a+b*coth(x))/b^3
```

Rubi [A]
time = 0.05, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3587, 711}

$$-\frac{(a^2-b^2)\log(a+b\coth(x))}{b^3}+\frac{a\coth(x)}{b^2}-\frac{\coth^2(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^4/(a + b*Coth[x]),x]
```

```
[Out] (a*Coth[x])/b^2 - Coth[x]^2/(2*b) - ((a^2 - b^2)*Log[a + b*Coth[x]])/b^3
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^4(x)}{a+b\coth(x)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1-\frac{x^2}{b^2}}{a+x}\,dx, x, b\coth(x)\right)}{b}\\
&= \frac{\operatorname{Subst}\left(\int \left(\frac{a}{b^2}-\frac{x}{b^2}+\frac{-a^2+b^2}{b^2(a+x)}\right)\,dx, x, b\coth(x)\right)}{b}\\
&= \frac{a\coth(x)}{b^2}-\frac{\coth^2(x)}{2b}-\frac{(a^2-b^2)\log(a+b\coth(x))}{b^3}
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 50, normalized size = 1.25

$$\frac{2ab\coth(x)-b^2\operatorname{csch}^2(x)+2(a^2-b^2)\left(\log(\sinh(x))-\log(b\cosh(x)+a\sinh(x))\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^4/(a + b*Coth[x]),x]
```

```
[Out] (2*a*b*Coth[x] - b^2*Csch[x]^2 + 2*(a^2 - b^2)*(Log[Sinh[x]] - Log[b*Cosh[x
] + a*Sinh[x]]))/(2*b^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(38) = 76.
time = 0.71, size = 102, normalized size = 2.55

method	result
default	$\frac{-\frac{b\left(\tanh^2\left(\frac{x}{2}\right)\right)}{2}+2a\tanh\left(\frac{x}{2}\right)}{4b^2}-\frac{1}{8b\tanh\left(\frac{x}{2}\right)^2}+\frac{(4a^2-4b^2)\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{4b^3}+\frac{a}{2b^2\tanh\left(\frac{x}{2}\right)}+\frac{(-4a^2+4b^2)\ln\left(b\left(\tanh^2\left(\frac{x}{2}\right)\right)+2a\tanh\right.}{4b^3}$
risch	$\frac{2\,e^{2x}a-2b\,e^{2x}-2a}{\left(e^{2x}-1\right)^2b^2}+\frac{\ln\left(e^{2x}-1\right)a^2}{b^3}-\frac{\ln\left(e^{2x}-1\right)}{b}-\frac{\ln\left(e^{2x}-\frac{a-b}{a+b}\right)a^2}{b^3}+\frac{\ln\left(e^{2x}-\frac{a-b}{a+b}\right)}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^4/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b^2*(-1/2*b*tanh(1/2*x)^2+2*a*tanh(1/2*x))-1/8/b/tanh(1/2*x)^2+1/4/b^3*
(4*a^2-4*b^2)*ln(tanh(1/2*x))+1/2*a/b^2/tanh(1/2*x)+1/4/b^3*(-4*a^2+4*b^2)*
ln(b*tanh(1/2*x)^2+2*a*tanh(1/2*x)+b)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(38) = 76.
time = 0.26, size = 110, normalized size = 2.75

$$\frac{2\left((a+b)e^{(-2x)}-a\right)}{2b^2e^{(-2x)}-b^2e^{(-4x)}-b^2}-\frac{(a^2-b^2)\log\left(-(a-b)e^{(-2x)}+a+b\right)}{b^3}+\frac{(a^2-b^2)\log\left(e^{(-x)}+1\right)}{b^3}+\frac{(a^2-b^2)\log\left(e^{(-x)}-1\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] 2*((a + b)*e^(-2*x) - a)/(2*b^2*e^(-2*x) - b^2*e^(-4*x) - b^2) - (a^2 - b^2
)*log(-(a - b)*e^(-2*x) + a + b)/b^3 + (a^2 - b^2)*log(e^(-x) + 1)/b^3 + (a
^2 - b^2)*log(e^(-x) - 1)/b^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 434 vs. 2(38) = 76.
time = 0.39, size = 434, normalized size = 10.85

$$\frac{2\,(ab-b^2)\cosh(x)^2+4\,(ab-b^2)\cosh(x)\sinh(x)+2\,(ab-b^2)\sinh(x)^2-2\,ab-\left((a^2-b^2)\cosh(x)^4+4\,(a^2-b^2)\cosh(x)\sinh(x)^3+(a^2-b^2)\sinh(x)^4-2\,(a^2-b^2)\cosh(x)^2+2\,\left(3\,(a^2-b^2)\cosh(x)^2-a^2+b^2\right)\sinh(x)^2+a^2-b^2+4\,\left((a^2-b^2)\cosh(x)^3-(a^2-b^2)\cosh(x)\right)\sinh(x)\right)\log\left(\frac{2\,(b\cosh(x)+a\sinh(x))}{\cosh(x)-\sinh(x)}\right)+\left((a^2-b^2)\cosh(x)^4+4\,(a^2-b^2)\cosh(x)\sinh(x)^3+(a^2-b^2)\sinh(x)^4-2\,(a^2-b^2)\cosh(x)^2+2\,\left(3\,(a^2-b^2)\cosh(x)^2-a^2+b^2\right)\sinh(x)^2+a^2-b^2+4\,\left((a^2-b^2)\cosh(x)^3-(a^2-b^2)\cosh(x)\right)\sinh(x)\right)\log\left(\frac{2\sinh(x)}{\cosh(x)-\sinh(x)}\right)}{b^3\cosh(x)^4+4\,b^3\cosh(x)\sinh(x)^3+b^3\sinh(x)^4-2\,b^3\cosh(x)^2+b^3+2\,\left(3\,b^3\cosh(x)^2-b^3\right)\sinh(x)^2+4\,\left(b^3\cosh(x)^3-b^3\cosh(x)\right)\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] (2*(a*b - b^2)*cosh(x)^2 + 4*(a*b - b^2)*cosh(x)*sinh(x) + 2*(a*b - b^2)*si
nh(x)^2 - 2*a*b - ((a^2 - b^2)*cosh(x)^4 + 4*(a^2 - b^2)*cosh(x)*sinh(x)^3
+ (a^2 - b^2)*sinh(x)^4 - 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*(a^2 - b^2)*cosh(x
)^2 - a^2 + b^2)*sinh(x)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(x)^3 - (a^2 -
b^2)*cosh(x))*sinh(x))*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) +
 ((a^2 - b^2)*cosh(x)^4 + 4*(a^2 - b^2)*cosh(x)*sinh(x)^3 + (a^2 - b^2)*sin
h(x)^4 - 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*(a^2 - b^2)*cosh(x)^2 - a^2 + b^2)*
sinh(x)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(x)^3 - (a^2 - b^2)*cosh(x))*sin
h(x))*log(2*sinh(x)/(cosh(x) - sinh(x))))/(b^3*cosh(x)^4 + 4*b^3*cosh(x)*si
nh(x)^3 + b^3*sinh(x)^4 - 2*b^3*cosh(x)^2 + b^3 + 2*(3*b^3*cosh(x)^2 - b^3)
*sinh(x)^2 + 4*(b^3*cosh(x)^3 - b^3*cosh(x))*sinh(x))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^4(x)}{a + b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**4/(a+b*coth(x)),x)
```

```
[Out] Integral(csch(x)**4/(a + b*coth(x)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(38) = 76.
time = 0.43, size = 106, normalized size = 2.65

$$-\frac{(a^3 + a^2b - ab^2 - b^3)\log\left(\left|ae^{(2x)} + be^{(2x)} - a + b\right|\right)}{ab^3 + b^4} + \frac{(a^2 - b^2)\log\left(\left|e^{(2x)} - 1\right|\right)}{b^3} - \frac{2\left(ab - (ab - b^2)e^{(2x)}\right)}{b^3(e^{(2x)} - 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -(a^3 + a^2*b - a*b^2 - b^3)*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a*b^3
 + b^4) + (a^2 - b^2)*log(abs(e^(2*x) - 1))/b^3 - 2*(a*b - (a*b - b^2)*e^(2
*x))/(b^3*(e^(2*x) - 1)^2)
```

Mupad [**B**]
time = 1.44, size = 88, normalized size = 2.20

$$\frac{2(a-b)}{b^2\left(e^{2x}-1\right)} - \frac{2}{b\left(e^{4x} - 2e^{2x} + 1\right)} - \frac{\ln\left(b - a + a\,e^{2x} + b\,e^{2x}\right)(a+b)(a-b)}{b^3} + \frac{\ln\left(e^{2x}-1\right)(a+b)(a-b)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^4*(a + b*coth(x))),x)
```

```
[Out] (2*(a - b))/(b^2*(exp(2*x) - 1)) - 2/(b*(exp(4*x) - 2*exp(2*x) + 1)) - (log
(b - a + a*exp(2*x) + b*exp(2*x))*(a + b)*(a - b))/b^3 + (log(exp(2*x) - 1)
*(a + b)*(a - b))/b^3
```

3.105 $\int \frac{\cosh^4(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=60

$$\frac{x}{16}+\frac{1}{32(1-\coth(x))^2}-\frac{1}{8(1-\coth(x))}-\frac{1}{24(1+\coth(x))^3}+\frac{5}{32(1+\coth(x))^2}-\frac{3}{16(1+\coth(x))}$$

```
[Out] 1/16*x+1/32/(1-coth(x))^2-1/8/(1-coth(x))-1/24/(1+coth(x))^3+5/32/(1+coth(x
))^2-3/16/(1+coth(x))
```

Rubi [A]
time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3597, 862, 90, 213}

$$\frac{x}{16}-\frac{1}{8(1-\coth(x))}-\frac{3}{16(\coth(x)+1)}+\frac{1}{32(1-\coth(x))^2}+\frac{5}{32(\coth(x)+1)^2}-\frac{1}{24(\coth(x)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^4/(1 + Coth[x]),x]
```

```
[Out] x/16 + 1/(32*(1 - Coth[x])^2) - 1/(8*(1 - Coth[x])) - 1/(24*(1 + Coth[x])^3
) + 5/(32*(1 + Coth[x])^2) - 3/(16*(1 + Coth[x]))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\cosh^4(x)}{1+\coth(x)}\,dx = -\text{Subst}\left(\int \frac{x^4}{(1+x)\left(-1+x^2\right)^3}\,dx, x, \coth(x)\right)$$
$$= -\text{Subst}\left(\int \frac{x^4}{(-1+x)^3(1+x)^4}\,dx, x, \coth(x)\right)$$
$$= -\text{Subst}\left(\int \left(\frac{1}{16(-1+x)^3} + \frac{1}{8(-1+x)^2} - \frac{1}{8(1+x)^4} + \frac{5}{16(1+x)^3} - \frac{3}{16(1+x)^2} + \frac{}{16}\right.\right.$$
$$= \frac{1}{32(1-\coth(x))^2} - \frac{1}{8(1-\coth(x))} - \frac{1}{24(1+\coth(x))^3} + \frac{5}{32(1+\coth(x))^2} - \frac{3}{16(1+\text{c}}$$
$$= \frac{x}{16} + \frac{1}{32(1-\coth(x))^2} - \frac{1}{8(1-\coth(x))} - \frac{1}{24(1+\coth(x))^3} + \frac{5}{32(1+\coth(x))^2} - \frac{}{16}$$

Mathematica [A]
time = 0.07, size = 42, normalized size = 0.70

$$\frac{1}{192}(12x + 15\cosh(2x) + 6\cosh(4x) + \cosh(6x) + 3\sinh(2x) - 3\sinh(4x) - \sinh(6x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^4/(1 + Coth[x]),x]
[Out] (12*x + 15*Cosh[2*x] + 6*Cosh[4*x] + Cosh[6*x] + 3*Sinh[2*x] - 3*Sinh[4*x]
- Sinh[6*x])/192
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(48) = 96.
time = 0.42, size = 118, normalized size = 1.97

method	result
risch	$\frac{x}{16}+\frac{e^{4x}}{128}+\frac{3\,e^{2x}}{64}+\frac{e^{-2x}}{32}+\frac{3\,e^{-4x}}{128}+\frac{e^{-6x}}{192}$
default	$\frac{1}{8\left(\tanh\left(\frac{x}{2}\right)-1\right)^4}+\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)-1\right)^3}+\frac{3}{8\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}+\frac{1}{4\tanh\left(\frac{x}{2}\right)-4}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{16}+\frac{1}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^6}-\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(1+coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/(tanh(1/2*x)-1)^4+1/4/(tanh(1/2*x)-1)^3+3/8/(tanh(1/2*x)-1)^2+1/4/(tanh
(1/2*x)-1)-1/16*ln(tanh(1/2*x)-1)+1/3/(tanh(1/2*x)+1)^6-1/(tanh(1/2*x)+1)^5
+13/8/(tanh(1/2*x)+1)^4-19/12/(tanh(1/2*x)+1)^3+1/(tanh(1/2*x)+1)^2-3/8/(ta
nh(1/2*x)+1)+1/16*ln(tanh(1/2*x)+1)
```

Maxima [A]
time = 0.26, size = 36, normalized size = 0.60

$$\frac{1}{128}\left(6e^{(-2x)}+1\right)e^{(4x)}+\frac{1}{16}x+\frac{1}{32}e^{(-2x)}+\frac{3}{128}e^{(-4x)}+\frac{1}{192}e^{(-6x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 1/128*(6*e^(-2*x) + 1)*e^(4*x) + 1/16*x + 1/32*e^(-2*x) + 3/128*e^(-4*x) +
1/192*e^(-6*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(44) = 88.
time = 0.39, size = 92, normalized size = 1.53

$$\frac{5\cosh(x)^5+25\cosh(x)\sinh(x)^4+\sinh(x)^5+\left(10\cosh(x)^2+9\right)\sinh(x)^3+27\cosh(x)^3+\left(50\cosh(x)^3+81\cosh(x)\right)\sinh(x)^2+12\,(2x+1)\cosh(x)+\left(5\cosh(x)^4+27\cosh(x)^2+24x-12\right)\sinh(x)}{384\,(\cosh(x)+\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] 1/384*(5*cosh(x)^5 + 25*cosh(x)*sinh(x)^4 + sinh(x)^5 + (10*cosh(x)^2 + 9)*
sinh(x)^3 + 27*cosh(x)^3 + (50*cosh(x)^3 + 81*cosh(x))*sinh(x)^2 + 12*(2*x
+ 1)*cosh(x) + (5*cosh(x)^4 + 27*cosh(x)^2 + 24*x - 12)*sinh(x))/(cosh(x) +
 sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cosh^4(x)}{\coth(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**4/(1+coth(x)),x)
```

```
[Out] Integral(cosh(x)**4/(coth(x) + 1), x)
```

Giac [A]
time = 0.42, size = 42, normalized size = 0.70

$$-\frac{1}{384}\left(22e^{(6x)}-12e^{(4x)}-9e^{(2x)}-2\right)e^{(-6x)}+\frac{1}{16}x+\frac{1}{128}e^{(4x)}+\frac{3}{64}e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(1+coth(x)),x, algorithm="giac")
```

```
[Out] -1/384*(22*e^(6*x) - 12*e^(4*x) - 9*e^(2*x) - 2)*e^(-6*x) + 1/16*x + 1/128*
e^(4*x) + 3/64*e^(2*x)
```

Mupad [B]
time = 1.42, size = 34, normalized size = 0.57

$$\frac{x}{16}+\frac{e^{-2x}}{32}+\frac{3e^{2x}}{64}+\frac{3e^{-4x}}{128}+\frac{e^{4x}}{128}+\frac{e^{-6x}}{192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(coth(x) + 1),x)
```

```
[Out] x/16 + exp(-2*x)/32 + (3*exp(2*x))/64 + (3*exp(-4*x))/128 + exp(4*x)/128 +
exp(-6*x)/192
```

3.106 $\int \frac{\cosh^3(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=25

$$\frac{\cosh^5(x)}{5}-\frac{\sinh^3(x)}{3}-\frac{\sinh^5(x)}{5}$$

```
[Out] 1/5*cosh(x)^5-1/3*sinh(x)^3-1/5*sinh(x)^5
```

Rubi [A]
time = 0.12, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.636$, Rules used = {3599, 3187, 3186, 2645, 30, 2644, 14}

$$-\frac{\sinh^5(x)}{5}-\frac{\sinh^3(x)}{3}+\frac{\cosh^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(1 + Coth[x]),x]
```

```
[Out] Cosh[x]^5/5 - Sinh[x]^3/3 - Sinh[x]^5/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 3186

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 3187

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
Q[p, 0]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^3(x)}{1+\coth(x)}\,dx &= -\left(i\int \frac{\cosh^3(x)\sinh(x)}{-i\cosh(x)-i\sinh(x)}\,dx\right)\\
&= -\int \cosh^3(x)\sinh(x)(-\cosh(x)+\sinh(x))\,dx\\
&= i\int \left(-i\cosh^4(x)\sinh(x)+i\cosh^3(x)\sinh^2(x)\right)\,dx\\
&= \int \cosh^4(x)\sinh(x)\,dx-\int \cosh^3(x)\sinh^2(x)\,dx\\
&= -\left(i\text{Subst}\left(\int x^2\left(1-x^2\right)\,dx,x,i\sinh(x)\right)\right)+\text{Subst}\left(\int x^4\,dx,x,\cosh(x)\right)\\
&= \frac{\cosh^5(x)}{5}-i\text{Subst}\left(\int \left(x^2-x^4\right)\,dx,x,i\sinh(x)\right)\\
&= \frac{\cosh^5(x)}{5}-\frac{\sinh^3(x)}{3}-\frac{\sinh^5(x)}{5}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 34, normalized size = 1.36

$$\frac{1}{120}(\cosh(x)-\sinh(x))(20\cosh(2x)+4\cosh(4x)+10\sinh(2x)+\sinh(4x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(1 + Coth[x]),x]
```

```
[Out] ((Cosh[x] - Sinh[x])*(20*Cosh[2*x] + 4*Cosh[4*x] + 10*Sinh[2*x] + Sinh[4*x]
))/120
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(19) = 38$.
time = 0.38, size = 82, normalized size = 3.28

method	result
risch	$\frac{e^{3x}}{48}+\frac{e^{x}}{8}+\frac{e^{-3x}}{24}+\frac{e^{-5x}}{80}$
default	$-\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{4}}+\frac{2}{5\left(\tanh\left(\frac{x}{2}\right)+1\right)^{5}}+\frac{4}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}-\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{3}{8\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{1}{6\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}-$ 4(tanh(

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(1+coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(tanh(1/2*x)+1)^4+2/5/(tanh(1/2*x)+1)^5+4/3/(tanh(1/2*x)+1)^3-1/(tanh(1/
2*x)+1)^2+3/8/(tanh(1/2*x)+1)-1/6/(tanh(1/2*x)-1)^3-1/4/(tanh(1/2*x)-1)^2-3
/8/(tanh(1/2*x)-1)
```

Maxima [A]
time = 0.27, size = 27, normalized size = 1.08

$$\frac{1}{48}\left(6\,e^{(-2\,x)}+1\right)e^{(3\,x)}+\frac{1}{24}\,e^{(-3\,x)}+\frac{1}{80}\,e^{(-5\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 1/48*(6*e^(-2*x) + 1)*e^(3*x) + 1/24*e^(-3*x) + 1/80*e^(-5*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(19) = 38$.
time = 0.36, size = 56, normalized size = 2.24

$$\frac{\cosh\left(x\right)^{4}+\cosh\left(x\right)\sinh\left(x\right)^{3}+\sinh\left(x\right)^{4}+\left(6\,\cosh\left(x\right)^{2}+5\right)\sinh\left(x\right)^{2}+5\,\cosh\left(x\right)^{2}+\left(\cosh\left(x\right)^{3}+5\,\cosh\left(x\right)\right)\sinh\left(x\right)}{30\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] 1/30*(cosh(x)^4 + cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 5)*sinh(x)
^2 + 5*cosh(x)^2 + (cosh(x)^3 + 5*cosh(x))*sinh(x))/(cosh(x) + sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^{3}(x)}{\coth(x) + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(1+coth(x)),x)
```

```
[Out] Integral(cosh(x)**3/(coth(x) + 1), x)
```

Giac [A]
time = 0.41, size = 25, normalized size = 1.00

$$\frac{1}{240}\left(10\,e^{(2\,x)} + 3\right)e^{(-5\,x)} + \frac{1}{48}\,e^{(3\,x)} + \frac{1}{8}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(1+coth(x)),x, algorithm="giac")
```

```
[Out] 1/240*(10*e^(2*x) + 3)*e^(-5*x) + 1/48*e^(3*x) + 1/8*e^x
```

Mupad [B]
time = 1.32, size = 23, normalized size = 0.92

$$\frac{\mathrm{e}^{-3\,x}}{24} + \frac{\mathrm{e}^{3\,x}}{48} + \frac{\mathrm{e}^{-5\,x}}{80} + \frac{\mathrm{e}^{x}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(coth(x) + 1),x)
```

```
[Out] exp(-3*x)/24 + exp(3*x)/48 + exp(-5*x)/80 + exp(x)/8
```

3.107 $\int \frac{\cosh^2(x)}{1+\coth(x)} dx$

Optimal. Leaf size=38

$$\frac{x}{8} - \frac{1}{8(1-\coth(x))} + \frac{1}{8(1+\coth(x))^2} - \frac{1}{4(1+\coth(x))}$$

```
[Out] 1/8*x-1/8/(1-coth(x))+1/8/(1+coth(x))^2-1/4/(1+coth(x))
```

Rubi [A]
time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3597, 862, 90, 213}

$$\frac{x}{8} - \frac{1}{8(1-\coth(x))} - \frac{1}{4(\coth(x)+1)} + \frac{1}{8(\coth(x)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(1 + Coth[x]),x]
```

```
[Out] x/8 - 1/(8*(1 - Coth[x])) + 1/(8*(1 + Coth[x])^2) - 1/(4*(1 + Coth[x]))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
```

```
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^2(x)}{1+\coth(x)}\,dx &= -\text{Subst}\left(\int \frac{x^2}{(1+x)\left(-1+x^2\right)^2}\,dx, x, \coth(x)\right)\\
&= -\text{Subst}\left(\int \frac{x^2}{(-1+x)^2(1+x)^3}\,dx, x, \coth(x)\right)\\
&= -\text{Subst}\left(\int \left(\frac{1}{8(-1+x)^2}+\frac{1}{4(1+x)^3}-\frac{1}{4(1+x)^2}+\frac{1}{8\left(-1+x^2\right)}\right)dx, x, \coth(x)\right)\\
&= -\frac{1}{8(1-\coth(x))}+\frac{1}{8(1+\coth(x))^2}-\frac{1}{4(1+\coth(x))}-\frac{1}{8}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \text{cot}\right.\\
&= \frac{x}{8}-\frac{1}{8(1-\coth(x))}+\frac{1}{8(1+\coth(x))^2}-\frac{1}{4(1+\coth(x))}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 24, normalized size = 0.63

$$\frac{1}{32}(4x+4\cosh(2x)+\cosh(4x)-\sinh(4x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(1 + Coth[x]),x]
```

```
[Out] (4*x + 4*Cosh[2*x] + Cosh[4*x] - Sinh[4*x])/32
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(30) = 60$.
time = 0.40, size = 78, normalized size = 2.05

method	result
risch	$\frac{x}{8}+\frac{e^{2x}}{16}+\frac{e^{-2x}}{16}+\frac{e^{-4x}}{32}$
default	$\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)^4}-\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^3}+\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{8}+\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}+\frac{1}{4\tanh\left(\frac{x}{2}\right)-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(1+coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(tanh(1/2*x)+1)^4-1/(tanh(1/2*x)+1)^3+1/(tanh(1/2*x)+1)^2-1/2/(tanh(1/2
*x)+1)+1/8*ln(tanh(1/2*x)+1)+1/4/(tanh(1/2*x)-1)^2+1/4/(tanh(1/2*x)-1)-1/8*
ln(tanh(1/2*x)-1)
```

Maxima [A]
time = 0.26, size = 22, normalized size = 0.58

$$\frac{1}{8}\,x + \frac{1}{16}\,e^{(2\,x)} + \frac{1}{16}\,e^{(-2\,x)} + \frac{1}{32}\,e^{(-4\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(1+coth(x)),x, algorithm="maxima")
[Out] 1/8*x + 1/16*e^(2*x) + 1/16*e^(-2*x) + 1/32*e^(-4*x)
```

Fricas [A]
time = 0.36, size = 51, normalized size = 1.34

$$\frac{3\,\cosh\left(x\right)^{3} + 9\,\cosh\left(x\right)\sinh\left(x\right)^{2} + \sinh\left(x\right)^{3} + 2\,(2\,x+1)\cosh\left(x\right) + \left(3\,\cosh\left(x\right)^{2} + 4\,x - 2\right)\sinh\left(x\right)}{32\,(\cosh\left(x\right) + \sinh\left(x\right))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(1+coth(x)),x, algorithm="fricas")
[Out] 1/32*(3*cosh(x)^3 + 9*cosh(x)*sinh(x)^2 + sinh(x)^3 + 2*(2*x + 1)*cosh(x) +
 (3*cosh(x)^2 + 4*x - 2)*sinh(x))/(cosh(x) + sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^{2}\left(x\right)}{\coth\left(x\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(1+coth(x)),x)
[Out] Integral(cosh(x)**2/(coth(x) + 1), x)
```

Giac [A]
time = 0.40, size = 30, normalized size = 0.79

$$-\frac{1}{32}\left(3\,e^{(4\,x)} - 2\,e^{(2\,x)} - 1\right)e^{(-4\,x)} + \frac{1}{8}\,x + \frac{1}{16}\,e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(1+coth(x)),x, algorithm="giac")
[Out] -1/32*(3*e^(4*x) - 2*e^(2*x) - 1)*e^(-4*x) + 1/8*x + 1/16*e^(2*x)
```

Mupad [B]
time = 0.11, size = 22, normalized size = 0.58

$$\frac{x}{8} + \frac{\mathrm{e}^{-2\,x}}{16} + \frac{\mathrm{e}^{2\,x}}{16} + \frac{\mathrm{e}^{-4\,x}}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(coth(x) + 1),x)
```

```
[Out] x/8 + exp(-2*x)/16 + exp(2*x)/16 + exp(-4*x)/32
```

3.108 $\int \frac{\cosh(x)}{1+\coth(x)} dx$

Optimal. Leaf size=17

$$\frac{\cosh^3(x)}{3} - \frac{\sinh^3(x)}{3}$$

```
[Out] 1/3*cosh(x)^3-1/3*sinh(x)^3
```

Rubi [A]
time = 0.08, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3599, 3187, 3186, 2645, 30, 2644}

$$\frac{\cosh^3(x)}{3} - \frac{\sinh^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/(1 + Coth[x]),x]
```

```
[Out] Cosh[x]^3/3 - Sinh[x]^3/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 3186

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 3187

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
Q[p, 0]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(x)}{1+\coth(x)}\,dx &= -\left(i\int \frac{\cosh(x)\sinh(x)}{-i\cosh(x)-i\sinh(x)}\,dx\right)\\
&= -\int \cosh(x)\sinh(x)(-\cosh(x)+\sinh(x))\,dx\\
&= i\int \left(-i\cosh^2(x)\sinh(x)+i\cosh(x)\sinh^2(x)\right)\,dx\\
&= \int \cosh^2(x)\sinh(x)\,dx-\int \cosh(x)\sinh^2(x)\,dx\\
&= -\left(i\text{Subst}\left(\int x^2\,dx,x,i\sinh(x)\right)\right)+\text{Subst}\left(\int x^2\,dx,x,\cosh(x)\right)\\
&= \frac{\cosh^3(x)}{3}-\frac{\sinh^3(x)}{3}
\end{aligned}$$

Mathematica [**A**]
time = 0.01, size = 19, normalized size = 1.12

$$\frac{1}{12}\left(3\cosh(x)+\cosh(3x)-4\sinh^3(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(1 + Coth[x]),x]
[Out] (3*Cosh[x] + Cosh[3*x] - 4*Sinh[x]^3)/12
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(13) = 26$.
time = 0.37, size = 42, normalized size = 2.47

method	result	size
risch	$\frac{e^{x}}{4}+\frac{e^{-3x}}{12}$	12
default	$-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{2}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}-\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{1}{2\tanh\left(\frac{x}{2}\right)+2}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(1+coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(tanh(1/2*x)-1)+2/3/(tanh(1/2*x)+1)^3-1/(tanh(1/2*x)+1)^2+1/2/(tanh(1/
2*x)+1)
```

Maxima [A]
time = 0.27, size = 11, normalized size = 0.65

$$\frac{1}{12}\,e^{(-3\,x)}+\frac{1}{4}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 1/12*e^(-3*x) + 1/4*e^x
```

Fricas [A]
time = 0.36, size = 23, normalized size = 1.35

$$\frac{\cosh\left(x\right)^{2}+\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}}{3\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] 1/3*(cosh(x)^2 + cosh(x)*sinh(x) + sinh(x)^2)/(cosh(x) + sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cosh\left(x\right)}{\coth\left(x\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(1+coth(x)),x)
```

```
[Out] Integral(cosh(x)/(coth(x) + 1), x)
```

Giac [**A**]
time = 0.41, size = 11, normalized size = 0.65

$$\frac{1}{12}\,e^{(-3\,x)} + \frac{1}{4}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(1+coth(x)),x, algorithm="giac")
```

```
[Out] 1/12*e^(-3*x) + 1/4*e^x
```

Mupad [**B**]
time = 1.19, size = 11, normalized size = 0.65

$$\frac{\mathrm{e}^{-3\,x}}{12} + \frac{\mathrm{e}^{x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(coth(x) + 1),x)
```

```
[Out] exp(-3*x)/12 + exp(x)/4
```

3.109 $\int \frac{\mathbf{sech}(x)}{1+\coth(x)}\, dx$

Optimal. Leaf size=10

$$\mathrm{ArcTan}(\sinh(x)) + \cosh(x) - \sinh(x)$$

```
[Out] arctan(sinh(x))+cosh(x)-sinh(x)
```

Rubi [A]
time = 0.08, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {3599, 3187, 3186, 2718, 2672, 327, 209}

$$\mathrm{ArcTan}(\sinh(x)) - \sinh(x) + \cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]/(1 + Coth[x]),x]
```

```
[Out] ArcTan[Sinh[x]] + Cosh[x] - Sinh[x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3186

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 3187

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
Q[p, 0]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}(x)}{1+\coth(x)}\,dx &= -\left(i\int \frac{\tanh(x)}{-i\cosh(x)-i\sinh(x)}\,dx\right)\\
&= -\int (-\cosh(x)+\sinh(x))\tanh(x)\,dx\\
&= i\int (-i\sinh(x)+i\sinh(x)\tanh(x))\,dx\\
&= \int \sinh(x)\,dx - \int \sinh(x)\tanh(x)\,dx\\
&= \cosh(x) - \operatorname{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, \sinh(x)\right)\\
&= \cosh(x) - \sinh(x) + \operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(x)\right)\\
&= \tan^{-1}(\sinh(x)) + \cosh(x) - \sinh(x)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 16, normalized size = 1.60

$$2\text{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)+\cosh(x)-\sinh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(1 + Coth[x]),x]
```

```
[Out] 2*ArcTan[Tanh[x/2]] + Cosh[x] - Sinh[x]
```

Maple [A]
time = 0.52, size = 19, normalized size = 1.90

method	result	size
default	$\frac{2}{\tanh\left(\frac{x}{2}\right)+1}+2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)$	19
risch	$\mathrm{e}^{-x}+i\ln\left(\mathrm{e}^{x}+i\right)-i\ln\left(\mathrm{e}^{x}-i\right)$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(1+coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(tanh(1/2*x)+1)+2*arctan(tanh(1/2*x))
```

Maxima [A]
time = 0.50, size = 12, normalized size = 1.20

$$-2\arctan\left(e^{(-x)}\right)+e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] -2*arctan(e^(-x)) + e^(-x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(10) = 20$.
time = 0.33, size = 23, normalized size = 2.30

$$\frac{2\left(\cosh\left(x\right)+\sinh\left(x\right)\right)\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)+1}{\cosh\left(x\right)+\sinh\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] (2*(cosh(x) + sinh(x))*arctan(cosh(x) + sinh(x)) + 1)/(cosh(x) + sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{sech}\left(x\right)}{\coth\left(x\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(1+coth(x)),x)
```

```
[Out] Integral(sech(x)/(coth(x) + 1), x)
```

Giac [**A**]
time = 0.40, size = 10, normalized size = 1.00

$$2 \arctan\left(e^{x}\right) + e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(1+coth(x)),x, algorithm="giac")
```

```
[Out] 2*arctan(e^x) + e^(-x)
```

Mupad [**B**]
time = 1.28, size = 10, normalized size = 1.00

$$\mathrm{e}^{-x} + 2\,\mathrm{atan}(\mathrm{e}^{x})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)*(coth(x) + 1)),x)
```

```
[Out] exp(-x) + 2*atan(exp(x))
```

3.110 $\int \frac{\text{sech}^2(x)}{1+\coth(x)} \, dx$

Optimal. Leaf size=15

$$-\log(1+\coth(x))-\log(\tanh(x))+\tanh(x)$$

```
[Out] -ln(1+coth(x))-ln(tanh(x))+tanh(x)
```

Rubi [A]
time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3597, 46}

$$\tanh(x)-\log(\tanh(x))-\log(\coth(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(1 + Coth[x]),x]
```

```
[Out] -Log[1 + Coth[x]] - Log[Tanh[x]] + Tanh[x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{sech}^2(x)}{1+\coth(x)} \, dx &= -\text{Subst}\left(\int \frac{1}{x^2(1+x)} \, dx, x, \coth(x)\right)\\
&= -\text{Subst}\left(\int \left(\frac{1}{x^2}-\frac{1}{x}+\frac{1}{1+x}\right) \, dx, x, \coth(x)\right)\\
&= -\log(1+\coth(x))-\log(\tanh(x))+\tanh(x)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 9, normalized size = 0.60

$$-x+\log(\cosh(x))+\tanh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(1 + Coth[x]),x]
```

```
[Out] -x + Log[Cosh[x]] + Tanh[x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(15) = 30$.
time = 0.58, size = 36, normalized size = 2.40

method	result	size
risch	$-2x-\frac{2}{1+{\mathrm e}^{2 x}}+\ln \left(1+{\mathrm e}^{2 x}\right)$	22
default	$-2 \ln \left(\tanh \left(\frac{x}{2}\right)+1\right)+\frac{2 \tanh \left(\frac{x}{2}\right)}{\tanh ^{2}\left(\frac{x}{2}\right)+1}+\ln \left(\tanh ^{2}\left(\frac{x}{2}\right)+1\right)$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(1+coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*ln(tanh(1/2*x)+1)+2*tanh(1/2*x)/(tanh(1/2*x)^2+1)+ln(tanh(1/2*x)^2+1)
```

Maxima [A]
time = 0.48, size = 18, normalized size = 1.20

$$\frac{2}{e^{(-2\,x)}+1}+\log\left(e^{(-2\,x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 2/(e^(-2*x) + 1) + log(e^(-2*x) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(15) = 30$.
time = 0.35, size = 78, normalized size = 5.20

$$-\frac{2\,x\cosh\left(x\right)^{2}+4\,x\cosh\left(x\right)\sinh\left(x\right)+2\,x\sinh\left(x\right)^{2}-\left(\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}+1\right)\log\left(\frac{2\,\cosh\left(x\right)}{\cosh\left(x\right)-\sinh\left(x\right)}\right)+2\,x+2}{\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] -(2*x*cosh(x)^2 + 4*x*cosh(x)*sinh(x) + 2*x*sinh(x)^2 - (cosh(x)^2 + 2*cosh
(x)*sinh(x) + sinh(x)^2 + 1)*log(2*cosh(x)/(cosh(x) - sinh(x))) + 2*x + 2)/
(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{sech}^{2}\left(x\right)}{\coth\left(x\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(1+coth(x)),x)
```

```
[Out] Integral(sech(x)**2/(coth(x) + 1), x)
```

Giac [A]
time = 0.41, size = 27, normalized size = 1.80

$$-2\,x - \frac{e^{(2\,x)} + 3}{e^{(2\,x)} + 1} + \log\left(e^{(2\,x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(1+coth(x)),x, algorithm="giac")
```

```
[Out] -2*x - (e^(2*x) + 3)/(e^(2*x) + 1) + log(e^(2*x) + 1)
```

Mupad [B]
time = 1.19, size = 21, normalized size = 1.40

$$\ln\left(\mathrm{e}^{2\,x} + 1\right) - 2\,x - \frac{2}{\mathrm{e}^{2\,x} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2*(coth(x) + 1)),x)
```

```
[Out] log(exp(2*x) + 1) - 2*x - 2/(exp(2*x) + 1)
```

3.111 $\int \frac{\operatorname{sech}^3(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=20

$$-\frac{1}{2}\text{ArcTan}(\sinh(x)) - \operatorname{sech}(x) + \frac{1}{2}\operatorname{sech}(x)\tanh(x)$$

```
[Out] -1/2*arctan(sinh(x))-sech(x)+1/2*sech(x)*tanh(x)
```

Rubi [A]
time = 0.11, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {3599, 3187, 3186, 2686, 8, 2691, 3855}

$$-\frac{1}{2}\text{ArcTan}(\sinh(x)) - \operatorname{sech}(x) + \frac{1}{2}\tanh(x)\operatorname{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^3/(1 + Coth[x]),x]
```

```
[Out] -1/2*ArcTan[Sinh[x]] - Sech[x] + (Sech[x]*Tanh[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3186

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
```

```
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 3187

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
Q[p, 0]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{sech}^3(x)}{1+\coth(x)}\,dx &= -\left(i\int \frac{\operatorname{sech}^2(x)\tanh(x)}{-i\cosh(x)-i\sinh(x)}\,dx\right)\\
&= -\int \operatorname{sech}^2(x)(-\cosh(x)+\sinh(x))\tanh(x)\,dx\\
&= i\int \left(-i\operatorname{sech}(x)\tanh(x)+i\operatorname{sech}(x)\tanh^2(x)\right)\,dx\\
&= \int \operatorname{sech}(x)\tanh(x)\,dx-\int \operatorname{sech}(x)\tanh^2(x)\,dx\\
&= \frac{1}{2}\operatorname{sech}(x)\tanh(x)-\frac{1}{2}\int \operatorname{sech}(x)\,dx-\operatorname{Subst}\left(\int 1\,dx,x,\operatorname{sech}(x)\right)\\
&= -\frac{1}{2}\tan^{-1}(\sinh(x))-\operatorname{sech}(x)+\frac{1}{2}\operatorname{sech}(x)\tanh(x)
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 20, normalized size = 1.00

$$-\operatorname{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{1}{2}\operatorname{sech}(x)(-2+\tanh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^3/(1 + Coth[x]),x]
```

```
[Out] -ArcTan[Tanh[x/2]] + (Sech[x]*(-2 + Tanh[x]))/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(16) = 32$.
time = 0.59, size = 45, normalized size = 2.25

method	result	size
risch	$-\frac{e^{x}\left(e^{2x}+3\right)}{\left(1+e^{2x}\right)^{2}}+\frac{i\ln\left(e^{x}-i\right)}{2}-\frac{i\ln\left(e^{x}+i\right)}{2}$	38
default	$\frac{-\left(\tanh^{3}\left(\frac{x}{2}\right)\right)-2\left(\tanh^{2}\left(\frac{x}{2}\right)\right)+\tanh\left(\frac{x}{2}\right)-2}{\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)^{2}}-\arctan\left(\tanh\left(\frac{x}{2}\right)\right)$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^3/(1+coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 4*(-1/4*tanh(1/2*x)^3-1/2*tanh(1/2*x)^2+1/4*tanh(1/2*x)-1/2)/(tanh(1/2*x)^2
+1)^2-arctan(tanh(1/2*x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(16) = 32$.
time = 0.48, size = 33, normalized size = 1.65

$$-\frac{e^{(-x)}+3\,e^{(-3\,x)}}{2\,e^{(-2\,x)}+e^{(-4\,x)}+1}+\arctan\left(e^{(-x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] -(e^(-x) + 3*e^(-3*x))/(2*e^(-2*x) + e^(-4*x) + 1) + arctan(e^(-x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(16) = 32$.
time = 0.35, size = 140, normalized size = 7.00

$$-\frac{\cosh\left(x\right)^{3}+3\,\cosh\left(x\right)\sinh\left(x\right)^{2}+\sinh\left(x\right)^{3}+\left(\cosh\left(x\right)^{4}+4\,\cosh\left(x\right)\sinh\left(x\right)^{3}+\sinh\left(x\right)^{4}+2\,\left(3\,\cosh\left(x\right)^{2}+1\right)\sinh\left(x\right)^{2}+2\,\cosh\left(x\right)^{2}+4\,\left(\cosh\left(x\right)^{3}+\cosh\left(x\right)\right)\sinh\left(x\right)+1\right)\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)+3\,\left(\cosh\left(x\right)^{2}+1\right)\sinh\left(x\right)+3\,\cosh\left(x\right)}{\cosh\left(x\right)^{4}+4\,\cosh\left(x\right)\sinh\left(x\right)^{3}+\sinh\left(x\right)^{4}+2\,\left(3\,\cosh\left(x\right)^{2}+1\right)\sinh\left(x\right)^{2}+2\,\cosh\left(x\right)^{2}+4\,\left(\cosh\left(x\right)^{3}+\cosh\left(x\right)\right)\sinh\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] -(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (cosh(x)^4 + 4*cosh(x)*sinh
(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x
)^3 + cosh(x))*sinh(x) + 1)*arctan(cosh(x) + sinh(x)) + 3*(cosh(x)^2 + 1)*s
```

```
inh(x) + 3*cosh(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cos
h(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^3(x)}{\coth(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**3/(1+coth(x)),x)
[Out] Integral(sech(x)**3/(coth(x) + 1), x)
```

Giac [**A**]
time = 0.41, size = 25, normalized size = 1.25

$$-\frac{e^{(3x)}+3e^{x}}{\left(e^{(2x)}+1\right)^{2}}-\arctan\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(1+coth(x)),x, algorithm="giac")
[Out] -(e^(3*x) + 3*e^x)/(e^(2*x) + 1)^2 - arctan(e^x)
```

Mupad [**B**]
time = 1.25, size = 22, normalized size = 1.10

$$-\mathrm{atan}(\mathrm{e}^{x})-\frac{1}{2\cosh(x)}-\frac{\mathrm{e}^{-x}}{2\cosh(x)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^3*(coth(x) + 1)),x)
[Out] - atan(exp(x)) - 1/(2*cosh(x)) - exp(-x)/(2*cosh(x)^2)
```

3.112 $\int \frac{\operatorname{sech}^4(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=17

$$\frac{\tanh^2(x)}{2} - \frac{\tanh^3(x)}{3}$$

[Out] 1/2*tanh(x)^2-1/3*tanh(x)^3

Rubi [A]

time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3597, 862, 45}

$$\frac{\tanh^2(x)}{2} - \frac{\tanh^3(x)}{3}$$

Antiderivative was successfully verified.

[In] Int[Sech[x]^4/(1 + Coth[x]),x]

[Out] Tanh[x]^2/2 - Tanh[x]^3/3

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\text{sech}^4(x)}{1+\coth(x)}\,dx = -\text{Subst}\left(\int \frac{-1+x^2}{x^4(1+x)}\,dx, x, \coth(x)\right)$$
$$= -\text{Subst}\left(\int \frac{-1+x}{x^4}\,dx, x, \coth(x)\right)$$
$$= -\text{Subst}\left(\int \left(-\frac{1}{x^4}+\frac{1}{x^3}\right)dx, x, \coth(x)\right)$$
$$= \frac{\tanh^2(x)}{2}-\frac{\tanh^3(x)}{3}$$

Mathematica [A]
time = 0.04, size = 17, normalized size = 1.00

$$\frac{1}{6}\left(-3\text{sech}^2(x)-2\tanh^3(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^4/(1 + Coth[x]),x]
[Out] (-3*Sech[x]^2 - 2*Tanh[x]^3)/6
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(13) = 26.
time = 0.59, size = 38, normalized size = 2.24

method	result	size
risch	$-\frac{2(3\,e^{2x}-1)}{3(1+e^{2x})^3}$	19
default	$-\frac{4\left(-\frac{\left(\tanh^4\left(\frac{x}{2}\right)\right)}{2}+\frac{2\left(\tanh^3\left(\frac{x}{2}\right)\right)}{3}-\frac{\left(\tanh^2\left(\frac{x}{2}\right)\right)}{2}\right)}{\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^3}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^4/(1+coth(x)),x,method=_RETURNVERBOSE)
[Out] -4*(-1/2*tanh(1/2*x)^4+2/3*tanh(1/2*x)^3-1/2*tanh(1/2*x)^2)/(tanh(1/2*x)^2+
1)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(13) = 26.
time = 0.26, size = 75, normalized size = 4.41

$$-\frac{2\,e^{(-2\,x)}}{3\,e^{(-2\,x)}+3\,e^{(-4\,x)}+e^{(-6\,x)}+1}-\frac{4\,e^{(-4\,x)}}{3\,e^{(-2\,x)}+3\,e^{(-4\,x)}+e^{(-6\,x)}+1}-\frac{2}{3\,(3\,e^{(-2\,x)}+3\,e^{(-4\,x)}+e^{(-6\,x)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] -2*e^(-2*x)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) - 4*e^(-4*x)/(3*e^(-2*
x) + 3*e^(-4*x) + e^(-6*x) + 1) - 2/3/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) +
 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(13) = 26$.
time = 0.36, size = 84, normalized size = 4.94

$$-\frac{4\left(\cosh\left(x\right)+2\sinh\left(x\right)\right)}{3\left(\cosh\left(x\right)^{5}+5\cosh\left(x\right)\sinh\left(x\right)^{4}+\sinh\left(x\right)^{5}+\left(10\cosh\left(x\right)^{2}+3\right)\sinh\left(x\right)^{3}+3\cosh\left(x\right)^{3}+\left(10\cosh\left(x\right)^{3}+9\cosh\left(x\right)\right)\sinh\left(x\right)^{2}+\left(5\cosh\left(x\right)^{4}+9\cosh\left(x\right)^{2}+2\right)\sinh\left(x\right)+4\cosh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] -4/3*(cosh(x) + 2*sinh(x))/(cosh(x)^5 + 5*cosh(x)*sinh(x)^4 + sinh(x)^5 + (
10*cosh(x)^2 + 3)*sinh(x)^3 + 3*cosh(x)^3 + (10*cosh(x)^3 + 9*cosh(x))*sinh
(x)^2 + (5*cosh(x)^4 + 9*cosh(x)^2 + 2)*sinh(x) + 4*cosh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{sech}^{4}\left(x\right)}{\coth\left(x\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**4/(1+coth(x)),x)
```

```
[Out] Integral(sech(x)**4/(coth(x) + 1), x)
```

Giac [A]
time = 0.41, size = 18, normalized size = 1.06

$$-\frac{2\left(3\,e^{(2\,x)}-1\right)}{3\left(e^{(2\,x)}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(1+coth(x)),x, algorithm="giac")
```

```
[Out] -2/3*(3*e^(2*x) - 1)/(e^(2*x) + 1)^3
```

Mupad [B]
time = 0.07, size = 18, normalized size = 1.06

$$-\frac{2\left(3\,\mathrm{e}^{2\,x}-1\right)}{3\left(\mathrm{e}^{2\,x}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^4*(coth(x) + 1)),x)
[Out] -(2*(3*exp(2*x) - 1))/(3*(exp(2*x) + 1)^3)
```

3.113 $\int \sqrt{1+\coth(x)}\,\text{sech}^2(x)\,dx$

Optimal. Leaf size=21

$$\tanh^{-1}\left(\sqrt{1+\coth(x)}\right)+\sqrt{1+\coth(x)}\,\tanh(x)$$

```
[Out] arctanh((1+coth(x))^(1/2))+(1+coth(x))^(1/2)*tanh(x)
```

Rubi [A]
time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3597, 43, 65, 213}

$$\tanh^{-1}\left(\sqrt{\coth(x)+1}\right)+\tanh(x)\sqrt{\coth(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Coth[x]]*Sech[x]^2,x]
```

```
[Out] ArcTanh[Sqrt[1 + Coth[x]]] + Sqrt[1 + Coth[x]]*Tanh[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
```

```
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{1+\coth(x)}\,\mathrm{sech}^2(x)\,dx &= -\mathrm{Subst}\left(\int \frac{\sqrt{1+x}}{x^2}\,dx, x, \coth(x)\right)\\
&= \sqrt{1+\coth(x)}\,\tanh(x) - \frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, \coth(x)\right)\\
&= \sqrt{1+\coth(x)}\,\tanh(x) - \mathrm{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+\coth(x)}\right)\\
&= \tanh^{-1}\left(\sqrt{1+\coth(x)}\right) + \sqrt{1+\coth(x)}\,\tanh(x)
\end{aligned}
$$

Mathematica [**C**] Result contains complex when optimal does not.
time = 13.80, size = 133, normalized size = 6.33

$$
\frac{1}{2}\sqrt{1+\coth(x)}\left(\frac{(1-i)\mathrm{ArcTan}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i(1+\coth(x))}\right)}{\sqrt{i(1+\coth(x))}} + \frac{2\left(2\tanh^{-1}\left(\sqrt{\tanh\left(\frac{x}{2}\right)}\right) - \sqrt{2}\tanh^{-1}\left(\frac{1+\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{\tanh\left(\frac{x}{2}\right)}}\right)\right)\sinh\left(\frac{x}{2}\right)\left(-\cosh\left(\frac{x}{2}\right)+\sinh\left(\frac{x}{2}\right)\right)}{\sqrt{\tanh\left(\frac{x}{2}\right)}} + 2\tanh(x)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Coth[x]]*Sech[x]^2,x]
[Out] (Sqrt[1 + Coth[x]]*(((1 - I)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Coth[x])]])/Sqr
t[I*(1 + Coth[x])] + (2*(2*ArcTanh[Sqrt[Tanh[x/2]]] - Sqrt[2]*ArcTanh[(1 +
Tanh[x/2])/(Sqrt[2]*Sqrt[Tanh[x/2]])])*Sinh[x/2]*(-Cosh[x/2] + Sinh[x/2]))/
Sqrt[Tanh[x/2]] + 2*Tanh[x]))/2
```

Maple [**F**]
time = 2.09, size = 0, normalized size = 0.00

$$\int \mathrm{sech}(x)^2\sqrt{1+\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2*(1+coth(x))^(1/2),x)
[Out] int(sech(x)^2*(1+coth(x))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(1+coth(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(coth(x) + 1)*sech(x)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(17) = 34$.
time = 0.39, size = 231, normalized size = 11.00

$$\frac{4\sqrt{2}\left(\sqrt{2}\cosh\left(x\right)+\sqrt{2}\sinh\left(x\right)\right)\sqrt{\frac{\sinh\left(x\right)}{\cosh\left(x\right)-\sinh\left(x\right)}}+\left(\cosh\left(x\right)^{2}+2\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}+1\right)\log\left(\frac{2\sqrt{2}\left(\sqrt{2}\cosh(x)+\sqrt{2}\sinh(x)\right)\sqrt{\frac{\sinh\left(x\right)}{\cosh\left(x\right)-\sinh\left(x\right)}}+3\cosh(x)^{2}+6\cosh(x)\sinh(x)+3\sinh(x)^{2}-1}{\cosh(x)^{2}+2\cosh(x)\sinh(x)+\sinh(x)^{2}}\right)-\left(\cosh\left(x\right)^{2}+2\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}+1\right)\log\left(-\frac{2\sqrt{2}\left(\sqrt{2}\cosh(x)+\sqrt{2}\sinh(x)\right)\sqrt{\frac{\sinh\left(x\right)}{\cosh\left(x\right)-\sinh\left(x\right)}}-3\cosh(x)^{2}-6\cosh(x)\sinh(x)-3\sinh(x)^{2}+1}{\cosh(x)^{2}+2\cosh(x)\sinh(x)+\sinh(x)^{2}}\right)}{4\left(\cosh\left(x\right)^{2}+2\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(1+coth(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(2)*(sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(sinh(x)/(cosh(x) -
sinh(x))) + (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log((2*sqrt(2)*
(sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(sinh(x)/(cosh(x) - sinh(x))) + 3*c
osh(x)^2 + 6*cosh(x)*sinh(x) + 3*sinh(x)^2 - 1)/(cosh(x)^2 + 2*cosh(x)*sinh
(x) + sinh(x)^2)) - (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log(-(2
*sqrt(2)*(sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(sinh(x)/(cosh(x) - sinh(x
))) - 3*cosh(x)^2 - 6*cosh(x)*sinh(x) - 3*sinh(x)^2 + 1)/(cosh(x)^2 + 2*cos
h(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\coth\left(x\right)+1}\,\operatorname{sech}^{2}\left(x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2*(1+coth(x))**(1/2),x)
```

```
[Out] Integral(sqrt(coth(x) + 1)*sech(x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(17) = 34$.
time = 0.41, size = 122, normalized size = 5.81

$$-\frac{1}{4}\sqrt{2}\left(\sqrt{2}\log\left(\frac{\left(\sqrt{e^{(2x)}-1}-e^{x}\right)^{2}-2\sqrt{2}+3}{\left(\sqrt{e^{(2x)}-1}-e^{x}\right)^{2}+2\sqrt{2}+3}\right)-\frac{8\left(3\left(\sqrt{e^{(2x)}-1}-e^{x}\right)^{2}+1\right)}{\left(\sqrt{e^{(2x)}-1}-e^{x}\right)^{4}+6\left(\sqrt{e^{(2x)}-1}-e^{x}\right)^{2}+1}\right)\operatorname{sgn}\left(e^{(2x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(1+coth(x))^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(sqrt(2)*log(((sqrt(e^(2*x) - 1) - e^x)^2 - 2*sqrt(2) + 3)/((s
qrt(e^(2*x) - 1) - e^x)^2 + 2*sqrt(2) + 3)) - 8*(3*(sqrt(e^(2*x) - 1) - e^x
)^2 + 1)/((sqrt(e^(2*x) - 1) - e^x)^4 + 6*(sqrt(e^(2*x) - 1) - e^x)^2 + 1))
*sgn(e^(2*x) - 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sqrt{\coth(x)+1}}{\cosh(x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(x) + 1)^(1/2)/cosh(x)^2,x)
```

```
[Out] int((coth(x) + 1)^(1/2)/cosh(x)^2, x)
```

3.114 $\int \frac{\cosh^4(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=147

$$-\frac{a(3a+b)\log(1-\coth(x))}{16(a+b)^3}+\frac{a(3a-b)\log(1+\coth(x))}{16(a-b)^3}-\frac{a^4b\log(a+b\coth(x))}{(a^2-b^2)^3}-\frac{(4b(2a^2-b^2)-a(5a^2-b^2)\coth(x))\sinh^2(x)}{8(a^2-b^2)^2}$$

```
[Out] -1/16*a*(3*a+b)*ln(1-coth(x))/(a+b)^3+1/16*a*(3*a-b)*ln(1+coth(x))/(a-b)^3-
a^4*b*ln(a+b*coth(x))/(a^2-b^2)^3-1/8*(4*b*(2*a^2-b^2)-a*(5*a^2-b^2)*coth(x
))*sinh(x)^2/(a^2-b^2)^2-1/4*(b-a*coth(x))*sinh(x)^4/(a^2-b^2)
```

Rubi [A]
time = 0.25, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3597, 1661, 815}

$$-\frac{\sinh^4(x)(b-a\coth(x))}{4(a^2-b^2)}-\frac{\sinh^2(x)\left(4b(2a^2-b^2)-a(5a^2-b^2)\coth(x)\right)}{8(a^2-b^2)^2}-\frac{a^4b\log(a+b\coth(x))}{(a^2-b^2)^3}-\frac{a(3a+b)\log(1-\coth(x))}{16(a+b)^3}+\frac{a(3a-b)\log(\coth(x)+1)}{16(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^4/(a + b*Coth[x]),x]
```

```
[Out] -1/16*(a*(3*a + b)*Log[1 - Coth[x]])/(a + b)^3 + (a*(3*a - b)*Log[1 + Coth[
x]])/(16*(a - b)^3) - (a^4*b*Log[a + b*Coth[x]])/(a^2 - b^2)^3 - ((4*b*(2*a
^2 - b^2) - a*(5*a^2 - b^2)*Coth[x])*Sinh[x]^2)/(8*(a^2 - b^2)^2) - ((b - a
*Coth[x])*Sinh[x]^4)/(4*(a^2 - b^2))
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\cosh^4(x)}{a+b\coth(x)}\,dx = -\left(b\text{Subst}\left(\int \frac{x^4}{(a+x)\left(-b^2+x^2\right)^3}\,dx, x, b\coth(x)\right)\right)$$

$$= -\frac{(b-a\coth(x))\sinh^4(x)}{4\left(a^2-b^2\right)} - \frac{\text{Subst}\left(\int \frac{\frac{a^2b^4}{a^2-b^2}-\frac{3ab^4x}{a^2-b^2}+4b^2x^2}{(a+x)\left(-b^2+x^2\right)^2}\,dx, x, b\coth(x)\right)}{4b}$$

$$= -\frac{\left(4b(2a^2-b^2)-a(5a^2-b^2)\coth(x)\right)\sinh^2(x)}{8\left(a^2-b^2\right)^2} - \frac{(b-a\coth(x))\sinh^4(x)}{4\left(a^2-b^2\right)} - \frac{\text{Subst}\left(\cdots\right.}{\cdots}$$

$$= -\frac{\left(4b(2a^2-b^2)-a(5a^2-b^2)\coth(x)\right)\sinh^2(x)}{8\left(a^2-b^2\right)^2} - \frac{(b-a\coth(x))\sinh^4(x)}{4\left(a^2-b^2\right)} - \frac{\text{Subst}\left(\cdots\right.}{\cdots}$$

$$= -\frac{a(3a+b)\log(1-\coth(x))}{16(a+b)^3} + \frac{a(3a-b)\log(1+\coth(x))}{16(a-b)^3} - \frac{a^4b\log(a+b\coth(x))}{\left(a^2-b^2\right)^3} \cdots$$

Mathematica [A]

time = 0.40, size = 144, normalized size = 0.98

$$\frac{12a^5x+24a^3b^2x-4ab^4x-4b(3a^4-4a^2b^2+b^4)\cosh(2x)-b(a^2-b^2)^2\cosh(4x)-32a^4b\log(b\cosh(x)+a\sinh(x))+8a^3(a^2-b^2)\sinh(2x)+a^5\sinh(4x)-2a^3b^2\sinh(4x)+ab^4\sinh(4x)}{32(a-b)^3(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^4/(a + b*Coth[x]),x]
```

```
[Out] (12*a^5*x + 24*a^3*b^2*x - 4*a*b^4*x - 4*b*(3*a^4 - 4*a^2*b^2 + b^4)*Cosh[2
*x] - b*(a^2 - b^2)^2*Cosh[4*x] - 32*a^4*b*Log[b*Cosh[x] + a*Sinh[x]] + 8*a
^3*(a^2 - b^2)*Sinh[2*x] + a^5*Sinh[4*x] - 2*a^3*b^2*Sinh[4*x] + a*b^4*Sinh
[4*x])/(32*(a - b)^3*(a + b)^3)
```

Maple [A]

time = 0.77, size = 249, normalized size = 1.69

method	result
risch	$\frac{3a^2x}{8(a+b)^3}+\frac{axb}{8(a+b)^3}+\frac{e^{4x}}{64a+64b}+\frac{e^{2x}a}{8(a+b)^2}+\frac{e^{2x}b}{16(a+b)^2}+\frac{e^{-2x}b}{16(a-b)^2}-\frac{e^{-2x}a}{8(a-b)^2}-\frac{e^{-4x}}{64(a-b)}+\frac{2a^4bx}{a^6-3a^4b^2+3a^2b^4-b^6}-$

default	$-\frac{a^4 b \ln\left(b\left(\tanh^2\left(\frac{x}{2}\right)\right)+2a\tanh\left(\frac{x}{2}\right)+b\right)}{(a-b)^3(a+b)^3}-\frac{1}{(4a-4b)\left(\tanh\left(\frac{x}{2}\right)+1\right)^4}+\frac{4}{(8a-8b)\left(\tanh\left(\frac{x}{2}\right)+1\right)^3}-\frac{-5a+3b}{8(a-b)^2\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{}{8(a-b)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -a^4*b/(a-b)^3/(a+b)^3*ln(b*tanh(1/2*x)^2+2*a*tanh(1/2*x)+b)-1/(4*a-4*b)/(t
anh(1/2*x)+1)^4+4/(8*a-8*b)/(tanh(1/2*x)+1)^3-1/8*(-5*a+3*b)/(a-b)^2/(tanh(
1/2*x)+1)-1/8*(7*a-5*b)/(a-b)^2/(tanh(1/2*x)+1)^2+1/8*a*(3*a-b)/(a-b)^3*ln(
tanh(1/2*x)+1)+1/(4*a+4*b)/(tanh(1/2*x)-1)^4+4/(8*a+8*b)/(tanh(1/2*x)-1)^3-
1/8*(-7*a-5*b)/(a+b)^2/(tanh(1/2*x)-1)^2-1/8*(-5*a-3*b)/(a+b)^2/(tanh(1/2*x
)-1)-1/8*a*(3*a+b)/(a+b)^3*ln(tanh(1/2*x)-1)
```

Maxima [A]
time = 0.28, size = 154, normalized size = 1.05

$$-\frac{a^4 b \log\left(-(a-b)e^{(-2x)}+a+b\right)}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{(3a^2+ab)x}{8(a^3+3a^2b+3ab^2+b^3)}+\frac{\left(4(2a+b)e^{(-2x)}+a+b\right)e^{(4x)}}{64(a^2+2ab+b^2)}-\frac{4(2a-b)e^{(-2x)}+(a-b)e^{(-4x)}}{64(a^2-2ab+b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] -a^4*b*log(-(a - b)*e^(-2*x) + a + b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) +
 1/8*(3*a^2 + a*b)*x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 1/64*(4*(2*a + b)*e^
(-2*x) + a + b)*e^(4*x)/(a^2 + 2*a*b + b^2) - 1/64*(4*(2*a - b)*e^(-2*x) +
(a - b)*e^(-4*x))/(a^2 - 2*a*b + b^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1229 vs. $2(139) = 278$.
time = 0.36, size = 1229, normalized size = 8.36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] 1/64*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^8 + 8*(a^
5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^7 + (a^5 -
 a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^8 + 4*(2*a^5 - 3*a^4*
b - 2*a^3*b^2 + 4*a^2*b^3 - b^5)*cosh(x)^6 + 4*(2*a^5 - 3*a^4*b - 2*a^3*b^2
 + 4*a^2*b^3 - b^5 + 7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*
cosh(x)^2)*sinh(x)^6 + 8*(3*a^5 + 8*a^4*b + 6*a^3*b^2 - a*b^4)*x*cosh(x)^4
+ 8*(7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 + 3*(2
*a^5 - 3*a^4*b - 2*a^3*b^2 + 4*a^2*b^3 - b^5)*cosh(x))*sinh(x)^5 - a^5 - a^
4*b + 2*a^3*b^2 + 2*a^2*b^3 - a*b^4 - b^5 + 2*(35*(a^5 - a^4*b - 2*a^3*b^2
+ 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 30*(2*a^5 - 3*a^4*b - 2*a^3*b^2 + 4*
```

```
a^2*b^3 - b^5)*cosh(x)^2 + 4*(3*a^5 + 8*a^4*b + 6*a^3*b^2 - a*b^4)*x)*sinh(
x)^4 + 8*(7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 +
 10*(2*a^5 - 3*a^4*b - 2*a^3*b^2 + 4*a^2*b^3 - b^5)*cosh(x)^3 + 4*(3*a^5 +
8*a^4*b + 6*a^3*b^2 - a*b^4)*x*cosh(x))*sinh(x)^3 - 4*(2*a^5 + 3*a^4*b - 2*
a^3*b^2 - 4*a^2*b^3 + b^5)*cosh(x)^2 + 4*(7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^
2*b^3 + a*b^4 - b^5)*cosh(x)^6 - 2*a^5 - 3*a^4*b + 2*a^3*b^2 + 4*a^2*b^3 -
b^5 + 15*(2*a^5 - 3*a^4*b - 2*a^3*b^2 + 4*a^2*b^3 - b^5)*cosh(x)^4 + 12*(3*
a^5 + 8*a^4*b + 6*a^3*b^2 - a*b^4)*x*cosh(x)^2)*sinh(x)^2 - 64*(a^4*b*cosh(
x)^4 + 4*a^4*b*cosh(x)^3*sinh(x) + 6*a^4*b*cosh(x)^2*sinh(x)^2 + 4*a^4*b*co
sh(x)*sinh(x)^3 + a^4*b*sinh(x)^4)*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) -
 sinh(x))) + 8*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)
^7 + 3*(2*a^5 - 3*a^4*b - 2*a^3*b^2 + 4*a^2*b^3 - b^5)*cosh(x)^5 + 4*(3*a^5
 + 8*a^4*b + 6*a^3*b^2 - a*b^4)*x*cosh(x)^3 - (2*a^5 + 3*a^4*b - 2*a^3*b^2
- 4*a^2*b^3 + b^5)*cosh(x))*sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*c
osh(x)^4 + 4*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3*sinh(x) + 6*(a^6
 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2*sinh(x)^2 + 4*(a^6 - 3*a^4*b^2 +
3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*si
nh(x)^4)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^4(x)}{a + b\coth(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**4/(a+b*coth(x)),x)
```

```
[Out] Integral(cosh(x)**4/(a + b*coth(x)), x)
```

Giac [**A**]
time = 0.41, size = 216, normalized size = 1.47

$$-\frac{a^4 b\log\left(\left|-ae^{(2x)} - be^{(2x)} + a - b\right|\right)}{a^6 - 3a^4b^2 + 3a^2b^4 - b^6} + \frac{(3a^2 - ab)x}{8(a^3 - 3a^2b + 3ab^2 - b^3)} - \frac{\left(18a^2e^{(4x)} - 6abe^{(4x)} + 8a^2e^{(2x)} - 12abe^{(2x)} + 4b^2e^{(2x)} + a^2 - 2ab + b^2\right)e^{(-4x)}}{64(a^3 - 3a^2b + 3ab^2 - b^3)} + \frac{ae^{(4x)} + be^{(4x)} + 8ae^{(2x)} + 4be^{(2x)}}{64(a^2 + 2ab + b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -a^4*b*log(abs(-a*e^(2*x) - b*e^(2*x) + a - b))/(a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6) + 1/8*(3*a^2 - a*b)*x/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - 1/64*(18*a
^2*e^(4*x) - 6*a*b*e^(4*x) + 8*a^2*e^(2*x) - 12*a*b*e^(2*x) + 4*b^2*e^(2*x)
 + a^2 - 2*a*b + b^2)*e^(-4*x)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 1/64*(a*e^
(4*x) + b*e^(4*x) + 8*a*e^(2*x) + 4*b*e^(2*x))/(a^2 + 2*a*b + b^2)
```

Mupad [**B**]
time = 1.74, size = 135, normalized size = 0.92

$$\frac{e^{4x}}{64a + 64b} - \frac{e^{-4x}}{64a - 64b} - \frac{e^{-2x}(2a - b)}{16(a - b)^2} + \frac{e^{2x}(2a + b)}{16(a + b)^2} - \frac{a^4 b\ln\left(b - a + a e^{2x} + b e^{2x}\right)}{a^6 - 3a^4b^2 + 3a^2b^4 - b^6} + \frac{a x(3a - b)}{8(a - b)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a + b*coth(x)),x)
```

```
[Out] exp(4*x)/(64*a + 64*b) - exp(-4*x)/(64*a - 64*b) - (exp(-2*x)*(2*a - b))/(1
6*(a - b)^2) + (exp(2*x)*(2*a + b))/(16*(a + b)^2) - (a^4*b*log(b - a + a*e
xp(2*x) + b*exp(2*x)))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) + (a*x*(3*a - b)
)/(8*(a - b)^3)
```

3.115 $\int \frac{\cosh^3(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=135

$$\frac{a^3b\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}-\frac{a^2b\cosh(x)}{(a^2-b^2)^2}-\frac{b\cosh^3(x)}{3(a^2-b^2)}+\frac{ab^2\sinh(x)}{(a^2-b^2)^2}+\frac{a\sinh(x)}{a^2-b^2}+\frac{a\sinh^3(x)}{3(a^2-b^2)}$$

```
[Out] a^3*b*arctanh((a*cosh(x)+b*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)-a^2*b*
cosh(x)/(a^2-b^2)^2-1/3*b*cosh(x)^3/(a^2-b^2)+a*b^2*sinh(x)/(a^2-b^2)^2+a*s
inh(x)/(a^2-b^2)+1/3*a*sinh(x)^3/(a^2-b^2)
```

Rubi [A]
time = 0.18, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {3599, 3188, 2713, 2645, 30, 3179, 2717, 3153, 212}

$$\frac{a\sinh^3(x)}{3(a^2-b^2)}+\frac{a\sinh(x)}{a^2-b^2}+\frac{ab^2\sinh(x)}{(a^2-b^2)^2}-\frac{b\cosh^3(x)}{3(a^2-b^2)}-\frac{a^2b\cosh(x)}{(a^2-b^2)^2}+\frac{a^3b\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(a + b*Coth[x]),x]
```

```
[Out] (a^3*b*ArcTanh[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
- (a^2*b*Cosh[x])/(a^2 - b^2)^2 - (b*Cosh[x]^3)/(3*(a^2 - b^2)) + (a*b^2*Si
nh[x])/(a^2 - b^2)^2 + (a*Sinh[x])/(a^2 - b^2) + (a*Sinh[x]^3)/(3*(a^2 - b^
2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3179

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(Cos[c + d*x]^(m - 1)/(d*(a^2 +
b^2)*(m - 1))), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rule 3188

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[a*(b/(a^2
+ b^2)), Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x] + b
*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rubi steps

$$\int \frac{\cosh^3(x)}{a+b\coth(x)}\,dx = -\left(i\int \frac{\cosh^3(x)\sinh(x)}{-ib\cosh(x)-ia\sinh(x)}\,dx\right)$$

$$= \frac{a\int \cosh^3(x)\,dx}{a^2-b^2} - \frac{b\int \cosh^2(x)\sinh(x)\,dx}{a^2-b^2} + \frac{(iab)\int \frac{\cosh^2(x)}{-ib\cosh(x)-ia\sinh(x)}\,dx}{a^2-b^2}$$

$$= -\frac{a^2b\cosh(x)}{(a^2-b^2)^2} + \frac{(ia^3b)\int \frac{1}{-ib\cosh(x)-ia\sinh(x)}\,dx}{(a^2-b^2)^2} + \frac{(ab^2)\int \cosh(x)\,dx}{(a^2-b^2)^2} + \frac{(ia)\text{Subst}\left(\int (1\right.}{}$$

$$= -\frac{a^2b\cosh(x)}{(a^2-b^2)^2} - \frac{b\cosh^3(x)}{3(a^2-b^2)} + \frac{ab^2\sinh(x)}{(a^2-b^2)^2} + \frac{a\sinh(x)}{a^2-b^2} + \frac{a\sinh^3(x)}{3(a^2-b^2)} - \frac{(a^3b)\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{a^3b\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} - \frac{a^2b\cosh(x)}{(a^2-b^2)^2} - \frac{b\cosh^3(x)}{3(a^2-b^2)} + \frac{ab^2\sinh(x)}{(a^2-b^2)^2} + \frac{a\sinh(x)}{a^2-b^2}$$

Mathematica [A]
time = 1.01, size = 167, normalized size = 1.24

$$\frac{1}{12}\left(-\frac{24a^3b\text{ArcTan}\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a+b}\sqrt{a+b}}\right)}{(-a+b)^{5/2}(a+b)^{5/2}} + \frac{3b(-5a^2+b^2)\cosh(x)}{(a-b)^2(a+b)^2} + \frac{b\cosh(3x)}{(-a+b)(a+b)} + \frac{3a(3a^2+b^2)\sinh(x)}{(a-b)^2(a+b)^2} + \frac{a^3\sinh(3x)}{(a-b)^2(a+b)^2} - \frac{ab^2\sinh(3x)}{(a-b)^2(a+b)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(a + b*Coth[x]),x]
```

```
[Out] ((-24*a^3*b*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a + b])])/((-a + b)
^(5/2)*(a + b)^(5/2)) + (3*b*(-5*a^2 + b^2)*Cosh[x])/((a - b)^2*(a + b)^2)
+ (b*Cosh[3*x])/((-a + b)*(a + b)) + (3*a*(3*a^2 + b^2)*Sinh[x])/((a - b)^2
*(a + b)^2) + (a^3*Sinh[3*x])/((a - b)^2*(a + b)^2) - (a*b^2*Sinh[3*x])/((a
 - b)^2*(a + b)^2))/12
```

Maple [A]
time = 0.73, size = 176, normalized size = 1.30

method	result
risch	$\frac{e^{3x}}{24a+24b} + \frac{3\,e^{x}a}{8(a+b)^2} + \frac{e^{x}b}{8(a+b)^2} - \frac{3\,e^{-x}a}{8(a-b)^2} + \frac{e^{-x}b}{8(a-b)^2} - \frac{e^{-3x}}{24(a-b)} + \frac{b\,a^3\ln\left(e^{x}+\frac{a-b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(a+b)^2(a-b)^2} - \frac{b\,a^3\ln\left(e^{x}-\frac{a-}{\sqrt{a^2-}}\right.}{\sqrt{a^2-b^2}\,(a+b)}$
default	$-\frac{4}{3\left(\tanh\left(\frac{x}{2}\right)-1\right)^3(4a+4b)} - \frac{2}{(4a+4b)\left(\tanh\left(\frac{x}{2}\right)-1\right)^2} - \frac{2a+b}{2(a+b)^2\left(\tanh\left(\frac{x}{2}\right)-1\right)} - \frac{2a^3b\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)b+2a}{2\sqrt{-a^2+b^2}}\right)}{(a-b)^2(a+b)^2\sqrt{-a^2+b^2}} - \frac{}{3(\tanh}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -4/3/(tanh(1/2*x)-1)^3/(4*a+4*b)-2/(4*a+4*b)/(tanh(1/2*x)-1)^2-1/2*(2*a+b)/
(a+b)^2/(tanh(1/2*x)-1)-2*a^3*b/(a-b)^2/(a+b)^2/(-a^2+b^2)^(1/2)*arctan(1/2
*(2*tanh(1/2*x)*b+2*a)/(-a^2+b^2)^(1/2))-4/3/(tanh(1/2*x)+1)^3/(4*a-4*b)+2/
(4*a-4*b)/(tanh(1/2*x)+1)^2-1/2*(-b+2*a)/(a-b)^2/(tanh(1/2*x)+1)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 909 vs. $2(127) = 254$.
time = 0.41, size = 1873, normalized size = 13.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] [1/24*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a
^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5
- a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 - a^5 - a^4*b + 2*
a^3*b^2 + 2*a^2*b^3 - a*b^4 - b^5 + 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*
b^3 - a*b^4 - b^5)*cosh(x)^4 + 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 -
 a*b^4 - b^5 + 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x
)^2)*sinh(x)^4 + 4*(5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*c
osh(x)^3 + 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x
))*sinh(x)^3 - 3*(3*a^5 + 5*a^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5)*co
sh(x)^2 - 3*(3*a^5 + 5*a^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5 - 5*(a^5
 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 - 6*(3*a^5 - 5*a^
4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x)^2)*sinh(x)^2 + 24*(a^3*b
*cosh(x)^3 + 3*a^3*b*cosh(x)^2*sinh(x) + 3*a^3*b*cosh(x)*sinh(x)^2 + a^3*b*
sinh(x)^3)*sqrt(a^2 - b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(
x) + (a + b)*sinh(x)^2 + 2*sqrt(a^2 - b^2)*(cosh(x) + sinh(x)) + a - b)/((a
 + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a + b)) +
 6*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 + 2*(3*a^
```

```
5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x)^3 - (3*a^5 + 5*a
^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5)*cosh(x))*sinh(x))/((a^6 - 3*a^4
*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*c
osh(x)^2*sinh(x) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^2
+ (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^3), 1/24*((a^5 - a^4*b - 2*a^
3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a^5 - a^4*b - 2*a^3*b^2 + 2
*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5 - a^4*b - 2*a^3*b^2 + 2*a^
2*b^3 + a*b^4 - b^5)*sinh(x)^6 - a^5 - a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - a*b^
4 - b^5 + 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x)
^4 + 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5 + 5*(a^5 - a^
4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^5
 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 + 3*(3*a^5 - 5*a^
4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x))*sinh(x)^3 - 3*(3*a^5 +
5*a^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5)*cosh(x)^2 - 3*(3*a^5 + 5*a^4
*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5 - 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a
^2*b^3 + a*b^4 - b^5)*cosh(x)^4 - 6*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^
3 - a*b^4 - b^5)*cosh(x)^2)*sinh(x)^2 - 48*(a^3*b*cosh(x)^3 + 3*a^3*b*cosh(
x)^2*sinh(x) + 3*a^3*b*cosh(x)*sinh(x)^2 + a^3*b*sinh(x)^3)*sqrt(-a^2 + b^2
)*arctan(sqrt(-a^2 + b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) + 6*((a^5 -
a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 + 2*(3*a^5 - 5*a^4*b
 - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x)^3 - (3*a^5 + 5*a^4*b - 2*a^
3*b^2 - 6*a^2*b^3 - a*b^4 + b^5)*cosh(x))*sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^
2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2*si
nh(x) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^2 + (a^6 - 3*
a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3(x)}{a + b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(a+b*coth(x)),x)
```

```
[Out] Integral(cosh(x)**3/(a + b*coth(x)), x)
```

Giac [A]
time = 0.41, size = 164, normalized size = 1.21

$$\frac{2\,a^3 b \arctan\left(-\frac{a e^x + b e^x}{\sqrt{-a^2+b^2}}\right)}{\left(a^4 - 2\,a^2 b^2 + b^4\right)\sqrt{-a^2+b^2}} - \frac{\left(9\,a e^{(2\,x)} - 3\,b e^{(2\,x)} + a - b\right) e^{(-3\,x)}}{24\,\left(a^2 - 2\,ab + b^2\right)} + \frac{a^2 e^{(3\,x)} + 2\,ab e^{(3\,x)} + b^2 e^{(3\,x)} + 9\,a^2 e^x + 12\,ab e^x + 3\,b^2 e^x}{24\,\left(a^3 + 3\,a^2 b + 3\,ab^2 + b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] 2*a^3*b*arctan(-(a*e^x + b*e^x)/sqrt(-a^2 + b^2))/((a^4 - 2*a^2*b^2 + b^4)*
sqrt(-a^2 + b^2)) - 1/24*(9*a*e^(2*x) - 3*b*e^(2*x) + a - b)*e^(-3*x)/(a^2
- 2*a*b + b^2) + 1/24*(a^2*e^(3*x) + 2*a*b*e^(3*x) + b^2*e^(3*x) + 9*a^2*e^
x + 12*a*b*e^x + 3*b^2*e^x)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
```

Mupad [B]
time = 2.04, size = 262, normalized size = 1.94

$$\frac{e^{3x}}{24a+24b}-\frac{e^{-3x}}{24a-24b}+\frac{e^{x}(3a+b)}{8(a+b)^2}-\frac{e^{-x}(3a-b)}{8(a-b)^2}+\frac{2\,\mathrm{atan}\left(\frac{a^3\,b\,e^x\,\sqrt{-a^{10}+5a^8b^2-10a^6b^4+10a^4b^6-5a^2b^8+b^{10}}}{a^5\sqrt{a^6b^2}-b^5\sqrt{a^6b^2}+2a^2b^3\sqrt{a^6b^2}-2a^3b^2\sqrt{a^6b^2}+ab^4\sqrt{a^6b^2}-a^4b\sqrt{a^6b^2}}\right)\sqrt{a^6b^2}}{\sqrt{-a^{10}+5a^8b^2-10a^6b^4+10a^4b^6-5a^2b^8+b^{10}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a + b*coth(x)),x)
```

```
[Out] exp(3*x)/(24*a + 24*b) - exp(-3*x)/(24*a - 24*b) + (exp(x)*(3*a + b))/(8*(a
 + b)^2) - (exp(-x)*(3*a - b))/(8*(a - b)^2) + (2*atan((a^3*b*exp(x)*(b^10
- a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2))/(a^5*(a^6*
b^2)^(1/2) - b^5*(a^6*b^2)^(1/2) + 2*a^2*b^3*(a^6*b^2)^(1/2) - 2*a^3*b^2*(a
^6*b^2)^(1/2) + a*b^4*(a^6*b^2)^(1/2) - a^4*b*(a^6*b^2)^(1/2)))*(a^6*b^2)^(
1/2))/(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)
```

3.116 $\int \frac{\cosh^2(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=85

$$-\frac{a\log(1-\coth(x))}{4(a+b)^2}+\frac{a\log(1+\coth(x))}{4(a-b)^2}-\frac{a^2b\log(a+b\coth(x))}{(a^2-b^2)^2}-\frac{(b-a\coth(x))\sinh^2(x)}{2(a^2-b^2)}$$

```
[Out] -1/4*a*ln(1-coth(x))/(a+b)^2+1/4*a*ln(1+coth(x))/(a-b)^2-a^2*b*ln(a+b*coth(
x))/(a^2-b^2)^2-1/2*(b-a*coth(x))*sinh(x)^2/(a^2-b^2)
```

Rubi [A]
time = 0.11, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3597, 1661, 815}

$$-\frac{a^2b\log(a+b\coth(x))}{(a^2-b^2)^2}-\frac{\sinh^2(x)(b-a\coth(x))}{2(a^2-b^2)}-\frac{a\log(1-\coth(x))}{4(a+b)^2}+\frac{a\log(\coth(x)+1)}{4(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(a + b*Coth[x]),x]
```

```
[Out] -1/4*(a*Log[1 - Coth[x]])/(a + b)^2 + (a*Log[1 + Coth[x]])/(4*(a - b)^2) -
(a^2*b*Log[a + b*Coth[x]])/(a^2 - b^2)^2 - ((b - a*Coth[x])*Sinh[x]^2)/(2*(
a^2 - b^2))
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
```

```
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\cosh^2(x)}{a+b\coth(x)}\,dx = -\left(b\text{Subst}\left(\int \frac{x^2}{(a+x)\left(-b^2+x^2\right)^2}\,dx, x, b\coth(x)\right)\right)$$
$$= -\frac{(b-a\coth(x))\sinh^2(x)}{2\left(a^2-b^2\right)} - \frac{\text{Subst}\left(\int \frac{\frac{a^2b^2}{a^2-b^2}-\frac{ab^2x}{a^2-b^2}}{(a+x)\left(-b^2+x^2\right)}\,dx, x, b\coth(x)\right)}{2b}$$
$$= -\frac{(b-a\coth(x))\sinh^2(x)}{2\left(a^2-b^2\right)} - \frac{\text{Subst}\left(\int \left(-\frac{ab}{2(a+b)^2(b-x)} + \frac{2a^2b^2}{(a-b)^2(a+b)^2(a+x)} - \frac{ab}{2(a-b)^2(b+x)}\right)\right.}{2b}$$
$$= -\frac{a\log(1-\coth(x))}{4(a+b)^2} + \frac{a\log(1+\coth(x))}{4(a-b)^2} - \frac{a^2b\log(a+b\coth(x))}{\left(a^2-b^2\right)^2} - \frac{(b-a\coth(x))\,s}{2\left(a^2-b^2\right.}$$

Mathematica [A]
time = 0.17, size = 73, normalized size = 0.86

$$\frac{\left(-a^2b+b^3\right)\cosh(2x)+a\left(2\left(a^2+b^2\right)x-4ab\log(b\cosh(x)+a\sinh(x))+\left(a^2-b^2\right)\sinh(2x)\right)}{4(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(a + b*Coth[x]),x]
```

```
[Out] ((-(a^2*b) + b^3)*Cosh[2*x] + a*(2*(a^2 + b^2)*x - 4*a*b*Log[b*Cosh[x] + a*
Sinh[x]] + (a^2 - b^2)*Sinh[2*x]))/(4*(a - b)^2*(a + b)^2)
```

Maple [A]
time = 0.73, size = 146, normalized size = 1.72

method	result
risch	$\frac{ax}{2(a+b)^2}+\frac{e^{2x}}{8a+8b}-\frac{e^{-2x}}{8(a-b)}+\frac{2a^2bx}{a^4-2a^2b^2+b^4}-\frac{a^2b\ln\left(e^{2x}-\frac{a-b}{a+b}\right)}{a^4-2a^2b^2+b^4}$
default	$\frac{2}{(4a+4b)\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}+\frac{4}{(8a+8b)\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2(a+b)^2}-\frac{a^2b\ln\left(b\left(\tanh^2\left(\frac{x}{2}\right)\right)+2a\tanh\left(\frac{x}{2}\right)+b\right)}{(a-b)^2(a+b)^2}-\frac{2}{(4a-4b)\left(\tanh\left(\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(4*a+4*b)/(tanh(1/2*x)-1)^2+4/(8*a+8*b)/(tanh(1/2*x)-1)-1/2*a/(a+b)^2*ln(
tanh(1/2*x)-1)-a^2*b/(a-b)^2/(a+b)^2*ln(b*tanh(1/2*x)^2+2*a*tanh(1/2*x)+b)-
```

```
2/(4*a-4*b)/(tanh(1/2*x)+1)^2+4/(8*a-8*b)/(tanh(1/2*x)+1)+1/2*a/(a-b)^2*ln(
tanh(1/2*x)+1)
```

Maxima [A]
time = 0.28, size = 80, normalized size = 0.94

$$-\frac{a^2 b \log\left(-(a-b)e^{(-2x)}+a+b\right)}{a^4-2a^2b^2+b^4}+\frac{ax}{2(a^2+2ab+b^2)}+\frac{e^{(2x)}}{8(a+b)}-\frac{e^{(-2x)}}{8(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] -a^2*b*log(-(a - b)*e^(-2*x) + a + b)/(a^4 - 2*a^2*b^2 + b^4) + 1/2*a*x/(a^
2 + 2*a*b + b^2) + 1/8*e^(2*x)/(a + b) - 1/8*e^(-2*x)/(a - b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(80) = 160$.
time = 0.36, size = 334, normalized size = 3.93

$$\frac{(a^3-a^2b-ab^2+b^3)\cosh(x)^4+4(a^3-a^2b-ab^2+b^3)\cosh(x)\sinh(x)^3+(a^3-a^2b-ab^2+b^3)\sinh(x)^4+4(a^3+2a^2b+ab^2)x\cosh(x)^2-a^3-a^2b+ab^2+b^3+2\left(3(a^3-a^2b-ab^2+b^3)\cosh(x)^2+2(a^3+2a^2b+ab^2)x\right)\sinh(x)^2-8\left(a^2b\cosh(x)^2+2a^2b\cosh(x)\sinh(x)+a^2b\sinh(x)^2\right)\log\left(\frac{2(b\cosh(x)+a\sinh(x))}{\cosh(x)-\sinh(x)}\right)+4\left((a^3-a^2b-ab^2+b^3)\cosh(x)^3+2(a^3+2a^2b+ab^2)x\cosh(x)\right)\sinh(x)}{8\left((a^4-2a^2b^2+b^4)\cosh(x)^2+2(a^4-2a^2b^2+b^4)\cosh(x)\sinh(x)+(a^4-2a^2b^2+b^4)\sinh(x)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] 1/8*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 - a^2*b - a*b^2 + b^3)*
cosh(x)*sinh(x)^3 + (a^3 - a^2*b - a*b^2 + b^3)*sinh(x)^4 + 4*(a^3 + 2*a^2*
b + a*b^2)*x*cosh(x)^2 - a^3 - a^2*b + a*b^2 + b^3 + 2*(3*(a^3 - a^2*b - a*
b^2 + b^3)*cosh(x)^2 + 2*(a^3 + 2*a^2*b + a*b^2)*x)*sinh(x)^2 - 8*(a^2*b*co
sh(x)^2 + 2*a^2*b*cosh(x)*sinh(x) + a^2*b*sinh(x)^2)*log(2*(b*cosh(x) + a*s
inh(x))/(cosh(x) - sinh(x))) + 4*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^3 + 2
*(a^3 + 2*a^2*b + a*b^2)*x*cosh(x))*sinh(x))/((a^4 - 2*a^2*b^2 + b^4)*cosh(
x)^2 + 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x) + (a^4 - 2*a^2*b^2 + b^4)*
sinh(x)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(x)}{a+b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(a+b*coth(x)),x)
```

```
[Out] Integral(cosh(x)**2/(a + b*coth(x)), x)
```

Giac [A]
time = 0.43, size = 104, normalized size = 1.22

$$-\frac{a^{2}b\log\left(\left|-ae^{(2\,x)}-be^{(2\,x)}+a-b\right|\right)}{a^{4}-2\,a^{2}b^{2}+b^{4}}+\frac{ax}{2\,(a^{2}-2\,ab+b^{2})}-\frac{\left(2\,ae^{(2\,x)}+a-b\right)e^{(-2\,x)}}{8\,(a^{2}-2\,ab+b^{2})}+\frac{e^{(2\,x)}}{8\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -a^2*b*log(abs(-a*e^(2*x) - b*e^(2*x) + a - b))/(a^4 - 2*a^2*b^2 + b^4) + 1
/2*a*x/(a^2 - 2*a*b + b^2) - 1/8*(2*a*e^(2*x) + a - b)*e^(-2*x)/(a^2 - 2*a*
b + b^2) + 1/8*e^(2*x)/(a + b)
```

Mupad [B]
time = 1.40, size = 82, normalized size = 0.96

$$\frac{\mathrm{e}^{2\,x}}{8\,a+8\,b}-\frac{\mathrm{e}^{-2\,x}}{8\,a-8\,b}+\frac{a\,x}{2\,{(a-b)}^2}-\frac{a^2\,b\,\ln\left(b-a+a\,\mathrm{e}^{2\,x}+b\,\mathrm{e}^{2\,x}\right)}{a^4-2\,a^2\,b^2+b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a + b*coth(x)),x)
```

```
[Out] exp(2*x)/(8*a + 8*b) - exp(-2*x)/(8*a - 8*b) + (a*x)/(2*(a - b)^2) - (a^2*b
*log(b - a + a*exp(2*x) + b*exp(2*x)))/(a^4 + b^4 - 2*a^2*b^2)
```

3.117 $\int \frac{\cosh(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=72

$$\frac{ab\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{b\cosh(x)}{a^2-b^2}+\frac{a\sinh(x)}{a^2-b^2}$$

```
[Out] a*b*arctanh((a*cosh(x)+b*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)-b*cosh(x
)/(a^2-b^2)+a*sinh(x)/(a^2-b^2)
```

Rubi [A]
time = 0.08, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {3599, 3188, 2717, 2718, 3153, 212}

$$\frac{a\sinh(x)}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}+\frac{ab\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/(a + b*Coth[x]),x]
```

```
[Out] (a*b*ArcTanh[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) -
(b*Cosh[x])/(a^2 - b^2) + (a*Sinh[x])/(a^2 - b^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
```

```
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3188

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[a*(b/(a^2
+ b^2)), Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x] + b
*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(x)}{a+b\coth(x)}\,dx &= -\left(i\int \frac{\cosh(x)\sinh(x)}{-ib\cosh(x)-ia\sinh(x)}\,dx\right)\\
&= \frac{a\int \cosh(x)\,dx}{a^2-b^2}-\frac{b\int \sinh(x)\,dx}{a^2-b^2}+\frac{(iab)\int \frac{1}{-ib\cosh(x)-ia\sinh(x)}\,dx}{a^2-b^2}\\
&= -\frac{b\cosh(x)}{a^2-b^2}+\frac{a\sinh(x)}{a^2-b^2}-\frac{(ab)\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx,x,-a\cosh(x)-b\sinh(x)\right)}{a^2-b^2}\\
&= \frac{ab\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}-\frac{b\cosh(x)}{a^2-b^2}+\frac{a\sinh(x)}{a^2-b^2}
\end{aligned}$$

Mathematica [**A**]
time = 0.21, size = 79, normalized size = 1.10

$$\frac{2ab\text{ArcTan}\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a+b}\sqrt{a+b}}\right)}{(-a+b)^{3/2}(a+b)^{3/2}}+\frac{b\cosh(x)}{-a^2+b^2}+\frac{a\sinh(x)}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(a + b*Coth[x]),x]
```

```
[Out] (2*a*b*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a + b])])/((-a + b)^(3/2
)*(a + b)^(3/2)) + (b*Cosh[x])/(-a^2 + b^2) + (a*Sinh[x])/(a^2 - b^2)
```

Maple [A]
time = 0.70, size = 92, normalized size = 1.28

method	result	size
default	$-\frac{4}{(4a+4b)\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{4}{(4a-4b)\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{2ab\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)b+2a}{2\sqrt{-a^2+b^2}}\right)}{(a+b)(a-b)\sqrt{-a^2+b^2}}$	92
risch	$\frac{e^{x}}{2b+2a}-\frac{e^{-x}}{2(a-b)}+\frac{ba\ln\left(e^{x}+\frac{a-b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)}-\frac{ba\ln\left(e^{x}-\frac{a-b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)}$	120

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -4/(4*a+4*b)/(tanh(1/2*x)-1)-4/(4*a-4*b)/(tanh(1/2*x)+1)-2*a*b/(a+b)/(a-b)/
(-a^2+b^2)^(1/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*a)/(-a^2+b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(68) = 136.
time = 0.40, size = 431, normalized size = 5.99

$$\left[-\frac{a^3+a^2b-ab^2-b^3-(a^3-a^2b-ab^2+b^3)\cosh(x)^2-2(a^3-a^2b-ab^2+b^3)\cosh(x)\sinh(x)-(a^3-a^2b-ab^2+b^3)\sinh(x)^2+2(ab\cosh(x)+ab\sinh(x))\sqrt{a^2-b^2}\log\left(\frac{(a+b)\cosh(x)^2+2(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2-2\sqrt{a^2-b^2}(\cosh(x)+\sinh(x))+a-b}{(a+b)\cosh(x)^2+2(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2-a+b}\right)}{2((a^4-2a^2b^2+b^4)\cosh(x)+(a^4-2a^2b^2+b^4)\sinh(x))}, -\frac{a^3+a^2b-ab^2-b^3-(a^3-a^2b-ab^2+b^3)\cosh(x)^2-2(a^3-a^2b-ab^2+b^3)\cosh(x)\sinh(x)-(a^3-a^2b-ab^2+b^3)\sinh(x)^2+4(ab\cosh(x)+ab\sinh(x))\sqrt{-a^2+b^2}\arctan\left(\frac{\sqrt{-a^2+b^2}}{(a+b)\cosh(x)+(a+b)\sinh(x)}\right)}{2((a^4-2a^2b^2+b^4)\cosh(x)+(a^4-2a^2b^2+b^4)\sinh(x))}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(a^3 + a^2*b - a*b^2 - b^3 - (a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^2 -
2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) - (a^3 - a^2*b - a*b^2 + b^3)
*sinh(x)^2 + 2*(a*b*cosh(x) + a*b*sinh(x))*sqrt(a^2 - b^2)*log(((a + b)*cos
```

```
h(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(a^2 - b^2)*
(cosh(x) + sinh(x)) + a - b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x)
 + (a + b)*sinh(x)^2 - a + b)))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 - 2
*a^2*b^2 + b^4)*sinh(x)), -1/2*(a^3 + a^2*b - a*b^2 - b^3 - (a^3 - a^2*b -
a*b^2 + b^3)*cosh(x)^2 - 2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) - (a
^3 - a^2*b - a*b^2 + b^3)*sinh(x)^2 + 4*(a*b*cosh(x) + a*b*sinh(x))*sqrt(-a
^2 + b^2)*arctan(sqrt(-a^2 + b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))))/((a
^4 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 - 2*a^2*b^2 + b^4)*sinh(x))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(x)}{a + b\coth(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*coth(x)),x)
```

```
[Out] Integral(cosh(x)/(a + b*coth(x)), x)
```

Giac [**A**]
time = 0.41, size = 71, normalized size = 0.99

$$-\frac{2\,ab\arctan\left(\frac{ae^x+be^x}{\sqrt{-a^2+b^2}}\right)}{(a^2-b^2)\sqrt{-a^2+b^2}} - \frac{e^{(-x)}}{2\,(a-b)} + \frac{e^x}{2\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -2*a*b*arctan((a*e^x + b*e^x)/sqrt(-a^2 + b^2))/((a^2 - b^2)*sqrt(-a^2 + b^
2)) - 1/2*e^(-x)/(a - b) + 1/2*e^x/(a + b)
```

Mupad [**B**]
time = 1.53, size = 158, normalized size = 2.19

$$\frac{e^x}{2\,a+2\,b} - \frac{e^{-x}}{2\,a-2\,b} + \frac{2\,\mathrm{atan}\left(\frac{a\,b\,e^x\,\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}}{a^3\,\sqrt{a^2\,b^2}+b^3\,\sqrt{a^2\,b^2}-a\,b^2\,\sqrt{a^2\,b^2}-a^2\,b\,\sqrt{a^2\,b^2}}\right)\,\sqrt{a^2\,b^2}}{\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a + b*coth(x)),x)
```

```
[Out] exp(x)/(2*a + 2*b) - exp(-x)/(2*a - 2*b) + (2*atan((a*b*exp(x)*(b^6 - a^6 -
 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(a^3*(a^2*b^2)^(1/2) + b^3*(a^2*b^2)^(1/2) -
 a*b^2*(a^2*b^2)^(1/2) - a^2*b*(a^2*b^2)^(1/2)))*(a^2*b^2)^(1/2))/(b^6 - a^
6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)
```

3.118 $\int \frac{\operatorname{sech}(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=50

$$\frac{\operatorname{ArcTan}(\sinh(x))}{a}+\frac{b\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}$$

```
[Out] arctan(sinh(x))/a+b*arctanh((a*cosh(x)+b*sinh(x))/(a^2-b^2)^(1/2))/a/(a^2-b
^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {3599, 3189, 3855, 3153, 212}

$$\frac{b\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}+\frac{\operatorname{ArcTan}(\sinh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]/(a + b*Coth[x]),x]
```

```
[Out] ArcTan[Sinh[x]]/a + (b*ArcTanh[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]])/(a
*Sqrt[a^2 - b^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3189

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[cos[c + d*x]^m*(sin[c + d*x]^n/(a*cos[c + d*x] + b*sin[c + d*x])),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}(x)}{a+b\coth(x)}\,dx &= -\left(i\int \frac{\tanh(x)}{-ib\cosh(x)-ia\sinh(x)}\,dx\right)\\ &= -\int \left(-\frac{\operatorname{sech}(x)}{a}+\frac{ib}{a(ib\cosh(x)+ia\sinh(x))}\right)dx\\ &= \frac{\int \operatorname{sech}(x)\,dx}{a}-\frac{(ib)\int \frac{1}{ib\cosh(x)+ia\sinh(x)}\,dx}{a}\\ &= \frac{\tan^{-1}(\sinh(x))}{a}+\frac{b\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx,x,a\cosh(x)+b\sinh(x)\right)}{a}\\ &= \frac{\tan^{-1}(\sinh(x))}{a}+\frac{b\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 60, normalized size = 1.20

$$\frac{2\left(\operatorname{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{b\operatorname{ArcTan}\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a+b}\sqrt{a+b}}\right)}{\sqrt{-a+b}\sqrt{a+b}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(a + b*Coth[x]),x]
```

```
[Out] (2*(ArcTan[Tanh[x/2]] - (b*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a +
b])])/(Sqrt[-a + b]*Sqrt[a + b])))/a
```

Maple [A]
time = 0.80, size = 54, normalized size = 1.08

method	result	size
default	$-\frac{2b\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)b+2a}{2\sqrt{-a^{2}+b^{2}}}\right)}{a\sqrt{-a^{2}+b^{2}}}+\frac{2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$	54
risch	$\frac{i\ln\left(\mathrm{e}^{x}+i\right)}{a}-\frac{i\ln\left(\mathrm{e}^{x}-i\right)}{a}+\frac{b\ln\left(\mathrm{e}^{x}+\frac{a-b}{\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,a}-\frac{b\ln\left(\mathrm{e}^{x}-\frac{a-b}{\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,a}$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/a*b/(-a^2+b^2)^(1/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*a)/(-a^2+b^2)^(1/2))+
2/a*arctan(tanh(1/2*x))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]

time = 0.42, size = 200, normalized size = 4.00

$$\left[\frac{\sqrt{a^{2}-b^{2}}\,b\log\left(\frac{(a+b)\cosh(x)^{2}+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^{2}+2\,\sqrt{a^{2}-b^{2}}\,(\cosh(x)+\sinh(x))+a-b}{(a+b)\cosh(x)^{2}+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^{2}-a+b}\right)+2\,(a^{2}-b^{2})\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}{a^{3}-ab^{2}},\,-\frac{2\left(\sqrt{-a^{2}+b^{2}}\,b\arctan\left(\frac{\sqrt{-a^{2}+b^{2}}}{(a+b)\cosh(x)+(a+b)\sinh(x)}\right)-(a^{2}-b^{2})\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)\right)}{a^{3}-ab^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] [(sqrt(a^2 - b^2)*b*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a
 + b)*sinh(x)^2 + 2*sqrt(a^2 - b^2)*(cosh(x) + sinh(x)) + a - b)/((a + b)*c
osh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a + b)) + 2*(a^2
 - b^2)*arctan(cosh(x) + sinh(x)))/(a^3 - a*b^2), -2*(sqrt(-a^2 + b^2)*b*ar
ctan(sqrt(-a^2 + b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) - (a^2 - b^2)*ar
ctan(cosh(x) + sinh(x)))/(a^3 - a*b^2)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{sech}\left(x\right)}{a+b\coth\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*coth(x)),x)
```

```
[Out] Integral(sech(x)/(a + b*coth(x)), x)
```

Giac [A]
time = 0.40, size = 48, normalized size = 0.96

$$-\frac{2\,b\arctan\left(\frac{ae^{x}+be^{x}}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}\,a}+\frac{2\,\arctan\left(e^{x}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -2*b*arctan((a*e^x + b*e^x)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*a) + 2*arct
an(e^x)/a
```

Mupad [B]
time = 3.22, size = 164, normalized size = 3.28

$$\frac{b\,\ln\left(32\,a\,b^2\,\mathrm{e}^x+32\,a^2\,b\,\mathrm{e}^x+32\,a\,b\,\sqrt{a^2-b^2}\right)}{a\,\sqrt{a^2-b^2}}-\frac{b\,\ln\left(32\,a\,b^2\,\mathrm{e}^x+32\,a^2\,b\,\mathrm{e}^x-32\,a\,b\,\sqrt{a^2-b^2}\right)}{a\,\sqrt{a^2-b^2}}+\frac{\ln\left(32\,a\,b\,\mathrm{e}^x-32\,a^2\,\mathrm{e}^x+a\,b\,32\mathrm{i}-a^2\,32\mathrm{i}\right)\,1\mathrm{i}}{a}-\frac{\ln\left(32\,a^2\,\mathrm{e}^x-32\,a\,b\,\mathrm{e}^x+a\,b\,32\mathrm{i}-a^2\,32\mathrm{i}\right)\,1\mathrm{i}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)*(a + b*coth(x))),x)
```

```
[Out] (log(a*b*32i - a^2*32i - 32*a^2*exp(x) + 32*a*b*exp(x))*1i)/a - (log(a*b*32
i - a^2*32i + 32*a^2*exp(x) - 32*a*b*exp(x))*1i)/a - (b*log(32*a*b^2*exp(x)
 + 32*a^2*b*exp(x) - 32*a*b*(a^2 - b^2)^(1/2)))/(a*(a^2 - b^2)^(1/2)) + (b*
log(32*a*b^2*exp(x) + 32*a^2*b*exp(x) + 32*a*b*(a^2 - b^2)^(1/2)))/(a*(a^2
- b^2)^(1/2))
```

3.119 $\int \frac{\text{sech}^2(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=29

$$-\frac{b\log(a+b\coth(x))}{a^2}-\frac{b\log(\tanh(x))}{a^2}+\frac{\tanh(x)}{a}$$

```
[Out] -b*ln(a+b*coth(x))/a^2-b*ln(tanh(x))/a^2+tanh(x)/a
```

Rubi [A]
time = 0.04, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3597, 46}

$$-\frac{b\log(\tanh(x))}{a^2}-\frac{b\log(a+b\coth(x))}{a^2}+\frac{\tanh(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(a + b*Coth[x]),x]
```

```
[Out] -((b*Log[a + b*Coth[x]])/a^2) - (b*Log[Tanh[x]])/a^2 + Tanh[x]/a
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{sech}^2(x)}{a+b\coth(x)}\,dx &= -\left(b\text{Subst}\left(\int \frac{1}{x^2(a+x)}\,dx,x,b\coth(x)\right)\right)\\
&= -\left(b\text{Subst}\left(\int \left(\frac{1}{ax^2}-\frac{1}{a^2x}+\frac{1}{a^2(a+x)}\right)\,dx,x,b\coth(x)\right)\right)\\
&= -\frac{b\log(a+b\coth(x))}{a^2}-\frac{b\log(\tanh(x))}{a^2}+\frac{\tanh(x)}{a}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 27, normalized size = 0.93

$$\frac{b\log(\cosh(x)) - b\log(b\cosh(x) + a\sinh(x)) + a\tanh(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(a + b*Coth[x]),x]
[Out] (b*Log[Cosh[x]] - b*Log[b*Cosh[x] + a*Sinh[x]] + a*Tanh[x])/a^2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(29) = 58$.
time = 0.78, size = 61, normalized size = 2.10

method	result	size
risch	$-\frac{2}{a(1+e^{2x})} - \frac{b\ln\left(e^{2x} - \frac{a-b}{a+b}\right)}{a^2} + \frac{b\ln(1+e^{2x})}{a^2}$	51
default	$-\frac{2\left(-\frac{a\tanh\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)+1} - \frac{b\ln\left(\tanh^2\left(\frac{x}{2}\right)+1\right)}{2}\right)}{a^2} - \frac{b\ln\left(b\left(\tanh^2\left(\frac{x}{2}\right)\right)+2a\tanh\left(\frac{x}{2}\right)+b\right)}{a^2}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(a+b*coth(x)),x,method=_RETURNVERBOSE)
[Out] -2/a^2*(-a*tanh(1/2*x)/(tanh(1/2*x)^2+1)-1/2*b*ln(tanh(1/2*x)^2+1))-b/a^2*l
n(b*tanh(1/2*x)^2+2*a*tanh(1/2*x)+b)
```

Maxima [A]
time = 0.50, size = 46, normalized size = 1.59

$$-\frac{b\log\left(-(a-b)e^{(-2x)} + a + b\right)}{a^2} + \frac{b\log\left(e^{(-2x)} + 1\right)}{a^2} + \frac{2}{ae^{(-2x)} + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*coth(x)),x, algorithm="maxima")
[Out] -b*log(-(a - b)*e^(-2*x) + a + b)/a^2 + b*log(e^(-2*x) + 1)/a^2 + 2/(a*e^(-
2*x) + a)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(29) = 58$.
time = 0.35, size = 117, normalized size = 4.03

$$-\frac{\left(b\cosh(x)^2 + 2b\cosh(x)\sinh(x) + b\sinh(x)^2 + b\right)\log\left(\frac{2(b\cosh(x)+a\sinh(x))}{\cosh(x)-\sinh(x)}\right) - \left(b\cosh(x)^2 + 2b\cosh(x)\sinh(x) + b\sinh(x)^2 + b\right)\log\left(\frac{2\cosh(x)}{\cosh(x)-\sinh(x)}\right) + 2a}{a^2\cosh(x)^2 + 2a^2\cosh(x)\sinh(x) + a^2\sinh(x)^2 + a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] -((b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)*log(2*(b*cosh(x) +
a*sinh(x))/(cosh(x) - sinh(x))) - (b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*si
nh(x)^2 + b)*log(2*cosh(x)/(cosh(x) - sinh(x))) + 2*a)/(a^2*cosh(x)^2 + 2*a
^2*cosh(x)*sinh(x) + a^2*sinh(x)^2 + a^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(x)}{a + b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(a+b*coth(x)),x)
```

```
[Out] Integral(sech(x)**2/(a + b*coth(x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(29) = 58.
time = 0.40, size = 76, normalized size = 2.62

$$-\frac{(ab+b^2)\log\left(\left|ae^{(2x)}+be^{(2x)}-a+b\right|\right)}{a^3+a^2b}+\frac{b\log\left(e^{(2x)}+1\right)}{a^2}-\frac{be^{(2x)}+2a+b}{a^2(e^{(2x)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -(a*b + b^2)*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^3 + a^2*b) + b*log(
e^(2*x) + 1)/a^2 - (b*e^(2*x) + 2*a + b)/(a^2*(e^(2*x) + 1))
```

Mupad [B]
time = 1.58, size = 323, normalized size = 11.14

$$\frac{2\,\mathrm{atan}\left(\frac{b\left(a^4\,{\left(b^2\right)}^{3/2}-a^6\,\sqrt{b^2}\right)\left(b^6\,\sqrt{-a^4}-a\,b^5\,\sqrt{-a^4}-a^2\,b^4\,\sqrt{-a^4}+a^3\,b^3\,\sqrt{-a^4}+b^6\,e^{2\,x}\,\sqrt{-a^4}-2\,a^2\,b^4\,e^{2\,x}\,\sqrt{-a^4}+a^4\,b^2\,e^{2\,x}\,\sqrt{-a^4}\right)+b^2\left(a^3\,{\left(b^2\right)}^{3/2}-a^5\,\sqrt{b^2}\right)\left(b^6\,\sqrt{-a^4}-a\,b^5\,\sqrt{-a^4}-a^2\,b^4\,\sqrt{-a^4}+a^3\,b^3\,\sqrt{-a^4}+b^6\,e^{2\,x}\,\sqrt{-a^4}-2\,a^2\,b^4\,e^{2\,x}\,\sqrt{-a^4}+a^4\,b^2\,e^{2\,x}\,\sqrt{-a^4}\right)}{-a^{12}\,b^4+3\,a^{10}\,b^6-3\,a^8\,b^8+a^6\,b^{10}}\right)\sqrt{b^2}}{\sqrt{-a^4}}-\frac{2}{a\,\left(e^{2\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2*(a + b*coth(x))),x)
```

```
[Out] (2*atan((b*(a^4*(b^2)^(3/2) - a^6*(b^2)^(1/2))*(b^6*(-a^4)^(1/2) - a*b^5*(-
a^4)^(1/2) - a^2*b^4*(-a^4)^(1/2) + a^3*b^3*(-a^4)^(1/2) + b^6*exp(2*x)*(-a
^4)^(1/2) - 2*a^2*b^4*exp(2*x)*(-a^4)^(1/2) + a^4*b^2*exp(2*x)*(-a^4)^(1/2)
) + b^2*(a^3*(b^2)^(3/2) - a^5*(b^2)^(1/2))*(b^6*(-a^4)^(1/2) - a*b^5*(-a^4
)^(1/2) - a^2*b^4*(-a^4)^(1/2) + a^3*b^3*(-a^4)^(1/2) + b^6*exp(2*x)*(-a^4)
^(1/2) - 2*a^2*b^4*exp(2*x)*(-a^4)^(1/2) + a^4*b^2*exp(2*x)*(-a^4)^(1/2)))/
(a^6*b^10 - 3*a^8*b^8 + 3*a^10*b^6 - a^12*b^4))*(b^2)^(1/2))/(-a^4)^(1/2) -
 2/(a*(exp(2*x) + 1))
```

3.120 $\int \frac{\text{sech}^3(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=83

$$\frac{\text{ArcTan}(\sinh(x))}{2a}-\frac{b^2\text{ArcTan}(\sinh(x))}{a^3}+\frac{b\sqrt{a^2-b^2}\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a^3}-\frac{b\text{sech}(x)}{a^2}+\frac{\text{sech}(x)\tanh(x)}{2a}$$

```
[Out] 1/2*arctan(sinh(x))/a-b^2*arctan(sinh(x))/a^3-b*sech(x)/a^2+b*arctanh((a*co
sh(x)+b*sinh(x))/(a^2-b^2)^(1/2))*(a^2-b^2)^(1/2)/a^3+1/2*sech(x)*tanh(x)/a
```

Rubi [A]
time = 0.17, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3599, 3189, 3853, 3855, 3183, 3153, 212}

$$-\frac{b^2\text{ArcTan}(\sinh(x))}{a^3}-\frac{b\text{sech}(x)}{a^2}+\frac{b\sqrt{a^2-b^2}\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a^3}+\frac{\text{ArcTan}(\sinh(x))}{2a}+\frac{\tanh(x)\text{sech}(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^3/(a + b*Coth[x]),x]
```

```
[Out] ArcTan[Sinh[x]]/(2*a) - (b^2*ArcTan[Sinh[x]])/a^3 + (b*Sqrt[a^2 - b^2]*ArcT
anh[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]])/a^3 - (b*Sech[x])/a^2 + (Sech
[x]*Tanh[x])/(2*a)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3183

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Dist[a/b^2, Int[Cos[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/b
^2, Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /;
FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3189

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[cos[c + d*x]^m*(sin[c + d*x]^n/(a*cos[c + d*x] + b*sin[c + d*x])),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{sech}^3(x)}{a+b\coth(x)}\,dx &= -\left(i\int \frac{\operatorname{sech}^2(x)\tanh(x)}{-ib\cosh(x)-ia\sinh(x)}\,dx\right)\\
&= -\int \left(-\frac{\operatorname{sech}^3(x)}{a}+\frac{ib\operatorname{sech}^2(x)}{a(ib\cosh(x)+ia\sinh(x))}\right)\,dx\\
&= \frac{\int \operatorname{sech}^3(x)\,dx}{a}-\frac{(ib)\int \frac{\operatorname{sech}^2(x)}{ib\cosh(x)+ia\sinh(x)}\,dx}{a}\\
&= -\frac{b\operatorname{sech}(x)}{a^2}+\frac{\operatorname{sech}(x)\tanh(x)}{2a}+\frac{\int \operatorname{sech}(x)\,dx}{2a}-\frac{b^2\int \operatorname{sech}(x)\,dx}{a^3}-\frac{(ib(a^2-b^2))\int \frac{}{ib\,\mathrm{c}}}{a^3}\\
&= \frac{\tan^{-1}(\sinh(x))}{2a}-\frac{b^2\tan^{-1}(\sinh(x))}{a^3}-\frac{b\operatorname{sech}(x)}{a^2}+\frac{\operatorname{sech}(x)\tanh(x)}{2a}+\frac{(b(a^2-b^2))\,\mathrm{Su}}{}\\
&= \frac{\tan^{-1}(\sinh(x))}{2a}-\frac{b^2\tan^{-1}(\sinh(x))}{a^3}+\frac{b\sqrt{a^2-b^2}\,\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a^3}-\frac{b\operatorname{sech}}{a^2}
\end{aligned}
$$

Mathematica [A]
time = 0.13, size = 85, normalized size = 1.02

$$\frac{2(a^2-2b^2)\,\text{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)+4b\sqrt{-a+b}\,\sqrt{a+b}\,\text{ArcTan}\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a+b}\,\sqrt{a+b}}\right)+a\,\text{sech}(x)(-2b+a\tanh(x))}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^3/(a + b*Coth[x]),x]
```

```
[Out] (2*(a^2 - 2*b^2)*ArcTan[Tanh[x/2]] + 4*b*Sqrt[-a + b]*Sqrt[a + b]*ArcTan[(a
 + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a + b])] + a*Sech[x]*(-2*b + a*Tanh[x]))
/(2*a^3)
```

Maple [A]
time = 0.93, size = 121, normalized size = 1.46

method	result
default	$\frac{\frac{2\left(-\frac{a^2\left(\tanh^3\left(\frac{x}{2}\right)\right)}{2}-ab\left(\tanh^2\left(\frac{x}{2}\right)\right)+\frac{a^2\tanh\left(\frac{x}{2}\right)}{2}-ab\right)}{\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^2}+(a^2-2b^2)\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a^3}-\frac{2b(a^2-b^2)\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)b+2a}{2\sqrt{-a^2+b^2}}\right)}{a^3\sqrt{-a^2+b^2}}$
risch	$\frac{e^x\left(e^{2x}a-2b\,e^{2x}-a-2b\right)}{\left(1+e^{2x}\right)^2a^2}+\frac{\sqrt{a^2-b^2}\,b\ln\left(e^x+\frac{\sqrt{a^2-b^2}}{a+b}\right)}{a^3}-\frac{\sqrt{a^2-b^2}\,b\ln\left(e^x-\frac{\sqrt{a^2-b^2}}{a+b}\right)}{a^3}+\frac{i\ln(e^x+i)}{2a}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^3/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/a^3*((-1/2*a^2*tanh(1/2*x)^3-a*b*tanh(1/2*x)^2+1/2*a^2*tanh(1/2*x)-a*b)/(
tanh(1/2*x)^2+1)^2+1/2*(a^2-2*b^2)*arctan(tanh(1/2*x)))-2*b*(a^2-b^2)/a^3/(
-a^2+b^2)^(1/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*a)/(-a^2+b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 401 vs. $2(75) = 150$.
time = 0.38, size = 856, normalized size = 10.31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] [((a^2 - 2*a*b)*cosh(x)^3 + 3*(a^2 - 2*a*b)*cosh(x)*sinh(x)^2 + (a^2 - 2*a*
b)*sinh(x)^3 + (b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cos
h(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh
(x) + b)*sqrt(a^2 - b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x)
 + (a + b)*sinh(x)^2 + 2*sqrt(a^2 - b^2)*(cosh(x) + sinh(x)) + a - b)/((a +
 b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a + b)) + (
(a^2 - 2*b^2)*cosh(x)^4 + 4*(a^2 - 2*b^2)*cosh(x)*sinh(x)^3 + (a^2 - 2*b^2)
*sinh(x)^4 + 2*(a^2 - 2*b^2)*cosh(x)^2 + 2*(3*(a^2 - 2*b^2)*cosh(x)^2 + a^2
 - 2*b^2)*sinh(x)^2 + a^2 - 2*b^2 + 4*((a^2 - 2*b^2)*cosh(x)^3 + (a^2 - 2*b
^2)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) - (a^2 + 2*a*b)*cosh(x) + (
3*(a^2 - 2*a*b)*cosh(x)^2 - a^2 - 2*a*b)*sinh(x))/(a^3*cosh(x)^4 + 4*a^3*co
sh(x)*sinh(x)^3 + a^3*sinh(x)^4 + 2*a^3*cosh(x)^2 + a^3 + 2*(3*a^3*cosh(x)^
2 + a^3)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + a^3*cosh(x))*sinh(x)), ((a^2 - 2*a*
b)*cosh(x)^3 + 3*(a^2 - 2*a*b)*cosh(x)*sinh(x)^2 + (a^2 - 2*a*b)*sinh(x)^3
- 2*(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*
(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sq
rt(-a^2 + b^2)*arctan(sqrt(-a^2 + b^2)/((a + b)*cosh(x) + (a + b)*sinh(x)))
 + ((a^2 - 2*b^2)*cosh(x)^4 + 4*(a^2 - 2*b^2)*cosh(x)*sinh(x)^3 + (a^2 - 2*
b^2)*sinh(x)^4 + 2*(a^2 - 2*b^2)*cosh(x)^2 + 2*(3*(a^2 - 2*b^2)*cosh(x)^2 +
 a^2 - 2*b^2)*sinh(x)^2 + a^2 - 2*b^2 + 4*((a^2 - 2*b^2)*cosh(x)^3 + (a^2 -
 2*b^2)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) - (a^2 + 2*a*b)*cosh(x)
 + (3*(a^2 - 2*a*b)*cosh(x)^2 - a^2 - 2*a*b)*sinh(x))/(a^3*cosh(x)^4 + 4*a^
3*cosh(x)*sinh(x)^3 + a^3*sinh(x)^4 + 2*a^3*cosh(x)^2 + a^3 + 2*(3*a^3*cosh
(x)^2 + a^3)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + a^3*cosh(x))*sinh(x))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^3(x)}{a + b\coth(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**3/(a+b*coth(x)),x)
```

```
[Out] Integral(sech(x)**3/(a + b*coth(x)), x)
```

Giac [A]
time = 0.40, size = 102, normalized size = 1.23

$$\frac{(a^2-2\,b^2)\arctan\left(e^x\right)}{a^3}-\frac{2\,(a^2b-b^3)\arctan\left(\frac{ae^x+be^x}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,a^3}+\frac{ae^{(3\,x)}-2\,be^{(3\,x)}-ae^x-2\,be^x}{a^2\left(e^{(2\,x)}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] (a^2 - 2*b^2)*arctan(e^x)/a^3 - 2*(a^2*b - b^3)*arctan((a*e^x + b*e^x)/sqrt
(-a^2 + b^2))/(sqrt(-a^2 + b^2)*a^3) + (a*e^(3*x) - 2*b*e^(3*x) - a*e^x - 2
*b*e^x)/(a^2*(e^(2*x) + 1)^2)
```

Mupad [B]
time = 3.94, size = 166, normalized size = 2.00

$$\frac{e^x\,(a-2\,b)}{a^2\,\left(e^{2\,x}+1\right)}+\frac{\ln\left(e^x+1\mathrm{i}\right)\,\left(a^2\,1\mathrm{i}-b^2\,2\mathrm{i}\right)}{2\,a^3}-\frac{2\,e^x}{a\,\left(2\,e^{2\,x}+e^{4\,x}+1\right)}-\frac{\ln\left(e^x-\mathrm{i}\right)\,\left(a^2\,1\mathrm{i}-b^2\,2\mathrm{i}\right)}{2\,a^3}+\frac{b\,\ln\left(a\,e^x+b\,e^x+\sqrt{a^2-b^2}\right)\,\sqrt{(a+b)\,(a-b)}}{a^3}-\frac{b\,\ln\left(a\,e^x+b\,e^x-\sqrt{a^2-b^2}\right)\,\sqrt{(a+b)\,(a-b)}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^3*(a + b*coth(x))),x)
```

```
[Out] (log(exp(x) + 1i)*(a^2*1i - b^2*2i))/(2*a^3) - (log(exp(x) - 1i)*(a^2*1i -
b^2*2i))/(2*a^3) - (2*exp(x))/(a*(2*exp(2*x) + exp(4*x) + 1)) + (exp(x)*(a
- 2*b))/(a^2*(exp(2*x) + 1)) + (b*log(a*exp(x) + b*exp(x) + (a^2 - b^2)^(1/
2))*((a + b)*(a - b))^(1/2))/a^3 - (b*log(a*exp(x) + b*exp(x) - (a^2 - b^2)
^(1/2))*((a + b)*(a - b))^(1/2))/a^3
```

3.121 $\int \frac{\text{sech}^4(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=79

$$-\frac{b(a^2-b^2)\log(a+b\coth(x))}{a^4}-\frac{b(a^2-b^2)\log(\tanh(x))}{a^4}+\frac{(a^2-b^2)\tanh(x)}{a^3}+\frac{b\tanh^2(x)}{2a^2}-\frac{\tanh^3(x)}{3a}$$

```
[Out] -b*(a^2-b^2)*ln(a+b*coth(x))/a^4-b*(a^2-b^2)*ln(tanh(x))/a^4+(a^2-b^2)*tanh
(x)/a^3+1/2*b*tanh(x)^2/a^2-1/3*tanh(x)^3/a
```

Rubi [A]
time = 0.07, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3597, 908}

$$\frac{b\tanh^2(x)}{2a^2}-\frac{b(a^2-b^2)\log(\tanh(x))}{a^4}-\frac{b(a^2-b^2)\log(a+b\coth(x))}{a^4}+\frac{(a^2-b^2)\tanh(x)}{a^3}-\frac{\tanh^3(x)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^4/(a + b*Coth[x]),x]
```

```
[Out] -((b*(a^2 - b^2)*Log[a + b*Coth[x]])/a^4) - (b*(a^2 - b^2)*Log[Tanh[x]])/a^
4 + ((a^2 - b^2)*Tanh[x])/a^3 + (b*Tanh[x]^2)/(2*a^2) - Tanh[x]^3/(3*a)
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\text{sech}^4(x)}{a+b\coth(x)}\,dx &= -\left(b\text{Subst}\left(\int \frac{-b^2+x^2}{x^4(a+x)}\,dx,x,b\coth(x)\right)\right)\\ &= -\left(b\text{Subst}\left(\int \left(-\frac{b^2}{ax^4}+\frac{b^2}{a^2x^3}+\frac{a^2-b^2}{a^3x^2}+\frac{-a^2+b^2}{a^4x}+\frac{a^2-b^2}{a^4(a+x)}\right)dx,x,b\coth(x)\right.\\ &= -\frac{b(a^2-b^2)\log(a+b\coth(x))}{a^4}-\frac{b(a^2-b^2)\log(\tanh(x))}{a^4}+\frac{(a^2-b^2)\tanh(x)}{a^3}+\end{aligned}$$

Mathematica [A]
time = 0.21, size = 68, normalized size = 0.86

$$\frac{-6b(-a^2+b^2)\left(\log(\cosh(x))-\log(b\cosh(x)+a\sinh(x))\right)+(4a^3-6ab^2)\tanh(x)+a^2\text{sech}^2(x)(-3b+2a\tanh(x))}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^4/(a + b*Coth[x]),x]
```

```
[Out] (-6*b*(-a^2 + b^2)*(Log[Cosh[x]] - Log[b*Cosh[x] + a*Sinh[x]]) + (4*a^3 - 6
*a*b^2)*Tanh[x] + a^2*Sech[x]^2*(-3*b + 2*a*Tanh[x]))/(6*a^4)
```

Maple [A]
time = 0.83, size = 151, normalized size = 1.91

method	result
risch	$-\frac{2(3ab\,e^{4x}-3b^2e^{4x}+6a^2e^{2x}+3ab\,e^{2x}-6b^2e^{2x}+2a^2-3b^2)}{3a^3(1+e^{2x})^3}-\frac{b\ln\left(e^{2x}-\frac{a-b}{a+b}\right)}{a^2}+\frac{b^3\ln\left(e^{2x}-\frac{a-b}{a+b}\right)}{a^4}+\frac{b\ln(1+e^{2x})}{a^2}-\frac{b^3\ln(1+e^{2x}}{a^4}$
default	$-\frac{b(a^2-b^2)\ln\left(b\left(\tanh^2\left(\frac{x}{2}\right)\right)+2a\tanh\left(\frac{x}{2}\right)+b\right)}{a^4}-\frac{2\left(\frac{\left(-a^3+a\,b^2\right)\left(\tanh^5\left(\frac{x}{2}\right)\right)-a^2b\left(\tanh^4\left(\frac{x}{2}\right)\right)+\left(-\frac{2}{3}a^3+2a\,b^2\right)\left(\tanh^3\left(\frac{x}{2}\right)\right)-a^2b\left(\tanh^2\left(\frac{x}{2}\right.\right.}{\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^3}\right.}{a^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^4/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -b*(a^2-b^2)/a^4*ln(b*tanh(1/2*x)^2+2*a*tanh(1/2*x)+b)-2/a^4*(((-a^3+a*b^2)
*tanh(1/2*x)^5-a^2*b*tanh(1/2*x)^4+(-2/3*a^3+2*a*b^2)*tanh(1/2*x)^3-a^2*b*t
anh(1/2*x)^2+(-a^3+a*b^2)*tanh(1/2*x))/(tanh(1/2*x)^2+1)^3-1/2*b*(a^2-b^2)*
ln(tanh(1/2*x)^2+1))
```

Maxima [A]
time = 0.48, size = 133, normalized size = 1.68

$$\frac{2\,(2\,a^2-3\,b^2+3\,(2\,a^2-ab-2\,b^2)e^{(-2\,x)}-3\,(ab+b^2)e^{(-4\,x)})}{3\,(3\,a^3e^{(-2\,x)}+3\,a^3e^{(-4\,x)}+a^3e^{(-6\,x)}+a^3)}-\frac{(a^2b-b^3)\log\left(-(a-b)e^{(-2\,x)}+a+b\right)}{a^4}+\frac{(a^2b-b^3)\log\left(e^{(-2\,x)}+1\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] 2/3*(2*a^2 - 3*b^2 + 3*(2*a^2 - a*b - 2*b^2)*e^(-2*x) - 3*(a*b + b^2)*e^(-4
*x))/(3*a^3*e^(-2*x) + 3*a^3*e^(-4*x) + a^3*e^(-6*x) + a^3) - (a^2*b - b^3)
*log(-(a - b)*e^(-2*x) + a + b)/a^4 + (a^2*b - b^3)*log(e^(-2*x) + 1)/a^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 909 vs. 2(75) = 150.
time = 0.37, size = 909, normalized size = 11.51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] -1/3*(6*(a^2*b - a*b^2)*cosh(x)^4 + 24*(a^2*b - a*b^2)*cosh(x)*sinh(x)^3 +
6*(a^2*b - a*b^2)*sinh(x)^4 + 4*a^3 - 6*a*b^2 + 6*(2*a^3 + a^2*b - 2*a*b^2)
*cosh(x)^2 + 6*(2*a^3 + a^2*b - 2*a*b^2 + 6*(a^2*b - a*b^2)*cosh(x)^2)*sinh
(x)^2 + 3*((a^2*b - b^3)*cosh(x)^6 + 6*(a^2*b - b^3)*cosh(x)*sinh(x)^5 + (a
^2*b - b^3)*sinh(x)^6 + 3*(a^2*b - b^3)*cosh(x)^4 + 3*(a^2*b - b^3 + 5*(a^2
*b - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^2*b - b^3)*cosh(x)^3 + 3*(a^2*b -
b^3)*cosh(x))*sinh(x)^3 + a^2*b - b^3 + 3*(a^2*b - b^3)*cosh(x)^2 + 3*(5*(a
^2*b - b^3)*cosh(x)^4 + a^2*b - b^3 + 6*(a^2*b - b^3)*cosh(x)^2)*sinh(x)^2
+ 6*((a^2*b - b^3)*cosh(x)^5 + 2*(a^2*b - b^3)*cosh(x)^3 + (a^2*b - b^3)*co
sh(x))*sinh(x))*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) - 3*((a^
2*b - b^3)*cosh(x)^6 + 6*(a^2*b - b^3)*cosh(x)*sinh(x)^5 + (a^2*b - b^3)*si
nh(x)^6 + 3*(a^2*b - b^3)*cosh(x)^4 + 3*(a^2*b - b^3 + 5*(a^2*b - b^3)*cosh
(x)^2)*sinh(x)^4 + 4*(5*(a^2*b - b^3)*cosh(x)^3 + 3*(a^2*b - b^3)*cosh(x))*
sinh(x)^3 + a^2*b - b^3 + 3*(a^2*b - b^3)*cosh(x)^2 + 3*(5*(a^2*b - b^3)*co
sh(x)^4 + a^2*b - b^3 + 6*(a^2*b - b^3)*cosh(x)^2)*sinh(x)^2 + 6*((a^2*b -
b^3)*cosh(x)^5 + 2*(a^2*b - b^3)*cosh(x)^3 + (a^2*b - b^3)*cosh(x))*sinh(x)
)*log(2*cosh(x)/(cosh(x) - sinh(x))) + 12*(2*(a^2*b - a*b^2)*cosh(x)^3 + (2
*a^3 + a^2*b - 2*a*b^2)*cosh(x))*sinh(x))/(a^4*cosh(x)^6 + 6*a^4*cosh(x)*si
nh(x)^5 + a^4*sinh(x)^6 + 3*a^4*cosh(x)^4 + 3*a^4*cosh(x)^2 + 3*(5*a^4*cosh
(x)^2 + a^4)*sinh(x)^4 + a^4 + 4*(5*a^4*cosh(x)^3 + 3*a^4*cosh(x))*sinh(x)^
3 + 3*(5*a^4*cosh(x)^4 + 6*a^4*cosh(x)^2 + a^4)*sinh(x)^2 + 6*(a^4*cosh(x)^
5 + 2*a^4*cosh(x)^3 + a^4*cosh(x))*sinh(x))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^4(x)}{a + b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**4/(a+b*coth(x)),x)
```

```
[Out] Integral(sech(x)**4/(a + b*coth(x)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(75) = 150.
time = 0.42, size = 201, normalized size = 2.54

$$-\frac{(a^3b + a^2b^2 - ab^3 - b^4)\log\left(\left|ae^{(2x)} + be^{(2x)} - a + b\right|\right)}{a^5 + a^4b} + \frac{(a^2b - b^3)\log\left(e^{(2x)} + 1\right)}{a^4} - \frac{11\,a^2be^{(6x)} - 11\,b^3e^{(6x)} + 45\,a^2be^{(4x)} - 12\,ab^2e^{(4x)} - 33\,b^3e^{(4x)} + 24\,a^3e^{(2x)} + 45\,a^2be^{(2x)} - 24\,ab^2e^{(2x)} - 33\,b^3e^{(2x)} + 8\,a^3 + 11\,a^2b - 12\,ab^2 - 11\,b^3}{6\,a^4\left(e^{(2x)} + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -(a^3*b + a^2*b^2 - a*b^3 - b^4)*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a
^5 + a^4*b) + (a^2*b - b^3)*log(e^(2*x) + 1)/a^4 - 1/6*(11*a^2*b*e^(6*x) -
11*b^3*e^(6*x) + 45*a^2*b*e^(4*x) - 12*a*b^2*e^(4*x) - 33*b^3*e^(4*x) + 24*
a^3*e^(2*x) + 45*a^2*b*e^(2*x) - 24*a*b^2*e^(2*x) - 33*b^3*e^(2*x) + 8*a^3
+ 11*a^2*b - 12*a*b^2 - 11*b^3)/(a^4*(e^(2*x) + 1)^3)
```

Mupad [B]
time = 1.45, size = 123, normalized size = 1.56

$$\frac{8}{3\,a\,\left(3\,{\mathrm{e}}^{2\,x}+3\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{6\,x}+1\right)}-\frac{2\,\left(2\,a-b\right)}{a^2\,\left(2\,{\mathrm{e}}^{2\,x}+{\mathrm{e}}^{4\,x}+1\right)}-\frac{2\,b\,\left(a-b\right)}{a^3\,\left({\mathrm{e}}^{2\,x}+1\right)}-\frac{b\,\ln\left(b-a+a\,{\mathrm{e}}^{2\,x}+b\,{\mathrm{e}}^{2\,x}\right)\,\left(a+b\right)\,\left(a-b\right)}{a^4}+\frac{b\,\ln\left({\mathrm{e}}^{2\,x}+1\right)\,\left(a+b\right)\,\left(a-b\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^4*(a + b*coth(x))),x)
```

```
[Out] 8/(3*a*(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1)) - (2*(2*a - b))/(a^2*(2*ex
p(2*x) + exp(4*x) + 1)) - (2*b*(a - b))/(a^3*(exp(2*x) + 1)) - (b*log(b - a
 + a*exp(2*x) + b*exp(2*x))*(a + b)*(a - b))/a^4 + (b*log(exp(2*x) + 1)*(a
+ b)*(a - b))/a^4
```

3.122 $\int \frac{\mathrm{sech}(x)}{i+2\coth(x)}\,dx$

Optimal. Leaf size=31

$$-i\mathrm{ArcTan}(\sinh(x)) - \frac{2\tanh^{-1}\left(\frac{\cosh(x)-2i\sinh(x)}{\sqrt{5}}\right)}{\sqrt{5}}$$

```
[Out] -I*arctan(sinh(x))-2/5*arctanh(1/5*(cosh(x)-2*I*sinh(x))*5^(1/2))*5^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3599, 3189, 3855, 3153, 212}

$$-i\mathrm{ArcTan}(\sinh(x)) - \frac{2\tanh^{-1}\left(\frac{\cosh(x)-2i\sinh(x)}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]/(I + 2*Coth[x]),x]
```

```
[Out] (-I)*ArcTan[Sinh[x]] - (2*ArcTanh[(Cosh[x] - (2*I)*Sinh[x])/Sqrt[5]])/Sqrt[
5]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3189

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[cos[c + d*x]^m*(sin[c + d*x]^n/(a*cos[c + d*x] + b*sin[c + d*x])),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}(x)}{i+2\coth(x)}\,dx &= -\left(i\int \frac{\tanh(x)}{-2i\cosh(x)+\sinh(x)}\,dx\right)\\ &= -\int \left(i\operatorname{sech}(x)-\frac{2i}{2\cosh(x)+i\sinh(x)}\right)\,dx\\ &= -(i\int \operatorname{sech}(x)\,dx)+2i\int \frac{1}{2\cosh(x)+i\sinh(x)}\,dx\\ &= -i\tan^{-1}(\sinh(x))-2\text{Subst}\left(\int \frac{1}{5-x^2}\,dx,x,\cosh(x)-2i\sinh(x)\right)\\ &= -i\tan^{-1}(\sinh(x))-\frac{2\tanh^{-1}\left(\frac{\cosh(x)-2i\sinh(x)}{\sqrt{5}}\right)}{\sqrt{5}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 38, normalized size = 1.23

$$-2i\text{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{4\tanh^{-1}\left(\frac{1-2i\tanh\left(\frac{x}{2}\right)}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(I + 2*Coth[x]),x]
```

```
[Out] (-2*I)*ArcTan[Tanh[x/2]] - (4*ArcTanh[(1 - (2*I)*Tanh[x/2])/Sqrt[5]])/Sqrt[
5]
```

Maple [A]
time = 1.28, size = 41, normalized size = 1.32

method	result	size

default	$-\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)+\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)+\frac{4i\sqrt{5}\,\arctan\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)+i\right)\sqrt{5}}{5}\right)}{5}$	41
risch	$-\ln\left(e^{x}-i\right)+\ln\left(e^{x}+i\right)+\frac{2\sqrt{5}\,\ln\left(e^{x}-\frac{2i\sqrt{5}}{5}-\frac{\sqrt{5}}{5}\right)}{5}-\frac{2\sqrt{5}\,\ln\left(e^{x}+\frac{2i\sqrt{5}}{5}+\frac{\sqrt{5}}{5}\right)}{5}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(I+2*coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(tanh(1/2*x)-I)+ln(tanh(1/2*x)+I)+4/5*I*5^(1/2)*arctan(1/5*(2*tanh(1/2*x
)+I)*5^(1/2))
```

Maxima [A]
time = 0.50, size = 38, normalized size = 1.23

$$\frac{2}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-(2i+1)\,e^{(-x)}}{\sqrt{5}+(2i+1)\,e^{(-x)}}\right)+2i\arctan\left(e^{(-x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+2*coth(x)),x, algorithm="maxima")
```

```
[Out] 2/5*sqrt(5)*log(-(sqrt(5) - (2*I + 1)*e^(-x))/(sqrt(5) + (2*I + 1)*e^(-x)))
 + 2*I*arctan(e^(-x))
```

Fricas [A]
time = 0.35, size = 41, normalized size = 1.32

$$-\frac{2}{5}\sqrt{5}\log\left(\left(\frac{2}{5}i+\frac{1}{5}\right)\sqrt{5}+e^{x}\right)+\frac{2}{5}\sqrt{5}\log\left(-\left(\frac{2}{5}i+\frac{1}{5}\right)\sqrt{5}+e^{x}\right)+\log\left(e^{x}+i\right)-\log\left(e^{x}-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+2*coth(x)),x, algorithm="fricas")
```

```
[Out] -2/5*sqrt(5)*log((2/5*I + 1/5)*sqrt(5) + e^x) + 2/5*sqrt(5)*log(-(2/5*I + 1
/5)*sqrt(5) + e^x) + log(e^x + I) - log(e^x - I)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{sech}(x)}{2\coth(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+2*coth(x)),x)
```

```
[Out] Integral(sech(x)/(2*coth(x) + I), x)
```

Giac [A]
time = 0.41, size = 26, normalized size = 0.84

$$\frac{4}{5}i\sqrt{5}\arctan\left(\left(\frac{1}{5}i+\frac{2}{5}\right)\sqrt{5}e^{x}\right)+\log\left(e^{x}+i\right)-\log\left(e^{x}-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+2*coth(x)),x, algorithm="giac")
```

```
[Out] 4/5*I*sqrt(5)*arctan((1/5*I + 2/5)*sqrt(5)*e^x) + log(e^x + I) - log(e^x -
I)
```

Mupad [B]
time = 0.51, size = 65, normalized size = 2.10

$$\ln\left(\mathrm{e}^{x}\left(32+64\mathrm{i}\right)-64+32\mathrm{i}\right)-\ln\left(\mathrm{e}^{x}\left(32+64\mathrm{i}\right)+64-32\mathrm{i}\right)-\frac{2\sqrt{5}\,\ln\left(\mathrm{e}^{x}\left(-\frac{256}{5}+\frac{192}{5}\mathrm{i}\right)+\sqrt{5}\left(-\frac{128}{5}-\frac{64}{5}\mathrm{i}\right)\right)}{5}+\frac{2\sqrt{5}\,\ln\left(\mathrm{e}^{x}\left(-\frac{256}{5}+\frac{192}{5}\mathrm{i}\right)+\sqrt{5}\left(\frac{128}{5}+\frac{64}{5}\mathrm{i}\right)\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)*(2*coth(x) + 1i)),x)
```

```
[Out] log(exp(x)*(32 + 64i) - (64 - 32i)) - log(exp(x)*(32 + 64i) + (64 - 32i)) -
 (2*5^(1/2)*log(- exp(x)*(256/5 - 192i/5) - 5^(1/2)*(128/5 + 64i/5)))/5 + (
2*5^(1/2)*log(5^(1/2)*(128/5 + 64i/5) - exp(x)*(256/5 - 192i/5)))/5
```

3.123 $\int \frac{\tanh^4(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=43

$$\frac{5x}{2}-2\log(\cosh(x))-\frac{5\tanh(x)}{2}+\tanh^2(x)-\frac{5\tanh^3(x)}{6}+\frac{\tanh^3(x)}{2(1+\coth(x))}$$

```
[Out] 5/2*x-2*ln(cosh(x))-5/2*tanh(x)+tanh(x)^2-5/6*tanh(x)^3+1/2*tanh(x)^3/(1+co
th(x))
```

Rubi [A]
time = 0.08, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3633, 3610, 3612, 3556}

$$\frac{5x}{2}-\frac{5\tanh^3(x)}{6}+\tanh^2(x)-\frac{5\tanh(x)}{2}-2\log(\cosh(x))+\frac{\tanh^3(x)}{2(\coth(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/(1 + Coth[x]),x]
```

```
[Out] (5*x)/2 - 2*Log[Cosh[x]] - (5*Tanh[x])/2 + Tanh[x]^2 - (5*Tanh[x]^3)/6 + Ta
nh[x]^3/(2*(1 + Coth[x]))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3633

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-a)*((c + d*Tan[e + f*x])^(n + 1)/(2*f*(b*
c - a*d)*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a*(b*c - a*d)), Int[(c + d*
Tan[e + f*x])^n*Simp[b*c + a*d*(n - 1) - b*d*n*Tan[e + f*x], x], x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &
& NeQ[c^2 + d^2, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^4(x)}{1+\coth(x)}\,dx &= \frac{\tanh^3(x)}{2(1+\coth(x))}-\frac{1}{2}\int(-5+4\coth(x))\tanh^4(x)\,dx\\
&= -\frac{5}{6}\tanh^3(x)+\frac{\tanh^3(x)}{2(1+\coth(x))}-\frac{1}{2}i\int(-4i+5i\coth(x))\tanh^3(x)\,dx\\
&= \tanh^2(x)-\frac{5\tanh^3(x)}{6}+\frac{\tanh^3(x)}{2(1+\coth(x))}+\frac{1}{2}\int(5-4\coth(x))\tanh^2(x)\,dx\\
&= -\frac{5\tanh(x)}{2}+\tanh^2(x)-\frac{5\tanh^3(x)}{6}+\frac{\tanh^3(x)}{2(1+\coth(x))}+\frac{1}{2}i\int(4i-5i\coth(x))\tanh(x
\\
&= \frac{5x}{2}-\frac{5\tanh(x)}{2}+\tanh^2(x)-\frac{5\tanh^3(x)}{6}+\frac{\tanh^3(x)}{2(1+\coth(x))}-2\int\tanh(x)\,dx\\
&= \frac{5x}{2}-2\log(\cosh(x))-\frac{5\tanh(x)}{2}+\tanh^2(x)-\frac{5\tanh^3(x)}{6}+\frac{\tanh^3(x)}{2(1+\coth(x))}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 40, normalized size = 0.93

$$\frac{1}{12}(30x+3\cosh(2x)-24\log(\cosh(x))-3\sinh(2x)-28\tanh(x)+\operatorname{sech}^2(x)(-6+4\tanh(x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(1 + Coth[x]),x]
```

```
[Out] (30*x + 3*Cosh[2*x] - 24*Log[Cosh[x]] - 3*Sinh[2*x] - 28*Tanh[x] + Sech[x]^
2*(-6 + 4*Tanh[x]))/12
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(35) = 70.
time = 0.57, size = 96, normalized size = 2.23

method	result
risch	$\frac{9x}{2}+\frac{e^{-2x}}{4}+\frac{4\,e^{4x}+6\,e^{2x}+\frac{14}{3}}{(1+e^{2x})^3}-2\ln\left(1+e^{2x}\right)$

default	$\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}-\frac{1}{\tanh\left(\frac{x}{2}\right)+1}+\frac{9\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2}-\frac{4\left(\tanh^{5}\left(\frac{x}{2}\right)-\frac{\left(\tanh^{4}\left(\frac{x}{2}\right)\right)}{2}+\frac{8\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{3}-\frac{\left(\tanh^{2}\left(\frac{x}{2}\right)\right)}{2}+\tanh\left(\frac{x}{2}\right)\right)}{\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)^{3}}-2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(1+coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(tanh(1/2*x)+1)^2-1/(tanh(1/2*x)+1)+9/2*ln(tanh(1/2*x)+1)-4*(tanh(1/2*x)^
5-1/2*tanh(1/2*x)^4+8/3*tanh(1/2*x)^3-1/2*tanh(1/2*x)^2+tanh(1/2*x))/(tanh(
1/2*x)^2+1)^3-2*ln(tanh(1/2*x)^2+1)-1/2*ln(tanh(1/2*x)-1)
```

Maxima [A]
time = 0.48, size = 55, normalized size = 1.28

$$\frac{1}{2}\,x-\frac{2\,\left(15\,e^{(-2\,x)}+12\,e^{(-4\,x)}+7\right)}{3\,\left(3\,e^{(-2\,x)}+3\,e^{(-4\,x)}+e^{(-6\,x)}+1\right)}+\frac{1}{4}\,e^{(-2\,x)}-2\,\log\left(e^{(-2\,x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x - 2/3*(15*e^(-2*x) + 12*e^(-4*x) + 7)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-
6*x) + 1) + 1/4*e^(-2*x) - 2*log(e^(-2*x) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 571 vs. $2(35) = 70$.
time = 0.36, size = 571, normalized size = 13.28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] 1/12*(54*x*cosh(x)^8 + 432*x*cosh(x)*sinh(x)^7 + 54*x*sinh(x)^8 + 3*(54*x +
 17)*cosh(x)^6 + 3*(504*x*cosh(x)^2 + 54*x + 17)*sinh(x)^6 + 18*(168*x*cosh
(x)^3 + (54*x + 17)*cosh(x))*sinh(x)^5 + 81*(2*x + 1)*cosh(x)^4 + 9*(420*x*
cosh(x)^4 + 5*(54*x + 17)*cosh(x)^2 + 18*x + 9)*sinh(x)^4 + 12*(252*x*cosh(
x)^5 + 5*(54*x + 17)*cosh(x)^3 + 27*(2*x + 1)*cosh(x))*sinh(x)^3 + (54*x +
65)*cosh(x)^2 + (1512*x*cosh(x)^6 + 45*(54*x + 17)*cosh(x)^4 + 486*(2*x + 1
)*cosh(x)^2 + 54*x + 65)*sinh(x)^2 - 24*(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 +
sinh(x)^8 + (28*cosh(x)^2 + 3)*sinh(x)^6 + 3*cosh(x)^6 + 2*(28*cosh(x)^3 +
9*cosh(x))*sinh(x)^5 + (70*cosh(x)^4 + 45*cosh(x)^2 + 3)*sinh(x)^4 + 3*cosh
(x)^4 + 4*(14*cosh(x)^5 + 15*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + (28*cosh(x)
^6 + 45*cosh(x)^4 + 18*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(4*cosh(x)^
7 + 9*cosh(x)^5 + 6*cosh(x)^3 + cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) -
sinh(x))) + 2*(216*x*cosh(x)^7 + 9*(54*x + 17)*cosh(x)^5 + 162*(2*x + 1)*co
```

```
sh(x)^3 + (54*x + 65)*cosh(x))*sinh(x) + 3)/(cosh(x)^8 + 8*cosh(x)*sinh(x)^
7 + sinh(x)^8 + (28*cosh(x)^2 + 3)*sinh(x)^6 + 3*cosh(x)^6 + 2*(28*cosh(x)^
3 + 9*cosh(x))*sinh(x)^5 + (70*cosh(x)^4 + 45*cosh(x)^2 + 3)*sinh(x)^4 + 3*
cosh(x)^4 + 4*(14*cosh(x)^5 + 15*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + (28*cos
h(x)^6 + 45*cosh(x)^4 + 18*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(4*cosh
(x)^7 + 9*cosh(x)^5 + 6*cosh(x)^3 + cosh(x))*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(x)}{\coth(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(1+coth(x)),x)
```

```
[Out] Integral(tanh(x)**4/(coth(x) + 1), x)
```

Giac [A]
time = 0.41, size = 47, normalized size = 1.09

$$\frac{9}{2}\,x + \frac{\left(51\,e^{(6\,x)} + 81\,e^{(4\,x)} + 65\,e^{(2\,x)} + 3\right)e^{(-2\,x)}}{12\,\left(e^{(2\,x)} + 1\right)^3} - 2\,\log\left(e^{(2\,x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(1+coth(x)),x, algorithm="giac")
```

```
[Out] 9/2*x + 1/12*(51*e^(6*x) + 81*e^(4*x) + 65*e^(2*x) + 3)*e^(-2*x)/(e^(2*x) +
 1)^3 - 2*log(e^(2*x) + 1)
```

Mupad [B]
time = 1.30, size = 69, normalized size = 1.60

$$\frac{9\,x}{2} - 2\,\ln\left(\mathrm{e}^{2\,x}+1\right) + \frac{\mathrm{e}^{-2\,x}}{4} + \frac{8}{3\,\left(3\,\mathrm{e}^{2\,x}+3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}+1\right)} - \frac{2}{2\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1} + \frac{4}{\mathrm{e}^{2\,x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(coth(x) + 1),x)
```

```
[Out] (9*x)/2 - 2*log(exp(2*x) + 1) + exp(-2*x)/4 + 8/(3*(3*exp(2*x) + 3*exp(4*x)
 + exp(6*x) + 1)) - 2/(2*exp(2*x) + exp(4*x) + 1) + 4/(exp(2*x) + 1)
```

3.124 $\int \frac{\tanh^3(x)}{1+\coth(x)} dx$

Optimal. Leaf size=37

$$-\frac{3x}{2} + 2\log(\cosh(x)) + \frac{3\tanh(x)}{2} - \tanh^2(x) + \frac{\tanh^2(x)}{2(1+\coth(x))}$$

```
[Out] -3/2*x+2*ln(cosh(x))+3/2*tanh(x)-tanh(x)^2+1/2*tanh(x)^2/(1+coth(x))
```

Rubi [A]
time = 0.07, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3633, 3610, 3612, 3556}

$$-\frac{3x}{2} - \tanh^2(x) + \frac{3\tanh(x)}{2} + 2\log(\cosh(x)) + \frac{\tanh^2(x)}{2(\coth(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/(1 + Coth[x]),x]
```

```
[Out] (-3*x)/2 + 2*Log[Cosh[x]] + (3*Tanh[x])/2 - Tanh[x]^2 + Tanh[x]^2/(2*(1 + C
oth[x]))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3633

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-a)*((c + d*Tan[e + f*x])^(n + 1)/(2*f*(b*
c - a*d)*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a*(b*c - a*d)), Int[(c + d*
Tan[e + f*x])^n*Simp[b*c + a*d*(n - 1) - b*d*n*Tan[e + f*x], x], x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &
& NeQ[c^2 + d^2, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^3(x)}{1+\coth(x)}\,dx &= \frac{\tanh^2(x)}{2(1+\coth(x))}-\frac{1}{2}\int(-4+3\coth(x))\tanh^3(x)\,dx\\
&= -\tanh^2(x)+\frac{\tanh^2(x)}{2(1+\coth(x))}-\frac{1}{2}i\int(-3i+4i\coth(x))\tanh^2(x)\,dx\\
&= \frac{3\tanh(x)}{2}-\tanh^2(x)+\frac{\tanh^2(x)}{2(1+\coth(x))}+\frac{1}{2}\int(4-3\coth(x))\tanh(x)\,dx\\
&= -\frac{3x}{2}+\frac{3\tanh(x)}{2}-\tanh^2(x)+\frac{\tanh^2(x)}{2(1+\coth(x))}+2\int\tanh(x)\,dx\\
&= -\frac{3x}{2}+2\log(\cosh(x))+\frac{3\tanh(x)}{2}-\tanh^2(x)+\frac{\tanh^2(x)}{2(1+\coth(x))}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 33, normalized size = 0.89

$$\frac{1}{4}\left(-6x-\cosh(2x)+8\log(\cosh(x))+2\mathrm{sech}^2(x)+\sinh(2x)+4\tanh(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(1 + Coth[x]),x]
[Out] (-6*x - Cosh[2*x] + 8*Log[Cosh[x]] + 2*Sech[x]^2 + Sinh[2*x] + 4*Tanh[x])/4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(31) = 62.
time = 0.58, size = 80, normalized size = 2.16

method	result
risch	$-\frac{7x}{2}-\frac{e^{-2x}}{4}-\frac{2}{(1+e^{2x})^2}+2\ln\left(1+e^{2x}\right)$
default	$-\frac{\ln(\tanh(\frac{x}{2})-1)}{2}+\frac{2(\tanh^3(\frac{x}{2}))-2(\tanh^2(\frac{x}{2}))+2\tanh(\frac{x}{2})}{(\tanh^2(\frac{x}{2})+1)^2}+2\ln\left(\tanh^2\left(\frac{x}{2}\right)+1\right)-\frac{1}{(\tanh(\frac{x}{2})+1)^2}+\frac{1}{\tanh(\frac{x}{2})+1}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(1+coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(tanh(1/2*x)-1)+2*(tanh(1/2*x)^3-tanh(1/2*x)^2+tanh(1/2*x))/(tanh(1/
2*x)^2+1)^2+2*ln(tanh(1/2*x)^2+1)-1/(tanh(1/2*x)+1)^2+1/(tanh(1/2*x)+1)-7/2
*ln(tanh(1/2*x)+1)
```

Maxima [A]
time = 0.49, size = 43, normalized size = 1.16

$$\frac{1}{2}\,x + \frac{2\left(2\,e^{(-2\,x)} + 1\right)}{2\,e^{(-2\,x)} + e^{(-4\,x)} + 1} - \frac{1}{4}\,e^{(-2\,x)} + 2\,\log\left(e^{(-2\,x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x + 2*(2*e^(-2*x) + 1)/(2*e^(-2*x) + e^(-4*x) + 1) - 1/4*e^(-2*x) + 2*l
og(e^(-2*x) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(31) = 62$.
time = 0.36, size = 354, normalized size = 9.57

$$-\frac{14\,x\cosh\left(x\right)^{6} + 84\,x\cosh\left(x\right)\sinh\left(x\right)^{5} + 14\,x\sinh\left(x\right)^{6} + \left(28\,x + 1\right)\cosh\left(x\right)^{4} + \left(210\,x\cosh\left(x\right)^{2} + 28\,x + 1\right)\sinh\left(x\right)^{4} + 4\,\left(70\,x\cosh\left(x\right)^{3} + \left(28\,x + 1\right)\cosh\left(x\right)\right)\sinh\left(x\right)^{3} + 2\,\left(7\,x + 5\right)\cosh\left(x\right)^{2} + 2\,\left(105\,x\cosh\left(x\right)^{4} + 3\,\left(28\,x + 1\right)\cosh\left(x\right)^{2} + 7\,x + 5\right)\sinh\left(x\right)^{2} - 8\,\left(\cosh\left(x\right)^{6} + 6\,\cosh\left(x\right)\sinh\left(x\right)^{5} + \sinh\left(x\right)^{6} + \left(15\,\cosh\left(x\right)^{2} + 2\right)\sinh\left(x\right)^{4} + 2\,\cosh\left(x\right)^{4} + 4\,\left(5\,\cosh\left(x\right)^{3} + 2\,\cosh\left(x\right)\right)\sinh\left(x\right)^{3} + \left(15\,\cosh\left(x\right)^{4} + 12\,\cosh\left(x\right)^{2} + 1\right)\sinh\left(x\right)^{2} + \cosh\left(x\right)^{2} + 2\,\left(3\,\cosh\left(x\right)^{5} + 4\,\cosh\left(x\right)^{3} + \cosh\left(x\right)\right)\sinh\left(x\right)\right)\log\left(\frac{2\,\cosh\left(x\right)}{\cosh\left(x\right) - \sinh\left(x\right)}\right) + 4\,\left(21\,x\cosh\left(x\right)^{5} + \left(28\,x + 1\right)\cosh\left(x\right)^{3} + \left(7\,x + 5\right)\cosh\left(x\right)\right)\sinh\left(x\right) + 1}{4\,\left(\cosh\left(x\right)^{6} + 6\,\cosh\left(x\right)\sinh\left(x\right)^{5} + \sinh\left(x\right)^{6} + \left(15\,\cosh\left(x\right)^{2} + 2\right)\sinh\left(x\right)^{4} + 2\,\cosh\left(x\right)^{4} + 4\,\left(5\,\cosh\left(x\right)^{3} + 2\,\cosh\left(x\right)\right)\sinh\left(x\right)^{3} + \left(15\,\cosh\left(x\right)^{4} + 12\,\cosh\left(x\right)^{2} + 1\right)\sinh\left(x\right)^{2} + \cosh\left(x\right)^{2} + 2\,\left(3\,\cosh\left(x\right)^{5} + 4\,\cosh\left(x\right)^{3} + \cosh\left(x\right)\right)\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] -1/4*(14*x*cosh(x)^6 + 84*x*cosh(x)*sinh(x)^5 + 14*x*sinh(x)^6 + (28*x + 1)
*cosh(x)^4 + (210*x*cosh(x)^2 + 28*x + 1)*sinh(x)^4 + 4*(70*x*cosh(x)^3 + (
28*x + 1)*cosh(x))*sinh(x)^3 + 2*(7*x + 5)*cosh(x)^2 + 2*(105*x*cosh(x)^4 +
 3*(28*x + 1)*cosh(x)^2 + 7*x + 5)*sinh(x)^2 - 8*(cosh(x)^6 + 6*cosh(x)*sin
h(x)^5 + sinh(x)^6 + (15*cosh(x)^2 + 2)*sinh(x)^4 + 2*cosh(x)^4 + 4*(5*cosh
(x)^3 + 2*cosh(x))*sinh(x)^3 + (15*cosh(x)^4 + 12*cosh(x)^2 + 1)*sinh(x)^2
+ cosh(x)^2 + 2*(3*cosh(x)^5 + 4*cosh(x)^3 + cosh(x))*sinh(x))*log(2*cosh(x
)/(cosh(x) - sinh(x))) + 4*(21*x*cosh(x)^5 + (28*x + 1)*cosh(x)^3 + (7*x +
5)*cosh(x))*sinh(x) + 1)/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + (15
*cosh(x)^2 + 2)*sinh(x)^4 + 2*cosh(x)^4 + 4*(5*cosh(x)^3 + 2*cosh(x))*sinh(
x)^3 + (15*cosh(x)^4 + 12*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(3*cosh(
x)^5 + 4*cosh(x)^3 + cosh(x))*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{3}\left(x\right)}{\coth\left(x\right) + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(1+coth(x)),x)
```

```
[Out] Integral(tanh(x)**3/(coth(x) + 1), x)
```

Giac [A]
time = 0.41, size = 39, normalized size = 1.05

$$-\frac{7}{2}\,x-\frac{\left(e^{(4\,x)}+10\,e^{(2\,x)}+1\right)e^{(-2\,x)}}{4\left(e^{(2\,x)}+1\right)^{2}}+2\,\log\left(e^{(2\,x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(1+coth(x)),x, algorithm="giac")
```

```
[Out] -7/2*x - 1/4*(e^(4*x) + 10*e^(2*x) + 1)*e^(-2*x)/(e^(2*x) + 1)^2 + 2*log(e^
(2*x) + 1)
```

Mupad [B]
time = 1.21, size = 35, normalized size = 0.95

$$2\,\ln\left(\mathrm{e}^{2\,x}+1\right)-\frac{7\,x}{2}-\frac{\mathrm{e}^{-2\,x}}{4}-\frac{2}{2\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(coth(x) + 1),x)
```

```
[Out] 2*log(exp(2*x) + 1) - (7*x)/2 - exp(-2*x)/4 - 2/(2*exp(2*x) + exp(4*x) + 1)
```

3.125 $\int \frac{\tanh^2(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=29

$$\frac{3x}{2}-\log(\cosh(x))-\frac{3\tanh(x)}{2}+\frac{\tanh(x)}{2(1+\coth(x))}$$

```
[Out] 3/2*x-ln(cosh(x))-3/2*tanh(x)+1/2*tanh(x)/(1+coth(x))
```

Rubi [A]
time = 0.06, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3633, 3610, 3612, 3556}

$$\frac{3x}{2}-\frac{3\tanh(x)}{2}-\log(\cosh(x))+\frac{\tanh(x)}{2(\coth(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(1 + Coth[x]),x]
```

```
[Out] (3*x)/2 - Log[Cosh[x]] - (3*Tanh[x])/2 + Tanh[x]/(2*(1 + Coth[x]))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3633

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-a)*((c + d*Tan[e + f*x])^(n + 1)/(2*f*(b*
```

```
c - a*d)*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a*(b*c - a*d)), Int[(c + d*
Tan[e + f*x])^n*Simp[b*c + a*d*(n - 1) - b*d*n*Tan[e + f*x], x], x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &
& NeQ[c^2 + d^2, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^2(x)}{1+\coth(x)}\,dx &= \frac{\tanh(x)}{2(1+\coth(x))}-\frac{1}{2}\int(-3+2\coth(x))\tanh^2(x)\,dx\\
&= -\frac{3\tanh(x)}{2}+\frac{\tanh(x)}{2(1+\coth(x))}-\frac{1}{2}i\int(-2i+3i\coth(x))\tanh(x)\,dx\\
&= \frac{3x}{2}-\frac{3\tanh(x)}{2}+\frac{\tanh(x)}{2(1+\coth(x))}-\int\tanh(x)\,dx\\
&= \frac{3x}{2}-\log(\cosh(x))-\frac{3\tanh(x)}{2}+\frac{\tanh(x)}{2(1+\coth(x))}
\end{aligned}$$

Mathematica [**A**]
time = 0.03, size = 27, normalized size = 0.93

$$\frac{1}{4}(6x+\cosh(2x)-4\log(\cosh(x))-\sinh(2x)-4\tanh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(1 + Coth[x]),x]
```

```
[Out] (6*x + Cosh[2*x] - 4*Log[Cosh[x]] - Sinh[2*x] - 4*Tanh[x])/4
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(23) = 46$.
time = 0.49, size = 65, normalized size = 2.24

method	result	size
risch	$\frac{5x}{2}+\frac{e^{-2x}}{4}+\frac{2}{1+e^{2x}}-\ln\left(1+e^{2x}\right)$	30
default	$-\frac{2\tanh\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)+1}-\ln\left(\tanh^2\left(\frac{x}{2}\right)+1\right)+\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}-\frac{1}{\tanh\left(\frac{x}{2}\right)+1}+\frac{5\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(1+coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*tanh(1/2*x)/(tanh(1/2*x)^2+1)-ln(tanh(1/2*x)^2+1)+1/(tanh(1/2*x)+1)^2-1/
(tanh(1/2*x)+1)+5/2*ln(tanh(1/2*x)+1)-1/2*ln(tanh(1/2*x)-1)
```

Maxima [A]
time = 0.48, size = 29, normalized size = 1.00

$$\frac{1}{2}\,x - \frac{2}{e^{(-2\,x)} + 1} + \frac{1}{4}\,e^{(-2\,x)} - \log\left(e^{(-2\,x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x - 2/(e^(-2*x) + 1) + 1/4*e^(-2*x) - log(e^(-2*x) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(23) = 46.
time = 0.35, size = 186, normalized size = 6.41

$$\frac{10\,x\cosh\left(x\right)^{4} + 40\,x\cosh\left(x\right)\sinh\left(x\right)^{3} + 10\,x\sinh\left(x\right)^{4} + \left(10\,x + 9\right)\cosh\left(x\right)^{2} + \left(60\,x\cosh\left(x\right)^{2} + 10\,x + 9\right)\sinh\left(x\right)^{2} - 4\,\left(\cosh\left(x\right)^{4} + 4\,\cosh\left(x\right)\sinh\left(x\right)^{3} + \sinh\left(x\right)^{4} + \left(6\,\cosh\left(x\right)^{2} + 1\right)\sinh\left(x\right)^{2} + \cosh\left(x\right)^{2} + 2\,\left(2\,\cosh\left(x\right)^{3} + \cosh\left(x\right)\right)\sinh\left(x\right)\right)\log\left(\frac{2\,\cosh\left(x\right)}{\cosh\left(x\right) - \sinh\left(x\right)}\right) + 2\,\left(20\,x\cosh\left(x\right)^{3} + \left(10\,x + 9\right)\cosh\left(x\right)\right)\sinh\left(x\right) + 1}{4\,\left(\cosh\left(x\right)^{4} + 4\,\cosh\left(x\right)\sinh\left(x\right)^{3} + \sinh\left(x\right)^{4} + \left(6\,\cosh\left(x\right)^{2} + 1\right)\sinh\left(x\right)^{2} + \cosh\left(x\right)^{2} + 2\,\left(2\,\cosh\left(x\right)^{3} + \cosh\left(x\right)\right)\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] 1/4*(10*x*cosh(x)^4 + 40*x*cosh(x)*sinh(x)^3 + 10*x*sinh(x)^4 + (10*x + 9)*
cosh(x)^2 + (60*x*cosh(x)^2 + 10*x + 9)*sinh(x)^2 - 4*(cosh(x)^4 + 4*cosh(x
)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(2*co
sh(x)^3 + cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) + 2*(20*x*co
sh(x)^3 + (10*x + 9)*cosh(x))*sinh(x) + 1)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3
 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(2*cosh(x)^3 + c
osh(x))*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{2}\left(x\right)}{\coth\left(x\right) + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(1+coth(x)),x)
```

```
[Out] Integral(tanh(x)**2/(coth(x) + 1), x)
```

Giac [A]
time = 0.40, size = 35, normalized size = 1.21

$$\frac{5}{2}\,x + \frac{\left(9\,e^{(2\,x)} + 1\right)e^{(-2\,x)}}{4\,\left(e^{(2\,x)} + 1\right)} - \log\left(e^{(2\,x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(1+coth(x)),x, algorithm="giac")
[Out] 5/2*x + 1/4*(9*e^(2*x) + 1)*e^(-2*x)/(e^(2*x) + 1) - log(e^(2*x) + 1)
```

Mupad [B]
time = 1.22, size = 29, normalized size = 1.00

$$\frac{5\,x}{2}-\ln\left(\mathrm{e}^{2\,x}+1\right)+\frac{\mathrm{e}^{-2\,x}}{4}+\frac{2}{\mathrm{e}^{2\,x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(coth(x) + 1),x)
[Out] (5*x)/2 - log(exp(2*x) + 1) + exp(-2*x)/4 + 2/(exp(2*x) + 1)
```

3.126 $\int \frac{\tanh(x)}{1+\coth(x)} dx$

Optimal. Leaf size=19

$$-\frac{x}{2}+\frac{1}{2(1+\coth(x))}+\log(\cosh(x))$$

```
[Out] -1/2*x+1/2/(1+coth(x))+ln(cosh(x))
```

Rubi [A]
time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3632, 3560, 8, 3556}

$$-\frac{x}{2}+\frac{1}{2(\coth(x)+1)}+\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(1 + Coth[x]),x]
```

```
[Out] -1/2*x + 1/(2*(1 + Coth[x])) + Log[Cosh[x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3632

```
Int[1/(((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Tan[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Tan[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh(x)}{1+\coth(x)}\,dx &= -\int \frac{1}{1+\coth(x)}\,dx + \int \tanh(x)\,dx\\ &= \frac{1}{2(1+\coth(x))} + \log(\cosh(x)) - \frac{\int 1\,dx}{2}\\ &= -\frac{x}{2} + \frac{1}{2(1+\coth(x))} + \log(\cosh(x))\end{aligned}$$

Mathematica [A]
time = 0.02, size = 23, normalized size = 1.21

$$\frac{1}{4}(-2x - \cosh(2x) + 4\log(\cosh(x)) + \sinh(2x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(1 + Coth[x]),x]
```

```
[Out] (-2*x - Cosh[2*x] + 4*Log[Cosh[x]] + Sinh[2*x])/4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(15) = 30$.
time = 0.51, size = 47, normalized size = 2.47

method	result	size
risch	$-\frac{3x}{2} - \frac{e^{-2x}}{4} + \ln(1 + e^{2x})$	18
default	$-\frac{1}{(\tanh(\frac{x}{2})+1)^2} + \frac{1}{\tanh(\frac{x}{2})+1} - \frac{3\ln(\tanh(\frac{x}{2})+1)}{2} - \frac{\ln(\tanh(\frac{x}{2})-1)}{2} + \ln\left(\tanh^2\left(\frac{x}{2}\right) + 1\right)$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(1+coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(tanh(1/2*x)+1)^2+1/(tanh(1/2*x)+1)-3/2*ln(tanh(1/2*x)+1)-1/2*ln(tanh(1/
2*x)-1)+ln(tanh(1/2*x)^2+1)
```

Maxima [A]
time = 0.50, size = 17, normalized size = 0.89

$$\frac{1}{2}\,x - \frac{1}{4}\,e^{(-2\,x)} + \log\left(e^{(-2\,x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x - 1/4*e^(-2*x) + log(e^(-2*x) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(15) = 30$.
time = 0.36, size = 73, normalized size = 3.84

$$-\frac{6\,x\cosh\left(x\right)^{2}+12\,x\cosh\left(x\right)\sinh\left(x\right)+6\,x\sinh\left(x\right)^{2}-4\,\left(\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}\right)\log\left(\frac{2\,\cosh\left(x\right)}{\cosh\left(x\right)-\sinh\left(x\right)}\right)+1}{4\,\left(\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] -1/4*(6*x*cosh(x)^2 + 12*x*cosh(x)*sinh(x) + 6*x*sinh(x)^2 - 4*(cosh(x)^2 +
 2*cosh(x)*sinh(x) + sinh(x)^2)*log(2*cosh(x)/(cosh(x) - sinh(x))) + 1)/(co
sh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh\left(x\right)}{\coth\left(x\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(1+coth(x)),x)
```

```
[Out] Integral(tanh(x)/(coth(x) + 1), x)
```

Giac [A]
time = 0.42, size = 17, normalized size = 0.89

$$-\frac{3}{2}\,x-\frac{1}{4}\,e^{\left(-2\,x\right)}+\log\left(e^{\left(2\,x\right)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(1+coth(x)),x, algorithm="giac")
```

```
[Out] -3/2*x - 1/4*e^(-2*x) + log(e^(2*x) + 1)
```

Mupad [B]
time = 1.17, size = 17, normalized size = 0.89

$$\ln\left({\mathrm{e}}^{2\,x}+1\right)-\frac{3\,x}{2}-\frac{{\mathrm{e}}^{-2\,x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(coth(x) + 1),x)
```

```
[Out] log(exp(2*x) + 1) - (3*x)/2 - exp(-2*x)/4
```

3.127 $\int \frac{1}{1+\coth(x)}\,dx$

Optimal. Leaf size=16

$$\frac{x}{2}-\frac{1}{2(1+\coth(x))}$$

```
[Out] 1/2*x-1/2/(1+coth(x))
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3560, 8}

$$\frac{x}{2}-\frac{1}{2(\coth(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Coth[x])^(-1),x]
```

```
[Out] x/2 - 1/(2*(1 + Coth[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+\coth(x)}\,dx &= -\frac{1}{2(1+\coth(x))}+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}-\frac{1}{2(1+\coth(x))}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 18, normalized size = 1.12

$$\frac{1}{4}(2x+\cosh(2x)-\sinh(2x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Coth[x])^(-1),x]
```

```
[Out] (2*x + Cosh[2*x] - Sinh[2*x])/4
```

Maple [A]
time = 0.24, size = 24, normalized size = 1.50

method	result	size
risch	$\frac{x}{2}+\frac{{\rm e}^{-2x}}{4}$	11
derivativedivides	$-\frac{\ln(\coth(x)-1)}{4}-\frac{1}{2(1+\coth(x))}+\frac{\ln(1+\coth(x))}{4}$	24
default	$-\frac{\ln(\coth(x)-1)}{4}-\frac{1}{2(1+\coth(x))}+\frac{\ln(1+\coth(x))}{4}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*ln(coth(x)-1)-1/2/(1+coth(x))+1/4*ln(1+coth(x))
```

Maxima [A]
time = 0.26, size = 10, normalized size = 0.62

$$\frac{1}{2}\,x+\frac{1}{4}\,e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/4*e^(-2*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.
time = 0.37, size = 26, normalized size = 1.62

$$\frac{(2\,x+1)\cosh\left(x\right)+(2\,x-1)\sinh\left(x\right)}{4\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] 1/4*((2*x + 1)*cosh(x) + (2*x - 1)*sinh(x))/(cosh(x) + sinh(x))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(10) = 20$.
time = 0.27, size = 27, normalized size = 1.69

$$\frac{x\tanh\left(x\right)}{2\tanh\left(x\right)+2}+\frac{x}{2\tanh\left(x\right)+2}+\frac{1}{2\tanh\left(x\right)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x)),x)
```

```
[Out] x*tanh(x)/(2*tanh(x) + 2) + x/(2*tanh(x) + 2) + 1/(2*tanh(x) + 2)
```

Giac [A]
time = 0.42, size = 10, normalized size = 0.62

$$\frac{1}{2}x + \frac{1}{4}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x)),x, algorithm="giac")
```

```
[Out] 1/2*x + 1/4*e^(-2*x)
```

Mupad [B]
time = 0.00, size = 14, normalized size = 0.88

$$\frac{x}{2} - \frac{1}{2(\coth(x) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x) + 1),x)
```

```
[Out] x/2 - 1/(2*(coth(x) + 1))
```

3.128 $\int \frac{\coth(x)}{1+\coth(x)} \, dx$

Optimal. Leaf size=16

$$\frac{x}{2} + \frac{1}{2(1 + \coth(x))}$$

```
[Out] 1/2*x+1/2/(1+coth(x))
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3607, 8}

$$\frac{x}{2} + \frac{1}{2(\coth(x) + 1)}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(1 + Coth[x]),x]
```

```
[Out] x/2 + 1/(2*(1 + Coth[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3607

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && LtQ[m, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\coth(x)}{1+\coth(x)} \, dx &= \frac{1}{2(1+\coth(x))} + \frac{\int 1 \, dx}{2} \\ &= \frac{x}{2} + \frac{1}{2(1+\coth(x))} \end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.12

$$\frac{1}{4}(2x - \cosh(2x) + \sinh(2x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(1 + Coth[x]),x]
[Out] (2*x - Cosh[2*x] + Sinh[2*x])/4
```

Maple [A]
time = 0.27, size = 24, normalized size = 1.50

method	result	size
risch	$\frac{x}{2}-\frac{{\mathrm e}^{-2 x}}{4}$	11
derivativedivides	$\frac{1}{2+2 \coth \left(x \right)}+\frac{\ln \left(1+\coth \left(x \right)\right)}{4}-\frac{\ln \left(\coth \left(x \right)-1\right)}{4}$	24
default	$\frac{1}{2+2 \coth \left(x \right)}+\frac{\ln \left(1+\coth \left(x \right)\right)}{4}-\frac{\ln \left(\coth \left(x \right)-1\right)}{4}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(1+coth(x)),x,method=_RETURNVERBOSE)
[Out] 1/2/(1+coth(x))+1/4*ln(1+coth(x))-1/4*ln(coth(x)-1)
```

Maxima [A]
time = 0.27, size = 10, normalized size = 0.62

$$\frac{1}{2}\,x-\frac{1}{4}\,e^{\left(-2\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1+coth(x)),x, algorithm="maxima")
[Out] 1/2*x - 1/4*e^(-2*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.
time = 0.33, size = 26, normalized size = 1.62

$$\frac{\left(2\,x-1\right)\cosh\left(x\right)+\left(2\,x+1\right)\sinh\left(x\right)}{4\,\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1+coth(x)),x, algorithm="fricas")
[Out] 1/4*((2*x - 1)*cosh(x) + (2*x + 1)*sinh(x))/(cosh(x) + sinh(x))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(10) = 20$.
time = 0.27, size = 27, normalized size = 1.69

$$\frac{x\tanh\left(x\right)}{2\tanh\left(x\right)+2}+\frac{x}{2\tanh\left(x\right)+2}-\frac{1}{2\tanh\left(x\right)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1+coth(x)),x)
```

```
[Out] x*tanh(x)/(2*tanh(x) + 2) + x/(2*tanh(x) + 2) - 1/(2*tanh(x) + 2)
```

Giac [A]
time = 0.41, size = 10, normalized size = 0.62

$$\frac{1}{2}\,x-\frac{1}{4}\,e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1+coth(x)),x, algorithm="giac")
```

```
[Out] 1/2*x - 1/4*e^(-2*x)
```

Mupad [B]
time = 0.05, size = 12, normalized size = 0.75

$$\frac{x}{2}+\frac{1}{2\,\left(\mathrm{coth}\left(x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(coth(x) + 1),x)
```

```
[Out] x/2 + 1/(2*(coth(x) + 1))
```

3.129 $\int \frac{\coth^2(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=19

$$-\frac{x}{2}-\frac{1}{2(1+\coth(x))}+\log(\sinh(x))$$

```
[Out] -1/2*x-1/2/(1+coth(x))+ln(sinh(x))
```

Rubi [A]
time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3621, 3556}

$$-\frac{x}{2}-\frac{1}{2(\coth(x)+1)}+\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(1 + Coth[x]),x]
```

```
[Out] -1/2*x - 1/(2*(1 + Coth[x])) + Log[Sinh[x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3621

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-b)*(a*c + b*d)^2*((a + b*Tan[e + f*x])^
m/(2*a^3*f*m)), x] + Dist[1/(2*a^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[
a*c^2 - 2*b*c*d + a*d^2 - 2*b*d^2*Tan[e + f*x], x], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LeQ[m, -1] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^2(x)}{1+\coth(x)}\,dx &= -\frac{1}{2(1+\coth(x))}-\frac{1}{2}\int (1-2\coth(x))\,dx\\ &= -\frac{x}{2}-\frac{1}{2(1+\coth(x))}+\int \coth(x)\,dx\\ &= -\frac{x}{2}-\frac{1}{2(1+\coth(x))}+\log(\sinh(x))\end{aligned}$$

Mathematica [A]
time = 0.02, size = 23, normalized size = 1.21

$$\frac{1}{4}(-2x + \cosh(2x) + 4\log(\sinh(x)) - \sinh(2x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(1 + Coth[x]),x]
[Out] (-2*x + Cosh[2*x] + 4*Log[Sinh[x]] - Sinh[2*x])/4
```

Maple [A]
time = 0.26, size = 24, normalized size = 1.26

method	result	size
risch	$-\frac{3x}{2} + \frac{e^{-2x}}{4} + \ln\left(e^{2x} - 1\right)$	18
derivativedivides	$-\frac{\ln(\coth(x)-1)}{4} - \frac{1}{2(1+\coth(x))} - \frac{3\ln(1+\coth(x))}{4}$	24
default	$-\frac{\ln(\coth(x)-1)}{4} - \frac{1}{2(1+\coth(x))} - \frac{3\ln(1+\coth(x))}{4}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(1+coth(x)),x,method=_RETURNVERBOSE)
[Out] -1/4*ln(coth(x)-1)-1/2/(1+coth(x))-3/4*ln(1+coth(x))
```

Maxima [A]
time = 0.28, size = 24, normalized size = 1.26

$$\frac{1}{2}\,x + \frac{1}{4}\,e^{(-2\,x)} + \log\left(e^{(-x)} + 1\right) + \log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(1+coth(x)),x, algorithm="maxima")
[Out] 1/2*x + 1/4*e^(-2*x) + log(e^(-x) + 1) + log(e^(-x) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(15) = 30$.
time = 0.36, size = 73, normalized size = 3.84

$$-\frac{6\,x\cosh\left(x\right)^{2} + 12\,x\cosh\left(x\right)\sinh\left(x\right) + 6\,x\sinh\left(x\right)^{2} - 4\left(\cosh\left(x\right)^{2} + 2\,\cosh\left(x\right)\sinh\left(x\right) + \sinh\left(x\right)^{2}\right)\log\left(\frac{2\,\sinh\left(x\right)}{\cosh\left(x\right)-\sinh\left(x\right)}\right) - 1}{4\left(\cosh\left(x\right)^{2} + 2\,\cosh\left(x\right)\sinh\left(x\right) + \sinh\left(x\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] -1/4*(6*x*cosh(x)^2 + 12*x*cosh(x)*sinh(x) + 6*x*sinh(x)^2 - 4*(cosh(x)^2 +
 2*cosh(x)*sinh(x) + sinh(x)^2)*log(2*sinh(x)/(cosh(x) - sinh(x))) - 1)/(co
sh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(15) = 30$.
time = 0.36, size = 92, normalized size = 4.84

$$\frac{x\tanh(x)}{2\tanh(x)+2}+\frac{x}{2\tanh(x)+2}-\frac{2\log(\tanh(x)+1)\tanh(x)}{2\tanh(x)+2}-\frac{2\log(\tanh(x)+1)}{2\tanh(x)+2}+\frac{2\log(\tanh(x))\tanh(x)}{2\tanh(x)+2}+\frac{2\log(\tanh(x))}{2\tanh(x)+2}+\frac{1}{2\tanh(x)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(1+coth(x)),x)
```

```
[Out] x*tanh(x)/(2*tanh(x) + 2) + x/(2*tanh(x) + 2) - 2*log(tanh(x) + 1)*tanh(x)/
(2*tanh(x) + 2) - 2*log(tanh(x) + 1)/(2*tanh(x) + 2) + 2*log(tanh(x))*tanh(
x)/(2*tanh(x) + 2) + 2*log(tanh(x))/(2*tanh(x) + 2) + 1/(2*tanh(x) + 2)
```

Giac [A]
time = 0.41, size = 18, normalized size = 0.95

$$-\frac{3}{2}x+\frac{1}{4}e^{(-2x)}+\log\left(\left|e^{(2x)}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(1+coth(x)),x, algorithm="giac")
```

```
[Out] -3/2*x + 1/4*e^(-2*x) + log(abs(e^(2*x) - 1))
```

Mupad [B]
time = 0.06, size = 21, normalized size = 1.11

$$\frac{x}{2}-\ln\left(\coth(x)+1\right)-\frac{1}{2\left(\coth\left(x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(coth(x) + 1),x)
```

```
[Out] x/2 - log(coth(x) + 1) - 1/(2*(coth(x) + 1))
```

3.130 $\int \frac{\coth^3(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=31

$$\frac{3x}{2}-\frac{3\coth(x)}{2}+\frac{\coth^2(x)}{2(1+\coth(x))}-\log(\sinh(x))$$

```
[Out] 3/2*x-3/2*coth(x)+1/2*coth(x)^2/(1+coth(x))-ln(sinh(x))
```

Rubi [A]
time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3631, 3606, 3556}

$$\frac{3x}{2}+\frac{\coth^2(x)}{2(\coth(x)+1)}-\frac{3\coth(x)}{2}-\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/(1 + Coth[x]),x]
[Out] (3*x)/2 - (3*Coth[x])/2 + Coth[x]^2/(2*(1 + Coth[x])) - Log[Sinh[x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3631

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((c + d*Tan[e + f*x])^(n - 1)/(
2*a*f*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a^2), Int[(c + d*Tan[e + f*x])
^(n - 2)*Simp[a*c^2 + a*d^2*(n - 1) - b*c*d*n - d*(a*c*(n - 2) + b*d*n)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^3(x)}{1+\coth(x)}\,dx &= \frac{\coth^2(x)}{2(1+\coth(x))} - \frac{1}{2}\int (2-3\coth(x))\coth(x)\,dx\\ &= \frac{3x}{2} - \frac{3\coth(x)}{2} + \frac{\coth^2(x)}{2(1+\coth(x))} - \int \coth(x)\,dx\\ &= \frac{3x}{2} - \frac{3\coth(x)}{2} + \frac{\coth^2(x)}{2(1+\coth(x))} - \log(\sinh(x))\end{aligned}$$

Mathematica [A]
time = 0.03, size = 27, normalized size = 0.87

$$\frac{1}{4}(6x - \cosh(2x) - 4\coth(x) - 4\log(\sinh(x)) + \sinh(2x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/(1 + Coth[x]),x]
[Out] (6*x - Cosh[2*x] - 4*Coth[x] - 4*Log[Sinh[x]] + Sinh[2*x])/4
```

Maple [A]
time = 0.35, size = 28, normalized size = 0.90

method	result	size
derivativedivides	$-\coth\left(x\right)-\frac{\ln\left(\coth\left(x\right)-1\right)}{4}+\frac{1}{2+2\coth\left(x\right)}+\frac{5\ln\left(1+\coth\left(x\right)\right)}{4}$	28
default	$-\coth\left(x\right)-\frac{\ln\left(\coth\left(x\right)-1\right)}{4}+\frac{1}{2+2\coth\left(x\right)}+\frac{5\ln\left(1+\coth\left(x\right)\right)}{4}$	28
risch	$\frac{5x}{2}-\frac{{\mathrm e}^{-2x}}{4}-\frac{2}{{\mathrm e}^{2x}-1}-\ln\left({\mathrm e}^{2x}-1\right)$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(1+coth(x)),x,method=_RETURNVERBOSE)
[Out] -coth(x)-1/4*ln(coth(x)-1)+1/2/(1+coth(x))+5/4*ln(1+coth(x))
```

Maxima [A]
time = 0.26, size = 38, normalized size = 1.23

$$\frac{1}{2}\,x + \frac{2}{e^{(-2\,x)} - 1} - \frac{1}{4}\,e^{(-2\,x)} - \log\left(e^{(-x)} + 1\right) - \log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(1+coth(x)),x, algorithm="maxima")
[Out] 1/2*x + 2/(e^(-2*x) - 1) - 1/4*e^(-2*x) - log(e^(-x) + 1) - log(e^(-x) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(25) = 50$.

time = 0.38, size = 196, normalized size = 6.32

$$\frac{10\,x\cosh\left(x\right)^{4} + 40\,x\cosh\left(x\right)\sinh\left(x\right)^{3} + 10\,x\sinh\left(x\right)^{4} - (10\,x + 9)\cosh\left(x\right)^{2} + \left(60\,x\cosh\left(x\right)^{2} - 10\,x - 9\right)\sinh\left(x\right)^{2} - 4\left(\cosh\left(x\right)^{4} + 4\,\cosh\left(x\right)\sinh\left(x\right)^{3} + \sinh\left(x\right)^{4} + \left(6\,\cosh\left(x\right)^{2} - 1\right)\sinh\left(x\right)^{2} - \cosh\left(x\right)^{2} + 2\left(2\,\cosh\left(x\right)^{3} - \cosh\left(x\right)\right)\sinh\left(x\right)\right)\log\left(\frac{2\,\sinh(x)}{\cosh(x)-\sinh(x)}\right) + 2\left(20\,x\cosh\left(x\right)^{3} - (10\,x + 9)\cosh\left(x\right)\right)\sinh\left(x\right) + 1}{4\left(\cosh\left(x\right)^{4} + 4\,\cosh\left(x\right)\sinh\left(x\right)^{3} + \sinh\left(x\right)^{4} + \left(6\,\cosh\left(x\right)^{2} - 1\right)\sinh\left(x\right)^{2} - \cosh\left(x\right)^{2} + 2\left(2\,\cosh\left(x\right)^{3} - \cosh\left(x\right)\right)\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] 1/4*(10*x*cosh(x)^4 + 40*x*cosh(x)*sinh(x)^3 + 10*x*sinh(x)^4 - (10*x + 9)*
cosh(x)^2 + (60*x*cosh(x)^2 - 10*x - 9)*sinh(x)^2 - 4*(cosh(x)^4 + 4*cosh(x
)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 - 1)*sinh(x)^2 - cosh(x)^2 + 2*(2*co
sh(x)^3 - cosh(x))*sinh(x))*log(2*sinh(x)/(cosh(x) - sinh(x))) + 2*(20*x*co
sh(x)^3 - (10*x + 9)*cosh(x))*sinh(x) + 1)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3
 + sinh(x)^4 + (6*cosh(x)^2 - 1)*sinh(x)^2 - cosh(x)^2 + 2*(2*cosh(x)^3 - c
osh(x))*sinh(x))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(27) = 54$.

time = 0.56, size = 160, normalized size = 5.16

$$\frac{x\tanh^{2}\left(x\right)}{2\tanh^{2}\left(x\right) + 2\tanh\left(x\right)} + \frac{x\tanh\left(x\right)}{2\tanh^{2}\left(x\right) + 2\tanh\left(x\right)} + \frac{2\log\left(\tanh\left(x\right) + 1\right)\tanh^{2}\left(x\right)}{2\tanh^{2}\left(x\right) + 2\tanh\left(x\right)} + \frac{2\log\left(\tanh\left(x\right) + 1\right)\tanh\left(x\right)}{2\tanh^{2}\left(x\right) + 2\tanh\left(x\right)} - \frac{2\log\left(\tanh\left(x\right)\right)\tanh^{2}\left(x\right)}{2\tanh^{2}\left(x\right) + 2\tanh\left(x\right)} - \frac{2\log\left(\tanh\left(x\right)\right)\tanh\left(x\right)}{2\tanh^{2}\left(x\right) + 2\tanh\left(x\right)} - \frac{3\tanh\left(x\right)}{2\tanh^{2}\left(x\right) + 2\tanh\left(x\right)} - \frac{2}{2\tanh^{2}\left(x\right) + 2\tanh\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(1+coth(x)),x)
```

```
[Out] x*tanh(x)**2/(2*tanh(x)**2 + 2*tanh(x)) + x*tanh(x)/(2*tanh(x)**2 + 2*tanh(
x)) + 2*log(tanh(x) + 1)*tanh(x)**2/(2*tanh(x)**2 + 2*tanh(x)) + 2*log(tanh
(x) + 1)*tanh(x)/(2*tanh(x)**2 + 2*tanh(x)) - 2*log(tanh(x))*tanh(x)**2/(2*
tanh(x)**2 + 2*tanh(x)) - 2*log(tanh(x))*tanh(x)/(2*tanh(x)**2 + 2*tanh(x))
 - 3*tanh(x)/(2*tanh(x)**2 + 2*tanh(x)) - 2/(2*tanh(x)**2 + 2*tanh(x))
```

Giac [A]

time = 0.42, size = 36, normalized size = 1.16

$$\frac{5}{2}\,x - \frac{\left(9\,e^{(2\,x)} - 1\right)e^{(-2\,x)}}{4\left(e^{(2\,x)} - 1\right)} - \log\left(\left|e^{(2\,x)} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(1+coth(x)),x, algorithm="giac")
```

```
[Out] 5/2*x - 1/4*(9*e^(2*x) - 1)*e^(-2*x)/(e^(2*x) - 1) - log(abs(e^(2*x) - 1))
```

Mupad [B]

time = 1.16, size = 21, normalized size = 0.68

$$\frac{x}{2} + \ln\left(\mathrm{coth}(x) + 1\right) - \mathrm{coth}(x) + \frac{1}{2\,\left(\mathrm{coth}\left(x\right) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(coth(x) + 1),x)
[Out] x/2 + log(coth(x) + 1) - coth(x) + 1/(2*(coth(x) + 1))
```

3.131 $\int \frac{\coth^4(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=37

$$-\frac{3x}{2}+\frac{3\coth(x)}{2}-\coth^2(x)+\frac{\coth^3(x)}{2(1+\coth(x))}+2\log(\sinh(x))$$

```
[Out] -3/2*x+3/2*coth(x)-coth(x)^2+1/2*coth(x)^3/(1+coth(x))+2*ln(sinh(x))
```

Rubi [A]
time = 0.05, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3631, 3609, 3606, 3556}

$$-\frac{3x}{2}+\frac{\coth^3(x)}{2(\coth(x)+1)}-\coth^2(x)+\frac{3\coth(x)}{2}+2\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^4/(1 + Coth[x]),x]
```

```
[Out] (-3*x)/2 + (3*Coth[x])/2 - Coth[x]^2 + Coth[x]^3/(2*(1 + Coth[x])) + 2*Log[
Sinh[x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3631

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((c + d*Tan[e + f*x])^(n - 1)/(
```

```
2*a*f*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a^2), Int[(c + d*Tan[e + f*x])
^(n - 2)*Simp[a*c^2 + a*d^2*(n - 1) - b*c*d*n - d*(a*c*(n - 2) + b*d*n)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^4(x)}{1+\coth(x)}\,dx &= \frac{\coth^3(x)}{2(1+\coth(x))}-\frac{1}{2}\int (3-4\coth(x))\coth^2(x)\,dx\\ &= -\coth^2(x)+\frac{\coth^3(x)}{2(1+\coth(x))}+\frac{1}{2}i\int (-4i+3i\coth(x))\coth(x)\,dx\\ &= -\frac{3x}{2}+\frac{3\coth(x)}{2}-\coth^2(x)+\frac{\coth^3(x)}{2(1+\coth(x))}+2\int \coth(x)\,dx\\ &= -\frac{3x}{2}+\frac{3\coth(x)}{2}-\coth^2(x)+\frac{\coth^3(x)}{2(1+\coth(x))}+2\log(\sinh(x))\end{aligned}$$

Mathematica [A]
time = 0.04, size = 33, normalized size = 0.89

$$\frac{1}{4}\left(-6x+\cosh(2x)+4\coth(x)-2\text{csch}^2(x)+8\log(\sinh(x))-\sinh(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4/(1 + Coth[x]),x]
[Out] (-6*x + Cosh[2*x] + 4*Coth[x] - 2*Csch[x]^2 + 8*Log[Sinh[x]] - Sinh[2*x])/4
```

Maple [A]
time = 0.42, size = 32, normalized size = 0.86

method	result	size
risch	$-\frac{7x}{2}+\frac{e^{-2x}}{4}-\frac{2}{(e^{2x}-1)^2}+2\ln\left(e^{2x}-1\right)$	30
derivativedivides	$-\frac{(\coth^2(x))}{2}+\coth(x)-\frac{\ln(\coth(x)-1)}{4}-\frac{1}{2(1+\coth(x))}-\frac{7\ln(1+\coth(x))}{4}$	32
default	$-\frac{(\coth^2(x))}{2}+\coth(x)-\frac{\ln(\coth(x)-1)}{4}-\frac{1}{2(1+\coth(x))}-\frac{7\ln(1+\coth(x))}{4}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(1+coth(x)),x,method=_RETURNVERBOSE)
[Out] -1/2*coth(x)^2+coth(x)-1/4*ln(coth(x)-1)-1/2/(1+coth(x))-7/4*ln(1+coth(x))
```

Maxima [A]
time = 0.27, size = 54, normalized size = 1.46

$$\frac{1}{2}\,x + \frac{2\,\left(2\,e^{(-2\,x)} - 1\right)}{2\,e^{(-2\,x)} - e^{(-4\,x)} - 1} + \frac{1}{4}\,e^{(-2\,x)} + 2\,\log\left(e^{(-x)} + 1\right) + 2\,\log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x + 2*(2*e^(-2*x) - 1)/(2*e^(-2*x) - e^(-4*x) - 1) + 1/4*e^(-2*x) + 2*l
og(e^(-x) + 1) + 2*log(e^(-x) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(31) = 62$.
time = 0.39, size = 357, normalized size = 9.65

$$-\frac{14\,x\cosh\left(x\right)^{6} + 84\,x\cosh\left(x\right)\sinh\left(x\right)^{5} + 14\,x\sinh\left(x\right)^{6} - \left(28\,x + 1\right)\cosh\left(x\right)^{4} + \left(210\,x\cosh\left(x\right)^{2} - 28\,x - 1\right)\sinh\left(x\right)^{4} + 4\,\left(70\,x\cosh\left(x\right)^{3} - \left(28\,x + 1\right)\cosh\left(x\right)\right)\sinh\left(x\right)^{3} + 2\,\left(7\,x + 5\right)\cosh\left(x\right)^{2} + 2\,\left(105\,x\cosh\left(x\right)^{4} - 3\,\left(28\,x + 1\right)\cosh\left(x\right)^{2} + 7\,x + 5\right)\sinh\left(x\right)^{2} - 8\,\left(\cosh\left(x\right)^{6} + 6\,\cosh\left(x\right)\sinh\left(x\right)^{5} + \sinh\left(x\right)^{6} + \left(15\,\cosh\left(x\right)^{2} - 2\right)\sinh\left(x\right)^{4} - 2\,\cosh\left(x\right)^{4} + 4\,\left(5\,\cosh\left(x\right)^{3} - 2\,\cosh\left(x\right)\right)\sinh\left(x\right)^{3} + \left(15\,\cosh\left(x\right)^{4} - 12\,\cosh\left(x\right)^{2} + 1\right)\sinh\left(x\right)^{2} + \cosh\left(x\right)^{2} + 2\,\left(3\,\cosh\left(x\right)^{5} - 4\,\cosh\left(x\right)^{3} + \cosh\left(x\right)\right)\sinh\left(x\right)\right)\log\left(\frac{2\,\sinh\left(x\right)}{\cosh\left(x\right) - \sinh\left(x\right)}\right) + 4\,\left(21\,x\cosh\left(x\right)^{5} - \left(28\,x + 1\right)\cosh\left(x\right)^{3} + \left(7\,x + 5\right)\cosh\left(x\right)\right)\sinh\left(x\right) - 1}{4\,\left(\cosh\left(x\right)^{6} + 6\,\cosh\left(x\right)\sinh\left(x\right)^{5} + \sinh\left(x\right)^{6} + \left(15\,\cosh\left(x\right)^{2} - 2\right)\sinh\left(x\right)^{4} - 2\,\cosh\left(x\right)^{4} + 4\,\left(5\,\cosh\left(x\right)^{3} - 2\,\cosh\left(x\right)\right)\sinh\left(x\right)^{3} + \left(15\,\cosh\left(x\right)^{4} - 12\,\cosh\left(x\right)^{2} + 1\right)\sinh\left(x\right)^{2} + \cosh\left(x\right)^{2} + 2\,\left(3\,\cosh\left(x\right)^{5} - 4\,\cosh\left(x\right)^{3} + \cosh\left(x\right)\right)\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] -1/4*(14*x*cosh(x)^6 + 84*x*cosh(x)*sinh(x)^5 + 14*x*sinh(x)^6 - (28*x + 1)
*cosh(x)^4 + (210*x*cosh(x)^2 - 28*x - 1)*sinh(x)^4 + 4*(70*x*cosh(x)^3 - (
28*x + 1)*cosh(x))*sinh(x)^3 + 2*(7*x + 5)*cosh(x)^2 + 2*(105*x*cosh(x)^4 -
 3*(28*x + 1)*cosh(x)^2 + 7*x + 5)*sinh(x)^2 - 8*(cosh(x)^6 + 6*cosh(x)*sin
h(x)^5 + sinh(x)^6 + (15*cosh(x)^2 - 2)*sinh(x)^4 - 2*cosh(x)^4 + 4*(5*cosh
(x)^3 - 2*cosh(x))*sinh(x)^3 + (15*cosh(x)^4 - 12*cosh(x)^2 + 1)*sinh(x)^2
+ cosh(x)^2 + 2*(3*cosh(x)^5 - 4*cosh(x)^3 + cosh(x))*sinh(x))*log(2*sinh(x
)/(cosh(x) - sinh(x))) + 4*(21*x*cosh(x)^5 - (28*x + 1)*cosh(x)^3 + (7*x +
5)*cosh(x))*sinh(x) - 1)/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + (15
*cosh(x)^2 - 2)*sinh(x)^4 - 2*cosh(x)^4 + 4*(5*cosh(x)^3 - 2*cosh(x))*sinh(
x)^3 + (15*cosh(x)^4 - 12*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(3*cosh(
x)^5 - 4*cosh(x)^3 + cosh(x))*sinh(x))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(34) = 68$.
time = 0.76, size = 197, normalized size = 5.32

$$\frac{x\tanh^{3}\left(x\right)}{2\tanh^{3}\left(x\right) + 2\tanh^{2}\left(x\right)} + \frac{x\tanh^{2}\left(x\right)}{2\tanh^{3}\left(x\right) + 2\tanh^{2}\left(x\right)} - \frac{4\log\left(\tanh\left(x\right) + 1\right)\tanh^{3}\left(x\right)}{2\tanh^{3}\left(x\right) + 2\tanh^{2}\left(x\right)} - \frac{4\log\left(\tanh\left(x\right) + 1\right)\tanh^{2}\left(x\right)}{2\tanh^{3}\left(x\right) + 2\tanh^{2}\left(x\right)} + \frac{4\log\left(\tanh\left(x\right)\right)\tanh^{3}\left(x\right)}{2\tanh^{3}\left(x\right) + 2\tanh^{2}\left(x\right)} + \frac{4\log\left(\tanh\left(x\right)\right)\tanh^{2}\left(x\right)}{2\tanh^{3}\left(x\right) + 2\tanh^{2}\left(x\right)} + \frac{3\tanh^{2}\left(x\right)}{2\tanh^{3}\left(x\right) + 2\tanh^{2}\left(x\right)} + \frac{\tanh\left(x\right)}{2\tanh^{3}\left(x\right) + 2\tanh^{2}\left(x\right)} - \frac{1}{2\tanh^{3}\left(x\right) + 2\tanh^{2}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4/(1+coth(x)),x)
```

```
[Out] x*tanh(x)**3/(2*tanh(x)**3 + 2*tanh(x)**2) + x*tanh(x)**2/(2*tanh(x)**3 + 2
*tanh(x)**2) - 4*log(tanh(x) + 1)*tanh(x)**3/(2*tanh(x)**3 + 2*tanh(x)**2)
```

```
- 4*log(tanh(x) + 1)*tanh(x)**2/(2*tanh(x)**3 + 2*tanh(x)**2) + 4*log(tanh(
x))*tanh(x)**3/(2*tanh(x)**3 + 2*tanh(x)**2) + 4*log(tanh(x))*tanh(x)**2/(2
*tanh(x)**3 + 2*tanh(x)**2) + 3*tanh(x)**2/(2*tanh(x)**3 + 2*tanh(x)**2) +
tanh(x)/(2*tanh(x)**3 + 2*tanh(x)**2) - 1/(2*tanh(x)**3 + 2*tanh(x)**2)
```

Giac [A]
time = 0.40, size = 40, normalized size = 1.08

$$-\frac{7}{2}x + \frac{\left(e^{(4x)} - 10e^{(2x)} + 1\right)e^{(-2x)}}{4\left(e^{(2x)} - 1\right)^2} + 2\log\left(\left|e^{(2x)} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(1+coth(x)),x, algorithm="giac")
```

```
[Out] -7/2*x + 1/4*(e^(4*x) - 10*e^(2*x) + 1)*e^(-2*x)/(e^(2*x) - 1)^2 + 2*log(ab
s(e^(2*x) - 1))
```

Mupad [B]
time = 0.07, size = 29, normalized size = 0.78

$$\frac{x}{2} - 2\ln\left(\coth(x) + 1\right) + \coth(x) - \frac{\coth(x)^2}{2} - \frac{1}{2\left(\coth\left(x\right) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(coth(x) + 1),x)
```

```
[Out] x/2 - 2*log(coth(x) + 1) + coth(x) - coth(x)^2/2 - 1/(2*(coth(x) + 1))
```

3.132 $\int \coth(x)(1+\coth(x))^{3/2}\,dx$

Optimal. Leaf size=45

$$2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)-2\sqrt{1+\coth(x)}-\frac{2}{3}(1+\coth(x))^{3/2}$$

```
[Out] -2/3*(1+coth(x))^(3/2)+2*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-2*(
1+coth(x))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3608, 3559, 3561, 212}

$$-\frac{2}{3}(\coth(x)+1)^{3/2}-2\sqrt{\coth(x)+1}+2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]*(1 + Coth[x])^(3/2),x]
```

```
[Out] 2*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 2*Sqrt[1 + Coth[x]] - (2*(1
+ Coth[x])^(3/2))/3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3608

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Dist
```

```
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \coth(x)(1+\coth(x))^{3/2}\,dx &= -\frac{2}{3}(1+\coth(x))^{3/2}+\int (1+\coth(x))^{3/2}\,dx\\ &= -2\sqrt{1+\coth(x)}-\frac{2}{3}(1+\coth(x))^{3/2}+2\int \sqrt{1+\coth(x)}\,dx\\ &= -2\sqrt{1+\coth(x)}-\frac{2}{3}(1+\coth(x))^{3/2}+4\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1+\coth(x)}\right)\\ &= 2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)-2\sqrt{1+\coth(x)}-\frac{2}{3}(1+\coth(x))^{3/2}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.10, size = 90, normalized size = 2.00

$$-\frac{2(1+\coth(x))^{3/2}\left(\cosh(x)\sqrt{i(1+\coth(x))}-(3-3i)\text{ArcTan}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i(1+\coth(x))}\right)\sinh(x)+4\sqrt{i(1+\coth(x))}\sinh(x)\right)}{3\sqrt{i(1+\coth(x))}(\cosh(x)+\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]*(1 + Coth[x])^(3/2),x]
[Out] (-2*(1 + Coth[x])^(3/2)*(Cosh[x]*Sqrt[I*(1 + Coth[x])] - (3 - 3*I)*ArcTan[(
1/2 + I/2)*Sqrt[I*(1 + Coth[x])]]*Sinh[x] + 4*Sqrt[I*(1 + Coth[x])]*Sinh[x]
))/(3*Sqrt[I*(1 + Coth[x])]*(Cosh[x] + Sinh[x]))
```

Maple [A]
time = 0.60, size = 35, normalized size = 0.78

method	result	size
derivativedivides	$-\frac{2(1+\coth(x))^{\frac{3}{2}}}{3}+2\,\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}-2\sqrt{1+\coth(x)}$	35
default	$-\frac{2(1+\coth(x))^{\frac{3}{2}}}{3}+2\,\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}-2\sqrt{1+\coth(x)}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(1+coth(x))^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/3*(1+coth(x))^(3/2)+2*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-2*(
1+coth(x))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(1+coth(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((coth(x) + 1)^(3/2)*coth(x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 259 vs. 2(34) = 68.
time = 0.34, size = 259, normalized size = 5.76

$$-\frac{2\sqrt{2}\left(5\sqrt{2}\cosh(x)^3+15\sqrt{2}\cosh(x)\sinh(x)^2+5\sqrt{2}\sinh(x)^3+3\left(5\sqrt{2}\cosh(x)^2-\sqrt{2}\right)\sinh(x)-3\sqrt{2}\cosh(x)\right)\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}}-3\left(\sqrt{2}\cosh(x)^4+4\sqrt{2}\cosh(x)\sinh(x)^3+\sqrt{2}\sinh(x)^4+2\left(3\sqrt{2}\cosh(x)^2-\sqrt{2}\right)\sinh(x)^2-2\sqrt{2}\cosh(x)^2+4\left(\sqrt{2}\cosh(x)^3-\sqrt{2}\cosh(x)\right)\sinh(x)+\sqrt{2}\right)\log\left(2\sqrt{2}\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}}(\cosh(x)+\sinh(x))+2\cosh(x)^2+4\cosh(x)\sinh(x)+2\sinh(x)^2-1\right)}{3\left(\cosh(x)^4+4\cosh(x)\sinh(x)^3+\sinh(x)^4+2\left(3\cosh(x)^2-1\right)\sinh(x)^2-2\cosh(x)^2+4\left(\cosh(x)^3-\cosh(x)\right)\sinh(x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(1+coth(x))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*sqrt(2)*(5*sqrt(2)*cosh(x)^3 + 15*sqrt(2)*cosh(x)*sinh(x)^2 + 5*sqr
t(2)*sinh(x)^3 + 3*(5*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x) - 3*sqrt(2)*cosh
(x))*sqrt(sinh(x)/(cosh(x) - sinh(x))) - 3*(sqrt(2)*cosh(x)^4 + 4*sqrt(2)*c
osh(x)*sinh(x)^3 + sqrt(2)*sinh(x)^4 + 2*(3*sqrt(2)*cosh(x)^2 - sqrt(2))*si
nh(x)^2 - 2*sqrt(2)*cosh(x)^2 + 4*(sqrt(2)*cosh(x)^3 - sqrt(2)*cosh(x))*sin
h(x) + sqrt(2))*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x) - sinh(x)))*(cosh(x) +
sinh(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - 1))/(cosh(x)^4 +
 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x
)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (\coth(x)+1)^{\frac{3}{2}}\coth(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(1+coth(x))**(3/2),x)
```

```
[Out] Integral((coth(x) + 1)**(3/2)*coth(x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 135 vs. 2(34) = 68.
time = 0.41, size = 135, normalized size = 3.00

$$-\frac{1}{3}\sqrt{2}\left(3\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x)}+1\right|\right)\operatorname{sgn}\left(e^{(2x)}-1\right)+\frac{2\left(9\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^2\operatorname{sgn}\left(e^{(2x)}-1\right)+12\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)\operatorname{sgn}\left(e^{(2x)}-1\right)+5\operatorname{sgn}\left(e^{(2x)}-1\right)\right)}{\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}+1\right)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(1+coth(x))^(3/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(2)*(3*log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1))*sgn(e^(
2*x) - 1) + 2*(9*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^2*sgn(e^(2*x) - 1) + 1
2*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))*sgn(e^(2*x) - 1) + 5*sgn(e^(2*x) - 1)
)/(sqrt(e^(4*x) - e^(2*x)) - e^(2*x) + 1)^3)
```

Mupad [B]
time = 1.24, size = 34, normalized size = 0.76

$$2\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{\coth\left(x\right)+1}}{2}\right)-2\sqrt{\coth\left(x\right)+1}-\frac{2\left(\coth(x)+1\right)^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(coth(x) + 1)^(3/2),x)
```

```
[Out] 2*2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2) - 2*(coth(x) + 1)^(1/2) -
(2*(coth(x) + 1)^(3/2))/3
```

3.133 $\int \coth(x)\sqrt{1+\coth(x)}\,dx$

Optimal. Leaf size=32

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)-2\sqrt{1+\coth(x)}$$

```
[Out] arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-2*(1+coth(x))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.273$, Rules used = {3608, 3561, 212}

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)-2\sqrt{\coth(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]*Sqrt[1 + Coth[x]],x]
```

```
[Out] Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 2*Sqrt[1 + Coth[x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3608

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Dist
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Rubi steps

$$\int \coth(x)\sqrt{1+\coth(x)}\,dx = -2\sqrt{1+\coth(x)} + \int \sqrt{1+\coth(x)}\,dx$$
$$= -2\sqrt{1+\coth(x)} + 2\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+\coth(x)}\right)$$
$$= \sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right) - 2\sqrt{1+\coth(x)}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.08, size = 53, normalized size = 1.66

$$(1+i)\sqrt{1+\coth(x)}\left((-1+i) - \frac{i\text{ArcTan}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i(1+\coth(x))}\right)}{\sqrt{i(1+\coth(x))}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]*Sqrt[1 + Coth[x]],x]
```

```
[Out] (1 + I)*Sqrt[1 + Coth[x]]*((-1 + I) - (I*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Cot
h[x])]])/Sqrt[I*(1 + Coth[x])])
```

Maple [A]
time = 0.68, size = 26, normalized size = 0.81

method	result	size
derivativedivides	$\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}\sqrt{2}}{2}\right)\sqrt{2} - 2\sqrt{1+\coth(x)}$	26
default	$\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}\sqrt{2}}{2}\right)\sqrt{2} - 2\sqrt{1+\coth(x)}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(1+coth(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-2*(1+coth(x))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(1+coth(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(coth(x) + 1)*coth(x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(25) = 50$.
time = 0.36, size = 131, normalized size = 4.09

$$-\frac{4\sqrt{2}\left(\sqrt{2}\cosh\left(x\right)+\sqrt{2}\sinh\left(x\right)\right)\sqrt{\frac{\sinh\left(x\right)}{\cosh\left(x\right)-\sinh\left(x\right)}}-\left(\sqrt{2}\cosh\left(x\right)^{2}+2\sqrt{2}\cosh\left(x\right)\sinh\left(x\right)+\sqrt{2}\sinh\left(x\right)^{2}-\sqrt{2}\right)\log\left(2\sqrt{2}\sqrt{\frac{\sinh\left(x\right)}{\cosh\left(x\right)-\sinh\left(x\right)}}\left(\cosh\left(x\right)+\sinh\left(x\right)\right)+2\cosh\left(x\right)^{2}+4\cosh\left(x\right)\sinh\left(x\right)+2\sinh\left(x\right)^{2}-1\right)}{2\left(\cosh\left(x\right)^{2}+2\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(1+coth(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(4*sqrt(2)*(sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(sinh(x)/(cosh(x) -
 sinh(x))) - (sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(
x)^2 - sqrt(2))*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x) - sinh(x)))*(cosh(x) +
sinh(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - 1))/(cosh(x)^2 +
 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\coth\left(x\right)+1}\,\coth\left(x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(1+coth(x))**(1/2),x)
```

```
[Out] Integral(sqrt(coth(x) + 1)*coth(x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(25) = 50$.
time = 0.41, size = 71, normalized size = 2.22

$$-\frac{1}{2}\sqrt{2}\left(\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x)}+1\right|\right)\operatorname{sgn}\left(e^{(2x)}-1\right)+\frac{4\operatorname{sgn}\left(e^{(2x)}-1\right)}{\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(1+coth(x))^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1))*sgn(e^(2*
x) - 1) + 4*sgn(e^(2*x) - 1)/(sqrt(e^(4*x) - e^(2*x)) - e^(2*x) + 1))
```

Mupad [B]
time = 1.20, size = 25, normalized size = 0.78

$$\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{\coth\left(x\right)+1}}{2}\right)-2\sqrt{\coth\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(coth(x) + 1)^(1/2),x)
[Out] 2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2) - 2*(coth(x) + 1)^(1/2)
```

3.134 $\int \frac{\coth(x)}{\sqrt{1+\coth(x)}}\,dx$

Optimal. Leaf size=30

$$\frac{\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{\sqrt{1+\coth(x)}}$$

```
[Out] 1/2*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)+1/(1+coth(x))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3607, 3561, 212}

$$\frac{1}{\sqrt{\coth(x)+1}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/Sqrt[1 + Coth[x]],x]
```

```
[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/Sqrt[2] + 1/Sqrt[1 + Coth[x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3607

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && LtQ[m, 0]
```

Rubi steps

$$\int \frac{\coth(x)}{\sqrt{1+\coth(x)}}\,dx = \frac{1}{\sqrt{1+\coth(x)}} + \frac{1}{2}\int \sqrt{1+\coth(x)}\,dx$$
$$= \frac{1}{\sqrt{1+\coth(x)}} + \text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+\coth(x)}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{\sqrt{2}} + \frac{1}{\sqrt{1+\coth(x)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.14, size = 97, normalized size = 3.23

$$\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\text{ArcTan}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i+i\coth(x)}\right)\text{csch}(x)(\cosh(x)+\sinh(x))}{\sqrt{i+i\coth(x)}\sqrt{1+\coth(x)}} + \frac{\text{csch}(x)(\cosh(x)+\sinh(x))\left(\frac{1}{2}-\frac{1}{2}\cosh(2x)+\frac{1}{2}\sinh(2x)\right)}{\sqrt{1+\coth(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/Sqrt[1 + Coth[x]],x]
```

```
[Out] ((1/2 - I/2)*ArcTan[(1/2 + I/2)*Sqrt[I + I*Coth[x]]]*Csch[x]*(Cosh[x] + Sin
h[x]))/(Sqrt[I + I*Coth[x]]*Sqrt[1 + Coth[x]]) + (Csch[x]*(Cosh[x] + Sinh[x
])*(1/2 - Cosh[2*x]/2 + Sinh[2*x]/2))/Sqrt[1 + Coth[x]]
```

Maple [A]
time = 0.75, size = 25, normalized size = 0.83

method	result	size
derivativedivides	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{2}+\frac{1}{\sqrt{1+\coth(x)}}$	25
default	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{2}+\frac{1}{\sqrt{1+\coth(x)}}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(1+coth(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)+1/(1+coth(x))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1+coth(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)/sqrt(coth(x) + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(24) = 48$.
time = 0.35, size = 85, normalized size = 2.83

$$\frac{\left(\sqrt{2}\cosh\left(x\right)+\sqrt{2}\sinh\left(x\right)\right)\log\left(2\sqrt{2}\sqrt{\frac{\sinh\left(x\right)}{\cosh\left(x\right)-\sinh\left(x\right)}}\left(\cosh\left(x\right)+\sinh\left(x\right)\right)+2\cosh\left(x\right)^{2}+4\cosh\left(x\right)\sinh\left(x\right)+2\sinh\left(x\right)^{2}-1\right)+4\sqrt{\frac{\sinh\left(x\right)}{\cosh\left(x\right)-\sinh\left(x\right)}}}{4\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1+coth(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*((sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x
) - sinh(x)))*(cosh(x) + sinh(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sin
h(x)^2 - 1) + 4*sqrt(sinh(x)/(cosh(x) - sinh(x))))/(cosh(x) + sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\coth\left(x\right)}{\sqrt{\coth\left(x\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1+coth(x))**(1/2),x)
```

```
[Out] Integral(coth(x)/sqrt(coth(x) + 1), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(24) = 48$.
time = 0.41, size = 64, normalized size = 2.13

$$-\frac{\sqrt{2}\left(\frac{2}{\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}}+\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x)}+1\right|\right)\right)}{4\,\mathrm{sgn}\left(e^{(2x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1+coth(x))^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(2/(sqrt(e^(4*x) - e^(2*x)) - e^(2*x)) + log(abs(2*sqrt(e^(4*x
) - e^(2*x)) - 2*e^(2*x) + 1)))/sgn(e^(2*x) - 1)
```

Mupad [B]
time = 1.23, size = 24, normalized size = 0.80

$$\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\sqrt{\coth\left(x\right)+1}}{2}\right)}{2}+\frac{1}{\sqrt{\coth\left(x\right)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(coth(x) + 1)^(1/2),x)
[Out] (2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2))/2 + 1/(coth(x) + 1)^(1/2)
```

3.135 $\int \frac{\coth(x)}{(1+\coth(x))^{3/2}}\,dx$

Optimal. Leaf size=49

$$\frac{\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{2\sqrt{2}}+\frac{1}{3(1+\coth(x))^{3/2}}-\frac{1}{2\sqrt{1+\coth(x)}}$$

```
[Out] 1/3/(1+coth(x))^(3/2)+1/4*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-1/
2/(1+coth(x))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3607, 3560, 3561, 212}

$$-\frac{1}{2\sqrt{\coth(x)+1}}+\frac{1}{3(\coth(x)+1)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(1 + Coth[x])^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/(2*Sqrt[2]) + 1/(3*(1 + Coth[x])^(3/2))
- 1/(2*Sqrt[1 + Coth[x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3607

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && LtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth(x)}{(1+\coth(x))^{3/2}}\,dx &= \frac{1}{3(1+\coth(x))^{3/2}}+\frac{1}{2}\int \frac{1}{\sqrt{1+\coth(x)}}\,dx\\ &= \frac{1}{3(1+\coth(x))^{3/2}}-\frac{1}{2\sqrt{1+\coth(x)}}+\frac{1}{4}\int \sqrt{1+\coth(x)}\,dx\\ &= \frac{1}{3(1+\coth(x))^{3/2}}-\frac{1}{2\sqrt{1+\coth(x)}}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1+\coth(x)}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{2\sqrt{2}}+\frac{1}{3(1+\coth(x))^{3/2}}-\frac{1}{2\sqrt{1+\coth(x)}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.23, size = 84, normalized size = 1.71

$$\left(\frac{1}{4}+\frac{i}{4}\right)\sqrt{1+\coth(x)}\left(-\frac{i\text{ArcTan}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i(1+\coth(x))}\right)}{\sqrt{i(1+\coth(x))}}+\left(\frac{1}{6}-\frac{i}{6}\right)(-2+\cosh(2x)+\cosh(4x)-\sinh(2x)-\sinh(4x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(1 + Coth[x])^(3/2),x]
```

```
[Out] (1/4 + I/4)*Sqrt[1 + Coth[x]]*(((-I)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Coth[x]
)]])/Sqrt[I*(1 + Coth[x])] + (1/6 - I/6)*(-2 + Cosh[2*x] + Cosh[4*x] - Sinh
[2*x] - Sinh[4*x]))
```

Maple [A]
time = 0.69, size = 35, normalized size = 0.71

method	result	size
derivativedivides	$\frac{1}{3(1+\coth(x))^{\frac{3}{2}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{4}-\frac{1}{2\sqrt{1+\coth(x)}}$	35

default	$\frac{1}{3(1+\coth(x))^{\frac{3}{2}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{4}-\frac{1}{2\sqrt{1+\coth(x)}}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(1+coth(x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/(1+coth(x))^(3/2)+1/4*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-1/
2/(1+coth(x))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1+coth(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)/(coth(x) + 1)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(34) = 68$.
time = 0.36, size = 166, normalized size = 3.39

$$-\frac{2\sqrt{2}\left(2\sqrt{2}\cosh(x)^2+4\sqrt{2}\cosh(x)\sinh(x)+2\sqrt{2}\sinh(x)^2+\sqrt{2}\right)\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}}-3\left(\sqrt{2}\cosh(x)^3+3\sqrt{2}\cosh(x)^2\sinh(x)+3\sqrt{2}\cosh(x)\sinh(x)^2+\sqrt{2}\sinh(x)^3\right)\log\left(2\sqrt{2}\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}}(\cosh(x)+\sinh(x))+2\cosh(x)^2+4\cosh(x)\sinh(x)+2\sinh(x)^2-1\right)}{24\left(\cosh(x)^3+3\cosh(x)^2\sinh(x)+3\cosh(x)\sinh(x)^2+\sinh(x)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1+coth(x))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/24*(2*sqrt(2)*(2*sqrt(2)*cosh(x)^2 + 4*sqrt(2)*cosh(x)*sinh(x) + 2*sqrt(
2)*sinh(x)^2 + sqrt(2))*sqrt(sinh(x)/(cosh(x) - sinh(x))) - 3*(sqrt(2)*cosh
(x)^3 + 3*sqrt(2)*cosh(x)^2*sinh(x) + 3*sqrt(2)*cosh(x)*sinh(x)^2 + sqrt(2)
*sinh(x)^3)*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sinh
(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - 1))/(cosh(x)^3 + 3*c
osh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2 + sinh(x)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{(\coth(x)+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1+coth(x))**(3/2),x)
```

```
[Out] Integral(coth(x)/(coth(x) + 1)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(34) = 68$.
time = 0.40, size = 89, normalized size = 1.82

$$-\frac{\sqrt{2}\left(\frac{2\left(3\sqrt{e^{(4x)}-e^{(2x)}}-3e^{(2x)}+1\right)}{\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^{3}}+3\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x)}+1\right|\right)\right)}{24\,\mathrm{sgn}\left(e^{(2x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1+coth(x))^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*sqrt(2)*(2*(3*sqrt(e^(4*x) - e^(2*x)) - 3*e^(2*x) + 1)/(sqrt(e^(4*x)
- e^(2*x)) - e^(2*x))^3 + 3*log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) +
 1)))/sgn(e^(2*x) - 1)
```

Mupad [B]
time = 1.22, size = 32, normalized size = 0.65

$$\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{\coth(x)+1}}{2}\right)}{4}-\frac{\frac{\coth(x)}{2}+\frac{1}{6}}{\left(\coth(x)+1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(coth(x) + 1)^(3/2),x)
```

```
[Out] (2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2))/4 - (coth(x)/2 + 1/6)/(cot
h(x) + 1)^(3/2)
```

3.136 $\int \coth^2(x)(1+\coth(x))^{3/2}\,dx$

Optimal. Leaf size=45

$$2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)-2\sqrt{1+\coth(x)}-\frac{2}{5}(1+\coth(x))^{5/2}$$

```
[Out] -2/5*(1+coth(x))^(5/2)+2*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-2*(
1+coth(x))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3624, 3559, 3561, 212}

$$-\frac{2}{5}(\coth(x)+1)^{5/2}-2\sqrt{\coth(x)+1}+2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2*(1 + Coth[x])^(3/2),x]
```

```
[Out] 2*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 2*Sqrt[1 + Coth[x]] - (2*(1
+ Coth[x])^(5/2))/5
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
```

```
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \coth^2(x)(1+\coth(x))^{3/2}\,dx &= -\frac{2}{5}(1+\coth(x))^{5/2}+\int (1+\coth(x))^{3/2}\,dx\\ &= -2\sqrt{1+\coth(x)}-\frac{2}{5}(1+\coth(x))^{5/2}+2\int \sqrt{1+\coth(x)}\,dx\\ &= -2\sqrt{1+\coth(x)}-\frac{2}{5}(1+\coth(x))^{5/2}+4\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1+\text{co}\right.\\ &= 2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)-2\sqrt{1+\coth(x)}-\frac{2}{5}(1+\coth(x))^{5/2}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.18, size = 70, normalized size = 1.56

$$-\frac{2\left(7+2\coth^2(x)+(5+5i)\text{ArcTan}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i(1+\coth(x))}\right)\sqrt{i(1+\coth(x))}+\text{csch}^2(x)+\coth(x)\left(9+\text{csch}^2(x)\right)\right)}{5\sqrt{1+\coth(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2*(1 + Coth[x])^(3/2),x]
```

```
[Out] (-2*(7 + 2*Coth[x]^2 + (5 + 5*I)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Coth[x])]]*
Sqrt[I*(1 + Coth[x])] + Csch[x]^2 + Coth[x]*(9 + Csch[x]^2)))/(5*Sqrt[1 + C
oth[x]])
```

Maple [A]
time = 0.68, size = 35, normalized size = 0.78

method	result	size
derivativedivides	$-\frac{2(1+\coth(x))^{\frac{5}{2}}}{5}+2\,\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}-2\sqrt{1+\coth(x)}$	35
default	$-\frac{2(1+\coth(x))^{\frac{5}{2}}}{5}+2\,\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}-2\sqrt{1+\coth(x)}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2*(1+coth(x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(1+coth(x))^(5/2)+2*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-2*(
1+coth(x))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(1+coth(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((coth(x) + 1)^(3/2)*coth(x)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 436 vs. $2(34) = 68$.
time = 0.38, size = 436, normalized size = 9.69

$$-\frac{2\sqrt{2}\left(9\sqrt{2}\cosh(x)^5+45\sqrt{2}\cosh(x)\sinh(x)^4+9\sqrt{2}\sinh(x)^5+10\left(9\sqrt{2}\cosh(x)^2-\sqrt{2}\right)\sinh(x)^3-10\sqrt{2}\cosh(x)^3+30\left(3\sqrt{2}\cosh(x)^3-\sqrt{2}\cosh(x)\right)\sinh(x)^2+5\left(9\sqrt{2}\cosh(x)^4-6\sqrt{2}\cosh(x)^2+\sqrt{2}\right)\sinh(x)+5\sqrt{2}\cosh(x)\right)\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}}-5\left(\sqrt{2}\cosh(x)^6+6\sqrt{2}\cosh(x)\sinh(x)^5+\sqrt{2}\sinh(x)^6+3\left(5\sqrt{2}\cosh(x)^2-\sqrt{2}\right)\sinh(x)^4-3\sqrt{2}\cosh(x)^4+4\left(5\sqrt{2}\cosh(x)^3-3\sqrt{2}\cosh(x)\right)\sinh(x)^3+3\left(5\sqrt{2}\cosh(x)^4-6\sqrt{2}\cosh(x)^2+\sqrt{2}\right)\sinh(x)^2+3\sqrt{2}\cosh(x)^2+6\left(\sqrt{2}\cosh(x)^5-2\sqrt{2}\cosh(x)^3+\sqrt{2}\cosh(x)\right)\sinh(x)-\sqrt{2}\right)\log\left(2\sqrt{2}\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}}(\cosh(x)+\sinh(x))+2\cosh(x)^2+4\cosh(x)\sinh(x)+2\sinh(x)^2-1\right)}{5\left(\cosh(x)^6+6\cosh(x)\sinh(x)^5+\sinh(x)^6+3\left(5\cosh(x)^2-1\right)\sinh(x)^4-3\cosh(x)^4+4\left(5\cosh(x)^3-3\cosh(x)\right)\sinh(x)^3+3\left(5\cosh(x)^4-6\cosh(x)^2+1\right)\sinh(x)^2+3\cosh(x)^2+6\left(\cosh(x)^5-2\cosh(x)^3+\cosh(x)\right)\sinh(x)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(1+coth(x))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/5*(2*sqrt(2)*(9*sqrt(2)*cosh(x)^5 + 45*sqrt(2)*cosh(x)*sinh(x)^4 + 9*sqr
t(2)*sinh(x)^5 + 10*(9*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x)^3 - 10*sqrt(2)*
cosh(x)^3 + 30*(3*sqrt(2)*cosh(x)^3 - sqrt(2)*cosh(x))*sinh(x)^2 + 5*(9*sqr
t(2)*cosh(x)^4 - 6*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x) + 5*sqrt(2)*cosh(x)
)*sqrt(sinh(x)/(cosh(x) - sinh(x))) - 5*(sqrt(2)*cosh(x)^6 + 6*sqrt(2)*cosh
(x)*sinh(x)^5 + sqrt(2)*sinh(x)^6 + 3*(5*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(
x)^4 - 3*sqrt(2)*cosh(x)^4 + 4*(5*sqrt(2)*cosh(x)^3 - 3*sqrt(2)*cosh(x))*si
nh(x)^3 + 3*(5*sqrt(2)*cosh(x)^4 - 6*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x)^2
 + 3*sqrt(2)*cosh(x)^2 + 6*(sqrt(2)*cosh(x)^5 - 2*sqrt(2)*cosh(x)^3 + sqrt(
2)*cosh(x))*sinh(x) - sqrt(2))*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x) - sinh(x
)))*(cosh(x) + sinh(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - 1
))/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 - 1)*sinh(
x)^4 - 3*cosh(x)^4 + 4*(5*cosh(x)^3 - 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4
 - 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 - 2*cosh(x)^3 +
cosh(x))*sinh(x) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (\coth(x)+1)^{\frac{3}{2}}\coth^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2*(1+coth(x))**(3/2),x)
```

```
[Out] Integral((coth(x) + 1)**(3/2)*coth(x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(34) = 68.
time = 0.42, size = 197, normalized size = 4.38

$$-\frac{1}{5}\sqrt{2}\left(5\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x)}+1\right|\right)\operatorname{sgn}\left(e^{(2x)}-1\right)+\frac{2\left(25\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^{4}\operatorname{sgn}\left(e^{(2x)}-1\right)+60\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^{3}\operatorname{sgn}\left(e^{(2x)}-1\right)+70\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^{2}\operatorname{sgn}\left(e^{(2x)}-1\right)+40\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)\operatorname{sgn}\left(e^{(2x)}-1\right)+9\operatorname{sgn}\left(e^{(2x)}-1\right)\right)}{\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}+1\right)^{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(1+coth(x))^(3/2),x, algorithm="giac")
```

```
[Out] -1/5*sqrt(2)*(5*log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1))*sgn(e^(
2*x) - 1) + 2*(25*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^4*sgn(e^(2*x) - 1) +
60*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^3*sgn(e^(2*x) - 1) + 70*(sqrt(e^(4*x
) - e^(2*x)) - e^(2*x))^2*sgn(e^(2*x) - 1) + 40*(sqrt(e^(4*x) - e^(2*x)) -
e^(2*x))*sgn(e^(2*x) - 1) + 9*sgn(e^(2*x) - 1))/(sqrt(e^(4*x) - e^(2*x)) -
e^(2*x) + 1)^5)
```

Mupad [B]
time = 1.25, size = 34, normalized size = 0.76

$$2\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{\coth\left(x\right)+1}}{2}\right)-2\sqrt{\coth\left(x\right)+1}-\frac{2\left(\coth(x)+1\right)^{5/2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2*(coth(x) + 1)^(3/2),x)
```

```
[Out] 2*2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2) - 2*(coth(x) + 1)^(1/2) -
(2*(coth(x) + 1)^(5/2))/5
```

3.137 $\int \coth^2(x)\sqrt{1+\coth(x)}\,dx$

Optimal. Leaf size=34

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)-\frac{2}{3}(1+\coth(x))^{3/2}$$

```
[Out] -2/3*(1+coth(x))^(3/2)+arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3624, 3561, 212}

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)-\frac{2}{3}(\coth(x)+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2*Sqrt[1 + Coth[x]],x]
```

```
[Out] Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - (2*(1 + Coth[x])^(3/2))/3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \coth^2(x)\sqrt{1+\coth(x)}\,dx &= -\frac{2}{3}(1+\coth(x))^{3/2}+\int \sqrt{1+\coth(x)}\,dx\\ &= -\frac{2}{3}(1+\coth(x))^{3/2}+2\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1+\coth(x)}\right)\\ &= \sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)-\frac{2}{3}(1+\coth(x))^{3/2}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.12, size = 61, normalized size = 1.79

$$\frac{-2-4\coth(x)-2\coth^2(x)-(3+3i)\text{ArcTan}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i(1+\coth(x))}\right)\sqrt{i(1+\coth(x))}}{3\sqrt{1+\coth(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2*Sqrt[1 + Coth[x]],x]
```

```
[Out] (-2 - 4*Coth[x] - 2*Coth[x]^2 - (3 + 3*I)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Co
th[x])]]*Sqrt[I*(1 + Coth[x])])/(3*Sqrt[1 + Coth[x]])
```

Maple [A]
time = 0.79, size = 26, normalized size = 0.76

method	result	size
derivativedivides	$-\frac{2(1+\coth(x))^{\frac{3}{2}}}{3}+\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}$	26
default	$-\frac{2(1+\coth(x))^{\frac{3}{2}}}{3}+\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2*(1+coth(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(1+coth(x))^(3/2)+arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(1+coth(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(coth(x) + 1)*coth(x)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(25) = 50$.
time = 0.36, size = 242, normalized size = 7.12

$$-\frac{8\sqrt{2}\left(\sqrt{2}\cosh(x)^3+3\sqrt{2}\cosh(x)^2\sinh(x)+3\sqrt{2}\cosh(x)\sinh(x)^2+\sqrt{2}\sinh(x)^3\right)\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}}-3\left(\sqrt{2}\cosh(x)^4+4\sqrt{2}\cosh(x)\sinh(x)^3+\sqrt{2}\sinh(x)^4+2\left(3\sqrt{2}\cosh(x)^2-\sqrt{2}\right)\sinh(x)^2-2\sqrt{2}\cosh(x)^2+4\left(\sqrt{2}\cosh(x)^3-\sqrt{2}\cosh(x)\right)\sinh(x)+\sqrt{2}\right)\log\left(2\sqrt{2}\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}}(\cosh(x)+\sinh(x))+2\cosh(x)^2+4\cosh(x)\sinh(x)+2\sinh(x)^2-1\right)}{6\left(\cosh(x)^4+4\cosh(x)\sinh(x)^3+\sinh(x)^4+2\left(3\cosh(x)^2-1\right)\sinh(x)^2-2\cosh(x)^2+4\left(\cosh(x)^3-\cosh(x)\right)\sinh(x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(1+coth(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(8*sqrt(2)*(sqrt(2)*cosh(x)^3 + 3*sqrt(2)*cosh(x)^2*sinh(x) + 3*sqrt(2
)*cosh(x)*sinh(x)^2 + sqrt(2)*sinh(x)^3)*sqrt(sinh(x)/(cosh(x) - sinh(x)))
- 3*(sqrt(2)*cosh(x)^4 + 4*sqrt(2)*cosh(x)*sinh(x)^3 + sqrt(2)*sinh(x)^4 +
2*(3*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x)^2 - 2*sqrt(2)*cosh(x)^2 + 4*(sqrt
(2)*cosh(x)^3 - sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*log(2*sqrt(2)*sqrt(sinh
(x)/(cosh(x) - sinh(x)))*(cosh(x) + sinh(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh
(x) + 2*sinh(x)^2 - 1))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3
*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) +
 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\coth(x)+1}\,\coth^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2*(1+coth(x))**(1/2),x)
```

```
[Out] Integral(sqrt(coth(x) + 1)*coth(x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(25) = 50$.
time = 0.43, size = 133, normalized size = 3.91

$$-\frac{1}{6}\sqrt{2}\left(3\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x)}+1\right|\right)\operatorname{sgn}\left(e^{(2x)}-1\right)+\frac{8\left(3\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^2\operatorname{sgn}\left(e^{(2x)}-1\right)+3\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)\operatorname{sgn}\left(e^{(2x)}-1\right)+\operatorname{sgn}\left(e^{(2x)}-1\right)\right)}{\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}+1\right)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(1+coth(x))^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(2)*(3*log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1))*sgn(e^(
2*x) - 1) + 8*(3*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^2*sgn(e^(2*x) - 1) + 3
*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))*sgn(e^(2*x) - 1) + sgn(e^(2*x) - 1))/(
sqrt(e^(4*x) - e^(2*x)) - e^(2*x) + 1)^3)
```

Mupad [B]
time = 1.19, size = 25, normalized size = 0.74

$$\sqrt{2}\ \mathrm{atanh}\left(\frac{\sqrt{2}\ \sqrt{\coth\left(x\right)+1}}{2}\right)-\frac{2\left(\coth(x)+1\right)^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2*(coth(x) + 1)^(1/2),x)
```

```
[Out] 2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2) - (2*(coth(x) + 1)^(3/2))/3
```

3.138 $\int \frac{\coth^2(x)}{\sqrt{1+\coth(x)}}\,dx$

Optimal. Leaf size=42

$$\frac{\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{\sqrt{1+\coth(x)}}-2\sqrt{1+\coth(x)}$$

```
[Out] 1/2*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-1/(1+coth(x))^(1/2)-2*(1
+coth(x))^(1/2)
```

Rubi [**A**]
time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3624, 3560, 3561, 212}

$$-2\sqrt{\coth(x)+1}-\frac{1}{\sqrt{\coth(x)+1}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/Sqrt[1 + Coth[x]],x]
```

```
[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/Sqrt[2] - 1/Sqrt[1 + Coth[x]] - 2*Sqrt[1
 + Coth[x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^2(x)}{\sqrt{1+\coth(x)}}\,dx &= -2\sqrt{1+\coth(x)}+\int \frac{1}{\sqrt{1+\coth(x)}}\,dx\\ &= -\frac{1}{\sqrt{1+\coth(x)}}-2\sqrt{1+\coth(x)}+\frac{1}{2}\int \sqrt{1+\coth(x)}\,dx\\ &= -\frac{1}{\sqrt{1+\coth(x)}}-2\sqrt{1+\coth(x)}+\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1+\coth(x)}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{\sqrt{1+\coth(x)}}-2\sqrt{1+\coth(x)}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.25, size = 81, normalized size = 1.93

$$\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\text{csch}(x)(\cosh(x)+\sinh(x))\left(-\frac{i\text{ArcTan}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i(1+\coth(x))}\right)}{\sqrt{i(1+\coth(x))}}+\left(\frac{1}{2}-\frac{i}{2}\right)(-5+\cosh(2x)-\sinh(2x))\right)}{\sqrt{1+\coth(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/Sqrt[1 + Coth[x]],x]
```

```
[Out] ((1/2 + I/2)*Csch[x]*(Cosh[x] + Sinh[x])*(((-I)*ArcTan[(1/2 + I/2)*Sqrt[I*(
1 + Coth[x])]])/Sqrt[I*(1 + Coth[x])] + (1/2 - I/2)*(-5 + Cosh[2*x] - Sinh[
2*x])))/Sqrt[1 + Coth[x]]
```

Maple [A]
time = 0.77, size = 35, normalized size = 0.83

method	result	size

derivativedivides	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth\left(x\right)}\,\sqrt{2}}{2}\right)\sqrt{2}}{2}-\frac{1}{\sqrt{1+\coth\left(x\right)}}-2\sqrt{1+\coth\left(x\right)}$	35
default	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth\left(x\right)}\,\sqrt{2}}{2}\right)\sqrt{2}}{2}-\frac{1}{\sqrt{1+\coth\left(x\right)}}-2\sqrt{1+\coth\left(x\right)}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(1+coth(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-1/(1+coth(x))^(1/2)-2*(1
+coth(x))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(1+coth(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)^2/sqrt(coth(x) + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(34) = 68$.
time = 0.36, size = 189, normalized size = 4.50

$$-\frac{2\sqrt{2}\left(5\sqrt{2}\cosh\left(x\right)^{2}+10\sqrt{2}\cosh\left(x\right)\sinh\left(x\right)+5\sqrt{2}\sinh\left(x\right)^{2}-\sqrt{2}\right)\sqrt{\frac{\sinh\left(x\right)}{\cosh\left(x\right)-\sinh\left(x\right)}}-\left(\sqrt{2}\cosh\left(x\right)^{3}+3\sqrt{2}\cosh\left(x\right)\sinh\left(x\right)^{2}+\sqrt{2}\sinh\left(x\right)^{3}+\left(3\sqrt{2}\cosh\left(x\right)^{2}-\sqrt{2}\right)\sinh\left(x\right)-\sqrt{2}\cosh\left(x\right)\right)\log\left(2\sqrt{2}\sqrt{\frac{\sinh\left(x\right)}{\cosh\left(x\right)-\sinh\left(x\right)}}\left(\cosh\left(x\right)+\sinh\left(x\right)\right)+2\cosh\left(x\right)^{2}+4\cosh\left(x\right)\sinh\left(x\right)+2\sinh\left(x\right)^{2}-1\right)}{4\left(\cosh\left(x\right)^{3}+3\cosh\left(x\right)\sinh\left(x\right)^{2}+\sinh\left(x\right)^{3}+\left(3\cosh\left(x\right)^{2}-1\right)\sinh\left(x\right)-\cosh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(1+coth(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(2)*(5*sqrt(2)*cosh(x)^2 + 10*sqrt(2)*cosh(x)*sinh(x) + 5*sqrt(
2)*sinh(x)^2 - sqrt(2))*sqrt(sinh(x)/(cosh(x) - sinh(x))) - (sqrt(2)*cosh(x
)^3 + 3*sqrt(2)*cosh(x)*sinh(x)^2 + sqrt(2)*sinh(x)^3 + (3*sqrt(2)*cosh(x)^
2 - sqrt(2))*sinh(x) - sqrt(2)*cosh(x))*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x)
 - sinh(x)))*(cosh(x) + sinh(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh
(x)^2 - 1))/(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 - 1
)*sinh(x) - cosh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^{2}\left(x\right)}{\sqrt{\coth\left(x\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(1+coth(x))**(1/2),x)
```

```
[Out] Integral(coth(x)**2/sqrt(coth(x) + 1), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(34) = 68$.
time = 0.42, size = 88, normalized size = 2.10

$$-\frac{\frac{5\sqrt{2}e^{(2x)}}{\operatorname{sgn}(e^{(2x)}-1)} - \frac{\sqrt{2}}{\operatorname{sgn}(e^{(2x)}-1)}}{2\sqrt{e^{(4x)} - e^{(2x)}}} - \frac{\sqrt{2}\log\left(\left|4\sqrt{e^{(4x)} - e^{(2x)}} - 4e^{(2x)} + 2\right|\right)}{4\operatorname{sgn}\left(e^{(2x)} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(1+coth(x))^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(5*sqrt(2)*e^(2*x)/sgn(e^(2*x) - 1) - sqrt(2)/sgn(e^(2*x) - 1))/sqrt(e
^(4*x) - e^(2*x)) - 1/4*sqrt(2)*log(abs(4*sqrt(e^(4*x) - e^(2*x)) - 4*e^(2*
x) + 2))/sgn(e^(2*x) - 1)
```

Mupad [B]
time = 1.26, size = 36, normalized size = 0.86

$$\frac{\sqrt{2}\,\operatorname{atanh}\left(\frac{\sqrt{2}\,\sqrt{\coth(x)+1}}{2}\right)}{2} - \frac{3}{\sqrt{\coth(x)+1}} - \frac{2\coth(x)}{\sqrt{\coth(x)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(coth(x) + 1)^(1/2),x)
```

```
[Out] (2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2))/2 - 3/(coth(x) + 1)^(1/2)
- (2*coth(x))/(coth(x) + 1)^(1/2)
```

3.139 $\int \frac{\coth^2(x)}{(1+\coth(x))^{3/2}}\,dx$

Optimal. Leaf size=49

$$\frac{\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{1}{3(1+\coth(x))^{3/2}}+\frac{3}{2\sqrt{1+\coth(x)}}$$

```
[Out] -1/3/(1+coth(x))^(3/2)+1/4*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)+3
/2/(1+coth(x))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3621, 3607, 3561, 212}

$$\frac{3}{2\sqrt{\coth(x)+1}}-\frac{1}{3(\coth(x)+1)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(1 + Coth[x])^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/(2*Sqrt[2]) - 1/(3*(1 + Coth[x])^(3/2))
+ 3/(2*Sqrt[1 + Coth[x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3607

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && LtQ[m, 0]
```

Rule 3621

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-b)*(a*c + b*d)^2*((a + b*Tan[e + f*x])^
m/(2*a^3*f*m)), x] + Dist[1/(2*a^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[
a*c^2 - 2*b*c*d + a*d^2 - 2*b*d^2*Tan[e + f*x], x], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LeQ[m, -1] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^2(x)}{(1+\coth(x))^{3/2}}\,dx &= -\frac{1}{3(1+\coth(x))^{3/2}}-\frac{1}{2}\int \frac{1-2\coth(x)}{\sqrt{1+\coth(x)}}\,dx\\
&= -\frac{1}{3(1+\coth(x))^{3/2}}+\frac{3}{2\sqrt{1+\coth(x)}}+\frac{1}{4}\int \sqrt{1+\coth(x)}\,dx\\
&= -\frac{1}{3(1+\coth(x))^{3/2}}+\frac{3}{2\sqrt{1+\coth(x)}}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1+\coth(x)}\right)\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{1}{3(1+\coth(x))^{3/2}}+\frac{3}{2\sqrt{1+\coth(x)}}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.24, size = 86, normalized size = 1.76

$$\left(\frac{1}{4}+\frac{i}{4}\right)\sqrt{1+\coth(x)}\left(-\frac{i\text{ArcTan}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i(1+\coth(x))}\right)}{\sqrt{i(1+\coth(x))}}-\left(\frac{1}{6}-\frac{i}{6}\right)(-8+7\cosh(2x)+\cosh(4x)-7\sinh(2x)-\sinh(4x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(1 + Coth[x])^(3/2),x]
```

```
[Out] (1/4 + I/4)*Sqrt[1 + Coth[x]]*(((-I)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Coth[x]
)]])/Sqrt[I*(1 + Coth[x])] - (1/6 - I/6)*(-8 + 7*Cosh[2*x] + Cosh[4*x] - 7*
Sinh[2*x] - Sinh[4*x]))
```

Maple [A]
time = 0.76, size = 35, normalized size = 0.71

method	result	size
derivativedivides	$-\frac{1}{3(1+\coth(x))^{\frac{3}{2}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{4}+\frac{3}{2\sqrt{1+\coth(x)}}$	35

default	$-\frac{1}{3(1+\coth(x))^{\frac{3}{2}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{4}+\frac{3}{2\sqrt{1+\coth(x)}}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(1+coth(x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/(1+coth(x))^(3/2)+1/4*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)+3
/2/(1+coth(x))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(1+coth(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)^2/(coth(x) + 1)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(34) = 68$.
time = 0.36, size = 166, normalized size = 3.39

$$\frac{2\sqrt{2}\left(8\sqrt{2}\cosh(x)^2+16\sqrt{2}\cosh(x)\sinh(x)+8\sqrt{2}\sinh(x)^2+\sqrt{2}\right)\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}}+3\left(\sqrt{2}\cosh(x)^3+3\sqrt{2}\cosh(x)^2\sinh(x)+3\sqrt{2}\cosh(x)\sinh(x)^2+\sqrt{2}\sinh(x)^3\right)\log\left(2\sqrt{2}\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}}(\cosh(x)+\sinh(x))+2\cosh(x)^2+4\cosh(x)\sinh(x)+2\sinh(x)^2-1\right)}{24\left(\cosh(x)^3+3\cosh(x)^2\sinh(x)+3\cosh(x)\sinh(x)^2+\sinh(x)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(1+coth(x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*(2*sqrt(2)*(8*sqrt(2)*cosh(x)^2 + 16*sqrt(2)*cosh(x)*sinh(x) + 8*sqrt(
2)*sinh(x)^2 + sqrt(2))*sqrt(sinh(x)/(cosh(x) - sinh(x))) + 3*(sqrt(2)*cosh
(x)^3 + 3*sqrt(2)*cosh(x)^2*sinh(x) + 3*sqrt(2)*cosh(x)*sinh(x)^2 + sqrt(2)
*sinh(x)^3)*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sinh
(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - 1))/(cosh(x)^3 + 3*c
osh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2 + sinh(x)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(x)}{(\coth(x)+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(1+coth(x))**(3/2),x)
```

[Out] Integral(coth(x)**2/(coth(x) + 1)**(3/2), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(34) = 68.
time = 0.42, size = 113, normalized size = 2.31

$$-\frac{\sqrt{2}\left(\frac{2\left(6\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^{2}-3\sqrt{e^{(4x)}-e^{(2x)}}+3e^{(2x)}-1\right)}{\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^{3}}+3\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x)}+1\right|\right)\right)}{24\,\mathrm{sgn}\left(e^{(2x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(1+coth(x))^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*sqrt(2)*(2*(6*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^2 - 3*sqrt(e^(4*x)
- e^(2*x)) + 3*e^(2*x) - 1)/(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^3 + 3*log(a
bs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1)))/sgn(e^(2*x) - 1)
```

Mupad [B]
time = 1.23, size = 31, normalized size = 0.63

$$\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{\coth(x)+1}}{2}\right)}{4}+\frac{\frac{3\coth(x)}{2}+\frac{7}{6}}{(\coth(x)+1)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(coth(x) + 1)^(3/2),x)
```

```
[Out] (2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2))/4 + ((3*coth(x))/2 + 7/6)/
(coth(x) + 1)^(3/2)
```

3.140 $\int \frac{\tanh^4(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=97

$$\frac{ax}{a^2-b^2}-\frac{b(a^2+b^2)\log(\cosh(x))}{a^4}-\frac{b^5\log(b\cosh(x)+a\sinh(x))}{a^4(a^2-b^2)}-\frac{(a^2+b^2)\tanh(x)}{a^3}+\frac{b\tanh^2(x)}{2a^2}-\frac{\tanh^3(x)}{3a}$$

```
[Out] a*x/(a^2-b^2)-b*(a^2+b^2)*ln(cosh(x))/a^4-b^5*ln(b*cosh(x)+a*sinh(x))/a^4/(
a^2-b^2)-(a^2+b^2)*tanh(x)/a^3+1/2*b*tanh(x)^2/a^2-1/3*tanh(x)^3/a
```

Rubi [A]
time = 0.36, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3650, 3730, 3731, 3732, 3611, 3556}

$$\frac{ax}{a^2-b^2}+\frac{b\tanh^2(x)}{2a^2}-\frac{b(a^2+b^2)\log(\cosh(x))}{a^4}-\frac{b^5\log(a\sinh(x)+b\cosh(x))}{a^4(a^2-b^2)}-\frac{(a^2+b^2)\tanh(x)}{a^3}-\frac{\tanh^3(x)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/(a + b*Coth[x]),x]
```

```
[Out] (a*x)/(a^2 - b^2) - (b*(a^2 + b^2)*Log[Cosh[x]])/a^4 - (b^5*Log[b*Cosh[x] +
 a*Sinh[x]])/(a^4*(a^2 - b^2)) - ((a^2 + b^2)*Tanh[x])/a^3 + (b*Tanh[x]^2)/
(2*a^2) - Tanh[x]^3/(3*a)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
```

```
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3731

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(A*b^2 + a^2*C)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)
*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp
[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)) - a*C*(b*c*(m + 1) + a*d*(n
+ 1)) - (m + 1)*(b*c - a*d)*(A*b - b*C)*Tan[e + f*x] - d*(A*b^2 + a^2*C)*(m
 + n + 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m,
 -1] &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3732

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{\tanh^4(x)}{a+b\coth(x)}\,dx = -\frac{\tanh^3(x)}{3a} - \frac{i\int \frac{(-3ib+3ia\coth(x)+3ib\coth^2(x))\tanh^3(x)}{a+b\coth(x)}dx}{3a}$$
$$= \frac{b\tanh^2(x)}{2a^2} - \frac{\tanh^3(x)}{3a} - \frac{\int \frac{(-6(a^2+b^2)+6b^2\coth^2(x))\tanh^2(x)}{a+b\coth(x)}dx}{6a^2}$$
$$= -\frac{(a^2+b^2)\tanh(x)}{a^3} + \frac{b\tanh^2(x)}{2a^2} - \frac{\tanh^3(x)}{3a} + \frac{i\int \frac{(6ib(a^2+b^2)-6ia^3\coth(x)-6ib(a^2+b^2)\coth}{a+b\coth(x)}}{6a^3}$$
$$= \frac{ax}{a^2-b^2} - \frac{(a^2+b^2)\tanh(x)}{a^3} + \frac{b\tanh^2(x)}{2a^2} - \frac{\tanh^3(x)}{3a} - \frac{(ib^5)\int \frac{-ib-ia\coth(x)}{a+b\coth(x)}dx}{a^4(a^2-b^2)} - (b($$
$$= \frac{ax}{a^2-b^2} - \frac{b(a^2+b^2)\log(\cosh(x))}{a^4} - \frac{b^5\log(b\cosh(x)+a\sinh(x))}{a^4(a^2-b^2)} - \frac{(a^2+b^2)\tanh(x}{a^3}$$

Mathematica [A]
time = 0.25, size = 105, normalized size = 1.08

$$\frac{6a^5x+6(-a^4b+b^5)\log(\cosh(x))-6b^5\log(b\cosh(x)+a\sinh(x))+(-8a^5+2a^3b^2+6ab^4)\tanh(x)+a^2(a^2-b^2)\operatorname{sech}^2(x)(-3b+2a\tanh(x))}{6a^4(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(a + b*Coth[x]),x]
```

```
[Out] (6*a^5*x + 6*(-(a^4*b) + b^5)*Log[Cosh[x]] - 6*b^5*Log[b*Cosh[x] + a*Sinh[x
]] + (-8*a^5 + 2*a^3*b^2 + 6*a*b^4)*Tanh[x] + a^2*(a^2 - b^2)*Sech[x]^2*(-3
*b + 2*a*Tanh[x]))/(6*a^4*(a - b)*(a + b))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(93) = 186.
time = 0.68, size = 190, normalized size = 1.96

method	result
risch	$\frac{x}{a+b}+\frac{2bx}{a^2}+\frac{2x\,b^3}{a^4}+\frac{2x\,b^5}{a^4(a^2-b^2)}+\frac{4a^2e^{4x}-2ab\,e^{4x}+2b^2e^{4x}+4a^2e^{2x}-2ab\,e^{2x}+4b^2e^{2x}+\frac{8a^2}{3}+2b^2}{a^3(1+e^{2x})^3}-\frac{b\ln(1+e^{2x})}{a^2}-\frac{b^3\ln(1}{a}$
default	$-\frac{b^5\ln\left(b\left(\tanh^2\left(\frac{x}{2}\right)\right)+2a\tanh\left(\frac{x}{2}\right)+b\right)}{(a+b)(a-b)a^4}+\frac{64\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{64a-64b}-\frac{64\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{64a+64b}+\frac{2\left(\left(-a^3-a\,b^2\right)\left(\tanh^5\left(\frac{x}{2}\right)\right)+a^2b\left(\tanh^4\left(\frac{x}{2}\right)\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -b^5/(a+b)/(a-b)/a^4*ln(b*tanh(1/2*x)^2+2*a*tanh(1/2*x)+b)+64/(64*a-64*b)*l
n(tanh(1/2*x)+1)-64/(64*a+64*b)*ln(tanh(1/2*x)-1)+2/a^4*(((-a^3-a*b^2)*tanh
```

```
(1/2*x)^5+a^2*b*tanh(1/2*x)^4+(-10/3*a^3-2*a*b^2)*tanh(1/2*x)^3+a^2*b*tanh(
1/2*x)^2+(-a^3-a*b^2)*tanh(1/2*x))/(tanh(1/2*x)^2+1)^3-1/2*b*(a^2+b^2)*ln(t
anh(1/2*x)^2+1))
```

Maxima [A]
time = 0.51, size = 146, normalized size = 1.51

$$-\frac{b^5\log\left(-(a-b)e^{(-2x)}+a+b\right)}{a^6-a^4b^2}-\frac{2\left(4a^2+3b^2+3\left(2a^2+ab+2b^2\right)e^{(-2x)}+3\left(2a^2+ab+b^2\right)e^{(-4x)}\right)}{3\left(3a^3e^{(-2x)}+3a^3e^{(-4x)}+a^3e^{(-6x)}+a^3\right)}+\frac{x}{a+b}-\frac{(a^2b+b^3)\log\left(e^{(-2x)}+1\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] -b^5*log(-(a - b)*e^(-2*x) + a + b)/(a^6 - a^4*b^2) - 2/3*(4*a^2 + 3*b^2 +
3*(2*a^2 + a*b + 2*b^2)*e^(-2*x) + 3*(2*a^2 + a*b + b^2)*e^(-4*x))/(3*a^3*e
^(-2*x) + 3*a^3*e^(-4*x) + a^3*e^(-6*x) + a^3) + x/(a + b) - (a^2*b + b^3)*
log(e^(-2*x) + 1)/a^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1294 vs. $2(93) = 186$.
time = 0.40, size = 1294, normalized size = 13.34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a^5 + a^4*b)*x*cosh(x)^6 + 18*(a^5 + a^4*b)*x*cosh(x)*sinh(x)^5 + 3
*(a^5 + a^4*b)*x*sinh(x)^6 + 8*a^5 - 2*a^3*b^2 - 6*a*b^4 + 3*(4*a^5 - 2*a^4
*b - 2*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 + 3*(a^5 + a^4*b)*x)*cosh(x)^4 + 3*(4*
a^5 - 2*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 + 15*(a^5 + a^4*b)*x*cosh(x
)^2 + 3*(a^5 + a^4*b)*x)*sinh(x)^4 + 12*(5*(a^5 + a^4*b)*x*cosh(x)^3 + (4*a
^5 - 2*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 + 3*(a^5 + a^4*b)*x)*cosh(x)
)*sinh(x)^3 + 3*(4*a^5 - 2*a^4*b + 2*a^2*b^3 - 4*a*b^4 + 3*(a^5 + a^4*b)*x)
*cosh(x)^2 + 3*(15*(a^5 + a^4*b)*x*cosh(x)^4 + 4*a^5 - 2*a^4*b + 2*a^2*b^3
- 4*a*b^4 + 6*(4*a^5 - 2*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 + 3*(a^5 +
 a^4*b)*x)*cosh(x)^2 + 3*(a^5 + a^4*b)*x)*sinh(x)^2 + 3*(a^5 + a^4*b)*x - 3
*(b^5*cosh(x)^6 + 6*b^5*cosh(x)*sinh(x)^5 + b^5*sinh(x)^6 + 3*b^5*cosh(x)^4
 + 3*b^5*cosh(x)^2 + b^5 + 3*(5*b^5*cosh(x)^2 + b^5)*sinh(x)^4 + 4*(5*b^5*c
osh(x)^3 + 3*b^5*cosh(x))*sinh(x)^3 + 3*(5*b^5*cosh(x)^4 + 6*b^5*cosh(x)^2
+ b^5)*sinh(x)^2 + 6*(b^5*cosh(x)^5 + 2*b^5*cosh(x)^3 + b^5*cosh(x))*sinh(x
))*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) - 3*((a^4*b - b^5)*co
sh(x)^6 + 6*(a^4*b - b^5)*cosh(x)*sinh(x)^5 + (a^4*b - b^5)*sinh(x)^6 + a^4
*b - b^5 + 3*(a^4*b - b^5)*cosh(x)^4 + 3*(a^4*b - b^5 + 5*(a^4*b - b^5)*cos
h(x)^2)*sinh(x)^4 + 4*(5*(a^4*b - b^5)*cosh(x)^3 + 3*(a^4*b - b^5)*cosh(x))
*sinh(x)^3 + 3*(a^4*b - b^5)*cosh(x)^2 + 3*(a^4*b - b^5 + 5*(a^4*b - b^5)*c
```

```
osh(x)^4 + 6*(a^4*b - b^5)*cosh(x)^2)*sinh(x)^2 + 6*((a^4*b - b^5)*cosh(x)^
5 + 2*(a^4*b - b^5)*cosh(x)^3 + (a^4*b - b^5)*cosh(x))*sinh(x))*log(2*cosh(
x)/(cosh(x) - sinh(x))) + 6*(3*(a^5 + a^4*b)*x*cosh(x)^5 + 2*(4*a^5 - 2*a^4
*b - 2*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 + 3*(a^5 + a^4*b)*x)*cosh(x)^3 + (4*a^
5 - 2*a^4*b + 2*a^2*b^3 - 4*a*b^4 + 3*(a^5 + a^4*b)*x)*cosh(x))*sinh(x))/((
a^6 - a^4*b^2)*cosh(x)^6 + 6*(a^6 - a^4*b^2)*cosh(x)*sinh(x)^5 + (a^6 - a^4
*b^2)*sinh(x)^6 + a^6 - a^4*b^2 + 3*(a^6 - a^4*b^2)*cosh(x)^4 + 3*(a^6 - a^
4*b^2 + 5*(a^6 - a^4*b^2)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^6 - a^4*b^2)*cosh(
x)^3 + 3*(a^6 - a^4*b^2)*cosh(x))*sinh(x)^3 + 3*(a^6 - a^4*b^2)*cosh(x)^2 +
 3*(a^6 - a^4*b^2 + 5*(a^6 - a^4*b^2)*cosh(x)^4 + 6*(a^6 - a^4*b^2)*cosh(x)
^2)*sinh(x)^2 + 6*((a^6 - a^4*b^2)*cosh(x)^5 + 2*(a^6 - a^4*b^2)*cosh(x)^3
+ (a^6 - a^4*b^2)*cosh(x))*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(x)}{a + b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(a+b*coth(x)),x)
```

```
[Out] Integral(tanh(x)**4/(a + b*coth(x)), x)
```

Giac [A]
time = 0.41, size = 141, normalized size = 1.45

$$-\frac{b^5\log\left(\left|ae^{(2x)} + be^{(2x)} - a + b\right|\right)}{a^6 - a^4b^2} + \frac{x}{a - b} - \frac{(a^2b + b^3)\log\left(e^{(2x)} + 1\right)}{a^4} + \frac{2\left(4a^3 + 3ab^2 + 3(2a^3 - a^2b + ab^2)e^{(4x)} + 3(2a^3 - a^2b + 2ab^2)e^{(2x)}\right)}{3a^4\left(e^{(2x)} + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -b^5*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^6 - a^4*b^2) + x/(a - b) -
(a^2*b + b^3)*log(e^(2*x) + 1)/a^4 + 2/3*(4*a^3 + 3*a*b^2 + 3*(2*a^3 - a^2*
b + a*b^2)*e^(4*x) + 3*(2*a^3 - a^2*b + 2*a*b^2)*e^(2*x))/(a^4*(e^(2*x) + 1
)^3)
```

Mupad [B]
time = 1.61, size = 163, normalized size = 1.68

$$\frac{8}{3a\left(3e^{2x} + 3e^{4x} + e^{6x} + 1\right)} + \frac{x}{a - b} - \frac{b^5\ln\left(b - a + a\,e^{2x} + b\,e^{2x}\right)}{a^6 - a^4b^2} - \frac{\ln\left(e^{2x} + 1\right)\left(a^2b + b^3\right)}{a^4} + \frac{2\left(2a^3 + a^2b + b^3\right)}{a^3\left(a + b\right)\left(e^{2x} + 1\right)} - \frac{2\left(2a^2 + ab - b^2\right)}{a^2\left(a + b\right)\left(2e^{2x} + e^{4x} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a + b*coth(x)),x)
```

```
[Out] 8/(3*a*(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1)) + x/(a - b) - (b^5*log(b -
 a + a*exp(2*x) + b*exp(2*x)))/(a^6 - a^4*b^2) - (log(exp(2*x) + 1)*(a^2*b
+ b^3))/a^4 + (2*(a^2*b + 2*a^3 + b^3))/(a^3*(a + b)*(exp(2*x) + 1)) - (2*(
a*b + 2*a^2 - b^2))/(a^2*(a + b)*(2*exp(2*x) + exp(4*x) + 1))
```

3.141 $\int \frac{\tanh^3(x)}{a+b\coth(x)} dx$

Optimal. Leaf size=76

$$-\frac{bx}{a^2-b^2}+\frac{(a^2+b^2)\log(\cosh(x))}{a^3}+\frac{b^4\log(b\cosh(x)+a\sinh(x))}{a^3(a^2-b^2)}+\frac{b\tanh(x)}{a^2}-\frac{\tanh^2(x)}{2a}$$

```
[Out] -b*x/(a^2-b^2)+(a^2+b^2)*ln(cosh(x))/a^3+b^4*ln(b*cosh(x)+a*sinh(x))/a^3/(a
^2-b^2)+b*tanh(x)/a^2-1/2*tanh(x)^2/a
```

Rubi [A]
time = 0.22, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3650, 3730, 3733, 3611, 3556}

$$-\frac{bx}{a^2-b^2}+\frac{b\tanh(x)}{a^2}+\frac{(a^2+b^2)\log(\cosh(x))}{a^3}+\frac{b^4\log(a\sinh(x)+b\cosh(x))}{a^3(a^2-b^2)}-\frac{\tanh^2(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/(a + b*Coth[x]),x]
```

```
[Out] -((b*x)/(a^2 - b^2)) + ((a^2 + b^2)*Log[Cosh[x]])/a^3 + (b^4*Log[b*Cosh[x]
+ a*Sinh[x]])/(a^3*(a^2 - b^2)) + (b*Tanh[x])/a^2 - Tanh[x]^2/(2*a)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
```

```
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3733

```
Int[((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2)/(((a_) + (b_.)*tan[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(a*(
A*c - c*C) - b*(A*d - C*d))*(x/((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^
2 + a^2*C)/((b*c - a*d)*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e
 + f*x]), x], x] - Dist[(c^2*C + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d -
 c*Tan[e + f*x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f,
A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^3(x)}{a+b\coth(x)}\,dx &= -\frac{\tanh^2(x)}{2a}-\frac{i\int \frac{(-2ib+2ia\coth(x)+2ib\coth^2(x))\tanh^2(x)}{a+b\coth(x)}\,dx}{2a}\\ &= \frac{b\tanh(x)}{a^2}-\frac{\tanh^2(x)}{2a}-\frac{\int \frac{(-2(a^2+b^2)+2b^2\coth^2(x))\tanh(x)}{a+b\coth(x)}\,dx}{2a^2}\\ &= -\frac{bx}{a^2-b^2}+\frac{b\tanh(x)}{a^2}-\frac{\tanh^2(x)}{2a}+\frac{(ib^4)\int \frac{-ib-ia\coth(x)}{a+b\coth(x)}\,dx}{a^3(a^2-b^2)}+\frac{(a^2+b^2)\int \tanh(x)\,dx}{a^3}\\ &= -\frac{bx}{a^2-b^2}+\frac{(a^2+b^2)\log(\cosh(x))}{a^3}+\frac{b^4\log(b\cosh(x)+a\sinh(x))}{a^3(a^2-b^2)}+\frac{b\tanh(x)}{a^2}-\frac{\tanh^2(x)}{2a}\end{aligned}$$

Mathematica [A]
time = 0.23, size = 88, normalized size = 1.16

$$\frac{a^2(a^2-b^2)\operatorname{sech}^2(x)+2(-a^3bx+(a^4-b^4)\log(\cosh(x))+b^4\log(b\cosh(x)+a\sinh(x))+ab(a^2-b^2)\tanh(x))}{2a^3(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(a + b*Coth[x]),x]
```

```
[Out] (a^2*(a^2 - b^2)*Sech[x]^2 + 2*(-(a^3*b*x) + (a^4 - b^4)*Log[Cosh[x]] + b^4
*Log[b*Cosh[x] + a*Sinh[x]] + a*b*(a^2 - b^2)*Tanh[x]))/(2*a^3*(a - b)*(a +
 b))
```

Maple [A]
time = 0.66, size = 138, normalized size = 1.82

method	result
risch	$\frac{x}{a+b}-\frac{2x}{a}-\frac{2x\,b^{2}}{a^{3}}-\frac{2x\,b^{4}}{a^{3}(a^{2}-b^{2})}+\frac{2\,\mathrm{e}^{2x}a-2b\,\mathrm{e}^{2x}-2b}{(1+\mathrm{e}^{2x})^{2}a^{2}}+\frac{\ln(1+\mathrm{e}^{2x})}{a}+\frac{\ln(1+\mathrm{e}^{2x})b^{2}}{a^{3}}+\frac{b^{4}\ln\left(\mathrm{e}^{2x}-\frac{a-b}{a+b}\right)}{a^{3}(a^{2}-b^{2})}$
default	$-\frac{32\ln(\tanh(\frac{x}{2})+1)}{32a-32b}-\frac{32\ln(\tanh(\frac{x}{2})-1)}{32a+32b}+\frac{b^{4}\ln(b(\tanh^{2}(\frac{x}{2}))+2a\tanh(\frac{x}{2})+b)}{(a+b)(a-b)a^{3}}+\frac{\frac{2\left(ab\left(\tanh^{3}(\frac{x}{2})\right)-a^{2}\left(\tanh^{2}(\frac{x}{2})\right)+\tanh(\frac{x}{2})ba\right)}{\left(\tanh^{2}(\frac{x}{2})+1\right)^{2}}}{a^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -32/(32*a-32*b)*ln(tanh(1/2*x)+1)-32/(32*a+32*b)*ln(tanh(1/2*x)-1)+b^4/(a+b
)/(a-b)/a^3*ln(b*tanh(1/2*x)^2+2*a*tanh(1/2*x)+b)+2/a^3*((a*b*tanh(1/2*x)^3
-a^2*tanh(1/2*x)^2+tanh(1/2*x)*b*a)/(tanh(1/2*x)^2+1)^2+1/2*(a^2+b^2)*ln(ta
nh(1/2*x)^2+1))
```

Maxima [A]
time = 0.51, size = 94, normalized size = 1.24

$$\frac{b^{4}\log\left(-(a-b)e^{(-2\,x)}+a+b\right)}{a^{5}-a^{3}b^{2}}+\frac{2\left((a+b)e^{(-2\,x)}+b\right)}{2\,a^{2}e^{(-2\,x)}+a^{2}e^{(-4\,x)}+a^{2}}+\frac{x}{a+b}+\frac{(a^{2}+b^{2})\log\left(e^{(-2\,x)}+1\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] b^4*log(-(a - b)*e^(-2*x) + a + b)/(a^5 - a^3*b^2) + 2*((a + b)*e^(-2*x) +
b)/(2*a^2*e^(-2*x) + a^2*e^(-4*x) + a^2) + x/(a + b) + (a^2 + b^2)*log(e^(-
2*x) + 1)/a^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 637 vs. $2(74) = 148$.
time = 0.40, size = 637, normalized size = 8.38

$$-\frac{(a^{4}+a^{3}b)x\cosh(x)^{4}+4(a^{4}+a^{3}b)x\cosh(x)\sinh(x)^{3}+(a^{4}+a^{3}b)x\sinh(x)^{4}+2a^{3}b-2ab^{3}-2(a^{4}-a^{3}b-a^{2}b^{2}+ab^{3}-(a^{4}+a^{3}b)x)\cosh(x)^{2}-2(a^{4}-a^{3}b-a^{2}b^{2}+ab^{3}-3(a^{4}+a^{3}b)x\cosh(x)^{2}-(a^{4}+a^{3}b)x)\sinh(x)^{2}+(a^{4}+a^{3}b)x-(b^{4}\cosh(x)^{4}+4b^{4}\cosh(x)\sinh(x)^{3}+b^{4}\sinh(x)^{4}+2b^{4}\cosh(x)^{2}+b^{4}+2(3b^{4}\cosh(x)^{2}+b^{4})\sinh(x)^{2}+4(b^{4}\cosh(x)^{3}+b^{4}\cosh(x))\sinh(x))\log\left(\frac{2(b\cosh(x)+a\sinh(x))}{\cosh(x)-\sinh(x)}\right)-((a^{4}-b^{4})\cosh(x)^{4}+4(a^{4}-b^{4})\cosh(x)\sinh(x)^{3}+(a^{4}-b^{4})\sinh(x)^{4}+a^{4}-b^{4}+2(a^{4}-b^{4})\cosh(x)^{2}+2(a^{4}-b^{4}+3(a^{4}-b^{4})\cosh(x)^{2})\sinh(x)^{2}+4((a^{4}-b^{4})\cosh(x)^{3}+(a^{4}-b^{4})\cosh(x))\sinh(x))\log\left(\frac{2\cosh(x)}{\cosh(x)-\sinh(x)}\right)+4((a^{4}+a^{3}b)x\cosh(x)^{3}-(a^{4}-a^{3}b-a^{2}b^{2}+ab^{3}-(a^{4}+a^{3}b)x)\cosh(x))\sinh(x)}{a^{5}-a^{3}b^{2}+(a^{5}-a^{3}b^{2})\cosh(x)^{4}+4(a^{5}-a^{3}b^{2})\cosh(x)\sinh(x)^{3}+(a^{5}-a^{3}b^{2})\sinh(x)^{4}+2(a^{5}-a^{3}b^{2})\cosh(x)^{2}+2(a^{5}-a^{3}b^{2}+3(a^{5}-a^{3}b^{2})\cosh(x)^{2})\sinh(x)^{2}+4((a^{5}-a^{3}b^{2})\cosh(x)^{3}+(a^{5}-a^{3}b^{2})\cosh(x))\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] -((a^4 + a^3*b)*x*cosh(x)^4 + 4*(a^4 + a^3*b)*x*cosh(x)*sinh(x)^3 + (a^4 +
a^3*b)*x*sinh(x)^4 + 2*a^3*b - 2*a*b^3 - 2*(a^4 - a^3*b - a^2*b^2 + a*b^3 -
 (a^4 + a^3*b)*x)*cosh(x)^2 - 2*(a^4 - a^3*b - a^2*b^2 + a*b^3 - 3*(a^4 + a
^3*b)*x*cosh(x)^2 - (a^4 + a^3*b)*x)*sinh(x)^2 + (a^4 + a^3*b)*x - (b^4*cos
h(x)^4 + 4*b^4*cosh(x)*sinh(x)^3 + b^4*sinh(x)^4 + 2*b^4*cosh(x)^2 + b^4 +
2*(3*b^4*cosh(x)^2 + b^4)*sinh(x)^2 + 4*(b^4*cosh(x)^3 + b^4*cosh(x))*sinh(
x))*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) - ((a^4 - b^4)*cosh(
x)^4 + 4*(a^4 - b^4)*cosh(x)*sinh(x)^3 + (a^4 - b^4)*sinh(x)^4 + a^4 - b^4
+ 2*(a^4 - b^4)*cosh(x)^2 + 2*(a^4 - b^4 + 3*(a^4 - b^4)*cosh(x)^2)*sinh(x)
^2 + 4*((a^4 - b^4)*cosh(x)^3 + (a^4 - b^4)*cosh(x))*sinh(x))*log(2*cosh(x)
/(cosh(x) - sinh(x))) + 4*((a^4 + a^3*b)*x*cosh(x)^3 - (a^4 - a^3*b - a^2*b
^2 + a*b^3 - (a^4 + a^3*b)*x)*cosh(x))*sinh(x))/(a^5 - a^3*b^2 + (a^5 - a^3
*b^2)*cosh(x)^4 + 4*(a^5 - a^3*b^2)*cosh(x)*sinh(x)^3 + (a^5 - a^3*b^2)*sin
h(x)^4 + 2*(a^5 - a^3*b^2)*cosh(x)^2 + 2*(a^5 - a^3*b^2 + 3*(a^5 - a^3*b^2)
*cosh(x)^2)*sinh(x)^2 + 4*((a^5 - a^3*b^2)*cosh(x)^3 + (a^5 - a^3*b^2)*cosh
(x))*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(x)}{a + b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(a+b*coth(x)),x)
```

```
[Out] Integral(tanh(x)**3/(a + b*coth(x)), x)
```

Giac [A]
time = 0.41, size = 97, normalized size = 1.28

$$\frac{b^4\log\left(\left|ae^{(2x)} + be^{(2x)} - a + b\right|\right)}{a^5 - a^3b^2} - \frac{x}{a - b} + \frac{(a^2 + b^2)\log\left(e^{(2x)} + 1\right)}{a^3} - \frac{2\left(ab - (a^2 - ab)e^{(2x)}\right)}{a^3\left(e^{(2x)} + 1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] b^4*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^5 - a^3*b^2) - x/(a - b) + (
a^2 + b^2)*log(e^(2*x) + 1)/a^3 - 2*(a*b - (a^2 - a*b)*e^(2*x))/(a^3*(e^(2*
x) + 1)^2)
```

Mupad [B]
time = 1.51, size = 111, normalized size = 1.46

$$\frac{\ln\left(e^{2x} + 1\right)\,\left(a^2 + b^2\right)}{a^3} - \frac{x}{a - b} - \frac{2}{a\,\left(2\,e^{2x} + e^{4x} + 1\right)} + \frac{b^4\,\ln\left(b - a + a\,e^{2x} + b\,e^{2x}\right)}{a^5 - a^3\,b^2} + \frac{2\,\left(a^2 - b^2\right)}{a^2\,\left(a + b\right)\,\left(e^{2x} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a + b*coth(x)),x)
```

```
[Out] (log(exp(2*x) + 1)*(a^2 + b^2))/a^3 - x/(a - b) - 2/(a*(2*exp(2*x) + exp(4*
x) + 1)) + (b^4*log(b - a + a*exp(2*x) + b*exp(2*x)))/(a^5 - a^3*b^2) + (2*
(a^2 - b^2))/(a^2*(a + b)*(exp(2*x) + 1))
```

3.142 $\int \frac{\tanh^2(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=60

$$\frac{ax}{a^2-b^2}-\frac{b\log(\cosh(x))}{a^2}-\frac{b^3\log(b\cosh(x)+a\sinh(x))}{a^2\,(a^2-b^2)}-\frac{\tanh(x)}{a}$$

```
[Out] a*x/(a^2-b^2)-b*ln(cosh(x))/a^2-b^3*ln(b*cosh(x)+a*sinh(x))/a^2/(a^2-b^2)-t
anh(x)/a
```

Rubi [A]
time = 0.13, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3650, 3732, 3611, 3556}

$$\frac{ax}{a^2-b^2}-\frac{b^3\log(a\sinh(x)+b\cosh(x))}{a^2\,(a^2-b^2)}-\frac{b\log(\cosh(x))}{a^2}-\frac{\tanh(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(a + b*Coth[x]),x]
```

```
[Out] (a*x)/(a^2 - b^2) - (b*Log[Cosh[x]])/a^2 - (b^3*Log[b*Cosh[x] + a*Sinh[x]])
/(a^2*(a^2 - b^2)) - Tanh[x]/a
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
```

```
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0]))
```

Rule 3732

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^2(x)}{a+b\coth(x)}\,dx &= -\frac{\tanh(x)}{a}-\frac{i\int \frac{(-ib+ia\coth(x)+ib\coth^2(x))\tanh(x)}{a+b\coth(x)}\,dx}{a}\\ &= \frac{ax}{a^2-b^2}-\frac{\tanh(x)}{a}-\frac{b\int \tanh(x)\,dx}{a^2}-\frac{(ib^3)\int \frac{-ib-ia\coth(x)}{a+b\coth(x)}\,dx}{a^2\,(a^2-b^2)}\\ &= \frac{ax}{a^2-b^2}-\frac{b\log(\cosh(x))}{a^2}-\frac{b^3\log(b\cosh(x)+a\sinh(x))}{a^2\,(a^2-b^2)}-\frac{\tanh(x)}{a}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 64, normalized size = 1.07

$$\frac{a^3x+(-a^2b+b^3)\log(\cosh(x))-b^3\log(b\cosh(x)+a\sinh(x))+(-a^3+ab^2)\tanh(x)}{a^4-a^2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(a + b*Coth[x]),x]
```

```
[Out] (a^3*x + (-(a^2*b) + b^3)*Log[Cosh[x]] - b^3*Log[b*Cosh[x] + a*Sinh[x]] + (
-a^3 + a*b^2)*Tanh[x])/(a^4 - a^2*b^2)
```

Maple [A]
time = 0.64, size = 111, normalized size = 1.85

method	result	size
risch	$\frac{x}{a+b}+\frac{2bx}{a^2}+\frac{2x\,b^3}{a^2(a^2-b^2)}+\frac{2}{a(1+e^{2x})}-\frac{b\ln(1+e^{2x})}{a^2}-\frac{b^3\ln\left(e^{2x}-\frac{a-b}{a+b}\right)}{a^2(a^2-b^2)}$	99

default	$\frac{16\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{16a-16b}+\frac{-\frac{2a\tanh\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)+1}-b\ln\left(\tanh^2\left(\frac{x}{2}\right)+1\right)}{a^2}-\frac{16\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{16a+16b}-\frac{b^3\ln\left(b\left(\tanh^2\left(\frac{x}{2}\right)\right)+2a\tanh\left(\frac{x}{2}\right)+b\right)}{(a+b)(a-b)a^2}$	111

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 16/(16*a-16*b)*ln(tanh(1/2*x)+1)+2/a^2*(-a*tanh(1/2*x)/(tanh(1/2*x)^2+1)-1/
2*b*ln(tanh(1/2*x)^2+1))-16/(16*a+16*b)*ln(tanh(1/2*x)-1)-b^3/(a+b)/(a-b)/a
^2*ln(b*tanh(1/2*x)^2+2*a*tanh(1/2*x)+b)
```

Maxima [**A**]
time = 0.48, size = 67, normalized size = 1.12

$$-\frac{b^3\log\left(-(a-b)e^{(-2x)}+a+b\right)}{a^4-a^2b^2}+\frac{x}{a+b}-\frac{b\log\left(e^{(-2x)}+1\right)}{a^2}-\frac{2}{ae^{(-2x)}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] -b^3*log(-(a - b)*e^(-2*x) + a + b)/(a^4 - a^2*b^2) + x/(a + b) - b*log(e^(
-2*x) + 1)/a^2 - 2/(a*e^(-2*x) + a)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(60) = 120$.
time = 0.37, size = 264, normalized size = 4.40

$$\frac{(a^3+a^2b)x\cosh(x)^2+2(a^3+a^2b)x\cosh(x)\sinh(x)+(a^3+a^2b)x\sinh(x)^2+2a^3-2ab^2+(a^3+a^2b)x-\left(b^3\cosh(x)^2+2b^3\cosh(x)\sinh(x)+b^3\sinh(x)^2+b^3\right)\log\left(\frac{2(b\cosh(x)+a\sinh(x))}{\cosh(x)-\sinh(x)}\right)-\left(a^2b-b^3+(a^2b-b^3)\cosh(x)^2+2(a^2b-b^3)\cosh(x)\sinh(x)+(a^2b-b^3)\sinh(x)^2\right)\log\left(\frac{2\cosh(x)}{\cosh(x)-\sinh(x)}\right)}{a^4-a^2b^2+(a^4-a^2b^2)\cosh(x)^2+2(a^4-a^2b^2)\cosh(x)\sinh(x)+(a^4-a^2b^2)\sinh(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] ((a^3 + a^2*b)*x*cosh(x)^2 + 2*(a^3 + a^2*b)*x*cosh(x)*sinh(x) + (a^3 + a^2
*b)*x*sinh(x)^2 + 2*a^3 - 2*a*b^2 + (a^3 + a^2*b)*x - (b^3*cosh(x)^2 + 2*b^
3*cosh(x)*sinh(x) + b^3*sinh(x)^2 + b^3)*log(2*(b*cosh(x) + a*sinh(x))/(cos
h(x) - sinh(x))) - (a^2*b - b^3 + (a^2*b - b^3)*cosh(x)^2 + 2*(a^2*b - b^3)
*cosh(x)*sinh(x) + (a^2*b - b^3)*sinh(x)^2)*log(2*cosh(x)/(cosh(x) - sinh(x
))))/(a^4 - a^2*b^2 + (a^4 - a^2*b^2)*cosh(x)^2 + 2*(a^4 - a^2*b^2)*cosh(x)
*sinh(x) + (a^4 - a^2*b^2)*sinh(x)^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tanh^2(x)}{a+b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(a+b*coth(x)),x)
```

```
[Out] Integral(tanh(x)**2/(a + b*coth(x)), x)
```

Giac [A]
time = 0.42, size = 74, normalized size = 1.23

$$-\frac{b^3 \log \left(\left|ae^{(2x)} + be^{(2x)} - a + b\right|\right)}{a^4 - a^2b^2} + \frac{x}{a-b} - \frac{b \log \left(e^{(2x)} + 1\right)}{a^2} + \frac{2}{a(e^{(2x)} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -b^3*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^4 - a^2*b^2) + x/(a - b) -
b*log(e^(2*x) + 1)/a^2 + 2/(a*(e^(2*x) + 1))
```

Mupad [B]
time = 1.45, size = 73, normalized size = 1.22

$$\frac{2}{a\,\left(\mathrm{e}^{2\,x}+1\right)}+\frac{x}{a-b}-\frac{b^3\,\ln\left(b-a+a\,\mathrm{e}^{2\,x}+b\,\mathrm{e}^{2\,x}\right)}{a^4-a^2\,b^2}-\frac{b\,\ln\left(\mathrm{e}^{2\,x}+1\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a + b*coth(x)),x)
```

```
[Out] 2/(a*(exp(2*x) + 1)) + x/(a - b) - (b^3*log(b - a + a*exp(2*x) + b*exp(2*x)
))/(a^4 - a^2*b^2) - (b*log(exp(2*x) + 1))/a^2
```

3.143 $\int \frac{\tanh(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=51

$$-\frac{bx}{a^2-b^2}+\frac{\log(\cosh(x))}{a}+\frac{b^2\log(b\cosh(x)+a\sinh(x))}{a\,(a^2-b^2)}$$

```
[Out] -b*x/(a^2-b^2)+ln(cosh(x))/a+b^2*ln(b*cosh(x)+a*sinh(x))/a/(a^2-b^2)
```

Rubi [A]
time = 0.06, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3652, 3611, 3556}

$$-\frac{bx}{a^2-b^2}+\frac{b^2\log(a\sinh(x)+b\cosh(x))}{a\,(a^2-b^2)}+\frac{\log(\cosh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(a + b*Coth[x]),x]
```

```
[Out] -((b*x)/(a^2 - b^2)) + Log[Cosh[x]]/a + (b^2*Log[b*Cosh[x] + a*Sinh[x]])/(a
*(a^2 - b^2))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3652

```
Int[1/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f
_.)*(x_)])), x_Symbol] :> Simp[(a*c - b*d)*(x/((a^2 + b^2)*(c^2 + d^2))), x
] + (Dist[b^2/((b*c - a*d)*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Ta
n[e + f*x]), x], x] - Dist[d^2/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e
+ f*x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{\tanh(x)}{a+b\coth(x)}\,dx = -\frac{bx}{a^2-b^2}+\frac{\int \tanh(x)\,dx}{a}+\frac{(ib^2)\int \frac{-ib-ia\coth(x)}{a+b\coth(x)}\,dx}{a\,(a^2-b^2)}$$
$$= -\frac{bx}{a^2-b^2}+\frac{\log(\cosh(x))}{a}+\frac{b^2\log(b\cosh(x)+a\sinh(x))}{a\,(a^2-b^2)}$$

Mathematica [A]
time = 0.07, size = 46, normalized size = 0.90

$$\frac{(a^2-b^2)\log(\cosh(x))+b(-ax+b\log(b\cosh(x)+a\sinh(x)))}{a^3-ab^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(a + b*Coth[x]),x]
[Out] ((a^2 - b^2)*Log[Cosh[x]] + b*(-(a*x) + b*Log[b*Cosh[x] + a*Sinh[x]]))/(a^3
 - a*b^2)
```

Maple [A]
time = 0.64, size = 88, normalized size = 1.73

method	result	size
risch	$\frac{x}{a+b}-\frac{2x}{a}-\frac{2x\,b^2}{a(a^2-b^2)}+\frac{\ln(1+\mathrm{e}^{2x})}{a}+\frac{b^2\ln\left(\mathrm{e}^{2x}-\frac{a-b}{a+b}\right)}{a(a^2-b^2)}$	82
default	$-\frac{8\ln(\tanh(\frac{x}{2})+1)}{8a-8b}+\frac{\ln(\tanh^2(\frac{x}{2})+1)}{a}+\frac{b^2\ln(b(\tanh^2(\frac{x}{2}))+2a\tanh(\frac{x}{2})+b)}{(a+b)(a-b)a}-\frac{8\ln(\tanh(\frac{x}{2})-1)}{8a+8b}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*coth(x)),x,method=_RETURNVERBOSE)
[Out] -8/(8*a-8*b)*ln(tanh(1/2*x)+1)+1/a*ln(tanh(1/2*x)^2+1)+b^2/(a+b)/(a-b)/a*ln
(b*tanh(1/2*x)^2+2*a*tanh(1/2*x)+b)-8/(8*a+8*b)*ln(tanh(1/2*x)-1)
```

Maxima [A]
time = 0.47, size = 50, normalized size = 0.98

$$\frac{b^2\log\left(-(a-b)e^{(-2\,x)}+a+b\right)}{a^3-ab^2}+\frac{x}{a+b}+\frac{\log\left(e^{(-2\,x)}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*coth(x)),x, algorithm="maxima")
[Out] b^2*log(-(a - b)*e^(-2*x) + a + b)/(a^3 - a*b^2) + x/(a + b) + log(e^(-2*x)
 + 1)/a
```

Fricas [A]
time = 0.38, size = 73, normalized size = 1.43

$$\frac{b^2\log\left(\frac{2\,(b\cosh(x)+a\sinh(x))}{\cosh(x)-\sinh(x)}\right)-(a^2+ab)x+(a^2-b^2)\log\left(\frac{2\,\cosh(x)}{\cosh(x)-\sinh(x)}\right)}{a^3-ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] (b^2*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) - (a^2 + a*b)*x + (
a^2 - b^2)*log(2*cosh(x)/(cosh(x) - sinh(x))))/(a^3 - a*b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tanh(x)}{a+b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*coth(x)),x)
```

```
[Out] Integral(tanh(x)/(a + b*coth(x)), x)
```

Giac [A]
time = 0.41, size = 57, normalized size = 1.12

$$\frac{b^2\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}-a+b\right|\right)}{a^3-ab^2}-\frac{x}{a-b}+\frac{\log\left(e^{(2\,x)}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] b^2*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^3 - a*b^2) - x/(a - b) + log
(e^(2*x) + 1)/a
```

Mupad [B]
time = 0.32, size = 58, normalized size = 1.14

$$\frac{\ln\left(\mathrm{e}^{2\,x}+1\right)}{a}-\frac{x}{a-b}-\frac{b^2\,\ln\left(b-a+a\,\mathrm{e}^{2\,x}+b\,\mathrm{e}^{2\,x}\right)}{a\,b^2-a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a + b*coth(x)),x)
```

```
[Out] log(exp(2*x) + 1)/a - x/(a - b) - (b^2*log(b - a + a*exp(2*x) + b*exp(2*x))
)/(a*b^2 - a^3)
```

3.144 $\int \frac{1}{a+b\coth(x)}\,dx$

Optimal. Leaf size=39

$$\frac{ax}{a^2-b^2}-\frac{b\log(b\cosh(x)+a\sinh(x))}{a^2-b^2}$$

[Out] a*x/(a^2-b^2)-b*ln(b*cosh(x)+a*sinh(x))/(a^2-b^2)

Rubi [A]
time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3565, 3611}

$$\frac{ax}{a^2-b^2}-\frac{b\log(a\sinh(x)+b\cosh(x))}{a^2-b^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Coth[x])^(-1),x]

[Out] (a*x)/(a^2 - b^2) - (b*Log[b*Cosh[x] + a*Sinh[x]])/(a^2 - b^2)

Rule 3565

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[a*(x/(a^
2 + b^2)), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\coth(x)}\,dx &= \frac{ax}{a^2-b^2}-\frac{(ib)\int \frac{-ib-ia\coth(x)}{a+b\coth(x)}\,dx}{a^2-b^2}\\ &= \frac{ax}{a^2-b^2}-\frac{b\log(b\cosh(x)+a\sinh(x))}{a^2-b^2}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 29, normalized size = 0.74

$$\frac{ax-b\log(b\cosh(x)+a\sinh(x))}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Coth[x])^(-1),x]
```

```
[Out] (a*x - b*Log[b*Cosh[x] + a*Sinh[x]])/(a^2 - b^2)
```

Maple [A]
time = 0.28, size = 55, normalized size = 1.41

method	result	size
derivativedivides	$-\frac{b\ln(a+b\coth(x))}{(a-b)(a+b)}+\frac{\ln(1+\coth(x))}{2a-2b}-\frac{\ln(\coth(x)-1)}{2b+2a}$	55
default	$-\frac{b\ln(a+b\coth(x))}{(a-b)(a+b)}+\frac{\ln(1+\coth(x))}{2a-2b}-\frac{\ln(\coth(x)-1)}{2b+2a}$	55
risch	$\frac{x}{a+b}+\frac{2bx}{a^2-b^2}-\frac{b\ln\left(e^{2x}-\frac{a-b}{a+b}\right)}{a^2-b^2}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -b/(a-b)/(a+b)*ln(a+b*coth(x))+1/(2*a-2*b)*ln(1+coth(x))-1/(2*b+2*a)*ln(cot
h(x)-1)
```

Maxima [A]
time = 0.28, size = 37, normalized size = 0.95

$$-\frac{b\log\left(-(a-b)e^{(-2\,x)}+a+b\right)}{a^2-b^2}+\frac{x}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] -b*log(-(a - b)*e^(-2*x) + a + b)/(a^2 - b^2) + x/(a + b)
```

Fricas [A]
time = 0.36, size = 42, normalized size = 1.08

$$\frac{(a+b)x-b\log\left(\frac{2\,(b\cosh(x)+a\sinh(x))}{\cosh(x)-\sinh(x)}\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] ((a + b)*x - b*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))))/(a^2 - b
^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(29) = 58$.
time = 0.50, size = 148, normalized size = 3.79

$$\begin{cases} \tilde{\infty}(x - \log(\tanh(x) + 1)) & \text{for } a = 0 \wedge b = 0 \\ \frac{x - \log(\tanh(x)+1)}{b} & \text{for } a = 0 \\ -\frac{x\tanh(x)}{2b\tanh(x)-2b} + \frac{x}{2b\tanh(x)-2b} - \frac{1}{2b\tanh(x)-2b} & \text{for } a = -b \\ \frac{x\tanh(x)}{2b\tanh(x)+2b} + \frac{x}{2b\tanh(x)+2b} + \frac{1}{2b\tanh(x)+2b} & \text{for } a = b \\ \frac{ax}{a^2-b^2} - \frac{bx}{a^2-b^2} + \frac{b\log(\tanh(x)+1)}{a^2-b^2} - \frac{b\log\left(\tanh(x)+\frac{b}{a}\right)}{a^2-b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(x)),x)
```

```
[Out] Piecewise((zoo*(x - log(tanh(x) + 1)), Eq(a, 0) & Eq(b, 0)), ((x - log(tanh
(x) + 1))/b, Eq(a, 0)), (-x*tanh(x)/(2*b*tanh(x) - 2*b) + x/(2*b*tanh(x) -
2*b) - 1/(2*b*tanh(x) - 2*b), Eq(a, -b)), (x*tanh(x)/(2*b*tanh(x) + 2*b) +
x/(2*b*tanh(x) + 2*b) + 1/(2*b*tanh(x) + 2*b), Eq(a, b)), (a*x/(a**2 - b**2
) - b*x/(a**2 - b**2) + b*log(tanh(x) + 1)/(a**2 - b**2) - b*log(tanh(x) +
b/a)/(a**2 - b**2), True))
```

Giac [A]
time = 0.41, size = 43, normalized size = 1.10

$$-\frac{b\log\left(\left|ae^{(2x)} + be^{(2x)} - a + b\right|\right)}{a^2 - b^2} + \frac{x}{a - b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -b*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^2 - b^2) + x/(a - b)
```

Mupad [B]
time = 0.08, size = 42, normalized size = 1.08

$$\frac{x}{a - b} - \frac{b\ln\left(b - a + a\,\mathrm{e}^{2x} + b\,\mathrm{e}^{2x}\right)}{a^2 - b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*coth(x)),x)
```

```
[Out] x/(a - b) - (b*log(b - a + a*exp(2*x) + b*exp(2*x)))/(a^2 - b^2)
```

3.145 $\int \frac{\coth(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=39

$$-\frac{bx}{a^2-b^2}+\frac{a\log(b\cosh(x)+a\sinh(x))}{a^2-b^2}$$

```
[Out] -b*x/(a^2-b^2)+a*ln(b*cosh(x)+a*sinh(x))/(a^2-b^2)
```

Rubi [A]
time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3612, 3611}

$$\frac{a\log(a\sinh(x)+b\cosh(x))}{a^2-b^2}-\frac{bx}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(a + b*Coth[x]),x]
```

```
[Out] -((b*x)/(a^2 - b^2)) + (a*Log[b*Cosh[x] + a*Sinh[x]])/(a^2 - b^2)
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth(x)}{a+b\coth(x)}\,dx &= -\frac{bx}{a^2-b^2}+\frac{(ia)\int \frac{-ib-ia\coth(x)}{a+b\coth(x)}\,dx}{a^2-b^2}\\ &= -\frac{bx}{a^2-b^2}+\frac{a\log(b\cosh(x)+a\sinh(x))}{a^2-b^2}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 29, normalized size = 0.74

$$\frac{-bx + a\log(b\cosh(x) + a\sinh(x))}{a^2 - b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(a + b*Coth[x]),x]
```

```
[Out] (-(b*x) + a*Log[b*Cosh[x] + a*Sinh[x]])/(a^2 - b^2)
```

Maple [A]
time = 0.29, size = 55, normalized size = 1.41

method	result	size
derivativedivides	$\frac{a\ln(a+b\coth(x))}{(a+b)(a-b)} - \frac{\ln(\coth(x)-1)}{2b+2a} - \frac{\ln(1+\coth(x))}{2a-2b}$	55
default	$\frac{a\ln(a+b\coth(x))}{(a+b)(a-b)} - \frac{\ln(\coth(x)-1)}{2b+2a} - \frac{\ln(1+\coth(x))}{2a-2b}$	55
risch	$\frac{x}{a+b} - \frac{2ax}{a^2-b^2} + \frac{a\ln\left(\mathrm{e}^{2x} - \frac{a-b}{a+b}\right)}{a^2-b^2}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] a/(a+b)/(a-b)*ln(a+b*coth(x))-1/(2*b+2*a)*ln(coth(x)-1)-1/(2*a-2*b)*ln(1+co
th(x))
```

Maxima [A]
time = 0.27, size = 36, normalized size = 0.92

$$\frac{a\log\left(-(a-b)e^{(-2x)} + a + b\right)}{a^2 - b^2} + \frac{x}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] a*log(-(a - b)*e^(-2*x) + a + b)/(a^2 - b^2) + x/(a + b)
```

Fricas [A]
time = 0.38, size = 43, normalized size = 1.10

$$-\frac{(a+b)x - a\log\left(\frac{2\,(b\cosh(x)+a\sinh(x))}{\cosh(x)-\sinh(x)}\right)}{a^2 - b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] -((a + b)*x - a*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))))/(a^2 -
b^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(29) = 58$.
time = 0.50, size = 134, normalized size = 3.44

$$\begin{cases} \tilde{\infty}x & \text{for } a = 0 \wedge b = 0 \\ \frac{x}{b} & \text{for } a = 0 \\ \frac{x\tanh(x)}{2b\tanh(x)-2b} - \frac{x}{2b\tanh(x)-2b} - \frac{1}{2b\tanh(x)-2b} & \text{for } a = -b \\ \frac{x\tanh(x)}{2b\tanh(x)+2b} + \frac{x}{2b\tanh(x)+2b} - \frac{1}{2b\tanh(x)+2b} & \text{for } a = b \\ \frac{ax}{a^2-b^2} - \frac{a\log(\tanh(x)+1)}{a^2-b^2} + \frac{a\log\left(\tanh(x)+\frac{b}{a}\right)}{a^2-b^2} - \frac{bx}{a^2-b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*coth(x)),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x/b, Eq(a, 0)), (x*tanh(x)/(2*b*ta
nh(x) - 2*b) - x/(2*b*tanh(x) - 2*b) - 1/(2*b*tanh(x) - 2*b), Eq(a, -b)), (
x*tanh(x)/(2*b*tanh(x) + 2*b) + x/(2*b*tanh(x) + 2*b) - 1/(2*b*tanh(x) + 2*
b), Eq(a, b)), (a*x/(a**2 - b**2) - a*log(tanh(x) + 1)/(a**2 - b**2) + a*lo
g(tanh(x) + b/a)/(a**2 - b**2) - b*x/(a**2 - b**2), True))
```

Giac [A]
time = 0.41, size = 43, normalized size = 1.10

$$\frac{a\log\left(\left|ae^{(2x)} + be^{(2x)} - a + b\right|\right)}{a^2 - b^2} - \frac{x}{a - b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] a*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^2 - b^2) - x/(a - b)
```

Mupad [B]
time = 0.06, size = 42, normalized size = 1.08

$$\frac{a\ln\left(b - a + a\,e^{2x} + b\,e^{2x}\right)}{a^2 - b^2} - \frac{x}{a - b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a + b*coth(x)),x)
```

```
[Out] (a*log(b - a + a*exp(2*x) + b*exp(2*x)))/(a^2 - b^2) - x/(a - b)
```

3.146 $\int \frac{\coth^2(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=63

$$-\frac{ax}{b^2}+\frac{a^3x}{b^2\,(a^2-b^2)}+\frac{\log(\sinh(x))}{b}-\frac{a^2\log(b\cosh(x)+a\sinh(x))}{b\,(a^2-b^2)}$$

```
[Out] -a*x/b^2+a^3*x/b^2/(a^2-b^2)+ln(sinh(x))/b-a^2*ln(b*cosh(x)+a*sinh(x))/b/(a
^2-b^2)
```

Rubi [A]
time = 0.07, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3622, 3556, 3565, 3611}

$$-\frac{a^2\log(a\sinh(x)+b\cosh(x))}{b\,(a^2-b^2)}+\frac{a^3x}{b^2\,(a^2-b^2)}-\frac{ax}{b^2}+\frac{\log(\sinh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(a + b*Coth[x]),x]
```

```
[Out] -((a*x)/b^2) + (a^3*x)/(b^2*(a^2 - b^2)) + Log[Sinh[x]]/b - (a^2*Log[b*Cosh
[x] + a*Sinh[x]])/(b*(a^2 - b^2))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3565

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[a*(x/(a^
2 + b^2)), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3622

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^2/((a_.) + (b_.)*tan[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[d*(2*b*c - a*d)*(x/b^2), x] + (Dist[d^2/b, In
```

```
t[Tan[e + f*x], x], x] + Dist[(b*c - a*d)^2/b^2, Int[1/(a + b*Tan[e + f*x])
, x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 +
 b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^2(x)}{a+b\coth(x)}\,dx &= -\frac{ax}{b^2}+\frac{a^2\int \frac{1}{a+b\coth(x)}\,dx}{b^2}+\frac{\int \coth(x)\,dx}{b}\\ &= -\frac{ax}{b^2}+\frac{a^3x}{b^2(a^2-b^2)}+\frac{\log(\sinh(x))}{b}-\frac{(ia^2)\int \frac{-ib-ia\coth(x)}{a+b\coth(x)}\,dx}{b(a^2-b^2)}\\ &= -\frac{ax}{b^2}+\frac{a^3x}{b^2(a^2-b^2)}+\frac{\log(\sinh(x))}{b}-\frac{a^2\log(b\cosh(x)+a\sinh(x))}{b(a^2-b^2)}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 49, normalized size = 0.78

$$\frac{abx+a^2\log(\sinh(x))-b^2\log(\sinh(x))-a^2\log(b\cosh(x)+a\sinh(x))}{a^2b-b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(a + b*Coth[x]),x]
```

```
[Out] (a*b*x + a^2*Log[Sinh[x]] - b^2*Log[Sinh[x]] - a^2*Log[b*Cosh[x] + a*Sinh[x
]])/(a^2*b - b^3)
```

Maple [A]
time = 0.29, size = 60, normalized size = 0.95

method	result	size
derivativedivides	$\frac{\ln(1+\coth(x))}{2a-2b}-\frac{a^2\ln(a+b\coth(x))}{(a+b)(a-b)b}-\frac{\ln(\coth(x)-1)}{2b+2a}$	60
default	$\frac{\ln(1+\coth(x))}{2a-2b}-\frac{a^2\ln(a+b\coth(x))}{(a+b)(a-b)b}-\frac{\ln(\coth(x)-1)}{2b+2a}$	60
risch	$\frac{x}{a+b}+\frac{2x\,a^2}{b(a^2-b^2)}-\frac{2x}{b}-\frac{a^2\ln\left(e^{2x}-\frac{a-b}{a+b}\right)}{b(a^2-b^2)}+\frac{\ln(e^{2x}-1)}{b}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(2*a-2*b)*ln(1+coth(x))-a^2/(a+b)/(a-b)/b*ln(a+b*coth(x))-1/(2*b+2*a)*ln(
coth(x)-1)
```

Maxima [A]
time = 0.28, size = 63, normalized size = 1.00

$$-\frac{a^2 \log\left(-(a-b)e^{(-2x)} + a + b\right)}{a^2 b - b^3} + \frac{x}{a+b} + \frac{\log\left(e^{(-x)} + 1\right)}{b} + \frac{\log\left(e^{(-x)} - 1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] -a^2*log(-(a - b)*e^(-2*x) + a + b)/(a^2*b - b^3) + x/(a + b) + log(e^(-x)
+ 1)/b + log(e^(-x) - 1)/b
```

Fricas [A]
time = 0.40, size = 76, normalized size = 1.21

$$-\frac{a^2 \log\left(\frac{2\,(b\cosh(x)+a\sinh(x))}{\cosh(x)-\sinh(x)}\right) - (ab+b^2)x - (a^2-b^2)\log\left(\frac{2\,\sinh(x)}{\cosh(x)-\sinh(x)}\right)}{a^2 b - b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] -(a^2*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) - (a*b + b^2)*x -
(a^2 - b^2)*log(2*sinh(x)/(cosh(x) - sinh(x))))/(a^2*b - b^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(51) = 102$.
time = 0.86, size = 372, normalized size = 5.90

$$\begin{cases} \tilde{\infty}(x - \log(\tanh(x)+1) + \log(\tanh(x))) & \text{for } a=0 \wedge b=0 \\ \frac{x-\log(\tanh(x)+1)+\log(\tanh(x))}{b} & \text{for } a=0 \\ \frac{3x\tanh(x)}{2b\tanh(x)-2b} - \frac{3x}{2b\tanh(x)-2b} - \frac{2\log(\tanh(x)+1)\tanh(x)}{2b\tanh(x)-2b} + \frac{2\log(\tanh(x)+1)}{2b\tanh(x)-2b} + \frac{2\log(\tanh(x))\tanh(x)}{2b\tanh(x)-2b} - \frac{2\log(\tanh(x))}{2b\tanh(x)-2b} - \frac{1}{2b\tanh(x)-2b} & \text{for } a=-b \\ \frac{x\tanh(x)}{2b\tanh(x)+2b} + \frac{x}{2b\tanh(x)+2b} - \frac{2\log(\tanh(x)+1)\tanh(x)}{2b\tanh(x)+2b} - \frac{2\log(\tanh(x)+1)}{2b\tanh(x)+2b} + \frac{2\log(\tanh(x))\tanh(x)}{2b\tanh(x)+2b} + \frac{2\log(\tanh(x))}{2b\tanh(x)+2b} + \frac{1}{2b\tanh(x)+2b} & \text{for } a=b \\ \frac{x-\frac{1}{\tanh(x)}}{a} & \text{for } b=0 \\ -\frac{a^2\log\left(\tanh(x)+\frac{b}{a}\right)}{a^2b-b^3} + \frac{a^2\log(\tanh(x))}{a^2b-b^3} + \frac{abx}{a^2b-b^3} - \frac{b^2x}{a^2b-b^3} + \frac{b^2\log(\tanh(x)+1)}{a^2b-b^3} - \frac{b^2\log(\tanh(x))}{a^2b-b^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(a+b*coth(x)),x)
```

```
[Out] Piecewise((zoo*(x - log(tanh(x) + 1) + log(tanh(x))), Eq(a, 0) & Eq(b, 0)),
 ((x - log(tanh(x) + 1) + log(tanh(x)))/b, Eq(a, 0)), (3*x*tanh(x)/(2*b*tan
h(x) - 2*b) - 3*x/(2*b*tanh(x) - 2*b) - 2*log(tanh(x) + 1)*tanh(x)/(2*b*tan
h(x) - 2*b) + 2*log(tanh(x) + 1)/(2*b*tanh(x) - 2*b) + 2*log(tanh(x))*tanh(
x)/(2*b*tanh(x) - 2*b) - 2*log(tanh(x))/(2*b*tanh(x) - 2*b) - 1/(2*b*tanh(x
) - 2*b), Eq(a, -b)), (x*tanh(x)/(2*b*tanh(x) + 2*b) + x/(2*b*tanh(x) + 2*b
) - 2*log(tanh(x) + 1)*tanh(x)/(2*b*tanh(x) + 2*b) - 2*log(tanh(x) + 1)/(2*
```

```
b*tanh(x) + 2*b) + 2*log(tanh(x))*tanh(x)/(2*b*tanh(x) + 2*b) + 2*log(tanh(
x))/(2*b*tanh(x) + 2*b) + 1/(2*b*tanh(x) + 2*b), Eq(a, b)), ((x - 1/tanh(x)
)/a, Eq(b, 0)), (-a**2*log(tanh(x) + b/a)/(a**2*b - b**3) + a**2*log(tanh(x
))/(a**2*b - b**3) + a*b*x/(a**2*b - b**3) - b**2*x/(a**2*b - b**3) + b**2*
log(tanh(x) + 1)/(a**2*b - b**3) - b**2*log(tanh(x))/(a**2*b - b**3), True)
)
```

Giac [**A**]
time = 0.41, size = 59, normalized size = 0.94

$$-\frac{a^2\log\left(\left|ae^{(2x)}+be^{(2x)}-a+b\right|\right)}{a^2b-b^3}+\frac{x}{a-b}+\frac{\log\left(\left|e^{(2x)}-1\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -a^2*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^2*b - b^3) + x/(a - b) + lo
g(abs(e^(2*x) - 1))/b
```

Mupad [**B**]
time = 1.48, size = 57, normalized size = 0.90

$$\frac{\ln\left(\mathrm{e}^{2x}-1\right)}{b}+\frac{x}{a-b}-\frac{a^2\,\ln\left(b-a+a\,\mathrm{e}^{2x}+b\,\mathrm{e}^{2x}\right)}{a^2\,b-b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a + b*coth(x)),x)
```

```
[Out] log(exp(2*x) - 1)/b + x/(a - b) - (a^2*log(b - a + a*exp(2*x) + b*exp(2*x))
)/(a^2*b - b^3)
```

3.147 $\int \frac{\coth^3(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=64

$$-\frac{bx}{a^2-b^2}-\frac{\coth(x)}{b}+\frac{a^3\log(a+b\coth(x))}{b^2\,(a^2-b^2)}+\frac{a\log(\sinh(x))}{a^2-b^2}$$

```
[Out] -b*x/(a^2-b^2)-coth(x)/b+a^3*ln(a+b*coth(x))/b^2/(a^2-b^2)+a*ln(sinh(x))/(a
^2-b^2)
```

Rubi [A]
time = 0.09, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3647, 3707, 3698, 31, 3556}

$$-\frac{bx}{a^2-b^2}+\frac{a\log(\sinh(x))}{a^2-b^2}+\frac{a^3\log(a+b\coth(x))}{b^2\,(a^2-b^2)}-\frac{\coth(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/(a + b*Coth[x]),x]
```

```
[Out] -((b*x)/(a^2 - b^2)) - Coth[x]/b + (a^3*Log[a + b*Coth[x]])/(b^2*(a^2 - b^2
)) + (a*Log[Sinh[x]])/(a^2 - b^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\coth^3(x)}{a+b\coth(x)}\,dx &= -\frac{\coth(x)}{b}-\frac{\int \frac{-a-b\coth(x)+a\coth^2(x)}{a+b\coth(x)}\,dx}{b}\\
&= -\frac{bx}{a^2-b^2}-\frac{\coth(x)}{b}+\frac{a\int \coth(x)\,dx}{a^2-b^2}+\frac{a^3\int \frac{1-\coth^2(x)}{a+b\coth(x)}\,dx}{b\,(a^2-b^2)}\\
&= -\frac{bx}{a^2-b^2}-\frac{\coth(x)}{b}+\frac{a\log(\sinh(x))}{a^2-b^2}+\frac{a^3\mathrm{Subst}\left(\int \frac{1}{a+x}\,dx,x,b\coth(x)\right)}{b^2\,(a^2-b^2)}\\
&= -\frac{bx}{a^2-b^2}-\frac{\coth(x)}{b}+\frac{a^3\log(a+b\coth(x))}{b^2\,(a^2-b^2)}+\frac{a\log(\sinh(x))}{a^2-b^2}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 64, normalized size = 1.00

$$\frac{b^3x+b(a^2-b^2)\coth(x)+a(a^2-b^2)\log(\sinh(x))-a^3\log(b\cosh(x)+a\sinh(x))}{b^2(-a+b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/(a + b*Coth[x]),x]
```

```
[Out] (b^3*x + b*(a^2 - b^2)*Coth[x] + a*(a^2 - b^2)*Log[Sinh[x]] - a^3*Log[b*Cos
h[x] + a*Sinh[x]])/(b^2*(-a + b)*(a + b))
```

Maple [A]
time = 0.44, size = 67, normalized size = 1.05

method	result	size
derivativedivides	$-\frac{\coth(x)}{b}+\frac{a^3\ln(a+b\coth(x))}{b^2(a+b)(a-b)}-\frac{\ln(1+\coth(x))}{2a-2b}-\frac{\ln(\coth(x)-1)}{2b+2a}$	67
default	$-\frac{\coth(x)}{b}+\frac{a^3\ln(a+b\coth(x))}{b^2(a+b)(a-b)}-\frac{\ln(1+\coth(x))}{2a-2b}-\frac{\ln(\coth(x)-1)}{2b+2a}$	67
risch	$\frac{x}{a+b}+\frac{2ax}{b^2}-\frac{2a^3x}{b^2(a^2-b^2)}-\frac{2}{b(e^{2x}-1)}-\frac{a\ln(e^{2x}-1)}{b^2}+\frac{a^3\ln\left(e^{2x}-\frac{a-b}{a+b}\right)}{b^2(a^2-b^2)}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -coth(x)/b+1/b^2*a^3/(a+b)/(a-b)*ln(a+b*coth(x))-1/(2*a-2*b)*ln(1+coth(x))-
1/(2*b+2*a)*ln(coth(x)-1)
```

Maxima [A]
time = 0.28, size = 82, normalized size = 1.28

$$\frac{a^3\log\left(-(a-b)e^{(-2x)}+a+b\right)}{a^2b^2-b^4}+\frac{x}{a+b}-\frac{a\log\left(e^{(-x)}+1\right)}{b^2}-\frac{a\log\left(e^{(-x)}-1\right)}{b^2}+\frac{2}{be^{(-2x)}-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] a^3*log(-(a - b)*e^(-2*x) + a + b)/(a^2*b^2 - b^4) + x/(a + b) - a*log(e^(-
x) + 1)/b^2 - a*log(e^(-x) - 1)/b^2 + 2/(b*e^(-2*x) - b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(64) = 128.
time = 0.38, size = 271, normalized size = 4.23

$$\frac{(ab^2+b^3)x\cosh(x)^2+2(ab^2+b^3)x\cosh(x)\sinh(x)+(ab^2+b^3)x\sinh(x)^2+2a^2b-2b^3-(ab^2+b^3)x-\left(a^3\cosh(x)^2+2a^3\cosh(x)\sinh(x)+a^3\sinh(x)^2-a^3\right)\log\left(\frac{2(b\cosh(x)+a\sinh(x))}{\cosh(x)-\sinh(x)}\right)-\left(a^3-ab^2-(a^3-ab^2)\cosh(x)^2-2(a^3-ab^2)\cosh(x)\sinh(x)-(a^3-ab^2)\sinh(x)^2\right)\log\left(\frac{2\sinh(x)}{\cosh(x)-\sinh(x)}\right)}{a^2b^2-b^4-(a^2b^2-b^4)\cosh(x)^2-2(a^2b^2-b^4)\cosh(x)\sinh(x)-(a^2b^2-b^4)\sinh(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] ((a*b^2 + b^3)*x*cosh(x)^2 + 2*(a*b^2 + b^3)*x*cosh(x)*sinh(x) + (a*b^2 + b
^3)*x*sinh(x)^2 + 2*a^2*b - 2*b^3 - (a*b^2 + b^3)*x - (a^3*cosh(x)^2 + 2*a^
3*cosh(x)*sinh(x) + a^3*sinh(x)^2 - a^3)*log(2*(b*cosh(x) + a*sinh(x))/(cos
h(x) - sinh(x))) - (a^3 - a*b^2 - (a^3 - a*b^2)*cosh(x)^2 - 2*(a^3 - a*b^2)
*cosh(x)*sinh(x) - (a^3 - a*b^2)*sinh(x)^2)*log(2*sinh(x)/(cosh(x) - sinh(x
))))/(a^2*b^2 - b^4 - (a^2*b^2 - b^4)*cosh(x)^2 - 2*(a^2*b^2 - b^4)*cosh(x)
*sinh(x) - (a^2*b^2 - b^4)*sinh(x)^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 636 vs. 2(49) = 98.

time = 1.32, size = 636, normalized size = 9.94

$$\begin{cases} \tilde{\infty}\left(x - \frac{1}{\tanh(x)}\right) & \text{for } a = 0 \wedge b = 0 \\ \frac{5x\tanh^2(x)}{2b\tanh^2(x)-2b\tanh(x)} - \frac{5x\tanh(x)}{2b\tanh^2(x)-2b\tanh(x)} - \frac{2\log(\tanh(x)+1)\tanh^2(x)}{2b\tanh^2(x)-2b\tanh(x)} + \frac{2\log(\tanh(x)+1)\tanh(x)}{2b\tanh^2(x)-2b\tanh(x)} + \frac{2\log(\tanh(x))\tanh^2(x)}{2b\tanh^2(x)-2b\tanh(x)} - \frac{2\log(\tanh(x))\tanh(x)}{2b\tanh^2(x)-2b\tanh(x)} - \frac{3\tanh(x)}{2b\tanh^2(x)-2b\tanh(x)} + \frac{2}{2b\tanh^2(x)-2b\tanh(x)} & \text{for } a = -b \\ \frac{x\tanh^2(x)}{2b\tanh^2(x)+2b\tanh(x)} + \frac{x\tanh(x)}{2b\tanh^2(x)+2b\tanh(x)} + \frac{2\log(\tanh(x)+1)\tanh^2(x)}{2b\tanh^2(x)+2b\tanh(x)} + \frac{2\log(\tanh(x)+1)\tanh(x)}{2b\tanh^2(x)+2b\tanh(x)} - \frac{2\log(\tanh(x))\tanh^2(x)}{2b\tanh^2(x)+2b\tanh(x)} - \frac{2\log(\tanh(x))\tanh(x)}{2b\tanh^2(x)+2b\tanh(x)} - \frac{3\tanh(x)}{2b\tanh^2(x)+2b\tanh(x)} - \frac{2}{2b\tanh^2(x)+2b\tanh(x)} & \text{for } a = b \\ \frac{x - \log(\tanh(x)+1) + \log(\tanh(x)) - \frac{1}{2\tanh^2(x)}}{a} & \text{for } b = 0 \\ \frac{x - \frac{1}{\tanh(x)}}{b} & \text{for } a = 0 \\ \frac{a^3\log\left(\tanh(x)+\frac{b}{a}\right)\tanh(x)}{a^2b^2\tanh(x)-b^4\tanh(x)} - \frac{a^3\log(\tanh(x))\tanh(x)}{a^2b^2\tanh(x)-b^4\tanh(x)} - \frac{a^2b}{a^2b^2\tanh(x)-b^4\tanh(x)} + \frac{ab^2x\tanh(x)}{a^2b^2\tanh(x)-b^4\tanh(x)} - \frac{ab^2\log(\tanh(x)+1)\tanh(x)}{a^2b^2\tanh(x)-b^4\tanh(x)} + \frac{ab^2\log(\tanh(x))\tanh(x)}{a^2b^2\tanh(x)-b^4\tanh(x)} - \frac{b^3x\tanh(x)}{a^2b^2\tanh(x)-b^4\tanh(x)} + \frac{b^3}{a^2b^2\tanh(x)-b^4\tanh(x)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(a+b*coth(x)),x)
```

```
[Out] Piecewise((zoo*(x - 1/tanh(x)), Eq(a, 0) & Eq(b, 0)), (5*x*tanh(x)**2/(2*b*
tanh(x)**2 - 2*b*tanh(x)) - 5*x*tanh(x)/(2*b*tanh(x)**2 - 2*b*tanh(x)) - 2*
log(tanh(x) + 1)*tanh(x)**2/(2*b*tanh(x)**2 - 2*b*tanh(x)) + 2*log(tanh(x)
+ 1)*tanh(x)/(2*b*tanh(x)**2 - 2*b*tanh(x)) + 2*log(tanh(x))*tanh(x)**2/(2*
b*tanh(x)**2 - 2*b*tanh(x)) - 2*log(tanh(x))*tanh(x)/(2*b*tanh(x)**2 - 2*b*
tanh(x)) - 3*tanh(x)/(2*b*tanh(x)**2 - 2*b*tanh(x)) + 2/(2*b*tanh(x)**2 - 2
*b*tanh(x)), Eq(a, -b)), (x*tanh(x)**2/(2*b*tanh(x)**2 + 2*b*tanh(x)) + x*t
anh(x)/(2*b*tanh(x)**2 + 2*b*tanh(x)) + 2*log(tanh(x) + 1)*tanh(x)**2/(2*b*
tanh(x)**2 + 2*b*tanh(x)) + 2*log(tanh(x) + 1)*tanh(x)/(2*b*tanh(x)**2 + 2*
b*tanh(x)) - 2*log(tanh(x))*tanh(x)**2/(2*b*tanh(x)**2 + 2*b*tanh(x)) - 2*l
og(tanh(x))*tanh(x)/(2*b*tanh(x)**2 + 2*b*tanh(x)) - 3*tanh(x)/(2*b*tanh(x)
**2 + 2*b*tanh(x)) - 2/(2*b*tanh(x)**2 + 2*b*tanh(x)), Eq(a, b)), ((x - log
(tanh(x) + 1) + log(tanh(x)) - 1/(2*tanh(x)**2))/a, Eq(b, 0)), ((x - 1/tanh
(x))/b, Eq(a, 0)), (a**3*log(tanh(x) + b/a)*tanh(x)/(a**2*b**2*tanh(x) - b*
*4*tanh(x)) - a**3*log(tanh(x))*tanh(x)/(a**2*b**2*tanh(x) - b**4*tanh(x))
- a**2*b/(a**2*b**2*tanh(x) - b**4*tanh(x)) + a*b**2*x*tanh(x)/(a**2*b**2*t
anh(x) - b**4*tanh(x)) - a*b**2*log(tanh(x) + 1)*tanh(x)/(a**2*b**2*tanh(x)
 - b**4*tanh(x)) + a*b**2*log(tanh(x))*tanh(x)/(a**2*b**2*tanh(x) - b**4*ta
nh(x)) - b**3*x*tanh(x)/(a**2*b**2*tanh(x) - b**4*tanh(x)) + b**3/(a**2*b**
2*tanh(x) - b**4*tanh(x)), True))
```

Giac [A]
time = 0.43, size = 76, normalized size = 1.19

$$\frac{a^3\log\left(\left|ae^{(2x)} + be^{(2x)} - a + b\right|\right)}{a^2b^2 - b^4} - \frac{x}{a-b} - \frac{a\log\left(\left|e^{(2x)} - 1\right|\right)}{b^2} - \frac{2}{b(e^{(2x)} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] a^3*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^2*b^2 - b^4) - x/(a - b) - a
*log(abs(e^(2*x) - 1))/b^2 - 2/(b*(e^(2*x) - 1))
```

Mupad [B]
time = 1.51, size = 74, normalized size = 1.16

$$-\frac{2}{b\,\left({\mathrm{e}}^{2\,x}-1\right)}-\frac{x}{a-b}-\frac{a^3\,\ln\left(b-a+a\,{\mathrm{e}}^{2\,x}+b\,{\mathrm{e}}^{2\,x}\right)}{b^4-a^2\,b^2}-\frac{a\,\ln\left({\mathrm{e}}^{2\,x}-1\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a + b*coth(x)),x)
```

```
[Out] - 2/(b*(exp(2*x) - 1)) - x/(a - b) - (a^3*log(b - a + a*exp(2*x) + b*exp(2*
x)))/(b^4 - a^2*b^2) - (a*log(exp(2*x) - 1))/b^2
```

3.148 $\int \frac{\coth^4(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=76

$$\frac{ax}{a^2-b^2}+\frac{a\coth(x)}{b^2}-\frac{\coth^2(x)}{2b}-\frac{a^4\log(a+b\coth(x))}{b^3\,(a^2-b^2)}-\frac{b\log(\sinh(x))}{a^2-b^2}$$

```
[Out] a*x/(a^2-b^2)+a*coth(x)/b^2-1/2*coth(x)^2/b-a^4*ln(a+b*coth(x))/b^3/(a^2-b^
2)-b*ln(sinh(x))/(a^2-b^2)
```

Rubi [A]
time = 0.16, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3647, 3728, 3708, 3698, 31, 3556}

$$\frac{ax}{a^2-b^2}-\frac{b\log(\sinh(x))}{a^2-b^2}-\frac{a^4\log(a+b\coth(x))}{b^3\,(a^2-b^2)}+\frac{a\coth(x)}{b^2}-\frac{\coth^2(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^4/(a + b*Coth[x]),x]
```

```
[Out] (a*x)/(a^2 - b^2) + (a*Coth[x])/b^2 - Coth[x]^2/(2*b) - (a^4*Log[a + b*Coth
[x]])/(b^3*(a^2 - b^2)) - (b*Log[Sinh[x]])/(a^2 - b^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3708

```
Int[((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[a*(A - C)*(x/(a^2 + b^2)), x] + (Dist[(a^2*C +
A*b^2)/(a^2 + b^2), Int[(1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] -
 Dist[b*((A - C)/(a^2 + b^2)), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e,
 f, A, C}, x] && NeQ[a^2*C + A*b^2, 0] && NeQ[a^2 + b^2, 0] && NeQ[A, C]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\coth^4(x)}{a+b\coth(x)}\,dx &= -\frac{\coth^2(x)}{2b}-\frac{\int \frac{\coth(x)\left(-2a-2b\coth(x)+2a\coth^2(x)\right)}{a+b\coth(x)}\,dx}{2b}\\
&= \frac{a\coth(x)}{b^2}-\frac{\coth^2(x)}{2b}-\frac{\int \frac{2a^2-2\left(a^2+b^2\right)\coth^2(x)}{a+b\coth(x)}\,dx}{2b^2}\\
&= \frac{ax}{a^2-b^2}+\frac{a\coth(x)}{b^2}-\frac{\coth^2(x)}{2b}-\frac{a^4\int \frac{1-\coth^2(x)}{a+b\coth(x)}\,dx}{b^2\left(a^2-b^2\right)}-\frac{b\int \coth(x)\,dx}{a^2-b^2}\\
&= \frac{ax}{a^2-b^2}+\frac{a\coth(x)}{b^2}-\frac{\coth^2(x)}{2b}-\frac{b\log(\sinh(x))}{a^2-b^2}-\frac{a^4\text{Subst}\left(\int \frac{1}{a+x}\,dx,x,b\coth(x)\right)}{b^3\left(a^2-b^2\right)}\\
&= \frac{ax}{a^2-b^2}+\frac{a\coth(x)}{b^2}-\frac{\coth^2(x)}{2b}-\frac{a^4\log(a+b\coth(x))}{b^3\left(a^2-b^2\right)}-\frac{b\log(\sinh(x))}{a^2-b^2}
\end{aligned}
$$

Mathematica [A]

time = 0.15, size = 88, normalized size = 1.16

$$\frac{2ab^3x + 2ab(a^2 - b^2)\coth(x) + (-a^2b^2 + b^4)\operatorname{csch}^2(x) + 2(a^4 - b^4)\log(\sinh(x)) - 2a^4\log(b\cosh(x) + a\sinh(x))}{2(a - b)b^3(a + b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4/(a + b*Coth[x]),x]
```

```
[Out] (2*a*b^3*x + 2*a*b*(a^2 - b^2)*Coth[x] + (-(a^2*b^2) + b^4)*Csch[x]^2 + 2*(
a^4 - b^4)*Log[Sinh[x]] - 2*a^4*Log[b*Cosh[x] + a*Sinh[x]])/(2*(a - b)*b^3*
(a + b))
```

Maple [A]
time = 0.39, size = 76, normalized size = 1.00

method	result	s
derivativedivides	$\frac{-\frac{b\left(\coth^2(x)\right)}{2}+a\coth(x)}{b^2} - \frac{a^4\ln(a+b\coth(x))}{b^3(a+b)(a-b)} - \frac{\ln(\coth(x)-1)}{2b+2a} + \frac{\ln(1+\coth(x))}{2a-2b}$	7
default	$\frac{-\frac{b\left(\coth^2(x)\right)}{2}+a\coth(x)}{b^2} - \frac{a^4\ln(a+b\coth(x))}{b^3(a+b)(a-b)} - \frac{\ln(\coth(x)-1)}{2b+2a} + \frac{\ln(1+\coth(x))}{2a-2b}$	7
risch	$\frac{x}{a+b} - \frac{2x\,a^2}{b^3} - \frac{2x}{b} + \frac{2x\,a^4}{b^3(a^2-b^2)} + \frac{2\,e^{2x}a-2b\,e^{2x}-2a}{(e^{2x}-1)^2b^2} + \frac{\ln(e^{2x}-1)a^2}{b^3} + \frac{\ln(e^{2x}-1)}{b} - \frac{a^4\ln\left(e^{2x}-\frac{a-b}{a+b}\right)}{b^3(a^2-b^2)}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(-1/2*b*coth(x)^2+a*coth(x))-1/b^3*a^4/(a+b)/(a-b)*ln(a+b*coth(x))-1/
(2*b+2*a)*ln(coth(x)-1)+1/(2*a-2*b)*ln(1+coth(x))
```

Maxima [A]
time = 0.29, size = 119, normalized size = 1.57

$$-\frac{a^4\log\left(-(a-b)e^{(-2x)} + a + b\right)}{a^2b^3 - b^5} + \frac{2\left((a+b)e^{(-2x)} - a\right)}{2\,b^2e^{(-2x)} - b^2e^{(-4x)} - b^2} + \frac{x}{a+b} + \frac{(a^2+b^2)\log\left(e^{(-x)}+1\right)}{b^3} + \frac{(a^2+b^2)\log\left(e^{(-x)}-1\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] -a^4*log(-(a - b)*e^(-2*x) + a + b)/(a^2*b^3 - b^5) + 2*((a + b)*e^(-2*x) -
 a)/(2*b^2*e^(-2*x) - b^2*e^(-4*x) - b^2) + x/(a + b) + (a^2 + b^2)*log(e^(
-x) + 1)/b^3 + (a^2 + b^2)*log(e^(-x) - 1)/b^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 648 vs. 2(74) = 148.
time = 0.38, size = 648, normalized size = 8.53

$$\frac{(ab^3+b^4)x\cosh(x)^4 + 4(ab^3+b^4)x\cosh(x)\sinh(x)^3 + (ab^3+b^4)x\sinh(x)^4 - 2a^3b + 2ab^3 + 2\left(a^3b - a^2b^2 - ab^3 + b^4 - (ab^3+b^4)x\right)\cosh(x)^2 + 2\left(a^3b - a^2b^2 - ab^3 + b^4 + 3(ab^3+b^4)x\cosh(x)^2 - (ab^3+b^4)x\right)\sinh(x)^2 + (ab^3+b^4)x - \left(a^4\cosh(x)^4 + 4a^4\cosh(x)\sinh(x)^3 + a^4\sinh(x)^4 - 2a^4\cosh(x)^2 + a^4 + 2\left(3a^4\cosh(x)^2 - a^4\right)\sinh(x)^2 + 4\left(a^4\cosh(x)^3 - a^4\cosh(x)\right)\sinh(x)\right)\log\left(\frac{2(b\cosh(x)+a\sinh(x))}{\cosh(x)-\sinh(x)}\right) + \left((a^4-b^4)\cosh(x)^4 + 4(a^4-b^4)\cosh(x)\sinh(x)^3 + (a^4-b^4)\sinh(x)^4 + a^4 - b^4 - 2(a^4-b^4)\cosh(x)^2 - 2\left(a^4 - b^4 - 3(a^4-b^4)\cosh(x)^2\right)\sinh(x)^2 + 4\left((a^4-b^4)\cosh(x)^3 - (a^4-b^4)\cosh(x)\right)\sinh(x)\right)\log\left(\frac{2\sinh(x)}{\cosh(x)-\sinh(x)}\right) + 4\left((ab^3+b^4)x\cosh(x)^3 + \left(a^3b - a^2b^2 - ab^3 + b^4 - (ab^3+b^4)x\right)\cosh(x)\right)\sinh(x)}{a^2b^3 - b^5 + (a^2b^3-b^5)\cosh(x)^4 + 4(a^2b^3-b^5)\cosh(x)\sinh(x)^3 + (a^2b^3-b^5)\sinh(x)^4 - 2(a^2b^3-b^5)\cosh(x)^2 - 2\left(a^2b^3 - b^5 - 3(a^2b^3-b^5)\cosh(x)^2\right)\sinh(x)^2 + 4\left((a^2b^3-b^5)\cosh(x)^3 - (a^2b^3-b^5)\cosh(x)\right)\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] ((a*b^3 + b^4)*x*cosh(x)^4 + 4*(a*b^3 + b^4)*x*cosh(x)*sinh(x)^3 + (a*b^3 +
 b^4)*x*sinh(x)^4 - 2*a^3*b + 2*a*b^3 + 2*(a^3*b - a^2*b^2 - a*b^3 + b^4 -
(a*b^3 + b^4)*x)*cosh(x)^2 + 2*(a^3*b - a^2*b^2 - a*b^3 + b^4 + 3*(a*b^3 +
b^4)*x*cosh(x)^2 - (a*b^3 + b^4)*x)*sinh(x)^2 + (a*b^3 + b^4)*x - (a^4*cosh
(x)^4 + 4*a^4*cosh(x)*sinh(x)^3 + a^4*sinh(x)^4 - 2*a^4*cosh(x)^2 + a^4 + 2
*(3*a^4*cosh(x)^2 - a^4)*sinh(x)^2 + 4*(a^4*cosh(x)^3 - a^4*cosh(x))*sinh(x
))*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) + ((a^4 - b^4)*cosh(x
)^4 + 4*(a^4 - b^4)*cosh(x)*sinh(x)^3 + (a^4 - b^4)*sinh(x)^4 + a^4 - b^4 -
 2*(a^4 - b^4)*cosh(x)^2 - 2*(a^4 - b^4 - 3*(a^4 - b^4)*cosh(x)^2)*sinh(x)^
2 + 4*((a^4 - b^4)*cosh(x)^3 - (a^4 - b^4)*cosh(x))*sinh(x))*log(2*sinh(x)/
(cosh(x) - sinh(x))) + 4*((a*b^3 + b^4)*x*cosh(x)^3 + (a^3*b - a^2*b^2 - a*
b^3 + b^4 - (a*b^3 + b^4)*x)*cosh(x))*sinh(x))/(a^2*b^3 - b^5 + (a^2*b^3 -
b^5)*cosh(x)^4 + 4*(a^2*b^3 - b^5)*cosh(x)*sinh(x)^3 + (a^2*b^3 - b^5)*sinh
(x)^4 - 2*(a^2*b^3 - b^5)*cosh(x)^2 - 2*(a^2*b^3 - b^5 - 3*(a^2*b^3 - b^5)*
cosh(x)^2)*sinh(x)^2 + 4*((a^2*b^3 - b^5)*cosh(x)^3 - (a^2*b^3 - b^5)*cosh(
x))*sinh(x))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 882 vs. $2(61) = 122$.
time = 1.82, size = 882, normalized size = 11.61

$$\begin{cases} \tilde{\infty}\left(x - \log\left(\tanh\left(x\right) + 1\right) + \log\left(\tanh\left(x\right)\right) - \frac{1}{2\tanh^{2}\left(x\right)}\right) & \text{for } a = 0 \wedge b = 0 \\ \frac{x - \log\left(\tanh\left(x\right)+1\right) + \log\left(\tanh\left(x\right)\right) - \frac{1}{2\tanh^{2}\left(x\right)}}{b} & \text{for } a = 0 \\ \frac{7x\tanh^{3}\left(x\right)}{2b\tanh^{3}\left(x\right)-2b\tanh^{2}\left(x\right)} - \frac{7x\tanh^{2}\left(x\right)}{2b\tanh^{3}\left(x\right)-2b\tanh^{2}\left(x\right)} - \frac{4\log\left(\tanh\left(x\right)+1\right)\tanh^{3}\left(x\right)}{2b\tanh^{3}\left(x\right)-2b\tanh^{2}\left(x\right)} + \frac{4\log\left(\tanh\left(x\right)+1\right)\tanh^{2}\left(x\right)}{2b\tanh^{3}\left(x\right)-2b\tanh^{2}\left(x\right)} + \frac{4\log\left(\tanh\left(x\right)\right)\tanh^{3}\left(x\right)}{2b\tanh^{3}\left(x\right)-2b\tanh^{2}\left(x\right)} - \frac{4\log\left(\tanh\left(x\right)\right)\tanh^{2}\left(x\right)}{2b\tanh^{3}\left(x\right)-2b\tanh^{2}\left(x\right)} - \frac{3\tanh^{2}\left(x\right)}{2b\tanh^{3}\left(x\right)-2b\tanh^{2}\left(x\right)} + \frac{\tanh\left(x\right)}{2b\tanh^{3}\left(x\right)-2b\tanh^{2}\left(x\right)} + \frac{1}{2b\tanh^{3}\left(x\right)-2b\tanh^{2}\left(x\right)} & \text{for } a = -b \\ \frac{x\tanh^{3}\left(x\right)}{2b\tanh^{3}\left(x\right)+2b\tanh^{2}\left(x\right)} + \frac{x\tanh^{2}\left(x\right)}{2b\tanh^{3}\left(x\right)+2b\tanh^{2}\left(x\right)} - \frac{4\log\left(\tanh\left(x\right)+1\right)\tanh^{3}\left(x\right)}{2b\tanh^{3}\left(x\right)+2b\tanh^{2}\left(x\right)} - \frac{4\log\left(\tanh\left(x\right)+1\right)\tanh^{2}\left(x\right)}{2b\tanh^{3}\left(x\right)+2b\tanh^{2}\left(x\right)} + \frac{4\log\left(\tanh\left(x\right)\right)\tanh^{3}\left(x\right)}{2b\tanh^{3}\left(x\right)+2b\tanh^{2}\left(x\right)} + \frac{4\log\left(\tanh\left(x\right)\right)\tanh^{2}\left(x\right)}{2b\tanh^{3}\left(x\right)+2b\tanh^{2}\left(x\right)} + \frac{3\tanh^{2}\left(x\right)}{2b\tanh^{3}\left(x\right)+2b\tanh^{2}\left(x\right)} + \frac{\tanh\left(x\right)}{2b\tanh^{3}\left(x\right)+2b\tanh^{2}\left(x\right)} - \frac{1}{2b\tanh^{3}\left(x\right)+2b\tanh^{2}\left(x\right)} & \text{for } a = b \\ \frac{x - \frac{1}{\tanh\left(x\right)} - \frac{1}{3\tanh^{3}\left(x\right)}}{a} & \text{for } b = 0 \\ -\frac{2a^{4}\log\left(\tanh\left(x\right)+\frac{b}{a}\right)\tanh^{2}\left(x\right)}{2a^{2}b^{3}\tanh^{2}\left(x\right)-2b^{5}\tanh^{2}\left(x\right)} + \frac{2a^{4}\log\left(\tanh\left(x\right)\right)\tanh^{2}\left(x\right)}{2a^{2}b^{3}\tanh^{2}\left(x\right)-2b^{5}\tanh^{2}\left(x\right)} + \frac{2a^{3}b\tanh\left(x\right)}{2a^{2}b^{3}\tanh^{2}\left(x\right)-2b^{5}\tanh^{2}\left(x\right)} - \frac{a^{2}b^{2}}{2a^{2}b^{3}\tanh^{2}\left(x\right)-2b^{5}\tanh^{2}\left(x\right)} + \frac{2ab^{3}x\tanh^{2}\left(x\right)}{2a^{2}b^{3}\tanh^{2}\left(x\right)-2b^{5}\tanh^{2}\left(x\right)} - \frac{2ab^{3}\tanh\left(x\right)}{2a^{2}b^{3}\tanh^{2}\left(x\right)-2b^{5}\tanh^{2}\left(x\right)} - \frac{2b^{4}x\tanh^{2}\left(x\right)}{2a^{2}b^{3}\tanh^{2}\left(x\right)-2b^{5}\tanh^{2}\left(x\right)} + \frac{2b^{4}\log\left(\tanh\left(x\right)+1\right)\tanh^{2}\left(x\right)}{2a^{2}b^{3}\tanh^{2}\left(x\right)-2b^{5}\tanh^{2}\left(x\right)} - \frac{2b^{4}\log\left(\tanh\left(x\right)\right)\tanh^{2}\left(x\right)}{2a^{2}b^{3}\tanh^{2}\left(x\right)-2b^{5}\tanh^{2}\left(x\right)} + \frac{b^{4}}{2a^{2}b^{3}\tanh^{2}\left(x\right)-2b^{5}\tanh^{2}\left(x\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4/(a+b*coth(x)),x)
```

```
[Out] Piecewise((zoo*(x - log(tanh(x) + 1) + log(tanh(x)) - 1/(2*tanh(x)**2)), Eq
(a, 0) & Eq(b, 0)), ((x - log(tanh(x) + 1) + log(tanh(x)) - 1/(2*tanh(x)**2
))/b, Eq(a, 0)), (7*x*tanh(x)**3/(2*b*tanh(x)**3 - 2*b*tanh(x)**2) - 7*x*ta
nh(x)**2/(2*b*tanh(x)**3 - 2*b*tanh(x)**2) - 4*log(tanh(x) + 1)*tanh(x)**3/
(2*b*tanh(x)**3 - 2*b*tanh(x)**2) + 4*log(tanh(x) + 1)*tanh(x)**2/(2*b*tanh
(x)**3 - 2*b*tanh(x)**2) + 4*log(tanh(x))*tanh(x)**3/(2*b*tanh(x)**3 - 2*b*
tanh(x)**2) - 4*log(tanh(x))*tanh(x)**2/(2*b*tanh(x)**3 - 2*b*tanh(x)**2) -
 3*tanh(x)**2/(2*b*tanh(x)**3 - 2*b*tanh(x)**2) + tanh(x)/(2*b*tanh(x)**3 -
 2*b*tanh(x)**2) + 1/(2*b*tanh(x)**3 - 2*b*tanh(x)**2), Eq(a, -b)), (x*tanh
(x)**3/(2*b*tanh(x)**3 + 2*b*tanh(x)**2) + x*tanh(x)**2/(2*b*tanh(x)**3 + 2
*b*tanh(x)**2) - 4*log(tanh(x) + 1)*tanh(x)**3/(2*b*tanh(x)**3 + 2*b*tanh(x
)**2) - 4*log(tanh(x) + 1)*tanh(x)**2/(2*b*tanh(x)**3 + 2*b*tanh(x)**2) + 4
*log(tanh(x))*tanh(x)**3/(2*b*tanh(x)**3 + 2*b*tanh(x)**2) + 4*log(tanh(x))
```

```
*tanh(x)**2/(2*b*tanh(x)**3 + 2*b*tanh(x)**2) + 3*tanh(x)**2/(2*b*tanh(x)**
3 + 2*b*tanh(x)**2) + tanh(x)/(2*b*tanh(x)**3 + 2*b*tanh(x)**2) - 1/(2*b*ta
nh(x)**3 + 2*b*tanh(x)**2), Eq(a, b)), ((x - 1/tanh(x) - 1/(3*tanh(x)**3))/
a, Eq(b, 0)), (-2*a**4*log(tanh(x) + b/a)*tanh(x)**2/(2*a**2*b**3*tanh(x)**
2 - 2*b**5*tanh(x)**2) + 2*a**4*log(tanh(x))*tanh(x)**2/(2*a**2*b**3*tanh(x
)**2 - 2*b**5*tanh(x)**2) + 2*a**3*b*tanh(x)/(2*a**2*b**3*tanh(x)**2 - 2*b*
*5*tanh(x)**2) - a**2*b**2/(2*a**2*b**3*tanh(x)**2 - 2*b**5*tanh(x)**2) + 2
*a*b**3*x*tanh(x)**2/(2*a**2*b**3*tanh(x)**2 - 2*b**5*tanh(x)**2) - 2*a*b**
3*tanh(x)/(2*a**2*b**3*tanh(x)**2 - 2*b**5*tanh(x)**2) - 2*b**4*x*tanh(x)**
2/(2*a**2*b**3*tanh(x)**2 - 2*b**5*tanh(x)**2) + 2*b**4*log(tanh(x) + 1)*ta
nh(x)**2/(2*a**2*b**3*tanh(x)**2 - 2*b**5*tanh(x)**2) - 2*b**4*log(tanh(x))
*tanh(x)**2/(2*a**2*b**3*tanh(x)**2 - 2*b**5*tanh(x)**2) + b**4/(2*a**2*b**
3*tanh(x)**2 - 2*b**5*tanh(x)**2), True))
```

Giac [A]
time = 0.41, size = 100, normalized size = 1.32

$$-\frac{a^4\log\left(\left|ae^{(2x)}+be^{(2x)}-a+b\right|\right)}{a^2b^3-b^5}+\frac{x}{a-b}+\frac{(a^2+b^2)\log\left(\left|e^{(2x)}-1\right|\right)}{b^3}-\frac{2\left(ab-(ab-b^2)e^{(2x)}\right)}{b^3(e^{(2x)}-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -a^4*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^2*b^3 - b^5) + x/(a - b) +
(a^2 + b^2)*log(abs(e^(2*x) - 1))/b^3 - 2*(a*b - (a*b - b^2)*e^(2*x))/(b^3*
(e^(2*x) - 1)^2)
```

Mupad [B]
time = 1.63, size = 110, normalized size = 1.45

$$\frac{x}{a-b}-\frac{2}{b\,\left(\mathrm{e}^{4x}-2\,\mathrm{e}^{2x}+1\right)}+\frac{\ln\left(\mathrm{e}^{2x}-1\right)\,\left(a^2+b^2\right)}{b^3}+\frac{a^4\,\ln\left(b-a+a\,\mathrm{e}^{2x}+b\,\mathrm{e}^{2x}\right)}{b^5-a^2\,b^3}+\frac{2\,\left(a^2-b^2\right)}{b^2\,\left(a+b\right)\,\left(\mathrm{e}^{2x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(a + b*coth(x)),x)
```

```
[Out] x/(a - b) - 2/(b*(exp(4*x) - 2*exp(2*x) + 1)) + (log(exp(2*x) - 1)*(a^2 + b
^2))/b^3 + (a^4*log(b - a + a*exp(2*x) + b*exp(2*x)))/(b^5 - a^2*b^3) + (2*
(a^2 - b^2))/(b^2*(a + b)*(exp(2*x) - 1))
```

3.149 $\int \frac{\coth^5(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=94

$$-\frac{bx}{a^2-b^2}-\frac{(a^2+b^2)\coth(x)}{b^3}+\frac{a\coth^2(x)}{2b^2}-\frac{\coth^3(x)}{3b}+\frac{a^5\log(a+b\coth(x))}{b^4(a^2-b^2)}+\frac{a\log(\sinh(x))}{a^2-b^2}$$

```
[Out] -b*x/(a^2-b^2)-(a^2+b^2)*coth(x)/b^3+1/2*a*coth(x)^2/b^2-1/3*coth(x)^3/b+a^
5*ln(a+b*coth(x))/b^4/(a^2-b^2)+a*ln(sinh(x))/(a^2-b^2)
```

Rubi [A]
time = 0.27, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3647, 3728, 3729, 3707, 3698, 31, 3556}

$$-\frac{bx}{a^2-b^2}+\frac{a\log(\sinh(x))}{a^2-b^2}-\frac{(a^2+b^2)\coth(x)}{b^3}+\frac{a^5\log(a+b\coth(x))}{b^4(a^2-b^2)}+\frac{a\coth^2(x)}{2b^2}-\frac{\coth^3(x)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^5/(a + b*Coth[x]),x]
```

```
[Out] -((b*x)/(a^2 - b^2)) - ((a^2 + b^2)*Coth[x])/b^3 + (a*Coth[x]^2)/(2*b^2) -
Coth[x]^3/(3*b) + (a^5*Log[a + b*Coth[x]])/(b^4*(a^2 - b^2)) + (a*Log[Sinh[
x]])/(a^2 - b^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3729

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m + n +
1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*
Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*(b*c*m + a*d*(n + 1)) + d*(A*b -
 b*C)*(m + n + 1)*Tan[e + f*x] - C*m*(b*c - a*d)*Tan[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 +
 b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[
m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{\coth^5(x)}{a+b\coth(x)}\,dx = -\frac{\coth^3(x)}{3b} - \frac{\int \frac{\coth^2(x)(-3a-3b\coth(x)+3a\coth^2(x))}{a+b\coth(x)}\,dx}{3b}$$

$$= \frac{a\coth^2(x)}{2b^2} - \frac{\coth^3(x)}{3b} - \frac{\int \frac{\coth(x)(6a^2-6(a^2+b^2)\coth^2(x))}{a+b\coth(x)}\,dx}{6b^2}$$

$$= -\frac{(a^2+b^2)\coth(x)}{b^3} + \frac{a\coth^2(x)}{2b^2} - \frac{\coth^3(x)}{3b} - \frac{\int \frac{-6a(a^2+b^2)-6b^3\coth(x)+6a(a^2+b^2)\coth^2(x)}{a+b\coth(x)}\,d}{6b^3}$$

$$= -\frac{bx}{a^2-b^2} - \frac{(a^2+b^2)\coth(x)}{b^3} + \frac{a\coth^2(x)}{2b^2} - \frac{\coth^3(x)}{3b} + \frac{a\int \coth(x)\,dx}{a^2-b^2} + \frac{a^5\int \frac{1-\coth}{a+b\coth}}{b^3(a^2-}$$

$$= -\frac{bx}{a^2-b^2} - \frac{(a^2+b^2)\coth(x)}{b^3} + \frac{a\coth^2(x)}{2b^2} - \frac{\coth^3(x)}{3b} + \frac{a\log(\sinh(x))}{a^2-b^2} + \frac{a^5\text{Subst}(\int}{}$$

$$= -\frac{bx}{a^2-b^2} - \frac{(a^2+b^2)\coth(x)}{b^3} + \frac{a\coth^2(x)}{2b^2} - \frac{\coth^3(x)}{3b} + \frac{a^5\log(a+b\coth(x))}{b^4(a^2-b^2)} + \frac{a\log}{}$$

Mathematica [A]
time = 0.23, size = 108, normalized size = 1.15

$$\frac{6b^5x - 3ab^2(a^2-b^2)\operatorname{csch}^2(x) + 2b(a^2-b^2)\coth(x)\left(3a^2+4b^2+b^2\operatorname{csch}^2(x)\right) + 6a(a^4-b^4)\log(\sinh(x)) - 6a^5\log(b\cosh(x)+a\sinh(x))}{6b^4(-a+b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^5/(a + b*Coth[x]),x]
```

```
[Out] (6*b^5*x - 3*a*b^2*(a^2 - b^2)*Csch[x]^2 + 2*b*(a^2 - b^2)*Coth[x]*(3*a^2 +
 4*b^2 + b^2*Csch[x]^2) + 6*a*(a^4 - b^4)*Log[Sinh[x]] - 6*a^5*Log[b*Cosh[x
] + a*Sinh[x]])/(6*b^4*(-a + b)*(a + b))
```

Maple [A]
time = 0.38, size = 95, normalized size = 1.01

method	result
derivativedivides	$-\frac{\frac{b^2(\coth^3(x))}{3}-\frac{a(\coth^2(x))b}{2}+a^2\coth(x)+b^2\coth(x)}{b^3} - \frac{\ln(1+\coth(x))}{2a-2b} + \frac{a^5\ln(a+b\coth(x))}{b^4(a+b)(a-b)} - \frac{\ln(\coth(x)-1)}{2b+2a}$
default	$-\frac{\frac{b^2(\coth^3(x))}{3}-\frac{a(\coth^2(x))b}{2}+a^2\coth(x)+b^2\coth(x)}{b^3} - \frac{\ln(1+\coth(x))}{2a-2b} + \frac{a^5\ln(a+b\coth(x))}{b^4(a+b)(a-b)} - \frac{\ln(\coth(x)-1)}{2b+2a}$
risch	$\frac{x}{a+b} + \frac{2a^3x}{b^4} + \frac{2ax}{b^2} - \frac{2x\,a^5}{b^4(a^2-b^2)} - \frac{2(3a^2e^{4x}-3ab\,e^{4x}+6b^2e^{4x}-6a^2e^{2x}+3ab\,e^{2x}-6b^2e^{2x}+3a^2+4b^2)}{3b^3(e^{2x}-1)^3} - \frac{a^3\ln(e^{2x}}{b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^5/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

[Out] -1/b^3*(1/3*b^2*coth(x)^3-1/2*a*coth(x)^2*b+a^2*coth(x)+b^2*coth(x))-1/(2*a-2*b)*ln(1+coth(x))+1/b^4*a^5/(a+b)/(a-b)*ln(a+b*coth(x))-1/(2*b+2*a)*ln(coth(x)-1)

Maxima [A]
time = 0.29, size = 169, normalized size = 1.80

$$\frac{a^5\log\left(-(a-b)e^{(-2x)}+a+b\right)}{a^2b^4-b^6}+\frac{2\left(3a^2+4b^2-3\left(2a^2+ab+2b^2\right)e^{(-2x)}+3\left(a^2+ab+2b^2\right)e^{(-4x)}\right)}{3\left(3b^3e^{(-2x)}-3b^3e^{(-4x)}+b^3e^{(-6x)}-b^3\right)}+\frac{x}{a+b}-\frac{\left(a^3+ab^2\right)\log\left(e^{(-x)}+1\right)}{b^4}-\frac{\left(a^3+ab^2\right)\log\left(e^{(-x)}-1\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^5/(a+b*coth(x)),x, algorithm="maxima")

[Out] a^5*log(-(a - b)*e^(-2*x) + a + b)/(a^2*b^4 - b^6) + 2/3*(3*a^2 + 4*b^2 - 3*(2*a^2 + a*b + 2*b^2)*e^(-2*x) + 3*(a^2 + a*b + 2*b^2)*e^(-4*x))/(3*b^3*e^(-2*x) - 3*b^3*e^(-4*x) + b^3*e^(-6*x) - b^3) + x/(a + b) - (a^3 + a*b^2)*log(e^(-x) + 1)/b^4 - (a^3 + a*b^2)*log(e^(-x) - 1)/b^4

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1299 vs. 2(90) = 180.
time = 0.42, size = 1299, normalized size = 13.82

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^5/(a+b*coth(x)),x, algorithm="fricas")

[Out] -1/3*(3*(a*b^4 + b^5)*x*cosh(x)^6 + 18*(a*b^4 + b^5)*x*cosh(x)*sinh(x)^5 + 3*(a*b^4 + b^5)*x*sinh(x)^6 + 6*a^4*b + 2*a^2*b^3 - 8*b^5 + 3*(2*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 - 4*b^5 - 3*(a*b^4 + b^5)*x)*cosh(x)^4 + 3*(2*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 - 4*b^5 + 15*(a*b^4 + b^5)*x*cosh(x)^2 - 3*(a*b^4 + b^5)*x)*sinh(x)^4 + 12*(5*(a*b^4 + b^5)*x*cosh(x)^3 + (2*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 - 4*b^5 - 3*(a*b^4 + b^5)*x)*cosh(x))*sinh(x)^3 - 3*(4*a^4*b - 2*a^3*b^2 + 2*a*b^4 - 4*b^5 - 3*(a*b^4 + b^5)*x)*cosh(x)^2 + 3*(15*(a*b^4 + b^5)*x*cosh(x)^4 - 4*a^4*b + 2*a^3*b^2 - 2*a*b^4 + 4*b^5 + 6*(2*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 - 4*b^5 - 3*(a*b^4 + b^5)*x)*cosh(x)^2 + 3*(a*b^4 + b^5)*x)*sinh(x)^2 - 3*(a*b^4 + b^5)*x - 3*(a^5*cosh(x)^6 + 6*a^5*cosh(x)*sinh(x)^5 + a^5*sinh(x)^6 - 3*a^5*cosh(x)^4 + 3*a^5*cosh(x)^2 - a^5 + 3*(5*a^5*cosh(x)^2 - a^5)*sinh(x)^4 + 4*(5*a^5*cosh(x)^3 - 3*a^5*cosh(x))*sinh(x)^3 + 3*(5*a^5*cosh(x)^4 - 6*a^5*cosh(x)^2 + a^5)*sinh(x)^2 + 6*(a^5*cosh(x)^5 - 2*a^5*cosh(x)^3 + a^5*cosh(x))*sinh(x))*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) + 3*((a^5 - a*b^4)*cosh(x)^6 + 6*(a^5 - a*b^4)*cosh(x)*sinh(x)^5 + (a^5 - a*b^4)*sinh(x)^6 - a^5 + a*b^4 - 3*(a^5 - a*b^4)*cosh(x)^4 - 3*(a^5 - a*b^4 - 5*(a^5 - a*b^4)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^5 - a*b^4)*cosh(x)^3 - 3*(a^5 - a*b^4)*cosh(x))*sinh(x)^3 + 3*(a^5 - a*b^4)*cosh(x)^2 + 3*(a^5 - a*b^4 + 5*(a^5 - a*b^4)*cosh(x)^4 - 6*(a^5 - a*b^4)*cosh(x)^2)*sinh(x)^2 + 6*((a^5 - a*b^4)*cosh(x)

```
^5 - 2*(a^5 - a*b^4)*cosh(x)^3 + (a^5 - a*b^4)*cosh(x))*sinh(x))*log(2*sinh
(x)/(cosh(x) - sinh(x))) + 6*(3*(a*b^4 + b^5)*x*cosh(x)^5 + 2*(2*a^4*b - 2*
a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 - 4*b^5 - 3*(a*b^4 + b^5)*x)*cosh(x)^3 - (4*a
^4*b - 2*a^3*b^2 + 2*a*b^4 - 4*b^5 - 3*(a*b^4 + b^5)*x)*cosh(x))*sinh(x))/(
(a^2*b^4 - b^6)*cosh(x)^6 + 6*(a^2*b^4 - b^6)*cosh(x)*sinh(x)^5 + (a^2*b^4
- b^6)*sinh(x)^6 - a^2*b^4 + b^6 - 3*(a^2*b^4 - b^6)*cosh(x)^4 - 3*(a^2*b^4
 - b^6 - 5*(a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^2*b^4 - b^6)*cosh
(x)^3 - 3*(a^2*b^4 - b^6)*cosh(x))*sinh(x)^3 + 3*(a^2*b^4 - b^6)*cosh(x)^2
+ 3*(a^2*b^4 - b^6 + 5*(a^2*b^4 - b^6)*cosh(x)^4 - 6*(a^2*b^4 - b^6)*cosh(x
)^2)*sinh(x)^2 + 6*((a^2*b^4 - b^6)*cosh(x)^5 - 2*(a^2*b^4 - b^6)*cosh(x)^3
 + (a^2*b^4 - b^6)*cosh(x))*sinh(x))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1013 vs. $2(78) = 156$.
time = 3.02, size = 1013, normalized size = 10.78

$$\begin{cases} \tilde{\infty}\left(x - \frac{1}{\tanh(x)} - \frac{1}{3\tanh^3(x)}\right) & \text{for } a = 0 \wedge b = 0 \\ \frac{x - \frac{1}{\tanh(x)} - \frac{1}{3\tanh^3(x)}}{b} & \text{for } a = 0 \\ \frac{27x\tanh^4(x)}{6b\tanh^4(x)-6b\tanh^3(x)} - \frac{27x\tanh^3(x)}{6b\tanh^4(x)-6b\tanh^3(x)} - \frac{12\log(\tanh(x)+1)\tanh^4(x)}{6b\tanh^4(x)-6b\tanh^3(x)} + \frac{12\log(\tanh(x)+1)\tanh^3(x)}{6b\tanh^4(x)-6b\tanh^3(x)} + \frac{12\log(\tanh(x))\tanh^4(x)}{6b\tanh^4(x)-6b\tanh^3(x)} - \frac{12\log(\tanh(x))\tanh^3(x)}{6b\tanh^4(x)-6b\tanh^3(x)} - \frac{15\tanh^3(x)}{6b\tanh^4(x)-6b\tanh^3(x)} + \frac{9\tanh^2(x)}{6b\tanh^4(x)-6b\tanh^3(x)} + \frac{\tanh(x)}{6b\tanh^4(x)-6b\tanh^3(x)} + \frac{2}{6b\tanh^4(x)-6b\tanh^3(x)} & \text{for } a = -b \\ \frac{3x\tanh^4(x)}{6b\tanh^4(x)+6b\tanh^3(x)} + \frac{3x\tanh^3(x)}{6b\tanh^4(x)+6b\tanh^3(x)} + \frac{12\log(\tanh(x)+1)\tanh^4(x)}{6b\tanh^4(x)+6b\tanh^3(x)} + \frac{12\log(\tanh(x)+1)\tanh^3(x)}{6b\tanh^4(x)+6b\tanh^3(x)} - \frac{12\log(\tanh(x))\tanh^4(x)}{6b\tanh^4(x)+6b\tanh^3(x)} - \frac{12\log(\tanh(x))\tanh^3(x)}{6b\tanh^4(x)+6b\tanh^3(x)} - \frac{15\tanh^3(x)}{6b\tanh^4(x)+6b\tanh^3(x)} - \frac{9\tanh^2(x)}{6b\tanh^4(x)+6b\tanh^3(x)} + \frac{\tanh(x)}{6b\tanh^4(x)+6b\tanh^3(x)} - \frac{2}{6b\tanh^4(x)+6b\tanh^3(x)} & \text{for } a = b \\ \frac{x - \log(\tanh(x)+1) + \log(\tanh(x)) - \frac{1}{2\tanh^2(x)} - \frac{1}{4\tanh^4(x)}}{a} & \text{for } b = 0 \\ \frac{6a^5\log\left(\tanh(x)+\frac{b}{a}\right)\tanh^3(x)}{6a^2b^4\tanh^3(x)-6b^6\tanh^3(x)} - \frac{6a^5\log(\tanh(x))\tanh^3(x)}{6a^2b^4\tanh^3(x)-6b^6\tanh^3(x)} - \frac{6a^4b\tanh^2(x)}{6a^2b^4\tanh^3(x)-6b^6\tanh^3(x)} + \frac{3a^3b^2\tanh(x)}{6a^2b^4\tanh^3(x)-6b^6\tanh^3(x)} - \frac{2a^2b^3}{6a^2b^4\tanh^3(x)-6b^6\tanh^3(x)} + \frac{6ab^4x\tanh^3(x)}{6a^2b^4\tanh^3(x)-6b^6\tanh^3(x)} - \frac{6ab^4\log(\tanh(x)+1)\tanh^3(x)}{6a^2b^4\tanh^3(x)-6b^6\tanh^3(x)} + \frac{6ab^4\log(\tanh(x))\tanh^3(x)}{6a^2b^4\tanh^3(x)-6b^6\tanh^3(x)} - \frac{3ab^4\tanh(x)}{6a^2b^4\tanh^3(x)-6b^6\tanh^3(x)} - \frac{6b^5x\tanh^3(x)}{6a^2b^4\tanh^3(x)-6b^6\tanh^3(x)} + \frac{6b^5\tanh^2(x)}{6a^2b^4\tanh^3(x)-6b^6\tanh^3(x)} + \frac{2b^5}{6a^2b^4\tanh^3(x)-6b^6\tanh^3(x)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**5/(a+b*coth(x)),x)
```

```
[Out] Piecewise((zoo*(x - 1/tanh(x) - 1/(3*tanh(x)**3)), Eq(a, 0) & Eq(b, 0)), ((
x - 1/tanh(x) - 1/(3*tanh(x)**3))/b, Eq(a, 0)), (27*x*tanh(x)**4/(6*b*tanh(
x)**4 - 6*b*tanh(x)**3) - 27*x*tanh(x)**3/(6*b*tanh(x)**4 - 6*b*tanh(x)**3)
 - 12*log(tanh(x) + 1)*tanh(x)**4/(6*b*tanh(x)**4 - 6*b*tanh(x)**3) + 12*lo
g(tanh(x) + 1)*tanh(x)**3/(6*b*tanh(x)**4 - 6*b*tanh(x)**3) + 12*log(tanh(x
))*tanh(x)**4/(6*b*tanh(x)**4 - 6*b*tanh(x)**3) - 12*log(tanh(x))*tanh(x)**
3/(6*b*tanh(x)**4 - 6*b*tanh(x)**3) - 15*tanh(x)**3/(6*b*tanh(x)**4 - 6*b*t
anh(x)**3) + 9*tanh(x)**2/(6*b*tanh(x)**4 - 6*b*tanh(x)**3) + tanh(x)/(6*b*
tanh(x)**4 - 6*b*tanh(x)**3) + 2/(6*b*tanh(x)**4 - 6*b*tanh(x)**3), Eq(a, -
b)), (3*x*tanh(x)**4/(6*b*tanh(x)**4 + 6*b*tanh(x)**3) + 3*x*tanh(x)**3/(6*
b*tanh(x)**4 + 6*b*tanh(x)**3) + 12*log(tanh(x) + 1)*tanh(x)**4/(6*b*tanh(x
)**4 + 6*b*tanh(x)**3) + 12*log(tanh(x) + 1)*tanh(x)**3/(6*b*tanh(x)**4 + 6
*b*tanh(x)**3) - 12*log(tanh(x))*tanh(x)**4/(6*b*tanh(x)**4 + 6*b*tanh(x)**
3) - 12*log(tanh(x))*tanh(x)**3/(6*b*tanh(x)**4 + 6*b*tanh(x)**3) - 15*tanh
(x)**3/(6*b*tanh(x)**4 + 6*b*tanh(x)**3) - 9*tanh(x)**2/(6*b*tanh(x)**4 + 6
*b*tanh(x)**3) + tanh(x)/(6*b*tanh(x)**4 + 6*b*tanh(x)**3) - 2/(6*b*tanh(x)
**4 + 6*b*tanh(x)**3), Eq(a, b)), ((x - log(tanh(x) + 1) + log(tanh(x)) - 1
/(2*tanh(x)**2) - 1/(4*tanh(x)**4))/a, Eq(b, 0)), (6*a**5*log(tanh(x) + b/a
)*tanh(x)**3/(6*a**2*b**4*tanh(x)**3 - 6*b**6*tanh(x)**3) - 6*a**5*log(tanh
(x))*tanh(x)**3/(6*a**2*b**4*tanh(x)**3 - 6*b**6*tanh(x)**3) - 6*a**4*b*tan
h(x)**2/(6*a**2*b**4*tanh(x)**3 - 6*b**6*tanh(x)**3) + 3*a**3*b**2*tanh(x)/
(6*a**2*b**4*tanh(x)**3 - 6*b**6*tanh(x)**3) - 2*a**2*b**3/(6*a**2*b**4*tan
```

```
h(x)**3 - 6*b**6*tanh(x)**3) + 6*a*b**4*x*tanh(x)**3/(6*a**2*b**4*tanh(x)**
3 - 6*b**6*tanh(x)**3) - 6*a*b**4*log(tanh(x) + 1)*tanh(x)**3/(6*a**2*b**4*
tanh(x)**3 - 6*b**6*tanh(x)**3) + 6*a*b**4*log(tanh(x))*tanh(x)**3/(6*a**2*
b**4*tanh(x)**3 - 6*b**6*tanh(x)**3) - 3*a*b**4*tanh(x)/(6*a**2*b**4*tanh(x
)**3 - 6*b**6*tanh(x)**3) - 6*b**5*x*tanh(x)**3/(6*a**2*b**4*tanh(x)**3 - 6
*b**6*tanh(x)**3) + 6*b**5*tanh(x)**2/(6*a**2*b**4*tanh(x)**3 - 6*b**6*tanh
(x)**3) + 2*b**5/(6*a**2*b**4*tanh(x)**3 - 6*b**6*tanh(x)**3), True))
```

Giac [A]
time = 0.44, size = 143, normalized size = 1.52

$$\frac{a^5\log\left(\left|ae^{(2x)}+be^{(2x)}-a+b\right|\right)}{a^2b^4-b^6}-\frac{x}{a-b}-\frac{(a^3+ab^2)\log\left(\left|e^{(2x)}-1\right|\right)}{b^4}-\frac{2\left(3a^2b+4b^3+3(a^2b-ab^2+2b^3)e^{(4x)}-3(2a^2b-ab^2+2b^3)e^{(2x)}\right)}{3b^4(e^{(2x)}-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] a^5*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^2*b^4 - b^6) - x/(a - b) - (
a^3 + a*b^2)*log(abs(e^(2*x) - 1))/b^4 - 2/3*(3*a^2*b + 4*b^3 + 3*(a^2*b -
a*b^2 + 2*b^3)*e^(4*x) - 3*(2*a^2*b - a*b^2 + 2*b^3)*e^(2*x))/(b^4*(e^(2*x)
 - 1)^3)
```

Mupad [B]
time = 1.63, size = 164, normalized size = 1.74

$$-\frac{8}{3b\left(3e^{2x}-3e^{4x}+e^{6x}-1\right)}-\frac{x}{a-b}-\frac{a^5\ln\left(b-a+a\,e^{2x}+b\,e^{2x}\right)}{b^6-a^2b^4}-\frac{\ln\left(e^{2x}-1\right)\left(a^3+a\,b^2\right)}{b^4}-\frac{2\left(a^3+a\,b^2+2b^3\right)}{b^3\,(a+b)\,\left(e^{2x}-1\right)}-\frac{2\left(-a^2+a\,b+2b^2\right)}{b^2\,(a+b)\,\left(e^{4x}-2e^{2x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^5/(a + b*coth(x)),x)
```

```
[Out] - 8/(3*b*(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1)) - x/(a - b) - (a^5*log(b
 - a + a*exp(2*x) + b*exp(2*x)))/(b^6 - a^2*b^4) - (log(exp(2*x) - 1)*(a*b^
2 + a^3))/b^4 - (2*(a*b^2 + a^3 + 2*b^3))/(b^3*(a + b)*(exp(2*x) - 1)) - (2
*(a*b - a^2 + 2*b^2))/(b^2*(a + b)*(exp(4*x) - 2*exp(2*x) + 1))
```

3.150 $\int \frac{x\operatorname{csch}^2(x)}{(a+b\coth(x))^2}\,dx$

Optimal. Leaf size=54

$$-\frac{ax}{b\left(a^2-b^2\right)}+\frac{x}{b(a+b\coth(x))}+\frac{\log(b\cosh(x)+a\sinh(x))}{a^2-b^2}$$

```
[Out] -a*x/b/(a^2-b^2)+x/b/(a+b*coth(x))+ln(b*cosh(x)+a*sinh(x))/(a^2-b^2)
```

Rubi [A]
time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5575, 3565, 3611}

$$-\frac{ax}{b\left(a^2-b^2\right)}+\frac{\log(a\sinh(x)+b\cosh(x))}{a^2-b^2}+\frac{x}{b(a+b\coth(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Csch[x]^2)/(a + b*Coth[x])^2,x]
```

```
[Out] -((a*x)/(b*(a^2 - b^2))) + x/(b*(a + b*Coth[x])) + Log[b*Cosh[x] + a*Sinh[x
]]/(a^2 - b^2)
```

Rule 3565

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[a*(x/(a^
2 + b^2)), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 5575

```
Int[Csch[(c_.) + (d_.)*(x_)]^2*(Coth[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.
)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(e + f*x)^m)*((a + b*Coth
[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[f*(m/(b*d*(n + 1))), Int[(e +
f*x)^(m - 1)*(a + b*Coth[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{x\operatorname{csch}^2(x)}{(a+b\coth(x))^2}\,dx = \frac{x}{b(a+b\coth(x))} - \frac{\int \frac{1}{a+b\coth(x)}\,dx}{b}$$
$$= -\frac{ax}{b\left(a^2-b^2\right)} + \frac{x}{b(a+b\coth(x))} + \frac{i\int \frac{-ib-ia\coth(x)}{a+b\coth(x)}\,dx}{a^2-b^2}$$
$$= -\frac{ax}{b\left(a^2-b^2\right)} + \frac{x}{b(a+b\coth(x))} + \frac{\log(b\cosh(x)+a\sinh(x))}{a^2-b^2}$$

Mathematica [A]
time = 0.14, size = 49, normalized size = 0.91

$$\frac{ax-b\log(b\cosh(x)+a\sinh(x))}{-a^2b+b^3} + \frac{x\sinh(x)}{b^2\cosh(x)+ab\sinh(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Csch[x]^2)/(a + b*Coth[x])^2,x]
```

```
[Out] (a*x - b*Log[b*Cosh[x] + a*Sinh[x]])/(-(a^2*b) + b^3) + (x*Sinh[x])/(b^2*Co
sh[x] + a*b*Sinh[x])
```

Maple [A]
time = 1.30, size = 73, normalized size = 1.35

method	result	size
risch	$-\frac{2x}{a^2-b^2} - \frac{2x}{(e^{2x}a+b\,e^{2x}-a+b)(a+b)} + \frac{\ln\left(e^{2x}-\frac{a-b}{a+b}\right)}{a^2-b^2}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csch(x)^2/(a+b*coth(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/(a^2-b^2)*x-2*x/(exp(2*x)*a+b*exp(2*x)-a+b)/(a+b)+1/(a^2-b^2)*ln(exp(2*x
)-(a-b)/(a+b))
```

Maxima [A]
time = 0.41, size = 68, normalized size = 1.26

$$\frac{2\,xe^{(2\,x)}}{a^2-2\,ab+b^2-(a^2-b^2)e^{(2\,x)}} + \frac{\log\left(\frac{(a+b)e^{(2\,x)}-a+b}{a+b}\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(x)^2/(a+b*coth(x))^2,x, algorithm="maxima")
```

```
[Out] 2*x*e^(2*x)/(a^2 - 2*a*b + b^2 - (a^2 - b^2)*e^(2*x)) + log(((a + b)*e^(2*x
) - a + b)/(a + b))/(a^2 - b^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(54) = 108$.
time = 0.42, size = 184, normalized size = 3.41

$$\frac{2\,(a+b)x\cosh\left(x\right)^{2}+4\,(a+b)x\cosh\left(x\right)\sinh\left(x\right)+2\,(a+b)x\sinh\left(x\right)^{2}-\left((a+b)\cosh\left(x\right)^{2}+2\,(a+b)\cosh\left(x\right)\sinh\left(x\right)+(a+b)\sinh\left(x\right)^{2}-a+b\right)\log\left(\frac{2\,(b\cosh(x)+a\sinh(x))}{\cosh(x)-\sinh(x)}\right)}{a^{3}-a^{2}b-ab^{2}+b^{3}-(a^{3}+a^{2}b-ab^{2}-b^{3})\cosh\left(x\right)^{2}-2\,(a^{3}+a^{2}b-ab^{2}-b^{3})\cosh\left(x\right)\sinh\left(x\right)-(a^{3}+a^{2}b-ab^{2}-b^{3})\sinh\left(x\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(x)^2/(a+b*coth(x))^2,x, algorithm="fricas")
```

```
[Out] (2*(a + b)*x*cosh(x)^2 + 4*(a + b)*x*cosh(x)*sinh(x) + 2*(a + b)*x*sinh(x)^
2 - ((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a
+ b)*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))))/(a^3 - a^2*b - a*b
^2 + b^3 - (a^3 + a^2*b - a*b^2 - b^3)*cosh(x)^2 - 2*(a^3 + a^2*b - a*b^2 -
 b^3)*cosh(x)*sinh(x) - (a^3 + a^2*b - a*b^2 - b^3)*sinh(x)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\operatorname{csch}^{2}\left(x\right)}{\left(a+b\coth\left(x\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(x)**2/(a+b*coth(x))**2,x)
```

```
[Out] Integral(x*csch(x)**2/(a + b*coth(x))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(54) = 108$.
time = 0.42, size = 169, normalized size = 3.13

$$-\frac{2\,axe^{(2\,x)}+2\,bxe^{(2\,x)}-ae^{(2\,x)}\log\left(ae^{(2\,x)}+be^{(2\,x)}-a+b\right)-be^{(2\,x)}\log\left(ae^{(2\,x)}+be^{(2\,x)}-a+b\right)+a\log\left(ae^{(2\,x)}+be^{(2\,x)}-a+b\right)-b\log\left(ae^{(2\,x)}+be^{(2\,x)}-a+b\right)}{a^{3}e^{(2\,x)}+a^{2}be^{(2\,x)}-ab^{2}e^{(2\,x)}-b^{3}e^{(2\,x)}-a^{3}+a^{2}b+ab^{2}-b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(x)^2/(a+b*coth(x))^2,x, algorithm="giac")
```

```
[Out] -(2*a*x*e^(2*x) + 2*b*x*e^(2*x) - a*e^(2*x)*log(a*e^(2*x) + b*e^(2*x) - a +
 b) - b*e^(2*x)*log(a*e^(2*x) + b*e^(2*x) - a + b) + a*log(a*e^(2*x) + b*e^
(2*x) - a + b) - b*log(a*e^(2*x) + b*e^(2*x) - a + b))/(a^3*e^(2*x) + a^2*b
*e^(2*x) - a*b^2*e^(2*x) - b^3*e^(2*x) - a^3 + a^2*b + a*b^2 - b^3)
```

Mupad [B]
time = 1.28, size = 68, normalized size = 1.26

$$\frac{\ln\left(b-a+a\,{\mathrm{e}}^{2\,x}+b\,{\mathrm{e}}^{2\,x}\right)}{a^2-b^2}-\frac{2\,x}{a^2-b^2}-\frac{2\,x}{\left(a+b\right)\,\left(b-a+{\mathrm{e}}^{2\,x}\,\left(a+b\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(sinh(x)^2*(a + b*coth(x))^2),x)
```

```
[Out] log(b - a + a*exp(2*x) + b*exp(2*x))/(a^2 - b^2) - (2*x)/(a^2 - b^2) - (2*x
)/((a + b)*(b - a + exp(2*x)*(a + b)))
```

3.151 $\int x^3 \coth(a + 2\log(x))\,dx$

Optimal. Leaf size=30

$$\frac{x^4}{4} + \frac{1}{2}e^{-2a}\log\left(1 - e^{2a}x^4\right)$$

```
[Out] 1/4*x^4+1/2*ln(1-exp(2*a)*x^4)/exp(2*a)
```

Rubi [A]
time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {5657, 455, 45}

$$\frac{1}{2}e^{-2a}\log\left(1 - e^{2a}x^4\right) + \frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Coth[a + 2*Log[x]],x]
[Out] x^4/4 + Log[1 - E^(2*a)*x^4]/(2*E^(2*a))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int x^3 \coth(a + 2\log(x))\,dx = \int x^3 \coth(a + 2\log(x))\,dx$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 64 vs. 2(30) = 60.
time = 0.02, size = 64, normalized size = 2.13

$$\frac{x^4}{4}+\frac{1}{2}\cosh(2a)\log\left(-\cosh(a)+x^4\cosh(a)+\sinh(a)+x^4\sinh(a)\right)-\frac{1}{2}\log\left(-\cosh(a)+x^4\cosh(a)+\sinh(a)+x^4\sinh(a)\right)\sinh(2a)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Coth[a + 2*Log[x]],x]
```

```
[Out] x^4/4 + (Cosh[2*a]*Log[-Cosh[a] + x^4*Cosh[a] + Sinh[a] + x^4*Sinh[a]])/2 -
 (Log[-Cosh[a] + x^4*Cosh[a] + Sinh[a] + x^4*Sinh[a]]*Sinh[2*a])/2
```

Maple [A]
time = 0.44, size = 24, normalized size = 0.80

method	result	size
risch	$\frac{x^4}{4}+\frac{e^{-2a}\ln\left(-1+e^{2a}x^4\right)}{2}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*coth(a+2*ln(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4+1/2*exp(-2*a)*ln(-1+exp(2*a)*x^4)
```

Maxima [A]
time = 0.27, size = 36, normalized size = 1.20

$$\frac{1}{4}x^4+\frac{1}{2}e^{(-2a)}\log\left(x^2e^a+1\right)+\frac{1}{2}e^{(-2a)}\log\left(x^2e^a-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*coth(a+2*log(x)),x, algorithm="maxima")
```

```
[Out] 1/4*x^4 + 1/2*e^(-2*a)*log(x^2*e^a + 1) + 1/2*e^(-2*a)*log(x^2*e^a - 1)
```

Fricas [A]
time = 0.37, size = 28, normalized size = 0.93

$$\frac{1}{4}\left(x^4e^{(2a)}+2\log\left(x^4e^{(2a)}-1\right)\right)e^{(-2a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*coth(a+2*log(x)),x, algorithm="fricas")
```

```
[Out] 1/4*(x^4*e^(2*a) + 2*log(x^4*e^(2*a) - 1))*e^(-2*a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \coth\left(a + 2\log\left(x\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*coth(a+2*ln(x)),x)
```

```
[Out] Integral(x**3*coth(a + 2*log(x)), x)
```

Giac [A]
time = 0.41, size = 24, normalized size = 0.80

$$\frac{1}{4}\,x^4 + \frac{1}{2}\,e^{(-2\,a)}\log\left(\left|x^4 e^{(2\,a)} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*coth(a+2*log(x)),x, algorithm="giac")
```

```
[Out] 1/4*x^4 + 1/2*e^(-2*a)*log(abs(x^4*e^(2*a) - 1))
```

Mupad [B]
time = 1.25, size = 23, normalized size = 0.77

$$\frac{\ln\left(x^4 - \mathrm{e}^{-2\,a}\right)\,\mathrm{e}^{-2\,a}}{2} + \frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*coth(a + 2*log(x)),x)
```

```
[Out] (log(x^4 - exp(-2*a))*exp(-2*a))/2 + x^4/4
```

3.152 $\int x^2 \coth(a + 2\log(x))\,dx$

Optimal. Leaf size=45

$$\frac{x^3}{3} + e^{-3a/2}\text{ArcTan}\left(e^{a/2}x\right) - e^{-3a/2}\tanh^{-1}\left(e^{a/2}x\right)$$

```
[Out] 1/3*x^3+arctan(exp(1/2*a)*x)/exp(3/2*a)-arctanh(exp(1/2*a)*x)/exp(3/2*a)
```

Rubi [A]
time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {5657, 470, 304, 209, 212}

$$e^{-3a/2}\text{ArcTan}\left(e^{a/2}x\right) - e^{-3a/2}\tanh^{-1}\left(e^{a/2}x\right) + \frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Coth[a + 2*Log[x]],x]
```

```
[Out] x^3/3 + ArcTan[E^(a/2)*x]/E^((3*a)/2) - ArcTanh[E^(a/2)*x]/E^((3*a)/2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
```

```
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int x^2 \coth(a+2\log(x))\,dx = \int x^2 \coth(a+2\log(x))\,dx$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.17, size = 64, normalized size = 1.42

$$\frac{1}{6}\left(2x^3+3\text{RootSum}\left[-\cosh(a)+\sinh(a)+\cosh(a)\#1^4+\sinh(a)\#1^4\&,\frac{\log(x)-\log(x-\#1)}{\#1}\&\right](-\cosh(2a)+\sinh(2a))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Coth[a + 2*Log[x]],x]
[Out] (2*x^3 + 3*RootSum[-Cosh[a] + Sinh[a] + Cosh[a]*#1^4 + Sinh[a]*#1^4 & , (Lo
g[x] - Log[x - #1])/#1 & ]*(-Cosh[2*a] + Sinh[2*a]))/6
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(35) = 70.
time = 0.71, size = 83, normalized size = 1.84

method	result	size
risch	$\frac{x^3}{3}+\frac{\ln\left(-\sqrt{\mathrm{e}^{a}}\,x+1\right)}{2(\mathrm{e}^{a})^{\frac{3}{2}}}-\frac{\ln\left(\sqrt{\mathrm{e}^{a}}\,x+1\right)}{2(\mathrm{e}^{a})^{\frac{3}{2}}}+\frac{\ln\left(-\mathrm{e}^{2a}x+(-\mathrm{e}^{a})^{\frac{3}{2}}\right)}{2(-\mathrm{e}^{a})^{\frac{3}{2}}}-\frac{\ln\left(\mathrm{e}^{2a}x+(-\mathrm{e}^{a})^{\frac{3}{2}}\right)}{2(-\mathrm{e}^{a})^{\frac{3}{2}}}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*coth(a+2*ln(x)),x,method=_RETURNVERBOSE)
[Out] 1/3*x^3+1/2/exp(a)^(3/2)*ln(-exp(a)^(1/2)*x+1)-1/2/exp(a)^(3/2)*ln(exp(a)^(
1/2)*x+1)+1/2/(-exp(a))^(3/2)*ln(-exp(2*a)*x+(-exp(a))^(3/2))-1/2/(-exp(a))
^(3/2)*ln(exp(2*a)*x+(-exp(a))^(3/2))
```

Maxima [A]
time = 0.51, size = 48, normalized size = 1.07

$$\frac{1}{3}\,x^{3}+\arctan\left(xe^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(-\frac{3}{2}\,a\right)}+\frac{1}{2}\,e^{\left(-\frac{3}{2}\,a\right)}\log\left(\frac{xe^{a}-e^{\left(\frac{1}{2}\,a\right)}}{xe^{a}+e^{\left(\frac{1}{2}\,a\right)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*coth(a+2*log(x)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + arctan(x*e^(1/2*a))*e^(-3/2*a) + 1/2*e^(-3/2*a)*log((x*e^a - e^(1
/2*a))/(x*e^a + e^(1/2*a)))
```

Fricas [A]
time = 0.36, size = 62, normalized size = 1.38

$$\frac{1}{6}\left(2\,x^{3}e^{(2\,a)}+6\,\arctan\left(xe^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(\frac{1}{2}\,a\right)}+3\,e^{\left(\frac{1}{2}\,a\right)}\log\left(\frac{x^{2}e^{a}-2\,xe^{\left(\frac{1}{2}\,a\right)}+1}{x^{2}e^{a}-1}\right)\right)e^{(-2\,a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*coth(a+2*log(x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*x^3*e^(2*a) + 6*arctan(x*e^(1/2*a))*e^(1/2*a) + 3*e^(1/2*a)*log((x^2
*e^a - 2*x*e^(1/2*a) + 1)/(x^2*e^a - 1)))*e^(-2*a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\coth\left(a+2\log\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*coth(a+2*ln(x)),x)
```

```
[Out] Integral(x**2*coth(a + 2*log(x)), x)
```

Giac [A]
time = 0.40, size = 54, normalized size = 1.20

$$\frac{1}{3}\,x^{3}+\arctan\left(xe^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(-\frac{3}{2}\,a\right)}+\frac{1}{2}\,e^{\left(-\frac{3}{2}\,a\right)}\log\left(\frac{\left|2\,xe^{a}-2\,e^{\left(\frac{1}{2}\,a\right)}\right|}{\left|2\,xe^{a}+2\,e^{\left(\frac{1}{2}\,a\right)}\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*coth(a+2*log(x)),x, algorithm="giac")
```

```
[Out] 1/3*x^3 + arctan(x*e^(1/2*a))*e^(-3/2*a) + 1/2*e^(-3/2*a)*log(abs(2*x*e^a -
 2*e^(1/2*a))/abs(2*x*e^a + 2*e^(1/2*a)))
```

Mupad [B]
time = 1.21, size = 39, normalized size = 0.87

$$\frac{\operatorname{atan}\left(x\,{\left({\mathrm{e}}^{2\,a}\right)}^{1/4}\right)}{{\left({\mathrm{e}}^{2\,a}\right)}^{3/4}}-\frac{\operatorname{atanh}\left(x\,{\left({\mathrm{e}}^{2\,a}\right)}^{1/4}\right)}{{\left({\mathrm{e}}^{2\,a}\right)}^{3/4}}+\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*coth(a + 2*log(x)),x)
```

```
[Out] atan(x*exp(2*a)^(1/4))/exp(2*a)^(3/4) - atanh(x*exp(2*a)^(1/4))/exp(2*a)^(3
/4) + x^3/3
```

3.153 $\int x \coth(a+2\log(x))\,dx$

Optimal. Leaf size=23

$$\frac{x^2}{2}-e^{-a}\tanh^{-1}\left(e^ax^2\right)$$

```
[Out] 1/2*x^2-arctanh(exp(a)*x^2)/exp(a)
```

Rubi [A]
time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {5657, 470, 281, 212}

$$\frac{x^2}{2}-e^{-a}\tanh^{-1}\left(e^ax^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Coth[a + 2*Log[x]],x]
```

```
[Out] x^2/2 - ArcTanh[E^a*x^2]/E^a
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int x \coth(a+2\log(x))\,dx = \int x \coth(a+2\log(x))\,dx$$

Mathematica [A]
time = 0.14, size = 26, normalized size = 1.13

$$\frac{x^2}{2} + \tanh^{-1}\left(x^2(\cosh(a)+\sinh(a))\right)(-\cosh(a)+\sinh(a))$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Coth[a + 2*Log[x]],x]
```

```
[Out] x^2/2 + ArcTanh[x^2*(Cosh[a] + Sinh[a])]*(-Cosh[a] + Sinh[a])
```

Maple [A]
time = 0.57, size = 37, normalized size = 1.61

method	result	size
risch	$\frac{x^2}{2} - \frac{e^{-a}\ln(e^a x^2+1)}{2} + \frac{e^{-a}\ln(e^a x^2-1)}{2}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*coth(a+2*ln(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2-1/2*exp(-a)*ln(exp(a)*x^2+1)+1/2*exp(-a)*ln(exp(a)*x^2-1)
```

Maxima [A]
time = 0.28, size = 36, normalized size = 1.57

$$\frac{1}{2}x^2 - \frac{1}{2}e^{(-a)}\log\left(x^2e^a+1\right) + \frac{1}{2}e^{(-a)}\log\left(x^2e^a-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(a+2*log(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/2*e^(-a)*log(x^2*e^a + 1) + 1/2*e^(-a)*log(x^2*e^a - 1)
```

Fricas [A]
time = 0.35, size = 33, normalized size = 1.43

$$\frac{1}{2}\left(x^2e^a - \log\left(x^2e^a+1\right) + \log\left(x^2e^a-1\right)\right)e^{(-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(a+2*log(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(x^2*e^a - log(x^2*e^a + 1) + log(x^2*e^a - 1))*e^(-a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \coth \left(a + 2 \log \left(x\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(a+2*ln(x)),x)
```

```
[Out] Integral(x*coth(a + 2*log(x)), x)
```

Giac [A]
time = 0.41, size = 37, normalized size = 1.61

$$\frac{1}{2}\,x^2 - \frac{1}{2}\,e^{(-a)}\log\left(x^2e^a + 1\right) + \frac{1}{2}\,e^{(-a)}\log\left(\left|x^2e^a - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(a+2*log(x)),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/2*e^(-a)*log(x^2*e^a + 1) + 1/2*e^(-a)*log(abs(x^2*e^a - 1))
```

Mupad [B]
time = 1.23, size = 25, normalized size = 1.09

$$\frac{x^2}{2} - \frac{\mathrm{atanh}\left(x^2\,\sqrt{\mathrm{e}^{2\,a}}\right)}{\sqrt{\mathrm{e}^{2\,a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*coth(a + 2*log(x)),x)
```

```
[Out] x^2/2 - atanh(x^2*exp(2*a)^(1/2))/exp(2*a)^(1/2)
```

3.154 $\int \coth(a + 2\log(x))\,dx$

Optimal. Leaf size=40

$$x - e^{-a/2}\text{ArcTan}\left(e^{a/2}x\right) - e^{-a/2}\tanh^{-1}\left(e^{a/2}x\right)$$

```
[Out] x-arctan(exp(1/2*a)*x)/exp(1/2*a)-arctanh(exp(1/2*a)*x)/exp(1/2*a)
```

Rubi [A]
time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5653, 396, 218, 212, 209}

$$-e^{-a/2}\text{ArcTan}\left(e^{a/2}x\right) - e^{-a/2}\tanh^{-1}\left(e^{a/2}x\right) + x$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + 2*Log[x]],x]
```

```
[Out] x - ArcTan[E^(a/2)*x]/E^(a/2) - ArcTanh[E^(a/2)*x]/E^(a/2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 5653

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[(-1 - E^(2*
a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p, x] /; FreeQ[{a, b, d, p}, x]
```

Rubi steps

$$\int \coth(a+2\log(x))\,dx = \int \coth(a+2\log(x))\,dx$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.13, size = 58, normalized size = 1.45

$$x+\frac{1}{2}\text{RootSum}\left[-\cosh(a)+\sinh(a)+\cosh(a)\#1^4+\sinh(a)\#1^4\&,\frac{\log(x)-\log(x-\#1)}{\#1^3}\&\right](-\cosh(2a)+\sinh(2a))$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + 2*Log[x]],x]
[Out] x + (RootSum[-Cosh[a] + Sinh[a] + Cosh[a]*#1^4 + Sinh[a]*#1^4 & , (Log[x] -
 Log[x - #1])/#1^3 & ]*(-Cosh[2*a] + Sinh[2*a]))/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(32) = 64.
time = 0.71, size = 71, normalized size = 1.78

method	result	size
risch	$x+\frac{\ln\left(\sqrt{e^{a}}\,x-1\right)}{2\sqrt{e^{a}}}-\frac{\ln\left(\sqrt{e^{a}}\,x+1\right)}{2\sqrt{e^{a}}}-\frac{\ln\left(x\sqrt{-e^{a}}+1\right)}{2\sqrt{-e^{a}}}+\frac{\ln\left(x\sqrt{-e^{a}}-1\right)}{2\sqrt{-e^{a}}}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+2*ln(x)),x,method=_RETURNVERBOSE)
[Out] x+1/2/exp(a)^(1/2)*ln(exp(a)^(1/2)*x-1)-1/2/exp(a)^(1/2)*ln(exp(a)^(1/2)*x+
1)-1/2/(-exp(a))^(1/2)*ln(x*(-exp(a))^(1/2)+1)+1/2/(-exp(a))^(1/2)*ln(x*(-e
xp(a))^(1/2)-1)
```

Maxima [A]
time = 0.49, size = 45, normalized size = 1.12

$$-\arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(-\frac{1}{2}a\right)}+\frac{1}{2}e^{\left(-\frac{1}{2}a\right)}\log\left(\frac{xe^{a}-e^{\left(\frac{1}{2}a\right)}}{xe^{a}+e^{\left(\frac{1}{2}a\right)}}\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x)),x, algorithm="maxima")
```

```
[Out] -arctan(x*e^(1/2*a))*e^(-1/2*a) + 1/2*e^(-1/2*a)*log((x*e^a - e^(1/2*a))/(x
*e^a + e^(1/2*a))) + x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(28) = 56$.
time = 0.36, size = 58, normalized size = 1.45

$$-\frac{1}{2}\left(2\arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(\frac{1}{2}a\right)}-2xe^{a}-e^{\left(\frac{1}{2}a\right)}\log\left(\frac{x^{2}e^{a}-2xe^{\left(\frac{1}{2}a\right)}+1}{x^{2}e^{a}-1}\right)\right)e^{(-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*arctan(x*e^(1/2*a))*e^(1/2*a) - 2*x*e^a - e^(1/2*a)*log((x^2*e^a -
2*x*e^(1/2*a) + 1)/(x^2*e^a - 1)))*e^(-a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\coth\left(a+2\log\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*ln(x)),x)
```

```
[Out] Integral(coth(a + 2*log(x)), x)
```

Giac [A]
time = 0.42, size = 51, normalized size = 1.28

$$-\arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(-\frac{1}{2}a\right)}+\frac{1}{2}e^{\left(-\frac{1}{2}a\right)}\log\left(\frac{\left|2xe^{a}-2e^{\left(\frac{1}{2}a\right)}\right|}{\left|2xe^{a}+2e^{\left(\frac{1}{2}a\right)}\right|}\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x)),x, algorithm="giac")
```

```
[Out] -arctan(x*e^(1/2*a))*e^(-1/2*a) + 1/2*e^(-1/2*a)*log(abs(2*x*e^a - 2*e^(1/2
*a))/abs(2*x*e^a + 2*e^(1/2*a))) + x
```

Mupad [B]
time = 1.19, size = 36, normalized size = 0.90

$$x-\frac{\mathrm{atan}\left(x\,{\left({\mathrm{e}}^{2\,a}\right)}^{1/4}\right)}{{\left({\mathrm{e}}^{2\,a}\right)}^{1/4}}-\frac{\mathrm{atanh}\left(x\,{\left({\mathrm{e}}^{2\,a}\right)}^{1/4}\right)}{{\left({\mathrm{e}}^{2\,a}\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + 2*log(x)),x)
```

```
[Out] x - atan(x*exp(2*a)^(1/4))/exp(2*a)^(1/4) - atanh(x*exp(2*a)^(1/4))/exp(2*a
)^(1/4)
```

3.155 $\int \frac{\coth(a+2\log(x))}{x} \, dx$

Optimal. Leaf size=12

$$\frac{1}{2}\log(\sinh(a+2\log(x)))$$

[Out] 1/2*ln(sinh(a+2*ln(x)))

Rubi [A]
time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3556}

$$\frac{1}{2}\log(\sinh(a+2\log(x)))$$

Antiderivative was successfully verified.

[In] Int[Coth[a + 2*Log[x]]/x,x]

[Out] Log[Sinh[a + 2*Log[x]]]/2

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth(a+2\log(x))}{x} \, dx &= \text{Subst}\left(\int \coth(a+2x) \, dx, x, \log(x)\right)\\ &= \frac{1}{2}\log(\sinh(a+2\log(x)))\end{aligned}$$

Mathematica [A]
time = 0.02, size = 21, normalized size = 1.75

$$\frac{1}{2}(\log(\cosh(a+2\log(x)))+\log(\tanh(a+2\log(x))))$$

Antiderivative was successfully verified.

[In] Integrate[Coth[a + 2*Log[x]]/x,x]

[Out] (Log[Cosh[a + 2*Log[x]]] + Log[Tanh[a + 2*Log[x]]])/2

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(10) = 20$.
time = 0.79, size = 26, normalized size = 2.17

method	result	size
risch	$-\ln(x) + \frac{\ln(1-e^{2a}x^4)}{2}$	20
derivativedivides	$-\frac{\ln(\coth(a+2\ln(x))-1)}{4} - \frac{\ln(\coth(a+2\ln(x))+1)}{4}$	26
default	$-\frac{\ln(\coth(a+2\ln(x))-1)}{4} - \frac{\ln(\coth(a+2\ln(x))+1)}{4}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+2*ln(x))/x,x,method=_RETURNVERBOSE)
[Out] -1/4*ln(coth(a+2*ln(x))-1)-1/4*ln(coth(a+2*ln(x))+1)
```

Maxima [A]
time = 0.27, size = 10, normalized size = 0.83

$$\frac{1}{2}\log\left(\sinh\left(a+2\log\left(x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))/x,x, algorithm="maxima")
[Out] 1/2*log(sinh(a + 2*log(x)))
```

Fricas [A]
time = 0.37, size = 18, normalized size = 1.50

$$\frac{1}{2}\log\left(x^4e^{(2a)}-1\right)-\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))/x,x, algorithm="fricas")
[Out] 1/2*log(x^4*e^(2*a) - 1) - log(x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(10) = 20$.
time = 0.53, size = 27, normalized size = 2.25

$$\log\left(x\right)-\frac{\log\left(\tanh\left(a+2\log\left(x\right)\right)+1\right)}{2}+\frac{\log\left(\tanh\left(a+2\log\left(x\right)\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*ln(x))/x,x)
```

```
[Out] log(x) - log(tanh(a + 2*log(x)) + 1)/2 + log(tanh(a + 2*log(x)))/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(10) = 20.
time = 0.39, size = 21, normalized size = 1.75

$$-\frac{1}{4}\log\left(x^4\right)+\frac{1}{2}\log\left(\left|x^4e^{(2a)}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))/x,x, algorithm="giac")
```

```
[Out] -1/4*log(x^4) + 1/2*log(abs(x^4*e^(2*a) - 1))
```

Mupad [B]
time = 1.21, size = 18, normalized size = 1.50

$$\frac{\ln\left(x^4-\mathrm{e}^{-2a}\right)}{2}-\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + 2*log(x))/x,x)
```

```
[Out] log(x^4 - exp(-2*a))/2 - log(x)
```

3.156 $\int \frac{\coth(a+2\log(x))}{x^2} \, dx$

Optimal. Leaf size=41

$$\frac{1}{x} + e^{a/2}\text{ArcTan}\left(e^{a/2}x\right) - e^{a/2}\tanh^{-1}\left(e^{a/2}x\right)$$

```
[Out] 1/x+exp(1/2*a)*arctan(exp(1/2*a)*x)-exp(1/2*a)*arctanh(exp(1/2*a)*x)
```

Rubi [A]
time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {5657, 464, 304, 209, 212}

$$e^{a/2}\text{ArcTan}\left(e^{a/2}x\right) - e^{a/2}\tanh^{-1}\left(e^{a/2}x\right) + \frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + 2*Log[x]]/x^2,x]
```

```
[Out] x^(-1) + E^(a/2)*ArcTan[E^(a/2)*x] - E^(a/2)*ArcTanh[E^(a/2)*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeqQ[b*c
```

```
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int \frac{\coth(a+2\log(x))}{x^2}\,dx = \int \frac{\coth(a+2\log(x))}{x^2}\,dx$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.12, size = 62, normalized size = 1.51

$$\frac{2+x\text{RootSum}\left[-\cosh(a)-\sinh(a)+\cosh(a)\#1^4-\sinh(a)\#1^4\&,\frac{\log(x)+\log\left(\frac{1}{x}-\#1\right)}{\#1^3}\&\right](\cosh(a)+\sinh(a))^2}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + 2*Log[x]]/x^2,x]
[Out] (2 + x*RootSum[-Cosh[a] - Sinh[a] + Cosh[a]*#1^4 - Sinh[a]*#1^4 & , (Log[x]
 + Log[x^(-1) - #1])/#1^3 & ]*(Cosh[a] + Sinh[a])^2)/(2*x)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.70, size = 88, normalized size = 2.15

method	result
risch	$\frac{1}{x}+\frac{\sqrt{-e^{a}}\,\ln\left(-e^{2a}x+(-e^{a})^{\frac{3}{2}}\right)}{2}-\frac{\sqrt{-e^{a}}\,\ln\left(-e^{2a}x-(-e^{a})^{\frac{3}{2}}\right)}{2}+\frac{\left(\sum_{\_R=\text{RootOf}(\_Z^2-e^{a})}\_R\ln\left(\left(-5\_R^4+4e^{2a}\right)x+\_\right.\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+2*ln(x))/x^2,x,method=_RETURNVERBOSE)
[Out] 1/x+1/2*(-exp(a))^(1/2)*ln(-exp(2*a)*x+(-exp(a))^(3/2))-1/2*(-exp(a))^(1/2)
*ln(-exp(2*a)*x-(-exp(a))^(3/2))+1/2*sum(_R*ln((-5*_R^4+4*exp(2*a))*x+_R^3)
,_R=RootOf(_Z^2-exp(a)))
```

Maxima [A]
time = 0.47, size = 47, normalized size = 1.15

$$-\arctan\left(\frac{e^{\left(-\frac{1}{2}a\right)}}{x}\right)e^{\left(\frac{1}{2}a\right)} + \frac{1}{2}e^{\left(\frac{1}{2}a\right)}\log\left(\frac{\frac{1}{x} - e^{\left(\frac{1}{2}a\right)}}{\frac{1}{x} + e^{\left(\frac{1}{2}a\right)}}\right) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))/x^2,x, algorithm="maxima")
```

```
[Out] -arctan(e^(-1/2*a)/x)*e^(1/2*a) + 1/2*e^(1/2*a)*log((1/x - e^(1/2*a))/(1/x
+ e^(1/2*a))) + 1/x
```

Fricas [A]
time = 0.39, size = 54, normalized size = 1.32

$$\frac{2\,x\arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(\frac{1}{2}a\right)} + xe^{\left(\frac{1}{2}a\right)}\log\left(\frac{x^2e^a - 2\,xe^{\left(\frac{1}{2}a\right)} + 1}{x^2e^a - 1}\right) + 2}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*x*arctan(x*e^(1/2*a))*e^(1/2*a) + x*e^(1/2*a)*log((x^2*e^a - 2*x*e^(
1/2*a) + 1)/(x^2*e^a - 1)) + 2)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(a + 2\log\left(x\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*ln(x))/x**2,x)
```

```
[Out] Integral(coth(a + 2*log(x))/x**2, x)
```

Giac [A]
time = 0.41, size = 52, normalized size = 1.27

$$\arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(\frac{1}{2}a\right)} + \frac{1}{2}e^{\left(\frac{1}{2}a\right)}\log\left(\frac{\left|2\,xe^a - 2\,e^{\left(\frac{1}{2}a\right)}\right|}{\left|2\,xe^a + 2\,e^{\left(\frac{1}{2}a\right)}\right|}\right) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))/x^2,x, algorithm="giac")
```

```
[Out] arctan(x*e^(1/2*a))*e^(1/2*a) + 1/2*e^(1/2*a)*log(abs(2*x*e^a - 2*e^(1/2*a)
)/abs(2*x*e^a + 2*e^(1/2*a))) + 1/x
```

Mupad [B]
time = 1.20, size = 37, normalized size = 0.90

$$\left(\mathrm{e}^{2\,a}\right)^{1/4}\,\mathrm{atan}\left(x\,\left(\mathrm{e}^{2\,a}\right)^{1/4}\right)-\left(\mathrm{e}^{2\,a}\right)^{1/4}\,\mathrm{atanh}\left(x\,\left(\mathrm{e}^{2\,a}\right)^{1/4}\right)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + 2*log(x))/x^2,x)
```

```
[Out] exp(2*a)^(1/4)*atan(x*exp(2*a)^(1/4)) - exp(2*a)^(1/4)*atanh(x*exp(2*a)^(1/
4)) + 1/x
```

3.157 $\int \frac{\coth(a+2\log(x))}{x^3}\,dx$

Optimal. Leaf size=21

$$\frac{1}{2x^2}-e^a\tanh^{-1}\left(e^ax^2\right)$$

```
[Out] 1/2/x^2-exp(a)*arctanh(exp(a)*x^2)
```

Rubi [A]
time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5657, 464, 281, 212}

$$\frac{1}{2x^2}-e^a\tanh^{-1}\left(e^ax^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + 2*Log[x]]/x^3,x]
```

```
[Out] 1/(2*x^2) - E^a*ArcTanh[E^a*x^2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int \frac{\coth(a+2\log(x))}{x^3}\,dx = \int \frac{\coth(a+2\log(x))}{x^3}\,dx$$

Mathematica [A]
time = 0.11, size = 27, normalized size = 1.29

$$\frac{1}{2x^2} - \tanh^{-1}\left(\frac{\cosh(a)-\sinh(a)}{x^2}\right)(\cosh(a)+\sinh(a))$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + 2*Log[x]]/x^3,x]
[Out] 1/(2*x^2) - ArcTanh[(Cosh[a] - Sinh[a])/x^2]*(Cosh[a] + Sinh[a])
```

Maple [A]
time = 0.51, size = 35, normalized size = 1.67

method	result	size
risch	$\frac{1}{2x^2}+\frac{e^a \ln(-e^a x^2+1)}{2}-\frac{e^a \ln(-e^a x^2-1)}{2}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+2*ln(x))/x^3,x,method=_RETURNVERBOSE)
[Out] 1/2/x^2+1/2*exp(a)*ln(-exp(a)*x^2+1)-1/2*exp(a)*ln(-exp(a)*x^2-1)
```

Maxima [A]
time = 0.26, size = 30, normalized size = 1.43

$$-\frac{1}{2}\,e^a\log\left(\frac{1}{x^2}+e^a\right)+\frac{1}{2}\,e^a\log\left(\frac{1}{x^2}-e^a\right)+\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))/x^3,x, algorithm="maxima")
[Out] -1/2*e^a*log(1/x^2 + e^a) + 1/2*e^a*log(1/x^2 - e^a) + 1/2/x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(17) = 34$.
time = 0.34, size = 38, normalized size = 1.81

$$-\frac{x^2e^a\log\left(x^2e^a+1\right)-x^2e^a\log\left(x^2e^a-1\right)-1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))/x^3,x, algorithm="fricas")
[Out] -1/2*(x^2*e^a*log(x^2*e^a + 1) - x^2*e^a*log(x^2*e^a - 1) - 1)/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(a + 2\log(x))}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*ln(x))/x**3,x)
[Out] Integral(coth(a + 2*log(x))/x**3, x)
```

Giac [A]
time = 0.41, size = 33, normalized size = 1.57

$$-\frac{1}{2}\, e^{a} \log\left(x^{2} e^{a} + 1\right) + \frac{1}{2}\, e^{a} \log\left(\left|x^{2} e^{a} - 1\right|\right) + \frac{1}{2\, x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))/x^3,x, algorithm="giac")
[Out] -1/2*e^a*log(x^2*e^a + 1) + 1/2*e^a*log(abs(x^2*e^a - 1)) + 1/2/x^2
```

Mupad [B]
time = 1.21, size = 25, normalized size = 1.19

$$\frac{1}{2\, x^2} - \mathrm{atanh}\left(x^2\, \sqrt{\mathrm{e}^{2\,a}}\right)\, \sqrt{\mathrm{e}^{2\,a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + 2*log(x))/x^3,x)
[Out] 1/(2*x^2) - atanh(x^2*exp(2*a)^(1/2))*exp(2*a)^(1/2)
```

3.158 $\int x^3 \coth^2(a+2\log(x))\,dx$

Optimal. Leaf size=47

$$\frac{x^4}{4}+\frac{e^{-2a}}{1-e^{2a}x^4}+e^{-2a}\log\left(1-e^{2a}x^4\right)$$

```
[Out] 1/4*x^4+1/exp(2*a)/(1-exp(2*a)*x^4)+ln(1-exp(2*a)*x^4)/exp(2*a)
```

Rubi [A]
time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5657, 455, 45}

$$\frac{e^{-2a}}{1-e^{2a}x^4}+e^{-2a}\log\left(1-e^{2a}x^4\right)+\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Coth[a + 2*Log[x]]^2,x]
```

```
[Out] x^4/4 + 1/(E^(2*a)*(1 - E^(2*a)*x^4)) + Log[1 - E^(2*a)*x^4]/E^(2*a)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int x^3\coth^2(a+2\log(x))\,dx=\int x^3\coth^2(a+2\log(x))\,dx$$

Mathematica [A]
time = 0.08, size = 86, normalized size = 1.83

$$\frac{x^4}{4}+\cosh(2a)\log\left(\left(-1+x^4\right)\cosh(a)+\left(1+x^4\right)\sinh(a)\right)-\log\left(\left(-1+x^4\right)\cosh(a)+\left(1+x^4\right)\sinh(a)\right)\sinh(2a)+\frac{-\cosh(3a)+\sinh(3a)}{\left(-1+x^4\right)\cosh(a)+\left(1+x^4\right)\sinh(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Coth[a + 2*Log[x]]^2,x]
```

```
[Out] x^4/4 + Cosh[2*a]*Log[(-1 + x^4)*Cosh[a] + (1 + x^4)*Sinh[a]] - Log[(-1 + x
^4)*Cosh[a] + (1 + x^4)*Sinh[a]]*Sinh[2*a] + (-Cosh[3*a] + Sinh[3*a])/((-1
+ x^4)*Cosh[a] + (1 + x^4)*Sinh[a])
```

Maple [A]
time = 0.42, size = 41, normalized size = 0.87

method	result	size
risch	$\frac{x^4}{4}-\frac{e^{-2a}}{-1+e^{2a}x^4}+e^{-2a}\ln\left(-1+e^{2a}x^4\right)$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*coth(a+2*ln(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4-exp(-2*a)/(-1+exp(2*a)*x^4)+exp(-2*a)*ln(-1+exp(2*a)*x^4)
```

Maxima [A]
time = 0.27, size = 53, normalized size = 1.13

$$\frac{1}{4}x^4+e^{(-2a)}\log\left(x^2e^a+1\right)+e^{(-2a)}\log\left(x^2e^a-1\right)-\frac{1}{x^4e^{(4a)}-e^{(2a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*coth(a+2*log(x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^4 + e^(-2*a)*log(x^2*e^a + 1) + e^(-2*a)*log(x^2*e^a - 1) - 1/(x^4*e^
(4*a) - e^(2*a))
```

Fricas [A]
time = 0.36, size = 61, normalized size = 1.30

$$\frac{x^8e^{(4a)}-x^4e^{(2a)}+4\left(x^4e^{(2a)}-1\right)\log\left(x^4e^{(2a)}-1\right)-4}{4\left(x^4e^{(4a)}-e^{(2a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*coth(a+2*log(x))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(x^8*e^(4*a) - x^4*e^(2*a) + 4*(x^4*e^(2*a) - 1)*log(x^4*e^(2*a) - 1) -
 4)/(x^4*e^(4*a) - e^(2*a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \coth^2 (a + 2 \log (x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*coth(a+2*ln(x))**2,x)
```

```
[Out] Integral(x**3*coth(a + 2*log(x))**2, x)
```

Giac [A]
time = 0.41, size = 40, normalized size = 0.85

$$\frac{1}{4}\, x^4 - \frac{x^4}{x^4 e^{(2\, a)} - 1} + e^{(-2\, a)} \log \left(\left| x^4 e^{(2\, a)} - 1 \right| \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*coth(a+2*log(x))^2,x, algorithm="giac")
```

```
[Out] 1/4*x^4 - x^4/(x^4*e^(2*a) - 1) + e^(-2*a)*log(abs(x^4*e^(2*a) - 1))
```

Mupad [B]
time = 1.25, size = 40, normalized size = 0.85

$$\ln \left(x^4 - \mathrm{e}^{-2\, a} \right)\, \mathrm{e}^{-2\, a} - \frac{\mathrm{e}^{-2\, a}}{x^4\, \mathrm{e}^{2\, a} - 1} + \frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*coth(a + 2*log(x))^2,x)
```

```
[Out] log(x^4 - exp(-2*a))*exp(-2*a) - exp(-2*a)/(x^4*exp(2*a) - 1) + x^4/4
```

3.159 $\int x^2 \coth^2(a+2\log(x))\,dx$

Optimal. Leaf size=68

$$\frac{x^3}{3}+\frac{x^3}{1-e^{2a}x^4}+\frac{3}{2}e^{-3a/2}\text{ArcTan}\left(e^{a/2}x\right)-\frac{3}{2}e^{-3a/2}\tanh^{-1}\left(e^{a/2}x\right)$$

```
[Out] 1/3*x^3+x^3/(1-exp(2*a)*x^4)+3/2*arctan(exp(1/2*a)*x)/exp(3/2*a)-3/2*arctan
h(exp(1/2*a)*x)/exp(3/2*a)
```

Rubi [A]
time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {5657, 474, 470, 304, 209, 212}

$$\frac{3}{2}e^{-3a/2}\text{ArcTan}\left(e^{a/2}x\right)+\frac{x^3}{1-e^{2a}x^4}-\frac{3}{2}e^{-3a/2}\tanh^{-1}\left(e^{a/2}x\right)+\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Coth[a + 2*Log[x]]^2,x]
```

```
[Out] x^3/3 + x^3/(1 - E^(2*a)*x^4) + (3*ArcTan[E^(a/2)*x])/(2*E^((3*a)/2)) - (3*
ArcTanh[E^(a/2)*x])/(2*E^((3*a)/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
```

```
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int x^2 \coth^2(a+2\log(x))\,dx = \int x^2 \coth^2(a+2\log(x))\,dx$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 2.12, size = 154, normalized size = 2.26

$$\frac{e^{-4a}\left(-9317-17825e^{2a}x^4-4787e^{4a}x^8+1481e^{6a}x^{12}+7(1331+1976e^{2a}x^4-398e^{4a}x^8-632e^{6a}x^{12}+27e^{8a}x^{16})\ {}_2F_1\left(\frac{3}{4},1;\frac{7}{4};e^{2a}x^4\right)\right)}{2688x^5}+\frac{16e^{2a}x^7\left(1+e^{2a}x^4\right)^2\ {}_4F_3\left(\frac{7}{4},2,2,2;1,1,\frac{19}{4};e^{2a}x^4\right)}{1155}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Coth[a + 2*Log[x]]^2,x]
```

```
[Out] (-9317 - 17825*E^(2*a)*x^4 - 4787*E^(4*a)*x^8 + 1481*E^(6*a)*x^12 + 7*(1331
 + 1976*E^(2*a)*x^4 - 398*E^(4*a)*x^8 - 632*E^(6*a)*x^12 + 27*E^(8*a)*x^16)
*Hypergeometric2F1[3/4, 1, 7/4, E^(2*a)*x^4])/(2688*E^(4*a)*x^5) + (16*E^(2
*a)*x^7*(1 + E^(2*a)*x^4)^2*HypergeometricPFQ[{7/4, 2, 2, 2}, {1, 1, 19/4},
 E^(2*a)*x^4])/1155
```

Maple [A]
time = 0.59, size = 100, normalized size = 1.47

method	result	size
risch	$\frac{x^3}{3}-\frac{x^3}{-1+e^{2a}x^4}+\frac{3\ln\left(-\sqrt{e^a}\,x+1\right)}{4(e^a)^{\frac{3}{2}}}-\frac{3\ln\left(\sqrt{e^a}\,x+1\right)}{4(e^a)^{\frac{3}{2}}}+\frac{3\ln\left(-e^{2a}x+(-e^a)^{\frac{3}{2}}\right)}{4(-e^a)^{\frac{3}{2}}}-\frac{3\ln\left(e^{2a}x+(-e^a)^{\frac{3}{2}}\right)}{4(-e^a)^{\frac{3}{2}}}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*coth(a+2*ln(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3-x^3/(-1+exp(2*a)*x^4)+3/4/exp(a)^(3/2)*ln(-exp(a)^(1/2)*x+1)-3/4/ex
p(a)^(3/2)*ln(exp(a)^(1/2)*x+1)+3/4/(-exp(a))^(3/2)*ln(-exp(2*a)*x+(-exp(a)
)^(3/2))-3/4/(-exp(a))^(3/2)*ln(exp(2*a)*x+(-exp(a))^(3/2))
```

Maxima [A]
time = 0.48, size = 66, normalized size = 0.97

$$\frac{1}{3}x^3-\frac{x^3}{x^4e^{(2a)}-1}+\frac{3}{2}\arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(-\frac{3}{2}a\right)}+\frac{3}{4}e^{\left(-\frac{3}{2}a\right)}\log\left(\frac{xe^a-e^{\left(\frac{1}{2}a\right)}}{xe^a+e^{\left(\frac{1}{2}a\right)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*coth(a+2*log(x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3 - x^3/(x^4*e^(2*a) - 1) + 3/2*arctan(x*e^(1/2*a))*e^(-3/2*a) + 3/4*
e^(-3/2*a)*log((x*e^a - e^(1/2*a))/(x*e^a + e^(1/2*a)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(49) = 98.
time = 0.36, size = 104, normalized size = 1.53

$$\frac{4\,x^7e^{(4\,a)}-16\,x^3e^{(2\,a)}+18\left(x^4e^{(2\,a)}-1\right)\arctan\left(xe^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(\frac{1}{2}\,a\right)}+9\left(x^4e^{(2\,a)}-1\right)e^{\left(\frac{1}{2}\,a\right)}\log\left(\frac{x^2e^a-2\,xe^{\left(\frac{1}{2}\,a\right)}+1}{x^2e^a-1}\right)}{12\left(x^4e^{(4\,a)}-e^{(2\,a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*coth(a+2*log(x))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(4*x^7*e^(4*a) - 16*x^3*e^(2*a) + 18*(x^4*e^(2*a) - 1)*arctan(x*e^(1/2
*a))*e^(1/2*a) + 9*(x^4*e^(2*a) - 1)*e^(1/2*a)*log((x^2*e^a - 2*x*e^(1/2*a)
 + 1)/(x^2*e^a - 1)))/(x^4*e^(4*a) - e^(2*a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\coth^2\left(a+2\log\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*coth(a+2*ln(x))**2,x)
```

```
[Out] Integral(x**2*coth(a + 2*log(x))**2, x)
```

Giac [A]
time = 0.42, size = 72, normalized size = 1.06

$$\frac{1}{3}\,x^3-\frac{x^3}{x^4e^{(2\,a)}-1}+\frac{3}{2}\,\arctan\left(xe^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(-\frac{3}{2}\,a\right)}+\frac{3}{4}\,e^{\left(-\frac{3}{2}\,a\right)}\log\left(\frac{\left|2\,xe^a-2\,e^{\left(\frac{1}{2}\,a\right)}\right|}{\left|2\,xe^a+2\,e^{\left(\frac{1}{2}\,a\right)}\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*coth(a+2*log(x))^2,x, algorithm="giac")
```

```
[Out] 1/3*x^3 - x^3/(x^4*e^(2*a) - 1) + 3/2*arctan(x*e^(1/2*a))*e^(-3/2*a) + 3/4*
e^(-3/2*a)*log(abs(2*x*e^a - 2*e^(1/2*a))/abs(2*x*e^a + 2*e^(1/2*a)))
```

Mupad [B]
time = 1.24, size = 60, normalized size = 0.88

$$\frac{3\,\mathrm{atan}\left(x\,{\left(\mathrm{e}^{2\,a}\right)}^{1/4}\right)}{2\,{\left(\mathrm{e}^{2\,a}\right)}^{3/4}}-\frac{x^3}{x^4\,\mathrm{e}^{2\,a}-1}+\frac{x^3}{3}+\frac{\mathrm{atan}\left(x\,{\left(\mathrm{e}^{2\,a}\right)}^{1/4}\,1{}\mathrm{i}\right)\,3{}\mathrm{i}}{2\,{\left(\mathrm{e}^{2\,a}\right)}^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*coth(a + 2*log(x))^2,x)
```

```
[Out] (3*atan(x*exp(2*a)^(1/4)))/(2*exp(2*a)^(3/4)) - x^3/(x^4*exp(2*a) - 1) + (a
tan(x*exp(2*a)^(1/4)*1i)*3i)/(2*exp(2*a)^(3/4)) + x^3/3
```

3.160 $\int x \coth^2(a+2\log(x))\,dx$

Optimal. Leaf size=41

$$\frac{x^2}{2}+\frac{x^2}{1-e^{2a}x^4}-e^{-a}\tanh^{-1}\left(e^a x^2\right)$$

```
[Out] 1/2*x^2+x^2/(1-exp(2*a)*x^4)-arctanh(exp(a)*x^2)/exp(a)
```

Rubi [A]
time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {5657, 474, 470, 281, 212}

$$-e^{-a}\tanh^{-1}\left(e^a x^2\right)+\frac{x^2}{1-e^{2a}x^4}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Coth[a + 2*Log[x]]^2,x]
```

```
[Out] x^2/2 + x^2/(1 - E^(2*a)*x^4) - ArcTanh[E^a*x^2]/E^a
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
```

```
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int x\coth^2(a+2\log(x))\,dx = \int x\coth^2(a+2\log(x))\,dx$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 2.14, size = 163, normalized size = 3.98

$$\frac{e^{-4a}\left(-375-713e^{2a}x^4-181e^{4a}x^8+61e^{6a}x^{12}+\frac{3(125+196e^{2a}x^4-14e^{4a}x^8-52e^{6a}x^{12}+e^{8a}x^{16})\tanh^{-1}\left(\sqrt{e^{2a}x^4}\right)}{\sqrt{e^{2a}x^4}}\right)}{96x^6}+\frac{2}{105}e^{2a}x^6\left(1+e^{2a}x^4\right)^2\,{}_4F_3\left(\frac{3}{2},2,2,2;1,1,\frac{9}{2};e^{2a}x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Coth[a + 2*Log[x]]^2,x]
```

```
[Out] (-375 - 713*E^(2*a)*x^4 - 181*E^(4*a)*x^8 + 61*E^(6*a)*x^12 + (3*(125 + 196
*E^(2*a)*x^4 - 14*E^(4*a)*x^8 - 52*E^(6*a)*x^12 + E^(8*a)*x^16)*ArcTanh[Sqr
t[E^(2*a)*x^4]])/Sqrt[E^(2*a)*x^4])/(96*E^(4*a)*x^6) + (2*E^(2*a)*x^6*(1 +
E^(2*a)*x^4)^2*HypergeometricPFQ[{3/2, 2, 2, 2}, {1, 1, 9/2}, E^(2*a)*x^4])
/105
```

Maple [A]
time = 0.43, size = 54, normalized size = 1.32

method	result	size
risch	$\frac{x^2}{2}-\frac{x^2}{-1+e^{2a}x^4}-\frac{e^{-a}\ln(e^a x^2+1)}{2}+\frac{e^{-a}\ln(e^a x^2-1)}{2}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*coth(a+2*ln(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2-x^2/(-1+exp(2*a)*x^4)-1/2*exp(-a)*ln(exp(a)*x^2+1)+1/2*exp(-a)*ln(e
xp(a)*x^2-1)
```

Maxima [A]
time = 0.26, size = 53, normalized size = 1.29

$$\frac{1}{2}\,x^2 - \frac{1}{2}\,e^{(-a)}\log\left(x^2e^a + 1\right) + \frac{1}{2}\,e^{(-a)}\log\left(x^2e^a - 1\right) - \frac{x^2}{x^4e^{(2\,a)} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(a+2*log(x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/2*e^(-a)*log(x^2*e^a + 1) + 1/2*e^(-a)*log(x^2*e^a - 1) - x^2/(
x^4*e^(2*a) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(36) = 72.
time = 0.35, size = 74, normalized size = 1.80

$$\frac{x^6e^{(3\,a)} - 3\,x^2e^a - \left(x^4e^{(2\,a)} - 1\right)\log\left(x^2e^a + 1\right) + \left(x^4e^{(2\,a)} - 1\right)\log\left(x^2e^a - 1\right)}{2\left(x^4e^{(3\,a)} - e^a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(a+2*log(x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(x^6*e^(3*a) - 3*x^2*e^a - (x^4*e^(2*a) - 1)*log(x^2*e^a + 1) + (x^4*e^
(2*a) - 1)*log(x^2*e^a - 1))/(x^4*e^(3*a) - e^a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\coth^2\left(a + 2\log\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(a+2*ln(x))**2,x)
```

```
[Out] Integral(x*coth(a + 2*log(x))**2, x)
```

Giac [A]
time = 0.42, size = 54, normalized size = 1.32

$$\frac{1}{2}\,x^2 - \frac{1}{2}\,e^{(-a)}\log\left(x^2e^a + 1\right) + \frac{1}{2}\,e^{(-a)}\log\left(\left|x^2e^a - 1\right|\right) - \frac{x^2}{x^4e^{(2\,a)} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(a+2*log(x))^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/2*e^(-a)*log(x^2*e^a + 1) + 1/2*e^(-a)*log(abs(x^2*e^a - 1)) -
x^2/(x^4*e^(2*a) - 1)
```

Mupad [B]
time = 1.23, size = 42, normalized size = 1.02

$$\frac{x^2}{2}-\frac{x^2}{x^4\,{\mathrm{e}}^{2\,a}-1}-\frac{\mathrm{atanh}\left(x^2\,\sqrt{{\mathrm{e}}^{2\,a}}\right)}{\sqrt{{\mathrm{e}}^{2\,a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*coth(a + 2*log(x))^2,x)
[Out] x^2/2 - x^2/(x^4*exp(2*a) - 1) - atanh(x^2*exp(2*a)^(1/2))/exp(2*a)^(1/2)
```

3.161 $\int \coth^2(a+2\log(x))\,dx$

Optimal. Leaf size=60

$$x+\frac{x}{1-e^{2a}x^4}-\frac{1}{2}e^{-a/2}\text{ArcTan}\left(e^{a/2}x\right)-\frac{1}{2}e^{-a/2}\tanh^{-1}\left(e^{a/2}x\right)$$

```
[Out] x+x/(1-exp(2*a)*x^4)-1/2*arctan(exp(1/2*a)*x)/exp(1/2*a)-1/2*arctanh(exp(1/
2*a)*x)/exp(1/2*a)
```

Rubi [A]
time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5653, 398, 294, 218, 212, 209}

$$-\frac{1}{2}e^{-a/2}\text{ArcTan}\left(e^{a/2}x\right)+\frac{x}{1-e^{2a}x^4}-\frac{1}{2}e^{-a/2}\tanh^{-1}\left(e^{a/2}x\right)+x$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + 2*Log[x]]^2,x]
```

```
[Out] x + x/(1 - E^(2*a)*x^4) - ArcTan[E^(a/2)*x]/(2*E^(a/2)) - ArcTanh[E^(a/2)*x
]/(2*E^(a/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
```

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 5653

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[(-1 - E^(2*
a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p, x] /; FreeQ[{a, b, d, p}, x]
```

Rubi steps

$$\int \coth^2(a+2\log(x))\,dx = \int \coth^2(a+2\log(x))\,dx$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.62, size = 153, normalized size = 2.55

$$\frac{e^{-4a}\left(-3645-6769e^{2a}x^4-1483e^{4a}x^8+681e^{6a}x^{12}+5(729+1208e^{2a}x^4+102e^{4a}x^8-248e^{6a}x^{12}+e^{8a}x^{16})\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};e^{2a}x^4\right)\right)}{640x^7}+\frac{16}{585}e^{2a}x^5\left(1+e^{2a}x^4\right)^2\,{}_4F_3\left(\frac{5}{4},2,2,2;1,1,\frac{17}{4};e^{2a}x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + 2*Log[x]]^2,x]
```

```
[Out] (-3645 - 6769*E^(2*a)*x^4 - 1483*E^(4*a)*x^8 + 681*E^(6*a)*x^12 + 5*(729 +
1208*E^(2*a)*x^4 + 102*E^(4*a)*x^8 - 248*E^(6*a)*x^12 + E^(8*a)*x^16)*Hyper
geometric2F1[1/4, 1, 5/4, E^(2*a)*x^4])/(640*E^(4*a)*x^7) + (16*E^(2*a)*x^5
*(1 + E^(2*a)*x^4)^2*HypergeometricPFQ[{5/4, 2, 2, 2}, {1, 1, 17/4}, E^(2*a
)*x^4])/585
```

Maple [A]
time = 0.60, size = 86, normalized size = 1.43

method	result	size

risch	$x-\frac{x}{-1+e^{2a}x^4}-\frac{\ln\left(x\sqrt{-e^a}+1\right)}{4\sqrt{-e^a}}+\frac{\ln\left(x\sqrt{-e^a}-1\right)}{4\sqrt{-e^a}}+\frac{\ln\left(\sqrt{e^a}\,x-1\right)}{4\sqrt{e^a}}-\frac{\ln\left(\sqrt{e^a}\,x+1\right)}{4\sqrt{e^a}}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+2*ln(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] x-x/(-1+exp(2*a)*x^4)-1/4/(-exp(a))^(1/2)*ln(x*(-exp(a))^(1/2)+1)+1/4/(-exp
(a))^(1/2)*ln(x*(-exp(a))^(1/2)-1)+1/4/exp(a)^(1/2)*ln(exp(a)^(1/2)*x-1)-1/
4/exp(a)^(1/2)*ln(exp(a)^(1/2)*x+1)
```

Maxima [A]
time = 0.48, size = 60, normalized size = 1.00

$$-\frac{1}{2}\arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(-\frac{1}{2}a\right)}+\frac{1}{4}e^{\left(-\frac{1}{2}a\right)}\log\left(\frac{xe^a-e^{\left(\frac{1}{2}a\right)}}{xe^a+e^{\left(\frac{1}{2}a\right)}}\right)+x-\frac{x}{x^4e^{(2a)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^2,x, algorithm="maxima")
```

```
[Out] -1/2*arctan(x*e^(1/2*a))*e^(-1/2*a) + 1/4*e^(-1/2*a)*log((x*e^a - e^(1/2*a)
)/(x*e^a + e^(1/2*a))) + x - x/(x^4*e^(2*a) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(43) = 86.
time = 0.36, size = 97, normalized size = 1.62

$$\frac{4x^5e^{(3a)}-2\left(x^4e^{(2a)}-1\right)\arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(\frac{1}{2}a\right)}+\left(x^4e^{(2a)}-1\right)e^{\left(\frac{1}{2}a\right)}\log\left(\frac{x^2e^a-2xe^{\left(\frac{1}{2}a\right)}+1}{x^2e^a-1}\right)-8xe^a}{4\left(x^4e^{(3a)}-e^a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*x^5*e^(3*a) - 2*(x^4*e^(2*a) - 1)*arctan(x*e^(1/2*a))*e^(1/2*a) + (x
^4*e^(2*a) - 1)*e^(1/2*a)*log((x^2*e^a - 2*x*e^(1/2*a) + 1)/(x^2*e^a - 1))
- 8*x*e^a)/(x^4*e^(3*a) - e^a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\coth^2\left(a+2\log\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*ln(x))**2,x)
```

```
[Out] Integral(coth(a + 2*log(x))**2, x)
```

Giac [A]
time = 0.40, size = 66, normalized size = 1.10

$$-\frac{1}{2}\arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(-\frac{1}{2}a\right)}+\frac{1}{4}e^{\left(-\frac{1}{2}a\right)}\log\left(\frac{\left|2xe^{a}-2e^{\left(\frac{1}{2}a\right)}\right|}{\left|2xe^{a}+2e^{\left(\frac{1}{2}a\right)}\right|}\right)+x-\frac{x}{x^{4}e^{(2a)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^2,x, algorithm="giac")
```

```
[Out] -1/2*arctan(x*e^(1/2*a))*e^(-1/2*a) + 1/4*e^(-1/2*a)*log(abs(2*x*e^a - 2*e^
(1/2*a))/abs(2*x*e^a + 2*e^(1/2*a))) + x - x/(x^4*e^(2*a) - 1)
```

Mupad [B]
time = 1.21, size = 54, normalized size = 0.90

$$x-\frac{\mathrm{atan}\left(x\,{\left({\mathrm{e}}^{2\,a}\right)}^{1/4}\right)}{2\,{\left({\mathrm{e}}^{2\,a}\right)}^{1/4}}-\frac{x}{x^4\,{\mathrm{e}}^{2\,a}-1}+\frac{\mathrm{atan}\left(x\,{\left({\mathrm{e}}^{2\,a}\right)}^{1/4}\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{2\,{\left({\mathrm{e}}^{2\,a}\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + 2*log(x))^2,x)
```

```
[Out] x - atan(x*exp(2*a)^(1/4))/(2*exp(2*a)^(1/4)) + (atan(x*exp(2*a)^(1/4)*1i)*
1i)/(2*exp(2*a)^(1/4)) - x/(x^4*exp(2*a) - 1)
```

3.162 $\int \frac{\coth^2(a+2\log(x))}{x}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{2}\coth(a+2\log(x))+\log(x)$$

```
[Out] -1/2*coth(a+2*ln(x))+ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3554, 8}

$$\log(x)-\frac{1}{2}\coth(a+2\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + 2*Log[x]]^2/x,x]
```

```
[Out] -1/2*Coth[a + 2*Log[x]] + Log[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^2(a+2\log(x))}{x}\,dx &= \text{Subst}\left(\int \coth^2(a+2x)\,dx,x,\log(x)\right)\\ &= -\frac{1}{2}\coth(a+2\log(x))+\text{Subst}\left(\int 1\,dx,x,\log(x)\right)\\ &= -\frac{1}{2}\coth(a+2\log(x))+\log(x)\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.04, size = 28, normalized size = 2.00

$$-\frac{1}{2}\coth(a+2\log(x))\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\tanh^2(a+2\log(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + 2*Log[x]]^2/x,x]
```

```
[Out] -1/2*(Coth[a + 2*Log[x]]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[a + 2*Log[x]]
^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(12) = 24$.
time = 0.79, size = 35, normalized size = 2.50

method	result	size
risch	$-\frac{1}{-1+e^{2a}x^4}+\ln(x)$	18
derivativedivides	$-\frac{\coth(a+2\ln(x))}{2}-\frac{\ln(\coth(a+2\ln(x))-1)}{4}+\frac{\ln(\coth(a+2\ln(x))+1)}{4}$	35
default	$-\frac{\coth(a+2\ln(x))}{2}-\frac{\ln(\coth(a+2\ln(x))-1)}{4}+\frac{\ln(\coth(a+2\ln(x))+1)}{4}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+2*ln(x))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*coth(a+2*ln(x))-1/4*ln(coth(a+2*ln(x))-1)+1/4*ln(coth(a+2*ln(x))+1)
```

Maxima [A]
time = 0.27, size = 19, normalized size = 1.36

$$\frac{1}{2}a+\frac{1}{e^{(-2a-4\log(x))}-1}+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^2/x,x, algorithm="maxima")
```

```
[Out] 1/2*a + 1/(e^(-2*a - 4*log(x)) - 1) + log(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(12) = 24$.
time = 0.33, size = 28, normalized size = 2.00

$$\frac{\left(x^4e^{(2a)}-1\right)\log(x)-1}{x^4e^{(2a)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^2/x,x, algorithm="fricas")
```

```
[Out] ((x^4*e^(2*a) - 1)*log(x) - 1)/(x^4*e^(2*a) - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(12) = 24$.
time = 2.65, size = 78, normalized size = 5.57

$$\begin{cases} -\frac{\log\left(-\frac{1}{x^2}\right)}{2\tanh^2\left(\log\left(-\frac{1}{x^2}\right)+2\log(x)\right)} & \text{for } a = \log\left(-\frac{1}{x^2}\right) \\ -\frac{\log\left(\frac{1}{x^2}\right)}{2\tanh^2\left(\log\left(\frac{1}{x^2}\right)+2\log(x)\right)} & \text{for } a = \log\left(\frac{1}{x^2}\right) \\ \log(x) - \frac{1}{2\tanh(a+2\log(x))} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*ln(x))**2/x,x)
```

```
[Out] Piecewise((-log(-1/x**2)/(2*tanh(log(-1/x**2) + 2*log(x))**2), Eq(a, log(-1
/x**2))), (-log(x**(-2))/(2*tanh(log(x**(-2)) + 2*log(x))**2), Eq(a, log(x*
*(-2)))), (log(x) - 1/(2*tanh(a + 2*log(x))), True))
```

Giac [A]
time = 0.42, size = 21, normalized size = 1.50

$$-\frac{1}{x^4 e^{(2a)} - 1} + \frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^2/x,x, algorithm="giac")
```

```
[Out] -1/(x^4*e^(2*a) - 1) + 1/4*log(x^4)
```

Mupad [B]
time = 1.19, size = 28, normalized size = 2.00

$$\ln(x) - \frac{e^{2a}\,x^4 + 1}{2\,\left(x^4\,e^{2a} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + 2*log(x))^2/x,x)
```

```
[Out] log(x) - (x^4*exp(2*a) + 1)/(2*(x^4*exp(2*a) - 1))
```

3.163 $\int \frac{\coth^2(a+2\log(x))}{x^2} dx$

Optimal. Leaf size=86

$$-\frac{1}{x(1-e^{2a}x^4)}+\frac{2e^{2a}x^3}{1-e^{2a}x^4}-\frac{1}{2}e^{a/2}\mathrm{ArcTan}\left(e^{a/2}x\right)+\frac{1}{2}e^{a/2}\tanh^{-1}\left(e^{a/2}x\right)$$

```
[Out] -1/x/(1-exp(2*a)*x^4)+2*exp(2*a)*x^3/(1-exp(2*a)*x^4)-1/2*exp(1/2*a)*arctan
(exp(1/2*a)*x)+1/2*exp(1/2*a)*arctanh(exp(1/2*a)*x)
```

Rubi [A]
time = 0.04, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {5657, 473, 468, 304, 209, 212}

$$-\frac{1}{2}e^{a/2}\mathrm{ArcTan}\left(e^{a/2}x\right)-\frac{1}{x(1-e^{2a}x^4)}+\frac{2e^{2a}x^3}{1-e^{2a}x^4}+\frac{1}{2}e^{a/2}\tanh^{-1}\left(e^{a/2}x\right)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + 2*Log[x]]^2/x^2,x]
```

```
[Out] -(1/(x*(1 - E^(2*a)*x^4))) + (2*E^(2*a)*x^3)/(1 - E^(2*a)*x^4) - (E^(a/2)*A
rcTan[E^(a/2)*x])/2 + (E^(a/2)*ArcTanh[E^(a/2)*x])/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
```

```
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int \frac{\coth^2(a+2\log(x))}{x^2}\,dx = \int \frac{\coth^2(a+2\log(x))}{x^2}\,dx$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.93, size = 153, normalized size = 1.78

$$\frac{e^{-2a}\left(-343-1163e^{2a}x^4-241e^{4a}x^8+3e^{6a}x^{12}+\left(343+632e^{2a}x^4+362e^{4a}x^8-56e^{6a}x^{12}-e^{8a}x^{16}\right)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};e^{2a}x^4\right)\right)}{384x^5}+\frac{16}{231}e^{2a}x^3\left(1+e^{2a}x^4\right)^2\,{}_4F_3\left(\frac{3}{4},2,2,2;1,1,\frac{15}{4};e^{2a}x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + 2*Log[x]]^2/x^2,x]
```

```
[Out] (-343 - 1163*E^(2*a)*x^4 - 241*E^(4*a)*x^8 + 3*E^(6*a)*x^12 + (343 + 632*E^
(2*a)*x^4 + 362*E^(4*a)*x^8 - 56*E^(6*a)*x^12 - E^(8*a)*x^16)*Hypergeometri
c2F1[3/4, 1, 7/4, E^(2*a)*x^4])/(384*E^(2*a)*x^5) + (16*E^(2*a)*x^3*(1 + E^
(2*a)*x^4)^2*HypergeometricPFQ[{3/4, 2, 2, 2}, {1, 1, 15/4}, E^(2*a)*x^4])/
231
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.61, size = 104, normalized size = 1.21

method	result
risch	$\frac{-2\,\mathrm{e}^{2a}x^4+1}{x(-1+\mathrm{e}^{2a}x^4)}+\frac{\sqrt{\mathrm{e}^a}\,\ln\left(-(\mathrm{e}^a)^{\frac{3}{2}}-\mathrm{e}^{2a}x\right)}{4}-\frac{\sqrt{\mathrm{e}^a}\,\ln\left((\mathrm{e}^a)^{\frac{3}{2}}-\mathrm{e}^{2a}x\right)}{4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^2+\mathrm{e}^a\right)}\_R\ln\left(\left(-5\_R^4+4\,\mathrm{e}^{2a}\right)x-\right.\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+2*ln(x))^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-2*exp(2*a)*x^4+1)/x/(-1+exp(2*a)*x^4)+1/4*exp(a)^(1/2)*ln(-exp(a)^(3/2)-e
xp(2*a)*x)-1/4*exp(a)^(1/2)*ln(exp(a)^(3/2)-exp(2*a)*x)+1/4*sum(_R*ln((-5*_
R^4+4*exp(2*a))*x-_R^3),_R=RootOf(_Z^2+exp(a)))
```

Maxima [**A**]
time = 0.48, size = 69, normalized size = 0.80

$$\frac{1}{2}\arctan\left(\frac{e^{\left(-\frac{1}{2}a\right)}}{x}\right)e^{\left(\frac{1}{2}a\right)}-\frac{1}{4}e^{\left(\frac{1}{2}a\right)}\log\left(\frac{\frac{1}{x}-e^{\left(\frac{1}{2}a\right)}}{\frac{1}{x}+e^{\left(\frac{1}{2}a\right)}}\right)-\frac{1}{x}+\frac{e^{(2a)}}{x\left(\frac{1}{x^4}-e^{(2a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*arctan(e^(-1/2*a)/x)*e^(1/2*a) - 1/4*e^(1/2*a)*log((1/x - e^(1/2*a))/(1
/x + e^(1/2*a))) - 1/x + e^(2*a)/(x*(1/x^4 - e^(2*a)))
```

Fricas [**A**]
time = 0.35, size = 97, normalized size = 1.13

$$-\frac{8\,x^4e^{(2\,a)}+2\left(x^5e^{(2\,a)}-x\right)\arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(\frac{1}{2}a\right)}-\left(x^5e^{(2\,a)}-x\right)e^{\left(\frac{1}{2}a\right)}\log\left(\frac{x^2e^a+2\,xe^{\left(\frac{1}{2}a\right)}+1}{x^2e^a-1}\right)-4}{4\left(x^5e^{(2\,a)}-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^2/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(8*x^4*e^(2*a) + 2*(x^5*e^(2*a) - x)*arctan(x*e^(1/2*a))*e^(1/2*a) - (
x^5*e^(2*a) - x)*e^(1/2*a)*log((x^2*e^a + 2*x*e^(1/2*a) + 1)/(x^2*e^a - 1))
 - 4)/(x^5*e^(2*a) - x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\coth^2\left(a+2\log\left(x\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*ln(x))**2/x**2,x)
```

```
[Out] Integral(coth(a + 2*log(x))**2/x**2, x)
```

Giac [A]
time = 0.40, size = 77, normalized size = 0.90

$$-\frac{1}{2}\arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(\frac{1}{2}a\right)}-\frac{1}{4}e^{\left(\frac{1}{2}a\right)}\log\left(\frac{\left|2\,xe^{a}-2\,e^{\left(\frac{1}{2}a\right)}\right|}{\left|2\,xe^{a}+2\,e^{\left(\frac{1}{2}a\right)}\right|}\right)-\frac{2\,x^{4}e^{(2\,a)}-1}{x^{5}e^{(2\,a)}-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^2/x^2,x, algorithm="giac")
```

```
[Out] -1/2*arctan(x*e^(1/2*a))*e^(1/2*a) - 1/4*e^(1/2*a)*log(abs(2*x*e^a - 2*e^(1
/2*a))/abs(2*x*e^a + 2*e^(1/2*a))) - (2*x^4*e^(2*a) - 1)/(x^5*e^(2*a) - x)
```

Mupad [B]
time = 1.21, size = 60, normalized size = 0.70

$$\frac{\left(\mathrm{e}^{2\,a}\right)^{1/4}\,\mathrm{atanh}\left(x\,\left(\mathrm{e}^{2\,a}\right)^{1/4}\right)}{2}-\frac{\left(\mathrm{e}^{2\,a}\right)^{1/4}\,\mathrm{atan}\left(x\,\left(\mathrm{e}^{2\,a}\right)^{1/4}\right)}{2}+\frac{2\,x^{4}\,\mathrm{e}^{2\,a}-1}{x-x^{5}\,\mathrm{e}^{2\,a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + 2*log(x))^2/x^2,x)
```

```
[Out] (exp(2*a)^(1/4)*atanh(x*exp(2*a)^(1/4)))/2 - (exp(2*a)^(1/4)*atan(x*exp(2*a
)^(1/4)))/2 + (2*x^4*exp(2*a) - 1)/(x - x^5*exp(2*a))
```

3.164 $\int \frac{\coth^2(a+2\log(x))}{x^3}\,dx$

Optimal. Leaf size=60

$$-\frac{1}{2x^2(1-e^{2a}x^4)}+\frac{3e^{2a}x^2}{2(1-e^{2a}x^4)}+e^a\tanh^{-1}\left(e^ax^2\right)$$

```
[Out] -1/2/x^2/(1-exp(2*a)*x^4)+3/2*exp(2*a)*x^2/(1-exp(2*a)*x^4)+exp(a)*arctanh(
exp(a)*x^2)
```

Rubi [A]
time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5657, 473, 468, 281, 212}

$$e^a\tanh^{-1}\left(e^ax^2\right)+\frac{3e^{2a}x^2}{2(1-e^{2a}x^4)}-\frac{1}{2x^2(1-e^{2a}x^4)}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + 2*Log[x]]^2/x^3,x]
```

```
[Out] -1/2*1/(x^2*(1 - E^(2*a)*x^4)) + (3*E^(2*a)*x^2)/(2*(1 - E^(2*a)*x^4)) + E^
a*ArcTanh[E^a*x^2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int \frac{\coth^2(a+2\log(x))}{x^3}\,dx = \int \frac{\coth^2(a+2\log(x))}{x^3}\,dx$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 2.12, size = 155, normalized size = 2.58

$$\frac{15\left(-77-\frac{27e^{-2a}}{x^4}-17e^{2a}x^4+e^{4a}x^8\right)-\frac{15\left(-27-52e^{2a}x^4-54e^{4a}x^8+4e^{6a}x^{12}+e^{8a}x^{16}\right)\tanh^{-1}\left(\sqrt{e^{2a}x^4}\right)}{\left(e^{2a}x^4\right)^{3/2}}+64\left(e^ax^2+e^{3a}x^6\right)^2\,{}_4F_3\left(\frac{1}{2},2,2,2;1,1,\frac{7}{2};e^{2a}x^4\right)}{480x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + 2*Log[x]]^2/x^3,x]
```

```
[Out] (15*(-77 - 27/(E^(2*a)*x^4) - 17*E^(2*a)*x^4 + E^(4*a)*x^8) - (15*(-27 - 52
*E^(2*a)*x^4 - 54*E^(4*a)*x^8 + 4*E^(6*a)*x^12 + E^(8*a)*x^16)*ArcTanh[Sqrt
[E^(2*a)*x^4]])/(E^(2*a)*x^4)^(3/2) + 64*(E^a*x^2 + E^(3*a)*x^6)^2*Hypergeo
metricPFQ[{1/2, 2, 2, 2}, {1, 1, 7/2}, E^(2*a)*x^4])/(480*x^2)
```

Maple [A]
time = 0.38, size = 55, normalized size = 0.92

method	result	size
risch	$\frac{-\frac{3\,\mathrm{e}^{2a}x^4}{2}+\frac{1}{2}}{x^2\left(-1+\mathrm{e}^{2a}x^4\right)}-\frac{\mathrm{e}^a\ln\left(\mathrm{e}^a x^2-1\right)}{2}+\frac{\mathrm{e}^a\ln\left(\mathrm{e}^a x^2+1\right)}{2}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+2*ln(x))^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-3/2*exp(2*a)*x^4+1/2)/x^2/(-1+exp(2*a)*x^4)-1/2*exp(a)*ln(exp(a)*x^2-1)+1
/2*exp(a)*ln(exp(a)*x^2+1)
```

Maxima [A]
time = 0.26, size = 50, normalized size = 0.83

$$\frac{1}{2}\,e^{a}\log\left(\frac{1}{x^{2}}+e^{a}\right)-\frac{1}{2}\,e^{a}\log\left(\frac{1}{x^{2}}-e^{a}\right)-\frac{1}{2\,x^{2}}+\frac{e^{(2\,a)}}{x^{2}\left(\frac{1}{x^{4}}-e^{(2\,a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*e^a*log(1/x^2 + e^a) - 1/2*e^a*log(1/x^2 - e^a) - 1/2/x^2 + e^(2*a)/(x^
2*(1/x^4 - e^(2*a)))
```

Fricas [A]
time = 0.36, size = 82, normalized size = 1.37

$$-\frac{3\,x^{4}e^{(2\,a)}-\left(x^{6}e^{(3\,a)}-x^{2}e^{a}\right)\log\left(x^{2}e^{a}+1\right)+\left(x^{6}e^{(3\,a)}-x^{2}e^{a}\right)\log\left(x^{2}e^{a}-1\right)-1}{2\left(x^{6}e^{(2\,a)}-x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(3*x^4*e^(2*a) - (x^6*e^(3*a) - x^2*e^a)*log(x^2*e^a + 1) + (x^6*e^(3*
a) - x^2*e^a)*log(x^2*e^a - 1) - 1)/(x^6*e^(2*a) - x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\coth^{2}\left(a+2\log\left(x\right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*ln(x))**2/x**3,x)
```

```
[Out] Integral(coth(a + 2*log(x))**2/x**3, x)
```

Giac [A]
time = 0.41, size = 57, normalized size = 0.95

$$\frac{1}{2}\,e^{a}\log\left(x^{2}e^{a}+1\right)-\frac{1}{2}\,e^{a}\log\left(\left|x^{2}e^{a}-1\right|\right)-\frac{3\,x^{4}e^{(2\,a)}-1}{2\left(x^{6}e^{(2\,a)}-x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^2/x^3,x, algorithm="giac")
```

```
[Out] 1/2*e^a*log(x^2*e^a + 1) - 1/2*e^a*log(abs(x^2*e^a - 1)) - 1/2*(3*x^4*e^(2*
a) - 1)/(x^6*e^(2*a) - x^2)
```

Mupad [B]
time = 1.23, size = 48, normalized size = 0.80

$$\operatorname{atanh}\left(x^2\,\sqrt{\mathrm{e}^{2\,a}}\right)\,\sqrt{\mathrm{e}^{2\,a}}-\frac{\frac{3\,x^4\,\mathrm{e}^{2\,a}}{2}-\frac{1}{2}}{x^6\,\mathrm{e}^{2\,a}-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + 2*log(x))^2/x^3,x)
```

```
[Out] atanh(x^2*exp(2*a)^(1/2))*exp(2*a)^(1/2) - ((3*x^4*exp(2*a))/2 - 1/2)/(x^6*
exp(2*a) - x^2)
```

3.165 $\int (ex)^m \coth(a+2\log(x))\,dx$

Optimal. Leaf size=59

$$\frac{(ex)^{1+m}}{e(1+m)}-\frac{2(ex)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};e^{2a}x^4\right)}{e(1+m)}$$

```
[Out] (e*x)^(1+m)/e/(1+m)-2*(e*x)^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],exp(
2*a)*x^4)/e/(1+m)
```

Rubi [A]
time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5657, 470, 371}

$$\frac{(ex)^{m+1}}{e(m+1)}-\frac{2(ex)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};e^{2a}x^4\right)}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Coth[a + 2*Log[x]],x]
```

```
[Out] (e*x)^(1 + m)/(e*(1 + m)) - (2*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/4
, (5 + m)/4, E^(2*a)*x^4])/(e*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int (ex)^m \coth(a+2\log(x))\,dx = \int (ex)^m \coth(a+2\log(x))\,dx$$

Mathematica [A]
time = 0.07, size = 46, normalized size = 0.78

$$-\frac{x(ex)^m \left(-1+2\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};x^4(\cosh(2a)+\sinh(2a))\right)\right)}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Coth[a + 2*Log[x]],x]
[Out] -((x*(e*x)^m*(-1 + 2*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, x^4*(Cosh[2
*a] + Sinh[2*a])]))/(1 + m))
```

Maple [F]
time = 0.46, size = 0, normalized size = 0.00

$$\int (ex)^m \coth\left(a+2\ln\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*coth(a+2*ln(x)),x)
[Out] int((e*x)^m*coth(a+2*ln(x)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(a+2*log(x)),x, algorithm="maxima")
[Out] integrate((x*e)^m*coth(a + 2*log(x)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(a+2*log(x)),x, algorithm="fricas")
```

```
[Out] integral((x*e)^m*coth(a + 2*log(x)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \coth (a + 2\log (x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*coth(a+2*ln(x)),x)
[Out] Integral((e*x)**m*coth(a + 2*log(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(a+2*log(x)),x, algorithm="giac")
[Out] integrate((e*x)^m*coth(a + 2*log(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \coth(a + 2\,\ln(x))\,\left(e\,x\right)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + 2*log(x))*(e*x)^m,x)
[Out] int(coth(a + 2*log(x))*(e*x)^m, x)
```

3.166 $\int (ex)^m \coth^2(a+2\log(x))\,dx$

Optimal. Leaf size=79

$$\frac{(ex)^{1+m}}{e(1+m)}+\frac{(ex)^{1+m}}{e\left(1-e^{2a}x^4\right)}-\frac{(ex)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};e^{2a}x^4\right)}{e}$$

```
[Out] (e*x)^(1+m)/e/(1+m)+(e*x)^(1+m)/e/(1-exp(2*a)*x^4)-(e*x)^(1+m)*hypergeom([1
, 1/4+1/4*m],[5/4+1/4*m],exp(2*a)*x^4)/e
```

Rubi [A]
time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {5657, 474, 470, 371}

$$-\frac{(ex)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};e^{2a}x^4\right)}{e}+\frac{(ex)^{m+1}}{e\left(1-e^{2a}x^4\right)}+\frac{(ex)^{m+1}}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Coth[a + 2*Log[x]]^2,x]
```

```
[Out] (e*x)^(1 + m)/(e*(1 + m)) + (e*x)^(1 + m)/(e*(1 - E^(2*a)*x^4)) - ((e*x)^(1
 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, E^(2*a)*x^4])/e
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
```

```
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int (ex)^m \coth^2(a+2\log(x))\,dx=\int (ex)^m \coth^2(a+2\log(x))\,dx$$

Mathematica [A]
time = 0.12, size = 77, normalized size = 0.97

$$-\frac{x(ex)^m\left(-1+4\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};x^4(\cosh(2a)+\sinh(2a))\right)-4\,{}_2F_1\left(2,\frac{1+m}{4};\frac{5+m}{4};x^4(\cosh(2a)+\sinh(2a))\right)\right)}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Coth[a + 2*Log[x]]^2,x]
[Out] -((x*(e*x)^m*(-1 + 4*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, x^4*(Cosh[2
*a] + Sinh[2*a])] - 4*Hypergeometric2F1[2, (1 + m)/4, (5 + m)/4, x^4*(Cosh[
2*a] + Sinh[2*a])]))/(1 + m))
```

Maple [F]
time = 0.45, size = 0, normalized size = 0.00

$$\int (ex)^m \left(\coth^2\left(a+2\ln\left(x\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*coth(a+2*ln(x))^2,x)
[Out] int((e*x)^m*coth(a+2*ln(x))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(a+2*log(x))^2,x, algorithm="maxima")
[Out] integrate((x*e)^m*coth(a + 2*log(x))^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(a+2*log(x))^2,x, algorithm="fricas")
[Out] integral((x*e)^m*coth(a + 2*log(x))^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \coth^2 (a + 2 \log (x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*coth(a+2*ln(x))**2,x)
[Out] Integral((e*x)**m*coth(a + 2*log(x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(a+2*log(x))^2,x, algorithm="giac")
[Out] integrate((e*x)^m*coth(a + 2*log(x))^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{coth}(a + 2\,\ln (x))^2\, (e\,x)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + 2*log(x))^2*(e*x)^m,x)
[Out] int(coth(a + 2*log(x))^2*(e*x)^m, x)
```

3.167 $\int (ex)^m \coth^3(a + 2\log(x))\,dx$

Optimal. Leaf size=177

$$\frac{(3+m)(5+m)(ex)^{1+m}}{8e(1+m)}-\frac{(ex)^{1+m}\left(1+e^{2a}x^4\right)^2}{4e\left(1-e^{2a}x^4\right)^2}-\frac{e^{-2a}(ex)^{1+m}\left(e^{2a}(3-m)-e^{4a}(5+m)x^4\right)}{8e\left(1-e^{2a}x^4\right)}-\frac{(9+2m+m^2)}{}$$

```
[Out] 1/8*(3+m)*(5+m)*(e*x)^(1+m)/e/(1+m)-1/4*(e*x)^(1+m)*(1+exp(2*a)*x^4)^2/e/(1
-exp(2*a)*x^4)^2-1/8*(e*x)^(1+m)*(exp(2*a)*(3-m)-exp(4*a)*(5+m)*x^4)/e/exp(
2*a)/(1-exp(2*a)*x^4)-1/4*(m^2+2*m+9)*(e*x)^(1+m)*hypergeom([1, 1/4+1/4*m],
[5/4+1/4*m],exp(2*a)*x^4)/e/(1+m)
```

Rubi [A]

time = 0.13, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5657, 479, 591, 470, 371}

$$-\frac{(m^2+2m+9)\,(ex)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};e^{2a}x^4\right)}{4e(m+1)}-\frac{\left(e^{2a}x^4+1\right)^2(ex)^{m+1}}{4e\left(1-e^{2a}x^4\right)^2}-\frac{e^{-2a}\left(e^{2a}(3-m)-e^{4a}(m+5)x^4\right)(ex)^{m+1}}{8e\left(1-e^{2a}x^4\right)}+\frac{(m+3)(m+5)(ex)^{m+1}}{8e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Coth[a + 2*Log[x]]^3,x]
```

```
[Out] ((3 + m)*(5 + m)*(e*x)^(1 + m))/(8*e*(1 + m)) - ((e*x)^(1 + m)*(1 + E^(2*a)
*x^4)^2)/(4*e*(1 - E^(2*a)*x^4)^2) - ((e*x)^(1 + m)*(E^(2*a)*(3 - m) - E^(4
*a)*(5 + m)*x^4))/(8*e*E^(2*a)*(1 - E^(2*a)*x^4)) - ((9 + 2*m + m^2)*(e*x)^
(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, E^(2*a)*x^4])/(4*e*(1 +
m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
```

```
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 591

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(
m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*b*g*n*(p + 1))), x] + Dist[1/(
a*b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*
(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m +
 n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n,
 0] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e -
 a*f])
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int (ex)^m \coth^3(a+2\log(x))\,dx = \int (ex)^m \coth^3(a+2\log(x))\,dx$$

Mathematica [A]
time = 0.15, size = 108, normalized size = 0.61

$$-\frac{x(ex)^m \left(-1+6\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};x^4(\cosh(2a)+\sinh(2a))\right)-12\,{}_2F_1\left(2,\frac{1+m}{4};\frac{5+m}{4};x^4(\cosh(2a)+\sinh(2a))\right)+8\,{}_2F_1\left(3,\frac{1+m}{4};\frac{5+m}{4};x^4(\cosh(2a)+\sinh(2a))\right)\right)}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Coth[a + 2*Log[x]]^3,x]
```

```
[Out] -((x*(e*x)^m*(-1 + 6*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, x^4*(Cosh[2
*a] + Sinh[2*a])] - 12*Hypergeometric2F1[2, (1 + m)/4, (5 + m)/4, x^4*(Cosh
[2*a] + Sinh[2*a])] + 8*Hypergeometric2F1[3, (1 + m)/4, (5 + m)/4, x^4*(Cos
h[2*a] + Sinh[2*a])]))/(1 + m))
```

Maple [F]
time = 0.60, size = 0, normalized size = 0.00

$$\int (ex)^m \left(\coth^3 \left(a + 2\ln \left(x\right)\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*coth(a+2*ln(x))^3,x)
```

```
[Out] int((e*x)^m*coth(a+2*ln(x))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(a+2*log(x))^3,x, algorithm="maxima")
```

```
[Out] integrate((x*e)^m*coth(a + 2*log(x))^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(a+2*log(x))^3,x, algorithm="fricas")
```

```
[Out] integral((x*e)^m*coth(a + 2*log(x))^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \coth^3 \left(a + 2\log \left(x\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*coth(a+2*ln(x))**3,x)
```

```
[Out] Integral((e*x)**m*coth(a + 2*log(x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(a+2*log(x))^3,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*coth(a + 2*log(x))^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \coth(a+2\ln(x))^3\,(e\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + 2*log(x))^3*(e*x)^m,x)
```

```
[Out] int(coth(a + 2*log(x))^3*(e*x)^m, x)
```

3.168 $\int \coth^p(a+b\log(x))\,dx$

Optimal. Leaf size=79

$$x\left(-1-e^{2a}x^{2b}\right)^p\left(1+e^{2a}x^{2b}\right)^{-p}F_1\left(\frac{1}{2b};p,-p;\frac{1}{2}\left(2+\frac{1}{b}\right);e^{2a}x^{2b},-e^{2a}x^{2b}\right)$$

```
[Out] x*(-1-exp(2*a)*x^(2*b))^p*AppellF1(1/2/b,p,-p,1+1/2/b,exp(2*a)*x^(2*b),-exp
(2*a)*x^(2*b))/((1+exp(2*a)*x^(2*b))^p)
```

Rubi [A]
time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5653, 441, 440}

$$x\left(-e^{2a}x^{2b}-1\right)^p\left(e^{2a}x^{2b}+1\right)^{-p}F_1\left(\frac{1}{2b};p,-p;\frac{1}{2}\left(2+\frac{1}{b}\right);e^{2a}x^{2b},-e^{2a}x^{2b}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*Log[x]]^p,x]
```

```
[Out] (x*(-1 - E^(2*a)*x^(2*b))^p*AppellF1[1/(2*b), p, -p, (2 + b^(-1))/2, E^(2*a
)*x^(2*b), -(E^(2*a)*x^(2*b))])/(1 + E^(2*a)*x^(2*b))^p
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 5653

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[(-1 - E^(2*
a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p, x] /; FreeQ[{a, b, d, p}, x]
```

Rubi steps

$$\int \coth^p(a+b\log(x))\,dx = \int \coth^p(a+b\log(x))\,dx$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 259 vs. 2(79) = 158.
time = 1.36, size = 259, normalized size = 3.28

$$\frac{(1+2b)x\left(\frac{1+e^{2a}x^{2b}}{-1+e^{2a}x^{2b}}\right)^p F_1\left(\frac{1}{2b};p,-p;1+\frac{1}{2b};e^{2a}x^{2b},-e^{2a}x^{2b}\right)}{2be^{2a}px^{2b}F_1\left(1+\frac{1}{2b};p,1-p;2+\frac{1}{2b};e^{2a}x^{2b},-e^{2a}x^{2b}\right)+2be^{2a}px^{2b}F_1\left(1+\frac{1}{2b};1+p,-p;2+\frac{1}{2b};e^{2a}x^{2b},-e^{2a}x^{2b}\right)+(1+2b)F_1\left(\frac{1}{2b};p,-p;1+\frac{1}{2b};e^{2a}x^{2b},-e^{2a}x^{2b}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[a + b*Log[x]]^p,x]
```

```
[Out] ((1 + 2*b)*x*((1 + E^(2*a)*x^(2*b))/(-1 + E^(2*a)*x^(2*b)))^p*AppellF1[1/(2
*b), p, -p, 1 + 1/(2*b), E^(2*a)*x^(2*b), -(E^(2*a)*x^(2*b))])/(2*b*E^(2*a)
*p*x^(2*b)*AppellF1[1 + 1/(2*b), p, 1 - p, 2 + 1/(2*b), E^(2*a)*x^(2*b), -(
E^(2*a)*x^(2*b))] + 2*b*E^(2*a)*p*x^(2*b)*AppellF1[1 + 1/(2*b), 1 + p, -p,
2 + 1/(2*b), E^(2*a)*x^(2*b), -(E^(2*a)*x^(2*b))] + (1 + 2*b)*AppellF1[1/(2
*b), p, -p, 1 + 1/(2*b), E^(2*a)*x^(2*b), -(E^(2*a)*x^(2*b))])
```

Maple [F]
time = 0.85, size = 0, normalized size = 0.00

$$\int \coth^p(a+b\ln(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+b*ln(x))^p,x)
```

```
[Out] int(coth(a+b*ln(x))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(x))^p,x, algorithm="maxima")
```

```
[Out] integrate(coth(b*log(x) + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(x))^p,x, algorithm="fricas")
```

```
[Out] integral(coth(b*log(x) + a)^p, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \coth^p(a + b\log(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*ln(x))**p,x)
```

```
[Out] Integral(coth(a + b*log(x))**p, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(x))^p,x, algorithm="giac")
```

```
[Out] integrate(coth(b*log(x) + a)^p, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \coth(a + b\,\ln(x))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + b*log(x))^p,x)
```

```
[Out] int(coth(a + b*log(x))^p, x)
```

3.169 $\int (ex)^m \coth^p(a + b\log(x))\,dx$

Optimal. Leaf size=99

$$\frac{(ex)^{1+m}\left(-1-e^{2a}x^{2b}\right)^p\left(1+e^{2a}x^{2b}\right)^{-p}F_1\left(\frac{1+m}{2b};p,-p;1+\frac{1+m}{2b};e^{2a}x^{2b},-e^{2a}x^{2b}\right)}{e(1+m)}$$

```
[Out] (e*x)^(1+m)*(-1-exp(2*a)*x^(2*b))^p*AppellF1(1/2*(1+m)/b,p,-p,1+1/2*(1+m)/b
,exp(2*a)*x^(2*b),-exp(2*a)*x^(2*b))/e/(1+m)/((1+exp(2*a)*x^(2*b))^p)
```

Rubi [A]

time = 0.06, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5657, 525, 524}

$$\frac{(ex)^{m+1}\left(-e^{2a}x^{2b}-1\right)^p\left(e^{2a}x^{2b}+1\right)^{-p}F_1\left(\frac{m+1}{2b};p,-p;\frac{m+1}{2b}+1;e^{2a}x^{2b},-e^{2a}x^{2b}\right)}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Coth[a + b*Log[x]]^p,x]
```

```
[Out] ((e*x)^(1 + m)*(-1 - E^(2*a)*x^(2*b))^p*AppellF1[(1 + m)/(2*b), p, -p, 1 +
(1 + m)/(2*b), E^(2*a)*x^(2*b), -(E^(2*a)*x^(2*b))])/(e*(1 + m)*(1 + E^(2*a
)*x^(2*b))^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int (ex)^m \coth^p(a+b\log(x))\,dx = \int (ex)^m \coth^p(a+b\log(x))\,dx$$

Mathematica [A]
time = 2.15, size = 126, normalized size = 1.27

$$\frac{x(ex)^m \left(1-e^{2a}x^{2b}\right)^p \left(1+e^{2a}x^{2b}\right)^{-p} \left(\frac{1+e^{2a}x^{2b}}{-1+e^{2a}x^{2b}}\right)^p F_1\left(\frac{1+m}{2b};p,-p;1+\frac{1+m}{2b};e^{2a}x^{2b},-e^{2a}x^{2b}\right)}{1+m}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*x)^m*Coth[a + b*Log[x]]^p,x]
```

```
[Out] (x*(e*x)^m*(1 - E^(2*a)*x^(2*b))^p*((1 + E^(2*a)*x^(2*b))/(-1 + E^(2*a)*x^(
2*b)))^p*AppellF1[(1 + m)/(2*b), p, -p, 1 + (1 + m)/(2*b), E^(2*a)*x^(2*b),
 -(E^(2*a)*x^(2*b))])/((1 + m)*(1 + E^(2*a)*x^(2*b))^p)
```

Maple [F]
time = 0.45, size = 0, normalized size = 0.00

$$\int (ex)^m \left(\coth^p\left(a+b\ln\left(x\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*coth(a+b*ln(x))^p,x)
```

```
[Out] int((e*x)^m*coth(a+b*ln(x))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(a+b*log(x))^p,x, algorithm="maxima")
```

```
[Out] integrate((x*e)^m*coth(b*log(x) + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(a+b*log(x))^p,x, algorithm="fricas")
```

```
[Out] integral((x*e)^m*coth(b*log(x) + a)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \coth^p(a + b\log(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*coth(a+b*ln(x))**p,x)
```

```
[Out] Integral((e*x)**m*coth(a + b*log(x))**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(a+b*log(x))^p,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*coth(b*log(x) + a)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \coth(a + b\,\ln(x))^p\,(e\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + b*log(x))^p*(e*x)^m,x)
```

```
[Out] int(coth(a + b*log(x))^p*(e*x)^m, x)
```

3.170 $\int \coth^p\left(a+\frac{\log(x)}{2}\right)\,dx$

Optimal. Leaf size=52

$$-\frac{2^{-p}e^{-2a}(-1-e^{2a}x)^{1+p}\ {}_2F_1\left(p,1+p;2+p;\frac{1}{2}(1+e^{2a}x)\right)}{1+p}$$

```
[Out] -(-1-exp(2*a)*x)^(1+p)*hypergeom([p, 1+p],[2+p],1/2+1/2*exp(2*a)*x)/(2^p)/e
xp(2*a)/(1+p)
```

Rubi [A]
time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {5653, 71}

$$-\frac{e^{-2a}2^{-p}(-e^{2a}x-1)^{p+1}\ {}_2F_1\left(p,p+1;p+2;\frac{1}{2}(e^{2a}x+1)\right)}{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + Log[x]/2]^p,x]
```

```
[Out] -(((-1 - E^(2*a)*x)^(1 + p)*Hypergeometric2F1[p, 1 + p, 2 + p, (1 + E^(2*a)
*x)/2])/(2^p*E^(2*a)*(1 + p)))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 5653

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[(-1 - E^(2*
a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p, x] /; FreeQ[{a, b, d, p}, x]
```

Rubi steps

$$\int \coth^p\left(a+\frac{\log(x)}{2}\right)\,dx=\int \coth^p\left(\frac{1}{2}(2a+\log(x))\right)\,dx$$

Mathematica [A]
time = 0.38, size = 83, normalized size = 1.60

$$-\frac{2^p e^{-2a}(1+e^{2a}x)^{1-p}\left(\frac{1+e^{2a}x}{-1+e^{2a}x}\right)^{-1+p} {}_2F_1\left(1-p,-p;2-p;\frac{1}{2}-\frac{1}{2}e^{2a}x\right)}{-1+p}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + Log[x]/2]^p,x]
```

```
[Out] -((2^p*(1 + E^(2*a)*x)^(1 - p)*((1 + E^(2*a)*x)/(-1 + E^(2*a)*x))^(-1 + p)*
Hypergeometric2F1[1 - p, -p, 2 - p, 1/2 - (E^(2*a)*x)/2])/(E^(2*a)*(-1 + p)
))
```

Maple [F]
time = 0.85, size = 0, normalized size = 0.00

$$\int \coth^p\left(a+\frac{\ln(x)}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+1/2*ln(x))^p,x)
```

```
[Out] int(coth(a+1/2*ln(x))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/2*log(x))^p,x, algorithm="maxima")
```

```
[Out] integrate(coth(a + 1/2*log(x))^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/2*log(x))^p,x, algorithm="fricas")
```

```
[Out] integral(coth(a + 1/2*log(x))^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \coth^{p}\left(a + \frac{\log\left(x\right)}{2}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/2*ln(x))**p,x)
```

```
[Out] Integral(coth(a + log(x)/2)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/2*log(x))^p,x, algorithm="giac")
```

```
[Out] integrate(coth(a + 1/2*log(x))^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\mathrm{coth}\left(a+\frac{\ln\left(x\right)}{2}\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + log(x)/2)^p,x)
```

```
[Out] int(coth(a + log(x)/2)^p, x)
```

3.171 $\int \coth^p\left(a+\frac{\log(x)}{4}\right)\,dx$

Optimal. Leaf size=108

$$e^{-4a}\left(-1-e^{2a}\sqrt{x}\right)^{1+p}\left(1-e^{2a}\sqrt{x}\right)^{1-p}-\frac{2^{1-p}e^{-4a}p\left(-1-e^{2a}\sqrt{x}\right)^{1+p}\,{}_2F_1\left(p,1+p;2+p;\frac{1}{2}\left(1+e^{2a}\sqrt{x}\right)\right)}{1+p}$$

```
[Out] -2^(1-p)*p*hypergeom([p, 1+p],[2+p],1/2+1/2*exp(2*a)*x^(1/2))*(-1-exp(2*a)*
x^(1/2))^(1+p)/exp(4*a)/(1+p)+(-1-exp(2*a)*x^(1/2))^(1+p)*(1-exp(2*a)*x^(1/
2))^(1-p)/exp(4*a)
```

Rubi [A]
time = 0.06, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {5653, 383, 81, 71}

$$e^{-4a}\left(-e^{2a}\sqrt{x}-1\right)^{p+1}\left(1-e^{2a}\sqrt{x}\right)^{1-p}-\frac{e^{-4a}2^{1-p}p\left(-e^{2a}\sqrt{x}-1\right)^{p+1}\,{}_2F_1\left(p,p+1;p+2;\frac{1}{2}\left(e^{2a}\sqrt{x}+1\right)\right)}{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + Log[x]/4]^p,x]
```

```
[Out] ((-1 - E^(2*a)*Sqrt[x])^(1 + p)*(1 - E^(2*a)*Sqrt[x])^(1 - p))/E^(4*a) - (2
^(1 - p)*p*(-1 - E^(2*a)*Sqrt[x])^(1 + p)*Hypergeometric2F1[p, 1 + p, 2 + p
, (1 + E^(2*a)*Sqrt[x])/2])/(E^(4*a)*(1 + p))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
```

```
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 5653

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[(-1 - E^(2*
a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p, x] /; FreeQ[{a, b, d, p}, x]
```

Rubi steps

$$\int \coth^p\left(a+\frac{\log(x)}{4}\right)\,dx=\int \coth^p\left(\frac{1}{4}(4a+\log(x))\right)\,dx$$

Mathematica [A]
time = 0.49, size = 125, normalized size = 1.16

$$\frac{e^{-4a}\left(1+e^{2a}\sqrt{x}\right)^{1-p}\left(\frac{1+e^{2a}\sqrt{x}}{-1+e^{2a}\sqrt{x}}\right)^{-1+p}\left((-1+p)\left(1+e^{2a}\sqrt{x}\right)^{1+p}-2^{1+p}p\,{}_2F_1\left(1-p,-p;2-p;\frac{1}{2}-\frac{1}{2}e^{2a}\sqrt{x}\right)\right)}{-1+p}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + Log[x]/4]^p,x]
```

```
[Out] ((1 + E^(2*a)*Sqrt[x])^(1 - p)*((1 + E^(2*a)*Sqrt[x])/(-1 + E^(2*a)*Sqrt[x]
))^(-1 + p)*((-1 + p)*(1 + E^(2*a)*Sqrt[x])^(1 + p) - 2^(1 + p)*p*Hypergeom
etric2F1[1 - p, -p, 2 - p, 1/2 - (E^(2*a)*Sqrt[x])/2]))/(E^(4*a)*(-1 + p))
```

Maple [F]
time = 0.84, size = 0, normalized size = 0.00

$$\int \coth^p\left(a+\frac{\ln(x)}{4}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+1/4*ln(x))^p,x)
```

```
[Out] int(coth(a+1/4*ln(x))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/4*log(x))^p,x, algorithm="maxima")
[Out] integrate(coth(a + 1/4*log(x))^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/4*log(x))^p,x, algorithm="fricas")
[Out] integral(coth(a + 1/4*log(x))^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \coth^{p}\left(a + \frac{\log\left(x\right)}{4}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/4*ln(x))**p,x)
[Out] Integral(coth(a + log(x)/4)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/4*log(x))^p,x, algorithm="giac")
[Out] integrate(coth(a + 1/4*log(x))^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\mathrm{coth}\left(a+\frac{\ln\left(x\right)}{4}\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + log(x)/4)^p,x)
[Out] int(coth(a + log(x)/4)^p, x)
```

3.172 $\int \coth^p\left(a+\frac{\log(x)}{6}\right)\,dx$

Optimal. Leaf size=162

$$e^{-6a}p\left(-1-e^{2a}\sqrt[3]{x}\right)^{1+p}\left(1-e^{2a}\sqrt[3]{x}\right)^{1-p}+e^{-4a}\left(-1-e^{2a}\sqrt[3]{x}\right)^{1+p}\left(1-e^{2a}\sqrt[3]{x}\right)^{1-p}\sqrt[3]{x}-\frac{2^{-p}e^{-6a}(1+2p^2)\left(\cdots\right.}{}$$

```
[Out] p*(-1-exp(2*a)*x^(1/3))^(1+p)*(1-exp(2*a)*x^(1/3))^(1-p)/exp(6*a)+(-1-exp(2
*a)*x^(1/3))^(1+p)*(1-exp(2*a)*x^(1/3))^(1-p)*x^(1/3)/exp(4*a)-(2*p^2+1)*(-
1-exp(2*a)*x^(1/3))^(1+p)*hypergeom([p, 1+p],[2+p],1/2+1/2*exp(2*a)*x^(1/3)
)/(2^p)/exp(6*a)/(1+p)
```

Rubi [A]

time = 0.09, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {5653, 383, 92, 81, 71}

$$-\frac{e^{-6a}2^{-p}(2p^2+1)\left(-e^{2a}\sqrt[3]{x}-1\right)^{p+1}{}_2F_1\left(p,p+1;p+2;\frac{1}{2}\left(e^{2a}\sqrt[3]{x}+1\right)\right)}{p+1}+e^{-6a}p\left(-e^{2a}\sqrt[3]{x}-1\right)^{p+1}\left(1-e^{2a}\sqrt[3]{x}\right)^{1-p}+e^{-4a}\sqrt[3]{x}\left(-e^{2a}\sqrt[3]{x}-1\right)^{p+1}\left(1-e^{2a}\sqrt[3]{x}\right)^{1-p}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + Log[x]/6]^p,x]
```

```
[Out] (p*(-1 - E^(2*a)*x^(1/3))^(1 + p)*(1 - E^(2*a)*x^(1/3))^(1 - p))/E^(6*a) +
((-1 - E^(2*a)*x^(1/3))^(1 + p)*(1 - E^(2*a)*x^(1/3))^(1 - p)*x^(1/3))/E^(4
*a) - ((1 + 2*p^2)*(-1 - E^(2*a)*x^(1/3))^(1 + p)*Hypergeometric2F1[p, 1 +
p, 2 + p, (1 + E^(2*a)*x^(1/3))/2])/(2^p*E^(6*a)*(1 + p))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 5653

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[(-1 - E^(2*
a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p, x] /; FreeQ[{a, b, d, p}, x]
```

Rubi steps

$$\int \coth^p\left(a+\frac{\log(x)}{6}\right)\,dx=\int \coth^p\left(\frac{1}{6}(6a+\log(x))\right)\,dx$$

Mathematica [A]
time = 0.50, size = 142, normalized size = 0.88

$$\frac{e^{-6a}\left(1+e^{2a}\sqrt[3]{x}\right)^{1-p}\left(\frac{1+e^{2a}\sqrt[3]{x}}{-1+e^{2a}\sqrt[3]{x}}\right)^{-1+p}\left((-1+p)\left(1+e^{2a}\sqrt[3]{x}\right)^{1+p}\left(p+e^{2a}\sqrt[3]{x}\right)-2^p(1+2p^2)\,{}_2F_1\left(1-p,-p;2-p;\frac{1}{2}-\frac{1}{2}e^{2a}\sqrt[3]{x}\right)\right)}{-1+p}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[a + Log[x]/6]^p,x]
[Out] ((1 + E^(2*a)*x^(1/3))^(1 - p)*((1 + E^(2*a)*x^(1/3))/(-1 + E^(2*a)*x^(1/3)
))^(-1 + p)*((-1 + p)*(1 + E^(2*a)*x^(1/3))^(1 + p)*(p + E^(2*a)*x^(1/3)) -
 2^p*(1 + 2*p^2)*Hypergeometric2F1[1 - p, -p, 2 - p, 1/2 - (E^(2*a)*x^(1/3)
)/2]))/(E^(6*a)*(-1 + p))
```

Maple [F]
time = 0.85, size = 0, normalized size = 0.00

$$\int \coth^p\left(a+\frac{\ln(x)}{6}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+1/6*ln(x))^p,x)
```

```
[Out] int(coth(a+1/6*ln(x))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/6*log(x))^p,x, algorithm="maxima")
```

```
[Out] integrate(coth(a + 1/6*log(x))^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/6*log(x))^p,x, algorithm="fricas")
```

```
[Out] integral(coth(a + 1/6*log(x))^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \coth^{p}\left(a + \frac{\log\left(x\right)}{6}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/6*ln(x))**p,x)
```

```
[Out] Integral(coth(a + log(x)/6)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/6*log(x))^p,x, algorithm="giac")
```

```
[Out] integrate(coth(a + 1/6*log(x))^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \coth\left(a+\frac{\ln(x)}{6}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + log(x)/6)^p,x)
[Out] int(coth(a + log(x)/6)^p, x)
```

3.173 $\int \coth^p\left(a+\frac{\log(x)}{8}\right)\,dx$

Optimal. Leaf size=194

$$\frac{1}{3}e^{-12a}\left(-1-e^{2a}\sqrt[4]{x}\right)^{1+p}\left(1-e^{2a}\sqrt[4]{x}\right)^{1-p}\left(e^{4a}(3+2p^2)+2e^{6a}p\sqrt[4]{x}\right)+e^{-4a}\left(-1-e^{2a}\sqrt[4]{x}\right)^{1+p}\left(1-e^{2a}\sqrt[4]{x}\right)$$

```
[Out] 1/3*(-1-exp(2*a)*x^(1/4))^(1+p)*(1-exp(2*a)*x^(1/4))^(1-p)*(exp(4*a)*(2*p^2
+3)+2*exp(6*a)*p*x^(1/4))/exp(12*a)-1/3*2^(2-p)*p*(p^2+2)*(-1-exp(2*a)*x^(1
/4))^(1+p)*hypergeom([p, 1+p],[2+p],1/2+1/2*exp(2*a)*x^(1/4))/exp(8*a)/(1+p
)+(-1-exp(2*a)*x^(1/4))^(1+p)*(1-exp(2*a)*x^(1/4))^(1-p)*x^(1/2)/exp(4*a)
```

Rubi [A]
time = 0.11, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {5653, 383, 102, 152, 71}

$$-\frac{e^{-8a}2^{2-p}p(p^2+2)\left(-e^{2a}\sqrt[4]{x}-1\right)^{p+1}{}_2F_1\left(p,p+1;p+2;\frac{1}{2}\left(e^{2a}\sqrt[4]{x}+1\right)\right)}{3(p+1)}+\frac{1}{3}e^{-12a}\left(-e^{2a}\sqrt[4]{x}-1\right)^{p+1}\left(e^{4a}(2p^2+3)+2e^{6a}p\sqrt[4]{x}\right)\left(1-e^{2a}\sqrt[4]{x}\right)^{1-p}+e^{-4a}\sqrt{x}\left(-e^{2a}\sqrt[4]{x}-1\right)^{p+1}\left(1-e^{2a}\sqrt[4]{x}\right)^{1-p}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + Log[x]/8]^p,x]
```

```
[Out] ((-1 - E^(2*a)*x^(1/4))^(1 + p)*(1 - E^(2*a)*x^(1/4))^(1 - p)*(E^(4*a)*(3 +
 2*p^2) + 2*E^(6*a)*p*x^(1/4)))/(3*E^(12*a)) + ((-1 - E^(2*a)*x^(1/4))^(1 +
 p)*(1 - E^(2*a)*x^(1/4))^(1 - p)*Sqrt[x])/E^(4*a) - (2^(2 - p)*p*(2 + p^2)
*(-1 - E^(2*a)*x^(1/4))^(1 + p)*Hypergeometric2F1[p, 1 + p, 2 + p, (1 + E^(
2*a)*x^(1/4))/2])/(3*E^(8*a)*(1 + p))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 5653

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[(-1 - E^(2*
a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p, x] /; FreeQ[{a, b, d, p}, x]
```

Rubi steps

$$\int \coth^p\left(a+\frac{\log(x)}{8}\right) dx = \int \coth^p\left(\frac{1}{8}(8a+\log(x))\right) dx$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 2.04, size = 176, normalized size = 0.91

$$\frac{5\left(\frac{1+e^{2a}\sqrt[4]{x}}{-1+e^{2a}\sqrt[4]{x}}\right)^p xF_1\left(4;p,-p;5;e^{2a}\sqrt[4]{x},-e^{2a}\sqrt[4]{x}\right)}{5F_1\left(4;p,-p;5;e^{2a}\sqrt[4]{x},-e^{2a}\sqrt[4]{x}\right)+e^{2a}p\sqrt[4]{x}\left(F_1\left(5;p,1-p;6;e^{2a}\sqrt[4]{x},-e^{2a}\sqrt[4]{x}\right)+F_1\left(5;1+p,-p;6;e^{2a}\sqrt[4]{x},-e^{2a}\sqrt[4]{x}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[a + Log[x]/8]^p,x]
```

```
[Out] (5*((1 + E^(2*a)*x^(1/4))/(-1 + E^(2*a)*x^(1/4)))^p*x*AppellF1[4, p, -p, 5,
 E^(2*a)*x^(1/4), -(E^(2*a)*x^(1/4))])/(5*AppellF1[4, p, -p, 5, E^(2*a)*x^(
1/4), -(E^(2*a)*x^(1/4))] + E^(2*a)*p*x^(1/4)*(AppellF1[5, p, 1 - p, 6, E^(
```

```
2*a)*x^(1/4), -(E^(2*a)*x^(1/4))] + AppellF1[5, 1 + p, -p, 6, E^(2*a)*x^(1/
4), -(E^(2*a)*x^(1/4))]))
```

Maple [F]
time = 0.85, size = 0, normalized size = 0.00

$$\int \coth^{p}\left(a+\frac{\ln\left(x\right)}{8}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+1/8*ln(x))^p,x)
[Out] int(coth(a+1/8*ln(x))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/8*log(x))^p,x, algorithm="maxima")
[Out] integrate(coth(a + 1/8*log(x))^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/8*log(x))^p,x, algorithm="fricas")
[Out] integral(coth(a + 1/8*log(x))^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \coth^{p}\left(a+\frac{\log\left(x\right)}{8}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/8*ln(x))**p,x)
[Out] Integral(coth(a + log(x)/8)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/8*log(x))^p,x, algorithm="giac")
```

```
[Out] integrate(coth(a + 1/8*log(x))^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \coth\left(a+\frac{\ln(x)}{8}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + log(x)/8)^p,x)
```

```
[Out] int(coth(a + log(x)/8)^p, x)
```

3.174 $\int \coth^p(a+\log(x))\,dx$

Optimal. Leaf size=61

$$x\left(-1-e^{2a}x^2\right)^p\left(1+e^{2a}x^2\right)^{-p}F_1\left(\frac{1}{2};p,-p;\frac{3}{2};e^{2a}x^2,-e^{2a}x^2\right)$$

```
[Out] x*(-1-exp(2*a)*x^2)^p*AppellF1(1/2,p,-p,3/2,exp(2*a)*x^2,-exp(2*a)*x^2)/((1
+exp(2*a)*x^2)^p)
```

Rubi [A]
time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5653, 441, 440}

$$x\left(-e^{2a}x^2-1\right)^p\left(e^{2a}x^2+1\right)^{-p}F_1\left(\frac{1}{2};p,-p;\frac{3}{2};e^{2a}x^2,-e^{2a}x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + Log[x]]^p,x]
```

```
[Out] (x*(-1 - E^(2*a)*x^2)^p*AppellF1[1/2, p, -p, 3/2, E^(2*a)*x^2, -(E^(2*a)*x^
2)])/(1 + E^(2*a)*x^2)^p
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 5653

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[(-1 - E^(2*
a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p, x] /; FreeQ[{a, b, d, p}, x]
```

Rubi steps

$$\int \coth^p(a+\log(x))\,dx = \int \coth^p(a+\log(x))\,dx$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 171 vs. 2(61) = 122.
time = 1.11, size = 171, normalized size = 2.80

$$\frac{3x\left(\frac{1+e^{2a}x^2}{-1+e^{2a}x^2}\right)^p F_1\left(\frac{1}{2};p,-p;\frac{3}{2};e^{2a}x^2,-e^{2a}x^2\right)}{3F_1\left(\frac{1}{2};p,-p;\frac{3}{2};e^{2a}x^2,-e^{2a}x^2\right)+2e^{2a}px^2\left(F_1\left(\frac{3}{2};p,1-p;\frac{5}{2};e^{2a}x^2,-e^{2a}x^2\right)+F_1\left(\frac{3}{2};1+p,-p;\frac{5}{2};e^{2a}x^2,-e^{2a}x^2\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[a + Log[x]]^p,x]
[Out] (3*x*((1 + E^(2*a)*x^2)/(-1 + E^(2*a)*x^2))^p*AppellF1[1/2, p, -p, 3/2, E^(
2*a)*x^2, -(E^(2*a)*x^2)])/(3*AppellF1[1/2, p, -p, 3/2, E^(2*a)*x^2, -(E^(2
*a)*x^2)] + 2*E^(2*a)*p*x^2*(AppellF1[3/2, p, 1 - p, 5/2, E^(2*a)*x^2, -(E^
(2*a)*x^2)] + AppellF1[3/2, 1 + p, -p, 5/2, E^(2*a)*x^2, -(E^(2*a)*x^2)]))
```

Maple [F]
time = 0.84, size = 0, normalized size = 0.00

$$\int \coth^{p}\left(a+\ln\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+ln(x))^p,x)
[Out] int(coth(a+ln(x))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+log(x))^p,x, algorithm="maxima")
[Out] integrate(coth(a + log(x))^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+log(x))^p,x, algorithm="fricas")
[Out] integral(coth(a + log(x))^p, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \coth^p\left(a + \log\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+ln(x))**p,x)
[Out] Integral(coth(a + log(x))**p, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+log(x))^p,x, algorithm="giac")
[Out] integrate(coth(a + log(x))^p, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{coth}(a + \ln\left(x\right))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + log(x))^p,x)
[Out] int(coth(a + log(x))^p, x)
```

3.175 $\int \coth^p(a+2\log(x))\,dx$

Optimal. Leaf size=61

$$x\left(-1-e^{2a}x^4\right)^p\left(1+e^{2a}x^4\right)^{-p}F_1\left(\frac{1}{4};p,-p;\frac{5}{4};e^{2a}x^4,-e^{2a}x^4\right)$$

```
[Out] x*(-1-exp(2*a)*x^4)^p*AppellF1(1/4,p,-p,5/4,exp(2*a)*x^4,-exp(2*a)*x^4)/((1
+exp(2*a)*x^4)^p)
```

Rubi [A]
time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5653, 441, 440}

$$x\left(-e^{2a}x^4-1\right)^p\left(e^{2a}x^4+1\right)^{-p}F_1\left(\frac{1}{4};p,-p;\frac{5}{4};e^{2a}x^4,-e^{2a}x^4\right)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + 2*Log[x]]^p,x]
```

```
[Out] (x*(-1 - E^(2*a)*x^4)^p*AppellF1[1/4, p, -p, 5/4, E^(2*a)*x^4, -(E^(2*a)*x^
4)])/(1 + E^(2*a)*x^4)^p
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 5653

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[(-1 - E^(2*
a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p, x] /; FreeQ[{a, b, d, p}, x]
```

Rubi steps

$$\int \coth^p(a+2\log(x))\,dx=\int \coth^p(a+2\log(x))\,dx$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 171 vs. $2(61) = 122$.
time = 1.25, size = 171, normalized size = 2.80

$$\frac{5x\left(\frac{1+e^{2a}x^4}{-1+e^{2a}x^4}\right)^p F_1\left(\frac{1}{4};p,-p;\frac{5}{4};e^{2a}x^4,-e^{2a}x^4\right)}{5F_1\left(\frac{1}{4};p,-p;\frac{5}{4};e^{2a}x^4,-e^{2a}x^4\right)+4e^{2a}px^4\left(F_1\left(\frac{5}{4};p,1-p;\frac{9}{4};e^{2a}x^4,-e^{2a}x^4\right)+F_1\left(\frac{5}{4};1+p,-p;\frac{9}{4};e^{2a}x^4,-e^{2a}x^4\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[a + 2*Log[x]]^p,x]
```

```
[Out] (5*x*((1 + E^(2*a)*x^4)/(-1 + E^(2*a)*x^4))^p*AppellF1[1/4, p, -p, 5/4, E^(
2*a)*x^4, -(E^(2*a)*x^4)])/(5*AppellF1[1/4, p, -p, 5/4, E^(2*a)*x^4, -(E^(2
*a)*x^4)] + 4*E^(2*a)*p*x^4*(AppellF1[5/4, p, 1 - p, 9/4, E^(2*a)*x^4, -(E^
(2*a)*x^4)] + AppellF1[5/4, 1 + p, -p, 9/4, E^(2*a)*x^4, -(E^(2*a)*x^4)]))
```

Maple [F]
time = 0.85, size = 0, normalized size = 0.00

$$\int \coth^p\left(a+2\ln\left(x\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+2*ln(x))^p,x)
```

```
[Out] int(coth(a+2*ln(x))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^p,x, algorithm="maxima")
```

```
[Out] integrate(coth(a + 2*log(x))^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^p,x, algorithm="fricas")
```

```
[Out] integral(coth(a + 2*log(x))^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \coth^{p}\left(a + 2\log\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*ln(x))**p,x)
[Out] Integral(coth(a + 2*log(x))**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^p,x, algorithm="giac")
[Out] integrate(coth(a + 2*log(x))^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{coth}(a + 2\,\ln\left(x\right))^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + 2*log(x))^p,x)
[Out] int(coth(a + 2*log(x))^p, x)
```

3.176 $\int \coth^p(a+3\log(x))\,dx$

Optimal. Leaf size=61

$$x\left(-1-e^{2a}x^6\right)^p\left(1+e^{2a}x^6\right)^{-p}F_1\left(\frac{1}{6};p,-p;\frac{7}{6};e^{2a}x^6,-e^{2a}x^6\right)$$

```
[Out] x*(-1-exp(2*a)*x^6)^p*AppellF1(1/6,p,-p,7/6,exp(2*a)*x^6,-exp(2*a)*x^6)/((1
+exp(2*a)*x^6)^p)
```

Rubi [A]
time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5653, 441, 440}

$$x\left(-e^{2a}x^6-1\right)^p\left(e^{2a}x^6+1\right)^{-p}F_1\left(\frac{1}{6};p,-p;\frac{7}{6};e^{2a}x^6,-e^{2a}x^6\right)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + 3*Log[x]]^p,x]
```

```
[Out] (x*(-1 - E^(2*a)*x^6)^p*AppellF1[1/6, p, -p, 7/6, E^(2*a)*x^6, -(E^(2*a)*x^
6)])/(1 + E^(2*a)*x^6)^p
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 5653

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[(-1 - E^(2*
a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p, x] /; FreeQ[{a, b, d, p}, x]
```

Rubi steps

$$\int \coth^p(a+3\log(x))\,dx=\int \coth^p(a+3\log(x))\,dx$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 171 vs. 2(61) = 122.
time = 1.32, size = 171, normalized size = 2.80

$$\frac{7x\left(\frac{1+e^{2a}x^6}{-1+e^{2a}x^6}\right)^p F_1\left(\frac{1}{6};p,-p;\frac{7}{6};e^{2a}x^6,-e^{2a}x^6\right)}{7F_1\left(\frac{1}{6};p,-p;\frac{7}{6};e^{2a}x^6,-e^{2a}x^6\right)+6e^{2a}px^6\left(F_1\left(\frac{7}{6};p,1-p;\frac{13}{6};e^{2a}x^6,-e^{2a}x^6\right)+F_1\left(\frac{7}{6};1+p,-p;\frac{13}{6};e^{2a}x^6,-e^{2a}x^6\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[a + 3*Log[x]]^p,x]
```

```
[Out] (7*x*((1 + E^(2*a)*x^6)/(-1 + E^(2*a)*x^6))^p*AppellF1[1/6, p, -p, 7/6, E^(
2*a)*x^6, -(E^(2*a)*x^6)])/(7*AppellF1[1/6, p, -p, 7/6, E^(2*a)*x^6, -(E^(2
*a)*x^6)] + 6*E^(2*a)*p*x^6*(AppellF1[7/6, p, 1 - p, 13/6, E^(2*a)*x^6, -(E
^(2*a)*x^6)] + AppellF1[7/6, 1 + p, -p, 13/6, E^(2*a)*x^6, -(E^(2*a)*x^6)])
)
```

Maple [F]
time = 0.86, size = 0, normalized size = 0.00

$$\int \coth^p(a+3\ln(x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+3*ln(x))^p,x)
```

```
[Out] int(coth(a+3*ln(x))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+3*log(x))^p,x, algorithm="maxima")
```

```
[Out] integrate(coth(a + 3*log(x))^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+3*log(x))^p,x, algorithm="fricas")
```

```
[Out] integral(coth(a + 3*log(x))^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \coth^{p}\left(a + 3\log\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+3*ln(x))**p,x)
```

```
[Out] Integral(coth(a + 3*log(x))**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+3*log(x))^p,x, algorithm="giac")
```

```
[Out] integrate(coth(a + 3*log(x))^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{coth}(a + 3\,\ln\left(x\right))^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + 3*log(x))^p,x)
```

```
[Out] int(coth(a + 3*log(x))^p, x)
```

3.177 $\int x^3 \coth(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=58

$$\frac{x^4}{4}-\frac{1}{2}x^4\,{}_2F_1\left(1,\frac{2}{bdn};1+\frac{2}{bdn};e^{2ad}(cx^n)^{2bd}\right)$$

[Out] 1/4*x^4-1/2*x^4*hypergeom([1, 2/b/d/n],[1+2/b/d/n],exp(2*a*d)*(c*x^n)^(2*b*d))

Rubi [A]
time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {5659, 5657, 470, 371}

$$\frac{x^4}{4}-\frac{1}{2}x^4\,{}_2F_1\left(1,\frac{2}{bdn};1+\frac{2}{bdn};e^{2ad}(cx^n)^{2bd}\right)$$

Antiderivative was successfully verified.

[In] Int[x^3*Coth[d*(a + b*Log[c*x^n])],x]

[Out] x^4/4 - (x^4*Hypergeometric2F1[1, 2/(b*d*n), 1 + 2/(b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d)])/2

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 5659

```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Coth[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x^3 \coth(d(a+b\log(cx^n)))\,dx = \int x^3 \coth(d(a+b\log(cx^n)))\,dx$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 198 vs. 2(58) = 116.
time = 4.87, size = 198, normalized size = 3.41

$$-\frac{x^4\left(2e^{2d(a+b\log(cx^n))}\,{}_2F_1\left(1,1+\frac{2}{bdn};2+\frac{2}{bdn};e^{2d(a+b\log(cx^n))}\right)+(2+bdn)\left(\coth(d(a+b\log(cx^n)))-\coth(d(a-bn\log(x)+b\log(cx^n)))+{}_2F_1\left(1,\frac{2}{bdn};1+\frac{2}{bdn};e^{2d(a+b\log(cx^n))}\right)+\operatorname{csch}(d(a+b\log(cx^n)))\operatorname{csch}(d(a-bn\log(x)+b\log(cx^n)))\sinh(bdn\log(x))\right)\right)}{8+4bdn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Coth[d*(a + b*Log[c*x^n])],x]
```

```
[Out] -((x^4*(2*E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 2/(b*d*n), 2
+ 2/(b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + (2 + b*d*n)*(Coth[d*(a + b*Log[c
*x^n])] - Coth[d*(a - b*n*Log[x] + b*Log[c*x^n])] + Hypergeometric2F1[1, 2/
(b*d*n), 1 + 2/(b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + Csch[d*(a + b*Log[c*x
^n])]*Csch[d*(a - b*n*Log[x] + b*Log[c*x^n])]*Sinh[b*d*n*Log[x]])))/(8 + 4*
b*d*n))
```

Maple [F]
time = 0.91, size = 0, normalized size = 0.00

$$\int x^3 \coth(d(a+b\ln(c\,x^n)))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*coth(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(x^3*coth(d*(a+b*ln(c*x^n))),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*coth(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] 1/4*x^4 - integrate(x^3/(c^(b*d)*e^(b*d*log(x^n) + a*d) + 1), x) + integrat
e(x^3/(c^(b*d)*e^(b*d*log(x^n) + a*d) - 1), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*coth(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] integral(x^3*coth(b*d*log(c*x^n) + a*d), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \coth\left(ad + bd\log\left(cx^n\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*coth(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral(x**3*coth(a*d + b*d*log(c*x**n)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*coth(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] integrate(x^3*coth((b*log(c*x^n) + a)*d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^3 \coth(d\left(a + b\,\ln\left(c\,x^n\right)\right))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*coth(d*(a + b*log(c*x^n))),x)
```

```
[Out] int(x^3*coth(d*(a + b*log(c*x^n))), x)
```

3.178 $\int x^2 \coth(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=62

$$\frac{x^3}{3}-\frac{2}{3}x^3\,{}_2F_1\left(1,\frac{3}{2bdn};1+\frac{3}{2bdn};e^{2ad}(cx^n)^{2bd}\right)$$

```
[Out] 1/3*x^3-2/3*x^3*hypergeom([1, 3/2/b/d/n],[1+3/2/b/d/n],exp(2*a*d)*(c*x^n)^(
2*b*d))
```

Rubi [A]
time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {5659, 5657, 470, 371}

$$\frac{x^3}{3}-\frac{2}{3}x^3\,{}_2F_1\left(1,\frac{3}{2bdn};1+\frac{3}{2bdn};e^{2ad}(cx^n)^{2bd}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Coth[d*(a + b*Log[c*x^n])],x]
```

```
[Out] x^3/3 - (2*x^3*Hypergeometric2F1[1, 3/(2*b*d*n), 1 + 3/(2*b*d*n), E^(2*a*d)
*(c*x^n)^(2*b*d)])/3
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 5659

```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Coth[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x^2 \coth(d(a+b\log(cx^n)))\,dx = \int x^2 \coth(d(a+b\log(cx^n)))\,dx$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 207 vs. 2(62) = 124.
time = 3.51, size = 207, normalized size = 3.34

$$-\frac{x^3\left(3e^{2d(a+b\log(cx^n))}\,{}_2F_1\left(1,1+\frac{3}{2bdn};2+\frac{3}{2bdn};e^{2d(a+b\log(cx^n))}\right)+(3+2bdn)\left(\coth(d(a+b\log(cx^n)))-\coth(d(a-bn\log(x)+b\log(cx^n)))+{}_2F_1\left(1,\frac{3}{2bdn};1+\frac{3}{2bdn};e^{2d(a+b\log(cx^n))}\right)+\operatorname{csch}(d(a+b\log(cx^n)))\operatorname{csch}(d(a-bn\log(x)+b\log(cx^n)))\sinh(bdn\log(x))\right)\right)}{9+6bdn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Coth[d*(a + b*Log[c*x^n])],x]
```

```
[Out] -((x^3*(3*E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 3/(2*b*d*n),
2 + 3/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + (3 + 2*b*d*n)*(Coth[d*(a + b
*Log[c*x^n])] - Coth[d*(a - b*n*Log[x] + b*Log[c*x^n])] + Hypergeometric2F1
[1, 3/(2*b*d*n), 1 + 3/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + Csch[d*(a +
 b*Log[c*x^n])]*Csch[d*(a - b*n*Log[x] + b*Log[c*x^n])]*Sinh[b*d*n*Log[x]])
))/(9 + 6*b*d*n))
```

Maple [F]
time = 0.94, size = 0, normalized size = 0.00

$$\int x^2 \coth(d(a+b\ln(c\,x^n)))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*coth(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(x^2*coth(d*(a+b*ln(c*x^n))),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*coth(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 - integrate(x^2/(c^(b*d)*e^(b*d*log(x^n) + a*d) + 1), x) + integrat
e(x^2/(c^(b*d)*e^(b*d*log(x^n) + a*d) - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*coth(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] integral(x^2*coth(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \coth\left(ad + bd\log\left(cx^n\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*coth(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral(x**2*coth(a*d + b*d*log(c*x**n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*coth(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] integrate(x^2*coth((b*log(c*x^n) + a)*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^2 \coth(d\left(a + b\,\ln\left(c\,x^n\right)\right))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*coth(d*(a + b*log(c*x^n))),x)
```

```
[Out] int(x^2*coth(d*(a + b*log(c*x^n))), x)
```

3.179 $\int x \coth(d(a + b \log(cx^n)))\, dx$

Optimal. Leaf size=54

$$\frac{x^2}{2} - x^2\,{}_2F_1\left(1, \frac{1}{bdn}; 1 + \frac{1}{bdn}; e^{2ad}(cx^n)^{2bd}\right)$$

```
[Out] 1/2*x^2-x^2*hypergeom([1, 1/b/d/n],[1+1/b/d/n],exp(2*a*d)*(c*x^n)^(2*b*d))
```

Rubi [A]
time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {5659, 5657, 470, 371}

$$\frac{x^2}{2} - x^2\,{}_2F_1\left(1, \frac{1}{bdn}; 1 + \frac{1}{bdn}; e^{2ad}(cx^n)^{2bd}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Coth[d*(a + b*Log[c*x^n])],x]
```

```
[Out] x^2/2 - x^2*Hypergeometric2F1[1, 1/(b*d*n), 1 + 1/(b*d*n), E^(2*a*d)*(c*x^n
)^(2*b*d)]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 5659

```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
```

```
^((m + 1)/n - 1)*Coth[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x\coth\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int x\coth\left(d(a+b\log\left(cx^n\right))\right)\,dx$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 193 vs. 2(54) = 108.
time = 4.75, size = 193, normalized size = 3.57

$$-\frac{x^2\left(e^{2d(a+b\log(cx^n))}\,{}_2F_1\left(1,1+\frac{1}{bdn};2+\frac{1}{bdn};e^{2d(a+b\log(cx^n))}\right)+(1+bdn)\left(\coth\left(d(a+b\log\left(cx^n\right))\right)-\coth\left(d(a-bn\log(x)+b\log\left(cx^n\right))\right)+{}_2F_1\left(1,\frac{1}{bdn};1+\frac{1}{bdn};e^{2d(a+b\log(cx^n))}\right)+\text{csch}(d(a+b\log\left(cx^n\right)))\,\text{csch}(d(a-bn\log(x)+b\log\left(cx^n\right)))\sinh(bdn\log(x))\right)\right)}{2+2bdn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Coth[d*(a + b*Log[c*x^n])],x]
```

```
[Out] -((x^2*(E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 1/(b*d*n), 2 +
1/(b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + (1 + b*d*n)*(Coth[d*(a + b*Log[c*x
^n])] - Coth[d*(a - b*n*Log[x] + b*Log[c*x^n])] + Hypergeometric2F1[1, 1/(b
*d*n), 1 + 1/(b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + Csch[d*(a + b*Log[c*x^n
])]*Csch[d*(a - b*n*Log[x] + b*Log[c*x^n])]*Sinh[b*d*n*Log[x]])))/(2 + 2*b*
d*n))
```

Maple [F]
time = 0.96, size = 0, normalized size = 0.00

$$\int x\coth\left(d(a+b\ln\left(c\,x^n\right))\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*coth(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(x*coth(d*(a+b*ln(c*x^n))),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - integrate(x/(c^(b*d)*e^(b*d*log(x^n) + a*d) + 1), x) + integrate(
x/(c^(b*d)*e^(b*d*log(x^n) + a*d) - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(d*(a+b*log(c*x^n))),x, algorithm="fricas")
[Out] integral(x*coth(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\coth\left(ad+bd\log\left(cx^{n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(d*(a+b*ln(c*x**n))),x)
[Out] Integral(x*coth(a*d + b*d*log(c*x**n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(d*(a+b*log(c*x^n))),x, algorithm="giac")
[Out] integrate(x*coth((b*log(c*x^n) + a)*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\mathrm{coth}\left(d\,\left(a+b\,\ln\left(c\,x^n\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*coth(d*(a + b*log(c*x^n))),x)
[Out] int(x*coth(d*(a + b*log(c*x^n))), x)
```

3.180 $\int \coth(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=52

$$x-2x\,{}_2F_1\left(1,\frac{1}{2bdn};1+\frac{1}{2bdn};e^{2ad}(cx^n)^{2bd}\right)$$

```
[Out] x-2*x*hypergeom([1, 1/2/b/d/n],[1+1/2/b/d/n],exp(2*a*d)*(c*x^n)^(2*b*d))
```

Rubi [A]
time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5655, 5657, 470, 371}

$$x-2x\,{}_2F_1\left(1,\frac{1}{2bdn};1+\frac{1}{2bdn};e^{2ad}(cx^n)^{2bd}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[d*(a + b*Log[c*x^n])],x]
```

```
[Out] x - 2*x*Hypergeometric2F1[1, 1/(2*b*d*n), 1 + 1/(2*b*d*n), E^(2*a*d)*(c*x^n
)^(2*b*d)]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5655

```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Coth[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
```

```
x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int \coth (d(a+b\log (cx^n)))\ dx=\int \coth (d(a+b\log (cx^n)))\ dx$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 198 vs. 2(52) = 104.
time = 6.16, size = 198, normalized size = 3.81

$$-\frac{e^{2d(a+b\log(cx^n))}x\,{}_2F_1\left(1,1+\frac{1}{2bdn};2+\frac{1}{2bdn};e^{2d(a+b\log(cx^n))}\right)}{1+2bdn}-x\left(\coth (d(a+b\log (cx^n)))-\coth (d(a-bn\log(x)+b\log (cx^n)))+{}_2F_1\left(1,\frac{1}{2bdn};1+\frac{1}{2bdn};e^{2d(a+b\log(cx^n))}\right)+\operatorname{csch}(d(a+b\log (cx^n)))\operatorname{csch}(d(a-bn\log(x)+b\log (cx^n)))\sinh(bdn\log(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[d*(a + b*Log[c*x^n])],x]
```

```
[Out] -((E^(2*d*(a + b*Log[c*x^n]))*x*Hypergeometric2F1[1, 1 + 1/(2*b*d*n), 2 + 1
/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))])/(1 + 2*b*d*n)) - x*(Coth[d*(a + b*
Log[c*x^n])] - Coth[d*(a - b*n*Log[x] + b*Log[c*x^n])] + Hypergeometric2F1[
1, 1/(2*b*d*n), 1 + 1/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + Csch[d*(a +
b*Log[c*x^n])]*Csch[d*(a - b*n*Log[x] + b*Log[c*x^n])]*Sinh[b*d*n*Log[x]])
```

Maple [F]
time = 0.66, size = 0, normalized size = 0.00

$$\int \coth (d(a+b\ln (c\,x^n)))\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(coth(d*(a+b*ln(c*x^n))),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] x - integrate(1/(c^(b*d)*e^(b*d*log(x^n) + a*d) + 1), x) + integrate(1/(c^(
b*d)*e^(b*d*log(x^n) + a*d) - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n))),x, algorithm="fricas")
[Out] integral(coth(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \coth\left(d(a+b\log\left(cx^{n}\right))\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*ln(c*x**n))),x)
[Out] Integral(coth(d*(a + b*log(c*x**n))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n))),x, algorithm="giac")
[Out] integrate(coth((b*log(c*x^n) + a)*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{coth}(d\,\left(a+b\,\ln\left(c\,x^n\right)\right))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a + b*log(c*x^n))),x)
[Out] int(coth(d*(a + b*log(c*x^n))), x)
```

3.181 $\int \frac{\coth(d(a+b\log(cx^n)))}{x}\,dx$

Optimal. Leaf size=25

$$\frac{\log\left(\sinh\left(ad+bd\log\left(cx^n\right)\right)\right)}{bdn}$$

```
[Out] ln(sinh(a*d+b*d*ln(c*x^n)))/b/d/n
```

Rubi [**A**]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{3556\}$

$$\frac{\log\left(\sinh\left(ad+bd\log\left(cx^n\right)\right)\right)}{bdn}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[d*(a + b*Log[c*x^n])]/x,x]
```

```
[Out] Log[Sinh[a*d + b*d*Log[c*x^n]]]/(b*d*n)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth\left(d(a+b\log\left(cx^n\right))\right)}{x}\,dx &= \frac{\operatorname{Subst}\left(\int \coth(d(a+bx))\,dx, x, \log\left(cx^n\right)\right)}{n}\\ &= \frac{\log\left(\sinh\left(ad+bd\log\left(cx^n\right)\right)\right)}{bdn}\end{aligned}$$

Mathematica [**A**]
time = 0.05, size = 40, normalized size = 1.60

$$\frac{\log\left(\cosh\left(d(a+b\log\left(cx^n\right))\right)\right)+\log\left(\tanh\left(ad+bd\log\left(cx^n\right)\right)\right)}{bdn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[d*(a + b*Log[c*x^n])]/x,x]
```

```
[Out] (Log[Cosh[d*(a + b*Log[c*x^n])]] + Log[Tanh[a*d + b*d*Log[c*x^n]]])/(b*d*n)
```

Maple [A]
time = 2.29, size = 48, normalized size = 1.92

method	result
derivativedivides	$\frac{-\frac{\ln(\coth(d(a+b\ln(c\,x^{n})))-1)}{2}-\frac{\ln(\coth(d(a+b\ln(c\,x^{n})))+1)}{2}}{nbd}$
default	$\frac{-\frac{\ln(\coth(d(a+b\ln(c\,x^{n})))-1)}{2}-\frac{\ln(\coth(d(a+b\ln(c\,x^{n})))+1)}{2}}{nbd}$
risch	$\ln(x)-\frac{2a}{nb}-\frac{2\ln(c)}{n}-\frac{2\ln(x^{n})}{n}+\frac{i\pi\mathrm{csgn}(ic\,x^{n})^{3}}{n}-\frac{i\pi\mathrm{csgn}(ic\,x^{n})^{2}\mathrm{csgn}(ic)}{n}-\frac{i\pi\mathrm{csgn}(ic\,x^{n})^{2}\mathrm{csgn}(ix^{n})}{n}+i\pi$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a+b*ln(c*x^n)))/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b/d*(-1/2*ln(coth(d*(a+b*ln(c*x^n)))-1)-1/2*ln(coth(d*(a+b*ln(c*x^n)))+
1))
```

Maxima [A]
time = 0.26, size = 24, normalized size = 0.96

$$\frac{\log\left(\sinh\left((b\log\left(cx^{n}\right)+a)d\right)\right)}{bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))/x,x, algorithm="maxima")
```

```
[Out] log(sinh((b*log(c*x^n) + a)*d))/(b*d*n)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(25) = 50.
time = 0.37, size = 76, normalized size = 3.04

$$-\frac{bdn\log\left(x\right)-\log\left(\frac{2\sinh(bdn\log(x)+bd\log(c)+ad)}{\cosh(bdn\log(x)+bd\log(c)+ad)-\sinh(bdn\log(x)+bd\log(c)+ad)}\right)}{bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))/x,x, algorithm="fricas")
```

```
[Out] -(b*d*n*log(x) - log(2*sinh(b*d*n*log(x) + b*d*log(c) + a*d)/(cosh(b*d*n*lo
g(x) + b*d*log(c) + a*d) - sinh(b*d*n*log(x) + b*d*log(c) + a*d))))/(b*d*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\coth\left(ad+bd\log\left(cx^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*ln(c*x**n)))/x,x)
```

```
[Out] Integral(coth(a*d + b*d*log(c*x**n))/x, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(25) = 50.
time = 0.49, size = 74, normalized size = 2.96

$$\frac{\log\left(\sqrt{-2\,x^{2\,bdn}|c|^{2\,bd}\cos\left(\pi bd\mathrm{sgn}\left(c\right)-\pi bd\right)e^{(2\,ad)}+x^{4\,bdn}|c|^{4\,bd}e^{(4\,ad)}+1}\right)}{bdn}-\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))/x,x, algorithm="giac")
```

```
[Out] log(sqrt(-2*x^(2*b*d*n)*abs(c)^(2*b*d)*cos(pi*b*d*sgn(c) - pi*b*d)*e^(2*a*d
) + x^(4*b*d*n)*abs(c)^(4*b*d)*e^(4*a*d) + 1))/(b*d*n) - log(x)
```

Mupad [B]
time = 1.19, size = 34, normalized size = 1.36

$$\frac{\ln\left(\mathrm{e}^{2\,a\,d}\,{\left(c\,x^n\right)}^{2\,b\,d}-1\right)}{b\,d\,n}-\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a + b*log(c*x^n)))/x,x)
```

```
[Out] log(exp(2*a*d)*(c*x^n)^(2*b*d) - 1)/(b*d*n) - log(x)
```

3.182 $\int \frac{\coth(d(a+b\log(cx^n)))}{x^2} dx$

Optimal. Leaf size=58

$$-\frac{1}{x}+\frac{2\,{}_2F_1\left(1,-\frac{1}{2bdn};1-\frac{1}{2bdn};e^{2ad}(cx^n)^{2bd}\right)}{x}$$

```
[Out] -1/x+2*hypergeom([1, -1/2/b/d/n],[1-1/2/b/d/n],exp(2*a*d)*(c*x^n)^(2*b*d))/
x
```

Rubi [A]
time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {5659, 5657, 470, 371}

$$\frac{2\,{}_2F_1\left(1,-\frac{1}{2bdn};1-\frac{1}{2bdn};e^{2ad}(cx^n)^{2bd}\right)}{x}-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[d*(a + b*Log[c*x^n])]/x^2,x]
```

```
[Out] -x^(-1) + (2*Hypergeometric2F1[1, -1/2*1/(b*d*n), 1 - 1/(2*b*d*n), E^(2*a*d
)*(c*x^n)^(2*b*d)])/x
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 5659

```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Coth[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{\coth (d(a+b\log (cx^n)))}{x^2}\,dx=\int \frac{\coth (d(a+b\log (cx^n)))}{x^2}\,dx$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 197 vs. 2(58) = 116.
time = 2.61, size = 197, normalized size = 3.40

$$\frac{\coth (d(a+b\log (cx^n)))-\coth (d(a-bn\log(x)+b\log (cx^n)))-\frac{e^{2d(a+b\log(cx^n))}\,{}_2F_1\left(1,1-\frac{1}{2bdn};2-\frac{1}{2bdn};e^{2d(a+b\log(cx^n))}\right)}{-1+2bdn}+{}_2F_1\left(1,-\frac{1}{2bdn};1-\frac{1}{2bdn};e^{2d(a+b\log(cx^n))}\right)+\operatorname{csch}(d(a+b\log (cx^n)))\operatorname{csch}(d(a-bn\log(x)+b\log (cx^n)))\sinh(bdn\log(x))}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[d*(a + b*Log[c*x^n])]/x^2,x]
```

```
[Out] (Coth[d*(a + b*Log[c*x^n])] - Coth[d*(a - b*n*Log[x] + b*Log[c*x^n])] - (E^
(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - 1/(2*b*d*n), 2 - 1/(2*b*d
*n), E^(2*d*(a + b*Log[c*x^n]))])/(-1 + 2*b*d*n) + Hypergeometric2F1[1, -1/
2*1/(b*d*n), 1 - 1/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + Csch[d*(a + b*L
og[c*x^n])]*Csch[d*(a - b*n*Log[x] + b*Log[c*x^n])]*Sinh[b*d*n*Log[x]])/x
```

Maple [F]
time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{\coth (d(a+b\ln (c\,x^n)))}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a+b*ln(c*x^n)))/x^2,x)
```

```
[Out] int(coth(d*(a+b*ln(c*x^n)))/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))/x^2,x, algorithm="maxima")
```

```
[Out] -1/x - integrate(1/(c^(b*d)*x^2*e^(b*d*log(x^n) + a*d) + x^2), x) + integra
te(1/(c^(b*d)*x^2*e^(b*d*log(x^n) + a*d) - x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))/x^2,x, algorithm="fricas")
```

```
[Out] integral(coth(b*d*log(c*x^n) + a*d)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(ad + bd\log\left(cx^{n}\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*ln(c*x**n)))/x**2,x)
```

```
[Out] Integral(coth(a*d + b*d*log(c*x**n))/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))/x^2,x, algorithm="giac")
```

```
[Out] integrate(coth((b*log(c*x^n) + a)*d)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{coth}\left(d\left(a+b\,\ln\left(c\,x^{n}\right)\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a + b*log(c*x^n)))/x^2,x)
```

```
[Out] int(coth(d*(a + b*log(c*x^n)))/x^2, x)
```

3.183 $\int \frac{\coth(d(a+b\log(cx^n)))}{x^3}\,dx$

Optimal. Leaf size=55

$$-\frac{1}{2x^2}+\frac{{}_2F_1\left(1,-\frac{1}{bdn};1-\frac{1}{bdn};e^{2ad}(cx^n)^{2bd}\right)}{x^2}$$

```
[Out] -1/2/x^2+hypergeom([1, -1/b/d/n],[1-1/b/d/n],exp(2*a*d)*(c*x^n)^(2*b*d))/x^
2
```

Rubi [A]
time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {5659, 5657, 470, 371}

$$\frac{{}_2F_1\left(1,-\frac{1}{bdn};1-\frac{1}{bdn};e^{2ad}(cx^n)^{2bd}\right)}{x^2}-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[d*(a + b*Log[c*x^n])]/x^3,x]
```

```
[Out] -1/2*1/x^2 + Hypergeometric2F1[1, -(1/(b*d*n)), 1 - 1/(b*d*n), E^(2*a*d)*(c
*x^n)^(2*b*d)]/x^2
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 5659

```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Coth[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{\coth(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\coth(d(a+b\log(cx^n)))}{x^3}\,dx$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 191 vs. 2(55) = 110.
time = 2.59, size = 191, normalized size = 3.47

$$\frac{\coth(d(a+b\log(cx^n))) - \coth(d(a-bn\log(x)+b\log(cx^n))) - \frac{e^{2d(a+b\log(cx^n))}\,{}_2F_1\left(1,1-\frac{1}{bdn};2-\frac{1}{bdn};e^{2d(a+b\log(cx^n))}\right)}{-1+bdn} + {}_2F_1\left(1,-\frac{1}{bdn};1-\frac{1}{bdn};e^{2d(a+b\log(cx^n))}\right) + \operatorname{csch}(d(a+b\log(cx^n)))\operatorname{csch}(d(a-bn\log(x)+b\log(cx^n)))\sinh(bdn\log(x))}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[d*(a + b*Log[c*x^n])]/x^3,x]
```

```
[Out] (Coth[d*(a + b*Log[c*x^n])] - Coth[d*(a - b*n*Log[x] + b*Log[c*x^n])] - (E^
(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - 1/(b*d*n), 2 - 1/(b*d*n),
 E^(2*d*(a + b*Log[c*x^n]))])/(-1 + b*d*n) + Hypergeometric2F1[1, -(1/(b*d*
n)), 1 - 1/(b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + Csch[d*(a + b*Log[c*x^n])
]*Csch[d*(a - b*n*Log[x] + b*Log[c*x^n])]*Sinh[b*d*n*Log[x]])/(2*x^2)
```

Maple [**F**]
time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{\coth(d(a+b\ln(c\,x^n)))}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a+b*ln(c*x^n)))/x^3,x)
```

```
[Out] int(coth(d*(a+b*ln(c*x^n)))/x^3,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2/x^2 - integrate(1/(c^(b*d)*x^3*e^(b*d*log(x^n) + a*d) + x^3), x) + int
egrate(1/(c^(b*d)*x^3*e^(b*d*log(x^n) + a*d) - x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))/x^3,x, algorithm="fricas")
```

```
[Out] integral(coth(b*d*log(c*x^n) + a*d)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(ad + bd\log\left(cx^{n}\right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*ln(c*x**n)))/x**3,x)
```

```
[Out] Integral(coth(a*d + b*d*log(c*x**n))/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))/x^3,x, algorithm="giac")
```

```
[Out] integrate(coth((b*log(c*x^n) + a)*d)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\coth\left(d\left(a+b\,\ln\left(c\,x^{n}\right)\right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a + b*log(c*x^n)))/x^3,x)
```

```
[Out] int(coth(d*(a + b*log(c*x^n)))/x^3, x)
```

3.184 $\int x^3 \coth^2(d(a + b\log(cx^n)))\,dx$

Optimal. Leaf size=132

$$\frac{1}{4}\left(1+\frac{4}{bdn}\right)x^4+\frac{x^4\left(1+e^{2ad}(cx^n)^{2bd}\right)}{bdn\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)}-\frac{2x^4\,{}_2F_1\left(1,\frac{2}{bdn};1+\frac{2}{bdn};e^{2ad}(cx^n)^{2bd}\right)}{bdn}$$

```
[Out] 1/4*(1+4/b/d/n)*x^4+x^4*(1+exp(2*a*d)*(c*x^n)^(2*b*d))/b/d/n/(1-exp(2*a*d)*
(c*x^n)^(2*b*d))-2*x^4*hypergeom([1, 2/b/d/n],[1+2/b/d/n],exp(2*a*d)*(c*x^n
)^(2*b*d))/b/d/n
```

Rubi [A]
time = 0.12, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5659, 5657, 516, 470, 371}

$$-\frac{2x^4\,{}_2F_1\left(1,\frac{2}{bdn};1+\frac{2}{bdn};e^{2ad}(cx^n)^{2bd}\right)}{bdn}+\frac{x^4\left(e^{2ad}(cx^n)^{2bd}+1\right)}{bdn\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)}+\frac{1}{4}x^4\left(\frac{4}{bdn}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Coth[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] ((1 + 4/(b*d*n))*x^4)/4 + (x^4*(1 + E^(2*a*d)*(c*x^n)^(2*b*d)))/(b*d*n*(1 -
 E^(2*a*d)*(c*x^n)^(2*b*d))) - (2*x^4*Hypergeometric2F1[1, 2/(b*d*n), 1 + 2
/(b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d)])/(b*d*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 516

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
```

```
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ
[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 5659

```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Coth[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x^3 \coth^2\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int x^3 \coth^2\left(d(a+b\log\left(cx^n\right))\right)\,dx$$

Mathematica [A]
time = 4.93, size = 155, normalized size = 1.17

$$\frac{x^4\left(-8e^{2d(a+b\log(cx^n))}\,{}_2F_1\left(1,1+\frac{2}{bdn};2+\frac{2}{bdn};e^{2d(a+b\log(cx^n))}\right)+(2+bdn)\left(bdn-4\coth\left(d(a+b\log\left(cx^n\right))\right)-4\,{}_2F_1\left(1,\frac{2}{bdn};1+\frac{2}{bdn};e^{2d(a+b\log(cx^n))}\right)\right)\right)}{4bdn(2+bdn)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Coth[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] (x^4*(-8*E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 2/(b*d*n), 2 +
 2/(b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + (2 + b*d*n)*(b*d*n - 4*Coth[d*(a
+ b*Log[c*x^n])] - 4*Hypergeometric2F1[1, 2/(b*d*n), 1 + 2/(b*d*n), E^(2*d*
(a + b*Log[c*x^n]))])))/(4*b*d*n*(2 + b*d*n))
```

Maple [F]
time = 0.55, size = 0, normalized size = 0.00

$$\int x^3\left(\coth^2\left(d(a+b\ln\left(c\,x^n\right))\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*coth(d*(a+b*ln(c*x^n)))^2,x)
```

```
[Out] int(x^3*coth(d*(a+b*ln(c*x^n)))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

```
[Out] 1/4*(b*c^(2*b*d)*d*n*x^4*e^(2*b*d*log(x^n) + 2*a*d) - (b*d*n + 8)*x^4)/(b*c
^(2*b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n) - 4*integrate(x^3/(b*c^(b*
d)*d*n*e^(b*d*log(x^n) + a*d) + b*d*n), x) + 4*integrate(x^3/(b*c^(b*d)*d*n
*e^(b*d*log(x^n) + a*d) - b*d*n), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

```
[Out] integral(x^3*coth(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \coth^2 (ad + bd \log (cx^n))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*coth(d*(a+b*ln(c*x**n)))**2,x)
```

```
[Out] Integral(x**3*coth(a*d + b*d*log(c*x**n))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*coth((b*log(c*x^n) + a)*d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \coth(d\,(a+b\,\ln(c\,x^n)))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*coth(d*(a + b*log(c*x^n)))^2,x)
[Out] int(x^3*coth(d*(a + b*log(c*x^n)))^2, x)
```

3.185 $\int x^2 \coth^2(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=136

$$\frac{1}{3}\left(1+\frac{3}{bdn}\right)x^3+\frac{x^3\left(1+e^{2ad}(cx^n)^{2bd}\right)}{bdn\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)}-\frac{2x^3\,{}_2F_1\left(1,\frac{3}{2bdn};1+\frac{3}{2bdn};e^{2ad}(cx^n)^{2bd}\right)}{bdn}$$

```
[Out] 1/3*(1+3/b/d/n)*x^3+x^3*(1+exp(2*a*d)*(c*x^n)^(2*b*d))/b/d/n/(1-exp(2*a*d)*
(c*x^n)^(2*b*d))-2*x^3*hypergeom([1, 3/2/b/d/n],[1+3/2/b/d/n],exp(2*a*d)*(c
*x^n)^(2*b*d))/b/d/n
```

Rubi [A]
time = 0.12, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5659, 5657, 516, 470, 371}

$$-\frac{2x^3\,{}_2F_1\left(1,\frac{3}{2bdn};1+\frac{3}{2bdn};e^{2ad}(cx^n)^{2bd}\right)}{bdn}+\frac{x^3\left(e^{2ad}(cx^n)^{2bd}+1\right)}{bdn\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)}+\frac{1}{3}x^3\left(\frac{3}{bdn}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Coth[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] ((1 + 3/(b*d*n))*x^3)/3 + (x^3*(1 + E^(2*a*d)*(c*x^n)^(2*b*d)))/(b*d*n*(1 -
 E^(2*a*d)*(c*x^n)^(2*b*d))) - (2*x^3*Hypergeometric2F1[1, 3/(2*b*d*n), 1 +
 3/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d)])/(b*d*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 516

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
```

```
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ
[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 5659

```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Coth[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x^2 \coth^2\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int x^2 \coth^2\left(d(a+b\log\left(cx^n\right))\right)\,dx$$

Mathematica [A]
time = 3.09, size = 165, normalized size = 1.21

$$\frac{x^3\left(-9e^{2d(a+b\log(cx^n))}\,{}_2F_1\left(1,1+\frac{3}{2bdn};2+\frac{3}{2bdn};e^{2d(a+b\log(cx^n))}\right)+(3+2bdn)\left(bdn-3\coth\left(d(a+b\log\left(cx^n\right))\right)-3\,{}_2F_1\left(1,\frac{3}{2bdn};1+\frac{3}{2bdn};e^{2d(a+b\log(cx^n))}\right)\right)\right)}{3bdn(3+2bdn)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Coth[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] (x^3*(-9*E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 3/(2*b*d*n), 2
 + 3/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + (3 + 2*b*d*n)*(b*d*n - 3*Coth
[d*(a + b*Log[c*x^n])] - 3*Hypergeometric2F1[1, 3/(2*b*d*n), 1 + 3/(2*b*d*n
), E^(2*d*(a + b*Log[c*x^n]))])))/(3*b*d*n*(3 + 2*b*d*n))
```

Maple [F]
time = 0.55, size = 0, normalized size = 0.00

$$\int x^2\left(\coth^2\left(d(a+b\ln\left(c\,x^n\right))\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*coth(d*(a+b*ln(c*x^n)))^2,x)
```

```
[Out] int(x^2*coth(d*(a+b*ln(c*x^n)))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b*c^(2*b*d)*d*n*x^3*e^(2*b*d*log(x^n) + 2*a*d) - (b*d*n + 6)*x^3)/(b*c
^(2*b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n) - 3*integrate(x^2/(b*c^(b*
d)*d*n*e^(b*d*log(x^n) + a*d) + b*d*n), x) + 3*integrate(x^2/(b*c^(b*d)*d*n
*e^(b*d*log(x^n) + a*d) - b*d*n), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*coth(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \coth^2 (ad + bd \log (cx^n))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*coth(d*(a+b*ln(c*x**n)))**2,x)
```

```
[Out] Integral(x**2*coth(a*d + b*d*log(c*x**n))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*coth((b*log(c*x^n) + a)*d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \coth(d\,(a+b\,\ln\,(c\,x^n)))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*coth(d*(a + b*log(c*x^n)))^2,x)
[Out] int(x^2*coth(d*(a + b*log(c*x^n)))^2, x)
```

3.186 $\int x \coth^2(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=130

$$\frac{1}{2}\left(1+\frac{2}{bdn}\right)x^2+\frac{x^2\left(1+e^{2ad}(cx^n)^{2bd}\right)}{bdn\left(1-e^{2ad}(cx^n)^{2bd}\right)}-\frac{2x^2\,{}_2F_1\left(1,\frac{1}{bdn};1+\frac{1}{bdn};e^{2ad}(cx^n)^{2bd}\right)}{bdn}$$

```
[Out] 1/2*(1+2/b/d/n)*x^2+x^2*(1+exp(2*a*d)*(c*x^n)^(2*b*d))/b/d/n/(1-exp(2*a*d)*
(c*x^n)^(2*b*d))-2*x^2*hypergeom([1, 1/b/d/n],[1+1/b/d/n],exp(2*a*d)*(c*x^n
)^(2*b*d))/b/d/n
```

Rubi [A]
time = 0.11, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {5659, 5657, 516, 470, 371}

$$-\frac{2x^2\,{}_2F_1\left(1,\frac{1}{bdn};1+\frac{1}{bdn};e^{2ad}(cx^n)^{2bd}\right)}{bdn}+\frac{x^2\left(e^{2ad}(cx^n)^{2bd}+1\right)}{bdn\left(1-e^{2ad}(cx^n)^{2bd}\right)}+\frac{1}{2}x^2\left(\frac{2}{bdn}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Coth[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] ((1 + 2/(b*d*n))*x^2)/2 + (x^2*(1 + E^(2*a*d)*(c*x^n)^(2*b*d)))/(b*d*n*(1 -
 E^(2*a*d)*(c*x^n)^(2*b*d))) - (2*x^2*Hypergeometric2F1[1, 1/(b*d*n), 1 + 1
/(b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d)])/(b*d*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 516

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
```

```
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ
[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 5659

```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Coth[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int x \coth^2(d(a+b\log(cx^n)))\,dx = \int x \coth^2(d(a+b\log(cx^n)))\,dx$$

Mathematica [A]
time = 4.50, size = 151, normalized size = 1.16

$$\frac{x^2\left(-2e^{2d(a+b\log(cx^n))}\,{}_2F_1\left(1,1+\frac{1}{bdn};2+\frac{1}{bdn};e^{2d(a+b\log(cx^n))}\right)+(1+bdn)\left(bdn-2\coth\left(d(a+b\log(cx^n))\right)-2\,{}_2F_1\left(1,\frac{1}{bdn};1+\frac{1}{bdn};e^{2d(a+b\log(cx^n))}\right)\right)\right)}{2bdn(1+bdn)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Coth[d*(a + b*Log[c*x^n])]^2,x]
[Out] (x^2*(-2*E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 1/(b*d*n), 2 +
 1/(b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + (1 + b*d*n)*(b*d*n - 2*Coth[d*(a
+ b*Log[c*x^n])] - 2*Hypergeometric2F1[1, 1/(b*d*n), 1 + 1/(b*d*n), E^(2*d*
(a + b*Log[c*x^n]))])))/(2*b*d*n*(1 + b*d*n))
```

Maple [F]
time = 0.57, size = 0, normalized size = 0.00

$$\int x\left(\coth^2\left(d(a+b\ln(c\,x^n))\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*coth(d*(a+b*ln(c*x^n)))^2,x)
```

```
[Out] int(x*coth(d*(a+b*ln(c*x^n)))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c^(2*b*d)*d*n*x^2*e^(2*b*d*log(x^n) + 2*a*d) - (b*d*n + 4)*x^2)/(b*c
^(2*b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n) - 2*integrate(x/(b*c^(b*d)
*d*n*e^(b*d*log(x^n) + a*d) + b*d*n), x) + 2*integrate(x/(b*c^(b*d)*d*n*e^(
b*d*log(x^n) + a*d) - b*d*n), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

```
[Out] integral(x*coth(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\coth^{2}\left(ad+bd\log\left(cx^{n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(d*(a+b*ln(c*x**n)))**2,x)
```

```
[Out] Integral(x*coth(a*d + b*d*log(c*x**n))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

```
[Out] integrate(x*coth((b*log(c*x^n) + a)*d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x \coth(d(a+b\ln(c x^n)))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*coth(d*(a + b*log(c*x^n)))^2,x)
```

```
[Out] int(x*coth(d*(a + b*log(c*x^n)))^2, x)
```

3.187 $\int \coth^2(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=126

$$\left(1+\frac{1}{bdn}\right)x+\frac{x\left(1+e^{2ad}(cx^n)^{2bd}\right)}{bdn\left(1-e^{2ad}(cx^n)^{2bd}\right)}-\frac{2x\,{}_2F_1\left(1,\frac{1}{2bdn};1+\frac{1}{2bdn};e^{2ad}(cx^n)^{2bd}\right)}{bdn}$$

```
[Out] (1+1/b/d/n)*x+x*(1+exp(2*a*d)*(c*x^n)^(2*b*d))/b/d/n/(1-exp(2*a*d)*(c*x^n)^
(2*b*d))-2*x*hypergeom([1, 1/2/b/d/n],[1+1/2/b/d/n],exp(2*a*d)*(c*x^n)^(2*b
*d))/b/d/n
```

Rubi [A]
time = 0.11, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5655, 5657, 516, 470, 371}

$$-\frac{2x\,{}_2F_1\left(1,\frac{1}{2bdn};1+\frac{1}{2bdn};e^{2ad}(cx^n)^{2bd}\right)}{bdn}+\frac{x\left(e^{2ad}(cx^n)^{2bd}+1\right)}{bdn\left(1-e^{2ad}(cx^n)^{2bd}\right)}+x\left(\frac{1}{bdn}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] (1 + 1/(b*d*n))*x + (x*(1 + E^(2*a*d)*(c*x^n)^(2*b*d)))/(b*d*n*(1 - E^(2*a*
d)*(c*x^n)^(2*b*d))) - (2*x*Hypergeometric2F1[1, 1/(2*b*d*n), 1 + 1/(2*b*d*
n), E^(2*a*d)*(c*x^n)^(2*b*d)])/(b*d*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 516

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
```

```
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ
[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 5655

```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Coth[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int \coth^2(d(a+b\log(cx^n)))\,dx = \int \coth^2(d(a+b\log(cx^n)))\,dx$$

Mathematica [A]
time = 5.70, size = 160, normalized size = 1.27

$$\frac{x\left(-e^{2d(a+b\log(cx^n))}\,{}_2F_1\left(1,1+\frac{1}{2bdn};2+\frac{1}{2bdn};e^{2d(a+b\log(cx^n))}\right)+(1+2bdn)\left(bdn-\coth(d(a+b\log(cx^n)))-{}_2F_1\left(1,\frac{1}{2bdn};1+\frac{1}{2bdn};e^{2d(a+b\log(cx^n))}\right)\right)\right)}{bdn(1+2bdn)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[d*(a + b*Log[c*x^n])]^2,x]
[Out] (x*(-(E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 1/(2*b*d*n), 2 +
1/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))]) + (1 + 2*b*d*n)*(b*d*n - Coth[d*(
a + b*Log[c*x^n])] - Hypergeometric2F1[1, 1/(2*b*d*n), 1 + 1/(2*b*d*n), E^(
2*d*(a + b*Log[c*x^n]))])))/(b*d*n*(1 + 2*b*d*n))
```

Maple [F]
time = 0.54, size = 0, normalized size = 0.00

$$\int \coth^2(d(a+b\ln(cx^n)))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a+b*ln(c*x^n)))^2,x)
```

```
[Out] int(coth(d*(a+b*ln(c*x^n)))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

```
[Out] (b*c^(2*b*d)*d*n*x*e^(2*b*d*log(x^n) + 2*a*d) - (b*d*n + 2)*x)/(b*c^(2*b*d)
*d*n*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n) - integrate(1/(b*c^(b*d)*d*n*e^(b*
d*log(x^n) + a*d) + b*d*n), x) + integrate(1/(b*c^(b*d)*d*n*e^(b*d*log(x^n)
 + a*d) - b*d*n), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

```
[Out] integral(coth(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \coth^2\left(d(a + b\log\left(cx^n\right))\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*ln(c*x**n)))**2,x)
```

```
[Out] Integral(coth(d*(a + b*log(c*x**n)))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

```
[Out] integrate(coth((b*log(c*x^n) + a)*d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \coth(d\,(a+b\,\ln\left(c\,x^n\right)))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a + b*log(c*x^n)))^2,x)
[Out] int(coth(d*(a + b*log(c*x^n)))^2, x)
```

3.188 $\int \frac{\coth^2(d(a+b\log(cx^n)))}{x} dx$

Optimal. Leaf size=28

$$-\frac{\coth(ad+bd\log(cx^n))}{bdn}+\log(x)$$

[Out] -coth(a*d+b*d*ln(c*x^n))/b/d/n+ln(x)

Rubi [A]
time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3554, 8}

$$\log(x)-\frac{\coth(ad+bd\log(cx^n))}{bdn}$$

Antiderivative was successfully verified.

[In] Int[Coth[d*(a + b*Log[c*x^n])]^2/x,x]

[Out] -(Coth[a*d + b*d*Log[c*x^n]]/(b*d*n)) + Log[x]

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^2(d(a+b\log(cx^n)))}{x}\,dx &= \frac{\text{Subst}\left(\int \coth^2(d(a+bx))\,dx,x,\log(cx^n)\right)}{n}\\ &= -\frac{\coth(ad+bd\log(cx^n))}{bdn}+\frac{\text{Subst}\left(\int 1\,dx,x,\log(cx^n)\right)}{n}\\ &= -\frac{\coth(ad+bd\log(cx^n))}{bdn}+\log(x)\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.08, size = 49, normalized size = 1.75

$$-\frac{\coth(ad+bd\log(cx^n))\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\tanh^2(ad+bd\log(cx^n))\right)}{bdn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[d*(a + b*Log[c*x^n])]^2/x,x]
```

```
[Out] -((Coth[a*d + b*d*Log[c*x^n]]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[a*d + b*
d*Log[c*x^n]]^2])/(b*d*n))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(28) = 56$.
time = 2.32, size = 63, normalized size = 2.25

method	result
derivativedivides	$\frac{-\coth(d(a+b\ln(c\,x^n)))-\frac{\ln(\coth(d(a+b\ln(c\,x^n)))-1)}{2}+\frac{\ln(\coth(d(a+b\ln(c\,x^n)))+1)}{2}}{nbd}$
default	$\frac{-\coth(d(a+b\ln(c\,x^n)))-\frac{\ln(\coth(d(a+b\ln(c\,x^n)))-1)}{2}+\frac{\ln(\coth(d(a+b\ln(c\,x^n)))+1)}{2}}{nbd}$
risch	$\ln(x)-\frac{2}{dbn\left(e^{d\left(-ib\pi\mathrm{csgn}(ic\,x^n)^3+ib\pi\mathrm{csgn}(ic\,x^n)^2\mathrm{csgn}(ic)+ib\pi\mathrm{csgn}(ic\,x^n)^2\mathrm{csgn}(ix^n)-ib\pi\,\mathrm{csgn}(ic\,x^n)\mathrm{csgn}(ic)\mathrm{csgn}(ix^n)+2b\ln\right.}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a+b*ln(c*x^n)))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b/d*(-coth(d*(a+b*ln(c*x^n)))-1/2*ln(coth(d*(a+b*ln(c*x^n)))-1)+1/2*ln(
coth(d*(a+b*ln(c*x^n)))+1))
```

Maxima [A]
time = 0.32, size = 37, normalized size = 1.32

$$-\frac{2}{bc^{2\,bd}dne^{(2\,bd\log(x^n)+2\,ad)}-bdn}+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2/x,x, algorithm="maxima")
```

```
[Out] -2/(b*c^(2*b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n) + log(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(28) = 56$.
time = 0.35, size = 72, normalized size = 2.57

$$\frac{(bdn\log(x)+1)\sinh(bdn\log(x)+bd\log(c)+ad)-\cosh(bdn\log(x)+bd\log(c)+ad)}{bdn\sinh(bdn\log(x)+bd\log(c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2/x,x, algorithm="fricas")
```

```
[Out] ((b*d*n*log(x) + 1)*sinh(b*d*n*log(x) + b*d*log(c) + a*d) - cosh(b*d*n*log(
x) + b*d*log(c) + a*d))/(b*d*n*sinh(b*d*n*log(x) + b*d*log(c) + a*d))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(ad + bd\log(cx^n))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*ln(c*x**n)))**2/x,x)
```

```
[Out] Integral(coth(a*d + b*d*log(c*x**n))**2/x, x)
```

Giac [A]
time = 0.48, size = 37, normalized size = 1.32

$$-\frac{2}{(c^{2\,bd}x^{2\,bdn}e^{(2\,ad)} - 1)bdn} + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2/x,x, algorithm="giac")
```

```
[Out] -2/((c^(2*b*d)*x^(2*b*d*n)*e^(2*a*d) - 1)*b*d*n) + log(x)
```

Mupad [B]
time = 1.19, size = 34, normalized size = 1.21

$$\ln(x) - \frac{2}{b\,d\,n\,\left(\mathrm{e}^{2\,a\,d}\,(c\,x^n)^{2\,b\,d} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a + b*log(c*x^n)))^2/x,x)
```

```
[Out] log(x) - 2/(b*d*n*(exp(2*a*d)*(c*x^n)^(2*b*d) - 1))
```

3.189 $\int \frac{\coth^2(d(a+b\log(cx^n)))}{x^2}\,dx$

Optimal. Leaf size=134

$$-\frac{1-\frac{1}{bdn}}{x}+\frac{1+e^{2ad}(cx^n)^{2bd}}{bdnx\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)}-\frac{2\,{}_2F_1\left(1,-\frac{1}{2bdn};1-\frac{1}{2bdn};e^{2ad}(cx^n)^{2bd}\right)}{bdnx}$$

```
[Out] (-1+1/b/d/n)/x+(1+exp(2*a*d)*(c*x^n)^(2*b*d))/b/d/n/x/(1-exp(2*a*d)*(c*x^n)
^(2*b*d))-2*hypergeom([1, -1/2/b/d/n],[1-1/2/b/d/n],exp(2*a*d)*(c*x^n)^(2*b
*d))/b/d/n/x
```

Rubi [A]
time = 0.12, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5659, 5657, 516, 470, 371}

$$-\frac{2\,{}_2F_1\left(1,-\frac{1}{2bdn};1-\frac{1}{2bdn};e^{2ad}(cx^n)^{2bd}\right)}{bdnx}+\frac{e^{2ad}(cx^n)^{2bd}+1}{bdnx\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)}-\frac{1-\frac{1}{bdn}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[d*(a + b*Log[c*x^n])]^2/x^2,x]
```

```
[Out] -((1 - 1/(b*d*n))/x) + (1 + E^(2*a*d)*(c*x^n)^(2*b*d))/(b*d*n*x*(1 - E^(2*a
*d)*(c*x^n)^(2*b*d))) - (2*Hypergeometric2F1[1, -1/2*1/(b*d*n), 1 - 1/(2*b*
d*n), E^(2*a*d)*(c*x^n)^(2*b*d)])/(b*d*n*x)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 516

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
```

```
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ
[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 5659

```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Coth[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{\coth^2(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\coth^2(d(a+b\log(cx^n)))}{x^2}\,dx$$

Mathematica [A]
time = 2.46, size = 158, normalized size = 1.18

$$\frac{e^{2d(a+b\log(cx^n))}\,{}_2F_1\left(1,1-\frac{1}{2bdn};2-\frac{1}{2bdn};e^{2d(a+b\log(cx^n))}\right)-(-1+2bdn)\left(bdn+\coth(d(a+b\log(cx^n)))+{}_2F_1\left(1,-\frac{1}{2bdn};1-\frac{1}{2bdn};e^{2d(a+b\log(cx^n))}\right)\right)}{bdn(-1+2bdn)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[d*(a + b*Log[c*x^n])]^2/x^2,x]
```

```
[Out] (E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - 1/(2*b*d*n), 2 - 1/(2*
b*d*n), E^(2*d*(a + b*Log[c*x^n]))] - (-1 + 2*b*d*n)*(b*d*n + Coth[d*(a + b
*Log[c*x^n])] + Hypergeometric2F1[1, -1/2*1/(b*d*n), 1 - 1/(2*b*d*n), E^(2*
d*(a + b*Log[c*x^n]))]))/(b*d*n*(-1 + 2*b*d*n)*x)
```

Maple [F]
time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(d(a+b\ln(c\,x^n)))}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a+b*ln(c*x^n)))^2/x^2,x)
```

```
[Out] int(coth(d*(a+b*ln(c*x^n)))^2/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2/x^2,x, algorithm="maxima")
```

```
[Out] -(b*c^(2*b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n + 2)/(b*c^(2*b*d)*d*n*
x*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n*x) + integrate(1/(b*c^(b*d)*d*n*x^2*e^
(b*d*log(x^n) + a*d) + b*d*n*x^2), x) - integrate(1/(b*c^(b*d)*d*n*x^2*e^(b
*d*log(x^n) + a*d) - b*d*n*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(coth(b*d*log(c*x^n) + a*d)^2/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(ad + bd\log(cx^n))}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*ln(c*x**n)))**2/x**2,x)
```

```
[Out] Integral(coth(a*d + b*d*log(c*x**n))**2/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(coth((b*log(c*x^n) + a)*d)^2/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(d\,(a+b\,\ln\left(c\,x^n\right)))^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a + b*log(c*x^n)))^2/x^2,x)
```

```
[Out] int(coth(d*(a + b*log(c*x^n)))^2/x^2, x)
```

3.190 $\int \frac{\coth^2(d(a+b\log(cx^n)))}{x^3} dx$

Optimal. Leaf size=135

$$\frac{2-bdn}{2bdnx^2}+\frac{1+e^{2ad}(cx^n)^{2bd}}{bdnx^2\left(1-e^{2ad}(cx^n)^{2bd}\right)}-\frac{2\,{}_2F_1\left(1,-\frac{1}{bdn};1-\frac{1}{bdn};e^{2ad}(cx^n)^{2bd}\right)}{bdnx^2}$$

```
[Out] 1/2*(-b*d*n+2)/b/d/n/x^2+(1+exp(2*a*d)*(c*x^n)^(2*b*d))/b/d/n/x^2/(1-exp(2*
a*d)*(c*x^n)^(2*b*d))-2*hypergeom([1, -1/b/d/n],[1-1/b/d/n],exp(2*a*d)*(c*x
^n)^(2*b*d))/b/d/n/x^2
```

Rubi [A]
time = 0.12, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5659, 5657, 516, 470, 371}

$$-\frac{2\,{}_2F_1\left(1,-\frac{1}{bdn};1-\frac{1}{bdn};e^{2ad}(cx^n)^{2bd}\right)}{bdnx^2}+\frac{e^{2ad}(cx^n)^{2bd}+1}{bdnx^2\left(1-e^{2ad}(cx^n)^{2bd}\right)}+\frac{2-bdn}{2bdnx^2}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[d*(a + b*Log[c*x^n])]^2/x^3,x]
```

```
[Out] (2 - b*d*n)/(2*b*d*n*x^2) + (1 + E^(2*a*d)*(c*x^n)^(2*b*d))/(b*d*n*x^2*(1 -
 E^(2*a*d)*(c*x^n)^(2*b*d))) - (2*Hypergeometric2F1[1, -(1/(b*d*n)), 1 - 1/
(b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d)])/(b*d*n*x^2)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 516

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
```

```
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ
[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 5659

```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Coth[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{\coth^2(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\coth^2(d(a+b\log(cx^n)))}{x^3}\,dx$$

Mathematica [A]
time = 2.43, size = 156, normalized size = 1.16

$$\frac{2e^{2d(a+b\log(cx^n))}\,{}_2F_1\left(1,1-\frac{1}{bdn};2-\frac{1}{bdn};e^{2d(a+b\log(cx^n))}\right)-(-1+bdn)\left(bdn+2\coth\left(d(a+b\log(cx^n))\right)+2\,{}_2F_1\left(1,-\frac{1}{bdn};1-\frac{1}{bdn};e^{2d(a+b\log(cx^n))}\right)\right)}{2bdn(-1+bdn)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[d*(a + b*Log[c*x^n])]^2/x^3,x]
```

```
[Out] (2*E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - 1/(b*d*n), 2 - 1/(b*
d*n), E^(2*d*(a + b*Log[c*x^n]))] - (-1 + b*d*n)*(b*d*n + 2*Coth[d*(a + b*L
og[c*x^n])] + 2*Hypergeometric2F1[1, -(1/(b*d*n)), 1 - 1/(b*d*n), E^(2*d*(a
 + b*Log[c*x^n]))]))/(2*b*d*n*(-1 + b*d*n)*x^2)
```

Maple [F]
time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(d(a+b\ln(c\,x^n)))}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a+b*ln(c*x^n)))^2/x^3,x)
```

```
[Out] int(coth(d*(a+b*ln(c*x^n)))^2/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b*c^(2*b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n + 4)/(b*c^(2*b*d)*
d*n*x^2*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n*x^2) + 2*integrate(1/(b*c^(b*d)*
d*n*x^3*e^(b*d*log(x^n) + a*d) + b*d*n*x^3), x) - 2*integrate(1/(b*c^(b*d)*
d*n*x^3*e^(b*d*log(x^n) + a*d) - b*d*n*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral(coth(b*d*log(c*x^n) + a*d)^2/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^{2}\left(ad+bd\log\left(cx^{n}\right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*ln(c*x**n)))**2/x**3,x)
```

```
[Out] Integral(coth(a*d + b*d*log(c*x**n))**2/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(coth((b*log(c*x^n) + a)*d)^2/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(d\,(a+b\,\ln\left(c\,x^n\right)))^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a + b*log(c*x^n)))^2/x^3,x)
[Out] int(coth(d*(a + b*log(c*x^n)))^2/x^3, x)
```

3.191 $\int \frac{\coth^3(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=43

$$-\frac{\coth^2\left(a+b\log\left(cx^n\right)\right)}{2bn}+\frac{\log\left(\sinh\left(a+b\log\left(cx^n\right)\right)\right)}{bn}$$

```
[Out] -1/2*coth(a+b*ln(c*x^n))^2/b/n+ln(sinh(a+b*ln(c*x^n)))/b/n
```

Rubi [A]
time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3554, 3556}

$$\frac{\log\left(\sinh\left(a+b\log\left(cx^n\right)\right)\right)}{bn}-\frac{\coth^2\left(a+b\log\left(cx^n\right)\right)}{2bn}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] -1/2*Coth[a + b*Log[c*x^n]]^2/(b*n) + Log[Sinh[a + b*Log[c*x^n]]]/(b*n)
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^3\left(a+b\log\left(cx^n\right)\right)}{x}\,dx &= \frac{\text{Subst}\left(\int \coth^3(a+bx)\,dx,x,\log\left(cx^n\right)\right)}{n}\\ &= -\frac{\coth^2\left(a+b\log\left(cx^n\right)\right)}{2bn}+\frac{\text{Subst}\left(\int \coth(a+bx)\,dx,x,\log\left(cx^n\right)\right)}{n}\\ &= -\frac{\coth^2\left(a+b\log\left(cx^n\right)\right)}{2bn}+\frac{\log\left(\sinh\left(a+b\log\left(cx^n\right)\right)\right)}{bn}\end{aligned}$$

Mathematica [A]
time = 0.16, size = 52, normalized size = 1.21

$$-\frac{\coth^2\left(a+b\log\left(cx^n\right)\right)-2\log\left(\cosh\left(a+b\log\left(cx^n\right)\right)\right)-2\log\left(\tanh\left(a+b\log\left(cx^n\right)\right)\right)}{2bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] -1/2*(Coth[a + b*Log[c*x^n]]^2 - 2*Log[Cosh[a + b*Log[c*x^n]]] - 2*Log[Tanh
[a + b*Log[c*x^n]]])/(b*n)
```

Maple [A]
time = 2.38, size = 56, normalized size = 1.30

method	result
derivativedivides	$\frac{-\frac{\left(\coth^{2}\left(a+b\ln\left(c\,x^{n}\right)\right)\right)}{2}-\frac{\ln\left(\coth\left(a+b\ln\left(c\,x^{n}\right)\right)-1\right)}{2}-\frac{\ln\left(\coth\left(a+b\ln\left(c\,x^{n}\right)\right)+1\right)}{2}}{nb}$
default	$\frac{-\frac{\left(\coth^{2}\left(a+b\ln\left(c\,x^{n}\right)\right)\right)}{2}-\frac{\ln\left(\coth\left(a+b\ln\left(c\,x^{n}\right)\right)-1\right)}{2}-\frac{\ln\left(\coth\left(a+b\ln\left(c\,x^{n}\right)\right)+1\right)}{2}}{nb}$
risch	$\ln\left(x\right)-\frac{2a}{nb}-\frac{2\ln(c)}{n}-\frac{2\ln(x^{n})}{n}+\frac{i\pi\mathrm{csgn}(ic\,x^{n})^{3}}{n}-\frac{i\pi\mathrm{csgn}(ic\,x^{n})^{2}\mathrm{csgn}(ic)}{n}-\frac{i\pi\mathrm{csgn}(ic\,x^{n})^{2}\mathrm{csgn}(ix^{n})}{n}+i\pi$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+b*ln(c*x^n))^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b*(-1/2*coth(a+b*ln(c*x^n))^2-1/2*ln(coth(a+b*ln(c*x^n))-1)-1/2*ln(coth
(a+b*ln(c*x^n))+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 330 vs. 2(41) = 82.
time = 0.33, size = 330, normalized size = 7.67

$$-\frac{4\,c^{2\,b}e^{\left(2\,b\log\left(x^{n}\right)+2\,a\right)}-3}{4\,\left(bc^{4\,b}ne^{\left(4\,b\log\left(x^{n}\right)+4\,a\right)}-2\,bc^{2\,b}ne^{\left(2\,b\log\left(x^{n}\right)+2\,a\right)}+bn\right)}-\frac{3\,\left(2\,c^{2\,b}e^{\left(2\,b\log\left(x^{n}\right)+2\,a\right)}-1\right)}{4\,\left(bc^{4\,b}ne^{\left(4\,b\log\left(x^{n}\right)+4\,a\right)}-2\,bc^{2\,b}ne^{\left(2\,b\log\left(x^{n}\right)+2\,a\right)}+bn\right)}+\frac{2\,c^{2\,b}e^{\left(2\,b\log\left(x^{n}\right)+2\,a\right)}-3}{4\,\left(bc^{4\,b}ne^{\left(4\,b\log\left(x^{n}\right)+4\,a\right)}-2\,bc^{2\,b}ne^{\left(2\,b\log\left(x^{n}\right)+2\,a\right)}+bn\right)}-\frac{3}{4\,\left(bc^{4\,b}ne^{\left(4\,b\log\left(x^{n}\right)+4\,a\right)}-2\,bc^{2\,b}ne^{\left(2\,b\log\left(x^{n}\right)+2\,a\right)}+bn\right)}+\frac{\log\left(\frac{\left(c^{b}e^{\left(b\log\left(x^{n}\right)+a\right)}+1\right)e^{\left(-a\right)}}{c^{b}}\right)}{bn}+\frac{\log\left(\frac{\left(c^{b}e^{\left(b\log\left(x^{n}\right)+a\right)}-1\right)e^{\left(-a\right)}}{c^{b}}\right)}{bn}-\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^3/x,x, algorithm="maxima")
```

```
[Out] -1/4*(4*c^(2*b)*e^(2*b*log(x^n) + 2*a) - 3)/(b*c^(4*b)*n*e^(4*b*log(x^n) +
4*a) - 2*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n) - 3/4*(2*c^(2*b)*e^(2*b*
log(x^n) + 2*a) - 1)/(b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) - 2*b*c^(2*b)*n*e^
(2*b*log(x^n) + 2*a) + b*n) + 1/4*(2*c^(2*b)*e^(2*b*log(x^n) + 2*a) - 3)/(b
*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) - 2*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) +
b*n) - 3/4/(b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) - 2*b*c^(2*b)*n*e^(2*b*log(x
^n) + 2*a) + b*n) + log((c^b*e^(b*log(x^n) + a) + 1)*e^(-a)/c^b)/(b*n) + lo
g((c^b*e^(b*log(x^n) + a) - 1)*e^(-a)/c^b)/(b*n) - log(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 572 vs. 2(41) = 82.
time = 0.35, size = 572, normalized size = 13.30

$$-\frac{bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{4}\log\left(x\right)+4\,bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\log\left(x\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}+bn\log\left(x\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{4}-2\,\left(bn\log\left(x\right)-1\right)\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+bn\log\left(x\right)+2\,\left(3\,bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}\log\left(x\right)-bn\log\left(x\right)+1\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}-\left(\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{4}+4\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{4}+2\,\left(3\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}-1\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}-2\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+4\,\left(\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}-\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+1\right)\log\left(\frac{2\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}\right)+4\,\left(bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}\log\left(x\right)-\left(bn\log\left(x\right)-1\right)\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{4}+4\,bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}+bn\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{4}-2\,bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2\,\left(3\,bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}-bn\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+bn+4\,\left(bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}-bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^3/x,x, algorithm="fricas")
```

```
[Out] -(b*n*cosh(b*n*log(x) + b*log(c) + a)^4*log(x) + 4*b*n*cosh(b*n*log(x) + b*
log(c) + a)*log(x)*sinh(b*n*log(x) + b*log(c) + a)^3 + b*n*log(x)*sinh(b*n*
log(x) + b*log(c) + a)^4 - 2*(b*n*log(x) - 1)*cosh(b*n*log(x) + b*log(c) +
a)^2 + b*n*log(x) + 2*(3*b*n*cosh(b*n*log(x) + b*log(c) + a)^2*log(x) - b*n
*log(x) + 1)*sinh(b*n*log(x) + b*log(c) + a)^2 - (cosh(b*n*log(x) + b*log(c
) + a)^4 + 4*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a
)^3 + sinh(b*n*log(x) + b*log(c) + a)^4 + 2*(3*cosh(b*n*log(x) + b*log(c) +
 a)^2 - 1)*sinh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(c)
 + a)^2 + 4*(cosh(b*n*log(x) + b*log(c) + a)^3 - cosh(b*n*log(x) + b*log(c)
 + a))*sinh(b*n*log(x) + b*log(c) + a) + 1)*log(2*sinh(b*n*log(x) + b*log(c
) + a)/(cosh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)))
 + 4*(b*n*cosh(b*n*log(x) + b*log(c) + a)^3*log(x) - (b*n*log(x) - 1)*cosh(
b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a))/(b*n*cosh(b*n*
log(x) + b*log(c) + a)^4 + 4*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*l
og(x) + b*log(c) + a)^3 + b*n*sinh(b*n*log(x) + b*log(c) + a)^4 - 2*b*n*cos
h(b*n*log(x) + b*log(c) + a)^2 + 2*(3*b*n*cosh(b*n*log(x) + b*log(c) + a)^2
 - b*n)*sinh(b*n*log(x) + b*log(c) + a)^2 + b*n + 4*(b*n*cosh(b*n*log(x) +
b*log(c) + a)^3 - b*n*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*
log(c) + a))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*ln(c*x**n))**3/x,x)
```

```
[Out] Exception raised: TypeError >> Invalid NaN comparison
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(41) = 82.
time = 0.47, size = 127, normalized size = 2.95

$$\frac{\log\left(\sqrt{-2\,x^{2\,bn}|c|^{2\,b}\cos\left(\pi b\mathrm{sgn}\left(c\right)-\pi b\right)e^{\left(2\,a\right)}+x^{4\,bn}|c|^{4\,b}e^{\left(4\,a\right)}+1}\right)}{bn}-\frac{3\,c^{4\,b}x^{4\,bn}e^{\left(4\,a\right)}-2\,c^{2\,b}x^{2\,bn}e^{\left(2\,a\right)}+3}{2\,\left(c^{2\,b}x^{2\,bn}e^{\left(2\,a\right)}-1\right)^{2}bn}-\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^3/x,x, algorithm="giac")
```

```
[Out] log(sqrt(-2*x^(2*b*n)*abs(c)^(2*b)*cos(pi*b*sgn(c) - pi*b)*e^(2*a) + x^(4*b
*n)*abs(c)^(4*b)*e^(4*a) + 1))/(b*n) - 1/2*(3*c^(4*b)*x^(4*b*n)*e^(4*a) - 2
```

```
*c^(2*b)*x^(2*b*n)*e^(2*a) + 3)/((c^(2*b)*x^(2*b*n)*e^(2*a) - 1)^2*b*n) - l
og(x)
```

Mupad [B]
time = 1.23, size = 95, normalized size = 2.21

$$\frac{2}{b\,n-b\,n\,\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}}-\ln\left(x\right)-\frac{2}{b\,n-2\,b\,n\,\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}+b\,n\,\mathrm{e}^{4\,a}\,{\left(c\,x^n\right)}^{4\,b}}+\frac{\ln\left(\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}-1\right)}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + b*log(c*x^n))^3/x,x)
```

```
[Out] 2/(b*n - b*n*exp(2*a)*(c*x^n)^(2*b)) - log(x) - 2/(b*n - 2*b*n*exp(2*a)*(c*
x^n)^(2*b) + b*n*exp(4*a)*(c*x^n)^(4*b)) + log(exp(2*a)*(c*x^n)^(2*b) - 1)/
(b*n)
```

3.192 $\int \frac{\coth^4(a+b\log(cx^n))}{x} dx$

Optimal. Leaf size=45

$$-\frac{\coth(a+b\log(cx^n))}{bn}-\frac{\coth^3(a+b\log(cx^n))}{3bn}+\log(x)$$

```
[Out] -coth(a+b*ln(c*x^n))/b/n-1/3*coth(a+b*ln(c*x^n))^3/b/n+ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3554, 8}

$$-\frac{\coth^3(a+b\log(cx^n))}{3bn}-\frac{\coth(a+b\log(cx^n))}{bn}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] -(Coth[a + b*Log[c*x^n]]/(b*n)) - Coth[a + b*Log[c*x^n]]^3/(3*b*n) + Log[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^4(a+b\log(cx^n))}{x} dx &= \frac{\text{Subst}\left(\int \coth^4(a+bx)\,dx, x, \log(cx^n)\right)}{n}\\
&= -\frac{\coth^3(a+b\log(cx^n))}{3bn}+\frac{\text{Subst}\left(\int \coth^2(a+bx)\,dx, x, \log(cx^n)\right)}{n}\\
&= -\frac{\coth(a+b\log(cx^n))}{bn}-\frac{\coth^3(a+b\log(cx^n))}{3bn}+\frac{\text{Subst}\left(\int 1\,dx, x, \log(cx^n)\right)}{n}\\
&= -\frac{\coth(a+b\log(cx^n))}{bn}-\frac{\coth^3(a+b\log(cx^n))}{3bn}+\log(x)
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.08, size = 44, normalized size = 0.98

$$-\frac{\coth^3(a+b\log(cx^n))\ {}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\tanh^2(a+b\log(cx^n))\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] -1/3*(Coth[a + b*Log[c*x^n]]^3*Hypergeometric2F1[-3/2, 1, -1/2, Tanh[a + b*
Log[c*x^n]]^2])/(b*n)
```

Maple [A]
time = 2.36, size = 69, normalized size = 1.53

method	result
derivativedivides	$\frac{-\frac{\left(\coth^3(a+b\ln(c\,x^n))\right)}{3}-\coth(a+b\ln(c\,x^n))-\frac{\ln(\coth(a+b\ln(c\,x^n))-1)}{2}+\frac{\ln(\coth(a+b\ln(c\,x^n))+1)}{2}}{nb}$
default	$\frac{-\frac{\left(\coth^3(a+b\ln(c\,x^n))\right)}{3}-\coth(a+b\ln(c\,x^n))-\frac{\ln(\coth(a+b\ln(c\,x^n))-1)}{2}+\frac{\ln(\coth(a+b\ln(c\,x^n))+1)}{2}}{nb}$
risch	$\ln(x)-\frac{4\left(3(x^n)^{4b}c^{4b}e^{4a}e^{-2ib\pi\mathrm{csgn}(ic\,x^n)^3}e^{2ib\pi\mathrm{csgn}(ic\,x^n)^2\mathrm{csgn}(ic)}e^{2ib\pi\mathrm{csgn}(ic\,x^n)^2\mathrm{csgn}(ix^n)}e^{-2ib\pi\,\mathrm{csgn}(ic\,x^n)\mathrm{csgn}(ic)c\cdots}\right.}{3bn\left((x^n)^{2b}c^{2b}e^{2a}e^{-ib\pi\mathrm{csgn}(ic\,x^n)^3}e^{ib\pi\mathrm{csgn}(ic\,x^n)^2\mathrm{cs}\cdots}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+b*ln(c*x^n))^4/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b*(-1/3*coth(a+b*ln(c*x^n))^3-coth(a+b*ln(c*x^n))-1/2*ln(coth(a+b*ln(c*
x^n))-1)+1/2*ln(coth(a+b*ln(c*x^n))+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 499 vs. 2(43) = 86.
time = 0.35, size = 499, normalized size = 11.09

$$-\frac{18c^{4b}e^{(4b\log(x^n)+4a)}-27c^{2b}e^{(2b\log(x^n)+2a)}+11}{12\left(bc^{6b}ne^{(6b\log(x^n)+6a)}-3bc^{4b}ne^{(4b\log(x^n)+4a)}+3bc^{2b}ne^{(2b\log(x^n)+2a)}-bn\right)}-\frac{6c^{4b}e^{(4b\log(x^n)+4a)}-15c^{2b}e^{(2b\log(x^n)+2a)}+11}{12\left(bc^{6b}ne^{(6b\log(x^n)+6a)}-3bc^{4b}ne^{(4b\log(x^n)+4a)}+3bc^{2b}ne^{(2b\log(x^n)+2a)}-bn\right)}-\frac{2\left(3c^{4b}e^{(4b\log(x^n)+4a)}-3c^{2b}e^{(2b\log(x^n)+2a)}+1\right)}{3\left(bc^{6b}ne^{(6b\log(x^n)+6a)}-3bc^{4b}ne^{(4b\log(x^n)+4a)}+3bc^{2b}ne^{(2b\log(x^n)+2a)}-bn\right)}-\frac{3c^{2b}e^{(2b\log(x^n)+2a)}-1}{2\left(bc^{6b}ne^{(6b\log(x^n)+6a)}-3bc^{4b}ne^{(4b\log(x^n)+4a)}+3bc^{2b}ne^{(2b\log(x^n)+2a)}-bn\right)}-\frac{2}{3\left(bc^{6b}ne^{(6b\log(x^n)+6a)}-3bc^{4b}ne^{(4b\log(x^n)+4a)}+3bc^{2b}ne^{(2b\log(x^n)+2a)}-bn\right)}+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^4/x,x, algorithm="maxima")
```

```
[Out] -1/12*(18*c^(4*b)*e^(4*b*log(x^n) + 4*a) - 27*c^(2*b)*e^(2*b*log(x^n) + 2*a
) + 11)/(b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a) - 3*b*c^(4*b)*n*e^(4*b*log(x^n)
 + 4*a) + 3*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) - b*n) - 1/12*(6*c^(4*b)*e^(
4*b*log(x^n) + 4*a) - 15*c^(2*b)*e^(2*b*log(x^n) + 2*a) + 11)/(b*c^(6*b)*n*
e^(6*b*log(x^n) + 6*a) - 3*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 3*b*c^(2*b)
*n*e^(2*b*log(x^n) + 2*a) - b*n) - 2/3*(3*c^(4*b)*e^(4*b*log(x^n) + 4*a) -
```

```
3*c^(2*b)*e^(2*b*log(x^n) + 2*a) + 1)/(b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a) -
 3*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 3*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a
) - b*n) - 1/2*(3*c^(2*b)*e^(2*b*log(x^n) + 2*a) - 1)/(b*c^(6*b)*n*e^(6*b*l
og(x^n) + 6*a) - 3*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 3*b*c^(2*b)*n*e^(2*
b*log(x^n) + 2*a) - b*n) - 2/3/(b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a) - 3*b*c^
(4*b)*n*e^(4*b*log(x^n) + 4*a) + 3*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) - b*n
) + log(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(43) = 86$.
time = 0.36, size = 171, normalized size = 3.80

$$\frac{(3\,bn\log(x)+4)\sinh(bn\log(x)+b\log(c)+a)^3-4\,\cosh(bn\log(x)+b\log(c)+a)^3-12\,\cosh(bn\log(x)+b\log(c)+a)\sinh(bn\log(x)+b\log(c)+a)^2+3\left((3\,bn\log(x)+4)\cosh(bn\log(x)+b\log(c)+a)^2-3\,bn\log(x)-4\right)\sinh(bn\log(x)+b\log(c)+a)}{3\left(bn\sinh(bn\log(x)+b\log(c)+a)^3+3\left(bn\cosh(bn\log(x)+b\log(c)+a)^2-bn\right)\sinh(bn\log(x)+b\log(c)+a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^4/x,x, algorithm="fricas")
```

```
[Out] 1/3*((3*b*n*log(x) + 4)*sinh(b*n*log(x) + b*log(c) + a)^3 - 4*cosh(b*n*log(
x) + b*log(c) + a)^3 - 12*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) +
 b*log(c) + a)^2 + 3*((3*b*n*log(x) + 4)*cosh(b*n*log(x) + b*log(c) + a)^2
- 3*b*n*log(x) - 4)*sinh(b*n*log(x) + b*log(c) + a))/(b*n*sinh(b*n*log(x) +
 b*log(c) + a)^3 + 3*(b*n*cosh(b*n*log(x) + b*log(c) + a)^2 - b*n)*sinh(b*n
*log(x) + b*log(c) + a))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*ln(c*x**n))**4/x,x)
```

```
[Out] Exception raised: TypeError >> Invalid NaN comparison
```

Giac [A]
time = 0.50, size = 67, normalized size = 1.49

$$-\frac{4\left(3\,c^{4\,b}x^{4\,bn}e^{(4\,a)}-3\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+2\right)}{3\left(c^{2\,b}x^{2\,bn}e^{(2\,a)}-1\right)^3bn}+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^4/x,x, algorithm="giac")
```

```
[Out] -4/3*(3*c^(4*b)*x^(4*b*n)*e^(4*a) - 3*c^(2*b)*x^(2*b*n)*e^(2*a) + 2)/((c^(2
*b)*x^(2*b*n)*e^(2*a) - 1)^3*b*n) + log(x)
```

Mupad [B]
time = 1.21, size = 163, normalized size = 3.62

$$\ln(x)-\frac{\frac{4}{3\,b\,n}+\frac{4\,{\mathrm{e}}^{4\,a}\,{\left(c\,x^n\right)}^{4\,b}}{3\,b\,n}}{3\,{\mathrm{e}}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}-3\,{\mathrm{e}}^{4\,a}\,{\left(c\,x^n\right)}^{4\,b}+{\mathrm{e}}^{6\,a}\,{\left(c\,x^n\right)}^{6\,b}-1}-\frac{4}{3\,b\,n\,\left({\mathrm{e}}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}-1\right)}-\frac{4\,{\mathrm{e}}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}}{3\,b\,n\,\left({\mathrm{e}}^{4\,a}\,{\left(c\,x^n\right)}^{4\,b}-2\,{\mathrm{e}}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + b*log(c*x^n))^4/x,x)
```

```
[Out] log(x) - (4/(3*b*n) + (4*exp(4*a)*(c*x^n)^(4*b))/(3*b*n))/(3*exp(2*a)*(c*x^
n)^(2*b) - 3*exp(4*a)*(c*x^n)^(4*b) + exp(6*a)*(c*x^n)^(6*b) - 1) - 4/(3*b*
n*(exp(2*a)*(c*x^n)^(2*b) - 1)) - (4*exp(2*a)*(c*x^n)^(2*b))/(3*b*n*(exp(4*
a)*(c*x^n)^(4*b) - 2*exp(2*a)*(c*x^n)^(2*b) + 1))
```

3.193 $\int \frac{\coth^5(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=66

$$-\frac{\coth^2(a+b\log(cx^n))}{2bn}-\frac{\coth^4(a+b\log(cx^n))}{4bn}+\frac{\log(\sinh(a+b\log(cx^n)))}{bn}$$

```
[Out] -1/2*coth(a+b*ln(c*x^n))^2/b/n-1/4*coth(a+b*ln(c*x^n))^4/b/n+ln(sinh(a+b*ln
(c*x^n)))/b/n
```

Rubi [A]
time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3554, 3556}

$$\frac{\log(\sinh(a+b\log(cx^n)))}{bn}-\frac{\coth^4(a+b\log(cx^n))}{4bn}-\frac{\coth^2(a+b\log(cx^n))}{2bn}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*Log[c*x^n]]^5/x,x]
```

```
[Out] -1/2*Coth[a + b*Log[c*x^n]]^2/(b*n) - Coth[a + b*Log[c*x^n]]^4/(4*b*n) + Lo
g[Sinh[a + b*Log[c*x^n]]]/(b*n)
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^5(a+b\log(cx^n))}{x}\,dx &= \frac{\text{Subst}\left(\int \coth^5(a+bx)\,dx, x, \log(cx^n)\right)}{n}\\
&= -\frac{\coth^4(a+b\log(cx^n))}{4bn}+\frac{\text{Subst}\left(\int \coth^3(a+bx)\,dx, x, \log(cx^n)\right)}{n}\\
&= -\frac{\coth^2(a+b\log(cx^n))}{2bn}-\frac{\coth^4(a+b\log(cx^n))}{4bn}+\frac{\text{Subst}\left(\int \coth(a+bx)\,d\right.}{n}\\
&= -\frac{\coth^2(a+b\log(cx^n))}{2bn}-\frac{\coth^4(a+b\log(cx^n))}{4bn}+\frac{\log(\sinh(a+b\log(cx^n}{bn}
\end{aligned}$$

Mathematica [A]
time = 0.20, size = 67, normalized size = 1.02

$$-\frac{2\coth^2\left(a+b\log\left(cx^n\right)\right)+\coth^4\left(a+b\log\left(cx^n\right)\right)-4\log\left(\cosh\left(a+b\log\left(cx^n\right)\right)\right)-4\log\left(\tanh\left(a+b\log\left(cx^n\right)\right)\right)}{4bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*Log[c*x^n]]^5/x,x]
```

```
[Out] -1/4*(2*Coth[a + b*Log[c*x^n]]^2 + Coth[a + b*Log[c*x^n]]^4 - 4*Log[Cosh[a
+ b*Log[c*x^n]]] - 4*Log[Tanh[a + b*Log[c*x^n]]])/(b*n)
```

Maple [A]
time = 2.47, size = 71, normalized size = 1.08

method	result
derivativedivides	$\frac{-\frac{\left(\coth^4(a+b\ln(c\,x^n))\right)}{4}-\frac{\left(\coth^2(a+b\ln(c\,x^n))\right)}{2}-\frac{\ln(\coth(a+b\ln(c\,x^n))-1)}{2}-\frac{\ln(\coth(a+b\ln(c\,x^n))+1)}{2}}{nb}$
default	$\frac{-\frac{\left(\coth^4(a+b\ln(c\,x^n))\right)}{4}-\frac{\left(\coth^2(a+b\ln(c\,x^n))\right)}{2}-\frac{\ln(\coth(a+b\ln(c\,x^n))-1)}{2}-\frac{\ln(\coth(a+b\ln(c\,x^n))+1)}{2}}{nb}$
risch	$\ln(x)-\frac{2a}{nb}-\frac{2\ln(c)}{n}-\frac{2\ln(x^n)}{n}+\frac{i\pi\mathrm{csgn}(ic\,x^n)^3}{n}-\frac{i\pi\mathrm{csgn}(ic\,x^n)^2\mathrm{csgn}(ic)}{n}-\frac{i\pi\mathrm{csgn}(ic\,x^n)^2\mathrm{csgn}(ix^n)}{n}+i\pi$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+b*ln(c*x^n))^5/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b*(-1/4*coth(a+b*ln(c*x^n))^4-1/2*coth(a+b*ln(c*x^n))^2-1/2*ln(coth(a+b
*ln(c*x^n))-1)-1/2*ln(coth(a+b*ln(c*x^n))+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 855 vs. 2(62) = 124.
time = 0.40, size = 855, normalized size = 12.95

$$-\frac{48\,c^{6b}e^{(6b\log(x^n)+6a)}-108\,c^{4b}e^{(4b\log(x^n)+4a)}+88\,c^{2b}e^{(2b\log(x^n)+2a)}-25}{24\left(bc^{8b}ne^{(8b\log(x^n)+8a)}-4\,bc^{6b}ne^{(6b\log(x^n)+6a)}+6\,bc^{4b}ne^{(4b\log(x^n)+4a)}-4\,bc^{2b}ne^{(2b\log(x^n)+2a)}+bn\right)}+\frac{12\,c^{6b}e^{(6b\log(x^n)+6a)}-42\,c^{4b}e^{(4b\log(x^n)+4a)}+52\,c^{2b}e^{(2b\log(x^n)+2a)}-25}{24\left(bc^{8b}ne^{(8b\log(x^n)+8a)}-4\,bc^{6b}ne^{(6b\log(x^n)+6a)}+6\,bc^{4b}ne^{(4b\log(x^n)+4a)}-4\,bc^{2b}ne^{(2b\log(x^n)+2a)}+bn\right)}-\frac{5\left(4\,c^{6b}e^{(6b\log(x^n)+6a)}-6\,c^{4b}e^{(4b\log(x^n)+4a)}+4\,c^{2b}e^{(2b\log(x^n)+2a)}-1\right)}{8\left(bc^{8b}ne^{(8b\log(x^n)+8a)}-4\,bc^{6b}ne^{(6b\log(x^n)+6a)}+6\,bc^{4b}ne^{(4b\log(x^n)+4a)}-4\,bc^{2b}ne^{(2b\log(x^n)+2a)}+bn\right)}-\frac{5\left(6\,c^{4b}e^{(4b\log(x^n)+4a)}-4\,c^{2b}e^{(2b\log(x^n)+2a)}+1\right)}{12\left(bc^{8b}ne^{(8b\log(x^n)+8a)}-4\,bc^{6b}ne^{(6b\log(x^n)+6a)}+6\,bc^{4b}ne^{(4b\log(x^n)+4a)}-4\,bc^{2b}ne^{(2b\log(x^n)+2a)}+bn\right)}-\frac{5\left(4\,c^{2b}e^{(2b\log(x^n)+2a)}-1\right)}{12\left(bc^{8b}ne^{(8b\log(x^n)+8a)}-4\,bc^{6b}ne^{(6b\log(x^n)+6a)}+6\,bc^{4b}ne^{(4b\log(x^n)+4a)}-4\,bc^{2b}ne^{(2b\log(x^n)+2a)}+bn\right)}-\frac{5}{8\left(bc^{8b}ne^{(8b\log(x^n)+8a)}-4\,bc^{6b}ne^{(6b\log(x^n)+6a)}+6\,bc^{4b}ne^{(4b\log(x^n)+4a)}-4\,bc^{2b}ne^{(2b\log(x^n)+2a)}+bn\right)}+\frac{\log\left(\frac{\left(c^be^{(b\log(x^n)+a)}+1\right)e^{(-a)}}{c^b}\right)}{bn}+\frac{\log\left(\frac{\left(c^be^{(b\log(x^n)+a)}-1\right)e^{(-a)}}{c^b}\right)}{bn}-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^5/x,x, algorithm="maxima")
```

```
[Out] -1/24*(48*c^(6*b)*e^(6*b*log(x^n) + 6*a) - 108*c^(4*b)*e^(4*b*log(x^n) + 4*
a) + 88*c^(2*b)*e^(2*b*log(x^n) + 2*a) - 25)/(b*c^(8*b)*n*e^(8*b*log(x^n) +
 8*a) - 4*b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a) + 6*b*c^(4*b)*n*e^(4*b*log(x^n
) + 4*a) - 4*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n) + 1/24*(12*c^(6*b)*e
^(6*b*log(x^n) + 6*a) - 42*c^(4*b)*e^(4*b*log(x^n) + 4*a) + 52*c^(2*b)*e^(2
*b*log(x^n) + 2*a) - 25)/(b*c^(8*b)*n*e^(8*b*log(x^n) + 8*a) - 4*b*c^(6*b)*
n*e^(6*b*log(x^n) + 6*a) + 6*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) - 4*b*c^(2*
```

```
b)*n*e^(2*b*log(x^n) + 2*a) + b*n) - 5/8*(4*c^(6*b)*e^(6*b*log(x^n) + 6*a)
- 6*c^(4*b)*e^(4*b*log(x^n) + 4*a) + 4*c^(2*b)*e^(2*b*log(x^n) + 2*a) - 1)/
(b*c^(8*b)*n*e^(8*b*log(x^n) + 8*a) - 4*b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a)
+ 6*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) - 4*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*
a) + b*n) - 5/12*(6*c^(4*b)*e^(4*b*log(x^n) + 4*a) - 4*c^(2*b)*e^(2*b*log(x
^n) + 2*a) + 1)/(b*c^(8*b)*n*e^(8*b*log(x^n) + 8*a) - 4*b*c^(6*b)*n*e^(6*b*
log(x^n) + 6*a) + 6*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) - 4*b*c^(2*b)*n*e^(2
*b*log(x^n) + 2*a) + b*n) - 5/12*(4*c^(2*b)*e^(2*b*log(x^n) + 2*a) - 1)/(b*
c^(8*b)*n*e^(8*b*log(x^n) + 8*a) - 4*b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a) + 6
*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) - 4*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a)
+ b*n) - 5/8/(b*c^(8*b)*n*e^(8*b*log(x^n) + 8*a) - 4*b*c^(6*b)*n*e^(6*b*log
(x^n) + 6*a) + 6*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) - 4*b*c^(2*b)*n*e^(2*b*
log(x^n) + 2*a) + b*n) + log((c^b*e^(b*log(x^n) + a) + 1)*e^(-a)/c^b)/(b*n)
 + log((c^b*e^(b*log(x^n) + a) - 1)*e^(-a)/c^b)/(b*n) - log(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1576 vs. $2(62) = 124$.
time = 0.36, size = 1576, normalized size = 23.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^5/x,x, algorithm="fricas")
```

```
[Out] -(b*n*cosh(b*n*log(x) + b*log(c) + a)^8*log(x) + 8*b*n*cosh(b*n*log(x) + b*
log(c) + a)*log(x)*sinh(b*n*log(x) + b*log(c) + a)^7 + b*n*log(x)*sinh(b*n*
log(x) + b*log(c) + a)^8 - 4*(b*n*log(x) - 1)*cosh(b*n*log(x) + b*log(c) +
a)^6 + 4*(7*b*n*cosh(b*n*log(x) + b*log(c) + a)^2*log(x) - b*n*log(x) + 1)*
sinh(b*n*log(x) + b*log(c) + a)^6 + 8*(7*b*n*cosh(b*n*log(x) + b*log(c) + a
)^3*log(x) - 3*(b*n*log(x) - 1)*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*l
og(x) + b*log(c) + a)^5 + 2*(3*b*n*log(x) - 2)*cosh(b*n*log(x) + b*log(c) +
 a)^4 + 2*(35*b*n*cosh(b*n*log(x) + b*log(c) + a)^4*log(x) - 30*(b*n*log(x)
 - 1)*cosh(b*n*log(x) + b*log(c) + a)^2 + 3*b*n*log(x) - 2)*sinh(b*n*log(x)
 + b*log(c) + a)^4 + 8*(7*b*n*cosh(b*n*log(x) + b*log(c) + a)^5*log(x) - 10
*(b*n*log(x) - 1)*cosh(b*n*log(x) + b*log(c) + a)^3 + (3*b*n*log(x) - 2)*co
sh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^3 - 4*(b*n*l
og(x) - 1)*cosh(b*n*log(x) + b*log(c) + a)^2 + b*n*log(x) + 4*(7*b*n*cosh(b
*n*log(x) + b*log(c) + a)^6*log(x) - 15*(b*n*log(x) - 1)*cosh(b*n*log(x) +
b*log(c) + a)^4 + 3*(3*b*n*log(x) - 2)*cosh(b*n*log(x) + b*log(c) + a)^2 -
b*n*log(x) + 1)*sinh(b*n*log(x) + b*log(c) + a)^2 - (cosh(b*n*log(x) + b*lo
g(c) + a)^8 + 8*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c)
+ a)^7 + sinh(b*n*log(x) + b*log(c) + a)^8 + 4*(7*cosh(b*n*log(x) + b*log(c
) + a)^2 - 1)*sinh(b*n*log(x) + b*log(c) + a)^6 - 4*cosh(b*n*log(x) + b*log
(c) + a)^6 + 8*(7*cosh(b*n*log(x) + b*log(c) + a)^3 - 3*cosh(b*n*log(x) + b
*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^5 + 2*(35*cosh(b*n*log(x) + b
```

```
*log(c) + a)^4 - 30*cosh(b*n*log(x) + b*log(c) + a)^2 + 3)*sinh(b*n*log(x)
+ b*log(c) + a)^4 + 6*cosh(b*n*log(x) + b*log(c) + a)^4 + 8*(7*cosh(b*n*log
(x) + b*log(c) + a)^5 - 10*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*cosh(b*n*l
og(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^3 + 4*(7*cosh(b*n*lo
g(x) + b*log(c) + a)^6 - 15*cosh(b*n*log(x) + b*log(c) + a)^4 + 9*cosh(b*n*
log(x) + b*log(c) + a)^2 - 1)*sinh(b*n*log(x) + b*log(c) + a)^2 - 4*cosh(b*
n*log(x) + b*log(c) + a)^2 + 8*(cosh(b*n*log(x) + b*log(c) + a)^7 - 3*cosh(
b*n*log(x) + b*log(c) + a)^5 + 3*cosh(b*n*log(x) + b*log(c) + a)^3 - cosh(b
*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a) + 1)*log(2*sinh(
b*n*log(x) + b*log(c) + a)/(cosh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(
x) + b*log(c) + a))) + 8*(b*n*cosh(b*n*log(x) + b*log(c) + a)^7*log(x) - 3*
(b*n*log(x) - 1)*cosh(b*n*log(x) + b*log(c) + a)^5 + (3*b*n*log(x) - 2)*cos
h(b*n*log(x) + b*log(c) + a)^3 - (b*n*log(x) - 1)*cosh(b*n*log(x) + b*log(c
) + a))*sinh(b*n*log(x) + b*log(c) + a))/(b*n*cosh(b*n*log(x) + b*log(c) +
a)^8 + 8*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a
)^7 + b*n*sinh(b*n*log(x) + b*log(c) + a)^8 - 4*b*n*cosh(b*n*log(x) + b*log
(c) + a)^6 + 4*(7*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 - b*n)*sinh(b*n*log
(x) + b*log(c) + a)^6 + 6*b*n*cosh(b*n*log(x) + b*log(c) + a)^4 + 8*(7*b*n*
cosh(b*n*log(x) + b*log(c) + a)^3 - 3*b*n*cosh(b*n*log(x) + b*log(c) + a))*
sinh(b*n*log(x) + b*log(c) + a)^5 + 2*(35*b*n*cosh(b*n*log(x) + b*log(c) +
a)^4 - 30*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 3*b*n)*sinh(b*n*log(x) +
b*log(c) + a)^4 - 4*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 8*(7*b*n*cosh(b
*n*log(x) + b*log(c) + a)^5 - 10*b*n*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*
b*n*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^3 + 4*
(7*b*n*cosh(b*n*log(x) + b*log(c) + a)^6 - 15*b*n*cosh(b*n*log(x) + b*log(c
) + a)^4 + 9*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 - b*n)*sinh(b*n*log(x) +
 b*log(c) + a)^2 + b*n + 8*(b*n*cosh(b*n*log(x) + b*log(c) + a)^7 - 3*b*n*c
osh(b*n*log(x) + b*log(c) + a)^5 + 3*b*n*cosh(b*n*log(x) + b*log(c) + a)^3
- b*n*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*ln(c*x**n))**5/x,x)
```

```
[Out] Exception raised: TypeError >> Invalid NaN comparison
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 161 vs. 2(62) = 124.
time = 0.49, size = 161, normalized size = 2.44

$$\frac{\log\left(\sqrt{-2\,x^{2\,bn}|c|^{2\,b}\cos\left(\pi b\mathrm{sgn}\left(c\right)-\pi b\right)e^{\left(2\,a\right)}+x^{4\,bn}|c|^{4\,b}e^{\left(4\,a\right)}+1}\right)}{bn}-\frac{25\,c^{8\,b}x^{8\,bn}e^{\left(8\,a\right)}-52\,c^{6\,b}x^{6\,bn}e^{\left(6\,a\right)}+102\,c^{4\,b}x^{4\,bn}e^{\left(4\,a\right)}-52\,c^{2\,b}x^{2\,bn}e^{\left(2\,a\right)}+25}{12\,\left(c^{2\,b}x^{2\,bn}e^{\left(2\,a\right)}-1\right)^{4}bn}-\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^5/x,x, algorithm="giac")
```

```
[Out] log(sqrt(-2*x^(2*b*n)*abs(c)^(2*b)*cos(pi*b*sgn(c) - pi*b)*e^(2*a) + x^(4*b
*n)*abs(c)^(4*b)*e^(4*a) + 1))/(b*n) - 1/12*(25*c^(8*b)*x^(8*b*n)*e^(8*a) -
 52*c^(6*b)*x^(6*b*n)*e^(6*a) + 102*c^(4*b)*x^(4*b*n)*e^(4*a) - 52*c^(2*b)*
x^(2*b*n)*e^(2*a) + 25)/((c^(2*b)*x^(2*b*n)*e^(2*a) - 1)^4*b*n) - log(x)
```

Mupad [B]
time = 1.20, size = 229, normalized size = 3.47

$$\frac{8}{b\,n-3\,b\,n\,{\mathrm{e}}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}+3\,b\,n\,{\mathrm{e}}^{4\,a}\,{\left(c\,x^n\right)}^{4\,b}-b\,n\,{\mathrm{e}}^{6\,a}\,{\left(c\,x^n\right)}^{6\,b}}-\ln\left(x\right)+\frac{4}{b\,n-b\,n\,{\mathrm{e}}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}}-\frac{4}{b\,n-4\,b\,n\,{\mathrm{e}}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}+6\,b\,n\,{\mathrm{e}}^{4\,a}\,{\left(c\,x^n\right)}^{4\,b}-4\,b\,n\,{\mathrm{e}}^{6\,a}\,{\left(c\,x^n\right)}^{6\,b}+b\,n\,{\mathrm{e}}^{8\,a}\,{\left(c\,x^n\right)}^{8\,b}}-\frac{8}{b\,n-2\,b\,n\,{\mathrm{e}}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}+b\,n\,{\mathrm{e}}^{4\,a}\,{\left(c\,x^n\right)}^{4\,b}}+\frac{\ln\left({\mathrm{e}}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}-1\right)}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + b*log(c*x^n))^5/x,x)
```

```
[Out] 8/(b*n - 3*b*n*exp(2*a)*(c*x^n)^(2*b) + 3*b*n*exp(4*a)*(c*x^n)^(4*b) - b*n*
exp(6*a)*(c*x^n)^(6*b)) - log(x) + 4/(b*n - b*n*exp(2*a)*(c*x^n)^(2*b)) - 4
/(b*n - 4*b*n*exp(2*a)*(c*x^n)^(2*b) + 6*b*n*exp(4*a)*(c*x^n)^(4*b) - 4*b*n
*exp(6*a)*(c*x^n)^(6*b) + b*n*exp(8*a)*(c*x^n)^(8*b)) - 8/(b*n - 2*b*n*exp(
2*a)*(c*x^n)^(2*b) + b*n*exp(4*a)*(c*x^n)^(4*b)) + log(exp(2*a)*(c*x^n)^(2*
b) - 1)/(b*n)
```

3.194 $\int (ex)^m \coth (d(a+b\log (cx^n))) \, dx$

Optimal. Leaf size=87

$$\frac{(ex)^{1+m}}{e(1+m)}-\frac{2(ex)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2bdn};1+\frac{1+m}{2bdn};e^{2ad}(cx^n)^{2bd}\right)}{e(1+m)}$$

```
[Out] (e*x)^(1+m)/e/(1+m)-2*(e*x)^(1+m)*hypergeom([1, 1/2*(1+m)/b/d/n],[1+1/2*(1+
m)/b/d/n],exp(2*a*d)*(c*x^n)^(2*b*d))/e/(1+m)
```

Rubi [A]
time = 0.05, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {5659, 5657, 470, 371}

$$\frac{(ex)^{m+1}}{e(m+1)}-\frac{2(ex)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2bdn};\frac{m+1}{2bdn}+1;e^{2ad}(cx^n)^{2bd}\right)}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Coth[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (e*x)^(1 + m)/(e*(1 + m)) - (2*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/(
2*b*d*n), 1 + (1 + m)/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d)])/(e*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 5659

```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Coth[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int (ex)^m \coth (d(a+b\log (cx^n)))\ dx = \int (ex)^m \coth (d(a+b\log (cx^n)))\ dx$$

Mathematica [A]
time = 9.10, size = 158, normalized size = 1.82

$$\frac{x(ex)^m\left(-{}_2F_1\left(1,\frac{1+m}{2bdn};1+\frac{1+m}{2bdn};e^{2d(a+b\log(cx^n))}\right)-\frac{e^{2ad}(1+m)(cx^n)^{2bd}\,{}_2F_1\left(1,\frac{1+m+2bdn}{2bdn};\frac{1+m+4bdn}{2bdn};e^{2ad}(cx^n)^{2bd}\right)}{1+m+2bdn}\right)}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Coth[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (x*(e*x)^m*(-Hypergeometric2F1[1, (1 + m)/(2*b*d*n), 1 + (1 + m)/(2*b*d*n),
 E^(2*d*(a + b*Log[c*x^n]))] - (E^(2*a*d)*(1 + m)*(c*x^n)^(2*b*d)*Hypergeom
etric2F1[1, (1 + m + 2*b*d*n)/(2*b*d*n), (1 + m + 4*b*d*n)/(2*b*d*n), E^(2*
a*d)*(c*x^n)^(2*b*d)])/(1 + m + 2*b*d*n)))/(1 + m)
```

Maple [F]
time = 0.57, size = 0, normalized size = 0.00

$$\int (ex)^m \coth (d(a+b\ln (c\,x^n)))\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*coth(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int((e*x)^m*coth(d*(a+b*ln(c*x^n))),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] x*e^(m*log(x) + m)/(m + 1) - integrate(e^(m*log(x) + m)/(c^(b*d)*e^(b*d*log
(x^n) + a*d) + 1), x) + integrate(e^(m*log(x) + m)/(c^(b*d)*e^(b*d*log(x^n)
 + a*d) - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] integral((x*e)^m*coth(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \coth (ad + bd \log (cx^n))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*coth(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral((e*x)**m*coth(a*d + b*d*log(c*x**n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*coth((b*log(c*x^n) + a)*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{coth}(d\,(a + b\,\ln (c\,x^n)))\,(e\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a + b*log(c*x^n)))*(e*x)^m,x)
```

```
[Out] int(coth(d*(a + b*log(c*x^n)))*(e*x)^m, x)
```

3.195 $\int (ex)^m \coth^2(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=168

$$\frac{(1+m+bdn)(ex)^{1+m}}{bde(1+m)n}+\frac{(ex)^{1+m}\left(1+e^{2ad}(cx^n)^{2bd}\right)}{bden\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)}-\frac{2(ex)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2bdn};1+\frac{1+m}{2bdn};e^{2ad}(cx^n)^{2bd}\right)}{bden}$$

```
[Out] (b*d*n+m+1)*(e*x)^(1+m)/b/d/e/(1+m)/n+(e*x)^(1+m)*(1+exp(2*a*d)*(c*x^n)^(2*
b*d))/b/d/e/n/(1-exp(2*a*d)*(c*x^n)^(2*b*d))-2*(e*x)^(1+m)*hypergeom([1, 1/
2*(1+m)/b/d/n],[1+1/2*(1+m)/b/d/n],exp(2*a*d)*(c*x^n)^(2*b*d))/b/d/e/n
```

Rubi [A]
time = 0.15, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5659, 5657, 516, 470, 371}

$$-\frac{2(ex)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2bdn};\frac{m+1}{2bdn}+1;e^{2ad}(cx^n)^{2bd}\right)}{bden}+\frac{(ex)^{m+1}\left(e^{2ad}(cx^n)^{2bd}+1\right)}{bden\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)}+\frac{(ex)^{m+1}(bdn+m+1)}{bde(m+1)n}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Coth[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] ((1 + m + b*d*n)*(e*x)^(1 + m))/(b*d*e*(1 + m)*n) + ((e*x)^(1 + m)*(1 + E^(
2*a*d)*(c*x^n)^(2*b*d)))/(b*d*e*n*(1 - E^(2*a*d)*(c*x^n)^(2*b*d))) - (2*(e*
x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/(2*b*d*n), 1 + (1 + m)/(2*b*d*n), E
^(2*a*d)*(c*x^n)^(2*b*d)])/(b*d*e*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 516

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ
[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 5659

```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Coth[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int (ex)^m \coth^2(d(a+b\log(cx^n)))\,dx = \int (ex)^m \coth^2(d(a+b\log(cx^n)))\,dx$$

Mathematica [A]
time = 9.90, size = 312, normalized size = 1.86

$$(ex)^m\left(\frac{x}{1+m}-\frac{e^{-\frac{(1+2m)(a-bn\log(x)+b\log(cx^n))}{bn}}x^{-2m}\left(e^{\frac{(1+2m)(a+b\log(cx^n))}{bn}}(1+m+2bdn)\coth(d(a+b\log(cx^n)))+e^{\frac{(1+2m)(a+b\log(cx^n))}{bn}}(1+m+2bdn)\,{}_2F_1\left(1,\frac{1+m}{2bdn};1+\frac{1+m}{2bdn};e^{2d(a+b\log(cx^n))}\right)+e^{\frac{(1+2m+2bdn)(a-bn\log(x)+b\log(cx^n))}{bn}}(1+m)x^{1+2m+2bdn}\,{}_2F_1\left(1,\frac{1+m+2bdn}{2bdn};\frac{1+m+4bdn}{2bdn};e^{2d(a+b\log(cx^n))}\right)\right)}{bdn(1+m+2bdn)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Coth[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] (e*x)^m*(x/(1 + m) - (E^(((1 + 2*m)*(a + b*Log[c*x^n]))/(b*n))*(1 + m + 2*b
*d*n)*Coth[d*(a + b*Log[c*x^n])] + E^(((1 + 2*m)*(a + b*Log[c*x^n]))/(b*n))
*(1 + m + 2*b*d*n)*Hypergeometric2F1[1, (1 + m)/(2*b*d*n), 1 + (1 + m)/(2*b
*d*n), E^(2*d*(a + b*Log[c*x^n]))] + E^(((1 + 2*m + 2*b*d*n)*(a - b*n*Log[x
] + b*Log[c*x^n]))/(b*n))*(1 + m)*x^(1 + 2*m + 2*b*d*n)*Hypergeometric2F1[1
, (1 + m + 2*b*d*n)/(2*b*d*n), (1 + m + 4*b*d*n)/(2*b*d*n), E^(2*d*(a + b*L
og[c*x^n]))])/(b*d*E^(((1 + 2*m)*(a - b*n*Log[x] + b*Log[c*x^n]))/(b*n))*n*
(1 + m + 2*b*d*n)*x^(2*m)))
```

Maple [F]
time = 0.43, size = 0, normalized size = 0.00

$$\int (ex)^m \left(\coth^2 \left(d(a + b \ln (c\, x^n))\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*coth(d*(a+b*ln(c*x^n)))^2,x)
[Out] int((e*x)^m*coth(d*(a+b*ln(c*x^n)))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
[Out] -(m*e^m + e^m)*integrate(x^m/(b*c^(b*d)*d*n*e^(b*d*log(x^n) + a*d) + b*d*n)
, x) + (m*e^m + e^m)*integrate(x^m/(b*c^(b*d)*d*n*e^(b*d*log(x^n) + a*d) -
b*d*n), x) + (b*c^(2*b*d)*d*n*x*e^(2*b*d*log(x^n) + 2*a*d + m*log(x) + m) -
 (b*d*n*e^m + 2*m*e^m + 2*e^m)*x*x^m)/((m*n + n)*b*c^(2*b*d)*d*e^(2*b*d*log
(x^n) + 2*a*d) - (m*n + n)*b*d)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
[Out] integral((x*e)^m*coth(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \coth^2 \left(ad + bd \log \left(cx^n\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*coth(d*(a+b*ln(c*x**n)))**2,x)
[Out] Integral((e*x)**m*coth(a*d + b*d*log(c*x**n))**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
[Out] integrate((e*x)^m*coth((b*log(c*x^n) + a)*d)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \coth(d\,(a+b\,\ln\left(c\,x^n\right)))^2\,(e\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a + b*log(c*x^n)))^2*(e*x)^m,x)
[Out] int(coth(d*(a + b*log(c*x^n)))^2*(e*x)^m, x)
```

3.196 $\int (ex)^m \coth^3 (d(a + b \log (cx^n))) \, dx$

Optimal. Leaf size=306

$$\frac{(1+m+bdn)(1+m+2bdn)(ex)^{1+m}}{2b^2d^2e(1+m)n^2}-\frac{(ex)^{1+m}\left(1+e^{2ad}(cx^n)^{2bd}\right)^2}{2bden\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)^2}+\frac{e^{-2ad}(ex)^{1+m}\left(\frac{e^{2ad}(1+m-2bdn)}{n}+\frac{e^{4ad}(1}{}\right.}{2b^2d^2en\left(1-e^{2ad}\left(cx^n\right.\right.}$$

```
[Out] 1/2*(b*d*n+m+1)*(2*b*d*n+m+1)*(e*x)^(1+m)/b^2/d^2/e/(1+m)/n^2-1/2*(e*x)^(1+
m)*(1+exp(2*a*d)*(c*x^n)^(2*b*d))^2/b/d/e/n/(1-exp(2*a*d)*(c*x^n)^(2*b*d))^
2+1/2*(e*x)^(1+m)*(exp(2*a*d)*(-2*b*d*n+m+1)/n+exp(4*a*d)*(2*b*d*n+m+1)*(c*
x^n)^(2*b*d)/n)/b^2/d^2/e/exp(2*a*d)/n/(1-exp(2*a*d)*(c*x^n)^(2*b*d))-(2*b^
2*d^2*n^2+m^2+2*m+1)*(e*x)^(1+m)*hypergeom([1, 1/2*(1+m)/b/d/n],[1+1/2*(1+m
)/b/d/n],exp(2*a*d)*(c*x^n)^(2*b*d))/b^2/d^2/e/(1+m)/n^2
```

Rubi [A]
time = 0.33, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5659, 5657, 516, 608, 470, 371}

$$-\frac{(ex)^{m+1}\left(2b^2d^2n^2+m^2+2m+1\right)\,{}_2F_1\left(1,\frac{m+1}{2bdn};\frac{m+1}{2bdn}+1;e^{2ad}(cx^n)^{2bd}\right)}{b^2d^2e(m+1)n^2}+\frac{e^{-2ad}(ex)^{m+1}\left(\frac{e^{4ad}(2bdn+m+1)(cx^n)^{2bd}}{n}+\frac{e^{2ad}(-2bdn+m+1)}{n}\right)}{2b^2d^2en\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)}-\frac{(ex)^{m+1}\left(e^{2ad}(cx^n)^{2bd}+1\right)^2}{2bden\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)^2}+\frac{(ex)^{m+1}(bdn+m+1)(2bdn+m+1)}{2b^2d^2e(m+1)n^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Coth[d*(a + b*Log[c*x^n])]^3,x]
```

```
[Out] ((1 + m + b*d*n)*(1 + m + 2*b*d*n)*(e*x)^(1 + m))/(2*b^2*d^2*e*(1 + m)*n^2)
 - ((e*x)^(1 + m)*(1 + E^(2*a*d)*(c*x^n)^(2*b*d))^2)/(2*b*d*e*n*(1 - E^(2*a
*d)*(c*x^n)^(2*b*d))^2) + ((e*x)^(1 + m)*((E^(2*a*d)*(1 + m - 2*b*d*n))/n +
 (E^(4*a*d)*(1 + m + 2*b*d*n)*(c*x^n)^(2*b*d))/n))/(2*b^2*d^2*e*E^(2*a*d)*n
*(1 - E^(2*a*d)*(c*x^n)^(2*b*d))) - ((1 + 2*m + m^2 + 2*b^2*d^2*n^2)*(e*x)^
(1 + m)*Hypergeometric2F1[1, (1 + m)/(2*b*d*n), 1 + (1 + m)/(2*b*d*n), E^(2
*a*d)*(c*x^n)^(2*b*d)])/(b^2*d^2*e*(1 + m)*n^2)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
```

```
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 516

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ
[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 608

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(
m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*b*g*n*(p + 1))), x] + Dist[1/(
a*b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*
(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m +
 n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && LtQ[
p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 5659

```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Coth[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int (ex)^m \coth^3 (d(a + b\log (cx^n)))\ dx = \int (ex)^m \coth^3 (d(a + b\log (cx^n)))\ dx$$

Mathematica [A]
time = 13.21, size = 600, normalized size = 1.96

$$\frac{x(ex)^m \coth (d(a+b(-n\log(x)+\log (cx^n))))}{1+m}-\frac{x(ex)^m \text{csch}^2(bdn\log(x)+d(a+b(-n\log(x)+\log (cx^n))))}{2bdn}+\frac{(1+m)x(ex)^m \text{csch}(d(a+b(-n\log(x)+\log (cx^n))))\text{csch}(bdn\log(x)+d(a+b(-n\log(x)+\log (cx^n))))\sinh(bdn\log(x))}{2b^2d^2n^2}-\frac{(1+2m+m^2+2b^2d^2n^2)\,x^{-m}(ex)^m \text{csch}(d(a+b(-n\log(x)+\log (cx^n))))\left(\text{[illegible]}\right)}{2b^2d^2n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Coth[d*(a + b*Log[c*x^n])]^3,x]
```

```
[Out] (x*(e*x)^m*Coth[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))])/(1 + m) - (x*(e*x)^m
*Csch[b*d*n*Log[x] + d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]^2)/(2*b*d*n) + (
(1 + m)*x*(e*x)^m*Csch[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*Csch[b*d*n*Log
[x] + d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*Sinh[b*d*n*Log[x]])/(2*b^2*d^2*
n^2) - ((1 + 2*m + m^2 + 2*b^2*d^2*n^2)*(e*x)^m*Csch[d*(a + b*(-(n*Log[x])
+ Log[c*x^n]))]*((x^(1 + m)*Csch[d*(a + b*Log[c*x^n])]*Sinh[b*d*n*Log[x]])/
(1 + m) + ((E^((a + 2*a*m + b*(1 + m)*n*Log[x] + b*(1 + 2*m)*(-(n*Log[x]) +
 Log[c*x^n]))/(b*n))*(1 + m + 2*b*d*n)*Coth[d*(a + b*Log[c*x^n])] + E^((a +
 2*a*m + b*(1 + m)*n*Log[x] + b*(1 + 2*m)*(-(n*Log[x]) + Log[c*x^n]))/(b*n)
)*(1 + m + 2*b*d*n)*Hypergeometric2F1[1, (1 + m)/(2*b*d*n), 1 + (1 + m)/(2*
b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + E^((a*(1 + 2*m + 2*b*d*n))/(b*n) + (1
 + m + 2*b*d*n)*Log[x] + ((1 + 2*m + 2*b*d*n)*(-(n*Log[x]) + Log[c*x^n]))/n
)*(1 + m)*Hypergeometric2F1[1, (1 + m + 2*b*d*n)/(2*b*d*n), (1 + m + 4*b*d*
n)/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))])*Sinh[d*(a + b*(-(n*Log[x]) + Log
[c*x^n]))])/(E^(((1 + 2*m)*(a + b*(-(n*Log[x]) + Log[c*x^n])))/(b*n))*(1 +
m)*(1 + m + 2*b*d*n))))/(2*b^2*d^2*n^2*x^m)
```

Maple [F]
time = 1.40, size = 0, normalized size = 0.00

$$\int (ex)^m \left(\coth^3\left(d(a + b\ln\left(c\,x^n\right))\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*coth(d*(a+b*ln(c*x^n)))^3,x)
```

```
[Out] int((e*x)^m*coth(d*(a+b*ln(c*x^n)))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^3,x, algorithm="maxima")
```

```
[Out] -(2*b^2*d^2*n^2*e^m + m^2*e^m + 2*m*e^m + e^m)*integrate(1/2*x^m/(b^2*c^(b*
d)*d^2*n^2*e^(b*d*log(x^n) + a*d) + b^2*d^2*n^2), x) + (2*b^2*d^2*n^2*e^m +
 m^2*e^m + 2*m*e^m + e^m)*integrate(1/2*x^m/(b^2*c^(b*d)*d^2*n^2*e^(b*d*log
(x^n) + a*d) - b^2*d^2*n^2), x) + (b^2*c^(4*b*d)*d^2*n^2*x*e^(4*b*d*log(x^n
) + 4*a*d + m*log(x) + m) + (b^2*d^2*n^2*e^m + m^2*e^m + 2*m*e^m + e^m)*x*x
^m - (2*b^2*c^(2*b*d)*d^2*n^2*e^(2*a*d + m) + 2*(m*n*e^m + n*e^m)*b*c^(2*b*
d)*d*e^(2*a*d) + (m^2*e^m + 2*m*e^m + e^m)*c^(2*b*d)*e^(2*a*d))*x*e^(2*b*d*
```

```
log(x^n) + m*log(x)))/((m*n^2 + n^2)*b^2*c^(4*b*d)*d^2*e^(4*b*d*log(x^n) +
4*a*d) - 2*(m*n^2 + n^2)*b^2*c^(2*b*d)*d^2*e^(2*b*d*log(x^n) + 2*a*d) + (m*
n^2 + n^2)*b^2*d^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^3,x, algorithm="fricas")
```

```
[Out] integral((x*e)^m*coth(b*d*log(c*x^n) + a*d)^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*coth(d*(a+b*ln(c*x**n)))**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^3,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*coth((b*log(c*x^n) + a)*d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \coth(d\,(a+b\,\ln\left(c\,x^n\right)))^3\,{(e\,x)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a + b*log(c*x^n)))^3*(e*x)^m,x)
```

```
[Out] int(coth(d*(a + b*log(c*x^n)))^3*(e*x)^m, x)
```

3.197 $\int \coth^p(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=115

$$x\left(-1-e^{2ad}(cx^n)^{2bd}\right)^p\left(1+e^{2ad}(cx^n)^{2bd}\right)^{-p}F_1\left(\frac{1}{2bdn};p,-p;1+\frac{1}{2bdn};e^{2ad}(cx^n)^{2bd},-e^{2ad}(cx^n)^{2bd}\right)$$

```
[Out] x*(-1-exp(2*a*d)*(c*x^n)^(2*b*d))^p*AppellF1(1/2/b/d/n,p,-p,1+1/2/b/d/n,exp
(2*a*d)*(c*x^n)^(2*b*d),-exp(2*a*d)*(c*x^n)^(2*b*d))/((1+exp(2*a*d)*(c*x^n)
^(2*b*d))^p)
```

Rubi [A]
time = 0.09, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {5655, 5657, 525, 524}

$$x\left(-e^{2ad}(cx^n)^{2bd}-1\right)^p\left(e^{2ad}(cx^n)^{2bd}+1\right)^{-p}F_1\left(\frac{1}{2bdn};p,-p;1+\frac{1}{2bdn};e^{2ad}(cx^n)^{2bd},-e^{2ad}(cx^n)^{2bd}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[d*(a + b*Log[c*x^n])]^p,x]
```

```
[Out] (x*(-1 - E^(2*a*d)*(c*x^n)^(2*b*d))^p*AppellF1[1/(2*b*d*n), p, -p, 1 + 1/(2
*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d), -(E^(2*a*d)*(c*x^n)^(2*b*d))])/(1 + E^(
2*a*d)*(c*x^n)^(2*b*d))^p
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 5655

```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Coth[d*(a + b*Log[x])]^p, x]
```

```
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\int \coth^p(d(a+b\log(cx^n)))\,dx = \int \coth^p(d(a+b\log(cx^n)))\,dx$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 387 vs. 2(115) = 230.
time = 2.46, size = 387, normalized size = 3.37

$$\frac{(1+2bdn)x\left(\frac{1+e^{2ad}(cx^n)^{2bd}}{-1+e^{2ad}(cx^n)^{2bd}}\right)^p F_1\left(\frac{1}{2bdn};p,-p;1+\frac{1}{2bdn};e^{2ad}(cx^n)^{2bd},-e^{2ad}(cx^n)^{2bd}\right)}{2bde^{2ad}np(cx^n)^{2bd}F_1\left(1+\frac{1}{2bdn};p,1-p;2+\frac{1}{2bdn};e^{2ad}(cx^n)^{2bd},-e^{2ad}(cx^n)^{2bd}\right)+2bde^{2ad}np(cx^n)^{2bd}F_1\left(1+\frac{1}{2bdn};1+p,-p;2+\frac{1}{2bdn};e^{2ad}(cx^n)^{2bd},-e^{2ad}(cx^n)^{2bd}\right)+(1+2bdn)F_1\left(\frac{1}{2bdn};p,-p;1+\frac{1}{2bdn};e^{2ad}(cx^n)^{2bd},-e^{2ad}(cx^n)^{2bd}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[d*(a + b*Log[c*x^n])]^p,x]
```

```
[Out] ((1 + 2*b*d*n)*x*((1 + E^(2*a*d)*(c*x^n)^(2*b*d))/(-1 + E^(2*a*d)*(c*x^n)^(
2*b*d)))^p*AppellF1[1/(2*b*d*n), p, -p, 1 + 1/(2*b*d*n), E^(2*a*d)*(c*x^n)^
(2*b*d), -(E^(2*a*d)*(c*x^n)^(2*b*d))])/(2*b*d*E^(2*a*d)*n*p*(c*x^n)^(2*b*d
)*AppellF1[1 + 1/(2*b*d*n), p, 1 - p, 2 + 1/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2
*b*d), -(E^(2*a*d)*(c*x^n)^(2*b*d))] + 2*b*d*E^(2*a*d)*n*p*(c*x^n)^(2*b*d)*
AppellF1[1 + 1/(2*b*d*n), 1 + p, -p, 2 + 1/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2*
b*d), -(E^(2*a*d)*(c*x^n)^(2*b*d))] + (1 + 2*b*d*n)*AppellF1[1/(2*b*d*n), p
, -p, 1 + 1/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d), -(E^(2*a*d)*(c*x^n)^(2*b*
d))])
```

Maple [**F**]
time = 1.18, size = 0, normalized size = 0.00

$$\int \coth^p(d(a+b\ln(c\,x^n)))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a+b*ln(c*x^n)))^p,x)
```

```
[Out] int(coth(d*(a+b*ln(c*x^n)))^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^p,x, algorithm="maxima")
[Out] integrate(coth((b*log(c*x^n) + a)*d)^p, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^p,x, algorithm="fricas")
[Out] integral(coth(b*d*log(c*x^n) + a*d)^p, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*ln(c*x**n)))**p,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^p,x, algorithm="giac")
[Out] integrate(coth((b*log(c*x^n) + a)*d)^p, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \coth(d\,(a+b\,\ln\left(c\,x^n\right)))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a + b*log(c*x^n)))^p,x)
[Out] int(coth(d*(a + b*log(c*x^n)))^p, x)
```

3.198 $\int (ex)^m \coth^p(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=135

$$\frac{(ex)^{1+m}\left(-1-e^{2ad}(cx^n)^{2bd}\right)^p\left(1+e^{2ad}(cx^n)^{2bd}\right)^{-p}F_1\left(\frac{1+m}{2bdn};p,-p;1+\frac{1+m}{2bdn};e^{2ad}(cx^n)^{2bd},-e^{2ad}(cx^n)^{2bd}\right)}{e(1+m)}$$

```
[Out] (e*x)^(1+m)*(-1-exp(2*a*d)*(c*x^n)^(2*b*d))^p*AppellF1(1/2*(1+m)/b/d/n,p,-p
,1+1/2*(1+m)/b/d/n,exp(2*a*d)*(c*x^n)^(2*b*d),-exp(2*a*d)*(c*x^n)^(2*b*d))/
e/(1+m)/((1+exp(2*a*d)*(c*x^n)^(2*b*d))^p)
```

Rubi [A]
time = 0.11, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5659, 5657, 525, 524}

$$\frac{(ex)^{m+1}\left(-e^{2ad}(cx^n)^{2bd}-1\right)^p\left(e^{2ad}(cx^n)^{2bd}+1\right)^{-p}F_1\left(\frac{m+1}{2bdn};p,-p;\frac{m+1}{2bdn}+1;e^{2ad}(cx^n)^{2bd},-e^{2ad}(cx^n)^{2bd}\right)}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Coth[d*(a + b*Log[c*x^n])]^p,x]
```

```
[Out] ((e*x)^(1 + m)*(-1 - E^(2*a*d)*(c*x^n)^(2*b*d))^p*AppellF1[(1 + m)/(2*b*d*n
), p, -p, 1 + (1 + m)/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d), -(E^(2*a*d)*(c*
x^n)^(2*b*d))])/(e*(1 + m)*(1 + E^(2*a*d)*(c*x^n)^(2*b*d))^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
```

```
x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 5659

```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Coth[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int (ex)^m \coth^p(d(a+b\log(cx^n)))\,dx = \int (ex)^m \coth^p(d(a+b\log(cx^n)))\,dx$$

Mathematica [A]
time = 3.47, size = 174, normalized size = 1.29

$$\frac{x(ex)^m\left(1-e^{2ad}(cx^n)^{2bd}\right)^p\left(1+e^{2ad}(cx^n)^{2bd}\right)^{-p}\left(\frac{1+e^{2ad}(cx^n)^{2bd}}{-1+e^{2ad}(cx^n)^{2bd}}\right)^p F_1\left(\frac{1+m}{2bdn};p,-p;1+\frac{1+m}{2bdn};e^{2ad}(cx^n)^{2bd},-e^{2ad}(cx^n)^{2bd}\right)}{1+m}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*x)^m*Coth[d*(a + b*Log[c*x^n])]^p,x]
```

```
[Out] (x*(e*x)^m*(1 - E^(2*a*d)*(c*x^n)^(2*b*d))^p*((1 + E^(2*a*d)*(c*x^n)^(2*b*d
))/(-1 + E^(2*a*d)*(c*x^n)^(2*b*d)))^p*AppellF1[(1 + m)/(2*b*d*n), p, -p, 1
 + (1 + m)/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d), -(E^(2*a*d)*(c*x^n)^(2*b*d
))])/((1 + m)*(1 + E^(2*a*d)*(c*x^n)^(2*b*d))^p)
```

Maple [F]
time = 1.06, size = 0, normalized size = 0.00

$$\int (ex)^m \left(\coth^p(d(a+b\ln(c\,x^n)))\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*coth(d*(a+b*ln(c*x^n)))^p,x)
```

```
[Out] int((e*x)^m*coth(d*(a+b*ln(c*x^n)))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^p,x, algorithm="maxima")
```

```
[Out] integrate((x*e)^m*coth((b*log(c*x^n) + a)*d)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^p,x, algorithm="fricas")
```

```
[Out] integral((x*e)^m*coth(b*d*log(c*x^n) + a*d)^p, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*coth(d*(a+b*ln(c*x**n)))**p,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^p,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*coth((b*log(c*x^n) + a)*d)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \coth(d\,(a+b\,\ln\left(c\,x^n\right)))^p\,{(e\,x)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a + b*log(c*x^n)))^p*(e*x)^m,x)
```

```
[Out] int(coth(d*(a + b*log(c*x^n)))^p*(e*x)^m, x)
```

3.199 $\int \frac{\coth^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=73

$$-\frac{\text{ArcTan}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}+\frac{\tanh^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}-\frac{2\coth^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{3bn}$$

```
[Out] -arctan(coth(a+b*ln(c*x^n))^(1/2))/b/n+arctanh(coth(a+b*ln(c*x^n))^(1/2))/b
/n-2/3*coth(a+b*ln(c*x^n))^(3/2)/b/n
```

Rubi [A]
time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3554, 3557, 335, 304, 209, 212}

$$-\frac{\text{ArcTan}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}-\frac{2\coth^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{3bn}+\frac{\tanh^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] -(ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n)) + ArcTanh[Sqrt[Coth[a + b*Log
[c*x^n]]]]/(b*n) - (2*Coth[a + b*Log[c*x^n]]^(3/2))/(3*b*n)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx &= \frac{\text{Subst}\left(\int \coth^{\frac{5}{2}}(a+bx)\,dx, x, \log(cx^n)\right)}{n} \\
&= -\frac{2\coth^{\frac{3}{2}}(a+b\log(cx^n))}{3bn} + \frac{\text{Subst}\left(\int \sqrt{\coth(a+bx)}\,dx, x, \log(cx^n)\right)}{n} \\
&= -\frac{2\coth^{\frac{3}{2}}(a+b\log(cx^n))}{3bn} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{-1+x^2}\,dx, x, \coth(a+b\log(cx^n))\right)}{bn} \\
&= -\frac{2\coth^{\frac{3}{2}}(a+b\log(cx^n))}{3bn} - \frac{2\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt{\coth(a+b\log(cx^n))}\right.}{bn} \\
&= -\frac{2\coth^{\frac{3}{2}}(a+b\log(cx^n))}{3bn} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\coth(a+b\log(cx^n))}\right)}{bn} - \\
&= -\frac{\tan^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} + \frac{\tanh^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} -
\end{aligned}$$

Mathematica [A]
time = 0.21, size = 64, normalized size = 0.88

$$-\frac{3\text{ArcTan}\left(\sqrt{\coth(a+b\log(cx^n))}\right) - 3\tanh^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right) + 2\coth^{\frac{3}{2}}(a+b\log(cx^n))}{3bn}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[a + b*Log[c*x^n]]^(5/2)/x,x]

[Out] -1/3*(3*ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]] - 3*ArcTanh[Sqrt[Coth[a + b*Lo
g[c*x^n]]]] + 2*Coth[a + b*Log[c*x^n]]^(3/2))/(b*n)

Maple [A]
time = 5.12, size = 76, normalized size = 1.04

method	result
derivativedivides	$\frac{-\frac{2\left(\coth^{\frac{3}{2}}\left(a+b\ln\left(c\,x^{n}\right)\right)\right)}{3}-\frac{\ln\left(\sqrt{\coth}\left(a+b\ln\left(c\,x^{n}\right)\right)-1\right)}{2}+\frac{\ln\left(\sqrt{\coth}\left(a+b\ln\left(c\,x^{n}\right)\right)+1\right)}{2}-\arctan\left(\sqrt{\coth}\left(a+b\ln\right.\right.}{nb}$
default	$\frac{-\frac{2\left(\coth^{\frac{3}{2}}\left(a+b\ln\left(c\,x^{n}\right)\right)\right)}{3}-\frac{\ln\left(\sqrt{\coth}\left(a+b\ln\left(c\,x^{n}\right)\right)-1\right)}{2}+\frac{\ln\left(\sqrt{\coth}\left(a+b\ln\left(c\,x^{n}\right)\right)+1\right)}{2}-\arctan\left(\sqrt{\coth}\left(a+b\ln\right.\right.}{nb}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(a+b*ln(c*x^n))^(5/2)/x,x,method=_RETURNVERBOSE)

[Out] 1/n/b*(-2/3*coth(a+b*ln(c*x^n))^(3/2)-1/2*ln(coth(a+b*ln(c*x^n))^(1/2)-1)+1
/2*ln(coth(a+b*ln(c*x^n))^(1/2)+1)-arctan(coth(a+b*ln(c*x^n))^(1/2)))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+b*log(c*x^n))^(5/2)/x,x, algorithm="maxima")

[Out] integrate(coth(b*log(c*x^n) + a)^(5/2)/x, x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 626 vs. $2(65) = 130$.
time = 0.38, size = 626, normalized size = 8.58

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")

[Out] 1/6*(6*(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) +
a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)
*arctan(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(c) +
a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)^2 + (
cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(

```
b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*sqrt(co
sh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a))) - 4*cosh(b*
n*log(x) + b*log(c) + a)^2 - 3*(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(
b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x
) + b*log(c) + a)^2 - 1)*log(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*
n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x)
+ b*log(c) + a)^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x)
+ b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c
) + a)^2 - 1)*sqrt(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(
c) + a))) - 8*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) +
a) - 4*sinh(b*n*log(x) + b*log(c) + a)^2 - 4*(cosh(b*n*log(x) + b*log(c) +
a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) +
sinh(b*n*log(x) + b*log(c) + a)^2 + 1)*sqrt(cosh(b*n*log(x) + b*log(c) + a)
/sinh(b*n*log(x) + b*log(c) + a)) + 4)/(b*n*cosh(b*n*log(x) + b*log(c) + a)
^2 + 2*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)
+ b*n*sinh(b*n*log(x) + b*log(c) + a)^2 - b*n)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*ln(c*x**n))**(5/2)/x,x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^(5/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 2.24, size = 65, normalized size = 0.89

$$\frac{\operatorname{atanh}\left(\sqrt{\coth\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)}{b\,n}-\frac{\operatorname{atan}\left(\sqrt{\coth\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)}{b\,n}-\frac{2\coth\left(a+b\,\ln\left(c\,x^n\right)\right)^{3/2}}{3\,b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + b*log(c*x^n))^(5/2)/x,x)
```

```
[Out] atanh(coth(a + b*log(c*x^n))^(1/2))/(b*n) - atan(coth(a + b*log(c*x^n))^(1/
2))/(b*n) - (2*coth(a + b*log(c*x^n))^(3/2))/(3*b*n)
```

3.200 $\int \frac{\coth^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=70

$$\frac{\text{ArcTan}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}+\frac{\tanh^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}-\frac{2\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}}{bn}$$

```
[Out] arctan(coth(a+b*ln(c*x^n))^(1/2))/b/n+arctanh(coth(a+b*ln(c*x^n))^(1/2))/b/
n-2*coth(a+b*ln(c*x^n))^(1/2)/b/n
```

Rubi [A]
time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3554, 3557, 335, 218, 212, 209}

$$\frac{\text{ArcTan}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}-\frac{2\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}}{bn}+\frac{\tanh^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n) + ArcTanh[Sqrt[Coth[a + b*Log[c*
x^n]]]]/(b*n) - (2*Sqrt[Coth[a + b*Log[c*x^n]]])/(b*n)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{\coth^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{\text{Subst}\left(\int \coth^{\frac{3}{2}}(a+bx)\,dx, x, \log(cx^n)\right)}{n}$$
$$= -\frac{2\sqrt{\coth(a+b\log(cx^n))}}{bn} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\coth(a+bx)}}\,dx, x, \log(cx^n)\right)}{n}$$
$$= -\frac{2\sqrt{\coth(a+b\log(cx^n))}}{bn} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(-1+x^2)}\,dx, x, \coth(a+b\log(cx^n\right.}{bn}$$
$$= -\frac{2\sqrt{\coth(a+b\log(cx^n))}}{bn} - \frac{2\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt{\coth(a+b\log(cx^n))}\right.}{bn}$$
$$= -\frac{2\sqrt{\coth(a+b\log(cx^n))}}{bn} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\coth(a+b\log(cx^n))}\right)}{bn}$$
$$= \frac{\tan^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} + \frac{\tanh^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} - \frac{2}{}$$

Mathematica [A]
time = 0.11, size = 57, normalized size = 0.81

$$\frac{\text{ArcTan}\left(\sqrt{\coth(a+b\log(cx^n))}\right) + \tanh^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right) - 2\sqrt{\coth(a+b\log(cx^n))}}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] (ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]] + ArcTanh[Sqrt[Coth[a + b*Log[c*x^n]]
]] - 2*Sqrt[Coth[a + b*Log[c*x^n]]])/(b*n)
```

Maple [A]
time = 4.65, size = 74, normalized size = 1.06

method	result
derivativedivides	$\frac{-2\left(\sqrt{\coth}\,(a+b\ln(c\,x^{n}))\right)-\frac{\ln\left(\sqrt{\coth}\,(a+b\ln(c\,x^{n}))-1\right)}{2}+\frac{\ln\left(\sqrt{\coth}\,(a+b\ln(c\,x^{n}))+1\right)}{2}+\arctan\left(\sqrt{\coth}\,(a\right.}{nb}$
default	$\frac{-2\left(\sqrt{\coth}\,(a+b\ln(c\,x^{n}))\right)-\frac{\ln\left(\sqrt{\coth}\,(a+b\ln(c\,x^{n}))-1\right)}{2}+\frac{\ln\left(\sqrt{\coth}\,(a+b\ln(c\,x^{n}))+1\right)}{2}+\arctan\left(\sqrt{\coth}\,(a\right.}{nb}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+b*ln(c*x^n))^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b*(-2*coth(a+b*ln(c*x^n))^(1/2)-1/2*ln(coth(a+b*ln(c*x^n))^(1/2)-1)+1/2
*ln(coth(a+b*ln(c*x^n))^(1/2)+1)+arctan(coth(a+b*ln(c*x^n))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(coth(b*log(c*x^n) + a)^(3/2)/x, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(64) = 128$.
time = 0.37, size = 334, normalized size = 4.77

$$-\frac{4\sqrt{\frac{\cosh(bn\log(x)+b\log(c)+a)}{\sinh(bn\log(x)+b\log(c)+a)}}+2\arctan\left(-\cosh(bn\log(x)+b\log(c)+a)^2-2\cosh(bn\log(x)+b\log(c)+a)\sinh(bn\log(x)+b\log(c)+a)-\sinh(bn\log(x)+b\log(c)+a)^2+\left(\cosh(bn\log(x)+b\log(c)+a)^2+2\cosh(bn\log(x)+b\log(c)+a)\sinh(bn\log(x)+b\log(c)+a)+\sinh(bn\log(x)+b\log(c)+a)^2-1\right)\sqrt{\frac{\cosh(bn\log(x)+b\log(c)+a)}{\sinh(bn\log(x)+b\log(c)+a)}}\right)+\log\left(-\cosh(bn\log(x)+b\log(c)+a)^2-2\cosh(bn\log(x)+b\log(c)+a)\sinh(bn\log(x)+b\log(c)+a)-\sinh(bn\log(x)+b\log(c)+a)^2+\left(\cosh(bn\log(x)+b\log(c)+a)^2+2\cosh(bn\log(x)+b\log(c)+a)\sinh(bn\log(x)+b\log(c)+a)+\sinh(bn\log(x)+b\log(c)+a)^2-1\right)\sqrt{\frac{\cosh(bn\log(x)+b\log(c)+a)}{\sinh(bn\log(x)+b\log(c)+a)}}\right)}{2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] -1/2*(4*sqrt(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a
)) + 2*arctan(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*lo
g(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)
^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)
```

```
*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*s
qrt(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a))) + log
(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(c) + a)*sin
h(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)^2 + (cosh(b*
n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log
(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*sqrt(cosh(b*n*
log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a))))/(b*n)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*ln(c*x**n))**(3/2)/x,x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^(3/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 1.86, size = 51, normalized size = 0.73

$$\frac{\mathrm{atan}\left(\sqrt{\mathrm{coth}\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)+\mathrm{atanh}\left(\sqrt{\mathrm{coth}\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)-2\,\sqrt{\mathrm{coth}\left(a+b\,\ln\left(c\,x^n\right)\right)}}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + b*log(c*x^n))^(3/2)/x,x)
```

```
[Out] (atan(coth(a + b*log(c*x^n))^(1/2)) + atanh(coth(a + b*log(c*x^n))^(1/2)) -
 2*coth(a + b*log(c*x^n))^(1/2))/(b*n)
```

3.201 $\int \frac{\sqrt{\coth(a+b\log(cx^n))}}{x}\,dx$

Optimal. Leaf size=48

$$-\frac{\operatorname{ArcTan}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}+\frac{\tanh^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}$$

```
[Out] -arctan(coth(a+b*ln(c*x^n))^(1/2))/b/n+arctanh(coth(a+b*ln(c*x^n))^(1/2))/b
/n
```

Rubi [A]
time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3557, 335, 304, 209, 212}

$$\frac{\tanh^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}-\frac{\operatorname{ArcTan}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Coth[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] -(ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n)) + ArcTanh[Sqrt[Coth[a + b*Log
[c*x^n]]]]/(b*n)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\coth(a+b\log(cx^n))}}{x}\,dx &= \frac{\text{Subst}\left(\int \sqrt{\coth(a+bx)}\,dx, x, \log(cx^n)\right)}{n}\\
&= -\frac{\text{Subst}\left(\int \frac{\sqrt{x}}{-1+x^2}\,dx, x, \coth(a+b\log(cx^n))\right)}{bn}\\
&= -\frac{2\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt{\coth(a+b\log(cx^n))}\right)}{bn}\\
&= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\coth(a+b\log(cx^n))}\right)}{bn} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\text{cot}}\right.}{bn}\\
&= -\frac{\tan^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} + \frac{\tanh^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 48, normalized size = 1.00

$$-\frac{\text{ArcTan}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} + \frac{\tanh^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Coth[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] -(ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n)) + ArcTanh[Sqrt[Coth[a + b*Log
[c*x^n]]]]/(b*n)
```

Maple [A]
time = 4.79, size = 61, normalized size = 1.27

method	result	size
derivativedivides	$\frac{-\frac{\ln\left(\sqrt{\coth}\,(a+b\ln(c\,x^n))-1\right)}{2}+\frac{\ln\left(\sqrt{\coth}\,(a+b\ln(c\,x^n))+1\right)}{2}-\arctan\left(\sqrt{\coth}\,(a+b\ln(c\,x^n))\right)}{nb}$	61
default	$\frac{-\frac{\ln\left(\sqrt{\coth}\,(a+b\ln(c\,x^n))-1\right)}{2}+\frac{\ln\left(\sqrt{\coth}\,(a+b\ln(c\,x^n))+1\right)}{2}-\arctan\left(\sqrt{\coth}\,(a+b\ln(c\,x^n))\right)}{nb}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+b*ln(c*x^n))^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b*(-1/2*ln(coth(a+b*ln(c*x^n))^(1/2)-1)+1/2*ln(coth(a+b*ln(c*x^n))^(1/2
)+1)-arctan(coth(a+b*ln(c*x^n))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(coth(b*log(c*x^n) + a))/x, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(44) = 88$.
time = 0.38, size = 305, normalized size = 6.35

$$\frac{2\arctan\left(-\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2-2\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2+\left(\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2+2\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2-1\right)\sqrt{\frac{\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}}\right)-\log\left(-\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2-2\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2+\left(\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2+2\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2-1\right)\sqrt{\frac{\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}}\right)}{2\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(2*arctan(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*lo
g(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)
^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)
*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*s
qrt(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a))) - log
(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(c) + a)*sin
h(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)^2 + (cosh(b*
n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log
(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*sqrt(cosh(b*n*
log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a))))/(b*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\coth\left(a + b\log\left(cx^n\right)\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*ln(c*x**n))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(coth(a + b*log(c*x**n)))/x, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^(1/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 1.50, size = 39, normalized size = 0.81

$$-\frac{\operatorname{atan}\left(\sqrt{\coth\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)-\operatorname{atanh}\left(\sqrt{\coth\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + b*log(c*x^n))^(1/2)/x,x)
```

```
[Out] -(atan(coth(a + b*log(c*x^n))^(1/2)) - atanh(coth(a + b*log(c*x^n))^(1/2)))
/(b*n)
```

3.202 $\int \frac{1}{x\sqrt{\coth(a+b\log(cx^n))}}\,dx$

Optimal. Leaf size=47

$$\frac{\text{ArcTan}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}+\frac{\tanh^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}$$

```
[Out] arctan(coth(a+b*ln(c*x^n))^(1/2))/b/n+arctanh(coth(a+b*ln(c*x^n))^(1/2))/b/
n
```

Rubi [A]
time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3557, 335, 218, 212, 209}

$$\frac{\text{ArcTan}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}+\frac{\tanh^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[Coth[a + b*Log[c*x^n]]]),x]
```

```
[Out] ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n) + ArcTanh[Sqrt[Coth[a + b*Log[c*
x^n]]]]/(b*n)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\sqrt{\coth(a+b\log(cx^n))}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\coth(a+bx)}}\,dx, x, \log(cx^n)\right)}{n}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(-1+x^2)}\,dx, x, \coth(a+b\log(cx^n))\right)}{bn}\\
&= -\frac{2\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt{\coth(a+b\log(cx^n))}\right)}{bn}\\
&= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\coth(a+b\log(cx^n))}\right)}{bn} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\text{co}\right.}{bn}\\
&= \frac{\tan^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} + \frac{\tanh^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 47, normalized size = 1.00

$$
\frac{\text{ArcTan}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} + \frac{\tanh^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[Coth[a + b*Log[c*x^n]]]),x]
```

```
[Out] ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n) + ArcTanh[Sqrt[Coth[a + b*Log[c*
x^n]]]]/(b*n)
```

Maple [A]
time = 4.69, size = 37, normalized size = 0.79

method	result	size
derivativedivides	$\frac{\operatorname{arctanh}\left(\sqrt{\coth}\,(a+b\ln(c\,x^{n}))\right)+\arctan\left(\sqrt{\coth}\,(a+b\ln(c\,x^{n}))\right)}{nb}$	37
default	$\frac{\operatorname{arctanh}\left(\sqrt{\coth}\,(a+b\ln(c\,x^{n}))\right)+\arctan\left(\sqrt{\coth}\,(a+b\ln(c\,x^{n}))\right)}{nb}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/coth(a+b*ln(c*x^n))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b*(arctanh(coth(a+b*ln(c*x^n))^(1/2))+arctan(coth(a+b*ln(c*x^n))^(1/2))
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/coth(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*sqrt(coth(b*log(c*x^n) + a))), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(43) = 86$.
time = 0.37, size = 303, normalized size = 6.45

$$-\frac{2\arctan\left(-\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}-2\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+\left(\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}-1\right)\sqrt{\frac{\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}}\right)+\log\left(-\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}-2\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+\left(\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}-1\right)\sqrt{\frac{\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}}\right)}{2\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/coth(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*arctan(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*l
og(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a
)^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a
)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*
sqrt(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a))) + lo
g(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(c) + a)*si
nh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)^2 + (cosh(b
*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*lo
g(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*sqrt(cosh(b*n
*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a))))/(b*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\coth\left(a+b\log\left(cx^{n}\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/coth(a+b*ln(c*x**n))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(coth(a + b*log(c*x**n)))), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/coth(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 1.64, size = 36, normalized size = 0.77

$$\frac{\mathrm{atan}\left(\sqrt{\mathrm{coth}\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)+\mathrm{atanh}\left(\sqrt{\mathrm{coth}\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*coth(a + b*log(c*x^n))^(1/2)),x)
```

```
[Out] (atan(coth(a + b*log(c*x^n))^(1/2)) + atanh(coth(a + b*log(c*x^n))^(1/2)))/
(b*n)
```

3.203 $\int \frac{1}{x \coth^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=71

$$-\frac{\text{ArcTan}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}+\frac{\tanh^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}-\frac{2}{bn\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}}$$

```
[Out] -arctan(coth(a+b*ln(c*x^n))^(1/2))/b/n+arctanh(coth(a+b*ln(c*x^n))^(1/2))/b
/n-2/b/n/coth(a+b*ln(c*x^n))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3555, 3557, 335, 304, 209, 212}

$$-\frac{\text{ArcTan}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}-\frac{2}{bn\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}}+\frac{\tanh^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Coth[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] -(ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n)) + ArcTanh[Sqrt[Coth[a + b*Log
[c*x^n]]]]/(b*n) - 2/(b*n*Sqrt[Coth[a + b*Log[c*x^n]]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{x\coth^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\coth^{\frac{3}{2}}(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$

$$= -\frac{2}{bn\sqrt{\coth(a+b\log(cx^n))}} + \frac{\text{Subst}\left(\int \sqrt{\coth(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$

$$= -\frac{2}{bn\sqrt{\coth(a+b\log(cx^n))}} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{-1+x^2}\,dx, x, \coth(a+b\log(cx^n))\right)}{bn}$$

$$= -\frac{2}{bn\sqrt{\coth(a+b\log(cx^n))}} - \frac{2\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt{\coth(a+b\log(cx}\right.}{bn}$$

$$= -\frac{2}{bn\sqrt{\coth(a+b\log(cx^n))}} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\coth(a+b\log(cx^n)}\right.}{bn}$$

$$= -\frac{\tan^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} + \frac{\tanh^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} -$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.11, size = 44, normalized size = 0.62

$$-\frac{2\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};\coth^2\left(a+b\log\left(cx^n\right)\right)\right)}{bn\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Coth[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/4, 1, 3/4, Coth[a + b*Log[c*x^n]]^2])/(b*n*Sqrt[Co
th[a + b*Log[c*x^n]]])
```

Maple [A]
time = 4.71, size = 76, normalized size = 1.07

method	result
derivativedivides	$\frac{-\arctan\left(\sqrt{\coth}\left(a+b\ln\left(c\,x^n\right)\right)\right)-\frac{2}{\sqrt{\coth\left(a+b\ln\left(c\,x^n\right)\right)}}-\frac{\ln\left(\sqrt{\coth}\left(a+b\ln\left(c\,x^n\right)\right)-1\right)}{2}+\frac{\ln\left(\sqrt{\coth}\right.}{}}{nb}$
default	$\frac{-\arctan\left(\sqrt{\coth}\left(a+b\ln\left(c\,x^n\right)\right)\right)-\frac{2}{\sqrt{\coth\left(a+b\ln\left(c\,x^n\right)\right)}}-\frac{\ln\left(\sqrt{\coth}\left(a+b\ln\left(c\,x^n\right)\right)-1\right)}{2}+\frac{\ln\left(\sqrt{\coth}\right.}{}}{nb}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/coth(a+b*ln(c*x^n))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b*(-arctan(coth(a+b*ln(c*x^n))^(1/2))-2/coth(a+b*ln(c*x^n))^(1/2)-1/2*l
n(coth(a+b*ln(c*x^n))^(1/2)-1)+1/2*ln(coth(a+b*ln(c*x^n))^(1/2)+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/coth(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*coth(b*log(c*x^n) + a)^(3/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 625 vs. $2(65) = 130$.
time = 0.36, size = 625, normalized size = 8.80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/coth(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) +
a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 + 1)
*arctan(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(c) +
 a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)^2 + (
cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(
b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*sqrt(co
sh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a))) - 4*cosh(b*
n*log(x) + b*log(c) + a)^2 - (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*
n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x)
+ b*log(c) + a)^2 + 1)*log(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*
log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) +
b*log(c) + a)^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) +
b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c)
+ a)^2 - 1)*sqrt(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c)
 + a))) - 8*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)
 - 4*sinh(b*n*log(x) + b*log(c) + a)^2 - 4*(cosh(b*n*log(x) + b*log(c) + a)
^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + si
nh(b*n*log(x) + b*log(c) + a)^2 - 1)*sqrt(cosh(b*n*log(x) + b*log(c) + a)/s
inh(b*n*log(x) + b*log(c) + a)) - 4)/(b*n*cosh(b*n*log(x) + b*log(c) + a)^2
 + 2*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) +
b*n*sinh(b*n*log(x) + b*log(c) + a)^2 + b*n)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/coth(a+b*ln(c*x**n))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/coth(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 1.74, size = 65, normalized size = 0.92

$$\frac{\mathrm{atanh}\left(\sqrt{\mathrm{coth}\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)}{b\,n}-\frac{\mathrm{atan}\left(\sqrt{\mathrm{coth}\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)}{b\,n}-\frac{2}{b\,n\,\sqrt{\mathrm{coth}\left(a+b\,\ln\left(c\,x^n\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*coth(a + b*log(c*x^n))^(3/2)),x)
```

```
[Out] atanh(coth(a + b*log(c*x^n))^(1/2))/(b*n) - atan(coth(a + b*log(c*x^n))^(1/
2))/(b*n) - 2/(b*n*coth(a + b*log(c*x^n))^(1/2))
```

3.204 $\int \frac{1}{x \coth^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=72

$$\frac{\operatorname{ArcTan}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}+\frac{\tanh^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}-\frac{2}{3bn\coth^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}$$

```
[Out] arctan(coth(a+b*ln(c*x^n))^(1/2))/b/n+arctanh(coth(a+b*ln(c*x^n))^(1/2))/b/
n-2/3/b/n/coth(a+b*ln(c*x^n))^(3/2)
```

Rubi [A]
time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3555, 3557, 335, 218, 212, 209}

$$\frac{\operatorname{ArcTan}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}-\frac{2}{3bn\coth^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}+\frac{\tanh^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Coth[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n) + ArcTanh[Sqrt[Coth[a + b*Log[c*
x^n]]]]/(b*n) - 2/(3*b*n*Coth[a + b*Log[c*x^n]]^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{x\coth^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\coth^{\frac{5}{2}}(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$

$$= -\frac{2}{3bn\coth^{\frac{3}{2}}(a+b\log(cx^n))} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\coth(a+bx)}}\,dx, x, \log(cx^n)\right)}{n}$$

$$= -\frac{2}{3bn\coth^{\frac{3}{2}}(a+b\log(cx^n))} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(-1+x^2)}\,dx, x, \coth(a+b\log\right.}{bn}$$

$$= -\frac{2}{3bn\coth^{\frac{3}{2}}(a+b\log(cx^n))} - \frac{2\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt{\coth(a+b\log(c}\right.}{bn}$$

$$= -\frac{2}{3bn\coth^{\frac{3}{2}}(a+b\log(cx^n))} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\coth(a+b\log(cx^n}\right.}{bn}$$

$$= \frac{\tan^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} + \frac{\tanh^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} -$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.14, size = 46, normalized size = 0.64

$$-\frac{2\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};\coth^2\left(a+b\log\left(cx^n\right)\right)\right)}{3bn\coth^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Coth[a + b*Log[c*x^n]]^(5/2)),x]
[Out] (-2*Hypergeometric2F1[-3/4, 1, 1/4, Coth[a + b*Log[c*x^n]]^2])/(3*b*n*Coth[
a + b*Log[c*x^n]]^(3/2))
```

Maple [A]
time = 4.70, size = 74, normalized size = 1.03

method	result
derivativedivides	$\frac{\frac{\ln\left(\sqrt{\coth}\left(a+b\ln\left(c\,x^{n}\right)\right)+1\right)}{2}-\frac{\ln\left(\sqrt{\coth}\left(a+b\ln\left(c\,x^{n}\right)\right)-1\right)}{2}-\frac{2}{3\coth\left(a+b\ln\left(c\,x^{n}\right)\right)^{\frac{3}{2}}}+\arctan\left(\sqrt{\coth}\left(a+b\ln\left(c\,x^{n}\right)\right)\right.}{nb}$
default	$\frac{\frac{\ln\left(\sqrt{\coth}\left(a+b\ln\left(c\,x^{n}\right)\right)+1\right)}{2}-\frac{\ln\left(\sqrt{\coth}\left(a+b\ln\left(c\,x^{n}\right)\right)-1\right)}{2}-\frac{2}{3\coth\left(a+b\ln\left(c\,x^{n}\right)\right)^{\frac{3}{2}}}+\arctan\left(\sqrt{\coth}\left(a+b\ln\left(c\,x^{n}\right)\right)\right.}{nb}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/coth(a+b*ln(c*x^n))^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/n/b*(1/2*ln(coth(a+b*ln(c*x^n))^(1/2)+1)-1/2*ln(coth(a+b*ln(c*x^n))^(1/2)
-1)-2/3/coth(a+b*ln(c*x^n))^(3/2)+arctan(coth(a+b*ln(c*x^n))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/coth(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
[Out] integrate(1/(x*coth(b*log(c*x^n) + a)^(5/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1104 vs. 2(64) = 128.
time = 0.43, size = 1104, normalized size = 15.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/coth(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/6*(4*cosh(b*n*log(x) + b*log(c) + a)^4 + 16*cosh(b*n*log(x) + b*log(c) +
 a)*sinh(b*n*log(x) + b*log(c) + a)^3 + 4*sinh(b*n*log(x) + b*log(c) + a)^4
 + 8*(3*cosh(b*n*log(x) + b*log(c) + a)^2 + 1)*sinh(b*n*log(x) + b*log(c) +
 a)^2 + 6*(cosh(b*n*log(x) + b*log(c) + a)^4 + 4*cosh(b*n*log(x) + b*log(c)
 + a)*sinh(b*n*log(x) + b*log(c) + a)^3 + sinh(b*n*log(x) + b*log(c) + a)^4
 + 2*(3*cosh(b*n*log(x) + b*log(c) + a)^2 + 1)*sinh(b*n*log(x) + b*log(c) +
 a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)^2 + 4*(cosh(b*n*log(x) + b*log(c)
 + a)^3 + cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)
+ 1)*arctan(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(
c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)^2
 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*s
inh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*sqr
t(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a))) + 8*cos
h(b*n*log(x) + b*log(c) + a)^2 + 3*(cosh(b*n*log(x) + b*log(c) + a)^4 + 4*c
osh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3 + sinh(b*n
*log(x) + b*log(c) + a)^4 + 2*(3*cosh(b*n*log(x) + b*log(c) + a)^2 + 1)*sin
h(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)^2 + 4*(c
osh(b*n*log(x) + b*log(c) + a)^3 + cosh(b*n*log(x) + b*log(c) + a))*sinh(b*
n*log(x) + b*log(c) + a) + 1)*log(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*co
sh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*lo
g(x) + b*log(c) + a)^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*lo
g(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*
log(c) + a)^2 - 1)*sqrt(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b
*log(c) + a))) + 16*(cosh(b*n*log(x) + b*log(c) + a)^3 + cosh(b*n*log(x) +
b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a) + 4*(cosh(b*n*log(x) + b*log
(c) + a)^4 + 4*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) +
 a)^3 + sinh(b*n*log(x) + b*log(c) + a)^4 + 2*(3*cosh(b*n*log(x) + b*log(c)
 + a)^2 - 1)*sinh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(
c) + a)^2 + 4*(cosh(b*n*log(x) + b*log(c) + a)^3 - cosh(b*n*log(x) + b*log(
c) + a))*sinh(b*n*log(x) + b*log(c) + a) + 1)*sqrt(cosh(b*n*log(x) + b*log(
c) + a)/sinh(b*n*log(x) + b*log(c) + a)) + 4)/(b*n*cosh(b*n*log(x) + b*log(
c) + a)^4 + 4*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c
) + a)^3 + b*n*sinh(b*n*log(x) + b*log(c) + a)^4 + 2*b*n*cosh(b*n*log(x) +
b*log(c) + a)^2 + 2*(3*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + b*n)*sinh(b*
n*log(x) + b*log(c) + a)^2 + b*n + 4*(b*n*cosh(b*n*log(x) + b*log(c) + a)^3
 + b*n*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/coth(a+b*ln(c*x**n))**(5/2),x)
```

[Out] Timed out

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/coth(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 2.39, size = 64, normalized size = 0.89

$$\frac{\operatorname{atan}\left(\sqrt{\coth\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)}{b\,n}+\frac{\operatorname{atanh}\left(\sqrt{\coth\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)}{b\,n}-\frac{2}{3\,b\,n\,{\coth\left(a+b\,\ln\left(c\,x^n\right)\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*coth(a + b*log(c*x^n))^(5/2)),x)
```

```
[Out] atan(coth(a + b*log(c*x^n))^(1/2))/(b*n) + atanh(coth(a + b*log(c*x^n))^(1/
2))/(b*n) - 2/(3*b*n*coth(a + b*log(c*x^n))^(3/2))
```

3.205 $\int \frac{\coth^5(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx$

Optimal. Leaf size=135

$$\frac{(b-2c)\tanh^{-1}\left(\frac{b+2c\coth^2(x)}{2\sqrt{c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{4c^{3/2}}+\frac{\tanh^{-1}\left(\frac{2a+b+(b+2c)\coth^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4}}\right.}{2\sqrt{a+b+c}}$$

```
[Out] 1/4*(b-2*c)*arctanh(1/2*(b+2*c*coth(x)^2)/c^(1/2)/(a+b*coth(x)^2+c*coth(x)^
4)^(1/2))/c^(3/2)+1/2*arctanh(1/2*(2*a+b+(b+2*c)*coth(x)^2)/(a+b+c)^(1/2)/(
a+b*coth(x)^2+c*coth(x)^4)^(1/2))/(a+b+c)^(1/2)-1/2*(a+b*coth(x)^2+c*coth(x
)^4)^(1/2)/c
```

Rubi [A]
time = 0.26, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3782, 1265, 1667, 857, 635, 212, 738}

$$\frac{(b-2c)\tanh^{-1}\left(\frac{b+2c\coth^2(x)}{2\sqrt{c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{4c^{3/2}}-\frac{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}{2c}+\frac{\tanh^{-1}\left(\frac{2a+(b+2c)\coth^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^5/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] ((b - 2*c)*ArcTanh[(b + 2*c*Coth[x]^2)/(2*Sqrt[c]*Sqrt[a + b*Coth[x]^2 + c*
Coth[x]^4])])/(4*c^(3/2)) + ArcTanh[(2*a + b + (b + 2*c)*Coth[x]^2)/(2*Sqrt
[a + b + c]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4])]/(2*Sqrt[a + b + c]) - Sqr
t[a + b*Coth[x]^2 + c*Coth[x]^4]/(2*c)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 3782

```
Int[cot[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*(cot[(d_.) + (e_.)*(x_)]*(
f_.))^(n_.) + (c_.)*(cot[(d_.) + (e_.)*(x_)]*(f_.))^(n2_.))^(p_), x_Symbol]
 :> Dist[-f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x
], x, f*Cot[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n
2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\coth^5(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = -\text{Subst}\left(\int \frac{x^5}{(1+x^2)\sqrt{a-bx^2+cx^4}}\,dx, x, -i\coth(x)\right)$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(1+x)\sqrt{a-bx+cx^2}}\,dx, x, -\coth^2(x)\right)\right)$$

$$= -\frac{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}{2c} - \frac{\text{Subst}\left(\int \frac{\frac{b}{2}+\frac{1}{2}(b-2c)x}{(1+x)\sqrt{a-bx+cx^2}}\,d\right.}{2c}$$

$$= -\frac{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}{2c} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a-bx+c}}\right.$$

$$= -\frac{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}{2c} - \frac{(b-2c)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, -\right.}{2}$$

$$= \frac{(b-2c)\tanh^{-1}\left(\frac{b+2c\coth^2(x)}{2\sqrt{c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{4c^{3/2}} + \frac{\tanh^{-1}\left(\right.}{}$$

Mathematica [A]
time = 7.09, size = 266, normalized size = 1.97

$$\frac{\frac{2\left(2c^{3/2}\tanh^{-1}\left(\frac{-a+c+(a+b+c)\cosh(2x)}{2\sqrt{a+b+c}\sqrt{c+(b+2c)\sinh^2(x)+(a+b+c)\sinh^4(x)}}\right)+(b-2c)\sqrt{a+b+c}\tanh^{-1}\left(\frac{2c+(b+2c)\sinh^2(x)}{2\sqrt{c}\sqrt{c+(b+2c)\sinh^2(x)+(a+b+c)\sinh^4(x)}}\right)\right)\sqrt{3a-b+3c-4(a-c)\cosh(2x)+(a+b+c)\cosh(4x)}\,\text{csch}^2(x)}{\sqrt{a+b+c}}-\sqrt{2}\sqrt{c}\,(3a-b+3c-4(a-c)\cosh(2x)+(a+b+c)\cosh(4x))\text{csch}^4(x)}{8c^{3/2}\sqrt{(3a-b+3c-4(a-c)\cosh(2x)+(a+b+c)\cosh(4x))\text{csch}^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^5/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] ((2*(2*c^(3/2)*ArcTanh[(-a + c + (a + b + c)*Cosh[2*x])/(2*Sqrt[a + b + c]*
Sqrt[c + (b + 2*c)*Sinh[x]^2 + (a + b + c)*Sinh[x]^4])] + (b - 2*c)*Sqrt[a
+ b + c]*ArcTanh[(2*c + (b + 2*c)*Sinh[x]^2)/(2*Sqrt[c]*Sqrt[c + (b + 2*c)*
Sinh[x]^2 + (a + b + c)*Sinh[x]^4])])*Sqrt[3*a - b + 3*c - 4*(a - c)*Cosh[2
*x] + (a + b + c)*Cosh[4*x]]*Csch[x]^2)/Sqrt[a + b + c] - Sqrt[2]*Sqrt[c]*(
3*a - b + 3*c - 4*(a - c)*Cosh[2*x] + (a + b + c)*Cosh[4*x])*Csch[x]^4)/(8*
c^(3/2)*Sqrt[(3*a - b + 3*c - 4*(a - c)*Cosh[2*x] + (a + b + c)*Cosh[4*x])*
Csch[x]^4])
```

Maple [A]
time = 1.29, size = 149, normalized size = 1.10

method	result
derivativedivides	$-\frac{\sqrt{a+b\left(\coth^2(x)\right)+c\left(\coth^4(x)\right)}}{2c}+\frac{b\ln\left(\frac{\frac{b}{2}+c\left(\coth^2(x)\right)}{\sqrt{c}}+\sqrt{a+b\left(\coth^2(x)\right)+c\left(\coth^4(\right.}\right.}{4c^{\frac{3}{2}}}$
default	$-\frac{\sqrt{a+b\left(\coth^2(x)\right)+c\left(\coth^4(x)\right)}}{2c}+\frac{b\ln\left(\frac{\frac{b}{2}+c\left(\coth^2(x)\right)}{\sqrt{c}}+\sqrt{a+b\left(\coth^2(x)\right)+c\left(\coth^4(\right.}\right.}{4c^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^5/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(a+b*coth(x)^2+c*coth(x)^4)^(1/2)/c+1/4*b/c^(3/2)*ln((1/2*b+c*coth(x)^
2)/c^(1/2)+(a+b*coth(x)^2+c*coth(x)^4)^(1/2))-1/2*ln((1/2*b+c*coth(x)^2)/c^
(1/2)+(a+b*coth(x)^2+c*coth(x)^4)^(1/2))/c^(1/2)+1/2/(a+b+c)^(1/2)*arctanh(
1/2*(b*coth(x)^2+2*c*coth(x)^2+2*a+b)/(a+b+c)^(1/2)/(a+b*coth(x)^2+c*coth(x
)^4)^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(coth(x)^5/sqrt(c*coth(x)^4 + b*coth(x)^2 + a), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2086 vs. $2(111) = 222$.
time = 1.25, size = 8951, normalized size = 66.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/8*(((a*b + b^2 - (2*a + b)*c - 2*c^2)*cosh(x)^4 + 4*(a*b + b^2 - (2*a +
 b)*c - 2*c^2)*cosh(x)*sinh(x)^3 + (a*b + b^2 - (2*a + b)*c - 2*c^2)*sinh(x
)^4 - 2*(a*b + b^2 - (2*a + b)*c - 2*c^2)*cosh(x)^2 + 2*(3*(a*b + b^2 - (2*
```

```
a + b)*c - 2*c^2)*cosh(x)^2 - a*b - b^2 + (2*a + b)*c + 2*c^2)*sinh(x)^2 +
a*b + b^2 - (2*a + b)*c - 2*c^2 + 4*((a*b + b^2 - (2*a + b)*c - 2*c^2)*cosh
(x)^3 - (a*b + b^2 - (2*a + b)*c - 2*c^2)*cosh(x))*sinh(x))*sqrt(c)*log(((b
^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^8 + 8*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cos
h(x)*sinh(x)^7 + (b^2 + 4*(a + 2*b)*c + 8*c^2)*sinh(x)^8 - 4*(b^2 + 4*a*c -
 8*c^2)*cosh(x)^6 + 4*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^2 - b^2 - 4*
a*c + 8*c^2)*sinh(x)^6 + 8*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^3 - 3*(
b^2 + 4*a*c - 8*c^2)*cosh(x))*sinh(x)^5 + 2*(3*b^2 + 4*(3*a - 2*b)*c + 24*c
^2)*cosh(x)^4 + 2*(35*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^4 - 30*(b^2 + 4
*a*c - 8*c^2)*cosh(x)^2 + 3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*sinh(x)^4 + 8*(
7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^5 - 10*(b^2 + 4*a*c - 8*c^2)*cosh(x
)^3 + (3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*cosh(x))*sinh(x)^3 - 4*(b^2 + 4*a*
c - 8*c^2)*cosh(x)^2 + 4*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^6 - 15*(b
^2 + 4*a*c - 8*c^2)*cosh(x)^4 + 3*(3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*cosh(x
)^2 - b^2 - 4*a*c + 8*c^2)*sinh(x)^2 - 4*sqrt(2)*((b + 2*c)*cosh(x)^4 + 4*(
b + 2*c)*cosh(x)*sinh(x)^3 + (b + 2*c)*sinh(x)^4 - 2*(b - 2*c)*cosh(x)^2 +
2*(3*(b + 2*c)*cosh(x)^2 - b + 2*c)*sinh(x)^2 + 4*((b + 2*c)*cosh(x)^3 - (b
 - 2*c)*cosh(x))*sinh(x) + b + 2*c)*sqrt(c)*sqrt(((a + b + c)*cosh(x)^4 + (
a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2
*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*c
osh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + b^2 + 4*(a + 2*b)*
c + 8*c^2 + 8*((b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^7 - 3*(b^2 + 4*a*c - 8
*c^2)*cosh(x)^5 + (3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*cosh(x)^3 - (b^2 + 4*a
*c - 8*c^2)*cosh(x))*sinh(x))/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8
 + 4*(7*cosh(x)^2 - 1)*sinh(x)^6 - 4*cosh(x)^6 + 8*(7*cosh(x)^3 - 3*cosh(x))
*sinh(x)^5 + 2*(35*cosh(x)^4 - 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 +
8*(7*cosh(x)^5 - 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 - 15*
cosh(x)^4 + 9*cosh(x)^2 - 1)*sinh(x)^2 - 4*cosh(x)^2 + 8*(cosh(x)^7 - 3*cos
h(x)^5 + 3*cosh(x)^3 - cosh(x))*sinh(x) + 1)) - 2*(c^2*cosh(x)^4 + 4*c^2*co
sh(x)*sinh(x)^3 + c^2*sinh(x)^4 - 2*c^2*cosh(x)^2 + 2*(3*c^2*cosh(x)^2 - c^
2)*sinh(x)^2 + c^2 + 4*(c^2*cosh(x)^3 - c^2*cosh(x))*sinh(x))*sqrt(a + b +
c)*log(((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*b
+ b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a +
b)*c + c^2)*sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 + 2
*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x)^
6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3 - 3*(a^2 + a*b -
 b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*co
sh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4 - 30*(a^2
 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*sinh(x
)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 - 10*(a^2 + a*
b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x))*s
inh(x)^3 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 +
2*(a + b)*c + c^2)*cosh(x)^6 - 15*(a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(3*
a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(
x)^2 + sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 + (
```

```
a + b + c)*sinh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - a
 + c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 - (a - c)*cosh(x))*sinh(x) + a +
 b + c)*sqrt(a + b + c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4
 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2
+ 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 -
 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2
+ 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7 - 3*(a^2 + a*b - b*c
 - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^3 - (a^2
+ a*b - b*c - c^2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*c
osh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + 4*sqrt(2)*((a + b)
*c + c^2)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*c
osh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*
c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sin
h(x)^3 + sinh(x)^4)))/(((a + b)*c^2 + c^3)*cosh(x)^4 + 4*((a + b)*c^2 + c^3
)*cosh(x)*sinh(x)^3 + ((a + b)*c^2 + c^3)*sinh(x)^4 + (a + b)*c^2 + c^3 - 2
*((a + b)*c^2 + c^3)*cosh(x)^2 - 2*((a + b)*c^2 + c^3 - 3*((a + b)*c^2 + c^
3)*cosh(x)^2)*sinh(x)^2 + 4*(((a + b)*c^2 + c^3)*cosh(x)^3 - ((a + b)*c^2 +
 c^3)*cosh(x))*sinh(x)), -1/8*(4*(c^2*cosh(x)^4 + 4*c^2*cosh(x)*sinh(x)^3 +
 c^2*sinh(x)^4 - 2*c^2*cosh(x)^2 + 2*(3*c^2*cos...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^5(x)}{\sqrt{a + b\coth^2(x) + c\coth^4(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**5/(a+b*coth(x)**2+c*coth(x)**4)**(1/2),x)
[Out] Integral(coth(x)**5/sqrt(a + b*coth(x)**2 + c*coth(x)**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="giac")
[Out] integrate(coth(x)^5/sqrt(c*coth(x)^4 + b*coth(x)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(x)^5}{\sqrt{c\coth(x)^4 + b\coth(x)^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^5/(a + b*coth(x)^2 + c*coth(x)^4)^(1/2),x)
[Out] int(coth(x)^5/(a + b*coth(x)^2 + c*coth(x)^4)^(1/2), x)
```

3.206 $\int \frac{\coth^3(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx$

Optimal. Leaf size=105

$$-\frac{\tanh^{-1}\left(\frac{b+2c\coth^2(x)}{2\sqrt{c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{c}}+\frac{\tanh^{-1}\left(\frac{2a+b+(b+2c)\coth^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}}$$

```
[Out] -1/2*arctanh(1/2*(b+2*c*coth(x)^2)/c^(1/2)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2
))/c^(1/2)+1/2*arctanh(1/2*(2*a+b+(b+2*c)*coth(x)^2)/(a+b+c)^(1/2)/(a+b*cot
h(x)^2+c*coth(x)^4)^(1/2))/(a+b+c)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3782, 1265, 857, 635, 212, 738}

$$\frac{\tanh^{-1}\left(\frac{2a+(b+2c)\coth^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}}-\frac{\tanh^{-1}\left(\frac{b+2c\coth^2(x)}{2\sqrt{c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] -1/2*ArcTanh[(b + 2*c*Coth[x]^2)/(2*Sqrt[c]*Sqrt[a + b*Coth[x]^2 + c*Coth[x
]^4])]/Sqrt[c] + ArcTanh[(2*a + b + (b + 2*c)*Coth[x]^2)/(2*Sqrt[a + b + c]
*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4])]/(2*Sqrt[a + b + c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 3782

```
Int[cot[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*(cot[(d_.) + (e_.)*(x_)]*(
f_.))^(n_.) + (c_.)*(cot[(d_.) + (e_.)*(x_)]*(f_.))^(n2_.))^(p_), x_Symbol]
 :> Dist[-f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x
], x, f*Cot[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n
2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\coth^3(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = \text{Subst}\left(\int \frac{x^3}{(1+x^2)\sqrt{a-bx^2+cx^4}}\,dx, x, -i\coth(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(1+x)\sqrt{a-bx+cx^2}}\,dx, x, -\coth^2(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{a-bx+cx^2}}\,dx, x, -\coth^2(x)\right) - \frac{1}{2}\text{Subst}\left(\int \frac{}{(1+}\right.$$

$$= \text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{-b-2c\coth^2(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right) + \text{Subst}$$

$$= \frac{\tanh^{-1}\left(\frac{-b-2c\coth^2(x)}{2\sqrt{c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{c}} + \frac{\tanh^{-1}\left(\frac{}{2\sqrt{a+}}\right.}{}$$

Mathematica [A]
time = 98.24, size = 199, normalized size = 1.90

$$\frac{\left(\frac{\tanh^{-1}\left(\frac{-a+c+(a+b+c)\cosh(2x)}{2\sqrt{a+b+c}\sqrt{c+(b+2c)\sinh^2(x)+(a+b+c)\sinh^4(x)}}\right)}{\sqrt{a+b+c}}-\frac{\tanh^{-1}\left(\frac{2c+(b+2c)\sinh^2(x)}{2\sqrt{c}\sqrt{c+(b+2c)\sinh^2(x)+(a+b+c)\sinh^4(x)}}\right)}{\sqrt{c}}\right)\sqrt{3a-b+3c-4(a-c)\cosh(2x)+(a+b+c)\cosh(4x)}\,\text{csch}^2(x)}{2\sqrt{(3a-b+3c-4(a-c)\cosh(2x)+(a+b+c)\cosh(4x))\text{csch}^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] ((ArcTanh[(-a + c + (a + b + c)*Cosh[2*x])/(2*Sqrt[a + b + c]*Sqrt[c + (b + 
 2*c)*Sinh[x]^2 + (a + b + c)*Sinh[x]^4])]/Sqrt[a + b + c] - ArcTanh[(2*c + 
 (b + 2*c)*Sinh[x]^2)/(2*Sqrt[c]*Sqrt[c + (b + 2*c)*Sinh[x]^2 + (a + b + c)
*Sinh[x]^4])]/Sqrt[c])*Sqrt[3*a - b + 3*c - 4*(a - c)*Cosh[2*x] + (a + b + 
c)*Cosh[4*x]]*Csch[x]^2)/(2*Sqrt[(3*a - b + 3*c - 4*(a - c)*Cosh[2*x] + (a 
+ b + c)*Cosh[4*x])*Csch[x]^4])
```

Maple [A]
time = 1.20, size = 90, normalized size = 0.86

method	result
derivativedivides	$-\frac{\ln\left(\frac{\frac{b}{2}+c\left(\coth^2(x)\right)}{\sqrt{c}}+\sqrt{a+b\left(\coth^2(x)\right)+c\left(\coth^4(x)\right)}\right)}{2\sqrt{c}}+\frac{\text{arctanh}\left(\frac{b\left(\coth^2(x)\right)+2c}{2\sqrt{a+b+c}\sqrt{a+b\left(c\right.}}\right.}{2\sqrt{a+b}}$
default	$-\frac{\ln\left(\frac{\frac{b}{2}+c\left(\coth^2(x)\right)}{\sqrt{c}}+\sqrt{a+b\left(\coth^2(x)\right)+c\left(\coth^4(x)\right)}\right)}{2\sqrt{c}}+\frac{\text{arctanh}\left(\frac{b\left(\coth^2(x)\right)+2c}{2\sqrt{a+b+c}\sqrt{a+b\left(c\right.}}\right.}{2\sqrt{a+b}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln((1/2*b+c*coth(x)^2)/c^(1/2)+(a+b*coth(x)^2+c*coth(x)^4)^(1/2))/c^(1
/2)+1/2/(a+b+c)^(1/2)*arctanh(1/2*(b*coth(x)^2+2*c*coth(x)^2+2*a+b)/(a+b+c)
^(1/2)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(coth(x)^3/sqrt(c*coth(x)^4 + b*coth(x)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1522 vs. $2(85) = 170$.
time = 1.01, size = 6695, normalized size = 63.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/4*((a + b + c)*sqrt(c)*log(((b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^8 + 8*
(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)*sinh(x)^7 + (b^2 + 4*(a + 2*b)*c + 8*
c^2)*sinh(x)^8 - 4*(b^2 + 4*a*c - 8*c^2)*cosh(x)^6 + 4*(7*(b^2 + 4*(a + 2*b
)*c + 8*c^2)*cosh(x)^2 - b^2 - 4*a*c + 8*c^2)*sinh(x)^6 + 8*(7*(b^2 + 4*(a
+ 2*b)*c + 8*c^2)*cosh(x)^3 - 3*(b^2 + 4*a*c - 8*c^2)*cosh(x))*sinh(x)^5 +
2*(3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*cosh(x)^4 + 2*(35*(b^2 + 4*(a + 2*b)*c
 + 8*c^2)*cosh(x)^4 - 30*(b^2 + 4*a*c - 8*c^2)*cosh(x)^2 + 3*b^2 + 4*(3*a -
 2*b)*c + 24*c^2)*sinh(x)^4 + 8*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^5
- 10*(b^2 + 4*a*c - 8*c^2)*cosh(x)^3 + (3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*c
osh(x))*sinh(x)^3 - 4*(b^2 + 4*a*c - 8*c^2)*cosh(x)^2 + 4*(7*(b^2 + 4*(a +
2*b)*c + 8*c^2)*cosh(x)^6 - 15*(b^2 + 4*a*c - 8*c^2)*cosh(x)^4 + 3*(3*b^2 +
 4*(3*a - 2*b)*c + 24*c^2)*cosh(x)^2 - b^2 - 4*a*c + 8*c^2)*sinh(x)^2 - 4*s
qrt(2)*((b + 2*c)*cosh(x)^4 + 4*(b + 2*c)*cosh(x)*sinh(x)^3 + (b + 2*c)*sin
h(x)^4 - 2*(b - 2*c)*cosh(x)^2 + 2*(3*(b + 2*c)*cosh(x)^2 - b + 2*c)*sinh(x
)^2 + 4*((b + 2*c)*cosh(x)^3 - (b - 2*c)*cosh(x))*sinh(x) + b + 2*c)*sqrt(c
)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2
 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh
(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 +
 sinh(x)^4)) + b^2 + 4*(a + 2*b)*c + 8*c^2 + 8*((b^2 + 4*(a + 2*b)*c + 8*c^
2)*cosh(x)^7 - 3*(b^2 + 4*a*c - 8*c^2)*cosh(x)^5 + (3*b^2 + 4*(3*a - 2*b)*c
 + 24*c^2)*cosh(x)^3 - (b^2 + 4*a*c - 8*c^2)*cosh(x))*sinh(x))/(cosh(x)^8 +
 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 - 1)*sinh(x)^6 - 4*cosh(x
)^6 + 8*(7*cosh(x)^3 - 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 - 30*cosh(x)^
2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cosh(x)^5 - 10*cosh(x)^3 + 3*cosh(x))
*sinh(x)^3 + 4*(7*cosh(x)^6 - 15*cosh(x)^4 + 9*cosh(x)^2 - 1)*sinh(x)^2 - 4
*cosh(x)^2 + 8*(cosh(x)^7 - 3*cosh(x)^5 + 3*cosh(x)^3 - cosh(x))*sinh(x) +
1)) + sqrt(a + b + c)*c*log(((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x
)^8 + 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 +
2*a*b + b^2 + 2*(a + b)*c + c^2)*sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*cosh
(x)^6 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 - a^2 - a*b
+ b*c + c^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(
```

```
x)^3 - 3*(a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 2*
(a + b)*c + 3*c^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2
)*cosh(x)^4 - 30*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b + 2*(a +
 b)*c + 3*c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cos
h(x)^5 - 10*(a^2 + a*b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a + b)*
c + 3*c^2)*cosh(x))*sinh(x)^3 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*
(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^6 - 15*(a^2 + a*b - b*c - c
^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^2 - a^2 - a
*b + b*c + c^2)*sinh(x)^2 + sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*
cosh(x)*sinh(x)^3 + (a + b + c)*sinh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(3*(a +
 b + c)*cosh(x)^2 - a + c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 - (a - c)*c
osh(x))*sinh(x) + a + b + c)*sqrt(a + b + c)*sqrt(((a + b + c)*cosh(x)^4 +
(a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 -
2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*
cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b + b^2
 + 2*(a + b)*c + c^2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7
 - 3*(a^2 + a*b - b*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c
^2)*cosh(x)^3 - (a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*co
sh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4))
)/((a + b)*c + c^2), -1/4*(2*sqrt(-a - b - c)*c*arctan(sqrt(2)*((a + b + c)
*cosh(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 + (a + b + c)*sinh(x)^4 - 2*(a
 - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - a + c)*sinh(x)^2 + 4*((a + b
 + c)*cosh(x)^3 - (a - c)*cosh(x))*sinh(x) + a + b + c)*sqrt(-a - b - c)*sq
rt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2
*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^
4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sin
h(x)^4))/((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*
b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a
+ b)*c + c^2)*sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 +
 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x
)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3 - 3*(a^2 + a*b
 - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c +
 3*c^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4
 - 30*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b - b^2 + 2*(3*a + b)
*c + 3*c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x
)^5 - 10*(a^2 + a*b - b*c - c^2)*cosh(x)^3 + (3...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(x)}{\sqrt{a + b\coth^2(x) + c\coth^4(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(a+b*coth(x)**2+c*coth(x)**4)**(1/2),x)
```

```
[Out] Integral(coth(x)**3/sqrt(a + b*coth(x)**2 + c*coth(x)**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="giac")
[Out] integrate(coth(x)^3/sqrt(c*coth(x)^4 + b*coth(x)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(x)^3}{\sqrt{c\coth\left(x\right)^4+b\coth\left(x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a + b*coth(x)^2 + c*coth(x)^4)^(1/2),x)
[Out] int(coth(x)^3/(a + b*coth(x)^2 + c*coth(x)^4)^(1/2), x)
```

3.207 $\int \frac{\coth(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx$

Optimal. Leaf size=58

$$\frac{\tanh^{-1}\left(\frac{2a+b+(b+2c)\coth^2(x)}{2\sqrt{a+b+c}\,\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}}$$

```
[Out] 1/2*arctanh(1/2*(2*a+b+(b+2*c)*coth(x)^2)/(a+b+c)^(1/2)/(a+b*coth(x)^2+c*co
th(x)^4)^(1/2))/(a+b+c)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3782, 1261, 738, 212}

$$\frac{\tanh^{-1}\left(\frac{2a+(b+2c)\coth^2(x)+b}{2\sqrt{a+b+c}\,\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] ArcTanh[(2*a + b + (b + 2*c)*Coth[x]^2)/(2*Sqrt[a + b + c]*Sqrt[a + b*Coth[
x]^2 + c*Coth[x]^4])]/(2*Sqrt[a + b + c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
```

```
x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 3782

```
Int[cot[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*(cot[(d_.) + (e_.)*(x_)]*(
f_.))^(n_.) + (c_.)*(cot[(d_.) + (e_.)*(x_)]*(f_.))^(n2_.))^(p_), x_Symbol]
 :> Dist[-f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x
], x, f*Cot[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n
2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx &= -\text{Subst}\left(\int \frac{x}{(1+x^2)\sqrt{a-bx^2+cx^4}}\,dx, x, -i\coth(x)\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a-bx+cx^2}}\,dx, x, -\coth^2(x)\right)\right)\\ &= \text{Subst}\left(\int \frac{1}{4a+4b+4c-x^2}\,dx, x, \frac{2a+b+(b+2c)\coth^2(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{2a+b+(b+2c)\coth^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 141 vs. 2(58) = 116.
time = 53.72, size = 141, normalized size = 2.43

$$\frac{\tanh^{-1}\left(\frac{-a+c+(a+b+c)\cosh(2x)}{2\sqrt{a+b+c}\sqrt{c+(b+2c)\sinh^2(x)+(a+b+c)\sinh^4(x)}}\right)\sqrt{3a-b+3c-4(a-c)\cosh(2x)+(a+b+c)\cosh(4x)}\,\text{csch}^2(x)}{2\sqrt{a+b+c}\sqrt{(3a-b+3c-4(a-c)\cosh(2x)+(a+b+c)\cosh(4x))\text{csch}^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] (ArcTanh[(-a + c + (a + b + c)*Cosh[2*x])/(2*Sqrt[a + b + c]*Sqrt[c + (b +
2*c)*Sinh[x]^2 + (a + b + c)*Sinh[x]^4])]*Sqrt[3*a - b + 3*c - 4*(a - c)*Co
sh[2*x] + (a + b + c)*Cosh[4*x]]*Csch[x]^2)/(2*Sqrt[a + b + c]*Sqrt[(3*a -
b + 3*c - 4*(a - c)*Cosh[2*x] + (a + b + c)*Cosh[4*x])*Csch[x]^4])
```

Maple [A]
time = 1.18, size = 52, normalized size = 0.90

method	result	size
derivativedivides	$\frac{\operatorname{arctanh}\left(\frac{b\left(\coth^2(x)\right)+2c\left(\coth^2(x)\right)+2a+b}{2\sqrt{a+b+c}\,\sqrt{a+b\left(\coth^2(x)\right)+c\left(\coth^4(x)\right)}}\right)}{2\sqrt{a+b+c}}$	52
default	$\frac{\operatorname{arctanh}\left(\frac{b\left(\coth^2(x)\right)+2c\left(\coth^2(x)\right)+2a+b}{2\sqrt{a+b+c}\,\sqrt{a+b\left(\coth^2(x)\right)+c\left(\coth^4(x)\right)}}\right)}{2\sqrt{a+b+c}}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(a+b+c)^(1/2)*arctanh(1/2*(b*coth(x)^2+2*c*coth(x)^2+2*a+b)/(a+b+c)^(1/
2)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)/sqrt(c*coth(x)^4 + b*coth(x)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 884 vs. $2(48) = 96$.
time = 0.78, size = 1752, normalized size = 30.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*
b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a
+ b)*c + c^2)*sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 +
 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x
)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3 - 3*(a^2 + a*b
 - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*
```

```
cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4 - 30*(a
^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*sinh
(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 - 10*(a^2 +
a*b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x))
*sinh(x)^3 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2
+ 2*(a + b)*c + c^2)*cosh(x)^6 - 15*(a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(
3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sin
h(x)^2 + sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 +
 (a + b + c)*sinh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 -
 a + c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 - (a - c)*cosh(x))*sinh(x) + a
 + b + c)*sqrt(a + b + c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)
^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^
2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2
 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^
2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7 - 3*(a^2 + a*b - b
*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^3 - (a^
2 + a*b - b*c - c^2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6
*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/sqrt(a + b + c), -
1/2*sqrt(-a - b - c)*arctan(sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*
cosh(x)*sinh(x)^3 + (a + b + c)*sinh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(3*(a +
 b + c)*cosh(x)^2 - a + c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 - (a - c)*c
osh(x))*sinh(x) + a + b + c)*sqrt(-a - b - c)*sqrt(((a + b + c)*cosh(x)^4 +
 (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 -
 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6
*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/((a^2 + 2*a*b + b^
2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2
)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*sinh(x)^8 - 4
*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c
+ c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b
^2 + 2*(a + b)*c + c^2)*cosh(x)^3 - 3*(a^2 + a*b - b*c - c^2)*cosh(x))*sinh
(x)^5 + 2*(3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^4 + 2*(35*(
a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4 - 30*(a^2 + a*b - b*c - c^
2)*cosh(x)^2 + 3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*sinh(x)^4 + 8*(
7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 - 10*(a^2 + a*b - b*c -
 c^2)*cosh(x)^3 + (3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x))*si
nh(x)^3 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 + 2
*(a + b)*c + c^2)*cosh(x)^6 - 15*(a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(3*a
^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2
)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2 + 8*((a^2 + 2*a*b + b^2
 + 2*(a + b)*c + c^2)*cosh(x)^7 - 3*(a^2 + a*b - b*c - c^2)*cosh(x)^5 + (3*
a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^3 - (a^2 + a*b - b*c - c
^2)*cosh(x))*sinh(x)))/(a + b + c)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*coth(x)**2+c*coth(x)**4)**(1/2),x)
```

```
[Out] Integral(coth(x)/sqrt(a + b*coth(x)**2 + c*coth(x)**4), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\coth(x)}{\sqrt{c\coth(x)^4+b\coth(x)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a + b*coth(x)^2 + c*coth(x)^4)^(1/2),x)
```

```
[Out] int(coth(x)/(a + b*coth(x)^2 + c*coth(x)^4)^(1/2), x)
```

3.208 $\int \frac{\tanh(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx$

Optimal. Leaf size=106

$$-\frac{\tanh^{-1}\left(\frac{2a+b\coth^2(x)}{2\sqrt{a}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a}}+\frac{\tanh^{-1}\left(\frac{2a+b+(b+2c)\coth^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}}$$

```
[Out] -1/2*arctanh(1/2*(2*a+b*coth(x)^2)/a^(1/2)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2
))/a^(1/2)+1/2*arctanh(1/2*(2*a+b+(b+2*c)*coth(x)^2)/(a+b+c)^(1/2)/(a+b*cot
h(x)^2+c*coth(x)^4)^(1/2))/(a+b+c)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3782, 1265, 974, 738, 212}

$$\frac{\tanh^{-1}\left(\frac{2a+(b+2c)\coth^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}}-\frac{\tanh^{-1}\left(\frac{2a+b\coth^2(x)}{2\sqrt{a}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] -1/2*ArcTanh[(2*a + b*Coth[x]^2)/(2*Sqrt[a]*Sqrt[a + b*Coth[x]^2 + c*Coth[x
]^4])]/Sqrt[a] + ArcTanh[(2*a + b + (b + 2*c)*Coth[x]^2)/(2*Sqrt[a + b + c]
*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4])]/(2*Sqrt[a + b + c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 3782

```
Int[cot[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*(cot[(d_.) + (e_.)*(x_)]*(
f_.))^(n_.) + (c_.)*(cot[(d_.) + (e_.)*(x_)]*(f_.))^(n2_.))^(p_), x_Symbol]
 :> Dist[-f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x
], x, f*Cot[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n
2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\tanh(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = \text{Subst}\left(\int \frac{1}{x\left(1+x^2\right)\sqrt{a-bx^2+cx^4}}\,dx, x, -i\coth(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+x)\sqrt{a-bx+cx^2}}\,dx, x, -\coth^2(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{(-1-x)\sqrt{a-bx+cx^2}}+\frac{1}{x\sqrt{a-bx+cx^2}}\right)dx, x,$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(-1-x)\sqrt{a-bx+cx^2}}\,dx, x, -\coth^2(x)\right)+\frac{1}{2}\text{Subst}\Bigg($$

$$= -\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+b\coth^2(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right) - \text{Subst}$$

$$= -\frac{\tanh^{-1}\left(\frac{2a+b\coth^2(x)}{2\sqrt{a}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a}} - \frac{\tanh^{-1}\Bigg(\frac{}{2\sqrt{a+}}}{}$$

Mathematica [A]
time = 6.29, size = 203, normalized size = 1.92

$$-\frac{\left(-\frac{\tanh^{-1}\left(\frac{2a-(2a+b)\cosh^2(x)}{2\sqrt{a}\sqrt{a-(2a+b)\cosh^2(x)+(a+b+c)\cosh^4(x)}}\right)}{\sqrt{a}}-\frac{\tanh^{-1}\left(\frac{-a+c+(a+b+c)\cosh(2x)}{2\sqrt{a+b+c}\sqrt{a-(2a+b)\cosh^2(x)+(a+b+c)\cosh^4(x)}}\right)}{\sqrt{a+b+c}}\right)\sqrt{3a-b+3c-4(a-c)\cosh(2x)+(a+b+c)\cosh(4x)}\operatorname{csch}^2(x)}{2\sqrt{(3a-b+3c-4(a-c)\cosh(2x)+(a+b+c)\cosh(4x))\operatorname{csch}^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] -1/2*((-(ArcTanh[(2*a - (2*a + b)*Cosh[x]^2)/(2*Sqrt[a]*Sqrt[a - (2*a + b)*
Cosh[x]^2 + (a + b + c)*Cosh[x]^4])]/Sqrt[a]) - ArcTanh[(-a + c + (a + b +
c)*Cosh[2*x])/(2*Sqrt[a + b + c]*Sqrt[a - (2*a + b)*Cosh[x]^2 + (a + b + c)
*Cosh[x]^4])]/Sqrt[a + b + c])*Sqrt[3*a - b + 3*c - 4*(a - c)*Cosh[2*x] + (
a + b + c)*Cosh[4*x]]*Csch[x]^2)/Sqrt[(3*a - b + 3*c - 4*(a - c)*Cosh[2*x]
+ (a + b + c)*Cosh[4*x])*Csch[x]^4]
```

Maple [F]
time = 2.55, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{\sqrt{a+b\left(\coth^2(x)\right)+c\left(\coth^4(x)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x)
```

```
[Out] int(tanh(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)/sqrt(c*coth(x)^4 + b*coth(x)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1524 vs. 2(86) = 172.
time = 0.97, size = 6705, normalized size = 63.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="fricas")
[Out] [1/4*((a + b + c)*sqrt(a)*log(((8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^8 + 8*
(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)*sinh(x)^7 + (8*a^2 + 8*a*b + b^2 + 4*
a*c)*sinh(x)^8 - 4*(8*a^2 - b^2 - 4*a*c)*cosh(x)^6 + 4*(7*(8*a^2 + 8*a*b +
b^2 + 4*a*c)*cosh(x)^2 - 8*a^2 + b^2 + 4*a*c)*sinh(x)^6 + 8*(7*(8*a^2 + 8*a
*b + b^2 + 4*a*c)*cosh(x)^3 - 3*(8*a^2 - b^2 - 4*a*c)*cosh(x))*sinh(x)^5 +
2*(24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*cosh(x)^4 + 2*(35*(8*a^2 + 8*a*b + b^2
+ 4*a*c)*cosh(x)^4 - 30*(8*a^2 - b^2 - 4*a*c)*cosh(x)^2 + 24*a^2 - 8*a*b +
3*b^2 + 12*a*c)*sinh(x)^4 + 8*(7*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^5 -
10*(8*a^2 - b^2 - 4*a*c)*cosh(x)^3 + (24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*cosh
(x))*sinh(x)^3 - 4*(8*a^2 - b^2 - 4*a*c)*cosh(x)^2 + 4*(7*(8*a^2 + 8*a*b +
b^2 + 4*a*c)*cosh(x)^6 - 15*(8*a^2 - b^2 - 4*a*c)*cosh(x)^4 + 3*(24*a^2 - 8
*a*b + 3*b^2 + 12*a*c)*cosh(x)^2 - 8*a^2 + b^2 + 4*a*c)*sinh(x)^2 - 4*sqrt(
2)*((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)
^4 - 2*(2*a - b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 - 2*a + b)*sinh(x)^2
+ 4*((2*a + b)*cosh(x)^3 - (2*a - b)*cosh(x))*sinh(x) + 2*a + b)*sqrt(a)*sq
rt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2
*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^
4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sin
h(x)^4)) + 8*a^2 + 8*a*b + b^2 + 4*a*c + 8*((8*a^2 + 8*a*b + b^2 + 4*a*c)*c
osh(x)^7 - 3*(8*a^2 - b^2 - 4*a*c)*cosh(x)^5 + (24*a^2 - 8*a*b + 3*b^2 + 12
*a*c)*cosh(x)^3 - (8*a^2 - b^2 - 4*a*c)*cosh(x))*sinh(x))/(cosh(x)^8 + 8*co
sh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 + 1)*sinh(x)^6 + 4*cosh(x)^6 +
 8*(7*cosh(x)^3 + 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 + 30*cosh(x)^2 + 3
)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cosh(x)^5 + 10*cosh(x)^3 + 3*cosh(x))*sinh
(x)^3 + 4*(7*cosh(x)^6 + 15*cosh(x)^4 + 9*cosh(x)^2 + 1)*sinh(x)^2 + 4*cosh
(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^5 + 3*cosh(x)^3 + cosh(x))*sinh(x) + 1)) +
 sqrt(a + b + c)*a*log(((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 +
 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b
 + b^2 + 2*(a + b)*c + c^2)*sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6
 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 - a^2 - a*b + b*c
 + c^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3
- 3*(a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 2*(a +
b)*c + 3*c^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cos
h(x)^4 - 30*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b + 2*(a + b)*c
 + 3*c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^
5 - 10*(a^2 + a*b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3
*c^2)*cosh(x))*sinh(x)^3 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2
+ 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^6 - 15*(a^2 + a*b - b*c - c^2)*c
osh(x)^4 + 3*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^2 - a^2 - a*b +
b*c + c^2)*sinh(x)^2 + sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(
x)*sinh(x)^3 + (a + b + c)*sinh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(3*(a + b +
c)*cosh(x)^2 - a + c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 - (a - c)*cosh(x
))*sinh(x) + a + b + c)*sqrt(a + b + c)*sqrt(((a + b + c)*cosh(x)^4 + (a +
```

```
b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a +
 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(
x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b + b^2 + 2*
(a + b)*c + c^2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7 - 3*
(a^2 + a*b - b*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*c
osh(x)^3 - (a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)
^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)))/(a^
2 + a*b + a*c), -1/4*(2*a*sqrt(-a - b - c)*arctan(sqrt(2)*((a + b + c)*cosh
(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 + (a + b + c)*sinh(x)^4 - 2*(a - c)
*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - a + c)*sinh(x)^2 + 4*((a + b + c)
*cosh(x)^3 - (a - c)*cosh(x))*sinh(x) + a + b + c)*sqrt(-a - b - c)*sqrt(((
a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(
a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4
*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^
4))/((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b
^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a + b)*
c + c^2)*sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*
b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x)^6 +
 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3 - 3*(a^2 + a*b - b*
c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^
2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4 - 30
*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c +
3*c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 -
 10*(a^2 + a*b - b*c - c^2)*cosh(x)^3 + (3*a^2 ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{\sqrt{a + b\coth^2(x) + c\coth^4(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*coth(x)**2+c*coth(x)**4)**(1/2),x)
[Out] Integral(tanh(x)/sqrt(a + b*coth(x)**2 + c*coth(x)**4), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(x)}{\sqrt{c \coth(x)^4 + b \coth(x)^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a + b*coth(x)^2 + c*coth(x)^4)^(1/2),x)
[Out] int(tanh(x)/(a + b*coth(x)^2 + c*coth(x)^4)^(1/2), x)
```

3.209 $\int \frac{\tanh^3(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx$

Optimal. Leaf size=183

$$-\frac{\tanh^{-1}\left(\frac{2a+b\coth^2(x)}{2\sqrt{a}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a}}+\frac{b\tanh^{-1}\left(\frac{2a+b\coth^2(x)}{2\sqrt{a}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{4a^{3/2}}+\tanh$$

```
[Out] 1/4*b*arctanh(1/2*(2*a+b*coth(x)^2)/a^(1/2)/(a+b*coth(x)^2+c*coth(x)^4)^(1/
2))/a^(3/2)-1/2*arctanh(1/2*(2*a+b*coth(x)^2)/a^(1/2)/(a+b*coth(x)^2+c*coth
(x)^4)^(1/2))/a^(1/2)+1/2*arctanh(1/2*(2*a+b+(b+2*c)*coth(x)^2)/(a+b+c)^(1/
2)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2))/(a+b+c)^(1/2)-1/2*(a+b*coth(x)^2+c*co
th(x)^4)^(1/2)*tanh(x)^2/a
```

Rubi [A]
time = 0.24, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3782, 1265, 974, 744, 738, 212}

$$\frac{b\tanh^{-1}\left(\frac{2a+b\coth^2(x)}{2\sqrt{a}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{4a^{3/2}}-\frac{\tanh^{-1}\left(\frac{2a+b\coth^2(x)}{2\sqrt{a}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a}}+\frac{\tanh^{-1}\left(\frac{2a+(b+2c)\coth^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}}-\frac{\tanh^2(x)\sqrt{a+b\coth^2(x)+c\coth^4(x)}}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] -1/2*ArcTanh[(2*a + b*Coth[x]^2)/(2*Sqrt[a]*Sqrt[a + b*Coth[x]^2 + c*Coth[x
]^4])]/Sqrt[a] + (b*ArcTanh[(2*a + b*Coth[x]^2)/(2*Sqrt[a]*Sqrt[a + b*Coth[
x]^2 + c*Coth[x]^4])])/(4*a^(3/2)) + ArcTanh[(2*a + b + (b + 2*c)*Coth[x]^2
)/(2*Sqrt[a + b + c]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4])]/(2*Sqrt[a + b +
c]) - (Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4]*Tanh[x]^2)/(2*a)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 3782

```
Int[cot[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*(cot[(d_.) + (e_.)*(x_)]*(
f_.))^(n_.) + (c_.)*(cot[(d_.) + (e_.)*(x_)]*(f_.))^(n2_.))^(p_), x_Symbol]
 :> Dist[-f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x
], x, f*Cot[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n
2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^3(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx &= -\text{Subst}\left(\int \frac{1}{x^3\left(1+x^2\right)\sqrt{a-bx^2+cx^4}}\,dx,x,-i\coth(x)\right) \\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(1+x)\sqrt{a-bx+cx^2}}\,dx,x,-\coth^2(x)\right)\right) \\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x^2\sqrt{a-bx+cx^2}}-\frac{1}{x\sqrt{a-bx+cx^2}}+\frac{}{(1+x)}\right.\right.\right. \\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\sqrt{a-bx+cx^2}}\,dx,x,-\coth^2(x)\right)\right)+\frac{1}{2}\text{Subst}\Bigg(\\
&= -\frac{\sqrt{a+b\coth^2(x)+c\coth^4(x)}\,\tanh^2(x)}{2a}-\frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{a-bx+}}\right.}{} \\
&= -\frac{\tanh^{-1}\left(\frac{2a+b\coth^2(x)}{2\sqrt{a}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a}}+\frac{\tanh^{-1}\left(\frac{}{2\sqrt{a}}\right.}{} \\
&= -\frac{\tanh^{-1}\left(\frac{2a+b\coth^2(x)}{2\sqrt{a}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a}}+\frac{b\tanh^{-1}\left(\frac{}{2\sqrt{}}\right.}{}
\end{aligned}
$$

Mathematica [A]
time = 8.39, size = 278, normalized size = 1.52

$$
\frac{\left((2a-b)\sqrt{a+b+c}\tanh^{-1}\left(\frac{2a-(2a+b)\cosh^2(x)}{2\sqrt{a}\sqrt{a-(2a+b)\cosh^2(x)+(a+b+c)\cosh^4(x)}}\right)+2a^{3/2}\tanh^{-1}\left(\frac{-a+c+(a+b+c)\cosh(2x)}{2\sqrt{a+b+c}\sqrt{a-(2a+b)\cosh^2(x)+(a+b+c)\cosh^4(x)}}\right)\right)\sqrt{3a-b+3c-4(a-c)\cosh(2x)+(a+b+c)\cosh(4x)}\,\text{csch}^2(x)}{4a^{3/2}\sqrt{a+b+c}\sqrt{(3a-b+3c-4(a-c)\cosh(2x)+(a+b+c)\cosh(4x))\text{csch}^4(x)}}-\frac{\sqrt{(3a-b+3c-4(a-c)\cosh(2x)+(a+b+c)\cosh(4x))\text{csch}^4(x)}\,\tanh^2(x)}{4\sqrt{2}\,a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] (((2*a - b)*Sqrt[a + b + c]*ArcTanh[(2*a - (2*a + b)*Cosh[x]^2)/(2*Sqrt[a]*
Sqrt[a - (2*a + b)*Cosh[x]^2 + (a + b + c)*Cosh[x]^4])] + 2*a^(3/2)*ArcTanh
[(-a + c + (a + b + c)*Cosh[2*x])/(2*Sqrt[a + b + c]*Sqrt[a - (2*a + b)*Cos
h[x]^2 + (a + b + c)*Cosh[x]^4])])*Sqrt[3*a - b + 3*c - 4*(a - c)*Cosh[2*x]
 + (a + b + c)*Cosh[4*x]]*Csch[x]^2)/(4*a^(3/2)*Sqrt[a + b + c]*Sqrt[(3*a -
 b + 3*c - 4*(a - c)*Cosh[2*x] + (a + b + c)*Cosh[4*x])*Csch[x]^4]) - (Sqrt
[(3*a - b + 3*c - 4*(a - c)*Cosh[2*x] + (a + b + c)*Cosh[4*x])*Csch[x]^4]*T
anh[x]^2)/(4*Sqrt[2]*a)
```

Maple [F]
time = 2.53, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\left(\coth^2(x)\right)+c\left(\coth^4(x)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x)
```

```
[Out] int(tanh(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(tanh(x)^3/sqrt(c*coth(x)^4 + b*coth(x)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2135 vs. 2(149) = 298.
time = 1.18, size = 9148, normalized size = 49.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/8*(((2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x)^4 + 4*(2*a^2 + a*b - b^2
+ (2*a - b)*c)*cosh(x)*sinh(x)^3 + (2*a^2 + a*b - b^2 + (2*a - b)*c)*sinh(x
)^4 + 2*(2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x)^2 + 2*(3*(2*a^2 + a*b - b
^2 + (2*a - b)*c)*cosh(x)^2 + 2*a^2 + a*b - b^2 + (2*a - b)*c)*sinh(x)^2 +
2*a^2 + a*b - b^2 + (2*a - b)*c + 4*((2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh
(x)^3 + (2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x))*sinh(x))*sqrt(a)*log(((8
*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^8 + 8*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cos
h(x)*sinh(x)^7 + (8*a^2 + 8*a*b + b^2 + 4*a*c)*sinh(x)^8 - 4*(8*a^2 - b^2 -
 4*a*c)*cosh(x)^6 + 4*(7*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^2 - 8*a^2 +
b^2 + 4*a*c)*sinh(x)^6 + 8*(7*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^3 - 3*(
8*a^2 - b^2 - 4*a*c)*cosh(x))*sinh(x)^5 + 2*(24*a^2 - 8*a*b + 3*b^2 + 12*a*
c)*cosh(x)^4 + 2*(35*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^4 - 30*(8*a^2 -
b^2 - 4*a*c)*cosh(x)^2 + 24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*sinh(x)^4 + 8*(7*
```

```
(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^5 - 10*(8*a^2 - b^2 - 4*a*c)*cosh(x)^
3 + (24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*cosh(x))*sinh(x)^3 - 4*(8*a^2 - b^2 -
 4*a*c)*cosh(x)^2 + 4*(7*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^6 - 15*(8*a^
2 - b^2 - 4*a*c)*cosh(x)^4 + 3*(24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*cosh(x)^2
- 8*a^2 + b^2 + 4*a*c)*sinh(x)^2 + 4*sqrt(2)*((2*a + b)*cosh(x)^4 + 4*(2*a
+ b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 - 2*(2*a - b)*cosh(x)^2 + 2*(3
*(2*a + b)*cosh(x)^2 - 2*a + b)*sinh(x)^2 + 4*((2*a + b)*cosh(x)^3 - (2*a -
 b)*cosh(x))*sinh(x) + 2*a + b)*sqrt(a)*sqrt(((a + b + c)*cosh(x)^4 + (a +
b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a +
 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(
x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + 8*a^2 + 8*a*b + b^2 +
4*a*c + 8*((8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^7 - 3*(8*a^2 - b^2 - 4*a*c
)*cosh(x)^5 + (24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*cosh(x)^3 - (8*a^2 - b^2 -
4*a*c)*cosh(x))*sinh(x))/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(
7*cosh(x)^2 + 1)*sinh(x)^6 + 4*cosh(x)^6 + 8*(7*cosh(x)^3 + 3*cosh(x))*sinh
(x)^5 + 2*(35*cosh(x)^4 + 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*
cosh(x)^5 + 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 + 15*cosh(
x)^4 + 9*cosh(x)^2 + 1)*sinh(x)^2 + 4*cosh(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^
5 + 3*cosh(x)^3 + cosh(x))*sinh(x) + 1)) - 2*(a^2*cosh(x)^4 + 4*a^2*cosh(x)
*sinh(x)^3 + a^2*sinh(x)^4 + 2*a^2*cosh(x)^2 + 2*(3*a^2*cosh(x)^2 + a^2)*si
nh(x)^2 + a^2 + 4*(a^2*cosh(x)^3 + a^2*cosh(x))*sinh(x))*sqrt(a + b + c)*lo
g(((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2
 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a + b)*c
+ c^2)*sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b
+ b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x)^6 + 8
*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3 - 3*(a^2 + a*b - b*c
- c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)
^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4 - 30*(a^2 + a*
b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*sinh(x)^4 +
 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 - 10*(a^2 + a*b - b
*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x))*sinh(x
)^3 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a
+ b)*c + c^2)*cosh(x)^6 - 15*(a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(3*a^2 +
 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x)^2
+ sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 + (a + b
 + c)*sinh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - a + c)
*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 - (a - c)*cosh(x))*sinh(x) + a + b +
c)*sqrt(a + b + c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*
(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a
 - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*co
sh(x)*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2 + 8*(
(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7 - 3*(a^2 + a*b - b*c - c^
2)*cosh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^3 - (a^2 + a*b
 - b*c - c^2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x
)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + 4*sqrt(2)*(a^2 + a*b +
```

```
a*c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x
)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(c
osh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^
3 + sinh(x)^4)))/((a^3 + a^2*b + a^2*c)*cosh(x)^4 + 4*(a^3 + a^2*b + a^2*c)
*cosh(x)*sinh(x)^3 + (a^3 + a^2*b + a^2*c)*sinh(x)^4 + a^3 + a^2*b + a^2*c
+ 2*(a^3 + a^2*b + a^2*c)*cosh(x)^2 + 2*(a^3 + a^2*b + a^2*c + 3*(a^3 + a^2
*b + a^2*c)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + a^2*b + a^2*c)*cosh(x)^3 + (a^
3 + a^2*b + a^2*c)*cosh(x))*sinh(x)), -1/8*(4*(a^2*cosh(x)^4 + 4*a^2*cosh(x
)*sinh(x)^3 + a^2*sinh(x)^4 + 2*a^2*cosh(x)^2 +...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(x)}{\sqrt{a + b\coth^2(x) + c\coth^4(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(a+b*coth(x)**2+c*coth(x)**4)**(1/2),x)
[Out] Integral(tanh(x)**3/sqrt(a + b*coth(x)**2 + c*coth(x)**4), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(x)^3}{\sqrt{c\coth(x)^4 + b\coth(x)^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a + b*coth(x)^2 + c*coth(x)^4)^(1/2),x)
[Out] int(tanh(x)^3/(a + b*coth(x)^2 + c*coth(x)^4)^(1/2), x)
```

3.210 $\int \coth(x)\sqrt{a+b\coth^2(x)+c\coth^4(x)}\,dx$

Optimal. Leaf size=132

$$-\frac{(b+2c)\tanh^{-1}\left(\frac{b+2c\coth^2(x)}{2\sqrt{c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{4\sqrt{c}}+\frac{1}{2}\sqrt{a+b+c}\,\tanh^{-1}\left(\frac{2a+b+(b+2}{2\sqrt{a+b+c}\,\sqrt{a+b\,\mathrm{co}}}\right.$$

```
[Out] -1/4*(b+2*c)*arctanh(1/2*(b+2*c*coth(x)^2)/c^(1/2)/(a+b*coth(x)^2+c*coth(x)
^4)^(1/2))/c^(1/2)+1/2*arctanh(1/2*(2*a+b+(b+2*c)*coth(x)^2)/(a+b+c)^(1/2)/
(a+b*coth(x)^2+c*coth(x)^4)^(1/2))*(a+b+c)^(1/2)-1/2*(a+b*coth(x)^2+c*coth(
x)^4)^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3782, 1261, 748, 857, 635, 212, 738}

$$-\frac{1}{2}\sqrt{a+b\coth^2(x)+c\coth^4(x)}-\frac{(b+2c)\tanh^{-1}\left(\frac{b+2c\coth^2(x)}{2\sqrt{c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{4\sqrt{c}}+\frac{1}{2}\sqrt{a+b+c}\,\tanh^{-1}\left(\frac{2a+(b+2c)\coth^2(x)+b}{2\sqrt{a+b+c}\,\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] -1/4*((b + 2*c)*ArcTanh[(b + 2*c*Coth[x]^2)/(2*Sqrt[c]*Sqrt[a + b*Coth[x]^2
 + c*Coth[x]^4])])/Sqrt[c] + (Sqrt[a + b + c]*ArcTanh[(2*a + b + (b + 2*c)*
Coth[x]^2)/(2*Sqrt[a + b + c]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4])])/2 - Sq
rt[a + b*Coth[x]^2 + c*Coth[x]^4]/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 3782

```
Int[cot[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*(cot[(d_.) + (e_.)*(x_)]*(
f_.))^(n_.) + (c_.)*(cot[(d_.) + (e_.)*(x_)]*(f_.))^(n2_.))^(p_), x_Symbol]
 :> Dist[-f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x
], x, f*Cot[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n
2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \coth(x)\sqrt{a+b\coth^2(x)+c\coth^4(x)}\,dx &= -\text{Subst}\left(\int \frac{x\sqrt{a-bx^2+cx^4}}{1+x^2}\,dx, x, -i\coth(x)\right) \\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a-bx+cx^2}}{1+x}\,dx, x, -\coth^2(x)\right)\right) \\
&= -\frac{1}{2}\sqrt{a+b\coth^2(x)+c\coth^4(x)} + \frac{1}{4}\text{Subst}\left(\int \frac{-2a-b+}{(1+x)\sqrt{a}}\right. \\
&= -\frac{1}{2}\sqrt{a+b\coth^2(x)+c\coth^4(x)} + \frac{1}{2}(-a-b-c)\text{Subst}\left(\int\right. \\
&= -\frac{1}{2}\sqrt{a+b\coth^2(x)+c\coth^4(x)} + (a+b+c)\text{Subst}\left(\int \frac{}{4c}\right. \\
&= -\frac{(b+2c)\tanh^{-1}\left(\frac{b+2c\coth^2(x)}{2\sqrt{c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{4\sqrt{c}} +
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 304 vs. 2(132) = 264.
time = 5.84, size = 304, normalized size = 2.30

$$
\frac{\operatorname{csch}^2(x)\left(4\sqrt{c}(a+b+c)\tanh^{-1}\left(\frac{-a+c+(a+b+c)\cosh(2x)}{2\sqrt{a+b+c}\sqrt{c+(b+2c)\sinh^2(x)+(a+b+c)\sinh^4(x)}}\right)\sqrt{3a-b+3c-4(a-c)\cosh(2x)+(a+b+c)\cosh(4x)}+\sqrt{a+b+c}\left(-2(b+2c)\tanh^{-1}\left(\frac{2c+(b+2c)\sinh^2(x)}{2\sqrt{c}\sqrt{c+(b+2c)\sinh^2(x)+(a+b+c)\sinh^4(x)}}\right)\sqrt{3a-b+3c-4(a-c)\cosh(2x)+(a+b+c)\cosh(4x)}-\sqrt{2}\sqrt{c}(3a-b+3c-4(a-c)\cosh(2x)+(a+b+c)\cosh(4x))\operatorname{csch}^2(x)\right)\right)}{8\sqrt{c}\sqrt{a+b+c}\sqrt{(3a-b+3c-4(a-c)\cosh(2x)+(a+b+c)\cosh(4x))\operatorname{csch}^4(x)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] (Csch[x]^2*(4*Sqrt[c]*(a + b + c)*ArcTanh[(-a + c + (a + b + c)*Cosh[2*x])/
(2*Sqrt[a + b + c]*Sqrt[c + (b + 2*c)*Sinh[x]^2 + (a + b + c)*Sinh[x]^4])]*
Sqrt[3*a - b + 3*c - 4*(a - c)*Cosh[2*x] + (a + b + c)*Cosh[4*x]] + Sqrt[a
+ b + c]*(-2*(b + 2*c)*ArcTanh[(2*c + (b + 2*c)*Sinh[x]^2)/(2*Sqrt[c]*Sqrt[
c + (b + 2*c)*Sinh[x]^2 + (a + b + c)*Sinh[x]^4])]*Sqrt[3*a - b + 3*c - 4*(
a - c)*Cosh[2*x] + (a + b + c)*Cosh[4*x]] - Sqrt[2]*Sqrt[c]*(3*a - b + 3*c
- 4*(a - c)*Cosh[2*x] + (a + b + c)*Cosh[4*x])*Csch[x]^2)))/(8*Sqrt[c]*Sqrt
[a + b + c]*Sqrt[(3*a - b + 3*c - 4*(a - c)*Cosh[2*x] + (a + b + c)*Cosh[4*
x])*Csch[x]^4])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 1.01, size = 850, normalized size = 6.44 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(a+b*coth(x)^2+c*coth(x)^4)^(1/2)-1/8*(b+c)*2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*coth(x)^2)^(1/2)*(4+2*(b+(-4
*a*c+b^2)^(1/2))/a*coth(x)^2)^(1/2)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2)*Ellip
ticF(1/2*coth(x)*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(
-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(c+1/2*b)*ln((b+2*c*coth(x)^2)/c^(1/2)+2
*(a+b*coth(x)^2+c*coth(x)^4)^(1/2))/c^(1/2)-1/2*(a+b+c)*(-1/2/(a+b+c)^(1/2)
*arctanh(1/2*(b*coth(x)^2+2*c*coth(x)^2+2*a+b)/(a+b+c)^(1/2)/(a+b*coth(x)^2
+c*coth(x)^4)^(1/2))-2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(1-1/2*(-b+(
-4*a*c+b^2)^(1/2))/a*coth(x)^2)^(1/2)*(1+1/2*(b+(-4*a*c+b^2)^(1/2))/a*coth(
x)^2)^(1/2)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2)*EllipticPi(1/2*coth(x)*2^(1/2
)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),2/(-b+(-4*a*c+b^2)^(1/2))*a,(-1/2*(b+(-
4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)))-1/8*
(-b-c)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2
))/a*coth(x)^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*coth(x)^2)^(1/2)/(a+b*c
oth(x)^2+c*coth(x)^4)^(1/2)*EllipticF(1/2*coth(x)*2^(1/2)*((-b+(-4*a*c+b^2)
^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(a+b+c)
*(-1/2/(a+b+c)^(1/2)*arctanh(1/2*(b*coth(x)^2+2*c*coth(x)^2+2*a+b)/(a+b+c)^
(1/2)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2))+2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2)*(1-1/2*(-b+(-4*a*c+b^2)^(1/2))/a*coth(x)^2)^(1/2)*(1+1/2*(b+(-4*a*c
+b^2)^(1/2))/a*coth(x)^2)^(1/2)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2)*EllipticP
i(1/2*coth(x)*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),2/(-b+(-4*a*c+b^2)^
(1/2))*a,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))/a)^(1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*coth(x)^4 + b*coth(x)^2 + a)*coth(x), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1840 vs. $2(108) = 216$.
time = 1.44, size = 7964, normalized size = 60.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(((b + 2*c)*cosh(x)^4 + 4*(b + 2*c)*cosh(x)*sinh(x)^3 + (b + 2*c)*sinh
(x)^4 - 2*(b + 2*c)*cosh(x)^2 + 2*(3*(b + 2*c)*cosh(x)^2 - b - 2*c)*sinh(x)
```

```
^2 + 4*((b + 2*c)*cosh(x)^3 - (b + 2*c)*cosh(x))*sinh(x) + b + 2*c)*sqrt(c)
*log(((b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^8 + 8*(b^2 + 4*(a + 2*b)*c + 8*
c^2)*cosh(x)*sinh(x)^7 + (b^2 + 4*(a + 2*b)*c + 8*c^2)*sinh(x)^8 - 4*(b^2 +
 4*a*c - 8*c^2)*cosh(x)^6 + 4*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^2 -
b^2 - 4*a*c + 8*c^2)*sinh(x)^6 + 8*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)
^3 - 3*(b^2 + 4*a*c - 8*c^2)*cosh(x))*sinh(x)^5 + 2*(3*b^2 + 4*(3*a - 2*b)*
c + 24*c^2)*cosh(x)^4 + 2*(35*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^4 - 30*
(b^2 + 4*a*c - 8*c^2)*cosh(x)^2 + 3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*sinh(x)
^4 + 8*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^5 - 10*(b^2 + 4*a*c - 8*c^2
)*cosh(x)^3 + (3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*cosh(x))*sinh(x)^3 - 4*(b^
2 + 4*a*c - 8*c^2)*cosh(x)^2 + 4*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^6
 - 15*(b^2 + 4*a*c - 8*c^2)*cosh(x)^4 + 3*(3*b^2 + 4*(3*a - 2*b)*c + 24*c^2
)*cosh(x)^2 - b^2 - 4*a*c + 8*c^2)*sinh(x)^2 - 4*sqrt(2)*((b + 2*c)*cosh(x)
^4 + 4*(b + 2*c)*cosh(x)*sinh(x)^3 + (b + 2*c)*sinh(x)^4 - 2*(b - 2*c)*cosh
(x)^2 + 2*(3*(b + 2*c)*cosh(x)^2 - b + 2*c)*sinh(x)^2 + 4*((b + 2*c)*cosh(x
)^3 - (b - 2*c)*cosh(x))*sinh(x) + b + 2*c)*sqrt(c)*sqrt(((a + b + c)*cosh(
x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(
x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(
x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + b^2 + 4*(a
 + 2*b)*c + 8*c^2 + 8*((b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^7 - 3*(b^2 + 4
*a*c - 8*c^2)*cosh(x)^5 + (3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*cosh(x)^3 - (b
^2 + 4*a*c - 8*c^2)*cosh(x))*sinh(x))/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + si
nh(x)^8 + 4*(7*cosh(x)^2 - 1)*sinh(x)^6 - 4*cosh(x)^6 + 8*(7*cosh(x)^3 - 3*
cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 - 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh
(x)^4 + 8*(7*cosh(x)^5 - 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)
^6 - 15*cosh(x)^4 + 9*cosh(x)^2 - 1)*sinh(x)^2 - 4*cosh(x)^2 + 8*(cosh(x)^7
 - 3*cosh(x)^5 + 3*cosh(x)^3 - cosh(x))*sinh(x) + 1)) + 2*(c*cosh(x)^4 + 4*
c*cosh(x)*sinh(x)^3 + c*sinh(x)^4 - 2*c*cosh(x)^2 + 2*(3*c*cosh(x)^2 - c)*s
inh(x)^2 + 4*(c*cosh(x)^3 - c*cosh(x))*sinh(x) + c)*sqrt(a + b + c)*log(((a
^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2 + 2*
(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2
)*sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2
 + 2*(a + b)*c + c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x)^6 + 8*(7*(
a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3 - 3*(a^2 + a*b - b*c - c^2
)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^4 +
2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4 - 30*(a^2 + a*b - b
*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*sinh(x)^4 + 8*(7
*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 - 10*(a^2 + a*b - b*c -
c^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x))*sinh(x)^3 -
 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*
c + c^2)*cosh(x)^6 - 15*(a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*
b + 2*(a + b)*c + 3*c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x)^2 + sqr
t(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 + (a + b + c)
*sinh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - a + c)*sinh
(x)^2 + 4*((a + b + c)*cosh(x)^3 - (a - c)*cosh(x))*sinh(x) + a + b + c)*sq
```

```
rt(a + b + c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a -
c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b
+ 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)
*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2 + 8*((a^2
+ 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7 - 3*(a^2 + a*b - b*c - c^2)*co
sh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^3 - (a^2 + a*b - b*
c - c^2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*s
inh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) - 4*sqrt(2)*c*sqrt(((a + b + c
)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c
)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^
3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)))/(c*c
osh(x)^4 + 4*c*cosh(x)*sinh(x)^3 + c*sinh(x)^4 - 2*c*cosh(x)^2 + 2*(3*c*cos
h(x)^2 - c)*sinh(x)^2 + 4*(c*cosh(x)^3 - c*cosh(x))*sinh(x) + c), -1/8*(4*(
c*cosh(x)^4 + 4*c*cosh(x)*sinh(x)^3 + c*sinh(x)^4 - 2*c*cosh(x)^2 + 2*(3*c*
cosh(x)^2 - c)*sinh(x)^2 + 4*(c*cosh(x)^3 - c*cosh(x))*sinh(x) + c)*sqrt(-a
 - b - c)*arctan(sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(x)*sin
h(x)^3 + (a + b + c)*sinh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cos
h(x)^2 - a + c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 - (a - c)*cosh(x))*sin
h(x) + a + b + c)*sqrt(-a - b - c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c
)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\coth^2(x) + c\coth^4(x)}\,\coth(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*coth(x)**2+c*coth(x)**4)**(1/2),x)
[Out] Integral(sqrt(a + b*coth(x)**2 + c*coth(x)**4)*coth(x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*coth(x)^4 + b*coth(x)^2 + a)*coth(x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \coth(x)\,\sqrt{c\coth(x)^4 + b\coth(x)^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(a + b*coth(x)^2 + c*coth(x)^4)^(1/2),x)
[Out] int(coth(x)*(a + b*coth(x)^2 + c*coth(x)^4)^(1/2), x)
```

3.211 $\int e^{c(a+bx)} \coth^2(ac+bcx)^{5/2}\, dx$

Optimal. Leaf size=319

$$\frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\ \tanh(ac+bcx)}{bc}-\frac{4e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\ \tanh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^4}+\frac{26e^{c(a+bx)}\sqrt{\coth^2(a}{3bc\,(1}$$

```
[Out] exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2)*tanh(b*c*x+a*c)/b/c-4*exp(c*(b*x+a
))*(coth(b*c*x+a*c)^2)^(1/2)*tanh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^4+26/
3*exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2)*tanh(b*c*x+a*c)/b/c/(1-exp(2*c*(
b*x+a)))^3-55/6*exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2)*tanh(b*c*x+a*c)/b/
c/(1-exp(2*c*(b*x+a)))^2+25/4*exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2)*tanh
(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))-15/4*arctanh(exp(c*(b*x+a)))*(coth(b*c
*x+a*c)^2)^(1/2)*tanh(b*c*x+a*c)/b/c
```

Rubi [A]

time = 0.64, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {6852, 2320, 398, 1828, 1171, 393, 213}

$$-\frac{15\tanh^{-1}\left(e^{c(a+bx)}\right)\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{4bc}+\frac{e^{c(a+bx)}\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{bc}+\frac{25e^{c(a+bx)}\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{4bc\left(1-e^{2c(a+bx)}\right)}-\frac{55e^{c(a+bx)}\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{6bc\left(1-e^{2c(a+bx)}\right)^2}+\frac{26e^{c(a+bx)}\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{3bc\left(1-e^{2c(a+bx)}\right)^3}-\frac{4e^{c(a+bx)}\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{bc\left(1-e^{2c(a+bx)}\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*(Coth[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] (E^(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c) - (4*E^
(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c*(1 - E^(2*c
*(a + b*x)))^4) + (26*E^(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c +
b*c*x])/(3*b*c*(1 - E^(2*c*(a + b*x)))^3) - (55*E^(c*(a + b*x))*Sqrt[Coth[a
*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(6*b*c*(1 - E^(2*c*(a + b*x)))^2) + (25*E
^(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(4*b*c*(1 - E^(
2*c*(a + b*x)))) - (15*ArcTanh[E^(c*(a + b*x))]*Sqrt[Coth[a*c + b*c*x]^2]*T
anh[a*c + b*c*x])/(4*b*c)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
```

```
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int e^{c(a+bx)} \coth^2(ac+bcx)^{5/2}\,dx = \left(\sqrt{\coth^2(ac+bcx)}\,\tanh(ac+bcx)\right)\int e^{c(a+bx)}\coth^5(ac+bcx)\,dx$$

$$= \frac{\left(\sqrt{\coth^2(ac+bcx)}\,\tanh(ac+bcx)\right)\operatorname{Subst}\left(\int \frac{(1+x^2)^5}{(-1+x^2)^5}\,dx, x, e^{c(a+bx)}\right)}{bc}$$

$$= \frac{\left(\sqrt{\coth^2(ac+bcx)}\,\tanh(ac+bcx)\right)\operatorname{Subst}\left(\int \left(1+\frac{2(1+10x^4+5x^8)}{(-1+x^2)^5}\right)\,dx, x,$$

$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,\tanh(ac+bcx)}{bc} + \frac{\left(2\sqrt{\coth^2(ac+bcx)}\,\tanh(\right.}{}$$

$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,\tanh(ac+bcx)}{bc} - \frac{4e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,\mathrm{t}}{bc\,(1-e^{2c(a+bx)}}$$

$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,\tanh(ac+bcx)}{bc} - \frac{4e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,\mathrm{t}}{bc\,(1-e^{2c(a+bx)}}$$

$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,\tanh(ac+bcx)}{bc} - \frac{4e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,\mathrm{t}}{bc\,(1-e^{2c(a+bx)}}$$

$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,\tanh(ac+bcx)}{bc} - \frac{4e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,\mathrm{t}}{bc\,(1-e^{2c(a+bx)}}$$

$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,\tanh(ac+bcx)}{bc} - \frac{4e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,\mathrm{t}}{bc\,(1-e^{2c(a+bx)}}$$

Mathematica [A]
time = 10.19, size = 164, normalized size = 0.51

$$\frac{\sqrt{\coth^2(c(a+bx))}\left(66e^{c(a+bx)} - 314e^{3c(a+bx)} + 374e^{5c(a+bx)} - 246e^{7c(a+bx)} + 24e^{9c(a+bx)} + 45\left(-1+e^{2c(a+bx)}\right)^4\log\left(1-e^{c(a+bx)}\right) - 45\left(-1+e^{2c(a+bx)}\right)^4\log\left(1+e^{c(a+bx)}\right)\right)\tanh(c(a+bx))}{24bc\left(-1+e^{2c(a+bx)}\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*(Coth[a*c + b*c*x]^2)^(5/2),x]
[Out] (Sqrt[Coth[c*(a + b*x)]^2]*(66*E^(c*(a + b*x)) - 314*E^(3*c*(a + b*x)) + 37
4*E^(5*c*(a + b*x)) - 246*E^(7*c*(a + b*x)) + 24*E^(9*c*(a + b*x)) + 45*(-1
 + E^(2*c*(a + b*x)))^4*Log[1 - E^(c*(a + b*x))] - 45*(-1 + E^(2*c*(a + b*x
)))^4*Log[1 + E^(c*(a + b*x))])*Tanh[c*(a + b*x)])/(24*b*c*(-1 + E^(2*c*(a
+ b*x)))^4)
```

Maple [A]
time = 5.64, size = 320, normalized size = 1.00

method	result
risch	$\frac{\left(\mathrm{e}^{2c(bx+a)}-1\right)\sqrt{\frac{\left(1+\mathrm{e}^{2c(bx+a)}\right)^{2}}{\left(\mathrm{e}^{2c(bx+a)}-1\right)^{2}}}\,\mathrm{e}^{c(bx+a)}}{\left(1+\mathrm{e}^{2c(bx+a)}\right)bc}-\frac{\sqrt{\frac{\left(1+\mathrm{e}^{2c(bx+a)}\right)^{2}}{\left(\mathrm{e}^{2c(bx+a)}-1\right)^{2}}}\,\mathrm{e}^{c(bx+a)}\left(75\,\mathrm{e}^{6c(bx+a)}-115\,\mathrm{e}^{4c(bx+a)}+109\,\mathrm{e}^{2c(bx+a)}-21\right)}{12\left(1+\mathrm{e}^{2c(bx+a)}\right)\left(\mathrm{e}^{2c(bx+a)}-1\right)^{3}bc}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(1+exp(2*c*(b*x+a)))*(exp(2*c*(b*x+a))-1)*((1+exp(2*c*(b*x+a)))^2/(exp(2*
c*(b*x+a))-1)^2)^(1/2)*exp(c*(b*x+a))/b/c-1/12/(1+exp(2*c*(b*x+a)))/(exp(2*
c*(b*x+a))-1)^3*((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)*exp(c
*(b*x+a))*(75*exp(6*c*(b*x+a))-115*exp(4*c*(b*x+a))+109*exp(2*c*(b*x+a))-21
)/b/c-15/8/(1+exp(2*c*(b*x+a)))*(exp(2*c*(b*x+a))-1)*((1+exp(2*c*(b*x+a)))^
2/(exp(2*c*(b*x+a))-1)^2)^(1/2)/b/c*ln(exp(c*(b*x+a))+1)+15/8/(1+exp(2*c*(b
*x+a)))*(exp(2*c*(b*x+a))-1)*((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2
)^(1/2)/b/c*ln(exp(c*(b*x+a))-1)
```

Maxima [A]
time = 0.50, size = 167, normalized size = 0.52

$$-\frac{15\,\log\left(e^{(bcx+ac)}+1\right)}{8\,bc}+\frac{15\,\log\left(e^{(bcx+ac)}-1\right)}{8\,bc}+\frac{12\,e^{(9\,bcx+9\,ac)}-123\,e^{(7\,bcx+7\,ac)}+187\,e^{(5\,bcx+5\,ac)}-157\,e^{(3\,bcx+3\,ac)}+33\,e^{(bcx+ac)}}{12\,bc\left(e^{(8\,bcx+8\,ac)}-4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}-4\,e^{(2\,bcx+2\,ac)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -15/8*log(e^(b*c*x + a*c) + 1)/(b*c) + 15/8*log(e^(b*c*x + a*c) - 1)/(b*c)
+ 1/12*(12*e^(9*b*c*x + 9*a*c) - 123*e^(7*b*c*x + 7*a*c) + 187*e^(5*b*c*x +
 5*a*c) - 157*e^(3*b*c*x + 3*a*c) + 33*e^(b*c*x + a*c))/(b*c*(e^(8*b*c*x +
8*a*c) - 4*e^(6*b*c*x + 6*a*c) + 6*e^(4*b*c*x + 4*a*c) - 4*e^(2*b*c*x + 2*a
*c) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1617 vs. $2(281) = 562$.
time = 0.37, size = 1617, normalized size = 5.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24*(24*cosh(b*c*x + a*c)^9 + 216*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^8 +
24*sinh(b*c*x + a*c)^9 + 6*(144*cosh(b*c*x + a*c)^2 - 41)*sinh(b*c*x + a*c)
```

```
^7 - 246*cosh(b*c*x + a*c)^7 + 42*(48*cosh(b*c*x + a*c)^3 - 41*cosh(b*c*x +
 a*c))*sinh(b*c*x + a*c)^6 + 2*(1512*cosh(b*c*x + a*c)^4 - 2583*cosh(b*c*x
+ a*c)^2 + 187)*sinh(b*c*x + a*c)^5 + 374*cosh(b*c*x + a*c)^5 + 2*(1512*cos
h(b*c*x + a*c)^5 - 4305*cosh(b*c*x + a*c)^3 + 935*cosh(b*c*x + a*c))*sinh(b
*c*x + a*c)^4 + 2*(1008*cosh(b*c*x + a*c)^6 - 4305*cosh(b*c*x + a*c)^4 + 18
70*cosh(b*c*x + a*c)^2 - 157)*sinh(b*c*x + a*c)^3 - 314*cosh(b*c*x + a*c)^3
 + 2*(432*cosh(b*c*x + a*c)^7 - 2583*cosh(b*c*x + a*c)^5 + 1870*cosh(b*c*x
+ a*c)^3 - 471*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^2 - 45*(cosh(b*c*x + a*
c)^8 + 8*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^7 + sinh(b*c*x + a*c)^8 + 4*(7
*cosh(b*c*x + a*c)^2 - 1)*sinh(b*c*x + a*c)^6 - 4*cosh(b*c*x + a*c)^6 + 8*(
7*cosh(b*c*x + a*c)^3 - 3*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^5 + 2*(35*co
sh(b*c*x + a*c)^4 - 30*cosh(b*c*x + a*c)^2 + 3)*sinh(b*c*x + a*c)^4 + 6*cos
h(b*c*x + a*c)^4 + 8*(7*cosh(b*c*x + a*c)^5 - 10*cosh(b*c*x + a*c)^3 + 3*co
sh(b*c*x + a*c))*sinh(b*c*x + a*c)^3 + 4*(7*cosh(b*c*x + a*c)^6 - 15*cosh(b
*c*x + a*c)^4 + 9*cosh(b*c*x + a*c)^2 - 1)*sinh(b*c*x + a*c)^2 - 4*cosh(b*c
*x + a*c)^2 + 8*(cosh(b*c*x + a*c)^7 - 3*cosh(b*c*x + a*c)^5 + 3*cosh(b*c*x
 + a*c)^3 - cosh(b*c*x + a*c))*sinh(b*c*x + a*c) + 1)*log(cosh(b*c*x + a*c)
 + sinh(b*c*x + a*c) + 1) + 45*(cosh(b*c*x + a*c)^8 + 8*cosh(b*c*x + a*c)*s
inh(b*c*x + a*c)^7 + sinh(b*c*x + a*c)^8 + 4*(7*cosh(b*c*x + a*c)^2 - 1)*si
nh(b*c*x + a*c)^6 - 4*cosh(b*c*x + a*c)^6 + 8*(7*cosh(b*c*x + a*c)^3 - 3*co
sh(b*c*x + a*c))*sinh(b*c*x + a*c)^5 + 2*(35*cosh(b*c*x + a*c)^4 - 30*cosh(
b*c*x + a*c)^2 + 3)*sinh(b*c*x + a*c)^4 + 6*cosh(b*c*x + a*c)^4 + 8*(7*cosh
(b*c*x + a*c)^5 - 10*cosh(b*c*x + a*c)^3 + 3*cosh(b*c*x + a*c))*sinh(b*c*x
+ a*c)^3 + 4*(7*cosh(b*c*x + a*c)^6 - 15*cosh(b*c*x + a*c)^4 + 9*cosh(b*c*x
 + a*c)^2 - 1)*sinh(b*c*x + a*c)^2 - 4*cosh(b*c*x + a*c)^2 + 8*(cosh(b*c*x
+ a*c)^7 - 3*cosh(b*c*x + a*c)^5 + 3*cosh(b*c*x + a*c)^3 - cosh(b*c*x + a*c
))*sinh(b*c*x + a*c) + 1)*log(cosh(b*c*x + a*c) + sinh(b*c*x + a*c) - 1) +
2*(108*cosh(b*c*x + a*c)^8 - 861*cosh(b*c*x + a*c)^6 + 935*cosh(b*c*x + a*c
)^4 - 471*cosh(b*c*x + a*c)^2 + 33)*sinh(b*c*x + a*c) + 66*cosh(b*c*x + a*c
))/(b*c*cosh(b*c*x + a*c)^8 + 8*b*c*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^7 +
 b*c*sinh(b*c*x + a*c)^8 - 4*b*c*cosh(b*c*x + a*c)^6 + 4*(7*b*c*cosh(b*c*x
+ a*c)^2 - b*c)*sinh(b*c*x + a*c)^6 + 6*b*c*cosh(b*c*x + a*c)^4 + 8*(7*b*c*
cosh(b*c*x + a*c)^3 - 3*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^5 + 2*(35*
b*c*cosh(b*c*x + a*c)^4 - 30*b*c*cosh(b*c*x + a*c)^2 + 3*b*c)*sinh(b*c*x +
a*c)^4 - 4*b*c*cosh(b*c*x + a*c)^2 + 8*(7*b*c*cosh(b*c*x + a*c)^5 - 10*b*c*
cosh(b*c*x + a*c)^3 + 3*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^3 + 4*(7*b
*c*cosh(b*c*x + a*c)^6 - 15*b*c*cosh(b*c*x + a*c)^4 + 9*b*c*cosh(b*c*x + a*
c)^2 - b*c)*sinh(b*c*x + a*c)^2 + b*c + 8*(b*c*cosh(b*c*x + a*c)^7 - 3*b*c*
cosh(b*c*x + a*c)^5 + 3*b*c*cosh(b*c*x + a*c)^3 - b*c*cosh(b*c*x + a*c))*si
nh(b*c*x + a*c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.42, size = 181, normalized size = 0.57

$$\frac{\frac{24\,e^{(bcx+ac)}}{\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)}-\frac{45\,\log\left(e^{(bcx+ac)}+1\right)}{\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)}+\frac{45\,\log\left(\left|e^{(bcx+ac)}-1\right|\right)}{\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)}-\frac{2\,\left(75\,e^{(7\,bcx+7\,ac)}-115\,e^{(5\,bcx+5\,ac)}+109\,e^{(3\,bcx+3\,ac)}-21\,e^{(bcx+ac)}\right)}{\left(e^{(2\,bcx+2\,ac)}-1\right)^{4}\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)}}{24\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24*(24*e^(b*c*x + a*c)/sgn(e^(2*b*c*x + 2*a*c) - 1) - 45*log(e^(b*c*x + a
*c) + 1)/sgn(e^(2*b*c*x + 2*a*c) - 1) + 45*log(abs(e^(b*c*x + a*c) - 1))/sg
n(e^(2*b*c*x + 2*a*c) - 1) - 2*(75*e^(7*b*c*x + 7*a*c) - 115*e^(5*b*c*x + 5
*a*c) + 109*e^(3*b*c*x + 3*a*c) - 21*e^(b*c*x + a*c))/((e^(2*b*c*x + 2*a*c)
 - 1)^4*sgn(e^(2*b*c*x + 2*a*c) - 1)))/(b*c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{e}^{c\,(a+b\,x)}\,{\left({\mathrm{coth}\left(a\,c+b\,c\,x\right)}^2\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*(coth(a*c + b*c*x)^2)^(5/2),x)
```

```
[Out] int(exp(c*(a + b*x))*(coth(a*c + b*c*x)^2)^(5/2), x)
```

3.212 $\int e^{c(a+bx)} \coth^2(ac+bcx)^{3/2}\,dx$

Optimal. Leaf size=197

$$\frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,\tanh(ac+bcx)}{bc}-\frac{2e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,\tanh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2}+\frac{3e^{c(a+bx)}\sqrt{\coth^2(ac}{bc\,(1-}$$

```
[Out] exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2)*tanh(b*c*x+a*c)/b/c-2*exp(c*(b*x+a
))*(coth(b*c*x+a*c)^2)^(1/2)*tanh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^2+3*e
xp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2)*tanh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x
+a)))-3*arctanh(exp(c*(b*x+a)))*(coth(b*c*x+a*c)^2)^(1/2)*tanh(b*c*x+a*c)/b
/c
```

Rubi [A]
time = 0.21, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {6852, 2320, 398, 1172, 12, 294, 213}

$$-\frac{3\tanh^{-1}\left(e^{c(a+bx)}\right)\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{bc}+\frac{e^{c(a+bx)}\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{bc}+\frac{3e^{c(a+bx)}\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{bc\left(1-e^{2c(a+bx)}\right)}-\frac{2e^{c(a+bx)}\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{bc\left(1-e^{2c(a+bx)}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*(Coth[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] (E^(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c) - (2*E^
(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c*(1 - E^(2*c
*(a + b*x)))^2) + (3*E^(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b
*c*x])/(b*c*(1 - E^(2*c*(a + b*x)))) - (3*ArcTanh[E^(c*(a + b*x))]*Sqrt[Cot
h[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1172

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int e^{c(a+bx)} \coth^2(ac+bcx)^{3/2}\,dx = \left(\sqrt{\coth^2(ac+bcx)}\,\tanh(ac+bcx)\right)\int e^{c(a+bx)}\coth^3(ac+bcx)\,dx$$

$$= \frac{\left(\sqrt{\coth^2(ac+bcx)}\,\tanh(ac+bcx)\right)\text{Subst}\left(\int \frac{(1+x^2)^3}{(-1+x^2)^3}\,dx, x, e^{c(a+bx)}\right)}{bc}$$

$$= \frac{\left(\sqrt{\coth^2(ac+bcx)}\,\tanh(ac+bcx)\right)\text{Subst}\left(\int \left(1+\frac{2(1+3x^4)}{(-1+x^2)^3}\right)\,dx, x, e^{c(a+}\right.}{bc}$$

$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,\tanh(ac+bcx)}{bc} + \frac{\left(2\sqrt{\coth^2(ac+bcx)}\,\tanh(\right.}{}$$

$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,\tanh(ac+bcx)}{bc} - \frac{2e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,\mathrm{t}}{bc\left(1-e^{2c(a+bx)}\right.}$$

$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,\tanh(ac+bcx)}{bc} - \frac{2e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,\mathrm{t}}{bc\left(1-e^{2c(a+bx)}\right.}$$

$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,\tanh(ac+bcx)}{bc} - \frac{2e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,\mathrm{t}}{bc\left(1-e^{2c(a+bx)}\right.}$$

$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,\tanh(ac+bcx)}{bc} - \frac{2e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,\mathrm{t}}{bc\left(1-e^{2c(a+bx)}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 2.75, size = 334, normalized size = 1.70

$$-\frac{e^{-5c(a+bx)}\coth^2(c(a+bx))^{3/2}\left(-21\left(252105+507305e^{2c(a+bx)}+173916e^{4c(a+bx)}-154296e^{6c(a+bx)}-73885e^{8c(a+bx)}+4887e^{10c(a+bx)}\right)-\frac{315\left(-16807-28218e^{2c(a+bx)}+1173e^{4c(a+bx)}+17748e^{6c(a+bx)}+4299e^{8c(a+bx)}-1434e^{10c(a+bx)}+7e^{12c(a+bx)}\right)\tanh^{-1}\left(\sqrt{e^{2c(a+bx)}}\right)}{\sqrt{e^{2c(a+bx)}}}+384e^{8c(a+bx)}\left(1+e^{2c(a+bx)}\right)^2\left(7+5e^{2c(a+bx)}\right){}_5F_4\left(\frac{3}{2},2,2,2,2;1,1,1,\frac{11}{2};e^{2c(a+bx)}\right)+256e^{8c(a+bx)}\left(1+e^{2c(a+bx)}\right)^3{}_6F_5\left(\frac{3}{2},2,2,2,2,2;1,1,1,1,\frac{11}{2};e^{2c(a+bx)}\right)\right)\tanh^3(c(a+bx))}{60480bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*(Coth[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] -1/60480*((Coth[c*(a + b*x)]^2)^(3/2)*(-21*(252105 + 507305*E^(2*c*(a + b*x
)) + 173916*E^(4*c*(a + b*x)) - 154296*E^(6*c*(a + b*x)) - 73885*E^(8*c*(a
+ b*x)) + 4887*E^(10*c*(a + b*x))) - (315*(-16807 - 28218*E^(2*c*(a + b*x))
 + 1173*E^(4*c*(a + b*x)) + 17748*E^(6*c*(a + b*x)) + 4299*E^(8*c*(a + b*x)
) - 1434*E^(10*c*(a + b*x)) + 7*E^(12*c*(a + b*x)))*ArcTanh[Sqrt[E^(2*c*(a
+ b*x))]])/Sqrt[E^(2*c*(a + b*x))] + 384*E^(8*c*(a + b*x))*(1 + E^(2*c*(a +
 b*x)))^2*(7 + 5*E^(2*c*(a + b*x)))*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1
```

```
, 1, 1, 11/2}, E^(2*c*(a + b*x))] + 256*E^(8*c*(a + b*x))*(1 + E^(2*c*(a +
b*x)))^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 11/2}, E^(2*c
*(a + b*x))])*Tanh[c*(a + b*x)]^3)/(b*c*E^(5*c*(a + b*x)))
```

Maple [A]
time = 5.13, size = 298, normalized size = 1.51

method	result
risch	$\frac{\left(e^{2c(bx+a)}-1\right)\sqrt{\frac{\left(1+e^{2c(bx+a)}\right)^2}{\left(e^{2c(bx+a)}-1\right)^2}}\, e^{c(bx+a)}}{\left(1+e^{2c(bx+a)}\right)bc} - \frac{\sqrt{\frac{\left(1+e^{2c(bx+a)}\right)^2}{\left(e^{2c(bx+a)}-1\right)^2}}\, e^{c(bx+a)}\left(3\,e^{2c(bx+a)}-1\right)}{\left(1+e^{2c(bx+a)}\right)\left(e^{2c(bx+a)}-1\right)bc} - \frac{3\left(e^{2c(bx+a)}-1\right)\sqrt{\frac{(1+e^{2c(bx+a}}{(e^{2c(bx+a)}-}}}{2(1+e^{2c(bx+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(1+exp(2*c*(b*x+a)))*(exp(2*c*(b*x+a))-1)*((1+exp(2*c*(b*x+a)))^2/(exp(2*
c*(b*x+a))-1)^2)^(1/2)*exp(c*(b*x+a))/b/c-1/(1+exp(2*c*(b*x+a)))/(exp(2*c*(
b*x+a))-1)*((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)*exp(c*(b*x
+a))*(3*exp(2*c*(b*x+a))-1)/b/c-3/2/(1+exp(2*c*(b*x+a)))*(exp(2*c*(b*x+a))-
1)*((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)/b/c*ln(exp(c*(b*x+
a))+1)+3/2/(1+exp(2*c*(b*x+a)))*(exp(2*c*(b*x+a))-1)*((1+exp(2*c*(b*x+a)))^
2/(exp(2*c*(b*x+a))-1)^2)^(1/2)/b/c*ln(exp(c*(b*x+a))-1)
```

Maxima [A]
time = 0.49, size = 112, normalized size = 0.57

$$-\frac{3\,\log\left(e^{(bcx+ac)}+1\right)}{2\,bc}+\frac{3\,\log\left(e^{(bcx+ac)}-1\right)}{2\,bc}+\frac{e^{(5\,bcx+5\,ac)}-5\,e^{(3\,bcx+3\,ac)}+2\,e^{(bcx+ac)}}{bc\left(e^{(4\,bcx+4\,ac)}-2\,e^{(2\,bcx+2\,ac)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -3/2*log(e^(b*c*x + a*c) + 1)/(b*c) + 3/2*log(e^(b*c*x + a*c) - 1)/(b*c) +
(e^(5*b*c*x + 5*a*c) - 5*e^(3*b*c*x + 3*a*c) + 2*e^(b*c*x + a*c))/(b*c*(e^(
4*b*c*x + 4*a*c) - 2*e^(2*b*c*x + 2*a*c) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 613 vs. 2(179) = 358.
time = 0.38, size = 613, normalized size = 3.11

$$\frac{2\cosh(bcx+ac)^5+10\cosh(bcx+ac)\sinh(bcx+ac)^4+2\sinh(bcx+ac)^5+10\left(2\cosh(bcx+ac)^2-1\right)\sinh(bcx+ac)^3-10\cosh(bcx+ac)^3+10\left(2\cosh(bcx+ac)^3-3\cosh(bcx+ac)\right)\sinh(bcx+ac)^2-3\left(\cosh(bcx+ac)^4+4\cosh(bcx+ac)\sinh(bcx+ac)^3+\sinh(bcx+ac)^4+2\left(3\cosh(bcx+ac)^2-1\right)\sinh(bcx+ac)^2-2\cosh(bcx+ac)^2+4\left(\cosh(bcx+ac)^3-\cosh(bcx+ac)\right)\sinh(bcx+ac)+1\right)\log\left(\cosh(bcx+ac)+\sinh(bcx+ac)+1\right)+3\left(\cosh(bcx+ac)^4+4\cosh(bcx+ac)\sinh(bcx+ac)^3+\sinh(bcx+ac)^4+2\left(3\cosh(bcx+ac)^2-1\right)\sinh(bcx+ac)^2-2\cosh(bcx+ac)^2+4\left(\cosh(bcx+ac)^3-\cosh(bcx+ac)\right)\sinh(bcx+ac)+1\right)\log\left(\cosh(bcx+ac)+\sinh(bcx+ac)-1\right)+2\left(5\cosh(bcx+ac)^4-15\cosh(bcx+ac)^2+2\right)\sinh(bcx+ac)+4\cosh(bcx+ac)}{2\left(bc\cosh(bcx+ac)^4+4\,bc\cosh(bcx+ac)\sinh(bcx+ac)^3+bc\sinh(bcx+ac)^4-2\,bc\cosh(bcx+ac)^2+2\left(3\,bc\cosh(bcx+ac)^2-bc\right)\sinh(bcx+ac)^2+bc+4\left(bc\cosh(bcx+ac)^3-bc\cosh(bcx+ac)\right)\sinh(bcx+ac)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*cosh(b*c*x + a*c)^5 + 10*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^4 + 2*s
inh(b*c*x + a*c)^5 + 10*(2*cosh(b*c*x + a*c)^2 - 1)*sinh(b*c*x + a*c)^3 - 1
0*cosh(b*c*x + a*c)^3 + 10*(2*cosh(b*c*x + a*c)^3 - 3*cosh(b*c*x + a*c))*si
nh(b*c*x + a*c)^2 - 3*(cosh(b*c*x + a*c)^4 + 4*cosh(b*c*x + a*c)*sinh(b*c*x
 + a*c)^3 + sinh(b*c*x + a*c)^4 + 2*(3*cosh(b*c*x + a*c)^2 - 1)*sinh(b*c*x
+ a*c)^2 - 2*cosh(b*c*x + a*c)^2 + 4*(cosh(b*c*x + a*c)^3 - cosh(b*c*x + a*
c))*sinh(b*c*x + a*c) + 1)*log(cosh(b*c*x + a*c) + sinh(b*c*x + a*c) + 1) +
 3*(cosh(b*c*x + a*c)^4 + 4*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^3 + sinh(b*
c*x + a*c)^4 + 2*(3*cosh(b*c*x + a*c)^2 - 1)*sinh(b*c*x + a*c)^2 - 2*cosh(b
*c*x + a*c)^2 + 4*(cosh(b*c*x + a*c)^3 - cosh(b*c*x + a*c))*sinh(b*c*x + a*
c) + 1)*log(cosh(b*c*x + a*c) + sinh(b*c*x + a*c) - 1) + 2*(5*cosh(b*c*x +
a*c)^4 - 15*cosh(b*c*x + a*c)^2 + 2)*sinh(b*c*x + a*c) + 4*cosh(b*c*x + a*c
))/(b*c*cosh(b*c*x + a*c)^4 + 4*b*c*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^3 +
 b*c*sinh(b*c*x + a*c)^4 - 2*b*c*cosh(b*c*x + a*c)^2 + 2*(3*b*c*cosh(b*c*x
+ a*c)^2 - b*c)*sinh(b*c*x + a*c)^2 + b*c + 4*(b*c*cosh(b*c*x + a*c)^3 - b*
c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.41, size = 155, normalized size = 0.79

$$\frac{\frac{2\,e^{(bcx+ac)}}{\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)}-\frac{3\,\log\left(e^{(bcx+ac)}+1\right)}{\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)}+\frac{3\,\log\left(\left|e^{(bcx+ac)}-1\right|\right)}{\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)}-\frac{2\,\left(3\,e^{(3\,bcx+3\,ac)}-e^{(bcx+ac)}\right)}{\left(e^{(2\,bcx+2\,ac)}-1\right)^{2}\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)}}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*e^(b*c*x + a*c)/sgn(e^(2*b*c*x + 2*a*c) - 1) - 3*log(e^(b*c*x + a*c)
 + 1)/sgn(e^(2*b*c*x + 2*a*c) - 1) + 3*log(abs(e^(b*c*x + a*c) - 1))/sgn(e^
(2*b*c*x + 2*a*c) - 1) - 2*(3*e^(3*b*c*x + 3*a*c) - e^(b*c*x + a*c))/((e^(2
*b*c*x + 2*a*c) - 1)^2*sgn(e^(2*b*c*x + 2*a*c) - 1)))/(b*c)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{c\,(a+b\,x)}\,{\left({\mathrm{coth}\left(a\,c+b\,c\,x\right)}^2\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*(coth(a*c + b*c*x)^2)^(3/2),x)
[Out] int(exp(c*(a + b*x))*(coth(a*c + b*c*x)^2)^(3/2), x)
```

3.213 $\int e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,dx$

Optimal. Leaf size=83

$$\frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc}-\frac{2\tanh^{-1}\left(e^{c(a+bx)}\right)\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc}$$

```
[Out] exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2)*tanh(b*c*x+a*c)/b/c-2*arctanh(exp(
c*(b*x+a)))*(coth(b*c*x+a*c)^2)^(1/2)*tanh(b*c*x+a*c)/b/c
```

Rubi [A]
time = 0.10, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6852, 2320, 396, 212}

$$\frac{e^{c(a+bx)}\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{bc}-\frac{2\tanh^{-1}\left(e^{c(a+bx)}\right)\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2],x]
```

```
[Out] (E^(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c) - (2*Ar
cTanh[E^(c*(a + b*x))]*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,dx = \left(\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)\right)\int e^{c(a+bx)}\coth(ac+bcx)\,dx$$
$$= \frac{\left(\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)\right)\text{Subst}\left(\int \frac{-1-x^2}{1-x^2}\,dx, x, e^{c(a+bx)}\right)}{bc}$$
$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc} - \frac{\left(2\sqrt{\coth^2(ac+bcx)}\ \text{tanh}\right.}{}$$
$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc} - \frac{2\tanh^{-1}\left(e^{c(a+bx)}\right)\sqrt{\coth^2}}{b}$$

Mathematica [A]
time = 0.04, size = 51, normalized size = 0.61

$$\frac{\left(e^{c(a+bx)}-2\tanh^{-1}\left(e^{c(a+bx)}\right)\right)\sqrt{\coth^2(c(a+bx))}\tanh(c(a+bx))}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2],x]
```

```
[Out] ((E^(c*(a + b*x)) - 2*ArcTanh[E^(c*(a + b*x))])*Sqrt[Coth[c*(a + b*x)]^2]*T
anh[c*(a + b*x)])/(b*c)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(77) = 154$.
time = 5.89, size = 213, normalized size = 2.57

method	result
risch	$\frac{\left(e^{2c(bx+a)}-1\right)\sqrt{\frac{\left(1+e^{2c(bx+a)}\right)^2}{\left(e^{2c(bx+a)}-1\right)^2}}\,e^{c(bx+a)}}{\left(1+e^{2c(bx+a)}\right)bc}+\frac{\left(e^{2c(bx+a)}-1\right)\sqrt{\frac{\left(1+e^{2c(bx+a)}\right)^2}{\left(e^{2c(bx+a)}-1\right)^2}}\,\ln\left(e^{c(bx+a)}-1\right)}{\left(1+e^{2c(bx+a)}\right)bc}-\frac{\left(e^{2c(bx+a)}-1\right)\sqrt{\frac{(1+e^{2c}}{(e^{2c(bx-}}}}{(1+e^{2c(}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(1+exp(2*c*(b*x+a)))*(exp(2*c*(b*x+a))-1)*((1+exp(2*c*(b*x+a)))^2/(exp(2*
c*(b*x+a))-1)^2)^(1/2)*exp(c*(b*x+a))/b/c+1/(1+exp(2*c*(b*x+a)))*(exp(2*c*(
b*x+a))-1)*((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)/b/c*ln(exp
(c*(b*x+a))-1)-1/(1+exp(2*c*(b*x+a)))*(exp(2*c*(b*x+a))-1)*((1+exp(2*c*(b*x
+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)/b/c*ln(exp(c*(b*x+a))+1)
```

Maxima [A]
time = 0.48, size = 56, normalized size = 0.67

$$\frac{e^{(bcx+ac)}}{bc}-\frac{\log\left(e^{(bcx+ac)}+1\right)}{bc}+\frac{\log\left(e^{(bcx+ac)}-1\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] e^(b*c*x + a*c)/(b*c) - log(e^(b*c*x + a*c) + 1)/(b*c) + log(e^(b*c*x + a*c
) - 1)/(b*c)
```

Fricas [A]
time = 0.35, size = 70, normalized size = 0.84

$$\frac{\cosh\left(bcx+ac\right)-\log\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)+1\right)+\log\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)-1\right)+\sinh\left(bcx+ac\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (cosh(b*c*x + a*c) - log(cosh(b*c*x + a*c) + sinh(b*c*x + a*c) + 1) + log(c
osh(b*c*x + a*c) + sinh(b*c*x + a*c) - 1) + sinh(b*c*x + a*c))/(b*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int\sqrt{\coth^{2}\left(ac+bcx\right)}\,e^{bcx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)**2)**(1/2),x)
```

```
[Out] exp(a*c)*Integral(sqrt(coth(a*c + b*c*x)**2)*exp(b*c*x), x)
```

Giac [A]
time = 0.41, size = 94, normalized size = 1.13

$$\frac{\frac{e^{(bcx+ac)}}{\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)}-\frac{\log\left(e^{(bcx+ac)}+1\right)}{\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)}+\frac{\log\left(\left|e^{(bcx+ac)}-1\right|\right)}{\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] (e^(b*c*x + a*c)/sgn(e^(2*b*c*x + 2*a*c) - 1) - log(e^(b*c*x + a*c) + 1)/sg
n(e^(2*b*c*x + 2*a*c) - 1) + log(abs(e^(b*c*x + a*c) - 1))/sgn(e^(2*b*c*x +
 2*a*c) - 1))/(b*c)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int e^{c(a+bx)}\sqrt{\coth(ac+bcx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*(coth(a*c + b*c*x)^2)^(1/2),x)
```

```
[Out] int(exp(c*(a + b*x))*(coth(a*c + b*c*x)^2)^(1/2), x)
```

3.214 $\int \frac{e^{c(a+bx)}}{\sqrt{\coth^2(ac+bcx)}} dx$

Optimal. Leaf size=83

$$\frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{2\text{ArcTan}\left(e^{c(a+bx)}\right)\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}}$$

```
[Out] exp(c*(b*x+a))*coth(b*c*x+a*c)/b/c/(coth(b*c*x+a*c)^2)^(1/2)-2*arctan(exp(c
*(b*x+a)))*coth(b*c*x+a*c)/b/c/(coth(b*c*x+a*c)^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6852, 2320, 396, 209}

$$\frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{2\text{ArcTan}\left(e^{c(a+bx)}\right)\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/Sqrt[Coth[a*c + b*c*x]^2],x]
```

```
[Out] (E^(c*(a + b*x))*Coth[a*c + b*c*x])/(b*c*Sqrt[Coth[a*c + b*c*x]^2]) - (2*Ar
cTan[E^(c*(a + b*x))]*Coth[a*c + b*c*x])/(b*c*Sqrt[Coth[a*c + b*c*x]^2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{c(a+bx)}}{\sqrt{\coth^2(ac+bcx)}}\,dx &= \frac{\coth(ac+bcx)\int e^{c(a+bx)}\tanh(ac+bcx)\,dx}{\sqrt{\coth^2(ac+bcx)}}\\ &= \frac{\coth(ac+bcx)\text{Subst}\left(\int \frac{-1+x^2}{1+x^2}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\coth^2(ac+bcx)}}\\ &= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{(2\coth(ac+bcx))\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\coth^2(ac+bcx)}}\\ &= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{2\tan^{-1}\left(e^{c(a+bx)}\right)\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 51, normalized size = 0.61

$$\frac{\left(e^{c(a+bx)} - 2\text{ArcTan}\left(e^{c(a+bx)}\right)\right)\coth(c(a+bx))}{bc\sqrt{\coth^2(c(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/Sqrt[Coth[a*c + b*c*x]^2],x]
```

```
[Out] ((E^(c*(a + b*x)) - 2*ArcTan[E^(c*(a + b*x))])*Coth[c*(a + b*x)])/(b*c*Sqrt
[Coth[c*(a + b*x)]^2])
```

Maple [C] Result contains complex when optimal does not.
time = 5.76, size = 218, normalized size = 2.63

method	result	size

risch	$\frac{(1+e^{2c(bx+a)})e^{c(bx+a)}}{\sqrt{\frac{(1+e^{2c(bx+a)})^2}{(e^{2c(bx+a)}-1)^2}}\,(e^{2c(bx+a)}-1)bc}+\frac{i(1+e^{2c(bx+a)})\ln(e^{c(bx+a)}-i)}{\sqrt{\frac{(1+e^{2c(bx+a)})^2}{(e^{2c(bx+a)}-1)^2}}\,(e^{2c(bx+a)}-1)cb}-\frac{i(1+e^{2c(bx+a)})\ln(e^{c(bx+a)}+i)}{\sqrt{\frac{(1+e^{2c(bx+a)})^2}{(e^{2c(bx+a)}-1)^2}}\,(e^{2c(bx+a)}-1)cb}$	218

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)/(exp(2*c*(b*x+a))-1
)*(1+exp(2*c*(b*x+a)))*exp(c*(b*x+a))/b/c+I/((1+exp(2*c*(b*x+a)))^2/(exp(2*
c*(b*x+a))-1)^2)^(1/2)/(exp(2*c*(b*x+a))-1)*(1+exp(2*c*(b*x+a)))/c/b*ln(exp
(c*(b*x+a))-I)-I/((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)/(exp
(2*c*(b*x+a))-1)*(1+exp(2*c*(b*x+a)))/c/b*ln(exp(c*(b*x+a))+I)
```

Maxima [A]
time = 0.48, size = 35, normalized size = 0.42

$$-\frac{2\,\arctan\left(e^{(bcx+ac)}\right)}{bc}+\frac{e^{(bcx+ac)}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*arctan(e^(b*c*x + a*c))/(b*c) + e^(b*c*x + a*c)/(b*c)
```

Fricas [A]
time = 0.35, size = 53, normalized size = 0.64

$$-\frac{2\,\arctan\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)-\cosh\left(bcx+ac\right)-\sinh\left(bcx+ac\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*arctan(cosh(b*c*x + a*c) + sinh(b*c*x + a*c)) - cosh(b*c*x + a*c) - sin
h(b*c*x + a*c))/(b*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int\frac{e^{bcx}}{\sqrt{\coth^2\left(ac+bcx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)**2)**(1/2),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)/sqrt(coth(a*c + b*c*x)**2), x)
```

Giac [A]
time = 0.41, size = 60, normalized size = 0.72

$$-\frac{2\arctan\left(e^{(bcx+ac)}\right)\operatorname{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)-e^{(bcx+ac)}\operatorname{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(2*arctan(e^(b*c*x + a*c))*sgn(e^(2*b*c*x + 2*a*c) - 1) - e^(b*c*x + a*c)*
sgn(e^(2*b*c*x + 2*a*c) - 1))/(b*c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e^{c\,(a+b\,x)}}{\sqrt{\operatorname{coth}\left(a\,c+b\,c\,x\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))/(coth(a*c + b*c*x)^2)^(1/2),x)
```

```
[Out] int(exp(c*(a + b*x))/(coth(a*c + b*c*x)^2)^(1/2), x)
```

3.215 $\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{3/2}}\,dx$

Optimal. Leaf size=193

$$\frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}}-\frac{2e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^2\sqrt{\coth^2(ac+bcx)}}+\frac{3e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)\sqrt{\coth^2(ac+bcx)}}-\frac{3\mathrm{ArcT}}{}$$

```
[Out] exp(c*(b*x+a))*coth(b*c*x+a*c)/b/c/(coth(b*c*x+a*c)^2)^(1/2)-2*exp(c*(b*x+a
))*coth(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^2/(coth(b*c*x+a*c)^2)^(1/2)+3*e
xp(c*(b*x+a))*coth(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))/(coth(b*c*x+a*c)^2)^
(1/2)-3*arctan(exp(c*(b*x+a)))*coth(b*c*x+a*c)/b/c/(coth(b*c*x+a*c)^2)^(1/2
)
```

Rubi [A]
time = 0.60, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {6852, 2320, 398, 1172, 12, 294, 209}

$$-\frac{3\,\mathrm{ArcTan}\left(e^{c(a+bx)}\right)\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}}+\frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}}+\frac{3e^{c(a+bx)}\coth(ac+bcx)}{bc\left(e^{2c(a+bx)}+1\right)\sqrt{\coth^2(ac+bcx)}}-\frac{2e^{c(a+bx)}\coth(ac+bcx)}{bc\left(e^{2c(a+bx)}+1\right)^2\sqrt{\coth^2(ac+bcx)}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/(Coth[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] (E^(c*(a + b*x))*Coth[a*c + b*c*x])/(b*c*Sqrt[Coth[a*c + b*c*x]^2]) - (2*E^
(c*(a + b*x))*Coth[a*c + b*c*x])/(b*c*(1 + E^(2*c*(a + b*x)))^2*Sqrt[Coth[a
*c + b*c*x]^2]) + (3*E^(c*(a + b*x))*Coth[a*c + b*c*x])/(b*c*(1 + E^(2*c*(a
 + b*x)))*Sqrt[Coth[a*c + b*c*x]^2]) - (3*ArcTan[E^(c*(a + b*x))]*Coth[a*c
+ b*c*x])/(b*c*Sqrt[Coth[a*c + b*c*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
```

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1172

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{3/2}}\,dx = \frac{\coth(ac+bcx)\int e^{c(a+bx)}\tanh^3(ac+bcx)\,dx}{\sqrt{\coth^2(ac+bcx)}}$$

$$= \frac{\coth(ac+bcx)\text{Subst}\left(\int \frac{(-1+x^2)^3}{(1+x^2)^3}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\coth^2(ac+bcx)}}$$

$$= \frac{\coth(ac+bcx)\text{Subst}\left(\int \left(1-\frac{2(1+3x^4)}{(1+x^2)^3}\right)\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\coth^2(ac+bcx)}}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{(2\coth(ac+bcx))\text{Subst}\left(\int \frac{1+3x^4}{(1+x^2)^3}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\coth^2(ac+bcx)}}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{2e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^2\sqrt{\coth^2(ac+bcx)}} + \frac{\coth(ac+bcx)}{2b}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{2e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^2\sqrt{\coth^2(ac+bcx)}} - \frac{(6\coth(ac+bc}{b}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{2e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^2\sqrt{\coth^2(ac+bcx)}} + \frac{3e^{c(a+bx)}}{bc\left(1+e^{2c(a+bx}\right.}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{2e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^2\sqrt{\coth^2(ac+bcx)}} + \frac{3e^{c(a+bx)}}{bc\left(1+e^{2c(a+bx}\right.}$$

Mathematica [A]
time = 0.21, size = 104, normalized size = 0.54

$$\frac{\left(e^{c(a+bx)}\left(2+5e^{2c(a+bx)}+e^{4c(a+bx)}\right)-3\left(1+e^{2c(a+bx)}\right)^2\text{ArcTan}\left(e^{c(a+bx)}\right)\right)\coth(c(a+bx))}{bc\left(1+e^{2c(a+bx)}\right)^2\sqrt{\coth^2(c(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/(Coth[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] ((E^(c*(a + b*x))*(2 + 5*E^(2*c*(a + b*x)) + E^(4*c*(a + b*x))) - 3*(1 + E^
(2*c*(a + b*x)))^2*ArcTan[E^(c*(a + b*x))])*Coth[c*(a + b*x)])/(b*c*(1 + E^
(2*c*(a + b*x)))^2*Sqrt[Coth[c*(a + b*x)]^2])
```

Maple [C] Result contains complex when optimal does not.
time = 5.25, size = 301, normalized size = 1.56

method	result
risch	$\frac{(1+e^{2c(bx+a)})e^{c(bx+a)}}{\sqrt{\frac{(1+e^{2c(bx+a)})^2}{(e^{2c(bx+a)}-1)^2}}\,(e^{2c(bx+a)}-1)bc}+\frac{e^{c(bx+a)}(3\,e^{2c(bx+a)}+1)}{(1+e^{2c(bx+a)})(e^{2c(bx+a)}-1)\sqrt{\frac{(1+e^{2c(bx+a)})^2}{(e^{2c(bx+a)}-1)^2}}\,bc}+\frac{3i(1+e^{2c(bx+a)})\ln(e^{c(bx+a)}}{2\sqrt{\frac{(1+e^{2c(bx+a)})^2}{(e^{2c(bx+a)}-1)^2}}\,(e^{2c(bx+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)/(exp(2*c*(b*x+a))-1
)*(1+exp(2*c*(b*x+a)))*exp(c*(b*x+a))/b/c+1/(1+exp(2*c*(b*x+a)))/(exp(2*c*(
b*x+a))-1)/((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)*exp(c*(b*x
+a))*(3*exp(2*c*(b*x+a))+1)/b/c+3/2*I*(1+exp(2*c*(b*x+a)))/(exp(2*c*(b*x+a)
)-1)/((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)/c/b*ln(exp(c*(b*
x+a))-I)-3/2*I*(1+exp(2*c*(b*x+a)))/(exp(2*c*(b*x+a))-1)/((1+exp(2*c*(b*x+a
)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)/c/b*ln(exp(c*(b*x+a))+I)
```

Maxima [A]
time = 0.48, size = 90, normalized size = 0.47

$$-\frac{3\,\arctan\left(e^{(bcx+ac)}\right)}{bc}+\frac{e^{(5\,bcx+5\,ac)}+5\,e^{(3\,bcx+3\,ac)}+2\,e^{(bcx+ac)}}{bc\left(e^{(4\,bcx+4\,ac)}+2\,e^{(2\,bcx+2\,ac)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -3*arctan(e^(b*c*x + a*c))/(b*c) + (e^(5*b*c*x + 5*a*c) + 5*e^(3*b*c*x + 3*
a*c) + 2*e^(b*c*x + a*c))/(b*c*(e^(4*b*c*x + 4*a*c) + 2*e^(2*b*c*x + 2*a*c)
 + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 458 vs. 2(179) = 358.
time = 0.35, size = 458, normalized size = 2.37

$$\frac{\cosh(bcx+ac)^5+5\cosh(bcx+ac)\sinh(bcx+ac)^4+\sinh(bcx+ac)^5+5\left(2\cosh(bcx+ac)^2+1\right)\sinh(bcx+ac)^3+5\cosh(bcx+ac)^3+5\left(2\cosh(bcx+ac)^3+3\cosh(bcx+ac)\right)\sinh(bcx+ac)^2-3\left(\cosh(bcx+ac)^4+4\cosh(bcx+ac)\sinh(bcx+ac)^3+\sinh(bcx+ac)^4+2\left(3\cosh(bcx+ac)^2+1\right)\sinh(bcx+ac)^2+2\cosh(bcx+ac)^2+4\left(\cosh(bcx+ac)^3+\cosh(bcx+ac)\right)\sinh(bcx+ac)+1\right)\arctan\left(\cosh(bcx+ac)+\sinh(bcx+ac)\right)+\left(5\cosh(bcx+ac)^4+15\cosh(bcx+ac)^2+2\right)\sinh(bcx+ac)+2\cosh(bcx+ac)}{bc\cosh(bcx+ac)^4+4\,bc\cosh(bcx+ac)\sinh(bcx+ac)^3+bc\sinh(bcx+ac)^4+2\,bc\cosh(bcx+ac)^2+2\left(3\,bc\cosh(bcx+ac)^2+bc\right)\sinh(bcx+ac)^2+bc+4\left(bc\cosh(bcx+ac)^3+bc\cosh(bcx+ac)\right)\sinh(bcx+ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (cosh(b*c*x + a*c)^5 + 5*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^4 + sinh(b*c*x
 + a*c)^5 + 5*(2*cosh(b*c*x + a*c)^2 + 1)*sinh(b*c*x + a*c)^3 + 5*cosh(b*c*
x + a*c)^3 + 5*(2*cosh(b*c*x + a*c)^3 + 3*cosh(b*c*x + a*c))*sinh(b*c*x + a
*c)^2 - 3*(cosh(b*c*x + a*c)^4 + 4*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^3 +
sinh(b*c*x + a*c)^4 + 2*(3*cosh(b*c*x + a*c)^2 + 1)*sinh(b*c*x + a*c)^2 + 2
*cosh(b*c*x + a*c)^2 + 4*(cosh(b*c*x + a*c)^3 + cosh(b*c*x + a*c))*sinh(b*c
```

```
*x + a*c) + 1)*arctan(cosh(b*c*x + a*c) + sinh(b*c*x + a*c)) + (5*cosh(b*c*
x + a*c)^4 + 15*cosh(b*c*x + a*c)^2 + 2)*sinh(b*c*x + a*c) + 2*cosh(b*c*x +
 a*c))/(b*c*cosh(b*c*x + a*c)^4 + 4*b*c*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)
^3 + b*c*sinh(b*c*x + a*c)^4 + 2*b*c*cosh(b*c*x + a*c)^2 + 2*(3*b*c*cosh(b*
c*x + a*c)^2 + b*c)*sinh(b*c*x + a*c)^2 + b*c + 4*(b*c*cosh(b*c*x + a*c)^3
+ b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int\frac{e^{bcx}}{\left(\coth^{2}\left(ac+bcx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)**2)**(3/2),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)/(coth(a*c + b*c*x)**2)**(3/2), x)
```

Giac [**A**]
time = 0.41, size = 130, normalized size = 0.67

$$-\frac{\left(3\arctan\left(e^{(bcx+ac)}\right)e^{(-ac)}\mathrm{sgn}\left(e^{(2bcx+2ac)}-1\right)-e^{(bcx)}\mathrm{sgn}\left(e^{(2bcx+2ac)}-1\right)-\frac{3e^{(3bcx+2ac)}\mathrm{sgn}\left(e^{(2bcx+2ac)}-1\right)+e^{(bcx)}\mathrm{sgn}\left(e^{(2bcx+2ac)}-1\right)}{\left(e^{(2bcx+2ac)}+1\right)^{2}}\right)e^{(ac)}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -(3*arctan(e^(b*c*x + a*c))*e^(-a*c)*sgn(e^(2*b*c*x + 2*a*c) - 1) - e^(b*c*
x)*sgn(e^(2*b*c*x + 2*a*c) - 1) - (3*e^(3*b*c*x + 2*a*c)*sgn(e^(2*b*c*x + 2
*a*c) - 1) + e^(b*c*x)*sgn(e^(2*b*c*x + 2*a*c) - 1))/(e^(2*b*c*x + 2*a*c) +
 1)^2)*e^(a*c)/(b*c)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\mathrm{e}^{c\,\left(a+b\,x\right)}}{{\left({\mathrm{coth}\left(a\,c+b\,c\,x\right)}^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))/(coth(a*c + b*c*x)^2)^(3/2),x)
```

```
[Out] int(exp(c*(a + b*x))/(coth(a*c + b*c*x)^2)^(3/2), x)
```

3.216 $\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{5/2}} \, dx$

Optimal. Leaf size=311

$$\frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}}-\frac{4e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^4\sqrt{\coth^2(ac+bcx)}}+\frac{26e^{c(a+bx)}\coth(ac+bcx)}{3bc\left(1+e^{2c(a+bx)}\right)^3\sqrt{\coth^2(ac+bcx)}}-\frac{}{6bc}$$

```
[Out] exp(c*(b*x+a))*coth(b*c*x+a*c)/b/c/(coth(b*c*x+a*c)^2)^(1/2)-4*exp(c*(b*x+a
))*coth(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^4/(coth(b*c*x+a*c)^2)^(1/2)+26/
3*exp(c*(b*x+a))*coth(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^3/(coth(b*c*x+a*c
)^2)^(1/2)-55/6*exp(c*(b*x+a))*coth(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^2/(
coth(b*c*x+a*c)^2)^(1/2)+25/4*exp(c*(b*x+a))*coth(b*c*x+a*c)/b/c/(1+exp(2*c
*(b*x+a)))/(coth(b*c*x+a*c)^2)^(1/2)-15/4*arctan(exp(c*(b*x+a)))*coth(b*c*x
+a*c)/b/c/(coth(b*c*x+a*c)^2)^(1/2)
```

Rubi [A]

time = 1.22, antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {6852, 2320, 398, 1828, 1171, 393, 209}

$$-\frac{15\text{ArcTan}\left(e^{c(a+bx)}\right)\coth(ac+bcx)}{4bc\sqrt{\coth^2(ac+bcx)}}+\frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}}+\frac{25e^{c(a+bx)}\coth(ac+bcx)}{4bc\left(e^{2c(a+bx)}+1\right)\sqrt{\coth^2(ac+bcx)}}-\frac{55e^{c(a+bx)}\coth(ac+bcx)}{6bc\left(e^{2c(a+bx)}+1\right)^2\sqrt{\coth^2(ac+bcx)}}+\frac{26e^{c(a+bx)}\coth(ac+bcx)}{3bc\left(e^{2c(a+bx)}+1\right)^3\sqrt{\coth^2(ac+bcx)}}-\frac{4e^{c(a+bx)}\coth(ac+bcx)}{bc\left(e^{2c(a+bx)}+1\right)^4\sqrt{\coth^2(ac+bcx)}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/(Coth[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] (E^(c*(a + b*x))*Coth[a*c + b*c*x])/(b*c*Sqrt[Coth[a*c + b*c*x]^2]) - (4*E^
(c*(a + b*x))*Coth[a*c + b*c*x])/(b*c*(1 + E^(2*c*(a + b*x)))^4*Sqrt[Coth[a
*c + b*c*x]^2]) + (26*E^(c*(a + b*x))*Coth[a*c + b*c*x])/(3*b*c*(1 + E^(2*c
*(a + b*x)))^3*Sqrt[Coth[a*c + b*c*x]^2]) - (55*E^(c*(a + b*x))*Coth[a*c +
b*c*x])/(6*b*c*(1 + E^(2*c*(a + b*x)))^2*Sqrt[Coth[a*c + b*c*x]^2]) + (25*E
^(c*(a + b*x))*Coth[a*c + b*c*x])/(4*b*c*(1 + E^(2*c*(a + b*x)))*Sqrt[Coth[
a*c + b*c*x]^2]) - (15*ArcTan[E^(c*(a + b*x))]*Coth[a*c + b*c*x])/(4*b*c*Sq
rt[Coth[a*c + b*c*x]^2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
```

```
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{5/2}}\,dx = \frac{\coth(ac+bcx)\int e^{c(a+bx)}\tanh^5(ac+bcx)\,dx}{\sqrt{\coth^2(ac+bcx)}}$$

$$= \frac{\coth(ac+bcx)\text{Subst}\left(\int \frac{(-1+x^2)^5}{(1+x^2)^5}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\coth^2(ac+bcx)}}$$

$$= \frac{\coth(ac+bcx)\text{Subst}\left(\int \left(1-\frac{2(1+10x^4+5x^8)}{(1+x^2)^5}\right)\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\coth^2(ac+bcx)}}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{(2\coth(ac+bcx))\text{Subst}\left(\int \frac{1+10x^4+5x^8}{(1+x^2)^5}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\coth^2(ac+bcx)}}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{4e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^4\sqrt{\coth^2(ac+bcx)}} + \frac{\coth(ac+bc}{}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{4e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^4\sqrt{\coth^2(ac+bcx)}} + \frac{26e^{c(a+}}{3bc\left(1+e^{2c(a}\right.}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{4e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^4\sqrt{\coth^2(ac+bcx)}} + \frac{26e^{c(a+}}{3bc\left(1+e^{2c(a}\right.}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{4e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^4\sqrt{\coth^2(ac+bcx)}} + \frac{26e^{c(a+}}{3bc\left(1+e^{2c(a}\right.}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{4e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^4\sqrt{\coth^2(ac+bcx)}} + \frac{26e^{c(a+}}{3bc\left(1+e^{2c(a}\right.}$$

Mathematica [A]
time = 0.33, size = 133, normalized size = 0.43

$$\frac{\left(e^{c(a+bx)}\left(33+157e^{2c(a+bx)}+187e^{4c(a+bx)}+123e^{6c(a+bx)}+12e^{8c(a+bx)}\right)-45\left(1+e^{2c(a+bx)}\right)^4\text{ArcTan}\left(e^{c(a+bx)}\right)\right)\coth(c(a+bx))}{12bc\left(1+e^{2c(a+bx)}\right)^4\sqrt{\coth^2(c(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/(Coth[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] ((E^(c*(a + b*x))*(33 + 157*E^(2*c*(a + b*x)) + 187*E^(4*c*(a + b*x)) + 123
*E^(6*c*(a + b*x)) + 12*E^(8*c*(a + b*x))) - 45*(1 + E^(2*c*(a + b*x)))^4*A
```

```
rcTan[E^(c*(a + b*x))]]*Coth[c*(a + b*x)])/(12*b*c*(1 + E^(2*c*(a + b*x)))^
4*Sqrt[Coth[c*(a + b*x)]^2])
```

Maple [C] Result contains complex when optimal does not.
time = 5.32, size = 324, normalized size = 1.04

method	result
risch	$\frac{\left(1+{\mathrm e}^{2 c \left(b x +a \right)}\right) {\mathrm e}^{c \left(b x +a \right)}}{\sqrt{\frac{\left(1+{\mathrm e}^{2 c \left(b x +a \right)}\right)^{2}}{\left({\mathrm e}^{2 c \left(b x +a \right)}-1\right)^{2}}}\, \left({\mathrm e}^{2 c \left(b x +a \right)}-1\right) b c}+\frac{{\mathrm e}^{c \left(b x +a \right)} \left(75 \,{\mathrm e}^{6 c \left(b x +a \right)}+115 \,{\mathrm e}^{4 c \left(b x +a \right)}+109 \,{\mathrm e}^{2 c \left(b x +a \right)}+21\right)}{12 \left(1+{\mathrm e}^{2 c \left(b x +a \right)}\right)^{3} \left({\mathrm e}^{2 c \left(b x +a \right)}-1\right) \sqrt{\frac{\left(1+{\mathrm e}^{2 c \left(b x +a \right)}\right)^{2}}{\left({\mathrm e}^{2 c \left(b x +a \right)}-1\right)^{2}}}\, b c}+\frac{15 i \left(1+{\mathrm e}^{2 c \left(b x +a \right)}\right) \ln \left({\mathrm e}^{c \left(b x\right.}\right.}{8 \sqrt{\frac{\left(1+{\mathrm e}^{2 c \left(b x +a \right)}\right)^{2}}{\left({\mathrm e}^{2 c \left(b x +a \right)}-1\right)^{2}}}\, \left({\mathrm e}^{2 c \left(b x\right.}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)/(exp(2*c*(b*x+a))-1
)*(1+exp(2*c*(b*x+a)))*exp(c*(b*x+a))/b/c+1/12/(1+exp(2*c*(b*x+a)))^3/(exp(
2*c*(b*x+a))-1)/((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)*exp(c
*(b*x+a))*(75*exp(6*c*(b*x+a))+115*exp(4*c*(b*x+a))+109*exp(2*c*(b*x+a))+21
)/b/c+15/8*I*(1+exp(2*c*(b*x+a)))/(exp(2*c*(b*x+a))-1)/((1+exp(2*c*(b*x+a))
)^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)/c/b*ln(exp(c*(b*x+a))-I)-15/8*I*(1+exp(2*
c*(b*x+a)))/(exp(2*c*(b*x+a))-1)/((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-
1)^2)^(1/2)/c/b*ln(exp(c*(b*x+a))+I)
```

Maxima [A]
time = 0.48, size = 145, normalized size = 0.47

$$-\frac{15 \arctan\left(e^{(bcx+ac)}\right)}{4\,bc}+\frac{12\,e^{(9\,bcx+9\,ac)}+123\,e^{(7\,bcx+7\,ac)}+187\,e^{(5\,bcx+5\,ac)}+157\,e^{(3\,bcx+3\,ac)}+33\,e^{(bcx+ac)}}{12\,bc\left(e^{(8\,bcx+8\,ac)}+4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}+4\,e^{(2\,bcx+2\,ac)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -15/4*arctan(e^(b*c*x + a*c))/(b*c) + 1/12*(12*e^(9*b*c*x + 9*a*c) + 123*e^
(7*b*c*x + 7*a*c) + 187*e^(5*b*c*x + 5*a*c) + 157*e^(3*b*c*x + 3*a*c) + 33*
e^(b*c*x + a*c))/(b*c*(e^(8*b*c*x + 8*a*c) + 4*e^(6*b*c*x + 6*a*c) + 6*e^(4
*b*c*x + 4*a*c) + 4*e^(2*b*c*x + 2*a*c) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1226 vs. $2(281) = 562$.
time = 0.36, size = 1226, normalized size = 3.94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12*(12*cosh(b*c*x + a*c)^9 + 108*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^8 +
12*sinh(b*c*x + a*c)^9 + 3*(144*cosh(b*c*x + a*c)^2 + 41)*sinh(b*c*x + a*c)
^7 + 123*cosh(b*c*x + a*c)^7 + 21*(48*cosh(b*c*x + a*c)^3 + 41*cosh(b*c*x +
 a*c))*sinh(b*c*x + a*c)^6 + (1512*cosh(b*c*x + a*c)^4 + 2583*cosh(b*c*x +
a*c)^2 + 187)*sinh(b*c*x + a*c)^5 + 187*cosh(b*c*x + a*c)^5 + (1512*cosh(b*
c*x + a*c)^5 + 4305*cosh(b*c*x + a*c)^3 + 935*cosh(b*c*x + a*c))*sinh(b*c*x
 + a*c)^4 + (1008*cosh(b*c*x + a*c)^6 + 4305*cosh(b*c*x + a*c)^4 + 1870*cos
h(b*c*x + a*c)^2 + 157)*sinh(b*c*x + a*c)^3 + 157*cosh(b*c*x + a*c)^3 + (43
2*cosh(b*c*x + a*c)^7 + 2583*cosh(b*c*x + a*c)^5 + 1870*cosh(b*c*x + a*c)^3
 + 471*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^2 - 45*(cosh(b*c*x + a*c)^8 + 8
*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^7 + sinh(b*c*x + a*c)^8 + 4*(7*cosh(b*
c*x + a*c)^2 + 1)*sinh(b*c*x + a*c)^6 + 4*cosh(b*c*x + a*c)^6 + 8*(7*cosh(b
*c*x + a*c)^3 + 3*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^5 + 2*(35*cosh(b*c*x
 + a*c)^4 + 30*cosh(b*c*x + a*c)^2 + 3)*sinh(b*c*x + a*c)^4 + 6*cosh(b*c*x
+ a*c)^4 + 8*(7*cosh(b*c*x + a*c)^5 + 10*cosh(b*c*x + a*c)^3 + 3*cosh(b*c*x
 + a*c))*sinh(b*c*x + a*c)^3 + 4*(7*cosh(b*c*x + a*c)^6 + 15*cosh(b*c*x + a
*c)^4 + 9*cosh(b*c*x + a*c)^2 + 1)*sinh(b*c*x + a*c)^2 + 4*cosh(b*c*x + a*c
)^2 + 8*(cosh(b*c*x + a*c)^7 + 3*cosh(b*c*x + a*c)^5 + 3*cosh(b*c*x + a*c)^
3 + cosh(b*c*x + a*c))*sinh(b*c*x + a*c) + 1)*arctan(cosh(b*c*x + a*c) + si
nh(b*c*x + a*c)) + (108*cosh(b*c*x + a*c)^8 + 861*cosh(b*c*x + a*c)^6 + 935
*cosh(b*c*x + a*c)^4 + 471*cosh(b*c*x + a*c)^2 + 33)*sinh(b*c*x + a*c) + 33
*cosh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^8 + 8*b*c*cosh(b*c*x + a*c)*sinh
(b*c*x + a*c)^7 + b*c*sinh(b*c*x + a*c)^8 + 4*b*c*cosh(b*c*x + a*c)^6 + 4*(
7*b*c*cosh(b*c*x + a*c)^2 + b*c)*sinh(b*c*x + a*c)^6 + 6*b*c*cosh(b*c*x + a
*c)^4 + 8*(7*b*c*cosh(b*c*x + a*c)^3 + 3*b*c*cosh(b*c*x + a*c))*sinh(b*c*x
+ a*c)^5 + 2*(35*b*c*cosh(b*c*x + a*c)^4 + 30*b*c*cosh(b*c*x + a*c)^2 + 3*b
*c)*sinh(b*c*x + a*c)^4 + 4*b*c*cosh(b*c*x + a*c)^2 + 8*(7*b*c*cosh(b*c*x +
 a*c)^5 + 10*b*c*cosh(b*c*x + a*c)^3 + 3*b*c*cosh(b*c*x + a*c))*sinh(b*c*x
+ a*c)^3 + 4*(7*b*c*cosh(b*c*x + a*c)^6 + 15*b*c*cosh(b*c*x + a*c)^4 + 9*b*
c*cosh(b*c*x + a*c)^2 + b*c)*sinh(b*c*x + a*c)^2 + b*c + 8*(b*c*cosh(b*c*x
+ a*c)^7 + 3*b*c*cosh(b*c*x + a*c)^5 + 3*b*c*cosh(b*c*x + a*c)^3 + b*c*cosh
(b*c*x + a*c))*sinh(b*c*x + a*c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.41, size = 185, normalized size = 0.59

$$-\frac{\left(45 \arctan\left(e^{(bcx+ac)}\right) e^{(-ac)}\operatorname{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right) - 12\,e^{(bcx)}\operatorname{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right) - \frac{75\,e^{(7\,bcx+6\,ac)}\operatorname{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)+115\,e^{(5\,bcx+4\,ac)}\operatorname{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)+109\,e^{(3\,bcx+2\,ac)}\operatorname{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)+21\,e^{(bcx)}\operatorname{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)}{\left(e^{(2\,bcx+2\,ac)}+1\right)^{4}}\right)e^{(ac)}}{12\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/12*(45*arctan(e^(b*c*x + a*c))*e^(-a*c)*sgn(e^(2*b*c*x + 2*a*c) - 1) - 1
2*e^(b*c*x)*sgn(e^(2*b*c*x + 2*a*c) - 1) - (75*e^(7*b*c*x + 6*a*c)*sgn(e^(2
*b*c*x + 2*a*c) - 1) + 115*e^(5*b*c*x + 4*a*c)*sgn(e^(2*b*c*x + 2*a*c) - 1)
 + 109*e^(3*b*c*x + 2*a*c)*sgn(e^(2*b*c*x + 2*a*c) - 1) + 21*e^(b*c*x)*sgn(
e^(2*b*c*x + 2*a*c) - 1))/(e^(2*b*c*x + 2*a*c) + 1)^4)*e^(a*c)/(b*c)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e^{c\,(a+b\,x)}}{\left(\coth\left(a\,c+b\,c\,x\right)^2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))/(coth(a*c + b*c*x)^2)^(5/2),x)
```

```
[Out] int(exp(c*(a + b*x))/(coth(a*c + b*c*x)^2)^(5/2), x)
```

3.217 $\int \sin^3(\coth(a+bx))\,dx$

Optimal. Leaf size=157

$$-\frac{3\text{CosIntegral}(1-\coth(a+bx))\sin(1)}{8b}-\frac{3\text{CosIntegral}(1+\coth(a+bx))\sin(1)}{8b}+\frac{\text{CosIntegral}(3-3\coth(a$$

```
[Out] 1/8*cos(3)*Si(-3+3*coth(b*x+a))/b-3/8*cos(1)*Si(-1+coth(b*x+a))/b+3/8*cos(1
)*Si(1+coth(b*x+a))/b-1/8*cos(3)*Si(3+3*coth(b*x+a))/b-3/8*Ci(1-coth(b*x+a)
)*sin(1)/b-3/8*Ci(1+coth(b*x+a))*sin(1)/b+1/8*Ci(3-3*coth(b*x+a))*sin(3)/b+
1/8*Ci(3+3*coth(b*x+a))*sin(3)/b
```

Rubi [A]
time = 0.28, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {6857, 3393, 3384, 3380, 3383}

$$\frac{\sin(3)\text{CosIntegral}(3-3\coth(a+bx))}{8b}+\frac{\sin(3)\text{CosIntegral}(3\coth(a+bx)+3)}{8b}-\frac{3\sin(1)\text{CosIntegral}(1-\coth(a+bx))}{8b}-\frac{3\sin(1)\text{CosIntegral}(\coth(a+bx)+1)}{8b}-\frac{\cos(3)\text{Si}(3-3\coth(a+bx))}{8b}+\frac{3\cos(1)\text{Si}(1-\coth(a+bx))}{8b}+\frac{3\cos(1)\text{Si}(\coth(a+bx)+1)}{8b}-\frac{\cos(3)\text{Si}(3\coth(a+bx)+3)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Coth[a + b*x]]^3,x]
```

```
[Out] (-3*CosIntegral[1 - Coth[a + b*x]]*Sin[1])/(8*b) - (3*CosIntegral[1 + Coth[
a + b*x]]*Sin[1])/(8*b) + (CosIntegral[3 - 3*Coth[a + b*x]]*Sin[3])/(8*b) +
 (CosIntegral[3 + 3*Coth[a + b*x]]*Sin[3])/(8*b) - (Cos[3]*SinIntegral[3 -
3*Coth[a + b*x]])/(8*b) + (3*Cos[1]*SinIntegral[1 - Coth[a + b*x]])/(8*b) +
 (3*Cos[1]*SinIntegral[1 + Coth[a + b*x]])/(8*b) - (Cos[3]*SinIntegral[3 +
3*Coth[a + b*x]])/(8*b)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \sin^3(\coth(a+bx))\,dx &= \frac{\text{Subst}\left(\int \frac{\sin^3(x)}{1-x^2}\,dx, x, \coth(a+bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{\sin^3(x)}{2(-1+x)}+\frac{\sin^3(x)}{2(1+x)}\right)\,dx, x, \coth(a+bx)\right)}{b} \\
&= -\frac{\text{Subst}\left(\int \frac{\sin^3(x)}{-1+x}\,dx, x, \coth(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \frac{\sin^3(x)}{1+x}\,dx, x, \coth(a+bx)\right)}{2b} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{3\sin(x)}{4(-1+x)}-\frac{\sin(3x)}{4(-1+x)}\right)\,dx, x, \coth(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \left(\frac{3\sin(x)}{4(1+x)}-\frac{\sin(3x)}{4(1+x)}\right. \cdots}{2b} \\
&= \frac{\text{Subst}\left(\int \frac{\sin(3x)}{-1+x}\,dx, x, \coth(a+bx)\right)}{8b}-\frac{\text{Subst}\left(\int \frac{\sin(3x)}{1+x}\,dx, x, \coth(a+bx)\right)}{8b}-\frac{3\text{Su}\cdots}{\cdots} \\
&= \frac{(3\cos(1))\text{Subst}\left(\int \frac{\sin(1-x)}{-1+x}\,dx, x, \coth(a+bx)\right)}{8b}+\frac{(3\cos(1))\text{Subst}\left(\int \frac{\sin(1+x)}{1+x}\,dx, x \cdots\right.}{8b} \\
&= -\frac{3\text{Ci}(1-\coth(a+bx))\sin(1)}{8b}-\frac{3\text{Ci}(1+\coth(a+bx))\sin(1)}{8b}+\frac{\text{Ci}(3-3\coth(a \cdots}{8b}
\end{aligned}$$

Mathematica [A]
time = 0.19, size = 124, normalized size = 0.79

$$\frac{-6\text{CosIntegral}(1-\coth(a+bx))\sin(1)-6\text{CosIntegral}(1+\coth(a+bx))\sin(1)+2\text{CosIntegral}(3-3\coth(a+bx))\sin(3)+2\text{CosIntegral}(3+3\coth(a+bx))\sin(3)-2\cos(3)\text{Si}(3-3\coth(a+bx))+6\cos(1)\text{Si}(1-\coth(a+bx))+6\cos(1)\text{Si}(1+\coth(a+bx))-2\cos(3)\text{Si}(3+3\coth(a+bx))}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Coth[a + b*x]]^3,x]
```

```
[Out] (-6*CosIntegral[1 - Coth[a + b*x]]*Sin[1] - 6*CosIntegral[1 + Coth[a + b*x]
]*Sin[1] + 2*CosIntegral[3 - 3*Coth[a + b*x]]*Sin[3] + 2*CosIntegral[3 + 3*
```

```
Coth[a + b*x]]*Sin[3] - 2*Cos[3]*SinIntegral[3 - 3*Coth[a + b*x]] + 6*Cos[1
]*SinIntegral[1 - Coth[a + b*x]] + 6*Cos[1]*SinIntegral[1 + Coth[a + b*x]]
- 2*Cos[3]*SinIntegral[3 + 3*Coth[a + b*x]])/(16*b)
```

Maple [A]
time = 4.29, size = 118, normalized size = 0.75

method	result
derivativedivides	$-\frac{\mathrm{sinIntegral}(3+3\coth(bx+a))\cos(3)}{8}+\frac{\mathrm{cosineIntegral}(3+3\coth(bx+a))\sin(3)}{8}+\frac{\mathrm{sinIntegral}(-3+3\coth(bx+a))\cos(3)}{8}+\frac{\mathrm{cosineIntegral}}{}$
default	$-\frac{\mathrm{sinIntegral}(3+3\coth(bx+a))\cos(3)}{8}+\frac{\mathrm{cosineIntegral}(3+3\coth(bx+a))\sin(3)}{8}+\frac{\mathrm{sinIntegral}(-3+3\coth(bx+a))\cos(3)}{8}+\frac{\mathrm{cosineIntegral}}{}$
risch	$\frac{ie^{3i}\,\mathrm{expIntegral}\left(1,-\frac{6ie^{-a}}{e^{2bx+a}-e^{-a}}\right)}{16b}-\frac{ie^{-3i}\,\mathrm{expIntegral}\left(1,-\frac{6ie^{-a}}{e^{2bx+a}-e^{-a}}-6i\right)}{16b}+\frac{\pi\,e^{-3i}\mathrm{csgn}\left(\frac{e^{-a}}{-e^{2bx+a}+e^{-a}}\right)}{16b}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(coth(b*x+a))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/8*Si(3+3*coth(b*x+a))*cos(3)+1/8*Ci(3+3*coth(b*x+a))*sin(3)+1/8*Si(
-3+3*coth(b*x+a))*cos(3)+1/8*Ci(-3+3*coth(b*x+a))*sin(3)+3/8*Si(coth(b*x+a)
+1)*cos(1)-3/8*Ci(coth(b*x+a)+1)*sin(1)-3/8*Si(-1+coth(b*x+a))*cos(1)-3/8*C
i(-1+coth(b*x+a))*sin(1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(coth(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] integrate(sin(coth(b*x + a))^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(139) = 278$.
time = 0.42, size = 296, normalized size = 1.89

$$\frac{\mathrm{Ci}\left(\frac{6e^{(2bx+2a)}}{e^{(2bx+2a)}-1}\right)\sin(3)+\mathrm{Ci}\left(-\frac{6e^{(2bx+2a)}}{e^{(2bx+2a)}-1}\right)\sin(3)+\mathrm{Ci}\left(\frac{6}{e^{(2bx+2a)}-1}\right)\sin(3)+\mathrm{Ci}\left(-\frac{6}{e^{(2bx+2a)}-1}\right)\sin(3)-3\,\mathrm{Ci}\left(\frac{2e^{(2bx+2a)}}{e^{(2bx+2a)}-1}\right)\sin(1)-3\,\mathrm{Ci}\left(-\frac{2e^{(2bx+2a)}}{e^{(2bx+2a)}-1}\right)\sin(1)-3\,\mathrm{Ci}\left(\frac{2}{e^{(2bx+2a)}-1}\right)\sin(1)-3\,\mathrm{Ci}\left(-\frac{2}{e^{(2bx+2a)}-1}\right)\sin(1)-2\cos(3)\,\mathrm{Si}\left(\frac{6e^{(2bx+2a)}}{e^{(2bx+2a)}-1}\right)+6\cos(1)\,\mathrm{Si}\left(\frac{2e^{(2bx+2a)}}{e^{(2bx+2a)}-1}\right)+2\cos(3)\,\mathrm{Si}\left(\frac{6}{e^{(2bx+2a)}-1}\right)-6\cos(1)\,\mathrm{Si}\left(\frac{2}{e^{(2bx+2a)}-1}\right)}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(coth(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/16*(cos_integral(6*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1))*sin(3) + cos_in
tegral(-6*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1))*sin(3) + cos_integral(6/(e
^(2*b*x + 2*a) - 1))*sin(3) + cos_integral(-6/(e^(2*b*x + 2*a) - 1))*sin(3)
 - 3*cos_integral(2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1))*sin(1) - 3*cos_i
```

```
ntegral(-2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1))*sin(1) - 3*cos_integral(2
/(e^(2*b*x + 2*a) - 1))*sin(1) - 3*cos_integral(-2/(e^(2*b*x + 2*a) - 1))*s
in(1) - 2*cos(3)*sin_integral(6*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) + 6*
cos(1)*sin_integral(2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) + 2*cos(3)*sin
_integral(6/(e^(2*b*x + 2*a) - 1)) - 6*cos(1)*sin_integral(2/(e^(2*b*x + 2*
a) - 1)))/b
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^3(\coth(a+bx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(coth(b*x+a))**3,x)
```

```
[Out] Integral(sin(coth(a + b*x))**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(coth(b*x+a))^3,x, algorithm="giac")
```

```
[Out] integrate(sin(coth(b*x + a))^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(\mathrm{coth}(a+b\,x)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(coth(a + b*x))^3,x)
```

```
[Out] int(sin(coth(a + b*x))^3, x)
```

3.218 $\int \sin^2(\coth(a+bx))\,dx$

Optimal. Leaf size=115

$$\frac{\cos(2)\text{CosIntegral}(2-2\coth(a+bx))}{4b}-\frac{\cos(2)\text{CosIntegral}(2+2\coth(a+bx))}{4b}-\frac{\log(1-\coth(a+bx))}{4b}+\frac{\text{lo}}{}$$

[Out] 1/4*Ci(2-2*coth(b*x+a))*cos(2)/b-1/4*Ci(2+2*coth(b*x+a))*cos(2)/b-1/4*ln(1-coth(b*x+a))/b+1/4*ln(1+coth(b*x+a))/b-1/4*Si(-2+2*coth(b*x+a))*sin(2)/b-1/4*Si(2+2*coth(b*x+a))*sin(2)/b

Rubi [A]
time = 0.19, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {6857, 3393, 3384, 3380, 3383}

$$\frac{\cos(2)\text{CosIntegral}(2-2\coth(a+bx))}{4b}-\frac{\cos(2)\text{CosIntegral}(2\coth(a+bx)+2)}{4b}+\frac{\sin(2)\text{Si}(2-2\coth(a+bx))}{4b}-\frac{\sin(2)\text{Si}(2\coth(a+bx)+2)}{4b}-\frac{\log(1-\coth(a+bx))}{4b}+\frac{\log(\coth(a+bx)+1)}{4b}$$

Antiderivative was successfully verified.

[In] Int[Sin[Coth[a + b*x]]^2,x]

[Out] (Cos[2]*CosIntegral[2 - 2*Coth[a + b*x]])/(4*b) - (Cos[2]*CosIntegral[2 + 2*Coth[a + b*x]])/(4*b) - Log[1 - Coth[a + b*x]]/(4*b) + Log[1 + Coth[a + b*x]]/(4*b) + (Sin[2]*SinIntegral[2 - 2*Coth[a + b*x]])/(4*b) - (Sin[2]*SinIntegral[2 + 2*Coth[a + b*x]])/(4*b)

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \sin^2(\coth(a+bx))\,dx &= \frac{\text{Subst}\left(\int \frac{\sin^2(x)}{1-x^2}\,dx, x, \coth(a+bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{\sin^2(x)}{2(-1+x)}+\frac{\sin^2(x)}{2(1+x)}\right)\,dx, x, \coth(a+bx)\right)}{b} \\
&= -\frac{\text{Subst}\left(\int \frac{\sin^2(x)}{-1+x}\,dx, x, \coth(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \frac{\sin^2(x)}{1+x}\,dx, x, \coth(a+bx)\right)}{2b} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{2(-1+x)}-\frac{\cos(2x)}{2(-1+x)}\right)\,dx, x, \coth(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \left(\frac{1}{2(1+x)}-\frac{\cos(2x)}{2(1+x)}\right.\right.}{2b} \\
&= -\frac{\log(1-\coth(a+bx))}{4b}+\frac{\log(1+\coth(a+bx))}{4b}+\frac{\text{Subst}\left(\int \frac{\cos(2x)}{-1+x}\,dx, x, \coth(a\right.}{4b} \\
&= -\frac{\log(1-\coth(a+bx))}{4b}+\frac{\log(1+\coth(a+bx))}{4b}+\frac{\cos(2)\text{Subst}\left(\int \frac{\cos(2-2x)}{-1+x}\,dx, x\right.}{4b} \\
&= \frac{\cos(2)\text{Ci}(2-2\coth(a+bx))}{4b}-\frac{\cos(2)\text{Ci}(2+2\coth(a+bx))}{4b}-\frac{\log(1-\coth(a+}{4b}
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 88, normalized size = 0.77

$$\frac{\cos(2)\text{CosIntegral}(2-2\coth(a+bx))-\cos(2)\text{CosIntegral}(2(1+\coth(a+bx)))-\log(1-\coth(a+bx))+\log(1+\coth(a+bx))+\sin(2)\text{Si}(2-2\coth(a+bx))-\sin(2)\text{Si}(2(1+\coth(a+bx)))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Coth[a + b*x]]^2,x]
[Out] (Cos[2]*CosIntegral[2 - 2*Coth[a + b*x]] - Cos[2]*CosIntegral[2*(1 + Coth[a
 + b*x])] - Log[1 - Coth[a + b*x]] + Log[1 + Coth[a + b*x]] + Sin[2]*SinInt
egral[2 - 2*Coth[a + b*x]] - Sin[2]*SinIntegral[2*(1 + Coth[a + b*x])])/(4*
b)
```

Maple [A]
time = 3.46, size = 88, normalized size = 0.77

method	result
derivativedivides	$\frac{-\frac{\ln(-1+\coth(bx+a))}{4}+\frac{\ln(\coth(bx+a)+1)}{4}-\frac{\text{sinIntegral}(-2+2\coth(bx+a))\sin(2)}{4}+\frac{\text{cosineIntegral}(-2+2\coth(bx+a))\cos(2)}{4}-\frac{\text{sinInte}}{}}{b}$
default	$\frac{-\frac{\ln(-1+\coth(bx+a))}{4}+\frac{\ln(\coth(bx+a)+1)}{4}-\frac{\text{sinIntegral}(-2+2\coth(bx+a))\sin(2)}{4}+\frac{\text{cosineIntegral}(-2+2\coth(bx+a))\cos(2)}{4}-\frac{\text{sinInte}}{}}{b}$
risch	$-\frac{i\mathrm{e}^{-2i}\,\text{csgn}\left(\frac{\mathrm{e}^{-a}}{-\mathrm{e}^{2bx+a}+\mathrm{e}^{-a}}\right)\pi}{8b}-\frac{i\mathrm{e}^{-2i}\,\text{sinIntegral}\left(\frac{4\,\mathrm{e}^{-a}}{\mathrm{e}^{2bx+a}-\mathrm{e}^{-a}}\right)}{4b}-\frac{\mathrm{e}^{-2i}\,\text{expIntegral}\left(1,-\frac{4i\mathrm{e}^{-a}}{\mathrm{e}^{2bx+a}-\mathrm{e}^{-a}}\right)}{8b}+\frac{\mathrm{e}^{2i}\,\text{ex}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(coth(b*x+a))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/4*ln(-1+coth(b*x+a))+1/4*ln(coth(b*x+a)+1)-1/4*Si(-2+2*coth(b*x+a))
*sin(2)+1/4*Ci(-2+2*coth(b*x+a))*cos(2)-1/4*Si(2+2*coth(b*x+a))*sin(2)-1/4*
Ci(2+2*coth(b*x+a))*cos(2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(coth(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 1/2*x - 1/2*integrate(cos(2*(e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1)), x
)
```

Fricas [A]
time = 0.35, size = 155, normalized size = 1.35

$$\frac{4\,bx-\cos(2)\,\mathrm{Ci}\left(\frac{4\,e^{(2\,bx+2\,a)}}{e^{(2\,bx+2\,a)}-1}\right)-\cos(2)\,\mathrm{Ci}\left(-\frac{4\,e^{(2\,bx+2\,a)}}{e^{(2\,bx+2\,a)}-1}\right)+\cos(2)\,\mathrm{Ci}\left(\frac{4}{e^{(2\,bx+2\,a)}-1}\right)+\cos(2)\,\mathrm{Ci}\left(-\frac{4}{e^{(2\,bx+2\,a)}-1}\right)-2\,\sin(2)\,\mathrm{Si}\left(\frac{4\,e^{(2\,bx+2\,a)}}{e^{(2\,bx+2\,a)}-1}\right)-2\,\sin(2)\,\mathrm{Si}\left(\frac{4}{e^{(2\,bx+2\,a)}-1}\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(coth(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x - cos(2)*cos_integral(4*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) -
 cos(2)*cos_integral(-4*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) + cos(2)*cos
_integral(4/(e^(2*b*x + 2*a) - 1)) + cos(2)*cos_integral(-4/(e^(2*b*x + 2*a
) - 1)) - 2*sin(2)*sin_integral(4*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) -
2*sin(2)*sin_integral(4/(e^(2*b*x + 2*a) - 1)))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^2\left(\coth\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(coth(b*x+a))**2,x)
```

```
[Out] Integral(sin(coth(a + b*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(coth(b*x+a))^2,x, algorithm="giac")
```

```
[Out] integrate(sin(coth(b*x + a))^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(\coth(a+b\,x)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(coth(a + b*x))^2,x)
```

```
[Out] int(sin(coth(a + b*x))^2, x)
```

3.219 $\int \sin(\coth(a+bx))\,dx$

Optimal. Leaf size=77

$$-\frac{\text{CosIntegral}(1-\coth(a+bx))\sin(1)}{2b}-\frac{\text{CosIntegral}(1+\coth(a+bx))\sin(1)}{2b}+\frac{\cos(1)\text{Si}(1-\coth(a+bx))}{2b}$$

```
[Out] -1/2*cos(1)*Si(-1+coth(b*x+a))/b+1/2*cos(1)*Si(1+coth(b*x+a))/b-1/2*Ci(1-co
th(b*x+a))*sin(1)/b-1/2*Ci(1+coth(b*x+a))*sin(1)/b
```

Rubi [A]
time = 0.10, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3414, 3384, 3380, 3383}

$$-\frac{\sin(1)\text{CosIntegral}(1-\coth(a+bx))}{2b}-\frac{\sin(1)\text{CosIntegral}(\coth(a+bx)+1)}{2b}+\frac{\cos(1)\text{Si}(1-\coth(a+bx))}{2b}+\frac{\cos(1)\text{Si}(\coth(a+bx)+1)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Coth[a + b*x]],x]
```

```
[Out] -1/2*(CosIntegral[1 - Coth[a + b*x]]*Sin[1])/b - (CosIntegral[1 + Coth[a +
b*x]]*Sin[1])/(2*b) + (Cos[1]*SinIntegral[1 - Coth[a + b*x]])/(2*b) + (Cos[
1]*SinIntegral[1 + Coth[a + b*x]])/(2*b)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$
\begin{aligned}
\int \sin(\coth(a+bx))\,dx &= \frac{\text{Subst}\left(\int \frac{\sin(x)}{1-x^2}\,dx, x, \coth(a+bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int \left(\frac{\sin(x)}{2(1-x)}+\frac{\sin(x)}{2(1+x)}\right)\,dx, x, \coth(a+bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int \frac{\sin(x)}{1-x}\,dx, x, \coth(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \frac{\sin(x)}{1+x}\,dx, x, \coth(a+bx)\right)}{2b} \\
&= -\frac{\cos(1)\text{Subst}\left(\int \frac{\sin(1-x)}{1-x}\,dx, x, \coth(a+bx)\right)}{2b}+\frac{\cos(1)\text{Subst}\left(\int \frac{\sin(1+x)}{1+x}\,dx, x, \text{coth}(\right.}{2b} \\
&= -\frac{\text{Ci}(1-\coth(a+bx))\sin(1)}{2b}-\frac{\text{Ci}(1+\coth(a+bx))\sin(1)}{2b}+\frac{\cos(1)\text{Si}(1-\text{coth}(}{2b}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 59, normalized size = 0.77

$$
-\frac{\text{CosIntegral}(1-\coth(a+bx))\sin(1)+\text{CosIntegral}(1+\coth(a+bx))\sin(1)-\cos(1)(\text{Si}(1-\coth(a+bx))+\text{Si}(1+\coth(a+bx)))}{2b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Coth[a + b*x]],x]
```

```
[Out] -1/2*(CosIntegral[1 - Coth[a + b*x]]*Sin[1] + CosIntegral[1 + Coth[a + b*x]
]*Sin[1] - Cos[1]*(SinIntegral[1 - Coth[a + b*x]] + SinIntegral[1 + Coth[a
+ b*x]]))/b
```

Maple [A]
time = 3.32, size = 58, normalized size = 0.75

method	result
derivativedivides	$\frac{\frac{\text{sinIntegral}(\coth(bx+a)+1)\cos(1)}{2}-\frac{\text{cosineIntegral}(\coth(bx+a)+1)\sin(1)}{2}-\frac{\text{sinIntegral}(-1+\coth(bx+a))\cos(1)}{2}-\frac{\text{cosineIntegral}(-1+\text{cotl}}{2}}{b}$
default	$\frac{\frac{\text{sinIntegral}(\coth(bx+a)+1)\cos(1)}{2}-\frac{\text{cosineIntegral}(\coth(bx+a)+1)\sin(1)}{2}-\frac{\text{sinIntegral}(-1+\coth(bx+a))\cos(1)}{2}-\frac{\text{cosineIntegral}(-1+\text{cotl}}{2}}{b}$
risch	$\frac{i\mathrm{e}^{-i}\,\text{expIntegral}\left(1,-\frac{2i\mathrm{e}^{-a}}{\mathrm{e}^{2bx+a}-\mathrm{e}^{-a}}-2i\right)}{4b}-\frac{\pi\,\text{csgn}\left(\frac{\mathrm{e}^{-a}}{-\mathrm{e}^{2bx+a}+\mathrm{e}^{-a}}\right)\mathrm{e}^{i}}{4b}-\frac{\text{sinIntegral}\left(\frac{2\,\mathrm{e}^{-a}}{\mathrm{e}^{2bx+a}-\mathrm{e}^{-a}}\right)\mathrm{e}^{i}}{2b}-\frac{i\,\text{expIntegral}(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/2*Si(coth(b*x+a)+1)*cos(1)-1/2*Ci(coth(b*x+a)+1)*sin(1)-1/2*Si(-1+co
th(b*x+a))*cos(1)-1/2*Ci(-1+coth(b*x+a))*sin(1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] integrate(sin(coth(b*x + a)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(67) = 134$.
time = 0.39, size = 149, normalized size = 1.94

$$-\frac{\operatorname{Ci}\left(\frac{2\,e^{(2\,bx+2\,a)}}{e^{(2\,bx+2\,a)}-1}\right)\sin\left(1\right)+\operatorname{Ci}\left(-\frac{2\,e^{(2\,bx+2\,a)}}{e^{(2\,bx+2\,a)}-1}\right)\sin\left(1\right)+\operatorname{Ci}\left(\frac{2}{e^{(2\,bx+2\,a)}-1}\right)\sin\left(1\right)+\operatorname{Ci}\left(-\frac{2}{e^{(2\,bx+2\,a)}-1}\right)\sin\left(1\right)-2\,\cos\left(1\right)\operatorname{Si}\left(\frac{2\,e^{(2\,bx+2\,a)}}{e^{(2\,bx+2\,a)}-1}\right)+2\,\cos\left(1\right)\operatorname{Si}\left(\frac{2}{e^{(2\,bx+2\,a)}-1}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/4*(cos_integral(2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1))*sin(1) + cos_in
tegral(-2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1))*sin(1) + cos_integral(2/(e
^(2*b*x + 2*a) - 1))*sin(1) + cos_integral(-2/(e^(2*b*x + 2*a) - 1))*sin(1)
 - 2*cos(1)*sin_integral(2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) + 2*cos(1
)*sin_integral(2/(e^(2*b*x + 2*a) - 1)))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(\coth\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(coth(b*x+a)),x)
```

```
[Out] Integral(sin(coth(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(sin(coth(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin (\coth(a + b\,x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(coth(a + b*x)),x)
```

```
[Out] int(sin(coth(a + b*x)), x)
```

3.220 $\int \csc(\coth(a+bx))\,dx$

Optimal. Leaf size=67

$$\frac{1}{2}\text{Int}\left(\frac{\csc(\coth(a+bx))\text{csch}^2(a+bx)}{-1+\coth(a+bx)},x\right)-\frac{1}{2}\text{Int}\left(\frac{\csc(\coth(a+bx))\text{csch}^2(a+bx)}{1+\coth(a+bx)},x\right)$$

```
[Out] 1/2*Unintegrable(csc(coth(b*x+a))*csch(b*x+a)^2/(-1+coth(b*x+a)),x)-1/2*Uni
ntegrable(csc(coth(b*x+a))*csch(b*x+a)^2/(1+coth(b*x+a)),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \csc(\coth(a+bx))\,dx$$

Verification is not applicable to the result.

```
[In] Int[Csc[Coth[a + b*x]],x]
```

```
[Out] -1/2*Defer[Subst][Defer[Int][Csc[x]/(-1 + x), x], x, Coth[a + b*x]]/b + Def
er[Subst][Defer[Int][Csc[x]/(1 + x), x], x, Coth[a + b*x]]/(2*b)
```

Rubi steps

$$\begin{aligned}\int \csc(\coth(a+bx))\,dx &= \frac{\text{Subst}\left(\int \frac{\csc(x)}{1-x^2}\,dx, x, \coth(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{\csc(x)}{2(-1+x)}+\frac{\csc(x)}{2(1+x)}\right)dx, x, \coth(a+bx)\right)}{b}\\ &= -\frac{\text{Subst}\left(\int \frac{\csc(x)}{-1+x}\,dx, x, \coth(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \frac{\csc(x)}{1+x}\,dx, x, \coth(a+bx)\right)}{2b}\end{aligned}$$

Mathematica [A]
time = 1.94, size = 0, normalized size = 0.00

$$\int \csc(\coth(a+bx))\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Csc[Coth[a + b*x]],x]
```

```
[Out] Integrate[Csc[Coth[a + b*x]], x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \csc \left(\coth \left(bx+a\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(coth(b*x+a)),x)
[Out] int(csc(coth(b*x+a)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(coth(b*x+a)),x, algorithm="maxima")
[Out] integrate(csc(coth(b*x + a)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(coth(b*x+a)),x, algorithm="fricas")
[Out] integral(csc(coth(b*x + a)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \csc \left(\coth \left(a+bx\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(coth(b*x+a)),x)
[Out] Integral(csc(coth(a + b*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(csc(coth(b*x + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(\coth\left(a+b\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(coth(a + b*x)),x)
```

```
[Out] int(1/sin(coth(a + b*x)), x)
```

3.221 $\int \cos^3(\coth(a+bx))\,dx$

Optimal. Leaf size=157

$$-\frac{\cos(3)\text{CosIntegral}(3-3\coth(a+bx))}{8b}-\frac{3\cos(1)\text{CosIntegral}(1-\coth(a+bx))}{8b}+\frac{3\cos(1)\text{CosIntegral}(1+c}{8b}$$

```
[Out] -3/8*Ci(1-coth(b*x+a))*cos(1)/b+3/8*Ci(1+coth(b*x+a))*cos(1)/b-1/8*Ci(3-3*c
oth(b*x+a))*cos(3)/b+1/8*Ci(3+3*coth(b*x+a))*cos(3)/b+3/8*Si(-1+coth(b*x+a)
)*sin(1)/b+3/8*Si(1+coth(b*x+a))*sin(1)/b+1/8*Si(-3+3*coth(b*x+a))*sin(3)/b
+1/8*Si(3+3*coth(b*x+a))*sin(3)/b
```

Rubi [A]
time = 0.27, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {6857, 3393, 3384, 3380, 3383}

$$-\frac{\cos(3)\text{CosIntegral}(3-3\coth(a+bx))}{8b}-\frac{3\cos(1)\text{CosIntegral}(1-\coth(a+bx))}{8b}+\frac{3\cos(1)\text{CosIntegral}(\coth(a+bx)+1)}{8b}+\frac{\cos(3)\text{CosIntegral}(3\coth(a+bx)+3)}{8b}-\frac{\sin(3)\text{Si}(3-3\coth(a+bx))}{8b}-\frac{3\sin(1)\text{Si}(1-\coth(a+bx))}{8b}+\frac{3\sin(1)\text{Si}(\coth(a+bx)+1)}{8b}+\frac{\sin(3)\text{Si}(3\coth(a+bx)+3)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Coth[a + b*x]]^3,x]
```

```
[Out] -1/8*(Cos[3]*CosIntegral[3 - 3*Coth[a + b*x]])/b - (3*Cos[1]*CosIntegral[1
 - Coth[a + b*x]])/(8*b) + (3*Cos[1]*CosIntegral[1 + Coth[a + b*x]])/(8*b) +
 (Cos[3]*CosIntegral[3 + 3*Coth[a + b*x]])/(8*b) - (Sin[3]*SinIntegral[3 -
3*Coth[a + b*x]])/(8*b) - (3*Sin[1]*SinIntegral[1 - Coth[a + b*x]])/(8*b) +
 (3*Sin[1]*SinIntegral[1 + Coth[a + b*x]])/(8*b) + (Sin[3]*SinIntegral[3 +
3*Coth[a + b*x]])/(8*b)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \cos^3(\coth(a+bx))\,dx &= \frac{\text{Subst}\left(\int \frac{\cos^3(x)}{1-x^2}\,dx, x, \coth(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{\cos^3(x)}{2(-1+x)}+\frac{\cos^3(x)}{2(1+x)}\right)\,dx, x, \coth(a+bx)\right)}{b}\\
&= -\frac{\text{Subst}\left(\int \frac{\cos^3(x)}{-1+x}\,dx, x, \coth(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \frac{\cos^3(x)}{1+x}\,dx, x, \coth(a+bx)\right)}{2b}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{3\cos(x)}{4(-1+x)}+\frac{\cos(3x)}{4(-1+x)}\right)\,dx, x, \coth(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \left(\frac{3\cos(x)}{4(1+x)}+\frac{\cos(3
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\text{Subst}\left(\int \frac{\cos(3x)}{-1+x}\,dx, x, \coth(a+bx)\right)}{8b}+\frac{\text{Subst}\left(\int \frac{\cos(3x)}{1+x}\,dx, x, \coth(a+bx)\right)}{8b}-\\
&= -\frac{(3\cos(1))\text{Subst}\left(\int \frac{\cos(1-x)}{-1+x}\,dx, x, \coth(a+bx)\right)}{8b}+\frac{(3\cos(1))\text{Subst}\left(\int \frac{\cos(1+x)}{1+x}\,d\right.}{8b}\\
&= -\frac{\cos(3)\text{Ci}(3-3\coth(a+bx))}{8b}-\frac{3\cos(1)\text{Ci}(1-\coth(a+bx))}{8b}+\frac{3\cos(1)\text{Ci}(1
\end{aligned}
$$

Mathematica [A]
time = 0.20, size = 124, normalized size = 0.79

$$
\frac{-2\cos(3)\text{CosIntegral}(3-3\coth(a+bx))-6\cos(1)\text{CosIntegral}(1-\coth(a+bx))+6\cos(1)\text{CosIntegral}(1+\coth(a+bx))+2\cos(3)\text{CosIntegral}(3+3\coth(a+bx))-2\sin(3)\text{Si}(3-3\coth(a+bx))-6\sin(1)\text{Si}(1-\coth(a+bx))+6\sin(1)\text{Si}(1+\coth(a+bx))+2\sin(3)\text{Si}(3+3\coth(a+bx))}{16b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Coth[a + b*x]]^3,x]

[Out] (-2*Cos[3]*CosIntegral[3 - 3*Coth[a + b*x]] - 6*Cos[1]*CosIntegral[1 - Coth
[a + b*x]] + 6*Cos[1]*CosIntegral[1 + Coth[a + b*x]] + 2*Cos[3]*CosIntegral
```

```
[3 + 3*Coth[a + b*x]] - 2*Sin[3]*SinIntegral[3 - 3*Coth[a + b*x]] - 6*Sin[1
]*SinIntegral[1 - Coth[a + b*x]] + 6*Sin[1]*SinIntegral[1 + Coth[a + b*x]]
+ 2*Sin[3]*SinIntegral[3 + 3*Coth[a + b*x]])/(16*b)
```

Maple [A]
time = 3.47, size = 118, normalized size = 0.75

method	result
derivativedivides	$\frac{\text{sinIntegral}(3+3\coth(bx+a))\sin(3)}{8}+\frac{\text{cosineIntegral}(3+3\coth(bx+a))\cos(3)}{8}+\frac{\text{sinIntegral}(-3+3\coth(bx+a))\sin(3)}{8}-\frac{\text{cosineIntegral}(-3-$
default	$\frac{\text{sinIntegral}(3+3\coth(bx+a))\sin(3)}{8}+\frac{\text{cosineIntegral}(3+3\coth(bx+a))\cos(3)}{8}+\frac{\text{sinIntegral}(-3+3\coth(bx+a))\sin(3)}{8}-\frac{\text{cosineIntegral}(-3-$
risch	$-\frac{e^{3i}\,\text{expIntegral}\left(1,\frac{6ie^{-a}}{e^{2bx+a}-e^{-a}}+6i\right)}{16b}+\frac{i\pi\,e^{-3i}\text{csgn}\left(\frac{e^{-a}}{-e^{2bx+a}+e^{-a}}\right)}{16b}+\frac{ie^{-3i}\,\text{sinIntegral}\left(\frac{6\,e^{-a}}{e^{2bx+a}-e^{-a}}\right)}{8b}+\frac{e^{-3i}\,\text{ex}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(coth(b*x+a))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/8*Si(3+3*coth(b*x+a))*sin(3)+1/8*Ci(3+3*coth(b*x+a))*cos(3)+1/8*Si(-
3+3*coth(b*x+a))*sin(3)-1/8*Ci(-3+3*coth(b*x+a))*cos(3)+3/8*Si(coth(b*x+a)+
1)*sin(1)+3/8*Ci(coth(b*x+a)+1)*cos(1)+3/8*Si(-1+coth(b*x+a))*sin(1)-3/8*Ci
(-1+coth(b*x+a))*cos(1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(coth(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] integrate(cos(coth(b*x + a))^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(139) = 278$.
time = 0.38, size = 298, normalized size = 1.90

$$\frac{\cos(3)\,\mathrm{Ci}\left(\frac{6\,e^{(2\,bx+2\,a)}}{e^{(2\,bx+2\,a)}-1}\right)+3\,\cos(1)\,\mathrm{Ci}\left(\frac{2\,e^{(2\,bx+2\,a)}}{e^{(2\,bx+2\,a)}-1}\right)+3\,\cos(1)\,\mathrm{Ci}\left(-\frac{2\,e^{(2\,bx+2\,a)}}{e^{(2\,bx+2\,a)}-1}\right)+\cos(3)\,\mathrm{Ci}\left(-\frac{6\,e^{(2\,bx+2\,a)}}{e^{(2\,bx+2\,a)}-1}\right)-\cos(3)\,\mathrm{Ci}\left(\frac{6}{e^{(2\,bx+2\,a)}-1}\right)-3\,\cos(1)\,\mathrm{Ci}\left(\frac{2}{e^{(2\,bx+2\,a)}-1}\right)-3\,\cos(1)\,\mathrm{Ci}\left(-\frac{2}{e^{(2\,bx+2\,a)}-1}\right)-\cos(3)\,\mathrm{Ci}\left(-\frac{6}{e^{(2\,bx+2\,a)}-1}\right)+2\,\sin(3)\,\mathrm{Si}\left(\frac{6\,e^{(2\,bx+2\,a)}}{e^{(2\,bx+2\,a)}-1}\right)+6\,\sin(1)\,\mathrm{Si}\left(\frac{2\,e^{(2\,bx+2\,a)}}{e^{(2\,bx+2\,a)}-1}\right)+2\,\sin(3)\,\mathrm{Si}\left(\frac{6}{e^{(2\,bx+2\,a)}-1}\right)+6\,\sin(1)\,\mathrm{Si}\left(\frac{2}{e^{(2\,bx+2\,a)}-1}\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(coth(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/16*(cos(3)*cos_integral(6*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) + 3*cos(
1)*cos_integral(2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) + 3*cos(1)*cos_int
egral(-2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) + cos(3)*cos_integral(-6*e^
(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) - cos(3)*cos_integral(6/(e^(2*b*x + 2*
```

```
a) - 1)) - 3*cos(1)*cos_integral(2/(e^(2*b*x + 2*a) - 1)) - 3*cos(1)*cos_in
tegral(-2/(e^(2*b*x + 2*a) - 1)) - cos(3)*cos_integral(-6/(e^(2*b*x + 2*a)
- 1)) + 2*sin(3)*sin_integral(6*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) + 6*
sin(1)*sin_integral(2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) + 2*sin(3)*sin
_integral(6/(e^(2*b*x + 2*a) - 1)) + 6*sin(1)*sin_integral(2/(e^(2*b*x + 2*
a) - 1)))/b
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \cos^3\left(\coth\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(coth(b*x+a))**3,x)
```

```
[Out] Integral(cos(coth(a + b*x))**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(coth(b*x+a))^3,x, algorithm="giac")
```

```
[Out] integrate(cos(coth(b*x + a))^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(\mathrm{coth}(a+b\,x)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(coth(a + b*x))^3,x)
```

```
[Out] int(cos(coth(a + b*x))^3, x)
```

3.222 $\int \cos^2(\coth(a+bx))\,dx$

Optimal. Leaf size=115

$$-\frac{\cos(2)\text{CosIntegral}(2-2\coth(a+bx))}{4b}+\frac{\cos(2)\text{CosIntegral}(2+2\coth(a+bx))}{4b}-\frac{\log(1-\coth(a+bx))}{4b}+\text{lo}$$

```
[Out] -1/4*Ci(2-2*coth(b*x+a))*cos(2)/b+1/4*Ci(2+2*coth(b*x+a))*cos(2)/b-1/4*ln(1
-coth(b*x+a))/b+1/4*ln(1+coth(b*x+a))/b+1/4*Si(-2+2*coth(b*x+a))*sin(2)/b+1
/4*Si(2+2*coth(b*x+a))*sin(2)/b
```

Rubi [A]
time = 0.19, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {6857, 3393, 3384, 3380, 3383}

$$-\frac{\cos(2)\text{CosIntegral}(2-2\coth(a+bx))}{4b}+\frac{\cos(2)\text{CosIntegral}(2\coth(a+bx)+2)}{4b}-\frac{\sin(2)\text{Si}(2-2\coth(a+bx))}{4b}+\frac{\sin(2)\text{Si}(2\coth(a+bx)+2)}{4b}-\frac{\log(1-\coth(a+bx))}{4b}+\frac{\log(\coth(a+bx)+1)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Coth[a + b*x]]^2,x]
```

```
[Out] -1/4*(Cos[2]*CosIntegral[2 - 2*Coth[a + b*x]])/b + (Cos[2]*CosIntegral[2 +
2*Coth[a + b*x]])/(4*b) - Log[1 - Coth[a + b*x]]/(4*b) + Log[1 + Coth[a + b
*x]]/(4*b) - (Sin[2]*SinIntegral[2 - 2*Coth[a + b*x]])/(4*b) + (Sin[2]*SinI
ntegral[2 + 2*Coth[a + b*x]])/(4*b)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos^2(\coth(a+bx))\,dx &= \frac{\text{Subst}\left(\int \frac{\cos^2(x)}{1-x^2}\,dx, x, \coth(a+bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{\cos^2(x)}{2(-1+x)}+\frac{\cos^2(x)}{2(1+x)}\right)\,dx, x, \coth(a+bx)\right)}{b} \\
&= -\frac{\text{Subst}\left(\int \frac{\cos^2(x)}{-1+x}\,dx, x, \coth(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \frac{\cos^2(x)}{1+x}\,dx, x, \coth(a+bx)\right)}{2b} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{2(-1+x)}+\frac{\cos(2x)}{2(-1+x)}\right)\,dx, x, \coth(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \left(\frac{1}{2(1+x)}+\frac{\cos(2x}{2(1+x}\right.\right.}{2} \\
&= -\frac{\log(1-\coth(a+bx))}{4b}+\frac{\log(1+\coth(a+bx))}{4b}-\frac{\text{Subst}\left(\int \frac{\cos(2x)}{-1+x}\,dx, x, \coth(\right.}{4b} \\
&= -\frac{\log(1-\coth(a+bx))}{4b}+\frac{\log(1+\coth(a+bx))}{4b}-\frac{\cos(2)\text{Subst}\left(\int \frac{\cos(2-2x)}{-1+x}\,dx,\right.}{4b} \\
&= -\frac{\cos(2)\text{Ci}(2-2\coth(a+bx))}{4b}+\frac{\cos(2)\text{Ci}(2+2\coth(a+bx))}{4b}-\frac{\log(1-\coth(}{4b}
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 88, normalized size = 0.77

$$\frac{-\cos(2)\text{CosIntegral}(2-2\coth(a+bx))+\cos(2)\text{CosIntegral}(2(1+\coth(a+bx)))-\log(1-\coth(a+bx))+\log(1+\coth(a+bx))-\sin(2)\text{Si}(2-2\coth(a+bx))+\sin(2)\text{Si}(2(1+\coth(a+bx)))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Coth[a + b*x]]^2,x]
```

```
[Out] (-(Cos[2]*CosIntegral[2 - 2*Coth[a + b*x]]) + Cos[2]*CosIntegral[2*(1 + Cot
h[a + b*x])] - Log[1 - Coth[a + b*x]] + Log[1 + Coth[a + b*x]] - Sin[2]*Sin
Integral[2 - 2*Coth[a + b*x]] + Sin[2]*SinIntegral[2*(1 + Coth[a + b*x])])/
(4*b)
```

Maple [A]
time = 2.43, size = 88, normalized size = 0.77

method	result
derivativedivides	$\frac{\frac{\mathrm{sinIntegral}(-2+2\coth(bx+a))\sin(2)}{4}-\frac{\mathrm{cosineIntegral}(-2+2\coth(bx+a))\cos(2)}{4}+\frac{\mathrm{sinIntegral}(2+2\coth(bx+a))\sin(2)}{4}+\mathrm{cosineIntegral}(2-}{b}$
default	$\frac{\frac{\mathrm{sinIntegral}(-2+2\coth(bx+a))\sin(2)}{4}-\frac{\mathrm{cosineIntegral}(-2+2\coth(bx+a))\cos(2)}{4}+\frac{\mathrm{sinIntegral}(2+2\coth(bx+a))\sin(2)}{4}+\mathrm{cosineIntegral}(2-}{b}$
risch	$\frac{\mathrm{e}^{-2i}\,\mathrm{expIntegral}\left(1,\frac{4i\mathrm{e}^{-a}}{\mathrm{e}^{2bx+a}-\mathrm{e}^{-a}}\right)}{8b}-\frac{\mathrm{e}^{2i}\,\mathrm{expIntegral}\left(1,\frac{4i\mathrm{e}^{-a}}{\mathrm{e}^{2bx+a}-\mathrm{e}^{-a}}+4i\right)}{8b}-\frac{i\pi\,\mathrm{e}^{2i}\mathrm{csgn}\left(\frac{\mathrm{e}^{-a}}{-\mathrm{e}^{2bx+a}+\mathrm{e}^{-a}}\right)}{8b}-\frac{i\mathrm{e}^{2i}\,\mathrm{sinIn}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(coth(b*x+a))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/4*Si(-2+2*coth(b*x+a))*sin(2)-1/4*Ci(-2+2*coth(b*x+a))*cos(2)+1/4*Si
(2+2*coth(b*x+a))*sin(2)+1/4*Ci(2+2*coth(b*x+a))*cos(2)-1/4*ln(-1+coth(b*x+
a))+1/4*ln(coth(b*x+a)+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(coth(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/2*integrate(cos(2*(e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1)), x
)
```

Fricas [A]
time = 0.42, size = 155, normalized size = 1.35

$$\frac{4\,bx+\cos\left(2\right)\mathrm{Ci}\left(\frac{4\,e^{\left(2\,bx+2\,a\right)}}{e^{\left(2\,bx+2\,a\right)}-1}\right)+\cos\left(2\right)\mathrm{Ci}\left(-\frac{4\,e^{\left(2\,bx+2\,a\right)}}{e^{\left(2\,bx+2\,a\right)}-1}\right)-\cos\left(2\right)\mathrm{Ci}\left(\frac{4}{e^{\left(2\,bx+2\,a\right)}-1}\right)-\cos\left(2\right)\mathrm{Ci}\left(-\frac{4}{e^{\left(2\,bx+2\,a\right)}-1}\right)+2\,\sin\left(2\right)\mathrm{Si}\left(\frac{4\,e^{\left(2\,bx+2\,a\right)}}{e^{\left(2\,bx+2\,a\right)}-1}\right)+2\,\sin\left(2\right)\mathrm{Si}\left(\frac{4}{e^{\left(2\,bx+2\,a\right)}-1}\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(coth(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x + cos(2)*cos_integral(4*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) +
 cos(2)*cos_integral(-4*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) - cos(2)*cos
_integral(4/(e^(2*b*x + 2*a) - 1)) - cos(2)*cos_integral(-4/(e^(2*b*x + 2*a
) - 1)) + 2*sin(2)*sin_integral(4*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) +
2*sin(2)*sin_integral(4/(e^(2*b*x + 2*a) - 1)))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \cos^{2}\left(\coth\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(coth(b*x+a))**2,x)
```

```
[Out] Integral(cos(coth(a + b*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(coth(b*x+a))^2,x, algorithm="giac")
```

```
[Out] integrate(cos(coth(b*x + a))^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(\coth(a+b\,x)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(coth(a + b*x))^2,x)
```

```
[Out] int(cos(coth(a + b*x))^2, x)
```

3.223 $\int \cos(\coth(a+bx))\,dx$

Optimal. Leaf size=77

$$-\frac{\cos(1)\text{CosIntegral}(1-\coth(a+bx))}{2b}+\frac{\cos(1)\text{CosIntegral}(1+\coth(a+bx))}{2b}-\frac{\sin(1)\text{Si}(1-\coth(a+bx))}{2b}+$$

```
[Out] -1/2*Ci(1-coth(b*x+a))*cos(1)/b+1/2*Ci(1+coth(b*x+a))*cos(1)/b+1/2*Si(-1+co
th(b*x+a))*sin(1)/b+1/2*Si(1+coth(b*x+a))*sin(1)/b
```

Rubi [A]

time = 0.10, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3415, 3384, 3380, 3383}

$$-\frac{\cos(1)\text{CosIntegral}(1-\coth(a+bx))}{2b}+\frac{\cos(1)\text{CosIntegral}(\coth(a+bx)+1)}{2b}-\frac{\sin(1)\text{Si}(1-\coth(a+bx))}{2b}+\frac{\sin(1)\text{Si}(\coth(a+bx)+1)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Coth[a + b*x]],x]
```

```
[Out] -1/2*(Cos[1]*CosIntegral[1 - Coth[a + b*x]])/b + (Cos[1]*CosIntegral[1 + Co
th[a + b*x]])/(2*b) - (Sin[1]*SinIntegral[1 - Coth[a + b*x]])/(2*b) + (Sin[
1]*SinIntegral[1 + Coth[a + b*x]])/(2*b)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3415

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$
\begin{aligned}
\int \cos(\coth(a+bx))\,dx &= \frac{\text{Subst}\left(\int \frac{\cos(x)}{1-x^2}\,dx, x, \coth(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \left(\frac{\cos(x)}{2(1-x)}+\frac{\cos(x)}{2(1+x)}\right)\,dx, x, \coth(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{\cos(x)}{1-x}\,dx, x, \coth(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \frac{\cos(x)}{1+x}\,dx, x, \coth(a+bx)\right)}{2b}\\
&= \frac{\cos(1)\text{Subst}\left(\int \frac{\cos(1-x)}{1-x}\,dx, x, \coth(a+bx)\right)}{2b}+\frac{\cos(1)\text{Subst}\left(\int \frac{\cos(1+x)}{1+x}\,dx, x, \coth\right.}{2b}\\
&= -\frac{\cos(1)\text{Ci}(1-\coth(a+bx))}{2b}+\frac{\cos(1)\text{Ci}(1+\coth(a+bx))}{2b}-\frac{\sin(1)\text{Si}(1-\coth}{2b}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 62, normalized size = 0.81

$$
-\frac{\cos(1)\text{CosIntegral}(1-\coth(a+bx))-\cos(1)\text{CosIntegral}(1+\coth(a+bx))+\sin(1)\text{Si}(1-\coth(a+bx))-\sin(1)\text{Si}(1+\coth(a+bx))}{2b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Coth[a + b*x]],x]
```

```
[Out] -1/2*(Cos[1]*CosIntegral[1 - Coth[a + b*x]] - Cos[1]*CosIntegral[1 + Coth[a
 + b*x]] + Sin[1]*SinIntegral[1 - Coth[a + b*x]] - Sin[1]*SinIntegral[1 + C
oth[a + b*x]])/b
```

Maple [A]
time = 3.01, size = 58, normalized size = 0.75

method	result
derivativedivides	$\frac{\frac{\text{sinIntegral}(\coth(bx+a)+1)\sin(1)}{2}+\frac{\text{cosineIntegral}(\coth(bx+a)+1)\cos(1)}{2}+\frac{\text{sinIntegral}(-1+\coth(bx+a))\sin(1)}{2}-\frac{\text{cosineIntegral}(-1+\text{co}}{2}}{b}$
default	$\frac{\frac{\text{sinIntegral}(\coth(bx+a)+1)\sin(1)}{2}+\frac{\text{cosineIntegral}(\coth(bx+a)+1)\cos(1)}{2}+\frac{\text{sinIntegral}(-1+\coth(bx+a))\sin(1)}{2}-\frac{\text{cosineIntegral}(-1+\text{co}}{2}}{b}$
risch	$\frac{e^{-i}\,\text{expIntegral}\left(1,\frac{2ie^{-a}}{e^{2bx+a}-e^{-a}}\right)}{4b}-\frac{e^{i}\,\text{expIntegral}\left(1,\frac{2ie^{-a}}{e^{2bx+a}-e^{-a}}+2i\right)}{4b}-\frac{e^{-i}\,\text{expIntegral}\left(1,-\frac{2ie^{-a}}{e^{2bx+a}-e^{-a}}-2i\right)}{4b}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/2*Si(coth(b*x+a)+1)*sin(1)+1/2*Ci(coth(b*x+a)+1)*cos(1)+1/2*Si(-1+co
th(b*x+a))*sin(1)-1/2*Ci(-1+coth(b*x+a))*cos(1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(coth(b*x+a)),x, algorithm="maxima")
[Out] integrate(cos(coth(b*x + a)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(67) = 134.
time = 0.36, size = 151, normalized size = 1.96

$$\frac{\cos(1)\operatorname{Ci}\left(\frac{2e^{(2bx+2a)}}{e^{(2bx+2a)}-1}\right)+\cos(1)\operatorname{Ci}\left(-\frac{2e^{(2bx+2a)}}{e^{(2bx+2a)}-1}\right)-\cos(1)\operatorname{Ci}\left(\frac{2}{e^{(2bx+2a)}-1}\right)-\cos(1)\operatorname{Ci}\left(-\frac{2}{e^{(2bx+2a)}-1}\right)+2\sin(1)\operatorname{Si}\left(\frac{2e^{(2bx+2a)}}{e^{(2bx+2a)}-1}\right)+2\sin(1)\operatorname{Si}\left(\frac{2}{e^{(2bx+2a)}-1}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(coth(b*x+a)),x, algorithm="fricas")
[Out] 1/4*(cos(1)*cos_integral(2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) + cos(1)*
cos_integral(-2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) - cos(1)*cos_integra
l(2/(e^(2*b*x + 2*a) - 1)) - cos(1)*cos_integral(-2/(e^(2*b*x + 2*a) - 1))
+ 2*sin(1)*sin_integral(2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) + 2*sin(1)
*sin_integral(2/(e^(2*b*x + 2*a) - 1)))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\cos\left(\coth\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(coth(b*x+a)),x)
[Out] Integral(cos(coth(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(coth(b*x+a)),x, algorithm="giac")
[Out] integrate(cos(coth(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(\coth(a+b\,x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(coth(a + b*x)),x)
```

```
[Out] int(cos(coth(a + b*x)), x)
```

3.224 $\int \sec(\coth(a+bx))\,dx$

Optimal. Leaf size=67

$$\frac{1}{2}\text{Int}\left(\frac{\text{csch}^2(a+bx)\sec(\coth(a+bx))}{-1+\coth(a+bx)},x\right)-\frac{1}{2}\text{Int}\left(\frac{\text{csch}^2(a+bx)\sec(\coth(a+bx))}{1+\coth(a+bx)},x\right)$$

```
[Out] 1/2*Unintegrable(csch(b*x+a)^2*sec(coth(b*x+a))/(-1+coth(b*x+a)),x)-1/2*Uni
ntegrable(csch(b*x+a)^2*sec(coth(b*x+a))/(1+coth(b*x+a)),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sec(\coth(a+bx))\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[Coth[a + b*x]],x]
```

```
[Out] -1/2*Defer[Subst][Defer[Int][Sec[x]/(-1 + x), x], x, Coth[a + b*x]]/b + Def
er[Subst][Defer[Int][Sec[x]/(1 + x), x], x, Coth[a + b*x]]/(2*b)
```

Rubi steps

$$\begin{aligned}
\int \sec(\coth(a+bx))\,dx &= \frac{\text{Subst}\left(\int \frac{\sec(x)}{1-x^2}\,dx,x,\coth(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{\sec(x)}{2(-1+x)}+\frac{\sec(x)}{2(1+x)}\right)dx,x,\coth(a+bx)\right)}{b}\\
&= -\frac{\text{Subst}\left(\int \frac{\sec(x)}{-1+x}\,dx,x,\coth(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \frac{\sec(x)}{1+x}\,dx,x,\coth(a+bx)\right)}{2b}
\end{aligned}$$

Mathematica [A]
time = 4.15, size = 0, normalized size = 0.00

$$\int \sec(\coth(a+bx))\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[Coth[a + b*x]],x]
```

```
[Out] Integrate[Sec[Coth[a + b*x]], x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \sec \left(\coth \left(bx+a\right)\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(coth(b*x+a)),x)
[Out] int(sec(coth(b*x+a)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(coth(b*x+a)),x, algorithm="maxima")
[Out] integrate(sec(coth(b*x + a)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(coth(b*x+a)),x, algorithm="fricas")
[Out] integral(sec(coth(b*x + a)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sec \left(\coth \left(a+bx\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(coth(b*x+a)),x)
[Out] Integral(sec(coth(a + b*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(sec(coth(b*x + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(\coth\left(a+b\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(coth(a + b*x)),x)
```

```
[Out] int(1/cos(coth(a + b*x)), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#            Albert Rich to use with Sagemath. This is used to
#            grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#            'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#            issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])   #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```