Computer algebra independent integration tests

Summer 2022 edition

6-Hyperbolic-functions/6.2-Hyperbolic-cosine/169-6.2.5-Hyperbolic-cosine-functions

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [336]. This is test number [169].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (336)	0.00 (0)
Mathematica	100.00 (336)	0.00 (0)
Fricas	96.73 (325)	3.27 (11)
Maple	88.39 (297)	11.61 (39)
Giac	76.79 (258)	23.21 (78)
Maxima	61.90 (208)	38.10 (128)
Mupad	56.55 (190)	43.45 (146)
Sympy	30.65 (103)	69.35 (233)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

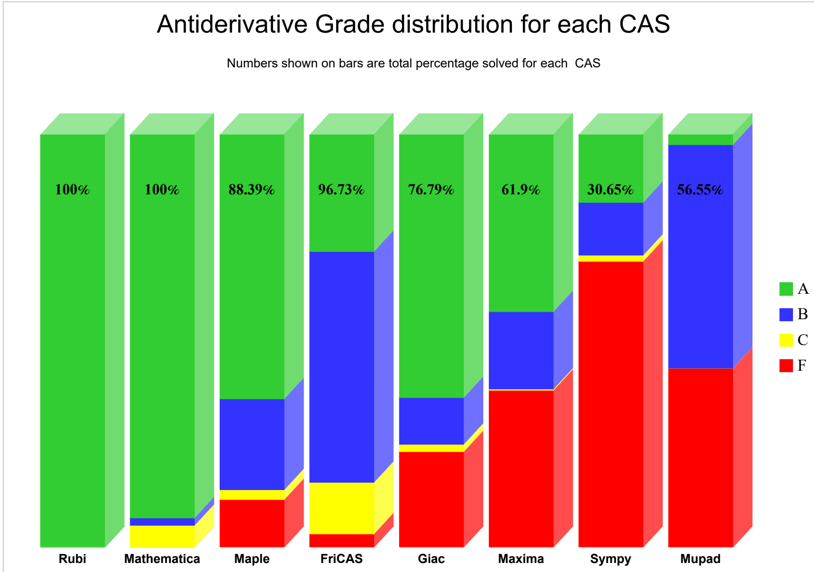
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

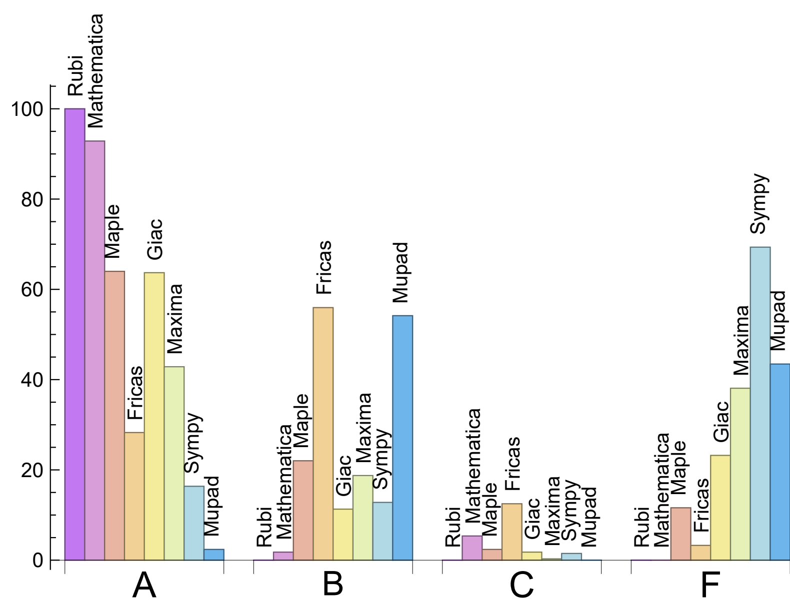
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	92.86	1.79	5.36	0.00
Maple	63.99	22.02	2.38	11.61
Giac	63.69	11.31	1.79	23.21
Maxima	42.86	18.75	0.30	38.10
Fricas	28.27	55.95	12.50	3.27
Sympy	16.37	12.80	1.49	69.35
Mupad	N/A	54.17	0.00	43.45

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	39	100.00 %	0.00 %	0.00 %
Fricas	11	72.73 %	0.00 %	27.27 %
Giac	78	93.59 %	3.85 %	2.56 %
Maxima	128	63.28 %	0.00 %	36.72 %
Sympy	233	72.96 %	18.45 %	8.58 %
Mupad	146	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

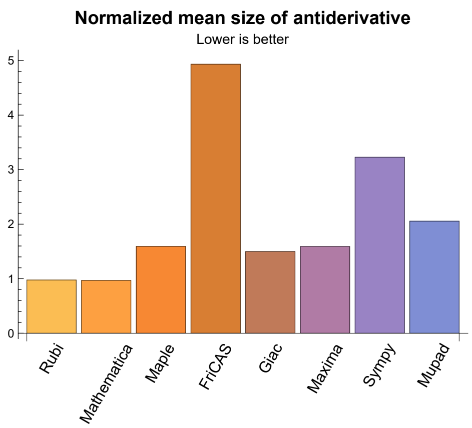
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

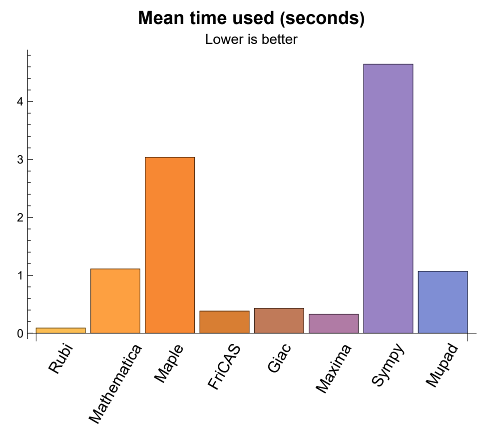
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.09	89.27	0.98	63.50	1.00
Mathematica	1.11	92.43	0.97	55.00	0.92
Maple	3.04	132.55	1.59	78.00	1.15
Maxima	0.33	104.94	1.59	80.00	1.23
Fricas	0.38	533.78	4.93	203.00	2.84
Sympy	4.65	172.57	3.23	49.00	1.52
Giac	0.43	152.47	1.50	64.50	1.14
Mupad	1.07	137.66	2.06	67.50	1.29

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
1
2
3
4
5
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
1
2
3
4
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{216, 217, 221, 226, 227, 232, 233, 238\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi $\{\}$

Mathematica $\{319, 325, 326, 327, 329, 331\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

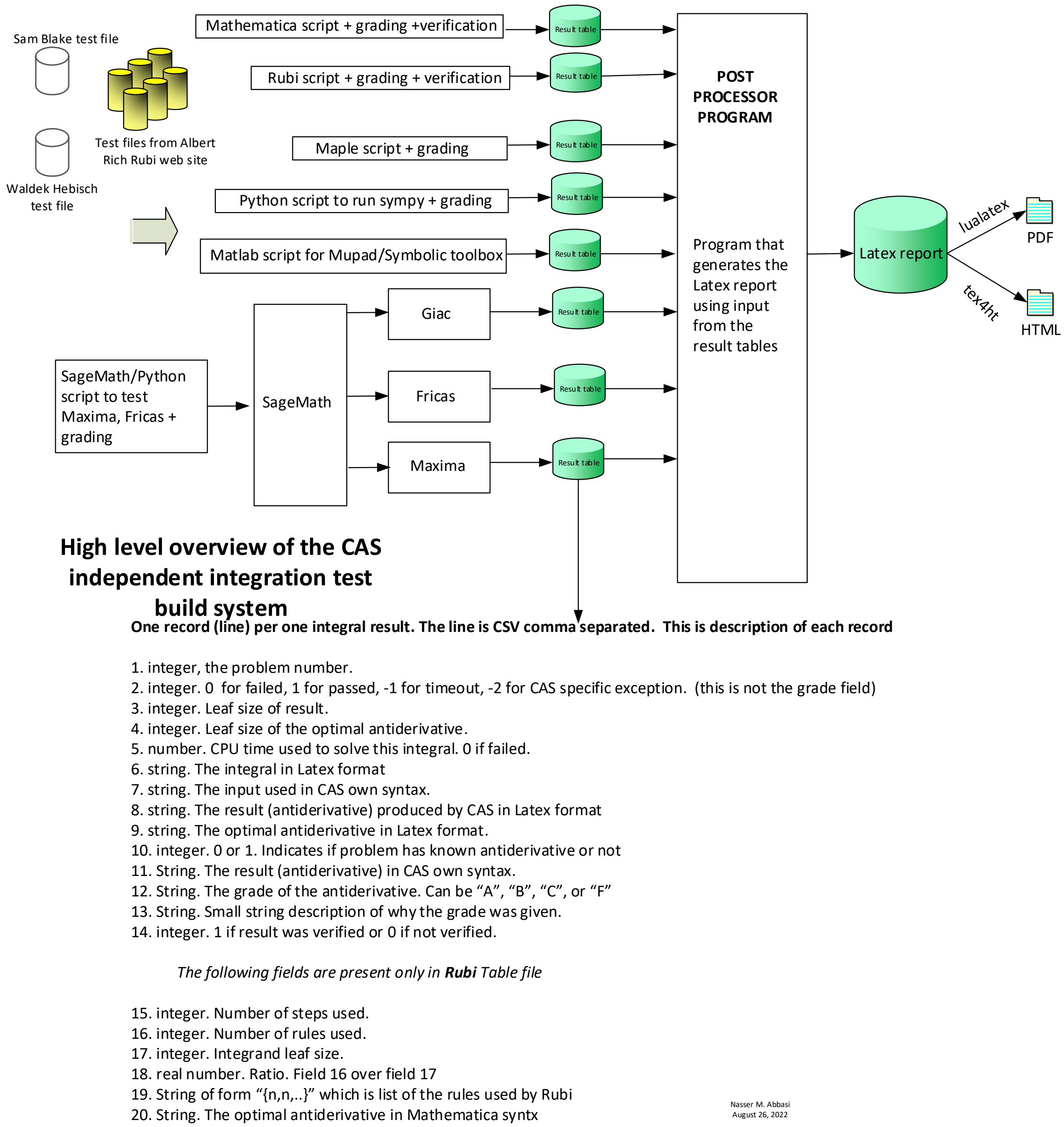
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 14, 15, 16, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 248, 249, 250, 251, 252, 254, 255, 256, 259, 260, 261, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 276, 277, 278, 281, 282, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 330, 331, 333, 334 }

B grade: { 1, 75, 247, 262, 328, 329 }

C grade: { 9, 13, 17, 21, 130, 143, 210, 253, 257, 258, 275, 279, 280, 283, 284, 332, 335, 336 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 12, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 104, 105, 106, 110, 111, 112, 113, 114, 115, 117, 118, 121, 122, 123, 124, 134, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 159, 160, 161, 162, 163, 164, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 200, 201, 202, 203, 204, 205, 207, 208, 209, 216, 217, 220, 221, 225, 226, 227, 231, 232, 233, 238, 247, 248, 256, 264, 265, 266, 267, 268, 269, 270, 272, 273, 274, 277, 278, 281, 282, 285, 286, 287, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 333, 334, 335, 336 }

B grade: { 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 40, 45, 46, 47, 54, 55, 79, 80, 81, 82, 83, 84, 85, 101, 102, 103, 107, 108, 109, 116, 119, 120, 125, 126, 127, 135, 136, 137, 152, 153, 154, 155, 156, 157, 158, 165, 166, 167, 168, 169, 170, 199, 206, 210, 211, 212, 218, 219, 224, 230, 236, 237, 252, 253, 254, 255, 257, 262, 263 }

C grade: { 271, 275, 276, 279, 280, 283, 284, 302 }

F grade: { 23, 128, 129, 130, 131, 132, 133, 213, 214, 215, 222, 223, 228, 229, 234, 235, 239, 240, 241, 242, 243, 244, 245, 246, 249, 250, 251, 258, 259, 260, 261, 288, 289, 290, 291, 329, 330, 331, 332 }

2.1.4 Maxima

A grade: { 1, 2, 4, 6, 24, 25, 26, 27, 28, 29, 30, 32, 36, 42, 43, 44, 62, 63, 64, 65, 66, 71, 72, 73, 74, 75, 76, 77, 78, 93, 97, 117, 121, 122, 123, 124, 125, 126, 127, 134, 135, 136, 137, 140, 141, 142, 143, 145, 146, 147, 151, 153, 155, 158, 159, 171, 172, 174, 180, 182, 183, 185, 191, 192, 193, 200, 201, 216, 217, 220, 221, 225, 226, 227, 232, 233, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 250, 264, 265, 266, 267, 268, 269, 271, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 285, 286, 287, 292, 293, 294, 295, 296, 297, 300, 301, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 333, 334 }

B grade: { 3, 5, 31, 33, 34, 35, 37, 38, 39, 40, 45, 46, 47, 48, 49, 50, 87, 88, 89, 90, 91, 92, 94, 95, 96, 98, 99, 100, 101, 102, 103, 138, 139, 144, 148, 149, 150, 152, 154, 156, 157, 160, 161, 162, 163, 164, 165, 167, 169, 176, 187, 188, 189, 190, 194, 195, 196, 237, 249, 251, 270, 272, 298 }

C grade: { 302 }

F grade: { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 41, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 67, 68, 69, 70, 79, 80, 81, 82, 83, 84, 85, 86, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 119, 120, 128, 129, 130, 131, 132, 133, 166, 168, 170, 173, 175, 177, 178, 179, 181, 184, 186, 197, 198, 199, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 218, 219, 222, 223, 224, 228, 229, 230, 231, 234, 235, 236, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 283, 284, 288, 289, 290, 291, 299, 329, 330, 331, 332, 335, 336 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 25, 26, 27, 28, 32, 36, 40, 44, 45, 50, 51, 57, 58, 62, 63, 64, 65, 66, 67, 71, 75, 93, 94, 97, 98, 101, 110, 114, 115, 117, 142, 143, 154, 155, 156, 157, 158, 159, 172, 182, 183, 192, 203, 204, 205, 216, 217, 220, 221, 226, 227, 232, 233, 238, 239, 240, 241, 242, 243, 244, 247, 248, 249, 250, 251, 258, 259, 260, 261, 262, 264, 266, 268, 274, 275, 279, 280, 292, 293, 294, 295, 299, 300, 301, 302, 303, 304, 333, 334 }

B grade: { 24, 29, 30, 31, 33, 34, 35, 37, 38, 39, 41, 42, 43, 46, 47, 48, 49, 52, 53, 54, 55, 56, 59, 60, 61, 68, 69, 70, 72, 73, 74, 76, 77, 78, 87, 88, 89, 90, 91, 92, 95, 96, 99, 100, 102, 103, 104, 105, 106, 111, 112, 113, 116, 121, 122, 123, 124, 125, 126, 127, 134, 135, 136, 137, 138, 139, 140, 141, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 173, 174, 175, 176, 177, 178, 179, 180, 181, 184, 185, 186, 187, 188, 189, 190, 191, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 206, 207, 208, 209, 210, 211, 212, 218, 219, 222, 223, 224, 225, 228, 229, 230, 231, 234, 235, 236, 237, 245, 246, 263, 265, 267, 269, 270, 271, 272, 273, 276, 277, 278, 281, 282, 283, 284, 285, 286, 287, 296, 297, 298, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 330, 335, 336 }

C grade: { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 79, 80, 81, 82, 83, 84, 85, 86, 107, 108, 109, 118, 119, 120, 128, 129, 130, 131, 132, 133, 252, 253, 254, 255, 256, 257 }

F grade: { 23, 213, 214, 215, 288, 289, 290, 291, 329, 331, 332 }

2.1.6 Sympy

A grade: { 1, 3, 5, 27, 32, 33, 34, 35, 36, 37, 38, 39, 62, 63, 64, 65, 66, 71, 75, 93, 94, 95, 96, 97, 98, 99, 100, 115, 117, 124, 140, 141, 142, 143, 146, 147, 148, 149, 159, 171, 200, 201, 216, 217, 221, 226, 227, 232, 233, 238, 274, 278, 282, 333, 334 }

B grade: { 2, 4, 6, 24, 25, 26, 56, 57, 67, 76, 77, 78, 110, 114, 116, 144, 145, 150, 151, 152, 153, 154, 155, 156, 157, 158, 170, 199, 206, 225, 231, 247, 249, 251, 264, 265, 266, 267, 273, 277, 281, 286, 299 }

C grade: { 72, 73, 74, 287, 294 }

F grade: { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 28, 29, 30, 31, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 58, 59, 60, 61, 68, 69, 70, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 101, 102, 103, 104, 105, 106, 107, 108, 109, 111, 112, 113, 118, 119, 120, 121, 122, 123, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 202, 203, 204, 205, 207, 208, 209, 210, 211, 212, 213, 214, 215, 218, 219, 220, 222, 223, 224, 228, 229, 230, 234, 235, 236, 237, 239, 240, 241, 242, 243, 244, 245, 246, 248, 250, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 268, 269, 270, 271, 272, 275, 276, 279, 280, 283, 284, 285, 288, 289, 290, 291, 292, 293, 295, 296, 297, 298, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 335, 336 }

2.1.7 Giac

A grade: { 2, 4, 6, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 87, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 110, 111, 114, 115, 117, 121, 122, 123, 124, 126, 127, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 196, 199, 200, 201, 202, 203, 204, 205, 206, 207, 216, 217, 220, 221, 225, 226, 227, 231, 232, 233, 237, 238, 239, 240, 250, 251, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 303, 304, 305, 307, 308, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 333, 334 }

B grade: { 1, 3, 5, 48, 49, 50, 66, 88, 89, 90, 91, 92, 105, 106, 112, 113, 116, 144, 160, 162, 174, 176, 177, 180, 195, 208, 209, 241, 242, 243, 244, 245, 246, 247, 248, 249, 262, 263 }

C grade: { 285, 286, 287, 302, 306, 309 }

F grade: { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 47, 79, 80, 81, 82, 83, 84, 85, 86, 107, 108, 109, 118, 119, 120, 125, 128, 129, 130, 131, 132, 133, 197, 198, 210, 211, 212, 213, 214, 215, 218, 219, 222, 223, 224, 228, 229, 230, 234, 235, 236, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 288, 289, 290, 291, 329, 330, 331, 332, 335, 336 }

2.1.8 Mupad

A grade: { 216, 217, 221, 226, 227, 232, 233, 238 }

B grade: { 1, 2, 3, 4, 5, 6, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 44, 50, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 71, 72, 73, 74, 75, 76, 77, 78, 93, 94, 95, 96, 97, 98, 99, 100, 110, 111, 114, 115, 116, 117, 124, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 199, 200, 201, 202, 203, 204, 205, 206, 207, 218, 225, 231, 237, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 294, 296, 297, 298, 299, 329, 330, 331, 333, 334 }

C grade: { }

F grade: { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 40, 41, 42, 43, 45, 46, 47, 48, 49, 51, 52, 53, 69, 70, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 101, 102, 103, 104, 105, 106, 107, 108, 109, 112, 113, 118, 119, 120, 121, 122, 123, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 197, 198, 208, 209, 210, 211, 212, 213, 214, 215, 219, 220, 222, 223, 224, 228, 229, 230, 234, 235, 236, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 288, 289, 290, 291, 292, 293, 295, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 332, 335, 336 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	21	11	10	10	12	26	10
N.S.	1	1.00	2.10	1.10	1.00	1.00	1.20	2.60	1.00
time (sec)	N/A	0.003	0.007	0.394	0.262	0.357	0.051	0.411	0.045

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	27	32	22	46	32	18
N.S.	1	1.00	0.92	1.08	1.28	0.88	1.84	1.28	0.72
time (sec)	N/A	0.007	0.020	0.643	0.273	0.431	0.076	0.391	0.882

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	27	54	32	36	54	22
N.S.	1	1.00	1.00	1.04	2.08	1.23	1.38	2.08	0.85
time (sec)	N/A	0.008	0.007	1.178	0.254	0.389	0.117	0.418	0.885

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	33	33	60	49	95	60	31
N.S.	1	1.00	0.72	0.72	1.30	1.07	2.07	1.30	0.67
time (sec)	N/A	0.014	0.032	1.253	0.264	0.462	0.175	0.408	0.083

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	44	41	82	66	58	82	31
N.S.	1	1.00	1.07	1.00	2.00	1.61	1.41	2.00	0.76
time (sec)	N/A	0.011	0.013	1.197	0.252	0.375	0.267	0.423	0.918

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	43	47	86	90	139	88	42
N.S.	1	1.00	0.64	0.70	1.28	1.34	2.07	1.31	0.63
time (sec)	N/A	0.024	0.031	1.275	0.260	0.422	0.422	0.416	0.966

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	55	201	0	326	0	0	-1
N.S.	1	1.00	0.80	2.91	0.00	4.72	0.00	0.00	-0.01
time (sec)	N/A	0.024	0.082	1.188	0.000	0.142	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	44	188	0	203	0	0	-1
N.S.	1	1.00	0.96	4.09	0.00	4.41	0.00	0.00	-0.02
time (sec)	N/A	0.016	0.042	1.023	0.000	0.134	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	81	174	0	102	0	0	-1
N.S.	1	1.00	1.76	3.78	0.00	2.22	0.00	0.00	-0.02
time (sec)	N/A	0.015	0.078	1.116	0.000	0.161	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	135	0	37	0	0	-1
N.S.	1	1.00	1.00	6.75	0.00	1.85	0.00	0.00	-0.05
time (sec)	N/A	0.008	0.026	1.035	0.000	0.097	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	135	0	24	0	0	-1
N.S.	1	1.00	1.00	6.75	0.00	1.20	0.00	0.00	-0.05
time (sec)	N/A	0.007	0.028	0.862	0.000	0.142	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	103	0	148	0	0	-1
N.S.	1	1.00	1.00	2.45	0.00	3.52	0.00	0.00	-0.02
time (sec)	N/A	0.015	0.046	1.184	0.000	0.084	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	84	217	0	310	0	0	-1
N.S.	1	1.00	1.83	4.72	0.00	6.74	0.00	0.00	-0.02
time (sec)	N/A	0.016	0.052	1.074	0.000	0.122	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	63	363	0	613	0	0	-1
N.S.	1	1.00	0.91	5.26	0.00	8.88	0.00	0.00	-0.01
time (sec)	N/A	0.024	0.103	1.389	0.000	0.092	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	53	145	0	256	0	0	-1
N.S.	1	1.00	0.82	2.23	0.00	3.94	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.037	1.246	0.000	0.125	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	41	184	0	161	0	0	-1
N.S.	1	1.00	0.85	3.83	0.00	3.35	0.00	0.00	-0.02
time (sec)	N/A	0.019	0.032	1.185	0.000	0.103	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	57	130	0	68	0	0	-1
N.S.	1	1.00	1.19	2.71	0.00	1.42	0.00	0.00	-0.02
time (sec)	N/A	0.020	0.043	1.114	0.000	0.142	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	118	0	28	0	0	-1
N.S.	1	1.00	1.00	4.37	0.00	1.04	0.00	0.00	-0.04
time (sec)	N/A	0.011	0.008	1.345	0.000	0.086	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	100	0	16	0	0	-1
N.S.	1	1.00	1.00	3.70	0.00	0.59	0.00	0.00	-0.04
time (sec)	N/A	0.012	0.011	0.902	0.000	0.139	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	34	159	0	101	0	0	-1
N.S.	1	1.00	0.74	3.46	0.00	2.20	0.00	0.00	-0.02
time (sec)	N/A	0.018	0.024	1.263	0.000	0.081	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	56	177	0	213	0	0	-1
N.S.	1	1.00	1.12	3.54	0.00	4.26	0.00	0.00	-0.02
time (sec)	N/A	0.019	0.030	1.071	0.000	0.131	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	43	254	0	422	0	0	-1
N.S.	1	1.00	0.64	3.79	0.00	6.30	0.00	0.00	-0.01
time (sec)	N/A	0.028	0.034	1.355	0.000	0.101	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	65	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.014	0.045	0.710	0.000	0.000	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	53	86	66	100	337	70	70
N.S.	1	1.00	0.98	1.59	1.22	1.85	6.24	1.30	1.30
time (sec)	N/A	0.056	0.060	0.442	0.267	0.367	0.724	0.399	0.961

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	45	70	56	70	189	51	52
N.S.	1	1.00	1.05	1.63	1.30	1.63	4.40	1.19	1.21
time (sec)	N/A	0.038	0.038	0.446	0.264	0.371	0.408	0.400	0.917

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	32	46	41	47	63	35	34
N.S.	1	1.00	1.28	1.84	1.64	1.88	2.52	1.40	1.36
time (sec)	N/A	0.054	0.041	0.430	0.265	0.396	0.250	0.400	0.901

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	14	28	18	24	8	17	17
N.S.	1	1.00	0.78	1.56	1.00	1.33	0.44	0.94	0.94
time (sec)	N/A	0.024	0.021	0.435	0.269	0.350	0.136	0.394	0.875

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	19	23	29	0	20	31
N.S.	1	1.00	1.10	0.95	1.15	1.45	0.00	1.00	1.55
time (sec)	N/A	0.032	0.020	0.485	0.501	0.404	0.000	0.407	0.878

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	43	33	45	127	0	36	58
N.S.	1	1.00	1.54	1.18	1.61	4.54	0.00	1.29	2.07
time (sec)	N/A	0.048	0.062	0.539	0.469	0.343	0.000	0.374	0.895

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	49	46	73	325	0	48	73
N.S.	1	1.00	1.14	1.07	1.70	7.56	0.00	1.12	1.70
time (sec)	N/A	0.058	0.063	0.528	0.470	0.420	0.000	0.388	0.909

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	60	52	101	600	0	57	107
N.S.	1	1.00	1.07	0.93	1.80	10.71	0.00	1.02	1.91
time (sec)	N/A	0.058	0.133	0.556	0.515	0.391	0.000	0.388	0.897

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	14	14	18	22	17	15	15
N.S.	1	1.00	0.70	0.70	0.90	1.10	0.85	0.75	0.75
time (sec)	N/A	0.008	0.014	0.815	0.295	0.404	0.268	0.394	0.895

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	34	30	90	113	36	25	25
N.S.	1	1.00	0.72	0.64	1.91	2.40	0.77	0.53	0.53
time (sec)	N/A	0.018	0.024	0.804	0.274	0.450	0.498	0.392	0.059

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	44	43	205	174	51	36	36
N.S.	1	1.00	0.63	0.61	2.93	2.49	0.73	0.51	0.51
time (sec)	N/A	0.028	0.043	0.796	0.264	0.372	1.024	0.391	0.934

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	54	56	364	347	68	47	283
N.S.	1	1.00	0.58	0.60	3.91	3.73	0.73	0.51	3.04
time (sec)	N/A	0.039	0.057	0.819	0.278	0.483	2.625	0.387	0.919

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	14	16	18	24	19	15	15
N.S.	1	1.00	0.61	0.70	0.78	1.04	0.83	0.65	0.65
time (sec)	N/A	0.009	0.022	0.873	0.261	0.381	0.385	0.383	0.893

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	31	32	90	117	39	25	25
N.S.	1	1.00	0.61	0.63	1.76	2.29	0.76	0.49	0.49
time (sec)	N/A	0.019	0.024	0.947	0.261	0.412	0.660	0.386	0.059

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	41	45	205	174	56	36	36
N.S.	1	1.00	0.54	0.59	2.70	2.29	0.74	0.47	0.47
time (sec)	N/A	0.032	0.042	0.912	0.278	0.369	1.389	0.397	0.911

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	51	58	364	347	73	47	283
N.S.	1	1.00	0.50	0.57	3.60	3.44	0.72	0.47	2.80
time (sec)	N/A	0.044	0.057	0.909	0.276	0.382	3.028	0.396	0.085

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	34	92	114	62	0	37	-1
N.S.	1	1.00	0.67	1.80	2.24	1.22	0.00	0.73	-0.02
time (sec)	N/A	0.035	0.017	1.176	0.524	0.374	0.000	0.390	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	35	40	0	92	0	74	-1
N.S.	1	1.00	0.66	0.75	0.00	1.74	0.00	1.40	-0.02
time (sec)	N/A	0.040	0.023	1.119	0.000	0.353	0.000	0.395	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	71	73	121	327	0	105	-1
N.S.	1	1.00	0.80	0.82	1.36	3.67	0.00	1.18	-0.01
time (sec)	N/A	0.035	0.094	0.938	0.485	0.374	0.000	0.424	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	55	58	81	140	0	75	-1
N.S.	1	1.00	0.93	0.98	1.37	2.37	0.00	1.27	-0.02
time (sec)	N/A	0.022	0.048	0.943	0.468	0.358	0.000	0.428	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	29	43	40	41	0	35	26
N.S.	1	1.00	1.12	1.65	1.54	1.58	0.00	1.35	1.00
time (sec)	N/A	0.010	0.025	0.884	0.479	0.373	0.000	0.407	0.115

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	40	103	86	149	0	21	-1
N.S.	1	1.00	0.87	2.24	1.87	3.24	0.00	0.46	-0.02
time (sec)	N/A	0.016	0.013	0.941	0.532	0.424	0.000	0.417	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	63	144	170	219	0	70	-1
N.S.	1	1.00	0.82	1.87	2.21	2.84	0.00	0.91	-0.01
time (sec)	N/A	0.031	0.062	1.084	0.543	0.352	0.000	0.417	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	91	178	250	525	0	0	-1
N.S.	1	1.00	0.85	1.66	2.34	4.91	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.192	1.120	0.567	0.370	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	72	71	190	328	0	194	-1
N.S.	1	1.00	0.78	0.77	2.07	3.57	0.00	2.11	-0.01
time (sec)	N/A	0.040	0.098	1.164	0.484	0.415	0.000	0.423	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	56	56	124	139	0	127	-1
N.S.	1	1.00	0.92	0.92	2.03	2.28	0.00	2.08	-0.02
time (sec)	N/A	0.025	0.068	1.010	0.479	0.350	0.000	0.417	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	30	41	58	42	0	63	27
N.S.	1	1.00	1.11	1.52	2.15	1.56	0.00	2.33	1.00
time (sec)	N/A	0.013	0.025	1.026	0.482	0.366	0.000	0.418	0.941

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	41	41	0	154	0	40	-1
N.S.	1	1.00	0.85	0.85	0.00	3.21	0.00	0.83	-0.02
time (sec)	N/A	0.017	0.026	0.928	0.000	0.341	0.000	0.438	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	85	87	0	274	0	115	-1
N.S.	1	1.00	1.08	1.10	0.00	3.47	0.00	1.46	-0.01
time (sec)	N/A	0.034	0.126	1.060	0.000	0.423	0.000	0.424	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	115	137	0	580	0	177	-1
N.S.	1	1.00	1.05	1.25	0.00	5.27	0.00	1.61	-0.01
time (sec)	N/A	0.047	0.128	1.083	0.000	0.359	0.000	0.449	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	99	203	0	1625	0	133	209
N.S.	1	1.00	0.88	1.81	0.00	14.51	0.00	1.19	1.87
time (sec)	N/A	0.220	0.135	0.456	0.000	0.399	0.000	0.410	1.263

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	78	153	0	903	0	92	167
N.S.	1	1.00	0.92	1.80	0.00	10.62	0.00	1.08	1.96
time (sec)	N/A	0.126	0.094	0.464	0.000	0.415	0.000	0.402	1.117

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	57	94	0	449	1275	62	139
N.S.	1	1.00	0.92	1.52	0.00	7.24	20.56	1.00	2.24
time (sec)	N/A	0.076	0.081	0.441	0.000	0.436	71.858	0.431	1.041

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	48	64	0	218	241	42	109
N.S.	1	1.00	0.92	1.23	0.00	4.19	4.63	0.81	2.10
time (sec)	N/A	0.037	0.035	0.414	0.000	0.368	16.262	0.414	0.219

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	51	0	227	0	45	286
N.S.	1	1.00	1.00	0.94	0.00	4.20	0.00	0.83	5.30
time (sec)	N/A	0.048	0.040	0.556	0.000	0.370	0.000	0.412	3.456

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	63	73	0	515	0	61	294
N.S.	1	1.00	0.98	1.14	0.00	8.05	0.00	0.95	4.59
time (sec)	N/A	0.084	0.081	0.712	0.000	0.406	0.000	0.411	3.098

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	82	109	0	1370	0	89	476
N.S.	1	1.00	0.94	1.25	0.00	15.75	0.00	1.02	5.47
time (sec)	N/A	0.211	0.146	0.762	0.000	0.447	0.000	0.408	4.126

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	101	145	0	2483	0	123	547
N.S.	1	1.00	0.89	1.27	0.00	21.78	0.00	1.08	4.80
time (sec)	N/A	0.334	0.313	0.800	0.000	0.446	0.000	0.398	4.533

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	133	145	273	190	314	263	160
N.S.	1	1.00	0.73	0.79	1.49	1.04	1.72	1.44	0.87
time (sec)	N/A	0.179	0.270	1.875	0.258	0.361	0.331	0.403	1.144

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	104	108	183	123	240	196	114
N.S.	1	1.00	0.76	0.79	1.34	0.90	1.75	1.43	0.83
time (sec)	N/A	0.106	0.168	1.252	0.262	0.410	0.221	0.412	0.191

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	80	71	116	78	128	131	73
N.S.	1	1.00	0.89	0.79	1.29	0.87	1.42	1.46	0.81
time (sec)	N/A	0.048	0.102	1.108	0.266	0.385	0.134	0.419	0.955

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	46	51	55	40	78	75	41
N.S.	1	1.00	0.92	1.02	1.10	0.80	1.56	1.50	0.82
time (sec)	N/A	0.011	0.064	0.745	0.266	0.344	0.091	0.410	0.923

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	26	16	15	17	17	32	15
N.S.	1	1.00	1.73	1.07	1.00	1.13	1.13	2.13	1.00
time (sec)	N/A	0.006	0.007	0.753	0.260	0.350	0.052	0.405	0.057

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	48	44	0	237	163	39	53
N.S.	1	1.00	0.98	0.90	0.00	4.84	3.33	0.80	1.08
time (sec)	N/A	0.026	0.041	0.872	0.000	0.430	2.472	0.400	1.225

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	84	118	0	743	0	99	215
N.S.	1	1.00	0.98	1.37	0.00	8.64	0.00	1.15	2.50
time (sec)	N/A	0.059	0.172	0.907	0.000	0.393	0.000	0.406	1.301

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	113	186	0	2591	0	195	-1
N.S.	1	1.00	0.85	1.40	0.00	19.48	0.00	1.47	-0.01
time (sec)	N/A	0.109	0.317	0.942	0.000	0.415	0.000	0.403	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	160	284	0	5705	0	329	-1
N.S.	1	1.00	0.87	1.54	0.00	31.01	0.00	1.79	-0.01
time (sec)	N/A	0.187	0.744	1.001	0.000	0.394	0.000	0.421	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	23	18	19	24	24	16	34
N.S.	1	1.00	1.05	0.82	0.86	1.09	1.09	0.73	1.55
time (sec)	N/A	0.010	0.028	0.871	0.475	0.440	0.441	0.412	0.125

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	45	46	64	147	354	54	74
N.S.	1	1.00	0.94	0.96	1.33	3.06	7.38	1.12	1.54
time (sec)	N/A	0.023	0.086	0.879	0.474	0.381	1.220	0.405	0.939

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	55	62	108	408	600	76	137
N.S.	1	1.00	0.75	0.85	1.48	5.59	8.22	1.04	1.88
time (sec)	N/A	0.043	0.122	0.875	0.470	0.451	2.361	0.413	0.959

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	65	75	152	793	908	98	223
N.S.	1	1.00	0.66	0.77	1.55	8.09	9.27	1.00	2.28
time (sec)	N/A	0.069	0.194	0.865	0.482	0.374	4.952	0.412	0.949

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	77	34	37	42	41	28	40
N.S.	1	1.00	2.48	1.10	1.19	1.35	1.32	0.90	1.29
time (sec)	N/A	0.010	0.025	0.848	0.274	0.444	0.348	0.402	0.943

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	45	64	81	212	199	65	77
N.S.	1	1.00	0.80	1.14	1.45	3.79	3.55	1.16	1.38
time (sec)	N/A	0.026	0.094	0.848	0.283	0.480	0.815	0.403	0.938

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	58	94	125	563	445	87	141
N.S.	1	1.00	0.72	1.16	1.54	6.95	5.49	1.07	1.74
time (sec)	N/A	0.043	0.143	0.868	0.280	0.413	1.712	0.412	0.954

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	68	124	169	1078	784	109	226
N.S.	1	1.00	0.64	1.17	1.59	10.17	7.40	1.03	2.13
time (sec)	N/A	0.066	0.192	0.859	0.275	0.436	3.745	0.409	0.114

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	150	685	0	464	0	0	-1
N.S.	1	1.00	0.98	4.48	0.00	3.03	0.00	0.00	-0.01
time (sec)	N/A	0.165	0.377	1.361	0.000	0.163	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	111	466	0	265	0	0	-1
N.S.	1	1.00	0.90	3.76	0.00	2.14	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.173	1.194	0.000	0.103	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	276	0	196	0	0	-1
N.S.	1	1.00	1.00	4.52	0.00	3.21	0.00	0.00	-0.02
time (sec)	N/A	0.028	0.068	1.532	0.000	0.084	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	146	0	61	0	0	-1
N.S.	1	1.00	1.00	3.17	0.00	1.33	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.029	0.925	0.000	0.080	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	68	298	0	413	0	0	-1
N.S.	1	1.00	0.81	3.55	0.00	4.92	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.094	1.418	0.000	0.127	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	135	459	0	1281	0	0	-1
N.S.	1	1.00	0.76	2.59	0.00	7.24	0.00	0.00	-0.01
time (sec)	N/A	0.146	0.387	1.606	0.000	0.191	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	165	566	0	3315	0	0	-1
N.S.	1	1.00	0.73	2.49	0.00	14.60	0.00	0.00	-0.00
time (sec)	N/A	0.216	0.514	1.866	0.000	0.160	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	73	181	0	174	0	0	-1
N.S.	1	1.00	0.73	1.81	0.00	1.74	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.276	1.339	0.000	0.144	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	60	71	237	563	0	153	-1
N.S.	1	1.00	0.64	0.76	2.52	5.99	0.00	1.63	-0.01
time (sec)	N/A	0.063	0.102	1.078	0.513	0.433	0.000	0.423	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	46	57	163	279	0	113	-1
N.S.	1	1.00	0.68	0.84	2.40	4.10	0.00	1.66	-0.01
time (sec)	N/A	0.051	0.068	1.072	0.500	0.370	0.000	0.412	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	31	39	90	100	0	71	-1
N.S.	1	1.00	0.78	0.98	2.25	2.50	0.00	1.78	-0.02
time (sec)	N/A	0.034	0.029	1.069	0.492	0.415	0.000	0.415	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	61	69	288	564	0	295	-1
N.S.	1	1.00	0.62	0.70	2.94	5.76	0.00	3.01	-0.01
time (sec)	N/A	0.069	0.102	1.225	0.519	0.438	0.000	0.419	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	47	55	199	279	0	212	-1
N.S.	1	1.00	0.66	0.77	2.80	3.93	0.00	2.99	-0.01
time (sec)	N/A	0.056	0.100	1.174	0.513	0.364	0.000	0.434	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	32	39	109	107	0	131	-1
N.S.	1	1.00	0.73	0.89	2.48	2.43	0.00	2.98	-0.02
time (sec)	N/A	0.039	0.040	1.025	0.502	0.401	0.000	0.407	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	23	34	26	29	15	17	19
N.S.	1	1.00	1.28	1.89	1.44	1.61	0.83	0.94	1.06
time (sec)	N/A	0.025	0.043	0.347	0.277	0.420	0.135	0.402	0.046

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	25	34	129	50	36	30	30
N.S.	1	1.00	0.71	0.97	3.69	1.43	1.03	0.86	0.86
time (sec)	N/A	0.025	0.045	0.302	0.264	0.440	0.245	0.418	0.076

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	42	38	263	127	46	46	141
N.S.	1	1.00	0.75	0.68	4.70	2.27	0.82	0.82	2.52
time (sec)	N/A	0.035	0.072	0.448	0.269	0.358	0.477	0.401	0.920

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	57	55	449	175	78	60	231
N.S.	1	1.00	0.76	0.73	5.99	2.33	1.04	0.80	3.08
time (sec)	N/A	0.040	0.078	0.458	0.291	0.375	0.952	0.409	0.917

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	35	36	27	31	15	16	19
N.S.	1	1.00	1.75	1.80	1.35	1.55	0.75	0.80	0.95
time (sec)	N/A	0.029	0.043	0.416	0.269	0.389	0.225	0.413	0.053

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	25	26	131	48	36	32	32
N.S.	1	1.00	0.68	0.70	3.54	1.30	0.97	0.86	0.86
time (sec)	N/A	0.028	0.043	0.419	0.271	0.453	0.369	0.420	0.933

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	42	39	267	127	46	46	143
N.S.	1	1.00	0.70	0.65	4.45	2.12	0.77	0.77	2.38
time (sec)	N/A	0.038	0.061	0.375	0.282	0.346	0.662	0.402	0.081

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	57	56	451	175	78	60	233
N.S.	1	1.00	0.70	0.69	5.57	2.16	0.96	0.74	2.88
time (sec)	N/A	0.048	0.074	0.428	0.276	0.410	1.190	0.420	0.917

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	41	128	174	72	0	44	-1
N.S.	1	1.00	0.73	2.29	3.11	1.29	0.00	0.79	-0.02
time (sec)	N/A	0.042	0.026	1.312	0.563	0.443	0.000	0.421	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	44	159	300	192	0	78	-1
N.S.	1	1.00	0.68	2.45	4.62	2.95	0.00	1.20	-0.02
time (sec)	N/A	0.048	0.059	1.328	0.568	0.444	0.000	0.420	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	57	209	427	512	0	118	-1
N.S.	1	1.00	0.61	2.25	4.59	5.51	0.00	1.27	-0.01
time (sec)	N/A	0.065	0.119	1.361	0.601	0.338	0.000	0.425	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	40	63	0	99	0	84	-1
N.S.	1	1.00	0.70	1.11	0.00	1.74	0.00	1.47	-0.02
time (sec)	N/A	0.047	0.041	1.200	0.000	0.360	0.000	0.426	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	71	83	0	217	0	111	-1
N.S.	1	1.00	1.09	1.28	0.00	3.34	0.00	1.71	-0.02
time (sec)	N/A	0.051	0.115	1.257	0.000	0.443	0.000	0.433	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	108	118	0	548	0	189	-1
N.S.	1	1.00	1.15	1.26	0.00	5.83	0.00	2.01	-0.01
time (sec)	N/A	0.066	0.261	1.250	0.000	0.362	0.000	0.417	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	203	1365	0	1141	0	0	-1
N.S.	1	1.00	0.87	5.86	0.00	4.90	0.00	0.00	-0.00
time (sec)	N/A	0.314	0.421	1.511	0.000	0.167	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	124	973	0	635	0	0	-1
N.S.	1	1.00	0.69	5.38	0.00	3.51	0.00	0.00	-0.01
time (sec)	N/A	0.223	0.486	1.474	0.000	0.139	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	123	613	0	325	0	0	-1
N.S.	1	1.00	0.89	4.44	0.00	2.36	0.00	0.00	-0.01
time (sec)	N/A	0.141	0.246	1.420	0.000	0.147	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	59	73	0	240	403	50	242
N.S.	1	1.00	0.98	1.22	0.00	4.00	6.72	0.83	4.03
time (sec)	N/A	0.047	0.072	0.450	0.000	0.385	17.213	0.409	1.123

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	81	108	0	828	0	107	246
N.S.	1	1.00	0.99	1.32	0.00	10.10	0.00	1.30	3.00
time (sec)	N/A	0.056	0.142	0.418	0.000	0.388	0.000	0.405	1.420

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	134	207	0	3166	0	249	-1
N.S.	1	1.00	0.99	1.53	0.00	23.45	0.00	1.84	-0.01
time (sec)	N/A	0.129	0.308	0.477	0.000	0.451	0.000	0.408	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	196	342	0	7603	0	453	-1
N.S.	1	1.00	0.99	1.74	0.00	38.59	0.00	2.30	-0.01
time (sec)	N/A	0.257	0.612	0.549	0.000	0.468	0.000	0.423	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	81	0	190	168	57	205
N.S.	1	1.00	1.00	1.45	0.00	3.39	3.00	1.02	3.66
time (sec)	N/A	0.056	0.056	0.635	0.000	0.404	18.331	0.415	0.494

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	0	6	3	6	6
N.S.	1	1.00	1.00	1.17	0.00	1.00	0.50	1.00	1.00
time (sec)	N/A	0.001	0.001	0.344	0.000	0.386	0.157	0.406	0.020

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	29	0	54	26	26	51
N.S.	1	1.00	1.00	2.64	0.00	4.91	2.36	2.36	4.64
time (sec)	N/A	0.022	0.041	0.361	0.000	0.370	102.952	0.412	1.020

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	24	32	34	45	44	37	48
N.S.	1	1.00	0.67	0.89	0.94	1.25	1.22	1.03	1.33
time (sec)	N/A	0.033	0.063	0.462	0.478	0.365	0.331	0.415	0.111

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	80	218	0	183	0	0	-1
N.S.	1	1.00	0.74	2.02	0.00	1.69	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.348	1.831	0.000	0.085	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	133	485	0	639	0	0	-1
N.S.	1	1.00	0.88	3.19	0.00	4.20	0.00	0.00	-0.01
time (sec)	N/A	0.143	0.285	1.655	0.000	0.137	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	172	797	0	2153	0	0	-1
N.S.	1	1.00	0.74	3.45	0.00	9.32	0.00	0.00	-0.00
time (sec)	N/A	0.232	0.662	2.110	0.000	0.145	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	42	38	71	817	0	79	-1
N.S.	1	1.00	0.58	0.53	0.99	11.35	0.00	1.10	-0.01
time (sec)	N/A	0.036	0.020	0.821	0.498	0.384	0.000	0.416	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	36	32	53	501	0	61	-1
N.S.	1	1.00	0.68	0.60	1.00	9.45	0.00	1.15	-0.02
time (sec)	N/A	0.025	0.013	0.779	0.490	0.356	0.000	0.401	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	26	24	35	222	0	29	-1
N.S.	1	1.00	0.76	0.71	1.03	6.53	0.00	0.85	-0.03
time (sec)	N/A	0.016	0.008	0.786	0.477	0.423	0.000	0.414	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	15	17	69	15	14	17
N.S.	1	1.00	1.00	1.15	1.31	5.31	1.15	1.08	1.31
time (sec)	N/A	0.008	0.004	0.813	0.476	0.335	0.187	0.426	0.053

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	21	55	8	186	0	0	-1
N.S.	1	1.00	1.31	3.44	0.50	11.62	0.00	0.00	-0.06
time (sec)	N/A	0.009	0.008	0.981	0.509	0.348	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	31	82	41	299	0	56	-1
N.S.	1	1.00	0.74	1.95	0.98	7.12	0.00	1.33	-0.02
time (sec)	N/A	0.019	0.012	0.982	0.507	0.402	0.000	0.409	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	40	102	75	837	0	67	-1
N.S.	1	1.00	0.66	1.67	1.23	13.72	0.00	1.10	-0.02
time (sec)	N/A	0.028	0.026	1.029	0.512	0.399	0.000	0.404	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	65	0	0	823	0	0	-1
N.S.	1	1.00	0.54	0.00	0.00	6.80	0.00	0.00	-0.01
time (sec)	N/A	0.038	0.087	0.980	0.000	0.132	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	54	0	0	317	0	0	-1
N.S.	1	1.00	0.76	0.00	0.00	4.46	0.00	0.00	-0.01
time (sec)	N/A	0.025	0.055	0.927	0.000	0.123	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	59	0	0	59	0	0	-1
N.S.	1	1.00	1.23	0.00	0.00	1.23	0.00	0.00	-0.02
time (sec)	N/A	0.018	0.041	1.009	0.000	0.133	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	36	0	0	93	0	0	-1
N.S.	1	1.00	0.78	0.00	0.00	2.02	0.00	0.00	-0.02
time (sec)	N/A	0.017	0.019	1.004	0.000	0.109	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	48	0	0	629	0	0	-1
N.S.	1	1.00	0.64	0.00	0.00	8.39	0.00	0.00	-0.01
time (sec)	N/A	0.025	0.045	0.946	0.000	0.162	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	61	0	0	1668	0	0	-1
N.S.	1	1.00	0.50	0.00	0.00	13.79	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.083	0.925	0.000	0.140	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	53	177	100	1597	0	114	-1
N.S.	1	1.00	0.40	1.34	0.76	12.10	0.00	0.86	-0.01
time (sec)	N/A	0.037	0.084	1.925	0.490	0.401	0.000	0.412	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	38	131	62	659	0	52	-1
N.S.	1	1.00	0.49	1.68	0.79	8.45	0.00	0.67	-0.01
time (sec)	N/A	0.024	0.048	1.864	0.492	0.423	0.000	0.407	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	25	89	27	180	0	28	-1
N.S.	1	1.00	0.69	2.47	0.75	5.00	0.00	0.78	-0.03
time (sec)	N/A	0.014	0.012	1.866	0.475	0.369	0.000	0.414	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	56	16	116	0	13	39
N.S.	1	1.00	1.00	3.73	1.07	7.73	0.00	0.87	2.60
time (sec)	N/A	0.012	0.005	1.487	0.482	0.361	0.000	0.410	0.065

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	30	80	165	1137	0	27	48
N.S.	1	1.00	0.45	1.19	2.46	16.97	0.00	0.40	0.72
time (sec)	N/A	0.017	0.020	1.573	0.490	0.425	0.000	0.418	0.970

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	47	96	457	3065	0	39	256
N.S.	1	1.00	0.40	0.82	3.91	26.20	0.00	0.33	2.19
time (sec)	N/A	0.023	0.035	1.578	0.503	0.425	0.000	0.413	0.986

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	12	9	8	31	7	10	8
N.S.	1	1.00	1.50	1.12	1.00	3.88	0.88	1.25	1.00
time (sec)	N/A	0.014	0.009	0.239	0.258	0.352	0.154	0.403	0.074

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	12	9	8	31	7	10	8
N.S.	1	1.00	1.50	1.12	1.00	3.88	0.88	1.25	1.00
time (sec)	N/A	0.015	0.009	0.303	0.259	0.375	0.161	0.412	0.908

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	18	24	12	20	7	10	10
N.S.	1	1.00	1.50	2.00	1.00	1.67	0.58	0.83	0.83
time (sec)	N/A	0.021	0.006	0.378	0.258	0.408	0.197	0.406	0.897

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	24	26	12	22	7	10	10
N.S.	1	1.00	1.71	1.86	0.86	1.57	0.50	0.71	0.71
time (sec)	N/A	0.021	0.010	0.501	0.273	0.356	0.352	0.415	0.045

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	13	11	23	48	58	21	10
N.S.	1	1.00	1.30	1.10	2.30	4.80	5.80	2.10	1.00
time (sec)	N/A	0.026	0.015	0.325	0.284	0.348	0.198	0.407	0.940

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	13	11	23	54	58	22	10
N.S.	1	1.00	1.08	0.92	1.92	4.50	4.83	1.83	0.83
time (sec)	N/A	0.026	0.014	0.477	0.263	0.397	0.214	0.397	0.944

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	9	8	55	15	12	8
N.S.	1	1.00	1.20	0.90	0.80	5.50	1.50	1.20	0.80
time (sec)	N/A	0.015	0.008	0.244	0.275	0.417	0.255	0.402	0.920

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	8	55	14	12	8
N.S.	1	1.00	1.00	0.92	0.67	4.58	1.17	1.00	0.67
time (sec)	N/A	0.016	0.009	0.244	0.278	0.334	0.271	0.411	0.076

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	12	9	49	33	7	16	16
N.S.	1	1.00	0.86	0.64	3.50	2.36	0.50	1.14	1.14
time (sec)	N/A	0.022	0.020	0.357	0.264	0.358	0.329	0.408	0.925

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	12	9	49	33	8	16	16
N.S.	1	1.00	0.75	0.56	3.06	2.06	0.50	1.00	1.00
time (sec)	N/A	0.022	0.024	0.412	0.281	0.408	0.546	0.404	0.922

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	20	15	31	89	126	21	14
N.S.	1	1.00	1.43	1.07	2.21	6.36	9.00	1.50	1.00
time (sec)	N/A	0.027	0.008	0.336	0.261	0.356	0.267	0.400	0.984

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	27	17	35	90	126	20	16
N.S.	1	1.00	1.35	0.85	1.75	4.50	6.30	1.00	0.80
time (sec)	N/A	0.028	0.011	0.444	0.265	0.345	0.285	0.406	0.994

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	51	165	102	101	1253	90	131
N.S.	1	1.00	0.89	2.89	1.79	1.77	21.98	1.58	2.30
time (sec)	N/A	0.042	0.050	0.521	0.263	0.409	3.543	0.412	1.262

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	27	107	84	94	284	75	107
N.S.	1	1.00	0.59	2.33	1.83	2.04	6.17	1.63	2.33
time (sec)	N/A	0.046	0.023	0.489	0.270	0.380	2.307	0.404	1.133

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	39	125	78	57	692	66	95
N.S.	1	1.00	0.89	2.84	1.77	1.30	15.73	1.50	2.16
time (sec)	N/A	0.038	0.041	0.510	0.254	0.328	1.393	0.414	1.045

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	21	87	60	52	150	51	71
N.S.	1	1.00	0.64	2.64	1.82	1.58	4.55	1.55	2.15
time (sec)	N/A	0.039	0.015	0.483	0.265	0.379	0.892	0.401	0.979

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	25	85	54	27	294	40	59
N.S.	1	1.00	0.81	2.74	1.74	0.87	9.48	1.29	1.90
time (sec)	N/A	0.033	0.027	0.501	0.271	0.385	0.537	0.408	0.942

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	13	47	36	18	49	27	35
N.S.	1	1.00	0.68	2.47	1.89	0.95	2.58	1.42	1.84
time (sec)	N/A	0.031	0.011	0.473	0.265	0.368	0.303	0.414	0.936

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	17	45	23	11	46	17	23
N.S.	1	1.00	1.31	3.46	1.77	0.85	3.54	1.31	1.77
time (sec)	N/A	0.028	0.010	0.481	0.286	0.416	0.174	0.406	0.913

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	12	12	11	16	7	17	9
N.S.	1	1.00	1.33	1.33	1.22	1.78	0.78	1.89	1.00
time (sec)	N/A	0.017	0.006	0.325	0.294	0.433	0.050	0.421	0.888

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	42	20	47	103	0	52	51
N.S.	1	1.00	1.83	0.87	2.04	4.48	0.00	2.26	2.22
time (sec)	N/A	0.037	0.024	0.519	0.256	0.340	0.000	0.415	0.928

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	30	29	59	94	0	35	89
N.S.	1	1.00	1.25	1.21	2.46	3.92	0.00	1.46	3.71
time (sec)	N/A	0.032	0.035	0.582	0.266	0.360	0.000	0.418	0.919

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	60	38	103	631	0	94	114
N.S.	1	1.00	1.22	0.78	2.10	12.88	0.00	1.92	2.33
time (sec)	N/A	0.057	0.118	0.598	0.264	0.369	0.000	0.408	0.933

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	38	45	233	250	0	59	263
N.S.	1	1.00	1.03	1.22	6.30	6.76	0.00	1.59	7.11
time (sec)	N/A	0.033	0.042	0.589	0.264	0.321	0.000	0.413	0.953

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	89	54	155	1551	0	116	244
N.S.	1	1.00	1.14	0.69	1.99	19.88	0.00	1.49	3.13
time (sec)	N/A	0.076	0.199	0.592	0.278	0.342	0.000	0.405	1.048

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	144	547	310	2134	0	229	289
N.S.	1	1.00	1.03	3.91	2.21	15.24	0.00	1.64	2.06
time (sec)	N/A	0.119	0.131	0.453	0.273	0.458	0.000	0.426	1.715

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	154	409	0	2913	0	266	348
N.S.	1	1.00	1.00	2.66	0.00	18.92	0.00	1.73	2.26
time (sec)	N/A	0.295	0.170	0.480	0.000	0.395	0.000	0.404	1.701

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	84	328	178	866	0	124	169
N.S.	1	1.00	1.01	3.95	2.14	10.43	0.00	1.49	2.04
time (sec)	N/A	0.076	0.085	0.580	0.263	0.376	0.000	0.405	1.310

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	95	223	0	1099	0	146	222
N.S.	1	1.00	0.91	2.14	0.00	10.57	0.00	1.40	2.13
time (sec)	N/A	0.175	0.136	0.452	0.000	0.397	0.000	0.410	1.310

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	161	84	234	0	56	79
N.S.	1	1.00	1.00	4.02	2.10	5.85	0.00	1.40	1.98
time (sec)	N/A	0.048	0.046	0.436	0.264	0.392	0.000	0.412	1.044

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	54	100	0	279	892	68	139
N.S.	1	1.00	0.92	1.69	0.00	4.73	15.12	1.15	2.36
time (sec)	N/A	0.076	0.059	0.458	0.000	0.371	68.477	0.435	1.045

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	27	14	19	11
N.S.	1	1.00	1.00	1.09	1.00	2.45	1.27	1.73	1.00
time (sec)	N/A	0.018	0.013	0.276	0.263	0.518	0.158	0.394	0.058

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	37	52	59	58	0	67	160
N.S.	1	1.00	0.70	0.98	1.11	1.09	0.00	1.26	3.02
time (sec)	N/A	0.054	0.051	0.520	0.273	0.474	0.000	0.409	1.286

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	77	78	0	470	0	76	327
N.S.	1	1.00	1.15	1.16	0.00	7.01	0.00	1.13	4.88
time (sec)	N/A	0.070	0.145	0.557	0.000	0.470	0.000	0.405	1.479

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	100	90	154	818	0	179	291
N.S.	1	1.00	1.10	0.99	1.69	8.99	0.00	1.97	3.20
time (sec)	N/A	0.115	0.202	0.613	0.273	0.382	0.000	0.426	1.477

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	141	127	0	2339	0	156	642
N.S.	1	1.00	1.28	1.15	0.00	21.26	0.00	1.42	5.84
time (sec)	N/A	0.173	0.413	0.596	0.000	0.431	0.000	0.415	1.975

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	148	138	348	3450	0	338	559
N.S.	1	1.00	0.98	0.91	2.30	22.85	0.00	2.24	3.70
time (sec)	N/A	0.186	0.622	0.875	0.299	0.512	0.000	0.424	1.831

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	201	213	0	6381	0	303	1031
N.S.	1	1.00	1.26	1.34	0.00	40.13	0.00	1.91	6.48
time (sec)	N/A	0.340	1.247	0.677	0.000	0.493	0.000	0.422	2.605

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	61	99	0	700	0	68	139
N.S.	1	1.00	0.91	1.48	0.00	10.45	0.00	1.01	2.07
time (sec)	N/A	0.082	0.078	0.457	0.000	0.452	0.000	0.415	1.123

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	100	154	0	2003	0	144	722
N.S.	1	1.00	0.88	1.36	0.00	17.73	0.00	1.27	6.39
time (sec)	N/A	0.272	0.298	0.718	0.000	0.496	0.000	0.403	5.941

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	46	95	96	450	0	115	1221
N.S.	1	1.00	0.81	1.67	1.68	7.89	0.00	2.02	21.42
time (sec)	N/A	0.064	0.072	0.531	0.527	0.388	0.000	0.408	1.610

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	78	0	326	0	67	285
N.S.	1	1.00	1.00	1.28	0.00	5.34	0.00	1.10	4.67
time (sec)	N/A	0.152	0.080	0.613	0.000	0.491	0.000	0.415	3.533

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	33	40	0	33	201
N.S.	1	1.00	1.00	1.05	1.65	2.00	0.00	1.65	10.05
time (sec)	N/A	0.028	0.008	0.553	0.496	0.417	0.000	0.410	0.425

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	38	53	59	60	0	67	148
N.S.	1	1.00	0.70	0.98	1.09	1.11	0.00	1.24	2.74
time (sec)	N/A	0.049	0.054	0.648	0.266	0.448	0.000	0.408	0.427

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	78	0	470	0	76	337
N.S.	1	1.00	1.00	1.01	0.00	6.10	0.00	0.99	4.38
time (sec)	N/A	0.071	0.147	0.634	0.000	0.451	0.000	0.404	1.338

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	101	91	156	839	0	178	291
N.S.	1	1.00	1.07	0.97	1.66	8.93	0.00	1.89	3.10
time (sec)	N/A	0.141	0.156	0.684	0.284	0.463	0.000	0.404	1.519

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	131	127	0	2417	0	172	666
N.S.	1	1.00	0.96	0.93	0.00	17.64	0.00	1.26	4.86
time (sec)	N/A	0.142	0.371	0.694	0.000	0.446	0.000	0.418	1.806

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	58	64	89	750	0	58	183
N.S.	1	1.00	1.26	1.39	1.93	16.30	0.00	1.26	3.98
time (sec)	N/A	0.067	0.064	0.605	0.485	0.522	0.000	0.418	1.062

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	25	30	223	174	0	48	117
N.S.	1	1.00	0.83	1.00	7.43	5.80	0.00	1.60	3.90
time (sec)	N/A	0.063	0.021	0.582	0.264	0.419	0.000	0.408	1.013

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	46	48	57	315	0	39	95
N.S.	1	1.00	1.39	1.45	1.73	9.55	0.00	1.18	2.88
time (sec)	N/A	0.056	0.047	0.577	0.466	0.433	0.000	0.425	0.962

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	17	18	70	66	0	22	25
N.S.	1	1.00	0.89	0.95	3.68	3.47	0.00	1.16	1.32
time (sec)	N/A	0.046	0.018	0.567	0.271	0.378	0.000	0.413	0.919

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	18	30	23	50	0	22	33
N.S.	1	1.00	1.20	2.00	1.53	3.33	0.00	1.47	2.20
time (sec)	N/A	0.035	0.035	0.589	0.495	0.447	0.000	0.406	0.921

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	12	16	24	28	0	22	26
N.S.	1	1.00	0.67	0.89	1.33	1.56	0.00	1.22	1.44
time (sec)	N/A	0.028	0.014	0.516	0.269	0.374	0.000	0.409	0.076

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	42	20	48	103	0	52	51
N.S.	1	1.00	1.27	0.61	1.45	3.12	0.00	1.58	1.55
time (sec)	N/A	0.044	0.031	0.576	0.275	0.450	0.000	0.406	0.922

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	25	29	121	91	0	35	92
N.S.	1	1.00	0.83	0.97	4.03	3.03	0.00	1.17	3.07
time (sec)	N/A	0.059	0.037	0.586	0.276	0.368	0.000	0.414	0.927

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	60	38	103	631	0	94	132
N.S.	1	1.00	1.30	0.83	2.24	13.72	0.00	2.04	2.87
time (sec)	N/A	0.076	0.104	0.589	0.264	0.484	0.000	0.425	0.962

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	45	469	224	0	59	263
N.S.	1	1.00	1.00	1.10	11.44	5.46	0.00	1.44	6.41
time (sec)	N/A	0.059	0.061	0.604	0.268	0.411	0.000	0.394	1.029

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	30	0	376	0	0	-1
N.S.	1	1.00	1.00	0.81	0.00	10.16	0.00	0.00	-0.03
time (sec)	N/A	0.044	0.016	0.926	0.000	0.596	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	19	0	356	0	0	-1
N.S.	1	1.00	1.00	0.79	0.00	14.83	0.00	0.00	-0.04
time (sec)	N/A	0.042	0.011	1.013	0.000	0.462	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	55	112	0	291	741	60	197
N.S.	1	1.00	0.98	2.00	0.00	5.20	13.23	1.07	3.52
time (sec)	N/A	0.091	0.062	0.566	0.000	0.362	16.848	0.414	2.982

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	19	28	19	46	20	22	22
N.S.	1	1.00	1.06	1.56	1.06	2.56	1.11	1.22	1.22
time (sec)	N/A	0.056	0.026	0.421	0.256	0.484	0.144	0.400	0.056

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	19	36	20	48	31	22	21
N.S.	1	1.00	0.79	1.50	0.83	2.00	1.29	0.92	0.88
time (sec)	N/A	0.066	0.035	0.474	0.260	0.493	0.230	0.435	0.909

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	61	100	0	315	0	66	160
N.S.	1	1.00	0.94	1.54	0.00	4.85	0.00	1.02	2.46
time (sec)	N/A	0.111	0.123	0.765	0.000	0.380	0.000	0.439	12.141

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	81	101	0	303	0	90	974
N.S.	1	1.00	0.81	1.01	0.00	3.03	0.00	0.90	9.74
time (sec)	N/A	0.127	0.183	0.774	0.000	2.133	0.000	0.420	3.679

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	63	59	0	249	0	53	636
N.S.	1	1.00	1.02	0.95	0.00	4.02	0.00	0.85	10.26
time (sec)	N/A	0.090	0.086	0.718	0.000	0.928	0.000	0.416	6.413

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	81	100	0	298	0	90	983
N.S.	1	1.00	0.82	1.01	0.00	3.01	0.00	0.91	9.93
time (sec)	N/A	0.218	0.163	0.645	0.000	1.967	0.000	0.420	3.328

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	81	156	0	585	695	94	653
N.S.	1	1.00	0.94	1.81	0.00	6.80	8.08	1.09	7.59
time (sec)	N/A	0.117	0.185	2.267	0.000	0.493	16.993	0.452	2.188

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	115	144	0	1526	0	155	301
N.S.	1	1.00	0.95	1.19	0.00	12.61	0.00	1.28	2.49
time (sec)	N/A	0.130	0.319	2.106	0.000	0.456	0.000	0.442	1.588

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	175	273	0	5158	0	370	-1
N.S.	1	1.00	0.94	1.46	0.00	27.58	0.00	1.98	-0.01
time (sec)	N/A	0.206	0.580	2.251	0.000	0.511	0.000	0.477	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	245	459	0	11677	0	657	-1
N.S.	1	1.00	0.94	1.77	0.00	44.91	0.00	2.53	-0.00
time (sec)	N/A	0.333	1.746	2.224	0.000	0.674	0.000	0.465	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	536	487	0	780	0	0	-1
N.S.	1	1.00	2.81	2.55	0.00	4.08	0.00	0.00	-0.01
time (sec)	N/A	0.257	0.427	0.888	0.000	0.418	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	221	686	0	1162	0	0	-1
N.S.	1	1.00	0.76	2.36	0.00	3.99	0.00	0.00	-0.00
time (sec)	N/A	0.390	0.482	0.848	0.000	0.615	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	295	889	0	1542	0	0	-1
N.S.	1	1.00	0.75	2.27	0.00	3.94	0.00	0.00	-0.00
time (sec)	N/A	0.412	0.431	0.862	0.000	0.523	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.074	0.060	180.000	0.000	0.000	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	57	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.058	0.027	180.000	0.000	0.000	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.028	0.028	180.000	0.000	0.000	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.026	3.903	180.000	0.000	0.000	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.054	16.794	180.000	0.000	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	59	138	0	480	0	0	110
N.S.	1	1.00	0.98	2.30	0.00	8.00	0.00	0.00	1.83
time (sec)	N/A	0.041	0.093	0.935	0.000	0.485	0.000	0.000	1.091

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	231	0	1692	0	0	-1
N.S.	1	1.00	1.00	2.66	0.00	19.45	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.200	1.060	0.000	0.500	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	41	47	42	67	0	42	-1
N.S.	1	1.00	0.87	1.00	0.89	1.43	0.00	0.89	-0.02
time (sec)	N/A	0.346	0.098	3.654	0.359	0.404	0.000	0.400	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.026	3.528	0.339	0.000	0.000	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	326	0	0	624	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	1.91	0.00	0.00	-0.00
time (sec)	N/A	0.339	0.027	0.361	0.000	0.523	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	244	0	0	497	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	2.03	0.00	0.00	-0.00
time (sec)	N/A	0.284	0.027	0.344	0.000	0.426	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	160	368	0	354	0	0	-1
N.S.	1	1.00	0.99	2.29	0.00	2.20	0.00	0.00	-0.01
time (sec)	N/A	0.164	0.016	1.438	0.000	0.366	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	44	41	31	18
N.S.	1	1.00	1.00	1.06	1.00	2.44	2.28	1.72	1.00
time (sec)	N/A	0.021	0.046	0.372	0.268	0.443	0.426	0.417	0.072

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.025	9.415	180.000	0.000	0.000	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.038	17.610	0.913	0.000	0.000	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	495	495	386	0	0	1174	0	0	-1
N.S.	1	1.00	0.78	0.00	0.00	2.37	0.00	0.00	-0.00
time (sec)	N/A	0.584	1.088	1.084	0.000	0.448	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	293	0	0	937	0	0	-1
N.S.	1	1.00	0.79	0.00	0.00	2.53	0.00	0.00	-0.00
time (sec)	N/A	0.509	0.904	1.087	0.000	0.455	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	187	862	0	669	0	0	-1
N.S.	1	1.00	0.77	3.53	0.00	2.74	0.00	0.00	-0.00
time (sec)	N/A	0.292	0.689	2.133	0.000	0.540	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	69	129	0	415	1122	89	176
N.S.	1	1.00	0.95	1.77	0.00	5.68	15.37	1.22	2.41
time (sec)	N/A	0.094	0.145	0.997	0.000	0.410	62.612	0.418	1.115

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.038	79.993	1.097	0.000	0.000	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.040	22.279	1.060	0.000	0.000	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	586	586	935	0	0	2025	0	0	-1
N.S.	1	1.00	1.60	0.00	0.00	3.46	0.00	0.00	-0.00
time (sec)	N/A	0.502	2.686	0.810	0.000	0.476	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	432	432	697	0	0	1622	0	0	-1
N.S.	1	1.00	1.61	0.00	0.00	3.75	0.00	0.00	-0.00
time (sec)	N/A	0.400	2.189	0.787	0.000	0.498	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	414	860	0	1196	0	0	-1
N.S.	1	1.00	1.44	2.99	0.00	4.15	0.00	0.00	-0.00
time (sec)	N/A	0.241	1.933	2.101	0.000	0.369	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	55	205	130	340	0	88	122
N.S.	1	1.00	0.90	3.36	2.13	5.57	0.00	1.44	2.00
time (sec)	N/A	0.055	0.084	0.991	0.282	0.513	0.000	0.430	1.065

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.039	145.307	1.010	0.000	0.000	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	41	0	51	44	0	47	44
N.S.	1	1.00	0.76	0.00	0.94	0.81	0.00	0.87	0.81
time (sec)	N/A	0.008	0.043	0.499	0.277	0.400	0.000	0.423	0.995

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	56	0	67	90	0	169	53
N.S.	1	1.00	0.64	0.00	0.76	1.02	0.00	1.92	0.60
time (sec)	N/A	0.015	0.075	3.486	0.290	0.438	0.000	0.426	1.006

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	117	0	115	199	0	665	94
N.S.	1	1.00	0.79	0.00	0.77	1.34	0.00	4.46	0.63
time (sec)	N/A	0.031	0.387	3.346	0.299	0.482	0.000	0.450	1.047

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	167	0	129	293	0	777	102
N.S.	1	1.00	0.87	0.00	0.68	1.53	0.00	4.07	0.53
time (sec)	N/A	0.037	0.297	3.544	0.303	0.375	0.000	0.456	1.038

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	74	54	0	64	99	0	235	55
N.S.	1	1.01	0.74	0.00	0.88	1.36	0.00	3.22	0.75
time (sec)	N/A	0.019	0.094	0.406	0.280	0.429	0.000	0.427	1.045

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	122	87	0	87	250	0	759	73
N.S.	1	1.02	0.72	0.00	0.72	2.08	0.00	6.32	0.61
time (sec)	N/A	0.034	0.204	2.899	0.283	0.368	0.000	0.440	1.088

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	292	0	138	584	0	3225	117
N.S.	1	1.00	1.44	0.00	0.68	2.88	0.00	15.89	0.58
time (sec)	N/A	0.073	0.932	2.518	0.303	0.405	0.000	0.526	1.182

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	267	311	0	161	1123	0	6880	134
N.S.	1	1.00	1.17	0.00	0.61	4.22	0.00	25.86	0.50
time (sec)	N/A	0.102	2.425	2.759	0.310	0.472	0.000	0.555	1.195

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	37	19	18	19	37	42	18
N.S.	1	1.00	2.06	1.06	1.00	1.06	2.06	2.33	1.00
time (sec)	N/A	0.011	0.010	2.003	0.261	0.348	0.280	0.416	1.027

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	36	45	49	39	0	80	32
N.S.	1	1.00	0.92	1.15	1.26	1.00	0.00	2.05	0.82
time (sec)	N/A	0.021	0.025	2.590	0.277	0.546	0.000	0.422	1.046

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	0	86	53	76	81	35
N.S.	1	1.00	1.00	0.00	2.05	1.26	1.81	1.93	0.83
time (sec)	N/A	0.024	0.009	180.000	0.292	0.392	1.713	0.427	1.049

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	51	0	93	84	0	114	50
N.S.	1	1.00	0.70	0.00	1.27	1.15	0.00	1.56	0.68
time (sec)	N/A	0.036	0.037	180.000	0.269	0.539	0.000	0.432	1.107

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	68	0	130	105	110	116	49
N.S.	1	1.00	1.05	0.00	2.00	1.62	1.69	1.78	0.75
time (sec)	N/A	0.026	0.018	180.000	0.264	0.459	10.334	0.417	1.165

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	62	256	0	332	0	0	-1
N.S.	1	1.00	0.93	3.82	0.00	4.96	0.00	0.00	-0.01
time (sec)	N/A	0.031	0.044	4.873	0.000	0.100	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	114	237	0	170	0	0	-1
N.S.	1	1.00	1.70	3.54	0.00	2.54	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.093	3.599	0.000	0.186	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	183	0	58	0	0	-1
N.S.	1	1.00	1.00	6.54	0.00	2.07	0.00	0.00	-0.04
time (sec)	N/A	0.019	0.019	3.388	0.000	0.114	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	183	0	39	0	0	-1
N.S.	1	1.00	1.00	6.54	0.00	1.39	0.00	0.00	-0.04
time (sec)	N/A	0.020	0.018	3.161	0.000	0.096	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	58	141	0	243	0	0	-1
N.S.	1	1.00	0.92	2.24	0.00	3.86	0.00	0.00	-0.02
time (sec)	N/A	0.032	0.045	3.627	0.000	0.108	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	122	295	0	501	0	0	-1
N.S.	1	1.00	1.82	4.40	0.00	7.48	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.065	3.489	0.000	0.156	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	85	0	0	187	0	0	-1
N.S.	1	1.00	0.41	0.00	0.00	0.91	0.00	0.00	-0.00
time (sec)	N/A	0.109	0.244	2.579	0.000	0.466	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	74	0	0	141	0	0	-1
N.S.	1	1.00	0.73	0.00	0.00	1.38	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.217	2.479	0.000	0.406	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	61	0	0	68	0	0	-1
N.S.	1	1.00	1.45	0.00	0.00	1.62	0.00	0.00	-0.02
time (sec)	N/A	0.036	0.083	2.385	0.000	0.457	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	121	0	0	128	0	0	-1
N.S.	1	1.00	1.20	0.00	0.00	1.27	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.146	2.418	0.000	0.399	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	373	347	0	171	0	764	-1
N.S.	1	1.00	3.69	3.44	0.00	1.69	0.00	7.56	-0.01
time (sec)	N/A	0.129	0.247	1.911	0.000	0.461	0.000	1.915	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	111	358	0	366	0	749	-1
N.S.	1	1.00	1.04	3.35	0.00	3.42	0.00	7.00	-0.01
time (sec)	N/A	0.153	0.204	6.855	0.000	0.416	0.000	6.483	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	62	80	68	113	139	60	58
N.S.	1	1.00	0.75	0.96	0.82	1.36	1.67	0.72	0.70
time (sec)	N/A	0.027	0.031	1.303	0.264	0.430	3.061	0.390	0.502

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	47	61	53	95	182	57	42
N.S.	1	1.00	0.82	1.07	0.93	1.67	3.19	1.00	0.74
time (sec)	N/A	0.025	0.027	1.145	0.267	0.355	1.242	0.400	0.262

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	39	52	40	54	78	34	34
N.S.	1	1.00	0.80	1.06	0.82	1.10	1.59	0.69	0.69
time (sec)	N/A	0.020	0.017	1.001	0.266	0.395	0.500	0.391	0.965

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	37	24	50	80	22	18
N.S.	1	1.00	1.00	1.61	1.04	2.17	3.48	0.96	0.78
time (sec)	N/A	0.010	0.010	0.664	0.270	0.419	0.208	0.382	0.921

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	17	16	30	0	16	16
N.S.	1	1.00	1.00	1.00	0.94	1.76	0.00	0.94	0.94
time (sec)	N/A	0.013	0.012	0.371	0.480	0.409	0.000	0.382	0.922

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	36	25	37	105	0	35	48
N.S.	1	1.00	0.90	0.62	0.92	2.62	0.00	0.88	1.20
time (sec)	N/A	0.022	0.045	0.632	0.471	0.433	0.000	0.392	0.079

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	32	68	86	0	31	31
N.S.	1	1.00	1.00	1.10	2.34	2.97	0.00	1.07	1.07
time (sec)	N/A	0.019	0.015	1.422	0.261	0.390	0.000	0.406	0.927

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	64	82	83	513	0	60	130
N.S.	1	1.00	0.67	0.86	0.87	5.40	0.00	0.63	1.37
time (sec)	N/A	0.033	0.063	1.538	0.511	0.393	0.000	0.409	0.956

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	44	43	172	233	0	42	42
N.S.	1	1.00	0.73	0.72	2.87	3.88	0.00	0.70	0.70
time (sec)	N/A	0.034	0.027	1.553	0.279	0.406	0.000	0.393	0.955

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	34	17	47	42	17	17
N.S.	1	1.00	1.00	1.31	0.65	1.81	1.62	0.65	0.65
time (sec)	N/A	0.013	0.012	0.542	0.257	0.426	0.172	0.388	0.076

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	22	13	26	20	13	12
N.S.	1	1.00	0.84	1.16	0.68	1.37	1.05	0.68	0.63
time (sec)	N/A	0.009	0.008	0.362	0.265	0.346	0.095	0.393	0.052

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	24	25	76	113	0	76	77
N.S.	1	1.00	0.26	0.27	0.83	1.23	0.00	0.83	0.84
time (sec)	N/A	0.049	0.009	0.871	0.473	0.429	0.000	0.402	1.114

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	106	36	88	162	0	88	85
N.S.	1	1.00	0.95	0.32	0.79	1.46	0.00	0.79	0.77
time (sec)	N/A	0.060	0.066	0.924	0.460	0.480	0.000	0.383	1.088

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	34	17	67	42	17	17
N.S.	1	1.00	1.00	1.31	0.65	2.58	1.62	0.65	0.65
time (sec)	N/A	0.014	0.012	0.482	0.258	0.336	0.174	0.404	0.076

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	26	13	38	20	13	12
N.S.	1	1.00	0.84	1.37	0.68	2.00	1.05	0.68	0.63
time (sec)	N/A	0.008	0.008	0.627	0.266	0.356	0.103	0.410	0.926

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	24	79	71	83	0	44	65
N.S.	1	1.00	0.44	1.44	1.29	1.51	0.00	0.80	1.18
time (sec)	N/A	0.041	0.010	0.760	0.467	0.391	0.000	0.386	1.039

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	34	59	79	154	0	79	84
N.S.	1	1.00	0.31	0.54	0.72	1.40	0.00	0.72	0.76
time (sec)	N/A	0.153	0.017	1.009	0.505	0.382	0.000	0.400	0.309

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	34	17	87	42	17	17
N.S.	1	1.00	1.00	1.31	0.65	3.35	1.62	0.65	0.65
time (sec)	N/A	0.013	0.013	0.598	0.266	0.471	0.170	0.397	0.961

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	26	13	46	20	13	14
N.S.	1	1.00	1.00	1.37	0.68	2.42	1.05	0.68	0.74
time (sec)	N/A	0.008	0.010	0.572	0.264	0.357	0.092	0.395	0.049

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	24	25	0	1087	0	249	479
N.S.	1	1.00	0.06	0.07	0.00	2.93	0.00	0.67	1.29
time (sec)	N/A	0.231	0.009	0.879	0.000	0.515	0.000	0.460	4.560

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	379	379	34	36	0	1367	0	261	473
N.S.	1	1.00	0.09	0.09	0.00	3.61	0.00	0.69	1.25
time (sec)	N/A	0.233	0.016	1.012	0.000	0.483	0.000	0.397	3.467

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	159	326	142	4701	0	1211	154
N.S.	1	1.00	0.79	1.61	0.70	23.27	0.00	6.00	0.76
time (sec)	N/A	0.056	0.445	2.293	0.280	0.433	0.000	0.411	1.811

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	85	143	98	1111	1052	889	100
N.S.	1	1.00	0.64	1.08	0.74	8.42	7.97	6.73	0.76
time (sec)	N/A	0.038	0.142	1.502	0.282	0.389	13.857	0.406	1.249

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	C	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	50	74	67	382	442	597	74
N.S.	1	1.00	0.67	0.99	0.89	5.09	5.89	7.96	0.99
time (sec)	N/A	0.013	0.069	0.465	0.274	0.448	2.388	0.397	1.007

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	70	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.016	0.015	0.282	0.000	0.000	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.021	0.016	0.402	0.000	0.000	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	96	0	0	0	0	0	-1
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.038	0.177	0.513	0.000	0.000	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	101	0	0	0	0	0	-1
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.138	0.519	0.000	0.000	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	106	326	112	218	0	101	-1
N.S.	1	1.00	0.42	1.30	0.45	0.87	0.00	0.40	-0.00
time (sec)	N/A	0.175	0.070	7.299	0.267	0.432	0.000	0.392	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	78	216	74	126	0	73	-1
N.S.	1	1.00	0.48	1.33	0.46	0.78	0.00	0.45	-0.01
time (sec)	N/A	0.091	0.076	6.760	0.275	0.397	0.000	0.397	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	48	106	29	66	235	23	76
N.S.	1	1.00	0.65	1.43	0.39	0.89	3.18	0.31	1.03
time (sec)	N/A	0.070	0.030	6.809	0.273	0.358	2.489	0.395	0.121

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	42	66	21	42	0	20	-1
N.S.	1	1.00	0.95	1.50	0.48	0.95	0.00	0.45	-0.02
time (sec)	N/A	0.082	0.040	6.467	0.489	0.412	0.000	0.386	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	46	69	84	120	0	38	76
N.S.	1	1.00	0.82	1.23	1.50	2.14	0.00	0.68	1.36
time (sec)	N/A	0.091	0.054	6.510	0.302	0.404	0.000	0.387	0.944

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	72	80	209	315	0	51	89
N.S.	1	1.00	0.51	0.57	1.48	2.23	0.00	0.36	0.63
time (sec)	N/A	0.138	0.050	6.894	0.292	0.361	0.000	0.402	0.104

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	84	91	386	589	0	64	345
N.S.	1	1.00	0.44	0.48	2.02	3.08	0.00	0.34	1.81
time (sec)	N/A	0.184	0.057	7.017	0.288	0.346	0.000	0.444	0.111

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	28	62	0	45	99	32	45
N.S.	1	1.00	0.68	1.51	0.00	1.10	2.41	0.78	1.10
time (sec)	N/A	0.010	0.037	0.857	0.000	0.458	0.170	0.395	0.087

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	79	72	65	104	0	73	-1
N.S.	1	1.00	0.93	0.85	0.76	1.22	0.00	0.86	-0.01
time (sec)	N/A	0.056	0.059	1.452	0.297	0.381	0.000	0.425	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	91	97	81	130	0	91	-1
N.S.	1	1.00	0.90	0.96	0.80	1.29	0.00	0.90	-0.01
time (sec)	N/A	0.107	0.107	3.361	0.296	0.383	0.000	0.407	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	51	52	45	44	0	45	-1
N.S.	1	1.00	0.78	0.80	0.69	0.68	0.00	0.69	-0.02
time (sec)	N/A	0.057	0.049	1.449	0.303	0.521	0.000	0.423	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	71	48	47	76	0	49	-1
N.S.	1	1.00	1.09	0.74	0.72	1.17	0.00	0.75	-0.02
time (sec)	N/A	0.058	0.069	5.958	0.288	0.370	0.000	0.400	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	122	105	89	165	0	101	-1
N.S.	1	1.00	1.06	0.91	0.77	1.43	0.00	0.88	-0.01
time (sec)	N/A	0.125	0.254	5.888	0.279	0.384	0.000	0.428	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	102	100	90	211	0	106	-1
N.S.	1	1.00	0.93	0.91	0.82	1.92	0.00	0.96	-0.01
time (sec)	N/A	0.111	0.092	0.729	0.286	0.369	0.000	0.410	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	149	126	127	278	0	355	-1
N.S.	1	1.00	1.01	0.85	0.86	1.88	0.00	2.40	-0.01
time (sec)	N/A	0.138	0.525	2.020	0.527	0.442	0.000	0.446	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	286	207	200	443	0	223	-1
N.S.	1	1.00	1.20	0.87	0.84	1.85	0.00	0.93	-0.00
time (sec)	N/A	0.208	0.294	2.638	0.502	0.386	0.000	0.425	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	123	126	106	335	0	132	-1
N.S.	1	1.00	1.07	1.10	0.92	2.91	0.00	1.15	-0.01
time (sec)	N/A	0.160	0.190	0.762	0.304	0.383	0.000	0.415	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	220	158	147	434	0	387	-1
N.S.	1	1.00	1.37	0.98	0.91	2.70	0.00	2.40	-0.01
time (sec)	N/A	0.208	0.446	2.153	0.496	0.393	0.000	0.425	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	353	265	236	723	0	281	-1
N.S.	1	1.00	1.37	1.03	0.92	2.81	0.00	1.09	-0.00
time (sec)	N/A	0.357	0.620	2.852	0.512	0.467	0.000	0.423	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	104	117	105	276	0	132	-1
N.S.	1	1.00	0.78	0.88	0.79	2.08	0.00	0.99	-0.01
time (sec)	N/A	0.158	0.106	0.766	0.276	0.373	0.000	0.414	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	131	139	131	302	0	150	-1
N.S.	1	1.00	0.81	0.86	0.81	1.88	0.00	0.93	-0.01
time (sec)	N/A	0.168	0.163	1.342	0.301	0.445	0.000	0.407	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	214	234	211	548	0	264	-1
N.S.	1	1.00	0.79	0.86	0.78	2.02	0.00	0.97	-0.00
time (sec)	N/A	0.265	0.335	2.270	0.298	0.412	0.000	0.413	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	75	70	69	145	0	75	-1
N.S.	1	1.00	0.93	0.86	0.85	1.79	0.00	0.93	-0.01
time (sec)	N/A	0.138	0.222	0.725	0.286	0.402	0.000	0.417	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	179	101	100	254	0	107	-1
N.S.	1	1.00	1.40	0.79	0.78	1.98	0.00	0.84	-0.01
time (sec)	N/A	0.160	0.393	1.602	0.301	0.550	0.000	0.420	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	270	144	143	491	0	155	-1
N.S.	1	1.00	1.58	0.84	0.84	2.87	0.00	0.91	-0.01
time (sec)	N/A	0.254	0.882	2.462	0.301	0.528	0.000	0.409	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	165	147	127	381	0	172	-1
N.S.	1	1.00	1.18	1.05	0.91	2.72	0.00	1.23	-0.01
time (sec)	N/A	0.262	0.466	0.789	0.307	0.499	0.000	0.435	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	258	177	161	480	0	198	-1
N.S.	1	1.00	1.41	0.97	0.88	2.62	0.00	1.08	-0.01
time (sec)	N/A	0.251	1.008	1.666	0.312	0.432	0.000	0.429	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	478	302	263	969	0	352	-1
N.S.	1	1.00	1.59	1.01	0.88	3.23	0.00	1.17	-0.00
time (sec)	N/A	0.477	4.061	2.703	0.303	0.503	0.000	0.430	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	134	156	133	338	0	167	-1
N.S.	1	1.00	0.88	1.02	0.87	2.21	0.00	1.09	-0.01
time (sec)	N/A	0.222	0.188	0.853	0.327	0.492	0.000	0.404	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	183	211	189	427	0	223	-1
N.S.	1	1.00	0.84	0.96	0.86	1.95	0.00	1.02	-0.00
time (sec)	N/A	0.284	0.327	1.345	0.281	0.516	0.000	0.424	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	262	316	271	688	0	339	-1
N.S.	1	1.00	0.83	1.00	0.86	2.18	0.00	1.08	-0.00
time (sec)	N/A	0.365	0.613	2.356	0.292	0.474	0.000	0.416	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	185	160	139	324	0	181	-1
N.S.	1	1.00	1.20	1.04	0.90	2.10	0.00	1.18	-0.01
time (sec)	N/A	0.272	0.472	0.881	0.275	0.565	0.000	0.423	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	257	217	199	466	0	239	-1
N.S.	1	1.00	1.14	0.96	0.88	2.07	0.00	1.06	-0.00
time (sec)	N/A	0.285	1.518	1.726	0.274	0.427	0.000	0.423	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	501	326	287	851	0	369	-1
N.S.	1	1.00	1.55	1.01	0.89	2.63	0.00	1.14	-0.00
time (sec)	N/A	0.426	4.722	2.492	0.278	0.666	0.000	0.413	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	251	186	155	436	0	207	-1
N.S.	1	1.00	1.56	1.16	0.96	2.71	0.00	1.29	-0.01
time (sec)	N/A	0.355	1.042	0.881	0.275	0.522	0.000	0.420	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	339	249	219	602	0	271	-1
N.S.	1	1.00	1.42	1.04	0.92	2.52	0.00	1.13	-0.00
time (sec)	N/A	0.413	4.258	1.807	0.285	0.465	0.000	0.430	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	2991	384	323	1099	0	427	-1
N.S.	1	1.00	8.69	1.12	0.94	3.19	0.00	1.24	-0.00
time (sec)	N/A	0.603	6.439	2.779	0.286	0.537	0.000	0.439	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	46	0	0	0	0	0	39
N.S.	1	1.00	2.30	0.00	0.00	0.00	0.00	0.00	1.95
time (sec)	N/A	0.033	0.253	0.053	0.000	0.000	0.000	0.000	0.148

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	16	0	0	109	0	0	42
N.S.	1	1.00	0.67	0.00	0.00	4.54	0.00	0.00	1.75
time (sec)	N/A	0.034	0.046	0.052	0.000	0.520	0.000	0.000	0.967

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	64	0	0	0	0	0	110
N.S.	1	1.00	1.36	0.00	0.00	0.00	0.00	0.00	2.34
time (sec)	N/A	0.046	0.407	0.051	0.000	0.000	0.000	0.000	1.106

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	76	0	0	0	0	0	-1
N.S.	1	1.00	2.11	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.067	0.117	0.053	0.000	0.000	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	26	25	36	23	41	36	24
N.S.	1	1.00	0.87	0.83	1.20	0.77	1.37	1.20	0.80
time (sec)	N/A	0.027	0.040	0.247	0.265	0.393	0.063	0.406	0.049

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	51	73	81	54	85	75	48
N.S.	1	1.00	0.91	1.30	1.45	0.96	1.52	1.34	0.86
time (sec)	N/A	0.052	0.053	0.392	0.275	0.407	0.090	0.426	0.069

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	180	212	0	316	0	0	-1
N.S.	1	1.00	0.85	1.00	0.00	1.48	0.00	0.00	-0.00
time (sec)	N/A	0.396	0.209	0.811	0.000	0.399	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	248	376	0	1346	0	0	-1
N.S.	1	1.00	0.92	1.39	0.00	4.97	0.00	0.00	-0.00
time (sec)	N/A	0.554	0.239	0.826	0.000	0.468	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [213] had the largest ratio of [36]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	6	0.167
2	A	2	2	1.00	8	0.250
3	A	2	1	1.00	8	0.125
4	A	3	2	1.00	8	0.250
5	A	2	1	1.00	8	0.125
6	A	4	2	1.00	8	0.250
7	A	3	2	1.00	10	0.200
8	A	2	2	1.00	10	0.200
9	A	2	2	1.00	10	0.200
10	A	1	1	1.00	10	0.100
11	A	1	1	1.00	10	0.100
12	A	2	2	1.00	10	0.200
13	A	2	2	1.00	10	0.200
14	A	3	2	1.00	10	0.200
15	A	4	3	1.00	8	0.375
16	A	3	3	1.00	8	0.375
17	A	3	3	1.00	8	0.375
18	A	2	2	1.00	8	0.250
19	A	2	2	1.00	8	0.250
20	A	3	3	1.00	8	0.375
21	A	3	3	1.00	8	0.375
22	A	4	3	1.00	8	0.375
23	A	1	1	1.00	10	0.100
24	A	6	5	1.00	13	0.385
25	A	2	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	4	4	1.00	13	0.308
27	A	2	2	1.00	11	0.182
28	A	3	3	1.00	11	0.273
29	A	5	5	1.00	13	0.385
30	A	6	6	1.00	13	0.462
31	A	6	5	1.00	13	0.385
32	A	1	1	1.00	10	0.100
33	A	2	2	1.00	10	0.200
34	A	3	2	1.00	10	0.200
35	A	4	2	1.00	10	0.200
36	A	1	1	1.00	12	0.083
37	A	2	2	1.00	12	0.167
38	A	3	2	1.00	12	0.167
39	A	4	2	1.00	12	0.167
40	A	3	3	1.00	13	0.231
41	A	3	3	1.00	14	0.214
42	A	3	2	1.00	14	0.143
43	A	2	2	1.00	14	0.143
44	A	1	1	1.00	14	0.071
45	A	2	2	1.00	14	0.143
46	A	3	3	1.00	14	0.214
47	A	4	3	1.00	14	0.214
48	A	3	2	1.00	15	0.133
49	A	2	2	1.00	15	0.133
50	A	1	1	1.00	15	0.067
51	A	2	2	1.00	15	0.133
52	A	3	3	1.00	15	0.200
53	A	4	3	1.00	15	0.200
54	A	6	6	1.00	13	0.462
55	A	5	5	1.00	13	0.385
56	A	5	5	1.00	13	0.385
57	A	3	3	1.00	11	0.273
58	A	4	4	1.00	11	0.364
59	A	6	6	1.00	13	0.462
60	A	6	6	1.00	13	0.462

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	7	6	1.00	13	0.462
62	A	4	3	1.00	12	0.250
63	A	3	3	1.00	12	0.250
64	A	2	2	1.00	12	0.167
65	A	1	1	1.00	12	0.083
66	A	2	1	1.00	10	0.100
67	A	2	2	1.00	12	0.167
68	A	4	4	1.00	12	0.333
69	A	5	5	1.00	12	0.417
70	A	6	5	1.00	12	0.417
71	A	2	2	1.00	12	0.167
72	A	4	4	1.00	12	0.333
73	A	5	5	1.00	12	0.417
74	A	6	5	1.00	12	0.417
75	A	1	1	1.00	12	0.083
76	A	3	3	1.00	12	0.250
77	A	4	4	1.00	12	0.333
78	A	5	4	1.00	12	0.333
79	A	7	7	1.00	10	0.700
80	A	6	6	1.00	10	0.600
81	A	2	2	1.00	14	0.143
82	A	2	2	1.00	10	0.200
83	A	4	4	1.00	10	0.400
84	A	7	7	1.00	10	0.700
85	A	8	7	1.00	10	0.700
86	A	5	5	1.00	13	0.385
87	A	4	3	1.00	17	0.176
88	A	3	3	1.00	17	0.176
89	A	2	2	1.00	17	0.118
90	A	4	3	1.00	18	0.167
91	A	3	3	1.00	18	0.167
92	A	2	2	1.00	18	0.111
93	A	2	2	1.00	13	0.154
94	A	2	2	1.00	13	0.154
95	A	3	3	1.00	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	4	3	1.00	13	0.231
97	A	2	2	1.00	15	0.133
98	A	2	2	1.00	15	0.133
99	A	3	3	1.00	15	0.200
100	A	4	3	1.00	15	0.200
101	A	3	3	1.00	17	0.176
102	A	3	3	1.00	17	0.176
103	A	4	4	1.00	17	0.235
104	A	3	3	1.00	18	0.167
105	A	3	3	1.00	18	0.167
106	A	4	4	1.00	18	0.222
107	A	8	6	1.00	17	0.353
108	A	7	6	1.00	17	0.353
109	A	6	6	1.00	17	0.353
110	A	3	3	1.00	15	0.200
111	A	4	4	1.00	15	0.267
112	A	5	4	1.00	15	0.267
113	A	6	4	1.00	15	0.267
114	A	3	3	1.00	20	0.150
115	A	2	2	1.00	20	0.100
116	A	2	2	1.00	15	0.133
117	A	2	2	1.00	13	0.154
118	A	5	5	1.00	17	0.294
119	A	6	6	1.00	17	0.353
120	A	7	6	1.00	17	0.353
121	A	5	3	1.00	10	0.300
122	A	4	3	1.00	10	0.300
123	A	3	3	1.00	10	0.300
124	A	2	2	1.00	10	0.200
125	A	2	2	1.00	10	0.200
126	A	3	3	1.00	10	0.300
127	A	4	3	1.00	10	0.300
128	A	6	3	1.00	10	0.300
129	A	4	3	1.00	10	0.300
130	A	3	3	1.00	10	0.300

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	3	3	1.00	10	0.300
132	A	4	3	1.00	10	0.300
133	A	6	3	1.00	10	0.300
134	A	7	3	1.00	10	0.300
135	A	5	3	1.00	10	0.300
136	A	3	3	1.00	10	0.300
137	A	3	3	1.00	10	0.300
138	A	3	2	1.00	10	0.200
139	A	3	2	1.00	10	0.200
140	A	2	2	1.00	9	0.222
141	A	2	2	1.00	11	0.182
142	A	2	2	1.00	11	0.182
143	A	2	2	1.00	13	0.154
144	A	3	2	1.00	11	0.182
145	A	3	2	1.00	13	0.154
146	A	2	2	1.00	9	0.222
147	A	2	2	1.00	11	0.182
148	A	1	1	1.00	11	0.091
149	A	1	1	1.00	13	0.077
150	A	3	2	1.00	11	0.182
151	A	3	2	1.00	13	0.154
152	A	5	3	1.00	13	0.231
153	A	3	2	1.00	13	0.154
154	A	4	3	1.00	13	0.231
155	A	3	2	1.00	13	0.154
156	A	3	3	1.00	13	0.231
157	A	2	1	1.00	13	0.077
158	A	2	2	1.00	13	0.154
159	A	2	2	1.00	11	0.182
160	A	4	3	1.00	11	0.273
161	A	3	3	1.00	13	0.231
162	A	4	3	1.00	13	0.231
163	A	3	2	1.00	13	0.154
164	A	4	3	1.00	13	0.231
165	A	3	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	6	5	1.00	13	0.385
167	A	3	2	1.00	13	0.154
168	A	5	5	1.00	13	0.385
169	A	3	2	1.00	13	0.154
170	A	4	4	1.00	13	0.308
171	A	2	2	1.00	11	0.182
172	A	6	4	1.00	11	0.364
173	A	4	4	1.00	13	0.308
174	A	4	3	1.00	13	0.231
175	A	5	5	1.00	13	0.385
176	A	5	4	1.00	13	0.308
177	A	6	5	1.00	13	0.385
178	A	4	4	1.00	13	0.308
179	A	6	6	1.00	13	0.462
180	A	3	2	1.00	13	0.154
181	A	6	6	1.00	13	0.462
182	A	4	4	1.00	11	0.364
183	A	3	2	1.00	11	0.182
184	A	7	6	1.00	13	0.462
185	A	4	3	1.00	13	0.231
186	A	12	8	1.00	13	0.615
187	A	6	5	1.00	13	0.385
188	A	5	4	1.00	13	0.308
189	A	5	5	1.00	13	0.385
190	A	5	4	1.00	13	0.308
191	A	4	4	1.00	13	0.308
192	A	4	4	1.00	11	0.364
193	A	5	5	1.00	11	0.454
194	A	5	4	1.00	13	0.308
195	A	6	5	1.00	13	0.385
196	A	6	5	1.00	13	0.385
197	A	4	4	1.00	13	0.308
198	A	3	3	1.00	13	0.231
199	A	6	5	1.00	15	0.333
200	A	5	4	1.00	13	0.308

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	5	4	1.00	15	0.267
202	A	8	7	1.00	15	0.467
203	A	7	5	1.00	15	0.333
204	A	5	5	1.00	15	0.333
205	A	11	8	1.00	15	0.533
206	A	6	6	1.00	31	0.194
207	A	7	7	1.00	31	0.226
208	A	8	7	1.00	31	0.226
209	A	9	7	1.00	31	0.226
210	A	9	6	1.00	12	0.500
211	A	11	7	1.00	14	0.500
212	A	13	8	1.00	14	0.571
213	A	5	3	1.00	36	0.083
214	A	4	3	1.00	36	0.083
215	A	2	2	1.00	34	0.059
216	A	0	0	0.00	0	0.000
217	A	0	0	0.00	0	0.000
218	A	3	3	1.00	12	0.250
219	A	5	5	1.00	12	0.417
220	A	13	5	1.00	20	0.250
221	A	0	0	0.00	0	0.000
222	A	11	6	1.00	22	0.273
223	A	9	5	1.00	22	0.227
224	A	7	4	1.00	20	0.200
225	A	2	2	1.00	19	0.105
226	A	0	0	0.00	0	0.000
227	A	0	0	0.00	0	0.000
228	A	18	11	1.00	24	0.458
229	A	15	10	1.00	24	0.417
230	A	12	9	1.00	22	0.409
231	A	4	4	1.00	21	0.190
232	A	0	0	0.00	0	0.000
233	A	0	0	0.00	0	0.000
234	A	21	14	1.00	24	0.583
235	A	16	11	1.00	24	0.458

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	13	10	1.00	22	0.454
237	A	3	2	1.00	21	0.095
238	A	0	0	0.00	0	0.000
239	A	1	1	1.00	11	0.091
240	A	2	2	1.00	13	0.154
241	A	2	2	1.00	13	0.154
242	A	3	2	1.00	13	0.154
243	A	1	1	1.01	15	0.067
244	A	2	2	1.02	17	0.118
245	A	2	2	1.00	17	0.118
246	A	3	2	1.00	17	0.118
247	A	2	1	1.00	15	0.067
248	A	3	2	1.00	17	0.118
249	A	3	1	1.00	17	0.059
250	A	4	2	1.00	17	0.118
251	A	3	1	1.00	17	0.059
252	A	3	2	1.00	19	0.105
253	A	3	2	1.00	19	0.105
254	A	2	1	1.00	19	0.053
255	A	2	1	1.00	19	0.053
256	A	3	2	1.00	19	0.105
257	A	3	2	1.00	19	0.105
258	A	8	8	1.00	18	0.444
259	A	6	6	1.00	18	0.333
260	A	3	3	1.00	18	0.167
261	A	4	4	1.00	18	0.222
262	A	5	5	1.00	14	0.357
263	A	6	6	1.00	16	0.375
264	A	4	3	1.00	16	0.188
265	A	5	4	1.00	16	0.250
266	A	4	3	1.00	16	0.188
267	A	4	3	1.00	14	0.214
268	A	3	3	1.00	14	0.214
269	A	4	4	1.00	16	0.250
270	A	3	3	1.00	16	0.188

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	6	5	1.00	16	0.312
272	A	5	4	1.00	16	0.250
273	A	4	3	1.00	10	0.300
274	A	4	3	1.00	8	0.375
275	A	11	8	1.00	8	1.000
276	A	12	9	1.00	10	0.900
277	A	4	3	1.00	10	0.300
278	A	4	3	1.00	8	0.375
279	A	9	9	1.00	8	1.125
280	A	13	9	1.00	10	0.900
281	A	4	3	1.00	10	0.300
282	A	4	3	1.00	8	0.375
283	A	21	9	1.00	8	1.125
284	A	22	9	1.00	10	0.900
285	A	2	2	1.00	18	0.111
286	A	2	2	1.00	18	0.111
287	A	1	1	1.00	16	0.062
288	A	1	1	1.00	16	0.062
289	A	1	1	1.00	18	0.056
290	A	2	2	1.00	18	0.111
291	A	2	2	1.00	18	0.111
292	A	6	5	1.00	25	0.200
293	A	6	5	1.00	25	0.200
294	A	5	4	1.00	25	0.160
295	A	4	4	1.00	25	0.160
296	A	4	4	1.00	25	0.160
297	A	6	5	1.00	25	0.200
298	A	6	5	1.00	25	0.200
299	A	1	1	1.00	10	0.100
300	A	6	4	1.00	12	0.333
301	A	6	4	1.00	15	0.267
302	A	6	3	1.00	12	0.250
303	A	4	2	1.00	14	0.143
304	A	6	3	1.00	17	0.176
305	A	8	5	1.00	16	0.312

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	9	6	1.00	18	0.333
307	A	14	5	1.00	18	0.278
308	A	8	5	1.00	19	0.263
309	A	9	6	1.00	21	0.286
310	A	14	5	1.00	21	0.238
311	A	8	4	1.00	16	0.250
312	A	9	4	1.00	18	0.222
313	A	14	4	1.00	18	0.222
314	A	6	4	1.00	18	0.222
315	A	7	4	1.00	20	0.200
316	A	10	4	1.00	20	0.200
317	A	8	5	1.00	21	0.238
318	A	9	5	1.00	23	0.217
319	A	14	5	1.00	23	0.217
320	A	8	4	1.00	19	0.210
321	A	10	4	1.00	21	0.190
322	A	14	4	1.00	21	0.190
323	A	8	5	1.00	21	0.238
324	A	10	5	1.00	23	0.217
325	A	14	5	1.00	23	0.217
326	A	8	5	1.00	24	0.208
327	A	10	5	1.00	26	0.192
328	A	14	5	1.00	26	0.192
329	A	2	1	1.00	17	0.059
330	A	2	1	1.00	20	0.050
331	A	3	1	1.00	20	0.050
332	A	3	2	1.00	21	0.095
333	A	6	5	1.00	6	0.833
334	A	9	6	1.00	6	1.000
335	A	8	4	1.00	16	0.250
336	A	8	4	1.00	19	0.210

Chapter 3

Listing of integrals

Local contents

3.1 $\int \cosh(a+bx)\,dx$

Optimal. Leaf size=10

$$\frac{\sinh(a+bx)}{b}$$

```
[Out] sinh(b*x+a)/b
```

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2717}

$$\frac{\sinh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x],x]
```

```
[Out] Sinh[a + b*x]/b
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cosh(a+bx)\,dx = \frac{\sinh(a+bx)}{b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 21 vs. 2(10) = 20.
time = 0.01, size = 21, normalized size = 2.10

$$\frac{\cosh(bx)\sinh(a)}{b} + \frac{\cosh(a)\sinh(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x],x]
```

```
[Out] (Cosh[b*x]*Sinh[a])/b + (Cosh[a]*Sinh[b*x])/b
```

Maple [A]
time = 0.39, size = 11, normalized size = 1.10

method	result	size
derivativedivides	$\frac{\sinh(bx+a)}{b}$	11
default	$\frac{\sinh(bx+a)}{b}$	11
risch	$\frac{e^{bx+a}}{2b}-\frac{e^{-bx-a}}{2b}$	27
meijerg	$\frac{\cosh(a)\sinh(bx)}{b}-\frac{\sinh(a)\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cosh(bx)}{\sqrt{\pi}}\right)}{b}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] sinh(b*x+a)/b
```

Maxima [A]
time = 0.26, size = 10, normalized size = 1.00

$$\frac{\sinh(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] sinh(b*x + a)/b
```

Fricas [A]
time = 0.36, size = 10, normalized size = 1.00

$$\frac{\sinh(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] sinh(b*x + a)/b
```

Sympy [A]
time = 0.05, size = 12, normalized size = 1.20

$$\begin{cases} \frac{\sinh(a+bx)}{b} & \text{for } b \neq 0 \\ x\cosh(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a),x)
```

[Out] Piecewise((sinh(a + b*x)/b, Ne(b, 0)), (x*cosh(a), True))

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(10) = 20$.
time = 0.41, size = 26, normalized size = 2.60

$$\frac{e^{(bx+a)}}{2\,b} - \frac{e^{(-bx-a)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*e^(b*x + a)/b - 1/2*e^(-b*x - a)/b
```

Mupad [B]
time = 0.05, size = 10, normalized size = 1.00

$$\frac{\sinh(a + b\,x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x),x)
```

```
[Out] sinh(a + b*x)/b
```

3.2 $\int \cosh^2(a+bx)\,dx$

Optimal. Leaf size=25

$$\frac{x}{2}+\frac{\cosh(a+bx)\sinh(a+bx)}{2b}$$

```
[Out] 1/2*x+1/2*cosh(b*x+a)*sinh(b*x+a)/b
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = {2715, 8}

$$\frac{\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^2,x]
```

```
[Out] x/2 + (Cosh[a + b*x]*Sinh[a + b*x])/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}\int \cosh^2(a+bx)\,dx &= \frac{\cosh(a+bx)\sinh(a+bx)}{2b}+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}+\frac{\cosh(a+bx)\sinh(a+bx)}{2b}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 23, normalized size = 0.92

$$\frac{2(a+bx)+\sinh(2(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2,x]
[Out] (2*(a + b*x) + Sinh[2*(a + b*x)])/(4*b)
```

Maple [A]
time = 0.64, size = 27, normalized size = 1.08

method	result	size
derivativedivides	$\frac{\frac{\cosh(bx+a)\sinh(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}}{b}$	27
default	$\frac{\frac{\cosh(bx+a)\sinh(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}}{b}$	27
risch	$\frac{x}{2}+\frac{e^{2bx+2a}}{8b}-\frac{e^{-2bx-2a}}{8b}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/2*cosh(b*x+a)*sinh(b*x+a)+1/2*b*x+1/2*a)
```

Maxima [A]
time = 0.27, size = 32, normalized size = 1.28

$$\frac{1}{2}\,x+\frac{e^{(2\,bx+2\,a)}}{8\,b}-\frac{e^{(-2\,bx-2\,a)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2,x, algorithm="maxima")
[Out] 1/2*x + 1/8*e^(2*b*x + 2*a)/b - 1/8*e^(-2*b*x - 2*a)/b
```

Fricas [A]
time = 0.43, size = 22, normalized size = 0.88

$$\frac{bx+\cosh\left(bx+a\right)\sinh\left(bx+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2,x, algorithm="fricas")
[Out] 1/2*(b*x + cosh(b*x + a)*sinh(b*x + a))/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(19) = 38$.
time = 0.08, size = 46, normalized size = 1.84

$$\begin{cases} -\frac{x\sinh^2\left(a+bx\right)}{2}+\frac{x\cosh^2\left(a+bx\right)}{2}+\frac{\sinh\left(a+bx\right)\cosh\left(a+bx\right)}{2b} & \text{for}\ b\neq 0\\ x\cosh^2\left(a\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2,x)
```

```
[Out] Piecewise((-x*sinh(a + b*x)**2/2 + x*cosh(a + b*x)**2/2 + sinh(a + b*x)*cos
h(a + b*x)/(2*b), Ne(b, 0)), (x*cosh(a)**2, True))
```

Giac [A]
time = 0.39, size = 32, normalized size = 1.28

$$\frac{1}{2}\,x + \frac{e^{(2\,bx+2\,a)}}{8\,b} - \frac{e^{(-2\,bx-2\,a)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*x + 1/8*e^(2*b*x + 2*a)/b - 1/8*e^(-2*b*x - 2*a)/b
```

Mupad [B]
time = 0.88, size = 18, normalized size = 0.72

$$\frac{x}{2} + \frac{\sinh(2\,a + 2\,b\,x)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2,x)
```

```
[Out] x/2 + sinh(2*a + 2*b*x)/(4*b)
```

3.3 $\int \cosh^3(a+bx)\,dx$

Optimal. Leaf size=26

$$\frac{\sinh(a+bx)}{b}+\frac{\sinh^3(a+bx)}{3b}$$

```
[Out] sinh(b*x+a)/b+1/3*sinh(b*x+a)^3/b
```

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used = {2713}

$$\frac{\sinh^3(a+bx)}{3b}+\frac{\sinh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^3,x]
```

```
[Out] Sinh[a + b*x]/b + Sinh[a + b*x]^3/(3*b)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cosh^3(a+bx)\,dx &= \frac{i\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-i\sinh(a+bx)\right)}{b}\\ &= \frac{\sinh(a+bx)}{b}+\frac{\sinh^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 1.00

$$\frac{\sinh(a+bx)}{b}+\frac{\sinh^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^3,x]
```

```
[Out] Sinh[a + b*x]/b + Sinh[a + b*x]^3/(3*b)
```

Maple [A]
time = 1.18, size = 27, normalized size = 1.04

method	result	size
default	$\frac{3\sinh(bx+a)}{4b}+\frac{\sinh(3bx+3a)}{12b}$	27
risch	$\frac{e^{3bx+3a}}{24b}+\frac{3\,e^{bx+a}}{8b}-\frac{3\,e^{-bx-a}}{8b}-\frac{e^{-3bx-3a}}{24b}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3/4*sinh(b*x+a)/b+1/12/b*sinh(3*b*x+3*a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(24) = 48.
time = 0.25, size = 54, normalized size = 2.08

$$\frac{e^{(3\,bx+3\,a)}}{24\,b}+\frac{3\,e^{(bx+a)}}{8\,b}-\frac{3\,e^{(-bx-a)}}{8\,b}-\frac{e^{(-3\,bx-3\,a)}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/24*e^(3*b*x + 3*a)/b + 3/8*e^(b*x + a)/b - 3/8*e^(-b*x - a)/b - 1/24*e^(-
3*b*x - 3*a)/b
```

Fricas [A]
time = 0.39, size = 32, normalized size = 1.23

$$\frac{\sinh\left(bx+a\right)^3+3\left(\cosh\left(bx+a\right)^2+3\right)\sinh\left(bx+a\right)}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(sinh(b*x + a)^3 + 3*(cosh(b*x + a)^2 + 3)*sinh(b*x + a))/b
```

Sympy [A]
time = 0.12, size = 36, normalized size = 1.38

$$\begin{cases} -\frac{2\sinh^3\left(a+bx\right)}{3b}+\frac{\sinh\left(a+bx\right)\cosh^2\left(a+bx\right)}{b} & \text{for } b\neq 0\\ x\cosh^3\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3,x)
```

[Out] Piecewise((-2*sinh(a + b*x)**3/(3*b) + sinh(a + b*x)*cosh(a + b*x)**2/b, Ne(b, 0)), (x*cosh(a)**3, True))

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(24) = 48$.
time = 0.42, size = 54, normalized size = 2.08

$$\frac{e^{(3\,bx+3\,a)}}{24\,b} + \frac{3\,e^{(bx+a)}}{8\,b} - \frac{3\,e^{(-bx-a)}}{8\,b} - \frac{e^{(-3\,bx-3\,a)}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b*x+a)^3,x, algorithm="giac")

[Out] 1/24*e^(3*b*x + 3*a)/b + 3/8*e^(b*x + a)/b - 3/8*e^(-b*x - a)/b - 1/24*e^(-3*b*x - 3*a)/b

Mupad [B]
time = 0.89, size = 22, normalized size = 0.85

$$\frac{\sinh(a + b\,x)^3 + 3\sinh(a + b\,x)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(a + b*x)^3,x)

[Out] (3*sinh(a + b*x) + sinh(a + b*x)^3)/(3*b)

3.4 $\int \cosh^4(a+bx)\,dx$

Optimal. Leaf size=46

$$\frac{3x}{8}+\frac{3\cosh(a+bx)\sinh(a+bx)}{8b}+\frac{\cosh^3(a+bx)\sinh(a+bx)}{4b}$$

```
[Out] 3/8*x+3/8*cosh(b*x+a)*sinh(b*x+a)/b+1/4*cosh(b*x+a)^3*sinh(b*x+a)/b
```

Rubi [A]
time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2715, 8}

$$\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3\sinh(a+bx)\cosh(a+bx)}{8b}+\frac{3x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^4,x]
```

```
[Out] (3*x)/8 + (3*Cosh[a + b*x]*Sinh[a + b*x])/(8*b) + (Cosh[a + b*x]^3*Sinh[a + b*x])/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}\int \cosh^4(a+bx)\,dx &= \frac{\cosh^3(a+bx)\sinh(a+bx)}{4b}+\frac{3}{4}\int \cosh^2(a+bx)\,dx\\ &= \frac{3\cosh(a+bx)\sinh(a+bx)}{8b}+\frac{\cosh^3(a+bx)\sinh(a+bx)}{4b}+\frac{3\int 1\,dx}{8}\\ &= \frac{3x}{8}+\frac{3\cosh(a+bx)\sinh(a+bx)}{8b}+\frac{\cosh^3(a+bx)\sinh(a+bx)}{4b}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 33, normalized size = 0.72

$$\frac{12(a+bx)+8\sinh(2(a+bx))+\sinh(4(a+bx))}{32b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^4,x]
```

```
[Out] (12*(a + b*x) + 8*Sinh[2*(a + b*x)] + Sinh[4*(a + b*x)])/(32*b)
```

Maple [A]
time = 1.25, size = 33, normalized size = 0.72

method	result	size
default	$\frac{3x}{8}+\frac{\sinh(2bx+2a)}{4b}+\frac{\sinh(4bx+4a)}{32b}$	33
risch	$\frac{3x}{8}+\frac{e^{4bx+4a}}{64b}+\frac{e^{2bx+2a}}{8b}-\frac{e^{-2bx-2a}}{8b}-\frac{e^{-4bx-4a}}{64b}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*x+1/4/b*sinh(2*b*x+2*a)+1/32/b*sinh(4*b*x+4*a)
```

Maxima [A]
time = 0.26, size = 60, normalized size = 1.30

$$\frac{3}{8}x+\frac{e^{(4\,bx+4\,a)}}{64\,b}+\frac{e^{(2\,bx+2\,a)}}{8\,b}-\frac{e^{(-2\,bx-2\,a)}}{8\,b}-\frac{e^{(-4\,bx-4\,a)}}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 3/8*x + 1/64*e^(4*b*x + 4*a)/b + 1/8*e^(2*b*x + 2*a)/b - 1/8*e^(-2*b*x - 2*
a)/b - 1/64*e^(-4*b*x - 4*a)/b
```

Fricas [A]
time = 0.46, size = 49, normalized size = 1.07

$$\frac{\cosh(bx+a)\sinh(bx+a)^3+3\,bx+\left(\cosh(bx+a)^3+4\,\cosh(bx+a)\right)\sinh(bx+a)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/8*(cosh(b*x + a)*sinh(b*x + a)^3 + 3*b*x + (cosh(b*x + a)^3 + 4*cosh(b*x
+ a))*sinh(b*x + a))/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(41) = 82$.
time = 0.18, size = 95, normalized size = 2.07

$$\begin{cases} \frac{3x\sinh^4(a+bx)}{8} - \frac{3x\sinh^2(a+bx)\cosh^2(a+bx)}{4} + \frac{3x\cosh^4(a+bx)}{8} - \frac{3\sinh^3(a+bx)\cosh(a+bx)}{8b} + \frac{5\sinh(a+bx)\cosh^3(a+bx)}{8b} & \text{for } b \neq 0 \\ x\cosh^4(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**4,x)
```

```
[Out] Piecewise((3*x*sinh(a + b*x)**4/8 - 3*x*sinh(a + b*x)**2*cosh(a + b*x)**2/4
 + 3*x*cosh(a + b*x)**4/8 - 3*sinh(a + b*x)**3*cosh(a + b*x)/(8*b) + 5*sinh
(a + b*x)*cosh(a + b*x)**3/(8*b), Ne(b, 0)), (x*cosh(a)**4, True))
```

Giac [A]
time = 0.41, size = 60, normalized size = 1.30

$$\frac{3}{8}x + \frac{e^{(4bx+4a)}}{64b} + \frac{e^{(2bx+2a)}}{8b} - \frac{e^{(-2bx-2a)}}{8b} - \frac{e^{(-4bx-4a)}}{64b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 3/8*x + 1/64*e^(4*b*x + 4*a)/b + 1/8*e^(2*b*x + 2*a)/b - 1/8*e^(-2*b*x - 2*
a)/b - 1/64*e^(-4*b*x - 4*a)/b
```

Mupad [B]
time = 0.08, size = 31, normalized size = 0.67

$$\frac{3x}{8} + \frac{\frac{\sinh(2a+2bx)}{4} + \frac{\sinh(4a+4bx)}{32}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^4,x)
```

```
[Out] (3*x)/8 + (sinh(2*a + 2*b*x)/4 + sinh(4*a + 4*b*x)/32)/b
```

3.5 $\int \cosh^5(a+bx)\,dx$

Optimal. Leaf size=41

$$\frac{\sinh(a+bx)}{b}+\frac{2\sinh^3(a+bx)}{3b}+\frac{\sinh^5(a+bx)}{5b}$$

```
[Out] sinh(b*x+a)/b+2/3*sinh(b*x+a)^3/b+1/5*sinh(b*x+a)^5/b
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = $\{2713\}$

$$\frac{\sinh^5(a+bx)}{5b}+\frac{2\sinh^3(a+bx)}{3b}+\frac{\sinh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^5,x]
```

```
[Out] Sinh[a + b*x]/b + (2*Sinh[a + b*x]^3)/(3*b) + Sinh[a + b*x]^5/(5*b)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cosh^5(a+bx)\,dx &= \frac{i\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,-i\sinh(a+bx)\right)}{b}\\ &= \frac{\sinh(a+bx)}{b}+\frac{2\sinh^3(a+bx)}{3b}+\frac{\sinh^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 44, normalized size = 1.07

$$\frac{5\sinh(a+bx)}{8b}+\frac{5\sinh(3(a+bx))}{48b}+\frac{\sinh(5(a+bx))}{80b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^5,x]
```

[Out] (5*Sinh[a + b*x])/(8*b) + (5*Sinh[3*(a + b*x)])/(48*b) + Sinh[5*(a + b*x)]/(80*b)

Maple [A]
time = 1.20, size = 41, normalized size = 1.00

method	result	size
default	$\frac{5\sinh(bx+a)}{8b}+\frac{5\sinh(3bx+3a)}{48b}+\frac{\sinh(5bx+5a)}{80b}$	41
risch	$\frac{e^{5bx+5a}}{160b}+\frac{5\,e^{3bx+3a}}{96b}+\frac{5\,e^{bx+a}}{16b}-\frac{5\,e^{-bx-a}}{16b}-\frac{5\,e^{-3bx-3a}}{96b}-\frac{e^{-5bx-5a}}{160b}$	83

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(b*x+a)^5,x,method=_RETURNVERBOSE)

[Out] 5/8*sinh(b*x+a)/b+5/48/b*sinh(3*b*x+3*a)+1/80/b*sinh(5*b*x+5*a)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(37) = 74$.
time = 0.25, size = 82, normalized size = 2.00

$$\frac{e^{(5\,bx+5\,a)}}{160\,b}+\frac{5\,e^{(3\,bx+3\,a)}}{96\,b}+\frac{5\,e^{(bx+a)}}{16\,b}-\frac{5\,e^{(-bx-a)}}{16\,b}-\frac{5\,e^{(-3\,bx-3\,a)}}{96\,b}-\frac{e^{(-5\,bx-5\,a)}}{160\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b*x+a)^5,x, algorithm="maxima")

[Out] 1/160*e^(5*b*x + 5*a)/b + 5/96*e^(3*b*x + 3*a)/b + 5/16*e^(b*x + a)/b - 5/16*e^(-b*x - a)/b - 5/96*e^(-3*b*x - 3*a)/b - 1/160*e^(-5*b*x - 5*a)/b

Fricas [A]
time = 0.37, size = 66, normalized size = 1.61

$$\frac{3\,\sinh\left(bx+a\right)^{5}+5\left(6\,\cosh\left(bx+a\right)^{2}+5\right)\sinh\left(bx+a\right)^{3}+15\left(\cosh\left(bx+a\right)^{4}+5\,\cosh\left(bx+a\right)^{2}+10\right)\sinh\left(bx+a\right)}{240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b*x+a)^5,x, algorithm="fricas")

[Out] 1/240*(3*sinh(b*x + a)^5 + 5*(6*cosh(b*x + a)^2 + 5)*sinh(b*x + a)^3 + 15*(cosh(b*x + a)^4 + 5*cosh(b*x + a)^2 + 10)*sinh(b*x + a))/b

Sympy [A]
time = 0.27, size = 58, normalized size = 1.41

$$\begin{cases} \frac{8\sinh^{5}\left(a+bx\right)}{15b}-\frac{4\sinh^{3}\left(a+bx\right)\cosh^{2}\left(a+bx\right)}{3b}+\frac{\sinh\left(a+bx\right)\cosh^{4}\left(a+bx\right)}{b} & \text{for}\: b\neq 0\\ x\cosh^{5}\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**5,x)
```

```
[Out] Piecewise((8*sinh(a + b*x)**5/(15*b) - 4*sinh(a + b*x)**3*cosh(a + b*x)**2/
(3*b) + sinh(a + b*x)*cosh(a + b*x)**4/b, Ne(b, 0)), (x*cosh(a)**5, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(37) = 74$.
time = 0.42, size = 82, normalized size = 2.00

$$\frac{e^{(5\,bx+5\,a)}}{160\,b} + \frac{5\,e^{(3\,bx+3\,a)}}{96\,b} + \frac{5\,e^{(bx+a)}}{16\,b} - \frac{5\,e^{(-bx-a)}}{16\,b} - \frac{5\,e^{(-3\,bx-3\,a)}}{96\,b} - \frac{e^{(-5\,bx-5\,a)}}{160\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/160*e^(5*b*x + 5*a)/b + 5/96*e^(3*b*x + 3*a)/b + 5/16*e^(b*x + a)/b - 5/1
6*e^(-b*x - a)/b - 5/96*e^(-3*b*x - 3*a)/b - 1/160*e^(-5*b*x - 5*a)/b
```

Mupad [B]
time = 0.92, size = 31, normalized size = 0.76

$$\frac{\frac{\sinh(a+b\,x)^5}{5} + \frac{2\,\sinh(a+b\,x)^3}{3} + \sinh(a+b\,x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^5,x)
```

```
[Out] (sinh(a + b*x) + (2*sinh(a + b*x)^3)/3 + sinh(a + b*x)^5/5)/b
```

3.6 $\int \cosh^6(a+bx)\,dx$

Optimal. Leaf size=67

$$\frac{5x}{16}+\frac{5\cosh(a+bx)\sinh(a+bx)}{16b}+\frac{5\cosh^3(a+bx)\sinh(a+bx)}{24b}+\frac{\cosh^5(a+bx)\sinh(a+bx)}{6b}$$

```
[Out] 5/16*x+5/16*cosh(b*x+a)*sinh(b*x+a)/b+5/24*cosh(b*x+a)^3*sinh(b*x+a)/b+1/6*
cosh(b*x+a)^5*sinh(b*x+a)/b
```

Rubi [A]
time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2715, 8}

$$\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}+\frac{5\sinh(a+bx)\cosh^3(a+bx)}{24b}+\frac{5\sinh(a+bx)\cosh(a+bx)}{16b}+\frac{5x}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^6,x]
```

```
[Out] (5*x)/16 + (5*Cosh[a + b*x]*Sinh[a + b*x])/(16*b) + (5*Cosh[a + b*x]^3*Sinh
[a + b*x])/(24*b) + (Cosh[a + b*x]^5*Sinh[a + b*x])/(6*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}\int \cosh^6(a+bx)\,dx &= \frac{\cosh^5(a+bx)\sinh(a+bx)}{6b}+\frac{5}{6}\int \cosh^4(a+bx)\,dx\\ &= \frac{5\cosh^3(a+bx)\sinh(a+bx)}{24b}+\frac{\cosh^5(a+bx)\sinh(a+bx)}{6b}+\frac{5}{8}\int \cosh^2(a+bx)\,dx\\ &= \frac{5\cosh(a+bx)\sinh(a+bx)}{16b}+\frac{5\cosh^3(a+bx)\sinh(a+bx)}{24b}+\frac{\cosh^5(a+bx)\sinh(a+bx)}{6b}\\ &= \frac{5x}{16}+\frac{5\cosh(a+bx)\sinh(a+bx)}{16b}+\frac{5\cosh^3(a+bx)\sinh(a+bx)}{24b}+\frac{\cosh^5(a+bx)\sinh(a+bx)}{6b}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 43, normalized size = 0.64

$$\frac{60a+60bx+45\sinh(2(a+bx))+9\sinh(4(a+bx))+\sinh(6(a+bx))}{192b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^6,x]
```

```
[Out] (60*a + 60*b*x + 45*Sinh[2*(a + b*x)] + 9*Sinh[4*(a + b*x)] + Sinh[6*(a + b
*x)])/(192*b)
```

Maple [A]
time = 1.28, size = 47, normalized size = 0.70

method	result	size
default	$\frac{5x}{16}+\frac{15\sinh(2bx+2a)}{64b}+\frac{3\sinh(4bx+4a)}{64b}+\frac{\sinh(6bx+6a)}{192b}$	47
risch	$\frac{5x}{16}+\frac{e^{6bx+6a}}{384b}+\frac{3\,e^{4bx+4a}}{128b}+\frac{15\,e^{2bx+2a}}{128b}-\frac{15\,e^{-2bx-2a}}{128b}-\frac{3\,e^{-4bx-4a}}{128b}-\frac{e^{-6bx-6a}}{384b}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 5/16*x+15/64/b*sinh(2*b*x+2*a)+3/64/b*sinh(4*b*x+4*a)+1/192/b*sinh(6*b*x+6*
a)
```

Maxima [A]
time = 0.26, size = 86, normalized size = 1.28

$$\frac{\left(9\,e^{(-2\,bx-2\,a)}+45\,e^{(-4\,bx-4\,a)}+1\right)e^{(6\,bx+6\,a)}}{384\,b}+\frac{5\,(bx+a)}{16\,b}-\frac{45\,e^{(-2\,bx-2\,a)}+9\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] 1/384*(9*e^(-2*b*x - 2*a) + 45*e^(-4*b*x - 4*a) + 1)*e^(6*b*x + 6*a)/b + 5/
16*(b*x + a)/b - 1/384*(45*e^(-2*b*x - 2*a) + 9*e^(-4*b*x - 4*a) + e^(-6*b*
x - 6*a))/b
```

Fricas [A]
time = 0.42, size = 90, normalized size = 1.34

$$\frac{3\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{5}+2\,\left(5\,\cosh\left(bx+a\right)^{3}+9\,\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right)^{3}+30\,bx+3\,\left(\cosh\left(bx+a\right)^{5}+6\,\cosh\left(bx+a\right)^{3}+15\,\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right)}{96\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] 1/96*(3*cosh(b*x + a)*sinh(b*x + a)^5 + 2*(5*cosh(b*x + a)^3 + 9*cosh(b*x +
 a))*sinh(b*x + a)^3 + 30*b*x + 3*(cosh(b*x + a)^5 + 6*cosh(b*x + a)^3 + 15
*cosh(b*x + a))*sinh(b*x + a))/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(61) = 122$.
time = 0.42, size = 139, normalized size = 2.07

$$\begin{cases} -\frac{5x\sinh^6(a+bx)}{16} + \frac{15x\sinh^4(a+bx)\cosh^2(a+bx)}{16} - \frac{15x\sinh^2(a+bx)\cosh^4(a+bx)}{16} + \frac{5x\cosh^6(a+bx)}{16} + \frac{5\sinh^5(a+bx)\cosh(a+bx)}{16b} - \frac{5\sinh^3(a+bx)\cosh^3(a+bx)}{6b} + \frac{11\sinh(a+bx)\cosh^5(a+bx)}{16b} & \text{for } b \neq 0 \\ x\cosh^6(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**6,x)
```

```
[Out] Piecewise((-5*x*sinh(a + b*x)**6/16 + 15*x*sinh(a + b*x)**4*cosh(a + b*x)**
2/16 - 15*x*sinh(a + b*x)**2*cosh(a + b*x)**4/16 + 5*x*cosh(a + b*x)**6/16
+ 5*sinh(a + b*x)**5*cosh(a + b*x)/(16*b) - 5*sinh(a + b*x)**3*cosh(a + b*x
)**3/(6*b) + 11*sinh(a + b*x)*cosh(a + b*x)**5/(16*b), Ne(b, 0)), (x*cosh(a
)**6, True))
```

Giac [A]
time = 0.42, size = 88, normalized size = 1.31

$$\frac{5}{16}\,x + \frac{e^{(6\,bx+6\,a)}}{384\,b} + \frac{3\,e^{(4\,bx+4\,a)}}{128\,b} + \frac{15\,e^{(2\,bx+2\,a)}}{128\,b} - \frac{15\,e^{(-2\,bx-2\,a)}}{128\,b} - \frac{3\,e^{(-4\,bx-4\,a)}}{128\,b} - \frac{e^{(-6\,bx-6\,a)}}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^6,x, algorithm="giac")
```

```
[Out] 5/16*x + 1/384*e^(6*b*x + 6*a)/b + 3/128*e^(4*b*x + 4*a)/b + 15/128*e^(2*b*
x + 2*a)/b - 15/128*e^(-2*b*x - 2*a)/b - 3/128*e^(-4*b*x - 4*a)/b - 1/384*e
^(-6*b*x - 6*a)/b
```

Mupad [B]
time = 0.97, size = 42, normalized size = 0.63

$$\frac{5\,x}{16} + \frac{\frac{15\,\sinh(2\,a+2\,b\,x)}{64} + \frac{3\,\sinh(4\,a+4\,b\,x)}{64} + \frac{\sinh(6\,a+6\,b\,x)}{192}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^6,x)
```

```
[Out] (5*x)/16 + ((15*sinh(2*a + 2*b*x))/64 + (3*sinh(4*a + 4*b*x))/64 + sinh(6*a
 + 6*b*x)/192)/b
```

3.7 $\int \cosh^{\frac{7}{2}}(a+bx)\,dx$

Optimal. Leaf size=69

$$-\frac{10iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{21b}+\frac{10\sqrt{\cosh(a+bx)}\,\sinh(a+bx)}{21b}+\frac{2\cosh^{\frac{5}{2}}(a+bx)\sinh(a+bx)}{7b}$$

```
[Out] -10/21*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticF(I*sinh
(1/2*a+1/2*b*x),2^(1/2))/b+2/7*cosh(b*x+a)^(5/2)*sinh(b*x+a)/b+10/21*sinh(b
*x+a)*cosh(b*x+a)^(1/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2715, 2720}

$$-\frac{10iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{21b}+\frac{2\sinh(a+bx)\cosh^{\frac{5}{2}}(a+bx)}{7b}+\frac{10\sinh(a+bx)\sqrt{\cosh(a+bx)}}{21b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^(7/2),x]
```

```
[Out] (((-10*I)/21)*EllipticF[(I/2)*(a + b*x), 2])/b + (10*Sqrt[Cosh[a + b*x]]*Si
nh[a + b*x])/(21*b) + (2*Cosh[a + b*x]^(5/2)*Sinh[a + b*x])/(7*b)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cosh^{\frac{7}{2}}(a+bx)\,dx &= \frac{2\cosh^{\frac{5}{2}}(a+bx)\sinh(a+bx)}{7b}+\frac{5}{7}\int \cosh^{\frac{3}{2}}(a+bx)\,dx\\ &= \frac{10\sqrt{\cosh(a+bx)}\,\sinh(a+bx)}{21b}+\frac{2\cosh^{\frac{5}{2}}(a+bx)\sinh(a+bx)}{7b}+\frac{5}{21}\int \frac{1}{\sqrt{\cosh(a+b}}\\ &= -\frac{10iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{21b}+\frac{10\sqrt{\cosh(a+bx)}\,\sinh(a+bx)}{21b}+\frac{2\cosh^{\frac{5}{2}}(a+bx)\sinh(a+}{7b}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 55, normalized size = 0.80

$$\frac{-20iF\left(\frac{1}{2}i(a+bx)\middle|2\right)+\sqrt{\cosh(a+bx)}\,(23\sinh(a+bx)+3\sinh(3(a+bx)))}{42b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^(7/2),x]
```

```
[Out] ((-20*I)*EllipticF[(I/2)*(a + b*x), 2] + Sqrt[Cosh[a + b*x]]*(23*Sinh[a + b
*x] + 3*Sinh[3*(a + b*x)]))/(42*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(85) = 170$.
time = 1.19, size = 201, normalized size = 2.91

method	result
default	$\frac{2\sqrt{\left(2\left(\cosh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,\left(48\left(\cosh^9\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-120\left(\cosh^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+128\left(\cosh^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\right.}{21\sqrt{2\left(\sinh^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sinh^2\left(\frac{bx}{2}\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21*((2*cosh(1/2*b*x+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(48*cosh(1/2
*b*x+1/2*a)^9-120*cosh(1/2*b*x+1/2*a)^7+128*cosh(1/2*b*x+1/2*a)^5-72*cosh(1
/2*b*x+1/2*a)^3+5*(-sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-2*cosh(1/2*b*x+1/2*a)^2+
1)^(1/2)*EllipticF(cosh(1/2*b*x+1/2*a),2^(1/2))+16*cosh(1/2*b*x+1/2*a))/(2*
sinh(1/2*b*x+1/2*a)^4+sinh(1/2*b*x+1/2*a)^2)^(1/2)/sinh(1/2*b*x+1/2*a)/(2*c
osh(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(b*x + a)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 326, normalized size = 4.72

$$\frac{40\left(\sqrt{2}\cosh(bx+a)^3+3\sqrt{2}\cosh(bx+a)^2\sinh(bx+a)+3\sqrt{2}\cosh(bx+a)\sinh(bx+a)^2+\sqrt{2}\sinh(bx+a)^3\right)\mathrm{weierstrassPInverse}(-4,0,\cosh(bx+a)+\sinh(bx+a))+\left(3\cosh(bx+a)^6+18\cosh(bx+a)\sinh(bx+a)^5+3\sinh(bx+a)^6+(45\cosh(bx+a)^2+23)\sinh(bx+a)^4+23\cosh(bx+a)^4+4(15\cosh(bx+a)^3+23\cosh(bx+a))\sinh(bx+a)^3+(45\cosh(bx+a)^4+138\cosh(bx+a)^2-23)\sinh(bx+a)^2-23\cosh(bx+a)^2+2(9\cosh(bx+a)^5+46\cosh(bx+a)^3-23\cosh(bx+a))\sinh(bx+a)-3\right)\sqrt{\cosh(bx+a)}}{84\left(b\cosh(bx+a)^3+3b\cosh(bx+a)^2\sinh(bx+a)+3b\cosh(bx+a)\sinh(bx+a)^2+b\sinh(bx+a)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/84*(40*(sqrt(2)*cosh(b*x + a)^3 + 3*sqrt(2)*cosh(b*x + a)^2*sinh(b*x + a)
 + 3*sqrt(2)*cosh(b*x + a)*sinh(b*x + a)^2 + sqrt(2)*sinh(b*x + a)^3)*weier
strassPInverse(-4, 0, cosh(b*x + a) + sinh(b*x + a)) + (3*cosh(b*x + a)^6 +
 18*cosh(b*x + a)*sinh(b*x + a)^5 + 3*sinh(b*x + a)^6 + (45*cosh(b*x + a)^2
 + 23)*sinh(b*x + a)^4 + 23*cosh(b*x + a)^4 + 4*(15*cosh(b*x + a)^3 + 23*co
sh(b*x + a))*sinh(b*x + a)^3 + (45*cosh(b*x + a)^4 + 138*cosh(b*x + a)^2 -
23)*sinh(b*x + a)^2 - 23*cosh(b*x + a)^2 + 2*(9*cosh(b*x + a)^5 + 46*cosh(b
*x + a)^3 - 23*cosh(b*x + a))*sinh(b*x + a) - 3)*sqrt(cosh(b*x + a)))/(b*co
sh(b*x + a)^3 + 3*b*cosh(b*x + a)^2*sinh(b*x + a) + 3*b*cosh(b*x + a)*sinh(
b*x + a)^2 + b*sinh(b*x + a)^3)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3060 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cosh(a+b\,x)^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^(7/2),x)
```

```
[Out] int(cosh(a + b*x)^(7/2), x)
```

3.8 $\int \cosh^{\frac{5}{2}}(a+bx)\,dx$

Optimal. Leaf size=46

$$-\frac{6iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{5b}+\frac{2\cosh^{\frac{3}{2}}(a+bx)\sinh(a+bx)}{5b}$$

```
[Out] -6/5*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticE(I*sinh(1
/2*a+1/2*b*x),2^(1/2))/b+2/5*cosh(b*x+a)^(3/2)*sinh(b*x+a)/b
```

Rubi [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2715, 2719}

$$\frac{2\sinh(a+bx)\cosh^{\frac{3}{2}}(a+bx)}{5b}-\frac{6iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^(5/2),x]
```

```
[Out] (((-6*I)/5)*EllipticE[(I/2)*(a + b*x), 2])/b + (2*Cosh[a + b*x]^(3/2)*Sinh[
a + b*x])/(5*b)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cosh^{\frac{5}{2}}(a+bx)\,dx &= \frac{2\cosh^{\frac{3}{2}}(a+bx)\sinh(a+bx)}{5b}+\frac{3}{5}\int \sqrt{\cosh(a+bx)}\,dx\\ &= -\frac{6iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{5b}+\frac{2\cosh^{\frac{3}{2}}(a+bx)\sinh(a+bx)}{5b}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 44, normalized size = 0.96

$$\frac{-6iE\left(\frac{1}{2}i(a+bx)\middle|2\right)+\sqrt{\cosh(a+bx)}\,\sinh(2(a+bx))}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^(5/2),x]
```

```
[Out] ((-6*I)*EllipticE[(I/2)*(a + b*x), 2] + Sqrt[Cosh[a + b*x]]*Sinh[2*(a + b*x
)])/(5*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(66) = 132$.
time = 1.02, size = 188, normalized size = 4.09

method	result
default	$\frac{2\sqrt{\left(2\left(\cosh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\left(8\left(\cosh^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-16\left(\cosh^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+10\left(\cosh^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-3\sqrt{\ }\right.}{5\sqrt{2\left(\sinh^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)}\,\sinh(} $

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*((2*cosh(1/2*b*x+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(8*cosh(1/2*b
*x+1/2*a)^7-16*cosh(1/2*b*x+1/2*a)^5+10*cosh(1/2*b*x+1/2*a)^3-3*(-sinh(1/2*
b*x+1/2*a)^2)^(1/2)*(-2*cosh(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticE(cosh(1/2*b
*x+1/2*a),2^(1/2))-2*cosh(1/2*b*x+1/2*a))/(2*sinh(1/2*b*x+1/2*a)^4+sinh(1/2
*b*x+1/2*a)^2)^(1/2)/sinh(1/2*b*x+1/2*a)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(1/2)/
b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(b*x + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 203, normalized size = 4.41

$$-\frac{12\left(\sqrt{2}\cosh(bx+a)^2+2\sqrt{2}\cosh(bx+a)\sinh(bx+a)+\sqrt{2}\sinh(bx+a)^2\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cosh(bx+a)+\sinh(bx+a)))-\left(\cosh(bx+a)^4+4\cosh(bx+a)\sinh(bx+a)^3+\sinh(bx+a)^4+6\left(\cosh(bx+a)^2-2\right)\sinh(bx+a)^2-12\cosh(bx+a)^2+4\left(\cosh(bx+a)^3-6\cosh(bx+a)\right)\sinh(bx+a)-1\right)\sqrt{\cosh(bx+a)}}{10\left(b\cosh(bx+a)^2+2b\cosh(bx+a)\sinh(bx+a)+b\sinh(bx+a)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/10*(12*(sqrt(2)*cosh(b*x + a)^2 + 2*sqrt(2)*cosh(b*x + a)*sinh(b*x + a)
+ sqrt(2)*sinh(b*x + a)^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cosh(b*x + a) + sinh(b*x + a))) - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh
(b*x + a)^3 + sinh(b*x + a)^4 + 6*(cosh(b*x + a)^2 - 2)*sinh(b*x + a)^2 - 1
2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - 6*cosh(b*x + a))*sinh(b*x + a) - 1
)*sqrt(cosh(b*x + a)))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a)
 + b*sinh(b*x + a)^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cosh(a+b\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^(5/2),x)
```

```
[Out] int(cosh(a + b*x)^(5/2), x)
```

3.9 $\int \cosh^{\frac{3}{2}}(a+bx)\,dx$

Optimal. Leaf size=46

$$-\frac{2iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{3b}+\frac{2\sqrt{\cosh(a+bx)}\,\sinh(a+bx)}{3b}$$

```
[Out] -2/3*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticF(I*sinh(1
/2*a+1/2*b*x),2^(1/2))/b+2/3*sinh(b*x+a)*cosh(b*x+a)^(1/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2715, 2720}

$$\frac{2\sinh(a+bx)\sqrt{\cosh(a+bx)}}{3b}-\frac{2iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^(3/2),x]
```

```
[Out] (((-2*I)/3)*EllipticF[(I/2)*(a + b*x), 2])/b + (2*Sqrt[Cosh[a + b*x]]*Sinh[
a + b*x])/(3*b)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cosh^{\frac{3}{2}}(a+bx)\,dx &= \frac{2\sqrt{\cosh(a+bx)}\,\sinh(a+bx)}{3b}+\frac{1}{3}\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx\\ &= -\frac{2iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{3b}+\frac{2\sqrt{\cosh(a+bx)}\,\sinh(a+bx)}{3b}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.08, size = 81, normalized size = 1.76

$$\frac{\sinh(2(a+bx))+2\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\cosh(2(a+bx))-\sinh(2(a+bx))\right)\sqrt{1+\cosh(2(a+bx))+\sinh(2(a+bx))}}{3b\sqrt{\cosh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^(3/2),x]
```

```
[Out] (Sinh[2*(a + b*x)] + 2*Hypergeometric2F1[1/4, 1/2, 5/4, -Cosh[2*(a + b*x)]
- Sinh[2*(a + b*x)]]*Sqrt[1 + Cosh[2*(a + b*x)] + Sinh[2*(a + b*x)]])/(3*b*
Sqrt[Cosh[a + b*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(66) = 132.
time = 1.12, size = 174, normalized size = 3.78

method	result
default	$\frac{2\sqrt{\left(2\left(\cosh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\left(4\left(\cosh^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-6\left(\cosh^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sqrt{-\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\right.}{3\sqrt{2\left(\sinh^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)}\,\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*((2*cosh(1/2*b*x+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(4*cosh(1/2*b
*x+1/2*a)^5-6*cosh(1/2*b*x+1/2*a)^3+(-sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-2*cosh
(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticF(cosh(1/2*b*x+1/2*a),2^(1/2))+2*cosh(1/
2*b*x+1/2*a))/(2*sinh(1/2*b*x+1/2*a)^4+sinh(1/2*b*x+1/2*a)^2)^(1/2)/sinh(1/
2*b*x+1/2*a)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(b*x + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.16, size = 102, normalized size = 2.22

$$\frac{2\left(\sqrt{2}\cosh(bx+a)+\sqrt{2}\sinh(bx+a)\right)\mathrm{weierstrassPInverse}(-4,0,\cosh(bx+a)+\sinh(bx+a))+\left(\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2-1\right)\sqrt{\cosh(bx+a)}}{3\left(b\cosh(bx+a)+b\sinh(bx+a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*(sqrt(2)*cosh(b*x + a) + sqrt(2)*sinh(b*x + a))*weierstrassPInverse(
-4, 0, cosh(b*x + a) + sinh(b*x + a)) + (cosh(b*x + a)^2 + 2*cosh(b*x + a)*
sinh(b*x + a) + sinh(b*x + a)^2 - 1)*sqrt(cosh(b*x + a)))/(b*cosh(b*x + a)
+ b*sinh(b*x + a))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh^{\frac{3}{2}}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**(3/2),x)
```

```
[Out] Integral(cosh(a + b*x)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cosh(a+b\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^(3/2),x)
```

```
[Out] int(cosh(a + b*x)^(3/2), x)
```

3.10 $\int \sqrt{\cosh(a+bx)}\,dx$

Optimal. Leaf size=20

$$-\frac{2iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

```
[Out] -2*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticE(I*sinh(1/2
*a+1/2*b*x),2^(1/2))/b
```

Rubi [A]
time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{2719\}$

$$-\frac{2iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cosh[a + b*x]],x]
[Out] ((-2*I)*EllipticE[(I/2)*(a + b*x), 2])/b
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cosh(a+bx)}\,dx = -\frac{2iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

Mathematica [A]
time = 0.03, size = 20, normalized size = 1.00

$$-\frac{2iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cosh[a + b*x]],x]
[Out] ((-2*I)*EllipticE[(I/2)*(a + b*x), 2])/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(46) = 92$.
time = 1.04, size = 135, normalized size = 6.75

method	result
default	$-\frac{2\sqrt{\left(2\left(\cosh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,\sqrt{-\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,\sqrt{-2\left(\cosh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+1}}{\sqrt{2\left(\sinh^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)}\,\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{2\left(\cosh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-}}$
risch	$\frac{\sqrt{2}\,\sqrt{\left(e^{2bx+2a}+1\right)e^{-bx-a}}}{b}+\frac{\left(-\frac{2\left(e^{2bx+2a}+1\right)}{\sqrt{\left(e^{2bx+2a}+1\right)e^{bx+a}}}+\frac{i\sqrt{-i\left(e^{bx+a}+i\right)}\,\sqrt{2}\,\sqrt{i\left(e^{bx+a}-i\right)}\,\sqrt{i}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*((2*cosh(1/2*b*x+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-sinh(1/2*b*x
+1/2*a)^2)^(1/2)*(-2*cosh(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticE(cosh(1/2*b*x+
1/2*a),2^(1/2))/(2*sinh(1/2*b*x+1/2*a)^4+sinh(1/2*b*x+1/2*a)^2)^(1/2)/sinh(
1/2*b*x+1/2*a)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cosh(b*x + a)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 37, normalized size = 1.85

$$-\frac{2\left(\sqrt{2}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cosh(bx+a)+\sinh(bx+a)))+\sqrt{\cosh(bx+a)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(sqrt(2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cosh(b*x + a)
 + sinh(b*x + a))) + sqrt(cosh(b*x + a)))/b
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\cosh{(a + bx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**(1/2),x)
[Out] Integral(sqrt(cosh(a + b*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(cosh(b*x + a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.05

$$\int \sqrt{\cosh\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^(1/2),x)
[Out] int(cosh(a + b*x)^(1/2), x)
```

3.11 $\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx$

Optimal. Leaf size=20

$$-\frac{2iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

```
[Out] -2*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticF(I*sinh(1/2
*a+1/2*b*x),2^(1/2))/b
```

Rubi [A]
time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{2720\}$

$$-\frac{2iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Cosh[a + b*x]],x]
```

```
[Out] ((-2*I)*EllipticF[(I/2)*(a + b*x), 2])/b
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx = -\frac{2iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

Mathematica [A]
time = 0.03, size = 20, normalized size = 1.00

$$-\frac{2iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Cosh[a + b*x]],x]
```

```
[Out] ((-2*I)*EllipticF[(I/2)*(a + b*x), 2])/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(46) = 92$.
time = 0.86, size = 135, normalized size = 6.75

method	result
default	$\frac{2\sqrt{\left(2\left(\cosh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,\sqrt{-\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,\sqrt{-2\left(\cosh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+1}}{\sqrt{2\left(\sinh^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)}\,\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{2\left(\cosh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*((2*cosh(1/2*b*x+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-sinh(1/2*b*x+
1/2*a)^2)^(1/2)*(-2*cosh(1/2*b*x+1/2*a)^2+1)^(1/2)/(2*sinh(1/2*b*x+1/2*a)^4
+sinh(1/2*b*x+1/2*a)^2)^(1/2)*EllipticF(cosh(1/2*b*x+1/2*a),2^(1/2))/sinh(1
/2*b*x+1/2*a)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(cosh(b*x + a)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 24, normalized size = 1.20

$$\frac{2\sqrt{2}\,\mathrm{weierstrassPInverse}(-4,0,\cosh(bx+a)+\sinh(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*weierstrassPInverse(-4, 0, cosh(b*x + a) + sinh(b*x + a))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)**(1/2),x)
```

```
[Out] Integral(1/sqrt(cosh(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(cosh(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\sqrt{\cosh\left(a+b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(a + b*x)^(1/2),x)
```

```
[Out] int(1/cosh(a + b*x)^(1/2), x)
```

3.12 $\int \frac{1}{\cosh^{\frac{3}{2}}(a+bx)}\,dx$

Optimal. Leaf size=42

$$\frac{2iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}+\frac{2\sinh(a+bx)}{b\sqrt{\cosh(a+bx)}}$$

```
[Out] 2*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticE(I*sinh(1/2*
a+1/2*b*x),2^(1/2))/b+2*sinh(b*x+a)/b/cosh(b*x+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2716, 2719}

$$\frac{2\sinh(a+bx)}{b\sqrt{\cosh(a+bx)}}+\frac{2iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^(-3/2),x]
```

```
[Out] ((2*I)*EllipticE[(I/2)*(a + b*x), 2])/b + (2*Sinh[a + b*x])/(b*Sqrt[Cosh[a
+ b*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\cosh^{\frac{3}{2}}(a+bx)}\,dx &= \frac{2\sinh(a+bx)}{b\sqrt{\cosh(a+bx)}}-\int \sqrt{\cosh(a+bx)}\,dx\\ &= \frac{2iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}+\frac{2\sinh(a+bx)}{b\sqrt{\cosh(a+bx)}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 42, normalized size = 1.00

$$\frac{2iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}+\frac{2\sinh(a+bx)}{b\sqrt{\cosh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^(-3/2),x]
```

```
[Out] ((2*I)*EllipticE[(I/2)*(a + b*x), 2])/b + (2*Sinh[a + b*x])/(b*Sqrt[Cosh[a
+ b*x]])
```

Maple [A]
time = 1.18, size = 103, normalized size = 2.45

method	result
default	$\frac{4\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+2\sqrt{-2\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\ \mathrm{EllipticE}\left(\cosh\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)\sqrt{-\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right.}}{\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{2\left(\cosh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\ b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(2*cosh(1/2*b*x+1/2*a)*sinh(1/2*b*x+1/2*a)^2+(-2*sinh(1/2*b*x+1/2*a)^2-1)
^(1/2)*EllipticE(cosh(1/2*b*x+1/2*a),2^(1/2))*(-sinh(1/2*b*x+1/2*a)^2)^(1/2
))/sinh(1/2*b*x+1/2*a)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(b*x + a)^(-3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 148, normalized size = 3.52

$$\frac{2\left(\left(\sqrt{2}\cosh(bx+a)^2+2\sqrt{2}\cosh(bx+a)\sinh(bx+a)+\sqrt{2}\sinh(bx+a)^2+\sqrt{2}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cosh(bx+a)+\sinh(bx+a)))+2\left(\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2\right)\sqrt{\cosh(bx+a)}\right)}{b\cosh(bx+a)^2+2b\cosh(bx+a)\sinh(bx+a)+b\sinh(bx+a)^2+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*((sqrt(2)*cosh(b*x + a)^2 + 2*sqrt(2)*cosh(b*x + a)*sinh(b*x + a) + sqrt(
2)*sinh(b*x + a)^2 + sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cosh(b*x + a) + sinh(b*x + a))) + 2*(cosh(b*x + a)^2 + 2*cosh(b*x + a)
*sinh(b*x + a) + sinh(b*x + a)^2)*sqrt(cosh(b*x + a)))/(b*cosh(b*x + a)^2 +
 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 + b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\cosh^{\frac{3}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)**(3/2),x)
```

```
[Out] Integral(cosh(a + b*x)**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\mathrm{cosh}\left(a+b\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(a + b*x)^(3/2),x)
```

```
[Out] int(1/cosh(a + b*x)^(3/2), x)
```

3.13 $\int \frac{1}{\cosh^{\frac{5}{2}}(a+bx)}\,dx$

Optimal. Leaf size=46

$$-\frac{2iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{3b}+\frac{2\sinh(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}$$

```
[Out] -2/3*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticF(I*sinh(1
/2*a+1/2*b*x),2^(1/2))/b+2/3*sinh(b*x+a)/b/cosh(b*x+a)^(3/2)
```

Rubi [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2716, 2720}

$$\frac{2\sinh(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}-\frac{2iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^(-5/2),x]
```

```
[Out] (((-2*I)/3)*EllipticF[(I/2)*(a + b*x), 2])/b + (2*Sinh[a + b*x])/(3*b*Cosh[
a + b*x]^(3/2))
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\cosh^{\frac{5}{2}}(a+bx)}\,dx&=\frac{2\sinh(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}+\frac{1}{3}\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx\\&=-\frac{2iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{3b}+\frac{2\sinh(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.05, size = 84, normalized size = 1.83

$$\frac{2\left(\sinh(a+bx)+\cosh(a+bx)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\cosh(2(a+bx))-\sinh(2(a+bx))\right)\sqrt{1+\cosh(2(a+bx))+\sinh(2(a+bx))}\right)}{3b\cosh^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^(-5/2),x]
```

```
[Out] (2*(Sinh[a + b*x] + Cosh[a + b*x]*Hypergeometric2F1[1/4, 1/2, 5/4, -Cosh[2*
(a + b*x)] - Sinh[2*(a + b*x)]]*Sqrt[1 + Cosh[2*(a + b*x)] + Sinh[2*(a + b*
x)]]))/(3*b*Cosh[a + b*x]^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(66) = 132$.
time = 1.07, size = 217, normalized size = 4.72

method	result
default	$\frac{2\left(2\sqrt{-\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\sqrt{-2\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\,\mathrm{EllipticF}\left(\cosh\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+2\cosh(}{3\sqrt{2\left(\sinh^4\left(\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(2*(-sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-2*sinh(1/2*b*x+1/2*a)^2-1)^(1/2)*El
lipticF(cosh(1/2*b*x+1/2*a),2^(1/2))*sinh(1/2*b*x+1/2*a)^2+2*cosh(1/2*b*x+1
/2*a)*sinh(1/2*b*x+1/2*a)^2+(-sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-2*sinh(1/2*b*x
+1/2*a)^2-1)^(1/2)*EllipticF(cosh(1/2*b*x+1/2*a),2^(1/2)))*((2*cosh(1/2*b*x
+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)/(2*sinh(1/2*b*x+1/2*a)^4+sinh(1/2
*b*x+1/2*a)^2)^(1/2)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(3/2)/sinh(1/2*b*x+1/2*a)/
b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(b*x + a)^(-5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 310, normalized size = 6.74

$$\frac{2\left(\left(\sqrt{2}\cosh\left(bx+a\right)^{4}+4\sqrt{2}\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3}+\sqrt{2}\sinh\left(bx+a\right)^{4}+2\left(3\sqrt{2}\cosh\left(bx+a\right)^{2}+\sqrt{2}\right)\sinh\left(bx+a\right)^{2}+2\sqrt{2}\cosh\left(bx+a\right)^{2}+4\left(\sqrt{2}\cosh\left(bx+a\right)^{3}+\sqrt{2}\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right)+\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cosh\left(bx+a\right)+\sinh\left(bx+a\right))+2\left(\cosh\left(bx+a\right)^{3}+3\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}+\sinh\left(bx+a\right)^{3}+\left(3\cosh\left(bx+a\right)^{2}-1\right)\sinh\left(bx+a\right)-\cosh\left(bx+a\right)\right)\sqrt{\cosh\left(bx+a\right)}\right)}{3\left(b\cosh\left(bx+a\right)^{4}+4b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3}+b\sinh\left(bx+a\right)^{4}+2b\cosh\left(bx+a\right)^{2}+2\left(3b\cosh\left(bx+a\right)^{2}+b\right)\sinh\left(bx+a\right)^{2}+4\left(b\cosh\left(bx+a\right)^{3}+b\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right)+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*((sqrt(2)*cosh(b*x + a)^4 + 4*sqrt(2)*cosh(b*x + a)*sinh(b*x + a)^3 + s
qrt(2)*sinh(b*x + a)^4 + 2*(3*sqrt(2)*cosh(b*x + a)^2 + sqrt(2))*sinh(b*x +
 a)^2 + 2*sqrt(2)*cosh(b*x + a)^2 + 4*(sqrt(2)*cosh(b*x + a)^3 + sqrt(2)*co
sh(b*x + a))*sinh(b*x + a) + sqrt(2))*weierstrassPInverse(-4, 0, cosh(b*x +
 a) + sinh(b*x + a)) + 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2
 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))
*sqrt(cosh(b*x + a)))/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^
3 + b*sinh(b*x + a)^4 + 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 + b)*s
inh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\cosh^{\frac{5}{2}}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)**(5/2),x)
```

```
[Out] Integral(cosh(a + b*x)**(-5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^(-5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\cosh\left(a+b\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(a + b*x)^(5/2),x)
```

```
[Out] int(1/cosh(a + b*x)^(5/2), x)
```

3.14 $\int \frac{1}{\cosh^{\frac{7}{2}}(a+bx)} dx$

Optimal. Leaf size=69

$$\frac{6iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{5b}+\frac{2\sinh(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}+\frac{6\sinh(a+bx)}{5b\sqrt{\cosh(a+bx)}}$$

```
[Out] 6/5*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticE(I*sinh(1/
2*a+1/2*b*x),2^(1/2))/b+2/5*sinh(b*x+a)/b/cosh(b*x+a)^(5/2)+6/5*sinh(b*x+a)
/b/cosh(b*x+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2716, 2719}

$$\frac{6iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{5b}+\frac{2\sinh(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}+\frac{6\sinh(a+bx)}{5b\sqrt{\cosh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^(-7/2),x]
```

```
[Out] (((6*I)/5)*EllipticE[(I/2)*(a + b*x), 2])/b + (2*Sinh[a + b*x])/(5*b*Cosh[a
 + b*x]^(5/2)) + (6*Sinh[a + b*x])/(5*b*Sqrt[Cosh[a + b*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\cosh^{\frac{7}{2}}(a+bx)}\,dx = \frac{2\sinh(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)} + \frac{3}{5}\int \frac{1}{\cosh^{\frac{3}{2}}(a+bx)}\,dx$$
$$= \frac{2\sinh(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)} + \frac{6\sinh(a+bx)}{5b\sqrt{\cosh(a+bx)}} - \frac{3}{5}\int \sqrt{\cosh(a+bx)}\,dx$$
$$= \frac{6iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{5b} + \frac{2\sinh(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)} + \frac{6\sinh(a+bx)}{5b\sqrt{\cosh(a+bx)}}$$

Mathematica [A]
time = 0.10, size = 63, normalized size = 0.91

$$\frac{6i\cosh^{\frac{3}{2}}(a+bx)E\left(\frac{1}{2}i(a+bx)\middle|2\right) + 3\sinh(2(a+bx)) + 2\tanh(a+bx)}{5b\cosh^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^(-7/2),x]
```

```
[Out] ((6*I)*Cosh[a + b*x]^(3/2)*EllipticE[(I/2)*(a + b*x), 2] + 3*Sinh[2*(a + b*
x)] + 2*Tanh[a + b*x])/(5*b*Cosh[a + b*x]^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 362 vs. 2(85) = 170.
time = 1.39, size = 363, normalized size = 5.26

method	result
default	$2\sqrt{\left(2\left(\cosh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\left(24\left(\sinh^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)+12\,\mathrm{EllipticE}\left(\cosh\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(b*x+a)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*((2*cosh(1/2*b*x+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)/(8*sinh(1/2*b
*x+1/2*a)^6+12*sinh(1/2*b*x+1/2*a)^4+6*sinh(1/2*b*x+1/2*a)^2+1)/sinh(1/2*b*
x+1/2*a)^3*(24*sinh(1/2*b*x+1/2*a)^6*cosh(1/2*b*x+1/2*a)+12*EllipticE(cosh(
1/2*b*x+1/2*a),2^(1/2))*(-2*sinh(1/2*b*x+1/2*a)^2-1)^(1/2)*(-sinh(1/2*b*x+1
/2*a)^2)^(1/2)*sinh(1/2*b*x+1/2*a)^4+24*sinh(1/2*b*x+1/2*a)^4*cosh(1/2*b*x+
1/2*a)+12*EllipticE(cosh(1/2*b*x+1/2*a),2^(1/2))*(-2*sinh(1/2*b*x+1/2*a)^2-
1)^(1/2)*(-sinh(1/2*b*x+1/2*a)^2)^(1/2)*sinh(1/2*b*x+1/2*a)^2+8*cosh(1/2*b*
x+1/2*a)*sinh(1/2*b*x+1/2*a)^2+3*(-2*sinh(1/2*b*x+1/2*a)^2-1)^(1/2)*Ellipti
cE(cosh(1/2*b*x+1/2*a),2^(1/2))*(-sinh(1/2*b*x+1/2*a)^2)^(1/2))*(2*sinh(1/2
```

```
*b*x+1/2*a)^4+sinh(1/2*b*x+1/2*a)^2)^(1/2)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(1/2
)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(b*x + a)^(-7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 613, normalized size = 8.88

$$\frac{2\left(3\left(\sqrt{2}\cosh\left(bx+a\right)^{6}+6\sqrt{2}\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{5}+\sqrt{2}\sinh\left(bx+a\right)^{6}+3\left(5\sqrt{2}\cosh\left(bx+a\right)^{2}+\sqrt{2}\right)\sinh\left(bx+a\right)^{4}+3\sqrt{2}\cosh\left(bx+a\right)^{4}+4\left(5\sqrt{2}\cosh\left(bx+a\right)^{3}+3\sqrt{2}\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right)^{3}+3\left(5\sqrt{2}\cosh\left(bx+a\right)^{4}+6\sqrt{2}\cosh\left(bx+a\right)^{2}+\sqrt{2}\right)\sinh\left(bx+a\right)^{2}+3\sqrt{2}\cosh\left(bx+a\right)^{2}+6\left(\sqrt{2}\cosh\left(bx+a\right)^{5}+2\sqrt{2}\cosh\left(bx+a\right)^{3}+\sqrt{2}\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right)+\sqrt{2}\right){\rm weierstrassZeta}\left(-4,0,{\rm weierstrassPInverse}\left(-4,0,\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)+2\left(3\cosh\left(bx+a\right)^{6}+18\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{5}+3\sinh\left(bx+a\right)^{6}+\left(45\cosh\left(bx+a\right)^{2}+8\right)\sinh\left(bx+a\right)^{4}+8\cosh\left(bx+a\right)^{4}+4\left(15\cosh\left(bx+a\right)^{3}+8\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right)^{3}+\left(45\cosh\left(bx+a\right)^{4}+48\cosh\left(bx+a\right)^{2}+1\right)\sinh\left(bx+a\right)^{2}+\cosh\left(bx+a\right)^{2}+2\left(9\cosh\left(bx+a\right)^{5}+16\cosh\left(bx+a\right)^{3}+\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right)\right)\sqrt{\cosh\left(bx+a\right)}\right)}{5\left(b\cosh\left(bx+a\right)^{6}+6b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{5}+b\sinh\left(bx+a\right)^{6}+3b\cosh\left(bx+a\right)^{4}+3\left(5b\cosh\left(bx+a\right)^{2}+b\right)\sinh\left(bx+a\right)^{4}+4\left(5b\cosh\left(bx+a\right)^{3}+3b\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right)^{3}+3b\cosh\left(bx+a\right)^{2}+3\left(5b\cosh\left(bx+a\right)^{4}+6b\cosh\left(bx+a\right)^{2}+b\right)\sinh\left(bx+a\right)^{2}+6\left(b\cosh\left(bx+a\right)^{5}+2b\cosh\left(bx+a\right)^{3}+b\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right)+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/5*(3*(sqrt(2)*cosh(b*x + a)^6 + 6*sqrt(2)*cosh(b*x + a)*sinh(b*x + a)^5 +
 sqrt(2)*sinh(b*x + a)^6 + 3*(5*sqrt(2)*cosh(b*x + a)^2 + sqrt(2))*sinh(b*x
 + a)^4 + 3*sqrt(2)*cosh(b*x + a)^4 + 4*(5*sqrt(2)*cosh(b*x + a)^3 + 3*sqrt
(2)*cosh(b*x + a))*sinh(b*x + a)^3 + 3*(5*sqrt(2)*cosh(b*x + a)^4 + 6*sqrt(
2)*cosh(b*x + a)^2 + sqrt(2))*sinh(b*x + a)^2 + 3*sqrt(2)*cosh(b*x + a)^2 +
 6*(sqrt(2)*cosh(b*x + a)^5 + 2*sqrt(2)*cosh(b*x + a)^3 + sqrt(2)*cosh(b*x
+ a))*sinh(b*x + a) + sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cosh(b*x + a) + sinh(b*x + a))) + 2*(3*cosh(b*x + a)^6 + 18*cosh(b*x
+ a)*sinh(b*x + a)^5 + 3*sinh(b*x + a)^6 + (45*cosh(b*x + a)^2 + 8)*sinh(b*
x + a)^4 + 8*cosh(b*x + a)^4 + 4*(15*cosh(b*x + a)^3 + 8*cosh(b*x + a))*sin
h(b*x + a)^3 + (45*cosh(b*x + a)^4 + 48*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^
2 + cosh(b*x + a)^2 + 2*(9*cosh(b*x + a)^5 + 16*cosh(b*x + a)^3 + cosh(b*x
+ a))*sinh(b*x + a))*sqrt(cosh(b*x + a)))/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x
 + a)*sinh(b*x + a)^5 + b*sinh(b*x + a)^6 + 3*b*cosh(b*x + a)^4 + 3*(5*b*co
sh(b*x + a)^2 + b)*sinh(b*x + a)^4 + 4*(5*b*cosh(b*x + a)^3 + 3*b*cosh(b*x
+ a))*sinh(b*x + a)^3 + 3*b*cosh(b*x + a)^2 + 3*(5*b*cosh(b*x + a)^4 + 6*b*
cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 6*(b*cosh(b*x + a)^5 + 2*b*cosh(b*x
+ a)^3 + b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3880 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^(-7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(a+b\,x\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(a + b*x)^(7/2),x)
```

```
[Out] int(1/cosh(a + b*x)^(7/2), x)
```

3.15 $\int (a\cosh(x))^{7/2}\,dx$

Optimal. Leaf size=65

$$-\frac{10ia^4\sqrt{\cosh(x)}\,F\left(\frac{ix}{2}\middle|2\right)}{21\sqrt{a\cosh(x)}}+\frac{10}{21}a^3\sqrt{a\cosh(x)}\,\sinh(x)+\frac{2}{7}a(a\cosh(x))^{5/2}\sinh(x)$$

```
[Out] 2/7*a*(a*cosh(x))^(5/2)*sinh(x)-10/21*I*a^4*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*
x)*EllipticF(I*sinh(1/2*x),2^(1/2))*cosh(x)^(1/2)/(a*cosh(x))^(1/2)+10/21*a
^3*sinh(x)*(a*cosh(x))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2715, 2721, 2720}

$$-\frac{10ia^4\sqrt{\cosh(x)}\,F\left(\frac{ix}{2}\middle|2\right)}{21\sqrt{a\cosh(x)}}+\frac{10}{21}a^3\sinh(x)\sqrt{a\cosh(x)}+\frac{2}{7}a\sinh(x)(a\cosh(x))^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x])^(7/2),x]
```

```
[Out] (((-10*I)/21)*a^4*Sqrt[Cosh[x]]*EllipticF[(I/2)*x, 2])/Sqrt[a*Cosh[x]] + (1
0*a^3*Sqrt[a*Cosh[x]]*Sinh[x])/21 + (2*a*(a*Cosh[x])^(5/2)*Sinh[x])/7
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\int (a\cosh(x))^{7/2}\,dx &= \frac{2}{7}a(a\cosh(x))^{5/2}\sinh(x) + \frac{1}{7}\left(5a^2\right)\int (a\cosh(x))^{3/2}\,dx \\
&= \frac{10}{21}a^3\sqrt{a\cosh(x)}\,\sinh(x) + \frac{2}{7}a(a\cosh(x))^{5/2}\sinh(x) + \frac{1}{21}\left(5a^4\right)\int \frac{1}{\sqrt{a\cosh(x)}}\,dx \\
&= \frac{10}{21}a^3\sqrt{a\cosh(x)}\,\sinh(x) + \frac{2}{7}a(a\cosh(x))^{5/2}\sinh(x) + \frac{\left(5a^4\sqrt{\cosh(x)}\right)\int \frac{1}{\sqrt{\cosh(x)}}}{21\sqrt{a\cosh(x)}} \\
&= -\frac{10ia^4\sqrt{\cosh(x)}\,F\left(\frac{ix}{2}\middle|2\right)}{21\sqrt{a\cosh(x)}} + \frac{10}{21}a^3\sqrt{a\cosh(x)}\,\sinh(x) + \frac{2}{7}a(a\cosh(x))^{5/2}\sinh(x)
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 53, normalized size = 0.82

$$
\frac{a^3\sqrt{a\cosh(x)}\left(-20iF\left(\frac{ix}{2}\middle|2\right) + \sqrt{\cosh(x)}\,(23\sinh(x) + 3\sinh(3x))\right)}{42\sqrt{\cosh(x)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x])^(7/2),x]
```

```
[Out] (a^3*Sqrt[a*Cosh[x]]*((-20*I)*EllipticF[(I/2)*x, 2] + Sqrt[Cosh[x]]*(23*Sin
h[x] + 3*Sinh[3*x])))/(42*Sqrt[Cosh[x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(66) = 132.
time = 1.25, size = 145, normalized size = 2.23

method	result
default	$\frac{\sqrt{a\left(2\left(\cosh^2\left(\frac{x}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{x}{2}\right)\right)}\,a^4\left(96\left(\cosh^9\left(\frac{x}{2}\right)\right)-240\left(\cosh^7\left(\frac{x}{2}\right)\right)+256\left(\cosh^5\left(\frac{x}{2}\right)\right)+5\sqrt{2}\,\sqrt{-2\left(\cosh^2\left(\frac{x}{2}\right.\right.}\right.}{21\sqrt{a\left(2\left(\sinh^4\left(\frac{x}{2}\right)\right)+\sinh^2\left(\frac{x}{2}\right)\right)}\,\sinh\left(\frac{x}{2}\right)\sqrt{a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/21*(a*(2*cosh(1/2*x)^2-1)*sinh(1/2*x)^2)^(1/2)*a^4*(96*cosh(1/2*x)^9-240*
cosh(1/2*x)^7+256*cosh(1/2*x)^5+5*2^(1/2)*(-2*cosh(1/2*x)^2+1)^(1/2)*(-sinh
(1/2*x)^2)^(1/2)*EllipticF(2^(1/2)*cosh(1/2*x),1/2*2^(1/2))-144*cosh(1/2*x)
^3+32*cosh(1/2*x))/(a*(2*sinh(1/2*x)^4+sinh(1/2*x)^2))^(1/2)/sinh(1/2*x)/(a
*(2*cosh(1/2*x)^2-1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))^(7/2),x, algorithm="maxima")
[Out] integrate((a*cosh(x))^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 256, normalized size = 3.94

$$\frac{40\left(\sqrt{2}\,a^3\cosh(x)^3+3\sqrt{2}\,a^3\cosh(x)^2\sinh(x)+3\sqrt{2}\,a^3\cosh(x)\sinh(x)^2+\sqrt{2}\,a^3\sinh(x)^3\right)\sqrt{a}\,\mathrm{weierstrassPInverse}(-4,0,\cosh(x)+\sinh(x))+\left(3a^3\cosh(x)^6+18a^3\cosh(x)\sinh(x)^5+3a^3\sinh(x)^6+23a^3\cosh(x)^4-23a^3\cosh(x)^2+\left(45a^3\cosh(x)^2+23a^3\right)\sinh(x)^4+4\left(15a^3\cosh(x)^3+23a^3\cosh(x)\right)\sinh(x)^3-3a^3+\left(45a^3\cosh(x)^4+138a^3\cosh(x)^2-23a^3\right)\sinh(x)^2+2\left(9a^3\cosh(x)^5+46a^3\cosh(x)^3-23a^3\cosh(x)\right)\sinh(x)\right)\sqrt{a\cosh(x)}}{84\left(\cosh(x)^3+3\cosh(x)^2\sinh(x)+3\cosh(x)\sinh(x)^2+\sinh(x)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))^(7/2),x, algorithm="fricas")
[Out] 1/84*(40*(sqrt(2)*a^3*cosh(x)^3 + 3*sqrt(2)*a^3*cosh(x)^2*sinh(x) + 3*sqrt(
2)*a^3*cosh(x)*sinh(x)^2 + sqrt(2)*a^3*sinh(x)^3)*sqrt(a)*weierstrassPInver
se(-4, 0, cosh(x) + sinh(x)) + (3*a^3*cosh(x)^6 + 18*a^3*cosh(x)*sinh(x)^5
+ 3*a^3*sinh(x)^6 + 23*a^3*cosh(x)^4 - 23*a^3*cosh(x)^2 + (45*a^3*cosh(x)^2
 + 23*a^3)*sinh(x)^4 + 4*(15*a^3*cosh(x)^3 + 23*a^3*cosh(x))*sinh(x)^3 - 3*
a^3 + (45*a^3*cosh(x)^4 + 138*a^3*cosh(x)^2 - 23*a^3)*sinh(x)^2 + 2*(9*a^3*
cosh(x)^5 + 46*a^3*cosh(x)^3 - 23*a^3*cosh(x))*sinh(x))*sqrt(a*cosh(x)))/(c
osh(x)^3 + 3*cosh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2 + sinh(x)^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))**(7/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))^(7/2),x, algorithm="giac")
[Out] integrate((a*cosh(x))^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int (a\cosh(x))^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x))^(7/2),x)
```

```
[Out] int((a*cosh(x))^(7/2), x)
```

3.16 $\int (a \cosh(x))^{5/2} \, dx$

Optimal. Leaf size=48

$$-\frac{6ia^2\sqrt{a\cosh(x)}\,E\left(\frac{ix}{2}\middle|2\right)}{5\sqrt{\cosh(x)}}+\frac{2}{5}a(a\cosh(x))^{3/2}\sinh(x)$$

```
[Out] 2/5*a*(a*cosh(x))^(3/2)*sinh(x)-6/5*I*a^2*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)
*EllipticE(I*sinh(1/2*x),2^(1/2))*(a*cosh(x))^(1/2)/cosh(x)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2715, 2721, 2719}

$$\frac{2}{5}a\sinh(x)(a\cosh(x))^{3/2}-\frac{6ia^2E\left(\frac{ix}{2}\middle|2\right)\sqrt{a\cosh(x)}}{5\sqrt{\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x])^(5/2),x]
```

```
[Out] (((-6*I)/5)*a^2*Sqrt[a*Cosh[x]]*EllipticE[(I/2)*x, 2])/Sqrt[Cosh[x]] + (2*a
*(a*Cosh[x])^(3/2)*Sinh[x])/5
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int (a\cosh(x))^{5/2}\,dx = \frac{2}{5}a(a\cosh(x))^{3/2}\sinh(x) + \frac{1}{5}(3a^2)\int \sqrt{a\cosh(x)}\,dx$$
$$= \frac{2}{5}a(a\cosh(x))^{3/2}\sinh(x) + \frac{\left(3a^2\sqrt{a\cosh(x)}\right)\int \sqrt{\cosh(x)}\,dx}{5\sqrt{\cosh(x)}}$$
$$= -\frac{6ia^2\sqrt{a\cosh(x)}\,E\left(\frac{ix}{2}\middle|2\right)}{5\sqrt{\cosh(x)}} + \frac{2}{5}a(a\cosh(x))^{3/2}\sinh(x)$$

Mathematica [A]
time = 0.03, size = 41, normalized size = 0.85

$$\frac{2(a\cosh(x))^{5/2}\left(-3iE\left(\frac{ix}{2}\middle|2\right)+\cosh^{\frac{3}{2}}(x)\sinh(x)\right)}{5\cosh^{\frac{5}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x])^(5/2),x]
```

```
[Out] (2*(a*Cosh[x])^(5/2)*((-3*I)*EllipticE[(I/2)*x, 2] + Cosh[x]^(3/2)*Sinh[x])
)/(5*Cosh[x]^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(53) = 106.
time = 1.18, size = 184, normalized size = 3.83

method	result
default	$\frac{\sqrt{a\left(2\left(\cosh^2\left(\frac{x}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{x}{2}\right)\right)}\,a^3\left(16\cosh\left(\frac{x}{2}\right)\left(\sinh^6\left(\frac{x}{2}\right)\right)+16\left(\sinh^4\left(\frac{x}{2}\right)\right)\cosh\left(\frac{x}{2}\right)+3\sqrt{2}\,\sqrt{-2\left(\sinh^2\left(\frac{x}{2}\right)\right.}\right.}{5\sqrt{a\left(2\left(\sin\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*(a*(2*cosh(1/2*x)^2-1)*sinh(1/2*x)^2)^(1/2)*a^3*(16*cosh(1/2*x)*sinh(1/
2*x)^6+16*sinh(1/2*x)^4*cosh(1/2*x)+3*2^(1/2)*(-2*sinh(1/2*x)^2-1)^(1/2)*(-
sinh(1/2*x)^2)^(1/2)*EllipticF(2^(1/2)*cosh(1/2*x),1/2*2^(1/2))-6*2^(1/2)*(
-2*sinh(1/2*x)^2-1)^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(2^(1/2)*cosh(1/2
*x),1/2*2^(1/2))+4*sinh(1/2*x)^2*cosh(1/2*x))/(a*(2*sinh(1/2*x)^4+sinh(1/2*
x)^2))^(1/2)/sinh(1/2*x)/(a*(2*cosh(1/2*x)^2-1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cosh(x))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 161, normalized size = 3.35

$$-\frac{12\left(\sqrt{2}\,a^2\cosh\left(x\right)^2+2\,\sqrt{2}\,a^2\cosh\left(x\right)\sinh\left(x\right)+\sqrt{2}\,a^2\sinh\left(x\right)^2\right)\sqrt{a}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cosh\left(x\right)+\sinh\left(x\right)))-\left(a^2\cosh\left(x\right)^4+4\,a^2\cosh\left(x\right)\sinh\left(x\right)^3+a^2\sinh\left(x\right)^4-12\,a^2\cosh\left(x\right)^2+6\left(a^2\cosh\left(x\right)^2-2\,a^2\right)\sinh\left(x\right)^2-a^2+4\left(a^2\cosh\left(x\right)^3-6\,a^2\cosh\left(x\right)\right)\sinh\left(x\right)\right)\sqrt{a\cosh\left(x\right)}}{10\left(\cosh\left(x\right)^2+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/10*(12*(sqrt(2)*a^2*cosh(x)^2 + 2*sqrt(2)*a^2*cosh(x)*sinh(x) + sqrt(2)*
a^2*sinh(x)^2)*sqrt(a)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, co
sh(x) + sinh(x))) - (a^2*cosh(x)^4 + 4*a^2*cosh(x)*sinh(x)^3 + a^2*sinh(x)^
4 - 12*a^2*cosh(x)^2 + 6*(a^2*cosh(x)^2 - 2*a^2)*sinh(x)^2 - a^2 + 4*(a^2*c
osh(x)^3 - 6*a^2*cosh(x))*sinh(x))*sqrt(a*cosh(x)))/(cosh(x)^2 + 2*cosh(x)*
sinh(x) + sinh(x)^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cosh(x))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int (a\cosh(x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x))^(5/2),x)
```

```
[Out] int((a*cosh(x))^(5/2), x)
```

3.17 $\int (a\cosh(x))^{3/2}\,dx$

Optimal. Leaf size=48

$$-\frac{2ia^2\sqrt{\cosh(x)}\,F\left(\frac{ix}{2}\middle|2\right)}{3\sqrt{a\cosh(x)}}+\frac{2}{3}a\sqrt{a\cosh(x)}\,\sinh(x)$$

```
[Out] -2/3*I*a^2*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh(1/2*x),2^(1/2
))*cosh(x)^(1/2)/(a*cosh(x))^(1/2)+2/3*a*sinh(x)*(a*cosh(x))^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2715, 2721, 2720}

$$\frac{2}{3}a\sinh(x)\sqrt{a\cosh(x)}-\frac{2ia^2\sqrt{\cosh(x)}\,F\left(\frac{ix}{2}\middle|2\right)}{3\sqrt{a\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x])^(3/2),x]
```

```
[Out] (((-2*I)/3)*a^2*Sqrt[Cosh[x]]*EllipticF[(I/2)*x, 2])/Sqrt[a*Cosh[x]] + (2*a
*Sqrt[a*Cosh[x]]*Sinh[x])/3
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int (a\cosh(x))^{3/2}\,dx = \frac{2}{3}a\sqrt{a\cosh(x)}\,\sinh(x) + \frac{1}{3}a^2\int \frac{1}{\sqrt{a\cosh(x)}}\,dx$$
$$= \frac{2}{3}a\sqrt{a\cosh(x)}\,\sinh(x) + \frac{\left(a^2\sqrt{\cosh(x)}\right)\int \frac{1}{\sqrt{\cosh(x)}}\,dx}{3\sqrt{a\cosh(x)}}$$
$$= -\frac{2ia^2\sqrt{\cosh(x)}\,F\left(\frac{ix}{2}\middle|\,2\right)}{3\sqrt{a\cosh(x)}} + \frac{2}{3}a\sqrt{a\cosh(x)}\,\sinh(x)$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.04, size = 57, normalized size = 1.19

$$\frac{2}{3}(a\cosh(x))^{3/2}\left({}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\cosh(2x)-\sinh(2x)\right)\operatorname{sech}^2(x)\sqrt{1+\cosh(2x)+\sinh(2x)}+\tanh(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x])^(3/2),x]
```

```
[Out] (2*(a*Cosh[x])^(3/2)*(Hypergeometric2F1[1/4, 1/2, 5/4, -Cosh[2*x] - Sinh[2*
x]]*Sech[x]^2*Sqrt[1 + Cosh[2*x] + Sinh[2*x]] + Tanh[x]))/3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(53) = 106.
time = 1.11, size = 130, normalized size = 2.71

method	result
default	$\frac{\sqrt{a\left(2\left(\cosh^2\left(\frac{x}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{x}{2}\right)\right)}\,a^2\left(8\left(\sinh^4\left(\frac{x}{2}\right)\right)\cosh\left(\frac{x}{2}\right)+\sqrt{2}\,\sqrt{-2\left(\sinh^2\left(\frac{x}{2}\right)\right)-1}\,\sqrt{-\left(\sinh^2\left(\frac{x}{2}\right.\right.}\right.}{3\sqrt{a\left(2\left(\sinh^4\left(\frac{x}{2}\right)\right)+\sinh^2\left(\frac{x}{2}\right)\right)}\,\sinh\left(\frac{x}{2}\right)\sqrt{a\left(2\left(\cosh^2\left(\frac{x}{2}\right.\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(a*(2*cosh(1/2*x)^2-1)*sinh(1/2*x)^2)^(1/2)*a^2*(8*sinh(1/2*x)^4*cosh(1
/2*x)+2^(1/2)*(-2*sinh(1/2*x)^2-1)^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(2
^(1/2)*cosh(1/2*x),1/2*2^(1/2))+4*sinh(1/2*x)^2*cosh(1/2*x))/(a*(2*sinh(1/2
*x)^4+sinh(1/2*x)^2))^(1/2)/sinh(1/2*x)/(a*(2*cosh(1/2*x)^2-1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cosh(x))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 68, normalized size = 1.42

$$\frac{2\left(\sqrt{2}\,a\cosh\left(x\right)+\sqrt{2}\,a\sinh\left(x\right)\right)\sqrt{a}\ \mathrm{weierstrassPInverse}\left(-4,0,\cosh\left(x\right)+\sinh\left(x\right)\right)+\left(a\cosh\left(x\right)^{2}+2\,a\cosh\left(x\right)\sinh\left(x\right)+a\sinh\left(x\right)^{2}-a\right)\sqrt{a\cosh\left(x\right)}}{3\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*(sqrt(2)*a*cosh(x) + sqrt(2)*a*sinh(x))*sqrt(a)*weierstrassPInverse(
-4, 0, cosh(x) + sinh(x)) + (a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^
2 - a)*sqrt(a*cosh(x)))/(cosh(x) + sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\cosh\left(x\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))**(3/2),x)
```

```
[Out] Integral((a*cosh(x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cosh(x))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int{\left(a\,\mathrm{cosh}\left(x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x))^(3/2),x)
```

```
[Out] int((a*cosh(x))^(3/2), x)
```

3.18 $\int \sqrt{a \cosh(x)}\, dx$

Optimal. Leaf size=27

$$-\frac{2 i \sqrt{a \cosh(x)}\, E\left(\left.\frac{i x}{2}\right|2\right)}{\sqrt{\cosh(x)}}$$

```
[Out] -2*I*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*sinh(1/2*x),2^(1/2))*(a*
cosh(x))^(1/2)/cosh(x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$,
Rules used = {2721, 2719}

$$-\frac{2 i E\left(\left.\frac{i x}{2}\right|2\right) \sqrt{a \cosh(x)}}{\sqrt{\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Cosh[x]],x]
```

```
[Out] ((-2*I)*Sqrt[a*Cosh[x]]*EllipticE[(I/2)*x, 2])/Sqrt[Cosh[x]]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \int \sqrt{a \cosh(x)}\, dx &= \frac{\sqrt{a \cosh(x)} \int \sqrt{\cosh(x)}\, dx}{\sqrt{\cosh(x)}} \\ &= -\frac{2 i \sqrt{a \cosh(x)}\, E\left(\left.\frac{i x}{2}\right|2\right)}{\sqrt{\cosh(x)}} \end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 1.00

$$-\frac{2 i \sqrt{a \cosh(x)}\, E\left(\left.\frac{i x}{2}\right|2\right)}{\sqrt{\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Cosh[x]],x]
```

```
[Out] ((-2*I)*Sqrt[a*Cosh[x]]*EllipticE[(I/2)*x, 2])/Sqrt[Cosh[x]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(38) = 76$.
time = 1.34, size = 118, normalized size = 4.37

method	result
default	$\frac{\sqrt{a\left(2\left(\cosh^{2}\left(\frac{x}{2}\right)\right)-1\right)\left(\sinh^{2}\left(\frac{x}{2}\right)\right)}\, a\sqrt{2}\,\sqrt{-2\left(\cosh^{2}\left(\frac{x}{2}\right)\right)+1}\,\sqrt{-\left(\sinh^{2}\left(\frac{x}{2}\right)\right)}\,\left(\mathrm{EllipticF}\left(\sqrt{2}\right.\right.}{\sqrt{a\left(2\left(\sinh^{4}\left(\frac{x}{2}\right)\right)+\sinh^{2}\left(\frac{x}{2}\right)\right)}\,\sinh\left(\frac{x}{2}\right)\sqrt{a\left(2\left(\cosh^{2}\left(\frac{x}{2}\right)\right)-\right.}}$
risch	$\sqrt{2}\,\sqrt{a\left(1+\mathrm{e}^{2x}\right)\mathrm{e}^{-x}}+\frac{\left(-\frac{4\left(\mathrm{e}^{2x}a+a\right)}{a\sqrt{\mathrm{e}^{x}\left(\mathrm{e}^{2x}a+a\right)}}+\frac{2i\sqrt{-i\left(\mathrm{e}^{x}+i\right)}\,\sqrt{2}\,\sqrt{i\left(\mathrm{e}^{x}-i\right)}\,\sqrt{i\mathrm{e}^{x}}\,\left(-2i\,\mathrm{EllipticE}\left(\right.\right.}{\sqrt{a\,\mathrm{e}^{3x}}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (a*(2*cosh(1/2*x)^2-1)*sinh(1/2*x)^2)^(1/2)*a*2^(1/2)*(-2*cosh(1/2*x)^2+1)^
(1/2)*(-sinh(1/2*x)^2)^(1/2)*(EllipticF(2^(1/2)*cosh(1/2*x),1/2*2^(1/2))-2*
EllipticE(2^(1/2)*cosh(1/2*x),1/2*2^(1/2)))/(a*(2*sinh(1/2*x)^4+sinh(1/2*x)
^2))^(1/2)/sinh(1/2*x)/(a*(2*cosh(1/2*x)^2-1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*cosh(x)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 28, normalized size = 1.04

$$-2\sqrt{2}\,\sqrt{a}\ \mathrm{weierstrassZeta}(-4, 0, \mathrm{weierstrassPInverse}(-4, 0, \cosh(x) + \sinh(x))) - 2\sqrt{a\cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(2)*sqrt(a)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cosh(x
) + sinh(x))) - 2*sqrt(a*cosh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\cosh{(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))**(1/2),x)
```

```
[Out] Integral(sqrt(a*cosh(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cosh(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{a\cosh{(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x))^(1/2),x)
```

```
[Out] int((a*cosh(x))^(1/2), x)
```

3.19 $\int \frac{1}{\sqrt{a\cosh(x)}}\,dx$

Optimal. Leaf size=27

$$-\frac{2i\sqrt{\cosh(x)}\,F\left(\frac{ix}{2}\middle|2\right)}{\sqrt{a\cosh(x)}}$$

```
[Out] -2*I*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh(1/2*x),2^(1/2))*cos
h(x)^(1/2)/(a*cosh(x))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2721, 2720}

$$-\frac{2i\sqrt{\cosh(x)}\,F\left(\frac{ix}{2}\middle|2\right)}{\sqrt{a\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Cosh[x]],x]
```

```
[Out] ((-2*I)*Sqrt[Cosh[x]]*EllipticF[(I/2)*x, 2])/Sqrt[a*Cosh[x]]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a\cosh(x)}}\,dx &= \frac{\sqrt{\cosh(x)}\int \frac{1}{\sqrt{\cosh(x)}}\,dx}{\sqrt{a\cosh(x)}}\\ &= -\frac{2i\sqrt{\cosh(x)}\,F\left(\frac{ix}{2}\middle|2\right)}{\sqrt{a\cosh(x)}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 1.00

$$-\frac{2i\sqrt{\cosh(x)}\,F\left(\frac{ix}{2}\middle|2\right)}{\sqrt{a\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Cosh[x]],x]
[Out] ((-2*I)*Sqrt[Cosh[x]]*EllipticF[(I/2)*x, 2])/Sqrt[a*Cosh[x]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(38) = 76.
time = 0.90, size = 100, normalized size = 3.70

method	result
default	$\frac{\sqrt{a\left(2\left(\cosh^2\left(\frac{x}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{x}{2}\right)\right)}\,\sqrt{2}\,\sqrt{-2\left(\cosh^2\left(\frac{x}{2}\right)\right)+1}\,\sqrt{-\left(\sinh^2\left(\frac{x}{2}\right)\right)}\,\mathrm{EllipticF}\left(\sqrt{2}\,\cosh\right.}{\sqrt{a\left(2\left(\sinh^4\left(\frac{x}{2}\right)\right)+\sinh^2\left(\frac{x}{2}\right)\right)}\,\sinh\left(\frac{x}{2}\right)\sqrt{a\left(2\left(\cosh^2\left(\frac{x}{2}\right)\right)-1\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x))^(1/2),x,method=_RETURNVERBOSE)
[Out] (a*(2*cosh(1/2*x)^2-1)*sinh(1/2*x)^2)^(1/2)*2^(1/2)*(-2*cosh(1/2*x)^2+1)^(1
/2)*(-sinh(1/2*x)^2)^(1/2)/(a*(2*sinh(1/2*x)^4+sinh(1/2*x)^2))^(1/2)*Ellipt
icF(2^(1/2)*cosh(1/2*x),1/2*2^(1/2))/sinh(1/2*x)/(a*(2*cosh(1/2*x)^2-1))^(1
/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(a*cosh(x)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 16, normalized size = 0.59

$$\frac{2\sqrt{2}\,\mathrm{weierstrassPInverse}(-4,0,\cosh(x)+\sinh(x))}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*weierstrassPInverse(-4, 0, cosh(x) + sinh(x))/sqrt(a)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\cosh{(x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*cosh(x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(a*cosh(x)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\sqrt{a\cosh{(x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x))^(1/2),x)
```

```
[Out] int(1/(a*cosh(x))^(1/2), x)
```

3.20 $\int \frac{1}{(a\cosh(x))^{3/2}}\,dx$

Optimal. Leaf size=46

$$\frac{2i\sqrt{a\cosh(x)}\,E\left(\frac{ix}{2}\middle|2\right)}{a^2\sqrt{\cosh(x)}}+\frac{2\sinh(x)}{a\sqrt{a\cosh(x)}}$$

```
[Out] 2*sinh(x)/a/(a*cosh(x))^(1/2)+2*I*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*Ellipti
cE(I*sinh(1/2*x),2^(1/2))*(a*cosh(x))^(1/2)/a^2/cosh(x)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2716, 2721, 2719}

$$\frac{2\sinh(x)}{a\sqrt{a\cosh(x)}}+\frac{2iE\left(\frac{ix}{2}\middle|2\right)\sqrt{a\cosh(x)}}{a^2\sqrt{\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x])^(-3/2),x]
```

```
[Out] ((2*I)*Sqrt[a*Cosh[x]]*EllipticE[(I/2)*x, 2])/(a^2*Sqrt[Cosh[x]]) + (2*Sinh
[x])/(a*Sqrt[a*Cosh[x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(a\cosh(x))^{3/2}}\,dx = \frac{2\sinh(x)}{a\sqrt{a\cosh(x)}} - \frac{\int \sqrt{a\cosh(x)}\,dx}{a^2}$$
$$= \frac{2\sinh(x)}{a\sqrt{a\cosh(x)}} - \frac{\sqrt{a\cosh(x)}\int \sqrt{\cosh(x)}\,dx}{a^2\sqrt{\cosh(x)}}$$
$$= \frac{2i\sqrt{a\cosh(x)}\,E\left(\frac{ix}{2}\middle|2\right)}{a^2\sqrt{\cosh(x)}} + \frac{2\sinh(x)}{a\sqrt{a\cosh(x)}}$$

Mathematica [A]
time = 0.02, size = 34, normalized size = 0.74

$$\frac{2\cosh(x)\left(i\sqrt{\cosh(x)}\,E\left(\frac{ix}{2}\middle|2\right)+\sinh(x)\right)}{(a\cosh(x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x])^(-3/2),x]
```

```
[Out] (2*Cosh[x]*(I*Sqrt[Cosh[x]]*EllipticE[(I/2)*x, 2] + Sinh[x]))/(a*Cosh[x])^(
3/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(55) = 110$.
time = 1.26, size = 159, normalized size = 3.46

method	result
default	$\frac{\sqrt{2\left(\sinh^4\left(\frac{x}{2}\right)\right)a+\left(\sinh^2\left(\frac{x}{2}\right)\right)a}\,\left(-\sqrt{2}\,\sqrt{-2\left(\sinh^2\left(\frac{x}{2}\right)\right)-1}\,\sqrt{-\left(\sinh^2\left(\frac{x}{2}\right)\right)}\,\mathrm{EllipticF}\left(\sqrt{2}\,\mathrm{co}\right.\right.}{a\sqrt{a\left(2\left(\sinh^4\left(\frac{x}{2}\right)\right)+\sinh^2\left(\frac{x}{2}\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(2*sinh(1/2*x)^4*a+sinh(1/2*x)^2*a)^(1/2)*(-2^(1/2)*(-2*sinh(1/2*x)^2-1
)^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(2^(1/2)*cosh(1/2*x),1/2*2^(1/2))+2
*2^(1/2)*(-2*sinh(1/2*x)^2-1)^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(2^(1/2
)*cosh(1/2*x),1/2*2^(1/2))+4*sinh(1/2*x)^2*cosh(1/2*x))/(a*(2*sinh(1/2*x)^4
+sinh(1/2*x)^2))^(1/2)/sinh(1/2*x)/(a*(2*cosh(1/2*x)^2-1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cosh(x))^(-3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 101, normalized size = 2.20

$$\frac{2\left(\left(\sqrt{2}\cosh\left(x\right)^{2}+2\sqrt{2}\cosh\left(x\right)\sinh\left(x\right)+\sqrt{2}\sinh\left(x\right)^{2}+\sqrt{2}\right)\sqrt{a}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cosh\left(x\right)+\sinh\left(x\right)))+2\sqrt{a\cosh\left(x\right)}\left(\cosh\left(x\right)^{2}+2\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}\right)\right)}{a^{2}\cosh\left(x\right)^{2}+2a^{2}\cosh\left(x\right)\sinh\left(x\right)+a^{2}\sinh\left(x\right)^{2}+a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] 2*((sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2 + sqr
t(2))*sqrt(a)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cosh(x) + s
inh(x))) + 2*sqrt(a*cosh(x))*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2))/(
a^2*cosh(x)^2 + 2*a^2*cosh(x)*sinh(x) + a^2*sinh(x)^2 + a^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(a\cosh\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))**(3/2),x)
```

```
[Out] Integral((a*cosh(x))**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cosh(x))^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{{\left(a\,\cosh\left(x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x))^(3/2),x)
```

```
[Out] int(1/(a*cosh(x))^(3/2), x)
```

3.21 $\int \frac{1}{(a\cosh(x))^{5/2}}\,dx$

Optimal. Leaf size=50

$$-\frac{2i\sqrt{\cosh(x)}\,F\left(\frac{ix}{2}\middle|2\right)}{3a^2\sqrt{a\cosh(x)}}+\frac{2\sinh(x)}{3a(a\cosh(x))^{3/2}}$$

```
[Out] 2/3*sinh(x)/a/(a*cosh(x))^(3/2)-2/3*I*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh(1/2*x),2^(1/2))*cosh(x)^(1/2)/a^2/(a*cosh(x))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2716, 2721, 2720}

$$\frac{2\sinh(x)}{3a(a\cosh(x))^{3/2}}-\frac{2i\sqrt{\cosh(x)}\,F\left(\frac{ix}{2}\middle|2\right)}{3a^2\sqrt{a\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x])^(-5/2),x]
```

```
[Out] (((-2*I)/3)*Sqrt[Cosh[x]]*EllipticF[(I/2)*x, 2])/(a^2*Sqrt[a*Cosh[x]]) + (2*Sinh[x])/(3*a*(a*Cosh[x])^(3/2))
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a\cosh(x))^{5/2}}\,dx &= \frac{2\sinh(x)}{3a(a\cosh(x))^{3/2}} + \frac{\int \frac{1}{\sqrt{a\cosh(x)}}\,dx}{3a^2} \\
&= \frac{2\sinh(x)}{3a(a\cosh(x))^{3/2}} + \frac{\sqrt{\cosh(x)}\int \frac{1}{\sqrt{\cosh(x)}}\,dx}{3a^2\sqrt{a\cosh(x)}} \\
&= -\frac{2i\sqrt{\cosh(x)}\,F\left(\frac{ix}{2}\middle|2\right)}{3a^2\sqrt{a\cosh(x)}} + \frac{2\sinh(x)}{3a(a\cosh(x))^{3/2}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.03, size = 56, normalized size = 1.12

$$
\frac{2\left({}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\cosh(2x)-\sinh(2x)\right)\sqrt{1+\cosh(2x)+\sinh(2x)}+\tanh(x)\right)}{3a^2\sqrt{a\cosh(x)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x])^(-5/2),x]
```

```
[Out] (2*(Hypergeometric2F1[1/4, 1/2, 5/4, -Cosh[2*x] - Sinh[2*x]]*Sqrt[1 + Cosh[
2*x] + Sinh[2*x]] + Tanh[x]))/(3*a^2*Sqrt[a*Cosh[x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 176 vs. 2(55) = 110.
time = 1.07, size = 177, normalized size = 3.54

method	result
default	$\frac{\left(2\sqrt{2}\,\sqrt{-2\left(\sinh^2\left(\frac{x}{2}\right)\right)-1}\,\sqrt{-\left(\sinh^2\left(\frac{x}{2}\right)\right)}\,\mathrm{EllipticF}\left(\sqrt{2}\,\cosh\left(\frac{x}{2}\right),\frac{\sqrt{2}}{2}\right)\left(\sinh^2\left(\frac{x}{2}\right)\right)+\sqrt{2}\,\sqrt{-2\left(\sinh\right.}\right.}{3a^2\sqrt{a\left(2\left(\sinh^4\left(\frac{x}{2}\right)\right)+\sinh^2\left(\frac{x}{2}\right)\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(2*2^(1/2)*(-2*sinh(1/2*x)^2-1)^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(
2^(1/2)*cosh(1/2*x),1/2*2^(1/2))*sinh(1/2*x)^2+2^(1/2)*(-2*sinh(1/2*x)^2-1)
^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(2^(1/2)*cosh(1/2*x),1/2*2^(1/2))+4*
sinh(1/2*x)^2*cosh(1/2*x))/a^2*(a*(2*cosh(1/2*x)^2-1)*sinh(1/2*x)^2)^(1/2)/
(a*(2*sinh(1/2*x)^4+sinh(1/2*x)^2))^(1/2)/(2*cosh(1/2*x)^2-1)/sinh(1/2*x)/(
a*(2*cosh(1/2*x)^2-1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cosh(x))^(-5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 213, normalized size = 4.26

$$\frac{2\left(\left(\sqrt{2}\cosh\left(x\right)^{4}+4\sqrt{2}\cosh\left(x\right)\sinh\left(x\right)^{3}+\sqrt{2}\sinh\left(x\right)^{4}+2\left(3\sqrt{2}\cosh\left(x\right)^{2}+\sqrt{2}\right)\sinh\left(x\right)^{2}+2\sqrt{2}\cosh\left(x\right)^{2}+4\left(\sqrt{2}\cosh\left(x\right)^{3}+\sqrt{2}\cosh\left(x\right)\right)\sinh\left(x\right)+\sqrt{2}\right)\sqrt{a}\,\mathrm{weierstrassPInverse}(-4,0,\cosh\left(x\right)+\sinh\left(x\right))+2\left(\cosh\left(x\right)^{3}+3\cosh\left(x\right)\sinh\left(x\right)^{2}+\sinh\left(x\right)^{3}+\left(3\cosh\left(x\right)^{2}-1\right)\sinh\left(x\right)-\cosh\left(x\right)\right)\sqrt{a\cosh\left(x\right)}\right)}{3\left(a^{3}\cosh\left(x\right)^{4}+4a^{3}\cosh\left(x\right)\sinh\left(x\right)^{3}+a^{3}\sinh\left(x\right)^{4}+2a^{3}\cosh\left(x\right)^{2}+a^{3}+2\left(3a^{3}\cosh\left(x\right)^{2}+a^{3}\right)\sinh\left(x\right)^{2}+4\left(a^{3}\cosh\left(x\right)^{3}+a^{3}\cosh\left(x\right)\right)\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*((sqrt(2)*cosh(x)^4 + 4*sqrt(2)*cosh(x)*sinh(x)^3 + sqrt(2)*sinh(x)^4 +
 2*(3*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x)^2 + 2*sqrt(2)*cosh(x)^2 + 4*(sqr
t(2)*cosh(x)^3 + sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*sqrt(a)*weierstrassPIn
verse(-4, 0, cosh(x) + sinh(x)) + 2*(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh
(x)^3 + (3*cosh(x)^2 - 1)*sinh(x) - cosh(x))*sqrt(a*cosh(x)))/(a^3*cosh(x)^
4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*sinh(x)^4 + 2*a^3*cosh(x)^2 + a^3 + 2*(3*
a^3*cosh(x)^2 + a^3)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + a^3*cosh(x))*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(a\cosh\left(x\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))**(5/2),x)
```

```
[Out] Integral((a*cosh(x))**(-5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cosh(x))^(-5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(a \cosh \left(x\right)\right)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x))^(5/2),x)
[Out] int(1/(a*cosh(x))^(5/2), x)
```

3.22 $\int \frac{1}{(a\cosh(x))^{7/2}}\,dx$

Optimal. Leaf size=67

$$\frac{6i\sqrt{a\cosh(x)}\,E\left(\frac{ix}{2}\middle|2\right)}{5a^4\sqrt{\cosh(x)}}+\frac{2\sinh(x)}{5a(a\cosh(x))^{5/2}}+\frac{6\sinh(x)}{5a^3\sqrt{a\cosh(x)}}$$

```
[Out] 2/5*sinh(x)/a/(a*cosh(x))^(5/2)+6/5*sinh(x)/a^3/(a*cosh(x))^(1/2)+6/5*I*(co
sh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*sinh(1/2*x),2^(1/2))*(a*cosh(x))
^(1/2)/a^4/cosh(x)^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2716, 2721, 2719}

$$\frac{6iE\left(\frac{ix}{2}\middle|2\right)\sqrt{a\cosh(x)}}{5a^4\sqrt{\cosh(x)}}+\frac{6\sinh(x)}{5a^3\sqrt{a\cosh(x)}}+\frac{2\sinh(x)}{5a(a\cosh(x))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x])^(-7/2),x]
```

```
[Out] (((6*I)/5)*Sqrt[a*Cosh[x]]*EllipticE[(I/2)*x, 2])/(a^4*Sqrt[Cosh[x]]) + (2*
Sinh[x])/(5*a*(a*Cosh[x])^(5/2)) + (6*Sinh[x])/(5*a^3*Sqrt[a*Cosh[x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a\cosh(x))^{7/2}}\,dx &= \frac{2\sinh(x)}{5a(a\cosh(x))^{5/2}} + \frac{3\int \frac{1}{(a\cosh(x))^{3/2}}\,dx}{5a^2} \\
&= \frac{2\sinh(x)}{5a(a\cosh(x))^{5/2}} + \frac{6\sinh(x)}{5a^3\sqrt{a\cosh(x)}} - \frac{3\int \sqrt{a\cosh(x)}\,dx}{5a^4} \\
&= \frac{2\sinh(x)}{5a(a\cosh(x))^{5/2}} + \frac{6\sinh(x)}{5a^3\sqrt{a\cosh(x)}} - \frac{\left(3\sqrt{a\cosh(x)}\right)\int \sqrt{\cosh(x)}\,dx}{5a^4\sqrt{\cosh(x)}} \\
&= \frac{6i\sqrt{a\cosh(x)}\,E\left(\frac{ix}{2}\middle|2\right)}{5a^4\sqrt{\cosh(x)}} + \frac{2\sinh(x)}{5a(a\cosh(x))^{5/2}} + \frac{6\sinh(x)}{5a^3\sqrt{a\cosh(x)}}
\end{aligned}
$$

Mathematica [**A**]
time = 0.03, size = 43, normalized size = 0.64

$$
\frac{2\left(3i\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right) + 3\cosh(x)\sinh(x) + \tanh(x)\right)}{5a^2(a\cosh(x))^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x])^(-7/2),x]
```

```
[Out] (2*((3*I)*Cosh[x]^(3/2)*EllipticE[(I/2)*x, 2] + 3*Cosh[x]*Sinh[x] + Tanh[x]
))/(5*a^2*(a*Cosh[x])^(3/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(68) = 136$.
time = 1.36, size = 254, normalized size = 3.79

method	result
default	$\frac{2\sqrt{a\left(2\left(\cosh^2\left(\frac{x}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{x}{2}\right)\right)}\left(\frac{\cosh\left(\frac{x}{2}\right)\sqrt{a\left(2\left(\sinh^4\left(\frac{x}{2}\right)\right)+\sinh^2\left(\frac{x}{2}\right)\right)}}{20a\left(\cosh^2\left(\frac{x}{2}\right)-\frac{1}{2}\right)^3}+\frac{6\left(\sinh^2\left(\frac{x}{2}\right.\right.}{5\sqrt{a\left(2\left(\cosh^2\left(\frac{x}{2}\right)\right.\right.}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(a*(2*cosh(1/2*x)^2-1)*sinh(1/2*x)^2)^(1/2)/a^3*(1/20*cosh(1/2*x)/a*(a*(2
*sinh(1/2*x)^4+sinh(1/2*x)^2))^(1/2)/(cosh(1/2*x)^2-1/2)^3+6/5*sinh(1/2*x)^
2*cosh(1/2*x)/(a*(2*cosh(1/2*x)^2-1)*sinh(1/2*x)^2)^(1/2)+3/10*2^(1/2)*(-2*
```

```
cosh(1/2*x)^2+1)^(1/2)*(-sinh(1/2*x)^2)^(1/2)/(a*(2*sinh(1/2*x)^4+sinh(1/2*
x)^2))^(1/2)*EllipticF(2^(1/2)*cosh(1/2*x),1/2*2^(1/2))-3/5*2^(1/2)*(-2*cos
h(1/2*x)^2+1)^(1/2)*(-sinh(1/2*x)^2)^(1/2)/(a*(2*sinh(1/2*x)^4+sinh(1/2*x)^
2))^(1/2)*(EllipticF(2^(1/2)*cosh(1/2*x),1/2*2^(1/2))-EllipticE(2^(1/2)*cos
h(1/2*x),1/2*2^(1/2))))/sinh(1/2*x)/(a*(2*cosh(1/2*x)^2-1))^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cosh(x))^(-7/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 422, normalized size = 6.30

$$\frac{2\left(3\left(\sqrt{2}\cosh(x)^6+6\sqrt{2}\cosh(x)\sinh(x)^5+\sqrt{2}\sinh(x)^6+3\left(5\sqrt{2}\cosh(x)^2+\sqrt{2}\right)\sinh(x)^4+3\sqrt{2}\cosh(x)^4+4\left(5\sqrt{2}\cosh(x)^3+3\sqrt{2}\cosh(x)\right)\sinh(x)^3+3\left(5\sqrt{2}\cosh(x)^4+6\sqrt{2}\cosh(x)^2+\sqrt{2}\right)\sinh(x)^2+3\sqrt{2}\cosh(x)^2+6\left(\sqrt{2}\cosh(x)^5+2\sqrt{2}\cosh(x)^3+\sqrt{2}\cosh(x)\right)\sinh(x)+\sqrt{2}\right)\sqrt{a}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cosh(x)+\sinh(x)))+2\left(3\cosh(x)^6+18\cosh(x)\sinh(x)^5+3\sinh(x)^6+(45\cosh(x)^2+8)\sinh(x)^4+8\cosh(x)^4+4(15\cosh(x)^3+8\cosh(x))\sinh(x)^3+(45\cosh(x)^4+48\cosh(x)^2+1)\sinh(x)^2+\cosh(x)^2+2(9\cosh(x)^5+16\cosh(x)^3+\cosh(x))\sinh(x)\right)\sqrt{a\cosh(x)}\right)}{5\left(a^4\cosh(x)^6+6a^4\cosh(x)\sinh(x)^5+a^4\sinh(x)^6+3a^4\cosh(x)^4+3a^4\cosh(x)^2+3(5a^4\cosh(x)^2+a^4)\sinh(x)^4+a^4+4(5a^4\cosh(x)^3+3a^4\cosh(x))\sinh(x)^3+3(5a^4\cosh(x)^4+6a^4\cosh(x)^2+a^4)\sinh(x)^2+6(a^4\cosh(x)^5+2a^4\cosh(x)^3+a^4\cosh(x))\sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/5*(3*(sqrt(2)*cosh(x)^6 + 6*sqrt(2)*cosh(x)*sinh(x)^5 + sqrt(2)*sinh(x)^6
 + 3*(5*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x)^4 + 3*sqrt(2)*cosh(x)^4 + 4*(5
*sqrt(2)*cosh(x)^3 + 3*sqrt(2)*cosh(x))*sinh(x)^3 + 3*(5*sqrt(2)*cosh(x)^4
+ 6*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x)^2 + 3*sqrt(2)*cosh(x)^2 + 6*(sqrt(
2)*cosh(x)^5 + 2*sqrt(2)*cosh(x)^3 + sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*sq
rt(a)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cosh(x) + sinh(x)))
 + 2*(3*cosh(x)^6 + 18*cosh(x)*sinh(x)^5 + 3*sinh(x)^6 + (45*cosh(x)^2 + 8)
*sinh(x)^4 + 8*cosh(x)^4 + 4*(15*cosh(x)^3 + 8*cosh(x))*sinh(x)^3 + (45*cos
h(x)^4 + 48*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(9*cosh(x)^5 + 16*cosh
(x)^3 + cosh(x))*sinh(x))*sqrt(a*cosh(x)))/(a^4*cosh(x)^6 + 6*a^4*cosh(x)*s
inh(x)^5 + a^4*sinh(x)^6 + 3*a^4*cosh(x)^4 + 3*a^4*cosh(x)^2 + 3*(5*a^4*cos
h(x)^2 + a^4)*sinh(x)^4 + a^4 + 4*(5*a^4*cosh(x)^3 + 3*a^4*cosh(x))*sinh(x)
^3 + 3*(5*a^4*cosh(x)^4 + 6*a^4*cosh(x)^2 + a^4)*sinh(x)^2 + 6*(a^4*cosh(x)
^5 + 2*a^4*cosh(x)^3 + a^4*cosh(x))*sinh(x))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3880 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*cosh(x))^(-7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a\cosh(x))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x))^(7/2),x)
```

```
[Out] int(1/(a*cosh(x))^(7/2), x)
```

3.23 $\int (b\cosh(c+dx))^n\,dx$

Optimal. Leaf size=71

$$-\frac{(b\cosh(c+dx))^{1+n}\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\cosh^2(c+dx)\right)\sinh(c+dx)}{bd(1+n)\sqrt{-\sinh^2(c+dx)}}$$

```
[Out] -(b*cosh(d*x+c))^(1+n)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cosh(d*x+c)^2
)*sinh(d*x+c)/b/d/(1+n)/(-sinh(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2722}

$$-\frac{\sinh(c+dx)(b\cosh(c+dx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cosh^2(c+dx)\right)}{bd(n+1)\sqrt{-\sinh^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cosh[c + d*x])^n,x]
```

```
[Out] -(((b*Cosh[c + d*x])^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, C
osh[c + d*x]^2]*Sinh[c + d*x])/(b*d*(1 + n)*Sqrt[-Sinh[c + d*x]^2]))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (b\cosh(c+dx))^n\,dx = -\frac{(b\cosh(c+dx))^{1+n}\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\cosh^2(c+dx)\right)\sinh(c+dx)}{bd(1+n)\sqrt{-\sinh^2(c+dx)}}$$

Mathematica [A]
time = 0.05, size = 65, normalized size = 0.92

$$\frac{(b\cosh(c+dx))^n\coth(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\cosh^2(c+dx)\right)\sqrt{-\sinh^2(c+dx)}}{d(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cosh[c + d*x])^n,x]
```

```
[Out] ((b*Cosh[c + d*x])^n*Coth[c + d*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n
)/2, Cosh[c + d*x]^2]*Sqrt[-Sinh[c + d*x]^2])/(d*(1 + n))
```

Maple [F]
time = 0.71, size = 0, normalized size = 0.00

$$\int (b \cosh (dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cosh(d*x+c))^n,x)
```

```
[Out] int((b*cosh(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*cosh(d*x + c))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*cosh(d*x + c))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b \cosh (c+dx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(d*x+c))**n,x)
```

```
[Out] Integral((b*cosh(c + d*x))**n, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*cosh(d*x + c))^n, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (b\cosh(c+d\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cosh(c + d*x))^n,x)
```

```
[Out] int((b*cosh(c + d*x))^n, x)
```

3.24 $\int \frac{\cosh^4(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=54

$$-\frac{3x}{2a}+\frac{4\sinh(x)}{a}-\frac{3\cosh(x)\sinh(x)}{2a}-\frac{\cosh^3(x)\sinh(x)}{a+a\cosh(x)}+\frac{4\sinh^3(x)}{3a}$$

```
[Out] -3/2*x/a+4*sinh(x)/a-3/2*cosh(x)*sinh(x)/a-cosh(x)^3*sinh(x)/(a+a*cosh(x))+
4/3*sinh(x)^3/a
```

Rubi [A]
time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2846, 2827, 2715, 8, 2713}

$$-\frac{3x}{2a}+\frac{4\sinh^3(x)}{3a}+\frac{4\sinh(x)}{a}-\frac{\sinh(x)\cosh^3(x)}{a\cosh(x)+a}-\frac{3\sinh(x)\cosh(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^4/(a + a*Cosh[x]),x]
```

```
[Out] (-3*x)/(2*a) + (4*Sinh[x])/a - (3*Cosh[x]*Sinh[x])/(2*a) - (Cosh[x]^3*Sinh[
x])/(a + a*Cosh[x]) + (4*Sinh[x]^3)/(3*a)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2846

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((c + d*Sin[e +
 f*x])^(n - 1)/(a*f*(a + b*Sin[e + f*x]))), x] - Dist[d/(a*b), Int[(c + d*S
in[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e
 + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || E
qQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^4(x)}{a+a\cosh(x)}\,dx &= -\frac{\cosh^3(x)\sinh(x)}{a+a\cosh(x)}-\frac{\int \cosh^2(x)(3a-4a\cosh(x))\,dx}{a^2}\\ &= -\frac{\cosh^3(x)\sinh(x)}{a+a\cosh(x)}-\frac{3\int \cosh^2(x)\,dx}{a}+\frac{4\int \cosh^3(x)\,dx}{a}\\ &= -\frac{3\cosh(x)\sinh(x)}{2a}-\frac{\cosh^3(x)\sinh(x)}{a+a\cosh(x)}+\frac{(4i)\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-i\sinh(x)\right)}{a}-\\ &= -\frac{3x}{2a}+\frac{4\sinh(x)}{a}-\frac{3\cosh(x)\sinh(x)}{2a}-\frac{\cosh^3(x)\sinh(x)}{a+a\cosh(x)}+\frac{4\sinh^3(x)}{3a}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 53, normalized size = 0.98

$$\frac{\text{sech}\left(\frac{x}{2}\right)\left(-36x\cosh\left(\frac{x}{2}\right)+45\sinh\left(\frac{x}{2}\right)+18\sinh\left(\frac{3x}{2}\right)-2\sinh\left(\frac{5x}{2}\right)+\sinh\left(\frac{7x}{2}\right)\right)}{24a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^4/(a + a*Cosh[x]),x]
[Out] (Sech[x/2]*(-36*x*Cosh[x/2] + 45*Sinh[x/2] + 18*Sinh[(3*x)/2] - 2*Sinh[(5*x
)/2] + Sinh[(7*x)/2]))/(24*a)
```

Maple [A]
time = 0.44, size = 86, normalized size = 1.59

method	result
risch	$-\frac{3x}{2a}+\frac{e^{3x}}{24a}-\frac{e^{2x}}{8a}+\frac{7\,e^{x}}{8a}-\frac{7\,e^{-x}}{8a}+\frac{e^{-2x}}{8a}-\frac{e^{-3x}}{24a}-\frac{2}{(e^{x}+1)a}$
default	$\frac{\tanh\left(\frac{x}{2}\right)-\frac{1}{3\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}-\frac{1}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{5}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2}-\frac{1}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}+\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}-\frac{5}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)}-}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(tanh(1/2*x)-1/3/(tanh(1/2*x)-1)^3-1/(tanh(1/2*x)-1)^2-5/2/(tanh(1/2*x)
-1)+3/2*ln(tanh(1/2*x)-1)-1/3/(tanh(1/2*x)+1)^3+1/(tanh(1/2*x)+1)^2-5/2/(ta
nh(1/2*x)+1)-3/2*ln(tanh(1/2*x)+1))
```

Maxima [A]
time = 0.27, size = 66, normalized size = 1.22

$$-\frac{3\,x}{2\,a}-\frac{21\,e^{(-x)}-3\,e^{(-2\,x)}+e^{(-3\,x)}}{24\,a}-\frac{2\,e^{(-x)}-18\,e^{(-2\,x)}-69\,e^{(-3\,x)}-1}{24\,\left(ae^{(-3\,x)}+ae^{(-4\,x)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -3/2*x/a - 1/24*(21*e^(-x) - 3*e^(-2*x) + e^(-3*x))/a - 1/24*(2*e^(-x) - 18
*e^(-2*x) - 69*e^(-3*x) - 1)/(a*e^(-3*x) + a*e^(-4*x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(48) = 96$.
time = 0.37, size = 100, normalized size = 1.85

$$\frac{\cosh\left(x\right)^{4}+\left(4\,\cosh\left(x\right)-1\right)\sinh\left(x\right)^{3}+\sinh\left(x\right)^{4}-3\,\cosh\left(x\right)^{3}+\left(6\,\cosh\left(x\right)^{2}-9\,\cosh\left(x\right)+20\right)\sinh\left(x\right)^{2}-3\,\left(12\,x-1\right)\cosh\left(x\right)+20\,\cosh\left(x\right)^{2}+\left(4\,\cosh\left(x\right)^{3}-3\,\cosh\left(x\right)^{2}-36\,x+32\,\cosh\left(x\right)+39\right)\sinh\left(x\right)-36\,x-69}{24\,\left(a\cosh\left(x\right)+a\sinh\left(x\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/24*(cosh(x)^4 + (4*cosh(x) - 1)*sinh(x)^3 + sinh(x)^4 - 3*cosh(x)^3 + (6*
cosh(x)^2 - 9*cosh(x) + 20)*sinh(x)^2 - 3*(12*x - 1)*cosh(x) + 20*cosh(x)^2
 + (4*cosh(x)^3 - 3*cosh(x)^2 - 36*x + 32*cosh(x) + 39)*sinh(x) - 36*x - 69
)/(a*cosh(x) + a*sinh(x) + a)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(49) = 98$.
time = 0.72, size = 337, normalized size = 6.24

$$-\frac{9x\tanh^{6}\left(\frac{x}{2}\right)}{6a\tanh^{6}\left(\frac{x}{2}\right)-18a\tanh^{4}\left(\frac{x}{2}\right)+18a\tanh^{2}\left(\frac{x}{2}\right)-6a}+\frac{27x\tanh^{4}\left(\frac{x}{2}\right)}{6a\tanh^{6}\left(\frac{x}{2}\right)-18a\tanh^{4}\left(\frac{x}{2}\right)+18a\tanh^{2}\left(\frac{x}{2}\right)-6a}-\frac{27x\tanh^{2}\left(\frac{x}{2}\right)}{6a\tanh^{6}\left(\frac{x}{2}\right)-18a\tanh^{4}\left(\frac{x}{2}\right)+18a\tanh^{2}\left(\frac{x}{2}\right)-6a}+\frac{9x}{6a\tanh^{6}\left(\frac{x}{2}\right)-18a\tanh^{4}\left(\frac{x}{2}\right)+18a\tanh^{2}\left(\frac{x}{2}\right)-6a}+\frac{6\tanh^{7}\left(\frac{x}{2}\right)}{6a\tanh^{6}\left(\frac{x}{2}\right)-18a\tanh^{4}\left(\frac{x}{2}\right)+18a\tanh^{2}\left(\frac{x}{2}\right)-6a}-\frac{48\tanh^{5}\left(\frac{x}{2}\right)}{6a\tanh^{6}\left(\frac{x}{2}\right)-18a\tanh^{4}\left(\frac{x}{2}\right)+18a\tanh^{2}\left(\frac{x}{2}\right)-6a}+\frac{50\tanh^{3}\left(\frac{x}{2}\right)}{6a\tanh^{6}\left(\frac{x}{2}\right)-18a\tanh^{4}\left(\frac{x}{2}\right)+18a\tanh^{2}\left(\frac{x}{2}\right)-6a}-\frac{24\tanh\left(\frac{x}{2}\right)}{6a\tanh^{6}\left(\frac{x}{2}\right)-18a\tanh^{4}\left(\frac{x}{2}\right)+18a\tanh^{2}\left(\frac{x}{2}\right)-6a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**4/(a+a*cosh(x)),x)
```

```
[Out] -9*x*tanh(x/2)**6/(6*a*tanh(x/2)**6 - 18*a*tanh(x/2)**4 + 18*a*tanh(x/2)**2
 - 6*a) + 27*x*tanh(x/2)**4/(6*a*tanh(x/2)**6 - 18*a*tanh(x/2)**4 + 18*a*ta
```

```
nh(x/2)**2 - 6*a) - 27*x*tanh(x/2)**2/(6*a*tanh(x/2)**6 - 18*a*tanh(x/2)**4
 + 18*a*tanh(x/2)**2 - 6*a) + 9*x/(6*a*tanh(x/2)**6 - 18*a*tanh(x/2)**4 + 1
8*a*tanh(x/2)**2 - 6*a) + 6*tanh(x/2)**7/(6*a*tanh(x/2)**6 - 18*a*tanh(x/2)
**4 + 18*a*tanh(x/2)**2 - 6*a) - 48*tanh(x/2)**5/(6*a*tanh(x/2)**6 - 18*a*t
anh(x/2)**4 + 18*a*tanh(x/2)**2 - 6*a) + 50*tanh(x/2)**3/(6*a*tanh(x/2)**6
- 18*a*tanh(x/2)**4 + 18*a*tanh(x/2)**2 - 6*a) - 24*tanh(x/2)/(6*a*tanh(x/2
)**6 - 18*a*tanh(x/2)**4 + 18*a*tanh(x/2)**2 - 6*a)
```

Giac [A]
time = 0.40, size = 70, normalized size = 1.30

$$-\frac{3\,x}{2\,a}-\frac{\left(69\,e^{(3\,x)}+18\,e^{(2\,x)}-2\,e^{x}+1\right)e^{(-3\,x)}}{24\,a(e^{x}+1)}+\frac{a^{2}e^{(3\,x)}-3\,a^{2}e^{(2\,x)}+21\,a^{2}e^{x}}{24\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -3/2*x/a - 1/24*(69*e^(3*x) + 18*e^(2*x) - 2*e^x + 1)*e^(-3*x)/(a*(e^x + 1)
) + 1/24*(a^2*e^(3*x) - 3*a^2*e^(2*x) + 21*a^2*e^x)/a^3
```

Mupad [B]
time = 0.96, size = 70, normalized size = 1.30

$$\frac{\mathrm{e}^{-2\,x}}{8\,a}-\frac{7\,\mathrm{e}^{-x}}{8\,a}-\frac{\mathrm{e}^{2\,x}}{8\,a}-\frac{\mathrm{e}^{-3\,x}}{24\,a}+\frac{\mathrm{e}^{3\,x}}{24\,a}-\frac{3\,x}{2\,a}-\frac{2}{a\,\left(\mathrm{e}^{x}+1\right)}+\frac{7\,\mathrm{e}^{x}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a + a*cosh(x)),x)
```

```
[Out] exp(-2*x)/(8*a) - (7*exp(-x))/(8*a) - exp(2*x)/(8*a) - exp(-3*x)/(24*a) + e
xp(3*x)/(24*a) - (3*x)/(2*a) - 2/(a*(exp(x) + 1)) + (7*exp(x))/(8*a)
```

3.25 $\int \frac{\cosh^3(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=43

$$\frac{3x}{2a}-\frac{2\sinh(x)}{a}+\frac{3\cosh(x)\sinh(x)}{2a}-\frac{\cosh^2(x)\sinh(x)}{a+a\cosh(x)}$$

```
[Out] 3/2*x/a-2*sinh(x)/a+3/2*cosh(x)*sinh(x)/a-cosh(x)^2*sinh(x)/(a+a*cosh(x))
```

Rubi [A]
time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2846, 2813}

$$\frac{3x}{2a}-\frac{2\sinh(x)}{a}-\frac{\sinh(x)\cosh^2(x)}{a\cosh(x)+a}+\frac{3\sinh(x)\cosh(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(a + a*Cosh[x]),x]
```

```
[Out] (3*x)/(2*a) - (2*Sinh[x])/a + (3*Cosh[x]*Sinh[x])/(2*a) - (Cosh[x]^2*Sinh[x
])/(a + a*Cosh[x])
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2846

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((c + d*Sin[e +
 f*x])^(n - 1)/(a*f*(a + b*Sin[e + f*x]))), x] - Dist[d/(a*b), Int[(c + d*S
in[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e
 + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || E
qQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^3(x)}{a+a\cosh(x)}\,dx &= -\frac{\cosh^2(x)\sinh(x)}{a+a\cosh(x)}-\frac{\int \cosh(x)(2a-3a\cosh(x))\,dx}{a^2}\\ &= \frac{3x}{2a}-\frac{2\sinh(x)}{a}+\frac{3\cosh(x)\sinh(x)}{2a}-\frac{\cosh^2(x)\sinh(x)}{a+a\cosh(x)}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 45, normalized size = 1.05

$$\frac{\operatorname{sech}\left(\frac{x}{2}\right)\left(12x\cosh\left(\frac{x}{2}\right)-12\sinh\left(\frac{x}{2}\right)-3\sinh\left(\frac{3x}{2}\right)+\sinh\left(\frac{5x}{2}\right)\right)}{8a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(a + a*Cosh[x]),x]
[Out] (Sech[x/2]*(12*x*Cosh[x/2] - 12*Sinh[x/2] - 3*Sinh[(3*x)/2] + Sinh[(5*x)/2]
))/(8*a)
```

Maple [A]
time = 0.45, size = 70, normalized size = 1.63

method	result	size
risch	$\frac{3x}{2a}+\frac{e^{2x}}{8a}-\frac{e^{x}}{2a}+\frac{e^{-x}}{2a}-\frac{e^{-2x}}{8a}+\frac{2}{(e^{x}+1)a}$	53
default	$\frac{-\tanh\left(\frac{x}{2}\right)+\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{3}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2}-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{3}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2}}{a}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
[Out] 1/a*(-tanh(1/2*x)+1/2/(tanh(1/2*x)-1)^2+3/2/(tanh(1/2*x)-1)-3/2*ln(tanh(1/2
*x)-1)-1/2/(tanh(1/2*x)+1)^2+3/2/(tanh(1/2*x)+1)+3/2*ln(tanh(1/2*x)+1))
```

Maxima [A]
time = 0.26, size = 56, normalized size = 1.30

$$\frac{3\,x}{2\,a}+\frac{4\,e^{(-x)}-e^{(-2\,x)}}{8\,a}-\frac{3\,e^{(-x)}+20\,e^{(-2\,x)}-1}{8\,\left(ae^{(-2\,x)}+ae^{(-3\,x)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+a*cosh(x)),x, algorithm="maxima")
[Out] 3/2*x/a + 1/8*(4*e^(-x) - e^(-2*x))/a - 1/8*(3*e^(-x) + 20*e^(-2*x) - 1)/(a
*e^(-2*x) + a*e^(-3*x))
```

Fricas [A]
time = 0.37, size = 70, normalized size = 1.63

$$\frac{\cosh\left(x\right)^{3}+\left(3\,\cosh\left(x\right)-4\right)\sinh\left(x\right)^{2}+\sinh\left(x\right)^{3}+\left(12\,x-1\right)\cosh\left(x\right)-4\,\cosh\left(x\right)^{2}+\left(3\,\cosh\left(x\right)^{2}+12\,x-4\,\cosh\left(x\right)-7\right)\sinh\left(x\right)+12\,x+20}{8\left(a\cosh\left(x\right)+a\sinh\left(x\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/8*(cosh(x)^3 + (3*cosh(x) - 4)*sinh(x)^2 + sinh(x)^3 + (12*x - 1)*cosh(x)
 - 4*cosh(x)^2 + (3*cosh(x)^2 + 12*x - 4*cosh(x) - 7)*sinh(x) + 12*x + 20)/
(a*cosh(x) + a*sinh(x) + a)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(39) = 78.
time = 0.41, size = 189, normalized size = 4.40

$$\frac{3x\tanh^4\left(\frac{x}{2}\right)}{2a\tanh^4\left(\frac{x}{2}\right)-4a\tanh^2\left(\frac{x}{2}\right)+2a}-\frac{6x\tanh^2\left(\frac{x}{2}\right)}{2a\tanh^4\left(\frac{x}{2}\right)-4a\tanh^2\left(\frac{x}{2}\right)+2a}+\frac{3x}{2a\tanh^4\left(\frac{x}{2}\right)-4a\tanh^2\left(\frac{x}{2}\right)+2a}-\frac{2\tanh^5\left(\frac{x}{2}\right)}{2a\tanh^4\left(\frac{x}{2}\right)-4a\tanh^2\left(\frac{x}{2}\right)+2a}+\frac{10\tanh^3\left(\frac{x}{2}\right)}{2a\tanh^4\left(\frac{x}{2}\right)-4a\tanh^2\left(\frac{x}{2}\right)+2a}-\frac{4\tanh\left(\frac{x}{2}\right)}{2a\tanh^4\left(\frac{x}{2}\right)-4a\tanh^2\left(\frac{x}{2}\right)+2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(a+a*cosh(x)),x)
```

```
[Out] 3*x*tanh(x/2)**4/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 + 2*a) - 6*x*tanh(x/2
)**2/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 + 2*a) + 3*x/(2*a*tanh(x/2)**4 -
4*a*tanh(x/2)**2 + 2*a) - 2*tanh(x/2)**5/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)*
*2 + 2*a) + 10*tanh(x/2)**3/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 + 2*a) - 4
*tanh(x/2)/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 + 2*a)
```

Giac [A]
time = 0.40, size = 51, normalized size = 1.19

$$\frac{3\,x}{2\,a}+\frac{\left(20\,e^{(2\,x)}+3\,e^{x}-1\right)e^{(-2\,x)}}{8\,a(e^{x}+1)}+\frac{ae^{(2\,x)}-4\,ae^{x}}{8\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] 3/2*x/a + 1/8*(20*e^(2*x) + 3*e^x - 1)*e^(-2*x)/(a*(e^x + 1)) + 1/8*(a*e^(2
*x) - 4*a*e^x)/a^2
```

Mupad [B]
time = 0.92, size = 52, normalized size = 1.21

$$\frac{\mathrm{e}^{-x}}{2\,a}-\frac{\mathrm{e}^{-2\,x}}{8\,a}+\frac{\mathrm{e}^{2\,x}}{8\,a}+\frac{3\,x}{2\,a}+\frac{2}{a\,\left(\mathrm{e}^{x}+1\right)}-\frac{\mathrm{e}^{x}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a + a*cosh(x)),x)
```

```
[Out] exp(-x)/(2*a) - exp(-2*x)/(8*a) + exp(2*x)/(8*a) + (3*x)/(2*a) + 2/(a*(exp(
x) + 1)) - exp(x)/(2*a)
```

3.26 $\int \frac{\cosh^2(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=25

$$-\frac{x}{a}+\frac{\sinh(x)}{a}+\frac{\sinh(x)}{a(1+\cosh(x))}$$

```
[Out] -x/a+sinh(x)/a+sinh(x)/a/(1+cosh(x))
```

Rubi [A]
time = 0.05, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2825, 12, 2814, 2727}

$$-\frac{x}{a}+\frac{\sinh(x)}{a}+\frac{\sinh(x)}{a(\cosh(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(a + a*Cosh[x]),x]
```

```
[Out] -(x/a) + Sinh[x]/a + Sinh[x]/(a*(1 + Cosh[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2825

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*(Cos[e + f*x]/(d*f)), x] + Dist[1/d, I
nt[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\cosh^2(x)}{a+a\cosh(x)}\,dx = \frac{\sinh(x)}{a} - \frac{\int \frac{a\cosh(x)}{a+a\cosh(x)}\,dx}{a}$$
$$= \frac{\sinh(x)}{a} - \int \frac{\cosh(x)}{a+a\cosh(x)}\,dx$$
$$= -\frac{x}{a} + \frac{\sinh(x)}{a} + \int \frac{1}{a+a\cosh(x)}\,dx$$
$$= -\frac{x}{a} + \frac{\sinh(x)}{a} + \frac{\sinh(x)}{a+a\cosh(x)}$$

Mathematica [A]
time = 0.04, size = 32, normalized size = 1.28

$$\frac{-2x + \operatorname{sech}\left(\frac{x}{2}\right)\sinh\left(\frac{3x}{2}\right) + 3\tanh\left(\frac{x}{2}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(a + a*Cosh[x]),x]
[Out] (-2*x + Sech[x/2]*Sinh[(3*x)/2] + 3*Tanh[x/2])/(2*a)
```

Maple [A]
time = 0.43, size = 46, normalized size = 1.84

method	result	size
risch	$-\frac{x}{a}+\frac{e^{x}}{2a}-\frac{e^{-x}}{2a}-\frac{2}{(e^{x}+1)a}$	35
default	$\frac{\tanh\left(\frac{x}{2}\right)-\frac{1}{\tanh\left(\frac{x}{2}\right)-1}+\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\frac{1}{\tanh\left(\frac{x}{2}\right)+1}-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
[Out] 1/a*(tanh(1/2*x)-1/(tanh(1/2*x)-1)+ln(tanh(1/2*x)-1)-1/(tanh(1/2*x)+1)-ln(t
anh(1/2*x)+1))
```

Maxima [A]
time = 0.26, size = 41, normalized size = 1.64

$$-\frac{x}{a} + \frac{5\,e^{(-x)} + 1}{2\left(ae^{(-x)} + ae^{(-2\,x)}\right)} - \frac{e^{(-x)}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -x/a + 1/2*(5*e^(-x) + 1)/(a*e^(-x) + a*e^(-2*x)) - 1/2*e^(-x)/a
```

Fricas [A]
time = 0.40, size = 47, normalized size = 1.88

$$-\frac{2\,x\cosh\left(x\right)-\cosh\left(x\right)^{2}+2\,\left(x-\cosh\left(x\right)-1\right)\sinh\left(x\right)-\sinh\left(x\right)^{2}+2\,x+5}{2\,\left(a\cosh\left(x\right)+a\sinh\left(x\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*x*cosh(x) - cosh(x)^2 + 2*(x - cosh(x) - 1)*sinh(x) - sinh(x)^2 + 2
*x + 5)/(a*cosh(x) + a*sinh(x) + a)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(17) = 34$.
time = 0.25, size = 63, normalized size = 2.52

$$-\frac{x\tanh^{2}\left(\frac{x}{2}\right)}{a\tanh^{2}\left(\frac{x}{2}\right)-a}+\frac{x}{a\tanh^{2}\left(\frac{x}{2}\right)-a}+\frac{\tanh^{3}\left(\frac{x}{2}\right)}{a\tanh^{2}\left(\frac{x}{2}\right)-a}-\frac{3\tanh\left(\frac{x}{2}\right)}{a\tanh^{2}\left(\frac{x}{2}\right)-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(a+a*cosh(x)),x)
```

```
[Out] -x*tanh(x/2)**2/(a*tanh(x/2)**2 - a) + x/(a*tanh(x/2)**2 - a) + tanh(x/2)**
3/(a*tanh(x/2)**2 - a) - 3*tanh(x/2)/(a*tanh(x/2)**2 - a)
```

Giac [A]
time = 0.40, size = 35, normalized size = 1.40

$$-\frac{x}{a}-\frac{(5\,e^{x}+1)e^{(-x)}}{2\,a(e^{x}+1)}+\frac{e^{x}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -x/a - 1/2*(5*e^x + 1)*e^(-x)/(a*(e^x + 1)) + 1/2*e^x/a
```

Mupad [B]
time = 0.90, size = 34, normalized size = 1.36

$$\frac{\mathrm{e}^{x}}{2\,a}-\frac{x}{a}-\frac{2}{a\,\left(\mathrm{e}^{x}+1\right)}-\frac{\mathrm{e}^{-x}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a + a*cosh(x)),x)
```

```
[Out] exp(x)/(2*a) - x/a - 2/(a*(exp(x) + 1)) - exp(-x)/(2*a)
```

3.27 $\int \frac{\cosh(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=18

$$\frac{x}{a}-\frac{\sinh(x)}{a+a\cosh(x)}$$

```
[Out] x/a-sinh(x)/(a+a*cosh(x))
```

Rubi [A]
time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2814, 2727}

$$\frac{x}{a}-\frac{\sinh(x)}{a\cosh(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/(a + a*Cosh[x]),x]
```

```
[Out] x/a - Sinh[x]/(a + a*Cosh[x])
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(x)}{a+a\cosh(x)}\,dx &= \frac{x}{a}-\int \frac{1}{a+a\cosh(x)}\,dx\\ &= \frac{x}{a}-\frac{\sinh(x)}{a+a\cosh(x)}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 14, normalized size = 0.78

$$\frac{x-\tanh\left(\frac{x}{2}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(a + a*Cosh[x]),x]
[Out] (x - Tanh[x/2])/a
```

Maple [A]
time = 0.44, size = 28, normalized size = 1.56

method	result	size
risch	$\frac{x}{a}+\frac{2}{(e^{x}+1)a}$	18
default	$\frac{-\tanh\left(\frac{x}{2}\right)-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
[Out] 1/a*(-tanh(1/2*x)-ln(tanh(1/2*x)-1)+ln(tanh(1/2*x)+1))
```

Maxima [A]
time = 0.27, size = 18, normalized size = 1.00

$$\frac{x}{a}-\frac{2}{ae^{(-x)}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*cosh(x)),x, algorithm="maxima")
[Out] x/a - 2/(a*e^(-x) + a)
```

Fricas [A]
time = 0.35, size = 24, normalized size = 1.33

$$\frac{x\cosh(x)+x\sinh(x)+x+2}{a\cosh(x)+a\sinh(x)+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*cosh(x)),x, algorithm="fricas")
[Out] (x*cosh(x) + x*sinh(x) + x + 2)/(a*cosh(x) + a*sinh(x) + a)
```

Sympy [A]
time = 0.14, size = 8, normalized size = 0.44

$$\frac{x}{a}-\frac{\tanh\left(\frac{x}{2}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*cosh(x)),x)
```

```
[Out] x/a - tanh(x/2)/a
```

Giac [A]
time = 0.39, size = 17, normalized size = 0.94

$$\frac{x}{a} + \frac{2}{a(e^x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] x/a + 2/(a*(e^x + 1))
```

Mupad [B]
time = 0.87, size = 17, normalized size = 0.94

$$\frac{x}{a} + \frac{2}{a\,\left(\mathrm{e}^x + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a + a*cosh(x)),x)
```

```
[Out] x/a + 2/(a*(exp(x) + 1))
```

3.28 $\int \frac{\text{sech}(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=20

$$\frac{\text{ArcTan}(\sinh(x))}{a}-\frac{\sinh(x)}{a+a\cosh(x)}$$

```
[Out] arctan(sinh(x))/a-sinh(x)/(a+a*cosh(x))
```

Rubi [A]

time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2826, 3855, 2727}

$$\frac{\text{ArcTan}(\sinh(x))}{a}-\frac{\sinh(x)}{a\cosh(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]/(a + a*Cosh[x]),x]
```

```
[Out] ArcTan[Sinh[x]]/a - Sinh[x]/(a + a*Cosh[x])
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2826

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\text{sech}(x)}{a+a\cosh(x)}\,dx&=\frac{\int \text{sech}(x)\,dx}{a}-\int \frac{1}{a+a\cosh(x)}\,dx\\&=\frac{\tan^{-1}(\sinh(x))}{a}-\frac{\sinh(x)}{a+a\cosh(x)}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 1.10

$$\frac{2\text{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)-\tanh\left(\frac{x}{2}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(a + a*Cosh[x]),x]
[Out] (2*ArcTan[Tanh[x/2]] - Tanh[x/2])/a
```

Maple [A]
time = 0.48, size = 19, normalized size = 0.95

method	result	size
default	$\frac{-\tanh\left(\frac{x}{2}\right)+2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$	19
risch	$\frac{2}{\left(e^{x}+1\right)a}+\frac{i\ln\left(e^{x}+i\right)}{a}-\frac{i\ln\left(e^{x}-i\right)}{a}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
[Out] 1/a*(-tanh(1/2*x)+2*arctan(tanh(1/2*x)))
```

Maxima [A]
time = 0.50, size = 23, normalized size = 1.15

$$-\frac{2\arctan\left(e^{(-x)}\right)}{a}-\frac{2}{ae^{(-x)}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+a*cosh(x)),x, algorithm="maxima")
[Out] -2*arctan(e^(-x))/a - 2/(a*e^(-x) + a)
```

Fricas [A]
time = 0.40, size = 29, normalized size = 1.45

$$\frac{2\left(\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)+1\right)}{a\cosh\left(x\right)+a\sinh\left(x\right)+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+a*cosh(x)),x, algorithm="fricas")
[Out] 2*((cosh(x) + sinh(x) + 1)*arctan(cosh(x) + sinh(x)) + 1)/(a*cosh(x) + a*si
nh(x) + a)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{sech}(x)}{\cosh(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+a*cosh(x)),x)
[Out] Integral(sech(x)/(cosh(x) + 1), x)/a
```

Giac [**A**]
time = 0.41, size = 20, normalized size = 1.00

$$\frac{2\arctan(e^x)}{a} + \frac{2}{a(e^x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+a*cosh(x)),x, algorithm="giac")
[Out] 2*arctan(e^x)/a + 2/(a*(e^x + 1))
```

Mupad [**B**]
time = 0.88, size = 31, normalized size = 1.55

$$\frac{2}{a\,(e^x + 1)} + \frac{2\,\mathrm{atan}\left(\frac{e^x\,\sqrt{a^2}}{a}\right)}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)*(a + a*cosh(x))),x)
[Out] 2/(a*(exp(x) + 1)) + (2*atan((exp(x)*(a^2)^(1/2))/a))/(a^2)^(1/2)
```

3.29 $\int \frac{\operatorname{sech}^2(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=28

$$-\frac{\operatorname{ArcTan}(\sinh(x))}{a}+\frac{2\tanh(x)}{a}-\frac{\tanh(x)}{a+a\cosh(x)}$$

```
[Out] -arctan(sinh(x))/a+2*tanh(x)/a-tanh(x)/(a+a*cosh(x))
```

Rubi [A]
time = 0.05, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2847, 2827, 3852, 8, 3855}

$$-\frac{\operatorname{ArcTan}(\sinh(x))}{a}+\frac{2\tanh(x)}{a}-\frac{\tanh(x)}{a\cosh(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(a + a*Cosh[x]),x]
```

```
[Out] -(ArcTan[Sinh[x]]/a) + (2*Tanh[x])/a - Tanh[x]/(a + a*Cosh[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2847

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Dist[d/(a*(b*c - a*d))
, Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c
^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^2(x)}{a+a\cosh(x)}\,dx &= -\frac{\tanh(x)}{a+a\cosh(x)}-\frac{\int (-2a+a\cosh(x))\operatorname{sech}^2(x)\,dx}{a^2}\\
&= -\frac{\tanh(x)}{a+a\cosh(x)}-\frac{\int \operatorname{sech}(x)\,dx}{a}+\frac{2\int \operatorname{sech}^2(x)\,dx}{a}\\
&= -\frac{\tan^{-1}(\sinh(x))}{a}-\frac{\tanh(x)}{a+a\cosh(x)}+\frac{(2i)\text{Subst}\left(\int 1\,dx,x,-i\tanh(x)\right)}{a}\\
&= -\frac{\tan^{-1}(\sinh(x))}{a}+\frac{2\tanh(x)}{a}-\frac{\tanh(x)}{a+a\cosh(x)}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 43, normalized size = 1.54

$$\frac{2\cosh\left(\frac{x}{2}\right)\left(\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)\left(-2\text{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)+\tanh(x)\right)\right)}{a(1+\cosh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(a + a*Cosh[x]),x]
[Out] (2*Cosh[x/2]*(Sinh[x/2] + Cosh[x/2]*(-2*ArcTan[Tanh[x/2]] + Tanh[x])))/(a*(
1 + Cosh[x]))
```

Maple [A]
time = 0.54, size = 33, normalized size = 1.18

method	result	size
default	$\frac{\tanh\left(\frac{x}{2}\right)+\frac{2\tanh\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)+1}-2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$	33
risch	$-\frac{2\left(e^{2x}+e^x+2\right)}{\left(e^x+1\right)a\left(1+e^{2x}\right)}+\frac{i\ln\left(e^x-i\right)}{a}-\frac{i\ln\left(e^x+i\right)}{a}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
[Out] 1/a*(tanh(1/2*x)+2*tanh(1/2*x)/(tanh(1/2*x)^2+1)-2*arctan(tanh(1/2*x)))
```

Maxima [A]
time = 0.47, size = 45, normalized size = 1.61

$$\frac{2\left(e^{(-x)} + e^{(-2x)} + 2\right)}{ae^{(-x)} + ae^{(-2x)} + ae^{(-3x)} + a} + \frac{2\arctan\left(e^{(-x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] 2*(e^(-x) + e^(-2*x) + 2)/(a*e^(-x) + a*e^(-2*x) + a*e^(-3*x) + a) + 2*arct
an(e^(-x))/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(28) = 56$.
time = 0.34, size = 127, normalized size = 4.54

$$-\frac{2\left(\left(\cosh(x)^3 + (3\cosh(x) + 1)\sinh(x)^2 + \sinh(x)^3 + \cosh(x)^2 + \left(3\cosh(x)^2 + 2\cosh(x) + 1\right)\sinh(x) + \cosh(x) + 1\right)\arctan\left(\cosh(x) + \sinh(x)\right) + \cosh(x)^2 + (2\cosh(x) + 1)\sinh(x) + \sinh(x)^2 + \cosh(x) + 2\right)}{a\cosh(x)^3 + a\sinh(x)^3 + a\cosh(x)^2 + (3a\cosh(x) + a)\sinh(x)^2 + a\cosh(x) + \left(3a\cosh(x)^2 + 2a\cosh(x) + a\right)\sinh(x) + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] -2*((cosh(x)^3 + (3*cosh(x) + 1)*sinh(x)^2 + sinh(x)^3 + cosh(x)^2 + (3*cos
h(x)^2 + 2*cosh(x) + 1)*sinh(x) + cosh(x) + 1)*arctan(cosh(x) + sinh(x)) +
cosh(x)^2 + (2*cosh(x) + 1)*sinh(x) + sinh(x)^2 + cosh(x) + 2)/(a*cosh(x)^3
 + a*sinh(x)^3 + a*cosh(x)^2 + (3*a*cosh(x) + a)*sinh(x)^2 + a*cosh(x) + (3
*a*cosh(x)^2 + 2*a*cosh(x) + a)*sinh(x) + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{sech}^2(x)}{\cosh(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(a+a*cosh(x)),x)
```

```
[Out] Integral(sech(x)**2/(cosh(x) + 1), x)/a
```

Giac [A]
time = 0.37, size = 36, normalized size = 1.29

$$-\frac{2\arctan\left(e^x\right)}{a} - \frac{2\left(e^{(2x)} + e^x + 2\right)}{a\left(e^{(3x)} + e^{(2x)} + e^x + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -2*arctan(e^x)/a - 2*(e^(2*x) + e^x + 2)/(a*(e^(3*x) + e^(2*x) + e^x + 1))
```

Mupad [B]
time = 0.89, size = 58, normalized size = 2.07

$$-\frac{\frac{2\,{\mathrm{e}}^{2\,x}}{a}+\frac{4}{a}+\frac{2\,{\mathrm{e}}^x}{a}}{{\mathrm{e}}^{2\,x}+{\mathrm{e}}^{3\,x}+{\mathrm{e}}^x+1}-\frac{2\,\mathrm{atan}\left(\frac{{\mathrm{e}}^x\,\sqrt{a^2}}{a}\right)}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2*(a + a*cosh(x))),x)
```

```
[Out] - ((2*exp(2*x))/a + 4/a + (2*exp(x))/a)/(exp(2*x) + exp(3*x) + exp(x) + 1)
- (2*atan((exp(x)*(a^2)^(1/2))/a))/(a^2)^(1/2)
```

3.30 $\int \frac{\text{sech}^3(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=43

$$\frac{3\text{ArcTan}(\sinh(x))}{2a}-\frac{2\tanh(x)}{a}+\frac{3\text{sech}(x)\tanh(x)}{2a}-\frac{\text{sech}(x)\tanh(x)}{a+a\cosh(x)}$$

```
[Out] 3/2*arctan(sinh(x))/a-2*tanh(x)/a+3/2*sech(x)*tanh(x)/a-sech(x)*tanh(x)/(a+
a*cosh(x))
```

Rubi [A]
time = 0.06, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2847, 2827, 3853, 3855, 3852, 8}

$$\frac{3\text{ArcTan}(\sinh(x))}{2a}-\frac{2\tanh(x)}{a}+\frac{3\tanh(x)\text{sech}(x)}{2a}-\frac{\tanh(x)\text{sech}(x)}{a\cosh(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^3/(a + a*Cosh[x]),x]
```

```
[Out] (3*ArcTan[Sinh[x]])/(2*a) - (2*Tanh[x])/a + (3*Sech[x]*Tanh[x])/(2*a) - (Se
ch[x]*Tanh[x])/(a + a*Cosh[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2847

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Dist[d/(a*(b*c - a*d))
, Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c
^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^3(x)}{a+a\cosh(x)}\,dx &= -\frac{\operatorname{sech}(x)\tanh(x)}{a+a\cosh(x)}-\frac{\int(-3a+2a\cosh(x))\operatorname{sech}^3(x)\,dx}{a^2}\\ &= -\frac{\operatorname{sech}(x)\tanh(x)}{a+a\cosh(x)}-\frac{2\int \operatorname{sech}^2(x)\,dx}{a}+\frac{3\int \operatorname{sech}^3(x)\,dx}{a}\\ &= \frac{3\operatorname{sech}(x)\tanh(x)}{2a}-\frac{\operatorname{sech}(x)\tanh(x)}{a+a\cosh(x)}-\frac{(2i)\operatorname{Subst}\left(\int 1\,dx,x,-i\tanh(x)\right)}{a}+\frac{3\int \operatorname{sech}(}{2a}\\ &= \frac{3\tan^{-1}(\sinh(x))}{2a}-\frac{2\tanh(x)}{a}+\frac{3\operatorname{sech}(x)\tanh(x)}{2a}-\frac{\operatorname{sech}(x)\tanh(x)}{a+a\cosh(x)}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 49, normalized size = 1.14

$$\frac{\cosh\left(\frac{x}{2}\right)\left(-2\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)\left(6\operatorname{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)+(-2+\operatorname{sech}(x))\tanh(x)\right)\right)}{a(1+\cosh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^3/(a + a*Cosh[x]),x]
```

```
[Out] (Cosh[x/2]*(-2*Sinh[x/2] + Cosh[x/2]*(6*ArcTan[Tanh[x/2]] + (-2 + Sech[x])*
Tanh[x])))/(a*(1 + Cosh[x]))
```

Maple [A]
time = 0.53, size = 46, normalized size = 1.07

method	result	size

default	$\frac{-\tanh\left(\frac{x}{2}\right)+\frac{-3\left(\tanh^{3}\left(\frac{x}{2}\right)\right)-\tanh\left(\frac{x}{2}\right)}{\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)^{2}}+3\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$	46
risch	$\frac{3\,e^{4x}+3\,e^{3x}+5\,e^{2x}+e^{x}+4}{\left(1+e^{2x}\right)^{2}a\left(e^{x}+1\right)}+\frac{3i\ln\left(e^{x}+i\right)}{2a}-\frac{3i\ln\left(e^{x}-i\right)}{2a}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^3/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-tanh(1/2*x)+2*(-3/2*tanh(1/2*x)^3-1/2*tanh(1/2*x))/(tanh(1/2*x)^2+1)^
2+3*arctan(tanh(1/2*x)))
```

Maxima [**A**]
time = 0.47, size = 73, normalized size = 1.70

$$-\frac{e^{(-x)}+5\,e^{(-2\,x)}+3\,e^{(-3\,x)}+3\,e^{(-4\,x)}+4}{ae^{(-x)}+2\,ae^{(-2\,x)}+2\,ae^{(-3\,x)}+ae^{(-4\,x)}+ae^{(-5\,x)}+a}-\frac{3\,\arctan\left(e^{(-x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -(e^(-x) + 5*e^(-2*x) + 3*e^(-3*x) + 3*e^(-4*x) + 4)/(a*e^(-x) + 2*a*e^(-2*
x) + 2*a*e^(-3*x) + a*e^(-4*x) + a*e^(-5*x) + a) - 3*arctan(e^(-x))/a
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(39) = 78$.
time = 0.42, size = 325, normalized size = 7.56

$$\frac{3\,\cosh\left(x\right)^{4}+3\,\left(4\,\cosh\left(x\right)+1\right)\sinh\left(x\right)^{3}+3\,\sinh\left(x\right)^{4}+3\,\cosh\left(x\right)^{3}+\left(18\,\cosh\left(x\right)^{2}+9\,\cosh\left(x\right)+5\right)\sinh\left(x\right)^{2}+3\,\left(\cosh\left(x\right)^{5}+\left(5\,\cosh\left(x\right)+1\right)\sinh\left(x\right)^{4}+\sinh\left(x\right)^{5}+\cosh\left(x\right)^{4}+2\,\left(5\,\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)+1\right)\sinh\left(x\right)^{3}+2\,\cosh\left(x\right)^{3}+2\,\left(5\,\cosh\left(x\right)^{3}+3\,\cosh\left(x\right)^{2}+3\,\cosh\left(x\right)+1\right)\sinh\left(x\right)^{2}+2\,\cosh\left(x\right)^{2}+\left(5\,\cosh\left(x\right)^{4}+4\,\cosh\left(x\right)^{3}+6\,\cosh\left(x\right)^{2}+4\,\cosh\left(x\right)+1\right)\sinh\left(x\right)+\cosh\left(x\right)+1\right)\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)+5\,\cosh\left(x\right)^{2}+\left(12\,\cosh\left(x\right)^{3}+9\,\cosh\left(x\right)^{2}+10\,\cosh\left(x\right)+1\right)\sinh\left(x\right)+\cosh\left(x\right)+4}{a\cosh\left(x\right)^{5}+a\sinh\left(x\right)^{5}+a\cosh\left(x\right)^{4}+\left(5\,a\cosh\left(x\right)+a\right)\sinh\left(x\right)^{4}+2\,a\cosh\left(x\right)^{3}+2\,\left(5\,a\cosh\left(x\right)^{2}+2\,a\cosh\left(x\right)+a\right)\sinh\left(x\right)^{3}+2\,a\cosh\left(x\right)^{2}+2\,\left(5\,a\cosh\left(x\right)^{3}+3\,a\cosh\left(x\right)^{2}+3\,a\cosh\left(x\right)+a\right)\sinh\left(x\right)^{2}+a\cosh\left(x\right)+\left(5\,a\cosh\left(x\right)^{4}+4\,a\cosh\left(x\right)^{3}+6\,a\cosh\left(x\right)^{2}+4\,a\cosh\left(x\right)+a\right)\sinh\left(x\right)+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] (3*cosh(x)^4 + 3*(4*cosh(x) + 1)*sinh(x)^3 + 3*sinh(x)^4 + 3*cosh(x)^3 + (1
8*cosh(x)^2 + 9*cosh(x) + 5)*sinh(x)^2 + 3*(cosh(x)^5 + (5*cosh(x) + 1)*sin
h(x)^4 + sinh(x)^5 + cosh(x)^4 + 2*(5*cosh(x)^2 + 2*cosh(x) + 1)*sinh(x)^3
+ 2*cosh(x)^3 + 2*(5*cosh(x)^3 + 3*cosh(x)^2 + 3*cosh(x) + 1)*sinh(x)^2 + 2
*cosh(x)^2 + (5*cosh(x)^4 + 4*cosh(x)^3 + 6*cosh(x)^2 + 4*cosh(x) + 1)*sinh
(x) + cosh(x) + 1)*arctan(cosh(x) + sinh(x)) + 5*cosh(x)^2 + (12*cosh(x)^3
+ 9*cosh(x)^2 + 10*cosh(x) + 1)*sinh(x) + cosh(x) + 4)/(a*cosh(x)^5 + a*sin
h(x)^5 + a*cosh(x)^4 + (5*a*cosh(x) + a)*sinh(x)^4 + 2*a*cosh(x)^3 + 2*(5*a
*cosh(x)^2 + 2*a*cosh(x) + a)*sinh(x)^3 + 2*a*cosh(x)^2 + 2*(5*a*cosh(x)^3
+ 3*a*cosh(x)^2 + 3*a*cosh(x) + a)*sinh(x)^2 + a*cosh(x) + (5*a*cosh(x)^4 +
 4*a*cosh(x)^3 + 6*a*cosh(x)^2 + 4*a*cosh(x) + a)*sinh(x) + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{sech}^3(x)}{\cosh(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**3/(a+a*cosh(x)),x)
```

```
[Out] Integral(sech(x)**3/(cosh(x) + 1), x)/a
```

Giac [A]
time = 0.39, size = 48, normalized size = 1.12

$$\frac{3 \arctan(e^x)}{a} + \frac{e^{(3x)} + 2e^{(2x)} - e^x + 2}{a\left(e^{(2x)} + 1\right)^2} + \frac{2}{a(e^x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] 3*arctan(e^x)/a + (e^(3*x) + 2*e^(2*x) - e^x + 2)/(a*(e^(2*x) + 1)^2) + 2/(
a*(e^x + 1))
```

Mupad [B]
time = 0.91, size = 73, normalized size = 1.70

$$\frac{2}{a\,(e^x+1)} + \frac{\frac{2}{a} + \frac{e^x}{a}}{e^{2x}+1} + \frac{3\,\operatorname{atan}\left(\frac{e^x\sqrt{a^2}}{a}\right)}{\sqrt{a^2}} - \frac{2\,e^x}{a\,(2\,e^{2x}+e^{4x}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^3*(a + a*cosh(x))),x)
```

```
[Out] 2/(a*(exp(x) + 1)) + (2/a + exp(x)/a)/(exp(2*x) + 1) + (3*atan((exp(x)*(a^2
)^(1/2))/a))/(a^2)^(1/2) - (2*exp(x))/(a*(2*exp(2*x) + exp(4*x) + 1))
```

3.31 $\int \frac{\operatorname{sech}^4(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=56

$$-\frac{3\text{ArcTan}(\sinh(x))}{2a}+\frac{4\tanh(x)}{a}-\frac{3\text{sech}(x)\tanh(x)}{2a}-\frac{\text{sech}^2(x)\tanh(x)}{a+a\cosh(x)}-\frac{4\tanh^3(x)}{3a}$$

```
[Out] -3/2*arctan(sinh(x))/a+4*tanh(x)/a-3/2*sech(x)*tanh(x)/a-sech(x)^2*tanh(x)/
(a+a*cosh(x))-4/3*tanh(x)^3/a
```

Rubi [A]
time = 0.06, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2847, 2827, 3852, 3853, 3855}

$$-\frac{3\text{ArcTan}(\sinh(x))}{2a}-\frac{4\tanh^3(x)}{3a}+\frac{4\tanh(x)}{a}-\frac{3\tanh(x)\text{sech}(x)}{2a}-\frac{\tanh(x)\text{sech}^2(x)}{a\cosh(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^4/(a + a*Cosh[x]),x]
```

```
[Out] (-3*ArcTan[Sinh[x]])/(2*a) + (4*Tanh[x])/a - (3*Sech[x]*Tanh[x])/(2*a) - (S
ech[x]^2*Tanh[x])/(a + a*Cosh[x]) - (4*Tanh[x]^3)/(3*a)
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2847

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Dist[d/(a*(b*c - a*d))
, Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c
^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^4(x)}{a+a\cosh(x)}\,dx &= -\frac{\operatorname{sech}^2(x)\tanh(x)}{a+a\cosh(x)}-\frac{\int(-4a+3a\cosh(x))\operatorname{sech}^4(x)\,dx}{a^2}\\ &= -\frac{\operatorname{sech}^2(x)\tanh(x)}{a+a\cosh(x)}-\frac{3\int \operatorname{sech}^3(x)\,dx}{a}+\frac{4\int \operatorname{sech}^4(x)\,dx}{a}\\ &= -\frac{3\operatorname{sech}(x)\tanh(x)}{2a}-\frac{\operatorname{sech}^2(x)\tanh(x)}{a+a\cosh(x)}+\frac{(4i)\operatorname{Subst}\left(\int (1+x^2)\,dx,x,-i\tanh(x)\right)}{a}-\\ &= -\frac{3\tan^{-1}(\sinh(x))}{2a}+\frac{4\tanh(x)}{a}-\frac{3\operatorname{sech}(x)\tanh(x)}{2a}-\frac{\operatorname{sech}^2(x)\tanh(x)}{a+a\cosh(x)}-\frac{4\tanh^3(x}{3a}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 60, normalized size = 1.07

$$\frac{\cosh\left(\frac{x}{2}\right)\left(6\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)\left(-18\operatorname{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)+\left(10-3\operatorname{sech}(x)+2\operatorname{sech}^2(x)\right)\tanh(x)\right)\right)}{3a(1+\cosh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^4/(a + a*Cosh[x]),x]
```

```
[Out] (Cosh[x/2]*(6*Sinh[x/2] + Cosh[x/2]*(-18*ArcTan[Tanh[x/2]] + (10 - 3*Sech[x
] + 2*Sech[x]^2)*Tanh[x])))/(3*a*(1 + Cosh[x]))
```

Maple [A]
time = 0.56, size = 52, normalized size = 0.93

method	result	size
default	$\frac{\tanh\left(\frac{x}{2}\right)-\frac{8\left(-\frac{5\left(\tanh^5\left(\frac{x}{2}\right)\right)}{8}-\frac{2\left(\tanh^3\left(\frac{x}{2}\right)\right)}{3}-\frac{3\tanh\left(\frac{x}{2}\right)}{8}\right)}{\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^3}-3\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$	52

risch	$-\frac{9\,\mathrm{e}^{6x}+9\,\mathrm{e}^{5x}+24\,\mathrm{e}^{4x}+24\,\mathrm{e}^{3x}+39\,\mathrm{e}^{2x}+7\,\mathrm{e}^{x}+16}{3(1+\mathrm{e}^{2x})^{3}a(\mathrm{e}^{x}+1)}+\frac{3i\ln(\mathrm{e}^{x}-i)}{2a}-\frac{3i\ln(\mathrm{e}^{x}+i)}{2a}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^4/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(tanh(1/2*x)-8*(-5/8*tanh(1/2*x)^5-2/3*tanh(1/2*x)^3-3/8*tanh(1/2*x))/(
tanh(1/2*x)^2+1)^3-3*arctan(tanh(1/2*x)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(50) = 100$.
time = 0.51, size = 101, normalized size = 1.80

$$\frac{7\,e^{(-x)}+39\,e^{(-2\,x)}+24\,e^{(-3\,x)}+24\,e^{(-4\,x)}+9\,e^{(-5\,x)}+9\,e^{(-6\,x)}+16}{3\left(ae^{(-x)}+3\,ae^{(-2\,x)}+3\,ae^{(-3\,x)}+3\,ae^{(-4\,x)}+3\,ae^{(-5\,x)}+ae^{(-6\,x)}+ae^{(-7\,x)}+a\right)}+\frac{3\,\arctan\left(e^{(-x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] 1/3*(7*e^(-x) + 39*e^(-2*x) + 24*e^(-3*x) + 24*e^(-4*x) + 9*e^(-5*x) + 9*e^
(-6*x) + 16)/(a*e^(-x) + 3*a*e^(-2*x) + 3*a*e^(-3*x) + 3*a*e^(-4*x) + 3*a*e
^(-5*x) + a*e^(-6*x) + a*e^(-7*x) + a) + 3*arctan(e^(-x))/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(50) = 100$.
time = 0.39, size = 600, normalized size = 10.71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] -1/3*(9*cosh(x)^6 + 9*(6*cosh(x) + 1)*sinh(x)^5 + 9*sinh(x)^6 + 9*cosh(x)^5
 + 3*(45*cosh(x)^2 + 15*cosh(x) + 8)*sinh(x)^4 + 24*cosh(x)^4 + 6*(30*cosh(
x)^3 + 15*cosh(x)^2 + 16*cosh(x) + 4)*sinh(x)^3 + 24*cosh(x)^3 + 3*(45*cosh
(x)^4 + 30*cosh(x)^3 + 48*cosh(x)^2 + 24*cosh(x) + 13)*sinh(x)^2 + 9*(cosh(
x)^7 + (7*cosh(x) + 1)*sinh(x)^6 + sinh(x)^7 + cosh(x)^6 + 3*(7*cosh(x)^2 +
 2*cosh(x) + 1)*sinh(x)^5 + 3*cosh(x)^5 + (35*cosh(x)^3 + 15*cosh(x)^2 + 15
*cosh(x) + 3)*sinh(x)^4 + 3*cosh(x)^4 + (35*cosh(x)^4 + 20*cosh(x)^3 + 30*c
osh(x)^2 + 12*cosh(x) + 3)*sinh(x)^3 + 3*cosh(x)^3 + 3*(7*cosh(x)^5 + 5*cos
h(x)^4 + 10*cosh(x)^3 + 6*cosh(x)^2 + 3*cosh(x) + 1)*sinh(x)^2 + 3*cosh(x)^
2 + (7*cosh(x)^6 + 6*cosh(x)^5 + 15*cosh(x)^4 + 12*cosh(x)^3 + 9*cosh(x)^2
+ 6*cosh(x) + 1)*sinh(x) + cosh(x) + 1)*arctan(cosh(x) + sinh(x)) + 39*cosh
(x)^2 + (54*cosh(x)^5 + 45*cosh(x)^4 + 96*cosh(x)^3 + 72*cosh(x)^2 + 78*cos
h(x) + 7)*sinh(x) + 7*cosh(x) + 16)/(a*cosh(x)^7 + a*sinh(x)^7 + a*cosh(x)^
```

```
6 + (7*a*cosh(x) + a)*sinh(x)^6 + 3*a*cosh(x)^5 + 3*(7*a*cosh(x)^2 + 2*a*co
sh(x) + a)*sinh(x)^5 + 3*a*cosh(x)^4 + (35*a*cosh(x)^3 + 15*a*cosh(x)^2 + 1
5*a*cosh(x) + 3*a)*sinh(x)^4 + 3*a*cosh(x)^3 + (35*a*cosh(x)^4 + 20*a*cosh(
x)^3 + 30*a*cosh(x)^2 + 12*a*cosh(x) + 3*a)*sinh(x)^3 + 3*a*cosh(x)^2 + 3*(
7*a*cosh(x)^5 + 5*a*cosh(x)^4 + 10*a*cosh(x)^3 + 6*a*cosh(x)^2 + 3*a*cosh(x
) + a)*sinh(x)^2 + a*cosh(x) + (7*a*cosh(x)^6 + 6*a*cosh(x)^5 + 15*a*cosh(x
)^4 + 12*a*cosh(x)^3 + 9*a*cosh(x)^2 + 6*a*cosh(x) + a)*sinh(x) + a)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{sech}^{4}(x)}{\cosh(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**4/(a+a*cosh(x)),x)
```

```
[Out] Integral(sech(x)**4/(cosh(x) + 1), x)/a
```

Giac [**A**]
time = 0.39, size = 57, normalized size = 1.02

$$-\frac{3\arctan\left(e^{x}\right)}{a}-\frac{2}{a(e^{x}+1)}-\frac{3\,e^{(5\,x)}+6\,e^{(4\,x)}+24\,e^{(2\,x)}-3\,e^{x}+10}{3\,a\left(e^{(2\,x)}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -3*arctan(e^x)/a - 2/(a*(e^x + 1)) - 1/3*(3*e^(5*x) + 6*e^(4*x) + 24*e^(2*x
) - 3*e^x + 10)/(a*(e^(2*x) + 1)^3)
```

Mupad [**B**]
time = 0.90, size = 107, normalized size = 1.91

$$\frac{8}{3\,a\,\left(3\,\mathrm{e}^{2\,x}+3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}+1\right)}-\frac{\frac{4}{a}-\frac{2\,\mathrm{e}^{x}}{a}}{2\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1}-\frac{2}{a\,\left(\mathrm{e}^{x}+1\right)}-\frac{\frac{2}{a}+\frac{\mathrm{e}^{x}}{a}}{\mathrm{e}^{2\,x}+1}-\frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^{x}\,\sqrt{a^2}}{a}\right)}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^4*(a + a*cosh(x))),x)
```

```
[Out] 8/(3*a*(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1)) - (4/a - (2*exp(x))/a)/(2*
exp(2*x) + exp(4*x) + 1) - 2/(a*(exp(x) + 1)) - (2/a + exp(x)/a)/(exp(2*x)
+ 1) - (3*atan((exp(x)*(a^2)^(1/2))/a))/(a^2)^(1/2)
```

3.32 $\int \frac{1}{1+\cosh(c+dx)}\,dx$

Optimal. Leaf size=20

$$\frac{\sinh(c+dx)}{d(1+\cosh(c+dx))}$$

```
[Out] sinh(d*x+c)/d/(1+cosh(d*x+c))
```

Rubi [A]
time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2727}

$$\frac{\sinh(c+dx)}{d(\cosh(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cosh[c + d*x])^(-1),x]
```

```
[Out] Sinh[c + d*x]/(d*(1 + Cosh[c + d*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{1+\cosh(c+dx)}\,dx = \frac{\sinh(c+dx)}{d(1+\cosh(c+dx))}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 0.70

$$\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cosh[c + d*x])^(-1),x]
```

```
[Out] Tanh[(c + d*x)/2]/d
```

Maple [A]
time = 0.82, size = 14, normalized size = 0.70

method	result	size
derivativedivides	$\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}$	14
default	$\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}$	14
risch	$-\frac{2}{d\left(e^{dx+c}+1\right)}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cosh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*tanh(1/2*d*x+1/2*c)
```

Maxima [A]
time = 0.30, size = 18, normalized size = 0.90

$$\frac{2}{d(e^{(-dx-c)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2/(d*(e^(-d*x - c) + 1))
```

Fricas [A]
time = 0.40, size = 22, normalized size = 1.10

$$-\frac{2}{d\cosh(dx+c)+d\sinh(dx+c)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -2/(d*cosh(d*x + c) + d*sinh(d*x + c) + d)
```

Sympy [A]
time = 0.27, size = 17, normalized size = 0.85

$$\begin{cases} \frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d} & \text{for } d \neq 0 \\ \frac{x}{\cosh(c)+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(d*x+c)),x)
```

```
[Out] Piecewise((tanh(c/2 + d*x/2)/d, Ne(d, 0)), (x/(cosh(c) + 1), True))
```

Giac [A]
time = 0.39, size = 15, normalized size = 0.75

$$-\frac{2}{d(e^{(dx+c)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] -2/(d*(e^(d*x + c) + 1))
```

Mupad [B]
time = 0.89, size = 15, normalized size = 0.75

$$-\frac{2}{d\left(e^{c+dx}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x) + 1),x)
```

```
[Out] -2/(d*(exp(c + d*x) + 1))
```

3.33 $\int \frac{1}{(1+\cosh(c+dx))^2}\,dx$

Optimal. Leaf size=47

$$\frac{\sinh(c+dx)}{3d(1+\cosh(c+dx))^2}+\frac{\sinh(c+dx)}{3d(1+\cosh(c+dx))}$$

```
[Out] 1/3*sinh(d*x+c)/d/(1+cosh(d*x+c))^2+1/3*sinh(d*x+c)/d/(1+cosh(d*x+c))
```

Rubi [A]
time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.200$, Rules used = {2729, 2727}

$$\frac{\sinh(c+dx)}{3d(\cosh(c+dx)+1)}+\frac{\sinh(c+dx)}{3d(\cosh(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cosh[c + d*x])^(-2),x]
```

```
[Out] Sinh[c + d*x]/(3*d*(1 + Cosh[c + d*x])^2) + Sinh[c + d*x]/(3*d*(1 + Cosh[c
+ d*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+\cosh(c+dx))^2}\,dx&=\frac{\sinh(c+dx)}{3d(1+\cosh(c+dx))^2}+\frac{1}{3}\int \frac{1}{1+\cosh(c+dx)}\,dx\\&=\frac{\sinh(c+dx)}{3d(1+\cosh(c+dx))^2}+\frac{\sinh(c+dx)}{3d(1+\cosh(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 34, normalized size = 0.72

$$\frac{4\sinh(c+dx)+\sinh(2(c+dx))}{6d(1+\cosh(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cosh[c + d*x])^(-2),x]
```

```
[Out] (4*Sinh[c + d*x] + Sinh[2*(c + d*x)])/(6*d*(1 + Cosh[c + d*x])^2)
```

Maple [A]
time = 0.80, size = 30, normalized size = 0.64

method	result	size
risch	$-\frac{2\left(3\,e^{dx+c}+1\right)}{3d\left(e^{dx+c}+1\right)^{3}}$	26
derivativedivides	$\frac{-\frac{\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6}+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}}{d}$	30
default	$\frac{-\frac{\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6}+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}}{d}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cosh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/6*tanh(1/2*d*x+1/2*c)^3+1/2*tanh(1/2*d*x+1/2*c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(43) = 86$.
time = 0.27, size = 90, normalized size = 1.91

$$\frac{2\,e^{(-dx-c)}}{d(3\,e^{(-dx-c)}+3\,e^{(-2\,dx-2\,c)}+e^{(-3\,dx-3\,c)}+1)}+\frac{2}{3\,d(3\,e^{(-dx-c)}+3\,e^{(-2\,dx-2\,c)}+e^{(-3\,dx-3\,c)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 2*e^(-d*x - c)/(d*(3*e^(-d*x - c) + 3*e^(-2*d*x - 2*c) + e^(-3*d*x - 3*c) +
 1)) + 2/3/(d*(3*e^(-d*x - c) + 3*e^(-2*d*x - 2*c) + e^(-3*d*x - 3*c) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(43) = 86$.
time = 0.45, size = 113, normalized size = 2.40

$$-\frac{2\,(3\,\cosh\left(dx+c\right)+3\,\sinh\left(dx+c\right)+1)}{3\,\left(d\cosh\left(dx+c\right)^{3}+d\sinh\left(dx+c\right)^{3}+3\,d\cosh\left(dx+c\right)^{2}+3\,(d\cosh\left(dx+c\right)+d)\sinh\left(dx+c\right)^{2}+3\,d\cosh\left(dx+c\right)+3\,\left(d\cosh\left(dx+c\right)^{2}+2\,d\cosh\left(dx+c\right)+d\right)\sinh\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2/3*(3*cosh(d*x + c) + 3*sinh(d*x + c) + 1)/(d*cosh(d*x + c)^3 + d*sinh(d*
x + c)^3 + 3*d*cosh(d*x + c)^2 + 3*(d*cosh(d*x + c) + d)*sinh(d*x + c)^2 +
3*d*cosh(d*x + c) + 3*(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c) + d)*sinh(d*x
+ c) + d)
```

Sympy [A]
time = 0.50, size = 36, normalized size = 0.77

$$\begin{cases} -\frac{\tanh^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{6d}+\frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{2d} & \text{for } d\neq 0\\ \frac{x}{(\cosh(c)+1)^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(d*x+c))**2,x)
```

```
[Out] Piecewise((-tanh(c/2 + d*x/2)**3/(6*d) + tanh(c/2 + d*x/2)/(2*d), Ne(d, 0))
, (x/(cosh(c) + 1)**2, True))
```

Giac [A]
time = 0.39, size = 25, normalized size = 0.53

$$-\frac{2\left(3\,e^{(dx+c)}+1\right)}{3\,d(e^{(dx+c)}+1)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*e^(d*x + c) + 1)/(d*(e^(d*x + c) + 1)^3)
```

Mupad [B]
time = 0.06, size = 25, normalized size = 0.53

$$-\frac{2\left(3\,\mathrm{e}^{c+d\,x}+1\right)}{3\,d\left(\mathrm{e}^{c+d\,x}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x) + 1)^2,x)
```

```
[Out] -(2*(3*exp(c + d*x) + 1))/(3*d*(exp(c + d*x) + 1)^3)
```

3.34 $\int \frac{1}{(1+\cosh(c+dx))^3} \, dx$

Optimal. Leaf size=70

$$\frac{\sinh(c+dx)}{5d(1+\cosh(c+dx))^3}+\frac{2\sinh(c+dx)}{15d(1+\cosh(c+dx))^2}+\frac{2\sinh(c+dx)}{15d(1+\cosh(c+dx))}$$

```
[Out] 1/5*sinh(d*x+c)/d/(1+cosh(d*x+c))^3+2/15*sinh(d*x+c)/d/(1+cosh(d*x+c))^2+2/
15*sinh(d*x+c)/d/(1+cosh(d*x+c))
```

Rubi [A]
time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2729, 2727}

$$\frac{2\sinh(c+dx)}{15d(\cosh(c+dx)+1)}+\frac{2\sinh(c+dx)}{15d(\cosh(c+dx)+1)^2}+\frac{\sinh(c+dx)}{5d(\cosh(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cosh[c + d*x])^(-3),x]
```

```
[Out] Sinh[c + d*x]/(5*d*(1 + Cosh[c + d*x])^3) + (2*Sinh[c + d*x])/(15*d*(1 + Co
sh[c + d*x])^2) + (2*Sinh[c + d*x])/(15*d*(1 + Cosh[c + d*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+\cosh(c+dx))^3}\,dx &= \frac{\sinh(c+dx)}{5d(1+\cosh(c+dx))^3}+\frac{2}{5}\int \frac{1}{(1+\cosh(c+dx))^2}\,dx\\ &= \frac{\sinh(c+dx)}{5d(1+\cosh(c+dx))^3}+\frac{2\sinh(c+dx)}{15d(1+\cosh(c+dx))^2}+\frac{2}{15}\int \frac{1}{1+\cosh(c+dx)}\,dx\\ &= \frac{\sinh(c+dx)}{5d(1+\cosh(c+dx))^3}+\frac{2\sinh(c+dx)}{15d(1+\cosh(c+dx))^2}+\frac{2\sinh(c+dx)}{15d(1+\cosh(c+dx))}\end{aligned}$$

Mathematica [**A**]
time = 0.04, size = 44, normalized size = 0.63

$$\frac{15\sinh(c+dx)+6\sinh(2(c+dx))+\sinh(3(c+dx))}{30d(1+\cosh(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cosh[c + d*x])^(-3),x]
```

```
[Out] (15*Sinh[c + d*x] + 6*Sinh[2*(c + d*x)] + Sinh[3*(c + d*x)])/(30*d*(1 + Cos
h[c + d*x])^3)
```

Maple [**A**]
time = 0.80, size = 43, normalized size = 0.61

method	result	size
risch	$-\frac{4\left(10\,e^{2dx+2c}+5\,e^{dx+c}+1\right)}{15d\left(e^{dx+c}+1\right)^5}$	37
derivativedivides	$\frac{\frac{\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20}-\frac{\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6}+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}}{d}$	43
default	$\frac{\frac{\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20}-\frac{\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6}+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}}{d}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cosh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/20*tanh(1/2*d*x+1/2*c)^5-1/6*tanh(1/2*d*x+1/2*c)^3+1/4*tanh(1/2*d*x+
1/2*c))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(64) = 128.
time = 0.26, size = 205, normalized size = 2.93

$$\frac{4\,e^{(-dx-c)}}{3\,d(5\,e^{(-dx-c)}+10\,e^{(-2\,dx-2\,c)}+10\,e^{(-3\,dx-3\,c)}+5\,e^{(-4\,dx-4\,c)}+e^{(-5\,dx-5\,c)}+1)}+\frac{8\,e^{(-2\,dx-2\,c)}}{3\,d(5\,e^{(-dx-c)}+10\,e^{(-2\,dx-2\,c)}+10\,e^{(-3\,dx-3\,c)}+5\,e^{(-4\,dx-4\,c)}+e^{(-5\,dx-5\,c)}+1)}+\frac{4}{15\,d(5\,e^{(-dx-c)}+10\,e^{(-2\,dx-2\,c)}+10\,e^{(-3\,dx-3\,c)}+5\,e^{(-4\,dx-4\,c)}+e^{(-5\,dx-5\,c)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 4/3*e^(-d*x - c)/(d*(5*e^(-d*x - c) + 10*e^(-2*d*x - 2*c) + 10*e^(-3*d*x -
3*c) + 5*e^(-4*d*x - 4*c) + e^(-5*d*x - 5*c) + 1)) + 8/3*e^(-2*d*x - 2*c)/(
d*(5*e^(-d*x - c) + 10*e^(-2*d*x - 2*c) + 10*e^(-3*d*x - 3*c) + 5*e^(-4*d*x
 - 4*c) + e^(-5*d*x - 5*c) + 1)) + 4/15/(d*(5*e^(-d*x - c) + 10*e^(-2*d*x -
 2*c) + 10*e^(-3*d*x - 3*c) + 5*e^(-4*d*x - 4*c) + e^(-5*d*x - 5*c) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(64) = 128$.
time = 0.37, size = 174, normalized size = 2.49

$$-\frac{4\,(11\,\cosh\left(dx+c\right)+9\,\sinh\left(dx+c\right)+5)}{15\,\left(d\cosh\left(dx+c\right)^{4}+d\sinh\left(dx+c\right)^{4}+5\,d\cosh\left(dx+c\right)^{3}+\left(4\,d\cosh\left(dx+c\right)+5\,d\right)\sinh\left(dx+c\right)^{3}+10\,d\cosh\left(dx+c\right)^{2}+\left(6\,d\cosh\left(dx+c\right)^{2}+15\,d\cosh\left(dx+c\right)+10\,d\right)\sinh\left(dx+c\right)^{2}+11\,d\cosh\left(dx+c\right)+\left(4\,d\cosh\left(dx+c\right)^{3}+15\,d\cosh\left(dx+c\right)^{2}+20\,d\cosh\left(dx+c\right)+9\,d\right)\sinh\left(dx+c\right)+5\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -4/15*(11*cosh(d*x + c) + 9*sinh(d*x + c) + 5)/(d*cosh(d*x + c)^4 + d*sinh(
d*x + c)^4 + 5*d*cosh(d*x + c)^3 + (4*d*cosh(d*x + c) + 5*d)*sinh(d*x + c)^
3 + 10*d*cosh(d*x + c)^2 + (6*d*cosh(d*x + c)^2 + 15*d*cosh(d*x + c) + 10*d
)*sinh(d*x + c)^2 + 11*d*cosh(d*x + c) + (4*d*cosh(d*x + c)^3 + 15*d*cosh(d
*x + c)^2 + 20*d*cosh(d*x + c) + 9*d)*sinh(d*x + c) + 5*d)
```

Sympy [A]
time = 1.02, size = 51, normalized size = 0.73

$$\begin{cases} \frac{\tanh^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{20d}-\frac{\tanh^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{6d}+\frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{4d} & \text{for}\ d\neq 0\\ \frac{x}{\left(\cosh\left(c\right)+1\right)^{3}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(d*x+c))**3,x)
```

```
[Out] Piecewise((tanh(c/2 + d*x/2)**5/(20*d) - tanh(c/2 + d*x/2)**3/(6*d) + tanh(
c/2 + d*x/2)/(4*d), Ne(d, 0)), (x/(cosh(c) + 1)**3, True))
```

Giac [A]
time = 0.39, size = 36, normalized size = 0.51

$$-\frac{4\,\left(10\,e^{\left(2\,dx+2\,c\right)}+5\,e^{\left(dx+c\right)}+1\right)}{15\,d\left(e^{\left(dx+c\right)}+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -4/15*(10*e^(2*d*x + 2*c) + 5*e^(d*x + c) + 1)/(d*(e^(d*x + c) + 1)^5)
```

Mupad [B]
time = 0.93, size = 36, normalized size = 0.51

$$-\frac{4\,\left(5\,\mathrm{e}^{c+d\,x}+10\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}{15\,d\,\left(\mathrm{e}^{c+d\,x}+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x) + 1)^3,x)
```

```
[Out] -(4*(5*exp(c + d*x) + 10*exp(2*c + 2*d*x) + 1))/(15*d*(exp(c + d*x) + 1)^5)
```

3.35 $\int \frac{1}{(1+\cosh(c+dx))^4}\,dx$

Optimal. Leaf size=93

$$\frac{\sinh(c+dx)}{7d(1+\cosh(c+dx))^4}+\frac{3\sinh(c+dx)}{35d(1+\cosh(c+dx))^3}+\frac{2\sinh(c+dx)}{35d(1+\cosh(c+dx))^2}+\frac{2\sinh(c+dx)}{35d(1+\cosh(c+dx))}$$

```
[Out] 1/7*sinh(d*x+c)/d/(1+cosh(d*x+c))^4+3/35*sinh(d*x+c)/d/(1+cosh(d*x+c))^3+2/
35*sinh(d*x+c)/d/(1+cosh(d*x+c))^2+2/35*sinh(d*x+c)/d/(1+cosh(d*x+c))
```

Rubi [A]
time = 0.04, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2729, 2727}

$$\frac{2\sinh(c+dx)}{35d(\cosh(c+dx)+1)}+\frac{2\sinh(c+dx)}{35d(\cosh(c+dx)+1)^2}+\frac{3\sinh(c+dx)}{35d(\cosh(c+dx)+1)^3}+\frac{\sinh(c+dx)}{7d(\cosh(c+dx)+1)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cosh[c + d*x])^(-4),x]
```

```
[Out] Sinh[c + d*x]/(7*d*(1 + Cosh[c + d*x])^4) + (3*Sinh[c + d*x])/(35*d*(1 + Co
sh[c + d*x])^3) + (2*Sinh[c + d*x])/(35*d*(1 + Cosh[c + d*x])^2) + (2*Sinh[
c + d*x])/(35*d*(1 + Cosh[c + d*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+\cosh(c+dx))^4}\,dx &= \frac{\sinh(c+dx)}{7d(1+\cosh(c+dx))^4} + \frac{3}{7}\int \frac{1}{(1+\cosh(c+dx))^3}\,dx \\ &= \frac{\sinh(c+dx)}{7d(1+\cosh(c+dx))^4} + \frac{3\sinh(c+dx)}{35d(1+\cosh(c+dx))^3} + \frac{6}{35}\int \frac{1}{(1+\cosh(c+dx))^2}\,d \\ &= \frac{\sinh(c+dx)}{7d(1+\cosh(c+dx))^4} + \frac{3\sinh(c+dx)}{35d(1+\cosh(c+dx))^3} + \frac{2\sinh(c+dx)}{35d(1+\cosh(c+dx))^2} + \\ &= \frac{\sinh(c+dx)}{7d(1+\cosh(c+dx))^4} + \frac{3\sinh(c+dx)}{35d(1+\cosh(c+dx))^3} + \frac{2\sinh(c+dx)}{35d(1+\cosh(c+dx))^2} + \end{aligned}$$

Mathematica [A]
time = 0.06, size = 54, normalized size = 0.58

$$\frac{56\sinh(c+dx)+28\sinh(2(c+dx))+8\sinh(3(c+dx))+\sinh(4(c+dx))}{140d(1+\cosh(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cosh[c + d*x])^(-4),x]
```

```
[Out] (56*Sinh[c + d*x] + 28*Sinh[2*(c + d*x)] + 8*Sinh[3*(c + d*x)] + Sinh[4*(c
+ d*x)])/(140*d*(1 + Cosh[c + d*x])^4)
```

Maple [A]
time = 0.82, size = 56, normalized size = 0.60

method	result	size
risch	$-\frac{4\left(35\,\mathrm{e}^{3dx+3c}+21\,\mathrm{e}^{2dx+2c}+7\,\mathrm{e}^{dx+c}+1\right)}{35d\left(\mathrm{e}^{dx+c}+1\right)^{7}}$	48
derivativedivides	$\frac{-\frac{\left(\tanh^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{56}+\frac{3\left(\tanh^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{40}-\frac{\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8}}{d}$	56
default	$\frac{-\frac{\left(\tanh^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{56}+\frac{3\left(\tanh^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{40}-\frac{\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8}}{d}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cosh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/56*tanh(1/2*d*x+1/2*c)^7+3/40*tanh(1/2*d*x+1/2*c)^5-1/8*tanh(1/2*d*
x+1/2*c)^3+1/8*tanh(1/2*d*x+1/2*c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(85) = 170$.
time = 0.28, size = 364, normalized size = 3.91

$$\frac{4e^{(-dx-c)}}{5d(7e^{(-dx-c)}+21e^{(-2dx-2c)}+35e^{(-3dx-3c)}+35e^{(-4dx-4c)}+21e^{(-5dx-5c)}+7e^{(-6dx-6c)}+e^{(-7dx-7c)}+1)}+\frac{12e^{(-2dx-2c)}}{5d(7e^{(-dx-c)}+21e^{(-2dx-2c)}+35e^{(-3dx-3c)}+35e^{(-4dx-4c)}+21e^{(-5dx-5c)}+7e^{(-6dx-6c)}+e^{(-7dx-7c)}+1)}+\frac{4e^{(-3dx-3c)}}{d(7e^{(-dx-c)}+21e^{(-2dx-2c)}+35e^{(-3dx-3c)}+35e^{(-4dx-4c)}+21e^{(-5dx-5c)}+7e^{(-6dx-6c)}+e^{(-7dx-7c)}+1)}+\frac{4}{35d(7e^{(-dx-c)}+21e^{(-2dx-2c)}+35e^{(-3dx-3c)}+35e^{(-4dx-4c)}+21e^{(-5dx-5c)}+7e^{(-6dx-6c)}+e^{(-7dx-7c)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+cosh(d*x+c))^4,x, algorithm="maxima")

[Out] 4/5*e^(-d*x - c)/(d*(7*e^(-d*x - c) + 21*e^(-2*d*x - 2*c) + 35*e^(-3*d*x - 3*c) + 35*e^(-4*d*x - 4*c) + 21*e^(-5*d*x - 5*c) + 7*e^(-6*d*x - 6*c) + e^(-7*d*x - 7*c) + 1)) + 12/5*e^(-2*d*x - 2*c)/(d*(7*e^(-d*x - c) + 21*e^(-2*d*x - 2*c) + 35*e^(-3*d*x - 3*c) + 35*e^(-4*d*x - 4*c) + 21*e^(-5*d*x - 5*c) + 7*e^(-6*d*x - 6*c) + e^(-7*d*x - 7*c) + 1)) + 4*e^(-3*d*x - 3*c)/(d*(7*e^(-d*x - c) + 21*e^(-2*d*x - 2*c) + 35*e^(-3*d*x - 3*c) + 35*e^(-4*d*x - 4*c) + 21*e^(-5*d*x - 5*c) + 7*e^(-6*d*x - 6*c) + e^(-7*d*x - 7*c) + 1)) + 4/35/(d*(7*e^(-d*x - c) + 21*e^(-2*d*x - 2*c) + 35*e^(-3*d*x - 3*c) + 35*e^(-4*d*x - 4*c) + 21*e^(-5*d*x - 5*c) + 7*e^(-6*d*x - 6*c) + e^(-7*d*x - 7*c) + 1))

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(85) = 170$.
time = 0.48, size = 347, normalized size = 3.73

$$-\frac{4\left(35\cosh\left(dx+c\right)^{2}+10\left(7\cosh\left(dx+c\right)+2\right)\sinh\left(dx+c\right)+35\sinh\left(dx+c\right)^{2}+22\cosh\left(dx+c\right)+7\right)}{35\left(d\cosh\left(dx+c\right)^{6}+d\sinh\left(dx+c\right)^{6}+7\,d\cosh\left(dx+c\right)^{5}+\left(6\,d\cosh\left(dx+c\right)+7\,d\right)\sinh\left(dx+c\right)^{5}+21\,d\cosh\left(dx+c\right)^{4}+\left(15\,d\cosh\left(dx+c\right)^{2}+35\,d\cosh\left(dx+c\right)+21\,d\right)\sinh\left(dx+c\right)^{4}+35\,d\cosh\left(dx+c\right)^{3}+\left(20\,d\cosh\left(dx+c\right)^{3}+70\,d\cosh\left(dx+c\right)^{2}+84\,d\cosh\left(dx+c\right)+35\,d\right)\sinh\left(dx+c\right)^{3}+35\,d\cosh\left(dx+c\right)^{2}+\left(15\,d\cosh\left(dx+c\right)^{4}+70\,d\cosh\left(dx+c\right)^{3}+126\,d\cosh\left(dx+c\right)^{2}+105\,d\cosh\left(dx+c\right)+35\,d\right)\sinh\left(dx+c\right)^{2}+22\,d\cosh\left(dx+c\right)+\left(6\,d\cosh\left(dx+c\right)^{5}+35\,d\cosh\left(dx+c\right)^{4}+84\,d\cosh\left(dx+c\right)^{3}+105\,d\cosh\left(dx+c\right)^{2}+70\,d\cosh\left(dx+c\right)+20\,d\right)\sinh\left(dx+c\right)+7\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+cosh(d*x+c))^4,x, algorithm="fricas")

[Out] -4/35*(35*cosh(d*x + c)^2 + 10*(7*cosh(d*x + c) + 2)*sinh(d*x + c) + 35*sinh(d*x + c)^2 + 22*cosh(d*x + c) + 7)/(d*cosh(d*x + c)^6 + d*sinh(d*x + c)^6 + 7*d*cosh(d*x + c)^5 + (6*d*cosh(d*x + c) + 7*d)*sinh(d*x + c)^5 + 21*d*cosh(d*x + c)^4 + (15*d*cosh(d*x + c)^2 + 35*d*cosh(d*x + c) + 21*d)*sinh(d*x + c)^4 + 35*d*cosh(d*x + c)^3 + (20*d*cosh(d*x + c)^3 + 70*d*cosh(d*x + c)^2 + 84*d*cosh(d*x + c) + 35*d)*sinh(d*x + c)^3 + 35*d*cosh(d*x + c)^2 + (15*d*cosh(d*x + c)^4 + 70*d*cosh(d*x + c)^3 + 126*d*cosh(d*x + c)^2 + 105*d*cosh(d*x + c) + 35*d)*sinh(d*x + c)^2 + 22*d*cosh(d*x + c) + (6*d*cosh(d*x + c)^5 + 35*d*cosh(d*x + c)^4 + 84*d*cosh(d*x + c)^3 + 105*d*cosh(d*x + c)^2 + 70*d*cosh(d*x + c) + 20*d)*sinh(d*x + c) + 7*d)

Sympy [A]
time = 2.62, size = 68, normalized size = 0.73

$$\begin{cases} -\frac{\tanh^{7}\left(\frac{c}{2}+\frac{dx}{2}\right)}{56d}+\frac{3\tanh^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{40d}-\frac{\tanh^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{8d}+\frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{8d} & \text{for } d\neq 0 \\ \frac{x}{\left(\cosh\left(c\right)+1\right)^{4}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+cosh(d*x+c))**4,x)

```
[Out] Piecewise((-tanh(c/2 + d*x/2)**7/(56*d) + 3*tanh(c/2 + d*x/2)**5/(40*d) - t
anh(c/2 + d*x/2)**3/(8*d) + tanh(c/2 + d*x/2)/(8*d), Ne(d, 0)), (x/(cosh(c)
 + 1)**4, True))
```

Giac [A]
time = 0.39, size = 47, normalized size = 0.51

$$-\frac{4\left(35\,e^{(3\,dx+3\,c)}+21\,e^{(2\,dx+2\,c)}+7\,e^{(dx+c)}+1\right)}{35\,d\left(e^{(dx+c)}+1\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -4/35*(35*e^(3*d*x + 3*c) + 21*e^(2*d*x + 2*c) + 7*e^(d*x + c) + 1)/(d*(e^(
d*x + c) + 1)^7)
```

Mupad [B]
time = 0.92, size = 283, normalized size = 3.04

$$-\frac{4}{35\,d\,\left(4\,{\mathrm{e}}^{c+d\,x}+6\,{\mathrm{e}}^{2\,c+2\,d\,x}+4\,{\mathrm{e}}^{3\,c+3\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1\right)}-\frac{16\,{\mathrm{e}}^{c+d\,x}}{35\,d\,\left(5\,{\mathrm{e}}^{c+d\,x}+10\,{\mathrm{e}}^{2\,c+2\,d\,x}+10\,{\mathrm{e}}^{3\,c+3\,d\,x}+5\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{5\,c+5\,d\,x}+1\right)}-\frac{8\,{\mathrm{e}}^{2\,c+2\,d\,x}}{7\,d\,\left(6\,{\mathrm{e}}^{c+d\,x}+15\,{\mathrm{e}}^{2\,c+2\,d\,x}+20\,{\mathrm{e}}^{3\,c+3\,d\,x}+15\,{\mathrm{e}}^{4\,c+4\,d\,x}+6\,{\mathrm{e}}^{5\,c+5\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}+1\right)}-\frac{16\,{\mathrm{e}}^{3\,c+3\,d\,x}}{7\,d\,\left(7\,{\mathrm{e}}^{c+d\,x}+21\,{\mathrm{e}}^{2\,c+2\,d\,x}+35\,{\mathrm{e}}^{3\,c+3\,d\,x}+35\,{\mathrm{e}}^{4\,c+4\,d\,x}+21\,{\mathrm{e}}^{5\,c+5\,d\,x}+7\,{\mathrm{e}}^{6\,c+6\,d\,x}+{\mathrm{e}}^{7\,c+7\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x) + 1)^4,x)
```

```
[Out] - 4/(35*d*(4*exp(c + d*x) + 6*exp(2*c + 2*d*x) + 4*exp(3*c + 3*d*x) + exp(4
*c + 4*d*x) + 1)) - (16*exp(c + d*x))/(35*d*(5*exp(c + d*x) + 10*exp(2*c +
2*d*x) + 10*exp(3*c + 3*d*x) + 5*exp(4*c + 4*d*x) + exp(5*c + 5*d*x) + 1))
- (8*exp(2*c + 2*d*x))/(7*d*(6*exp(c + d*x) + 15*exp(2*c + 2*d*x) + 20*exp(
3*c + 3*d*x) + 15*exp(4*c + 4*d*x) + 6*exp(5*c + 5*d*x) + exp(6*c + 6*d*x)
+ 1)) - (16*exp(3*c + 3*d*x))/(7*d*(7*exp(c + d*x) + 21*exp(2*c + 2*d*x) +
35*exp(3*c + 3*d*x) + 35*exp(4*c + 4*d*x) + 21*exp(5*c + 5*d*x) + 7*exp(6*c
 + 6*d*x) + exp(7*c + 7*d*x) + 1))
```

3.36 $\int \frac{1}{1-\cosh(c+dx)}\,dx$

Optimal. Leaf size=23

$$-\frac{\sinh(c+dx)}{d(1-\cosh(c+dx))}$$

```
[Out] -sinh(d*x+c)/d/(1-cosh(d*x+c))
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2727}

$$-\frac{\sinh(c+dx)}{d(1-\cosh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cosh[c + d*x])^(-1),x]
```

```
[Out] -(Sinh[c + d*x]/(d*(1 - Cosh[c + d*x])))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{1-\cosh(c+dx)}\,dx = -\frac{\sinh(c+dx)}{d(1-\cosh(c+dx))}$$

Mathematica [A]
time = 0.02, size = 14, normalized size = 0.61

$$\frac{\coth\left(\frac{1}{2}(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cosh[c + d*x])^(-1),x]
```

```
[Out] Coth[(c + d*x)/2]/d
```

Maple [A]
time = 0.87, size = 16, normalized size = 0.70

method	result	size
derivativedivides	$\frac{1}{d\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}$	16
default	$\frac{1}{d\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}$	16
risch	$\frac{2}{d\left(e^{dx+c}-1\right)}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cosh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/tanh(1/2*d*x+1/2*c)
```

Maxima [A]
time = 0.26, size = 18, normalized size = 0.78

$$-\frac{2}{d(e^{(-dx-c)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -2/(d*(e^(-d*x - c) - 1))
```

Fricas [A]
time = 0.38, size = 24, normalized size = 1.04

$$\frac{2}{d\cosh(dx+c)+d\sinh(dx+c)-d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 2/(d*cosh(d*x + c) + d*sinh(d*x + c) - d)
```

Sympy [A]
time = 0.39, size = 19, normalized size = 0.83

$$\begin{cases} \frac{1}{d\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)} & \text{for}\ d\neq 0 \\ \frac{x}{1-\cosh(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c)),x)
```

```
[Out] Piecewise((1/(d*tanh(c/2 + d*x/2)), Ne(d, 0)), (x/(1 - cosh(c)), True))
```

Giac [A]
time = 0.38, size = 15, normalized size = 0.65

$$\frac{2}{d(e^{(dx+c)} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] 2/(d*(e^(d*x + c) - 1))
```

Mupad [B]
time = 0.89, size = 15, normalized size = 0.65

$$\frac{2}{d\ (\mathrm{e}^{c+d\,x} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cosh(c + d*x) - 1),x)
```

```
[Out] 2/(d*(exp(c + d*x) - 1))
```

3.37 $\int \frac{1}{(1-\cosh(c+dx))^2} dx$

Optimal. Leaf size=51

$$-\frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))^2}-\frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))}$$

```
[Out] -1/3*sinh(d*x+c)/d/(1-cosh(d*x+c))^2-1/3*sinh(d*x+c)/d/(1-cosh(d*x+c))
```

Rubi [A]
time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2729, 2727}

$$-\frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))}-\frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cosh[c + d*x])^(-2),x]
```

```
[Out] -1/3*Sinh[c + d*x]/(d*(1 - Cosh[c + d*x])^2) - Sinh[c + d*x]/(3*d*(1 - Cosh
[c + d*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-\cosh(c+dx))^2}\,dx &= -\frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))^2}+\frac{1}{3}\int \frac{1}{1-\cosh(c+dx)}\,dx\\ &= -\frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))^2}-\frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 31, normalized size = 0.61

$$\frac{(-2+\cosh(c+dx))\sinh(c+dx)}{3d(-1+\cosh(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cosh[c + d*x])^(-2),x]
```

```
[Out] ((-2 + Cosh[c + d*x])*Sinh[c + d*x])/(3*d*(-1 + Cosh[c + d*x])^2)
```

Maple [A]
time = 0.95, size = 32, normalized size = 0.63

method	result	size
risch	$-\frac{2\left(-1+3\,\mathrm{e}^{dx+c}\right)}{3d\left(\mathrm{e}^{dx+c}-1\right)^{3}}$	26
derivativedivides	$\frac{\frac{1}{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{1}{6\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}}{d}$	32
default	$\frac{\frac{1}{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{1}{6\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}}{d}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cosh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/tanh(1/2*d*x+1/2*c)-1/6/tanh(1/2*d*x+1/2*c)^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(43) = 86.
time = 0.26, size = 90, normalized size = 1.76

$$\frac{2\,e^{(-dx-c)}}{d(3\,e^{(-dx-c)}-3\,e^{(-2\,dx-2\,c)}+e^{(-3\,dx-3\,c)}-1)}-\frac{2}{3\,d(3\,e^{(-dx-c)}-3\,e^{(-2\,dx-2\,c)}+e^{(-3\,dx-3\,c)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 2*e^(-d*x - c)/(d*(3*e^(-d*x - c) - 3*e^(-2*d*x - 2*c) + e^(-3*d*x - 3*c) -
 1)) - 2/3/(d*(3*e^(-d*x - c) - 3*e^(-2*d*x - 2*c) + e^(-3*d*x - 3*c) - 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(43) = 86.
time = 0.41, size = 117, normalized size = 2.29

$$-\frac{2\,(3\,\cosh\left(dx+c\right)+3\,\sinh\left(dx+c\right)-1)}{3\,\left(d\cosh\left(dx+c\right)^{3}+d\sinh\left(dx+c\right)^{3}-3\,d\cosh\left(dx+c\right)^{2}+3\,(d\cosh\left(dx+c\right)-d)\sinh\left(dx+c\right)^{2}+3\,d\cosh\left(dx+c\right)+3\,\left(d\cosh\left(dx+c\right)^{2}-2\,d\cosh\left(dx+c\right)+d\right)\sinh\left(dx+c\right)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2/3*(3*cosh(d*x + c) + 3*sinh(d*x + c) - 1)/(d*cosh(d*x + c)^3 + d*sinh(d*
x + c)^3 - 3*d*cosh(d*x + c)^2 + 3*(d*cosh(d*x + c) - d)*sinh(d*x + c)^2 +
3*d*cosh(d*x + c) + 3*(d*cosh(d*x + c)^2 - 2*d*cosh(d*x + c) + d)*sinh(d*x
+ c) - d)
```

Sympy [A]
time = 0.66, size = 39, normalized size = 0.76

$$\begin{cases} \frac{1}{2d\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)} - \frac{1}{6d\tanh^3\left(\frac{c}{2}+\frac{dx}{2}\right)} & \text{for } d \neq 0 \\ \frac{x}{(1-\cosh(c))^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c))**2,x)
```

```
[Out] Piecewise((1/(2*d*tanh(c/2 + d*x/2)) - 1/(6*d*tanh(c/2 + d*x/2)**3), Ne(d,
0)), (x/(1 - cosh(c))**2, True))
```

Giac [A]
time = 0.39, size = 25, normalized size = 0.49

$$-\frac{2\left(3\,e^{(dx+c)}-1\right)}{3\,d(e^{(dx+c)}-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*e^(d*x + c) - 1)/(d*(e^(d*x + c) - 1)^3)
```

Mupad [B]
time = 0.06, size = 25, normalized size = 0.49

$$-\frac{2\left(3\,\mathrm{e}^{c+d\,x}-1\right)}{3\,d\left(\mathrm{e}^{c+d\,x}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x) - 1)^2,x)
```

```
[Out] -(2*(3*exp(c + d*x) - 1))/(3*d*(exp(c + d*x) - 1)^3)
```

3.38 $\int \frac{1}{(1-\cosh(c+dx))^3}\,dx$

Optimal. Leaf size=76

$$-\frac{\sinh(c+dx)}{5d(1-\cosh(c+dx))^3}-\frac{2\sinh(c+dx)}{15d(1-\cosh(c+dx))^2}-\frac{2\sinh(c+dx)}{15d(1-\cosh(c+dx))}$$

```
[Out] -1/5*sinh(d*x+c)/d/(1-cosh(d*x+c))^3-2/15*sinh(d*x+c)/d/(1-cosh(d*x+c))^2-2
/15*sinh(d*x+c)/d/(1-cosh(d*x+c))
```

Rubi [A]
time = 0.03, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {2729, 2727}

$$-\frac{2\sinh(c+dx)}{15d(1-\cosh(c+dx))}-\frac{2\sinh(c+dx)}{15d(1-\cosh(c+dx))^2}-\frac{\sinh(c+dx)}{5d(1-\cosh(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cosh[c + d*x])^(-3),x]
```

```
[Out] -1/5*Sinh[c + d*x]/(d*(1 - Cosh[c + d*x])^3) - (2*Sinh[c + d*x])/(15*d*(1 -
 Cosh[c + d*x])^2) - (2*Sinh[c + d*x])/(15*d*(1 - Cosh[c + d*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-\cosh(c+dx))^3}\,dx &= -\frac{\sinh(c+dx)}{5d(1-\cosh(c+dx))^3}+\frac{2}{5}\int \frac{1}{(1-\cosh(c+dx))^2}\,dx\\ &= -\frac{\sinh(c+dx)}{5d(1-\cosh(c+dx))^3}-\frac{2\sinh(c+dx)}{15d(1-\cosh(c+dx))^2}+\frac{2}{15}\int \frac{1}{1-\cosh(c+dx)}\,d\\ &= -\frac{\sinh(c+dx)}{5d(1-\cosh(c+dx))^3}-\frac{2\sinh(c+dx)}{15d(1-\cosh(c+dx))^2}-\frac{2\sinh(c+dx)}{15d(1-\cosh(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 41, normalized size = 0.54

$$\frac{(8-6\cosh(c+dx)+\cosh(2(c+dx)))\sinh(c+dx)}{15d(-1+\cosh(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cosh[c + d*x])^(-3),x]
```

```
[Out] ((8 - 6*Cosh[c + d*x] + Cosh[2*(c + d*x)])*Sinh[c + d*x])/(15*d*(-1 + Cosh[
c + d*x])^3)
```

Maple [A]
time = 0.91, size = 45, normalized size = 0.59

method	result	size
risch	$\frac{\frac{8\,e^{2dx+2c}}{3}-\frac{4\,e^{dx+c}}{3}+\frac{4}{15}}{d\left(e^{dx+c}-1\right)^5}$	37
derivativedivides	$\frac{-\frac{1}{6\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{1}{20\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}+\frac{1}{4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}}{d}$	45
default	$\frac{-\frac{1}{6\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{1}{20\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}+\frac{1}{4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}}{d}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cosh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/6/tanh(1/2*d*x+1/2*c)^3+1/20/tanh(1/2*d*x+1/2*c)^5+1/4/tanh(1/2*d*x
+1/2*c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(64) = 128$.
time = 0.28, size = 205, normalized size = 2.70

$$\frac{4\,e^{(-dx-c)}}{3\,d\left(5\,e^{(-dx-c)}-10\,e^{(-2\,dx-2\,c)}+10\,e^{(-3\,dx-3\,c)}-5\,e^{(-4\,dx-4\,c)}+e^{(-5\,dx-5\,c)}-1\right)}-\frac{8\,e^{(-2\,dx-2\,c)}}{3\,d\left(5\,e^{(-dx-c)}-10\,e^{(-2\,dx-2\,c)}+10\,e^{(-3\,dx-3\,c)}-5\,e^{(-4\,dx-4\,c)}+e^{(-5\,dx-5\,c)}-1\right)}-\frac{4}{15\,d\left(5\,e^{(-dx-c)}-10\,e^{(-2\,dx-2\,c)}+10\,e^{(-3\,dx-3\,c)}-5\,e^{(-4\,dx-4\,c)}+e^{(-5\,dx-5\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 4/3*e^(-d*x - c)/(d*(5*e^(-d*x - c) - 10*e^(-2*d*x - 2*c) + 10*e^(-3*d*x -
3*c) - 5*e^(-4*d*x - 4*c) + e^(-5*d*x - 5*c) - 1)) - 8/3*e^(-2*d*x - 2*c)/(
d*(5*e^(-d*x - c) - 10*e^(-2*d*x - 2*c) + 10*e^(-3*d*x - 3*c) - 5*e^(-4*d*x
 - 4*c) + e^(-5*d*x - 5*c) - 1)) - 4/15/(d*(5*e^(-d*x - c) - 10*e^(-2*d*x -
 2*c) + 10*e^(-3*d*x - 3*c) - 5*e^(-4*d*x - 4*c) + e^(-5*d*x - 5*c) - 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(64) = 128$.
time = 0.37, size = 174, normalized size = 2.29

$$\frac{4\,(11\,\cosh\left(dx+c\right)+9\,\sinh\left(dx+c\right)-5)}{15\,\left(d\cosh\left(dx+c\right)^{4}+d\sinh\left(dx+c\right)^{4}-5\,d\cosh\left(dx+c\right)^{3}+\left(4\,d\cosh\left(dx+c\right)-5\,d\right)\sinh\left(dx+c\right)^{3}+10\,d\cosh\left(dx+c\right)^{2}+\left(6\,d\cosh\left(dx+c\right)^{2}-15\,d\cosh\left(dx+c\right)+10\,d\right)\sinh\left(dx+c\right)^{2}-11\,d\cosh\left(dx+c\right)+\left(4\,d\cosh\left(dx+c\right)^{3}-15\,d\cosh\left(dx+c\right)^{2}+20\,d\cosh\left(dx+c\right)-9\,d\right)\sinh\left(dx+c\right)+5\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 4/15*(11*cosh(d*x + c) + 9*sinh(d*x + c) - 5)/(d*cosh(d*x + c)^4 + d*sinh(d
*x + c)^4 - 5*d*cosh(d*x + c)^3 + (4*d*cosh(d*x + c) - 5*d)*sinh(d*x + c)^3
 + 10*d*cosh(d*x + c)^2 + (6*d*cosh(d*x + c)^2 - 15*d*cosh(d*x + c) + 10*d)
*sinh(d*x + c)^2 - 11*d*cosh(d*x + c) + (4*d*cosh(d*x + c)^3 - 15*d*cosh(d*
x + c)^2 + 20*d*cosh(d*x + c) - 9*d)*sinh(d*x + c) + 5*d)
```

Sympy [A]
time = 1.39, size = 56, normalized size = 0.74

$$\begin{cases} \frac{1}{4d\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)} - \frac{1}{6d\tanh^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)} + \frac{1}{20d\tanh^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)} & \text{for}\: d \neq 0 \\ \frac{x}{(1-\cosh\left(c\right))^{3}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c))**3,x)
```

```
[Out] Piecewise((1/(4*d*tanh(c/2 + d*x/2)) - 1/(6*d*tanh(c/2 + d*x/2)**3) + 1/(20
*d*tanh(c/2 + d*x/2)**5), Ne(d, 0)), (x/(1 - cosh(c))**3, True))
```

Giac [A]
time = 0.40, size = 36, normalized size = 0.47

$$\frac{4\,\left(10\,e^{(2\,dx+2\,c)}-5\,e^{(dx+c)}+1\right)}{15\,d(e^{(dx+c)}-1)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 4/15*(10*e^(2*d*x + 2*c) - 5*e^(d*x + c) + 1)/(d*(e^(d*x + c) - 1)^5)
```

Mupad [B]
time = 0.91, size = 36, normalized size = 0.47

$$\frac{4\,\left(10\,{\mathrm{e}}^{2\,c+2\,d\,x}-5\,{\mathrm{e}}^{c+d\,x}+1\right)}{15\,d\,{\left({\mathrm{e}}^{c+d\,x}-1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cosh(c + d*x) - 1)^3,x)
```

```
[Out] (4*(10*exp(2*c + 2*d*x) - 5*exp(c + d*x) + 1))/(15*d*(exp(c + d*x) - 1)^5)
```

3.39 $\int \frac{1}{(1-\cosh(c+dx))^4}\,dx$

Optimal. Leaf size=101

$$-\frac{\sinh(c+dx)}{7d(1-\cosh(c+dx))^4}-\frac{3\sinh(c+dx)}{35d(1-\cosh(c+dx))^3}-\frac{2\sinh(c+dx)}{35d(1-\cosh(c+dx))^2}-\frac{2\sinh(c+dx)}{35d(1-\cosh(c+dx))}$$

```
[Out] -1/7*sinh(d*x+c)/d/(1-cosh(d*x+c))^4-3/35*sinh(d*x+c)/d/(1-cosh(d*x+c))^3-2
/35*sinh(d*x+c)/d/(1-cosh(d*x+c))^2-2/35*sinh(d*x+c)/d/(1-cosh(d*x+c))
```

Rubi [A]

time = 0.04, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2729, 2727}

$$-\frac{2\sinh(c+dx)}{35d(1-\cosh(c+dx))}-\frac{2\sinh(c+dx)}{35d(1-\cosh(c+dx))^2}-\frac{3\sinh(c+dx)}{35d(1-\cosh(c+dx))^3}-\frac{\sinh(c+dx)}{7d(1-\cosh(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cosh[c + d*x])^(-4),x]
```

```
[Out] -1/7*Sinh[c + d*x]/(d*(1 - Cosh[c + d*x])^4) - (3*Sinh[c + d*x])/(35*d*(1 -
 Cosh[c + d*x])^3) - (2*Sinh[c + d*x])/(35*d*(1 - Cosh[c + d*x])^2) - (2*Si
nh[c + d*x])/(35*d*(1 - Cosh[c + d*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(1-\cosh(c+dx))^4}\,dx &= -\frac{\sinh(c+dx)}{7d(1-\cosh(c+dx))^4} + \frac{3}{7}\int \frac{1}{(1-\cosh(c+dx))^3}\,dx \\
&= -\frac{\sinh(c+dx)}{7d(1-\cosh(c+dx))^4} - \frac{3\sinh(c+dx)}{35d(1-\cosh(c+dx))^3} + \frac{6}{35}\int \frac{1}{(1-\cosh(c+dx)} \\
&= -\frac{\sinh(c+dx)}{7d(1-\cosh(c+dx))^4} - \frac{3\sinh(c+dx)}{35d(1-\cosh(c+dx))^3} - \frac{2\sinh(c+dx)}{35d(1-\cosh(c+dx))^2} \\
&= -\frac{\sinh(c+dx)}{7d(1-\cosh(c+dx))^4} - \frac{3\sinh(c+dx)}{35d(1-\cosh(c+dx))^3} - \frac{2\sinh(c+dx)}{35d(1-\cosh(c+dx))^2}
\end{aligned}
$$

Mathematica [**A**]
time = 0.06, size = 51, normalized size = 0.50

$$\frac{(-32+29\cosh(c+dx)-8\cosh(2(c+dx))+\cosh(3(c+dx)))\sinh(c+dx)}{70d(-1+\cosh(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cosh[c + d*x])^(-4),x]
```

```
[Out] ((-32 + 29*Cosh[c + d*x] - 8*Cosh[2*(c + d*x)] + Cosh[3*(c + d*x)])*Sinh[c
+ d*x])/(70*d*(-1 + Cosh[c + d*x])^4)
```

Maple [**A**]
time = 0.91, size = 58, normalized size = 0.57

method	result	size
risch	$-\frac{4\left(35\,e^{3dx+3c}-21\,e^{2dx+2c}+7\,e^{dx+c}-1\right)}{35d\left(e^{dx+c}-1\right)^7}$	48
derivativedivides	$\frac{\frac{3}{40\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}+\frac{1}{8\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{1}{8\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{1}{56\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}}{d}$	58
default	$\frac{\frac{3}{40\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}+\frac{1}{8\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{1}{8\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{1}{56\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}}{d}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cosh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(3/40/tanh(1/2*d*x+1/2*c)^5+1/8/tanh(1/2*d*x+1/2*c)-1/8/tanh(1/2*d*x+1/
2*c)^3-1/56/tanh(1/2*d*x+1/2*c)^7)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 364 vs. 2(85) = 170.
time = 0.28, size = 364, normalized size = 3.60

$$\frac{4\,e^{(-dx-c)}}{5\,d(7\,e^{(-dx-c)}-21\,e^{(-2\,dx-2\,c)}+35\,e^{(-3\,dx-3\,c)}-35\,e^{(-4\,dx-4\,c)}+21\,e^{(-5\,dx-5\,c)}-7\,e^{(-6\,dx-6\,c)}+e^{(-7\,dx-7\,c)}-1)}-\frac{12\,e^{(-2\,dx-2\,c)}}{5\,d(7\,e^{(-dx-c)}-21\,e^{(-2\,dx-2\,c)}+35\,e^{(-3\,dx-3\,c)}-35\,e^{(-4\,dx-4\,c)}+21\,e^{(-5\,dx-5\,c)}-7\,e^{(-6\,dx-6\,c)}+e^{(-7\,dx-7\,c)}-1)}+\frac{4\,e^{(-3\,dx-3\,c)}}{d(7\,e^{(-dx-c)}-21\,e^{(-2\,dx-2\,c)}+35\,e^{(-3\,dx-3\,c)}-35\,e^{(-4\,dx-4\,c)}+21\,e^{(-5\,dx-5\,c)}-7\,e^{(-6\,dx-6\,c)}+e^{(-7\,dx-7\,c)}-1)}-\frac{4}{35\,d(7\,e^{(-dx-c)}-21\,e^{(-2\,dx-2\,c)}+35\,e^{(-3\,dx-3\,c)}-35\,e^{(-4\,dx-4\,c)}+21\,e^{(-5\,dx-5\,c)}-7\,e^{(-6\,dx-6\,c)}+e^{(-7\,dx-7\,c)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 4/5*e^(-d*x - c)/(d*(7*e^(-d*x - c) - 21*e^(-2*d*x - 2*c) + 35*e^(-3*d*x -
3*c) - 35*e^(-4*d*x - 4*c) + 21*e^(-5*d*x - 5*c) - 7*e^(-6*d*x - 6*c) + e^(
-7*d*x - 7*c) - 1)) - 12/5*e^(-2*d*x - 2*c)/(d*(7*e^(-d*x - c) - 21*e^(-2*d
*x - 2*c) + 35*e^(-3*d*x - 3*c) - 35*e^(-4*d*x - 4*c) + 21*e^(-5*d*x - 5*c)
 - 7*e^(-6*d*x - 6*c) + e^(-7*d*x - 7*c) - 1)) + 4*e^(-3*d*x - 3*c)/(d*(7*e
^(-d*x - c) - 21*e^(-2*d*x - 2*c) + 35*e^(-3*d*x - 3*c) - 35*e^(-4*d*x - 4*
c) + 21*e^(-5*d*x - 5*c) - 7*e^(-6*d*x - 6*c) + e^(-7*d*x - 7*c) - 1)) - 4/
35/(d*(7*e^(-d*x - c) - 21*e^(-2*d*x - 2*c) + 35*e^(-3*d*x - 3*c) - 35*e^(-
4*d*x - 4*c) + 21*e^(-5*d*x - 5*c) - 7*e^(-6*d*x - 6*c) + e^(-7*d*x - 7*c)
- 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(85) = 170$.
time = 0.38, size = 347, normalized size = 3.44

$$-\frac{4\left(35\cosh\left(dx+c\right)^{2}+10\left(7\cosh\left(dx+c\right)-2\right)\sinh\left(dx+c\right)+35\sinh\left(dx+c\right)^{2}-22\cosh\left(dx+c\right)+7\right)}{35\left(d\cosh\left(dx+c\right)^{6}+d\sinh\left(dx+c\right)^{6}-7d\cosh\left(dx+c\right)^{5}+\left(6d\cosh\left(dx+c\right)-7d\right)\sinh\left(dx+c\right)^{5}+21d\cosh\left(dx+c\right)^{4}+\left(15d\cosh\left(dx+c\right)^{2}-35d\cosh\left(dx+c\right)+21d\right)\sinh\left(dx+c\right)^{4}-35d\cosh\left(dx+c\right)^{3}+\left(20d\cosh\left(dx+c\right)^{3}-70d\cosh\left(dx+c\right)^{2}+84d\cosh\left(dx+c\right)-35d\right)\sinh\left(dx+c\right)^{3}+35d\cosh\left(dx+c\right)^{2}+\left(15d\cosh\left(dx+c\right)^{4}-70d\cosh\left(dx+c\right)^{3}+126d\cosh\left(dx+c\right)^{2}-105d\cosh\left(dx+c\right)+35d\right)\sinh\left(dx+c\right)^{2}-22d\cosh\left(dx+c\right)+\left(6d\cosh\left(dx+c\right)^{5}-35d\cosh\left(dx+c\right)^{4}+84d\cosh\left(dx+c\right)^{3}-105d\cosh\left(dx+c\right)^{2}+70d\cosh\left(dx+c\right)-20d\right)\sinh\left(dx+c\right)+7d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -4/35*(35*cosh(d*x + c)^2 + 10*(7*cosh(d*x + c) - 2)*sinh(d*x + c) + 35*sin
h(d*x + c)^2 - 22*cosh(d*x + c) + 7)/(d*cosh(d*x + c)^6 + d*sinh(d*x + c)^6
 - 7*d*cosh(d*x + c)^5 + (6*d*cosh(d*x + c) - 7*d)*sinh(d*x + c)^5 + 21*d*c
osh(d*x + c)^4 + (15*d*cosh(d*x + c)^2 - 35*d*cosh(d*x + c) + 21*d)*sinh(d*
x + c)^4 - 35*d*cosh(d*x + c)^3 + (20*d*cosh(d*x + c)^3 - 70*d*cosh(d*x + c
)^2 + 84*d*cosh(d*x + c) - 35*d)*sinh(d*x + c)^3 + 35*d*cosh(d*x + c)^2 + (
15*d*cosh(d*x + c)^4 - 70*d*cosh(d*x + c)^3 + 126*d*cosh(d*x + c)^2 - 105*d
*cosh(d*x + c) + 35*d)*sinh(d*x + c)^2 - 22*d*cosh(d*x + c) + (6*d*cosh(d*x
 + c)^5 - 35*d*cosh(d*x + c)^4 + 84*d*cosh(d*x + c)^3 - 105*d*cosh(d*x + c)
^2 + 70*d*cosh(d*x + c) - 20*d)*sinh(d*x + c) + 7*d)
```

Sympy [A]
time = 3.03, size = 73, normalized size = 0.72

$$\begin{cases} \frac{1}{8d\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}-\frac{1}{8d\tanh^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}+\frac{3}{40d\tanh^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}-\frac{1}{56d\tanh^{7}\left(\frac{c}{2}+\frac{dx}{2}\right)} & \text{for}\ d\neq 0 \\ \frac{x}{\left(1-\cosh\left(c\right)\right)^{4}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c))**4,x)
```

```
[Out] Piecewise((1/(8*d*tanh(c/2 + d*x/2)) - 1/(8*d*tanh(c/2 + d*x/2)**3) + 3/(40
*d*tanh(c/2 + d*x/2)**5) - 1/(56*d*tanh(c/2 + d*x/2)**7), Ne(d, 0)), (x/(1
- cosh(c))**4, True))
```

Giac [A]
time = 0.40, size = 47, normalized size = 0.47

$$-\frac{4\left(35\,e^{(3\,dx+3\,c)}-21\,e^{(2\,dx+2\,c)}+7\,e^{(dx+c)}-1\right)}{35\,d\left(e^{(dx+c)}-1\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -4/35*(35*e^(3*d*x + 3*c) - 21*e^(2*d*x + 2*c) + 7*e^(d*x + c) - 1)/(d*(e^(
d*x + c) - 1)^7)
```

Mupad [B]
time = 0.08, size = 283, normalized size = 2.80

$$-\frac{4}{35\,d\,\left(6\,\mathrm{e}^{2\,c+2\,d\,x}-4\,\mathrm{e}^{c+d\,x}-4\,\mathrm{e}^{3\,c+3\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}-\frac{16\,\mathrm{e}^{c+d\,x}}{35\,d\,\left(5\,\mathrm{e}^{c+d\,x}-10\,\mathrm{e}^{2\,c+2\,d\,x}+10\,\mathrm{e}^{3\,c+3\,d\,x}-5\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{5\,c+5\,d\,x}-1\right)}-\frac{8\,\mathrm{e}^{2\,c+2\,d\,x}}{7\,d\,\left(15\,\mathrm{e}^{2\,c+2\,d\,x}-6\,\mathrm{e}^{c+d\,x}-20\,\mathrm{e}^{3\,c+3\,d\,x}+15\,\mathrm{e}^{4\,c+4\,d\,x}-6\,\mathrm{e}^{5\,c+5\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1\right)}-\frac{16\,\mathrm{e}^{3\,c+3\,d\,x}}{7\,d\,\left(7\,\mathrm{e}^{c+d\,x}-21\,\mathrm{e}^{2\,c+2\,d\,x}+35\,\mathrm{e}^{3\,c+3\,d\,x}-35\,\mathrm{e}^{4\,c+4\,d\,x}+21\,\mathrm{e}^{5\,c+5\,d\,x}-7\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{7\,c+7\,d\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x) - 1)^4,x)
```

```
[Out] - 4/(35*d*(6*exp(2*c + 2*d*x) - 4*exp(c + d*x) - 4*exp(3*c + 3*d*x) + exp(4
*c + 4*d*x) + 1)) - (16*exp(c + d*x))/(35*d*(5*exp(c + d*x) - 10*exp(2*c +
2*d*x) + 10*exp(3*c + 3*d*x) - 5*exp(4*c + 4*d*x) + exp(5*c + 5*d*x) - 1))
- (8*exp(2*c + 2*d*x))/(7*d*(15*exp(2*c + 2*d*x) - 6*exp(c + d*x) - 20*exp(
3*c + 3*d*x) + 15*exp(4*c + 4*d*x) - 6*exp(5*c + 5*d*x) + exp(6*c + 6*d*x)
+ 1)) - (16*exp(3*c + 3*d*x))/(7*d*(7*exp(c + d*x) - 21*exp(2*c + 2*d*x) +
35*exp(3*c + 3*d*x) - 35*exp(4*c + 4*d*x) + 21*exp(5*c + 5*d*x) - 7*exp(6*c
 + 6*d*x) + exp(7*c + 7*d*x) - 1))
```

3.40 $\int \frac{\cosh(x)}{\sqrt{a+a\cosh(x)}}\,dx$

Optimal. Leaf size=51

$$-\frac{\sqrt{2}\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\sinh(x)}{\sqrt{2}\,\sqrt{a+a\cosh(x)}}\right)}{\sqrt{a}}+\frac{2\sinh(x)}{\sqrt{a+a\cosh(x)}}$$

```
[Out] -arctan(1/2*sinh(x)*a^(1/2)*2^(1/2)/(a+a*cosh(x))^(1/2))*2^(1/2)/a^(1/2)+2*
sinh(x)/(a+a*cosh(x))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2830, 2728, 212}

$$\frac{2\sinh(x)}{\sqrt{a\cosh(x)+a}}-\frac{\sqrt{2}\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\sinh(x)}{\sqrt{2}\,\sqrt{a\cosh(x)+a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/Sqrt[a + a*Cosh[x]],x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Sinh[x])/(Sqrt[2]*Sqrt[a + a*Cosh[x]])])/Sqrt[a]
) + (2*Sinh[x])/Sqrt[a + a*Cosh[x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
```

```
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(x)}{\sqrt{a+a\cosh(x)}}\,dx &= \frac{2\sinh(x)}{\sqrt{a+a\cosh(x)}} - \int \frac{1}{\sqrt{a+a\cosh(x)}}\,dx \\
&= \frac{2\sinh(x)}{\sqrt{a+a\cosh(x)}} - 2i\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{ia\sinh(x)}{\sqrt{a+a\cosh(x)}}\right) \\
&= -\frac{\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sinh(x)}{\sqrt{2}\,\sqrt{a+a\cosh(x)}}\right)}{\sqrt{a}} + \frac{2\sinh(x)}{\sqrt{a+a\cosh(x)}}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 34, normalized size = 0.67

$$-\frac{2\cosh\left(\frac{x}{2}\right)\left(\text{ArcTan}\left(\sinh\left(\frac{x}{2}\right)\right)-2\sinh\left(\frac{x}{2}\right)\right)}{\sqrt{a(1+\cosh(x))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/Sqrt[a + a*Cosh[x]],x]
[Out] (-2*Cosh[x/2]*(ArcTan[Sinh[x/2]] - 2*Sinh[x/2]))/Sqrt[a*(1 + Cosh[x])]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(40) = 80.
time = 1.18, size = 92, normalized size = 1.80

method	result	size
default	$\frac{\cosh\left(\frac{x}{2}\right)\sqrt{\left(\sinh^2\left(\frac{x}{2}\right)\right)a}\,\left(\ln\left(\frac{2\sqrt{\left(\sinh^2\left(\frac{x}{2}\right)\right)a}\,\sqrt{-a}-2a}{\cosh\left(\frac{x}{2}\right)}\right)a+2\sqrt{\left(\sinh^2\left(\frac{x}{2}\right)\right)a}\,\sqrt{-a}\right)\sqrt{2}}{a\sqrt{-a}\,\sinh\left(\frac{x}{2}\right)\sqrt{a\left(\cosh^2\left(\frac{x}{2}\right)\right)}}$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a+a*cosh(x))^(1/2),x,method=_RETURNVERBOSE)
[Out] cosh(1/2*x)*(sinh(1/2*x)^2*a)^(1/2)*(ln(2/cosh(1/2*x)*((sinh(1/2*x)^2*a)^(1
/2)*(-a)^(1/2)-a))*a+2*(sinh(1/2*x)^2*a)^(1/2)*(-a)^(1/2))/a/(-a)^(1/2)/sin
h(1/2*x)*2^(1/2)/(a*cosh(1/2*x)^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(40) = 80$.
time = 0.52, size = 114, normalized size = 2.24

$$-\sqrt{2}\left(\frac{\arctan\left(e^{\left(\frac{1}{2}x\right)}\right)}{\sqrt{a}} - \frac{e^{\left(\frac{1}{2}x\right)}}{\sqrt{a}\,e^{x} + \sqrt{a}}\right) + \frac{1}{3}\sqrt{2}\left(\frac{3\arctan\left(e^{\left(-\frac{1}{2}x\right)}\right)}{\sqrt{a}} - \frac{2\,e^{\left(-\frac{1}{2}x\right)}}{\sqrt{a}} - \frac{e^{\left(-\frac{1}{2}x\right)}}{\sqrt{a}\,e^{(-x)} + \sqrt{a}}\right) + \frac{3\sqrt{2}\sqrt{a}\,e^{\left(\frac{3}{2}x\right)} - \sqrt{2}\sqrt{a}\,e^{\left(-\frac{1}{2}x\right)}}{3\,(ae^{x} + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*cosh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(2)*(arctan(e^(1/2*x))/sqrt(a) - e^(1/2*x)/(sqrt(a)*e^x + sqrt(a))) +
1/3*sqrt(2)*(3*arctan(e^(-1/2*x))/sqrt(a) - 2*e^(-1/2*x)/sqrt(a) - e^(-1/2*
x)/(sqrt(a)*e^(-x) + sqrt(a))) + 1/3*(3*sqrt(2)*sqrt(a)*e^(3/2*x) - sqrt(2)
*sqrt(a)*e^(-1/2*x))/(a*e^x + a)
```

Fricas [A]
time = 0.37, size = 62, normalized size = 1.22

$$\frac{2\left(\sqrt{\frac{1}{2}}\sqrt{\frac{a}{\cosh(x)+\sinh(x)}}\,(\cosh(x)+\sinh(x)-1) - \sqrt{2}\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{\cosh(x)+\sinh(x)}}\,(\cosh(x)+\sinh(x))}{\sqrt{a}}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*cosh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(1/2)*sqrt(a/(cosh(x) + sinh(x)))*(cosh(x) + sinh(x) - 1) - sqrt(2)*
sqrt(a)*arctan(sqrt(2)*sqrt(1/2)*sqrt(a/(cosh(x) + sinh(x)))*(cosh(x) + sin
h(x))/sqrt(a)))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(x)}{\sqrt{a\,(\cosh(x)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*cosh(x))**(1/2),x)
```

```
[Out] Integral(cosh(x)/sqrt(a*(cosh(x) + 1)), x)
```

Giac [A]
time = 0.39, size = 37, normalized size = 0.73

$$-\frac{2\sqrt{2}\,\arctan\left(e^{\left(\frac{1}{2}x\right)}\right)}{\sqrt{a}} + \frac{\sqrt{2}\,e^{\left(\frac{1}{2}x\right)}}{\sqrt{a}} - \frac{\sqrt{2}\,e^{\left(-\frac{1}{2}x\right)}}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*cosh(x))^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(2)*arctan(e^(1/2*x))/sqrt(a) + sqrt(2)*e^(1/2*x)/sqrt(a) - sqrt(2)*
e^(-1/2*x)/sqrt(a)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cosh(x)}{\sqrt{a + a\cosh\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a + a*cosh(x))^(1/2),x)
```

```
[Out] int(cosh(x)/(a + a*cosh(x))^(1/2), x)
```

3.41 $\int \frac{\cosh(x)}{\sqrt{a-a\cosh(x)}}\,dx$

Optimal. Leaf size=53

$$-\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sinh(x)}{\sqrt{2}\,\sqrt{a-a\cosh(x)}}\right)}{\sqrt{a}}+\frac{2\sinh(x)}{\sqrt{a-a\cosh(x)}}$$

```
[Out] -arctan(1/2*sinh(x)*a^(1/2)*2^(1/2)/(a-a*cosh(x))^(1/2))*2^(1/2)/a^(1/2)+2*
sinh(x)/(a-a*cosh(x))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2830, 2728, 212}

$$\frac{2\sinh(x)}{\sqrt{a-a\cosh(x)}}-\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sinh(x)}{\sqrt{2}\,\sqrt{a-a\cosh(x)}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/Sqrt[a - a*Cosh[x]],x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Sinh[x])/(Sqrt[2]*Sqrt[a - a*Cosh[x]])])/Sqrt[a]
) + (2*Sinh[x])/Sqrt[a - a*Cosh[x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
```

```
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(x)}{\sqrt{a-a\cosh(x)}}\,dx &= \frac{2\sinh(x)}{\sqrt{a-a\cosh(x)}}+\int \frac{1}{\sqrt{a-a\cosh(x)}}\,dx\\ &= \frac{2\sinh(x)}{\sqrt{a-a\cosh(x)}}+2i\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{ia\sinh(x)}{\sqrt{a-a\cosh(x)}}\right)\\ &= -\frac{\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sinh(x)}{\sqrt{2}\,\sqrt{a-a\cosh(x)}}\right)}{\sqrt{a}}+\frac{2\sinh(x)}{\sqrt{a-a\cosh(x)}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 0.66

$$\frac{2\left(2\cosh\left(\frac{x}{2}\right)+\log\left(\tanh\left(\frac{x}{4}\right)\right)\right)\sinh\left(\frac{x}{2}\right)}{\sqrt{a-a\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/Sqrt[a - a*Cosh[x]],x]
```

```
[Out] (2*(2*Cosh[x/2] + Log[Tanh[x/4]])*Sinh[x/2])/Sqrt[a - a*Cosh[x]]
```

Maple [A]
time = 1.12, size = 40, normalized size = 0.75

method	result	size
default	$\frac{\sinh\left(\frac{x}{2}\right)\left(4\cosh\left(\frac{x}{2}\right)+\ln\left(\cosh\left(\frac{x}{2}\right)-1\right)-\ln\left(\cosh\left(\frac{x}{2}\right)+1\right)\right)}{\sqrt{-2\left(\sinh^2\left(\frac{x}{2}\right)\right)a}}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a-a*cosh(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] sinh(1/2*x)*(4*cosh(1/2*x)+ln(cosh(1/2*x)-1)-ln(cosh(1/2*x)+1))/(-2*sinh(1/
2*x)^2*a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a-a*cosh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(x)/sqrt(-a*cosh(x) + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(42) = 84$.
time = 0.35, size = 92, normalized size = 1.74

$$\frac{\sqrt{2}\,a\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a}{\cosh(x)+\sinh(x)}}\sqrt{-\frac{1}{a}}(\cosh(x)+\sinh(x))-\cosh(x)-\sinh(x)-1}{\cosh(x)+\sinh(x)-1}\right)-2\sqrt{\frac{1}{2}}\sqrt{-\frac{a}{\cosh(x)+\sinh(x)}}(\cosh(x)+\sinh(x)+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a-a*cosh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*a*sqrt(-1/a)*log((2*sqrt(2)*sqrt(1/2)*sqrt(-a/(cosh(x) + sinh(x)))
*sqrt(-1/a)*(cosh(x) + sinh(x)) - cosh(x) - sinh(x) - 1)/(cosh(x) + sinh(x)
 - 1)) - 2*sqrt(1/2)*sqrt(-a/(cosh(x) + sinh(x)))*(cosh(x) + sinh(x) + 1))/
a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cosh(x)}{\sqrt{-a(\cosh(x)-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a-a*cosh(x))**(1/2),x)
```

```
[Out] Integral(cosh(x)/sqrt(-a*(cosh(x) - 1)), x)
```

Giac [A]
time = 0.40, size = 74, normalized size = 1.40

$$-\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{-ae^x}}{\sqrt{a}}\right)}{\sqrt{a}\,\mathrm{sgn}(-e^x+1)}-\frac{\sqrt{2}}{\sqrt{-ae^x}\,\mathrm{sgn}(-e^x+1)}+\frac{\sqrt{2}\sqrt{-ae^x}}{a\mathrm{sgn}(-e^x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a-a*cosh(x))^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(2)*arctan(sqrt(-a*e^x)/sqrt(a))/(sqrt(a)*sgn(-e^x + 1)) - sqrt(2)/(
sqrt(-a*e^x)*sgn(-e^x + 1)) + sqrt(2)*sqrt(-a*e^x)/(a*sgn(-e^x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cosh(x)}{\sqrt{a - a\cosh(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a - a*cosh(x))^(1/2),x)
[Out] int(cosh(x)/(a - a*cosh(x))^(1/2), x)
```

3.42 $\int (a+a\cosh(c+dx))^{5/2}\,dx$

Optimal. Leaf size=89

$$\frac{64a^3\sinh(c+dx)}{15d\sqrt{a+a\cosh(c+dx)}}+\frac{16a^2\sqrt{a+a\cosh(c+dx)}\,\sinh(c+dx)}{15d}+\frac{2a(a+a\cosh(c+dx))^{3/2}\sinh(c+dx)}{5d}$$

```
[Out] 2/5*a*(a+a*cosh(d*x+c))^(3/2)*sinh(d*x+c)/d+64/15*a^3*sinh(d*x+c)/d/(a+a*co
sh(d*x+c))^(1/2)+16/15*a^2*sinh(d*x+c)*(a+a*cosh(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.03, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2726, 2725}

$$\frac{64a^3\sinh(c+dx)}{15d\sqrt{a\cosh(c+dx)+a}}+\frac{16a^2\sinh(c+dx)\sqrt{a\cosh(c+dx)+a}}{15d}+\frac{2a\sinh(c+dx)(a\cosh(c+dx)+a)^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[c + d*x])^(5/2),x]
```

```
[Out] (64*a^3*Sinh[c + d*x])/(15*d*Sqrt[a + a*Cosh[c + d*x]]) + (16*a^2*Sqrt[a +
a*Cosh[c + d*x]]*Sinh[c + d*x])/(15*d) + (2*a*(a + a*Cosh[c + d*x])^(3/2)*S
inh[c + d*x])/(5*d)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\cosh(c+dx))^{5/2}\,dx &= \frac{2a(a+a\cosh(c+dx))^{3/2}\sinh(c+dx)}{5d}+\frac{1}{5}(8a)\int (a+a\cosh(c+dx))^{3/2}\,dx\\ &= \frac{16a^2\sqrt{a+a\cosh(c+dx)}\,\sinh(c+dx)}{15d}+\frac{2a(a+a\cosh(c+dx))^{3/2}\sinh(c+\\ &= \frac{64a^3\sinh(c+dx)}{15d\sqrt{a+a\cosh(c+dx)}}+\frac{16a^2\sqrt{a+a\cosh(c+dx)}\,\sinh(c+dx)}{15d}+\frac{2a(a\end{aligned}$$

Mathematica [A]
time = 0.09, size = 71, normalized size = 0.80

$$\frac{a^2\sqrt{a(1+\cosh(c+dx))}\,\mathrm{sech}\left(\frac{1}{2}(c+dx)\right)\left(150\sinh\left(\frac{1}{2}(c+dx)\right)+25\sinh\left(\frac{3}{2}(c+dx)\right)+3\sinh\left(\frac{5}{2}(c+dx)\right)\right)}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[c + d*x])^(5/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cosh[c + d*x])]*Sech[(c + d*x)/2]*(150*Sinh[(c + d*x)/2] +
 25*Sinh[(3*(c + d*x))/2] + 3*Sinh[(5*(c + d*x))/2]))/(30*d)
```

Maple [A]
time = 0.94, size = 73, normalized size = 0.82

method	result	size
default	$\frac{8a^3\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\left(3\left(\cosh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\left(\cosh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\right)\sqrt{2}}{15\sqrt{a\left(\cosh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 8/15*a^3*cosh(1/2*d*x+1/2*c)*sinh(1/2*d*x+1/2*c)*(3*cosh(1/2*d*x+1/2*c)^4+4
*cosh(1/2*d*x+1/2*c)^2+8)*2^(1/2)/(a*cosh(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [A]
time = 0.48, size = 121, normalized size = 1.36

$$\frac{\sqrt{2}\,a^{\frac{5}{2}}e^{\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)}}{20\,d}+\frac{5\,\sqrt{2}\,a^{\frac{5}{2}}e^{\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)}}{12\,d}+\frac{5\,\sqrt{2}\,a^{\frac{5}{2}}e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}-\frac{5\,\sqrt{2}\,a^{\frac{5}{2}}e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}}{2\,d}-\frac{5\,\sqrt{2}\,a^{\frac{5}{2}}e^{\left(-\frac{3}{2}\,dx-\frac{3}{2}\,c\right)}}{12\,d}-\frac{\sqrt{2}\,a^{\frac{5}{2}}e^{\left(-\frac{5}{2}\,dx-\frac{5}{2}\,c\right)}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/20*sqrt(2)*a^(5/2)*e^(5/2*d*x + 5/2*c)/d + 5/12*sqrt(2)*a^(5/2)*e^(3/2*d*
x + 3/2*c)/d + 5/2*sqrt(2)*a^(5/2)*e^(1/2*d*x + 1/2*c)/d - 5/2*sqrt(2)*a^(5
/2)*e^(-1/2*d*x - 1/2*c)/d - 5/12*sqrt(2)*a^(5/2)*e^(-3/2*d*x - 3/2*c)/d -
1/20*sqrt(2)*a^(5/2)*e^(-5/2*d*x - 5/2*c)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(77) = 154$.
time = 0.37, size = 327, normalized size = 3.67

$$\frac{\sqrt{\frac{1}{2}}\left(3\,a^2\cosh\left(dx+c\right)^5+3\,a^2\sinh\left(dx+c\right)^5+25\,a^2\cosh\left(dx+c\right)^4+150\,a^2\cosh\left(dx+c\right)^3+5\left(3\,a^2\cosh\left(dx+c\right)+5\,a^2\right)\sinh\left(dx+c\right)^4-150\,a^2\cosh\left(dx+c\right)^2+10\left(3\,a^2\cosh\left(dx+c\right)^2+10\,a^2\cosh\left(dx+c\right)+15\,a^2\right)\sinh\left(dx+c\right)^3-25\,a^2\cosh\left(dx+c\right)+30\left(a^2\cosh\left(dx+c\right)^3+5\,a^2\cosh\left(dx+c\right)^2+15\,a^2\cosh\left(dx+c\right)-5\,a^2\right)\sinh\left(dx+c\right)^2-3\,a^2+5\left(3\,a^2\cosh\left(dx+c\right)^4+20\,a^2\cosh\left(dx+c\right)^3+90\,a^2\cosh\left(dx+c\right)^2-60\,a^2\cosh\left(dx+c\right)-5\,a^2\right)\sinh\left(dx+c\right)\right)\sqrt{\frac{a}{\cosh\left(dx+c\right)+\sinh\left(dx+c\right)}}}{30\left(d\cosh\left(dx+c\right)^2+2\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)+d\sinh\left(dx+c\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/30*sqrt(1/2)*(3*a^2*cosh(d*x + c)^5 + 3*a^2*sinh(d*x + c)^5 + 25*a^2*cosh
(d*x + c)^4 + 150*a^2*cosh(d*x + c)^3 + 5*(3*a^2*cosh(d*x + c) + 5*a^2)*sin
h(d*x + c)^4 - 150*a^2*cosh(d*x + c)^2 + 10*(3*a^2*cosh(d*x + c)^2 + 10*a^2
*cosh(d*x + c) + 15*a^2)*sinh(d*x + c)^3 - 25*a^2*cosh(d*x + c) + 30*(a^2*c
osh(d*x + c)^3 + 5*a^2*cosh(d*x + c)^2 + 15*a^2*cosh(d*x + c) - 5*a^2)*sinh
(d*x + c)^2 - 3*a^2 + 5*(3*a^2*cosh(d*x + c)^4 + 20*a^2*cosh(d*x + c)^3 + 9
0*a^2*cosh(d*x + c)^2 - 60*a^2*cosh(d*x + c) - 5*a^2)*sinh(d*x + c))*sqrt(a
/(cosh(d*x + c) + sinh(d*x + c)))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*si
nh(d*x + c) + d*sinh(d*x + c)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.42, size = 105, normalized size = 1.18

$$-\frac{\sqrt{2}\left(\left(150\,a^{\frac{5}{2}}e^{\left(2\,dx+\frac{5}{2}\,c\right)}+25\,a^{\frac{5}{2}}e^{\left(dx+\frac{3}{2}\,c\right)}+3\,a^{\frac{5}{2}}e^{\left(\frac{1}{2}\,c\right)}\right)e^{\left(-\frac{5}{2}\,dx-3\,c\right)}-\left(3\,a^{\frac{5}{2}}e^{\left(\frac{5}{2}\,dx+\frac{35}{2}\,c\right)}+25\,a^{\frac{5}{2}}e^{\left(\frac{3}{2}\,dx+\frac{33}{2}\,c\right)}+150\,a^{\frac{5}{2}}e^{\left(\frac{1}{2}\,dx+\frac{31}{2}\,c\right)}\right)e^{\left(-15\,c\right)}\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/60*sqrt(2)*((150*a^(5/2)*e^(2*d*x + 5/2*c) + 25*a^(5/2)*e^(d*x + 3/2*c)
+ 3*a^(5/2)*e^(1/2*c))*e^(-5/2*d*x - 3*c) - (3*a^(5/2)*e^(5/2*d*x + 35/2*c)
 + 25*a^(5/2)*e^(3/2*d*x + 33/2*c) + 150*a^(5/2)*e^(1/2*d*x + 31/2*c))*e^(-
15*c))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+a\cosh(c+d\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cosh(c + d*x))^(5/2),x)
```

```
[Out] int((a + a*cosh(c + d*x))^(5/2), x)
```

3.43 $\int (a+a\cosh (c+dx))^{3/2}\,dx$

Optimal. Leaf size=59

$$\frac{8a^2\sinh(c+dx)}{3d\sqrt{a+a\cosh(c+dx)}}+\frac{2a\sqrt{a+a\cosh(c+dx)}\,\sinh(c+dx)}{3d}$$

```
[Out] 8/3*a^2*sinh(d*x+c)/d/(a+a*cosh(d*x+c))^(1/2)+2/3*a*sinh(d*x+c)*(a+a*cosh(d
*x+c))^(1/2)/d
```

Rubi [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2726, 2725}

$$\frac{8a^2\sinh(c+dx)}{3d\sqrt{a\cosh(c+dx)+a}}+\frac{2a\sinh(c+dx)\sqrt{a\cosh(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[c + d*x])^(3/2),x]
```

```
[Out] (8*a^2*Sinh[c + d*x])/(3*d*Sqrt[a + a*Cosh[c + d*x]]) + (2*a*Sqrt[a + a*Cos
h[c + d*x]]*Sinh[c + d*x])/(3*d)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\cosh(c+dx))^{3/2}\,dx &= \frac{2a\sqrt{a+a\cosh(c+dx)}\,\sinh(c+dx)}{3d}+\frac{1}{3}(4a)\int \sqrt{a+a\cosh(c+dx)}\,dx\\ &= \frac{8a^2\sinh(c+dx)}{3d\sqrt{a+a\cosh(c+dx)}}+\frac{2a\sqrt{a+a\cosh(c+dx)}\,\sinh(c+dx)}{3d}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 55, normalized size = 0.93

$$\frac{a\sqrt{a(1+\cosh(c+dx))}\,\mathrm{sech}\left(\frac{1}{2}(c+dx)\right)\left(9\sinh\left(\frac{1}{2}(c+dx)\right)+\sinh\left(\frac{3}{2}(c+dx)\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[c + d*x])^(3/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cosh[c + d*x])]*Sech[(c + d*x)/2]*(9*Sinh[(c + d*x)/2] + Sin
h[(3*(c + d*x))/2]))/(3*d)
```

Maple [A]
time = 0.94, size = 58, normalized size = 0.98

method	result	size
default	$\frac{4a^2\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\cosh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)\sqrt{2}}{3\sqrt{a\left(\cosh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4/3*a^2*cosh(1/2*d*x+1/2*c)*sinh(1/2*d*x+1/2*c)*(cosh(1/2*d*x+1/2*c)^2+2)*2
^(1/2)/(a*cosh(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [A]
time = 0.47, size = 81, normalized size = 1.37

$$\frac{\sqrt{2}\,a^{\frac{3}{2}}e^{\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)}}{6\,d}+\frac{3\,\sqrt{2}\,a^{\frac{3}{2}}e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}-\frac{3\,\sqrt{2}\,a^{\frac{3}{2}}e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}}{2\,d}-\frac{\sqrt{2}\,a^{\frac{3}{2}}e^{\left(-\frac{3}{2}\,dx-\frac{3}{2}\,c\right)}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*a^(3/2)*e^(3/2*d*x + 3/2*c)/d + 3/2*sqrt(2)*a^(3/2)*e^(1/2*d*x
+ 1/2*c)/d - 3/2*sqrt(2)*a^(3/2)*e^(-1/2*d*x - 1/2*c)/d - 1/6*sqrt(2)*a^(3/
2)*e^(-3/2*d*x - 3/2*c)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(51) = 102.
time = 0.36, size = 140, normalized size = 2.37

$$\frac{\sqrt{\frac{1}{2}}\left(a\cosh\left(dx+c\right)^3+a\sinh\left(dx+c\right)^3+9\,a\cosh\left(dx+c\right)^2+3\left(a\cosh\left(dx+c\right)+3\,a\right)\sinh\left(dx+c\right)^2-9\,a\cosh\left(dx+c\right)+3\left(a\cosh\left(dx+c\right)^2+6\,a\cosh\left(dx+c\right)-3\,a\right)\sinh\left(dx+c\right)-a\right)\sqrt{\frac{a}{\cosh\left(dx+c\right)+\sinh\left(dx+c\right)}}}{3\left(d\cosh\left(dx+c\right)+d\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(1/2)*(a*cosh(d*x + c)^3 + a*sinh(d*x + c)^3 + 9*a*cosh(d*x + c)^2
+ 3*(a*cosh(d*x + c) + 3*a)*sinh(d*x + c)^2 - 9*a*cosh(d*x + c) + 3*(a*cosh
(d*x + c)^2 + 6*a*cosh(d*x + c) - 3*a)*sinh(d*x + c) - a)*sqrt(a/(cosh(d*x
+ c) + sinh(d*x + c)))/(d*cosh(d*x + c) + d*sinh(d*x + c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cosh(c+dx)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*cosh(c + d*x) + a)**(3/2), x)
```

Giac [**A**]
time = 0.43, size = 75, normalized size = 1.27

$$-\frac{\sqrt{2}\left(\left(9\,a^{\frac{3}{2}}e^{\left(dx+\frac{3}{2}\,c\right)}+a^{\frac{3}{2}}e^{\left(\frac{1}{2}\,c\right)}\right)e^{\left(-\frac{3}{2}\,dx-2\,c\right)}-\left(a^{\frac{3}{2}}e^{\left(\frac{3}{2}\,dx+\frac{15}{2}\,c\right)}+9\,a^{\frac{3}{2}}e^{\left(\frac{1}{2}\,dx+\frac{13}{2}\,c\right)}\right)e^{\left(-6\,c\right)}\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(2)*((9*a^(3/2)*e^(d*x + 3/2*c) + a^(3/2)*e^(1/2*c))*e^(-3/2*d*x -
 2*c) - (a^(3/2)*e^(3/2*d*x + 15/2*c) + 9*a^(3/2)*e^(1/2*d*x + 13/2*c))*e^(
-6*c))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int (a+a\cosh(c+d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cosh(c + d*x))^(3/2),x)
```

```
[Out] int((a + a*cosh(c + d*x))^(3/2), x)
```

3.44 $\int \sqrt{a+a\cosh(c+dx)}\,dx$

Optimal. Leaf size=26

$$\frac{2a\sinh(c+dx)}{d\sqrt{a+a\cosh(c+dx)}}$$

```
[Out] 2*a*sinh(d*x+c)/d/(a+a*cosh(d*x+c))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = $\{2725\}$

$$\frac{2a\sinh(c+dx)}{d\sqrt{a\cosh(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cosh[c + d*x]],x]
```

```
[Out] (2*a*Sinh[c + d*x])/(d*Sqrt[a + a*Cosh[c + d*x]])
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\cosh(c+dx)}\,dx = \frac{2a\sinh(c+dx)}{d\sqrt{a+a\cosh(c+dx)}}$$

Mathematica [A]
time = 0.03, size = 29, normalized size = 1.12

$$\frac{2\sqrt{a(1+\cosh(c+dx))}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cosh[c + d*x]],x]
```

```
[Out] (2*Sqrt[a*(1 + Cosh[c + d*x])]*Tanh[(c + d*x)/2])/d
```

Maple [A]
time = 0.88, size = 43, normalized size = 1.65

method	result	size
default	$\frac{2a\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2}}{\sqrt{a\left(\cosh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$	43
risch	$\frac{\sqrt{2}\,\sqrt{a\left(\mathrm{e}^{dx+c}+1\right)^{2}\mathrm{e}^{-dx-c}}\,\left(\mathrm{e}^{dx+c}-1\right)}{\left(\mathrm{e}^{dx+c}+1\right)d}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a*cosh(1/2*d*x+1/2*c)*sinh(1/2*d*x+1/2*c)*2^(1/2)/(a*cosh(1/2*d*x+1/2*c)^
2)^(1/2)/d
```

Maxima [A]
time = 0.48, size = 40, normalized size = 1.54

$$\frac{\sqrt{2}\,\sqrt{a}\,e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{d}-\frac{\sqrt{2}\,\sqrt{a}\,e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(2)*sqrt(a)*e^(1/2*d*x + 1/2*c)/d - sqrt(2)*sqrt(a)*e^(-1/2*d*x - 1/2*c
)/d
```

Fricas [A]
time = 0.37, size = 41, normalized size = 1.58

$$\frac{2\sqrt{\frac{1}{2}}\sqrt{\frac{a}{\cosh\left(dx+c\right)+\sinh\left(dx+c\right)}}\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)-1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(1/2)*sqrt(a/(cosh(d*x + c) + sinh(d*x + c)))*(cosh(d*x + c) + sinh(d
*x + c) - 1)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\cosh\left(c+dx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*cosh(c + d*x) + a), x)
```

Giac [A]
time = 0.41, size = 35, normalized size = 1.35

$$\frac{\sqrt{2}\left(\sqrt{a}\,e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\sqrt{a}\,e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*(sqrt(a)*e^(1/2*d*x + 1/2*c) - sqrt(a)*e^(-1/2*d*x - 1/2*c))/d
```

Mupad [B]
time = 0.11, size = 26, normalized size = 1.00

$$\frac{2\tanh\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a+a\cosh\left(c+d\,x\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cosh(c + d*x))^(1/2),x)
```

```
[Out] (2*tanh(c/2 + (d*x)/2)*(a + a*cosh(c + d*x))^(1/2))/d
```

3.45 $\int \frac{1}{\sqrt{a+a\cosh(c+dx)}}\,dx$

Optimal. Leaf size=46

$$\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{2}\,\sqrt{a+a\cosh(c+dx)}}\right)}{\sqrt{a}\,d}$$

```
[Out] arctan(1/2*sinh(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cosh(d*x+c))^(1/2))*2^(1/2)/d/a
^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2728, 212}

$$\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{2}\,\sqrt{a\cosh(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + a*Cosh[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcTan[(Sqrt[a]*Sinh[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cosh[c + d*x]])
])/(Sqrt[a]*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\cosh(c+dx)}}\,dx = \frac{(2i)\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{ia\sinh(c+dx)}{\sqrt{a+a\cosh(c+dx)}}\right)}{d}$$
$$= \frac{\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{2}\,\sqrt{a+a\cosh(c+dx)}}\right)}{\sqrt{a}\,d}$$

Mathematica [A]
time = 0.01, size = 40, normalized size = 0.87

$$\frac{2\mathrm{ArcTan}\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)\cosh\left(\frac{1}{2}(c+dx)\right)}{d\sqrt{a(1+\cosh(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + a*Cosh[c + d*x]],x]
```

```
[Out] (2*ArcTan[Sinh[(c + d*x)/2]]*Cosh[(c + d*x)/2])/(d*Sqrt[a*(1 + Cosh[c + d*x
])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(37) = 74$.
time = 0.94, size = 103, normalized size = 2.24

method	result	size
default	$-\frac{\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(\sinh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}\,\ln\left(\frac{2\sqrt{-a}\,\sqrt{\left(\sinh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}-2a}{\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\sqrt{2}}{\sqrt{-a}\,\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cosh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$	103

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cosh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -cosh(1/2*d*x+1/2*c)*(sinh(1/2*d*x+1/2*c)^2*a)^(1/2)/(-a)^(1/2)*ln(2*((-a)^
(1/2)*(sinh(1/2*d*x+1/2*c)^2*a)^(1/2)-a)/cosh(1/2*d*x+1/2*c))/sinh(1/2*d*x+
1/2*c)*2^(1/2)/(a*cosh(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(37) = 74$.
time = 0.53, size = 86, normalized size = 1.87

$$2\sqrt{2}\left(\frac{\arctan\left(e^{\left(\frac{1}{2}dx+\frac{1}{2}c\right)}\right)}{\sqrt{a}\,d} + \frac{e^{\left(\frac{1}{2}dx+\frac{1}{2}c\right)}}{\left(\sqrt{a}\,e^{(dx+c)}+\sqrt{a}\right)d}\right) - \frac{2\sqrt{2}\,e^{\left(\frac{1}{2}dx+\frac{1}{2}c\right)}}{\sqrt{a}\,de^{(dx+c)}+\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(2)*(arctan(e^(1/2*d*x + 1/2*c))/(sqrt(a)*d) + e^(1/2*d*x + 1/2*c)/((
sqrt(a)*e^(d*x + c) + sqrt(a))*d)) - 2*sqrt(2)*e^(1/2*d*x + 1/2*c)/(sqrt(a)
*d*e^(d*x + c) + sqrt(a)*d)
```

Fricas [A]
time = 0.42, size = 149, normalized size = 3.24

$$\left[\frac{\sqrt{2}\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{\cosh\left(dx+c\right)+\sinh\left(dx+c\right)}}\sqrt{-\frac{1}{a}}\left(\cosh(dx+c)+\sinh(dx+c)\right)+\cosh(dx+c)+\sinh(dx+c)-1}{\cosh(dx+c)+\sinh(dx+c)+1}\right)}{d}, \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{\cosh\left(dx+c\right)+\sinh\left(dx+c\right)}}\left(\cosh(dx+c)+\sinh(dx+c)\right)}{\sqrt{a}}\right)}{\sqrt{a}\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(2)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt(1/2)*sqrt(a/(cosh(d*x + c) + sinh(
d*x + c)))*sqrt(-1/a)*(cosh(d*x + c) + sinh(d*x + c)) + cosh(d*x + c) + sin
h(d*x + c) - 1)/(cosh(d*x + c) + sinh(d*x + c) + 1))/d, 2*sqrt(2)*arctan(sq
rt(2)*sqrt(1/2)*sqrt(a/(cosh(d*x + c) + sinh(d*x + c)))*(cosh(d*x + c) + si
nh(d*x + c))/sqrt(a))/(sqrt(a)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\cosh\left(c+dx\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*cosh(c + d*x) + a), x)
```

Giac [A]
time = 0.42, size = 21, normalized size = 0.46

$$\frac{2\sqrt{2}\arctan\left(e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}\right)}{\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(2)*arctan(e^(1/2*d*x + 1/2*c))/(sqrt(a)*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{a+a\cosh\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*cosh(c + d*x))^(1/2),x)
[Out] int(1/(a + a*cosh(c + d*x))^(1/2), x)
```

3.46 $\int \frac{1}{(a+a\cosh(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{2}\,\sqrt{a+a\cosh(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{\sinh(c+dx)}{2d(a+a\cosh(c+dx))^{3/2}}$$

```
[Out] 1/2*sinh(d*x+c)/d/(a+a*cosh(d*x+c))^(3/2)+1/4*arctan(1/2*sinh(d*x+c)*a^(1/2
)*2^(1/2)/(a+a*cosh(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2729, 2728, 212}

$$\frac{\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{2}\,\sqrt{a\cosh(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{\sinh(c+dx)}{2d(a\cosh(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[c + d*x])^(-3/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Sinh[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cosh[c + d*x]])]/(2*Sqrt
[2]*a^(3/2)*d) + Sinh[c + d*x]/(2*d*(a + a*Cosh[c + d*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(a+a\cosh(c+dx))^{3/2}}\,dx = \frac{\sinh(c+dx)}{2d(a+a\cosh(c+dx))^{3/2}} + \frac{\int \frac{1}{\sqrt{a+a\cosh(c+dx)}}\,dx}{4a}$$

$$= \frac{\sinh(c+dx)}{2d(a+a\cosh(c+dx))^{3/2}} + \frac{i\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{ia\sinh(c+dx)}{\sqrt{a+a\cosh(c+dx)}}\right)}{2ad}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a+a\cosh(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{\sinh(c+dx)}{2d(a+a\cosh(c+dx))^{3/2}}$$

Mathematica [A]
time = 0.06, size = 63, normalized size = 0.82

$$\frac{\cosh^2\left(\frac{1}{2}(c+dx)\right)\left(\text{ArcTan}\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)\cosh\left(\frac{1}{2}(c+dx)\right)+\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d(a(1+\cosh(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[c + d*x])^(-3/2),x]
```

```
[Out] (Cosh[(c + d*x)/2]^2*(ArcTan[Sinh[(c + d*x)/2]]*Cosh[(c + d*x)/2] + Tanh[(c
 + d*x)/2]))/(d*(a*(1 + Cosh[c + d*x]))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(62) = 124.
time = 1.08, size = 144, normalized size = 1.87

method	result
default	$-\frac{\sqrt{\left(\sinh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}\left(\ln\left(\frac{2\sqrt{-a}\sqrt{\left(\sinh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}-2a}{\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\left(\cosh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\sqrt{-a}\sqrt{\left(\sinh^2\left(\frac{dx}{2}+\right.\right.}\right.}{4a^2\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-a}\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cosh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cosh(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(sinh(1/2*d*x+1/2*c)^2*a)^(1/2)*(ln(2*((-a)^(1/2)*(sinh(1/2*d*x+1/2*c)
^2*a)^(1/2)-a)/cosh(1/2*d*x+1/2*c))*a*cosh(1/2*d*x+1/2*c)^2-(-a)^(1/2)*(sin
h(1/2*d*x+1/2*c)^2*a)^(1/2))/a^2/cosh(1/2*d*x+1/2*c)/(-a)^(1/2)/sinh(1/2*d*
x+1/2*c)*2^(1/2)/(a*cosh(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(62) = 124$.
time = 0.54, size = 170, normalized size = 2.21

$$\frac{1}{6}\sqrt{2}\left(\frac{3e^{\left(\frac{5}{2}dx+\frac{5}{2}c\right)}+8e^{\left(\frac{3}{2}dx+\frac{3}{2}c\right)}-3e^{\left(\frac{1}{2}dx+\frac{1}{2}c\right)}}{\left(a^{\frac{3}{2}}e^{(3dx+3c)}+3a^{\frac{3}{2}}e^{(2dx+2c)}+3a^{\frac{3}{2}}e^{(dx+c)}+a^{\frac{3}{2}}\right)d}+\frac{3\arctan\left(e^{\left(\frac{1}{2}dx+\frac{1}{2}c\right)}\right)}{a^{\frac{3}{2}}d}\right)-\frac{4\sqrt{2}e^{\left(\frac{3}{2}dx+\frac{3}{2}c\right)}}{3\left(a^{\frac{3}{2}}de^{(3dx+3c)}+3a^{\frac{3}{2}}de^{(2dx+2c)}+3a^{\frac{3}{2}}de^{(dx+c)}+a^{\frac{3}{2}}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*((3*e^(5/2*d*x + 5/2*c) + 8*e^(3/2*d*x + 3/2*c) - 3*e^(1/2*d*x
+ 1/2*c))/((a^(3/2)*e^(3*d*x + 3*c) + 3*a^(3/2)*e^(2*d*x + 2*c) + 3*a^(3/2)
*e^(d*x + c) + a^(3/2))*d) + 3*arctan(e^(1/2*d*x + 1/2*c))/(a^(3/2)*d)) - 4
/3*sqrt(2)*e^(3/2*d*x + 3/2*c)/(a^(3/2)*d*e^(3*d*x + 3*c) + 3*a^(3/2)*d*e^(
2*d*x + 2*c) + 3*a^(3/2)*d*e^(d*x + c) + a^(3/2)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(62) = 124$.
time = 0.35, size = 219, normalized size = 2.84

$$-\frac{\sqrt{2}\left(\cosh(dx+c)^2+2(\cosh(dx+c)+1)\sinh(dx+c)+\sinh(dx+c)^2+2\cosh(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{\cosh(dx+c)+\sinh(dx+c)}}}{\sqrt{a}}\right)-2\sqrt{\frac{1}{2}}\left(\cosh(dx+c)^2+(2\cosh(dx+c)-1)\sinh(dx+c)+\sinh(dx+c)^2-\cosh(dx+c)\right)\sqrt{\frac{a}{\cosh(dx+c)+\sinh(dx+c)}}}{2\left(a^2d\cosh(dx+c)^2+a^2d\sinh(dx+c)^2+2a^2d\cosh(dx+c)+a^2d+2\left(a^2d\cosh(dx+c)+a^2d\right)\sinh(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*(cosh(d*x + c)^2 + 2*(cosh(d*x + c) + 1)*sinh(d*x + c) + sinh
(d*x + c)^2 + 2*cosh(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(1/2)*sqrt(a/
(cosh(d*x + c) + sinh(d*x + c)))/sqrt(a)) - 2*sqrt(1/2)*(cosh(d*x + c)^2 +
(2*cosh(d*x + c) - 1)*sinh(d*x + c) + sinh(d*x + c)^2 - cosh(d*x + c))*sqrt
(a/(cosh(d*x + c) + sinh(d*x + c))))/(a^2*d*cosh(d*x + c)^2 + a^2*d*sinh(d*
x + c)^2 + 2*a^2*d*cosh(d*x + c) + a^2*d + 2*(a^2*d*cosh(d*x + c) + a^2*d)*
sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cosh(c+dx)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*cosh(c + d*x) + a)**(-3/2), x)
```

Giac [A]
time = 0.42, size = 70, normalized size = 0.91

$$\frac{\frac{\sqrt{2}\arctan\left(e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}\right)}{a^{\frac{3}{2}}}+\frac{\sqrt{2}\left(\sqrt{a}\,e^{\left(\frac{3}{2}\,dx+2\,c\right)}-\sqrt{a}\,e^{\left(\frac{1}{2}\,dx+c\right)}\right)e^{\left(-\frac{1}{2}\,c\right)}}{a^2\left(e^{(dx+c)}+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(2)*arctan(e^(1/2*d*x + 1/2*c))/a^(3/2) + sqrt(2)*(sqrt(a)*e^(3/2*
d*x + 2*c) - sqrt(a)*e^(1/2*d*x + c))*e^(-1/2*c)/(a^2*(e^(d*x + c) + 1)^2))
/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+a\cosh\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*cosh(c + d*x))^(3/2),x)
```

```
[Out] int(1/(a + a*cosh(c + d*x))^(3/2), x)
```

3.47 $\int \frac{1}{(a+a\cosh(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=107

$$\frac{3\text{ArcTan}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a+a\cosh(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{\sinh(c+dx)}{4d(a+a\cosh(c+dx))^{5/2}}+\frac{3\sinh(c+dx)}{16ad(a+a\cosh(c+dx))^{3/2}}$$

```
[Out] 1/4*sinh(d*x+c)/d/(a+a*cosh(d*x+c))^(5/2)+3/16*sinh(d*x+c)/a/d/(a+a*cosh(d*
x+c))^(3/2)+3/32*arctan(1/2*sinh(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cosh(d*x+c))^(
1/2))/a^(5/2)/d*2^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2729, 2728, 212}

$$\frac{3\text{ArcTan}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a\cosh(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{3\sinh(c+dx)}{16ad(a\cosh(c+dx)+a)^{3/2}}+\frac{\sinh(c+dx)}{4d(a\cosh(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[c + d*x])^(-5/2),x]
```

```
[Out] (3*ArcTan[(Sqrt[a]*Sinh[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cosh[c + d*x]])])/(16
*Sqrt[2]*a^(5/2)*d) + Sinh[c + d*x]/(4*d*(a + a*Cosh[c + d*x])^(5/2)) + (3*
Sinh[c + d*x])/(16*a*d*(a + a*Cosh[c + d*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(a+a\cosh(c+dx))^{5/2}}\,dx = \frac{\sinh(c+dx)}{4d(a+a\cosh(c+dx))^{5/2}} + \frac{3\int \frac{1}{(a+a\cosh(c+dx))^{3/2}}\,dx}{8a}$$

$$= \frac{\sinh(c+dx)}{4d(a+a\cosh(c+dx))^{5/2}} + \frac{3\sinh(c+dx)}{16ad(a+a\cosh(c+dx))^{3/2}} + \frac{3\int \frac{1}{\sqrt{a+a\cosh}}}{32a}$$

$$= \frac{\sinh(c+dx)}{4d(a+a\cosh(c+dx))^{5/2}} + \frac{3\sinh(c+dx)}{16ad(a+a\cosh(c+dx))^{3/2}} + \frac{(3i)\text{Subst}\left(\int \frac{}{2a-}\right.}{}$$

$$= \frac{3\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a+a\cosh(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} + \frac{\sinh(c+dx)}{4d(a+a\cosh(c+dx))^{5/2}} + \frac{}{16ad(}$$

Mathematica [A]
time = 0.19, size = 91, normalized size = 0.85

$$\frac{\cosh^5\left(\frac{1}{2}(c+dx)\right)\left(32\text{csch}^4(c+dx)\sinh^5\left(\frac{1}{2}(c+dx)\right)+3\left(\text{ArcTan}\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)+\text{sech}\left(\frac{1}{2}(c+dx)\right)\tanh\left(\frac{1}{2}(c+dx)\right)\right)\right)}{4d(a(1+\cosh(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[c + d*x])^(-5/2),x]
[Out] (Cosh[(c + d*x)/2]^5*(32*Csch[c + d*x]^4*Sinh[(c + d*x)/2]^5 + 3*(ArcTan[Si
nh[(c + d*x)/2]] + Sech[(c + d*x)/2]*Tanh[(c + d*x)/2])))/(4*d*(a*(1 + Cosh
[c + d*x]))^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(88) = 176.
time = 1.12, size = 178, normalized size = 1.66

method	result
default	$-\frac{\sqrt{\left(\sinh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}\left(3\ln\left(\frac{2\sqrt{-a}\sqrt{\left(\sinh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}-2a}{\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\left(\cosh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-3\sqrt{\left(\sinh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}\right.}{32a^3\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)^3\sqrt{-a}\,\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cosh^2\left(\frac{dx}{2}\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cosh(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/32*(sinh(1/2*d*x+1/2*c)^2*a)^(1/2)*(3*ln(2*((-a)^(1/2)*(sinh(1/2*d*x+1/2
*c)^2*a)^(1/2)-a)/cosh(1/2*d*x+1/2*c))*a*cosh(1/2*d*x+1/2*c)^4-3*(sinh(1/2*
```

```
d*x+1/2*c)^2*a)^(1/2)*cosh(1/2*d*x+1/2*c)^2*(-a)^(1/2)-2*(-a)^(1/2)*(sinh(1
/2*d*x+1/2*c)^2*a)^(1/2))/a^3/cosh(1/2*d*x+1/2*c)^3/(-a)^(1/2)/sinh(1/2*d*x
+1/2*c)*2^(1/2)/(a*cosh(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(88) = 176$.
time = 0.57, size = 250, normalized size = 2.34

$$\frac{1}{80}\sqrt{2}\left(\frac{15\,e^{\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)}+70\,e^{\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)}+128\,e^{\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)}-70\,e^{\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)}-15\,e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{\left(a^{\frac{5}{2}}e^{(5\,dx+5\,c)}+5\,a^{\frac{5}{2}}e^{(4\,dx+4\,c)}+10\,a^{\frac{5}{2}}e^{(3\,dx+3\,c)}+10\,a^{\frac{5}{2}}e^{(2\,dx+2\,c)}+5\,a^{\frac{5}{2}}e^{(dx+c)}+a^{\frac{5}{2}}\right)d}+\frac{15\arctan\left(e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}\right)}{a^{\frac{5}{2}}d}\right)-\frac{8\sqrt{2}\,e^{\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)}}{5\left(a^{\frac{5}{2}}de^{(5\,dx+5\,c)}+5\,a^{\frac{5}{2}}de^{(4\,dx+4\,c)}+10\,a^{\frac{5}{2}}de^{(3\,dx+3\,c)}+10\,a^{\frac{5}{2}}de^{(2\,dx+2\,c)}+5\,a^{\frac{5}{2}}de^{(dx+c)}+a^{\frac{5}{2}}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/80*sqrt(2)*((15*e^(9/2*d*x + 9/2*c) + 70*e^(7/2*d*x + 7/2*c) + 128*e^(5/2
*d*x + 5/2*c) - 70*e^(3/2*d*x + 3/2*c) - 15*e^(1/2*d*x + 1/2*c))/((a^(5/2)*
e^(5*d*x + 5*c) + 5*a^(5/2)*e^(4*d*x + 4*c) + 10*a^(5/2)*e^(3*d*x + 3*c) +
10*a^(5/2)*e^(2*d*x + 2*c) + 5*a^(5/2)*e^(d*x + c) + a^(5/2))*d) + 15*arcta
n(e^(1/2*d*x + 1/2*c))/(a^(5/2)*d)) - 8/5*sqrt(2)*e^(5/2*d*x + 5/2*c)/(a^(5
/2)*d*e^(5*d*x + 5*c) + 5*a^(5/2)*d*e^(4*d*x + 4*c) + 10*a^(5/2)*d*e^(3*d*x
 + 3*c) + 10*a^(5/2)*d*e^(2*d*x + 2*c) + 5*a^(5/2)*d*e^(d*x + c) + a^(5/2)*
d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 525 vs. $2(88) = 176$.
time = 0.37, size = 525, normalized size = 4.91

$$-\frac{3\sqrt{2}\left(\cosh\left(dx+c\right)^{4}+4\left(\cosh\left(dx+c\right)+1\right)\sinh\left(dx+c\right)^{3}+\sinh\left(dx+c\right)^{4}+4\cosh\left(dx+c\right)^{3}+6\left(\cosh\left(dx+c\right)^{2}+2\cosh\left(dx+c\right)+1\right)\sinh\left(dx+c\right)^{2}+6\cosh\left(dx+c\right)^{2}+4\left(\cosh\left(dx+c\right)^{3}+3\cosh\left(dx+c\right)^{2}+3\cosh\left(dx+c\right)+1\right)\sinh\left(dx+c\right)+4\cosh\left(dx+c\right)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{a}\sqrt{\frac{a}{\cosh\left(dx+c\right)+\sinh\left(dx+c\right)}}}{a}\right)-2\sqrt{\frac{1}{2}}\left(3\cosh\left(dx+c\right)^{4}+\left(12\cosh\left(dx+c\right)+11\right)\sinh\left(dx+c\right)^{3}+3\sinh\left(dx+c\right)^{4}+11\cosh\left(dx+c\right)^{3}+\left(18\cosh\left(dx+c\right)^{2}+33\cosh\left(dx+c\right)-11\right)\sinh\left(dx+c\right)^{2}-11\cosh\left(dx+c\right)^{2}+\left(12\cosh\left(dx+c\right)^{3}+33\cosh\left(dx+c\right)^{2}-22\cosh\left(dx+c\right)-3\right)\sinh\left(dx+c\right)-3\cosh\left(dx+c\right)\right)\sqrt{\frac{a}{\cosh\left(dx+c\right)+\sinh\left(dx+c\right)}}}{16\left(a^{3}d\cosh\left(dx+c\right)^{4}+a^{3}d\sinh\left(dx+c\right)^{4}+4a^{3}d\cosh\left(dx+c\right)^{3}+6a^{3}d\cosh\left(dx+c\right)^{2}+4a^{3}d\cosh\left(dx+c\right)+a^{3}d+4\left(a^{3}d\cosh\left(dx+c\right)+a^{3}d\right)\sinh\left(dx+c\right)^{3}+6\left(a^{3}d\cosh\left(dx+c\right)^{2}+2a^{3}d\cosh\left(dx+c\right)+a^{3}d\right)\sinh\left(dx+c\right)^{2}+4\left(a^{3}d\cosh\left(dx+c\right)^{3}+3a^{3}d\cosh\left(dx+c\right)^{2}+3a^{3}d\cosh\left(dx+c\right)+a^{3}d\right)\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/16*(3*sqrt(2)*(cosh(d*x + c)^4 + 4*(cosh(d*x + c) + 1)*sinh(d*x + c)^3 +
 sinh(d*x + c)^4 + 4*cosh(d*x + c)^3 + 6*(cosh(d*x + c)^2 + 2*cosh(d*x + c)
 + 1)*sinh(d*x + c)^2 + 6*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + 3*cosh(d*x
 + c)^2 + 3*cosh(d*x + c) + 1)*sinh(d*x + c) + 4*cosh(d*x + c) + 1)*sqrt(a)
*arctan(sqrt(2)*sqrt(1/2)*sqrt(a)*sqrt(a/(cosh(d*x + c) + sinh(d*x + c)))/a
) - 2*sqrt(1/2)*(3*cosh(d*x + c)^4 + (12*cosh(d*x + c) + 11)*sinh(d*x + c)^
3 + 3*sinh(d*x + c)^4 + 11*cosh(d*x + c)^3 + (18*cosh(d*x + c)^2 + 33*cosh(
d*x + c) - 11)*sinh(d*x + c)^2 - 11*cosh(d*x + c)^2 + (12*cosh(d*x + c)^3 +
 33*cosh(d*x + c)^2 - 22*cosh(d*x + c) - 3)*sinh(d*x + c) - 3*cosh(d*x + c)
)*sqrt(a/(cosh(d*x + c) + sinh(d*x + c))))/(a^3*d*cosh(d*x + c)^4 + a^3*d*s
inh(d*x + c)^4 + 4*a^3*d*cosh(d*x + c)^3 + 6*a^3*d*cosh(d*x + c)^2 + 4*a^3*
d*cosh(d*x + c) + a^3*d + 4*(a^3*d*cosh(d*x + c) + a^3*d)*sinh(d*x + c)^3 +
 6*(a^3*d*cosh(d*x + c)^2 + 2*a^3*d*cosh(d*x + c) + a^3*d)*sinh(d*x + c)^2
+ 4*(a^3*d*cosh(d*x + c)^3 + 3*a^3*d*cosh(d*x + c)^2 + 3*a^3*d*cosh(d*x + c
) + a^3*d)*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a \cosh{(c + dx)} + a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(d*x+c))**(5/2),x)
```

```
[Out] Integral((a*cosh(c + d*x) + a)**(-5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%{[%%%{%%{[147456,0]:[1,0,-2]%%},[0]%%%},0]:[1,0,%%%{-1,[1]%
%%}]%%
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a + a\,\mathrm{cosh}\left(c + d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*cosh(c + d*x))^(5/2),x)
```

```
[Out] int(1/(a + a*cosh(c + d*x))^(5/2), x)
```

3.48 $\int (a - a\cosh(c+dx))^{5/2}\,dx$

Optimal. Leaf size=92

$$-\frac{64a^3\sinh(c+dx)}{15d\sqrt{a-a\cosh(c+dx)}}-\frac{16a^2\sqrt{a-a\cosh(c+dx)}\,\sinh(c+dx)}{15d}-\frac{2a(a-a\cosh(c+dx))^{3/2}\sinh(c+}{5d}$$

```
[Out] -2/5*a*(a-a*cosh(d*x+c))^(3/2)*sinh(d*x+c)/d-64/15*a^3*sinh(d*x+c)/d/(a-a*c
osh(d*x+c))^(1/2)-16/15*a^2*sinh(d*x+c)*(a-a*cosh(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2726, 2725}

$$-\frac{64a^3\sinh(c+dx)}{15d\sqrt{a-a\cosh(c+dx)}}-\frac{16a^2\sinh(c+dx)\sqrt{a-a\cosh(c+dx)}}{15d}-\frac{2a\sinh(c+dx)(a-a\cosh(c+dx))^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Cosh[c + d*x])^(5/2),x]
```

```
[Out] (-64*a^3*Sinh[c + d*x])/(15*d*Sqrt[a - a*Cosh[c + d*x]]) - (16*a^2*Sqrt[a -
 a*Cosh[c + d*x]]*Sinh[c + d*x])/(15*d) - (2*a*(a - a*Cosh[c + d*x])^(3/2)*
Sinh[c + d*x])/(5*d)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rubi steps

$$\begin{aligned}\int (a-a\cosh(c+dx))^{5/2}\,dx &= -\frac{2a(a-a\cosh(c+dx))^{3/2}\sinh(c+dx)}{5d}+\frac{1}{5}(8a)\int (a-a\cosh(c+dx))^{3/2} \\ &= -\frac{16a^2\sqrt{a-a\cosh(c+dx)}\,\sinh(c+dx)}{15d}-\frac{2a(a-a\cosh(c+dx))^{3/2}\sinh(}{5d} \\ &= -\frac{64a^3\sinh(c+dx)}{15d\sqrt{a-a\cosh(c+dx)}}-\frac{16a^2\sqrt{a-a\cosh(c+dx)}\,\sinh(c+dx)}{15d}-2a\end{aligned}$$

Mathematica [A]
time = 0.10, size = 72, normalized size = 0.78

$$\frac{a^2\sqrt{a-a\cosh(c+dx)}\left(150\cosh\left(\frac{1}{2}(c+dx)\right)-25\cosh\left(\frac{3}{2}(c+dx)\right)+3\cosh\left(\frac{5}{2}(c+dx)\right)\right)\operatorname{csch}\left(\frac{1}{2}(c+dx)\right)}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Cosh[c + d*x])^(5/2),x]
```

```
[Out] (a^2*Sqrt[a - a*Cosh[c + d*x]]*(150*Cosh[(c + d*x)/2] - 25*Cosh[(3*(c + d*x
))/2] + 3*Cosh[(5*(c + d*x))/2])*Csch[(c + d*x)/2])/(30*d)
```

Maple [A]
time = 1.16, size = 71, normalized size = 0.77

method	result	size
default	$-\frac{16\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)a^3\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\left(3\left(\sinh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4\left(\sinh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\right)}{15\sqrt{-2\left(\sinh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}\,d}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*cosh(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -16/15*sinh(1/2*d*x+1/2*c)*a^3*cosh(1/2*d*x+1/2*c)*(3*sinh(1/2*d*x+1/2*c)^4
-4*sinh(1/2*d*x+1/2*c)^2+8)/(-2*sinh(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(80) = 160.
time = 0.48, size = 190, normalized size = 2.07

$$\frac{5\sqrt{2}\,a^{\frac{5}{2}}e^{(-dx-c)}}{12\,d\left(-e^{(-dx-c)}\right)^{\frac{5}{2}}}-\frac{5\sqrt{2}\,a^{\frac{5}{2}}e^{(-2\,dx-2\,c)}}{2\,d\left(-e^{(-dx-c)}\right)^{\frac{5}{2}}}-\frac{5\sqrt{2}\,a^{\frac{5}{2}}e^{(-3\,dx-3\,c)}}{2\,d\left(-e^{(-dx-c)}\right)^{\frac{5}{2}}}+\frac{5\sqrt{2}\,a^{\frac{5}{2}}e^{(-4\,dx-4\,c)}}{12\,d\left(-e^{(-dx-c)}\right)^{\frac{5}{2}}}-\frac{\sqrt{2}\,a^{\frac{5}{2}}e^{(-5\,dx-5\,c)}}{20\,d\left(-e^{(-dx-c)}\right)^{\frac{5}{2}}}-\frac{\sqrt{2}\,a^{\frac{5}{2}}}{20\,d\left(-e^{(-dx-c)}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 5/12*sqrt(2)*a^(5/2)*e^(-d*x - c)/(d*(-e^(-d*x - c))^(5/2)) - 5/2*sqrt(2)*a
^(5/2)*e^(-2*d*x - 2*c)/(d*(-e^(-d*x - c))^(5/2)) - 5/2*sqrt(2)*a^(5/2)*e^(
-3*d*x - 3*c)/(d*(-e^(-d*x - c))^(5/2)) + 5/12*sqrt(2)*a^(5/2)*e^(-4*d*x -
4*c)/(d*(-e^(-d*x - c))^(5/2)) - 1/20*sqrt(2)*a^(5/2)*e^(-5*d*x - 5*c)/(d*(
-e^(-d*x - c))^(5/2)) - 1/20*sqrt(2)*a^(5/2)/(d*(-e^(-d*x - c))^(5/2))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 328 vs. 2(80) = 160.
time = 0.42, size = 328, normalized size = 3.57

$$\frac{\sqrt{\frac{1}{2}}\left(3\,a^2\cosh\left(dx+c\right)^5+3\,a^2\sinh\left(dx+c\right)^5-25\,a^2\cosh\left(dx+c\right)^4+150\,a^2\cosh\left(dx+c\right)^3+5\left(3\,a^2\cosh\left(dx+c\right)-5\,a^2\right)\sinh\left(dx+c\right)^4+150\,a^2\cosh\left(dx+c\right)^2+10\left(3\,a^2\cosh\left(dx+c\right)^2-10\,a^2\cosh\left(dx+c\right)+15\,a^2\right)\sinh\left(dx+c\right)^3-25\,a^2\cosh\left(dx+c\right)+30\left(a^2\cosh\left(dx+c\right)^3-5\,a^2\cosh\left(dx+c\right)^2+15\,a^2\cosh\left(dx+c\right)+5\,a^2\right)\sinh\left(dx+c\right)^2+3\,a^2+5\left(3\,a^2\cosh\left(dx+c\right)^4-20\,a^2\cosh\left(dx+c\right)^3+90\,a^2\cosh\left(dx+c\right)^2+60\,a^2\cosh\left(dx+c\right)-5\,a^2\right)\sinh\left(dx+c\right)\right)\sqrt{-\frac{a}{\cosh\left(dx+c\right)+\sinh\left(dx+c\right)}}}{30\left(d\cosh\left(dx+c\right)^2+2\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)+d\sinh\left(dx+c\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/30*sqrt(1/2)*(3*a^2*cosh(d*x + c)^5 + 3*a^2*sinh(d*x + c)^5 - 25*a^2*cosh
(d*x + c)^4 + 150*a^2*cosh(d*x + c)^3 + 5*(3*a^2*cosh(d*x + c) - 5*a^2)*sin
h(d*x + c)^4 + 150*a^2*cosh(d*x + c)^2 + 10*(3*a^2*cosh(d*x + c)^2 - 10*a^2
*cosh(d*x + c) + 15*a^2)*sinh(d*x + c)^3 - 25*a^2*cosh(d*x + c) + 30*(a^2*c
osh(d*x + c)^3 - 5*a^2*cosh(d*x + c)^2 + 15*a^2*cosh(d*x + c) + 5*a^2)*sinh
(d*x + c)^2 + 3*a^2 + 5*(3*a^2*cosh(d*x + c)^4 - 20*a^2*cosh(d*x + c)^3 + 9
0*a^2*cosh(d*x + c)^2 + 60*a^2*cosh(d*x + c) - 5*a^2)*sinh(d*x + c))*sqrt(-
a/(cosh(d*x + c) + sinh(d*x + c)))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*s
inh(d*x + c) + d*sinh(d*x + c)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(80) = 160.
time = 0.42, size = 194, normalized size = 2.11

$$-\frac{\sqrt{2}\left(3\,\sqrt{-ae^{(dx+c)}}\,a^{2}e^{(2\,dx+2\,c)}\mathrm{sgn}\left(-e^{(dx+c)}+1\right)-25\,\sqrt{-ae^{(dx+c)}}\,a^{2}e^{(dx+c)}\mathrm{sgn}\left(-e^{(dx+c)}+1\right)+150\,\sqrt{-ae^{(dx+c)}}\,a^{2}\mathrm{sgn}\left(-e^{(dx+c)}+1\right)-\frac{\left(150\,a^{5}e^{(2\,dx+2\,c)}\mathrm{sgn}\left(-e^{(dx+c)}+1\right)-25\,a^{5}e^{(dx+c)}\mathrm{sgn}\left(-e^{(dx+c)}+1\right)+3\,a^{5}\mathrm{sgn}\left(-e^{(dx+c)}+1\right)\right)e^{(-2\,dx-2\,c)}}{\sqrt{-ae^{(dx+c)}}\,a^{2}}\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/60*sqrt(2)*(3*sqrt(-a*e^(d*x + c))*a^2*e^(2*d*x + 2*c)*sgn(-e^(d*x + c)
+ 1) - 25*sqrt(-a*e^(d*x + c))*a^2*e^(d*x + c)*sgn(-e^(d*x + c) + 1) + 150*
sqrt(-a*e^(d*x + c))*a^2*sgn(-e^(d*x + c) + 1) - (150*a^5*e^(2*d*x + 2*c)*s
gn(-e^(d*x + c) + 1) - 25*a^5*e^(d*x + c)*sgn(-e^(d*x + c) + 1) + 3*a^5*sgn
(-e^(d*x + c) + 1))*e^(-2*d*x - 2*c)/(sqrt(-a*e^(d*x + c))*a^2))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a-a\cosh(c+d\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*cosh(c + d*x))^(5/2),x)
```

```
[Out] int((a - a*cosh(c + d*x))^(5/2), x)
```

3.49 $\int (a-a\cosh(c+dx))^{3/2}\,dx$

Optimal. Leaf size=61

$$-\frac{8a^2\sinh(c+dx)}{3d\sqrt{a-a\cosh(c+dx)}}-\frac{2a\sqrt{a-a\cosh(c+dx)}\,\sinh(c+dx)}{3d}$$

```
[Out] -8/3*a^2*sinh(d*x+c)/d/(a-a*cosh(d*x+c))^(1/2)-2/3*a*sinh(d*x+c)*(a-a*cosh(
d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.133$, Rules used = {2726, 2725}

$$-\frac{8a^2\sinh(c+dx)}{3d\sqrt{a-a\cosh(c+dx)}}-\frac{2a\sinh(c+dx)\sqrt{a-a\cosh(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Cosh[c + d*x])^(3/2),x]
```

```
[Out] (-8*a^2*Sinh[c + d*x])/(3*d*Sqrt[a - a*Cosh[c + d*x]]) - (2*a*Sqrt[a - a*Co
sh[c + d*x]]*Sinh[c + d*x])/(3*d)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rubi steps

$$\begin{aligned}\int (a-a\cosh(c+dx))^{3/2}\,dx&=-\frac{2a\sqrt{a-a\cosh(c+dx)}\,\sinh(c+dx)}{3d}+\frac{1}{3}(4a)\int\sqrt{a-a\cosh(c+dx)}\,dx\\&=-\frac{8a^2\sinh(c+dx)}{3d\sqrt{a-a\cosh(c+dx)}}-\frac{2a\sqrt{a-a\cosh(c+dx)}\,\sinh(c+dx)}{3d}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 56, normalized size = 0.92

$$-\frac{a\sqrt{a-a\cosh(c+dx)}\left(-9\cosh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{3}{2}(c+dx)\right)\right)\operatorname{csch}\left(\frac{1}{2}(c+dx)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Cosh[c + d*x])^(3/2),x]
```

```
[Out] -1/3*(a*Sqrt[a - a*Cosh[c + d*x]]*(-9*Cosh[(c + d*x)/2] + Cosh[(3*(c + d*x)
)/2])*Csch[(c + d*x)/2])/d
```

Maple [A]
time = 1.01, size = 56, normalized size = 0.92

method	result	size
default	$\frac{8\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)a^{2}\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\cosh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)}{3\sqrt{-2\left(\sinh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}\,d}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*cosh(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 8/3*sinh(1/2*d*x+1/2*c)*a^2*cosh(1/2*d*x+1/2*c)*(cosh(1/2*d*x+1/2*c)^2-3)/(
-2*sinh(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(53) = 106.
time = 0.48, size = 124, normalized size = 2.03

$$\frac{3\sqrt{2}\,a^{\frac{3}{2}}e^{(-dx-c)}}{2\,d\left(-e^{(-dx-c)}\right)^{\frac{3}{2}}}+\frac{3\sqrt{2}\,a^{\frac{3}{2}}e^{(-2\,dx-2\,c)}}{2\,d\left(-e^{(-dx-c)}\right)^{\frac{3}{2}}}-\frac{\sqrt{2}\,a^{\frac{3}{2}}e^{(-3\,dx-3\,c)}}{6\,d\left(-e^{(-dx-c)}\right)^{\frac{3}{2}}}-\frac{\sqrt{2}\,a^{\frac{3}{2}}}{6\,d\left(-e^{(-dx-c)}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 3/2*sqrt(2)*a^(3/2)*e^(-d*x - c)/(d*(-e^(-d*x - c))^(3/2)) + 3/2*sqrt(2)*a^
(3/2)*e^(-2*d*x - 2*c)/(d*(-e^(-d*x - c))^(3/2)) - 1/6*sqrt(2)*a^(3/2)*e^(-
3*d*x - 3*c)/(d*(-e^(-d*x - c))^(3/2)) - 1/6*sqrt(2)*a^(3/2)/(d*(-e^(-d*x -
 c))^(3/2))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(53) = 106.
time = 0.35, size = 139, normalized size = 2.28

$$-\frac{\sqrt{\frac{1}{2}}\left(a\cosh\left(dx+c\right)^{3}+a\sinh\left(dx+c\right)^{3}-9\,a\cosh\left(dx+c\right)^{2}+3\left(a\cosh\left(dx+c\right)-3\,a\right)\sinh\left(dx+c\right)^{2}-9\,a\cosh\left(dx+c\right)+3\left(a\cosh\left(dx+c\right)^{2}-6\,a\cosh\left(dx+c\right)-3\,a\right)\sinh\left(dx+c\right)+a\right)\sqrt{-\frac{a}{\cosh\left(dx+c\right)+\sinh\left(dx+c\right)}}}{3\left(d\cosh\left(dx+c\right)+d\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(1/2)*(a*cosh(d*x + c)^3 + a*sinh(d*x + c)^3 - 9*a*cosh(d*x + c)^2
 + 3*(a*cosh(d*x + c) - 3*a)*sinh(d*x + c)^2 - 9*a*cosh(d*x + c) + 3*(a*cos
h(d*x + c)^2 - 6*a*cosh(d*x + c) - 3*a)*sinh(d*x + c) + a)*sqrt(-a/(cosh(d*
x + c) + sinh(d*x + c)))/(d*cosh(d*x + c) + d*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-a\cosh\left(c+dx\right)+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(d*x+c))**(3/2),x)
```

```
[Out] Integral((-a*cosh(c + d*x) + a)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(53) = 106.
time = 0.42, size = 127, normalized size = 2.08

$$\frac{\sqrt{2}\left(\sqrt{-ae^{(dx+c)}}\,ae^{(dx+c)}\mathrm{sgn}\left(-e^{(dx+c)}+1\right)-9\sqrt{-ae^{(dx+c)}}\,a\mathrm{sgn}\left(-e^{(dx+c)}+1\right)+\frac{\left(9\,a^{3}e^{(dx+c)}\mathrm{sgn}\left(-e^{(dx+c)}+1\right)-a^{3}\mathrm{sgn}\left(-e^{(dx+c)}+1\right)\right)e^{(-dx-c)}}{\sqrt{-ae^{(dx+c)}}\,a}\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*(sqrt(-a*e^(d*x + c))*a*e^(d*x + c)*sgn(-e^(d*x + c) + 1) - 9*s
qrt(-a*e^(d*x + c))*a*sgn(-e^(d*x + c) + 1) + (9*a^3*e^(d*x + c)*sgn(-e^(d*
x + c) + 1) - a^3*sgn(-e^(d*x + c) + 1))*e^(-d*x - c)/(sqrt(-a*e^(d*x + c))
*a))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(a-a\,\mathrm{cosh}\left(c+d\,x\right)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*cosh(c + d*x))^(3/2),x)
```

```
[Out] int((a - a*cosh(c + d*x))^(3/2), x)
```

3.50 $\int \sqrt{a-a\cosh(c+dx)}\,dx$

Optimal. Leaf size=27

$$-\frac{2a\sinh(c+dx)}{d\sqrt{a-a\cosh(c+dx)}}$$

```
[Out] -2*a*sinh(d*x+c)/d/(a-a*cosh(d*x+c))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2725}

$$-\frac{2a\sinh(c+dx)}{d\sqrt{a-a\cosh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Cosh[c + d*x]],x]
```

```
[Out] (-2*a*Sinh[c + d*x])/(d*Sqrt[a - a*Cosh[c + d*x]])
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a-a\cosh(c+dx)}\,dx = -\frac{2a\sinh(c+dx)}{d\sqrt{a-a\cosh(c+dx)}}$$

Mathematica [A]
time = 0.03, size = 30, normalized size = 1.11

$$\frac{2\sqrt{a-a\cosh(c+dx)}\coth\left(\frac{1}{2}(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Cosh[c + d*x]],x]
```

```
[Out] (2*Sqrt[a - a*Cosh[c + d*x]]*Coth[(c + d*x)/2])/d
```

Maple [A]
time = 1.03, size = 41, normalized size = 1.52

method	result	size
default	$-\frac{4 \sinh\left(\frac{dx}{2}+\frac{c}{2}\right) a \cosh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{-2 \left(\sinh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right) a}\, d}$	41
risch	$\frac{\sqrt{2}\,\sqrt{-a \left(\mathrm{e}^{dx+c}-1\right)^{2} \mathrm{e}^{-dx-c}}\,\left(\mathrm{e}^{dx+c}+1\right)}{\left(\mathrm{e}^{dx+c}-1\right) d}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*cosh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4*sinh(1/2*d*x+1/2*c)*a*cosh(1/2*d*x+1/2*c)/(-2*sinh(1/2*d*x+1/2*c)^2*a)^(
1/2)/d
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(25) = 50.
time = 0.48, size = 58, normalized size = 2.15

$$-\frac{\sqrt{2}\,\sqrt{a}\,e^{(-dx-c)}}{d\sqrt{-e^{(-dx-c)}}}-\frac{\sqrt{2}\,\sqrt{a}}{d\sqrt{-e^{(-dx-c)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(2)*sqrt(a)*e^(-d*x - c)/(d*sqrt(-e^(-d*x - c))) - sqrt(2)*sqrt(a)/(d*
sqrt(-e^(-d*x - c)))
```

Fricas [A]
time = 0.37, size = 42, normalized size = 1.56

$$\frac{2\sqrt{\frac{1}{2}}\,\sqrt{-\frac{a}{\cosh(dx+c)+\sinh(dx+c)}}\,(\cosh(dx+c)+\sinh(dx+c)+1)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(1/2)*sqrt(-a/(cosh(d*x + c) + sinh(d*x + c)))*(cosh(d*x + c) + sinh(
d*x + c) + 1)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a\cosh(c+dx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(d*x+c))**(1/2),x)
[Out] Integral(sqrt(-a*cosh(c + d*x) + a), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(25) = 50$.
time = 0.42, size = 63, normalized size = 2.33

$$-\frac{\sqrt{2}\left(\sqrt{-ae^{(dx+c)}}\,a\mathrm{sgn}\left(-e^{(dx+c)}+1\right)-\frac{a^2\mathrm{sgn}\left(-e^{(dx+c)}+1\right)}{\sqrt{-ae^{(dx+c)}}}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(d*x+c))^(1/2),x, algorithm="giac")
[Out] -sqrt(2)*(sqrt(-a*e^(d*x + c))*a*sgn(-e^(d*x + c) + 1) - a^2*sgn(-e^(d*x +
c) + 1)/sqrt(-a*e^(d*x + c)))/(a*d)
```

Mupad [B]
time = 0.94, size = 27, normalized size = 1.00

$$\frac{2\coth\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a-a\cosh\left(c+d\,x\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*cosh(c + d*x))^(1/2),x)
[Out] (2*coth(c/2 + (d*x)/2)*(a - a*cosh(c + d*x))^(1/2))/d
```

3.51 $\int \frac{1}{\sqrt{a-a\cosh(c+dx)}}\,dx$

Optimal. Leaf size=48

$$-\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{2}\,\sqrt{a-a\cosh(c+dx)}}\right)}{\sqrt{a}\,d}$$

```
[Out] -arctan(1/2*sinh(d*x+c)*a^(1/2)*2^(1/2)/(a-a*cosh(d*x+c))^(1/2))*2^(1/2)/d/
a^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2728, 212}

$$-\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{2}\,\sqrt{a-a\cosh(c+dx)}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a - a*Cosh[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Sinh[c + d*x])/(Sqrt[2]*Sqrt[a - a*Cosh[c + d*x]
])])/(Sqrt[a]*d))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a - a\cosh(c + dx)}}\,dx = \frac{(2i)\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{ia\sinh(c+dx)}{\sqrt{a - a\cosh(c + dx)}}\right)}{d}$$
$$= -\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a - a\cosh(c + dx)}}\right)}{\sqrt{a}\,d}$$

Mathematica [**A**]
time = 0.03, size = 41, normalized size = 0.85

$$\frac{2\log\left(\tanh\left(\frac{1}{4}(c + dx)\right)\right)\sinh\left(\frac{1}{2}(c + dx)\right)}{d\sqrt{a - a\cosh(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a - a*Cosh[c + d*x]],x]
[Out] (2*Log[Tanh[(c + d*x)/4]]*Sinh[(c + d*x)/2])/(d*Sqrt[a - a*Cosh[c + d*x]])
```

Maple [**A**]
time = 0.93, size = 41, normalized size = 0.85

method	result	size
default	$-\frac{2\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\operatorname{arctanh}\left(\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\sqrt{-2\left(\sinh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}\ d}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*cosh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*sinh(1/2*d*x+1/2*c)*arctanh(cosh(1/2*d*x+1/2*c))/(-2*sinh(1/2*d*x+1/2*c)
^2*a)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*cosh(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-a*cosh(d*x + c) + a), x)
```

Fricas [A]
time = 0.34, size = 154, normalized size = 3.21

$$\left[\frac{\sqrt{2}\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a}{\cosh\left(dx+c\right)+\sinh\left(dx+c\right)}}\sqrt{-\frac{1}{a}}\left(\cosh(dx+c)+\sinh(dx+c)\right)-\cosh(dx+c)-\sinh(dx+c)-1}{\cosh(dx+c)+\sinh(dx+c)-1}\right)}{d},\ \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a}{\cosh\left(dx+c\right)+\sinh\left(dx+c\right)}}\left(\cosh(dx+c)+\sinh(dx+c)\right)}{\sqrt{a}}\right)}{\sqrt{a}\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(2)*sqrt(-1/a)*log((2*sqrt(2)*sqrt(1/2)*sqrt(-a/(cosh(d*x + c) + sinh(
d*x + c)))*sqrt(-1/a)*(cosh(d*x + c) + sinh(d*x + c)) - cosh(d*x + c) - sin
h(d*x + c) - 1)/(cosh(d*x + c) + sinh(d*x + c) - 1))/d, 2*sqrt(2)*arctan(sq
rt(2)*sqrt(1/2)*sqrt(-a/(cosh(d*x + c) + sinh(d*x + c)))*(cosh(d*x + c) + s
inh(d*x + c))/sqrt(a))/(sqrt(a)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a\cosh\left(c+dx\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*cosh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(-a*cosh(c + d*x) + a), x)
```

Giac [A]
time = 0.44, size = 40, normalized size = 0.83

$$-\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{-ae^{(dx+c)}}}{\sqrt{a}}\right)}{\sqrt{a}\,d\mathrm{sgn}\left(-e^{(dx+c)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(2)*arctan(sqrt(-a*e^(d*x + c))/sqrt(a))/(sqrt(a)*d*sgn(-e^(d*x + c)
 + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{a-a\cosh\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - a*cosh(c + d*x))^(1/2),x)
```

```
[Out] int(1/(a - a*cosh(c + d*x))^(1/2), x)
```

3.52 $\int \frac{1}{(a-a\cosh(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=79

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{2}\,\sqrt{a-a\cosh(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{\sinh(c+dx)}{2d(a-a\cosh(c+dx))^{3/2}}$$

```
[Out] -1/2*sinh(d*x+c)/d/(a-a*cosh(d*x+c))^(3/2)-1/4*arctan(1/2*sinh(d*x+c)*a^(1/
2)*2^(1/2)/(a-a*cosh(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2729, 2728, 212}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{2}\,\sqrt{a-a\cosh(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{\sinh(c+dx)}{2d(a-a\cosh(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Cosh[c + d*x])^(-3/2),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[a]*Sinh[c + d*x])/(Sqrt[2]*Sqrt[a - a*Cosh[c + d*x]])]/(S
qrt[2]*a^(3/2)*d) - Sinh[c + d*x]/(2*d*(a - a*Cosh[c + d*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(a-a\cosh(c+dx))^{3/2}}\,dx = -\frac{\sinh(c+dx)}{2d(a-a\cosh(c+dx))^{3/2}} + \frac{\int \frac{1}{\sqrt{a-a\cosh(c+dx)}}\,dx}{4a}$$

$$= -\frac{\sinh(c+dx)}{2d(a-a\cosh(c+dx))^{3/2}} + \frac{i\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{ia\sinh(c+dx)}{\sqrt{a-a\cosh(c+dx)}}\right)}{2ad}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a-a\cosh(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sinh(c+dx)}{2d(a-a\cosh(c+dx))^{3/2}}$$

Mathematica [A]
time = 0.13, size = 85, normalized size = 1.08

$$\frac{\left(\operatorname{csch}^2\left(\frac{1}{4}(c+dx)\right) + 4\log\left(\tanh\left(\frac{1}{4}(c+dx)\right)\right) + \operatorname{sech}^2\left(\frac{1}{4}(c+dx)\right)\right)\sinh^3\left(\frac{1}{2}(c+dx)\right)}{4ad(-1+\cosh(c+dx))\sqrt{a-a\cosh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Cosh[c + d*x])^(-3/2),x]
```

```
[Out] ((Csch[(c + d*x)/4]^2 + 4*Log[Tanh[(c + d*x)/4]] + Sech[(c + d*x)/4]^2)*Sin
h[(c + d*x)/2]^3)/(4*a*d*(-1 + Cosh[c + d*x])*Sqrt[a - a*Cosh[c + d*x]])
```

Maple [A]
time = 1.06, size = 87, normalized size = 1.10

method	result	size
default	$\frac{2\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)+\left(\ln\left(\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\ln\left(\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\right)\left(\sinh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sinh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}\,d}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*cosh(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a*(2*cosh(1/2*d*x+1/2*c)+(ln(cosh(1/2*d*x+1/2*c)-1)-ln(cosh(1/2*d*x+1/2
*c)+1))*sinh(1/2*d*x+1/2*c)^2)/sinh(1/2*d*x+1/2*c)/(-2*sinh(1/2*d*x+1/2*c)^
2*a)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*cosh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-a*cosh(d*x + c) + a)^(-3/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(64) = 128$.
time = 0.42, size = 274, normalized size = 3.47

$$-\frac{\sqrt{2}\left(\cosh\left(dx+c\right)^{2}+2\left(\cosh\left(dx+c\right)-1\right)\sinh\left(dx+c\right)+\sinh\left(dx+c\right)^{2}-2\cosh\left(dx+c\right)+1\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-a}\sqrt{-\frac{a}{\cosh\left(dx+c\right)+\sinh\left(dx+c\right)}}\left(\cosh(dx+c)+\sinh(dx+c)\right)+a\cosh(dx+c)+a\sinh(dx+c)+a}{\cosh(dx+c)+\sinh(dx+c)-1}\right)+4\sqrt{\frac{1}{2}}\left(\cosh\left(dx+c\right)^{2}+\left(2\cosh\left(dx+c\right)+1\right)\sinh\left(dx+c\right)+\sinh\left(dx+c\right)^{2}+\cosh\left(dx+c\right)\right)\sqrt{-\frac{a}{\cosh\left(dx+c\right)+\sinh\left(dx+c\right)}}}{4\left(a^{2}d\cosh\left(dx+c\right)^{2}+a^{2}d\sinh\left(dx+c\right)^{2}-2a^{2}d\cosh\left(dx+c\right)+a^{2}d+2\left(a^{2}d\cosh\left(dx+c\right)-a^{2}d\right)\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*cosh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(2)*(cosh(d*x + c)^2 + 2*(cosh(d*x + c) - 1)*sinh(d*x + c) + sinh
(d*x + c)^2 - 2*cosh(d*x + c) + 1)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(1/2)*sqrt(
-a)*sqrt(-a/(cosh(d*x + c) + sinh(d*x + c)))*(cosh(d*x + c) + sinh(d*x + c)
) + a*cosh(d*x + c) + a*sinh(d*x + c) + a)/(cosh(d*x + c) + sinh(d*x + c) -
 1)) + 4*sqrt(1/2)*(cosh(d*x + c)^2 + (2*cosh(d*x + c) + 1)*sinh(d*x + c) +
 sinh(d*x + c)^2 + cosh(d*x + c))*sqrt(-a/(cosh(d*x + c) + sinh(d*x + c))))
/(a^2*d*cosh(d*x + c)^2 + a^2*d*sinh(d*x + c)^2 - 2*a^2*d*cosh(d*x + c) + a
^2*d + 2*(a^2*d*cosh(d*x + c) - a^2*d)*sinh(d*x + c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a\cosh\left(c+dx\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*cosh(d*x+c))**(3/2),x)
```

```
[Out] Integral((-a*cosh(c + d*x) + a)**(-3/2), x)
```

Giac [**A**]
time = 0.42, size = 115, normalized size = 1.46

$$-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{-ae^{(dx+c)}}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}d\mathrm{sgn}\left(-e^{(dx+c)}+1\right)}+\frac{\sqrt{2}\sqrt{-ae^{(dx+c)}}ae^{(dx+c)}+\sqrt{2}\sqrt{-ae^{(dx+c)}}a}{2\left(ae^{(dx+c)}-a\right)^{2}ad\mathrm{sgn}\left(-e^{(dx+c)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*cosh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(sqrt(-a*e^(d*x + c))/sqrt(a))/(a^(3/2)*d*sgn(-e^(d*x +
c) + 1)) + 1/2*(sqrt(2)*sqrt(-a*e^(d*x + c))*a*e^(d*x + c) + sqrt(2)*sqrt(-
a*e^(d*x + c))*a)/((a*e^(d*x + c) - a)^2*a*d*sgn(-e^(d*x + c) + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a - a\cosh(c + d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - a*cosh(c + d*x))^(3/2),x)
```

```
[Out] int(1/(a - a*cosh(c + d*x))^(3/2), x)
```

3.53 $\int \frac{1}{(a-a\cosh(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=110

$$-\frac{3\text{ArcTan}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a-a\cosh(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{\sinh(c+dx)}{4d(a-a\cosh(c+dx))^{5/2}}-\frac{3\sinh(c+dx)}{16ad(a-a\cosh(c+dx))^{3/2}}$$

```
[Out] -1/4*sinh(d*x+c)/d/(a-a*cosh(d*x+c))^(5/2)-3/16*sinh(d*x+c)/a/d/(a-a*cosh(d
*x+c))^(3/2)-3/32*arctan(1/2*sinh(d*x+c)*a^(1/2)*2^(1/2)/(a-a*cosh(d*x+c))^
(1/2))/a^(5/2)/d*2^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2729, 2728, 212}

$$-\frac{3\text{ArcTan}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a-a\cosh(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{3\sinh(c+dx)}{16ad(a-a\cosh(c+dx))^{3/2}}-\frac{\sinh(c+dx)}{4d(a-a\cosh(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Cosh[c + d*x])^(-5/2),x]
```

```
[Out] (-3*ArcTan[(Sqrt[a]*Sinh[c + d*x])/(Sqrt[2]*Sqrt[a - a*Cosh[c + d*x]])])/(1
6*Sqrt[2]*a^(5/2)*d) - Sinh[c + d*x]/(4*d*(a - a*Cosh[c + d*x])^(5/2)) - (3
*Sinh[c + d*x])/(16*a*d*(a - a*Cosh[c + d*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(a-a\cosh(c+dx))^{5/2}}\,dx = -\frac{\sinh(c+dx)}{4d(a-a\cosh(c+dx))^{5/2}} + \frac{3\int \frac{1}{(a-a\cosh(c+dx))^{3/2}}\,dx}{8a}$$

$$= -\frac{\sinh(c+dx)}{4d(a-a\cosh(c+dx))^{5/2}} - \frac{3\sinh(c+dx)}{16ad(a-a\cosh(c+dx))^{3/2}} + \frac{3\int \frac{}{\sqrt{a-a\,\mathrm{co}}}}{32}$$

$$= -\frac{\sinh(c+dx)}{4d(a-a\cosh(c+dx))^{5/2}} - \frac{3\sinh(c+dx)}{16ad(a-a\cosh(c+dx))^{3/2}} + \frac{(3i)\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{3\tan^{-1}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{2}\,\sqrt{a-a\cosh(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} - \frac{\sinh(c+dx)}{4d(a-a\cosh(c+dx))^{5/2}} - \frac{}{16a\text{d}}$$

Mathematica [A]
time = 0.13, size = 115, normalized size = 1.05

$$\frac{\left(6\mathrm{csch}^2\left(\frac{1}{4}(c+dx)\right)-\mathrm{csch}^4\left(\frac{1}{4}(c+dx)\right)+24\log\left(\tanh\left(\frac{1}{4}(c+dx)\right)\right)+6\mathrm{sech}^2\left(\frac{1}{4}(c+dx)\right)+\mathrm{sech}^4\left(\frac{1}{4}(c+dx)\right)\right)\sinh^5\left(\frac{1}{2}(c+dx)\right)}{32a^2d(-1+\cosh(c+dx))^2\sqrt{a-a\cosh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Cosh[c + d*x])^(-5/2),x]
```

```
[Out] ((6*Csch[(c + d*x)/4]^2 - Csch[(c + d*x)/4]^4 + 24*Log[Tanh[(c + d*x)/4]] +
 6*Sech[(c + d*x)/4]^2 + Sech[(c + d*x)/4]^4)*Sinh[(c + d*x)/2]^5)/(32*a^2*
d*(-1 + Cosh[c + d*x])^2*Sqrt[a - a*Cosh[c + d*x]])
```

Maple [A]
time = 1.08, size = 137, normalized size = 1.25

method	result	size
default	$\frac{6\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sinh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)+\left(3\ln\left(\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-3\ln\left(\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\right)\left(\sinh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32a^2\left(\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\left(\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sinh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}\;d}$	137

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*cosh(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/a^2*(6*cosh(1/2*d*x+1/2*c)*sinh(1/2*d*x+1/2*c)^2-4*cosh(1/2*d*x+1/2*c)
+(3*ln(cosh(1/2*d*x+1/2*c)-1)-3*ln(cosh(1/2*d*x+1/2*c)+1))*sinh(1/2*d*x+1/2
*c)^4)/(cosh(1/2*d*x+1/2*c)+1)/(cosh(1/2*d*x+1/2*c)-1)/sinh(1/2*d*x+1/2*c)/
(-2*sinh(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*cosh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((-a*cosh(d*x + c) + a)^(-5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 580 vs. $2(91) = 182$.
time = 0.36, size = 580, normalized size = 5.27

$$-\frac{3\sqrt{2}\left(\cosh\left(dx+c\right)^{4}+4\left(\cosh\left(dx+c\right)-1\right)\sinh\left(dx+c\right)^{3}+\sinh\left(dx+c\right)^{4}-4\cosh\left(dx+c\right)^{3}+6\left(\cosh\left(dx+c\right)^{2}-2\cosh\left(dx+c\right)+1\right)\sinh\left(dx+c\right)^{2}+6\cosh\left(dx+c\right)^{2}+4\left(\cosh\left(dx+c\right)^{3}-3\cosh\left(dx+c\right)^{2}+3\cosh\left(dx+c\right)-1\right)\sinh\left(dx+c\right)-4\cosh\left(dx+c\right)+1\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-a}\sqrt{-\frac{a}{\cosh\left(dx+c\right)+\sinh\left(dx+c\right)}}\left(\cosh(dx+c)+\sinh(dx+c)\right)+a\cosh(dx+c)+a\sinh(dx+c)+a}{\cosh(dx+c)+\sinh(dx+c)-1}\right)+4\sqrt{\frac{1}{2}}\left(3\cosh\left(dx+c\right)^{4}+\left(12\cosh\left(dx+c\right)-11\right)\sinh\left(dx+c\right)^{3}+3\sinh\left(dx+c\right)^{4}-11\cosh\left(dx+c\right)^{3}+\left(18\cosh\left(dx+c\right)^{2}-33\cosh\left(dx+c\right)-11\right)\sinh\left(dx+c\right)^{2}-11\cosh\left(dx+c\right)^{2}+\left(12\cosh\left(dx+c\right)^{3}-33\cosh\left(dx+c\right)^{2}-22\cosh\left(dx+c\right)+3\right)\sinh\left(dx+c\right)+3\cosh\left(dx+c\right)\right)\sqrt{-\frac{a}{\cosh\left(dx+c\right)+\sinh\left(dx+c\right)}}}{32\left(a^{3}d\cosh\left(dx+c\right)^{4}+a^{3}d\sinh\left(dx+c\right)^{4}-4a^{3}d\cosh\left(dx+c\right)^{3}+6a^{3}d\cosh\left(dx+c\right)^{2}-4a^{3}d\cosh\left(dx+c\right)+a^{3}d+4\left(a^{3}d\cosh\left(dx+c\right)-a^{3}d\right)\sinh\left(dx+c\right)^{3}+6\left(a^{3}d\cosh\left(dx+c\right)^{2}-2a^{3}d\cosh\left(dx+c\right)+a^{3}d\right)\sinh\left(dx+c\right)^{2}+4\left(a^{3}d\cosh\left(dx+c\right)^{3}-3a^{3}d\cosh\left(dx+c\right)^{2}+3a^{3}d\cosh\left(dx+c\right)-a^{3}d\right)\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*cosh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/32*(3*sqrt(2)*(cosh(d*x + c)^4 + 4*(cosh(d*x + c) - 1)*sinh(d*x + c)^3 +
 sinh(d*x + c)^4 - 4*cosh(d*x + c)^3 + 6*(cosh(d*x + c)^2 - 2*cosh(d*x + c)
 + 1)*sinh(d*x + c)^2 + 6*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - 3*cosh(d*x
 + c)^2 + 3*cosh(d*x + c) - 1)*sinh(d*x + c) - 4*cosh(d*x + c) + 1)*sqrt(-a
)*log(-(2*sqrt(2)*sqrt(1/2)*sqrt(-a)*sqrt(-a/(cosh(d*x + c) + sinh(d*x + c)
))*(cosh(d*x + c) + sinh(d*x + c)) + a*cosh(d*x + c) + a*sinh(d*x + c) + a)
/(cosh(d*x + c) + sinh(d*x + c) - 1)) + 4*sqrt(1/2)*(3*cosh(d*x + c)^4 + (1
2*cosh(d*x + c) - 11)*sinh(d*x + c)^3 + 3*sinh(d*x + c)^4 - 11*cosh(d*x + c
)^3 + (18*cosh(d*x + c)^2 - 33*cosh(d*x + c) - 11)*sinh(d*x + c)^2 - 11*cos
h(d*x + c)^2 + (12*cosh(d*x + c)^3 - 33*cosh(d*x + c)^2 - 22*cosh(d*x + c)
+ 3)*sinh(d*x + c) + 3*cosh(d*x + c))*sqrt(-a/(cosh(d*x + c) + sinh(d*x + c
))))/(a^3*d*cosh(d*x + c)^4 + a^3*d*sinh(d*x + c)^4 - 4*a^3*d*cosh(d*x + c)
^3 + 6*a^3*d*cosh(d*x + c)^2 - 4*a^3*d*cosh(d*x + c) + a^3*d + 4*(a^3*d*cos
h(d*x + c) - a^3*d)*sinh(d*x + c)^3 + 6*(a^3*d*cosh(d*x + c)^2 - 2*a^3*d*co
sh(d*x + c) + a^3*d)*sinh(d*x + c)^2 + 4*(a^3*d*cosh(d*x + c)^3 - 3*a^3*d*c
osh(d*x + c)^2 + 3*a^3*d*cosh(d*x + c) - a^3*d)*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a\cosh\left(c+dx\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*cosh(d*x+c))**(5/2),x)
```

```
[Out] Integral((-a*cosh(c + d*x) + a)**(-5/2), x)
```

Giac [A]
time = 0.45, size = 177, normalized size = 1.61

$$-\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{-ae^{(dx+c)}}}{\sqrt{a}}\right)}{16\,a^{\frac{5}{2}}d\mathrm{sgn}\left(-e^{(dx+c)}+1\right)}+\frac{3\sqrt{2}\sqrt{-ae^{(dx+c)}}a^3e^{(3\,dx+3\,c)}-11\sqrt{2}\sqrt{-ae^{(dx+c)}}a^3e^{(2\,dx+2\,c)}-11\sqrt{2}\sqrt{-ae^{(dx+c)}}a^3e^{(dx+c)}+3\sqrt{2}\sqrt{-ae^{(dx+c)}}a^3}{16\left(ae^{(dx+c)}-a\right)^4a^2d\mathrm{sgn}\left(-e^{(dx+c)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*cosh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -3/16*sqrt(2)*arctan(sqrt(-a*e^(d*x + c))/sqrt(a))/(a^(5/2)*d*sgn(-e^(d*x +
 c) + 1)) + 1/16*(3*sqrt(2)*sqrt(-a*e^(d*x + c))*a^3*e^(3*d*x + 3*c) - 11*s
qrt(2)*sqrt(-a*e^(d*x + c))*a^3*e^(2*d*x + 2*c) - 11*sqrt(2)*sqrt(-a*e^(d*x
 + c))*a^3*e^(d*x + c) + 3*sqrt(2)*sqrt(-a*e^(d*x + c))*a^3)/((a*e^(d*x + c
) - a)^4*a^2*d*sgn(-e^(d*x + c) + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a-a\cosh\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - a*cosh(c + d*x))^(5/2),x)
```

```
[Out] int(1/(a - a*cosh(c + d*x))^(5/2), x)
```

3.54 $\int \frac{\cosh^4(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=112

$$-\frac{a(2a^2+b^2)\,x}{2b^4}+\frac{2a^4\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^4\sqrt{a+b}}+\frac{(3a^2+2b^2)\sinh(x)}{3b^3}-\frac{a\cosh(x)\sinh(x)}{2b^2}+\frac{\cosh^2(x)\sinh(x)}{3b}$$

```
[Out] -1/2*a*(2*a^2+b^2)*x/b^4+1/3*(3*a^2+2*b^2)*sinh(x)/b^3-1/2*a*cosh(x)*sinh(x
)/b^2+1/3*cosh(x)^2*sinh(x)/b+2*a^4*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(
1/2))/b^4/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2872, 3128, 3102, 2814, 2738, 214}

$$\frac{2a^4\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^4\sqrt{a-b}\,\sqrt{a+b}}-\frac{ax(2a^2+b^2)}{2b^4}+\frac{(3a^2+2b^2)\sinh(x)}{3b^3}-\frac{a\sinh(x)\cosh(x)}{2b^2}+\frac{\sinh(x)\cosh^2(x)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^4/(a + b*Cosh[x]),x]
```

```
[Out] -1/2*(a*(2*a^2 + b^2)*x)/b^4 + (2*a^4*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[
a + b]])/(Sqrt[a - b]*b^4*Sqrt[a + b]) + ((3*a^2 + 2*b^2)*Sinh[x])/(3*b^3)
- (a*Cosh[x]*Sinh[x])/(2*b^2) + (Cosh[x]^2*Sinh[x])/(3*b)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2872

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
 + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*
(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m]
 || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &
& NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cosh^4(x)}{a+b\cosh(x)}\,dx = \frac{\cosh^2(x)\sinh(x)}{3b} + \frac{\int \frac{\cosh(x)(2a+2b\cosh(x)-3a\cosh^2(x))}{a+b\cosh(x)}\,dx}{3b}$$

$$= -\frac{a\cosh(x)\sinh(x)}{2b^2} + \frac{\cosh^2(x)\sinh(x)}{3b} + \frac{\int \frac{-3a^2+ab\cosh(x)+2(3a^2+2b^2)\cosh^2(x)}{a+b\cosh(x)}\,dx}{6b^2}$$

$$= \frac{(3a^2+2b^2)\sinh(x)}{3b^3} - \frac{a\cosh(x)\sinh(x)}{2b^2} + \frac{\cosh^2(x)\sinh(x)}{3b} + \frac{\int \frac{-3a^2b-3a(2a^2+b^2)\cosh(x}{a+b\cosh(x)}}{6b^3}$$

$$= -\frac{a(2a^2+b^2)\,x}{2b^4} + \frac{(3a^2+2b^2)\sinh(x)}{3b^3} - \frac{a\cosh(x)\sinh(x)}{2b^2} + \frac{\cosh^2(x)\sinh(x)}{3b} + \frac{a^4\int}{}$$

$$= -\frac{a(2a^2+b^2)\,x}{2b^4} + \frac{(3a^2+2b^2)\sinh(x)}{3b^3} - \frac{a\cosh(x)\sinh(x)}{2b^2} + \frac{\cosh^2(x)\sinh(x)}{3b} + \frac{(2a}{}$$

$$= -\frac{a(2a^2+b^2)\,x}{2b^4} + \frac{2a^4\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^4\sqrt{a+b}} + \frac{(3a^2+2b^2)\sinh(x)}{3b^3} - \frac{a\cosh(x)\,s}{2b^2}$$

Mathematica [A]
time = 0.14, size = 99, normalized size = 0.88

$$\frac{-6a(2a^2+b^2)\,x - \frac{24a^4\text{ArcTan}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} + 3b(4a^2+3b^2)\sinh(x) - 3ab^2\sinh(2x) + b^3\sinh(3x)}{12b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^4/(a + b*Cosh[x]),x]
```

```
[Out] (-6*a*(2*a^2 + b^2)*x - (24*a^4*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]
])/Sqrt[-a^2 + b^2] + 3*b*(4*a^2 + 3*b^2)*Sinh[x] - 3*a*b^2*Sinh[2*x] + b^3
*Sinh[3*x])/(12*b^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(94) = 188.
time = 0.46, size = 203, normalized size = 1.81

method	result
default	$-\frac{1}{3b(\tanh(\frac{x}{2})+1)^3} - \frac{-a-b}{2b^2(\tanh(\frac{x}{2})+1)^2} - \frac{2a^2+ab+2b^2}{2b^3(\tanh(\frac{x}{2})+1)} - \frac{a(2a^2+b^2)\ln(\tanh(\frac{x}{2})+1)}{2b^4} + \frac{2a^4\operatorname{arctanh}\left(\frac{(a-b)\tanh(}{\sqrt{(a+b)(a}}\right.}{b^4\sqrt{(a+b)(a-}}$

risch	$-\frac{a^{3} x}{b^{4}}-\frac{a x}{2 b^{2}}+\frac{\mathrm{e}^{3 x}}{24 b}-\frac{a\,\mathrm{e}^{2 x}}{8 b^{2}}+\frac{\mathrm{e}^{x} a^{2}}{2 b^{3}}+\frac{3\,\mathrm{e}^{x}}{8 b}-\frac{\mathrm{e}^{-x} a^{2}}{2 b^{3}}-\frac{3\,\mathrm{e}^{-x}}{8 b}+\frac{a\,\mathrm{e}^{-2 x}}{8 b^{2}}-\frac{\mathrm{e}^{-3 x}}{24 b}+\frac{a^{4} \ln \left(\mathrm{e}^{x}+\frac{a \sqrt{a^{2}-b^{2}}-a^{2}+b^{2}}{b \sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\, b^{4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/b/(tanh(1/2*x)+1)^3-1/2*(-a-b)/b^2/(tanh(1/2*x)+1)^2-1/2*(2*a^2+a*b+2*
b^2)/b^3/(tanh(1/2*x)+1)-1/2*a*(2*a^2+b^2)/b^4*ln(tanh(1/2*x)+1)+2*a^4/b^4/
((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))-1/3/b/(t
anh(1/2*x)-1)^3-1/2*(a+b)/b^2/(tanh(1/2*x)-1)^2-1/2*(2*a^2+a*b+2*b^2)/b^3/(
tanh(1/2*x)-1)+1/2*a*(2*a^2+b^2)/b^4*ln(tanh(1/2*x)-1)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 778 vs. $2(94) = 188$.
time = 0.40, size = 1625, normalized size = 14.51

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [1/24*((a^2*b^3 - b^5)*cosh(x)^6 + (a^2*b^3 - b^5)*sinh(x)^6 - 3*(a^3*b^2 -
 a*b^4)*cosh(x)^5 - 3*(a^3*b^2 - a*b^4 - 2*(a^2*b^3 - b^5)*cosh(x))*sinh(x)
^5 - a^2*b^3 + b^5 - 12*(2*a^5 - a^3*b^2 - a*b^4)*x*cosh(x)^3 + 3*(4*a^4*b
- a^2*b^3 - 3*b^5)*cosh(x)^4 + 3*(4*a^4*b - a^2*b^3 - 3*b^5 + 5*(a^2*b^3 -
b^5)*cosh(x)^2 - 5*(a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^4 + 2*(10*(a^2*b^3 -
b^5)*cosh(x)^3 - 15*(a^3*b^2 - a*b^4)*cosh(x)^2 - 6*(2*a^5 - a^3*b^2 - a*b^
4)*x + 6*(4*a^4*b - a^2*b^3 - 3*b^5)*cosh(x))*sinh(x)^3 - 3*(4*a^4*b - a^2*
b^3 - 3*b^5)*cosh(x)^2 - 3*(4*a^4*b - a^2*b^3 - 3*b^5 - 5*(a^2*b^3 - b^5)*c
osh(x)^4 + 10*(a^3*b^2 - a*b^4)*cosh(x)^3 + 12*(2*a^5 - a^3*b^2 - a*b^4)*x*
cosh(x) - 6*(4*a^4*b - a^2*b^3 - 3*b^5)*cosh(x)^2)*sinh(x)^2 + 24*(a^4*cosh
```

```
(x)^3 + 3*a^4*cosh(x)^2*sinh(x) + 3*a^4*cosh(x)*sinh(x)^2 + a^4*sinh(x)^3)*
sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2
- b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*si
nh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*si
nh(x) + b)) + 3*(a^3*b^2 - a*b^4)*cosh(x) + 3*(2*(a^2*b^3 - b^5)*cosh(x)^5
+ a^3*b^2 - a*b^4 - 5*(a^3*b^2 - a*b^4)*cosh(x)^4 - 12*(2*a^5 - a^3*b^2 - a
*b^4)*x*cosh(x)^2 + 4*(4*a^4*b - a^2*b^3 - 3*b^5)*cosh(x)^3 - 2*(4*a^4*b -
a^2*b^3 - 3*b^5)*cosh(x))*sinh(x))/((a^2*b^4 - b^6)*cosh(x)^3 + 3*(a^2*b^4
- b^6)*cosh(x)^2*sinh(x) + 3*(a^2*b^4 - b^6)*cosh(x)*sinh(x)^2 + (a^2*b^4 -
 b^6)*sinh(x)^3), 1/24*((a^2*b^3 - b^5)*cosh(x)^6 + (a^2*b^3 - b^5)*sinh(x)
^6 - 3*(a^3*b^2 - a*b^4)*cosh(x)^5 - 3*(a^3*b^2 - a*b^4 - 2*(a^2*b^3 - b^5)
*cosh(x))*sinh(x)^5 - a^2*b^3 + b^5 - 12*(2*a^5 - a^3*b^2 - a*b^4)*x*cosh(x
)^3 + 3*(4*a^4*b - a^2*b^3 - 3*b^5)*cosh(x)^4 + 3*(4*a^4*b - a^2*b^3 - 3*b^
5 + 5*(a^2*b^3 - b^5)*cosh(x)^2 - 5*(a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^4 +
2*(10*(a^2*b^3 - b^5)*cosh(x)^3 - 15*(a^3*b^2 - a*b^4)*cosh(x)^2 - 6*(2*a^5
 - a^3*b^2 - a*b^4)*x + 6*(4*a^4*b - a^2*b^3 - 3*b^5)*cosh(x))*sinh(x)^3 -
3*(4*a^4*b - a^2*b^3 - 3*b^5)*cosh(x)^2 - 3*(4*a^4*b - a^2*b^3 - 3*b^5 - 5*
(a^2*b^3 - b^5)*cosh(x)^4 + 10*(a^3*b^2 - a*b^4)*cosh(x)^3 + 12*(2*a^5 - a^
3*b^2 - a*b^4)*x*cosh(x) - 6*(4*a^4*b - a^2*b^3 - 3*b^5)*cosh(x)^2)*sinh(x)
^2 - 48*(a^4*cosh(x)^3 + 3*a^4*cosh(x)^2*sinh(x) + 3*a^4*cosh(x)*sinh(x)^2
+ a^4*sinh(x)^3)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*s
inh(x) + a)/(a^2 - b^2)) + 3*(a^3*b^2 - a*b^4)*cosh(x) + 3*(2*(a^2*b^3 - b^
5)*cosh(x)^5 + a^3*b^2 - a*b^4 - 5*(a^3*b^2 - a*b^4)*cosh(x)^4 - 12*(2*a^5
- a^3*b^2 - a*b^4)*x*cosh(x)^2 + 4*(4*a^4*b - a^2*b^3 - 3*b^5)*cosh(x)^3 -
2*(4*a^4*b - a^2*b^3 - 3*b^5)*cosh(x))*sinh(x))/((a^2*b^4 - b^6)*cosh(x)^3
+ 3*(a^2*b^4 - b^6)*cosh(x)^2*sinh(x) + 3*(a^2*b^4 - b^6)*cosh(x)*sinh(x)^2
 + (a^2*b^4 - b^6)*sinh(x)^3)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**4/(a+b*cosh(x)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.41, size = 133, normalized size = 1.19

$$\frac{2\,a^4\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,b^4}+\frac{b^2e^{(3\,x)}-3\,abe^{(2\,x)}+12\,a^2e^x+9\,b^2e^x}{24\,b^3}-\frac{(2\,a^3+ab^2)x}{2\,b^4}+\frac{(3\,ab^2e^x-b^3-3\,(4\,a^2b+3\,b^3)e^{(2\,x)})e^{(-3\,x)}}{24\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*a^4*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b^4) + 1/24*(b
^2*e^(3*x) - 3*a*b*e^(2*x) + 12*a^2*e^x + 9*b^2*e^x)/b^3 - 1/2*(2*a^3 + a*b
^2)*x/b^4 + 1/24*(3*a*b^2*e^x - b^3 - 3*(4*a^2*b + 3*b^3)*e^(2*x))*e^(-3*x)
/b^4
```

Mupad [B]
time = 1.26, size = 209, normalized size = 1.87

$$\frac{e^{3x}}{24b}-\frac{e^{-3x}}{24b}-\frac{x\,(2\,a^3+a\,b^2)}{2\,b^4}+\frac{e^x\,(4\,a^2+3\,b^2)}{8\,b^3}+\frac{a\,e^{-2x}}{8\,b^2}-\frac{a\,e^{2x}}{8\,b^2}-\frac{e^{-x}\,(4\,a^2+3\,b^2)}{8\,b^3}+\frac{a^4\,\ln\left(-\frac{2\,a^4\,e^x}{b^5}-\frac{2\,a^4\,(b+a\,e^x)}{b^5\,\sqrt{a+b}\,\sqrt{a-b}}\right)}{b^4\,\sqrt{a+b}\,\sqrt{a-b}}-\frac{a^4\,\ln\left(\frac{2\,a^4\,(b+a\,e^x)}{b^5\,\sqrt{a+b}\,\sqrt{a-b}}-\frac{2\,a^4\,e^x}{b^5}\right)}{b^4\,\sqrt{a+b}\,\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a + b*cosh(x)),x)
```

```
[Out] exp(3*x)/(24*b) - exp(-3*x)/(24*b) - (x*(a*b^2 + 2*a^3))/(2*b^4) + (exp(x)*
(4*a^2 + 3*b^2))/(8*b^3) + (a*exp(-2*x))/(8*b^2) - (a*exp(2*x))/(8*b^2) - (
exp(-x)*(4*a^2 + 3*b^2))/(8*b^3) + (a^4*log(- (2*a^4*exp(x))/b^5 - (2*a^4*(
b + a*exp(x)))/(b^5*(a + b)^(1/2)*(a - b)^(1/2))))/(b^4*(a + b)^(1/2)*(a -
b)^(1/2)) - (a^4*log((2*a^4*(b + a*exp(x)))/(b^5*(a + b)^(1/2)*(a - b)^(1/2
)) - (2*a^4*exp(x))/b^5))/(b^4*(a + b)^(1/2)*(a - b)^(1/2))
```

3.55 $\int \frac{\cosh^3(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=85

$$\frac{(2a^2+b^2)\,x}{2b^3}-\frac{2a^3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^3\sqrt{a+b}}-\frac{a\sinh(x)}{b^2}+\frac{\cosh(x)\sinh(x)}{2b}$$

```
[Out] 1/2*(2*a^2+b^2)*x/b^3-a*sinh(x)/b^2+1/2*cosh(x)*sinh(x)/b-2*a^3*arctanh((a-
b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/b^3/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2872, 3102, 2814, 2738, 214}

$$-\frac{2a^3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^3\sqrt{a-b}\,\sqrt{a+b}}+\frac{x(2a^2+b^2)}{2b^3}-\frac{a\sinh(x)}{b^2}+\frac{\sinh(x)\cosh(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(a + b*Cosh[x]),x]
```

```
[Out] ((2*a^2 + b^2)*x)/(2*b^3) - (2*a^3*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a +
 b]])/(Sqrt[a - b]*b^3*Sqrt[a + b]) - (a*Sinh[x])/b^2 + (Cosh[x]*Sinh[x])/(
2*b)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2872

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
 + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*
(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m]
 || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &
& NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^3(x)}{a+b\cosh(x)}\,dx &= \frac{\cosh(x)\sinh(x)}{2b}+\frac{\int \frac{a+b\cosh(x)-2a\cosh^2(x)}{a+b\cosh(x)}\,dx}{2b}\\
&= -\frac{a\sinh(x)}{b^2}+\frac{\cosh(x)\sinh(x)}{2b}+\frac{\int \frac{ab+(2a^2+b^2)\cosh(x)}{a+b\cosh(x)}\,dx}{2b^2}\\
&= \frac{(2a^2+b^2)\,x}{2b^3}-\frac{a\sinh(x)}{b^2}+\frac{\cosh(x)\sinh(x)}{2b}-\frac{a^3\int \frac{1}{a+b\cosh(x)}\,dx}{b^3}\\
&= \frac{(2a^2+b^2)\,x}{2b^3}-\frac{a\sinh(x)}{b^2}+\frac{\cosh(x)\sinh(x)}{2b}-\frac{(2a^3)\,\mathrm{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b^3}\\
&= \frac{(2a^2+b^2)\,x}{2b^3}-\frac{2a^3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^3\sqrt{a+b}}-\frac{a\sinh(x)}{b^2}+\frac{\cosh(x)\sinh(x)}{2b}
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 78, normalized size = 0.92

$$
\frac{4a^2x+2b^2x+\frac{8a^3\mathrm{ArcTan}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}-4ab\sinh(x)+b^2\sinh(2x)}{4b^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(a + b*Cosh[x]),x]
```

```
[Out] (4*a^2*x + 2*b^2*x + (8*a^3*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/S
qrt[-a^2 + b^2] - 4*a*b*Sinh[x] + b^2*Sinh[2*x])/(4*b^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(71) = 142$.
time = 0.46, size = 153, normalized size = 1.80

method	result
default	$-\frac{1}{2b\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}-\frac{-b-2a}{2b^{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{\left(2a^{2}+b^{2}\right)\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2b^{3}}-\frac{2a^{3}\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^{3}\sqrt{(a+b)(a-b)}}+\frac{1}{2b(\tanh(}$
risch	$\frac{x\,a^{2}}{b^{3}}+\frac{x}{2b}+\frac{e^{2x}}{8b}-\frac{a\,e^{x}}{2b^{2}}+\frac{a\,e^{-x}}{2b^{2}}-\frac{e^{-2x}}{8b}+\frac{a^{3}\ln\left(e^{x}+\frac{a\sqrt{a^{2}-b^{2}}+a^{2}-b^{2}}{b\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,b^{3}}-\frac{a^{3}\ln\left(e^{x}+\frac{a\sqrt{a^{2}-b^{2}}-a^{2}+b^{2}}{b\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,b^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b/(tanh(1/2*x)+1)^2-1/2*(-b-2*a)/b^2/(tanh(1/2*x)+1)+1/2*(2*a^2+b^2)/b
^3*ln(tanh(1/2*x)+1)-2*a^3/b^3/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x
)/((a+b)*(a-b))^(1/2))+1/2/b/(tanh(1/2*x)-1)^2-1/2*(-b-2*a)/b^2/(tanh(1/2*x
)-1)+1/2/b^3*(-2*a^2-b^2)*ln(tanh(1/2*x)-1)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 417 vs. $2(71) = 142$.
time = 0.42, size = 903, normalized size = 10.62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [1/8*((a^2*b^2 - b^4)*cosh(x)^4 + (a^2*b^2 - b^4)*sinh(x)^4 - a^2*b^2 + b^4
 + 4*(2*a^4 - a^2*b^2 - b^4)*x*cosh(x)^2 - 4*(a^3*b - a*b^3)*cosh(x)^3 - 4*
(a^3*b - a*b^3 - (a^2*b^2 - b^4)*cosh(x))*sinh(x)^3 + 2*(3*(a^2*b^2 - b^4)*
cosh(x)^2 + 2*(2*a^4 - a^2*b^2 - b^4)*x - 6*(a^3*b - a*b^3)*cosh(x))*sinh(x
)^2 + 8*(a^3*cosh(x)^2 + 2*a^3*cosh(x)*sinh(x) + a^3*sinh(x)^2)*sqrt(a^2 -
b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(
b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))
/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b))
 + 4*(a^3*b - a*b^3)*cosh(x) + 4*(a^3*b - a*b^3 + (a^2*b^2 - b^4)*cosh(x)^3
 + 2*(2*a^4 - a^2*b^2 - b^4)*x*cosh(x) - 3*(a^3*b - a*b^3)*cosh(x)^2)*sinh(
x))/((a^2*b^3 - b^5)*cosh(x)^2 + 2*(a^2*b^3 - b^5)*cosh(x)*sinh(x) + (a^2*b
^3 - b^5)*sinh(x)^2), 1/8*((a^2*b^2 - b^4)*cosh(x)^4 + (a^2*b^2 - b^4)*sinh
(x)^4 - a^2*b^2 + b^4 + 4*(2*a^4 - a^2*b^2 - b^4)*x*cosh(x)^2 - 4*(a^3*b -
a*b^3)*cosh(x)^3 - 4*(a^3*b - a*b^3 - (a^2*b^2 - b^4)*cosh(x))*sinh(x)^3 +
2*(3*(a^2*b^2 - b^4)*cosh(x)^2 + 2*(2*a^4 - a^2*b^2 - b^4)*x - 6*(a^3*b - a
*b^3)*cosh(x))*sinh(x)^2 + 16*(a^3*cosh(x)^2 + 2*a^3*cosh(x)*sinh(x) + a^3*
sinh(x)^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x)
 + a)/(a^2 - b^2)) + 4*(a^3*b - a*b^3)*cosh(x) + 4*(a^3*b - a*b^3 + (a^2*b^
2 - b^4)*cosh(x)^3 + 2*(2*a^4 - a^2*b^2 - b^4)*x*cosh(x) - 3*(a^3*b - a*b^3
)*cosh(x)^2)*sinh(x))/((a^2*b^3 - b^5)*cosh(x)^2 + 2*(a^2*b^3 - b^5)*cosh(x
)*sinh(x) + (a^2*b^3 - b^5)*sinh(x)^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(a+b*cosh(x)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.40, size = 92, normalized size = 1.08

$$-\frac{2\,a^3\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,b^3}+\frac{be^{(2\,x)}-4\,ae^x}{8\,b^2}+\frac{(2\,a^2+b^2)x}{2\,b^3}+\frac{(4\,abe^x-b^2)e^{(-2\,x)}}{8\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] -2*a^3*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b^3) + 1/8*(b
*e^(2*x) - 4*a*e^x)/b^2 + 1/2*(2*a^2 + b^2)*x/b^3 + 1/8*(4*a*b*e^x - b^2)*e
^(-2*x)/b^3
```

Mupad [B]

time = 1.12, size = 167, normalized size = 1.96

$$\frac{e^{2x}}{8b}-\frac{e^{-2x}}{8b}-\frac{a\,e^{x}}{2b^2}+\frac{a\,e^{-x}}{2b^2}+\frac{x\,(2a^2+b^2)}{2b^3}+\frac{a^3\ln\left(\frac{2a^3e^x}{b^4}-\frac{2a^3(b+a\,e^x)}{b^4\sqrt{a+b}\,\sqrt{a-b}}\right)}{b^3\sqrt{a+b}\,\sqrt{a-b}}-\frac{a^3\ln\left(\frac{2a^3e^x}{b^4}+\frac{2a^3(b+a\,e^x)}{b^4\sqrt{a+b}\,\sqrt{a-b}}\right)}{b^3\sqrt{a+b}\,\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a + b*cosh(x)),x)
```

```
[Out] exp(2*x)/(8*b) - exp(-2*x)/(8*b) - (a*exp(x))/(2*b^2) + (a*exp(-x))/(2*b^2)
 + (x*(2*a^2 + b^2))/(2*b^3) + (a^3*log((2*a^3*exp(x))/b^4 - (2*a^3*(b + a*
exp(x)))/(b^4*(a + b)^(1/2)*(a - b)^(1/2))))/(b^3*(a + b)^(1/2)*(a - b)^(1/
2)) - (a^3*log((2*a^3*exp(x))/b^4 + (2*a^3*(b + a*exp(x)))/(b^4*(a + b)^(1/
2)*(a - b)^(1/2))))/(b^3*(a + b)^(1/2)*(a - b)^(1/2))
```

3.56 $\int \frac{\cosh^2(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=62

$$-\frac{ax}{b^2}+\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^2\sqrt{a+b}}+\frac{\sinh(x)}{b}$$

```
[Out] -a*x/b^2+sinh(x)/b+2*a^2*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/b^2/(
a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2825, 12, 2814, 2738, 214}

$$\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^2\sqrt{a-b}\,\sqrt{a+b}}-\frac{ax}{b^2}+\frac{\sinh(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(a + b*Cosh[x]),x]
```

```
[Out] -((a*x)/b^2) + (2*a^2*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a
 - b]*b^2*Sqrt[a + b]) + Sinh[x]/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2825

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*(Cos[e + f*x]/(d*f)), x] + Dist[1/d, I
nt[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2(x)}{a+b\cosh(x)}\,dx &= \frac{\sinh(x)}{b}-\frac{\int \frac{a\cosh(x)}{a+b\cosh(x)}\,dx}{b}\\ &= \frac{\sinh(x)}{b}-\frac{a\int \frac{\cosh(x)}{a+b\cosh(x)}\,dx}{b}\\ &= -\frac{ax}{b^2}+\frac{\sinh(x)}{b}+\frac{a^2\int \frac{1}{a+b\cosh(x)}\,dx}{b^2}\\ &= -\frac{ax}{b^2}+\frac{\sinh(x)}{b}+\frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b^2}\\ &= -\frac{ax}{b^2}+\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^2\sqrt{a+b}}+\frac{\sinh(x)}{b}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 57, normalized size = 0.92

$$\frac{a\left(-x-\frac{2a\text{ArcTan}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}\right)+b\sinh(x)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(a + b*Cosh[x]),x]
```

```
[Out] (a*(-x - (2*a*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2
]) + b*Sinh[x])/b^2
```

Maple [A]
time = 0.44, size = 94, normalized size = 1.52

method	result	size
default	$\frac{2a^{2}\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^{2}\sqrt{(a+b)(a-b)}}-\frac{1}{b\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b^{2}}-\frac{1}{b\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b^{2}}$	94
risch	$-\frac{ax}{b^{2}}+\frac{e^{x}}{2b}-\frac{e^{-x}}{2b}+\frac{a^{2}\ln\left(e^{x}+\frac{a\sqrt{a^{2}-b^{2}}-a^{2}+b^{2}}{b\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,b^{2}}-\frac{a^{2}\ln\left(e^{x}+\frac{a\sqrt{a^{2}-b^{2}}+a^{2}-b^{2}}{b\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,b^{2}}$	144

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a^2/b^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2)
)-1/b/(tanh(1/2*x)-1)+a/b^2*ln(tanh(1/2*x)-1)-1/b/(tanh(1/2*x)+1)-a/b^2*ln(
tanh(1/2*x)+1)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(52) = 104.
time = 0.44, size = 449, normalized size = 7.24

$$\left[-\frac{a^{2}b-b^{3}+2\,(a^{3}-ab^{2})x\cosh(x)-(a^{2}b-b^{3})\cosh(x)^{2}-(a^{2}b-b^{3})\sinh(x)^{2}-2\,(a^{2}\cosh(x)+a^{2}\sinh(x))\sqrt{a^{2}-b^{2}}\log\left(\frac{b^{2}\cosh(x)^{2}+b^{2}\sinh(x)^{2}+2\,ab\cosh(x)+2\,a^{2}-b^{2}+2\,(b^{2}\cosh(x)+ab)\sinh(x)-2\,\sqrt{a^{2}-b^{2}}\,(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^{2}+b\sinh(x)^{2}+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right)+2\,((a^{3}-ab^{2})x-(a^{2}b-b^{3})\cosh(x))\sinh(x)}{2\,((a^{2}b^{2}-b^{4})\cosh(x)+(a^{2}b^{2}-b^{4})\sinh(x))},\ -\frac{a^{2}b-b^{3}+2\,(a^{3}-ab^{2})x\cosh(x)-(a^{2}b-b^{3})\cosh(x)^{2}-(a^{2}b-b^{3})\sinh(x)^{2}+4\,(a^{2}\cosh(x)+a^{2}\sinh(x))\sqrt{-a^{2}+b^{2}}\arctan\left(-\frac{\sqrt{-a^{2}+b^{2}}\,(b\cosh(x)+b\sinh(x)+a)}{a^{2}-b^{2}}\right)+2\,((a^{3}-ab^{2})x-(a^{2}b-b^{3})\cosh(x))\sinh(x)}{2\,((a^{2}b^{2}-b^{4})\cosh(x)+(a^{2}b^{2}-b^{4})\sinh(x))}\right]$$

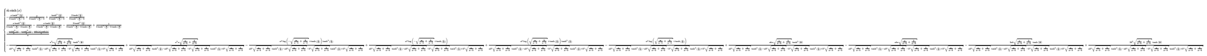
Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(a^2*b - b^3 + 2*(a^3 - a*b^2)*x*cosh(x) - (a^2*b - b^3)*cosh(x)^2 -
(a^2*b - b^3)*sinh(x)^2 - 2*(a^2*cosh(x) + a^2*sinh(x))*sqrt(a^2 - b^2)*log
((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh
(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh
(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)) + 2*((a
^3 - a*b^2)*x - (a^2*b - b^3)*cosh(x))*sinh(x))/((a^2*b^2 - b^4)*cosh(x) +
```

```
(a^2*b^2 - b^4)*sinh(x)), -1/2*(a^2*b - b^3 + 2*(a^3 - a*b^2)*x*cosh(x) - (
a^2*b - b^3)*cosh(x)^2 - (a^2*b - b^3)*sinh(x)^2 + 4*(a^2*cosh(x) + a^2*sin
h(x))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)
/(a^2 - b^2)) + 2*((a^3 - a*b^2)*x - (a^2*b - b^3)*cosh(x))*sinh(x))/((a^2*
b^2 - b^4)*cosh(x) + (a^2*b^2 - b^4)*sinh(x))]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1275 vs. $2(53) = 106$.
time = 71.86, size = 1275, normalized size = 20.56



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(a+b*cosh(x)),x)
```

```
[Out] Piecewise((zoo*sinh(x), Eq(a, 0) & Eq(b, 0)), (-x*tanh(x/2)**2/(b*tanh(x/2)
**2 - b) + x/(b*tanh(x/2)**2 - b) + tanh(x/2)**3/(b*tanh(x/2)**2 - b) - 3*t
anh(x/2)/(b*tanh(x/2)**2 - b), Eq(a, b)), (x*tanh(x/2)**3/(b*tanh(x/2)**3 -
 b*tanh(x/2)) - x*tanh(x/2)/(b*tanh(x/2)**3 - b*tanh(x/2)) - 3*tanh(x/2)**2
/(b*tanh(x/2)**3 - b*tanh(x/2)) + 1/(b*tanh(x/2)**3 - b*tanh(x/2)), Eq(a, -
b)), ((-x*sinh(x)**2/2 + x*cosh(x)**2/2 + sinh(x)*cosh(x)/2)/a, Eq(b, 0)),
(-a**2*x*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2/(a*b**2*sqrt(a/(a - b) +
b/(a - b))*tanh(x/2)**2 - a*b**2*sqrt(a/(a - b) + b/(a - b)) - b**3*sqrt(a/
(a - b) + b/(a - b))*tanh(x/2)**2 + b**3*sqrt(a/(a - b) + b/(a - b))) + a**
2*x*sqrt(a/(a - b) + b/(a - b))/(a*b**2*sqrt(a/(a - b) + b/(a - b))*tanh(x/
2)**2 - a*b**2*sqrt(a/(a - b) + b/(a - b)) - b**3*sqrt(a/(a - b) + b/(a - b
))*tanh(x/2)**2 + b**3*sqrt(a/(a - b) + b/(a - b))) - a**2*log(-sqrt(a/(a -
 b) + b/(a - b)) + tanh(x/2))*tanh(x/2)**2/(a*b**2*sqrt(a/(a - b) + b/(a -
b))*tanh(x/2)**2 - a*b**2*sqrt(a/(a - b) + b/(a - b)) - b**3*sqrt(a/(a - b)
 + b/(a - b))*tanh(x/2)**2 + b**3*sqrt(a/(a - b) + b/(a - b))) + a**2*log(-
sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b**2*sqrt(a/(a - b) + b/(a - b)
)*tanh(x/2)**2 - a*b**2*sqrt(a/(a - b) + b/(a - b)) - b**3*sqrt(a/(a - b) +
 b/(a - b))*tanh(x/2)**2 + b**3*sqrt(a/(a - b) + b/(a - b))) + a**2*log(sqr
t(a/(a - b) + b/(a - b)) + tanh(x/2))*tanh(x/2)**2/(a*b**2*sqrt(a/(a - b) +
 b/(a - b))*tanh(x/2)**2 - a*b**2*sqrt(a/(a - b) + b/(a - b)) - b**3*sqrt(a
/(a - b) + b/(a - b))*tanh(x/2)**2 + b**3*sqrt(a/(a - b) + b/(a - b))) - a*
*2*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b**2*sqrt(a/(a - b) + b/
(a - b))*tanh(x/2)**2 - a*b**2*sqrt(a/(a - b) + b/(a - b)) - b**3*sqrt(a/(a
 - b) + b/(a - b))*tanh(x/2)**2 + b**3*sqrt(a/(a - b) + b/(a - b))) + a*b*x
*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2/(a*b**2*sqrt(a/(a - b) + b/(a - b
))*tanh(x/2)**2 - a*b**2*sqrt(a/(a - b) + b/(a - b)) - b**3*sqrt(a/(a - b)
+ b/(a - b))*tanh(x/2)**2 + b**3*sqrt(a/(a - b) + b/(a - b))) - a*b*x*sqrt(
a/(a - b) + b/(a - b))/(a*b**2*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 - a
*b**2*sqrt(a/(a - b) + b/(a - b)) - b**3*sqrt(a/(a - b) + b/(a - b))*tanh(x
/2)**2 + b**3*sqrt(a/(a - b) + b/(a - b))) - 2*a*b*sqrt(a/(a - b) + b/(a -
```

```
b))*tanh(x/2)/(a*b**2*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 - a*b**2*sqr
t(a/(a - b) + b/(a - b)) - b**3*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 +
b**3*sqrt(a/(a - b) + b/(a - b))) + 2*b**2*sqrt(a/(a - b) + b/(a - b))*tanh
(x/2)/(a*b**2*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 - a*b**2*sqrt(a/(a -
 b) + b/(a - b)) - b**3*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 + b**3*sqr
t(a/(a - b) + b/(a - b))), True))
```

Giac [A]
time = 0.43, size = 62, normalized size = 1.00

$$\frac{2\,a^2\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,b^2}-\frac{ax}{b^2}-\frac{e^{(-x)}}{2\,b}+\frac{e^x}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*a^2*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b^2) - a*x/b^2
 - 1/2*e^(-x)/b + 1/2*e^x/b
```

Mupad [B]
time = 1.04, size = 139, normalized size = 2.24

$$\frac{e^x}{2\,b}-\frac{e^{-x}}{2\,b}-\frac{a\,x}{b^2}+\frac{a^2\,\ln\left(-\frac{2\,a^2\,e^x}{b^3}-\frac{2\,a^2\,(b+a\,e^x)}{b^3\,\sqrt{a+b}\,\sqrt{a-b}}\right)}{b^2\,\sqrt{a+b}\,\sqrt{a-b}}-\frac{a^2\,\ln\left(\frac{2\,a^2\,(b+a\,e^x)}{b^3\,\sqrt{a+b}\,\sqrt{a-b}}-\frac{2\,a^2\,e^x}{b^3}\right)}{b^2\,\sqrt{a+b}\,\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a + b*cosh(x)),x)
```

```
[Out] exp(x)/(2*b) - exp(-x)/(2*b) - (a*x)/b^2 + (a^2*log(- (2*a^2*exp(x))/b^3 -
(2*a^2*(b + a*exp(x)))/(b^3*(a + b)^(1/2)*(a - b)^(1/2))))/(b^2*(a + b)^(1/
2)*(a - b)^(1/2)) - (a^2*log((2*a^2*(b + a*exp(x)))/(b^3*(a + b)^(1/2)*(a -
 b)^(1/2)) - (2*a^2*exp(x))/b^3))/(b^2*(a + b)^(1/2)*(a - b)^(1/2))
```

3.57 $\int \frac{\cosh(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=52

$$\frac{x}{b}-\frac{2a\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b\sqrt{a+b}}$$

```
[Out] x/b-2*a*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/b/(a-b)^(1/2)/(a+b)^(1
/2)
```

Rubi [A]
time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2814, 2738, 214}

$$\frac{x}{b}-\frac{2a\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\,\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/(a + b*Cosh[x]),x]
```

```
[Out] x/b - (2*a*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqr
t[a + b])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh(x)}{a+b\cosh(x)}\,dx &= \frac{x}{b}-\frac{a\int \frac{1}{a+b\cosh(x)}\,dx}{b}\\
&= \frac{x}{b}-\frac{(2a)\text{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b}\\
&= \frac{x}{b}-\frac{2a\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b\sqrt{a+b}}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 48, normalized size = 0.92

$$
\frac{x+\frac{2a\text{ArcTan}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}}{b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(a + b*Cosh[x]),x]
```

```
[Out] (x + (2*a*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2])/b
```

Maple [A]
time = 0.41, size = 64, normalized size = 1.23

method	result	size
default	$\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b}-\frac{2a\,\text{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b\sqrt{(a+b)(a-b)}}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b}$	64
risch	$\frac{x}{b}+\frac{a\ln\left(e^x+\frac{a\sqrt{a^2-b^2}+a^2-b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,b}-\frac{a\ln\left(e^x+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,b}$	122

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*ln(tanh(1/2*x)+1)-2*a/b/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/(
(a+b)*(a-b))^(1/2))-1/b*ln(tanh(1/2*x)-1)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.37, size = 218, normalized size = 4.19

$$\left[\frac{\sqrt{a^2-b^2}\,a\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\,(b^2\cosh(x)+ab)\sinh(x)+2\,\sqrt{a^2-b^2}\,(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right)+(a^2-b^2)x}{a^2b-b^3},\ \frac{2\,\sqrt{-a^2+b^2}\,a\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\cosh(x)+b\sinh(x)+a)}{a^2-b^2}\right)+(a^2-b^2)x}{a^2b-b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [(sqrt(a^2 - b^2)*a*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*
a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*cosh(x) +
b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a
)*sinh(x) + b)) + (a^2 - b^2)*x)/(a^2*b - b^3), (2*sqrt(-a^2 + b^2)*a*arcta
n(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) + (a^2 - b^2)*
x)/(a^2*b - b^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(41) = 82.
time = 16.26, size = 241, normalized size = 4.63

$$\begin{cases} \tilde{\infty}x & \text{for}\: a=0 \wedge b=0 \\ \frac{x}{b}-\frac{1}{b\tanh\left(\frac{x}{2}\right)} & \text{for}\: a=-b \\ \frac{\sinh\left(x\right)}{a} & \text{for}\: b=0 \\ \frac{x}{b}-\frac{\tanh\left(\frac{x}{2}\right)}{b} & \text{for}\: a=b \\ \frac{ax\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}+\frac{a\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}-\frac{a\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}-\frac{bx\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*cosh(x)),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x/b - 1/(b*tanh(x/2)), Eq(a, -b)),
 (sinh(x)/a, Eq(b, 0)), (x/b - tanh(x/2)/b, Eq(a, b)), (a*x*sqrt(a/(a - b)
+ b/(a - b))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a
- b))) + a*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b
) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) - a*log(sqrt(a/(a - b) +
 b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a
- b) + b/(a - b))) - b*x*sqrt(a/(a - b) + b/(a - b))/(a*b*sqrt(a/(a - b) +
b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))), True))
```

Giac [A]
time = 0.41, size = 42, normalized size = 0.81

$$-\frac{2\,a\arctan\left(\frac{be^{x}+a}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}\,b}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] -2*a*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b) + x/b
```

Mupad [B]
time = 0.22, size = 109, normalized size = 2.10

$$\frac{x}{b}+\frac{a\,\ln\left(\frac{2\,a\,\mathrm{e}^{x}}{b^{2}}-\frac{2\,a\,(b+a\,\mathrm{e}^{x})}{b^{2}\,\sqrt{a+b}\,\sqrt{a-b}}\right)}{b\,\sqrt{a+b}\,\sqrt{a-b}}-\frac{a\,\ln\left(\frac{2\,a\,\mathrm{e}^{x}}{b^{2}}+\frac{2\,a\,(b+a\,\mathrm{e}^{x})}{b^{2}\,\sqrt{a+b}\,\sqrt{a-b}}\right)}{b\,\sqrt{a+b}\,\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a + b*cosh(x)),x)
```

```
[Out] x/b + (a*log((2*a*exp(x))/b^2 - (2*a*(b + a*exp(x)))/(b^2*(a + b)^(1/2)*(a
- b)^(1/2))))/(b*(a + b)^(1/2)*(a - b)^(1/2)) - (a*log((2*a*exp(x))/b^2 + (
2*a*(b + a*exp(x)))/(b^2*(a + b)^(1/2)*(a - b)^(1/2))))/(b*(a + b)^(1/2)*(a
 - b)^(1/2))
```

3.58 $\int \frac{\text{sech}(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=54

$$\frac{\text{ArcTan}(\sinh(x))}{a}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\,\sqrt{a+b}}$$

```
[Out] arctan(sinh(x))/a-2*b*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/a/(a-b)^
(1/2)/(a+b)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2826, 3855, 2738, 214}

$$\frac{\text{ArcTan}(\sinh(x))}{a}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\,\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]/(a + b*Cosh[x]),x]
```

```
[Out] ArcTan[Sinh[x]]/a - (2*b*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(a*S
qrt[a - b]*Sqrt[a + b])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2826

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}(x)}{a+b\cosh(x)}\,dx &= \frac{\int \operatorname{sech}(x)\,dx}{a}-\frac{b\int \frac{1}{a+b\cosh(x)}\,dx}{a}\\ &= \frac{\tan^{-1}(\sinh(x))}{a}-\frac{(2b)\text{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a}\\ &= \frac{\tan^{-1}(\sinh(x))}{a}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\,\sqrt{a+b}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 54, normalized size = 1.00

$$\frac{2\left(\text{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{b\text{ArcTan}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(a + b*Cosh[x]),x]
```

```
[Out] (2*(ArcTan[Tanh[x/2]] + (b*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sq
rt[-a^2 + b^2]))/a
```

Maple [A]
time = 0.56, size = 51, normalized size = 0.94

method	result	size
default	$-\frac{2b\,\text{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a\sqrt{(a+b)(a-b)}}+\frac{2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$	51
risch	$\frac{b\ln\left(e^{x}+\frac{a\sqrt{a^2-b^2}+a^2-b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,a}-\frac{b\ln\left(e^{x}+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,a}+\frac{i\ln(e^{x}+i)}{a}-\frac{i\ln(e^{x}-i)}{a}$	141

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/a*b/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))+2
/a*arctan(tanh(1/2*x))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.37, size = 227, normalized size = 4.20

$$\left[\frac{\sqrt{a^2-b^2}\, b \log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\,(b^2\cosh(x)+ab)\sinh(x)+2\,\sqrt{a^2-b^2}\,(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right)+2\,(a^2-b^2)\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}{a^3-ab^2},\ \frac{2\left(\sqrt{-a^2+b^2}\, b\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\cosh(x)+b\sinh(x)+a)}{a^2-b^2}\right)+(a^2-b^2)\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)\right)}{a^3-ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [(sqrt(a^2 - b^2)*b*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*
a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*cosh(x) +
b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a
)*sinh(x) + b)) + 2*(a^2 - b^2)*arctan(cosh(x) + sinh(x)))/(a^3 - a*b^2), 2
*(sqrt(-a^2 + b^2)*b*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(
a^2 - b^2)) + (a^2 - b^2)*arctan(cosh(x) + sinh(x)))/(a^3 - a*b^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}\left(x\right)}{a+b\cosh\left(x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*cosh(x)),x)
```

```
[Out] Integral(sech(x)/(a + b*cosh(x)), x)
```

Giac [A]
time = 0.41, size = 45, normalized size = 0.83

$$-\frac{2\,b\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,a}+\frac{2\,\arctan\left(e^x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] -2*b*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*a) + 2*arctan(e
^x)/a
```

Mupad [B]
time = 3.46, size = 286, normalized size = 5.30

$$\frac{b\,\ln\left(64\,a^4\,b-64\,a^2\,b^3+128\,a^5\,\mathrm{e}^x+32\,a\,b^3\,\sqrt{a^2-b^2}-64\,a^3\,b\,\sqrt{a^2-b^2}+32\,a\,b^4\,\mathrm{e}^x-128\,a^4\,\mathrm{e}^x\,\sqrt{a^2-b^2}-160\,a^3\,b^2\,\mathrm{e}^x+96\,a^2\,b^2\,\mathrm{e}^x\,\sqrt{a^2-b^2}\right)}{a\,\sqrt{a^2-b^2}}-\frac{b\,\ln\left(64\,a^4\,b-64\,a^2\,b^3+128\,a^5\,\mathrm{e}^x-32\,a\,b^3\,\sqrt{a^2-b^2}+64\,a^3\,b\,\sqrt{a^2-b^2}+32\,a\,b^4\,\mathrm{e}^x+128\,a^4\,\mathrm{e}^x\,\sqrt{a^2-b^2}-160\,a^3\,b^2\,\mathrm{e}^x-96\,a^2\,b^2\,\mathrm{e}^x\,\sqrt{a^2-b^2}\right)}{a\,\sqrt{a^2-b^2}}-\frac{\ln\left(\mathrm{e}^x-\mathrm{i}\right)\,1\mathrm{i}-\ln\left(\mathrm{e}^x+1\mathrm{i}\right)\,1\mathrm{i}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)*(a + b*cosh(x))),x)
```

```
[Out] (b*log(64*a^4*b - 64*a^2*b^3 + 128*a^5*exp(x) + 32*a*b^3*(a^2 - b^2)^(1/2)
- 64*a^3*b*(a^2 - b^2)^(1/2) + 32*a*b^4*exp(x) - 128*a^4*exp(x)*(a^2 - b^2)
^(1/2) - 160*a^3*b^2*exp(x) + 96*a^2*b^2*exp(x)*(a^2 - b^2)^(1/2)))/(a*(a^2
 - b^2)^(1/2)) - (b*log(64*a^4*b - 64*a^2*b^3 + 128*a^5*exp(x) - 32*a*b^3*(
a^2 - b^2)^(1/2) + 64*a^3*b*(a^2 - b^2)^(1/2) + 32*a*b^4*exp(x) + 128*a^4*e
xp(x)*(a^2 - b^2)^(1/2) - 160*a^3*b^2*exp(x) - 96*a^2*b^2*exp(x)*(a^2 - b^2
)^(1/2)))/(a*(a^2 - b^2)^(1/2)) - (log(exp(x) - 1i)*1i - log(exp(x) + 1i)*1
i)/a
```

3.59 $\int \frac{\operatorname{sech}^2(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=64

$$-\frac{b\text{ArcTan}(\sinh(x))}{a^2}+\frac{2b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\,\sqrt{a+b}}+\frac{\tanh(x)}{a}$$

```
[Out] -b*arctan(sinh(x))/a^2+2*b^2*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/a
^2/(a-b)^(1/2)/(a+b)^(1/2)+tanh(x)/a
```

Rubi [A]
time = 0.08, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2881, 12, 2826, 3855, 2738, 214}

$$-\frac{b\text{ArcTan}(\sinh(x))}{a^2}+\frac{2b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\,\sqrt{a+b}}+\frac{\tanh(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(a + b*Cosh[x]),x]
```

```
[Out] -((b*ArcTan[Sinh[x]])/a^2) + (2*b^2*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a
+ b]])/(a^2*Sqrt[a - b]*Sqrt[a + b]) + Tanh[x]/a
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2826

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{sech}^2(x)}{a+b\cosh(x)}\,dx &= \frac{\tanh(x)}{a}-\frac{\int \frac{b\operatorname{sech}(x)}{a+b\cosh(x)}\,dx}{a}\\
&= \frac{\tanh(x)}{a}-\frac{b\int \frac{\operatorname{sech}(x)}{a+b\cosh(x)}\,dx}{a}\\
&= \frac{\tanh(x)}{a}-\frac{b\int \operatorname{sech}(x)\,dx}{a^2}+\frac{b^2\int \frac{1}{a+b\cosh(x)}\,dx}{a^2}\\
&= -\frac{b\tan^{-1}(\sinh(x))}{a^2}+\frac{\tanh(x)}{a}+\frac{(2b^2)\operatorname{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2}\\
&= -\frac{b\tan^{-1}(\sinh(x))}{a^2}+\frac{2b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\,\sqrt{a+b}}+\frac{\tanh(x)}{a}
\end{aligned}
$$

Mathematica [A]

time = 0.08, size = 63, normalized size = 0.98

$$\frac{-2b\text{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{2b^2\text{ArcTan}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}+a\tanh(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(a + b*Cosh[x]),x]
```

```
[Out] (-2*b*ArcTan[Tanh[x/2]] - (2*b^2*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2
]])/Sqrt[-a^2 + b^2] + a*Tanh[x])/a^2
```

Maple [A]
time = 0.71, size = 73, normalized size = 1.14

method	result	size
default	$-\frac{2\left(-\frac{a\tanh\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)+1}+b\arctan\left(\tanh\left(\frac{x}{2}\right)\right)\right)}{a^2}+\frac{2b^2\,\mathrm{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^2\sqrt{(a+b)(a-b)}}$	73
risch	$-\frac{2}{a(1+e^{2x})}+\frac{b^2\ln\left(e^x+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,a^2}-\frac{b^2\ln\left(e^x+\frac{a\sqrt{a^2-b^2}+a^2-b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,a^2}+\frac{ib\ln(e^x-i)}{a^2}-\frac{ib\ln(e^x+i)}{a^2}$	160

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/a^2*(-a*tanh(1/2*x)/(tanh(1/2*x)^2+1)+b*arctan(tanh(1/2*x)))+2*b^2/a^2/(
(a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(54) = 108.
time = 0.41, size = 515, normalized size = 8.05

$$\left[-\frac{2a^3-2ab^2-\left(b^2\cosh(x)^2+2b^2\cosh(x)\sinh(x)+b^2\sinh(x)^2+b^2\right)\sqrt{a^2-b^2}\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2ab\cosh(x)+2a^2-b^2+2\left(b^2\cosh(x)+ab\right)\sinh(x)-2\sqrt{a^2-b^2}\left(b\cosh(x)+b\sinh(x)+a\right)}{b\cosh(x)^2+b\sinh(x)^2+2a\cosh(x)+2\left(b\cosh(x)+a\right)\sinh(x)+b}\right)+2\left(a^2b-b^3+\left(a^2b-b^3\right)\cosh(x)^2+2\left(a^2b-b^3\right)\cosh(x)\sinh(x)+\left(a^2b-b^3\right)\sinh(x)^2\right)\arctan\left(\cosh(x)+\sinh(x)\right)}{a^4-a^2b^2+\left(a^4-a^2b^2\right)\cosh(x)^2+2\left(a^4-a^2b^2\right)\cosh(x)\sinh(x)+\left(a^4-a^2b^2\right)\sinh(x)^2},\ -\frac{2\left(a^3-ab^2+\left(b^2\cosh(x)^2+2b^2\cosh(x)\sinh(x)+b^2\sinh(x)^2+b^2\right)\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}\left(b\cosh(x)+b\sinh(x)+a\right)}{a^2-b^2}\right)+\left(a^2b-b^3+\left(a^2b-b^3\right)\cosh(x)^2+2\left(a^2b-b^3\right)\cosh(x)\sinh(x)+\left(a^2b-b^3\right)\sinh(x)^2\right)\arctan\left(\cosh(x)+\sinh(x)\right)\right)}{a^4-a^2b^2+\left(a^4-a^2b^2\right)\cosh(x)^2+2\left(a^4-a^2b^2\right)\cosh(x)\sinh(x)+\left(a^4-a^2b^2\right)\sinh(x)^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [-(2*a^3 - 2*a*b^2 - (b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2
 + b^2)*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x)
+ 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x
) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x)
 + a)*sinh(x) + b)) + 2*(a^2*b - b^3 + (a^2*b - b^3)*cosh(x)^2 + 2*(a^2*b -
 b^3)*cosh(x)*sinh(x) + (a^2*b - b^3)*sinh(x)^2)*arctan(cosh(x) + sinh(x)))
/(a^4 - a^2*b^2 + (a^4 - a^2*b^2)*cosh(x)^2 + 2*(a^4 - a^2*b^2)*cosh(x)*sin
h(x) + (a^4 - a^2*b^2)*sinh(x)^2), -2*(a^3 - a*b^2 + (b^2*cosh(x)^2 + 2*b^2
*cosh(x)*sinh(x) + b^2*sinh(x)^2 + b^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2
+ b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) + (a^2*b - b^3 + (a^2*b - b
^3)*cosh(x)^2 + 2*(a^2*b - b^3)*cosh(x)*sinh(x) + (a^2*b - b^3)*sinh(x)^2)*
arctan(cosh(x) + sinh(x)))/(a^4 - a^2*b^2 + (a^4 - a^2*b^2)*cosh(x)^2 + 2*(
a^4 - a^2*b^2)*cosh(x)*sinh(x) + (a^4 - a^2*b^2)*sinh(x)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(x)}{a + b\cosh(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(a+b*cosh(x)),x)
```

```
[Out] Integral(sech(x)**2/(a + b*cosh(x)), x)
```

Giac [A]
time = 0.41, size = 61, normalized size = 0.95

$$\frac{2\, b^2 \arctan\left(\frac{b e^x + a}{\sqrt{-a^2 + b^2}}\right)}{\sqrt{-a^2 + b^2}\, a^2} - \frac{2\, b \arctan\left(e^x\right)}{a^2} - \frac{2}{a(e^{(2\,x)} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*b^2*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*a^2) - 2*b*arc
tan(e^x)/a^2 - 2/(a*(e^(2*x) + 1))
```

Mupad [B]
time = 3.10, size = 294, normalized size = 4.59

$$\frac{b^2\,\ln\left(64\,a\,b^3-64\,a^3\,b+32\,b^3\,\sqrt{a^2-b^2}-128\,a^4\,\mathrm{e}^x-32\,b^4\,\mathrm{e}^x-64\,a^2\,b\,\sqrt{a^2-b^2}-128\,a^3\,\mathrm{e}^x\,\sqrt{a^2-b^2}+160\,a^2\,b^2\,\mathrm{e}^x+96\,a\,b^2\,\mathrm{e}^x\,\sqrt{a^2-b^2}\right)}{a^2\,\sqrt{a^2-b^2}}+\frac{b\,\left(\ln\left(32\,\mathrm{e}^x-32{}\mathrm{i}\right)\,1{}\mathrm{i}-\ln\left(32\,\mathrm{e}^x+32{}\mathrm{i}\right)\,1{}\mathrm{i}\right)}{a^2}-\frac{b^2\,\ln\left(64\,a^3\,b-64\,a\,b^3+32\,b^3\,\sqrt{a^2-b^2}+128\,a^4\,\mathrm{e}^x+32\,b^4\,\mathrm{e}^x-64\,a^2\,b\,\sqrt{a^2-b^2}-128\,a^3\,\mathrm{e}^x\,\sqrt{a^2-b^2}-160\,a^2\,b^2\,\mathrm{e}^x+96\,a\,b^2\,\mathrm{e}^x\,\sqrt{a^2-b^2}\right)}{a^2\,\sqrt{a^2-b^2}}-\frac{2}{a+a\,\mathrm{e}^{2\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2*(a + b*cosh(x))),x)
```

```
[Out] (b*(log(32*exp(x) - 32i)*1i - log(32*exp(x) + 32i)*1i))/a^2 - 2/(a + a*exp(
2*x)) - (b^2*log(64*a^3*b - 64*a*b^3 + 32*b^3*(a^2 - b^2)^(1/2) + 128*a^4*e
xp(x) + 32*b^4*exp(x) - 64*a^2*b*(a^2 - b^2)^(1/2) - 128*a^3*exp(x)*(a^2 -
b^2)^(1/2) - 160*a^2*b^2*exp(x) + 96*a*b^2*exp(x)*(a^2 - b^2)^(1/2)))/(a^2*
(a^2 - b^2)^(1/2)) + (b^2*log(64*a*b^3 - 64*a^3*b + 32*b^3*(a^2 - b^2)^(1/2
) - 128*a^4*exp(x) - 32*b^4*exp(x) - 64*a^2*b*(a^2 - b^2)^(1/2) - 128*a^3*e
xp(x)*(a^2 - b^2)^(1/2) + 160*a^2*b^2*exp(x) + 96*a*b^2*exp(x)*(a^2 - b^2)^
(1/2)))/(a^2*(a^2 - b^2)^(1/2))
```

3.60 $\int \frac{\operatorname{sech}^3(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=87

$$\frac{(a^2+2b^2)\operatorname{ArcTan}(\sinh(x))}{2a^3}-\frac{2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\,\sqrt{a+b}}-\frac{b\tanh(x)}{a^2}+\frac{\operatorname{sech}(x)\tanh(x)}{2a}$$

```
[Out] 1/2*(a^2+2*b^2)*arctan(sinh(x))/a^3-2*b^3*arctanh((a-b)^(1/2)*tanh(1/2*x)/(
a+b)^(1/2))/a^3/(a-b)^(1/2)/(a+b)^(1/2)-b*tanh(x)/a^2+1/2*sech(x)*tanh(x)/a
```

Rubi [A]
time = 0.21, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2881, 3134, 3080, 3855, 2738, 214}

$$-\frac{2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\,\sqrt{a+b}}-\frac{b\tanh(x)}{a^2}+\frac{(a^2+2b^2)\operatorname{ArcTan}(\sinh(x))}{2a^3}+\frac{\tanh(x)\operatorname{sech}(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^3/(a + b*Cosh[x]),x]
```

```
[Out] ((a^2 + 2*b^2)*ArcTan[Sinh[x]])/(2*a^3) - (2*b^3*ArcTanh[(Sqrt[a - b]*Tanh[
x/2])/Sqrt[a + b]])/(a^3*Sqrt[a - b]*Sqrt[a + b]) - (b*Tanh[x])/a^2 + (Sech
[x]*Tanh[x])/(2*a)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
```

```
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[
n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(x)}{a+b\cosh(x)}\,dx = \frac{\operatorname{sech}(x)\tanh(x)}{2a} + \frac{\int \frac{(-2b+a\cosh(x)+b\cosh^2(x))\operatorname{sech}^2(x)}{a+b\cosh(x)}\,dx}{2a}$$

$$= -\frac{b\tanh(x)}{a^2} + \frac{\operatorname{sech}(x)\tanh(x)}{2a} + \frac{\int \frac{(a^2+2b^2+ab\cosh(x))\operatorname{sech}(x)}{a+b\cosh(x)}\,dx}{2a^2}$$

$$= -\frac{b\tanh(x)}{a^2} + \frac{\operatorname{sech}(x)\tanh(x)}{2a} - \frac{b^3\int \frac{1}{a+b\cosh(x)}\,dx}{a^3} + \frac{(a^2+2b^2)\int \operatorname{sech}(x)\,dx}{2a^3}$$

$$= \frac{(a^2+2b^2)\tan^{-1}(\sinh(x))}{2a^3} - \frac{b\tanh(x)}{a^2} + \frac{\operatorname{sech}(x)\tanh(x)}{2a} - \frac{(2b^3)\operatorname{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\right.}{a^3}$$

$$= \frac{(a^2+2b^2)\tan^{-1}(\sinh(x))}{2a^3} - \frac{2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\sqrt{a+b}} - \frac{b\tanh(x)}{a^2} + \frac{\operatorname{sech}(x)\tan}{2a}$$

Mathematica [A]
time = 0.15, size = 82, normalized size = 0.94

$$\frac{2(a^2+2b^2)\operatorname{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right) + \frac{4b^3\operatorname{ArcTan}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} + a(-2b+a\operatorname{sech}(x))\tanh(x)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^3/(a + b*Cosh[x]),x]
```

```
[Out] (2*(a^2 + 2*b^2)*ArcTan[Tanh[x/2]] + (4*b^3*ArcTan[((a - b)*Tanh[x/2])/Sqrt
[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + a*(-2*b + a*Sech[x])*Tanh[x])/(2*a^3)
```

Maple [A]
time = 0.76, size = 109, normalized size = 1.25

method	result
default	$-\frac{2b^3\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^3\sqrt{(a+b)(a-b)}} + \frac{\frac{2\left(\left(-\frac{1}{2}a^2-ab\right)\left(\tanh^3\left(\frac{x}{2}\right)\right)+\left(\frac{1}{2}a^2-ab\right)\tanh\left(\frac{x}{2}\right)\right)}{\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^2}+(a^2+2b^2)\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a^3}$
risch	$\frac{a\,e^{3x}+2b\,e^{2x}-a\,e^{x}+2b}{(1+e^{2x})^2a^2} + \frac{i\ln(e^x+i)}{2a} + \frac{i\ln(e^x+i)b^2}{a^3} - \frac{i\ln(e^x-i)}{2a} - \frac{i\ln(e^x-i)b^2}{a^3} + \frac{b^3\ln\left(e^x+\frac{a\sqrt{a^2-b^2}+a^2-b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,a^3} - \frac{b^3\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^3/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*b^3/a^3/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2
))+2/a^3*(((-1/2*a^2-a*b)*tanh(1/2*x)^3+(1/2*a^2-a*b)*tanh(1/2*x))/(tanh(1/
2*x)^2+1)^2+1/2*(a^2+2*b^2)*arctan(tanh(1/2*x)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 651 vs. $2(73) = 146$.
time = 0.45, size = 1370, normalized size = 15.75

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [(2*a^3*b - 2*a*b^3 + (a^4 - a^2*b^2)*cosh(x)^3 + (a^4 - a^2*b^2)*sinh(x)^3
 + 2*(a^3*b - a*b^3)*cosh(x)^2 + (2*a^3*b - 2*a*b^3 + 3*(a^4 - a^2*b^2)*cos
h(x))*sinh(x)^2 + (b^3*cosh(x)^4 + 4*b^3*cosh(x)*sinh(x)^3 + b^3*sinh(x)^4
+ 2*b^3*cosh(x)^2 + b^3 + 2*(3*b^3*cosh(x)^2 + b^3)*sinh(x)^2 + 4*(b^3*cosh
(x)^3 + b^3*cosh(x))*sinh(x))*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh
(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqr
t(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*
cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)) + ((a^4 + a^2*b^2 - 2*b^4)*cosh(x
)^4 + 4*(a^4 + a^2*b^2 - 2*b^4)*cosh(x)*sinh(x)^3 + (a^4 + a^2*b^2 - 2*b^4)
*sinh(x)^4 + a^4 + a^2*b^2 - 2*b^4 + 2*(a^4 + a^2*b^2 - 2*b^4)*cosh(x)^2 +
2*(a^4 + a^2*b^2 - 2*b^4 + 3*(a^4 + a^2*b^2 - 2*b^4)*cosh(x)^2)*sinh(x)^2 +
 4*((a^4 + a^2*b^2 - 2*b^4)*cosh(x)^3 + (a^4 + a^2*b^2 - 2*b^4)*cosh(x))*si
nh(x))*arctan(cosh(x) + sinh(x)) - (a^4 - a^2*b^2)*cosh(x) - (a^4 - a^2*b^2
 - 3*(a^4 - a^2*b^2)*cosh(x)^2 - 4*(a^3*b - a*b^3)*cosh(x))*sinh(x))/(a^5 -
 a^3*b^2 + (a^5 - a^3*b^2)*cosh(x)^4 + 4*(a^5 - a^3*b^2)*cosh(x)*sinh(x)^3
+ (a^5 - a^3*b^2)*sinh(x)^4 + 2*(a^5 - a^3*b^2)*cosh(x)^2 + 2*(a^5 - a^3*b^
2 + 3*(a^5 - a^3*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^5 - a^3*b^2)*cosh(x)^3 +
 (a^5 - a^3*b^2)*cosh(x))*sinh(x)), (2*a^3*b - 2*a*b^3 + (a^4 - a^2*b^2)*co
sh(x)^3 + (a^4 - a^2*b^2)*sinh(x)^3 + 2*(a^3*b - a*b^3)*cosh(x)^2 + (2*a^3*
b - 2*a*b^3 + 3*(a^4 - a^2*b^2)*cosh(x))*sinh(x)^2 + 2*(b^3*cosh(x)^4 + 4*b
```

```
^3*cosh(x)*sinh(x)^3 + b^3*sinh(x)^4 + 2*b^3*cosh(x)^2 + b^3 + 2*(3*b^3*cos
h(x)^2 + b^3)*sinh(x)^2 + 4*(b^3*cosh(x)^3 + b^3*cosh(x))*sinh(x))*sqrt(-a^
2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2))
+ ((a^4 + a^2*b^2 - 2*b^4)*cosh(x)^4 + 4*(a^4 + a^2*b^2 - 2*b^4)*cosh(x)*si
nh(x)^3 + (a^4 + a^2*b^2 - 2*b^4)*sinh(x)^4 + a^4 + a^2*b^2 - 2*b^4 + 2*(a^
4 + a^2*b^2 - 2*b^4)*cosh(x)^2 + 2*(a^4 + a^2*b^2 - 2*b^4 + 3*(a^4 + a^2*b^
2 - 2*b^4)*cosh(x)^2)*sinh(x)^2 + 4*((a^4 + a^2*b^2 - 2*b^4)*cosh(x)^3 + (a
^4 + a^2*b^2 - 2*b^4)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) - (a^4 -
a^2*b^2)*cosh(x) - (a^4 - a^2*b^2 - 3*(a^4 - a^2*b^2)*cosh(x)^2 - 4*(a^3*b
- a*b^3)*cosh(x))*sinh(x))/(a^5 - a^3*b^2 + (a^5 - a^3*b^2)*cosh(x)^4 + 4*(
a^5 - a^3*b^2)*cosh(x)*sinh(x)^3 + (a^5 - a^3*b^2)*sinh(x)^4 + 2*(a^5 - a^3
*b^2)*cosh(x)^2 + 2*(a^5 - a^3*b^2 + 3*(a^5 - a^3*b^2)*cosh(x)^2)*sinh(x)^2
 + 4*((a^5 - a^3*b^2)*cosh(x)^3 + (a^5 - a^3*b^2)*cosh(x))*sinh(x))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^3(x)}{a + b\cosh(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**3/(a+b*cosh(x)),x)
```

```
[Out] Integral(sech(x)**3/(a + b*cosh(x)), x)
```

Giac [A]
time = 0.41, size = 89, normalized size = 1.02

$$-\frac{2\,b^3\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,a^3}+\frac{(a^2+2\,b^2)\arctan(e^x)}{a^3}+\frac{ae^{(3\,x)}+2\,be^{(2\,x)}-ae^x+2\,b}{a^2(e^{(2\,x)}+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] -2*b^3*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*a^3) + (a^2 +
 2*b^2)*arctan(e^x)/a^3 + (a*e^(3*x) + 2*b*e^(2*x) - a*e^x + 2*b)/(a^2*(e^(
2*x) + 1)^2)
```

Mupad [B]
time = 4.13, size = 476, normalized size = 5.47

$$\frac{e^x}{a+a\,e^{2\,x}}-\frac{2\,e^x}{a+2\,a\,e^{2\,x}+a\,e^{4\,x}}+\frac{2\,b}{a^2\,e^{2\,x}+a^2}-\frac{\ln(1+e^x\,1i)\,1i-\ln(e^x+1i)\,1i}{2\,a}-\frac{b^2\,(\ln(1+e^x\,1i)\,1i-\ln(e^x+1i)\,1i)}{a^3}-\frac{b^3\,\ln\left(16\,a^5\,b-48\,a\,b^5-24\,b^5\sqrt{a^2-b^2}+32\,a^3\,b^3+32\,a^6\,e^x+24\,b^6\,e^x+16\,a^4\,b\sqrt{a^2-b^2}+40\,a^2\,b^3\sqrt{a^2-b^2}+32\,a^5\,e^x\sqrt{a^2-b^2}-112\,a^2\,b^4\,e^x+56\,a^4\,b^2\,e^x+72\,a^3\,b^2\,e^x\sqrt{a^2-b^2}-72\,a\,b^4\,e^x\sqrt{a^2-b^2}\right)}{a^3\sqrt{a^2-b^2}}+\frac{b^3\,\ln\left(16\,a^5\,b-48\,a\,b^5+24\,b^5\sqrt{a^2-b^2}+32\,a^3\,b^3+32\,a^6\,e^x+24\,b^6\,e^x-16\,a^4\,b\sqrt{a^2-b^2}-40\,a^2\,b^3\sqrt{a^2-b^2}-32\,a^5\,e^x\sqrt{a^2-b^2}-112\,a^2\,b^4\,e^x+56\,a^4\,b^2\,e^x-72\,a^3\,b^2\,e^x\sqrt{a^2-b^2}+72\,a\,b^4\,e^x\sqrt{a^2-b^2}\right)}{a^3\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^3*(a + b*cosh(x))),x)
```

```
[Out] exp(x)/(a + a*exp(2*x)) - (2*exp(x))/(a + 2*a*exp(2*x) + a*exp(4*x)) + (2*b
)/(a^2*exp(2*x) + a^2) - (log(exp(x)*1i + 1)*1i - log(exp(x) + 1i)*1i)/(2*a
) - (b^2*(log(exp(x)*1i + 1)*1i - log(exp(x) + 1i)*1i))/a^3 - (b^3*log(16*a
^5*b - 48*a*b^5 - 24*b^5*(a^2 - b^2)^(1/2) + 32*a^3*b^3 + 32*a^6*exp(x) + 2
4*b^6*exp(x) + 16*a^4*b*(a^2 - b^2)^(1/2) + 40*a^2*b^3*(a^2 - b^2)^(1/2) +
32*a^5*exp(x)*(a^2 - b^2)^(1/2) - 112*a^2*b^4*exp(x) + 56*a^4*b^2*exp(x) +
72*a^3*b^2*exp(x)*(a^2 - b^2)^(1/2) - 72*a*b^4*exp(x)*(a^2 - b^2)^(1/2)))/(
a^3*(a^2 - b^2)^(1/2)) + (b^3*log(16*a^5*b - 48*a*b^5 + 24*b^5*(a^2 - b^2)^
(1/2) + 32*a^3*b^3 + 32*a^6*exp(x) + 24*b^6*exp(x) - 16*a^4*b*(a^2 - b^2)^(
1/2) - 40*a^2*b^3*(a^2 - b^2)^(1/2) - 32*a^5*exp(x)*(a^2 - b^2)^(1/2) - 112
*a^2*b^4*exp(x) + 56*a^4*b^2*exp(x) - 72*a^3*b^2*exp(x)*(a^2 - b^2)^(1/2) +
 72*a*b^4*exp(x)*(a^2 - b^2)^(1/2)))/(a^3*(a^2 - b^2)^(1/2))
```

3.61 $\int \frac{\mathbf{sech}^4(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=114

$$-\frac{b(a^2+2b^2)\,\mathrm{ArcTan}(\sinh(x))}{2a^4}+\frac{2b^4\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\,\sqrt{a+b}}+\frac{(2a^2+3b^2)\tanh(x)}{3a^3}-\frac{b\mathrm{sech}(x)\tanh(x)}{2a^2}+\frac{\mathrm{sec}}{}$$

```
[Out] -1/2*b*(a^2+2*b^2)*arctan(sinh(x))/a^4+2*b^4*arctanh((a-b)^(1/2)*tanh(1/2*x
)/(a+b)^(1/2))/a^4/(a-b)^(1/2)/(a+b)^(1/2)+1/3*(2*a^2+3*b^2)*tanh(x)/a^3-1/
2*b*sech(x)*tanh(x)/a^2+1/3*sech(x)^2*tanh(x)/a
```

Rubi [A]
time = 0.33, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2881, 3134, 3080, 3855, 2738, 214}

$$\frac{2b^4\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\,\sqrt{a+b}}-\frac{b\tanh(x)\mathrm{sech}(x)}{2a^2}-\frac{b(a^2+2b^2)\,\mathrm{ArcTan}(\sinh(x))}{2a^4}+\frac{(2a^2+3b^2)\tanh(x)}{3a^3}+\frac{\tanh(x)\mathrm{sech}^2(x)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^4/(a + b*Cosh[x]),x]
```

```
[Out] -1/2*(b*(a^2 + 2*b^2)*ArcTan[Sinh[x]])/a^4 + (2*b^4*ArcTanh[(Sqrt[a - b]*Ta
nh[x/2])/Sqrt[a + b]])/(a^4*Sqrt[a - b]*Sqrt[a + b]) + ((2*a^2 + 3*b^2)*Tan
h[x])/(3*a^3) - (b*Sech[x]*Tanh[x])/(2*a^2) + (Sech[x]^2*Tanh[x])/(3*a)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
```

```
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[
n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\operatorname{sech}^4(x)}{a+b\cosh(x)}\,dx = \frac{\operatorname{sech}^2(x)\tanh(x)}{3a} + \frac{\int \frac{(-3b+2a\cosh(x)+2b\cosh^2(x))\operatorname{sech}^3(x)}{a+b\cosh(x)}\,dx}{3a}$$
$$= -\frac{b\operatorname{sech}(x)\tanh(x)}{2a^2} + \frac{\operatorname{sech}^2(x)\tanh(x)}{3a} + \frac{\int \frac{(2(2a^2+3b^2)+ab\cosh(x)-3b^2\cosh^2(x))\operatorname{sech}^2(x)}{a+b\cosh(x)}\,dx}{6a^2}$$
$$= \frac{(2a^2+3b^2)\tanh(x)}{3a^3} - \frac{b\operatorname{sech}(x)\tanh(x)}{2a^2} + \frac{\operatorname{sech}^2(x)\tanh(x)}{3a} + \frac{\int \frac{(-3b(a^2+2b^2)-3ab^2\cosh(x}{a+b\cosh(x)}}{6a^3}$$
$$= \frac{(2a^2+3b^2)\tanh(x)}{3a^3} - \frac{b\operatorname{sech}(x)\tanh(x)}{2a^2} + \frac{\operatorname{sech}^2(x)\tanh(x)}{3a} + \frac{b^4\int \frac{1}{a+b\cosh(x)}\,dx}{a^4} - \frac{(b(}{}$$
$$= -\frac{b(a^2+2b^2)\tan^{-1}(\sinh(x))}{2a^4} + \frac{(2a^2+3b^2)\tanh(x)}{3a^3} - \frac{b\operatorname{sech}(x)\tanh(x)}{2a^2} + \frac{\operatorname{sech}^2(x)\tan}{3a}$$
$$= -\frac{b(a^2+2b^2)\tan^{-1}(\sinh(x))}{2a^4} + \frac{2b^4\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\sqrt{a+b}} + \frac{(2a^2+3b^2)\tanh(x)}{3a^3} -$$

Mathematica [A]
time = 0.31, size = 101, normalized size = 0.89

$$\frac{-6b(a^2+2b^2)\operatorname{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right) - \frac{12b^4\operatorname{ArcTan}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} + a\left(4a^2+6b^2-3ab\operatorname{sech}(x)+2a^2\operatorname{sech}^2(x)\right)\tanh(x)}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^4/(a + b*Cosh[x]),x]
```

```
[Out] (-6*b*(a^2 + 2*b^2)*ArcTan[Tanh[x/2]] - (12*b^4*ArcTan[((a - b)*Tanh[x/2])/
Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + a*(4*a^2 + 6*b^2 - 3*a*b*Sech[x] + 2*
a^2*Sech[x]^2)*Tanh[x])/(6*a^4)
```

Maple [A]
time = 0.80, size = 145, normalized size = 1.27

method	result
default	$\frac{2b^4\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^4\sqrt{(a+b)(a-b)}} - \frac{2\left(\frac{\left(-a^3-\frac{1}{2}a^2b-ab^2\right)\left(\tanh^5\left(\frac{x}{2}\right)\right)+\left(-\frac{2}{3}a^3-2ab^2\right)\left(\tanh^3\left(\frac{x}{2}\right)\right)+\left(-a^3-ab^2+\frac{1}{2}a^2b\right)\tanh\left(\frac{x}{2}\right)}{\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^3}\right.}{a^4}$
risch	$-\frac{3ab\,e^{5x}+6b^2e^{4x}+12a^2e^{2x}+12b^2e^{2x}-3b\,e^{x}a+4a^2+6b^2}{3a^3(1+e^{2x})^3} + \frac{ib\ln(e^x-i)}{2a^2} + \frac{ib^3\ln(e^x-i)}{a^4} - \frac{ib\ln(e^x+i)}{2a^2} - \frac{ib^3\ln(e^x+i)}{a^4} + \frac{b^4\ln\left(e\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^4/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^4/a^4/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2)
)-2/a^4*(((-a^3-1/2*a^2*b-a*b^2)*tanh(1/2*x)^5+(-2/3*a^3-2*a*b^2)*tanh(1/2*
x)^3+(-a^3-a*b^2+1/2*a^2*b)*tanh(1/2*x))/(tanh(1/2*x)^2+1)^3+1/2*b*(a^2+2*b
^2)*arctan(tanh(1/2*x)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1207 vs. 2(96) = 192.
time = 0.45, size = 2483, normalized size = 21.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [-1/3*(3*(a^4*b - a^2*b^3)*cosh(x)^5 + 3*(a^4*b - a^2*b^3)*sinh(x)^5 + 4*a^
5 + 2*a^3*b^2 - 6*a*b^4 + 6*(a^3*b^2 - a*b^4)*cosh(x)^4 + 3*(2*a^3*b^2 - 2*
a*b^4 + 5*(a^4*b - a^2*b^3)*cosh(x))*sinh(x)^4 + 6*(5*(a^4*b - a^2*b^3)*cos
h(x)^2 + 4*(a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^3 + 12*(a^5 - a*b^4)*cosh(x)^
2 + 6*(2*a^5 - 2*a*b^4 + 5*(a^4*b - a^2*b^3)*cosh(x)^3 + 6*(a^3*b^2 - a*b^4
)*cosh(x)^2)*sinh(x)^2 - 3*(b^4*cosh(x)^6 + 6*b^4*cosh(x)*sinh(x)^5 + b^4*s
inh(x)^6 + 3*b^4*cosh(x)^4 + 3*b^4*cosh(x)^2 + 3*(5*b^4*cosh(x)^2 + b^4)*si
nh(x)^4 + b^4 + 4*(5*b^4*cosh(x)^3 + 3*b^4*cosh(x))*sinh(x)^3 + 3*(5*b^4*co
sh(x)^4 + 6*b^4*cosh(x)^2 + b^4)*sinh(x)^2 + 6*(b^4*cosh(x)^5 + 2*b^4*cosh(
x)^3 + b^4*cosh(x))*sinh(x))*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(
x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt
(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*c
osh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)) + 3*((a^4*b + a^2*b^3 - 2*b^5)*cos
```

```
h(x)^6 + 6*(a^4*b + a^2*b^3 - 2*b^5)*cosh(x)*sinh(x)^5 + (a^4*b + a^2*b^3 -
 2*b^5)*sinh(x)^6 + a^4*b + a^2*b^3 - 2*b^5 + 3*(a^4*b + a^2*b^3 - 2*b^5)*c
osh(x)^4 + 3*(a^4*b + a^2*b^3 - 2*b^5 + 5*(a^4*b + a^2*b^3 - 2*b^5)*cosh(x)
^2)*sinh(x)^4 + 4*(5*(a^4*b + a^2*b^3 - 2*b^5)*cosh(x)^3 + 3*(a^4*b + a^2*b
^3 - 2*b^5)*cosh(x))*sinh(x)^3 + 3*(a^4*b + a^2*b^3 - 2*b^5)*cosh(x)^2 + 3*
(a^4*b + a^2*b^3 - 2*b^5 + 5*(a^4*b + a^2*b^3 - 2*b^5)*cosh(x)^4 + 6*(a^4*b
 + a^2*b^3 - 2*b^5)*cosh(x)^2)*sinh(x)^2 + 6*((a^4*b + a^2*b^3 - 2*b^5)*cos
h(x)^5 + 2*(a^4*b + a^2*b^3 - 2*b^5)*cosh(x)^3 + (a^4*b + a^2*b^3 - 2*b^5)*
cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) - 3*(a^4*b - a^2*b^3)*cosh(x) -
 3*(a^4*b - a^2*b^3 - 5*(a^4*b - a^2*b^3)*cosh(x)^4 - 8*(a^3*b^2 - a*b^4)*c
osh(x)^3 - 8*(a^5 - a*b^4)*cosh(x))*sinh(x))/((a^6 - a^4*b^2)*cosh(x)^6 + 6
*(a^6 - a^4*b^2)*cosh(x)*sinh(x)^5 + (a^6 - a^4*b^2)*sinh(x)^6 + a^6 - a^4*
b^2 + 3*(a^6 - a^4*b^2)*cosh(x)^4 + 3*(a^6 - a^4*b^2 + 5*(a^6 - a^4*b^2)*co
sh(x)^2)*sinh(x)^4 + 4*(5*(a^6 - a^4*b^2)*cosh(x)^3 + 3*(a^6 - a^4*b^2)*cos
h(x))*sinh(x)^3 + 3*(a^6 - a^4*b^2)*cosh(x)^2 + 3*(a^6 - a^4*b^2 + 5*(a^6 -
 a^4*b^2)*cosh(x)^4 + 6*(a^6 - a^4*b^2)*cosh(x)^2)*sinh(x)^2 + 6*((a^6 - a^
4*b^2)*cosh(x)^5 + 2*(a^6 - a^4*b^2)*cosh(x)^3 + (a^6 - a^4*b^2)*cosh(x))*s
inh(x)), -1/3*(3*(a^4*b - a^2*b^3)*cosh(x)^5 + 3*(a^4*b - a^2*b^3)*sinh(x)^
5 + 4*a^5 + 2*a^3*b^2 - 6*a*b^4 + 6*(a^3*b^2 - a*b^4)*cosh(x)^4 + 3*(2*a^3*
b^2 - 2*a*b^4 + 5*(a^4*b - a^2*b^3)*cosh(x))*sinh(x)^4 + 6*(5*(a^4*b - a^2*
b^3)*cosh(x)^2 + 4*(a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^3 + 12*(a^5 - a*b^4)*
cosh(x)^2 + 6*(2*a^5 - 2*a*b^4 + 5*(a^4*b - a^2*b^3)*cosh(x)^3 + 6*(a^3*b^2
 - a*b^4)*cosh(x)^2)*sinh(x)^2 + 6*(b^4*cosh(x)^6 + 6*b^4*cosh(x)*sinh(x)^5
 + b^4*sinh(x)^6 + 3*b^4*cosh(x)^4 + 3*b^4*cosh(x)^2 + 3*(5*b^4*cosh(x)^2 +
 b^4)*sinh(x)^4 + b^4 + 4*(5*b^4*cosh(x)^3 + 3*b^4*cosh(x))*sinh(x)^3 + 3*(
5*b^4*cosh(x)^4 + 6*b^4*cosh(x)^2 + b^4)*sinh(x)^2 + 6*(b^4*cosh(x)^5 + 2*b
^4*cosh(x)^3 + b^4*cosh(x))*sinh(x))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b
^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) + 3*((a^4*b + a^2*b^3 - 2*b^5)
*cosh(x)^6 + 6*(a^4*b + a^2*b^3 - 2*b^5)*cosh(x)*sinh(x)^5 + (a^4*b + a^2*b
^3 - 2*b^5)*sinh(x)^6 + a^4*b + a^2*b^3 - 2*b^5 + 3*(a^4*b + a^2*b^3 - 2*b^
5)*cosh(x)^4 + 3*(a^4*b + a^2*b^3 - 2*b^5 + 5*(a^4*b + a^2*b^3 - 2*b^5)*cos
h(x)^2)*sinh(x)^4 + 4*(5*(a^4*b + a^2*b^3 - 2*b^5)*cosh(x)^3 + 3*(a^4*b + a
^2*b^3 - 2*b^5)*cosh(x))*sinh(x)^3 + 3*(a^4*b + a^2*b^3 - 2*b^5)*cosh(x)^2
 + 3*(a^4*b + a^2*b^3 - 2*b^5 + 5*(a^4*b + a^2*b^3 - 2*b^5)*cosh(x)^4 + 6*(a
^4*b + a^2*b^3 - 2*b^5)*cosh(x)^2)*sinh(x)^2 + 6*((a^4*b + a^2*b^3 - 2*b^5)
*cosh(x)^5 + 2*(a^4*b + a^2*b^3 - 2*b^5)*cosh(x)^3 + (a^4*b + a^2*b^3 - 2*b
^5)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) - 3*(a^4*b - a^2*b^3)*cosh(
x) - 3*(a^4*b - a^2*b^3 - 5*(a^4*b - a^2*b^3)*cosh(x)^4 - 8*(a^3*b^2 - a*b^
4)*cosh(x)^3 - 8*(a^5 - a*b^4)*cosh(x))*sinh(x))/((a^6 - a^4*b^2)*cosh(x)^6
 + 6*(a^6 - a^4*b^2)*cosh(x)*sinh(x)^5 + (a^6 - a^4*b^2)*sinh(x)^6 + a^6 -
a^4*b^2 + 3*(a^6 - a^4*b^2)*cosh(x)^4 + 3*(a^6 - a^4*b^2 + 5*(a^6 - a^4*b^2
)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^6 - a^4*b^2)*cosh(x)^3 + 3*(a^6 - a^4*b^2)
*cosh(x))*sinh(x)^3 + 3*(a^6 - a^4*b^2)*cosh(x)^2 + 3*(a^6 - a^4*b^2 + 5*(a
^6 - a^4*b^2)*cosh(x)^4 + 6*(a^6 - a^4*b^2)*cosh(x)^2)*sinh(x)^2 + 6*((a^6
- a^4*b^2)*cosh(x)^5 + 2*(a^6 - a^4*b^2)*cosh(x)^3 + (a^6 - a^4*b^2)*cosh(x
```

```
))*sinh(x))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^4(x)}{a + b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**4/(a+b*cosh(x)),x)
[Out] Integral(sech(x)**4/(a + b*cosh(x)), x)
```

Giac [A]
time = 0.40, size = 123, normalized size = 1.08

$$\frac{2\,b^4\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,a^4} - \frac{(a^2b+2\,b^3)\arctan(e^x)}{a^4} - \frac{3\,abe^{(5\,x)}+6\,b^2e^{(4\,x)}+12\,a^2e^{(2\,x)}+12\,b^2e^{(2\,x)}-3\,abe^x+4\,a^2+6\,b^2}{3\,a^3\left(e^{(2\,x)}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*cosh(x)),x, algorithm="giac")
[Out] 2*b^4*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*a^4) - (a^2*b
+ 2*b^3)*arctan(e^x)/a^4 - 1/3*(3*a*b*e^(5*x) + 6*b^2*e^(4*x) + 12*a^2*e^(2
*x) + 12*b^2*e^(2*x) - 3*a*b*e^x + 4*a^2 + 6*b^2)/(a^3*(e^(2*x) + 1)^3)
```

Mupad [B]
time = 4.53, size = 547, normalized size = 4.80

$$\frac{8}{3\,(a+3\,a\,e^{2\,x}+3\,a\,e^{4\,x}+a\,e^{6\,x})} - \frac{4}{a+2\,a\,e^{2\,x}+a\,e^{4\,x}} - \frac{2\,b^2}{a^3\,e^{2\,x}+a^3} + \frac{b^3\,(\ln(e^x-i)\,1i-\ln(e^x+1i)\,1i)}{a^4} - \frac{b\,e^x}{a^2\,e^{2\,x}+a^2} + \frac{2\,b\,e^x}{2\,a^2\,e^{2\,x}+a^2\,e^{4\,x}+a^2} + \frac{b\,(\ln(e^x-i)\,1i-\ln(e^x+1i)\,1i)}{2\,a^2} + \frac{b^4\,\ln\left(32\,a^3\,b^4-24\,b^6\,\sqrt{a^2-b^2}-48\,a\,b^6+16\,a^5\,b^2+24\,b^7\,e^x+32\,a^6\,b\,e^x+40\,a^2\,b^4\,\sqrt{a^2-b^2}+16\,a^4\,b^2\,\sqrt{a^2-b^2}-112\,a^2\,b^5\,e^x+56\,a^4\,b^3\,e^x+72\,a^3\,b^3\,e^x\,\sqrt{a^2-b^2}-72\,a\,b^5\,e^x\,\sqrt{a^2-b^2}+32\,a^5\,b\,e^x\,\sqrt{a^2-b^2}\right)}{a^4\,\sqrt{a^2-b^2}} - \frac{b^4\,\ln\left(24\,b^6\,\sqrt{a^2-b^2}-48\,a\,b^6+32\,a^3\,b^4+16\,a^5\,b^2+24\,b^7\,e^x+32\,a^6\,b\,e^x-40\,a^2\,b^4\,\sqrt{a^2-b^2}-16\,a^4\,b^2\,\sqrt{a^2-b^2}-112\,a^2\,b^5\,e^x+56\,a^4\,b^3\,e^x-72\,a^3\,b^3\,e^x\,\sqrt{a^2-b^2}+72\,a\,b^5\,e^x\,\sqrt{a^2-b^2}-32\,a^5\,b\,e^x\,\sqrt{a^2-b^2}\right)}{a^4\,\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^4*(a + b*cosh(x))),x)
[Out] 8/(3*(a + 3*a*exp(2*x) + 3*a*exp(4*x) + a*exp(6*x))) - 4/(a + 2*a*exp(2*x)
+ a*exp(4*x)) - (2*b^2)/(a^3*exp(2*x) + a^3) + (b^3*(log(exp(x) - 1i)*1i -
log(exp(x) + 1i)*1i))/a^4 - (b*exp(x))/(a^2*exp(2*x) + a^2) + (2*b*exp(x))/
(2*a^2*exp(2*x) + a^2*exp(4*x) + a^2) + (b*(log(exp(x) - 1i)*1i - log(exp(x
) + 1i)*1i))/(2*a^2) + (b^4*log(32*a^3*b^4 - 24*b^6*(a^2 - b^2)^(1/2) - 48*
a*b^6 + 16*a^5*b^2 + 24*b^7*exp(x) + 32*a^6*b*exp(x) + 40*a^2*b^4*(a^2 - b^
2)^(1/2) + 16*a^4*b^2*(a^2 - b^2)^(1/2) - 112*a^2*b^5*exp(x) + 56*a^4*b^3*e
xp(x) + 72*a^3*b^3*exp(x)*(a^2 - b^2)^(1/2) - 72*a*b^5*exp(x)*(a^2 - b^2)^(
1/2) + 32*a^5*b*exp(x)*(a^2 - b^2)^(1/2)))/(a^4*(a^2 - b^2)^(1/2)) - (b^4*l
og(24*b^6*(a^2 - b^2)^(1/2) - 48*a*b^6 + 32*a^3*b^4 + 16*a^5*b^2 + 24*b^7*e
xp(x) + 32*a^6*b*exp(x) - 40*a^2*b^4*(a^2 - b^2)^(1/2) - 16*a^4*b^2*(a^2 -
b^2)^(1/2) - 112*a^2*b^5*exp(x) + 56*a^4*b^3*exp(x) - 72*a^3*b^3*exp(x)*(a^
2 - b^2)^(1/2) + 72*a*b^5*exp(x)*(a^2 - b^2)^(1/2) - 32*a^5*b*exp(x)*(a^2 -
 b^2)^(1/2)))/(a^4*(a^2 - b^2)^(1/2))
```

3.62 $\int (a+b\cosh(c+dx))^5\,dx$

Optimal. Leaf size=183

$$\frac{1}{8}a\left(8a^4+40a^2b^2+15b^4\right)x+\frac{b(107a^4+192a^2b^2+16b^4)\sinh(c+dx)}{30d}+\frac{7ab^2(22a^2+23b^2)\cosh(c+dx)\sinh(c$$

```
[Out] 1/8*a*(8*a^4+40*a^2*b^2+15*b^4)*x+1/30*b*(107*a^4+192*a^2*b^2+16*b^4)*sinh(
d*x+c)/d+7/120*a*b^2*(22*a^2+23*b^2)*cosh(d*x+c)*sinh(d*x+c)/d+1/60*b*(47*a
^2+16*b^2)*(a+b*cosh(d*x+c))^2*sinh(d*x+c)/d+9/20*a*b*(a+b*cosh(d*x+c))^3*s
inh(d*x+c)/d+1/5*b*(a+b*cosh(d*x+c))^4*sinh(d*x+c)/d
```

Rubi [A]
time = 0.18, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2735, 2832, 2813}

$$\frac{b(47a^2+16b^2)\sinh(c+dx)(a+b\cosh(c+dx))^2}{60d}+\frac{7ab^2(22a^2+23b^2)\sinh(c+dx)\cosh(c+dx)}{120d}+\frac{b(107a^4+192a^2b^2+16b^4)\sinh(c+dx)}{30d}+\frac{1}{8}ax\left(8a^4+40a^2b^2+15b^4\right)+\frac{b\sinh(c+dx)(a+b\cosh(c+dx))^4}{5d}+\frac{9ab\sinh(c+dx)(a+b\cosh(c+dx))^3}{20d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[c + d*x])^5,x]
```

```
[Out] (a*(8*a^4 + 40*a^2*b^2 + 15*b^4)*x)/8 + (b*(107*a^4 + 192*a^2*b^2 + 16*b^4)
*Sinh[c + d*x])/(30*d) + (7*a*b^2*(22*a^2 + 23*b^2)*Cosh[c + d*x]*Sinh[c +
d*x])/(120*d) + (b*(47*a^2 + 16*b^2)*(a + b*Cosh[c + d*x])^2*Sinh[c + d*x])
/(60*d) + (9*a*b*(a + b*Cosh[c + d*x])^3*Sinh[c + d*x])/(20*d) + (b*(a + b*
Cosh[c + d*x])^4*Sinh[c + d*x])/(5*d)
```

Rule 2735

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[1/n, Int[(a + b*S
in[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x],
 x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
```

```
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\cosh(c+dx))^5\,dx = \frac{b(a+b\cosh(c+dx))^4\sinh(c+dx)}{5d}+\frac{1}{5}\int (a+b\cosh(c+dx))^3\left(5a^2+4b^2+\right.$$

$$= \frac{9ab(a+b\cosh(c+dx))^3\sinh(c+dx)}{20d}+\frac{b(a+b\cosh(c+dx))^4\sinh(c+dx)}{5d}$$

$$= \frac{b(47a^2+16b^2)(a+b\cosh(c+dx))^2\sinh(c+dx)}{60d}+\frac{9ab(a+b\cosh(c+dx))^3}{20d}$$

$$= \frac{1}{8}a\left(8a^4+40a^2b^2+15b^4\right)x+\frac{b(107a^4+192a^2b^2+16b^4)\sinh(c+dx)}{30d}+\frac{7ab^2(}{}$$

Mathematica [A]
time = 0.27, size = 133, normalized size = 0.73

$$\frac{60a(8a^4+40a^2b^2+15b^4)(c+dx)+300b(8a^4+12a^2b^2+b^4)\sinh(c+dx)+600ab^2(2a^2+b^2)\sinh(2(c+dx))+50b^3(8a^2+b^2)\sinh(3(c+dx))+75ab^4\sinh(4(c+dx))+6b^5\sinh(5(c+dx))}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[c + d*x])^5,x]
```

```
[Out] (60*a*(8*a^4 + 40*a^2*b^2 + 15*b^4)*(c + d*x) + 300*b*(8*a^4 + 12*a^2*b^2 +
 b^4)*Sinh[c + d*x] + 600*a*b^2*(2*a^2 + b^2)*Sinh[2*(c + d*x)] + 50*b^3*(8
*a^2 + b^2)*Sinh[3*(c + d*x)] + 75*a*b^4*Sinh[4*(c + d*x)] + 6*b^5*Sinh[5*(
c + d*x)])/(480*d)
```

Maple [A]
time = 1.88, size = 145, normalized size = 0.79

method	result
default	$a^5x+\frac{\left(\frac{5}{16}b^5+\frac{5}{2}a^2b^3\right)\sinh(3dx+3c)}{3d}+\frac{\left(\frac{5}{2}ab^4+5a^3b^2\right)\sinh(2dx+2c)}{2d}+\frac{\left(\frac{5}{8}b^5+\frac{15}{2}a^2b^3+5a^4b\right)\sinh(dx+c)}{d}+\frac{15xab^4}{8}+5x\,a$
risch	$a^5x+5x\,a^3b^2+\frac{15xab^4}{8}+\frac{b^5e^{5dx+5c}}{160d}+\frac{5ab^4e^{4dx+4c}}{64d}+\frac{5b^3e^{3dx+3c}a^2}{12d}+\frac{5b^5e^{3dx+3c}}{96d}+\frac{5a^3b^2e^{2dx+2c}}{4d}+\frac{5ab^4e^{2dx+2}}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] a^5*x+1/3*(5/16*b^5+5/2*a^2*b^3)/d*sinh(3*d*x+3*c)+1/2*(5/2*a*b^4+5*a^3*b^2
)/d*sinh(2*d*x+2*c)+(5/8*b^5+15/2*a^2*b^3+5*a^4*b)/d*sinh(d*x+c)+15/8*x*a*b
^4+5*x*a^3*b^2+1/80*b^5/d*sinh(5*d*x+5*c)+5/32*a*b^4/d*sinh(4*d*x+4*c)
```

Maxima [A]
time = 0.26, size = 273, normalized size = 1.49

$$\frac{5}{64}\,ab^4\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}+\frac{8\,e^{(2\,dx+2\,c)}}{d}-\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)+\frac{5}{4}\,a^3b^2\left(4\,x+\frac{e^{(2\,dx+2\,c)}}{d}-\frac{e^{(-2\,dx-2\,c)}}{d}\right)+a^5x+\frac{1}{480}\,b^5\left(\frac{3\,e^{(5\,dx+5\,c)}}{d}+\frac{25\,e^{(3\,dx+3\,c)}}{d}+\frac{150\,e^{(dx+c)}}{d}-\frac{150\,e^{(-dx-c)}}{d}-\frac{25\,e^{(-3\,dx-3\,c)}}{d}-\frac{3\,e^{(-5\,dx-5\,c)}}{d}\right)+\frac{5}{12}\,a^2b^3\left(\frac{e^{(3\,dx+3\,c)}}{d}+\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}-\frac{e^{(-3\,dx-3\,c)}}{d}\right)+\frac{5\,a^4b\sinh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 5/64*a*b^4*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x -
2*c)/d - e^(-4*d*x - 4*c)/d) + 5/4*a^3*b^2*(4*x + e^(2*d*x + 2*c)/d - e^(-2
*d*x - 2*c)/d) + a^5*x + 1/480*b^5*(3*e^(5*d*x + 5*c)/d + 25*e^(3*d*x + 3*c
)/d + 150*e^(d*x + c)/d - 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d - 3*e^
(-5*d*x - 5*c)/d) + 5/12*a^2*b^3*(e^(3*d*x + 3*c)/d + 9*e^(d*x + c)/d - 9*e
^(-d*x - c)/d - e^(-3*d*x - 3*c)/d) + 5*a^4*b*sinh(d*x + c)/d
```

Fricas [A]
time = 0.36, size = 190, normalized size = 1.04

$$\frac{3\,b^5\sinh\left(dx+c\right)^5+5\left(6\,b^5\cosh\left(dx+c\right)^2+30\,ab^4\cosh\left(dx+c\right)+40\,a^2b^3+5\,b^5\right)\sinh\left(dx+c\right)^3+30\left(8\,a^5+40\,a^3b^2+15\,ab^4\right)dx+15\left(b^5\cosh\left(dx+c\right)^4+10\,ab^4\cosh\left(dx+c\right)^3+80\,a^4b+120\,a^2b^3+10\,b^5+5\left(8\,a^2b^3+b^5\right)\cosh\left(dx+c\right)^2+40\left(2\,a^3b^2+ab^4\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/240*(3*b^5*sinh(d*x + c)^5 + 5*(6*b^5*cosh(d*x + c)^2 + 30*a*b^4*cosh(d*x
 + c) + 40*a^2*b^3 + 5*b^5)*sinh(d*x + c)^3 + 30*(8*a^5 + 40*a^3*b^2 + 15*a
*b^4)*d*x + 15*(b^5*cosh(d*x + c)^4 + 10*a*b^4*cosh(d*x + c)^3 + 80*a^4*b +
 120*a^2*b^3 + 10*b^5 + 5*(8*a^2*b^3 + b^5)*cosh(d*x + c)^2 + 40*(2*a^3*b^2
 + a*b^4)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [A]
time = 0.33, size = 314, normalized size = 1.72

$$\begin{cases} a^5x+\frac{5a^4b\sinh\left(c+dx\right)}{d}-5a^3b^2x\sinh^2\left(c+dx\right)+5a^3b^2x\cosh^2\left(c+dx\right)+\frac{5a^3b^2\sinh\left(c+dx\right)\cosh\left(c+dx\right)}{d}-\frac{20a^2b^3\sinh^3\left(c+dx\right)}{3d}+\frac{10a^2b^3\sinh\left(c+dx\right)\cosh^2\left(c+dx\right)}{d}+\frac{15ab^4x\sinh^4\left(c+dx\right)}{8}-\frac{15ab^4x\sinh^2\left(c+dx\right)\cosh^2\left(c+dx\right)}{4}+\frac{15ab^4x\cosh^4\left(c+dx\right)}{8}-\frac{15ab^4\sinh^3\left(c+dx\right)\cosh\left(c+dx\right)}{8d}+\frac{25ab^4\sinh\left(c+dx\right)\cosh^3\left(c+dx\right)}{8d}+\frac{8b^5\sinh^5\left(c+dx\right)}{15d}-\frac{4b^5\sinh^3\left(c+dx\right)\cosh^2\left(c+dx\right)}{3d}+\frac{b^5\sinh\left(c+dx\right)\cosh^4\left(c+dx\right)}{d} & \text{for}\: d \neq 0 \\ x\left(a+b\cosh\left(c\right)\right)^5 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))**5,x)
```

```
[Out] Piecewise((a**5*x + 5*a**4*b*sinh(c + d*x)/d - 5*a**3*b**2*x*sinh(c + d*x)*
*2 + 5*a**3*b**2*x*cosh(c + d*x)**2 + 5*a**3*b**2*sinh(c + d*x)*cosh(c + d*
x)/d - 20*a**2*b**3*sinh(c + d*x)**3/(3*d) + 10*a**2*b**3*sinh(c + d*x)*cos
h(c + d*x)**2/d + 15*a*b**4*x*sinh(c + d*x)**4/8 - 15*a*b**4*x*sinh(c + d*x
)**2*cosh(c + d*x)**2/4 + 15*a*b**4*x*cosh(c + d*x)**4/8 - 15*a*b**4*sinh(c
 + d*x)**3*cosh(c + d*x)/(8*d) + 25*a*b**4*sinh(c + d*x)*cosh(c + d*x)**3/(
8*d) + 8*b**5*sinh(c + d*x)**5/(15*d) - 4*b**5*sinh(c + d*x)**3*cosh(c + d*
x)**2/(3*d) + b**5*sinh(c + d*x)*cosh(c + d*x)**4/d, Ne(d, 0)), (x*(a + b*c
osh(c))**5, True))
```

Giac [A]
time = 0.40, size = 263, normalized size = 1.44

$$\frac{b^5e^{(5\,dx+5\,c)}}{160\,d}+\frac{5\,ab^4e^{(4\,dx+4\,c)}}{64\,d}-\frac{5\,ab^4e^{(-4\,dx-4\,c)}}{64\,d}-\frac{b^5e^{(-5\,dx-5\,c)}}{160\,d}+\frac{1}{8}\,(8\,a^5+40\,a^3b^2+15\,ab^4)x+\frac{5\,(8\,a^2b^3+b^5)e^{(3\,dx+3\,c)}}{96\,d}+\frac{5\,(2\,a^3b^2+ab^4)e^{(2\,dx+2\,c)}}{8\,d}+\frac{5\,(8\,a^4b+12\,a^2b^3+b^5)e^{(dx+c)}}{16\,d}-\frac{5\,(8\,a^4b+12\,a^2b^3+b^5)e^{(-dx-c)}}{16\,d}-\frac{5\,(2\,a^3b^2+ab^4)e^{(-2\,dx-2\,c)}}{8\,d}-\frac{5\,(8\,a^2b^3+b^5)e^{(-3\,dx-3\,c)}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/160*b^5*e^(5*d*x + 5*c)/d + 5/64*a*b^4*e^(4*d*x + 4*c)/d - 5/64*a*b^4*e^(
-4*d*x - 4*c)/d - 1/160*b^5*e^(-5*d*x - 5*c)/d + 1/8*(8*a^5 + 40*a^3*b^2 +
15*a*b^4)*x + 5/96*(8*a^2*b^3 + b^5)*e^(3*d*x + 3*c)/d + 5/8*(2*a^3*b^2 + a
*b^4)*e^(2*d*x + 2*c)/d + 5/16*(8*a^4*b + 12*a^2*b^3 + b^5)*e^(d*x + c)/d -
 5/16*(8*a^4*b + 12*a^2*b^3 + b^5)*e^(-d*x - c)/d - 5/8*(2*a^3*b^2 + a*b^4)
*e^(-2*d*x - 2*c)/d - 5/96*(8*a^2*b^3 + b^5)*e^(-3*d*x - 3*c)/d
```

Mupad [B]
time = 1.14, size = 160, normalized size = 0.87

$$\frac{75\,b^5\,\sinh(c+d\,x)+\frac{25\,b^5\,\sinh(3\,c+3\,d\,x)}{2}+\frac{3\,b^5\,\sinh(5\,c+5\,d\,x)}{2}+150\,a\,b^4\,\sinh(2\,c+2\,d\,x)+\frac{75\,a\,b^4\,\sinh(4\,c+4\,d\,x)}{4}+900\,a^2\,b^3\,\sinh(c+d\,x)+300\,a^3\,b^2\,\sinh(2\,c+2\,d\,x)+100\,a^2\,b^3\,\sinh(3\,c+3\,d\,x)+600\,a^4\,b\,\sinh(c+d\,x)+120\,a^5\,d\,x+225\,a\,b^4\,d\,x+600\,a^3\,b^2\,d\,x}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(c + d*x))^5,x)
```

```
[Out] (75*b^5*sinh(c + d*x) + (25*b^5*sinh(3*c + 3*d*x))/2 + (3*b^5*sinh(5*c + 5*
d*x))/2 + 150*a*b^4*sinh(2*c + 2*d*x) + (75*a*b^4*sinh(4*c + 4*d*x))/4 + 90
0*a^2*b^3*sinh(c + d*x) + 300*a^3*b^2*sinh(2*c + 2*d*x) + 100*a^2*b^3*sinh(
3*c + 3*d*x) + 600*a^4*b*sinh(c + d*x) + 120*a^5*d*x + 225*a*b^4*d*x + 600*
a^3*b^2*d*x)/(120*d)
```

3.63 $\int (a + b\cosh(c + dx))^4 \, dx$

Optimal. Leaf size=137

$$\frac{1}{8}\left(8a^4 + 24a^2b^2 + 3b^4\right)x + \frac{ab(19a^2 + 16b^2)\sinh(c + dx)}{6d} + \frac{b^2(26a^2 + 9b^2)\cosh(c + dx)\sinh(c + dx)}{24d} + \frac{7ab(a +$$

```
[Out] 1/8*(8*a^4+24*a^2*b^2+3*b^4)*x+1/6*a*b*(19*a^2+16*b^2)*sinh(d*x+c)/d+1/24*b
^2*(26*a^2+9*b^2)*cosh(d*x+c)*sinh(d*x+c)/d+7/12*a*b*(a+b*cosh(d*x+c))^2*si
nh(d*x+c)/d+1/4*b*(a+b*cosh(d*x+c))^3*sinh(d*x+c)/d
```

Rubi [A]
time = 0.11, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2735, 2832, 2813}

$$\frac{ab(19a^2 + 16b^2)\sinh(c + dx)}{6d} + \frac{b^2(26a^2 + 9b^2)\sinh(c + dx)\cosh(c + dx)}{24d} + \frac{1}{8}x\left(8a^4 + 24a^2b^2 + 3b^4\right) + \frac{b\sinh(c + dx)(a + b\cosh(c + dx))^3}{4d} + \frac{7ab\sinh(c + dx)(a + b\cosh(c + dx))^2}{12d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[c + d*x])^4,x]
```

```
[Out] ((8*a^4 + 24*a^2*b^2 + 3*b^4)*x)/8 + (a*b*(19*a^2 + 16*b^2)*Sinh[c + d*x])/
(6*d) + (b^2*(26*a^2 + 9*b^2)*Cosh[c + d*x]*Sinh[c + d*x])/(24*d) + (7*a*b*
(a + b*Cosh[c + d*x])^2*Sinh[c + d*x])/(12*d) + (b*(a + b*Cosh[c + d*x])^3*
Sinh[c + d*x])/(4*d)
```

Rule 2735

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[1/n, Int[(a + b*S
in[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x],
 x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
```

```
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\cosh(c+dx))^4\,dx = \frac{b(a+b\cosh(c+dx))^3\sinh(c+dx)}{4d} + \frac{1}{4}\int (a+b\cosh(c+dx))^2\left(4a^2+3b^2+\right.$$
$$= \frac{7ab(a+b\cosh(c+dx))^2\sinh(c+dx)}{12d} + \frac{b(a+b\cosh(c+dx))^3\sinh(c+dx)}{4d}$$
$$= \frac{1}{8}\left(8a^4+24a^2b^2+3b^4\right)x + \frac{ab(19a^2+16b^2)\sinh(c+dx)}{6d} + \frac{b^2(26a^2+9b^2)\cos}{}$$

Mathematica [A]
time = 0.17, size = 104, normalized size = 0.76

$$\frac{12(8a^4+24a^2b^2+3b^4)(c+dx)+96ab(4a^2+3b^2)\sinh(c+dx)+24b^2(6a^2+b^2)\sinh(2(c+dx))+32ab^3\sinh(3(c+dx))+3b^4\sinh(4(c+dx))}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[c + d*x])^4,x]
```

```
[Out] (12*(8*a^4 + 24*a^2*b^2 + 3*b^4)*(c + d*x) + 96*a*b*(4*a^2 + 3*b^2)*Sinh[c
+ d*x] + 24*b^2*(6*a^2 + b^2)*Sinh[2*(c + d*x)] + 32*a*b^3*Sinh[3*(c + d*x)
] + 3*b^4*Sinh[4*(c + d*x)])/(96*d)
```

Maple [A]
time = 1.25, size = 108, normalized size = 0.79

method	result
default	$x\,a^4+\frac{\left(\frac{1}{2}b^4+3a^2b^2\right)\sinh(2dx+2c)}{2d}+\frac{\left(4a^3b+3a\,b^3\right)\sinh(dx+c)}{d}+\frac{a\,b^3\sinh(3dx+3c)}{3d}+\frac{3x\,b^4}{8}+3x\,a^2b^2+\frac{b^4\sinh(4dx+4}{32d}$
risch	$x\,a^4+\frac{3x\,b^4}{8}+3x\,a^2b^2+\frac{b^4\mathrm{e}^{4dx+4c}}{64d}+\frac{a\,b^3\mathrm{e}^{3dx+3c}}{6d}+\frac{3b^2\mathrm{e}^{2dx+2c}a^2}{4d}+\frac{b^4\mathrm{e}^{2dx+2c}}{8d}+\frac{2a^3b\,\mathrm{e}^{dx+c}}{d}+\frac{3a\,b^3\mathrm{e}^{dx+c}}{2d}-\frac{2a^3}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] x*a^4+1/2*(1/2*b^4+3*a^2*b^2)/d*sinh(2*d*x+2*c)+(4*a^3*b+3*a*b^3)/d*sinh(d*
x+c)+1/3*a*b^3/d*sinh(3*d*x+3*c)+3/8*x*b^4+3*x*a^2*b^2+1/32*b^4/d*sinh(4*d*
x+4*c)
```

Maxima [A]
time = 0.26, size = 183, normalized size = 1.34

$$\frac{1}{64}b^4\left(24x+\frac{e^{(4dx+4c)}}{d}+\frac{8e^{(2dx+2c)}}{d}-\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)+\frac{3}{4}a^2b^2\left(4x+\frac{e^{(2dx+2c)}}{d}-\frac{e^{(-2dx-2c)}}{d}\right)+a^4x+\frac{1}{6}ab^3\left(\frac{e^{(3dx+3c)}}{d}+\frac{9e^{(dx+c)}}{d}-\frac{9e^{(-dx-c)}}{d}-\frac{e^{(-3dx-3c)}}{d}\right)+\frac{4a^3b\sinh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/64*b^4*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + 3/4*a^2*b^2*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d
*x - 2*c)/d) + a^4*x + 1/6*a*b^3*(e^(3*d*x + 3*c)/d + 9*e^(d*x + c)/d - 9*e
^(-d*x - c)/d - e^(-3*d*x - 3*c)/d) + 4*a^3*b*sinh(d*x + c)/d
```

Fricas [A]
time = 0.41, size = 123, normalized size = 0.90

$$\frac{(3\,b^4\cosh(dx+c)+8\,ab^3)\sinh(dx+c)^3+3\,(8\,a^4+24\,a^2b^2+3\,b^4)dx+3\,(b^4\cosh(dx+c)^3+8\,ab^3\cosh(dx+c)^2+32\,a^3b+24\,ab^3+4\,(6\,a^2b^2+b^4)\cosh(dx+c))\sinh(dx+c)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/24*((3*b^4*cosh(d*x + c) + 8*a*b^3)*sinh(d*x + c)^3 + 3*(8*a^4 + 24*a^2*b
^2 + 3*b^4)*d*x + 3*(b^4*cosh(d*x + c)^3 + 8*a*b^3*cosh(d*x + c)^2 + 32*a^3
*b + 24*a*b^3 + 4*(6*a^2*b^2 + b^4)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [A]
time = 0.22, size = 240, normalized size = 1.75

$$\begin{cases} a^4x+\frac{4a^3b\sinh(c+dx)}{d}-3a^2b^2x\sinh^2(c+dx)+3a^2b^2x\cosh^2(c+dx)+\frac{3a^2b^2\sinh(c+dx)\cosh(c+dx)}{d}-\frac{8ab^3\sinh^3(c+dx)}{3d}+\frac{4ab^3\sinh(c+dx)\cosh^2(c+dx)}{d}+\frac{3b^4x\sinh^4(c+dx)}{8}-\frac{3b^4x\sinh^2(c+dx)\cosh^2(c+dx)}{4}+\frac{3b^4x\cosh^4(c+dx)}{8}-\frac{3b^4\sinh^3(c+dx)\cosh(c+dx)}{8d}+\frac{5b^4\sinh(c+dx)\cosh^3(c+dx)}{8d} & \text{for } d\neq 0 \\ x(a+b\cosh(c))^4 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*x + 4*a**3*b*sinh(c + d*x)/d - 3*a**2*b**2*x*sinh(c + d*x)*
*2 + 3*a**2*b**2*x*cosh(c + d*x)**2 + 3*a**2*b**2*sinh(c + d*x)*cosh(c + d*
x)/d - 8*a*b**3*sinh(c + d*x)**3/(3*d) + 4*a*b**3*sinh(c + d*x)*cosh(c + d*
x)**2/d + 3*b**4*x*sinh(c + d*x)**4/8 - 3*b**4*x*sinh(c + d*x)**2*cosh(c +
d*x)**2/4 + 3*b**4*x*cosh(c + d*x)**4/8 - 3*b**4*sinh(c + d*x)**3*cosh(c +
d*x)/(8*d) + 5*b**4*sinh(c + d*x)*cosh(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a
+ b*cosh(c))**4, True))
```

Giac [A]
time = 0.41, size = 196, normalized size = 1.43

$$\frac{b^4e^{(4\,dx+4\,c)}}{64\,d}+\frac{ab^3e^{(3\,dx+3\,c)}}{6\,d}-\frac{ab^3e^{(-3\,dx-3\,c)}}{6\,d}-\frac{b^4e^{(-4\,dx-4\,c)}}{64\,d}+\frac{1}{8}\,(8\,a^4+24\,a^2b^2+3\,b^4)x+\frac{(6\,a^2b^2+b^4)e^{(2\,dx+2\,c)}}{8\,d}+\frac{(4\,a^3b+3\,ab^3)e^{(dx+c)}}{2\,d}-\frac{(4\,a^3b+3\,ab^3)e^{(-dx-c)}}{2\,d}-\frac{(6\,a^2b^2+b^4)e^{(-2\,dx-2\,c)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/64*b^4*e^(4*d*x + 4*c)/d + 1/6*a*b^3*e^(3*d*x + 3*c)/d - 1/6*a*b^3*e^(-3*
d*x - 3*c)/d - 1/64*b^4*e^(-4*d*x - 4*c)/d + 1/8*(8*a^4 + 24*a^2*b^2 + 3*b^
```

```
4)*x + 1/8*(6*a^2*b^2 + b^4)*e^(2*d*x + 2*c)/d + 1/2*(4*a^3*b + 3*a*b^3)*e^
(d*x + c)/d - 1/2*(4*a^3*b + 3*a*b^3)*e^(-d*x - c)/d - 1/8*(6*a^2*b^2 + b^4
)*e^(-2*d*x - 2*c)/d
```

Mupad [B]
time = 0.19, size = 114, normalized size = 0.83

$$\frac{6\,b^4\sinh(2\,c+2\,d\,x)+\frac{3\,b^4\sinh(4\,c+4\,d\,x)}{4}+8\,a\,b^3\sinh(3\,c+3\,d\,x)+36\,a^2\,b^2\sinh(2\,c+2\,d\,x)+72\,a\,b^3\sinh(c+d\,x)+96\,a^3\,b\sinh(c+d\,x)+24\,a^4\,d\,x+9\,b^4\,d\,x+72\,a^2\,b^2\,d\,x}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(c + d*x))^4,x)
```

```
[Out] (6*b^4*sinh(2*c + 2*d*x) + (3*b^4*sinh(4*c + 4*d*x))/4 + 8*a*b^3*sinh(3*c +
 3*d*x) + 36*a^2*b^2*sinh(2*c + 2*d*x) + 72*a*b^3*sinh(c + d*x) + 96*a^3*b*
sinh(c + d*x) + 24*a^4*d*x + 9*b^4*d*x + 72*a^2*b^2*d*x)/(24*d)
```

3.64 $\int (a+b\cosh(c+dx))^3\,dx$

Optimal. Leaf size=90

$$\frac{1}{2}a\left(2a^2+3b^2\right)x+\frac{2b(4a^2+b^2)\sinh(c+dx)}{3d}+\frac{5ab^2\cosh(c+dx)\sinh(c+dx)}{6d}+\frac{b(a+b\cosh(c+dx))^2\sinh(c+$$

[Out] 1/2*a*(2*a^2+3*b^2)*x+2/3*b*(4*a^2+b^2)*sinh(d*x+c)/d+5/6*a*b^2*cosh(d*x+c)*sinh(d*x+c)/d+1/3*b*(a+b*cosh(d*x+c))^2*sinh(d*x+c)/d

Rubi [A]

time = 0.05, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2735, 2813}

$$\frac{2b(4a^2+b^2)\sinh(c+dx)}{3d}+\frac{1}{2}ax\left(2a^2+3b^2\right)+\frac{5ab^2\sinh(c+dx)\cosh(c+dx)}{6d}+\frac{b\sinh(c+dx)(a+b\cosh(c+dx))^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[c + d*x])^3,x]
```

```
[Out] (a*(2*a^2 + 3*b^2)*x)/2 + (2*b*(4*a^2 + b^2)*Sinh[c + d*x])/(3*d) + (5*a*b^2*Cosh[c + d*x]*Sinh[c + d*x])/(6*d) + (b*(a + b*Cosh[c + d*x])^2*Sinh[c + d*x])/(3*d)
```

Rule 2735

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[1/n, Int[(a + b*Sin[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Cos[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int (a+b\cosh(c+dx))^3\,dx &= \frac{b(a+b\cosh(c+dx))^2\sinh(c+dx)}{3d}+\frac{1}{3}\int (a+b\cosh(c+dx))\left(3a^2+2b^2+5a\right.\\ &= \frac{1}{2}a\left(2a^2+3b^2\right)x+\frac{2b(4a^2+b^2)\sinh(c+dx)}{3d}+\frac{5ab^2\cosh(c+dx)\sinh(c+dx)}{6d}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 80, normalized size = 0.89

$$\frac{12a^3c+18ab^2c+12a^3dx+18ab^2dx+9b(4a^2+b^2)\sinh(c+dx)+9ab^2\sinh(2(c+dx))+b^3\sinh(3(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[c + d*x])^3,x]
```

```
[Out] (12*a^3*c + 18*a*b^2*c + 12*a^3*d*x + 18*a*b^2*d*x + 9*b*(4*a^2 + b^2)*Sinh
[c + d*x] + 9*a*b^2*Sinh[2*(c + d*x)] + b^3*Sinh[3*(c + d*x)])/(12*d)
```

Maple [A]
time = 1.11, size = 71, normalized size = 0.79

method	result
default	$a^3x+\frac{(\frac{3}{4}b^3+3a^2b)\sinh(dx+c)}{d}+\frac{3a\,b^2x}{2}+\frac{b^3\sinh(3dx+3c)}{12d}+\frac{3a\,b^2\sinh(2dx+2c)}{4d}$
risch	$a^3x+\frac{3a\,b^2x}{2}+\frac{b^3\mathrm{e}^{3dx+3c}}{24d}+\frac{3\,\mathrm{e}^{2dx+2c}a\,b^2}{8d}+\frac{3b\,\mathrm{e}^{dx+c}a^2}{2d}+\frac{3b^3\mathrm{e}^{dx+c}}{8d}-\frac{3b\,\mathrm{e}^{-dx-c}a^2}{2d}-\frac{3b^3\mathrm{e}^{-dx-c}}{8d}-\frac{3\,\mathrm{e}^{-2dx-2c}a\,b^2}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*x+(3/4*b^3+3*a^2*b)/d*sinh(d*x+c)+3/2*a*b^2*x+1/12*b^3/d*sinh(3*d*x+3*c
)+3/4*a*b^2/d*sinh(2*d*x+2*c)
```

Maxima [A]
time = 0.27, size = 116, normalized size = 1.29

$$\frac{3}{8}\,ab^2\left(4\,x+\frac{e^{(2\,dx+2\,c)}}{d}-\frac{e^{(-2\,dx-2\,c)}}{d}\right)+a^3x+\frac{1}{24}\,b^3\left(\frac{e^{(3\,dx+3\,c)}}{d}+\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}-\frac{e^{(-3\,dx-3\,c)}}{d}\right)+\frac{3\,a^2b\sinh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 3/8*a*b^2*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) + a^3*x + 1/24*b^3
*(e^(3*d*x + 3*c)/d + 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d - e^(-3*d*x - 3*c)
/d) + 3*a^2*b*sinh(d*x + c)/d
```

Fricas [A]
time = 0.39, size = 78, normalized size = 0.87

$$\frac{b^3\sinh\left(dx+c\right)^3+6\left(2\,a^3+3\,ab^2\right)dx+3\left(b^3\cosh\left(dx+c\right)^2+6\,ab^2\cosh\left(dx+c\right)+12\,a^2b+3\,b^3\right)\sinh\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(b^3*sinh(d*x + c)^3 + 6*(2*a^3 + 3*a*b^2)*d*x + 3*(b^3*cosh(d*x + c)^
2 + 6*a*b^2*cosh(d*x + c) + 12*a^2*b + 3*b^3)*sinh(d*x + c))/d
```

Sympy [A]
time = 0.13, size = 128, normalized size = 1.42

$$\begin{cases} a^3x + \frac{3a^2b\sinh{(c+dx)}}{d} - \frac{3ab^2x\sinh^2{(c+dx)}}{2} + \frac{3ab^2x\cosh^2{(c+dx)}}{2} + \frac{3ab^2\sinh{(c+dx)}\cosh{(c+dx)}}{2d} - \frac{2b^3\sinh^3{(c+dx)}}{3d} + \frac{b^3\sinh{(c+dx)}\cosh^2{(c+dx)}}{d} & \text{for } d \neq 0 \\ x(a+b\cosh{(c)})^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*x + 3*a**2*b*sinh(c + d*x)/d - 3*a*b**2*x*sinh(c + d*x)**2/
2 + 3*a*b**2*x*cosh(c + d*x)**2/2 + 3*a*b**2*sinh(c + d*x)*cosh(c + d*x)/(2
*d) - 2*b**3*sinh(c + d*x)**3/(3*d) + b**3*sinh(c + d*x)*cosh(c + d*x)**2/d
, Ne(d, 0)), (x*(a + b*cosh(c))**3, True))
```

Giac [A]
time = 0.42, size = 131, normalized size = 1.46

$$\frac{b^3e^{(3\,dx+3\,c)}}{24\,d} + \frac{3\,ab^2e^{(2\,dx+2\,c)}}{8\,d} - \frac{3\,ab^2e^{(-2\,dx-2\,c)}}{8\,d} - \frac{b^3e^{(-3\,dx-3\,c)}}{24\,d} + \frac{1}{2}\,(2\,a^3+3\,ab^2)x + \frac{3\,(4\,a^2b+b^3)e^{(dx+c)}}{8\,d} - \frac{3\,(4\,a^2b+b^3)e^{(-dx-c)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/24*b^3*e^(3*d*x + 3*c)/d + 3/8*a*b^2*e^(2*d*x + 2*c)/d - 3/8*a*b^2*e^(-2*
d*x - 2*c)/d - 1/24*b^3*e^(-3*d*x - 3*c)/d + 1/2*(2*a^3 + 3*a*b^2)*x + 3/8*
(4*a^2*b + b^3)*e^(d*x + c)/d - 3/8*(4*a^2*b + b^3)*e^(-d*x - c)/d
```

Mupad [B]
time = 0.95, size = 73, normalized size = 0.81

$$\frac{\frac{9\,b^3\,\sinh(c+d\,x)}{2} + \frac{b^3\,\sinh(3\,c+3\,d\,x)}{2} + \frac{9\,a\,b^2\,\sinh(2\,c+2\,d\,x)}{2} + 18\,a^2\,b\,\sinh(c+d\,x) + 6\,a^3\,d\,x + 9\,a\,b^2\,d\,x}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(c + d*x))^3,x)
```

```
[Out] ((9*b^3*sinh(c + d*x))/2 + (b^3*sinh(3*c + 3*d*x))/2 + (9*a*b^2*sinh(2*c +
2*d*x))/2 + 18*a^2*b*sinh(c + d*x) + 6*a^3*d*x + 9*a*b^2*d*x)/(6*d)
```

3.65 $\int (a+b\cosh(c+dx))^2\,dx$

Optimal. Leaf size=50

$$\frac{1}{2}\left(2a^2+b^2\right)x+\frac{2ab\sinh(c+dx)}{d}+\frac{b^2\cosh(c+dx)\sinh(c+dx)}{2d}$$

```
[Out] 1/2*(2*a^2+b^2)*x+2*a*b*sinh(d*x+c)/d+1/2*b^2*cosh(d*x+c)*sinh(d*x+c)/d
```

Rubi [A]
time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2723}

$$\frac{1}{2}x\left(2a^2+b^2\right)+\frac{2ab\sinh(c+dx)}{d}+\frac{b^2\sinh(c+dx)\cosh(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[c + d*x])^2,x]
[Out] ((2*a^2 + b^2)*x)/2 + (2*a*b*Sinh[c + d*x])/d + (b^2*Cosh[c + d*x]*Sinh[c + d*x])/(2*d)
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(2*a^2 + b^
2)*(x/2), x] + (-Simp[2*a*b*(Cos[c + d*x]/d), x] - Simp[b^2*Cos[c + d*x]*(S
in[c + d*x]/(2*d)), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int (a+b\cosh(c+dx))^2\,dx=\frac{1}{2}\left(2a^2+b^2\right)x+\frac{2ab\sinh(c+dx)}{d}+\frac{b^2\cosh(c+dx)\sinh(c+dx)}{2d}$$

Mathematica [A]
time = 0.06, size = 46, normalized size = 0.92

$$\frac{2\left(2a^2+b^2\right)(c+dx)+8ab\sinh(c+dx)+b^2\sinh(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[c + d*x])^2,x]
[Out] (2*(2*a^2 + b^2)*(c + d*x) + 8*a*b*Sinh[c + d*x] + b^2*Sinh[2*(c + d*x)])/(
4*d)
```

Maple [A]
time = 0.74, size = 51, normalized size = 1.02

method	result	size
derivativedivides	$\frac{b^{2}\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2ab\sinh(dx+c)+a^{2}(dx+c)}{d}$	51
default	$\frac{b^{2}\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2ab\sinh(dx+c)+a^{2}(dx+c)}{d}$	51
risch	$a^{2}x+\frac{b^{2}x}{2}+\frac{e^{2dx+2c}b^{2}}{8d}+\frac{ab\,e^{dx+c}}{d}-\frac{ab\,e^{-dx-c}}{d}-\frac{e^{-2dx-2c}b^{2}}{8d}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*(1/2*cosh(d*x+c)*sinh(d*x+c)+1/2*d*x+1/2*c)+2*a*b*sinh(d*x+c)+a^2*
(d*x+c))
```

Maxima [A]
time = 0.27, size = 55, normalized size = 1.10

$$\frac{1}{8}\,b^{2}\left(4\,x+\frac{e^{(2\,dx+2\,c)}}{d}-\frac{e^{(-2\,dx-2\,c)}}{d}\right)+a^{2}x+\frac{2\,ab\sinh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) + a^2*x + 2*a*b*sinh
(d*x + c)/d
```

Fricas [A]
time = 0.34, size = 40, normalized size = 0.80

$$\frac{(2\,a^{2}+b^{2})dx+(b^{2}\cosh\left(dx+c\right)+4\,ab)\sinh\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*((2*a^2 + b^2)*d*x + (b^2*cosh(d*x + c) + 4*a*b)*sinh(d*x + c))/d
```

Sympy [A]
time = 0.09, size = 78, normalized size = 1.56

$$\begin{cases} a^{2}x+\frac{2ab\sinh\left(c+dx\right)}{d}-\frac{b^{2}x\sinh^{2}\left(c+dx\right)}{2}+\frac{b^{2}x\cosh^{2}\left(c+dx\right)}{2}+\frac{b^{2}\sinh\left(c+dx\right)\cosh\left(c+dx\right)}{2d} & \text{for } d\neq 0 \\ x(a+b\cosh\left(c\right))^{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*sinh(c + d*x)/d - b**2*x*sinh(c + d*x)**2/2 + b**
2*x*cosh(c + d*x)**2/2 + b**2*sinh(c + d*x)*cosh(c + d*x)/(2*d), Ne(d, 0)),
 (x*(a + b*cosh(c))**2, True))
```

Giac [A]
time = 0.41, size = 75, normalized size = 1.50

$$\frac{1}{2}\left(2\,a^2+b^2\right)x+\frac{b^2e^{(2\,dx+2\,c)}}{8\,d}+\frac{abe^{(dx+c)}}{d}-\frac{abe^{(-dx-c)}}{d}-\frac{b^2e^{(-2\,dx-2\,c)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*a^2 + b^2)*x + 1/8*b^2*e^(2*d*x + 2*c)/d + a*b*e^(d*x + c)/d - a*b*e
^(-d*x - c)/d - 1/8*b^2*e^(-2*d*x - 2*c)/d
```

Mupad [B]
time = 0.92, size = 41, normalized size = 0.82

$$\frac{\frac{\sinh(2\,c+2\,d\,x)\,b^2}{4}+2\,a\,\sinh(c+d\,x)\,b}{d}+a^2\,x+\frac{b^2\,x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(c + d*x))^2,x)
```

```
[Out] ((b^2*sinh(2*c + 2*d*x))/4 + 2*a*b*sinh(c + d*x))/d + a^2*x + (b^2*x)/2
```

3.66 $\int (a+b\cosh (c+dx))\,dx$

Optimal. Leaf size=15

$$ax+\frac{b\sinh (c+dx)}{d}$$

```
[Out] a*x+b*sinh(d*x+c)/d
```

Rubi [A]

time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2717}

$$ax+\frac{b\sinh (c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Cosh[c + d*x],x]
[Out] a*x + (b*Sinh[c + d*x])/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\cosh (c+dx))\,dx &= ax+b\int \cosh (c+dx)\,dx\\ &= ax+\frac{b\sinh (c+dx)}{d}\end{aligned}$$

Mathematica [A]

time = 0.01, size = 26, normalized size = 1.73

$$ax+\frac{b\cosh (dx)\sinh (c)}{d}+\frac{b\cosh (c)\sinh (dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Cosh[c + d*x],x]
[Out] a*x + (b*Cosh[d*x]*Sinh[c])/d + (b*Cosh[c]*Sinh[d*x])/d
```

Maple [A]

time = 0.75, size = 16, normalized size = 1.07

method	result	size
default	$ax+\frac{b\sinh(dx+c)}{d}$	16
derivativedivides	$\frac{(dx+c)a+b\sinh(dx+c)}{d}$	21
risch	$ax+\frac{b\,\mathrm{e}^{dx+c}}{2d}-\frac{b\,\mathrm{e}^{-dx-c}}{2d}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+b*sinh(d*x+c)/d
```

Maxima [A]
time = 0.26, size = 15, normalized size = 1.00

$$ax+\frac{b\sinh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] a*x + b*sinh(d*x + c)/d
```

Fricas [A]
time = 0.35, size = 17, normalized size = 1.13

$$\frac{adx+b\sinh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] (a*d*x + b*sinh(d*x + c))/d
```

Sympy [A]
time = 0.05, size = 17, normalized size = 1.13

$$ax+b\left(\begin{cases}\frac{\sinh\left(c+dx\right)}{d} & \text{for}\ d\neq 0\\ x\cosh\left(c\right) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cosh(d*x+c),x)
```

```
[Out] a*x + b*Piecewise((sinh(c + d*x)/d, Ne(d, 0)), (x*cosh(c), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(15) = 30$.
time = 0.40, size = 32, normalized size = 2.13

$$ax + \frac{1}{2}b\left(\frac{e^{(dx+c)}}{d} - \frac{e^{(-dx-c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] a*x + 1/2*b*(e^(d*x + c)/d - e^(-d*x - c)/d)
```

Mupad [B]
time = 0.06, size = 15, normalized size = 1.00

$$a\,x + \frac{b\sinh(c + d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*cosh(c + d*x),x)
```

```
[Out] a*x + (b*sinh(c + d*x))/d
```

3.67 $\int \frac{1}{a+b\cosh(c+dx)}\,dx$

Optimal. Leaf size=49

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,\sqrt{a+b}\,d}$$

```
[Out] 2*arctanh((a-b)^(1/2)*tanh(1/2*d*x+1/2*c)/(a+b)^(1/2))/d/(a-b)^(1/2)/(a+b)^
(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2738, 211}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\,\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[c + d*x])^(-1),x]
```

```
[Out] (2*ArcTanh[(Sqrt[a - b]*Tanh[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[
a + b]*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\cosh(c+dx)}\,dx &= -\frac{(2i)\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{d}\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,\sqrt{a+b}\,d}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 48, normalized size = 0.98

$$-\frac{2\text{ArcTan}\left(\frac{(a-b)\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[c + d*x])^(-1),x]
```

```
[Out] (-2*ArcTan[((a - b)*Tanh[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(Sqrt[-a^2 + b^2]
*d)
```

Maple [A]
time = 0.87, size = 44, normalized size = 0.90

method	result	size
derivativedivides	$\frac{2\,\mathrm{arctanh}\left(\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{d\sqrt{(a+b)(a-b)}}$	44
default	$\frac{2\,\mathrm{arctanh}\left(\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{d\sqrt{(a+b)(a-b)}}$	44
risch	$\frac{\ln\left(\mathrm{e}^{dx+c}+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d}-\frac{\ln\left(\mathrm{e}^{dx+c}+\frac{a\sqrt{a^2-b^2}+a^2-b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d}$	123

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/
2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.43, size = 237, normalized size = 4.84

$$\left[\frac{\log\left(\frac{b^2\cosh(dx+c)^2+b^2\sinh(dx+c)^2+2\,ab\cosh(dx+c)+2\,a^2-b^2+2\,(b^2\cosh(dx+c)+ab)\sinh(dx+c)-2\,\sqrt{a^2-b^2}\,(b\cosh(dx+c)+b\sinh(dx+c)+a)}{b\cosh(dx+c)^2+b\sinh(dx+c)^2+2\,a\cosh(dx+c)+2\,(b\cosh(dx+c)+a)\sinh(dx+c)+b}\right)}{\sqrt{a^2-b^2}\,d}, -\frac{2\,\sqrt{-a^2+b^2}\,\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\cosh(dx+c)+b\sinh(dx+c)+a)}{a^2-b^2}\right)}{(a^2-b^2)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] [log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a
^2 - b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 - b^2)*(b
*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)
^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) + b))/(sqrt(
a^2 - b^2)*d), -2*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(d*x + c
) + b*sinh(d*x + c) + a)/(a^2 - b^2))/((a^2 - b^2)*d)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(41) = 82$.
time = 2.47, size = 163, normalized size = 3.33

$$\begin{cases} \frac{\tilde{\infty} x}{\cosh(c)} & \text{for}\: a = 0 \wedge b = 0 \wedge d = 0 \\ -\frac{1}{bd\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)} & \text{for}\: a = -b \\ \frac{x}{a+b\cosh(c)} & \text{for}\: d = 0 \\ \frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{bd} & \text{for}\: a = b \\ -\frac{\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{ad\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-bd\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} + \frac{\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{ad\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-bd\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x/cosh(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (-1/(b*d*tanh(c/
2 + d*x/2)), Eq(a, -b)), (x/(a + b*cosh(c)), Eq(d, 0)), (tanh(c/2 + d*x/2)/
(b*d), Eq(a, b)), (-log(-sqrt(a/(a - b) + b/(a - b)) + tanh(c/2 + d*x/2))/(
a*d*sqrt(a/(a - b) + b/(a - b)) - b*d*sqrt(a/(a - b) + b/(a - b))) + log(sq
rt(a/(a - b) + b/(a - b)) + tanh(c/2 + d*x/2))/(a*d*sqrt(a/(a - b) + b/(a -
 b)) - b*d*sqrt(a/(a - b) + b/(a - b))), True))
```

Giac [A]

time = 0.40, size = 39, normalized size = 0.80

$$\frac{2\arctan\left(\frac{be^{(dx+c)}+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] 2*arctan((b*e^(d*x + c) + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*d)
```

Mupad [B]
time = 1.22, size = 53, normalized size = 1.08

$$\frac{2\,\mathrm{atan}\left(\frac{a\,d+b\,d\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}}{\sqrt{b^2\,d^2-a^2\,d^2}}\right)}{\sqrt{b^2\,d^2-a^2\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cosh(c + d*x)),x)
```

```
[Out] (2*atan((a*d + b*d*exp(d*x)*exp(c))/(b^2*d^2 - a^2*d^2)^(1/2)))/(b^2*d^2 -
a^2*d^2)^(1/2)
```

3.68 $\int \frac{1}{(a+b\cosh(c+dx))^2}\,dx$

Optimal. Leaf size=86

$$\frac{2a\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d}-\frac{b\sinh(c+dx)}{(a^2-b^2)\,d(a+b\cosh(c+dx))}$$

```
[Out] 2*a*arctanh((a-b)^(1/2)*tanh(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(3/2)/(a+b)^
(3/2)/d-b*sinh(d*x+c)/(a^2-b^2)/d/(a+b*cosh(d*x+c))
```

Rubi [A]
time = 0.06, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2743, 12, 2738, 211}

$$\frac{2a\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}-\frac{b\sinh(c+dx)}{d\,(a^2-b^2)\,(a+b\cosh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[c + d*x])^(-2),x]
```

```
[Out] (2*a*ArcTanh[(Sqrt[a - b]*Tanh[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*(
a + b)^(3/2)*d) - (b*Sinh[c + d*x])/((a^2 - b^2)*d*(a + b*Cosh[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
```

```
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+b\cosh(c+dx))^2}\,dx &= -\frac{b\sinh(c+dx)}{(a^2-b^2)\,d(a+b\cosh(c+dx))}-\frac{\int \frac{a}{a+b\cosh(c+dx)}\,dx}{-a^2+b^2}\\ &= -\frac{b\sinh(c+dx)}{(a^2-b^2)\,d(a+b\cosh(c+dx))}+\frac{a\int \frac{1}{a+b\cosh(c+dx)}\,dx}{a^2-b^2}\\ &= -\frac{b\sinh(c+dx)}{(a^2-b^2)\,d(a+b\cosh(c+dx))}-\frac{(2ia)\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx,x,\tan\left(\frac{1}{2}(ic+\right.\right.}{(a^2-b^2)\,d}\\ &= \frac{2a\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d}-\frac{b\sinh(c+dx)}{(a^2-b^2)\,d(a+b\cosh(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.17, size = 84, normalized size = 0.98

$$\frac{\frac{2a\text{ArcTan}\left(\frac{(a-b)\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{3/2}}-\frac{b\sinh(c+dx)}{(a-b)(a+b)(a+b\cosh(c+dx))}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[c + d*x])^(-2),x]
```

```
[Out] ((2*a*ArcTan[((a - b)*Tanh[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3
/2) - (b*Sinh[c + d*x])/((a - b)*(a + b)*(a + b*Cosh[c + d*x])))/d
```

Maple [A]
time = 0.91, size = 118, normalized size = 1.37

method	result
derivativedivides	$\frac{\frac{2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a^2-b^2)\left(a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)}+\frac{2a\,\text{arctanh}\left(\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}}{d}$

default	$\frac{\frac{2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)}+\frac{2a\,\mathrm{arctanh}\left(\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}}{d}$
risch	$\frac{2a\,\mathrm{e}^{dx+c}+2b}{d(a^2-b^2)\left(b\,\mathrm{e}^{2dx+2c}+2a\,\mathrm{e}^{dx+c}+b\right)}+\frac{a\ln\left(\mathrm{e}^{dx+c}+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)d}-\frac{a\ln\left(\mathrm{e}^{dx+c}+\frac{a\sqrt{a^2-b^2}+a^2-b^2}{b\sqrt{a^2-b^2}}\right.}{\sqrt{a^2-b^2}\,(a+b)(a-b)d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*b/(a^2-b^2)*tanh(1/2*d*x+1/2*c)/(a*tanh(1/2*d*x+1/2*c)^2-b*tanh(1/2*
d*x+1/2*c)^2-a-b)+2*a/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/
2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(77) = 154$.
time = 0.39, size = 743, normalized size = 8.64

$$\left[\frac{2a^2b-2b^3-\left(ab\cosh(dx+c)^2+ab\sinh(dx+c)^2+2a^2\cosh(dx+c)+ab+2\left(ab\cosh(dx+c)+a^2\right)\sinh(dx+c)\right)\sqrt{a^2-b^2}\log\left(\frac{b^2\cosh(dx+c)^2+b^2\sinh(dx+c)^2+2ab\cosh(dx+c)+2a^2-b^2+2\left(b^2\cosh(dx+c)+ab\right)\sinh(dx+c)+2\sqrt{a^2-b^2}\left(b\cosh(dx+c)+b\sinh(dx+c)+a\right)}{b\cosh(dx+c)^2+b\sinh(dx+c)^2+2a\cosh(dx+c)+2\left(b\cosh(dx+c)+a\right)\sinh(dx+c)+b}\right)+2\left(a^3-ab^2\right)\cosh(dx+c)+2\left(a^3-ab^2\right)\sinh(dx+c)}{(a^4b-2a^2b^3+b^5)d\cosh(dx+c)^2+(a^4b-2a^2b^3+b^5)d\sinh(dx+c)^2+2(a^5-2a^3b^2+ab^4)d\cosh(dx+c)+(a^4b-2a^2b^3+b^5)d+2\left((a^4b-2a^2b^3+b^5)d\cosh(dx+c)+(a^5-2a^3b^2+ab^4)d\right)\sinh(dx+c)},\ \frac{2\left(a^2b-b^3-\left(ab\cosh(dx+c)^2+ab\sinh(dx+c)^2+2a^2\cosh(dx+c)+ab+2\left(ab\cosh(dx+c)+a^2\right)\sinh(dx+c)\right)\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}\left(b\cosh(dx+c)+b\sinh(dx+c)+a\right)}{a^2-b^2}\right)+\left(a^3-ab^2\right)\cosh(dx+c)+\left(a^3-ab^2\right)\sinh(dx+c)\right)}{(a^4b-2a^2b^3+b^5)d\cosh(dx+c)^2+(a^4b-2a^2b^3+b^5)d\sinh(dx+c)^2+2(a^5-2a^3b^2+ab^4)d\cosh(dx+c)+(a^4b-2a^2b^3+b^5)d+2\left((a^4b-2a^2b^3+b^5)d\cosh(dx+c)+(a^5-2a^3b^2+ab^4)d\right)\sinh(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [(2*a^2*b - 2*b^3 - (a*b*cosh(d*x + c)^2 + a*b*sinh(d*x + c)^2 + 2*a^2*cosh
(d*x + c) + a*b + 2*(a*b*cosh(d*x + c) + a^2)*sinh(d*x + c))*sqrt(a^2 - b^2
)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*
a^2 - b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 - b^2)*(
b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c
)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) + b)) + 2*(
a^3 - a*b^2)*cosh(d*x + c) + 2*(a^3 - a*b^2)*sinh(d*x + c))/((a^4*b - 2*a^2
*b^3 + b^5)*d*cosh(d*x + c)^2 + (a^4*b - 2*a^2*b^3 + b^5)*d*sinh(d*x + c)^2
```

```
 + 2*(a^5 - 2*a^3*b^2 + a*b^4)*d*cosh(d*x + c) + (a^4*b - 2*a^2*b^3 + b^5)*
d + 2*((a^4*b - 2*a^2*b^3 + b^5)*d*cosh(d*x + c) + (a^5 - 2*a^3*b^2 + a*b^4
)*d)*sinh(d*x + c)), 2*(a^2*b - b^3 - (a*b*cosh(d*x + c)^2 + a*b*sinh(d*x +
 c)^2 + 2*a^2*cosh(d*x + c) + a*b + 2*(a*b*cosh(d*x + c) + a^2)*sinh(d*x +
c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x
 + c) + a)/(a^2 - b^2)) + (a^3 - a*b^2)*cosh(d*x + c) + (a^3 - a*b^2)*sinh(
d*x + c))/((a^4*b - 2*a^2*b^3 + b^5)*d*cosh(d*x + c)^2 + (a^4*b - 2*a^2*b^3
 + b^5)*d*sinh(d*x + c)^2 + 2*(a^5 - 2*a^3*b^2 + a*b^4)*d*cosh(d*x + c) + (
a^4*b - 2*a^2*b^3 + b^5)*d + 2*((a^4*b - 2*a^2*b^3 + b^5)*d*cosh(d*x + c) +
 (a^5 - 2*a^3*b^2 + a*b^4)*d)*sinh(d*x + c))]
```

Sympy **[F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac **[A]**
time = 0.41, size = 99, normalized size = 1.15

$$\frac{2\left(\frac{a\arctan\left(\frac{be^{(dx+c)}+a}{\sqrt{-a^2+b^2}}\right)}{(a^2-b^2)\sqrt{-a^2+b^2}}+\frac{ae^{(dx+c)}+b}{(a^2-b^2)\left(be^{(2\,dx+2\,c)}+2\,ae^{(dx+c)}+b\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*(a*arctan((b*e^(d*x + c) + a)/sqrt(-a^2 + b^2))/((a^2 - b^2)*sqrt(-a^2 +
b^2)) + (a*e^(d*x + c) + b)/((a^2 - b^2)*(b*e^(2*d*x + 2*c) + 2*a*e^(d*x +
c) + b)))/d
```

Mupad **[B]**
time = 1.30, size = 215, normalized size = 2.50

$$\frac{\frac{2\,b^2}{d\,\left(a^2\,b-b^3\right)}+\frac{2\,a\,b\,\mathrm{e}^{c+d\,x}}{d\,\left(a^2\,b-b^3\right)}}{b+2\,a\,\mathrm{e}^{c+d\,x}+b\,\mathrm{e}^{2\,c+2\,d\,x}}+\frac{a\,\ln\left(-\frac{2\,a\,\mathrm{e}^{c+d\,x}}{b\,\left(a^2-b^2\right)}-\frac{2\,a\,\left(b+a\,\mathrm{e}^{c+d\,x}\right)}{b\,{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}\right)}{d\,{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}-\frac{a\,\ln\left(\frac{2\,a\,\left(b+a\,\mathrm{e}^{c+d\,x}\right)}{b\,{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}-\frac{2\,a\,\mathrm{e}^{c+d\,x}}{b\,\left(a^2-b^2\right)}\right)}{d\,{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cosh(c + d*x))^2,x)
```

```
[Out] ((2*b^2)/(d*(a^2*b - b^3)) + (2*a*b*exp(c + d*x))/(d*(a^2*b - b^3)))/(b + 2
*a*exp(c + d*x) + b*exp(2*c + 2*d*x)) + (a*log(- (2*a*exp(c + d*x))/(b*(a^2
 - b^2)) - (2*a*(b + a*exp(c + d*x)))/(b*(a + b)^(3/2)*(a - b)^(3/2))))/(d*
(a + b)^(3/2)*(a - b)^(3/2)) - (a*log((2*a*(b + a*exp(c + d*x)))/(b*(a + b)
^(3/2)*(a - b)^(3/2)) - (2*a*exp(c + d*x))/(b*(a^2 - b^2))))/(d*(a + b)^(3/
2)*(a - b)^(3/2))
```

3.69 $\int \frac{1}{(a+b\cosh(c+dx))^3} \, dx$

Optimal. Leaf size=133

$$\frac{(2a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d}-\frac{b\sinh(c+dx)}{2(a^2-b^2)\,d(a+b\cosh(c+dx))^2}-\frac{3ab\sinh(c+dx)}{2(a^2-b^2)^2\,d(a+b\cosh(c+$$

```
[Out] (2*a^2+b^2)*arctanh((a-b)^(1/2)*tanh(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2
)/(a+b)^(5/2)/d-1/2*b*sinh(d*x+c)/(a^2-b^2)/d/(a+b*cosh(d*x+c))^2-3/2*a*b*s
inh(d*x+c)/(a^2-b^2)^2/d/(a+b*cosh(d*x+c))
```

Rubi [A]
time = 0.11, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$,
Rules used = {2743, 2833, 12, 2738, 211}

$$\frac{(2a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{3ab\sinh(c+dx)}{2d(a^2-b^2)^2(a+b\cosh(c+dx))}-\frac{b\sinh(c+dx)}{2d(a^2-b^2)(a+b\cosh(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[c + d*x])^(-3),x]
```

```
[Out] ((2*a^2 + b^2)*ArcTanh[(Sqrt[a - b]*Tanh[(c + d*x)/2])/Sqrt[a + b]])/((a -
b)^(5/2)*(a + b)^(5/2)*d) - (b*Sinh[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Cosh[
c + d*x])^2) - (3*a*b*Sinh[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Cosh[c + d*x
]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{1}{(a+b\cosh(c+dx))^3}\,dx = -\frac{b\sinh(c+dx)}{2\left(a^2-b^2\right)d(a+b\cosh(c+dx))^2} - \frac{\int \frac{-2a+b\cosh(c+dx)}{(a+b\cosh(c+dx))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= -\frac{b\sinh(c+dx)}{2\left(a^2-b^2\right)d(a+b\cosh(c+dx))^2} - \frac{3ab\sinh(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\cosh(c+dx))} + \frac{\int}{}$$

$$= -\frac{b\sinh(c+dx)}{2\left(a^2-b^2\right)d(a+b\cosh(c+dx))^2} - \frac{3ab\sinh(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\cosh(c+dx))} + \frac{(2}{}$$

$$= -\frac{b\sinh(c+dx)}{2\left(a^2-b^2\right)d(a+b\cosh(c+dx))^2} - \frac{3ab\sinh(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\cosh(c+dx))} - \frac{(a}{}$$

$$= \frac{\left(2a^2+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{b\sinh(c+dx)}{2\left(a^2-b^2\right)d(a+b\cosh(c+dx)}$$

Mathematica [A]
time = 0.32, size = 113, normalized size = 0.85

$$\frac{-\frac{2\left(2a^2+b^2\right)\text{ArcTan}\left(\frac{(a-b)\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\left(-a^2+b^2\right)^{5/2}} + \frac{b\left(-4a^2+b^2-3ab\cosh(c+dx)\right)\sinh(c+dx)}{(a-b)^2(a+b)^2(a+b\cosh(c+dx))^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[c + d*x])^(-3),x]
```

```
[Out] ((-2*(2*a^2 + b^2)*ArcTan[((a - b)*Tanh[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-
a^2 + b^2)^(5/2) + (b*(-4*a^2 + b^2 - 3*a*b*Cosh[c + d*x])*Sinh[c + d*x])/(
(a - b)^2*(a + b)^2*(a + b*Cosh[c + d*x])^2))/(2*d)
```

Maple [A]
time = 0.94, size = 186, normalized size = 1.40

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{(4a+b)b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^2+2ab+b^2\right)}+\frac{(4a-b)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ab+b^2\right)}\right)}{\left(a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)^2}+\frac{\left(2a^2+b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\left(a^4-2a^2b^2+b^4\right)\sqrt{(a+b)(a-b)}}}{d}$
default	$\frac{-\frac{2\left(-\frac{(4a+b)b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^2+2ab+b^2\right)}+\frac{(4a-b)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ab+b^2\right)}\right)}{\left(a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)^2}+\frac{\left(2a^2+b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\left(a^4-2a^2b^2+b^4\right)\sqrt{(a+b)(a-b)}}}{d}$
risch	$\frac{2a^2b\,e^{3dx+3c}+b^3e^{3dx+3c}+6a^3e^{2dx+2c}+3a\,b^2e^{2dx+2c}+10a^2b\,e^{dx+c}-e^{dx+c}b^3+3a\,b^2}{d(a^2-b^2)^2\left(b\,e^{2dx+2c}+2a\,e^{dx+c}+b\right)^2}+\frac{\ln\left(e^{dx+c}+\frac{a\sqrt{a^2-b^2}-a^2+b}{b\sqrt{a^2-b^2}}\right.}{\sqrt{a^2-b^2}\,(a+b)^2(a-b)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(-1/2*(4*a+b)*b/(a-b)/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3+1/2*(4*
a-b)*b/(a+b)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^2-
b*tanh(1/2*d*x+1/2*c)^2-a-b)^2+(2*a^2+b^2)/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b)
)^(1/2)*arctanh((a-b)*tanh(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1239 vs. 2(120) = 240.
time = 0.42, size = 2591, normalized size = 19.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cosh(d*x+c))^3,x, algorithm="fricas")

[Out] [1/2*(6*a^3*b^2 - 6*a*b^4 + 2*(2*a^4*b - a^2*b^3 - b^5)*cosh(d*x + c)^3 + 2*(2*a^4*b - a^2*b^3 - b^5)*sinh(d*x + c)^3 + 6*(2*a^5 - a^3*b^2 - a*b^4)*cosh(d*x + c)^2 + 6*(2*a^5 - a^3*b^2 - a*b^4 + (2*a^4*b - a^2*b^3 - b^5)*cosh(d*x + c))*sinh(d*x + c)^2 + ((2*a^2*b^2 + b^4)*cosh(d*x + c)^4 + (2*a^2*b^2 + b^4)*sinh(d*x + c)^4 + 2*a^2*b^2 + b^4 + 4*(2*a^3*b + a*b^3)*cosh(d*x + c)^3 + 4*(2*a^3*b + a*b^3 + (2*a^2*b^2 + b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(4*a^4 + 4*a^2*b^2 + b^4)*cosh(d*x + c)^2 + 2*(4*a^4 + 4*a^2*b^2 + b^4 + 3*(2*a^2*b^2 + b^4)*cosh(d*x + c)^2 + 6*(2*a^3*b + a*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 4*(2*a^3*b + a*b^3)*cosh(d*x + c) + 4*(2*a^3*b + a*b^3 + (2*a^2*b^2 + b^4)*cosh(d*x + c)^3 + 3*(2*a^3*b + a*b^3)*cosh(d*x + c)^2 + (4*a^4 + 4*a^2*b^2 + b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - b^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 - b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 - b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) + b)) + 2*(10*a^4*b - 11*a^2*b^3 + b^5)*cosh(d*x + c) + 2*(10*a^4*b - 11*a^2*b^3 + b^5 + 3*(2*a^4*b - a^2*b^3 - b^5)*cosh(d*x + c)^2 + 6*(2*a^5 - a^3*b^2 - a*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cosh(d*x + c)^4 + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*sinh(d*x + c)^4 + 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cosh(d*x + c)^3 + 2*(2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8)*d*cosh(d*x + c)^2 + 4*((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cosh(d*x + c) + (a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d)*sinh(d*x + c)^3 + 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cosh(d*x + c) + 2*(3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cosh(d*x + c)^2 + 6*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cosh(d*x + c) + (2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8)*d)*sinh(d*x + c)^2 + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d + 4*((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cosh(d*x + c)^3 + 3*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cosh(d*x + c)^2 + (2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8)*d*cosh(d*x + c) + (a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d)*sinh(d*x + c)), (3*a^3*b^2 - 3*a*b^4 + (2*a^4*b - a^2*b^3 - b^5)*cosh(d*x + c)^3 + (2*a^4*b - a^2*b^3 - b^5)*sinh(d*x + c)^3 + 3*(2*a^5 - a^3*b^2 - a*b^4)*cosh(d*x + c)^2 + 3*(2*a^5 - a^3*b^2 - a*b^4 + (2*a^4*b - a^2*b^3 - b^5)*cosh(d*x + c))*sinh(d*x + c)^2 - ((2*a^2*b^2 + b^4)*cosh(d*x + c)^4 + (2*a^2*b^2 + b^4)*sinh(d*x + c)^4 + 2*a^2*b^2 + b^4 + 4*(2*a^3*b + a*b^3)*cosh(d*x + c)^3 + 4*(2*a^3*b + a*b^3 + (2*a^2*b^2 + b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(4*a^4 + 4*a^2*b^2 + b^4)*cosh(d*x + c)^2 + 2*(4*a^4 + 4*a^2*b^2 + b^4 + 3*(2*a^2*b^2 + b^4)*cosh(d*x + c)^2 + 6*(2*a^3*b + a*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 4*(2*a^3*b + a*b^3)*cosh(d*x + c) + 4*(2*a^3*b + a*b^3 + (2*a^2*b^2 + b^4)*cosh(d*x + c)^3 + 3*(2*a^3*b + a*b^3)*cosh(d*x + c)^2 + (4*a^4 + 4*a^2*b^2 + b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh

```
(d*x + c) + b*sinh(d*x + c) + a)/(a^2 - b^2)) + (10*a^4*b - 11*a^2*b^3 + b^
5)*cosh(d*x + c) + (10*a^4*b - 11*a^2*b^3 + b^5 + 3*(2*a^4*b - a^2*b^3 - b^
5)*cosh(d*x + c)^2 + 6*(2*a^5 - a^3*b^2 - a*b^4)*cosh(d*x + c))*sinh(d*x +
c))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cosh(d*x + c)^4 + (a^6*b^2 -
 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*sinh(d*x + c)^4 + 4*(a^7*b - 3*a^5*b^3 + 3*
a^3*b^5 - a*b^7)*d*cosh(d*x + c)^3 + 2*(2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2
*b^6 - b^8)*d*cosh(d*x + c)^2 + 4*((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*
d*cosh(d*x + c) + (a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d)*sinh(d*x + c)^
3 + 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cosh(d*x + c) + 2*(3*(a^6*b
^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cosh(d*x + c)^2 + 6*(a^7*b - 3*a^5*b^3
+ 3*a^3*b^5 - a*b^7)*d*cosh(d*x + c) + (2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2
*b^6 - b^8)*d)*sinh(d*x + c)^2 + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d
+ 4*((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cosh(d*x + c)^3 + 3*(a^7*b -
 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cosh(d*x + c)^2 + (2*a^8 - 5*a^6*b^2 + 3*
a^4*b^4 + a^2*b^6 - b^8)*d*cosh(d*x + c) + (a^7*b - 3*a^5*b^3 + 3*a^3*b^5 -
 a*b^7)*d)*sinh(d*x + c))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.40, size = 195, normalized size = 1.47

$$\frac{\frac{(2\,a^2+b^2)\arctan\left(\frac{be^{(dx+c)}+a}{\sqrt{-a^2+b^2}}\right)}{(a^4-2\,a^2b^2+b^4)\sqrt{-a^2+b^2}}+\frac{2\,a^2be^{(3\,dx+3\,c)}+b^3e^{(3\,dx+3\,c)}+6\,a^3e^{(2\,dx+2\,c)}+3\,ab^2e^{(2\,dx+2\,c)}+10\,a^2be^{(dx+c)}-b^3e^{(dx+c)}+3\,ab^2}{(a^4-2\,a^2b^2+b^4)\left(be^{(2\,dx+2\,c)}+2\,ae^{(dx+c)}+b\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] ((2*a^2 + b^2)*arctan((b*e^(d*x + c) + a)/sqrt(-a^2 + b^2))/((a^4 - 2*a^2*b
^2 + b^4)*sqrt(-a^2 + b^2)) + (2*a^2*b*e^(3*d*x + 3*c) + b^3*e^(3*d*x + 3*c
) + 6*a^3*e^(2*d*x + 2*c) + 3*a*b^2*e^(2*d*x + 2*c) + 10*a^2*b*e^(d*x + c)
- b^3*e^(d*x + c) + 3*a*b^2)/((a^4 - 2*a^2*b^2 + b^4)*(b*e^(2*d*x + 2*c) +
2*a*e^(d*x + c) + b)^2))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\cosh\left(c+d\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cosh(c + d*x))^3,x)
```

```
[Out] int(1/(a + b*cosh(c + d*x))^3, x)
```

3.70 $\int \frac{1}{(a+b\cosh(c+dx))^4}\,dx$

Optimal. Leaf size=184

$$\frac{a(2a^2+3b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d}-\frac{b\sinh(c+dx)}{3\left(a^2-b^2\right)d(a+b\cosh(c+dx))^3}-\frac{5ab\sinh(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\cosh(c$$

```
[Out] a*(2*a^2+3*b^2)*arctanh((a-b)^(1/2)*tanh(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^
(7/2)/(a+b)^(7/2)/d-1/3*b*sinh(d*x+c)/(a^2-b^2)/d/(a+b*cosh(d*x+c))^3-5/6*a
*b*sinh(d*x+c)/(a^2-b^2)^2/d/(a+b*cosh(d*x+c))^2-1/6*b*(11*a^2+4*b^2)*sinh(
d*x+c)/(a^2-b^2)^3/d/(a+b*cosh(d*x+c))
```

Rubi [A]
time = 0.19, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2743, 2833, 12, 2738, 211}

$$\frac{a(2a^2+3b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{b(11a^2+4b^2)\sinh(c+dx)}{6d\left(a^2-b^2\right)^3(a+b\cosh(c+dx))}-\frac{5ab\sinh(c+dx)}{6d\left(a^2-b^2\right)^2(a+b\cosh(c+dx))^2}-\frac{b\sinh(c+dx)}{3d\left(a^2-b^2\right)(a+b\cosh(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[c + d*x])^(-4),x]
```

```
[Out] (a*(2*a^2 + 3*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[(c + d*x)/2])/Sqrt[a + b]])/((
a - b)^(7/2)*(a + b)^(7/2)*d) - (b*Sinh[c + d*x])/(3*(a^2 - b^2)*d*(a + b*C
osh[c + d*x])^3) - (5*a*b*Sinh[c + d*x])/(6*(a^2 - b^2)^2*d*(a + b*Cosh[c +
 d*x])^2) - (b*(11*a^2 + 4*b^2)*Sinh[c + d*x])/(6*(a^2 - b^2)^3*d*(a + b*Co
sh[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+b\cosh(c+dx))^4}\,dx &= -\frac{b\sinh(c+dx)}{3\left(a^2-b^2\right)d(a+b\cosh(c+dx))^3}-\frac{\int \frac{-3a+2b\cosh(c+dx)}{(a+b\cosh(c+dx))^3}\,dx}{3\left(a^2-b^2\right)}\\
&= -\frac{b\sinh(c+dx)}{3\left(a^2-b^2\right)d(a+b\cosh(c+dx))^3}-\frac{5ab\sinh(c+dx)}{6\left(a^2-b^2\right)^2d(a+b\cosh(c+dx))^2}+\\
&= -\frac{b\sinh(c+dx)}{3\left(a^2-b^2\right)d(a+b\cosh(c+dx))^3}-\frac{5ab\sinh(c+dx)}{6\left(a^2-b^2\right)^2d(a+b\cosh(c+dx))^2}-\\
&= -\frac{b\sinh(c+dx)}{3\left(a^2-b^2\right)d(a+b\cosh(c+dx))^3}-\frac{5ab\sinh(c+dx)}{6\left(a^2-b^2\right)^2d(a+b\cosh(c+dx))^2}-\\
&= -\frac{b\sinh(c+dx)}{3\left(a^2-b^2\right)d(a+b\cosh(c+dx))^3}-\frac{5ab\sinh(c+dx)}{6\left(a^2-b^2\right)^2d(a+b\cosh(c+dx))^2}-\\
&= \frac{a\left(2a^2+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d}-\frac{b\sinh(c+dx)}{3\left(a^2-b^2\right)d(a+b\cosh(c+d
\end{aligned}$$

Mathematica [A]
time = 0.74, size = 160, normalized size = 0.87

$$\frac{\frac{6a\left(2a^2+3b^2\right)\mathrm{ArcTan}\left(\frac{(a-b)\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\left(-a^2+b^2\right)^{7/2}}-\frac{b\left(36a^4+a^2b^2+8b^4+6ab\left(9a^2+b^2\right)\cosh(c+dx)+\left(11a^2b^2+4b^4\right)\cosh(2(c+dx))\right)\sinh(c+dx)}{2(a-b)^3(a+b)^3(a+b\cosh(c+dx))^3}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[c + d*x])^(-4),x]
```

```
[Out] ((6*a*(2*a^2 + 3*b^2)*ArcTan[((a - b)*Tanh[(c + d*x)/2])/Sqrt[-a^2 + b^2]])
/(-a^2 + b^2)^(7/2) - (b*(36*a^4 + a^2*b^2 + 8*b^4 + 6*a*b*(9*a^2 + b^2)*Co
sh[c + d*x] + (11*a^2*b^2 + 4*b^4)*Cosh[2*(c + d*x)])*Sinh[c + d*x])/(2*(a
- b)^3*(a + b)^3*(a + b*Cosh[c + d*x])^3))/(6*d)
```

Maple [A]
time = 1.00, size = 284, normalized size = 1.54

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{(6a^2+3ab+2b^2)b\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)(a^3+3a^2b+3ab^2+b^3)}+\frac{2(9a^2+b^2)b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3(a^2+2ab+b^2)(a^2-2ab+b^2)}-\frac{(6a^2-3ab+2b^2)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)(a^3-3a^2b+3ab^2-b^3)}\right)}{\left(a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)^3}+\frac{a(2a^2+3b^2)\,\mathrm{arctanh}}{(a^6-3a^4b^2+3a^2b^4}}{d}$
default	$\frac{-\frac{2\left(-\frac{(6a^2+3ab+2b^2)b\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)(a^3+3a^2b+3ab^2+b^3)}+\frac{2(9a^2+b^2)b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3(a^2+2ab+b^2)(a^2-2ab+b^2)}-\frac{(6a^2-3ab+2b^2)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)(a^3-3a^2b+3ab^2-b^3)}\right)}{\left(a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)^3}+\frac{a(2a^2+3b^2)\,\mathrm{arctanh}}{(a^6-3a^4b^2+3a^2b^4}}{d}$
risch	$\frac{6a^3b^2e^{5dx+5c}+9a\,b^4e^{5dx+5c}+30a^4b\,e^{4dx+4c}+45a^2b^3e^{4dx+4c}+44a^5e^{3dx+3c}+82a^3b^2e^{3dx+3c}+24a\,b^4e^{3dx+3c}+102a^4b\,e^{2dx}}{3d(a^2-b^2)^3(b\,e^{2dx+2c}+2a\,e^{dx+c}+b)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(-1/2*(6*a^2+3*a*b+2*b^2)*b/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tanh(1/
2*d*x+1/2*c)^5+2/3*(9*a^2+b^2)*b/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tanh(1/2*d
*x+1/2*c)^3-1/2*(6*a^2-3*a*b+2*b^2)*b/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tanh(
1/2*d*x+1/2*c))/(a*tanh(1/2*d*x+1/2*c)^2-b*tanh(1/2*d*x+1/2*c)^2-a-b)^3+a*(
2*a^2+3*b^2)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b
)*tanh(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2798 vs. $2(169) = 338$.
time = 0.39, size = 5705, normalized size = 31.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [1/6*(22*a^4*b^3 - 14*a^2*b^5 - 8*b^7 + 6*(2*a^5*b^2 + a^3*b^4 - 3*a*b^6)*c
osh(d*x + c)^5 + 6*(2*a^5*b^2 + a^3*b^4 - 3*a*b^6)*sinh(d*x + c)^5 + 30*(2*
a^6*b + a^4*b^3 - 3*a^2*b^5)*cosh(d*x + c)^4 + 30*(2*a^6*b + a^4*b^3 - 3*a^
2*b^5 + (2*a^5*b^2 + a^3*b^4 - 3*a*b^6)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*
(22*a^7 + 19*a^5*b^2 - 29*a^3*b^4 - 12*a*b^6)*cosh(d*x + c)^3 + 4*(22*a^7 +
 19*a^5*b^2 - 29*a^3*b^4 - 12*a*b^6 + 15*(2*a^5*b^2 + a^3*b^4 - 3*a*b^6)*co
sh(d*x + c)^2 + 30*(2*a^6*b + a^4*b^3 - 3*a^2*b^5)*cosh(d*x + c))*sinh(d*x
+ c)^3 + 12*(17*a^6*b - 11*a^4*b^3 - 4*a^2*b^5 - 2*b^7)*cosh(d*x + c)^2 + 1
2*(17*a^6*b - 11*a^4*b^3 - 4*a^2*b^5 - 2*b^7 + 5*(2*a^5*b^2 + a^3*b^4 - 3*a
*b^6)*cosh(d*x + c)^3 + 15*(2*a^6*b + a^4*b^3 - 3*a^2*b^5)*cosh(d*x + c)^2
+ (22*a^7 + 19*a^5*b^2 - 29*a^3*b^4 - 12*a*b^6)*cosh(d*x + c))*sinh(d*x + c
)^2 - 3*((2*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^6 + (2*a^3*b^3 + 3*a*b^5)*sinh
(d*x + c)^6 + 2*a^3*b^3 + 3*a*b^5 + 6*(2*a^4*b^2 + 3*a^2*b^4)*cosh(d*x + c)
^5 + 6*(2*a^4*b^2 + 3*a^2*b^4 + (2*a^3*b^3 + 3*a*b^5)*cosh(d*x + c))*sinh(d
*x + c)^5 + 3*(8*a^5*b + 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^4 + 3*(8*a^5*b
 + 14*a^3*b^3 + 3*a*b^5 + 5*(2*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^2 + 10*(2*a
^4*b^2 + 3*a^2*b^4)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(4*a^6 + 12*a^4*b^2
+ 9*a^2*b^4)*cosh(d*x + c)^3 + 4*(4*a^6 + 12*a^4*b^2 + 9*a^2*b^4 + 5*(2*a^3
*b^3 + 3*a*b^5)*cosh(d*x + c)^3 + 15*(2*a^4*b^2 + 3*a^2*b^4)*cosh(d*x + c)^
2 + 3*(8*a^5*b + 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(
8*a^5*b + 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^2 + 3*(8*a^5*b + 14*a^3*b^3 +
 3*a*b^5 + 5*(2*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^4 + 20*(2*a^4*b^2 + 3*a^2*
b^4)*cosh(d*x + c)^3 + 6*(8*a^5*b + 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^2 +
 4*(4*a^6 + 12*a^4*b^2 + 9*a^2*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 + 6*(2*a
^4*b^2 + 3*a^2*b^4)*cosh(d*x + c) + 6*(2*a^4*b^2 + 3*a^2*b^4 + (2*a^3*b^3 +
 3*a*b^5)*cosh(d*x + c)^5 + 5*(2*a^4*b^2 + 3*a^2*b^4)*cosh(d*x + c)^4 + 2*(
8*a^5*b + 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^3 + 2*(4*a^6 + 12*a^4*b^2 + 9
*a^2*b^4)*cosh(d*x + c)^2 + (8*a^5*b + 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c))
*sinh(d*x + c))*sqrt(a^2 - b^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c
)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 - b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(
d*x + c) + 2*sqrt(a^2 - b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*co
sh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c)
+ a)*sinh(d*x + c) + b)) + 30*(4*a^5*b^2 - 3*a^3*b^4 - a*b^6)*cosh(d*x + c)
```

```
 + 6*(20*a^5*b^2 - 15*a^3*b^4 - 5*a*b^6 + 5*(2*a^5*b^2 + a^3*b^4 - 3*a*b^6)
*cosh(d*x + c)^4 + 20*(2*a^6*b + a^4*b^3 - 3*a^2*b^5)*cosh(d*x + c)^3 + 2*(
22*a^7 + 19*a^5*b^2 - 29*a^3*b^4 - 12*a*b^6)*cosh(d*x + c)^2 + 4*(17*a^6*b
- 11*a^4*b^3 - 4*a^2*b^5 - 2*b^7)*cosh(d*x + c))*sinh(d*x + c))/((a^8*b^3 -
 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cosh(d*x + c)^6 + (a^8*b^3 - 4
*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*sinh(d*x + c)^6 + 6*(a^9*b^2 - 4
*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cosh(d*x + c)^5 + 3*(4*a^10*b
- 15*a^8*b^3 + 20*a^6*b^5 - 10*a^4*b^7 + b^11)*d*cosh(d*x + c)^4 + 6*((a^8*
b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cosh(d*x + c) + (a^9*b^2
- 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d)*sinh(d*x + c)^5 + 4*(2*a^1
1 - 5*a^9*b^2 + 10*a^5*b^6 - 10*a^3*b^8 + 3*a*b^10)*d*cosh(d*x + c)^3 + 3*(
5*(a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cosh(d*x + c)^2 +
10*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cosh(d*x + c) +
 (4*a^10*b - 15*a^8*b^3 + 20*a^6*b^5 - 10*a^4*b^7 + b^11)*d)*sinh(d*x + c)^
4 + 3*(4*a^10*b - 15*a^8*b^3 + 20*a^6*b^5 - 10*a^4*b^7 + b^11)*d*cosh(d*x +
 c)^2 + 4*(5*(a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cosh(d*
x + c)^3 + 15*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cosh
(d*x + c)^2 + 3*(4*a^10*b - 15*a^8*b^3 + 20*a^6*b^5 - 10*a^4*b^7 + b^11)*d*
cosh(d*x + c) + (2*a^11 - 5*a^9*b^2 + 10*a^5*b^6 - 10*a^3*b^8 + 3*a*b^10)*d
)*sinh(d*x + c)^3 + 6*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10
)*d*cosh(d*x + c) + 3*(5*(a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^1
1)*d*cosh(d*x + c)^4 + 20*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*
b^10)*d*cosh(d*x + c)^3 + 6*(4*a^10*b - 15*a^8*b^3 + 20*a^6*b^5 - 10*a^4*b^
7 + b^11)*d*cosh(d*x + c)^2 + 4*(2*a^11 - 5*a^9*b^2 + 10*a^5*b^6 - 10*a^3*b
^8 + 3*a*b^10)*d*cosh(d*x + c) + (4*a^10*b - 15*a^8*b^3 + 20*a^6*b^5 - 10*a
^4*b^7 + b^11)*d)*sinh(d*x + c)^2 + (a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^
2*b^9 + b^11)*d + 6*((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d
*cosh(d*x + c)^5 + 5*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)
*d*cosh(d*x + c)^4 + 2*(4*a^10*b - 15*a^8*b^3 + 20*a^6*b^5 - 10*a^4*b^7 + b
^11)*d*cosh(d*x + c)^3 + 2*(2*a^11 - 5*a^9*b^2 + 10*a^5*b^6 - 10*a^3*b^8 +
3*a*b^10)*d*cosh(d*x + c)^2 + (4*a^10*b - 15*a^8*b^3 + 20*a^6*b^5 - 10*a^4*
b^7 + b^11)*d*cosh(d*x + c) + (a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8
+ a*b^10)*d)*sinh(d*x + c)), 1/3*(11*a^4*b^3 - 7*a^2*b^5 - 4*b^7 + 3*(2*a^5
*b^2 + a^3*b^4 - 3*a*b^6)*cosh(d*x + c)^5 + 3*(...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [**A**]

time = 0.42, size = 329, normalized size = 1.79

$$\frac{\frac{3\,(2\,a^3+3\,ab^2)\arctan\left(\frac{be^{(dx+c)}+a}{\sqrt{-a^2+b^2}}\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\sqrt{-a^2+b^2}}+\frac{6\,a^3b^2e^{(5\,dx+5\,c)}+9\,ab^4e^{(5\,dx+5\,c)}+30\,a^4be^{(4\,dx+4\,c)}+45\,a^2b^3e^{(4\,dx+4\,c)}+44\,a^5e^{(3\,dx+3\,c)}+82\,a^3b^2e^{(3\,dx+3\,c)}+24\,ab^4e^{(3\,dx+3\,c)}+102\,a^4be^{(2\,dx+2\,c)}+36\,a^2b^3e^{(2\,dx+2\,c)}+12\,b^5e^{(2\,dx+2\,c)}+60\,a^3b^2e^{(dx+c)}+15\,ab^4e^{(dx+c)}+11\,a^2b^3+4\,b^5}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\left(be^{(2\,dx+2\,c)}+2\,ae^{(dx+c)}+b\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(2*a^3 + 3*a*b^2)*arctan((b*e^(d*x + c) + a)/sqrt(-a^2 + b^2))/((a^6
 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) + (6*a^3*b^2*e^(5*d*x + 5
*c) + 9*a*b^4*e^(5*d*x + 5*c) + 30*a^4*b*e^(4*d*x + 4*c) + 45*a^2*b^3*e^(4*
d*x + 4*c) + 44*a^5*e^(3*d*x + 3*c) + 82*a^3*b^2*e^(3*d*x + 3*c) + 24*a*b^4
*e^(3*d*x + 3*c) + 102*a^4*b*e^(2*d*x + 2*c) + 36*a^2*b^3*e^(2*d*x + 2*c) +
 12*b^5*e^(2*d*x + 2*c) + 60*a^3*b^2*e^(d*x + c) + 15*a*b^4*e^(d*x + c) + 1
1*a^2*b^3 + 4*b^5)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(b*e^(2*d*x + 2*c)
+ 2*a*e^(d*x + c) + b)^3))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\cosh(c+d\,x))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cosh(c + d*x))^4,x)
```

```
[Out] int(1/(a + b*cosh(c + d*x))^4, x)
```

3.71 $\int \frac{1}{3+5\cosh(c+dx)}\,dx$

Optimal. Leaf size=22

$$\frac{\text{ArcTan}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}$$

```
[Out] 1/2*arctan(1/2*tanh(1/2*d*x+1/2*c))/d
```

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2738, 212}

$$\frac{\text{ArcTan}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 5*Cosh[c + d*x])^(-1),x]
[Out] ArcTan[Tanh[(c + d*x)/2]/2]/(2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{3+5\cosh(c+dx)}\,dx &= -\frac{(2i)\text{Subst}\left(\int \frac{1}{8-2x^2}\,dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{d}\\ &= \frac{\tan^{-1}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 23, normalized size = 1.05

$$-\frac{\text{ArcTan}\left(2\coth\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 5*Cosh[c + d*x])^(-1),x]
```

```
[Out] -1/2*ArcTan[2*Coth[c/2 + (d*x)/2]]/d
```

Maple [A]
time = 0.87, size = 18, normalized size = 0.82

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{2d}$	18
default	$\frac{\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{2d}$	18
risch	$\frac{i\ln\left(e^{dx+c}+\frac{3}{5}+\frac{4i}{5}\right)}{4d}-\frac{i\ln\left(e^{dx+c}+\frac{3}{5}-\frac{4i}{5}\right)}{4d}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+5*cosh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arctan(1/2*tanh(1/2*d*x+1/2*c))/d
```

Maxima [A]
time = 0.48, size = 19, normalized size = 0.86

$$-\frac{\arctan\left(\frac{5}{4}\,e^{(-dx-c)}+\frac{3}{4}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*arctan(5/4*e^(-d*x - c) + 3/4)/d
```

Fricas [A]
time = 0.44, size = 24, normalized size = 1.09

$$\frac{\arctan\left(\frac{5}{4}\,\cosh\left(dx+c\right)+\frac{5}{4}\,\sinh\left(dx+c\right)+\frac{3}{4}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(5/4*cosh(d*x + c) + 5/4*sinh(d*x + c) + 3/4)/d
```

Sympy [A]
time = 0.44, size = 24, normalized size = 1.09

$$\begin{cases} \frac{\operatorname{atan}\left(\frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}\right)}{2d} & \text{for } d \neq 0 \\ \frac{x}{5\cosh(c)+3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c)),x)
```

```
[Out] Piecewise((atan(tanh(c/2 + d*x/2)/2)/(2*d), Ne(d, 0)), (x/(5*cosh(c) + 3),
True))
```

Giac [A]
time = 0.41, size = 16, normalized size = 0.73

$$\frac{\arctan\left(\frac{5}{4}e^{(dx+c)}+\frac{3}{4}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*arctan(5/4*e^(d*x + c) + 3/4)/d
```

Mupad [B]
time = 0.13, size = 34, normalized size = 1.55

$$\frac{\operatorname{atan}\left(\frac{3\sqrt{d^2}+5\,e^{d\,x}\,e^c\,\sqrt{d^2}}{4\,d}\right)}{2\sqrt{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*cosh(c + d*x) + 3),x)
```

```
[Out] atan((3*(d^2)^(1/2) + 5*exp(d*x)*exp(c)*(d^2)^(1/2))/(4*d))/(2*(d^2)^(1/2))
```

3.72 $\int \frac{1}{(3+5\cosh(c+dx))^2}\,dx$

Optimal. Leaf size=48

$$-\frac{3\text{ArcTan}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{32d}+\frac{5\sinh(c+dx)}{16d(3+5\cosh(c+dx))}$$

```
[Out] -3/32*arctan(1/2*tanh(1/2*d*x+1/2*c))/d+5/16*sinh(d*x+c)/d/(3+5*cosh(d*x+c)
)
```

Rubi [A]
time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2743, 12, 2738, 212}

$$\frac{5\sinh(c+dx)}{16d(5\cosh(c+dx)+3)}-\frac{3\text{ArcTan}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{32d}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 5*Cosh[c + d*x])^(-2),x]
```

```
[Out] (-3*ArcTan[Tanh[(c + d*x)/2]/2])/(32*d) + (5*Sinh[c + d*x])/(16*d*(3 + 5*Co
sh[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
```

```
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(3+5\cosh(c+dx))^2}\,dx &= \frac{5\sinh(c+dx)}{16d(3+5\cosh(c+dx))}+\frac{1}{16}\int -\frac{3}{3+5\cosh(c+dx)}\,dx\\ &= \frac{5\sinh(c+dx)}{16d(3+5\cosh(c+dx))}-\frac{3}{16}\int \frac{1}{3+5\cosh(c+dx)}\,dx\\ &= \frac{5\sinh(c+dx)}{16d(3+5\cosh(c+dx))}+\frac{(3i)\text{Subst}\left(\int \frac{1}{8-2x^2}\,dx,x,\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{8d}\\ &= -\frac{3\tan^{-1}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{32d}+\frac{5\sinh(c+dx)}{16d(3+5\cosh(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 45, normalized size = 0.94

$$\frac{-3\text{ArcTan}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)+\frac{10\sinh(c+dx)}{3+5\cosh(c+dx)}}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 5*Cosh[c + d*x])^(-2),x]
```

```
[Out] (-3*ArcTan[Tanh[(c + d*x)/2]/2] + (10*Sinh[c + d*x])/(3 + 5*Cosh[c + d*x]))
/(32*d)
```

Maple [A]
time = 0.88, size = 46, normalized size = 0.96

method	result	size
derivativedivides	$\frac{\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{16\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+4\right)}-\frac{3\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{32}}{d}$	46
default	$\frac{\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{16\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+4\right)}-\frac{3\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{32}}{d}$	46
risch	$-\frac{3\,e^{dx+c}+5}{8d\left(5\,e^{2dx+2c}+6\,e^{dx+c}+5\right)}+\frac{3i\ln\left(e^{dx+c}+\frac{3}{5}-\frac{4i}{5}\right)}{64d}-\frac{3i\ln\left(e^{dx+c}+\frac{3}{5}+\frac{4i}{5}\right)}{64d}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+5*cosh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(5/16*tanh(1/2*d*x+1/2*c)/(tanh(1/2*d*x+1/2*c)^2+4)-3/32*arctan(1/2*tan
h(1/2*d*x+1/2*c)))
```

Maxima [A]
time = 0.47, size = 64, normalized size = 1.33

$$\frac{3\,\arctan\left(\frac{5}{4}\,e^{(-dx-c)}+\frac{3}{4}\right)}{32\,d}+\frac{3\,e^{(-dx-c)}+5}{8\,d(6\,e^{(-dx-c)}+5\,e^{(-2\,dx-2\,c)}+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 3/32*arctan(5/4*e^(-d*x - c) + 3/4)/d + 1/8*(3*e^(-d*x - c) + 5)/(d*(6*e^(-
d*x - c) + 5*e^(-2*d*x - 2*c) + 5))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(41) = 82$.
time = 0.38, size = 147, normalized size = 3.06

$$-\frac{3\left(5\,\cosh\left(dx+c\right)^{2}+2\left(5\,\cosh\left(dx+c\right)+3\right)\sinh\left(dx+c\right)+5\,\sinh\left(dx+c\right)^{2}+6\,\cosh\left(dx+c\right)+5\right)\arctan\left(\frac{5}{4}\,\cosh\left(dx+c\right)+\frac{5}{4}\,\sinh\left(dx+c\right)+\frac{3}{4}\right)+12\,\cosh\left(dx+c\right)+12\,\sinh\left(dx+c\right)+20}{32\left(5\,d\cosh\left(dx+c\right)^{2}+5\,d\sinh\left(dx+c\right)^{2}+6\,d\cosh\left(dx+c\right)+2\left(5\,d\cosh\left(dx+c\right)+3\,d\right)\sinh\left(dx+c\right)+5\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/32*(3*(5*cosh(d*x + c)^2 + 2*(5*cosh(d*x + c) + 3)*sinh(d*x + c) + 5*sin
h(d*x + c)^2 + 6*cosh(d*x + c) + 5)*arctan(5/4*cosh(d*x + c) + 5/4*sinh(d*x
 + c) + 3/4) + 12*cosh(d*x + c) + 12*sinh(d*x + c) + 20)/(5*d*cosh(d*x + c)
^2 + 5*d*sinh(d*x + c)^2 + 6*d*cosh(d*x + c) + 2*(5*d*cosh(d*x + c) + 3*d)*
sinh(d*x + c) + 5*d)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.22, size = 354, normalized size = 7.38

$$\begin{cases} \frac{x}{25\cosh^{2}\left(dx+\log\left(-3e^{-dx}-4ie^{-dx}\right)-\log\left(5\right)\right)+30\cosh\left(dx+\log\left(-3e^{-dx}-4ie^{-dx}\right)-\log\left(5\right)\right)+9} & \text{for}\: c=\log\left(\left(-\frac{3}{5}-\frac{4i}{5}\right)e^{-dx}\right) \\ -\frac{\log\left(-3e^{-dx}+4ie^{-dx}\right)}{25d\cosh^{2}\left(dx+\log\left(-3e^{-dx}+4ie^{-dx}\right)-\log\left(5\right)\right)+30d\cosh\left(dx+\log\left(-3e^{-dx}+4ie^{-dx}\right)-\log\left(5\right)\right)+9d}+\frac{\log\left(5\right)}{25d\cosh^{2}\left(dx+\log\left(-3e^{-dx}+4ie^{-dx}\right)-\log\left(5\right)\right)+30d\cosh\left(dx+\log\left(-3e^{-dx}+4ie^{-dx}\right)-\log\left(5\right)\right)+9d} & \text{for}\: c=\log\left(\left(-\frac{3}{5}+\frac{4i}{5}\right)e^{-dx}\right) \\ \frac{x}{\left(5\cosh\left(c\right)+3\right)^{2}} & \text{for}\: d=0 \\ -\frac{3\tanh^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{atan}\left(\frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}\right)}{32d\tanh^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+128d}+\frac{10\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{32d\tanh^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+128d}-\frac{12\operatorname{atan}\left(\frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}\right)}{32d\tanh^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+128d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c))**2,x)
```

```
[Out] Piecewise((x/(25*cosh(d*x + log(-3*exp(-d*x) - 4*I*exp(-d*x)) - log(5))**2
+ 30*cosh(d*x + log(-3*exp(-d*x) - 4*I*exp(-d*x)) - log(5)) + 9), Eq(c, log
((-3/5 - 4*I/5)*exp(-d*x)))), (-log(-3*exp(-d*x) + 4*I*exp(-d*x))/(25*d*cos
```

```
h(d*x + log(-3*exp(-d*x) + 4*I*exp(-d*x)) - log(5))**2 + 30*d*cosh(d*x + lo
g(-3*exp(-d*x) + 4*I*exp(-d*x)) - log(5)) + 9*d) + log(5)/(25*d*cosh(d*x +
log(-3*exp(-d*x) + 4*I*exp(-d*x)) - log(5))**2 + 30*d*cosh(d*x + log(-3*exp
(-d*x) + 4*I*exp(-d*x)) - log(5)) + 9*d), Eq(c, log((-3/5 + 4*I/5)*exp(-d*x
)))), (x/(5*cosh(c) + 3)**2, Eq(d, 0)), (-3*tanh(c/2 + d*x/2)**2*atan(tanh(
c/2 + d*x/2)/2)/(32*d*tanh(c/2 + d*x/2)**2 + 128*d) + 10*tanh(c/2 + d*x/2)/
(32*d*tanh(c/2 + d*x/2)**2 + 128*d) - 12*atan(tanh(c/2 + d*x/2)/2)/(32*d*ta
nh(c/2 + d*x/2)**2 + 128*d), True))
```

Giac [A]
time = 0.41, size = 54, normalized size = 1.12

$$-\frac{\frac{4\left(3\,e^{(dx+c)}+5\right)}{5\,e^{(2\,dx+2\,c)}+6\,e^{(dx+c)}+5}+3\,\arctan\left(\frac{5}{4}\,e^{(dx+c)}+\frac{3}{4}\right)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/32*(4*(3*e^(d*x + c) + 5)/(5*e^(2*d*x + 2*c) + 6*e^(d*x + c) + 5) + 3*ar
ctan(5/4*e^(d*x + c) + 3/4))/d
```

Mupad [B]
time = 0.94, size = 74, normalized size = 1.54

$$-\frac{\frac{3\,\mathrm{e}^{c+d\,x}}{8\,d}+\frac{5}{8\,d}}{6\,\mathrm{e}^{c+d\,x}+5\,\mathrm{e}^{2\,c+2\,d\,x}+5}-\frac{3\,\mathrm{atan}\left(\left(\frac{3}{4\,d}+\frac{5\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}}{4\,d}\right)\,\sqrt{d^2}\right)}{32\,\sqrt{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*cosh(c + d*x) + 3)^2,x)
```

```
[Out] - ((3*exp(c + d*x))/(8*d) + 5/(8*d))/(6*exp(c + d*x) + 5*exp(2*c + 2*d*x) +
 5) - (3*atan((3/(4*d) + (5*exp(d*x)*exp(c))/(4*d))*(d^2)^(1/2)))/(32*(d^2)
^(1/2))
```

3.73 $\int \frac{1}{(3+5\cosh(c+dx))^3}\,dx$

Optimal. Leaf size=73

$$\frac{43\text{ArcTan}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{1024d}+\frac{5\sinh(c+dx)}{32d(3+5\cosh(c+dx))^2}-\frac{45\sinh(c+dx)}{512d(3+5\cosh(c+dx))}$$

```
[Out] 43/1024*arctan(1/2*tanh(1/2*d*x+1/2*c))/d+5/32*sinh(d*x+c)/d/(3+5*cosh(d*x+
c))^2-45/512*sinh(d*x+c)/d/(3+5*cosh(d*x+c))
```

Rubi [A]
time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2743, 2833, 12, 2738, 212}

$$\frac{43\text{ArcTan}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{1024d}-\frac{45\sinh(c+dx)}{512d(5\cosh(c+dx)+3)}+\frac{5\sinh(c+dx)}{32d(5\cosh(c+dx)+3)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 5*Cosh[c + d*x])^(-3),x]
```

```
[Out] (43*ArcTan[Tanh[(c + d*x)/2]/2])/(1024*d) + (5*Sinh[c + d*x])/(32*d*(3 + 5*
Cosh[c + d*x])^2) - (45*Sinh[c + d*x])/(512*d*(3 + 5*Cosh[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
```

```
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(3+5\cosh(c+dx))^3}\,dx &= \frac{5\sinh(c+dx)}{32d(3+5\cosh(c+dx))^2}+\frac{1}{32}\int \frac{-6+5\cosh(c+dx)}{(3+5\cosh(c+dx))^2}\,dx\\ &= \frac{5\sinh(c+dx)}{32d(3+5\cosh(c+dx))^2}-\frac{45\sinh(c+dx)}{512d(3+5\cosh(c+dx))}+\frac{1}{512}\int \frac{43}{3+5\cosh(c+\\ &= \frac{5\sinh(c+dx)}{32d(3+5\cosh(c+dx))^2}-\frac{45\sinh(c+dx)}{512d(3+5\cosh(c+dx))}+\frac{43}{512}\int \frac{1}{3+5\cosh(c+\\ &= \frac{5\sinh(c+dx)}{32d(3+5\cosh(c+dx))^2}-\frac{45\sinh(c+dx)}{512d(3+5\cosh(c+dx))}-\frac{(43i)\text{Subst}\left(\int \frac{1}{8-2x^2}\,d\right.}{25}\\ &= \frac{43\tan^{-1}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{1024d}+\frac{5\sinh(c+dx)}{32d(3+5\cosh(c+dx))^2}-\frac{45\sinh(c+}{512d(3+5\cosh(}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 55, normalized size = 0.75

$$\frac{43\text{ArcTan}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)-\frac{10(11+45\cosh(c+dx))\sinh(c+dx)}{(3+5\cosh(c+dx))^2}}{1024d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 5*Cosh[c + d*x])^(-3),x]
```

```
[Out] (43*ArcTan[Tanh[(c + d*x)/2]/2] - (10*(11 + 45*Cosh[c + d*x])*Sinh[c + d*x]
)/(3 + 5*Cosh[c + d*x])^2)/(1024*d)
```

Maple [A]
time = 0.88, size = 62, normalized size = 0.85

method	result	size
derivativedivides	$\frac{\frac{-\frac{85\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128}-\frac{35\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{32}}{4\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)+4\right)^{2}}+\frac{43\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{1024}}{d}$	62
default	$\frac{\frac{-\frac{85\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128}-\frac{35\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{32}}{4\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)+4\right)^{2}}+\frac{43\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{1024}}{d}$	62
risch	$\frac{215\,\mathrm{e}^{3dx+3c}+387\,\mathrm{e}^{2dx+2c}+325\,\mathrm{e}^{dx+c}+225}{256d\left(5\,\mathrm{e}^{2dx+2c}+6\,\mathrm{e}^{dx+c}+5\right)^{2}}+\frac{43i\ln\left(\mathrm{e}^{dx+c}+\frac{3}{5}+\frac{4i}{5}\right)}{2048d}-\frac{43i\ln\left(\mathrm{e}^{dx+c}+\frac{3}{5}-\frac{4i}{5}\right)}{2048d}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+5*cosh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*(-85/128*tanh(1/2*d*x+1/2*c)^3-35/32*tanh(1/2*d*x+1/2*c))/(tanh(1/
2*d*x+1/2*c)^2+4)^2+43/1024*arctan(1/2*tanh(1/2*d*x+1/2*c)))
```

Maxima [A]
time = 0.47, size = 108, normalized size = 1.48

$$-\frac{43\arctan\left(\frac{5}{4}\,e^{(-dx-c)}+\frac{3}{4}\right)}{1024\,d}-\frac{325\,e^{(-dx-c)}+387\,e^{(-2\,dx-2\,c)}+215\,e^{(-3\,dx-3\,c)}+225}{256\,d(60\,e^{(-dx-c)}+86\,e^{(-2\,dx-2\,c)}+60\,e^{(-3\,dx-3\,c)}+25\,e^{(-4\,dx-4\,c)}+25)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -43/1024*arctan(5/4*e^(-d*x - c) + 3/4)/d - 1/256*(325*e^(-d*x - c) + 387*e
^(-2*d*x - 2*c) + 215*e^(-3*d*x - 3*c) + 225)/(d*(60*e^(-d*x - c) + 86*e^(-
2*d*x - 2*c) + 60*e^(-3*d*x - 3*c) + 25*e^(-4*d*x - 4*c) + 25))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 408 vs. 2(64) = 128.
time = 0.45, size = 408, normalized size = 5.59

$$\frac{860\cosh(dx+c)^3+516\,(5\cosh(dx+c)+3)\sinh(dx+c)^2+860\sinh(dx+c)^3+43\,(25\cosh(dx+c)^4+20\,(5\cosh(dx+c)+3)\sinh(dx+c)^3+25\sinh(dx+c)^4+60\cosh(dx+c)^3+2\,(75\cosh(dx+c)^2+90\cosh(dx+c)+43)\sinh(dx+c)^2+86\cosh(dx+c)^2+4\,(25\cosh(dx+c)^3+45\cosh(dx+c)^2+43\cosh(dx+c)+15)\sinh(dx+c)+60\cosh(dx+c)+25)\arctan\left(\frac{5}{4}\cosh(dx+c)+\frac{5}{4}\sinh(dx+c)+\frac{3}{4}\right)+1548\cosh(dx+c)^2+4\,(645\cosh(dx+c)^2+774\cosh(dx+c)+325)\sinh(dx+c)+1300\cosh(dx+c)+900}{1024\,(25\,d\cosh(dx+c)^4+25\,d\sinh(dx+c)^4+60\,d\cosh(dx+c)^3+20\,(5\,d\cosh(dx+c)+3\,d)\sinh(dx+c)^3+86\,d\cosh(dx+c)^2+2\,(75\,d\cosh(dx+c)^2+90\,d\cosh(dx+c)+43\,d)\sinh(dx+c)^2+60\,d\cosh(dx+c)+4\,(25\,d\cosh(dx+c)^3+45\,d\cosh(dx+c)^2+43\,d\cosh(dx+c)+15\,d)\sinh(dx+c)+25\,d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/1024*(860*cosh(d*x + c)^3 + 516*(5*cosh(d*x + c) + 3)*sinh(d*x + c)^2 + 8
60*sinh(d*x + c)^3 + 43*(25*cosh(d*x + c)^4 + 20*(5*cosh(d*x + c) + 3)*sinh
(d*x + c)^3 + 25*sinh(d*x + c)^4 + 60*cosh(d*x + c)^3 + 2*(75*cosh(d*x + c)
^2 + 90*cosh(d*x + c) + 43)*sinh(d*x + c)^2 + 86*cosh(d*x + c)^2 + 4*(25*co
sh(d*x + c)^3 + 45*cosh(d*x + c)^2 + 43*cosh(d*x + c) + 15)*sinh(d*x + c) +
 60*cosh(d*x + c) + 25)*arctan(5/4*cosh(d*x + c) + 5/4*sinh(d*x + c) + 3/4)
```

```
 + 1548*cosh(d*x + c)^2 + 4*(645*cosh(d*x + c)^2 + 774*cosh(d*x + c) + 325)
*sinh(d*x + c) + 1300*cosh(d*x + c) + 900)/(25*d*cosh(d*x + c)^4 + 25*d*sin
h(d*x + c)^4 + 60*d*cosh(d*x + c)^3 + 20*(5*d*cosh(d*x + c) + 3*d)*sinh(d*x
 + c)^3 + 86*d*cosh(d*x + c)^2 + 2*(75*d*cosh(d*x + c)^2 + 90*d*cosh(d*x +
c) + 43*d)*sinh(d*x + c)^2 + 60*d*cosh(d*x + c) + 4*(25*d*cosh(d*x + c)^3 +
 45*d*cosh(d*x + c)^2 + 43*d*cosh(d*x + c) + 15*d)*sinh(d*x + c) + 25*d)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.36, size = 600, normalized size = 8.22

$$\begin{cases} \frac{x}{125\cosh^3\left(dx+\log\left(-3e^{-dx}-4ie^{-dx}\right)-\log(5)\right)+225\cosh^2\left(dx+\log\left(-3e^{-dx}-4ie^{-dx}\right)-\log(5)\right)+135\cosh\left(dx+\log\left(-3e^{-dx}-4ie^{-dx}\right)-\log(5)\right)+27} & \text{for}\ c=\log\left(\left(-\frac{3}{5}-\frac{4i}{5}\right)e^{-dx}\right) \\ -\frac{\log\left(-3e^{-dx}+4ie^{-dx}\right)}{125d\cosh^3\left(dx+\log\left(-3e^{-dx}+4ie^{-dx}\right)-\log(5)\right)+225d\cosh^2\left(dx+\log\left(-3e^{-dx}+4ie^{-dx}\right)-\log(5)\right)+135d\cosh\left(dx+\log\left(-3e^{-dx}+4ie^{-dx}\right)-\log(5)\right)+27d}+\frac{\log(5)}{125d\cosh^3\left(dx+\log\left(-3e^{-dx}+4ie^{-dx}\right)-\log(5)\right)+225d\cosh^2\left(dx+\log\left(-3e^{-dx}+4ie^{-dx}\right)-\log(5)\right)+135d\cosh\left(dx+\log\left(-3e^{-dx}+4ie^{-dx}\right)-\log(5)\right)+27d} & \text{for}\ c=\log\left(\left(-\frac{3}{5}+\frac{4i}{5}\right)e^{-dx}\right) \\ \frac{x}{(5\cosh(c)+3)^3} & \text{for}\ d=0 \\ \frac{43\tanh^4\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{atan}\left(\frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}\right)}{1024d\tanh^4\left(\frac{c}{2}+\frac{dx}{2}\right)+8192d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)+16384d}-\frac{170\tanh^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{1024d\tanh^4\left(\frac{c}{2}+\frac{dx}{2}\right)+8192d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)+16384d}+\frac{344\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{atan}\left(\frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}\right)}{1024d\tanh^4\left(\frac{c}{2}+\frac{dx}{2}\right)+8192d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)+16384d}-\frac{280\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{1024d\tanh^4\left(\frac{c}{2}+\frac{dx}{2}\right)+8192d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)+16384d}+\frac{688\operatorname{atan}\left(\frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}\right)}{1024d\tanh^4\left(\frac{c}{2}+\frac{dx}{2}\right)+8192d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)+16384d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c))**3,x)
```

```
[Out] Piecewise((x/(125*cosh(d*x + log(-3*exp(-d*x) - 4*I*exp(-d*x)) - log(5))**3
 + 225*cosh(d*x + log(-3*exp(-d*x) - 4*I*exp(-d*x)) - log(5))**2 + 135*cosh
(d*x + log(-3*exp(-d*x) - 4*I*exp(-d*x)) - log(5)) + 27), Eq(c, log((-3/5 -
 4*I/5)*exp(-d*x)))), (-log(-3*exp(-d*x) + 4*I*exp(-d*x))/(125*d*cosh(d*x +
 log(-3*exp(-d*x) + 4*I*exp(-d*x)) - log(5))**3 + 225*d*cosh(d*x + log(-3*e
xp(-d*x) + 4*I*exp(-d*x)) - log(5))**2 + 135*d*cosh(d*x + log(-3*exp(-d*x)
+ 4*I*exp(-d*x)) - log(5)) + 27*d) + log(5)/(125*d*cosh(d*x + log(-3*exp(-d
*x) + 4*I*exp(-d*x)) - log(5))**3 + 225*d*cosh(d*x + log(-3*exp(-d*x) + 4*I
*exp(-d*x)) - log(5))**2 + 135*d*cosh(d*x + log(-3*exp(-d*x) + 4*I*exp(-d*x
)) - log(5)) + 27*d), Eq(c, log((-3/5 + 4*I/5)*exp(-d*x)))), (x/(5*cosh(c)
+ 3)**3, Eq(d, 0)), (43*tanh(c/2 + d*x/2)**4*atan(tanh(c/2 + d*x/2)/2)/(102
4*d*tanh(c/2 + d*x/2)**4 + 8192*d*tanh(c/2 + d*x/2)**2 + 16384*d) - 170*tan
h(c/2 + d*x/2)**3/(1024*d*tanh(c/2 + d*x/2)**4 + 8192*d*tanh(c/2 + d*x/2)**
2 + 16384*d) + 344*tanh(c/2 + d*x/2)**2*atan(tanh(c/2 + d*x/2)/2)/(1024*d*t
anh(c/2 + d*x/2)**4 + 8192*d*tanh(c/2 + d*x/2)**2 + 16384*d) - 280*tanh(c/2
 + d*x/2)/(1024*d*tanh(c/2 + d*x/2)**4 + 8192*d*tanh(c/2 + d*x/2)**2 + 1638
4*d) + 688*atan(tanh(c/2 + d*x/2)/2)/(1024*d*tanh(c/2 + d*x/2)**4 + 8192*d*
tanh(c/2 + d*x/2)**2 + 16384*d), True))
```

Giac [A]
time = 0.41, size = 76, normalized size = 1.04

$$\frac{\frac{4\left(215\,e^{(3\,dx+3\,c)}+387\,e^{(2\,dx+2\,c)}+325\,e^{(dx+c)}+225\right)}{\left(5\,e^{(2\,dx+2\,c)}+6\,e^{(dx+c)}+5\right)^2}+43\,\arctan\left(\frac{5}{4}\,e^{(dx+c)}+\frac{3}{4}\right)}{1024\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/1024*(4*(215*e^(3*d*x + 3*c) + 387*e^(2*d*x + 2*c) + 325*e^(d*x + c) + 22
5)/(5*e^(2*d*x + 2*c) + 6*e^(d*x + c) + 5)^2 + 43*arctan(5/4*e^(d*x + c) +
3/4))/d
```

Mupad [B]
time = 0.96, size = 137, normalized size = 1.88

$$\frac{\frac{43\,e^{c+d\,x}}{256\,d}+\frac{129}{1280\,d}}{6\,e^{c+d\,x}+5\,e^{2\,c+2\,d\,x}+5}-\frac{\frac{7\,e^{c+d\,x}}{40\,d}-\frac{3}{8\,d}}{60\,e^{c+d\,x}+86\,e^{2\,c+2\,d\,x}+60\,e^{3\,c+3\,d\,x}+25\,e^{4\,c+4\,d\,x}+25}+\frac{43\,\mathrm{atan}\left(\left(\frac{3}{4\,d}+\frac{5\,e^{d\,x}\,e^{c}}{4\,d}\right)\,\sqrt{d^2}\right)}{1024\,\sqrt{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*cosh(c + d*x) + 3)^3,x)
```

```
[Out] ((43*exp(c + d*x))/(256*d) + 129/(1280*d))/(6*exp(c + d*x) + 5*exp(2*c + 2*
d*x) + 5) - ((7*exp(c + d*x))/(40*d) - 3/(8*d))/(60*exp(c + d*x) + 86*exp(2
*c + 2*d*x) + 60*exp(3*c + 3*d*x) + 25*exp(4*c + 4*d*x) + 25) + (43*atan((3
/(4*d) + (5*exp(d*x)*exp(c))/(4*d))*(d^2)^(1/2)))/(1024*(d^2)^(1/2))
```

3.74 $\int \frac{1}{(3+5\cosh(c+dx))^4}\,dx$

Optimal. Leaf size=98

$$-\frac{279\text{ArcTan}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{16384d}+\frac{5\sinh(c+dx)}{48d(3+5\cosh(c+dx))^3}-\frac{25\sinh(c+dx)}{512d(3+5\cosh(c+dx))^2}+\frac{995\sinh(c+dx)}{24576d(3+5\cosh(c+dx))}$$

```
[Out] -279/16384*arctan(1/2*tanh(1/2*d*x+1/2*c))/d+5/48*sinh(d*x+c)/d/(3+5*cosh(d
*x+c))^3-25/512*sinh(d*x+c)/d/(3+5*cosh(d*x+c))^2+995/24576*sinh(d*x+c)/d/(
3+5*cosh(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2743, 2833, 12, 2738, 212}

$$-\frac{279\text{ArcTan}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{16384d}+\frac{995\sinh(c+dx)}{24576d(5\cosh(c+dx)+3)}-\frac{25\sinh(c+dx)}{512d(5\cosh(c+dx)+3)^2}+\frac{5\sinh(c+dx)}{48d(5\cosh(c+dx)+3)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 5*Cosh[c + d*x])^(-4),x]
```

```
[Out] (-279*ArcTan[Tanh[(c + d*x)/2]/2])/(16384*d) + (5*Sinh[c + d*x])/(48*d*(3 +
 5*Cosh[c + d*x])^3) - (25*Sinh[c + d*x])/(512*d*(3 + 5*Cosh[c + d*x])^2) +
 (995*Sinh[c + d*x])/(24576*d*(3 + 5*Cosh[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
```

```
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(3+5\cosh(c+dx))^4}\,dx &= \frac{5\sinh(c+dx)}{48d(3+5\cosh(c+dx))^3}+\frac{1}{48}\int \frac{-9+10\cosh(c+dx)}{(3+5\cosh(c+dx))^3}\,dx \\
&= \frac{5\sinh(c+dx)}{48d(3+5\cosh(c+dx))^3}-\frac{25\sinh(c+dx)}{512d(3+5\cosh(c+dx))^2}+\frac{\int \frac{154-75\cosh(c+dx)}{(3+5\cosh(c+dx))^2}\,d}{1536} \\
&= \frac{5\sinh(c+dx)}{48d(3+5\cosh(c+dx))^3}-\frac{25\sinh(c+dx)}{512d(3+5\cosh(c+dx))^2}+\frac{995\sinh(c+}{24576d(3+5\cosh(} \\
&= \frac{5\sinh(c+dx)}{48d(3+5\cosh(c+dx))^3}-\frac{25\sinh(c+dx)}{512d(3+5\cosh(c+dx))^2}+\frac{995\sinh(c+}{24576d(3+5\cosh(} \\
&= \frac{5\sinh(c+dx)}{48d(3+5\cosh(c+dx))^3}-\frac{25\sinh(c+dx)}{512d(3+5\cosh(c+dx))^2}+\frac{995\sinh(c+}{24576d(3+5\cosh(} \\
&= -\frac{279\tan^{-1}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{16384d}+\frac{5\sinh(c+dx)}{48d(3+5\cosh(c+dx))^3}-\frac{25\sinh}{512d(3+5c}
\end{aligned}
$$

Mathematica [A]
time = 0.19, size = 65, normalized size = 0.66

$$
\frac{-837\text{ArcTan}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)+\frac{5(8141+9540\cosh(c+dx)+4975\cosh(2(c+dx)))\sinh(c+dx)}{(3+5\cosh(c+dx))^3}}{49152d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 5*Cosh[c + d*x])^(-4),x]
```

```
[Out] (-837*ArcTan[Tanh[(c + d*x)/2]/2] + (5*(8141 + 9540*Cosh[c + d*x] + 4975*Co
sh[2*(c + d*x)])*Sinh[c + d*x])/(3 + 5*Cosh[c + d*x])^3)/(49152*d)
```

Maple [A]
time = 0.86, size = 75, normalized size = 0.77

method	result
derivativedivides	$-\frac{-\frac{-\frac{745\left(\tanh^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1024}-\frac{265\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96}-\frac{295\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{64}}{8\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)+4\right)^{3}}-\frac{279\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{16384}}{d}$
default	$-\frac{-\frac{-\frac{745\left(\tanh^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1024}-\frac{265\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96}-\frac{295\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{64}}{8\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)+4\right)^{3}}-\frac{279\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{16384}}{d}$
risch	$-\frac{20925\,e^{5dx+5c}+62775\,e^{4dx+4c}+111042\,e^{3dx+3c}+119310\,e^{2dx+2c}+68625\,e^{dx+c}+24875}{12288d\left(5\,e^{2dx+2c}+6\,e^{dx+c}+5\right)^{3}}+\frac{279i\ln\left(e^{dx+c}+\frac{3}{5}-\frac{4i}{5}\right)}{32768d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+5*cosh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/8*(-745/1024*tanh(1/2*d*x+1/2*c)^5-265/96*tanh(1/2*d*x+1/2*c)^3-295
/64*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^2+4)^3-279/16384*arctan(1/2*t
anh(1/2*d*x+1/2*c)))
```

Maxima [A]
time = 0.48, size = 152, normalized size = 1.55

$$\frac{279\arctan\left(\frac{5}{4}e^{(-dx-c)}+\frac{3}{4}\right)}{16384\,d}+\frac{68625\,e^{(-dx-c)}+119310\,e^{(-2\,dx-2\,c)}+111042\,e^{(-3\,dx-3\,c)}+62775\,e^{(-4\,dx-4\,c)}+20925\,e^{(-5\,dx-5\,c)}+24875}{12288\,d(450\,e^{(-dx-c)}+915\,e^{(-2\,dx-2\,c)}+1116\,e^{(-3\,dx-3\,c)}+915\,e^{(-4\,dx-4\,c)}+450\,e^{(-5\,dx-5\,c)}+125\,e^{(-6\,dx-6\,c)}+125)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 279/16384*arctan(5/4*e^(-d*x - c) + 3/4)/d + 1/12288*(68625*e^(-d*x - c) +
119310*e^(-2*d*x - 2*c) + 111042*e^(-3*d*x - 3*c) + 62775*e^(-4*d*x - 4*c)
+ 20925*e^(-5*d*x - 5*c) + 24875)/(d*(450*e^(-d*x - c) + 915*e^(-2*d*x - 2*
c) + 1116*e^(-3*d*x - 3*c) + 915*e^(-4*d*x - 4*c) + 450*e^(-5*d*x - 5*c) +
125*e^(-6*d*x - 6*c) + 125))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 793 vs. $2(87) = 174$.
time = 0.37, size = 793, normalized size = 8.09

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/49152*(83700*cosh(d*x + c)^5 + 83700*(5*cosh(d*x + c) + 3)*sinh(d*x + c)
^4 + 83700*sinh(d*x + c)^5 + 251100*cosh(d*x + c)^4 + 2232*(375*cosh(d*x +
```

c)^2 + 450*cosh(d*x + c) + 199)*sinh(d*x + c)^3 + 444168*cosh(d*x + c)^3 + 24*(34875*cosh(d*x + c)^3 + 62775*cosh(d*x + c)^2 + 55521*cosh(d*x + c) + 19885)*sinh(d*x + c)^2 + 837*(125*cosh(d*x + c)^6 + 150*(5*cosh(d*x + c) + 3)*sinh(d*x + c)^5 + 125*sinh(d*x + c)^6 + 450*cosh(d*x + c)^5 + 15*(125*cosh(d*x + c)^2 + 150*cosh(d*x + c) + 61)*sinh(d*x + c)^4 + 915*cosh(d*x + c)^4 + 4*(625*cosh(d*x + c)^3 + 1125*cosh(d*x + c)^2 + 915*cosh(d*x + c) + 279)*sinh(d*x + c)^3 + 1116*cosh(d*x + c)^3 + 3*(625*cosh(d*x + c)^4 + 1500*cosh(d*x + c)^3 + 1830*cosh(d*x + c)^2 + 1116*cosh(d*x + c) + 305)*sinh(d*x + c)^2 + 915*cosh(d*x + c)^2 + 6*(125*cosh(d*x + c)^5 + 375*cosh(d*x + c)^4 + 610*cosh(d*x + c)^3 + 558*cosh(d*x + c)^2 + 305*cosh(d*x + c) + 75)*sinh(d*x + c) + 450*cosh(d*x + c) + 125)*arctan(5/4*cosh(d*x + c) + 5/4*sinh(d*x + c) + 3/4) + 477240*cosh(d*x + c)^2 + 12*(34875*cosh(d*x + c)^4 + 83700*cosh(d*x + c)^3 + 111042*cosh(d*x + c)^2 + 79540*cosh(d*x + c) + 22875)*sinh(d*x + c) + 274500*cosh(d*x + c) + 99500)/(125*d*cosh(d*x + c)^6 + 125*d*sinh(d*x + c)^6 + 450*d*cosh(d*x + c)^5 + 150*(5*d*cosh(d*x + c) + 3*d)*sinh(d*x + c)^5 + 915*d*cosh(d*x + c)^4 + 15*(125*d*cosh(d*x + c)^2 + 150*d*cosh(d*x + c) + 61*d)*sinh(d*x + c)^4 + 1116*d*cosh(d*x + c)^3 + 4*(625*d*cosh(d*x + c)^3 + 1125*d*cosh(d*x + c)^2 + 915*d*cosh(d*x + c) + 279*d)*sinh(d*x + c)^3 + 915*d*cosh(d*x + c)^2 + 3*(625*d*cosh(d*x + c)^4 + 1500*d*cosh(d*x + c)^3 + 1830*d*cosh(d*x + c)^2 + 1116*d*cosh(d*x + c) + 305*d)*sinh(d*x + c)^2 + 450*d*cosh(d*x + c) + 6*(125*d*cosh(d*x + c)^5 + 375*d*cosh(d*x + c)^4 + 610*d*cosh(d*x + c)^3 + 558*d*cosh(d*x + c)^2 + 305*d*cosh(d*x + c) + 75*d)*sinh(d*x + c) + 125*d)

Sympy [C] Result contains complex when optimal does not.
time = 4.95, size = 908, normalized size = 9.27

$$\begin{cases} \frac{x}{625\cosh^4\left(dx+\log\left(-3e^{-dx}-4ie^{-dx}\right)-\log(5)\right)+1500\cosh^3\left(dx+\log\left(-3e^{-dx}-4ie^{-dx}\right)-\log(5)\right)+1350\cosh^2\left(dx+\log\left(-3e^{-dx}-4ie^{-dx}\right)-\log(5)\right)+540\cosh\left(dx+\log\left(-3e^{-dx}-4ie^{-dx}\right)-\log(5)\right)+81} & \text{for}\: c=\log\left(\left(-\frac{3}{5}-\frac{4i}{5}\right)e^{-dx}\right) \\ -\frac{\log\left(-3e^{-dx}+4ie^{-dx}\right)}{625d\cosh^4\left(dx+\log\left(-3e^{-dx}+4ie^{-dx}\right)-\log(5)\right)+1500d\cosh^3\left(dx+\log\left(-3e^{-dx}+4ie^{-dx}\right)-\log(5)\right)+1350d\cosh^2\left(dx+\log\left(-3e^{-dx}+4ie^{-dx}\right)-\log(5)\right)+540d\cosh\left(dx+\log\left(-3e^{-dx}+4ie^{-dx}\right)-\log(5)\right)+81d}+\frac{\log(5)}{625d\cosh^4\left(dx+\log\left(-3e^{-dx}+4ie^{-dx}\right)-\log(5)\right)+1500d\cosh^3\left(dx+\log\left(-3e^{-dx}+4ie^{-dx}\right)-\log(5)\right)+1350d\cosh^2\left(dx+\log\left(-3e^{-dx}+4ie^{-dx}\right)-\log(5)\right)+540d\cosh\left(dx+\log\left(-3e^{-dx}+4ie^{-dx}\right)-\log(5)\right)+81d} & \text{for}\: c=\log\left(\left(-\frac{3}{5}+\frac{4i}{5}\right)e^{-dx}\right) \\ \frac{x}{\left(5\cosh(c)+3\right)^4} & \text{for}\: d=0 \\ -\frac{837\tanh^6\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{atan}\left(\frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}\right)}{49152d\tanh^6\left(\frac{c}{2}+\frac{dx}{2}\right)+589824d\tanh^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2359296d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3145728d}+\frac{4470\tanh^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{49152d\tanh^6\left(\frac{c}{2}+\frac{dx}{2}\right)+589824d\tanh^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2359296d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3145728d}-\frac{10044\tanh^4\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{atan}\left(\frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}\right)}{49152d\tanh^6\left(\frac{c}{2}+\frac{dx}{2}\right)+589824d\tanh^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2359296d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3145728d}+\frac{16960\tanh^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{49152d\tanh^6\left(\frac{c}{2}+\frac{dx}{2}\right)+589824d\tanh^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2359296d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3145728d}-\frac{40176\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{atan}\left(\frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}\right)}{49152d\tanh^6\left(\frac{c}{2}+\frac{dx}{2}\right)+589824d\tanh^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2359296d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3145728d}+\frac{28320\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{49152d\tanh^6\left(\frac{c}{2}+\frac{dx}{2}\right)+589824d\tanh^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2359296d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3145728d}-\frac{53568\operatorname{atan}\left(\frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}\right)}{49152d\tanh^6\left(\frac{c}{2}+\frac{dx}{2}\right)+589824d\tanh^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2359296d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3145728d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c))**4,x)
```

```
[Out] Piecewise((x/(625*cosh(d*x + log(-3*exp(-d*x) - 4*I*exp(-d*x)) - log(5))**4
 + 1500*cosh(d*x + log(-3*exp(-d*x) - 4*I*exp(-d*x)) - log(5))**3 + 1350*co
sh(d*x + log(-3*exp(-d*x) - 4*I*exp(-d*x)) - log(5))**2 + 540*cosh(d*x + lo
g(-3*exp(-d*x) - 4*I*exp(-d*x)) - log(5)) + 81), Eq(c, log((-3/5 - 4*I/5)*e
xp(-d*x)))), (-log(-3*exp(-d*x) + 4*I*exp(-d*x))/(625*d*cosh(d*x + log(-3*e
xp(-d*x) + 4*I*exp(-d*x)) - log(5))**4 + 1500*d*cosh(d*x + log(-3*exp(-d*x)
 + 4*I*exp(-d*x)) - log(5))**3 + 1350*d*cosh(d*x + log(-3*exp(-d*x) + 4*I*e
xp(-d*x)) - log(5))**2 + 540*d*cosh(d*x + log(-3*exp(-d*x) + 4*I*exp(-d*x))
 - log(5)) + 81*d) + log(5)/(625*d*cosh(d*x + log(-3*exp(-d*x) + 4*I*exp(-d
*x)) - log(5))**4 + 1500*d*cosh(d*x + log(-3*exp(-d*x) + 4*I*exp(-d*x)) - l
og(5))**3 + 1350*d*cosh(d*x + log(-3*exp(-d*x) + 4*I*exp(-d*x)) - log(5))**
2 + 540*d*cosh(d*x + log(-3*exp(-d*x) + 4*I*exp(-d*x)) - log(5)) + 81*d), E
q(c, log((-3/5 + 4*I/5)*exp(-d*x)))), (x/(5*cosh(c) + 3)**4, Eq(d, 0)), (-8
```

```
37*tanh(c/2 + d*x/2)**6*atan(tanh(c/2 + d*x/2)/2)/(49152*d*tanh(c/2 + d*x/2
)**6 + 589824*d*tanh(c/2 + d*x/2)**4 + 2359296*d*tanh(c/2 + d*x/2)**2 + 314
5728*d) + 4470*tanh(c/2 + d*x/2)**5/(49152*d*tanh(c/2 + d*x/2)**6 + 589824*
d*tanh(c/2 + d*x/2)**4 + 2359296*d*tanh(c/2 + d*x/2)**2 + 3145728*d) - 1004
4*tanh(c/2 + d*x/2)**4*atan(tanh(c/2 + d*x/2)/2)/(49152*d*tanh(c/2 + d*x/2)
**6 + 589824*d*tanh(c/2 + d*x/2)**4 + 2359296*d*tanh(c/2 + d*x/2)**2 + 3145
728*d) + 16960*tanh(c/2 + d*x/2)**3/(49152*d*tanh(c/2 + d*x/2)**6 + 589824*
d*tanh(c/2 + d*x/2)**4 + 2359296*d*tanh(c/2 + d*x/2)**2 + 3145728*d) - 4017
6*tanh(c/2 + d*x/2)**2*atan(tanh(c/2 + d*x/2)/2)/(49152*d*tanh(c/2 + d*x/2)
**6 + 589824*d*tanh(c/2 + d*x/2)**4 + 2359296*d*tanh(c/2 + d*x/2)**2 + 3145
728*d) + 28320*tanh(c/2 + d*x/2)/(49152*d*tanh(c/2 + d*x/2)**6 + 589824*d*t
anh(c/2 + d*x/2)**4 + 2359296*d*tanh(c/2 + d*x/2)**2 + 3145728*d) - 53568*a
tan(tanh(c/2 + d*x/2)/2)/(49152*d*tanh(c/2 + d*x/2)**6 + 589824*d*tanh(c/2
+ d*x/2)**4 + 2359296*d*tanh(c/2 + d*x/2)**2 + 3145728*d), True))
```

Giac [A]
time = 0.41, size = 98, normalized size = 1.00

$$-\frac{\frac{4\,\left(20925\,e^{\left(5\,dx+5\,c\right)}+62775\,e^{\left(4\,dx+4\,c\right)}+111042\,e^{\left(3\,dx+3\,c\right)}+119310\,e^{\left(2\,dx+2\,c\right)}+68625\,e^{\left(dx+c\right)}+24875\right)}{\left(5\,e^{\left(2\,dx+2\,c\right)}+6\,e^{\left(dx+c\right)}+5\right)^{3}}+837\,\arctan\left(\frac{5}{4}\,e^{\left(dx+c\right)}+\frac{3}{4}\right)}{49152\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/49152*(4*(20925*e^(5*d*x + 5*c) + 62775*e^(4*d*x + 4*c) + 111042*e^(3*d*
x + 3*c) + 119310*e^(2*d*x + 2*c) + 68625*e^(d*x + c) + 24875)/(5*e^(2*d*x
+ 2*c) + 6*e^(d*x + c) + 5)^3 + 837*arctan(5/4*e^(d*x + c) + 3/4))/d
```

Mupad [B]
time = 0.95, size = 223, normalized size = 2.28

$$\frac{\frac{39\,\mathrm{e}^{c+d\,x}}{50\,d}+\frac{7}{30\,d}}{450\,\mathrm{e}^{c+d\,x}+915\,\mathrm{e}^{2\,c+2\,d\,x}+1116\,\mathrm{e}^{3\,c+3\,d\,x}+915\,\mathrm{e}^{4\,c+4\,d\,x}+450\,\mathrm{e}^{5\,c+5\,d\,x}+125\,\mathrm{e}^{6\,c+6\,d\,x}+125}-\frac{\frac{93\,\mathrm{e}^{c+d\,x}}{640\,d}+\frac{791}{3200\,d}}{60\,\mathrm{e}^{c+d\,x}+86\,\mathrm{e}^{2\,c+2\,d\,x}+60\,\mathrm{e}^{3\,c+3\,d\,x}+25\,\mathrm{e}^{4\,c+4\,d\,x}+25}-\frac{279\,\mathrm{atan}\left(\left(\frac{3}{4\,d}+\frac{5\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}}{4\,d}\right)\,\sqrt{d^2}\right)}{16384\,\sqrt{d^2}}-\frac{\frac{279\,\mathrm{e}^{c+d\,x}}{4096\,d}+\frac{837}{20480\,d}}{6\,\mathrm{e}^{c+d\,x}+5\,\mathrm{e}^{2\,c+2\,d\,x}+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*cosh(c + d*x) + 3)^4,x)
```

```
[Out] ((39*exp(c + d*x))/(50*d) + 7/(30*d))/(450*exp(c + d*x) + 915*exp(2*c + 2*d
*x) + 1116*exp(3*c + 3*d*x) + 915*exp(4*c + 4*d*x) + 450*exp(5*c + 5*d*x) +
 125*exp(6*c + 6*d*x) + 125) - ((93*exp(c + d*x))/(640*d) + 791/(3200*d))/(
60*exp(c + d*x) + 86*exp(2*c + 2*d*x) + 60*exp(3*c + 3*d*x) + 25*exp(4*c +
4*d*x) + 25) - (279*atan((3/(4*d) + (5*exp(d*x)*exp(c))/(4*d))*(d^2)^(1/2))
)/(16384*(d^2)^(1/2)) - ((279*exp(c + d*x))/(4096*d) + 837/(20480*d))/(6*ex
p(c + d*x) + 5*exp(2*c + 2*d*x) + 5)
```

3.75 $\int \frac{1}{5+3\cosh(c+dx)}\,dx$

Optimal. Leaf size=31

$$\frac{x}{4}-\frac{\tanh^{-1}\left(\frac{\sinh(c+dx)}{3+\cosh(c+dx)}\right)}{2d}$$

```
[Out] 1/4*x-1/2*arctanh(sinh(d*x+c)/(3+cosh(d*x+c)))/d
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{2736\}$

$$\frac{x}{4}-\frac{\tanh^{-1}\left(\frac{\sinh(c+dx)}{\cosh(c+dx)+3}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 3*Cosh[c + d*x])^(-1),x]
```

```
[Out] x/4 - ArcTanh[Sinh[c + d*x]/(3 + Cosh[c + d*x])]/(2*d)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rubi steps

$$\int \frac{1}{5+3\cosh(c+dx)}\,dx=\frac{x}{4}-\frac{\tanh^{-1}\left(\frac{\sinh(c+dx)}{3+\cosh(c+dx)}\right)}{2d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 77 vs. 2(31) = 62.
time = 0.02, size = 77, normalized size = 2.48

$$-\frac{\log\left(2\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)-\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{4d}+\frac{\log\left(2\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)+\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 3*Cosh[c + d*x])^(-1),x]
```

```
[Out] -1/4*Log[2*Cosh[c/2 + (d*x)/2] - Sinh[c/2 + (d*x)/2]]/d + Log[2*Cosh[c/2 +
(d*x)/2] + Sinh[c/2 + (d*x)/2]]/(4*d)
```

Maple [A]
time = 0.85, size = 34, normalized size = 1.10

method	result	size
risch	$-\frac{\ln(3+{\mathrm e}^{d x +c})}{4 d}+\frac{\ln({\mathrm e}^{d x +c}+\frac{1}{3})}{4 d}$	30
derivativedivides	$\frac{\frac{\ln\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+2\right)}{4}-\frac{\ln\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-2\right)}{4}}{d}$	34
default	$\frac{\frac{\ln\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+2\right)}{4}-\frac{\ln\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-2\right)}{4}}{d}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+3*cosh(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(1/4*ln(tanh(1/2*d*x+1/2*c)+2)-1/4*ln(tanh(1/2*d*x+1/2*c)-2))
```

Maxima [A]
time = 0.27, size = 37, normalized size = 1.19

$$-\frac{\log\left(3\,e^{(-dx-c)}+1\right)}{4\,d}+\frac{\log\left(e^{(-dx-c)}+3\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c)),x, algorithm="maxima")
[Out] -1/4*log(3*e^(-d*x - c) + 1)/d + 1/4*log(e^(-d*x - c) + 3)/d
```

Fricas [A]
time = 0.44, size = 42, normalized size = 1.35

$$\frac{\log\left(3\,\cosh\left(dx+c\right)+3\,\sinh\left(dx+c\right)+1\right)-\log\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)+3\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c)),x, algorithm="fricas")
[Out] 1/4*(log(3*cosh(d*x + c) + 3*sinh(d*x + c) + 1) - log(cosh(d*x + c) + sinh(
d*x + c) + 3))/d
```

Sympy [A]
time = 0.35, size = 41, normalized size = 1.32

$$\begin{cases} -\frac{\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-2\right)}{4d}+\frac{\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)+2\right)}{4d} & \text{for}\: d\neq 0\\ \frac{x}{3\cosh\left(c\right)+5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c)),x)
```

```
[Out] Piecewise((-log(tanh(c/2 + d*x/2) - 2)/(4*d) + log(tanh(c/2 + d*x/2) + 2)/(
4*d), Ne(d, 0)), (x/(3*cosh(c) + 5), True))
```

Giac [A]
time = 0.40, size = 28, normalized size = 0.90

$$\frac{\log\left(3\,e^{(dx+c)}+1\right)-\log\left(e^{(dx+c)}+3\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(log(3*e^(d*x + c) + 1) - log(e^(d*x + c) + 3))/d
```

Mupad [B]
time = 0.94, size = 40, normalized size = 1.29

$$-\frac{\mathrm{atan}\left(\frac{5\,\sqrt{-d^2}+3\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{-d^2}}{4\,d}\right)}{2\,\sqrt{-d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*cosh(c + d*x) + 5),x)
```

```
[Out] -atan((5*(-d^2)^(1/2) + 3*exp(d*x)*exp(c)*(-d^2)^(1/2))/(4*d))/(2*(-d^2)^(1
/2))
```

3.76 $\int \frac{1}{(5+3\cosh(c+dx))^2}\,dx$

Optimal. Leaf size=56

$$\frac{5x}{64}-\frac{5\tanh^{-1}\left(\frac{\sinh(c+dx)}{3+\cosh(c+dx)}\right)}{32d}-\frac{3\sinh(c+dx)}{16d(5+3\cosh(c+dx))}$$

```
[Out] 5/64*x-5/32*arctanh(sinh(d*x+c)/(3+cosh(d*x+c)))/d-3/16*sinh(d*x+c)/d/(5+3*
cosh(d*x+c))
```

Rubi [A]
time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = {2743, 12, 2736}

$$-\frac{3\sinh(c+dx)}{16d(3\cosh(c+dx)+5)}-\frac{5\tanh^{-1}\left(\frac{\sinh(c+dx)}{\cosh(c+dx)+3}\right)}{32d}+\frac{5x}{64}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 3*Cosh[c + d*x])^(-2),x]
```

```
[Out] (5*x)/64 - (5*ArcTanh[Sinh[c + d*x]/(3 + Cosh[c + d*x])])/(32*d) - (3*Sinh[
c + d*x])/(16*d*(5 + 3*Cosh[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(5+3\cosh(c+dx))^2}\,dx = -\frac{3\sinh(c+dx)}{16d(5+3\cosh(c+dx))} - \frac{1}{16}\int -\frac{5}{5+3\cosh(c+dx)}\,dx$$
$$= -\frac{3\sinh(c+dx)}{16d(5+3\cosh(c+dx))} + \frac{5}{16}\int \frac{1}{5+3\cosh(c+dx)}\,dx$$
$$= \frac{5x}{64} - \frac{5\tanh^{-1}\left(\frac{\sinh(c+dx)}{3+\cosh(c+dx)}\right)}{32d} - \frac{3\sinh(c+dx)}{16d(5+3\cosh(c+dx))}$$

Mathematica [A]
time = 0.09, size = 45, normalized size = 0.80

$$\frac{5\tanh^{-1}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right) - \frac{6\sinh(c+dx)}{5+3\cosh(c+dx)}}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 3*Cosh[c + d*x])^(-2),x]
```

```
[Out] (5*ArcTanh[Tanh[(c + d*x)/2]/2] - (6*Sinh[c + d*x])/(5 + 3*Cosh[c + d*x]))/
(32*d)
```

Maple [A]
time = 0.85, size = 64, normalized size = 1.14

method	result	size
derivativedivides	$\frac{\frac{3}{32\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}+\frac{5\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{64}+\frac{3}{32\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}-\frac{5\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{64}}{d}$	64
default	$\frac{\frac{3}{32\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}+\frac{5\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{64}+\frac{3}{32\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}-\frac{5\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{64}}{d}$	64
risch	$\frac{5\,e^{dx+c}+3}{8d\left(3\,e^{2dx+2c}+10\,e^{dx+c}+3\right)} - \frac{5\ln\left(3+e^{dx+c}\right)}{64d} + \frac{5\ln\left(e^{dx+c}+\frac{1}{3}\right)}{64d}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+3*cosh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(3/32/(tanh(1/2*d*x+1/2*c)+2)+5/64*ln(tanh(1/2*d*x+1/2*c)+2)+3/32/(tanh
(1/2*d*x+1/2*c)-2)-5/64*ln(tanh(1/2*d*x+1/2*c)-2))
```

Maxima [A]
time = 0.28, size = 81, normalized size = 1.45

$$-\frac{5\,\log\left(3\,e^{(-dx-c)}+1\right)}{64\,d} + \frac{5\,\log\left(e^{(-dx-c)}+3\right)}{64\,d} - \frac{5\,e^{(-dx-c)}+3}{8\,d\left(10\,e^{(-dx-c)}+3\,e^{(-2\,dx-2\,c)}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -5/64*log(3*e^(-d*x - c) + 1)/d + 5/64*log(e^(-d*x - c) + 3)/d - 1/8*(5*e^(
-d*x - c) + 3)/(d*(10*e^(-d*x - c) + 3*e^(-2*d*x - 2*c) + 3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(50) = 100$.
time = 0.48, size = 212, normalized size = 3.79

$$\frac{5\left(3\cosh\left(dx+c\right)^{2}+2\left(3\cosh\left(dx+c\right)+5\right)\sinh\left(dx+c\right)+3\sinh\left(dx+c\right)^{2}+10\cosh\left(dx+c\right)+3\right)\log\left(3\cosh\left(dx+c\right)+3\sinh\left(dx+c\right)+1\right)-5\left(3\cosh\left(dx+c\right)^{2}+2\left(3\cosh\left(dx+c\right)+5\right)\sinh\left(dx+c\right)+3\sinh\left(dx+c\right)^{2}+10\cosh\left(dx+c\right)+3\right)\log\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)+3\right)+40\cosh\left(dx+c\right)+40\sinh\left(dx+c\right)+24}{64\left(3d\cosh\left(dx+c\right)^{2}+3d\sinh\left(dx+c\right)^{2}+10d\cosh\left(dx+c\right)+2\left(3d\cosh\left(dx+c\right)+5d\right)\sinh\left(dx+c\right)+3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/64*(5*(3*cosh(d*x + c)^2 + 2*(3*cosh(d*x + c) + 5)*sinh(d*x + c) + 3*sinh
(d*x + c)^2 + 10*cosh(d*x + c) + 3)*log(3*cosh(d*x + c) + 3*sinh(d*x + c) +
 1) - 5*(3*cosh(d*x + c)^2 + 2*(3*cosh(d*x + c) + 5)*sinh(d*x + c) + 3*sinh
(d*x + c)^2 + 10*cosh(d*x + c) + 3)*log(cosh(d*x + c) + sinh(d*x + c) + 3)
+ 40*cosh(d*x + c) + 40*sinh(d*x + c) + 24)/(3*d*cosh(d*x + c)^2 + 3*d*sinh
(d*x + c)^2 + 10*d*cosh(d*x + c) + 2*(3*d*cosh(d*x + c) + 5*d)*sinh(d*x + c
) + 3*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(48) = 96$.
time = 0.82, size = 199, normalized size = 3.55

$$\begin{cases} -\frac{5\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-2\right)\tanh^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)}{64d\tanh^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)-256d}+\frac{20\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-2\right)}{64d\tanh^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)-256d}+\frac{5\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)+2\right)\tanh^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)}{64d\tanh^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)-256d}-\frac{20\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)+2\right)}{64d\tanh^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)-256d}+\frac{12\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{64d\tanh^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)-256d} & \text{for}\ d\neq 0\\ \frac{x}{\left(3\cosh\left(c\right)+5\right)^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c))**2,x)
```

```
[Out] Piecewise((-5*log(tanh(c/2 + d*x/2) - 2)*tanh(c/2 + d*x/2)**2/(64*d*tanh(c/
2 + d*x/2)**2 - 256*d) + 20*log(tanh(c/2 + d*x/2) - 2)/(64*d*tanh(c/2 + d*x
/2)**2 - 256*d) + 5*log(tanh(c/2 + d*x/2) + 2)*tanh(c/2 + d*x/2)**2/(64*d*t
anh(c/2 + d*x/2)**2 - 256*d) - 20*log(tanh(c/2 + d*x/2) + 2)/(64*d*tanh(c/2
 + d*x/2)**2 - 256*d) + 12*tanh(c/2 + d*x/2)/(64*d*tanh(c/2 + d*x/2)**2 - 2
56*d), Ne(d, 0)), (x/(3*cosh(c) + 5)**2, True))
```

Giac [A]
time = 0.40, size = 65, normalized size = 1.16

$$\frac{\frac{8\left(5e^{(dx+c)}+3\right)}{3e^{(2dx+2c)}+10e^{(dx+c)}+3}+5\log\left(3e^{(dx+c)}+1\right)-5\log\left(e^{(dx+c)}+3\right)}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/64*(8*(5*e^(d*x + c) + 3)/(3*e^(2*d*x + 2*c) + 10*e^(d*x + c) + 3) + 5*lo
g(3*e^(d*x + c) + 1) - 5*log(e^(d*x + c) + 3))/d
```

Mupad [B]
time = 0.94, size = 77, normalized size = 1.38

$$\frac{\frac{5\,e^{c+d\,x}}{8\,d}+\frac{3}{8\,d}}{10\,e^{c+d\,x}+3\,e^{2\,c+2\,d\,x}+3}-\frac{5\,\mathrm{atan}\left(\left(\frac{5}{4\,d}+\frac{3\,e^{d\,x}\,e^{c}}{4\,d}\right)\,\sqrt{-d^2}\right)}{32\,\sqrt{-d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*cosh(c + d*x) + 5)^2,x)
```

```
[Out] ((5*exp(c + d*x))/(8*d) + 3/(8*d))/(10*exp(c + d*x) + 3*exp(2*c + 2*d*x) +
3) - (5*atan((5/(4*d) + (3*exp(d*x)*exp(c))/(4*d))*(-d^2)^(1/2)))/(32*(-d^2
)^(1/2))
```

3.77 $\int \frac{1}{(5+3\cosh(c+dx))^3}\,dx$

Optimal. Leaf size=81

$$\frac{59x}{2048}-\frac{59\tanh^{-1}\left(\frac{\sinh(c+dx)}{3+\cosh(c+dx)}\right)}{1024d}-\frac{3\sinh(c+dx)}{32d(5+3\cosh(c+dx))^2}-\frac{45\sinh(c+dx)}{512d(5+3\cosh(c+dx))}$$

```
[Out] 59/2048*x-59/1024*arctanh(sinh(d*x+c)/(3+cosh(d*x+c)))/d-3/32*sinh(d*x+c)/d
/(5+3*cosh(d*x+c))^2-45/512*sinh(d*x+c)/d/(5+3*cosh(d*x+c))
```

Rubi [A]
time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2743, 2833, 12, 2736}

$$-\frac{45\sinh(c+dx)}{512d(3\cosh(c+dx)+5)}-\frac{3\sinh(c+dx)}{32d(3\cosh(c+dx)+5)^2}-\frac{59\tanh^{-1}\left(\frac{\sinh(c+dx)}{\cosh(c+dx)+3}\right)}{1024d}+\frac{59x}{2048}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 3*Cosh[c + d*x])^(-3),x]
```

```
[Out] (59*x)/2048 - (59*ArcTanh[Sinh[c + d*x]/(3 + Cosh[c + d*x])])/(1024*d) - (3
*Sinh[c + d*x])/(32*d*(5 + 3*Cosh[c + d*x])^2) - (45*Sinh[c + d*x])/(512*d*
(5 + 3*Cosh[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(5+3\cosh(c+dx))^3}\,dx &= -\frac{3\sinh(c+dx)}{32d(5+3\cosh(c+dx))^2}-\frac{1}{32}\int \frac{-10+3\cosh(c+dx)}{(5+3\cosh(c+dx))^2}\,dx\\ &= -\frac{3\sinh(c+dx)}{32d(5+3\cosh(c+dx))^2}-\frac{45\sinh(c+dx)}{512d(5+3\cosh(c+dx))}+\frac{1}{512}\int \frac{59}{5+3\cosh(\\ &= -\frac{3\sinh(c+dx)}{32d(5+3\cosh(c+dx))^2}-\frac{45\sinh(c+dx)}{512d(5+3\cosh(c+dx))}+\frac{59}{512}\int \frac{1}{5+3\cosh(\\ &= \frac{59x}{2048}-\frac{59\tanh^{-1}\left(\frac{\sinh(c+dx)}{3+\cosh(c+dx)}\right)}{1024d}-\frac{3\sinh(c+dx)}{32d(5+3\cosh(c+dx))^2}-\frac{45\sinh(}{512d(5+3\mathrm{c}}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 58, normalized size = 0.72

$$\frac{59\tanh^{-1}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)-\frac{3(182\sinh(c+dx)+45\sinh(2(c+dx)))}{(5+3\cosh(c+dx))^2}}{1024d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 3*Cosh[c + d*x])^(-3),x]
```

```
[Out] (59*ArcTanh[Tanh[(c + d*x)/2]/2] - (3*(182*Sinh[c + d*x] + 45*Sinh[2*(c + d
*x)]))/(5 + 3*Cosh[c + d*x])^2)/(1024*d)
```

Maple [A]
time = 0.87, size = 94, normalized size = 1.16

method	result
risch	$\frac{\frac{177\,\mathrm{e}^{3dx+3c}}{256}+\frac{885\,\mathrm{e}^{2dx+2c}}{256}+\frac{723\,\mathrm{e}^{dx+c}}{256}+\frac{135}{256}}{d\left(3\,\mathrm{e}^{2dx+2c}+10\,\mathrm{e}^{dx+c}+3\right)^2}-\frac{59\ln\left(3+\mathrm{e}^{dx+c}\right)}{2048d}+\frac{59\ln\left(\mathrm{e}^{dx+c}+\frac{1}{3}\right)}{2048d}$
derivativedivides	$\frac{\frac{9}{512\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)^2}+\frac{69}{1024\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}-\frac{59\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{2048}-\frac{9}{512\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^2}+\frac{69}{1024\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}-}{d}$

default	$\frac{\frac{9}{512\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)^{2}}+\frac{69}{1024\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}-\frac{59\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{2048}-\frac{9}{512\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^{2}}+\frac{69}{1024\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}+}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+3*cosh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(9/512/(tanh(1/2*d*x+1/2*c)-2)^2+69/1024/(tanh(1/2*d*x+1/2*c)-2)-59/204
8*ln(tanh(1/2*d*x+1/2*c)-2)-9/512/(tanh(1/2*d*x+1/2*c)+2)^2+69/1024/(tanh(1
/2*d*x+1/2*c)+2)+59/2048*ln(tanh(1/2*d*x+1/2*c)+2))
```

Maxima [A]
time = 0.28, size = 125, normalized size = 1.54

$$-\frac{59\,\log\left(3\,e^{(-dx-c)}+1\right)}{2048\,d}+\frac{59\,\log\left(e^{(-dx-c)}+3\right)}{2048\,d}-\frac{3\,\left(241\,e^{(-dx-c)}+295\,e^{(-2\,dx-2\,c)}+59\,e^{(-3\,dx-3\,c)}+45\right)}{256\,d(60\,e^{(-dx-c)}+118\,e^{(-2\,dx-2\,c)}+60\,e^{(-3\,dx-3\,c)}+9\,e^{(-4\,dx-4\,c)}+9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -59/2048*log(3*e^(-d*x - c) + 1)/d + 59/2048*log(e^(-d*x - c) + 3)/d - 3/25
6*(241*e^(-d*x - c) + 295*e^(-2*d*x - 2*c) + 59*e^(-3*d*x - 3*c) + 45)/(d*(
60*e^(-d*x - c) + 118*e^(-2*d*x - 2*c) + 60*e^(-3*d*x - 3*c) + 9*e^(-4*d*x
- 4*c) + 9))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 563 vs. $2(73) = 146$.
time = 0.41, size = 563, normalized size = 6.95

$$\frac{1416\,\cosh\left(dx+c\right)^{3}+1416\left(3\,\cosh\left(dx+c\right)+5\right)\sinh\left(dx+c\right)^{2}+1416\,\sinh\left(dx+c\right)^{3}+7080\,\cosh\left(dx+c\right)^{2}+59\left(9\,\cosh\left(dx+c\right)^{4}+12\left(3\,\cosh\left(dx+c\right)+5\right)\sinh\left(dx+c\right)^{3}+9\,\sinh\left(dx+c\right)^{4}+60\,\cosh\left(dx+c\right)^{3}+2\left(27\,\cosh\left(dx+c\right)^{2}+90\,\cosh\left(dx+c\right)+59\right)\sinh\left(dx+c\right)^{2}+118\,\cosh\left(dx+c\right)^{2}+4\left(9\,\cosh\left(dx+c\right)^{3}+45\,\cosh\left(dx+c\right)^{2}+59\,\cosh\left(dx+c\right)+15\right)\sinh\left(dx+c\right)+60\,\cosh\left(dx+c\right)+9\right)\log\left(3\,\cosh\left(dx+c\right)+3\,\sinh\left(dx+c\right)+1\right)-59\left(9\,\cosh\left(dx+c\right)^{4}+12\left(3\,\cosh\left(dx+c\right)+5\right)\sinh\left(dx+c\right)^{3}+9\,\sinh\left(dx+c\right)^{4}+60\,\cosh\left(dx+c\right)^{3}+2\left(27\,\cosh\left(dx+c\right)^{2}+90\,\cosh\left(dx+c\right)+59\right)\sinh\left(dx+c\right)^{2}+118\,\cosh\left(dx+c\right)^{2}+4\left(9\,\cosh\left(dx+c\right)^{3}+45\,\cosh\left(dx+c\right)^{2}+59\,\cosh\left(dx+c\right)+15\right)\sinh\left(dx+c\right)+60\,\cosh\left(dx+c\right)+9\right)\log\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)+3\right)+24\left(177\,\cosh\left(dx+c\right)^{2}+590\,\cosh\left(dx+c\right)+241\right)\sinh\left(dx+c\right)+5784\,\cosh\left(dx+c\right)+1080}{2048\left(9\,d\cosh\left(dx+c\right)^{4}+9\,d\sinh\left(dx+c\right)^{4}+60\,d\cosh\left(dx+c\right)^{3}+12\left(3\,d\cosh\left(dx+c\right)+5\,d\right)\sinh\left(dx+c\right)^{3}+118\,d\cosh\left(dx+c\right)^{2}+2\left(27\,d\cosh\left(dx+c\right)^{2}+90\,d\cosh\left(dx+c\right)+59\,d\right)\sinh\left(dx+c\right)^{2}+60\,d\cosh\left(dx+c\right)+4\left(9\,d\cosh\left(dx+c\right)^{3}+45\,d\cosh\left(dx+c\right)^{2}+59\,d\cosh\left(dx+c\right)+15\,d\right)\sinh\left(dx+c\right)+9\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2048*(1416*cosh(d*x + c)^3 + 1416*(3*cosh(d*x + c) + 5)*sinh(d*x + c)^2 +
 1416*sinh(d*x + c)^3 + 7080*cosh(d*x + c)^2 + 59*(9*cosh(d*x + c)^4 + 12*(
3*cosh(d*x + c) + 5)*sinh(d*x + c)^3 + 9*sinh(d*x + c)^4 + 60*cosh(d*x + c)
^3 + 2*(27*cosh(d*x + c)^2 + 90*cosh(d*x + c) + 59)*sinh(d*x + c)^2 + 118*c
osh(d*x + c)^2 + 4*(9*cosh(d*x + c)^3 + 45*cosh(d*x + c)^2 + 59*cosh(d*x +
c) + 15)*sinh(d*x + c) + 60*cosh(d*x + c) + 9)*log(3*cosh(d*x + c) + 3*sinh
(d*x + c) + 1) - 59*(9*cosh(d*x + c)^4 + 12*(3*cosh(d*x + c) + 5)*sinh(d*x
+ c)^3 + 9*sinh(d*x + c)^4 + 60*cosh(d*x + c)^3 + 2*(27*cosh(d*x + c)^2 + 9
0*cosh(d*x + c) + 59)*sinh(d*x + c)^2 + 118*cosh(d*x + c)^2 + 4*(9*cosh(d*x
 + c)^3 + 45*cosh(d*x + c)^2 + 59*cosh(d*x + c) + 15)*sinh(d*x + c) + 60*co
sh(d*x + c) + 9)*log(cosh(d*x + c) + sinh(d*x + c) + 3) + 24*(177*cosh(d*x
+ c)^2 + 590*cosh(d*x + c) + 241)*sinh(d*x + c) + 5784*cosh(d*x + c) + 1080
```

```
)/(9*d*cosh(d*x + c)^4 + 9*d*sinh(d*x + c)^4 + 60*d*cosh(d*x + c)^3 + 12*(3
*d*cosh(d*x + c) + 5*d)*sinh(d*x + c)^3 + 118*d*cosh(d*x + c)^2 + 2*(27*d*c
osh(d*x + c)^2 + 90*d*cosh(d*x + c) + 59*d)*sinh(d*x + c)^2 + 60*d*cosh(d*x
 + c) + 4*(9*d*cosh(d*x + c)^3 + 45*d*cosh(d*x + c)^2 + 59*d*cosh(d*x + c)
+ 15*d)*sinh(d*x + c) + 9*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(71) = 142$.
time = 1.71, size = 445, normalized size = 5.49

$$\begin{cases} -\frac{59\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-2\right)\tanh^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)}{2048d\tanh^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)-16384d\tanh^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+32768d}+\frac{472\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-2\right)\tanh^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)}{2048d\tanh^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)-16384d\tanh^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+32768d}-\frac{944\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-2\right)}{2048d\tanh^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)-16384d\tanh^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+32768d}+\frac{59\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)+2\right)\tanh^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)}{2048d\tanh^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)-16384d\tanh^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+32768d}-\frac{472\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)+2\right)\tanh^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)}{2048d\tanh^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)-16384d\tanh^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+32768d}+\frac{944\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)+2\right)}{2048d\tanh^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)-16384d\tanh^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+32768d}+\frac{276\tanh^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{2048d\tanh^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)-16384d\tanh^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+32768d}-\frac{816\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{2048d\tanh^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)-16384d\tanh^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+32768d} & \text{for}\ d\neq 0 \\ \frac{x}{(3\cosh(c)+5)^{3}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c))**3,x)
```

```
[Out] Piecewise((-59*log(tanh(c/2 + d*x/2) - 2)*tanh(c/2 + d*x/2)**4/(2048*d*tanh
(c/2 + d*x/2)**4 - 16384*d*tanh(c/2 + d*x/2)**2 + 32768*d) + 472*log(tanh(c
/2 + d*x/2) - 2)*tanh(c/2 + d*x/2)**2/(2048*d*tanh(c/2 + d*x/2)**4 - 16384*
d*tanh(c/2 + d*x/2)**2 + 32768*d) - 944*log(tanh(c/2 + d*x/2) - 2)/(2048*d*
tanh(c/2 + d*x/2)**4 - 16384*d*tanh(c/2 + d*x/2)**2 + 32768*d) + 59*log(tan
h(c/2 + d*x/2) + 2)*tanh(c/2 + d*x/2)**4/(2048*d*tanh(c/2 + d*x/2)**4 - 163
84*d*tanh(c/2 + d*x/2)**2 + 32768*d) - 472*log(tanh(c/2 + d*x/2) + 2)*tanh(
c/2 + d*x/2)**2/(2048*d*tanh(c/2 + d*x/2)**4 - 16384*d*tanh(c/2 + d*x/2)**2
 + 32768*d) + 944*log(tanh(c/2 + d*x/2) + 2)/(2048*d*tanh(c/2 + d*x/2)**4 -
 16384*d*tanh(c/2 + d*x/2)**2 + 32768*d) + 276*tanh(c/2 + d*x/2)**3/(2048*d
*tanh(c/2 + d*x/2)**4 - 16384*d*tanh(c/2 + d*x/2)**2 + 32768*d) - 816*tanh(
c/2 + d*x/2)/(2048*d*tanh(c/2 + d*x/2)**4 - 16384*d*tanh(c/2 + d*x/2)**2 +
32768*d), Ne(d, 0)), (x/(3*cosh(c) + 5)**3, True))
```

Giac [A]
time = 0.41, size = 87, normalized size = 1.07

$$\frac{\frac{24\,\left(59\,e^{(3\,dx+3\,c)}+295\,e^{(2\,dx+2\,c)}+241\,e^{(dx+c)}+45\right)}{\left(3\,e^{(2\,dx+2\,c)}+10\,e^{(dx+c)}+3\right)^{2}}+59\,\log\left(3\,e^{(dx+c)}+1\right)-59\,\log\left(e^{(dx+c)}+3\right)}{2048\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2048*(24*(59*e^(3*d*x + 3*c) + 295*e^(2*d*x + 2*c) + 241*e^(d*x + c) + 45
)/(3*e^(2*d*x + 2*c) + 10*e^(d*x + c) + 3)^2 + 59*log(3*e^(d*x + c) + 1) -
59*log(e^(d*x + c) + 3))/d
```

Mupad [B]
time = 0.95, size = 141, normalized size = 1.74

$$\frac{\frac{59\,{\mathrm{e}}^{c+d\,x}}{256\,d}+\frac{295}{768\,d}}{10\,{\mathrm{e}}^{c+d\,x}+3\,{\mathrm{e}}^{2\,c+2\,d\,x}+3}-\frac{59\,\mathrm{atan}\left(\left(\frac{5}{4\,d}+\frac{3\,{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^{c}}{4\,d}\right)\,\sqrt{-d^2}\right)}{1024\,\sqrt{-d^2}}-\frac{\frac{41\,{\mathrm{e}}^{c+d\,x}}{24\,d}+\frac{5}{8\,d}}{60\,{\mathrm{e}}^{c+d\,x}+118\,{\mathrm{e}}^{2\,c+2\,d\,x}+60\,{\mathrm{e}}^{3\,c+3\,d\,x}+9\,{\mathrm{e}}^{4\,c+4\,d\,x}+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*cosh(c + d*x) + 5)^3,x)
```

```
[Out] ((59*exp(c + d*x))/(256*d) + 295/(768*d))/(10*exp(c + d*x) + 3*exp(2*c + 2*
d*x) + 3) - (59*atan((5/(4*d) + (3*exp(d*x)*exp(c))/(4*d))*(-d^2)^(1/2)))/(
1024*(-d^2)^(1/2)) - ((41*exp(c + d*x))/(24*d) + 5/(8*d))/(60*exp(c + d*x)
+ 118*exp(2*c + 2*d*x) + 60*exp(3*c + 3*d*x) + 9*exp(4*c + 4*d*x) + 9)
```

3.78 $\int \frac{1}{(5+3\cosh(c+dx))^4}\,dx$

Optimal. Leaf size=106

$$\frac{385x}{32768}-\frac{385\tanh^{-1}\left(\frac{\sinh(c+dx)}{3+\cosh(c+dx)}\right)}{16384d}-\frac{\sinh(c+dx)}{16d(5+3\cosh(c+dx))^3}-\frac{25\sinh(c+dx)}{512d(5+3\cosh(c+dx))^2}-\frac{311\sinh(c}{8192d(5+3\cos}$$

```
[Out] 385/32768*x-385/16384*arctanh(sinh(d*x+c)/(3+cosh(d*x+c)))/d-1/16*sinh(d*x+
c)/d/(5+3*cosh(d*x+c))^3-25/512*sinh(d*x+c)/d/(5+3*cosh(d*x+c))^2-311/8192*
sinh(d*x+c)/d/(5+3*cosh(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2743, 2833, 12, 2736}

$$-\frac{311\sinh(c+dx)}{8192d(3\cosh(c+dx)+5)}-\frac{25\sinh(c+dx)}{512d(3\cosh(c+dx)+5)^2}-\frac{\sinh(c+dx)}{16d(3\cosh(c+dx)+5)^3}-\frac{385\tanh^{-1}\left(\frac{\sinh(c+dx)}{\cosh(c+dx)+3}\right)}{16384d}+\frac{385x}{32768}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 3*Cosh[c + d*x])^(-4),x]
```

```
[Out] (385*x)/32768 - (385*ArcTanh[Sinh[c + d*x]/(3 + Cosh[c + d*x])])/(16384*d)
- Sinh[c + d*x]/(16*d*(5 + 3*Cosh[c + d*x])^3) - (25*Sinh[c + d*x])/(512*d*
(5 + 3*Cosh[c + d*x])^2) - (311*Sinh[c + d*x])/(8192*d*(5 + 3*Cosh[c + d*x]
))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(5+3\cosh(c+dx))^4}\,dx &= -\frac{\sinh(c+dx)}{16d(5+3\cosh(c+dx))^3}-\frac{1}{48}\int \frac{-15+6\cosh(c+dx)}{(5+3\cosh(c+dx))^3}\,dx \\
&= -\frac{\sinh(c+dx)}{16d(5+3\cosh(c+dx))^3}-\frac{25\sinh(c+dx)}{512d(5+3\cosh(c+dx))^2}+\frac{\int \frac{186-75\cosh(c+dx)}{(5+3\cosh(c+dx))^2}\,d}{1536} \\
&= -\frac{\sinh(c+dx)}{16d(5+3\cosh(c+dx))^3}-\frac{25\sinh(c+dx)}{512d(5+3\cosh(c+dx))^2}-\frac{311\sinh(c+}{8192d(5+3\cosh(} \\
&= -\frac{\sinh(c+dx)}{16d(5+3\cosh(c+dx))^3}-\frac{25\sinh(c+dx)}{512d(5+3\cosh(c+dx))^2}-\frac{311\sinh(c+}{8192d(5+3\cosh(} \\
&= \frac{385x}{32768}-\frac{385\tanh^{-1}\left(\frac{\sinh(c+dx)}{3+\cosh(c+dx)}\right)}{16384d}-\frac{\sinh(c+dx)}{16d(5+3\cosh(c+dx))^3}-\frac{25\sinh}{512d(5+3}
\end{aligned}$$

Mathematica [A]
time = 0.19, size = 68, normalized size = 0.64

$$\frac{770\tanh^{-1}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)-\frac{9(4883\sinh(c+dx)+2340\sinh(2(c+dx))+311\sinh(3(c+dx)))}{(5+3\cosh(c+dx))^3}}{32768d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 3*Cosh[c + d*x])^(-4),x]
```

```
[Out] (770*ArcTanh[Tanh[(c + d*x)/2]/2] - (9*(4883*Sinh[c + d*x] + 2340*Sinh[2*(c
 + d*x)] + 311*Sinh[3*(c + d*x)]))/(5 + 3*Cosh[c + d*x])^3)/(32768*d)
```

Maple [A]
time = 0.86, size = 124, normalized size = 1.17

method	result
risch	$\frac{10395\,e^{5dx+5c}+86625\,e^{4dx+4c}+239470\,e^{3dx+3c}+218466\,e^{2dx+2c}+73575\,e^{dx+c}+8397}{12288d\left(3\,e^{2dx+2c}+10\,e^{dx+c}+3\right)^{3}}+\frac{385\ln\left(e^{dx+c}+\frac{1}{3}\right)}{32768d}-\frac{385\ln(}{32}$
derivativedivides	$\frac{\frac{9}{2048\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)^{3}}+\frac{81}{4096\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)^{2}}+\frac{639}{16384\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}-\frac{385\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{32768}+\frac{9}{2048\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}}{d}$
default	$\frac{\frac{9}{2048\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)^{3}}+\frac{81}{4096\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)^{2}}+\frac{639}{16384\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}-\frac{385\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{32768}+\frac{9}{2048\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+3*cosh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(9/2048/(tanh(1/2*d*x+1/2*c)-2)^3+81/4096/(tanh(1/2*d*x+1/2*c)-2)^2+639
/16384/(tanh(1/2*d*x+1/2*c)-2)-385/32768*ln(tanh(1/2*d*x+1/2*c)-2)+9/2048/(
tanh(1/2*d*x+1/2*c)+2)^3-81/4096/(tanh(1/2*d*x+1/2*c)+2)^2+639/16384/(tanh(
1/2*d*x+1/2*c)+2)+385/32768*ln(tanh(1/2*d*x+1/2*c)+2))
```

Maxima [A]
time = 0.28, size = 169, normalized size = 1.59

$$-\frac{385\,\log\left(3\,e^{(-dx-c)}+1\right)}{32768\,d}+\frac{385\,\log\left(e^{(-dx-c)}+3\right)}{32768\,d}-\frac{73575\,e^{(-dx-c)}+218466\,e^{(-2\,dx-2\,c)}+239470\,e^{(-3\,dx-3\,c)}+86625\,e^{(-4\,dx-4\,c)}+10395\,e^{(-5\,dx-5\,c)}+8397}{12288\,d(270\,e^{(-dx-c)}+981\,e^{(-2\,dx-2\,c)}+1540\,e^{(-3\,dx-3\,c)}+981\,e^{(-4\,dx-4\,c)}+270\,e^{(-5\,dx-5\,c)}+27\,e^{(-6\,dx-6\,c)}+27)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -385/32768*log(3*e^(-d*x - c) + 1)/d + 385/32768*log(e^(-d*x - c) + 3)/d -
1/12288*(73575*e^(-d*x - c) + 218466*e^(-2*d*x - 2*c) + 239470*e^(-3*d*x -
3*c) + 86625*e^(-4*d*x - 4*c) + 10395*e^(-5*d*x - 5*c) + 8397)/(d*(270*e^(-
d*x - c) + 981*e^(-2*d*x - 2*c) + 1540*e^(-3*d*x - 3*c) + 981*e^(-4*d*x - 4
*c) + 270*e^(-5*d*x - 5*c) + 27*e^(-6*d*x - 6*c) + 27))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1078 vs. $2(96) = 192$.
time = 0.44, size = 1078, normalized size = 10.17

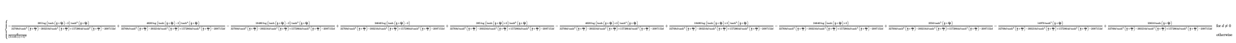
Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/98304*(83160*cosh(d*x + c)^5 + 138600*(3*cosh(d*x + c) + 5)*sinh(d*x + c)
^4 + 83160*sinh(d*x + c)^5 + 693000*cosh(d*x + c)^4 + 6160*(135*cosh(d*x +
c)^2 + 450*cosh(d*x + c) + 311)*sinh(d*x + c)^3 + 1915760*cosh(d*x + c)^3 +
 48*(17325*cosh(d*x + c)^3 + 86625*cosh(d*x + c)^2 + 119735*cosh(d*x + c) +
```

```
 36411)*sinh(d*x + c)^2 + 1747728*cosh(d*x + c)^2 + 1155*(27*cosh(d*x + c)^
6 + 54*(3*cosh(d*x + c) + 5)*sinh(d*x + c)^5 + 27*sinh(d*x + c)^6 + 270*cos
h(d*x + c)^5 + 9*(45*cosh(d*x + c)^2 + 150*cosh(d*x + c) + 109)*sinh(d*x +
c)^4 + 981*cosh(d*x + c)^4 + 4*(135*cosh(d*x + c)^3 + 675*cosh(d*x + c)^2 +
 981*cosh(d*x + c) + 385)*sinh(d*x + c)^3 + 1540*cosh(d*x + c)^3 + 3*(135*c
osh(d*x + c)^4 + 900*cosh(d*x + c)^3 + 1962*cosh(d*x + c)^2 + 1540*cosh(d*x
 + c) + 327)*sinh(d*x + c)^2 + 981*cosh(d*x + c)^2 + 6*(27*cosh(d*x + c)^5
+ 225*cosh(d*x + c)^4 + 654*cosh(d*x + c)^3 + 770*cosh(d*x + c)^2 + 327*cos
h(d*x + c) + 45)*sinh(d*x + c) + 270*cosh(d*x + c) + 27)*log(3*cosh(d*x + c
) + 3*sinh(d*x + c) + 1) - 1155*(27*cosh(d*x + c)^6 + 54*(3*cosh(d*x + c) +
 5)*sinh(d*x + c)^5 + 27*sinh(d*x + c)^6 + 270*cosh(d*x + c)^5 + 9*(45*cosh
(d*x + c)^2 + 150*cosh(d*x + c) + 109)*sinh(d*x + c)^4 + 981*cosh(d*x + c)^
4 + 4*(135*cosh(d*x + c)^3 + 675*cosh(d*x + c)^2 + 981*cosh(d*x + c) + 385)
*sinh(d*x + c)^3 + 1540*cosh(d*x + c)^3 + 3*(135*cosh(d*x + c)^4 + 900*cosh
(d*x + c)^3 + 1962*cosh(d*x + c)^2 + 1540*cosh(d*x + c) + 327)*sinh(d*x + c
)^2 + 981*cosh(d*x + c)^2 + 6*(27*cosh(d*x + c)^5 + 225*cosh(d*x + c)^4 + 6
54*cosh(d*x + c)^3 + 770*cosh(d*x + c)^2 + 327*cosh(d*x + c) + 45)*sinh(d*x
 + c) + 270*cosh(d*x + c) + 27)*log(cosh(d*x + c) + sinh(d*x + c) + 3) + 24
*(17325*cosh(d*x + c)^4 + 115500*cosh(d*x + c)^3 + 239470*cosh(d*x + c)^2 +
 145644*cosh(d*x + c) + 24525)*sinh(d*x + c) + 588600*cosh(d*x + c) + 67176
)/(27*d*cosh(d*x + c)^6 + 27*d*sinh(d*x + c)^6 + 270*d*cosh(d*x + c)^5 + 54
*(3*d*cosh(d*x + c) + 5*d)*sinh(d*x + c)^5 + 981*d*cosh(d*x + c)^4 + 9*(45*
d*cosh(d*x + c)^2 + 150*d*cosh(d*x + c) + 109*d)*sinh(d*x + c)^4 + 1540*d*c
osh(d*x + c)^3 + 4*(135*d*cosh(d*x + c)^3 + 675*d*cosh(d*x + c)^2 + 981*d*c
osh(d*x + c) + 385*d)*sinh(d*x + c)^3 + 981*d*cosh(d*x + c)^2 + 3*(135*d*co
sh(d*x + c)^4 + 900*d*cosh(d*x + c)^3 + 1962*d*cosh(d*x + c)^2 + 1540*d*cos
h(d*x + c) + 327*d)*sinh(d*x + c)^2 + 270*d*cosh(d*x + c) + 6*(27*d*cosh(d*
x + c)^5 + 225*d*cosh(d*x + c)^4 + 654*d*cosh(d*x + c)^3 + 770*d*cosh(d*x +
 c)^2 + 327*d*cosh(d*x + c) + 45*d)*sinh(d*x + c) + 27*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 784 vs. $2(94) = 188$.
time = 3.75, size = 784, normalized size = 7.40



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c))**4,x)
```

```
[Out] Piecewise((-385*log(tanh(c/2 + d*x/2) - 2)*tanh(c/2 + d*x/2)**6/(32768*d*ta
nh(c/2 + d*x/2)**6 - 393216*d*tanh(c/2 + d*x/2)**4 + 1572864*d*tanh(c/2 + d
*x/2)**2 - 2097152*d) + 4620*log(tanh(c/2 + d*x/2) - 2)*tanh(c/2 + d*x/2)**
4/(32768*d*tanh(c/2 + d*x/2)**6 - 393216*d*tanh(c/2 + d*x/2)**4 + 1572864*d
*tanh(c/2 + d*x/2)**2 - 2097152*d) - 18480*log(tanh(c/2 + d*x/2) - 2)*tanh(
c/2 + d*x/2)**2/(32768*d*tanh(c/2 + d*x/2)**6 - 393216*d*tanh(c/2 + d*x/2)*
```

*4 + 1572864*d*tanh(c/2 + d*x/2)**2 - 2097152*d) + 24640*log(tanh(c/2 + d*x
/2) - 2)/(32768*d*tanh(c/2 + d*x/2)**6 - 393216*d*tanh(c/2 + d*x/2)**4 + 15
72864*d*tanh(c/2 + d*x/2)**2 - 2097152*d) + 385*log(tanh(c/2 + d*x/2) + 2)*
tanh(c/2 + d*x/2)**6/(32768*d*tanh(c/2 + d*x/2)**6 - 393216*d*tanh(c/2 + d*
x/2)**4 + 1572864*d*tanh(c/2 + d*x/2)**2 - 2097152*d) - 4620*log(tanh(c/2 +
 d*x/2) + 2)*tanh(c/2 + d*x/2)**4/(32768*d*tanh(c/2 + d*x/2)**6 - 393216*d*
tanh(c/2 + d*x/2)**4 + 1572864*d*tanh(c/2 + d*x/2)**2 - 2097152*d) + 18480*
log(tanh(c/2 + d*x/2) + 2)*tanh(c/2 + d*x/2)**2/(32768*d*tanh(c/2 + d*x/2)*
*6 - 393216*d*tanh(c/2 + d*x/2)**4 + 1572864*d*tanh(c/2 + d*x/2)**2 - 20971
52*d) - 24640*log(tanh(c/2 + d*x/2) + 2)/(32768*d*tanh(c/2 + d*x/2)**6 - 39
3216*d*tanh(c/2 + d*x/2)**4 + 1572864*d*tanh(c/2 + d*x/2)**2 - 2097152*d) +
 2556*tanh(c/2 + d*x/2)**5/(32768*d*tanh(c/2 + d*x/2)**6 - 393216*d*tanh(c/
2 + d*x/2)**4 + 1572864*d*tanh(c/2 + d*x/2)**2 - 2097152*d) - 14976*tanh(c/
2 + d*x/2)**3/(32768*d*tanh(c/2 + d*x/2)**6 - 393216*d*tanh(c/2 + d*x/2)**4
 + 1572864*d*tanh(c/2 + d*x/2)**2 - 2097152*d) + 23616*tanh(c/2 + d*x/2)/(3
2768*d*tanh(c/2 + d*x/2)**6 - 393216*d*tanh(c/2 + d*x/2)**4 + 1572864*d*tan
h(c/2 + d*x/2)**2 - 2097152*d), Ne(d, 0)), (x/(3*cosh(c) + 5)**4, True))

Giac [A]
time = 0.41, size = 109, normalized size = 1.03

$$\frac{\frac{8\left(10395\,e^{(5\,dx+5\,c)}+86625\,e^{(4\,dx+4\,c)}+239470\,e^{(3\,dx+3\,c)}+218466\,e^{(2\,dx+2\,c)}+73575\,e^{(dx+c)}+8397\right)}{\left(3\,e^{(2\,dx+2\,c)}+10\,e^{(dx+c)}+3\right)^3}+1155\,\log\left(3\,e^{(dx+c)}+1\right)-1155\,\log\left(e^{(dx+c)}+3\right)}{98304\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5+3*cosh(d*x+c))^4,x, algorithm="giac")

[Out] 1/98304*(8*(10395*e^(5*d*x + 5*c) + 86625*e^(4*d*x + 4*c) + 239470*e^(3*d*x
 + 3*c) + 218466*e^(2*d*x + 2*c) + 73575*e^(d*x + c) + 8397)/(3*e^(2*d*x +
2*c) + 10*e^(d*x + c) + 3)^3 + 1155*log(3*e^(d*x + c) + 1) - 1155*log(e^(d*
x + c) + 3))/d

Mupad [B]
time = 0.11, size = 226, normalized size = 2.13

$$\frac{\frac{385\,e^{c+d\,x}}{4096\,d}+\frac{1925}{12288\,d}}{10\,e^{c+d\,x}+3\,e^{2\,c+2\,d\,x}+3}-\frac{385\,\mathrm{atan}\left(\left(\frac{5}{4\,d}+\frac{3\,e^{d\,x}\,e^{c}}{4\,d}\right)\,\sqrt{-d^2}\right)}{16384\,\sqrt{-d^2}}-\frac{\frac{385\,e^{c+d\,x}}{1152\,d}+\frac{3461}{3456\,d}}{60\,e^{c+d\,x}+118\,e^{2\,c+2\,d\,x}+60\,e^{3\,c+3\,d\,x}+9\,e^{4\,c+4\,d\,x}+9}+\frac{\frac{365\,e^{c+d\,x}}{54\,d}+\frac{41}{18\,d}}{270\,e^{c+d\,x}+981\,e^{2\,c+2\,d\,x}+1540\,e^{3\,c+3\,d\,x}+981\,e^{4\,c+4\,d\,x}+270\,e^{5\,c+5\,d\,x}+27\,e^{6\,c+6\,d\,x}+27}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3*cosh(c + d*x) + 5)^4,x)

[Out] ((385*exp(c + d*x))/(4096*d) + 1925/(12288*d))/(10*exp(c + d*x) + 3*exp(2*c
 + 2*d*x) + 3) - (385*atan((5/(4*d) + (3*exp(d*x)*exp(c))/(4*d))*(-d^2)^(1/
2)))/(16384*(-d^2)^(1/2)) - ((385*exp(c + d*x))/(1152*d) + 3461/(3456*d))/(
60*exp(c + d*x) + 118*exp(2*c + 2*d*x) + 60*exp(3*c + 3*d*x) + 9*exp(4*c +
4*d*x) + 9) + ((365*exp(c + d*x))/(54*d) + 41/(18*d))/(270*exp(c + d*x) + 9
81*exp(2*c + 2*d*x) + 1540*exp(3*c + 3*d*x) + 981*exp(4*c + 4*d*x) + 270*ex
p(5*c + 5*d*x) + 27*exp(6*c + 6*d*x) + 27)

3.79 $\int (a+b\cosh(x))^{5/2}\,dx$

Optimal. Leaf size=153

$$-\frac{2i(23a^2+9b^2)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{15\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\frac{16ia(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{15\sqrt{a+b\cosh(x)}}+\frac{16}{15}ab\sqrt{a+b\cosh(x}$$

```
[Out] 2/5*b*(a+b*cosh(x))^(3/2)*sinh(x)+16/15*a*b*sinh(x)*(a+b*cosh(x))^(1/2)-2/1
5*I*(23*a^2+9*b^2)*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*sinh(1/2*x
),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cosh(x))^(1/2)/((a+b*cosh(x))/(a+b))^(1/2)+
16/15*I*a*(a^2-b^2)*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh(1/2*
x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cosh(x))/(a+b))^(1/2)/(a+b*cosh(x))^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {2735, 2832, 2831, 2742, 2740, 2734, 2732}

$$\frac{16ia(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{15\sqrt{a+b\cosh(x)}}-\frac{2i(23a^2+9b^2)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{15\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\frac{2}{5}b\sinh(x)(a+b\cosh(x))^{3/2}+\frac{16}{15}ab\sinh(x)\sqrt{a+b\cosh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x])^(5/2),x]
```

```
[Out] (((-2*I)/15)*(23*a^2 + 9*b^2)*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/
(a + b)])/Sqrt[(a + b*Cosh[x])/(a + b)] + (((16*I)/15)*a*(a^2 - b^2)*Sqrt[(
a + b*Cosh[x])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/Sqrt[a + b*Cosh[
x]] + (16*a*b*Sqrt[a + b*Cosh[x]]*Sinh[x])/15 + (2*b*(a + b*Cosh[x])^(3/2)*
Sinh[x])/5
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2735

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[1/n, Int[(a + b*S
in[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x],
 x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\cosh(x))^{5/2}\,dx = \frac{2}{5}b(a+b\cosh(x))^{3/2}\sinh(x)+\frac{2}{5}\int\sqrt{a+b\cosh(x)}\left(\frac{1}{2}\left(5a^2+3b^2\right)+4ab\cosh(x)\right.$$

$$=\frac{16}{15}ab\sqrt{a+b\cosh(x)}\,\sinh(x)+\frac{2}{5}b(a+b\cosh(x))^{3/2}\sinh(x)+\frac{4}{15}\int\frac{\frac{1}{4}a(15a^2+17$$

$$=\frac{16}{15}ab\sqrt{a+b\cosh(x)}\,\sinh(x)+\frac{2}{5}b(a+b\cosh(x))^{3/2}\sinh(x)-\frac{1}{15}\left(8a\left(a^2-b^2\right)\right)\int$$

$$=\frac{16}{15}ab\sqrt{a+b\cosh(x)}\,\sinh(x)+\frac{2}{5}b(a+b\cosh(x))^{3/2}\sinh(x)+\frac{\left(\left(23a^2+9b^2\right)\sqrt{a\cdot}\right.}{}$$

$$=-\frac{2i\left(23a^2+9b^2\right)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{15\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\frac{16ia\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\right.}{15\sqrt{a+b\cosh(x)}}$$

Mathematica [A]
time = 0.38, size = 150, normalized size = 0.98

$$\frac{-2i\left(23a^3+23a^2b+9ab^2+9b^3\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)+16ia\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)+b\left(22a^2+3b^2+28ab\cosh(x)+3b^2\cosh(2x)\right)\sinh(x)}{15\sqrt{a+b\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x])^(5/2),x]
```

```
[Out] ((-2*I)*(23*a^3 + 23*a^2*b + 9*a*b^2 + 9*b^3)*Sqrt[(a + b*Cosh[x])/(a + b)]
*EllipticE[(I/2)*x, (2*b)/(a + b)] + (16*I)*a*(a^2 - b^2)*Sqrt[(a + b*Cosh[
x])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)] + b*(22*a^2 + 3*b^2 + 28*a*b
*Cosh[x] + 3*b^2*Cosh[2*x])*Sinh[x])/(15*Sqrt[a + b*Cosh[x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 684 vs. 2(169) = 338.
time = 1.36, size = 685, normalized size = 4.48

method	result

default	$\frac{2\left(24\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}}\,\left(\sinh^{6}\left(\frac{x}{2}\right)\right)b^{3}+\left(56\sqrt{-\frac{2b}{a-b}}\,a\,b^{2}+24\sqrt{-\frac{2b}{a-b}}\,b^{3}\right)\left(\sinh^{4}\left(\frac{x}{2}\right)\right)\cosh\left(\frac{x}{2}\right)+\left(22\sqrt{-\frac{2b}{a-b}}\,a^{2}b+28\sqrt{\ldots}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(x))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*(24*cosh(1/2*x)*(-2*b/(a-b))^(1/2)*sinh(1/2*x)^6*b^3+(56*(-2*b/(a-b))^
(1/2)*a*b^2+24*(-2*b/(a-b))^(1/2)*b^3)*sinh(1/2*x)^4*cosh(1/2*x)+(22*(-2*b/
(a-b))^(1/2)*a^2*b+28*(-2*b/(a-b))^(1/2)*a*b^2+6*(-2*b/(a-b))^(1/2)*b^3)*si
nh(1/2*x)^2*cosh(1/2*x)+15*a^3*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*
(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-
b)/b)^(1/2))+23*a^2*b*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/
2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/
2))+17*a*b^2*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(
1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))+9*b^3
*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*Ellipti
cF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))-46*(2*b/(a-b)*sin
h(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(cosh(1/2*x)*
(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))*a^2*b-18*(2*b/(a-b)*sinh(1/2*x)^
2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(cosh(1/2*x)*(-2*b/(a-
b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))*b^3)*((2*b*cosh(1/2*x)^2+a-b)*sinh(1/2*x)
^2)^(1/2)/(-2*b/(a-b))^(1/2)/(2*sinh(1/2*x)^4*b+(a+b)*sinh(1/2*x)^2)^(1/2)/
sinh(1/2*x)/(2*b*sinh(1/2*x)^2+a+b)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cosh(x) + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.16, size = 464, normalized size = 3.03

$$-\frac{4\left(\sqrt{2}\left(a^{3}-33ab^{2}\right)\cosh\left(x\right)^{2}+2\sqrt{2}\left(a^{3}-33ab^{2}\right)\cosh\left(x\right)\sinh\left(x\right)+\sqrt{2}\left(a^{3}-33ab^{2}\right)\sinh\left(x\right)^{2}\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(\frac{4\left(4a^{2}-3b^{2}\right)}{3b^{2}},-\frac{8\left(8a^{3}-9ab^{2}\right)}{27b^{3}},\frac{3b\cosh\left(x\right)+3b\sinh\left(x\right)+2a}{3b}\right)+12\left(\sqrt{2}\left(23a^{2}b+9b^{3}\right)\cosh\left(x\right)^{2}+2\sqrt{2}\left(23a^{2}b+9b^{3}\right)\cosh\left(x\right)\sinh\left(x\right)+\sqrt{2}\left(23a^{2}b+9b^{3}\right)\sinh\left(x\right)^{2}\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(\frac{4\left(4a^{2}-3b^{2}\right)}{3b^{2}},-\frac{8\left(8a^{3}-9ab^{2}\right)}{27b^{3}},\mathrm{weierstrassPInverse}\left(\frac{4\left(4a^{2}-3b^{2}\right)}{3b^{2}},-\frac{8\left(8a^{3}-9ab^{2}\right)}{27b^{3}},\frac{3b\cosh\left(x\right)+3b\sinh\left(x\right)+2a}{3b}\right)\right)-3\left(3b^{3}\cosh\left(x\right)^{4}+3b^{3}\sinh\left(x\right)^{4}+22ab^{2}\cosh\left(x\right)^{3}-22ab^{2}\cosh\left(x\right)+2\left(6b^{3}\cosh\left(x\right)+11ab^{2}\right)\sinh\left(x\right)^{3}-3b^{3}-4\left(23a^{2}b+9b^{3}\right)\cosh\left(x\right)^{2}+2\left(9b^{3}\cosh\left(x\right)^{2}+33ab^{2}\cosh\left(x\right)-46a^{2}b-18b^{3}\right)\sinh\left(x\right)^{2}+2\left(6b^{3}\cosh\left(x\right)^{3}+33ab^{2}\cosh\left(x\right)^{2}-11ab^{2}-4\left(23a^{2}b+9b^{3}\right)\cosh\left(x\right)\right)\sinh\left(x\right)\right)\sqrt{b\cosh\left(x\right)+a}}{90\left(b\cosh\left(x\right)^{2}+2b\cosh\left(x\right)\sinh\left(x\right)+b\sinh\left(x\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/90*(4*(sqrt(2)*(a^3 - 33*a*b^2)*cosh(x)^2 + 2*sqrt(2)*(a^3 - 33*a*b^2)*c
osh(x)*sinh(x) + sqrt(2)*(a^3 - 33*a*b^2)*sinh(x)^2)*sqrt(b)*weierstrassPIn
verse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cosh(x
) + 3*b*sinh(x) + 2*a)/b) + 12*(sqrt(2)*(23*a^2*b + 9*b^3)*cosh(x)^2 + 2*sq
rt(2)*(23*a^2*b + 9*b^3)*cosh(x)*sinh(x) + sqrt(2)*(23*a^2*b + 9*b^3)*sinh(
x)^2)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b
^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^
2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)) - 3*(3*b^3*cosh(x)^4 + 3*
b^3*sinh(x)^4 + 22*a*b^2*cosh(x)^3 - 22*a*b^2*cosh(x) + 2*(6*b^3*cosh(x) +
11*a*b^2)*sinh(x)^3 - 3*b^3 - 4*(23*a^2*b + 9*b^3)*cosh(x)^2 + 2*(9*b^3*cos
h(x)^2 + 33*a*b^2*cosh(x) - 46*a^2*b - 18*b^3)*sinh(x)^2 + 2*(6*b^3*cosh(x)
^3 + 33*a*b^2*cosh(x)^2 - 11*a*b^2 - 4*(23*a^2*b + 9*b^3)*cosh(x))*sinh(x))
*sqrt(b*cosh(x) + a))/(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cosh(x) + a)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\cosh(x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(x))^(5/2),x)
```

```
[Out] int((a + b*cosh(x))^(5/2), x)
```

3.80 $\int (a+b\cosh(x))^{3/2}\,dx$

Optimal. Leaf size=124

$$-\frac{8ia\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{3\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\frac{2i(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{3\sqrt{a+b\cosh(x)}}+\frac{2}{3}b\sqrt{a+b\cosh(x)}\,\sinh(x)$$

```
[Out] 2/3*b*sinh(x)*(a+b*cosh(x))^(1/2)-8/3*I*a*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)
*EllipticE(I*sinh(1/2*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cosh(x))^(1/2)/((a+b
*cosh(x))/(a+b))^(1/2)+2/3*I*(a^2-b^2)*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*El
lipticF(I*sinh(1/2*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cosh(x))/(a+b))^(1/2)/
(a+b*cosh(x))^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {2735, 2831, 2742, 2740, 2734, 2732}

$$\frac{2i(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{3\sqrt{a+b\cosh(x)}}+\frac{2}{3}b\sinh(x)\sqrt{a+b\cosh(x)}-\frac{8ia\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{3\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x])^(3/2),x]
```

```
[Out] (((-8*I)/3)*a*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)])/Sqrt[(
a + b*Cosh[x])/(a + b)] + (((2*I)/3)*(a^2 - b^2)*Sqrt[(a + b*Cosh[x])/(a +
b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/Sqrt[a + b*Cosh[x]] + (2*b*Sqrt[a +
b*Cosh[x]]*Sinh[x])/3
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2735

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[1/n, Int[(a + b*S
in[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x],
 x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+b\cosh(x))^{3/2}\,dx = \frac{2}{3}b\sqrt{a+b\cosh(x)}\,\sinh(x)+\frac{2}{3}\int \frac{\frac{1}{2}(3a^2+b^2)+2ab\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx$$

$$= \frac{2}{3}b\sqrt{a+b\cosh(x)}\,\sinh(x)+\frac{1}{3}(4a)\int \sqrt{a+b\cosh(x)}\,dx+\frac{1}{3}(-a^2+b^2)\int \frac{\quad}{\sqrt{a\ \cdots}}$$

$$= \frac{2}{3}b\sqrt{a+b\cosh(x)}\,\sinh(x)+\frac{\left(4a\sqrt{a+b\cosh(x)}\right)\int \sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}\,dx}{3\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\cdots$$

$$= -\frac{8ia\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{3\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\frac{2i(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{3\sqrt{a+b\cosh(x)}}+\frac{2}{3}b\sqrt{\cdots}$$

Mathematica [A]
time = 0.17, size = 111, normalized size = 0.90

$$\frac{-8ia(a+b)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)+2i(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)+2b(a+b\cosh(x))\sinh(x)}{3\sqrt{a+b\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x])^(3/2),x]
```

```
[Out] ((-8*I)*a*(a + b)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticE[(I/2)*x, (2*b)/(a
 + b)] + (2*I)*(a^2 - b^2)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticF[(I/2)*x,
 (2*b)/(a + b)] + 2*b*(a + b*Cosh[x])*Sinh[x])/(3*Sqrt[a + b*Cosh[x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(144) = 288$.
time = 1.19, size = 466, normalized size = 3.76

method	result
default	$2\left(4\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}}\left(\sinh^4\left(\frac{x}{2}\right)\right)b^2+2\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}}\left(\sinh^2\left(\frac{x}{2}\right)\right)ab+2\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}}\left(\sinh^2\left(\frac{x}{2}\right)\right)b^2+3a^2\sqrt{\frac{2b(\sinh^2}{a-}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(4*cosh(1/2*x)*(-2*b/(a-b))^(1/2)*sinh(1/2*x)^4*b^2+2*cosh(1/2*x)*(-2*b
/(a-b))^(1/2)*sinh(1/2*x)^2*a*b+2*cosh(1/2*x)*(-2*b/(a-b))^(1/2)*sinh(1/2*x
)^2*b^2+3*a^2*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^
(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))+4*a*
b*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*Ellipt
icF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))+b^2*(2*b/(a-b)*s
inh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x
)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))-8*(2*b/(a-b)*sinh(1/2*x)^2+(a+
b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(cosh(1/2*x)*(-2*b/(a-b))^(
1/2),1/2*(-2*(a-b)/b)^(1/2))*a*b)*((2*b*cosh(1/2*x)^2+a-b)*sinh(1/2*x)^2)^(
1/2)/(-2*b/(a-b))^(1/2)/(2*sinh(1/2*x)^4*b+(a+b)*sinh(1/2*x)^2)^(1/2)/sinh(
1/2*x)/(2*b*sinh(1/2*x)^2+a+b)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cosh(x) + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 265, normalized size = 2.14

$$\frac{2\left(\sqrt{2}\left(a^2+3b^2\right)\cosh(x)+\sqrt{2}\left(a^2+3b^2\right)\sinh(x)\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8a^3-9ab^2)}{27b^3},\frac{3b\cosh(x)+3b\sinh(x)+2a}{3b}\right)-24\left(\sqrt{2}\,ab\cosh(x)+\sqrt{2}\,ab\sinh(x)\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8a^3-9ab^2)}{27b^3},\mathrm{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8a^3-9ab^2)}{27b^3},\frac{3b\cosh(x)+3b\sinh(x)+2a}{3b}\right)\right)+3\left(b^2\cosh(x)^2+b^2\sinh(x)^2-8ab\cosh(x)-b^2+2\left(b^2\cosh(x)-4ab\right)\sinh(x)\right)\sqrt{b\cosh(x)+a}}{9\left(b\cosh(x)+b\sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/9*(2*(sqrt(2)*(a^2 + 3*b^2)*cosh(x) + sqrt(2)*(a^2 + 3*b^2)*sinh(x))*sqrt
(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^
3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) - 24*(sqrt(2)*a*b*cosh(x) + sqr
t(2)*a*b*sinh(x))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8
*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*
a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)) + 3*(b^2*cosh
(x)^2 + b^2*sinh(x)^2 - 8*a*b*cosh(x) - b^2 + 2*(b^2*cosh(x) - 4*a*b)*sinh(
x))*sqrt(b*cosh(x) + a))/(b*cosh(x) + b*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\cosh(x))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))**(3/2),x)
```

```
[Out] Integral((a + b*cosh(x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cosh(x) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\cosh(x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(x))^(3/2),x)
```

```
[Out] int((a + b*cosh(x))^(3/2), x)
```

3.81 $\int \sqrt{a+b\cosh(c+dx)}\,dx$

Optimal. Leaf size=61

$$-\frac{2i\sqrt{a+b\cosh(c+dx)}\,E\left(\frac{1}{2}i(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cosh(c+dx)}{a+b}}}$$

```
[Out] -2*I*(cosh(1/2*d*x+1/2*c)^2)^(1/2)/cosh(1/2*d*x+1/2*c)*EllipticE(I*sinh(1/2
*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cosh(d*x+c))^(1/2)/d/((a+b*cosh(d
*x+c))/(a+b))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2734, 2732}

$$-\frac{2i\sqrt{a+b\cosh(c+dx)}\,E\left(\frac{1}{2}i(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cosh(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cosh[c + d*x]],x]
```

```
[Out] ((-2*I)*Sqrt[a + b*Cosh[c + d*x]]*EllipticE[(I/2)*(c + d*x), (2*b)/(a + b)]
)/(d*Sqrt[(a + b*Cosh[c + d*x])/(a + b)])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] && !GtQ[a + b, 0]
```

Rubi steps

$$\int \sqrt{a+b\cosh(c+dx)}\,dx = \frac{\sqrt{a+b\cosh(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cosh(c+dx)}{a+b}}\,dx}{\sqrt{\frac{a+b\cosh(c+dx)}{a+b}}}$$
$$= -\frac{2i\sqrt{a+b\cosh(c+dx)}\,E\left(\frac{1}{2}i(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cosh(c+dx)}{a+b}}}$$

Mathematica [A]
time = 0.07, size = 61, normalized size = 1.00

$$-\frac{2i\sqrt{a+b\cosh(c+dx)}\,E\left(\frac{1}{2}i(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cosh(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cosh[c + d*x]],x]
[Out] ((-2*I)*Sqrt[a + b*Cosh[c + d*x]]*EllipticE[(I/2)*(c + d*x), (2*b)/(a + b)]
)/(d*Sqrt[(a + b*Cosh[c + d*x])/(a + b)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 275 vs. 2(86) = 172.
time = 1.53, size = 276, normalized size = 4.52

method	result
default	$\frac{2\left(a\,\mathrm{EllipticF}\left(\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-\frac{2b}{a-b}},\frac{\sqrt{-\frac{2(a-b)}{b}}}{2}\right)+b\,\mathrm{EllipticF}\left(\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-\frac{2b}{a-b}},\frac{\sqrt{-\frac{2(a-b)}{b}}}{2}\right)-2b\,\mathrm{EllipticE}\right.}{\sqrt{-\frac{2b}{a-b}}\sqrt{2\left(\sinh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+}}$

risch	$\frac{\sqrt{2}\,\sqrt{\left(b\,\mathrm{e}^{2dx+2c}+2a\,\mathrm{e}^{dx+c}+b\right)\mathrm{e}^{-dx-c}}}{d}+\frac{\left(\frac{2a\left(a+\sqrt{a^2-b^2}\right)\sqrt{\frac{\left(\mathrm{e}^{dx+c}+\frac{a+\sqrt{a^2-b^2}}{b}\right)b}{a+\sqrt{a^2-b^2}}}\,\sqrt{\frac{\mathrm{e}^{dx+c}}{-\frac{a+\sqrt{a^2}}{b}}}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(a*EllipticF(cosh(1/2*d*x+1/2*c)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2
))+b*EllipticF(cosh(1/2*d*x+1/2*c)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2
))-2*b*EllipticE(cosh(1/2*d*x+1/2*c)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1
/2)))*(-sinh(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cosh(1/2*d*x+1/2*c)^2+a-b)/(a-b)
)^(1/2)*((2*b*cosh(1/2*d*x+1/2*c)^2+a-b)*sinh(1/2*d*x+1/2*c)^2)^(1/2)/(-2*b
/(a-b))^(1/2)/(2*sinh(1/2*d*x+1/2*c)^4*b+(a+b)*sinh(1/2*d*x+1/2*c)^2)^(1/2)
/sinh(1/2*d*x+1/2*c)/(2*b*sinh(1/2*d*x+1/2*c)^2+a+b)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cosh(d*x + c) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 196, normalized size = 3.21

$$\frac{2\left(\sqrt{2}\,a\sqrt{b}\,\mathrm{weierstrassPInverse}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3},\frac{3\,b\cosh(dx+c)+3\,b\sinh(dx+c)+2\,a}{3\,b}\right)-3\,\sqrt{2}\,b^{\frac{3}{2}}\mathrm{weierstrassZeta}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3},\mathrm{weierstrassPInverse}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3},\frac{3\,b\cosh(dx+c)+3\,b\sinh(dx+c)+2\,a}{3\,b}\right)\right)-3\,\sqrt{b\cosh\left(dx+c\right)+a}\,b\right)}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(sqrt(2)*a*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(
8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cosh(d*x + c) + 3*b*sinh(d*x + c) + 2*a)/b)
- 3*sqrt(2)*b^(3/2)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 -
 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 -
9*a*b^2)/b^3, 1/3*(3*b*cosh(d*x + c) + 3*b*sinh(d*x + c) + 2*a)/b)) - 3*sqr
t(b*cosh(d*x + c) + a)*b)/(b*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \cosh (c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*cosh(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cosh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{a + b \cosh (c + d\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(c + d*x))^(1/2),x)
```

```
[Out] int((a + b*cosh(c + d*x))^(1/2), x)
```

3.82 $\int \frac{1}{\sqrt{a+b\cosh(x)}}\,dx$

Optimal. Leaf size=46

$$-\frac{2i\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{\sqrt{a+b\cosh(x)}}$$

```
[Out] -2*I*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh(1/2*x),2^(1/2)*(b/(
a+b))^(1/2))*((a+b*cosh(x))/(a+b))^(1/2)/(a+b*cosh(x))^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2742, 2740}

$$-\frac{2i\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{\sqrt{a+b\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Cosh[x]],x]
```

```
[Out] ((-2*I)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/Sq
rt[a + b*Cosh[x]]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\cosh(x)}}\,dx = \frac{\sqrt{\frac{a+b\cosh(x)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}}\,dx}{\sqrt{a+b\cosh(x)}}$$

$$= -\frac{2i\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{\sqrt{a+b\cosh(x)}}$$

Mathematica [A]
time = 0.03, size = 46, normalized size = 1.00

$$-\frac{2i\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{\sqrt{a+b\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Cosh[x]],x]
```

```
[Out] ((-2*I)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/Sq
rt[a + b*Cosh[x]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(60) = 120$.
time = 0.92, size = 146, normalized size = 3.17

method	result
default	$\frac{2\sqrt{\left(2b\left(\cosh^2\left(\frac{x}{2}\right)\right)+a-b\right)\left(\sinh^2\left(\frac{x}{2}\right)\right)}\,\sqrt{\frac{2b\left(\cosh^2\left(\frac{x}{2}\right)\right)+a-b}{a-b}}\,\sqrt{-\left(\sinh^2\left(\frac{x}{2}\right)\right)}\,\mathrm{EllipticF}\left(\cosh\left(\frac{x}{2}\right)\sqrt{-\right.}{\sqrt{-\frac{2b}{a-b}}\,\sqrt{2\left(\sinh^4\left(\frac{x}{2}\right)\right)b+(a+b)\left(\sinh^2\left(\frac{x}{2}\right)\right)}\,\sinh\left(\frac{x}{2}\right)\sqrt{2b\left(\sinh^2\left(\frac{x}{2}\right)\right)+a+b}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*((2*b*cosh(1/2*x)^2+a-b)*sinh(1/2*x)^2)^(1/2)/(-2*b/(a-b))^(1/2)*((2*b*co
sh(1/2*x)^2+a-b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)/(2*sinh(1/2*x)^4*b+(a+
b)*sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a
-b)/b)^(1/2))/sinh(1/2*x)/(2*b*sinh(1/2*x)^2+a+b)^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*cosh(x) + a), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 61, normalized size = 1.33

$$\frac{2\sqrt{2}\,\text{weierstrassPInverse}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},\,-\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3},\,\frac{3\,b\cosh(x)+3\,b\sinh(x)+2\,a}{3\,b}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x))^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(2)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b
^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)/sqrt(b)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\cosh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x))**(1/2),x)
[Out] Integral(1/sqrt(a + b*cosh(x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(b*cosh(x) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{a+b\cosh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cosh(x))^(1/2),x)
[Out] int(1/(a + b*cosh(x))^(1/2), x)
```

3.83 $\int \frac{1}{(a+b\cosh(x))^{3/2}}\,dx$

Optimal. Leaf size=84

$$-\frac{2i\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}}-\frac{2b\sinh(x)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}$$

```
[Out] -2*b*sinh(x)/(a^2-b^2)/(a+b*cosh(x))^(1/2)-2*I*(cosh(1/2*x)^2)^(1/2)/cosh(1
/2*x)*EllipticE(I*sinh(1/2*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cosh(x))^(1/2)/
(a^2-b^2)/((a+b*cosh(x))/(a+b))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2743, 21, 2734, 2732}

$$-\frac{2b\sinh(x)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}-\frac{2i\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x])^(-3/2),x]
```

```
[Out] ((-2*I)*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)])/((a^2 - b^2)
*Sqrt[(a + b*Cosh[x])/(a + b)]) - (2*b*Sinh[x])/((a^2 - b^2)*Sqrt[a + b*Cos
h[x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+b\cosh(x))^{3/2}}\,dx &= -\frac{2b\sinh(x)}{(a^2-b^2)\sqrt{a+b\cosh(x)}} - \frac{2\int \frac{-\frac{a}{2}-\frac{1}{2}b\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx}{a^2-b^2} \\
&= -\frac{2b\sinh(x)}{(a^2-b^2)\sqrt{a+b\cosh(x)}} + \frac{\int \sqrt{a+b\cosh(x)}\,dx}{a^2-b^2} \\
&= -\frac{2b\sinh(x)}{(a^2-b^2)\sqrt{a+b\cosh(x)}} + \frac{\sqrt{a+b\cosh(x)}\int \sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}\,dx}{(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}} \\
&= -\frac{2i\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}} - \frac{2b\sinh(x)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}
\end{aligned}$$

Mathematica [**A**]
time = 0.09, size = 68, normalized size = 0.81

$$-\frac{2\left(i(a+b)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)+b\sinh(x)\right)}{(a-b)(a+b)\sqrt{a+b\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x])^(-3/2),x]
```

```
[Out] (-2*(I*(a + b)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticE[(I/2)*x, (2*b)/(a +
b)] + b*Sinh[x]))/((a - b)*(a + b)*Sqrt[a + b*Cosh[x]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 297 vs. 2(96) = 192.
time = 1.42, size = 298, normalized size = 3.55

method	result
default	$-\frac{2\left(2\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}}\,(\sinh^{2}(\frac{x}{2}))b-\sqrt{-\left(\sinh^{2}\left(\frac{x}{2}\right)\right)}\,\sqrt{\frac{2b(\sinh^{2}(\frac{x}{2}))}{a-b}+\frac{a+b}{a-b}}\ \mathrm{EllipticF}\left(\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}},\frac{\sqrt{-}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*cosh(x))^(3/2),x,method=_RETURNVERBOSE)

[Out] -2*(2*cosh(1/2*x)*(-2*b/(a-b))^(1/2)*sinh(1/2*x)^2*b-(-sinh(1/2*x)^2)^(1/2)*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))*a-(-sinh(1/2*x)^2)^(1/2)*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))*b+2*(-sinh(1/2*x)^2)^(1/2)*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*EllipticE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))*b)/(-2*b/(a-b))^(1/2)/(a-b)/(a+b)/sinh(1/2*x)/(2*b*sinh(1/2*x)^2+a+b)^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cosh(x))^(3/2),x, algorithm="maxima")

[Out] integrate((b*cosh(x) + a)^(-3/2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 413, normalized size = 4.92

$$\frac{2\left(\left(\sqrt{2}\,ab\cosh(x)^{2}+\sqrt{2}\,ab\sinh(x)^{2}+2\sqrt{2}\,a^{2}\cosh(x)+\sqrt{2}\,ab+2\left(\sqrt{2}\,ab\cosh(x)+\sqrt{2}\,a^{2}\right)\sinh(x)\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(\frac{4(4a^{2}-3b^{2})}{3b^{2}},-\frac{8(8a^{3}-9ab^{2})}{27b^{3}},\frac{3b\cosh(x)+3b\sinh(x)+2a}{3b}\right)-3\left(\sqrt{2}\,b^{2}\cosh(x)^{2}+\sqrt{2}\,b^{2}\sinh(x)^{2}+2\sqrt{2}\,ab\cosh(x)+\sqrt{2}\,b^{2}+2\left(\sqrt{2}\,b^{2}\cosh(x)+\sqrt{2}\,ab\right)\sinh(x)\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(\frac{4(4a^{2}-3b^{2})}{3b^{2}},-\frac{8(8a^{3}-9ab^{2})}{27b^{3}},\mathrm{weierstrassPInverse}\left(\frac{4(4a^{2}-3b^{2})}{3b^{2}},-\frac{8(8a^{3}-9ab^{2})}{27b^{3}},\frac{3b\cosh(x)+3b\sinh(x)+2a}{3b}\right)\right)-6\left(b^{2}\cosh(x)^{2}+b^{2}\sinh(x)^{2}+ab\cosh(x)+(2b^{2}\cosh(x)+ab)\sinh(x)\right)\sqrt{b\cosh(x)+a}\right)}{3\left(a^{2}b^{2}-b^{4}+(a^{2}b^{2}-b^{4})\cosh(x)^{2}+(a^{2}b^{2}-b^{4})\sinh(x)^{2}+2(a^{3}b-ab^{3})\cosh(x)+2(a^{3}b-ab^{3}+(a^{2}b^{2}-b^{4})\cosh(x))\sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cosh(x))^(3/2),x, algorithm="fricas")

[Out] 2/3*((sqrt(2)*a*b*cosh(x)^2 + sqrt(2)*a*b*sinh(x)^2 + 2*sqrt(2)*a^2*cosh(x) + sqrt(2)*a*b + 2*(sqrt(2)*a*b*cosh(x) + sqrt(2)*a^2)*sinh(x))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) - 3*(sqrt(2)*b^2*cosh(x)^2 + sqrt(2)*b^2*sinh(x)^2 + 2*sqrt(2)*a*b*cosh(x) + sqrt(2)*b^2 + 2*(sqrt(2)*b^2*cosh(x) + sqrt(2)*a*b)*sinh(x))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8

```
/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)) - 6*(b
^2*cosh(x)^2 + b^2*sinh(x)^2 + a*b*cosh(x) + (2*b^2*cosh(x) + a*b)*sinh(x))
*sqrt(b*cosh(x) + a))/(a^2*b^2 - b^4 + (a^2*b^2 - b^4)*cosh(x)^2 + (a^2*b^2
 - b^4)*sinh(x)^2 + 2*(a^3*b - a*b^3)*cosh(x) + 2*(a^3*b - a*b^3 + (a^2*b^2
 - b^4)*cosh(x))*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\cosh(x))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x))**(3/2),x)
[Out] Integral((a + b*cosh(x))**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x))^(3/2),x, algorithm="giac")
[Out] integrate((b*cosh(x) + a)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\cosh(x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cosh(x))^(3/2),x)
[Out] int(1/(a + b*cosh(x))^(3/2), x)
```

3.84 $\int \frac{1}{(a+b\cosh(x))^{5/2}}\,dx$

Optimal. Leaf size=177

$$-\frac{8ia\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{3(a^2-b^2)^2\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\frac{2i\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{3(a^2-b^2)\sqrt{a+b\cosh(x)}}-\frac{2b\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}-\frac{8ab\cdots}{3(a^2-b^2)^2\cdots}$$

```
[Out] -2/3*b*sinh(x)/(a^2-b^2)/(a+b*cosh(x))^(3/2)-8/3*a*b*sinh(x)/(a^2-b^2)^2/(a
+b*cosh(x))^(1/2)-8/3*I*a*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*sin
h(1/2*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cosh(x))^(1/2)/(a^2-b^2)^2/((a+b*cos
h(x))/(a+b))^(1/2)+2/3*I*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh
(1/2*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cosh(x))/(a+b))^(1/2)/(a^2-b^2)/(a+b
*cosh(x))^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {2743, 2833, 2831, 2742, 2740, 2734, 2732}

$$-\frac{8ab\sinh(x)}{3(a^2-b^2)^2\sqrt{a+b\cosh(x)}}-\frac{2b\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}+\frac{2i\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{3(a^2-b^2)\sqrt{a+b\cosh(x)}}-\frac{8ia\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{3(a^2-b^2)^2\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x])^(-5/2),x]
```

```
[Out] (((-8*I)/3)*a*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)])/((a^2
- b^2)^2*Sqrt[(a + b*Cosh[x])/(a + b)]) + (((2*I)/3)*Sqrt[(a + b*Cosh[x])/(
a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/((a^2 - b^2)*Sqrt[a + b*Cosh[x]]
) - (2*b*Sinh[x])/(3*(a^2 - b^2)*(a + b*Cosh[x])^(3/2)) - (8*a*b*Sinh[x])/(
3*(a^2 - b^2)^2*Sqrt[a + b*Cosh[x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{1}{(a+b\cosh(x))^{5/2}}\,dx = -\frac{2b\sinh(x)}{3\,(a^2-b^2)\,(a+b\cosh(x))^{3/2}} - \frac{2\int \frac{-\frac{3a}{2}+\frac{1}{2}b\cosh(x)}{(a+b\cosh(x))^{3/2}}\,dx}{3\,(a^2-b^2)}$$

$$= -\frac{2b\sinh(x)}{3\,(a^2-b^2)\,(a+b\cosh(x))^{3/2}} - \frac{8ab\sinh(x)}{3\,(a^2-b^2)^2\,\sqrt{a+b\cosh(x)}} + \frac{4\int \frac{\frac{1}{4}(3a^2+b^2)+ab\,c}{\sqrt{a+b\cosh}}}{3\,(a^2-b^2}$$

$$= -\frac{2b\sinh(x)}{3\,(a^2-b^2)\,(a+b\cosh(x))^{3/2}} - \frac{8ab\sinh(x)}{3\,(a^2-b^2)^2\,\sqrt{a+b\cosh(x)}} + \frac{(4a)\int \sqrt{a+b\,c}}{3\,(a^2-b}$$

$$= -\frac{2b\sinh(x)}{3\,(a^2-b^2)\,(a+b\cosh(x))^{3/2}} - \frac{8ab\sinh(x)}{3\,(a^2-b^2)^2\,\sqrt{a+b\cosh(x)}} + \frac{\left(4a\sqrt{a+b\cosh}\right.}{3\,(a^2}$$

$$= -\frac{8ia\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{3\,(a^2-b^2)^2\sqrt{\frac{a+b\cosh(x)}{a+b}}} + \frac{2i\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{3\,(a^2-b^2)\,\sqrt{a+b\cosh(x)}} - \frac{2b\,\mathrm{si}}{3\,(a^2-b^2)\,(a}$$

Mathematica [A]
time = 0.39, size = 135, normalized size = 0.76

$$\frac{-8ia(a+b)^2\left(\frac{a+b\cosh(x)}{a+b}\right)^{3/2}E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)+2i(a-b)(a+b)^2\left(\frac{a+b\cosh(x)}{a+b}\right)^{3/2}F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)+2b(-5a^2+b^2-4ab\cosh(x))\sinh(x)}{3(a-b)^2(a+b)^2(a+b\cosh(x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x])^(-5/2),x]
```

```
[Out] ((-8*I)*a*(a + b)^2*((a + b*Cosh[x])/(a + b))^(3/2)*EllipticE[(I/2)*x, (2*b
)/(a + b)] + (2*I)*(a - b)*(a + b)^2*((a + b*Cosh[x])/(a + b))^(3/2)*Ellipt
icF[(I/2)*x, (2*b)/(a + b)] + 2*b*(-5*a^2 + b^2 - 4*a*b*Cosh[x])*Sinh[x])/(
3*(a - b)^2*(a + b)^2*(a + b*Cosh[x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(193) = 386$.
time = 1.61, size = 459, normalized size = 2.59

method	result

default	$$\frac{\sqrt{\left(2b\left(\cosh^2\left(\frac{x}{2}\right)\right)+a-b\right)\left(\sinh^2\left(\frac{x}{2}\right)\right)}\left(-\frac{\cosh\left(\frac{x}{2}\right)\sqrt{2\left(\sinh^4\left(\frac{x}{2}\right)\right)b+(a+b)\left(\sinh^2\left(\frac{x}{2}\right)\right)}}{3b(a-b)(a+b)\left(\cosh^2\left(\frac{x}{2}\right)+\frac{a-b}{2b}\right)^2}-\frac{}{3(a-b)^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(x))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((2*b*cosh(1/2*x)^2+a-b)*sinh(1/2*x)^2)^(1/2)*(-1/3/b/(a-b)/(a+b)*cosh(1/2*
x)*(2*sinh(1/2*x)^4*b+(a+b)*sinh(1/2*x)^2)^(1/2)/(cosh(1/2*x)^2+1/2*(a-b)/b
)^2-16/3*sinh(1/2*x)^2*b/(a-b)^2/(a+b)^2*cosh(1/2*x)*a/((2*b*cosh(1/2*x)^2+
a-b)*sinh(1/2*x)^2)^(1/2)+2*(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)/(-2*b/(a-
b))^(1/2)*((2*b*cosh(1/2*x)^2+a-b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)/(2*s
inh(1/2*x)^4*b+(a+b)*sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b)
)^(1/2),1/2*((-2*a+2*b)/b)^(1/2))-32/3*a*b/(a+b)^2/(a-b)^2*(-a+b)/(-2*b/(a-
b))^(1/2)*((2*b*cosh(1/2*x)^2+a-b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)/(2*s
inh(1/2*x)^4*b+(a+b)*sinh(1/2*x)^2)^(1/2)/(2*a-2*b)*(EllipticF(cosh(1/2*x)*
(-2*b/(a-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))-EllipticE(cosh(1/2*x)*(-2*b/(a
-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))))/sinh(1/2*x)/(2*b*sinh(1/2*x)^2+a+b)^
(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cosh(x) + a)^(-5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.19, size = 1281, normalized size = 7.24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/9*((sqrt(2)*(a^2*b^2 + 3*b^4)*cosh(x)^4 + sqrt(2)*(a^2*b^2 + 3*b^4)*sinh(
x)^4 + 4*sqrt(2)*(a^3*b + 3*a*b^3)*cosh(x)^3 + 4*(sqrt(2)*(a^2*b^2 + 3*b^4)
```

```
*cosh(x) + sqrt(2)*(a^3*b + 3*a*b^3))*sinh(x)^3 + 2*sqrt(2)*(2*a^4 + 7*a^2*
b^2 + 3*b^4)*cosh(x)^2 + 2*(3*sqrt(2)*(a^2*b^2 + 3*b^4)*cosh(x)^2 + 6*sqrt(
2)*(a^3*b + 3*a*b^3)*cosh(x) + sqrt(2)*(2*a^4 + 7*a^2*b^2 + 3*b^4))*sinh(x)
^2 + 4*sqrt(2)*(a^3*b + 3*a*b^3)*cosh(x) + 4*(sqrt(2)*(a^2*b^2 + 3*b^4)*cos
h(x)^3 + 3*sqrt(2)*(a^3*b + 3*a*b^3)*cosh(x)^2 + sqrt(2)*(2*a^4 + 7*a^2*b^2
 + 3*b^4)*cosh(x) + sqrt(2)*(a^3*b + 3*a*b^3))*sinh(x) + sqrt(2)*(a^2*b^2 +
 3*b^4))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3
- 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) - 12*(sqrt(2)*a*b^
3*cosh(x)^4 + sqrt(2)*a*b^3*sinh(x)^4 + 4*sqrt(2)*a^2*b^2*cosh(x)^3 + 4*sqr
t(2)*a^2*b^2*cosh(x) + sqrt(2)*a*b^3 + 4*(sqrt(2)*a*b^3*cosh(x) + sqrt(2)*a
^2*b^2)*sinh(x)^3 + 2*sqrt(2)*(2*a^3*b + a*b^3)*cosh(x)^2 + 2*(3*sqrt(2)*a*
b^3*cosh(x)^2 + 6*sqrt(2)*a^2*b^2*cosh(x) + sqrt(2)*(2*a^3*b + a*b^3))*sinh
(x)^2 + 4*(sqrt(2)*a*b^3*cosh(x)^3 + 3*sqrt(2)*a^2*b^2*cosh(x)^2 + sqrt(2)*
a^2*b^2 + sqrt(2)*(2*a^3*b + a*b^3)*cosh(x))*sinh(x))*sqrt(b)*weierstrassZe
ta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInvers
e(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) +
3*b*sinh(x) + 2*a)/b)) - 6*(4*a*b^3*cosh(x)^4 + 4*a*b^3*sinh(x)^4 + (13*a^2
*b^2 - b^4)*cosh(x)^3 + (16*a*b^3*cosh(x) + 13*a^2*b^2 - b^4)*sinh(x)^3 + 4
*(2*a^3*b + a*b^3)*cosh(x)^2 + (24*a*b^3*cosh(x)^2 + 8*a^3*b + 4*a*b^3 + 3*
(13*a^2*b^2 - b^4)*cosh(x))*sinh(x)^2 + (3*a^2*b^2 + b^4)*cosh(x) + (16*a*b
^3*cosh(x)^3 + 3*a^2*b^2 + b^4 + 3*(13*a^2*b^2 - b^4)*cosh(x)^2 + 8*(2*a^3*
b + a*b^3)*cosh(x))*sinh(x))*sqrt(b*cosh(x) + a))/(a^4*b^3 - 2*a^2*b^5 + b^
7 + (a^4*b^3 - 2*a^2*b^5 + b^7)*cosh(x)^4 + (a^4*b^3 - 2*a^2*b^5 + b^7)*sin
h(x)^4 + 4*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cosh(x)^3 + 4*(a^5*b^2 - 2*a^3*b^4
 + a*b^6 + (a^4*b^3 - 2*a^2*b^5 + b^7)*cosh(x))*sinh(x)^3 + 2*(2*a^6*b - 3*
a^4*b^3 + b^7)*cosh(x)^2 + 2*(2*a^6*b - 3*a^4*b^3 + b^7 + 3*(a^4*b^3 - 2*a^
2*b^5 + b^7)*cosh(x)^2 + 6*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cosh(x))*sinh(x)^2
 + 4*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cosh(x) + 4*(a^5*b^2 - 2*a^3*b^4 + a*b^6
 + (a^4*b^3 - 2*a^2*b^5 + b^7)*cosh(x)^3 + 3*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*
cosh(x)^2 + (2*a^6*b - 3*a^4*b^3 + b^7)*cosh(x))*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\cosh(x))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x))**(5/2),x)
```

```
[Out] Integral((a + b*cosh(x))**(-5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cosh(x) + a)^(-5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\cosh(x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cosh(x))^(5/2),x)
```

```
[Out] int(1/(a + b*cosh(x))^(5/2), x)
```

3.85 $\int \frac{1}{(a+b\cosh(x))^{7/2}}\,dx$

Optimal. Leaf size=227

$$-\frac{2i(23a^2+9b^2)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{15\left(a^2-b^2\right)^3\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\frac{16ia\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{15\left(a^2-b^2\right)^2\sqrt{a+b\cosh(x)}}-\frac{2b\sinh(x)}{5\left(a^2-b^2\right)(a+b\cosh(x))^{5/2}}$$

```
[Out] -2/5*b*sinh(x)/(a^2-b^2)/(a+b*cosh(x))^(5/2)-16/15*a*b*sinh(x)/(a^2-b^2)^2/
(a+b*cosh(x))^(3/2)-2/15*b*(23*a^2+9*b^2)*sinh(x)/(a^2-b^2)^3/(a+b*cosh(x))
^(1/2)-2/15*I*(23*a^2+9*b^2)*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*
sinh(1/2*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cosh(x))^(1/2)/(a^2-b^2)^3/((a+b*
cosh(x))/(a+b))^(1/2)+16/15*I*a*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF
(I*sinh(1/2*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cosh(x))/(a+b))^(1/2)/(a^2-b^
2)^2/(a+b*cosh(x))^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {2743, 2833, 2831, 2742, 2740, 2734, 2732}

$$-\frac{2b(23a^2+9b^2)\sinh(x)}{15\left(a^2-b^2\right)^3\sqrt{a+b\cosh(x)}}-\frac{16ab\sinh(x)}{15\left(a^2-b^2\right)^2(a+b\cosh(x))^{3/2}}-\frac{2b\sinh(x)}{5\left(a^2-b^2\right)(a+b\cosh(x))^{5/2}}+\frac{16ia\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{15\left(a^2-b^2\right)^2\sqrt{a+b\cosh(x)}}-\frac{2i(23a^2+9b^2)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{15\left(a^2-b^2\right)^3\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x])^(-7/2),x]
```

```
[Out] (((-2*I)/15)*(23*a^2 + 9*b^2)*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/
(a + b)])/((a^2 - b^2)^3*Sqrt[(a + b*Cosh[x])/(a + b)]) + (((16*I)/15)*a*Sq
rt[(a + b*Cosh[x])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/((a^2 - b^2)
^2*Sqrt[a + b*Cosh[x]]) - (2*b*Sinh[x])/(5*(a^2 - b^2)*(a + b*Cosh[x])^(5/2
)) - (16*a*b*Sinh[x])/(15*(a^2 - b^2)^2*(a + b*Cosh[x])^(3/2)) - (2*b*(23*a
^2 + 9*b^2)*Sinh[x])/(15*(a^2 - b^2)^3*Sqrt[a + b*Cosh[x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{1}{(a+b\cosh(x))^{7/2}}\,dx = -\frac{2b\sinh(x)}{5\,(a^2-b^2)\,(a+b\cosh(x))^{5/2}} - \frac{2\int \frac{-\frac{5a}{2}+\frac{3}{2}b\cosh(x)}{(a+b\cosh(x))^{5/2}}\,dx}{5\,(a^2-b^2)}$$

$$= -\frac{2b\sinh(x)}{5\,(a^2-b^2)\,(a+b\cosh(x))^{5/2}} - \frac{16ab\sinh(x)}{15\,(a^2-b^2)^2\,(a+b\cosh(x))^{3/2}} + \frac{4\int \frac{\frac{3}{4}(5a^2+3b^2)\cdots}{(a+b\cosh\cdots}}{15\,(a^2\cdots}$$

$$= -\frac{2b\sinh(x)}{5\,(a^2-b^2)\,(a+b\cosh(x))^{5/2}} - \frac{16ab\sinh(x)}{15\,(a^2-b^2)^2\,(a+b\cosh(x))^{3/2}} - \frac{2b(23a^2+\cdots}{15\,(a^2-b^2)^3\cdots}$$

$$= -\frac{2b\sinh(x)}{5\,(a^2-b^2)\,(a+b\cosh(x))^{5/2}} - \frac{16ab\sinh(x)}{15\,(a^2-b^2)^2\,(a+b\cosh(x))^{3/2}} - \frac{2b(23a^2+\cdots}{15\,(a^2-b^2)^3\cdots}$$

$$= -\frac{2b\sinh(x)}{5\,(a^2-b^2)\,(a+b\cosh(x))^{5/2}} - \frac{16ab\sinh(x)}{15\,(a^2-b^2)^2\,(a+b\cosh(x))^{3/2}} - \frac{2b(23a^2+\cdots}{15\,(a^2-b^2)^3\cdots}$$

$$= -\frac{2i(23a^2+9b^2)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{15\,(a^2-b^2)^3\sqrt{\frac{a+b\cosh(x)}{a+b}}} + \frac{16ia\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{15\,(a^2-b^2)^2\sqrt{a+b\cosh(x)}} - \cdots$$

Mathematica [A]
time = 0.51, size = 165, normalized size = 0.73

$$\frac{2\left(-\frac{i\left(\frac{a+b\cosh(x)}{a+b}\right)^{5/2}\left((23a^2+9b^2)E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)+8a(-a+b)F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)\right)}{(a-b)^3}+\frac{b(34a^4-5a^2b^2+3b^4+2ab(27a^2+5b^2)\cosh(x)+b^2(23a^2+9b^2)\cosh^2(x))\sinh(x)}{(-a^2+b^2)^3}\right)}{15(a+b\cosh(x))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x])^(-7/2),x]
```

```
[Out] (2*(((-I)*((a + b*Cosh[x])/(a + b))^(5/2)*((23*a^2 + 9*b^2)*EllipticE[(I/2)
*x, (2*b)/(a + b)] + 8*a*(-a + b)*EllipticF[(I/2)*x, (2*b)/(a + b)]))/(a -
b)^3 + (b*(34*a^4 - 5*a^2*b^2 + 3*b^4 + 2*a*b*(27*a^2 + 5*b^2)*Cosh[x] + b^
2*(23*a^2 + 9*b^2)*Cosh[x]^2)*Sinh[x])/(-a^2 + b^2)^3))/(15*(a + b*Cosh[x])
^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 565 vs. $2(239) = 478$.
time = 1.87, size = 566, normalized size = 2.49

method	result
default	$\frac{\sqrt{\left(2b\left(\cosh^2\left(\frac{x}{2}\right)\right)+a-b\right)\left(\sinh^2\left(\frac{x}{2}\right)\right)}\left(-\frac{\cosh\left(\frac{x}{2}\right)\sqrt{2\left(\sinh^4\left(\frac{x}{2}\right)\right)b+(a+b)\left(\sinh^2\left(\frac{x}{2}\right)\right)}}{10b^2(a-b)(a+b)\left(\cosh^2\left(\frac{x}{2}\right)+\frac{a-b}{2b}\right)^3}-\frac{8a\cosh(}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(x))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((2*b*cosh(1/2*x)^2+a-b)*sinh(1/2*x)^2)^(1/2)*(-1/10/b^2/(a-b)/(a+b)*cosh(1
/2*x)*(2*sinh(1/2*x)^4*b+(a+b)*sinh(1/2*x)^2)^(1/2)/(cosh(1/2*x)^2+1/2*(a-b
)/b)^3-8/15*a/b/(a+b)^2/(a-b)^2*cosh(1/2*x)*(2*sinh(1/2*x)^4*b+(a+b)*sinh(1
/2*x)^2)^(1/2)/(cosh(1/2*x)^2+1/2*(a-b)/b)^2-4/15*sinh(1/2*x)^2*b/(a-b)^3/(
a+b)^3*cosh(1/2*x)*(23*a^2+9*b^2)/((2*b*cosh(1/2*x)^2+a-b)*sinh(1/2*x)^2)^(
1/2)+2*(15*a^2-8*a*b+9*b^2)/(15*a^5+15*a^4*b-30*a^3*b^2-30*a^2*b^3+15*a*b^4
+15*b^5)/(-2*b/(a-b))^(1/2)*((2*b*cosh(1/2*x)^2+a-b)/(a-b))^(1/2)*(-sinh(1/
2*x)^2)^(1/2)/(2*sinh(1/2*x)^4*b+(a+b)*sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(
1/2*x)*(-2*b/(a-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))-8/15*b*(23*a^2+9*b^2)/(
a+b)^3/(a-b)^3*(-a+b)/(-2*b/(a-b))^(1/2)*((2*b*cosh(1/2*x)^2+a-b)/(a-b))^(1
/2)*(-sinh(1/2*x)^2)^(1/2)/(2*sinh(1/2*x)^4*b+(a+b)*sinh(1/2*x)^2)^(1/2)/(2
*a-2*b)*(EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))
-EllipticE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))))/sinh(
1/2*x)/(2*b*sinh(1/2*x)^2+a+b)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cosh(x) + a)^(-7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.16, size = 3315, normalized size = 14.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x))^(7/2),x, algorithm="fricas")
```

```
[Out] -2/45*((sqrt(2)*(a^3*b^3 - 33*a*b^5)*cosh(x)^6 + sqrt(2)*(a^3*b^3 - 33*a*b^
5)*sinh(x)^6 + 6*sqrt(2)*(a^4*b^2 - 33*a^2*b^4)*cosh(x)^5 + 6*(sqrt(2)*(a^3
*b^3 - 33*a*b^5)*cosh(x) + sqrt(2)*(a^4*b^2 - 33*a^2*b^4))*sinh(x)^5 + 3*sq
rt(2)*(4*a^5*b - 131*a^3*b^3 - 33*a*b^5)*cosh(x)^4 + 3*(5*sqrt(2)*(a^3*b^3
- 33*a*b^5)*cosh(x)^2 + 10*sqrt(2)*(a^4*b^2 - 33*a^2*b^4)*cosh(x) + sqrt(2)
*(4*a^5*b - 131*a^3*b^3 - 33*a*b^5))*sinh(x)^4 + 4*sqrt(2)*(2*a^6 - 63*a^4*
b^2 - 99*a^2*b^4)*cosh(x)^3 + 4*(5*sqrt(2)*(a^3*b^3 - 33*a*b^5)*cosh(x)^3 +
 15*sqrt(2)*(a^4*b^2 - 33*a^2*b^4)*cosh(x)^2 + 3*sqrt(2)*(4*a^5*b - 131*a^3
*b^3 - 33*a*b^5)*cosh(x) + sqrt(2)*(2*a^6 - 63*a^4*b^2 - 99*a^2*b^4))*sinh(
x)^3 + 3*sqrt(2)*(4*a^5*b - 131*a^3*b^3 - 33*a*b^5)*cosh(x)^2 + 3*(5*sqrt(2
)*(a^3*b^3 - 33*a*b^5)*cosh(x)^4 + 20*sqrt(2)*(a^4*b^2 - 33*a^2*b^4)*cosh(x
)^3 + 6*sqrt(2)*(4*a^5*b - 131*a^3*b^3 - 33*a*b^5)*cosh(x)^2 + 4*sqrt(2)*(2
*a^6 - 63*a^4*b^2 - 99*a^2*b^4)*cosh(x) + sqrt(2)*(4*a^5*b - 131*a^3*b^3 -
33*a*b^5))*sinh(x)^2 + 6*sqrt(2)*(a^4*b^2 - 33*a^2*b^4)*cosh(x) + 6*(sqrt(2
)*(a^3*b^3 - 33*a*b^5)*cosh(x)^5 + 5*sqrt(2)*(a^4*b^2 - 33*a^2*b^4)*cosh(x)
^4 + 2*sqrt(2)*(4*a^5*b - 131*a^3*b^3 - 33*a*b^5)*cosh(x)^3 + 2*sqrt(2)*(2*
a^6 - 63*a^4*b^2 - 99*a^2*b^4)*cosh(x)^2 + sqrt(2)*(4*a^5*b - 131*a^3*b^3 -
 33*a*b^5)*cosh(x) + sqrt(2)*(a^4*b^2 - 33*a^2*b^4))*sinh(x) + sqrt(2)*(a^3
*b^3 - 33*a*b^5))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/2
7*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) + 3*(sqrt
(2)*(23*a^2*b^4 + 9*b^6)*cosh(x)^6 + sqrt(2)*(23*a^2*b^4 + 9*b^6)*sinh(x)^6
 + 6*sqrt(2)*(23*a^3*b^3 + 9*a*b^5)*cosh(x)^5 + 6*(sqrt(2)*(23*a^2*b^4 + 9*
b^6)*cosh(x) + sqrt(2)*(23*a^3*b^3 + 9*a*b^5))*sinh(x)^5 + 3*sqrt(2)*(92*a^
4*b^2 + 59*a^2*b^4 + 9*b^6)*cosh(x)^4 + 3*(5*sqrt(2)*(23*a^2*b^4 + 9*b^6)*c
osh(x)^2 + 10*sqrt(2)*(23*a^3*b^3 + 9*a*b^5)*cosh(x) + sqrt(2)*(92*a^4*b^2
+ 59*a^2*b^4 + 9*b^6))*sinh(x)^4 + 4*sqrt(2)*(46*a^5*b + 87*a^3*b^3 + 27*a*
b^5)*cosh(x)^3 + 4*(5*sqrt(2)*(23*a^2*b^4 + 9*b^6)*cosh(x)^3 + 15*sqrt(2)*(
23*a^3*b^3 + 9*a*b^5)*cosh(x)^2 + 3*sqrt(2)*(92*a^4*b^2 + 59*a^2*b^4 + 9*b^
6)*cosh(x) + sqrt(2)*(46*a^5*b + 87*a^3*b^3 + 27*a*b^5))*sinh(x)^3 + 3*sqrt
(2)*(92*a^4*b^2 + 59*a^2*b^4 + 9*b^6)*cosh(x)^2 + 3*(5*sqrt(2)*(23*a^2*b^4
+ 9*b^6)*cosh(x)^4 + 20*sqrt(2)*(23*a^3*b^3 + 9*a*b^5)*cosh(x)^3 + 6*sqrt(2
)*(92*a^4*b^2 + 59*a^2*b^4 + 9*b^6)*cosh(x)^2 + 4*sqrt(2)*(46*a^5*b + 87*a^
3*b^3 + 27*a*b^5)*cosh(x) + sqrt(2)*(92*a^4*b^2 + 59*a^2*b^4 + 9*b^6))*sinh
(x)^2 + 6*sqrt(2)*(23*a^3*b^3 + 9*a*b^5)*cosh(x) + 6*(sqrt(2)*(23*a^2*b^4 +
 9*b^6)*cosh(x)^5 + 5*sqrt(2)*(23*a^3*b^3 + 9*a*b^5)*cosh(x)^4 + 2*sqrt(2)*
(92*a^4*b^2 + 59*a^2*b^4 + 9*b^6)*cosh(x)^3 + 2*sqrt(2)*(46*a^5*b + 87*a^3*
b^3 + 27*a*b^5)*cosh(x)^2 + sqrt(2)*(92*a^4*b^2 + 59*a^2*b^4 + 9*b^6)*cosh(
x) + sqrt(2)*(23*a^3*b^3 + 9*a*b^5))*sinh(x) + sqrt(2)*(23*a^2*b^4 + 9*b^6)
)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/
b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b
^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)) + 6*((23*a^2*b^4 + 9*b^6)*cos
h(x)^6 + (23*a^2*b^4 + 9*b^6)*sinh(x)^6 + (123*a^3*b^3 + 37*a*b^5)*cosh(x)^
5 + (123*a^3*b^3 + 37*a*b^5 + 6*(23*a^2*b^4 + 9*b^6)*cosh(x))*sinh(x)^5 + 2
```

```
*(103*a^4*b^2 + 45*a^2*b^4 + 12*b^6)*cosh(x)^4 + (206*a^4*b^2 + 90*a^2*b^4
+ 24*b^6 + 15*(23*a^2*b^4 + 9*b^6)*cosh(x)^2 + 5*(123*a^3*b^3 + 37*a*b^5)*c
osh(x))*sinh(x)^4 + 2*(46*a^5*b + 87*a^3*b^3 + 27*a*b^5)*cosh(x)^3 + 2*(46*
a^5*b + 87*a^3*b^3 + 27*a*b^5 + 10*(23*a^2*b^4 + 9*b^6)*cosh(x)^3 + 5*(123*
a^3*b^3 + 37*a*b^5)*cosh(x)^2 + 4*(103*a^4*b^2 + 45*a^2*b^4 + 12*b^6)*cosh(
x))*sinh(x)^3 + (70*a^4*b^2 + 87*a^2*b^4 + 3*b^6)*cosh(x)^2 + (70*a^4*b^2 +
 87*a^2*b^4 + 3*b^6 + 15*(23*a^2*b^4 + 9*b^6)*cosh(x)^4 + 10*(123*a^3*b^3 +
 37*a*b^5)*cosh(x)^3 + 12*(103*a^4*b^2 + 45*a^2*b^4 + 12*b^6)*cosh(x)^2 + 6
*(46*a^5*b + 87*a^3*b^3 + 27*a*b^5)*cosh(x))*sinh(x)^2 + (15*a^3*b^3 + 17*a
*b^5)*cosh(x) + (15*a^3*b^3 + 17*a*b^5 + 6*(23*a^2*b^4 + 9*b^6)*cosh(x)^5 +
 5*(123*a^3*b^3 + 37*a*b^5)*cosh(x)^4 + 8*(103*a^4*b^2 + 45*a^2*b^4 + 12*b^
6)*cosh(x)^3 + 6*(46*a^5*b + 87*a^3*b^3 + 27*a*b^5)*cosh(x)^2 + 2*(70*a^4*b
^2 + 87*a^2*b^4 + 3*b^6)*cosh(x))*sinh(x))*sqrt(b*cosh(x) + a))/(a^6*b^4 -
3*a^4*b^6 + 3*a^2*b^8 - b^10 + (a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10)*cos
h(x)^6 + (a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10)*sinh(x)^6 + 6*(a^7*b^3 -
3*a^5*b^5 + 3*a^3*b^7 - a*b^9)*cosh(x)^5 + 6*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b
^7 - a*b^9 + (a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10)*cosh(x))*sinh(x)^5 +
3*(4*a^8*b^2 - 11*a^6*b^4 + 9*a^4*b^6 - a^2*b^8 - b^10)*cosh(x)^4 + 3*(4*a^
8*b^2 - 11*a^6*b^4 + 9*a^4*b^6 - a^2*b^8 - b^10 + 5*(a^6*b^4 - 3*a^4*b^6 +
3*a^2*b^8 - b^10)*cosh(x)^2 + 10*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9)*
cosh(x))*sinh(x)^4 + 4*(2*a^9*b - 3*a^7*b^3 - 3*a^5*b^5 + 7*a^3*b^7 - 3*a*b
^9)*cosh(x)^3 + 4*(2*a^9*b - 3*a^7*b^3 - 3*a^5*b^5 + 7*a^3*b^7 - 3*a*b^9 +
5*(a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10)*cosh(x)^3 + 15*(a^7*b^3 - 3*a^5*
b^5 + 3*a^3*b^7 - a*b^9)*cosh(x)^2 + 3*(4*a^8*b...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x))**(7/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x))^(7/2),x, algorithm="giac")
[Out] integrate((b*cosh(x) + a)^(-7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b\cosh{(x)})^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cosh(x))^(7/2),x)
```

```
[Out] int(1/(a + b*cosh(x))^(7/2), x)
```

3.86 $\int \frac{\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx$

Optimal. Leaf size=100

$$-\frac{2i\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\frac{2ia\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{b\sqrt{a+b\cosh(x)}}$$

```
[Out] -2*I*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*sinh(1/2*x),2^(1/2)*(b/(
a+b))^(1/2))*(a+b*cosh(x))^(1/2)/b/((a+b*cosh(x))/(a+b))^(1/2)+2*I*a*(cosh(
1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh(1/2*x),2^(1/2)*(b/(a+b))^(1/2)
)*((a+b*cosh(x))/(a+b))^(1/2)/b/(a+b*cosh(x))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2831, 2742, 2740, 2734, 2732}

$$\frac{2ia\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{b\sqrt{a+b\cosh(x)}}-\frac{2i\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/Sqrt[a + b*Cosh[x]],x]
```

```
[Out] ((-2*I)*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)])/(b*Sqrt[(a +
 b*Cosh[x])/(a + b)]) + ((2*I)*a*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticF[(I
/2)*x, (2*b)/(a + b)])/(b*Sqrt[a + b*Cosh[x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx = \frac{\int \sqrt{a+b\cosh(x)}\,dx}{b} - \frac{a\int \frac{1}{\sqrt{a+b\cosh(x)}}\,dx}{b}$$

$$= \frac{\sqrt{a+b\cosh(x)}\int \sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}\,dx}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}} - \frac{\left(a\sqrt{\frac{a+b\cosh(x)}{a+b}}\right)\int \frac{1}{\sqrt{\frac{a}{a+b}+\cdots}}}{b\sqrt{a+b\cosh(x)}}$$

$$= -\frac{2i\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}} + \frac{2ia\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{b\sqrt{a+b\cosh(x)}}$$

Mathematica [A]
time = 0.28, size = 73, normalized size = 0.73

$$-\frac{2i\sqrt{\frac{a+b\cosh(x)}{a+b}}\left((a+b)E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)-aF\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)\right)}{b\sqrt{a+b\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/Sqrt[a + b*Cosh[x]],x]
```

```
[Out] ((-2*I)*Sqrt[(a + b*Cosh[x])/(a + b)]*((a + b)*EllipticE[(I/2)*x, (2*b)/(a
+ b)] - a*EllipticF[(I/2)*x, (2*b)/(a + b)]))/(b*Sqrt[a + b*Cosh[x]])
```

Maple [A]
time = 1.34, size = 181, normalized size = 1.81

method	result
default	$\frac{2\left(\mathrm{EllipticF}\left(\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}}, \frac{\sqrt{-\frac{2(a-b)}{b}}}{2}\right)-2\,\mathrm{EllipticE}\left(\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}}, \frac{\sqrt{-\frac{2(a-b)}{b}}}{2}\right)\right)\sqrt{-\left(\sinh^2\left(\frac{x}{2}\right)\right)} \cdots}{\sqrt{-\frac{2b}{a-b}}\,\sqrt{2\left(\sinh^4\left(\frac{x}{2}\right)\right)b+(a+b)\left(\sinh^2\left(\frac{x}{2}\right)\right)}\,\sinh\left(\frac{x}{2}\right)\sqrt{2 \cdots}}$
risch	$\frac{\left(b\,\mathrm{e}^{2x}+2a\,\mathrm{e}^{x}+b\right)\sqrt{2}\,\mathrm{e}^{-x}}{b\sqrt{\left(b\,\mathrm{e}^{2x}+2a\,\mathrm{e}^{x}+b\right)\mathrm{e}^{-x}}}+\frac{\left(-\frac{4\left(b\,\mathrm{e}^{2x}+2a\,\mathrm{e}^{x}+b\right)}{b\sqrt{\left(b\,\mathrm{e}^{2x}+2a\,\mathrm{e}^{x}+b\right)\mathrm{e}^{x}}}+\frac{4\left(a+\sqrt{a^2-b^2}\right)\sqrt{\frac{\left(\mathrm{e}^{x}+\frac{a+\sqrt{a^2-b^2}}{b}\right)b}{a+\sqrt{a^2-b^2}} \cdots}}{\cdots}\right) \cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a+b*cosh(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))-2*Ellip
ticE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2)))*(-sinh(1/2*x)^
2)^(1/2)*((2*b*cosh(1/2*x)^2+a-b)/(a-b))^(1/2)*((2*b*cosh(1/2*x)^2+a-b)*sin
h(1/2*x)^2)^(1/2)/(-2*b/(a-b))^(1/2)/(2*sinh(1/2*x)^4*b+(a+b)*sinh(1/2*x)^2
)^(1/2)/sinh(1/2*x)/(2*b*sinh(1/2*x)^2+a+b)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*cosh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(x)/sqrt(b*cosh(x) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 174, normalized size = 1.74

$$-\frac{2\left(2\sqrt{2}\,a\sqrt{b}\,\mathrm{weierstrassPInverse}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3},\frac{3\,b\cosh(x)+3\,b\sinh(x)+2\,a}{3\,b}\right)+3\sqrt{2}\,b^{\frac{3}{2}}\mathrm{weierstrassZeta}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3},\mathrm{weierstrassPInverse}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3},\frac{3\,b\cosh(x)+3\,b\sinh(x)+2\,a}{3\,b}\right)\right)+3\sqrt{b\cosh(x)+a}\,b\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*cosh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(2*sqrt(2)*a*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/2
7*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) + 3*sqrt(
2)*b^(3/2)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)
/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/
b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)) + 3*sqrt(b*cosh(x) + a)*b)/b
^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(x)}{\sqrt{a + b\cosh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*cosh(x))**(1/2),x)
```

```
[Out] Integral(cosh(x)/sqrt(a + b*cosh(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*cosh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(x)/sqrt(b*cosh(x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(x)}{\sqrt{a + b\cosh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a + b*cosh(x))^(1/2),x)
```

```
[Out] int(cosh(x)/(a + b*cosh(x))^(1/2), x)
```

3.87 $\int (a+a\cosh(x))^{5/2}(A+B\cosh(x))\,dx$

Optimal. Leaf size=94

$$\frac{64a^3(7A+5B)\sinh(x)}{105\sqrt{a+a\cosh(x)}}+\frac{16}{105}a^2(7A+5B)\sqrt{a+a\cosh(x)}\,\sinh(x)+\frac{2}{35}a(7A+5B)(a+a\cosh(x))^{3/2}\sinh(x)+$$

```
[Out] 2/35*a*(7*A+5*B)*(a+a*cosh(x))^(3/2)*sinh(x)+2/7*B*(a+a*cosh(x))^(5/2)*sinh
(x)+64/105*a^3*(7*A+5*B)*sinh(x)/(a+a*cosh(x))^(1/2)+16/105*a^2*(7*A+5*B)*s
inh(x)*(a+a*cosh(x))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2830, 2726, 2725}

$$\frac{64a^3(7A+5B)\sinh(x)}{105\sqrt{a\cosh(x)+a}}+\frac{16}{105}a^2(7A+5B)\sinh(x)\sqrt{a\cosh(x)+a}+\frac{2}{35}a(7A+5B)\sinh(x)(a\cosh(x)+a)^{3/2}+\frac{2}{7}B\sinh(x)(a\cosh(x)+a)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[x])^(5/2)*(A + B*Cosh[x]),x]
```

```
[Out] (64*a^3*(7*A + 5*B)*Sinh[x])/(105*Sqrt[a + a*Cosh[x]]) + (16*a^2*(7*A + 5*B
)*Sqrt[a + a*Cosh[x]]*Sinh[x])/105 + (2*a*(7*A + 5*B)*(a + a*Cosh[x])^(3/2)
*Sinh[x])/35 + (2*B*(a + a*Cosh[x])^(5/2)*Sinh[x])/7
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int (a+a\cosh(x))^{5/2}(A+B\cosh(x))\,dx = \frac{2}{7}B(a+a\cosh(x))^{5/2}\sinh(x)+\frac{1}{7}(7A+5B)\int (a+a\cosh(x))^{5/2}$$
$$= \frac{2}{35}a(7A+5B)(a+a\cosh(x))^{3/2}\sinh(x)+\frac{2}{7}B(a+a\cosh(x))^{5/2}\,\mathrm{s}$$
$$= \frac{16}{105}a^2(7A+5B)\sqrt{a+a\cosh(x)}\,\sinh(x)+\frac{2}{35}a(7A+5B)(a+a$$
$$= \frac{64a^3(7A+5B)\sinh(x)}{105\sqrt{a+a\cosh(x)}}+\frac{16}{105}a^2(7A+5B)\sqrt{a+a\cosh(x)}\,\sinh($$

Mathematica [**A**]
time = 0.10, size = 60, normalized size = 0.64

$$\frac{1}{210}a^2\sqrt{a(1+\cosh(x))}\,(1246A+1040B+(392A+505B)\cosh(x)+6(7A+20B)\cosh(2x)+15B\cosh(3x))\tanh\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[x])^(5/2)*(A + B*Cosh[x]),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cosh[x])]*(1246*A + 1040*B + (392*A + 505*B)*Cosh[x] + 6*(
7*A + 20*B)*Cosh[2*x] + 15*B*Cosh[3*x])*Tanh[x/2])/210
```

Maple [**A**]
time = 1.08, size = 71, normalized size = 0.76

method	result	size
default	$\frac{8\cosh\left(\frac{x}{2}\right)a^3\sinh\left(\frac{x}{2}\right)\left(30B\left(\sinh^6\left(\frac{x}{2}\right)\right)+(21A+105B)\left(\sinh^4\left(\frac{x}{2}\right)\right)+(70A+140B)\left(\sinh^2\left(\frac{x}{2}\right)\right)+105A+105B\right)\sqrt{2}}{105\sqrt{a\left(\cosh^2\left(\frac{x}{2}\right)\right)}}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(x))^(5/2)*(A+B*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 8/105*cosh(1/2*x)*a^3*sinh(1/2*x)*(30*B*sinh(1/2*x)^6+(21*A+105*B)*sinh(1/2
*x)^4+(70*A+140*B)*sinh(1/2*x)^2+105*A+105*B)*2^(1/2)/(a*cosh(1/2*x)^2)^(1/
2)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(78) = 156.
time = 0.51, size = 237, normalized size = 2.52

$$\frac{1}{60}\left(3\sqrt{2}\,a^{\frac{5}{2}}e^{\left(\frac{5}{2}x\right)}+25\sqrt{2}\,a^{\frac{5}{2}}e^{\left(\frac{3}{2}x\right)}+150\sqrt{2}\,a^{\frac{5}{2}}e^{\left(\frac{1}{2}x\right)}-150\sqrt{2}\,a^{\frac{5}{2}}e^{\left(-\frac{1}{2}x\right)}-25\sqrt{2}\,a^{\frac{5}{2}}e^{\left(-\frac{3}{2}x\right)}-3\sqrt{2}\,a^{\frac{5}{2}}e^{\left(-\frac{5}{2}x\right)}\right)A+\frac{1}{168}\left(\left(3\sqrt{2}\,a^{\frac{5}{2}}e^{(-x)}+21\sqrt{2}\,a^{\frac{5}{2}}e^{(-2x)}+70\sqrt{2}\,a^{\frac{5}{2}}e^{(-3x)}+210\sqrt{2}\,a^{\frac{5}{2}}e^{(-4x)}-105\sqrt{2}\,a^{\frac{5}{2}}e^{(-5x)}-7\sqrt{2}\,a^{\frac{5}{2}}e^{(-6x)}\right)e^{\left(\frac{9}{2}x\right)}+\left(7\sqrt{2}\,a^{\frac{5}{2}}e^{(-x)}+105\sqrt{2}\,a^{\frac{5}{2}}e^{(-2x)}-210\sqrt{2}\,a^{\frac{5}{2}}e^{(-3x)}-70\sqrt{2}\,a^{\frac{5}{2}}e^{(-4x)}-21\sqrt{2}\,a^{\frac{5}{2}}e^{(-5x)}-3\sqrt{2}\,a^{\frac{5}{2}}e^{(-6x)}\right)e^{\left(\frac{5}{2}x\right)}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(5/2)*(A+B*cosh(x)),x, algorithm="maxima")
```

```
[Out] 1/60*(3*sqrt(2)*a^(5/2)*e^(5/2*x) + 25*sqrt(2)*a^(5/2)*e^(3/2*x) + 150*sqrt
(2)*a^(5/2)*e^(1/2*x) - 150*sqrt(2)*a^(5/2)*e^(-1/2*x) - 25*sqrt(2)*a^(5/2)
*e^(-3/2*x) - 3*sqrt(2)*a^(5/2)*e^(-5/2*x))*A + 1/168*((3*sqrt(2)*a^(5/2)*e
^(-x) + 21*sqrt(2)*a^(5/2)*e^(-2*x) + 70*sqrt(2)*a^(5/2)*e^(-3*x) + 210*sqr
t(2)*a^(5/2)*e^(-4*x) - 105*sqrt(2)*a^(5/2)*e^(-5*x) - 7*sqrt(2)*a^(5/2)*e^
(-6*x))*e^(9/2*x) + (7*sqrt(2)*a^(5/2)*e^(-x) + 105*sqrt(2)*a^(5/2)*e^(-2*x
) - 210*sqrt(2)*a^(5/2)*e^(-3*x) - 70*sqrt(2)*a^(5/2)*e^(-4*x) - 21*sqrt(2)
*a^(5/2)*e^(-5*x) - 3*sqrt(2)*a^(5/2)*e^(-6*x))*e^(5/2*x))*B
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 563 vs. $2(78) = 156$.
time = 0.43, size = 563, normalized size = 5.99

$$\frac{\sqrt{\frac{1}{2}}\left(15Ba^2\cosh(x)^7 + 15Ba^2\sinh(x)^7 + 21(2A+5B)a^2\cosh(x)^6 + 35(10A+11B)a^2\cosh(x)^5 + 525(4A+3B)a^2\cosh(x)^4 + 21(5Ba^2\cosh(x)+(2A+5B)a^2)\sinh(x)^6 - 525(4A+3B)a^2\cosh(x)^3 + 7(45Ba^2\cosh(x)^2+18(2A+5B)a^2\cosh(x)+5(10A+11B)a^2)\sinh(x)^5 - 35(10A+11B)a^2\cosh(x)^2 + 35(15Ba^2\cosh(x)^3+9(2A+5B)a^2\cosh(x)^2+5(10A+11B)a^2\cosh(x)+15(4A+3B)a^2)\sinh(x)^4 - 21(2A+5B)a^2\cosh(x) + 35(15Ba^2\cosh(x)^4+12(2A+5B)a^2\cosh(x)^3+10(10A+11B)a^2\cosh(x)^2+60(4A+3B)a^2\cosh(x)-15(4A+3B)a^2)\sinh(x)^3 - 15Ba^2 + 35(9Ba^2\cosh(x)^5+9(2A+5B)a^2\cosh(x)^4+10(10A+11B)a^2\cosh(x)^3+90(4A+3B)a^2\cosh(x)^2-45(4A+3B)a^2\cosh(x)-(10A+11B)a^2)\sinh(x)^2 + 7(15Ba^2\cosh(x)^6+18(2A+5B)a^2\cosh(x)^5+25(10A+11B)a^2\cosh(x)^4+300(4A+3B)a^2\cosh(x)^3-225(4A+3B)a^2\cosh(x)^2-10(10A+11B)a^2\cosh(x)-3(2A+5B)a^2)\sinh(x)\right)\sqrt{\frac{a}{\cosh(x)+\sinh(x)}}}{420\left(\cosh(x)^3+3\cosh(x)^2\sinh(x)+3\cosh(x)\sinh(x)^2+\sinh(x)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(5/2)*(A+B*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/420*sqrt(1/2)*(15*B*a^2*cosh(x)^7 + 15*B*a^2*sinh(x)^7 + 21*(2*A + 5*B)*a
^2*cosh(x)^6 + 35*(10*A + 11*B)*a^2*cosh(x)^5 + 525*(4*A + 3*B)*a^2*cosh(x)
^4 + 21*(5*B*a^2*cosh(x) + (2*A + 5*B)*a^2)*sinh(x)^6 - 525*(4*A + 3*B)*a^2
*cosh(x)^3 + 7*(45*B*a^2*cosh(x)^2 + 18*(2*A + 5*B)*a^2*cosh(x) + 5*(10*A +
 11*B)*a^2)*sinh(x)^5 - 35*(10*A + 11*B)*a^2*cosh(x)^2 + 35*(15*B*a^2*cosh(
x)^3 + 9*(2*A + 5*B)*a^2*cosh(x)^2 + 5*(10*A + 11*B)*a^2*cosh(x) + 15*(4*A
+ 3*B)*a^2)*sinh(x)^4 - 21*(2*A + 5*B)*a^2*cosh(x) + 35*(15*B*a^2*cosh(x)^4
 + 12*(2*A + 5*B)*a^2*cosh(x)^3 + 10*(10*A + 11*B)*a^2*cosh(x)^2 + 60*(4*A
+ 3*B)*a^2*cosh(x) - 15*(4*A + 3*B)*a^2)*sinh(x)^3 - 15*B*a^2 + 35*(9*B*a^2
*cosh(x)^5 + 9*(2*A + 5*B)*a^2*cosh(x)^4 + 10*(10*A + 11*B)*a^2*cosh(x)^3 +
 90*(4*A + 3*B)*a^2*cosh(x)^2 - 45*(4*A + 3*B)*a^2*cosh(x) - (10*A + 11*B)*
a^2)*sinh(x)^2 + 7*(15*B*a^2*cosh(x)^6 + 18*(2*A + 5*B)*a^2*cosh(x)^5 + 25*
(10*A + 11*B)*a^2*cosh(x)^4 + 300*(4*A + 3*B)*a^2*cosh(x)^3 - 225*(4*A + 3*
B)*a^2*cosh(x)^2 - 10*(10*A + 11*B)*a^2*cosh(x) - 3*(2*A + 5*B)*a^2)*sinh(x
))*sqrt(a/(cosh(x) + sinh(x)))/(cosh(x)^3 + 3*cosh(x)^2*sinh(x) + 3*cosh(x)
*sinh(x)^2 + sinh(x)^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))**(5/2)*(A+B*cosh(x)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [A]
time = 0.42, size = 153, normalized size = 1.63

$$-\frac{1}{840}\sqrt{2}\left(\frac{\left(2100\,Aa^6e^{(3\,x)}+1575\,Ba^6e^{(3\,x)}+350\,Aa^6e^{(2\,x)}+385\,Ba^6e^{(2\,x)}+42\,Aa^6e^x+105\,Ba^6e^x+15\,Ba^6\right)e^{\left(-\frac{7}{2}\,x\right)}}{a^{\frac{7}{2}}}-\frac{15\,Ba^{\frac{19}{2}}e^{\left(\frac{7}{2}\,x\right)}+42\,Aa^{\frac{19}{2}}e^{\left(\frac{5}{2}\,x\right)}+105\,Ba^{\frac{19}{2}}e^{\left(\frac{5}{2}\,x\right)}+350\,Aa^{\frac{19}{2}}e^{\left(\frac{3}{2}\,x\right)}+385\,Ba^{\frac{19}{2}}e^{\left(\frac{3}{2}\,x\right)}+2100\,Aa^{\frac{19}{2}}e^{\left(\frac{1}{2}\,x\right)}+1575\,Ba^{\frac{19}{2}}e^{\left(\frac{1}{2}\,x\right)}}{a^7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(5/2)*(A+B*cosh(x)),x, algorithm="giac")
```

```
[Out] -1/840*sqrt(2)*((2100*A*a^6*e^(3*x) + 1575*B*a^6*e^(3*x) + 350*A*a^6*e^(2*x
) + 385*B*a^6*e^(2*x) + 42*A*a^6*e^x + 105*B*a^6*e^x + 15*B*a^6)*e^(-7/2*x)
/a^(7/2) - (15*B*a^(19/2)*e^(7/2*x) + 42*A*a^(19/2)*e^(5/2*x) + 105*B*a^(19
/2)*e^(5/2*x) + 350*A*a^(19/2)*e^(3/2*x) + 385*B*a^(19/2)*e^(3/2*x) + 2100*
A*a^(19/2)*e^(1/2*x) + 1575*B*a^(19/2)*e^(1/2*x))/a^7)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\cosh(x))\,\left(a+a\cosh(x)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))*(a + a*cosh(x))^(5/2),x)
```

```
[Out] int((A + B*cosh(x))*(a + a*cosh(x))^(5/2), x)
```

3.88 $\int (a+a\cosh(x))^{3/2}(A+B\cosh(x))\,dx$

Optimal. Leaf size=68

$$\frac{8a^2(5A+3B)\sinh(x)}{15\sqrt{a+a\cosh(x)}}+\frac{2}{15}a(5A+3B)\sqrt{a+a\cosh(x)}\,\sinh(x)+\frac{2}{5}B(a+a\cosh(x))^{3/2}\sinh(x)$$

```
[Out] 2/5*B*(a+a*cosh(x))^(3/2)*sinh(x)+8/15*a^2*(5*A+3*B)*sinh(x)/(a+a*cosh(x))^
(1/2)+2/15*a*(5*A+3*B)*sinh(x)*(a+a*cosh(x))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2830, 2726, 2725}

$$\frac{8a^2(5A+3B)\sinh(x)}{15\sqrt{a\cosh(x)+a}}+\frac{2}{15}a(5A+3B)\sinh(x)\sqrt{a\cosh(x)+a}+\frac{2}{5}B\sinh(x)(a\cosh(x)+a)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[x])^(3/2)*(A + B*Cosh[x]),x]
```

```
[Out] (8*a^2*(5*A + 3*B)*Sinh[x])/(15*Sqrt[a + a*Cosh[x]]) + (2*a*(5*A + 3*B)*Sqr
t[a + a*Cosh[x]]*Sinh[x])/15 + (2*B*(a + a*Cosh[x])^(3/2)*Sinh[x])/5
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int (a+a\cosh(x))^{3/2}(A+B\cosh(x))\,dx = \frac{2}{5}B(a+a\cosh(x))^{3/2}\sinh(x)+\frac{1}{5}(5A+3B)\int (a+a\cosh(x))^{3/2}$$
$$= \frac{2}{15}a(5A+3B)\sqrt{a+a\cosh(x)}\,\sinh(x)+\frac{2}{5}B(a+a\cosh(x))^{3/2}\,\text{si}$$
$$= \frac{8a^2(5A+3B)\sinh(x)}{15\sqrt{a+a\cosh(x)}}+\frac{2}{15}a(5A+3B)\sqrt{a+a\cosh(x)}\,\sinh(x)$$

Mathematica [A]
time = 0.07, size = 46, normalized size = 0.68

$$\frac{1}{15}a\sqrt{a(1+\cosh(x))}\,(50A+39B+2(5A+9B)\cosh(x)+3B\cosh(2x))\tanh\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[x])^(3/2)*(A + B*Cosh[x]),x]
```

```
[Out] (a*Sqrt[a*(1 + Cosh[x])]*(50*A + 39*B + 2*(5*A + 9*B)*Cosh[x] + 3*B*Cosh[2*
x])*Tanh[x/2])/15
```

Maple [A]
time = 1.07, size = 57, normalized size = 0.84

method	result	size
default	$\frac{4\cosh\left(\frac{x}{2}\right)a^2\sinh\left(\frac{x}{2}\right)\left(6B\left(\sinh^4\left(\frac{x}{2}\right)\right)+(5A+15B)\left(\sinh^2\left(\frac{x}{2}\right)\right)+15A+15B\right)\sqrt{2}}{15\sqrt{a\left(\cosh^2\left(\frac{x}{2}\right)\right)}}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(x))^(3/2)*(A+B*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 4/15*cosh(1/2*x)*a^2*sinh(1/2*x)*(6*B*sinh(1/2*x)^4+(5*A+15*B)*sinh(1/2*x)^
2+15*A+15*B)*2^(1/2)/(a*cosh(1/2*x)^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(56) = 112.
time = 0.50, size = 163, normalized size = 2.40

$$\frac{1}{6}\left(\sqrt{2}a^{\frac{3}{2}}e^{\left(\frac{3}{2}x\right)}+9\sqrt{2}a^{\frac{3}{2}}e^{\left(\frac{1}{2}x\right)}-9\sqrt{2}a^{\frac{3}{2}}e^{\left(-\frac{1}{2}x\right)}-\sqrt{2}a^{\frac{3}{2}}e^{\left(-\frac{3}{2}x\right)}\right)A+\frac{1}{20}\left(\left(\sqrt{2}a^{\frac{3}{2}}e^{(-x)}+5\sqrt{2}a^{\frac{3}{2}}e^{(-2x)}+15\sqrt{2}a^{\frac{3}{2}}e^{(-3x)}-5\sqrt{2}a^{\frac{3}{2}}e^{(-4x)}\right)e^{\left(\frac{7}{2}x\right)}+\left(5\sqrt{2}a^{\frac{3}{2}}e^{(-x)}-15\sqrt{2}a^{\frac{3}{2}}e^{(-2x)}-5\sqrt{2}a^{\frac{3}{2}}e^{(-3x)}-\sqrt{2}a^{\frac{3}{2}}e^{(-4x)}\right)e^{\left(\frac{3}{2}x\right)}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(3/2)*(A+B*cosh(x)),x, algorithm="maxima")
```

```
[Out] 1/6*(sqrt(2)*a^(3/2)*e^(3/2*x) + 9*sqrt(2)*a^(3/2)*e^(1/2*x) - 9*sqrt(2)*a^
(3/2)*e^(-1/2*x) - sqrt(2)*a^(3/2)*e^(-3/2*x))*A + 1/20*((sqrt(2)*a^(3/2)*e
^(-x) + 5*sqrt(2)*a^(3/2)*e^(-2*x) + 15*sqrt(2)*a^(3/2)*e^(-3*x) - 5*sqrt(2
)*a^(3/2)*e^(-4*x))*e^(7/2*x) + (5*sqrt(2)*a^(3/2)*e^(-x) - 15*sqrt(2)*a^(3
/2)*e^(-2*x) - 5*sqrt(2)*a^(3/2)*e^(-3*x) - sqrt(2)*a^(3/2)*e^(-4*x))*e^(3/
2*x))*B
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(56) = 112$.
time = 0.37, size = 279, normalized size = 4.10

$$\frac{\sqrt{\frac{1}{2}}\left(3\,Ba\cosh\left(x\right)^{5}+3\,Ba\sinh\left(x\right)^{5}+5\,(2\,A+3\,B)a\cosh\left(x\right)^{4}+30\,(3\,A+2\,B)a\cosh\left(x\right)^{3}+5\,\left(3\,Ba\cosh\left(x\right)+(2\,A+3\,B)a\right)\sinh\left(x\right)^{4}-30\,(3\,A+2\,B)a\cosh\left(x\right)^{2}+10\,\left(3\,Ba\cosh\left(x\right)^{2}+2\,(2\,A+3\,B)a\cosh\left(x\right)+3\,(3\,A+2\,B)a\right)\sinh\left(x\right)^{3}-5\,(2\,A+3\,B)a\cosh\left(x\right)+30\,\left(Ba\cosh\left(x\right)^{3}+(2\,A+3\,B)a\cosh\left(x\right)^{2}+3\,(3\,A+2\,B)a\cosh\left(x\right)-(3\,A+2\,B)a\right)\sinh\left(x\right)^{2}-3\,Ba+5\,\left(3\,Ba\cosh\left(x\right)^{4}+4\,(2\,A+3\,B)a\cosh\left(x\right)^{3}+18\,(3\,A+2\,B)a\cosh\left(x\right)^{2}-12\,(3\,A+2\,B)a\cosh\left(x\right)-(2\,A+3\,B)a\right)\sinh\left(x\right)\right)\sqrt{\frac{a}{\cosh\left(x\right)+\sinh\left(x\right)}}}{30\,\left(\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(3/2)*(A+B*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/30*sqrt(1/2)*(3*B*a*cosh(x)^5 + 3*B*a*sinh(x)^5 + 5*(2*A + 3*B)*a*cosh(x)
^4 + 30*(3*A + 2*B)*a*cosh(x)^3 + 5*(3*B*a*cosh(x) + (2*A + 3*B)*a)*sinh(x)
^4 - 30*(3*A + 2*B)*a*cosh(x)^2 + 10*(3*B*a*cosh(x)^2 + 2*(2*A + 3*B)*a*cos
h(x) + 3*(3*A + 2*B)*a)*sinh(x)^3 - 5*(2*A + 3*B)*a*cosh(x) + 30*(B*a*cosh(
x)^3 + (2*A + 3*B)*a*cosh(x)^2 + 3*(3*A + 2*B)*a*cosh(x) - (3*A + 2*B)*a)*s
inh(x)^2 - 3*B*a + 5*(3*B*a*cosh(x)^4 + 4*(2*A + 3*B)*a*cosh(x)^3 + 18*(3*A
 + 2*B)*a*cosh(x)^2 - 12*(3*A + 2*B)*a*cosh(x) - (2*A + 3*B)*a)*sinh(x))*sq
rt(a/(cosh(x) + sinh(x)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\left(\cosh\left(x\right)+1\right)\right)^{\frac{3}{2}}\left(A+B\cosh\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))**(3/2)*(A+B*cosh(x)),x)
```

```
[Out] Integral((a*(cosh(x) + 1))**(3/2)*(A + B*cosh(x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(56) = 112$.
time = 0.41, size = 113, normalized size = 1.66

$$-\frac{1}{60}\,\sqrt{2}\left(\frac{\left(90\,Aa^{4}e^{\left(2\,x\right)}+60\,Ba^{4}e^{\left(2\,x\right)}+10\,Aa^{4}e^{x}+15\,Ba^{4}e^{x}+3\,Ba^{4}\right)e^{\left(-\frac{5}{2}\,x\right)}}{a^{\frac{5}{2}}}-\frac{3\,Ba^{\frac{13}{2}}e^{\left(\frac{5}{2}\,x\right)}+10\,Aa^{\frac{13}{2}}e^{\left(\frac{3}{2}\,x\right)}+15\,Ba^{\frac{13}{2}}e^{\left(\frac{3}{2}\,x\right)}+90\,Aa^{\frac{13}{2}}e^{\left(\frac{1}{2}\,x\right)}+60\,Ba^{\frac{13}{2}}e^{\left(\frac{1}{2}\,x\right)}}{a^{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(3/2)*(A+B*cosh(x)),x, algorithm="giac")
```

```
[Out] -1/60*sqrt(2)*((90*A*a^4*e^(2*x) + 60*B*a^4*e^(2*x) + 10*A*a^4*e^x + 15*B*a
^4*e^x + 3*B*a^4)*e^(-5/2*x)/a^(5/2) - (3*B*a^(13/2)*e^(5/2*x) + 10*A*a^(13
```

```
/2)*e^(3/2*x) + 15*B*a^(13/2)*e^(3/2*x) + 90*A*a^(13/2)*e^(1/2*x) + 60*B*a^
(13/2)*e^(1/2*x))/a^5)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B\cosh(x))\,(a + a\cosh(x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))*(a + a*cosh(x))^(3/2),x)
[Out] int((A + B*cosh(x))*(a + a*cosh(x))^(3/2), x)
```

3.89 $\int \sqrt{a+a\cosh(x)}\,(A+B\cosh(x))\,dx$

Optimal. Leaf size=40

$$\frac{2a(3A+B)\sinh(x)}{3\sqrt{a+a\cosh(x)}}+\frac{2}{3}B\sqrt{a+a\cosh(x)}\,\sinh(x)$$

```
[Out] 2/3*a*(3*A+B)*sinh(x)/(a+a*cosh(x))^(1/2)+2/3*B*sinh(x)*(a+a*cosh(x))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2830, 2725}

$$\frac{2a(3A+B)\sinh(x)}{3\sqrt{a\cosh(x)+a}}+\frac{2}{3}B\sinh(x)\sqrt{a\cosh(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cosh[x]]*(A + B*Cosh[x]),x]
```

```
[Out] (2*a*(3*A + B)*Sinh[x])/(3*Sqrt[a + a*Cosh[x]]) + (2*B*Sqrt[a + a*Cosh[x]]*
Sinh[x])/3
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+a\cosh(x)}\,(A+B\cosh(x))\,dx &= \frac{2}{3}B\sqrt{a+a\cosh(x)}\,\sinh(x)+\frac{1}{3}(3A+B)\int \sqrt{a+a\cosh(x)}\,dx\\ &= \frac{2a(3A+B)\sinh(x)}{3\sqrt{a+a\cosh(x)}}+\frac{2}{3}B\sqrt{a+a\cosh(x)}\,\sinh(x)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 31, normalized size = 0.78

$$\frac{2}{3}\sqrt{a(1+\cosh(x))}\,(3A+2B+B\cosh(x))\tanh\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cosh[x]]*(A + B*Cosh[x]),x]
```

```
[Out] (2*Sqrt[a*(1 + Cosh[x])]*(3*A + 2*B + B*Cosh[x])*Tanh[x/2])/3
```

Maple [A]
time = 1.07, size = 39, normalized size = 0.98

method	result	size
default	$\frac{2\cosh\left(\frac{x}{2}\right)a\sinh\left(\frac{x}{2}\right)\left(2B\left(\cosh^{2}\left(\frac{x}{2}\right)\right)+3A+B\right)\sqrt{2}}{3\sqrt{a\left(\cosh^{2}\left(\frac{x}{2}\right)\right)}}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(x))^(1/2)*(A+B*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*cosh(1/2*x)*a*sinh(1/2*x)*(2*B*cosh(1/2*x)^2+3*A+B)*2^(1/2)/(a*cosh(1/2
*x)^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(32) = 64$.
time = 0.49, size = 90, normalized size = 2.25

$$\left(\sqrt{2}\sqrt{a}e^{\left(\frac{1}{2}x\right)}-\sqrt{2}\sqrt{a}e^{\left(-\frac{1}{2}x\right)}\right)A+\frac{1}{6}\left(\left(\sqrt{2}\sqrt{a}e^{(-x)}+3\sqrt{2}\sqrt{a}e^{(-2x)}\right)e^{\left(\frac{5}{2}x\right)}-\left(3\sqrt{2}\sqrt{a}e^{(-x)}+\sqrt{2}\sqrt{a}e^{(-2x)}\right)e^{\left(\frac{1}{2}x\right)}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(1/2)*(A+B*cosh(x)),x, algorithm="maxima")
```

```
[Out] (sqrt(2)*sqrt(a)*e^(1/2*x) - sqrt(2)*sqrt(a)*e^(-1/2*x))*A + 1/6*((sqrt(2)*
sqrt(a)*e^(-x) + 3*sqrt(2)*sqrt(a)*e^(-2*x))*e^(5/2*x) - (3*sqrt(2)*sqrt(a)
*e^(-x) + sqrt(2)*sqrt(a)*e^(-2*x))*e^(1/2*x))*B
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(32) = 64$.
time = 0.41, size = 100, normalized size = 2.50

$$\frac{\sqrt{\frac{1}{2}}\left(B\cosh(x)^{3}+B\sinh(x)^{3}+3(2A+B)\cosh(x)^{2}+3(B\cosh(x)+2A+B)\sinh(x)^{2}-3(2A+B)\cosh(x)+3\left(B\cosh(x)^{2}+2(2A+B)\cosh(x)-2A-B\right)\sinh(x)-B\right)\sqrt{\frac{a}{\cosh(x)+\sinh(x)}}}{3(\cosh(x)+\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(1/2)*(A+B*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(1/2)*(B*cosh(x)^3 + B*sinh(x)^3 + 3*(2*A + B)*cosh(x)^2 + 3*(B*cos
h(x) + 2*A + B)*sinh(x)^2 - 3*(2*A + B)*cosh(x) + 3*(B*cosh(x)^2 + 2*(2*A +
 B)*cosh(x) - 2*A - B)*sinh(x) - B)*sqrt(a/(cosh(x) + sinh(x)))/(cosh(x) +
sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\cosh\left(x\right)+1\right)}\left(A+B\cosh\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))**(1/2)*(A+B*cosh(x)),x)
```

```
[Out] Integral(sqrt(a*(cosh(x) + 1))*(A + B*cosh(x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(32) = 64.
time = 0.42, size = 71, normalized size = 1.78

$$-\frac{1}{6}\sqrt{2}\left(\frac{\left(6\,Aa^{2}e^{x}+3\,Ba^{2}e^{x}+Ba^{2}\right)e^{\left(-\frac{3}{2}\,x\right)}}{a^{\frac{3}{2}}}-\frac{Ba^{\frac{7}{2}}e^{\left(\frac{3}{2}\,x\right)}+6\,Aa^{\frac{7}{2}}e^{\left(\frac{1}{2}\,x\right)}+3\,Ba^{\frac{7}{2}}e^{\left(\frac{1}{2}\,x\right)}}{a^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(1/2)*(A+B*cosh(x)),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(2)*((6*A*a^2*e^x + 3*B*a^2*e^x + B*a^2)*e^(-3/2*x)/a^(3/2) - (B*a
^(7/2)*e^(3/2*x) + 6*A*a^(7/2)*e^(1/2*x) + 3*B*a^(7/2)*e^(1/2*x))/a^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \left(A+B\cosh(x)\right)\,\sqrt{a+a\cosh\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))*(a + a*cosh(x))^(1/2),x)
```

```
[Out] int((A + B*cosh(x))*(a + a*cosh(x))^(1/2), x)
```

3.90 $\int (a-a\cosh(x))^{5/2}(A+B\cosh(x))\,dx$

Optimal. Leaf size=98

$$-\frac{64a^3(7A-5B)\sinh(x)}{105\sqrt{a-a\cosh(x)}}-\frac{16}{105}a^2(7A-5B)\sqrt{a-a\cosh(x)}\,\sinh(x)-\frac{2}{35}a(7A-5B)(a-a\cosh(x))^{3/2}\sinh(x)$$

```
[Out] -2/35*a*(7*A-5*B)*(a-a*cosh(x))^(3/2)*sinh(x)+2/7*B*(a-a*cosh(x))^(5/2)*sin
h(x)-64/105*a^3*(7*A-5*B)*sinh(x)/(a-a*cosh(x))^(1/2)-16/105*a^2*(7*A-5*B)*
sinh(x)*(a-a*cosh(x))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2830, 2726, 2725}

$$-\frac{64a^3(7A-5B)\sinh(x)}{105\sqrt{a-a\cosh(x)}}-\frac{16}{105}a^2(7A-5B)\sinh(x)\sqrt{a-a\cosh(x)}-\frac{2}{35}a(7A-5B)\sinh(x)(a-a\cosh(x))^{3/2}+\frac{2}{7}B\sinh(x)(a-a\cosh(x))^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Cosh[x])^(5/2)*(A + B*Cosh[x]),x]
```

```
[Out] (-64*a^3*(7*A - 5*B)*Sinh[x])/(105*Sqrt[a - a*Cosh[x]]) - (16*a^2*(7*A - 5*
B)*Sqrt[a - a*Cosh[x]]*Sinh[x])/105 - (2*a*(7*A - 5*B)*(a - a*Cosh[x])^(3/2
)*Sinh[x])/35 + (2*B*(a - a*Cosh[x])^(5/2)*Sinh[x])/7
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int (a - a\cosh(x))^{5/2}(A + B\cosh(x))\,dx = \frac{2}{7}B(a - a\cosh(x))^{5/2}\sinh(x) - \frac{1}{7}(-7A + 5B)\int (a - a\cosh(x))$$
$$= -\frac{2}{35}a(7A - 5B)(a - a\cosh(x))^{3/2}\sinh(x) + \frac{2}{7}B(a - a\cosh(x))$$
$$= -\frac{16}{105}a^2(7A - 5B)\sqrt{a - a\cosh(x)}\,\sinh(x) - \frac{2}{35}a(7A - 5B)(a$$
$$= -\frac{64a^3(7A - 5B)\sinh(x)}{105\sqrt{a - a\cosh(x)}} - \frac{16}{105}a^2(7A - 5B)\sqrt{a - a\cosh(x)}\,\text{si}$$

Mathematica [A]
time = 0.10, size = 61, normalized size = 0.62

$$\frac{1}{210}a^2\sqrt{a - a\cosh(x)}\,(1246A - 1040B + (-392A + 505B)\cosh(x) + 6(7A - 20B)\cosh(2x) + 15B\cosh(3x))\coth\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Cosh[x])^(5/2)*(A + B*Cosh[x]),x]
```

```
[Out] (a^2*Sqrt[a - a*Cosh[x]]*(1246*A - 1040*B + (-392*A + 505*B)*Cosh[x] + 6*(7
*A - 20*B)*Cosh[2*x] + 15*B*Cosh[3*x])*Coth[x/2])/210
```

Maple [A]
time = 1.22, size = 69, normalized size = 0.70

method	result	size
default	$-\frac{16\sinh\left(\frac{x}{2}\right)a^3\cosh\left(\frac{x}{2}\right)\left(30B\left(\sinh^6\left(\frac{x}{2}\right)\right)+(21A-15B)\left(\sinh^4\left(\frac{x}{2}\right)\right)+(-28A+20B)\left(\sinh^2\left(\frac{x}{2}\right)\right)+56A-40B\right)}{105\sqrt{-2\left(\sinh^2\left(\frac{x}{2}\right)\right)a}}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*cosh(x))^(5/2)*(A+B*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -16/105*sinh(1/2*x)*a^3*cosh(1/2*x)*(30*B*sinh(1/2*x)^6+(21*A-15*B)*sinh(1/
2*x)^4+(-28*A+20*B)*sinh(1/2*x)^2+56*A-40*B)/(-2*sinh(1/2*x)^2*a)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 288 vs. 2(82) = 164.
time = 0.52, size = 288, normalized size = 2.94

$$\frac{1}{60}\left(\frac{25\sqrt{2}a^{\frac{5}{2}}e^{(-x)}}{(-e^{(-x)})^{\frac{5}{2}}} - \frac{150\sqrt{2}a^{\frac{5}{2}}e^{(-2x)}}{(-e^{(-x)})^{\frac{5}{2}}} - \frac{150\sqrt{2}a^{\frac{5}{2}}e^{(-3x)}}{(-e^{(-x)})^{\frac{5}{2}}} + \frac{25\sqrt{2}a^{\frac{5}{2}}e^{(-4x)}}{(-e^{(-x)})^{\frac{5}{2}}} - \frac{3\sqrt{2}a^{\frac{5}{2}}e^{(-5x)}}{(-e^{(-x)})^{\frac{5}{2}}} - \frac{3\sqrt{2}a^{\frac{5}{2}}}{(-e^{(-x)})^{\frac{5}{2}}}\right)A + \frac{1}{168}B\left(\frac{\left(21\sqrt{2}a^{\frac{5}{2}}e^{(-x)} - 70\sqrt{2}a^{\frac{5}{2}}e^{(-2x)} + 210\sqrt{2}a^{\frac{5}{2}}e^{(-3x)} + 105\sqrt{2}a^{\frac{5}{2}}e^{(-4x)} - 7\sqrt{2}a^{\frac{5}{2}}e^{(-5x)} - 3\sqrt{2}a^{\frac{5}{2}}\right)e^x}{(-e^{(-x)})^{\frac{5}{2}}} - \frac{7\sqrt{2}a^{\frac{5}{2}}e^{(-x)} - 105\sqrt{2}a^{\frac{5}{2}}e^{(-2x)} - 210\sqrt{2}a^{\frac{5}{2}}e^{(-3x)} + 70\sqrt{2}a^{\frac{5}{2}}e^{(-4x)} - 21\sqrt{2}a^{\frac{5}{2}}e^{(-5x)} + 3\sqrt{2}a^{\frac{5}{2}}e^{(-6x)}}{(-e^{(-x)})^{\frac{5}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(x))^(5/2)*(A+B*cosh(x)),x, algorithm="maxima")
```

```
[Out] 1/60*(25*sqrt(2)*a^(5/2)*e^(-x)/(-e^(-x))^(5/2) - 150*sqrt(2)*a^(5/2)*e^(-2
*x)/(-e^(-x))^(5/2) - 150*sqrt(2)*a^(5/2)*e^(-3*x)/(-e^(-x))^(5/2) + 25*sqr
t(2)*a^(5/2)*e^(-4*x)/(-e^(-x))^(5/2) - 3*sqrt(2)*a^(5/2)*e^(-5*x)/(-e^(-x)
)^(5/2) - 3*sqrt(2)*a^(5/2)/(-e^(-x))^(5/2))*A + 1/168*B*((21*sqrt(2)*a^(5/
2)*e^(-x) - 70*sqrt(2)*a^(5/2)*e^(-2*x) + 210*sqrt(2)*a^(5/2)*e^(-3*x) + 10
5*sqrt(2)*a^(5/2)*e^(-4*x) - 7*sqrt(2)*a^(5/2)*e^(-5*x) - 3*sqrt(2)*a^(5/2)
)*e^x/(-e^(-x))^(5/2) - (7*sqrt(2)*a^(5/2)*e^(-x) - 105*sqrt(2)*a^(5/2)*e^(
-2*x) - 210*sqrt(2)*a^(5/2)*e^(-3*x) + 70*sqrt(2)*a^(5/2)*e^(-4*x) - 21*sqr
t(2)*a^(5/2)*e^(-5*x) + 3*sqrt(2)*a^(5/2)*e^(-6*x))/(-e^(-x))^(5/2))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 564 vs. $2(82) = 164$.
time = 0.44, size = 564, normalized size = 5.76

$$\frac{\sqrt{\frac{1}{2}}\left(15\,Ba^{2}\cosh\left(x\right)^{7}+15\,Ba^{2}\sinh\left(x\right)^{7}+21\,(2\,A-5\,B)a^{2}\cosh\left(x\right)^{6}-35\,(10\,A-11\,B)a^{2}\cosh\left(x\right)^{5}+525\,(4\,A-3\,B)a^{2}\cosh\left(x\right)^{4}+21\left(5\,Ba^{2}\cosh\left(x\right)+(2\,A-5\,B)a^{2}\right)\sinh\left(x\right)^{6}+525\,(4\,A-3\,B)a^{2}\cosh\left(x\right)^{3}+7\left(45\,Ba^{2}\cosh\left(x\right)^{2}+18\,(2\,A-5\,B)a^{2}\cosh\left(x\right)-5\,(10\,A-11\,B)a^{2}\right)\sinh\left(x\right)^{5}-35\,(10\,A-11\,B)a^{2}\cosh\left(x\right)^{2}+35\left(15\,Ba^{2}\cosh\left(x\right)^{3}+9\,(2\,A-5\,B)a^{2}\cosh\left(x\right)^{2}-5\,(10\,A-11\,B)a^{2}\cosh\left(x\right)+15\,(4\,A-3\,B)a^{2}\right)\sinh\left(x\right)^{4}+21\,(2\,A-5\,B)a^{2}\cosh\left(x\right)+35\left(15\,Ba^{2}\cosh\left(x\right)^{4}+12\,(2\,A-5\,B)a^{2}\cosh\left(x\right)^{3}-10\,(10\,A-11\,B)a^{2}\cosh\left(x\right)^{2}+60\,(4\,A-3\,B)a^{2}\cosh\left(x\right)+15\,(4\,A-3\,B)a^{2}\right)\sinh\left(x\right)^{3}+15\,Ba^{2}+35\left(9\,Ba^{2}\cosh\left(x\right)^{5}+9\,(2\,A-5\,B)a^{2}\cosh\left(x\right)^{4}-10\,(10\,A-11\,B)a^{2}\cosh\left(x\right)^{3}+90\,(4\,A-3\,B)a^{2}\cosh\left(x\right)^{2}+45\,(4\,A-3\,B)a^{2}\cosh\left(x\right)-(10\,A-11\,B)a^{2}\right)\sinh\left(x\right)^{2}+7\left(15\,Ba^{2}\cosh\left(x\right)^{6}+18\,(2\,A-5\,B)a^{2}\cosh\left(x\right)^{5}-25\,(10\,A-11\,B)a^{2}\cosh\left(x\right)^{4}+300\,(4\,A-3\,B)a^{2}\cosh\left(x\right)^{3}+225\,(4\,A-3\,B)a^{2}\cosh\left(x\right)^{2}-10\,(10\,A-11\,B)a^{2}\cosh\left(x\right)+3\,(2\,A-5\,B)a^{2}\right)\sinh\left(x\right)\right)\sqrt{-\frac{a}{\cosh\left(x\right)+\sinh\left(x\right)}}}{420\left(\cosh\left(x\right)^{3}+3\cosh\left(x\right)^{2}\sinh\left(x\right)+3\cosh\left(x\right)\sinh\left(x\right)^{2}+\sinh\left(x\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(x))^(5/2)*(A+B*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/420*sqrt(1/2)*(15*B*a^2*cosh(x)^7 + 15*B*a^2*sinh(x)^7 + 21*(2*A - 5*B)*a
^2*cosh(x)^6 - 35*(10*A - 11*B)*a^2*cosh(x)^5 + 525*(4*A - 3*B)*a^2*cosh(x)
^4 + 21*(5*B*a^2*cosh(x) + (2*A - 5*B)*a^2)*sinh(x)^6 + 525*(4*A - 3*B)*a^2
*cosh(x)^3 + 7*(45*B*a^2*cosh(x)^2 + 18*(2*A - 5*B)*a^2*cosh(x) - 5*(10*A -
 11*B)*a^2)*sinh(x)^5 - 35*(10*A - 11*B)*a^2*cosh(x)^2 + 35*(15*B*a^2*cosh(
x)^3 + 9*(2*A - 5*B)*a^2*cosh(x)^2 - 5*(10*A - 11*B)*a^2*cosh(x) + 15*(4*A
- 3*B)*a^2)*sinh(x)^4 + 21*(2*A - 5*B)*a^2*cosh(x) + 35*(15*B*a^2*cosh(x)^4
 + 12*(2*A - 5*B)*a^2*cosh(x)^3 - 10*(10*A - 11*B)*a^2*cosh(x)^2 + 60*(4*A
- 3*B)*a^2*cosh(x) + 15*(4*A - 3*B)*a^2)*sinh(x)^3 + 15*B*a^2 + 35*(9*B*a^2
*cosh(x)^5 + 9*(2*A - 5*B)*a^2*cosh(x)^4 - 10*(10*A - 11*B)*a^2*cosh(x)^3 +
 90*(4*A - 3*B)*a^2*cosh(x)^2 + 45*(4*A - 3*B)*a^2*cosh(x) - (10*A - 11*B)*
a^2)*sinh(x)^2 + 7*(15*B*a^2*cosh(x)^6 + 18*(2*A - 5*B)*a^2*cosh(x)^5 - 25*
(10*A - 11*B)*a^2*cosh(x)^4 + 300*(4*A - 3*B)*a^2*cosh(x)^3 + 225*(4*A - 3*
B)*a^2*cosh(x)^2 - 10*(10*A - 11*B)*a^2*cosh(x) + 3*(2*A - 5*B)*a^2)*sinh(x
))*sqrt(-a/(cosh(x) + sinh(x)))/(cosh(x)^3 + 3*cosh(x)^2*sinh(x) + 3*cosh(x
)*sinh(x)^2 + sinh(x)^3)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(x))**(5/2)*(A+B*cosh(x)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 295 vs. 2(82) = 164.
time = 0.42, size = 295, normalized size = 3.01

$$\frac{1}{840}\sqrt{2}\left(\frac{\left(2100\,Aa^6e^{(3\,x)}\mathrm{sgn}\left(-e^x+1\right)-1575\,Ba^6e^{(3\,x)}\mathrm{sgn}\left(-e^x+1\right)-350\,Aa^6e^{(2\,x)}\mathrm{sgn}\left(-e^x+1\right)+385\,Ba^6e^{(2\,x)}\mathrm{sgn}\left(-e^x+1\right)+42\,Aa^6e^x\mathrm{sgn}\left(-e^x+1\right)-105\,Ba^6e^x\mathrm{sgn}\left(-e^x+1\right)+15\,Ba^6\mathrm{sgn}\left(-e^x+1\right)\right)e^{(-3\,x)}}{\sqrt{-ae^x}a^3}-\frac{15\,\sqrt{-ae^x}Ba^9e^{(3\,x)}\mathrm{sgn}\left(-e^x+1\right)+42\,\sqrt{-ae^x}Aa^9e^{(2\,x)}\mathrm{sgn}\left(-e^x+1\right)-105\,\sqrt{-ae^x}Ba^9e^{(2\,x)}\mathrm{sgn}\left(-e^x+1\right)-350\,\sqrt{-ae^x}Aa^9e^x\mathrm{sgn}\left(-e^x+1\right)+385\,\sqrt{-ae^x}Ba^9e^x\mathrm{sgn}\left(-e^x+1\right)+2100\,\sqrt{-ae^x}Aa^9\mathrm{sgn}\left(-e^x+1\right)-1575\,\sqrt{-ae^x}Ba^9\mathrm{sgn}\left(-e^x+1\right)}{a^7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(x))^(5/2)*(A+B*cosh(x)),x, algorithm="giac")
```

```
[Out] 1/840*sqrt(2)*((2100*A*a^6*e^(3*x)*sgn(-e^x + 1) - 1575*B*a^6*e^(3*x)*sgn(-
e^x + 1) - 350*A*a^6*e^(2*x)*sgn(-e^x + 1) + 385*B*a^6*e^(2*x)*sgn(-e^x + 1
) + 42*A*a^6*e^x*sgn(-e^x + 1) - 105*B*a^6*e^x*sgn(-e^x + 1) + 15*B*a^6*sgn
(-e^x + 1))*e^(-3*x)/(sqrt(-a*e^x)*a^3) - (15*sqrt(-a*e^x)*B*a^9*e^(3*x)*sg
n(-e^x + 1) + 42*sqrt(-a*e^x)*A*a^9*e^(2*x)*sgn(-e^x + 1) - 105*sqrt(-a*e^x
)*B*a^9*e^(2*x)*sgn(-e^x + 1) - 350*sqrt(-a*e^x)*A*a^9*e^x*sgn(-e^x + 1) +
385*sqrt(-a*e^x)*B*a^9*e^x*sgn(-e^x + 1) + 2100*sqrt(-a*e^x)*A*a^9*sgn(-e^x
 + 1) - 1575*sqrt(-a*e^x)*B*a^9*sgn(-e^x + 1))/a^7)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\cosh(x))\,\left(a-a\cosh(x)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))*(a - a*cosh(x))^(5/2),x)
```

```
[Out] int((A + B*cosh(x))*(a - a*cosh(x))^(5/2), x)
```

3.91 $\int (a-a\cosh(x))^{3/2}(A+B\cosh(x))\,dx$

Optimal. Leaf size=71

$$-\frac{8a^2(5A-3B)\sinh(x)}{15\sqrt{a-a\cosh(x)}}-\frac{2}{15}a(5A-3B)\sqrt{a-a\cosh(x)}\,\sinh(x)+\frac{2}{5}B(a-a\cosh(x))^{3/2}\sinh(x)$$

```
[Out] 2/5*B*(a-a*cosh(x))^(3/2)*sinh(x)-8/15*a^2*(5*A-3*B)*sinh(x)/(a-a*cosh(x))^
(1/2)-2/15*a*(5*A-3*B)*sinh(x)*(a-a*cosh(x))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2830, 2726, 2725}

$$-\frac{8a^2(5A-3B)\sinh(x)}{15\sqrt{a-a\cosh(x)}}-\frac{2}{15}a(5A-3B)\sinh(x)\sqrt{a-a\cosh(x)}+\frac{2}{5}B\sinh(x)(a-a\cosh(x))^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Cosh[x])^(3/2)*(A + B*Cosh[x]),x]
```

```
[Out] (-8*a^2*(5*A - 3*B)*Sinh[x])/(15*Sqrt[a - a*Cosh[x]]) - (2*a*(5*A - 3*B)*Sq
rt[a - a*Cosh[x]]*Sinh[x])/15 + (2*B*(a - a*Cosh[x])^(3/2)*Sinh[x])/5
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int (a-a\cosh(x))^{3/2}(A+B\cosh(x))\,dx = \frac{2}{5}B(a-a\cosh(x))^{3/2}\sinh(x) - \frac{1}{5}(-5A+3B)\int (a-a\cosh(x))$$

$$= -\frac{2}{15}a(5A-3B)\sqrt{a-a\cosh(x)}\,\sinh(x) + \frac{2}{5}B(a-a\cosh(x))^{3/}$$

$$= -\frac{8a^2(5A-3B)\sinh(x)}{15\sqrt{a-a\cosh(x)}} - \frac{2}{15}a(5A-3B)\sqrt{a-a\cosh(x)}\,\sinh($$

Mathematica [A]
time = 0.10, size = 47, normalized size = 0.66

$$-\frac{1}{15}a\sqrt{a-a\cosh(x)}\,(-50A+39B+2(5A-9B)\cosh(x)+3B\cosh(2x))\coth\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Cosh[x])^(3/2)*(A + B*Cosh[x]),x]
```

```
[Out] -1/15*(a*Sqrt[a - a*Cosh[x]]*(-50*A + 39*B + 2*(5*A - 9*B)*Cosh[x] + 3*B*Co
sh[2*x])*Coth[x/2])
```

Maple [A]
time = 1.17, size = 55, normalized size = 0.77

method	result	size
default	$\frac{8\sinh\left(\frac{x}{2}\right)a^2\cosh\left(\frac{x}{2}\right)\left(6B\left(\sinh^4\left(\frac{x}{2}\right)\right)+(5A-3B)\left(\sinh^2\left(\frac{x}{2}\right)\right)-10A+6B\right)}{15\sqrt{-2\left(\sinh^2\left(\frac{x}{2}\right)\right)a}}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*cosh(x))^(3/2)*(A+B*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 8/15*sinh(1/2*x)*a^2*cosh(1/2*x)*(6*B*sinh(1/2*x)^4+(5*A-3*B)*sinh(1/2*x)^2
-10*A+6*B)/(-2*sinh(1/2*x)^2*a)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(59) = 118.
time = 0.51, size = 199, normalized size = 2.80

$$\frac{1}{6}\left(\frac{9\sqrt{2}a^{\frac{3}{2}}e^{(-x)}}{(-e^{(-x)})^{\frac{3}{2}}} + \frac{9\sqrt{2}a^{\frac{3}{2}}e^{(-2x)}}{(-e^{(-x)})^{\frac{3}{2}}} - \frac{\sqrt{2}a^{\frac{3}{2}}e^{(-3x)}}{(-e^{(-x)})^{\frac{3}{2}}} - \frac{\sqrt{2}a^{\frac{3}{2}}}{(-e^{(-x)})^{\frac{3}{2}}}\right)A + \frac{1}{20}B\left(\frac{\left(5\sqrt{2}a^{\frac{3}{2}}e^{(-x)} - 15\sqrt{2}a^{\frac{3}{2}}e^{(-2x)} - 5\sqrt{2}a^{\frac{3}{2}}e^{(-3x)} - \sqrt{2}a^{\frac{3}{2}}\right)e^{x}}{(-e^{(-x)})^{\frac{3}{2}}} - \frac{5\sqrt{2}a^{\frac{3}{2}}e^{(-x)} + 15\sqrt{2}a^{\frac{3}{2}}e^{(-2x)} - 5\sqrt{2}a^{\frac{3}{2}}e^{(-3x)} + \sqrt{2}a^{\frac{3}{2}}e^{(-4x)}}{(-e^{(-x)})^{\frac{3}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(x))^(3/2)*(A+B*cosh(x)),x, algorithm="maxima")
```

```
[Out] 1/6*(9*sqrt(2)*a^(3/2)*e^(-x)/(-e^(-x))^(3/2) + 9*sqrt(2)*a^(3/2)*e^(-2*x)/
(-e^(-x))^(3/2) - sqrt(2)*a^(3/2)*e^(-3*x)/(-e^(-x))^(3/2) - sqrt(2)*a^(3/2
)/(-e^(-x))^(3/2))*A + 1/20*B*((5*sqrt(2)*a^(3/2)*e^(-x) - 15*sqrt(2)*a^(3/
2)*e^(-2*x) - 5*sqrt(2)*a^(3/2)*e^(-3*x) - sqrt(2)*a^(3/2))*e^x/(-e^(-x))^(
3/2) - (5*sqrt(2)*a^(3/2)*e^(-x) + 15*sqrt(2)*a^(3/2)*e^(-2*x) - 5*sqrt(2)*
a^(3/2)*e^(-3*x) + sqrt(2)*a^(3/2)*e^(-4*x))/(-e^(-x))^(3/2))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(59) = 118$.
time = 0.36, size = 279, normalized size = 3.93

$$-\frac{\sqrt{\frac{1}{2}}\left(3\,Ba\cosh\left(x\right)^{5}+3\,Ba\sinh\left(x\right)^{5}+5\,(2\,A-3\,B)a\cosh\left(x\right)^{4}-30\,(3\,A-2\,B)a\cosh\left(x\right)^{3}+5\,\left(3\,Ba\cosh\left(x\right)+(2\,A-3\,B)a\right)\sinh\left(x\right)^{4}-30\,(3\,A-2\,B)a\cosh\left(x\right)^{2}+10\,\left(3\,Ba\cosh\left(x\right)^{2}+2\,(2\,A-3\,B)a\cosh\left(x\right)-3\,(3\,A-2\,B)a\right)\sinh\left(x\right)^{3}+5\,(2\,A-3\,B)a\cosh\left(x\right)+30\,\left(Ba\cosh\left(x\right)^{3}+(2\,A-3\,B)a\cosh\left(x\right)^{2}-3\,(3\,A-2\,B)a\cosh\left(x\right)-(3\,A-2\,B)a\right)\sinh\left(x\right)^{2}+3\,Ba+5\,\left(3\,Ba\cosh\left(x\right)^{4}+4\,(2\,A-3\,B)a\cosh\left(x\right)^{3}-18\,(3\,A-2\,B)a\cosh\left(x\right)^{2}-12\,(3\,A-2\,B)a\cosh\left(x\right)+(2\,A-3\,B)a\right)\sinh\left(x\right)\right)\sqrt{-\frac{a}{\cosh\left(x\right)+\sinh\left(x\right)}}}{30\,\left(\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(x))^(3/2)*(A+B*cosh(x)),x, algorithm="fricas")
```

```
[Out] -1/30*sqrt(1/2)*(3*B*a*cosh(x)^5 + 3*B*a*sinh(x)^5 + 5*(2*A - 3*B)*a*cosh(x
)^4 - 30*(3*A - 2*B)*a*cosh(x)^3 + 5*(3*B*a*cosh(x) + (2*A - 3*B)*a)*sinh(x
)^4 - 30*(3*A - 2*B)*a*cosh(x)^2 + 10*(3*B*a*cosh(x)^2 + 2*(2*A - 3*B)*a*co
sh(x) - 3*(3*A - 2*B)*a)*sinh(x)^3 + 5*(2*A - 3*B)*a*cosh(x) + 30*(B*a*cosh
(x)^3 + (2*A - 3*B)*a*cosh(x)^2 - 3*(3*A - 2*B)*a*cosh(x) - (3*A - 2*B)*a)*
sinh(x)^2 + 3*B*a + 5*(3*B*a*cosh(x)^4 + 4*(2*A - 3*B)*a*cosh(x)^3 - 18*(3*
A - 2*B)*a*cosh(x)^2 - 12*(3*A - 2*B)*a*cosh(x) + (2*A - 3*B)*a)*sinh(x))*s
qrt(-a/(cosh(x) + sinh(x)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-a\left(\cosh\left(x\right)-1\right)\right)^{\frac{3}{2}}\left(A+B\cosh\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(x))**(3/2)*(A+B*cosh(x)),x)
```

```
[Out] Integral((-a*(cosh(x) - 1))**(3/2)*(A + B*cosh(x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(59) = 118$.
time = 0.43, size = 212, normalized size = 2.99

$$\frac{1}{60}\,\sqrt{2}\left(\frac{\left(90\,Aa^{4}e^{\left(2\,x\right)}\mathrm{sgn}\left(-e^{x}+1\right)-60\,Ba^{4}e^{\left(2\,x\right)}\mathrm{sgn}\left(-e^{x}+1\right)-10\,Aa^{4}e^{x}\mathrm{sgn}\left(-e^{x}+1\right)+15\,Ba^{4}e^{x}\mathrm{sgn}\left(-e^{x}+1\right)-3\,Ba^{4}\mathrm{sgn}\left(-e^{x}+1\right)\right)e^{\left(-2\,x\right)}}{\sqrt{-ae^{x}}\,a^{2}}+\frac{3\,\sqrt{-ae^{x}}\,Ba^{6}e^{\left(2\,x\right)}\mathrm{sgn}\left(-e^{x}+1\right)+10\,\sqrt{-ae^{x}}\,Aa^{6}e^{x}\mathrm{sgn}\left(-e^{x}+1\right)-15\,\sqrt{-ae^{x}}\,Ba^{6}e^{x}\mathrm{sgn}\left(-e^{x}+1\right)-90\,\sqrt{-ae^{x}}\,Aa^{6}\mathrm{sgn}\left(-e^{x}+1\right)+60\,\sqrt{-ae^{x}}\,Ba^{6}\mathrm{sgn}\left(-e^{x}+1\right)}{a^{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(x))^(3/2)*(A+B*cosh(x)),x, algorithm="giac")
```

```
[Out] 1/60*sqrt(2)*((90*A*a^4*e^(2*x)*sgn(-e^x + 1) - 60*B*a^4*e^(2*x)*sgn(-e^x +
 1) - 10*A*a^4*e^x*sgn(-e^x + 1) + 15*B*a^4*e^x*sgn(-e^x + 1) - 3*B*a^4*sgn
```

```
(-e^x + 1))*e^(-2*x)/(sqrt(-a*e^x)*a^2) + (3*sqrt(-a*e^x)*B*a^6*e^(2*x)*sgn
(-e^x + 1) + 10*sqrt(-a*e^x)*A*a^6*e^x*sgn(-e^x + 1) - 15*sqrt(-a*e^x)*B*a^
6*e^x*sgn(-e^x + 1) - 90*sqrt(-a*e^x)*A*a^6*sgn(-e^x + 1) + 60*sqrt(-a*e^x)
*B*a^6*sgn(-e^x + 1))/a^5)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\cosh(x))\ (a-a\cosh(x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))*(a - a*cosh(x))^(3/2),x)
```

```
[Out] int((A + B*cosh(x))*(a - a*cosh(x))^(3/2), x)
```

3.92 $\int \sqrt{a-a\cosh(x)}\,(A+B\cosh(x))\,dx$

Optimal. Leaf size=44

$$-\frac{2a(3A-B)\sinh(x)}{3\sqrt{a-a\cosh(x)}}+\frac{2}{3}B\sqrt{a-a\cosh(x)}\,\sinh(x)$$

```
[Out] -2/3*a*(3*A-B)*sinh(x)/(a-a*cosh(x))^(1/2)+2/3*B*sinh(x)*(a-a*cosh(x))^(1/2
)
```

Rubi [A]
time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2830, 2725}

$$\frac{2}{3}B\sinh(x)\sqrt{a-a\cosh(x)}-\frac{2a(3A-B)\sinh(x)}{3\sqrt{a-a\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Cosh[x]]*(A + B*Cosh[x]),x]
```

```
[Out] (-2*a*(3*A - B)*Sinh[x])/(3*Sqrt[a - a*Cosh[x]]) + (2*B*Sqrt[a - a*Cosh[x]]
*Sinh[x])/3
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a-a\cosh(x)}\,(A+B\cosh(x))\,dx &= \frac{2}{3}B\sqrt{a-a\cosh(x)}\,\sinh(x)-\frac{1}{3}(-3A+B)\int \sqrt{a-a\cosh(x)}\,dx\\ &= -\frac{2a(3A-B)\sinh(x)}{3\sqrt{a-a\cosh(x)}}+\frac{2}{3}B\sqrt{a-a\cosh(x)}\,\sinh(x)\end{aligned}$$

Mathematica [A]
time = 0.04, size = 32, normalized size = 0.73

$$\frac{2}{3}\sqrt{a-a\cosh(x)}\,(3A-2B+B\cosh(x))\coth\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Cosh[x]]*(A + B*Cosh[x]),x]
[Out] (2*Sqrt[a - a*Cosh[x]]*(3*A - 2*B + B*Cosh[x])*Coth[x/2])/3
```

Maple [A]
time = 1.02, size = 39, normalized size = 0.89

method	result	size
default	$-\frac{4\sinh\left(\frac{x}{2}\right)a\cosh\left(\frac{x}{2}\right)\left(2B\left(\cosh^{2}\left(\frac{x}{2}\right)\right)+3A-3B\right)}{3\sqrt{-2\left(\sinh^{2}\left(\frac{x}{2}\right)\right)a}}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*cosh(x))^(1/2)*(A+B*cosh(x)),x,method=_RETURNVERBOSE)
[Out] -4/3*sinh(1/2*x)*a*cosh(1/2*x)*(2*B*cosh(1/2*x)^2+3*A-3*B)/(-2*sinh(1/2*x)^
2*a)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(36) = 72.
time = 0.50, size = 109, normalized size = 2.48

$$-\left(\frac{\sqrt{2}\,\sqrt{a}\,e^{(-x)}}{\sqrt{-e^{(-x)}}}+\frac{\sqrt{2}\,\sqrt{a}}{\sqrt{-e^{(-x)}}}\right)A+\frac{1}{6}\left(\frac{\left(3\sqrt{2}\,\sqrt{a}\,e^{(-x)}-\sqrt{2}\,\sqrt{a}\right)e^{x}}{\sqrt{-e^{(-x)}}}+\frac{3\sqrt{2}\,\sqrt{a}\,e^{(-x)}-\sqrt{2}\,\sqrt{a}\,e^{(-2x)}}{\sqrt{-e^{(-x)}}}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(x))^(1/2)*(A+B*cosh(x)),x, algorithm="maxima")
[Out] -(sqrt(2)*sqrt(a)*e^(-x)/sqrt(-e^(-x)) + sqrt(2)*sqrt(a)/sqrt(-e^(-x)))*A +
 1/6*((3*sqrt(2)*sqrt(a)*e^(-x) - sqrt(2)*sqrt(a))*e^x/sqrt(-e^(-x)) + (3*s
qrt(2)*sqrt(a)*e^(-x) - sqrt(2)*sqrt(a)*e^(-2*x))/sqrt(-e^(-x)))*B
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(36) = 72.
time = 0.40, size = 107, normalized size = 2.43

$$\frac{\sqrt{\frac{1}{2}}\left(B\cosh\left(x\right)^{3}+B\sinh\left(x\right)^{3}+3\left(2A-B\right)\cosh\left(x\right)^{2}+3\left(B\cosh\left(x\right)+2A-B\right)\sinh\left(x\right)^{2}+3\left(2A-B\right)\cosh\left(x\right)+3\left(B\cosh\left(x\right)^{2}+2\left(2A-B\right)\cosh\left(x\right)+2A-B\right)\sinh\left(x\right)+B\right)\sqrt{-\frac{a}{\cosh\left(x\right)+\sinh\left(x\right)}}}{3\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(x))^(1/2)*(A+B*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(1/2)*(B*cosh(x)^3 + B*sinh(x)^3 + 3*(2*A - B)*cosh(x)^2 + 3*(B*cos
h(x) + 2*A - B)*sinh(x)^2 + 3*(2*A - B)*cosh(x) + 3*(B*cosh(x)^2 + 2*(2*A -
 B)*cosh(x) + 2*A - B)*sinh(x) + B)*sqrt(-a/(cosh(x) + sinh(x)))/(cosh(x) +
 sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a\left(\cosh\left(x\right)-1\right)}\left(A+B\cosh\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(x))**(1/2)*(A+B*cosh(x)),x)
```

```
[Out] Integral(sqrt(-a*(cosh(x) - 1))*(A + B*cosh(x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(36) = 72.
time = 0.41, size = 131, normalized size = 2.98

$$\frac{1}{6}\sqrt{2}\left(\frac{\left(6\,Aa^{2}e^{x}\mathrm{sgn}\left(-e^{x}+1\right)-3\,Ba^{2}e^{x}\mathrm{sgn}\left(-e^{x}+1\right)+Ba^{2}\mathrm{sgn}\left(-e^{x}+1\right)\right)e^{\left(-x\right)}}{\sqrt{-ae^{x}}\,a}-\frac{\sqrt{-ae^{x}}\,Ba^{3}e^{x}\mathrm{sgn}\left(-e^{x}+1\right)+6\,\sqrt{-ae^{x}}\,Aa^{3}\mathrm{sgn}\left(-e^{x}+1\right)-3\,\sqrt{-ae^{x}}\,Ba^{3}\mathrm{sgn}\left(-e^{x}+1\right)}{a^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(x))^(1/2)*(A+B*cosh(x)),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*((6*A*a^2*e^x*sgn(-e^x + 1) - 3*B*a^2*e^x*sgn(-e^x + 1) + B*a^2
*sgn(-e^x + 1))*e^(-x)/(sqrt(-a*e^x)*a) - (sqrt(-a*e^x)*B*a^3*e^x*sgn(-e^x
+ 1) + 6*sqrt(-a*e^x)*A*a^3*sgn(-e^x + 1) - 3*sqrt(-a*e^x)*B*a^3*sgn(-e^x +
 1))/a^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \left(A+B\cosh(x)\right)\,\sqrt{a-a\cosh\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))*(a - a*cosh(x))^(1/2),x)
```

```
[Out] int((A + B*cosh(x))*(a - a*cosh(x))^(1/2), x)
```

3.93 $\int \frac{A+B\cosh(x)}{1+\cosh(x)}\,dx$

Optimal. Leaf size=18

$$Bx+\frac{(A-B)\sinh(x)}{1+\cosh(x)}$$

```
[Out] B*x+(A-B)*sinh(x)/(1+cosh(x))
```

Rubi [A]

time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2814, 2727}

$$\frac{(A-B)\sinh(x)}{\cosh(x)+1}+Bx$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(1 + Cosh[x]),x]
```

```
[Out] B*x + ((A - B)*Sinh[x])/(1 + Cosh[x])
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cosh(x)}{1+\cosh(x)}\,dx &= Bx-(-A+B)\int \frac{1}{1+\cosh(x)}\,dx\\ &= Bx+\frac{(A-B)\sinh(x)}{1+\cosh(x)}\end{aligned}$$

Mathematica [A]

time = 0.04, size = 23, normalized size = 1.28

$$\frac{\left(A-B+Bx\coth\left(\frac{x}{2}\right)\right)\sinh(x)}{1+\cosh(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(1 + Cosh[x]),x]
```

```
[Out] ((A - B + B*x*Coth[x/2])*Sinh[x])/(1 + Cosh[x])
```

Maple [A]
time = 0.35, size = 34, normalized size = 1.89

method	result	size
risch	$Bx-\frac{2A}{e^{x}+1}+\frac{2B}{e^{x}+1}$	23
default	$A\tanh\left(\frac{x}{2}\right)-B\tanh\left(\frac{x}{2}\right)-B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(cosh(x)+1),x,method=_RETURNVERBOSE)
```

```
[Out] A*tanh(1/2*x)-B*tanh(1/2*x)-B*ln(tanh(1/2*x)-1)+B*ln(tanh(1/2*x)+1)
```

Maxima [A]
time = 0.28, size = 26, normalized size = 1.44

$$B\left(x-\frac{2}{e^{(-x)}+1}\right)+\frac{2\,A}{e^{(-x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x)),x, algorithm="maxima")
```

```
[Out] B*(x - 2/(e^(-x) + 1)) + 2*A/(e^(-x) + 1)
```

Fricas [A]
time = 0.42, size = 29, normalized size = 1.61

$$\frac{Bx\cosh\left(x\right)+Bx\sinh\left(x\right)+Bx-2\,A+2\,B}{\cosh\left(x\right)+\sinh\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x)),x, algorithm="fricas")
```

```
[Out] (B*x*cosh(x) + B*x*sinh(x) + B*x - 2*A + 2*B)/(cosh(x) + sinh(x) + 1)
```

Sympy [A]
time = 0.13, size = 15, normalized size = 0.83

$$A\tanh\left(\frac{x}{2}\right)+Bx-B\tanh\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x)),x)
```

```
[Out] A*tanh(x/2) + B*x - B*tanh(x/2)
```

Giac [A]
time = 0.40, size = 17, normalized size = 0.94

$$Bx - \frac{2\,(A-B)}{e^x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x)),x, algorithm="giac")
```

```
[Out] B*x - 2*(A - B)/(e^x + 1)
```

Mupad [B]
time = 0.05, size = 19, normalized size = 1.06

$$B\,x - \frac{2\,A-2\,B}{\mathrm{e}^x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(cosh(x) + 1),x)
```

```
[Out] B*x - (2*A - 2*B)/(exp(x) + 1)
```

3.94 $\int \frac{A+B\cosh(x)}{(1+\cosh(x))^2}\,dx$

Optimal. Leaf size=35

$$\frac{(A-B)\sinh(x)}{3(1+\cosh(x))^2}+\frac{(A+2B)\sinh(x)}{3(1+\cosh(x))}$$

```
[Out] 1/3*(A-B)*sinh(x)/(1+cosh(x))^2+1/3*(A+2*B)*sinh(x)/(1+cosh(x))
```

Rubi [A]
time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2829, 2727}

$$\frac{(A+2B)\sinh(x)}{3(\cosh(x)+1)}+\frac{(A-B)\sinh(x)}{3(\cosh(x)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(1 + Cosh[x])^2,x]
```

```
[Out] ((A - B)*Sinh[x])/(3*(1 + Cosh[x])^2) + ((A + 2*B)*Sinh[x])/(3*(1 + Cosh[x]
))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cosh(x)}{(1+\cosh(x))^2}\,dx &= \frac{(A-B)\sinh(x)}{3(1+\cosh(x))^2}+\frac{1}{3}(A+2B)\int \frac{1}{1+\cosh(x)}\,dx\\ &= \frac{(A-B)\sinh(x)}{3(1+\cosh(x))^2}+\frac{(A+2B)\sinh(x)}{3(1+\cosh(x))}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 25, normalized size = 0.71

$$\frac{(2A+B+(A+2B)\cosh(x))\sinh(x)}{3(1+\cosh(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(1 + Cosh[x])^2,x]
```

```
[Out] ((2*A + B + (A + 2*B)*Cosh[x])*Sinh[x])/(3*(1 + Cosh[x])^2)
```

Maple [A]
time = 0.30, size = 34, normalized size = 0.97

method	result	size
risch	$-\frac{2\left(3B\,\mathrm{e}^{2x}+3A\,\mathrm{e}^{x}+3B\,\mathrm{e}^{x}+A+2B\right)}{3\left(\mathrm{e}^{x}+1\right)^{3}}$	31
default	$-\frac{A\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{6}+\frac{B\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{6}+\frac{A\tanh\left(\frac{x}{2}\right)}{2}+\frac{B\tanh\left(\frac{x}{2}\right)}{2}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(cosh(x)+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*A*tanh(1/2*x)^3+1/6*B*tanh(1/2*x)^3+1/2*A*tanh(1/2*x)+1/2*B*tanh(1/2*x
)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(31) = 62$.
time = 0.26, size = 129, normalized size = 3.69

$$\frac{2}{3}\,B\left(\frac{3\,e^{(-x)}}{3\,e^{(-x)}+3\,e^{(-2\,x)}+e^{(-3\,x)}+1}+\frac{3\,e^{(-2\,x)}}{3\,e^{(-x)}+3\,e^{(-2\,x)}+e^{(-3\,x)}+1}+\frac{2}{3\,e^{(-x)}+3\,e^{(-2\,x)}+e^{(-3\,x)}+1}\right)+\frac{2}{3}\,A\left(\frac{3\,e^{(-x)}}{3\,e^{(-x)}+3\,e^{(-2\,x)}+e^{(-3\,x)}+1}+\frac{1}{3\,e^{(-x)}+3\,e^{(-2\,x)}+e^{(-3\,x)}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x))^2,x, algorithm="maxima")
```

```
[Out] 2/3*B*(3*e^(-x)/(3*e^(-x) + 3*e^(-2*x) + e^(-3*x) + 1) + 3*e^(-2*x)/(3*e^(-
x) + 3*e^(-2*x) + e^(-3*x) + 1) + 2/(3*e^(-x) + 3*e^(-2*x) + e^(-3*x) + 1))
 + 2/3*A*(3*e^(-x)/(3*e^(-x) + 3*e^(-2*x) + e^(-3*x) + 1) + 1/(3*e^(-x) + 3
*e^(-2*x) + e^(-3*x) + 1))
```

Fricas [A]
time = 0.44, size = 50, normalized size = 1.43

$$-\frac{2\left((A+5\,B)\cosh\left(x\right)-(A-B)\sinh\left(x\right)+3\,A+3\,B\right)}{3\left(\cosh\left(x\right)^{2}+2\left(\cosh\left(x\right)+1\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}+4\,\cosh\left(x\right)+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x))^2,x, algorithm="fricas")
```

```
[Out] -2/3*((A + 5*B)*cosh(x) - (A - B)*sinh(x) + 3*A + 3*B)/(cosh(x)^2 + 2*(cosh
(x) + 1)*sinh(x) + sinh(x)^2 + 4*cosh(x) + 3)
```

Sympy [A]
time = 0.25, size = 36, normalized size = 1.03

$$-\frac{A\tanh^3\left(\frac{x}{2}\right)}{6}+\frac{A\tanh\left(\frac{x}{2}\right)}{2}+\frac{B\tanh^3\left(\frac{x}{2}\right)}{6}+\frac{B\tanh\left(\frac{x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x))**2,x)
```

```
[Out] -A*tanh(x/2)**3/6 + A*tanh(x/2)/2 + B*tanh(x/2)**3/6 + B*tanh(x/2)/2
```

Giac [A]
time = 0.42, size = 30, normalized size = 0.86

$$-\frac{2\left(3\,Be^{(2\,x)}+3\,Ae^{x}+3\,Be^{x}+A+2\,B\right)}{3\left(e^{x}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*B*e^(2*x) + 3*A*e^x + 3*B*e^x + A + 2*B)/(e^x + 1)^3
```

Mupad [B]
time = 0.08, size = 30, normalized size = 0.86

$$-\frac{2\left(A+2\,B+3\,A\,\mathrm{e}^{x}+3\,B\,\mathrm{e}^{x}+3\,B\,\mathrm{e}^{2\,x}\right)}{3\left(\mathrm{e}^{x}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(cosh(x) + 1)^2,x)
```

```
[Out] -(2*(A + 2*B + 3*A*exp(x) + 3*B*exp(x) + 3*B*exp(2*x)))/(3*(exp(x) + 1)^3)
```

3.95 $\int \frac{A+B\cosh(x)}{(1+\cosh(x))^3}\,dx$

Optimal. Leaf size=56

$$\frac{(A-B)\sinh(x)}{5(1+\cosh(x))^3}+\frac{(2A+3B)\sinh(x)}{15(1+\cosh(x))^2}+\frac{(2A+3B)\sinh(x)}{15(1+\cosh(x))}$$

```
[Out] 1/5*(A-B)*sinh(x)/(1+cosh(x))^3+1/15*(2*A+3*B)*sinh(x)/(1+cosh(x))^2+1/15*(
2*A+3*B)*sinh(x)/(1+cosh(x))
```

Rubi [A]
time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2829, 2729, 2727}

$$\frac{(2A+3B)\sinh(x)}{15(\cosh(x)+1)}+\frac{(2A+3B)\sinh(x)}{15(\cosh(x)+1)^2}+\frac{(A-B)\sinh(x)}{5(\cosh(x)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(1 + Cosh[x])^3,x]
```

```
[Out] ((A - B)*Sinh[x])/(5*(1 + Cosh[x])^3) + ((2*A + 3*B)*Sinh[x])/(15*(1 + Cosh
[x])^2) + ((2*A + 3*B)*Sinh[x])/(15*(1 + Cosh[x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cosh(x)}{(1+\cosh(x))^3}\,dx &= \frac{(A-B)\sinh(x)}{5(1+\cosh(x))^3}+\frac{1}{5}(2A+3B)\int \frac{1}{(1+\cosh(x))^2}\,dx\\ &= \frac{(A-B)\sinh(x)}{5(1+\cosh(x))^3}+\frac{(2A+3B)\sinh(x)}{15(1+\cosh(x))^2}+\frac{1}{15}(2A+3B)\int \frac{1}{1+\cosh(x)}\,dx\\ &= \frac{(A-B)\sinh(x)}{5(1+\cosh(x))^3}+\frac{(2A+3B)\sinh(x)}{15(1+\cosh(x))^2}+\frac{(2A+3B)\sinh(x)}{15(1+\cosh(x))}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 42, normalized size = 0.75

$$\frac{(16A+9B+6(2A+3B)\cosh(x)+(2A+3B)\cosh(2x))\sinh(x)}{30(1+\cosh(x))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(1 + Cosh[x])^3,x]
```

```
[Out] ((16*A + 9*B + 6*(2*A + 3*B)*Cosh[x] + (2*A + 3*B)*Cosh[2*x])*Sinh[x])/(30*
(1 + Cosh[x])^3)
```

Maple [A]
time = 0.45, size = 38, normalized size = 0.68

method	result	size
default	$\frac{(A-B)(\tanh^5(\frac{x}{2}))}{20}-\frac{A(\tanh^3(\frac{x}{2}))}{6}+\frac{A\tanh(\frac{x}{2})}{4}+\frac{B\tanh(\frac{x}{2})}{4}$	38
risch	$-\frac{2(15B\,e^{3x}+20A\,e^{2x}+15B\,e^{2x}+10A\,e^{x}+15B\,e^{x}+2A+3B)}{15(e^{x}+1)^5}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(cosh(x)+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/20*(A-B)*tanh(1/2*x)^5-1/6*A*tanh(1/2*x)^3+1/4*A*tanh(1/2*x)+1/4*B*tanh(1
/2*x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 263 vs. 2(50) = 100.
time = 0.27, size = 263, normalized size = 4.70

$$\frac{4}{15}A\left(\frac{5e^{(-x)}}{5e^{(-x)}+10e^{(-2x)}+10e^{(-3x)}+5e^{(-4x)}+e^{(-5x)}+1}+\frac{10e^{(-2x)}}{5e^{(-x)}+10e^{(-2x)}+10e^{(-3x)}+5e^{(-4x)}+e^{(-5x)}+1}+\frac{1}{5e^{(-x)}+10e^{(-2x)}+10e^{(-3x)}+5e^{(-4x)}+e^{(-5x)}+1}\right)+\frac{2}{5}B\left(\frac{5e^{(-x)}}{5e^{(-x)}+10e^{(-2x)}+10e^{(-3x)}+5e^{(-4x)}+e^{(-5x)}+1}+\frac{5e^{(-2x)}}{5e^{(-x)}+10e^{(-2x)}+10e^{(-3x)}+5e^{(-4x)}+e^{(-5x)}+1}+\frac{5e^{(-3x)}}{5e^{(-x)}+10e^{(-2x)}+10e^{(-3x)}+5e^{(-4x)}+e^{(-5x)}+1}+\frac{1}{5e^{(-x)}+10e^{(-2x)}+10e^{(-3x)}+5e^{(-4x)}+e^{(-5x)}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x))^3,x, algorithm="maxima")
```

```
[Out] 4/15*A*(5*e^(-x)/(5*e^(-x) + 10*e^(-2*x) + 10*e^(-3*x) + 5*e^(-4*x) + e^(-5
*x) + 1) + 10*e^(-2*x)/(5*e^(-x) + 10*e^(-2*x) + 10*e^(-3*x) + 5*e^(-4*x) +
 e^(-5*x) + 1) + 1/(5*e^(-x) + 10*e^(-2*x) + 10*e^(-3*x) + 5*e^(-4*x) + e^(
-5*x) + 1)) + 2/5*B*(5*e^(-x)/(5*e^(-x) + 10*e^(-2*x) + 10*e^(-3*x) + 5*e^(
-4*x) + e^(-5*x) + 1) + 5*e^(-2*x)/(5*e^(-x) + 10*e^(-2*x) + 10*e^(-3*x) +
5*e^(-4*x) + e^(-5*x) + 1) + 5*e^(-3*x)/(5*e^(-x) + 10*e^(-2*x) + 10*e^(-3*
x) + 5*e^(-4*x) + e^(-5*x) + 1) + 1/(5*e^(-x) + 10*e^(-2*x) + 10*e^(-3*x) +
 5*e^(-4*x) + e^(-5*x) + 1))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(50) = 100$.
time = 0.36, size = 127, normalized size = 2.27

$$-\frac{2\,(15\,B\cosh\left(x\right)^{2} + 15\,B\sinh\left(x\right)^{2} + 2\,(11\,A + 9\,B)\cosh\left(x\right) + 6\,(5\,B\cosh\left(x\right) + 3\,A + 2\,B)\sinh\left(x\right) + 10\,A + 15\,B)}{15\,(\cosh\left(x\right)^{4} + (4\,\cosh\left(x\right) + 5)\sinh\left(x\right)^{3} + \sinh\left(x\right)^{4} + 5\,\cosh\left(x\right)^{3} + (6\,\cosh\left(x\right)^{2} + 15\,\cosh\left(x\right) + 10)\sinh\left(x\right)^{2} + 10\,\cosh\left(x\right)^{2} + (4\,\cosh\left(x\right)^{3} + 15\,\cosh\left(x\right)^{2} + 20\,\cosh\left(x\right) + 9)\sinh\left(x\right) + 11\,\cosh\left(x\right) + 5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x))^3,x, algorithm="fricas")
```

```
[Out] -2/15*(15*B*cosh(x)^2 + 15*B*sinh(x)^2 + 2*(11*A + 9*B)*cosh(x) + 6*(5*B*co
sh(x) + 3*A + 2*B)*sinh(x) + 10*A + 15*B)/(cosh(x)^4 + (4*cosh(x) + 5)*sinh
(x)^3 + sinh(x)^4 + 5*cosh(x)^3 + (6*cosh(x)^2 + 15*cosh(x) + 10)*sinh(x)^2
 + 10*cosh(x)^2 + (4*cosh(x)^3 + 15*cosh(x)^2 + 20*cosh(x) + 9)*sinh(x) + 1
1*cosh(x) + 5)
```

Sympy [**A**]
time = 0.48, size = 46, normalized size = 0.82

$$\frac{A\tanh^{5}\left(\frac{x}{2}\right)}{20} - \frac{A\tanh^{3}\left(\frac{x}{2}\right)}{6} + \frac{A\tanh\left(\frac{x}{2}\right)}{4} - \frac{B\tanh^{5}\left(\frac{x}{2}\right)}{20} + \frac{B\tanh\left(\frac{x}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x))**3,x)
```

```
[Out] A*tanh(x/2)**5/20 - A*tanh(x/2)**3/6 + A*tanh(x/2)/4 - B*tanh(x/2)**5/20 +
B*tanh(x/2)/4
```

Giac [**A**]
time = 0.40, size = 46, normalized size = 0.82

$$-\frac{2\,\left(15\,Be^{(3\,x)} + 20\,Ae^{(2\,x)} + 15\,Be^{(2\,x)} + 10\,Ae^{x} + 15\,Be^{x} + 2\,A + 3\,B\right)}{15\,{\left(e^{x} + 1\right)}^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x))^3,x, algorithm="giac")
```

```
[Out] -2/15*(15*B*e^(3*x) + 20*A*e^(2*x) + 15*B*e^(2*x) + 10*A*e^x + 15*B*e^x + 2
*A + 3*B)/(e^x + 1)^5
```

Mupad [B]
time = 0.92, size = 141, normalized size = 2.52

$$-\frac{\frac{4\,B\,\mathrm{e}^x}{5}+\frac{8\,A\,\mathrm{e}^{2\,x}}{5}+\frac{4\,B\,\mathrm{e}^{3\,x}}{5}}{10\,\mathrm{e}^{2\,x}+10\,\mathrm{e}^{3\,x}+5\,\mathrm{e}^{4\,x}+\mathrm{e}^{5\,x}+5\,\mathrm{e}^x+1}-\frac{\frac{B}{5}+\frac{4\,A\,\mathrm{e}^x}{5}+\frac{3\,B\,\mathrm{e}^{2\,x}}{5}}{6\,\mathrm{e}^{2\,x}+4\,\mathrm{e}^{3\,x}+\mathrm{e}^{4\,x}+4\,\mathrm{e}^x+1}-\frac{\frac{4\,A}{15}+\frac{2\,B\,\mathrm{e}^x}{5}}{3\,\mathrm{e}^{2\,x}+\mathrm{e}^{3\,x}+3\,\mathrm{e}^x+1}-\frac{B}{5\,\left(\mathrm{e}^{2\,x}+2\,\mathrm{e}^x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(cosh(x) + 1)^3,x)
```

```
[Out] - ((4*B*exp(x))/5 + (8*A*exp(2*x))/5 + (4*B*exp(3*x))/5)/(10*exp(2*x) + 10*
exp(3*x) + 5*exp(4*x) + exp(5*x) + 5*exp(x) + 1) - (B/5 + (4*A*exp(x))/5 +
(3*B*exp(2*x))/5)/(6*exp(2*x) + 4*exp(3*x) + exp(4*x) + 4*exp(x) + 1) - ((4
*A)/15 + (2*B*exp(x))/5)/(3*exp(2*x) + exp(3*x) + 3*exp(x) + 1) - B/(5*(exp
(2*x) + 2*exp(x) + 1))
```

3.96 $\int \frac{A+B\cosh(x)}{(1+\cosh(x))^4} dx$

Optimal. Leaf size=75

$$\frac{(A-B)\sinh(x)}{7(1+\cosh(x))^4}+\frac{(3A+4B)\sinh(x)}{35(1+\cosh(x))^3}+\frac{2(3A+4B)\sinh(x)}{105(1+\cosh(x))^2}+\frac{2(3A+4B)\sinh(x)}{105(1+\cosh(x))}$$

```
[Out] 1/7*(A-B)*sinh(x)/(1+cosh(x))^4+1/35*(3*A+4*B)*sinh(x)/(1+cosh(x))^3+2/105*
(3*A+4*B)*sinh(x)/(1+cosh(x))^2+2/105*(3*A+4*B)*sinh(x)/(1+cosh(x))
```

Rubi [A]
time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2829, 2729, 2727}

$$\frac{2(3A+4B)\sinh(x)}{105(\cosh(x)+1)}+\frac{2(3A+4B)\sinh(x)}{105(\cosh(x)+1)^2}+\frac{(3A+4B)\sinh(x)}{35(\cosh(x)+1)^3}+\frac{(A-B)\sinh(x)}{7(\cosh(x)+1)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(1 + Cosh[x])^4,x]
```

```
[Out] ((A - B)*Sinh[x])/(7*(1 + Cosh[x])^4) + ((3*A + 4*B)*Sinh[x])/(35*(1 + Cosh
[x])^3) + (2*(3*A + 4*B)*Sinh[x])/(105*(1 + Cosh[x])^2) + (2*(3*A + 4*B)*Si
nh[x])/(105*(1 + Cosh[x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cosh(x)}{(1+\cosh(x))^4}\,dx &= \frac{(A-B)\sinh(x)}{7(1+\cosh(x))^4}+\frac{1}{7}(3A+4B)\int \frac{1}{(1+\cosh(x))^3}\,dx\\ &= \frac{(A-B)\sinh(x)}{7(1+\cosh(x))^4}+\frac{(3A+4B)\sinh(x)}{35(1+\cosh(x))^3}+\frac{1}{35}(2(3A+4B))\int \frac{1}{(1+\cosh(x))^2}\,dx\\ &= \frac{(A-B)\sinh(x)}{7(1+\cosh(x))^4}+\frac{(3A+4B)\sinh(x)}{35(1+\cosh(x))^3}+\frac{2(3A+4B)\sinh(x)}{105(1+\cosh(x))^2}+\frac{1}{105}(2(3A+4B))\int\\ &= \frac{(A-B)\sinh(x)}{7(1+\cosh(x))^4}+\frac{(3A+4B)\sinh(x)}{35(1+\cosh(x))^3}+\frac{2(3A+4B)\sinh(x)}{105(1+\cosh(x))^2}+\frac{2(3A+4B)\sinh(x)}{105(1+\cosh(x))}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 57, normalized size = 0.76

$$\frac{(96A+58B+29(3A+4B)\cosh(x)+8(3A+4B)\cosh(2x)+3A\cosh(3x)+4B\cosh(3x))\sinh(x)}{210(1+\cosh(x))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(1 + Cosh[x])^4,x]
```

```
[Out] ((96*A + 58*B + 29*(3*A + 4*B)*Cosh[x] + 8*(3*A + 4*B)*Cosh[2*x] + 3*A*Cosh
[3*x] + 4*B*Cosh[3*x])*Sinh[x])/(210*(1 + Cosh[x])^4)
```

Maple [A]
time = 0.46, size = 55, normalized size = 0.73

method	result	size
default	$-\frac{(A-B)\left(\tanh^7\left(\frac{x}{2}\right)\right)}{56}-\frac{(-3A+B)\left(\tanh^5\left(\frac{x}{2}\right)\right)}{40}-\frac{(3A+B)\left(\tanh^3\left(\frac{x}{2}\right)\right)}{24}+\frac{A\tanh\left(\frac{x}{2}\right)}{8}+\frac{B\tanh\left(\frac{x}{2}\right)}{8}$	55
risch	$-\frac{4\left(70B\,e^{4x}+105A\,e^{3x}+70B\,e^{3x}+63A\,e^{2x}+84B\,e^{2x}+21A\,e^{x}+28B\,e^{x}+3A+4B\right)}{105\left(e^{x}+1\right)^7}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(cosh(x)+1)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/56*(A-B)*tanh(1/2*x)^7-1/40*(-3*A+B)*tanh(1/2*x)^5-1/24*(3*A+B)*tanh(1/2
*x)^3+1/8*A*tanh(1/2*x)+1/8*B*tanh(1/2*x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 449 vs. $2(67) = 134$.
time = 0.29, size = 449, normalized size = 5.99

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x))^4,x, algorithm="maxima")
```

```
[Out] 8/105*B*(14*e^(-x)/(7*e^(-x) + 21*e^(-2*x) + 35*e^(-3*x) + 35*e^(-4*x) + 21
*e^(-5*x) + 7*e^(-6*x) + e^(-7*x) + 1) + 42*e^(-2*x)/(7*e^(-x) + 21*e^(-2*x
) + 35*e^(-3*x) + 35*e^(-4*x) + 21*e^(-5*x) + 7*e^(-6*x) + e^(-7*x) + 1) +
35*e^(-3*x)/(7*e^(-x) + 21*e^(-2*x) + 35*e^(-3*x) + 35*e^(-4*x) + 21*e^(-5*
x) + 7*e^(-6*x) + e^(-7*x) + 1) + 35*e^(-4*x)/(7*e^(-x) + 21*e^(-2*x) + 35*
e^(-3*x) + 35*e^(-4*x) + 21*e^(-5*x) + 7*e^(-6*x) + e^(-7*x) + 1) + 2/(7*e^
(-x) + 21*e^(-2*x) + 35*e^(-3*x) + 35*e^(-4*x) + 21*e^(-5*x) + 7*e^(-6*x) +
 e^(-7*x) + 1)) + 4/35*A*(7*e^(-x)/(7*e^(-x) + 21*e^(-2*x) + 35*e^(-3*x) +
35*e^(-4*x) + 21*e^(-5*x) + 7*e^(-6*x) + e^(-7*x) + 1) + 21*e^(-2*x)/(7*e^(
-x) + 21*e^(-2*x) + 35*e^(-3*x) + 35*e^(-4*x) + 21*e^(-5*x) + 7*e^(-6*x) +
e^(-7*x) + 1) + 35*e^(-3*x)/(7*e^(-x) + 21*e^(-2*x) + 35*e^(-3*x) + 35*e^(-
4*x) + 21*e^(-5*x) + 7*e^(-6*x) + e^(-7*x) + 1) + 1/(7*e^(-x) + 21*e^(-2*x)
 + 35*e^(-3*x) + 35*e^(-4*x) + 21*e^(-5*x) + 7*e^(-6*x) + e^(-7*x) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(67) = 134$.
time = 0.37, size = 175, normalized size = 2.33

$$-\frac{4\left((3A+74B)\cosh(x)^2+(3A+74B)\sinh(x)^2+14(9A+7B)\cosh(x)-6((A-22B)\cosh(x)-14A-7B)\sinh(x)+63A+84B\right)}{105\left(\cosh(x)^5+(5\cosh(x)+7)\sinh(x)^4+\sinh(x)^5+7\cosh(x)^4+\left(10\cosh(x)^2+28\cosh(x)+21\right)\sinh(x)^3+21\cosh(x)^3+\left(10\cosh(x)^3+42\cosh(x)^2+63\cosh(x)+36\right)\sinh(x)^2+36\cosh(x)^2+\left(5\cosh(x)^4+28\cosh(x)^3+63\cosh(x)^2+68\cosh(x)+28\right)\sinh(x)+42\cosh(x)+21\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x))^4,x, algorithm="fricas")
```

```
[Out] -4/105*((3*A + 74*B)*cosh(x)^2 + (3*A + 74*B)*sinh(x)^2 + 14*(9*A + 7*B)*co
sh(x) - 6*((A - 22*B)*cosh(x) - 14*A - 7*B)*sinh(x) + 63*A + 84*B)/(cosh(x)
^5 + (5*cosh(x) + 7)*sinh(x)^4 + sinh(x)^5 + 7*cosh(x)^4 + (10*cosh(x)^2 +
28*cosh(x) + 21)*sinh(x)^3 + 21*cosh(x)^3 + (10*cosh(x)^3 + 42*cosh(x)^2 +
63*cosh(x) + 36)*sinh(x)^2 + 36*cosh(x)^2 + (5*cosh(x)^4 + 28*cosh(x)^3 + 6
3*cosh(x)^2 + 68*cosh(x) + 28)*sinh(x) + 42*cosh(x) + 21)
```

Sympy [A]
time = 0.95, size = 78, normalized size = 1.04

$$-\frac{A\tanh^7\left(\frac{x}{2}\right)}{56}+\frac{3A\tanh^5\left(\frac{x}{2}\right)}{40}-\frac{A\tanh^3\left(\frac{x}{2}\right)}{8}+\frac{A\tanh\left(\frac{x}{2}\right)}{8}+\frac{B\tanh^7\left(\frac{x}{2}\right)}{56}-\frac{B\tanh^5\left(\frac{x}{2}\right)}{40}-\frac{B\tanh^3\left(\frac{x}{2}\right)}{24}+\frac{B\tanh\left(\frac{x}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x))**4,x)
```

```
[Out] -A*tanh(x/2)**7/56 + 3*A*tanh(x/2)**5/40 - A*tanh(x/2)**3/8 + A*tanh(x/2)/8
 + B*tanh(x/2)**7/56 - B*tanh(x/2)**5/40 - B*tanh(x/2)**3/24 + B*tanh(x/2)/
8
```

Giac [A]
time = 0.41, size = 60, normalized size = 0.80

$$-\frac{4\left(70\,Be^{(4\,x)}+105\,Ae^{(3\,x)}+70\,Be^{(3\,x)}+63\,Ae^{(2\,x)}+84\,Be^{(2\,x)}+21\,Ae^{x}+28\,Be^{x}+3\,A+4\,B\right)}{105\,\left(e^{x}+1\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x))^4,x, algorithm="giac")
```

```
[Out] -4/105*(70*B*e^(4*x) + 105*A*e^(3*x) + 70*B*e^(3*x) + 63*A*e^(2*x) + 84*B*e
^(2*x) + 21*A*e^x + 28*B*e^x + 3*A + 4*B)/(e^x + 1)^7
```

Mupad [B]
time = 0.92, size = 231, normalized size = 3.08

$$-\frac{\frac{4\,A}{35}+\frac{8\,B\,\mathrm{e}^{x}}{35}}{6\,\mathrm{e}^{2\,x}+4\,\mathrm{e}^{3\,x}+\mathrm{e}^{4\,x}+4\,\mathrm{e}^{x}+1}-\frac{8\,B}{105\,\left(3\,\mathrm{e}^{2\,x}+\mathrm{e}^{3\,x}+3\,\mathrm{e}^{x}+1\right)}-\frac{\frac{16\,A\,\mathrm{e}^{3\,x}}{7}+\frac{8\,B\,\mathrm{e}^{2\,x}}{7}+\frac{8\,B\,\mathrm{e}^{4\,x}}{7}}{21\,\mathrm{e}^{2\,x}+35\,\mathrm{e}^{3\,x}+35\,\mathrm{e}^{4\,x}+21\,\mathrm{e}^{5\,x}+7\,\mathrm{e}^{6\,x}+\mathrm{e}^{7\,x}+7\,\mathrm{e}^{x}+1}-\frac{\frac{8\,B\,\mathrm{e}^{x}}{21}+\frac{8\,A\,\mathrm{e}^{2\,x}}{7}+\frac{16\,B\,\mathrm{e}^{3\,x}}{21}}{15\,\mathrm{e}^{2\,x}+20\,\mathrm{e}^{3\,x}+15\,\mathrm{e}^{4\,x}+6\,\mathrm{e}^{5\,x}+\mathrm{e}^{6\,x}+6\,\mathrm{e}^{x}+1}-\frac{\frac{8\,B}{105}+\frac{16\,A\,\mathrm{e}^{x}}{35}+\frac{16\,B\,\mathrm{e}^{2\,x}}{35}}{10\,\mathrm{e}^{2\,x}+10\,\mathrm{e}^{3\,x}+5\,\mathrm{e}^{4\,x}+\mathrm{e}^{5\,x}+5\,\mathrm{e}^{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(cosh(x) + 1)^4,x)
```

```
[Out] - ((4*A)/35 + (8*B*exp(x))/35)/(6*exp(2*x) + 4*exp(3*x) + exp(4*x) + 4*exp(
x) + 1) - (8*B)/(105*(3*exp(2*x) + exp(3*x) + 3*exp(x) + 1)) - ((16*A*exp(3
*x))/7 + (8*B*exp(2*x))/7 + (8*B*exp(4*x))/7)/(21*exp(2*x) + 35*exp(3*x) +
35*exp(4*x) + 21*exp(5*x) + 7*exp(6*x) + exp(7*x) + 7*exp(x) + 1) - ((8*B*e
xp(x))/21 + (8*A*exp(2*x))/7 + (16*B*exp(3*x))/21)/(15*exp(2*x) + 20*exp(3*
x) + 15*exp(4*x) + 6*exp(5*x) + exp(6*x) + 6*exp(x) + 1) - ((8*B)/105 + (16
*A*exp(x))/35 + (16*B*exp(2*x))/35)/(10*exp(2*x) + 10*exp(3*x) + 5*exp(4*x)
 + exp(5*x) + 5*exp(x) + 1)
```

3.97 $\int \frac{A+B\cosh(x)}{1-\cosh(x)}\,dx$

Optimal. Leaf size=20

$$-Bx - \frac{(A+B)\sinh(x)}{1-\cosh(x)}$$

```
[Out] -B*x-(A+B)*sinh(x)/(1-cosh(x))
```

Rubi [A]
time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2814, 2727}

$$-\frac{(A+B)\sinh(x)}{1-\cosh(x)} - Bx$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(1 - Cosh[x]),x]
```

```
[Out] -(B*x) - ((A + B)*Sinh[x])/(1 - Cosh[x])
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\cosh(x)}{1-\cosh(x)}\,dx &= -Bx-(-A-B)\int \frac{1}{1-\cosh(x)}\,dx \\
&= -Bx-\frac{(A+B)\sinh(x)}{1-\cosh(x)}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 35, normalized size = 1.75

$$\frac{2\sinh\left(\frac{x}{2}\right)\left((A+B)\cosh\left(\frac{x}{2}\right)-Bx\sinh\left(\frac{x}{2}\right)\right)}{-1+\cosh(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(1 - Cosh[x]),x]
[Out] (2*Sinh[x/2]*((A + B)*Cosh[x/2] - B*x*Sinh[x/2]))/(-1 + Cosh[x])
```

Maple [A]
time = 0.42, size = 36, normalized size = 1.80

method	result	size
risch	$-Bx+\frac{2A}{e^{x}-1}+\frac{2B}{e^{x}-1}$	24
default	$-\frac{-A-B}{\tanh\left(\frac{x}{2}\right)}+B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(1-cosh(x)),x,method=_RETURNVERBOSE)
[Out] -(-A-B)/tanh(1/2*x)+B*ln(tanh(1/2*x)-1)-B*ln(tanh(1/2*x)+1)
```

Maxima [A]
time = 0.27, size = 27, normalized size = 1.35

$$-B\left(x+\frac{2}{e^{(-x)}-1}\right)-\frac{2\,A}{e^{(-x)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x)),x, algorithm="maxima")
[Out] -B*(x + 2/(e^(-x) - 1)) - 2*A/(e^(-x) - 1)
```

Fricas [A]
time = 0.39, size = 31, normalized size = 1.55

$$-\frac{Bx\cosh\left(x\right)+Bx\sinh\left(x\right)-Bx-2\,A-2\,B}{\cosh\left(x\right)+\sinh\left(x\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x)),x, algorithm="fricas")
[Out] -(B*x*cosh(x) + B*x*sinh(x) - B*x - 2*A - 2*B)/(cosh(x) + sinh(x) - 1)
```

Sympy [A]
time = 0.23, size = 15, normalized size = 0.75

$$\frac{A}{\tanh\left(\frac{x}{2}\right)}-Bx+\frac{B}{\tanh\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x)),x)
[Out] A/tanh(x/2) - B*x + B/tanh(x/2)
```

Giac [A]
time = 0.41, size = 16, normalized size = 0.80

$$-Bx + \frac{2(A+B)}{e^x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x)),x, algorithm="giac")
[Out] -B*x + 2*(A + B)/(e^x - 1)
```

Mupad [B]
time = 0.05, size = 19, normalized size = 0.95

$$\frac{2A + 2B}{e^x - 1} - Bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(A + B*cosh(x))/(cosh(x) - 1),x)
[Out] (2*A + 2*B)/(exp(x) - 1) - B*x
```

3.98 $\int \frac{A+B\cosh(x)}{(1-\cosh(x))^2}\,dx$

Optimal. Leaf size=37

$$-\frac{(A+B)\sinh(x)}{3(1-\cosh(x))^2}-\frac{(A-2B)\sinh(x)}{3(1-\cosh(x))}$$

```
[Out] -1/3*(A+B)*sinh(x)/(1-cosh(x))^2-1/3*(A-2*B)*sinh(x)/(1-cosh(x))
```

Rubi [A]
time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.133$, Rules used = {2829, 2727}

$$-\frac{(A-2B)\sinh(x)}{3(1-\cosh(x))}-\frac{(A+B)\sinh(x)}{3(1-\cosh(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(1 - Cosh[x])^2,x]
```

```
[Out] -1/3*((A + B)*Sinh[x])/(1 - Cosh[x])^2 - ((A - 2*B)*Sinh[x])/(3*(1 - Cosh[x
]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cosh(x)}{(1-\cosh(x))^2}\,dx &= -\frac{(A+B)\sinh(x)}{3(1-\cosh(x))^2}+\frac{1}{3}(A-2B)\int \frac{1}{1-\cosh(x)}\,dx\\ &= -\frac{(A+B)\sinh(x)}{3(1-\cosh(x))^2}-\frac{(A-2B)\sinh(x)}{3(1-\cosh(x))}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 25, normalized size = 0.68

$$\frac{(-2A+B+(A-2B)\cosh(x))\sinh(x)}{3(-1+\cosh(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(1 - Cosh[x])^2,x]
```

```
[Out] ((-2*A + B + (A - 2*B)*Cosh[x])*Sinh[x])/(3*(-1 + Cosh[x])^2)
```

Maple [A]
time = 0.42, size = 26, normalized size = 0.70

method	result	size
default	$-\frac{-A+B}{2\tanh\left(\frac{x}{2}\right)}-\frac{A+B}{6\tanh\left(\frac{x}{2}\right)^{3}}$	26
risch	$-\frac{2\left(3B\,e^{2x}+3A\,e^{x}-3B\,e^{x}-A+2B\right)}{3\left(e^{x}-1\right)^{3}}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(1-cosh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-A+B)/tanh(1/2*x)-1/6*(A+B)/tanh(1/2*x)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(29) = 58.
time = 0.27, size = 131, normalized size = 3.54

$$-\frac{2}{3}B\left(\frac{3e^{(-x)}}{3e^{(-x)}-3e^{(-2x)}+e^{(-3x)}-1}-\frac{3e^{(-2x)}}{3e^{(-x)}-3e^{(-2x)}+e^{(-3x)}-1}-\frac{2}{3e^{(-x)}-3e^{(-2x)}+e^{(-3x)}-1}\right)+\frac{2}{3}A\left(\frac{3e^{(-x)}}{3e^{(-x)}-3e^{(-2x)}+e^{(-3x)}-1}-\frac{1}{3e^{(-x)}-3e^{(-2x)}+e^{(-3x)}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x))^2,x, algorithm="maxima")
```

```
[Out] -2/3*B*(3*e^(-x)/(3*e^(-x) - 3*e^(-2*x) + e^(-3*x) - 1) - 3*e^(-2*x)/(3*e^(
-x) - 3*e^(-2*x) + e^(-3*x) - 1) - 2/(3*e^(-x) - 3*e^(-2*x) + e^(-3*x) - 1)
) + 2/3*A*(3*e^(-x)/(3*e^(-x) - 3*e^(-2*x) + e^(-3*x) - 1) - 1/(3*e^(-x) -
3*e^(-2*x) + e^(-3*x) - 1))
```

Fricas [A]
time = 0.45, size = 48, normalized size = 1.30

$$\frac{2\left((A-5\,B)\cosh\left(x\right)-(A+B)\sinh\left(x\right)-3\,A+3\,B\right)}{3\left(\cosh\left(x\right)^{2}+2\left(\cosh\left(x\right)-1\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}-4\,\cosh\left(x\right)+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x))^2,x, algorithm="fricas")
```

```
[Out] 2/3*((A - 5*B)*cosh(x) - (A + B)*sinh(x) - 3*A + 3*B)/(cosh(x)^2 + 2*(cosh(
x) - 1)*sinh(x) + sinh(x)^2 - 4*cosh(x) + 3)
```

Sympy [A]
time = 0.37, size = 36, normalized size = 0.97

$$\frac{A}{2\tanh\left(\frac{x}{2}\right)} - \frac{A}{6\tanh^3\left(\frac{x}{2}\right)} - \frac{B}{2\tanh\left(\frac{x}{2}\right)} - \frac{B}{6\tanh^3\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x))**2,x)
```

```
[Out] A/(2*tanh(x/2)) - A/(6*tanh(x/2)**3) - B/(2*tanh(x/2)) - B/(6*tanh(x/2)**3)
```

Giac [A]
time = 0.42, size = 32, normalized size = 0.86

$$-\frac{2\left(3\,Be^{(2\,x)} + 3\,Ae^{x} - 3\,Be^{x} - A + 2\,B\right)}{3\,(e^{x} - 1)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*B*e^(2*x) + 3*A*e^x - 3*B*e^x - A + 2*B)/(e^x - 1)^3
```

Mupad [B]
time = 0.93, size = 32, normalized size = 0.86

$$-\frac{2\,(2\,B - A + 3\,A\,\mathrm{e}^{x} - 3\,B\,\mathrm{e}^{x} + 3\,B\,\mathrm{e}^{2\,x})}{3\,(\mathrm{e}^{x} - 1)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(cosh(x) - 1)^2,x)
```

```
[Out] -(2*(2*B - A + 3*A*exp(x) - 3*B*exp(x) + 3*B*exp(2*x)))/(3*(exp(x) - 1)^3)
```

3.99 $\int \frac{A+B\cosh(x)}{(1-\cosh(x))^3} dx$

Optimal. Leaf size=60

$$-\frac{(A+B)\sinh(x)}{5(1-\cosh(x))^3}-\frac{(2A-3B)\sinh(x)}{15(1-\cosh(x))^2}-\frac{(2A-3B)\sinh(x)}{15(1-\cosh(x))}$$

```
[Out] -1/5*(A+B)*sinh(x)/(1-cosh(x))^3-1/15*(2*A-3*B)*sinh(x)/(1-cosh(x))^2-1/15*
(2*A-3*B)*sinh(x)/(1-cosh(x))
```

Rubi [A]
time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2829, 2729, 2727}

$$-\frac{(2A-3B)\sinh(x)}{15(1-\cosh(x))}-\frac{(2A-3B)\sinh(x)}{15(1-\cosh(x))^2}-\frac{(A+B)\sinh(x)}{5(1-\cosh(x))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(1 - Cosh[x])^3,x]
```

```
[Out] -1/5*((A + B)*Sinh[x])/(1 - Cosh[x])^3 - ((2*A - 3*B)*Sinh[x])/(15*(1 - Cos
h[x])^2) - ((2*A - 3*B)*Sinh[x])/(15*(1 - Cosh[x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cosh(x)}{(1-\cosh(x))^3}\,dx &= -\frac{(A+B)\sinh(x)}{5(1-\cosh(x))^3}+\frac{1}{5}(2A-3B)\int \frac{1}{(1-\cosh(x))^2}\,dx\\ &= -\frac{(A+B)\sinh(x)}{5(1-\cosh(x))^3}-\frac{(2A-3B)\sinh(x)}{15(1-\cosh(x))^2}+\frac{1}{15}(2A-3B)\int \frac{1}{1-\cosh(x)}\,dx\\ &= -\frac{(A+B)\sinh(x)}{5(1-\cosh(x))^3}-\frac{(2A-3B)\sinh(x)}{15(1-\cosh(x))^2}-\frac{(2A-3B)\sinh(x)}{15(1-\cosh(x))}\end{aligned}$$

Mathematica [**A**]
time = 0.06, size = 42, normalized size = 0.70

$$\frac{(16A-9B-6(2A-3B)\cosh(x)+(2A-3B)\cosh(2x))\sinh(x)}{30(-1+\cosh(x))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(1 - Cosh[x])^3,x]
```

```
[Out] ((16*A - 9*B - 6*(2*A - 3*B)*Cosh[x] + (2*A - 3*B)*Cosh[2*x])*Sinh[x])/(30*
(-1 + Cosh[x])^3)
```

Maple [**A**]
time = 0.38, size = 39, normalized size = 0.65

method	result	size
default	$-\frac{-A+B}{4\tanh\left(\frac{x}{2}\right)}-\frac{-A-B}{20\tanh\left(\frac{x}{2}\right)^{5}}-\frac{A}{6\tanh\left(\frac{x}{2}\right)^{3}}$	39
risch	$\frac{2B\,{\rm e}^{3x}+\frac{8A\,{\rm e}^{2x}}{3}-2B\,{\rm e}^{2x}-\frac{4A\,{\rm e}^{x}}{3}+2B\,{\rm e}^{x}+\frac{4A}{15}-\frac{2B}{5}}{\left({\rm e}^{x}-1\right)^{5}}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(1-cosh(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(-A+B)/tanh(1/2*x)-1/20*(-A-B)/tanh(1/2*x)^5-1/6*A/tanh(1/2*x)^3
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(48) = 96$.
time = 0.28, size = 267, normalized size = 4.45

$$-\frac{2}{5}B\left(\frac{5e^{(-x)}}{5e^{(-x)}-10e^{(-2x)}+10e^{(-3x)}-5e^{(-4x)}+e^{(-5x)}-1}-\frac{5e^{(-2x)}}{5e^{(-x)}-10e^{(-2x)}+10e^{(-3x)}-5e^{(-4x)}+e^{(-5x)}-1}+\frac{5e^{(-3x)}}{5e^{(-x)}-10e^{(-2x)}+10e^{(-3x)}-5e^{(-4x)}+e^{(-5x)}-1}-\frac{1}{5e^{(-x)}-10e^{(-2x)}+10e^{(-3x)}-5e^{(-4x)}+e^{(-5x)}-1}\right)+\frac{4}{15}A\left(\frac{5e^{(-x)}}{5e^{(-x)}-10e^{(-2x)}+10e^{(-3x)}-5e^{(-4x)}+e^{(-5x)}-1}-\frac{10e^{(-2x)}}{5e^{(-x)}-10e^{(-2x)}+10e^{(-3x)}-5e^{(-4x)}+e^{(-5x)}-1}-\frac{1}{5e^{(-x)}-10e^{(-2x)}+10e^{(-3x)}-5e^{(-4x)}+e^{(-5x)}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x))^3,x, algorithm="maxima")
```

```
[Out] -2/5*B*(5*e^(-x)/(5*e^(-x) - 10*e^(-2*x) + 10*e^(-3*x) - 5*e^(-4*x) + e^(-5
*x) - 1) - 5*e^(-2*x)/(5*e^(-x) - 10*e^(-2*x) + 10*e^(-3*x) - 5*e^(-4*x) +
e^(-5*x) - 1) + 5*e^(-3*x)/(5*e^(-x) - 10*e^(-2*x) + 10*e^(-3*x) - 5*e^(-4*
x) + e^(-5*x) - 1) - 1/(5*e^(-x) - 10*e^(-2*x) + 10*e^(-3*x) - 5*e^(-4*x) +
 e^(-5*x) - 1)) + 4/15*A*(5*e^(-x)/(5*e^(-x) - 10*e^(-2*x) + 10*e^(-3*x) -
5*e^(-4*x) + e^(-5*x) - 1) - 10*e^(-2*x)/(5*e^(-x) - 10*e^(-2*x) + 10*e^(-3
*x) - 5*e^(-4*x) + e^(-5*x) - 1) - 1/(5*e^(-x) - 10*e^(-2*x) + 10*e^(-3*x)
- 5*e^(-4*x) + e^(-5*x) - 1))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(48) = 96$.
time = 0.35, size = 127, normalized size = 2.12

$$\frac{2\,(15\,B\cosh(x)^2 + 15\,B\sinh(x)^2 + 2\,(11\,A - 9\,B)\cosh(x) + 6\,(5\,B\cosh(x) + 3\,A - 2\,B)\sinh(x) - 10\,A + 15\,B)}{15\,(\cosh(x)^4 + (4\,\cosh(x) - 5)\sinh(x)^3 + \sinh(x)^4 - 5\,\cosh(x)^3 + (6\,\cosh(x)^2 - 15\,\cosh(x) + 10)\sinh(x)^2 + 10\,\cosh(x)^2 + (4\,\cosh(x)^3 - 15\,\cosh(x)^2 + 20\,\cosh(x) - 9)\sinh(x) - 11\,\cosh(x) + 5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x))^3,x, algorithm="fricas")
```

```
[Out] 2/15*(15*B*cosh(x)^2 + 15*B*sinh(x)^2 + 2*(11*A - 9*B)*cosh(x) + 6*(5*B*cos
h(x) + 3*A - 2*B)*sinh(x) - 10*A + 15*B)/(cosh(x)^4 + (4*cosh(x) - 5)*sinh(
x)^3 + sinh(x)^4 - 5*cosh(x)^3 + (6*cosh(x)^2 - 15*cosh(x) + 10)*sinh(x)^2
+ 10*cosh(x)^2 + (4*cosh(x)^3 - 15*cosh(x)^2 + 20*cosh(x) - 9)*sinh(x) - 11
*cosh(x) + 5)
```

Sympy [**A**]
time = 0.66, size = 46, normalized size = 0.77

$$\frac{A}{4\tanh\left(\frac{x}{2}\right)} - \frac{A}{6\tanh^3\left(\frac{x}{2}\right)} + \frac{A}{20\tanh^5\left(\frac{x}{2}\right)} - \frac{B}{4\tanh\left(\frac{x}{2}\right)} + \frac{B}{20\tanh^5\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x))**3,x)
```

```
[Out] A/(4*tanh(x/2)) - A/(6*tanh(x/2)**3) + A/(20*tanh(x/2)**5) - B/(4*tanh(x/2)
) + B/(20*tanh(x/2)**5)
```

Giac [**A**]
time = 0.40, size = 46, normalized size = 0.77

$$\frac{2\left(15\,Be^{(3\,x)} + 20\,Ae^{(2\,x)} - 15\,Be^{(2\,x)} - 10\,Ae^{x} + 15\,Be^{x} + 2\,A - 3\,B\right)}{15\,(e^{x} - 1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x))^3,x, algorithm="giac")
```

```
[Out] 2/15*(15*B*e^(3*x) + 20*A*e^(2*x) - 15*B*e^(2*x) - 10*A*e^x + 15*B*e^x + 2*
A - 3*B)/(e^x - 1)^5
```

Mupad [B]
time = 0.08, size = 143, normalized size = 2.38

$$\frac{\frac{B}{5}+\frac{4\,A\,{\mathrm{e}}^x}{5}+\frac{3\,B\,{\mathrm{e}}^{2\,x}}{5}}{6\,{\mathrm{e}}^{2\,x}-4\,{\mathrm{e}}^{3\,x}+{\mathrm{e}}^{4\,x}-4\,{\mathrm{e}}^x+1}-\frac{\frac{4\,A}{15}+\frac{2\,B\,{\mathrm{e}}^x}{5}}{3\,{\mathrm{e}}^{2\,x}-{\mathrm{e}}^{3\,x}-3\,{\mathrm{e}}^x+1}-\frac{\frac{4\,B\,{\mathrm{e}}^x}{5}+\frac{8\,A\,{\mathrm{e}}^{2\,x}}{5}+\frac{4\,B\,{\mathrm{e}}^{3\,x}}{5}}{10\,{\mathrm{e}}^{2\,x}-10\,{\mathrm{e}}^{3\,x}+5\,{\mathrm{e}}^{4\,x}-{\mathrm{e}}^{5\,x}-5\,{\mathrm{e}}^x+1}+\frac{B}{5\,\left({\mathrm{e}}^{2\,x}-2\,{\mathrm{e}}^x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(A + B*cosh(x))/(cosh(x) - 1)^3,x)
```

```
[Out] (B/5 + (4*A*exp(x))/5 + (3*B*exp(2*x))/5)/(6*exp(2*x) - 4*exp(3*x) + exp(4*
x) - 4*exp(x) + 1) - ((4*A)/15 + (2*B*exp(x))/5)/(3*exp(2*x) - exp(3*x) - 3
*exp(x) + 1) - ((4*B*exp(x))/5 + (8*A*exp(2*x))/5 + (4*B*exp(3*x))/5)/(10*e
xp(2*x) - 10*exp(3*x) + 5*exp(4*x) - exp(5*x) - 5*exp(x) + 1) + B/(5*(exp(2
*x) - 2*exp(x) + 1))
```

3.100 $\int \frac{A+B\cosh(x)}{(1-\cosh(x))^4}\,dx$

Optimal. Leaf size=81

$$-\frac{(A+B)\sinh(x)}{7(1-\cosh(x))^4}-\frac{(3A-4B)\sinh(x)}{35(1-\cosh(x))^3}-\frac{2(3A-4B)\sinh(x)}{105(1-\cosh(x))^2}-\frac{2(3A-4B)\sinh(x)}{105(1-\cosh(x))}$$

```
[Out] -1/7*(A+B)*sinh(x)/(1-cosh(x))^4-1/35*(3*A-4*B)*sinh(x)/(1-cosh(x))^3-2/105
*(3*A-4*B)*sinh(x)/(1-cosh(x))^2-2/105*(3*A-4*B)*sinh(x)/(1-cosh(x))
```

Rubi [A]
time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2829, 2729, 2727}

$$-\frac{2(3A-4B)\sinh(x)}{105(1-\cosh(x))}-\frac{2(3A-4B)\sinh(x)}{105(1-\cosh(x))^2}-\frac{(3A-4B)\sinh(x)}{35(1-\cosh(x))^3}-\frac{(A+B)\sinh(x)}{7(1-\cosh(x))^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(1 - Cosh[x])^4,x]
```

```
[Out] -1/7*((A + B)*Sinh[x])/(1 - Cosh[x])^4 - ((3*A - 4*B)*Sinh[x])/(35*(1 - Cos
h[x])^3) - (2*(3*A - 4*B)*Sinh[x])/(105*(1 - Cosh[x])^2) - (2*(3*A - 4*B)*S
inh[x])/(105*(1 - Cosh[x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+B\cosh(x)}{(1-\cosh(x))^4}\,dx &= -\frac{(A+B)\sinh(x)}{7(1-\cosh(x))^4}+\frac{1}{7}(3A-4B)\int\frac{1}{(1-\cosh(x))^3}\,dx\\
&= -\frac{(A+B)\sinh(x)}{7(1-\cosh(x))^4}-\frac{(3A-4B)\sinh(x)}{35(1-\cosh(x))^3}+\frac{1}{35}(2(3A-4B))\int\frac{1}{(1-\cosh(x))^2}\,dx\\
&= -\frac{(A+B)\sinh(x)}{7(1-\cosh(x))^4}-\frac{(3A-4B)\sinh(x)}{35(1-\cosh(x))^3}-\frac{2(3A-4B)\sinh(x)}{105(1-\cosh(x))^2}+\frac{1}{105}(2(3A-4B))\\
&= -\frac{(A+B)\sinh(x)}{7(1-\cosh(x))^4}-\frac{(3A-4B)\sinh(x)}{35(1-\cosh(x))^3}-\frac{2(3A-4B)\sinh(x)}{105(1-\cosh(x))^2}-\frac{2(3A-4B)\sinh(}{105(1-\cosh(x)}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 57, normalized size = 0.70

$$\frac{(-96A+58B+29(3A-4B)\cosh(x)-8(3A-4B)\cosh(2x)+3A\cosh(3x)-4B\cosh(3x))\sinh(x)}{210(-1+\cosh(x))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(1 - Cosh[x])^4,x]
```

```
[Out] ((-96*A + 58*B + 29*(3*A - 4*B)*Cosh[x] - 8*(3*A - 4*B)*Cosh[2*x] + 3*A*Cos
h[3*x] - 4*B*Cosh[3*x])*Sinh[x])/(210*(-1 + Cosh[x])^4)
```

Maple [A]
time = 0.43, size = 56, normalized size = 0.69

method	result	size
default	$-\frac{3A-B}{24\tanh\left(\frac{x}{2}\right)^{3}}-\frac{-A+B}{8\tanh\left(\frac{x}{2}\right)}-\frac{-3A-B}{40\tanh\left(\frac{x}{2}\right)^{5}}-\frac{A+B}{56\tanh\left(\frac{x}{2}\right)^{7}}$	56
risch	$-\frac{4\left(70B\,\mathrm{e}^{4x}+105A\,\mathrm{e}^{3x}-70B\,\mathrm{e}^{3x}-63A\,\mathrm{e}^{2x}+84B\,\mathrm{e}^{2x}+21A\,\mathrm{e}^{x}-28B\,\mathrm{e}^{x}-3A+4B\right)}{105\left(\mathrm{e}^{x}-1\right)^{7}}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(1-cosh(x))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(3*A-B)/tanh(1/2*x)^3-1/8*(-A+B)/tanh(1/2*x)-1/40*(-3*A-B)/tanh(1/2*x
)^5-1/56*(A+B)/tanh(1/2*x)^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 451 vs. 2(65) = 130.
time = 0.28, size = 451, normalized size = 5.57

$$-\frac{8}{105}B\left(\frac{14e^{(-x)}}{7e^{(-x)}-21e^{(-2x)}+35e^{(-3x)}-35e^{(-4x)}+21e^{(-5x)}-7e^{(-6x)}+e^{(-7x)}-1}-\frac{42e^{(-2x)}}{7e^{(-x)}-21e^{(-2x)}+35e^{(-3x)}-35e^{(-4x)}+21e^{(-5x)}-7e^{(-6x)}+e^{(-7x)}-1}+\frac{35e^{(-3x)}}{7e^{(-x)}-21e^{(-2x)}+35e^{(-3x)}-35e^{(-4x)}+21e^{(-5x)}-7e^{(-6x)}+e^{(-7x)}-1}-\frac{35e^{(-4x)}}{7e^{(-x)}-21e^{(-2x)}+35e^{(-3x)}-35e^{(-4x)}+21e^{(-5x)}-7e^{(-6x)}+e^{(-7x)}-1}-\frac{2}{7e^{(-x)}-21e^{(-2x)}+35e^{(-3x)}-35e^{(-4x)}+21e^{(-5x)}-7e^{(-6x)}+e^{(-7x)}-1}\right)+\frac{4}{35}A\left(\frac{7e^{(-x)}}{7e^{(-x)}-21e^{(-2x)}+35e^{(-3x)}-35e^{(-4x)}+21e^{(-5x)}-7e^{(-6x)}+e^{(-7x)}-1}-\frac{21e^{(-2x)}}{7e^{(-x)}-21e^{(-2x)}+35e^{(-3x)}-35e^{(-4x)}+21e^{(-5x)}-7e^{(-6x)}+e^{(-7x)}-1}+\frac{35e^{(-3x)}}{7e^{(-x)}-21e^{(-2x)}+35e^{(-3x)}-35e^{(-4x)}+21e^{(-5x)}-7e^{(-6x)}+e^{(-7x)}-1}-\frac{1}{7e^{(-x)}-21e^{(-2x)}+35e^{(-3x)}-35e^{(-4x)}+21e^{(-5x)}-7e^{(-6x)}+e^{(-7x)}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x))^4,x, algorithm="maxima")
```

```
[Out] -8/105*B*(14*e^(-x)/(7*e^(-x) - 21*e^(-2*x) + 35*e^(-3*x) - 35*e^(-4*x) + 2
1*e^(-5*x) - 7*e^(-6*x) + e^(-7*x) - 1) - 42*e^(-2*x)/(7*e^(-x) - 21*e^(-2*
x) + 35*e^(-3*x) - 35*e^(-4*x) + 21*e^(-5*x) - 7*e^(-6*x) + e^(-7*x) - 1) +
 35*e^(-3*x)/(7*e^(-x) - 21*e^(-2*x) + 35*e^(-3*x) - 35*e^(-4*x) + 21*e^(-5
*x) - 7*e^(-6*x) + e^(-7*x) - 1) - 35*e^(-4*x)/(7*e^(-x) - 21*e^(-2*x) + 35
*e^(-3*x) - 35*e^(-4*x) + 21*e^(-5*x) - 7*e^(-6*x) + e^(-7*x) - 1) - 2/(7*e
^(-x) - 21*e^(-2*x) + 35*e^(-3*x) - 35*e^(-4*x) + 21*e^(-5*x) - 7*e^(-6*x)
+ e^(-7*x) - 1)) + 4/35*A*(7*e^(-x)/(7*e^(-x) - 21*e^(-2*x) + 35*e^(-3*x) -
 35*e^(-4*x) + 21*e^(-5*x) - 7*e^(-6*x) + e^(-7*x) - 1) - 21*e^(-2*x)/(7*e^
(-x) - 21*e^(-2*x) + 35*e^(-3*x) - 35*e^(-4*x) + 21*e^(-5*x) - 7*e^(-6*x) +
 e^(-7*x) - 1) + 35*e^(-3*x)/(7*e^(-x) - 21*e^(-2*x) + 35*e^(-3*x) - 35*e^(
-4*x) + 21*e^(-5*x) - 7*e^(-6*x) + e^(-7*x) - 1) - 1/(7*e^(-x) - 21*e^(-2*x
) + 35*e^(-3*x) - 35*e^(-4*x) + 21*e^(-5*x) - 7*e^(-6*x) + e^(-7*x) - 1))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(65) = 130$.
time = 0.41, size = 175, normalized size = 2.16

$$\frac{4\left((3A-74B)\cosh(x)^2+(3A-74B)\sinh(x)^2-14(9A-7B)\cosh(x)-6((A+22B)\cosh(x)+14A-7B)\sinh(x)+63A-84B\right)}{105\left(\cosh(x)^5+(5\cosh(x)-7)\sinh(x)^4+\sinh(x)^5-7\cosh(x)^4+\left(10\cosh(x)^2-28\cosh(x)+21\right)\sinh(x)^3+21\cosh(x)^3+\left(10\cosh(x)^3-42\cosh(x)^2+63\cosh(x)-36\right)\sinh(x)^2-36\cosh(x)^2+\left(5\cosh(x)^4-28\cosh(x)^3+63\cosh(x)^2-68\cosh(x)+28\right)\sinh(x)+42\cosh(x)-21\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x))^4,x, algorithm="fricas")
```

```
[Out] 4/105*((3*A - 74*B)*cosh(x)^2 + (3*A - 74*B)*sinh(x)^2 - 14*(9*A - 7*B)*cos
h(x) - 6*((A + 22*B)*cosh(x) + 14*A - 7*B)*sinh(x) + 63*A - 84*B)/(cosh(x)^
5 + (5*cosh(x) - 7)*sinh(x)^4 + sinh(x)^5 - 7*cosh(x)^4 + (10*cosh(x)^2 - 2
8*cosh(x) + 21)*sinh(x)^3 + 21*cosh(x)^3 + (10*cosh(x)^3 - 42*cosh(x)^2 + 6
3*cosh(x) - 36)*sinh(x)^2 - 36*cosh(x)^2 + (5*cosh(x)^4 - 28*cosh(x)^3 + 63
*cosh(x)^2 - 68*cosh(x) + 28)*sinh(x) + 42*cosh(x) - 21)
```

Sympy [**A**]
time = 1.19, size = 78, normalized size = 0.96

$$\frac{A}{8\tanh\left(\frac{x}{2}\right)}-\frac{A}{8\tanh^3\left(\frac{x}{2}\right)}+\frac{3A}{40\tanh^5\left(\frac{x}{2}\right)}-\frac{A}{56\tanh^7\left(\frac{x}{2}\right)}-\frac{B}{8\tanh\left(\frac{x}{2}\right)}+\frac{B}{24\tanh^3\left(\frac{x}{2}\right)}+\frac{B}{40\tanh^5\left(\frac{x}{2}\right)}-\frac{B}{56\tanh^7\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x))**4,x)
```

```
[Out] A/(8*tanh(x/2)) - A/(8*tanh(x/2)**3) + 3*A/(40*tanh(x/2)**5) - A/(56*tanh(x
/2)**7) - B/(8*tanh(x/2)) + B/(24*tanh(x/2)**3) + B/(40*tanh(x/2)**5) - B/(
56*tanh(x/2)**7)
```

Giac [A]
time = 0.42, size = 60, normalized size = 0.74

$$-\frac{4\left(70\,Be^{(4\,x)}+105\,Ae^{(3\,x)}-70\,Be^{(3\,x)}-63\,Ae^{(2\,x)}+84\,Be^{(2\,x)}+21\,Ae^{x}-28\,Be^{x}-3\,A+4\,B\right)}{105\,(e^{x}-1)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x))^4,x, algorithm="giac")
```

```
[Out] -4/105*(70*B*e^(4*x) + 105*A*e^(3*x) - 70*B*e^(3*x) - 63*A*e^(2*x) + 84*B*e
^(2*x) + 21*A*e^x - 28*B*e^x - 3*A + 4*B)/(e^x - 1)^7
```

Mupad [B]
time = 0.92, size = 233, normalized size = 2.88

$$\frac{\frac{8\,B}{105}+\frac{16\,A\,e^{x}}{35}+\frac{16\,B\,e^{2\,x}}{35}}{10\,e^{2\,x}-10\,e^{3\,x}+5\,e^{4\,x}-e^{5\,x}-5\,e^{x}+1}-\frac{\frac{4\,A}{35}+\frac{8\,B\,e^{x}}{35}}{6\,e^{2\,x}-4\,e^{3\,x}+e^{4\,x}-4\,e^{x}+1}-\frac{\frac{8\,B\,e^{x}}{21}+\frac{8\,A\,e^{2\,x}}{7}+\frac{16\,B\,e^{3\,x}}{21}}{15\,e^{2\,x}-20\,e^{3\,x}+15\,e^{4\,x}-6\,e^{5\,x}+e^{6\,x}-6\,e^{x}+1}+\frac{8\,B}{105\,\left(3\,e^{2\,x}-e^{3\,x}-3\,e^{x}+1\right)}+\frac{\frac{16\,A\,e^{3\,x}}{7}+\frac{8\,B\,e^{2\,x}}{7}+\frac{8\,B\,e^{4\,x}}{7}}{21\,e^{2\,x}-35\,e^{3\,x}+35\,e^{4\,x}-21\,e^{5\,x}+7\,e^{6\,x}-e^{7\,x}-7\,e^{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(cosh(x) - 1)^4,x)
```

```
[Out] ((8*B)/105 + (16*A*exp(x))/35 + (16*B*exp(2*x))/35)/(10*exp(2*x) - 10*exp(3
*x) + 5*exp(4*x) - exp(5*x) - 5*exp(x) + 1) - ((4*A)/35 + (8*B*exp(x))/35)/
(6*exp(2*x) - 4*exp(3*x) + exp(4*x) - 4*exp(x) + 1) - ((8*B*exp(x))/21 + (8
*A*exp(2*x))/7 + (16*B*exp(3*x))/21)/(15*exp(2*x) - 20*exp(3*x) + 15*exp(4*
x) - 6*exp(5*x) + exp(6*x) - 6*exp(x) + 1) + (8*B)/(105*(3*exp(2*x) - exp(3
*x) - 3*exp(x) + 1)) + ((16*A*exp(3*x))/7 + (8*B*exp(2*x))/7 + (8*B*exp(4*x
))/7)/(21*exp(2*x) - 35*exp(3*x) + 35*exp(4*x) - 21*exp(5*x) + 7*exp(6*x) -
 exp(7*x) - 7*exp(x) + 1)
```

3.101 $\int \frac{A+B\cosh(x)}{\sqrt{a+a\cosh(x)}}\,dx$

Optimal. Leaf size=56

$$\frac{\sqrt{2}\,(A-B)\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\sinh(x)}{\sqrt{2}\,\sqrt{a+a\cosh(x)}}\right)}{\sqrt{a}}+\frac{2B\sinh(x)}{\sqrt{a+a\cosh(x)}}$$

```
[Out] (A-B)*arctan(1/2*sinh(x)*a^(1/2)*2^(1/2)/(a+a*cosh(x))^(1/2))*2^(1/2)/a^(1/
2)+2*B*sinh(x)/(a+a*cosh(x))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2830, 2728, 212}

$$\frac{\sqrt{2}\,(A-B)\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\sinh(x)}{\sqrt{2}\,\sqrt{a\cosh(x)+a}}\right)}{\sqrt{a}}+\frac{2B\sinh(x)}{\sqrt{a\cosh(x)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/Sqrt[a + a*Cosh[x]],x]
```

```
[Out] (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sinh[x])/(Sqrt[2]*Sqrt[a + a*Cosh[x]])])/S
qrt[a] + (2*B*Sinh[x])/Sqrt[a + a*Cosh[x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
```

```
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cosh(x)}{\sqrt{a+a\cosh(x)}}\,dx &= \frac{2B\sinh(x)}{\sqrt{a+a\cosh(x)}}+(A-B)\int \frac{1}{\sqrt{a+a\cosh(x)}}\,dx\\ &= \frac{2B\sinh(x)}{\sqrt{a+a\cosh(x)}}+(2i(A-B))\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,-\frac{ia\sinh(x)}{\sqrt{a+a\cosh(x)}}\right)\\ &= \frac{\sqrt{2}\,(A-B)\tan^{-1}\left(\frac{\sqrt{a}\,\sinh(x)}{\sqrt{2}\,\sqrt{a+a\cosh(x)}}\right)}{\sqrt{a}}+\frac{2B\sinh(x)}{\sqrt{a+a\cosh(x)}}\end{aligned}$$

Mathematica [**A**]
time = 0.03, size = 41, normalized size = 0.73

$$\frac{2\cosh\left(\frac{x}{2}\right)\left((A-B)\mathrm{ArcTan}\left(\sinh\left(\frac{x}{2}\right)\right)+2B\sinh\left(\frac{x}{2}\right)\right)}{\sqrt{a(1+\cosh(x))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/Sqrt[a + a*Cosh[x]],x]
```

```
[Out] (2*Cosh[x/2]*((A - B)*ArcTan[Sinh[x/2]] + 2*B*Sinh[x/2]))/Sqrt[a*(1 + Cosh[
x])]
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(45) = 90$.
time = 1.31, size = 128, normalized size = 2.29

method	result
default	$-\frac{\cosh\left(\frac{x}{2}\right)\sqrt{\left(\sinh^2\left(\frac{x}{2}\right)\right)a}\left(\ln\left(\frac{2\sqrt{-a}\,\sqrt{\left(\sinh^2\left(\frac{x}{2}\right)\right)a}\,-2a}{\cosh\left(\frac{x}{2}\right)}\right)aA-2B\sqrt{\left(\sinh^2\left(\frac{x}{2}\right)\right)a}\,\sqrt{-a}\,-\ln\left(\frac{2\sqrt{-a}}{}\right.\right.}{a\sqrt{-a}\,\sinh\left(\frac{x}{2}\right)\sqrt{a\left(\cosh^2\left(\frac{x}{2}\right)\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a+a*cosh(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -cosh(1/2*x)*(sinh(1/2*x)^2*a)^(1/2)*(ln(2/cosh(1/2*x)*((-a)^(1/2)*(sinh(1/
2*x)^2*a)^(1/2)-a))*a*A-2*B*(sinh(1/2*x)^2*a)^(1/2)*(-a)^(1/2)-ln(2/cosh(1/
```

```
2*x)*((-a)^(1/2)*(sinh(1/2*x)^2*a)^(1/2)-a))*a*B)/a/(-a)^(1/2)/sinh(1/2*x)*
2^(1/2)/(a*cosh(1/2*x)^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(45) = 90$.
time = 0.56, size = 174, normalized size = 3.11

$$2\left(\sqrt{2}\left(\frac{\arctan\left(e^{\left(\frac{1}{2}x\right)}\right)}{\sqrt{a}}+\frac{e^{\left(\frac{1}{2}x\right)}}{\sqrt{a}\,e^{x}+\sqrt{a}}\right)-\frac{\sqrt{2}\,e^{\left(\frac{1}{2}x\right)}}{\sqrt{a}\,e^{x}+\sqrt{a}}\right)A-\frac{1}{3}\left(3\sqrt{2}\left(\frac{\arctan\left(e^{\left(\frac{1}{2}x\right)}\right)}{\sqrt{a}}-\frac{e^{\left(\frac{1}{2}x\right)}}{\sqrt{a}\,e^{x}+\sqrt{a}}\right)-\sqrt{2}\left(\frac{3\arctan\left(e^{\left(-\frac{1}{2}x\right)}\right)}{\sqrt{a}}-\frac{2\,e^{\left(-\frac{1}{2}x\right)}}{\sqrt{a}}-\frac{e^{\left(-\frac{1}{2}x\right)}}{\sqrt{a}\,e^{\left(-x\right)}+\sqrt{a}}\right)-\frac{3\sqrt{2}\sqrt{a}\,e^{\left(\frac{3}{2}x\right)}-\sqrt{2}\sqrt{a}\,e^{\left(-\frac{1}{2}x\right)}}{ae^{x}+a}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+a*cosh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*(sqrt(2)*(arctan(e^(1/2*x))/sqrt(a) + e^(1/2*x)/(sqrt(a)*e^x + sqrt(a)))
- sqrt(2)*e^(1/2*x)/(sqrt(a)*e^x + sqrt(a)))*A - 1/3*(3*sqrt(2)*(arctan(e^(
1/2*x))/sqrt(a) - e^(1/2*x)/(sqrt(a)*e^x + sqrt(a))) - sqrt(2)*(3*arctan(e^
(-1/2*x))/sqrt(a) - 2*e^(-1/2*x)/sqrt(a) - e^(-1/2*x)/(sqrt(a)*e^(-x) + sqr
t(a))) - (3*sqrt(2)*sqrt(a)*e^(3/2*x) - sqrt(2)*sqrt(a)*e^(-1/2*x))/(a*e^x
+ a))*B
```

Fricas [A]
time = 0.44, size = 72, normalized size = 1.29

$$\frac{2\left(\sqrt{2}\,(A-B)\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{\cosh\left(x\right)+\sinh\left(x\right)}}\,(\cosh(x)+\sinh(x))}{\sqrt{a}}\right)+\sqrt{\frac{1}{2}}\,(B\cosh\left(x\right)+B\sinh\left(x\right)-B)\sqrt{\frac{a}{\cosh\left(x\right)+\sinh\left(x\right)}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+a*cosh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(2)*(A - B)*sqrt(a)*arctan(sqrt(2)*sqrt(1/2)*sqrt(a/(cosh(x) + sinh(
x)))*(cosh(x) + sinh(x))/sqrt(a)) + sqrt(1/2)*(B*cosh(x) + B*sinh(x) - B)*s
qrt(a/(cosh(x) + sinh(x))))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\cosh\left(x\right)}{\sqrt{a\left(\cosh\left(x\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+a*cosh(x))**(1/2),x)
```

```
[Out] Integral((A + B*cosh(x))/sqrt(a*(cosh(x) + 1)), x)
```

Giac [**A**]
time = 0.42, size = 44, normalized size = 0.79

$$\frac{2\sqrt{2}(A-B)\arctan\left(e^{\left(\frac{1}{2}x\right)}\right)}{\sqrt{a}}+\frac{\sqrt{2}Be^{\left(\frac{1}{2}x\right)}}{\sqrt{a}}-\frac{\sqrt{2}Be^{\left(-\frac{1}{2}x\right)}}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+a*cosh(x))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(2)*(A - B)*arctan(e^(1/2*x))/sqrt(a) + sqrt(2)*B*e^(1/2*x)/sqrt(a) -
 sqrt(2)*B*e^(-1/2*x)/sqrt(a)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{A+B\cosh(x)}{\sqrt{a+a\cosh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(a + a*cosh(x))^(1/2),x)
```

```
[Out] int((A + B*cosh(x))/(a + a*cosh(x))^(1/2), x)
```

3.102 $\int \frac{A+B\cosh(x)}{(a+a\cosh(x))^{3/2}}\,dx$

Optimal. Leaf size=65

$$\frac{(A+3B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sinh(x)}{\sqrt{2}\,\sqrt{a+a\cosh(x)}}\right)}{2\sqrt{2}\,a^{3/2}}+\frac{(A-B)\sinh(x)}{2(a+a\cosh(x))^{3/2}}$$

```
[Out] 1/2*(A-B)*sinh(x)/(a+a*cosh(x))^(3/2)+1/4*(A+3*B)*arctan(1/2*sinh(x)*a^(1/2
)*2^(1/2)/(a+a*cosh(x))^(1/2))/a^(3/2)*2^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2829, 2728, 212}

$$\frac{(A+3B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sinh(x)}{\sqrt{2}\,\sqrt{a\cosh(x)+a}}\right)}{2\sqrt{2}\,a^{3/2}}+\frac{(A-B)\sinh(x)}{2(a\cosh(x)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(a + a*Cosh[x])^(3/2),x]
```

```
[Out] ((A + 3*B)*ArcTan[(Sqrt[a]*Sinh[x])/(Sqrt[2]*Sqrt[a + a*Cosh[x]])])/(2*Sqrt
[2]*a^(3/2)) + ((A - B)*Sinh[x])/(2*(a + a*Cosh[x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{A+B\cosh(x)}{(a+a\cosh(x))^{3/2}}\,dx = \frac{(A-B)\sinh(x)}{2(a+a\cosh(x))^{3/2}} + \frac{(A+3B)\int \frac{1}{\sqrt{a+a\cosh(x)}}\,dx}{4a}$$

$$= \frac{(A-B)\sinh(x)}{2(a+a\cosh(x))^{3/2}} + \frac{(i(A+3B))\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{ia\sinh(x)}{\sqrt{a+a\cosh(x)}}\right)}{2a}$$

$$= \frac{(A+3B)\tan^{-1}\left(\frac{\sqrt{a}\,\sinh(x)}{\sqrt{2}\,\sqrt{a+a\cosh(x)}}\right)}{2\sqrt{2}\,a^{3/2}} + \frac{(A-B)\sinh(x)}{2(a+a\cosh(x))^{3/2}}$$

Mathematica [**A**]
time = 0.06, size = 44, normalized size = 0.68

$$\frac{(A+3B)\text{ArcTan}\left(\sinh\left(\frac{x}{2}\right)\right)\cosh^3\left(\frac{x}{2}\right) + \frac{1}{2}(A-B)\sinh(x)}{(a(1+\cosh(x)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(a + a*Cosh[x])^(3/2),x]
```

```
[Out] ((A + 3*B)*ArcTan[Sinh[x/2]]*Cosh[x/2]^3 + ((A - B)*Sinh[x])/2)/(a*(1 + Cos
h[x]))^(3/2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(50) = 100$.
time = 1.33, size = 159, normalized size = 2.45

method	result
default	$-\frac{\sqrt{\left(\sinh^2\left(\frac{x}{2}\right)\right)a}\left(A\ln\left(\frac{2\sqrt{-a}\,\sqrt{\left(\sinh^2\left(\frac{x}{2}\right)\right)a}-2a}{\cosh\left(\frac{x}{2}\right)}\right)\left(\cosh^2\left(\frac{x}{2}\right)\right)a+3B\ln\left(\frac{2\sqrt{-a}\,\sqrt{\left(\sinh^2\left(\frac{x}{2}\right)\right)a}-2a}{\cosh\left(\frac{x}{2}\right)}\right)\cdots}{4\cosh\left(\frac{x}{2}\right)a^2\sqrt{-a}\,\sinh\left(\frac{x}{2}\right)\sqrt{a\left(\cosh^2\left(\cdots\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a+a*cosh(x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(sinh(1/2*x)^2*a)^(1/2)*(A*ln(2/cosh(1/2*x)*((-a)^(1/2)*(sinh(1/2*x)^2
*a)^(1/2)-a))*cosh(1/2*x)^2*a+3*B*ln(2/cosh(1/2*x)*((-a)^(1/2)*(sinh(1/2*x)
^2*a)^(1/2)-a))*a*cosh(1/2*x)^2-A*(sinh(1/2*x)^2*a)^(1/2)*(-a)^(1/2)+B*(sin
```

```
h(1/2*x)^2*a)^(1/2)*(-a)^(1/2))/cosh(1/2*x)/a^2/(-a)^(1/2)/sinh(1/2*x)*2^(1
/2)/(a*cosh(1/2*x)^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(50) = 100$.
time = 0.57, size = 300, normalized size = 4.62

$$\frac{1}{6}\left(\sqrt{2}\left(\frac{3e^{(\frac{5}{2}x)}+8e^{(\frac{3}{2}x)}-3e^{(\frac{1}{2}x)}}{a^{\frac{3}{2}}e^{(3x)}+3a^{\frac{3}{2}}e^{(2x)}+3a^{\frac{3}{2}}e^{x}+a^{\frac{3}{2}}}+\frac{3\arctan\left(e^{(\frac{1}{2}x)}\right)}{a^{\frac{3}{2}}}\right)-\frac{8\sqrt{2}e^{(\frac{3}{2}x)}}{a^{\frac{3}{2}}e^{(3x)}+3a^{\frac{3}{2}}e^{(2x)}+3a^{\frac{3}{2}}e^{x}+a^{\frac{3}{2}}}\right)A+\frac{1}{20}\left(\sqrt{2}\left(\frac{15e^{(\frac{5}{2}x)}+40e^{(\frac{3}{2}x)}+33e^{(\frac{1}{2}x)}}{a^{\frac{3}{2}}e^{(3x)}+3a^{\frac{3}{2}}e^{(2x)}+3a^{\frac{3}{2}}e^{x}+a^{\frac{3}{2}}}+\frac{15\arctan\left(e^{(\frac{1}{2}x)}\right)}{a^{\frac{3}{2}}}\right)+5\sqrt{2}\left(\frac{3e^{(\frac{5}{2}x)}-8e^{(\frac{3}{2}x)}-3e^{(\frac{1}{2}x)}}{a^{\frac{3}{2}}e^{(3x)}+3a^{\frac{3}{2}}e^{(2x)}+3a^{\frac{3}{2}}e^{x}+a^{\frac{3}{2}}}+\frac{3\arctan\left(e^{(\frac{1}{2}x)}\right)}{a^{\frac{3}{2}}}\right)-\frac{8\left(5\sqrt{2}\sqrt{a}e^{(\frac{5}{2}x)}+\sqrt{2}\sqrt{a}e^{(\frac{1}{2}x)}\right)}{a^{2}e^{(3x)}+3a^{2}e^{(2x)}+3a^{2}e^{x}+a^{2}}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+a*cosh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*(sqrt(2)*((3*e^(5/2*x) + 8*e^(3/2*x) - 3*e^(1/2*x))/(a^(3/2)*e^(3*x) +
3*a^(3/2)*e^(2*x) + 3*a^(3/2)*e^x + a^(3/2)) + 3*arctan(e^(1/2*x))/a^(3/2))
 - 8*sqrt(2)*e^(3/2*x)/(a^(3/2)*e^(3*x) + 3*a^(3/2)*e^(2*x) + 3*a^(3/2)*e^x
 + a^(3/2)))*A + 1/20*(sqrt(2)*((15*e^(5/2*x) + 40*e^(3/2*x) + 33*e^(1/2*x)
)/(a^(3/2)*e^(3*x) + 3*a^(3/2)*e^(2*x) + 3*a^(3/2)*e^x + a^(3/2)) + 15*arct
an(e^(1/2*x))/a^(3/2)) + 5*sqrt(2)*((3*e^(5/2*x) - 8*e^(3/2*x) - 3*e^(1/2*x
))/(a^(3/2)*e^(3*x) + 3*a^(3/2)*e^(2*x) + 3*a^(3/2)*e^x + a^(3/2)) + 3*arct
an(e^(1/2*x))/a^(3/2)) - 8*(5*sqrt(2)*sqrt(a)*e^(5/2*x) + sqrt(2)*sqrt(a)*e
^(1/2*x))/(a^2*e^(3*x) + 3*a^2*e^(2*x) + 3*a^2*e^x + a^2))*B
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(50) = 100$.
time = 0.44, size = 192, normalized size = 2.95

$$-\frac{\sqrt{2}\left((A+3B)\cosh(x)^{2}+(A+3B)\sinh(x)^{2}+2(A+3B)\cosh(x)+2((A+3B)\cosh(x)+A+3B)\sinh(x)+A+3B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{a}\sqrt{\frac{a}{\cosh(x)+\sinh(x)}}}{a}\right)-2\sqrt{\frac{1}{2}}\left((A-B)\cosh(x)^{2}+(A-B)\sinh(x)^{2}-(A-B)\cosh(x)+(2(A-B)\cosh(x)-A+B)\sinh(x)\right)\sqrt{\frac{a}{\cosh(x)+\sinh(x)}}}{2\left(a^{2}\cosh(x)^{2}+a^{2}\sinh(x)^{2}+2a^{2}\cosh(x)+a^{2}+2\left(a^{2}\cosh(x)+a^{2}\right)\sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+a*cosh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*((A + 3*B)*cosh(x)^2 + (A + 3*B)*sinh(x)^2 + 2*(A + 3*B)*cosh
(x) + 2*((A + 3*B)*cosh(x) + A + 3*B)*sinh(x) + A + 3*B)*sqrt(a)*arctan(sqr
t(2)*sqrt(1/2)*sqrt(a)*sqrt(a/(cosh(x) + sinh(x)))/a) - 2*sqrt(1/2)*((A - B
)*cosh(x)^2 + (A - B)*sinh(x)^2 - (A - B)*cosh(x) + (2*(A - B)*cosh(x) - A
+ B)*sinh(x))*sqrt(a/(cosh(x) + sinh(x))))/(a^2*cosh(x)^2 + a^2*sinh(x)^2 +
 2*a^2*cosh(x) + a^2 + 2*(a^2*cosh(x) + a^2)*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\cosh(x)}{(a(\cosh(x)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+a*cosh(x))**(3/2),x)
```

```
[Out] Integral((A + B*cosh(x))/(a*(cosh(x) + 1))**(3/2), x)
```

Giac [A]
time = 0.42, size = 78, normalized size = 1.20

$$\frac{\left(\sqrt{2}\,A+3\,\sqrt{2}\,B\right)\arctan\left(e^{\left(\frac{1}{2}\,x\right)}\right)}{2\,a^{\frac{3}{2}}}+\frac{\sqrt{2}\,\left(Aa^{\frac{3}{2}}e^{\left(\frac{3}{2}\,x\right)}-Ba^{\frac{3}{2}}e^{\left(\frac{3}{2}\,x\right)}-Aa^{\frac{3}{2}}e^{\left(\frac{1}{2}\,x\right)}+Ba^{\frac{3}{2}}e^{\left(\frac{1}{2}\,x\right)}\right)}{2\,(ae^{x}+a)^{2}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+a*cosh(x))^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(2)*A + 3*sqrt(2)*B)*arctan(e^(1/2*x))/a^(3/2) + 1/2*sqrt(2)*(A*a^
(3/2)*e^(3/2*x) - B*a^(3/2)*e^(3/2*x) - A*a^(3/2)*e^(1/2*x) + B*a^(3/2)*e^(
1/2*x))/((a*e^x + a)^2*a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{A+B\cosh(x)}{\left(a+a\cosh\left(x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(a + a*cosh(x))^(3/2),x)
```

```
[Out] int((A + B*cosh(x))/(a + a*cosh(x))^(3/2), x)
```

3.103 $\int \frac{A+B\cosh(x)}{(a+a\cosh(x))^{5/2}}\,dx$

Optimal. Leaf size=93

$$\frac{(3A+5B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sinh(x)}{\sqrt{2}\,\sqrt{a+a\cosh(x)}}\right)}{16\sqrt{2}\,a^{5/2}}+\frac{(A-B)\sinh(x)}{4(a+a\cosh(x))^{5/2}}+\frac{(3A+5B)\sinh(x)}{16a(a+a\cosh(x))^{3/2}}$$

```
[Out] 1/4*(A-B)*sinh(x)/(a+a*cosh(x))^(5/2)+1/16*(3*A+5*B)*sinh(x)/a/(a+a*cosh(x)
)^(3/2)+1/32*(3*A+5*B)*arctan(1/2*sinh(x)*a^(1/2)*2^(1/2)/(a+a*cosh(x))^(1/
2))/a^(5/2)*2^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2829, 2729, 2728, 212}

$$\frac{(3A+5B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sinh(x)}{\sqrt{2}\,\sqrt{a\cosh(x)+a}}\right)}{16\sqrt{2}\,a^{5/2}}+\frac{(3A+5B)\sinh(x)}{16a(a\cosh(x)+a)^{3/2}}+\frac{(A-B)\sinh(x)}{4(a\cosh(x)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(a + a*Cosh[x])^(5/2),x]
```

```
[Out] ((3*A + 5*B)*ArcTan[(Sqrt[a]*Sinh[x])/(Sqrt[2]*Sqrt[a + a*Cosh[x]])])/(16*S
qrt[2]*a^(5/2)) + ((A - B)*Sinh[x])/(4*(a + a*Cosh[x])^(5/2)) + ((3*A + 5*B
)*Sinh[x])/(16*a*(a + a*Cosh[x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{A+B\cosh(x)}{(a+a\cosh(x))^{5/2}}\,dx = \frac{(A-B)\sinh(x)}{4(a+a\cosh(x))^{5/2}} + \frac{(3A+5B)\int \frac{1}{(a+a\cosh(x))^{3/2}}\,dx}{8a}$$

$$= \frac{(A-B)\sinh(x)}{4(a+a\cosh(x))^{5/2}} + \frac{(3A+5B)\sinh(x)}{16a(a+a\cosh(x))^{3/2}} + \frac{(3A+5B)\int \frac{1}{\sqrt{a+a\cosh(x)}}\,dx}{32a^2}$$

$$= \frac{(A-B)\sinh(x)}{4(a+a\cosh(x))^{5/2}} + \frac{(3A+5B)\sinh(x)}{16a(a+a\cosh(x))^{3/2}} + \frac{(i(3A+5B))\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x$$

$$16a^2$$

$$= \frac{(3A+5B)\tan^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a+a\cosh(x)}}\right)}{16\sqrt{2}\,a^{5/2}} + \frac{(A-B)\sinh(x)}{4(a+a\cosh(x))^{5/2}} + \frac{(3A+5B)\,\mathrm{s}}{16a(a+a\cos}$$

Mathematica [A]
time = 0.12, size = 57, normalized size = 0.61

$$\frac{4(3A+5B)\text{ArcTan}\left(\sinh\left(\frac{x}{2}\right)\right)\cosh^5\left(\frac{x}{2}\right)+(7A+B+(3A+5B)\cosh(x))\sinh(x)}{16(a(1+\cosh(x)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(a + a*Cosh[x])^(5/2),x]
```

```
[Out] (4*(3*A + 5*B)*ArcTan[Sinh[x/2]]*Cosh[x/2]^5 + (7*A + B + (3*A + 5*B)*Cosh[
x])*Sinh[x])/(16*(a*(1 + Cosh[x]))^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 208 vs. 2(74) = 148.
time = 1.36, size = 209, normalized size = 2.25

method	result

default	$-\frac{\sqrt{\left(\sinh^2\left(\frac{x}{2}\right)\right) a}\,\left(3A\ln\left(\frac{2\sqrt{-a}\,\sqrt{\left(\sinh^2\left(\frac{x}{2}\right)\right) a}-2a}{\cosh\left(\frac{x}{2}\right)}\right)\left(\cosh^4\left(\frac{x}{2}\right)\right)a+5B\ln\left(\frac{2\sqrt{-a}\,\sqrt{\left(\sinh^2\left(\frac{x}{2}\right)\right) a}-2a}{\cosh\left(\frac{x}{2}\right)}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a+a*cosh(x))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/32*(sinh(1/2*x)^2*a)^(1/2)*(3*A*ln(2/cosh(1/2*x)*((-a)^(1/2)*(sinh(1/2*x
)^2*a)^(1/2)-a))*cosh(1/2*x)^4*a+5*B*ln(2/cosh(1/2*x)*((-a)^(1/2)*(sinh(1/2
*x)^2*a)^(1/2)-a))*cosh(1/2*x)^4*a-3*A*(sinh(1/2*x)^2*a)^(1/2)*(-a)^(1/2)*c
osh(1/2*x)^2-5*B*(sinh(1/2*x)^2*a)^(1/2)*(-a)^(1/2)*cosh(1/2*x)^2-2*A*(sinh
(1/2*x)^2*a)^(1/2)*(-a)^(1/2)+2*B*(sinh(1/2*x)^2*a)^(1/2)*(-a)^(1/2))/cosh(
1/2*x)^3/a^3/(-a)^(1/2)/sinh(1/2*x)*2^(1/2)/(a*cosh(1/2*x)^2)^(1/2)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(74) = 148$.
time = 0.60, size = 427, normalized size = 4.59

$$\frac{1}{80}\left(\sqrt{2}\left(\frac{15e^{\left(\frac{9}{2}x\right)}+70e^{\left(\frac{7}{2}x\right)}+128e^{\left(\frac{5}{2}x\right)}-70e^{\left(\frac{3}{2}x\right)}-15e^{\left(\frac{1}{2}x\right)}}{a^{\frac{5}{2}}e^{(5x)}+5a^{\frac{5}{2}}e^{(4x)}+10a^{\frac{5}{2}}e^{(3x)}+10a^{\frac{5}{2}}e^{(2x)}+5a^{\frac{5}{2}}e^{x}+a^{\frac{5}{2}}}+\frac{15\arctan\left(e^{\left(\frac{1}{2}x\right)}\right)}{a^{\frac{5}{2}}}\right)-\frac{128\sqrt{2}e^{\left(\frac{5}{2}x\right)}}{a^{\frac{5}{2}}e^{(5x)}+5a^{\frac{5}{2}}e^{(4x)}+10a^{\frac{5}{2}}e^{(3x)}+10a^{\frac{5}{2}}e^{(2x)}+5a^{\frac{5}{2}}e^{x}+a^{\frac{5}{2}}}\right)A+\frac{1}{672}\left(\sqrt{2}\left(\frac{105e^{\left(\frac{9}{2}x\right)}+490e^{\left(\frac{7}{2}x\right)}+896e^{\left(\frac{5}{2}x\right)}+790e^{\left(\frac{3}{2}x\right)}-105e^{\left(\frac{1}{2}x\right)}}{a^{\frac{5}{2}}e^{(5x)}+5a^{\frac{5}{2}}e^{(4x)}+10a^{\frac{5}{2}}e^{(3x)}+10a^{\frac{5}{2}}e^{(2x)}+5a^{\frac{5}{2}}e^{x}+a^{\frac{5}{2}}}+\frac{105\arctan\left(e^{\left(\frac{1}{2}x\right)}\right)}{a^{\frac{5}{2}}}\right)+7\sqrt{2}\left(\frac{15e^{\left(\frac{9}{2}x\right)}+70e^{\left(\frac{7}{2}x\right)}-128e^{\left(\frac{5}{2}x\right)}-70e^{\left(\frac{3}{2}x\right)}-15e^{\left(\frac{1}{2}x\right)}}{a^{\frac{5}{2}}e^{(5x)}+5a^{\frac{5}{2}}e^{(4x)}+10a^{\frac{5}{2}}e^{(3x)}+10a^{\frac{5}{2}}e^{(2x)}+5a^{\frac{5}{2}}e^{x}+a^{\frac{5}{2}}}+\frac{15\arctan\left(e^{\left(\frac{1}{2}x\right)}\right)}{a^{\frac{5}{2}}}\right)-\frac{128\left(7\sqrt{2}\sqrt{a}e^{\left(\frac{7}{2}x\right)}+3\sqrt{2}\sqrt{a}e^{\left(\frac{3}{2}x\right)}\right)}{a^{3}e^{(5x)}+5a^{3}e^{(4x)}+10a^{3}e^{(3x)}+10a^{3}e^{(2x)}+5a^{3}e^{x}+a^{3}}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+a*cosh(x))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/80*(sqrt(2)*((15*e^(9/2*x) + 70*e^(7/2*x) + 128*e^(5/2*x) - 70*e^(3/2*x)
- 15*e^(1/2*x))/(a^(5/2)*e^(5*x) + 5*a^(5/2)*e^(4*x) + 10*a^(5/2)*e^(3*x) +
 10*a^(5/2)*e^(2*x) + 5*a^(5/2)*e^x + a^(5/2)) + 15*arctan(e^(1/2*x))/a^(5/
2)) - 128*sqrt(2)*e^(5/2*x)/(a^(5/2)*e^(5*x) + 5*a^(5/2)*e^(4*x) + 10*a^(5/
2)*e^(3*x) + 10*a^(5/2)*e^(2*x) + 5*a^(5/2)*e^x + a^(5/2)))*A + 1/672*(sqrt
(2)*((105*e^(9/2*x) + 490*e^(7/2*x) + 896*e^(5/2*x) + 790*e^(3/2*x) - 105*e
^(1/2*x))/(a^(5/2)*e^(5*x) + 5*a^(5/2)*e^(4*x) + 10*a^(5/2)*e^(3*x) + 10*a^
(5/2)*e^(2*x) + 5*a^(5/2)*e^x + a^(5/2)) + 105*arctan(e^(1/2*x))/a^(5/2)) +
 7*sqrt(2)*((15*e^(9/2*x) + 70*e^(7/2*x) - 128*e^(5/2*x) - 70*e^(3/2*x) - 1
5*e^(1/2*x))/(a^(5/2)*e^(5*x) + 5*a^(5/2)*e^(4*x) + 10*a^(5/2)*e^(3*x) + 10
*a^(5/2)*e^(2*x) + 5*a^(5/2)*e^x + a^(5/2)) + 15*arctan(e^(1/2*x))/a^(5/2))
 - 128*(7*sqrt(2)*sqrt(a)*e^(7/2*x) + 3*sqrt(2)*sqrt(a)*e^(3/2*x))/(a^3*e^(
5*x) + 5*a^3*e^(4*x) + 10*a^3*e^(3*x) + 10*a^3*e^(2*x) + 5*a^3*e^x + a^3))*
B
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 512 vs. $2(74) = 148$.
time = 0.34, size = 512, normalized size = 5.51

$$-\frac{\sqrt{2}\left((3A+5B)\cosh(x)^4+(3A+5B)\sinh(x)^4+4(3A+5B)\cosh(x)^3+4\left((3A+5B)\cosh(x)+3A+5B\right)\sinh(x)^3+6(3A+5B)\cosh(x)^2+6\left((3A+5B)\cosh(x)^2+2(3A+5B)\cosh(x)+3A+5B\right)\sinh(x)^2+4(3A+5B)\cosh(x)+4\left((3A+5B)\cosh(x)^3+3(3A+5B)\cosh(x)^2+3(3A+5B)\cosh(x)+3A+5B\right)\sinh(x)+3A+5B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{a}\sqrt{\frac{a}{\cosh(x)+\sinh(x)}}}{a}\right)-2\sqrt{\frac{1}{2}}\left((3A+5B)\cosh(x)^4+(3A+5B)\sinh(x)^4+(11A-3B)\cosh(x)^3+\left(4(3A+5B)\cosh(x)+11A-3B\right)\sinh(x)^3-(11A-3B)\cosh(x)^2+\left(6(3A+5B)\cosh(x)^2+3(11A-3B)\cosh(x)-11A+3B\right)\sinh(x)^2-(3A+5B)\cosh(x)+\left(4(3A+5B)\cosh(x)^3+3(11A-3B)\cosh(x)^2-2(11A-3B)\cosh(x)-3A-5B\right)\sinh(x)\right)\sqrt{\frac{a}{\cosh(x)+\sinh(x)}}}{16\left(a^3\cosh(x)^4+a^3\sinh(x)^4+4a^3\cosh(x)^3+6a^3\cosh(x)^2+4a^3\cosh(x)+4\left(a^3\cosh(x)+a^3\right)\sinh(x)^3+a^3+6\left(a^3\cosh(x)^2+2a^3\cosh(x)+a^3\right)\sinh(x)^2+4\left(a^3\cosh(x)^3+3a^3\cosh(x)^2+3a^3\cosh(x)+a^3\right)\sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+a*cosh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(2)*((3*A + 5*B)*cosh(x)^4 + (3*A + 5*B)*sinh(x)^4 + 4*(3*A + 5*
B)*cosh(x)^3 + 4*((3*A + 5*B)*cosh(x) + 3*A + 5*B)*sinh(x)^3 + 6*(3*A + 5*B
)*cosh(x)^2 + 6*((3*A + 5*B)*cosh(x)^2 + 2*(3*A + 5*B)*cosh(x) + 3*A + 5*B)
*sinh(x)^2 + 4*(3*A + 5*B)*cosh(x) + 4*((3*A + 5*B)*cosh(x)^3 + 3*(3*A + 5*
B)*cosh(x)^2 + 3*(3*A + 5*B)*cosh(x) + 3*A + 5*B)*sinh(x) + 3*A + 5*B)*sqrt
(a)*arctan(sqrt(2)*sqrt(1/2)*sqrt(a)*sqrt(a/(cosh(x) + sinh(x)))/a) - 2*sqr
t(1/2)*((3*A + 5*B)*cosh(x)^4 + (3*A + 5*B)*sinh(x)^4 + (11*A - 3*B)*cosh(x
)^3 + (4*(3*A + 5*B)*cosh(x) + 11*A - 3*B)*sinh(x)^3 - (11*A - 3*B)*cosh(x)
^2 + (6*(3*A + 5*B)*cosh(x)^2 + 3*(11*A - 3*B)*cosh(x) - 11*A + 3*B)*sinh(x
)^2 - (3*A + 5*B)*cosh(x) + (4*(3*A + 5*B)*cosh(x)^3 + 3*(11*A - 3*B)*cosh(
x)^2 - 2*(11*A - 3*B)*cosh(x) - 3*A - 5*B)*sinh(x))*sqrt(a/(cosh(x) + sinh(
x))))/(a^3*cosh(x)^4 + a^3*sinh(x)^4 + 4*a^3*cosh(x)^3 + 6*a^3*cosh(x)^2 +
4*a^3*cosh(x) + 4*(a^3*cosh(x) + a^3)*sinh(x)^3 + a^3 + 6*(a^3*cosh(x)^2 +
2*a^3*cosh(x) + a^3)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + 3*a^3*cosh(x)^2 + 3*a^3
*cosh(x) + a^3)*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\cosh{\left(x \right)}}{\left(a \left(\cosh{\left(x \right)} + 1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+a*cosh(x))**(5/2),x)
```

```
[Out] Integral((A + B*cosh(x))/(a*(cosh(x) + 1))**(5/2), x)
```

Giac [A]
time = 0.43, size = 118, normalized size = 1.27

$$\frac{\sqrt{2} \left(3 \, A + 5 \, B\right) \arctan\left(e^{\left(\frac{1}{2} \, x\right)}\right)}{16 \, a^{\frac{5}{2}}} + \frac{\sqrt{2} \left(3 \, A a^{\frac{7}{2}} e^{\left(\frac{7}{2} \, x\right)} + 5 \, B a^{\frac{7}{2}} e^{\left(\frac{7}{2} \, x\right)} + 11 \, A a^{\frac{7}{2}} e^{\left(\frac{5}{2} \, x\right)} - 3 \, B a^{\frac{7}{2}} e^{\left(\frac{5}{2} \, x\right)} - 11 \, A a^{\frac{7}{2}} e^{\left(\frac{3}{2} \, x\right)} + 3 \, B a^{\frac{7}{2}} e^{\left(\frac{3}{2} \, x\right)} - 3 \, A a^{\frac{7}{2}} e^{\left(\frac{1}{2} \, x\right)} - 5 \, B a^{\frac{7}{2}} e^{\left(\frac{1}{2} \, x\right)}\right)}{16 \, {\left(a e^{x} + a\right)}^{4} a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+a*cosh(x))^(5/2),x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*(3*A + 5*B)*arctan(e^(1/2*x))/a^(5/2) + 1/16*sqrt(2)*(3*A*a^(7
/2)*e^(7/2*x) + 5*B*a^(7/2)*e^(7/2*x) + 11*A*a^(7/2)*e^(5/2*x) - 3*B*a^(7/2
)*e^(5/2*x) - 11*A*a^(7/2)*e^(3/2*x) + 3*B*a^(7/2)*e^(3/2*x) - 3*A*a^(7/2)*
e^(1/2*x) - 5*B*a^(7/2)*e^(1/2*x))/((a*e^x + a)^4*a^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\mathrm{cosh}\left(x\right)}{{\left(a+a\,\mathrm{cosh}\left(x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(a + a*cosh(x))^(5/2),x)
[Out] int((A + B*cosh(x))/(a + a*cosh(x))^(5/2), x)
```

3.104 $\int \frac{A+B\cosh(x)}{\sqrt{a-a\cosh(x)}}\,dx$

Optimal. Leaf size=57

$$-\frac{\sqrt{2}\,(A+B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sinh(x)}{\sqrt{2}\,\sqrt{a-a\cosh(x)}}\right)}{\sqrt{a}}+\frac{2B\sinh(x)}{\sqrt{a-a\cosh(x)}}$$

```
[Out] -(A+B)*arctan(1/2*sinh(x)*a^(1/2)*2^(1/2)/(a-a*cosh(x))^(1/2))*2^(1/2)/a^(1
/2)+2*B*sinh(x)/(a-a*cosh(x))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2830, 2728, 212}

$$\frac{2B\sinh(x)}{\sqrt{a-a\cosh(x)}}-\frac{\sqrt{2}\,(A+B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sinh(x)}{\sqrt{2}\,\sqrt{a-a\cosh(x)}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/Sqrt[a - a*Cosh[x]],x]
```

```
[Out] -((Sqrt[2]*(A + B)*ArcTan[(Sqrt[a]*Sinh[x])/(Sqrt[2]*Sqrt[a - a*Cosh[x]])])
/Sqrt[a]) + (2*B*Sinh[x])/Sqrt[a - a*Cosh[x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
```

```
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cosh(x)}{\sqrt{a-a\cosh(x)}}\,dx &= \frac{2B\sinh(x)}{\sqrt{a-a\cosh(x)}}+(A+B)\int \frac{1}{\sqrt{a-a\cosh(x)}}\,dx\\ &= \frac{2B\sinh(x)}{\sqrt{a-a\cosh(x)}}+(2i(A+B))\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,\frac{ia\sinh(x)}{\sqrt{a-a\cosh(x)}}\right)\\ &= -\frac{\sqrt{2}\,(A+B)\tan^{-1}\left(\frac{\sqrt{a}\,\sinh(x)}{\sqrt{2}\,\sqrt{a-a\cosh(x)}}\right)}{\sqrt{a}}+\frac{2B\sinh(x)}{\sqrt{a-a\cosh(x)}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 40, normalized size = 0.70

$$\frac{2\left(2B\cosh\left(\frac{x}{2}\right)+(A+B)\log\left(\tanh\left(\frac{x}{4}\right)\right)\right)\sinh\left(\frac{x}{2}\right)}{\sqrt{a-a\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/Sqrt[a - a*Cosh[x]],x]
[Out] (2*(2*B*Cosh[x/2] + (A + B)*Log[Tanh[x/4]])*Sinh[x/2])/Sqrt[a - a*Cosh[x]]
```

Maple [A]
time = 1.20, size = 63, normalized size = 1.11

method	result	size
default	$\frac{\sinh\left(\frac{x}{2}\right)\left(\ln\left(\cosh\left(\frac{x}{2}\right)-1\right)A-\ln\left(\cosh\left(\frac{x}{2}\right)+1\right)A+\ln\left(\cosh\left(\frac{x}{2}\right)-1\right)B-\ln\left(\cosh\left(\frac{x}{2}\right)+1\right)B+4B\cosh\left(\frac{x}{2}\right)\right)}{\sqrt{-2\left(\sinh^2\left(\frac{x}{2}\right)\right)a}}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a-a*cosh(x))^(1/2),x,method=_RETURNVERBOSE)
[Out] sinh(1/2*x)*(ln(cosh(1/2*x)-1)*A-ln(cosh(1/2*x)+1)*A+ln(cosh(1/2*x)-1)*B-ln
(cosh(1/2*x)+1)*B+4*B*cosh(1/2*x))/(-2*sinh(1/2*x)^2*a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a-a*cosh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cosh(x) + A)/sqrt(-a*cosh(x) + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(46) = 92$.
time = 0.36, size = 99, normalized size = 1.74

$$\frac{\sqrt{2}\,(A+B)a\sqrt{-\frac{1}{a}}\,\log\left(\frac{2\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a}{\cosh(x)+\sinh(x)}}\sqrt{-\frac{1}{a}}(\cosh(x)+\sinh(x))-\cosh(x)-\sinh(x)-1}{\cosh(x)+\sinh(x)-1}\right)-2\sqrt{\frac{1}{2}}\,(B\cosh(x)+B\sinh(x)+B)\sqrt{-\frac{a}{\cosh(x)+\sinh(x)}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a-a*cosh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*(A + B)*a*sqrt(-1/a)*log((2*sqrt(2)*sqrt(1/2)*sqrt(-a/(cosh(x) + s
inh(x)))*sqrt(-1/a)*(cosh(x) + sinh(x)) - cosh(x) - sinh(x) - 1)/(cosh(x) +
 sinh(x) - 1)) - 2*sqrt(1/2)*(B*cosh(x) + B*sinh(x) + B)*sqrt(-a/(cosh(x) +
 sinh(x))))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{A+B\cosh(x)}{\sqrt{-a(\cosh(x)-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a-a*cosh(x))**(1/2),x)
```

```
[Out] Integral((A + B*cosh(x))/sqrt(-a*(cosh(x) - 1)), x)
```

Giac [A]
time = 0.43, size = 84, normalized size = 1.47

$$-\frac{2\left(\sqrt{2}\,A+\sqrt{2}\,B\right)\arctan\left(\frac{\sqrt{-ae^x}}{\sqrt{a}}\right)}{\sqrt{a}\,\mathrm{sgn}\left(-e^x+1\right)}-\frac{\sqrt{2}\,B}{\sqrt{-ae^x}\,\mathrm{sgn}\left(-e^x+1\right)}+\frac{\sqrt{2}\sqrt{-ae^x}\,B}{a\mathrm{sgn}\left(-e^x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a-a*cosh(x))^(1/2),x, algorithm="giac")
```

```
[Out] -2*(sqrt(2)*A + sqrt(2)*B)*arctan(sqrt(-a*e^x)/sqrt(a))/(sqrt(a)*sgn(-e^x +
 1)) - sqrt(2)*B/(sqrt(-a*e^x)*sgn(-e^x + 1)) + sqrt(2)*sqrt(-a*e^x)*B/(a*s
gn(-e^x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{A + B\cosh(x)}{\sqrt{a - a\cosh(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(a - a*cosh(x))^(1/2),x)
[Out] int((A + B*cosh(x))/(a - a*cosh(x))^(1/2), x)
```

3.105 $\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{3/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{(A-3B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sinh(x)}{\sqrt{2}\,\sqrt{a-a\cosh(x)}}\right)}{2\sqrt{2}\,a^{3/2}}-\frac{(A+B)\sinh(x)}{2(a-a\cosh(x))^{3/2}}$$

```
[Out] -1/2*(A+B)*sinh(x)/(a-a*cosh(x))^(3/2)-1/4*(A-3*B)*arctan(1/2*sinh(x)*a^(1/
2)*2^(1/2)/(a-a*cosh(x))^(1/2))/a^(3/2)*2^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2829, 2728, 212}

$$-\frac{(A-3B)\text{ArcTan}\left(\frac{\sqrt{a}\,\sinh(x)}{\sqrt{2}\,\sqrt{a-a\cosh(x)}}\right)}{2\sqrt{2}\,a^{3/2}}-\frac{(A+B)\sinh(x)}{2(a-a\cosh(x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(a - a*Cosh[x])^(3/2),x]
```

```
[Out] -1/2*((A - 3*B)*ArcTan[(Sqrt[a]*Sinh[x])/(Sqrt[2]*Sqrt[a - a*Cosh[x]])])/(S
qrt[2]*a^(3/2)) - ((A + B)*Sinh[x])/(2*(a - a*Cosh[x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{3/2}}\,dx &= -\frac{(A+B)\sinh(x)}{2(a-a\cosh(x))^{3/2}}+\frac{(A-3B)\int \frac{1}{\sqrt{a-a\cosh(x)}}\,dx}{4a}\\ &= -\frac{(A+B)\sinh(x)}{2(a-a\cosh(x))^{3/2}}+\frac{(i(A-3B))\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,\frac{ia\sinh(x)}{\sqrt{a-a\cosh(x)}}\right)}{2a}\\ &= -\frac{(A-3B)\tan^{-1}\left(\frac{\sqrt{a}\,\sinh(x)}{\sqrt{2}\,\sqrt{a-a\cosh(x)}}\right)}{2\sqrt{2}\,a^{3/2}}-\frac{(A+B)\sinh(x)}{2(a-a\cosh(x))^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 71, normalized size = 1.09

$$\frac{\left((A+B)\text{csch}^2\left(\frac{x}{4}\right)+4(A-3B)\log\left(\tanh\left(\frac{x}{4}\right)\right)+(A+B)\text{sech}^2\left(\frac{x}{4}\right)\right)\sinh^3\left(\frac{x}{2}\right)}{4a(-1+\cosh(x))\sqrt{a-a\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(a - a*Cosh[x])^(3/2),x]
```

```
[Out] (((A + B)*Csch[x/4]^2 + 4*(A - 3*B)*Log[Tanh[x/4]] + (A + B)*Sech[x/4]^2)*S
inh[x/2]^3)/(4*a*(-1 + Cosh[x])*Sqrt[a - a*Cosh[x]])
```

Maple [A]
time = 1.26, size = 83, normalized size = 1.28

method	result	size
default	$\frac{\cosh\left(\frac{x}{2}\right)(2A+2B)+\left(\ln\left(\cosh\left(\frac{x}{2}\right)-1\right)A-\ln\left(\cosh\left(\frac{x}{2}\right)+1\right)A-3\ln\left(\cosh\left(\frac{x}{2}\right)-1\right)B+3\ln\left(\cosh\left(\frac{x}{2}\right)+1\right)B\right)\left(\sinh^2\left(\frac{x}{2}\right)\right)}{4a\sinh\left(\frac{x}{2}\right)\sqrt{-2\left(\sinh^2\left(\frac{x}{2}\right)\right)a}}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a-a*cosh(x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a*(cosh(1/2*x)*(2*A+2*B)+(ln(cosh(1/2*x)-1)*A-ln(cosh(1/2*x)+1)*A-3*ln(
cosh(1/2*x)-1)*B+3*ln(cosh(1/2*x)+1)*B)*sinh(1/2*x)^2)/sinh(1/2*x)/(-2*sinh
(1/2*x)^2*a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a-a*cosh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cosh(x) + A)/(-a*cosh(x) + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(50) = 100$.
time = 0.44, size = 217, normalized size = 3.34

$$\frac{\sqrt{2}\left((A-3B)\cosh(x)^2+(A-3B)\sinh(x)^2-2(A-3B)\cosh(x)+2((A-3B)\cosh(x)-A+3B)\sinh(x)+A-3B\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-a}\sqrt{-\frac{a}{\cosh(x)+\sinh(x)}}(\cosh(x)+\sinh(x))-a\cosh(x)-a\sinh(x)-a}{\cosh(x)+\sinh(x)-1}\right)-4\sqrt{\frac{1}{2}}\left((A+B)\cosh(x)^2+(A+B)\sinh(x)^2+(A+B)\cosh(x)+(2(A+B)\cosh(x)+A+B)\sinh(x)\right)\sqrt{-\frac{a}{\cosh(x)+\sinh(x)}}}{4\left(a^2\cosh(x)^2+a^2\sinh(x)^2-2a^2\cosh(x)+a^2+2(a^2\cosh(x)-a^2)\sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a-a*cosh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*((A - 3*B)*cosh(x)^2 + (A - 3*B)*sinh(x)^2 - 2*(A - 3*B)*cosh(
x) + 2*((A - 3*B)*cosh(x) - A + 3*B)*sinh(x) + A - 3*B)*sqrt(-a)*log((2*sqr
t(2)*sqrt(1/2)*sqrt(-a)*sqrt(-a/(cosh(x) + sinh(x)))*(cosh(x) + sinh(x)) -
a*cosh(x) - a*sinh(x) - a)/(cosh(x) + sinh(x) - 1)) - 4*sqrt(1/2)*((A + B)*
cosh(x)^2 + (A + B)*sinh(x)^2 + (A + B)*cosh(x) + (2*(A + B)*cosh(x) + A +
B)*sinh(x))*sqrt(-a/(cosh(x) + sinh(x))))/(a^2*cosh(x)^2 + a^2*sinh(x)^2 -
2*a^2*cosh(x) + a^2 + 2*(a^2*cosh(x) - a^2)*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\cosh(x)}{(-a(\cosh(x)-1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a-a*cosh(x))**(3/2),x)
```

```
[Out] Integral((A + B*cosh(x))/(-a*(cosh(x) - 1))**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(50) = 100$.
time = 0.43, size = 111, normalized size = 1.71

$$-\frac{\left(\sqrt{2}A-3\sqrt{2}B\right)\arctan\left(\frac{\sqrt{-ae^x}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}\mathrm{sgn}(-e^x+1)}+\frac{\sqrt{2}\left(\sqrt{-ae^x}Aae^x+\sqrt{-ae^x}Bae^x+\sqrt{-ae^x}Aa+\sqrt{-ae^x}Ba\right)}{2(ae^x-a)^2a\,\mathrm{sgn}(-e^x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a-a*cosh(x))^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(2)*A - 3*sqrt(2)*B)*arctan(sqrt(-a*e^x)/sqrt(a))/(a^(3/2)*sgn(-e
^x + 1)) + 1/2*sqrt(2)*(sqrt(-a*e^x)*A*a*e^x + sqrt(-a*e^x)*B*a*e^x + sqrt(
-a*e^x)*A*a + sqrt(-a*e^x)*B*a)/((a*e^x - a)^2*a*sgn(-e^x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{A+B\cosh(x)}{\left(a-a\cosh\left(x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(a - a*cosh(x))^(3/2),x)
[Out] int((A + B*cosh(x))/(a - a*cosh(x))^(3/2), x)
```

3.106 $\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{5/2}}\,dx$

Optimal. Leaf size=94

$$-\frac{(3A-5B)\text{ArcTan}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a-a\cosh(x)}}\right)}{16\sqrt{2}\,a^{5/2}}-\frac{(A+B)\sinh(x)}{4(a-a\cosh(x))^{5/2}}-\frac{(3A-5B)\sinh(x)}{16a(a-a\cosh(x))^{3/2}}$$

```
[Out] -1/4*(A+B)*sinh(x)/(a-a*cosh(x))^(5/2)-1/16*(3*A-5*B)*sinh(x)/a/(a-a*cosh(x
))^(3/2)-1/32*(3*A-5*B)*arctan(1/2*sinh(x)*a^(1/2)*2^(1/2)/(a-a*cosh(x))^(1
/2))/a^(5/2)*2^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2829, 2729, 2728, 212}

$$-\frac{(3A-5B)\text{ArcTan}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a-a\cosh(x)}}\right)}{16\sqrt{2}\,a^{5/2}}-\frac{(3A-5B)\sinh(x)}{16a(a-a\cosh(x))^{3/2}}-\frac{(A+B)\sinh(x)}{4(a-a\cosh(x))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(a - a*Cosh[x])^(5/2),x]
```

```
[Out] -1/16*((3*A - 5*B)*ArcTan[(Sqrt[a]*Sinh[x])/(Sqrt[2]*Sqrt[a - a*Cosh[x]])])
/(Sqrt[2]*a^(5/2)) - ((A + B)*Sinh[x])/(4*(a - a*Cosh[x])^(5/2)) - ((3*A -
5*B)*Sinh[x])/(16*a*(a - a*Cosh[x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{5/2}}\,dx = -\frac{(A+B)\sinh(x)}{4(a-a\cosh(x))^{5/2}}+\frac{(3A-5B)\int \frac{1}{(a-a\cosh(x))^{3/2}}\,dx}{8a}$$

$$= -\frac{(A+B)\sinh(x)}{4(a-a\cosh(x))^{5/2}}-\frac{(3A-5B)\sinh(x)}{16a(a-a\cosh(x))^{3/2}}+\frac{(3A-5B)\int \frac{1}{\sqrt{a-a\cosh(x)}}$$

$$= -\frac{(A+B)\sinh(x)}{4(a-a\cosh(x))^{5/2}}-\frac{(3A-5B)\sinh(x)}{16a(a-a\cosh(x))^{3/2}}+\frac{(i(3A-5B))\text{Subst}\left(\int \frac{1}{2a-x^2}\,d\right.}{16a^2}$$

$$= -\frac{(3A-5B)\tan^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a-a\cosh(x)}}\right)}{16\sqrt{2}\,a^{5/2}}-\frac{(A+B)\sinh(x)}{4(a-a\cosh(x))^{5/2}}-\frac{(3A-5}{16a(a-a}$$

Mathematica [A]
time = 0.26, size = 108, normalized size = 1.15

$$\frac{\left(2(3A-5B)\text{csch}^2\left(\frac{x}{4}\right)-(A+B)\text{csch}^4\left(\frac{x}{4}\right)+8(3A-5B)\log\left(\tanh\left(\frac{x}{4}\right)\right)+2(3A-5B)\text{sech}^2\left(\frac{x}{4}\right)+(A+B)\text{sech}^4\left(\frac{x}{4}\right)\right)\sinh^5\left(\frac{x}{2}\right)}{32a^2(-1+\cosh(x))^2\sqrt{a-a\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(a - a*Cosh[x])^(5/2),x]
[Out] ((2*(3*A - 5*B)*Csch[x/4]^2 - (A + B)*Csch[x/4]^4 + 8*(3*A - 5*B)*Log[Tanh[
x/4]] + 2*(3*A - 5*B)*Sech[x/4]^2 + (A + B)*Sech[x/4]^4)*Sinh[x/2]^5)/(32*a
^2*(-1 + Cosh[x])^2*Sqrt[a - a*Cosh[x]])
```

Maple [A]
time = 1.25, size = 118, normalized size = 1.26

method	result

default	$\frac{(6A-10B)\cosh\left(\frac{x}{2}\right)\left(\sinh^2\left(\frac{x}{2}\right)\right)+(-4A-4B)\cosh\left(\frac{x}{2}\right)+\left(3\ln\left(\cosh\left(\frac{x}{2}\right)-1\right)A-3\ln\left(\cosh\left(\frac{x}{2}\right)+1\right)A-5\ln\left(\cosh\left(\frac{x}{2}\right)-1\right)B+5\ln\left(\cosh\left(\frac{x}{2}\right)+\right.\right.}{32a^2\left(\cosh\left(\frac{x}{2}\right)+1\right)\left(\cosh\left(\frac{x}{2}\right)-1\right)\sinh\left(\frac{x}{2}\right)\sqrt{-2\left(\sinh^2\left(\frac{x}{2}\right)\right)a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a-a*cosh(x))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/a^2*((6*A-10*B)*cosh(1/2*x)*sinh(1/2*x)^2+(-4*A-4*B)*cosh(1/2*x)+(3*ln
(cosh(1/2*x)-1)*A-3*ln(cosh(1/2*x)+1)*A-5*ln(cosh(1/2*x)-1)*B+5*ln(cosh(1/2
*x)+1)*B)*sinh(1/2*x)^4)/(cosh(1/2*x)+1)/(cosh(1/2*x)-1)/sinh(1/2*x)/(-2*si
nh(1/2*x)^2*a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a-a*cosh(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cosh(x) + A)/(-a*cosh(x) + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 548 vs. 2(75) = 150.
time = 0.36, size = 548, normalized size = 5.83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a-a*cosh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/32*(sqrt(2)*((3*A - 5*B)*cosh(x)^4 + (3*A - 5*B)*sinh(x)^4 - 4*(3*A - 5*B
)*cosh(x)^3 + 4*((3*A - 5*B)*cosh(x) - 3*A + 5*B)*sinh(x)^3 + 6*(3*A - 5*B)
*cosh(x)^2 + 6*((3*A - 5*B)*cosh(x)^2 - 2*(3*A - 5*B)*cosh(x) + 3*A - 5*B)*
sinh(x)^2 - 4*(3*A - 5*B)*cosh(x) + 4*((3*A - 5*B)*cosh(x)^3 - 3*(3*A - 5*B
)*cosh(x)^2 + 3*(3*A - 5*B)*cosh(x) - 3*A + 5*B)*sinh(x) + 3*A - 5*B)*sqrt(
-a)*log((2*sqrt(2)*sqrt(1/2)*sqrt(-a)*sqrt(-a/(cosh(x) + sinh(x)))*(cosh(x)
 + sinh(x)) - a*cosh(x) - a*sinh(x) - a)/(cosh(x) + sinh(x) - 1)) - 4*sqrt(
1/2)*((3*A - 5*B)*cosh(x)^4 + (3*A - 5*B)*sinh(x)^4 - (11*A + 3*B)*cosh(x)^
3 + (4*(3*A - 5*B)*cosh(x) - 11*A - 3*B)*sinh(x)^3 - (11*A + 3*B)*cosh(x)^2
 + (6*(3*A - 5*B)*cosh(x)^2 - 3*(11*A + 3*B)*cosh(x) - 11*A - 3*B)*sinh(x)^
2 + (3*A - 5*B)*cosh(x) + (4*(3*A - 5*B)*cosh(x)^3 - 3*(11*A + 3*B)*cosh(x)
^2 - 2*(11*A + 3*B)*cosh(x) + 3*A - 5*B)*sinh(x))*sqrt(-a/(cosh(x) + sinh(x
))))/(a^3*cosh(x)^4 + a^3*sinh(x)^4 - 4*a^3*cosh(x)^3 + 6*a^3*cosh(x)^2 - 4
```

```
*a^3*cosh(x) + 4*(a^3*cosh(x) - a^3)*sinh(x)^3 + a^3 + 6*(a^3*cosh(x)^2 - 2
*a^3*cosh(x) + a^3)*sinh(x)^2 + 4*(a^3*cosh(x)^3 - 3*a^3*cosh(x)^2 + 3*a^3*
cosh(x) - a^3)*sinh(x))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a-a*cosh(x))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(75) = 150.
time = 0.42, size = 189, normalized size = 2.01

$$-\frac{\sqrt{2}\,(3\,A-5\,B)\arctan\left(\frac{\sqrt{-ae^{x}}}{\sqrt{a}}\right)}{16\,a^{\frac{5}{2}}\mathrm{sgn}\left(-e^{x}+1\right)}+\frac{\sqrt{2}\,\left(3\,\sqrt{-ae^{x}}\,Aa^{3}e^{(3\,x)}-5\,\sqrt{-ae^{x}}\,Ba^{3}e^{(3\,x)}-11\,\sqrt{-ae^{x}}\,Aa^{3}e^{(2\,x)}-3\,\sqrt{-ae^{x}}\,Ba^{3}e^{(2\,x)}-11\,\sqrt{-ae^{x}}\,Aa^{3}e^{x}-3\,\sqrt{-ae^{x}}\,Ba^{3}e^{x}+3\,\sqrt{-ae^{x}}\,Aa^{3}-5\,\sqrt{-ae^{x}}\,Ba^{3}\right)}{16\,(ae^{x}-a)^{4}a^{2}\mathrm{sgn}\left(-e^{x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a-a*cosh(x))^(5/2),x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*(3*A - 5*B)*arctan(sqrt(-a*e^x)/sqrt(a))/(a^(5/2)*sgn(-e^x +
1)) + 1/16*sqrt(2)*(3*sqrt(-a*e^x)*A*a^3*e^(3*x) - 5*sqrt(-a*e^x)*B*a^3*e^(
3*x) - 11*sqrt(-a*e^x)*A*a^3*e^(2*x) - 3*sqrt(-a*e^x)*B*a^3*e^(2*x) - 11*sq
rt(-a*e^x)*A*a^3*e^x - 3*sqrt(-a*e^x)*B*a^3*e^x + 3*sqrt(-a*e^x)*A*a^3 - 5*
sqrt(-a*e^x)*B*a^3)/((a*e^x - a)^4*a^2*sgn(-e^x + 1))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\cosh(x)}{\left(a-a\cosh\left(x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(a - a*cosh(x))^(5/2),x)
```

```
[Out] int((A + B*cosh(x))/(a - a*cosh(x))^(5/2), x)
```

3.107 $\int (a+b\cosh(x))^{5/2}(A+B\cosh(x))\,dx$

Optimal. Leaf size=233

$$-\frac{2i(161a^2Ab+63Ab^3+15a^3B+145ab^2B)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{105b\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\frac{2i(a^2-b^2)\left(56aAb+15a^2B+25b^2\right.}{105b\sqrt{a+b}}$$

```
[Out] 2/35*(7*A*b+5*B*a)*(a+b*cosh(x))^(3/2)*sinh(x)+2/7*B*(a+b*cosh(x))^(5/2)*si
nh(x)+2/105*(56*A*a*b+15*B*a^2+25*B*b^2)*sinh(x)*(a+b*cosh(x))^(1/2)-2/105*
I*(161*A*a^2*b+63*A*b^3+15*B*a^3+145*B*a*b^2)*(cosh(1/2*x)^2)^(1/2)/cosh(1/
2*x)*EllipticE(I*sinh(1/2*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cosh(x))^(1/2)/b
/((a+b*cosh(x))/(a+b))^(1/2)+2/105*I*(a^2-b^2)*(56*A*a*b+15*B*a^2+25*B*b^2)
*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh(1/2*x),2^(1/2)*(b/(a+b)
)^(1/2))*((a+b*cosh(x))/(a+b))^(1/2)/b/(a+b*cosh(x))^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2832, 2831, 2742, 2740, 2734, 2732}

$$\frac{2}{105}\sinh(x)\left(15a^2B+56aAb+25b^2B\right)\sqrt{a+b\cosh(x)}+\frac{2i(a^2-b^2)\left(15a^2B+56aAb+25b^2B\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{105b\sqrt{a+b\cosh(x)}}-\frac{2i(15a^3B+161a^2Ab+145ab^2B+63Ab^3)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{105b\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\frac{2}{35}\sinh(x)(5aB+7Ab)(a+b\cosh(x))^{3/2}+\frac{2}{7}B\sinh(x)(a+b\cosh(x))^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x])^(5/2)*(A + B*Cosh[x]),x]
```

```
[Out] (((-2*I)/105)*(161*a^2*A*b + 63*A*b^3 + 15*a^3*B + 145*a*b^2*B)*Sqrt[a + b*
Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)])/(b*Sqrt[(a + b*Cosh[x])/(a + b)
]) + (((2*I)/105)*(a^2 - b^2)*(56*a*A*b + 15*a^2*B + 25*b^2*B)*Sqrt[(a + b*
Cosh[x])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/(b*Sqrt[a + b*Cosh[x]]
) + (2*(56*a*A*b + 15*a^2*B + 25*b^2*B)*Sqrt[a + b*Cosh[x]]*Sinh[x])/105 +
(2*(7*A*b + 5*a*B)*(a + b*Cosh[x])^(3/2)*Sinh[x])/35 + (2*B*(a + b*Cosh[x])
^(5/2)*Sinh[x])/7
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\cosh(x))^{5/2}(A+B\cosh(x))\,dx = \frac{2}{7}B(a+b\cosh(x))^{5/2}\sinh(x)+\frac{2}{7}\int (a+b\cosh(x))^{3/2}\left(\frac{1}{2}(7aA+\right.$$

$$=\frac{2}{35}(7Ab+5aB)(a+b\cosh(x))^{3/2}\sinh(x)+\frac{2}{7}B(a+b\cosh(x))^{5/2}$$

$$=\frac{2}{105}\left(56aAb+15a^2B+25b^2B\right)\sqrt{a+b\cosh(x)}\,\sinh(x)+\frac{2}{35}(7Ab$$

$$=\frac{2}{105}\left(56aAb+15a^2B+25b^2B\right)\sqrt{a+b\cosh(x)}\,\sinh(x)+\frac{2}{35}(7Ab$$

$$=\frac{2}{105}\left(56aAb+15a^2B+25b^2B\right)\sqrt{a+b\cosh(x)}\,\sinh(x)+\frac{2}{35}(7Ab$$

$$=-\frac{2i(161a^2Ab+63Ab^3+15a^3B+145ab^2B)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\right.}{105b\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

Mathematica [A]
time = 0.42, size = 203, normalized size = 0.87

$$\frac{-\frac{2i\sqrt{\frac{a+b\cosh(x)}{a+b}}\left(b(105a^3A+119aAb^2+135a^2bB+25b^3B)F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)+(161a^2Ab+63Ab^3+15a^3B+145ab^2B)\left((a+b)E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)-aF\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)\right)\right)}{b}+(a+b\cosh(x))\left(154aAb+90a^2B+65b^2B+6b(7Ab+15aB)\cosh(x)+15b^2B\cosh(2x)\right)\sinh(x)}{105\sqrt{a+b\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x])^(5/2)*(A + B*Cosh[x]),x]
```

```
[Out] (((-2*I)*Sqrt[(a + b*Cosh[x])/(a + b)]*(b*(105*a^3*A + 119*a*A*b^2 + 135*a^
2*b*B + 25*b^3*B)*EllipticF[(I/2)*x, (2*b)/(a + b)] + (161*a^2*A*b + 63*A*b
^3 + 15*a^3*B + 145*a*b^2*B)*((a + b)*EllipticE[(I/2)*x, (2*b)/(a + b)] - a
*EllipticF[(I/2)*x, (2*b)/(a + b)])))/b + (a + b*Cosh[x])*(154*a*A*b + 90*a
^2*B + 65*b^2*B + 6*b*(7*A*b + 15*a*B)*Cosh[x] + 15*b^2*B*Cosh[2*x])*Sinh[x
])/(105*Sqrt[a + b*Cosh[x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1364 vs. $2(245) = 490$.
time = 1.51, size = 1365, normalized size = 5.86

method	result	size

default	Expression too large to display	1365

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cosh(x))^(5/2)*(A+B*cosh(x)),x,method=_RETURNVERBOSE)

[Out] 2/105*(240*B*cosh(1/2*x)*(-2*b/(a-b))^(1/2)*sinh(1/2*x)^8*b^3+(168*A*(-2*b/(a-b))^(1/2)*b^3+480*B*(-2*b/(a-b))^(1/2)*a*b^2+360*B*(-2*b/(a-b))^(1/2)*b^3)*sinh(1/2*x)^6*cosh(1/2*x)+(392*A*(-2*b/(a-b))^(1/2)*a*b^2+168*A*(-2*b/(a-b))^(1/2)*b^3+360*B*(-2*b/(a-b))^(1/2)*a^2*b+480*B*(-2*b/(a-b))^(1/2)*a*b^2+280*B*(-2*b/(a-b))^(1/2)*b^3)*sinh(1/2*x)^4*cosh(1/2*x)+(154*A*(-2*b/(a-b))^(1/2)*a^2*b+196*A*(-2*b/(a-b))^(1/2)*a*b^2+42*A*(-2*b/(a-b))^(1/2)*b^3+90*B*(-2*b/(a-b))^(1/2)*a^3+180*B*(-2*b/(a-b))^(1/2)*a^2*b+170*B*(-2*b/(a-b))^(1/2)*a*b^2+80*B*(-2*b/(a-b))^(1/2)*b^3)*sinh(1/2*x)^2*cosh(1/2*x)+105*A*a^3*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))+161*A*a^2*b*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))+119*A*a*b^2*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))+63*A*b^3*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))-322*A*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))*a^2*b-126*A*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))*b^3+15*B*a^3*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))+135*B*a^2*b*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))+145*B*a*b^2*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))+25*B*b^3*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))-30*B*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))*a^3-290*B*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))*a*b^2)*((2*b*cosh(1/2*x)^2+a-b)*sinh(1/2*x)^2)^(1/2)/(-2*b/(a-b))^(1/2)/(2*sinh(1/2*x)^4*b+(a+b)*sinh(1/2*x)^2)^(1/2)/sinh(1/2*x)/(2*b*sinh(1/2*x)^2+a+b)^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cosh(x))^(5/2)*(A+B*cosh(x)),x, algorithm="maxima")

[Out] integrate((B*cosh(x) + A)*(b*cosh(x) + a)^(5/2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.17, size = 1141, normalized size = 4.90

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cosh(x))^(5/2)*(A+B*cosh(x)),x, algorithm="fricas")

[Out] -1/1260*(8*(sqrt(2)*(30*B*a^4 + 7*A*a^3*b - 115*B*a^2*b^2 - 231*A*a*b^3 - 75*B*b^4)*cosh(x)^3 + 3*sqrt(2)*(30*B*a^4 + 7*A*a^3*b - 115*B*a^2*b^2 - 231*A*a*b^3 - 75*B*b^4)*cosh(x)^2*sinh(x) + 3*sqrt(2)*(30*B*a^4 + 7*A*a^3*b - 115*B*a^2*b^2 - 231*A*a*b^3 - 75*B*b^4)*cosh(x)*sinh(x)^2 + sqrt(2)*(30*B*a^4 + 7*A*a^3*b - 115*B*a^2*b^2 - 231*A*a*b^3 - 75*B*b^4)*sinh(x)^3)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) + 24*(sqrt(2)*(15*B*a^3*b + 161*A*a^2*b^2 + 145*B*a*b^3 + 63*A*b^4)*cosh(x)^3 + 3*sqrt(2)*(15*B*a^3*b + 161*A*a^2*b^2 + 145*B*a*b^3 + 63*A*b^4)*cosh(x)^2*sinh(x) + 3*sqrt(2)*(15*B*a^3*b + 161*A*a^2*b^2 + 145*B*a*b^3 + 63*A*b^4)*cosh(x)*sinh(x)^2 + sqrt(2)*(15*B*a^3*b + 161*A*a^2*b^2 + 145*B*a*b^3 + 63*A*b^4)*sinh(x)^3)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)) - 3*(15*B*b^4*cosh(x)^6 + 15*B*b^4*sinh(x)^6 + 6*(15*B*a*b^3 + 7*A*b^4)*cosh(x)^5 + 6*(15*B*b^4*cosh(x) + 15*B*a*b^3 + 7*A*b^4)*sinh(x)^5 - 15*B*b^4 + (180*B*a^2*b^2 + 308*A*a*b^3 + 115*B*b^4)*cosh(x)^4 + (225*B*b^4*cosh(x)^2 + 180*B*a^2*b^2 + 308*A*a*b^3 + 115*B*b^4 + 30*(15*B*a*b^3 + 7*A*b^4)*cosh(x))*sinh(x)^4 - 8*(15*B*a^3*b + 161*A*a^2*b^2 + 145*B*a*b^3 + 63*A*b^4)*cosh(x)^3 + 4*(75*B*b^4*cosh(x)^3 - 30*B*a^3*b - 322*A*a^2*b^2 - 290*B*a*b^3 - 126*A*b^4 + 15*(15*B*a*b^3 + 7*A*b^4)*cosh(x)^2 + (180*B*a^2*b^2 + 308*A*a*b^3 + 115*B*b^4)*cosh(x))*sinh(x)^3 - (180*B*a^2*b^2 + 308*A*a*b^3 + 115*B*b^4)*cosh(x)^2 + (225*B*b^4*cosh(x)^4 - 180*B*a^2*b^2 - 308*A*a*b^3 - 115*B*b^4 + 60*(15*B*a*b^3 + 7*A*b^4)*cosh(x)^3 + 6*(180*B*a^2*b^2 + 308*A*a*b^3 + 115*B*b^4)*cosh(x)^2 - 24*(15*B*a^3*b + 161*A*a^2*b^2 + 145*B*a*b^3 + 63*A*b^4)*cosh(x))*sinh(x)^2 - 6*(15*B*a*b^3 + 7*A*b^4)*cosh(x) + 2*(45*B*b^4*cosh(x)^5 - 45*B*a*b^3 - 21*A*b^4 + 15*(15*B*a*b^3 + 7*A*b^4)*cosh(x)^4 + 2*(180*B*a^2*b^2 + 308*A*a*b^3 + 115*B*b^4)*cosh(x)^3 - 12*(15*B*a^3*b + 161*A*a^2*b^2 + 145*B*a*b^3 + 63*A*b^4)*cosh(x)^2 - (180*B*a^2*b^2 + 308*A*a*b^3 + 115*B*b^4)*cosh(x))*sinh(x))*sqrt(b*cosh(x) + a))/(b^2*cosh(x)^3 + 3*b^2*cosh(x)^2*sinh(x) + 3*b^2*cosh(x)*sinh(x)^2 + b^2*sinh(x)^3)

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))**(5/2)*(A+B*cosh(x)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(5/2)*(A+B*cosh(x)),x, algorithm="giac")
```

```
[Out] integrate((B*cosh(x) + A)*(b*cosh(x) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (A + B\cosh(x))\,(a + b\cosh(x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))*(a + b*cosh(x))^(5/2),x)
```

```
[Out] int((A + B*cosh(x))*(a + b*cosh(x))^(5/2), x)
```

3.108 $\int (a+b\cosh(x))^{3/2}(A+B\cosh(x))\,dx$

Optimal. Leaf size=181

$$-\frac{2i(20aAb+3a^2B+9b^2B)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{15b\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\frac{2i(a^2-b^2)(5Ab+3aB)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{15b\sqrt{a+b\cosh(x)}}$$

```
[Out] 2/5*B*(a+b*cosh(x))^(3/2)*sinh(x)+2/15*(5*A*b+3*B*a)*sinh(x)*(a+b*cosh(x))^
(1/2)-2/15*I*(20*A*a*b+3*B*a^2+9*B*b^2)*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*E
llipticE(I*sinh(1/2*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cosh(x))^(1/2)/b/((a+b
*cosh(x))/(a+b))^(1/2)+2/15*I*(a^2-b^2)*(5*A*b+3*B*a)*(cosh(1/2*x)^2)^(1/2)
/cosh(1/2*x)*EllipticF(I*sinh(1/2*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cosh(x)
)/(a+b))^(1/2)/b/(a+b*cosh(x))^(1/2)
```

Rubi [A]

time = 0.22, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2832, 2831, 2742, 2740, 2734, 2732}

$$\frac{2i(a^2-b^2)(3aB+5Ab)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{15b\sqrt{a+b\cosh(x)}}-\frac{2i(3a^2B+20aAb+9b^2B)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{15b\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\frac{2}{15}\sinh(x)(3aB+5Ab)\sqrt{a+b\cosh(x)}+\frac{2}{5}B\sinh(x)(a+b\cosh(x))^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x])^(3/2)*(A + B*Cosh[x]),x]
```

```
[Out] (((-2*I)/15)*(20*a*A*b + 3*a^2*B + 9*b^2*B)*Sqrt[a + b*Cosh[x]]*EllipticE[(
I/2)*x, (2*b)/(a + b)])/(b*Sqrt[(a + b*Cosh[x])/(a + b)]) + (((2*I)/15)*(a^
2 - b^2)*(5*A*b + 3*a*B)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticF[(I/2)*x, (
2*b)/(a + b)])/(b*Sqrt[a + b*Cosh[x]]) + (2*(5*A*b + 3*a*B)*Sqrt[a + b*Cosh
[x]]*Sinh[x])/15 + (2*B*(a + b*Cosh[x])^(3/2)*Sinh[x])/5
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\cosh(x))^{3/2}(A+B\cosh(x))\,dx = \frac{2}{5}B(a+b\cosh(x))^{3/2}\sinh(x)+\frac{2}{5}\int \sqrt{a+b\cosh(x)}\,\left(\frac{1}{2}(5aA+3\right.$$

$$= \frac{2}{15}(5Ab+3aB)\sqrt{a+b\cosh(x)}\,\sinh(x)+\frac{2}{5}B(a+b\cosh(x))^{3/2}\,\mathrm{si}$$

$$= \frac{2}{15}(5Ab+3aB)\sqrt{a+b\cosh(x)}\,\sinh(x)+\frac{2}{5}B(a+b\cosh(x))^{3/2}\,\mathrm{si}$$

$$= \frac{2}{15}(5Ab+3aB)\sqrt{a+b\cosh(x)}\,\sinh(x)+\frac{2}{5}B(a+b\cosh(x))^{3/2}\,\mathrm{si}$$

$$= -\frac{2i(20aAb+3a^2B+9b^2B)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{15b\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\frac{2i(a^2-}{}$$

Mathematica [A]
time = 0.49, size = 124, normalized size = 0.69

$$\frac{2}{15}\sqrt{a+b\cosh(x)}\left(-\frac{i\left((20aAb+3a^2B+9b^2B)\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)-(a-b)(5Ab+3aB)F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}+(5Ab+6aB+3bB\cosh(x))\sinh(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x])^(3/2)*(A + B*Cosh[x]),x]
```

```
[Out] (2*Sqrt[a + b*Cosh[x]]*(((-I)*((20*a*A*b + 3*a^2*B + 9*b^2*B)*EllipticE[(I/
2)*x, (2*b)/(a + b)] - (a - b)*(5*A*b + 3*a*B)*EllipticF[(I/2)*x, (2*b)/(a
+ b)]))/(b*Sqrt[(a + b*Cosh[x])/(a + b)]) + (5*A*b + 6*a*B + 3*b*B*Cosh[x])
*Sinh[x]))/15
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 972 vs. $2(197) = 394$.
time = 1.47, size = 973, normalized size = 5.38

method	result	size
default	Expression too large to display	973

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cosh(x))^(3/2)*(A+B*cosh(x)),x,method=_RETURNVERBOSE)

[Out] 2/15*(24*B*cosh(1/2*x)*(-2*b/(a-b))^(1/2)*sinh(1/2*x)^6*b^2+(20*A*(-2*b/(a-b))^(1/2)*b^2+36*B*(-2*b/(a-b))^(1/2)*a*b+24*B*(-2*b/(a-b))^(1/2)*b^2)*sinh(1/2*x)^4*cosh(1/2*x)+(10*A*(-2*b/(a-b))^(1/2)*a*b+10*A*(-2*b/(a-b))^(1/2)*b^2+12*B*(-2*b/(a-b))^(1/2)*a^2+18*B*(-2*b/(a-b))^(1/2)*a*b+6*B*(-2*b/(a-b))^(1/2)*b^2)*sinh(1/2*x)^2*cosh(1/2*x)+15*a^2*A*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))+20*A*a*b*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))+5*A*b^2*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))-40*A*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))*a*b+3*B*a^2*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))+12*B*a*b*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))+9*B*b^2*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))-6*B*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))*a^2-18*B*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))*b^2)*((2*b*cosh(1/2*x)^2+a-b)*sinh(1/2*x)^2)^(1/2)/(-2*b/(a-b))^(1/2)/(2*sinh(1/2*x)^4*b+(a+b)*sinh(1/2*x)^2)^(1/2)/sinh(1/2*x)/(2*b*sinh(1/2*x)^2+a+b)^(1/2)

<u>**Maxima**</u> [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cosh(x))^(3/2)*(A+B*cosh(x)),x, algorithm="maxima")

[Out] integrate((B*cosh(x) + A)*(b*cosh(x) + a)^(3/2), x)

<u>**Fricas**</u> [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 635, normalized size = 3.51

$$-\frac{4\left(\sqrt{2}\left(6Ba^3-5Aa^2b-18Bab^2-15Ab^3\right)\cosh(x)^2+2\sqrt{2}\left(6Ba^3-5Aa^2b-18Bab^2-15Ab^3\right)\cosh(x)\sinh(x)+\sqrt{2}\left(6Ba^3-5Aa^2b-18Bab^2-15Ab^3\right)\sinh(x)^2\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8a^3-9ab^2)}{27b^3},\frac{3b\cosh(x)+3b\sinh(x)+2a}{3b}\right)+12\left(\sqrt{2}\left(3Ba^2b+20Aab^2+9Bb^3\right)\cosh(x)^2+2\sqrt{2}\left(3Ba^2b+20Aab^2+9Bb^3\right)\cosh(x)\sinh(x)+\sqrt{2}\left(3Ba^2b+20Aab^2+9Bb^3\right)\sinh(x)^2\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8a^3-9ab^2)}{27b^3},\mathrm{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8a^3-9ab^2)}{27b^3},\frac{3b\cosh(x)+3b\sinh(x)+2a}{3b}\right)\right)-3\left(3Bb^3\cosh(x)^4+3Bb^3\sinh(x)^4-3Bb^3+2(6Bab^2+5Ab^3)\cosh(x)^3+2(6Bb^3\cosh(x)+6Bab^2+5Ab^3)\sinh(x)^3-4(3Ba^2b+20Aab^2+9Bb^3)\cosh(x)^2+2(9Bb^3\cosh(x)^2-6Ba^2b-40Aab^2-18Bb^3+3(6Bab^2+5Ab^3)\cosh(x))\sinh(x)^2-2(6Bab^2+5Ab^3)\cosh(x)+2(6Bb^3\cosh(x)^3-6Bab^2-5Ab^3+3(6Bab^2+5Ab^3)\cosh(x)^2-4(3Ba^2b+20Aab^2+9Bb^3)\cosh(x))\sinh(x)\right)\sqrt{b\cosh(x)+a}}{90\left(b^2\cosh(x)^2+2b^2\cosh(x)\sinh(x)+b^2\sinh(x)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cosh(x))^(3/2)*(A+B*cosh(x)),x, algorithm="fricas")

[Out] -1/90*(4*(sqrt(2)*(6*B*a^3 - 5*A*a^2*b - 18*B*a*b^2 - 15*A*b^3)*cosh(x)^2 +
 2*sqrt(2)*(6*B*a^3 - 5*A*a^2*b - 18*B*a*b^2 - 15*A*b^3)*cosh(x)*sinh(x) +

```
sqrt(2)*(6*B*a^3 - 5*A*a^2*b - 18*B*a*b^2 - 15*A*b^3)*sinh(x)^2)*sqrt(b)*we
ierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3
*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) + 12*(sqrt(2)*(3*B*a^2*b + 20*A*a*b^2
 + 9*B*b^3)*cosh(x)^2 + 2*sqrt(2)*(3*B*a^2*b + 20*A*a*b^2 + 9*B*b^3)*cosh(x
)*sinh(x) + sqrt(2)*(3*B*a^2*b + 20*A*a*b^2 + 9*B*b^3)*sinh(x)^2)*sqrt(b)*w
eierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weiers
trassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*
b*cosh(x) + 3*b*sinh(x) + 2*a)/b)) - 3*(3*B*b^3*cosh(x)^4 + 3*B*b^3*sinh(x)
^4 - 3*B*b^3 + 2*(6*B*a*b^2 + 5*A*b^3)*cosh(x)^3 + 2*(6*B*b^3*cosh(x) + 6*B
*a*b^2 + 5*A*b^3)*sinh(x)^3 - 4*(3*B*a^2*b + 20*A*a*b^2 + 9*B*b^3)*cosh(x)^
2 + 2*(9*B*b^3*cosh(x)^2 - 6*B*a^2*b - 40*A*a*b^2 - 18*B*b^3 + 3*(6*B*a*b^2
 + 5*A*b^3)*cosh(x))*sinh(x)^2 - 2*(6*B*a*b^2 + 5*A*b^3)*cosh(x) + 2*(6*B*b
^3*cosh(x)^3 - 6*B*a*b^2 - 5*A*b^3 + 3*(6*B*a*b^2 + 5*A*b^3)*cosh(x)^2 - 4*
(3*B*a^2*b + 20*A*a*b^2 + 9*B*b^3)*cosh(x))*sinh(x))*sqrt(b*cosh(x) + a))/(
b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\cosh{(x)})\,(a + b\cosh{(x)})^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))**(3/2)*(A+B*cosh(x)),x)
[Out] Integral((A + B*cosh(x))*(a + b*cosh(x))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(3/2)*(A+B*cosh(x)),x, algorithm="giac")
[Out] integrate((B*cosh(x) + A)*(b*cosh(x) + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+B\cosh(x)\right)\,{\left(a+b\cosh(x)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))*(a + b*cosh(x))^(3/2),x)
[Out] int((A + B*cosh(x))*(a + b*cosh(x))^(3/2), x)
```

3.109 $\int \sqrt{a+b\cosh(x)}\,(A+B\cosh(x))\,dx$

Optimal. Leaf size=138

$$-\frac{2i(3Ab+aB)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{3b\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\frac{2i(a^2-b^2)\,B\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{3b\sqrt{a+b\cosh(x)}}+\frac{2}{3}B\sqrt{a+b\cosh(x)}$$

```
[Out] 2/3*B*sinh(x)*(a+b*cosh(x))^(1/2)-2/3*I*(3*A*b+B*a)*(cosh(1/2*x)^2)^(1/2)/c
osh(1/2*x)*EllipticE(I*sinh(1/2*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cosh(x))^(
1/2)/b/((a+b*cosh(x))/(a+b))^(1/2)+2/3*I*(a^2-b^2)*B*(cosh(1/2*x)^2)^(1/2)/
cosh(1/2*x)*EllipticF(I*sinh(1/2*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cosh(x))
/(a+b))^(1/2)/b/(a+b*cosh(x))^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2832, 2831, 2742, 2740, 2734, 2732}

$$\frac{2iB(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{3b\sqrt{a+b\cosh(x)}}-\frac{2i(aB+3Ab)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{3b\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\frac{2}{3}B\sinh(x)\sqrt{a+b\cosh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cosh[x]]*(A + B*Cosh[x]),x]
```

```
[Out] (((-2*I)/3)*(3*A*b + a*B)*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a +
 b)])/(b*Sqrt[(a + b*Cosh[x])/(a + b)]) + (((2*I)/3)*(a^2 - b^2)*B*Sqrt[(a
+ b*Cosh[x])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/(b*Sqrt[a + b*Cosh
[x]]) + (2*B*Sqrt[a + b*Cosh[x]]*Sinh[x])/3
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] && !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\int \sqrt{a+b\cosh(x)}\,(A+B\cosh(x))\,dx = \frac{2}{3}B\sqrt{a+b\cosh(x)}\,\sinh(x)+\frac{2}{3}\int \frac{\frac{1}{2}(3aA+bB)+\frac{1}{2}(3Ab+aB)}{\sqrt{a+b\cosh(x)}}$$

$$= \frac{2}{3}B\sqrt{a+b\cosh(x)}\,\sinh(x)-\frac{\left(\left(a^2-b^2\right)B\right)\int \frac{1}{\sqrt{a+b\cosh(x)}}\,d}{3b}$$

$$= \frac{2}{3}B\sqrt{a+b\cosh(x)}\,\sinh(x)+\frac{\left((3Ab+aB)\sqrt{a+b\cosh(x)}\right)\int}{3b\sqrt{\frac{a+b\cosh}{a+b}}}$$

$$= -\frac{2i(3Ab+aB)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{3b\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\frac{2i\left(a^2-b^2\right)B\sqrt{\frac{a+}{}}}{3b\sqrt{a+}}$$

Mathematica [A]
time = 0.25, size = 123, normalized size = 0.89

$$\frac{-2i(a+b)(3Ab+aB)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)+2i\left(a^2-b^2\right)B\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)+2bB(a+b\cosh(x))\sinh(x)}{3b\sqrt{a+b\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cosh[x]]*(A + B*Cosh[x]),x]
```

```
[Out] ((-2*I)*(a + b)*(3*A*b + a*B)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticE[(I/2)
*x, (2*b)/(a + b)] + (2*I)*(a^2 - b^2)*B*Sqrt[(a + b*Cosh[x])/(a + b)]*Elli
pticF[(I/2)*x, (2*b)/(a + b)] + 2*b*B*(a + b*Cosh[x])*Sinh[x])/(3*b*Sqrt[a
+ b*Cosh[x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 612 vs. 2(158) = 316.
time = 1.42, size = 613, normalized size = 4.44

method	result
default	$\frac{2\left(4B\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}}\,\left(\sinh^4\left(\frac{x}{2}\right)\right)b+2B\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}}\,\left(\sinh^2\left(\frac{x}{2}\right)\right)a+2B\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}}\,\left(\sinh^2\left(\frac{x}{2}\right)\right)b+3Aa\sqrt{\frac{2b(\mathrm{si}}{}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(x))^(1/2)*(A+B*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(4*B*cosh(1/2*x)*(-2*b/(a-b))^(1/2)*sinh(1/2*x)^4*b+2*B*cosh(1/2*x)*(-2
*b/(a-b))^(1/2)*sinh(1/2*x)^2*a+2*B*cosh(1/2*x)*(-2*b/(a-b))^(1/2)*sinh(1/2
*x)^2*b+3*A*a*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^
(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))+3*A*
b*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*Ellipt
icF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))-6*A*(2*b/(a-b)*s
inh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(cosh(1/2*x
)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))*b+B*a*(2*b/(a-b)*sinh(1/2*x)^2
+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b
))^(1/2),1/2*(-2*(a-b)/b)^(1/2))+B*b*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^
(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(
-2*(a-b)/b)^(1/2))-2*B*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1
/2*x)^2)^(1/2)*EllipticE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1
/2))*a)*((2*b*cosh(1/2*x)^2+a-b)*sinh(1/2*x)^2)^(1/2)/(-2*b/(a-b))^(1/2)/(2
*sinh(1/2*x)^4*b+(a+b)*sinh(1/2*x)^2)^(1/2)/sinh(1/2*x)/(2*b*sinh(1/2*x)^2+
a+b)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(1/2)*(A+B*cosh(x)),x, algorithm="maxima")
```

```
[Out] integrate((B*cosh(x) + A)*sqrt(b*cosh(x) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 325, normalized size = 2.36

$$-\frac{2\left(\sqrt{2}\,(2\,Ba^2-3\,Aab-3\,Bb^2)\cosh\left(x\right)+\sqrt{2}\,(2\,Ba^2-3\,Aab-3\,Bb^2)\sinh\left(x\right)\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3},\frac{3\,b\cosh(x)+3\,b\sinh(x)+2\,a}{3\,b}\right)+6\left(\sqrt{2}\,(Bab+3\,Ab^2)\cosh\left(x\right)+\sqrt{2}\,(Bab+3\,Ab^2)\sinh\left(x\right)\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3},\mathrm{weierstrassPInverse}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3},\frac{3\,b\cosh(x)+3\,b\sinh(x)+2\,a}{3\,b}\right)\right)-3\left(Bb^2\cosh\left(x\right)^2+Bb^2\sinh\left(x\right)^2-Bb^2-2\,(Bab+3\,Ab^2)\cosh\left(x\right)+2\,(Bb^2\cosh\left(x\right)-Bab-3\,Ab^2)\sinh\left(x\right)\right)\sqrt{b\cosh\left(x\right)+a}}{9\,(b^2\cosh\left(x\right)+b^2\sinh\left(x\right))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(1/2)*(A+B*cosh(x)),x, algorithm="fricas")
```

```
[Out] -1/9*(2*(sqrt(2)*(2*B*a^2 - 3*A*a*b - 3*B*b^2)*cosh(x) + sqrt(2)*(2*B*a^2 -
 3*A*a*b - 3*B*b^2)*sinh(x))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2
)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b
) + 6*(sqrt(2)*(B*a*b + 3*A*b^2)*cosh(x) + sqrt(2)*(B*a*b + 3*A*b^2)*sinh(x
))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)
```

```
/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/
b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)) - 3*(B*b^2*cosh(x)^2 + B*b^2
*sinh(x)^2 - B*b^2 - 2*(B*a*b + 3*A*b^2)*cosh(x) + 2*(B*b^2*cosh(x) - B*a*b
 - 3*A*b^2)*sinh(x))*sqrt(b*cosh(x) + a))/(b^2*cosh(x) + b^2*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\cosh{(x)})\sqrt{a + b\cosh{(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))**(1/2)*(A+B*cosh(x)),x)
[Out] Integral((A + B*cosh(x))*sqrt(a + b*cosh(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(1/2)*(A+B*cosh(x)),x, algorithm="giac")
[Out] integrate((B*cosh(x) + A)*sqrt(b*cosh(x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B\cosh(x))\,\sqrt{a + b\cosh{(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))*(a + b*cosh(x))^(1/2),x)
[Out] int((A + B*cosh(x))*(a + b*cosh(x))^(1/2), x)
```

3.110 $\int \frac{A+B\cosh(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=60

$$\frac{Bx}{b}+\frac{2(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b\sqrt{a+b}}$$

```
[Out] B*x/b+2*(A*b-B*a)*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/b/(a-b)^(1/2
)/(a+b)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.200$, Rules used = {2814, 2738, 214}

$$\frac{2(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\,\sqrt{a+b}}+\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(a + b*Cosh[x]),x]
```

```
[Out] (B*x)/b + (2*(A*b - a*B)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqr
t[a - b]*b*Sqrt[a + b])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cosh(x)}{a+b\cosh(x)}\,dx &= \frac{Bx}{b} - \frac{(-Ab+aB)\int \frac{1}{a+b\cosh(x)}\,dx}{b}\\ &= \frac{Bx}{b} - \frac{(2(-Ab+aB))\text{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b}\\ &= \frac{Bx}{b} + \frac{2(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b\sqrt{a+b}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 59, normalized size = 0.98

$$\frac{Bx}{b} + \frac{2(-Ab+aB)\text{ArcTan}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{b\sqrt{-a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(a + b*Cosh[x]),x]
[Out] (B*x)/b + (2*(-(A*b) + a*B)*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/(
b*Sqrt[-a^2 + b^2])
```

Maple [A]
time = 0.45, size = 73, normalized size = 1.22

method	result
default	$-\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b} - \frac{2(-Ab+Ba)\,\text{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b\sqrt{(a+b)(a-b)}} + \frac{B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b}$
risch	$\frac{Bx}{b} + \frac{\ln\left(e^x+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)A}{\sqrt{a^2-b^2}} - \frac{\ln\left(e^x+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)Ba}{\sqrt{a^2-b^2}\,b} - \frac{\ln\left(e^x+\frac{a\sqrt{a^2-b^2}+a^2-b^2}{b\sqrt{a^2-b^2}}\right)A}{\sqrt{a^2-b^2}} + \ln\Big($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
[Out] -B/b*ln(tanh(1/2*x)-1)-2/b*(-A*b+B*a)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan
h(1/2*x)/((a+b)*(a-b))^(1/2))+B/b*ln(tanh(1/2*x)+1)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.39, size = 240, normalized size = 4.00

$$\left[-\frac{(Ba-Ab)\sqrt{a^2-b^2}\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\,(b^2\cosh(x)+ab)\sinh(x)-2\,\sqrt{a^2-b^2}\,(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right)-(Ba^2-Bb^2)x}{a^2b-b^3},\ \frac{2\,(Ba-Ab)\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\cosh(x)+b\sinh(x)+a)}{a^2-b^2}\right)+(Ba^2-Bb^2)x}{a^2b-b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [-((B*a - A*b)*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*c
osh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b
*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*
cosh(x) + a)*sinh(x) + b)) - (B*a^2 - B*b^2)*x)/(a^2*b - b^3), (2*(B*a - A*
b)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a
^2 - b^2)) + (B*a^2 - B*b^2)*x)/(a^2*b - b^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 403 vs. 2(49) = 98.
time = 17.21, size = 403, normalized size = 6.72

$$\begin{cases} \tilde{\infty}\left(2A\operatorname{atan}\left(\tanh\left(\frac{x}{2}\right)\right)+Bx\right) & \text{for}\: a=0\wedge b=0 \\ -\frac{A}{b\tanh\left(\frac{x}{2}\right)}+\frac{Bx}{b}-\frac{B}{b\tanh\left(\frac{x}{2}\right)} & \text{for}\: a=-b \\ \frac{Ax+B\sinh(x)}{a} & \text{for}\: b=0 \\ \frac{A\tanh\left(\frac{x}{2}\right)}{b}+\frac{Bx}{b}-\frac{B\tanh\left(\frac{x}{2}\right)}{b} & \text{for}\: a=b \\ -\frac{Ab\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}+\frac{Ab\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}+\frac{Bax\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}+\frac{Ba\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}-\frac{Ba\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}-\frac{Bbx\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)),x)
```

```
[Out] Piecewise((zoo*(2*A*atan(tanh(x/2)) + B*x), Eq(a, 0) & Eq(b, 0)), (-A/(b*ta
nh(x/2)) + B*x/b - B/(b*tanh(x/2)), Eq(a, -b)), ((A*x + B*sinh(x))/a, Eq(b,
 0)), (A*tanh(x/2)/b + B*x/b - B*tanh(x/2)/b, Eq(a, b)), (-A*b*log(-sqrt(a/
(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*s
qrt(a/(a - b) + b/(a - b))) + A*b*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/
2))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) +
B*a*x*sqrt(a/(a - b) + b/(a - b))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*s
qrt(a/(a - b) + b/(a - b))) + B*a*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x
/2))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) -
 B*a*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b) + b/(
```

```
a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) - B*b*x*sqrt(a/(a - b) + b/(a -
 b))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))),
True))
```

Giac [A]
time = 0.41, size = 50, normalized size = 0.83

$$\frac{Bx}{b} - \frac{2\,(Ba - Ab)\arctan\left(\frac{be^{x}+a}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] B*x/b - 2*(B*a - A*b)*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2
)*b)
```

Mupad [B]
time = 1.12, size = 242, normalized size = 4.03

$$\frac{2\,\mathrm{atan}\left(\frac{b^{2}\,\mathrm{e}^{x}\,\sqrt{b^{4}-a^{2}\,b^{2}}\,\left(\frac{2\,\left(A\,b\,\sqrt{b^{4}-a^{2}\,b^{2}}-B\,a\,\sqrt{b^{4}-a^{2}\,b^{2}}\right)}{b^{4}\,\sqrt{b^{4}-a^{2}\,b^{2}}\,\sqrt{{\left(A\,b-B\,a\right)}^{2}}}+\frac{2\,a^{2}\,\sqrt{A^{2}\,b^{2}-2\,A\,B\,a\,b+B^{2}\,a^{2}}}{b^{2}\,\left(b^{4}-a^{2}\,b^{2}\right)\,\left(A\,b-B\,a\right)}\right)}{2}+\frac{a\,b\,\sqrt{A^{2}\,b^{2}-2\,A\,B\,a\,b+B^{2}\,a^{2}}}{\sqrt{b^{4}-a^{2}\,b^{2}}\,\left(A\,b-B\,a\right)}\right)\,\sqrt{A^{2}\,b^{2}-2\,A\,B\,a\,b+B^{2}\,a^{2}}}{\sqrt{b^{4}-a^{2}\,b^{2}}}+\frac{B\,x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(a + b*cosh(x)),x)
```

```
[Out] (2*atan((b^2*exp(x)*(b^4 - a^2*b^2)^(1/2)*((2*(A*b*(b^4 - a^2*b^2)^(1/2) -
B*a*(b^4 - a^2*b^2)^(1/2)))/(b^4*(b^4 - a^2*b^2)^(1/2)*((A*b - B*a)^2)^(1/2
)) + (2*a^2*(A^2*b^2 + B^2*a^2 - 2*A*B*a*b)^(1/2))/(b^2*(b^4 - a^2*b^2)*(A*
b - B*a))))/2 + (a*b*(A^2*b^2 + B^2*a^2 - 2*A*B*a*b)^(1/2))/((b^4 - a^2*b^2
)^(1/2)*(A*b - B*a)))*(A^2*b^2 + B^2*a^2 - 2*A*B*a*b)^(1/2))/(b^4 - a^2*b^2
)^(1/2) + (B*x)/b
```

3.111 $\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^2}\,dx$

Optimal. Leaf size=82

$$\frac{2(aA-bB)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}-\frac{(Ab-aB)\sinh(x)}{(a^2-b^2)\,(a+b\cosh(x))}$$

```
[Out] 2*(A*a-B*b)*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(3/2)/(a+b)^
(3/2)-(A*b-B*a)*sinh(x)/(a^2-b^2)/(a+b*cosh(x))
```

Rubi [A]
time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2833, 12, 2738, 214}

$$\frac{2(aA-bB)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}-\frac{\sinh(x)(Ab-aB)}{(a^2-b^2)\,(a+b\cosh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(a + b*Cosh[x])^2,x]
```

```
[Out] (2*(a*A - b*B)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/((a - b)^(3/2)
*(a + b)^(3/2)) - ((A*b - a*B)*Sinh[x])/((a^2 - b^2)*(a + b*Cosh[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
```

```
f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^2}\,dx &= -\frac{(Ab-aB)\sinh(x)}{(a^2-b^2)(a+b\cosh(x))}+\frac{\int \frac{-aA+bB}{a+b\cosh(x)}\,dx}{-a^2+b^2}\\
&= -\frac{(Ab-aB)\sinh(x)}{(a^2-b^2)(a+b\cosh(x))}+\frac{(aA-bB)\int \frac{1}{a+b\cosh(x)}\,dx}{a^2-b^2}\\
&= -\frac{(Ab-aB)\sinh(x)}{(a^2-b^2)(a+b\cosh(x))}+\frac{(2(aA-bB))\text{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2-b^2}\\
&= \frac{2(aA-bB)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}-\frac{(Ab-aB)\sinh(x)}{(a^2-b^2)(a+b\cosh(x))}
\end{aligned}
$$

Mathematica [A]
time = 0.14, size = 81, normalized size = 0.99

$$
\frac{2(aA-bB)\text{ArcTan}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\left(-a^2+b^2\right)^{3/2}}+\frac{(-Ab+aB)\sinh(x)}{(a-b)(a+b)(a+b\cosh(x))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(a + b*Cosh[x])^2,x]
```

```
[Out] (2*(a*A - b*B)*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(
3/2) + ((-(A*b) + a*B)*Sinh[x])/((a - b)*(a + b)*(a + b*Cosh[x]))
```

Maple [A]
time = 0.42, size = 108, normalized size = 1.32

method	result
default	$\frac{2(Ab-Ba)\tanh\left(\frac{x}{2}\right)}{(a^2-b^2)\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-b\left(\tanh^2\left(\frac{x}{2}\right)\right)-a-b\right)}+\frac{2(Aa-Bb)\,\text{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}$

risch	$\frac{2(Ab-Ba)(a\,e^{x}+b)}{b(a^2-b^2)(b\,e^{2x}+2a\,e^{x}+b)}+\frac{\ln\left(e^{x}+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)Aa}{\sqrt{a^2-b^2}\,(a+b)(a-b)}-\frac{\ln\left(e^{x}+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)Bb}{\sqrt{a^2-b^2}\,(a+b)(a-b)}-\frac{\ln\left(e^{x}+\frac{a\sqrt{a^2-b^2}}{b\sqrt{a^2-}}\right.}{\sqrt{a^2-b^2}\,(a+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a+b*cosh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(A*b-B*a)/(a^2-b^2)*tanh(1/2*x)/(a*tanh(1/2*x)^2-b*tanh(1/2*x)^2-a-b)+2*(
A*a-B*b)/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(
a-b))^(1/2))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(71) = 142$.
time = 0.39, size = 828, normalized size = 10.10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^2,x, algorithm="fricas")
```

```
[Out] [-(2*B*a^3*b - 2*A*a^2*b^2 - 2*B*a*b^3 + 2*A*b^4 - (A*a*b^2 - B*b^3 + (A*a*
b^2 - B*b^3)*cosh(x)^2 + (A*a*b^2 - B*b^3)*sinh(x)^2 + 2*(A*a^2*b - B*a*b^2
)*cosh(x) + 2*(A*a^2*b - B*a*b^2 + (A*a*b^2 - B*b^3)*cosh(x))*sinh(x))*sqrt
(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^
2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x
) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x
) + b)) + 2*(B*a^4 - A*a^3*b - B*a^2*b^2 + A*a*b^3)*cosh(x) + 2*(B*a^4 - A*
a^3*b - B*a^2*b^2 + A*a*b^3)*sinh(x))/(a^4*b^2 - 2*a^2*b^4 + b^6 + (a^4*b^2
 - 2*a^2*b^4 + b^6)*cosh(x)^2 + (a^4*b^2 - 2*a^2*b^4 + b^6)*sinh(x)^2 + 2*(
a^5*b - 2*a^3*b^3 + a*b^5)*cosh(x) + 2*(a^5*b - 2*a^3*b^3 + a*b^5 + (a^4*b^
2 - 2*a^2*b^4 + b^6)*cosh(x))*sinh(x)), -2*(B*a^3*b - A*a^2*b^2 - B*a*b^3 +
 A*b^4 + (A*a*b^2 - B*b^3 + (A*a*b^2 - B*b^3)*cosh(x)^2 + (A*a*b^2 - B*b^3)
```

```
*sinh(x)^2 + 2*(A*a^2*b - B*a*b^2)*cosh(x) + 2*(A*a^2*b - B*a*b^2 + (A*a*b^
2 - B*b^3)*cosh(x))*sinh(x))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*c
osh(x) + b*sinh(x) + a)/(a^2 - b^2)) + (B*a^4 - A*a^3*b - B*a^2*b^2 + A*a*b
^3)*cosh(x) + (B*a^4 - A*a^3*b - B*a^2*b^2 + A*a*b^3)*sinh(x))/(a^4*b^2 - 2
*a^2*b^4 + b^6 + (a^4*b^2 - 2*a^2*b^4 + b^6)*cosh(x)^2 + (a^4*b^2 - 2*a^2*b
^4 + b^6)*sinh(x)^2 + 2*(a^5*b - 2*a^3*b^3 + a*b^5)*cosh(x) + 2*(a^5*b - 2*
a^3*b^3 + a*b^5 + (a^4*b^2 - 2*a^2*b^4 + b^6)*cosh(x))*sinh(x))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.41, size = 107, normalized size = 1.30

$$\frac{2\,(Aa-Bb)\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{(a^2-b^2)\sqrt{-a^2+b^2}}-\frac{2\,(Ba^2e^x-Aabe^x+Bab-Ab^2)}{(a^2b-b^3)(be^{(2\,x)}+2\,ae^x+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^2,x, algorithm="giac")
```

```
[Out] 2*(A*a - B*b)*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/((a^2 - b^2)*sqrt(-a^2 +
 b^2)) - 2*(B*a^2*e^x - A*a*b*e^x + B*a*b - A*b^2)/((a^2*b - b^3)*(b*e^(2*x
) + 2*a*e^x + b))
```

Mupad [**B**]
time = 1.42, size = 246, normalized size = 3.00

$$\frac{\frac{2\,(A\,b^3-B\,a\,b^2)}{b\,(a^2\,b-b^3)}-\frac{2\,\mathrm{e}^x\,(B\,a^2\,b^2-A\,a\,b^3)}{b^2\,(a^2\,b-b^3)}}{b+2\,a\,\mathrm{e}^x+b\,\mathrm{e}^{2\,x}}+\frac{\ln\left(-\frac{2\,\mathrm{e}^x\,(A\,a-B\,b)}{b\,(a^2-b^2)}-\frac{2\,(b+a\,\mathrm{e}^x)\,(A\,a-B\,b)}{b\,(a+b)^{3/2}\,(a-b)^{3/2}}\right)\,(A\,a-B\,b)}{(a+b)^{3/2}\,(a-b)^{3/2}}-\frac{\ln\left(\frac{2\,(b+a\,\mathrm{e}^x)\,(A\,a-B\,b)}{b\,(a+b)^{3/2}\,(a-b)^{3/2}}-\frac{2\,\mathrm{e}^x\,(A\,a-B\,b)}{b\,(a^2-b^2)}\right)\,(A\,a-B\,b)}{(a+b)^{3/2}\,(a-b)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(a + b*cosh(x))^2,x)
```

```
[Out] ((2*(A*b^3 - B*a*b^2))/(b*(a^2*b - b^3)) - (2*exp(x)*(B*a^2*b^2 - A*a*b^3))
/(b^2*(a^2*b - b^3)))/(b + 2*a*exp(x) + b*exp(2*x)) + (log(- (2*exp(x)*(A*a
 - B*b))/(b*(a^2 - b^2)) - (2*(b + a*exp(x))*(A*a - B*b))/(b*(a + b)^(3/2)*
(a - b)^(3/2)))*(A*a - B*b))/((a + b)^(3/2)*(a - b)^(3/2)) - (log((2*(b + a
*exp(x))*(A*a - B*b))/(b*(a + b)^(3/2)*(a - b)^(3/2)) - (2*exp(x)*(A*a - B*
b))/(b*(a^2 - b^2)))*(A*a - B*b))/((a + b)^(3/2)*(a - b)^(3/2))
```

3.112 $\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^3}\,dx$

Optimal. Leaf size=135

$$\frac{(2a^2A+Ab^2-3abB)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}}-\frac{(Ab-aB)\sinh(x)}{2\left(a^2-b^2\right)(a+b\cosh(x))^2}-\frac{(3aAb-a^2B-2b^2B)\sinh(x}{2\left(a^2-b^2\right)^2(a+b\cosh(x))}$$

```
[Out] (2*A*a^2+A*b^2-3*B*a*b)*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^
(5/2)/(a+b)^(5/2)-1/2*(A*b-B*a)*sinh(x)/(a^2-b^2)/(a+b*cosh(x))^2-1/2*(3*A*
a*b-B*a^2-2*B*b^2)*sinh(x)/(a^2-b^2)^2/(a+b*cosh(x))
```

Rubi [A]
time = 0.13, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2833, 12, 2738, 214}

$$\frac{(2a^2A-3abB+Ab^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}}-\frac{\sinh(x)\left(a^2(-B)+3aAb-2b^2B\right)}{2\left(a^2-b^2\right)^2(a+b\cosh(x))}-\frac{\sinh(x)(Ab-aB)}{2\left(a^2-b^2\right)(a+b\cosh(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(a + b*Cosh[x])^3,x]
```

```
[Out] ((2*a^2*A + A*b^2 - 3*a*b*B)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/
((a - b)^(5/2)*(a + b)^(5/2)) - ((A*b - a*B)*Sinh[x])/(2*(a^2 - b^2)*(a + b
*Cosh[x])^2) - ((3*a*A*b - a^2*B - 2*b^2*B)*Sinh[x])/(2*(a^2 - b^2)^2*(a +
b*Cosh[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^3}\,dx = -\frac{(Ab-aB)\sinh(x)}{2(a^2-b^2)(a+b\cosh(x))^2} - \frac{\int \frac{-2(aA-bB)+(Ab-aB)\cosh(x)}{(a+b\cosh(x))^2}\,dx}{2(a^2-b^2)}$$

$$= -\frac{(Ab-aB)\sinh(x)}{2(a^2-b^2)(a+b\cosh(x))^2} - \frac{(3aAb-a^2B-2b^2B)\sinh(x)}{2(a^2-b^2)^2(a+b\cosh(x))} + \frac{\int \frac{2a^2A+Ab^2-3abB}{a+b\cosh(x)}\,}{2(a^2-b^2)^2}$$

$$= -\frac{(Ab-aB)\sinh(x)}{2(a^2-b^2)(a+b\cosh(x))^2} - \frac{(3aAb-a^2B-2b^2B)\sinh(x)}{2(a^2-b^2)^2(a+b\cosh(x))} + \frac{(2a^2A+Ab^2-3}{2(a}$$

$$= -\frac{(Ab-aB)\sinh(x)}{2(a^2-b^2)(a+b\cosh(x))^2} - \frac{(3aAb-a^2B-2b^2B)\sinh(x)}{2(a^2-b^2)^2(a+b\cosh(x))} + \frac{(2a^2A+Ab^2-3}{}$$

$$= \frac{(2a^2A+Ab^2-3abB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}} - \frac{(Ab-aB)\sinh(x)}{2(a^2-b^2)(a+b\cosh(x))^2} -$$

Mathematica [A]
time = 0.31, size = 134, normalized size = 0.99

$$\frac{1}{2}\left(-\frac{2(2a^2A+Ab^2-3abB)\,\mathrm{ArcTan}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{5/2}} + \frac{(-Ab+aB)\sinh(x)}{(a-b)(a+b)(a+b\cosh(x))^2} + \frac{(-3aAb+a^2B+2b^2B)\sinh(x)}{(a-b)^2(a+b)^2(a+b\cosh(x))}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(a + b*Cosh[x])^3,x]
```

```
[Out] ((-2*(2*a^2*A + A*b^2 - 3*a*b*B)*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2
]])/(-a^2 + b^2)^(5/2) + ((-(A*b) + a*B)*Sinh[x])/((a - b)*(a + b)*(a + b*C
osh[x])^2) + ((-3*a*A*b + a^2*B + 2*b^2*B)*Sinh[x])/((a - b)^2*(a + b)^2*(a
 + b*Cosh[x])))/2
```

Maple [A]
time = 0.48, size = 207, normalized size = 1.53

method	result
default	$-\frac{2\left(-\frac{\left(4Aab+A\,b^{2}-2B\,a^{2}-Bab-2B\,b^{2}\right)\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{2(a-b)\left(a^{2}+2ab+b^{2}\right)}+\frac{\left(4Aab-A\,b^{2}-2B\,a^{2}+Bab-2B\,b^{2}\right)\tanh\left(\frac{x}{2}\right)}{2(a+b)\left(a^{2}-2ab+b^{2}\right)}\right)}{\left(a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-b\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-a-b\right)^{2}}+\frac{\left(2a^{2}A+A\,b^{2}-3Bab\right)\operatorname{arctanh}\left(\frac{}{\sqrt{}}\right.}{\left(a^{4}-2a^{2}b^{2}+b^{4}\right)\sqrt{(a+}}$
risch	$\frac{2A\,a^{2}b^{2}e^{3x}+A\,b^{4}e^{3x}-3Ba\,b^{3}e^{3x}+6A\,a^{3}b\,e^{2x}+3Aa\,b^{3}e^{2x}-2B\,a^{4}e^{2x}-5B\,a^{2}b^{2}e^{2x}-2B\,b^{4}e^{2x}+10A\,a^{2}b^{2}e^{x}-A\,b^{4}e^{x}-4B\,a^{3}b\,e^{x}-5Ba\,b^{3}}{b\left(a^{2}-b^{2}\right)^{2}\left(b\,e^{2x}+2a\,e^{x}+b\right)^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a+b*cosh(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-1/2*(4*A*a*b+A*b^2-2*B*a^2-B*a*b-2*B*b^2)/(a-b)/(a^2+2*a*b+b^2)*tanh(1
/2*x)^3+1/2*(4*A*a*b-A*b^2-2*B*a^2+B*a*b-2*B*b^2)/(a+b)/(a^2-2*a*b+b^2)*tan
h(1/2*x))/(a*tanh(1/2*x)^2-b*tanh(1/2*x)^2-a-b)^2+(2*A*a^2+A*b^2-3*B*a*b)/(
a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-
b))^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1545 vs. 2(120) = 240.
time = 0.45, size = 3166, normalized size = 23.45

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*B*a^4*b^2 - 6*A*a^3*b^3 + 2*B*a^2*b^4 + 6*A*a*b^5 - 4*B*b^6 - 2*(2
*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6)*cosh(x)^3 - 2*(2*
A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6)*sinh(x)^3 + 2*(2*B
*a^6 - 6*A*a^5*b + 3*B*a^4*b^2 + 3*A*a^3*b^3 - 3*B*a^2*b^4 + 3*A*a*b^5 - 2*
```

```
B*b^6)*cosh(x)^2 + 2*(2*B*a^6 - 6*A*a^5*b + 3*B*a^4*b^2 + 3*A*a^3*b^3 - 3*B
*a^2*b^4 + 3*A*a*b^5 - 2*B*b^6 - 3*(2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 +
 3*B*a*b^5 - A*b^6)*cosh(x))*sinh(x)^2 - (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 +
 (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*cosh(x)^4 + (2*A*a^2*b^3 - 3*B*a*b^4 + A
*b^5)*sinh(x)^4 + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*cosh(x)^3 + 4*(2*
A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*cosh(
x))*sinh(x)^3 + 2*(4*A*a^4*b - 6*B*a^3*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^
5)*cosh(x)^2 + 2*(4*A*a^4*b - 6*B*a^3*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5
 + 3*(2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*cosh(x)^2 + 6*(2*A*a^3*b^2 - 3*B*a^2
*b^3 + A*a*b^4)*cosh(x))*sinh(x)^2 + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4
)*cosh(x) + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 + (2*A*a^2*b^3 - 3*B*a*b
^4 + A*b^5)*cosh(x)^3 + 3*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*cosh(x)^2 +
 (4*A*a^4*b - 6*B*a^3*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*cosh(x))*sinh(
x))*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*
a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) +
b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a
)*sinh(x) + b)) + 2*(4*B*a^5*b - 10*A*a^4*b^2 + B*a^3*b^3 + 11*A*a^2*b^4 -
5*B*a*b^5 - A*b^6)*cosh(x) + 2*(4*B*a^5*b - 10*A*a^4*b^2 + B*a^3*b^3 + 11*A
*a^2*b^4 - 5*B*a*b^5 - A*b^6 - 3*(2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3
*B*a*b^5 - A*b^6)*cosh(x)^2 + 2*(2*B*a^6 - 6*A*a^5*b + 3*B*a^4*b^2 + 3*A*a^
3*b^3 - 3*B*a^2*b^4 + 3*A*a*b^5 - 2*B*b^6)*cosh(x))*sinh(x))/(a^6*b^3 - 3*a
^4*b^5 + 3*a^2*b^7 - b^9 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*cosh(x)^
4 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*sinh(x)^4 + 4*(a^7*b^2 - 3*a^5*
b^4 + 3*a^3*b^6 - a*b^8)*cosh(x)^3 + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a
*b^8 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*cosh(x))*sinh(x)^3 + 2*(2*a^
8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9)*cosh(x)^2 + 2*(2*a^8*b - 5*a^6
*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9 + 3*(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9
)*cosh(x)^2 + 6*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*cosh(x))*sinh(x)^
2 + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*cosh(x) + 4*(a^7*b^2 - 3*a^
5*b^4 + 3*a^3*b^6 - a*b^8 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*cosh(x)
^3 + 3*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*cosh(x)^2 + (2*a^8*b - 5*a
^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9)*cosh(x))*sinh(x)), -(B*a^4*b^2 - 3*A*a^
3*b^3 + B*a^2*b^4 + 3*A*a*b^5 - 2*B*b^6 - (2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^
2*b^4 + 3*B*a*b^5 - A*b^6)*cosh(x)^3 - (2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b
^4 + 3*B*a*b^5 - A*b^6)*sinh(x)^3 + (2*B*a^6 - 6*A*a^5*b + 3*B*a^4*b^2 + 3*
A*a^3*b^3 - 3*B*a^2*b^4 + 3*A*a*b^5 - 2*B*b^6)*cosh(x)^2 + (2*B*a^6 - 6*A*a
^5*b + 3*B*a^4*b^2 + 3*A*a^3*b^3 - 3*B*a^2*b^4 + 3*A*a*b^5 - 2*B*b^6 - 3*(2
*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6)*cosh(x))*sinh(x)^
2 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*co
sh(x)^4 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*sinh(x)^4 + 4*(2*A*a^3*b^2 - 3*
B*a^2*b^3 + A*a*b^4)*cosh(x)^3 + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 + (
2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*cosh(x))*sinh(x)^3 + 2*(4*A*a^4*b - 6*B*a^
3*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*cosh(x)^2 + 2*(4*A*a^4*b - 6*B*a^3
*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 + 3*(2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5
)*cosh(x)^2 + 6*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*cosh(x))*sinh(x)^2 +
```

```
4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*cosh(x) + 4*(2*A*a^3*b^2 - 3*B*a^2*
b^3 + A*a*b^4 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*cosh(x)^3 + 3*(2*A*a^3*b^
2 - 3*B*a^2*b^3 + A*a*b^4)*cosh(x)^2 + (4*A*a^4*b - 6*B*a^3*b^2 + 4*A*a^2*b
^3 - 3*B*a*b^4 + A*b^5)*cosh(x))*sinh(x))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^
2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) + (4*B*a^5*b - 10*A*a^4*b
^2 + B*a^3*b^3 + 11*A*a^2*b^4 - 5*B*a*b^5 - A*b^6)*cosh(x) + (4*B*a^5*b - 1
0*A*a^4*b^2 + B*a^3*b^3 + 11*A*a^2*b^4 - 5*B*a*b^5 - A*b^6 - 3*(2*A*a^4*b^2
 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6)*cosh(x)^2 + 2*(2*B*a^6 - 6*
A*a^5*b + 3*B*a^4*b^2 + 3*A*a^3*b^3 - 3*B*a^2*b^4 + 3*A*a*b^5 - 2*B*b^6)*co
sh(x))*sinh(x))/(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + (a^6*b^3 - 3*a^4*b
^5 + 3*a^2*b^7 - b^9)*cosh(x)^4 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*s
inh(x)^4 + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*cosh(x)^3 + 4*(a^7*b
^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9
)*cosh(x))*sinh(x)^3 + 2*(2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9)*
cosh(x)^2 + 2*(2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9 + 3*(a^6*b^3
 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*cosh(x)^2 + 6*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*
b^6 - a*b^8)*cosh(x))*sinh(x)^2 + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^
8)*cosh(x) + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 249 vs. 2(120) = 240.
time = 0.41, size = 249, normalized size = 1.84

$$\frac{(2\,Aa^2-3\,Bab+Ab^2)\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{(a^4-2\,a^2b^2+b^4)\sqrt{-a^2+b^2}}+\frac{2\,Aa^2b^2e^{(3\,x)}-3\,Bab^3e^{(3\,x)}+Ab^4e^{(3\,x)}-2\,Ba^4e^{(2\,x)}+6\,Aa^3be^{(2\,x)}-5\,Ba^2b^2e^{(2\,x)}+3\,Aab^3e^{(2\,x)}-2\,Bb^4e^{(2\,x)}-4\,Ba^3be^x+10\,Aa^2b^2e^x-5\,Bab^3e^x-Ab^4e^x-Ba^2b^2+3\,Aab^3-2\,Bb^4}{(a^4b-2\,a^2b^3+b^5)(be^{(2\,x)}+2\,ae^x+b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^3,x, algorithm="giac")
```

```
[Out] (2*A*a^2 - 3*B*a*b + A*b^2)*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/((a^4 - 2*
a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) + (2*A*a^2*b^2*e^(3*x) - 3*B*a*b^3*e^(3*x)
 + A*b^4*e^(3*x) - 2*B*a^4*e^(2*x) + 6*A*a^3*b*e^(2*x) - 5*B*a^2*b^2*e^(2*x
) + 3*A*a*b^3*e^(2*x) - 2*B*b^4*e^(2*x) - 4*B*a^3*b*e^x + 10*A*a^2*b^2*e^x
- 5*B*a*b^3*e^x - A*b^4*e^x - B*a^2*b^2 + 3*A*a*b^3 - 2*B*b^4)/((a^4*b - 2*
a^2*b^3 + b^5)*(b*e^(2*x) + 2*a*e^x + b)^2)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\cosh(x)}{\left(a+b\cosh\left(x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(a + b*cosh(x))^3,x)
```

```
[Out] int((A + B*cosh(x))/(a + b*cosh(x))^3, x)
```

3.113 $\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^4}\,dx$

Optimal. Leaf size=197

$$\frac{(2a^3A+3aAb^2-4a^2bB-b^3B)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}}-\frac{(Ab-aB)\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^3}-\frac{(5aAb-2a^2B-3}{6(a^2-b^2)^2(a+}$$

```
[Out] (2*A*a^3+3*A*a*b^2-4*B*a^2*b-B*b^3)*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(
1/2))/(a-b)^(7/2)/(a+b)^(7/2)-1/3*(A*b-B*a)*sinh(x)/(a^2-b^2)/(a+b*cosh(x))
^3-1/6*(5*A*a*b-2*B*a^2-3*B*b^2)*sinh(x)/(a^2-b^2)^2/(a+b*cosh(x))^2-1/6*(1
1*A*a^2*b+4*A*b^3-2*B*a^3-13*B*a*b^2)*sinh(x)/(a^2-b^2)^3/(a+b*cosh(x))
```

Rubi [A]
time = 0.26, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2833, 12, 2738, 214}

$$-\frac{\sinh(x)\left(-2a^2B+5aAb-3b^2B\right)}{6\left(a^2-b^2\right)^2(a+b\cosh(x))^2}-\frac{\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^3}+\frac{(2a^3A-4a^2bB+3aAb^2-b^3B)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}}-\frac{\sinh(x)\left(-2a^3B+11a^2Ab-13ab^2B+4Ab^3\right)}{6\left(a^2-b^2\right)^3(a+b\cosh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(a + b*Cosh[x])^4,x]
```

```
[Out] ((2*a^3*A + 3*a*A*b^2 - 4*a^2*b*B - b^3*B)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/
Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)) - ((A*b - a*B)*Sinh[x])/(3*(a^2
 - b^2)*(a + b*Cosh[x])^3) - ((5*a*A*b - 2*a^2*B - 3*b^2*B)*Sinh[x])/(6*(a^
2 - b^2)^2*(a + b*Cosh[x])^2) - ((11*a^2*A*b + 4*A*b^3 - 2*a^3*B - 13*a*b^2
*B)*Sinh[x])/(6*(a^2 - b^2)^3*(a + b*Cosh[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^4}\,dx = -\frac{(Ab-aB)\sinh(x)}{3\,(a^2-b^2)\,(a+b\cosh(x))^3} - \frac{\int \frac{-3(aA-bB)+2(Ab-aB)\cosh(x)}{(a+b\cosh(x))^3}\,dx}{3\,(a^2-b^2)}$$

$$= -\frac{(Ab-aB)\sinh(x)}{3\,(a^2-b^2)\,(a+b\cosh(x))^3} - \frac{(5aAb-2a^2B-3b^2B)\sinh(x)}{6\,(a^2-b^2)^2\,(a+b\cosh(x))^2} + \frac{\int \frac{2(3a^2A+2Ab^2-5\ldots}{\ldots}}{\ldots}$$

$$= -\frac{(Ab-aB)\sinh(x)}{3\,(a^2-b^2)\,(a+b\cosh(x))^3} - \frac{(5aAb-2a^2B-3b^2B)\sinh(x)}{6\,(a^2-b^2)^2\,(a+b\cosh(x))^2} - \frac{(11a^2Ab+4Ab\ldots}{6\,(a^2-\ldots}$$

$$= -\frac{(Ab-aB)\sinh(x)}{3\,(a^2-b^2)\,(a+b\cosh(x))^3} - \frac{(5aAb-2a^2B-3b^2B)\sinh(x)}{6\,(a^2-b^2)^2\,(a+b\cosh(x))^2} - \frac{(11a^2Ab+4Ab\ldots}{6\,(a^2-\ldots}$$

$$= -\frac{(Ab-aB)\sinh(x)}{3\,(a^2-b^2)\,(a+b\cosh(x))^3} - \frac{(5aAb-2a^2B-3b^2B)\sinh(x)}{6\,(a^2-b^2)^2\,(a+b\cosh(x))^2} - \frac{(11a^2Ab+4Ab\ldots}{6\,(a^2-\ldots}$$

$$= \frac{(2a^3A+3aAb^2-4a^2bB-b^3B)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}} - \frac{(Ab-aB)\sinh\ldots}{3\,(a^2-b^2)\,(a+b\cos\ldots}$$

Mathematica [A]

time = 0.61, size = 196, normalized size = 0.99

$$\frac{1}{6}\left(\frac{6(2a^3A+3aAb^2-4a^2bB-b^3B)\,\mathrm{ArcTan}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{7/2}} + \frac{2(-Ab+aB)\sinh(x)}{(a-b)(a+b)(a+b\cosh(x))^3} + \frac{(-5aAb+2a^2B+3b^2B)\sinh(x)}{(a-b)^2(a+b)^2(a+b\cosh(x))^2} + \frac{(-11a^2Ab-4Ab^3+2a^3B+13ab^2B)\sinh(x)}{(a-b)^3(a+b)^3(a+b\cosh(x))}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(a + b*Cosh[x])^4,x]
```

```
[Out] ((6*(2*a^3*A + 3*a*A*b^2 - 4*a^2*b*B - b^3*B)*ArcTan[((a - b)*Tanh[x/2])/Sq
rt[-a^2 + b^2]])/(-a^2 + b^2)^(7/2) + (2*(-(A*b) + a*B)*Sinh[x])/((a - b)*(
a + b)*(a + b*Cosh[x])^3) + ((-5*a*A*b + 2*a^2*B + 3*b^2*B)*Sinh[x])/((a -
b)^2*(a + b)^2*(a + b*Cosh[x])^2) + ((-11*a^2*A*b - 4*A*b^3 + 2*a^3*B + 13*
a*b^2*B)*Sinh[x])/((a - b)^3*(a + b)^3*(a + b*Cosh[x])))/6
```

Maple [A]
time = 0.55, size = 342, normalized size = 1.74

method	result
default	$-\frac{2\left(-\frac{\left(6A\,a^{2}b+3Aa\,b^{2}+2A\,b^{3}-2B\,a^{3}-2B\,a^{2}b-6Ba\,b^{2}-B\,b^{3}\right)\left(\tanh^{5}\left(\frac{x}{2}\right)\right)}{2(a-b)\left(a^{3}+3a^{2}b+3a\,b^{2}+b^{3}\right)}+\frac{2\left(9A\,a^{2}b+A\,b^{3}-3B\,a^{3}-7Ba\,b^{2}\right)\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{3\left(a^{2}+2ab+b^{2}\right)\left(a^{2}-2ab+b^{2}\right)}-\frac{\left(6A\,a^{2}b-3Aa\,b^{2}+2}{2}\right.}{\left(a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-b\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-a-b\right)^{3}}$
risch	$\frac{-8B\,a^{6}e^{3x}+12A\,b^{6}e^{2x}-3B\,b^{6}e^{5x}-13Ba\,b^{5}+11A\,a^{2}b^{4}-2B\,a^{3}b^{3}-66B\,a^{2}b^{4}e^{x}+4A\,b^{6}-24B\,a^{5}b\,e^{2x}-102B\,a^{3}b^{3}e^{2x}-24Ba\,b^{5}e^{2x}+3B\,b}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a+b*cosh(x))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-1/2*(6*A*a^2*b+3*A*a*b^2+2*A*b^3-2*B*a^3-2*B*a^2*b-6*B*a*b^2-B*b^3)/(a
-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tanh(1/2*x)^5+2/3*(9*A*a^2*b+A*b^3-3*B*a^3-7*
B*a*b^2)/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tanh(1/2*x)^3-1/2*(6*A*a^2*b-3*A*a
*b^2+2*A*b^3-2*B*a^3+2*B*a^2*b-6*B*a*b^2+B*b^3)/(a+b)/(a^3-3*a^2*b+3*a*b^2-
b^3)*tanh(1/2*x))/(a*tanh(1/2*x)^2-b*tanh(1/2*x)^2-a-b)^3+(2*A*a^3+3*A*a*b^
2-4*B*a^2*b-B*b^3)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan
h((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3767 vs. 2(181) = 362.
time = 0.47, size = 7603, normalized size = 38.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^4,x, algorithm="fricas")
```

[Out] [-1/6*(4*B*a^5*b^3 - 22*A*a^4*b^4 + 22*B*a^3*b^5 + 14*A*a^2*b^6 - 26*B*a*b^7 + 8*A*b^8 - 6*(2*A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 + B*b^8)*cosh(x)^5 - 6*(2*A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 + B*b^8)*sinh(x)^5 - 30*(2*A*a^6*b^2 - 4*B*a^5*b^3 + A*a^4*b^4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 + B*a*b^7)*cosh(x)^4 - 30*(2*A*a^6*b^2 - 4*B*a^5*b^3 + A*a^4*b^4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 + B*a*b^7 + (2*A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 + B*b^8)*cosh(x))*sinh(x)^4 + 4*(4*B*a^8 - 22*A*a^7*b + 28*B*a^6*b^2 - 19*A*a^5*b^3 + 7*B*a^4*b^4 + 29*A*a^3*b^5 - 39*B*a^2*b^6 + 12*A*a*b^7)*cosh(x)^3 + 4*(4*B*a^8 - 22*A*a^7*b + 28*B*a^6*b^2 - 19*A*a^5*b^3 + 7*B*a^4*b^4 + 29*A*a^3*b^5 - 39*B*a^2*b^6 + 12*A*a*b^7 - 15*(2*A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 + B*b^8)*cosh(x)^2 - 30*(2*A*a^6*b^2 - 4*B*a^5*b^3 + A*a^4*b^4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 + B*a*b^7)*cosh(x))*sinh(x)^3 + 12*(4*B*a^7*b - 17*A*a^6*b^2 + 13*B*a^5*b^3 + 11*A*a^4*b^4 - 13*B*a^3*b^5 + 4*A*a^2*b^6 - 4*B*a*b^7 + 2*A*b^8)*cosh(x)^2 + 12*(4*B*a^7*b - 17*A*a^6*b^2 + 13*B*a^5*b^3 + 11*A*a^4*b^4 - 13*B*a^3*b^5 + 4*A*a^2*b^6 - 4*B*a*b^7 + 2*A*b^8 - 5*(2*A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 + B*b^8)*cosh(x)^3 - 15*(2*A*a^6*b^2 - 4*B*a^5*b^3 + A*a^4*b^4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 + B*a*b^7)*cosh(x)^2 + (4*B*a^8 - 22*A*a^7*b + 28*B*a^6*b^2 - 19*A*a^5*b^3 + 7*B*a^4*b^4 + 29*A*a^3*b^5 - 39*B*a^2*b^6 + 12*A*a*b^7)*cosh(x))*sinh(x)^2 - 3*(2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7 + (2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x)^6 + (2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*sinh(x)^6 + 6*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6)*cosh(x)^5 + 6*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6 + (2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x))*sinh(x)^5 + 3*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x)^4 + 3*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7 + 5*(2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x)^2 + 10*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6)*cosh(x))*sinh(x)^4 + 4*(4*A*a^6*b - 8*B*a^5*b^2 + 12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*a^2*b^5 - 3*B*a*b^6)*cosh(x)^3 + 4*(4*A*a^6*b - 8*B*a^5*b^2 + 12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*a^2*b^5 - 3*B*a*b^6 + 5*(2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x)^3 + 15*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6)*cosh(x)^2 + 3*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x))*sinh(x)^3 + 3*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x)^2 + 3*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7 + 5*(2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x)^4 + 20*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6)*cosh(x)^3 + 6*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x)^2 + 4*(4*A*a^6*b - 8*B*a^5*b^2 + 12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*a^2*b^5 - 3*B*a*b^6)*cosh(x))*sinh(x)^2 + 6*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6)*cosh(x) + 6*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6 + (2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x)^5 + 5*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6)*cosh(x)^4 + 2*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a

```
^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x)^3 + 2*(4*A*a^6*b - 8*B*a^
5*b^2 + 12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*a^2*b^5 - 3*B*a*b^6)*cosh(x)^2 +
(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^
7)*cosh(x))*sinh(x))*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2
*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b
^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) +
 2*(b*cosh(x) + a)*sinh(x) + b)) + 6*(4*B*a^6*b^2 - 20*A*a^5*b^3 + 18*B*a^4
*b^4 + 15*A*a^3*b^5 - 23*B*a^2*b^6 + 5*A*a*b^7 + B*b^8)*cosh(x) + 6*(4*B*a^
6*b^2 - 20*A*a^5*b^3 + 18*B*a^4*b^4 + 15*A*a^3*b^5 - 23*B*a^2*b^6 + 5*A*a*b
^7 + B*b^8 - 5*(2*A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a
*b^7 + B*b^8)*cosh(x)^4 - 20*(2*A*a^6*b^2 - 4*B*a^5*b^3 + A*a^4*b^4 + 3*B*a
^3*b^5 - 3*A*a^2*b^6 + B*a*b^7)*cosh(x)^3 + 2*(4*B*a^8 - 22*A*a^7*b + 28*B*
a^6*b^2 - 19*A*a^5*b^3 + 7*B*a^4*b^4 + 29*A*a^3*b^5 - 39*B*a^2*b^6 + 12*A*a
*b^7)*cosh(x)^2 + 4*(4*B*a^7*b - 17*A*a^6*b^2 + 13*B*a^5*b^3 + 11*A*a^4*b^4
 - 13*B*a^3*b^5 + 4*A*a^2*b^6 - 4*B*a*b^7 + 2*A*b^8)*cosh(x))*sinh(x))/(a^8
*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12 + (a^8*b^4 - 4*a^6*b^6 + 6
*a^4*b^8 - 4*a^2*b^10 + b^12)*cosh(x)^6 + (a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8
- 4*a^2*b^10 + b^12)*sinh(x)^6 + 6*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3
*b^9 + a*b^11)*cosh(x)^5 + 6*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 +
 a*b^11 + (a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*cosh(x))*si
nh(x)^5 + 3*(4*a^10*b^2 - 15*a^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8 + b^12)*cosh
(x)^4 + 3*(4*a^10*b^2 - 15*a^8*b^4 + 20*a^6*b^6...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))**4,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 453 vs. 2(181) = 362.
time = 0.42, size = 453, normalized size = 2.30

$$\frac{\left(2\,Aa^{3}-4\,Ba^{2}b+3\,Aab^{2}-Bb^{3}\right)\arctan\left(\frac{be^{x}+a}{\sqrt{-a^{2}+b^{2}}}\right)}{\left(a^{6}-3\,a^{4}b^{2}+3\,a^{2}b^{4}-b^{6}\right)\sqrt{-a^{2}+b^{2}}}+\frac{6\,Aa^{3}b^{3}e^{(5\,x)}-12\,Ba^{2}b^{4}e^{(5\,x)}+9\,Aab^{5}e^{(5\,x)}-3\,Bb^{6}e^{(5\,x)}+30\,Aa^{4}b^{2}e^{(4\,x)}-60\,Ba^{3}b^{3}e^{(4\,x)}+45\,Aa^{2}b^{4}e^{(4\,x)}-15\,Bab^{5}e^{(4\,x)}-8\,Ba^{6}e^{(3\,x)}+44\,Aa^{5}be^{(3\,x)}-64\,Ba^{4}b^{2}e^{(3\,x)}+82\,Aa^{3}b^{3}e^{(3\,x)}-78\,Ba^{2}b^{4}e^{(3\,x)}+24\,Aab^{5}e^{(3\,x)}-24\,Ba^{5}be^{(2\,x)}+102\,Aa^{4}b^{2}e^{(2\,x)}-102\,Ba^{3}b^{3}e^{(2\,x)}+36\,Aa^{2}b^{4}e^{(2\,x)}-24\,Bab^{5}e^{(2\,x)}+12\,Ab^{6}e^{(2\,x)}-12\,Ba^{4}b^{2}e^{x}+60\,Aa^{3}b^{3}e^{x}-66\,Ba^{2}b^{4}e^{x}+15\,Aab^{5}e^{x}+3\,Bb^{6}e^{x}-2\,Ba^{3}b^{3}+11\,Aa^{2}b^{4}-13\,Bab^{5}+4\,Ab^{6}}{3\,\left(a^{6}b-3\,a^{4}b^{3}+3\,a^{2}b^{5}-b^{7}\right)\left(be^{(2\,x)}+2\,ae^{x}+b\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^4,x, algorithm="giac")
```

```
[Out] (2*A*a^3 - 4*B*a^2*b + 3*A*a*b^2 - B*b^3)*arctan((b*e^x + a)/sqrt(-a^2 + b^
2))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) + 1/3*(6*A*a^3*b
^3*e^(5*x) - 12*B*a^2*b^4*e^(5*x) + 9*A*a*b^5*e^(5*x) - 3*B*b^6*e^(5*x) + 3
0*A*a^4*b^2*e^(4*x) - 60*B*a^3*b^3*e^(4*x) + 45*A*a^2*b^4*e^(4*x) - 15*B*a*
```

```
b^5*e^(4*x) - 8*B*a^6*e^(3*x) + 44*A*a^5*b*e^(3*x) - 64*B*a^4*b^2*e^(3*x) +
 82*A*a^3*b^3*e^(3*x) - 78*B*a^2*b^4*e^(3*x) + 24*A*a*b^5*e^(3*x) - 24*B*a^
5*b*e^(2*x) + 102*A*a^4*b^2*e^(2*x) - 102*B*a^3*b^3*e^(2*x) + 36*A*a^2*b^4*
e^(2*x) - 24*B*a*b^5*e^(2*x) + 12*A*b^6*e^(2*x) - 12*B*a^4*b^2*e^x + 60*A*a
^3*b^3*e^x - 66*B*a^2*b^4*e^x + 15*A*a*b^5*e^x + 3*B*b^6*e^x - 2*B*a^3*b^3
+ 11*A*a^2*b^4 - 13*B*a*b^5 + 4*A*b^6)/((a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^
7)*(b*e^(2*x) + 2*a*e^x + b)^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\cosh(x)}{\left(a+b\cosh\left(x\right)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(a + b*cosh(x))^4,x)
```

```
[Out] int((A + B*cosh(x))/(a + b*cosh(x))^4, x)
```

3.114 $\int \frac{\frac{bB}{a}+B\cosh(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=56

$$\frac{Bx}{b}-\frac{2\sqrt{a-b}\,\sqrt{a+b}\,B\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{ab}$$

```
[Out] B*x/b-2*B*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))*(a-b)^(1/2)*(a+b)^(1
/2)/a/b
```

Rubi [A]
time = 0.06, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2814, 2738, 214}

$$\frac{Bx}{b}-\frac{2B\sqrt{a-b}\,\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Int[((b*B)/a + B*Cosh[x])/(a + b*Cosh[x]),x]
```

```
[Out] (B*x)/b - (2*Sqrt[a - b]*Sqrt[a + b]*B*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt
[a + b]])/(a*b)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\frac{bB}{a}+B\cosh(x)}{a+b\cosh(x)}\,dx = \frac{Bx}{b}-\frac{\left(aB-\frac{b^2B}{a}\right)\int \frac{1}{a+b\cosh(x)}\,dx}{b}$$
$$= \frac{Bx}{b}-\frac{\left(2\left(aB-\frac{b^2B}{a}\right)\right)\mathrm{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b}$$
$$= \frac{Bx}{b}-\frac{2\sqrt{a-b}\,\sqrt{a+b}\,B\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{ab}$$

Mathematica [A]
time = 0.06, size = 56, normalized size = 1.00

$$\frac{B\left(\frac{ax}{b}+\frac{2\sqrt{-a^2+b^2}\,\mathrm{ArcTan}\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{b}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*B)/a + B*Cosh[x])/(a + b*Cosh[x]),x]
```

```
[Out] (B*((a*x)/b + (2*Sqrt[-a^2 + b^2]*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[-a^2 + b
^2]])/b))/a
```

Maple [A]
time = 0.64, size = 81, normalized size = 1.45

method	result	size
default	$\frac{2B\left(\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2b}-\frac{\left(a^2-b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b\sqrt{(a+b)(a-b)}}-\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b}\right)}{a}$	81
risch	$\frac{Bx}{b}+\frac{\sqrt{a^2-b^2}\,B\ln\left(e^x+\frac{a+\sqrt{a^2-b^2}}{b}\right)}{ba}-\frac{\sqrt{a^2-b^2}\,B\ln\left(e^x-\frac{-a+\sqrt{a^2-b^2}}{b}\right)}{ba}$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*B/a+B*cosh(x))/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*B/a*(1/2*a/b*ln(tanh(1/2*x)+1)-(a^2-b^2)/b/((a+b)*(a-b))^(1/2)*arctanh((a
-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))-1/2*a/b*ln(tanh(1/2*x)-1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*cosh(x))/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.40, size = 190, normalized size = 3.39

$$\left[\frac{Bax + \sqrt{a^2-b^2}\,B\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\,(b^2\cosh(x)+ab)\sinh(x)+2\,\sqrt{a^2-b^2}\,(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right)}{ab}, \frac{Bax+2\,\sqrt{-a^2+b^2}\,B\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\cosh(x)+b\sinh(x)+a)}{a^2-b^2}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*cosh(x))/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [(B*a*x + sqrt(a^2 - b^2)*B*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh
(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*co
sh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cos
h(x) + a)*sinh(x) + b)))/(a*b), (B*a*x + 2*sqrt(-a^2 + b^2)*B*arctan(-sqrt(
-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)))/(a*b)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(44) = 88$.
time = 18.33, size = 168, normalized size = 3.00

$$\begin{cases} \text{NaN} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{B\sinh(x)}{a} & \text{for}\: b = 0 \\ \frac{Bx}{b} & \text{for}\: a = -b \vee a = b \\ \frac{Bx}{b} + \frac{B\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{b\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} - \frac{B\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{b\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} + \frac{B\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{a\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} - \frac{B\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{a\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*cosh(x))/(a+b*cosh(x)),x)
```

```
[Out] Piecewise((nan, Eq(a, 0) & Eq(b, 0)), (B*sinh(x)/a, Eq(b, 0)), (B*x/b, Eq(a
, b) | Eq(a, -b)), (B*x/b + B*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))
/(b*sqrt(a/(a - b) + b/(a - b))) - B*log(sqrt(a/(a - b) + b/(a - b)) + tanh
(x/2))/(b*sqrt(a/(a - b) + b/(a - b))) + B*log(-sqrt(a/(a - b) + b/(a - b))
```

```
+ tanh(x/2))/(a*sqrt(a/(a - b) + b/(a - b))) - B*log(sqrt(a/(a - b) + b/(a
 - b)) + tanh(x/2))/(a*sqrt(a/(a - b) + b/(a - b))), True))
```

Giac [A]
time = 0.41, size = 57, normalized size = 1.02

$$\frac{Bx}{b} - \frac{2\,(Ba^2 - Bb^2)\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*cosh(x))/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] B*x/b - 2*(B*a^2 - B*b^2)*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 +
 b^2)*a*b)
```

Mupad [B]
time = 0.49, size = 205, normalized size = 3.66

$$\frac{2\,\mathrm{atan}\left(\left(\frac{b\,\sqrt{a^2\,b^2}\,\sqrt{B^2\,b^2-B^2\,a^2}}{B\,\left(b^4-a^2\,b^2\right)}+\frac{a\,b^2\,\mathrm{e}^x\,\left(\frac{2\,\sqrt{B^2\,b^2-B^2\,a^2}}{B\,b^2\,\left(b^4-a^2\,b^2\right)}-\frac{2\,\left(B\,a^2\,\sqrt{a^2\,b^2}-B\,b^2\,\sqrt{a^2\,b^2}\right)}{a^2\,b^4\,\sqrt{-B^2\,\left(a^2-b^2\right)}\,\sqrt{a^2\,b^2}}\right)\,\sqrt{a^2\,b^2}}{2}\right)\,\sqrt{B^2\,b^2-B^2\,a^2}\right)}{\sqrt{a^2\,b^2}}+\frac{B\,x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cosh(x) + (B*b)/a)/(a + b*cosh(x)),x)
```

```
[Out] (2*atan((b*(a^2*b^2)^(1/2)*(B^2*b^2 - B^2*a^2)^(1/2))/(B*(b^4 - a^2*b^2)) +
 (a*b^2*exp(x)*((2*(B^2*b^2 - B^2*a^2)^(1/2))/(B*b^2*(b^4 - a^2*b^2)) - (2*
(B*a^2*(a^2*b^2)^(1/2) - B*b^2*(a^2*b^2)^(1/2)))/(a^2*b^4*(-B^2*(a^2 - b^2)
)^(1/2)*(a^2*b^2)^(1/2)))*(a^2*b^2)^(1/2))/2)*(B^2*b^2 - B^2*a^2)^(1/2))/(a
^2*b^2)^(1/2) + (B*x)/b
```

3.115 $\int \frac{\frac{aB}{b}+B\cosh(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=6

$$\frac{Bx}{b}$$

```
[Out] B*x/b
```

Rubi [A]
time = 0.00, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 8}

$$\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B)/b + B*Cosh[x])/(a + b*Cosh[x]),x]
```

```
[Out] (B*x)/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rubi steps

$$\begin{aligned}\int \frac{\frac{aB}{b}+B\cosh(x)}{a+b\cosh(x)}\,dx &= \frac{B\int 1\,dx}{b}\\ &= \frac{Bx}{b}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 6, normalized size = 1.00

$$\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B)/b + B*Cosh[x])/(a + b*Cosh[x]),x]
```

```
[Out] (B*x)/b
```

Maple [A]
time = 0.34, size = 7, normalized size = 1.17

method	result	size
default	$\frac{Bx}{b}$	7
risch	$\frac{Bx}{b}$	7

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B/b+B*cosh(x))/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] B*x/b
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B/b+B*cosh(x))/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.39, size = 6, normalized size = 1.00

$$\frac{Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B/b+B*cosh(x))/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] B*x/b
```

Sympy [A]
time = 0.16, size = 3, normalized size = 0.50

$$\frac{Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B/b+B*cosh(x))/(a+b*cosh(x)),x)
```

```
[Out] B*x/b
```

Giac [A]
time = 0.41, size = 6, normalized size = 1.00

$$\frac{Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B/b+B*cosh(x))/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] B*x/b
```

Mupad [B]
time = 0.02, size = 6, normalized size = 1.00

$$\frac{B\,x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cosh(x) + (B*a)/b)/(a + b*cosh(x)),x)
```

```
[Out] (B*x)/b
```

3.116 $\int \frac{a+b\cosh(x)}{(b+a\cosh(x))^2} \, dx$

Optimal. Leaf size=11

$$\frac{\sinh(x)}{b+a\cosh(x)}$$

```
[Out] sinh(x)/(b+a*cosh(x))
```

Rubi [A]
time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2833, 8}

$$\frac{\sinh(x)}{a\cosh(x)+b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x])/(b + a*Cosh[x])^2,x]
```

```
[Out] Sinh[x]/(b + a*Cosh[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\cosh(x)}{(b+a\cosh(x))^2}\,dx &= \frac{\sinh(x)}{b+a\cosh(x)}+\frac{\int 0\,dx}{a^2-b^2}\\ &= \frac{\sinh(x)}{b+a\cosh(x)}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 11, normalized size = 1.00

$$\frac{\sinh(x)}{b+a\cosh(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x])/(b + a*Cosh[x])^2,x]
```

```
[Out] Sinh[x]/(b + a*Cosh[x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(11) = 22$.
time = 0.36, size = 29, normalized size = 2.64

method	result	size
risch	$-\frac{2(e^{x}b+a)}{a(e^{2x}a+2\,e^{x}b+a)}$	27
default	$\frac{2\tanh\left(\frac{x}{2}\right)}{a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-b\left(\tanh^{2}\left(\frac{x}{2}\right)\right)+a+b}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(x))/(b+a*cosh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*tanh(1/2*x)/(a*tanh(1/2*x)^2-b*tanh(1/2*x)^2+a+b)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))/(b+a*cosh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(11) = 22$.
time = 0.37, size = 54, normalized size = 4.91

$$-\frac{2\left(b\cosh\left(x\right)+b\sinh\left(x\right)+a\right)}{a^{2}\cosh\left(x\right)^{2}+a^{2}\sinh\left(x\right)^{2}+2\,ab\cosh\left(x\right)+a^{2}+2\left(a^{2}\cosh\left(x\right)+ab\right)\sinh\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))/(b+a*cosh(x))^2,x, algorithm="fricas")
```

```
[Out] -2*(b*cosh(x) + b*sinh(x) + a)/(a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(
x) + a^2 + 2*(a^2*cosh(x) + a*b)*sinh(x))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(8) = 16$.
time = 102.95, size = 26, normalized size = 2.36

$$\frac{2\tanh\left(\frac{x}{2}\right)}{a\tanh^{2}\left(\frac{x}{2}\right) + a - b\tanh^{2}\left(\frac{x}{2}\right) + b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))/(b+a*cosh(x))**2,x)
```

```
[Out] 2*tanh(x/2)/(a*tanh(x/2)**2 + a - b*tanh(x/2)**2 + b)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(11) = 22$.
time = 0.41, size = 26, normalized size = 2.36

$$-\frac{2\,(be^{x} + a)}{(ae^{(2\,x)} + 2\,be^{x} + a)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))/(b+a*cosh(x))^2,x, algorithm="giac")
```

```
[Out] -2*(b*e^x + a)/((a*e^(2*x) + 2*b*e^x + a)*a)
```

Mupad [B]
time = 1.02, size = 51, normalized size = 4.64

$$-\frac{\frac{2\,\mathrm{e}^{x}\,\left(a\,b^{3}-a^{3}\,b\right)}{a\,\left(a\,b^{2}-a^{3}\right)}+2}{a+2\,b\,\mathrm{e}^{x}+a\,\mathrm{e}^{2\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(x))/(b + a*cosh(x))^2,x)
```

```
[Out] -((2*exp(x)*(a*b^3 - a^3*b))/(a*(a*b^2 - a^3)) + 2)/(a + 2*b*exp(x) + a*exp
(2*x))
```

3.117 $\int \frac{3+\cosh(x)}{2-\cosh(x)}\,dx$

Optimal. Leaf size=36

$$-x+\frac{5x}{\sqrt{3}}+\frac{10\tanh^{-1}\left(\frac{\sinh(x)}{2+\sqrt{3}-\cosh(x)}\right)}{\sqrt{3}}$$

```
[Out] -x+5/3*x*3^(1/2)+10/3*arctanh(sinh(x)/(2-cosh(x)+3^(1/2)))*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2814, 2736}

$$\frac{5x}{\sqrt{3}}-x+\frac{10\tanh^{-1}\left(\frac{\sinh(x)}{-\cosh(x)+\sqrt{3}+2}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + Cosh[x])/(2 - Cosh[x]),x]
```

```
[Out] -x + (5*x)/Sqrt[3] + (10*ArcTanh[Sinh[x]/(2 + Sqrt[3] - Cosh[x])])/Sqrt[3]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{3+\cosh(x)}{2-\cosh(x)}\,dx &= -x+5\int \frac{1}{2-\cosh(x)}\,dx\\ &= -x+\frac{5x}{\sqrt{3}}+\frac{10\tanh^{-1}\left(\frac{\sinh(x)}{2+\sqrt{3}-\cosh(x)}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 24, normalized size = 0.67

$$-x+\frac{10\tanh^{-1}\left(\sqrt{3}\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + Cosh[x])/(2 - Cosh[x]),x]
[Out] -x + (10*ArcTanh[Sqrt[3]*Tanh[x/2]])/Sqrt[3]
```

Maple [A]
time = 0.46, size = 32, normalized size = 0.89

method	result	size
default	$\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{10\sqrt{3}\,\operatorname{arctanh}\left(\tanh\left(\frac{x}{2}\right)\sqrt{3}\right)}{3}$	32
risch	$-x+\frac{5\sqrt{3}\,\ln\left(e^{x}-2+\sqrt{3}\right)}{3}-\frac{5\sqrt{3}\,\ln\left(e^{x}-2-\sqrt{3}\right)}{3}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+cosh(x))/(2-cosh(x)),x,method=_RETURNVERBOSE)
[Out] ln(tanh(1/2*x)-1)-ln(tanh(1/2*x)+1)+10/3*3^(1/2)*arctanh(tanh(1/2*x)*3^(1/2
))
```

Maxima [A]
time = 0.48, size = 34, normalized size = 0.94

$$\frac{5}{3}\sqrt{3}\log\left(-\frac{\sqrt{3}-e^{(-x)}+2}{\sqrt{3}+e^{(-x)}-2}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+cosh(x))/(2-cosh(x)),x, algorithm="maxima")
[Out] 5/3*sqrt(3)*log(-(sqrt(3) - e^(-x) + 2)/(sqrt(3) + e^(-x) - 2)) - x
```

Fricas [A]
time = 0.36, size = 45, normalized size = 1.25

$$\frac{5}{3}\sqrt{3}\log\left(-\frac{2\left(\sqrt{3}-2\right)\cosh\left(x\right)-\left(2\sqrt{3}-3\right)\sinh\left(x\right)-\sqrt{3}+2}{\cosh\left(x\right)-2}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+cosh(x))/(2-cosh(x)),x, algorithm="fricas")
```

```
[Out] 5/3*sqrt(3)*log(-(2*(sqrt(3) - 2)*cosh(x) - (2*sqrt(3) - 3)*sinh(x) - sqrt(
3) + 2)/(cosh(x) - 2)) - x
```

Sympy [A]
time = 0.33, size = 44, normalized size = 1.22

$$-x - \frac{5\sqrt{3}\log\left(\tanh\left(\frac{x}{2}\right) - \frac{\sqrt{3}}{3}\right)}{3} + \frac{5\sqrt{3}\log\left(\tanh\left(\frac{x}{2}\right) + \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+cosh(x))/(2-cosh(x)),x)
```

```
[Out] -x - 5*sqrt(3)*log(tanh(x/2) - sqrt(3)/3)/3 + 5*sqrt(3)*log(tanh(x/2) + sqr
t(3)/3)/3
```

Giac [A]
time = 0.41, size = 37, normalized size = 1.03

$$-\frac{5}{3}\sqrt{3}\log\left(\frac{\left|-2\sqrt{3} + 2e^{x} - 4\right|}{\left|2\sqrt{3} + 2e^{x} - 4\right|}\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+cosh(x))/(2-cosh(x)),x, algorithm="giac")
```

```
[Out] -5/3*sqrt(3)*log(abs(-2*sqrt(3) + 2*e^x - 4)/abs(2*sqrt(3) + 2*e^x - 4)) -
x
```

Mupad [B]
time = 0.11, size = 48, normalized size = 1.33

$$\frac{5\sqrt{3}\ln\left(10\,e^{x} + \frac{5\sqrt{3}\,(4\,e^{x}-2)}{3}\right)}{3} - \frac{5\sqrt{3}\ln\left(10\,e^{x} - \frac{5\sqrt{3}\,(4\,e^{x}-2)}{3}\right)}{3} - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(cosh(x) + 3)/(cosh(x) - 2),x)
```

```
[Out] (5*3^(1/2)*log(10*exp(x) + (5*3^(1/2)*(4*exp(x) - 2))/3))/3 - (5*3^(1/2)*lo
g(10*exp(x) - (5*3^(1/2)*(4*exp(x) - 2))/3))/3 - x
```

3.118 $\int \frac{A+B\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx$

Optimal. Leaf size=108

$$-\frac{2iB\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}-\frac{2i(Ab-aB)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{b\sqrt{a+b\cosh(x)}}$$

```
[Out] -2*I*B*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*sinh(1/2*x),2^(1/2)*(b
/(a+b))^(1/2))*(a+b*cosh(x))^(1/2)/b/((a+b*cosh(x))/(a+b))^(1/2)-2*I*(A*b-B
*a)*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh(1/2*x),2^(1/2)*(b/(a
+b))^(1/2))*((a+b*cosh(x))/(a+b))^(1/2)/b/(a+b*cosh(x))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2831, 2742, 2740, 2734, 2732}

$$-\frac{2i(Ab-aB)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{b\sqrt{a+b\cosh(x)}}-\frac{2iB\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/Sqrt[a + b*Cosh[x]],x]
```

```
[Out] ((-2*I)*B*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)])/(b*Sqrt[(a
 + b*Cosh[x])/(a + b)]) - ((2*I)*(A*b - a*B)*Sqrt[(a + b*Cosh[x])/(a + b)]*
EllipticF[(I/2)*x, (2*b)/(a + b)])/(b*Sqrt[a + b*Cosh[x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx = \frac{B\int \sqrt{a+b\cosh(x)}\,dx}{b} + \frac{(Ab-aB)\int \frac{1}{\sqrt{a+b\cosh(x)}}\,dx}{b}$$

$$= \frac{\left(B\sqrt{a+b\cosh(x)}\right)\int \sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}\,dx}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}} + \frac{\left((Ab-aB)\sqrt{\frac{a+b\cosh(x)}{a+b}}\right)}{b\sqrt{a+b\,c}}$$

$$= -\frac{2iB\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}} - \frac{2i(Ab-aB)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{b\sqrt{a+b\cosh(x)}}$$

Mathematica [**A**]
time = 0.35, size = 80, normalized size = 0.74

$$-\frac{2i\sqrt{\frac{a+b\cosh(x)}{a+b}}\left((a+b)BE\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)+(Ab-aB)F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)\right)}{b\sqrt{a+b\cosh(x)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Cosh[x])/Sqrt[a + b*Cosh[x]],x]

[Out] ((-2*I)*Sqrt[(a + b*Cosh[x])/(a + b)]*((a + b)*B*EllipticE[(I/2)*x, (2*b)/(
a + b)] + (A*b - a*B)*EllipticF[(I/2)*x, (2*b)/(a + b)]))/(b*Sqrt[a + b*Cos
h[x]])

Maple [A]
time = 1.83, size = 218, normalized size = 2.02

method	result
default	$\frac{2\left(A\,\mathrm{EllipticF}\left(\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}},\frac{\sqrt{-\frac{2(a-b)}{b}}}{2}\right)+B\,\mathrm{EllipticF}\left(\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}},\frac{\sqrt{-\frac{2(a-b)}{b}}}{2}\right)-2B\,\mathrm{EllipticE}\left(\cosh\left(\frac{x}{2}\right.\right.\right.\cdots}{\sqrt{-\frac{2b}{a-b}}\,\sqrt{2\left(\sinh^4\left(\frac{x}{2}\right)\right)b+(a+b)\left(\mathrm{s}\cdots\right.}}$
risch	$\frac{B\left(b\,\mathrm{e}^{2x}+2a\,\mathrm{e}^{x}+b\right)\sqrt{2}\,\mathrm{e}^{-x}}{b\sqrt{\left(b\,\mathrm{e}^{2x}+2a\,\mathrm{e}^{x}+b\right)\mathrm{e}^{-x}}}+\frac{\left(\frac{4A\left(a+\sqrt{a^2-b^2}\right)\sqrt{\frac{\left(\mathrm{e}^{x}+\frac{a+\sqrt{a^2-b^2}}{b}\right)b}{a+\sqrt{a^2-b^2}}}\sqrt{\frac{\mathrm{e}^{x}-\frac{-a+\sqrt{a^2-b^2}}{b}}{-\frac{a+\sqrt{a^2-b^2}}{b}-\frac{-a+\sqrt{a^2\cdots}}{b}}}\cdots}{\cdots}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cosh(x))/(a+b*cosh(x))^(1/2),x,method=_RETURNVERBOSE)

[Out] 2*(A*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))+B*Ell
ipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))-2*B*EllipticE
(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2)))*(-sinh(1/2*x)^2)^(
1/2)*((2*b*cosh(1/2*x)^2+a-b)/(a-b))^(1/2)*((2*b*cosh(1/2*x)^2+a-b)*sinh(1/
2*x)^2)^(1/2)/(-2*b/(a-b))^(1/2)/(2*sinh(1/2*x)^4*b+(a+b)*sinh(1/2*x)^2)^(1
/2)/sinh(1/2*x)/(2*b*sinh(1/2*x)^2+a+b)^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cosh(x) + A)/sqrt(b*cosh(x) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 183, normalized size = 1.69

$$-\frac{2\left(3\sqrt{2}\,Bb^{\frac{3}{2}}\,\mathrm{weierstrassZeta}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3},\mathrm{weierstrassPInverse}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3},\frac{3\,b\cosh(x)+3\,b\sinh(x)+2\,a}{3\,b}\right)\right)+\sqrt{2}\,(2\,Ba-3\,Ab)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3},\frac{3\,b\cosh(x)+3\,b\sinh(x)+2\,a}{3\,b}\right)+3\,\sqrt{b\cosh\left(x\right)+a}\,Bb\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(3*sqrt(2)*B*b^(3/2)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8
*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*
a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)) + sqrt(2)*(2*
B*a - 3*A*b)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*
a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) + 3*sqrt(b*cos
h(x) + a)*B*b)/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\cosh\left(x\right)}{\sqrt{a+b\cosh\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))**(1/2),x)
```

```
[Out] Integral((A + B*cosh(x))/sqrt(a + b*cosh(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*cosh(x) + A)/sqrt(b*cosh(x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\cosh(x)}{\sqrt{a+b\cosh\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(a + b*cosh(x))^(1/2),x)
```

```
[Out] int((A + B*cosh(x))/(a + b*cosh(x))^(1/2), x)
```

3.119 $\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{3/2}}\,dx$

Optimal. Leaf size=152

$$-\frac{2i(Ab-aB)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{b\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}}-\frac{2iB\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{b\sqrt{a+b\cosh(x)}}-\frac{2(Ab-aB)\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}$$

```
[Out] -2*(A*b-B*a)*sinh(x)/(a^2-b^2)/(a+b*cosh(x))^(1/2)-2*I*(A*b-B*a)*(cosh(1/2*
x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*sinh(1/2*x),2^(1/2)*(b/(a+b))^(1/2))*(a
+b*cosh(x))^(1/2)/b/(a^2-b^2)/((a+b*cosh(x))/(a+b))^(1/2)-2*I*B*(cosh(1/2*x
)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh(1/2*x),2^(1/2)*(b/(a+b))^(1/2))*((a
+b*cosh(x))/(a+b))^(1/2)/b/(a+b*cosh(x))^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2833, 2831, 2742, 2740, 2734, 2732}

$$-\frac{2\sinh(x)(Ab-aB)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}-\frac{2i(Ab-aB)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{b\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}}-\frac{2iB\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{b\sqrt{a+b\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(a + b*Cosh[x])^(3/2),x]
```

```
[Out] ((-2*I)*(A*b - a*B)*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)])/
(b*(a^2 - b^2)*Sqrt[(a + b*Cosh[x])/(a + b)]) - ((2*I)*B*Sqrt[(a + b*Cosh[x
])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/(b*Sqrt[a + b*Cosh[x]]) - (2
*(A*b - a*B)*Sinh[x])/((a^2 - b^2)*Sqrt[a + b*Cosh[x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{3/2}}\,dx = -\frac{2(Ab-aB)\sinh(x)}{(a^2-b^2)\sqrt{a+b\cosh(x)}} - \frac{2\int \frac{\frac{1}{2}(-aA+bB)-\frac{1}{2}(Ab-aB)\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx}{a^2-b^2}$$

$$= -\frac{2(Ab-aB)\sinh(x)}{(a^2-b^2)\sqrt{a+b\cosh(x)}} + \frac{B\int \frac{1}{\sqrt{a+b\cosh(x)}}\,dx}{b} + \frac{(Ab-aB)\int \sqrt{a+b}}{b\,(a^2-b^2)}$$

$$= -\frac{2(Ab-aB)\sinh(x)}{(a^2-b^2)\sqrt{a+b\cosh(x)}} + \frac{\left((Ab-aB)\sqrt{a+b\cosh(x)}\right)\int \sqrt{\frac{a}{a+b}+\frac{b\cosh}{a+}}}{b\,(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

$$= -\frac{2i(Ab-aB)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{b\,(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}} - \frac{2iB\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{b\sqrt{a+b\cosh(x)}} -$$

Mathematica [A]
time = 0.29, size = 133, normalized size = 0.88

$$\frac{2i(a+b)(-Ab+aB)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right) - 2i(a^2-b^2)\,B\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right) + 2b(-Ab+aB)\sinh(x)}{(a-b)b(a+b)\sqrt{a+b\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(a + b*Cosh[x])^(3/2),x]
```

```
[Out] ((2*I)*(a + b)*(-(A*b) + a*B)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticE[(I/2)
*x, (2*b)/(a + b)] - (2*I)*(a^2 - b^2)*B*Sqrt[(a + b*Cosh[x])/(a + b)]*Elli
pticF[(I/2)*x, (2*b)/(a + b)] + 2*b*(-(A*b) + a*B)*Sinh[x])/((a - b)*b*(a +
 b)*Sqrt[a + b*Cosh[x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 484 vs. $2(178) = 356$.
time = 1.66, size = 485, normalized size = 3.19

method	result

default	$$\frac{\sqrt{\left(2b\left(\cosh^2\left(\frac{x}{2}\right)\right)+a-b\right)\left(\sinh^2\left(\frac{x}{2}\right)\right)}\left(\frac{2B\sqrt{\frac{2b\left(\cosh^2\left(\frac{x}{2}\right)\right)+a-b}{a-b}}\sqrt{-\left(\sinh^2\left(\frac{x}{2}\right)\right)}\,\text{EllipticF}\left(\cosh\left(\frac{x}{2}\right)\sqrt{-\dots}\right.}{b\sqrt{-\frac{2b}{a-b}}\sqrt{2\left(\sinh^4\left(\frac{x}{2}\right)\right)b+(a+b)\left(\sinh^2\left(\dots\right.\right.}}\right.}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a+b*cosh(x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((2*b*cosh(1/2*x)^2+a-b)*sinh(1/2*x)^2)^(1/2)*(2*B/b/(-2*b/(a-b))^(1/2)*((2
*b*cosh(1/2*x)^2+a-b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)/(2*sinh(1/2*x)^4*
b+(a+b)*sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(
(-2*a+2*b)/b)^(1/2))-2*(A*b-B*a)/b/sinh(1/2*x)^2/(2*b*sinh(1/2*x)^2+a+b)/(-
2*b/(a-b))^(1/2)/(a^2-b^2)*(2*sinh(1/2*x)^4*b+(a+b)*sinh(1/2*x)^2)^(1/2)*(2
*cosh(1/2*x)*(-2*b/(a-b))^(1/2)*sinh(1/2*x)^2*b-(-sinh(1/2*x)^2)^(1/2)*(2*b
/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^
(1/2),1/2*((-2*a+2*b)/b)^(1/2))*a-(-sinh(1/2*x)^2)^(1/2)*(2*b/(a-b)*sinh(1/
2*x)^2+(a+b)/(a-b))^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*((-2
*a+2*b)/b)^(1/2))*b+2*(-sinh(1/2*x)^2)^(1/2)*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)
/(a-b))^(1/2)*EllipticE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*((-2*a+2*b)/b)^(
1/2))*b))/sinh(1/2*x)/(2*b*sinh(1/2*x)^2+a+b)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cosh(x) + A)/(b*cosh(x) + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 639, normalized size = 4.20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*((sqrt(2)*(2*B*a^2*b + A*a*b^2 - 3*B*b^3)*cosh(x)^2 + sqrt(2)*(2*B*a^2*
b + A*a*b^2 - 3*B*b^3)*sinh(x)^2 + 2*sqrt(2)*(2*B*a^3 + A*a^2*b - 3*B*a*b^2
```

```
)*cosh(x) + 2*(sqrt(2)*(2*B*a^2*b + A*a*b^2 - 3*B*b^3)*cosh(x) + sqrt(2)*(2
*B*a^3 + A*a^2*b - 3*B*a*b^2))*sinh(x) + sqrt(2)*(2*B*a^2*b + A*a*b^2 - 3*B
*b^3))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 -
9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) + 3*(sqrt(2)*(B*a*b^
2 - A*b^3)*cosh(x)^2 + sqrt(2)*(B*a*b^2 - A*b^3)*sinh(x)^2 + 2*sqrt(2)*(B*a
^2*b - A*a*b^2)*cosh(x) + 2*(sqrt(2)*(B*a*b^2 - A*b^3)*cosh(x) + sqrt(2)*(B
*a^2*b - A*a*b^2))*sinh(x) + sqrt(2)*(B*a*b^2 - A*b^3))*sqrt(b)*weierstrass
Zeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInve
rse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cosh(x)
+ 3*b*sinh(x) + 2*a)/b)) + 6*((B*a*b^2 - A*b^3)*cosh(x)^2 + (B*a*b^2 - A*b^
3)*sinh(x)^2 + (B*a^2*b - A*a*b^2)*cosh(x) + (B*a^2*b - A*a*b^2 + 2*(B*a*b^
2 - A*b^3)*cosh(x))*sinh(x))*sqrt(b*cosh(x) + a))/(a^2*b^3 - b^5 + (a^2*b^3
 - b^5)*cosh(x)^2 + (a^2*b^3 - b^5)*sinh(x)^2 + 2*(a^3*b^2 - a*b^4)*cosh(x)
 + 2*(a^3*b^2 - a*b^4 + (a^2*b^3 - b^5)*cosh(x))*sinh(x))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*cosh(x) + A)/(b*cosh(x) + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(a + b*cosh(x))^(3/2),x)
```

```
[Out] int((A + B*cosh(x))/(a + b*cosh(x))^(3/2), x)
```

3.120 $\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{5/2}}\,dx$

Optimal. Leaf size=231

$$-\frac{2i(4aAb-a^2B-3b^2B)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{3b\left(a^2-b^2\right)^2\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\frac{2i(Ab-aB)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{3b\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}-\frac{2(Ab-}{3\left(a^2-b^2\right)}$$

```
[Out] -2/3*(A*b-B*a)*sinh(x)/(a^2-b^2)/(a+b*cosh(x))^(3/2)-2/3*(4*A*a*b-B*a^2-3*B
*b^2)*sinh(x)/(a^2-b^2)^2/(a+b*cosh(x))^(1/2)-2/3*I*(4*A*a*b-B*a^2-3*B*b^2)
*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*sinh(1/2*x),2^(1/2)*(b/(a+b)
)^(1/2))*(a+b*cosh(x))^(1/2)/b/(a^2-b^2)^2/((a+b*cosh(x))/(a+b))^(1/2)+2/3*
I*(A*b-B*a)*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh(1/2*x),2^(1/
2)*(b/(a+b))^(1/2))*((a+b*cosh(x))/(a+b))^(1/2)/b/(a^2-b^2)/(a+b*cosh(x))^(
1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2833, 2831, 2742, 2740, 2734, 2732}

$$-\frac{2\sinh(x)\left(a^2(-B)+4aAb-3b^2B\right)}{3\left(a^2-b^2\right)^2\sqrt{a+b\cosh(x)}}-\frac{2\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}}+\frac{2i(Ab-aB)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{3b\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}-\frac{2i\left(a^2(-B)+4aAb-3b^2B\right)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{3b\left(a^2-b^2\right)^2\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(a + b*Cosh[x])^(5/2),x]
```

```
[Out] (((-2*I)/3)*(4*a*A*b - a^2*B - 3*b^2*B)*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)
*x, (2*b)/(a + b)])/(b*(a^2 - b^2)^2*Sqrt[(a + b*Cosh[x])/(a + b)]) + (((2*
I)/3)*(A*b - a*B)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a
 + b)])/(b*(a^2 - b^2)*Sqrt[a + b*Cosh[x]]) - (2*(A*b - a*B)*Sinh[x])/(3*(a
^2 - b^2)*(a + b*Cosh[x])^(3/2)) - (2*(4*a*A*b - a^2*B - 3*b^2*B)*Sinh[x])/
(3*(a^2 - b^2)^2*Sqrt[a + b*Cosh[x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{5/2}}\,dx = -\frac{2(Ab-aB)\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}} - \frac{2\int \frac{-\frac{3}{2}(aA-bB)+\frac{1}{2}(Ab-aB)\cosh(x)}{(a+b\cosh(x))^{3/2}}\,dx}{3\left(a^2-b^2\right)}$$

$$= -\frac{2(Ab-aB)\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}} - \frac{2(4aAb-a^2B-3b^2B)\sinh(x)}{3\left(a^2-b^2\right)^2\sqrt{a+b\cosh(x)}} + \frac{4\int \frac{1}{4}(3a^2A+Ab \cdots}{\cdots}$$

$$= -\frac{2(Ab-aB)\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}} - \frac{2(4aAb-a^2B-3b^2B)\sinh(x)}{3\left(a^2-b^2\right)^2\sqrt{a+b\cosh(x)}} - \frac{(Ab-aB)\int \cdots}{3b \cdots}$$

$$= -\frac{2(Ab-aB)\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}} - \frac{2(4aAb-a^2B-3b^2B)\sinh(x)}{3\left(a^2-b^2\right)^2\sqrt{a+b\cosh(x)}} + \frac{\left((4aAb-a^2 \cdots\right.}{\cdots}$$

$$= -\frac{2i(4aAb-a^2B-3b^2B)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{3b\left(a^2-b^2\right)^2\sqrt{\frac{a+b\cosh(x)}{a+b}}} + \frac{2i(Ab-aB)\sqrt{\frac{a+b\cosh \cdots}{a+b \cdots}}}{3b\left(a^2-b^2\right)\sqrt{a+b \cdots}}$$

Mathematica [A]
time = 0.66, size = 172, normalized size = 0.74

$$\frac{2\left(\frac{i\left(\frac{a+b\cosh(x)}{a+b}\right)^{3/2}\left(\left(-4aAb+a^2B+3b^2B\right)E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)-(a-b)(-Ab+aB)F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)\right)}{(a-b)^2b}+\frac{\left(-5a^2Ab+Ab^3+2a^3B+2ab^2B+b\left(-4aAb+a^2B+3b^2B\right)\cosh(x)\right)\sinh(x)}{\left(a^2-b^2\right)^2}\right)}{3(a+b\cosh(x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(a + b*Cosh[x])^(5/2),x]
```

```
[Out] (2*((I*((a + b*Cosh[x])/(a + b))^(3/2)*((-4*a*A*b + a^2*B + 3*b^2*B)*Ellipt
icE[(I/2)*x, (2*b)/(a + b)] - (a - b)*(-(A*b) + a*B)*EllipticF[(I/2)*x, (2*
b)/(a + b)]))/((a - b)^2*b) + ((-5*a^2*A*b + A*b^3 + 2*a^3*B + 2*a*b^2*B +
b*(-4*a*A*b + a^2*B + 3*b^2*B)*Cosh[x])*Sinh[x])/(a^2 - b^2)^2))/(3*(a + b*
Cosh[x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 796 vs. 2(247) = 494.
time = 2.11, size = 797, normalized size = 3.45

method	result

default	$\dfrac{\sqrt{\left(2b\left(\cosh^2\left(\frac{x}{2}\right)\right)+a-b\right)\left(\sinh^2\left(\frac{x}{2}\right)\right)}\left(-\dfrac{2B\sqrt{2\left(\sinh^4\left(\frac{x}{2}\right)\right)b+(a+b)\left(\sinh^2\left(\frac{x}{2}\right)\right)}\left(2\cosh\left(\frac{x}{2}\right)\sqrt{\cdots}\right.}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a+b*cosh(x))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((2*b*cosh(1/2*x)^2+a-b)*sinh(1/2*x)^2)^(1/2)*(-2*B/b/sinh(1/2*x)^2/(2*b*si
nh(1/2*x)^2+a+b)/(-2*b/(a-b))^(1/2)/(a^2-b^2)*(2*sinh(1/2*x)^4*b+(a+b)*sinh
(1/2*x)^2)^(1/2)*(2*cosh(1/2*x)*(-2*b/(a-b))^(1/2)*sinh(1/2*x)^2*b-(-sinh(1
/2*x)^2)^(1/2)*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*EllipticF(cosh(1
/2*x)*(-2*b/(a-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))*a-(-sinh(1/2*x)^2)^(1/2)
*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a
-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))*b+2*(-sinh(1/2*x)^2)^(1/2)*(2*b/(a-b)*
sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*EllipticE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1
/2*((-2*a+2*b)/b)^(1/2))*b)+2*(A*b-B*a)/b*(-1/6/b/(a-b)/(a+b)*cosh(1/2*x)*(
2*sinh(1/2*x)^4*b+(a+b)*sinh(1/2*x)^2)^(1/2)/(cosh(1/2*x)^2+1/2*(a-b)/b)^2-
8/3*sinh(1/2*x)^2*b/(a-b)^2/(a+b)^2*cosh(1/2*x)*a/((2*b*cosh(1/2*x)^2+a-b)*
sinh(1/2*x)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)/(-2*b/(a-b))^(1/
2)*((2*b*cosh(1/2*x)^2+a-b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)/(2*sinh(1/2
*x)^4*b+(a+b)*sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2)
,1/2*((-2*a+2*b)/b)^(1/2))-16/3*a*b/(a+b)^2/(a-b)^2*(-a+b)/(-2*b/(a-b))^(1/
2)*((2*b*cosh(1/2*x)^2+a-b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)/(2*sinh(1/2
*x)^4*b+(a+b)*sinh(1/2*x)^2)^(1/2)/(2*a-2*b)*(EllipticF(cosh(1/2*x)*(-2*b/(
a-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))-EllipticE(cosh(1/2*x)*(-2*b/(a-b))^(1
/2),1/2*((-2*a+2*b)/b)^(1/2)))))/sinh(1/2*x)/(2*b*sinh(1/2*x)^2+a+b)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cosh(x) + A)/(b*cosh(x) + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 2153, normalized size = 9.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/9*((sqrt(2)*(2*B*a^3*b^2 + A*a^2*b^3 - 6*B*a*b^4 + 3*A*b^5)*cosh(x)^4 + s
qrt(2)*(2*B*a^3*b^2 + A*a^2*b^3 - 6*B*a*b^4 + 3*A*b^5)*sinh(x)^4 + 4*sqrt(2
)*(2*B*a^4*b + A*a^3*b^2 - 6*B*a^2*b^3 + 3*A*a*b^4)*cosh(x)^3 + 4*(sqrt(2)*
(2*B*a^3*b^2 + A*a^2*b^3 - 6*B*a*b^4 + 3*A*b^5)*cosh(x) + sqrt(2)*(2*B*a^4*
b + A*a^3*b^2 - 6*B*a^2*b^3 + 3*A*a*b^4))*sinh(x)^3 + 2*sqrt(2)*(4*B*a^5 +
2*A*a^4*b - 10*B*a^3*b^2 + 7*A*a^2*b^3 - 6*B*a*b^4 + 3*A*b^5)*cosh(x)^2 + 2
*(3*sqrt(2)*(2*B*a^3*b^2 + A*a^2*b^3 - 6*B*a*b^4 + 3*A*b^5)*cosh(x)^2 + 6*s
qrt(2)*(2*B*a^4*b + A*a^3*b^2 - 6*B*a^2*b^3 + 3*A*a*b^4)*cosh(x) + sqrt(2)*
(4*B*a^5 + 2*A*a^4*b - 10*B*a^3*b^2 + 7*A*a^2*b^3 - 6*B*a*b^4 + 3*A*b^5))*s
inh(x)^2 + 4*sqrt(2)*(2*B*a^4*b + A*a^3*b^2 - 6*B*a^2*b^3 + 3*A*a*b^4)*cosh
(x) + 4*(sqrt(2)*(2*B*a^3*b^2 + A*a^2*b^3 - 6*B*a*b^4 + 3*A*b^5)*cosh(x)^3
+ 3*sqrt(2)*(2*B*a^4*b + A*a^3*b^2 - 6*B*a^2*b^3 + 3*A*a*b^4)*cosh(x)^2 + s
qrt(2)*(4*B*a^5 + 2*A*a^4*b - 10*B*a^3*b^2 + 7*A*a^2*b^3 - 6*B*a*b^4 + 3*A*
b^5)*cosh(x) + sqrt(2)*(2*B*a^4*b + A*a^3*b^2 - 6*B*a^2*b^3 + 3*A*a*b^4))*s
inh(x) + sqrt(2)*(2*B*a^3*b^2 + A*a^2*b^3 - 6*B*a*b^4 + 3*A*b^5))*sqrt(b)*w
eierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/
3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) + 3*(sqrt(2)*(B*a^2*b^3 - 4*A*a*b^4
+ 3*B*b^5)*cosh(x)^4 + sqrt(2)*(B*a^2*b^3 - 4*A*a*b^4 + 3*B*b^5)*sinh(x)^4
+ 4*sqrt(2)*(B*a^3*b^2 - 4*A*a^2*b^3 + 3*B*a*b^4)*cosh(x)^3 + 4*(sqrt(2)*(B
*a^2*b^3 - 4*A*a*b^4 + 3*B*b^5)*cosh(x) + sqrt(2)*(B*a^3*b^2 - 4*A*a^2*b^3
+ 3*B*a*b^4))*sinh(x)^3 + 2*sqrt(2)*(2*B*a^4*b - 8*A*a^3*b^2 + 7*B*a^2*b^3
- 4*A*a*b^4 + 3*B*b^5)*cosh(x)^2 + 2*(3*sqrt(2)*(B*a^2*b^3 - 4*A*a*b^4 + 3*
B*b^5)*cosh(x)^2 + 6*sqrt(2)*(B*a^3*b^2 - 4*A*a^2*b^3 + 3*B*a*b^4)*cosh(x)
+ sqrt(2)*(2*B*a^4*b - 8*A*a^3*b^2 + 7*B*a^2*b^3 - 4*A*a*b^4 + 3*B*b^5))*si
nh(x)^2 + 4*sqrt(2)*(B*a^3*b^2 - 4*A*a^2*b^3 + 3*B*a*b^4)*cosh(x) + 4*(sqrt
(2)*(B*a^2*b^3 - 4*A*a*b^4 + 3*B*b^5)*cosh(x)^3 + 3*sqrt(2)*(B*a^3*b^2 - 4*
A*a^2*b^3 + 3*B*a*b^4)*cosh(x)^2 + sqrt(2)*(2*B*a^4*b - 8*A*a^3*b^2 + 7*B*a
^2*b^3 - 4*A*a*b^4 + 3*B*b^5)*cosh(x) + sqrt(2)*(B*a^3*b^2 - 4*A*a^2*b^3 +
3*B*a*b^4))*sinh(x) + sqrt(2)*(B*a^2*b^3 - 4*A*a*b^4 + 3*B*b^5))*sqrt(b)*we
ierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierst
rassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b
*cosh(x) + 3*b*sinh(x) + 2*a)/b)) + 6*((B*a^2*b^3 - 4*A*a*b^4 + 3*B*b^5)*co
sh(x)^4 + (B*a^2*b^3 - 4*A*a*b^4 + 3*B*b^5)*sinh(x)^4 + (4*B*a^3*b^2 - 13*A
*a^2*b^3 + 8*B*a*b^4 + A*b^5)*cosh(x)^3 + (4*B*a^3*b^2 - 13*A*a^2*b^3 + 8*B
```

```
*a*b^4 + A*b^5 + 4*(B*a^2*b^3 - 4*A*a*b^4 + 3*B*b^5)*cosh(x))*sinh(x)^3 + (
2*B*a^4*b - 8*A*a^3*b^2 + 7*B*a^2*b^3 - 4*A*a*b^4 + 3*B*b^5)*cosh(x)^2 + (2
*B*a^4*b - 8*A*a^3*b^2 + 7*B*a^2*b^3 - 4*A*a*b^4 + 3*B*b^5 + 6*(B*a^2*b^3 -
 4*A*a*b^4 + 3*B*b^5)*cosh(x)^2 + 3*(4*B*a^3*b^2 - 13*A*a^2*b^3 + 8*B*a*b^4
 + A*b^5)*cosh(x))*sinh(x)^2 - (3*A*a^2*b^3 - 4*B*a*b^4 + A*b^5)*cosh(x) -
(3*A*a^2*b^3 - 4*B*a*b^4 + A*b^5 - 4*(B*a^2*b^3 - 4*A*a*b^4 + 3*B*b^5)*cosh
(x)^3 - 3*(4*B*a^3*b^2 - 13*A*a^2*b^3 + 8*B*a*b^4 + A*b^5)*cosh(x)^2 - 2*(2
*B*a^4*b - 8*A*a^3*b^2 + 7*B*a^2*b^3 - 4*A*a*b^4 + 3*B*b^5)*cosh(x))*sinh(x
))*sqrt(b*cosh(x) + a))/(a^4*b^4 - 2*a^2*b^6 + b^8 + (a^4*b^4 - 2*a^2*b^6 +
 b^8)*cosh(x)^4 + (a^4*b^4 - 2*a^2*b^6 + b^8)*sinh(x)^4 + 4*(a^5*b^3 - 2*a^
3*b^5 + a*b^7)*cosh(x)^3 + 4*(a^5*b^3 - 2*a^3*b^5 + a*b^7 + (a^4*b^4 - 2*a^
2*b^6 + b^8)*cosh(x))*sinh(x)^3 + 2*(2*a^6*b^2 - 3*a^4*b^4 + b^8)*cosh(x)^2
 + 2*(2*a^6*b^2 - 3*a^4*b^4 + b^8 + 3*(a^4*b^4 - 2*a^2*b^6 + b^8)*cosh(x)^2
 + 6*(a^5*b^3 - 2*a^3*b^5 + a*b^7)*cosh(x))*sinh(x)^2 + 4*(a^5*b^3 - 2*a^3*
b^5 + a*b^7)*cosh(x) + 4*(a^5*b^3 - 2*a^3*b^5 + a*b^7 + (a^4*b^4 - 2*a^2*b^
6 + b^8)*cosh(x)^3 + 3*(a^5*b^3 - 2*a^3*b^5 + a*b^7)*cosh(x)^2 + (2*a^6*b^2
 - 3*a^4*b^4 + b^8)*cosh(x))*sinh(x))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))**(5/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^(5/2),x, algorithm="giac")
[Out] integrate((B*cosh(x) + A)/(b*cosh(x) + a)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(a + b*cosh(x))^(5/2),x)
[Out] int((A + B*cosh(x))/(a + b*cosh(x))^(5/2), x)
```

3.121 $\int \left(a\cosh^2(x)\right)^{7/2}\,dx$

Optimal. Leaf size=72

$$\frac{16}{35}a^3\sqrt{a\cosh^2(x)}\,\tanh(x)+\frac{8}{35}a^2\left(a\cosh^2(x)\right)^{3/2}\tanh(x)+\frac{6}{35}a\left(a\cosh^2(x)\right)^{5/2}\tanh(x)+\frac{1}{7}\left(a\cosh^2(x)\right)^{7/2}\tan$$

```
[Out] 8/35*a^2*(a*cosh(x)^2)^(3/2)*tanh(x)+6/35*a*(a*cosh(x)^2)^(5/2)*tanh(x)+1/7
*(a*cosh(x)^2)^(7/2)*tanh(x)+16/35*a^3*(a*cosh(x)^2)^(1/2)*tanh(x)
```

Rubi [A]
time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3282, 3286, 2717}

$$\frac{16}{35}a^3\tanh(x)\sqrt{a\cosh^2(x)}+\frac{8}{35}a^2\tanh(x)\left(a\cosh^2(x)\right)^{3/2}+\frac{1}{7}\tanh(x)\left(a\cosh^2(x)\right)^{7/2}+\frac{6}{35}a\tanh(x)\left(a\cosh^2(x)\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x]^2)^(7/2),x]
```

```
[Out] (16*a^3*Sqrt[a*Cosh[x]^2]*Tanh[x])/35 + (8*a^2*(a*Cosh[x]^2)^(3/2)*Tanh[x])
/35 + (6*a*(a*Cosh[x]^2)^(5/2)*Tanh[x])/35 + ((a*Cosh[x]^2)^(7/2)*Tanh[x])/
7
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3282

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-Cot[e + f*x
])*((b*Sin[e + f*x]^2)^p/(2*f*p)), x] + Dist[b*((2*p - 1)/(2*p)), Int[(b*Si
n[e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && Gt
Q[p, 1]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \left(a\cosh^2(x)\right)^{7/2}\,dx &= \frac{1}{7}\left(a\cosh^2(x)\right)^{7/2}\tanh(x)+\frac{1}{7}(6a)\int \left(a\cosh^2(x)\right)^{5/2}\,dx\\ &= \frac{6}{35}a\left(a\cosh^2(x)\right)^{5/2}\tanh(x)+\frac{1}{7}\left(a\cosh^2(x)\right)^{7/2}\tanh(x)+\frac{1}{35}\left(24a^2\right)\int \left(a\cosh^2(x)\right.\\ &= \frac{8}{35}a^2\left(a\cosh^2(x)\right)^{3/2}\tanh(x)+\frac{6}{35}a\left(a\cosh^2(x)\right)^{5/2}\tanh(x)+\frac{1}{7}\left(a\cosh^2(x)\right)^{7/2}\tanh\\ &= \frac{8}{35}a^2\left(a\cosh^2(x)\right)^{3/2}\tanh(x)+\frac{6}{35}a\left(a\cosh^2(x)\right)^{5/2}\tanh(x)+\frac{1}{7}\left(a\cosh^2(x)\right)^{7/2}\tanh\\ &= \frac{16}{35}a^3\sqrt{a\cosh^2(x)}\,\tanh(x)+\frac{8}{35}a^2\left(a\cosh^2(x)\right)^{3/2}\tanh(x)+\frac{6}{35}a\left(a\cosh^2(x)\right)^{5/2}\text{ta}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 42, normalized size = 0.58

$$\frac{a^3\sqrt{a\cosh^2(x)}\,\operatorname{sech}(x)(1225\sinh(x)+245\sinh(3x)+49\sinh(5x)+5\sinh(7x))}{2240}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x]^2)^(7/2),x]
[Out] (a^3*Sqrt[a*Cosh[x]^2]*Sech[x]*(1225*Sinh[x] + 245*Sinh[3*x] + 49*Sinh[5*x]
 + 5*Sinh[7*x]))/2240
```

Maple [A]
time = 0.82, size = 38, normalized size = 0.53

method	result
default	$\frac{a^4\cosh(x)\sinh(x)\left(5\left(\cosh^6(x)\right)+6\left(\cosh^4(x)\right)+8\left(\cosh^2(x)\right)+16\right)}{35\sqrt{a\left(\cosh^2(x)\right)}}$
risch	$\frac{a^3 e^{8x}\sqrt{a\left(1+e^{2x}\right)^2 e^{-2x}}}{896+896\,e^{2x}}+\frac{7a^3 e^{6x}\sqrt{a\left(1+e^{2x}\right)^2 e^{-2x}}}{640\left(1+e^{2x}\right)}+\frac{7a^3 e^{4x}\sqrt{a\left(1+e^{2x}\right)^2 e^{-2x}}}{128\left(1+e^{2x}\right)}+\frac{35a^3 e^{2x}\sqrt{a\,(1}}{128(1}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^2)^(7/2),x,method=_RETURNVERBOSE)
[Out] 1/35*a^4*cosh(x)*sinh(x)*(5*cosh(x)^6+6*cosh(x)^4+8*cosh(x)^2+16)/(a*cosh(x
)^2)^(1/2)
```

Maxima [A]
time = 0.50, size = 71, normalized size = 0.99

$$\frac{1}{896}\,a^{\frac{7}{2}}e^{(7\,x)}+\frac{7}{640}\,a^{\frac{7}{2}}e^{(5\,x)}+\frac{7}{128}\,a^{\frac{7}{2}}e^{(3\,x)}-\frac{35}{128}\,a^{\frac{7}{2}}e^{(-x)}-\frac{7}{128}\,a^{\frac{7}{2}}e^{(-3\,x)}-\frac{7}{640}\,a^{\frac{7}{2}}e^{(-5\,x)}-\frac{1}{896}\,a^{\frac{7}{2}}e^{(-7\,x)}+\frac{35}{128}\,a^{\frac{7}{2}}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/896*a^(7/2)*e^(7*x) + 7/640*a^(7/2)*e^(5*x) + 7/128*a^(7/2)*e^(3*x) - 35/
128*a^(7/2)*e^(-x) - 7/128*a^(7/2)*e^(-3*x) - 7/640*a^(7/2)*e^(-5*x) - 1/89
6*a^(7/2)*e^(-7*x) + 35/128*a^(7/2)*e^x
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 817 vs. $2(56) = 112$.
time = 0.38, size = 817, normalized size = 11.35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/4480*(70*a^3*cosh(x)*e^x*sinh(x)^13 + 5*a^3*e^x*sinh(x)^14 + 7*(65*a^3*co
sh(x)^2 + 7*a^3)*e^x*sinh(x)^12 + 28*(65*a^3*cosh(x)^3 + 21*a^3*cosh(x))*e^
x*sinh(x)^11 + 7*(715*a^3*cosh(x)^4 + 462*a^3*cosh(x)^2 + 35*a^3)*e^x*sinh(
x)^10 + 70*(143*a^3*cosh(x)^5 + 154*a^3*cosh(x)^3 + 35*a^3*cosh(x))*e^x*sin
h(x)^9 + 35*(429*a^3*cosh(x)^6 + 693*a^3*cosh(x)^4 + 315*a^3*cosh(x)^2 + 35
*a^3)*e^x*sinh(x)^8 + 8*(2145*a^3*cosh(x)^7 + 4851*a^3*cosh(x)^5 + 3675*a^3
*cosh(x)^3 + 1225*a^3*cosh(x))*e^x*sinh(x)^7 + 7*(2145*a^3*cosh(x)^8 + 6468
*a^3*cosh(x)^6 + 7350*a^3*cosh(x)^4 + 4900*a^3*cosh(x)^2 - 175*a^3)*e^x*sin
h(x)^6 + 14*(715*a^3*cosh(x)^9 + 2772*a^3*cosh(x)^7 + 4410*a^3*cosh(x)^5 +
4900*a^3*cosh(x)^3 - 525*a^3*cosh(x))*e^x*sinh(x)^5 + 35*(143*a^3*cosh(x)^1
0 + 693*a^3*cosh(x)^8 + 1470*a^3*cosh(x)^6 + 2450*a^3*cosh(x)^4 - 525*a^3*c
osh(x)^2 - 7*a^3)*e^x*sinh(x)^4 + 140*(13*a^3*cosh(x)^11 + 77*a^3*cosh(x)^9
 + 210*a^3*cosh(x)^7 + 490*a^3*cosh(x)^5 - 175*a^3*cosh(x)^3 - 7*a^3*cosh(x
))*e^x*sinh(x)^3 + 7*(65*a^3*cosh(x)^12 + 462*a^3*cosh(x)^10 + 1575*a^3*cos
h(x)^8 + 4900*a^3*cosh(x)^6 - 2625*a^3*cosh(x)^4 - 210*a^3*cosh(x)^2 - 7*a^
3)*e^x*sinh(x)^2 + 14*(5*a^3*cosh(x)^13 + 42*a^3*cosh(x)^11 + 175*a^3*cosh(
x)^9 + 700*a^3*cosh(x)^7 - 525*a^3*cosh(x)^5 - 70*a^3*cosh(x)^3 - 7*a^3*cos
h(x))*e^x*sinh(x) + (5*a^3*cosh(x)^14 + 49*a^3*cosh(x)^12 + 245*a^3*cosh(x)
^10 + 1225*a^3*cosh(x)^8 - 1225*a^3*cosh(x)^6 - 245*a^3*cosh(x)^4 - 49*a^3*
cosh(x)^2 - 5*a^3)*e^x)*sqrt(a*e^(4*x) + 2*a*e^(2*x) + a)*e^(-x)/(cosh(x)^7
*e^(2*x) + (e^(2*x) + 1)*sinh(x)^7 + cosh(x)^7 + 7*(cosh(x)*e^(2*x) + cosh(
x))*sinh(x)^6 + 21*(cosh(x)^2*e^(2*x) + cosh(x)^2)*sinh(x)^5 + 35*(cosh(x)^
3*e^(2*x) + cosh(x)^3)*sinh(x)^4 + 35*(cosh(x)^4*e^(2*x) + cosh(x)^4)*sinh(
x)^3 + 21*(cosh(x)^5*e^(2*x) + cosh(x)^5)*sinh(x)^2 + 7*(cosh(x)^6*e^(2*x)
+ cosh(x)^6)*sinh(x))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)**2)**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep
```

Giac [A]
time = 0.42, size = 79, normalized size = 1.10

$$\frac{1}{4480}\left(5\,a^3e^{(7\,x)}+49\,a^3e^{(5\,x)}+245\,a^3e^{(3\,x)}+1225\,a^3e^x-\left(1225\,a^3e^{(6\,x)}+245\,a^3e^{(4\,x)}+49\,a^3e^{(2\,x)}+5\,a^3\right)e^{(-7\,x)}\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^2)^(7/2),x, algorithm="giac")
```

```
[Out] 1/4480*(5*a^3*e^(7*x) + 49*a^3*e^(5*x) + 245*a^3*e^(3*x) + 1225*a^3*e^x - (
1225*a^3*e^(6*x) + 245*a^3*e^(4*x) + 49*a^3*e^(2*x) + 5*a^3)*e^(-7*x))*sqrt
(a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a\,{\mathrm{cosh}\left(x\right)}^2\right)}^{7/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^2)^(7/2),x)
```

```
[Out] int((a*cosh(x)^2)^(7/2), x)
```

3.122 $\int \left(a \cosh^2(x)\right)^{5/2} dx$

Optimal. Leaf size=53

$$\frac{8}{15}a^2\sqrt{a\cosh^2(x)}\tanh(x)+\frac{4}{15}a\left(a\cosh^2(x)\right)^{3/2}\tanh(x)+\frac{1}{5}\left(a\cosh^2(x)\right)^{5/2}\tanh(x)$$

```
[Out] 4/15*a*(a*cosh(x)^2)^(3/2)*tanh(x)+1/5*(a*cosh(x)^2)^(5/2)*tanh(x)+8/15*a^2
*(a*cosh(x)^2)^(1/2)*tanh(x)
```

Rubi [A]
time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3282, 3286, 2717}

$$\frac{8}{15}a^2\tanh(x)\sqrt{a\cosh^2(x)}+\frac{1}{5}\tanh(x)\left(a\cosh^2(x)\right)^{5/2}+\frac{4}{15}a\tanh(x)\left(a\cosh^2(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x]^2)^(5/2),x]
```

```
[Out] (8*a^2*Sqrt[a*Cosh[x]^2]*Tanh[x])/15 + (4*a*(a*Cosh[x]^2)^(3/2)*Tanh[x])/15
 + ((a*Cosh[x]^2)^(5/2)*Tanh[x])/5
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3282

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-Cot[e + f*x
])*((b*Sin[e + f*x]^2)^p/(2*f*p)), x] + Dist[b*((2*p - 1)/(2*p)), Int[(b*Si
n[e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] && !IntegerQ[p] && Gt
Q[p, 1]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(a\cosh^2(x)\right)^{5/2}\,dx = \frac{1}{5}\left(a\cosh^2(x)\right)^{5/2}\tanh(x) + \frac{1}{5}(4a)\int \left(a\cosh^2(x)\right)^{3/2}\,dx$$
$$= \frac{4}{15}a\left(a\cosh^2(x)\right)^{3/2}\tanh(x) + \frac{1}{5}\left(a\cosh^2(x)\right)^{5/2}\tanh(x) + \frac{1}{15}\left(8a^2\right)\int \sqrt{a\cosh^2(x)}$$
$$= \frac{4}{15}a\left(a\cosh^2(x)\right)^{3/2}\tanh(x) + \frac{1}{5}\left(a\cosh^2(x)\right)^{5/2}\tanh(x) + \frac{1}{15}\left(8a^2\sqrt{a\cosh^2(x)}\,\text{sec}\right.$$
$$= \frac{8}{15}a^2\sqrt{a\cosh^2(x)}\,\tanh(x) + \frac{4}{15}a\left(a\cosh^2(x)\right)^{3/2}\tanh(x) + \frac{1}{5}\left(a\cosh^2(x)\right)^{5/2}\tanh($$

Mathematica [A]
time = 0.01, size = 36, normalized size = 0.68

$$\frac{1}{240}a^2\sqrt{a\cosh^2(x)}\,\text{sech}(x)(150\sinh(x) + 25\sinh(3x) + 3\sinh(5x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x]^2)^(5/2),x]
[Out] (a^2*Sqrt[a*Cosh[x]^2]*Sech[x]*(150*Sinh[x] + 25*Sinh[3*x] + 3*Sinh[5*x]))/
240
```

Maple [A]
time = 0.78, size = 32, normalized size = 0.60

method	result
default	$\frac{a^3\cosh(x)\sinh(x)\left(3\left(\cosh^4(x)\right)+4\left(\cosh^2(x)\right)+8\right)}{15\sqrt{a\left(\cosh^2(x)\right)}}$
risch	$\frac{a^2 e^{6x}\sqrt{a\left(1+e^{2x}\right)^2 e^{-2x}}}{160+160\,e^{2x}} + \frac{5a^2 e^{4x}\sqrt{a\left(1+e^{2x}\right)^2 e^{-2x}}}{96(1+e^{2x})} + \frac{5a^2 e^{2x}\sqrt{a\left(1+e^{2x}\right)^2 e^{-2x}}}{16(1+e^{2x})} - \frac{5\sqrt{a\left(1+e^{2x}\right.}}{16(1+e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/15*a^3*cosh(x)*sinh(x)*(3*cosh(x)^4+4*cosh(x)^2+8)/(a*cosh(x)^2)^(1/2)
```

Maxima [A]
time = 0.49, size = 53, normalized size = 1.00

$$\frac{1}{160}\,a^{\frac{5}{2}}e^{(5\,x)} + \frac{5}{96}\,a^{\frac{5}{2}}e^{(3\,x)} - \frac{5}{16}\,a^{\frac{5}{2}}e^{(-x)} - \frac{5}{96}\,a^{\frac{5}{2}}e^{(-3\,x)} - \frac{1}{160}\,a^{\frac{5}{2}}e^{(-5\,x)} + \frac{5}{16}\,a^{\frac{5}{2}}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/160*a^(5/2)*e^(5*x) + 5/96*a^(5/2)*e^(3*x) - 5/16*a^(5/2)*e^(-x) - 5/96*a
^(5/2)*e^(-3*x) - 1/160*a^(5/2)*e^(-5*x) + 5/16*a^(5/2)*e^x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 501 vs. $2(41) = 82$.
time = 0.36, size = 501, normalized size = 9.45

$$\frac{\left(30\,a^{2}\cosh(x)e^{x}\sinh(x)^{9}+3\,a^{2}e^{x}\sinh(x)^{10}+5\left(27\,a^{2}\cosh(x)^{2}+5\,a^{2}\right)e^{x}\sinh(x)^{8}+40\left(9\,a^{2}\cosh(x)^{3}+5\,a^{2}\cosh(x)\right)e^{x}\sinh(x)^{7}+10\left(63\,a^{2}\cosh(x)^{4}+70\,a^{2}\cosh(x)^{2}+15\,a^{2}\right)e^{x}\sinh(x)^{6}+4\left(189\,a^{2}\cosh(x)^{5}+350\,a^{2}\cosh(x)^{3}+225\,a^{2}\cosh(x)\right)e^{x}\sinh(x)^{5}+10\left(63\,a^{2}\cosh(x)^{6}+175\,a^{2}\cosh(x)^{4}+225\,a^{2}\cosh(x)^{2}-15\,a^{2}\right)e^{x}\sinh(x)^{4}+40\left(9\,a^{2}\cosh(x)^{7}+35\,a^{2}\cosh(x)^{5}+75\,a^{2}\cosh(x)^{3}-15\,a^{2}\cosh(x)\right)e^{x}\sinh(x)^{3}+5\left(27\,a^{2}\cosh(x)^{8}+140\,a^{2}\cosh(x)^{6}+450\,a^{2}\cosh(x)^{4}-180\,a^{2}\cosh(x)^{2}-5\,a^{2}\right)e^{x}\sinh(x)^{2}+10\left(3\,a^{2}\cosh(x)^{9}+20\,a^{2}\cosh(x)^{7}+90\,a^{2}\cosh(x)^{5}-60\,a^{2}\cosh(x)^{3}-5\,a^{2}\cosh(x)\right)e^{x}\sinh(x)+\left(3\,a^{2}\cosh(x)^{10}+25\,a^{2}\cosh(x)^{8}+150\,a^{2}\cosh(x)^{6}-150\,a^{2}\cosh(x)^{4}-25\,a^{2}\cosh(x)^{2}-3\,a^{2}\right)e^{x}\right)\sqrt{ae^{(4x)}+2\,ae^{(2x)}+a}\,e^{(-x)}}{480\left(\cosh(x)^{5}e^{(2x)}+\left(e^{(2x)}+1\right)\sinh(x)^{5}+\cosh(x)^{5}+5\left(\cosh(x)e^{(2x)}+\cosh(x)\right)\sinh(x)^{4}+10\left(\cosh(x)^{2}e^{(2x)}+\cosh(x)^{2}\right)\sinh(x)^{3}+10\left(\cosh(x)^{3}e^{(2x)}+\cosh(x)^{3}\right)\sinh(x)^{2}+5\left(\cosh(x)^{4}e^{(2x)}+\cosh(x)^{4}\right)\sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/480*(30*a^2*cosh(x)*e^x*sinh(x)^9 + 3*a^2*e^x*sinh(x)^10 + 5*(27*a^2*cosh
(x)^2 + 5*a^2)*e^x*sinh(x)^8 + 40*(9*a^2*cosh(x)^3 + 5*a^2*cosh(x))*e^x*sin
h(x)^7 + 10*(63*a^2*cosh(x)^4 + 70*a^2*cosh(x)^2 + 15*a^2)*e^x*sinh(x)^6 +
4*(189*a^2*cosh(x)^5 + 350*a^2*cosh(x)^3 + 225*a^2*cosh(x))*e^x*sinh(x)^5 +
 10*(63*a^2*cosh(x)^6 + 175*a^2*cosh(x)^4 + 225*a^2*cosh(x)^2 - 15*a^2)*e^x
*sinh(x)^4 + 40*(9*a^2*cosh(x)^7 + 35*a^2*cosh(x)^5 + 75*a^2*cosh(x)^3 - 15
*a^2*cosh(x))*e^x*sinh(x)^3 + 5*(27*a^2*cosh(x)^8 + 140*a^2*cosh(x)^6 + 450
*a^2*cosh(x)^4 - 180*a^2*cosh(x)^2 - 5*a^2)*e^x*sinh(x)^2 + 10*(3*a^2*cosh(
x)^9 + 20*a^2*cosh(x)^7 + 90*a^2*cosh(x)^5 - 60*a^2*cosh(x)^3 - 5*a^2*cosh(
x))*e^x*sinh(x) + (3*a^2*cosh(x)^10 + 25*a^2*cosh(x)^8 + 150*a^2*cosh(x)^6
- 150*a^2*cosh(x)^4 - 25*a^2*cosh(x)^2 - 3*a^2)*e^x)*sqrt(a*e^(4*x) + 2*a*e
^(2*x) + a)*e^(-x)/(cosh(x)^5*e^(2*x) + (e^(2*x) + 1)*sinh(x)^5 + cosh(x)^5
 + 5*(cosh(x)*e^(2*x) + cosh(x))*sinh(x)^4 + 10*(cosh(x)^2*e^(2*x) + cosh(x
)^2)*sinh(x)^3 + 10*(cosh(x)^3*e^(2*x) + cosh(x)^3)*sinh(x)^2 + 5*(cosh(x)^
4*e^(2*x) + cosh(x)^4)*sinh(x))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)**2)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4848 deep
```

Giac [A]
time = 0.40, size = 61, normalized size = 1.15

$$\frac{1}{480}\left(3\,a^{2}e^{(5\,x)}+25\,a^{2}e^{(3\,x)}+150\,a^{2}e^{x}-\left(150\,a^{2}e^{(4\,x)}+25\,a^{2}e^{(2\,x)}+3\,a^{2}\right)e^{(-5\,x)}\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/480*(3*a^2*e^(5*x) + 25*a^2*e^(3*x) + 150*a^2*e^x - (150*a^2*e^(4*x) + 25
*a^2*e^(2*x) + 3*a^2)*e^(-5*x))*sqrt(a)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \left(a\cosh(x)^2\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^2)^(5/2),x)
```

```
[Out] int((a*cosh(x)^2)^(5/2), x)
```

3.123 $\int \left(a\cosh^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=34

$$\frac{2}{3}a\sqrt{a\cosh^2(x)}\,\tanh(x)+\frac{1}{3}\left(a\cosh^2(x)\right)^{3/2}\tanh(x)$$

```
[Out] 1/3*(a*cosh(x)^2)^(3/2)*tanh(x)+2/3*a*(a*cosh(x)^2)^(1/2)*tanh(x)
```

Rubi [A]
time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.300$, Rules used = {3282, 3286, 2717}

$$\frac{1}{3}\tanh(x)\left(a\cosh^2(x)\right)^{3/2}+\frac{2}{3}a\tanh(x)\sqrt{a\cosh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x]^2)^(3/2),x]
```

```
[Out] (2*a*Sqrt[a*Cosh[x]^2]*Tanh[x])/3 + ((a*Cosh[x]^2)^(3/2)*Tanh[x])/3
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3282

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-Cot[e + f*x
])*((b*Sin[e + f*x]^2)^p/(2*f*p)), x] + Dist[b*((2*p - 1)/(2*p)), Int[(b*Si
n[e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && Gt
Q[p, 1]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \left(a\cosh^2(x)\right)^{3/2}\,dx &= \frac{1}{3}\left(a\cosh^2(x)\right)^{3/2}\tanh(x) + \frac{1}{3}(2a)\int \sqrt{a\cosh^2(x)}\,dx \\ &= \frac{1}{3}\left(a\cosh^2(x)\right)^{3/2}\tanh(x) + \frac{1}{3}\left(2a\sqrt{a\cosh^2(x)}\,\mathrm{sech}(x)\right)\int \cosh(x)\,dx \\ &= \frac{2}{3}a\sqrt{a\cosh^2(x)}\,\tanh(x) + \frac{1}{3}\left(a\cosh^2(x)\right)^{3/2}\tanh(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 0.76

$$\frac{1}{12}a\sqrt{a\cosh^2(x)}\,\mathrm{sech}(x)(9\sinh(x)+\sinh(3x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x]^2)^(3/2),x]
[Out] (a*Sqrt[a*Cosh[x]^2]*Sech[x]*(9*Sinh[x] + Sinh[3*x]))/12
```

Maple [A]
time = 0.79, size = 24, normalized size = 0.71

method	result
default	$\frac{a^2\cosh(x)\sinh(x)\left(\cosh^2(x)+2\right)}{3\sqrt{a\left(\cosh^2(x)\right)}}$
risch	$\frac{a\,\mathrm{e}^{4x}\sqrt{a\left(1+\mathrm{e}^{2x}\right)^2\mathrm{e}^{-2x}}}{24+24\,\mathrm{e}^{2x}} + \frac{3a\,\mathrm{e}^{2x}\sqrt{a\left(1+\mathrm{e}^{2x}\right)^2\mathrm{e}^{-2x}}}{8(1+\mathrm{e}^{2x})} - \frac{3\sqrt{a\left(1+\mathrm{e}^{2x}\right)^2\mathrm{e}^{-2x}}\,a}{8(1+\mathrm{e}^{2x})} - \frac{a\,\mathrm{e}^{-2x}\sqrt{a\left(1+\mathrm{e}^{2x}\right.}}{24(1+\mathrm{e}^{2x})}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/3*a^2*cosh(x)*sinh(x)*(cosh(x)^2+2)/(a*cosh(x)^2)^(1/2)
```

Maxima [A]
time = 0.48, size = 35, normalized size = 1.03

$$\frac{1}{24}\,a^{\frac{3}{2}}e^{(3\,x)} - \frac{3}{8}\,a^{\frac{3}{2}}e^{(-x)} - \frac{1}{24}\,a^{\frac{3}{2}}e^{(-3\,x)} + \frac{3}{8}\,a^{\frac{3}{2}}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/24*a^(3/2)*e^(3*x) - 3/8*a^(3/2)*e^(-x) - 1/24*a^(3/2)*e^(-3*x) + 3/8*a^(
3/2)*e^x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(26) = 52$.
time = 0.42, size = 222, normalized size = 6.53

$$\frac{\left(6\,a\cosh\left(x\right)e^{x}\sinh\left(x\right)^{5}+ae^{x}\sinh\left(x\right)^{6}+3\left(5\,a\cosh\left(x\right)^{2}+3\,a\right)e^{x}\sinh\left(x\right)^{4}+4\left(5\,a\cosh\left(x\right)^{3}+9\,a\cosh\left(x\right)\right)e^{x}\sinh\left(x\right)^{3}+3\left(5\,a\cosh\left(x\right)^{4}+18\,a\cosh\left(x\right)^{2}-3\,a\right)e^{x}\sinh\left(x\right)^{2}+6\left(a\cosh\left(x\right)^{5}+6\,a\cosh\left(x\right)^{3}-3\,a\cosh\left(x\right)\right)e^{x}\sinh\left(x\right)+\left(a\cosh\left(x\right)^{6}+9\,a\cosh\left(x\right)^{4}-9\,a\cosh\left(x\right)^{2}-a\right)e^{x}\right)\sqrt{ae^{(4\,x)}+2\,ae^{(2\,x)}+a}\,e^{(-x)}}{24\left(\cosh\left(x\right)^{3}e^{(2\,x)}+\left(e^{(2\,x)}+1\right)\sinh\left(x\right)^{3}+\cosh\left(x\right)^{3}+3\left(\cosh\left(x\right)e^{(2\,x)}+\cosh\left(x\right)\right)\sinh\left(x\right)^{2}+3\left(\cosh\left(x\right)^{2}e^{(2\,x)}+\cosh\left(x\right)^{2}\right)\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*(6*a*cosh(x)*e^x*sinh(x)^5 + a*e^x*sinh(x)^6 + 3*(5*a*cosh(x)^2 + 3*a)
*e^x*sinh(x)^4 + 4*(5*a*cosh(x)^3 + 9*a*cosh(x))*e^x*sinh(x)^3 + 3*(5*a*cos
h(x)^4 + 18*a*cosh(x)^2 - 3*a)*e^x*sinh(x)^2 + 6*(a*cosh(x)^5 + 6*a*cosh(x)
^3 - 3*a*cosh(x))*e^x*sinh(x) + (a*cosh(x)^6 + 9*a*cosh(x)^4 - 9*a*cosh(x)^
2 - a)*e^x)*sqrt(a*e^(4*x) + 2*a*e^(2*x) + a)*e^(-x)/(cosh(x)^3*e^(2*x) + (
e^(2*x) + 1)*sinh(x)^3 + cosh(x)^3 + 3*(cosh(x)*e^(2*x) + cosh(x))*sinh(x)^
2 + 3*(cosh(x)^2*e^(2*x) + cosh(x)^2)*sinh(x))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.41, size = 29, normalized size = 0.85

$$-\frac{1}{24}\left(\left(9\,e^{(2\,x)}+1\right)e^{(-3\,x)}-e^{(3\,x)}-9\,e^{x}\right)a^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*((9*e^(2*x) + 1)*e^(-3*x) - e^(3*x) - 9*e^x)*a^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int {\left(a\,{\cosh\left(x\right)}^2\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^2)^(3/2),x)
```

```
[Out] int((a*cosh(x)^2)^(3/2), x)
```

3.124 $\int \sqrt{a \cosh^2(x)}\,dx$

Optimal. Leaf size=13

$$\sqrt{a \cosh^2(x)}\,\tanh(x)$$

```
[Out] (a*cosh(x)^2)^(1/2)*tanh(x)
```

Rubi [A]
time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3286, 2717}

$$\tanh(x)\sqrt{a \cosh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Cosh[x]^2],x]
```

```
[Out] Sqrt[a*Cosh[x]^2]*Tanh[x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned} \int \sqrt{a \cosh^2(x)}\,dx &= \left(\sqrt{a \cosh^2(x)}\,\operatorname{sech}(x)\right)\int \cosh(x)\,dx \\ &= \sqrt{a \cosh^2(x)}\,\tanh(x) \end{aligned}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$\sqrt{a \cosh^2(x)}\,\tanh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Cosh[x]^2],x]
```

```
[Out] Sqrt[a*Cosh[x]^2]*Tanh[x]
```

Maple [A]
time = 0.81, size = 15, normalized size = 1.15

method	result	size
default	$\frac{a\cosh(x)\sinh(x)}{\sqrt{a\left(\cosh^{2}(x)\right)}}$	15
risch	$\frac{\sqrt{a\left(1+\mathrm{e}^{2x}\right)^{2}\mathrm{e}^{-2x}}\,\mathrm{e}^{2x}}{2+2\,\mathrm{e}^{2x}}-\frac{\sqrt{a\left(1+\mathrm{e}^{2x}\right)^{2}\mathrm{e}^{-2x}}}{2(1+\mathrm{e}^{2x})}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(a*cosh(x)^2)^(1/2)*a*cosh(x)*sinh(x)
```

Maxima [A]
time = 0.48, size = 17, normalized size = 1.31

$$-\frac{1}{2}\sqrt{a}\,e^{(-x)}+\frac{1}{2}\sqrt{a}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(a)*e^(-x) + 1/2*sqrt(a)*e^x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(11) = 22.
time = 0.33, size = 69, normalized size = 5.31

$$\frac{\left(2\cosh\left(x\right)e^{x}\sinh\left(x\right)+e^{x}\sinh\left(x\right)^{2}+\left(\cosh\left(x\right)^{2}-1\right)e^{x}\right)\sqrt{ae^{(4x)}+2ae^{(2x)}+a}\,e^{(-x)}}{2\left(\cosh\left(x\right)e^{(2x)}+\left(e^{(2x)}+1\right)\sinh\left(x\right)+\cosh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*cosh(x)*e^x*sinh(x) + e^x*sinh(x)^2 + (cosh(x)^2 - 1)*e^x)*sqrt(a*e^
(4*x) + 2*a*e^(2*x) + a)*e^(-x)/(cosh(x)*e^(2*x) + (e^(2*x) + 1)*sinh(x) +
cosh(x))
```

Sympy [A]
time = 0.19, size = 15, normalized size = 1.15

$$\frac{\sqrt{a \cosh^2(x)} \sinh(x)}{\cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)**2)**(1/2),x)
```

```
[Out] sqrt(a*cosh(x)**2)*sinh(x)/cosh(x)
```

Giac [A]
time = 0.43, size = 14, normalized size = 1.08

$$-\frac{1}{2} \sqrt{a} \left(e^{(-x)} - e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a)*(e^(-x) - e^x)
```

Mupad [B]
time = 0.05, size = 17, normalized size = 1.31

$$\sqrt{a} \tanh(x) \left(\frac{e^{-x}}{2} + \frac{e^{x}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^2)^(1/2),x)
```

```
[Out] a^(1/2)*tanh(x)*(exp(-x)/2 + exp(x)/2)
```

3.125 $\int \frac{1}{\sqrt{a\cosh^2(x)}}\,dx$

Optimal. Leaf size=16

$$\frac{\text{ArcTan}(\sinh(x))\cosh(x)}{\sqrt{a\cosh^2(x)}}$$

```
[Out] arctan(sinh(x))*cosh(x)/(a*cosh(x)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3286, 3855}

$$\frac{\cosh(x)\text{ArcTan}(\sinh(x))}{\sqrt{a\cosh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Cosh[x]^2],x]
[Out] (ArcTan[Sinh[x]]*Cosh[x])/Sqrt[a*Cosh[x]^2]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a\cosh^2(x)}}\,dx &= \frac{\cosh(x)\int \operatorname{sech}(x)\,dx}{\sqrt{a\cosh^2(x)}}\\
&= \frac{\tan^{-1}(\sinh(x))\cosh(x)}{\sqrt{a\cosh^2(x)}}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 21, normalized size = 1.31

$$\frac{2\text{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)\cosh(x)}{\sqrt{a\cosh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Cosh[x]^2],x]
[Out] (2*ArcTan[Tanh[x/2]]*Cosh[x])/Sqrt[a*Cosh[x]^2]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(14) = 28$.
time = 0.98, size = 55, normalized size = 3.44

method	result	size
default	$-\frac{\cosh(x)\sqrt{a\left(\sinh^2(x)\right)}\,\ln\left(\frac{2\sqrt{-a}\,\sqrt{a\left(\sinh^2(x)\right)}-2a}{\cosh(x)}\right)}{\sqrt{-a}\,\sinh(x)\sqrt{a\left(\cosh^2(x)\right)}}$	55
risch	$\frac{i\mathrm{e}^{-x}\left(1+\mathrm{e}^{2x}\right)\ln\left(\mathrm{e}^{x}+i\right)}{\sqrt{a\left(1+\mathrm{e}^{2x}\right)^2\mathrm{e}^{-2x}}}-\frac{i\mathrm{e}^{-x}\left(1+\mathrm{e}^{2x}\right)\ln\left(\mathrm{e}^{x}-i\right)}{\sqrt{a\left(1+\mathrm{e}^{2x}\right)^2\mathrm{e}^{-2x}}}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -cosh(x)*(a*sinh(x)^2)^(1/2)/(-a)^(1/2)*ln(2*((-a)^(1/2)*(a*sinh(x)^2)^(1/2
)-a)/cosh(x))/sinh(x)/(a*cosh(x)^2)^(1/2)
```

Maxima [A]
time = 0.51, size = 8, normalized size = 0.50

$$\frac{2\arctan\left(e^{x}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^2)^(1/2),x, algorithm="maxima")
[Out] 2*arctan(e^x)/sqrt(a)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(14) = 28$.
time = 0.35, size = 186, normalized size = 11.62

$$\left[-\frac{\sqrt{-a}\log\left(\frac{a\cosh(x)^2-2\sqrt{ae^{(4x)}+2ae^{(2x)}+a}\,(\cosh(x)e^{x}+e^{x}\sinh(x))\sqrt{-a}\,e^{(-x)}+(ae^{(2x)}+a)\sinh(x)^2+\left(a\cosh(x)^2-a\right)e^{(2x)}+2\left(a\cosh(x)e^{(2x)}+a\cosh(x)\right)\sinh(x)-a}{(e^{(2x)}+1)\sinh(x)^2+\cosh(x)^2+\left(\cosh(x)^2+1\right)e^{(2x)}+2\left(\cosh(x)e^{(2x)}+\cosh(x)\right)\sinh(x)+1}\right)}{a},\ \frac{2\sqrt{ae^{(4x)}+2ae^{(2x)}+a}\,\arctan\left(\cosh(x)+\sinh(x)\right)}{ae^{(2x)}+a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-sqrt(-a)*log((a*cosh(x)^2 - 2*sqrt(a*e^(4*x) + 2*a*e^(2*x) + a)*(cosh(x)*
e^x + e^x*sinh(x))*sqrt(-a)*e^(-x) + (a*e^(2*x) + a)*sinh(x)^2 + (a*cosh(x)
^2 - a)*e^(2*x) + 2*(a*cosh(x)*e^(2*x) + a*cosh(x))*sinh(x) - a)/((e^(2*x)
+ 1)*sinh(x)^2 + cosh(x)^2 + (cosh(x)^2 + 1)*e^(2*x) + 2*(cosh(x)*e^(2*x) +
 cosh(x))*sinh(x) + 1))/a, 2*sqrt(a*e^(4*x) + 2*a*e^(2*x) + a)*arctan(cosh(
x) + sinh(x))/(a*e^(2*x) + a)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\cosh^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*cosh(x)**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\sqrt{a\,{\cosh\left(x\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)^2)^(1/2),x)
```

```
[Out] int(1/(a*cosh(x)^2)^(1/2), x)
```

3.126 $\int \frac{1}{\left(a\cosh^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=42

$$\frac{\operatorname{ArcTan}(\sinh(x))\cosh(x)}{2a\sqrt{a\cosh^2(x)}}+\frac{\tanh(x)}{2a\sqrt{a\cosh^2(x)}}$$

```
[Out] 1/2*arctan(sinh(x))*cosh(x)/a/(a*cosh(x)^2)^(1/2)+1/2*tanh(x)/a/(a*cosh(x)^
2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.300$, Rules used = {3283, 3286, 3855}

$$\frac{\cosh(x)\operatorname{ArcTan}(\sinh(x))}{2a\sqrt{a\cosh^2(x)}}+\frac{\tanh(x)}{2a\sqrt{a\cosh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x]^2)^(-3/2),x]
```

```
[Out] (ArcTan[Sinh[x]]*Cosh[x])/(2*a*Sqrt[a*Cosh[x]^2]) + Tanh[x]/(2*a*Sqrt[a*Cos
h[x]^2])
```

Rule 3283

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[Cot[e + f*x]*
((b*Sin[e + f*x]^2)^(p + 1)/(b*f*(2*p + 1))), x] + Dist[2*((p + 1)/(b*(2*p
+ 1))), Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&  !
IntegerQ[p] && LtQ[p, -1]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a\cosh^2(x)\right)^{3/2}}\,dx = \frac{\tanh(x)}{2a\sqrt{a\cosh^2(x)}} + \frac{\int \frac{1}{\sqrt{a\cosh^2(x)}}\,dx}{2a}$$
$$= \frac{\tanh(x)}{2a\sqrt{a\cosh^2(x)}} + \frac{\cosh(x)\int \operatorname{sech}(x)\,dx}{2a\sqrt{a\cosh^2(x)}}$$
$$= \frac{\tan^{-1}(\sinh(x))\cosh(x)}{2a\sqrt{a\cosh^2(x)}} + \frac{\tanh(x)}{2a\sqrt{a\cosh^2(x)}}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 0.74

$$\frac{2\text{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)\cosh(x) + \tanh(x)}{2a\sqrt{a\cosh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x]^2)^(-3/2),x]
[Out] (2*ArcTan[Tanh[x/2]]*Cosh[x] + Tanh[x])/(2*a*Sqrt[a*Cosh[x]^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(34) = 68.
time = 0.98, size = 82, normalized size = 1.95

method	result	size
default	$\frac{\sqrt{a\left(\sinh^2(x)\right)}\left(-\ln\left(\frac{2\sqrt{-a}\,\sqrt{a\left(\sinh^2(x)\right)}-2a}{\cosh(x)}\right)a\left(\cosh^2(x)\right)+\sqrt{-a}\,\sqrt{a\left(\sinh^2(x)\right)}\right)}{2a^2\cosh(x)\sqrt{-a}\,\sinh(x)\sqrt{a\left(\cosh^2(x)\right)}}$	82
risch	$\frac{e^{2x}-1}{a(1+e^{2x})\sqrt{a\left(1+e^{2x}\right)^2 e^{-2x}}} + \frac{i(1+e^{2x})e^{-x}\ln(e^x+i)}{2a\sqrt{a\left(1+e^{2x}\right)^2 e^{-2x}}} - \frac{i(1+e^{2x})e^{-x}\ln(e^x-i)}{2a\sqrt{a\left(1+e^{2x}\right)^2 e^{-2x}}}$	112

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/2/a^2/cosh(x)*(a*sinh(x)^2)^(1/2)*(-ln(2*((-a)^(1/2)*(a*sinh(x)^2)^(1/2)-
a)/cosh(x))*a*cosh(x)^2+(-a)^(1/2)*(a*sinh(x)^2)^(1/2))/(-a)^(1/2)/sinh(x)/
(a*cosh(x)^2)^(1/2)
```

Maxima [A]
time = 0.51, size = 41, normalized size = 0.98

$$\frac{e^{(3\,x)} - e^{x}}{a^{\frac{3}{2}} e^{(4\,x)} + 2\,a^{\frac{3}{2}} e^{(2\,x)} + a^{\frac{3}{2}}} + \frac{\arctan\left(e^{x}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] (e^(3*x) - e^x)/(a^(3/2)*e^(4*x) + 2*a^(3/2)*e^(2*x) + a^(3/2)) + arctan(e^
x)/a^(3/2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(34) = 68.
time = 0.40, size = 299, normalized size = 7.12

$$\frac{\left(3\,\cosh\left(x\right) e^{x} \sinh\left(x\right)^{2} + e^{x} \sinh\left(x\right)^{3} + \left(3\,\cosh\left(x\right)^{2} - 1\right) e^{x} \sinh\left(x\right) + \left(4\,\cosh\left(x\right) e^{x} \sinh\left(x\right)^{3} + e^{x} \sinh\left(x\right)^{4} + 2\,\left(3\,\cosh\left(x\right)^{2} + 1\right) e^{x} \sinh\left(x\right)^{2} + 4\,\left(\cosh\left(x\right)^{3} + \cosh\left(x\right)\right) e^{x} \sinh\left(x\right) + \left(\cosh\left(x\right)^{4} + 2\,\cosh\left(x\right)^{2} + 1\right) e^{x}\right) \arctan\left(\cosh\left(x\right) + \sinh\left(x\right)\right) + \left(\cosh\left(x\right)^{3} - \cosh\left(x\right)\right) e^{x}\right) \sqrt{a e^{(4\,x)} + 2\,a e^{(2\,x)} + a}\, e^{(-x)}}{a^{2} \cosh\left(x\right)^{4} + \left(a^{2} e^{(2\,x)} + a^{2}\right) \sinh\left(x\right)^{4} + 2\,a^{2} \cosh\left(x\right)^{2} + 4\,\left(a^{2} \cosh\left(x\right) e^{(2\,x)} + a^{2} \cosh\left(x\right)\right) \sinh\left(x\right)^{3} + 2\,\left(3\,a^{2} \cosh\left(x\right)^{2} + a^{2} + \left(3\,a^{2} \cosh\left(x\right)^{2} + a^{2}\right) e^{(2\,x)}\right) \sinh\left(x\right)^{2} + a^{2} + \left(a^{2} \cosh\left(x\right)^{4} + 2\,a^{2} \cosh\left(x\right)^{2} + a^{2}\right) e^{(2\,x)} + 4\,\left(a^{2} \cosh\left(x\right)^{3} + a^{2} \cosh\left(x\right) + \left(a^{2} \cosh\left(x\right)^{3} + a^{2} \cosh\left(x\right)\right) e^{(2\,x)}\right) \sinh\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (3*cosh(x)*e^x*sinh(x)^2 + e^x*sinh(x)^3 + (3*cosh(x)^2 - 1)*e^x*sinh(x) +
(4*cosh(x)*e^x*sinh(x)^3 + e^x*sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*e^x*sinh(x)^
2 + 4*(cosh(x)^3 + cosh(x))*e^x*sinh(x) + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^x
)*arctan(cosh(x) + sinh(x)) + (cosh(x)^3 - cosh(x))*e^x)*sqrt(a*e^(4*x) + 2
*a*e^(2*x) + a)*e^(-x)/(a^2*cosh(x)^4 + (a^2*e^(2*x) + a^2)*sinh(x)^4 + 2*a
^2*cosh(x)^2 + 4*(a^2*cosh(x)*e^(2*x) + a^2*cosh(x))*sinh(x)^3 + 2*(3*a^2*c
osh(x)^2 + a^2 + (3*a^2*cosh(x)^2 + a^2)*e^(2*x))*sinh(x)^2 + a^2 + (a^2*co
sh(x)^4 + 2*a^2*cosh(x)^2 + a^2)*e^(2*x) + 4*(a^2*cosh(x)^3 + a^2*cosh(x) +
 (a^2*cosh(x)^3 + a^2*cosh(x))*e^(2*x))*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a \cosh^{2}\left(x\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)**2)**(3/2),x)
```

```
[Out] Integral((a*cosh(x)**2)**(-3/2), x)
```

Giac [A]
time = 0.41, size = 56, normalized size = 1.33

$$\frac{\frac{\pi + 2\,\arctan\left(\frac{1}{2}\,\left(e^{(2\,x)} - 1\right) e^{(-x)}\right)}{\sqrt{a}} - \frac{4\,\left(e^{(-x)} - e^{x}\right)}{\left(\left(e^{(-x)} - e^{x}\right)^{2} + 4\right) \sqrt{a}}}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*((pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))/sqrt(a) - 4*(e^(-x) - e^x)/(
((e^(-x) - e^x)^2 + 4)*sqrt(a)))/a
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(a\cosh\left(x\right)^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)^2)^(3/2),x)
```

```
[Out] int(1/(a*cosh(x)^2)^(3/2), x)
```

3.127 $\int \frac{1}{\left(a\cosh^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=61

$$\frac{3\mathrm{ArcTan}(\sinh(x))\cosh(x)}{8a^2\sqrt{a\cosh^2(x)}}+\frac{\tanh(x)}{4a\left(a\cosh^2(x)\right)^{3/2}}+\frac{3\tanh(x)}{8a^2\sqrt{a\cosh^2(x)}}$$

```
[Out] 3/8*arctan(sinh(x))*cosh(x)/a^2/(a*cosh(x)^2)^(1/2)+1/4*tanh(x)/a/(a*cosh(x
)^2)^(3/2)+3/8*tanh(x)/a^2/(a*cosh(x)^2)^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3283, 3286, 3855}

$$\frac{3\cosh(x)\mathrm{ArcTan}(\sinh(x))}{8a^2\sqrt{a\cosh^2(x)}}+\frac{3\tanh(x)}{8a^2\sqrt{a\cosh^2(x)}}+\frac{\tanh(x)}{4a\left(a\cosh^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x]^2)^(-5/2),x]
```

```
[Out] (3*ArcTan[Sinh[x]]*Cosh[x])/(8*a^2*Sqrt[a*Cosh[x]^2]) + Tanh[x]/(4*a*(a*Cos
h[x]^2)^(3/2)) + (3*Tanh[x])/(8*a^2*Sqrt[a*Cosh[x]^2])
```

Rule 3283

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[Cot[e + f*x]*
((b*Sin[e + f*x]^2)^(p + 1)/(b*f*(2*p + 1))), x] + Dist[2*((p + 1)/(b*(2*p
+ 1))), Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&  !
IntegerQ[p] && LtQ[p, -1]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a\cosh^2(x)\right)^{5/2}}\,dx &= \frac{\tanh(x)}{4a\left(a\cosh^2(x)\right)^{3/2}}+\frac{3\int \frac{1}{\left(a\cosh^2(x)\right)^{3/2}}\,dx}{4a}\\
&= \frac{\tanh(x)}{4a\left(a\cosh^2(x)\right)^{3/2}}+\frac{3\tanh(x)}{8a^2\sqrt{a\cosh^2(x)}}+\frac{3\int \frac{1}{\sqrt{a\cosh^2(x)}}\,dx}{8a^2}\\
&= \frac{\tanh(x)}{4a\left(a\cosh^2(x)\right)^{3/2}}+\frac{3\tanh(x)}{8a^2\sqrt{a\cosh^2(x)}}+\frac{(3\cosh(x))\int \operatorname{sech}(x)\,dx}{8a^2\sqrt{a\cosh^2(x)}}\\
&= \frac{3\tan^{-1}(\sinh(x))\cosh(x)}{8a^2\sqrt{a\cosh^2(x)}}+\frac{\tanh(x)}{4a\left(a\cosh^2(x)\right)^{3/2}}+\frac{3\tanh(x)}{8a^2\sqrt{a\cosh^2(x)}}
\end{aligned}
$$

Mathematica [**A**]
time = 0.03, size = 40, normalized size = 0.66

$$\frac{6\text{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)\cosh(x)+\left(3+2\text{sech}^2(x)\right)\tanh(x)}{8a^2\sqrt{a\cosh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x]^2)^(-5/2),x]
```

```
[Out] (6*ArcTan[Tanh[x/2]]*Cosh[x] + (3 + 2*Sech[x]^2)*Tanh[x])/(8*a^2*Sqrt[a*Cos
h[x]^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(49) = 98.
time = 1.03, size = 102, normalized size = 1.67

method	result
default	$\frac{\sqrt{a\left(\sinh^2(x)\right)}\left(-3\ln\left(\frac{2\sqrt{-a}\,\sqrt{a\left(\sinh^2(x)\right)}-2a}{\cosh(x)}\right)a\left(\cosh^4(x)\right)+3\sqrt{a\left(\sinh^2(x)\right)}\left(\cosh^2(x)\right)\sqrt{-a}+2\sqrt{-}\right.}{8a^3\cosh(x)^3\sqrt{-a}\,\sinh(x)\sqrt{a\left(\cosh^2(x)\right)}}$
risch	$\frac{3\,\mathrm{e}^{6x}+11\,\mathrm{e}^{4x}-11\,\mathrm{e}^{2x}-3}{4a^2(1+\mathrm{e}^{2x})^3\sqrt{a\left(1+\mathrm{e}^{2x}\right)^2\mathrm{e}^{-2x}}}+\frac{3i(1+\mathrm{e}^{2x})\mathrm{e}^{-x}\ln(\mathrm{e}^x+i)}{8a^2\sqrt{a\left(1+\mathrm{e}^{2x}\right)^2\mathrm{e}^{-2x}}}-\frac{3i(1+\mathrm{e}^{2x})\mathrm{e}^{-x}\ln(\mathrm{e}^x-i)}{8a^2\sqrt{a\left(1+\mathrm{e}^{2x}\right)^2\mathrm{e}^{-2x}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/a^3/cosh(x)^3*(a*sinh(x)^2)^(1/2)*(-3*ln(2*((-a)^(1/2)*(a*sinh(x)^2)^(1
/2)-a)/cosh(x))*a*cosh(x)^4+3*(a*sinh(x)^2)^(1/2)*cosh(x)^2*(-a)^(1/2)+2*(-
a)^(1/2)*(a*sinh(x)^2)^(1/2))/(-a)^(1/2)/sinh(x)/(a*cosh(x)^2)^(1/2)
```

Maxima [A]
time = 0.51, size = 75, normalized size = 1.23

$$\frac{3\,e^{(7\,x)}+11\,e^{(5\,x)}-11\,e^{(3\,x)}-3\,e^{x}}{4\left(a^{\frac{5}{2}}e^{(8\,x)}+4\,a^{\frac{5}{2}}e^{(6\,x)}+6\,a^{\frac{5}{2}}e^{(4\,x)}+4\,a^{\frac{5}{2}}e^{(2\,x)}+a^{\frac{5}{2}}\right)}+\frac{3\,\arctan\left(e^{x}\right)}{4\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*(3*e^(7*x) + 11*e^(5*x) - 11*e^(3*x) - 3*e^x)/(a^(5/2)*e^(8*x) + 4*a^(5
/2)*e^(6*x) + 6*a^(5/2)*e^(4*x) + 4*a^(5/2)*e^(2*x) + a^(5/2)) + 3/4*arctan
(e^x)/a^(5/2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 837 vs. 2(49) = 98.
time = 0.40, size = 837, normalized size = 13.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(21*cosh(x)*e^x*sinh(x)^6 + 3*e^x*sinh(x)^7 + (63*cosh(x)^2 + 11)*e^x*s
inh(x)^5 + 5*(21*cosh(x)^3 + 11*cosh(x))*e^x*sinh(x)^4 + (105*cosh(x)^4 + 1
10*cosh(x)^2 - 11)*e^x*sinh(x)^3 + (63*cosh(x)^5 + 110*cosh(x)^3 - 33*cosh(
x))*e^x*sinh(x)^2 + (21*cosh(x)^6 + 55*cosh(x)^4 - 33*cosh(x)^2 - 3)*e^x*si
nh(x) + 3*(8*cosh(x)*e^x*sinh(x)^7 + e^x*sinh(x)^8 + 4*(7*cosh(x)^2 + 1)*e^
x*sinh(x)^6 + 8*(7*cosh(x)^3 + 3*cosh(x))*e^x*sinh(x)^5 + 2*(35*cosh(x)^4 +
 30*cosh(x)^2 + 3)*e^x*sinh(x)^4 + 8*(7*cosh(x)^5 + 10*cosh(x)^3 + 3*cosh(x
))*e^x*sinh(x)^3 + 4*(7*cosh(x)^6 + 15*cosh(x)^4 + 9*cosh(x)^2 + 1)*e^x*sin
h(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^5 + 3*cosh(x)^3 + cosh(x))*e^x*sinh(x) +
(cosh(x)^8 + 4*cosh(x)^6 + 6*cosh(x)^4 + 4*cosh(x)^2 + 1)*e^x)*arctan(cosh(
x) + sinh(x)) + (3*cosh(x)^7 + 11*cosh(x)^5 - 11*cosh(x)^3 - 3*cosh(x))*e^x
)*sqrt(a*e^(4*x) + 2*a*e^(2*x) + a)*e^(-x)/(a^3*cosh(x)^8 + 4*a^3*cosh(x)^6
 + (a^3*e^(2*x) + a^3)*sinh(x)^8 + 8*(a^3*cosh(x)*e^(2*x) + a^3*cosh(x))*si
nh(x)^7 + 6*a^3*cosh(x)^4 + 4*(7*a^3*cosh(x)^2 + a^3 + (7*a^3*cosh(x)^2 + a
```

```
^3)*e^(2*x))*sinh(x)^6 + 8*(7*a^3*cosh(x)^3 + 3*a^3*cosh(x) + (7*a^3*cosh(x
)^3 + 3*a^3*cosh(x))*e^(2*x))*sinh(x)^5 + 4*a^3*cosh(x)^2 + 2*(35*a^3*cosh(
x)^4 + 30*a^3*cosh(x)^2 + 3*a^3 + (35*a^3*cosh(x)^4 + 30*a^3*cosh(x)^2 + 3*
a^3)*e^(2*x))*sinh(x)^4 + 8*(7*a^3*cosh(x)^5 + 10*a^3*cosh(x)^3 + 3*a^3*cos
h(x) + (7*a^3*cosh(x)^5 + 10*a^3*cosh(x)^3 + 3*a^3*cosh(x))*e^(2*x))*sinh(x
)^3 + a^3 + 4*(7*a^3*cosh(x)^6 + 15*a^3*cosh(x)^4 + 9*a^3*cosh(x)^2 + a^3 +
 (7*a^3*cosh(x)^6 + 15*a^3*cosh(x)^4 + 9*a^3*cosh(x)^2 + a^3)*e^(2*x))*sinh
(x)^2 + (a^3*cosh(x)^8 + 4*a^3*cosh(x)^6 + 6*a^3*cosh(x)^4 + 4*a^3*cosh(x)^
2 + a^3)*e^(2*x) + 8*(a^3*cosh(x)^7 + 3*a^3*cosh(x)^5 + 3*a^3*cosh(x)^3 + a
^3*cosh(x) + (a^3*cosh(x)^7 + 3*a^3*cosh(x)^5 + 3*a^3*cosh(x)^3 + a^3*cosh(
x))*e^(2*x))*sinh(x))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.40, size = 67, normalized size = 1.10

$$\frac{3\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2x)}-1\right)e^{(-x)}\right)\right)}{16\,a^{\frac{5}{2}}}-\frac{3\left(e^{(-x)}-e^{x}\right)^{3}+20\,e^{(-x)}-20\,e^{x}}{4\left(\left(e^{(-x)}-e^{x}\right)^{2}+4\right)^{2}a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 3/16*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))/a^(5/2) - 1/4*(3*(e^(-x) - e
^x)^3 + 20*e^(-x) - 20*e^x)/(((e^(-x) - e^x)^2 + 4)^2*a^(5/2))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(a\cosh\left(x\right)^{2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)^2)^(5/2),x)
```

```
[Out] int(1/(a*cosh(x)^2)^(5/2), x)
```

3.128 $\int \left(a\cosh^3(x)\right)^{5/2}\,dx$

Optimal. Leaf size=121

$$-\frac{26ia^2\sqrt{a\cosh^3(x)}\,F\left(\frac{ix}{2}\middle|2\right)}{77\cosh^{\frac{3}{2}}(x)}+\frac{78}{385}a^2\cosh(x)\sqrt{a\cosh^3(x)}\,\sinh(x)+\frac{26}{165}a^2\cosh^3(x)\sqrt{a\cosh^3(x)}\,\sinh(x)+$$

```
[Out] -26/77*I*a^2*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh(1/2*x),2^(1
/2))*(a*cosh(x)^3)^(1/2)/cosh(x)^(3/2)+78/385*a^2*cosh(x)*sinh(x)*(a*cosh(x
)^3)^(1/2)+26/165*a^2*cosh(x)^3*sinh(x)*(a*cosh(x)^3)^(1/2)+2/15*a^2*cosh(x
)^5*sinh(x)*(a*cosh(x)^3)^(1/2)+26/77*a^2*(a*cosh(x)^3)^(1/2)*tanh(x)
```

Rubi [A]
time = 0.04, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3286, 2715, 2720}

$$\frac{26}{165}a^2\sinh(x)\cosh^3(x)\sqrt{a\cosh^3(x)}+\frac{78}{385}a^2\sinh(x)\cosh(x)\sqrt{a\cosh^3(x)}+\frac{26}{77}a^2\tanh(x)\sqrt{a\cosh^3(x)}-\frac{26ia^2F\left(\frac{ix}{2}\middle|2\right)\sqrt{a\cosh^3(x)}}{77\cosh^{\frac{3}{2}}(x)}+\frac{2}{15}a^2\sinh(x)\cosh^5(x)\sqrt{a\cosh^3(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x]^3)^(5/2),x]
```

```
[Out] (((-26*I)/77)*a^2*Sqrt[a*Cosh[x]^3]*EllipticF[(I/2)*x, 2])/Cosh[x]^(3/2) +
(78*a^2*Cosh[x]*Sqrt[a*Cosh[x]^3]*Sinh[x])/385 + (26*a^2*Cosh[x]^3*Sqrt[a*C
osh[x]^3]*Sinh[x])/165 + (2*a^2*Cosh[x]^5*Sqrt[a*Cosh[x]^3]*Sinh[x])/15 + (
26*a^2*Sqrt[a*Cosh[x]^3]*Tanh[x])/77
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
```

```
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(a\cosh^3(x)\right)^{5/2}\,dx = \frac{\left(a^2\sqrt{a\cosh^3(x)}\right)\int \cosh^{\frac{15}{2}}(x)\,dx}{\cosh^{\frac{3}{2}}(x)}$$

$$= \frac{2}{15}a^2\cosh^5(x)\sqrt{a\cosh^3(x)}\,\sinh(x) + \frac{\left(13a^2\sqrt{a\cosh^3(x)}\right)\int \cosh^{\frac{11}{2}}(x)\,dx}{15\cosh^{\frac{3}{2}}(x)}$$

$$= \frac{26}{165}a^2\cosh^3(x)\sqrt{a\cosh^3(x)}\,\sinh(x) + \frac{2}{15}a^2\cosh^5(x)\sqrt{a\cosh^3(x)}\,\sinh(x) + \frac{\left(39a^2\right.}{}$$

$$= \frac{78}{385}a^2\cosh(x)\sqrt{a\cosh^3(x)}\,\sinh(x) + \frac{26}{165}a^2\cosh^3(x)\sqrt{a\cosh^3(x)}\,\sinh(x) + \frac{2}{15}a^2\,c$$

$$= \frac{78}{385}a^2\cosh(x)\sqrt{a\cosh^3(x)}\,\sinh(x) + \frac{26}{165}a^2\cosh^3(x)\sqrt{a\cosh^3(x)}\,\sinh(x) + \frac{2}{15}a^2\,c$$

$$= -\frac{26ia^2\sqrt{a\cosh^3(x)}\,F\left(\frac{ix}{2}\middle|2\right)}{77\cosh^{\frac{3}{2}}(x)} + \frac{78}{385}a^2\cosh(x)\sqrt{a\cosh^3(x)}\,\sinh(x) + \frac{26}{165}a^2\cosh^3($$

Mathematica [A]
time = 0.09, size = 65, normalized size = 0.54

$$\frac{a\left(a\cosh^3(x)\right)^{3/2}\left(-12480iF\left(\frac{ix}{2}\middle|2\right) + \sqrt{\cosh(x)}\,(15465\sinh(x) + 3657\sinh(3x) + 749\sinh(5x) + 77\sinh(7x))\right)}{36960\cosh^{\frac{9}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x]^3)^(5/2),x]
[Out] (a*(a*Cosh[x]^3)^(3/2)*((-12480*I)*EllipticF[(I/2)*x, 2] + Sqrt[Cosh[x]]*(1
5465*Sinh[x] + 3657*Sinh[3*x] + 749*Sinh[5*x] + 77*Sinh[7*x])))/(36960*Cosh
[x]^(9/2))
```

Maple [F]
time = 0.98, size = 0, normalized size = 0.00

$$\int \left(a\left(\cosh^3(x)\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*cosh(x)^3)^(5/2),x)

[Out] int((a*cosh(x)^3)^(5/2),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cosh(x)^3)^(5/2),x, algorithm="maxima")

[Out] integrate((a*cosh(x)^3)^(5/2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 823, normalized size = 6.80

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cosh(x)^3)^(5/2),x, algorithm="fricas")

[Out] 1/73920*(24960*(sqrt(2)*a^2*cosh(x)^7 + 7*sqrt(2)*a^2*cosh(x)^6*sinh(x) + 21*sqrt(2)*a^2*cosh(x)^5*sinh(x)^2 + 35*sqrt(2)*a^2*cosh(x)^4*sinh(x)^3 + 35*sqrt(2)*a^2*cosh(x)^3*sinh(x)^4 + 21*sqrt(2)*a^2*cosh(x)^2*sinh(x)^5 + 7*sqrt(2)*a^2*cosh(x)*sinh(x)^6 + sqrt(2)*a^2*sinh(x)^7)*sqrt(a)*weierstrassPInverse(-4, 0, cosh(x) + sinh(x)) + (77*a^2*cosh(x)^14 + 1078*a^2*cosh(x)*sinh(x)^13 + 77*a^2*sinh(x)^14 + 749*a^2*cosh(x)^12 + 7*(1001*a^2*cosh(x)^2 + 107*a^2)*sinh(x)^12 + 3657*a^2*cosh(x)^10 + 28*(1001*a^2*cosh(x)^3 + 321*a^2*cosh(x))*sinh(x)^11 + (77077*a^2*cosh(x)^4 + 49434*a^2*cosh(x)^2 + 3657*a^2)*sinh(x)^10 + 15465*a^2*cosh(x)^8 + 2*(77077*a^2*cosh(x)^5 + 82390*a^2*cosh(x)^3 + 18285*a^2*cosh(x))*sinh(x)^9 + 3*(77077*a^2*cosh(x)^6 + 123585*a^2*cosh(x)^4 + 54855*a^2*cosh(x)^2 + 5155*a^2)*sinh(x)^8 - 15465*a^2*cosh(x)^6 + 24*(11011*a^2*cosh(x)^7 + 24717*a^2*cosh(x)^5 + 18285*a^2*cosh(x)^3 + 5155*a^2*cosh(x))*sinh(x)^7 + 3*(77077*a^2*cosh(x)^8 + 230692*a^2*cosh(x)^6 + 255990*a^2*cosh(x)^4 + 144340*a^2*cosh(x)^2 - 5155*a^2)*sinh(x)^6 - 3657*a^2*cosh(x)^4 + 2*(77077*a^2*cosh(x)^9 + 296604*a^2*cosh(x)^7 + 460782*a^2*cosh(x)^5 + 433020*a^2*cosh(x)^3 - 46395*a^2*cosh(x))*sinh(x)^5 + (77077*a^2*cosh(x)^10 + 370755*a^2*cosh(x)^8 + 767970*a^2*cosh(x)^6 + 1082550*a^2*cosh(x)^4 - 231975*a^2*cosh(x)^2 - 3657*a^2)*sinh(x)^4 - 749*a^2*cosh(x)^2 + 4*(7007*a^2*cosh(x)^11 + 41195*a^2*cosh(x)^9 + 109710*a^2*cosh(x)^7 + 216510*a^2*cosh(x)^5 - 77325*a^2*cosh(x)^3 - 3657*a^2*cosh(x))*sinh(x)^3 + (7007*a^2*cosh(x)^12 + 49434*a^2*cosh(x)^10 + 164565*a^2*cosh(x)^8 + 433020*a^2*cosh(x)^6 - 231975*a^2*cosh(x)^4 - 21942*a^2*cosh(x)^2 - 749*a^2)*sinh(x

```
)^2 - 77*a^2 + 2*(539*a^2*cosh(x)^13 + 4494*a^2*cosh(x)^11 + 18285*a^2*cosh
(x)^9 + 61860*a^2*cosh(x)^7 - 46395*a^2*cosh(x)^5 - 7314*a^2*cosh(x)^3 - 74
9*a^2*cosh(x))*sinh(x))*sqrt(a*cosh(x)))/(cosh(x)^7 + 7*cosh(x)^6*sinh(x) +
 21*cosh(x)^5*sinh(x)^2 + 35*cosh(x)^4*sinh(x)^3 + 35*cosh(x)^3*sinh(x)^4 +
 21*cosh(x)^2*sinh(x)^5 + 7*cosh(x)*sinh(x)^6 + sinh(x)^7)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)**3)**(5/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^3)^(5/2),x, algorithm="giac")
[Out] integrate((a*cosh(x)^3)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a\cosh(x)^3\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^3)^(5/2),x)
[Out] int((a*cosh(x)^3)^(5/2), x)
```

3.129 $\int \left(a \cosh^3(x)\right)^{3/2} dx$

Optimal. Leaf size=71

$$-\frac{14ia\sqrt{a\cosh^3(x)}\,E\left(\frac{ix}{2}\middle|2\right)}{15\cosh^{\frac{3}{2}}(x)}+\frac{14}{45}a\sqrt{a\cosh^3(x)}\,\sinh(x)+\frac{2}{9}a\cosh^2(x)\sqrt{a\cosh^3(x)}\,\sinh(x)$$

```
[Out] -14/15*I*a*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*sinh(1/2*x),2^(1/2
))*(a*cosh(x)^3)^(1/2)/cosh(x)^(3/2)+14/45*a*sinh(x)*(a*cosh(x)^3)^(1/2)+2/
9*a*cosh(x)^2*sinh(x)*(a*cosh(x)^3)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3286, 2715, 2719}

$$\frac{14}{45}a\sinh(x)\sqrt{a\cosh^3(x)}-\frac{14iaE\left(\frac{ix}{2}\middle|2\right)\sqrt{a\cosh^3(x)}}{15\cosh^{\frac{3}{2}}(x)}+\frac{2}{9}a\sinh(x)\cosh^2(x)\sqrt{a\cosh^3(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x]^3)^(3/2),x]
```

```
[Out] (((-14*I)/15)*a*Sqrt[a*Cosh[x]^3]*EllipticE[(I/2)*x, 2])/Cosh[x]^(3/2) + (1
4*a*Sqrt[a*Cosh[x]^3]*Sinh[x])/45 + (2*a*Cosh[x]^2*Sqrt[a*Cosh[x]^3]*Sinh[x
])/9
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
```

```
FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(a\cosh^3(x)\right)^{3/2}\,dx = \frac{\left(a\sqrt{a\cosh^3(x)}\right)\int \cosh^{\frac{9}{2}}(x)\,dx}{\cosh^{\frac{3}{2}}(x)}$$
$$= \frac{2}{9}a\cosh^2(x)\sqrt{a\cosh^3(x)}\,\sinh(x) + \frac{\left(7a\sqrt{a\cosh^3(x)}\right)\int \cosh^{\frac{5}{2}}(x)\,dx}{9\cosh^{\frac{3}{2}}(x)}$$
$$= \frac{14}{45}a\sqrt{a\cosh^3(x)}\,\sinh(x) + \frac{2}{9}a\cosh^2(x)\sqrt{a\cosh^3(x)}\,\sinh(x) + \frac{\left(7a\sqrt{a\cosh^3(x)}\right)}{15\cosh}$$
$$= -\frac{14ia\sqrt{a\cosh^3(x)}\,E\left(\frac{ix}{2}\middle|2\right)}{15\cosh^{\frac{3}{2}}(x)} + \frac{14}{45}a\sqrt{a\cosh^3(x)}\,\sinh(x) + \frac{2}{9}a\cosh^2(x)\sqrt{a\cosh^3(x}$$

Mathematica [A]
time = 0.06, size = 54, normalized size = 0.76

$$\frac{\left(a\cosh^3(x)\right)^{3/2}\left(-168iE\left(\frac{ix}{2}\middle|2\right)+\sqrt{\cosh(x)}\,(38\sinh(2x)+5\sinh(4x))\right)}{180\cosh^{\frac{9}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x]^3)^(3/2),x]
[Out] ((a*Cosh[x]^3)^(3/2)*((-168*I)*EllipticE[(I/2)*x, 2] + Sqrt[Cosh[x]]*(38*Si
nh[2*x] + 5*Sinh[4*x])))/(180*Cosh[x]^(9/2))
```

Maple [F]
time = 0.93, size = 0, normalized size = 0.00

$$\int \left(a\left(\cosh^3(x)\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^3)^(3/2),x)
[Out] int((a*cosh(x)^3)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cosh(x)^3)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 317, normalized size = 4.46

$$-\frac{336\left(\sqrt{2}\,a\cosh\left(x\right)^{4}+4\sqrt{2}\,a\cosh\left(x\right)^{3}\sinh\left(x\right)+6\sqrt{2}\,a\cosh\left(x\right)^{2}\sinh\left(x\right)^{2}+4\sqrt{2}\,a\cosh\left(x\right)\sinh\left(x\right)^{3}+\sqrt{2}\,a\sinh\left(x\right)^{4}\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cosh\left(x\right)+\sinh\left(x\right)\right)\right)-\left(5\,a\cosh\left(x\right)^{8}+40\,a\cosh\left(x\right)\sinh\left(x\right)^{7}+5\,a\sinh\left(x\right)^{8}+38\,a\cosh\left(x\right)^{6}+2\left(70\,a\cosh\left(x\right)^{2}+19\,a\right)\sinh\left(x\right)^{6}+4\left(70\,a\cosh\left(x\right)^{3}+57\,a\cosh\left(x\right)\right)\sinh\left(x\right)^{5}-336\,a\cosh\left(x\right)^{4}+2\left(175\,a\cosh\left(x\right)^{4}+285\,a\cosh\left(x\right)^{2}-168\,a\right)\sinh\left(x\right)^{4}+8\left(35\,a\cosh\left(x\right)^{5}+95\,a\cosh\left(x\right)^{3}-168\,a\cosh\left(x\right)\right)\sinh\left(x\right)^{3}-38\,a\cosh\left(x\right)^{2}+2\left(70\,a\cosh\left(x\right)^{6}+285\,a\cosh\left(x\right)^{4}-1008\,a\cosh\left(x\right)^{2}-19\,a\right)\sinh\left(x\right)^{2}+4\left(10\,a\cosh\left(x\right)^{7}+57\,a\cosh\left(x\right)^{5}-336\,a\cosh\left(x\right)^{3}-19\,a\cosh\left(x\right)\right)\sinh\left(x\right)-5\,a\right)\sqrt{a\cosh\left(x\right)}}{360\left(\cosh\left(x\right)^{4}+4\cosh\left(x\right)^{3}\sinh\left(x\right)+6\cosh\left(x\right)^{2}\sinh\left(x\right)^{2}+4\cosh\left(x\right)\sinh\left(x\right)^{3}+\sinh\left(x\right)^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/360*(336*(sqrt(2)*a*cosh(x)^4 + 4*sqrt(2)*a*cosh(x)^3*sinh(x) + 6*sqrt(2
)*a*cosh(x)^2*sinh(x)^2 + 4*sqrt(2)*a*cosh(x)*sinh(x)^3 + sqrt(2)*a*sinh(x)
^4)*sqrt(a)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cosh(x) + sin
h(x))) - (5*a*cosh(x)^8 + 40*a*cosh(x)*sinh(x)^7 + 5*a*sinh(x)^8 + 38*a*cos
h(x)^6 + 2*(70*a*cosh(x)^2 + 19*a)*sinh(x)^6 + 4*(70*a*cosh(x)^3 + 57*a*cos
h(x))*sinh(x)^5 - 336*a*cosh(x)^4 + 2*(175*a*cosh(x)^4 + 285*a*cosh(x)^2 -
168*a)*sinh(x)^4 + 8*(35*a*cosh(x)^5 + 95*a*cosh(x)^3 - 168*a*cosh(x))*sinh
(x)^3 - 38*a*cosh(x)^2 + 2*(70*a*cosh(x)^6 + 285*a*cosh(x)^4 - 1008*a*cosh(
x)^2 - 19*a)*sinh(x)^2 + 4*(10*a*cosh(x)^7 + 57*a*cosh(x)^5 - 336*a*cosh(x)
^3 - 19*a*cosh(x))*sinh(x) - 5*a)*sqrt(a*cosh(x)))/(cosh(x)^4 + 4*cosh(x)^3
*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)**3)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^3)^(3/2),x, algorithm="giac")
[Out] integrate((a*cosh(x)^3)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a\cosh(x)^3\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^3)^(3/2),x)
[Out] int((a*cosh(x)^3)^(3/2), x)
```

3.130 $\int \sqrt{a\cosh^3(x)}\,dx$

Optimal. Leaf size=48

$$-\frac{2i\sqrt{a\cosh^3(x)}\,F\left(\frac{ix}{2}\middle|2\right)}{3\cosh^{\frac{3}{2}}(x)}+\frac{2}{3}\sqrt{a\cosh^3(x)}\,\tanh(x)$$

```
[Out] -2/3*I*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh(1/2*x),2^(1/2))*(
a*cosh(x)^3)^(1/2)/cosh(x)^(3/2)+2/3*(a*cosh(x)^3)^(1/2)*tanh(x)
```

Rubi [A]
time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.300$, Rules used = {3286, 2715, 2720}

$$\frac{2}{3}\tanh(x)\sqrt{a\cosh^3(x)}-\frac{2iF\left(\frac{ix}{2}\middle|2\right)\sqrt{a\cosh^3(x)}}{3\cosh^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Cosh[x]^3],x]
```

```
[Out] (((-2*I)/3)*Sqrt[a*Cosh[x]^3]*EllipticF[(I/2)*x, 2])/Cosh[x]^(3/2) + (2*Sqr
t[a*Cosh[x]^3]*Tanh[x])/3
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a\cosh^3(x)}\,dx &= \frac{\sqrt{a\cosh^3(x)}\int \cosh^{\frac{3}{2}}(x)\,dx}{\cosh^{\frac{3}{2}}(x)}\\
&= \frac{2}{3}\sqrt{a\cosh^3(x)}\,\tanh(x)+\frac{\sqrt{a\cosh^3(x)}\int \frac{1}{\sqrt{\cosh(x)}}\,dx}{3\cosh^{\frac{3}{2}}(x)}\\
&= -\frac{2i\sqrt{a\cosh^3(x)}\,F\left(\frac{ix}{2}\middle|2\right)}{3\cosh^{\frac{3}{2}}(x)}+\frac{2}{3}\sqrt{a\cosh^3(x)}\,\tanh(x)
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.04, size = 59, normalized size = 1.23

$$\frac{2}{3}\sqrt{a\cosh^3(x)}\left({}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\cosh(2x)-\sinh(2x)\right)\operatorname{sech}^2(x)\sqrt{1+\cosh(2x)+\sinh(2x)}+\tanh(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Cosh[x]^3],x]
```

```
[Out] (2*Sqrt[a*Cosh[x]^3]*(Hypergeometric2F1[1/4, 1/2, 5/4, -Cosh[2*x] - Sinh[2*
x]]*Sech[x]^2*Sqrt[1 + Cosh[2*x] + Sinh[2*x]] + Tanh[x]))/3
```

Maple [F]
time = 1.01, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\cosh^3(x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^3)^(1/2),x)
```

```
[Out] int((a*cosh(x)^3)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*cosh(x)^3), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 59, normalized size = 1.23

$$\frac{2\left(\sqrt{2}\cosh\left(x\right)+\sqrt{2}\sinh\left(x\right)\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-4,0,\cosh\left(x\right)+\sinh\left(x\right)\right)+\sqrt{a\cosh\left(x\right)}\left(\cosh\left(x\right)^{2}+2\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}-1\right)}{3\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*(sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(a)*weierstrassPInverse(-4,
0, cosh(x) + sinh(x)) + sqrt(a*cosh(x))*(cosh(x)^2 + 2*cosh(x)*sinh(x) + si
nh(x)^2 - 1))/(cosh(x) + sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\cosh^{3}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)**3)**(1/2),x)
```

```
[Out] Integral(sqrt(a*cosh(x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cosh(x)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\sqrt{a\,{\cosh\left(x\right)}^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^3)^(1/2),x)
```

```
[Out] int((a*cosh(x)^3)^(1/2), x)
```

3.131 $\int \frac{1}{\sqrt{a\cosh^3(x)}}\,dx$

Optimal. Leaf size=46

$$\frac{2i\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)}{\sqrt{a\cosh^3(x)}}+\frac{2\cosh(x)\sinh(x)}{\sqrt{a\cosh^3(x)}}$$

```
[Out] 2*I*cosh(x)^(3/2)*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*sinh(1/2*x)
,2^(1/2))/(a*cosh(x)^3)^(1/2)+2*cosh(x)*sinh(x)/(a*cosh(x)^3)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3286, 2716, 2719}

$$\frac{2\sinh(x)\cosh(x)}{\sqrt{a\cosh^3(x)}}+\frac{2i\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)}{\sqrt{a\cosh^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Cosh[x]^3],x]
```

```
[Out] ((2*I)*Cosh[x]^(3/2)*EllipticE[(I/2)*x, 2])/Sqrt[a*Cosh[x]^3] + (2*Cosh[x]*
Sinh[x])/Sqrt[a*Cosh[x]^3]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
```

```
FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a\cosh^3(x)}}\,dx &= \frac{\cosh^{\frac{3}{2}}(x)\int \frac{1}{\cosh^{\frac{3}{2}}(x)}\,dx}{\sqrt{a\cosh^3(x)}}\\ &= \frac{2\cosh(x)\sinh(x)}{\sqrt{a\cosh^3(x)}} - \frac{\cosh^{\frac{3}{2}}(x)\int \sqrt{\cosh(x)}\,dx}{\sqrt{a\cosh^3(x)}}\\ &= \frac{2i\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)}{\sqrt{a\cosh^3(x)}} + \frac{2\cosh(x)\sinh(x)}{\sqrt{a\cosh^3(x)}}\end{aligned}$$

Mathematica [**A**]
time = 0.02, size = 36, normalized size = 0.78

$$\frac{2\cosh(x)\left(i\sqrt{\cosh(x)}\,E\left(\frac{ix}{2}\middle|2\right)+\sinh(x)\right)}{\sqrt{a\cosh^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Cosh[x]^3],x]
[Out] (2*Cosh[x]*(I*Sqrt[Cosh[x]]*EllipticE[(I/2)*x, 2] + Sinh[x]))/Sqrt[a*Cosh[x
]^3]
```

Maple [**F**]
time = 1.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\left(\cosh^3(x)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)^3)^(1/2),x)
[Out] int(1/(a*cosh(x)^3)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*cosh(x)^3), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 93, normalized size = 2.02

$$\frac{2\left(\left(\sqrt{2}\cosh\left(x\right)^{2}+2\sqrt{2}\cosh\left(x\right)\sinh\left(x\right)+\sqrt{2}\sinh\left(x\right)^{2}+\sqrt{2}\right)\sqrt{a}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cosh\left(x\right)+\sinh\left(x\right)))+2\sqrt{a\cosh\left(x\right)}\left(\cosh\left(x\right)^{2}+2\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}\right)\right)}{a\cosh\left(x\right)^{2}+2\,a\cosh\left(x\right)\sinh\left(x\right)+a\sinh\left(x\right)^{2}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*((sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2 + sqr
t(2))*sqrt(a)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cosh(x) + s
inh(x))) + 2*sqrt(a*cosh(x))*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2))/(
a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{a\cosh^{3}\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)**3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*cosh(x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(a*cosh(x)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{\sqrt{a\cosh\left(x\right)^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)^3)^(1/2),x)
```

```
[Out] int(1/(a*cosh(x)^3)^(1/2), x)
```

3.132 $\int \frac{1}{\left(a\cosh^3(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=75

$$-\frac{10i\cosh^{\frac{3}{2}}(x)F\left(\frac{ix}{2}\middle|2\right)}{21a\sqrt{a\cosh^3(x)}}+\frac{10\sinh(x)}{21a\sqrt{a\cosh^3(x)}}+\frac{2\text{sech}(x)\tanh(x)}{7a\sqrt{a\cosh^3(x)}}$$

```
[Out] -10/21*I*cosh(x)^(3/2)*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh(1
/2*x),2^(1/2))/a/(a*cosh(x)^3)^(1/2)+10/21*sinh(x)/a/(a*cosh(x)^3)^(1/2)+2/
7*sech(x)*tanh(x)/a/(a*cosh(x)^3)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3286, 2716, 2720}

$$\frac{10\sinh(x)}{21a\sqrt{a\cosh^3(x)}}-\frac{10i\cosh^{\frac{3}{2}}(x)F\left(\frac{ix}{2}\middle|2\right)}{21a\sqrt{a\cosh^3(x)}}+\frac{2\tanh(x)\text{sech}(x)}{7a\sqrt{a\cosh^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x]^3)^(-3/2),x]
```

```
[Out] (((-10*I)/21)*Cosh[x]^(3/2)*EllipticF[(I/2)*x, 2])/(a*Sqrt[a*Cosh[x]^3]) +
(10*Sinh[x])/(21*a*Sqrt[a*Cosh[x]^3]) + (2*Sech[x]*Tanh[x])/(7*a*Sqrt[a*Cos
h[x]^3])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
```

```
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a\cosh^3(x)\right)^{3/2}}\,dx &= \frac{\cosh^{\frac{3}{2}}(x)\int \frac{1}{\cosh^{\frac{9}{2}}(x)}\,dx}{a\sqrt{a\cosh^3(x)}}\\ &= \frac{2\mathrm{sech}(x)\tanh(x)}{7a\sqrt{a\cosh^3(x)}}+\frac{\left(5\cosh^{\frac{3}{2}}(x)\right)\int \frac{1}{\cosh^{\frac{5}{2}}(x)}\,dx}{7a\sqrt{a\cosh^3(x)}}\\ &= \frac{10\sinh(x)}{21a\sqrt{a\cosh^3(x)}}+\frac{2\mathrm{sech}(x)\tanh(x)}{7a\sqrt{a\cosh^3(x)}}+\frac{\left(5\cosh^{\frac{3}{2}}(x)\right)\int \frac{1}{\sqrt{\cosh(x)}}\,dx}{21a\sqrt{a\cosh^3(x)}}\\ &= -\frac{10i\cosh^{\frac{3}{2}}(x)F\left(\frac{ix}{2}\middle|2\right)}{21a\sqrt{a\cosh^3(x)}}+\frac{10\sinh(x)}{21a\sqrt{a\cosh^3(x)}}+\frac{2\mathrm{sech}(x)\tanh(x)}{7a\sqrt{a\cosh^3(x)}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 48, normalized size = 0.64

$$\frac{2\cosh^2(x)\left(-5i\cosh^{\frac{5}{2}}(x)F\left(\frac{ix}{2}\middle|2\right)+5\cosh(x)\sinh(x)+3\tanh(x)\right)}{21\left(a\cosh^3(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x]^3)^(-3/2),x]
```

```
[Out] (2*Cosh[x]^2*((-5*I)*Cosh[x]^(5/2)*EllipticF[(I/2)*x, 2] + 5*Cosh[x]*Sinh[x
] + 3*Tanh[x]))/(21*(a*Cosh[x]^3)^(3/2))
```

Maple [F]
time = 0.95, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\left(\cosh^3(x)\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)^3)^(3/2),x)
```

```
[Out] int(1/(a*cosh(x)^3)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cosh(x)^3)^(-3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.16, size = 629, normalized size = 8.39

$$\frac{2\left(5\left(\sqrt{2}\cosh(x)^8+8\sqrt{2}\cosh(x)\sinh(x)^7+\sqrt{2}\sinh(x)^8+4\left(7\sqrt{2}\cosh(x)^2+\sqrt{2}\right)\sinh(x)^6+4\sqrt{2}\cosh(x)^6+8\left(7\sqrt{2}\cosh(x)^3+3\sqrt{2}\cosh(x)\right)\sinh(x)^5+2\left(35\sqrt{2}\cosh(x)^4+30\sqrt{2}\cosh(x)^2+3\sqrt{2}\right)\sinh(x)^4+6\sqrt{2}\cosh(x)^4+8\left(7\sqrt{2}\cosh(x)^5+10\sqrt{2}\cosh(x)^3+3\sqrt{2}\cosh(x)\right)\sinh(x)^3+4\left(7\sqrt{2}\cosh(x)^6+15\sqrt{2}\cosh(x)^4+9\sqrt{2}\cosh(x)^2+\sqrt{2}\right)\sinh(x)^2+4\sqrt{2}\cosh(x)^2+8\left(\sqrt{2}\cosh(x)^7+3\sqrt{2}\cosh(x)^5+3\sqrt{2}\cosh(x)^3+\sqrt{2}\cosh(x)\right)\sinh(x)+\sqrt{2}\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-4,0,\cosh(x)+\sinh(x)\right)+2\left(5\cosh(x)^7+35\cosh(x)\sinh(x)^6+5\sinh(x)^7+\left(105\cosh(x)^2+17\right)\sinh(x)^5+17\cosh(x)^5+5\left(35\cosh(x)^3+17\cosh(x)\right)\sinh(x)^4+\left(175\cosh(x)^4+170\cosh(x)^2-17\right)\sinh(x)^3-17\cosh(x)^3+\left(105\cosh(x)^5+170\cosh(x)^3-51\cosh(x)\right)\sinh(x)^2+\left(35\cosh(x)^6+85\cosh(x)^4-51\cosh(x)^2-5\right)\sinh(x)-5\cosh(x)\right)\sqrt{a\cosh(x)}\right)}{21\left(a^2\cosh(x)^8+8a^2\cosh(x)\sinh(x)^7+a^2\sinh(x)^8+4a^2\cosh(x)^6+4\left(7a^2\cosh(x)^2+a^2\right)\sinh(x)^6+6a^2\cosh(x)^4+8\left(7a^2\cosh(x)^3+3a^2\cosh(x)\right)\sinh(x)^5+2\left(35a^2\cosh(x)^4+30a^2\cosh(x)^2+3a^2\right)\sinh(x)^4+4a^2\cosh(x)^2+8\left(7a^2\cosh(x)^5+10a^2\cosh(x)^3+3a^2\cosh(x)\right)\sinh(x)^3+4\left(7a^2\cosh(x)^6+15a^2\cosh(x)^4+9a^2\cosh(x)^2+a^2\right)\sinh(x)^2+a^2+8\left(a^2\cosh(x)^7+3a^2\cosh(x)^5+3a^2\cosh(x)^3+a^2\cosh(x)\right)\sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/21*(5*(sqrt(2)*cosh(x)^8 + 8*sqrt(2)*cosh(x)*sinh(x)^7 + sqrt(2)*sinh(x)^
8 + 4*(7*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x)^6 + 4*sqrt(2)*cosh(x)^6 + 8*(
7*sqrt(2)*cosh(x)^3 + 3*sqrt(2)*cosh(x))*sinh(x)^5 + 2*(35*sqrt(2)*cosh(x)^
4 + 30*sqrt(2)*cosh(x)^2 + 3*sqrt(2))*sinh(x)^4 + 6*sqrt(2)*cosh(x)^4 + 8*(
7*sqrt(2)*cosh(x)^5 + 10*sqrt(2)*cosh(x)^3 + 3*sqrt(2)*cosh(x))*sinh(x)^3 +
 4*(7*sqrt(2)*cosh(x)^6 + 15*sqrt(2)*cosh(x)^4 + 9*sqrt(2)*cosh(x)^2 + sqrt
(2))*sinh(x)^2 + 4*sqrt(2)*cosh(x)^2 + 8*(sqrt(2)*cosh(x)^7 + 3*sqrt(2)*cos
h(x)^5 + 3*sqrt(2)*cosh(x)^3 + sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*sqrt(a)*
weierstrassPInverse(-4, 0, cosh(x) + sinh(x)) + 2*(5*cosh(x)^7 + 35*cosh(x)
*sinh(x)^6 + 5*sinh(x)^7 + (105*cosh(x)^2 + 17)*sinh(x)^5 + 17*cosh(x)^5 +
5*(35*cosh(x)^3 + 17*cosh(x))*sinh(x)^4 + (175*cosh(x)^4 + 170*cosh(x)^2 -
17)*sinh(x)^3 - 17*cosh(x)^3 + (105*cosh(x)^5 + 170*cosh(x)^3 - 51*cosh(x))
*sinh(x)^2 + (35*cosh(x)^6 + 85*cosh(x)^4 - 51*cosh(x)^2 - 5)*sinh(x) - 5*c
osh(x))*sqrt(a*cosh(x)))/(a^2*cosh(x)^8 + 8*a^2*cosh(x)*sinh(x)^7 + a^2*sin
h(x)^8 + 4*a^2*cosh(x)^6 + 4*(7*a^2*cosh(x)^2 + a^2)*sinh(x)^6 + 6*a^2*cosh
(x)^4 + 8*(7*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^5 + 2*(35*a^2*cosh(x)^4
 + 30*a^2*cosh(x)^2 + 3*a^2)*sinh(x)^4 + 4*a^2*cosh(x)^2 + 8*(7*a^2*cosh(x)
^5 + 10*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + 4*(7*a^2*cosh(x)^6 + 15*
a^2*cosh(x)^4 + 9*a^2*cosh(x)^2 + a^2)*sinh(x)^2 + a^2 + 8*(a^2*cosh(x)^7 +
 3*a^2*cosh(x)^5 + 3*a^2*cosh(x)^3 + a^2*cosh(x))*sinh(x))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)**3)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cosh(x)^3)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a\cosh\left(x\right)^3\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)^3)^(3/2),x)
```

```
[Out] int(1/(a*cosh(x)^3)^(3/2), x)
```

3.133 $\int \frac{1}{\left(a\cosh^3(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=121

$$\frac{154i\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)}{195a^2\sqrt{a\cosh^3(x)}}+\frac{154\cosh(x)\sinh(x)}{195a^2\sqrt{a\cosh^3(x)}}+\frac{154\tanh(x)}{585a^2\sqrt{a\cosh^3(x)}}+\frac{22\text{sech}^2(x)\tanh(x)}{117a^2\sqrt{a\cosh^3(x)}}+\frac{2\text{sech}^4(x)\tanh(x)}{13a^2\sqrt{a\cosh^3(x)}}$$

```
[Out] 154/195*I*cosh(x)^(3/2)*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*sinh(
1/2*x),2^(1/2))/a^2/(a*cosh(x)^3)^(1/2)+154/195*cosh(x)*sinh(x)/a^2/(a*cosh
(x)^3)^(1/2)+154/585*tanh(x)/a^2/(a*cosh(x)^3)^(1/2)+22/117*sech(x)^2*tanh(
x)/a^2/(a*cosh(x)^3)^(1/2)+2/13*sech(x)^4*tanh(x)/a^2/(a*cosh(x)^3)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3286, 2716, 2719}

$$\frac{154\sinh(x)\cosh(x)}{195a^2\sqrt{a\cosh^3(x)}}+\frac{154\tanh(x)}{585a^2\sqrt{a\cosh^3(x)}}+\frac{154i\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)}{195a^2\sqrt{a\cosh^3(x)}}+\frac{2\tanh(x)\text{sech}^4(x)}{13a^2\sqrt{a\cosh^3(x)}}+\frac{22\tanh(x)\text{sech}^2(x)}{117a^2\sqrt{a\cosh^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x]^3)^(-5/2),x]
```

```
[Out] (((154*I)/195)*Cosh[x]^(3/2)*EllipticE[(I/2)*x, 2])/(a^2*Sqrt[a*Cosh[x]^3])
 + (154*Cosh[x]*Sinh[x])/(195*a^2*Sqrt[a*Cosh[x]^3]) + (154*Tanh[x])/(585*a
^2*Sqrt[a*Cosh[x]^3]) + (22*Sech[x]^2*Tanh[x])/(117*a^2*Sqrt[a*Cosh[x]^3])
+ (2*Sech[x]^4*Tanh[x])/(13*a^2*Sqrt[a*Cosh[x]^3])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
```

```
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\left(a\cosh^3(x)\right)^{5/2}}\,dx = \frac{\cosh^{\frac{3}{2}}(x)\int \frac{1}{\cosh^{\frac{15}{2}}(x)}\,dx}{a^2\sqrt{a\cosh^3(x)}}$$

$$= \frac{2\text{sech}^4(x)\tanh(x)}{13a^2\sqrt{a\cosh^3(x)}} + \frac{\left(11\cosh^{\frac{3}{2}}(x)\right)\int \frac{1}{\cosh^{\frac{11}{2}}(x)}\,dx}{13a^2\sqrt{a\cosh^3(x)}}$$

$$= \frac{22\text{sech}^2(x)\tanh(x)}{117a^2\sqrt{a\cosh^3(x)}} + \frac{2\text{sech}^4(x)\tanh(x)}{13a^2\sqrt{a\cosh^3(x)}} + \frac{\left(77\cosh^{\frac{3}{2}}(x)\right)\int \frac{1}{\cosh^{\frac{7}{2}}(x)}\,dx}{117a^2\sqrt{a\cosh^3(x)}}$$

$$= \frac{154\tanh(x)}{585a^2\sqrt{a\cosh^3(x)}} + \frac{22\text{sech}^2(x)\tanh(x)}{117a^2\sqrt{a\cosh^3(x)}} + \frac{2\text{sech}^4(x)\tanh(x)}{13a^2\sqrt{a\cosh^3(x)}} + \frac{\left(77\cosh^{\frac{3}{2}}(x)\right)\int}{195a^2\sqrt{a\cos}}$$

$$= \frac{154\cosh(x)\sinh(x)}{195a^2\sqrt{a\cosh^3(x)}} + \frac{154\tanh(x)}{585a^2\sqrt{a\cosh^3(x)}} + \frac{22\text{sech}^2(x)\tanh(x)}{117a^2\sqrt{a\cosh^3(x)}} + \frac{2\text{sech}^4(x)\tanh(}{13a^2\sqrt{a\cosh^3(}}$$

$$= \frac{154i\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)}{195a^2\sqrt{a\cosh^3(x)}} + \frac{154\cosh(x)\sinh(x)}{195a^2\sqrt{a\cosh^3(x)}} + \frac{154\tanh(x)}{585a^2\sqrt{a\cosh^3(x)}} + \frac{22\text{sech}^2(x)\text{ta}}{117a^2\sqrt{a\cos}}$$

Mathematica [A]
time = 0.08, size = 61, normalized size = 0.50

$$\frac{462i\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right) + 462\cosh(x)\sinh(x) + 2\left(77 + 55\text{sech}^2(x) + 45\text{sech}^4(x)\right)\tanh(x)}{585a^2\sqrt{a\cosh^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x]^3)^(-5/2),x]
[Out] ((462*I)*Cosh[x]^(3/2)*EllipticE[(I/2)*x, 2] + 462*Cosh[x]*Sinh[x] + 2*(77
+ 55*Sech[x]^2 + 45*Sech[x]^4)*Tanh[x])/(585*a^2*Sqrt[a*Cosh[x]^3])
```

Maple [F]
time = 0.92, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\left(\cosh^3(x)\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)^3)^(5/2),x)
```

```
[Out] int(1/(a*cosh(x)^3)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cosh(x)^3)^(-5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 1668, normalized size = 13.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/585*(231*(sqrt(2)*cosh(x)^14 + 14*sqrt(2)*cosh(x)*sinh(x)^13 + sqrt(2)*si
nh(x)^14 + 7*(13*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x)^12 + 7*sqrt(2)*cosh(x
)^12 + 28*(13*sqrt(2)*cosh(x)^3 + 3*sqrt(2)*cosh(x))*sinh(x)^11 + 7*(143*sq
rt(2)*cosh(x)^4 + 66*sqrt(2)*cosh(x)^2 + 3*sqrt(2))*sinh(x)^10 + 21*sqrt(2)
*cosh(x)^10 + 14*(143*sqrt(2)*cosh(x)^5 + 110*sqrt(2)*cosh(x)^3 + 15*sqrt(2
)*cosh(x))*sinh(x)^9 + 7*(429*sqrt(2)*cosh(x)^6 + 495*sqrt(2)*cosh(x)^4 + 1
35*sqrt(2)*cosh(x)^2 + 5*sqrt(2))*sinh(x)^8 + 35*sqrt(2)*cosh(x)^8 + 8*(429
*sqrt(2)*cosh(x)^7 + 693*sqrt(2)*cosh(x)^5 + 315*sqrt(2)*cosh(x)^3 + 35*sqr
t(2)*cosh(x))*sinh(x)^7 + 7*(429*sqrt(2)*cosh(x)^8 + 924*sqrt(2)*cosh(x)^6
+ 630*sqrt(2)*cosh(x)^4 + 140*sqrt(2)*cosh(x)^2 + 5*sqrt(2))*sinh(x)^6 + 35
*sqrt(2)*cosh(x)^6 + 14*(143*sqrt(2)*cosh(x)^9 + 396*sqrt(2)*cosh(x)^7 + 37
8*sqrt(2)*cosh(x)^5 + 140*sqrt(2)*cosh(x)^3 + 15*sqrt(2)*cosh(x))*sinh(x)^5
 + 7*(143*sqrt(2)*cosh(x)^10 + 495*sqrt(2)*cosh(x)^8 + 630*sqrt(2)*cosh(x)^
6 + 350*sqrt(2)*cosh(x)^4 + 75*sqrt(2)*cosh(x)^2 + 3*sqrt(2))*sinh(x)^4 + 2
1*sqrt(2)*cosh(x)^4 + 28*(13*sqrt(2)*cosh(x)^11 + 55*sqrt(2)*cosh(x)^9 + 90
*sqrt(2)*cosh(x)^7 + 70*sqrt(2)*cosh(x)^5 + 25*sqrt(2)*cosh(x)^3 + 3*sqrt(2
)*cosh(x))*sinh(x)^3 + 7*(13*sqrt(2)*cosh(x)^12 + 66*sqrt(2)*cosh(x)^10 + 1
35*sqrt(2)*cosh(x)^8 + 140*sqrt(2)*cosh(x)^6 + 75*sqrt(2)*cosh(x)^4 + 18*sq
rt(2)*cosh(x)^2 + sqrt(2))*sinh(x)^2 + 7*sqrt(2)*cosh(x)^2 + 14*(sqrt(2)*co
sh(x)^13 + 6*sqrt(2)*cosh(x)^11 + 15*sqrt(2)*cosh(x)^9 + 20*sqrt(2)*cosh(x)
^7 + 15*sqrt(2)*cosh(x)^5 + 6*sqrt(2)*cosh(x)^3 + sqrt(2)*cosh(x))*sinh(x)
+ sqrt(2))*sqrt(a)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cosh(x
```

```
) + sinh(x))) + 2*(231*cosh(x)^14 + 3234*cosh(x)*sinh(x)^13 + 231*sinh(x)^1
4 + 77*(273*cosh(x)^2 + 20)*sinh(x)^12 + 1540*cosh(x)^12 + 924*(91*cosh(x)^
3 + 20*cosh(x))*sinh(x)^11 + 11*(21021*cosh(x)^4 + 9240*cosh(x)^2 + 397)*si
nh(x)^10 + 4367*cosh(x)^10 + 22*(21021*cosh(x)^5 + 15400*cosh(x)^3 + 1985*c
osh(x))*sinh(x)^9 + (693693*cosh(x)^6 + 762300*cosh(x)^4 + 196515*cosh(x)^2
 + 6808)*sinh(x)^8 + 6808*cosh(x)^8 + 8*(99099*cosh(x)^7 + 152460*cosh(x)^5
 + 65505*cosh(x)^3 + 6808*cosh(x))*sinh(x)^7 + (693693*cosh(x)^8 + 1422960*
cosh(x)^6 + 917070*cosh(x)^4 + 190624*cosh(x)^2 + 1277)*sinh(x)^6 + 1277*co
sh(x)^6 + 2*(231231*cosh(x)^9 + 609840*cosh(x)^7 + 550242*cosh(x)^5 + 19062
4*cosh(x)^3 + 3831*cosh(x))*sinh(x)^5 + (231231*cosh(x)^10 + 762300*cosh(x)
^8 + 917070*cosh(x)^6 + 476560*cosh(x)^4 + 19155*cosh(x)^2 + 484)*sinh(x)^4
 + 484*cosh(x)^4 + 4*(21021*cosh(x)^11 + 84700*cosh(x)^9 + 131010*cosh(x)^7
 + 95312*cosh(x)^5 + 6385*cosh(x)^3 + 484*cosh(x))*sinh(x)^3 + (21021*cosh(
x)^12 + 101640*cosh(x)^10 + 196515*cosh(x)^8 + 190624*cosh(x)^6 + 19155*cos
h(x)^4 + 2904*cosh(x)^2 + 77)*sinh(x)^2 + 77*cosh(x)^2 + 2*(1617*cosh(x)^13
 + 9240*cosh(x)^11 + 21835*cosh(x)^9 + 27232*cosh(x)^7 + 3831*cosh(x)^5 + 9
68*cosh(x)^3 + 77*cosh(x))*sinh(x))*sqrt(a*cosh(x)))/(a^3*cosh(x)^14 + 14*a
^3*cosh(x)*sinh(x)^13 + a^3*sinh(x)^14 + 7*a^3*cosh(x)^12 + 21*a^3*cosh(x)^
10 + 7*(13*a^3*cosh(x)^2 + a^3)*sinh(x)^12 + 28*(13*a^3*cosh(x)^3 + 3*a^3*c
osh(x))*sinh(x)^11 + 35*a^3*cosh(x)^8 + 7*(143*a^3*cosh(x)^4 + 66*a^3*cosh(
x)^2 + 3*a^3)*sinh(x)^10 + 14*(143*a^3*cosh(x)^5 + 110*a^3*cosh(x)^3 + 15*a
^3*cosh(x))*sinh(x)^9 + 35*a^3*cosh(x)^6 + 7*(429*a^3*cosh(x)^6 + 495*a^3*c
osh(x)^4 + 135*a^3*cosh(x)^2 + 5*a^3)*sinh(x)^8 + 8*(429*a^3*cosh(x)^7 + 69
3*a^3*cosh(x)^5 + 315*a^3*cosh(x)^3 + 35*a^3*cosh(x))*sinh(x)^7 + 21*a^3*co
sh(x)^4 + 7*(429*a^3*cosh(x)^8 + 924*a^3*cosh(x)^6 + 630*a^3*cosh(x)^4 + 14
0*a^3*cosh(x)^2 + 5*a^3)*sinh(x)^6 + 14*(143*a^3*cosh(x)^9 + 396*a^3*cosh(x
)^7 + 378*a^3*cosh(x)^5 + 140*a^3*cosh(x)^3 + 15*a^3*cosh(x))*sinh(x)^5 + 7
*a^3*cosh(x)^2 + 7*(143*a^3*cosh(x)^10 + 495*a^3*cosh(x)^8 + 630*a^3*cosh(x
)^6 + 350*a^3*cosh(x)^4 + 75*a^3*cosh(x)^2 + 3*a^3)*sinh(x)^4 + 28*(13*a^3*
cosh(x)^11 + 55*a^3*cosh(x)^9 + 90*a^3*cosh(x)^7 + 70*a^3*cosh(x)^5 + 25*a^
3*cosh(x)^3 + 3*a^3*cosh(x))*sinh(x)^3 + a^3 + 7*(13*a^3*cosh(x)^12 + 66*a^
3*cosh(x)^10 + 135*a^3*cosh(x)^8 + 140*a^3*cosh(x)^6 + 75*a^3*cosh(x)^4 + 1
8*a^3*cosh(x)^2 + a^3)*sinh(x)^2 + 14*(a^3*cosh(x)^13 + 6*a^3*cosh(x)^11 +
15*a^3*cosh(x)^9 + 20*a^3*cosh(x)^7 + 15*a^3*cosh(x)^5 + 6*a^3*cosh(x)^3 +
a^3*cosh(x))*sinh(x))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)**3)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3880 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^3)^(5/2),x, algorithm="giac")
[Out] integrate((a*cosh(x)^3)^(-5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a\cosh\left(x\right)^3\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)^3)^(5/2),x)
[Out] int(1/(a*cosh(x)^3)^(5/2), x)
```

3.134 $\int \left(a\cosh^4(x)\right)^{5/2}\,dx$

Optimal. Leaf size=132

$$\frac{63}{256}a^2x\sqrt{a\cosh^4(x)}\,\text{sech}^2(x)+\frac{21}{128}a^2\cosh(x)\sqrt{a\cosh^4(x)}\,\sinh(x)+\frac{21}{160}a^2\cosh^3(x)\sqrt{a\cosh^4(x)}\,\sinh(x)+\frac{9}{8}$$

```
[Out] 63/256*a^2*x*sech(x)^2*(a*cosh(x)^4)^(1/2)+21/128*a^2*cosh(x)*sinh(x)*(a*co
sh(x)^4)^(1/2)+21/160*a^2*cosh(x)^3*sinh(x)*(a*cosh(x)^4)^(1/2)+9/80*a^2*co
sh(x)^5*sinh(x)*(a*cosh(x)^4)^(1/2)+1/10*a^2*cosh(x)^7*sinh(x)*(a*cosh(x)^4
)^(1/2)+63/256*a^2*(a*cosh(x)^4)^(1/2)*tanh(x)
```

Rubi [A]
time = 0.04, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3286, 2715, 8}

$$\frac{21}{128}a^2\sinh(x)\cosh(x)\sqrt{a\cosh^4(x)}+\frac{63}{256}a^2\tanh(x)\sqrt{a\cosh^4(x)}+\frac{63}{256}a^2x\,\text{sech}^2(x)\sqrt{a\cosh^4(x)}+\frac{1}{10}a^2\sinh(x)\cosh^7(x)\sqrt{a\cosh^4(x)}+\frac{9}{80}a^2\sinh(x)\cosh^5(x)\sqrt{a\cosh^4(x)}+\frac{21}{160}a^2\sinh(x)\cosh^3(x)\sqrt{a\cosh^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x]^4)^(5/2),x]
```

```
[Out] (63*a^2*x*Sqrt[a*Cosh[x]^4]*Sech[x]^2)/256 + (21*a^2*Cosh[x]*Sqrt[a*Cosh[x]
^4]*Sinh[x])/128 + (21*a^2*Cosh[x]^3*Sqrt[a*Cosh[x]^4]*Sinh[x])/160 + (9*a^
2*Cosh[x]^5*Sqrt[a*Cosh[x]^4]*Sinh[x])/80 + (a^2*Cosh[x]^7*Sqrt[a*Cosh[x]^4
]*Sinh[x])/10 + (63*a^2*Sqrt[a*Cosh[x]^4]*Tanh[x])/256
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(a\cosh^4(x)\right)^{5/2}\,dx = \left(a^2\sqrt{a\cosh^4(x)}\,\operatorname{sech}^2(x)\right)\int \cosh^{10}(x)\,dx$$
$$= \frac{1}{10}a^2\cosh^7(x)\sqrt{a\cosh^4(x)}\,\sinh(x) + \frac{1}{10}\left(9a^2\sqrt{a\cosh^4(x)}\,\operatorname{sech}^2(x)\right)\int \cosh^8(x)\,$$
$$= \frac{9}{80}a^2\cosh^5(x)\sqrt{a\cosh^4(x)}\,\sinh(x) + \frac{1}{10}a^2\cosh^7(x)\sqrt{a\cosh^4(x)}\,\sinh(x) + \frac{1}{80}\left(6\right.$$
$$= \frac{21}{160}a^2\cosh^3(x)\sqrt{a\cosh^4(x)}\,\sinh(x) + \frac{9}{80}a^2\cosh^5(x)\sqrt{a\cosh^4(x)}\,\sinh(x) + \frac{1}{10}a^2$$
$$= \frac{21}{128}a^2\cosh(x)\sqrt{a\cosh^4(x)}\,\sinh(x) + \frac{21}{160}a^2\cosh^3(x)\sqrt{a\cosh^4(x)}\,\sinh(x) + \frac{9}{80}a^2$$
$$= \frac{21}{128}a^2\cosh(x)\sqrt{a\cosh^4(x)}\,\sinh(x) + \frac{21}{160}a^2\cosh^3(x)\sqrt{a\cosh^4(x)}\,\sinh(x) + \frac{9}{80}a^2$$
$$= \frac{63}{256}a^2x\sqrt{a\cosh^4(x)}\,\operatorname{sech}^2(x) + \frac{21}{128}a^2\cosh(x)\sqrt{a\cosh^4(x)}\,\sinh(x) + \frac{21}{160}a^2\cosh^3$$

Mathematica [A]
time = 0.08, size = 53, normalized size = 0.40

$$\frac{a\left(a\cosh^4(x)\right)^{3/2}\operatorname{sech}^6(x)(2520x + 2100\sinh(2x) + 600\sinh(4x) + 150\sinh(6x) + 25\sinh(8x) + 2\sinh(10x))}{10240}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x]^4)^(5/2),x]
[Out] (a*(a*Cosh[x]^4)^(3/2)*Sech[x]^6*(2520*x + 2100*Sinh[2*x] + 600*Sinh[4*x] +
 150*Sinh[6*x] + 25*Sinh[8*x] + 2*Sinh[10*x]))/10240
```

Maple [A]
time = 1.92, size = 177, normalized size = 1.34

method	result
default	$\frac{\sqrt{8}\,(1+\cosh(2x))\sqrt{a\left(-1+\cosh\left(2x\right)\right)\left(1+\cosh\left(2x\right)\right)}\,\sqrt{2}\,a^{\frac{3}{2}}\left(8\sqrt{a}\,\sqrt{a\left(\sinh^2\left(2x\right)\right)}\,(\sinh^4(2x))+50\right.}{}$
risch	$\frac{63a^2e^{2x}\sqrt{a\left(1+e^{2x}\right)^4e^{-4x}}\,x}{256(1+e^{2x})^2} + \frac{a^2e^{12x}\sqrt{a\left(1+e^{2x}\right)^4e^{-4x}}}{10240(1+e^{2x})^2} + \frac{5a^2e^{10x}\sqrt{a\left(1+e^{2x}\right)^4e^{-4x}}}{4096(1+e^{2x})^2} + \frac{15a^2e^{8x}\sqrt{a}}{20}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^4)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10240*8^(1/2)*(1+cosh(2*x))*(a*(-1+cosh(2*x))*(1+cosh(2*x)))^(1/2)*2^(1/2
)*a^(3/2)*(8*a^(1/2)*(a*sinh(2*x)^2)^(1/2)*sinh(2*x)^4+50*a^(1/2)*(a*sinh(2
*x)^2)^(1/2)*cosh(2*x)*sinh(2*x)^2+160*a^(1/2)*(a*sinh(2*x)^2)^(1/2)*sinh(2
*x)^2+325*cosh(2*x)*(a*sinh(2*x)^2)^(1/2)*a^(1/2)+640*(a*sinh(2*x)^2)^(1/2)
*a^(1/2)+315*ln(cosh(2*x)*a^(1/2)+(a*sinh(2*x)^2)^(1/2))*a)/sinh(2*x)/(a*(1
+cosh(2*x))^2)^(1/2)
```

Maxima [A]
time = 0.49, size = 100, normalized size = 0.76

$$\frac{63}{256}\,a^{\frac{5}{2}}x+\frac{1}{20480}\,\left(25\,a^{\frac{5}{2}}e^{(-2\,x)}+150\,a^{\frac{5}{2}}e^{(-4\,x)}+600\,a^{\frac{5}{2}}e^{(-6\,x)}+2100\,a^{\frac{5}{2}}e^{(-8\,x)}-2100\,a^{\frac{5}{2}}e^{(-12\,x)}-600\,a^{\frac{5}{2}}e^{(-14\,x)}-150\,a^{\frac{5}{2}}e^{(-16\,x)}-25\,a^{\frac{5}{2}}e^{(-18\,x)}-2\,a^{\frac{5}{2}}e^{(-20\,x)}+2\,a^{\frac{5}{2}}\right)e^{(10\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] 63/256*a^(5/2)*x + 1/20480*(25*a^(5/2)*e^(-2*x) + 150*a^(5/2)*e^(-4*x) + 60
0*a^(5/2)*e^(-6*x) + 2100*a^(5/2)*e^(-8*x) - 2100*a^(5/2)*e^(-12*x) - 600*a
^(5/2)*e^(-14*x) - 150*a^(5/2)*e^(-16*x) - 25*a^(5/2)*e^(-18*x) - 2*a^(5/2)
*e^(-20*x) + 2*a^(5/2))*e^(10*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1597 vs. $2(108) = 216$.
time = 0.40, size = 1597, normalized size = 12.10

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/20480*(40*a^2*cosh(x)*e^(2*x)*sinh(x)^19 + 2*a^2*e^(2*x)*sinh(x)^20 + 5*(
76*a^2*cosh(x)^2 + 5*a^2)*e^(2*x)*sinh(x)^18 + 30*(76*a^2*cosh(x)^3 + 15*a^
2*cosh(x))*e^(2*x)*sinh(x)^17 + 15*(646*a^2*cosh(x)^4 + 255*a^2*cosh(x)^2 +
 10*a^2)*e^(2*x)*sinh(x)^16 + 48*(646*a^2*cosh(x)^5 + 425*a^2*cosh(x)^3 + 5
0*a^2*cosh(x))*e^(2*x)*sinh(x)^15 + 60*(1292*a^2*cosh(x)^6 + 1275*a^2*cosh(
x)^4 + 300*a^2*cosh(x)^2 + 10*a^2)*e^(2*x)*sinh(x)^14 + 120*(1292*a^2*cosh(
x)^7 + 1785*a^2*cosh(x)^5 + 700*a^2*cosh(x)^3 + 70*a^2*cosh(x))*e^(2*x)*sin
h(x)^13 + 60*(4199*a^2*cosh(x)^8 + 7735*a^2*cosh(x)^6 + 4550*a^2*cosh(x)^4
+ 910*a^2*cosh(x)^2 + 35*a^2)*e^(2*x)*sinh(x)^12 + 80*(4199*a^2*cosh(x)^9 +
 9945*a^2*cosh(x)^7 + 8190*a^2*cosh(x)^5 + 2730*a^2*cosh(x)^3 + 315*a^2*cos
h(x))*e^(2*x)*sinh(x)^11 + 2*(184756*a^2*cosh(x)^10 + 546975*a^2*cosh(x)^8
+ 600600*a^2*cosh(x)^6 + 300300*a^2*cosh(x)^4 + 69300*a^2*cosh(x)^2 + 2520*
a^2*x)*e^(2*x)*sinh(x)^10 + 20*(16796*a^2*cosh(x)^11 + 60775*a^2*cosh(x)^9
+ 85800*a^2*cosh(x)^7 + 60060*a^2*cosh(x)^5 + 23100*a^2*cosh(x)^3 + 2520*a^
2*x*cosh(x))*e^(2*x)*sinh(x)^9 + 30*(8398*a^2*cosh(x)^12 + 36465*a^2*cosh(x
```

```
)^10 + 64350*a^2*cosh(x)^8 + 60060*a^2*cosh(x)^6 + 34650*a^2*cosh(x)^4 + 75
60*a^2*x*cosh(x)^2 - 70*a^2)*e^(2*x)*sinh(x)^8 + 240*(646*a^2*cosh(x)^13 +
3315*a^2*cosh(x)^11 + 7150*a^2*cosh(x)^9 + 8580*a^2*cosh(x)^7 + 6930*a^2*co
sh(x)^5 + 2520*a^2*x*cosh(x)^3 - 70*a^2*cosh(x))*e^(2*x)*sinh(x)^7 + 60*(12
92*a^2*cosh(x)^14 + 7735*a^2*cosh(x)^12 + 20020*a^2*cosh(x)^10 + 30030*a^2*
cosh(x)^8 + 32340*a^2*cosh(x)^6 + 17640*a^2*x*cosh(x)^4 - 980*a^2*cosh(x)^2
 - 10*a^2)*e^(2*x)*sinh(x)^6 + 24*(1292*a^2*cosh(x)^15 + 8925*a^2*cosh(x)^1
3 + 27300*a^2*cosh(x)^11 + 50050*a^2*cosh(x)^9 + 69300*a^2*cosh(x)^7 + 5292
0*a^2*x*cosh(x)^5 - 4900*a^2*cosh(x)^3 - 150*a^2*cosh(x))*e^(2*x)*sinh(x)^5
 + 30*(323*a^2*cosh(x)^16 + 2550*a^2*cosh(x)^14 + 9100*a^2*cosh(x)^12 + 200
20*a^2*cosh(x)^10 + 34650*a^2*cosh(x)^8 + 35280*a^2*x*cosh(x)^6 - 4900*a^2*
cosh(x)^4 - 300*a^2*cosh(x)^2 - 5*a^2)*e^(2*x)*sinh(x)^4 + 120*(19*a^2*cosh
(x)^17 + 170*a^2*cosh(x)^15 + 700*a^2*cosh(x)^13 + 1820*a^2*cosh(x)^11 + 38
50*a^2*cosh(x)^9 + 5040*a^2*x*cosh(x)^7 - 980*a^2*cosh(x)^5 - 100*a^2*cosh(
x)^3 - 5*a^2*cosh(x))*e^(2*x)*sinh(x)^3 + 5*(76*a^2*cosh(x)^18 + 765*a^2*co
sh(x)^16 + 3600*a^2*cosh(x)^14 + 10920*a^2*cosh(x)^12 + 27720*a^2*cosh(x)^1
0 + 45360*a^2*x*cosh(x)^8 - 11760*a^2*cosh(x)^6 - 1800*a^2*cosh(x)^4 - 180*
a^2*cosh(x)^2 - 5*a^2)*e^(2*x)*sinh(x)^2 + 10*(4*a^2*cosh(x)^19 + 45*a^2*co
sh(x)^17 + 240*a^2*cosh(x)^15 + 840*a^2*cosh(x)^13 + 2520*a^2*cosh(x)^11 +
5040*a^2*x*cosh(x)^9 - 1680*a^2*cosh(x)^7 - 360*a^2*cosh(x)^5 - 60*a^2*cosh
(x)^3 - 5*a^2*cosh(x))*e^(2*x)*sinh(x) + (2*a^2*cosh(x)^20 + 25*a^2*cosh(x)
^18 + 150*a^2*cosh(x)^16 + 600*a^2*cosh(x)^14 + 2100*a^2*cosh(x)^12 + 5040*
a^2*x*cosh(x)^10 - 2100*a^2*cosh(x)^8 - 600*a^2*cosh(x)^6 - 150*a^2*cosh(x)
^4 - 25*a^2*cosh(x)^2 - 2*a^2)*e^(2*x))*sqrt(a*e^(8*x) + 4*a*e^(6*x) + 6*a*
e^(4*x) + 4*a*e^(2*x) + a)*e^(-2*x)/(cosh(x)^10*e^(4*x) + 2*cosh(x)^10*e^(2
*x) + (e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^10 + cosh(x)^10 + 10*(cosh(x)*e^(4*
x) + 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^9 + 45*(cosh(x)^2*e^(4*x) + 2*cos
h(x)^2*e^(2*x) + cosh(x)^2)*sinh(x)^8 + 120*(cosh(x)^3*e^(4*x) + 2*cosh(x)^
3*e^(2*x) + cosh(x)^3)*sinh(x)^7 + 210*(cosh(x)^4*e^(4*x) + 2*cosh(x)^4*e^(
2*x) + cosh(x)^4)*sinh(x)^6 + 252*(cosh(x)^5*e^(4*x) + 2*cosh(x)^5*e^(2*x)
+ cosh(x)^5)*sinh(x)^5 + 210*(cosh(x)^6*e^(4*x) + 2*cosh(x)^6*e^(2*x) + cos
h(x)^6)*sinh(x)^4 + 120*(cosh(x)^7*e^(4*x) + 2*cosh(x)^7*e^(2*x) + cosh(x)^
7)*sinh(x)^3 + 45*(cosh(x)^8*e^(4*x) + 2*cosh(x)^8*e^(2*x) + cosh(x)^8)*sin
h(x)^2 + 10*(cosh(x)^9*e^(4*x) + 2*cosh(x)^9*e^(2*x) + cosh(x)^9)*sinh(x))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)**4)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]

time = 0.41, size = 114, normalized size = 0.86

$$\frac{1}{20480}\left(5040\,a^{2}x+2\,a^{2}e^{(10\,x)}+25\,a^{2}e^{(8\,x)}+150\,a^{2}e^{(6\,x)}+600\,a^{2}e^{(4\,x)}+2100\,a^{2}e^{(2\,x)}-\left(5754\,a^{2}e^{(10\,x)}+2100\,a^{2}e^{(8\,x)}+600\,a^{2}e^{(6\,x)}+150\,a^{2}e^{(4\,x)}+25\,a^{2}e^{(2\,x)}+2\,a^{2}\right)e^{(-10\,x)}\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] 1/20480*(5040*a^2*x + 2*a^2*e^(10*x) + 25*a^2*e^(8*x) + 150*a^2*e^(6*x) + 6
00*a^2*e^(4*x) + 2100*a^2*e^(2*x) - (5754*a^2*e^(10*x) + 2100*a^2*e^(8*x) +
 600*a^2*e^(6*x) + 150*a^2*e^(4*x) + 25*a^2*e^(2*x) + 2*a^2)*e^(-10*x))*sqr
t(a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a\cosh(x)^4\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^4)^(5/2),x)
```

```
[Out] int((a*cosh(x)^4)^(5/2), x)
```

3.135 $\int \left(a\cosh^4(x)\right)^{3/2}\,dx$

Optimal. Leaf size=78

$$\frac{5}{16}ax\sqrt{a\cosh^4(x)}\,\mathrm{sech}^2(x)+\frac{5}{24}a\cosh(x)\sqrt{a\cosh^4(x)}\,\sinh(x)+\frac{1}{6}a\cosh^3(x)\sqrt{a\cosh^4(x)}\,\sinh(x)+\frac{5}{16}a\sqrt{a$$

```
[Out] 5/16*a*x*sech(x)^2*(a*cosh(x)^4)^(1/2)+5/24*a*cosh(x)*sinh(x)*(a*cosh(x)^4)
^(1/2)+1/6*a*cosh(x)^3*sinh(x)*(a*cosh(x)^4)^(1/2)+5/16*a*(a*cosh(x)^4)^(1/
2)*tanh(x)
```

Rubi [A]
time = 0.02, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3286, 2715, 8}

$$\frac{5}{24}a\sinh(x)\cosh(x)\sqrt{a\cosh^4(x)}+\frac{5}{16}a\tanh(x)\sqrt{a\cosh^4(x)}+\frac{5}{16}ax\mathrm{sech}^2(x)\sqrt{a\cosh^4(x)}+\frac{1}{6}a\sinh(x)\cosh^3(x)\sqrt{a\cosh^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x]^4)^(3/2),x]
```

```
[Out] (5*a*x*Sqrt[a*Cosh[x]^4]*Sech[x]^2)/16 + (5*a*Cosh[x]*Sqrt[a*Cosh[x]^4]*Sin
h[x])/24 + (a*Cosh[x]^3*Sqrt[a*Cosh[x]^4]*Sinh[x])/6 + (5*a*Sqrt[a*Cosh[x]^
4]*Tanh[x])/16
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(a\cosh^4(x)\right)^{3/2}\,dx = \left(a\sqrt{a\cosh^4(x)}\,\operatorname{sech}^2(x)\right)\int \cosh^6(x)\,dx$$
$$= \frac{1}{6}a\cosh^3(x)\sqrt{a\cosh^4(x)}\,\sinh(x) + \frac{1}{6}\left(5a\sqrt{a\cosh^4(x)}\,\operatorname{sech}^2(x)\right)\int \cosh^4(x)\,dx$$
$$= \frac{5}{24}a\cosh(x)\sqrt{a\cosh^4(x)}\,\sinh(x) + \frac{1}{6}a\cosh^3(x)\sqrt{a\cosh^4(x)}\,\sinh(x) + \frac{1}{8}\Big(5a\sqrt{a\,\mathrm{co}}$$
$$= \frac{5}{24}a\cosh(x)\sqrt{a\cosh^4(x)}\,\sinh(x) + \frac{1}{6}a\cosh^3(x)\sqrt{a\cosh^4(x)}\,\sinh(x) + \frac{5}{16}a\sqrt{a\,\mathrm{cos}}$$
$$= \frac{5}{16}ax\sqrt{a\cosh^4(x)}\,\operatorname{sech}^2(x) + \frac{5}{24}a\cosh(x)\sqrt{a\cosh^4(x)}\,\sinh(x) + \frac{1}{6}a\cosh^3(x)\sqrt{a\,\mathrm{co}}$$

Mathematica [A]
time = 0.05, size = 38, normalized size = 0.49

$$\frac{1}{192}\left(a\cosh^4(x)\right)^{3/2}\operatorname{sech}^6(x)(60x + 45\sinh(2x) + 9\sinh(4x) + \sinh(6x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x]^4)^(3/2),x]
```

```
[Out] ((a*Cosh[x]^4)^(3/2)*Sech[x]^6*(60*x + 45*Sinh[2*x] + 9*Sinh[4*x] + Sinh[6*
x]))/192
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(62) = 124.
time = 1.86, size = 131, normalized size = 1.68

method	result
default	$\frac{\sqrt{8}\,(1+\cosh(2x))\sqrt{a(-1+\cosh(2x))(1+\cosh(2x))}\,\sqrt{2}\,\sqrt{a}\,\left(2\sqrt{a}\,\sqrt{a\left(\sinh^2(2x)\right)}\,(\sinh^2(2x))+9\,\mathrm{co}\right.}{384\sinh(2x)\sqrt{a\,(1+\mathrm{c}}}$
risch	$\frac{5a\,\mathrm{e}^{2x}\sqrt{a\left(1+\mathrm{e}^{2x}\right)^4\mathrm{e}^{-4x}}\,x}{16(1+\mathrm{e}^{2x})^2} + \frac{a\,\mathrm{e}^{8x}\sqrt{a\left(1+\mathrm{e}^{2x}\right)^4\mathrm{e}^{-4x}}}{384(1+\mathrm{e}^{2x})^2} + \frac{3a\,\mathrm{e}^{6x}\sqrt{a\left(1+\mathrm{e}^{2x}\right)^4\mathrm{e}^{-4x}}}{128(1+\mathrm{e}^{2x})^2} + \frac{15a\,\mathrm{e}^{4x}\sqrt{a\,(1+}}{128(1+\mathrm{e}^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/384*8^(1/2)*(1+cosh(2*x))*(a*(-1+cosh(2*x))*(1+cosh(2*x)))^(1/2)*2^(1/2)*
a^(1/2)*(2*a^(1/2)*(a*sinh(2*x)^2)^(1/2)*sinh(2*x)^2+9*cosh(2*x)*(a*sinh(2*
x)^2)^(1/2)*a^(1/2)+24*(a*sinh(2*x)^2)^(1/2)*a^(1/2)+15*ln(cosh(2*x)*a^(1/2
)+(a*sinh(2*x)^2)^(1/2))*a)/sinh(2*x)/(a*(1+cosh(2*x))^2)^(1/2)
```

Maxima [A]
time = 0.49, size = 62, normalized size = 0.79

$$\frac{5}{16}\,a^{\frac{3}{2}}x+\frac{1}{384}\left(9\,a^{\frac{3}{2}}e^{(-2\,x)}+45\,a^{\frac{3}{2}}e^{(-4\,x)}-45\,a^{\frac{3}{2}}e^{(-8\,x)}-9\,a^{\frac{3}{2}}e^{(-10\,x)}-a^{\frac{3}{2}}e^{(-12\,x)}+a^{\frac{3}{2}}\right)e^{(6\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] 5/16*a^(3/2)*x + 1/384*(9*a^(3/2)*e^(-2*x) + 45*a^(3/2)*e^(-4*x) - 45*a^(3/
2)*e^(-8*x) - 9*a^(3/2)*e^(-10*x) - a^(3/2)*e^(-12*x) + a^(3/2))*e^(6*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 659 vs. 2(62) = 124.
time = 0.42, size = 659, normalized size = 8.45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/384*(12*a*cosh(x)*e^(2*x)*sinh(x)^11 + a*e^(2*x)*sinh(x)^12 + 3*(22*a*cos
h(x)^2 + 3*a)*e^(2*x)*sinh(x)^10 + 10*(22*a*cosh(x)^3 + 9*a*cosh(x))*e^(2*x
)*sinh(x)^9 + 45*(11*a*cosh(x)^4 + 9*a*cosh(x)^2 + a)*e^(2*x)*sinh(x)^8 + 7
2*(11*a*cosh(x)^5 + 15*a*cosh(x)^3 + 5*a*cosh(x))*e^(2*x)*sinh(x)^7 + 6*(15
4*a*cosh(x)^6 + 315*a*cosh(x)^4 + 210*a*cosh(x)^2 + 20*a*x)*e^(2*x)*sinh(x)
^6 + 36*(22*a*cosh(x)^7 + 63*a*cosh(x)^5 + 70*a*cosh(x)^3 + 20*a*x*cosh(x))
*e^(2*x)*sinh(x)^5 + 45*(11*a*cosh(x)^8 + 42*a*cosh(x)^6 + 70*a*cosh(x)^4 +
 40*a*x*cosh(x)^2 - a)*e^(2*x)*sinh(x)^4 + 20*(11*a*cosh(x)^9 + 54*a*cosh(x
)^7 + 126*a*cosh(x)^5 + 120*a*x*cosh(x)^3 - 9*a*cosh(x))*e^(2*x)*sinh(x)^3
+ 3*(22*a*cosh(x)^10 + 135*a*cosh(x)^8 + 420*a*cosh(x)^6 + 600*a*x*cosh(x)^
4 - 90*a*cosh(x)^2 - 3*a)*e^(2*x)*sinh(x)^2 + 6*(2*a*cosh(x)^11 + 15*a*cosh
(x)^9 + 60*a*cosh(x)^7 + 120*a*x*cosh(x)^5 - 30*a*cosh(x)^3 - 3*a*cosh(x))*
e^(2*x)*sinh(x) + (a*cosh(x)^12 + 9*a*cosh(x)^10 + 45*a*cosh(x)^8 + 120*a*x
*cosh(x)^6 - 45*a*cosh(x)^4 - 9*a*cosh(x)^2 - a)*e^(2*x))*sqrt(a*e^(8*x) +
4*a*e^(6*x) + 6*a*e^(4*x) + 4*a*e^(2*x) + a)*e^(-2*x)/(cosh(x)^6*e^(4*x) +
2*cosh(x)^6*e^(2*x) + (e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^6 + cosh(x)^6 + 6*(
cosh(x)*e^(4*x) + 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^5 + 15*(cosh(x)^2*e^
(4*x) + 2*cosh(x)^2*e^(2*x) + cosh(x)^2)*sinh(x)^4 + 20*(cosh(x)^3*e^(4*x)
+ 2*cosh(x)^3*e^(2*x) + cosh(x)^3)*sinh(x)^3 + 15*(cosh(x)^4*e^(4*x) + 2*co
sh(x)^4*e^(2*x) + cosh(x)^4)*sinh(x)^2 + 6*(cosh(x)^5*e^(4*x) + 2*cosh(x)^5
*e^(2*x) + cosh(x)^5)*sinh(x))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)**4)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4848 deep
```

Giac [A]
time = 0.41, size = 52, normalized size = 0.67

$$-\frac{1}{384}\left(\left(110\,e^{(6\,x)}+45\,e^{(4\,x)}+9\,e^{(2\,x)}+1\right)e^{(-6\,x)}-120\,x-e^{(6\,x)}-9\,e^{(4\,x)}-45\,e^{(2\,x)}\right)a^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] -1/384*((110*e^(6*x) + 45*e^(4*x) + 9*e^(2*x) + 1)*e^(-6*x) - 120*x - e^(6*
x) - 9*e^(4*x) - 45*e^(2*x))*a^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a\,{\mathrm{cosh}\left(x\right)}^4\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^4)^(3/2),x)
```

```
[Out] int((a*cosh(x)^4)^(3/2), x)
```

3.136 $\int \sqrt{a\cosh^4(x)}\,dx$

Optimal. Leaf size=36

$$\frac{1}{2}x\sqrt{a\cosh^4(x)}\,\text{sech}^2(x)+\frac{1}{2}\sqrt{a\cosh^4(x)}\,\tanh(x)$$

```
[Out] 1/2*x*sech(x)^2*(a*cosh(x)^4)^(1/2)+1/2*(a*cosh(x)^4)^(1/2)*tanh(x)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3286, 2715, 8}

$$\frac{1}{2}\tanh(x)\sqrt{a\cosh^4(x)}+\frac{1}{2}x\text{sech}^2(x)\sqrt{a\cosh^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Cosh[x]^4],x]
```

```
[Out] (x*Sqrt[a*Cosh[x]^4]*Sech[x]^2)/2 + (Sqrt[a*Cosh[x]^4]*Tanh[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \sqrt{a\cosh^4(x)}\,dx = \left(\sqrt{a\cosh^4(x)}\,\operatorname{sech}^2(x)\right)\int \cosh^2(x)\,dx$$
$$= \frac{1}{2}\sqrt{a\cosh^4(x)}\,\tanh(x) + \frac{1}{2}\left(\sqrt{a\cosh^4(x)}\,\operatorname{sech}^2(x)\right)\int 1\,dx$$
$$= \frac{1}{2}x\sqrt{a\cosh^4(x)}\,\operatorname{sech}^2(x) + \frac{1}{2}\sqrt{a\cosh^4(x)}\,\tanh(x)$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 0.69

$$\frac{1}{2}\sqrt{a\cosh^4(x)}\,\operatorname{sech}^2(x)(x+\cosh(x)\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Cosh[x]^4],x]
[Out] (Sqrt[a*Cosh[x]^4]*Sech[x]^2*(x + Cosh[x]*Sinh[x]))/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(28) = 56$.
time = 1.87, size = 89, normalized size = 2.47

method	result
default	$\frac{\sqrt{8}\,(1+\cosh(2x))\sqrt{a\left(-1+\cosh\left(2x\right)\right)\left(1+\cosh\left(2x\right)\right)}\,\sqrt{2}\,\left(\sqrt{a\left(\sinh^2\left(2x\right)\right)}\,\sqrt{a}+\ln\left(\cosh(2x)\sqrt{a}+\sqrt{\ldots}\right.\right.}{16\sqrt{a}\,\sinh(2x)\sqrt{a\left(1+\cosh\left(2x\right)\right)^2}}$
risch	$\frac{\sqrt{a\left(1+e^{2x}\right)^4 e^{-4x}}\,e^{2x}x}{2(1+e^{2x})^2} + \frac{\sqrt{a\left(1+e^{2x}\right)^4 e^{-4x}}\,e^{4x}}{8(1+e^{2x})^2} - \frac{\sqrt{a\left(1+e^{2x}\right)^4 e^{-4x}}}{8(1+e^{2x})^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^4)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/16*8^(1/2)*(1+cosh(2*x))*(a*(-1+cosh(2*x))*(1+cosh(2*x)))^(1/2)*2^(1/2)*(
(a*sinh(2*x)^2)^(1/2)*a^(1/2)+ln(cosh(2*x)*a^(1/2)+(a*sinh(2*x)^2)^(1/2))*a
)/a^(1/2)/sinh(2*x)/(a*(1+cosh(2*x))^2)^(1/2)
```

Maxima [A]
time = 0.48, size = 27, normalized size = 0.75

$$-\frac{1}{8}\left(\sqrt{a}\,e^{(-4x)} - \sqrt{a}\right)e^{(2x)} + \frac{1}{2}\sqrt{a}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*(sqrt(a)*e^(-4*x) - sqrt(a))*e^(2*x) + 1/2*sqrt(a)*x
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(28) = 56$.
time = 0.37, size = 180, normalized size = 5.00

$$\frac{\left(4 \cosh\left(x\right) e^{\left(2 x\right)} \sinh\left(x\right)^{3} + e^{\left(2 x\right)} \sinh\left(x\right)^{4} + 2 \left(3 \cosh\left(x\right)^{2} + 2 x\right) e^{\left(2 x\right)} \sinh\left(x\right)^{2} + 4 \left(\cosh\left(x\right)^{3} + 2 x \cosh\left(x\right)\right) e^{\left(2 x\right)} \sinh\left(x\right) + \left(\cosh\left(x\right)^{4} + 4 x \cosh\left(x\right)^{2} - 1\right) e^{\left(2 x\right)}\right) \sqrt{a e^{\left(8 x\right)} + 4 a e^{\left(6 x\right)} + 6 a e^{\left(4 x\right)} + 4 a e^{\left(2 x\right)} + a}\, e^{\left(-2 x\right)}}{8 \left(\cosh\left(x\right)^{2} e^{\left(4 x\right)} + 2 \cosh\left(x\right)^{2} e^{\left(2 x\right)} + \left(e^{\left(4 x\right)} + 2 e^{\left(2 x\right)} + 1\right) \sinh\left(x\right)^{2} + \cosh\left(x\right)^{2} + 2 \left(\cosh\left(x\right) e^{\left(4 x\right)} + 2 \cosh\left(x\right) e^{\left(2 x\right)} + \cosh\left(x\right)\right) \sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(4*cosh(x)*e^(2*x)*sinh(x)^3 + e^(2*x)*sinh(x)^4 + 2*(3*cosh(x)^2 + 2*x
)*e^(2*x)*sinh(x)^2 + 4*(cosh(x)^3 + 2*x*cosh(x))*e^(2*x)*sinh(x) + (cosh(x
)^4 + 4*x*cosh(x)^2 - 1)*e^(2*x))*sqrt(a*e^(8*x) + 4*a*e^(6*x) + 6*a*e^(4*x
) + 4*a*e^(2*x) + a)*e^(-2*x)/(cosh(x)^2*e^(4*x) + 2*cosh(x)^2*e^(2*x) + (e
^(4*x) + 2*e^(2*x) + 1)*sinh(x)^2 + cosh(x)^2 + 2*(cosh(x)*e^(4*x) + 2*cosh
(x)*e^(2*x) + cosh(x))*sinh(x))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)**4)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.41, size = 28, normalized size = 0.78

$$-\frac{1}{8} \left(\left(2 e^{\left(2 x\right)} + 1\right) e^{\left(-2 x\right)} - 4 x - e^{\left(2 x\right)}\right) \sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*((2*e^(2*x) + 1)*e^(-2*x) - 4*x - e^(2*x))*sqrt(a)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{a \cosh\left(x\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^4)^(1/2),x)
```

```
[Out] int((a*cosh(x)^4)^(1/2), x)
```

3.137 $\int \frac{1}{\sqrt{a\cosh^4(x)}}\,dx$

Optimal. Leaf size=15

$$\frac{\cosh(x)\sinh(x)}{\sqrt{a\cosh^4(x)}}$$

[Out] cosh(x)*sinh(x)/(a*cosh(x)^4)^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3286, 3852, 8}

$$\frac{\sinh(x)\cosh(x)}{\sqrt{a\cosh^4(x)}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a*Cosh[x]^4],x]

[Out] (Cosh[x]*Sinh[x])/Sqrt[a*Cosh[x]^4]

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a\cosh^4(x)}}\,dx = \frac{\cosh^2(x)\int \operatorname{sech}^2(x)\,dx}{\sqrt{a\cosh^4(x)}}$$
$$= \frac{(i\cosh^2(x))\operatorname{Subst}(\int 1\,dx, x, -i\tanh(x))}{\sqrt{a\cosh^4(x)}}$$
$$= \frac{\cosh(x)\sinh(x)}{\sqrt{a\cosh^4(x)}}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$\frac{\cosh(x)\sinh(x)}{\sqrt{a\cosh^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Cosh[x]^4],x]
[Out] (Cosh[x]*Sinh[x])/Sqrt[a*Cosh[x]^4]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(13) = 26$.
time = 1.49, size = 56, normalized size = 3.73

method	result	size
risch	$-\frac{2\,e^{-2x}(1+e^{2x})}{\sqrt{a\,(1+e^{2x})^4\,e^{-4x}}}$	29
default	$\frac{\sqrt{8}\,\sqrt{2}\,\sqrt{a\,(-1+\cosh(2x))\,(1+\cosh(2x))}\,\sqrt{a\,(\sinh^2(2x))}}{4a\sinh(2x)\sqrt{a\,(1+\cosh(2x))^2}}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)^4)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/4*8^(1/2)*2^(1/2)*(a*(-1+cosh(2*x))*(1+cosh(2*x)))^(1/2)/a*(a*sinh(2*x)^2
)^(1/2)/sinh(2*x)/(a*(1+cosh(2*x))^2)^(1/2)
```

Maxima [A]
time = 0.48, size = 16, normalized size = 1.07

$$\frac{2}{\sqrt{a}\,e^{(-2\,x)} + \sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/(sqrt(a)*e^(-2*x) + sqrt(a))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(13) = 26$.
time = 0.36, size = 116, normalized size = 7.73

$$-\frac{2\sqrt{ae^{(8x)} + 4ae^{(6x)} + 6ae^{(4x)} + 4ae^{(2x)} + a}}{a\cosh(x)^2 + (ae^{(4x)} + 2ae^{(2x)} + a)\sinh(x)^2 + (a\cosh(x)^2 + a)e^{(4x)} + 2(a\cosh(x)^2 + a)e^{(2x)} + 2(a\cosh(x)e^{(4x)} + 2a\cosh(x)e^{(2x)} + a\cosh(x))\sinh(x) + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(a*e^(8*x) + 4*a*e^(6*x) + 6*a*e^(4*x) + 4*a*e^(2*x) + a)/(a*cosh(x)
^2 + (a*e^(4*x) + 2*a*e^(2*x) + a)*sinh(x)^2 + (a*cosh(x)^2 + a)*e^(4*x) +
2*(a*cosh(x)^2 + a)*e^(2*x) + 2*(a*cosh(x)*e^(4*x) + 2*a*cosh(x)*e^(2*x) +
a*cosh(x))*sinh(x) + a)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)**4)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.41, size = 13, normalized size = 0.87

$$-\frac{2}{\sqrt{a}\,(e^{(2x)} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] -2/(sqrt(a)*(e^(2*x) + 1))
```

Mupad [B]
time = 0.06, size = 39, normalized size = 2.60

$$-\frac{e^{-x}\sqrt{a\left(\frac{e^{-x}}{2} + \frac{e^{x}}{2}\right)^4}}{a\left(\frac{e^{-x}}{2} + \frac{e^{x}}{2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)^4)^(1/2),x)
```

```
[Out] -(exp(-x)*(a*(exp(-x)/2 + exp(x)/2)^4)^(1/2))/(a*(exp(-x)/2 + exp(x)/2)^3)
```

3.138 $\int \frac{1}{\left(a\cosh^4(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=67

$$\frac{\cosh(x)\sinh(x)}{a\sqrt{a\cosh^4(x)}}-\frac{2\sinh^2(x)\tanh(x)}{3a\sqrt{a\cosh^4(x)}}+\frac{\sinh^2(x)\tanh^3(x)}{5a\sqrt{a\cosh^4(x)}}$$

```
[Out] cosh(x)*sinh(x)/a/(a*cosh(x)^4)^(1/2)-2/3*sinh(x)^2*tanh(x)/a/(a*cosh(x)^4)
^(1/2)+1/5*sinh(x)^2*tanh(x)^3/a/(a*cosh(x)^4)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3286, 3852}

$$\frac{\sinh(x)\cosh(x)}{a\sqrt{a\cosh^4(x)}}+\frac{\sinh^2(x)\tanh^3(x)}{5a\sqrt{a\cosh^4(x)}}-\frac{2\sinh^2(x)\tanh(x)}{3a\sqrt{a\cosh^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x]^4)^(-3/2),x]
```

```
[Out] (Cosh[x]*Sinh[x])/(a*Sqrt[a*Cosh[x]^4]) - (2*Sinh[x]^2*Tanh[x])/(3*a*Sqrt[a
*Cosh[x]^4]) + (Sinh[x]^2*Tanh[x]^3)/(5*a*Sqrt[a*Cosh[x]^4])
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a\cosh^4(x)\right)^{3/2}}\,dx &= \frac{\cosh^2(x)\int \operatorname{sech}^6(x)\,dx}{a\sqrt{a\cosh^4(x)}} \\ &= \frac{\left(i\cosh^2(x)\right)\operatorname{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx, x, -i\tanh(x)\right)}{a\sqrt{a\cosh^4(x)}} \\ &= \frac{\cosh(x)\sinh(x)}{a\sqrt{a\cosh^4(x)}} - \frac{2\sinh^2(x)\tanh(x)}{3a\sqrt{a\cosh^4(x)}} + \frac{\sinh^2(x)\tanh^3(x)}{5a\sqrt{a\cosh^4(x)}}\end{aligned}$$

Mathematica [**A**]
time = 0.02, size = 30, normalized size = 0.45

$$\frac{\cosh(x)(8+6\cosh(2x)+\cosh(4x))\sinh(x)}{15\left(a\cosh^4(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x]^4)^(-3/2),x]
[Out] (Cosh[x]*(8 + 6*Cosh[2*x] + Cosh[4*x])*Sinh[x])/(15*(a*Cosh[x]^4)^(3/2))
```

Maple [**A**]
time = 1.57, size = 80, normalized size = 1.19

method	result	size
risch	$-\frac{16\,\mathrm{e}^{-2x}\left(10\,\mathrm{e}^{4x}+5\,\mathrm{e}^{2x}+1\right)}{15a\left(1+\mathrm{e}^{2x}\right)^3\sqrt{a\left(1+\mathrm{e}^{2x}\right)^4\mathrm{e}^{-4x}}}$	48
default	$\frac{\sqrt{8}\,\sqrt{2}\,\left(2\left(\cosh^2(2x)\right)+6\cosh(2x)+7\right)\sqrt{a\left(\sinh^2\left(2x\right)\right)}\,\sqrt{a\left(-1+\cosh\left(2x\right)\right)\left(1+\cosh\left(2x\right)\right)}}{15a^2(1+\cosh(2x))^2\sinh(2x)\sqrt{a\left(1+\cosh\left(2x\right)\right)^2}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)^4)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/15*8^(1/2)/a^2*2^(1/2)*(2*cosh(2*x)^2+6*cosh(2*x)+7)*(a*sinh(2*x)^2)^(1/2
)*(a*(-1+cosh(2*x))*(1+cosh(2*x)))^(1/2)/(1+cosh(2*x))^2/sinh(2*x)/(a*(1+co
sh(2*x))^2)^(1/2)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(57) = 114.
time = 0.49, size = 165, normalized size = 2.46

$$\frac{16\,e^{(-2\,x)}}{3\left(5\,a^{\frac{3}{2}}e^{(-2\,x)}+10\,a^{\frac{3}{2}}e^{(-4\,x)}+10\,a^{\frac{3}{2}}e^{(-6\,x)}+5\,a^{\frac{3}{2}}e^{(-8\,x)}+a^{\frac{3}{2}}e^{(-10\,x)}+a^{\frac{3}{2}}\right)}+\frac{32\,e^{(-4\,x)}}{3\left(5\,a^{\frac{3}{2}}e^{(-2\,x)}+10\,a^{\frac{3}{2}}e^{(-4\,x)}+10\,a^{\frac{3}{2}}e^{(-6\,x)}+5\,a^{\frac{3}{2}}e^{(-8\,x)}+a^{\frac{3}{2}}e^{(-10\,x)}+a^{\frac{3}{2}}\right)}+\frac{16}{15\left(5\,a^{\frac{3}{2}}e^{(-2\,x)}+10\,a^{\frac{3}{2}}e^{(-4\,x)}+10\,a^{\frac{3}{2}}e^{(-6\,x)}+5\,a^{\frac{3}{2}}e^{(-8\,x)}+a^{\frac{3}{2}}e^{(-10\,x)}+a^{\frac{3}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] 16/3*e^(-2*x)/(5*a^(3/2)*e^(-2*x) + 10*a^(3/2)*e^(-4*x) + 10*a^(3/2)*e^(-6*
x) + 5*a^(3/2)*e^(-8*x) + a^(3/2)*e^(-10*x) + a^(3/2)) + 32/3*e^(-4*x)/(5*a
^(3/2)*e^(-2*x) + 10*a^(3/2)*e^(-4*x) + 10*a^(3/2)*e^(-6*x) + 5*a^(3/2)*e^(
-8*x) + a^(3/2)*e^(-10*x) + a^(3/2)) + 16/15/(5*a^(3/2)*e^(-2*x) + 10*a^(3/
2)*e^(-4*x) + 10*a^(3/2)*e^(-6*x) + 5*a^(3/2)*e^(-8*x) + a^(3/2)*e^(-10*x)
+ a^(3/2))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1137 vs. $2(57) = 114$.
time = 0.42, size = 1137, normalized size = 16.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] -16/15*(40*cosh(x)*e^(2*x)*sinh(x)^3 + 10*e^(2*x)*sinh(x)^4 + 5*(12*cosh(x)
^2 + 1)*e^(2*x)*sinh(x)^2 + 10*(4*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x) + (1
0*cosh(x)^4 + 5*cosh(x)^2 + 1)*e^(2*x))*sqrt(a*e^(8*x) + 4*a*e^(6*x) + 6*a*
e^(4*x) + 4*a*e^(2*x) + a)*e^(-2*x)/(a^2*cosh(x)^10 + (a^2*e^(4*x) + 2*a^2*
e^(2*x) + a^2)*sinh(x)^10 + 5*a^2*cosh(x)^8 + 10*(a^2*cosh(x)*e^(4*x) + 2*a
^2*cosh(x)*e^(2*x) + a^2*cosh(x))*sinh(x)^9 + 5*(9*a^2*cosh(x)^2 + a^2 + (9
*a^2*cosh(x)^2 + a^2)*e^(4*x) + 2*(9*a^2*cosh(x)^2 + a^2)*e^(2*x))*sinh(x)^
8 + 10*a^2*cosh(x)^6 + 40*(3*a^2*cosh(x)^3 + a^2*cosh(x) + (3*a^2*cosh(x)^3
 + a^2*cosh(x))*e^(4*x) + 2*(3*a^2*cosh(x)^3 + a^2*cosh(x))*e^(2*x))*sinh(x
)^7 + 10*(21*a^2*cosh(x)^4 + 14*a^2*cosh(x)^2 + a^2 + (21*a^2*cosh(x)^4 + 1
4*a^2*cosh(x)^2 + a^2)*e^(4*x) + 2*(21*a^2*cosh(x)^4 + 14*a^2*cosh(x)^2 + a
^2)*e^(2*x))*sinh(x)^6 + 10*a^2*cosh(x)^4 + 4*(63*a^2*cosh(x)^5 + 70*a^2*co
sh(x)^3 + 15*a^2*cosh(x) + (63*a^2*cosh(x)^5 + 70*a^2*cosh(x)^3 + 15*a^2*co
sh(x))*e^(4*x) + 2*(63*a^2*cosh(x)^5 + 70*a^2*cosh(x)^3 + 15*a^2*cosh(x))*e
^(2*x))*sinh(x)^5 + 10*(21*a^2*cosh(x)^6 + 35*a^2*cosh(x)^4 + 15*a^2*cosh(x
)^2 + a^2 + (21*a^2*cosh(x)^6 + 35*a^2*cosh(x)^4 + 15*a^2*cosh(x)^2 + a^2)*
e^(4*x) + 2*(21*a^2*cosh(x)^6 + 35*a^2*cosh(x)^4 + 15*a^2*cosh(x)^2 + a^2)*
e^(2*x))*sinh(x)^4 + 5*a^2*cosh(x)^2 + 40*(3*a^2*cosh(x)^7 + 7*a^2*cosh(x)^
5 + 5*a^2*cosh(x)^3 + a^2*cosh(x) + (3*a^2*cosh(x)^7 + 7*a^2*cosh(x)^5 + 5*
a^2*cosh(x)^3 + a^2*cosh(x))*e^(4*x) + 2*(3*a^2*cosh(x)^7 + 7*a^2*cosh(x)^5
 + 5*a^2*cosh(x)^3 + a^2*cosh(x))*e^(2*x))*sinh(x)^3 + 5*(9*a^2*cosh(x)^8 +
 28*a^2*cosh(x)^6 + 30*a^2*cosh(x)^4 + 12*a^2*cosh(x)^2 + a^2 + (9*a^2*cosh
(x)^8 + 28*a^2*cosh(x)^6 + 30*a^2*cosh(x)^4 + 12*a^2*cosh(x)^2 + a^2)*e^(4*
x) + 2*(9*a^2*cosh(x)^8 + 28*a^2*cosh(x)^6 + 30*a^2*cosh(x)^4 + 12*a^2*cosh
(x)^2 + a^2)*e^(2*x))*sinh(x)^2 + a^2 + (a^2*cosh(x)^10 + 5*a^2*cosh(x)^8 +
 10*a^2*cosh(x)^6 + 10*a^2*cosh(x)^4 + 5*a^2*cosh(x)^2 + a^2)*e^(4*x) + 2*(
```

```
a^2*cosh(x)^10 + 5*a^2*cosh(x)^8 + 10*a^2*cosh(x)^6 + 10*a^2*cosh(x)^4 + 5*
a^2*cosh(x)^2 + a^2)*e^(2*x) + 10*(a^2*cosh(x)^9 + 4*a^2*cosh(x)^7 + 6*a^2*
cosh(x)^5 + 4*a^2*cosh(x)^3 + a^2*cosh(x) + (a^2*cosh(x)^9 + 4*a^2*cosh(x)^
7 + 6*a^2*cosh(x)^5 + 4*a^2*cosh(x)^3 + a^2*cosh(x))*e^(4*x) + 2*(a^2*cosh(
x)^9 + 4*a^2*cosh(x)^7 + 6*a^2*cosh(x)^5 + 4*a^2*cosh(x)^3 + a^2*cosh(x))*e
^(2*x))*sinh(x))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)**4)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3064 deep
```

Giac [A]
time = 0.42, size = 27, normalized size = 0.40

$$-\frac{16\left(10\,e^{(4\,x)}+5\,e^{(2\,x)}+1\right)}{15\,a^{\frac{3}{2}}\left(e^{(2\,x)}+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] -16/15*(10*e^(4*x) + 5*e^(2*x) + 1)/(a^(3/2)*(e^(2*x) + 1)^5)
```

Mupad [B]
time = 0.97, size = 48, normalized size = 0.72

$$-\frac{64\,\mathrm{e}^{2\,x}\,\sqrt{a\,{\left(\frac{\mathrm{e}^{-x}}{2}+\frac{\mathrm{e}^{x}}{2}\right)}^4}\,\left(5\,\mathrm{e}^{2\,x}+10\,\mathrm{e}^{4\,x}+1\right)}{15\,a^2\,{\left(\mathrm{e}^{2\,x}+1\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)^4)^(3/2),x)
```

```
[Out] -(64*exp(2*x)*(a*(exp(-x)/2 + exp(x)/2)^4)^(1/2)*(5*exp(2*x) + 10*exp(4*x)
+ 1))/(15*a^2*(exp(2*x) + 1)^7)
```

3.139 $\int \frac{1}{\left(a\cosh^4(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=117

$$\frac{\cosh(x)\sinh(x)}{a^2\sqrt{a\cosh^4(x)}}-\frac{4\sinh^2(x)\tanh(x)}{3a^2\sqrt{a\cosh^4(x)}}+\frac{6\sinh^2(x)\tanh^3(x)}{5a^2\sqrt{a\cosh^4(x)}}-\frac{4\sinh^2(x)\tanh^5(x)}{7a^2\sqrt{a\cosh^4(x)}}+\frac{\sinh^2(x)\tanh^7(x)}{9a^2\sqrt{a\cosh^4(x)}}$$

```
[Out] cosh(x)*sinh(x)/a^2/(a*cosh(x)^4)^(1/2)-4/3*sinh(x)^2*tanh(x)/a^2/(a*cosh(x
)^4)^(1/2)+6/5*sinh(x)^2*tanh(x)^3/a^2/(a*cosh(x)^4)^(1/2)-4/7*sinh(x)^2*ta
nh(x)^5/a^2/(a*cosh(x)^4)^(1/2)+1/9*sinh(x)^2*tanh(x)^7/a^2/(a*cosh(x)^4)^(
1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3286, 3852}

$$\frac{\sinh(x)\cosh(x)}{a^2\sqrt{a\cosh^4(x)}}+\frac{\sinh^2(x)\tanh^7(x)}{9a^2\sqrt{a\cosh^4(x)}}-\frac{4\sinh^2(x)\tanh^5(x)}{7a^2\sqrt{a\cosh^4(x)}}+\frac{6\sinh^2(x)\tanh^3(x)}{5a^2\sqrt{a\cosh^4(x)}}-\frac{4\sinh^2(x)\tanh(x)}{3a^2\sqrt{a\cosh^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x]^4)^(-5/2),x]
```

```
[Out] (Cosh[x]*Sinh[x])/(a^2*Sqrt[a*Cosh[x]^4]) - (4*Sinh[x]^2*Tanh[x])/(3*a^2*Sq
rt[a*Cosh[x]^4]) + (6*Sinh[x]^2*Tanh[x]^3)/(5*a^2*Sqrt[a*Cosh[x]^4]) - (4*S
inh[x]^2*Tanh[x]^5)/(7*a^2*Sqrt[a*Cosh[x]^4]) + (Sinh[x]^2*Tanh[x]^7)/(9*a^
2*Sqrt[a*Cosh[x]^4])
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{\left(a\cosh^4(x)\right)^{5/2}}\,dx = \frac{\cosh^2(x)\int \operatorname{sech}^{10}(x)\,dx}{a^2\sqrt{a\cosh^4(x)}}$$
$$= \frac{\left(i\cosh^2(x)\right)\operatorname{Subst}\left(\int \left(1+4x^2+6x^4+4x^6+x^8\right)\,dx, x, -i\tanh(x)\right)}{a^2\sqrt{a\cosh^4(x)}}$$
$$= \frac{\cosh(x)\sinh(x)}{a^2\sqrt{a\cosh^4(x)}} - \frac{4\sinh^2(x)\tanh(x)}{3a^2\sqrt{a\cosh^4(x)}} + \frac{6\sinh^2(x)\tanh^3(x)}{5a^2\sqrt{a\cosh^4(x)}} - \frac{4\sinh^2(x)\tanh^5(x)}{7a^2\sqrt{a\cosh^4(x)}}$$

Mathematica [A]
time = 0.03, size = 47, normalized size = 0.40

$$\frac{(128+130\cosh(2x)+46\cosh(4x)+10\cosh(6x)+\cosh(8x))\operatorname{sech}^6(x)\tanh(x)}{315a^2\sqrt{a\cosh^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x]^4)^(-5/2),x]
```

```
[Out] ((128 + 130*Cosh[2*x] + 46*Cosh[4*x] + 10*Cosh[6*x] + Cosh[8*x])*Sech[x]^6*
Tanh[x])/(315*a^2*Sqrt[a*Cosh[x]^4])
```

Maple [A]
time = 1.58, size = 96, normalized size = 0.82

method	result
risch	$-\frac{256\,e^{-2x}\left(126\,e^{8x}+84\,e^{6x}+36\,e^{4x}+9\,e^{2x}+1\right)}{315a^2(1+e^{2x})^7\sqrt{a\left(1+e^{2x}\right)^4 e^{-4x}}}$
default	$\frac{4\sqrt{8}\,\sqrt{2}\,\left(8(\cosh^4(2x))+40(\cosh^3(2x))+84(\cosh^2(2x))+100\cosh(2x)+83\right)\sqrt{a\left(\sinh^2(2x)\right)}\,\sqrt{a\left(-1+\cosh(2x)\right)}}{315a^3(1+\cosh(2x))^4\sinh(2x)\sqrt{a\left(1+\cosh(2x)\right)^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)^4)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4/315*8^(1/2)*2^(1/2)/a^3*(8*cosh(2*x)^4+40*cosh(2*x)^3+84*cosh(2*x)^2+100*
cosh(2*x)+83)*(a*sinh(2*x)^2)^(1/2)*(a*(-1+cosh(2*x))*(1+cosh(2*x)))^(1/2)/
(1+cosh(2*x))^4/sinh(2*x)/(a*(1+cosh(2*x))^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 457 vs. 2(99) = 198.

time = 0.50, size = 457, normalized size = 3.91

$$\frac{256\,e^{(-2x)}}{35\left(9a^{\frac{5}{2}}e^{(-2x)}+36a^{\frac{5}{2}}e^{(-4x)}+84a^{\frac{5}{2}}e^{(-6x)}+126a^{\frac{5}{2}}e^{(-8x)}+126a^{\frac{5}{2}}e^{(-10x)}+84a^{\frac{5}{2}}e^{(-12x)}+36a^{\frac{5}{2}}e^{(-14x)}+9a^{\frac{5}{2}}e^{(-16x)}+a^{\frac{5}{2}}e^{(-18x)}+a^{\frac{5}{2}}\right)}+\frac{1024\,e^{(-4x)}}{35\left(9a^{\frac{5}{2}}e^{(-2x)}+36a^{\frac{5}{2}}e^{(-4x)}+84a^{\frac{5}{2}}e^{(-6x)}+126a^{\frac{5}{2}}e^{(-8x)}+126a^{\frac{5}{2}}e^{(-10x)}+84a^{\frac{5}{2}}e^{(-12x)}+36a^{\frac{5}{2}}e^{(-14x)}+9a^{\frac{5}{2}}e^{(-16x)}+a^{\frac{5}{2}}e^{(-18x)}+a^{\frac{5}{2}}\right)}+\frac{1024\,e^{(-6x)}}{15\left(9a^{\frac{5}{2}}e^{(-2x)}+36a^{\frac{5}{2}}e^{(-4x)}+84a^{\frac{5}{2}}e^{(-6x)}+126a^{\frac{5}{2}}e^{(-8x)}+126a^{\frac{5}{2}}e^{(-10x)}+84a^{\frac{5}{2}}e^{(-12x)}+36a^{\frac{5}{2}}e^{(-14x)}+9a^{\frac{5}{2}}e^{(-16x)}+a^{\frac{5}{2}}e^{(-18x)}+a^{\frac{5}{2}}\right)}+\frac{512\,e^{(-8x)}}{5\left(9a^{\frac{5}{2}}e^{(-2x)}+36a^{\frac{5}{2}}e^{(-4x)}+84a^{\frac{5}{2}}e^{(-6x)}+126a^{\frac{5}{2}}e^{(-8x)}+126a^{\frac{5}{2}}e^{(-10x)}+84a^{\frac{5}{2}}e^{(-12x)}+36a^{\frac{5}{2}}e^{(-14x)}+9a^{\frac{5}{2}}e^{(-16x)}+a^{\frac{5}{2}}e^{(-18x)}+a^{\frac{5}{2}}\right)}+\frac{256}{315\left(9a^{\frac{5}{2}}e^{(-2x)}+36a^{\frac{5}{2}}e^{(-4x)}+84a^{\frac{5}{2}}e^{(-6x)}+126a^{\frac{5}{2}}e^{(-8x)}+126a^{\frac{5}{2}}e^{(-10x)}+84a^{\frac{5}{2}}e^{(-12x)}+36a^{\frac{5}{2}}e^{(-14x)}+9a^{\frac{5}{2}}e^{(-16x)}+a^{\frac{5}{2}}e^{(-18x)}+a^{\frac{5}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] 256/35*e^(-2*x)/(9*a^(5/2)*e^(-2*x) + 36*a^(5/2)*e^(-4*x) + 84*a^(5/2)*e^(-
6*x) + 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) + 84*a^(5/2)*e^(-12*x)
+ 36*a^(5/2)*e^(-14*x) + 9*a^(5/2)*e^(-16*x) + a^(5/2)*e^(-18*x) + a^(5/2))
 + 1024/35*e^(-4*x)/(9*a^(5/2)*e^(-2*x) + 36*a^(5/2)*e^(-4*x) + 84*a^(5/2)*
e^(-6*x) + 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) + 84*a^(5/2)*e^(-12
*x) + 36*a^(5/2)*e^(-14*x) + 9*a^(5/2)*e^(-16*x) + a^(5/2)*e^(-18*x) + a^(5
/2)) + 1024/15*e^(-6*x)/(9*a^(5/2)*e^(-2*x) + 36*a^(5/2)*e^(-4*x) + 84*a^(5
/2)*e^(-6*x) + 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) + 84*a^(5/2)*e^
(-12*x) + 36*a^(5/2)*e^(-14*x) + 9*a^(5/2)*e^(-16*x) + a^(5/2)*e^(-18*x) +
a^(5/2)) + 512/5*e^(-8*x)/(9*a^(5/2)*e^(-2*x) + 36*a^(5/2)*e^(-4*x) + 84*a^
(5/2)*e^(-6*x) + 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) + 84*a^(5/2)*
e^(-12*x) + 36*a^(5/2)*e^(-14*x) + 9*a^(5/2)*e^(-16*x) + a^(5/2)*e^(-18*x)
+ a^(5/2)) + 256/315/(9*a^(5/2)*e^(-2*x) + 36*a^(5/2)*e^(-4*x) + 84*a^(5/2)
*e^(-6*x) + 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) + 84*a^(5/2)*e^(-1
2*x) + 36*a^(5/2)*e^(-14*x) + 9*a^(5/2)*e^(-16*x) + a^(5/2)*e^(-18*x) + a^(
5/2))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3065 vs. $2(99) = 198$.
time = 0.42, size = 3065, normalized size = 26.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] -256/315*(1008*cosh(x)*e^(2*x)*sinh(x)^7 + 126*e^(2*x)*sinh(x)^8 + 84*(42*c
osh(x)^2 + 1)*e^(2*x)*sinh(x)^6 + 504*(14*cosh(x)^3 + cosh(x))*e^(2*x)*sinh
(x)^5 + 36*(245*cosh(x)^4 + 35*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^4 + 48*(147*c
osh(x)^5 + 35*cosh(x)^3 + 3*cosh(x))*e^(2*x)*sinh(x)^3 + 9*(392*cosh(x)^6 +
 140*cosh(x)^4 + 24*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^2 + 18*(56*cosh(x)^7 + 2
8*cosh(x)^5 + 8*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x) + (126*cosh(x)^8 + 84*
cosh(x)^6 + 36*cosh(x)^4 + 9*cosh(x)^2 + 1)*e^(2*x))*sqrt(a*e^(8*x) + 4*a*e
^(6*x) + 6*a*e^(4*x) + 4*a*e^(2*x) + a)*e^(-2*x)/(a^3*cosh(x)^18 + 9*a^3*co
sh(x)^16 + (a^3*e^(4*x) + 2*a^3*e^(2*x) + a^3)*sinh(x)^18 + 18*(a^3*cosh(x)
*e^(4*x) + 2*a^3*cosh(x)*e^(2*x) + a^3*cosh(x))*sinh(x)^17 + 36*a^3*cosh(x)
^14 + 9*(17*a^3*cosh(x)^2 + a^3 + (17*a^3*cosh(x)^2 + a^3)*e^(4*x) + 2*(17*
a^3*cosh(x)^2 + a^3)*e^(2*x))*sinh(x)^16 + 48*(17*a^3*cosh(x)^3 + 3*a^3*cos
```

h(x) + (17*a^3*cosh(x)^3 + 3*a^3*cosh(x))*e^(4*x) + 2*(17*a^3*cosh(x)^3 + 3
*a^3*cosh(x))*e^(2*x))*sinh(x)^15 + 84*a^3*cosh(x)^12 + 36*(85*a^3*cosh(x)^
4 + 30*a^3*cosh(x)^2 + a^3 + (85*a^3*cosh(x)^4 + 30*a^3*cosh(x)^2 + a^3)*e^
(4*x) + 2*(85*a^3*cosh(x)^4 + 30*a^3*cosh(x)^2 + a^3)*e^(2*x))*sinh(x)^14 +
504*(17*a^3*cosh(x)^5 + 10*a^3*cosh(x)^3 + a^3*cosh(x) + (17*a^3*cosh(x)^5
+ 10*a^3*cosh(x)^3 + a^3*cosh(x))*e^(4*x) + 2*(17*a^3*cosh(x)^5 + 10*a^3*c
osh(x)^3 + a^3*cosh(x))*e^(2*x))*sinh(x)^13 + 126*a^3*cosh(x)^10 + 84*(221*
a^3*cosh(x)^6 + 195*a^3*cosh(x)^4 + 39*a^3*cosh(x)^2 + a^3 + (221*a^3*cosh(
x)^6 + 195*a^3*cosh(x)^4 + 39*a^3*cosh(x)^2 + a^3)*e^(4*x) + 2*(221*a^3*cos
h(x)^6 + 195*a^3*cosh(x)^4 + 39*a^3*cosh(x)^2 + a^3)*e^(2*x))*sinh(x)^12 +
144*(221*a^3*cosh(x)^7 + 273*a^3*cosh(x)^5 + 91*a^3*cosh(x)^3 + 7*a^3*cosh(
x) + (221*a^3*cosh(x)^7 + 273*a^3*cosh(x)^5 + 91*a^3*cosh(x)^3 + 7*a^3*cosh
(x))*e^(4*x) + 2*(221*a^3*cosh(x)^7 + 273*a^3*cosh(x)^5 + 91*a^3*cosh(x)^3
+ 7*a^3*cosh(x))*e^(2*x))*sinh(x)^11 + 126*a^3*cosh(x)^8 + 18*(2431*a^3*cos
h(x)^8 + 4004*a^3*cosh(x)^6 + 2002*a^3*cosh(x)^4 + 308*a^3*cosh(x)^2 + 7*a^
3 + (2431*a^3*cosh(x)^8 + 4004*a^3*cosh(x)^6 + 2002*a^3*cosh(x)^4 + 308*a^3
*cosh(x)^2 + 7*a^3)*e^(4*x) + 2*(2431*a^3*cosh(x)^8 + 4004*a^3*cosh(x)^6 +
2002*a^3*cosh(x)^4 + 308*a^3*cosh(x)^2 + 7*a^3)*e^(2*x))*sinh(x)^10 + 4*(12
155*a^3*cosh(x)^9 + 25740*a^3*cosh(x)^7 + 18018*a^3*cosh(x)^5 + 4620*a^3*co
sh(x)^3 + 315*a^3*cosh(x) + (12155*a^3*cosh(x)^9 + 25740*a^3*cosh(x)^7 + 18
018*a^3*cosh(x)^5 + 4620*a^3*cosh(x)^3 + 315*a^3*cosh(x))*e^(4*x) + 2*(1215
5*a^3*cosh(x)^9 + 25740*a^3*cosh(x)^7 + 18018*a^3*cosh(x)^5 + 4620*a^3*cosh
(x)^3 + 315*a^3*cosh(x))*e^(2*x))*sinh(x)^9 + 84*a^3*cosh(x)^6 + 18*(2431*a
^3*cosh(x)^10 + 6435*a^3*cosh(x)^8 + 6006*a^3*cosh(x)^6 + 2310*a^3*cosh(x)^
4 + 315*a^3*cosh(x)^2 + 7*a^3 + (2431*a^3*cosh(x)^10 + 6435*a^3*cosh(x)^8 +
6006*a^3*cosh(x)^6 + 2310*a^3*cosh(x)^4 + 315*a^3*cosh(x)^2 + 7*a^3)*e^(4*
x) + 2*(2431*a^3*cosh(x)^10 + 6435*a^3*cosh(x)^8 + 6006*a^3*cosh(x)^6 + 231
0*a^3*cosh(x)^4 + 315*a^3*cosh(x)^2 + 7*a^3)*e^(2*x))*sinh(x)^8 + 144*(221*
a^3*cosh(x)^11 + 715*a^3*cosh(x)^9 + 858*a^3*cosh(x)^7 + 462*a^3*cosh(x)^5
+ 105*a^3*cosh(x)^3 + 7*a^3*cosh(x) + (221*a^3*cosh(x)^11 + 715*a^3*cosh(x)
^9 + 858*a^3*cosh(x)^7 + 462*a^3*cosh(x)^5 + 105*a^3*cosh(x)^3 + 7*a^3*cosh
(x))*e^(4*x) + 2*(221*a^3*cosh(x)^11 + 715*a^3*cosh(x)^9 + 858*a^3*cosh(x)^
7 + 462*a^3*cosh(x)^5 + 105*a^3*cosh(x)^3 + 7*a^3*cosh(x))*e^(2*x))*sinh(x)
^7 + 36*a^3*cosh(x)^4 + 84*(221*a^3*cosh(x)^12 + 858*a^3*cosh(x)^10 + 1287*
a^3*cosh(x)^8 + 924*a^3*cosh(x)^6 + 315*a^3*cosh(x)^4 + 42*a^3*cosh(x)^2 +
a^3 + (221*a^3*cosh(x)^12 + 858*a^3*cosh(x)^10 + 1287*a^3*cosh(x)^8 + 924*a
^3*cosh(x)^6 + 315*a^3*cosh(x)^4 + 42*a^3*cosh(x)^2 + a^3)*e^(4*x) + 2*(221
*a^3*cosh(x)^12 + 858*a^3*cosh(x)^10 + 1287*a^3*cosh(x)^8 + 924*a^3*cosh(x)
^6 + 315*a^3*cosh(x)^4 + 42*a^3*cosh(x)^2 + a^3)*e^(2*x))*sinh(x)^6 + 504*(
17*a^3*cosh(x)^13 + 78*a^3*cosh(x)^11 + 143*a^3*cosh(x)^9 + 132*a^3*cosh(x)
^7 + 63*a^3*cosh(x)^5 + 14*a^3*cosh(x)^3 + a^3*cosh(x) + (17*a^3*cosh(x)^13
+ 78*a^3*cosh(x)^11 + 143*a^3*cosh(x)^9 + 132*a^3*cosh(x)^7 + 63*a^3*cosh(
x)^5 + 14*a^3*cosh(x)^3 + a^3*cosh(x))*e^(4*x) + 2*(17*a^3*cosh(x)^13 + 78*
a^3*cosh(x)^11 + 143*a^3*cosh(x)^9 + 132*a^3*cosh(x)^7 + 63*a^3*cosh(x)^5 +
14*a^3*cosh(x)^3 + a^3*cosh(x))*e^(2*x))*sinh(x)^5 + 9*a^3*cosh(x)^2 + 36*

```
(85*a^3*cosh(x)^14 + 455*a^3*cosh(x)^12 + 1001*a^3*cosh(x)^10 + 1155*a^3*co
sh(x)^8 + 735*a^3*cosh(x)^6 + 245*a^3*cosh(x)^4 + 35*a^3*cosh(x)^2 + a^3 +
(85*a^3*cosh(x)^14 + 455*a^3*cosh(x)^12 + 1001*a^3*cosh(x)^10 + 1155*a^3*co
sh(x)^8 + 735*a^3*cosh(x)^6 + 245*a^3*cosh(x)^4 + 35*a^3*cosh(x)^2 + a^3)*e
^(4*x) + 2*(85*a^3*cosh(x)^14 + 455*a^3*cosh(x)^12 + 1001*a^3*cosh(x)^10 +
1155*a^3*cosh(x)^8 + 735*a^3*cosh(x)^6 + 245*a^3*cosh(x)^4 + 35*a^3*cosh(x)
^2 + a^3)*e^(2*x))*sinh(x)^4 + 48*(17*a^3*cosh(x)^15 + 105*a^3*cosh(x)^13 +
 273*a^3*cosh(x)^11 + 385*a^3*cosh(x)^9 + 315*a^3*cosh(x)^7 + 147*a^3*cosh(
x)^5 + 35*a^3*cosh(x)^3 + 3*a^3*cosh(x) + (17*a^3*cosh(x)^15 + 105*a^3*cosh
(x)^13 + 273*a^3*cosh(x)^11 + 385*a^3*cosh(x)^9...
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)**4)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5989 deep
```

Giac [**A**]
time = 0.41, size = 39, normalized size = 0.33

$$-\frac{256\left(126\,e^{(8\,x)}+84\,e^{(6\,x)}+36\,e^{(4\,x)}+9\,e^{(2\,x)}+1\right)}{315\,a^{\frac{5}{2}}\left(e^{(2\,x)}+1\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] -256/315*(126*e^(8*x) + 84*e^(6*x) + 36*e^(4*x) + 9*e^(2*x) + 1)/(a^(5/2)*(
e^(2*x) + 1)^9)
```

Mupad [**B**]
time = 0.99, size = 256, normalized size = 2.19

$$\frac{4096\,e^{4x}\sqrt{a\left(\frac{e^{-x}}{2}+\frac{e^{x}}{2}\right)^4}}{3\,a^3\,(e^{2x}+1)^6\,(e^{2x}+2\,e^{4x}+e^{6x})}-\frac{2048\,e^{4x}\sqrt{a\left(\frac{e^{-x}}{2}+\frac{e^{x}}{2}\right)^4}}{5\,a^3\,(e^{2x}+1)^5\,(e^{2x}+2\,e^{4x}+e^{6x})}-\frac{12288\,e^{4x}\sqrt{a\left(\frac{e^{-x}}{2}+\frac{e^{x}}{2}\right)^4}}{7\,a^3\,(e^{2x}+1)^7\,(e^{2x}+2\,e^{4x}+e^{6x})}+\frac{1024\,e^{4x}\sqrt{a\left(\frac{e^{-x}}{2}+\frac{e^{x}}{2}\right)^4}}{a^3\,(e^{2x}+1)^8\,(e^{2x}+2\,e^{4x}+e^{6x})}-\frac{2048\,e^{4x}\sqrt{a\left(\frac{e^{-x}}{2}+\frac{e^{x}}{2}\right)^4}}{9\,a^3\,(e^{2x}+1)^9\,(e^{2x}+2\,e^{4x}+e^{6x})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)^4)^(5/2),x)
```

```
[Out] (4096*exp(4*x)*(a*(exp(-x)/2 + exp(x)/2)^4)^(1/2))/(3*a^3*(exp(2*x) + 1)^6*
(exp(2*x) + 2*exp(4*x) + exp(6*x))) - (2048*exp(4*x)*(a*(exp(-x)/2 + exp(x)
/2)^4)^(1/2))/(5*a^3*(exp(2*x) + 1)^5*(exp(2*x) + 2*exp(4*x) + exp(6*x))) -
 (12288*exp(4*x)*(a*(exp(-x)/2 + exp(x)/2)^4)^(1/2))/(7*a^3*(exp(2*x) + 1)^
7*(exp(2*x) + 2*exp(4*x) + exp(6*x))) + (1024*exp(4*x)*(a*(exp(-x)/2 + exp(
x)/2)^4)^(1/2))/(a^3*(exp(2*x) + 1)^8*(exp(2*x) + 2*exp(4*x) + exp(6*x))) -
 (2048*exp(4*x)*(a*(exp(-x)/2 + exp(x)/2)^4)^(1/2))/(9*a^3*(exp(2*x) + 1)^9
*(exp(2*x) + 2*exp(4*x) + exp(6*x)))
```

3.140 $\int \frac{\sinh(x)}{(1+\cosh(x))^2} dx$

Optimal. Leaf size=8

$$-\frac{1}{1+\cosh(x)}$$

```
[Out] -1/(1+cosh(x))
```

Rubi [A]
time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2746, 32}

$$-\frac{1}{\cosh(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(1 + Cosh[x])^2,x]
```

```
[Out] -(1 + Cosh[x])^(-1)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(x)}{(1+\cosh(x))^2} dx &= \text{Subst}\left(\int \frac{1}{(1+x)^2} dx, x, \cosh(x)\right)\\ &= -\frac{1}{1+\cosh(x)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 1.50

$$-\frac{1}{2}\text{sech}^2\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(1 + Cosh[x])^2,x]
```

```
[Out] -1/2*Sech[x/2]^2
```

Maple [A]
time = 0.24, size = 9, normalized size = 1.12

method	result	size
derivativedivides	$-\frac{1}{\cosh(x)+1}$	9
default	$-\frac{1}{\cosh(x)+1}$	9
risch	$-\frac{2\,\mathrm{e}^{x}}{(\mathrm{e}^{x}+1)^{2}}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(cosh(x)+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/(cosh(x)+1)
```

Maxima [A]
time = 0.26, size = 8, normalized size = 1.00

$$-\frac{1}{\cosh\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+cosh(x))^2,x, algorithm="maxima")
```

```
[Out] -1/(cosh(x) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(8) = 16.
time = 0.35, size = 31, normalized size = 3.88

$$-\frac{2\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}{\cosh\left(x\right)^{2}+2\left(\cosh\left(x\right)+1\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}+2\,\cosh\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+cosh(x))^2,x, algorithm="fricas")
```

```
[Out] -2*(cosh(x) + sinh(x))/(cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2 + 2
*cosh(x) + 1)
```

Sympy [A]
time = 0.15, size = 7, normalized size = 0.88

$$-\frac{1}{\cosh\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+cosh(x))**2,x)
```

```
[Out] -1/(cosh(x) + 1)
```

Giac [A]
time = 0.40, size = 10, normalized size = 1.25

$$-\frac{2\,e^{x}}{\left(e^{x}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+cosh(x))^2,x, algorithm="giac")
```

```
[Out] -2*e^x/(e^x + 1)^2
```

Mupad [B]
time = 0.07, size = 8, normalized size = 1.00

$$-\frac{1}{\cosh\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(cosh(x) + 1)^2,x)
```

```
[Out] -1/(cosh(x) + 1)
```

3.141 $\int \frac{\sinh(x)}{(1-\cosh(x))^2} \, dx$

Optimal. Leaf size=8

$$\frac{1}{1-\cosh(x)}$$

```
[Out] 1/(1-cosh(x))
```

Rubi [A]
time = 0.02, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2746, 32}

$$\frac{1}{1-\cosh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(1 - Cosh[x])^2,x]
```

```
[Out] (1 - Cosh[x])^(-1)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned} \int \frac{\sinh(x)}{(1-\cosh(x))^2} \, dx &= -\text{Subst}\left(\int \frac{1}{(1+x)^2} \, dx, x, -\cosh(x)\right) \\ &= \frac{1}{1-\cosh(x)} \end{aligned}$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 1.50

$$-\frac{1}{2}\text{csch}^2\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(1 - Cosh[x])^2,x]
```

```
[Out] -1/2*Csch[x/2]^2
```

Maple [A]
time = 0.30, size = 9, normalized size = 1.12

method	result	size
derivativedivides	$\frac{1}{1-\cosh(x)}$	9
default	$\frac{1}{1-\cosh(x)}$	9
risch	$-\frac{2\,e^{x}}{(e^{x}-1)^{2}}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(1-cosh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/(1-cosh(x))
```

Maxima [A]
time = 0.26, size = 8, normalized size = 1.00

$$-\frac{1}{\cosh(x)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1-cosh(x))^2,x, algorithm="maxima")
```

```
[Out] -1/(cosh(x) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(8) = 16.
time = 0.38, size = 31, normalized size = 3.88

$$-\frac{2\,(\cosh(x)+\sinh(x))}{\cosh(x)^{2}+2\,(\cosh(x)-1)\sinh(x)+\sinh(x)^{2}-2\,\cosh(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1-cosh(x))^2,x, algorithm="fricas")
```

```
[Out] -2*(cosh(x) + sinh(x))/(cosh(x)^2 + 2*(cosh(x) - 1)*sinh(x) + sinh(x)^2 - 2
*cosh(x) + 1)
```

Sympy [A]
time = 0.16, size = 7, normalized size = 0.88

$$-\frac{1}{\cosh(x)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1-cosh(x))**2,x)
```

```
[Out] -1/(cosh(x) - 1)
```

Giac [A]
time = 0.41, size = 10, normalized size = 1.25

$$-\frac{2\,e^{x}}{\left(e^{x}-1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1-cosh(x))^2,x, algorithm="giac")
```

```
[Out] -2*e^x/(e^x - 1)^2
```

Mupad [B]
time = 0.91, size = 8, normalized size = 1.00

$$-\frac{1}{\cosh\left(x\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(cosh(x) - 1)^2,x)
```

```
[Out] -1/(cosh(x) - 1)
```

3.142 $\int \frac{\sinh^2(x)}{(1+\cosh(x))^2}\,dx$

Optimal. Leaf size=12

$$x-\frac{2\sinh(x)}{1+\cosh(x)}$$

```
[Out] x-2*sinh(x)/(1+cosh(x))
```

Rubi [A]
time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2759, 8}

$$x-\frac{2\sinh(x)}{\cosh(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(1 + Cosh[x])^2,x]
```

```
[Out] x - (2*Sinh[x])/(1 + Cosh[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^2(x)}{(1+\cosh(x))^2}\,dx &= -\frac{2\sinh(x)}{1+\cosh(x)}+\int 1\,dx\\ &= x-\frac{2\sinh(x)}{1+\cosh(x)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.50

$$2\tanh^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(1 + Cosh[x])^2,x]
```

```
[Out] 2*ArcTanh[Tanh[x/2]] - 2*Tanh[x/2]
```

Maple [A]
time = 0.38, size = 24, normalized size = 2.00

method	result	size
risch	$x+\frac{4}{e^{x}+1}$	11
default	$-2\tanh\left(\frac{x}{2}\right)-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(cosh(x)+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*tanh(1/2*x)-ln(tanh(1/2*x)-1)+ln(tanh(1/2*x)+1)
```

Maxima [A]
time = 0.26, size = 12, normalized size = 1.00

$$x-\frac{4}{e^{(-x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1+cosh(x))^2,x, algorithm="maxima")
```

```
[Out] x - 4/(e^(-x) + 1)
```

Fricas [A]
time = 0.41, size = 20, normalized size = 1.67

$$\frac{x\cosh\left(x\right)+x\sinh\left(x\right)+x+4}{\cosh\left(x\right)+\sinh\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1+cosh(x))^2,x, algorithm="fricas")
```

```
[Out] (x*cosh(x) + x*sinh(x) + x + 4)/(cosh(x) + sinh(x) + 1)
```

Sympy [A]
time = 0.20, size = 7, normalized size = 0.58

$$x-2\tanh\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(1+cosh(x))**2,x)
```

```
[Out] x - 2*tanh(x/2)
```

Giac [A]
time = 0.41, size = 10, normalized size = 0.83

$$x + \frac{4}{e^x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1+cosh(x))^2,x, algorithm="giac")
```

```
[Out] x + 4/(e^x + 1)
```

Mupad [B]
time = 0.90, size = 10, normalized size = 0.83

$$x + \frac{4}{\mathrm{e}^x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(cosh(x) + 1)^2,x)
```

```
[Out] x + 4/(exp(x) + 1)
```

3.143 $\int \frac{\sinh^2(x)}{(1-\cosh(x))^2}\,dx$

Optimal. Leaf size=14

$$x+\frac{2\sinh(x)}{1-\cosh(x)}$$

```
[Out] x+2*sinh(x)/(1-cosh(x))
```

Rubi [A]
time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2759, 8}

$$x+\frac{2\sinh(x)}{1-\cosh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(1 - Cosh[x])^2,x]
```

```
[Out] x + (2*Sinh[x])/(1 - Cosh[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^2(x)}{(1-\cosh(x))^2}\,dx &= \frac{2\sinh(x)}{1-\cosh(x)}+\int 1\,dx\\ &= x+\frac{2\sinh(x)}{1-\cosh(x)}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.01, size = 24, normalized size = 1.71

$$-2\coth\left(\frac{x}{2}\right)\ {}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\tanh^2\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(1 - Cosh[x])^2,x]
```

```
[Out] -2*Coth[x/2]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[x/2]^2]
```

Maple [A]
time = 0.50, size = 26, normalized size = 1.86

method	result	size
risch	$x-\frac{4}{e^{x}-1}$	11
default	$\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\frac{2}{\tanh\left(\frac{x}{2}\right)}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(1-cosh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] ln(tanh(1/2*x)+1)-ln(tanh(1/2*x)-1)-2/tanh(1/2*x)
```

Maxima [A]
time = 0.27, size = 12, normalized size = 0.86

$$x+\frac{4}{e^{(-x)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1-cosh(x))^2,x, algorithm="maxima")
```

```
[Out] x + 4/(e^(-x) - 1)
```

Fricas [A]
time = 0.36, size = 22, normalized size = 1.57

$$\frac{x\cosh\left(x\right)+x\sinh\left(x\right)-x-4}{\cosh\left(x\right)+\sinh\left(x\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1-cosh(x))^2,x, algorithm="fricas")
```

```
[Out] (x*cosh(x) + x*sinh(x) - x - 4)/(cosh(x) + sinh(x) - 1)
```

Sympy [A]
time = 0.35, size = 7, normalized size = 0.50

$$x - \frac{2}{\tanh\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(1-cosh(x))**2,x)
[Out] x - 2/tanh(x/2)
```

Giac [A]
time = 0.42, size = 10, normalized size = 0.71

$$x - \frac{4}{e^x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1-cosh(x))^2,x, algorithm="giac")
[Out] x - 4/(e^x - 1)
```

Mupad [B]
time = 0.05, size = 10, normalized size = 0.71

$$x - \frac{4}{e^x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(cosh(x) - 1)^2,x)
[Out] x - 4/(exp(x) - 1)
```

3.144 $\int \frac{\sinh^3(x)}{(1+\cosh(x))^2}\,dx$

Optimal. Leaf size=10

$$\cosh(x) - 2\log(1+\cosh(x))$$

[Out] cosh(x)-2*ln(1+cosh(x))

Rubi [A]
time = 0.03, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2746, 45}

$$\cosh(x) - 2\log(\cosh(x)+1)$$

Antiderivative was successfully verified.

[In] Int[Sinh[x]^3/(1 + Cosh[x])^2,x]

[Out] Cosh[x] - 2*Log[1 + Cosh[x]]

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^3(x)}{(1+\cosh(x))^2}\,dx &= -\text{Subst}\left(\int \frac{1-x}{1+x}\,dx, x, \cosh(x)\right)\\
&= -\text{Subst}\left(\int \left(-1+\frac{2}{1+x}\right)\,dx, x, \cosh(x)\right)\\
&= \cosh(x) - 2\log(1+\cosh(x))
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 13, normalized size = 1.30

$$-1+\cosh(x)-4\log\left(\cosh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(1 + Cosh[x])^2,x]
```

```
[Out] -1 + Cosh[x] - 4*Log[Cosh[x/2]]
```

Maple [A]
time = 0.32, size = 11, normalized size = 1.10

method	result	size
derivativedivides	$\cosh(x)-2\ln(\cosh(x)+1)$	11
default	$\cosh(x)-2\ln(\cosh(x)+1)$	11
risch	$2x+\frac{e^{x}}{2}+\frac{e^{-x}}{2}-4\ln(e^{x}+1)$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(cosh(x)+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] cosh(x)-2*ln(cosh(x)+1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(10) = 20$.
time = 0.28, size = 23, normalized size = 2.30

$$-2\,x+\frac{1}{2}\,e^{(-x)}+\frac{1}{2}\,e^{x}-4\,\log\left(e^{(-x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1+cosh(x))^2,x, algorithm="maxima")
```

```
[Out] -2*x + 1/2*e^(-x) + 1/2*e^x - 4*log(e^(-x) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(10) = 20$.
time = 0.35, size = 48, normalized size = 4.80

$$\frac{4\,x\cosh(x)+\cosh(x)^{2}-8\,(\cosh(x)+\sinh(x))\log(\cosh(x)+\sinh(x)+1)+2\,(2\,x+\cosh(x))\sinh(x)+\sinh(x)^{2}+1}{2\,(\cosh(x)+\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1+cosh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*x*cosh(x) + cosh(x)^2 - 8*(cosh(x) + sinh(x))*log(cosh(x) + sinh(x)
+ 1) + 2*(2*x + cosh(x))*sinh(x) + sinh(x)^2 + 1)/(cosh(x) + sinh(x))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(10) = 20$.
time = 0.20, size = 58, normalized size = 5.80

$$-\frac{2\log\left(\cosh\left(x\right)+1\right)\cosh\left(x\right)}{\cosh\left(x\right)+1}-\frac{2\log\left(\cosh\left(x\right)+1\right)}{\cosh\left(x\right)+1}-\frac{\sinh^{2}\left(x\right)}{\cosh\left(x\right)+1}+\frac{2\cosh^{2}\left(x\right)}{\cosh\left(x\right)+1}-\frac{2}{\cosh\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(1+cosh(x))**2,x)
```

```
[Out] -2*log(cosh(x) + 1)*cosh(x)/(cosh(x) + 1) - 2*log(cosh(x) + 1)/(cosh(x) + 1
) - sinh(x)**2/(cosh(x) + 1) + 2*cosh(x)**2/(cosh(x) + 1) - 2/(cosh(x) + 1)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(10) = 20$.
time = 0.41, size = 21, normalized size = 2.10

$$2\,x+\frac{1}{2}\,e^{(-x)}+\frac{1}{2}\,e^{x}-4\,\log\left(e^{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1+cosh(x))^2,x, algorithm="giac")
```

```
[Out] 2*x + 1/2*e^(-x) + 1/2*e^x - 4*log(e^x + 1)
```

Mupad [B]
time = 0.94, size = 10, normalized size = 1.00

$$\cosh(x)-2\,\ln\left(\cosh(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(cosh(x) + 1)^2,x)
```

```
[Out] cosh(x) - 2*log(cosh(x) + 1)
```

3.145 $\int \frac{\sinh^3(x)}{(1-\cosh(x))^2}\,dx$

Optimal. Leaf size=12

$$\cosh(x)+2\log(1-\cosh(x))$$

[Out] cosh(x)+2*ln(1-cosh(x))

Rubi [A]
time = 0.03, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.154$, Rules used = {2746, 45}

$$\cosh(x)+2\log(1-\cosh(x))$$

Antiderivative was successfully verified.

[In] Int[Sinh[x]^3/(1 - Cosh[x])^2,x]

[Out] Cosh[x] + 2*Log[1 - Cosh[x]]

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^3(x)}{(1-\cosh(x))^2}\,dx &= \text{Subst}\left(\int \frac{1-x}{1+x}\,dx,x,-\cosh(x)\right)\\ &= \text{Subst}\left(\int \left(-1+\frac{2}{1+x}\right)\,dx,x,-\cosh(x)\right)\\ &= \cosh(x)+2\log(1-\cosh(x))\end{aligned}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.08

$$-1+\cosh(x)+4\log\left(\sinh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(1 - Cosh[x])^2,x]
```

```
[Out] -1 + Cosh[x] + 4*Log[Sinh[x/2]]
```

Maple [A]
time = 0.48, size = 11, normalized size = 0.92

method	result	size
derivativedivides	$\cosh(x)+2\ln(\cosh(x)-1)$	11
default	$\cosh(x)+2\ln(\cosh(x)-1)$	11
risch	$-2x+\frac{e^{x}}{2}+\frac{e^{-x}}{2}+4\ln(e^{x}-1)$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(1-cosh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] cosh(x)+2*ln(cosh(x)-1)
```

Maxima [A]
time = 0.26, size = 23, normalized size = 1.92

$$2\,x+\frac{1}{2}\,e^{(-x)}+\frac{1}{2}\,e^{x}+4\,\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1-cosh(x))^2,x, algorithm="maxima")
```

```
[Out] 2*x + 1/2*e^(-x) + 1/2*e^x + 4*log(e^(-x) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(12) = 24.
time = 0.40, size = 54, normalized size = 4.50

$$-\frac{4\,x\cosh(x)-\cosh(x)^{2}-8\,(\cosh(x)+\sinh(x))\log(\cosh(x)+\sinh(x)-1)+2\,(2\,x-\cosh(x))\sinh(x)-\sinh(x)^{2}-1}{2\,(\cosh(x)+\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1-cosh(x))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(4*x*cosh(x) - cosh(x)^2 - 8*(cosh(x) + sinh(x))*log(cosh(x) + sinh(x)
 - 1) + 2*(2*x - cosh(x))*sinh(x) - sinh(x)^2 - 1)/(cosh(x) + sinh(x))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(10) = 20$.
time = 0.21, size = 58, normalized size = 4.83

$$\frac{2\log\left(\cosh\left(x\right)-1\right)\cosh\left(x\right)}{\cosh\left(x\right)-1}-\frac{2\log\left(\cosh\left(x\right)-1\right)}{\cosh\left(x\right)-1}-\frac{\sinh^{2}\left(x\right)}{\cosh\left(x\right)-1}+\frac{2\cosh^{2}\left(x\right)}{\cosh\left(x\right)-1}-\frac{2}{\cosh\left(x\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(1-cosh(x))**2,x)
```

```
[Out] 2*log(cosh(x) - 1)*cosh(x)/(cosh(x) - 1) - 2*log(cosh(x) - 1)/(cosh(x) - 1)
 - sinh(x)**2/(cosh(x) - 1) + 2*cosh(x)**2/(cosh(x) - 1) - 2/(cosh(x) - 1)
```

Giac [A]
time = 0.40, size = 22, normalized size = 1.83

$$-2\,x+\frac{1}{2}\,e^{(-x)}+\frac{1}{2}\,e^{x}+4\,\log\left(\left|e^{x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1-cosh(x))^2,x, algorithm="giac")
```

```
[Out] -2*x + 1/2*e^(-x) + 1/2*e^x + 4*log(abs(e^x - 1))
```

Mupad [B]
time = 0.94, size = 10, normalized size = 0.83

$$2\,\ln\left(\cosh(x)-1\right)+\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(cosh(x) - 1)^2,x)
```

```
[Out] 2*log(cosh(x) - 1) + cosh(x)
```

3.146 $\int \frac{\sinh(x)}{(1+\cosh(x))^3}\,dx$

Optimal. Leaf size=10

$$-\frac{1}{2(1+\cosh(x))^2}$$

```
[Out] -1/2/(1+cosh(x))^2
```

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2746, 32}

$$-\frac{1}{2(\cosh(x)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(1 + Cosh[x])^3,x]
```

```
[Out] -1/2*1/(1 + Cosh[x])^2
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2])
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(x)}{(1+\cosh(x))^3}\,dx &= \text{Subst}\left(\int \frac{1}{(1+x)^3}\,dx, x, \cosh(x)\right)\\ &= -\frac{1}{2(1+\cosh(x))^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 1.20

$$-\frac{1}{8}\text{sech}^4\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(1 + Cosh[x])^3,x]
```

```
[Out] -1/8*Sech[x/2]^4
```

Maple [A]
time = 0.24, size = 9, normalized size = 0.90

method	result	size
derivativedivides	$-\frac{1}{2(\cosh(x)+1)^2}$	9
default	$-\frac{1}{2(\cosh(x)+1)^2}$	9
risch	$-\frac{2\,e^{2x}}{(e^x+1)^4}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(cosh(x)+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(cosh(x)+1)^2
```

Maxima [A]
time = 0.28, size = 8, normalized size = 0.80

$$-\frac{1}{2\left(\cosh\left(x\right)+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+cosh(x))^3,x, algorithm="maxima")
```

```
[Out] -1/2/(cosh(x) + 1)^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(8) = 16.
time = 0.42, size = 55, normalized size = 5.50

$$-\frac{2\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}{\cosh\left(x\right)^{3}+\left(3\,\cosh\left(x\right)+4\right)\sinh\left(x\right)^{2}+\sinh\left(x\right)^{3}+4\,\cosh\left(x\right)^{2}+\left(3\,\cosh\left(x\right)^{2}+8\,\cosh\left(x\right)+5\right)\sinh\left(x\right)+7\,\cosh\left(x\right)+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+cosh(x))^3,x, algorithm="fricas")
```

```
[Out] -2*(cosh(x) + sinh(x))/(cosh(x)^3 + (3*cosh(x) + 4)*sinh(x)^2 + sinh(x)^3 +
 4*cosh(x)^2 + (3*cosh(x)^2 + 8*cosh(x) + 5)*sinh(x) + 7*cosh(x) + 4)
```

Sympy [A]
time = 0.26, size = 15, normalized size = 1.50

$$-\frac{1}{2\cosh^{2}\left(x\right)+4\cosh\left(x\right)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+cosh(x))**3,x)
```

```
[Out] -1/(2*cosh(x)**2 + 4*cosh(x) + 2)
```

Giac [A]
time = 0.40, size = 12, normalized size = 1.20

$$-\frac{2\,e^{(2\,x)}}{(e^x+1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+cosh(x))^3,x, algorithm="giac")
```

```
[Out] -2*e^(2*x)/(e^x + 1)^4
```

Mupad [B]
time = 0.92, size = 8, normalized size = 0.80

$$-\frac{1}{2\,(\cosh(x)+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(cosh(x) + 1)^3,x)
```

```
[Out] -1/(2*(cosh(x) + 1)^2)
```

3.147 $\int \frac{\sinh(x)}{(1-\cosh(x))^3} \, dx$

Optimal. Leaf size=12

$$\frac{1}{2(1-\cosh(x))^2}$$

[Out] 1/2/(1-cosh(x))^2

Rubi [A]
time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2746, 32}

$$\frac{1}{2(1-\cosh(x))^2}$$

Antiderivative was successfully verified.

[In] Int[Sinh[x]/(1 - Cosh[x])^3,x]

[Out] 1/(2*(1 - Cosh[x])^2)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(x)}{(1-\cosh(x))^3} \, dx &= -\text{Subst}\left(\int \frac{1}{(1+x)^3} \, dx, x, -\cosh(x)\right)\\ &= \frac{1}{2(1-\cosh(x))^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 1.00

$$\frac{1}{8}\text{csch}^4\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(1 - Cosh[x])^3,x]
[Out] Csch[x/2]^4/8
```

Maple [A]
time = 0.24, size = 11, normalized size = 0.92

method	result	size
derivativedivides	$\frac{1}{2(1-\cosh(x))^2}$	11
default	$\frac{1}{2(1-\cosh(x))^2}$	11
risch	$\frac{2\,e^{2x}}{(e^x-1)^4}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(1-cosh(x))^3,x,method=_RETURNVERBOSE)
[Out] 1/2/(1-cosh(x))^2
```

Maxima [A]
time = 0.28, size = 8, normalized size = 0.67

$$\frac{1}{2\left(\cosh\left(x\right)-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1-cosh(x))^3,x, algorithm="maxima")
[Out] 1/2/(cosh(x) - 1)^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(8) = 16.
time = 0.33, size = 55, normalized size = 4.58

$$\frac{2\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}{\cosh\left(x\right)^3+\left(3\,\cosh\left(x\right)-4\right)\sinh\left(x\right)^2+\sinh\left(x\right)^3-4\,\cosh\left(x\right)^2+\left(3\,\cosh\left(x\right)^2-8\,\cosh\left(x\right)+5\right)\sinh\left(x\right)+7\,\cosh\left(x\right)-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1-cosh(x))^3,x, algorithm="fricas")
[Out] 2*(cosh(x) + sinh(x))/(cosh(x)^3 + (3*cosh(x) - 4)*sinh(x)^2 + sinh(x)^3 -
4*cosh(x)^2 + (3*cosh(x)^2 - 8*cosh(x) + 5)*sinh(x) + 7*cosh(x) - 4)
```

Sympy [A]
time = 0.27, size = 14, normalized size = 1.17

$$\frac{1}{2\cosh^2\left(x\right)-4\cosh\left(x\right)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1-cosh(x))**3,x)
```

```
[Out] 1/(2*cosh(x)**2 - 4*cosh(x) + 2)
```

Giac [A]
time = 0.41, size = 12, normalized size = 1.00

$$\frac{2 e^{(2 x)}}{(e^x - 1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1-cosh(x))^3,x, algorithm="giac")
```

```
[Out] 2*e^(2*x)/(e^x - 1)^4
```

Mupad [B]
time = 0.08, size = 8, normalized size = 0.67

$$\frac{1}{2 (\cosh(x) - 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-sinh(x)/(cosh(x) - 1)^3,x)
```

```
[Out] 1/(2*(cosh(x) - 1)^2)
```

3.148 $\int \frac{\sinh^2(x)}{(1+\cosh(x))^3} \, dx$

Optimal. Leaf size=14

$$\frac{\sinh^3(x)}{3(1+\cosh(x))^3}$$

[Out] 1/3*sinh(x)^3/(1+cosh(x))^3

Rubi [A]
time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{2750\}$

$$\frac{\sinh^3(x)}{3(\cosh(x)+1)^3}$$

Antiderivative was successfully verified.

[In] Int[Sinh[x]^2/(1 + Cosh[x])^3,x]

[Out] Sinh[x]^3/(3*(1 + Cosh[x])^3)

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{\sinh^2(x)}{(1+\cosh(x))^3} \, dx = \frac{\sinh^3(x)}{3(1+\cosh(x))^3}$$

Mathematica [A]
time = 0.02, size = 12, normalized size = 0.86

$$\frac{1}{3} \tanh^3\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[x]^2/(1 + Cosh[x])^3,x]

[Out] Tanh[x/2]^3/3

Maple [A]
time = 0.36, size = 9, normalized size = 0.64

method	result	size
default	$\frac{\left(\tanh^3\left(\frac{x}{2}\right)\right)}{3}$	9
risch	$-\frac{2(3\,\mathrm{e}^{2x}+1)}{3(\mathrm{e}^{x}+1)^3}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(cosh(x)+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*tanh(1/2*x)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(12) = 24$.
time = 0.26, size = 49, normalized size = 3.50

$$\frac{2\,e^{(-2\,x)}}{3\,e^{(-x)}+3\,e^{(-2\,x)}+e^{(-3\,x)}+1}+\frac{2}{3\,(3\,e^{(-x)}+3\,e^{(-2\,x)}+e^{(-3\,x)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1+cosh(x))^3,x, algorithm="maxima")
```

```
[Out] 2*e^(-2*x)/(3*e^(-x) + 3*e^(-2*x) + e^(-3*x) + 1) + 2/3/(3*e^(-x) + 3*e^(-2
*x) + e^(-3*x) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(12) = 24$.
time = 0.36, size = 33, normalized size = 2.36

$$-\frac{4\,(2\,\cosh(x)+\sinh(x))}{3\,\left(\cosh(x)^2+2\,(\cosh(x)+1)\sinh(x)+\sinh(x)^2+4\,\cosh(x)+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1+cosh(x))^3,x, algorithm="fricas")
```

```
[Out] -4/3*(2*cosh(x) + sinh(x))/(cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2
 + 4*cosh(x) + 3)
```

Sympy [A]
time = 0.33, size = 7, normalized size = 0.50

$$\frac{\tanh^3\left(\frac{x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(1+cosh(x))**3,x)
```

```
[Out] tanh(x/2)**3/3
```

Giac [A]
time = 0.41, size = 16, normalized size = 1.14

$$-\frac{2\left(3e^{(2x)}+1\right)}{3\left(e^{x}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1+cosh(x))^3,x, algorithm="giac")
```

```
[Out] -2/3*(3*e^(2*x) + 1)/(e^x + 1)^3
```

Mupad [B]
time = 0.92, size = 16, normalized size = 1.14

$$-\frac{2\left(3\,\mathrm{e}^{2x}+1\right)}{3\left(\mathrm{e}^{x}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(cosh(x) + 1)^3,x)
```

```
[Out] -(2*(3*exp(2*x) + 1))/(3*(exp(x) + 1)^3)
```

3.149 $\int \frac{\sinh^2(x)}{(1-\cosh(x))^3}\,dx$

Optimal. Leaf size=16

$$-\frac{\sinh^3(x)}{3(1-\cosh(x))^3}$$

```
[Out] -1/3*sinh(x)^3/(1-cosh(x))^3
```

Rubi [A]
time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{2750\}$

$$-\frac{\sinh^3(x)}{3(1-\cosh(x))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(1 - Cosh[x])^3,x]
```

```
[Out] -1/3*Sinh[x]^3/(1 - Cosh[x])^3
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{\sinh^2(x)}{(1-\cosh(x))^3}\,dx = -\frac{\sinh^3(x)}{3(1-\cosh(x))^3}$$

Mathematica [A]
time = 0.02, size = 12, normalized size = 0.75

$$\frac{1}{3}\coth^3\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(1 - Cosh[x])^3,x]
```

```
[Out] Coth[x/2]^3/3
```

Maple [A]
time = 0.41, size = 9, normalized size = 0.56

method	result	size
default	$\frac{1}{3\tanh\left(\frac{x}{2}\right)^{3}}$	9
risch	$\frac{2\,\mathrm{e}^{2x}+\frac{2}{3}}{\left(\mathrm{e}^{x}-1\right)^{3}}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(1-cosh(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/tanh(1/2*x)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(12) = 24$.
time = 0.28, size = 49, normalized size = 3.06

$$-\frac{2\,e^{(-2\,x)}}{3\,e^{(-x)}-3\,e^{(-2\,x)}+e^{(-3\,x)}-1}-\frac{2}{3\,\left(3\,e^{(-x)}-3\,e^{(-2\,x)}+e^{(-3\,x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1-cosh(x))^3,x, algorithm="maxima")
```

```
[Out] -2*e^(-2*x)/(3*e^(-x) - 3*e^(-2*x) + e^(-3*x) - 1) - 2/3/(3*e^(-x) - 3*e^(-
2*x) + e^(-3*x) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(12) = 24$.
time = 0.41, size = 33, normalized size = 2.06

$$\frac{4\,(2\,\cosh\left(x\right)+\sinh\left(x\right))}{3\,\left(\cosh\left(x\right)^{2}+2\,(\cosh\left(x\right)-1)\sinh\left(x\right)+\sinh\left(x\right)^{2}-4\,\cosh\left(x\right)+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1-cosh(x))^3,x, algorithm="fricas")
```

```
[Out] 4/3*(2*cosh(x) + sinh(x))/(cosh(x)^2 + 2*(cosh(x) - 1)*sinh(x) + sinh(x)^2
- 4*cosh(x) + 3)
```

Sympy [A]
time = 0.55, size = 8, normalized size = 0.50

$$\frac{1}{3\tanh^{3}\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(1-cosh(x))**3,x)
```

```
[Out] 1/(3*tanh(x/2)**3)
```

Giac [A]
time = 0.40, size = 16, normalized size = 1.00

$$\frac{2\left(3\,e^{(2\,x)}+1\right)}{3\,(e^{x}-1)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1-cosh(x))^3,x, algorithm="giac")
```

```
[Out] 2/3*(3*e^(2*x) + 1)/(e^x - 1)^3
```

Mupad [B]
time = 0.92, size = 16, normalized size = 1.00

$$\frac{2\,(3\,\mathrm{e}^{2\,x}+1)}{3\,(\mathrm{e}^{x}-1)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-sinh(x)^2/(cosh(x) - 1)^3,x)
```

```
[Out] (2*(3*exp(2*x) + 1))/(3*(exp(x) - 1)^3)
```

3.150 $\int \frac{\sinh^3(x)}{(1+\cosh(x))^3} \, dx$

Optimal. Leaf size=14

$$\frac{2}{1+\cosh(x)}+\log(1+\cosh(x))$$

```
[Out] 2/(1+cosh(x))+ln(1+cosh(x))
```

Rubi [A]
time = 0.03, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2746, 45}

$$\frac{2}{\cosh(x)+1}+\log(\cosh(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(1 + Cosh[x])^3,x]
```

```
[Out] 2/(1 + Cosh[x]) + Log[1 + Cosh[x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^3(x)}{(1+\cosh(x))^3} \, dx &= -\text{Subst}\left(\int \frac{1-x}{(1+x)^2} \, dx,x,\cosh(x)\right)\\
&= -\text{Subst}\left(\int \left(\frac{1}{-1-x}+\frac{2}{(1+x)^2}\right) \, dx,x,\cosh(x)\right)\\
&= \frac{2}{1+\cosh(x)}+\log(1+\cosh(x))
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 1.43

$$2\log\left(\cosh\left(\frac{x}{2}\right)\right)-\tanh^{2}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(1 + Cosh[x])^3,x]
```

```
[Out] 2*Log[Cosh[x/2]] - Tanh[x/2]^2
```

Maple [A]
time = 0.34, size = 15, normalized size = 1.07

method	result	size
derivativedivides	$\frac{2}{\cosh(x)+1}+\ln\left(\cosh\left(x\right)+1\right)$	15
default	$\frac{2}{\cosh(x)+1}+\ln\left(\cosh\left(x\right)+1\right)$	15
risch	$-x+\frac{4\,\mathrm{e}^{x}}{\left(\mathrm{e}^{x}+1\right)^{2}}+2\ln\left(\mathrm{e}^{x}+1\right)$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(cosh(x)+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/(cosh(x)+1)+ln(cosh(x)+1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(14) = 28$.
time = 0.26, size = 31, normalized size = 2.21

$$x+\frac{4\,e^{(-x)}}{2\,e^{(-x)}+e^{(-2\,x)}+1}+2\,\log\left(e^{(-x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1+cosh(x))^3,x, algorithm="maxima")
```

```
[Out] x + 4*e^(-x)/(2*e^(-x) + e^(-2*x) + 1) + 2*log(e^(-x) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(14) = 28$.
time = 0.36, size = 89, normalized size = 6.36

$$-\frac{x\cosh\left(x\right)^{2}+x\sinh\left(x\right)^{2}+2\left(x-2\right)\cosh\left(x\right)-2\left(\cosh\left(x\right)^{2}+2\left(\cosh\left(x\right)+1\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}+2\,\cosh\left(x\right)+1\right)\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)+2\left(x\cosh\left(x\right)+x-2\right)\sinh\left(x\right)+x}{\cosh\left(x\right)^{2}+2\left(\cosh\left(x\right)+1\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}+2\,\cosh\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1+cosh(x))^3,x, algorithm="fricas")
```

```
[Out] -(x*cosh(x)^2 + x*sinh(x)^2 + 2*(x - 2)*cosh(x) - 2*(cosh(x)^2 + 2*(cosh(x)
 + 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)*log(cosh(x) + sinh(x) + 1) + 2*(
x*cosh(x) + x - 2)*sinh(x) + x)/(cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh
(x)^2 + 2*cosh(x) + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(12) = 24$.
time = 0.27, size = 126, normalized size = 9.00

$$\frac{2\log\left(\cosh\left(x\right)+1\right)\cosh^{2}\left(x\right)}{2\cosh^{2}\left(x\right)+4\cosh\left(x\right)+2}+\frac{4\log\left(\cosh\left(x\right)+1\right)\cosh\left(x\right)}{2\cosh^{2}\left(x\right)+4\cosh\left(x\right)+2}+\frac{2\log\left(\cosh\left(x\right)+1\right)}{2\cosh^{2}\left(x\right)+4\cosh\left(x\right)+2}-\frac{\sinh^{2}\left(x\right)}{2\cosh^{2}\left(x\right)+4\cosh\left(x\right)+2}+\frac{2\cosh\left(x\right)}{2\cosh^{2}\left(x\right)+4\cosh\left(x\right)+2}+\frac{2}{2\cosh^{2}\left(x\right)+4\cosh\left(x\right)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(1+cosh(x))**3,x)
```

```
[Out] 2*log(cosh(x) + 1)*cosh(x)**2/(2*cosh(x)**2 + 4*cosh(x) + 2) + 4*log(cosh(x
) + 1)*cosh(x)/(2*cosh(x)**2 + 4*cosh(x) + 2) + 2*log(cosh(x) + 1)/(2*cosh(
x)**2 + 4*cosh(x) + 2) - sinh(x)**2/(2*cosh(x)**2 + 4*cosh(x) + 2) + 2*cosh
(x)/(2*cosh(x)**2 + 4*cosh(x) + 2) + 2/(2*cosh(x)**2 + 4*cosh(x) + 2)
```

Giac [A]
time = 0.40, size = 21, normalized size = 1.50

$$-x+\frac{4\,e^{x}}{\left(e^{x}+1\right)^{2}}+2\,\log\left(e^{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1+cosh(x))^3,x, algorithm="giac")
```

```
[Out] -x + 4*e^x/(e^x + 1)^2 + 2*log(e^x + 1)
```

Mupad [B]
time = 0.98, size = 14, normalized size = 1.00

$$\ln\left(\cosh(x)+1\right)+\frac{2}{\cosh\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(cosh(x) + 1)^3,x)
```

```
[Out] log(cosh(x) + 1) + 2/(cosh(x) + 1)
```

3.151 $\int \frac{\sinh^3(x)}{(1-\cosh(x))^3} \, dx$

Optimal. Leaf size=20

$$-\frac{2}{1-\cosh(x)}-\log(1-\cosh(x))$$

```
[Out] -2/(1-cosh(x))-ln(1-cosh(x))
```

Rubi [A]

time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2746, 45}

$$-\frac{2}{1-\cosh(x)}-\log(1-\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(1 - Cosh[x])^3,x]
```

```
[Out] -2/(1 - Cosh[x]) - Log[1 - Cosh[x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^3(x)}{(1-\cosh(x))^3} \, dx &= \text{Subst}\left(\int \frac{1-x}{(1+x)^2} \, dx, x, -\cosh(x)\right)\\
&= \text{Subst}\left(\int \left(\frac{1}{-1-x}+\frac{2}{(1+x)^2}\right) \, dx, x, -\cosh(x)\right)\\
&= -\frac{2}{1-\cosh(x)}-\log(1-\cosh(x))
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 1.35

$$\coth^2\left(\frac{x}{2}\right) - 2\log\left(\cosh\left(\frac{x}{2}\right)\right) - 2\log\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(1 - Cosh[x])^3,x]
```

```
[Out] Coth[x/2]^2 - 2*Log[Cosh[x/2]] - 2*Log[Tanh[x/2]]
```

Maple [A]
time = 0.44, size = 17, normalized size = 0.85

method	result	size
derivativedivides	$-\ln\left(\cosh\left(x\right)-1\right)+\frac{2}{\cosh\left(x\right)-1}$	17
default	$-\ln\left(\cosh\left(x\right)-1\right)+\frac{2}{\cosh\left(x\right)-1}$	17
risch	$x+\frac{4\,\mathrm{e}^{x}}{\left(\mathrm{e}^{x}-1\right)^{2}}-2\ln\left(\mathrm{e}^{x}-1\right)$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(1-cosh(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -ln(cosh(x)-1)+2/(cosh(x)-1)
```

Maxima [A]
time = 0.27, size = 35, normalized size = 1.75

$$-x - \frac{4\,e^{(-x)}}{2\,e^{(-x)} - e^{(-2\,x)} - 1} - 2\,\log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1-cosh(x))^3,x, algorithm="maxima")
```

```
[Out] -x - 4*e^(-x)/(2*e^(-x) - e^(-2*x) - 1) - 2*log(e^(-x) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(18) = 36.
time = 0.34, size = 90, normalized size = 4.50

$$\frac{x\cosh\left(x\right)^{2} + x\sinh\left(x\right)^{2} - 2\left(x-2\right)\cosh\left(x\right) - 2\left(\cosh\left(x\right)^{2} + 2\left(\cosh\left(x\right)-1\right)\sinh\left(x\right) + \sinh\left(x\right)^{2} - 2\,\cosh\left(x\right) + 1\right)\log\left(\cosh\left(x\right) + \sinh\left(x\right) - 1\right) + 2\left(x\cosh\left(x\right) - x + 2\right)\sinh\left(x\right) + x}{\cosh\left(x\right)^{2} + 2\left(\cosh\left(x\right)-1\right)\sinh\left(x\right) + \sinh\left(x\right)^{2} - 2\,\cosh\left(x\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1-cosh(x))^3,x, algorithm="fricas")
```

```
[Out] (x*cosh(x)^2 + x*sinh(x)^2 - 2*(x - 2)*cosh(x) - 2*(cosh(x)^2 + 2*(cosh(x)
- 1)*sinh(x) + sinh(x)^2 - 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*(x
*cosh(x) - x + 2)*sinh(x) + x)/(cosh(x)^2 + 2*(cosh(x) - 1)*sinh(x) + sinh(
x)^2 - 2*cosh(x) + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(14) = 28$.
time = 0.28, size = 126, normalized size = 6.30

$$-\frac{2\log\left(\cosh\left(x\right)-1\right)\cosh^{2}\left(x\right)}{2\cosh^{2}\left(x\right)-4\cosh\left(x\right)+2}+\frac{4\log\left(\cosh\left(x\right)-1\right)\cosh\left(x\right)}{2\cosh^{2}\left(x\right)-4\cosh\left(x\right)+2}-\frac{2\log\left(\cosh\left(x\right)-1\right)}{2\cosh^{2}\left(x\right)-4\cosh\left(x\right)+2}+\frac{\sinh^{2}\left(x\right)}{2\cosh^{2}\left(x\right)-4\cosh\left(x\right)+2}+\frac{2\cosh\left(x\right)}{2\cosh^{2}\left(x\right)-4\cosh\left(x\right)+2}-\frac{2}{2\cosh^{2}\left(x\right)-4\cosh\left(x\right)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(1-cosh(x))**3,x)
```

```
[Out] -2*log(cosh(x) - 1)*cosh(x)**2/(2*cosh(x)**2 - 4*cosh(x) + 2) + 4*log(cosh(
x) - 1)*cosh(x)/(2*cosh(x)**2 - 4*cosh(x) + 2) - 2*log(cosh(x) - 1)/(2*cosh
(x)**2 - 4*cosh(x) + 2) + sinh(x)**2/(2*cosh(x)**2 - 4*cosh(x) + 2) + 2*cos
h(x)/(2*cosh(x)**2 - 4*cosh(x) + 2) - 2/(2*cosh(x)**2 - 4*cosh(x) + 2)
```

Giac [A]
time = 0.41, size = 20, normalized size = 1.00

$$x+\frac{4\,e^{x}}{\left(e^{x}-1\right)^{2}}-2\,\log\left(\left|e^{x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1-cosh(x))^3,x, algorithm="giac")
```

```
[Out] x + 4*e^x/(e^x - 1)^2 - 2*log(abs(e^x - 1))
```

Mupad [B]
time = 0.99, size = 16, normalized size = 0.80

$$\frac{2}{\cosh\left(x\right)-1}-\ln\left(\cosh(x)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-sinh(x)^3/(cosh(x) - 1)^3,x)
```

```
[Out] 2/(cosh(x) - 1) - log(cosh(x) - 1)
```

3.152 $\int \frac{\sinh^8(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=57

$$\frac{5x}{16a}-\frac{5\cosh(x)\sinh(x)}{16a}+\frac{5\cosh(x)\sinh^3(x)}{24a}-\frac{\cosh(x)\sinh^5(x)}{6a}+\frac{\sinh^7(x)}{7a}$$

```
[Out] 5/16*x/a-5/16*cosh(x)*sinh(x)/a+5/24*cosh(x)*sinh(x)^3/a-1/6*cosh(x)*sinh(x
)^5/a+1/7*sinh(x)^7/a
```

Rubi [A]
time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2761, 2715, 8}

$$\frac{5x}{16a}+\frac{\sinh^7(x)}{7a}-\frac{\sinh^5(x)\cosh(x)}{6a}+\frac{5\sinh^3(x)\cosh(x)}{24a}-\frac{5\sinh(x)\cosh(x)}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^8/(a + a*Cosh[x]),x]
```

```
[Out] (5*x)/(16*a) - (5*Cosh[x]*Sinh[x])/(16*a) + (5*Cosh[x]*Sinh[x]^3)/(24*a) -
(Cosh[x]*Sinh[x]^5)/(6*a) + Sinh[x]^7/(7*a)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^8(x)}{a+a\cosh(x)}\,dx &= \frac{\sinh^7(x)}{7a}-\frac{\int \sinh^6(x)\,dx}{a} \\
&= -\frac{\cosh(x)\sinh^5(x)}{6a}+\frac{\sinh^7(x)}{7a}+\frac{5\int \sinh^4(x)\,dx}{6a} \\
&= \frac{5\cosh(x)\sinh^3(x)}{24a}-\frac{\cosh(x)\sinh^5(x)}{6a}+\frac{\sinh^7(x)}{7a}-\frac{5\int \sinh^2(x)\,dx}{8a} \\
&= -\frac{5\cosh(x)\sinh(x)}{16a}+\frac{5\cosh(x)\sinh^3(x)}{24a}-\frac{\cosh(x)\sinh^5(x)}{6a}+\frac{\sinh^7(x)}{7a}+\frac{5\int 1\,dx}{16a} \\
&= \frac{5x}{16a}-\frac{5\cosh(x)\sinh(x)}{16a}+\frac{5\cosh(x)\sinh^3(x)}{24a}-\frac{\cosh(x)\sinh^5(x)}{6a}+\frac{\sinh^7(x)}{7a}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 51, normalized size = 0.89

$$\frac{420x-105\sinh(x)-315\sinh(2x)+63\sinh(3x)+63\sinh(4x)-21\sinh(5x)-7\sinh(6x)+3\sinh(7x)}{1344a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^8/(a + a*Cosh[x]),x]
```

```
[Out] (420*x - 105*Sinh[x] - 315*Sinh[2*x] + 63*Sinh[3*x] + 63*Sinh[4*x] - 21*Sin
h[5*x] - 7*Sinh[6*x] + 3*Sinh[7*x])/(1344*a)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(47) = 94$.
time = 0.52, size = 165, normalized size = 2.89

method	result
risch	$\frac{5x}{16a}+\frac{e^{7x}}{896a}-\frac{e^{6x}}{384a}-\frac{e^{5x}}{128a}+\frac{3\,e^{4x}}{128a}+\frac{3\,e^{3x}}{128a}-\frac{15\,e^{2x}}{128a}-\frac{5\,e^{x}}{128a}+\frac{5\,e^{-x}}{128a}+\frac{15\,e^{-2x}}{128a}-\frac{3\,e^{-3x}}{128a}-\frac{3\,e^{-4x}}{128a}+\frac{e^{-5x}}{128a}+$
default	$-\frac{1}{7\left(\tanh\left(\frac{x}{2}\right)+1\right)^7}+\frac{2}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^6}-\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^5}+\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)+1\right)^4}+\frac{11}{24\left(\tanh\left(\frac{x}{2}\right)+1\right)^3}+\frac{1}{8\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}-\frac{5}{16\left(\tanh\left(\frac{x}{2}\right)+1\right)}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^8/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 256/a*(-1/1792/(tanh(1/2*x)+1)^7+1/384/(tanh(1/2*x)+1)^6-1/256/(tanh(1/2*x)
+1)^5+1/1024/(tanh(1/2*x)+1)^4+11/6144/(tanh(1/2*x)+1)^3+1/2048/(tanh(1/2*x
)+1)^2-5/4096/(tanh(1/2*x)+1)+5/4096*ln(tanh(1/2*x)+1)-1/1792/(tanh(1/2*x)-
1)^7-1/384/(tanh(1/2*x)-1)^6-1/256/(tanh(1/2*x)-1)^5-1/1024/(tanh(1/2*x)-1)
^4+11/6144/(tanh(1/2*x)-1)^3-1/2048/(tanh(1/2*x)-1)^2-5/4096/(tanh(1/2*x)-1
)-5/4096*ln(tanh(1/2*x)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(47) = 94$.
time = 0.26, size = 102, normalized size = 1.79

$$-\frac{\left(7\,e^{(-x)} + 21\,e^{(-2\,x)} - 63\,e^{(-3\,x)} - 63\,e^{(-4\,x)} + 315\,e^{(-5\,x)} + 105\,e^{(-6\,x)} - 3\right)e^{(7\,x)}}{2688\,a} + \frac{5\,x}{16\,a} + \frac{105\,e^{(-x)} + 315\,e^{(-2\,x)} - 63\,e^{(-3\,x)} - 63\,e^{(-4\,x)} + 21\,e^{(-5\,x)} + 7\,e^{(-6\,x)} - 3\,e^{(-7\,x)}}{2688\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^8/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -1/2688*(7*e^(-x) + 21*e^(-2*x) - 63*e^(-3*x) - 63*e^(-4*x) + 315*e^(-5*x)
+ 105*e^(-6*x) - 3)*e^(7*x)/a + 5/16*x/a + 1/2688*(105*e^(-x) + 315*e^(-2*x
) - 63*e^(-3*x) - 63*e^(-4*x) + 21*e^(-5*x) + 7*e^(-6*x) - 3*e^(-7*x))/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(47) = 94$.
time = 0.41, size = 101, normalized size = 1.77

$$\frac{3\,\sinh\left(x\right)^{7} + 21\,\left(3\,\cosh\left(x\right)^{2} - 2\,\cosh\left(x\right) - 1\right)\sinh\left(x\right)^{5} + 7\,\left(15\,\cosh\left(x\right)^{4} - 20\,\cosh\left(x\right)^{3} - 30\,\cosh\left(x\right)^{2} + 36\,\cosh\left(x\right) + 9\right)\sinh\left(x\right)^{3} + 21\,\left(\cosh\left(x\right)^{6} - 2\,\cosh\left(x\right)^{5} - 5\,\cosh\left(x\right)^{4} + 12\,\cosh\left(x\right)^{3} + 9\,\cosh\left(x\right)^{2} - 30\,\cosh\left(x\right) - 5\right)\sinh\left(x\right) + 420\,x}{1344\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^8/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/1344*(3*sinh(x)^7 + 21*(3*cosh(x)^2 - 2*cosh(x) - 1)*sinh(x)^5 + 7*(15*co
sh(x)^4 - 20*cosh(x)^3 - 30*cosh(x)^2 + 36*cosh(x) + 9)*sinh(x)^3 + 21*(cos
h(x)^6 - 2*cosh(x)^5 - 5*cosh(x)^4 + 12*cosh(x)^3 + 9*cosh(x)^2 - 30*cosh(x
) - 5)*sinh(x) + 420*x)/a
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1253 vs. $2(51) = 102$.
time = 3.54, size = 1253, normalized size = 21.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**8/(a+a*cosh(x)),x)
```

```
[Out] 105*x*tanh(x/2)**14/(336*a*tanh(x/2)**14 - 2352*a*tanh(x/2)**12 + 7056*a*ta
nh(x/2)**10 - 11760*a*tanh(x/2)**8 + 11760*a*tanh(x/2)**6 - 7056*a*tanh(x/2
)**4 + 2352*a*tanh(x/2)**2 - 336*a) - 735*x*tanh(x/2)**12/(336*a*tanh(x/2)*
*14 - 2352*a*tanh(x/2)**12 + 7056*a*tanh(x/2)**10 - 11760*a*tanh(x/2)**8 +
11760*a*tanh(x/2)**6 - 7056*a*tanh(x/2)**4 + 2352*a*tanh(x/2)**2 - 336*a) +
 2205*x*tanh(x/2)**10/(336*a*tanh(x/2)**14 - 2352*a*tanh(x/2)**12 + 7056*a*
tanh(x/2)**10 - 11760*a*tanh(x/2)**8 + 11760*a*tanh(x/2)**6 - 7056*a*tanh(x
/2)**4 + 2352*a*tanh(x/2)**2 - 336*a) - 3675*x*tanh(x/2)**8/(336*a*tanh(x/2
)**14 - 2352*a*tanh(x/2)**12 + 7056*a*tanh(x/2)**10 - 11760*a*tanh(x/2)**8
```

```
+ 11760*a*tanh(x/2)**6 - 7056*a*tanh(x/2)**4 + 2352*a*tanh(x/2)**2 - 336*a)
 + 3675*x*tanh(x/2)**6/(336*a*tanh(x/2)**14 - 2352*a*tanh(x/2)**12 + 7056*a
*tanh(x/2)**10 - 11760*a*tanh(x/2)**8 + 11760*a*tanh(x/2)**6 - 7056*a*tanh(
x/2)**4 + 2352*a*tanh(x/2)**2 - 336*a) - 2205*x*tanh(x/2)**4/(336*a*tanh(x/
2)**14 - 2352*a*tanh(x/2)**12 + 7056*a*tanh(x/2)**10 - 11760*a*tanh(x/2)**8
 + 11760*a*tanh(x/2)**6 - 7056*a*tanh(x/2)**4 + 2352*a*tanh(x/2)**2 - 336*a
) + 735*x*tanh(x/2)**2/(336*a*tanh(x/2)**14 - 2352*a*tanh(x/2)**12 + 7056*a
*tanh(x/2)**10 - 11760*a*tanh(x/2)**8 + 11760*a*tanh(x/2)**6 - 7056*a*tanh(
x/2)**4 + 2352*a*tanh(x/2)**2 - 336*a) - 105*x/(336*a*tanh(x/2)**14 - 2352*
a*tanh(x/2)**12 + 7056*a*tanh(x/2)**10 - 11760*a*tanh(x/2)**8 + 11760*a*tan
h(x/2)**6 - 7056*a*tanh(x/2)**4 + 2352*a*tanh(x/2)**2 - 336*a) - 210*tanh(x
/2)**13/(336*a*tanh(x/2)**14 - 2352*a*tanh(x/2)**12 + 7056*a*tanh(x/2)**10
- 11760*a*tanh(x/2)**8 + 11760*a*tanh(x/2)**6 - 7056*a*tanh(x/2)**4 + 2352*
a*tanh(x/2)**2 - 336*a) + 1400*tanh(x/2)**11/(336*a*tanh(x/2)**14 - 2352*a*
tanh(x/2)**12 + 7056*a*tanh(x/2)**10 - 11760*a*tanh(x/2)**8 + 11760*a*tanh(
x/2)**6 - 7056*a*tanh(x/2)**4 + 2352*a*tanh(x/2)**2 - 336*a) - 3962*tanh(x/
2)**9/(336*a*tanh(x/2)**14 - 2352*a*tanh(x/2)**12 + 7056*a*tanh(x/2)**10 -
11760*a*tanh(x/2)**8 + 11760*a*tanh(x/2)**6 - 7056*a*tanh(x/2)**4 + 2352*a*
tanh(x/2)**2 - 336*a) - 6144*tanh(x/2)**7/(336*a*tanh(x/2)**14 - 2352*a*tan
h(x/2)**12 + 7056*a*tanh(x/2)**10 - 11760*a*tanh(x/2)**8 + 11760*a*tanh(x/2
)**6 - 7056*a*tanh(x/2)**4 + 2352*a*tanh(x/2)**2 - 336*a) + 3962*tanh(x/2)*
*5/(336*a*tanh(x/2)**14 - 2352*a*tanh(x/2)**12 + 7056*a*tanh(x/2)**10 - 117
60*a*tanh(x/2)**8 + 11760*a*tanh(x/2)**6 - 7056*a*tanh(x/2)**4 + 2352*a*tan
h(x/2)**2 - 336*a) - 1400*tanh(x/2)**3/(336*a*tanh(x/2)**14 - 2352*a*tanh(x
/2)**12 + 7056*a*tanh(x/2)**10 - 11760*a*tanh(x/2)**8 + 11760*a*tanh(x/2)**
6 - 7056*a*tanh(x/2)**4 + 2352*a*tanh(x/2)**2 - 336*a) + 210*tanh(x/2)/(336
*a*tanh(x/2)**14 - 2352*a*tanh(x/2)**12 + 7056*a*tanh(x/2)**10 - 11760*a*ta
nh(x/2)**8 + 11760*a*tanh(x/2)**6 - 7056*a*tanh(x/2)**4 + 2352*a*tanh(x/2)*
*2 - 336*a)
```

Giac [A]
time = 0.41, size = 90, normalized size = 1.58

$$\frac{\left(105\,e^{(6\,x)}+315\,e^{(5\,x)}-63\,e^{(4\,x)}-63\,e^{(3\,x)}+21\,e^{(2\,x)}+7\,e^{x}-3\right)e^{(-7\,x)}+840\,x+3\,e^{(7\,x)}-7\,e^{(6\,x)}-21\,e^{(5\,x)}+63\,e^{(4\,x)}+63\,e^{(3\,x)}-315\,e^{(2\,x)}-105\,e^{x}}{2688\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^8/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] 1/2688*((105*e^(6*x) + 315*e^(5*x) - 63*e^(4*x) - 63*e^(3*x) + 21*e^(2*x) +
 7*e^x - 3)*e^(-7*x) + 840*x + 3*e^(7*x) - 7*e^(6*x) - 21*e^(5*x) + 63*e^(4
*x) + 63*e^(3*x) - 315*e^(2*x) - 105*e^x)/a
```

Mupad [B]
time = 1.26, size = 131, normalized size = 2.30

$$\frac{5\,{\mathrm{e}}^{-x}}{128\,a}+\frac{15\,{\mathrm{e}}^{-2\,x}}{128\,a}-\frac{15\,{\mathrm{e}}^{2\,x}}{128\,a}-\frac{3\,{\mathrm{e}}^{-3\,x}}{128\,a}+\frac{3\,{\mathrm{e}}^{3\,x}}{128\,a}-\frac{3\,{\mathrm{e}}^{-4\,x}}{128\,a}+\frac{3\,{\mathrm{e}}^{4\,x}}{128\,a}+\frac{{\mathrm{e}}^{-5\,x}}{128\,a}-\frac{{\mathrm{e}}^{5\,x}}{128\,a}+\frac{{\mathrm{e}}^{-6\,x}}{384\,a}-\frac{{\mathrm{e}}^{6\,x}}{384\,a}-\frac{{\mathrm{e}}^{-7\,x}}{896\,a}+\frac{{\mathrm{e}}^{7\,x}}{896\,a}+\frac{5\,x}{16\,a}-\frac{5\,{\mathrm{e}}^{x}}{128\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^8/(a + a*cosh(x)),x)
```

```
[Out] (5*exp(-x))/(128*a) + (15*exp(-2*x))/(128*a) - (15*exp(2*x))/(128*a) - (3*e
xp(-3*x))/(128*a) + (3*exp(3*x))/(128*a) - (3*exp(-4*x))/(128*a) + (3*exp(4
*x))/(128*a) + exp(-5*x)/(128*a) - exp(5*x)/(128*a) + exp(-6*x)/(384*a) - e
xp(6*x)/(384*a) - exp(-7*x)/(896*a) + exp(7*x)/(896*a) + (5*x)/(16*a) - (5*
exp(x))/(128*a)
```

3.153 $\int \frac{\sinh^7(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=46

$$\frac{(a-a\cosh(x))^4}{a^5}-\frac{4(a-a\cosh(x))^5}{5a^6}+\frac{(a-a\cosh(x))^6}{6a^7}$$

```
[Out] (a-a*cosh(x))^4/a^5-4/5*(a-a*cosh(x))^5/a^6+1/6*(a-a*cosh(x))^6/a^7
```

Rubi [A]
time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2746, 45}

$$\frac{(a-a\cosh(x))^6}{6a^7}-\frac{4(a-a\cosh(x))^5}{5a^6}+\frac{(a-a\cosh(x))^4}{a^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^7/(a + a*Cosh[x]),x]
```

```
[Out] (a - a*Cosh[x])^4/a^5 - (4*(a - a*Cosh[x])^5)/(5*a^6) + (a - a*Cosh[x])^6/(
6*a^7)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^7(x)}{a+a\cosh(x)}\,dx &= -\frac{\text{Subst}\left(\int (a-x)^3(a+x)^2\,dx,x,a\cosh(x)\right)}{a^7}\\ &= -\frac{\text{Subst}\left(\int \left(4a^2(a-x)^3-4a(a-x)^4+(a-x)^5\right)\,dx,x,a\cosh(x)\right)}{a^7}\\ &= \frac{(a-a\cosh(x))^4}{a^5}-\frac{4(a-a\cosh(x))^5}{5a^6}+\frac{(a-a\cosh(x))^6}{6a^7}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 27, normalized size = 0.59

$$\frac{4(27+28\cosh(x)+5\cosh(2x))\sinh^8\left(\frac{x}{2}\right)}{15a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^7/(a + a*Cosh[x]),x]
```

```
[Out] (4*(27 + 28*Cosh[x] + 5*Cosh[2*x])*Sinh[x/2]^8)/(15*a)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(42) = 84.
time = 0.49, size = 107, normalized size = 2.33

method	result
default	$\frac{\frac{1}{6\left(\tanh\left(\frac{x}{2}\right)-1\right)^{6}}+\frac{7}{10\left(\tanh\left(\frac{x}{2}\right)-1\right)^{5}}+\frac{7}{8\left(\tanh\left(\frac{x}{2}\right)-1\right)^{4}}-\frac{7}{16\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{7}{16\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{1}{6\left(\tanh\left(\frac{x}{2}\right)+1\right)^{6}}-\frac{7}{10\left(\tanh\left(\frac{x}{2}\right)+1\right)^{5}}+\frac{}{8(\tanh}}{a}$
risch	$\frac{e^{6x}}{384a}-\frac{e^{5x}}{160a}-\frac{e^{4x}}{64a}+\frac{5\,e^{3x}}{96a}+\frac{5\,e^{2x}}{128a}-\frac{5\,e^{x}}{16a}-\frac{5\,e^{-x}}{16a}+\frac{5\,e^{-2x}}{128a}+\frac{5\,e^{-3x}}{96a}-\frac{e^{-4x}}{64a}-\frac{e^{-5x}}{160a}+\frac{e^{-6x}}{384a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^7/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 128/a*(1/768/(tanh(1/2*x)-1)^6+7/1280/(tanh(1/2*x)-1)^5+7/1024/(tanh(1/2*x)
-1)^4-7/2048/(tanh(1/2*x)-1)^2+7/2048/(tanh(1/2*x)-1)+1/768/(tanh(1/2*x)+1)
^6-7/1280/(tanh(1/2*x)+1)^5+7/1024/(tanh(1/2*x)+1)^4-7/2048/(tanh(1/2*x)+1)
^2-7/2048/(tanh(1/2*x)+1))
```

Maxima [A]
time = 0.27, size = 84, normalized size = 1.83

$$-\frac{\left(12\,e^{(-x)}+30\,e^{(-2\,x)}-100\,e^{(-3\,x)}-75\,e^{(-4\,x)}+600\,e^{(-5\,x)}-5\right)e^{(6\,x)}}{1920\,a}-\frac{600\,e^{(-x)}-75\,e^{(-2\,x)}-100\,e^{(-3\,x)}+30\,e^{(-4\,x)}+12\,e^{(-5\,x)}-5\,e^{(-6\,x)}}{1920\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^7/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -1/1920*(12*e^(-x) + 30*e^(-2*x) - 100*e^(-3*x) - 75*e^(-4*x) + 600*e^(-5*x
) - 5)*e^(6*x)/a - 1/1920*(600*e^(-x) - 75*e^(-2*x) - 100*e^(-3*x) + 30*e^(
-4*x) + 12*e^(-5*x) - 5*e^(-6*x))/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(45) = 90.
time = 0.38, size = 94, normalized size = 2.04

$$\frac{5\,\cosh\left(x\right)^{6}+5\,\sinh\left(x\right)^{6}-12\,\cosh\left(x\right)^{5}+15\,\left(5\,\cosh\left(x\right)^{2}-4\,\cosh\left(x\right)-2\right)\sinh\left(x\right)^{4}-30\,\cosh\left(x\right)^{4}+100\,\cosh\left(x\right)^{3}+15\,\left(5\,\cosh\left(x\right)^{4}-8\,\cosh\left(x\right)^{3}-12\,\cosh\left(x\right)^{2}+20\,\cosh\left(x\right)+5\right)\sinh\left(x\right)^{2}+75\,\cosh\left(x\right)^{2}-600\,\cosh\left(x\right)}{960\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^7/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/960*(5*cosh(x)^6 + 5*sinh(x)^6 - 12*cosh(x)^5 + 15*(5*cosh(x)^2 - 4*cosh(
x) - 2)*sinh(x)^4 - 30*cosh(x)^4 + 100*cosh(x)^3 + 15*(5*cosh(x)^4 - 8*cosh
(x)^3 - 12*cosh(x)^2 + 20*cosh(x) + 5)*sinh(x)^2 + 75*cosh(x)^2 - 600*cosh(
x))/a
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(39) = 78$.
time = 2.31, size = 284, normalized size = 6.17

$$\frac{320\tanh^6\left(\frac{x}{2}\right)}{15a\tanh^{12}\left(\frac{x}{2}\right)-90a\tanh^{10}\left(\frac{x}{2}\right)+225a\tanh^{8}\left(\frac{x}{2}\right)-300a\tanh^{6}\left(\frac{x}{2}\right)+225a\tanh^{4}\left(\frac{x}{2}\right)-90a\tanh^{2}\left(\frac{x}{2}\right)+15a}-\frac{240\tanh^4\left(\frac{x}{2}\right)}{15a\tanh^{12}\left(\frac{x}{2}\right)-90a\tanh^{10}\left(\frac{x}{2}\right)+225a\tanh^{8}\left(\frac{x}{2}\right)-300a\tanh^{6}\left(\frac{x}{2}\right)+225a\tanh^{4}\left(\frac{x}{2}\right)-90a\tanh^{2}\left(\frac{x}{2}\right)+15a}+\frac{96\tanh^2\left(\frac{x}{2}\right)}{15a\tanh^{12}\left(\frac{x}{2}\right)-90a\tanh^{10}\left(\frac{x}{2}\right)+225a\tanh^{8}\left(\frac{x}{2}\right)-300a\tanh^{6}\left(\frac{x}{2}\right)+225a\tanh^{4}\left(\frac{x}{2}\right)-90a\tanh^{2}\left(\frac{x}{2}\right)+15a}-\frac{16}{15a\tanh^{12}\left(\frac{x}{2}\right)-90a\tanh^{10}\left(\frac{x}{2}\right)+225a\tanh^{8}\left(\frac{x}{2}\right)-300a\tanh^{6}\left(\frac{x}{2}\right)+225a\tanh^{4}\left(\frac{x}{2}\right)-90a\tanh^{2}\left(\frac{x}{2}\right)+15a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**7/(a+a*cosh(x)),x)
```

```
[Out] 320*tanh(x/2)**6/(15*a*tanh(x/2)**12 - 90*a*tanh(x/2)**10 + 225*a*tanh(x/2)
**8 - 300*a*tanh(x/2)**6 + 225*a*tanh(x/2)**4 - 90*a*tanh(x/2)**2 + 15*a) -
 240*tanh(x/2)**4/(15*a*tanh(x/2)**12 - 90*a*tanh(x/2)**10 + 225*a*tanh(x/2
)**8 - 300*a*tanh(x/2)**6 + 225*a*tanh(x/2)**4 - 90*a*tanh(x/2)**2 + 15*a)
+ 96*tanh(x/2)**2/(15*a*tanh(x/2)**12 - 90*a*tanh(x/2)**10 + 225*a*tanh(x/2
)**8 - 300*a*tanh(x/2)**6 + 225*a*tanh(x/2)**4 - 90*a*tanh(x/2)**2 + 15*a)
- 16/(15*a*tanh(x/2)**12 - 90*a*tanh(x/2)**10 + 225*a*tanh(x/2)**8 - 300*a*
tanh(x/2)**6 + 225*a*tanh(x/2)**4 - 90*a*tanh(x/2)**2 + 15*a)
```

Giac [A]
time = 0.40, size = 75, normalized size = 1.63

$$-\frac{\left(600\,e^{(5\,x)}-75\,e^{(4\,x)}-100\,e^{(3\,x)}+30\,e^{(2\,x)}+12\,e^{x}-5\right)e^{(-6\,x)}-5\,e^{(6\,x)}+12\,e^{(5\,x)}+30\,e^{(4\,x)}-100\,e^{(3\,x)}-75\,e^{(2\,x)}+600\,e^{x}}{1920\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^7/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -1/1920*((600*e^(5*x) - 75*e^(4*x) - 100*e^(3*x) + 30*e^(2*x) + 12*e^x - 5)
*e^(-6*x) - 5*e^(6*x) + 12*e^(5*x) + 30*e^(4*x) - 100*e^(3*x) - 75*e^(2*x)
+ 600*e^x)/a
```

Mupad [B]
time = 1.13, size = 107, normalized size = 2.33

$$\frac{5\,\mathrm{e}^{-2\,x}}{128\,a}-\frac{5\,\mathrm{e}^{-x}}{16\,a}+\frac{5\,\mathrm{e}^{2\,x}}{128\,a}+\frac{5\,\mathrm{e}^{-3\,x}}{96\,a}+\frac{5\,\mathrm{e}^{3\,x}}{96\,a}-\frac{\mathrm{e}^{-4\,x}}{64\,a}-\frac{\mathrm{e}^{4\,x}}{64\,a}-\frac{\mathrm{e}^{-5\,x}}{160\,a}-\frac{\mathrm{e}^{5\,x}}{160\,a}+\frac{\mathrm{e}^{-6\,x}}{384\,a}+\frac{\mathrm{e}^{6\,x}}{384\,a}-\frac{5\,\mathrm{e}^{x}}{16\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^7/(a + a*cosh(x)),x)
```

```
[Out] (5*exp(-2*x))/(128*a) - (5*exp(-x))/(16*a) + (5*exp(2*x))/(128*a) + (5*exp(
-3*x))/(96*a) + (5*exp(3*x))/(96*a) - exp(-4*x)/(64*a) - exp(4*x)/(64*a) -
exp(-5*x)/(160*a) - exp(5*x)/(160*a) + exp(-6*x)/(384*a) + exp(6*x)/(384*a)
 - (5*exp(x))/(16*a)
```

3.154 $\int \frac{\sinh^6(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=44

$$-\frac{3x}{8a}+\frac{3\cosh(x)\sinh(x)}{8a}-\frac{\cosh(x)\sinh^3(x)}{4a}+\frac{\sinh^5(x)}{5a}$$

```
[Out] -3/8*x/a+3/8*cosh(x)*sinh(x)/a-1/4*cosh(x)*sinh(x)^3/a+1/5*sinh(x)^5/a
```

Rubi [A]
time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2761, 2715, 8}

$$-\frac{3x}{8a}+\frac{\sinh^5(x)}{5a}-\frac{\sinh^3(x)\cosh(x)}{4a}+\frac{3\sinh(x)\cosh(x)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^6/(a + a*Cosh[x]),x]
```

```
[Out] (-3*x)/(8*a) + (3*Cosh[x]*Sinh[x])/(8*a) - (Cosh[x]*Sinh[x]^3)/(4*a) + Sinh
[x]^5/(5*a)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^6(x)}{a+a\cosh(x)}\,dx &= \frac{\sinh^5(x)}{5a}-\frac{\int \sinh^4(x)\,dx}{a}\\
&= -\frac{\cosh(x)\sinh^3(x)}{4a}+\frac{\sinh^5(x)}{5a}+\frac{3\int \sinh^2(x)\,dx}{4a}\\
&= \frac{3\cosh(x)\sinh(x)}{8a}-\frac{\cosh(x)\sinh^3(x)}{4a}+\frac{\sinh^5(x)}{5a}-\frac{3\int 1\,dx}{8a}\\
&= -\frac{3x}{8a}+\frac{3\cosh(x)\sinh(x)}{8a}-\frac{\cosh(x)\sinh^3(x)}{4a}+\frac{\sinh^5(x)}{5a}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 39, normalized size = 0.89

$$\frac{-60x+20\sinh(x)+40\sinh(2x)-10\sinh(3x)-5\sinh(4x)+2\sinh(5x)}{160a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^6/(a + a*Cosh[x]),x]
```

```
[Out] (-60*x + 20*Sinh[x] + 40*Sinh[2*x] - 10*Sinh[3*x] - 5*Sinh[4*x] + 2*Sinh[5*
x])/(160*a)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(36) = 72$.
time = 0.51, size = 125, normalized size = 2.84

method	result
risch	$-\frac{3x}{8a}+\frac{e^{5x}}{160a}-\frac{e^{4x}}{64a}-\frac{e^{3x}}{32a}+\frac{e^{2x}}{8a}+\frac{e^{x}}{16a}-\frac{e^{-x}}{16a}-\frac{e^{-2x}}{8a}+\frac{e^{-3x}}{32a}+\frac{e^{-4x}}{64a}-\frac{e^{-5x}}{160a}$
default	$\frac{-\frac{1}{5\left(\tanh\left(\frac{x}{2}\right)+1\right)^5}+\frac{3}{4\left(\tanh\left(\frac{x}{2}\right)+1\right)^4}-\frac{3}{4\left(\tanh\left(\frac{x}{2}\right)+1\right)^3}-\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}+\frac{3}{8\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{8}-\frac{1}{5\left(\tanh\left(\frac{x}{2}\right)-1\right)^5}-\cdots}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^6/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 64/a*(-1/320/(tanh(1/2*x)+1)^5+3/256/(tanh(1/2*x)+1)^4-3/256/(tanh(1/2*x)+1
)^3-1/256/(tanh(1/2*x)+1)^2+3/512/(tanh(1/2*x)+1)-3/512*ln(tanh(1/2*x)+1)-1
/320/(tanh(1/2*x)-1)^5-3/256/(tanh(1/2*x)-1)^4-3/256/(tanh(1/2*x)-1)^3+1/25
6/(tanh(1/2*x)-1)^2+3/512/(tanh(1/2*x)-1)+3/512*ln(tanh(1/2*x)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(36) = 72$.
time = 0.25, size = 78, normalized size = 1.77

$$-\frac{\left(5\,e^{(-x)}+10\,e^{(-2\,x)}-40\,e^{(-3\,x)}-20\,e^{(-4\,x)}-2\right)e^{(5\,x)}}{320\,a}-\frac{3\,x}{8\,a}-\frac{20\,e^{(-x)}+40\,e^{(-2\,x)}-10\,e^{(-3\,x)}-5\,e^{(-4\,x)}+2\,e^{(-5\,x)}}{320\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^6/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -1/320*(5*e^(-x) + 10*e^(-2*x) - 40*e^(-3*x) - 20*e^(-4*x) - 2)*e^(5*x)/a -
 3/8*x/a - 1/320*(20*e^(-x) + 40*e^(-2*x) - 10*e^(-3*x) - 5*e^(-4*x) + 2*e^
(-5*x))/a
```

Fricas [A]
time = 0.33, size = 57, normalized size = 1.30

$$\frac{\sinh\left(x\right)^{5}+5\left(2\,\cosh\left(x\right)^{2}-2\,\cosh\left(x\right)-1\right)\sinh\left(x\right)^{3}+5\left(\cosh\left(x\right)^{4}-2\,\cosh\left(x\right)^{3}-3\,\cosh\left(x\right)^{2}+8\,\cosh\left(x\right)+2\right)\sinh\left(x\right)-30\,x}{80\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^6/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/80*(sinh(x)^5 + 5*(2*cosh(x)^2 - 2*cosh(x) - 1)*sinh(x)^3 + 5*(cosh(x)^4
- 2*cosh(x)^3 - 3*cosh(x)^2 + 8*cosh(x) + 2)*sinh(x) - 30*x)/a
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 692 vs. $2(37) = 74$.
time = 1.39, size = 692, normalized size = 15.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**6/(a+a*cosh(x)),x)
```

```
[Out] -15*x*tanh(x/2)**10/(40*a*tanh(x/2)**10 - 200*a*tanh(x/2)**8 + 400*a*tanh(x
/2)**6 - 400*a*tanh(x/2)**4 + 200*a*tanh(x/2)**2 - 40*a) + 75*x*tanh(x/2)**
8/(40*a*tanh(x/2)**10 - 200*a*tanh(x/2)**8 + 400*a*tanh(x/2)**6 - 400*a*tan
h(x/2)**4 + 200*a*tanh(x/2)**2 - 40*a) - 150*x*tanh(x/2)**6/(40*a*tanh(x/2)
**10 - 200*a*tanh(x/2)**8 + 400*a*tanh(x/2)**6 - 400*a*tanh(x/2)**4 + 200*a
*tanh(x/2)**2 - 40*a) + 150*x*tanh(x/2)**4/(40*a*tanh(x/2)**10 - 200*a*tanh
(x/2)**8 + 400*a*tanh(x/2)**6 - 400*a*tanh(x/2)**4 + 200*a*tanh(x/2)**2 - 4
0*a) - 75*x*tanh(x/2)**2/(40*a*tanh(x/2)**10 - 200*a*tanh(x/2)**8 + 400*a*t
anh(x/2)**6 - 400*a*tanh(x/2)**4 + 200*a*tanh(x/2)**2 - 40*a) + 15*x/(40*a*
tanh(x/2)**10 - 200*a*tanh(x/2)**8 + 400*a*tanh(x/2)**6 - 400*a*tanh(x/2)**
4 + 200*a*tanh(x/2)**2 - 40*a) + 30*tanh(x/2)**9/(40*a*tanh(x/2)**10 - 200*
a*tanh(x/2)**8 + 400*a*tanh(x/2)**6 - 400*a*tanh(x/2)**4 + 200*a*tanh(x/2)*
*2 - 40*a) - 140*tanh(x/2)**7/(40*a*tanh(x/2)**10 - 200*a*tanh(x/2)**8 + 40
0*a*tanh(x/2)**6 - 400*a*tanh(x/2)**4 + 200*a*tanh(x/2)**2 - 40*a) - 256*ta
nh(x/2)**5/(40*a*tanh(x/2)**10 - 200*a*tanh(x/2)**8 + 400*a*tanh(x/2)**6 -
400*a*tanh(x/2)**4 + 200*a*tanh(x/2)**2 - 40*a) + 140*tanh(x/2)**3/(40*a*ta
nh(x/2)**10 - 200*a*tanh(x/2)**8 + 400*a*tanh(x/2)**6 - 400*a*tanh(x/2)**4
+ 200*a*tanh(x/2)**2 - 40*a) - 30*tanh(x/2)/(40*a*tanh(x/2)**10 - 200*a*tan
```

```
h(x/2)**8 + 400*a*tanh(x/2)**6 - 400*a*tanh(x/2)**4 + 200*a*tanh(x/2)**2 -
40*a)
```

Giac [A]
time = 0.41, size = 66, normalized size = 1.50

$$-\frac{\left(20\,e^{(4\,x)}+40\,e^{(3\,x)}-10\,e^{(2\,x)}-5\,e^{x}+2\right)e^{(-5\,x)}+120\,x-2\,e^{(5\,x)}+5\,e^{(4\,x)}+10\,e^{(3\,x)}-40\,e^{(2\,x)}-20\,e^{x}}{320\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^6/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -1/320*((20*e^(4*x) + 40*e^(3*x) - 10*e^(2*x) - 5*e^x + 2)*e^(-5*x) + 120*x
 - 2*e^(5*x) + 5*e^(4*x) + 10*e^(3*x) - 40*e^(2*x) - 20*e^x)/a
```

Mupad [B]
time = 1.05, size = 95, normalized size = 2.16

$$\frac{\mathrm{e}^{2\,x}}{8\,a}-\frac{\mathrm{e}^{-2\,x}}{8\,a}-\frac{\mathrm{e}^{-x}}{16\,a}+\frac{\mathrm{e}^{-3\,x}}{32\,a}-\frac{\mathrm{e}^{3\,x}}{32\,a}+\frac{\mathrm{e}^{-4\,x}}{64\,a}-\frac{\mathrm{e}^{4\,x}}{64\,a}-\frac{\mathrm{e}^{-5\,x}}{160\,a}+\frac{\mathrm{e}^{5\,x}}{160\,a}-\frac{3\,x}{8\,a}+\frac{\mathrm{e}^{x}}{16\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^6/(a + a*cosh(x)),x)
```

```
[Out] exp(2*x)/(8*a) - exp(-2*x)/(8*a) - exp(-x)/(16*a) + exp(-3*x)/(32*a) - exp(
3*x)/(32*a) + exp(-4*x)/(64*a) - exp(4*x)/(64*a) - exp(-5*x)/(160*a) + exp(
5*x)/(160*a) - (3*x)/(8*a) + exp(x)/(16*a)
```

3.155 $\int \frac{\sinh^5(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=33

$$-\frac{2(a-a\cosh(x))^3}{3a^4}+\frac{(a-a\cosh(x))^4}{4a^5}$$

```
[Out] -2/3*(a-a*cosh(x))^3/a^4+1/4*(a-a*cosh(x))^4/a^5
```

Rubi [A]
time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2746, 45}

$$\frac{(a-a\cosh(x))^4}{4a^5}-\frac{2(a-a\cosh(x))^3}{3a^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^5/(a + a*Cosh[x]),x]
```

```
[Out] (-2*(a - a*Cosh[x])^3)/(3*a^4) + (a - a*Cosh[x])^4/(4*a^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^5(x)}{a+a\cosh(x)}\,dx &= \frac{\operatorname{Subst}\left(\int (a-x)^2(a+x)\,dx, x, a\cosh(x)\right)}{a^5}\\
&= \frac{\operatorname{Subst}\left(\int \left(2a(a-x)^2-(a-x)^3\right)\,dx, x, a\cosh(x)\right)}{a^5}\\
&= -\frac{2(a-a\cosh(x))^3}{3a^4}+\frac{(a-a\cosh(x))^4}{4a^5}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 21, normalized size = 0.64

$$\frac{2(5+3\cosh(x))\sinh^6\left(\frac{x}{2}\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^5/(a + a*Cosh[x]),x]
```

```
[Out] (2*(5 + 3*Cosh[x])*Sinh[x/2]^6)/(3*a)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(29) = 58.
time = 0.48, size = 87, normalized size = 2.64

method	result
risch	$\frac{e^{4x}}{64a}-\frac{e^{3x}}{24a}-\frac{e^{2x}}{16a}+\frac{3\,e^{x}}{8a}+\frac{3\,e^{-x}}{8a}-\frac{e^{-2x}}{16a}-\frac{e^{-3x}}{24a}+\frac{e^{-4x}}{64a}$
default	$\frac{\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)-1\right)^4}+\frac{5}{6\left(\tanh\left(\frac{x}{2}\right)-1\right)^3}+\frac{5}{8\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}-\frac{5}{8\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)+1\right)^4}-\frac{5}{6\left(\tanh\left(\frac{x}{2}\right)+1\right)^3}+\frac{5}{8\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}+\frac{5}{8\left(\tanh\left(\frac{x}{2}\right)\right.}}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^5/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 32/a*(1/128/(tanh(1/2*x)-1)^4+5/192/(tanh(1/2*x)-1)^3+5/256/(tanh(1/2*x)-1)
^2-5/256/(tanh(1/2*x)-1)+1/128/(tanh(1/2*x)+1)^4-5/192/(tanh(1/2*x)+1)^3+5/
256/(tanh(1/2*x)+1)^2+5/256/(tanh(1/2*x)+1))
```

Maxima [A]
time = 0.26, size = 60, normalized size = 1.82

$$-\frac{\left(8\,e^{(-x)}+12\,e^{(-2\,x)}-72\,e^{(-3\,x)}-3\right)e^{(4\,x)}}{192\,a}+\frac{72\,e^{(-x)}-12\,e^{(-2\,x)}-8\,e^{(-3\,x)}+3\,e^{(-4\,x)}}{192\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^5/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -1/192*(8*e^(-x) + 12*e^(-2*x) - 72*e^(-3*x) - 3)*e^(4*x)/a + 1/192*(72*e^(
-x) - 12*e^(-2*x) - 8*e^(-3*x) + 3*e^(-4*x))/a
```

Fricas [A]
time = 0.38, size = 52, normalized size = 1.58

$$\frac{3\,\cosh\left(x\right)^4+3\,\sinh\left(x\right)^4-8\,\cosh\left(x\right)^3+6\,\left(3\,\cosh\left(x\right)^2-4\,\cosh\left(x\right)-2\right)\sinh\left(x\right)^2-12\,\cosh\left(x\right)^2+72\,\cosh\left(x\right)}{96\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^5/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/96*(3*cosh(x)^4 + 3*sinh(x)^4 - 8*cosh(x)^3 + 6*(3*cosh(x)^2 - 4*cosh(x)
- 2)*sinh(x)^2 - 12*cosh(x)^2 + 72*cosh(x))/a
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(27) = 54$.
time = 0.89, size = 150, normalized size = 4.55

$$\frac{24\tanh^4\left(\frac{x}{2}\right)}{3a\tanh^8\left(\frac{x}{2}\right)-12a\tanh^6\left(\frac{x}{2}\right)+18a\tanh^4\left(\frac{x}{2}\right)-12a\tanh^2\left(\frac{x}{2}\right)+3a}-\frac{16\tanh^2\left(\frac{x}{2}\right)}{3a\tanh^8\left(\frac{x}{2}\right)-12a\tanh^6\left(\frac{x}{2}\right)+18a\tanh^4\left(\frac{x}{2}\right)-12a\tanh^2\left(\frac{x}{2}\right)+3a}+\frac{4}{3a\tanh^8\left(\frac{x}{2}\right)-12a\tanh^6\left(\frac{x}{2}\right)+18a\tanh^4\left(\frac{x}{2}\right)-12a\tanh^2\left(\frac{x}{2}\right)+3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**5/(a+a*cosh(x)),x)
```

```
[Out] 24*tanh(x/2)**4/(3*a*tanh(x/2)**8 - 12*a*tanh(x/2)**6 + 18*a*tanh(x/2)**4 -
 12*a*tanh(x/2)**2 + 3*a) - 16*tanh(x/2)**2/(3*a*tanh(x/2)**8 - 12*a*tanh(x
/2)**6 + 18*a*tanh(x/2)**4 - 12*a*tanh(x/2)**2 + 3*a) + 4/(3*a*tanh(x/2)**8
 - 12*a*tanh(x/2)**6 + 18*a*tanh(x/2)**4 - 12*a*tanh(x/2)**2 + 3*a)
```

Giac [A]
time = 0.40, size = 51, normalized size = 1.55

$$\frac{\left(72\,e^{(3\,x)}-12\,e^{(2\,x)}-8\,e^{x}+3\right)e^{(-4\,x)}+3\,e^{(4\,x)}-8\,e^{(3\,x)}-12\,e^{(2\,x)}+72\,e^{x}}{192\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^5/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] 1/192*((72*e^(3*x) - 12*e^(2*x) - 8*e^x + 3)*e^(-4*x) + 3*e^(4*x) - 8*e^(3*
x) - 12*e^(2*x) + 72*e^x)/a
```

Mupad [B]
time = 0.98, size = 71, normalized size = 2.15

$$\frac{3\,\mathrm{e}^{-x}}{8\,a}-\frac{\mathrm{e}^{-2\,x}}{16\,a}-\frac{\mathrm{e}^{2\,x}}{16\,a}-\frac{\mathrm{e}^{-3\,x}}{24\,a}-\frac{\mathrm{e}^{3\,x}}{24\,a}+\frac{\mathrm{e}^{-4\,x}}{64\,a}+\frac{\mathrm{e}^{4\,x}}{64\,a}+\frac{3\,\mathrm{e}^{x}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^5/(a + a*cosh(x)),x)
```

```
[Out] (3*exp(-x))/(8*a) - exp(-2*x)/(16*a) - exp(2*x)/(16*a) - exp(-3*x)/(24*a) -
 exp(3*x)/(24*a) + exp(-4*x)/(64*a) + exp(4*x)/(64*a) + (3*exp(x))/(8*a)
```

3.156 $\int \frac{\sinh^4(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=31

$$\frac{x}{2a} - \frac{\cosh(x)\sinh(x)}{2a} + \frac{\sinh^3(x)}{3a}$$

```
[Out] 1/2*x/a-1/2*cosh(x)*sinh(x)/a+1/3*sinh(x)^3/a
```

Rubi [A]

time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2761, 2715, 8}

$$\frac{x}{2a} + \frac{\sinh^3(x)}{3a} - \frac{\sinh(x)\cosh(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^4/(a + a*Cosh[x]),x]
```

```
[Out] x/(2*a) - (Cosh[x]*Sinh[x])/(2*a) + Sinh[x]^3/(3*a)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\sinh^4(x)}{a+a\cosh(x)}\,dx = \frac{\sinh^3(x)}{3a} - \frac{\int \sinh^2(x)\,dx}{a}$$
$$= -\frac{\cosh(x)\sinh(x)}{2a} + \frac{\sinh^3(x)}{3a} + \frac{\int 1\,dx}{2a}$$
$$= \frac{x}{2a} - \frac{\cosh(x)\sinh(x)}{2a} + \frac{\sinh^3(x)}{3a}$$

Mathematica [A]
time = 0.03, size = 25, normalized size = 0.81

$$\frac{6x - 3\sinh(x) - 3\sinh(2x) + \sinh(3x)}{12a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^4/(a + a*Cosh[x]),x]
```

```
[Out] (6*x - 3*Sinh[x] - 3*Sinh[2*x] + Sinh[3*x])/(12*a)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(25) = 50.
time = 0.50, size = 85, normalized size = 2.74

method	result
risch	$\frac{x}{2a}+\frac{e^{3x}}{24a}-\frac{e^{2x}}{8a}-\frac{e^{x}}{8a}+\frac{e^{-x}}{8a}+\frac{e^{-2x}}{8a}-\frac{e^{-3x}}{24a}$
default	$\frac{-\frac{1}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^3}+\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2}-\frac{1}{3\left(\tanh\left(\frac{x}{2}\right)-1\right)^3}-\frac{1}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right.\right.}{2}}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 16/a*(-1/48/(tanh(1/2*x)+1)^3+1/16/(tanh(1/2*x)+1)^2-1/32/(tanh(1/2*x)+1)+1
/32*ln(tanh(1/2*x)+1)-1/48/(tanh(1/2*x)-1)^3-1/16/(tanh(1/2*x)-1)^2-1/32/(t
anh(1/2*x)-1)-1/32*ln(tanh(1/2*x)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(25) = 50.
time = 0.27, size = 54, normalized size = 1.74

$$-\frac{\left(3\,e^{(-x)}+3\,e^{(-2\,x)}-1\right)e^{(3\,x)}}{24\,a}+\frac{x}{2\,a}+\frac{3\,e^{(-x)}+3\,e^{(-2\,x)}-e^{(-3\,x)}}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -1/24*(3*e^(-x) + 3*e^(-2*x) - 1)*e^(3*x)/a + 1/2*x/a + 1/24*(3*e^(-x) + 3*
e^(-2*x) - e^(-3*x))/a
```

Fricas [A]
time = 0.39, size = 27, normalized size = 0.87

$$\frac{\sinh\left(x\right)^{3} + 3\left(\cosh\left(x\right)^{2} - 2\,\cosh\left(x\right) - 1\right)\sinh\left(x\right) + 6\,x}{12\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/12*(sinh(x)^3 + 3*(cosh(x)^2 - 2*cosh(x) - 1)*sinh(x) + 6*x)/a
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(22) = 44.
time = 0.54, size = 294, normalized size = 9.48

$$\frac{3x\tanh^{6}\left(\frac{x}{2}\right)}{6a\tanh^{6}\left(\frac{x}{2}\right) - 18a\tanh^{4}\left(\frac{x}{2}\right) + 18a\tanh^{2}\left(\frac{x}{2}\right) - 6a} - \frac{9x\tanh^{4}\left(\frac{x}{2}\right)}{6a\tanh^{6}\left(\frac{x}{2}\right) - 18a\tanh^{4}\left(\frac{x}{2}\right) + 18a\tanh^{2}\left(\frac{x}{2}\right) - 6a} + \frac{9x\tanh^{2}\left(\frac{x}{2}\right)}{6a\tanh^{6}\left(\frac{x}{2}\right) - 18a\tanh^{4}\left(\frac{x}{2}\right) + 18a\tanh^{2}\left(\frac{x}{2}\right) - 6a} - \frac{3x}{6a\tanh^{6}\left(\frac{x}{2}\right) - 18a\tanh^{4}\left(\frac{x}{2}\right) + 18a\tanh^{2}\left(\frac{x}{2}\right) - 6a} - \frac{6\tanh^{5}\left(\frac{x}{2}\right)}{6a\tanh^{6}\left(\frac{x}{2}\right) - 18a\tanh^{4}\left(\frac{x}{2}\right) + 18a\tanh^{2}\left(\frac{x}{2}\right) - 6a} - \frac{16\tanh^{3}\left(\frac{x}{2}\right)}{6a\tanh^{6}\left(\frac{x}{2}\right) - 18a\tanh^{4}\left(\frac{x}{2}\right) + 18a\tanh^{2}\left(\frac{x}{2}\right) - 6a} + \frac{6\tanh\left(\frac{x}{2}\right)}{6a\tanh^{6}\left(\frac{x}{2}\right) - 18a\tanh^{4}\left(\frac{x}{2}\right) + 18a\tanh^{2}\left(\frac{x}{2}\right) - 6a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**4/(a+a*cosh(x)),x)
```

```
[Out] 3*x*tanh(x/2)**6/(6*a*tanh(x/2)**6 - 18*a*tanh(x/2)**4 + 18*a*tanh(x/2)**2
- 6*a) - 9*x*tanh(x/2)**4/(6*a*tanh(x/2)**6 - 18*a*tanh(x/2)**4 + 18*a*tanh
(x/2)**2 - 6*a) + 9*x*tanh(x/2)**2/(6*a*tanh(x/2)**6 - 18*a*tanh(x/2)**4 +
18*a*tanh(x/2)**2 - 6*a) - 3*x/(6*a*tanh(x/2)**6 - 18*a*tanh(x/2)**4 + 18*a
*tanh(x/2)**2 - 6*a) - 6*tanh(x/2)**5/(6*a*tanh(x/2)**6 - 18*a*tanh(x/2)**4
 + 18*a*tanh(x/2)**2 - 6*a) - 16*tanh(x/2)**3/(6*a*tanh(x/2)**6 - 18*a*tanh
(x/2)**4 + 18*a*tanh(x/2)**2 - 6*a) + 6*tanh(x/2)/(6*a*tanh(x/2)**6 - 18*a*
tanh(x/2)**4 + 18*a*tanh(x/2)**2 - 6*a)
```

Giac [A]
time = 0.41, size = 40, normalized size = 1.29

$$\frac{\left(3\,e^{\left(2\,x\right)} + 3\,e^{x} - 1\right)e^{\left(-3\,x\right)} + 12\,x + e^{\left(3\,x\right)} - 3\,e^{\left(2\,x\right)} - 3\,e^{x}}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] 1/24*((3*e^(2*x) + 3*e^x - 1)*e^(-3*x) + 12*x + e^(3*x) - 3*e^(2*x) - 3*e^x
)/a
```

Mupad [B]
time = 0.94, size = 59, normalized size = 1.90

$$\frac{e^{-x}}{8\,a}+\frac{e^{-2\,x}}{8\,a}-\frac{e^{2\,x}}{8\,a}-\frac{e^{-3\,x}}{24\,a}+\frac{e^{3\,x}}{24\,a}+\frac{x}{2\,a}-\frac{e^{x}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(a + a*cosh(x)),x)
```

```
[Out] exp(-x)/(8*a) + exp(-2*x)/(8*a) - exp(2*x)/(8*a) - exp(-3*x)/(24*a) + exp(3
*x)/(24*a) + x/(2*a) - exp(x)/(8*a)
```

3.157 $\int \frac{\sinh^3(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=19

$$-\frac{\cosh(x)}{a}+\frac{\cosh^2(x)}{2a}$$

[Out] -cosh(x)/a+1/2*cosh(x)^2/a

Rubi [A]
time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2746}

$$\frac{\cosh^2(x)}{2a}-\frac{\cosh(x)}{a}$$

Antiderivative was successfully verified.

[In] Int[Sinh[x]^3/(a + a*Cosh[x]),x]

[Out] -(Cosh[x]/a) + Cosh[x]^2/(2*a)

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^3(x)}{a+a\cosh(x)}\,dx &= -\frac{\text{Subst}\left(\int (a-x)\,dx, x, a\cosh(x)\right)}{a^3}\\ &= -\frac{\cosh(x)}{a}+\frac{\cosh^2(x)}{2a}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 0.68

$$\frac{2\sinh^4\left(\frac{x}{2}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(a + a*Cosh[x]),x]
```

```
[Out] (2*Sinh[x/2]^4)/a
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(17) = 34$.
time = 0.47, size = 47, normalized size = 2.47

method	result	size
risch	$\frac{e^{2x}}{8a} - \frac{e^{x}}{2a} - \frac{e^{-x}}{2a} + \frac{e^{-2x}}{8a}$	36
default	$\frac{\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}} - \frac{3}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)} + \frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}} + \frac{3}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)}}{a}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 8/a*(1/16/(tanh(1/2*x)+1)^2-3/16/(tanh(1/2*x)+1)+1/16/(tanh(1/2*x)-1)^2+3/1
6/(tanh(1/2*x)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(17) = 34$.
time = 0.26, size = 36, normalized size = 1.89

$$-\frac{\left(4\,e^{(-x)} - 1\right)e^{(2\,x)}}{8\,a} - \frac{4\,e^{(-x)} - e^{(-2\,x)}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*e^(-x) - 1)*e^(2*x)/a - 1/8*(4*e^(-x) - e^(-2*x))/a
```

Fricas [A]
time = 0.37, size = 18, normalized size = 0.95

$$\frac{\cosh\left(x\right)^{2} + \sinh\left(x\right)^{2} - 4\,\cosh\left(x\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/4*(cosh(x)^2 + sinh(x)^2 - 4*cosh(x))/a
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(12) = 24$.
time = 0.30, size = 49, normalized size = 2.58

$$\frac{4\tanh^{2}\left(\frac{x}{2}\right)}{a\tanh^{4}\left(\frac{x}{2}\right) - 2a\tanh^{2}\left(\frac{x}{2}\right) + a} - \frac{2}{a\tanh^{4}\left(\frac{x}{2}\right) - 2a\tanh^{2}\left(\frac{x}{2}\right) + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(a+a*cosh(x)),x)
```

```
[Out] 4*tanh(x/2)**2/(a*tanh(x/2)**4 - 2*a*tanh(x/2)**2 + a) - 2/(a*tanh(x/2)**4
- 2*a*tanh(x/2)**2 + a)
```

Giac [A]
time = 0.41, size = 27, normalized size = 1.42

$$-\frac{(4\,e^{x}-1)e^{(-2\,x)}-e^{(2\,x)}+4\,e^{x}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -1/8*((4*e^x - 1)*e^(-2*x) - e^(2*x) + 4*e^x)/a
```

Mupad [B]
time = 0.94, size = 35, normalized size = 1.84

$$\frac{\mathrm{e}^{-2\,x}}{8\,a}-\frac{\mathrm{e}^{-x}}{2\,a}+\frac{\mathrm{e}^{2\,x}}{8\,a}-\frac{\mathrm{e}^{x}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a + a*cosh(x)),x)
```

```
[Out] exp(-2*x)/(8*a) - exp(-x)/(2*a) + exp(2*x)/(8*a) - exp(x)/(2*a)
```

3.158 $\int \frac{\sinh^2(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=13

$$-\frac{x}{a}+\frac{\sinh(x)}{a}$$

```
[Out] -x/a+sinh(x)/a
```

Rubi [A]
time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2761, 8}

$$\frac{\sinh(x)}{a}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(a + a*Cosh[x]),x]
```

```
[Out] -(x/a) + Sinh[x]/a
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^2(x)}{a+a\cosh(x)}\,dx &= \frac{\sinh(x)}{a}-\frac{\int 1\,dx}{a}\\ &= -\frac{x}{a}+\frac{\sinh(x)}{a}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 1.31

$$\frac{2\left(-\frac{x}{2}+\frac{\sinh(x)}{2}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(a + a*Cosh[x]),x]
```

```
[Out] (2*(-1/2*x + Sinh[x]/2))/a
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(13) = 26$.
time = 0.48, size = 45, normalized size = 3.46

method	result	size
risch	$-\frac{x}{a}+\frac{e^{x}}{2a}-\frac{e^{-x}}{2a}$	24
default	$\frac{-\frac{1}{\tanh\left(\frac{x}{2}\right)-1}+\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\frac{1}{\tanh\left(\frac{x}{2}\right)+1}-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 4/a*(-1/4/(tanh(1/2*x)-1)+1/4*ln(tanh(1/2*x)-1)-1/4/(tanh(1/2*x)+1)-1/4*ln(
tanh(1/2*x)+1))
```

Maxima [A]
time = 0.29, size = 23, normalized size = 1.77

$$-\frac{x}{a}-\frac{e^{(-x)}}{2\,a}+\frac{e^{x}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -x/a - 1/2*e^(-x)/a + 1/2*e^x/a
```

Fricas [A]
time = 0.42, size = 11, normalized size = 0.85

$$-\frac{x-\sinh\left(x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] -(x - sinh(x))/a
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(7) = 14$.
time = 0.17, size = 46, normalized size = 3.54

$$-\frac{x\tanh^{2}\left(\frac{x}{2}\right)}{a\tanh^{2}\left(\frac{x}{2}\right)-a}+\frac{x}{a\tanh^{2}\left(\frac{x}{2}\right)-a}-\frac{2\tanh\left(\frac{x}{2}\right)}{a\tanh^{2}\left(\frac{x}{2}\right)-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(a+a*cosh(x)),x)
```

```
[Out] -x*tanh(x/2)**2/(a*tanh(x/2)**2 - a) + x/(a*tanh(x/2)**2 - a) - 2*tanh(x/2)
/(a*tanh(x/2)**2 - a)
```

Giac [A]
time = 0.41, size = 17, normalized size = 1.31

$$-\frac{2\,x + e^{(-x)} - e^{x}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -1/2*(2*x + e^(-x) - e^x)/a
```

Mupad [B]
time = 0.91, size = 23, normalized size = 1.77

$$\frac{\mathrm{e}^{x}}{2\,a} - \frac{x}{a} - \frac{\mathrm{e}^{-x}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a + a*cosh(x)),x)
```

```
[Out] exp(x)/(2*a) - x/a - exp(-x)/(2*a)
```

3.159 $\int \frac{\sinh(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=9

$$\frac{\log(1+\cosh(x))}{a}$$

```
[Out] ln(1+cosh(x))/a
```

Rubi [A]
time = 0.02, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2746, 31}

$$\frac{\log(\cosh(x)+1)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(a + a*Cosh[x]),x]
```

```
[Out] Log[1 + Cosh[x]]/a
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(x)}{a+a\cosh(x)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, a\cosh(x)\right)}{a}\\ &= \frac{\log(1+\cosh(x))}{a}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 1.33

$$\frac{2\log\left(\cosh\left(\frac{x}{2}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(a + a*Cosh[x]),x]
```

```
[Out] (2*Log[Cosh[x/2]])/a
```

Maple [A]
time = 0.32, size = 12, normalized size = 1.33

method	result	size
derivativedivides	$\frac{\ln(a+a\cosh(x))}{a}$	12
default	$\frac{\ln(a+a\cosh(x))}{a}$	12
risch	$-\frac{x}{a}+\frac{2\ln(e^x+1)}{a}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(a+a*cosh(x))/a
```

Maxima [A]
time = 0.29, size = 11, normalized size = 1.22

$$\frac{\log\left(a\cosh\left(x\right)+a\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] log(a*cosh(x) + a)/a
```

Fricas [A]
time = 0.43, size = 16, normalized size = 1.78

$$-\frac{x-2\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] -(x - 2*log(cosh(x) + sinh(x) + 1))/a
```

Sympy [A]
time = 0.05, size = 7, normalized size = 0.78

$$\frac{\log\left(\cosh\left(x\right)+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+a*cosh(x)),x)
```

```
[Out] log(cosh(x) + 1)/a
```

Giac [A]
time = 0.42, size = 17, normalized size = 1.89

$$-\frac{x}{a}+\frac{2\,\log\left(e^{x}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -x/a + 2*log(e^x + 1)/a
```

Mupad [B]
time = 0.89, size = 9, normalized size = 1.00

$$\frac{\ln\left(\cosh(x)+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a + a*cosh(x)),x)
```

```
[Out] log(cosh(x) + 1)/a
```

3.160 $\int \frac{\operatorname{csch}(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=23

$$-\frac{\tanh^{-1}(\cosh(x))}{2a}+\frac{1}{2(a+a\cosh(x))}$$

```
[Out] -1/2*arctanh(cosh(x))/a+1/2/(a+a*cosh(x))
```

Rubi [A]
time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2746, 46, 212}

$$\frac{1}{2(a\cosh(x)+a)}-\frac{\tanh^{-1}(\cosh(x))}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]/(a + a*Cosh[x]),x]
```

```
[Out] -1/2*ArcTanh[Cosh[x]]/a + 1/(2*(a + a*Cosh[x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\text{csch}(x)}{a+a\cosh(x)}\,dx = -\left(a\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^2}\,dx, x, a\cosh(x)\right)\right)$$
$$= -\left(a\text{Subst}\left(\int \left(\frac{1}{2a(a+x)^2}+\frac{1}{2a\,(a^2-x^2)}\right)dx, x, a\cosh(x)\right)\right)$$
$$= \frac{1}{2(a+a\cosh(x))}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, a\cosh(x)\right)$$
$$= -\frac{\tanh^{-1}(\cosh(x))}{2a}+\frac{1}{2(a+a\cosh(x))}$$

Mathematica [A]
time = 0.02, size = 42, normalized size = 1.83

$$\frac{1-2\cosh^2\left(\frac{x}{2}\right)\left(\log\left(\cosh\left(\frac{x}{2}\right)\right)-\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)}{2a(1+\cosh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]/(a + a*Cosh[x]),x]
```

```
[Out] (1 - 2*Cosh[x/2]^2*(Log[Cosh[x/2]] - Log[Sinh[x/2]]))/(2*a*(1 + Cosh[x]))
```

Maple [A]
time = 0.52, size = 20, normalized size = 0.87

method	result	size
default	$\frac{-\frac{\left(\tanh^2\left(\frac{x}{2}\right)\right)}{2}+\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2a}$	20
risch	$\frac{e^x}{(e^x+1)^2 a}-\frac{\ln(e^x+1)}{2a}+\frac{\ln(e^x-1)}{2a}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a*(-1/2*tanh(1/2*x)^2+ln(tanh(1/2*x)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(19) = 38$.
time = 0.26, size = 47, normalized size = 2.04

$$\frac{e^{(-x)}}{2\,ae^{(-x)}+ae^{(-2\,x)}+a}-\frac{\log\left(e^{(-x)}+1\right)}{2\,a}+\frac{\log\left(e^{(-x)}-1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] e^(-x)/(2*a*e^(-x) + a*e^(-2*x) + a) - 1/2*log(e^(-x) + 1)/a + 1/2*log(e^(-
x) - 1)/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(19) = 38$.
time = 0.34, size = 103, normalized size = 4.48

$$-\frac{(\cosh(x)^2+2(\cosh(x)+1)\sinh(x)+\sinh(x)^2+2\cosh(x)+1)\log(\cosh(x)+\sinh(x)+1)-(\cosh(x)^2+2(\cosh(x)+1)\sinh(x)+\sinh(x)^2+2\cosh(x)+1)\log(\cosh(x)+\sinh(x)-1)-2\cosh(x)-2\sinh(x)}{2\left(a\cosh(x)^2+a\sinh(x)^2+2a\cosh(x)+2(a\cosh(x)+a)\sinh(x)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] -1/2*((cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)*log
(cosh(x) + sinh(x) + 1) - (cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2
+ 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) - 2*cosh(x) - 2*sinh(x))/(a*cos
h(x)^2 + a*sinh(x)^2 + 2*a*cosh(x) + 2*(a*cosh(x) + a)*sinh(x) + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{csch}(x)}{\cosh(x)+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+a*cosh(x)),x)
```

```
[Out] Integral(csch(x)/(cosh(x) + 1), x)/a
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(19) = 38$.
time = 0.42, size = 52, normalized size = 2.26

$$-\frac{\log\left(e^{(-x)}+e^x+2\right)}{4\,a}+\frac{\log\left(e^{(-x)}+e^x-2\right)}{4\,a}+\frac{e^{(-x)}+e^x+6}{4\,a(e^{(-x)}+e^x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -1/4*log(e^(-x) + e^x + 2)/a + 1/4*log(e^(-x) + e^x - 2)/a + 1/4*(e^(-x) +
e^x + 6)/(a*(e^(-x) + e^x + 2))
```

Mupad [B]
time = 0.93, size = 51, normalized size = 2.22

$$\frac{1}{a\,\left(\mathrm{e}^x+1\right)}-\frac{1}{a\,\left(\mathrm{e}^{2\,x}+2\,\mathrm{e}^x+1\right)}-\frac{\mathrm{atan}\left(\frac{\mathrm{e}^x\,\sqrt{-a^2}}{a}\right)}{\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)*(a + a*cosh(x))),x)
```

```
[Out] 1/(a*(exp(x) + 1)) - 1/(a*(exp(2*x) + 2*exp(x) + 1)) - atan((exp(x)*(-a^2)^
(1/2))/a)/(-a^2)^(1/2)
```

3.161 $\int \frac{\operatorname{csch}^2(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=24

$$-\frac{2\coth(x)}{3a}+\frac{\operatorname{csch}(x)}{3(a+a\cosh(x))}$$

```
[Out] -2/3*coth(x)/a+1/3*csch(x)/(a+a*cosh(x))
```

Rubi [A]

time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.231$, Rules used = {2751, 3852, 8}

$$\frac{\operatorname{csch}(x)}{3(a\cosh(x)+a)}-\frac{2\coth(x)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^2/(a + a*Cosh[x]),x]
```

```
[Out] (-2*Coth[x])/(3*a) + Csch[x]/(3*(a + a*Cosh[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(x)}{a+a\cosh(x)}\,dx = \frac{\operatorname{csch}(x)}{3(a+a\cosh(x))} + \frac{2\int \operatorname{csch}^2(x)\,dx}{3a}$$
$$= \frac{\operatorname{csch}(x)}{3(a+a\cosh(x))} - \frac{(2i)\mathrm{Subst}(\int 1\,dx, x, -i\coth(x))}{3a}$$
$$= -\frac{2\coth(x)}{3a} + \frac{\operatorname{csch}(x)}{3(a+a\cosh(x))}$$

Mathematica [A]
time = 0.04, size = 30, normalized size = 1.25

$$-\frac{(2\cosh(x)+\cosh(2x))\operatorname{csch}\left(\frac{x}{2}\right)\operatorname{sech}^3\left(\frac{x}{2}\right)}{12a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^2/(a + a*Cosh[x]),x]
```

```
[Out] -1/12*((2*Cosh[x] + Cosh[2*x])*Csch[x/2]*Sech[x/2]^3)/a
```

Maple [A]
time = 0.58, size = 29, normalized size = 1.21

method	result	size
risch	$-\frac{4(1+2\,\mathrm{e}^{x})}{3(\mathrm{e}^{x}-1)a(\mathrm{e}^{x}+1)^{3}}$	24
default	$\frac{\frac{\left(\tanh^3\left(\frac{x}{2}\right)\right)}{3}-2\tanh\left(\frac{x}{2}\right)-\frac{1}{\tanh\left(\frac{x}{2}\right)}}{4a}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^2/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a*(1/3*tanh(1/2*x)^3-2*tanh(1/2*x)-1/tanh(1/2*x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(20) = 40$.
time = 0.27, size = 59, normalized size = 2.46

$$-\frac{8\,e^{(-x)}}{3\,(2\,ae^{(-x)}-2\,ae^{(-3\,x)}-ae^{(-4\,x)}+a)}-\frac{4}{3\,(2\,ae^{(-x)}-2\,ae^{(-3\,x)}-ae^{(-4\,x)}+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -8/3*e^(-x)/(2*a*e^(-x) - 2*a*e^(-3*x) - a*e^(-4*x) + a) - 4/3/(2*a*e^(-x)
- 2*a*e^(-3*x) - a*e^(-4*x) + a)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(20) = 40$.
time = 0.36, size = 94, normalized size = 3.92

$$-\frac{4\,(2\,\cosh\left(x\right)+2\,\sinh\left(x\right)+1)}{3\,\left(a\cosh\left(x\right)^{4}+a\sinh\left(x\right)^{4}+2\,a\cosh\left(x\right)^{3}+2\,(2\,a\cosh\left(x\right)+a)\sinh\left(x\right)^{3}+6\,\left(a\cosh\left(x\right)^{2}+a\cosh\left(x\right)\right)\sinh\left(x\right)^{2}-2\,a\cosh\left(x\right)+2\,\left(2\,a\cosh\left(x\right)^{3}+3\,a\cosh\left(x\right)^{2}-a\right)\sinh\left(x\right)-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] -4/3*(2*cosh(x) + 2*sinh(x) + 1)/(a*cosh(x)^4 + a*sinh(x)^4 + 2*a*cosh(x)^3
 + 2*(2*a*cosh(x) + a)*sinh(x)^3 + 6*(a*cosh(x)^2 + a*cosh(x))*sinh(x)^2 -
2*a*cosh(x) + 2*(2*a*cosh(x)^3 + 3*a*cosh(x)^2 - a)*sinh(x) - a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{csch}^{2}\left(x\right)}{\cosh\left(x\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2/(a+a*cosh(x)),x)
```

```
[Out] Integral(csch(x)**2/(cosh(x) + 1), x)/a
```

Giac [A]
time = 0.42, size = 35, normalized size = 1.46

$$-\frac{1}{2\,a(e^{x}-1)}+\frac{3\,e^{(2\,x)}+12\,e^{x}+5}{6\,a(e^{x}+1)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -1/2/(a*(e^x - 1)) + 1/6*(3*e^(2*x) + 12*e^x + 5)/(a*(e^x + 1)^3)
```

Mupad [B]
time = 0.92, size = 89, normalized size = 3.71

$$\frac{\frac{e^{2\,x}}{6\,a}+\frac{1}{6\,a}+\frac{e^{x}}{a}}{3\,e^{2\,x}+e^{3\,x}+3\,e^{x}+1}+\frac{\frac{1}{2\,a}+\frac{e^{x}}{6\,a}}{e^{2\,x}+2\,e^{x}+1}-\frac{1}{2\,a\,\left(e^{x}-1\right)}+\frac{1}{6\,a\,\left(e^{x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^2*(a + a*cosh(x))),x)
```

```
[Out] (exp(2*x)/(6*a) + 1/(6*a) + exp(x)/a)/(3*exp(2*x) + exp(3*x) + 3*exp(x) + 1
) + (1/(2*a) + exp(x)/(6*a))/(exp(2*x) + 2*exp(x) + 1) - 1/(2*a*(exp(x) - 1
)) + 1/(6*a*(exp(x) + 1))
```

3.162 $\int \frac{\operatorname{csch}^3(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=49

$$\frac{3\tanh^{-1}(\cosh(x))}{8a}+\frac{1}{8(a-a\cosh(x))}-\frac{a}{8(a+a\cosh(x))^2}-\frac{1}{4(a+a\cosh(x))}$$

```
[Out] 3/8*arctanh(cosh(x))/a+1/8/(a-a*cosh(x))-1/8*a/(a+a*cosh(x))^2-1/4/(a+a*cos
h(x))
```

Rubi [A]
time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2746, 46, 212}

$$-\frac{a}{8(a\cosh(x)+a)^2}+\frac{1}{8(a-a\cosh(x))}-\frac{1}{4(a\cosh(x)+a)}+\frac{3\tanh^{-1}(\cosh(x))}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^3/(a + a*Cosh[x]),x]
```

```
[Out] (3*ArcTanh[Cosh[x]])/(8*a) + 1/(8*(a - a*Cosh[x])) - a/(8*(a + a*Cosh[x])^2
) - 1/(4*(a + a*Cosh[x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^3(x)}{a+a\cosh(x)}\,dx &= a^3\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^3}\,dx, x, a\cosh(x)\right)\\ &= a^3\text{Subst}\left(\int \left(\frac{1}{8a^3(a-x)^2}+\frac{1}{4a^2(a+x)^3}+\frac{1}{4a^3(a+x)^2}+\frac{3}{8a^3\left(a^2-x^2\right)}\right)\,dx, x, a\,\mathrm{c}\right.\\ &= \frac{1}{8(a-a\cosh(x))}-\frac{a}{8(a+a\cosh(x))^2}-\frac{1}{4(a+a\cosh(x))}+\frac{3}{8}\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx,\right.\\ &= \frac{3\tanh^{-1}(\cosh(x))}{8a}+\frac{1}{8(a-a\cosh(x))}-\frac{a}{8(a+a\cosh(x))^2}-\frac{1}{4(a+a\cosh(x))}\end{aligned}$$

Mathematica [**A**]
time = 0.12, size = 60, normalized size = 1.22

$$-\frac{4+2\coth^2\left(\frac{x}{2}\right)-12\cosh^2\left(\frac{x}{2}\right)\left(\log\left(\cosh\left(\frac{x}{2}\right)\right)-\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)+\operatorname{sech}^2\left(\frac{x}{2}\right)}{16a(1+\cosh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^3/(a + a*Cosh[x]),x]
[Out] -1/16*(4 + 2*Coth[x/2]^2 - 12*Cosh[x/2]^2*(Log[Cosh[x/2]] - Log[Sinh[x/2]])
 + Sech[x/2]^2)/(a*(1 + Cosh[x]))
```

Maple [**A**]
time = 0.60, size = 38, normalized size = 0.78

method	result	size
default	$\frac{-\frac{\left(\tanh^4\left(\frac{x}{2}\right)\right)}{4}+\frac{3\left(\tanh^2\left(\frac{x}{2}\right)\right)}{2}-\frac{1}{2\tanh\left(\frac{x}{2}\right)^2}-3\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{8a}$	38
risch	$-\frac{e^x\left(3\,e^{4x}+6\,e^{3x}-2\,e^{2x}+6\,e^x+3\right)}{4\left(e^x-1\right)^2a\left(e^x+1\right)^4}+\frac{3\ln\left(e^x+1\right)}{8a}-\frac{3\ln\left(e^x-1\right)}{8a}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^3/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
[Out] 1/8/a*(-1/4*tanh(1/2*x)^4+3/2*tanh(1/2*x)^2-1/2/tanh(1/2*x)^2-3*ln(tanh(1/2
*x)))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(42) = 84$.
time = 0.26, size = 103, normalized size = 2.10

$$-\frac{3\,e^{(-x)}+6\,e^{(-2\,x)}-2\,e^{(-3\,x)}+6\,e^{(-4\,x)}+3\,e^{(-5\,x)}}{4\left(2\,ae^{(-x)}-ae^{(-2\,x)}-4\,ae^{(-3\,x)}-ae^{(-4\,x)}+2\,ae^{(-5\,x)}+ae^{(-6\,x)}+a\right)}+\frac{3\,\log\left(e^{(-x)}+1\right)}{8\,a}-\frac{3\,\log\left(e^{(-x)}-1\right)}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -1/4*(3*e^(-x) + 6*e^(-2*x) - 2*e^(-3*x) + 6*e^(-4*x) + 3*e^(-5*x))/(2*a*e^
(-x) - a*e^(-2*x) - 4*a*e^(-3*x) - a*e^(-4*x) + 2*a*e^(-5*x) + a*e^(-6*x) +
 a) + 3/8*log(e^(-x) + 1)/a - 3/8*log(e^(-x) - 1)/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 631 vs. $2(42) = 84$.
time = 0.37, size = 631, normalized size = 12.88

$$-\frac{6\cosh(x)^5 + 6(5\cosh(x)+2)\sinh(x)^4 + 6\sinh(x)^5 + 12\cosh(x)^4 + 4(15\cosh(x)^2+12\cosh(x)-1)\sinh(x)^3 - 4\cosh(x)^3 + 12(5\cosh(x)^3+6\cosh(x)^2-\cosh(x)+1)\sinh(x)^2 + 12\cosh(x)^2 - 3\left(\cosh(x)^6 + 2(3\cosh(x)+1)\sinh(x)^5 + \sinh(x)^6 + 2\cosh(x)^5 + (15\cosh(x)^2+10\cosh(x)-1)\sinh(x)^4 - \cosh(x)^4 + 4(5\cosh(x)^3+5\cosh(x)^2-\cosh(x)-1)\sinh(x)^3 - 4\cosh(x)^3 + (15\cosh(x)^4+20\cosh(x)^3-6\cosh(x)^2-12\cosh(x)-1)\sinh(x)^2 - \cosh(x)^2 + 2(3\cosh(x)^5+5\cosh(x)^4-2\cosh(x)^3-6\cosh(x)^2-\cosh(x)+1)\sinh(x) + 2\cosh(x)+1\right)\log(\cosh(x)+\sinh(x)+1) + 3\left(\cosh(x)^6 + 2(3\cosh(x)+1)\sinh(x)^5 + \sinh(x)^6 + 2\cosh(x)^5 + (15\cosh(x)^2+10\cosh(x)-1)\sinh(x)^4 - \cosh(x)^4 + 4(5\cosh(x)^3+5\cosh(x)^2-\cosh(x)-1)\sinh(x)^3 - 4\cosh(x)^3 + (15\cosh(x)^4+20\cosh(x)^3-6\cosh(x)^2-12\cosh(x)-1)\sinh(x)^2 - \cosh(x)^2 + 2(3\cosh(x)^5+5\cosh(x)^4-2\cosh(x)^3-6\cosh(x)^2-\cosh(x)+1)\sinh(x) + 2\cosh(x)+1\right)\log(\cosh(x)+\sinh(x)-1) + 6(5\cosh(x)^4+8\cosh(x)^3-2\cosh(x)^2+4\cosh(x)+1)\sinh(x) + 6\cosh(x)}{8\left(a\cosh(x)^6 + a\sinh(x)^6 + 2a\cosh(x)^5 + 2(3a\cosh(x)+a)\sinh(x)^5 - a\cosh(x)^4 + (15a\cosh(x)^2+10a\cosh(x)-a)\sinh(x)^4 - 4a\cosh(x)^3 + 4(5a\cosh(x)^3+5a\cosh(x)^2-a\cosh(x)-a)\sinh(x)^3 - a\cosh(x)^2 + (15a\cosh(x)^4+20a\cosh(x)^3-6a\cosh(x)^2-12a\cosh(x)-a)\sinh(x)^2 + 2a\cosh(x) + 2(3a\cosh(x)^5+5a\cosh(x)^4-2a\cosh(x)^3-6a\cosh(x)^2-a\cosh(x)+a)\sinh(x) + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] -1/8*(6*cosh(x)^5 + 6*(5*cosh(x) + 2)*sinh(x)^4 + 6*sinh(x)^5 + 12*cosh(x)^
4 + 4*(15*cosh(x)^2 + 12*cosh(x) - 1)*sinh(x)^3 - 4*cosh(x)^3 + 12*(5*cosh(
x)^3 + 6*cosh(x)^2 - cosh(x) + 1)*sinh(x)^2 + 12*cosh(x)^2 - 3*(cosh(x)^6 +
 2*(3*cosh(x) + 1)*sinh(x)^5 + sinh(x)^6 + 2*cosh(x)^5 + (15*cosh(x)^2 + 10
*cosh(x) - 1)*sinh(x)^4 - cosh(x)^4 + 4*(5*cosh(x)^3 + 5*cosh(x)^2 - cosh(x
) - 1)*sinh(x)^3 - 4*cosh(x)^3 + (15*cosh(x)^4 + 20*cosh(x)^3 - 6*cosh(x)^2
 - 12*cosh(x) - 1)*sinh(x)^2 - cosh(x)^2 + 2*(3*cosh(x)^5 + 5*cosh(x)^4 - 2
*cosh(x)^3 - 6*cosh(x)^2 - cosh(x) + 1)*sinh(x) + 2*cosh(x) + 1)*log(cosh(x
) + sinh(x) + 1) + 3*(cosh(x)^6 + 2*(3*cosh(x) + 1)*sinh(x)^5 + sinh(x)^6 +
 2*cosh(x)^5 + (15*cosh(x)^2 + 10*cosh(x) - 1)*sinh(x)^4 - cosh(x)^4 + 4*(5
*cosh(x)^3 + 5*cosh(x)^2 - cosh(x) - 1)*sinh(x)^3 - 4*cosh(x)^3 + (15*cosh(
x)^4 + 20*cosh(x)^3 - 6*cosh(x)^2 - 12*cosh(x) - 1)*sinh(x)^2 - cosh(x)^2 +
 2*(3*cosh(x)^5 + 5*cosh(x)^4 - 2*cosh(x)^3 - 6*cosh(x)^2 - cosh(x) + 1)*si
nh(x) + 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 6*(5*cosh(x)^4 + 8*cosh
(x)^3 - 2*cosh(x)^2 + 4*cosh(x) + 1)*sinh(x) + 6*cosh(x))/(a*cosh(x)^6 + a*
sinh(x)^6 + 2*a*cosh(x)^5 + 2*(3*a*cosh(x) + a)*sinh(x)^5 - a*cosh(x)^4 + (
15*a*cosh(x)^2 + 10*a*cosh(x) - a)*sinh(x)^4 - 4*a*cosh(x)^3 + 4*(5*a*cosh(
x)^3 + 5*a*cosh(x)^2 - a*cosh(x) - a)*sinh(x)^3 - a*cosh(x)^2 + (15*a*cosh(
x)^4 + 20*a*cosh(x)^3 - 6*a*cosh(x)^2 - 12*a*cosh(x) - a)*sinh(x)^2 + 2*a*c
osh(x) + 2*(3*a*cosh(x)^5 + 5*a*cosh(x)^4 - 2*a*cosh(x)^3 - 6*a*cosh(x)^2 -
 a*cosh(x) + a)*sinh(x) + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{csch}^{3}(x)}{\cosh(x)+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**3/(a+a*cosh(x)),x)
```

```
[Out] Integral(csch(x)**3/(cosh(x) + 1), x)/a
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(42) = 84$.
time = 0.41, size = 94, normalized size = 1.92

$$\frac{3\log\left(e^{(-x)}+e^{x}+2\right)}{16\,a}-\frac{3\log\left(e^{(-x)}+e^{x}-2\right)}{16\,a}+\frac{3\,e^{(-x)}+3\,e^{x}-10}{16\,a(e^{(-x)}+e^{x}-2)}-\frac{9\left(e^{(-x)}+e^{x}\right)^{2}+52\,e^{(-x)}+52\,e^{x}+84}{32\,a(e^{(-x)}+e^{x}+2)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] 3/16*log(e^(-x) + e^x + 2)/a - 3/16*log(e^(-x) + e^x - 2)/a + 1/16*(3*e^(-x
) + 3*e^x - 10)/(a*(e^(-x) + e^x - 2)) - 1/32*(9*(e^(-x) + e^x)^2 + 52*e^(-
x) + 52*e^x + 84)/(a*(e^(-x) + e^x + 2)^2)
```

Mupad [B]
time = 0.93, size = 114, normalized size = 2.33

$$\frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^{x}\,\sqrt{-a^2}}{a}\right)}{4\,\sqrt{-a^2}}-\frac{1}{2\,a\,\left(6\,\mathrm{e}^{2\,x}+4\,\mathrm{e}^{3\,x}+\mathrm{e}^{4\,x}+4\,\mathrm{e}^{x}+1\right)}-\frac{1}{4\,a\,\left(\mathrm{e}^{x}-1\right)}-\frac{1}{2\,a\,\left(\mathrm{e}^{x}+1\right)}-\frac{1}{4\,a\,\left(\mathrm{e}^{2\,x}-2\,\mathrm{e}^{x}+1\right)}+\frac{1}{a\,\left(3\,\mathrm{e}^{2\,x}+\mathrm{e}^{3\,x}+3\,\mathrm{e}^{x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^3*(a + a*cosh(x))),x)
```

```
[Out] (3*atan((exp(x)*(-a^2)^(1/2))/a))/(4*(-a^2)^(1/2)) - 1/(2*a*(6*exp(2*x) + 4
*exp(3*x) + exp(4*x) + 4*exp(x) + 1)) - 1/(4*a*(exp(x) - 1)) - 1/(2*a*(exp(
x) + 1)) - 1/(4*a*(exp(2*x) - 2*exp(x) + 1)) + 1/(a*(3*exp(2*x) + exp(3*x)
+ 3*exp(x) + 1))
```

3.163 $\int \frac{\operatorname{csch}^4(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=37

$$\frac{4\coth(x)}{5a}-\frac{4\coth^3(x)}{15a}+\frac{\operatorname{csch}^3(x)}{5(a+a\cosh(x))}$$

```
[Out] 4/5*coth(x)/a-4/15*coth(x)^3/a+1/5*csch(x)^3/(a+a*cosh(x))
```

Rubi [A]

time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.154$, Rules used = {2751, 3852}

$$-\frac{4\coth^3(x)}{15a}+\frac{4\coth(x)}{5a}+\frac{\operatorname{csch}^3(x)}{5(a\cosh(x)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^4/(a + a*Cosh[x]),x]
```

```
[Out] (4*Coth[x])/(5*a) - (4*Coth[x]^3)/(15*a) + Csch[x]^3/(5*(a + a*Cosh[x]))
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^4(x)}{a+a\cosh(x)}\,dx &= \frac{\operatorname{csch}^3(x)}{5(a+a\cosh(x))}+\frac{4\int \operatorname{csch}^4(x)\,dx}{5a}\\
&= \frac{\operatorname{csch}^3(x)}{5(a+a\cosh(x))}+\frac{(4i)\operatorname{Subst}\left(\int \left(1+x^2\right)\,dx,x,-i\coth(x)\right)}{5a}\\
&= \frac{4\coth(x)}{5a}-\frac{4\coth^3(x)}{15a}+\frac{\operatorname{csch}^3(x)}{5(a+a\cosh(x))}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 38, normalized size = 1.03

$$\frac{(-6\cosh(x)-2\cosh(2x)+2\cosh(3x)+\cosh(4x))\operatorname{csch}^3(x)}{15a(1+\cosh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^4/(a + a*Cosh[x]),x]
```

```
[Out] ((-6*Cosh[x] - 2*Cosh[2*x] + 2*Cosh[3*x] + Cosh[4*x])*Csch[x]^3)/(15*a*(1 +
 Cosh[x]))
```

Maple [A]
time = 0.59, size = 45, normalized size = 1.22

method	result	size
risch	$-\frac{16(6\,\mathrm{e}^{3x}+2\,\mathrm{e}^{2x}-2\,\mathrm{e}^{x}-1)}{15(\mathrm{e}^{x}+1)^{5}a(\mathrm{e}^{x}-1)^{3}}$	36
default	$\frac{\frac{\left(\tanh^{5}\left(\frac{x}{2}\right)\right)}{5}-\frac{4\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{3}+6\tanh\left(\frac{x}{2}\right)-\frac{1}{3\tanh\left(\frac{x}{2}\right)^{3}}+\frac{4}{\tanh\left(\frac{x}{2}\right)}}{16a}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^4/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/a*(1/5*tanh(1/2*x)^5-4/3*tanh(1/2*x)^3+6*tanh(1/2*x)-1/3/tanh(1/2*x)^3
+4/tanh(1/2*x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(31) = 62$.
time = 0.26, size = 233, normalized size = 6.30

$$\frac{32\,e^{(-x)}}{15\,(2\,ae^{(-x)}-2\,ae^{(-2x)}-6\,ae^{(-3x)}+6\,ae^{(-5x)}+2\,ae^{(-6x)}-2\,ae^{(-7x)}-ae^{(-8x)}+a)}-\frac{32\,e^{(-2x)}}{15\,(2\,ae^{(-x)}-2\,ae^{(-2x)}-6\,ae^{(-3x)}+6\,ae^{(-5x)}+2\,ae^{(-6x)}-2\,ae^{(-7x)}-ae^{(-8x)}+a)}-\frac{32\,e^{(-3x)}}{5\,(2\,ae^{(-x)}-2\,ae^{(-2x)}-6\,ae^{(-3x)}+6\,ae^{(-5x)}+2\,ae^{(-6x)}-2\,ae^{(-7x)}-ae^{(-8x)}+a)}+\frac{16}{15\,(2\,ae^{(-x)}-2\,ae^{(-2x)}-6\,ae^{(-3x)}+6\,ae^{(-5x)}+2\,ae^{(-6x)}-2\,ae^{(-7x)}-ae^{(-8x)}+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] 32/15*e^(-x)/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a
*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a) - 32/15*e^(-2*x)/(2*a*e^(-x) - 2
*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a
*e^(-8*x) + a) - 32/5*e^(-3*x)/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) +
6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a) + 16/15/(2*a*e
^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(
-7*x) - a*e^(-8*x) + a)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(31) = 62$.
time = 0.32, size = 250, normalized size = 6.76

$$-\frac{16 \left(6 \cosh\left(x\right)^{2} + 3 \left(4 \cosh\left(x\right) + 1\right) \sinh\left(x\right) + 6 \sinh\left(x\right)^{2} + \cosh\left(x\right) - 2\right)}{15 \left(a \cosh\left(x\right)^{7} + a \sinh\left(x\right)^{7} + 2 a \cosh\left(x\right)^{6} + \left(7 a \cosh\left(x\right) + 2 a\right) \sinh\left(x\right)^{6} - 2 a \cosh\left(x\right)^{5} + \left(21 a \cosh\left(x\right)^{2} + 12 a \cosh\left(x\right) - 2 a\right) \sinh\left(x\right)^{5} - 6 a \cosh\left(x\right)^{4} + \left(35 a \cosh\left(x\right)^{3} + 30 a \cosh\left(x\right)^{2} - 10 a \cosh\left(x\right) - 6 a\right) \sinh\left(x\right)^{4} + \left(35 a \cosh\left(x\right)^{4} + 40 a \cosh\left(x\right)^{3} - 20 a \cosh\left(x\right)^{2} - 24 a \cosh\left(x\right)\right) \sinh\left(x\right)^{3} + 6 a \cosh\left(x\right)^{2} + \left(21 a \cosh\left(x\right)^{5} + 30 a \cosh\left(x\right)^{4} - 20 a \cosh\left(x\right)^{3} - 36 a \cosh\left(x\right)^{2} + 6 a\right) \sinh\left(x\right)^{2} + a \cosh\left(x\right) + \left(7 a \cosh\left(x\right)^{6} + 12 a \cosh\left(x\right)^{5} - 10 a \cosh\left(x\right)^{4} - 24 a \cosh\left(x\right)^{3} + 12 a \cosh\left(x\right) + 3 a\right) \sinh\left(x\right) - 2 a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] -16/15*(6*cosh(x)^2 + 3*(4*cosh(x) + 1)*sinh(x) + 6*sinh(x)^2 + cosh(x) - 2
)/(a*cosh(x)^7 + a*sinh(x)^7 + 2*a*cosh(x)^6 + (7*a*cosh(x) + 2*a)*sinh(x)^
6 - 2*a*cosh(x)^5 + (21*a*cosh(x)^2 + 12*a*cosh(x) - 2*a)*sinh(x)^5 - 6*a*c
osh(x)^4 + (35*a*cosh(x)^3 + 30*a*cosh(x)^2 - 10*a*cosh(x) - 6*a)*sinh(x)^4
 + (35*a*cosh(x)^4 + 40*a*cosh(x)^3 - 20*a*cosh(x)^2 - 24*a*cosh(x))*sinh(x
)^3 + 6*a*cosh(x)^2 + (21*a*cosh(x)^5 + 30*a*cosh(x)^4 - 20*a*cosh(x)^3 - 3
6*a*cosh(x)^2 + 6*a)*sinh(x)^2 + a*cosh(x) + (7*a*cosh(x)^6 + 12*a*cosh(x)^
5 - 10*a*cosh(x)^4 - 24*a*cosh(x)^3 + 12*a*cosh(x) + 3*a)*sinh(x) - 2*a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{csch}^{4}\left(x\right)}{\cosh\left(x\right) + 1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**4/(a+a*cosh(x)),x)
```

```
[Out] Integral(csch(x)**4/(cosh(x) + 1), x)/a
```

Giac [A]
time = 0.41, size = 59, normalized size = 1.59

$$\frac{9 \, e^{\left(2 \, x\right)} - 24 \, e^{x} + 11}{24 \, a {\left(e^{x} - 1\right)}^{3}} - \frac{45 \, e^{\left(4 \, x\right)} + 240 \, e^{\left(3 \, x\right)} + 490 \, e^{\left(2 \, x\right)} + 320 \, e^{x} + 73}{120 \, a {\left(e^{x} + 1\right)}^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] 1/24*(9*e^(2*x) - 24*e^x + 11)/(a*(e^x - 1)^3) - 1/120*(45*e^(4*x) + 240*e^
(3*x) + 490*e^(2*x) + 320*e^x + 73)/(a*(e^x + 1)^5)
```

Mupad [B]
time = 0.95, size = 263, normalized size = 7.11

$$\frac{1}{6\,a\,\left(3\,{\mathrm{e}}^{2\,x}-{\mathrm{e}}^{3\,x}-3\,{\mathrm{e}}^x+1\right)}-\frac{\frac{3\,{\mathrm{e}}^{2\,x}}{8\,a}+\frac{3\,{\mathrm{e}}^{3\,x}}{40\,a}+\frac{1}{8\,a}+\frac{5\,{\mathrm{e}}^x}{8\,a}}{6\,{\mathrm{e}}^{2\,x}+4\,{\mathrm{e}}^{3\,x}+{\mathrm{e}}^{4\,x}+4\,{\mathrm{e}}^x+1}-\frac{\frac{3\,{\mathrm{e}}^{2\,x}}{40\,a}+\frac{5}{24\,a}+\frac{{\mathrm{e}}^x}{4\,a}}{3\,{\mathrm{e}}^{2\,x}+{\mathrm{e}}^{3\,x}+3\,{\mathrm{e}}^x+1}-\frac{\frac{1}{8\,a}+\frac{3\,{\mathrm{e}}^x}{40\,a}}{{\mathrm{e}}^{2\,x}+2\,{\mathrm{e}}^x+1}-\frac{\frac{5\,{\mathrm{e}}^{2\,x}}{4\,a}+\frac{{\mathrm{e}}^{3\,x}}{2\,a}+\frac{3\,{\mathrm{e}}^{4\,x}}{40\,a}+\frac{3}{40\,a}+\frac{{\mathrm{e}}^x}{2\,a}}{10\,{\mathrm{e}}^{2\,x}+10\,{\mathrm{e}}^{3\,x}+5\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{5\,x}+5\,{\mathrm{e}}^x+1}-\frac{1}{4\,a\,\left({\mathrm{e}}^{2\,x}-2\,{\mathrm{e}}^x+1\right)}+\frac{3}{8\,a\,\left({\mathrm{e}}^x-1\right)}-\frac{3}{40\,a\,\left({\mathrm{e}}^x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^4*(a + a*cosh(x))),x)
```

```
[Out] 1/(6*a*(3*exp(2*x) - exp(3*x) - 3*exp(x) + 1)) - ((3*exp(2*x))/(8*a) + (3*e
xp(3*x))/(40*a) + 1/(8*a) + (5*exp(x))/(8*a))/(6*exp(2*x) + 4*exp(3*x) + ex
p(4*x) + 4*exp(x) + 1) - ((3*exp(2*x))/(40*a) + 5/(24*a) + exp(x)/(4*a))/(3
*exp(2*x) + exp(3*x) + 3*exp(x) + 1) - (1/(8*a) + (3*exp(x))/(40*a))/(exp(2
*x) + 2*exp(x) + 1) - ((5*exp(2*x))/(4*a) + exp(3*x)/(2*a) + (3*exp(4*x))/(
40*a) + 3/(40*a) + exp(x)/(2*a))/(10*exp(2*x) + 10*exp(3*x) + 5*exp(4*x) +
exp(5*x) + 5*exp(x) + 1) - 1/(4*a*(exp(2*x) - 2*exp(x) + 1)) + 3/(8*a*(exp(
x) - 1)) - 3/(40*a*(exp(x) + 1))
```

3.164 $\int \frac{\operatorname{csch}^5(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=78

$$-\frac{5\tanh^{-1}(\cosh(x))}{16a}-\frac{a}{32(a-a\cosh(x))^2}-\frac{1}{8(a-a\cosh(x))}+\frac{a^2}{24(a+a\cosh(x))^3}+\frac{3a}{32(a+a\cosh(x))^2}+\frac{}{16(a}$$

```
[Out] -5/16*arctanh(cosh(x))/a-1/32*a/(a-a*cosh(x))^2-1/8/(a-a*cosh(x))+1/24*a^2/
(a+a*cosh(x))^3+3/32*a/(a+a*cosh(x))^2+3/16/(a+a*cosh(x))
```

Rubi [A]
time = 0.08, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2746, 46, 212}

$$\frac{a^2}{24(a\cosh(x)+a)^3}-\frac{a}{32(a-a\cosh(x))^2}+\frac{3a}{32(a\cosh(x)+a)^2}-\frac{1}{8(a-a\cosh(x))}+\frac{3}{16(a\cosh(x)+a)}-\frac{5\tanh^{-1}(\cosh(x))}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^5/(a + a*Cosh[x]),x]
```

```
[Out] (-5*ArcTanh[Cosh[x]])/(16*a) - a/(32*(a - a*Cosh[x])^2) - 1/(8*(a - a*Cosh[
x])) + a^2/(24*(a + a*Cosh[x])^3) + (3*a)/(32*(a + a*Cosh[x])^2) + 3/(16*(a
 + a*Cosh[x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\operatorname{csch}^5(x)}{a+a\cosh(x)}\,dx = -\left(a^5\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^4}\,dx, x, a\cosh(x)\right)\right)$$

$$= -\left(a^5\text{Subst}\left(\int \left(\frac{1}{16a^4(a-x)^3} + \frac{1}{8a^5(a-x)^2} + \frac{1}{8a^3(a+x)^4} + \frac{3}{16a^4(a+x)^3} + \frac{}{16a\cdots}\right.\right.\right.$$

$$= -\frac{a}{32(a-a\cosh(x))^2} - \frac{1}{8(a-a\cosh(x))} + \frac{a^2}{24(a+a\cosh(x))^3} + \frac{3a}{32(a+a\cosh(x))^2\cdots}$$

$$= -\frac{5\tanh^{-1}(\cosh(x))}{16a} - \frac{a}{32(a-a\cosh(x))^2} - \frac{1}{8(a-a\cosh(x))} + \frac{a^2}{24(a+a\cosh(x))^3\cdots}$$

Mathematica [A]
time = 0.20, size = 89, normalized size = 1.14

$$\frac{\cosh^2\left(\frac{x}{2}\right)\left(24\operatorname{csch}^2\left(\frac{x}{2}\right) - 3\operatorname{csch}^4\left(\frac{x}{2}\right) - 120\log\left(\cosh\left(\frac{x}{2}\right)\right) + 120\log\left(\sinh\left(\frac{x}{2}\right)\right) + 36\operatorname{sech}^2\left(\frac{x}{2}\right) + 9\operatorname{sech}^4\left(\frac{x}{2}\right) + 2\operatorname{sech}^6\left(\frac{x}{2}\right)\right)}{192(a+a\cosh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^5/(a + a*Cosh[x]),x]
```

```
[Out] (Cosh[x/2]^2*(24*Csch[x/2]^2 - 3*Csch[x/2]^4 - 120*Log[Cosh[x/2]] + 120*Log
[Sinh[x/2]] + 36*Sech[x/2]^2 + 9*Sech[x/2]^4 + 2*Sech[x/2]^6))/(192*(a + a*
Cosh[x]))
```

Maple [A]
time = 0.59, size = 54, normalized size = 0.69

method	result	size
default	$\frac{-\frac{\left(\tanh^6\left(\frac{x}{2}\right)\right)}{6}+\frac{5\left(\tanh^4\left(\frac{x}{2}\right)\right)}{4}-5\left(\tanh^2\left(\frac{x}{2}\right)\right)+\frac{5}{2\tanh\left(\frac{x}{2}\right)^2}-\frac{1}{4\tanh\left(\frac{x}{2}\right)^4}+10\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{32a}$	54
risch	$\frac{e^x\left(15e^{8x}+30e^{7x}-40e^{6x}-110e^{5x}+18e^{4x}-110e^{3x}-40e^{2x}+30e^x+15\right)}{24(e^x+1)^6 a(e^x-1)^4}-\frac{5\ln(e^x+1)}{16a}+\frac{5\ln(e^x-1)}{16a}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^5/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/a*(-1/6*tanh(1/2*x)^6+5/4*tanh(1/2*x)^4-5*tanh(1/2*x)^2+5/2/tanh(1/2*x
)^2-1/4/tanh(1/2*x)^4+10*ln(tanh(1/2*x)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 155 vs. 2(68) = 136.
time = 0.28, size = 155, normalized size = 1.99

$$\frac{15e^{(-x)}+30e^{(-2x)}-40e^{(-3x)}-110e^{(-4x)}+18e^{(-5x)}-110e^{(-6x)}-40e^{(-7x)}+30e^{(-8x)}+15e^{(-9x)}}{24\left(2ae^{(-x)}-3ae^{(-2x)}-8ae^{(-3x)}+2ae^{(-4x)}+12ae^{(-5x)}+2ae^{(-6x)}-8ae^{(-7x)}-3ae^{(-8x)}+2ae^{(-9x)}+ae^{(-10x)}+a\right)}-\frac{5\log\left(e^{(-x)}+1\right)}{16a}+\frac{5\log\left(e^{(-x)}-1\right)}{16a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^5/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] 1/24*(15*e^(-x) + 30*e^(-2*x) - 40*e^(-3*x) - 110*e^(-4*x) + 18*e^(-5*x) -
110*e^(-6*x) - 40*e^(-7*x) + 30*e^(-8*x) + 15*e^(-9*x))/(2*a*e^(-x) - 3*a*e
^(-2*x) - 8*a*e^(-3*x) + 2*a*e^(-4*x) + 12*a*e^(-5*x) + 2*a*e^(-6*x) - 8*a*
e^(-7*x) - 3*a*e^(-8*x) + 2*a*e^(-9*x) + a*e^(-10*x) + a) - 5/16*log(e^(-x)
 + 1)/a + 5/16*log(e^(-x) - 1)/a
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1551 vs. $2(68) = 136$.
time = 0.34, size = 1551, normalized size = 19.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^5/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/48*(30*cosh(x)^9 + 30*(9*cosh(x) + 2)*sinh(x)^8 + 30*sinh(x)^9 + 60*cosh(
x)^8 + 40*(27*cosh(x)^2 + 12*cosh(x) - 2)*sinh(x)^7 - 80*cosh(x)^7 + 20*(12
6*cosh(x)^3 + 84*cosh(x)^2 - 28*cosh(x) - 11)*sinh(x)^6 - 220*cosh(x)^6 + 1
2*(315*cosh(x)^4 + 280*cosh(x)^3 - 140*cosh(x)^2 - 110*cosh(x) + 3)*sinh(x)
^5 + 36*cosh(x)^5 + 20*(189*cosh(x)^5 + 210*cosh(x)^4 - 140*cosh(x)^3 - 165
*cosh(x)^2 + 9*cosh(x) - 11)*sinh(x)^4 - 220*cosh(x)^4 + 40*(63*cosh(x)^6 +
 84*cosh(x)^5 - 70*cosh(x)^4 - 110*cosh(x)^3 + 9*cosh(x)^2 - 22*cosh(x) - 2
)*sinh(x)^3 - 80*cosh(x)^3 + 60*(18*cosh(x)^7 + 28*cosh(x)^6 - 28*cosh(x)^5
 - 55*cosh(x)^4 + 6*cosh(x)^3 - 22*cosh(x)^2 - 4*cosh(x) + 1)*sinh(x)^2 + 6
0*cosh(x)^2 - 15*(cosh(x)^10 + 2*(5*cosh(x) + 1)*sinh(x)^9 + sinh(x)^10 + 2
*cosh(x)^9 + 3*(15*cosh(x)^2 + 6*cosh(x) - 1)*sinh(x)^8 - 3*cosh(x)^8 + 8*(
15*cosh(x)^3 + 9*cosh(x)^2 - 3*cosh(x) - 1)*sinh(x)^7 - 8*cosh(x)^7 + 2*(10
5*cosh(x)^4 + 84*cosh(x)^3 - 42*cosh(x)^2 - 28*cosh(x) + 1)*sinh(x)^6 + 2*c
osh(x)^6 + 12*(21*cosh(x)^5 + 21*cosh(x)^4 - 14*cosh(x)^3 - 14*cosh(x)^2 +
cosh(x) + 1)*sinh(x)^5 + 12*cosh(x)^5 + 2*(105*cosh(x)^6 + 126*cosh(x)^5 -
105*cosh(x)^4 - 140*cosh(x)^3 + 15*cosh(x)^2 + 30*cosh(x) + 1)*sinh(x)^4 +
2*cosh(x)^4 + 8*(15*cosh(x)^7 + 21*cosh(x)^6 - 21*cosh(x)^5 - 35*cosh(x)^4
+ 5*cosh(x)^3 + 15*cosh(x)^2 + cosh(x) - 1)*sinh(x)^3 - 8*cosh(x)^3 + 3*(15
*cosh(x)^8 + 24*cosh(x)^7 - 28*cosh(x)^6 - 56*cosh(x)^5 + 10*cosh(x)^4 + 40
*cosh(x)^3 + 4*cosh(x)^2 - 8*cosh(x) - 1)*sinh(x)^2 - 3*cosh(x)^2 + 2*(5*co
sh(x)^9 + 9*cosh(x)^8 - 12*cosh(x)^7 - 28*cosh(x)^6 + 6*cosh(x)^5 + 30*cosh
(x)^4 + 4*cosh(x)^3 - 12*cosh(x)^2 - 3*cosh(x) + 1)*sinh(x) + 2*cosh(x) + 1
)*log(cosh(x) + sinh(x) + 1) + 15*(cosh(x)^10 + 2*(5*cosh(x) + 1)*sinh(x)^9
 + sinh(x)^10 + 2*cosh(x)^9 + 3*(15*cosh(x)^2 + 6*cosh(x) - 1)*sinh(x)^8 -
3*cosh(x)^8 + 8*(15*cosh(x)^3 + 9*cosh(x)^2 - 3*cosh(x) - 1)*sinh(x)^7 - 8*
cosh(x)^7 + 2*(105*cosh(x)^4 + 84*cosh(x)^3 - 42*cosh(x)^2 - 28*cosh(x) + 1
)*sinh(x)^6 + 2*cosh(x)^6 + 12*(21*cosh(x)^5 + 21*cosh(x)^4 - 14*cosh(x)^3
```

```
- 14*cosh(x)^2 + cosh(x) + 1)*sinh(x)^5 + 12*cosh(x)^5 + 2*(105*cosh(x)^6 +
 126*cosh(x)^5 - 105*cosh(x)^4 - 140*cosh(x)^3 + 15*cosh(x)^2 + 30*cosh(x)
+ 1)*sinh(x)^4 + 2*cosh(x)^4 + 8*(15*cosh(x)^7 + 21*cosh(x)^6 - 21*cosh(x)^
5 - 35*cosh(x)^4 + 5*cosh(x)^3 + 15*cosh(x)^2 + cosh(x) - 1)*sinh(x)^3 - 8*
cosh(x)^3 + 3*(15*cosh(x)^8 + 24*cosh(x)^7 - 28*cosh(x)^6 - 56*cosh(x)^5 +
10*cosh(x)^4 + 40*cosh(x)^3 + 4*cosh(x)^2 - 8*cosh(x) - 1)*sinh(x)^2 - 3*co
sh(x)^2 + 2*(5*cosh(x)^9 + 9*cosh(x)^8 - 12*cosh(x)^7 - 28*cosh(x)^6 + 6*co
sh(x)^5 + 30*cosh(x)^4 + 4*cosh(x)^3 - 12*cosh(x)^2 - 3*cosh(x) + 1)*sinh(x
) + 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 10*(27*cosh(x)^8 + 48*cosh(
x)^7 - 56*cosh(x)^6 - 132*cosh(x)^5 + 18*cosh(x)^4 - 88*cosh(x)^3 - 24*cosh
(x)^2 + 12*cosh(x) + 3)*sinh(x) + 30*cosh(x))/(a*cosh(x)^10 + a*sinh(x)^10
+ 2*a*cosh(x)^9 + 2*(5*a*cosh(x) + a)*sinh(x)^9 - 3*a*cosh(x)^8 + 3*(15*a*c
osh(x)^2 + 6*a*cosh(x) - a)*sinh(x)^8 - 8*a*cosh(x)^7 + 8*(15*a*cosh(x)^3 +
 9*a*cosh(x)^2 - 3*a*cosh(x) - a)*sinh(x)^7 + 2*a*cosh(x)^6 + 2*(105*a*cosh
(x)^4 + 84*a*cosh(x)^3 - 42*a*cosh(x)^2 - 28*a*cosh(x) + a)*sinh(x)^6 + 12*
a*cosh(x)^5 + 12*(21*a*cosh(x)^5 + 21*a*cosh(x)^4 - 14*a*cosh(x)^3 - 14*a*c
osh(x)^2 + a*cosh(x) + a)*sinh(x)^5 + 2*a*cosh(x)^4 + 2*(105*a*cosh(x)^6 +
126*a*cosh(x)^5 - 105*a*cosh(x)^4 - 140*a*cosh(x)^3 + 15*a*cosh(x)^2 + 30*a
*cosh(x) + a)*sinh(x)^4 - 8*a*cosh(x)^3 + 8*(15*a*cosh(x)^7 + 21*a*cosh(x)^
6 - 21*a*cosh(x)^5 - 35*a*cosh(x)^4 + 5*a*cosh(x)^3 + 15*a*cosh(x)^2 + a*co
sh(x) - a)*sinh(x)^3 - 3*a*cosh(x)^2 + 3*(15*a*cosh(x)^8 + 24*a*cosh(x)^7 -
 28*a*cosh(x)^6 - 56*a*cosh(x)^5 + 10*a*cosh(x)^4 + 40*a*cosh(x)^3 + 4*a*co
sh(x)^2 - 8*a*cosh(x) - a)*sinh(x)^2 + 2*a*cosh(x) + 2*(5*a*cosh(x)^9 + 9*a
*cosh(x)^8 - 12*a*cosh(x)^7 - 28*a*cosh(x)^6 + 6*a*cosh(x)^5 + 30*a*cosh(x)
^4 + 4*a*cosh(x)^3 - 12*a*cosh(x)^2 - 3*a*cosh(x) + a)*sinh(x) + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{csch}^{5}(x)}{\cosh(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**5/(a+a*cosh(x)),x)
```

```
[Out] Integral(csch(x)**5/(cosh(x) + 1), x)/a
```

Giac [A]
time = 0.41, size = 116, normalized size = 1.49

$$-\frac{5\log\left(e^{(-x)}+e^{x}+2\right)}{32\,a}+\frac{5\log\left(e^{(-x)}+e^{x}-2\right)}{32\,a}-\frac{15\left(e^{(-x)}+e^{x}\right)^{2}-76\,e^{(-x)}-76\,e^{x}+100}{64\,a(e^{(-x)}+e^{x}-2)^{2}}+\frac{55\left(e^{(-x)}+e^{x}\right)^{3}+402\left(e^{(-x)}+e^{x}\right)^{2}+1020\,e^{(-x)}+1020\,e^{x}+936}{192\,a(e^{(-x)}+e^{x}+2)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^5/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -5/32*log(e^(-x) + e^x + 2)/a + 5/32*log(e^(-x) + e^x - 2)/a - 1/64*(15*(e^
(-x) + e^x)^2 - 76*e^(-x) - 76*e^x + 100)/(a*(e^(-x) + e^x - 2)^2) + 1/192*
(55*(e^(-x) + e^x)^3 + 402*(e^(-x) + e^x)^2 + 1020*e^(-x) + 1020*e^x + 936)
/(a*(e^(-x) + e^x + 2)^3)
```

Mupad [B]
time = 1.05, size = 244, normalized size = 3.13

$$\frac{1}{a\,\left(10\,{\mathrm{e}}^{2\,x}+10\,{\mathrm{e}}^{3\,x}+5\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{5\,x}+5\,{\mathrm{e}}^x+1\right)}+\frac{1}{4\,a\,\left(3\,{\mathrm{e}}^{2\,x}-{\mathrm{e}}^{3\,x}-3\,{\mathrm{e}}^x+1\right)}+\frac{1}{8\,a\,\left({\mathrm{e}}^{2\,x}-2\,{\mathrm{e}}^x+1\right)}-\frac{1}{8\,a\,\left(6\,{\mathrm{e}}^{2\,x}-4\,{\mathrm{e}}^{3\,x}+{\mathrm{e}}^{4\,x}-4\,{\mathrm{e}}^x+1\right)}-\frac{5}{8\,a\,\left(6\,{\mathrm{e}}^{2\,x}+4\,{\mathrm{e}}^{3\,x}+{\mathrm{e}}^{4\,x}+4\,{\mathrm{e}}^x+1\right)}+\frac{1}{4\,a\,\left({\mathrm{e}}^x-1\right)}+\frac{3}{8\,a\,\left({\mathrm{e}}^x+1\right)}-\frac{5\,\mathrm{atan}\left(\frac{{\mathrm{e}}^x\,\sqrt{-a^2}}{a}\right)}{8\,\sqrt{-a^2}}-\frac{1}{3\,a\,\left(15\,{\mathrm{e}}^{2\,x}+20\,{\mathrm{e}}^{3\,x}+15\,{\mathrm{e}}^{4\,x}+6\,{\mathrm{e}}^{5\,x}+{\mathrm{e}}^{6\,x}+6\,{\mathrm{e}}^x+1\right)}-\frac{5}{12\,a\,\left(3\,{\mathrm{e}}^{2\,x}+{\mathrm{e}}^{3\,x}+3\,{\mathrm{e}}^x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^5*(a + a*cosh(x))),x)
```

```
[Out] 1/(a*(10*exp(2*x) + 10*exp(3*x) + 5*exp(4*x) + exp(5*x) + 5*exp(x) + 1)) +
1/(4*a*(3*exp(2*x) - exp(3*x) - 3*exp(x) + 1)) + 1/(8*a*(exp(2*x) - 2*exp(x
) + 1)) - 1/(8*a*(6*exp(2*x) - 4*exp(3*x) + exp(4*x) - 4*exp(x) + 1)) - 5/(
8*a*(6*exp(2*x) + 4*exp(3*x) + exp(4*x) + 4*exp(x) + 1)) + 1/(4*a*(exp(x) -
 1)) + 3/(8*a*(exp(x) + 1)) - (5*atan((exp(x)*(-a^2)^(1/2))/a))/(8*(-a^2)^(
1/2)) - 1/(3*a*(15*exp(2*x) + 20*exp(3*x) + 15*exp(4*x) + 6*exp(5*x) + exp(
6*x) + 6*exp(x) + 1)) - 5/(12*a*(3*exp(2*x) + exp(3*x) + 3*exp(x) + 1))
```

3.165 $\int \frac{\sinh^7(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=140

$$-\frac{a(a^4-3a^2b^2+3b^4)\cosh(x)}{b^6}+\frac{(a^4-3a^2b^2+3b^4)\cosh^2(x)}{2b^5}-\frac{a(a^2-3b^2)\cosh^3(x)}{3b^4}+\frac{(a^2-3b^2)\cosh^4(x)}{4b^3}-\frac{a}{}$$

```
[Out] -a*(a^4-3*a^2*b^2+3*b^4)*cosh(x)/b^6+1/2*(a^4-3*a^2*b^2+3*b^4)*cosh(x)^2/b^
5-1/3*a*(a^2-3*b^2)*cosh(x)^3/b^4+1/4*(a^2-3*b^2)*cosh(x)^4/b^3-1/5*a*cosh(
x)^5/b^2+1/6*cosh(x)^6/b+(a^2-b^2)^3*ln(a+b*cosh(x))/b^7
```

Rubi [A]

time = 0.12, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2747, 711}

$$\frac{(a^2-b^2)^3\log(a+b\cosh(x))}{b^7}-\frac{a(a^2-3b^2)\cosh^3(x)}{3b^4}+\frac{(a^2-3b^2)\cosh^4(x)}{4b^3}-\frac{a(a^4-3a^2b^2+3b^4)\cosh(x)}{b^6}+\frac{(a^4-3a^2b^2+3b^4)\cosh^2(x)}{2b^5}-\frac{a\cosh^5(x)}{5b^2}+\frac{\cosh^6(x)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^7/(a + b*Cosh[x]),x]
```

```
[Out] -((a*(a^4 - 3*a^2*b^2 + 3*b^4)*Cosh[x])/b^6) + ((a^4 - 3*a^2*b^2 + 3*b^4)*C
osh[x]^2)/(2*b^5) - (a*(a^2 - 3*b^2)*Cosh[x]^3)/(3*b^4) + ((a^2 - 3*b^2)*Co
sh[x]^4)/(4*b^3) - (a*Cosh[x]^5)/(5*b^2) + Cosh[x]^6/(6*b) + ((a^2 - b^2)^3
*Log[a + b*Cosh[x]])/b^7
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sinh^7(x)}{a+b\cosh(x)}\,dx = -\frac{\text{Subst}\left(\int \frac{(b^2-x^2)^3}{a+x}\,dx, x, b\cosh(x)\right)}{b^7}$$

$$= -\frac{\text{Subst}\left(\int \left(a^5\left(1+\frac{3b^2(-a^2+b^2)}{a^4}\right) - (a^4-3a^2b^2+3b^4)\,x + a(a^2-3b^2)\,x^2 - (a^2-3b^2)\,x^3 \right.\right.}{b^7}$$

$$= -\frac{a(a^4-3a^2b^2+3b^4)\cosh(x)}{b^6} + \frac{(a^4-3a^2b^2+3b^4)\cosh^2(x)}{2b^5} - \frac{a(a^2-3b^2)\cosh^3(x)}{3b^4} +$$

Mathematica [A]
time = 0.13, size = 144, normalized size = 1.03

$$\frac{-120ab(8a^4-22a^2b^2+19b^4)\cosh(x)+15b^2(16a^4-40a^2b^2+29b^4)\cosh(2x)-20a(2a-3b)b^3(2a+3b)\cosh(3x)-30b^4(-a^2+2b^2)\cosh(4x)-12ab^5\cosh(5x)+5b^6\cosh(6x)+960(a^2-b^2)^3\log(a+b\cosh(x))}{960b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^7/(a + b*Cosh[x]),x]
```

```
[Out] (-120*a*b*(8*a^4 - 22*a^2*b^2 + 19*b^4)*Cosh[x] + 15*b^2*(16*a^4 - 40*a^2*b
^2 + 29*b^4)*Cosh[2*x] - 20*a*(2*a - 3*b)*b^3*(2*a + 3*b)*Cosh[3*x] - 30*b^
4*(-a^2 + 2*b^2)*Cosh[4*x] - 12*a*b^5*Cosh[5*x] + 5*b^6*Cosh[6*x] + 960*(a^
2 - b^2)^3*Log[a + b*Cosh[x]])/(960*b^7)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 546 vs. 2(130) = 260.
time = 0.45, size = 547, normalized size = 3.91

method	result
risch	$-\frac{19a\,e^{x}}{16b^2}+\frac{11a^3e^{x}}{8b^4}-\frac{a^5e^{-x}}{2b^6}+\frac{11a^3e^{-x}}{8b^4}-\frac{19a\,e^{-x}}{16b^2}-\frac{3x\,a^2}{b^3}+\frac{e^{-2x}a^4}{8b^5}-\frac{5\,e^{-2x}a^2}{16b^3}-\frac{a^3e^{-3x}}{24b^4}+\frac{3a\,e^{-3x}}{32b^2}+\frac{e^{-4x}a^2}{64b^3}-$
default	$\frac{1}{6b(\tanh(\frac{x}{2})-1)^6}-\frac{-5b-2a}{10b^2(\tanh(\frac{x}{2})-1)^5}-\frac{-2a^2-4ab-b^2}{8b^3(\tanh(\frac{x}{2})-1)^4}-\frac{-4a^3-6a^2b+3a\,b^2+7b^3}{12b^4(\tanh(\frac{x}{2})-1)^3}+\frac{(-a^6+3a^4b^2-3a^2b^4+b^6)\ln(\tanh(\frac{x}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^7/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/b/(tanh(1/2*x)-1)^6-1/10*(-5*b-2*a)/b^2/(tanh(1/2*x)-1)^5-1/8*(-2*a^2-4
*a*b-b^2)/b^3/(tanh(1/2*x)-1)^4-1/12*(-4*a^3-6*a^2*b+3*a*b^2+7*b^3)/b^4/(ta
nh(1/2*x)-1)^3+(-a^6+3*a^4*b^2-3*a^2*b^4+b^6)/b^7*ln(tanh(1/2*x)-1)-1/16*(-
8*a^4-8*a^3*b+14*a^2*b^2+14*a*b^3-5*b^4)/b^5/(tanh(1/2*x)-1)^2-1/16*(-16*a^
5-8*a^4*b+40*a^3*b^2+18*a^2*b^3-30*a*b^4-11*b^5)/b^6/(tanh(1/2*x)-1)+(a^7-a
^6*b-3*a^5*b^2+3*a^4*b^3+3*a^3*b^4-3*a^2*b^5-a*b^6+b^7)/b^7/(a-b)*ln(a*tanh
(1/2*x)^2-b*tanh(1/2*x)^2-a-b)+1/6/b/(tanh(1/2*x)+1)^6-1/10*(5*b+2*a)/b^2/(
```

```
tanh(1/2*x)+1)^5-1/8*(-2*a^2-4*a*b-b^2)/b^3/(tanh(1/2*x)+1)^4-1/12*(4*a^3+6
*a^2*b-3*a*b^2-7*b^3)/b^4/(tanh(1/2*x)+1)^3+(-a^6+3*a^4*b^2-3*a^2*b^4+b^6)/
b^7*ln(tanh(1/2*x)+1)-1/16*(-8*a^4-8*a^3*b+14*a^2*b^2+14*a*b^3-5*b^4)/b^5/(
tanh(1/2*x)+1)^2-1/16*(16*a^5+8*a^4*b-40*a^3*b^2-18*a^2*b^3+30*a*b^4+11*b^5
)/b^6/(tanh(1/2*x)+1)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(130) = 260$.
time = 0.27, size = 310, normalized size = 2.21

$$-\frac{\left(12\,ab^4e^{(-x)} - 5\,b^5 - 30\,(a^2b^3 - 2\,b^5)e^{(-2\,x)} + 20\,(4\,a^3b^2 - 9\,ab^4)e^{(-3\,x)} - 15\,(16\,a^4b - 40\,a^2b^3 + 29\,b^5)e^{(-4\,x)} + 120\,(8\,a^5 - 22\,a^3b^2 + 19\,ab^4)e^{(-5\,x)}\right)e^{(6\,x)}}{1920\,b^6} - \frac{12\,ab^4e^{(-5\,x)} - 5\,b^5e^{(-6\,x)} + 120\,(8\,a^5 - 22\,a^3b^2 + 19\,ab^4)e^{(-x)} - 15\,(16\,a^4b - 40\,a^2b^3 + 29\,b^5)e^{(-2\,x)} + 20\,(4\,a^3b^2 - 9\,ab^4)e^{(-3\,x)} - 30\,(a^2b^3 - 2\,b^5)e^{(-4\,x)}}{1920\,b^6} + \frac{(a^6 - 3\,a^4b^2 + 3\,a^2b^4 - b^6)x}{b^7} + \frac{(a^6 - 3\,a^4b^2 + 3\,a^2b^4 - b^6)\log\left(2\,ae^{(-x)} + be^{(-2\,x)} + b\right)}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^7/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] -1/1920*(12*a*b^4*e^(-x) - 5*b^5 - 30*(a^2*b^3 - 2*b^5)*e^(-2*x) + 20*(4*a^
3*b^2 - 9*a*b^4)*e^(-3*x) - 15*(16*a^4*b - 40*a^2*b^3 + 29*b^5)*e^(-4*x) +
120*(8*a^5 - 22*a^3*b^2 + 19*a*b^4)*e^(-5*x))*e^(6*x)/b^6 - 1/1920*(12*a*b^
4*e^(-5*x) - 5*b^5*e^(-6*x) + 120*(8*a^5 - 22*a^3*b^2 + 19*a*b^4)*e^(-x) -
15*(16*a^4*b - 40*a^2*b^3 + 29*b^5)*e^(-2*x) + 20*(4*a^3*b^2 - 9*a*b^4)*e^(
-3*x) - 30*(a^2*b^3 - 2*b^5)*e^(-4*x))/b^6 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 -
 b^6)*x/b^7 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*log(2*a*e^(-x) + b*e^(-2*
x) + b)/b^7
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2134 vs. $2(130) = 260$.
time = 0.46, size = 2134, normalized size = 15.24

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^7/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/1920*(5*b^6*cosh(x)^12 + 5*b^6*sinh(x)^12 - 12*a*b^5*cosh(x)^11 + 12*(5*b
^6*cosh(x) - a*b^5)*sinh(x)^11 + 30*(a^2*b^4 - 2*b^6)*cosh(x)^10 + 6*(55*b^
6*cosh(x)^2 - 22*a*b^5*cosh(x) + 5*a^2*b^4 - 10*b^6)*sinh(x)^10 - 20*(4*a^3
*b^3 - 9*a*b^5)*cosh(x)^9 + 20*(55*b^6*cosh(x)^3 - 33*a*b^5*cosh(x)^2 - 4*a
^3*b^3 + 9*a*b^5 + 15*(a^2*b^4 - 2*b^6)*cosh(x))*sinh(x)^9 + 15*(16*a^4*b^2
 - 40*a^2*b^4 + 29*b^6)*cosh(x)^8 + 15*(165*b^6*cosh(x)^4 - 132*a*b^5*cosh(
x)^3 + 16*a^4*b^2 - 40*a^2*b^4 + 29*b^6 + 90*(a^2*b^4 - 2*b^6)*cosh(x)^2 -
12*(4*a^3*b^3 - 9*a*b^5)*cosh(x))*sinh(x)^8 - 1920*(a^6 - 3*a^4*b^2 + 3*a^2
*b^4 - b^6)*x*cosh(x)^6 - 120*(8*a^5*b - 22*a^3*b^3 + 19*a*b^5)*cosh(x)^7 +
 120*(33*b^6*cosh(x)^5 - 33*a*b^5*cosh(x)^4 - 8*a^5*b + 22*a^3*b^3 - 19*a*b
^5 + 30*(a^2*b^4 - 2*b^6)*cosh(x)^3 - 6*(4*a^3*b^3 - 9*a*b^5)*cosh(x)^2 + (
16*a^4*b^2 - 40*a^2*b^4 + 29*b^6)*cosh(x))*sinh(x)^7 - 12*a*b^5*cosh(x) + 1
2*(385*b^6*cosh(x)^6 - 462*a*b^5*cosh(x)^5 + 525*(a^2*b^4 - 2*b^6)*cosh(x)^
```

4 - 140*(4*a^3*b^3 - 9*a*b^5)*cosh(x)^3 + 35*(16*a^4*b^2 - 40*a^2*b^4 + 29*b^6)*cosh(x)^2 - 160*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x - 70*(8*a^5*b - 22*a^3*b^3 + 19*a*b^5)*cosh(x))*sinh(x)^6 + 5*b^6 - 120*(8*a^5*b - 22*a^3*b^3 + 19*a*b^5)*cosh(x)^5 + 24*(165*b^6*cosh(x)^7 - 231*a*b^5*cosh(x)^6 - 40*a^5*b + 110*a^3*b^3 - 95*a*b^5 + 315*(a^2*b^4 - 2*b^6)*cosh(x)^5 - 105*(4*a^3*b^3 - 9*a*b^5)*cosh(x)^4 + 35*(16*a^4*b^2 - 40*a^2*b^4 + 29*b^6)*cosh(x)^3 - 480*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x) - 105*(8*a^5*b - 22*a^3*b^3 + 19*a*b^5)*cosh(x)^2)*sinh(x)^5 + 15*(16*a^4*b^2 - 40*a^2*b^4 + 29*b^6)*cosh(x)^4 + 15*(165*b^6*cosh(x)^8 - 264*a*b^5*cosh(x)^7 + 420*(a^2*b^4 - 2*b^6)*cosh(x)^6 + 16*a^4*b^2 - 40*a^2*b^4 + 29*b^6 - 168*(4*a^3*b^3 - 9*a*b^5)*cosh(x)^5 + 70*(16*a^4*b^2 - 40*a^2*b^4 + 29*b^6)*cosh(x)^4 - 1920*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^2 - 280*(8*a^5*b - 22*a^3*b^3 + 19*a*b^5)*cosh(x)^3 - 40*(8*a^5*b - 22*a^3*b^3 + 19*a*b^5)*cosh(x))*sinh(x)^4 - 20*(4*a^3*b^3 - 9*a*b^5)*cosh(x)^3 + 20*(55*b^6*cosh(x)^9 - 99*a*b^5*cosh(x)^8 + 180*(a^2*b^4 - 2*b^6)*cosh(x)^7 - 84*(4*a^3*b^3 - 9*a*b^5)*cosh(x)^6 - 4*a^3*b^3 + 9*a*b^5 + 42*(16*a^4*b^2 - 40*a^2*b^4 + 29*b^6)*cosh(x)^5 - 1920*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^3 - 210*(8*a^5*b - 22*a^3*b^3 + 19*a*b^5)*cosh(x)^4 - 60*(8*a^5*b - 22*a^3*b^3 + 19*a*b^5)*cosh(x)^2 + 3*(16*a^4*b^2 - 40*a^2*b^4 + 29*b^6)*cosh(x))*sinh(x)^3 + 30*(a^2*b^4 - 2*b^6)*cosh(x)^2 + 30*(11*b^6*cosh(x)^10 - 22*a*b^5*cosh(x)^9 + 45*(a^2*b^4 - 2*b^6)*cosh(x)^8 - 24*(4*a^3*b^3 - 9*a*b^5)*cosh(x)^7 + 14*(16*a^4*b^2 - 40*a^2*b^4 + 29*b^6)*cosh(x)^6 + a^2*b^4 - 2*b^6 - 960*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^4 - 84*(8*a^5*b - 22*a^3*b^3 + 19*a*b^5)*cosh(x)^5 - 40*(8*a^5*b - 22*a^3*b^3 + 19*a*b^5)*cosh(x)^3 + 3*(16*a^4*b^2 - 40*a^2*b^4 + 29*b^6)*cosh(x)^2 - 2*(4*a^3*b^3 - 9*a*b^5)*cosh(x))*sinh(x)^2 + 1920*((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^6 + 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^5*sinh(x) + 15*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4*sinh(x)^2 + 20*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3*sinh(x)^3 + 15*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2*sinh(x)^4 + 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^5 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^6)*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))) + 12*(5*b^6*cosh(x)^11 - 11*a*b^5*cosh(x)^10 + 25*(a^2*b^4 - 2*b^6)*cosh(x)^9 - 15*(4*a^3*b^3 - 9*a*b^5)*cosh(x)^8 + 10*(16*a^4*b^2 - 40*a^2*b^4 + 29*b^6)*cosh(x)^7 - 960*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^5 - 70*(8*a^5*b - 22*a^3*b^3 + 19*a*b^5)*cosh(x)^6 - a*b^5 - 50*(8*a^5*b - 22*a^3*b^3 + 19*a*b^5)*cosh(x)^4 + 5*(16*a^4*b^2 - 40*a^2*b^4 + 29*b^6)*cosh(x)^3 - 5*(4*a^3*b^3 - 9*a*b^5)*cosh(x)^2 + 5*(a^2*b^4 - 2*b^6)*cosh(x))*sinh(x))/(b^7*cosh(x)^6 + 6*b^7*cosh(x)^5*sinh(x) + 15*b^7*cosh(x)^4*sinh(x)^2 + 20*b^7*cosh(x)^3*sinh(x)^3 + 15*b^7*cosh(x)^2*sinh(x)^4 + 6*b^7*cosh(x)*sinh(x)^5 + b^7*sinh(x)^6)

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**7/(a+b*cosh(x)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.43, size = 229, normalized size = 1.64

$$\frac{5\,b^{5}\left(e^{(-x)}+e^{x}\right)^{6}-12\,ab^{4}\left(e^{(-x)}+e^{x}\right)^{5}+30\,a^{2}b^{3}\left(e^{(-x)}+e^{x}\right)^{4}-90\,b^{5}\left(e^{(-x)}+e^{x}\right)^{4}-80\,a^{3}b^{2}\left(e^{(-x)}+e^{x}\right)^{3}+240\,ab^{4}\left(e^{(-x)}+e^{x}\right)^{3}+240\,a^{4}b\left(e^{(-x)}+e^{x}\right)^{2}-720\,a^{2}b^{3}\left(e^{(-x)}+e^{x}\right)^{2}+720\,b^{5}\left(e^{(-x)}+e^{x}\right)^{2}-960\,a^{5}\left(e^{(-x)}+e^{x}\right)+2880\,a^{3}b^{2}\left(e^{(-x)}+e^{x}\right)-2880\,ab^{4}\left(e^{(-x)}+e^{x}\right)}{1920\,b^{6}}+\frac{\left(a^{6}-3\,a^{4}b^{2}+3\,a^{2}b^{4}-b^{6}\right)\log\left(\left|b\left(e^{(-x)}+e^{x}\right)+2\,a\right|\right)}{b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^7/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 1/1920*(5*b^5*(e^(-x) + e^x)^6 - 12*a*b^4*(e^(-x) + e^x)^5 + 30*a^2*b^3*(e^
(-x) + e^x)^4 - 90*b^5*(e^(-x) + e^x)^4 - 80*a^3*b^2*(e^(-x) + e^x)^3 + 240
*a*b^4*(e^(-x) + e^x)^3 + 240*a^4*b*(e^(-x) + e^x)^2 - 720*a^2*b^3*(e^(-x)
+ e^x)^2 + 720*b^5*(e^(-x) + e^x)^2 - 960*a^5*(e^(-x) + e^x) + 2880*a^3*b^2
*(e^(-x) + e^x) - 2880*a*b^4*(e^(-x) + e^x))/b^6 + (a^6 - 3*a^4*b^2 + 3*a^2
*b^4 - b^6)*log(abs(b*(e^(-x) + e^x) + 2*a))/b^7
```

Mupad [B]
time = 1.71, size = 289, normalized size = 2.06

$$\frac{\mathrm{e}^{-6\,x}}{384\,b}+\frac{\mathrm{e}^{6\,x}}{384\,b}-\frac{x\,{\left(a^2-b^2\right)}^3}{b^7}-\frac{\mathrm{e}^{-x}\,\left(8\,a^5-22\,a^3\,b^2+19\,a\,b^4\right)}{16\,b^6}+\frac{\mathrm{e}^{-3\,x}\,\left(9\,a\,b^2-4\,a^3\right)}{96\,b^4}+\frac{\mathrm{e}^{3\,x}\,\left(9\,a\,b^2-4\,a^3\right)}{96\,b^4}+\frac{\mathrm{e}^{-4\,x}\,\left(a^2-2\,b^2\right)}{64\,b^3}+\frac{\mathrm{e}^{4\,x}\,\left(a^2-2\,b^2\right)}{64\,b^3}-\frac{a\,\mathrm{e}^{-5\,x}}{160\,b^2}-\frac{a\,\mathrm{e}^{5\,x}}{160\,b^2}+\frac{\mathrm{e}^{-2\,x}\,\left(16\,a^4-40\,a^2\,b^2+29\,b^4\right)}{128\,b^5}+\frac{\mathrm{e}^{2\,x}\,\left(16\,a^4-40\,a^2\,b^2+29\,b^4\right)}{128\,b^5}-\frac{\mathrm{e}^{x}\,\left(8\,a^5-22\,a^3\,b^2+19\,a\,b^4\right)}{16\,b^6}+\frac{\ln\left(b+2\,a\,\mathrm{e}^{x}+b\,\mathrm{e}^{2\,x}\right)\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^7/(a + b*cosh(x)),x)
```

```
[Out] exp(-6*x)/(384*b) + exp(6*x)/(384*b) - (x*(a^2 - b^2)^3)/b^7 - (exp(-x)*(19
*a*b^4 + 8*a^5 - 22*a^3*b^2))/(16*b^6) + (exp(-3*x)*(9*a*b^2 - 4*a^3))/(96*
b^4) + (exp(3*x)*(9*a*b^2 - 4*a^3))/(96*b^4) + (exp(-4*x)*(a^2 - 2*b^2))/(6
4*b^3) + (exp(4*x)*(a^2 - 2*b^2))/(64*b^3) - (a*exp(-5*x))/(160*b^2) - (a*e
xp(5*x))/(160*b^2) + (exp(-2*x)*(16*a^4 + 29*b^4 - 40*a^2*b^2))/(128*b^5) +
 (exp(2*x)*(16*a^4 + 29*b^4 - 40*a^2*b^2))/(128*b^5) - (exp(x)*(19*a*b^4 +
8*a^5 - 22*a^3*b^2))/(16*b^6) + (log(b + 2*a*exp(x) + b*exp(2*x))*(a^6 - b^
6 + 3*a^2*b^4 - 3*a^4*b^2))/b^7
```

3.166 $\int \frac{\sinh^6(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=154

$$-\frac{a(8a^4-20a^2b^2+15b^4)\,x}{8b^6}+\frac{2(a-b)^{5/2}(a+b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^6}+\frac{\left(8(a^2-b^2)^2-ab(4a^2-7b^2)\,\mathrm{c}\right.}{8b^5}$$

```
[Out] -1/8*a*(8*a^4-20*a^2*b^2+15*b^4)*x/b^6+2*(a-b)^(5/2)*(a+b)^(5/2)*arctanh((a
-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/b^6+1/8*(8*(a^2-b^2)^2-a*b*(4*a^2-7*b^2)
*cosh(x))*sinh(x)/b^5+1/12*(4*a^2-4*b^2-3*a*b*cosh(x))*sinh(x)^3/b^3+1/5*si
nh(x)^5/b
```

Rubi [A]
time = 0.30, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2774, 2944, 2814, 2738, 214}

$$\frac{\sinh(x)\left(8(a^2-b^2)^2-ab(4a^2-7b^2)\cosh(x)\right)}{8b^5}+\frac{\sinh^3(x)\left(4(a^2-b^2)-3ab\cosh(x)\right)}{12b^3}-\frac{ax(8a^4-20a^2b^2+15b^4)}{8b^6}+\frac{2(a-b)^{5/2}(a+b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^6}+\frac{\sinh^5(x)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^6/(a + b*Cosh[x]),x]
```

```
[Out] -1/8*(a*(8*a^4 - 20*a^2*b^2 + 15*b^4)*x)/b^6 + (2*(a - b)^(5/2)*(a + b)^(5/
2)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/b^6 + ((8*(a^2 - b^2)^2 -
a*b*(4*a^2 - 7*b^2)*Cosh[x])*Sinh[x])/(8*b^5) + ((4*(a^2 - b^2) - 3*a*b*Cos
h[x])*Sinh[x]^3)/(12*b^3) + Sinh[x]^5/(5*b)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
```

```
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^6(x)}{a+b\cosh(x)}\,dx &= \frac{\sinh^5(x)}{5b} + \frac{\int \frac{(-b-a\cosh(x))\sinh^4(x)}{a+b\cosh(x)}\,dx}{b} \\
&= \frac{(4(a^2-b^2)-3ab\cosh(x))\sinh^3(x)}{12b^3} + \frac{\sinh^5(x)}{5b} - \frac{\int \frac{(b(a^2-4b^2)+a(4a^2-7b^2)\cosh(x))\sinh^2(x)}{a+b\cosh(x)}
\\
&= \frac{\left(8(a^2-b^2)^2-ab(4a^2-7b^2)\cosh(x)\right)\sinh(x)}{8b^5} + \frac{(4(a^2-b^2)-3ab\cosh(x))\sinh^3(x)
\\
&= -\frac{a(8a^4-20a^2b^2+15b^4)\,x}{8b^6} + \frac{\left(8(a^2-b^2)^2-ab(4a^2-7b^2)\cosh(x)\right)\sinh(x)}{8b^5} + \frac{(4(a^2
\\
&= -\frac{a(8a^4-20a^2b^2+15b^4)\,x}{8b^6} + \frac{\left(8(a^2-b^2)^2-ab(4a^2-7b^2)\cosh(x)\right)\sinh(x)}{8b^5} + \frac{(4(a^2
\\
&= -\frac{a(8a^4-20a^2b^2+15b^4)\,x}{8b^6} + \frac{2(a-b)^{5/2}(a+b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^6} + \frac{\left(8(a
\end{aligned}
$$

Mathematica [A]

time = 0.17, size = 154, normalized size = 1.00

$$\frac{-60a(8a^4-20a^2b^2+15b^4)x+960(-a^2+b^2)^{5/2}\,\text{ArcTan}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)+60b(8a^4-18a^2b^2+11b^4)\sinh(x)-120ab^2(a^2-2b^2)\sinh(2x)-10b^3(-4a^2+7b^2)\sinh(3x)-15ab^4\sinh(4x)+6b^5\sinh(5x)}{480b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^6/(a + b*Cosh[x]),x]
```

```
[Out] (-60*a*(8*a^4 - 20*a^2*b^2 + 15*b^4)*x + 960*(-a^2 + b^2)^(5/2)*ArcTan[((a
- b)*Tanh[x/2])/Sqrt[-a^2 + b^2]] + 60*b*(8*a^4 - 18*a^2*b^2 + 11*b^4)*Sinh
[x] - 120*a*b^2*(a^2 - 2*b^2)*Sinh[2*x] - 10*b^3*(-4*a^2 + 7*b^2)*Sinh[3*x]
 - 15*a*b^4*Sinh[4*x] + 6*b^5*Sinh[5*x])/(480*b^6)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(135) = 270$.
time = 0.48, size = 409, normalized size = 2.66

method	result
default	$-\frac{2(-a^6+3a^4b^2-3a^2b^4+b^6)\,\text{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^6\sqrt{(a+b)(a-b)}}-\frac{1}{5b\left(\tanh\left(\frac{x}{2}\right)-1\right)^5}-\frac{2b+a}{4b^2\left(\tanh\left(\frac{x}{2}\right)-1\right)^4}-\frac{4a^2+6ab-b^2}{12b^3\left(\tanh\left(\frac{x}{2}\right)-1\right.}$
risch	$-\frac{15ax}{8b^2}+\frac{e^{3x}a^2}{24b^3}-\frac{a^3e^{2x}}{8b^4}+\frac{a\,e^{2x}}{4b^2}-\frac{a^5x}{b^6}-\frac{a\,e^{4x}}{64b^2}+\frac{11\,e^{x}}{16b}+\frac{5a^3x}{2b^4}+\frac{a\,e^{-4x}}{64b^2}+\frac{e^{x}a^4}{2b^5}-\frac{9\,e^{x}a^2}{8b^3}-\frac{e^{-x}a^4}{2b^5}+\frac{9\,e^{-x}a^2}{8b^3}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^6/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/b^6*(-a^6+3*a^4*b^2-3*a^2*b^4+b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan
h(1/2*x)/((a+b)*(a-b))^(1/2))-1/5/b/(tanh(1/2*x)-1)^5-1/4*(2*b+a)/b^2/(tanh
(1/2*x)-1)^4-1/12*(4*a^2+6*a*b-b^2)/b^3/(tanh(1/2*x)-1)^3-1/8*(4*a^3+4*a^2*
b-5*a*b^2-5*b^3)/b^4/(tanh(1/2*x)-1)^2-1/8*(8*a^4+4*a^3*b-16*a^2*b^2-7*a*b^
3+8*b^4)/b^5/(tanh(1/2*x)-1)+1/8*a*(8*a^4-20*a^2*b^2+15*b^4)/b^6*ln(tanh(1/
2*x)-1)-1/5/b/(tanh(1/2*x)+1)^5-1/4*(-2*b-a)/b^2/(tanh(1/2*x)+1)^4-1/12*(4*
a^2+6*a*b-b^2)/b^3/(tanh(1/2*x)+1)^3-1/8*(-4*a^3-4*a^2*b+5*a*b^2+5*b^3)/b^4
/(tanh(1/2*x)+1)^2-1/8*(8*a^4+4*a^3*b-16*a^2*b^2-7*a*b^3+8*b^4)/b^5/(tanh(1
/2*x)+1)-1/8*a*(8*a^4-20*a^2*b^2+15*b^4)/b^6*ln(tanh(1/2*x)+1)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^6/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1422 vs. $2(134) = 268$.
time = 0.40, size = 2913, normalized size = 18.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^6/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [1/960*(6*b^5*cosh(x)^10 + 6*b^5*sinh(x)^10 - 15*a*b^4*cosh(x)^9 + 15*(4*b^
5*cosh(x) - a*b^4)*sinh(x)^9 + 10*(4*a^2*b^3 - 7*b^5)*cosh(x)^8 + 5*(54*b^5
*cosh(x)^2 - 27*a*b^4*cosh(x) + 8*a^2*b^3 - 14*b^5)*sinh(x)^8 - 120*(a^3*b^
2 - 2*a*b^4)*cosh(x)^7 + 20*(36*b^5*cosh(x)^3 - 27*a*b^4*cosh(x)^2 - 6*a^3*
b^2 + 12*a*b^4 + 4*(4*a^2*b^3 - 7*b^5)*cosh(x))*sinh(x)^7 - 120*(8*a^5 - 20
*a^3*b^2 + 15*a*b^4)*x*cosh(x)^5 + 60*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(
x)^6 + 20*(63*b^5*cosh(x)^4 - 63*a*b^4*cosh(x)^3 + 24*a^4*b - 54*a^2*b^3 +
33*b^5 + 14*(4*a^2*b^3 - 7*b^5)*cosh(x)^2 - 42*(a^3*b^2 - 2*a*b^4)*cosh(x))
*sinh(x)^6 + 15*a*b^4*cosh(x) + 2*(756*b^5*cosh(x)^5 - 945*a*b^4*cosh(x)^4
+ 280*(4*a^2*b^3 - 7*b^5)*cosh(x)^3 - 1260*(a^3*b^2 - 2*a*b^4)*cosh(x)^2 -
60*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*x + 180*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*
cosh(x))*sinh(x)^5 - 6*b^5 - 60*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(x)^4 +
 10*(126*b^5*cosh(x)^6 - 189*a*b^4*cosh(x)^5 - 48*a^4*b + 108*a^2*b^3 - 66*
b^5 + 70*(4*a^2*b^3 - 7*b^5)*cosh(x)^4 - 420*(a^3*b^2 - 2*a*b^4)*cosh(x)^3
- 60*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*x*cosh(x) + 90*(8*a^4*b - 18*a^2*b^3 +
 11*b^5)*cosh(x)^2)*sinh(x)^4 + 120*(a^3*b^2 - 2*a*b^4)*cosh(x)^3 + 20*(36*
b^5*cosh(x)^7 - 63*a*b^4*cosh(x)^6 + 28*(4*a^2*b^3 - 7*b^5)*cosh(x)^5 + 6*a
^3*b^2 - 12*a*b^4 - 210*(a^3*b^2 - 2*a*b^4)*cosh(x)^4 - 60*(8*a^5 - 20*a^3*
b^2 + 15*a*b^4)*x*cosh(x)^2 + 60*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(x)^3
- 12*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(x))*sinh(x)^3 - 10*(4*a^2*b^3 - 7
*b^5)*cosh(x)^2 + 10*(27*b^5*cosh(x)^8 - 54*a*b^4*cosh(x)^7 + 28*(4*a^2*b^3
 - 7*b^5)*cosh(x)^6 - 252*(a^3*b^2 - 2*a*b^4)*cosh(x)^5 - 4*a^2*b^3 + 7*b^5
 - 120*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*x*cosh(x)^3 + 90*(8*a^4*b - 18*a^2*b
^3 + 11*b^5)*cosh(x)^4 - 36*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(x)^2 + 36*
(a^3*b^2 - 2*a*b^4)*cosh(x))*sinh(x)^2 + 960*((a^4 - 2*a^2*b^2 + b^4)*cosh(
x)^5 + 5*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^4*sinh(x) + 10*(a^4 - 2*a^2*b^2 +
b^4)*cosh(x)^3*sinh(x)^2 + 10*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2*sinh(x)^3 +
 5*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^4 + (a^4 - 2*a^2*b^2 + b^4)*sinh
(x)^5)*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) +
 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x)
 + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x)
```

```
+ a)*sinh(x) + b)) + 5*(12*b^5*cosh(x)^9 - 27*a*b^4*cosh(x)^8 + 16*(4*a^2*b
^3 - 7*b^5)*cosh(x)^7 - 168*(a^3*b^2 - 2*a*b^4)*cosh(x)^6 - 120*(8*a^5 - 20
*a^3*b^2 + 15*a*b^4)*x*cosh(x)^4 + 72*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(
x)^5 + 3*a*b^4 - 48*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(x)^3 + 72*(a^3*b^2
 - 2*a*b^4)*cosh(x)^2 - 4*(4*a^2*b^3 - 7*b^5)*cosh(x))*sinh(x))/(b^6*cosh(x
)^5 + 5*b^6*cosh(x)^4*sinh(x) + 10*b^6*cosh(x)^3*sinh(x)^2 + 10*b^6*cosh(x)
^2*sinh(x)^3 + 5*b^6*cosh(x)*sinh(x)^4 + b^6*sinh(x)^5), 1/960*(6*b^5*cosh(
x)^10 + 6*b^5*sinh(x)^10 - 15*a*b^4*cosh(x)^9 + 15*(4*b^5*cosh(x) - a*b^4)*
sinh(x)^9 + 10*(4*a^2*b^3 - 7*b^5)*cosh(x)^8 + 5*(54*b^5*cosh(x)^2 - 27*a*b
^4*cosh(x) + 8*a^2*b^3 - 14*b^5)*sinh(x)^8 - 120*(a^3*b^2 - 2*a*b^4)*cosh(x
)^7 + 20*(36*b^5*cosh(x)^3 - 27*a*b^4*cosh(x)^2 - 6*a^3*b^2 + 12*a*b^4 + 4*
(4*a^2*b^3 - 7*b^5)*cosh(x))*sinh(x)^7 - 120*(8*a^5 - 20*a^3*b^2 + 15*a*b^4
)*x*cosh(x)^5 + 60*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(x)^6 + 20*(63*b^5*c
osh(x)^4 - 63*a*b^4*cosh(x)^3 + 24*a^4*b - 54*a^2*b^3 + 33*b^5 + 14*(4*a^2*
b^3 - 7*b^5)*cosh(x)^2 - 42*(a^3*b^2 - 2*a*b^4)*cosh(x))*sinh(x)^6 + 15*a*b
^4*cosh(x) + 2*(756*b^5*cosh(x)^5 - 945*a*b^4*cosh(x)^4 + 280*(4*a^2*b^3 -
7*b^5)*cosh(x)^3 - 1260*(a^3*b^2 - 2*a*b^4)*cosh(x)^2 - 60*(8*a^5 - 20*a^3*
b^2 + 15*a*b^4)*x + 180*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(x))*sinh(x)^5
- 6*b^5 - 60*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(x)^4 + 10*(126*b^5*cosh(x
)^6 - 189*a*b^4*cosh(x)^5 - 48*a^4*b + 108*a^2*b^3 - 66*b^5 + 70*(4*a^2*b^3
 - 7*b^5)*cosh(x)^4 - 420*(a^3*b^2 - 2*a*b^4)*cosh(x)^3 - 60*(8*a^5 - 20*a^
3*b^2 + 15*a*b^4)*x*cosh(x) + 90*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(x)^2)
*sinh(x)^4 + 120*(a^3*b^2 - 2*a*b^4)*cosh(x)^3 + 20*(36*b^5*cosh(x)^7 - 63*
a*b^4*cosh(x)^6 + 28*(4*a^2*b^3 - 7*b^5)*cosh(x)^5 + 6*a^3*b^2 - 12*a*b^4 -
 210*(a^3*b^2 - 2*a*b^4)*cosh(x)^4 - 60*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*x*c
osh(x)^2 + 60*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(x)^3 - 12*(8*a^4*b - 18*
a^2*b^3 + 11*b^5)*cosh(x))*sinh(x)^3 - 10*(4*a^2*b^3 - 7*b^5)*cosh(x)^2 + 1
0*(27*b^5*cosh(x)^8 - 54*a*b^4*cosh(x)^7 + 28*(4*a^2*b^3 - 7*b^5)*cosh(x)^6
 - 252*(a^3*b^2 - 2*a*b^4)*cosh(x)^5 - 4*a^2*b^3 + 7*b^5 - 120*(8*a^5 - 20*
a^3*b^2 + 15*a*b^4)*x*cosh(x)^3 + 90*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(x
)^4 - 36*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(x)^2 + 36*(a^3*b^2 - 2*a*b^4)
*cosh(x))*sinh(x)^2 - 1920*((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^5 + 5*(a^4 - 2*
a^2*b^2 + b^4)*cosh(x)^4*sinh(x) + 10*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^3*sin
h(x)^2 + 10*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2*sinh(x)^3 + 5*(a^4 - 2*a^2*b^
2 + b^4)*cosh(x)*sinh(x)^4 + (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^5)*sqrt(-a^2 +
 b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*s...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**6/(a+b*cosh(x)),x)
[Out] Timed out
```

Giac [A]
time = 0.40, size = 266, normalized size = 1.73

$$\frac{6\,b^4e^{(5\,x)}-15\,ab^3e^{(4\,x)}+40\,a^2b^2e^{(3\,x)}-70\,b^4e^{(3\,x)}-120\,a^3be^{(2\,x)}+240\,ab^3e^{(2\,x)}+480\,a^4e^x-1080\,a^2b^2e^x+660\,b^4e^x}{960\,b^5}-\frac{(8\,a^5-20\,a^3b^2+15\,ab^4)x}{8\,b^6}+\frac{(15\,ab^4e^x-6\,b^5-60\,(8\,a^4b-18\,a^2b^3+11\,b^5)e^{(4\,x)}+120\,(a^3b^2-2\,ab^4)e^{(3\,x)}-10\,(4\,a^2b^3-7\,b^5)e^{(2\,x)})e^{(-5\,x)}}{960\,b^6}+\frac{2\,(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^6/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 1/960*(6*b^4*e^(5*x) - 15*a*b^3*e^(4*x) + 40*a^2*b^2*e^(3*x) - 70*b^4*e^(3*
x) - 120*a^3*b*e^(2*x) + 240*a*b^3*e^(2*x) + 480*a^4*e^x - 1080*a^2*b^2*e^x
 + 660*b^4*e^x)/b^5 - 1/8*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*x/b^6 + 1/960*(15
*a*b^4*e^x - 6*b^5 - 60*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*e^(4*x) + 120*(a^3*
b^2 - 2*a*b^4)*e^(3*x) - 10*(4*a^2*b^3 - 7*b^5)*e^(2*x))*e^(-5*x)/b^6 + 2*(
a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sq
rt(-a^2 + b^2)*b^6)
```

Mupad [B]
time = 1.70, size = 348, normalized size = 2.26

$$\frac{e^{5\,x}}{160\,b}-\frac{e^{-5\,x}}{160\,b}-\frac{e^{-2\,x}\,(2\,a\,b^2-a^3)}{8\,b^4}+\frac{e^{2\,x}\,(2\,a\,b^2-a^3)}{8\,b^4}-\frac{x\,(8\,a^5-20\,a^3\,b^2+15\,a\,b^4)}{8\,b^6}+\frac{e^x\,(8\,a^4-18\,a^2\,b^2+11\,b^4)}{16\,b^5}+\frac{a\,e^{-4\,x}}{64\,b^2}-\frac{a\,e^{4\,x}}{64\,b^2}-\frac{e^{-x}\,(8\,a^4-18\,a^2\,b^2+11\,b^4)}{16\,b^5}-\frac{e^{-3\,x}\,(4\,a^2-7\,b^2)}{96\,b^3}+\frac{e^{3\,x}\,(4\,a^2-7\,b^2)}{96\,b^3}+\frac{\ln\left(-\frac{2\,e^x\,(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6)}{b^7}-\frac{2\,(a+b)^{5/2}\,(b+a\,e^x)\,(a-b)^{5/2}}{b^7}\right)\,(a+b)^{5/2}\,(a-b)^{5/2}}{b^6}-\frac{\ln\left(\frac{2\,(a+b)^{5/2}\,(b+a\,e^x)\,(a-b)^{5/2}}{b^7}-\frac{2\,e^x\,(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6)}{b^7}\right)\,(a+b)^{5/2}\,(a-b)^{5/2}}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^6/(a + b*cosh(x)),x)
```

```
[Out] exp(5*x)/(160*b) - exp(-5*x)/(160*b) - (exp(-2*x)*(2*a*b^2 - a^3))/(8*b^4)
+ (exp(2*x)*(2*a*b^2 - a^3))/(8*b^4) - (x*(15*a*b^4 + 8*a^5 - 20*a^3*b^2))/
(8*b^6) + (exp(x)*(8*a^4 + 11*b^4 - 18*a^2*b^2))/(16*b^5) + (a*exp(-4*x))/(
64*b^2) - (a*exp(4*x))/(64*b^2) - (exp(-x)*(8*a^4 + 11*b^4 - 18*a^2*b^2))/(
16*b^5) - (exp(-3*x)*(4*a^2 - 7*b^2))/(96*b^3) + (exp(3*x)*(4*a^2 - 7*b^2))
/(96*b^3) + (log(- (2*exp(x)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))/b^7 - (2*
(a + b)^(5/2)*(b + a*exp(x))*(a - b)^(5/2))/b^7)*(a + b)^(5/2)*(a - b)^(5/2
))/b^6 - (log((2*(a + b)^(5/2)*(b + a*exp(x))*(a - b)^(5/2))/b^7 - (2*exp(x
)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))/b^7)*(a + b)^(5/2)*(a - b)^(5/2))/b^
6
```

3.167 $\int \frac{\sinh^5(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=83

$$-\frac{a(a^2-2b^2)\cosh(x)}{b^4}+\frac{(a^2-2b^2)\cosh^2(x)}{2b^3}-\frac{a\cosh^3(x)}{3b^2}+\frac{\cosh^4(x)}{4b}+\frac{(a^2-b^2)^2\log(a+b\cosh(x))}{b^5}$$

```
[Out] -a*(a^2-2*b^2)*cosh(x)/b^4+1/2*(a^2-2*b^2)*cosh(x)^2/b^3-1/3*a*cosh(x)^3/b^
2+1/4*cosh(x)^4/b+(a^2-b^2)^2*ln(a+b*cosh(x))/b^5
```

Rubi [A]
time = 0.08, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = $\{2747, 711\}$

$$\frac{(a^2-b^2)^2\log(a+b\cosh(x))}{b^5}-\frac{a(a^2-2b^2)\cosh(x)}{b^4}+\frac{(a^2-2b^2)\cosh^2(x)}{2b^3}-\frac{a\cosh^3(x)}{3b^2}+\frac{\cosh^4(x)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^5/(a + b*Cosh[x]),x]
```

```
[Out] -((a*(a^2 - 2*b^2)*Cosh[x])/b^4) + ((a^2 - 2*b^2)*Cosh[x]^2)/(2*b^3) - (a*C
osh[x]^3)/(3*b^2) + Cosh[x]^4/(4*b) + ((a^2 - b^2)^2*Log[a + b*Cosh[x]])/b^
5
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sinh^5(x)}{a+b\cosh(x)}\,dx = \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{a+x}\,dx, x, b\cosh(x)\right)}{b^5}$$

$$= \frac{\text{Subst}\left(\int \left(-a^3\left(1-\frac{2b^2}{a^2}\right)+(a^2-2b^2)\,x-ax^2+x^3+\frac{(a^2-b^2)^2}{a+x}\right)dx, x, b\cosh(x)\right)}{b^5}$$

$$= -\frac{a(a^2-2b^2)\cosh(x)}{b^4}+\frac{(a^2-2b^2)\cosh^2(x)}{2b^3}-\frac{a\cosh^3(x)}{3b^2}+\frac{\cosh^4(x)}{4b}+\frac{(a^2-b^2)^2 \text{lo}}{}$$

Mathematica [**A**]
time = 0.09, size = 84, normalized size = 1.01

$$\frac{-24ab(4a^2-7b^2)\cosh(x)-12b^2(-2a^2+3b^2)\cosh(2x)-8ab^3\cosh(3x)+3b^4\cosh(4x)+96(a^2-b^2)^2\log(a+b\cosh(x))}{96b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^5/(a + b*Cosh[x]),x]
```

```
[Out] (-24*a*b*(4*a^2 - 7*b^2)*Cosh[x] - 12*b^2*(-2*a^2 + 3*b^2)*Cosh[2*x] - 8*a*
b^3*Cosh[3*x] + 3*b^4*Cosh[4*x] + 96*(a^2 - b^2)^2*Log[a + b*Cosh[x]])/(96*
b^5)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 327 vs. 2(77) = 154.
time = 0.58, size = 328, normalized size = 3.95

method	result
risch	$-\frac{x\,a^4}{b^5}+\frac{2x\,a^2}{b^3}-\frac{x}{b}+\frac{e^{4x}}{64b}-\frac{a\,e^{3x}}{24b^2}+\frac{e^{2x}a^2}{8b^3}-\frac{3\,e^{2x}}{16b}-\frac{a^3 e^{x}}{2b^4}+\frac{7a\,e^{x}}{8b^2}-\frac{a^3 e^{-x}}{2b^4}+\frac{7a\,e^{-x}}{8b^2}+\frac{e^{-2x}a^2}{8b^3}-\frac{3\,e^{-2x}}{16b}-$
default	$\frac{(a^5-a^4b-2a^3b^2+2a^2b^3+a\,b^4-b^5)\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-b\left(\tanh^2\left(\frac{x}{2}\right)\right)-a-b\right)}{b^5(a-b)}+\frac{1}{4b\left(\tanh\left(\frac{x}{2}\right)-1\right)^4}-\frac{-2a-3b}{6b^2\left(\tanh\left(\frac{x}{2}\right)-1\right)^3}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^5/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] (a^5-a^4*b-2*a^3*b^2+2*a^2*b^3+a*b^4-b^5)/b^5/(a-b)*ln(a*tanh(1/2*x)^2-b*ta
nh(1/2*x)^2-a-b)+1/4/b/(tanh(1/2*x)-1)^4-1/6*(-2*a-3*b)/b^2/(tanh(1/2*x)-1)
^3-1/8*(-4*a^2-4*a*b+3*b^2)/b^3/(tanh(1/2*x)-1)^2+(-a^4+2*a^2*b^2-b^4)/b^5*
ln(tanh(1/2*x)-1)-1/8*(-8*a^3-4*a^2*b+12*a*b^2+5*b^3)/b^4/(tanh(1/2*x)-1)+1
/4/b/(tanh(1/2*x)+1)^4-1/6*(2*a+3*b)/b^2/(tanh(1/2*x)+1)^3-1/8*(-4*a^2-4*a*
b+3*b^2)/b^3/(tanh(1/2*x)+1)^2+(-a^4+2*a^2*b^2-b^4)/b^5*ln(tanh(1/2*x)+1)-1
/8*(8*a^3+4*a^2*b-12*a*b^2-5*b^3)/b^4/(tanh(1/2*x)+1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(77) = 154$.
time = 0.26, size = 178, normalized size = 2.14

$$-\frac{\left(8\,ab^2e^{(-x)} - 3\,b^3 - 12\,(2\,a^2b - 3\,b^3)e^{(-2\,x)} + 24\,(4\,a^3 - 7\,ab^2)e^{(-3\,x)}\right)e^{(4\,x)}}{192\,b^4} - \frac{8\,ab^2e^{(-3\,x)} - 3\,b^3e^{(-4\,x)} + 24\,(4\,a^3 - 7\,ab^2)e^{(-x)} - 12\,(2\,a^2b - 3\,b^3)e^{(-2\,x)}}{192\,b^4} + \frac{(a^4 - 2\,a^2b^2 + b^4)x}{b^5} + \frac{(a^4 - 2\,a^2b^2 + b^4)\log\left(2\,ae^{(-x)} + be^{(-2\,x)} + b\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^5/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] -1/192*(8*a*b^2*e^(-x) - 3*b^3 - 12*(2*a^2*b - 3*b^3)*e^(-2*x) + 24*(4*a^3
- 7*a*b^2)*e^(-3*x))*e^(4*x)/b^4 - 1/192*(8*a*b^2*e^(-3*x) - 3*b^3*e^(-4*x)
 + 24*(4*a^3 - 7*a*b^2)*e^(-x) - 12*(2*a^2*b - 3*b^3)*e^(-2*x))/b^4 + (a^4
- 2*a^2*b^2 + b^4)*x/b^5 + (a^4 - 2*a^2*b^2 + b^4)*log(2*a*e^(-x) + b*e^(-2
*x) + b)/b^5
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 866 vs. $2(77) = 154$.
time = 0.38, size = 866, normalized size = 10.43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^5/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/192*(3*b^4*cosh(x)^8 + 3*b^4*sinh(x)^8 - 8*a*b^3*cosh(x)^7 + 8*(3*b^4*cos
h(x) - a*b^3)*sinh(x)^7 + 12*(2*a^2*b^2 - 3*b^4)*cosh(x)^6 + 4*(21*b^4*cosh
(x)^2 - 14*a*b^3*cosh(x) + 6*a^2*b^2 - 9*b^4)*sinh(x)^6 - 192*(a^4 - 2*a^2*
b^2 + b^4)*x*cosh(x)^4 - 24*(4*a^3*b - 7*a*b^3)*cosh(x)^5 + 24*(7*b^4*cosh(
x)^3 - 7*a*b^3*cosh(x)^2 - 4*a^3*b + 7*a*b^3 + 3*(2*a^2*b^2 - 3*b^4)*cosh(x
))*sinh(x)^5 - 8*a*b^3*cosh(x) + 2*(105*b^4*cosh(x)^4 - 140*a*b^3*cosh(x)^3
 + 90*(2*a^2*b^2 - 3*b^4)*cosh(x)^2 - 96*(a^4 - 2*a^2*b^2 + b^4)*x - 60*(4*
a^3*b - 7*a*b^3)*cosh(x))*sinh(x)^4 + 3*b^4 - 24*(4*a^3*b - 7*a*b^3)*cosh(x
)^3 + 8*(21*b^4*cosh(x)^5 - 35*a*b^3*cosh(x)^4 - 12*a^3*b + 21*a*b^3 + 30*(
2*a^2*b^2 - 3*b^4)*cosh(x)^3 - 96*(a^4 - 2*a^2*b^2 + b^4)*x*cosh(x) - 30*(4
*a^3*b - 7*a*b^3)*cosh(x)^2)*sinh(x)^3 + 12*(2*a^2*b^2 - 3*b^4)*cosh(x)^2 +
 12*(7*b^4*cosh(x)^6 - 14*a*b^3*cosh(x)^5 + 15*(2*a^2*b^2 - 3*b^4)*cosh(x)^
4 + 2*a^2*b^2 - 3*b^4 - 96*(a^4 - 2*a^2*b^2 + b^4)*x*cosh(x)^2 - 20*(4*a^3*
b - 7*a*b^3)*cosh(x)^3 - 6*(4*a^3*b - 7*a*b^3)*cosh(x))*sinh(x)^2 + 192*((a
^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + 4*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^3*sinh(
x) + 6*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2*sinh(x)^2 + 4*(a^4 - 2*a^2*b^2 + b
^4)*cosh(x)*sinh(x)^3 + (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^4)*log(2*(b*cosh(x)
 + a)/(cosh(x) - sinh(x))) + 8*(3*b^4*cosh(x)^7 - 7*a*b^3*cosh(x)^6 + 9*(2*
a^2*b^2 - 3*b^4)*cosh(x)^5 - 96*(a^4 - 2*a^2*b^2 + b^4)*x*cosh(x)^3 - 15*(4
*a^3*b - 7*a*b^3)*cosh(x)^4 - a*b^3 - 9*(4*a^3*b - 7*a*b^3)*cosh(x)^2 + 3*(
2*a^2*b^2 - 3*b^4)*cosh(x))*sinh(x))/(b^5*cosh(x)^4 + 4*b^5*cosh(x)^3*sinh(
x) + 6*b^5*cosh(x)^2*sinh(x)^2 + 4*b^5*cosh(x)*sinh(x)^3 + b^5*sinh(x)^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**5/(a+b*cosh(x)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.41, size = 124, normalized size = 1.49

$$\frac{3\,b^3\left(e^{(-x)}+e^x\right)^4-8\,ab^2\left(e^{(-x)}+e^x\right)^3+24\,a^2b\left(e^{(-x)}+e^x\right)^2-48\,b^3\left(e^{(-x)}+e^x\right)^2-96\,a^3\left(e^{(-x)}+e^x\right)+192\,ab^2\left(e^{(-x)}+e^x\right)}{192\,b^4}+\frac{\left(a^4-2\,a^2b^2+b^4\right)\log\left(\left|b\left(e^{(-x)}+e^x\right)+2\,a\right|\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^5/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 1/192*(3*b^3*(e^(-x) + e^x)^4 - 8*a*b^2*(e^(-x) + e^x)^3 + 24*a^2*b*(e^(-x)
 + e^x)^2 - 48*b^3*(e^(-x) + e^x)^2 - 96*a^3*(e^(-x) + e^x) + 192*a*b^2*(e^
(-x) + e^x))/b^4 + (a^4 - 2*a^2*b^2 + b^4)*log(abs(b*(e^(-x) + e^x) + 2*a))
/b^5
```

Mupad [**B**]
time = 1.31, size = 169, normalized size = 2.04

$$\frac{\mathrm{e}^{-4\,x}}{64\,b}+\frac{\mathrm{e}^{4\,x}}{64\,b}-\frac{x\,{\left(a^2-b^2\right)}^2}{b^5}+\frac{\mathrm{e}^{-x}\,\left(7\,a\,b^2-4\,a^3\right)}{8\,b^4}+\frac{\ln\left(b+2\,a\,\mathrm{e}^x+b\,\mathrm{e}^{2\,x}\right)\,\left(a^4-2\,a^2\,b^2+b^4\right)}{b^5}-\frac{a\,\mathrm{e}^{-3\,x}}{24\,b^2}-\frac{a\,\mathrm{e}^{3\,x}}{24\,b^2}+\frac{\mathrm{e}^{-2\,x}\,\left(2\,a^2-3\,b^2\right)}{16\,b^3}+\frac{\mathrm{e}^{2\,x}\,\left(2\,a^2-3\,b^2\right)}{16\,b^3}+\frac{\mathrm{e}^x\,\left(7\,a\,b^2-4\,a^3\right)}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^5/(a + b*cosh(x)),x)
```

```
[Out] exp(-4*x)/(64*b) + exp(4*x)/(64*b) - (x*(a^2 - b^2)^2)/b^5 + (exp(-x)*(7*a*
b^2 - 4*a^3))/(8*b^4) + (log(b + 2*a*exp(x) + b*exp(2*x))*(a^4 + b^4 - 2*a^
2*b^2))/b^5 - (a*exp(-3*x))/(24*b^2) - (a*exp(3*x))/(24*b^2) + (exp(-2*x)*(
2*a^2 - 3*b^2))/(16*b^3) + (exp(2*x)*(2*a^2 - 3*b^2))/(16*b^3) + (exp(x)*(7
*a*b^2 - 4*a^3))/(8*b^4)
```

3.168 $\int \frac{\sinh^4(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=104

$$-\frac{a(2a^2-3b^2)\,x}{2b^4}+\frac{2(a-b)^{3/2}(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^4}+\frac{(2(a^2-b^2)-ab\cosh(x))\sinh(x)}{2b^3}+\frac{\sinh^3(x)}{3b}$$

```
[Out] -1/2*a*(2*a^2-3*b^2)*x/b^4+2*(a-b)^(3/2)*(a+b)^(3/2)*arctanh((a-b)^(1/2)*ta
nh(1/2*x)/(a+b)^(1/2))/b^4+1/2*(2*a^2-2*b^2-a*b*cosh(x))*sinh(x)/b^3+1/3*si
nh(x)^3/b
```

Rubi [A]
time = 0.17, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2774, 2944, 2814, 2738, 214}

$$-\frac{ax(2a^2-3b^2)}{2b^4}+\frac{\sinh(x)\,(2(a^2-b^2)-ab\cosh(x))}{2b^3}+\frac{2(a-b)^{3/2}(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^4}+\frac{\sinh^3(x)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^4/(a + b*Cosh[x]),x]
```

```
[Out] -1/2*(a*(2*a^2 - 3*b^2)*x)/b^4 + (2*(a - b)^(3/2)*(a + b)^(3/2)*ArcTanh[(Sq
rt[a - b]*Tanh[x/2])/Sqrt[a + b]])/b^4 + ((2*(a^2 - b^2) - a*b*Cosh[x])*Sin
h[x])/(2*b^3) + Sinh[x]^3/(3*b)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
```

```
0] && IntegersQ[2*m, 2*p]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sinh^4(x)}{a+b\cosh(x)}\,dx = \frac{\sinh^3(x)}{3b}+\frac{\int \frac{(-b-a\cosh(x))\sinh^2(x)}{a+b\cosh(x)}\,dx}{b}$$

$$= \frac{(2(a^2-b^2)-ab\cosh(x))\sinh(x)}{2b^3}+\frac{\sinh^3(x)}{3b}-\frac{\int \frac{b(a^2-2b^2)+a(2a^2-3b^2)\cosh(x)}{a+b\cosh(x)}\,dx}{2b^3}$$

$$= -\frac{a(2a^2-3b^2)\,x}{2b^4}+\frac{(2(a^2-b^2)-ab\cosh(x))\sinh(x)}{2b^3}+\frac{\sinh^3(x)}{3b}+\frac{(a^2-b^2)^2\int \frac{1}{a+b\cosh(x)}}{b^4}$$

$$= -\frac{a(2a^2-3b^2)\,x}{2b^4}+\frac{(2(a^2-b^2)-ab\cosh(x))\sinh(x)}{2b^3}+\frac{\sinh^3(x)}{3b}+\frac{\left(2(a^2-b^2)^2\right)\mathrm{Su}}{}$$

$$= -\frac{a(2a^2-3b^2)\,x}{2b^4}+\frac{2(a-b)^{3/2}(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(\frac{x}{2})}{\sqrt{a+b}}\right)}{b^4}+\frac{(2(a^2-b^2)-}{}$$

Mathematica [A]

time = 0.14, size = 95, normalized size = 0.91

$$\frac{-12a^3x+18ab^2x-24(-a^2+b^2)^{3/2}\,\mathrm{ArcTan}\left(\frac{(a-b)\tanh(\frac{x}{2})}{\sqrt{-a^2+b^2}}\right)+12a^2b\sinh(x)-15b^3\sinh(x)-3ab^2\sinh(2x)+b^3\sinh(3x)}{12b^4}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[x]^4/(a + b*Cosh[x]),x]

[Out] (-12*a^3*x + 18*a*b^2*x - 24*(-a^2 + b^2)^(3/2)*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]] + 12*a^2*b*Sinh[x] - 15*b^3*Sinh[x] - 3*a*b^2*Sinh[2*x] + b^3*Sinh[3*x])/(12*b^4)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(87) = 174$.
time = 0.45, size = 223, normalized size = 2.14

method	result
default	$-\frac{1}{3b\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}-\frac{a+b}{2b^{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{2a^{2}+ab-2b^{2}}{2b^{3}\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{a\left(2a^{2}-3b^{2}\right)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b^{4}}-\frac{2\left(-a^{4}+2a^{2}b^{2}-b^{4}\right)\operatorname{arctanh}\left(\cdots\right.}{b^{4}\sqrt{(a+b)\left(\cdots\right.}}$
risch	$-\frac{a^{3}x}{b^{4}}+\frac{3ax}{2b^{2}}+\frac{e^{3x}}{24b}-\frac{a\,e^{2x}}{8b^{2}}+\frac{e^{x}a^{2}}{2b^{3}}-\frac{5\,e^{x}}{8b}-\frac{e^{-x}a^{2}}{2b^{3}}+\frac{5\,e^{-x}}{8b}+\frac{a\,e^{-2x}}{8b^{2}}-\frac{e^{-3x}}{24b}+\frac{\sqrt{a^{2}-b^{2}}\,\ln\left(e^{x}+\frac{a\sqrt{a^{2}-b^{2}}}{b\sqrt{a^{2}-\cdots}}\right.}{b^{4}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(x)^4/(a+b*cosh(x)),x,method=_RETURNVERBOSE)

[Out] -1/3/b/(tanh(1/2*x)-1)^3-1/2*(a+b)/b^2/(tanh(1/2*x)-1)^2-1/2*(2*a^2+a*b-2*b^2)/b^3/(tanh(1/2*x)-1)+1/2*a*(2*a^2-3*b^2)/b^4*ln(tanh(1/2*x)-1)-2/b^4*(-a^4+2*a^2*b^2-b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))-1/3/b/(tanh(1/2*x)+1)^3-1/2*(-a-b)/b^2/(tanh(1/2*x)+1)^2-1/2*(2*a^2+a*b-2*b^2)/b^3/(tanh(1/2*x)+1)-1/2*a*(2*a^2-3*b^2)/b^4*ln(tanh(1/2*x)+1)

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(x)^4/(a+b*cosh(x)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more de

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 515 vs. $2(86) = 172$.
time = 0.40, size = 1099, normalized size = 10.57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [1/24*(b^3*cosh(x)^6 + b^3*sinh(x)^6 - 3*a*b^2*cosh(x)^5 + 3*(2*b^3*cosh(x)
 - a*b^2)*sinh(x)^5 - 12*(2*a^3 - 3*a*b^2)*x*cosh(x)^3 + 3*(4*a^2*b - 5*b^3
)*cosh(x)^4 + 3*(5*b^3*cosh(x)^2 - 5*a*b^2*cosh(x) + 4*a^2*b - 5*b^3)*sinh(
x)^4 + 3*a*b^2*cosh(x) + 2*(10*b^3*cosh(x)^3 - 15*a*b^2*cosh(x)^2 - 6*(2*a^
3 - 3*a*b^2)*x + 6*(4*a^2*b - 5*b^3)*cosh(x))*sinh(x)^3 - b^3 - 3*(4*a^2*b
- 5*b^3)*cosh(x)^2 + 3*(5*b^3*cosh(x)^4 - 10*a*b^2*cosh(x)^3 - 4*a^2*b + 5*
b^3 - 12*(2*a^3 - 3*a*b^2)*x*cosh(x) + 6*(4*a^2*b - 5*b^3)*cosh(x)^2)*sinh(
x)^2 - 24*((a^2 - b^2)*cosh(x)^3 + 3*(a^2 - b^2)*cosh(x)^2*sinh(x) + 3*(a^2
 - b^2)*cosh(x)*sinh(x)^2 + (a^2 - b^2)*sinh(x)^3)*sqrt(a^2 - b^2)*log((b^2
*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) +
 a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2
 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)) + 3*(2*b^3*c
osh(x)^5 - 5*a*b^2*cosh(x)^4 - 12*(2*a^3 - 3*a*b^2)*x*cosh(x)^2 + 4*(4*a^2*
b - 5*b^3)*cosh(x)^3 + a*b^2 - 2*(4*a^2*b - 5*b^3)*cosh(x))*sinh(x))/(b^4*c
osh(x)^3 + 3*b^4*cosh(x)^2*sinh(x) + 3*b^4*cosh(x)*sinh(x)^2 + b^4*sinh(x)^
3), 1/24*(b^3*cosh(x)^6 + b^3*sinh(x)^6 - 3*a*b^2*cosh(x)^5 + 3*(2*b^3*cosh
(x) - a*b^2)*sinh(x)^5 - 12*(2*a^3 - 3*a*b^2)*x*cosh(x)^3 + 3*(4*a^2*b - 5*
b^3)*cosh(x)^4 + 3*(5*b^3*cosh(x)^2 - 5*a*b^2*cosh(x) + 4*a^2*b - 5*b^3)*si
nh(x)^4 + 3*a*b^2*cosh(x) + 2*(10*b^3*cosh(x)^3 - 15*a*b^2*cosh(x)^2 - 6*(2
*a^3 - 3*a*b^2)*x + 6*(4*a^2*b - 5*b^3)*cosh(x))*sinh(x)^3 - b^3 - 3*(4*a^2
*b - 5*b^3)*cosh(x)^2 + 3*(5*b^3*cosh(x)^4 - 10*a*b^2*cosh(x)^3 - 4*a^2*b +
 5*b^3 - 12*(2*a^3 - 3*a*b^2)*x*cosh(x) + 6*(4*a^2*b - 5*b^3)*cosh(x)^2)*si
nh(x)^2 - 48*((a^2 - b^2)*cosh(x)^3 + 3*(a^2 - b^2)*cosh(x)^2*sinh(x) + 3*(
a^2 - b^2)*cosh(x)*sinh(x)^2 + (a^2 - b^2)*sinh(x)^3)*sqrt(-a^2 + b^2)*arct
an(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) + 3*(2*b^3*co
sh(x)^5 - 5*a*b^2*cosh(x)^4 - 12*(2*a^3 - 3*a*b^2)*x*cosh(x)^2 + 4*(4*a^2*b
 - 5*b^3)*cosh(x)^3 + a*b^2 - 2*(4*a^2*b - 5*b^3)*cosh(x))*sinh(x))/(b^4*co
sh(x)^3 + 3*b^4*cosh(x)^2*sinh(x) + 3*b^4*cosh(x)*sinh(x)^2 + b^4*sinh(x)^3
)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**4/(a+b*cosh(x)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.41, size = 146, normalized size = 1.40

$$\frac{b^{2} e^{(3\,x)} - 3\,abe^{(2\,x)} + 12\,a^{2} e^{x} - 15\,b^{2} e^{x}}{24\,b^{3}} - \frac{(2\,a^{3} - 3\,ab^{2})x}{2\,b^{4}} + \frac{\left(3\,ab^{2} e^{x} - b^{3} - 3\,(4\,a^{2} b - 5\,b^{3})e^{(2\,x)}\right)e^{(-3\,x)}}{24\,b^{4}} + \frac{2\,(a^{4} - 2\,a^{2} b^{2} + b^{4})\arctan\left(\frac{be^{x}+a}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 1/24*(b^2*e^(3*x) - 3*a*b*e^(2*x) + 12*a^2*e^x - 15*b^2*e^x)/b^3 - 1/2*(2*a
^3 - 3*a*b^2)*x/b^4 + 1/24*(3*a*b^2*e^x - b^3 - 3*(4*a^2*b - 5*b^3)*e^(2*x)
)*e^(-3*x)/b^4 + 2*(a^4 - 2*a^2*b^2 + b^4)*arctan((b*e^x + a)/sqrt(-a^2 + b
^2))/(sqrt(-a^2 + b^2)*b^4)
```

Mupad [B]
time = 1.31, size = 222, normalized size = 2.13

$$\frac{e^{3x}}{24b}-\frac{e^{-3x}}{24b}+\frac{x\,(3\,a\,b^2-2\,a^3)}{2\,b^4}+\frac{e^{x}\,(4\,a^2-5\,b^2)}{8\,b^3}+\frac{a\,e^{-2x}}{8\,b^2}-\frac{a\,e^{2x}}{8\,b^2}-\frac{e^{-x}\,(4\,a^2-5\,b^2)}{8\,b^3}+\frac{\ln\left(-\frac{2\,e^{x}\,(a^4-2\,a^2\,b^2+b^4)}{b^5}-\frac{2\,(a+b)^{3/2}\,(b+a\,e^{x})\,(a-b)^{3/2}}{b^5}\right)\,(a+b)^{3/2}\,(a-b)^{3/2}}{b^4}-\frac{\ln\left(\frac{2\,(a+b)^{3/2}\,(b+a\,e^{x})\,(a-b)^{3/2}}{b^5}-\frac{2\,e^{x}\,(a^4-2\,a^2\,b^2+b^4)}{b^5}\right)\,(a+b)^{3/2}\,(a-b)^{3/2}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(a + b*cosh(x)),x)
```

```
[Out] exp(3*x)/(24*b) - exp(-3*x)/(24*b) + (x*(3*a*b^2 - 2*a^3))/(2*b^4) + (exp(x
)*(4*a^2 - 5*b^2))/(8*b^3) + (a*exp(-2*x))/(8*b^2) - (a*exp(2*x))/(8*b^2) -
 (exp(-x)*(4*a^2 - 5*b^2))/(8*b^3) + (log(- (2*exp(x)*(a^4 + b^4 - 2*a^2*b^
2))/b^5 - (2*(a + b)^(3/2)*(b + a*exp(x))*(a - b)^(3/2))/b^5)*(a + b)^(3/2)
*(a - b)^(3/2))/b^4 - (log((2*(a + b)^(3/2)*(b + a*exp(x))*(a - b)^(3/2))/b
^5 - (2*exp(x)*(a^4 + b^4 - 2*a^2*b^2))/b^5)*(a + b)^(3/2)*(a - b)^(3/2))/b
^4
```

3.169 $\int \frac{\sinh^3(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=40

$$-\frac{a\cosh(x)}{b^2}+\frac{\cosh^2(x)}{2b}+\frac{(a^2-b^2)\log(a+b\cosh(x))}{b^3}$$

```
[Out] -a*cosh(x)/b^2+1/2*cosh(x)^2/b+(a^2-b^2)*ln(a+b*cosh(x))/b^3
```

Rubi [A]
time = 0.05, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2747, 711}

$$\frac{(a^2-b^2)\log(a+b\cosh(x))}{b^3}-\frac{a\cosh(x)}{b^2}+\frac{\cosh^2(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(a + b*Cosh[x]),x]
```

```
[Out] -((a*Cosh[x])/b^2) + Cosh[x]^2/(2*b) + ((a^2 - b^2)*Log[a + b*Cosh[x]])/b^3
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^3(x)}{a+b\cosh(x)}\,dx &= -\frac{\text{Subst}\left(\int \frac{b^2-x^2}{a+x}\,dx,x,b\cosh(x)\right)}{b^3}\\
&= -\frac{\text{Subst}\left(\int \left(a-x+\frac{-a^2+b^2}{a+x}\right)\,dx,x,b\cosh(x)\right)}{b^3}\\
&= -\frac{a\cosh(x)}{b^2}+\frac{\cosh^2(x)}{2b}+\frac{(a^2-b^2)\log(a+b\cosh(x))}{b^3}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 40, normalized size = 1.00

$$-\frac{a\cosh(x)}{b^2}+\frac{\cosh(2x)}{4b}+\frac{(a^2-b^2)\log(a+b\cosh(x))}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(a + b*Cosh[x]),x]
```

```
[Out] -((a*Cosh[x])/b^2) + Cosh[2*x]/(4*b) + ((a^2 - b^2)*Log[a + b*Cosh[x]])/b^3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(38) = 76.
time = 0.44, size = 161, normalized size = 4.02

method	result
risch	$-\frac{x\,a^2}{b^3}+\frac{x}{b}+\frac{e^{2x}}{8b}-\frac{a\,e^{x}}{2b^2}-\frac{a\,e^{-x}}{2b^2}+\frac{e^{-2x}}{8b}+\frac{\ln\left(e^{2x}+\frac{2a\,e^{x}}{b}+1\right)a^2}{b^3}-\frac{\ln\left(e^{2x}+\frac{2a\,e^{x}}{b}+1\right)}{b}$
default	$\frac{(a^3-a^2b-a\,b^2+b^3)\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-b\left(\tanh^2\left(\frac{x}{2}\right)\right)-a-b\right)}{b^3(a-b)}+\frac{1}{2b\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}-\frac{2a+b}{2b^2\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{(-a^2+b^2)\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] (a^3-a^2*b-a*b^2+b^3)/b^3/(a-b)*ln(a*tanh(1/2*x)^2-b*tanh(1/2*x)^2-a-b)+1/2
/b/(tanh(1/2*x)+1)^2-1/2*(2*a+b)/b^2/(tanh(1/2*x)+1)+(-a^2+b^2)/b^3*ln(tanh
(1/2*x)+1)+1/2/b/(tanh(1/2*x)-1)^2-1/2*(-b-2*a)/b^2/(tanh(1/2*x)-1)+(-a^2+b
^2)/b^3*ln(tanh(1/2*x)-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(38) = 76.
time = 0.26, size = 84, normalized size = 2.10

$$-\frac{\left(4\,ae^{(-x)}-b\right)e^{(2\,x)}}{8\,b^2}-\frac{4\,ae^{(-x)}-be^{(-2\,x)}}{8\,b^2}+\frac{(a^2-b^2)x}{b^3}+\frac{(a^2-b^2)\log\left(2\,ae^{(-x)}+be^{(-2\,x)}+b\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*a*e^(-x) - b)*e^(2*x)/b^2 - 1/8*(4*a*e^(-x) - b*e^(-2*x))/b^2 + (a^
2 - b^2)*x/b^3 + (a^2 - b^2)*log(2*a*e^(-x) + b*e^(-2*x) + b)/b^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 234 vs. 2(38) = 76.
time = 0.39, size = 234, normalized size = 5.85

$$\frac{b^2\cosh(x)^4+b^2\sinh(x)^4-4\,ab\cosh(x)^3-8\,(a^2-b^2)x\cosh(x)^2+4\,(b^2\cosh(x)-ab)\sinh(x)^3-4\,ab\cosh(x)+2\,\left(3\,b^2\cosh(x)^2-6\,ab\cosh(x)-4\,(a^2-b^2)x\right)\sinh(x)^2+b^2+8\,\left((a^2-b^2)\cosh(x)^2+2\,(a^2-b^2)\cosh(x)\sinh(x)+(a^2-b^2)\sinh(x)^2\right)\log\left(\frac{2\,(b\cosh(x)+a)}{\cosh(x)-\sinh(x)}\right)+4\,\left(b^2\cosh(x)^3-3\,ab\cosh(x)^2-4\,(a^2-b^2)x\cosh(x)-ab\right)\sinh(x)}{8\,\left(b^3\cosh(x)^2+2\,b^3\cosh(x)\sinh(x)+b^3\sinh(x)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/8*(b^2*cosh(x)^4 + b^2*sinh(x)^4 - 4*a*b*cosh(x)^3 - 8*(a^2 - b^2)*x*cosh
(x)^2 + 4*(b^2*cosh(x) - a*b)*sinh(x)^3 - 4*a*b*cosh(x) + 2*(3*b^2*cosh(x)^
2 - 6*a*b*cosh(x) - 4*(a^2 - b^2)*x)*sinh(x)^2 + b^2 + 8*((a^2 - b^2)*cosh(
x)^2 + 2*(a^2 - b^2)*cosh(x)*sinh(x) + (a^2 - b^2)*sinh(x)^2)*log(2*(b*cosh
(x) + a)/(cosh(x) - sinh(x))) + 4*(b^2*cosh(x)^3 - 3*a*b*cosh(x)^2 - 4*(a^2
 - b^2)*x*cosh(x) - a*b)*sinh(x))/(b^3*cosh(x)^2 + 2*b^3*cosh(x)*sinh(x) +
b^3*sinh(x)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(a+b*cosh(x)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.41, size = 56, normalized size = 1.40

$$\frac{b\left(e^{(-x)}+e^{x}\right)^{2}-4\,a\left(e^{(-x)}+e^{x}\right)}{8\,b^{2}}+\frac{(a^{2}-b^{2})\log\left(\left|b\left(e^{(-x)}+e^{x}\right)+2\,a\right|\right)}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 1/8*(b*(e^(-x) + e^x)^2 - 4*a*(e^(-x) + e^x))/b^2 + (a^2 - b^2)*log(abs(b*(
e^(-x) + e^x) + 2*a))/b^3
```

Mupad [**B**]
time = 1.04, size = 79, normalized size = 1.98

$$\frac{\mathrm{e}^{-2\,x}}{8\,b}+\frac{\mathrm{e}^{2\,x}}{8\,b}+\frac{\ln\left(b+2\,a\,\mathrm{e}^{x}+b\,\mathrm{e}^{2\,x}\right)\,\left(a^{2}-b^{2}\right)}{b^{3}}-\frac{a\,\mathrm{e}^{x}}{2\,b^{2}}-\frac{a\,\mathrm{e}^{-x}}{2\,b^{2}}-\frac{x\,\left(a^{2}-b^{2}\right)}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a + b*cosh(x)),x)
```

```
[Out] exp(-2*x)/(8*b) + exp(2*x)/(8*b) + (log(b + 2*a*exp(x) + b*exp(2*x))*(a^2 -
 b^2))/b^3 - (a*exp(x))/(2*b^2) - (a*exp(-x))/(2*b^2) - (x*(a^2 - b^2))/b^3
```

3.170 $\int \frac{\sinh^2(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=59

$$-\frac{ax}{b^2}+\frac{2\sqrt{a-b}\,\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^2}+\frac{\sinh(x)}{b}$$

```
[Out] -a*x/b^2+sinh(x)/b+2*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))*(a-b)^(1/
2)*(a+b)^(1/2)/b^2
```

Rubi [A]

time = 0.08, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2774, 2814, 2738, 214}

$$-\frac{ax}{b^2}+\frac{2\sqrt{a-b}\,\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^2}+\frac{\sinh(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(a + b*Cosh[x]),x]
```

```
[Out] -((a*x)/b^2) + (2*Sqrt[a - b]*Sqrt[a + b]*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/S
qrt[a + b]])/b^2 + Sinh[x]/b
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^2(x)}{a+b\cosh(x)}\,dx &= \frac{\sinh(x)}{b}+\frac{\int \frac{-b-a\cosh(x)}{a+b\cosh(x)}\,dx}{b}\\ &= -\frac{ax}{b^2}+\frac{\sinh(x)}{b}-\left(1-\frac{a^2}{b^2}\right)\int \frac{1}{a+b\cosh(x)}\,dx\\ &= -\frac{ax}{b^2}+\frac{\sinh(x)}{b}-\left(2\left(1-\frac{a^2}{b^2}\right)\right)\text{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)\\ &= -\frac{ax}{b^2}+\frac{2\sqrt{a-b}\,\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^2}+\frac{\sinh(x)}{b}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 54, normalized size = 0.92

$$\frac{-ax+2\sqrt{-a^2+b^2}\,\text{ArcTan}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)+b\sinh(x)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(a + b*Cosh[x]),x]
```

```
[Out] (-(a*x) + 2*Sqrt[-a^2 + b^2]*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]] +
 b*Sinh[x])/b^2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(49) = 98$.
time = 0.46, size = 100, normalized size = 1.69

method	result
default	$-\frac{1}{b\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b^2}-\frac{1}{b\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b^2}-\frac{2\left(-a^2+b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right.}{b^2\sqrt{(a+b)(a-b)}}$

risch	$-\frac{ax}{b^2}+\frac{e^x}{2b}-\frac{e^{-x}}{2b}+\frac{\sqrt{a^2-b^2}\,\ln\left(e^x-\frac{-a+\sqrt{a^2-b^2}}{b}\right)}{b^2}-\frac{\sqrt{a^2-b^2}\,\ln\left(e^x+\frac{a+\sqrt{a^2-b^2}}{b}\right)}{b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b/(tanh(1/2*x)+1)-a/b^2*ln(tanh(1/2*x)+1)-1/b/(tanh(1/2*x)-1)+a/b^2*ln(t
anh(1/2*x)-1)-2/b^2*(-a^2+b^2)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x
)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(49) = 98$.
time = 0.37, size = 279, normalized size = 4.73

$$\left[-\frac{2\,ax\cosh(x)-b\cosh(x)^2-b\sinh(x)^2-2\sqrt{a^2-b^2}\,(\cosh(x)+\sinh(x))\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\,(b^2\cosh(x)+ab)\sinh(x)-2\sqrt{a^2-b^2}\,(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right)+2\,(ax-b\cosh(x))\sinh(x)+b}{2\,(b^2\cosh(x)+b^2\sinh(x))},\ -\frac{2\,ax\cosh(x)-b\cosh(x)^2-b\sinh(x)^2+4\sqrt{-a^2+b^2}\,(\cosh(x)+\sinh(x))\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\cosh(x)+b\sinh(x)+a)}{a^2-b^2}\right)+2\,(ax-b\cosh(x))\sinh(x)+b}{2\,(b^2\cosh(x)+b^2\sinh(x))}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*x*cosh(x) - b*cosh(x)^2 - b*sinh(x)^2 - 2*sqrt(a^2 - b^2)*(cosh(
x) + sinh(x))*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 -
b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh
(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh
(x) + b)) + 2*(a*x - b*cosh(x))*sinh(x) + b)/(b^2*cosh(x) + b^2*sinh(x)), -
1/2*(2*a*x*cosh(x) - b*cosh(x)^2 - b*sinh(x)^2 + 4*sqrt(-a^2 + b^2)*(cosh(x
) + sinh(x))*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^
2)) + 2*(a*x - b*cosh(x))*sinh(x) + b)/(b^2*cosh(x) + b^2*sinh(x))]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 892 vs. $2(49) = 98$.
time = 68.48, size = 892, normalized size = 15.12

$$\begin{cases} \tilde{\infty}\left(-\frac{2\tanh^2\left(\frac{x}{2}\right)\operatorname{atan}\left(\tanh\left(\frac{x}{2}\right)\right)}{\tanh^2\left(\frac{x}{2}\right)-1}-\frac{2\tanh\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)-1}+\frac{2\operatorname{atan}\left(\tanh\left(\frac{x}{2}\right)\right)}{\tanh^2\left(\frac{x}{2}\right)-1}\right) & \text{for}\ a=0\wedge b=0 \\ \frac{x\tanh^2\left(\frac{x}{2}\right)}{b\tanh^2\left(\frac{x}{2}\right)-b}-\frac{x}{b\tanh^2\left(\frac{x}{2}\right)-b}-\frac{2\tanh\left(\frac{x}{2}\right)}{b\tanh^2\left(\frac{x}{2}\right)-b} & \text{for}\ a=-b \\ \frac{\frac{x\sinh^2(x)}{2}-\frac{x\cosh^2(x)}{2}+\frac{\sinh(x)\cosh(x)}{2}}{a} & \text{for}\ b=0 \\ -\frac{x\tanh^2\left(\frac{x}{2}\right)}{b\tanh^2\left(\frac{x}{2}\right)-b}+\frac{x}{b\tanh^2\left(\frac{x}{2}\right)-b}-\frac{2\tanh\left(\frac{x}{2}\right)}{b\tanh^2\left(\frac{x}{2}\right)-b} & \text{for}\ a=b \\ -\frac{ax\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}\tanh^2\left(\frac{x}{2}\right)}{b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}\tanh^2\left(\frac{x}{2}\right)-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}+\frac{ax\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}{b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}\tanh^2\left(\frac{x}{2}\right)-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}-\frac{a\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)\tanh^2\left(\frac{x}{2}\right)}{b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}\tanh^2\left(\frac{x}{2}\right)-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}+\frac{a\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}\tanh^2\left(\frac{x}{2}\right)-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}+\frac{a\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)\tanh^2\left(\frac{x}{2}\right)}{b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}\tanh^2\left(\frac{x}{2}\right)-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}-\frac{a\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}\tanh^2\left(\frac{x}{2}\right)-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}-\frac{2b\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}\tanh\left(\frac{x}{2}\right)}{b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}\tanh^2\left(\frac{x}{2}\right)-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}-\frac{b\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)\tanh^2\left(\frac{x}{2}\right)}{b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}\tanh^2\left(\frac{x}{2}\right)-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}+\frac{b\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}\tanh^2\left(\frac{x}{2}\right)-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}+\frac{b\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)\tanh^2\left(\frac{x}{2}\right)}{b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}\tanh^2\left(\frac{x}{2}\right)-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}-\frac{b\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}\tanh^2\left(\frac{x}{2}\right)-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(a+b*cosh(x)),x)
```

```
[Out] Piecewise((zoo*(-2*tanh(x/2)**2*atan(tanh(x/2))/(tanh(x/2)**2 - 1) - 2*tanh
(x/2)/(tanh(x/2)**2 - 1) + 2*atan(tanh(x/2))/(tanh(x/2)**2 - 1)), Eq(a, 0)
& Eq(b, 0)), (x*tanh(x/2)**2/(b*tanh(x/2)**2 - b) - x/(b*tanh(x/2)**2 - b)
- 2*tanh(x/2)/(b*tanh(x/2)**2 - b), Eq(a, -b)), ((x*sinh(x)**2/2 - x*cosh(x
)**2/2 + sinh(x)*cosh(x)/2)/a, Eq(b, 0)), (-x*tanh(x/2)**2/(b*tanh(x/2)**2
- b) + x/(b*tanh(x/2)**2 - b) - 2*tanh(x/2)/(b*tanh(x/2)**2 - b), Eq(a, b))
, (-a*x*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2/(b**2*sqrt(a/(a - b) + b/(
a - b))*tanh(x/2)**2 - b**2*sqrt(a/(a - b) + b/(a - b))) + a*x*sqrt(a/(a -
b) + b/(a - b))/(b**2*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 - b**2*sqrt(
a/(a - b) + b/(a - b))) - a*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))*t
anh(x/2)**2/(b**2*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 - b**2*sqrt(a/(a
 - b) + b/(a - b))) + a*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(b**2
*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 - b**2*sqrt(a/(a - b) + b/(a - b)
)) + a*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))*tanh(x/2)**2/(b**2*sqrt
(a/(a - b) + b/(a - b))*tanh(x/2)**2 - b**2*sqrt(a/(a - b) + b/(a - b))) -
a*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(b**2*sqrt(a/(a - b) + b/(a
- b))*tanh(x/2)**2 - b**2*sqrt(a/(a - b) + b/(a - b))) - 2*b*sqrt(a/(a - b)
 + b/(a - b))*tanh(x/2)/(b**2*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 - b*
*2*sqrt(a/(a - b) + b/(a - b))) - b*log(-sqrt(a/(a - b) + b/(a - b)) + tanh
(x/2))*tanh(x/2)**2/(b**2*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 - b**2*s
qrt(a/(a - b) + b/(a - b))) + b*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2
))/(b**2*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 - b**2*sqrt(a/(a - b) + b
/(a - b))) + b*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))*tanh(x/2)**2/(b
**2*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 - b**2*sqrt(a/(a - b) + b/(a -
 b))) - b*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(b**2*sqrt(a/(a - b)
 + b/(a - b))*tanh(x/2)**2 - b**2*sqrt(a/(a - b) + b/(a - b))), True))
```

Giac [A]
time = 0.44, size = 68, normalized size = 1.15

$$-\frac{ax}{b^2} - \frac{e^{(-x)}}{2\,b} + \frac{e^x}{2\,b} + \frac{2\,(a^2-b^2)\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] -a*x/b^2 - 1/2*e^(-x)/b + 1/2*e^x/b + 2*(a^2 - b^2)*arctan((b*e^x + a)/sqrt
(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b^2)
```

Mupad [B]
time = 1.04, size = 139, normalized size = 2.36

$$\frac{\mathrm{e}^x}{2\,b} - \frac{\mathrm{e}^{-x}}{2\,b} - \frac{a\,x}{b^2} + \frac{\ln\left(-\frac{2\,\mathrm{e}^x\,(a^2-b^2)}{b^3} - \frac{2\,\sqrt{a+b}\,(b+a\,\mathrm{e}^x)\,\sqrt{a-b}}{b^3}\right)\,\sqrt{a+b}\,\sqrt{a-b}}{b^2} - \frac{\ln\left(\frac{2\,\sqrt{a+b}\,(b+a\,\mathrm{e}^x)\,\sqrt{a-b}}{b^3} - \frac{2\,\mathrm{e}^x\,(a^2-b^2)}{b^3}\right)\,\sqrt{a+b}\,\sqrt{a-b}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a + b*cosh(x)),x)
```

```
[Out] exp(x)/(2*b) - exp(-x)/(2*b) - (a*x)/b^2 + (log(- (2*exp(x)*(a^2 - b^2))/b^
3 - (2*(a + b)^(1/2)*(b + a*exp(x))*(a - b)^(1/2))/b^3)*(a + b)^(1/2)*(a -
b)^(1/2))/b^2 - (log((2*(a + b)^(1/2)*(b + a*exp(x))*(a - b)^(1/2))/b^3 - (
2*exp(x)*(a^2 - b^2))/b^3)*(a + b)^(1/2)*(a - b)^(1/2))/b^2
```

3.171 $\int \frac{\sinh(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=11

$$\frac{\log(a+b\cosh(x))}{b}$$

[Out] ln(a+b*cosh(x))/b

Rubi [A]
time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2747, 31}

$$\frac{\log(a+b\cosh(x))}{b}$$

Antiderivative was successfully verified.

[In] Int[Sinh[x]/(a + b*Cosh[x]),x]

[Out] Log[a + b*Cosh[x]]/b

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(x)}{a+b\cosh(x)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\cosh(x)\right)}{b}\\ &= \frac{\log(a+b\cosh(x))}{b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 11, normalized size = 1.00

$$\frac{\log(a+b\cosh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(a + b*Cosh[x]),x]
```

```
[Out] Log[a + b*Cosh[x]]/b
```

Maple [A]
time = 0.28, size = 12, normalized size = 1.09

method	result	size
derivativedivides	$\frac{\ln(a+b\cosh(x))}{b}$	12
default	$\frac{\ln(a+b\cosh(x))}{b}$	12
risch	$-\frac{x}{b}+\frac{\ln\left(\mathrm{e}^{2x}+\frac{2a\,\mathrm{e}^{x}}{b}+1\right)}{b}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(a+b*cosh(x))/b
```

Maxima [A]
time = 0.26, size = 11, normalized size = 1.00

$$\frac{\log\left(b\cosh\left(x\right)+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] log(b*cosh(x) + a)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(11) = 22$.
time = 0.52, size = 27, normalized size = 2.45

$$-\frac{x-\log\left(\frac{2\,(b\cosh\left(x\right)+a)}{\cosh\left(x\right)-\sinh\left(x\right)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] -(x - log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))))/b
```

Sympy [A]
time = 0.16, size = 14, normalized size = 1.27

$$\begin{cases}\frac{\log\left(\frac{a}{b}+\cosh\left(x\right)\right)}{b} & \text{for}\: b\neq 0\\ \frac{\cosh\left(x\right)}{a} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*cosh(x)),x)
```

```
[Out] Piecewise((log(a/b + cosh(x))/b, Ne(b, 0)), (cosh(x)/a, True))
```

Giac [A]
time = 0.39, size = 19, normalized size = 1.73

$$\frac{\log\left(\left|b\left(e^{(-x)}+e^{x}\right)+2\,a\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] log(abs(b*(e^(-x) + e^x) + 2*a))/b
```

Mupad [B]
time = 0.06, size = 11, normalized size = 1.00

$$\frac{\ln\left(a+b\cosh(x)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a + b*cosh(x)),x)
```

```
[Out] log(a + b*cosh(x))/b
```

3.172 $\int \frac{\operatorname{csch}(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=53

$$\frac{\log(1-\cosh(x))}{2(a+b)}-\frac{\log(1+\cosh(x))}{2(a-b)}+\frac{b\log(a+b\cosh(x))}{a^2-b^2}$$

```
[Out] 1/2*ln(1-cosh(x))/(a+b)-1/2*ln(1+cosh(x))/(a-b)+b*ln(a+b*cosh(x))/(a^2-b^2)
```

Rubi [A]
time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2747, 720, 31, 647}

$$\frac{b\log(a+b\cosh(x))}{a^2-b^2}+\frac{\log(1-\cosh(x))}{2(a+b)}-\frac{\log(\cosh(x)+1)}{2(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]/(a + b*Cosh[x]),x]
```

```
[Out] Log[1 - Cosh[x]]/(2*(a + b)) - Log[1 + Cosh[x]]/(2*(a - b)) + (b*Log[a + b*
Cosh[x]])/(a^2 - b^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
```

```
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{csch}(x)}{a+b\cosh(x)}\,dx &= -\left(b\text{Subst}\left(\int \frac{1}{(a+x)\left(b^2-x^2\right)}\,dx, x, b\cosh(x)\right)\right)\\
&= \frac{b\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\cosh(x)\right)}{a^2-b^2} + \frac{b\text{Subst}\left(\int \frac{-a+x}{b^2-x^2}\,dx, x, b\cosh(x)\right)}{a^2-b^2}\\
&= \frac{b\log(a+b\cosh(x))}{a^2-b^2} + \frac{\text{Subst}\left(\int \frac{1}{-b-x}\,dx, x, b\cosh(x)\right)}{2(a-b)} - \frac{\text{Subst}\left(\int \frac{1}{b-x}\,dx, x, b\cosh(x)\right)}{2(a+b)}\\
&= \frac{\log(1-\cosh(x))}{2(a+b)} - \frac{\log(1+\cosh(x))}{2(a-b)} + \frac{b\log(a+b\cosh(x))}{a^2-b^2}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 37, normalized size = 0.70

$$\frac{b\log(a+b\cosh(x)) - b\log(\sinh(x)) + a\log\left(\tanh\left(\frac{x}{2}\right)\right)}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]/(a + b*Cosh[x]),x]
[Out] (b*Log[a + b*Cosh[x]] - b*Log[Sinh[x]] + a*Log[Tanh[x/2]])/(a^2 - b^2)
```

Maple [A]
time = 0.52, size = 52, normalized size = 0.98

method	result	size
default	$\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a+b} + \frac{b\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-b\left(\tanh^2\left(\frac{x}{2}\right)\right)-a-b\right)}{(a+b)(a-b)}$	52
risch	$-\frac{x}{a+b} + \frac{x}{a-b} - \frac{2xb}{a^2-b^2} + \frac{\ln(e^x-1)}{a+b} - \frac{\ln(e^x+1)}{a-b} + \frac{b\ln\left(e^{2x}+\frac{2a\,e^x}{b}+1\right)}{a^2-b^2}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
[Out] 1/(a+b)*ln(tanh(1/2*x))+b/(a+b)/(a-b)*ln(a*tanh(1/2*x)^2-b*tanh(1/2*x)^2-a-
b)
```

Maxima [A]
time = 0.27, size = 59, normalized size = 1.11

$$\frac{b\log\left(2\,ae^{(-x)} + be^{(-2\,x)} + b\right)}{a^2-b^2} - \frac{\log\left(e^{(-x)}+1\right)}{a-b} + \frac{\log\left(e^{(-x)}-1\right)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] b*log(2*a*e^(-x) + b*e^(-2*x) + b)/(a^2 - b^2) - log(e^(-x) + 1)/(a - b) +
log(e^(-x) - 1)/(a + b)
```

Fricas [A]
time = 0.47, size = 58, normalized size = 1.09

$$\frac{b\log\left(\frac{2\,(b\cosh(x)+a)}{\cosh(x)-\sinh(x)}\right)-(a+b)\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)+(a-b)\log\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] (b*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))) - (a + b)*log(cosh(x) + sinh(
x) + 1) + (a - b)*log(cosh(x) + sinh(x) - 1))/(a^2 - b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{csch}\left(x\right)}{a+b\cosh\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*cosh(x)),x)
```

```
[Out] Integral(csch(x)/(a + b*cosh(x)), x)
```

Giac [A]
time = 0.41, size = 67, normalized size = 1.26

$$\frac{b^2\log\left(\left|b\left(e^{(-x)}+e^x\right)+2\,a\right|\right)}{a^2b-b^3}-\frac{\log\left(e^{(-x)}+e^x+2\right)}{2\,(a-b)}+\frac{\log\left(e^{(-x)}+e^x-2\right)}{2\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] b^2*log(abs(b*(e^(-x) + e^x) + 2*a))/(a^2*b - b^3) - 1/2*log(e^(-x) + e^x +
 2)/(a - b) + 1/2*log(e^(-x) + e^x - 2)/(a + b)
```

Mupad [B]
time = 1.29, size = 160, normalized size = 3.02

$$\frac{\ln\left(128\,a\,b^2-128\,a^2\,b+32\,a^3-32\,a^3\,\mathrm{e}^x-128\,a\,b^2\,\mathrm{e}^x+128\,a^2\,b\,\mathrm{e}^x\right)}{a+b}-\frac{\ln\left(128\,a\,b^2+128\,a^2\,b+32\,a^3+32\,a^3\,\mathrm{e}^x+128\,a\,b^2\,\mathrm{e}^x+128\,a^2\,b\,\mathrm{e}^x\right)}{a-b}+\frac{b\,\ln\left(16\,b^3\,\mathrm{e}^{2\,x}-4\,a^2\,b+16\,b^3-8\,a^3\,\mathrm{e}^x+32\,a\,b^2\,\mathrm{e}^x-4\,a^2\,b\,\mathrm{e}^{2\,x}\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)*(a + b*cosh(x))),x)
```

```
[Out] log(128*a*b^2 - 128*a^2*b + 32*a^3 - 32*a^3*exp(x) - 128*a*b^2*exp(x) + 128
*a^2*b*exp(x))/(a + b) - log(128*a*b^2 + 128*a^2*b + 32*a^3 + 32*a^3*exp(x)
 + 128*a*b^2*exp(x) + 128*a^2*b*exp(x))/(a - b) + (b*log(16*b^3*exp(2*x) -
4*a^2*b + 16*b^3 - 8*a^3*exp(x) + 32*a*b^2*exp(x) - 4*a^2*b*exp(2*x)))/(a^2
 - b^2)
```

3.173 $\int \frac{\operatorname{csch}^2(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=67

$$\frac{2b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}+\frac{(b-a\cosh(x))\operatorname{csch}(x)}{a^2-b^2}$$

```
[Out] 2*b^2*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(3/2)/(a+b)^(3/2)+
(b-a*cosh(x))*csch(x)/(a^2-b^2)
```

Rubi [A]
time = 0.07, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2775, 12, 2738, 214}

$$\frac{\operatorname{csch}(x)(b-a\cosh(x))}{a^2-b^2}+\frac{2b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^2/(a + b*Cosh[x]),x]
```

```
[Out] (2*b^2*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/((a - b)^(3/2)*(a + b)
^(3/2)) + ((b - a*Cosh[x])*Csch[x])/(a^2 - b^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2775

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^2(x)}{a+b\cosh(x)}\,dx &= \frac{(b-a\cosh(x))\operatorname{csch}(x)}{a^2-b^2}-\frac{\int \frac{b^2}{a+b\cosh(x)}\,dx}{-a^2+b^2}\\
&= \frac{(b-a\cosh(x))\operatorname{csch}(x)}{a^2-b^2}+\frac{b^2\int \frac{1}{a+b\cosh(x)}\,dx}{a^2-b^2}\\
&= \frac{(b-a\cosh(x))\operatorname{csch}(x)}{a^2-b^2}+\frac{(2b^2)\operatorname{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2-b^2}\\
&= \frac{2b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}+\frac{(b-a\cosh(x))\operatorname{csch}(x)}{a^2-b^2}
\end{aligned}$$

Mathematica [A]
time = 0.15, size = 77, normalized size = 1.15

$$\frac{2b^2\operatorname{ArcTan}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{3/2}}-\frac{\coth\left(\frac{x}{2}\right)}{2(a+b)}-\frac{\tanh\left(\frac{x}{2}\right)}{2(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^2/(a + b*Cosh[x]),x]
[Out] (2*b^2*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) - C
oth[x/2]/(2*(a + b)) - Tanh[x/2]/(2*(a - b))
```

Maple [A]
time = 0.56, size = 78, normalized size = 1.16

method	result	size
default	$-\frac{\tanh\left(\frac{x}{2}\right)}{2(a-b)}+\frac{2b^2\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}-\frac{1}{2(a+b)\tanh\left(\frac{x}{2}\right)}$	78

risch	$-\frac{2(-e^{x}b+a)}{(e^{2x}-1)(a^{2}-b^{2})}+\frac{b^{2}\ln\left(e^{x}+\frac{a\sqrt{a^{2}-b^{2}}-a^{2}+b^{2}}{b\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,(a+b)(a-b)}-\frac{b^{2}\ln\left(e^{x}+\frac{a\sqrt{a^{2}-b^{2}}+a^{2}-b^{2}}{b\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,(a+b)(a-b)}$	167

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^2/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(a-b)*tanh(1/2*x)+2/(a+b)/(a-b)*b^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*
tanh(1/2*x)/((a+b)*(a-b))^(1/2))-1/2/(a+b)/tanh(1/2*x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(59) = 118.
time = 0.47, size = 470, normalized size = 7.01

$$\left[\frac{2\,a^{3}-2\,ab^{2}+\left(b^{2}\cosh\left(x\right)^{2}+2\,b^{2}\cosh\left(x\right)\sinh\left(x\right)+b^{2}\sinh\left(x\right)^{2}-b^{2}\right)\sqrt{a^{2}-b^{2}}\log\left(\frac{b^{2}\cosh(x)^{2}+b^{2}\sinh(x)^{2}+2\,ab\cosh(x)+2\,a^{2}-b^{2}+2\,(b^{2}\cosh(x)+ab)\sinh(x)+2\,\sqrt{a^{2}-b^{2}}\,(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^{2}+b\sinh(x)^{2}+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right)-2\,(a^{2}b-b^{3})\cosh\left(x\right)-2\,(a^{2}b-b^{3})\sinh\left(x\right)}{a^{4}-2\,a^{2}b^{2}+b^{4}-(a^{4}-2\,a^{2}b^{2}+b^{4})\cosh\left(x\right)^{2}-2\,(a^{4}-2\,a^{2}b^{2}+b^{4})\cosh\left(x\right)\sinh\left(x\right)-(a^{4}-2\,a^{2}b^{2}+b^{4})\sinh\left(x\right)^{2}},\ \frac{2\left(a^{3}-ab^{2}+\left(b^{2}\cosh\left(x\right)^{2}+2\,b^{2}\cosh\left(x\right)\sinh\left(x\right)+b^{2}\sinh\left(x\right)^{2}-b^{2}\right)\sqrt{-a^{2}+b^{2}}\arctan\left(-\frac{\sqrt{-a^{2}+b^{2}}\,(b\cosh(x)+b\sinh(x)+a)}{a^{2}-b^{2}}\right)-(a^{2}b-b^{3})\cosh\left(x\right)-(a^{2}b-b^{3})\sinh\left(x\right)\right)}{a^{4}-2\,a^{2}b^{2}+b^{4}-(a^{4}-2\,a^{2}b^{2}+b^{4})\cosh\left(x\right)^{2}-2\,(a^{4}-2\,a^{2}b^{2}+b^{4})\cosh\left(x\right)\sinh\left(x\right)-(a^{4}-2\,a^{2}b^{2}+b^{4})\sinh\left(x\right)^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [(2*a^3 - 2*a*b^2 + (b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2
- b^2)*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) +
 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*cosh(x)
 + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x)
+ a)*sinh(x) + b)) - 2*(a^2*b - b^3)*cosh(x) - 2*(a^2*b - b^3)*sinh(x))/(a^
4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 - 2*(a^4 - 2*a^2*b^
2 + b^4)*cosh(x)*sinh(x) - (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^2), 2*(a^3 - a*b
^2 + (b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2 - b^2)*sqrt(-a^
2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2))
- (a^2*b - b^3)*cosh(x) - (a^2*b - b^3)*sinh(x))/(a^4 - 2*a^2*b^2 + b^4 - (
a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 - 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x
) - (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(x)}{a + b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2/(a+b*cosh(x)),x)
```

```
[Out] Integral(csch(x)**2/(a + b*cosh(x)), x)
```

Giac [A]
time = 0.40, size = 76, normalized size = 1.13

$$\frac{2\,b^2\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{(a^2-b^2)\sqrt{-a^2+b^2}} + \frac{2\,(be^x-a)}{(a^2-b^2)(e^{(2\,x)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*b^2*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/((a^2 - b^2)*sqrt(-a^2 + b^2)) +
 2*(b*e^x - a)/((a^2 - b^2)*(e^(2*x) - 1))
```

Mupad [B]
time = 1.48, size = 327, normalized size = 4.88

$$-\frac{\frac{2\,a}{a^2-b^2}-\frac{2\,b\,e^x}{a^2-b^2}}{e^{2\,x}-1}-\frac{2\,\mathrm{atan}\left(\left(\left(e^x\left(\frac{2}{(a^2-b^2)^2\,\sqrt{b^4}}+\frac{2\,a\left(a^3\,\sqrt{b^4}-a\,b^2\,\sqrt{b^4}\right)}{b^4\,(a^2-b^2)\,\sqrt{-(a^2-b^2)^3}\,\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}}\right)-\frac{2\,a\left(b^3\,\sqrt{b^4}-a^2\,b\,\sqrt{b^4}\right)}{b^4\,(a^2-b^2)\,\sqrt{-(a^2-b^2)^3}\,\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}}\right)\left(\frac{b^3\,\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}}{2}-\frac{a^2\,b\,\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}}{2}\right)\right)\sqrt{b^4}}{\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^2*(a + b*cosh(x))),x)
```

```
[Out] - ((2*a)/(a^2 - b^2) - (2*b*exp(x))/(a^2 - b^2))/(exp(2*x) - 1) - (2*atan((
exp(x)*(2/((a^2 - b^2)^2*(b^4)^(1/2)) + (2*a*(a^3*(b^4)^(1/2) - a*b^2*(b^4)
^(1/2)))/(b^4*(a^2 - b^2)*(-(a^2 - b^2)^3)^(1/2)*(b^6 - a^6 - 3*a^2*b^4 + 3
*a^4*b^2)^(1/2))) - (2*a*(b^3*(b^4)^(1/2) - a^2*b*(b^4)^(1/2)))/(b^4*(a^2 -
 b^2)*(-(a^2 - b^2)^3)^(1/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)))*((
b^3*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/2 - (a^2*b*(b^6 - a^6 - 3*a^
2*b^4 + 3*a^4*b^2)^(1/2))/2))*(b^4)^(1/2))/(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b
^2)^(1/2)
```

3.174 $\int \frac{\operatorname{csch}^3(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=91

$$\frac{(b-a\cosh(x))\operatorname{csch}^2(x)}{2\,(a^2-b^2)}-\frac{(a+2b)\log(1-\cosh(x))}{4(a+b)^2}+\frac{(a-2b)\log(1+\cosh(x))}{4(a-b)^2}+\frac{b^3\log(a+b\cosh(x))}{(a^2-b^2)^2}$$

```
[Out] 1/2*(b-a*cosh(x))*csch(x)^2/(a^2-b^2)-1/4*(a+2*b)*ln(1-cosh(x))/(a+b)^2+1/4
*(a-2*b)*ln(1+cosh(x))/(a-b)^2+b^3*ln(a+b*cosh(x))/(a^2-b^2)^2
```

Rubi [A]
time = 0.11, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2747, 755, 815}

$$\frac{\operatorname{csch}^2(x)(b-a\cosh(x))}{2\,(a^2-b^2)}+\frac{b^3\log(a+b\cosh(x))}{(a^2-b^2)^2}-\frac{(a+2b)\log(1-\cosh(x))}{4(a+b)^2}+\frac{(a-2b)\log(\cosh(x)+1)}{4(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^3/(a + b*Cosh[x]),x]
```

```
[Out] ((b - a*Cosh[x])*Csch[x]^2)/(2*(a^2 - b^2)) - ((a + 2*b)*Log[1 - Cosh[x]])/
(4*(a + b)^2) + ((a - 2*b)*Log[1 + Cosh[x]])/(4*(a - b)^2) + (b^3*Log[a + b
*Cosh[x]])/(a^2 - b^2)^2
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
```

```
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(x)}{a+b\cosh(x)}\,dx = b^3\operatorname{Subst}\left(\int \frac{1}{(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\cosh(x)\right)$$
$$= \frac{(b-a\cosh(x))\operatorname{csch}^2(x)}{2\left(a^2-b^2\right)} + \frac{b\operatorname{Subst}\left(\int \frac{a^2-2b^2+ax}{(a+x)(b^2-x^2)}\,dx, x, b\cosh(x)\right)}{2\left(a^2-b^2\right)}$$
$$= \frac{(b-a\cosh(x))\operatorname{csch}^2(x)}{2\left(a^2-b^2\right)} + \frac{b\operatorname{Subst}\left(\int \left(\frac{(a-b)(a+2b)}{2b(a+b)(b-x)} + \frac{2b^2}{(a-b)(a+b)(a+x)} + \frac{(a-2b)(a+b)}{2(a-b)b(b+x)}\right) dx,\right.}{2\left(a^2-b^2\right)}$$
$$= \frac{(b-a\cosh(x))\operatorname{csch}^2(x)}{2\left(a^2-b^2\right)} - \frac{(a+2b)\log(1-\cosh(x))}{4(a+b)^2} + \frac{(a-2b)\log(1+\cosh(x))}{4(a-b)^2} + \frac{b}{}$$

Mathematica [A]
time = 0.20, size = 100, normalized size = 1.10

$$-\frac{(a-b)^2(a+b)\operatorname{csch}^2\left(\frac{x}{2}\right) - 8b^3\log(a+b\cosh(x)) + 8b^3\log(\sinh(x)) + 4a^3\log\left(\tanh\left(\frac{x}{2}\right)\right) - 12ab^2\log\left(\tanh\left(\frac{x}{2}\right)\right) + (a-b)(a+b)^2\operatorname{sech}^2\left(\frac{x}{2}\right)}{8(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^3/(a + b*Cosh[x]),x]
[Out] -1/8*((a - b)^2*(a + b)*Csch[x/2]^2 - 8*b^3*Log[a + b*Cosh[x]] + 8*b^3*Log[
Sinh[x]] + 4*a^3*Log[Tanh[x/2]] - 12*a*b^2*Log[Tanh[x/2]] + (a - b)*(a + b)
^2*Sech[x/2]^2)/((a - b)^2*(a + b)^2)
```

Maple [A]
time = 0.61, size = 90, normalized size = 0.99

method	result
default	$\frac{\tanh^2\left(\frac{x}{2}\right)}{8a-8b} + \frac{b^3\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-b\left(\tanh^2\left(\frac{x}{2}\right)\right)-a-b\right)}{(a+b)^2(a-b)^2} - \frac{1}{8(a+b)\tanh\left(\frac{x}{2}\right)^2} + \frac{(-4b-2a)\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{4(a+b)^2}$
risch	$\frac{xb}{a^2-2ab+b^2} - \frac{xa}{2(a^2-2ab+b^2)} + \frac{xb}{a^2+2ab+b^2} + \frac{ax}{2a^2+4ab+2b^2} - \frac{2x\,b^3}{a^4-2a^2b^2+b^4} - \frac{e^x\left(e^{2x}a-2\,e^x b+a\right)}{\left(e^{2x}-1\right)^2\left(a^2-b^2\right)} - \frac{\ln(e^x+1)b}{a^2-2ab+b^2} + \frac{}{2a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^3/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
[Out] 1/8*tanh(1/2*x)^2/(a-b)+b^3/(a+b)^2/(a-b)^2*ln(a*tanh(1/2*x)^2-b*tanh(1/2*x
)^2-a-b)-1/8/(a+b)/tanh(1/2*x)^2+1/4/(a+b)^2*(-4*b-2*a)*ln(tanh(1/2*x))
```

Maxima [A]
time = 0.27, size = 154, normalized size = 1.69

$$\frac{b^3 \log\left(2\,ae^{(-x)} + be^{(-2\,x)} + b\right)}{a^4 - 2\,a^2b^2 + b^4} + \frac{(a-2\,b)\log\left(e^{(-x)}+1\right)}{2\,(a^2-2\,ab+b^2)} - \frac{(a+2\,b)\log\left(e^{(-x)}-1\right)}{2\,(a^2+2\,ab+b^2)} - \frac{ae^{(-x)} - 2\,be^{(-2\,x)} + ae^{(-3\,x)}}{a^2 - b^2 - 2\,(a^2-b^2)e^{(-2\,x)} + (a^2-b^2)e^{(-4\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] b^3*log(2*a*e^(-x) + b*e^(-2*x) + b)/(a^4 - 2*a^2*b^2 + b^4) + 1/2*(a - 2*b
)*log(e^(-x) + 1)/(a^2 - 2*a*b + b^2) - 1/2*(a + 2*b)*log(e^(-x) - 1)/(a^2
+ 2*a*b + b^2) - (a*e^(-x) - 2*b*e^(-2*x) + a*e^(-3*x))/(a^2 - b^2 - 2*(a^2
 - b^2)*e^(-2*x) + (a^2 - b^2)*e^(-4*x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 818 vs. 2(86) = 172.
time = 0.38, size = 818, normalized size = 8.99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^3 - a*b^2)*cosh(x)^3 + 2*(a^3 - a*b^2)*sinh(x)^3 - 4*(a^2*b - b^
3)*cosh(x)^2 - 2*(2*a^2*b - 2*b^3 - 3*(a^3 - a*b^2)*cosh(x))*sinh(x)^2 + 2*
(a^3 - a*b^2)*cosh(x) - 2*(b^3*cosh(x)^4 + 4*b^3*cosh(x)*sinh(x)^3 + b^3*si
nh(x)^4 - 2*b^3*cosh(x)^2 + b^3 + 2*(3*b^3*cosh(x)^2 - b^3)*sinh(x)^2 + 4*(
b^3*cosh(x)^3 - b^3*cosh(x))*sinh(x))*log(2*(b*cosh(x) + a)/(cosh(x) - sinh
(x))) - ((a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^4 + 4*(a^3 - 3*a*b^2 - 2*b^3)*cosh
(x)*sinh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*sinh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 -
 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 - 2*(a^3 - 3*a*b^2 - 2*b^3 - 3*(a^3 -
3*a*b^2 - 2*b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^
3 - (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) +
((a^3 - 3*a*b^2 + 2*b^3)*cosh(x)^4 + 4*(a^3 - 3*a*b^2 + 2*b^3)*cosh(x)*sinh
(x)^3 + (a^3 - 3*a*b^2 + 2*b^3)*sinh(x)^4 + a^3 - 3*a*b^2 + 2*b^3 - 2*(a^3
- 3*a*b^2 + 2*b^3)*cosh(x)^2 - 2*(a^3 - 3*a*b^2 + 2*b^3 - 3*(a^3 - 3*a*b^2
+ 2*b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 - 3*a*b^2 + 2*b^3)*cosh(x)^3 - (a^3
 - 3*a*b^2 + 2*b^3)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) + 2*(a^3 -
 a*b^2 + 3*(a^3 - a*b^2)*cosh(x)^2 - 4*(a^2*b - b^3)*cosh(x))*sinh(x))/((a^
4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + 4*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^
3 + (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*
a^2*b^2 + b^4)*cosh(x)^2 - 2*(a^4 - 2*a^2*b^2 + b^4 - 3*(a^4 - 2*a^2*b^2 +
b^4)*cosh(x)^2)*sinh(x)^2 + 4*((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^3 - (a^4 - 2
*a^2*b^2 + b^4)*cosh(x))*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(x)}{a+b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**3/(a+b*cosh(x)),x)
```

```
[Out] Integral(csch(x)**3/(a + b*cosh(x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(86) = 172.
time = 0.43, size = 179, normalized size = 1.97

$$\frac{b^4\log\left(\left|b\left(e^{(-x)}+e^x\right)+2\,a\right|\right)}{a^4b-2\,a^2b^3+b^5}+\frac{(a-2\,b)\log\left(e^{(-x)}+e^x+2\right)}{4\,(a^2-2\,ab+b^2)}-\frac{(a+2\,b)\log\left(e^{(-x)}+e^x-2\right)}{4\,(a^2+2\,ab+b^2)}+\frac{b^3\left(e^{(-x)}+e^x\right)^2-2\,a^3\left(e^{(-x)}+e^x\right)+2\,ab^2\left(e^{(-x)}+e^x\right)+4\,a^2b-8\,b^3}{2\,(a^4-2\,a^2b^2+b^4)\left(\left(e^{(-x)}+e^x\right)^2-4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] b^4*log(abs(b*(e^(-x) + e^x) + 2*a))/(a^4*b - 2*a^2*b^3 + b^5) + 1/4*(a - 2
*b)*log(e^(-x) + e^x + 2)/(a^2 - 2*a*b + b^2) - 1/4*(a + 2*b)*log(e^(-x) +
e^x - 2)/(a^2 + 2*a*b + b^2) + 1/2*(b^3*(e^(-x) + e^x)^2 - 2*a^3*(e^(-x) +
e^x) + 2*a*b^2*(e^(-x) + e^x) + 4*a^2*b - 8*b^3)/((a^4 - 2*a^2*b^2 + b^4)*(
(e^(-x) + e^x)^2 - 4))
```

Mupad [B]
time = 1.48, size = 291, normalized size = 3.20

$$\frac{\frac{2\,(a^2\,b-b^3)}{(a^2-b^2)^2}+\frac{e^x\,(a\,b^2-a^3)}{(a^2-b^2)^2}}{e^{2\,x}-1}+\frac{\frac{2\,b}{a^2-b^2}-\frac{2\,a\,e^x}{a^2-b^2}}{e^{4\,x}-2\,e^{2\,x}+1}+\frac{b^3\,\ln\left(16\,b^7\,e^{2\,x}-a^6\,b+16\,b^7-9\,a^2\,b^5+6\,a^4\,b^3-2\,a^7\,e^x-9\,a^2\,b^5\,e^{2\,x}+6\,a^4\,b^3\,e^{2\,x}+32\,a\,b^6\,e^x-a^6\,b\,e^{2\,x}-18\,a^3\,b^4\,e^x+12\,a^5\,b^2\,e^x\right)}{a^4-2\,a^2\,b^2+b^4}-\frac{\ln\left(e^x-1\right)\,(a+2\,b)}{2\,a^2+4\,a\,b+2\,b^2}+\frac{\ln\left(e^x+1\right)\,(a-2\,b)}{2\,a^2-4\,a\,b+2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^3*(a + b*cosh(x))),x)
```

```
[Out] ((2*(a^2*b - b^3))/(a^2 - b^2)^2 + (exp(x)*(a*b^2 - a^3))/(a^2 - b^2)^2)/(e
xp(2*x) - 1) + ((2*b)/(a^2 - b^2) - (2*a*exp(x))/(a^2 - b^2))/(exp(4*x) - 2
*exp(2*x) + 1) + (b^3*log(16*b^7*exp(2*x) - a^6*b + 16*b^7 - 9*a^2*b^5 + 6*
a^4*b^3 - 2*a^7*exp(x) - 9*a^2*b^5*exp(2*x) + 6*a^4*b^3*exp(2*x) + 32*a*b^6
*exp(x) - a^6*b*exp(2*x) - 18*a^3*b^4*exp(x) + 12*a^5*b^2*exp(x)))/(a^4 + b
^4 - 2*a^2*b^2) - (log(exp(x) - 1)*(a + 2*b))/(4*a*b + 2*a^2 + 2*b^2) + (lo
g(exp(x) + 1)*(a - 2*b))/(2*a^2 - 4*a*b + 2*b^2)
```

3.175 $\int \frac{\operatorname{csch}^4(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=110

$$\frac{2b^4\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}}+\frac{(3b^3+a(2a^2-5b^2)\cosh(x))\operatorname{csch}(x)}{3\left(a^2-b^2\right)^2}+\frac{(b-a\cosh(x))\operatorname{csch}^3(x)}{3\left(a^2-b^2\right)}$$

```
[Out] 2*b^4*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(5/2)/(a+b)^(5/2)+
1/3*(3*b^3+a*(2*a^2-5*b^2)*cosh(x))*csch(x)/(a^2-b^2)^2+1/3*(b-a*cosh(x))*c
sch(x)^3/(a^2-b^2)
```

Rubi [A]
time = 0.17, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2775, 2945, 12, 2738, 214}

$$\frac{\operatorname{csch}^3(x)(b-a\cosh(x))}{3\left(a^2-b^2\right)}+\frac{\operatorname{csch}(x)\left(a\left(2a^2-5b^2\right)\cosh(x)+3b^3\right)}{3\left(a^2-b^2\right)^2}+\frac{2b^4\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^4/(a + b*Cosh[x]),x]
```

```
[Out] (2*b^4*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/((a - b)^(5/2)*(a + b)
^(5/2)) + ((3*b^3 + a*(2*a^2 - 5*b^2)*Cosh[x])*Csch[x])/(3*(a^2 - b^2)^2) +
 ((b - a*Cosh[x])*Csch[x]^3)/(3*(a^2 - b^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2775

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\operatorname{csch}^4(x)}{a+b\cosh(x)}\,dx = \frac{(b-a\cosh(x))\operatorname{csch}^3(x)}{3(a^2-b^2)} + \frac{\int \frac{(-2a^2+3b^2-2ab\cosh(x))\operatorname{csch}^2(x)}{a+b\cosh(x)}\,dx}{3(a^2-b^2)}$$

$$= \frac{(3b^3+a(2a^2-5b^2)\cosh(x))\operatorname{csch}(x)}{3(a^2-b^2)^2} + \frac{(b-a\cosh(x))\operatorname{csch}^3(x)}{3(a^2-b^2)} + \frac{\int \frac{3b^4}{a+b\cosh(x)}\,dx}{3(a^2-b^2)^2}$$

$$= \frac{(3b^3+a(2a^2-5b^2)\cosh(x))\operatorname{csch}(x)}{3(a^2-b^2)^2} + \frac{(b-a\cosh(x))\operatorname{csch}^3(x)}{3(a^2-b^2)} + \frac{b^4\int \frac{1}{a+b\cosh(x)}\,dx}{(a^2-b^2)^2}$$

$$= \frac{(3b^3+a(2a^2-5b^2)\cosh(x))\operatorname{csch}(x)}{3(a^2-b^2)^2} + \frac{(b-a\cosh(x))\operatorname{csch}^3(x)}{3(a^2-b^2)} + \frac{(2b^4)\operatorname{Subst}\left(\int \frac{}{a+b\ldots}\right.}{(a\ldots}$$

$$= \frac{2b^4\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}} + \frac{(3b^3+a(2a^2-5b^2)\cosh(x))\operatorname{csch}(x)}{3(a^2-b^2)^2} + \frac{(b-a\cosh(\ldots}{3(a^2\ldots}$$

Mathematica [A]
time = 0.41, size = 141, normalized size = 1.28

$$\frac{1}{24}\left(-\frac{48b^4\operatorname{ArcTan}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{5/2}} + \frac{2(4a+7b)\coth\left(\frac{x}{2}\right)}{(a+b)^2} + \frac{8\operatorname{csch}^3(x)\sinh^4\left(\frac{x}{2}\right)}{a-b} - \frac{\operatorname{csch}^4\left(\frac{x}{2}\right)\sinh(x)}{2(a+b)} + \frac{8a\tanh\left(\frac{x}{2}\right)}{(a-b)^2} - \frac{14b\tanh\left(\frac{x}{2}\right)}{(a-b)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^4/(a + b*Cosh[x]),x]
```

```
[Out] ((-48*b^4*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5/2)
+ (2*(4*a + 7*b)*Coth[x/2])/(a + b)^2 + (8*Csch[x]^3*Sinh[x/2]^4)/(a - b) -
 (Csch[x/2]^4*Sinh[x])/(2*(a + b)) + (8*a*Tanh[x/2])/(a - b)^2 - (14*b*Tanh
[x/2])/(a - b)^2)/24
```

Maple [A]
time = 0.60, size = 127, normalized size = 1.15

method	result
default	$-\frac{\frac{a\left(\tanh^3\left(\frac{x}{2}\right)\right)}{3}-\frac{b\left(\tanh^3\left(\frac{x}{2}\right)\right)}{3}-3a\tanh\left(\frac{x}{2}\right)+5b\tanh\left(\frac{x}{2}\right)}{8(a-b)^2}+\frac{2b^4\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a-b)^2(a+b)^2\sqrt{(a+b)(a-b)}}-\frac{1}{24(a+b)\tanh\left(\frac{x}{2}\right)^3}-\frac{\ }{8(a}$
risch	$-\frac{2\left(-3b^3e^{5x}+3a\,b^2e^{4x}-4a^2b\,e^{3x}+10b^3e^{3x}+6a^3e^{2x}-12a\,b^2e^{2x}-3b^3e^x-2a^3+5a\,b^2\right)}{3(a^2-b^2)^2(e^{2x}-1)^3}+\frac{b^4\ln\left(e^x+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(a+b)^2(a-b)^2}-\frac{b^4\ln}{\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^4/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/(a-b)^2*(1/3*a*tanh(1/2*x)^3-1/3*b*tanh(1/2*x)^3-3*a*tanh(1/2*x)+5*b*t
anh(1/2*x))+2/(a-b)^2/(a+b)^2*b^4/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/
2*x)/((a+b)*(a-b))^(1/2))-1/24/(a+b)/tanh(1/2*x)^3-1/8/(a+b)^2*(-3*a-5*b)/t
anh(1/2*x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1135 vs. $2(97) = 194$.
time = 0.43, size = 2339, normalized size = 21.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [1/3*(6*(a^2*b^3 - b^5)*cosh(x)^5 + 6*(a^2*b^3 - b^5)*sinh(x)^5 + 4*a^5 - 1
4*a^3*b^2 + 10*a*b^4 - 6*(a^3*b^2 - a*b^4)*cosh(x)^4 - 6*(a^3*b^2 - a*b^4 -
 5*(a^2*b^3 - b^5)*cosh(x))*sinh(x)^4 + 4*(2*a^4*b - 7*a^2*b^3 + 5*b^5)*cos
h(x)^3 + 4*(2*a^4*b - 7*a^2*b^3 + 5*b^5 + 15*(a^2*b^3 - b^5)*cosh(x)^2 - 6*
(a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^3 - 12*(a^5 - 3*a^3*b^2 + 2*a*b^4)*cosh(
x)^2 - 12*(a^5 - 3*a^3*b^2 + 2*a*b^4 - 5*(a^2*b^3 - b^5)*cosh(x)^3 + 3*(a^3
*b^2 - a*b^4)*cosh(x)^2 - (2*a^4*b - 7*a^2*b^3 + 5*b^5)*cosh(x))*sinh(x)^2
+ 3*(b^4*cosh(x)^6 + 6*b^4*cosh(x)*sinh(x)^5 + b^4*sinh(x)^6 - 3*b^4*cosh(x
)^4 + 3*b^4*cosh(x)^2 + 3*(5*b^4*cosh(x)^2 - b^4)*sinh(x)^4 - b^4 + 4*(5*b^
4*cosh(x)^3 - 3*b^4*cosh(x))*sinh(x)^3 + 3*(5*b^4*cosh(x)^4 - 6*b^4*cosh(x)
^2 + b^4)*sinh(x)^2 + 6*(b^4*cosh(x)^5 - 2*b^4*cosh(x)^3 + b^4*cosh(x))*sin
h(x))*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) +
2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x)
+ b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) +
 a)*sinh(x) + b)) + 6*(a^2*b^3 - b^5)*cosh(x) + 6*(a^2*b^3 - b^5 + 5*(a^2*b
^3 - b^5)*cosh(x)^4 - 4*(a^3*b^2 - a*b^4)*cosh(x)^3 + 2*(2*a^4*b - 7*a^2*b^
3 + 5*b^5)*cosh(x)^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*cosh(x))*sinh(x))/((a^
6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^6 + 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4
 - b^6)*cosh(x)*sinh(x)^5 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^6 -
 a^6 + 3*a^4*b^2 - 3*a^2*b^4 + b^6 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*
cosh(x)^4 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 - 5*(a^6 - 3*a^4*b^2 + 3*a
^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^
6)*cosh(x)^3 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x)^3 + 3
*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2 + 3*(a^6 - 3*a^4*b^2 + 3*a^2
*b^4 - b^6 + 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 - 6*(a^6 - 3*a
^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^2 + 6*((a^6 - 3*a^4*b^2 + 3*a^
2*b^4 - b^6)*cosh(x)^5 - 2*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 +
(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x)), 2/3*(3*(a^2*b^3 - b^
5)*cosh(x)^5 + 3*(a^2*b^3 - b^5)*sinh(x)^5 + 2*a^5 - 7*a^3*b^2 + 5*a*b^4 -
3*(a^3*b^2 - a*b^4)*cosh(x)^4 - 3*(a^3*b^2 - a*b^4 - 5*(a^2*b^3 - b^5)*cosh
(x))*sinh(x)^4 + 2*(2*a^4*b - 7*a^2*b^3 + 5*b^5)*cosh(x)^3 + 2*(2*a^4*b - 7
*a^2*b^3 + 5*b^5 + 15*(a^2*b^3 - b^5)*cosh(x)^2 - 6*(a^3*b^2 - a*b^4)*cosh(
x))*sinh(x)^3 - 6*(a^5 - 3*a^3*b^2 + 2*a*b^4)*cosh(x)^2 - 6*(a^5 - 3*a^3*b^
2 + 2*a*b^4 - 5*(a^2*b^3 - b^5)*cosh(x)^3 + 3*(a^3*b^2 - a*b^4)*cosh(x)^2 -
 (2*a^4*b - 7*a^2*b^3 + 5*b^5)*cosh(x))*sinh(x)^2 - 3*(b^4*cosh(x)^6 + 6*b^
4*cosh(x)*sinh(x)^5 + b^4*sinh(x)^6 - 3*b^4*cosh(x)^4 + 3*b^4*cosh(x)^2 + 3
*(5*b^4*cosh(x)^2 - b^4)*sinh(x)^4 - b^4 + 4*(5*b^4*cosh(x)^3 - 3*b^4*cosh(
x))*sinh(x)^3 + 3*(5*b^4*cosh(x)^4 - 6*b^4*cosh(x)^2 + b^4)*sinh(x)^2 + 6*(
b^4*cosh(x)^5 - 2*b^4*cosh(x)^3 + b^4*cosh(x))*sinh(x))*sqrt(-a^2 + b^2)*ar
ctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) + 3*(a^2*b^
3 - b^5)*cosh(x) + 3*(a^2*b^3 - b^5 + 5*(a^2*b^3 - b^5)*cosh(x)^4 - 4*(a^3*
```

```
b^2 - a*b^4)*cosh(x)^3 + 2*(2*a^4*b - 7*a^2*b^3 + 5*b^5)*cosh(x)^2 - 4*(a^5
 - 3*a^3*b^2 + 2*a*b^4)*cosh(x))*sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b
^6)*cosh(x)^6 + 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^5 + (
a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^6 - a^6 + 3*a^4*b^2 - 3*a^2*b^4
+ b^6 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 - 3*(a^6 - 3*a^4*b^
2 + 3*a^2*b^4 - b^6 - 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh
(x)^4 + 4*(5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 - 3*(a^6 - 3*a^4
*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4
 - b^6)*cosh(x)^2 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 5*(a^6 - 3*a^4*b
^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 - 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cos
h(x)^2)*sinh(x)^2 + 6*((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^5 - 2*(a
^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4
- b^6)*cosh(x))*sinh(x))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^4(x)}{a + b\cosh(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**4/(a+b*cosh(x)),x)
```

```
[Out] Integral(csch(x)**4/(a + b*cosh(x)), x)
```

Giac [A]
time = 0.42, size = 156, normalized size = 1.42

$$\frac{2\,b^4 \arctan\left(\frac{be^x + a}{\sqrt{-a^2 + b^2}}\right)}{(a^4 - 2\,a^2b^2 + b^4)\sqrt{-a^2 + b^2}} + \frac{2\,\left(3\,b^3e^{(5\,x)} - 3\,ab^2e^{(4\,x)} + 4\,a^2be^{(3\,x)} - 10\,b^3e^{(3\,x)} - 6\,a^3e^{(2\,x)} + 12\,ab^2e^{(2\,x)} + 3\,b^3e^x + 2\,a^3 - 5\,ab^2\right)}{3\,(a^4 - 2\,a^2b^2 + b^4)(e^{(2\,x)} - 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*b^4*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(-a
^2 + b^2)) + 2/3*(3*b^3*e^(5*x) - 3*a*b^2*e^(4*x) + 4*a^2*b*e^(3*x) - 10*b^
3*e^(3*x) - 6*a^3*e^(2*x) + 12*a*b^2*e^(2*x) + 3*b^3*e^x + 2*a^3 - 5*a*b^2)
/((a^4 - 2*a^2*b^2 + b^4)*(e^(2*x) - 1)^3)
```

Mupad [B]
time = 1.97, size = 642, normalized size = 5.84

$$\frac{\frac{4\,\left(a\,b^2-a^3\right)}{{\left(a^2-b^2\right)}^2}+\frac{8\,{\mathrm{e}}^x\,\left(a^2\,b-b^3\right)}{3\,{\left(a^2-b^2\right)}^2}}{{\mathrm{e}}^{4\,x}-2\,{\mathrm{e}}^{2\,x}+1}-\frac{\frac{2\,a\,b^2}{{\left(a^2-b^2\right)}^2}-\frac{2\,b^3\,{\mathrm{e}}^x}{{\left(a^2-b^2\right)}^2}}{{\mathrm{e}}^{2\,x}-1}-\frac{\frac{8\,a}{3\,\left(a^2-b^2\right)}-\frac{8\,b\,{\mathrm{e}}^x}{3\,\left(a^2-b^2\right)}}{3\,{\mathrm{e}}^{2\,x}-3\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{6\,x}-1}+\frac{2\,\mathrm{atan}\left(\left({\mathrm{e}}^x\,\left(\frac{2\,b^2}{{\left(a^2-b^2\right)}^2\,\sqrt{b^8}\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{2\,a\,\left(a^5\,\sqrt{b^8}-2\,a^3\,b^2\,\sqrt{b^8}+a\,b^4\,\sqrt{b^8}\right)}{b^6\,\sqrt{-{\left(a^2-b^2\right)}^5}\,\left(a^4-2\,a^2\,b^2+b^4\right)\,\sqrt{-a^{10}+5\,a^8\,b^2-10\,a^6\,b^4+10\,a^4\,b^6-5\,a^2\,b^8+b^{10}}}\right)+\frac{2\,a\,\left(b^5\,\sqrt{b^8}-2\,a^2\,b^3\,\sqrt{b^8}+a^4\,b\,\sqrt{b^8}\right)}{b^6\,\sqrt{-{\left(a^2-b^2\right)}^5}\,\left(a^4-2\,a^2\,b^2+b^4\right)\,\sqrt{-a^{10}+5\,a^8\,b^2-10\,a^6\,b^4+10\,a^4\,b^6-5\,a^2\,b^8+b^{10}}}\right)\,\left(\frac{b^5\,\sqrt{-a^{10}+5\,a^8\,b^2-10\,a^6\,b^4+10\,a^4\,b^6-5\,a^2\,b^8+b^{10}}}{2}-a^2\,b^3\,\sqrt{-a^{10}+5\,a^8\,b^2-10\,a^6\,b^4+10\,a^4\,b^6-5\,a^2\,b^8+b^{10}}+\frac{a^4\,b\,\sqrt{-a^{10}+5\,a^8\,b^2-10\,a^6\,b^4+10\,a^4\,b^6-5\,a^2\,b^8+b^{10}}}{2}\right)\right)\,\sqrt{b^8}}{\sqrt{-a^{10}+5\,a^8\,b^2-10\,a^6\,b^4+10\,a^4\,b^6-5\,a^2\,b^8+b^{10}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^4*(a + b*cosh(x))),x)
```

```
[Out] ((4*(a*b^2 - a^3))/(a^2 - b^2)^2 + (8*exp(x)*(a^2*b - b^3))/(3*(a^2 - b^2)^
2))/(exp(4*x) - 2*exp(2*x) + 1) - ((2*a*b^2)/(a^2 - b^2)^2 - (2*b^3*exp(x))
/(a^2 - b^2)^2)/(exp(2*x) - 1) - ((8*a)/(3*(a^2 - b^2)) - (8*b*exp(x))/(3*(
a^2 - b^2)))/(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1) + (2*atan((exp(x)*((2
*b^2)/((a^2 - b^2)^2*(b^8)^(1/2)*(a^4 + b^4 - 2*a^2*b^2)) + (2*a*(a^5*(b^8)
^(1/2) - 2*a^3*b^2*(b^8)^(1/2) + a*b^4*(b^8)^(1/2)))/(b^6*(-(a^2 - b^2)^5)^
(1/2)*(a^4 + b^4 - 2*a^2*b^2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^
6*b^4 + 5*a^8*b^2)^(1/2))) + (2*a*(b^5*(b^8)^(1/2) - 2*a^2*b^3*(b^8)^(1/2)
+ a^4*b*(b^8)^(1/2)))/(b^6*(-(a^2 - b^2)^5)^(1/2)*(a^4 + b^4 - 2*a^2*b^2)*(
b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)))*((b^
5*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2))/2
- a^2*b^3*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(
1/2) + (a^4*b*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^
2)^(1/2))/2))*(b^8)^(1/2))/(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b
^4 + 5*a^8*b^2)^(1/2)
```

3.176 $\int \frac{\operatorname{csch}^5(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=151

$$\frac{(4b^3+a(3a^2-7b^2)\cosh(x))\operatorname{csch}^2(x)}{8\left(a^2-b^2\right)^2}+\frac{(b-a\cosh(x))\operatorname{csch}^4(x)}{4\left(a^2-b^2\right)}+\frac{(3a^2+9ab+8b^2)\log(1-\cosh(x))}{16(a+b)^3}-\frac{(3a^2-}{}$$

```
[Out] 1/8*(4*b^3+a*(3*a^2-7*b^2)*cosh(x))*csch(x)^2/(a^2-b^2)^2+1/4*(b-a*cosh(x))
*csch(x)^4/(a^2-b^2)+1/16*(3*a^2+9*a*b+8*b^2)*ln(1-cosh(x))/(a+b)^3-1/16*(3
*a^2-9*a*b+8*b^2)*ln(1+cosh(x))/(a-b)^3+b^5*ln(a+b*cosh(x))/(a^2-b^2)^3
```

Rubi [A]
time = 0.19, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2747, 755, 837, 815}

$$\frac{(3a^2+9ab+8b^2)\log(1-\cosh(x))}{16(a+b)^3}-\frac{(3a^2-9ab+8b^2)\log(\cosh(x)+1)}{16(a-b)^3}+\frac{\operatorname{csch}^4(x)(b-a\cosh(x))}{4\left(a^2-b^2\right)}+\frac{b^5\log(a+b\cosh(x))}{\left(a^2-b^2\right)^3}+\frac{\operatorname{csch}^2(x)\left(a(3a^2-7b^2)\cosh(x)+4b^3\right)}{8\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^5/(a + b*Cosh[x]),x]
```

```
[Out] ((4*b^3 + a*(3*a^2 - 7*b^2)*Cosh[x])*Csch[x]^2)/(8*(a^2 - b^2)^2) + ((b - a
*Cosh[x])*Csch[x]^4)/(4*(a^2 - b^2)) + ((3*a^2 + 9*a*b + 8*b^2)*Log[1 - Cos
h[x]])/(16*(a + b)^3) - ((3*a^2 - 9*a*b + 8*b^2)*Log[1 + Cosh[x]])/(16*(a -
 b)^3) + (b^5*Log[a + b*Cosh[x]])/(a^2 - b^2)^3
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
```

```
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\operatorname{csch}^5(x)}{a+b\cosh(x)}\,dx = -\left(b^5\operatorname{Subst}\left(\int \frac{1}{(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\cosh(x)\right)\right)$$

$$= \frac{(b-a\cosh(x))\operatorname{csch}^4(x)}{4\left(a^2-b^2\right)} - \frac{b^3\operatorname{Subst}\left(\int \frac{3a^2-4b^2+3ax}{(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\cosh(x)\right)}{4\left(a^2-b^2\right)}$$

$$= \frac{\left(4b^3+a(3a^2-7b^2)\cosh(x)\right)\operatorname{csch}^2(x)}{8\left(a^2-b^2\right)^2} + \frac{(b-a\cosh(x))\operatorname{csch}^4(x)}{4\left(a^2-b^2\right)} + \frac{b\operatorname{Subst}\left(\int \frac{-3a^4+7a\cdots}{\cdots}\right.}{\cdots}$$

$$= \frac{\left(4b^3+a(3a^2-7b^2)\cosh(x)\right)\operatorname{csch}^2(x)}{8\left(a^2-b^2\right)^2} + \frac{(b-a\cosh(x))\operatorname{csch}^4(x)}{4\left(a^2-b^2\right)} + \frac{b\operatorname{Subst}\left(\int \left(-\frac{(a-\cdots}{\cdots}\right.\right.}{\cdots}$$

$$= \frac{\left(4b^3+a(3a^2-7b^2)\cosh(x)\right)\operatorname{csch}^2(x)}{8\left(a^2-b^2\right)^2} + \frac{(b-a\cosh(x))\operatorname{csch}^4(x)}{4\left(a^2-b^2\right)} + \frac{(3a^2+9ab+8b^2\cdots}{16(a\cdots}$$

Mathematica [A]
time = 0.62, size = 148, normalized size = 0.98

$$\frac{1}{64}\left(\frac{2(3a+5b)\operatorname{csch}^2\left(\frac{x}{2}\right)}{(a+b)^2} - \frac{\operatorname{csch}^4\left(\frac{x}{2}\right)}{a+b} + \frac{\frac{8\left(8b^5\log(a+b\cosh(x))-8b^5\log(\sinh(x))+a\left(3a^4-10a^2b^2+15b^4\right)\log\left(\tanh\left(\frac{x}{2}\right)\right)\right)}{(a+b)^3} + 2(3a^2-8ab+5b^2)\operatorname{sech}^2\left(\frac{x}{2}\right) + (a-b)^2\operatorname{sech}^4\left(\frac{x}{2}\right)}{(a-b)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^5/(a + b*Cosh[x]),x]
```

```
[Out] ((2*(3*a + 5*b)*Csch[x/2]^2)/(a + b)^2 - Csch[x/2]^4/(a + b) + ((8*(8*b^5*L
og[a + b*Cosh[x]] - 8*b^5*Log[Sinh[x]] + a*(3*a^4 - 10*a^2*b^2 + 15*b^4)*Lo
g[Tanh[x/2]]))/(a + b)^3 + 2*(3*a^2 - 8*a*b + 5*b^2)*Sech[x/2]^2 + (a - b)^
2*Sech[x/2]^4)/(a - b)^3)/64
```

Maple [A]
time = 0.88, size = 138, normalized size = 0.91

method	result
default	$\frac{\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-b\left(\tanh^2\left(\frac{x}{2}\right)\right)-4a+6b\right)^2}{64(a-b)^3}+\frac{b^5\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-b\left(\tanh^2\left(\frac{x}{2}\right)\right)-a-b\right)}{(a-b)^3(a+b)^3}-\frac{1}{64(a+b)\tanh\left(\frac{x}{2}\right)^4}-\frac{-4a-6b}{32(a+b)^2\tanh\left(\frac{x}{2}\right)^2}$
risch	$-\frac{3x\,a^2}{8(a^3+3a^2b+3a\,b^2+b^3)}-\frac{9xab}{8(a^3+3a^2b+3a\,b^2+b^3)}-\frac{x\,b^2}{a^3+3a^2b+3a\,b^2+b^3}+\frac{3x\,a^2}{8(a^3-3a^2b+3a\,b^2-b^3)}-\frac{9xab}{8(a^3-3a^2b+3a\,b^2-b^3)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^5/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/64*(a*tanh(1/2*x)^2-b*tanh(1/2*x)^2-4*a+6*b)^2/(a-b)^3+1/(a-b)^3*b^5/(a+b
)^3*ln(a*tanh(1/2*x)^2-b*tanh(1/2*x)^2-a-b)-1/64/(a+b)/tanh(1/2*x)^4-1/32*(
-4*a-6*b)/(a+b)^2/tanh(1/2*x)^2+1/16/(a+b)^3*(6*a^2+18*a*b+16*b^2)*ln(tanh(
1/2*x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(144) = 288.
time = 0.30, size = 348, normalized size = 2.30

$$\frac{b^5\log\left(2\,ae^{(-x)}+be^{(-2\,x)}+b\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}-\frac{(3\,a^2-9\,ab+8\,b^2)\log\left(e^{(-x)}+1\right)}{8\,(a^3-3\,a^2b+3\,ab^2-b^3)}+\frac{(3\,a^2+9\,ab+8\,b^2)\log\left(e^{(-x)}-1\right)}{8\,(a^3+3\,a^2b+3\,ab^2+b^3)}+\frac{8\,b^3e^{(-2\,x)}+8\,b^3e^{(-6\,x)}+(3\,a^3-7\,ab^2)e^{(-x)}-(11\,a^3-15\,ab^2)e^{(-3\,x)}+16\,(a^2b-2\,b^3)e^{(-4\,x)}-(11\,a^3-15\,ab^2)e^{(-5\,x)}+(3\,a^3-7\,ab^2)e^{(-7\,x)}}{4\,(a^4-2\,a^2b^2+b^4-4\,(a^4-2\,a^2b^2+b^4)e^{(-2\,x)}+6\,(a^4-2\,a^2b^2+b^4)e^{(-4\,x)}-4\,(a^4-2\,a^2b^2+b^4)e^{(-6\,x)}+(a^4-2\,a^2b^2+b^4)e^{(-8\,x)})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^5/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] b^5*log(2*a*e^(-x) + b*e^(-2*x) + b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) -
1/8*(3*a^2 - 9*a*b + 8*b^2)*log(e^(-x) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3)
 + 1/8*(3*a^2 + 9*a*b + 8*b^2)*log(e^(-x) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b
^3) + 1/4*(8*b^3*e^(-2*x) + 8*b^3*e^(-6*x) + (3*a^3 - 7*a*b^2)*e^(-x) - (11
*a^3 - 15*a*b^2)*e^(-3*x) + 16*(a^2*b - 2*b^3)*e^(-4*x) - (11*a^3 - 15*a*b^
2)*e^(-5*x) + (3*a^3 - 7*a*b^2)*e^(-7*x))/(a^4 - 2*a^2*b^2 + b^4 - 4*(a^4 -
 2*a^2*b^2 + b^4)*e^(-2*x) + 6*(a^4 - 2*a^2*b^2 + b^4)*e^(-4*x) - 4*(a^4 -
2*a^2*b^2 + b^4)*e^(-6*x) + (a^4 - 2*a^2*b^2 + b^4)*e^(-8*x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3450 vs. 2(144) = 288.
time = 0.51, size = 3450, normalized size = 22.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^5/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/8*(2*(3*a^5 - 10*a^3*b^2 + 7*a*b^4)*cosh(x)^7 + 2*(3*a^5 - 10*a^3*b^2 + 7
*a*b^4)*sinh(x)^7 + 16*(a^2*b^3 - b^5)*cosh(x)^6 + 2*(8*a^2*b^3 - 8*b^5 + 7
```

*(3*a^5 - 10*a^3*b^2 + 7*a*b^4)*cosh(x))*sinh(x)^6 - 2*(11*a^5 - 26*a^3*b^2
 + 15*a*b^4)*cosh(x)^5 - 2*(11*a^5 - 26*a^3*b^2 + 15*a*b^4 - 21*(3*a^5 - 10
*a^3*b^2 + 7*a*b^4)*cosh(x)^2 - 48*(a^2*b^3 - b^5)*cosh(x))*sinh(x)^5 + 32*
(a^4*b - 3*a^2*b^3 + 2*b^5)*cosh(x)^4 + 2*(16*a^4*b - 48*a^2*b^3 + 32*b^5 +
 35*(3*a^5 - 10*a^3*b^2 + 7*a*b^4)*cosh(x)^3 + 120*(a^2*b^3 - b^5)*cosh(x)^
2 - 5*(11*a^5 - 26*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^4 - 2*(11*a^5 - 26*
a^3*b^2 + 15*a*b^4)*cosh(x)^3 - 2*(11*a^5 - 26*a^3*b^2 + 15*a*b^4 - 35*(3*a
^5 - 10*a^3*b^2 + 7*a*b^4)*cosh(x)^4 - 160*(a^2*b^3 - b^5)*cosh(x)^3 + 10*(
11*a^5 - 26*a^3*b^2 + 15*a*b^4)*cosh(x)^2 - 64*(a^4*b - 3*a^2*b^3 + 2*b^5)*
cosh(x))*sinh(x)^3 + 16*(a^2*b^3 - b^5)*cosh(x)^2 + 2*(21*(3*a^5 - 10*a^3*b
^2 + 7*a*b^4)*cosh(x)^5 + 8*a^2*b^3 - 8*b^5 + 120*(a^2*b^3 - b^5)*cosh(x)^4
 - 10*(11*a^5 - 26*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + 96*(a^4*b - 3*a^2*b^3 +
2*b^5)*cosh(x)^2 - 3*(11*a^5 - 26*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^2 +
2*(3*a^5 - 10*a^3*b^2 + 7*a*b^4)*cosh(x) + 8*(b^5*cosh(x)^8 + 8*b^5*cosh(x)
*sinh(x)^7 + b^5*sinh(x)^8 - 4*b^5*cosh(x)^6 + 6*b^5*cosh(x)^4 - 4*b^5*cosh
(x)^2 + 4*(7*b^5*cosh(x)^2 - b^5)*sinh(x)^6 + 8*(7*b^5*cosh(x)^3 - 3*b^5*co
sh(x))*sinh(x)^5 + b^5 + 2*(35*b^5*cosh(x)^4 - 30*b^5*cosh(x)^2 + 3*b^5)*si
nh(x)^4 + 8*(7*b^5*cosh(x)^5 - 10*b^5*cosh(x)^3 + 3*b^5*cosh(x))*sinh(x)^3
+ 4*(7*b^5*cosh(x)^6 - 15*b^5*cosh(x)^4 + 9*b^5*cosh(x)^2 - b^5)*sinh(x)^2
+ 8*(b^5*cosh(x)^7 - 3*b^5*cosh(x)^5 + 3*b^5*cosh(x)^3 - b^5*cosh(x))*sinh(
x))*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))) - ((3*a^5 - 10*a^3*b^2 + 15*
a*b^4 + 8*b^5)*cosh(x)^8 + 8*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*cosh(x
)*sinh(x)^7 + (3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*sinh(x)^8 - 4*(3*a^5
- 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*cosh(x)^6 - 4*(3*a^5 - 10*a^3*b^2 + 15*a*b
^4 + 8*b^5 - 7*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*cosh(x)^2)*sinh(x)^6
 + 8*(7*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*cosh(x)^3 - 3*(3*a^5 - 10*a
^3*b^2 + 15*a*b^4 + 8*b^5)*cosh(x))*sinh(x)^5 + 3*a^5 - 10*a^3*b^2 + 15*a*b
^4 + 8*b^5 + 6*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*cosh(x)^4 + 2*(9*a^5
 - 30*a^3*b^2 + 45*a*b^4 + 24*b^5 + 35*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b
^5)*cosh(x)^4 - 30*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*cosh(x)^2)*sinh(
x)^4 + 8*(7*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*cosh(x)^5 - 10*(3*a^5 -
 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*cosh(x)^3 + 3*(3*a^5 - 10*a^3*b^2 + 15*a*b^
4 + 8*b^5)*cosh(x))*sinh(x)^3 - 4*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*c
osh(x)^2 + 4*(7*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*cosh(x)^6 - 3*a^5 +
 10*a^3*b^2 - 15*a*b^4 - 8*b^5 - 15*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)
cosh(x)^4 + 9(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*cosh(x)^2)*sinh(x)^2
 + 8*((3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*cosh(x)^7 - 3*(3*a^5 - 10*a^3
*b^2 + 15*a*b^4 + 8*b^5)*cosh(x)^5 + 3*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b
^5)*cosh(x)^3 - (3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*cosh(x))*sinh(x))*l
og(cosh(x) + sinh(x) + 1) + ((3*a^5 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cosh(x
)^8 + 8*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cosh(x)*sinh(x)^7 + (3*a^5
- 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*sinh(x)^8 - 4*(3*a^5 - 10*a^3*b^2 + 15*a*b
^4 - 8*b^5)*cosh(x)^6 - 4*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5 - 7*(3*a^5
 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cosh(x)^2)*sinh(x)^6 + 8*(7*(3*a^5 - 10*a
^3*b^2 + 15*a*b^4 - 8*b^5)*cosh(x)^3 - 3*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 - 8

```
*b^5)*cosh(x))*sinh(x)^5 + 3*a^5 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5 + 6*(3*a^5
 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cosh(x)^4 + 2*(9*a^5 - 30*a^3*b^2 + 45*a*
b^4 - 24*b^5 + 35*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cosh(x)^4 - 30*(3
*a^5 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cosh(x)^2)*sinh(x)^4 + 8*(7*(3*a^5 -
10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cosh(x)^5 - 10*(3*a^5 - 10*a^3*b^2 + 15*a*b^
4 - 8*b^5)*cosh(x)^3 + 3*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cosh(x))*s
inh(x)^3 - 4*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cosh(x)^2 + 4*(7*(3*a^
5 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cosh(x)^6 - 3*a^5 + 10*a^3*b^2 - 15*a*b^
4 + 8*b^5 - 15*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cosh(x)^4 + 9*(3*a^5
 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cosh(x)^2)*sinh(x)^2 + 8*((3*a^5 - 10*a^3
*b^2 + 15*a*b^4 - 8*b^5)*cosh(x)^7 - 3*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 - 8*b
^5)*cosh(x)^5 + 3*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cosh(x)^3 - (3*a^
5 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x)
- 1) + 2*(7*(3*a^5 - 10*a^3*b^2 + 7*a*b^4)*cosh(x)^6 + 48*(a^2*b^3 - b^5)*c
osh(x)^5 + 3*a^5 - 10*a^3*b^2 + 7*a*b^4 - 5*(11*a^5 - 26*a^3*b^2 + 15*a*b^4
)*cosh(x)^4 + 64*(a^4*b - 3*a^2*b^3 + 2*b^5)*cosh(x)^3 - 3*(11*a^5 - 26*a^3
*b^2 + 15*a*b^4)*cosh(x)^2 + 16*(a^2*b^3 - b^5)*cosh(x))*sinh(x))/((a^6 - 3
*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^8 + 8*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^
6)*cosh(x)*sinh(x)^7 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^8 - 4*(a
^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^6 - 4...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^5(x)}{a+b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**5/(a+b*cosh(x)),x)
```

```
[Out] Integral(csch(x)**5/(a + b*cosh(x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 338 vs. 2(144) = 288.
time = 0.42, size = 338, normalized size = 2.24

$$\frac{b^6\log\left(\left|b\left(e^{(-x)}+e^x\right)+2a\right|\right)}{a^6b-3a^4b^3+3a^2b^5-b^7}-\frac{(3a^2-9ab+8b^2)\log\left(e^{(-x)}+e^x+2\right)}{16(a^3-3a^2b+3ab^2-b^3)}+\frac{(3a^2+9ab+8b^2)\log\left(e^{(-x)}+e^x-2\right)}{16(a^3+3a^2b+3ab^2+b^3)}+\frac{3b^5\left(e^{(-x)}+e^x\right)^4+3a^5\left(e^{(-x)}+e^x\right)^3-10a^3b^2\left(e^{(-x)}+e^x\right)^3+7ab^4\left(e^{(-x)}+e^x\right)^3+8a^2b^3\left(e^{(-x)}+e^x\right)^2-32b^5\left(e^{(-x)}+e^x\right)^2-20a^5\left(e^{(-x)}+e^x\right)+56a^3b^2\left(e^{(-x)}+e^x\right)-36ab^4\left(e^{(-x)}+e^x\right)+16a^4b-64a^2b^3+96b^5}{4(a^6-3a^4b^2+3a^2b^4-b^6)\left(\left(e^{(-x)}+e^x\right)^2-4\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^5/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] b^6*log(abs(b*(e^(-x) + e^x) + 2*a))/(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)
- 1/16*(3*a^2 - 9*a*b + 8*b^2)*log(e^(-x) + e^x + 2)/(a^3 - 3*a^2*b + 3*a*b
^2 - b^3) + 1/16*(3*a^2 + 9*a*b + 8*b^2)*log(e^(-x) + e^x - 2)/(a^3 + 3*a^2
*b + 3*a*b^2 + b^3) + 1/4*(3*b^5*(e^(-x) + e^x)^4 + 3*a^5*(e^(-x) + e^x)^3
- 10*a^3*b^2*(e^(-x) + e^x)^3 + 7*a*b^4*(e^(-x) + e^x)^3 + 8*a^2*b^3*(e^(-x
```

```
) + e^x)^2 - 32*b^5*(e^(-x) + e^x)^2 - 20*a^5*(e^(-x) + e^x) + 56*a^3*b^2*(
e^(-x) + e^x) - 36*a*b^4*(e^(-x) + e^x) + 16*a^4*b - 64*a^2*b^3 + 96*b^5)/(
(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*((e^(-x) + e^x)^2 - 4)^2)
```

Mupad [B]
time = 1.83, size = 559, normalized size = 3.70

$$\frac{\frac{4\,b}{a^2-b^2}-\frac{4\,a\,e^x}{a^2-b^2}}{6\,e^{4\,x}-4\,e^{2\,x}-4\,e^{6\,x}+e^{8\,x}+1}-\frac{\frac{2\,\left(b^5-a^2\,b^3\right)}{{\left(a^2-b^2\right)}^3}-\frac{e^x\,\left(3\,a^5-10\,a^3\,b^2+7\,a\,b^4\right)}{4\,{\left(a^2-b^2\right)}^3}}{e^{2\,x}-1}+\frac{\frac{8\,\left(a^2\,b-b^3\right)}{{\left(a^2-b^2\right)}^2}+\frac{6\,e^x\,\left(a\,b^2-a^3\right)}{{\left(a^2-b^2\right)}^2}}{3\,e^{2\,x}-3\,e^{4\,x}+e^{6\,x}-1}+\frac{\frac{2\,\left(2\,a^2\,b-b^3\right)}{{\left(a^2-b^2\right)}^2}-\frac{e^x\,\left(a^3+3\,a\,b^2\right)}{2\,{\left(a^2-b^2\right)}^2}}{e^{4\,x}-2\,e^{2\,x}+1}+\frac{b^5\,\ln\left(256\,b^{11}\,e^{2\,x}-9\,a^{10}\,b+256\,b^{11}-225\,a^2\,b^9+300\,a^4\,b^7-190\,a^6\,b^5+60\,a^8\,b^3-18\,a^{11}\,e^x-225\,a^2\,b^9\,e^{2\,x}+300\,a^4\,b^7\,e^{2\,x}-190\,a^6\,b^5\,e^{2\,x}+60\,a^8\,b^3\,e^{2\,x}+512\,a\,b^{10}\,e^x-9\,a^{10}\,b\,e^{2\,x}-450\,a^3\,b^8\,e^x+600\,a^5\,b^6\,e^x-380\,a^7\,b^4\,e^x+120\,a^9\,b^2\,e^x\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}+\frac{\ln\left(e^x-1\right)\,\left(3\,a^2+9\,a\,b+8\,b^2\right)}{8\,a^3+24\,a^2\,b+24\,a\,b^2+8\,b^3}-\frac{\ln\left(e^x+1\right)\,\left(3\,a^2-9\,a\,b+8\,b^2\right)}{8\,a^3-24\,a^2\,b+24\,a\,b^2-8\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^5*(a + b*cosh(x))),x)
```

```
[Out] ((4*b)/(a^2 - b^2) - (4*a*exp(x))/(a^2 - b^2))/(6*exp(4*x) - 4*exp(2*x) - 4
*exp(6*x) + exp(8*x) + 1) - ((2*(b^5 - a^2*b^3))/(a^2 - b^2)^3 - (exp(x)*(7
*a*b^4 + 3*a^5 - 10*a^3*b^2))/(4*(a^2 - b^2)^3))/(exp(2*x) - 1) + ((8*(a^2*
b - b^3))/(a^2 - b^2)^2 + (6*exp(x)*(a*b^2 - a^3))/(a^2 - b^2)^2)/(3*exp(2*
x) - 3*exp(4*x) + exp(6*x) - 1) + ((2*(2*a^2*b - b^3))/(a^2 - b^2)^2 - (exp
(x)*(3*a*b^2 + a^3))/(2*(a^2 - b^2)^2))/(exp(4*x) - 2*exp(2*x) + 1) + (b^5*
log(256*b^11*exp(2*x) - 9*a^10*b + 256*b^11 - 225*a^2*b^9 + 300*a^4*b^7 - 1
90*a^6*b^5 + 60*a^8*b^3 - 18*a^11*exp(x) - 225*a^2*b^9*exp(2*x) + 300*a^4*b
^7*exp(2*x) - 190*a^6*b^5*exp(2*x) + 60*a^8*b^3*exp(2*x) + 512*a*b^10*exp(x
) - 9*a^10*b*exp(2*x) - 450*a^3*b^8*exp(x) + 600*a^5*b^6*exp(x) - 380*a^7*b
^4*exp(x) + 120*a^9*b^2*exp(x)))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) + (log
(exp(x) - 1)*(9*a*b + 3*a^2 + 8*b^2))/(24*a*b^2 + 24*a^2*b + 8*a^3 + 8*b^3)
 - (log(exp(x) + 1)*(3*a^2 - 9*a*b + 8*b^2))/(24*a*b^2 - 24*a^2*b + 8*a^3 -
 8*b^3)
```

3.177 $\int \frac{\operatorname{csch}^6(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=159

$$\frac{2b^6\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}}+\frac{(15b^5-a(8a^4-26a^2b^2+33b^4)\cosh(x))\operatorname{csch}(x)}{15\left(a^2-b^2\right)^3}+\frac{(5b^3+a(4a^2-9b^2)\cosh(x}{15\left(a^2-b^2\right)^2}$$

```
[Out] 2*b^6*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(7/2)/(a+b)^(7/2)+
1/15*(15*b^5-a*(8*a^4-26*a^2*b^2+33*b^4)*cosh(x))*csch(x)/(a^2-b^2)^3+1/15*
(5*b^3+a*(4*a^2-9*b^2)*cosh(x))*csch(x)^3/(a^2-b^2)^2+1/5*(b-a*cosh(x))*csc
h(x)^5/(a^2-b^2)
```

Rubi [A]
time = 0.34, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2775, 2945, 12, 2738, 214}

$$\frac{\operatorname{csch}^5(x)(b-a\cosh(x))}{5\left(a^2-b^2\right)}+\frac{\operatorname{csch}^3(x)\left(a\left(4a^2-9b^2\right)\cosh(x)+5b^3\right)}{15\left(a^2-b^2\right)^2}+\frac{\operatorname{csch}(x)\left(15b^5-a\left(8a^4-26a^2b^2+33b^4\right)\cosh(x)\right)}{15\left(a^2-b^2\right)^3}+\frac{2b^6\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^6/(a + b*Cosh[x]),x]
```

```
[Out] (2*b^6*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)
^(7/2)) + ((15*b^5 - a*(8*a^4 - 26*a^2*b^2 + 33*b^4)*Cosh[x])*Csch[x])/(15*
(a^2 - b^2)^3) + ((5*b^3 + a*(4*a^2 - 9*b^2)*Cosh[x])*Csch[x]^3)/(15*(a^2 -
 b^2)^2) + ((b - a*Cosh[x])*Csch[x]^5)/(5*(a^2 - b^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2775

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\operatorname{csch}^6(x)}{a+b\cosh(x)}\,dx = \frac{(b-a\cosh(x))\operatorname{csch}^5(x)}{5\left(a^2-b^2\right)} + \frac{\int \frac{\left(-4a^2+5b^2-4ab\cosh(x)\right)\operatorname{csch}^4(x)}{a+b\cosh(x)}\,dx}{5\left(a^2-b^2\right)}$$

$$= \frac{\left(5b^3+a(4a^2-9b^2)\cosh(x)\right)\operatorname{csch}^3(x)}{15\left(a^2-b^2\right)^2} + \frac{(b-a\cosh(x))\operatorname{csch}^5(x)}{5\left(a^2-b^2\right)} + \frac{\int \frac{(8a^4-18a^2b^2+15b^4\cdots}{}}{1\cdots}$$

$$= \frac{\left(15b^5-a(8a^4-26a^2b^2+33b^4)\cosh(x)\right)\operatorname{csch}(x)}{15\left(a^2-b^2\right)^3} + \frac{\left(5b^3+a(4a^2-9b^2)\cosh(x)\right)\operatorname{csch}\cdots}{15\left(a^2-b^2\right)^2}$$

$$= \frac{\left(15b^5-a(8a^4-26a^2b^2+33b^4)\cosh(x)\right)\operatorname{csch}(x)}{15\left(a^2-b^2\right)^3} + \frac{\left(5b^3+a(4a^2-9b^2)\cosh(x)\right)\operatorname{csch}\cdots}{15\left(a^2-b^2\right)^2}$$

$$= \frac{\left(15b^5-a(8a^4-26a^2b^2+33b^4)\cosh(x)\right)\operatorname{csch}(x)}{15\left(a^2-b^2\right)^3} + \frac{\left(5b^3+a(4a^2-9b^2)\cosh(x)\right)\operatorname{csch}\cdots}{15\left(a^2-b^2\right)^2}$$

$$= \frac{2b^6\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}} + \frac{\left(15b^5-a(8a^4-26a^2b^2+33b^4)\cosh(x)\right)\operatorname{csch}(x)}{15\left(a^2-b^2\right)^3} + \cdots$$

Mathematica [A]

time = 1.25, size = 201, normalized size = 1.26

$$\frac{1}{480}\left(\frac{960b^6\text{ArcTan}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{7/2}}-\frac{2(64a^2+183ab+149b^2)\coth\left(\frac{x}{2}\right)}{(a+b)^3}-\frac{8(19a-29b)\text{csch}^3(x)\sinh^4\left(\frac{x}{2}\right)}{(a-b)^2}-\frac{96\text{csch}^5(x)\sinh^6\left(\frac{x}{2}\right)}{a-b}+\frac{(19a+29b)\text{csch}^4\left(\frac{x}{2}\right)\sinh(x)}{2(a+b)^2}-\frac{3\text{csch}^6\left(\frac{x}{2}\right)\sinh(x)}{2(a+b)}-\frac{2(64a^2-183ab+149b^2)\tanh\left(\frac{x}{2}\right)}{(a-b)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^6/(a + b*Cosh[x]),x]
```

```
[Out] ((960*b^6*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(7/2)
- (2*(64*a^2 + 183*a*b + 149*b^2)*Coth[x/2])/(a + b)^3 - (8*(19*a - 29*b)*C
sch[x]^3*Sinh[x/2]^4)/(a - b)^2 - (96*Csch[x]^5*Sinh[x/2]^6)/(a - b) + ((19
*a + 29*b)*Csch[x/2]^4*Sinh[x])/(2*(a + b)^2) - (3*Csch[x/2]^6*Sinh[x])/(2*
(a + b)) - (2*(64*a^2 - 183*a*b + 149*b^2)*Tanh[x/2])/(a - b)^3)/480
```

Maple [A]
time = 0.68, size = 213, normalized size = 1.34

method	result
default	$-\frac{\frac{a^2\left(\tanh^5\left(\frac{x}{2}\right)\right)}{5}-\frac{2ab\left(\tanh^5\left(\frac{x}{2}\right)\right)}{5}+\frac{b^2\left(\tanh^5\left(\frac{x}{2}\right)\right)}{5}-\frac{5a^2\left(\tanh^3\left(\frac{x}{2}\right)\right)}{3}+4ab\left(\tanh^3\left(\frac{x}{2}\right)\right)-\frac{7b^2\left(\tanh^3\left(\frac{x}{2}\right)\right)}{3}+10a^2\tanh\left(\frac{x}{2}\right)-28ab\tanh\left(\frac{x}{2}\right)}{32(a-b)^3}$
risch	$-\frac{2\left(-15b^5e^{9x}+15a\,b^4e^{8x}-20a^2b^3e^{7x}+80b^5e^{7x}+30a^3b^2e^{6x}-90a\,b^4e^{6x}-48a^4b\,e^{5x}+136a^2b^3e^{5x}-178b^5e^{5x}+80a^5e^{4x}-230a^3b^2e^{4x}+24\right.}{15(a^6-3a^4b^2+3a^2b^4-b^6)(e^{2x}-1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^6/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/32/(a-b)^3*(1/5*a^2*tanh(1/2*x)^5-2/5*a*b*tanh(1/2*x)^5+1/5*b^2*tanh(1/2
*x)^5-5/3*a^2*tanh(1/2*x)^3+4*a*b*tanh(1/2*x)^3-7/3*b^2*tanh(1/2*x)^3+10*a^
2*tanh(1/2*x)-28*a*b*tanh(1/2*x)+22*b^2*tanh(1/2*x))-1/160/(a+b)/tanh(1/2*x
)^5-1/96*(-5*a-7*b)/(a+b)^2/tanh(1/2*x)^3-1/32/(a+b)^3*(10*a^2+28*a*b+22*b^
2)/tanh(1/2*x)+2/(a-b)^3/(a+b)^3*b^6/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh
(1/2*x)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^6/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3156 vs. $2(144) = 288$.
time = 0.49, size = 6381, normalized size = 40.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^6/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [1/15*(30*(a^2*b^5 - b^7)*cosh(x)^9 + 30*(a^2*b^5 - b^7)*sinh(x)^9 - 30*(a^
3*b^4 - a*b^6)*cosh(x)^8 - 30*(a^3*b^4 - a*b^6 - 9*(a^2*b^5 - b^7)*cosh(x))
*sinh(x)^8 + 40*(a^4*b^3 - 5*a^2*b^5 + 4*b^7)*cosh(x)^7 + 40*(a^4*b^3 - 5*a
^2*b^5 + 4*b^7 + 27*(a^2*b^5 - b^7)*cosh(x)^2 - 6*(a^3*b^4 - a*b^6)*cosh(x)
)*sinh(x)^7 - 16*a^7 + 68*a^5*b^2 - 118*a^3*b^4 + 66*a*b^6 - 60*(a^5*b^2 -
4*a^3*b^4 + 3*a*b^6)*cosh(x)^6 - 20*(3*a^5*b^2 - 12*a^3*b^4 + 9*a*b^6 - 126
*(a^2*b^5 - b^7)*cosh(x)^3 + 42*(a^3*b^4 - a*b^6)*cosh(x)^2 - 14*(a^4*b^3 -
 5*a^2*b^5 + 4*b^7)*cosh(x))*sinh(x)^6 + 4*(24*a^6*b - 92*a^4*b^3 + 157*a^2
*b^5 - 89*b^7)*cosh(x)^5 + 4*(24*a^6*b - 92*a^4*b^3 + 157*a^2*b^5 - 89*b^7
+ 945*(a^2*b^5 - b^7)*cosh(x)^4 - 420*(a^3*b^4 - a*b^6)*cosh(x)^3 + 210*(a^
4*b^3 - 5*a^2*b^5 + 4*b^7)*cosh(x)^2 - 90*(a^5*b^2 - 4*a^3*b^4 + 3*a*b^6)*c
osh(x))*sinh(x)^5 - 20*(8*a^7 - 31*a^5*b^2 + 47*a^3*b^4 - 24*a*b^6)*cosh(x)
^4 - 20*(8*a^7 - 31*a^5*b^2 + 47*a^3*b^4 - 24*a*b^6 - 189*(a^2*b^5 - b^7)*c
osh(x)^5 + 105*(a^3*b^4 - a*b^6)*cosh(x)^4 - 70*(a^4*b^3 - 5*a^2*b^5 + 4*b^
7)*cosh(x)^3 + 45*(a^5*b^2 - 4*a^3*b^4 + 3*a*b^6)*cosh(x)^2 - (24*a^6*b - 9
2*a^4*b^3 + 157*a^2*b^5 - 89*b^7)*cosh(x))*sinh(x)^4 + 40*(a^4*b^3 - 5*a^2*
b^5 + 4*b^7)*cosh(x)^3 + 40*(a^4*b^3 - 5*a^2*b^5 + 4*b^7 + 63*(a^2*b^5 - b^
7)*cosh(x)^6 - 42*(a^3*b^4 - a*b^6)*cosh(x)^5 + 35*(a^4*b^3 - 5*a^2*b^5 + 4
*b^7)*cosh(x)^4 - 30*(a^5*b^2 - 4*a^3*b^4 + 3*a*b^6)*cosh(x)^3 + (24*a^6*b
- 92*a^4*b^3 + 157*a^2*b^5 - 89*b^7)*cosh(x)^2 - 2*(8*a^7 - 31*a^5*b^2 + 47
*a^3*b^4 - 24*a*b^6)*cosh(x))*sinh(x)^3 + 20*(4*a^7 - 17*a^5*b^2 + 28*a^3*b
^4 - 15*a*b^6)*cosh(x)^2 + 20*(54*(a^2*b^5 - b^7)*cosh(x)^7 + 4*a^7 - 17*a^
5*b^2 + 28*a^3*b^4 - 15*a*b^6 - 42*(a^3*b^4 - a*b^6)*cosh(x)^6 + 42*(a^4*b^
3 - 5*a^2*b^5 + 4*b^7)*cosh(x)^5 - 45*(a^5*b^2 - 4*a^3*b^4 + 3*a*b^6)*cosh(
x)^4 + 2*(24*a^6*b - 92*a^4*b^3 + 157*a^2*b^5 - 89*b^7)*cosh(x)^3 - 6*(8*a^
7 - 31*a^5*b^2 + 47*a^3*b^4 - 24*a*b^6)*cosh(x)^2 + 6*(a^4*b^3 - 5*a^2*b^5
+ 4*b^7)*cosh(x))*sinh(x)^2 - 15*(b^6*cosh(x)^10 + 10*b^6*cosh(x)*sinh(x)^9
 + b^6*sinh(x)^10 - 5*b^6*cosh(x)^8 + 10*b^6*cosh(x)^6 - 10*b^6*cosh(x)^4 +
 5*(9*b^6*cosh(x)^2 - b^6)*sinh(x)^8 + 5*b^6*cosh(x)^2 + 40*(3*b^6*cosh(x)^
3 - b^6*cosh(x))*sinh(x)^7 + 10*(21*b^6*cosh(x)^4 - 14*b^6*cosh(x)^2 + b^6)
*sinh(x)^6 - b^6 + 4*(63*b^6*cosh(x)^5 - 70*b^6*cosh(x)^3 + 15*b^6*cosh(x))
*sinh(x)^5 + 10*(21*b^6*cosh(x)^6 - 35*b^6*cosh(x)^4 + 15*b^6*cosh(x)^2 - b
^6)*sinh(x)^4 + 40*(3*b^6*cosh(x)^7 - 7*b^6*cosh(x)^5 + 5*b^6*cosh(x)^3 - b
```

```
^6*cosh(x))*sinh(x)^3 + 5*(9*b^6*cosh(x)^8 - 28*b^6*cosh(x)^6 + 30*b^6*cosh
(x)^4 - 12*b^6*cosh(x)^2 + b^6)*sinh(x)^2 + 10*(b^6*cosh(x)^9 - 4*b^6*cosh(
x)^7 + 6*b^6*cosh(x)^5 - 4*b^6*cosh(x)^3 + b^6*cosh(x))*sinh(x))*sqrt(a^2 -
 b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*
(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a)
)/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)
) + 30*(a^2*b^5 - b^7)*cosh(x) + 10*(27*(a^2*b^5 - b^7)*cosh(x)^8 - 24*(a^3
*b^4 - a*b^6)*cosh(x)^7 + 3*a^2*b^5 - 3*b^7 + 28*(a^4*b^3 - 5*a^2*b^5 + 4*b
^7)*cosh(x)^6 - 36*(a^5*b^2 - 4*a^3*b^4 + 3*a*b^6)*cosh(x)^5 + 2*(24*a^6*b
- 92*a^4*b^3 + 157*a^2*b^5 - 89*b^7)*cosh(x)^4 - 8*(8*a^7 - 31*a^5*b^2 + 47
*a^3*b^4 - 24*a*b^6)*cosh(x)^3 + 12*(a^4*b^3 - 5*a^2*b^5 + 4*b^7)*cosh(x)^2
 + 4*(4*a^7 - 17*a^5*b^2 + 28*a^3*b^4 - 15*a*b^6)*cosh(x))*sinh(x))/((a^8 -
 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^10 + 10*(a^8 - 4*a^6*b^2
+ 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)*sinh(x)^9 + (a^8 - 4*a^6*b^2 + 6*a^4
*b^4 - 4*a^2*b^6 + b^8)*sinh(x)^10 - 5*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2
*b^6 + b^8)*cosh(x)^8 - 5*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8 -
9*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^2)*sinh(x)^8 - a^
8 + 4*a^6*b^2 - 6*a^4*b^4 + 4*a^2*b^6 - b^8 + 40*(3*(a^8 - 4*a^6*b^2 + 6*a^
4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^3 - (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b
^6 + b^8)*cosh(x))*sinh(x)^7 + 10*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6
+ b^8)*cosh(x)^6 + 10*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8 + 21*(
a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^4 - 14*(a^8 - 4*a^6*
b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^2)*sinh(x)^6 + 4*(63*(a^8 - 4*a^
6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^5 - 70*(a^8 - 4*a^6*b^2 + 6*a^
4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^3 + 15*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^
2*b^6 + b^8)*cosh(x))*sinh(x)^5 - 10*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b
^6 + b^8)*cosh(x)^4 - 10*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8 - 2
1*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^6 + 35*(a^8 - 4*a
^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^4 - 15*(a^8 - 4*a^6*b^2 + 6*a
^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^2)*sinh(x)^4 + 40*(3*(a^8 - 4*a^6*b^2 + 6
*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^7 - 7*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*
a^2*b^6 + b^8)*cosh(x)^5 + 5*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8
)*cosh(x)^3 - (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x))*sinh
(x)^3 + 5*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^6(x)}{a + b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**6/(a+b*cosh(x)),x)
```

```
[Out] Integral(csch(x)**6/(a + b*cosh(x)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 303 vs. 2(144) = 288.

time = 0.42, size = 303, normalized size = 1.91

$$\frac{2\, b^{6} \arctan\left(\frac{b e^{x} + a}{\sqrt{-a^{2} + b^{2}}}\right)}{{\left(a^{6} - 3\, a^{4} b^{2} + 3\, a^{2} b^{4} - b^{6}\right)} \sqrt{-a^{2} + b^{2}}} + \frac{2\, {\left(15\, b^{5} e^{\left(9\, x\right)} - 15\, a b^{4} e^{\left(8\, x\right)} + 20\, a^{2} b^{3} e^{\left(7\, x\right)} - 80\, b^{5} e^{\left(7\, x\right)} - 30\, a^{3} b^{2} e^{\left(6\, x\right)} + 90\, a b^{4} e^{\left(6\, x\right)} + 48\, a^{4} b e^{\left(5\, x\right)} - 136\, a^{2} b^{3} e^{\left(5\, x\right)} + 178\, b^{5} e^{\left(5\, x\right)} - 80\, a^{5} e^{\left(4\, x\right)} + 230\, a^{3} b^{2} e^{\left(4\, x\right)} - 240\, a b^{4} e^{\left(4\, x\right)} + 20\, a^{2} b^{3} e^{\left(3\, x\right)} - 80\, b^{5} e^{\left(3\, x\right)} + 40\, a^{5} e^{\left(2\, x\right)} - 130\, a^{3} b^{2} e^{\left(2\, x\right)} + 150\, a b^{4} e^{\left(2\, x\right)} + 15\, b^{5} e^{x} - 8\, a^{5} + 26\, a^{3} b^{2} - 33\, a b^{4}\right)}}{15\, {\left(a^{6} - 3\, a^{4} b^{2} + 3\, a^{2} b^{4} - b^{6}\right)} {\left(e^{\left(2\, x\right)} - 1\right)}^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^6/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*b^6*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 -
b^6)*sqrt(-a^2 + b^2)) + 2/15*(15*b^5*e^(9*x) - 15*a*b^4*e^(8*x) + 20*a^2*b
^3*e^(7*x) - 80*b^5*e^(7*x) - 30*a^3*b^2*e^(6*x) + 90*a*b^4*e^(6*x) + 48*a^
4*b*e^(5*x) - 136*a^2*b^3*e^(5*x) + 178*b^5*e^(5*x) - 80*a^5*e^(4*x) + 230*
a^3*b^2*e^(4*x) - 240*a*b^4*e^(4*x) + 20*a^2*b^3*e^(3*x) - 80*b^5*e^(3*x) +
 40*a^5*e^(2*x) - 130*a^3*b^2*e^(2*x) + 150*a*b^4*e^(2*x) + 15*b^5*e^x - 8*
a^5 + 26*a^3*b^2 - 33*a*b^4)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(e^(2*x)
- 1)^5)
```

Mupad [**B**]

time = 2.61, size = 1031, normalized size = 6.48

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^6*(a + b*cosh(x))),x)
```

```
[Out] ((16*(a*b^2 - a^3))/(a^2 - b^2)^2 + (64*exp(x)*(a^2*b - b^3))/(5*(a^2 - b^2
)^2))/(6*exp(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1) - ((2*a*b^4)/(a
^2 - b^2)^3 - (2*b^5*exp(x))/(a^2 - b^2)^3)/(exp(2*x) - 1) - ((32*a)/(5*(a^
2 - b^2)) - (32*b*exp(x))/(5*(a^2 - b^2)))/(5*exp(2*x) - 10*exp(4*x) + 10*e
xp(6*x) - 5*exp(8*x) + exp(10*x) - 1) + ((8*(3*a*b^2 - 4*a^3))/(3*(a^2 - b^
2)^2) + (8*exp(x)*(12*a^2*b - 7*b^3))/(15*(a^2 - b^2)^2))/(3*exp(2*x) - 3*e
xp(4*x) + exp(6*x) - 1) + ((4*(a*b^4 - a^3*b^2))/(a^2 - b^2)^3 - (8*exp(x)*
(b^5 - a^2*b^3))/(3*(a^2 - b^2)^3))/(exp(4*x) - 2*exp(2*x) + 1) - (2*atan((
exp(x)*((2*b^4)/((a^2 - b^2)^3*(b^12)^(1/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*
b^2)) + (2*a*(a^7*(b^12)^(1/2) + 3*a^3*b^4*(b^12)^(1/2) - 3*a^5*b^2*(b^12)^
(1/2) - a*b^6*(b^12)^(1/2)))/(b^8*(-(a^2 - b^2)^7)^(1/2)*(a^6 - b^6 + 3*a^2
*b^4 - 3*a^4*b^2)*(b^14 - a^14 - 7*a^2*b^12 + 21*a^4*b^10 - 35*a^6*b^8 + 35
*a^8*b^6 - 21*a^10*b^4 + 7*a^12*b^2)^(1/2))) - (2*a*(b^7*(b^12)^(1/2) - 3*a
^2*b^5*(b^12)^(1/2) + 3*a^4*b^3*(b^12)^(1/2) - a^6*b*(b^12)^(1/2)))/(b^8*(-
(a^2 - b^2)^7)^(1/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)*(b^14 - a^14 - 7*a
^2*b^12 + 21*a^4*b^10 - 35*a^6*b^8 + 35*a^8*b^6 - 21*a^10*b^4 + 7*a^12*b^2)
^(1/2)))*((b^7*(b^14 - a^14 - 7*a^2*b^12 + 21*a^4*b^10 - 35*a^6*b^8 + 35*a^
8*b^6 - 21*a^10*b^4 + 7*a^12*b^2)^(1/2))/2 - (a^6*b*(b^14 - a^14 - 7*a^2*b^
12 + 21*a^4*b^10 - 35*a^6*b^8 + 35*a^8*b^6 - 21*a^10*b^4 + 7*a^12*b^2)^(1/2
```

```
))/2 - (3*a^2*b^5*(b^14 - a^14 - 7*a^2*b^12 + 21*a^4*b^10 - 35*a^6*b^8 + 35
*a^8*b^6 - 21*a^10*b^4 + 7*a^12*b^2)^(1/2))/2 + (3*a^4*b^3*(b^14 - a^14 - 7
*a^2*b^12 + 21*a^4*b^10 - 35*a^6*b^8 + 35*a^8*b^6 - 21*a^10*b^4 + 7*a^12*b^
2)^(1/2))/2))*(b^12)^(1/2))/(b^14 - a^14 - 7*a^2*b^12 + 21*a^4*b^10 - 35*a^
6*b^8 + 35*a^8*b^6 - 21*a^10*b^4 + 7*a^12*b^2)^(1/2)
```

3.178 $\int \frac{\sinh^2(x)}{(a+b\cosh(x))^2}\,dx$

Optimal. Leaf size=67

$$\frac{x}{b^2}-\frac{2a\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^2\sqrt{a+b}}-\frac{\sinh(x)}{b(a+b\cosh(x))}$$

```
[Out] x/b^2-sinh(x)/b/(a+b*cosh(x))-2*a*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/
2))/b^2/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2772, 2814, 2738, 214}

$$-\frac{2a\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^2\sqrt{a-b}\,\sqrt{a+b}}-\frac{\sinh(x)}{b(a+b\cosh(x))}+\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(a + b*Cosh[x])^2,x]
```

```
[Out] x/b^2 - (2*a*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^2
*Sqrt[a + b]) - Sinh[x]/(b*(a + b*Cosh[x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
```

```
tegersQ[2*m, 2*p]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^2(x)}{(a+b\cosh(x))^2}\,dx &= -\frac{\sinh(x)}{b(a+b\cosh(x))}+\frac{\int \frac{\cosh(x)}{a+b\cosh(x)}\,dx}{b} \\
&= \frac{x}{b^2}-\frac{\sinh(x)}{b(a+b\cosh(x))}-\frac{a\int \frac{1}{a+b\cosh(x)}\,dx}{b^2} \\
&= \frac{x}{b^2}-\frac{\sinh(x)}{b(a+b\cosh(x))}-\frac{(2a)\text{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b^2} \\
&= \frac{x}{b^2}-\frac{2a\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^2\sqrt{a+b}}-\frac{\sinh(x)}{b(a+b\cosh(x))}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 61, normalized size = 0.91

$$
\frac{x+\frac{2a\text{ArcTan}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}-\frac{b\sinh(x)}{a+b\cosh(x)}}{b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(a + b*Cosh[x])^2,x]
```

```
[Out] (x + (2*a*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] -
(b*Sinh[x])/(a + b*Cosh[x]))/b^2
```

Maple [A]
time = 0.46, size = 99, normalized size = 1.48

method	result	size

default	$\frac{\frac{2b\tanh\left(\frac{x}{2}\right)}{a\left(\tanh^2\left(\frac{x}{2}\right)\right)-b\left(\tanh^2\left(\frac{x}{2}\right)\right)-a-b}-\frac{2a\,\mathrm{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}}{b^2}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b^2}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b^2}$	99
risch	$\frac{x}{b^2}+\frac{2a\,e^x+2b}{b^2(b\,e^{2x}+2a\,e^x+b)}+\frac{a\ln\left(e^x+\frac{a\sqrt{a^2-b^2}+a^2-b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,b^2}-\frac{a\ln\left(e^x+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,b^2}$	148

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a+b*cosh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^2*(b*tanh(1/2*x)/(a*tanh(1/2*x)^2-b*tanh(1/2*x)^2-a-b)-a/((a+b)*(a-b))^
(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2)))+1/b^2*ln(tanh(1/2*x)+
1)-1/b^2*ln(tanh(1/2*x)-1)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*cosh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(57) = 114$.
time = 0.45, size = 700, normalized size = 10.45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*cosh(x))^2,x, algorithm="fricas")
```

```
[Out] [((a^2*b - b^3)*x*cosh(x)^2 + (a^2*b - b^3)*x*sinh(x)^2 + 2*a^2*b - 2*b^3 +
 (a*b*cosh(x)^2 + a*b*sinh(x)^2 + 2*a^2*cosh(x) + a*b + 2*(a*b*cosh(x) + a^
2)*sinh(x))*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh
(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*co
sh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cos
h(x) + a)*sinh(x) + b)) + (a^2*b - b^3)*x + 2*(a^3 - a*b^2 + (a^3 - a*b^2)*
x)*cosh(x) + 2*(a^3 - a*b^2 + (a^2*b - b^3)*x*cosh(x) + (a^3 - a*b^2)*x)*si
```

```
nh(x))/(a^2*b^3 - b^5 + (a^2*b^3 - b^5)*cosh(x)^2 + (a^2*b^3 - b^5)*sinh(x)
^2 + 2*(a^3*b^2 - a*b^4)*cosh(x) + 2*(a^3*b^2 - a*b^4 + (a^2*b^3 - b^5)*cos
h(x))*sinh(x)), ((a^2*b - b^3)*x*cosh(x)^2 + (a^2*b - b^3)*x*sinh(x)^2 + 2*
a^2*b - 2*b^3 + 2*(a*b*cosh(x)^2 + a*b*sinh(x)^2 + 2*a^2*cosh(x) + a*b + 2*
(a*b*cosh(x) + a^2)*sinh(x))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*c
osh(x) + b*sinh(x) + a)/(a^2 - b^2)) + (a^2*b - b^3)*x + 2*(a^3 - a*b^2 + (
a^3 - a*b^2)*x)*cosh(x) + 2*(a^3 - a*b^2 + (a^2*b - b^3)*x*cosh(x) + (a^3 -
 a*b^2)*x)*sinh(x))/(a^2*b^3 - b^5 + (a^2*b^3 - b^5)*cosh(x)^2 + (a^2*b^3 -
 b^5)*sinh(x)^2 + 2*(a^3*b^2 - a*b^4)*cosh(x) + 2*(a^3*b^2 - a*b^4 + (a^2*b
^3 - b^5)*cosh(x))*sinh(x))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(a+b*cosh(x))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.41, size = 68, normalized size = 1.01

$$-\frac{2\,a\arctan\left(\frac{be^{x}+a}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}\,b^{2}}+\frac{x}{b^{2}}+\frac{2\,(ae^{x}+b)}{(be^{(2\,x)}+2\,ae^{x}+b)b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*cosh(x))^2,x, algorithm="giac")
```

```
[Out] -2*a*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b^2) + x/b^2 +
2*(a*e^x + b)/((b*e^(2*x) + 2*a*e^x + b)*b^2)
```

Mupad [B]
time = 1.12, size = 139, normalized size = 2.07

$$\frac{x}{b^2}+\frac{\frac{2}{b}+\frac{2\,a\,\mathrm{e}^x}{b^2}}{b+2\,a\,\mathrm{e}^x+b\,\mathrm{e}^{2\,x}}+\frac{a\,\ln\left(\frac{2\,a\,\mathrm{e}^x}{b^3}-\frac{2\,a\,\left(b+a\,\mathrm{e}^x\right)}{b^3\,\sqrt{a+b}\,\sqrt{a-b}}\right)}{b^2\,\sqrt{a+b}\,\sqrt{a-b}}-\frac{a\,\ln\left(\frac{2\,a\,\mathrm{e}^x}{b^3}+\frac{2\,a\,\left(b+a\,\mathrm{e}^x\right)}{b^3\,\sqrt{a+b}\,\sqrt{a-b}}\right)}{b^2\,\sqrt{a+b}\,\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a + b*cosh(x))^2,x)
```

```
[Out] x/b^2 + (2/b + (2*a*exp(x))/b^2)/(b + 2*a*exp(x) + b*exp(2*x)) + (a*log((2*
a*exp(x))/b^3 - (2*a*(b + a*exp(x)))/(b^3*(a + b)^(1/2)*(a - b)^(1/2))))/(b
^2*(a + b)^(1/2)*(a - b)^(1/2)) - (a*log((2*a*exp(x))/b^3 + (2*a*(b + a*exp
(x)))/(b^3*(a + b)^(1/2)*(a - b)^(1/2))))/(b^2*(a + b)^(1/2)*(a - b)^(1/2))
```

3.179 $\int \frac{\tanh^4(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=113

$$\frac{b(3a^2-2b^2)\,\mathrm{ArcTan}(\sinh(x))}{2a^4}+\frac{2(a-b)^{3/2}(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4}-\frac{(4a^2-3b^2)\tanh(x)}{3a^3}-\frac{b\sec}{}$$

```
[Out] 1/2*b*(3*a^2-2*b^2)*arctan(sinh(x))/a^4+2*(a-b)^(3/2)*(a+b)^(3/2)*arctanh((
a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/a^4-1/3*(4*a^2-3*b^2)*tanh(x)/a^3-1/2*b
*sech(x)*tanh(x)/a^2+1/3*sech(x)^2*tanh(x)/a
```

Rubi [A]
time = 0.27, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2804, 3134, 3080, 3855, 2738, 214}

$$\frac{2(a-b)^{3/2}(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4}-\frac{b\tanh(x)\mathrm{sech}(x)}{2a^2}+\frac{b(3a^2-2b^2)\,\mathrm{ArcTan}(\sinh(x))}{2a^4}-\frac{(4a^2-3b^2)\tanh(x)}{3a^3}+\frac{\tanh(x)\mathrm{sech}^2(x)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/(a + b*Cosh[x]),x]
```

```
[Out] (b*(3*a^2 - 2*b^2)*ArcTan[Sinh[x]])/(2*a^4) + (2*(a - b)^(3/2)*(a + b)^(3/2
)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/a^4 - ((4*a^2 - 3*b^2)*Tanh
[x])/(3*a^3) - (b*Sech[x]*Tanh[x])/(2*a^2) + (Sech[x]^2*Tanh[x])/(3*a)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2804

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(3*a*f*Sin[
e + f*x]^3)), x] + (-Dist[1/(6*a^2), Int[((a + b*Sin[e + f*x])^m/Sin[e + f*
x]^2)*Simp[8*a^2 - b^2*(m - 1)*(m - 2) + a*b*m*Sin[e + f*x] - (6*a^2 - b^2*
m*(m - 2))*Sin[e + f*x]^2, x], x], x] - Simp[b*(m - 2)*Cos[e + f*x]*((a + b
```

```
*Sin[e + f*x])^(m + 1)/(6*a^2*f*Sin[e + f*x]^2)), x]) /; FreeQ[{a, b, e, f,
 m}, x] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\tanh^4(x)}{a+b\cosh(x)}\,dx = -\frac{b\mathrm{sech}(x)\tanh(x)}{2a^2}+\frac{\mathrm{sech}^2(x)\tanh(x)}{3a}-\frac{\int \frac{(2(4a^2-3b^2)-ab\cosh(x)-3(2a^2-b^2)\cosh^2(x))\mathrm{sech}^2}{a+b\cosh(x)}}{6a^2}$$

$$= -\frac{(4a^2-3b^2)\tanh(x)}{3a^3}-\frac{b\mathrm{sech}(x)\tanh(x)}{2a^2}+\frac{\mathrm{sech}^2(x)\tanh(x)}{3a}-\frac{\int \frac{(-3b(3a^2-2b^2)-3a(2a}{a+b\,c}}{6}$$

$$= -\frac{(4a^2-3b^2)\tanh(x)}{3a^3}-\frac{b\mathrm{sech}(x)\tanh(x)}{2a^2}+\frac{\mathrm{sech}^2(x)\tanh(x)}{3a}+\frac{(b(3a^2-2b^2))\int \mathrm{sec}}{2a^4}$$

$$= \frac{b(3a^2-2b^2)\tan^{-1}(\sinh(x))}{2a^4}-\frac{(4a^2-3b^2)\tanh(x)}{3a^3}-\frac{b\mathrm{sech}(x)\tanh(x)}{2a^2}+\frac{\mathrm{sech}^2(x)\,\mathrm{ta}}{3a}$$

$$= \frac{b(3a^2-2b^2)\tan^{-1}(\sinh(x))}{2a^4}+\frac{2(a-b)^{3/2}(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4}-\frac{(4a}{}$$

Mathematica [A]
time = 0.30, size = 100, normalized size = 0.88

$$\frac{6b(3a^2-2b^2)\,\mathrm{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)-12(-a^2+b^2)^{3/2}\,\mathrm{ArcTan}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)+a\left(-8a^2+6b^2-3ab\mathrm{sech}(x)+2a^2\mathrm{sech}^2(x)\right)\tanh(x)}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(a + b*Cosh[x]),x]
```

```
[Out] (6*b*(3*a^2 - 2*b^2)*ArcTan[Tanh[x/2]] - 12*(-a^2 + b^2)^(3/2)*ArcTan[((a -
 b)*Tanh[x/2])/Sqrt[-a^2 + b^2]] + a*(-8*a^2 + 6*b^2 - 3*a*b*Sech[x] + 2*a^
2*Sech[x]^2)*Tanh[x])/(6*a^4)
```

Maple [A]
time = 0.72, size = 154, normalized size = 1.36

method	result
default	$\frac{2(a-b)^2(a+b)^2\,\mathrm{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^4\sqrt{(a+b)(a-b)}}+\frac{\frac{2\left(\left(-a^3+\frac{1}{2}a^2b+a\,b^2\right)\left(\tanh^5\left(\frac{x}{2}\right)\right)+\left(-\frac{10}{3}a^3+2a\,b^2\right)\left(\tanh^3\left(\frac{x}{2}\right)\right)+\left(-a^3+a\,b^2-\frac{1}{2}\right.\right.}{\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^3}}{a^4}$
risch	$\frac{-3ab\,\mathrm{e}^{5x}+12a^2\mathrm{e}^{4x}-6b^2\mathrm{e}^{4x}+12a^2\mathrm{e}^{2x}-12b^2\mathrm{e}^{2x}+3b\,\mathrm{e}^{x}a+8a^2-6b^2}{3a^3\left(1+\mathrm{e}^{2x}\right)^3}+\frac{\sqrt{a^2-b^2}\,\ln\left(\mathrm{e}^{x}-\frac{-a+\sqrt{a^2-b^2}}{b}\right)}{a^2}-\frac{\sqrt{a^2-b^2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(a-b)^2/a^4*(a+b)^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*
(a-b))^(1/2))+2/a^4*(((-a^3+1/2*a^2*b+a*b^2)*tanh(1/2*x)^5+(-10/3*a^3+2*a*b
^2)*tanh(1/2*x)^3+(-a^3+a*b^2-1/2*a^2*b)*tanh(1/2*x))/(tanh(1/2*x)^2+1)^3+1
/2*b*(3*a^2-2*b^2)*arctan(tanh(1/2*x)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 967 vs. $2(95) = 190$.
time = 0.50, size = 2003, normalized size = 17.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [-1/3*(3*a^2*b*cosh(x)^5 + 3*a^2*b*sinh(x)^5 - 6*(2*a^3 - a*b^2)*cosh(x)^4
+ 3*(5*a^2*b*cosh(x) - 4*a^3 + 2*a*b^2)*sinh(x)^4 - 3*a^2*b*cosh(x) + 6*(5*
a^2*b*cosh(x)^2 - 4*(2*a^3 - a*b^2)*cosh(x))*sinh(x)^3 - 8*a^3 + 6*a*b^2 -
12*(a^3 - a*b^2)*cosh(x)^2 + 6*(5*a^2*b*cosh(x)^3 - 2*a^3 + 2*a*b^2 - 6*(2*
a^3 - a*b^2)*cosh(x)^2)*sinh(x)^2 + 3*((a^2 - b^2)*cosh(x)^6 + 6*(a^2 - b^2
)*cosh(x)*sinh(x)^5 + (a^2 - b^2)*sinh(x)^6 + 3*(a^2 - b^2)*cosh(x)^4 + 3*(
5*(a^2 - b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^4 + 4*(5*(a^2 - b^2)*cosh(x)^3
 + 3*(a^2 - b^2)*cosh(x))*sinh(x)^3 + 3*(a^2 - b^2)*cosh(x)^2 + 3*(5*(a^2 -
 b^2)*cosh(x)^4 + 6*(a^2 - b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + a^2 - b^
2 + 6*((a^2 - b^2)*cosh(x)^5 + 2*(a^2 - b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x
))*sinh(x))*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh
(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*co
sh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cos
h(x) + a)*sinh(x) + b)) - 3*((3*a^2*b - 2*b^3)*cosh(x)^6 + 6*(3*a^2*b - 2*b
^3)*cosh(x)*sinh(x)^5 + (3*a^2*b - 2*b^3)*sinh(x)^6 + 3*(3*a^2*b - 2*b^3)*c
osh(x)^4 + 3*(3*a^2*b - 2*b^3 + 5*(3*a^2*b - 2*b^3)*cosh(x)^2)*sinh(x)^4 +
4*(5*(3*a^2*b - 2*b^3)*cosh(x)^3 + 3*(3*a^2*b - 2*b^3)*cosh(x))*sinh(x)^3 +
 3*a^2*b - 2*b^3 + 3*(3*a^2*b - 2*b^3)*cosh(x)^2 + 3*(5*(3*a^2*b - 2*b^3)*c
osh(x)^4 + 3*a^2*b - 2*b^3 + 6*(3*a^2*b - 2*b^3)*cosh(x)^2)*sinh(x)^2 + 6*(
```

```
(3*a^2*b - 2*b^3)*cosh(x)^5 + 2*(3*a^2*b - 2*b^3)*cosh(x)^3 + (3*a^2*b - 2*
b^3)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) + 3*(5*a^2*b*cosh(x)^4 - 8
*(2*a^3 - a*b^2)*cosh(x)^3 - a^2*b - 8*(a^3 - a*b^2)*cosh(x))*sinh(x))/(a^4
*cosh(x)^6 + 6*a^4*cosh(x)*sinh(x)^5 + a^4*sinh(x)^6 + 3*a^4*cosh(x)^4 + 3*
a^4*cosh(x)^2 + 3*(5*a^4*cosh(x)^2 + a^4)*sinh(x)^4 + a^4 + 4*(5*a^4*cosh(x
)^3 + 3*a^4*cosh(x))*sinh(x)^3 + 3*(5*a^4*cosh(x)^4 + 6*a^4*cosh(x)^2 + a^4
)*sinh(x)^2 + 6*(a^4*cosh(x)^5 + 2*a^4*cosh(x)^3 + a^4*cosh(x))*sinh(x)), -
1/3*(3*a^2*b*cosh(x)^5 + 3*a^2*b*sinh(x)^5 - 6*(2*a^3 - a*b^2)*cosh(x)^4 +
3*(5*a^2*b*cosh(x) - 4*a^3 + 2*a*b^2)*sinh(x)^4 - 3*a^2*b*cosh(x) + 6*(5*a^
2*b*cosh(x)^2 - 4*(2*a^3 - a*b^2)*cosh(x))*sinh(x)^3 - 8*a^3 + 6*a*b^2 - 12
*(a^3 - a*b^2)*cosh(x)^2 + 6*(5*a^2*b*cosh(x)^3 - 2*a^3 + 2*a*b^2 - 6*(2*a^
3 - a*b^2)*cosh(x)^2)*sinh(x)^2 + 6*((a^2 - b^2)*cosh(x)^6 + 6*(a^2 - b^2)*
cosh(x)*sinh(x)^5 + (a^2 - b^2)*sinh(x)^6 + 3*(a^2 - b^2)*cosh(x)^4 + 3*(5*
(a^2 - b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^4 + 4*(5*(a^2 - b^2)*cosh(x)^3 +
 3*(a^2 - b^2)*cosh(x))*sinh(x)^3 + 3*(a^2 - b^2)*cosh(x)^2 + 3*(5*(a^2 - b
^2)*cosh(x)^4 + 6*(a^2 - b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + a^2 - b^2
+ 6*((a^2 - b^2)*cosh(x)^5 + 2*(a^2 - b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))
*sinh(x))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x)
+ a)/(a^2 - b^2)) - 3*((3*a^2*b - 2*b^3)*cosh(x)^6 + 6*(3*a^2*b - 2*b^3)*co
sh(x)*sinh(x)^5 + (3*a^2*b - 2*b^3)*sinh(x)^6 + 3*(3*a^2*b - 2*b^3)*cosh(x)
^4 + 3*(3*a^2*b - 2*b^3 + 5*(3*a^2*b - 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(
3*a^2*b - 2*b^3)*cosh(x)^3 + 3*(3*a^2*b - 2*b^3)*cosh(x))*sinh(x)^3 + 3*a^2
*b - 2*b^3 + 3*(3*a^2*b - 2*b^3)*cosh(x)^2 + 3*(5*(3*a^2*b - 2*b^3)*cosh(x)
^4 + 3*a^2*b - 2*b^3 + 6*(3*a^2*b - 2*b^3)*cosh(x)^2)*sinh(x)^2 + 6*((3*a^2
*b - 2*b^3)*cosh(x)^5 + 2*(3*a^2*b - 2*b^3)*cosh(x)^3 + (3*a^2*b - 2*b^3)*c
osh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) + 3*(5*a^2*b*cosh(x)^4 - 8*(2*a^
3 - a*b^2)*cosh(x)^3 - a^2*b - 8*(a^3 - a*b^2)*cosh(x))*sinh(x))/(a^4*cosh(
x)^6 + 6*a^4*cosh(x)*sinh(x)^5 + a^4*sinh(x)^6 + 3*a^4*cosh(x)^4 + 3*a^4*co
sh(x)^2 + 3*(5*a^4*cosh(x)^2 + a^4)*sinh(x)^4 + a^4 + 4*(5*a^4*cosh(x)^3 +
3*a^4*cosh(x))*sinh(x)^3 + 3*(5*a^4*cosh(x)^4 + 6*a^4*cosh(x)^2 + a^4)*sinh
(x)^2 + 6*(a^4*cosh(x)^5 + 2*a^4*cosh(x)^3 + a^4*cosh(x))*sinh(x))]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(x)}{a + b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(a+b*cosh(x)),x)
```

```
[Out] Integral(tanh(x)**4/(a + b*cosh(x)), x)
```

Giac [A]

time = 0.40, size = 144, normalized size = 1.27

$$\frac{(3a^2b - 2b^3)\arctan(e^x)}{a^4} + \frac{2(a^4 - 2a^2b^2 + b^4)\arctan\left(\frac{be^x + a}{\sqrt{-a^2 + b^2}}\right)}{\sqrt{-a^2 + b^2}\,a^4} - \frac{3abe^{(5x)} - 12a^2e^{(4x)} + 6b^2e^{(4x)} - 12a^2e^{(2x)} + 12b^2e^{(2x)} - 3abe^x - 8a^2 + 6b^2}{3a^3(e^{(2x)} + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] (3*a^2*b - 2*b^3)*arctan(e^x)/a^4 + 2*(a^4 - 2*a^2*b^2 + b^4)*arctan((b*e^x
 + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*a^4) - 1/3*(3*a*b*e^(5*x) - 12*a^
2*e^(4*x) + 6*b^2*e^(4*x) - 12*a^2*e^(2*x) + 12*b^2*e^(2*x) - 3*a*b*e^x - 8
*a^2 + 6*b^2)/(a^3*(e^(2*x) + 1)^3)
```

Mupad [B]
time = 5.94, size = 722, normalized size = 6.39

$$\frac{8}{3a\,(3e^{2x}+3e^{4x}+e^{6x}+1)}-\frac{\frac{4}{a}-\frac{2be^x}{a^2}}{2e^{2x}+e^{4x}+1}+\frac{\frac{2(2a^2-b^2)}{a^3}-\frac{be^x}{a^2}}{e^{2x}+1}+\frac{\ln\left(\left(\frac{\frac{32a^8-288e^xa^7b-272a^6b^2+600e^xa^5b^3+456a^4b^4-416e^xa^3b^5-288a^2b^6+96e^xab^7+64b^8}{a^6b^4}-\frac{\left(\frac{32\sqrt{(a+b)^3(a-b)^3}\,(4e^xa^3+3a^2b-3e^xab^2-2b^3)}{a^2b^5}+\frac{16(a^2-b^2)(8e^xa^3+4a^2b-7e^xab^2-4b^3)}{ab^5}\right)\sqrt{(a+b)^3(a-b)^3}}{a^4}}{a^4}\right)\sqrt{(a+b)^3(a-b)^3}-\frac{8(a^2-b^2)^2(3a^2-2b^2)(10e^xa^3+6a^2b-7e^xab^2-4b^3)}{a^9b^3}\right)\sqrt{(a+b)^3(a-b)^3}}{a^4}-\frac{\ln\left(-\left(\frac{\frac{32a^8-288e^xa^7b-272a^6b^2+600e^xa^5b^3+456a^4b^4-416e^xa^3b^5-288a^2b^6+96e^xab^7+64b^8}{a^6b^4}-\frac{\left(\frac{32\sqrt{(a+b)^3(a-b)^3}\,(4e^xa^3+3a^2b-3e^xab^2-2b^3)}{a^2b^5}-\frac{16(a^2-b^2)(8e^xa^3+4a^2b-7e^xab^2-4b^3)}{ab^5}\right)\sqrt{(a+b)^3(a-b)^3}}{a^4}}{a^4}\right)\sqrt{(a+b)^3(a-b)^3}-\frac{8(a^2-b^2)^2(3a^2-2b^2)(10e^xa^3+6a^2b-7e^xab^2-4b^3)}{a^9b^3}\right)\sqrt{(a+b)^3(a-b)^3}}{a^4}-\frac{b\ln(e^x-1i)\,(3a^2-2b^2)\,1i}{2a^4}+\frac{b\ln(e^x+1i)\,(3a^2-2b^2)\,1i}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a + b*cosh(x)),x)
```

```
[Out] 8/(3*a*(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1)) - (4/a - (2*b*exp(x))/a^2)
/(2*exp(2*x) + exp(4*x) + 1) + ((2*(2*a^2 - b^2))/a^3 - (b*exp(x))/a^2)/(ex
p(2*x) + 1) + (log((((32*a^8 + 64*b^8 - 288*a^2*b^6 + 456*a^4*b^4 - 272*a^6
*b^2 + 96*a*b^7*exp(x) - 288*a^7*b*exp(x) - 416*a^3*b^5*exp(x) + 600*a^5*b^
3*exp(x))/(a^6*b^4) - (((32*((a + b)^3*(a - b)^3)^(1/2)*(3*a^2*b - 2*b^3 +
4*a^3*exp(x) - 3*a*b^2*exp(x)))/(a^2*b^5) + (16*(a^2 - b^2)*(4*a^2*b - 4*b^
3 + 8*a^3*exp(x) - 7*a*b^2*exp(x)))/(a*b^5))*((a + b)^3*(a - b)^3)^(1/2))/a
^4)*((a + b)^3*(a - b)^3)^(1/2))/a^4 - (8*(a^2 - b^2)^2*(3*a^2 - 2*b^2)*(6*
a^2*b - 4*b^3 + 10*a^3*exp(x) - 7*a*b^2*exp(x)))/(a^9*b^3))*((a + b)^3*(a -
 b)^3)^(1/2))/a^4 - (log(- (((32*a^8 + 64*b^8 - 288*a^2*b^6 + 456*a^4*b^4 -
 272*a^6*b^2 + 96*a*b^7*exp(x) - 288*a^7*b*exp(x) - 416*a^3*b^5*exp(x) + 60
0*a^5*b^3*exp(x))/(a^6*b^4) - (((32*((a + b)^3*(a - b)^3)^(1/2)*(3*a^2*b -
2*b^3 + 4*a^3*exp(x) - 3*a*b^2*exp(x)))/(a^2*b^5) - (16*(a^2 - b^2)*(4*a^2*
b - 4*b^3 + 8*a^3*exp(x) - 7*a*b^2*exp(x)))/(a*b^5))*((a + b)^3*(a - b)^3)^
(1/2))/a^4)*((a + b)^3*(a - b)^3)^(1/2))/a^4 - (8*(a^2 - b^2)^2*(3*a^2 - 2*
b^2)*(6*a^2*b - 4*b^3 + 10*a^3*exp(x) - 7*a*b^2*exp(x)))/(a^9*b^3))*((a + b
)^3*(a - b)^3)^(1/2))/a^4 - (b*log(exp(x) - 1i)*(3*a^2 - 2*b^2)*1i)/(2*a^4)
 + (b*log(exp(x) + 1i)*(3*a^2 - 2*b^2)*1i)/(2*a^4)
```

3.180 $\int \frac{\tanh^3(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=57

$$\frac{(a^2-b^2)\log(\cosh(x))}{a^3}-\frac{(a^2-b^2)\log(a+b\cosh(x))}{a^3}-\frac{b\text{sech}(x)}{a^2}+\frac{\text{sech}^2(x)}{2a}$$

```
[Out] (a^2-b^2)*ln(cosh(x))/a^3-(a^2-b^2)*ln(a+b*cosh(x))/a^3-b*sech(x)/a^2+1/2*s
ech(x)^2/a
```

Rubi [A]
time = 0.06, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2800, 908}

$$-\frac{b\text{sech}(x)}{a^2}+\frac{(a^2-b^2)\log(\cosh(x))}{a^3}-\frac{(a^2-b^2)\log(a+b\cosh(x))}{a^3}+\frac{\text{sech}^2(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/(a + b*Cosh[x]),x]
```

```
[Out] ((a^2 - b^2)*Log[Cosh[x]])/a^3 - ((a^2 - b^2)*Log[a + b*Cosh[x]])/a^3 - (b*
Sech[x])/a^2 + Sech[x]^2/(2*a)
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tanh^3(x)}{a+b\cosh(x)}\,dx = -\text{Subst}\left(\int \frac{b^2-x^2}{x^3(a+x)}\,dx, x, b\cosh(x)\right)$$
$$= -\text{Subst}\left(\int \left(\frac{b^2}{ax^3}-\frac{b^2}{a^2x^2}+\frac{-a^2+b^2}{a^3x}+\frac{a^2-b^2}{a^3(a+x)}\right)\,dx, x, b\cosh(x)\right)$$
$$= \frac{(a^2-b^2)\log(\cosh(x))}{a^3}-\frac{(a^2-b^2)\log(a+b\cosh(x))}{a^3}-\frac{b\text{sech}(x)}{a^2}+\frac{\text{sech}^2(x)}{2a}$$

Mathematica [A]
time = 0.07, size = 46, normalized size = 0.81

$$\frac{2(a^2-b^2)\left(\log(\cosh(x))-\log(a+b\cosh(x))\right)-2ab\text{sech}(x)+a^2\text{sech}^2(x)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(a + b*Cosh[x]),x]
```

```
[Out] (2*(a^2 - b^2)*(Log[Cosh[x]] - Log[a + b*Cosh[x]]) - 2*a*b*Sech[x] + a^2*Se
ch[x]^2)/(2*a^3)
```

Maple [A]
time = 0.53, size = 95, normalized size = 1.67

method	result	size
default	$\frac{\frac{2a^2}{\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^2}+(a^2-b^2)\ln\left(\tanh^2\left(\frac{x}{2}\right)+1\right)-\frac{2a(a+b)}{\tanh^2\left(\frac{x}{2}\right)+1}}{a^3}-\frac{(a+b)(a-b)\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-b\left(\tanh^2\left(\frac{x}{2}\right)\right)-a-b\right)}{a^3}$	95
risch	$\frac{2\,e^x\left(-b\,e^{2x}+a\,e^x-b\right)}{\left(1+e^{2x}\right)^2a^2}-\frac{\ln\left(e^{2x}+\frac{2a\,e^x}{b}+1\right)}{a}+\frac{\ln\left(e^{2x}+\frac{2a\,e^x}{b}+1\right)b^2}{a^3}+\frac{\ln\left(1+e^{2x}\right)}{a}-\frac{\ln\left(1+e^{2x}\right)b^2}{a^3}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(2*a^2/(tanh(1/2*x)^2+1)^2+(a^2-b^2)*ln(tanh(1/2*x)^2+1)-2*a*(a+b)/(t
anh(1/2*x)^2+1))-(a+b)*(a-b)/a^3*ln(a*tanh(1/2*x)^2-b*tanh(1/2*x)^2-a-b)
```

Maxima [A]
time = 0.53, size = 96, normalized size = 1.68

$$-\frac{2\left(be^{(-x)}-ae^{(-2x)}+be^{(-3x)}\right)}{2\,a^2e^{(-2x)}+a^2e^{(-4x)}+a^2}-\frac{(a^2-b^2)\log\left(2\,ae^{(-x)}+be^{(-2x)}+b\right)}{a^3}+\frac{(a^2-b^2)\log\left(e^{(-2x)}+1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] -2*(b*e^(-x) - a*e^(-2*x) + b*e^(-3*x))/(2*a^2*e^(-2*x) + a^2*e^(-4*x) + a^
2) - (a^2 - b^2)*log(2*a*e^(-x) + b*e^(-2*x) + b)/a^3 + (a^2 - b^2)*log(e^(
-2*x) + 1)/a^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(55) = 110$.
time = 0.39, size = 450, normalized size = 7.89

$$-\frac{2\,ab\cosh\left(x\right)^{3}+2\,ab\sinh\left(x\right)^{3}-2\,a^{2}\cosh\left(x\right)^{2}+2\,ab\cosh\left(x\right)+2\,\left(3\,ab\cosh\left(x\right)-a^{2}\right)\sinh\left(x\right)^{2}+\left(\left(a^{2}-b^{2}\right)\cosh\left(x\right)^{4}+4\,\left(a^{2}-b^{2}\right)\cosh\left(x\right)\sinh\left(x\right)^{3}+\left(a^{2}-b^{2}\right)\sinh\left(x\right)^{4}+2\,\left(a^{2}-b^{2}\right)\cosh\left(x\right)^{2}+2\,\left(3\,\left(a^{2}-b^{2}\right)\cosh\left(x\right)^{2}+a^{2}-b^{2}\right)\sinh\left(x\right)^{2}+a^{2}-b^{2}+4\,\left(\left(a^{2}-b^{2}\right)\cosh\left(x\right)^{3}+\left(a^{2}-b^{2}\right)\cosh\left(x\right)\right)\sinh\left(x\right)\right)\log\left(\frac{2\,\left(b\cosh\left(x\right)+a\right)}{\cosh\left(x\right)-\sinh\left(x\right)}\right)-\left(\left(a^{2}-b^{2}\right)\cosh\left(x\right)^{4}+4\,\left(a^{2}-b^{2}\right)\cosh\left(x\right)\sinh\left(x\right)^{3}+\left(a^{2}-b^{2}\right)\sinh\left(x\right)^{4}+2\,\left(a^{2}-b^{2}\right)\cosh\left(x\right)^{2}+2\,\left(3\,\left(a^{2}-b^{2}\right)\cosh\left(x\right)^{2}+a^{2}-b^{2}\right)\sinh\left(x\right)^{2}+a^{2}-b^{2}+4\,\left(\left(a^{2}-b^{2}\right)\cosh\left(x\right)^{3}+\left(a^{2}-b^{2}\right)\cosh\left(x\right)\right)\sinh\left(x\right)\right)\log\left(\frac{2\,\cosh\left(x\right)}{\cosh\left(x\right)-\sinh\left(x\right)}\right)+2\,\left(3\,ab\cosh\left(x\right)^{2}-2\,a^{2}\cosh\left(x\right)+ab\right)\sinh\left(x\right)}{a^{3}\cosh\left(x\right)^{4}+4\,a^{3}\cosh\left(x\right)\sinh\left(x\right)^{3}+a^{3}\sinh\left(x\right)^{4}+2\,a^{3}\cosh\left(x\right)^{2}+a^{3}+2\,\left(3\,a^{3}\cosh\left(x\right)^{2}+a^{3}\right)\sinh\left(x\right)^{2}+4\,\left(a^{3}\cosh\left(x\right)^{3}+a^{3}\cosh\left(x\right)\right)\sinh\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] -(2*a*b*cosh(x)^3 + 2*a*b*sinh(x)^3 - 2*a^2*cosh(x)^2 + 2*a*b*cosh(x) + 2*(
3*a*b*cosh(x) - a^2)*sinh(x)^2 + ((a^2 - b^2)*cosh(x)^4 + 4*(a^2 - b^2)*cos
h(x)*sinh(x)^3 + (a^2 - b^2)*sinh(x)^4 + 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*(a^
2 - b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh
(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x))*log(2*(b*cosh(x) + a)/(cosh(x) - sinh
(x))) - ((a^2 - b^2)*cosh(x)^4 + 4*(a^2 - b^2)*cosh(x)*sinh(x)^3 + (a^2 - b
^2)*sinh(x)^4 + 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*(a^2 - b^2)*cosh(x)^2 + a^2
- b^2)*sinh(x)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(
x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) + 2*(3*a*b*cosh(x)^2 - 2*a^
2*cosh(x) + a*b)*sinh(x))/(a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*si
nh(x)^4 + 2*a^3*cosh(x)^2 + a^3 + 2*(3*a^3*cosh(x)^2 + a^3)*sinh(x)^2 + 4*(
a^3*cosh(x)^3 + a^3*cosh(x))*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{3}\left(x\right)}{a+b\cosh\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(a+b*cosh(x)),x)
```

```
[Out] Integral(tanh(x)**3/(a + b*cosh(x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(55) = 110$.
time = 0.41, size = 115, normalized size = 2.02

$$\frac{\left(a^{2}-b^{2}\right)\log\left(e^{\left(-x\right)}+e^{x}\right)}{a^{3}}-\frac{\left(a^{2}b-b^{3}\right)\log\left(\left|b\left(e^{\left(-x\right)}+e^{x}\right)+2\,a\right|\right)}{a^{3}b}-\frac{3\,a^{2}\left(e^{\left(-x\right)}+e^{x}\right)^{2}-3\,b^{2}\left(e^{\left(-x\right)}+e^{x}\right)^{2}+4\,ab\left(e^{\left(-x\right)}+e^{x}\right)-4\,a^{2}}{2\,a^{3}\left(e^{\left(-x\right)}+e^{x}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] (a^2 - b^2)*log(e^(-x) + e^x)/a^3 - (a^2*b - b^3)*log(abs(b*(e^(-x) + e^x)
+ 2*a))/(a^3*b) - 1/2*(3*a^2*(e^(-x) + e^x)^2 - 3*b^2*(e^(-x) + e^x)^2 + 4*
a*b*(e^(-x) + e^x) - 4*a^2)/(a^3*(e^(-x) + e^x)^2)
```

Mupad [B]
time = 1.61, size = 1221, normalized size = 21.42

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a + b*cosh(x)),x)
```

```
[Out] (2/a - (2*b*exp(x))/a^2)/(exp(2*x) + 1) - 2/(a*(2*exp(2*x) + exp(4*x) + 1))
 + ((2*atan((4*a^4*b^3*(a^2 - b^2)^2*(-a^6)^(1/2) - 4*a^6*b*(a^2 - b^2)^2*(
-a^6)^(1/2))*(exp(x)*(1/(16*a^4*b^2*(a^2 - b^2)^3*((a^2 - b^2)^2)^(1/2)) -
(a^2 - 2*b^2)^2/(16*a^8*b^2*(a^2 - b^2)^3*((a^2 - b^2)^2)^(1/2))) + 1/(8*a^
5*b*(a^2 - b^2)^3*((a^2 - b^2)^2)^(1/2)) + (a^2 - 2*b^2)/(8*a^7*b*(a^2 - b^
2)^3*((a^2 - b^2)^2)^(1/2)))) + 2*atan((a^2*(-a^6)^(1/2)*(a^4 + b^4 - 2*a^2
*b^2)^(1/2) - 2*b^2*(-a^6)^(1/2)*(a^4 + b^4 - 2*a^2*b^2)^(1/2))/(2*a^3*(a^2
 - b^2)^2) + ((a^7 - a^5*b^2)*(-a^6)^(1/2))/(2*a^6*(a^2 - b^2)*((a^2 - b^2)
^2)^(1/2)) + (a^6*b^2*exp(3*x)*((2*(a^7 - a^5*b^2)*(a^4 + b^4 - 2*a^2*b^2)^
(1/2))/(a^11*b^3*(a^2 - b^2)*((a^2 - b^2)^2)^(1/2)) - (2*(a^2 - 2*b^2)*(a^2
*(-a^6)^(1/2)*(a^4 + b^4 - 2*a^2*b^2)^(1/2) - 2*b^2*(-a^6)^(1/2)*(a^4 + b^4
 - 2*a^2*b^2)^(1/2))*(a^4 + b^4 - 2*a^2*b^2)^(1/2))/(a^10*b^3*(a^2 - b^2)^2
*(-a^6)^(1/2)))*(-a^6)^(1/2))/(8*(a^4 + b^4 - 2*a^2*b^2)^(1/2)) - (a^6*b^2*
exp(x)*(-a^6)^(1/2)*((8*(a^4 + b^4 - 2*a^2*b^2))/(a^8*b*(a^2 - b^2)^2) - (4
*(2*a^6*b - 2*a^4*b^3)*(a^4 + b^4 - 2*a^2*b^2)^(1/2))/(a^12*b^2*(a^2 - b^2)
*((a^2 - b^2)^2)^(1/2)) - (2*(a^7 - a^5*b^2)*(a^4 + b^4 - 2*a^2*b^2)^(1/2))
/(a^11*b^3*(a^2 - b^2)*((a^2 - b^2)^2)^(1/2)) + (2*(a^2 - 2*b^2)*(a^2*(-a^6
)^(1/2)*(a^4 + b^4 - 2*a^2*b^2)^(1/2) - 2*b^2*(-a^6)^(1/2)*(a^4 + b^4 - 2*a
^2*b^2)^(1/2))*(a^4 + b^4 - 2*a^2*b^2)^(1/2))/(a^10*b^3*(a^2 - b^2)^2*(-a^6
)^(1/2))))/(8*(a^4 + b^4 - 2*a^2*b^2)^(1/2)) + (a^6*b^2*exp(2*x)*(-a^6)^(1/
2)*((4*(a^2 - 2*b^2)*(a^4 + b^4 - 2*a^2*b^2))/(a^9*b^2*(a^2 - b^2)^2) + (4*
(a^2*(-a^6)^(1/2)*(a^4 + b^4 - 2*a^2*b^2)^(1/2) - 2*b^2*(-a^6)^(1/2)*(a^4 +
 b^4 - 2*a^2*b^2)^(1/2))*(a^4 + b^4 - 2*a^2*b^2)^(1/2))/(a^9*b^2*(a^2 - b^2
)^2*(-a^6)^(1/2)) + (2*(2*a^6*b - 2*a^4*b^3)*(a^4 + b^4 - 2*a^2*b^2)^(1/2))
/(a^11*b^3*(a^2 - b^2)*((a^2 - b^2)^2)^(1/2)) + (4*(a^7 - a^5*b^2)*(a^4 + b
^4 - 2*a^2*b^2)^(1/2))/(a^12*b^2*(a^2 - b^2)*((a^2 - b^2)^2)^(1/2))))/(8*(a
^4 + b^4 - 2*a^2*b^2)^(1/2))))*(a^4 + b^4 - 2*a^2*b^2)^(1/2))/(-a^6)^(1/2)
```

3.181 $\int \frac{\tanh^2(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=61

$$\frac{b\text{ArcTan}(\sinh(x))}{a^2}+\frac{2\sqrt{a-b}\,\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2}-\frac{\tanh(x)}{a}$$

```
[Out] b*arctan(sinh(x))/a^2+2*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))*(a-b)^
(1/2)*(a+b)^(1/2)/a^2-tanh(x)/a
```

Rubi [A]
time = 0.15, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2802, 3135, 3080, 3855, 2738, 214}

$$\frac{b\text{ArcTan}(\sinh(x))}{a^2}+\frac{2\sqrt{a-b}\,\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2}-\frac{\tanh(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(a + b*Cosh[x]),x]
```

```
[Out] (b*ArcTan[Sinh[x]])/a^2 + (2*Sqrt[a - b]*Sqrt[a + b]*ArcTanh[(Sqrt[a - b]*T
anh[x/2])/Sqrt[a + b]])/a^2 - Tanh[x]/a
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((1 - Sin[e + f*x]^2)/Sin[e + f*x]^
2), x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3135

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[
e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n +
 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*
(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
!(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^2(x)}{a+b\cosh(x)}\,dx &= -\int \frac{\left(1-\cosh^2(x)\right)\operatorname{sech}^2(x)}{a+b\cosh(x)}\,dx\\ &= -\frac{\tanh(x)}{a}-\frac{\int \frac{(-b-a\cosh(x))\operatorname{sech}(x)}{a+b\cosh(x)}\,dx}{a}\\ &= -\frac{\tanh(x)}{a}+\frac{b\int \operatorname{sech}(x)\,dx}{a^2}-\frac{\left(-a^2+b^2\right)\int \frac{1}{a+b\cosh(x)}\,dx}{a^2}\\ &= \frac{b\tan^{-1}(\sinh(x))}{a^2}-\frac{\tanh(x)}{a}-\frac{\left(2\left(-a^2+b^2\right)\right)\operatorname{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2}\\ &= \frac{b\tan^{-1}(\sinh(x))}{a^2}+\frac{2\sqrt{a-b}\,\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2}-\frac{\tanh(x)}{a}\end{aligned}$$

Mathematica [A]

time = 0.08, size = 61, normalized size = 1.00

$$\frac{2b\text{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)+2\sqrt{-a^2+b^2}\,\text{ArcTan}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)-a\tanh(x)}{a^2}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[x]^2/(a + b*Cosh[x]),x]

[Out] (2*b*ArcTan[Tanh[x/2]] + 2*Sqrt[-a^2 + b^2]*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]] - a*Tanh[x])/a^2

Maple [A]
time = 0.61, size = 78, normalized size = 1.28

method	result
default	$\frac{-\frac{2a\tanh\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)+1}+2b\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a^2}+\frac{2(a-b)(a+b)\,\text{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^2\sqrt{(a+b)(a-b)}}$
risch	$\frac{2}{a(1+e^{2x})}+\frac{\sqrt{a^2-b^2}\,\ln\left(e^x-\frac{-a+\sqrt{a^2-b^2}}{b}\right)}{a^2}-\frac{\sqrt{a^2-b^2}\,\ln\left(e^x+\frac{a+\sqrt{a^2-b^2}}{b}\right)}{a^2}+\frac{ib\ln(e^x+i)}{a^2}-\frac{ib\ln(e^x-}{a^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^2/(a+b*cosh(x)),x,method=_RETURNVERBOSE)

[Out] 2/a^2*(-a*tanh(1/2*x)/(tanh(1/2*x)^2+1)+b*arctan(tanh(1/2*x)))+2*(a-b)/a^2*(a+b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^2/(a+b*cosh(x)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more de

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(51) = 102.
time = 0.49, size = 326, normalized size = 5.34

$$\left[\frac{\sqrt{a^2-b^2}\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2+1\right)\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2ab\cosh(x)+2a^2-b^2+2\left(b^2\cosh(x)+ab\right)\sinh(x)-2\sqrt{a^2-b^2}\left(b\cosh(x)+b\sinh(x)+a\right)}{b\cosh(x)^2+b\sinh(x)^2+2a\cosh(x)+2\left(b\cosh(x)+a\right)\sinh(x)+b}\right)+2\left(b\cosh(x)^2+2b\cosh(x)\sinh(x)+b\sinh(x)^2+b\right)\arctan\left(\cosh(x)+\sinh(x)\right)+2a}{a^2\cosh(x)^2+2a^2\cosh(x)\sinh(x)+a^2\sinh(x)^2+a^2},\ -\frac{2\left(\sqrt{-a^2+b^2}\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2+1\right)\arctan\left(-\frac{\sqrt{-a^2+b^2}\left(b\cosh(x)+b\sinh(x)+a\right)}{a^2-b^2}\right)-\left(b\cosh(x)^2+2b\cosh(x)\sinh(x)+b\sinh(x)^2+b\right)\arctan\left(\cosh(x)+\sinh(x)\right)-a\right)}{a^2\cosh(x)^2+2a^2\cosh(x)\sinh(x)+a^2\sinh(x)^2+a^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [(sqrt(a^2 - b^2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log((b^2*
cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) +
a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2
+ b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)) + 2*(b*cosh(x
)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)*arctan(cosh(x) + sinh(x)) + 2*
a)/(a^2*cosh(x)^2 + 2*a^2*cosh(x)*sinh(x) + a^2*sinh(x)^2 + a^2), -2*(sqrt(
-a^2 + b^2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*arctan(-sqrt(-a
^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) - (b*cosh(x)^2 + 2*b*cos
h(x)*sinh(x) + b*sinh(x)^2 + b)*arctan(cosh(x) + sinh(x)) - a)/(a^2*cosh(x)
^2 + 2*a^2*cosh(x)*sinh(x) + a^2*sinh(x)^2 + a^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(x)}{a + b\cosh(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(a+b*cosh(x)),x)
```

```
[Out] Integral(tanh(x)**2/(a + b*cosh(x)), x)
```

Giac [A]
time = 0.41, size = 67, normalized size = 1.10

$$\frac{2\,b\arctan\left(e^{x}\right)}{a^2} + \frac{2\left(a^2-b^2\right)\arctan\left(\frac{be^{x}+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,a^2} + \frac{2}{a(e^{(2\,x)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*b*arctan(e^x)/a^2 + 2*(a^2 - b^2)*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(s
qrt(-a^2 + b^2)*a^2) + 2/(a*(e^(2*x) + 1))
```

Mupad [B]
time = 3.53, size = 285, normalized size = 4.67

$$\frac{2}{a+a\,\mathrm{e}^{2\,x}} + \frac{\ln\left(64\,a^3\,b-64\,a\,b^3-32\,b^3\,\sqrt{a^2-b^2}+128\,a^4\,\mathrm{e}^{x}+32\,b^4\,\mathrm{e}^{x}+64\,a^2\,b\,\sqrt{a^2-b^2}+128\,a^3\,\mathrm{e}^{x}\,\sqrt{a^2-b^2}-160\,a^2\,b^2\,\mathrm{e}^{x}-96\,a\,b^2\,\mathrm{e}^{x}\,\sqrt{a^2-b^2}\right)\,\sqrt{a^2-b^2}}{a^2} - \frac{\ln\left(64\,a^3\,b-64\,a\,b^3+32\,b^3\,\sqrt{a^2-b^2}+128\,a^4\,\mathrm{e}^{x}+32\,b^4\,\mathrm{e}^{x}-64\,a^2\,b\,\sqrt{a^2-b^2}-128\,a^3\,\mathrm{e}^{x}\,\sqrt{a^2-b^2}-160\,a^2\,b^2\,\mathrm{e}^{x}+96\,a\,b^2\,\mathrm{e}^{x}\,\sqrt{a^2-b^2}\right)\,\sqrt{a^2-b^2}}{a^2} - \frac{b\,\left(\ln\left(\mathrm{e}^{x}-\mathrm{i}\right)\,1\mathrm{i}-\ln\left(\mathrm{e}^{x}+1\mathrm{i}\right)\,1\mathrm{i}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a + b*cosh(x)),x)
```

```
[Out] 2/(a + a*exp(2*x)) + (log(64*a^3*b - 64*a*b^3 - 32*b^3*(a^2 - b^2)^(1/2) +
128*a^4*exp(x) + 32*b^4*exp(x) + 64*a^2*b*(a^2 - b^2)^(1/2) + 128*a^3*exp(x
)*(a^2 - b^2)^(1/2) - 160*a^2*b^2*exp(x) - 96*a*b^2*exp(x)*(a^2 - b^2)^(1/2
))*(a^2 - b^2)^(1/2))/a^2 - (log(64*a^3*b - 64*a*b^3 + 32*b^3*(a^2 - b^2)^(
1/2) + 128*a^4*exp(x) + 32*b^4*exp(x) - 64*a^2*b*(a^2 - b^2)^(1/2) - 128*a^
3*exp(x)*(a^2 - b^2)^(1/2) - 160*a^2*b^2*exp(x) + 96*a*b^2*exp(x)*(a^2 - b^
2)^(1/2))*(a^2 - b^2)^(1/2))/a^2 - (b*(log(exp(x) - 1i)*1i - log(exp(x) + 1
i)*1i))/a^2
```

3.182 $\int \frac{\tanh(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=20

$$\frac{\log(\cosh(x))}{a}-\frac{\log(a+b\cosh(x))}{a}$$

```
[Out] ln(cosh(x))/a-ln(a+b*cosh(x))/a
```

Rubi [A]
time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2800, 36, 29, 31}

$$\frac{\log(\cosh(x))}{a}-\frac{\log(a+b\cosh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(a + b*Cosh[x]),x]
```

```
[Out] Log[Cosh[x]]/a - Log[a + b*Cosh[x]]/a
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh(x)}{a+b\cosh(x)}\,dx &= \mathrm{Subst}\left(\int \frac{1}{x(a+x)}\,dx, x, b\cosh(x)\right)\\
&= \frac{\mathrm{Subst}\left(\int \frac{1}{x}\,dx, x, b\cosh(x)\right)}{a} - \frac{\mathrm{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\cosh(x)\right)}{a}\\
&= \frac{\log(\cosh(x))}{a} - \frac{\log(a+b\cosh(x))}{a}
\end{aligned}
$$

Mathematica [**A**]
time = 0.01, size = 20, normalized size = 1.00

$$
\frac{\log(\cosh(x))}{a} - \frac{\log(a+b\cosh(x))}{a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(a + b*Cosh[x]),x]
[Out] Log[Cosh[x]]/a - Log[a + b*Cosh[x]]/a
```

Maple [**A**]
time = 0.55, size = 21, normalized size = 1.05

method	result	size
derivativedivides	$\frac{\ln(\cosh(x))}{a} - \frac{\ln(a+b\cosh(x))}{a}$	21
default	$\frac{\ln(\cosh(x))}{a} - \frac{\ln(a+b\cosh(x))}{a}$	21
risch	$\frac{\ln(1+\mathrm{e}^{2x})}{a} - \frac{\ln\left(\mathrm{e}^{2x}+\frac{2a\,\mathrm{e}^{x}}{b}+1\right)}{a}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
[Out] ln(cosh(x))/a-ln(a+b*cosh(x))/a
```

Maxima [**A**]
time = 0.50, size = 33, normalized size = 1.65

$$
-\frac{\log\left(2\,ae^{(-x)} + be^{(-2\,x)} + b\right)}{a} + \frac{\log\left(e^{(-2\,x)} + 1\right)}{a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*cosh(x)),x, algorithm="maxima")
[Out] -log(2*a*e^(-x) + b*e^(-2*x) + b)/a + log(e^(-2*x) + 1)/a
```

Fricas [A]
time = 0.42, size = 40, normalized size = 2.00

$$-\frac{\log\left(\frac{2\,(b\cosh(x)+a)}{\cosh(x)-\sinh(x)}\right)-\log\left(\frac{2\,\cosh(x)}{\cosh(x)-\sinh(x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] -(log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))) - log(2*cosh(x)/(cosh(x) - sin
h(x))))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tanh\left(x\right)}{a+b\cosh\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*cosh(x)),x)
```

```
[Out] Integral(tanh(x)/(a + b*cosh(x)), x)
```

Giac [A]
time = 0.41, size = 33, normalized size = 1.65

$$\frac{\log\left(e^{(-x)}+e^{x}\right)}{a}-\frac{\log\left(\left|b\left(e^{(-x)}+e^{x}\right)+2\,a\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] log(e^(-x) + e^x)/a - log(abs(b*(e^(-x) + e^x) + 2*a))/a
```

Mupad [B]
time = 0.43, size = 201, normalized size = 10.05

$$\frac{2\,\mathrm{atan}\left(\frac{a\,\sqrt{-a^2}+b\,\mathrm{e}^x\,\sqrt{-a^2}+2\,a\,\mathrm{e}^{2\,x}\,\sqrt{-a^2}+b\,\mathrm{e}^{3\,x}\,\sqrt{-a^2}}{a^2}\right)}{\sqrt{-a^2}}-\frac{2\,\mathrm{atan}\left(\left(4\,a^4\,b\,\sqrt{-a^2}-4\,a^2\,b^3\,\sqrt{-a^2}\right)\,\left(\mathrm{e}^x\,\left(\frac{1}{16\,b^2\,{\left(a^2-b^2\right)}^2}-\frac{{\left(a^2-2\,b^2\right)}^2}{16\,a^4\,b^2\,{\left(a^2-b^2\right)}^2}\right)+\frac{1}{8\,a\,b\,{\left(a^2-b^2\right)}^2}+\frac{a^2-2\,b^2}{8\,a^3\,b\,{\left(a^2-b^2\right)}^2}\right)\right)}{\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a + b*cosh(x)),x)
```

```
[Out] (2*atan((a*(-a^2)^(1/2) + b*exp(x)*(-a^2)^(1/2) + 2*a*exp(2*x)*(-a^2)^(1/2)
 + b*exp(3*x)*(-a^2)^(1/2))/a^2))/(-a^2)^(1/2) - (2*atan((4*a^4*b*(-a^2)^(1
/2) - 4*a^2*b^3*(-a^2)^(1/2))*(exp(x)*(1/(16*b^2*(a^2 - b^2)^2) - (a^2 - 2*
b^2)^2/(16*a^4*b^2*(a^2 - b^2)^2)) + 1/(8*a*b*(a^2 - b^2)^2) + (a^2 - 2*b^2
)/(8*a^3*b*(a^2 - b^2)^2))))/(-a^2)^(1/2)
```

3.183 $\int \frac{\coth(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=54

$$\frac{\log(1-\cosh(x))}{2(a+b)}+\frac{\log(1+\cosh(x))}{2(a-b)}-\frac{a\log(a+b\cosh(x))}{a^2-b^2}$$

```
[Out] 1/2*ln(1-cosh(x))/(a+b)+1/2*ln(1+cosh(x))/(a-b)-a*ln(a+b*cosh(x))/(a^2-b^2)
```

Rubi [A]
time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2800, 815}

$$-\frac{a\log(a+b\cosh(x))}{a^2-b^2}+\frac{\log(1-\cosh(x))}{2(a+b)}+\frac{\log(\cosh(x)+1)}{2(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(a + b*Cosh[x]),x]
```

```
[Out] Log[1 - Cosh[x]]/(2*(a + b)) + Log[1 + Cosh[x]]/(2*(a - b)) - (a*Log[a + b*
Cosh[x]])/(a^2 - b^2)
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth(x)}{a+b\cosh(x)}\,dx &= -\text{Subst}\left(\int \frac{x}{(a+x)\left(b^2-x^2\right)}\,dx, x, b\cosh(x)\right)\\ &= -\text{Subst}\left(\int \left(\frac{1}{2(a+b)(b-x)}+\frac{a}{(a-b)(a+b)(a+x)}-\frac{1}{2(a-b)(b+x)}\right)dx, x, b\,\text{co}\right.\\ &= \frac{\log(1-\cosh(x))}{2(a+b)}+\frac{\log(1+\cosh(x))}{2(a-b)}-\frac{a\log(a+b\cosh(x))}{a^2-b^2}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 38, normalized size = 0.70

$$-\frac{a\log(a+b\cosh(x))-a\log(\sinh(x))+b\log\left(\tanh\left(\frac{x}{2}\right)\right)}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(a + b*Cosh[x]),x]
```

```
[Out] -((a*Log[a + b*Cosh[x]] - a*Log[Sinh[x]] + b*Log[Tanh[x/2]])/(a^2 - b^2))
```

Maple [A]
time = 0.65, size = 53, normalized size = 0.98

method	result	size
default	$\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a+b}-\frac{a\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-b\left(\tanh^2\left(\frac{x}{2}\right)\right)-a-b\right)}{(a+b)(a-b)}$	53
risch	$-\frac{x}{a+b}-\frac{x}{a-b}+\frac{2xa}{a^2-b^2}+\frac{\ln(\mathrm{e}^x-1)}{a+b}+\frac{\ln(\mathrm{e}^x+1)}{a-b}-\frac{a\ln\left(\mathrm{e}^{2x}+\frac{2a\,\mathrm{e}^x}{b}+1\right)}{a^2-b^2}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(a+b)*ln(tanh(1/2*x))-a/(a+b)/(a-b)*ln(a*tanh(1/2*x)^2-b*tanh(1/2*x)^2-a-
b)
```

Maxima [A]
time = 0.27, size = 59, normalized size = 1.09

$$-\frac{a\log\left(2\,ae^{(-x)}+be^{(-2\,x)}+b\right)}{a^2-b^2}+\frac{\log\left(e^{(-x)}+1\right)}{a-b}+\frac{\log\left(e^{(-x)}-1\right)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] -a*log(2*a*e^(-x) + b*e^(-2*x) + b)/(a^2 - b^2) + log(e^(-x) + 1)/(a - b) +
 log(e^(-x) - 1)/(a + b)
```

Fricas [A]
time = 0.45, size = 60, normalized size = 1.11

$$-\frac{a\log\left(\frac{2\,(b\cosh(x)+a)}{\cosh(x)-\sinh(x)}\right)-(a+b)\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)-(a-b)\log\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] -(a*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))) - (a + b)*log(cosh(x) + sinh
(x) + 1) - (a - b)*log(cosh(x) + sinh(x) - 1))/(a^2 - b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{a + b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*cosh(x)),x)
```

```
[Out] Integral(coth(x)/(a + b*cosh(x)), x)
```

Giac [A]
time = 0.41, size = 67, normalized size = 1.24

$$-\frac{ab\log\left(\left|b\left(e^{(-x)} + e^{x}\right) + 2\,a\right|\right)}{a^{2}b - b^{3}} + \frac{\log\left(e^{(-x)} + e^{x} + 2\right)}{2\,(a - b)} + \frac{\log\left(e^{(-x)} + e^{x} - 2\right)}{2\,(a + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] -a*b*log(abs(b*(e^(-x) + e^x) + 2*a))/(a^2*b - b^3) + 1/2*log(e^(-x) + e^x
+ 2)/(a - b) + 1/2*log(e^(-x) + e^x - 2)/(a + b)
```

Mupad [B]
time = 0.43, size = 148, normalized size = 2.74

$$\frac{\ln\left(128\,a\,b - 128\,a^2 - 32\,b^2 + 128\,a^2\,e^x + 32\,b^2\,e^x - 128\,a\,b\,e^x\right)}{a+b} + \frac{\ln\left(-128\,a\,b - 128\,a^2 - 32\,b^2 - 128\,a^2\,e^x - 32\,b^2\,e^x - 128\,a\,b\,e^x\right)}{a-b} - \frac{a\,\ln\left(16\,a^2\,b - 4\,b^3\,e^{2\,x} - 4\,b^3 + 32\,a^3\,e^x - 8\,a\,b^2\,e^x + 16\,a^2\,b\,e^{2\,x}\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a + b*cosh(x)),x)
```

```
[Out] log(128*a*b - 128*a^2 - 32*b^2 + 128*a^2*exp(x) + 32*b^2*exp(x) - 128*a*b*e
xp(x))/(a + b) + log(- 128*a*b - 128*a^2 - 32*b^2 - 128*a^2*exp(x) - 32*b^2
*exp(x) - 128*a*b*exp(x))/(a - b) - (a*log(16*a^2*b - 4*b^3*exp(2*x) - 4*b^
3 + 32*a^3*exp(x) - 8*a*b^2*exp(x) + 16*a^2*b*exp(2*x)))/(a^2 - b^2)
```

3.184 $\int \frac{\coth^2(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=77

$$\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}-\frac{a\coth(x)}{a^2-b^2}+\frac{b\text{csch}(x)}{a^2-b^2}$$

```
[Out] 2*a^2*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(3/2)/(a+b)^(3/2)-
a*coth(x)/(a^2-b^2)+b*csch(x)/(a^2-b^2)
```

Rubi [A]
time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2806, 3852, 8, 2686, 2738, 214}

$$-\frac{a\coth(x)}{a^2-b^2}+\frac{b\text{csch}(x)}{a^2-b^2}+\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(a + b*Cosh[x]),x]
```

```
[Out] (2*a^2*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/((a - b)^(3/2)*(a + b)
^(3/2)) - (a*Coth[x])/(a^2 - b^2) + (b*Csch[x])/(a^2 - b^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2806

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[a/(a^2 - b^2), Int[(g*Tan[e + f*x])^p/Sin[e + f*x]^
2, x], x] + (-Dist[b*(g/(a^2 - b^2)), Int[(g*Tan[e + f*x])^(p - 1)/Cos[e +
f*x], x], x] - Dist[a^2*(g^2/(a^2 - b^2)), Int[(g*Tan[e + f*x])^(p - 2)/(a
+ b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0
] && IntegersQ[2*p] && GtQ[p, 1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^2(x)}{a+b\cosh(x)}\,dx &= \frac{a\int \operatorname{csch}^2(x)\,dx}{a^2-b^2}+\frac{a^2\int \frac{1}{a+b\cosh(x)}\,dx}{a^2-b^2}-\frac{b\int \coth(x)\operatorname{csch}(x)\,dx}{a^2-b^2}\\ &= -\frac{(ia)\text{Subst}(\int 1\,dx,x,-i\coth(x))}{a^2-b^2}+\frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2-b^2}+\frac{(i\\ &= \frac{2a^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}-\frac{a\coth(x)}{a^2-b^2}+\frac{b\operatorname{csch}(x)}{a^2-b^2}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 77, normalized size = 1.00

$$\frac{2a^2\text{ArcTan}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\left(-a^2+b^2\right)^{3/2}}-\frac{\coth\left(\frac{x}{2}\right)}{2(a+b)}-\frac{\tanh\left(\frac{x}{2}\right)}{2(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(a + b*Cosh[x]),x]
```

```
[Out] (2*a^2*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) - C
oth[x/2]/(2*(a + b)) - Tanh[x/2]/(2*(a - b))
```

Maple [A]
time = 0.63, size = 78, normalized size = 1.01

method	result	size
default	$-\frac{\tanh\left(\frac{x}{2}\right)}{2(a-b)}-\frac{1}{2(a+b)\tanh\left(\frac{x}{2}\right)}+\frac{2a^{2}\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)\,(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)\,(a-b)}}$	78
risch	$-\frac{2(-\mathrm{e}^{x}b+a)}{(\mathrm{e}^{2x}-1)(a^{2}-b^{2})}+\frac{a^{2}\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^{2}-b^{2}}-a^{2}+b^{2}}{b\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,(a+b)(a-b)}-\frac{a^{2}\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^{2}-b^{2}}+a^{2}-b^{2}}{b\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,(a+b)(a-b)}$	167

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(a-b)*tanh(1/2*x)-1/2/(a+b)/tanh(1/2*x)+2/(a+b)/(a-b)*a^2/((a+b)*(a-b)
)^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(67) = 134$.
time = 0.45, size = 470, normalized size = 6.10

$$\left[\frac{2a^{3}-2ab^{2}+\left(a^{2}\cosh(x)^{2}+2a^{2}\cosh(x)\sinh(x)+a^{2}\sinh(x)^{2}-a^{2}\right)\sqrt{a^{2}-b^{2}}\log\left(\frac{b^{2}\cosh(x)^{2}+b^{2}\sinh(x)^{2}+2ab\cosh(x)+2a^{2}-b^{2}+2\left(b^{2}\cosh(x)+ab\right)\sinh(x)+2\sqrt{a^{2}-b^{2}}\left(b\cosh(x)+b\sinh(x)+a\right)}{b\cosh(x)^{2}+b\sinh(x)^{2}+2a\cosh(x)+2\left(b\cosh(x)+a\right)\sinh(x)+b}\right)-2\left(a^{2}b-b^{3}\right)\cosh(x)-2\left(a^{2}b-b^{3}\right)\sinh(x)}{a^{4}-2a^{2}b^{2}+b^{4}-\left(a^{4}-2a^{2}b^{2}+b^{4}\right)\cosh(x)^{2}-2\left(a^{4}-2a^{2}b^{2}+b^{4}\right)\cosh(x)\sinh(x)-\left(a^{4}-2a^{2}b^{2}+b^{4}\right)\sinh(x)^{2}},\ \frac{2\left(a^{3}-ab^{2}+\left(a^{2}\cosh(x)^{2}+2a^{2}\cosh(x)\sinh(x)+a^{2}\sinh(x)^{2}-a^{2}\right)\sqrt{-a^{2}+b^{2}}\arctan\left(-\frac{\sqrt{-a^{2}+b^{2}}\left(b\cosh(x)+b\sinh(x)+a\right)}{a^{2}-b^{2}}\right)-\left(a^{2}b-b^{3}\right)\cosh(x)-\left(a^{2}b-b^{3}\right)\sinh(x)\right)}{a^{4}-2a^{2}b^{2}+b^{4}-\left(a^{4}-2a^{2}b^{2}+b^{4}\right)\cosh(x)^{2}-2\left(a^{4}-2a^{2}b^{2}+b^{4}\right)\cosh(x)\sinh(x)-\left(a^{4}-2a^{2}b^{2}+b^{4}\right)\sinh(x)^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [(2*a^3 - 2*a*b^2 + (a^2*cosh(x)^2 + 2*a^2*cosh(x)*sinh(x) + a^2*sinh(x)^2
- a^2)*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) +
 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*cosh(x)
 + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x)
+ a)*sinh(x) + b)) - 2*(a^2*b - b^3)*cosh(x) - 2*(a^2*b - b^3)*sinh(x))/(a^
```

```
4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 - 2*(a^4 - 2*a^2*b^
2 + b^4)*cosh(x)*sinh(x) - (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^2), 2*(a^3 - a*b
^2 + (a^2*cosh(x)^2 + 2*a^2*cosh(x)*sinh(x) + a^2*sinh(x)^2 - a^2)*sqrt(-a^
2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2))
- (a^2*b - b^3)*cosh(x) - (a^2*b - b^3)*sinh(x))/(a^4 - 2*a^2*b^2 + b^4 - (
a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 - 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x
) - (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(x)}{a+b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(a+b*cosh(x)),x)
[Out] Integral(coth(x)**2/(a + b*cosh(x)), x)
```

Giac [A]
time = 0.40, size = 76, normalized size = 0.99

$$\frac{2\,a^2\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{(a^2-b^2)\sqrt{-a^2+b^2}}+\frac{2\,(be^x-a)}{(a^2-b^2)(e^{(2\,x)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*cosh(x)),x, algorithm="giac")
[Out] 2*a^2*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/((a^2 - b^2)*sqrt(-a^2 + b^2)) +
 2*(b*e^x - a)/((a^2 - b^2)*(e^(2*x) - 1))
```

Mupad [B]
time = 1.34, size = 337, normalized size = 4.38

$$-\frac{\frac{2\,a}{a^2-b^2}-\frac{2\,b\,e^x}{a^2-b^2}}{e^{2\,x}-1}-\frac{2\,\mathrm{atan}\left(\left(\left(e^x\left(\frac{2\,a^2}{b^2\,(a^2-b^2)^2\,\sqrt{a^4}}+\frac{2\left(a^3\,\sqrt{a^4}-a\,b^2\,\sqrt{a^4}\right)}{a\,b^2\,(a^2-b^2)\,\sqrt{-(a^2-b^2)^3}\,\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}}\right)-\frac{2\left(b^3\,\sqrt{a^4}-a^2\,b\,\sqrt{a^4}\right)}{a\,b^2\,(a^2-b^2)\,\sqrt{-(a^2-b^2)^3}\,\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}}\right)\left(\frac{b^3\,\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}}{2}-\frac{a^2\,b\,\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}}{2}\right)\right)\sqrt{a^4}}{\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a + b*cosh(x)),x)
[Out] - ((2*a)/(a^2 - b^2) - (2*b*exp(x))/(a^2 - b^2))/(exp(2*x) - 1) - (2*atan((
exp(x)*((2*a^2)/(b^2*(a^2 - b^2)^2*(a^4)^(1/2)) + (2*(a^3*(a^4)^(1/2) - a*b
^2*(a^4)^(1/2)))/(a*b^2*(a^2 - b^2)*(-(a^2 - b^2)^3)^(1/2)*(b^6 - a^6 - 3*a
^2*b^4 + 3*a^4*b^2)^(1/2))) - (2*(b^3*(a^4)^(1/2) - a^2*b*(a^4)^(1/2)))/(a*
b^2*(a^2 - b^2)*(-(a^2 - b^2)^3)^(1/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^
(1/2)))*((b^3*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/2 - (a^2*b*(b^6 -
a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/2))*(a^4)^(1/2))/(b^6 - a^6 - 3*a^2*b^4
 + 3*a^4*b^2)^(1/2)
```

3.185 $\int \frac{\coth^3(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=94

$$-\frac{(a-b\cosh(x))\operatorname{csch}^2(x)}{2(a^2-b^2)}+\frac{(2a+b)\log(1-\cosh(x))}{4(a+b)^2}+\frac{(2a-b)\log(1+\cosh(x))}{4(a-b)^2}-\frac{a^3\log(a+b\cosh(x))}{(a^2-b^2)^2}$$

```
[Out] -1/2*(a-b*cosh(x))*csch(x)^2/(a^2-b^2)+1/4*(2*a+b)*ln(1-cosh(x))/(a+b)^2+1/
4*(2*a-b)*ln(1+cosh(x))/(a-b)^2-a^3*ln(a+b*cosh(x))/(a^2-b^2)^2
```

Rubi [A]
time = 0.14, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2800, 1661, 815}

$$-\frac{\operatorname{csch}^2(x)(a-b\cosh(x))}{2(a^2-b^2)}-\frac{a^3\log(a+b\cosh(x))}{(a^2-b^2)^2}+\frac{(2a+b)\log(1-\cosh(x))}{4(a+b)^2}+\frac{(2a-b)\log(\cosh(x)+1)}{4(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/(a + b*Cosh[x]),x]
```

```
[Out] -1/2*((a - b*Cosh[x])*Csch[x]^2)/(a^2 - b^2) + ((2*a + b)*Log[1 - Cosh[x]])
/(4*(a + b)^2) + ((2*a - b)*Log[1 + Cosh[x]])/(4*(a - b)^2) - (a^3*Log[a +
b*Cosh[x]])/(a^2 - b^2)^2
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
```

```
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\coth^3(x)}{a+b\cosh(x)}\,dx = \text{Subst}\left(\int \frac{x^3}{(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\cosh(x)\right)$$
$$= -\frac{(a-b\cosh(x))\text{csch}^2(x)}{2\left(a^2-b^2\right)} + \frac{\text{Subst}\left(\int \frac{\frac{ab^4}{a^2-b^2}-\frac{b^2\left(2a^2-b^2\right)x}{a^2-b^2}}{(a+x)(b^2-x^2)}\,dx, x, b\cosh(x)\right)}{2b^2}$$
$$= -\frac{(a-b\cosh(x))\text{csch}^2(x)}{2\left(a^2-b^2\right)} + \frac{\text{Subst}\left(\int \left(-\frac{b^2(2a+b)}{2(a+b)^2(b-x)} - \frac{2a^3b^2}{(a-b)^2(a+b)^2(a+x)} + \frac{(2a-b)b^2}{2(a-b)^2(b+x)}\right.\right.}{2b^2}$$
$$= -\frac{(a-b\cosh(x))\text{csch}^2(x)}{2\left(a^2-b^2\right)} + \frac{(2a+b)\log(1-\cosh(x))}{4(a+b)^2} + \frac{(2a-b)\log(1+\cosh(x))}{4(a-b)^2} -$$

Mathematica [A]
time = 0.16, size = 101, normalized size = 1.07

$$\frac{-(a-b)^2(a+b)\text{csch}^2\left(\frac{x}{2}\right) - 8a^3\log(a+b\cosh(x)) + 8a^3\log(\sinh(x)) - 12a^2b\log\left(\tanh\left(\frac{x}{2}\right)\right) + 4b^3\log\left(\tanh\left(\frac{x}{2}\right)\right) + (a-b)(a+b)^2\text{sech}^2\left(\frac{x}{2}\right)}{8(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/(a + b*Cosh[x]),x]
```

```
[Out] (-((a - b)^2*(a + b)*Csch[x/2]^2) - 8*a^3*Log[a + b*Cosh[x]] + 8*a^3*Log[Si
nh[x]] - 12*a^2*b*Log[Tanh[x/2]] + 4*b^3*Log[Tanh[x/2]] + (a - b)*(a + b)^2
*Sech[x/2]^2)/(8*(a - b)^2*(a + b)^2)
```

Maple [A]
time = 0.68, size = 91, normalized size = 0.97

method	result
default	$-\frac{\tanh^2\left(\frac{x}{2}\right)}{8(a-b)} - \frac{a^3\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-b\left(\tanh^2\left(\frac{x}{2}\right)\right)-a-b\right)}{(a+b)^2(a-b)^2} - \frac{1}{8(a+b)\tanh\left(\frac{x}{2}\right)^2} + \frac{(4a+2b)\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{4(a+b)^2}$
risch	$-\frac{xa}{a^2+2ab+b^2} - \frac{xb}{2(a^2+2ab+b^2)} + \frac{bx}{2a^2-4ab+2b^2} - \frac{xa}{a^2-2ab+b^2} + \frac{2x\,a^3}{a^4-2a^2b^2+b^4} - \frac{e^x\left(-b\,e^{2x}+2a\,e^x-b\right)}{\left(e^{2x}-1\right)^2(a^2-b^2)} + \frac{\ln(e^x-1)a}{a^2+2ab+b^2} +$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*tanh(1/2*x)^2/(a-b)-a^3/(a+b)^2/(a-b)^2*ln(a*tanh(1/2*x)^2-b*tanh(1/2*
x)^2-a-b)-1/8/(a+b)/tanh(1/2*x)^2+1/4/(a+b)^2*(4*a+2*b)*ln(tanh(1/2*x))
```

Maxima [A]
time = 0.28, size = 156, normalized size = 1.66

$$-\frac{a^{3} \log \left(2 \, a e^{\left(-x\right)} + b e^{\left(-2 \, x\right)} + b\right)}{a^{4} - 2 \, a^{2} b^{2} + b^{4}} + \frac{{\left(2 \, a - b\right)} \log \left(e^{\left(-x\right)} + 1\right)}{2 \, {\left(a^{2} - 2 \, a b + b^{2}\right)}} + \frac{{\left(2 \, a + b\right)} \log \left(e^{\left(-x\right)} - 1\right)}{2 \, {\left(a^{2} + 2 \, a b + b^{2}\right)}} + \frac{b e^{\left(-x\right)} - 2 \, a e^{\left(-2 \, x\right)} + b e^{\left(-3 \, x\right)}}{a^{2} - b^{2} - 2 \, {\left(a^{2} - b^{2}\right)} e^{\left(-2 \, x\right)} + {\left(a^{2} - b^{2}\right)} e^{\left(-4 \, x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] -a^3*log(2*a*e^(-x) + b*e^(-2*x) + b)/(a^4 - 2*a^2*b^2 + b^4) + 1/2*(2*a -
b)*log(e^(-x) + 1)/(a^2 - 2*a*b + b^2) + 1/2*(2*a + b)*log(e^(-x) - 1)/(a^2
 + 2*a*b + b^2) + (b*e^(-x) - 2*a*e^(-2*x) + b*e^(-3*x))/(a^2 - b^2 - 2*(a^
2 - b^2)*e^(-2*x) + (a^2 - b^2)*e^(-4*x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 839 vs. 2(89) = 178.
time = 0.46, size = 839, normalized size = 8.93

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a^2*b - b^3)*cosh(x)^3 + 2*(a^2*b - b^3)*sinh(x)^3 - 4*(a^3 - a*b^2
)*cosh(x)^2 - 2*(2*a^3 - 2*a*b^2 - 3*(a^2*b - b^3)*cosh(x))*sinh(x)^2 + 2*(
a^2*b - b^3)*cosh(x) - 2*(a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*sin
h(x)^4 - 2*a^3*cosh(x)^2 + a^3 + 2*(3*a^3*cosh(x)^2 - a^3)*sinh(x)^2 + 4*(a
^3*cosh(x)^3 - a^3*cosh(x))*sinh(x))*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(
x))) + ((2*a^3 + 3*a^2*b - b^3)*cosh(x)^4 + 4*(2*a^3 + 3*a^2*b - b^3)*cosh(
x)*sinh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*sinh(x)^4 + 2*a^3 + 3*a^2*b - b^3 -
2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 - 2*(2*a^3 + 3*a^2*b - b^3 - 3*(2*a^3 +
 3*a^2*b - b^3)*cosh(x)^2)*sinh(x)^2 + 4*((2*a^3 + 3*a^2*b - b^3)*cosh(x)^3
 - (2*a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) + (
(2*a^3 - 3*a^2*b + b^3)*cosh(x)^4 + 4*(2*a^3 - 3*a^2*b + b^3)*cosh(x)*sinh(
x)^3 + (2*a^3 - 3*a^2*b + b^3)*sinh(x)^4 + 2*a^3 - 3*a^2*b + b^3 - 2*(2*a^3
 - 3*a^2*b + b^3)*cosh(x)^2 - 2*(2*a^3 - 3*a^2*b + b^3 - 3*(2*a^3 - 3*a^2*b
 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((2*a^3 - 3*a^2*b + b^3)*cosh(x)^3 - (2*a^
3 - 3*a^2*b + b^3)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) + 2*(a^2*b
- b^3 + 3*(a^2*b - b^3)*cosh(x)^2 - 4*(a^3 - a*b^2)*cosh(x))*sinh(x))/((a^4
 - 2*a^2*b^2 + b^4)*cosh(x)^4 + 4*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^3
 + (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a
^2*b^2 + b^4)*cosh(x)^2 - 2*(a^4 - 2*a^2*b^2 + b^4 - 3*(a^4 - 2*a^2*b^2 + b
^4)*cosh(x)^2)*sinh(x)^2 + 4*((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^3 - (a^4 - 2*
a^2*b^2 + b^4)*cosh(x))*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(x)}{a + b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(a+b*cosh(x)),x)
```

```
[Out] Integral(coth(x)**3/(a + b*cosh(x)), x)
```

Giac [A]
time = 0.40, size = 178, normalized size = 1.89

$$-\frac{a^3 b \log\left(\left|b\left(e^{(-x)} + e^x\right) + 2\,a\right|\right)}{a^4 b - 2\,a^2 b^3 + b^5} + \frac{(2\,a - b)\log\left(e^{(-x)} + e^x + 2\right)}{4\,(a^2 - 2\,ab + b^2)} + \frac{(2\,a + b)\log\left(e^{(-x)} + e^x - 2\right)}{4\,(a^2 + 2\,ab + b^2)} - \frac{a^3\left(e^{(-x)} + e^x\right)^2 - 2\,a^2 b\left(e^{(-x)} + e^x\right) + 2\,b^3\left(e^{(-x)} + e^x\right) - 4\,ab^2}{2\,(a^4 - 2\,a^2 b^2 + b^4)\left(\left(e^{(-x)} + e^x\right)^2 - 4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] -a^3*b*log(abs(b*(e^(-x) + e^x) + 2*a))/(a^4*b - 2*a^2*b^3 + b^5) + 1/4*(2*
a - b)*log(e^(-x) + e^x + 2)/(a^2 - 2*a*b + b^2) + 1/4*(2*a + b)*log(e^(-x)
 + e^x - 2)/(a^2 + 2*a*b + b^2) - 1/2*(a^3*(e^(-x) + e^x)^2 - 2*a^2*b*(e^(-
x) + e^x) + 2*b^3*(e^(-x) + e^x) - 4*a*b^2)/((a^4 - 2*a^2*b^2 + b^4)*((e^(-
x) + e^x)^2 - 4))
```

Mupad [B]
time = 1.52, size = 291, normalized size = 3.10

$$\frac{\frac{2\,(a\,b^2-a^3)}{(a^2-b^2)^2} + \frac{e^x\,(a^2\,b-b^3)}{(a^2-b^2)^2}}{e^{2\,x}-1} - \frac{\frac{2\,a}{a^2-b^2} - \frac{2\,b\,e^x}{a^2-b^2}}{e^{4\,x}-2\,e^{2\,x}+1} + \frac{\ln\left(e^x+1\right)\,(2\,a-b)}{2\,a^2-4\,a\,b+2\,b^2} - \frac{a^3\,\ln\left(b^7\,e^{2\,x} - 16\,a^6\,b + b^7 - 6\,a^2\,b^5 + 9\,a^4\,b^3 - 32\,a^7\,e^x - 6\,a^2\,b^5\,e^{2\,x} + 9\,a^4\,b^3\,e^{2\,x} + 2\,a\,b^6\,e^x - 16\,a^6\,b\,e^{2\,x} - 12\,a^3\,b^4\,e^x + 18\,a^5\,b^2\,e^x\right)}{a^4-2\,a^2\,b^2+b^4} + \frac{\ln\left(e^x-1\right)\,(2\,a+b)}{2\,a^2+4\,a\,b+2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a + b*cosh(x)),x)
```

```
[Out] ((2*(a*b^2 - a^3))/(a^2 - b^2)^2 + (exp(x)*(a^2*b - b^3))/(a^2 - b^2)^2)/(e
xp(2*x) - 1) - ((2*a)/(a^2 - b^2) - (2*b*exp(x))/(a^2 - b^2))/(exp(4*x) - 2
*exp(2*x) + 1) + (log(exp(x) + 1)*(2*a - b))/(2*a^2 - 4*a*b + 2*b^2) - (a^3
*log(b^7*exp(2*x) - 16*a^6*b + b^7 - 6*a^2*b^5 + 9*a^4*b^3 - 32*a^7*exp(x)
- 6*a^2*b^5*exp(2*x) + 9*a^4*b^3*exp(2*x) + 2*a*b^6*exp(x) - 16*a^6*b*exp(2
*x) - 12*a^3*b^4*exp(x) + 18*a^5*b^2*exp(x)))/(a^4 + b^4 - 2*a^2*b^2) + (lo
g(exp(x) - 1)*(2*a + b))/(4*a*b + 2*a^2 + 2*b^2)
```

3.186 $\int \frac{\coth^4(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=137

$$\frac{2a^4\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}}-\frac{a^3\coth(x)}{(a^2-b^2)^2}-\frac{a\coth^3(x)}{3(a^2-b^2)}+\frac{a^2b\text{csch}(x)}{(a^2-b^2)^2}+\frac{b\text{csch}(x)}{a^2-b^2}+\frac{b\text{csch}^3(x)}{3(a^2-b^2)}$$

```
[Out] 2*a^4*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(5/2)/(a+b)^(5/2)-
a^3*coth(x)/(a^2-b^2)^2-1/3*a*coth(x)^3/(a^2-b^2)+a^2*b*csch(x)/(a^2-b^2)^2
+b*csch(x)/(a^2-b^2)+1/3*b*csch(x)^3/(a^2-b^2)
```

Rubi [A]
time = 0.14, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {2806, 2687, 30, 2686, 3852, 8, 2738, 214}

$$\frac{2a^4\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}}-\frac{a\coth^3(x)}{3(a^2-b^2)}+\frac{b\text{csch}^3(x)}{3(a^2-b^2)}+\frac{a^2b\text{csch}(x)}{(a^2-b^2)^2}+\frac{b\text{csch}(x)}{a^2-b^2}-\frac{a^3\coth(x)}{(a^2-b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^4/(a + b*Cosh[x]),x]
```

```
[Out] (2*a^4*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/((a - b)^(5/2)*(a + b)
^(5/2)) - (a^3*Coth[x])/(a^2 - b^2)^2 - (a*Coth[x]^3)/(3*(a^2 - b^2)) + (a^
2*b*Csch[x])/(a^2 - b^2)^2 + (b*Csch[x])/(a^2 - b^2) + (b*Csch[x]^3)/(3*(a^
2 - b^2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
```

```
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2806

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[a/(a^2 - b^2), Int[(g*Tan[e + f*x])^p/Sin[e + f*x]^
2, x], x] + (-Dist[b*(g/(a^2 - b^2)), Int[(g*Tan[e + f*x])^(p - 1)/Cos[e +
f*x], x], x] - Dist[a^2*(g^2/(a^2 - b^2)), Int[(g*Tan[e + f*x])^(p - 2)/(a
+ b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0
] && IntegersQ[2*p] && GtQ[p, 1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\coth^4(x)}{a+b\cosh(x)}\,dx = \frac{a\int \coth^2(x)\mathrm{csch}^2(x)\,dx}{a^2-b^2} + \frac{a^2\int \frac{\coth^2(x)}{a+b\cosh(x)}\,dx}{a^2-b^2} - \frac{b\int \coth^3(x)\mathrm{csch}(x)\,dx}{a^2-b^2}$$

$$= \frac{a^3\int \mathrm{csch}^2(x)\,dx}{(a^2-b^2)^2} + \frac{a^4\int \frac{1}{a+b\cosh(x)}\,dx}{(a^2-b^2)^2} - \frac{(a^2b)\int \coth(x)\mathrm{csch}(x)\,dx}{(a^2-b^2)^2} - \frac{(ia)\mathrm{Subst}\left(\int x^2\,dx \right.}{a^2-}$$

$$= -\frac{a\coth^3(x)}{3(a^2-b^2)} + \frac{b\mathrm{csch}(x)}{a^2-b^2} + \frac{b\mathrm{csch}^3(x)}{3(a^2-b^2)} - \frac{(ia^3)\,\mathrm{Subst}(\int 1\,dx, x, -i\coth(x))}{(a^2-b^2)^2} + \frac{(2a^4)\,\mathrm{Su}}{}$$

$$= \frac{2a^4\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}} - \frac{a^3\coth(x)}{(a^2-b^2)^2} - \frac{a\coth^3(x)}{3(a^2-b^2)} + \frac{a^2b\mathrm{csch}(x)}{(a^2-b^2)^2} + \frac{b\mathrm{csch}(x)}{a^2-b^2} +$$

Mathematica [A]
time = 0.37, size = 131, normalized size = 0.96

$$\frac{1}{24}\left(-\frac{48a^4\mathrm{ArcTan}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{5/2}} - \frac{2(8a+5b)\coth\left(\frac{x}{2}\right)}{(a+b)^2} + \frac{8\mathrm{csch}^3(x)\sinh^4\left(\frac{x}{2}\right)}{a-b} - \frac{\mathrm{csch}^4\left(\frac{x}{2}\right)\sinh(x)}{2(a+b)} + \frac{2(-8a+5b)\tanh\left(\frac{x}{2}\right)}{(a-b)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4/(a + b*Cosh[x]),x]
```

```
[Out] ((-48*a^4*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5/2)
- (2*(8*a + 5*b)*Coth[x/2])/(a + b)^2 + (8*Csch[x]^3*Sinh[x/2]^4)/(a - b) -
 (Csch[x/2]^4*Sinh[x])/(2*(a + b)) + (2*(-8*a + 5*b)*Tanh[x/2])/(a - b)^2)/
24
```

Maple [A]
time = 0.69, size = 127, normalized size = 0.93

method	result
default	$-\frac{\frac{a\left(\tanh^3\left(\frac{x}{2}\right)\right)}{3}-\frac{b\left(\tanh^3\left(\frac{x}{2}\right)\right)}{3}+5a\tanh\left(\frac{x}{2}\right)-3b\tanh\left(\frac{x}{2}\right)}{8(a-b)^2} - \frac{1}{24(a+b)\tanh\left(\frac{x}{2}\right)^3} - \frac{5a+3b}{8(a+b)^2\tanh\left(\frac{x}{2}\right)} + \frac{2a^4\,\mathrm{arctanh}\left(\frac{(a-b)\,\mathrm{ta}}{\sqrt{(a+b}}\right.}{(a-b)^2(a+b)^2\sqrt{(a+}}$
risch	$-\frac{2(-6a^2b\,e^{5x}+3b^3e^{5x}+6a^3e^{4x}-3a\,b^2e^{4x}+8a^2b\,e^{3x}-2b^3e^{3x}-6a^3e^{2x}-6a^2b\,e^x+3b^3e^x+4a^3-a\,b^2)}{3(a^2-b^2)^2(e^{2x}-1)^3} + \frac{a^4\ln\left(e^x+\frac{a\sqrt{a^2-b^2}-a^2}{b\sqrt{a^2-b^2}}\right.}{\sqrt{a^2-b^2}\,(a+b)^2(a-b)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/(a-b)^2*(1/3*a*tanh(1/2*x)^3-1/3*b*tanh(1/2*x)^3+5*a*tanh(1/2*x)-3*b*t
anh(1/2*x))-1/24/(a+b)/tanh(1/2*x)^3-1/8*(5*a+3*b)/(a+b)^2/tanh(1/2*x)+2/(a
-b)^2/(a+b)^2*a^4/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b
))^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1174 vs. 2(123) = 246.
time = 0.45, size = 2417, normalized size = 17.64

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [1/3*(6*(2*a^4*b - 3*a^2*b^3 + b^5)*cosh(x)^5 + 6*(2*a^4*b - 3*a^2*b^3 + b^
5)*sinh(x)^5 - 8*a^5 + 10*a^3*b^2 - 2*a*b^4 - 6*(2*a^5 - 3*a^3*b^2 + a*b^4)
*cosh(x)^4 - 6*(2*a^5 - 3*a^3*b^2 + a*b^4 - 5*(2*a^4*b - 3*a^2*b^3 + b^5)*c
osh(x))*sinh(x)^4 - 4*(4*a^4*b - 5*a^2*b^3 + b^5)*cosh(x)^3 - 4*(4*a^4*b -
5*a^2*b^3 + b^5 - 15*(2*a^4*b - 3*a^2*b^3 + b^5)*cosh(x)^2 + 6*(2*a^5 - 3*a
^3*b^2 + a*b^4)*cosh(x))*sinh(x)^3 + 12*(a^5 - a^3*b^2)*cosh(x)^2 + 12*(a^5
 - a^3*b^2 + 5*(2*a^4*b - 3*a^2*b^3 + b^5)*cosh(x)^3 - 3*(2*a^5 - 3*a^3*b^2
 + a*b^4)*cosh(x)^2 - (4*a^4*b - 5*a^2*b^3 + b^5)*cosh(x))*sinh(x)^2 + 3*(a
^4*cosh(x)^6 + 6*a^4*cosh(x)*sinh(x)^5 + a^4*sinh(x)^6 - 3*a^4*cosh(x)^4 +
3*a^4*cosh(x)^2 + 3*(5*a^4*cosh(x)^2 - a^4)*sinh(x)^4 - a^4 + 4*(5*a^4*cosh
(x)^3 - 3*a^4*cosh(x))*sinh(x)^3 + 3*(5*a^4*cosh(x)^4 - 6*a^4*cosh(x)^2 + a
^4)*sinh(x)^2 + 6*(a^4*cosh(x)^5 - 2*a^4*cosh(x)^3 + a^4*cosh(x))*sinh(x))*
sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2
- b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*si
nh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*si
nh(x) + b)) + 6*(2*a^4*b - 3*a^2*b^3 + b^5)*cosh(x) + 6*(2*a^4*b - 3*a^2*b^
3 + b^5 + 5*(2*a^4*b - 3*a^2*b^3 + b^5)*cosh(x)^4 - 4*(2*a^5 - 3*a^3*b^2 +
a*b^4)*cosh(x)^3 - 2*(4*a^4*b - 5*a^2*b^3 + b^5)*cosh(x)^2 + 4*(a^5 - a^3*b
^2)*cosh(x))*sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^6 + 6*(a
```

```
^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^5 + (a^6 - 3*a^4*b^2 + 3*
a^2*b^4 - b^6)*sinh(x)^6 - a^6 + 3*a^4*b^2 - 3*a^2*b^4 + b^6 - 3*(a^6 - 3*a
^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6
- 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^6 -
3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b
^6)*cosh(x))*sinh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2 +
3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6
)*cosh(x)^4 - 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^2 +
6*((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^5 - 2*(a^6 - 3*a^4*b^2 + 3*a
^2*b^4 - b^6)*cosh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh
(x)), 2/3*(3*(2*a^4*b - 3*a^2*b^3 + b^5)*cosh(x)^5 + 3*(2*a^4*b - 3*a^2*b^3
 + b^5)*sinh(x)^5 - 4*a^5 + 5*a^3*b^2 - a*b^4 - 3*(2*a^5 - 3*a^3*b^2 + a*b^
4)*cosh(x)^4 - 3*(2*a^5 - 3*a^3*b^2 + a*b^4 - 5*(2*a^4*b - 3*a^2*b^3 + b^5)
*cosh(x))*sinh(x)^4 - 2*(4*a^4*b - 5*a^2*b^3 + b^5)*cosh(x)^3 - 2*(4*a^4*b
- 5*a^2*b^3 + b^5 - 15*(2*a^4*b - 3*a^2*b^3 + b^5)*cosh(x)^2 + 6*(2*a^5 - 3
*a^3*b^2 + a*b^4)*cosh(x))*sinh(x)^3 + 6*(a^5 - a^3*b^2)*cosh(x)^2 + 6*(a^5
 - a^3*b^2 + 5*(2*a^4*b - 3*a^2*b^3 + b^5)*cosh(x)^3 - 3*(2*a^5 - 3*a^3*b^2
 + a*b^4)*cosh(x)^2 - (4*a^4*b - 5*a^2*b^3 + b^5)*cosh(x))*sinh(x)^2 - 3*(a
^4*cosh(x)^6 + 6*a^4*cosh(x)*sinh(x)^5 + a^4*sinh(x)^6 - 3*a^4*cosh(x)^4 +
3*a^4*cosh(x)^2 + 3*(5*a^4*cosh(x)^2 - a^4)*sinh(x)^4 - a^4 + 4*(5*a^4*cosh
(x)^3 - 3*a^4*cosh(x))*sinh(x)^3 + 3*(5*a^4*cosh(x)^4 - 6*a^4*cosh(x)^2 + a
^4)*sinh(x)^2 + 6*(a^4*cosh(x)^5 - 2*a^4*cosh(x)^3 + a^4*cosh(x))*sinh(x))*
sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2
- b^2)) + 3*(2*a^4*b - 3*a^2*b^3 + b^5)*cosh(x) + 3*(2*a^4*b - 3*a^2*b^3 +
b^5 + 5*(2*a^4*b - 3*a^2*b^3 + b^5)*cosh(x)^4 - 4*(2*a^5 - 3*a^3*b^2 + a*b^
4)*cosh(x)^3 - 2*(4*a^4*b - 5*a^2*b^3 + b^5)*cosh(x)^2 + 4*(a^5 - a^3*b^2)*
cosh(x))*sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^6 + 6*(a^6 -
 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^5 + (a^6 - 3*a^4*b^2 + 3*a^2*
b^4 - b^6)*sinh(x)^6 - a^6 + 3*a^4*b^2 - 3*a^2*b^4 + b^6 - 3*(a^6 - 3*a^4*b
^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 - 5*
(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^6 - 3*a^
4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*
cosh(x))*sinh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2 + 3*(a
^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*co
sh(x)^4 - 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^2 + 6*((
a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^5 - 2*(a^6 - 3*a^4*b^2 + 3*a^2*b
^4 - b^6)*cosh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x))
]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^4(x)}{a + b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4/(a+b*cosh(x)),x)
```

```
[Out] Integral(coth(x)**4/(a + b*cosh(x)), x)
```

Giac [A]
time = 0.42, size = 172, normalized size = 1.26

$$\frac{2\,a^4\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{(a^4-2\,a^2b^2+b^4)\sqrt{-a^2+b^2}}+\frac{2\left(6\,a^2be^{(5\,x)}-3\,b^3e^{(5\,x)}-6\,a^3e^{(4\,x)}+3\,ab^2e^{(4\,x)}-8\,a^2be^{(3\,x)}+2\,b^3e^{(3\,x)}+6\,a^3e^{(2\,x)}+6\,a^2be^x-3\,b^3e^x-4\,a^3+ab^2\right)}{3\,(a^4-2\,a^2b^2+b^4)(e^{(2\,x)}-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*a^4*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(-a
^2 + b^2)) + 2/3*(6*a^2*b*e^(5*x) - 3*b^3*e^(5*x) - 6*a^3*e^(4*x) + 3*a*b^2
*e^(4*x) - 8*a^2*b*e^(3*x) + 2*b^3*e^(3*x) + 6*a^3*e^(2*x) + 6*a^2*b*e^x -
3*b^3*e^x - 4*a^3 + a*b^2)/((a^4 - 2*a^2*b^2 + b^4)*(e^(2*x) - 1)^3)
```

Mupad [B]
time = 1.81, size = 666, normalized size = 4.86

$$\frac{\frac{4\,(a\,b^2-a^3)}{(a^2-b^2)^2}+\frac{8\,e^x\,(a^2\,b-b^3)}{3\,(a^2-b^2)^2}}{e^{4\,x}-2\,e^{2\,x}+1}-\frac{\frac{8\,a}{3\,(a^2-b^2)}-\frac{8\,b\,e^x}{3\,(a^2-b^2)}}{3\,e^{2\,x}-3\,e^{4\,x}+e^{6\,x}-1}-\frac{\frac{2\,a\,(2\,a^2-b^2)}{(a^2-b^2)^2}-\frac{2\,b\,e^x\,(2\,a^2-b^2)}{(a^2-b^2)^2}}{e^{2\,x}-1}+\frac{2\,\mathrm{atan}\left(\left(\left(e^x\left(\frac{2\,a^4}{b^2\,(a^2-b^2)^2\,\sqrt{a^8}\,(a^4-2\,a^2\,b^2+b^4)}+\frac{2\left(a^5\,\sqrt{a^8}-2\,a^3\,b^2\,\sqrt{a^8}+a\,b^4\,\sqrt{a^8}\right)}{a^3\,b^2\,\sqrt{-(a^2-b^2)^5}\,(a^4-2\,a^2\,b^2+b^4)\,\sqrt{-a^{10}+5\,a^8\,b^2-10\,a^6\,b^4+10\,a^4\,b^6-5\,a^2\,b^8+b^{10}}}\right)+\frac{2\left(b^5\,\sqrt{a^8}-2\,a^2\,b^3\,\sqrt{a^8}+a^4\,b\,\sqrt{a^8}\right)}{a^3\,b^2\,\sqrt{-(a^2-b^2)^5}\,(a^4-2\,a^2\,b^2+b^4)\,\sqrt{-a^{10}+5\,a^8\,b^2-10\,a^6\,b^4+10\,a^4\,b^6-5\,a^2\,b^8+b^{10}}}\right)\left(\frac{b^5\,\sqrt{-a^{10}+5\,a^8\,b^2-10\,a^6\,b^4+10\,a^4\,b^6-5\,a^2\,b^8+b^{10}}}{2}-a^2\,b^3\,\sqrt{-a^{10}+5\,a^8\,b^2-10\,a^6\,b^4+10\,a^4\,b^6-5\,a^2\,b^8+b^{10}}+\frac{a^4\,b\,\sqrt{-a^{10}+5\,a^8\,b^2-10\,a^6\,b^4+10\,a^4\,b^6-5\,a^2\,b^8+b^{10}}}{2}\right)\right)\sqrt{a^8}\right)}{\sqrt{-a^{10}+5\,a^8\,b^2-10\,a^6\,b^4+10\,a^4\,b^6-5\,a^2\,b^8+b^{10}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(a + b*cosh(x)),x)
```

```
[Out] ((4*(a*b^2 - a^3))/(a^2 - b^2)^2 + (8*exp(x)*(a^2*b - b^3))/(3*(a^2 - b^2)^
2))/(exp(4*x) - 2*exp(2*x) + 1) - ((8*a)/(3*(a^2 - b^2)) - (8*b*exp(x))/(3*
(a^2 - b^2)))/(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1) - ((2*a*(2*a^2 - b^2
))/(a^2 - b^2)^2 - (2*b*exp(x)*(2*a^2 - b^2))/(a^2 - b^2)^2)/(exp(2*x) - 1)
 + (2*atan((exp(x)*((2*a^4)/(b^2*(a^2 - b^2)^2*(a^8)^(1/2)*(a^4 + b^4 - 2*a
^2*b^2)) + (2*(a^5*(a^8)^(1/2) - 2*a^3*b^2*(a^8)^(1/2) + a*b^4*(a^8)^(1/2))
)/(a^3*b^2*(-(a^2 - b^2)^5)^(1/2)*(a^4 + b^4 - 2*a^2*b^2)*(b^10 - a^10 - 5*
a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2))) + (2*(b^5*(a^8)^(1/2
) - 2*a^2*b^3*(a^8)^(1/2) + a^4*b*(a^8)^(1/2)))/(a^3*b^2*(-(a^2 - b^2)^5)^(
1/2)*(a^4 + b^4 - 2*a^2*b^2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6
*b^4 + 5*a^8*b^2)^(1/2)))*((b^5*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*
a^6*b^4 + 5*a^8*b^2)^(1/2))/2 - a^2*b^3*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b
^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) + (a^4*b*(b^10 - a^10 - 5*a^2*b^8 + 10*a
^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2))/2))*(a^8)^(1/2))/(b^10 - a^10 - 5*a
^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)
```

3.187 $\int \frac{\tanh^6(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=46

$$\frac{3\text{ArcTan}(\sinh(x))}{8a}-\frac{3\text{sech}(x)\tanh(x)}{8a}-\frac{\text{sech}(x)\tanh^3(x)}{4a}-\frac{\tanh^5(x)}{5a}$$

```
[Out] 3/8*arctan(sinh(x))/a-3/8*sech(x)*tanh(x)/a-1/4*sech(x)*tanh(x)^3/a-1/5*tan
h(x)^5/a
```

Rubi [A]
time = 0.07, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2785, 2687, 30, 2691, 3855}

$$\frac{3\text{ArcTan}(\sinh(x))}{8a}-\frac{\tanh^5(x)}{5a}-\frac{\tanh^3(x)\text{sech}(x)}{4a}-\frac{3\tanh(x)\text{sech}(x)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^6/(a + a*Cosh[x]),x]
```

```
[Out] (3*ArcTan[Sinh[x]])/(8*a) - (3*Sech[x]*Tanh[x])/(8*a) - (Sech[x]*Tanh[x]^3)
/(4*a) - Tanh[x]^5/(5*a)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^6(x)}{a+a\cosh(x)}\,dx &= \frac{\int \operatorname{sech}(x)\tanh^4(x)\,dx}{a}-\frac{\int \operatorname{sech}^2(x)\tanh^4(x)\,dx}{a}\\ &= -\frac{\operatorname{sech}(x)\tanh^3(x)}{4a}+\frac{i\text{Subst}\left(\int x^4\,dx,x,i\tanh(x)\right)}{a}+\frac{3\int \operatorname{sech}(x)\tanh^2(x)\,dx}{4a}\\ &= -\frac{3\operatorname{sech}(x)\tanh(x)}{8a}-\frac{\operatorname{sech}(x)\tanh^3(x)}{4a}-\frac{\tanh^5(x)}{5a}+\frac{3\int \operatorname{sech}(x)\,dx}{8a}\\ &= \frac{3\tan^{-1}(\sinh(x))}{8a}-\frac{3\operatorname{sech}(x)\tanh(x)}{8a}-\frac{\operatorname{sech}(x)\tanh^3(x)}{4a}-\frac{\tanh^5(x)}{5a}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 58, normalized size = 1.26

$$\frac{\cosh^2\left(\frac{x}{2}\right)\left(30\text{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)+\left(-8-25\text{sech}(x)+16\text{sech}^2(x)+10\text{sech}^3(x)-8\text{sech}^4(x)\right)\tanh(x)\right)}{20a(1+\cosh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^6/(a + a*Cosh[x]),x]
[Out] (Cosh[x/2]^2*(30*ArcTan[Tanh[x/2]] + (-8 - 25*Sech[x] + 16*Sech[x]^2 + 10*S
ech[x]^3 - 8*Sech[x]^4)*Tanh[x]))/(20*a*(1 + Cosh[x]))
```

Maple [A]
time = 0.60, size = 64, normalized size = 1.39

method	result	size
default	$\frac{\frac{64\left(\frac{3\left(\tanh^9\left(\frac{x}{2}\right)\right)}{256}+\frac{7\left(\tanh^7\left(\frac{x}{2}\right)\right)}{128}-\frac{\left(\tanh^5\left(\frac{x}{2}\right)\right)}{10}-\frac{7\left(\tanh^3\left(\frac{x}{2}\right)\right)}{128}-\frac{3\tanh\left(\frac{x}{2}\right)}{256}\right)}{\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^5}+\frac{3\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{4}}{a}$	64

risch	$-\frac{25\,\mathrm{e}^{9x}-40\,\mathrm{e}^{8x}+10\,\mathrm{e}^{7x}-80\,\mathrm{e}^{4x}-10\,\mathrm{e}^{3x}-25\,\mathrm{e}^{x}-8}{20(1+\mathrm{e}^{2x})^{5}a}+\frac{3i\ln(\mathrm{e}^{x}+i)}{8a}-\frac{3i\ln(\mathrm{e}^{x}-i)}{8a}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^6/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 64/a*((3/256*tanh(1/2*x)^9+7/128*tanh(1/2*x)^7-1/10*tanh(1/2*x)^5-7/128*tan
h(1/2*x)^3-3/256*tanh(1/2*x))/(tanh(1/2*x)^2+1)^5+3/256*arctan(tanh(1/2*x))
)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(38) = 76$.
time = 0.49, size = 89, normalized size = 1.93

$$-\frac{25\,e^{(-x)}+10\,e^{(-3\,x)}+80\,e^{(-4\,x)}-10\,e^{(-7\,x)}+40\,e^{(-8\,x)}-25\,e^{(-9\,x)}+8}{20\left(5\,ae^{(-2\,x)}+10\,ae^{(-4\,x)}+10\,ae^{(-6\,x)}+5\,ae^{(-8\,x)}+ae^{(-10\,x)}+a\right)}-\frac{3\,\arctan\left(e^{(-x)}\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^6/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -1/20*(25*e^(-x) + 10*e^(-3*x) + 80*e^(-4*x) - 10*e^(-7*x) + 40*e^(-8*x) -
25*e^(-9*x) + 8)/(5*a*e^(-2*x) + 10*a*e^(-4*x) + 10*a*e^(-6*x) + 5*a*e^(-8*
x) + a*e^(-10*x) + a) - 3/4*arctan(e^(-x))/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 750 vs. $2(38) = 76$.
time = 0.52, size = 750, normalized size = 16.30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^6/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] -1/20*(25*cosh(x)^9 + 5*(45*cosh(x) - 8)*sinh(x)^8 + 25*sinh(x)^9 - 40*cosh
(x)^8 + 10*(90*cosh(x)^2 - 32*cosh(x) + 1)*sinh(x)^7 + 10*cosh(x)^7 + 70*(3
0*cosh(x)^3 - 16*cosh(x)^2 + cosh(x))*sinh(x)^6 + 70*(45*cosh(x)^4 - 32*cos
h(x)^3 + 3*cosh(x)^2)*sinh(x)^5 + 10*(315*cosh(x)^5 - 280*cosh(x)^4 + 35*co
sh(x)^3 - 8)*sinh(x)^4 - 80*cosh(x)^4 + 10*(210*cosh(x)^6 - 224*cosh(x)^5 +
 35*cosh(x)^4 - 32*cosh(x) - 1)*sinh(x)^3 - 10*cosh(x)^3 + 10*(90*cosh(x)^7
 - 112*cosh(x)^6 + 21*cosh(x)^5 - 48*cosh(x)^2 - 3*cosh(x))*sinh(x)^2 - 15*
(cosh(x)^10 + 10*cosh(x)*sinh(x)^9 + sinh(x)^10 + 5*(9*cosh(x)^2 + 1)*sinh(
x)^8 + 5*cosh(x)^8 + 40*(3*cosh(x)^3 + cosh(x))*sinh(x)^7 + 10*(21*cosh(x)^
4 + 14*cosh(x)^2 + 1)*sinh(x)^6 + 10*cosh(x)^6 + 4*(63*cosh(x)^5 + 70*cosh(
x)^3 + 15*cosh(x))*sinh(x)^5 + 10*(21*cosh(x)^6 + 35*cosh(x)^4 + 15*cosh(x)
^2 + 1)*sinh(x)^4 + 10*cosh(x)^4 + 40*(3*cosh(x)^7 + 7*cosh(x)^5 + 5*cosh(x
```

```
)^3 + cosh(x))*sinh(x)^3 + 5*(9*cosh(x)^8 + 28*cosh(x)^6 + 30*cosh(x)^4 + 1
2*cosh(x)^2 + 1)*sinh(x)^2 + 5*cosh(x)^2 + 10*(cosh(x)^9 + 4*cosh(x)^7 + 6*
cosh(x)^5 + 4*cosh(x)^3 + cosh(x))*sinh(x) + 1)*arctan(cosh(x) + sinh(x)) +
 5*(45*cosh(x)^8 - 64*cosh(x)^7 + 14*cosh(x)^6 - 64*cosh(x)^3 - 6*cosh(x)^2
 - 5)*sinh(x) - 25*cosh(x) - 8)/(a*cosh(x)^10 + 10*a*cosh(x)*sinh(x)^9 + a*
sinh(x)^10 + 5*a*cosh(x)^8 + 5*(9*a*cosh(x)^2 + a)*sinh(x)^8 + 40*(3*a*cosh
(x)^3 + a*cosh(x))*sinh(x)^7 + 10*a*cosh(x)^6 + 10*(21*a*cosh(x)^4 + 14*a*c
osh(x)^2 + a)*sinh(x)^6 + 4*(63*a*cosh(x)^5 + 70*a*cosh(x)^3 + 15*a*cosh(x)
)*sinh(x)^5 + 10*a*cosh(x)^4 + 10*(21*a*cosh(x)^6 + 35*a*cosh(x)^4 + 15*a*c
osh(x)^2 + a)*sinh(x)^4 + 40*(3*a*cosh(x)^7 + 7*a*cosh(x)^5 + 5*a*cosh(x)^3
 + a*cosh(x))*sinh(x)^3 + 5*a*cosh(x)^2 + 5*(9*a*cosh(x)^8 + 28*a*cosh(x)^6
 + 30*a*cosh(x)^4 + 12*a*cosh(x)^2 + a)*sinh(x)^2 + 10*(a*cosh(x)^9 + 4*a*c
osh(x)^7 + 6*a*cosh(x)^5 + 4*a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tanh^6(x)}{\cosh(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**6/(a+a*cosh(x)),x)
```

```
[Out] Integral(tanh(x)**6/(cosh(x) + 1), x)/a
```

Giac [**A**]
time = 0.42, size = 58, normalized size = 1.26

$$\frac{3\arctan\left(e^{x}\right)}{4\,a}-\frac{25\,e^{(9\,x)}-40\,e^{(8\,x)}+10\,e^{(7\,x)}-80\,e^{(4\,x)}-10\,e^{(3\,x)}-25\,e^{x}-8}{20\,a\left(e^{(2\,x)}+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^6/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] 3/4*arctan(e^x)/a - 1/20*(25*e^(9*x) - 40*e^(8*x) + 10*e^(7*x) - 80*e^(4*x)
 - 10*e^(3*x) - 25*e^x - 8)/(a*(e^(2*x) + 1)^5)
```

Mupad [**B**]
time = 1.06, size = 183, normalized size = 3.98

$$\frac{\frac{16}{a}-\frac{6\,e^{x}}{a}}{3\,e^{2\,x}+3\,e^{4\,x}+e^{6\,x}+1}-\frac{\frac{8}{a}-\frac{9\,e^{x}}{2\,a}}{2\,e^{2\,x}+e^{4\,x}+1}+\frac{32}{5\,a\,\left(5\,e^{2\,x}+10\,e^{4\,x}+10\,e^{6\,x}+5\,e^{8\,x}+e^{10\,x}+1\right)}-\frac{\frac{16}{a}-\frac{4\,e^{x}}{a}}{4\,e^{2\,x}+6\,e^{4\,x}+4\,e^{6\,x}+e^{8\,x}+1}+\frac{\frac{2}{a}-\frac{5\,e^{x}}{4\,a}}{e^{2\,x}+1}+\frac{3\,\mathrm{atan}\left(\frac{e^{x}\,\sqrt{a^2}}{a}\right)}{4\,\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^6/(a + a*cosh(x)),x)
```

```
[Out] (16/a - (6*exp(x))/a)/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - (8/a - (9*
exp(x))/(2*a))/(2*exp(2*x) + exp(4*x) + 1) + 32/(5*a*(5*exp(2*x) + 10*exp(4
*x) + 10*exp(6*x) + 5*exp(8*x) + exp(10*x) + 1)) - (16/a - (4*exp(x))/a)/(4
*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1) + (2/a - (5*exp(x))/(4*
a))/(exp(2*x) + 1) + (3*atan((exp(x)*(a^2)^(1/2))/a))/(4*(a^2)^(1/2))
```

3.188 $\int \frac{\tanh^5(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=30

$$-\frac{\operatorname{sech}(x)}{a}+\frac{\operatorname{sech}^3(x)}{3a}-\frac{\tanh^4(x)}{4a}$$

[Out] -sech(x)/a+1/3*sech(x)^3/a-1/4*tanh(x)^4/a

Rubi [A]
time = 0.06, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.308$, Rules used = {2785, 2687, 30, 2686}

$$-\frac{\tanh^4(x)}{4a}+\frac{\operatorname{sech}^3(x)}{3a}-\frac{\operatorname{sech}(x)}{a}$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]^5/(a + a*Cosh[x]),x]

[Out] -(Sech[x]/a) + Sech[x]^3/(3*a) - Tanh[x]^4/(4*a)

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^5(x)}{a+a\cosh(x)}\,dx &= \frac{\int \operatorname{sech}(x)\tanh^3(x)\,dx}{a}-\frac{\int \operatorname{sech}^2(x)\tanh^3(x)\,dx}{a}\\ &= -\frac{\operatorname{Subst}\left(\int x^3\,dx,x,i\tanh(x)\right)}{a}+\frac{\operatorname{Subst}\left(\int \left(-1+x^2\right)\,dx,x,\operatorname{sech}(x)\right)}{a}\\ &= -\frac{\operatorname{sech}(x)}{a}+\frac{\operatorname{sech}^3(x)}{3a}-\frac{\tanh^4(x)}{4a}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 25, normalized size = 0.83

$$\frac{2(3+5\cosh(x))\operatorname{sech}^4(x)\sinh^6\left(\frac{x}{2}\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^5/(a + a*Cosh[x]),x]
[Out] (2*(3 + 5*Cosh[x])*Sech[x]^4*Sinh[x/2]^6)/(3*a)
```

Maple [A]
time = 0.58, size = 30, normalized size = 1.00

method	result	size
derivativedivides	$\frac{-\frac{1}{\cosh(x)}+\frac{1}{3\cosh(x)^3}+\frac{1}{2\cosh(x)^2}-\frac{1}{4\cosh(x)^4}}{a}$	30
default	$\frac{-\frac{1}{\cosh(x)}+\frac{1}{3\cosh(x)^3}+\frac{1}{2\cosh(x)^2}-\frac{1}{4\cosh(x)^4}}{a}$	30
risch	$-\frac{2\,e^{x}\left(3\,e^{6x}-3\,e^{5x}+5\,e^{4x}+5\,e^{2x}-3\,e^{x}+3\right)}{3\left(1+e^{2x}\right)^4 a}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
[Out] 1/a*(-1/cosh(x)+1/3/cosh(x)^3+1/2/cosh(x)^2-1/4/cosh(x)^4)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(26) = 52.
time = 0.26, size = 223, normalized size = 7.43

$$-\frac{2\,e^{(-x)}}{4\,ae^{(-2\,x)}+6\,ae^{(-4\,x)}+4\,ae^{(-6\,x)}+ae^{(-8\,x)}+a}+\frac{2\,e^{(-2\,x)}}{4\,ae^{(-2\,x)}+6\,ae^{(-4\,x)}+4\,ae^{(-6\,x)}+ae^{(-8\,x)}+a}-\frac{10\,e^{(-3\,x)}}{3\left(4\,ae^{(-2\,x)}+6\,ae^{(-4\,x)}+4\,ae^{(-6\,x)}+ae^{(-8\,x)}+a\right)}-\frac{10\,e^{(-5\,x)}}{3\left(4\,ae^{(-2\,x)}+6\,ae^{(-4\,x)}+4\,ae^{(-6\,x)}+ae^{(-8\,x)}+a\right)}+\frac{2\,e^{(-6\,x)}}{4\,ae^{(-2\,x)}+6\,ae^{(-4\,x)}+4\,ae^{(-6\,x)}+ae^{(-8\,x)}+a}-\frac{2\,e^{(-7\,x)}}{4\,ae^{(-2\,x)}+6\,ae^{(-4\,x)}+4\,ae^{(-6\,x)}+ae^{(-8\,x)}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -2*e^(-x)/(4*a*e^(-2*x) + 6*a*e^(-4*x) + 4*a*e^(-6*x) + a*e^(-8*x) + a) + 2
*e^(-2*x)/(4*a*e^(-2*x) + 6*a*e^(-4*x) + 4*a*e^(-6*x) + a*e^(-8*x) + a) - 1
0/3*e^(-3*x)/(4*a*e^(-2*x) + 6*a*e^(-4*x) + 4*a*e^(-6*x) + a*e^(-8*x) + a)
- 10/3*e^(-5*x)/(4*a*e^(-2*x) + 6*a*e^(-4*x) + 4*a*e^(-6*x) + a*e^(-8*x) +
a) + 2*e^(-6*x)/(4*a*e^(-2*x) + 6*a*e^(-4*x) + 4*a*e^(-6*x) + a*e^(-8*x) +
a) - 2*e^(-7*x)/(4*a*e^(-2*x) + 6*a*e^(-4*x) + 4*a*e^(-6*x) + a*e^(-8*x) +
a)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(26) = 52$.
time = 0.42, size = 174, normalized size = 5.80

$$-\frac{2\,\left(3\,\cosh\left(x\right)^{4}+3\,\left(4\,\cosh\left(x\right)-1\right)\sinh\left(x\right)^{3}+3\,\sinh\left(x\right)^{4}-3\,\cosh\left(x\right)^{3}+\left(18\,\cosh\left(x\right)^{2}-9\,\cosh\left(x\right)+8\right)\sinh\left(x\right)^{2}+8\,\cosh\left(x\right)^{2}+\left(12\,\cosh\left(x\right)^{3}-9\,\cosh\left(x\right)^{2}+4\,\cosh\left(x\right)+3\right)\sinh\left(x\right)-3\,\cosh\left(x\right)+5\right)}{3\,\left(a\cosh\left(x\right)^{5}+5\,a\cosh\left(x\right)\sinh\left(x\right)^{4}+a\sinh\left(x\right)^{5}+5\,a\cosh\left(x\right)^{3}+\left(10\,a\cosh\left(x\right)^{2}+3\,a\right)\sinh\left(x\right)^{3}+5\,\left(2\,a\cosh\left(x\right)^{3}+3\,a\cosh\left(x\right)\right)\sinh\left(x\right)^{2}+10\,a\cosh\left(x\right)+\left(5\,a\cosh\left(x\right)^{4}+9\,a\cosh\left(x\right)^{2}+2\,a\right)\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] -2/3*(3*cosh(x)^4 + 3*(4*cosh(x) - 1)*sinh(x)^3 + 3*sinh(x)^4 - 3*cosh(x)^3
 + (18*cosh(x)^2 - 9*cosh(x) + 8)*sinh(x)^2 + 8*cosh(x)^2 + (12*cosh(x)^3 -
 9*cosh(x)^2 + 4*cosh(x) + 3)*sinh(x) - 3*cosh(x) + 5)/(a*cosh(x)^5 + 5*a*c
osh(x)*sinh(x)^4 + a*sinh(x)^5 + 5*a*cosh(x)^3 + (10*a*cosh(x)^2 + 3*a)*sin
h(x)^3 + 5*(2*a*cosh(x)^3 + 3*a*cosh(x))*sinh(x)^2 + 10*a*cosh(x) + (5*a*co
sh(x)^4 + 9*a*cosh(x)^2 + 2*a)*sinh(x))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tanh^{5}\left(x\right)}{\cosh\left(x\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**5/(a+a*cosh(x)),x)
```

```
[Out] Integral(tanh(x)**5/(cosh(x) + 1), x)/a
```

Giac [**A**]
time = 0.41, size = 48, normalized size = 1.60

$$-\frac{2\,\left(3\,\left(e^{(-x)}+e^{x}\right)^{3}-3\,\left(e^{(-x)}+e^{x}\right)^{2}-4\,e^{(-x)}-4\,e^{x}+6\right)}{3\,a\left(e^{(-x)}+e^{x}\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -2/3*(3*(e^(-x) + e^x)^3 - 3*(e^(-x) + e^x)^2 - 4*e^(-x) - 4*e^x + 6)/(a*(e
^(-x) + e^x)^4)
```

Mupad [B]
time = 1.01, size = 117, normalized size = 3.90

$$\frac{\frac{8}{a}-\frac{8\,e^{x}}{3\,a}}{3\,e^{2\,x}+3\,e^{4\,x}+e^{6\,x}+1}-\frac{\frac{6}{a}-\frac{8\,e^{x}}{3\,a}}{2\,e^{2\,x}+e^{4\,x}+1}+\frac{\frac{2}{a}-\frac{2\,e^{x}}{a}}{e^{2\,x}+1}-\frac{4}{a\,\left(4\,e^{2\,x}+6\,e^{4\,x}+4\,e^{6\,x}+e^{8\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a + a*cosh(x)),x)
```

```
[Out] (8/a - (8*exp(x))/(3*a))/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - (6/a -
(8*exp(x))/(3*a))/(2*exp(2*x) + exp(4*x) + 1) + (2/a - (2*exp(x))/a)/(exp(2
*x) + 1) - 4/(a*(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1))
```

3.189 $\int \frac{\tanh^4(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=33

$$\frac{\text{ArcTan}(\sinh(x))}{2a}-\frac{\text{sech}(x)\tanh(x)}{2a}-\frac{\tanh^3(x)}{3a}$$

```
[Out] 1/2*arctan(sinh(x))/a-1/2*sech(x)*tanh(x)/a-1/3*tanh(x)^3/a
```

Rubi [A]
time = 0.06, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2785, 2687, 30, 2691, 3855}

$$\frac{\text{ArcTan}(\sinh(x))}{2a}-\frac{\tanh^3(x)}{3a}-\frac{\tanh(x)\text{sech}(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/(a + a*Cosh[x]),x]
```

```
[Out] ArcTan[Sinh[x]]/(2*a) - (Sech[x]*Tanh[x])/(2*a) - Tanh[x]^3/(3*a)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
```

```
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^4(x)}{a+a\cosh(x)}\,dx &= \frac{\int \operatorname{sech}(x)\tanh^2(x)\,dx}{a}-\frac{\int \operatorname{sech}^2(x)\tanh^2(x)\,dx}{a}\\ &= -\frac{\operatorname{sech}(x)\tanh(x)}{2a}-\frac{i\mathrm{Subst}\left(\int x^2\,dx,x,i\tanh(x)\right)}{a}+\frac{\int \operatorname{sech}(x)\,dx}{2a}\\ &= \frac{\tan^{-1}(\sinh(x))}{2a}-\frac{\operatorname{sech}(x)\tanh(x)}{2a}-\frac{\tanh^3(x)}{3a}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 46, normalized size = 1.39

$$\frac{\cosh^2\left(\frac{x}{2}\right)\left(6\mathrm{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)+\left(-2-3\operatorname{sech}(x)+2\operatorname{sech}^2(x)\right)\tanh(x)\right)}{3a(1+\cosh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(a + a*Cosh[x]),x]
```

```
[Out] (Cosh[x/2]^2*(6*ArcTan[Tanh[x/2]] + (-2 - 3*Sech[x] + 2*Sech[x]^2)*Tanh[x])
)/(3*a*(1 + Cosh[x]))
```

Maple [A]
time = 0.58, size = 48, normalized size = 1.45

method	result	size
default	$\frac{\frac{16\left(\frac{\left(\tanh^5\left(\frac{x}{2}\right)\right)}{16}-\frac{\left(\tanh^3\left(\frac{x}{2}\right)\right)}{6}-\frac{\tanh\left(\frac{x}{2}\right)}{16}\right)}{\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^3}+\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$	48
risch	$-\frac{3\,\mathrm{e}^{5x}-6\,\mathrm{e}^{4x}-3\,\mathrm{e}^{x}-2}{3\left(1+\mathrm{e}^{2x}\right)^3 a}+\frac{i\ln\left(\mathrm{e}^{x}+i\right)}{2a}-\frac{i\ln\left(\mathrm{e}^{x}-i\right)}{2a}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 16/a*((1/16*tanh(1/2*x)^5-1/6*tanh(1/2*x)^3-1/16*tanh(1/2*x))/(tanh(1/2*x)^
2+1)^3+1/16*arctan(tanh(1/2*x)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(27) = 54$.
time = 0.47, size = 57, normalized size = 1.73

$$-\frac{3\,e^{(-x)} + 6\,e^{(-4\,x)} - 3\,e^{(-5\,x)} + 2}{3\,(3\,ae^{(-2\,x)} + 3\,ae^{(-4\,x)} + ae^{(-6\,x)} + a)} - \frac{\arctan\left(e^{(-x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -1/3*(3*e^(-x) + 6*e^(-4*x) - 3*e^(-5*x) + 2)/(3*a*e^(-2*x) + 3*a*e^(-4*x)
+ a*e^(-6*x) + a) - arctan(e^(-x))/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(27) = 54$.
time = 0.43, size = 315, normalized size = 9.55

$$-\frac{3\,\cosh\left(x\right)^{5} + 3\,{\left(5\,\cosh\left(x\right) - 2\right)}\sinh\left(x\right)^{4} + 3\,\sinh\left(x\right)^{5} - 6\,\cosh\left(x\right)^{4} + 6\,{\left(5\,\cosh\left(x\right)^{2} - 4\,\cosh\left(x\right)\right)}\sinh\left(x\right)^{3} + 6\,{\left(5\,\cosh\left(x\right)^{3} - 6\,\cosh\left(x\right)^{2}\right)}\sinh\left(x\right)^{2} - 3\,{\left(\cosh\left(x\right)^{6} + 6\,\cosh\left(x\right)\sinh\left(x\right)^{5} + \sinh\left(x\right)^{6} + 3\,{\left(5\,\cosh\left(x\right)^{2} + 1\right)}\sinh\left(x\right)^{4} + 3\,\cosh\left(x\right)^{4} + 4\,{\left(5\,\cosh\left(x\right)^{3} + 3\,\cosh\left(x\right)\right)}\sinh\left(x\right)^{3} + 3\,{\left(5\,\cosh\left(x\right)^{4} + 6\,\cosh\left(x\right)^{2} + 1\right)}\sinh\left(x\right)^{2} + 3\,\cosh\left(x\right)^{2} + 6\,{\left(\cosh\left(x\right)^{5} + 2\,\cosh\left(x\right)^{3} + \cosh\left(x\right)\right)}\sinh\left(x\right) + 1\right)}\arctan\left(\cosh\left(x\right) + \sinh\left(x\right)\right) + 3\,{\left(5\,\cosh\left(x\right)^{4} - 8\,\cosh\left(x\right)^{3} - 1\right)}\sinh\left(x\right) - 3\,\cosh\left(x\right) - 2}{3\,{\left(a\cosh\left(x\right)^{6} + 6\,a\cosh\left(x\right)\sinh\left(x\right)^{5} + a\sinh\left(x\right)^{6} + 3\,a\cosh\left(x\right)^{4} + 3\,{\left(5\,a\cosh\left(x\right)^{2} + a\right)}\sinh\left(x\right)^{4} + 4\,{\left(5\,a\cosh\left(x\right)^{3} + 3\,a\cosh\left(x\right)\right)}\sinh\left(x\right)^{3} + 3\,a\cosh\left(x\right)^{2} + 3\,{\left(5\,a\cosh\left(x\right)^{4} + 6\,a\cosh\left(x\right)^{2} + a\right)}\sinh\left(x\right)^{2} + 6\,{\left(a\cosh\left(x\right)^{5} + 2\,a\cosh\left(x\right)^{3} + a\cosh\left(x\right)\right)}\sinh\left(x\right) + a\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] -1/3*(3*cosh(x)^5 + 3*(5*cosh(x) - 2)*sinh(x)^4 + 3*sinh(x)^5 - 6*cosh(x)^4
 + 6*(5*cosh(x)^2 - 4*cosh(x))*sinh(x)^3 + 6*(5*cosh(x)^3 - 6*cosh(x)^2)*si
nh(x)^2 - 3*(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 +
 1)*sinh(x)^4 + 3*cosh(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 3*(5*
cosh(x)^4 + 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 + 2*cos
h(x)^3 + cosh(x))*sinh(x) + 1)*arctan(cosh(x) + sinh(x)) + 3*(5*cosh(x)^4 -
 8*cosh(x)^3 - 1)*sinh(x) - 3*cosh(x) - 2)/(a*cosh(x)^6 + 6*a*cosh(x)*sinh(
x)^5 + a*sinh(x)^6 + 3*a*cosh(x)^4 + 3*(5*a*cosh(x)^2 + a)*sinh(x)^4 + 4*(5
*a*cosh(x)^3 + 3*a*cosh(x))*sinh(x)^3 + 3*a*cosh(x)^2 + 3*(5*a*cosh(x)^4 +
6*a*cosh(x)^2 + a)*sinh(x)^2 + 6*(a*cosh(x)^5 + 2*a*cosh(x)^3 + a*cosh(x))*
sinh(x) + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tanh^{4}{\left(x \right)}}{\cosh{\left(x \right)} + 1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(a+a*cosh(x)),x)
```

[Out] Integral(tanh(x)**4/(cosh(x) + 1), x)/a

Giac [A]
time = 0.42, size = 39, normalized size = 1.18

$$\frac{\arctan\left(e^{x}\right)}{a} - \frac{3\,e^{(5\,x)} - 6\,e^{(4\,x)} - 3\,e^{x} - 2}{3\,a{\left(e^{(2\,x)} + 1\right)}^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^4/(a+a*cosh(x)),x, algorithm="giac")

[Out] arctan(e^x)/a - 1/3*(3*e^(5*x) - 6*e^(4*x) - 3*e^x - 2)/(a*(e^(2*x) + 1)^3)

Mupad [B]
time = 0.96, size = 95, normalized size = 2.88

$$\frac{8}{3\,a\,\left(3\,\mathrm{e}^{2\,x}+3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}+1\right)}-\frac{\frac{4}{a}-\frac{2\,\mathrm{e}^{x}}{a}}{2\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1}+\frac{\frac{2}{a}-\frac{\mathrm{e}^{x}}{a}}{\mathrm{e}^{2\,x}+1}+\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{x}\,\sqrt{a^2}}{a}\right)}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^4/(a + a*cosh(x)),x)

[Out] 8/(3*a*(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1)) - (4/a - (2*exp(x))/a)/(2*exp(2*x) + exp(4*x) + 1) + (2/a - exp(x)/a)/(exp(2*x) + 1) + atan((exp(x)*(a^2)^(1/2))/a)/(a^2)^(1/2)

3.190 $\int \frac{\tanh^3(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=19

$$-\frac{\operatorname{sech}(x)}{a}+\frac{\operatorname{sech}^2(x)}{2a}$$

```
[Out] -sech(x)/a+1/2*sech(x)^2/a
```

Rubi [A]
time = 0.05, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.308$, Rules used = {2785, 2686, 30, 8}

$$\frac{\operatorname{sech}^2(x)}{2a}-\frac{\operatorname{sech}(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/(a + a*Cosh[x]),x]
```

```
[Out] -(Sech[x]/a) + Sech[x]^2/(2*a)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^3(x)}{a+a\cosh(x)}\,dx &= \frac{\int \operatorname{sech}(x)\tanh(x)\,dx}{a} - \frac{\int \operatorname{sech}^2(x)\tanh(x)\,dx}{a}\\ &= -\frac{\operatorname{Subst}(\int 1\,dx, x, \operatorname{sech}(x))}{a} + \frac{\operatorname{Subst}(\int x\,dx, x, \operatorname{sech}(x))}{a}\\ &= -\frac{\operatorname{sech}(x)}{a} + \frac{\operatorname{sech}^2(x)}{2a}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 17, normalized size = 0.89

$$\frac{2\operatorname{sech}^2(x)\sinh^4\left(\frac{x}{2}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(a + a*Cosh[x]),x]
```

```
[Out] (2*Sech[x]^2*Sinh[x/2]^4)/a
```

Maple [A]
time = 0.57, size = 18, normalized size = 0.95

method	result	size
derivativedivides	$\frac{\frac{1}{2\cosh(x)^2}-\frac{1}{\cosh(x)}}{a}$	18
default	$\frac{\frac{1}{2\cosh(x)^2}-\frac{1}{\cosh(x)}}{a}$	18
risch	$-\frac{2\,e^{x}\left(1-e^{x}+e^{2x}\right)}{\left(1+e^{2x}\right)^2 a}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(1/2/cosh(x)^2-1/cosh(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(17) = 34.
time = 0.27, size = 70, normalized size = 3.68

$$-\frac{2\,e^{(-x)}}{2\,ae^{(-2\,x)}+ae^{(-4\,x)}+a}+\frac{2\,e^{(-2\,x)}}{2\,ae^{(-2\,x)}+ae^{(-4\,x)}+a}-\frac{2\,e^{(-3\,x)}}{2\,ae^{(-2\,x)}+ae^{(-4\,x)}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -2*e^(-x)/(2*a*e^(-2*x) + a*e^(-4*x) + a) + 2*e^(-2*x)/(2*a*e^(-2*x) + a*e^
(-4*x) + a) - 2*e^(-3*x)/(2*a*e^(-2*x) + a*e^(-4*x) + a)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(17) = 34$.
time = 0.38, size = 66, normalized size = 3.47

$$-\frac{2\left(\cosh\left(x\right)^{2}+\left(2\cosh\left(x\right)-1\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}-\cosh\left(x\right)+1\right)}{a\cosh\left(x\right)^{3}+3\,a\cosh\left(x\right)\sinh\left(x\right)^{2}+a\sinh\left(x\right)^{3}+3\,a\cosh\left(x\right)+\left(3\,a\cosh\left(x\right)^{2}+a\right)\sinh\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] -2*(cosh(x)^2 + (2*cosh(x) - 1)*sinh(x) + sinh(x)^2 - cosh(x) + 1)/(a*cosh(
x)^3 + 3*a*cosh(x)*sinh(x)^2 + a*sinh(x)^3 + 3*a*cosh(x) + (3*a*cosh(x)^2 +
 a)*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\tanh^{3}\left(x\right)}{\cosh\left(x\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(a+a*cosh(x)),x)
```

```
[Out] Integral(tanh(x)**3/(cosh(x) + 1), x)/a
```

Giac [A]
time = 0.41, size = 22, normalized size = 1.16

$$-\frac{2\left(e^{(-x)}+e^{x}-1\right)}{a\left(e^{(-x)}+e^{x}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -2*(e^(-x) + e^x - 1)/(a*(e^(-x) + e^x)^2)
```

Mupad [B]
time = 0.92, size = 25, normalized size = 1.32

$$-\frac{2\,\mathrm{e}^{x}\,\left(\mathrm{e}^{2\,x}-\mathrm{e}^{x}+1\right)}{a\,\left(\mathrm{e}^{2\,x}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a + a*cosh(x)),x)
```

```
[Out] -(2*exp(x)*(exp(2*x) - exp(x) + 1))/(a*(exp(2*x) + 1)^2)
```

3.191 $\int \frac{\tanh^2(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=15

$$\frac{\operatorname{ArcTan}(\sinh(x))}{a}-\frac{\tanh(x)}{a}$$

[Out] arctan(sinh(x))/a-tanh(x)/a

Rubi [A]
time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2785, 3852, 8, 3855}

$$\frac{\operatorname{ArcTan}(\sinh(x))}{a}-\frac{\tanh(x)}{a}$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]^2/(a + a*Cosh[x]),x]

[Out] ArcTan[Sinh[x]]/a - Tanh[x]/a

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\tanh^2(x)}{a+a\cosh(x)}\,dx = \frac{\int \operatorname{sech}(x)\,dx}{a} - \frac{\int \operatorname{sech}^2(x)\,dx}{a}$$
$$= \frac{\tan^{-1}(\sinh(x))}{a} - \frac{i\text{Subst}(\int 1\,dx, x, -i\tanh(x))}{a}$$
$$= \frac{\tan^{-1}(\sinh(x))}{a} - \frac{\tanh(x)}{a}$$

Mathematica [A]
time = 0.04, size = 18, normalized size = 1.20

$$\frac{2\text{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right) - \tanh(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(a + a*Cosh[x]),x]
[Out] (2*ArcTan[Tanh[x/2]] - Tanh[x])/a
```

Maple [A]
time = 0.59, size = 30, normalized size = 2.00

method	result	size
default	$\frac{-\frac{2\tanh\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)+1}+2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$	30
risch	$\frac{2}{a(1+e^{2x})} + \frac{i\ln(e^x+i)}{a} - \frac{i\ln(e^x-i)}{a}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
[Out] 4/a*(-1/2*tanh(1/2*x)/(tanh(1/2*x)^2+1)+1/2*arctan(tanh(1/2*x)))
```

Maxima [A]
time = 0.50, size = 23, normalized size = 1.53

$$-\frac{2\arctan\left(e^{(-x)}\right)}{a} - \frac{2}{ae^{(-2x)} + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+a*cosh(x)),x, algorithm="maxima")
[Out] -2*arctan(e^(-x))/a - 2/(a*e^(-2*x) + a)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(15) = 30$.
time = 0.45, size = 50, normalized size = 3.33

$$\frac{2\left(\left(\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}+1\right)\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)+1\right)}{a\cosh\left(x\right)^{2}+2\,a\cosh\left(x\right)\sinh\left(x\right)+a\sinh\left(x\right)^{2}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] 2*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*arctan(cosh(x) + sinh(x)
) + 1)/(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tanh^{2}\left(x\right)}{\cosh\left(x\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(a+a*cosh(x)),x)
```

```
[Out] Integral(tanh(x)**2/(cosh(x) + 1), x)/a
```

Giac [A]
time = 0.41, size = 22, normalized size = 1.47

$$\frac{2\,\arctan\left(e^{x}\right)}{a}+\frac{2}{a(e^{(2\,x)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*arctan(e^x)/a + 2/(a*(e^(2*x) + 1))
```

Mupad [B]
time = 0.92, size = 33, normalized size = 2.20

$$\frac{2}{a\,\left(\mathrm{e}^{2\,x}+1\right)}+\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{x}\,\sqrt{a^{2}}}{a}\right)}{\sqrt{a^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a + a*cosh(x)),x)
```

```
[Out] 2/(a*(exp(2*x) + 1)) + (2*atan((exp(x)*(a^2)^(1/2))/a))/(a^2)^(1/2)
```

3.192 $\int \frac{\tanh(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=18

$$\frac{\log(\cosh(x))}{a}-\frac{\log(1+\cosh(x))}{a}$$

```
[Out] ln(cosh(x))/a-ln(1+cosh(x))/a
```

Rubi [A]
time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2786, 36, 29, 31}

$$\frac{\log(\cosh(x))}{a}-\frac{\log(\cosh(x)+1)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(a + a*Cosh[x]),x]
```

```
[Out] Log[Cosh[x]]/a - Log[1 + Cosh[x]]/a
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tanh(x)}{a+a\cosh(x)}\,dx = \text{Subst}\left(\int \frac{1}{x(a+x)}\,dx, x, a\cosh(x)\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, a\cosh(x)\right)}{a} - \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, a\cosh(x)\right)}{a}$$
$$= \frac{\log(\cosh(x))}{a} - \frac{\log(1+\cosh(x))}{a}$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 0.67

$$-\frac{2\tanh^{-1}(1+2\cosh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(a + a*Cosh[x]),x]
[Out] (-2*ArcTanh[1 + 2*Cosh[x]])/a
```

Maple [A]
time = 0.52, size = 16, normalized size = 0.89

method	result	size
derivativedivides	$\frac{\ln(\cosh(x))-\ln(\cosh(x)+1)}{a}$	16
default	$\frac{\ln(\cosh(x))-\ln(\cosh(x)+1)}{a}$	16
risch	$-\frac{2\ln(e^{x}+1)}{a}+\frac{\ln(1+e^{2x})}{a}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
[Out] 1/a*(ln(cosh(x))-ln(cosh(x)+1))
```

Maxima [A]
time = 0.27, size = 24, normalized size = 1.33

$$-\frac{2\,\log\left(e^{(-x)}+1\right)}{a}+\frac{\log\left(e^{(-2\,x)}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+a*cosh(x)),x, algorithm="maxima")
[Out] -2*log(e^(-x) + 1)/a + log(e^(-2*x) + 1)/a
```

Fricas [A]
time = 0.37, size = 28, normalized size = 1.56

$$\frac{\log\left(\frac{2\cosh(x)}{\cosh(x)-\sinh(x)}\right)-2\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+a*cosh(x)),x, algorithm="fricas")
[Out] (log(2*cosh(x)/(cosh(x) - sinh(x))) - 2*log(cosh(x) + sinh(x) + 1))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\tanh\left(x\right)}{\cosh\left(x\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+a*cosh(x)),x)
[Out] Integral(tanh(x)/(cosh(x) + 1), x)/a
```

Giac [A]
time = 0.41, size = 22, normalized size = 1.22

$$\frac{\log\left(e^{(2x)}+1\right)}{a}-\frac{2\log\left(e^{x}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+a*cosh(x)),x, algorithm="giac")
[Out] log(e^(2*x) + 1)/a - 2*log(e^x + 1)/a
```

Mupad [B]
time = 0.08, size = 26, normalized size = 1.44

$$-\frac{2\ln\left(36\,e^{x}+36\right)-\ln\left(3\,e^{2x}+3\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a + a*cosh(x)),x)
[Out] -(2*log(36*exp(x) + 36) - log(3*exp(2*x) + 3))/a
```

3.193 $\int \frac{\coth(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=33

$$-\frac{\tanh^{-1}(\cosh(x))}{2a}-\frac{\coth(x)\text{csch}(x)}{2a}+\frac{\text{csch}^2(x)}{2a}$$

```
[Out] -1/2*arctanh(cosh(x))/a-1/2*coth(x)*csch(x)/a+1/2*csch(x)^2/a
```

Rubi [A]
time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {2785, 2686, 30, 2691, 3855}

$$\frac{\text{csch}^2(x)}{2a}-\frac{\tanh^{-1}(\cosh(x))}{2a}-\frac{\coth(x)\text{csch}(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(a + a*Cosh[x]),x]
```

```
[Out] -1/2*ArcTanh[Cosh[x]]/a - (Coth[x]*Csch[x])/(2*a) + Csch[x]^2/(2*a)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
```

```
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth(x)}{a+a\cosh(x)}\,dx &= \frac{\int \coth^2(x)\operatorname{csch}(x)\,dx}{a}-\frac{\int \coth(x)\operatorname{csch}^2(x)\,dx}{a}\\ &= -\frac{\coth(x)\operatorname{csch}(x)}{2a}+\frac{\int \operatorname{csch}(x)\,dx}{2a}-\frac{\operatorname{Subst}(\int x\,dx,x,-i\operatorname{csch}(x))}{a}\\ &= -\frac{\tanh^{-1}(\cosh(x))}{2a}-\frac{\coth(x)\operatorname{csch}(x)}{2a}+\frac{\operatorname{csch}^2(x)}{2a}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 42, normalized size = 1.27

$$-\frac{1+2\cosh^2\left(\frac{x}{2}\right)\left(\log\left(\cosh\left(\frac{x}{2}\right)\right)-\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)}{2a(1+\cosh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(a + a*Cosh[x]),x]
[Out] -1/2*(1 + 2*Cosh[x/2]^2*(Log[Cosh[x/2]] - Log[Sinh[x/2]]))/(a*(1 + Cosh[x])
)
```

Maple [A]
time = 0.58, size = 20, normalized size = 0.61

method	result	size
default	$\frac{\frac{\left(\tanh^2\left(\frac{x}{2}\right)\right)}{2}+\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2a}$	20
risch	$-\frac{e^x}{\left(e^x+1\right)^2 a}+\frac{\ln\left(e^x-1\right)}{2a}-\frac{\ln\left(e^x+1\right)}{2a}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
[Out] 1/2/a*(1/2*tanh(1/2*x)^2+ln(tanh(1/2*x)))
```

Maxima [A]
time = 0.28, size = 48, normalized size = 1.45

$$-\frac{e^{(-x)}}{2\,ae^{(-x)} + ae^{(-2\,x)} + a} - \frac{\log\left(e^{(-x)} + 1\right)}{2\,a} + \frac{\log\left(e^{(-x)} - 1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -e^(-x)/(2*a*e^(-x) + a*e^(-2*x) + a) - 1/2*log(e^(-x) + 1)/a + 1/2*log(e^(
-x) - 1)/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(27) = 54.
time = 0.45, size = 103, normalized size = 3.12

$$-\frac{\left(\cosh\left(x\right)^{2} + 2\,\left(\cosh\left(x\right) + 1\right)\sinh\left(x\right) + \sinh\left(x\right)^{2} + 2\,\cosh\left(x\right) + 1\right)\log\left(\cosh\left(x\right) + \sinh\left(x\right) + 1\right) - \left(\cosh\left(x\right)^{2} + 2\,\left(\cosh\left(x\right) + 1\right)\sinh\left(x\right) + \sinh\left(x\right)^{2} + 2\,\cosh\left(x\right) + 1\right)\log\left(\cosh\left(x\right) + \sinh\left(x\right) - 1\right) + 2\,\cosh\left(x\right) + 2\,\sinh\left(x\right)}{2\,\left(a\cosh\left(x\right)^{2} + a\sinh\left(x\right)^{2} + 2\,a\cosh\left(x\right) + 2\,\left(a\cosh\left(x\right) + a\right)\sinh\left(x\right) + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] -1/2*((cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)*log
(cosh(x) + sinh(x) + 1) - (cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2
+ 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*cosh(x) + 2*sinh(x))/(a*cos
h(x)^2 + a*sinh(x)^2 + 2*a*cosh(x) + 2*(a*cosh(x) + a)*sinh(x) + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\coth\left(x\right)}{\cosh\left(x\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+a*cosh(x)),x)
```

```
[Out] Integral(coth(x)/(cosh(x) + 1), x)/a
```

Giac [A]
time = 0.41, size = 52, normalized size = 1.58

$$-\frac{\log\left(e^{(-x)} + e^{x} + 2\right)}{4\,a} + \frac{\log\left(e^{(-x)} + e^{x} - 2\right)}{4\,a} + \frac{e^{(-x)} + e^{x} - 2}{4\,a(e^{(-x)} + e^{x} + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -1/4*log(e^(-x) + e^x + 2)/a + 1/4*log(e^(-x) + e^x - 2)/a + 1/4*(e^(-x) +
e^x - 2)/(a*(e^(-x) + e^x + 2))
```

Mupad [B]
time = 0.92, size = 51, normalized size = 1.55

$$\frac{1}{a\,\left({\mathrm{e}}^{2\,x}+2\,{\mathrm{e}}^x+1\right)}-\frac{1}{a\,\left({\mathrm{e}}^x+1\right)}-\frac{\mathrm{atan}\left(\frac{{\mathrm{e}}^x\,\sqrt{-a^2}}{a}\right)}{\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a + a*cosh(x)),x)
```

```
[Out] 1/(a*(exp(2*x) + 2*exp(x) + 1)) - 1/(a*(exp(x) + 1)) - atan((exp(x)*(-a^2)^
(1/2))/a)/(-a^2)^(1/2)
```

3.194 $\int \frac{\coth^2(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=30

$$\frac{\coth^3(x)}{3a}-\frac{\operatorname{csch}(x)}{a}-\frac{\operatorname{csch}^3(x)}{3a}$$

```
[Out] 1/3*coth(x)^3/a-csch(x)/a-1/3*csch(x)^3/a
```

Rubi [A]
time = 0.06, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2785, 2687, 30, 2686}

$$\frac{\coth^3(x)}{3a}-\frac{\operatorname{csch}^3(x)}{3a}-\frac{\operatorname{csch}(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(a + a*Cosh[x]),x]
```

```
[Out] Coth[x]^3/(3*a) - Csch[x]/a - Csch[x]^3/(3*a)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\coth^2(x)}{a+a\cosh(x)}\,dx = \frac{\int \coth^3(x)\text{csch}(x)\,dx}{a} - \frac{\int \coth^2(x)\text{csch}^2(x)\,dx}{a}$$
$$= \frac{i\text{Subst}\left(\int x^2\,dx, x, i\coth(x)\right)}{a} + \frac{i\text{Subst}\left(\int (-1+x^2)\,dx, x, -i\text{csch}(x)\right)}{a}$$
$$= \frac{\coth^3(x)}{3a} - \frac{\text{csch}(x)}{a} - \frac{\text{csch}^3(x)}{3a}$$

Mathematica [**A**]
time = 0.04, size = 25, normalized size = 0.83

$$\frac{(-3-4\cosh(x)+\cosh(2x))\text{csch}(x)}{6a(1+\cosh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(a + a*Cosh[x]),x]
[Out] ((-3 - 4*Cosh[x] + Cosh[2*x])*Csch[x])/(6*a*(1 + Cosh[x]))
```

Maple [**A**]
time = 0.59, size = 29, normalized size = 0.97

method	result	size
default	$\frac{\frac{\left(\tanh^3\left(\frac{x}{2}\right)\right)}{3}+2\tanh\left(\frac{x}{2}\right)-\frac{1}{\tanh\left(\frac{x}{2}\right)}}{4a}$	29
risch	$-\frac{2\left(3\,\mathrm{e}^{3x}+3\,\mathrm{e}^{2x}+\mathrm{e}^{x}-1\right)}{3\left(\mathrm{e}^{x}+1\right)^3 a\left(\mathrm{e}^{x}-1\right)}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
[Out] 1/4/a*(1/3*tanh(1/2*x)^3+2*tanh(1/2*x)-1/tanh(1/2*x))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(26) = 52.
time = 0.28, size = 121, normalized size = 4.03

$$-\frac{2\,e^{(-x)}}{3\left(2\,ae^{(-x)}-2\,ae^{(-3\,x)}-ae^{(-4\,x)}+a\right)}-\frac{2\,e^{(-2\,x)}}{2\,ae^{(-x)}-2\,ae^{(-3\,x)}-ae^{(-4\,x)}+a}-\frac{2\,e^{(-3\,x)}}{2\,ae^{(-x)}-2\,ae^{(-3\,x)}-ae^{(-4\,x)}+a}+\frac{2}{3\left(2\,ae^{(-x)}-2\,ae^{(-3\,x)}-ae^{(-4\,x)}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -2/3*e^(-x)/(2*a*e^(-x) - 2*a*e^(-3*x) - a*e^(-4*x) + a) - 2*e^(-2*x)/(2*a*
e^(-x) - 2*a*e^(-3*x) - a*e^(-4*x) + a) - 2*e^(-3*x)/(2*a*e^(-x) - 2*a*e^(-
3*x) - a*e^(-4*x) + a) + 2/3/(2*a*e^(-x) - 2*a*e^(-3*x) - a*e^(-4*x) + a)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(26) = 52$.
time = 0.37, size = 91, normalized size = 3.03

$$-\frac{2\left(3\cosh\left(x\right)^{2}+2\left(3\cosh\left(x\right)+2\right)\sinh\left(x\right)+3\sinh\left(x\right)^{2}+2\cosh\left(x\right)+1\right)}{3\left(a\cosh\left(x\right)^{3}+a\sinh\left(x\right)^{3}+2\,a\cosh\left(x\right)^{2}+\left(3\,a\cosh\left(x\right)+2\,a\right)\sinh\left(x\right)^{2}-a\cosh\left(x\right)+\left(3\,a\cosh\left(x\right)^{2}+4\,a\cosh\left(x\right)+a\right)\sinh\left(x\right)-2\,a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+a*cosh(x)),x, algorithm="fricas")
[Out] -2/3*(3*cosh(x)^2 + 2*(3*cosh(x) + 2)*sinh(x) + 3*sinh(x)^2 + 2*cosh(x) + 1
)/(a*cosh(x)^3 + a*sinh(x)^3 + 2*a*cosh(x)^2 + (3*a*cosh(x) + 2*a)*sinh(x)^
2 - a*cosh(x) + (3*a*cosh(x)^2 + 4*a*cosh(x) + a)*sinh(x) - 2*a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\coth^{2}\left(x\right)}{\cosh\left(x\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(a+a*cosh(x)),x)
[Out] Integral(coth(x)**2/(cosh(x) + 1), x)/a
```

Giac [A]
time = 0.41, size = 35, normalized size = 1.17

$$-\frac{1}{2\,a(e^{x}-1)}-\frac{9\,e^{(2\,x)}+12\,e^{x}+7}{6\,a(e^{x}+1)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+a*cosh(x)),x, algorithm="giac")
[Out] -1/2/(a*(e^x - 1)) - 1/6*(9*e^(2*x) + 12*e^x + 7)/(a*(e^x + 1)^3)
```

Mupad [B]
time = 0.93, size = 92, normalized size = 3.07

$$-\frac{\frac{e^{2\,x}}{2\,a}+\frac{1}{2\,a}+\frac{e^{x}}{3\,a}}{3\,e^{2\,x}+e^{3\,x}+3\,e^{x}+1}-\frac{\frac{1}{6\,a}+\frac{e^{x}}{2\,a}}{e^{2\,x}+2\,e^{x}+1}-\frac{1}{2\,a\,\left(e^{x}-1\right)}-\frac{1}{2\,a\,\left(e^{x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a + a*cosh(x)),x)
[Out] - (exp(2*x)/(2*a) + 1/(2*a) + exp(x)/(3*a))/(3*exp(2*x) + exp(3*x) + 3*exp(
x) + 1) - (1/(6*a) + exp(x)/(2*a))/(exp(2*x) + 2*exp(x) + 1) - 1/(2*a*(exp(
x) - 1)) - 1/(2*a*(exp(x) + 1))
```

3.195 $\int \frac{\coth^3(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=46

$$-\frac{3\tanh^{-1}(\cosh(x))}{8a}+\frac{\coth^4(x)}{4a}-\frac{3\coth(x)\text{csch}(x)}{8a}-\frac{\coth^3(x)\text{csch}(x)}{4a}$$

```
[Out] -3/8*arctanh(cosh(x))/a+1/4*coth(x)^4/a-3/8*coth(x)*csch(x)/a-1/4*coth(x)^3
*csch(x)/a
```

Rubi [A]
time = 0.08, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2785, 2687, 30, 2691, 3855}

$$\frac{\coth^4(x)}{4a}-\frac{3\tanh^{-1}(\cosh(x))}{8a}-\frac{\coth^3(x)\text{csch}(x)}{4a}-\frac{3\coth(x)\text{csch}(x)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/(a + a*Cosh[x]),x]
```

```
[Out] (-3*ArcTanh[Cosh[x]])/(8*a) + Coth[x]^4/(4*a) - (3*Coth[x]*Csch[x])/(8*a) -
 (Coth[x]^3*Csch[x])/(4*a)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^3(x)}{a+a\cosh(x)}\,dx &= \frac{\int \coth^4(x)\operatorname{csch}(x)\,dx}{a}-\frac{\int \coth^3(x)\operatorname{csch}^2(x)\,dx}{a}\\ &= -\frac{\coth^3(x)\operatorname{csch}(x)}{4a}+\frac{3\int \coth^2(x)\operatorname{csch}(x)\,dx}{4a}+\frac{\operatorname{Subst}\left(\int x^3\,dx,x,i\coth(x)\right)}{a}\\ &= \frac{\coth^4(x)}{4a}-\frac{3\coth(x)\operatorname{csch}(x)}{8a}-\frac{\coth^3(x)\operatorname{csch}(x)}{4a}+\frac{3\int \operatorname{csch}(x)\,dx}{8a}\\ &= -\frac{3\tanh^{-1}(\cosh(x))}{8a}+\frac{\coth^4(x)}{4a}-\frac{3\coth(x)\operatorname{csch}(x)}{8a}-\frac{\coth^3(x)\operatorname{csch}(x)}{4a}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 60, normalized size = 1.30

$$\frac{-8-2\coth^2\left(\frac{x}{2}\right)-12\cosh^2\left(\frac{x}{2}\right)\left(\log\left(\cosh\left(\frac{x}{2}\right)\right)-\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)+\operatorname{sech}^2\left(\frac{x}{2}\right)}{16a(1+\cosh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/(a + a*Cosh[x]),x]
```

```
[Out] (-8 - 2*Coth[x/2]^2 - 12*Cosh[x/2]^2*(Log[Cosh[x/2]] - Log[Sinh[x/2]]) + Se
ch[x/2]^2)/(16*a*(1 + Cosh[x]))
```

Maple [A]
time = 0.59, size = 38, normalized size = 0.83

method	result	size
default	$\frac{\frac{\left(\tanh^4\left(\frac{x}{2}\right)\right)}{4}+\frac{3\left(\tanh^2\left(\frac{x}{2}\right)\right)}{2}-\frac{1}{2\tanh\left(\frac{x}{2}\right)^2}+3\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{8a}$	38
risch	$-\frac{e^x\left(5\,e^{4x}+2\,e^{3x}+2\,e^{2x}+2\,e^x+5\right)}{4(e^x-1)^2 a(e^x+1)^4}-\frac{3\ln(e^x+1)}{8a}+\frac{3\ln(e^x-1)}{8a}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/a*(1/4*tanh(1/2*x)^4+3/2*tanh(1/2*x)^2-1/2/tanh(1/2*x)^2+3*ln(tanh(1/2*
x)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(38) = 76$.
time = 0.26, size = 103, normalized size = 2.24

$$-\frac{5\,e^{(-x)} + 2\,e^{(-2\,x)} + 2\,e^{(-3\,x)} + 2\,e^{(-4\,x)} + 5\,e^{(-5\,x)}}{4\,(2\,ae^{(-x)} - ae^{(-2\,x)} - 4\,ae^{(-3\,x)} - ae^{(-4\,x)} + 2\,ae^{(-5\,x)} + ae^{(-6\,x)} + a)} - \frac{3\,\log\left(e^{(-x)} + 1\right)}{8\,a} + \frac{3\,\log\left(e^{(-x)} - 1\right)}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -1/4*(5*e^(-x) + 2*e^(-2*x) + 2*e^(-3*x) + 2*e^(-4*x) + 5*e^(-5*x))/(2*a*e^
(-x) - a*e^(-2*x) - 4*a*e^(-3*x) - a*e^(-4*x) + 2*a*e^(-5*x) + a*e^(-6*x) +
 a) - 3/8*log(e^(-x) + 1)/a + 3/8*log(e^(-x) - 1)/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 631 vs. $2(38) = 76$.
time = 0.48, size = 631, normalized size = 13.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] -1/8*(10*cosh(x)^5 + 2*(25*cosh(x) + 2)*sinh(x)^4 + 10*sinh(x)^5 + 4*cosh(x
)^4 + 4*(25*cosh(x)^2 + 4*cosh(x) + 1)*sinh(x)^3 + 4*cosh(x)^3 + 4*(25*cosh
(x)^3 + 6*cosh(x)^2 + 3*cosh(x) + 1)*sinh(x)^2 + 4*cosh(x)^2 + 3*(cosh(x)^6
 + 2*(3*cosh(x) + 1)*sinh(x)^5 + sinh(x)^6 + 2*cosh(x)^5 + (15*cosh(x)^2 +
10*cosh(x) - 1)*sinh(x)^4 - cosh(x)^4 + 4*(5*cosh(x)^3 + 5*cosh(x)^2 - cosh
(x) - 1)*sinh(x)^3 - 4*cosh(x)^3 + (15*cosh(x)^4 + 20*cosh(x)^3 - 6*cosh(x)
^2 - 12*cosh(x) - 1)*sinh(x)^2 - cosh(x)^2 + 2*(3*cosh(x)^5 + 5*cosh(x)^4 -
 2*cosh(x)^3 - 6*cosh(x)^2 - cosh(x) + 1)*sinh(x) + 2*cosh(x) + 1)*log(cosh
(x) + sinh(x) + 1) - 3*(cosh(x)^6 + 2*(3*cosh(x) + 1)*sinh(x)^5 + sinh(x)^6
 + 2*cosh(x)^5 + (15*cosh(x)^2 + 10*cosh(x) - 1)*sinh(x)^4 - cosh(x)^4 + 4*
(5*cosh(x)^3 + 5*cosh(x)^2 - cosh(x) - 1)*sinh(x)^3 - 4*cosh(x)^3 + (15*cos
h(x)^4 + 20*cosh(x)^3 - 6*cosh(x)^2 - 12*cosh(x) - 1)*sinh(x)^2 - cosh(x)^2
 + 2*(3*cosh(x)^5 + 5*cosh(x)^4 - 2*cosh(x)^3 - 6*cosh(x)^2 - cosh(x) + 1)*
sinh(x) + 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*(25*cosh(x)^4 + 8*c
osh(x)^3 + 6*cosh(x)^2 + 4*cosh(x) + 5)*sinh(x) + 10*cosh(x))/(a*cosh(x)^6
```

```
+ a*sinh(x)^6 + 2*a*cosh(x)^5 + 2*(3*a*cosh(x) + a)*sinh(x)^5 - a*cosh(x)^4
 + (15*a*cosh(x)^2 + 10*a*cosh(x) - a)*sinh(x)^4 - 4*a*cosh(x)^3 + 4*(5*a*c
osh(x)^3 + 5*a*cosh(x)^2 - a*cosh(x) - a)*sinh(x)^3 - a*cosh(x)^2 + (15*a*c
osh(x)^4 + 20*a*cosh(x)^3 - 6*a*cosh(x)^2 - 12*a*cosh(x) - a)*sinh(x)^2 + 2
*a*cosh(x) + 2*(3*a*cosh(x)^5 + 5*a*cosh(x)^4 - 2*a*cosh(x)^3 - 6*a*cosh(x)
^2 - a*cosh(x) + a)*sinh(x) + a)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\coth^3(x)}{\cosh(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(a+a*cosh(x)),x)
```

```
[Out] Integral(coth(x)**3/(cosh(x) + 1), x)/a
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(38) = 76.
time = 0.43, size = 94, normalized size = 2.04

$$-\frac{3\,\log\left(e^{(-x)}+e^x+2\right)}{16\,a}+\frac{3\,\log\left(e^{(-x)}+e^x-2\right)}{16\,a}-\frac{3\,e^{(-x)}+3\,e^x-2}{16\,a(e^{(-x)}+e^x-2)}+\frac{9\left(e^{(-x)}+e^x\right)^2+4\,e^{(-x)}+4\,e^x-12}{32\,a(e^{(-x)}+e^x+2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -3/16*log(e^(-x) + e^x + 2)/a + 3/16*log(e^(-x) + e^x - 2)/a - 1/16*(3*e^(-
x) + 3*e^x - 2)/(a*(e^(-x) + e^x - 2)) + 1/32*(9*(e^(-x) + e^x)^2 + 4*e^(-x
) + 4*e^x - 12)/(a*(e^(-x) + e^x + 2)^2)
```

Mupad [**B**]
time = 0.96, size = 132, normalized size = 2.87

$$\frac{3}{2\,a\,\left(\mathrm{e}^{2\,x}+2\,\mathrm{e}^x+1\right)}-\frac{1}{4\,a\,\left(\mathrm{e}^{2\,x}-2\,\mathrm{e}^x+1\right)}+\frac{1}{2\,a\,\left(6\,\mathrm{e}^{2\,x}+4\,\mathrm{e}^{3\,x}+\mathrm{e}^{4\,x}+4\,\mathrm{e}^x+1\right)}-\frac{1}{4\,a\,\left(\mathrm{e}^x-1\right)}-\frac{1}{a\,\left(\mathrm{e}^x+1\right)}-\frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^x\,\sqrt{-a^2}}{a}\right)}{4\,\sqrt{-a^2}}-\frac{1}{a\,\left(3\,\mathrm{e}^{2\,x}+\mathrm{e}^{3\,x}+3\,\mathrm{e}^x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a + a*cosh(x)),x)
```

```
[Out] 3/(2*a*(exp(2*x) + 2*exp(x) + 1)) - 1/(4*a*(exp(2*x) - 2*exp(x) + 1)) + 1/(
2*a*(6*exp(2*x) + 4*exp(3*x) + exp(4*x) + 4*exp(x) + 1)) - 1/(4*a*(exp(x) -
 1)) - 1/(a*(exp(x) + 1)) - (3*atan((exp(x)*(-a^2)^(1/2))/a))/(4*(-a^2)^(1/
2)) - 1/(a*(3*exp(2*x) + exp(3*x) + 3*exp(x) + 1))
```

3.196 $\int \frac{\coth^4(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=41

$$\frac{\coth^5(x)}{5a}-\frac{\operatorname{csch}(x)}{a}-\frac{2\operatorname{csch}^3(x)}{3a}-\frac{\operatorname{csch}^5(x)}{5a}$$

```
[Out] 1/5*coth(x)^5/a-csch(x)/a-2/3*csch(x)^3/a-1/5*csch(x)^5/a
```

Rubi [A]
time = 0.06, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2785, 2687, 30, 2686, 200}

$$\frac{\coth^5(x)}{5a}-\frac{\operatorname{csch}^5(x)}{5a}-\frac{2\operatorname{csch}^3(x)}{3a}-\frac{\operatorname{csch}(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^4/(a + a*Cosh[x]),x]
```

```
[Out] Coth[x]^5/(5*a) - Csch[x]/a - (2*Csch[x]^3)/(3*a) - Csch[x]^5/(5*a)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^4(x)}{a+a\cosh(x)}\,dx &= \frac{\int \coth^5(x)\text{csch}(x)\,dx}{a}-\frac{\int \coth^4(x)\text{csch}^2(x)\,dx}{a}\\ &= -\frac{i\text{Subst}\left(\int x^4\,dx,x,i\coth(x)\right)}{a}-\frac{i\text{Subst}\left(\int \left(-1+x^2\right)^2\,dx,x,-i\text{csch}(x)\right)}{a}\\ &= \frac{\coth^5(x)}{5a}-\frac{i\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,-i\text{csch}(x)\right)}{a}\\ &= \frac{\coth^5(x)}{5a}-\frac{\text{csch}(x)}{a}-\frac{2\text{csch}^3(x)}{3a}-\frac{\text{csch}^5(x)}{5a}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 41, normalized size = 1.00

$$-\frac{(-25+8\cosh(x)+36\cosh(2x)+24\cosh(3x)-3\cosh(4x))\text{csch}^3(x)}{120a(1+\cosh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4/(a + a*Cosh[x]),x]
```

```
[Out] -1/120*((-25 + 8*Cosh[x] + 36*Cosh[2*x] + 24*Cosh[3*x] - 3*Cosh[4*x])*Csch[
x]^3)/(a*(1 + Cosh[x]))
```

Maple [A]
time = 0.60, size = 45, normalized size = 1.10

method	result	size
default	$\frac{\frac{\left(\tanh^5\left(\frac{x}{2}\right)\right)}{5}+\frac{4\left(\tanh^3\left(\frac{x}{2}\right)\right)}{3}+6\tanh\left(\frac{x}{2}\right)-\frac{1}{3\tanh\left(\frac{x}{2}\right)^3}-\frac{4}{\tanh\left(\frac{x}{2}\right)}}{16a}$	45
risch	$-\frac{2\left(15\,e^{7x}+15\,e^{6x}-5\,e^{5x}-25\,e^{4x}+13\,e^{3x}+21\,e^{2x}+9\,e^{x}-3\right)}{15\left(e^{x}-1\right)^3 a\left(e^{x}+1\right)^5}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(a+a*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/a*(1/5*tanh(1/2*x)^5+4/3*tanh(1/2*x)^3+6*tanh(1/2*x)-1/3/tanh(1/2*x)^3
-4/tanh(1/2*x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 469 vs. $2(35) = 70$.
time = 0.27, size = 469, normalized size = 11.44

$$-\frac{6\,e^{(-x)}}{5\,(2\,ae^{(-x)} - 2\,ae^{(-2\,x)} - 6\,ae^{(-3\,x)} + 6\,ae^{(-5\,x)} + 2\,ae^{(-6\,x)} - 2\,ae^{(-7\,x)} - ae^{(-8\,x)} + a)} - \frac{14\,e^{(-2\,x)}}{5\,(2\,ae^{(-x)} - 2\,ae^{(-2\,x)} - 6\,ae^{(-3\,x)} + 6\,ae^{(-5\,x)} + 2\,ae^{(-6\,x)} - 2\,ae^{(-7\,x)} - ae^{(-8\,x)} + a)} - \frac{26\,e^{(-3\,x)}}{15\,(2\,ae^{(-x)} - 2\,ae^{(-2\,x)} - 6\,ae^{(-3\,x)} + 6\,ae^{(-5\,x)} + 2\,ae^{(-6\,x)} - 2\,ae^{(-7\,x)} - ae^{(-8\,x)} + a)} + \frac{10\,e^{(-4\,x)}}{3\,(2\,ae^{(-x)} - 2\,ae^{(-2\,x)} - 6\,ae^{(-3\,x)} + 6\,ae^{(-5\,x)} + 2\,ae^{(-6\,x)} - 2\,ae^{(-7\,x)} - ae^{(-8\,x)} + a)} + \frac{2\,e^{(-5\,x)}}{3\,(2\,ae^{(-x)} - 2\,ae^{(-2\,x)} - 6\,ae^{(-3\,x)} + 6\,ae^{(-5\,x)} + 2\,ae^{(-6\,x)} - 2\,ae^{(-7\,x)} - ae^{(-8\,x)} + a)} - \frac{2\,e^{(-6\,x)}}{2\,ae^{(-x)} - 2\,ae^{(-2\,x)} - 6\,ae^{(-3\,x)} + 6\,ae^{(-5\,x)} + 2\,ae^{(-6\,x)} - 2\,ae^{(-7\,x)} - ae^{(-8\,x)} + a} - \frac{2\,e^{(-7\,x)}}{2\,ae^{(-x)} - 2\,ae^{(-2\,x)} - 6\,ae^{(-3\,x)} + 6\,ae^{(-5\,x)} + 2\,ae^{(-6\,x)} - 2\,ae^{(-7\,x)} - ae^{(-8\,x)} + a} + \frac{2}{5\,(2\,ae^{(-x)} - 2\,ae^{(-2\,x)} - 6\,ae^{(-3\,x)} + 6\,ae^{(-5\,x)} + 2\,ae^{(-6\,x)} - 2\,ae^{(-7\,x)} - ae^{(-8\,x)} + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -6/5*e^(-x)/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*
e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a) - 14/5*e^(-2*x)/(2*a*e^(-x) - 2*a
*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e
^(-8*x) + a) - 26/15*e^(-3*x)/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6
*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a) + 10/3*e^(-4*x)
/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*e^(-6*x) -
2*a*e^(-7*x) - a*e^(-8*x) + a) + 2/3*e^(-5*x)/(2*a*e^(-x) - 2*a*e^(-2*x) -
6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a)
 - 2*e^(-6*x)/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*
a*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a) - 2*e^(-7*x)/(2*a*e^(-x) - 2*a*
e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e^
(-8*x) + a) + 2/5/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x)
+ 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(35) = 70$.
time = 0.41, size = 224, normalized size = 5.46

$$-\frac{2\,(15\,\cosh(x)^4 + 6\,(10\,\cosh(x) + 3)\sinh(x)^3 + 15\,\sinh(x)^4 + 12\,\cosh(x)^3 + 2\,(45\,\cosh(x)^2 + 18\,\cosh(x) + 2)\sinh(x)^2 + 4\,\cosh(x)^2 + 2\,(30\,\cosh(x)^3 + 27\,\cosh(x)^2 - 14\,\cosh(x) - 23)\sinh(x) - 4\,\cosh(x) + 13)}{15\,(a\cosh(x)^5 + a\sinh(x)^5 + 2\,a\cosh(x)^4 + (5\,a\cosh(x) + 2\,a)\sinh(x)^4 - 3\,a\cosh(x)^3 + (10\,a\cosh(x)^2 + 8\,a\cosh(x) - a)\sinh(x)^3 - 8\,a\cosh(x)^2 + (10\,a\cosh(x)^3 + 12\,a\cosh(x)^2 - 9\,a\cosh(x) - 8\,a)\sinh(x)^2 + 2\,a\cosh(x) + (5\,a\cosh(x)^4 + 8\,a\cosh(x)^3 - 3\,a\cosh(x)^2 - 8\,a\cosh(x) - 2\,a)\sinh(x) + 6\,a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] -2/15*(15*cosh(x)^4 + 6*(10*cosh(x) + 3)*sinh(x)^3 + 15*sinh(x)^4 + 12*cosh
(x)^3 + 2*(45*cosh(x)^2 + 18*cosh(x) + 2)*sinh(x)^2 + 4*cosh(x)^2 + 2*(30*c
osh(x)^3 + 27*cosh(x)^2 - 14*cosh(x) - 23)*sinh(x) - 4*cosh(x) + 13)/(a*cos
h(x)^5 + a*sinh(x)^5 + 2*a*cosh(x)^4 + (5*a*cosh(x) + 2*a)*sinh(x)^4 - 3*a*
cosh(x)^3 + (10*a*cosh(x)^2 + 8*a*cosh(x) - a)*sinh(x)^3 - 8*a*cosh(x)^2 +
(10*a*cosh(x)^3 + 12*a*cosh(x)^2 - 9*a*cosh(x) - 8*a)*sinh(x)^2 + 2*a*cosh(
x) + (5*a*cosh(x)^4 + 8*a*cosh(x)^3 - 3*a*cosh(x)^2 - 8*a*cosh(x) - 2*a)*si
nh(x) + 6*a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\coth^4(x)}{\cosh(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4/(a+a*cosh(x)),x)
```

```
[Out] Integral(coth(x)**4/(cosh(x) + 1), x)/a
```

Giac [A]
time = 0.39, size = 59, normalized size = 1.44

$$-\frac{15\,e^{(2\,x)}-24\,e^{x}+13}{24\,a{(e^{x}-1)}^{3}}-\frac{165\,e^{(4\,x)}+480\,e^{(3\,x)}+650\,e^{(2\,x)}+400\,e^{x}+113}{120\,a{(e^{x}+1)}^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -1/24*(15*e^(2*x) - 24*e^x + 13)/(a*(e^x - 1)^3) - 1/120*(165*e^(4*x) + 480
*e^(3*x) + 650*e^(2*x) + 400*e^x + 113)/(a*(e^x + 1)^5)
```

Mupad [B]
time = 1.03, size = 263, normalized size = 6.41

$$\frac{1}{6\,a\,\left(3\,{\mathrm{e}}^{2\,x}-{\mathrm{e}}^{3\,x}-3\,{\mathrm{e}}^{x}+1\right)}-\frac{\frac{3\,{\mathrm{e}}^{2\,x}}{8\,a}+\frac{11\,{\mathrm{e}}^{3\,x}}{40\,a}+\frac{1}{8\,a}+\frac{17\,{\mathrm{e}}^{x}}{40\,a}}{6\,{\mathrm{e}}^{2\,x}+4\,{\mathrm{e}}^{3\,x}+{\mathrm{e}}^{4\,x}+4\,{\mathrm{e}}^{x}+1}-\frac{\frac{11\,{\mathrm{e}}^{2\,x}}{40\,a}+\frac{17}{120\,a}+\frac{{\mathrm{e}}^{x}}{4\,a}}{3\,{\mathrm{e}}^{2\,x}+{\mathrm{e}}^{3\,x}+3\,{\mathrm{e}}^{x}+1}-\frac{\frac{1}{8\,a}+\frac{11\,{\mathrm{e}}^{x}}{40\,a}}{{\mathrm{e}}^{2\,x}+2\,{\mathrm{e}}^{x}+1}-\frac{\frac{17\,{\mathrm{e}}^{2\,x}}{20\,a}+\frac{{\mathrm{e}}^{3\,x}}{2\,a}+\frac{11\,{\mathrm{e}}^{4\,x}}{40\,a}+\frac{11}{40\,a}+\frac{{\mathrm{e}}^{x}}{2\,a}}{10\,{\mathrm{e}}^{2\,x}+10\,{\mathrm{e}}^{3\,x}+5\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{5\,x}+5\,{\mathrm{e}}^{x}+1}-\frac{1}{4\,a\,\left({\mathrm{e}}^{2\,x}-2\,{\mathrm{e}}^{x}+1\right)}-\frac{5}{8\,a\,\left({\mathrm{e}}^{x}-1\right)}-\frac{11}{40\,a\,\left({\mathrm{e}}^{x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(a + a*cosh(x)),x)
```

```
[Out] 1/(6*a*(3*exp(2*x) - exp(3*x) - 3*exp(x) + 1)) - ((3*exp(2*x))/(8*a) + (11*
exp(3*x))/(40*a) + 1/(8*a) + (17*exp(x))/(40*a))/(6*exp(2*x) + 4*exp(3*x) +
 exp(4*x) + 4*exp(x) + 1) - ((11*exp(2*x))/(40*a) + 17/(120*a) + exp(x)/(4*
a))/(3*exp(2*x) + exp(3*x) + 3*exp(x) + 1) - (1/(8*a) + (11*exp(x))/(40*a))
/(exp(2*x) + 2*exp(x) + 1) - ((17*exp(2*x))/(20*a) + exp(3*x)/(2*a) + (11*e
xp(4*x))/(40*a) + 11/(40*a) + exp(x)/(2*a))/(10*exp(2*x) + 10*exp(3*x) + 5*
exp(4*x) + exp(5*x) + 5*exp(x) + 1) - 1/(4*a*(exp(2*x) - 2*exp(x) + 1)) - 5
/(8*a*(exp(x) - 1)) - 11/(40*a*(exp(x) + 1))
```

3.197 $\int \sqrt{a+b\cosh(x)} \tanh(x)\, dx$

Optimal. Leaf size=37

$$-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\cosh(x)}}{\sqrt{a}}\right)+2\sqrt{a+b\cosh(x)}$$

```
[Out] -2*arctanh((a+b*cosh(x))^(1/2)/a^(1/2))*a^(1/2)+2*(a+b*cosh(x))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2800, 52, 65, 213}

$$2\sqrt{a+b\cosh(x)}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\cosh(x)}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cosh[x]]*Tanh[x],x]
```

```
[Out] -2*Sqrt[a]*ArcTanh[Sqrt[a + b*Cosh[x]]/Sqrt[a]] + 2*Sqrt[a + b*Cosh[x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+b\cosh(x)}\,\tanh(x)\,dx &= \text{Subst}\left(\int \frac{\sqrt{a+x}}{x}\,dx, x, b\cosh(x)\right)\\ &= 2\sqrt{a+b\cosh(x)} + a\text{Subst}\left(\int \frac{1}{x\sqrt{a+x}}\,dx, x, b\cosh(x)\right)\\ &= 2\sqrt{a+b\cosh(x)} + (2a)\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\cosh(x)}\right)\\ &= -2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cosh(x)}}{\sqrt{a}}\right)+2\sqrt{a+b\cosh(x)}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 37, normalized size = 1.00

$$-2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cosh(x)}}{\sqrt{a}}\right)+2\sqrt{a+b\cosh(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cosh[x]]*Tanh[x],x]
[Out] -2*Sqrt[a]*ArcTanh[Sqrt[a + b*Cosh[x]]/Sqrt[a]] + 2*Sqrt[a + b*Cosh[x]]
```

Maple [A]
time = 0.93, size = 30, normalized size = 0.81

method	result	size
derivativedivides	$-2\,\text{arctanh}\left(\frac{\sqrt{a+b\cosh\left(x\right)}}{\sqrt{a}}\right)\sqrt{a}+2\sqrt{a+b\cosh\left(x\right)}$	30
default	$-2\,\text{arctanh}\left(\frac{\sqrt{a+b\cosh\left(x\right)}}{\sqrt{a}}\right)\sqrt{a}+2\sqrt{a+b\cosh\left(x\right)}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(x))^(1/2)*tanh(x),x,method=_RETURNVERBOSE)
```

```
[Out] -2*arctanh((a+b*cosh(x))^(1/2)/a^(1/2))*a^(1/2)+2*(a+b*cosh(x))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(1/2)*tanh(x),x, algorithm="maxima")
[Out] integrate(sqrt(b*cosh(x) + a)*tanh(x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(29) = 58$.
time = 0.60, size = 376, normalized size = 10.16

$$\left[\frac{1}{2}\sqrt{a}\log\left(-\frac{b^2\cosh\left(x\right)^4+b^2\sinh\left(x\right)^4+16\,ab\cosh\left(x\right)^3+4\left(b^2\cosh\left(x\right)+4\,ab\right)\sinh\left(x\right)^3+16\,ab\cosh\left(x\right)+2\left(16\,a^2+b^2\right)\cosh\left(x\right)^2+2\left(3\,b^2\cosh\left(x\right)^2+24\,ab\cosh\left(x\right)+16\,a^2+b^2\right)\sinh\left(x\right)^2-8\left(b\cosh\left(x\right)^3+b\sinh\left(x\right)^3+4\,a\cosh\left(x\right)^2+\left(3\,b\cosh\left(x\right)+4\,a\right)\sinh\left(x\right)^2+b\cosh\left(x\right)+\left(3\,b\cosh\left(x\right)^2+8\,a\cosh\left(x\right)+b\right)\sinh\left(x\right)\right)\sqrt{b\cosh\left(x\right)+a}\sqrt{a}+b^2+4\left(b^2\cosh\left(x\right)^3+12\,ab\cosh\left(x\right)^2+4\,ab+\left(16\,a^2+b^2\right)\cosh\left(x\right)\right)\sinh\left(x\right)}{\cosh\left(x\right)^4+4\cosh\left(x\right)\sinh\left(x\right)^3+\sinh\left(x\right)^4+2\left(3\cosh\left(x\right)^2+1\right)\sinh\left(x\right)^2+2\cosh\left(x\right)^2+4\left(\cosh\left(x\right)^3+\cosh\left(x\right)\right)\sinh\left(x\right)+1}\right)+2\sqrt{b\cosh\left(x\right)+a},\ \sqrt{-a}\arctan\left(\frac{\left(b\cosh\left(x\right)^2+b\sinh\left(x\right)^2+4\,a\cosh\left(x\right)+2\left(b\cosh\left(x\right)+2\,a\right)\sinh\left(x\right)+b\right)\sqrt{b\cosh\left(x\right)+a}\sqrt{-a}}{2\left(ab\cosh\left(x\right)^2+ab\sinh\left(x\right)^2+2\,a^2\cosh\left(x\right)+ab+2\left(ab\cosh\left(x\right)+a^2\right)\sinh\left(x\right)\right)}\right)+2\sqrt{b\cosh\left(x\right)+a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(1/2)*tanh(x),x, algorithm="fricas")
[Out] [1/2*sqrt(a)*log(-(b^2*cosh(x)^4 + b^2*sinh(x)^4 + 16*a*b*cosh(x)^3 + 4*(b^
2*cosh(x) + 4*a*b)*sinh(x)^3 + 16*a*b*cosh(x) + 2*(16*a^2 + b^2)*cosh(x)^2
+ 2*(3*b^2*cosh(x)^2 + 24*a*b*cosh(x) + 16*a^2 + b^2)*sinh(x)^2 - 8*(b*cosh
(x)^3 + b*sinh(x)^3 + 4*a*cosh(x)^2 + (3*b*cosh(x) + 4*a)*sinh(x)^2 + b*cos
h(x) + (3*b*cosh(x)^2 + 8*a*cosh(x) + b)*sinh(x))*sqrt(b*cosh(x) + a)*sqrt(
a) + b^2 + 4*(b^2*cosh(x)^3 + 12*a*b*cosh(x)^2 + 4*a*b + (16*a^2 + b^2)*cos
h(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^
2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) + 2*
sqrt(b*cosh(x) + a), sqrt(-a)*arctan(1/2*(b*cosh(x)^2 + b*sinh(x)^2 + 4*a*c
osh(x) + 2*(b*cosh(x) + 2*a)*sinh(x) + b)*sqrt(b*cosh(x) + a)*sqrt(-a)/(a*b
*cosh(x)^2 + a*b*sinh(x)^2 + 2*a^2*cosh(x) + a*b + 2*(a*b*cosh(x) + a^2)*si
nh(x))) + 2*sqrt(b*cosh(x) + a)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\cosh\left(x\right)}\,\tanh\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))**(1/2)*tanh(x),x)
[Out] Integral(sqrt(a + b*cosh(x))*tanh(x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(1/2)*tanh(x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cosh(x) + a)*tanh(x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \tanh(x)\ \sqrt{a+b\cosh\left(x\right)}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)*(a + b*cosh(x))^(1/2),x)
```

```
[Out] int(tanh(x)*(a + b*cosh(x))^(1/2), x)
```

3.198 $\int \frac{\tanh(x)}{\sqrt{a+b\cosh(x)}}\,dx$

Optimal. Leaf size=24

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\cosh(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] -2*arctanh((a+b*cosh(x))^(1/2)/a^(1/2))/a^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2800, 65, 213}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\cosh(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/Sqrt[a + b*Cosh[x]],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Cosh[x]]/Sqrt[a]])/Sqrt[a]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tanh(x)}{\sqrt{a+b\cosh(x)}}\,dx = \text{Subst}\left(\int \frac{1}{x\sqrt{a+x}}\,dx, x, b\cosh(x)\right)$$
$$= 2\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\cosh(x)}\right)$$
$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\cosh(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Mathematica [**A**]
time = 0.01, size = 24, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\cosh(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/Sqrt[a + b*Cosh[x]],x]
[Out] (-2*ArcTanh[Sqrt[a + b*Cosh[x]]/Sqrt[a]])/Sqrt[a]
```

Maple [**A**]
time = 1.01, size = 19, normalized size = 0.79

method	result	size
derivativedivides	$-\frac{2\,\text{arctanh}\left(\frac{\sqrt{a+b\cosh(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$	19
default	$-\frac{2\,\text{arctanh}\left(\frac{\sqrt{a+b\cosh(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*cosh(x))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*arctanh((a+b*cosh(x))^(1/2)/a^(1/2))/a^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*cosh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)/sqrt(b*cosh(x) + a), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(18) = 36$.
time = 0.46, size = 356, normalized size = 14.83

$$\left[\frac{\log\left(\frac{b^{2}\cosh(x)^{4}+b^{2}\sinh(x)^{4}+16\,ab\cosh(x)^{3}+4\,\left(b^{2}\cosh(x)+4\,ab\right)\sinh(x)^{3}+16\,ab\cosh(x)+2\,\left(16\,a^{2}+b^{2}\right)\cosh(x)^{2}+2\,\left(3\,b^{2}\cosh(x)^{2}+24\,ab\cosh(x)+16\,a^{2}+b^{2}\right)\sinh(x)^{2}-8\,\left(b\cosh(x)^{3}+b\sinh(x)^{3}+4\,a\cosh(x)^{2}+\left(3\,b\cosh(x)+4\,a\right)\sinh(x)^{2}+b\cosh(x)+\left(3\,b\cosh(x)^{2}+8\,a\cosh(x)+b\right)\sinh(x)\right)\sqrt{b\cosh\left(x\right)+a}\,\sqrt{a}+b^{2}+4\,\left(b^{2}\cosh(x)^{3}+12\,ab\cosh(x)^{2}+4\,ab+\left(16\,a^{2}+b^{2}\right)\cosh(x)\right)\sinh(x)}{\cosh(x)^{4}+4\,\cosh(x)\sinh(x)^{3}+\sinh(x)^{4}+2\,\left(3\,\cosh(x)^{2}+1\right)\sinh(x)^{2}+2\,\cosh(x)^{2}+4\,\left(\cosh(x)^{3}+\cosh(x)\right)\sinh(x)+1}\right)}{2\,\sqrt{a}},\ \frac{\sqrt{-a}\,\arctan\left(\frac{\left(b\cosh(x)^{2}+b\sinh(x)^{2}+4\,a\cosh(x)+2\,\left(b\cosh(x)+2\,a\right)\sinh(x)+b\right)\sqrt{b\cosh\left(x\right)+a}\,\sqrt{-a}}{2\,\left(ab\cosh(x)^{2}+ab\sinh(x)^{2}+2\,a^{2}\cosh(x)+ab+2\,\left(ab\cosh(x)+a^{2}\right)\sinh(x)\right)}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*cosh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log((b^2*cosh(x)^4 + b^2*sinh(x)^4 + 16*a*b*cosh(x)^3 + 4*(b^2*cosh(x)
 + 4*a*b)*sinh(x)^3 + 16*a*b*cosh(x) + 2*(16*a^2 + b^2)*cosh(x)^2 + 2*(3*b^
2*cosh(x)^2 + 24*a*b*cosh(x) + 16*a^2 + b^2)*sinh(x)^2 - 8*(b*cosh(x)^3 + b
*sinh(x)^3 + 4*a*cosh(x)^2 + (3*b*cosh(x) + 4*a)*sinh(x)^2 + b*cosh(x) + (3
*b*cosh(x)^2 + 8*a*cosh(x) + b)*sinh(x))*sqrt(b*cosh(x) + a)*sqrt(a) + b^2
+ 4*(b^2*cosh(x)^3 + 12*a*b*cosh(x)^2 + 4*a*b + (16*a^2 + b^2)*cosh(x))*sin
h(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*si
nh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1))/sqrt(a), sqrt
(-a)*arctan(1/2*(b*cosh(x)^2 + b*sinh(x)^2 + 4*a*cosh(x) + 2*(b*cosh(x) + 2
*a)*sinh(x) + b)*sqrt(b*cosh(x) + a)*sqrt(-a)/(a*b*cosh(x)^2 + a*b*sinh(x)^
2 + 2*a^2*cosh(x) + a*b + 2*(a*b*cosh(x) + a^2)*sinh(x)))/a]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh{\left(x \right)}}{\sqrt{a + b \cosh{\left(x \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*cosh(x))**(1/2),x)
```

```
[Out] Integral(tanh(x)/sqrt(a + b*cosh(x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*cosh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(x)/sqrt(b*cosh(x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\tanh(x)}{\sqrt{a + b\cosh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a + b*cosh(x))^(1/2),x)
```

```
[Out] int(tanh(x)/(a + b*cosh(x))^(1/2), x)
```

3.199 $\int \frac{A+B\sinh(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=56

$$\frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,\sqrt{a+b}}+\frac{B\log(a+b\cosh(x))}{b}$$

```
[Out] B*ln(a+b*cosh(x))/b+2*A*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^
(1/2)/(a+b)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4486, 2738, 214, 2747, 31}

$$\frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,\sqrt{a+b}}+\frac{B\log(a+b\cosh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(a + b*Cosh[x]),x]
```

```
[Out] (2*A*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]
) + (B*Log[a + b*Cosh[x]])/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
```

```
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\sinh(x)}{a+b\cosh(x)}\,dx &= \int \left(\frac{A}{a+b\cosh(x)}+\frac{B\sinh(x)}{a+b\cosh(x)}\right)dx\\
&= A\int \frac{1}{a+b\cosh(x)}\,dx + B\int \frac{\sinh(x)}{a+b\cosh(x)}\,dx\\
&= (2A)\text{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)+\frac{B\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\cosh(x)\right)}{b}\\
&= \frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,\sqrt{a+b}}+\frac{B\log(a+b\cosh(x))}{b}
\end{aligned}$$

Mathematica [**A**]
time = 0.06, size = 55, normalized size = 0.98

$$-\frac{2A\text{ArcTan}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}+\frac{B\log(a+b\cosh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(a + b*Cosh[x]),x]
```

```
[Out] (-2*A*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + (B*L
og[a + b*Cosh[x]])/b
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(46) = 92$.
time = 0.57, size = 112, normalized size = 2.00

method	result

default	$-\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b}-\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b}+\frac{\frac{2(Ba-Bb)\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-b\left(\tanh^2\left(\frac{x}{2}\right)\right)-a-b\right)}{2a-2b}+\frac{2Ab\,\text{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b}}\right.}{\sqrt{(a+b)(a-b)}}}{b}$
risch	$\frac{Bx}{b}+\frac{2xB\,a^2b}{-a^2b^2+b^4}-\frac{2xB\,b^3}{-a^2b^2+b^4}+\frac{\ln\left(\mathrm{e}^x+\frac{Aab-\sqrt{A^2a^2b^2-A^2b^4}}{A\,b^2}\right)B\,a^2}{(a^2-b^2)b}-\frac{b\ln\left(\mathrm{e}^x+\frac{Aab-\sqrt{A^2a^2b^2-A^2b^4}}{A\,b^2}\right)B}{a^2-b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -B/b*ln(tanh(1/2*x)-1)-B/b*ln(tanh(1/2*x)+1)+2/b*(1/2*(B*a-B*b)/(a-b)*ln(a*
tanh(1/2*x)^2-b*tanh(1/2*x)^2-a-b)+A*b/((a+b)*(a-b))^(1/2)*arctanh((a-b)*ta
nh(1/2*x)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(46) = 92$.
time = 0.36, size = 291, normalized size = 5.20

$$\left[\frac{\sqrt{a^2-b^2}\,Ab\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\,(b^2\cosh(x)+ab)\sinh(x)-2\sqrt{a^2-b^2}\,(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right)-(Ba^2-Bb^2)x+(Ba^2-Bb^2)\log\left(\frac{2\,(b\cosh(x)+a)}{\cosh(x)-\sinh(x)}\right)}{a^2b-b^3},\ -\frac{2\sqrt{-a^2+b^2}\,Ab\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\cosh(x)+b\sinh(x)+a)}{a^2-b^2}\right)+(Ba^2-Bb^2)x-(Ba^2-Bb^2)\log\left(\frac{2\,(b\cosh(x)+a)}{\cosh(x)-\sinh(x)}\right)}{a^2b-b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [(sqrt(a^2 - b^2)*A*b*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) +
2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x)
+ b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) +
 a)*sinh(x) + b)) - (B*a^2 - B*b^2)*x + (B*a^2 - B*b^2)*log(2*(b*cosh(x) +
a)/(cosh(x) - sinh(x))))/(a^2*b - b^3), -(2*sqrt(-a^2 + b^2)*A*b*arctan(-sq
rt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) + (B*a^2 - B*b^2)*x
 - (B*a^2 - B*b^2)*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))))/(a^2*b - b^3
)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 741 vs. $2(48) = 96$.

time = 16.85, size = 741, normalized size = 13.23

$$\begin{cases} \tilde{\infty}\left(2A\operatorname{atan}\left(\tanh\left(\frac{x}{2}\right)\right)+Bx-2B\log\left(\tanh\left(\frac{x}{2}\right)+1\right)+B\log\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)\right) & \text{for } a=0 \wedge b=0 \\ -\frac{A}{b\tanh\left(\frac{x}{2}\right)}+\frac{Bx}{b}-\frac{2B\log\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b}+\frac{2B\log\left(\tanh\left(\frac{x}{2}\right)\right)}{b} & \text{for } a=-b \\ \frac{Ax+B\cosh(x)}{a} & \text{for } b=0 \\ \frac{A\tanh\left(\frac{x}{2}\right)}{b}+\frac{Bx}{b}-\frac{2B\log\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b} & \text{for } a=b \\ -\frac{Ab\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^{2}\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}+\frac{Ab\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^{2}\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}+\frac{Bax\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^{2}\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}+\frac{Ba\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^{2}\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}+\frac{Ba\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^{2}\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}-\frac{2Ba\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}\log\left(\tanh\left(\frac{x}{2}\right)+1\right)}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^{2}\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}-\frac{Bbx\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^{2}\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}-\frac{Bb\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^{2}\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}-\frac{Bb\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^{2}\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}+\frac{2Bb\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}\log\left(\tanh\left(\frac{x}{2}\right)+1\right)}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^{2}\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*cosh(x)),x)
```

```
[Out] Piecewise((zoo*(2*A*atan(tanh(x/2)) + B*x - 2*B*log(tanh(x/2) + 1) + B*log(
tanh(x/2)**2 + 1)), Eq(a, 0) & Eq(b, 0)), (-A/(b*tanh(x/2)) + B*x/b - 2*B*l
og(tanh(x/2) + 1)/b + 2*B*log(tanh(x/2))/b, Eq(a, -b)), ((A*x + B*cosh(x))/
a, Eq(b, 0)), (A*tanh(x/2)/b + B*x/b - 2*B*log(tanh(x/2) + 1)/b, Eq(a, b)),
 (-A*b*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b) +
b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) + A*b*log(sqrt(a/(a - b) + b
/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a -
b) + b/(a - b))) + B*a*x*sqrt(a/(a - b) + b/(a - b))/(a*b*sqrt(a/(a - b) +
b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) + B*a*sqrt(a/(a - b) + b/(a
- b))*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b) + b
/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) + B*a*sqrt(a/(a - b) + b/(a -
 b))*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b) + b/(
a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) - 2*B*a*sqrt(a/(a - b) + b/(a -
 b))*log(tanh(x/2) + 1)/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a -
 b) + b/(a - b))) - B*b*x*sqrt(a/(a - b) + b/(a - b))/(a*b*sqrt(a/(a - b) +
 b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) - B*b*sqrt(a/(a - b) + b/(a
 - b))*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b) +
b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) - B*b*sqrt(a/(a - b) + b/(a
- b))*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b) + b/
(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) + 2*B*b*sqrt(a/(a - b) + b/(a
- b))*log(tanh(x/2) + 1)/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a
- b) + b/(a - b))), True))
```

Giac [A]

time = 0.41, size = 60, normalized size = 1.07

$$\frac{2\,A\arctan\left(\frac{be^{x}+a}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}}-\frac{Bx}{b}+\frac{B\log\left(be^{(2\,x)}+2\,ae^{x}+b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*A*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/sqrt(-a^2 + b^2) - B*x/b + B*log(b
*e^(2*x) + 2*a*e^x + b)/b
```

Mupad [B]
time = 2.98, size = 197, normalized size = 3.52

$$\frac{2\,\mathrm{atan}\left(\frac{A^2\,b^2\,\mathrm{e}^x\,\sqrt{b^2-a^2}}{\left(A\,b^3-A\,a^2\,b\right)\sqrt{A^2}}+\frac{A^2\,a\,b\,\sqrt{b^2-a^2}}{\left(A\,b^3-A\,a^2\,b\right)\sqrt{A^2}}\right)\sqrt{A^2}}{\sqrt{b^2-a^2}}-\frac{B\,x}{b}+\frac{B\,b^3\,\ln\left(4\,A^2\,b+8\,A^2\,a\,\mathrm{e}^x+4\,A^2\,b\,\mathrm{e}^{2\,x}\right)}{b^4-a^2\,b^2}-\frac{B\,a^2\,b\,\ln\left(4\,A^2\,b+8\,A^2\,a\,\mathrm{e}^x+4\,A^2\,b\,\mathrm{e}^{2\,x}\right)}{b^4-a^2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))/(a + b*cosh(x)),x)
```

```
[Out] (2*atan((A^2*b^2*exp(x)*(b^2 - a^2)^(1/2))/((A*b^3 - A*a^2*b)*(A^2)^(1/2))
+ (A^2*a*b*(b^2 - a^2)^(1/2))/((A*b^3 - A*a^2*b)*(A^2)^(1/2)))*(A^2)^(1/2))
/(b^2 - a^2)^(1/2) - (B*x)/b + (B*b^3*log(4*A^2*b + 8*A^2*a*exp(x) + 4*A^2*
b*exp(2*x)))/(b^4 - a^2*b^2) - (B*a^2*b*log(4*A^2*b + 8*A^2*a*exp(x) + 4*A^
2*b*exp(2*x)))/(b^4 - a^2*b^2)
```

3.200 $\int \frac{A+B\sinh(x)}{1+\cosh(x)}\,dx$

Optimal. Leaf size=18

$$B\log(1+\cosh(x))+\frac{A\sinh(x)}{1+\cosh(x)}$$

```
[Out] B*ln(1+cosh(x))+A*sinh(x)/(1+cosh(x))
```

Rubi [A]
time = 0.06, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {4486, 2727, 2746, 31}

$$\frac{A\sinh(x)}{\cosh(x)+1}+B\log(\cosh(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(1 + Cosh[x]),x]
```

```
[Out] B*Log[1 + Cosh[x]] + (A*Sinh[x])/(1 + Cosh[x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sinh(x)}{1+\cosh(x)}\,dx &= \int \left(\frac{A}{1+\cosh(x)}+\frac{B\sinh(x)}{1+\cosh(x)}\right)\,dx\\ &= A\int \frac{1}{1+\cosh(x)}\,dx+B\int \frac{\sinh(x)}{1+\cosh(x)}\,dx\\ &= \frac{A\sinh(x)}{1+\cosh(x)}+B\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,\cosh(x)\right)\\ &= B\log(1+\cosh(x))+\frac{A\sinh(x)}{1+\cosh(x)}\end{aligned}$$

Mathematica [**A**]
time = 0.03, size = 19, normalized size = 1.06

$$2B\log\left(\cosh\left(\frac{x}{2}\right)\right)+A\tanh\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(1 + Cosh[x]),x]
```

```
[Out] 2*B*Log[Cosh[x/2]] + A*Tanh[x/2]
```

Maple [**A**]
time = 0.42, size = 28, normalized size = 1.56

method	result	size
risch	$-Bx-\frac{2A}{e^{x}+1}+2B\ln\left(e^{x}+1\right)$	23
default	$A\tanh\left(\frac{x}{2}\right)-B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(cosh(x)+1),x,method=_RETURNVERBOSE)
```

```
[Out] A*tanh(1/2*x)-B*ln(tanh(1/2*x)-1)-B*ln(tanh(1/2*x)+1)
```

Maxima [**A**]
time = 0.26, size = 19, normalized size = 1.06

$$B\log\left(\cosh\left(x\right)+1\right)+\frac{2\,A}{e^{(-x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(1+cosh(x)),x, algorithm="maxima")
```

```
[Out] B*log(cosh(x) + 1) + 2*A/(e^(-x) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(18) = 36$.
time = 0.48, size = 46, normalized size = 2.56

$$-\frac{Bx\cosh(x) + Bx\sinh(x) + Bx - 2(B\cosh(x) + B\sinh(x) + B)\log(\cosh(x) + \sinh(x) + 1) + 2A}{\cosh(x) + \sinh(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(1+cosh(x)),x, algorithm="fricas")
```

```
[Out] -(B*x*cosh(x) + B*x*sinh(x) + B*x - 2*(B*cosh(x) + B*sinh(x) + B)*log(cosh(
x) + sinh(x) + 1) + 2*A)/(cosh(x) + sinh(x) + 1)
```

Sympy [A]
time = 0.14, size = 20, normalized size = 1.11

$$A\tanh\left(\frac{x}{2}\right) + Bx - 2B\log\left(\tanh\left(\frac{x}{2}\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(1+cosh(x)),x)
```

```
[Out] A*tanh(x/2) + B*x - 2*B*log(tanh(x/2) + 1)
```

Giac [A]
time = 0.40, size = 22, normalized size = 1.22

$$-Bx + 2B\log(e^x + 1) - \frac{2A}{e^x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(1+cosh(x)),x, algorithm="giac")
```

```
[Out] -B*x + 2*B*log(e^x + 1) - 2*A/(e^x + 1)
```

Mupad [B]
time = 0.06, size = 22, normalized size = 1.22

$$2B\ln(\mathrm{e}^x + 1) - \frac{2A}{\mathrm{e}^x + 1} - Bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))/(cosh(x) + 1),x)
```

```
[Out] 2*B*log(exp(x) + 1) - (2*A)/(exp(x) + 1) - B*x
```

3.201 $\int \frac{A+B\sinh(x)}{1-\cosh(x)}\,dx$

Optimal. Leaf size=24

$$-B\log(1-\cosh(x)) - \frac{A\sinh(x)}{1-\cosh(x)}$$

```
[Out] -B*ln(1-cosh(x))-A*sinh(x)/(1-cosh(x))
```

Rubi [A]
time = 0.07, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {4486, 2727, 2746, 31}

$$-\frac{A\sinh(x)}{1-\cosh(x)} - B\log(1-\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(1 - Cosh[x]),x]
```

```
[Out] -(B*Log[1 - Cosh[x]]) - (A*Sinh[x])/(1 - Cosh[x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sinh(x)}{1-\cosh(x)}\,dx &= \int \left(-\frac{A}{-1+\cosh(x)}-\frac{B\sinh(x)}{-1+\cosh(x)}\right)dx\\ &= -\left(A\int \frac{1}{-1+\cosh(x)}\,dx\right)-B\int \frac{\sinh(x)}{-1+\cosh(x)}\,dx\\ &= -\frac{A\sinh(x)}{1-\cosh(x)}-B\text{Subst}\left(\int \frac{1}{-1+x}\,dx,x,\cosh(x)\right)\\ &= -B\log(1-\cosh(x))-\frac{A\sinh(x)}{1-\cosh(x)}\end{aligned}$$

Mathematica [**A**]
time = 0.03, size = 19, normalized size = 0.79

$$A\coth\left(\frac{x}{2}\right)-2B\log\left(\sinh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(1 - Cosh[x]),x]
[Out] A*Coth[x/2] - 2*B*Log[Sinh[x/2]]
```

Maple [**A**]
time = 0.47, size = 36, normalized size = 1.50

method	result	size
risch	$Bx+\frac{2A}{e^{x}-1}-2B\ln\left(e^{x}-1\right)$	22
default	$B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\frac{A}{\tanh\left(\frac{x}{2}\right)}-2B\ln\left(\tanh\left(\frac{x}{2}\right)\right)$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(1-cosh(x)),x,method=_RETURNVERBOSE)
[Out] B*ln(tanh(1/2*x)+1)+B*ln(tanh(1/2*x)-1)+A/tanh(1/2*x)-2*B*ln(tanh(1/2*x))
```

Maxima [**A**]
time = 0.26, size = 20, normalized size = 0.83

$$-B\log\left(\cosh\left(x\right)-1\right)-\frac{2\,A}{e^{\left(-x\right)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(1-cosh(x)),x, algorithm="maxima")
```

```
[Out] -B*log(cosh(x) - 1) - 2*A/(e^(-x) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(21) = 42$.
time = 0.49, size = 48, normalized size = 2.00

$$\frac{Bx\cosh(x) + Bx\sinh(x) - Bx - 2(B\cosh(x) + B\sinh(x) - B)\log(\cosh(x) + \sinh(x) - 1) + 2A}{\cosh(x) + \sinh(x) - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(1-cosh(x)),x, algorithm="fricas")
```

```
[Out] (B*x*cosh(x) + B*x*sinh(x) - B*x - 2*(B*cosh(x) + B*sinh(x) - B)*log(cosh(x
) + sinh(x) - 1) + 2*A)/(cosh(x) + sinh(x) - 1)
```

Sympy [A]
time = 0.23, size = 31, normalized size = 1.29

$$\frac{A}{\tanh\left(\frac{x}{2}\right)} - Bx + 2B\log\left(\tanh\left(\frac{x}{2}\right) + 1\right) - 2B\log\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(1-cosh(x)),x)
```

```
[Out] A/tanh(x/2) - B*x + 2*B*log(tanh(x/2) + 1) - 2*B*log(tanh(x/2))
```

Giac [A]
time = 0.43, size = 22, normalized size = 0.92

$$Bx - 2B\log\left(|e^{x} - 1|\right) + \frac{2A}{e^{x} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(1-cosh(x)),x, algorithm="giac")
```

```
[Out] B*x - 2*B*log(abs(e^x - 1)) + 2*A/(e^x - 1)
```

Mupad [B]
time = 0.91, size = 21, normalized size = 0.88

$$B\,x + \frac{2\,A}{e^{x} - 1} - 2\,B\,\ln\left(e^{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(A + B*sinh(x))/(cosh(x) - 1),x)
```

```
[Out] B*x + (2*A)/(exp(x) - 1) - 2*B*log(exp(x) - 1)
```

3.202 $\int \frac{A+B\tanh(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=65

$$\frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}+\frac{B\log(\cosh(x))}{a}-\frac{B\log(a+b\cosh(x))}{a}$$

```
[Out] B*ln(cosh(x))/a-B*ln(a+b*cosh(x))/a+2*A*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+
b)^(1/2))/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {4486, 2738, 214, 2800, 36, 29, 31}

$$\frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}-\frac{B\log(a+b\cosh(x))}{a}+\frac{B\log(\cosh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tanh[x])/(a + b*Cosh[x]),x]
```

```
[Out] (2*A*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]
) + (B*Log[Cosh[x]])/a - (B*Log[a + b*Cosh[x]])/a
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\tanh(x)}{a+b\cosh(x)}\,dx &= \int \left(\frac{A}{a+b\cosh(x)}+\frac{B\tanh(x)}{a+b\cosh(x)}\right)dx \\
&= A\int \frac{1}{a+b\cosh(x)}\,dx + B\int \frac{\tanh(x)}{a+b\cosh(x)}\,dx \\
&= (2A)\text{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right) + B\text{Subst}\left(\int \frac{1}{x(a+x)}\,dx, x, b \right. \\
&= \frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}} + \frac{B\text{Subst}\left(\int \frac{1}{x}\,dx, x, b\cosh(x)\right)}{a} - \frac{B\text{Subst}\left(\int \frac{1}{a+x}\,dx\right.}{a} \\
&= \frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}} + \frac{B\log(\cosh(x))}{a} - \frac{B\log(a+b\cosh(x))}{a}
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 61, normalized size = 0.94

$$-\frac{2A\text{ArcTan}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} + \frac{B(\log(\cosh(x)) - \log(a+b\cosh(x)))}{a}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Tanh[x])/(a + b*Cosh[x]),x]

[Out] (-2*A*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + (B*(
Log[Cosh[x]] - Log[a + b*Cosh[x]]))/a

Maple [A]
time = 0.76, size = 100, normalized size = 1.54

method	result
default	$\frac{\frac{2(-Ba+Bb)\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-b\left(\tanh^2\left(\frac{x}{2}\right)\right)-a-b\right)}{2a-2b}+\frac{2Aa\,\mathrm{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}}{a}+\frac{B\ln\left(\tanh^2\left(\frac{x}{2}\right)+1\right)}{a}$
risch	$-\frac{2xB}{a}-\frac{2xB\,a^3}{-a^4+a^2b^2}+\frac{2xBa\,b^2}{-a^4+a^2b^2}+\frac{B\ln(1+e^{2x})}{a}-\frac{a\ln\left(e^x+\frac{a^2A-\sqrt{A^2a^4-A^2a^2b^2}}{Aab}\right)B}{a^2-b^2}+\frac{\ln\left(e^x+\frac{a^2A-\sqrt{A^2a^4-}}{Aab}\right.}{(a^2-b^2)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*tanh(x))/(a+b*cosh(x)),x,method=_RETURNVERBOSE)

[Out] 2/a*(1/2*(-B*a+B*b)/(a-b)*ln(a*tanh(1/2*x)^2-b*tanh(1/2*x)^2-a-b)+A*a/((a+b
)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2)))+B/a*ln(tanh(
1/2*x)^2+1)

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*tanh(x))/(a+b*cosh(x)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(55) = 110.
time = 0.38, size = 315, normalized size = 4.85

$$\left[\frac{\sqrt{a^2-b^2}\,Aa\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\,(b^2\cosh(x)+ab)\sinh(x)-2\,\sqrt{a^2-b^2}\,(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right)-(Ba^2-Bb^2)\log\left(\frac{2\,(b\cosh(x)+a)}{\cosh(x)-\sinh(x)}\right)+(Ba^2-Bb^2)\log\left(\frac{2\cosh(x)}{\cosh(x)-\sinh(x)}\right)}{a^3-ab^2},-\frac{2\sqrt{-a^2+b^2}\,Aa\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\cosh(x)+b\sinh(x)+a)}{a^2-b^2}\right)+(Ba^2-Bb^2)\log\left(\frac{2\,(b\cosh(x)+a)}{\cosh(x)-\sinh(x)}\right)-(Ba^2-Bb^2)\log\left(\frac{2\cosh(x)}{\cosh(x)-\sinh(x)}\right)}{a^3-ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*tanh(x))/(a+b*cosh(x)),x, algorithm="fricas")

```
[Out] [(sqrt(a^2 - b^2)*A*a*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) +
2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x)
+ b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) +
 a)*sinh(x) + b)) - (B*a^2 - B*b^2)*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x
))) + (B*a^2 - B*b^2)*log(2*cosh(x)/(cosh(x) - sinh(x))))/(a^3 - a*b^2), -(
2*sqrt(-a^2 + b^2)*A*a*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)
/(a^2 - b^2)) + (B*a^2 - B*b^2)*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x)))
- (B*a^2 - B*b^2)*log(2*cosh(x)/(cosh(x) - sinh(x))))/(a^3 - a*b^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tanh{\left(x \right)}}{a + b \cosh{\left(x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tanh(x))/(a+b*cosh(x)),x)
```

```
[Out] Integral((A + B*tanh(x))/(a + b*cosh(x)), x)
```

Giac [**A**]
time = 0.44, size = 66, normalized size = 1.02

$$\frac{2 \, A \arctan\left(\frac{b e^{x} + a}{\sqrt{-a^{2} + b^{2}}}\right)}{\sqrt{-a^{2} + b^{2}}} - \frac{B \log\left(b e^{\left(2 \, x\right)} + 2 \, a e^{x} + b\right)}{a} + \frac{B \log\left(e^{\left(2 \, x\right)} + 1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tanh(x))/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*A*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/sqrt(-a^2 + b^2) - B*log(b*e^(2*x)
 + 2*a*e^x + b)/a + B*log(e^(2*x) + 1)/a
```

Mupad [**B**]
time = 12.14, size = 160, normalized size = 2.46

$$\frac{B\,\ln\left(16\,B^2\,b^2-16\,B^2\,a^2-16\,B^2\,a^2\,{\mathrm{e}}^{2\,x}+16\,B^2\,b^2\,{\mathrm{e}}^{2\,x}\right)}{a}-\frac{B\,\ln\left(16\,B^2\,b+32\,B^2\,a\,{\mathrm{e}}^x+16\,B^2\,b\,{\mathrm{e}}^{2\,x}\right)}{a}-\frac{2\,\mathrm{atan}\left(\frac{A^2\,b^2\,{\mathrm{e}}^x\,\sqrt{b^2-a^2}+A^2\,a\,b\,\sqrt{b^2-a^2}}{A\,b\,\left(a^2-b^2\right)\,\sqrt{A^2}}\right)\,\sqrt{A^2}}{\sqrt{b^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tanh(x))/(a + b*cosh(x)),x)
```

```
[Out] (B*log(16*B^2*b^2 - 16*B^2*a^2 - 16*B^2*a^2*exp(2*x) + 16*B^2*b^2*exp(2*x))
)/a - (B*log(16*B^2*b + 32*B^2*a*exp(x) + 16*B^2*b*exp(2*x)))/a - (2*atan((
A^2*b^2*exp(x)*(b^2 - a^2)^(1/2) + A^2*a*b*(b^2 - a^2)^(1/2))/(A*b*(a^2 - b
^2)*(A^2)^(1/2)))*(A^2)^(1/2))/(b^2 - a^2)^(1/2)
```

3.203 $\int \frac{A+B\coth(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=100

$$\frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,\sqrt{a+b}}+\frac{B\log(1-\cosh(x))}{2(a+b)}+\frac{B\log(1+\cosh(x))}{2(a-b)}-\frac{aB\log(a+b\cosh(x))}{a^2-b^2}$$

```
[Out] 1/2*B*ln(1-cosh(x))/(a+b)+1/2*B*ln(1+cosh(x))/(a-b)-a*B*ln(a+b*cosh(x))/(a^
2-b^2)+2*A*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(1/2)/(a+b)^(
1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4486, 2738, 214, 2800, 815}

$$-\frac{aB\log(a+b\cosh(x))}{a^2-b^2}+\frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,\sqrt{a+b}}+\frac{B\log(1-\cosh(x))}{2(a+b)}+\frac{B\log(\cosh(x)+1)}{2(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Coth[x])/(a + b*Cosh[x]),x]
```

```
[Out] (2*A*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]
) + (B*Log[1 - Cosh[x]])/(2*(a + b)) + (B*Log[1 + Cosh[x]])/(2*(a - b)) - (
a*B*Log[a + b*Cosh[x]])/(a^2 - b^2)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\int \frac{A+B\coth(x)}{a+b\cosh(x)}\,dx = \int \left(\frac{A}{a+b\cosh(x)}+\frac{B\coth(x)}{a+b\cosh(x)}\right)dx$$
$$= A\int \frac{1}{a+b\cosh(x)}\,dx + B\int \frac{\coth(x)}{a+b\cosh(x)}\,dx$$
$$= (2A)\text{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right) - B\text{Subst}\left(\int \frac{x}{(a+x)\left(b^2-x^2\right)}$$
$$= \frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}} - B\text{Subst}\left(\int \left(\frac{1}{2(a+b)(b-x)}+\frac{a}{(a-b)(a+b)(a-}\right.\right.$$
$$= \frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}} + \frac{B\log(1-\cosh(x))}{2(a+b)} + \frac{B\log(1+\cosh(x))}{2(a-b)} - \frac{aB\log}{}$$

Mathematica [A]
time = 0.18, size = 81, normalized size = 0.81

$$-\frac{2A\text{ArcTan}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} + \frac{B\left(a\log(a+b\cosh(x)) - a\log(\sinh(x)) + b\log\left(\tanh\left(\frac{x}{2}\right)\right)\right)}{-a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Coth[x])/(a + b*Cosh[x]),x]
```

```
[Out] (-2*A*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + (B*(
a*Log[a + b*Cosh[x]] - a*Log[Sinh[x]] + b*Log[Tanh[x/2]]))/(-a^2 + b^2)
```

Maple [A]
time = 0.77, size = 101, normalized size = 1.01

method	result
default	$\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a+b}+\frac{-\frac{Ba\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-b\left(\tanh^2\left(\frac{x}{2}\right)\right)-a-b\right)}{a-b}-\frac{(-2Aa-2Ab)\,\mathrm{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}}{a+b}$
risch	$-\frac{xB}{a-b}-\frac{xB}{a+b}-\frac{2xB\,a^3}{-a^4+2a^2b^2-b^4}+\frac{2xBa\,b^2}{-a^4+2a^2b^2-b^4}+\frac{B\ln(\mathrm{e}^x+1)}{a-b}+\frac{B\ln(\mathrm{e}^x-1)}{a+b}-\frac{\ln\left(\mathrm{e}^x+\frac{Aa-\sqrt{A^2a^2-A^2b^2}}{Ab}\right)Ba}{(a+b)(a-b)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*coth(x))/(a+b*cosh(x)),x,method=_RETURNVERBOSE)

[Out] B/(a+b)*ln(tanh(1/2*x))+1/(a+b)*(-B*a/(a-b)*ln(a*tanh(1/2*x)^2-b*tanh(1/2*x)^2-a-b)-(-2*A*a-2*A*b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2)))

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*coth(x))/(a+b*cosh(x)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more de

Fricas [**A**]
time = 2.13, size = 303, normalized size = 3.03

$$\left[-\frac{Ba\log\left(\frac{2\,(b\cosh(x)+a)}{\cosh(x)-\sinh(x)}\right)-\sqrt{a^2-b^2}\,A\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\,(b^2\cosh(x)+ab)\sinh(x)-2\,\sqrt{a^2-b^2}\,(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right)-(Ba+Bb)\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)-(Ba-Bb)\log\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)}{a^2-b^2},\ -\frac{Ba\log\left(\frac{2\,(b\cosh(x)+a)}{\cosh(x)-\sinh(x)}\right)+2\,\sqrt{-a^2+b^2}\,A\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\cosh(x)+b\sinh(x)+a)}{a^2-b^2}\right)-(Ba+Bb)\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)-(Ba-Bb)\log\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)}{a^2-b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*coth(x))/(a+b*cosh(x)),x, algorithm="fricas")

[Out] [-(B*a*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))) - sqrt(a^2 - b^2)*A*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)) - (B*a + B*b)*log(cosh(x) + sinh(x) + 1) - (B*a - B*b)*log(cosh(x) + sinh(x) - 1))/(a^2 - b^2), -(B*a*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))) + 2*sqrt(-a^2 + b^2)*A*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2))

```
- (B*a + B*b)*log(cosh(x) + sinh(x) + 1) - (B*a - B*b)*log(cosh(x) + sinh(x
) - 1))/(a^2 - b^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\coth\left(x\right)}{a + b\cosh\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*coth(x))/(a+b*cosh(x)),x)
[Out] Integral((A + B*coth(x))/(a + b*cosh(x)), x)
```

Giac [A]
time = 0.42, size = 90, normalized size = 0.90

$$-\frac{Ba\log\left(be^{(2\,x)} + 2\,ae^{x} + b\right)}{a^{2} - b^{2}} + \frac{2\,A\arctan\left(\frac{be^{x} + a}{\sqrt{-a^{2} + b^{2}}}\right)}{\sqrt{-a^{2} + b^{2}}} + \frac{B\log\left(e^{x} + 1\right)}{a - b} + \frac{B\log\left(|e^{x} - 1|\right)}{a + b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*coth(x))/(a+b*cosh(x)),x, algorithm="giac")
[Out] -B*a*log(b*e^(2*x) + 2*a*e^x + b)/(a^2 - b^2) + 2*A*arctan((b*e^x + a)/sqrt
(-a^2 + b^2))/sqrt(-a^2 + b^2) + B*log(e^x + 1)/(a - b) + B*log(abs(e^x - 1
))/(a + b)
```

Mupad [B]
time = 3.68, size = 974, normalized size = 9.74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*coth(x))/(a + b*cosh(x)),x)
[Out] (B*log(exp(x) + 1))/(a - b) + (log((((32*(A^2*b^3 + B^2*b^3 + A^2*a^2*b + 3
*B^2*a^2*b + 4*B^2*a^3*exp(x) + 5*B^2*a*b^2*exp(x) + 4*A*B*a^2*b + 8*A*B*a^
3*exp(x) + 2*A^2*a*b^2*exp(x) - 2*A*B*a*b^2*exp(x)))/b^5 + ((A*((a + b)^3*(
a - b)^3)^(1/2) - B*a^3 + B*a*b^2)*(128*exp(x)*(a^2 - b^2)^3*(A - 2*B) + a*
b^5*(64*A - 128*B) + a^5*b*(64*A - 128*B) + 96*b^6*exp(x)*(A - 3*B) - a^3*b
^3*(128*A - 256*B) - 192*a^2*b^4*exp(x)*(A - 3*B) + 96*a^4*b^2*exp(x)*(A -
3*B) + 128*A*a^3*exp(x)*((a^2 - b^2)^3)^(1/2) + 96*A*a^2*b*((a^2 - b^2)^3)^
(1/2) - 32*A*a*b^2*exp(x)*((a^2 - b^2)^3)^(1/2)))/((b^7 - a^2*b^5)*(a^2 - b
^2)^2))*(A*((a + b)^3*(a - b)^3)^(1/2) - B*a^3 + B*a*b^2))/(a^2 - b^2)^2 -
(32*B*(A^2*b^2*exp(x) + 4*B^2*a^2*exp(x) + A^2*a*b + B^2*a*b + 4*A*B*a^2*ex
```

```
p(x) - A*B*b^2*exp(x) + 2*A*B*a*b))/b^5)*(A*((a + b)^3*(a - b)^3)^(1/2) - B
*a^3 + B*a*b^2))/(a^4 + b^4 - 2*a^2*b^2) - (log(- (32*B*(A^2*b^2*exp(x) + 4
*B^2*a^2*exp(x) + A^2*a*b + B^2*a*b + 4*A*B*a^2*exp(x) - A*B*b^2*exp(x) + 2
*A*B*a*b))/b^5 - (((32*(A^2*b^3 + B^2*b^3 + A^2*a^2*b + 3*B^2*a^2*b + 4*B^2
*a^3*exp(x) + 5*B^2*a*b^2*exp(x) + 4*A*B*a^2*b + 8*A*B*a^3*exp(x) + 2*A^2*a
*b^2*exp(x) - 2*A*B*a*b^2*exp(x)))/b^5 - ((B*a^3 + A*((a + b)^3*(a - b)^3)^
(1/2) - B*a*b^2)*(128*exp(x)*(a^2 - b^2)^3*(A - 2*B) + a*b^5*(64*A - 128*B)
 + a^5*b*(64*A - 128*B) + 96*b^6*exp(x)*(A - 3*B) - a^3*b^3*(128*A - 256*B)
 - 192*a^2*b^4*exp(x)*(A - 3*B) + 96*a^4*b^2*exp(x)*(A - 3*B) - 128*A*a^3*e
xp(x)*((a^2 - b^2)^3)^(1/2) - 96*A*a^2*b*((a^2 - b^2)^3)^(1/2) + 32*A*a*b^2
*exp(x)*((a^2 - b^2)^3)^(1/2)))/((b^7 - a^2*b^5)*(a^2 - b^2)^2))*(B*a^3 + A
*((a + b)^3*(a - b)^3)^(1/2) - B*a*b^2))/(a^2 - b^2)^2)*(B*a^3 + A*((a + b)
^3*(a - b)^3)^(1/2) - B*a*b^2))/(a^4 + b^4 - 2*a^2*b^2) + (B*log(exp(x) - 1
))/(a + b)
```

3.204 $\int \frac{A+B\text{sech}(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=62

$$\frac{B\text{ArcTan}(\sinh(x))}{a}+\frac{2(aA-bB)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\,\sqrt{a+b}}$$

```
[Out] B*arctan(sinh(x))/a+2*(A*a-B*b)*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2)
)/a/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2907, 3080, 3855, 2738, 214}

$$\frac{2(aA-bB)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\,\sqrt{a+b}}+\frac{B\text{ArcTan}(\sinh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sech[x])/(a + b*Cosh[x]),x]
```

```
[Out] (B*ArcTan[Sinh[x]])/a + (2*(a*A - b*B)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt
[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2907

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((d + c*Sin[e +
 f*x])^n/Sin[e + f*x]^n), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\mathrm{sech}(x)}{a+b\cosh(x)}\,dx &= \int \frac{(B+A\cosh(x))\mathrm{sech}(x)}{a+b\cosh(x)}\,dx\\ &= \frac{B\int \mathrm{sech}(x)\,dx}{a}+\frac{(aA-bB)\int \frac{1}{a+b\cosh(x)}\,dx}{a}\\ &= \frac{B\tan^{-1}(\sinh(x))}{a}+\frac{(2(aA-bB))\mathrm{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a}\\ &= \frac{B\tan^{-1}(\sinh(x))}{a}+\frac{2(aA-bB)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\,\sqrt{a+b}}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 63, normalized size = 1.02

$$\frac{2\left(B\mathrm{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{(-aA+bB)\mathrm{ArcTan}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sech[x])/(a + b*Cosh[x]),x]
[Out] (2*(B*ArcTan[Tanh[x/2]] + ((-(a*A) + b*B)*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-
a^2 + b^2]])/Sqrt[-a^2 + b^2]))/a
```

Maple [A]
time = 0.72, size = 59, normalized size = 0.95

method	result
default	$\frac{2B\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}-\frac{2(-Aa+Bb)\,\mathrm{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a\sqrt{(a+b)(a-b)}}$
risch	$\frac{iB\ln(\mathrm{e}^{x}+i)}{a}-\frac{iB\ln(\mathrm{e}^{x}-i)}{a}+\frac{\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)A}{\sqrt{a^2-b^2}}-\frac{\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)Bb}{\sqrt{a^2-b^2}\,a}-\frac{\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^2-}}{b\sqrt{a^2}}\right.}{\sqrt{a^2-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sech(x))/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*B/a*arctan(tanh(1/2*x))-2*(-A*a+B*b)/a/((a+b)*(a-b))^(1/2)*arctanh((a-b)*
tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sech(x))/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.93, size = 249, normalized size = 4.02

$$\left[-\frac{(Aa-Bb)\sqrt{a^2-b^2}\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\,(b^2\cosh(x)+ab)\sinh(x)+2\sqrt{a^2-b^2}\,(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right)-2\,(Ba^2-Bb^2)\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}{a^3-ab^2},\ -\frac{2\left((Aa-Bb)\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\cosh(x)+b\sinh(x)+a)}{a^2-b^2}\right)-(Ba^2-Bb^2)\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)\right)}{a^3-ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sech(x))/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [-((A*a - B*b)*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*c
osh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b
*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*
cosh(x) + a)*sinh(x) + b)) - 2*(B*a^2 - B*b^2)*arctan(cosh(x) + sinh(x)))/(
a^3 - a*b^2), -2*((A*a - B*b)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*
cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) - (B*a^2 - B*b^2)*arctan(cosh(x) + si
nh(x)))/(a^3 - a*b^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\operatorname{sech}(x)}{a + b\cosh(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sech(x))/(a+b*cosh(x)),x)
```

```
[Out] Integral((A + B*sech(x))/(a + b*cosh(x)), x)
```

Giac [A]
time = 0.42, size = 53, normalized size = 0.85

$$\frac{2\,B\arctan\left(e^{x}\right)}{a} + \frac{2\,(Aa - Bb)\arctan\left(\frac{be^{x}+a}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sech(x))/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*B*arctan(e^x)/a + 2*(A*a - B*b)*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqr
t(-a^2 + b^2)*a)
```

Mupad [B]
time = 6.41, size = 636, normalized size = 10.26

$$\frac{\ln\left(\frac{\sqrt{(a+b)\,(a-b)}\,(Aa-Bb)\left(\frac{32\left(A^{2}a^{2}b-2ABab^{2}-4e^{x}B^{2}a^{3}-2B^{2}a^{2}b+3e^{x}B^{2}ab^{2}+2B^{2}b^{3}\right)}{b^{5}}+\frac{\sqrt{(a+b)\,(a-b)}\,(Aa-Bb)\left(\frac{32a^{2}\left(2Bb^{2}-4Aa^{2}e^{x}+Ab^{2}e^{x}-2Aab+3Babe^{x}\right)}{b^{5}}-\frac{32a^{2}\sqrt{(a+b)\,(a-b)}\,(Aa-Bb)\left(4e^{x}a^{3}+3a^{2}b-3e^{x}ab^{2}-2b^{3}\right)}{b^{5}\left(ab^{2}-a^{3}\right)}\right)}{ab^{2}-a^{3}}\right)}{ab^{2}-a^{3}}-\frac{32B(Aa-Bb)\left(2Bb-Abe^{x}+4Bae^{x}\right)}{b^{5}}\right)\sqrt{(a+b)\,(a-b)}\,(Aa-Bb)}{ab^{2}-a^{3}} - \frac{\ln\left(-\frac{\sqrt{(a+b)\,(a-b)}\,(Aa-Bb)\left(\frac{32\left(A^{2}a^{2}b-2ABab^{2}-4e^{x}B^{2}a^{3}-2B^{2}a^{2}b+3e^{x}B^{2}ab^{2}+2B^{2}b^{3}\right)}{b^{5}}-\frac{\sqrt{(a+b)\,(a-b)}\,(Aa-Bb)\left(\frac{32a^{2}\left(2Bb^{2}-4Aa^{2}e^{x}+Ab^{2}e^{x}-2Aab+3Babe^{x}\right)}{b^{5}}+\frac{32a^{2}\sqrt{(a+b)\,(a-b)}\,(Aa-Bb)\left(4e^{x}a^{3}+3a^{2}b-3e^{x}ab^{2}-2b^{3}\right)}{b^{5}\left(ab^{2}-a^{3}\right)}\right)}{ab^{2}-a^{3}}\right)}{ab^{2}-a^{3}}-\frac{32B(Aa-Bb)\left(2Bb-Abe^{x}+4Bae^{x}\right)}{b^{5}}\right)\sqrt{(a+b)\,(a-b)}\,(Aa-Bb)}{ab^{2}-a^{3}} - \frac{B\ln\left(e^{x}-1{}i\right)1{}i}{a} + \frac{B\ln\left(e^{x}+1{}i\right)1{}i}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cosh(x))/(a + b*cosh(x)),x)
```

```
[Out] (B*log(exp(x) + 1i)*1i)/a - (B*log(exp(x) - 1i)*1i)/a + (log((((a + b)*(a -
 b))^(1/2)*(A*a - B*b)*((32*(2*B^2*b^3 + A^2*a^2*b - 2*B^2*a^2*b - 4*B^2*a^
3*exp(x) + 3*B^2*a*b^2*exp(x) - 2*A*B*a*b^2))/b^5 + (((a + b)*(a - b))^(1/2
)*(A*a - B*b)*((32*a^2*(2*B*b^2 - 4*A*a^2*exp(x) + A*b^2*exp(x) - 2*A*a*b +
 3*B*a*b*exp(x)))/b^5 - (32*a^2*((a + b)*(a - b))^(1/2)*(A*a - B*b)*(3*a^2*
b - 2*b^3 + 4*a^3*exp(x) - 3*a*b^2*exp(x)))/(b^5*(a*b^2 - a^3))))/(a*b^2 -
a^3)))/(a*b^2 - a^3) - (32*B*(A*a - B*b)*(2*B*b - A*b*exp(x) + 4*B*a*exp(x)
))/b^5)*((a + b)*(a - b))^(1/2)*(A*a - B*b))/(a*b^2 - a^3) - (log(- (((a +
b)*(a - b))^(1/2)*(A*a - B*b)*((32*(2*B^2*b^3 + A^2*a^2*b - 2*B^2*a^2*b - 4
*B^2*a^3*exp(x) + 3*B^2*a*b^2*exp(x) - 2*A*B*a*b^2))/b^5 - (((a + b)*(a - b
))^(1/2)*(A*a - B*b)*((32*a^2*(2*B*b^2 - 4*A*a^2*exp(x) + A*b^2*exp(x) - 2*
A*a*b + 3*B*a*b*exp(x)))/b^5 + (32*a^2*((a + b)*(a - b))^(1/2)*(A*a - B*b)*
(3*a^2*b - 2*b^3 + 4*a^3*exp(x) - 3*a*b^2*exp(x)))/(b^5*(a*b^2 - a^3))))/(a
*b^2 - a^3)))/(a*b^2 - a^3) - (32*B*(A*a - B*b)*(2*B*b - A*b*exp(x) + 4*B*a
*exp(x)))/b^5)*((a + b)*(a - b))^(1/2)*(A*a - B*b))/(a*b^2 - a^3)
```

3.205 $\int \frac{A+B\,\mathrm{csch}(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=99

$$\frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,\sqrt{a+b}}+\frac{B\log(1-\cosh(x))}{2(a+b)}-\frac{B\log(1+\cosh(x))}{2(a-b)}+\frac{bB\log(a+b\cosh(x))}{a^2-b^2}$$

```
[Out] 1/2*B*ln(1-cosh(x))/(a+b)-1/2*B*ln(1+cosh(x))/(a-b)+b*B*ln(a+b*cosh(x))/(a^
2-b^2)+2*A*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(1/2)/(a+b)^(
1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {4310, 4486, 2738, 214, 2747, 720, 31, 647}

$$\frac{bB\log(a+b\cosh(x))}{a^2-b^2}+\frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,\sqrt{a+b}}+\frac{B\log(1-\cosh(x))}{2(a+b)}-\frac{B\log(\cosh(x)+1)}{2(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Csch[x])/(a + b*Cosh[x]),x]
```

```
[Out] (2*A*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]
) + (B*Log[1 - Cosh[x]])/(2*(a + b)) - (B*Log[1 + Cosh[x]])/(2*(a - b)) + (
b*B*Log[a + b*Cosh[x]])/(a^2 - b^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4310

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\int \frac{A+B\operatorname{csch}(x)}{a+b\cosh(x)}\,dx = -\left(i\int \frac{\operatorname{csch}(x)(iB+iA\sinh(x))}{a+b\cosh(x)}\,dx\right)$$
$$= \int \left(\frac{A}{a+b\cosh(x)}+\frac{B\operatorname{csch}(x)}{a+b\cosh(x)}\right)dx$$
$$= A\int \frac{1}{a+b\cosh(x)}\,dx + B\int \frac{\operatorname{csch}(x)}{a+b\cosh(x)}\,dx$$
$$= (2A)\text{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right) - (bB)\text{Subst}\left(\int \frac{1}{(a+x)\,(b^2-x$$
$$= \frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,\sqrt{a+b}} + \frac{(bB)\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\cosh(x)\right)}{a^2-b^2} + \frac{(bB)\text{Subst}(\int$$
$$= \frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,\sqrt{a+b}} + \frac{bB\log(a+b\cosh(x))}{a^2-b^2} + \frac{B\text{Subst}\left(\int \frac{1}{-b-x}\,dx, x, b\cos\right.}{2(a-b)}$$
$$= \frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,\sqrt{a+b}} + \frac{B\log(1-\cosh(x))}{2(a+b)} - \frac{B\log(1+\cosh(x))}{2(a-b)} + \frac{bB\log}{}$$

Mathematica [A]
time = 0.16, size = 81, normalized size = 0.82

$$-\frac{2A\text{ArcTan}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} + \frac{B\left(b\log(a+b\cosh(x)) - b\log(\sinh(x)) + a\log\left(\tanh\left(\frac{x}{2}\right)\right)\right)}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Csch[x])/(a + b*Cosh[x]),x]
```

```
[Out] (-2*A*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + (B*(
b*Log[a + b*Cosh[x]] - b*Log[Sinh[x]] + a*Log[Tanh[x/2]]))/(a^2 - b^2)
```

Maple [A]
time = 0.64, size = 100, normalized size = 1.01

method	result
default	$\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a+b} + \frac{\frac{Bb\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-b\left(\tanh^2\left(\frac{x}{2}\right)\right)-a-b\right)}{a-b} - \frac{(-2Aa-2Ab)\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)\,(a-b)}}\right)}{\sqrt{(a+b)\,(a-b)}}}{a+b}$

risch	$\frac{xB}{a-b} - \frac{xB}{a+b} + \frac{2xB\,a^2b}{-a^4+2a^2b^2-b^4} - \frac{2xB\,b^3}{-a^4+2a^2b^2-b^4} - \frac{B\ln(\mathrm{e}^x+1)}{a-b} + \frac{B\ln(\mathrm{e}^x-1)}{a+b} + \frac{\ln\left(\mathrm{e}^x+\frac{Aa-\sqrt{A^2a^2-A^2b^2}}{Ab}\right)Bb}{(a+b)(a-b)} +$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*csch(x))/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] B/(a+b)*ln(tanh(1/2*x))+1/(a+b)*(B*b/(a-b)*ln(a*tanh(1/2*x)^2-b*tanh(1/2*x)
^2-a-b)-(-2*A*a-2*A*b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)
*(a-b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*csch(x))/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 1.97, size = 298, normalized size = 3.01

$$\left[\frac{Bb\log\left(\frac{2\,(b\cosh(x)+a)}{\cosh(x)-\sinh(x)}\right)+\sqrt{a^2-b^2}\,A\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\,(b^2\cosh(x)+ab)\sinh(x)-2\,\sqrt{a^2-b^2}\,(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right)-(Ba+Bb)\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)+(Ba-Bb)\log\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)}{a^2-b^2},\ \frac{Bb\log\left(\frac{2\,(b\cosh(x)+a)}{\cosh(x)-\sinh(x)}\right)-2\,\sqrt{-a^2+b^2}\,A\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\cosh(x)+b\sinh(x)+a)}{a^2-b^2}\right)-(Ba+Bb)\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)+(Ba-Bb)\log\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)}{a^2-b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*csch(x))/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [(B*b*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))) + sqrt(a^2 - b^2)*A*log((b
^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x)
 + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)
^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)) - (B*a + B
*b)*log(cosh(x) + sinh(x) + 1) + (B*a - B*b)*log(cosh(x) + sinh(x) - 1))/(a
^2 - b^2), (B*b*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))) - 2*sqrt(-a^2 +
b^2)*A*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) -
(B*a + B*b)*log(cosh(x) + sinh(x) + 1) + (B*a - B*b)*log(cosh(x) + sinh(x)
- 1))/(a^2 - b^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{A+B\operatorname{csch}\left(x\right)}{a+b\cosh\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*csch(x))/(a+b*cosh(x)),x)

[Out] Integral((A + B*csch(x))/(a + b*cosh(x)), x)

Giac [A]
time = 0.42, size = 90, normalized size = 0.91

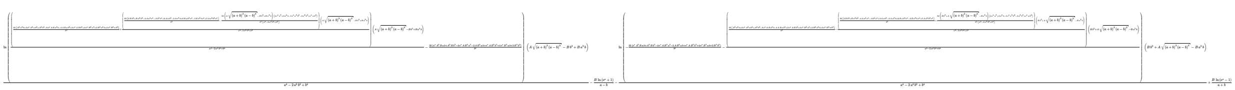
$$\frac{Bb\log\left(be^{(2x)}+2\,ae^{x}+b\right)}{a^2-b^2}+\frac{2\,A\arctan\left(\frac{be^{x}+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}-\frac{B\log\left(e^{x}+1\right)}{a-b}+\frac{B\log\left(|e^{x}-1|\right)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*csch(x))/(a+b*cosh(x)),x, algorithm="giac")

[Out] B*b*log(b*e^(2*x) + 2*a*e^x + b)/(a^2 - b^2) + 2*A*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/sqrt(-a^2 + b^2) - B*log(e^x + 1)/(a - b) + B*log(abs(e^x - 1))/(a + b)

Mupad [B]
time = 3.33, size = 983, normalized size = 9.93



Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/sinh(x))/(a + b*cosh(x)),x)

[Out] (log((((32*(A^2*b^3 + A^2*a^2*b + 2*B^2*a^2*b + 4*B^2*a^3*exp(x) + 3*B^2*a*b^2*exp(x) - 4*A*B*a*b^2 + 2*A*B*b^3*exp(x) + 2*A^2*a*b^2*exp(x) - 8*A*B*a^2*b*exp(x)))/b^5 + (((32*(2*B*b^4 + B*a^2*b^2 - 4*A*a^4*exp(x) - A*b^4*exp(x) + 2*A*a*b^3 - 2*A*a^3*b + 6*B*a*b^3*exp(x) - 3*B*a^3*b*exp(x) + 5*A*a^2*b^2*exp(x)))/b^5 - (32*(A*((a + b)^3*(a - b)^3)^(1/2) - B*b^3 + B*a^2*b)*(3*a^4*b - 3*a^2*b^3 + 4*a^5*exp(x) + a*b^4*exp(x) - 5*a^3*b^2*exp(x)))/(b^5*(a^4 + b^4 - 2*a^2*b^2)))*(A*((a + b)^3*(a - b)^3)^(1/2) - B*b^3 + B*a^2*b))/(a^4 + b^4 - 2*a^2*b^2))*(A*((a + b)^3*(a - b)^3)^(1/2) - B*b^3 + B*a^2*b))/(a^4 + b^4 - 2*a^2*b^2) - (32*(2*B^3*b^2 + A^2*B*b^2 - 2*A*B^2*a*b + 4*B^3*a*b*exp(x) - 4*A*B^2*a^2*exp(x) + A*B^2*b^2*exp(x) + A^2*B*a*b*exp(x)))/b^5)*(A*((a + b)^3*(a - b)^3)^(1/2) - B*b^3 + B*a^2*b))/(a^4 + b^4 - 2*a^2*b^2) - (B*log(exp(x) + 1))/(a - b) - (log(- (32*(2*B^3*b^2 + A^2*B*b^2 - 2*A*B^2*a*b + 4*B^3*a*b*exp(x) - 4*A*B^2*a^2*exp(x) + A*B^2*b^2*exp(x) + A^2*B*a*b*exp(x)))/b^5 - (((32*(A^2*b^3 + A^2*a^2*b + 2*B^2*a^2*b + 4*B^2*a^3*exp(x) + 3*B^2*a*b^2*exp(x) - 4*A*B*a*b^2 + 2*A*B*b^3*exp(x) + 2*A^2*a*b^2*exp(x) - 8*A*B*a^2*b*exp(x)))/b^5 - (((32*(2*B*b^4 + B*a^2*b^2 - 4*A*a^4*exp(x) - A*b^4*exp(x) + 2*A*a*b^3 - 2*A*a^3*b + 6*B*a*b^3*exp(x) - 3*B*a^3*b*e

```
xp(x) + 5*A*a^2*b^2*exp(x)))/b^5 + (32*(B*b^3 + A*((a + b)^3*(a - b)^3)^(1/
2) - B*a^2*b)*(3*a^4*b - 3*a^2*b^3 + 4*a^5*exp(x) + a*b^4*exp(x) - 5*a^3*b^
2*exp(x)))/(b^5*(a^4 + b^4 - 2*a^2*b^2)))*(B*b^3 + A*((a + b)^3*(a - b)^3)^
(1/2) - B*a^2*b))/(a^4 + b^4 - 2*a^2*b^2))*(B*b^3 + A*((a + b)^3*(a - b)^3)
^(1/2) - B*a^2*b))/(a^4 + b^4 - 2*a^2*b^2))*(B*b^3 + A*((a + b)^3*(a - b)^3
)^(1/2) - B*a^2*b))/(a^4 + b^4 - 2*a^2*b^2) + (B*log(exp(x) - 1))/(a + b)
```

3.206 $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{a+b\cosh(d+ex)}\,dx$

Optimal. Leaf size=86

$$\frac{Bx}{b}+\frac{2(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b\sqrt{a+b}\,e}+\frac{C\log(a+b\cosh(d+ex))}{be}$$

```
[Out] B*x/b+C*ln(a+b*cosh(e*x+d))/b/e+2*(A*b-B*a)*arctanh((a-b)^(1/2)*tanh(1/2*e*
x+1/2*d)/(a+b)^(1/2))/b/e/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4462, 2814, 2738, 211, 2747, 31}

$$\frac{2(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{be\sqrt{a-b}\,\sqrt{a+b}}+\frac{C\log(a+b\cosh(d+ex))}{be}+\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + b*Cosh[d + e*x]),x]
```

```
[Out] (B*x)/b + (2*(A*b - a*B)*ArcTanh[(Sqrt[a - b]*Tanh[(d + e*x)/2])/Sqrt[a + b
]])/(Sqrt[a - b]*b*Sqrt[a + b]*e) + (C*Log[a + b*Cosh[d + e*x]])/(b*e)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
```

```
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 4462

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{a+b\cosh(d+ex)}\,dx = C\int \frac{\sinh(d+ex)}{a+b\cosh(d+ex)}\,dx + \int \frac{A+B\cosh(d+ex)}{a+b\cosh(d+ex)}\,dx$$

$$= \frac{Bx}{b} - \frac{(-Ab+aB)\int \frac{1}{a+b\cosh(d+ex)}\,dx}{b} + \frac{C\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\right.}{be}$$

$$= \frac{Bx}{b} + \frac{C\log(a+b\cosh(d+ex))}{be} - \frac{(2i(Ab-aB))\text{Subst}\left(\int \frac{}{a+}\right.}{}$$

$$= \frac{Bx}{b} + \frac{2(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b\sqrt{a+b}\,e} + \frac{C\log(a}{}$$

Mathematica [**A**]
time = 0.19, size = 81, normalized size = 0.94

$$\frac{B(d+ex) + \frac{2(-Ab+aB)\text{ArcTan}\left(\frac{(a-b)\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} + C\log(a+b\cosh(d+ex))}{be}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + b*Cosh[d + e*x]),x]
```

```
[Out] (B*(d + e*x) + (2*(-(A*b) + a*B)*ArcTan[((a - b)*Tanh[(d + e*x)/2])/Sqrt[-a
^2 + b^2]])/Sqrt[-a^2 + b^2] + C*Log[a + b*Cosh[d + e*x]])/(b*e)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(77) = 154$.

time = 2.27, size = 156, normalized size = 1.81

method	result
derivativedivides	$\frac{\frac{\frac{2(aC-bC)\ln\left(a\left(\tanh^2\left(\frac{ex}{2}+\frac{d}{2}\right)\right)-b\left(\tanh^2\left(\frac{ex}{2}+\frac{d}{2}\right)\right)-a-b\right)}{2a-2b}-\frac{2(-Ab+Ba)\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}}{b}+\frac{(B-C)\ln(\tan}{}}{e}$
default	$\frac{\frac{\frac{2(aC-bC)\ln\left(a\left(\tanh^2\left(\frac{ex}{2}+\frac{d}{2}\right)\right)-b\left(\tanh^2\left(\frac{ex}{2}+\frac{d}{2}\right)\right)-a-b\right)}{2a-2b}-\frac{2(-Ab+Ba)\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}}{b}+\frac{(B-C)\ln(\tan}{}}{e}$
risch	$\frac{Bx}{b}+\frac{xC}{b}+\frac{2C\,a^2b\,e^2x}{-a^2b^2e^2+b^4e^2}-\frac{2C\,b^3e^2x}{-a^2b^2e^2+b^4e^2}+\frac{2C\,a^2bde}{-a^2b^2e^2+b^4e^2}-\frac{2C\,b^3de}{-a^2b^2e^2+b^4e^2}+\frac{\ln\left(e^{ex+d}+\frac{Aab-B\,a^2-\sqrt{}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d)),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/e*(2/b*(1/2*(C*a-C*b)/(a-b)*ln(a*tanh(1/2*e*x+1/2*d)^2-b*tanh(1/2*e*x+1/2
*d)^2-a-b)-(-A*b+B*a)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*e*x+1/2*d)
/((a+b)*(a-b))^(1/2)))+(B-C)/b*ln(tanh(1/2*e*x+1/2*d)+1)+(-B-C)/b*ln(tanh(1
/2*e*x+1/2*d)-1))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d)),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(77) = 154$.

time = 0.49, size = 585, normalized size = 6.80

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d)),x, algorithm="f
ricas")
```

```
[Out] [(((B - C)*a^2 - (B - C)*b^2)*x*cosh(1) + ((B - C)*a^2 - (B - C)*b^2)*x*sin
h(1) - (B*a - A*b)*sqrt(a^2 - b^2)*log((b^2*cosh(x*cosh(1) + x*sinh(1) + d)
^2 + b^2*sinh(x*cosh(1) + x*sinh(1) + d)^2 + 2*a*b*cosh(x*cosh(1) + x*sinh(
1) + d) + 2*a^2 - b^2 + 2*(b^2*cosh(x*cosh(1) + x*sinh(1) + d) + a*b)*sinh(
x*cosh(1) + x*sinh(1) + d) - 2*sqrt(a^2 - b^2)*(b*cosh(x*cosh(1) + x*sinh(1
) + d) + b*sinh(x*cosh(1) + x*sinh(1) + d) + a))/(b*cosh(x*cosh(1) + x*sinh
(1) + d)^2 + b*sinh(x*cosh(1) + x*sinh(1) + d)^2 + 2*a*cosh(x*cosh(1) + x*s
inh(1) + d) + 2*(b*cosh(x*cosh(1) + x*sinh(1) + d) + a)*sinh(x*cosh(1) + x*
sinh(1) + d) + b)) + (C*a^2 - C*b^2)*log(2*(b*cosh(x*cosh(1) + x*sinh(1) +
d) + a)/(cosh(x*cosh(1) + x*sinh(1) + d) - sinh(x*cosh(1) + x*sinh(1) + d))
))/((a^2*b - b^3)*cosh(1) + (a^2*b - b^3)*sinh(1)), (((B - C)*a^2 - (B - C)
*b^2)*x*cosh(1) + ((B - C)*a^2 - (B - C)*b^2)*x*sinh(1) + 2*(B*a - A*b)*sqr
t(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x*cosh(1) + x*sinh(1) + d) +
 b*sinh(x*cosh(1) + x*sinh(1) + d) + a)/(a^2 - b^2)) + (C*a^2 - C*b^2)*log(
2*(b*cosh(x*cosh(1) + x*sinh(1) + d) + a)/(cosh(x*cosh(1) + x*sinh(1) + d)
- sinh(x*cosh(1) + x*sinh(1) + d))))/((a^2*b - b^3)*cosh(1) + (a^2*b - b^3)
*sinh(1))]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 695 vs. $2(73) = 146$.
time = 16.99, size = 695, normalized size = 8.08

$$\begin{cases} \frac{\tilde{\infty} x (A + B\cosh(d) + C\sinh(d))}{\cosh(d)} & \text{for}\ a = 0 \wedge b = 0 \wedge e = 0 \\ \frac{A\tanh\left(\frac{d}{2}+\frac{ex}{2}\right)}{be} + \frac{Bx}{b} - \frac{B\tanh\left(\frac{d}{2}+\frac{ex}{2}\right)}{be} + \frac{Cx}{b} - \frac{2C\log\left(\tanh\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)}{be} & \text{for}\ a = b \\ -\frac{A}{be\tanh\left(\frac{d}{2}+\frac{ex}{2}\right)} + \frac{Bx}{b} - \frac{B}{be\tanh\left(\frac{d}{2}+\frac{ex}{2}\right)} + \frac{Cx}{b} - \frac{2C\log\left(\tanh\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)}{be} + \frac{2C\log\left(\tanh\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{be} & \text{for}\ a = -b \\ \frac{Ax + \frac{B\sinh(d+ex)}{e} + \frac{C\cosh(d+ex)}{e}}{a} & \text{for}\ b = 0 \\ \frac{x(A + B\cosh(d) + C\sinh(d))}{a + b\cosh(d)} & \text{for}\ e = 0 \\ -\frac{Ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{abe+b^2e} + \frac{Ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{abe+b^2e} + \frac{Baex}{abe+b^2e} + \frac{Ba\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{abe+b^2e} - \frac{Ba\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{abe+b^2e} + \frac{Bbex}{abe+b^2e} + \frac{Caex}{abe+b^2e} + \frac{Ca\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{abe+b^2e} + \frac{Ca\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{abe+b^2e} - \frac{2Ca\log\left(\tanh\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)}{abe+b^2e} + \frac{Cbex}{abe+b^2e} + \frac{Cb\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{abe+b^2e} + \frac{Cb\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{abe+b^2e} - \frac{2Cb\log\left(\tanh\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)}{abe+b^2e} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d)),x)
```

```
[Out] Piecewise((zoo*x*(A + B*cosh(d) + C*sinh(d))/cosh(d), Eq(a, 0) & Eq(b, 0) &
 Eq(e, 0)), (A*tanh(d/2 + e*x/2)/(b*e) + B*x/b - B*tanh(d/2 + e*x/2)/(b*e)
+ C*x/b - 2*C*log(tanh(d/2 + e*x/2) + 1)/(b*e), Eq(a, b)), (-A/(b*e*tanh(d/
2 + e*x/2)) + B*x/b - B/(b*e*tanh(d/2 + e*x/2)) + C*x/b - 2*C*log(tanh(d/2
+ e*x/2) + 1)/(b*e) + 2*C*log(tanh(d/2 + e*x/2))/(b*e), Eq(a, -b)), ((A*x +
 B*sinh(d + e*x)/e + C*cosh(d + e*x)/e)/a, Eq(b, 0)), (x*(A + B*cosh(d) + C
*sinh(d))/(a + b*cosh(d)), Eq(e, 0)), (-A*b*sqrt(a/(a - b) + b/(a - b))*log
(-sqrt(a/(a - b) + b/(a - b)) + tanh(d/2 + e*x/2))/(a*b*e + b**2*e) + A*b*s
qrt(a/(a - b) + b/(a - b))*log(sqrt(a/(a - b) + b/(a - b)) + tanh(d/2 + e*x
/2))/(a*b*e + b**2*e) + B*a*e*x/(a*b*e + b**2*e) + B*a*sqrt(a/(a - b) + b/(
a - b))*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(d/2 + e*x/2))/(a*b*e + b**2
*e) - B*a*sqrt(a/(a - b) + b/(a - b))*log(sqrt(a/(a - b) + b/(a - b)) + tan
h(d/2 + e*x/2))/(a*b*e + b**2*e) + B*b*e*x/(a*b*e + b**2*e) + C*a*e*x/(a*b*
```

```
e + b**2*e) + C*a*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(d/2 + e*x/2))/(a*
b*e + b**2*e) + C*a*log(sqrt(a/(a - b) + b/(a - b)) + tanh(d/2 + e*x/2))/(a
*b*e + b**2*e) - 2*C*a*log(tanh(d/2 + e*x/2) + 1)/(a*b*e + b**2*e) + C*b*e*
x/(a*b*e + b**2*e) + C*b*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(d/2 + e*x/
2))/(a*b*e + b**2*e) + C*b*log(sqrt(a/(a - b) + b/(a - b)) + tanh(d/2 + e*x
/2))/(a*b*e + b**2*e) - 2*C*b*log(tanh(d/2 + e*x/2) + 1)/(a*b*e + b**2*e),
True))
```

Giac [A]
time = 0.45, size = 94, normalized size = 1.09

$$\frac{\frac{(ex+d)(B-C)}{b}+\frac{C\log\left(be^{(2\,ex+2\,d)}+2\,ae^{(ex+d)}+b\right)}{b}-\frac{2\,(Ba-Ab)\arctan\left(\frac{be^{(ex+d)}+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,b}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d)),x, algorithm="g
iac")
```

```
[Out] ((e*x + d)*(B - C)/b + C*log(b*e^(2*e*x + 2*d) + 2*a*e^(e*x + d) + b)/b - 2
*(B*a - A*b)*arctan((b*e^(e*x + d) + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)
*b))/e
```

Mupad [B]
time = 2.19, size = 653, normalized size = 7.59

$$\frac{2\,\mathrm{atan}\left(\frac{a\,\sqrt{b^4\,e^2-a^2\,b^2\,e^2}\,\sqrt{A^2\,b^2-2\,A\,B\,a\,b+B^2\,a^2}}{B\,e\,a^3\,b-A\,e\,a^2\,b^2-B\,e\,a\,b^3+A\,e\,b^4}+\frac{a^2\,b^2\,e^{e\,x}\,e^d\,\sqrt{b^4\,e^2-a^2\,b^2\,e^2}\,\sqrt{A^2\,b^2-2\,A\,B\,a\,b+B^2\,a^2}}{B\,e\,a^3\,b^4-A\,e\,a^2\,b^5-B\,e\,a\,b^6+A\,e\,b^7}+\frac{A\,e^{e\,x}\,e^d\,\sqrt{b^4\,e^2-a^2\,b^2\,e^2}}{b\,e\,\sqrt{A^2\,b^2-2\,A\,B\,a\,b+B^2\,a^2}}-\frac{B\,a\,e^{e\,x}\,e^d\,\sqrt{b^4\,e^2-a^2\,b^2\,e^2}}{b^2\,e\,\sqrt{A^2\,b^2-2\,A\,B\,a\,b+B^2\,a^2}}\right)\,\sqrt{A^2\,b^2-2\,A\,B\,a\,b+B^2\,a^2}}{\sqrt{b^4\,e^2-a^2\,b^2\,e^2}}+\frac{B\,x}{b}-\frac{C\,x}{b}+\frac{C\,b^3\,e\,\ln\left(4\,A^2\,b^3+4\,B^2\,a^2\,b-8\,A\,B\,a\,b^2+8\,B^2\,a^3\,e^{e\,x}\,e^d+4\,A^2\,b^3\,e^{2\,d}\,e^{2\,e\,x}+8\,A^2\,a\,b^2\,e^{e\,x}\,e^d+4\,B^2\,a^2\,b\,e^{2\,d}\,e^{2\,e\,x}-16\,A\,B\,a^2\,b\,e^{e\,x}\,e^d-8\,A\,B\,a\,b^2\,e^{2\,d}\,e^{2\,e\,x}\right)}{b^4\,e^2-a^2\,b^2\,e^2}-\frac{C\,a^2\,b\,e\,\ln\left(4\,A^2\,b^3+4\,B^2\,a^2\,b-8\,A\,B\,a\,b^2+8\,B^2\,a^3\,e^{e\,x}\,e^d+4\,A^2\,b^3\,e^{2\,d}\,e^{2\,e\,x}+8\,A^2\,a\,b^2\,e^{e\,x}\,e^d+4\,B^2\,a^2\,b\,e^{2\,d}\,e^{2\,e\,x}-16\,A\,B\,a^2\,b\,e^{e\,x}\,e^d-8\,A\,B\,a\,b^2\,e^{2\,d}\,e^{2\,e\,x}\right)}{b^4\,e^2-a^2\,b^2\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + b*cosh(d + e*x)),x)
```

```
[Out] (2*atan((a*(b^4*e^2 - a^2*b^2*e^2)^(1/2)*(A^2*b^2 + B^2*a^2 - 2*A*B*a*b)^(1
/2))/(A*b^4*e - B*a*b^3*e + B*a^3*b*e - A*a^2*b^2*e) + (a^2*b^2*exp(e*x)*ex
p(d)*(b^4*e^2 - a^2*b^2*e^2)^(1/2)*(A^2*b^2 + B^2*a^2 - 2*A*B*a*b)^(1/2))/(
A*b^7*e - B*a*b^6*e - A*a^2*b^5*e + B*a^3*b^4*e) + (A*exp(e*x)*exp(d)*(b^4*
e^2 - a^2*b^2*e^2)^(1/2))/(b*e*(A^2*b^2 + B^2*a^2 - 2*A*B*a*b)^(1/2)) - (B*
a*exp(e*x)*exp(d)*(b^4*e^2 - a^2*b^2*e^2)^(1/2))/(b^2*e*(A^2*b^2 + B^2*a^2
- 2*A*B*a*b)^(1/2)))*(A^2*b^2 + B^2*a^2 - 2*A*B*a*b)^(1/2))/(b^4*e^2 - a^2*
b^2*e^2)^(1/2) + (B*x)/b - (C*x)/b + (C*b^3*e*log(4*A^2*b^3 + 4*B^2*a^2*b -
 8*A*B*a*b^2 + 8*B^2*a^3*exp(e*x)*exp(d) + 4*A^2*b^3*exp(2*d)*exp(2*e*x) +
8*A^2*a*b^2*exp(e*x)*exp(d) + 4*B^2*a^2*b*exp(2*d)*exp(2*e*x) - 16*A*B*a^2*
b*exp(e*x)*exp(d) - 8*A*B*a*b^2*exp(2*d)*exp(2*e*x)))/(b^4*e^2 - a^2*b^2*e^
2) - (C*a^2*b*e*log(4*A^2*b^3 + 4*B^2*a^2*b - 8*A*B*a*b^2 + 8*B^2*a^3*exp(e
*x)*exp(d) + 4*A^2*b^3*exp(2*d)*exp(2*e*x) + 8*A^2*a*b^2*exp(e*x)*exp(d) +
4*B^2*a^2*b*exp(2*d)*exp(2*e*x) - 16*A*B*a^2*b*exp(e*x)*exp(d) - 8*A*B*a*b^
2*exp(2*d)*exp(2*e*x)))/(b^4*e^2 - a^2*b^2*e^2)
```

3.207 $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^2}\,dx$

Optimal. Leaf size=121

$$\frac{2(aA-bB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}e}-\frac{C}{be(a+b\cosh(d+ex))}-\frac{(Ab-aB)\sinh(d+ex)}{(a^2-b^2)\,e(a+b\cosh(d+ex))}$$

```
[Out] 2*(A*a-B*b)*arctanh((a-b)^(1/2)*tanh(1/2*e*x+1/2*d)/(a+b)^(1/2))/(a-b)^(3/2
)/(a+b)^(3/2)/e-C/b/e/(a+b*cosh(e*x+d))-(A*b-B*a)*sinh(e*x+d)/(a^2-b^2)/e/(
a+b*cosh(e*x+d))
```

Rubi [A]
time = 0.13, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4462, 2833, 12, 2738, 211, 2747, 32}

$$-\frac{(Ab-aB)\sinh(d+ex)}{e\left(a^2-b^2\right)(a+b\cosh(d+ex))}+\frac{2(aA-bB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{e(a-b)^{3/2}(a+b)^{3/2}}-\frac{C}{be(a+b\cosh(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + b*Cosh[d + e*x])^2,x]
```

```
[Out] (2*(a*A - b*B)*ArcTanh[(Sqrt[a - b]*Tanh[(d + e*x)/2])/Sqrt[a + b]])/((a -
b)^(3/2)*(a + b)^(3/2)*e) - C/(b*e*(a + b*Cosh[d + e*x])) - ((A*b - a*B)*Si
nh[d + e*x])/((a^2 - b^2)*e*(a + b*Cosh[d + e*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 4462

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^2}\,dx = C\int \frac{\sinh(d+ex)}{(a+b\cosh(d+ex))^2}\,dx + \int \frac{A+B\cosh(d+ex)}{(a+b\cosh(d+ex))^2}\,dx$$

$$= -\frac{(Ab-aB)\sinh(d+ex)}{(a^2-b^2)\,e(a+b\cosh(d+ex))} + \frac{\int \frac{-aA+bB}{a+b\cosh(d+ex)}\,dx}{-a^2+b^2} + \frac{C\mathrm{S}}{}$$

$$= -\frac{C}{be(a+b\cosh(d+ex))} - \frac{(Ab-aB)\sinh(d+ex)}{(a^2-b^2)\,e(a+b\cosh(d+ex))} +$$

$$= -\frac{C}{be(a+b\cosh(d+ex))} - \frac{(Ab-aB)\sinh(d+ex)}{(a^2-b^2)\,e(a+b\cosh(d+ex))} -$$

$$= \frac{2(aA-bB)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}e} - \frac{C}{be(a+b\cosh}$$

Mathematica [A]
time = 0.32, size = 115, normalized size = 0.95

$$\frac{\frac{2(aA-bB)\text{ArcTan}\left(\frac{(a-b)\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{3/2}}+\frac{(-a^2+b^2)C-b(Ab-aB)\sinh(d+ex)}{(a-b)b(a+b)(a+b\cosh(d+ex))}}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + b*Cosh[d + e*x])^2,x
]
```

```
[Out] ((2*(a*A - b*B)*ArcTan[((a - b)*Tanh[(d + e*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2
 + b^2)^(3/2) + ((-a^2 + b^2)*C - b*(A*b - a*B)*Sinh[d + e*x])/((a - b)*b*(
a + b)*(a + b*Cosh[d + e*x])))/e
```

Maple [A]
time = 2.11, size = 144, normalized size = 1.19

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{(Ab-Ba)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)}{a^2-b^2}+\frac{C}{a-b}\right)}{a\left(\tanh^2\left(\frac{ex}{2}+\frac{d}{2}\right)\right)-b\left(\tanh^2\left(\frac{ex}{2}+\frac{d}{2}\right)\right)-a-b}+\frac{2(Aa-Bb)\,\mathrm{arctanh}\left(\frac{(a-b)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}}{e}$
default	$\frac{-\frac{2\left(-\frac{(Ab-Ba)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)}{a^2-b^2}+\frac{C}{a-b}\right)}{a\left(\tanh^2\left(\frac{ex}{2}+\frac{d}{2}\right)\right)-b\left(\tanh^2\left(\frac{ex}{2}+\frac{d}{2}\right)\right)-a-b}+\frac{2(Aa-Bb)\,\mathrm{arctanh}\left(\frac{(a-b)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}}{e}$
risch	$\frac{2Aab\,\mathrm{e}^{ex+d}-2B\,a^2\mathrm{e}^{ex+d}-2C\,a^2\mathrm{e}^{ex+d}+2C\,b^2\mathrm{e}^{ex+d}+2A\,b^2-2Bab}{be(a^2-b^2)\left(b\,\mathrm{e}^{2ex+2d}+2a\,\mathrm{e}^{ex+d}+b\right)}+\frac{\ln\left(\mathrm{e}^{ex+d}+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)Aa}{\sqrt{a^2-b^2}\,(a+b)(a-b)e}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^2,x,method=_RETURNVER
BOSE)
```

```
[Out] 1/e*(-2*(-(A*b-B*a)/(a^2-b^2)*tanh(1/2*e*x+1/2*d)+C/(a-b))/(a*tanh(1/2*e*x+
1/2*d)^2-b*tanh(1/2*e*x+1/2*d)^2-a-b)+2*(A*a-B*b)/(a+b)/(a-b)/((a+b)*(a-b))
^(1/2)*arctanh((a-b)*tanh(1/2*e*x+1/2*d)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^2,x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 681 vs. $2(112) = 224$.
time = 0.46, size = 1526, normalized size = 12.61

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^2,x, algorithm=
"fricas")
```

```
[Out] [-(2*B*a^3*b - 2*A*a^2*b^2 - 2*B*a*b^3 + 2*A*b^4 - (A*a*b^2 - B*b^3 + (A*a*
b^2 - B*b^3)*cosh(x*cosh(1) + x*sinh(1) + d)^2 + (A*a*b^2 - B*b^3)*sinh(x*c
osh(1) + x*sinh(1) + d)^2 + 2*(A*a^2*b - B*a*b^2)*cosh(x*cosh(1) + x*sinh(1
) + d) + 2*(A*a^2*b - B*a*b^2 + (A*a*b^2 - B*b^3)*cosh(x*cosh(1) + x*sinh(1
) + d))*sinh(x*cosh(1) + x*sinh(1) + d))*sqrt(a^2 - b^2)*log((b^2*cosh(x*co
sh(1) + x*sinh(1) + d)^2 + b^2*sinh(x*cosh(1) + x*sinh(1) + d)^2 + 2*a*b*co
sh(x*cosh(1) + x*sinh(1) + d) + 2*a^2 - b^2 + 2*(b^2*cosh(x*cosh(1) + x*sin
h(1) + d) + a*b)*sinh(x*cosh(1) + x*sinh(1) + d) - 2*sqrt(a^2 - b^2)*(b*cos
h(x*cosh(1) + x*sinh(1) + d) + b*sinh(x*cosh(1) + x*sinh(1) + d) + a))/(b*c
osh(x*cosh(1) + x*sinh(1) + d)^2 + b*sinh(x*cosh(1) + x*sinh(1) + d)^2 + 2*
a*cosh(x*cosh(1) + x*sinh(1) + d) + 2*(b*cosh(x*cosh(1) + x*sinh(1) + d) +
a)*sinh(x*cosh(1) + x*sinh(1) + d) + b)) + 2*((B + C)*a^4 - A*a^3*b - (B +
2*C)*a^2*b^2 + A*a*b^3 + C*b^4)*cosh(x*cosh(1) + x*sinh(1) + d) + 2*((B + C
)*a^4 - A*a^3*b - (B + 2*C)*a^2*b^2 + A*a*b^3 + C*b^4)*sinh(x*cosh(1) + x*s
inh(1) + d))/(((a^4*b^2 - 2*a^2*b^4 + b^6)*cosh(1) + (a^4*b^2 - 2*a^2*b^4 +
 b^6)*sinh(1))*cosh(x*cosh(1) + x*sinh(1) + d)^2 + ((a^4*b^2 - 2*a^2*b^4 +
b^6)*cosh(1) + (a^4*b^2 - 2*a^2*b^4 + b^6)*sinh(1))*sinh(x*cosh(1) + x*sinh
(1) + d)^2 + (a^4*b^2 - 2*a^2*b^4 + b^6)*cosh(1) + 2*((a^5*b - 2*a^3*b^3 +
a*b^5)*cosh(1) + (a^5*b - 2*a^3*b^3 + a*b^5)*sinh(1))*cosh(x*cosh(1) + x*si
nh(1) + d) + (a^4*b^2 - 2*a^2*b^4 + b^6)*sinh(1) + 2*((a^5*b - 2*a^3*b^3 +
a*b^5)*cosh(1) + ((a^4*b^2 - 2*a^2*b^4 + b^6)*cosh(1) + (a^4*b^2 - 2*a^2*b^
4 + b^6)*sinh(1))*cosh(x*cosh(1) + x*sinh(1) + d) + (a^5*b - 2*a^3*b^3 + a*
b^5)*sinh(1))*sinh(x*cosh(1) + x*sinh(1) + d)), -2*(B*a^3*b - A*a^2*b^2 - B
*a*b^3 + A*b^4 + (A*a*b^2 - B*b^3 + (A*a*b^2 - B*b^3)*cosh(x*cosh(1) + x*si
nh(1) + d)^2 + (A*a*b^2 - B*b^3)*sinh(x*cosh(1) + x*sinh(1) + d)^2 + 2*(A*a
^2*b - B*a*b^2)*cosh(x*cosh(1) + x*sinh(1) + d) + 2*(A*a^2*b - B*a*b^2 + (A
*a*b^2 - B*b^3)*cosh(x*cosh(1) + x*sinh(1) + d))*sinh(x*cosh(1) + x*sinh(1)
 + d))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x*cosh(1) + x*sinh
```

```
(1) + d) + b*sinh(x*cosh(1) + x*sinh(1) + d) + a)/(a^2 - b^2)) + ((B + C)*a
^4 - A*a^3*b - (B + 2*C)*a^2*b^2 + A*a*b^3 + C*b^4)*cosh(x*cosh(1) + x*sinh
(1) + d) + ((B + C)*a^4 - A*a^3*b - (B + 2*C)*a^2*b^2 + A*a*b^3 + C*b^4)*si
nh(x*cosh(1) + x*sinh(1) + d))/(((a^4*b^2 - 2*a^2*b^4 + b^6)*cosh(1) + (a^4
*b^2 - 2*a^2*b^4 + b^6)*sinh(1))*cosh(x*cosh(1) + x*sinh(1) + d)^2 + ((a^4*
b^2 - 2*a^2*b^4 + b^6)*cosh(1) + (a^4*b^2 - 2*a^2*b^4 + b^6)*sinh(1))*sinh(
x*cosh(1) + x*sinh(1) + d)^2 + (a^4*b^2 - 2*a^2*b^4 + b^6)*cosh(1) + 2*((a^
5*b - 2*a^3*b^3 + a*b^5)*cosh(1) + (a^5*b - 2*a^3*b^3 + a*b^5)*sinh(1))*cos
h(x*cosh(1) + x*sinh(1) + d) + (a^4*b^2 - 2*a^2*b^4 + b^6)*sinh(1) + 2*((a^
5*b - 2*a^3*b^3 + a*b^5)*cosh(1) + ((a^4*b^2 - 2*a^2*b^4 + b^6)*cosh(1) + (
a^4*b^2 - 2*a^2*b^4 + b^6)*sinh(1))*cosh(x*cosh(1) + x*sinh(1) + d) + (a^5*
b - 2*a^3*b^3 + a*b^5)*sinh(1))*sinh(x*cosh(1) + x*sinh(1) + d))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.44, size = 155, normalized size = 1.28

$$\frac{2\left(\frac{(Aa-Bb)\arctan\left(\frac{be^{(ex+d)}+a}{\sqrt{-a^2+b^2}}\right)}{(a^2-b^2)\sqrt{-a^2+b^2}}-\frac{Ba^2e^{(ex+d)}+Ca^2e^{(ex+d)}-Aabe^{(ex+d)}-Cb^2e^{(ex+d)}+Bab-Ab^2}{(a^2b-b^3)\left(be^{(2\,ex+2\,d)}+2\,ae^{(ex+d)}+b\right)}\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^2,x, algorithm=
"giac")
```

```
[Out] 2*((A*a - B*b)*arctan((b*e^(e*x + d) + a)/sqrt(-a^2 + b^2))/((a^2 - b^2)*sq
rt(-a^2 + b^2)) - (B*a^2*e^(e*x + d) + C*a^2*e^(e*x + d) - A*a*b*e^(e*x + d
) - C*b^2*e^(e*x + d) + B*a*b - A*b^2)/((a^2*b - b^3)*(b*e^(2*e*x + 2*d) +
2*a*e^(e*x + d) + b)))/e
```

Mupad [B]
time = 1.59, size = 301, normalized size = 2.49

$$\frac{\frac{2\,\left(A\,b^3-B\,a\,b^2\right)}{b\,e\,\left(a^2\,b-b^3\right)}+\frac{2\,{\mathrm{e}}^{d+e\,x}\,\left(C\,b^4-B\,a^2\,b^2-C\,a^2\,b^2+A\,a\,b^3\right)}{b^2\,e\,\left(a^2\,b-b^3\right)}}{b+2\,a\,{\mathrm{e}}^{d+e\,x}+b\,{\mathrm{e}}^{2\,d+2\,e\,x}}+\frac{\ln\left(-\frac{2\,{\mathrm{e}}^{d+e\,x}\,\left(A\,a-B\,b\right)}{b\,\left(a^2-b^2\right)}-\frac{2\,\left(A\,a-B\,b\right)\,\left(b+a\,{\mathrm{e}}^{d+e\,x}\right)}{b\,{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}\right)\,\left(A\,a-B\,b\right)}{e\,{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}-\frac{\ln\left(\frac{2\,\left(A\,a-B\,b\right)\,\left(b+a\,{\mathrm{e}}^{d+e\,x}\right)}{b\,{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}-\frac{2\,{\mathrm{e}}^{d+e\,x}\,\left(A\,a-B\,b\right)}{b\,\left(a^2-b^2\right)}\right)\,\left(A\,a-B\,b\right)}{e\,{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + b*cosh(d + e*x))^2,x)
```

```
[Out] ((2*(A*b^3 - B*a*b^2))/(b*e*(a^2*b - b^3)) + (2*exp(d + e*x)*(C*b^4 - B*a^2
*b^2 - C*a^2*b^2 + A*a*b^3))/(b^2*e*(a^2*b - b^3)))/(b + 2*a*exp(d + e*x) +
 b*exp(2*d + 2*e*x)) + (log(- (2*exp(d + e*x)*(A*a - B*b))/(b*(a^2 - b^2))
- (2*(A*a - B*b)*(b + a*exp(d + e*x)))/(b*(a + b)^(3/2)*(a - b)^(3/2)))*(A*
a - B*b))/(e*(a + b)^(3/2)*(a - b)^(3/2)) - (log((2*(A*a - B*b)*(b + a*exp(
d + e*x)))/(b*(a + b)^(3/2)*(a - b)^(3/2)) - (2*exp(d + e*x)*(A*a - B*b))/(
b*(a^2 - b^2)))*(A*a - B*b))/(e*(a + b)^(3/2)*(a - b)^(3/2))
```

3.208 $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^3}\,dx$

Optimal. Leaf size=187

$$\frac{(2a^2A+Ab^2-3abB)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}e}-\frac{C}{2be(a+b\cosh(d+ex))^2}-\frac{(Ab-aB)\sinh(d+e}{2\,(a^2-b^2)\,e(a+b\cosh(d}$$

```
[Out] (2*A*a^2+A*b^2-3*B*a*b)*arctanh((a-b)^(1/2)*tanh(1/2*e*x+1/2*d)/(a+b)^(1/2)
)/(a-b)^(5/2)/(a+b)^(5/2)/e-1/2*C/b/e/(a+b*cosh(e*x+d))^2-1/2*(A*b-B*a)*sin
h(e*x+d)/(a^2-b^2)/e/(a+b*cosh(e*x+d))^2-1/2*(3*A*a*b-B*a^2-2*B*b^2)*sinh(e
*x+d)/(a^2-b^2)^2/e/(a+b*cosh(e*x+d))
```

Rubi [A]
time = 0.21, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4462, 2833, 12, 2738, 211, 2747, 32}

$$\frac{(2a^2A-3abB+Ab^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{e(a-b)^{5/2}(a+b)^{5/2}}-\frac{(a^2(-B)+3aAb-2b^2B)\sinh(d+ex)}{2e\left(a^2-b^2\right)^2(a+b\cosh(d+ex))}-\frac{(Ab-aB)\sinh(d+ex)}{2e\left(a^2-b^2\right)(a+b\cosh(d+ex))^2}-\frac{C}{2be(a+b\cosh(d+ex))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + b*Cosh[d + e*x])^3,x]
```

```
[Out] ((2*a^2*A + A*b^2 - 3*a*b*B)*ArcTanh[(Sqrt[a - b]*Tanh[(d + e*x)/2])/Sqrt[a
 + b]])/((a - b)^(5/2)*(a + b)^(5/2)*e) - C/(2*b*e*(a + b*Cosh[d + e*x])^2)
 - ((A*b - a*B)*Sinh[d + e*x])/(2*(a^2 - b^2)*e*(a + b*Cosh[d + e*x])^2) -
((3*a*A*b - a^2*B - 2*b^2*B)*Sinh[d + e*x])/(2*(a^2 - b^2)^2*e*(a + b*Cosh[
d + e*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 4462

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^3}\,dx = C\int \frac{\sinh(d+ex)}{(a+b\cosh(d+ex))^3}\,dx + \int \frac{A+B\cosh(d+ex)}{(a+b\cosh(d+ex))^3}\,dx$$

$$= -\frac{(Ab-aB)\sinh(d+ex)}{2(a^2-b^2)\,e(a+b\cosh(d+ex))^2} - \frac{\int \frac{-2(aA-bB)+(Ab-aB)\cosh}{(a+b\cosh(d+ex))^2}}{2(a^2-b^2)}$$

$$= -\frac{C}{2be(a+b\cosh(d+ex))^2} - \frac{(Ab-aB)\sinh(d+ex)}{2(a^2-b^2)\,e(a+b\cosh(d+ex)}$$

$$= -\frac{C}{2be(a+b\cosh(d+ex))^2} - \frac{(Ab-aB)\sinh(d+ex)}{2(a^2-b^2)\,e(a+b\cosh(d+ex)}$$

$$= -\frac{C}{2be(a+b\cosh(d+ex))^2} - \frac{(Ab-aB)\sinh(d+ex)}{2(a^2-b^2)\,e(a+b\cosh(d+ex)}$$

$$= \frac{(2a^2A+Ab^2-3abB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}e} - \frac{}{2be(}$$

Mathematica [A]
time = 0.58, size = 175, normalized size = 0.94

$$\frac{-\frac{2(2a^2A+Ab^2-3abB)\operatorname{ArcTan}\left(\frac{(a-b)\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{5/2}} + \frac{(-3aAb+a^2B+2b^2B)\sinh(d+ex)}{(a-b)^2(a+b)^2(a+b\cosh(d+ex))} + \frac{(-a^2+b^2)C-b(Ab-aB)\sinh(d+ex)}{(a-b)b(a+b)(a+b\cosh(d+ex))^2}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + b*Cosh[d + e*x])^3,x
]
```

```
[Out] ((-2*(2*a^2*A + A*b^2 - 3*a*b*B)*ArcTan[((a - b)*Tanh[(d + e*x)/2])/Sqrt[-a
^2 + b^2]])/(-a^2 + b^2)^(5/2) + ((-3*a*A*b + a^2*B + 2*b^2*B)*Sinh[d + e*x
])/((a - b)^2*(a + b)^2*(a + b*Cosh[d + e*x])) + ((-a^2 + b^2)*C - b*(A*b -
 a*B)*Sinh[d + e*x])/((a - b)*b*(a + b)*(a + b*Cosh[d + e*x])^2))/(2*e)
```

Maple [A]
time = 2.25, size = 273, normalized size = 1.46

method	result
derivativedivides	$-\frac{\frac{2\left(-\frac{\left(4Aab+A\,b^2-2B\,a^2-Bab-2B\,b^2\right)\left(\tanh^3\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{2(a-b)\left(a^2+2ab+b^2\right)}+\frac{C\left(\tanh^2\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{a-b}+\frac{\left(4Aab-A\,b^2-2B\,a^2+Bab-2B\,b^2\right)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)}{2(a+b)\left(a^2-2ab+b^2\right)}\right.}{\left(a\left(\tanh^2\left(\frac{ex}{2}+\frac{d}{2}\right)\right)-b\left(\tanh^2\left(\frac{ex}{2}+\frac{d}{2}\right)\right)-a-b\right)^2}}{e}$

default	$-\frac{2\left(-\frac{\left(4Aab+A\,b^2-2B\,a^2-Bab-2B\,b^2\right)\left(\tanh^3\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{2(a-b)\left(a^2+2ab+b^2\right)}+\frac{C\left(\tanh^2\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{a-b}+\frac{\left(4Aab-A\,b^2-2B\,a^2+Bab-2B\,b^2\right)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)}{2(a+b)\left(a^2-2ab+b^2\right)}\right.}{\left(a\left(\tanh^2\left(\frac{ex}{2}+\frac{d}{2}\right)\right)-b\left(\tanh^2\left(\frac{ex}{2}+\frac{d}{2}\right)\right)-a-b\right)^2}}{e}$
risch	$\frac{2A\,a^2b^2\mathrm{e}^{3ex+3d}+A\,b^4\mathrm{e}^{3ex+3d}-3Ba\,b^3\mathrm{e}^{3ex+3d}+6A\,a^3b\,\mathrm{e}^{2ex+2d}+3Aa\,b^3\mathrm{e}^{2ex+2d}-2B\,a^4\mathrm{e}^{2ex+2d}-5B\,a^2b^2\mathrm{e}^{2ex+2d}-2B}{be(a^2-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^3,x,method=_RETURNVER
BOSE)
```

```
[Out] 1/e*(-2*(-1/2*(4*A*a*b+A*b^2-2*B*a^2-B*a*b-2*B*b^2)/(a-b)/(a^2+2*a*b+b^2)*t
anh(1/2*e*x+1/2*d)^3+C/(a-b)*tanh(1/2*e*x+1/2*d)^2+1/2*(4*A*a*b-A*b^2-2*B*a
^2+B*a*b-2*B*b^2)/(a+b)/(a^2-2*a*b+b^2)*tanh(1/2*e*x+1/2*d)-a*C/(a^2-2*a*b+
b^2))/(a*tanh(1/2*e*x+1/2*d)^2-b*tanh(1/2*e*x+1/2*d)^2-a-b)^2+(2*A*a^2+A*b^
2-3*B*a*b)/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*e
*x+1/2*d)/((a+b)*(a-b))^(1/2)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^3,x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2496 vs. 2(173) = 346.
time = 0.51, size = 5158, normalized size = 27.58

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^3,x, algorithm=
"fricas")
```

```
[Out] [-1/2*(2*B*a^4*b^2 - 6*A*a^3*b^3 + 2*B*a^2*b^4 + 6*A*a*b^5 - 4*B*b^6 - 2*(2
*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6)*cosh(x*cosh(1) +
```

x*sinh(1) + d)^3 - 2*(2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A
*b^6)*sinh(x*cosh(1) + x*sinh(1) + d)^3 + 2*(2*(B + C)*a^6 - 6*A*a^5*b + 3*
(B - 2*C)*a^4*b^2 + 3*A*a^3*b^3 - 3*(B - 2*C)*a^2*b^4 + 3*A*a*b^5 - 2*(B +
C)*b^6)*cosh(x*cosh(1) + x*sinh(1) + d)^2 + 2*(2*(B + C)*a^6 - 6*A*a^5*b +
3*(B - 2*C)*a^4*b^2 + 3*A*a^3*b^3 - 3*(B - 2*C)*a^2*b^4 + 3*A*a*b^5 - 2*(B
+ C)*b^6 - 3*(2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6)*co
sh(x*cosh(1) + x*sinh(1) + d))*sinh(x*cosh(1) + x*sinh(1) + d)^2 - (2*A*a^2
*b^3 - 3*B*a*b^4 + A*b^5 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*cosh(x*cosh(1)
 + x*sinh(1) + d)^4 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*sinh(x*cosh(1) + x*
sinh(1) + d)^4 + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*cosh(x*cosh(1) + x
sinh(1) + d)^3 + 4(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 + (2*A*a^2*b^3 - 3
*B*a*b^4 + A*b^5)*cosh(x*cosh(1) + x*sinh(1) + d))*sinh(x*cosh(1) + x*sinh(
1) + d)^3 + 2*(4*A*a^4*b - 6*B*a^3*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*c
osh(x*cosh(1) + x*sinh(1) + d)^2 + 2*(4*A*a^4*b - 6*B*a^3*b^2 + 4*A*a^2*b^3
 - 3*B*a*b^4 + A*b^5 + 3*(2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*cosh(x*cosh(1) +
 x*sinh(1) + d)^2 + 6*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*cosh(x*cosh(1)
+ x*sinh(1) + d))*sinh(x*cosh(1) + x*sinh(1) + d)^2 + 4*(2*A*a^3*b^2 - 3*B*
a^2*b^3 + A*a*b^4)*cosh(x*cosh(1) + x*sinh(1) + d) + 4*(2*A*a^3*b^2 - 3*B*a
^2*b^3 + A*a*b^4 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*cosh(x*cosh(1) + x*sin
h(1) + d)^3 + 3*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*cosh(x*cosh(1) + x*si
nh(1) + d)^2 + (4*A*a^4*b - 6*B*a^3*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*
cosh(x*cosh(1) + x*sinh(1) + d))*sinh(x*cosh(1) + x*sinh(1) + d))*sqrt(a^2
- b^2)*log((b^2*cosh(x*cosh(1) + x*sinh(1) + d)^2 + b^2*sinh(x*cosh(1) + x*
sinh(1) + d)^2 + 2*a*b*cosh(x*cosh(1) + x*sinh(1) + d) + 2*a^2 - b^2 + 2*(b
^2*cosh(x*cosh(1) + x*sinh(1) + d) + a*b)*sinh(x*cosh(1) + x*sinh(1) + d) -
 2*sqrt(a^2 - b^2)*(b*cosh(x*cosh(1) + x*sinh(1) + d) + b*sinh(x*cosh(1) +
x*sinh(1) + d) + a))/(b*cosh(x*cosh(1) + x*sinh(1) + d)^2 + b*sinh(x*cosh(1
) + x*sinh(1) + d)^2 + 2*a*cosh(x*cosh(1) + x*sinh(1) + d) + 2*(b*cosh(x*co
sh(1) + x*sinh(1) + d) + a)*sinh(x*cosh(1) + x*sinh(1) + d) + b)) + 2*(4*B*
a^5*b - 10*A*a^4*b^2 + B*a^3*b^3 + 11*A*a^2*b^4 - 5*B*a*b^5 - A*b^6)*cosh(x
*cosh(1) + x*sinh(1) + d) + 2*(4*B*a^5*b - 10*A*a^4*b^2 + B*a^3*b^3 + 11*A*
a^2*b^4 - 5*B*a*b^5 - A*b^6 - 3*(2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*
B*a*b^5 - A*b^6)*cosh(x*cosh(1) + x*sinh(1) + d)^2 + 2*(2*(B + C)*a^6 - 6*A
*a^5*b + 3*(B - 2*C)*a^4*b^2 + 3*A*a^3*b^3 - 3*(B - 2*C)*a^2*b^4 + 3*A*a*b^
5 - 2*(B + C)*b^6)*cosh(x*cosh(1) + x*sinh(1) + d))*sinh(x*cosh(1) + x*sinh
(1) + d))/(((a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*cosh(1) + (a^6*b^3 - 3*
a^4*b^5 + 3*a^2*b^7 - b^9)*sinh(1))*cosh(x*cosh(1) + x*sinh(1) + d)^4 + ((a
^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*cosh(1) + (a^6*b^3 - 3*a^4*b^5 + 3*a^
2*b^7 - b^9)*sinh(1))*sinh(x*cosh(1) + x*sinh(1) + d)^4 + 4*((a^7*b^2 - 3*a
^5*b^4 + 3*a^3*b^6 - a*b^8)*cosh(1) + (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*
b^8)*sinh(1))*cosh(x*cosh(1) + x*sinh(1) + d)^3 + 4*((a^7*b^2 - 3*a^5*b^4 +
 3*a^3*b^6 - a*b^8)*cosh(1) + ((a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*cosh
(1) + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*sinh(1))*cosh(x*cosh(1) + x*s
inh(1) + d) + (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*sinh(1))*sinh(x*cos
h(1) + x*sinh(1) + d)^3 + 2*((2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b

```
^9)*cosh(1) + (2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9)*sinh(1))*co
sh(x*cosh(1) + x*sinh(1) + d)^2 + 2*(3*((a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 -
b^9)*cosh(1) + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*sinh(1))*cosh(x*cosh
(1) + x*sinh(1) + d)^2 + (2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9)*
cosh(1) + 6*((a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*cosh(1) + (a^7*b^2 -
 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*sinh(1))*cosh(x*cosh(1) + x*sinh(1) + d) +
(2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9)*sinh(1))*sinh(x*cosh(1) +
 x*sinh(1) + d)^2 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*cosh(1) + 4*((a
^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*cosh(1) + (a^7*b^2 - 3*a^5*b^4 + 3*
a^3*b^6 - a*b^8)*sinh(1))*cosh(x*cosh(1) + x*sinh(1) + d) + (a^6*b^3 - 3*a^
4*b^5 + 3*a^2*b^7 - b^9)*sinh(1) + 4*(((a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b
^9)*cosh(1) + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*sinh(1))*cosh(x*cosh(
1) + x*sinh(1) + d)^3 + 3*((a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*cosh(1
) + (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*sinh(1))*cosh(x*cosh(1) + x*s
inh(1) + d)^2 + (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*cosh(1) + ((2*a^8
*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9)*cosh(1) + (2*a^8*b - 5*a^6*b^3
+ 3*a^4*b^5 + a^2*b^7 - b^9)*sinh(1))*cosh(x*cosh(1) + x*sinh(1) + d) + (a^
7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*sinh(1))*sinh(x*cosh(1) + x*sinh(1)
+ d)), -(B*a^4*b^2 - 3*A*a^3*b^3 + B*a^2*b^4 + 3*A*a*b^5 - 2*B*b^6 - (2*A*a
^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(171) = 342.
time = 0.48, size = 370, normalized size = 1.98

$$\frac{\frac{(2\,Aa^2-3\,Bab+Ab^2)\arctan\left(\frac{be^{(ex+d)}+a}{\sqrt{-a^2+b^2}}\right)}{(a^4-2\,a^2b^2+b^4)\sqrt{-a^2+b^2}}+\frac{2\,Aa^2b^2e^{(3\,ex+3\,d)}-3\,Bab^3e^{(3\,ex+3\,d)}+Ab^4e^{(3\,ex+3\,d)}-2\,Ba^4e^{(2\,ex+2\,d)}-2\,Ca^4e^{(2\,ex+2\,d)}+6\,Aa^3be^{(2\,ex+2\,d)}-5\,Ba^2b^2e^{(2\,ex+2\,d)}+4\,Ca^2b^2e^{(2\,ex+2\,d)}+3\,Aab^3e^{(2\,ex+2\,d)}-2\,Bb^4e^{(2\,ex+2\,d)}-2\,Cb^4e^{(2\,ex+2\,d)}-4\,Ba^3be^{(ex+d)}+10\,Aa^2b^2e^{(ex+d)}-5\,Bab^3e^{(ex+d)}-Ab^4e^{(ex+d)}-Ba^2b^2+3\,Aab^3-2\,Bb^4}{(a^4b-2\,a^2b^3+b^5)\left(be^{(2\,ex+2\,d)}+2\,ae^{(ex+d)}+b\right)^2}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^3,x, algorithm=
"giac")
```

```
[Out] ((2*A*a^2 - 3*B*a*b + A*b^2)*arctan((b*e^(e*x + d) + a)/sqrt(-a^2 + b^2))/(
(a^4 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) + (2*A*a^2*b^2*e^(3*e*x + 3*d) -
3*B*a*b^3*e^(3*e*x + 3*d) + A*b^4*e^(3*e*x + 3*d) - 2*B*a^4*e^(2*e*x + 2*d)
 - 2*C*a^4*e^(2*e*x + 2*d) + 6*A*a^3*b*e^(2*e*x + 2*d) - 5*B*a^2*b^2*e^(2*e
```

```
*x + 2*d) + 4*C*a^2*b^2*e^(2*e*x + 2*d) + 3*A*a*b^3*e^(2*e*x + 2*d) - 2*B*b
^4*e^(2*e*x + 2*d) - 2*C*b^4*e^(2*e*x + 2*d) - 4*B*a^3*b*e^(e*x + d) + 10*A
*a^2*b^2*e^(e*x + d) - 5*B*a*b^3*e^(e*x + d) - A*b^4*e^(e*x + d) - B*a^2*b^
2 + 3*A*a*b^3 - 2*B*b^4)/((a^4*b - 2*a^2*b^3 + b^5)*(b*e^(2*e*x + 2*d) + 2*
a*e^(e*x + d) + b)^2))/e
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\cosh(d+e\,x)+C\sinh(d+e\,x)}{\left(a+b\cosh\left(d+e\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + b*cosh(d + e*x))^3,x)
[Out] int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + b*cosh(d + e*x))^3, x)
```

3.209 $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^4}\,dx$

Optimal. Leaf size=260

$$\frac{(2a^3A+3aAb^2-4a^2bB-b^3B)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}e}-\frac{C}{3be(a+b\cosh(d+ex))^3}-\frac{(Ab-aB}{3\,(a^2-b^2)\,e(a}$$

```
[Out] (2*A*a^3+3*A*a*b^2-4*B*a^2*b-B*b^3)*arctanh((a-b)^(1/2)*tanh(1/2*e*x+1/2*d)
/(a+b)^(1/2))/(a-b)^(7/2)/(a+b)^(7/2)/e-1/3*C/b/e/(a+b*cosh(e*x+d))^3-1/3*(
A*b-B*a)*sinh(e*x+d)/(a^2-b^2)/e/(a+b*cosh(e*x+d))^3-1/6*(5*A*a*b-2*B*a^2-3
*B*b^2)*sinh(e*x+d)/(a^2-b^2)^2/e/(a+b*cosh(e*x+d))^2-1/6*(11*A*a^2*b+4*A*b
^3-2*B*a^3-13*B*a*b^2)*sinh(e*x+d)/(a^2-b^2)^3/e/(a+b*cosh(e*x+d))
```

Rubi [A]
time = 0.33, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4462, 2833, 12, 2738, 211, 2747, 32}

$$-\frac{(-2a^2B+5aAb-3b^2B)\sinh(d+ex)}{6e\,(a^2-b^2)^2\,(a+b\cosh(d+ex))^2}-\frac{(Ab-aB)\sinh(d+ex)}{3e\,(a^2-b^2)\,(a+b\cosh(d+ex))^3}+\frac{(2a^3A-4a^2bB+3aAb^2-b^3B)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{e(a-b)^{7/2}(a+b)^{7/2}}-\frac{(-2a^3B+11a^2Ab-13ab^2B+4Ab^3)\sinh(d+ex)}{6e\,(a^2-b^2)^3\,(a+b\cosh(d+ex))}-\frac{C}{3be(a+b\cosh(d+ex))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + b*Cosh[d + e*x])^4,x]
```

```
[Out] ((2*a^3*A + 3*a*A*b^2 - 4*a^2*b*B - b^3*B)*ArcTanh[(Sqrt[a - b]*Tanh[(d + e
*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*e) - C/(3*b*e*(a + b*Cos
h[d + e*x])^3) - ((A*b - a*B)*Sinh[d + e*x])/(3*(a^2 - b^2)*e*(a + b*Cosh[d
 + e*x])^3) - ((5*a*A*b - 2*a^2*B - 3*b^2*B)*Sinh[d + e*x])/(6*(a^2 - b^2)^
2*e*(a + b*Cosh[d + e*x])^2) - ((11*a^2*A*b + 4*A*b^3 - 2*a^3*B - 13*a*b^2*
B)*Sinh[d + e*x])/(6*(a^2 - b^2)^3*e*(a + b*Cosh[d + e*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 4462

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^4}\,dx = C\int \frac{\sinh(d+ex)}{(a+b\cosh(d+ex))^4}\,dx + \int \frac{A+B\cosh(d+ex)}{(a+b\cosh(d+ex))^4}\,dx$$

$$= -\frac{(Ab-aB)\sinh(d+ex)}{3(a^2-b^2)e(a+b\cosh(d+ex))^3} - \frac{\int \frac{-3(aA-bB)+2(Ab-aB)\,\mathrm{c}}{(a+b\cosh(d+ex))}}{3(a^2-b^2)}$$

$$= -\frac{C}{3be(a+b\cosh(d+ex))^3} - \frac{(Ab-aB)\sinh(d+ex)}{3(a^2-b^2)e(a+b\cosh(d+ex}$$

$$= -\frac{C}{3be(a+b\cosh(d+ex))^3} - \frac{(Ab-aB)\sinh(d+ex)}{3(a^2-b^2)e(a+b\cosh(d+ex}$$

$$= -\frac{C}{3be(a+b\cosh(d+ex))^3} - \frac{(Ab-aB)\sinh(d+ex)}{3(a^2-b^2)e(a+b\cosh(d+ex}$$

$$= -\frac{C}{3be(a+b\cosh(d+ex))^3} - \frac{(Ab-aB)\sinh(d+ex)}{3(a^2-b^2)e(a+b\cosh(d+ex}$$

$$= \frac{(2a^3A+3aAb^2-4a^2bB-b^3B)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(\frac{1}{2}(d+ex}{\sqrt{a+b}}\right.}{(a-b)^{7/2}(a+b)^{7/2}e}$$

Mathematica [A]

time = 1.75, size = 245, normalized size = 0.94

$$\frac{\frac{6(2a^3A+3aAb^2-4a^2bB-b^3B)\,\mathrm{ArcTan}\left(\frac{(a-b)\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{7/2}} + \frac{(-5aAb+2a^2B+3b^2B)\sinh(d+ex)}{(a-b)^2(a+b)^2(a+b\cosh(d+ex))^2} + \frac{(-11a^2Ab-4Ab^3+2a^3B+13ab^2B)\sinh(d+ex)}{(a-b)^3(a+b)^3(a+b\cosh(d+ex))} + \frac{2(-a^2+b^2)C-2b(Ab-aB)\sinh(d+ex)}{(a-b)b(a+b)(a+b\cosh(d+ex))^3}}{6e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + b*Cosh[d + e*x])^4,x
]
```

```
[Out] ((6*(2*a^3*A + 3*a*A*b^2 - 4*a^2*b*B - b^3*B)*ArcTan[((a - b)*Tanh[(d + e*x
)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(7/2) + ((-5*a*A*b + 2*a^2*B + 3*b^2*
B)*Sinh[d + e*x])/((a - b)^2*(a + b)^2*(a + b*Cosh[d + e*x])^2) + ((-11*a^2
*A*b - 4*A*b^3 + 2*a^3*B + 13*a*b^2*B)*Sinh[d + e*x])/((a - b)^3*(a + b)^3*
(a + b*Cosh[d + e*x])) + (2*(-a^2 + b^2)*C - 2*b*(A*b - a*B)*Sinh[d + e*x])
/((a - b)*b*(a + b)*(a + b*Cosh[d + e*x])^3))/(6*e)
```

Maple [A]

time = 2.22, size = 459, normalized size = 1.77 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^4,x,method=_RETURNVER
BOSE)
```

```
[Out] 1/e*(-2*(-1/2*(6*A*a^2*b+3*A*a*b^2+2*A*b^3-2*B*a^3-2*B*a^2*b-6*B*a*b^2-B*b^
3)/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tanh(1/2*e*x+1/2*d)^5+C/(a-b)*tanh(1/2*e
*x+1/2*d)^4+2/3*(9*A*a^2*b+A*b^3-3*B*a^3-7*B*a*b^2)/(a^2+2*a*b+b^2)/(a^2-2*
a*b+b^2)*tanh(1/2*e*x+1/2*d)^3-2*a*C/(a^2-2*a*b+b^2)*tanh(1/2*e*x+1/2*d)^2-
1/2*(6*A*a^2*b-3*A*a*b^2+2*A*b^3-2*B*a^3+2*B*a^2*b-6*B*a*b^2+B*b^3)/(a+b)/(
a^3-3*a^2*b+3*a*b^2-b^3)*tanh(1/2*e*x+1/2*d)+1/3*C*(3*a^2+b^2)/(a^3-3*a^2*b
+3*a*b^2-b^3))/(a*tanh(1/2*e*x+1/2*d)^2-b*tanh(1/2*e*x+1/2*d)^2-a-b)^3+(2*A
*a^3+3*A*a*b^2-4*B*a^2*b-B*b^3)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))
^(1/2)*arctanh((a-b)*tanh(1/2*e*x+1/2*d)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^4,x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5759 vs. 2(246) = 492.
time = 0.67, size = 11677, normalized size = 44.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^4,x, algorithm=
"fricas")
```

```
[Out] [-1/6*(4*B*a^5*b^3 - 22*A*a^4*b^4 + 22*B*a^3*b^5 + 14*A*a^2*b^6 - 26*B*a*b^
7 + 8*A*b^8 - 6*(2*A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*
a*b^7 + B*b^8)*cosh(x*cosh(1) + x*sinh(1) + d)^5 - 6*(2*A*a^5*b^3 - 4*B*a^4
*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 + B*b^8)*sinh(x*cosh(1) + x*sinh
(1) + d)^5 - 30*(2*A*a^6*b^2 - 4*B*a^5*b^3 + A*a^4*b^4 + 3*B*a^3*b^5 - 3*A*
a^2*b^6 + B*a*b^7)*cosh(x*cosh(1) + x*sinh(1) + d)^4 - 30*(2*A*a^6*b^2 - 4*
B*a^5*b^3 + A*a^4*b^4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 + B*a*b^7 + (2*A*a^5*b^3
- 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 + B*b^8)*cosh(x*cosh(1)
 + x*sinh(1) + d))*sinh(x*cosh(1) + x*sinh(1) + d)^4 + 4*(4*(B + C)*a^8 - 2
2*A*a^7*b + 4*(7*B - 4*C)*a^6*b^2 - 19*A*a^5*b^3 + (7*B + 24*C)*a^4*b^4 + 2
9*A*a^3*b^5 - (39*B + 16*C)*a^2*b^6 + 12*A*a*b^7 + 4*C*b^8)*cosh(x*cosh(1)
+ x*sinh(1) + d)^3 + 4*(4*(B + C)*a^8 - 22*A*a^7*b + 4*(7*B - 4*C)*a^6*b^2
```

- 19*A*a^5*b^3 + (7*B + 24*C)*a^4*b^4 + 29*A*a^3*b^5 - (39*B + 16*C)*a^2*b^
6 + 12*A*a*b^7 + 4*C*b^8 - 15*(2*A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*
a^2*b^6 - 3*A*a*b^7 + B*b^8)*cosh(x*cosh(1) + x*sinh(1) + d)^2 - 30*(2*A*a^
6*b^2 - 4*B*a^5*b^3 + A*a^4*b^4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 + B*a*b^7)*cosh
(x*cosh(1) + x*sinh(1) + d))*sinh(x*cosh(1) + x*sinh(1) + d)^3 + 12*(4*B*a^
7*b - 17*A*a^6*b^2 + 13*B*a^5*b^3 + 11*A*a^4*b^4 - 13*B*a^3*b^5 + 4*A*a^2*b
^6 - 4*B*a*b^7 + 2*A*b^8)*cosh(x*cosh(1) + x*sinh(1) + d)^2 + 12*(4*B*a^7*b
 - 17*A*a^6*b^2 + 13*B*a^5*b^3 + 11*A*a^4*b^4 - 13*B*a^3*b^5 + 4*A*a^2*b^6
- 4*B*a*b^7 + 2*A*b^8 - 5*(2*A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*
b^6 - 3*A*a*b^7 + B*b^8)*cosh(x*cosh(1) + x*sinh(1) + d)^3 - 15*(2*A*a^6*b^
2 - 4*B*a^5*b^3 + A*a^4*b^4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 + B*a*b^7)*cosh(x*c
osh(1) + x*sinh(1) + d)^2 + (4*(B + C)*a^8 - 22*A*a^7*b + 4*(7*B - 4*C)*a^6
*b^2 - 19*A*a^5*b^3 + (7*B + 24*C)*a^4*b^4 + 29*A*a^3*b^5 - (39*B + 16*C)*a
^2*b^6 + 12*A*a*b^7 + 4*C*b^8)*cosh(x*cosh(1) + x*sinh(1) + d))*sinh(x*cosh
(1) + x*sinh(1) + d)^2 - 3*(2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7 +
 (2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x*cosh(1) + x*sinh(1)
 + d)^6 + (2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*sinh(x*cosh(1) +
x*sinh(1) + d)^6 + 6*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6)*co
sh(x*cosh(1) + x*sinh(1) + d)^5 + 6*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^
5 - B*a*b^6 + (2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x*cosh(1
) + x*sinh(1) + d))*sinh(x*cosh(1) + x*sinh(1) + d)^5 + 3*(8*A*a^5*b^2 - 16
*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x*cosh(1)
 + x*sinh(1) + d)^4 + 3*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^
2*b^5 + 3*A*a*b^6 - B*b^7 + 5*(2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^
7)*cosh(x*cosh(1) + x*sinh(1) + d)^2 + 10*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*
a^2*b^5 - B*a*b^6)*cosh(x*cosh(1) + x*sinh(1) + d))*sinh(x*cosh(1) + x*sinh
(1) + d)^4 + 4*(4*A*a^6*b - 8*B*a^5*b^2 + 12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A
*a^2*b^5 - 3*B*a*b^6)*cosh(x*cosh(1) + x*sinh(1) + d)^3 + 4*(4*A*a^6*b - 8*
B*a^5*b^2 + 12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*a^2*b^5 - 3*B*a*b^6 + 5*(2*A*
a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x*cosh(1) + x*sinh(1) + d)^
3 + 15*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6)*cosh(x*cosh(1) +
 x*sinh(1) + d)^2 + 3*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*
b^5 + 3*A*a*b^6 - B*b^7)*cosh(x*cosh(1) + x*sinh(1) + d))*sinh(x*cosh(1) +
x*sinh(1) + d)^3 + 3*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b
^5 + 3*A*a*b^6 - B*b^7)*cosh(x*cosh(1) + x*sinh(1) + d)^2 + 3*(8*A*a^5*b^2
- 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7 + 5*(2*A*a^
3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x*cosh(1) + x*sinh(1) + d)^4
+ 20*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6)*cosh(x*cosh(1) + x
sinh(1) + d)^3 + 6(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^
5 + 3*A*a*b^6 - B*b^7)*cosh(x*cosh(1) + x*sinh(1) + d)^2 + 4*(4*A*a^6*b - 8
*B*a^5*b^2 + 12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*a^2*b^5 - 3*B*a*b^6)*cosh(x*
cosh(1) + x*sinh(1) + d))*sinh(x*cosh(1) + x*sinh(1) + d)^2 + 6*(2*A*a^4*b^
3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6)*cosh(x*cosh(1) + x*sinh(1) + d) +
6*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6 + (2*A*a^3*b^4 - 4*B*a
^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x*cosh(1) + x*sinh(1) + d)^5 + 5*(2*A*a^4*

```
b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6)*cosh(x*cosh(1) + x*sinh(1) + d)^
4 + 2*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6
- B*b^7)*cosh(x*cosh(1) + x*sinh(1) + d)^3 + 2*(4*A*a^6*b - 8*B*a^5*b^2 + 1
2*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*a^2*b^5 - 3*B*a*b^6)*cosh(x*cosh(1) + x*si
nh(1) + d)^2 + (8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3
*A*a*b^6 - B*b^7)*cosh(x*cosh(1) + x*sinh(1) + d))*sinh(x*cosh(1) + x*sinh(
1) + d))*sqrt(a^2 - b^2)*log((b^2*cosh(x*cosh(1) + x*sinh(1) + d)^2 + b^2*s
inh(x*cosh(1) + x*sinh(1) + d)^2 + 2*a*b*cosh(x*cosh(1) + x*sinh(1) + d) +
2*a^2 - b^2 + 2*(b^2*cosh(x*cosh(1) + x*sinh(1) + d) + a*b)*sinh(x*cosh(1)
+ x*sinh(1) + d) - 2*sqrt(a^2 - b^2)*(b*cosh(x*...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))**4,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 657 vs. 2(243) = 486.
time = 0.46, size = 657, normalized size = 2.53

$$\frac{\frac{3\,(2\,Aa^3-4\,Ba^2b+3\,Aab^2-Bb^3)\arctan\left(\frac{be^{(ex+d)}+a}{\sqrt{-a^2+b^2}}\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\sqrt{-a^2+b^2}}+\frac{6\,Aa^3b^3e^{(5\,ex+5\,d)}-12\,Ba^2b^4e^{(5\,ex+5\,d)}+9\,Aab^5e^{(5\,ex+5\,d)}-3\,Bb^6e^{(5\,ex+5\,d)}+30\,Aa^4b^2e^{(4\,ex+4\,d)}-60\,Ba^3b^3e^{(4\,ex+4\,d)}+45\,Aa^2b^4e^{(4\,ex+4\,d)}-15\,Bab^5e^{(4\,ex+4\,d)}-8\,Ba^6e^{(3\,ex+3\,d)}-8\,Ca^6e^{(3\,ex+3\,d)}+44\,Aa^5be^{(3\,ex+3\,d)}-64\,Ba^4b^2e^{(3\,ex+3\,d)}+24\,Ca^4b^2e^{(3\,ex+3\,d)}+82\,Aa^3b^3e^{(3\,ex+3\,d)}-78\,Ba^2b^4e^{(3\,ex+3\,d)}-24\,Ca^2b^4e^{(3\,ex+3\,d)}+24\,Aab^5e^{(3\,ex+3\,d)}+8\,Cb^6e^{(3\,ex+3\,d)}-24\,Ba^5be^{(2\,ex+2\,d)}+102\,Aa^4b^2e^{(2\,ex+2\,d)}-102\,Ba^3b^3e^{(2\,ex+2\,d)}+36\,Aa^2b^4e^{(2\,ex+2\,d)}-24\,Bab^5e^{(2\,ex+2\,d)}+12\,Ab^6e^{(2\,ex+2\,d)}-12\,Ba^4b^2e^{(ex+d)}+60\,Aa^3b^3e^{(ex+d)}-66\,Ba^2b^4e^{(ex+d)}+15\,Aab^5e^{(ex+d)}+3\,Bb^6e^{(ex+d)}-2\,Ba^3b^3+11\,Aa^2b^4-13\,Bab^5+4\,Ab^6}{(a^6b-3\,a^4b^3+3\,a^2b^5-b^7)\left(be^{(2\,ex+2\,d)}+2\,ae^{(ex+d)}+b\right)^3}}{3\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^4,x, algorithm=
"giac")
```

```
[Out] 1/3*(3*(2*A*a^3 - 4*B*a^2*b + 3*A*a*b^2 - B*b^3)*arctan((b*e^(e*x + d) + a)
/sqrt(-a^2 + b^2))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) +
 (6*A*a^3*b^3*e^(5*e*x + 5*d) - 12*B*a^2*b^4*e^(5*e*x + 5*d) + 9*A*a*b^5*e^
(5*e*x + 5*d) - 3*B*b^6*e^(5*e*x + 5*d) + 30*A*a^4*b^2*e^(4*e*x + 4*d) - 60
*B*a^3*b^3*e^(4*e*x + 4*d) + 45*A*a^2*b^4*e^(4*e*x + 4*d) - 15*B*a*b^5*e^(4
*e*x + 4*d) - 8*B*a^6*e^(3*e*x + 3*d) - 8*C*a^6*e^(3*e*x + 3*d) + 44*A*a^5*
b*e^(3*e*x + 3*d) - 64*B*a^4*b^2*e^(3*e*x + 3*d) + 24*C*a^4*b^2*e^(3*e*x +
3*d) + 82*A*a^3*b^3*e^(3*e*x + 3*d) - 78*B*a^2*b^4*e^(3*e*x + 3*d) - 24*C*a
^2*b^4*e^(3*e*x + 3*d) + 24*A*a*b^5*e^(3*e*x + 3*d) + 8*C*b^6*e^(3*e*x + 3*
d) - 24*B*a^5*b*e^(2*e*x + 2*d) + 102*A*a^4*b^2*e^(2*e*x + 2*d) - 102*B*a^3
*b^3*e^(2*e*x + 2*d) + 36*A*a^2*b^4*e^(2*e*x + 2*d) - 24*B*a*b^5*e^(2*e*x +
 2*d) + 12*A*b^6*e^(2*e*x + 2*d) - 12*B*a^4*b^2*e^(e*x + d) + 60*A*a^3*b^3*
e^(e*x + d) - 66*B*a^2*b^4*e^(e*x + d) + 15*A*a*b^5*e^(e*x + d) + 3*B*b^6*e
^(e*x + d) - 2*B*a^3*b^3 + 11*A*a^2*b^4 - 13*B*a*b^5 + 4*A*b^6)/((a^6*b - 3
*a^4*b^3 + 3*a^2*b^5 - b^7)*(b*e^(2*e*x + 2*d) + 2*a*e^(e*x + d) + b)^3))/e
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\cosh(d+e\,x)+C\sinh(d+e\,x)}{{\left(a+b\cosh\left(d+e\,x\right)\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + b*cosh(d + e*x))^4,x)
[Out] int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + b*cosh(d + e*x))^4, x)
```

3.210 $\int \frac{x}{a+b\cosh^2(x)}\,dx$

Optimal. Leaf size=191

$$\frac{x\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\,\sqrt{a+b}}\right)}{2\sqrt{a}\,\sqrt{a+b}}-\frac{x\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\,\sqrt{a+b}}\right)}{2\sqrt{a}\,\sqrt{a+b}}+\frac{\text{PolyLog}\left(2,-\frac{be^{2x}}{2a+b-2\sqrt{a}\,\sqrt{a+b}}\right)}{4\sqrt{a}\,\sqrt{a+b}}-\text{Poly}$$

```
[Out] 1/2*x*ln(1+b*exp(2*x)/(2*a+b-2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/(a+b)^(1/2)-1/
2*x*ln(1+b*exp(2*x)/(2*a+b+2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/(a+b)^(1/2)+1/4*
polylog(2,-b*exp(2*x)/(2*a+b-2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/(a+b)^(1/2)-1/
4*polylog(2,-b*exp(2*x)/(2*a+b+2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/(a+b)^(1/2)
```

Rubi [A]

time = 0.26, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5749, 3401, 2296, 2221, 2317, 2438}

$$\frac{\text{Li}_2\left(-\frac{be^{2x}}{2a-2\sqrt{a+b}\,\sqrt{a}+b}\right)}{4\sqrt{a}\,\sqrt{a+b}}-\frac{\text{Li}_2\left(-\frac{be^{2x}}{2a+2\sqrt{a+b}\,\sqrt{a}+b}\right)}{4\sqrt{a}\,\sqrt{a+b}}+\frac{x\log\left(\frac{be^{2x}}{-2\sqrt{a}\,\sqrt{a+b}+2a+b}+1\right)}{2\sqrt{a}\,\sqrt{a+b}}-\frac{x\log\left(\frac{be^{2x}}{2\sqrt{a}\,\sqrt{a+b}+2a+b}+1\right)}{2\sqrt{a}\,\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*Cosh[x]^2),x]
```

```
[Out] (x*Log[1 + (b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b])])/(2*Sqrt[a]*Sqrt[
a + b]) - (x*Log[1 + (b*E^(2*x))/(2*a + b + 2*Sqrt[a]*Sqrt[a + b])])/(2*Sqr
t[a]*Sqrt[a + b]) + PolyLog[2, -((b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a +
b]))]/(4*Sqrt[a]*Sqrt[a + b]) - PolyLog[2, -((b*E^(2*x))/(2*a + b + 2*Sqrt[
a]*Sqrt[a + b]))]/(4*Sqrt[a]*Sqrt[a + b])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5749

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^2*(b_.) + (a_))^(n_)*(x_)^(m_.), x_Symbol] :>
 Dist[1/2^n, Int[x^m*(2*a + b + b*Cosh[2*c + 2*d*x])^n, x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a - b, 0] && IGtQ[m, 0] && ILtQ[n, 0] && (EqQ[n, -1] |
| (EqQ[m, 1] && EqQ[n, -2]))
```

Rubi steps

$$\int \frac{x}{a+b\cosh^2(x)}\,dx = 2\int \frac{x}{2a+b+b\cosh(2x)}\,dx$$

$$= 4\int \frac{e^{2x}x}{b+2(2a+b)e^{2x}+be^{4x}}\,dx$$

$$= \frac{(2b)\int \frac{e^{2x}x}{-4\sqrt{a}\,\sqrt{a+b}+2(2a+b)+2be^{2x}}\,dx}{\sqrt{a}\,\sqrt{a+b}} - \frac{(2b)\int \frac{e^{2x}x}{4\sqrt{a}\,\sqrt{a+b}+2(2a+b)+2be^{2x}}\,dx}{\sqrt{a}\,\sqrt{a+b}}$$

$$= \frac{x\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\,\sqrt{a+b}}\right)}{2\sqrt{a}\,\sqrt{a+b}} - \frac{x\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\,\sqrt{a+b}}\right)}{2\sqrt{a}\,\sqrt{a+b}} - \frac{\int \log\left(1+\frac{}{-4\sqrt{}}\right.}{2\sqrt{}}$$

$$= \frac{x\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\,\sqrt{a+b}}\right)}{2\sqrt{a}\,\sqrt{a+b}} - \frac{x\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\,\sqrt{a+b}}\right)}{2\sqrt{a}\,\sqrt{a+b}} - \frac{\text{Subst}\left(\int \frac{\log\left(1-\right.}{}\right.}{}$$

$$= \frac{x\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\,\sqrt{a+b}}\right)}{2\sqrt{a}\,\sqrt{a+b}} - \frac{x\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\,\sqrt{a+b}}\right)}{2\sqrt{a}\,\sqrt{a+b}} + \frac{\text{Li}_2\left(-\frac{b}{2a+b-2\sqrt{}}\right.}{4\sqrt{a}\,\sqrt{a}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.43, size = 536, normalized size = 2.81

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*Cosh[x]^2),x]
```

```
[Out] -1/4*(4*x*ArcTan[((a + b)*Coth[x])/Sqrt[-(a*(a + b))]] + (2*I)*ArcCos[-1 -
(2*a)/b]*ArcTan[(a*Tanh[x])/Sqrt[-(a*(a + b))]] + (ArcCos[-1 - (2*a)/b] + 2
*ArcTan[((a + b)*Coth[x])/Sqrt[-(a*(a + b))]] - 2*ArcTan[(a*Tanh[x])/Sqrt[-
(a*(a + b))]])*Log[(Sqrt[2]*Sqrt[-(a*(a + b))])/(Sqrt[b]*E^x*Sqrt[2*a + b +
 b*Cosh[2*x]])] + (ArcCos[-1 - (2*a)/b] - 2*ArcTan[((a + b)*Coth[x])/Sqrt[-
(a*(a + b))]] + 2*ArcTan[(a*Tanh[x])/Sqrt[-(a*(a + b))]])*Log[(Sqrt[2]*Sqrt
[-(a*(a + b))]*E^x)/(Sqrt[b]*Sqrt[2*a + b + b*Cosh[2*x]])] - (ArcCos[-1 - (
2*a)/b] - 2*ArcTan[(a*Tanh[x])/Sqrt[-(a*(a + b))]])*Log[(2*(a + b)*(a + I*S
qrt[-(a*(a + b))])*(-1 + Tanh[x]))/(b*(a + b + I*Sqrt[-(a*(a + b))]*Tanh[x]
))] - (ArcCos[-1 - (2*a)/b] + 2*ArcTan[(a*Tanh[x])/Sqrt[-(a*(a + b))]])*Log
[((2*I)*(a + b)*(I*a + Sqrt[-(a*(a + b))])*(1 + Tanh[x]))/(b*(a + b + I*Sqr
t[-(a*(a + b))]*Tanh[x]))] + I*(PolyLog[2, ((2*a + b - (2*I)*Sqrt[-(a*(a +
b))])*(a + b - I*Sqrt[-(a*(a + b))]*Tanh[x]))/(b*(a + b + I*Sqrt[-(a*(a + b
```

```
))]*Tanh[x]))] - PolyLog[2, ((2*a + b + (2*I)*Sqrt[-(a*(a + b))])*(a + b -
I*Sqrt[-(a*(a + b))]*Tanh[x]))/(b*(a + b + I*Sqrt[-(a*(a + b))]*Tanh[x]))])
)/Sqrt[-(a*(a + b))]
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 486 vs. $2(147) = 294$.
time = 0.89, size = 487, normalized size = 2.55

method	result
risch	$\frac{\ln\left(1-\frac{b\,\mathrm{e}^{2x}}{-2\sqrt{a\,(a+b)}-2a-b}\right)x}{-2\sqrt{a\,(a+b)}-2a-b}+\frac{\ln\left(1-\frac{b\,\mathrm{e}^{2x}}{-2\sqrt{a\,(a+b)}-2a-b}\right)ax}{\sqrt{a\,(a+b)}\,\left(-2\sqrt{a\,(a+b)}-2a-b\right)}+\frac{\ln\left(1-\frac{b\,\mathrm{e}^{2x}}{-2\sqrt{a\,(a+b)}-2a-b}\right.}{2\sqrt{a\,(a+b)}\,\left(-2\sqrt{a\,(a+b)}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*cosh(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(-2*(a*(a+b))^(1/2)-2*a-b)*ln(1-b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))*x+
1/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*ln(1-b*exp(2*x)/(-2*(a*(a+b))^
(1/2)-2*a-b))*a*x+1/2/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*ln(1-b*exp
(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))*b*x-1/(-2*(a*(a+b))^(1/2)-2*a-b)*x^2-1/(a
*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*a*x^2-1/2/(a*(a+b))^(1/2)/(-2*(a*(
a+b))^(1/2)-2*a-b)*x^2*b+1/2/(-2*(a*(a+b))^(1/2)-2*a-b)*polylog(2,b*exp(2*x
)/(-2*(a*(a+b))^(1/2)-2*a-b))+1/2/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b
)*polylog(2,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))*a+1/4/(a*(a+b))^(1/2)/(-
2*(a*(a+b))^(1/2)-2*a-b)*polylog(2,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))*b
+1/2/(a*(a+b))^(1/2)*x*ln(1-b*exp(2*x)/(2*(a*(a+b))^(1/2)-2*a-b))-1/2/(a*(a
+b))^(1/2)*x^2+1/4/(a*(a+b))^(1/2)*polylog(2,b*exp(2*x)/(2*(a*(a+b))^(1/2)-
2*a-b))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*cosh(x)^2 + a), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 780 vs. $2(149) = 298$.
time = 0.42, size = 780, normalized size = 4.08

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(b*x*sqrt((a^2 + a*b)/b^2)*log((((2*a + b)*cosh(x) + (2*a + b)*sinh(x)
 - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 +
a*b)/b^2) + 2*a + b)/b) + b)/b) + b*x*sqrt((a^2 + a*b)/b^2)*log(-(((2*a + b
)*cosh(x) + (2*a + b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/
b^2))*sqrt(-(2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b) - b)/b) - b*x*sqrt((a^
2 + a*b)/b^2)*log((((2*a + b)*cosh(x) + (2*a + b)*sinh(x) + 2*(b*cosh(x) +
b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b
)/b) + b)/b) - b*x*sqrt((a^2 + a*b)/b^2)*log(-(((2*a + b)*cosh(x) + (2*a +
b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt((2*b*sqr
t((a^2 + a*b)/b^2) - 2*a - b)/b) - b)/b) + b*sqrt((a^2 + a*b)/b^2)*dilog(-(
((2*a + b)*cosh(x) + (2*a + b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^
2 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b) + b)/b + 1) +
 b*sqrt((a^2 + a*b)/b^2)*dilog((((2*a + b)*cosh(x) + (2*a + b)*sinh(x) - 2*
(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 + a*b)/
b^2) + 2*a + b)/b) - b)/b + 1) - b*sqrt((a^2 + a*b)/b^2)*dilog(-(((2*a + b)
*cosh(x) + (2*a + b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b
^2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b) + b)/b + 1) - b*sqrt((a^
2 + a*b)/b^2)*dilog((((2*a + b)*cosh(x) + (2*a + b)*sinh(x) + 2*(b*cosh(x)
+ b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a -
 b)/b) - b)/b + 1))/(a^2 + a*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a + b\cosh^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cosh(x)**2),x)
```

```
[Out] Integral(x/(a + b*cosh(x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cosh(x)^2),x, algorithm="giac")
```

```
[Out] integrate(x/(b*cosh(x)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{b\,{\cosh\left(x\right)}^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*cosh(x)^2),x)
```

```
[Out] int(x/(a + b*cosh(x)^2), x)
```

3.211 $\int \frac{x^2}{a+b\cosh^2(x)}\,dx$

Optimal. Leaf size=291

$$\frac{x^2\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}}-\frac{x^2\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}}+\frac{x\text{PolyLog}\left(2,-\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}}-\cdots$$

```
[Out] 1/2*x^2*ln(1+b*exp(2*x)/(2*a+b-2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/(a+b)^(1/2)-
1/2*x^2*ln(1+b*exp(2*x)/(2*a+b+2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/(a+b)^(1/2)+
1/2*x*polylog(2,-b*exp(2*x)/(2*a+b-2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/(a+b)^(1
/2)-1/2*x*polylog(2,-b*exp(2*x)/(2*a+b+2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/(a+b
)^(1/2)-1/4*polylog(3,-b*exp(2*x)/(2*a+b-2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/(a
+b)^(1/2)+1/4*polylog(3,-b*exp(2*x)/(2*a+b+2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/
(a+b)^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5749, 3401, 2296, 2221, 2611, 2320, 6724}

$$\frac{x\text{Li}_2\left(-\frac{be^{2x}}{2a-2\sqrt{a+b}\sqrt{a}+b}\right)}{2\sqrt{a}\sqrt{a+b}}-\frac{x\text{Li}_2\left(-\frac{be^{2x}}{2a+2\sqrt{a+b}\sqrt{a}+b}\right)}{2\sqrt{a}\sqrt{a+b}}-\frac{\text{Li}_3\left(-\frac{be^{2x}}{2a-2\sqrt{a+b}\sqrt{a}+b}\right)}{4\sqrt{a}\sqrt{a+b}}+\frac{\text{Li}_3\left(-\frac{be^{2x}}{2a+2\sqrt{a+b}\sqrt{a}+b}\right)}{4\sqrt{a}\sqrt{a+b}}+\frac{x^2\log\left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a+b}+2a+b}+1\right)}{2\sqrt{a}\sqrt{a+b}}-\frac{x^2\log\left(\frac{be^{2x}}{2\sqrt{a}\sqrt{a+b}+2a+b}+1\right)}{2\sqrt{a}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*Cosh[x]^2),x]
```

```
[Out] (x^2*Log[1 + (b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b])])/(2*Sqrt[a]*Sqr
t[a + b]) - (x^2*Log[1 + (b*E^(2*x))/(2*a + b + 2*Sqrt[a]*Sqrt[a + b])])/(2
*Sqrt[a]*Sqrt[a + b]) + (x*PolyLog[2, -((b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sq
rt[a + b]))])/(2*Sqrt[a]*Sqrt[a + b]) - (x*PolyLog[2, -((b*E^(2*x))/(2*a +
b + 2*Sqrt[a]*Sqrt[a + b]))])/(2*Sqrt[a]*Sqrt[a + b]) - PolyLog[3, -((b*E^(
2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b]))]/(4*Sqrt[a]*Sqrt[a + b]) + PolyLog
[3, -((b*E^(2*x))/(2*a + b + 2*Sqrt[a]*Sqrt[a + b]))]/(4*Sqrt[a]*Sqrt[a + b
])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5749

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^2*(b_.) + (a_))^(n_)*(x_)^(m_.), x_Symbol] :>
 Dist[1/2^n, Int[x^m*(2*a + b + b*Cosh[2*c + 2*d*x])^n, x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a - b, 0] && IGtQ[m, 0] && ILtQ[n, 0] && (EqQ[n, -1] |
| (EqQ[m, 1] && EqQ[n, -2]))
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2}{a+b\cosh^2(x)}\,dx = 2\int \frac{x^2}{2a+b+b\cosh(2x)}\,dx$$

$$= 4\int \frac{e^{2x}x^2}{b+2(2a+b)e^{2x}+be^{4x}}\,dx$$

$$= \frac{(2b)\int \frac{e^{2x}x^2}{-4\sqrt{a}\sqrt{a+b}+2(2a+b)+2be^{2x}}\,dx}{\sqrt{a}\sqrt{a+b}} - \frac{(2b)\int \frac{e^{2x}x^2}{4\sqrt{a}\sqrt{a+b}+2(2a+b)+2be^{2x}}\,dx}{\sqrt{a}\sqrt{a+b}}$$

$$= \frac{x^2\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{x^2\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{\int x\log\left(1+\right.}{}$$

$$= \frac{x^2\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{x^2\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} + \frac{x\text{Li}_2\left(-\frac{}{2a+b-}\right.}{2\sqrt{a}}$$

$$= \frac{x^2\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{x^2\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} + \frac{x\text{Li}_2\left(-\frac{}{2a+b-}\right.}{2\sqrt{a}}$$

$$= \frac{x^2\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{x^2\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} + \frac{x\text{Li}_2\left(-\frac{}{2a+b-}\right.}{2\sqrt{a}}$$

Mathematica [**A**]
time = 0.48, size = 221, normalized size = 0.76

$$\frac{2x^2\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)-2x^2\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)+2x\text{PolyLog}\left(2,-\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)-2x\text{PolyLog}\left(2,-\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)-\text{PolyLog}\left(3,-\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)+\text{PolyLog}\left(3,-\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{4\sqrt{a}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*Cosh[x]^2),x]
```

```
[Out] (2*x^2*Log[1 + (b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b])] - 2*x^2*Log[1
 + (b*E^(2*x))/(2*a + b + 2*Sqrt[a]*Sqrt[a + b])] + 2*x*PolyLog[2, -((b*E^(
2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b]))] - 2*x*PolyLog[2, -((b*E^(2*x))/(2
*a + b + 2*Sqrt[a]*Sqrt[a + b]))] - PolyLog[3, -((b*E^(2*x))/(2*a + b - 2*S
qrt[a]*Sqrt[a + b]))] + PolyLog[3, -((b*E^(2*x))/(2*a + b + 2*Sqrt[a]*Sqrt[
a + b]))])/(4*Sqrt[a]*Sqrt[a + b])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 685 vs. $2(225) = 450$.
time = 0.85, size = 686, normalized size = 2.36

method	result
risch	$-\frac{x^3}{3\sqrt{a(a+b)}}+\frac{x^2\ln\left(1-\frac{b\,e^{2x}}{2\sqrt{a(a+b)}-2a-b}\right)}{2\sqrt{a(a+b)}}+\frac{x\,\mathrm{polylog}\left(2,\frac{b\,e^{2x}}{2\sqrt{a(a+b)}-2a-b}\right)}{2\sqrt{a(a+b)}}-\frac{\mathrm{polylog}\left(3,\frac{b}{2\sqrt{a(a}\right.}{4\sqrt{a(a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*cosh(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/(a*(a+b))^(1/2)*x^3+1/2/(a*(a+b))^(1/2)*x^2*ln(1-b*exp(2*x)/(2*(a*(a+b
))^(1/2)-2*a-b))+1/2/(a*(a+b))^(1/2)*x*polylog(2,b*exp(2*x)/(2*(a*(a+b))^(1
/2)-2*a-b))-1/4/(a*(a+b))^(1/2)*polylog(3,b*exp(2*x)/(2*(a*(a+b))^(1/2)-2*a
-b))-2/3/(-2*(a*(a+b))^(1/2)-2*a-b)*x^3+1/(-2*(a*(a+b))^(1/2)-2*a-b)*x^2*ln
(1-b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))+1/(-2*(a*(a+b))^(1/2)-2*a-b)*x*po
lylog(2,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))-1/2/(-2*(a*(a+b))^(1/2)-2*a-
b)*polylog(3,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))-2/3/(a*(a+b))^(1/2)/(-2
*(a*(a+b))^(1/2)-2*a-b)*a*x^3+1/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*
a*x^2*ln(1-b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))+1/(a*(a+b))^(1/2)/(-2*(a*
(a+b))^(1/2)-2*a-b)*a*x*polylog(2,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))-1/
2/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*a*polylog(3,b*exp(2*x)/(-2*(a*
(a+b))^(1/2)-2*a-b))-1/3/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*b*x^3+1
/2/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*b*x^2*ln(1-b*exp(2*x)/(-2*(a*
(a+b))^(1/2)-2*a-b))+1/2/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*b*x*pol
ylog(2,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))-1/4/(a*(a+b))^(1/2)/(-2*(a*(a
+b))^(1/2)-2*a-b)*b*polylog(3,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*cosh(x)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1162 vs. $2(228) = 456$.
time = 0.62, size = 1162, normalized size = 3.99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(b*x^2*sqrt((a^2 + a*b)/b^2)*log((((2*a + b)*cosh(x) + (2*a + b)*sinh(
x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^2
+ a*b)/b^2) + 2*a + b)/b) + b)/b) + b*x^2*sqrt((a^2 + a*b)/b^2)*log(-(((2*a
 + b)*cosh(x) + (2*a + b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a
*b)/b^2))*sqrt(-(2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b) - b)/b) - b*x^2*sq
rt((a^2 + a*b)/b^2)*log((((2*a + b)*cosh(x) + (2*a + b)*sinh(x) + 2*(b*cosh
(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2
*a - b)/b) + b)/b) - b*x^2*sqrt((a^2 + a*b)/b^2)*log(-(((2*a + b)*cosh(x) +
 (2*a + b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(
(2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b) - b)/b) + 2*b*x*sqrt((a^2 + a*b)/b
^2)*dilog(-(((2*a + b)*cosh(x) + (2*a + b)*sinh(x) - 2*(b*cosh(x) + b*sinh(
x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b) +
 b)/b + 1) + 2*b*x*sqrt((a^2 + a*b)/b^2)*dilog((((2*a + b)*cosh(x) + (2*a +
 b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*s
qrt((a^2 + a*b)/b^2) + 2*a + b)/b) - b)/b + 1) - 2*b*x*sqrt((a^2 + a*b)/b^2
)*dilog(-(((2*a + b)*cosh(x) + (2*a + b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x)
)*sqrt((a^2 + a*b)/b^2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b) + b)
/b + 1) - 2*b*x*sqrt((a^2 + a*b)/b^2)*dilog((((2*a + b)*cosh(x) + (2*a + b)
*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt((2*b*sqrt(
(a^2 + a*b)/b^2) - 2*a - b)/b) - b)/b + 1) - 2*b*sqrt((a^2 + a*b)/b^2)*poly
log(3, ((2*a + b)*cosh(x) + (2*a + b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*s
qrt((a^2 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)/b) - 2
*b*sqrt((a^2 + a*b)/b^2)*polylog(3, -((2*a + b)*cosh(x) + (2*a + b)*sinh(x)
 - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 +
a*b)/b^2) + 2*a + b)/b)/b) + 2*b*sqrt((a^2 + a*b)/b^2)*polylog(3, ((2*a + b
)*cosh(x) + (2*a + b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/
b^2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b)/b) + 2*b*sqrt((a^2 + a*
b)/b^2)*polylog(3, -((2*a + b)*cosh(x) + (2*a + b)*sinh(x) + 2*(b*cosh(x) +
 b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a -
b)/b)/b))/(a^2 + a*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a + b\cosh^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*cosh(x)**2),x)
```

```
[Out] Integral(x**2/(a + b*cosh(x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*cosh(x)^2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*cosh(x)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{b\cosh(x)^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*cosh(x)^2),x)
```

```
[Out] int(x^2/(a + b*cosh(x)^2), x)
```

3.212 $\int \frac{x^3}{a+b\cosh^2(x)}\,dx$

Optimal. Leaf size=391

$$\frac{x^3\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}}-\frac{x^3\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}}+\frac{3x^2\text{PolyLog}\left(2,-\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{4\sqrt{a}\sqrt{a+b}}$$

```
[Out] 1/2*x^3*ln(1+b*exp(2*x)/(2*a+b-2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/(a+b)^(1/2)-
1/2*x^3*ln(1+b*exp(2*x)/(2*a+b+2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/(a+b)^(1/2)+
3/4*x^2*polylog(2,-b*exp(2*x)/(2*a+b-2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/(a+b)^
(1/2)-3/4*x^2*polylog(2,-b*exp(2*x)/(2*a+b+2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/
(a+b)^(1/2)-3/4*x*polylog(3,-b*exp(2*x)/(2*a+b-2*a^(1/2)*(a+b)^(1/2)))/a^(1
/2)/(a+b)^(1/2)+3/4*x*polylog(3,-b*exp(2*x)/(2*a+b+2*a^(1/2)*(a+b)^(1/2)))/
a^(1/2)/(a+b)^(1/2)+3/8*polylog(4,-b*exp(2*x)/(2*a+b-2*a^(1/2)*(a+b)^(1/2))
)/a^(1/2)/(a+b)^(1/2)-3/8*polylog(4,-b*exp(2*x)/(2*a+b+2*a^(1/2)*(a+b)^(1/2
)))/a^(1/2)/(a+b)^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5749, 3401, 2296, 2221, 2611, 6744, 2320, 6724}

$$\frac{3x^2\text{Li}_2\left(-\frac{be^{2x}}{2a-2\sqrt{a+b}\sqrt{a}+b}\right)}{4\sqrt{a}\sqrt{a+b}}-\frac{3x^2\text{Li}_2\left(-\frac{be^{2x}}{2a+2\sqrt{a+b}\sqrt{a}+b}\right)}{4\sqrt{a}\sqrt{a+b}}-\frac{3x\text{Li}_3\left(-\frac{be^{2x}}{2a-2\sqrt{a+b}\sqrt{a}+b}\right)}{4\sqrt{a}\sqrt{a+b}}+\frac{3x\text{Li}_3\left(-\frac{be^{2x}}{2a+2\sqrt{a+b}\sqrt{a}+b}\right)}{4\sqrt{a}\sqrt{a+b}}+\frac{3\text{Li}_4\left(-\frac{be^{2x}}{2a-2\sqrt{a+b}\sqrt{a}+b}\right)}{8\sqrt{a}\sqrt{a+b}}-\frac{3\text{Li}_4\left(-\frac{be^{2x}}{2a+2\sqrt{a+b}\sqrt{a}+b}\right)}{8\sqrt{a}\sqrt{a+b}}+\frac{x^3\log\left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a+b}+2a+b}+1\right)}{2\sqrt{a}\sqrt{a+b}}-\frac{x^3\log\left(\frac{be^{2x}}{2\sqrt{a}\sqrt{a+b}+2a+b}+1\right)}{2\sqrt{a}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*Cosh[x]^2),x]
```

```
[Out] (x^3*Log[1 + (b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b])])/(2*Sqrt[a]*Sqr
t[a + b]) - (x^3*Log[1 + (b*E^(2*x))/(2*a + b + 2*Sqrt[a]*Sqrt[a + b])])/(2
*Sqrt[a]*Sqrt[a + b]) + (3*x^2*PolyLog[2, -((b*E^(2*x))/(2*a + b - 2*Sqrt[a
]*Sqrt[a + b]))])/(4*Sqrt[a]*Sqrt[a + b]) - (3*x^2*PolyLog[2, -((b*E^(2*x))
/(2*a + b + 2*Sqrt[a]*Sqrt[a + b]))])/(4*Sqrt[a]*Sqrt[a + b]) - (3*x*PolyLo
g[3, -((b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b]))])/(4*Sqrt[a]*Sqrt[a +
 b]) + (3*x*PolyLog[3, -((b*E^(2*x))/(2*a + b + 2*Sqrt[a]*Sqrt[a + b]))])/(
4*Sqrt[a]*Sqrt[a + b]) + (3*PolyLog[4, -((b*E^(2*x))/(2*a + b - 2*Sqrt[a]*S
qrt[a + b]))])/(8*Sqrt[a]*Sqrt[a + b]) - (3*PolyLog[4, -((b*E^(2*x))/(2*a +
 b + 2*Sqrt[a]*Sqrt[a + b]))])/(8*Sqrt[a]*Sqrt[a + b])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5749

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^2*(b_.) + (a_))^(n_)*(x_)^(m_.), x_Symbol] :>
 Dist[1/2^n, Int[x^m*(2*a + b + b*Cosh[2*c + 2*d*x])^n, x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a - b, 0] && IGtQ[m, 0] && ILtQ[n, 0] && (EqQ[n, -1] |
| (EqQ[m, 1] && EqQ[n, -2]))
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3}{a+b\cosh^2(x)}\,dx = 2\int \frac{x^3}{2a+b+b\cosh(2x)}\,dx$$

$$= 4\int \frac{e^{2x}x^3}{b+2(2a+b)e^{2x}+be^{4x}}\,dx$$

$$= \frac{(2b)\int \frac{e^{2x}x^3}{-4\sqrt{a}\sqrt{a+b}+2(2a+b)+2be^{2x}}\,dx}{\sqrt{a}\sqrt{a+b}} - \frac{(2b)\int \frac{e^{2x}x^3}{4\sqrt{a}\sqrt{a+b}+2(2a+b)+2be^{2x}}\,dx}{\sqrt{a}\sqrt{a+b}}$$

$$= \frac{x^3\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{3\int x^2\log\left(1\right.}{}$$

$$= \frac{x^3\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} + \frac{3x^2\mathrm{Li}_2\left(-\frac{}{2a+}\right.}{4\sqrt{a}}$$

$$= \frac{x^3\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} + \frac{3x^2\mathrm{Li}_2\left(-\frac{}{2a+}\right.}{4\sqrt{a}}$$

$$= \frac{x^3\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} + \frac{3x^2\mathrm{Li}_2\left(-\frac{}{2a+}\right.}{4\sqrt{a}}$$

$$= \frac{x^3\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} + \frac{3x^2\mathrm{Li}_2\left(-\frac{}{2a+}\right.}{4\sqrt{a}}$$

Mathematica [A]
time = 0.43, size = 295, normalized size = 0.75

$$\frac{4x^3\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)-4x^3\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)+6x^2\mathrm{PolyLog}\left(2,-\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)-6x^2\mathrm{PolyLog}\left(2,-\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)-6x\mathrm{PolyLog}\left(3,-\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)+6x\mathrm{PolyLog}\left(3,-\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)+3\mathrm{PolyLog}\left(4,-\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)-3\mathrm{PolyLog}\left(4,-\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{8\sqrt{a}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*Cosh[x]^2),x]
```

```
[Out] (4*x^3*Log[1 + (b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b])] - 4*x^3*Log[1
 + (b*E^(2*x))/(2*a + b + 2*Sqrt[a]*Sqrt[a + b])] + 6*x^2*PolyLog[2, -((b*E
^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b]))] - 6*x^2*PolyLog[2, -((b*E^(2*x)
)/(2*a + b + 2*Sqrt[a]*Sqrt[a + b]))] - 6*x*PolyLog[3, -((b*E^(2*x))/(2*a +
 b - 2*Sqrt[a]*Sqrt[a + b]))] + 6*x*PolyLog[3, -((b*E^(2*x))/(2*a + b + 2*S
qrt[a]*Sqrt[a + b]))] + 3*PolyLog[4, -((b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqr
t[a + b]))] - 3*PolyLog[4, -((b*E^(2*x))/(2*a + b + 2*Sqrt[a]*Sqrt[a + b]))
])/(8*Sqrt[a]*Sqrt[a + b])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 888 vs. $2(303) = 606$.
time = 0.86, size = 889, normalized size = 2.27

method	result
risch	$\frac{x^{3} \ln \left(1-\frac{b\,{\mathrm e}^{2 x}}{2 \sqrt{a \left(a+b\right)}-2 a-b}\right)}{2 \sqrt{a \left(a+b\right)}}-\frac{x^{4}}{4 \sqrt{a \left(a+b\right)}}+\frac{3 x^{2} \operatorname{polylog}\left(2, \frac{b\,{\mathrm e}^{2 x}}{2 \sqrt{a \left(a+b\right)}-2 a-b}\right)}{4 \sqrt{a \left(a+b\right)}}-\frac{3 x \operatorname{polylog}\left(3, \frac{}{2 \sqrt{a \left(\right.}}\right.}{4 \sqrt{a \left(a\right.}}$ …

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*cosh(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(a*(a+b))^(1/2)*x^3*ln(1-b*exp(2*x)/(2*(a*(a+b))^(1/2)-2*a-b))-1/4/(a*(
a+b))^(1/2)*x^4+3/4/(a*(a+b))^(1/2)*x^2*polylog(2,b*exp(2*x)/(2*(a*(a+b))^(
1/2)-2*a-b))-3/4/(a*(a+b))^(1/2)*x*polylog(3,b*exp(2*x)/(2*(a*(a+b))^(1/2)-
2*a-b))+3/8/(a*(a+b))^(1/2)*polylog(4,b*exp(2*x)/(2*(a*(a+b))^(1/2)-2*a-b))
+1/(-2*(a*(a+b))^(1/2)-2*a-b)*ln(1-b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))*x
^3+1/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*ln(1-b*exp(2*x)/(-2*(a*(a+b
))^(1/2)-2*a-b))*a*x^3+1/2/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*ln(1-
b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))*b*x^3-1/2/(-2*(a*(a+b))^(1/2)-2*a-b)
*x^4-1/2/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*a*x^4-1/4/(a*(a+b))^(1/
2)/(-2*(a*(a+b))^(1/2)-2*a-b)*b*x^4+3/2/(-2*(a*(a+b))^(1/2)-2*a-b)*polylog(
2,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))*x^2+3/2/(a*(a+b))^(1/2)/(-2*(a*(a+
b))^(1/2)-2*a-b)*polylog(2,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))*a*x^2+3/4
/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*polylog(2,b*exp(2*x)/(-2*(a*(a+
b))^(1/2)-2*a-b))*b*x^2-3/2/(-2*(a*(a+b))^(1/2)-2*a-b)*polylog(3,b*exp(2*x)
/(-2*(a*(a+b))^(1/2)-2*a-b))*x-3/2/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-
b)*polylog(3,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))*a*x-3/4/(a*(a+b))^(1/2)
/(-2*(a*(a+b))^(1/2)-2*a-b)*polylog(3,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b)
)*b*x+3/4/(-2*(a*(a+b))^(1/2)-2*a-b)*polylog(4,b*exp(2*x)/(-2*(a*(a+b))^(1/
2)-2*a-b))+3/4/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*polylog(4,b*exp(2
*x)/(-2*(a*(a+b))^(1/2)-2*a-b))*a+3/8/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2
*a-b)*polylog(4,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))*b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(b*cosh(x)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1542 vs. $2(307) = 614$.
time = 0.52, size = 1542, normalized size = 3.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(b*x^3*sqrt((a^2 + a*b)/b^2)*log((((2*a + b)*cosh(x) + (2*a + b)*sinh(
x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^2
+ a*b)/b^2) + 2*a + b)/b) + b)/b) + b*x^3*sqrt((a^2 + a*b)/b^2)*log(-(((2*a
 + b)*cosh(x) + (2*a + b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a
*b)/b^2))*sqrt(-(2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b) - b)/b) - b*x^3*sq
rt((a^2 + a*b)/b^2)*log((((2*a + b)*cosh(x) + (2*a + b)*sinh(x) + 2*(b*cosh
(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2
*a - b)/b) + b)/b) - b*x^3*sqrt((a^2 + a*b)/b^2)*log(-(((2*a + b)*cosh(x) +
 (2*a + b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(
(2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b) - b)/b) + 3*b*x^2*sqrt((a^2 + a*b)
/b^2)*dilog(-(((2*a + b)*cosh(x) + (2*a + b)*sinh(x) - 2*(b*cosh(x) + b*sin
h(x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)
 + b)/b + 1) + 3*b*x^2*sqrt((a^2 + a*b)/b^2)*dilog((((2*a + b)*cosh(x) + (2
*a + b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2
*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b) - b)/b + 1) - 3*b*x^2*sqrt((a^2 + a*
b)/b^2)*dilog(-(((2*a + b)*cosh(x) + (2*a + b)*sinh(x) + 2*(b*cosh(x) + b*s
inh(x))*sqrt((a^2 + a*b)/b^2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b
) + b)/b + 1) - 3*b*x^2*sqrt((a^2 + a*b)/b^2)*dilog((((2*a + b)*cosh(x) + (
2*a + b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt((2
*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b) - b)/b + 1) - 6*b*x*sqrt((a^2 + a*b)
/b^2)*polylog(3, ((2*a + b)*cosh(x) + (2*a + b)*sinh(x) - 2*(b*cosh(x) + b*
sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)
/b)/b) - 6*b*x*sqrt((a^2 + a*b)/b^2)*polylog(3, -((2*a + b)*cosh(x) + (2*a
+ b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*
sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)/b) + 6*b*x*sqrt((a^2 + a*b)/b^2)*polylo
g(3, ((2*a + b)*cosh(x) + (2*a + b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqr
```

```
t((a^2 + a*b)/b^2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b)/b) + 6*b*
x*sqrt((a^2 + a*b)/b^2)*polylog(3, -((2*a + b)*cosh(x) + (2*a + b)*sinh(x)
+ 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt((2*b*sqrt((a^2 + a*
b)/b^2) - 2*a - b)/b)/b) + 6*b*sqrt((a^2 + a*b)/b^2)*polylog(4, ((2*a + b)*
cosh(x) + (2*a + b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^
2))*sqrt(-(2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)/b) + 6*b*sqrt((a^2 + a*b
)/b^2)*polylog(4, -((2*a + b)*cosh(x) + (2*a + b)*sinh(x) - 2*(b*cosh(x) +
b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 + a*b)/b^2) + 2*a +
b)/b)/b) - 6*b*sqrt((a^2 + a*b)/b^2)*polylog(4, ((2*a + b)*cosh(x) + (2*a +
 b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt((2*b*sq
rt((a^2 + a*b)/b^2) - 2*a - b)/b)/b) - 6*b*sqrt((a^2 + a*b)/b^2)*polylog(4,
 -((2*a + b)*cosh(x) + (2*a + b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((
a^2 + a*b)/b^2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b)/b))/(a^2 + a
*b)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{a + b\cosh^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*cosh(x)**2),x)
```

```
[Out] Integral(x**3/(a + b*cosh(x)**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*cosh(x)^2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(b*cosh(x)^2 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{b\cosh(x)^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*cosh(x)^2),x)
```

```
[Out] int(x^3/(a + b*cosh(x)^2), x)
```

3.213 $\int \frac{\cosh^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=58

$$-\frac{3\text{Chi}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}-\frac{\text{Chi}\left(\frac{3\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}$$

```
[Out] -3/4*Chi((-a*x+1)^(1/2)/(a*x+1)^(1/2))/a-1/4*Chi(3*(-a*x+1)^(1/2)/(a*x+1)^(
1/2))/a
```

Rubi [A]
time = 0.07, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {6813, 3393, 3382}

$$-\frac{3\text{Chi}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}-\frac{\text{Chi}\left(\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^3/(1 - a^2*x^2),x]
```

```
[Out] (-3*CoshIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]])/(4*a) - CoshIntegral[(3*Sqrt
[1 - a*x])/Sqrt[1 + a*x]]/(4*a)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6813

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[2*e*(g/(C*(e*f - d
*g))), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
```

```
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{\cosh^3(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{3\cosh(x)}{4x}+\frac{\cosh(3x)}{4x}\right)\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}\\
&= -\frac{\text{Subst}\left(\int \frac{\cosh(3x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}-\frac{3\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}\\
&= -\frac{3\text{Chi}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}-\frac{\text{Chi}\left(\frac{3\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 55, normalized size = 0.95

$$
\frac{-3\text{Chi}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)-\text{Chi}\left(\frac{3\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^3/(1 - a^2*x^2),x]
```

```
[Out] (-3*CoshIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]] - CoshIntegral[(3*Sqrt[1 - a*
x])/Sqrt[1 + a*x]])/(4*a)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$
\int \frac{\cosh^3\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-a^2x^2+1}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x)
```

```
[Out] int(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="m
axima")
```

```
[Out] -integrate(cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="f
ricas")
```

```
[Out] integral(-cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\cosh^{3}\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^{2}x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((-a*x+1)**(1/2)/(a*x+1)**(1/2))**3/(-a**2*x**2+1),x)
```

```
[Out] -Integral(cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))**3/(a**2*x**2 - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="g
iac")
```

```
[Out] integrate(-cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\cosh\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)^3}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cosh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^3/(a^2*x^2 - 1),x)
[Out] -int(cosh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^3/(a^2*x^2 - 1), x)
```

3.214 $\int \frac{\cosh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=58

$$-\frac{\mathrm{Chi}\left(\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}$$

```
[Out] -1/2*Chi(2*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a-1/2*ln((-a*x+1)^(1/2)/(a*x+1)^(1
/2))/a
```

Rubi [A]
time = 0.06, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {6813, 3393, 3382}

$$-\frac{\mathrm{Chi}\left(\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

```
[Out] -1/2*CoshIntegral[(2*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/a - Log[Sqrt[1 - a*x]/Sq
rt[1 + a*x]]/(2*a)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6813

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[2*e*(g/(C*(e*f - d
*g))), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
```

```
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{\cosh^2(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{2x}+\frac{\cosh(2x)}{2x}\right)\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a} \\
&= -\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a} - \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a} \\
&= -\frac{\text{Chi}\left(\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a} - \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 57, normalized size = 0.98

$$
-\frac{\text{Chi}\left(\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a} - \frac{\log(1-ax)}{4a} + \frac{\log(1+ax)}{4a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
[Out] -1/2*CoshIntegral[(2*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/a - Log[1 - a*x]/(4*a) +
 Log[1 + a*x]/(4*a)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$
\int \frac{\cosh^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-a^2x^2+1}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)
[Out] int(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="m
axima")
```

```
[Out] 1/4*log(a*x + 1)/a - 1/4*log(a*x - 1)/a - 1/4*integrate(e^(2*sqrt(-a*x + 1)
/sqrt(a*x + 1))/(a^2*x^2 - 1), x) - 1/4*integrate(e^(-2*sqrt(-a*x + 1)/sqrt
(a*x + 1))/(a^2*x^2 - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="f
ricas")
```

```
[Out] integral(-cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))^2/(a^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\cosh^{2}\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^{2}x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((-a*x+1)**(1/2)/(a*x+1)**(1/2))**2/(-a**2*x**2+1),x)
```

```
[Out] -Integral(cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))**2/(a**2*x**2 - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="g
iac")
```

```
[Out] integrate(-cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))^2/(a^2*x^2 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\cosh\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)^2}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cosh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2/(a^2*x^2 - 1),x)
[Out] -int(cosh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2/(a^2*x^2 - 1), x)
```

3.215 $\int \frac{\cosh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=26

$$-\frac{\text{Chi}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

```
[Out] -Chi((-a*x+1)^(1/2)/(a*x+1)^(1/2))/a
```

Rubi [A]
time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {6813, 3382}

$$-\frac{\text{Chi}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] -(CoshIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 6813

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[2*e*(g/(C*(e*f - d
*g))), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\cosh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$
$$= -\frac{\text{Chi}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [A]
time = 0.03, size = 26, normalized size = 1.00

$$-\frac{\text{Chi}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] -(CoshIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

```
[Out] int(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="max
ima")
```

```
[Out] -integrate(cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="fri
cas")
[Out] integral(-cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((-a*x+1)**(1/2)/(a*x+1)**(1/2))/(-a**2*x**2+1),x)
[Out] -Integral(cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a**2*x**2 - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="gia
c")
[Out] integrate(-cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$-\int \frac{\cosh\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cosh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))/(a^2*x^2 - 1),x)
[Out] -int(cosh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))/(a^2*x^2 - 1), x)
```

3.216 $\int \frac{\operatorname{sech}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=40

$$\operatorname{Int}\left(\frac{\operatorname{sech}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{(1-ax)(1+ax)},x\right)$$

```
[Out] Unintegrable(sech((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a*x+1)/(a*x+1),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{sech}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sech[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
[Out] -(Defer[Subst][Defer[Int][Sech[x]/x, x], x, Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

Rubi steps

$$\int \frac{\operatorname{sech}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{\operatorname{sech}(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [A]
time = 3.90, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sech[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
[Out] Integrate[Sech[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^{2} x^{2}+1\right) \cosh \left(\frac{\sqrt{-a x+1}}{\sqrt{a x+1}}\right)}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)/cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x)
```

```
[Out] int(1/(-a^2*x^2+1)/cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="m
axima")
```

```
[Out] -integrate(1/((a^2*x^2 - 1)*cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="f
ricas")
```

```
[Out] integral(-1/((a^2*x^2 - 1)*cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^{2} x^{2} \cosh \left(\frac{\sqrt{-a x+1}}{\sqrt{a x+1}}\right)-\cosh \left(\frac{\sqrt{-a x+1}}{\sqrt{a x+1}}\right)}\, d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)/cosh((-a*x+1)**(1/2)/(a*x+1)**(1/2)),x)
```

```
[Out] -Integral(1/(a**2*x**2*cosh(sqrt(-a*x + 1)/sqrt(a*x + 1)) - cosh(sqrt(-a*x
+ 1)/sqrt(a*x + 1))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="g
iac")
[Out] integrate(-1/((a^2*x^2 - 1)*cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{\cosh\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cosh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))*(a^2*x^2 - 1)),x)
[Out] -int(1/(cosh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))*(a^2*x^2 - 1)), x)
```

3.217 $\int \frac{\operatorname{sech}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=42

$$\text{Int}\left(\frac{\operatorname{sech}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{(1-ax)(1+ax)},x\right)$$

```
[Out] Unintegrable(sech((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a*x+1)/(a*x+1),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{sech}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sech[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

```
[Out] -(Defer[Subst][Defer[Int][Sech[x]^2/x, x], x, Sqrt[1 - a*x]/Sqrt[1 + a*x]]/
a)
```

Rubi steps

$$\int \frac{\operatorname{sech}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\operatorname{sech}^2(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [A]
time = 16.79, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sech[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

```
[Out] Integrate[Sech[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^{2} x^{2}+1\right) \cosh \left(\frac{\sqrt{-a x +1}}{\sqrt{a x +1}}\right)^{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)/cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x)
```

```
[Out] int(1/(-a^2*x^2+1)/cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm=
"maxima")
```

```
[Out] 2*sqrt(a*x + 1)/(sqrt(-a*x + 1)*a*e^(2*sqrt(-a*x + 1)/sqrt(a*x + 1)) + sqrt
(-a*x + 1)*a) + 2*integrate(sqrt(a*x + 1)/((a^2*x^2 - 1)*sqrt(-a*x + 1)*e^(
2*sqrt(-a*x + 1)/sqrt(a*x + 1)) + (a^2*x^2 - 1)*sqrt(-a*x + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm=
"fricas")
```

```
[Out] integral(-1/((a^2*x^2 - 1)*cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^{2} x^{2} \cosh^{2}{\left(\frac{\sqrt{- a x + 1}}{\sqrt{a x + 1}} \right)} - \cosh^{2}{\left(\frac{\sqrt{- a x + 1}}{\sqrt{a x + 1}} \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)/cosh((-a*x+1)**(1/2)/(a*x+1)**(1/2))**2,x)
```

```
[Out] -Integral(1/(a**2*x**2*cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))**2 - cosh(sqrt(-a
*x + 1)/sqrt(a*x + 1))**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm=
"giac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)*cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{\cosh\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)^2\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cosh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2*(a^2*x^2 - 1)),x)
```

```
[Out] -int(1/(cosh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2*(a^2*x^2 - 1)), x)
```

3.218 $\int \frac{x\sinh(x)}{(a+b\cosh(x))^2}\,dx$

Optimal. Leaf size=60

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b\sqrt{a+b}}-\frac{x}{b(a+b\cosh(x))}$$

```
[Out] -x/b/(a+b*cosh(x))+2*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/b/(a-b)^(
1/2)/(a+b)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5573, 2738, 214}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\,\sqrt{a+b}}-\frac{x}{b(a+b\cosh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sinh[x])/(a + b*Cosh[x])^2,x]
```

```
[Out] (2*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]
) - x/(b*(a + b*Cosh[x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 5573

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.
)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^m*((a + b*Cosh[c +
d*x])^(n + 1)/(b*d*(n + 1))), x] - Dist[f*(m/(b*d*(n + 1))), Int[(e + f*x)^
(m - 1)*(a + b*Cosh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\sinh(x)}{(a+b\cosh(x))^2}\,dx &= -\frac{x}{b(a+b\cosh(x))}+\frac{\int \frac{1}{a+b\cosh(x)}\,dx}{b}\\ &= -\frac{x}{b(a+b\cosh(x))}+\frac{2\mathrm{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b}\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b\sqrt{a+b}}-\frac{x}{b(a+b\cosh(x))}\end{aligned}$$

Mathematica [**A**]
time = 0.09, size = 59, normalized size = 0.98

$$-\frac{2\mathrm{ArcTan}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{b\sqrt{-a^2+b^2}}-\frac{x}{b(a+b\cosh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sinh[x])/(a + b*Cosh[x])^2,x]
```

```
[Out] (-2*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/(b*Sqrt[-a^2 + b^2]) - x/
(b*(a + b*Cosh[x]))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(50) = 100.
time = 0.94, size = 138, normalized size = 2.30

method	result	size
risch	$-\frac{2x\,e^{x}}{b(b\,e^{2x}+2a\,e^{x}+b)}+\frac{\ln\left(e^{x}+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,b}-\frac{\ln\left(e^{x}+\frac{a\sqrt{a^2-b^2}+a^2-b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,b}$	138

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(x)/(a+b*cosh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*x/b*exp(x)/(b*exp(2*x)+2*a*exp(x)+b)+1/(a^2-b^2)^(1/2)/b*ln(exp(x)+(a*(a
^2-b^2)^(1/2)-a^2+b^2)/b/(a^2-b^2)^(1/2))-1/(a^2-b^2)^(1/2)/b*ln(exp(x)+(a*
(a^2-b^2)^(1/2)+a^2-b^2)/b/(a^2-b^2)^(1/2))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(x)/(a+b*cosh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(50) = 100$.
time = 0.49, size = 480, normalized size = 8.00

$$\left[-\frac{2\,(a^2-b^2)x\cosh\left(x\right)+2\,(a^2-b^2)x\sinh\left(x\right)-\left(b\cosh\left(x\right)^2+b\sinh\left(x\right)^2+2\,a\cosh\left(x\right)+2\,(b\cosh\left(x\right)+a)\sinh\left(x\right)+b\right)\sqrt{a^2-b^2}\,\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\,(b^2\cosh(x)+ab)\sinh(x)-2\,\sqrt{a^2-b^2}\,(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right)}{a^2b^2-b^4+(a^2b^2-b^4)\cosh\left(x\right)^2+(a^2b^2-b^4)\sinh\left(x\right)^2+2\,(a^3b-ab^3)\cosh\left(x\right)+2\,(a^3b-ab^3+(a^2b^2-b^4)\cosh\left(x\right))\sinh\left(x\right)},\ -\frac{2\left((a^2-b^2)x\cosh\left(x\right)+(a^2-b^2)x\sinh\left(x\right)+\left(b\cosh\left(x\right)^2+b\sinh\left(x\right)^2+2\,a\cosh\left(x\right)+2\,(b\cosh\left(x\right)+a)\sinh\left(x\right)+b\right)\sqrt{-a^2+b^2}\,\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\cosh(x)+b\sinh(x)+a)}{a^2-b^2}\right)\right)}{a^2b^2-b^4+(a^2b^2-b^4)\cosh\left(x\right)^2+(a^2b^2-b^4)\sinh\left(x\right)^2+2\,(a^3b-ab^3)\cosh\left(x\right)+2\,(a^3b-ab^3+(a^2b^2-b^4)\cosh\left(x\right))\sinh\left(x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(x)/(a+b*cosh(x))^2,x, algorithm="fricas")
```

```
[Out] [-(2*(a^2 - b^2)*x*cosh(x) + 2*(a^2 - b^2)*x*sinh(x) - (b*cosh(x)^2 + b*sin
h(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)*sqrt(a^2 - b^2)*log((
b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x
) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x
)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)))/(a^2*b^2
 - b^4 + (a^2*b^2 - b^4)*cosh(x)^2 + (a^2*b^2 - b^4)*sinh(x)^2 + 2*(a^3*b -
 a*b^3)*cosh(x) + 2*(a^3*b - a*b^3 + (a^2*b^2 - b^4)*cosh(x))*sinh(x)), -2*
((a^2 - b^2)*x*cosh(x) + (a^2 - b^2)*x*sinh(x) + (b*cosh(x)^2 + b*sinh(x)^2
 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)*sqrt(-a^2 + b^2)*arctan(-sq
rt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)))/(a^2*b^2 - b^4 + (
a^2*b^2 - b^4)*cosh(x)^2 + (a^2*b^2 - b^4)*sinh(x)^2 + 2*(a^3*b - a*b^3)*co
sh(x) + 2*(a^3*b - a*b^3 + (a^2*b^2 - b^4)*cosh(x))*sinh(x))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sinh\left(x\right)}{\left(a+b\cosh\left(x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(x)/(a+b*cosh(x))**2,x)
```

```
[Out] Integral(x*sinh(x)/(a + b*cosh(x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(x)/(a+b*cosh(x))^2,x, algorithm="giac")
```

```
[Out] integrate(x*sinh(x)/(b*cosh(x) + a)^2, x)
```

Mupad [B]
time = 1.09, size = 110, normalized size = 1.83

$$\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^x\,\left(b^4-a^2\,b^2\right)+a\,b^3+a^2\,b^2\,\mathrm{e}^x}{b^2\,\sqrt{b^4-a^2\,b^2}}\right)}{\sqrt{b^4-a^2\,b^2}}-\frac{2\,\mathrm{e}^x\,\left(a^2\,x-b^2\,x\right)}{\left(a^2\,b-b^3\right)\,\left(b+2\,a\,\mathrm{e}^x+b\,\mathrm{e}^{2\,x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sinh(x))/(a + b*cosh(x))^2,x)
```

```
[Out] (2*atan((exp(x)*(b^4 - a^2*b^2) + a*b^3 + a^2*b^2*exp(x))/(b^2*(b^4 - a^2*b
^2)^(1/2))))/(b^4 - a^2*b^2)^(1/2) - (2*exp(x)*(a^2*x - b^2*x))/((a^2*b - b
^3)*(b + 2*a*exp(x) + b*exp(2*x)))
```

3.219 $\int \frac{x\sinh(x)}{(a+b\cosh(x))^3}\,dx$

Optimal. Leaf size=87

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b(a+b)^{3/2}}-\frac{x}{2b(a+b\cosh(x))^2}-\frac{\sinh(x)}{2\left(a^2-b^2\right)(a+b\cosh(x))}$$

```
[Out] a*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(3/2)/b/(a+b)^(3/2)-1/
2*x/b/(a+b*cosh(x))^2-1/2*sinh(x)/(a^2-b^2)/(a+b*cosh(x))
```

Rubi [A]
time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5573, 2743, 12, 2738, 214}

$$-\frac{\sinh(x)}{2\left(a^2-b^2\right)(a+b\cosh(x))}-\frac{x}{2b(a+b\cosh(x))^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sinh[x])/(a + b*Cosh[x])^3,x]
```

```
[Out] (a*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/((a - b)^(3/2)*b*(a + b)^(
3/2)) - x/(2*b*(a + b*Cosh[x])^2) - Sinh[x]/(2*(a^2 - b^2)*(a + b*Cosh[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
```

```
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 5573

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.
)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^m*((a + b*Cosh[c +
d*x])^(n + 1)/(b*d*(n + 1))), x] - Dist[f*(m/(b*d*(n + 1))), Int[(e + f*x)^
(m - 1)*(a + b*Cosh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\sinh(x)}{(a+b\cosh(x))^3}\,dx &= -\frac{x}{2b(a+b\cosh(x))^2}+\frac{\int \frac{1}{(a+b\cosh(x))^2}\,dx}{2b}\\ &= -\frac{x}{2b(a+b\cosh(x))^2}-\frac{\sinh(x)}{2\left(a^2-b^2\right)(a+b\cosh(x))}+\frac{\int \frac{a}{a+b\cosh(x)}\,dx}{2b\left(a^2-b^2\right)}\\ &= -\frac{x}{2b(a+b\cosh(x))^2}-\frac{\sinh(x)}{2\left(a^2-b^2\right)(a+b\cosh(x))}+\frac{a\int \frac{1}{a+b\cosh(x)}\,dx}{2b\left(a^2-b^2\right)}\\ &= -\frac{x}{2b(a+b\cosh(x))^2}-\frac{\sinh(x)}{2\left(a^2-b^2\right)(a+b\cosh(x))}+\frac{a\text{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx,x,\tan\right.}{b\left(a^2-b^2\right)}\\ &= \frac{a\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b(a+b)^{3/2}}-\frac{x}{2b(a+b\cosh(x))^2}-\frac{\sinh(x)}{2\left(a^2-b^2\right)(a+b\cosh(x))}\end{aligned}$$

Mathematica [A]
time = 0.20, size = 87, normalized size = 1.00

$$\frac{1}{2}\left(\frac{\frac{2a\text{ArcTan}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\left(-a^2+b^2\right)^{3/2}}-\frac{x}{(a+b\cosh(x))^2}}{b}-\frac{\sinh(x)}{(a-b)(a+b)(a+b\cosh(x))}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sinh[x])/(a + b*Cosh[x])^3,x]
```

```
[Out] (((2*a*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) - x
/(a + b*Cosh[x])^2)/b - Sinh[x]/((a - b)*(a + b)*(a + b*Cosh[x])))/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(73) = 146$.
time = 1.06, size = 231, normalized size = 2.66

method	result
risch	$-\frac{2a^2x\,\mathrm{e}^{2x}-ab\,\mathrm{e}^{3x}-2b^2x\,\mathrm{e}^{2x}-2a^2\mathrm{e}^{2x}-b^2\mathrm{e}^{2x}-3b\,\mathrm{e}^{x}a-b^2}{b(b\,\mathrm{e}^{2x}+2a\,\mathrm{e}^{x}+b)^2(a^2-b^2)}+\frac{a\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}\,(a+b)(a-b)b}-\frac{a\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^2-b^2}+a^2-}{b\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}\,(a+b)(a-b)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(x)/(a+b*cosh(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*(2*a^2*x*exp(2*x)-a*b*exp(3*x)-2*b^2*x*exp(2*x)-2*a^2*exp(2*x)-b^2*exp
(2*x)-3*b*exp(x)*a-b^2)/(b*exp(2*x)+2*a*exp(x)+b)^2/(a^2-b^2)+1/2/(a^2-b^2)
^(1/2)*a/(a+b)/(a-b)/b*ln(exp(x)+(a*(a^2-b^2)^(1/2)-a^2+b^2)/b/(a^2-b^2)^(1
/2))-1/2/(a^2-b^2)^(1/2)*a/(a+b)/(a-b)/b*ln(exp(x)+(a*(a^2-b^2)^(1/2)+a^2-b
^2)/b/(a^2-b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(x)/(a+b*cosh(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 808 vs. $2(73) = 146$.
time = 0.50, size = 1692, normalized size = 19.45

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(x)/(a+b*cosh(x))^3,x, algorithm="fricas")
```

```
[Out] [1/2*(2*a^2*b^2 - 2*b^4 + 2*(a^3*b - a*b^3)*cosh(x)^3 + 2*(a^3*b - a*b^3)*s
inh(x)^3 + 2*(2*a^4 - a^2*b^2 - b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*x)*cosh(x)^
2 + 2*(2*a^4 - a^2*b^2 - b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*x + 3*(a^3*b - a*b
^3)*cosh(x))*sinh(x)^2 - (a*b^2*cosh(x)^4 + a*b^2*sinh(x)^4 + 4*a^2*b*cosh(
x)^3 + 4*a^2*b*cosh(x) + 4*(a*b^2*cosh(x) + a^2*b)*sinh(x)^3 + a*b^2 + 2*(2
```

```
*a^3 + a*b^2)*cosh(x)^2 + 2*(3*a*b^2*cosh(x)^2 + 6*a^2*b*cosh(x) + 2*a^3 +
a*b^2)*sinh(x)^2 + 4*(a*b^2*cosh(x)^3 + 3*a^2*b*cosh(x)^2 + a^2*b + (2*a^3
+ a*b^2)*cosh(x))*sinh(x))*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)
^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a
^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cos
h(x) + 2*(b*cosh(x) + a)*sinh(x) + b)) + 6*(a^3*b - a*b^3)*cosh(x) + 2*(3*a
^3*b - 3*a*b^3 + 3*(a^3*b - a*b^3)*cosh(x)^2 + 2*(2*a^4 - a^2*b^2 - b^4 - 2
*(a^4 - 2*a^2*b^2 + b^4)*x)*cosh(x))*sinh(x))/(a^4*b^3 - 2*a^2*b^5 + b^7 +
(a^4*b^3 - 2*a^2*b^5 + b^7)*cosh(x)^4 + (a^4*b^3 - 2*a^2*b^5 + b^7)*sinh(x)
^4 + 4*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cosh(x)^3 + 4*(a^5*b^2 - 2*a^3*b^4 + a
*b^6 + (a^4*b^3 - 2*a^2*b^5 + b^7)*cosh(x))*sinh(x)^3 + 2*(2*a^6*b - 3*a^4*
b^3 + b^7)*cosh(x)^2 + 2*(2*a^6*b - 3*a^4*b^3 + b^7 + 3*(a^4*b^3 - 2*a^2*b^
5 + b^7)*cosh(x)^2 + 6*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cosh(x))*sinh(x)^2 + 4
*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cosh(x) + 4*(a^5*b^2 - 2*a^3*b^4 + a*b^6 + (
a^4*b^3 - 2*a^2*b^5 + b^7)*cosh(x)^3 + 3*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cosh
(x)^2 + (2*a^6*b - 3*a^4*b^3 + b^7)*cosh(x))*sinh(x)), (a^2*b^2 - b^4 + (a^
3*b - a*b^3)*cosh(x)^3 + (a^3*b - a*b^3)*sinh(x)^3 + (2*a^4 - a^2*b^2 - b^4
 - 2*(a^4 - 2*a^2*b^2 + b^4)*x)*cosh(x)^2 + (2*a^4 - a^2*b^2 - b^4 - 2*(a^4
 - 2*a^2*b^2 + b^4)*x + 3*(a^3*b - a*b^3)*cosh(x))*sinh(x)^2 - (a*b^2*cosh(
x)^4 + a*b^2*sinh(x)^4 + 4*a^2*b*cosh(x)^3 + 4*a^2*b*cosh(x) + 4*(a*b^2*cos
h(x) + a^2*b)*sinh(x)^3 + a*b^2 + 2*(2*a^3 + a*b^2)*cosh(x)^2 + 2*(3*a*b^2*
cosh(x)^2 + 6*a^2*b*cosh(x) + 2*a^3 + a*b^2)*sinh(x)^2 + 4*(a*b^2*cosh(x)^3
 + 3*a^2*b*cosh(x)^2 + a^2*b + (2*a^3 + a*b^2)*cosh(x))*sinh(x))*sqrt(-a^2
+ b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) +
3*(a^3*b - a*b^3)*cosh(x) + (3*a^3*b - 3*a*b^3 + 3*(a^3*b - a*b^3)*cosh(x)^
2 + 2*(2*a^4 - a^2*b^2 - b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*x)*cosh(x))*sinh(x
))/(a^4*b^3 - 2*a^2*b^5 + b^7 + (a^4*b^3 - 2*a^2*b^5 + b^7)*cosh(x)^4 + (a^
4*b^3 - 2*a^2*b^5 + b^7)*sinh(x)^4 + 4*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cosh(x
)^3 + 4*(a^5*b^2 - 2*a^3*b^4 + a*b^6 + (a^4*b^3 - 2*a^2*b^5 + b^7)*cosh(x))
*sinh(x)^3 + 2*(2*a^6*b - 3*a^4*b^3 + b^7)*cosh(x)^2 + 2*(2*a^6*b - 3*a^4*b
^3 + b^7 + 3*(a^4*b^3 - 2*a^2*b^5 + b^7)*cosh(x)^2 + 6*(a^5*b^2 - 2*a^3*b^4
 + a*b^6)*cosh(x))*sinh(x)^2 + 4*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cosh(x) + 4*
(a^5*b^2 - 2*a^3*b^4 + a*b^6 + (a^4*b^3 - 2*a^2*b^5 + b^7)*cosh(x)^3 + 3*(a
^5*b^2 - 2*a^3*b^4 + a*b^6)*cosh(x)^2 + (2*a^6*b - 3*a^4*b^3 + b^7)*cosh(x)
)*sinh(x))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(x)/(a+b*cosh(x))**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(x)/(a+b*cosh(x))^3,x, algorithm="giac")
[Out] integrate(x*sinh(x)/(b*cosh(x) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\sinh(x)}{\left(a+b\cosh\left(x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sinh(x))/(a + b*cosh(x))^3,x)
[Out] int((x*sinh(x))/(a + b*cosh(x))^3, x)
```

3.220 $\int \frac{(2+\cosh^2(a+bx))\sinh(a+bx)}{x}\,dx$

Optimal. Leaf size=47

$$\frac{9}{4}\text{Chi}(bx)\sinh(a)+\frac{1}{4}\text{Chi}(3bx)\sinh(3a)+\frac{9}{4}\cosh(a)\text{Shi}(bx)+\frac{1}{4}\cosh(3a)\text{Shi}(3bx)$$

```
[Out] 9/4*cosh(a)*Shi(b*x)+1/4*cosh(3*a)*Shi(3*b*x)+9/4*Chi(b*x)*sinh(a)+1/4*Chi(
3*b*x)*sinh(3*a)
```

Rubi [A]

time = 0.35, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6874, 3384, 3379, 3382, 5556}

$$\frac{9}{4}\sinh(a)\text{Chi}(bx)+\frac{1}{4}\sinh(3a)\text{Chi}(3bx)+\frac{9}{4}\cosh(a)\text{Shi}(bx)+\frac{1}{4}\cosh(3a)\text{Shi}(3bx)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + Cosh[a + b*x]^2)*Sinh[a + b*x])/x,x]
```

```
[Out] (9*CoshIntegral[b*x]*Sinh[a])/4 + (CoshIntegral[3*b*x]*Sinh[3*a])/4 + (9*Co
sh[a]*SinhIntegral[b*x])/4 + (Cosh[3*a]*SinhIntegral[3*b*x])/4
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
```

```
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+\cosh^2(a+bx))\sinh(a+bx)}{x}\,dx &= \int \left(\frac{2\sinh(a+bx)}{x}+\frac{\cosh^2(a+bx)\sinh(a+bx)}{x}\right)dx \\
&= 2\int \frac{\sinh(a+bx)}{x}\,dx+\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x}\,dx \\
&= (2\cosh(a))\int \frac{\sinh(bx)}{x}\,dx+(2\sinh(a))\int \frac{\cosh(bx)}{x}\,dx+\int \Bigg(\frac{\sin
\\
&= 2\mathrm{Chi}(bx)\sinh(a)+2\cosh(a)\mathrm{Shi}(bx)+\frac{1}{4}\int \frac{\sinh(a+bx)}{x}\,dx+\frac{1}{4}
\\
&= 2\mathrm{Chi}(bx)\sinh(a)+2\cosh(a)\mathrm{Shi}(bx)+\frac{1}{4}\cosh(a)\int \frac{\sinh(bx)}{x}\,dx
\\
&= \frac{9}{4}\mathrm{Chi}(bx)\sinh(a)+\frac{1}{4}\mathrm{Chi}(3bx)\sinh(3a)+\frac{9}{4}\cosh(a)\mathrm{Shi}(bx)+\frac{1}{4}
\end{aligned}$$

Mathematica [**A**]
time = 0.10, size = 41, normalized size = 0.87

$$\frac{1}{4}(9\mathrm{Chi}(bx)\sinh(a)+\mathrm{Chi}(3bx)\sinh(3a)+9\cosh(a)\mathrm{Shi}(bx)+\cosh(3a)\mathrm{Shi}(3bx))$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + Cosh[a + b*x]^2)*Sinh[a + b*x])/x,x]
```

```
[Out] (9*CoshIntegral[b*x]*Sinh[a] + CoshIntegral[3*b*x]*Sinh[3*a] + 9*Cosh[a]*Si
nhIntegral[b*x] + Cosh[3*a]*SinhIntegral[3*b*x])/4
```

Maple [**A**]
time = 3.65, size = 47, normalized size = 1.00

method	result	size
risch	$\frac{e^{-3a}\,\mathrm{expIntegral}(1,3bx)}{8}+\frac{9\,e^{-a}\,\mathrm{expIntegral}(1,bx)}{8}-\frac{9\,e^{a}\,\mathrm{expIntegral}(1,-bx)}{8}-\frac{e^{3a}\,\mathrm{expIntegral}(1,-3bx)}{8}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+cosh(b*x+a)^2)*sinh(b*x+a)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*exp(-3*a)*Ei(1,3*b*x)+9/8*exp(-a)*Ei(1,b*x)-9/8*exp(a)*Ei(1,-b*x)-1/8*e
xp(3*a)*Ei(1,-3*b*x)
```

Maxima [A]
time = 0.36, size = 42, normalized size = 0.89

$$\frac{1}{8}\,\mathrm{Ei}(3\,bx)\,e^{(3\,a)} - \frac{9}{8}\,\mathrm{Ei}(-bx)\,e^{(-a)} - \frac{1}{8}\,\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)} + \frac{9}{8}\,\mathrm{Ei}(bx)\,e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+cosh(b*x+a)^2)*sinh(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] 1/8*Ei(3*b*x)*e^(3*a) - 9/8*Ei(-b*x)*e^(-a) - 1/8*Ei(-3*b*x)*e^(-3*a) + 9/8
*Ei(b*x)*e^a
```

Fricas [A]
time = 0.40, size = 67, normalized size = 1.43

$$\frac{1}{8}\left(\mathrm{Ei}(3\,bx) - \mathrm{Ei}(-3\,bx)\right)\cosh\left(3\,a\right) + \frac{9}{8}\left(\mathrm{Ei}(bx) - \mathrm{Ei}(-bx)\right)\cosh\left(a\right) + \frac{1}{8}\left(\mathrm{Ei}(3\,bx) + \mathrm{Ei}(-3\,bx)\right)\sinh\left(3\,a\right) + \frac{9}{8}\left(\mathrm{Ei}(bx) + \mathrm{Ei}(-bx)\right)\sinh\left(a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+cosh(b*x+a)^2)*sinh(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] 1/8*(Ei(3*b*x) - Ei(-3*b*x))*cosh(3*a) + 9/8*(Ei(b*x) - Ei(-b*x))*cosh(a) +
 1/8*(Ei(3*b*x) + Ei(-3*b*x))*sinh(3*a) + 9/8*(Ei(b*x) + Ei(-b*x))*sinh(a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\cosh^{2}\left(a + bx\right) + 2\right)\sinh\left(a + bx\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+cosh(b*x+a)**2)*sinh(b*x+a)/x,x)
```

```
[Out] Integral((cosh(a + b*x)**2 + 2)*sinh(a + b*x)/x, x)
```

Giac [A]
time = 0.40, size = 42, normalized size = 0.89

$$\frac{1}{8}\,\mathrm{Ei}(3\,bx)\,e^{(3\,a)} - \frac{9}{8}\,\mathrm{Ei}(-bx)\,e^{(-a)} - \frac{1}{8}\,\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)} + \frac{9}{8}\,\mathrm{Ei}(bx)\,e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+cosh(b*x+a)^2)*sinh(b*x+a)/x,x, algorithm="giac")
```

```
[Out] 1/8*Ei(3*b*x)*e^(3*a) - 9/8*Ei(-b*x)*e^(-a) - 1/8*Ei(-3*b*x)*e^(-3*a) + 9/8
*Ei(b*x)*e^a
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sinh(a+b\,x)\,\left(\cosh(a+b\,x)^2+2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(a + b*x)*(cosh(a + b*x)^2 + 2))/x,x)
```

```
[Out] int((sinh(a + b*x)*(cosh(a + b*x)^2 + 2))/x, x)
```

3.221 $\int \frac{x^m \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x^m \sinh(c+dx)}{a+b\cosh(c+dx)},x\right)$$

```
[Out] Unintegrable(x^m*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*Sinh[c + d*x])/(a + b*Cosh[c + d*x]),x]
```

```
[Out] Defer[Int][(x^m*Sinh[c + d*x])/(a + b*Cosh[c + d*x]), x]
```

Rubi steps

$$\int \frac{x^m \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^m \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx$$

Mathematica [A]
time = 3.53, size = 0, normalized size = 0.00

$$\int \frac{x^m \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*Sinh[c + d*x])/(a + b*Cosh[c + d*x]),x]
```

```
[Out] Integrate[(x^m*Sinh[c + d*x])/(a + b*Cosh[c + d*x]), x]
```

Maple [A]
time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{x^m \sinh(dx+c)}{a+b\cosh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

```
[Out] int(x^m*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] x*e^(2*d*x + m*log(x) + 2*c)/(b*(m + 1)*e^(2*d*x + 2*c) + 2*a*(m + 1)*e^(d*
x + c) + b*(m + 1)) - 1/2*integrate(2*(2*a*d*x*e^(3*d*x + 3*c) + 2*a*(m + 1
)*e^(d*x + c) + b*(m + 1) + (2*b*d*x*e^(2*c) + b*(m + 1)*e^(2*c))*e^(2*d*x)
)*x^m/(b^2*(m + 1)*e^(4*d*x + 4*c) + 4*a*b*(m + 1)*e^(3*d*x + 3*c) + 4*a*b*
(m + 1)*e^(d*x + c) + b^2*(m + 1) + 2*(2*a^2*(m + 1)*e^(2*c) + b^2*(m + 1)*
e^(2*c))*e^(2*d*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(x^m*sinh(d*x + c)/(b*cosh(d*x + c) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \sinh(c + dx)}{a + b\cosh(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

```
[Out] Integral(x**m*sinh(c + d*x)/(a + b*cosh(c + d*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x^m*sinh(d*x + c)/(b*cosh(d*x + c) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m\,\sinh(c+d\,x)}{a+b\cosh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*sinh(c + d*x))/(a + b*cosh(c + d*x)),x)
[Out] int((x^m*sinh(c + d*x))/(a + b*cosh(c + d*x)), x)
```

3.222 $\int \frac{x^3 \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal. Leaf size=327

$$-\frac{x^4}{4b}+\frac{x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{x^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd}+\frac{3x^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{3x^2\text{PolyL}}{}$$

```
[Out] -1/4*x^4/b+x^3*ln(1+b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b/d+x^3*ln(1+b*exp(d*
x+c)/(a+(a^2-b^2)^(1/2)))/b/d+3*x^2*polylog(2,-b*exp(d*x+c)/(a-(a^2-b^2)^(1
/2)))/b/d^2+3*x^2*polylog(2,-b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))/b/d^2-6*x*po
lylog(3,-b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b/d^3-6*x*polylog(3,-b*exp(d*x+c
)/(a+(a^2-b^2)^(1/2)))/b/d^3+6*polylog(4,-b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))
/b/d^4+6*polylog(4,-b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))/b/d^4
```

Rubi [A]
time = 0.34, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$,
Rules used = {5681, 2221, 2611, 6744, 2320, 6724}

$$\frac{6\text{Li}_4\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^4}+\frac{6\text{Li}_4\left(-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^4}-\frac{6x\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^3}-\frac{6x\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^3}+\frac{3x^2\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{3x^2\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{x^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}+\frac{x^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{x^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sinh[c + d*x])/(a + b*Cosh[c + d*x]),x]
```

```
[Out] -1/4*x^4/b + (x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b*d) + (
x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b*d) + (3*x^2*PolyLog[
2, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b*d^2) + (3*x^2*PolyLog[2, -
((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b*d^2) - (6*x*PolyLog[3, -((b*E^
(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b*d^3) - (6*x*PolyLog[3, -((b*E^(c + d
*x))/(a + Sqrt[a^2 - b^2]))])/(b*d^3) + (6*PolyLog[4, -((b*E^(c + d*x))/(a
- Sqrt[a^2 - b^2]))])/(b*d^4) + (6*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a
^2 - b^2]))])/(b*d^4)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3 \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{x^4}{4b} + \int \frac{e^{c+dx}x^3}{a-\sqrt{a^2-b^2}+be^{c+dx}}\,dx + \int \frac{e^{c+dx}x^3}{a+\sqrt{a^2-b^2}+be^{c+dx}}\,dx$$

$$= -\frac{x^4}{4b} + \frac{x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{x^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd} - \frac{3\int x^2\log\left(1 \ldots\right.}{\ldots}$$

$$= -\frac{x^4}{4b} + \frac{x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{x^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd} + \frac{3x^2\mathrm{Li}_2\left(-\frac{\ldots}{a-\ldots}\right.}{b\ldots}$$

$$= -\frac{x^4}{4b} + \frac{x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{x^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd} + \frac{3x^2\mathrm{Li}_2\left(-\frac{\ldots}{a-\ldots}\right.}{b\ldots}$$

$$= -\frac{x^4}{4b} + \frac{x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{x^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd} + \frac{3x^2\mathrm{Li}_2\left(-\frac{\ldots}{a-\ldots}\right.}{b\ldots}$$

$$= -\frac{x^4}{4b} + \frac{x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{x^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd} + \frac{3x^2\mathrm{Li}_2\left(-\frac{\ldots}{a-\ldots}\right.}{b\ldots}$$

Mathematica [A]
time = 0.03, size = 326, normalized size = 1.00

$$-\frac{x^4}{4b} + \frac{x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{x^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd} + \frac{3x^2\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2} + \frac{3x^2\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^2} - \frac{6x\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^3} - \frac{6x\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^3} + \frac{6\mathrm{PolyLog}\left(4,\frac{be^{c+dx}}{-a+\sqrt{a^2-b^2}}\right)}{bd^4} + \frac{6\mathrm{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sinh[c + d*x])/(a + b*Cosh[c + d*x]),x]
```

```
[Out] -1/4*x^4/b + (x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b*d) + (
x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b*d) + (3*x^2*PolyLog[
2, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b*d^2) + (3*x^2*PolyLog[2, -
((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b*d^2) - (6*x*PolyLog[3, -((b*E^
(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b*d^3) - (6*x*PolyLog[3, -((b*E^(c + d
*x))/(a + Sqrt[a^2 - b^2]))])/(b*d^3) + (6*PolyLog[4, (b*E^(c + d*x))/(-a +
 Sqrt[a^2 - b^2])])/(b*d^4) + (6*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2
 - b^2]))])/(b*d^4)
```

Maple [F]
time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{x^3 \sinh\left(dx+c\right)}{a+b\cosh\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

```
[Out] int(x^3*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*x^4/b - 1/2*integrate(4*(a*x^3*e^(d*x + c) + b*x^3)/(b^2*e^(2*d*x + 2*c
) + 2*a*b*e^(d*x + c) + b^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 624 vs. 2(299) = 598.
time = 0.52, size = 624, normalized size = 1.91

$$-\frac{d^4x^4 - 12\,d^2x^2\,\mathrm{Li}_2\left(-\frac{a\cosh(dx+c)+a\sinh(dx+c)+(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2-b^2}{b^2}}+b}{b}+1\right) - 12\,d^2x^2\,\mathrm{Li}_2\left(-\frac{a\cosh(dx+c)+a\sinh(dx+c)-(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2-b^2}{b^2}}+b}{b}+1\right) + 4\,c^3\log\left(2\,b\cosh(dx+c)+2\,b\sinh(dx+c)+2\,b\sqrt{\frac{a^2-b^2}{b^2}}+2\,a\right) + 4\,c^3\log\left(2\,b\cosh(dx+c)+2\,b\sinh(dx+c)-2\,b\sqrt{\frac{a^2-b^2}{b^2}}+2\,a\right) + 24\,dx\,\mathrm{polylog}\left(3,-\frac{a\cosh(dx+c)+a\sinh(dx+c)+(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2-b^2}{b^2}}}{b}\right) + 24\,dx\,\mathrm{polylog}\left(3,-\frac{a\cosh(dx+c)+a\sinh(dx+c)-(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2-b^2}{b^2}}}{b}\right) - 4\,(d^3x^3+c^3)\log\left(\frac{a\cosh(dx+c)+a\sinh(dx+c)+(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2-b^2}{b^2}}+b}{b}\right) - 4\,(d^3x^3+c^3)\log\left(\frac{a\cosh(dx+c)+a\sinh(dx+c)-(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2-b^2}{b^2}}+b}{b}\right) - 24\,\mathrm{polylog}\left(4,-\frac{a\cosh(dx+c)+a\sinh(dx+c)+(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2-b^2}{b^2}}}{b}\right) - 24\,\mathrm{polylog}\left(4,-\frac{a\cosh(dx+c)+a\sinh(dx+c)-(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2-b^2}{b^2}}}{b}\right)}{4\,bd^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(d^4*x^4 - 12*d^2*x^2*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*c
osh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - 12*d^2*
x^2*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d
*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + 4*c^3*log(2*b*cosh(d*x + c) +
2*b*sinh(d*x + c) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) + 4*c^3*log(2*b*cosh(d
*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) + 24*d*x*pol
ylog(3, -(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x
 + c))*sqrt((a^2 - b^2)/b^2))/b) + 24*d*x*polylog(3, -(a*cosh(d*x + c) + a*
sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2))/
b) - 4*(d^3*x^3 + c^3)*log((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x
 + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) - 4*(d^3*x^3 + c^3)*
log((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c)
)*sqrt((a^2 - b^2)/b^2) + b)/b) - 24*polylog(4, -(a*cosh(d*x + c) + a*sinh(
d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2))/b) -
24*polylog(4, -(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*si
nh(d*x + c))*sqrt((a^2 - b^2)/b^2))/b))/(b*d^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

```
[Out] Integral(x**3*sinh(c + d*x)/(a + b*cosh(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x^3*sinh(d*x + c)/(b*cosh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \sinh(c+d\,x)}{a+b\cosh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*sinh(c + d*x))/(a + b*cosh(c + d*x)),x)
```

```
[Out] int((x^3*sinh(c + d*x))/(a + b*cosh(c + d*x)), x)
```

3.223 $\int \frac{x^2 \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal. Leaf size=245

$$-\frac{x^3}{3b}+\frac{x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{x^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd}+\frac{2x\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{2x\text{PolyLog}}{}$$

```
[Out] -1/3*x^3/b+x^2*ln(1+b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b/d+x^2*ln(1+b*exp(d*
x+c)/(a+(a^2-b^2)^(1/2)))/b/d+2*x*polylog(2,-b*exp(d*x+c)/(a-(a^2-b^2)^(1/2
)))/b/d^2+2*x*polylog(2,-b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))/b/d^2-2*polylog(
3,-b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b/d^3-2*polylog(3,-b*exp(d*x+c)/(a+(a^
2-b^2)^(1/2)))/b/d^3
```

Rubi [A]
time = 0.28, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {5681, 2221, 2611, 2320, 6724}

$$-\frac{2\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^3}-\frac{2\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^3}+\frac{2x\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{2x\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{x^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}+\frac{x^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{x^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sinh[c + d*x])/(a + b*Cosh[c + d*x]),x]
```

```
[Out] -1/3*x^3/b + (x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b*d) + (
x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b*d) + (2*x*PolyLog[2,
 -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b*d^2) + (2*x*PolyLog[2, -((b*
E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b*d^2) - (2*PolyLog[3, -((b*E^(c + d
*x))/(a - Sqrt[a^2 - b^2]))])/(b*d^3) - (2*PolyLog[3, -((b*E^(c + d*x))/(a
+ Sqrt[a^2 - b^2]))])/(b*d^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{x^3}{3b}+\int \frac{e^{c+dx}x^2}{a-\sqrt{a^2-b^2}+be^{c+dx}}\,dx+\int \frac{e^{c+dx}x^2}{a+\sqrt{a^2-b^2}+be^{c+dx}}\,dx$$

$$= -\frac{x^3}{3b}+\frac{x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{x^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd}-\frac{2\int x\log\left(1 \ldots\right.}{\ldots}$$

$$= -\frac{x^3}{3b}+\frac{x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{x^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd}+\frac{2x\mathrm{Li}_2\left(-\frac{\ldots}{a-\ldots}\right.}{bd\ldots}$$

$$= -\frac{x^3}{3b}+\frac{x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{x^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd}+\frac{2x\mathrm{Li}_2\left(-\frac{\ldots}{a-\ldots}\right.}{bd\ldots}$$

$$= -\frac{x^3}{3b}+\frac{x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{x^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd}+\frac{2x\mathrm{Li}_2\left(-\frac{\ldots}{a-\ldots}\right.}{bd\ldots}$$

Mathematica [A]
time = 0.03, size = 244, normalized size = 1.00

$$-\frac{x^3}{3b}+\frac{x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{x^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd}+\frac{2x\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{2x\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}-\frac{2\text{PolyLog}\left(3,\frac{be^{c+dx}}{-a+\sqrt{a^2-b^2}}\right)}{bd^3}-\frac{2\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sinh[c + d*x])/(a + b*Cosh[c + d*x]),x]
```

```
[Out] -1/3*x^3/b + (x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b*d) + (
x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b*d) + (2*x*PolyLog[2,
 -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b*d^2) + (2*x*PolyLog[2, -((b*
E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b*d^2) - (2*PolyLog[3, (b*E^(c + d*x
))/(-a + Sqrt[a^2 - b^2])])/(b*d^3) - (2*PolyLog[3, -((b*E^(c + d*x))/(a +
Sqrt[a^2 - b^2]))])/(b*d^3)
```

Maple [F]
time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{x^2\sinh(dx+c)}{a+b\cosh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

```
[Out] int(x^2*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3/b - 1/2*integrate(4*(a*x^2*e^(d*x + c) + b*x^2)/(b^2*e^(2*d*x + 2*c
) + 2*a*b*e^(d*x + c) + b^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 497 vs. 2(223) = 446.
time = 0.43, size = 497, normalized size = 2.03

$$-\frac{d^3x^3-6dx\text{Li}_2\left(-\frac{a\cosh(dx+c)+a\sinh(dx+c)+(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2-b^2}{b^2}}+b}{b}+1\right)-6dx\text{Li}_2\left(-\frac{a\cosh(dx+c)+a\sinh(dx+c)-(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2-b^2}{b^2}}+b}{b}+1\right)-3c^2\log\left(2b\cosh(dx+c)+2b\sinh(dx+c)+2b\sqrt{\frac{a^2-b^2}{b^2}}+2a\right)-3c^2\log\left(2b\cosh(dx+c)+2b\sinh(dx+c)-2b\sqrt{\frac{a^2-b^2}{b^2}}+2a\right)-3(d^2x^2-c^2)\log\left(\frac{a\cosh(dx+c)+a\sinh(dx+c)+(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2-b^2}{b^2}}+b}{b}\right)-3(d^2x^2-c^2)\log\left(\frac{a\cosh(dx+c)+a\sinh(dx+c)-(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2-b^2}{b^2}}+b}{b}\right)+6\,\text{polylog}\left(3,-\frac{a\cosh(dx+c)+a\sinh(dx+c)+(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2-b^2}{b^2}}}{b}\right)+6\,\text{polylog}\left(3,-\frac{a\cosh(dx+c)+a\sinh(dx+c)-(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2-b^2}{b^2}}}{b}\right)}{3bd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3*(d^3*x^3 - 6*d*x*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d
*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - 6*d*x*dilog(
-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*s
qrt((a^2 - b^2)/b^2) + b)/b + 1) - 3*c^2*log(2*b*cosh(d*x + c) + 2*b*sinh(d
*x + c) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) - 3*c^2*log(2*b*cosh(d*x + c) +
2*b*sinh(d*x + c) - 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) - 3*(d^2*x^2 - c^2)*lo
g((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*
sqrt((a^2 - b^2)/b^2) + b)/b) - 3*(d^2*x^2 - c^2)*log((a*cosh(d*x + c) + a*
sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) +
 b)/b) + 6*polylog(3, -(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c
) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2))/b) + 6*polylog(3, -(a*cosh(d*x
+ c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^
2)/b^2))/b))/(b*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sinh(c+dx)}{a+b\cosh(c+dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
[Out] Integral(x**2*sinh(c + d*x)/(a + b*cosh(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="giac")
[Out] integrate(x^2*sinh(d*x + c)/(b*cosh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \sinh(c+d\,x)}{a+b\cosh(c+d\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*sinh(c + d*x))/(a + b*cosh(c + d*x)),x)
[Out] int((x^2*sinh(c + d*x))/(a + b*cosh(c + d*x)), x)
```

3.224 $\int \frac{x\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal. Leaf size=161

$$-\frac{x^2}{2b}+\frac{x\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{x\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd}+\frac{\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a\ldots}\right)}{bd^2}$$

```
[Out] -1/2*x^2/b+x*ln(1+b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b/d+x*ln(1+b*exp(d*x+c)
/(a+(a^2-b^2)^(1/2)))/b/d+polylog(2,-b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b/d^
2+polylog(2,-b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))/b/d^2
```

Rubi [A]
time = 0.16, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5681, 2221, 2317, 2438}

$$\frac{\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{x\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}+\frac{x\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sinh[c + d*x])/(a + b*Cosh[c + d*x]),x]
```

```
[Out] -1/2*x^2/b + (x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b*d) + (x*
Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b*d) + PolyLog[2, -((b*E^(
c + d*x))/(a - Sqrt[a^2 - b^2]))]/(b*d^2) + PolyLog[2, -((b*E^(c + d*x))/(a
 + Sqrt[a^2 - b^2]))]/(b*d^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{x\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{x^2}{2b}+\int \frac{e^{c+dx}x}{a-\sqrt{a^2-b^2}+be^{c+dx}}\,dx+\int \frac{e^{c+dx}x}{a+\sqrt{a^2-b^2}+be^{c+dx}}\,dx$$

$$= -\frac{x^2}{2b}+\frac{x\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{x\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd}-\frac{\int\log\left(1+\frac{\quad}{a-\quad}\right.}{b}$$

$$= -\frac{x^2}{2b}+\frac{x\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{x\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd}-\frac{\text{Subst}\left(\int\frac{\log\left(\right.}{}\right.}{}$$

$$= -\frac{x^2}{2b}+\frac{x\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{x\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd}+\frac{\text{Li}_2\left(-\frac{be^{c+}}{a-\sqrt{a^2}}\right.}{bd^2}$$

Mathematica [**A**]
time = 0.02, size = 160, normalized size = 0.99

$$-\frac{x^2}{2b}+\frac{x\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{x\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd}+\frac{\text{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sinh[c + d*x])/(a + b*Cosh[c + d*x]),x]
```

```
[Out] -1/2*x^2/b + (x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b*d) + (x*
Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b*d) + PolyLog[2, (b*E^(c
+ d*x))/(-a + Sqrt[a^2 - b^2])]/(b*d^2) + PolyLog[2, -((b*E^(c + d*x))/(a +
 Sqrt[a^2 - b^2]))]/(b*d^2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(147) = 294$.
time = 1.44, size = 368, normalized size = 2.29

method	result
risch	$-\frac{x^2}{2b}-\frac{2cx}{db}-\frac{c^2}{d^2b}+\frac{\ln\left(\frac{-b\,e^{dx+c}+\sqrt{a^2-b^2}-a}{-a+\sqrt{a^2-b^2}}\right)x}{db}+\frac{\ln\left(\frac{-b\,e^{dx+c}+\sqrt{a^2-b^2}-a}{-a+\sqrt{a^2-b^2}}\right)c}{d^2b}+\frac{\ln\left(\frac{b\,e^{dx+c}+\sqrt{a^2-b^2}+a}{a+\sqrt{a^2-b^2}}\right.}{db}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(d*x+c)/(a+b*cosh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x^2/b-2/d/b*c*x-1/d^2/b*c^2+1/d/b*ln((-b*exp(d*x+c)+(a^2-b^2)^(1/2)-a)
/(-a+(a^2-b^2)^(1/2)))*x+1/d^2/b*ln((-b*exp(d*x+c)+(a^2-b^2)^(1/2)-a)/(-a+(
a^2-b^2)^(1/2)))*c+1/d/b*ln((b*exp(d*x+c)+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(
1/2)))*x+1/d^2/b*ln((b*exp(d*x+c)+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))*c
+1/d^2/b*dilog((-b*exp(d*x+c)+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))+1/d^
2/b*dilog((b*exp(d*x+c)+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))+2/d^2/b*c*l
n(exp(d*x+c))-1/d^2/b*c*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)+b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/b - 1/2*integrate(4*(a*x*e^(d*x + c) + b*x)/(b^2*e^(2*d*x + 2*c) +
2*a*b*e^(d*x + c) + b^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(145) = 290$.
time = 0.37, size = 354, normalized size = 2.20

$$-\frac{d^2x^2+2c\log\left(2b\cosh(dx+c)+2b\sinh(dx+c)+2b\sqrt{\frac{a^2-b^2}{b^2}}+2a\right)+2c\log\left(2b\cosh(dx+c)+2b\sinh(dx+c)-2b\sqrt{\frac{a^2-b^2}{b^2}}+2a\right)-2(dx+c)\log\left(\frac{a\cosh(dx+c)+a\sinh(dx+c)+(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2-b^2}{b^2}}+b}{b}\right)-2(dx+c)\log\left(\frac{a\cosh(dx+c)+a\sinh(dx+c)-(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2-b^2}{b^2}}+b}{b}\right)-2\,\mathrm{Li}_2\left(-\frac{a\cosh(dx+c)+a\sinh(dx+c)+(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2-b^2}{b^2}}+b}{b}+1\right)-2\,\mathrm{Li}_2\left(-\frac{a\cosh(dx+c)+a\sinh(dx+c)-(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2-b^2}{b^2}}+b}{b}+1\right)}{2bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(d^2*x^2 + 2*c*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a
^2 - b^2)/b^2) + 2*a) + 2*c*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b
*sqrt((a^2 - b^2)/b^2) + 2*a) - 2*(d*x + c)*log((a*cosh(d*x + c) + a*sinh(d
*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b)
 - 2*(d*x + c)*log((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) +
b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) - 2*dilog(-(a*cosh(d*x + c)
+ a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^
2) + b)/b + 1) - 2*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x
+ c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1))/(b*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sinh (c + dx)}{a + b \cosh (c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
[Out] Integral(x*sinh(c + d*x)/(a + b*cosh(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="giac")
[Out] integrate(x*sinh(d*x + c)/(b*cosh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x \sinh(c + d\,x)}{a + b \cosh (c + d\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sinh(c + d*x))/(a + b*cosh(c + d*x)),x)
[Out] int((x*sinh(c + d*x))/(a + b*cosh(c + d*x)), x)
```

3.225 $\int \frac{\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal. Leaf size=18

$$\frac{\log(a+b\cosh(c+dx))}{bd}$$

```
[Out] ln(a+b*cosh(d*x+c))/b/d
```

Rubi [A]
time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2747, 31}

$$\frac{\log(a+b\cosh(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]/(a + b*Cosh[c + d*x]),x]
```

```
[Out] Log[a + b*Cosh[c + d*x]]/(b*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\cosh(c+dx)\right)}{bd}\\ &= \frac{\log(a+b\cosh(c+dx))}{bd}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 18, normalized size = 1.00

$$\frac{\log(a+b\cosh(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]/(a + b*Cosh[c + d*x]),x]
```

```
[Out] Log[a + b*Cosh[c + d*x]]/(b*d)
```

Maple [A]
time = 0.37, size = 19, normalized size = 1.06

method	result	size
derivativedivides	$\frac{\ln(a+b\cosh(dx+c))}{bd}$	19
default	$\frac{\ln(a+b\cosh(dx+c))}{bd}$	19
risch	$-\frac{x}{b}-\frac{2c}{bd}+\frac{\ln\left(\mathrm{e}^{2dx+2c}+\frac{2a\,\mathrm{e}^{dx+c}}{b}+1\right)}{bd}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(a+b*cosh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(a+b*cosh(d*x+c))/b/d
```

Maxima [A]
time = 0.27, size = 18, normalized size = 1.00

$$\frac{\log\left(b\cosh\left(dx+c\right)+a\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] log(b*cosh(d*x + c) + a)/(b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(18) = 36$.
time = 0.44, size = 44, normalized size = 2.44

$$-\frac{dx-\log\left(\frac{2\left(b\cosh(dx+c)+a\right)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(d*x - log(2*(b*cosh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))))/(b*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(14) = 28$.

time = 0.43, size = 41, normalized size = 2.28

$$\begin{cases} \frac{x\sinh(c)}{a} & \text{for } b = 0 \wedge d = 0 \\ \frac{x\sinh(c)}{a+b\cosh(c)} & \text{for } d = 0 \\ \frac{\cosh(c+dx)}{ad} & \text{for } b = 0 \\ \frac{\log\left(\frac{a}{b}+\cosh(c+dx)\right)}{bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

```
[Out] Piecewise((x*sinh(c)/a, Eq(b, 0) & Eq(d, 0)), (x*sinh(c)/(a + b*cosh(c)), E
q(d, 0)), (cosh(c + d*x)/(a*d), Eq(b, 0)), (log(a/b + cosh(c + d*x))/(b*d),
 True))
```

Giac [A]
time = 0.42, size = 31, normalized size = 1.72

$$\frac{\log\left(\left|b\left(e^{(dx+c)} + e^{(-dx-c)}\right) + 2a\right|\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] log(abs(b*(e^(d*x + c) + e^(-d*x - c)) + 2*a))/(b*d)
```

Mupad [B]
time = 0.07, size = 18, normalized size = 1.00

$$\frac{\ln\left(a + b\cosh(c + d\,x)\right)}{b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)/(a + b*cosh(c + d*x)),x)
```

```
[Out] log(a + b*cosh(c + d*x))/(b*d)
```

3.226 $\int \frac{\sinh(c+dx)}{x(a+b\cosh(c+dx))} dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\sinh(c+dx)}{x(a+b\cosh(c+dx))},x\right)$$

```
[Out] Unintegrable(sinh(d*x+c)/x/(a+b*cosh(d*x+c)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sinh(c+dx)}{x(a+b\cosh(c+dx))} dx$$

Verification is not applicable to the result.

```
[In] Int[Sinh[c + d*x]/(x*(a + b*Cosh[c + d*x])),x]
[Out] Defer[Int][Sinh[c + d*x]/(x*(a + b*Cosh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sinh(c+dx)}{x(a+b\cosh(c+dx))} dx = \int \frac{\sinh(c+dx)}{x(a+b\cosh(c+dx))} dx$$

Mathematica [A]
time = 9.41, size = 0, normalized size = 0.00

$$\int \frac{\sinh(c+dx)}{x(a+b\cosh(c+dx))} dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sinh[c + d*x]/(x*(a + b*Cosh[c + d*x])),x]
[Out] Integrate[Sinh[c + d*x]/(x*(a + b*Cosh[c + d*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(dx+c)}{x(a+b\cosh(dx+c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/x/(a+b*cosh(d*x+c)),x)
```

```
[Out] int(sinh(d*x+c)/x/(a+b*cosh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/x/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] log(x)/b - 1/2*integrate(4*(a*e^(d*x + c) + b)/(b^2*x*e^(2*d*x + 2*c) + 2*a
*b*x*e^(d*x + c) + b^2*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/x/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sinh(d*x + c)/(b*x*cosh(d*x + c) + a*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(c + dx)}{x(a + b\cosh(c + dx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/x/(a+b*cosh(d*x+c)),x)
```

```
[Out] Integral(sinh(c + d*x)/(x*(a + b*cosh(c + d*x))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/x/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sinh(d*x + c)/((b*cosh(d*x + c) + a)*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sinh(c + d\,x)}{x\,\left(a + b\cosh\left(c + d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)/(x*(a + b*cosh(c + d*x))),x)
[Out] int(sinh(c + d*x)/(x*(a + b*cosh(c + d*x))), x)
```

3.227 $\int \frac{x^m \sinh^2(c+dx)}{a+b\cosh(c+dx)}\, dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m \sinh^2(c+dx)}{a+b\cosh(c+dx)},x\right)$$

```
[Out] Unintegrable(x^m*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \sinh^2(c+dx)}{a+b\cosh(c+dx)}\, dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]),x]
[Out] Defer[Int][(x^m*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]), x]
```

Rubi steps

$$\int \frac{x^m \sinh^2(c+dx)}{a+b\cosh(c+dx)}\, dx = \int \frac{x^m \sinh^2(c+dx)}{a+b\cosh(c+dx)}\, dx$$

Mathematica [A]
time = 17.61, size = 0, normalized size = 0.00

$$\int \frac{x^m \sinh^2(c+dx)}{a+b\cosh(c+dx)}\, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]),x]
[Out] Integrate[(x^m*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]), x]
```

Maple [A]
time = 0.91, size = 0, normalized size = 0.00

$$\int \frac{x^m \left(\sinh^2\left(dx+c\right)\right)}{a+b\cosh\left(dx+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x)
[Out] int(x^m*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="maxima")
[Out] integrate(x^m*sinh(d*x + c)^2/(b*cosh(d*x + c) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="fricas")
[Out] integral(x^m*sinh(d*x + c)^2/(b*cosh(d*x + c) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sinh(d*x+c)**2/(a+b*cosh(d*x+c)),x)
[Out] Integral(x**m*sinh(c + d*x)**2/(a + b*cosh(c + d*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="giac")
[Out] integrate(x^m*sinh(d*x + c)^2/(b*cosh(d*x + c) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m\,{\sinh\left(c+d\,x\right)}^2}{a+b\,\cosh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*sinh(c + d*x)^2)/(a + b*cosh(c + d*x)),x)
[Out] int((x^m*sinh(c + d*x)^2)/(a + b*cosh(c + d*x)), x)
```

3.228 $\int \frac{x^3 \sinh^2(c+dx)}{a+b \cosh(c+dx)} \, dx$

Optimal. Leaf size=495

$$-\frac{ax^4}{4b^2}-\frac{6\cosh(c+dx)}{bd^4}-\frac{3x^2\cosh(c+dx)}{bd^2}+\frac{\sqrt{a^2-b^2}\,x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d}-\frac{\sqrt{a^2-b^2}\,x^3\log\left(1+\frac{}{a+}\right.}{b^2d}$$

```
[Out] -1/4*a*x^4/b^2-6*cosh(d*x+c)/b/d^4-3*x^2*cosh(d*x+c)/b/d^2+6*x*sinh(d*x+c)/
b/d^3+x^3*sinh(d*x+c)/b/d+x^3*ln(1+b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))*(a^2-b
^2)^(1/2)/b^2/d-x^3*ln(1+b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/
b^2/d+3*x^2*polylog(2,-b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^
2/d^2-3*x^2*polylog(2,-b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^
2/d^2-6*x*polylog(3,-b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^2/
d^3+6*x*polylog(3,-b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^2/d^
3+6*polylog(4,-b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^2/d^4-6*
polylog(4,-b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^2/d^4
```

Rubi [A]
time = 0.58, antiderivative size = 495, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {5685, 30, 3377, 2718, 3401, 2296, 2221, 2611, 6744, 2320, 6724}

$$\frac{6\sqrt{a^2-b^2}\,\text{Li}_4\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^4}-\frac{6\sqrt{a^2-b^2}\,\text{Li}_4\left(-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^2d^4}-\frac{6x\sqrt{a^2-b^2}\,\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^3}+\frac{6x\sqrt{a^2-b^2}\,\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^2d^3}+\frac{3x^2\sqrt{a^2-b^2}\,\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^2}-\frac{3x^2\sqrt{a^2-b^2}\,\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^2d^2}+\frac{x^3\sqrt{a^2-b^2}\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{b^2d}-\frac{x^3\sqrt{a^2-b^2}\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{b^2d}-\frac{ax^4}{4b^2}-\frac{6\cosh(c+dx)}{bd^4}+\frac{6x\sinh(c+dx)}{bd^3}-\frac{3x^2\cosh(c+dx)}{bd^2}+\frac{x^3\sinh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]),x]
```

```
[Out] -1/4*(a*x^4)/b^2 - (6*Cosh[c + d*x])/(b*d^4) - (3*x^2*Cosh[c + d*x])/(b*d^2
) + (Sqrt[a^2 - b^2]*x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b
^2*d) - (Sqrt[a^2 - b^2]*x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])]
)/(b^2*d) + (3*Sqrt[a^2 - b^2]*x^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a
^2 - b^2]))])/(b^2*d^2) - (3*Sqrt[a^2 - b^2]*x^2*PolyLog[2, -((b*E^(c + d*x
))/(a + Sqrt[a^2 - b^2]))])/(b^2*d^2) - (6*Sqrt[a^2 - b^2]*x*PolyLog[3, -((
b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b^2*d^3) + (6*Sqrt[a^2 - b^2]*x*Po
lyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b^2*d^3) + (6*Sqrt[a^2
 - b^2]*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b^2*d^4) - (
6*Sqrt[a^2 - b^2]*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b^
2*d^4) + (6*x*Sinh[c + d*x])/(b*d^3) + (x^3*Sinh[c + d*x])/(b*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
```

```
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5685

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_))/(Cosh[(c_.)
+ (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Sinh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Sinh[c + d*x]^(n - 2
)*Cosh[c + d*x], x], x] + Dist[(a^2 - b^2)/b^2, Int[(e + f*x)^m*(Sinh[c + d
*x]^(n - 2)/(a + b*Cosh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{a\int x^3\,dx}{b^2}+\frac{\int x^3\cosh(c+dx)\,dx}{b}+\frac{(a^2-b^2)\int \frac{x^3}{a+b\cosh(c+dx)}\,dx}{b^2}$$

$$= -\frac{ax^4}{4b^2}+\frac{x^3\sinh(c+dx)}{bd}+\frac{(2(a^2-b^2))\int \frac{e^{c+dx}x^3}{b+2ae^{c+dx}+be^{2(c+dx)}}\,dx}{b^2}-\frac{3\int x^2\sinh(c+ \cdots}{bd}$$

$$= -\frac{ax^4}{4b^2}-\frac{3x^2\cosh(c+dx)}{bd^2}+\frac{x^3\sinh(c+dx)}{bd}+\frac{\left(2\sqrt{a^2-b^2}\right)\int \frac{e^{c+dx}x^3}{2a-2\sqrt{a^2-b^2}+2b \cdots}}{b}$$

$$= -\frac{ax^4}{4b^2}-\frac{3x^2\cosh(c+dx)}{bd^2}+\frac{\sqrt{a^2-b^2}\,x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d}-\frac{\sqrt{a^2-b^2}\,x^3 \cdots}{\cdots}$$

$$= -\frac{ax^4}{4b^2}-\frac{6\cosh(c+dx)}{bd^4}-\frac{3x^2\cosh(c+dx)}{bd^2}+\frac{\sqrt{a^2-b^2}\,x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2} \cdots}\right.}{b^2d}$$

$$= -\frac{ax^4}{4b^2}-\frac{6\cosh(c+dx)}{bd^4}-\frac{3x^2\cosh(c+dx)}{bd^2}+\frac{\sqrt{a^2-b^2}\,x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2} \cdots}\right.}{b^2d}$$

$$= -\frac{ax^4}{4b^2}-\frac{6\cosh(c+dx)}{bd^4}-\frac{3x^2\cosh(c+dx)}{bd^2}+\frac{\sqrt{a^2-b^2}\,x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2} \cdots}\right.}{b^2d}$$

$$= -\frac{ax^4}{4b^2}-\frac{6\cosh(c+dx)}{bd^4}-\frac{3x^2\cosh(c+dx)}{bd^2}+\frac{\sqrt{a^2-b^2}\,x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2} \cdots}\right.}{b^2d}$$

<u>**Mathematica**</u> [**A**]
time = 1.09, size = 386, normalized size = 0.78

$$\frac{-ad^4x^4+4\sqrt{a^2-b^2}\left(d^3x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)-d^3x^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)+3d^2x^2\text{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2-b^2}}\right)-3d^2x^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)-6dx\text{PolyLog}\left(3,\frac{be^{c+dx}}{-a+\sqrt{a^2-b^2}}\right)+6dx\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)+6\text{PolyLog}\left(4,\frac{be^{c+dx}}{-a+\sqrt{a^2-b^2}}\right)-6\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)\right)+4b\cosh(dx)\left(-3(2+d^2x^2)\cosh(c)+dx(6+d^2x^2)\sinh(c)\right)+4b\left(dx(6+d^2x^2)\cosh(c)-3(2+d^2x^2)\sinh(c)\right)\sinh(dx)}{4b^2d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]),x]
```

```
[Out] (-(a*d^4*x^4) + 4*Sqrt[a^2 - b^2]*(d^3*x^3*Log[1 + (b*E^(c + d*x))/(a - Sqr
t[a^2 - b^2])] - d^3*x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])] + 3
*d^2*x^2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 - b^2])] - 3*d^2*x^2*Pol
yLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))] - 6*d*x*PolyLog[3, (b*E^(
c + d*x))/(-a + Sqrt[a^2 - b^2])] + 6*d*x*PolyLog[3, -((b*E^(c + d*x))/(a +
 Sqrt[a^2 - b^2]))] + 6*PolyLog[4, (b*E^(c + d*x))/(-a + Sqrt[a^2 - b^2])]
- 6*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))]) + 4*b*Cosh[d*x]*(
-3*(2 + d^2*x^2)*Cosh[c] + d*x*(6 + d^2*x^2)*Sinh[c]) + 4*b*(d*x*(6 + d^2*x
^2)*Cosh[c] - 3*(2 + d^2*x^2)*Sinh[c])*Sinh[d*x])/(4*b^2*d^4)
```

Maple [F]
time = 1.08, size = 0, normalized size = 0.00

$$\int \frac{x^3 \left(\sinh^2 \left(dx+c\right)\right)}{a+b\cosh \left(dx+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x)
```

```
[Out] int(x^3*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-b>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1174 vs. $2(451) = 902$.
time = 0.45, size = 1174, normalized size = 2.37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(a*d^4*x^4*cosh(d*x + c) + 2*b*d^3*x^3 + 6*b*d^2*x^2 + 12*b*d*x - 2*(b
*d^3*x^3 - 3*b*d^2*x^2 + 6*b*d*x - 6*b)*cosh(d*x + c)^2 - 2*(b*d^3*x^3 - 3*
b*d^2*x^2 + 6*b*d*x - 6*b)*sinh(d*x + c)^2 - 12*(b*d^2*x^2*cosh(d*x + c) +
b*d^2*x^2*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cosh(d*x + c) + a*
sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) +
 b)/b + 1) + 12*(b*d^2*x^2*cosh(d*x + c) + b*d^2*x^2*sinh(d*x + c))*sqrt((a
^2 - b^2)/b^2)*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c)
 + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - 4*(b*c^3*cosh(d*x +
 c) + b*c^3*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*
b*sinh(d*x + c) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) + 4*(b*c^3*cosh(d*x + c)
 + b*c^3*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*s
inh(d*x + c) - 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) - 4*((b*d^3*x^3 + b*c^3)*co
```

```
sh(d*x + c) + (b*d^3*x^3 + b*c^3)*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*log(
(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sq
rt((a^2 - b^2)/b^2) + b)/b) + 4*((b*d^3*x^3 + b*c^3)*cosh(d*x + c) + (b*d^3
*x^3 + b*c^3)*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*log((a*cosh(d*x + c) + a
*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)
+ b)/b) - 24*(b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*poly
log(4, -(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x
+ c))*sqrt((a^2 - b^2)/b^2))/b) + 24*(b*cosh(d*x + c) + b*sinh(d*x + c))*sq
rt((a^2 - b^2)/b^2)*polylog(4, -(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cos
h(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2))/b) + 24*(b*d*x*cosh(d*
x + c) + b*d*x*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*polylog(3, -(a*cosh(d*x
 + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b
^2)/b^2))/b) - 24*(b*d*x*cosh(d*x + c) + b*d*x*sinh(d*x + c))*sqrt((a^2 - b
^2)/b^2)*polylog(3, -(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c)
+ b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2))/b) + (a*d^4*x^4 - 4*(b*d^3*x^3 -
3*b*d^2*x^2 + 6*b*d*x - 6*b)*cosh(d*x + c))*sinh(d*x + c) + 12*b)/(b^2*d^4*
cosh(d*x + c) + b^2*d^4*sinh(d*x + c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \sinh^2{(c + dx)}}{a + b \cosh{(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sinh(d*x+c)**2/(a+b*cosh(d*x+c)),x)
[Out] Integral(x**3*sinh(c + d*x)**2/(a + b*cosh(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="giac")
[Out] integrate(x^3*sinh(d*x + c)^2/(b*cosh(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\mathrm{sinh}\left(c+d\,x\right)}^2}{a+b\,\mathrm{cosh}\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*sinh(c + d*x)^2)/(a + b*cosh(c + d*x)),x)
[Out] int((x^3*sinh(c + d*x)^2)/(a + b*cosh(c + d*x)), x)
```

3.229 $\int \frac{x^2 \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal. Leaf size=370

$$-\frac{ax^3}{3b^2}-\frac{2x\cosh(c+dx)}{bd^2}+\frac{\sqrt{a^2-b^2}\,x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d}-\frac{\sqrt{a^2-b^2}\,x^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^2d}+\frac{2\sqrt{}}{}$$

```
[Out] -1/3*a*x^3/b^2-2*x*cosh(d*x+c)/b/d^2+2*sinh(d*x+c)/b/d^3+x^2*sinh(d*x+c)/b/
d+x^2*ln(1+b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^2/d-x^2*ln(1
+b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^2/d+2*x*polylog(2,-b*e
xp(d*x+c)/(a-(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^2/d^2-2*x*polylog(2,-b*exp
(d*x+c)/(a+(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^2/d^2-2*polylog(3,-b*exp(d*x
+c)/(a-(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^2/d^3+2*polylog(3,-b*exp(d*x+c)/
(a+(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^2/d^3
```

Rubi [A]
time = 0.51, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5685, 30, 3377, 2717, 3401, 2296, 2221, 2611, 2320, 6724}

$$-\frac{2\sqrt{a^2-b^2}\,\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^3}+\frac{2\sqrt{a^2-b^2}\,\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^2d^3}+\frac{2x\sqrt{a^2-b^2}\,\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^2}-\frac{2x\sqrt{a^2-b^2}\,\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^2d^2}+\frac{x^2\sqrt{a^2-b^2}\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{b^2d}-\frac{x^2\sqrt{a^2-b^2}\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{b^2d}-\frac{ax^3}{3b^2}+\frac{2\sinh(c+dx)}{bd^3}-\frac{2x\cosh(c+dx)}{bd^2}+\frac{x^2\sinh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]),x]
```

```
[Out] -1/3*(a*x^3)/b^2 - (2*x*Cosh[c + d*x])/(b*d^2) + (Sqrt[a^2 - b^2]*x^2*Log[1
 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b^2*d) - (Sqrt[a^2 - b^2]*x^2*L
og[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b^2*d) + (2*Sqrt[a^2 - b^2]
*x*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b^2*d^2) - (2*Sqr
t[a^2 - b^2]*x*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b^2*d
^2) - (2*Sqrt[a^2 - b^2]*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])
)])/(b^2*d^3) + (2*Sqrt[a^2 - b^2]*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a
^2 - b^2]))])/(b^2*d^3) + (2*Sinh[c + d*x])/(b*d^3) + (x^2*Sinh[c + d*x])/(
b*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5685

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_))/(Cosh[(c_.)
+ (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Sinh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Sinh[c + d*x]^(n - 2
)*Cosh[c + d*x], x], x] + Dist[(a^2 - b^2)/b^2, Int[(e + f*x)^m*(Sinh[c + d
*x]^(n - 2)/(a + b*Cosh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2 \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{a\int x^2\,dx}{b^2} + \frac{\int x^2\cosh(c+dx)\,dx}{b} + \frac{(a^2-b^2)\int \frac{x^2}{a+b\cosh(c+dx)}\,dx}{b^2}$$

$$= -\frac{ax^3}{3b^2} + \frac{x^2\sinh(c+dx)}{bd} + \frac{(2(a^2-b^2))\int \frac{e^{c+dx}x^2}{b+2ae^{c+dx}+be^{2(c+dx)}}\,dx}{b^2} - \frac{2\int x\sinh(c+}{bd}$$

$$= -\frac{ax^3}{3b^2} - \frac{2x\cosh(c+dx)}{bd^2} + \frac{x^2\sinh(c+dx)}{bd} + \frac{\left(2\sqrt{a^2-b^2}\right)\int \frac{e^{c+dx}x^2}{2a-2\sqrt{a^2-b^2}+2}}{b}$$

$$= -\frac{ax^3}{3b^2} - \frac{2x\cosh(c+dx)}{bd^2} + \frac{\sqrt{a^2-b^2}\,x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d} - \frac{\sqrt{a^2-b^2}\,x}{}$$

$$= -\frac{ax^3}{3b^2} - \frac{2x\cosh(c+dx)}{bd^2} + \frac{\sqrt{a^2-b^2}\,x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d} - \frac{\sqrt{a^2-b^2}\,x}{}$$

$$= -\frac{ax^3}{3b^2} - \frac{2x\cosh(c+dx)}{bd^2} + \frac{\sqrt{a^2-b^2}\,x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d} - \frac{\sqrt{a^2-b^2}\,x}{}$$

$$= -\frac{ax^3}{3b^2} - \frac{2x\cosh(c+dx)}{bd^2} + \frac{\sqrt{a^2-b^2}\,x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d} - \frac{\sqrt{a^2-b^2}\,x}{}$$

Mathematica [A]
time = 0.90, size = 293, normalized size = 0.79

$$\frac{-ad^3x^3+3\sqrt{a^2-b^2}\left(d^2x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)-d^2x^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)+2dx\,\text{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2-b^2}}\right)-2dx\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)-2\text{PolyLog}\left(3,\frac{be^{c+dx}}{-a+\sqrt{a^2-b^2}}\right)+2\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)\right)+3b\cosh(dx)\left(-2dx\cosh(c)+(2+d^2x^2)\sinh(c)\right)+3b\left((2+d^2x^2)\cosh(c)-2dx\sinh(c)\right)\sinh(dx)}{3b^2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]),x]
```

```
[Out] (-(a*d^3*x^3) + 3*Sqrt[a^2 - b^2]*(d^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqr
t[a^2 - b^2])] - d^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])] + 2
*d*x*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 - b^2])] - 2*d*x*PolyLog[2,
-((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))] - 2*PolyLog[3, (b*E^(c + d*x))/(-
a + Sqrt[a^2 - b^2])] + 2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]
))]) + 3*b*Cosh[d*x]*(-2*d*x*Cosh[c] + (2 + d^2*x^2)*Sinh[c]) + 3*b*((2 + d
^2*x^2)*Cosh[c] - 2*d*x*Sinh[c])*Sinh[d*x])/(3*b^2*d^3)
```

Maple [F]
time = 1.09, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(\sinh^2 (dx + c)\right)}{a + b \cosh (dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x)
```

```
[Out] int(x^2*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-b>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 937 vs. $2(336) = 672$.
time = 0.46, size = 937, normalized size = 2.53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(2*a*d^3*x^3*cosh(d*x + c) + 3*b*d^2*x^2 + 6*b*d*x - 3*(b*d^2*x^2 - 2*
b*d*x + 2*b)*cosh(d*x + c)^2 - 3*(b*d^2*x^2 - 2*b*d*x + 2*b)*sinh(d*x + c)^
```

```
2 - 12*(b*d*x*cosh(d*x + c) + b*d*x*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*di
log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c
))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + 12*(b*d*x*cosh(d*x + c) + b*d*x*sinh
(d*x + c))*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c)
- (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + 6
*(b*c^2*cosh(d*x + c) + b*c^2*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*log(2*b*
cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) - 6*(b
*c^2*cosh(d*x + c) + b*c^2*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*log(2*b*cos
h(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) - 6*((b*d
^2*x^2 - b*c^2)*cosh(d*x + c) + (b*d^2*x^2 - b*c^2)*sinh(d*x + c))*sqrt((a^
2 - b^2)/b^2)*log((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b
*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) + 6*((b*d^2*x^2 - b*c^2)*cosh
(d*x + c) + (b*d^2*x^2 - b*c^2)*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*log((a
*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt
((a^2 - b^2)/b^2) + b)/b) + 12*(b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^
2 - b^2)/b^2)*polylog(3, -(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x
+ c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2))/b) - 12*(b*cosh(d*x + c) + b
*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*polylog(3, -(a*cosh(d*x + c) + a*sinh
(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2))/b) +
 2*(a*d^3*x^3 - 3*(b*d^2*x^2 - 2*b*d*x + 2*b)*cosh(d*x + c))*sinh(d*x + c)
+ 6*b)/(b^2*d^3*cosh(d*x + c) + b^2*d^3*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sinh(d*x+c)**2/(a+b*cosh(d*x+c)),x)
[Out] Integral(x**2*sinh(c + d*x)**2/(a + b*cosh(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="giac")
[Out] integrate(x^2*sinh(d*x + c)^2/(b*cosh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \sinh(c+d\,x)^2}{a+b\cosh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*sinh(c + d*x)^2)/(a + b*cosh(c + d*x)),x)
[Out] int((x^2*sinh(c + d*x)^2)/(a + b*cosh(c + d*x)), x)
```

3.230 $\int \frac{x\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal. Leaf size=244

$$-\frac{ax^2}{2b^2}-\frac{\cosh(c+dx)}{bd^2}+\frac{\sqrt{a^2-b^2}\,x\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d}-\frac{\sqrt{a^2-b^2}\,x\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^2d}+\frac{\sqrt{a^2-b}}{}$$

```
[Out] -1/2*a*x^2/b^2-cosh(d*x+c)/b/d^2+x*sinh(d*x+c)/b/d+x*ln(1+b*exp(d*x+c)/(a-(
a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^2/d-x*ln(1+b*exp(d*x+c)/(a+(a^2-b^2)^(1/
2)))*(a^2-b^2)^(1/2)/b^2/d+polylog(2,-b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))*(a^
2-b^2)^(1/2)/b^2/d^2-polylog(2,-b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))*(a^2-b^2)
^(1/2)/b^2/d^2
```

Rubi [A]
time = 0.29, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {5685, 30, 3377, 2718, 3401, 2296, 2221, 2317, 2438}

$$\frac{\sqrt{a^2-b^2}\,\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^2}-\frac{\sqrt{a^2-b^2}\,\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^2d^2}+\frac{x\sqrt{a^2-b^2}\,\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{b^2d}-\frac{x\sqrt{a^2-b^2}\,\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{b^2d}-\frac{ax^2}{2b^2}-\frac{\cosh(c+dx)}{bd^2}+\frac{x\sinh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]),x]
```

```
[Out] -1/2*(a*x^2)/b^2 - Cosh[c + d*x]/(b*d^2) + (Sqrt[a^2 - b^2]*x*Log[1 + (b*E^
(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b^2*d) - (Sqrt[a^2 - b^2]*x*Log[1 + (b*
E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b^2*d) + (Sqrt[a^2 - b^2]*PolyLog[2,
-((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b^2*d^2) - (Sqrt[a^2 - b^2]*Pol
yLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b^2*d^2) + (x*Sinh[c +
d*x])/(b*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5685

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_))/(Cosh[(c_.)
+ (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Sinh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Sinh[c + d*x]^(n - 2
)*Cosh[c + d*x], x], x] + Dist[(a^2 - b^2)/b^2, Int[(e + f*x)^m*(Sinh[c + d
*x]^(n - 2)/(a + b*Cosh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{a\int x\,dx}{b^2} + \frac{\int x\cosh(c+dx)\,dx}{b} + \frac{(a^2-b^2)\int \frac{x}{a+b\cosh(c+dx)}\,dx}{b^2}$$

$$= -\frac{ax^2}{2b^2} + \frac{x\sinh(c+dx)}{bd} + \frac{(2(a^2-b^2))\int \frac{e^{c+dx}x}{b+2ae^{c+dx}+be^{2(c+dx)}}\,dx}{b^2} - \frac{\int \sinh(c+dx)\,}{bd}$$

$$= -\frac{ax^2}{2b^2} - \frac{\cosh(c+dx)}{bd^2} + \frac{x\sinh(c+dx)}{bd} + \frac{\left(2\sqrt{a^2-b^2}\right)\int \frac{e^{c+dx}x}{2a-2\sqrt{a^2-b^2}+2be^{c+}}}{b}$$

$$= -\frac{ax^2}{2b^2} - \frac{\cosh(c+dx)}{bd^2} + \frac{\sqrt{a^2-b^2}\,x\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d} - \frac{\sqrt{a^2-b^2}\,x\log}{}$$

$$= -\frac{ax^2}{2b^2} - \frac{\cosh(c+dx)}{bd^2} + \frac{\sqrt{a^2-b^2}\,x\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d} - \frac{\sqrt{a^2-b^2}\,x\log}{}$$

$$= -\frac{ax^2}{2b^2} - \frac{\cosh(c+dx)}{bd^2} + \frac{\sqrt{a^2-b^2}\,x\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d} - \frac{\sqrt{a^2-b^2}\,x\log}{}$$

Mathematica [A]
time = 0.69, size = 187, normalized size = 0.77

$$\frac{a(c-dx)(c+dx) - 2b\cosh(c+dx) + 2\sqrt{a^2-b^2}\left(dx\left(\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right) - \log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)\right) + \text{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2-b^2}}\right) - \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)\right) + 2bdx\sinh(c+dx)}{2b^2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]),x]
```

```
[Out] (a*(c - d*x)*(c + d*x) - 2*b*Cosh[c + d*x] + 2*Sqrt[a^2 - b^2]*(d*x*(Log[1
+ (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])] - Log[1 + (b*E^(c + d*x))/(a + Sqr
t[a^2 - b^2])]) + PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 - b^2])] - Poly
Log[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))]) + 2*b*d*x*Sinh[c + d*x])/
(2*b^2*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 861 vs. $2(222) = 444$.
time = 2.13, size = 862, normalized size = 3.53

method	result
risch	$-\frac{a\,x^2}{2b^2} + \frac{(dx-1)e^{dx+c}}{2d^2b} - \frac{(dx+1)e^{-dx-c}}{2d^2b} - \frac{\ln\left(\frac{b\,e^{dx+c}+\sqrt{a^2-b^2}+a}{a+\sqrt{a^2-b^2}}\right)x\,a^2}{b^2d\sqrt{a^2-b^2}} + \frac{\ln\left(\frac{b\,e^{dx+c}+\sqrt{a^2-b^2}+a}{a+\sqrt{a^2-b^2}}\right)x}{d\sqrt{a^2-b^2}} + \frac{\ln\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b^2*a*x^2+1/2*(d*x-1)/d^2/b*exp(d*x+c)-1/2*(d*x+1)/d^2/b*exp(-d*x-c)-1
/b^2/d/(a^2-b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/
2)))*x*a^2+1/d/(a^2-b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2-b^2)^(1/2)+a)/(a+(a^2-
b^2)^(1/2)))*x+1/b^2/d/(a^2-b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2-b^2)^(1/2)-a)
/(-a+(a^2-b^2)^(1/2)))*x*a^2-1/d/(a^2-b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2-b^2
)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))*x-1/b^2/d^2/(a^2-b^2)^(1/2)*ln((b*exp(d*x+
c)+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))*c*a^2+1/d^2/(a^2-b^2)^(1/2)*ln((
b*exp(d*x+c)+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))*c+1/b^2/d^2/(a^2-b^2)^
(1/2)*ln((-b*exp(d*x+c)+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))*c*a^2-1/d^
2/(a^2-b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2))
)*c+1/b^2/d^2/(a^2-b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2-b^2)^(1/2)-a)/(-a+(
a^2-b^2)^(1/2)))*a^2-1/d^2/(a^2-b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2-b^2)^(
1/2)-a)/(-a+(a^2-b^2)^(1/2)))-1/b^2/d^2/(a^2-b^2)^(1/2)*dilog((b*exp(d*x+c)
+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))*a^2+1/d^2/(a^2-b^2)^(1/2)*dilog((b
*exp(d*x+c)+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))-2/b^2/d^2*c/(-a^2+b^2)^
(1/2)*arctan(1/2*(2*b*exp(d*x+c)+2*a)/(-a^2+b^2)^(1/2))*a^2+2/d^2*c/(-a^2+b
^2)^(1/2)*arctan(1/2*(2*b*exp(d*x+c)+2*a)/(-a^2+b^2)^(1/2))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-b>0)', see `assume?` for more det
ails)Is
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 669 vs. $2(220) = 440$.
time = 0.54, size = 669, normalized size = 2.74

$$-\frac{ad^2x^2\cosh(dx+c) + bdx - (bdx-b)\cosh(dx+c)^2 - (bdx-b)\sinh(dx+c)^2 - 2(b\cosh(dx+c) + b\sinh(dx+c))\sqrt{\frac{a^2-b^2}{b^2}}\,\mathrm{Li}_2\left(-\frac{a\cosh(dx+c)+a\sinh(dx+c)+(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2-b^2}{b^2}}+b}{b}+1\right) + 2(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2-b^2}{b^2}}\,\mathrm{Li}_2\left(-\frac{a\cosh(dx+c)+a\sinh(dx+c)-(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2-b^2}{b^2}}+b}{b}+1\right) - 2(bc\cosh(dx+c)+bc\sinh(dx+c))\sqrt{\frac{a^2-b^2}{b^2}}\log\left(2b\cosh(dx+c)+2b\sinh(dx+c)+2b\sqrt{\frac{a^2-b^2}{b^2}}+2a\right) + 2(bc\cosh(dx+c)+bc\sinh(dx+c))\sqrt{\frac{a^2-b^2}{b^2}}\log\left(2b\cosh(dx+c)+2b\sinh(dx+c)-2b\sqrt{\frac{a^2-b^2}{b^2}}+2a\right) - 2((bdx+bc)\cosh(dx+c)+(bdx+bc)\sinh(dx+c))\sqrt{\frac{a^2-b^2}{b^2}}\log\left(\frac{a\cosh(dx+c)+a\sinh(dx+c)+(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2-b^2}{b^2}}+b}{b}\right) + 2((bdx+bc)\cosh(dx+c)+(bdx+bc)\sinh(dx+c))\sqrt{\frac{a^2-b^2}{b^2}}\log\left(\frac{a\cosh(dx+c)+a\sinh(dx+c)-(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2-b^2}{b^2}}+b}{b}\right) + (ad^2x^2 - 2(bdx-b)\cosh(dx+c))\sinh(dx+c) + b}{2(b^2d^2\cosh(dx+c) + b^2d^2\sinh(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(a*d^2*x^2*cosh(d*x + c) + b*d*x - (b*d*x - b)*cosh(d*x + c)^2 - (b*d*
x - b)*sinh(d*x + c)^2 - 2*(b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 -
b^2)/b^2)*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*
sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + 2*(b*cosh(d*x + c) + b*s
inh(d*x + c))*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cosh(d*x + c) + a*sinh(d*x +
c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1)
- 2*(b*c*cosh(d*x + c) + b*c*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*log(2*b*c
osh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) + 2*(b*
c*cosh(d*x + c) + b*c*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*log(2*b*cosh(d*x
 + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) - 2*((b*d*x +
b*c)*cosh(d*x + c) + (b*d*x + b*c)*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*log
((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*s
qrt((a^2 - b^2)/b^2) + b)/b) + 2*((b*d*x + b*c)*cosh(d*x + c) + (b*d*x + b*
c)*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*log((a*cosh(d*x + c) + a*sinh(d*x +
 c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) + (
a*d^2*x^2 - 2*(b*d*x - b)*cosh(d*x + c))*sinh(d*x + c) + b)/(b^2*d^2*cosh(d
*x + c) + b^2*d^2*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(d*x+c)**2/(a+b*cosh(d*x+c)),x)
[Out] Integral(x*sinh(c + d*x)**2/(a + b*cosh(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="giac")
[Out] integrate(x*sinh(d*x + c)^2/(b*cosh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,{\sinh(c+d\,x)}^2}{a+b\cosh(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sinh(c + d*x)^2)/(a + b*cosh(c + d*x)),x)
[Out] int((x*sinh(c + d*x)^2)/(a + b*cosh(c + d*x)), x)
```

3.231 $\int \frac{\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal. Leaf size=73

$$-\frac{ax}{b^2}+\frac{2\sqrt{a-b}\,\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d}+\frac{\sinh(c+dx)}{bd}$$

```
[Out] -a*x/b^2+sinh(d*x+c)/b/d+2*arctanh((a-b)^(1/2)*tanh(1/2*d*x+1/2*c)/(a+b)^(1
/2))*(a-b)^(1/2)*(a+b)^(1/2)/b^2/d
```

Rubi [A]
time = 0.09, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2774, 2814, 2738, 211}

$$\frac{2\sqrt{a-b}\,\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d}-\frac{ax}{b^2}+\frac{\sinh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2/(a + b*Cosh[c + d*x]),x]
```

```
[Out] -((a*x)/b^2) + (2*Sqrt[a - b]*Sqrt[a + b]*ArcTanh[(Sqrt[a - b]*Tanh[(c + d*
x)/2])/Sqrt[a + b]])/(b^2*d) + Sinh[c + d*x]/(b*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx &= \frac{\sinh(c+dx)}{bd}+\frac{\int \frac{-b-a\cosh(c+dx)}{a+b\cosh(c+dx)}\,dx}{b}\\
&= -\frac{ax}{b^2}+\frac{\sinh(c+dx)}{bd}-\left(1-\frac{a^2}{b^2}\right)\int \frac{1}{a+b\cosh(c+dx)}\,dx\\
&= -\frac{ax}{b^2}+\frac{\sinh(c+dx)}{bd}+\frac{\left(2i\left(1-\frac{a^2}{b^2}\right)\right)\operatorname{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx,x,\tan\left(\frac{1}{2}(ic+id\right.\right.}{d}\\
&= -\frac{ax}{b^2}+\frac{2\sqrt{a-b}\,\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d}+\frac{\sinh(c+dx)}{bd}
\end{aligned}$$

Mathematica [A]
time = 0.15, size = 69, normalized size = 0.95

$$\frac{-a(c+dx)+2\sqrt{-a^2+b^2}\,\operatorname{ArcTan}\left(\frac{(a-b)\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)+b\sinh(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2/(a + b*Cosh[c + d*x]),x]
```

```
[Out] (-(a*(c + d*x)) + 2*Sqrt[-a^2 + b^2]*ArcTan[((a - b)*Tanh[(c + d*x)/2])/Sqr
t[-a^2 + b^2]] + b*Sinh[c + d*x])/(b^2*d)
```

Maple [A]
time = 1.00, size = 129, normalized size = 1.77

method	result
derivativedivides	$\frac{-\frac{2\left(-a^2+b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^2\sqrt{(a+b)(a-b)}}-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln}{}}{d}$

default	$\frac{-\frac{2\left(-a^2+b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^2\sqrt{(a+b)(a-b)}}-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln\left(\mathrm{t}\cdots\right.}{}}{d}$
risch	$-\frac{ax}{b^2}+\frac{\mathrm{e}^{dx+c}}{2bd}-\frac{\mathrm{e}^{-dx-c}}{2bd}+\frac{\sqrt{a^2-b^2}\,\ln\left(\mathrm{e}^{dx+c}-\frac{-a+\sqrt{a^2-b^2}}{b}\right)}{d\,b^2}-\frac{\sqrt{a^2-b^2}\,\ln\left(\mathrm{e}^{dx+c}+\frac{a+\sqrt{a^2-\cdots}}{b}\right.}{d\,b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/b^2*(-a^2+b^2)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*d*x+1/2*c
)/((a+b)*(a-b))^(1/2))-1/b/(tanh(1/2*d*x+1/2*c)+1)-a/b^2*ln(tanh(1/2*d*x+1/
2*c)+1)-1/b/(tanh(1/2*d*x+1/2*c)-1)+a/b^2*ln(tanh(1/2*d*x+1/2*c)-1))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(64) = 128$.
time = 0.41, size = 415, normalized size = 5.68

$$\left[-\frac{2\,adx\cosh\left(dx+c\right)-b\cosh\left(dx+c\right)^2-b\sinh\left(dx+c\right)^2-2\sqrt{a^2-b^2}\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)\right)\log\left(\frac{b^2\cosh(dx+c)^2+b^2\sinh(dx+c)^2+2\,ab\cosh(dx+c)+2\,a^2-b^2+2\,(b^2\cosh(dx+c)+ab)\sinh(dx+c)-2\sqrt{a^2-b^2}\,(b\cosh(dx+c)+b\sinh(dx+c)+a)}{b\cosh(dx+c)^2+b\sinh(dx+c)^2+2\,a\cosh(dx+c)+2\,(b\cosh(dx+c)+a)\sinh(dx+c)+b}\right)+2\left(adx-b\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)+b}{2\left(b^2d\cosh\left(dx+c\right)+b^2d\sinh\left(dx+c\right)\right)},\,-\frac{2\,adx\cosh\left(dx+c\right)-b\cosh\left(dx+c\right)^2-b\sinh\left(dx+c\right)^2+4\sqrt{-a^2+b^2}\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)\right)\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\cosh(dx+c)+b\sinh(dx+c)+a)}{a^2-b^2}\right)+2\left(adx-b\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)+b}{2\left(b^2d\cosh\left(dx+c\right)+b^2d\sinh\left(dx+c\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*d*x*cosh(d*x + c) - b*cosh(d*x + c)^2 - b*sinh(d*x + c)^2 - 2*sq
rt(a^2 - b^2)*(cosh(d*x + c) + sinh(d*x + c))*log((b^2*cosh(d*x + c)^2 + b^
2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 - b^2 + 2*(b^2*cosh(d*x + c
) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 - b^2)*(b*cosh(d*x + c) + b*sinh(d*x +
c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*
cosh(d*x + c) + a)*sinh(d*x + c) + b)) + 2*(a*d*x - b*cosh(d*x + c))*sinh(d
*x + c) + b)/(b^2*d*cosh(d*x + c) + b^2*d*sinh(d*x + c)), -1/2*(2*a*d*x*cos
h(d*x + c) - b*cosh(d*x + c)^2 - b*sinh(d*x + c)^2 + 4*sqrt(-a^2 + b^2)*(co
```

```
sh(d*x + c) + sinh(d*x + c))*arctan(-sqrt(-a^2 + b^2)*(b*cosh(d*x + c) + b*
sinh(d*x + c) + a)/(a^2 - b^2)) + 2*(a*d*x - b*cosh(d*x + c))*sinh(d*x + c)
 + b)/(b^2*d*cosh(d*x + c) + b^2*d*sinh(d*x + c))]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1122 vs. $2(61) = 122$.

time = 62.61, size = 1122, normalized size = 15.37

$$\begin{cases} \frac{\tilde{\infty} x \sinh^2(c)}{\cosh(c)} & \text{for } a = 0 \wedge b = 0 \wedge d = 0 \\ -\frac{dx\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{bd\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)-bd} + \frac{dx}{bd\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)-bd} - \frac{2\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{bd\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)-bd} & \text{for } a = b \\ \frac{dx\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{bd\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)-bd} - \frac{dx}{bd\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)-bd} - \frac{2\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{bd\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)-bd} & \text{for } a = -b \\ \frac{\frac{x\sinh^2(c+dx)}{2} - \frac{x\cosh^2(c+dx)}{2} + \frac{\sinh(c+dx)\cosh(c+dx)}{2d}}{a} & \text{for } b = 0 \\ \frac{x\sinh^2(c)}{a+b\cosh(c)} & \text{for } d = 0 \\ -\frac{adx\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{b^2d\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)-b^2d\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} + \frac{adx\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}{b^2d\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)-b^2d\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} - \frac{a\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{b^2d\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)-b^2d\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} + \frac{a\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{b^2d\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)-b^2d\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} + \frac{a\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{b^2d\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)-b^2d\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} - \frac{a\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{b^2d\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)-b^2d\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} - \frac{2b\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{b^2d\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)-b^2d\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} - \frac{b\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{b^2d\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)-b^2d\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} + \frac{b\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{b^2d\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)-b^2d\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} + \frac{b\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{b^2d\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)-b^2d\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} - \frac{b\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{b^2d\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)-b^2d\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a+b*cosh(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x*sinh(c)**2/cosh(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (-d*x
*tanh(c/2 + d*x/2)**2/(b*d*tanh(c/2 + d*x/2)**2 - b*d) + d*x/(b*d*tanh(c/2
+ d*x/2)**2 - b*d) - 2*tanh(c/2 + d*x/2)/(b*d*tanh(c/2 + d*x/2)**2 - b*d),
Eq(a, b)), (d*x*tanh(c/2 + d*x/2)**2/(b*d*tanh(c/2 + d*x/2)**2 - b*d) - d*x
/(b*d*tanh(c/2 + d*x/2)**2 - b*d) - 2*tanh(c/2 + d*x/2)/(b*d*tanh(c/2 + d*x
/2)**2 - b*d), Eq(a, -b)), ((x*sinh(c + d*x)**2/2 - x*cosh(c + d*x)**2/2 +
sinh(c + d*x)*cosh(c + d*x)/(2*d))/a, Eq(b, 0)), (x*sinh(c)**2/(a + b*cosh(
c)), Eq(d, 0)), (-a*d*x*sqrt(a/(a - b) + b/(a - b))*tanh(c/2 + d*x/2)**2/(b
**2*d*sqrt(a/(a - b) + b/(a - b))*tanh(c/2 + d*x/2)**2 - b**2*d*sqrt(a/(a -
 b) + b/(a - b))) + a*d*x*sqrt(a/(a - b) + b/(a - b))/(b**2*d*sqrt(a/(a - b
) + b/(a - b))*tanh(c/2 + d*x/2)**2 - b**2*d*sqrt(a/(a - b) + b/(a - b))) -
 a*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(c/2 + d*x/2))*tanh(c/2 + d*x/2)*
*2/(b**2*d*sqrt(a/(a - b) + b/(a - b))*tanh(c/2 + d*x/2)**2 - b**2*d*sqrt(a
/(a - b) + b/(a - b))) + a*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(c/2 + d*
x/2))/(b**2*d*sqrt(a/(a - b) + b/(a - b))*tanh(c/2 + d*x/2)**2 - b**2*d*sqr
t(a/(a - b) + b/(a - b))) + a*log(sqrt(a/(a - b) + b/(a - b)) + tanh(c/2 +
d*x/2))*tanh(c/2 + d*x/2)**2/(b**2*d*sqrt(a/(a - b) + b/(a - b))*tanh(c/2 +
 d*x/2)**2 - b**2*d*sqrt(a/(a - b) + b/(a - b))) - a*log(sqrt(a/(a - b) + b
/(a - b)) + tanh(c/2 + d*x/2))/(b**2*d*sqrt(a/(a - b) + b/(a - b))*tanh(c/2
 + d*x/2)**2 - b**2*d*sqrt(a/(a - b) + b/(a - b))) - 2*b*sqrt(a/(a - b) + b
/(a - b))*tanh(c/2 + d*x/2)/(b**2*d*sqrt(a/(a - b) + b/(a - b))*tanh(c/2 +
d*x/2)**2 - b**2*d*sqrt(a/(a - b) + b/(a - b))) - b*log(-sqrt(a/(a - b) + b
/(a - b)) + tanh(c/2 + d*x/2))*tanh(c/2 + d*x/2)**2/(b**2*d*sqrt(a/(a - b)
+ b/(a - b))*tanh(c/2 + d*x/2)**2 - b**2*d*sqrt(a/(a - b) + b/(a - b))) + b
*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(c/2 + d*x/2))/(b**2*d*sqrt(a/(a -
b) + b/(a - b))*tanh(c/2 + d*x/2)**2 - b**2*d*sqrt(a/(a - b) + b/(a - b)))
+ b*log(sqrt(a/(a - b) + b/(a - b)) + tanh(c/2 + d*x/2))*tanh(c/2 + d*x/2)*
*2/(b**2*d*sqrt(a/(a - b) + b/(a - b))*tanh(c/2 + d*x/2)**2 - b**2*d*sqrt(a
/(a - b) + b/(a - b))) - b*log(sqrt(a/(a - b) + b/(a - b)) + tanh(c/2 + d*x
/2))/(b**2*d*sqrt(a/(a - b) + b/(a - b))*tanh(c/2 + d*x/2)**2 - b**2*d*sqrt
(a/(a - b) + b/(a - b))), True))
```

Giac [A]
time = 0.42, size = 89, normalized size = 1.22

$$-\frac{\frac{2\,(dx+c)a}{b^2}-\frac{e^{(dx+c)}}{b}+\frac{e^{(-dx-c)}}{b}-\frac{4\,(a^2-b^2)\arctan\left(\frac{be^{(dx+c)}+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*(d*x + c)*a/b^2 - e^(d*x + c)/b + e^(-d*x - c)/b - 4*(a^2 - b^2)*ar
ctan((b*e^(d*x + c) + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b^2))/d
```

Mupad [B]
time = 1.12, size = 176, normalized size = 2.41

$$\frac{e^{c+d\,x}}{2\,b\,d}-\frac{e^{-c-d\,x}}{2\,b\,d}-\frac{a\,x}{b^2}+\frac{\ln\left(-\frac{2\,e^{c+d\,x}\,(a^2-b^2)}{b^3}-\frac{2\,\sqrt{a+b}\,\sqrt{a-b}\,(b+a\,e^{c+d\,x})}{b^3}\right)\,\sqrt{a+b}\,\sqrt{a-b}}{b^2\,d}-\frac{\ln\left(\frac{2\,\sqrt{a+b}\,\sqrt{a-b}\,(b+a\,e^{c+d\,x})}{b^3}-\frac{2\,e^{c+d\,x}\,(a^2-b^2)}{b^3}\right)\,\sqrt{a+b}\,\sqrt{a-b}}{b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2/(a + b*cosh(c + d*x)),x)
```

```
[Out] exp(c + d*x)/(2*b*d) - exp(- c - d*x)/(2*b*d) - (a*x)/b^2 + (log(- (2*exp(c
 + d*x)*(a^2 - b^2))/b^3 - (2*(a + b)^(1/2)*(a - b)^(1/2)*(b + a*exp(c + d*
x)))/b^3)*(a + b)^(1/2)*(a - b)^(1/2))/(b^2*d) - (log((2*(a + b)^(1/2)*(a -
 b)^(1/2)*(b + a*exp(c + d*x)))/b^3 - (2*exp(c + d*x)*(a^2 - b^2))/b^3)*(a
+ b)^(1/2)*(a - b)^(1/2))/(b^2*d)
```

3.232 $\int \frac{\sinh^2(c+dx)}{x(a+b\cosh(c+dx))}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{\sinh^2(c+dx)}{x(a+b\cosh(c+dx))},x\right)$$

```
[Out] Unintegrable(sinh(d*x+c)^2/x/(a+b*cosh(d*x+c)),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sinh^2(c+dx)}{x(a+b\cosh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sinh[c + d*x]^2/(x*(a + b*Cosh[c + d*x])),x]
[Out] Defer[Int][Sinh[c + d*x]^2/(x*(a + b*Cosh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sinh^2(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \int \frac{\sinh^2(c+dx)}{x(a+b\cosh(c+dx))}\,dx$$

Mathematica [A]
time = 79.99, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(c+dx)}{x(a+b\cosh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sinh[c + d*x]^2/(x*(a + b*Cosh[c + d*x])),x]
[Out] Integrate[Sinh[c + d*x]^2/(x*(a + b*Cosh[c + d*x])), x]
```

Maple [A]
time = 1.10, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(dx+c)}{x\left(a+b\cosh\left(dx+c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/x/(a+b*cosh(d*x+c)),x)
```

```
[Out] int(sinh(d*x+c)^2/x/(a+b*cosh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/x/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2*(a^2*e^c - b^2*e^c)*integrate(e^(d*x)/(b^3*x*e^(2*d*x + 2*c) + 2*a*b^2*x*
e^(d*x + c) + b^3*x), x) + 1/2*Ei(-d*x)*e^(-c)/b + 1/2*Ei(d*x)*e^c/b - a*lo
g(x)/b^2
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/x/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sinh(d*x + c)^2/(b*x*cosh(d*x + c) + a*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(c+dx)}{x(a+b\cosh(c+dx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/x/(a+b*cosh(d*x+c)),x)
```

```
[Out] Integral(sinh(c + d*x)**2/(x*(a + b*cosh(c + d*x))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/x/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sinh(d*x + c)^2/((b*cosh(d*x + c) + a)*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sinh(c+d\,x)^2}{x\,\left(a+b\cosh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2/(x*(a + b*cosh(c + d*x))),x)
[Out] int(sinh(c + d*x)^2/(x*(a + b*cosh(c + d*x))), x)
```

3.233 $\int \frac{x^m \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m \sinh^3(c+dx)}{a+b\cosh(c+dx)},x\right)$$

```
[Out] Unintegrable(x^m*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]),x]
[Out] Defer[Int][(x^m*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]), x]
```

Rubi steps

$$\int \frac{x^m \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^m \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$$

Mathematica [A]
time = 22.28, size = 0, normalized size = 0.00

$$\int \frac{x^m \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]),x]
[Out] Integrate[(x^m*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]), x]
```

Maple [A]
time = 1.06, size = 0, normalized size = 0.00

$$\int \frac{x^m \left(\sinh^3(dx+c)\right)}{a+b\cosh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x)
[Out] int(x^m*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="maxima")
[Out] integrate(x^m*sinh(d*x + c)^3/(b*cosh(d*x + c) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="fricas")
[Out] integral(x^m*sinh(d*x + c)^3/(b*cosh(d*x + c) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \sinh^3(c+dx)}{a+b\cosh(c+dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sinh(d*x+c)**3/(a+b*cosh(d*x+c)),x)
[Out] Integral(x**m*sinh(c + d*x)**3/(a + b*cosh(c + d*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="giac")
[Out] integrate(x^m*sinh(d*x + c)^3/(b*cosh(d*x + c) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m \sinh(c+d\,x)^3}{a+b\cosh(c+d\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*sinh(c + d*x)^3)/(a + b*cosh(c + d*x)),x)
[Out] int((x^m*sinh(c + d*x)^3)/(a + b*cosh(c + d*x)), x)
```

3.234 $\int \frac{x^3 \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal. Leaf size=586

$$\frac{3x}{8bd^3}+\frac{x^3}{4bd}-\frac{(a^2-b^2)\,x^4}{4b^3}-\frac{6ax\cosh(c+dx)}{b^2d^3}-\frac{ax^3\cosh(c+dx)}{b^2d}+\frac{(a^2-b^2)\,x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d}+\frac{(a^2-}{}$$

```
[Out] 3/8*x/b/d^3+1/4*x^3/b/d-1/4*(a^2-b^2)*x^4/b^3-6*a*x*cosh(d*x+c)/b^2/d^3-a*x
^3*cosh(d*x+c)/b^2/d+(a^2-b^2)*x^3*ln(1+b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b
^3/d+(a^2-b^2)*x^3*ln(1+b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))/b^3/d+3*(a^2-b^2)
*x^2*polylog(2,-b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b^3/d^2+3*(a^2-b^2)*x^2*p
olylog(2,-b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))/b^3/d^2-6*(a^2-b^2)*x*polylog(3
,-b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b^3/d^3-6*(a^2-b^2)*x*polylog(3,-b*exp(
d*x+c)/(a+(a^2-b^2)^(1/2)))/b^3/d^3+6*(a^2-b^2)*polylog(4,-b*exp(d*x+c)/(a-
(a^2-b^2)^(1/2)))/b^3/d^4+6*(a^2-b^2)*polylog(4,-b*exp(d*x+c)/(a+(a^2-b^2)^
(1/2)))/b^3/d^4+6*a*sinh(d*x+c)/b^2/d^4+3*a*x^2*sinh(d*x+c)/b^2/d^2-3/8*cos
h(d*x+c)*sinh(d*x+c)/b/d^4-3/4*x^2*cosh(d*x+c)*sinh(d*x+c)/b/d^2+3/4*x*sinh
(d*x+c)^2/b/d^3+1/2*x^3*sinh(d*x+c)^2/b/d
```

Rubi [A]
time = 0.50, antiderivative size = 586, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 14, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {5685, 3377, 2717, 5480, 3392, 30, 2715, 8, 5681, 2221, 2611, 6744, 2320, 6724}

$$\frac{6(a^2-b^2)\,\text{Li}_4\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^4}+\frac{6(a^2-b^2)\,\text{Li}_4\left(-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^3d^4}-\frac{6x(a^2-b^2)\,\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^3}-\frac{6x(a^2-b^2)\,\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^3d^3}+\frac{3x^2(a^2-b^2)\,\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^2}+\frac{3x^2(a^2-b^2)\,\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^3d^2}+\frac{x^3(a^2-b^2)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{b^3d}+\frac{x^3(a^2-b^2)\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{b^3d}-\frac{x^4(a^2-b^2)}{4b^3}+\frac{6a\sinh(c+dx)}{b^2d^4}-\frac{6ax\cosh(c+dx)}{b^2d^3}+\frac{3ax^2\sinh(c+dx)}{b^2d^2}-\frac{ax^3\cosh(c+dx)}{b^2d}-\frac{3\sinh(c+dx)\cosh(c+dx)}{8bd^4}+\frac{3x\sinh^2(c+dx)}{4bd^3}-\frac{3x^2\sinh(c+dx)\cosh(c+dx)}{4bd^2}+\frac{x^3\sinh^2(c+dx)}{2bd}+\frac{3x}{8bd^3}+\frac{x^3}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]),x]
```

```
[Out] (3*x)/(8*b*d^3) + x^3/(4*b*d) - ((a^2 - b^2)*x^4)/(4*b^3) - (6*a*x*Cosh[c +
 d*x])/(b^2*d^3) - (a*x^3*Cosh[c + d*x])/(b^2*d) + ((a^2 - b^2)*x^3*Log[1 +
 (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b^3*d) + ((a^2 - b^2)*x^3*Log[1 +
 (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b^3*d) + (3*(a^2 - b^2)*x^2*PolyL
og[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b^3*d^2) + (3*(a^2 - b^2)
*x^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b^3*d^2) - (6*(
a^2 - b^2)*x*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b^3*d^3
) - (6*(a^2 - b^2)*x*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/
(b^3*d^3) + (6*(a^2 - b^2)*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2
]))])/(b^3*d^4) + (6*(a^2 - b^2)*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2
 - b^2]))])/(b^3*d^4) + (6*a*Sinh[c + d*x])/(b^2*d^4) + (3*a*x^2*Sinh[c + d
*x])/(b^2*d^2) - (3*Cosh[c + d*x]*Sinh[c + d*x])/(8*b*d^4) - (3*x^2*Cosh[c
+ d*x]*Sinh[c + d*x])/(4*b*d^2) + (3*x*Sinh[c + d*x]^2)/(4*b*d^3) + (x^3*Si
nh[c + d*x]^2)/(2*b*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 5480

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5685

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_))/(Cosh[(c_.)
+ (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Sinh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Sinh[c + d*x]^(n - 2
)*Cosh[c + d*x], x], x] + Dist[(a^2 - b^2)/b^2, Int[(e + f*x)^m*(Sinh[c + d
*x]^(n - 2)/(a + b*Cosh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
```

```
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3 \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{a\int x^3\sinh(c+dx)\,dx}{b^2} + \frac{\int x^3\cosh(c+dx)\sinh(c+dx)\,dx}{b} + \frac{(a^2-b^2)\int \frac{x^3\sinh}{a+b\cos}}{b^2}$$

$$= -\frac{(a^2-b^2)\,x^4}{4b^3} - \frac{ax^3\cosh(c+dx)}{b^2d} + \frac{x^3\sinh^2(c+dx)}{2bd} + \frac{(a^2-b^2)\int \frac{e^{c+dx}x^3}{a-\sqrt{a^2-b^2}}}{b^2}$$

$$= -\frac{(a^2-b^2)\,x^4}{4b^3} - \frac{ax^3\cosh(c+dx)}{b^2d} + \frac{(a^2-b^2)\,x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d} + \frac{(a^2-}{}$$

$$= \frac{x^3}{4bd} - \frac{(a^2-b^2)\,x^4}{4b^3} - \frac{6ax\cosh(c+dx)}{b^2d^3} - \frac{ax^3\cosh(c+dx)}{b^2d} + \frac{(a^2-b^2)\,x^3\log\left(1\right.}{b^3}$$

$$= \frac{3x}{8bd^3} + \frac{x^3}{4bd} - \frac{(a^2-b^2)\,x^4}{4b^3} - \frac{6ax\cosh(c+dx)}{b^2d^3} - \frac{ax^3\cosh(c+dx)}{b^2d} + \frac{(a^2-b^2)\,x}{}$$

$$= \frac{3x}{8bd^3} + \frac{x^3}{4bd} - \frac{(a^2-b^2)\,x^4}{4b^3} - \frac{6ax\cosh(c+dx)}{b^2d^3} - \frac{ax^3\cosh(c+dx)}{b^2d} + \frac{(a^2-b^2)\,x}{}$$

$$= \frac{3x}{8bd^3} + \frac{x^3}{4bd} - \frac{(a^2-b^2)\,x^4}{4b^3} - \frac{6ax\cosh(c+dx)}{b^2d^3} - \frac{ax^3\cosh(c+dx)}{b^2d} + \frac{(a^2-b^2)\,x}{}$$

Mathematica [A]
time = 2.69, size = 935, normalized size = 1.60

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]),x]
```

```
[Out] ((8*(-a^2 + b^2)*(d^4*E^(2*c)*x^4 - 2*d^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*
E^c - Sqrt[(a^2 - b^2)*E^(2*c)])] - 2*d^3*E^(2*c)*x^3*Log[1 + (b*E^(2*c + d
*x))/(a*E^c - Sqrt[(a^2 - b^2)*E^(2*c)])] - 2*d^3*x^3*Log[1 + (b*E^(2*c + d
*x))/(a*E^c + Sqrt[(a^2 - b^2)*E^(2*c)])] - 2*d^3*E^(2*c)*x^3*Log[1 + (b*E^
(2*c + d*x))/(a*E^c + Sqrt[(a^2 - b^2)*E^(2*c)])] - 6*d^2*(1 + E^(2*c))*x^2
```

```
*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 - b^2)*E^(2*c)]))] - 6*d
^2*(1 + E^(2*c))*x^2*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 - b^
2)*E^(2*c)]))] + 12*d*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 -
 b^2)*E^(2*c)]))] + 12*d*E^(2*c)*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c -
Sqrt[(a^2 - b^2)*E^(2*c)]))] + 12*d*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c
 + Sqrt[(a^2 - b^2)*E^(2*c)]))] + 12*d*E^(2*c)*x*PolyLog[3, -((b*E^(2*c + d
*x))/(a*E^c + Sqrt[(a^2 - b^2)*E^(2*c)]))] - 12*PolyLog[4, -((b*E^(2*c + d*
x))/(a*E^c - Sqrt[(a^2 - b^2)*E^(2*c)]))] - 12*E^(2*c)*PolyLog[4, -((b*E^(2
*c + d*x))/(a*E^c - Sqrt[(a^2 - b^2)*E^(2*c)]))] - 12*PolyLog[4, -((b*E^(2*
c + d*x))/(a*E^c + Sqrt[(a^2 - b^2)*E^(2*c)]))] - 12*E^(2*c)*PolyLog[4, -((
b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 - b^2)*E^(2*c)]))]))/(d^4*(1 + E^(2*c))
) - (16*a*b*Cosh[d*x]*(d*x*(6 + d^2*x^2)*Cosh[c] - 3*(2 + d^2*x^2)*Sinh[c])
)/d^4 + (b^2*Cosh[2*d*x]*(2*d*x*(3 + 2*d^2*x^2)*Cosh[2*c] - 3*(1 + 2*d^2*x^
2)*Sinh[2*c]))/d^4 - (16*a*b*(-3*(2 + d^2*x^2)*Cosh[c] + d*x*(6 + d^2*x^2)*
Sinh[c])*Sinh[d*x])/d^4 + (b^2*(-3*(1 + 2*d^2*x^2)*Cosh[2*c] + 2*d*x*(3 + 2
*d^2*x^2)*Sinh[2*c])*Sinh[2*d*x])/d^4 + 4*(a - b)*(a + b)*x^4*Tanh[c])/(16*
b^3)
```

Maple [F]
time = 0.81, size = 0, normalized size = 0.00

$$\int \frac{x^3 \left(\sinh^3 \left(dx+c\right)\right)}{a+b\cosh \left(dx+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x)
[Out] int(x^3*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="maxima")
[Out] 1/32*(8*(a^2*d^4*e^(2*c) - b^2*d^4*e^(2*c))*x^4 + (4*b^2*d^3*x^3*e^(4*c) -
6*b^2*d^2*x^2*e^(4*c) + 6*b^2*d*x*e^(4*c) - 3*b^2*e^(4*c))*e^(2*d*x) - 16*(
a*b*d^3*x^3*e^(3*c) - 3*a*b*d^2*x^2*e^(3*c) + 6*a*b*d*x*e^(3*c) - 6*a*b*e^(
3*c))*e^(d*x) - 16*(a*b*d^3*x^3*e^c + 3*a*b*d^2*x^2*e^c + 6*a*b*d*x*e^c + 6
*a*b*e^c)*e^(-d*x) + (4*b^2*d^3*x^3 + 6*b^2*d^2*x^2 + 6*b^2*d*x + 3*b^2)*e^
(-2*d*x))*e^(-2*c)/(b^3*d^4) - 1/8*integrate(16*((a^3*e^c - a*b^2*e^c)*x^3*
e^(d*x) + (a^2*b - b^3)*x^3)/(b^4*e^(2*d*x + 2*c) + 2*a*b^3*e^(d*x + c) + b
^4), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2025 vs. $2(546) = 1092$.
time = 0.48, size = 2025, normalized size = 3.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/32*(4*b^2*d^3*x^3 + 6*b^2*d^2*x^2 + (4*b^2*d^3*x^3 - 6*b^2*d^2*x^2 + 6*b^
2*d*x - 3*b^2)*cosh(d*x + c)^4 + (4*b^2*d^3*x^3 - 6*b^2*d^2*x^2 + 6*b^2*d*x
 - 3*b^2)*sinh(d*x + c)^4 + 6*b^2*d*x - 16*(a*b*d^3*x^3 - 3*a*b*d^2*x^2 + 6
*a*b*d*x - 6*a*b)*cosh(d*x + c)^3 - 4*(4*a*b*d^3*x^3 - 12*a*b*d^2*x^2 + 24*
a*b*d*x - 24*a*b - (4*b^2*d^3*x^3 - 6*b^2*d^2*x^2 + 6*b^2*d*x - 3*b^2)*cosh
(d*x + c))*sinh(d*x + c)^3 - 8*((a^2 - b^2)*d^4*x^4 - 2*(a^2 - b^2)*c^4)*co
sh(d*x + c)^2 - 2*(4*(a^2 - b^2)*d^4*x^4 - 8*(a^2 - b^2)*c^4 - 3*(4*b^2*d^3
*x^3 - 6*b^2*d^2*x^2 + 6*b^2*d*x - 3*b^2)*cosh(d*x + c)^2 + 24*(a*b*d^3*x^3
 - 3*a*b*d^2*x^2 + 6*a*b*d*x - 6*a*b)*cosh(d*x + c))*sinh(d*x + c)^2 + 3*b^
2 - 16*(a*b*d^3*x^3 + 3*a*b*d^2*x^2 + 6*a*b*d*x + 6*a*b)*cosh(d*x + c) + 96
*((a^2 - b^2)*d^2*x^2*cosh(d*x + c)^2 + 2*(a^2 - b^2)*d^2*x^2*cosh(d*x + c)
*sinh(d*x + c) + (a^2 - b^2)*d^2*x^2*sinh(d*x + c)^2)*dilog(-(a*cosh(d*x +
c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)
/b^2) + b)/b + 1) + 96*((a^2 - b^2)*d^2*x^2*cosh(d*x + c)^2 + 2*(a^2 - b^2)
*d^2*x^2*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*d^2*x^2*sinh(d*x + c)^2)
*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x
+ c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - 32*((a^2 - b^2)*c^3*cosh(d*x + c)
^2 + 2*(a^2 - b^2)*c^3*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*c^3*sinh(d
*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 - b^2)
/b^2) + 2*a) - 32*((a^2 - b^2)*c^3*cosh(d*x + c)^2 + 2*(a^2 - b^2)*c^3*cosh
(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*c^3*sinh(d*x + c)^2)*log(2*b*cosh(d*x
 + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) + 32*(((a^2 -
b^2)*d^3*x^3 + (a^2 - b^2)*c^3)*cosh(d*x + c)^2 + 2*((a^2 - b^2)*d^3*x^3 +
(a^2 - b^2)*c^3)*cosh(d*x + c)*sinh(d*x + c) + ((a^2 - b^2)*d^3*x^3 + (a^2
- b^2)*c^3)*sinh(d*x + c)^2)*log((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*co
sh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) + 32*(((a^2 -
b^2)*d^3*x^3 + (a^2 - b^2)*c^3)*cosh(d*x + c)^2 + 2*((a^2 - b^2)*d^3*x^3 +
(a^2 - b^2)*c^3)*cosh(d*x + c)*sinh(d*x + c) + ((a^2 - b^2)*d^3*x^3 + (a^2
- b^2)*c^3)*sinh(d*x + c)^2)*log((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*co
sh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) + 192*((a^2 -
b^2)*cosh(d*x + c)^2 + 2*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b
^2)*sinh(d*x + c)^2)*polylog(4, -(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*co
sh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2))/b) + 192*((a^2 - b^2)
*cosh(d*x + c)^2 + 2*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*
sinh(d*x + c)^2)*polylog(4, -(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d
*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2))/b) - 192*((a^2 - b^2)*d*x
```

```
*cosh(d*x + c)^2 + 2*(a^2 - b^2)*d*x*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b
^2)*d*x*sinh(d*x + c)^2)*polylog(3, -(a*cosh(d*x + c) + a*sinh(d*x + c) + (
b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2))/b) - 192*((a^2 -
b^2)*d*x*cosh(d*x + c)^2 + 2*(a^2 - b^2)*d*x*cosh(d*x + c)*sinh(d*x + c) +
(a^2 - b^2)*d*x*sinh(d*x + c)^2)*polylog(3, -(a*cosh(d*x + c) + a*sinh(d*x
+ c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2))/b) - 4*(4
*a*b*d^3*x^3 + 12*a*b*d^2*x^2 + 24*a*b*d*x - (4*b^2*d^3*x^3 - 6*b^2*d^2*x^2
 + 6*b^2*d*x - 3*b^2)*cosh(d*x + c)^3 + 12*(a*b*d^3*x^3 - 3*a*b*d^2*x^2 + 6
*a*b*d*x - 6*a*b)*cosh(d*x + c)^2 + 24*a*b + 4*((a^2 - b^2)*d^4*x^4 - 2*(a^
2 - b^2)*c^4)*cosh(d*x + c))*sinh(d*x + c))/(b^3*d^4*cosh(d*x + c)^2 + 2*b^
3*d^4*cosh(d*x + c)*sinh(d*x + c) + b^3*d^4*sinh(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sinh(d*x+c)**3/(a+b*cosh(d*x+c)),x)
[Out] Integral(x**3*sinh(c + d*x)**3/(a + b*cosh(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="giac")
[Out] integrate(x^3*sinh(d*x + c)^3/(b*cosh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\sinh(c+d\,x)}^3}{a+b\cosh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*sinh(c + d*x)^3)/(a + b*cosh(c + d*x)),x)
[Out] int((x^3*sinh(c + d*x)^3)/(a + b*cosh(c + d*x)), x)
```

3.235 $\int \frac{x^2 \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal. Leaf size=432

$$\frac{x^2}{4bd}-\frac{(a^2-b^2)\,x^3}{3b^3}-\frac{2a\cosh(c+dx)}{b^2d^3}-\frac{ax^2\cosh(c+dx)}{b^2d}+\frac{(a^2-b^2)\,x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d}+\frac{(a^2-b^2)\,x^2\,\mathrm{lo}}{}$$

```
[Out] 1/4*x^2/b/d-1/3*(a^2-b^2)*x^3/b^3-2*a*cosh(d*x+c)/b^2/d^3-a*x^2*cosh(d*x+c)
/b^2/d+(a^2-b^2)*x^2*ln(1+b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b^3/d+(a^2-b^2)
*x^2*ln(1+b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))/b^3/d+2*(a^2-b^2)*x*polylog(2,-
b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b^3/d^2+2*(a^2-b^2)*x*polylog(2,-b*exp(d*
x+c)/(a+(a^2-b^2)^(1/2)))/b^3/d^2-2*(a^2-b^2)*polylog(3,-b*exp(d*x+c)/(a-(a
^2-b^2)^(1/2)))/b^3/d^3-2*(a^2-b^2)*polylog(3,-b*exp(d*x+c)/(a+(a^2-b^2)^(1
/2)))/b^3/d^3+2*a*x*sinh(d*x+c)/b^2/d^2-1/2*x*cosh(d*x+c)*sinh(d*x+c)/b/d^2
+1/4*sinh(d*x+c)^2/b/d^3+1/2*x^2*sinh(d*x+c)^2/b/d
```

Rubi [A]
time = 0.40, antiderivative size = 432, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {5685, 3377, 2718, 5480, 3391, 30, 5681, 2221, 2611, 2320, 6724}

$$-\frac{2(a^2-b^2)\,\mathrm{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^3}-\frac{2(a^2-b^2)\,\mathrm{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^3d^3}+\frac{2x(a^2-b^2)\,\mathrm{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^2}+\frac{2x(a^2-b^2)\,\mathrm{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^3d^2}+\frac{x^2(a^2-b^2)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{b^3d}+\frac{x^2(a^2-b^2)\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{b^3d}-\frac{x^3(a^2-b^2)}{3b^3}-\frac{2a\cosh(c+dx)}{b^2d^3}+\frac{2ax\sinh(c+dx)}{b^2d^2}-\frac{ax^2\cosh(c+dx)}{b^2d}+\frac{\sinh^2(c+dx)}{4bd^3}-\frac{x\sinh(c+dx)\cosh(c+dx)}{2bd^2}+\frac{x^2\sinh^2(c+dx)}{2bd}+\frac{x^2}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]),x]
```

```
[Out] x^2/(4*b*d) - ((a^2 - b^2)*x^3)/(3*b^3) - (2*a*Cosh[c + d*x])/(b^2*d^3) - (
a*x^2*Cosh[c + d*x])/(b^2*d) + ((a^2 - b^2)*x^2*Log[1 + (b*E^(c + d*x))/(a
- Sqrt[a^2 - b^2])])/(b^3*d) + ((a^2 - b^2)*x^2*Log[1 + (b*E^(c + d*x))/(a
+ Sqrt[a^2 - b^2])])/(b^3*d) + (2*(a^2 - b^2)*x*PolyLog[2, -((b*E^(c + d*x)
)/(a - Sqrt[a^2 - b^2]))])/(b^3*d^2) + (2*(a^2 - b^2)*x*PolyLog[2, -((b*E^(
c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b^3*d^2) - (2*(a^2 - b^2)*PolyLog[3, -(
(b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b^3*d^3) - (2*(a^2 - b^2)*PolyLog
[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b^3*d^3) + (2*a*x*Sinh[c +
d*x])/(b^2*d^2) - (x*Cosh[c + d*x]*Sinh[c + d*x])/(2*b*d^2) + Sinh[c + d*x]
^2/(4*b*d^3) + (x^2*Sinh[c + d*x]^2)/(2*b*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5480

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5685

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_))/(Cosh[(c_.)
+ (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Sinh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Sinh[c + d*x]^(n - 2
)*Cosh[c + d*x], x], x] + Dist[(a^2 - b^2)/b^2, Int[(e + f*x)^m*(Sinh[c + d
*x]^(n - 2)/(a + b*Cosh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2 \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{a\int x^2\sinh(c+dx)\,dx}{b^2} + \frac{\int x^2\cosh(c+dx)\sinh(c+dx)\,dx}{b} + \frac{(a^2-b^2)\int \frac{x^2 \sinh}{a+b\cos}}{b^2}$$

$$= -\frac{(a^2-b^2)\,x^3}{3b^3} - \frac{ax^2\cosh(c+dx)}{b^2d} + \frac{x^2\sinh^2(c+dx)}{2bd} + \frac{(a^2-b^2)\int \frac{e^{c+dx}x^2}{a-\sqrt{a^2-b^2}}}{b^2}$$

$$= -\frac{(a^2-b^2)\,x^3}{3b^3} - \frac{ax^2\cosh(c+dx)}{b^2d} + \frac{(a^2-b^2)\,x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d} + \frac{(a^2}{}$$

$$= \frac{x^2}{4bd} - \frac{(a^2-b^2)\,x^3}{3b^3} - \frac{2a\cosh(c+dx)}{b^2d^3} - \frac{ax^2\cosh(c+dx)}{b^2d} + \frac{(a^2-b^2)\,x^2\log\left(1\right.}{b^3d}$$

$$= \frac{x^2}{4bd} - \frac{(a^2-b^2)\,x^3}{3b^3} - \frac{2a\cosh(c+dx)}{b^2d^3} - \frac{ax^2\cosh(c+dx)}{b^2d} + \frac{(a^2-b^2)\,x^2\log\left(1\right.}{b^3d}$$

$$= \frac{x^2}{4bd} - \frac{(a^2-b^2)\,x^3}{3b^3} - \frac{2a\cosh(c+dx)}{b^2d^3} - \frac{ax^2\cosh(c+dx)}{b^2d} + \frac{(a^2-b^2)\,x^2\log\left(1\right.}{b^3d}$$

Mathematica [A]
time = 2.19, size = 697, normalized size = 1.61

$$\frac{\frac{8(-a^2+b^2)\left(2d^3e^{2c}x^3-3d^2x^2\log\left(1+\frac{be^{2c+dx}}{ae^c-\sqrt{(a^2-b^2)e^{2c}}}\right)-3d^2e^{2c}x^2\log\left(1+\frac{be^{2c+dx}}{ae^c-\sqrt{(a^2-b^2)e^{2c}}}\right)-3d^2x^2\log\left(1+\frac{be^{2c+dx}}{ae^c+\sqrt{(a^2-b^2)e^{2c}}}\right)-3d^2e^{2c}x^2\log\left(1+\frac{be^{2c+dx}}{ae^c+\sqrt{(a^2-b^2)e^{2c}}}\right)-6d(1+e^{2c})x\,\mathrm{PolyLog}\left(2,-\frac{be^{2c+dx}}{ae^c-\sqrt{(a^2-b^2)e^{2c}}}\right)-6d(1+e^{2c})x\,\mathrm{PolyLog}\left(2,-\frac{be^{2c+dx}}{ae^c+\sqrt{(a^2-b^2)e^{2c}}}\right)+6\mathrm{PolyLog}\left(3,-\frac{be^{2c+dx}}{ae^c-\sqrt{(a^2-b^2)e^{2c}}}\right)+6e^{2c}\mathrm{PolyLog}\left(3,-\frac{be^{2c+dx}}{ae^c-\sqrt{(a^2-b^2)e^{2c}}}\right)+6\mathrm{PolyLog}\left(3,-\frac{be^{2c+dx}}{ae^c+\sqrt{(a^2-b^2)e^{2c}}}\right)+6e^{2c}\mathrm{PolyLog}\left(3,-\frac{be^{2c+dx}}{ae^c+\sqrt{(a^2-b^2)e^{2c}}}\right)\right)}{d^3(1+e^{2c})}-\frac{24ab\cosh(dx)((2+d^2x^2)\cosh(c)-2dx\sinh(c))}{d^3}+\frac{3b^2\cosh(2dx)((1+2d^2x^2)\cosh(2c)-2dx\sinh(2c))}{d^3}-\frac{24ab(-2dx\cosh(c)+(2+d^2x^2)\sinh(c))\sinh(dx)}{d^3}+\frac{3b^2(-2dx\cosh(2c)+(1+2d^2x^2)\sinh(2c))\sinh(2dx)}{d^3}+8(a^2-b^2)x^3\tanh(c)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]),x]
```

```
[Out] ((8*(-a^2 + b^2)*(2*d^3*E^(2*c)*x^3 - 3*d^2*x^2*Log[1 + (b*E^(2*c + d*x))/(
a*E^c - Sqrt[(a^2 - b^2)*E^(2*c)])] - 3*d^2*E^(2*c)*x^2*Log[1 + (b*E^(2*c +
 d*x))/(a*E^c - Sqrt[(a^2 - b^2)*E^(2*c)])] - 3*d^2*x^2*Log[1 + (b*E^(2*c +
 d*x))/(a*E^c + Sqrt[(a^2 - b^2)*E^(2*c)])] - 3*d^2*E^(2*c)*x^2*Log[1 + (b*
E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 - b^2)*E^(2*c)])] - 6*d*(1 + E^(2*c))*x*P
olyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 - b^2)*E^(2*c)]))] - 6*d*(
1 + E^(2*c))*x*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 - b^2)*E^(
2*c)]))] + 6*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 - b^2)*E^(2*
c)]))] + 6*E^(2*c)*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 - b^2)
*E^(2*c)]))] + 6*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 - b^2)*E
^(2*c)]))] + 6*E^(2*c)*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 -
b^2)*E^(2*c)]))]))/(d^3*(1 + E^(2*c))) - (24*a*b*Cosh[d*x]*((2 + d^2*x^2)*C
osh[c] - 2*d*x*Sinh[c]))/d^3 + (3*b^2*Cosh[2*d*x]*((1 + 2*d^2*x^2)*Cosh[2*c
] - 2*d*x*Sinh[2*c]))/d^3 - (24*a*b*(-2*d*x*Cosh[c] + (2 + d^2*x^2)*Sinh[c]
)*Sinh[d*x])/d^3 + (3*b^2*(-2*d*x*Cosh[2*c] + (1 + 2*d^2*x^2)*Sinh[2*c])*Si
nh[2*d*x])/d^3 + 8*(a^2 - b^2)*x^3*Tanh[c])/(24*b^3)
```

Maple [F]
time = 0.79, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(\sinh^3\left(dx+c\right)\right)}{a+b\cosh\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x)
```

```
[Out] int(x^2*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/48*(16*(a^2*d^3*e^(2*c) - b^2*d^3*e^(2*c))*x^3 + 3*(2*b^2*d^2*x^2*e^(4*c)
 - 2*b^2*d*x*e^(4*c) + b^2*e^(4*c))*e^(2*d*x) - 24*(a*b*d^2*x^2*e^(3*c) - 2
```

```
*a*b*d*x*e^(3*c) + 2*a*b*e^(3*c))*e^(d*x) - 24*(a*b*d^2*x^2*e^c + 2*a*b*d*x
*e^c + 2*a*b*e^c)*e^(-d*x) + 3*(2*b^2*d^2*x^2 + 2*b^2*d*x + b^2)*e^(-2*d*x)
)*e^(-2*c)/(b^3*d^3) - 1/8*integrate(16*((a^3*e^c - a*b^2*e^c)*x^2*e^(d*x)
+ (a^2*b - b^3)*x^2)/(b^4*e^(2*d*x + 2*c) + 2*a*b^3*e^(d*x + c) + b^4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1622 vs. $2(402) = 804$.
time = 0.50, size = 1622, normalized size = 3.75

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/48*(6*b^2*d^2*x^2 + 3*(2*b^2*d^2*x^2 - 2*b^2*d*x + b^2)*cosh(d*x + c)^4 +
 3*(2*b^2*d^2*x^2 - 2*b^2*d*x + b^2)*sinh(d*x + c)^4 + 6*b^2*d*x - 24*(a*b*
d^2*x^2 - 2*a*b*d*x + 2*a*b)*cosh(d*x + c)^3 - 12*(2*a*b*d^2*x^2 - 4*a*b*d*
x + 4*a*b - (2*b^2*d^2*x^2 - 2*b^2*d*x + b^2)*cosh(d*x + c))*sinh(d*x + c)^
3 - 16*((a^2 - b^2)*d^3*x^3 + 2*(a^2 - b^2)*c^3)*cosh(d*x + c)^2 - 2*(8*(a^
2 - b^2)*d^3*x^3 + 16*(a^2 - b^2)*c^3 - 9*(2*b^2*d^2*x^2 - 2*b^2*d*x + b^2)
*cosh(d*x + c)^2 + 36*(a*b*d^2*x^2 - 2*a*b*d*x + 2*a*b)*cosh(d*x + c))*sinh
(d*x + c)^2 + 3*b^2 - 24*(a*b*d^2*x^2 + 2*a*b*d*x + 2*a*b)*cosh(d*x + c) +
96*((a^2 - b^2)*d*x*cosh(d*x + c)^2 + 2*(a^2 - b^2)*d*x*cosh(d*x + c)*sinh(
d*x + c) + (a^2 - b^2)*d*x*sinh(d*x + c)^2)*dilog(-(a*cosh(d*x + c) + a*sin
h(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)
/b + 1) + 96*((a^2 - b^2)*d*x*cosh(d*x + c)^2 + 2*(a^2 - b^2)*d*x*cosh(d*x
+ c)*sinh(d*x + c) + (a^2 - b^2)*d*x*sinh(d*x + c)^2)*dilog(-(a*cosh(d*x +
c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)
/b^2) + b)/b + 1) + 48*((a^2 - b^2)*c^2*cosh(d*x + c)^2 + 2*(a^2 - b^2)*c^2
*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*c^2*sinh(d*x + c)^2)*log(2*b*cos
h(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) + 48*((a^
2 - b^2)*c^2*cosh(d*x + c)^2 + 2*(a^2 - b^2)*c^2*cosh(d*x + c)*sinh(d*x + c
) + (a^2 - b^2)*c^2*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x +
 c) - 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) + 48*(((a^2 - b^2)*d^2*x^2 - (a^2 -
b^2)*c^2)*cosh(d*x + c)^2 + 2*((a^2 - b^2)*d^2*x^2 - (a^2 - b^2)*c^2)*cosh(
d*x + c)*sinh(d*x + c) + ((a^2 - b^2)*d^2*x^2 - (a^2 - b^2)*c^2)*sinh(d*x +
 c)^2)*log((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d
*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) + 48*(((a^2 - b^2)*d^2*x^2 - (a^2 -
b^2)*c^2)*cosh(d*x + c)^2 + 2*((a^2 - b^2)*d^2*x^2 - (a^2 - b^2)*c^2)*cosh(
d*x + c)*sinh(d*x + c) + ((a^2 - b^2)*d^2*x^2 - (a^2 - b^2)*c^2)*sinh(d*x +
 c)^2)*log((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d
*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) - 96*((a^2 - b^2)*cosh(d*x + c)^2 +
2*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*sinh(d*x + c)^2)*po
lylog(3, -(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*
x + c))*sqrt((a^2 - b^2)/b^2))/b) - 96*((a^2 - b^2)*cosh(d*x + c)^2 + 2*(a^
```

```
2 - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*sinh(d*x + c)^2)*polylog
(3, -(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c
))*sqrt((a^2 - b^2)/b^2))/b) - 4*(6*a*b*d^2*x^2 + 12*a*b*d*x - 3*(2*b^2*d^2
*x^2 - 2*b^2*d*x + b^2)*cosh(d*x + c)^3 + 18*(a*b*d^2*x^2 - 2*a*b*d*x + 2*a
*b)*cosh(d*x + c)^2 + 12*a*b + 8*((a^2 - b^2)*d^3*x^3 + 2*(a^2 - b^2)*c^3)*
cosh(d*x + c))*sinh(d*x + c))/(b^3*d^3*cosh(d*x + c)^2 + 2*b^3*d^3*cosh(d*x
 + c)*sinh(d*x + c) + b^3*d^3*sinh(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sinh(d*x+c)**3/(a+b*cosh(d*x+c)),x)
[Out] Integral(x**2*sinh(c + d*x)**3/(a + b*cosh(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="giac")
[Out] integrate(x^2*sinh(d*x + c)^3/(b*cosh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\sinh\left(c+d\,x\right)}^3}{a+b\,\cosh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*sinh(c + d*x)^3)/(a + b*cosh(c + d*x)),x)
[Out] int((x^2*sinh(c + d*x)^3)/(a + b*cosh(c + d*x)), x)
```

3.236 $\int \frac{x \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal. Leaf size=288

$$\frac{x}{4bd}-\frac{(a^2-b^2)\,x^2}{2b^3}-\frac{ax\cosh(c+dx)}{b^2d}+\frac{(a^2-b^2)\,x\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d}+\frac{(a^2-b^2)\,x\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^3d}$$

```
[Out] 1/4*x/b/d-1/2*(a^2-b^2)*x^2/b^3-a*x*cosh(d*x+c)/b^2/d+(a^2-b^2)*x*ln(1+b*ex
p(d*x+c)/(a-(a^2-b^2)^(1/2)))/b^3/d+(a^2-b^2)*x*ln(1+b*exp(d*x+c)/(a+(a^2-b
^2)^(1/2)))/b^3/d+(a^2-b^2)*polylog(2,-b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b^
3/d^2+(a^2-b^2)*polylog(2,-b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))/b^3/d^2+a*sinh
(d*x+c)/b^2/d^2-1/4*cosh(d*x+c)*sinh(d*x+c)/b/d^2+1/2*x*sinh(d*x+c)^2/b/d
```

Rubi [A]
time = 0.24, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {5685, 3377, 2717, 5480, 2715, 8, 5681, 2221, 2317, 2438}

$$\frac{(a^2-b^2)\,\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^2}+\frac{(a^2-b^2)\,\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^3d^2}+\frac{x(a^2-b^2)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{b^3d}+\frac{x(a^2-b^2)\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{b^3d}-\frac{x^2(a^2-b^2)}{2b^3}+\frac{a\sinh(c+dx)}{b^2d^2}-\frac{ax\cosh(c+dx)}{b^2d}-\frac{\sinh(c+dx)\cosh(c+dx)}{4bd^2}+\frac{x\sinh^2(c+dx)}{2bd}+\frac{x}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]),x]
```

```
[Out] x/(4*b*d) - ((a^2 - b^2)*x^2)/(2*b^3) - (a*x*Cosh[c + d*x])/(b^2*d) + ((a^2
 - b^2)*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b^3*d) + ((a^2 -
 b^2)*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b^3*d) + ((a^2 - b
^2)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b^3*d^2) + ((a^2
 - b^2)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b^3*d^2) + (
a*Sinh[c + d*x])/(b^2*d^2) - (Cosh[c + d*x]*Sinh[c + d*x])/(4*b*d^2) + (x*S
inh[c + d*x]^2)/(2*b*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5480

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5685

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_))/(Cosh[(c_.)
+ (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Sinh
```

```
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Sinh[c + d*x]^(n - 2
)*Cosh[c + d*x], x], x] + Dist[(a^2 - b^2)/b^2, Int[(e + f*x)^m*(Sinh[c + d
*x]^(n - 2)/(a + b*Cosh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx &= -\frac{a\int x\sinh(c+dx)\,dx}{b^2}+\frac{\int x\cosh(c+dx)\sinh(c+dx)\,dx}{b}+\frac{(a^2-b^2)\int \frac{x\sinh(c\cdots}{a+b\cosh(\cdots}}{b^2}\\
&= -\frac{(a^2-b^2)\,x^2}{2b^3}-\frac{ax\cosh(c+dx)}{b^2d}+\frac{x\sinh^2(c+dx)}{2bd}+\frac{(a^2-b^2)\int \frac{e^{c+dx}x}{a-\sqrt{a^2-b^2}+b\cdots}}{b^2}\\
&= -\frac{(a^2-b^2)\,x^2}{2b^3}-\frac{ax\cosh(c+dx)}{b^2d}+\frac{(a^2-b^2)\,x\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d}+\frac{(a^2-\cdots}{\cdots}\\
&= \frac{x}{4bd}-\frac{(a^2-b^2)\,x^2}{2b^3}-\frac{ax\cosh(c+dx)}{b^2d}+\frac{(a^2-b^2)\,x\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d}+\frac{(a\cdots}{\cdots}\\
&= \frac{x}{4bd}-\frac{(a^2-b^2)\,x^2}{2b^3}-\frac{ax\cosh(c+dx)}{b^2d}+\frac{(a^2-b^2)\,x\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d}+\frac{(a\cdots}{\cdots}
\end{aligned}$$

Mathematica [A]
time = 1.93, size = 414, normalized size = 1.44

$$\frac{-8abdx\cosh(c+dx)+2b^2dx\cosh(2(c+dx))+4(a^2-b^2)\left(2c(c+dx)-(c+dx)^2+\frac{4a\sqrt{-(a^2-b^2)^2}\,c\,\text{ArcTan}\left(\frac{a+b\cosh(c+dx)+b\sinh(c+dx)}{\sqrt{-a^2+b^2}}\right)}{(a^2-b^2)^{3/2}}+\frac{4a\sqrt{-(a^2-b^2)^2}\,c\tanh^{-1}\left(\frac{a+b\cosh(c+dx)+b\sinh(c+dx)}{\sqrt{a^2-b^2}}\right)}{(-a^2+b^2)^{3/2}}-2c\log(2(a+b\cosh(c+dx))(\cosh(c+dx)+\sinh(c+dx)))+2(c+dx)\log\left(1+\frac{b(\cosh(c+dx)+\sinh(c+dx))}{a-\sqrt{a^2-b^2}}\right)+2(c+dx)\log\left(1+\frac{b(\cosh(c+dx)+\sinh(c+dx))}{a+\sqrt{a^2-b^2}}\right)+2\text{PolyLog}\left(2,\frac{b(\cosh(c+dx)+\sinh(c+dx))}{-a+\sqrt{a^2-b^2}}\right)+2\text{PolyLog}\left(2,-\frac{b(\cosh(c+dx)+\sinh(c+dx))}{a+\sqrt{a^2-b^2}}\right)\right)+8ab\sinh(c+dx)-b^2\sinh(2(c+dx))}{8b^3d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]),x]
```

```
[Out] (-8*a*b*d*x*Cosh[c + d*x] + 2*b^2*d*x*Cosh[2*(c + d*x)] + 4*(a^2 - b^2)*(2*
c*(c + d*x) - (c + d*x)^2 + (4*a*Sqrt[-(a^2 - b^2)^2]*c*ArcTan[(a + b*Cosh[
c + d*x] + b*Sinh[c + d*x])/Sqrt[-a^2 + b^2]])/(a^2 - b^2)^(3/2) + (4*a*Sqr
t[-(a^2 - b^2)^2]*c*ArcTanh[(a + b*Cosh[c + d*x] + b*Sinh[c + d*x])/Sqrt[a^
2 - b^2]])/(-a^2 + b^2)^(3/2) - 2*c*Log[2*(a + b*Cosh[c + d*x])*(Cosh[c + d
*x] + Sinh[c + d*x])] + 2*(c + d*x)*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*
x]))/(a - Sqrt[a^2 - b^2])] + 2*(c + d*x)*Log[1 + (b*(Cosh[c + d*x] + Sinh[
c + d*x]))/(a + Sqrt[a^2 - b^2])] + 2*PolyLog[2, (b*(Cosh[c + d*x] + Sinh[c
 + d*x]))/(-a + Sqrt[a^2 - b^2])] + 2*PolyLog[2, -((b*(Cosh[c + d*x] + Sinh
```

[c + d*x]))/(a + Sqrt[a^2 - b^2]))]) + 8*a*b*Sinh[c + d*x] - b^2*Sinh[2*(c + d*x)])/(8*b^3*d^2)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 859 vs. $2(268) = 536$.
time = 2.10, size = 860, normalized size = 2.99

method	result
risch	$-\frac{\ln\left(\frac{-b\,e^{dx+c}+\sqrt{a^2-b^2}-a}{-a+\sqrt{a^2-b^2}}\right)x}{db}-\frac{a^2c^2}{d^2b^3}+\frac{2cx}{db}-\frac{2c\ln\left(e^{dx+c}\right)}{d^2b}-\frac{a(dx+1)e^{-dx-c}}{2b^2d^2}-\frac{\ln\left(\frac{b\,e^{dx+c}+\sqrt{a^2-b^2}+a}{a+\sqrt{a^2-b^2}}\right)c}{d^2b}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d/b*ln((-b*exp(d*x+c)+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))*x-1/d^2/b
^3*a^2*c^2+2/d/b*c*x-2/d^2/b*c*ln(exp(d*x+c))-1/2*a*(d*x+1)/b^2/d^2*exp(-d*
x-c)-1/d^2/b*ln((b*exp(d*x+c)+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))*c+1/d
^2/b*c*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)+b)+1/d^2/b*c^2-1/d^2/b*dilog((-b*
exp(d*x+c)+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))-1/d^2/b*dilog((b*exp(d*
x+c)+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))-1/2*x^2/b^3*a^2+1/16*(2*d*x-1)
/d^2/b*exp(2*d*x+2*c)-2/d/b^3*a^2*c*x+1/d/b^3*ln((-b*exp(d*x+c)+(a^2-b^2)^(
1/2)-a)/(-a+(a^2-b^2)^(1/2)))*a^2*x+1/d^2/b^3*ln((-b*exp(d*x+c)+(a^2-b^2)^(
1/2)-a)/(-a+(a^2-b^2)^(1/2)))*a^2*c+1/d/b^3*ln((b*exp(d*x+c)+(a^2-b^2)^(1/2
)+a)/(a+(a^2-b^2)^(1/2)))*a^2*x+1/d^2/b^3*ln((b*exp(d*x+c)+(a^2-b^2)^(1/2)+
a)/(a+(a^2-b^2)^(1/2)))*a^2*c+2/d^2/b^3*c*a^2*ln(exp(d*x+c))-1/d^2/b^3*c*a^
2*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)+b)-1/2*a*(d*x-1)/b^2/d^2*exp(d*x+c)+1/
16*(2*d*x+1)/d^2/b*exp(-2*d*x-2*c)+1/d^2/b^3*a^2*dilog((-b*exp(d*x+c)+(a^2-
b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))+1/d^2/b^3*a^2*dilog((b*exp(d*x+c)+(a^2-
b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))-1/d^2/b*ln((-b*exp(d*x+c)+(a^2-b^2)^(1/2
)-a)/(-a+(a^2-b^2)^(1/2)))*c-1/d/b*ln((b*exp(d*x+c)+(a^2-b^2)^(1/2)+a)/(a+(
a^2-b^2)^(1/2)))*x+1/2*x^2/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(8*(a^2*d^2*e^(2*c) - b^2*d^2*e^(2*c))*x^2 + (2*b^2*d*x*e^(4*c) - b^2*
e^(4*c))*e^(2*d*x) - 8*(a*b*d*x*e^(3*c) - a*b*e^(3*c))*e^(d*x) - 8*(a*b*d*x
*e^c + a*b*e^c)*e^(-d*x) + (2*b^2*d*x + b^2)*e^(-2*d*x))*e^(-2*c)/(b^3*d^2)
 - 1/8*integrate(16*((a^3*e^c - a*b^2*e^c)*x*e^(d*x) + (a^2*b - b^3)*x)/(b^
4*e^(2*d*x + 2*c) + 2*a*b^3*e^(d*x + c) + b^4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1196 vs. $2(266) = 532$.

time = 0.37, size = 1196, normalized size = 4.15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*((2*b^2*d*x - b^2)*cosh(d*x + c)^4 + (2*b^2*d*x - b^2)*sinh(d*x + c)^4
 + 2*b^2*d*x - 8*(a*b*d*x - a*b)*cosh(d*x + c)^3 - 4*(2*a*b*d*x - 2*a*b - (
2*b^2*d*x - b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 8*((a^2 - b^2)*d^2*x^2 -
2*(a^2 - b^2)*c^2)*cosh(d*x + c)^2 - 2*(4*(a^2 - b^2)*d^2*x^2 - 8*(a^2 - b^
2)*c^2 - 3*(2*b^2*d*x - b^2)*cosh(d*x + c)^2 + 12*(a*b*d*x - a*b)*cosh(d*x
+ c))*sinh(d*x + c)^2 + b^2 - 8*(a*b*d*x + a*b)*cosh(d*x + c) + 16*((a^2 -
b^2)*cosh(d*x + c)^2 + 2*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b
^2)*sinh(d*x + c)^2)*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*
x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + 16*((a^2 - b^
2)*cosh(d*x + c)^2 + 2*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2
)*sinh(d*x + c)^2)*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x
+ c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - 16*((a^2 - b^2)
*c*cosh(d*x + c)^2 + 2*(a^2 - b^2)*c*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b
^2)*c*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt
((a^2 - b^2)/b^2) + 2*a) - 16*((a^2 - b^2)*c*cosh(d*x + c)^2 + 2*(a^2 - b^2
)*c*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*c*sinh(d*x + c)^2)*log(2*b*co
sh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) + 16*(((
a^2 - b^2)*d*x + (a^2 - b^2)*c)*cosh(d*x + c)^2 + 2*((a^2 - b^2)*d*x + (a^2
 - b^2)*c)*cosh(d*x + c)*sinh(d*x + c) + ((a^2 - b^2)*d*x + (a^2 - b^2)*c)*
sinh(d*x + c)^2)*log((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
+ b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) + 16*(((a^2 - b^2)*d*x + (
a^2 - b^2)*c)*cosh(d*x + c)^2 + 2*((a^2 - b^2)*d*x + (a^2 - b^2)*c)*cosh(d*
x + c)*sinh(d*x + c) + ((a^2 - b^2)*d*x + (a^2 - b^2)*c)*sinh(d*x + c)^2)*l
og((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))
*sqrt((a^2 - b^2)/b^2) + b)/b) - 4*(2*a*b*d*x - (2*b^2*d*x - b^2)*cosh(d*x
+ c)^3 + 6*(a*b*d*x - a*b)*cosh(d*x + c)^2 + 2*a*b + 4*((a^2 - b^2)*d^2*x^2
 - 2*(a^2 - b^2)*c^2)*cosh(d*x + c))*sinh(d*x + c))/(b^3*d^2*cosh(d*x + c)^
2 + 2*b^3*d^2*cosh(d*x + c)*sinh(d*x + c) + b^3*d^2*sinh(d*x + c)^2)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sinh^3(c + dx)}{a + b \cosh(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(d*x+c)**3/(a+b*cosh(d*x+c)),x)
[Out] Integral(x*sinh(c + d*x)**3/(a + b*cosh(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="giac")
[Out] integrate(x*sinh(d*x + c)^3/(b*cosh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,{\mathrm{sinh}\left(c+d\,x\right)}^3}{a+b\,\mathrm{cosh}\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sinh(c + d*x)^3)/(a + b*cosh(c + d*x)),x)
[Out] int((x*sinh(c + d*x)^3)/(a + b*cosh(c + d*x)), x)
```

3.237 $\int \frac{\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal. Leaf size=61

$$-\frac{a\cosh(c+dx)}{b^2d}+\frac{\cosh^2(c+dx)}{2bd}+\frac{(a^2-b^2)\log(a+b\cosh(c+dx))}{b^3d}$$

```
[Out] -a*cosh(d*x+c)/b^2/d+1/2*cosh(d*x+c)^2/b/d+(a^2-b^2)*ln(a+b*cosh(d*x+c))/b^
3/d
```

Rubi [A]
time = 0.06, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2747, 711}

$$\frac{(a^2-b^2)\log(a+b\cosh(c+dx))}{b^3d}-\frac{a\cosh(c+dx)}{b^2d}+\frac{\cosh^2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3/(a + b*Cosh[c + d*x]),x]
```

```
[Out] -((a*Cosh[c + d*x])/(b^2*d)) + Cosh[c + d*x]^2/(2*b*d) + ((a^2 - b^2)*Log[a
 + b*Cosh[c + d*x]])/(b^3*d)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{b^2-x^2}{a+x}\,dx,x,b\cosh(c+dx)\right)}{b^3d}\\ &= -\frac{\text{Subst}\left(\int \left(a-x+\frac{-a^2+b^2}{a+x}\right)\,dx,x,b\cosh(c+dx)\right)}{b^3d}\\ &= -\frac{a\cosh(c+dx)}{b^2d}+\frac{\cosh^2(c+dx)}{2bd}+\frac{(a^2-b^2)\log(a+b\cosh(c+dx))}{b^3d}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 55, normalized size = 0.90

$$\frac{-4ab\cosh(c+dx)+b^2\cosh(2(c+dx))+4(a^2-b^2)\log(a+b\cosh(c+dx))}{4b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3/(a + b*Cosh[c + d*x]),x]
```

```
[Out] (-4*a*b*Cosh[c + d*x] + b^2*Cosh[2*(c + d*x)] + 4*(a^2 - b^2)*Log[a + b*Cos
h[c + d*x]])/(4*b^3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(59) = 118.
time = 0.99, size = 205, normalized size = 3.36

method	result
risch	$-\frac{x a^2}{b^3}+\frac{x}{b}+\frac{e^{2dx+2c}}{8bd}-\frac{a\,e^{dx+c}}{2b^2d}-\frac{e^{-dx-c}a}{2b^2d}+\frac{e^{-2dx-2c}}{8bd}-\frac{2a^2c}{b^3d}+\frac{2c}{bd}+\frac{\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{b}+1\right)a^2}{b^3d}-$
derivativedivides	$\frac{\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-b-2a}{2b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\left(-a^2+b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^3}+\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{2a+b}{2b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}}{d}$
default	$\frac{\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-b-2a}{2b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\left(-a^2+b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^3}+\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{2a+b}{2b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/b/(tanh(1/2*d*x+1/2*c)-1)^2-1/2*(-b-2*a)/b^2/(tanh(1/2*d*x+1/2*c)-
1)+(-a^2+b^2)/b^3*ln(tanh(1/2*d*x+1/2*c)-1)+1/2/b/(tanh(1/2*d*x+1/2*c)+1)^2
-1/2*(2*a+b)/b^2/(tanh(1/2*d*x+1/2*c)+1)+(-a^2+b^2)/b^3*ln(tanh(1/2*d*x+1/2
*c)+1)+(a^3-a^2*b-a*b^2+b^3)/b^3/(a-b)*ln(a*tanh(1/2*d*x+1/2*c)^2-b*tanh(1/
2*d*x+1/2*c)^2-a-b))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(59) = 118.
time = 0.28, size = 130, normalized size = 2.13

$$-\frac{\left(4\,ae^{(-dx-c)}-b\right)e^{(2\,dx+2\,c)}}{8\,b^2d}+\frac{(a^2-b^2)(dx+c)}{b^3d}-\frac{4\,ae^{(-dx-c)}-be^{(-2\,dx-2\,c)}}{8\,b^2d}+\frac{(a^2-b^2)\log\left(2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}+b\right)}{b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/(b^2*d) + (a^2 - b^2)*(d*x + c)
/(b^3*d) - 1/8*(4*a*e^(-d*x - c) - b*e^(-2*d*x - 2*c))/(b^2*d) + (a^2 - b^2
)*log(2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) + b)/(b^3*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(59) = 118$.

time = 0.51, size = 340, normalized size = 5.57

$$\frac{b^2\cosh(dx+c)^4 + b^2\sinh(dx+c)^4 - 8(a^2-b^2)dx\cosh(dx+c)^2 - 4ab\cosh(dx+c)^3 + 4(b^2\cosh(dx+c)-ab)\sinh(dx+c)^3 - 4ab\cosh(dx+c) + 2(3b^2\cosh(dx+c)^2 - 4(a^2-b^2)dx - 6ab\cosh(dx+c))\sinh(dx+c)^2 + b^2 + 8((a^2-b^2)\cosh(dx+c)^2 + 2(a^2-b^2)\cosh(dx+c)\sinh(dx+c) + (a^2-b^2)\sinh(dx+c)^2)\log\left(\frac{2(b\cosh(dx+c)+a)}{\cosh(dx+c)-\sinh(dx+c)}\right) + 4(b^2\cosh(dx+c)^3 - 4(a^2-b^2)dx\cosh(dx+c) - 3ab\cosh(dx+c)^2 - ab)\sinh(dx+c)}{8(b^3d\cosh(dx+c)^2 + 2b^3d\cosh(dx+c)\sinh(dx+c) + b^3d\sinh(dx+c)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/8*(b^2*cosh(d*x + c)^4 + b^2*sinh(d*x + c)^4 - 8*(a^2 - b^2)*d*x*cosh(d*x
 + c)^2 - 4*a*b*cosh(d*x + c)^3 + 4*(b^2*cosh(d*x + c) - a*b)*sinh(d*x + c)
^3 - 4*a*b*cosh(d*x + c) + 2*(3*b^2*cosh(d*x + c)^2 - 4*(a^2 - b^2)*d*x - 6
*a*b*cosh(d*x + c))*sinh(d*x + c)^2 + b^2 + 8*((a^2 - b^2)*cosh(d*x + c)^2
+ 2*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*sinh(d*x + c)^2)*
log(2*(b*cosh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(b^2*cosh(
d*x + c)^3 - 4*(a^2 - b^2)*d*x*cosh(d*x + c) - 3*a*b*cosh(d*x + c)^2 - a*b)
*sinh(d*x + c))/(b^3*d*cosh(d*x + c)^2 + 2*b^3*d*cosh(d*x + c)*sinh(d*x + c
) + b^3*d*sinh(d*x + c)^2)
```

Sympy [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a+b*cosh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]

time = 0.43, size = 88, normalized size = 1.44

$$\frac{\frac{b\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-4\,a\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{b^2}+\frac{8\,(a^2-b^2)\log\left(\left|b\left(e^{(dx+c)}+e^{(-dx-c)}\right)+2\,a\right|\right)}{b^3}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*((b*(e^(d*x + c) + e^(-d*x - c))^2 - 4*a*(e^(d*x + c) + e^(-d*x - c)))/
b^2 + 8*(a^2 - b^2)*log(abs(b*(e^(d*x + c) + e^(-d*x - c)) + 2*a))/b^3)/d
```

Mupad [B]

time = 1.06, size = 122, normalized size = 2.00

$$\frac{e^{-2\,c-2\,d\,x}}{8\,b\,d}+\frac{e^{2\,c+2\,d\,x}}{8\,b\,d}-\frac{x\,(a^2-b^2)}{b^3}+\frac{\ln\left(b+2\,a\,e^{d\,x}\,e^{c}+b\,e^{2\,c}\,e^{2\,d\,x}\right)\,(a^2-b^2)}{b^3\,d}-\frac{a\,e^{-c-d\,x}}{2\,b^2\,d}-\frac{a\,e^{c+d\,x}}{2\,b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3/(a + b*cosh(c + d*x)),x)
```

```
[Out] exp(- 2*c - 2*d*x)/(8*b*d) + exp(2*c + 2*d*x)/(8*b*d) - (x*(a^2 - b^2))/b^3
 + (log(b + 2*a*exp(d*x)*exp(c) + b*exp(2*c)*exp(2*d*x))*(a^2 - b^2))/(b^3*
d) - (a*exp(- c - d*x))/(2*b^2*d) - (a*exp(c + d*x))/(2*b^2*d)
```

3.238 $\int \frac{\sinh^3(c+dx)}{x(a+b\cosh(c+dx))}\,dx$

Optimal. Leaf size=27

$$\mathrm{Int}\left(\frac{\sinh^3(c+dx)}{x(a+b\cosh(c+dx))},x\right)$$

```
[Out] Unintegrable(sinh(d*x+c)^3/x/(a+b*cosh(d*x+c)),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sinh^3(c+dx)}{x(a+b\cosh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sinh[c + d*x]^3/(x*(a + b*Cosh[c + d*x])),x]
```

```
[Out] Defer[Int][Sinh[c + d*x]^3/(x*(a + b*Cosh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \int \frac{\sinh^3(c+dx)}{x(a+b\cosh(c+dx))}\,dx$$

Mathematica [A]
time = 145.31, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3(c+dx)}{x(a+b\cosh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sinh[c + d*x]^3/(x*(a + b*Cosh[c + d*x])),x]
```

```
[Out] Integrate[Sinh[c + d*x]^3/(x*(a + b*Cosh[c + d*x])), x]
```

Maple [A]
time = 1.01, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3(dx+c)}{x\,(a+b\cosh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/x/(a+b*cosh(d*x+c)),x)
[Out] int(sinh(d*x+c)^3/x/(a+b*cosh(d*x+c)),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/x/(a+b*cosh(d*x+c)),x, algorithm="maxima")
[Out] 1/4*Ei(2*d*x)*e^(2*c)/b + 1/2*a*Ei(-d*x)*e^(-c)/b^2 - 1/4*Ei(-2*d*x)*e^(-2*
c)/b - 1/2*a*Ei(d*x)*e^c/b^2 + (a^2 - b^2)*log(x)/b^3 - 1/8*integrate(16*(a
^2*b - b^3 + (a^3*e^c - a*b^2*e^c)*e^(d*x))/(b^4*x*e^(2*d*x + 2*c) + 2*a*b^
3*x*e^(d*x + c) + b^4*x), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/x/(a+b*cosh(d*x+c)),x, algorithm="fricas")
[Out] integral(sinh(d*x + c)^3/(b*x*cosh(d*x + c) + a*x), x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3(c+dx)}{x(a+b\cosh(c+dx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/x/(a+b*cosh(d*x+c)),x)
[Out] Integral(sinh(c + d*x)**3/(x*(a + b*cosh(c + d*x))), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/x/(a+b*cosh(d*x+c)),x, algorithm="giac")
[Out] integrate(sinh(d*x + c)^3/((b*cosh(d*x + c) + a)*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sinh(c+d\,x)^3}{x\,\left(a+b\cosh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3/(x*(a + b*cosh(c + d*x))),x)
[Out] int(sinh(c + d*x)^3/(x*(a + b*cosh(c + d*x))), x)
```

3.239 $\int \cosh(a+b\log(cx^n))\,dx$

Optimal. Leaf size=54

$$\frac{x\cosh(a+b\log(cx^n))}{1-b^2n^2}-\frac{bnx\sinh(a+b\log(cx^n))}{1-b^2n^2}$$

```
[Out] x*cosh(a+b*ln(c*x^n))/(-b^2*n^2+1)-b*n*x*sinh(a+b*ln(c*x^n))/(-b^2*n^2+1)
```

Rubi [A]
time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = {5629}

$$\frac{x\cosh(a+b\log(cx^n))}{1-b^2n^2}-\frac{bnx\sinh(a+b\log(cx^n))}{1-b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*Log[c*x^n]],x]
```

```
[Out] (x*Cosh[a + b*Log[c*x^n]])/(1 - b^2*n^2) - (b*n*x*Sinh[a + b*Log[c*x^n]])/(
1 - b^2*n^2)
```

Rule 5629

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[(-
x)*(Cosh[d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 - 1)), x] + Simp[b*d*n*x*(Sinh[
d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 - 1)), x] /; FreeQ[{a, b, c, d, n}, x] &
& NeQ[b^2*d^2*n^2 - 1, 0]
```

Rubi steps

$$\int \cosh(a+b\log(cx^n))\,dx=\frac{x\cosh(a+b\log(cx^n))}{1-b^2n^2}-\frac{bnx\sinh(a+b\log(cx^n))}{1-b^2n^2}$$

Mathematica [A]
time = 0.04, size = 41, normalized size = 0.76

$$\frac{x(-\cosh(a+b\log(cx^n))+bn\sinh(a+b\log(cx^n)))}{-1+b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*Log[c*x^n]],x]
```

```
[Out] (x*(-Cosh[a + b*Log[c*x^n]] + b*n*Sinh[a + b*Log[c*x^n]]))/(-1 + b^2*n^2)
```

Maple [F]
time = 0.50, size = 0, normalized size = 0.00

$$\int \cosh\left(a+b\ln\left(c\,x^{n}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*ln(c*x^n)),x)
[Out] int(cosh(a+b*ln(c*x^n)),x)
```

Maxima [A]
time = 0.28, size = 51, normalized size = 0.94

$$\frac{c^{b}xe^{(b\log(x^{n})+a)}}{2\,(bn+1)}-\frac{xe^{(-a)}}{2\,(bc^{b}n-c^{b})(x^{n})^{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n)),x, algorithm="maxima")
[Out] 1/2*c^b*x*e^(b*log(x^n) + a)/(b*n + 1) - 1/2*x*e^(-a)/((b*c^b*n - c^b)*(x^n
)^b)
```

Fricas [A]
time = 0.40, size = 44, normalized size = 0.81

$$\frac{bnx\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-x\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{b^{2}n^{2}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] (b*n*x*sinh(b*n*log(x) + b*log(c) + a) - x*cosh(b*n*log(x) + b*log(c) + a))
/(b^2*n^2 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \int \cosh\left(a-\frac{\log\left(cx^{n}\right)}{n}\right)dx & \text{for}\: b=-\frac{1}{n} \\ \int \cosh\left(a+\frac{\log\left(cx^{n}\right)}{n}\right)dx & \text{for}\: b=\frac{1}{n} \\ \frac{bnx\sinh\left(a+b\log\left(cx^{n}\right)\right)}{b^{2}n^{2}-1}-\frac{x\cosh\left(a+b\log\left(cx^{n}\right)\right)}{b^{2}n^{2}-1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*ln(c*x**n)),x)
```

```
[Out] Piecewise((Integral(cosh(a - log(c*x**n)/n), x), Eq(b, -1/n)), (Integral(co
sh(a + log(c*x**n)/n), x), Eq(b, 1/n)), (b*n*x*sinh(a + b*log(c*x**n))/(b**
2*n**2 - 1) - x*cosh(a + b*log(c*x**n))/(b**2*n**2 - 1), True))
```

Giac [A]
time = 0.42, size = 47, normalized size = 0.87

$$\frac{c^b x x^{bn} e^a}{2\,(bn+1)} - \frac{x e^{(-a)}}{2\,(bn-1) c^b x^{bn}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/2*c^b*x*x^(b*n)*e^a/(b*n + 1) - 1/2*x*e^(-a)/((b*n - 1)*c^b*x^(b*n))
```

Mupad [B]
time = 1.00, size = 44, normalized size = 0.81

$$\frac{x\,\mathrm{e}^a\,{\left(c\,x^n\right)}^b}{2\,b\,n+2} - \frac{x\,\mathrm{e}^{-a}}{{\left(c\,x^n\right)}^b\,\left(2\,b\,n-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*log(c*x^n)),x)
```

```
[Out] (x*exp(a)*(c*x^n)^b)/(2*b*n + 2) - (x*exp(-a))/((c*x^n)^b*(2*b*n - 2))
```

3.240 $\int \cosh^2(a+b\log(cx^n))\,dx$

Optimal. Leaf size=88

$$-\frac{2b^2n^2x}{1-4b^2n^2}+\frac{x\cosh^2(a+b\log(cx^n))}{1-4b^2n^2}-\frac{2bnx\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{1-4b^2n^2}$$

```
[Out] -2*b^2*n^2*x/(-4*b^2*n^2+1)+x*cosh(a+b*ln(c*x^n))^2/(-4*b^2*n^2+1)-2*b*n*x*
cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))/(-4*b^2*n^2+1)
```

Rubi [A]
time = 0.02, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5631, 8}

$$\frac{x\cosh^2(a+b\log(cx^n))}{1-4b^2n^2}-\frac{2bnx\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{1-4b^2n^2}-\frac{2b^2n^2x}{1-4b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (-2*b^2*n^2*x)/(1 - 4*b^2*n^2) + (x*Cosh[a + b*Log[c*x^n]]^2)/(1 - 4*b^2*n^
2) - (2*b*n*x*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]])/(1 - 4*b^2*n^2
)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5631

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_), x_Symbol] :> Si
mp[(-x)*(Cosh[d*(a + b*Log[c*x^n])]^p/(b^2*d^2*n^2*p^2 - 1)), x] + (Dist[b^
2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 - 1)), Int[Cosh[d*(a + b*Log[c*x^n])]
^(p - 2), x], x] + Simp[b*d*n*p*x*Cosh[d*(a + b*Log[c*x^n])]^(p - 1)*(Sinh[
d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2*p^2 - 1)), x]) /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - 1, 0]
```

Rubi steps

$$\begin{aligned}\int \cosh^2(a+b\log(cx^n))\,dx &= \frac{x\cosh^2(a+b\log(cx^n))}{1-4b^2n^2}-\frac{2bnx\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{1-4b^2n^2}-\\ &= -\frac{2b^2n^2x}{1-4b^2n^2}+\frac{x\cosh^2(a+b\log(cx^n))}{1-4b^2n^2}-\frac{2bnx\cosh(a+b\log(cx^n))\sinh(a+\end{aligned}$$

Mathematica [A]
time = 0.08, size = 56, normalized size = 0.64

$$\frac{x(-1+4b^2n^2-\cosh\left(2(a+b\log\left(cx^n\right))\right)+2bn\sinh\left(2(a+b\log\left(cx^n\right))\right))}{-2+8b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (x*(-1 + 4*b^2*n^2 - Cosh[2*(a + b*Log[c*x^n])] + 2*b*n*Sinh[2*(a + b*Log[c
*x^n])]))/(-2 + 8*b^2*n^2)
```

Maple [F]
time = 3.49, size = 0, normalized size = 0.00

$$\int \cosh^2\left(a+b\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*ln(c*x^n))^2,x)
```

```
[Out] int(cosh(a+b*ln(c*x^n))^2,x)
```

Maxima [A]
time = 0.29, size = 67, normalized size = 0.76

$$\frac{c^{2\,b}xe^{\left(2\,b\log\left(x^n\right)+2\,a\right)}}{4\,(2\,bn+1)}+\frac{1}{2}\,x-\frac{xe^{\left(-2\,a\right)}}{4\,(2\,bc^{2\,b}n-c^{2\,b})(x^n)^{2\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 1/4*c^(2*b)*x*e^(2*b*log(x^n) + 2*a)/(2*b*n + 1) + 1/2*x - 1/4*x*e^(-2*a)/(
(2*b*c^(2*b)*n - c^(2*b))*(x^n)^(2*b))
```

Fricas [A]
time = 0.44, size = 90, normalized size = 1.02

$$\frac{4\,bnx\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-x\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2-x\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2+\left(4\,b^2n^2-1\right)x}{2\,(4\,b^2n^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*b*n*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a
) - x*cosh(b*n*log(x) + b*log(c) + a)^2 - x*sinh(b*n*log(x) + b*log(c) + a)
^2 + (4*b^2*n^2 - 1)*x)/(4*b^2*n^2 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \int \cosh^2\left(a - \frac{\log(cx^n)}{2n}\right) dx & \text{for } b = -\frac{1}{2n} \\ \int \cosh^2\left(a + \frac{\log(cx^n)}{2n}\right) dx & \text{for } b = \frac{1}{2n} \\ -\frac{2b^2n^2x\sinh^2(a+b\log(cx^n))}{4b^2n^2-1} + \frac{2b^2n^2x\cosh^2(a+b\log(cx^n))}{4b^2n^2-1} + \frac{2bnx\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{4b^2n^2-1} - \frac{x\cosh^2(a+b\log(cx^n))}{4b^2n^2-1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*ln(c*x**n))**2,x)
```

```
[Out] Piecewise((Integral(cosh(a - log(c*x**n)/(2*n))**2, x), Eq(b, -1/(2*n))), (
Integral(cosh(a + log(c*x**n)/(2*n))**2, x), Eq(b, 1/(2*n))), (-2*b**2*n**2
*x*sinh(a + b*log(c*x**n))**2/(4*b**2*n**2 - 1) + 2*b**2*n**2*x*cosh(a + b*
log(c*x**n))**2/(4*b**2*n**2 - 1) + 2*b*n*x*sinh(a + b*log(c*x**n))*cosh(a
+ b*log(c*x**n))/(4*b**2*n**2 - 1) - x*cosh(a + b*log(c*x**n))**2/(4*b**2*n
**2 - 1), True))
```

Giac [A]
time = 0.43, size = 169, normalized size = 1.92

$$\frac{bc^{2b}nxx^{2bn}e^{(2a)}}{2(4b^2n^2-1)} + \frac{2b^2n^2x}{4b^2n^2-1} - \frac{c^{2b}xx^{2bn}e^{(2a)}}{4(4b^2n^2-1)} - \frac{bnxe^{(-2a)}}{2(4b^2n^2-1)c^{2b}x^{2bn}} - \frac{x}{2(4b^2n^2-1)} - \frac{xe^{(-2a)}}{4(4b^2n^2-1)c^{2b}x^{2bn}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] 1/2*b*c^(2*b)*n*x*x^(2*b*n)*e^(2*a)/(4*b^2*n^2 - 1) + 2*b^2*n^2*x/(4*b^2*n^
2 - 1) - 1/4*c^(2*b)*x*x^(2*b*n)*e^(2*a)/(4*b^2*n^2 - 1) - 1/2*b*n*x*e^(-2*
a)/((4*b^2*n^2 - 1)*c^(2*b)*x^(2*b*n)) - 1/2*x/(4*b^2*n^2 - 1) - 1/4*x*e^(-
2*a)/((4*b^2*n^2 - 1)*c^(2*b)*x^(2*b*n))
```

Mupad [B]
time = 1.01, size = 53, normalized size = 0.60

$$\frac{x}{2} - \frac{x\,\mathrm{e}^{-2\,a}}{{\left(c\,x^n\right)}^{2\,b}\,\left(8\,b\,n-4\right)} + \frac{x\,\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}}{8\,b\,n+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*log(c*x^n))^2,x)
```

```
[Out] x/2 - (x*exp(-2*a))/((c*x^n)^(2*b)*(8*b*n - 4)) + (x*exp(2*a)*(c*x^n)^(2*b)
)/(8*b*n + 4)
```

3.241 $\int \cosh^3(a+b\log(cx^n))\,dx$

Optimal. Leaf size=149

$$-\frac{6b^2n^2x\cosh(a+b\log(cx^n))}{1-10b^2n^2+9b^4n^4}+\frac{x\cosh^3(a+b\log(cx^n))}{1-9b^2n^2}+\frac{6b^3n^3x\sinh(a+b\log(cx^n))}{1-10b^2n^2+9b^4n^4}-\frac{3bnx\cosh^2(a+b\log$$

```
[Out] -6*b^2*n^2*x*cosh(a+b*ln(c*x^n))/(9*b^4*n^4-10*b^2*n^2+1)+x*cosh(a+b*ln(c*x
^n))^3/(-9*b^2*n^2+1)+6*b^3*n^3*x*sinh(a+b*ln(c*x^n))/(9*b^4*n^4-10*b^2*n^2
+1)-3*b*n*x*cosh(a+b*ln(c*x^n))^2*sinh(a+b*ln(c*x^n))/(-9*b^2*n^2+1)
```

Rubi [A]
time = 0.03, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5631, 5629}

$$\frac{x\cosh^3(a+b\log(cx^n))}{1-9b^2n^2}-\frac{3bnx\sinh(a+b\log(cx^n))\cosh^2(a+b\log(cx^n))}{1-9b^2n^2}-\frac{6b^2n^2x\cosh(a+b\log(cx^n))}{9b^4n^4-10b^2n^2+1}+\frac{6b^3n^3x\sinh(a+b\log(cx^n))}{9b^4n^4-10b^2n^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (-6*b^2*n^2*x*Cosh[a + b*Log[c*x^n]])/(1 - 10*b^2*n^2 + 9*b^4*n^4) + (x*Cos
h[a + b*Log[c*x^n]]^3)/(1 - 9*b^2*n^2) + (6*b^3*n^3*x*Sinh[a + b*Log[c*x^n]
])/(1 - 10*b^2*n^2 + 9*b^4*n^4) - (3*b*n*x*Cosh[a + b*Log[c*x^n]]^2*Sinh[a
+ b*Log[c*x^n]])/(1 - 9*b^2*n^2)
```

Rule 5629

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[(-
x)*(Cosh[d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 - 1)), x] + Simp[b*d*n*x*(Sinh[
d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 - 1)), x] /; FreeQ[{a, b, c, d, n}, x] &
& NeQ[b^2*d^2*n^2 - 1, 0]
```

Rule 5631

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_), x_Symbol] :> Si
mp[(-x)*(Cosh[d*(a + b*Log[c*x^n])]^p/(b^2*d^2*n^2*p^2 - 1)), x] + (Dist[b^
2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 - 1)), Int[Cosh[d*(a + b*Log[c*x^n])]
^(p - 2), x], x] + Simp[b*d*n*p*x*Cosh[d*(a + b*Log[c*x^n])]^(p - 1)*(Sinh[
d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2*p^2 - 1)), x]) /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - 1, 0]
```

Rubi steps

$$\int \cosh^3(a+b\log(cx^n))\,dx = \frac{x\cosh^3(a+b\log(cx^n))}{1-9b^2n^2} - \frac{3bnx\cosh^2(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{1-9b^2n^2}$$

$$= -\frac{6b^2n^2x\cosh(a+b\log(cx^n))}{1-10b^2n^2+9b^4n^4} + \frac{x\cosh^3(a+b\log(cx^n))}{1-9b^2n^2} + \frac{6b^3n^3x\sinh(a+}{1-10b^2n^2}$$

Mathematica [A]
time = 0.39, size = 117, normalized size = 0.79

$$\frac{x((3-27b^2n^2)\cosh(a+b\log(cx^n))+(1-b^2n^2)\cosh(3(a+b\log(cx^n)))+6bn(-1+5b^2n^2+(-1+b^2n^2)\cosh(2(a+b\log(cx^n))))\sinh(a+b\log(cx^n)))}{4-40b^2n^2+36b^4n^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (x*((3 - 27*b^2*n^2)*Cosh[a + b*Log[c*x^n]] + (1 - b^2*n^2)*Cosh[3*(a + b*L
og[c*x^n])] + 6*b*n*(-1 + 5*b^2*n^2 + (-1 + b^2*n^2)*Cosh[2*(a + b*Log[c*x^
n])])*Sinh[a + b*Log[c*x^n]]))/(4 - 40*b^2*n^2 + 36*b^4*n^4)
```

Maple [F]
time = 3.35, size = 0, normalized size = 0.00

$$\int \cosh^3(a+b\ln(c\,x^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*ln(c*x^n))^3,x)
```

```
[Out] int(cosh(a+b*ln(c*x^n))^3,x)
```

Maxima [A]
time = 0.30, size = 115, normalized size = 0.77

$$\frac{c^{3\,b}xe^{(3\,b\log(x^n)+3\,a)}}{8\,(3\,bn+1)} + \frac{3\,c^bxe^{(b\log(x^n)+a)}}{8\,(bn+1)} - \frac{3\,xe^{(-b\log(x^n)-a)}}{8\,(bc^bn-c^b)} - \frac{xe^{(-3\,a)}}{8\,(3\,bc^{3\,b}n-c^{3\,b})(x^n)^{3\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] 1/8*c^(3*b)*x*e^(3*b*log(x^n) + 3*a)/(3*b*n + 1) + 3/8*c^b*x*e^(b*log(x^n)
+ a)/(b*n + 1) - 3/8*x*e^(-b*log(x^n) - a)/(b*c^b*n - c^b) - 1/8*x*e^(-3*a)
/((3*b*c^(3*b)*n - c^(3*b))*(x^n)^(3*b))
```

Fricas [A]
time = 0.48, size = 199, normalized size = 1.34

$$-\frac{(b^2n^2-1)x\cosh(bn\log(x)+b\log(c)+a)^3+3\,(b^2n^2-1)x\cosh(bn\log(x)+b\log(c)+a)\sinh(bn\log(x)+b\log(c)+a)^2-3\,(b^3n^3-bn)x\sinh(bn\log(x)+b\log(c)+a)^3+3\,(9\,b^2n^2-1)x\cosh(bn\log(x)+b\log(c)+a)-3\left(3\,(b^3n^3-bn)x\cosh(bn\log(x)+b\log(c)+a)^2+(9\,b^3n^3-bn)x\right)\sinh(bn\log(x)+b\log(c)+a)}{4\,(9\,b^4n^4-10\,b^2n^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] -1/4*((b^2*n^2 - 1)*x*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*(b^2*n^2 - 1)*x
*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^2 - 3*(b^3
*n^3 - b*n)*x*sinh(b*n*log(x) + b*log(c) + a)^3 + 3*(9*b^2*n^2 - 1)*x*cosh(
b*n*log(x) + b*log(c) + a) - 3*(3*(b^3*n^3 - b*n)*x*cosh(b*n*log(x) + b*log
(c) + a)^2 + (9*b^3*n^3 - b*n)*x)*sinh(b*n*log(x) + b*log(c) + a))/(9*b^4*n
^4 - 10*b^2*n^2 + 1)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \int \cosh^3\left(a - \frac{\log(cx^n)}{n}\right) dx & \text{for } b = -\frac{1}{n} \\ \int \cosh^3\left(a - \frac{\log(cx^n)}{3n}\right) dx & \text{for } b = -\frac{1}{3n} \\ \int \cosh^3\left(a + \frac{\log(cx^n)}{3n}\right) dx & \text{for } b = \frac{1}{3n} \\ \int \cosh^3\left(a + \frac{\log(cx^n)}{n}\right) dx & \text{for } b = \frac{1}{n} \\ -\frac{6b^3n^3x\sinh^3(a+b\log(cx^n))}{9b^4n^4-10b^2n^2+1} + \frac{9b^3n^3x\sinh(a+b\log(cx^n))\cosh^2(a+b\log(cx^n))}{9b^4n^4-10b^2n^2+1} + \frac{6b^2n^2x\sinh^2(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{9b^4n^4-10b^2n^2+1} - \frac{7b^2n^2x\cosh^3(a+b\log(cx^n))}{9b^4n^4-10b^2n^2+1} - \frac{3bnx\sinh(a+b\log(cx^n))\cosh^2(a+b\log(cx^n))}{9b^4n^4-10b^2n^2+1} + \frac{x\cosh^3(a+b\log(cx^n))}{9b^4n^4-10b^2n^2+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*ln(c*x**n))**3,x)
```

```
[Out] Piecewise((Integral(cosh(a - log(c*x**n)/n)**3, x), Eq(b, -1/n)), (Integral
(cosh(a - log(c*x**n)/(3*n))**3, x), Eq(b, -1/(3*n))), (Integral(cosh(a + l
og(c*x**n)/(3*n))**3, x), Eq(b, 1/(3*n))), (Integral(cosh(a + log(c*x**n)/n
)**3, x), Eq(b, 1/n)), (-6*b**3*n**3*x*sinh(a + b*log(c*x**n))**3/(9*b**4*n
**4 - 10*b**2*n**2 + 1) + 9*b**3*n**3*x*sinh(a + b*log(c*x**n))*cosh(a + b*
log(c*x**n))**2/(9*b**4*n**4 - 10*b**2*n**2 + 1) + 6*b**2*n**2*x*sinh(a + b
*log(c*x**n))**2*cosh(a + b*log(c*x**n))/(9*b**4*n**4 - 10*b**2*n**2 + 1) -
 7*b**2*n**2*x*cosh(a + b*log(c*x**n))**3/(9*b**4*n**4 - 10*b**2*n**2 + 1)
- 3*b*n*x*sinh(a + b*log(c*x**n))*cosh(a + b*log(c*x**n))**2/(9*b**4*n**4 -
 10*b**2*n**2 + 1) + x*cosh(a + b*log(c*x**n))**3/(9*b**4*n**4 - 10*b**2*n*
*2 + 1), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 665 vs. 2(150) = 300.
time = 0.45, size = 665, normalized size = 4.46

$$\frac{3\,b^3c^{3\,b}n^3xx^{3\,bn}e^{(3\,a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)} + \frac{27\,b^3c^bn^3xx^{bn}e^a}{8\,(9\,b^4n^4-10\,b^2n^2+1)} - \frac{b^2c^{3\,b}n^2xx^{3\,bn}e^{(3\,a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)} - \frac{27\,b^2c^bn^2xx^{bn}e^a}{8\,(9\,b^4n^4-10\,b^2n^2+1)} - \frac{3\,bc^{3\,b}nxx^{3\,bn}e^{(3\,a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)} - \frac{27\,b^3n^3xe^{(-a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)c^bx^{bn}} - \frac{3\,b^3n^3xe^{(-3\,a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)c^{3\,b}x^{3\,bn}} - \frac{3\,bc^bnxx^{bn}e^a}{8\,(9\,b^4n^4-10\,b^2n^2+1)} + \frac{c^{3\,b}xx^{3\,bn}e^{(3\,a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)} - \frac{27\,b^2n^2xe^{(-a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)c^bx^{bn}} - \frac{b^2n^2xe^{(-3\,a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)c^{3\,b}x^{3\,bn}} + \frac{3\,c^bxx^{bn}e^a}{8\,(9\,b^4n^4-10\,b^2n^2+1)} + \frac{3\,bnxe^{(-a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)c^bx^{bn}} + \frac{3\,bnxe^{(-3\,a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)c^{3\,b}x^{3\,bn}} + \frac{3\,xe^{(-a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)c^bx^{bn}} + \frac{xe^{(-3\,a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)c^{3\,b}x^{3\,bn}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] 3/8*b^3*c^(3*b)*n^3*x*x^(3*b*n)*e^(3*a)/(9*b^4*n^4 - 10*b^2*n^2 + 1) + 27/8
*b^3*c^b*n^3*x*x^(b*n)*e^a/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 1/8*b^2*c^(3*b)*n
^2*x*x^(3*b*n)*e^(3*a)/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 27/8*b^2*c^b*n^2*x*x^
```

```
(b*n)*e^a/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 3/8*b*c^(3*b)*n*x*x^(3*b*n)*e^(3*a
)/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 27/8*b^3*n^3*x*e^(-a)/((9*b^4*n^4 - 10*b^2
*n^2 + 1)*c^b*x^(b*n)) - 3/8*b^3*n^3*x*e^(-3*a)/((9*b^4*n^4 - 10*b^2*n^2 +
1)*c^(3*b)*x^(3*b*n)) - 3/8*b*c^b*n*x*x^(b*n)*e^a/(9*b^4*n^4 - 10*b^2*n^2 +
 1) + 1/8*c^(3*b)*x*x^(3*b*n)*e^(3*a)/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 27/8*b
^2*n^2*x*e^(-a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)*c^b*x^(b*n)) - 1/8*b^2*n^2*x*
e^(-3*a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)*c^(3*b)*x^(3*b*n)) + 3/8*c^b*x*x^(b*
n)*e^a/(9*b^4*n^4 - 10*b^2*n^2 + 1) + 3/8*b*n*x*e^(-a)/((9*b^4*n^4 - 10*b^2
*n^2 + 1)*c^b*x^(b*n)) + 3/8*b*n*x*e^(-3*a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)*c
^(3*b)*x^(3*b*n)) + 3/8*x*e^(-a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)*c^b*x^(b*n))
 + 1/8*x*e^(-3*a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)*c^(3*b)*x^(3*b*n))
```

Mupad [B]
time = 1.05, size = 94, normalized size = 0.63

$$\frac{x\,\mathrm{e}^{3\,a}\,{\left(c\,x^n\right)}^{3\,b}}{24\,b\,n+8}-\frac{x\,\mathrm{e}^{-3\,a}}{{\left(c\,x^n\right)}^{3\,b}\,\left(24\,b\,n-8\right)}-\frac{3\,x\,\mathrm{e}^{-a}}{{\left(c\,x^n\right)}^b\,\left(8\,b\,n-8\right)}+\frac{3\,x\,\mathrm{e}^a\,{\left(c\,x^n\right)}^b}{8\,b\,n+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*log(c*x^n))^3,x)
```

```
[Out] (x*exp(3*a)*(c*x^n)^(3*b))/(24*b*n + 8) - (x*exp(-3*a))/((c*x^n)^(3*b)*(24*
b*n - 8)) - (3*x*exp(-a))/((c*x^n)^b*(8*b*n - 8)) + (3*x*exp(a)*(c*x^n)^b)/
(8*b*n + 8)
```

3.242 $\int \cosh^4(a+b\log(cx^n))\,dx$

Optimal. Leaf size=191

$$\frac{24b^4n^4x}{1-20b^2n^2+64b^4n^4}-\frac{12b^2n^2x\cosh^2(a+b\log(cx^n))}{1-20b^2n^2+64b^4n^4}+\frac{x\cosh^4(a+b\log(cx^n))}{1-16b^2n^2}+\frac{24b^3n^3x\cosh(a+b\log(c\ldots}{1-20b^2n^2\ldots}$$

```
[Out] 24*b^4*n^4*x/(64*b^4*n^4-20*b^2*n^2+1)-12*b^2*n^2*x*cosh(a+b*ln(c*x^n))^2/(
64*b^4*n^4-20*b^2*n^2+1)+x*cosh(a+b*ln(c*x^n))^4/(-16*b^2*n^2+1)+24*b^3*n^3
*x*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))/(64*b^4*n^4-20*b^2*n^2+1)-4*b*n*
x*cosh(a+b*ln(c*x^n))^3*sinh(a+b*ln(c*x^n))/(-16*b^2*n^2+1)
```

Rubi [A]
time = 0.04, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5631, 8}

$$\frac{x\cosh^4(a+b\log(cx^n))}{1-16b^2n^2}-\frac{4bnx\sinh(a+b\log(cx^n))\cosh^3(a+b\log(cx^n))}{1-16b^2n^2}-\frac{12b^2n^2x\cosh^2(a+b\log(cx^n))}{64b^4n^4-20b^2n^2+1}+\frac{24b^3n^3x\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{64b^4n^4-20b^2n^2+1}+\frac{24b^4n^4x}{64b^4n^4-20b^2n^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (24*b^4*n^4*x)/(1 - 20*b^2*n^2 + 64*b^4*n^4) - (12*b^2*n^2*x*Cosh[a + b*Log
[c*x^n]]^2)/(1 - 20*b^2*n^2 + 64*b^4*n^4) + (x*Cosh[a + b*Log[c*x^n]]^4)/(1
 - 16*b^2*n^2) + (24*b^3*n^3*x*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]
])/(1 - 20*b^2*n^2 + 64*b^4*n^4) - (4*b*n*x*Cosh[a + b*Log[c*x^n]]^3*Sinh[a
 + b*Log[c*x^n]])/(1 - 16*b^2*n^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5631

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_), x_Symbol] :> Si
mp[(-x)*(Cosh[d*(a + b*Log[c*x^n])]^p/(b^2*d^2*n^2*p^2 - 1)), x] + (Dist[b^
2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 - 1)), Int[Cosh[d*(a + b*Log[c*x^n])]
^(p - 2), x], x] + Simp[b*d*n*p*x*Cosh[d*(a + b*Log[c*x^n])]^(p - 1)*(Sinh[
d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2*p^2 - 1)), x]) /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - 1, 0]
```

Rubi steps

$$\int \cosh^4(a+b\log(cx^n))\,dx = \frac{x\cosh^4(a+b\log(cx^n))}{1-16b^2n^2} - \frac{4bnx\cosh^3(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{1-16b^2n^2}$$
$$= -\frac{12b^2n^2x\cosh^2(a+b\log(cx^n))}{1-20b^2n^2+64b^4n^4} + \frac{x\cosh^4(a+b\log(cx^n))}{1-16b^2n^2} + \frac{24b^3n^3x\cosh(}{}$$
$$= \frac{24b^4n^4x}{1-20b^2n^2+64b^4n^4} - \frac{12b^2n^2x\cosh^2(a+b\log(cx^n))}{1-20b^2n^2+64b^4n^4} + \frac{x\cosh^4(a+b\log(c}{1-16b^2n^2}$$

Mathematica [A]
time = 0.30, size = 167, normalized size = 0.87

$$\frac{x(3-60b^2n^2+192b^4n^4+(4-64b^2n^2)\cosh(2(a+b\log(cx^n)))+(1-4b^2n^2)\cosh(4(a+b\log(cx^n)))-8bn\sinh(2(a+b\log(cx^n)))+128b^3n^3\sinh(2(a+b\log(cx^n)))-4bn\sinh(4(a+b\log(cx^n)))+16b^3n^3\sinh(4(a+b\log(cx^n))))}{8(1-20b^2n^2+64b^4n^4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (x*(3 - 60*b^2*n^2 + 192*b^4*n^4 + (4 - 64*b^2*n^2)*Cosh[2*(a + b*Log[c*x^n
])] + (1 - 4*b^2*n^2)*Cosh[4*(a + b*Log[c*x^n])] - 8*b*n*Sinh[2*(a + b*Log[
c*x^n])] + 128*b^3*n^3*Sinh[2*(a + b*Log[c*x^n])] - 4*b*n*Sinh[4*(a + b*Log
[c*x^n])] + 16*b^3*n^3*Sinh[4*(a + b*Log[c*x^n])]))/(8*(1 - 20*b^2*n^2 + 64
*b^4*n^4))
```

Maple [F]
time = 3.54, size = 0, normalized size = 0.00

$$\int \cosh^4(a+b\ln(c\,x^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*ln(c*x^n))^4,x)
```

```
[Out] int(cosh(a+b*ln(c*x^n))^4,x)
```

Maxima [A]
time = 0.30, size = 129, normalized size = 0.68

$$\frac{c^{4\,b}xe^{(4\,b\log(x^n)+4\,a)}}{16\,(4\,bn+1)} + \frac{c^{2\,b}xe^{(2\,b\log(x^n)+2\,a)}}{4\,(2\,bn+1)} + \frac{3}{8}\,x - \frac{xe^{(-2\,b\log(x^n)-2\,a)}}{4\,(2\,bc^{2\,b}n-c^{2\,b})} - \frac{xe^{(-4\,a)}}{16\,(4\,bc^{4\,b}n-c^{4\,b})(x^n)^{4\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^4,x, algorithm="maxima")
```

```
[Out] 1/16*c^(4*b)*x*e^(4*b*log(x^n) + 4*a)/(4*b*n + 1) + 1/4*c^(2*b)*x*e^(2*b*lo
g(x^n) + 2*a)/(2*b*n + 1) + 3/8*x - 1/4*x*e^(-2*b*log(x^n) - 2*a)/(2*b*c^(2
*b)*n - c^(2*b)) - 1/16*x*e^(-4*a)/((4*b*c^(4*b)*n - c^(4*b))*(x^n)^(4*b))
```

Fricas [A]
time = 0.38, size = 293, normalized size = 1.53

$$-\frac{{\left(4 \, b^{2} n^{2} - 1\right)} x \cosh\left(b n \log\left(x\right) + b \log\left(c\right) + a\right)^{4} - 16 \, {\left(4 \, b^{3} n^{3} - b n\right)} x \cosh\left(b n \log\left(x\right) + b \log\left(c\right) + a\right) \sinh\left(b n \log\left(x\right) + b \log\left(c\right) + a\right)^{3} + {\left(4 \, b^{2} n^{2} - 1\right)} x \sinh\left(b n \log\left(x\right) + b \log\left(c\right) + a\right)^{4} + 4 \, {\left(16 \, b^{2} n^{2} - 1\right)} x \cosh\left(b n \log\left(x\right) + b \log\left(c\right) + a\right)^{2} + 2 \, {\left(3 \, {\left(4 \, b^{2} n^{2} - 1\right)} x \cosh\left(b n \log\left(x\right) + b \log\left(c\right) + a\right)^{2} + 2 \, {\left(16 \, b^{2} n^{2} - 1\right)} x\right)} \sinh\left(b n \log\left(x\right) + b \log\left(c\right) + a\right)^{2} - 3 \, {\left(64 \, b^{4} n^{4} - 20 \, b^{2} n^{2} + 1\right)} x - 16 \, {\left({\left(4 \, b^{3} n^{3} - b n\right)} x \cosh\left(b n \log\left(x\right) + b \log\left(c\right) + a\right)^{3} + {\left(16 \, b^{3} n^{3} - b n\right)} x \cosh\left(b n \log\left(x\right) + b \log\left(c\right) + a\right)\right)} \sinh\left(b n \log\left(x\right) + b \log\left(c\right) + a\right)}{8 \, {\left(64 \, b^{4} n^{4} - 20 \, b^{2} n^{2} + 1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

```
[Out] -1/8*((4*b^2*n^2 - 1)*x*cosh(b*n*log(x) + b*log(c) + a)^4 - 16*(4*b^3*n^3 -
 b*n)*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3 +
 (4*b^2*n^2 - 1)*x*sinh(b*n*log(x) + b*log(c) + a)^4 + 4*(16*b^2*n^2 - 1)*x
*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*(3*(4*b^2*n^2 - 1)*x*cosh(b*n*log(x)
 + b*log(c) + a)^2 + 2*(16*b^2*n^2 - 1)*x)*sinh(b*n*log(x) + b*log(c) + a)^
2 - 3*(64*b^4*n^4 - 20*b^2*n^2 + 1)*x - 16*((4*b^3*n^3 - b*n)*x*cosh(b*n*lo
g(x) + b*log(c) + a)^3 + (16*b^3*n^3 - b*n)*x*cosh(b*n*log(x) + b*log(c) +
a))*sinh(b*n*log(x) + b*log(c) + a))/(64*b^4*n^4 - 20*b^2*n^2 + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \int \cosh^{4}{\left(a - \frac{\log{\left(c x^{n} \right)}}{2 n} \right)}\, dx & \text{for}\: b = - \frac{1}{2 n} \\ \int \cosh^{4}{\left(a - \frac{\log{\left(c x^{n} \right)}}{4 n} \right)}\, dx & \text{for}\: b = - \frac{1}{4 n} \\ \int \cosh^{4}{\left(a + \frac{\log{\left(c x^{n} \right)}}{4 n} \right)}\, dx & \text{for}\: b = \frac{1}{4 n} \\ \int \cosh^{4}{\left(a + \frac{\log{\left(c x^{n} \right)}}{2 n} \right)}\, dx & \text{for}\: b = \frac{1}{2 n} \\ \frac{24 b^{4} n^{4} x \sinh^{4}{\left(a + b \log{\left(c x^{n} \right)} \right)}}{64 b^{4} n^{4} - 20 b^{2} n^{2} + 1} - \frac{48 b^{4} n^{4} x \sinh^{2}{\left(a + b \log{\left(c x^{n} \right)} \right)} \cosh^{2}{\left(a + b \log{\left(c x^{n} \right)} \right)}}{64 b^{4} n^{4} - 20 b^{2} n^{2} + 1} + \frac{24 b^{4} n^{4} x \cosh^{4}{\left(a + b \log{\left(c x^{n} \right)} \right)}}{64 b^{4} n^{4} - 20 b^{2} n^{2} + 1} - \frac{24 b^{3} n^{3} x \sinh^{3}{\left(a + b \log{\left(c x^{n} \right)} \right)} \cosh{\left(a + b \log{\left(c x^{n} \right)} \right)}}{64 b^{4} n^{4} - 20 b^{2} n^{2} + 1} + \frac{40 b^{3} n^{3} x \sinh{\left(a + b \log{\left(c x^{n} \right)} \right)} \cosh^{3}{\left(a + b \log{\left(c x^{n} \right)} \right)}}{64 b^{4} n^{4} - 20 b^{2} n^{2} + 1} + \frac{12 b^{2} n^{2} x \sinh^{2}{\left(a + b \log{\left(c x^{n} \right)} \right)} \cosh^{2}{\left(a + b \log{\left(c x^{n} \right)} \right)}}{64 b^{4} n^{4} - 20 b^{2} n^{2} + 1} - \frac{16 b^{2} n^{2} x \cosh^{4}{\left(a + b \log{\left(c x^{n} \right)} \right)}}{64 b^{4} n^{4} - 20 b^{2} n^{2} + 1} - \frac{4 b n x \sinh{\left(a + b \log{\left(c x^{n} \right)} \right)} \cosh^{3}{\left(a + b \log{\left(c x^{n} \right)} \right)}}{64 b^{4} n^{4} - 20 b^{2} n^{2} + 1} + \frac{x \cosh^{4}{\left(a + b \log{\left(c x^{n} \right)} \right)}}{64 b^{4} n^{4} - 20 b^{2} n^{2} + 1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*ln(c*x**n))**4,x)
```

```
[Out] Piecewise((Integral(cosh(a - log(c*x**n)/(2*n))**4, x), Eq(b, -1/(2*n))), (
Integral(cosh(a - log(c*x**n)/(4*n))**4, x), Eq(b, -1/(4*n))), (Integral(co
sh(a + log(c*x**n)/(4*n))**4, x), Eq(b, 1/(4*n))), (Integral(cosh(a + log(c
*x**n)/(2*n))**4, x), Eq(b, 1/(2*n))), (24*b**4*n**4*x*sinh(a + b*log(c*x**
n))**4/(64*b**4*n**4 - 20*b**2*n**2 + 1) - 48*b**4*n**4*x*sinh(a + b*log(c*
x**n))**2*cosh(a + b*log(c*x**n))**2/(64*b**4*n**4 - 20*b**2*n**2 + 1) + 24
*b**4*n**4*x*cosh(a + b*log(c*x**n))**4/(64*b**4*n**4 - 20*b**2*n**2 + 1) -
 24*b**3*n**3*x*sinh(a + b*log(c*x**n))**3*cosh(a + b*log(c*x**n))/(64*b**4
*n**4 - 20*b**2*n**2 + 1) + 40*b**3*n**3*x*sinh(a + b*log(c*x**n))*cosh(a +
 b*log(c*x**n))**3/(64*b**4*n**4 - 20*b**2*n**2 + 1) + 12*b**2*n**2*x*sinh(
a + b*log(c*x**n))**2*cosh(a + b*log(c*x**n))**2/(64*b**4*n**4 - 20*b**2*n*
*2 + 1) - 16*b**2*n**2*x*cosh(a + b*log(c*x**n))**4/(64*b**4*n**4 - 20*b**2
*n**2 + 1) - 4*b*n*x*sinh(a + b*log(c*x**n))*cosh(a + b*log(c*x**n))**3/(64
*b**4*n**4 - 20*b**2*n**2 + 1) + x*cosh(a + b*log(c*x**n))**4/(64*b**4*n**4
 - 20*b**2*n**2 + 1), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 777 vs. 2(192) = 384.
time = 0.46, size = 777, normalized size = 4.07

$$\frac{b^{3} c^{4 b} n^{3} x x^{4 b n} e^{\left(4 a\right)}}{64 b^{4} n^{4} - 20 b^{2} n^{2} + 1} + \frac{8 b^{3} c^{2 b} n^{3} x x^{2 b n} e^{\left(2 a\right)}}{64 b^{4} n^{4} - 20 b^{2} n^{2} + 1} + \frac{24 b^{4} n^{4} x}{64 b^{4} n^{4} - 20 b^{2} n^{2} + 1} - \frac{b^{2} c^{4 b} n^{2} x x^{4 b n} e^{\left(4 a\right)}}{4 \left(64 b^{4} n^{4} - 20 b^{2} n^{2} + 1\right)} - \frac{4 b^{2} c^{2 b} n^{2} x x^{2 b n} e^{\left(2 a\right)}}{64 b^{4} n^{4} - 20 b^{2} n^{2} + 1} - \frac{b c^{4 b} n x x^{4 b n} e^{\left(4 a\right)}}{4 \left(64 b^{4} n^{4} - 20 b^{2} n^{2} + 1\right)} - \frac{b c^{2 b} n x x^{2 b n} e^{\left(2 a\right)}}{2 \left(64 b^{4} n^{4} - 20 b^{2} n^{2} + 1\right)} - \frac{8 b^{3} n^{3} x e^{\left(-2 a\right)}}{\left(64 b^{4} n^{4} - 20 b^{2} n^{2} + 1\right) c^{2 b} x^{2 b n}} - \frac{b^{3} n^{3} x e^{\left(-4 a\right)}}{\left(64 b^{4} n^{4} - 20 b^{2} n^{2} + 1\right) c^{4 b} x^{4 b n}} - \frac{15 b^{2} n^{2} x}{2 \left(64 b^{4} n^{4} - 20 b^{2} n^{2} + 1\right)} + \frac{c^{4 b} x x^{4 b n} e^{\left(4 a\right)}}{16 \left(64 b^{4} n^{4} - 20 b^{2} n^{2} + 1\right)} + \frac{c^{2 b} x x^{2 b n} e^{\left(2 a\right)}}{4 \left(64 b^{4} n^{4} - 20 b^{2} n^{2} + 1\right)} - \frac{4 b^{2} n^{2} x e^{\left(-2 a\right)}}{\left(64 b^{4} n^{4} - 20 b^{2} n^{2} + 1\right) c^{2 b} x^{2 b n}} - \frac{b^{2} n^{2} x e^{\left(-4 a\right)}}{4 \left(64 b^{4} n^{4} - 20 b^{2} n^{2} + 1\right) c^{4 b} x^{4 b n}} + \frac{b n x e^{\left(-2 a\right)}}{2 \left(64 b^{4} n^{4} - 20 b^{2} n^{2} + 1\right) c^{2 b} x^{2 b n}} + \frac{b n x e^{\left(-4 a\right)}}{4 \left(64 b^{4} n^{4} - 20 b^{2} n^{2} + 1\right) c^{4 b} x^{4 b n}} + \frac{3 x}{8 \left(64 b^{4} n^{4} - 20 b^{2} n^{2} + 1\right)} + \frac{x e^{\left(-2 a\right)}}{4 \left(64 b^{4} n^{4} - 20 b^{2} n^{2} + 1\right) c^{2 b} x^{2 b n}} + \frac{x e^{\left(-4 a\right)}}{16 \left(64 b^{4} n^{4} - 20 b^{2} n^{2} + 1\right) c^{4 b} x^{4 b n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^4,x, algorithm="giac")
```

```
[Out] b^3*c^(4*b)*n^3*x*x^(4*b*n)*e^(4*a)/(64*b^4*n^4 - 20*b^2*n^2 + 1) + 8*b^3*c
^(2*b)*n^3*x*x^(2*b*n)*e^(2*a)/(64*b^4*n^4 - 20*b^2*n^2 + 1) + 24*b^4*n^4*x
/(64*b^4*n^4 - 20*b^2*n^2 + 1) - 1/4*b^2*c^(4*b)*n^2*x*x^(4*b*n)*e^(4*a)/(6
4*b^4*n^4 - 20*b^2*n^2 + 1) - 4*b^2*c^(2*b)*n^2*x*x^(2*b*n)*e^(2*a)/(64*b^4
*n^4 - 20*b^2*n^2 + 1) - 1/4*b*c^(4*b)*n*x*x^(4*b*n)*e^(4*a)/(64*b^4*n^4 -
20*b^2*n^2 + 1) - 1/2*b*c^(2*b)*n*x*x^(2*b*n)*e^(2*a)/(64*b^4*n^4 - 20*b^2*
n^2 + 1) - 8*b^3*n^3*x*e^(-2*a)/((64*b^4*n^4 - 20*b^2*n^2 + 1)*c^(2*b)*x^(2
*b*n)) - b^3*n^3*x*e^(-4*a)/((64*b^4*n^4 - 20*b^2*n^2 + 1)*c^(4*b)*x^(4*b*n
)) - 15/2*b^2*n^2*x/(64*b^4*n^4 - 20*b^2*n^2 + 1) + 1/16*c^(4*b)*x*x^(4*b*n
)*e^(4*a)/(64*b^4*n^4 - 20*b^2*n^2 + 1) + 1/4*c^(2*b)*x*x^(2*b*n)*e^(2*a)/(
64*b^4*n^4 - 20*b^2*n^2 + 1) - 4*b^2*n^2*x*e^(-2*a)/((64*b^4*n^4 - 20*b^2*n
^2 + 1)*c^(2*b)*x^(2*b*n)) - 1/4*b^2*n^2*x*e^(-4*a)/((64*b^4*n^4 - 20*b^2*n
^2 + 1)*c^(4*b)*x^(4*b*n)) + 1/2*b*n*x*e^(-2*a)/((64*b^4*n^4 - 20*b^2*n^2 +
 1)*c^(2*b)*x^(2*b*n)) + 1/4*b*n*x*e^(-4*a)/((64*b^4*n^4 - 20*b^2*n^2 + 1)*
c^(4*b)*x^(4*b*n)) + 3/8*x/(64*b^4*n^4 - 20*b^2*n^2 + 1) + 1/4*x*e^(-2*a)/(
(64*b^4*n^4 - 20*b^2*n^2 + 1)*c^(2*b)*x^(2*b*n)) + 1/16*x*e^(-4*a)/((64*b^4
*n^4 - 20*b^2*n^2 + 1)*c^(4*b)*x^(4*b*n))
```

Mupad [B]
time = 1.04, size = 102, normalized size = 0.53

$$\frac{3\,x}{8}-\frac{x\,\mathrm{e}^{-2\,a}}{{\left(c\,x^n\right)}^{2\,b}\,\left(8\,b\,n-4\right)}+\frac{x\,\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}}{8\,b\,n+4}-\frac{x\,\mathrm{e}^{-4\,a}}{{\left(c\,x^n\right)}^{4\,b}\,\left(64\,b\,n-16\right)}+\frac{x\,\mathrm{e}^{4\,a}\,{\left(c\,x^n\right)}^{4\,b}}{64\,b\,n+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*log(c*x^n))^4,x)
```

```
[Out] (3*x)/8 - (x*exp(-2*a))/((c*x^n)^(2*b)*(8*b*n - 4)) + (x*exp(2*a)*(c*x^n)^(
2*b))/(8*b*n + 4) - (x*exp(-4*a))/((c*x^n)^(4*b)*(64*b*n - 16)) + (x*exp(4*
a)*(c*x^n)^(4*b))/(64*b*n + 16)
```

3.243 $\int x^m \cosh(a + b\log(cx^n))\,dx$

Optimal. Leaf size=73

$$\frac{(1+m)x^{1+m}\cosh(a+b\log(cx^n))}{(1+m)^2-b^2n^2}-\frac{bnx^{1+m}\sinh(a+b\log(cx^n))}{(1+m)^2-b^2n^2}$$

```
[Out] (1+m)*x^(1+m)*cosh(a+b*ln(c*x^n))/((1+m)^2-b^2*n^2)-b*n*x^(1+m)*sinh(a+b*ln
(c*x^n))/((1+m)^2-b^2*n^2)
```

Rubi [A]
time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.01, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {5639}

$$\frac{(m+1)x^{m+1}\cosh(a+b\log(cx^n))}{(-bn+m+1)(bn+m+1)}-\frac{bnx^{m+1}\sinh(a+b\log(cx^n))}{(m+1)^2-b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cosh[a + b*Log[c*x^n]],x]
```

```
[Out] ((1 + m)*x^(1 + m)*Cosh[a + b*Log[c*x^n]])/((1 + m - b*n)*(1 + m + b*n)) -
(b*n*x^(1 + m)*Sinh[a + b*Log[c*x^n]])/((1 + m)^2 - b^2*n^2)
```

Rule 5639

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-(m + 1))*(e*x)^(m + 1)*(Cosh[d*(a + b*Log[c*x^n])]/(b^2*
d^2*e*n^2 - e*(m + 1)^2)), x] + Simp[b*d*n*(e*x)^(m + 1)*(Sinh[d*(a + b*Log
[c*x^n])]/(b^2*d^2*e*n^2 - e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}
, x] && NeQ[b^2*d^2*n^2 - (m + 1)^2, 0]
```

Rubi steps

$$\int x^m\cosh(a+b\log(cx^n))\,dx=\frac{(1+m)x^{1+m}\cosh(a+b\log(cx^n))}{(1+m)^2-b^2n^2}-\frac{bnx^{1+m}\sinh(a+b\log(cx^n))}{(1+m)^2-b^2n^2}$$

Mathematica [A]
time = 0.09, size = 54, normalized size = 0.74

$$\frac{x^{1+m}((1+m)\cosh(a+b\log(cx^n))-bn\sinh(a+b\log(cx^n)))}{(1+m-bn)(1+m+bn)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cosh[a + b*Log[c*x^n]],x]
[Out] (x^(1 + m)*((1 + m)*Cosh[a + b*Log[c*x^n]] - b*n*Sinh[a + b*Log[c*x^n]]))/(
(1 + m - b*n)*(1 + m + b*n))
```

Maple [F]
time = 0.41, size = 0, normalized size = 0.00

$$\int x^{m} \cosh \left(a +b \ln \left(c \,x^{n}\right)\right)d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(a+b*ln(c*x^n)),x)
[Out] int(x^m*cosh(a+b*ln(c*x^n)),x)
```

Maxima [A]
time = 0.28, size = 64, normalized size = 0.88

$$\frac{c^{b} x e^{\left(b \log \left(x^{n}\right)+m \log \left(x\right)+a\right)}}{2 \left(b n+m+1\right)}-\frac{x e^{\left(-b \log \left(x^{n}\right)+m \log \left(x\right)-a\right)}}{2 \left(b c^{b} n-c^{b} (m+1)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(a+b*log(c*x^n)),x, algorithm="maxima")
[Out] 1/2*c^b*x*e^(b*log(x^n) + m*log(x) + a)/(b*n + m + 1) - 1/2*x*e^(-b*log(x^n
) + m*log(x) - a)/(b*c^b*n - c^b*(m + 1))
```

Fricas [A]
time = 0.43, size = 99, normalized size = 1.36

$$-\frac{(m+1)x \cosh \left(bn \log \left(x\right)+b \log \left(c\right)+a\right) \cosh \left(m \log \left(x\right)\right)+(m+1)x \cosh \left(bn \log \left(x\right)+b \log \left(c\right)+a\right) \sinh \left(m \log \left(x\right)\right)-\left(bnx \cosh \left(m \log \left(x\right)\right)+bnx \sinh \left(m \log \left(x\right)\right)\right) \sinh \left(bn \log \left(x\right)+b \log \left(c\right)+a\right)}{b^{2} n^{2}-m^{2}-2\, m-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] -((m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + (m + 1)*x*cosh
(b*n*log(x) + b*log(c) + a)*sinh(m*log(x)) - (b*n*x*cosh(m*log(x)) + b*n*x*
sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a))/(b^2*n^2 - m^2 - 2*m - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \log \left(x\right) \cosh \left(a\right) & \text{for}\: b = 0 \wedge m = -1 \\ \int x^{m} \cosh \left(- a + \frac{m \log \left(c x^{n}\right)}{n} + \frac{\log \left(c x^{n}\right)}{n}\right)\, dx & \text{for}\: b = \frac{- m - 1}{n} \\ \int x^{m} \cosh \left(a + \frac{m \log \left(c x^{n}\right)}{n} + \frac{\log \left(c x^{n}\right)}{n}\right)\, dx & \text{for}\: b = \frac{m + 1}{n} \\ \frac{b n x x^{m} \sinh \left(a + b \log \left(c x^{n}\right)\right)}{b^{2} n^{2} - m^{2} - 2 m - 1} - \frac{m x x^{m} \cosh \left(a + b \log \left(c x^{n}\right)\right)}{b^{2} n^{2} - m^{2} - 2 m - 1} - \frac{x x^{m} \cosh \left(a + b \log \left(c x^{n}\right)\right)}{b^{2} n^{2} - m^{2} - 2 m - 1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(a+b*ln(c*x**n)),x)
```

```
[Out] Piecewise((log(x)*cosh(a), Eq(b, 0) & Eq(m, -1)), (Integral(x**m*cosh(-a +
m*log(c*x**n)/n + log(c*x**n)/n), x), Eq(b, (-m - 1)/n)), (Integral(x**m*co
sh(a + m*log(c*x**n)/n + log(c*x**n)/n), x), Eq(b, (m + 1)/n)), (b*n*x*x**m
*sinh(a + b*log(c*x**n))/(b**2*n**2 - m**2 - 2*m - 1) - m*x*x**m*cosh(a + b
*log(c*x**n))/(b**2*n**2 - m**2 - 2*m - 1) - x*x**m*cosh(a + b*log(c*x**n))
/(b**2*n**2 - m**2 - 2*m - 1), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(75) = 150$.
time = 0.43, size = 235, normalized size = 3.22

$$\frac{bc^b nxx^{bn}x^m e^a}{2\,(b^2n^2 - m^2 - 2\,m - 1)} - \frac{c^b mxx^{bn}x^m e^a}{2\,(b^2n^2 - m^2 - 2\,m - 1)} - \frac{c^b xx^{bn}x^m e^a}{2\,(b^2n^2 - m^2 - 2\,m - 1)} - \frac{bnxx^m e^{(-a)}}{2\,(b^2n^2 - m^2 - 2\,m - 1)c^b x^{bn}} - \frac{mxx^m e^{(-a)}}{2\,(b^2n^2 - m^2 - 2\,m - 1)c^b x^{bn}} - \frac{xx^m e^{(-a)}}{2\,(b^2n^2 - m^2 - 2\,m - 1)c^b x^{bn}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/2*b*c^b*n*x*x^(b*n)*x^m*e^a/(b^2*n^2 - m^2 - 2*m - 1) - 1/2*c^b*m*x*x^(b*
n)*x^m*e^a/(b^2*n^2 - m^2 - 2*m - 1) - 1/2*c^b*x*x^(b*n)*x^m*e^a/(b^2*n^2 -
 m^2 - 2*m - 1) - 1/2*b*n*x*x^m*e^(-a)/((b^2*n^2 - m^2 - 2*m - 1)*c^b*x^(b*
n)) - 1/2*m*x*x^m*e^(-a)/((b^2*n^2 - m^2 - 2*m - 1)*c^b*x^(b*n)) - 1/2*x*x^
m*e^(-a)/((b^2*n^2 - m^2 - 2*m - 1)*c^b*x^(b*n))
```

Mupad [**B**]
time = 1.05, size = 55, normalized size = 0.75

$$\frac{x\,x^m\,\mathrm{e}^{-a}}{{\left(c\,x^n\right)}^b\,\left(2\,m-2\,b\,n+2\right)}+\frac{x\,x^m\,\mathrm{e}^a\,{\left(c\,x^n\right)}^b}{2\,m+2\,b\,n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(a + b*log(c*x^n)),x)
```

```
[Out] (x*x^m*exp(-a))/((c*x^n)^b*(2*m - 2*b*n + 2)) + (x*x^m*exp(a)*(c*x^n)^b)/(2
*m + 2*b*n + 2)
```

3.244 $\int x^m \cosh^2(a+b\log(cx^n))\,dx$

Optimal. Leaf size=120

$$-\frac{2b^2n^2x^{1+m}}{(1+m)\left((1+m)^2-4b^2n^2\right)}+\frac{(1+m)x^{1+m}\cosh^2(a+b\log(cx^n))}{(1+m)^2-4b^2n^2}-\frac{2bnx^{1+m}\cosh(a+b\log(cx^n))\sinh(a+}{(1+m)^2-4b^2n^2}$$

```
[Out] -2*b^2*n^2*x^(1+m)/(1+m)/((1+m)^2-4*b^2*n^2)+(1+m)*x^(1+m)*cosh(a+b*ln(c*x^
n))^2/((1+m)^2-4*b^2*n^2)-2*b*n*x^(1+m)*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x
^n))/((1+m)^2-4*b^2*n^2)
```

Rubi [A]
time = 0.03, antiderivative size = 122, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5641, 30}

$$\frac{(m+1)x^{m+1}\cosh^2(a+b\log(cx^n))}{-4b^2n^2+m^2+2m+1}-\frac{2bnx^{m+1}\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{(m+1)^2-4b^2n^2}-\frac{2b^2n^2x^{m+1}}{(m+1)\left((m+1)^2-4b^2n^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cosh[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (-2*b^2*n^2*x^(1 + m))/((1 + m)*((1 + m)^2 - 4*b^2*n^2)) + ((1 + m)*x^(1 +
m)*Cosh[a + b*Log[c*x^n]]^2)/(1 + 2*m + m^2 - 4*b^2*n^2) - (2*b*n*x^(1 + m)
*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]])/((1 + m)^2 - 4*b^2*n^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5641

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-(m + 1))*(e*x)^(m + 1)*(Cosh[d*(a + b*Log[c*x^n])]^
p/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2)), x] + (Dist[b^2*d^2*n^2*p*((p - 1)/(b^
2*d^2*n^2*p^2 - (m + 1)^2)), Int[(e*x)^m*Cosh[d*(a + b*Log[c*x^n])]^(p - 2)
, x], x] + Simp[b*d*n*p*(e*x)^(m + 1)*Sinh[d*(a + b*Log[c*x^n])]*(Cosh[d*(a
 + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2)), x]) /; FreeQ[
{a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - (m + 1)^2,
 0]
```

Rubi steps

$$\int x^m \cosh^2(a+b\log(cx^n))\,dx = \frac{(1+m)x^{1+m}\cosh^2(a+b\log(cx^n))}{(1+m)^2-4b^2n^2} - \frac{2bnx^{1+m}\cosh(a+b\log(cx^n))\sin}{(1+m)^2-4b^2n}$$
$$= -\frac{2b^2n^2x^{1+m}}{(1+m)\left((1+m)^2-4b^2n^2\right)} + \frac{(1+m)x^{1+m}\cosh^2(a+b\log(cx^n))}{(1+m)^2-4b^2n^2} - $$

Mathematica [A]
time = 0.20, size = 87, normalized size = 0.72

$$\frac{x^{1+m}(1+2m+m^2-4b^2n^2+(1+m)^2\cosh(2(a+b\log(cx^n)))-2b(1+m)n\sinh(2(a+b\log(cx^n))))}{2(1+m)(1+m-2bn)(1+m+2bn)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cosh[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (x^(1 + m)*(1 + 2*m + m^2 - 4*b^2*n^2 + (1 + m)^2*Cosh[2*(a + b*Log[c*x^n])
] - 2*b*(1 + m)*n*Sinh[2*(a + b*Log[c*x^n])]))/(2*(1 + m)*(1 + m - 2*b*n)*(
1 + m + 2*b*n))
```

Maple [F]
time = 2.90, size = 0, normalized size = 0.00

$$\int x^m\left(\cosh^2(a+b\ln(c\,x^n))\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(a+b*ln(c*x^n))^2,x)
```

```
[Out] int(x^m*cosh(a+b*ln(c*x^n))^2,x)
```

Maxima [A]
time = 0.28, size = 87, normalized size = 0.72

$$\frac{c^{2\,b}xe^{(2\,b\log(x^n)+m\log(x)+2\,a)}}{4\,(2\,bn+m+1)} - \frac{xe^{(-2\,b\log(x^n)+m\log(x)-2\,a)}}{4\,(2\,bc^{2\,b}n-c^{2\,b}(m+1))} + \frac{x^{m+1}}{2\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 1/4*c^(2*b)*x*e^(2*b*log(x^n) + m*log(x) + 2*a)/(2*b*n + m + 1) - 1/4*x*e^(
-2*b*log(x^n) + m*log(x) - 2*a)/(2*b*c^(2*b)*n - c^(2*b)*(m + 1)) + 1/2*x^(
m + 1)/(m + 1)
```

Fricas [A]
time = 0.37, size = 250, normalized size = 2.08

$$\frac{(m^2+2m+1)x\cosh(bn\log(x)+b\log(c)+a)^2\cosh(m\log(x)) - (4b^2n^2-m^2-2m-1)x\cosh(m\log(x)) + ((m^2+2m+1)x\cosh(m\log(x)) + (m^2+2m+1)x\sinh(m\log(x)))\sinh(bn\log(x)+b\log(c)+a)^2 - 4((bm+b)nx\cosh(bn\log(x)+b\log(c)+a)\cosh(m\log(x)) + (bm+b)nx\cosh(bn\log(x)+b\log(c)+a)\sinh(m\log(x)))\sinh(bn\log(x)+b\log(c)+a) + ((m^2+2m+1)x\cosh(bn\log(x)+b\log(c)+a)^2 - (4b^2n^2-m^2-2m-1)x)\sinh(m\log(x))}{2(m^3-4(b^2m+b^2)n^2+3m^2+3m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] 1/2*((m^2 + 2*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2*cosh(m*log(x)) - (
4*b^2*n^2 - m^2 - 2*m - 1)*x*cosh(m*log(x)) + ((m^2 + 2*m + 1)*x*cosh(m*log
(x)) + (m^2 + 2*m + 1)*x*sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a)^2
- 4*((b*m + b)*n*x*cosh(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + (b*m +
b)*n*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(m*log(x)))*sinh(b*n*log(x) + b*
log(c) + a) + ((m^2 + 2*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2 - (4*b^2
*n^2 - m^2 - 2*m - 1)*x)*sinh(m*log(x)))/(m^3 - 4*(b^2*m + b^2)*n^2 + 3*m^2
 + 3*m + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \log(x)\cosh^2(a) & \text{for}\: b = 0 \wedge m = -1 \\ \int x^m \cosh^2\left(-a + \frac{m\log(cx^n)}{2n} + \frac{\log(cx^n)}{2n}\right) dx & \text{for}\: b = \frac{-m-1}{2n} \\ \int x^m \cosh^2\left(a + \frac{m\log(cx^n)}{2n} + \frac{\log(cx^n)}{2n}\right) dx & \text{for}\: b = \frac{m+1}{2n} \\ \int \frac{\cosh^2(a+b\log(cx^n))}{x}\, dx & \text{for}\: m = -1 \\ -\frac{2b^2n^2xx^m\sinh^2(a+b\log(cx^n))}{4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1} + \frac{2b^2n^2xx^m\cosh^2(a+b\log(cx^n))}{4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1} + \frac{2bmnxx^m\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1} + \frac{2bnxx^m\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1} - \frac{m^2xx^m\cosh^2(a+b\log(cx^n))}{4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1} - \frac{2mxx^m\cosh^2(a+b\log(cx^n))}{4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1} - \frac{xx^m\cosh^2(a+b\log(cx^n))}{4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(a+b*ln(c*x**n))**2,x)
```

```
[Out] Piecewise((log(x)*cosh(a)**2, Eq(b, 0) & Eq(m, -1)), (Integral(x**m*cosh(-a
 + m*log(c*x**n)/(2*n) + log(c*x**n)/(2*n))**2, x), Eq(b, (-m - 1)/(2*n))),
 (Integral(x**m*cosh(a + m*log(c*x**n)/(2*n) + log(c*x**n)/(2*n))**2, x), E
q(b, (m + 1)/(2*n))), (Integral(cosh(a + b*log(c*x**n))**2/x, x), Eq(m, -1)
), (-2*b**2*n**2*x*x**m*sinh(a + b*log(c*x**n))**2/(4*b**2*m*n**2 + 4*b**2*
n**2 - m**3 - 3*m**2 - 3*m - 1) + 2*b**2*n**2*x*x**m*cosh(a + b*log(c*x**n)
)**2/(4*b**2*m*n**2 + 4*b**2*n**2 - m**3 - 3*m**2 - 3*m - 1) + 2*b*m*n*x*x*
*m*sinh(a + b*log(c*x**n))*cosh(a + b*log(c*x**n))/(4*b**2*m*n**2 + 4*b**2*
n**2 - m**3 - 3*m**2 - 3*m - 1) + 2*b*n*x*x**m*sinh(a + b*log(c*x**n))*cosh
(a + b*log(c*x**n))/(4*b**2*m*n**2 + 4*b**2*n**2 - m**3 - 3*m**2 - 3*m - 1)
 - m**2*x*x**m*cosh(a + b*log(c*x**n))**2/(4*b**2*m*n**2 + 4*b**2*n**2 - m*
*3 - 3*m**2 - 3*m - 1) - 2*m*x*x**m*cosh(a + b*log(c*x**n))**2/(4*b**2*m*n*
*2 + 4*b**2*n**2 - m**3 - 3*m**2 - 3*m - 1) - x*x**m*cosh(a + b*log(c*x**n)
)**2/(4*b**2*m*n**2 + 4*b**2*n**2 - m**3 - 3*m**2 - 3*m - 1), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 759 vs. $2(127) = 254$.
time = 0.44, size = 759, normalized size = 6.32

$$\frac{bc^{2b}mnxx^{2bn}x^me^{(2a)}}{2(4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1)} - \frac{c^{2b}m^2xx^{2bn}x^me^{(2a)}}{4(4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1)} + \frac{bc^{2b}nxx^{2bn}x^me^{(2a)}}{2(4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1)} + \frac{2b^2n^2xx^m}{4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1} - \frac{c^{2b}mxx^{2bn}x^me^{(2a)}}{2(4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1)} - \frac{c^{2b}xx^{2bn}x^me^{(2a)}}{4(4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1)} - \frac{m^2xx^m}{2(4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1)} - \frac{bmnxx^me^{(-2a)}}{2(4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1)c^{2b}x^{2bn}} - \frac{mxx^m}{4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1} - \frac{m^2xx^me^{(-2a)}}{4(4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1)c^{2b}x^{2bn}} - \frac{bnxx^me^{(-2a)}}{2(4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1)c^{2b}x^{2bn}} - \frac{xx^m}{2(4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1)} - \frac{mxx^me^{(-2a)}}{2(4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1)c^{2b}x^{2bn}} - \frac{xx^me^{(-2a)}}{4(4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1)c^{2b}x^{2bn}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] 1/2*b*c^(2*b)*m*n*x*x^(2*b*n)*x^m*e^(2*a)/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 -
3*m^2 - 3*m - 1) - 1/4*c^(2*b)*m^2*x*x^(2*b*n)*x^m*e^(2*a)/(4*b^2*m*n^2 + 4
*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) + 1/2*b*c^(2*b)*n*x*x^(2*b*n)*x^m*e^(2*a)
/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) + 2*b^2*n^2*x*x^m/(4*b^2
*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1/2*c^(2*b)*m*x*x^(2*b*n)*x^m
*e^(2*a)/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1/4*c^(2*b)*x*
x^(2*b*n)*x^m*e^(2*a)/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1
/2*m^2*x*x^m/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1/2*b*m*n*
x*x^m*e^(-2*a)/((4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1)*c^(2*b)*x
^(2*b*n)) - m*x*x^m/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1/4
*m^2*x*x^m*e^(-2*a)/((4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1)*c^(2
*b)*x^(2*b*n)) - 1/2*b*n*x*x^m*e^(-2*a)/((4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3
*m^2 - 3*m - 1)*c^(2*b)*x^(2*b*n)) - 1/2*x*x^m/(4*b^2*m*n^2 + 4*b^2*n^2 - m
^3 - 3*m^2 - 3*m - 1) - 1/2*m*x*x^m*e^(-2*a)/((4*b^2*m*n^2 + 4*b^2*n^2 - m^
3 - 3*m^2 - 3*m - 1)*c^(2*b)*x^(2*b*n)) - 1/4*x*x^m*e^(-2*a)/((4*b^2*m*n^2
+ 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1)*c^(2*b)*x^(2*b*n))
```

Mupad [B]
time = 1.09, size = 73, normalized size = 0.61

$$\frac{x\,x^m}{2\,m+2}+\frac{x\,x^m\,\mathrm{e}^{-2\,a}}{{\left(c\,x^n\right)}^{2\,b}\,\left(4\,m-8\,b\,n+4\right)}+\frac{x\,x^m\,\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}}{4\,m+8\,b\,n+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(a + b*log(c*x^n))^2,x)
```

```
[Out] (x*x^m)/(2*m + 2) + (x*x^m*exp(-2*a))/((c*x^n)^(2*b)*(4*m - 8*b*n + 4)) + (
x*x^m*exp(2*a)*(c*x^n)^(2*b))/(4*m + 8*b*n + 4)
```

3.245 $\int x^m \cosh^3(a + b\log(cx^n))\,dx$

Optimal. Leaf size=203

$$-\frac{6b^2(1+m)n^2x^{1+m}\cosh(a+b\log(cx^n))}{((1+m)^2-9b^2n^2)((1+m)^2-b^2n^2)}+\frac{(1+m)x^{1+m}\cosh^3(a+b\log(cx^n))}{(1+m)^2-9b^2n^2}+\frac{6b^3n^3x^{1+m}\sinh(a+b\log}{((1+m)^2-9b^2n^2)((1+m}$$

```
[Out] -6*b^2*(1+m)*n^2*x^(1+m)*cosh(a+b*ln(c*x^n))/((1+m)^2-9*b^2*n^2)/((1+m)^2-b
^2*n^2)+(1+m)*x^(1+m)*cosh(a+b*ln(c*x^n))^3/((1+m)^2-9*b^2*n^2)+6*b^3*n^3*x
^(1+m)*sinh(a+b*ln(c*x^n))/((1+m)^2-9*b^2*n^2)/((1+m)^2-b^2*n^2)-3*b*n*x^(1
+m)*cosh(a+b*ln(c*x^n))^2*sinh(a+b*ln(c*x^n))/((1+m)^2-9*b^2*n^2)
```

Rubi [A]
time = 0.07, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5641, 5639}

$$\frac{(m+1)x^{m+1}\cosh^3(a+b\log(cx^n))}{-9b^2n^2+m^2+2m+1}-\frac{6b^2(m+1)n^2x^{m+1}\cosh(a+b\log(cx^n))}{(-bn+m+1)(bn+m+1)((m+1)^2-9b^2n^2)}-\frac{3bnx^{m+1}\sinh(a+b\log(cx^n))\cosh^2(a+b\log(cx^n))}{(m+1)^2-9b^2n^2}+\frac{6b^3n^3x^{m+1}\sinh(a+b\log(cx^n))}{9b^4n^4-10b^2(m+1)^2n^2+(m+1)^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cosh[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (-6*b^2*(1 + m)*n^2*x^(1 + m)*Cosh[a + b*Log[c*x^n]])/((1 + m - b*n)*(1 + m
 + b*n)*((1 + m)^2 - 9*b^2*n^2)) + ((1 + m)*x^(1 + m)*Cosh[a + b*Log[c*x^n]
]^3)/(1 + 2*m + m^2 - 9*b^2*n^2) + (6*b^3*n^3*x^(1 + m)*Sinh[a + b*Log[c*x^
n]])/((1 + m)^4 - 10*b^2*(1 + m)^2*n^2 + 9*b^4*n^4) - (3*b*n*x^(1 + m)*Cosh
[a + b*Log[c*x^n]]^2*Sinh[a + b*Log[c*x^n]])/((1 + m)^2 - 9*b^2*n^2)
```

Rule 5639

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-(m + 1))*(e*x)^(m + 1)*(Cosh[d*(a + b*Log[c*x^n])]/(b^2*
d^2*e*n^2 - e*(m + 1)^2)), x] + Simp[b*d*n*(e*x)^(m + 1)*(Sinh[d*(a + b*Log
[c*x^n])]/(b^2*d^2*e*n^2 - e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}
, x] && NeQ[b^2*d^2*n^2 - (m + 1)^2, 0]
```

Rule 5641

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-(m + 1))*(e*x)^(m + 1)*(Cosh[d*(a + b*Log[c*x^n])]^
p/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2)), x] + (Dist[b^2*d^2*n^2*p*((p - 1)/(b^
2*d^2*n^2*p^2 - (m + 1)^2)), Int[(e*x)^m*Cosh[d*(a + b*Log[c*x^n])]^(p - 2)
, x], x] + Simp[b*d*n*p*(e*x)^(m + 1)*Sinh[d*(a + b*Log[c*x^n])]*(Cosh[d*(a
 + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2)), x]) /; FreeQ[
{a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - (m + 1)^2,
 0]
```

Rubi steps

$$\int x^m \cosh^3(a+b\log(cx^n))\,dx = \frac{(1+m)x^{1+m}\cosh^3(a+b\log(cx^n))}{(1+m)^2-9b^2n^2} - \frac{3bnx^{1+m}\cosh^2(a+b\log(cx^n))\sin}{(1+m)^2-9b^2n}$$
$$= -\frac{6b^2(1+m)n^2x^{1+m}\cosh(a+b\log(cx^n))}{(1+m)^4-10b^2(1+m)^2n^2+9b^4n^4} + \frac{(1+m)x^{1+m}\cosh^3(a+b\log}{(1+m)^2-9b^2n^2}$$

Mathematica [A]
time = 0.93, size = 292, normalized size = 1.44

$$\frac{1}{4}x^{1+m}\left(\frac{3\sinh(bn\log(x))\left(-bn\cosh(a-bn\log(x)+b\log(cx^n))+(1+m)\sinh(a-bn\log(x)+b\log(cx^n))\right)}{(1+m-bn)(1+m+bn)}+\frac{3\cosh(bn\log(x))\left((1+m)\cosh(a-bn\log(x)+b\log(cx^n))-bn\sinh(a-bn\log(x)+b\log(cx^n))\right)}{(1+m-bn)(1+m+bn)}+\frac{\sinh(3bn\log(x))\left(-3bn\cosh(3(a-bn\log(x)+b\log(cx^n)))+(1+m)\sinh(3(a-bn\log(x)+b\log(cx^n)))\right)}{(1+m-3bn)(1+m+3bn)}+\frac{\cosh(3bn\log(x))\left((1+m)\cosh(3(a-bn\log(x)+b\log(cx^n)))-3bn\sinh(3(a-bn\log(x)+b\log(cx^n)))\right)}{(1+m-3bn)(1+m+3bn)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cosh[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (x^(1 + m)*((3*Sinh[b*n*Log[x]]*(-(b*n*Cosh[a - b*n*Log[x] + b*Log[c*x^n]])
 + (1 + m)*Sinh[a - b*n*Log[x] + b*Log[c*x^n]]))/((1 + m - b*n)*(1 + m + b*
n)) + (3*Cosh[b*n*Log[x]]*((1 + m)*Cosh[a - b*n*Log[x] + b*Log[c*x^n]] - b*
n*Sinh[a - b*n*Log[x] + b*Log[c*x^n]]))/((1 + m - b*n)*(1 + m + b*n)) + (Si
nh[3*b*n*Log[x]]*(-3*b*n*Cosh[3*(a - b*n*Log[x] + b*Log[c*x^n])] + (1 + m)*
Sinh[3*(a - b*n*Log[x] + b*Log[c*x^n])]))/((1 + m - 3*b*n)*(1 + m + 3*b*n))
 + (Cosh[3*b*n*Log[x]]*((1 + m)*Cosh[3*(a - b*n*Log[x] + b*Log[c*x^n])] - 3
*b*n*Sinh[3*(a - b*n*Log[x] + b*Log[c*x^n])]))/((1 + m - 3*b*n)*(1 + m + 3*
b*n))))/4
```

Maple [F]
time = 2.52, size = 0, normalized size = 0.00

$$\int x^m \left(\cosh^3(a+b\ln(c\,x^n))\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(a+b*ln(c*x^n))^3,x)
```

```
[Out] int(x^m*cosh(a+b*ln(c*x^n))^3,x)
```

Maxima [A]
time = 0.30, size = 138, normalized size = 0.68

$$\frac{c^{3\,b}xe^{(3\,b\log(x^n)+m\log(x)+3\,a)}}{8\,(3\,bn+m+1)}+\frac{3\,c^bxe^{(b\log(x^n)+m\log(x)+a)}}{8\,(bn+m+1)}-\frac{3\,xe^{(-b\log(x^n)+m\log(x)-a)}}{8\,(bc^bn-c^b(m+1))}-\frac{xe^{(-3\,b\log(x^n)+m\log(x)-3\,a)}}{8\,(3\,bc^{3\,b}n-c^{3\,b}(m+1))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] 1/8*c^(3*b)*x*e^(3*b*log(x^n) + m*log(x) + 3*a)/(3*b*n + m + 1) + 3/8*c^b*x
*e^(b*log(x^n) + m*log(x) + a)/(b*n + m + 1) - 3/8*x*e^(-b*log(x^n) + m*log
(x) - a)/(b*c^b*n - c^b*(m + 1)) - 1/8*x*e^(-3*b*log(x^n) + m*log(x) - 3*a)
/(3*b*c^(3*b)*n - c^(3*b)*(m + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 584 vs. $2(214) = 428$.
time = 0.40, size = 584, normalized size = 2.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] 1/4*((m^3 - (b^2*m + b^2)*n^2 + 3*m^2 + 3*m + 1)*x*cosh(b*n*log(x) + b*log(
c) + a)^3*cosh(m*log(x)) + 3*(m^3 - 9*(b^2*m + b^2)*n^2 + 3*m^2 + 3*m + 1)*
x*cosh(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + 3*((b^3*n^3 - (b*m^2 + 2
*b*m + b)*n)*x*cosh(m*log(x)) + (b^3*n^3 - (b*m^2 + 2*b*m + b)*n)*x*sinh(m*
log(x)))*sinh(b*n*log(x) + b*log(c) + a)^3 + 3*((m^3 - (b^2*m + b^2)*n^2 +
3*m^2 + 3*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + (m^3 -
(b^2*m + b^2)*n^2 + 3*m^2 + 3*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)*sinh
(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a)^2 + 3*(3*(b^3*n^3 - (b*m^2 + 2*
b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)^2*cosh(m*log(x)) + (9*b^3*n^3
 - (b*m^2 + 2*b*m + b)*n)*x*cosh(m*log(x)) + (3*(b^3*n^3 - (b*m^2 + 2*b*m +
 b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)^2 + (9*b^3*n^3 - (b*m^2 + 2*b*m +
b)*n)*x)*sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a) + ((m^3 - (b^2*m +
 b^2)*n^2 + 3*m^2 + 3*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*(m^3 -
 9*(b^2*m + b^2)*n^2 + 3*m^2 + 3*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a))*
sinh(m*log(x)))/(9*b^4*n^4 + m^4 + 4*m^3 - 10*(b^2*m^2 + 2*b^2*m + b^2)*n^2
 + 6*m^2 + 4*m + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(a+b*ln(c*x**n))**3,x)
```

```
[Out] Piecewise((log(x)*cosh(a)**3, Eq(b, 0) & Eq(m, -1)), (Integral(x**m*cosh(-a
 + m*log(c*x**n)/(3*n) + log(c*x**n)/(3*n))**3, x), Eq(b, (-m - 1)/(3*n))),
 (Integral(x**m*cosh(-a + m*log(c*x**n)/n + log(c*x**n)/n)**3, x), Eq(b, (-
m - 1)/n)), (Integral(x**m*cosh(a + m*log(c*x**n)/(3*n) + log(c*x**n)/(3*n)
)**3, x), Eq(b, (m + 1)/(3*n))), (Integral(x**m*cosh(a + m*log(c*x**n)/n +
```

```
log(c*x**n)/n)**3, x), Eq(b, (m + 1)/n)), (-6*b**3*n**3*x*x**m*sinh(a + b*l
og(c*x**n))**3/(9*b**4*n**4 - 10*b**2*m**2*n**2 - 20*b**2*m*n**2 - 10*b**2*
n**2 + m**4 + 4*m**3 + 6*m**2 + 4*m + 1) + 9*b**3*n**3*x*x**m*sinh(a + b*lo
g(c*x**n))*cosh(a + b*log(c*x**n))**2/(9*b**4*n**4 - 10*b**2*m**2*n**2 - 20
*b**2*m*n**2 - 10*b**2*n**2 + m**4 + 4*m**3 + 6*m**2 + 4*m + 1) + 6*b**2*m*
n**2*x*x**m*sinh(a + b*log(c*x**n))**2*cosh(a + b*log(c*x**n))/(9*b**4*n**4
 - 10*b**2*m**2*n**2 - 20*b**2*m*n**2 - 10*b**2*n**2 + m**4 + 4*m**3 + 6*m*
*2 + 4*m + 1) - 7*b**2*m*n**2*x*x**m*cosh(a + b*log(c*x**n))**3/(9*b**4*n**
4 - 10*b**2*m**2*n**2 - 20*b**2*m*n**2 - 10*b**2*n**2 + m**4 + 4*m**3 + 6*m
**2 + 4*m + 1) + 6*b**2*n**2*x*x**m*sinh(a + b*log(c*x**n))**2*cosh(a + b*l
og(c*x**n))/(9*b**4*n**4 - 10*b**2*m**2*n**2 - 20*b**2*m*n**2 - 10*b**2*n**
2 + m**4 + 4*m**3 + 6*m**2 + 4*m + 1) - 7*b**2*n**2*x*x**m*cosh(a + b*log(c
*x**n))**3/(9*b**4*n**4 - 10*b**2*m**2*n**2 - 20*b**2*m*n**2 - 10*b**2*n**2
 + m**4 + 4*m**3 + 6*m**2 + 4*m + 1) - 3*b*m**2*n*x*x**m*sinh(a + b*log(c*x
**n))*cosh(a + b*log(c*x**n))**2/(9*b**4*n**4 - 10*b**2*m**2*n**2 - 20*b**2
*m*n**2 - 10*b**2*n**2 + m**4 + 4*m**3 + 6*m**2 + 4*m + 1) - 6*b*m*n*x*x**m
*sinh(a + b*log(c*x**n))*cosh(a + b*log(c*x**n))**2/(9*b**4*n**4 - 10*b**2*
m**2*n**2 - 20*b**2*m*n**2 - 10*b**2*n**2 + m**4 + 4*m**3 + 6*m**2 + 4*m +
1) - 3*b*n*x*x**m*sinh(a + b*log(c*x**n))*cosh(a + b*log(c*x**n))**2/(9*b**
4*n**4 - 10*b**2*m**2*n**2 - 20*b**2*m*n**2 - 10*b**2*n**2 + m**4 + 4*m**3
+ 6*m**2 + 4*m + 1) + m**3*x*x**m*cosh(a + b*log(c*x**n))**3/(9*b**4*n**4 -
 10*b**2*m**2*n**2 - 20*b**2*m*n**2 - 10*b**2*n**2 + m**4 + 4*m**3 + 6*m**2
 + 4*m + 1) + 3*m**2*x*x**m*cosh(a + b*log(c*x**n))**3/(9*b**4*n**4 - 10*b*
*2*m**2*n**2 - 20*b**2*m*n**2 - 10*b**2*n**2 + m**4 + 4*m**3 + 6*m**2 + 4*m
 + 1) + 3*m*x*x**m*cosh(a + b*log(c*x**n))**3/(9*b**4*n**4 - 10*b**2*m**2*n
**2 - 20*b**2*m*n**2 - 10*b**2*n**2 + m**4 + 4*m**3 + 6*m**2 + 4*m + 1) + x
*x**m*cosh(a + b*log(c*x**n))**3/(9*b**4*n**4 - 10*b**2*m**2*n**2 - 20*b**2
*m*n**2 - 10*b**2*n**2 + m**4 + 4*m**3 + 6*m**2 + 4*m + 1), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 3225 vs. $2(214) = 428$.
time = 0.53, size = 3225, normalized size = 15.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] 3/8*b^3*c^(3*b)*n^3*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 2
0*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 27/8*b^3*c^b*n^
3*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b
^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 1/8*b^2*c^(3*b)*m*n^2*x*x^(3*b*n)*x^m*e
^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^
3 + 6*m^2 + 4*m + 1) - 27/8*b^2*c^b*m*n^2*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10
*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) -
```

3/8*b*c^(3*b)*m^2*n*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 1/8*b^2*c^(3*b)*n^2*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 3/8*b*c^b*m^2*n*x*x^(b*n)*x^m *e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 27/8*b^2*c^b*n^2*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2 *m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 1/8 *c^(3*b)*m^3*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m *n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 3/4*b*c^(3*b)*m*n*x*x^ (3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b ^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 3/8*c^b*m^3*x*x^(b*n)*x^m*e^a/(9*b^4*n^ 4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 3/4*b*c^b*m*n*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2 *m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 3/8*c^(3*b)*m^2*x*x^ (3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b ^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 3/8*b*c^(3*b)*n*x*x^(3*b*n)*x^m*e^(3*a) /(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6* m^2 + 4*m + 1) - 27/8*b^3*n^3*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 2 0*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n)) - 3/ 8*b^3*n^3*x*x^m*e^(-3*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) + 9/8*c^b*m^2*x* x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n ^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 3/8*b*c^b*n*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 3/8*c^(3*b)*m*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b ^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 27/8*b^2*m*n^2*x*x ^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n)) - 1/8*b^2*m*n^2*x*x^m*e^(-3*a)/((9*b^ 4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) + 9/8*c^b*m*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b ^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 1 /8*c^(3*b)*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n ^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 3/8*b*m^2*n*x*x^m*e^(-a) /((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6 *m^2 + 4*m + 1)*c^b*x^(b*n)) - 27/8*b^2*n^2*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b ^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b *x^(b*n)) + 3/8*b*m^2*n*x*x^m*e^(-3*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^ 2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) - 1/8*b^2*n^2*x*x^m*e^(-3*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^ 4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) + 3/8*c^b*x*x^ (b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 3/8*m^3*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2 *n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b* n)) + 3/4*b*m*n*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n)) + 1/8*m^3*x*x^m*e^

```
(-3*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m
^3 + 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) + 3/4*b*m*n*x*x^m*e^(-3*a)/((9*b^4
*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4
*m + 1)*c^(3*b)*x^(3*b*n)) + 9/8*m^2*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*
n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n
)) + 3/8*b*n*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4
 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n)) + 3/8*m^2*x*x^m*e^(-3
*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3
+ 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) + 3/8*b*n*x*x^m*e^(-3*a)/((9*b^4*n^4
- 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m +
1)*c^(3*b)*x^(3*b*n)) + 9/8*m*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 2
0*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n)) + 3/
8*m*x*x^m*e^(-3*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b
^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) + 3/8*x*x^m*e^(-a)/((9
*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 ...
```

Mupad [B]
time = 1.18, size = 117, normalized size = 0.58

$$\frac{3\,x\,x^m\,\mathrm{e}^{-a}}{{\left(c\,x^n\right)}^b\,\left(8\,m-8\,b\,n+8\right)}+\frac{x\,x^m\,\mathrm{e}^{-3\,a}}{{\left(c\,x^n\right)}^{3\,b}\,\left(8\,m-24\,b\,n+8\right)}+\frac{x\,x^m\,\mathrm{e}^{3\,a}\,{\left(c\,x^n\right)}^{3\,b}}{8\,m+24\,b\,n+8}+\frac{3\,x\,x^m\,\mathrm{e}^a\,{\left(c\,x^n\right)}^b}{8\,m+8\,b\,n+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(a + b*log(c*x^n))^3,x)
```

```
[Out] (3*x*x^m*exp(-a))/((c*x^n)^b*(8*m - 8*b*n + 8)) + (x*x^m*exp(-3*a))/((c*x^n
)^(3*b)*(8*m - 24*b*n + 8)) + (x*x^m*exp(3*a)*(c*x^n)^(3*b))/(8*m + 24*b*n
+ 8) + (3*x*x^m*exp(a)*(c*x^n)^b)/(8*m + 8*b*n + 8)
```

3.246 $\int x^m \cosh^4(a + b\log(cx^n))\,dx$

Optimal. Leaf size=266

$$\frac{24b^4n^4x^{1+m}}{(1+m)\left((1+m)^2-16b^2n^2\right)\left((1+m)^2-4b^2n^2\right)}-\frac{12b^2(1+m)n^2x^{1+m}\cosh^2(a+b\log(cx^n))}{\left((1+m)^2-16b^2n^2\right)\left((1+m)^2-4b^2n^2\right)}+\frac{(1+m)x^{1+m}}{(1+}$$

```
[Out] 24*b^4*n^4*x^(1+m)/(1+m)/((1+m)^2-16*b^2*n^2)/((1+m)^2-4*b^2*n^2)-12*b^2*(1
+m)*n^2*x^(1+m)*cosh(a+b*ln(c*x^n))^2/((1+m)^2-16*b^2*n^2)/((1+m)^2-4*b^2*n
^2)+(1+m)*x^(1+m)*cosh(a+b*ln(c*x^n))^4/((1+m)^2-16*b^2*n^2)+24*b^3*n^3*x^(
1+m)*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))/((1+m)^2-16*b^2*n^2)/((1+m)^2-
4*b^2*n^2)-4*b*n*x^(1+m)*cosh(a+b*ln(c*x^n))^3*sinh(a+b*ln(c*x^n))/((1+m)^2
-16*b^2*n^2)
```

Rubi [A]
time = 0.10, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5641, 30}

$$\frac{(m+1)x^{m+1}\cosh^4(a+b\log(cx^n))}{-16b^2n^2+m^2+2m+1}-\frac{12b^2(m+1)n^2x^{m+1}\cosh^2(a+b\log(cx^n))}{\left((m+1)^2-16b^2n^2\right)\left(-4b^2n^2+m^2+2m+1\right)}-\frac{4bnx^{m+1}\sinh(a+b\log(cx^n))\cosh^3(a+b\log(cx^n))}{(m+1)^2-16b^2n^2}+\frac{24b^3n^3x^{m+1}\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{64b^4n^4-20b^2(m+1)^2n^2+(m+1)^4}+\frac{24b^4n^4x^{m+1}}{(m+1)\left((m+1)^2-16b^2n^2\right)\left((m+1)^2-4b^2n^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cosh[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (24*b^4*n^4*x^(1 + m))/((1 + m)*((1 + m)^2 - 16*b^2*n^2)*((1 + m)^2 - 4*b^2
*n^2)) - (12*b^2*(1 + m)*n^2*x^(1 + m)*Cosh[a + b*Log[c*x^n]]^2)/(((1 + m)^
2 - 16*b^2*n^2)*(1 + 2*m + m^2 - 4*b^2*n^2)) + ((1 + m)*x^(1 + m)*Cosh[a +
b*Log[c*x^n]]^4)/(1 + 2*m + m^2 - 16*b^2*n^2) + (24*b^3*n^3*x^(1 + m)*Cosh[
a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]])/((1 + m)^4 - 20*b^2*(1 + m)^2*n^2
 + 64*b^4*n^4) - (4*b*n*x^(1 + m)*Cosh[a + b*Log[c*x^n]]^3*Sinh[a + b*Log[c
*x^n]])/((1 + m)^2 - 16*b^2*n^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5641

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-(m + 1))*(e*x)^(m + 1)*(Cosh[d*(a + b*Log[c*x^n])]^
p/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2)), x] + (Dist[b^2*d^2*n^2*p*((p - 1)/(b^
2*d^2*n^2*p^2 - (m + 1)^2)), Int[(e*x)^m*Cosh[d*(a + b*Log[c*x^n])]^(p - 2)
, x], x] + Simp[b*d*n*p*(e*x)^(m + 1)*Sinh[d*(a + b*Log[c*x^n])]*(Cosh[d*(a
 + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2)), x]) /; FreeQ[
{a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - (m + 1)^2,
```

```
0]
```

Rubi steps

$$\int x^m \cosh^4(a+b\log(cx^n))\,dx = \frac{(1+m)x^{1+m}\cosh^4(a+b\log(cx^n))}{(1+m)^2-16b^2n^2} - \frac{4bnx^{1+m}\cosh^3(a+b\log(cx^n))\sin}{(1+m)^2-16b^2n}$$

$$= -\frac{12b^2(1+m)n^2x^{1+m}\cosh^2(a+b\log(cx^n))}{(1+m)^4-20b^2(1+m)^2n^2+64b^4n^4} + \frac{(1+m)x^{1+m}\cosh^4(a+b}{(1+m)^2-16b^2}$$

$$= \frac{24b^4n^4x^{1+m}}{(1+m)\left((1+m)^4-20b^2(1+m)^2n^2+64b^4n^4\right)} - \frac{12b^2(1+m)n^2x^{1+m}\mathrm{co}}{(1+m)^4-20b^2(1}$$

Mathematica [A]
time = 2.43, size = 311, normalized size = 1.17

$$\frac{1}{8}x^{1+m}\left(\frac{3}{1+m}+\frac{4\sinh(2bn\log(x))\left(-2bn\cosh\left(2(a-bn\log(x)+b\log\left(cx^n\right))\right)+(1+m)\sinh\left(2(a-bn\log(x)+b\log\left(cx^n\right))\right)\right)}{(1+m-2bn)(1+m+2bn)}+\frac{4\cosh(2bn\log(x))\left((1+m)\cosh\left(2(a-bn\log(x)+b\log\left(cx^n\right))\right)-2bn\sinh\left(2(a-bn\log(x)+b\log\left(cx^n\right))\right)\right)}{(1+m-2bn)(1+m+2bn)}+\frac{\sinh(4bn\log(x))\left(-4bn\cosh\left(4(a-bn\log(x)+b\log\left(cx^n\right))\right)+(1+m)\sinh\left(4(a-bn\log(x)+b\log\left(cx^n\right))\right)\right)}{(1+m-4bn)(1+m+4bn)}+\frac{\cosh(4bn\log(x))\left((1+m)\cosh\left(4(a-bn\log(x)+b\log\left(cx^n\right))\right)-4bn\sinh\left(4(a-bn\log(x)+b\log\left(cx^n\right))\right)\right)}{(1+m-4bn)(1+m+4bn)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cosh[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (x^(1 + m)*(3/(1 + m) + (4*Sinh[2*b*n*Log[x]]*(-2*b*n*Cosh[2*(a - b*n*Log[x
] + b*Log[c*x^n])] + (1 + m)*Sinh[2*(a - b*n*Log[x] + b*Log[c*x^n])]))/((1
+ m - 2*b*n)*(1 + m + 2*b*n)) + (4*Cosh[2*b*n*Log[x]]*((1 + m)*Cosh[2*(a -
b*n*Log[x] + b*Log[c*x^n])] - 2*b*n*Sinh[2*(a - b*n*Log[x] + b*Log[c*x^n])]
))/((1 + m - 2*b*n)*(1 + m + 2*b*n)) + (Sinh[4*b*n*Log[x]]*(-4*b*n*Cosh[4*(
a - b*n*Log[x] + b*Log[c*x^n])] + (1 + m)*Sinh[4*(a - b*n*Log[x] + b*Log[c*
x^n])]))/((1 + m - 4*b*n)*(1 + m + 4*b*n)) + (Cosh[4*b*n*Log[x]]*((1 + m)*C
osh[4*(a - b*n*Log[x] + b*Log[c*x^n])] - 4*b*n*Sinh[4*(a - b*n*Log[x] + b*L
og[c*x^n])]))/((1 + m - 4*b*n)*(1 + m + 4*b*n))))/8
```

Maple [F]
time = 2.76, size = 0, normalized size = 0.00

$$\int x^m\left(\cosh^4\left(a+b\ln\left(c\,x^n\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(a+b*ln(c*x^n))^4,x)
```

```
[Out] int(x^m*cosh(a+b*ln(c*x^n))^4,x)
```

Maxima [A]
time = 0.31, size = 161, normalized size = 0.61

$$\frac{c^{4\,b}xe^{\left(4\,b\log\left(x^n\right)+m\log\left(x\right)+4\,a\right)}}{16\,\left(4\,bn+m+1\right)}+\frac{c^{2\,b}xe^{\left(2\,b\log\left(x^n\right)+m\log\left(x\right)+2\,a\right)}}{4\,\left(2\,bn+m+1\right)}-\frac{xe^{\left(-2\,b\log\left(x^n\right)+m\log\left(x\right)-2\,a\right)}}{4\,\left(2\,bc^{2\,b}n-c^{2\,b}(m+1)\right)}-\frac{xe^{\left(-4\,b\log\left(x^n\right)+m\log\left(x\right)-4\,a\right)}}{16\,\left(4\,bc^{4\,b}n-c^{4\,b}(m+1)\right)}+\frac{3\,x^{m+1}}{8\,\left(m+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(a+b*log(c*x^n))^4,x, algorithm="maxima")
```

```
[Out] 1/16*c^(4*b)*x*e^(4*b*log(x^n) + m*log(x) + 4*a)/(4*b*n + m + 1) + 1/4*c^(2
*b)*x*e^(2*b*log(x^n) + m*log(x) + 2*a)/(2*b*n + m + 1) - 1/4*x*e^(-2*b*log
(x^n) + m*log(x) - 2*a)/(2*b*c^(2*b)*n - c^(2*b)*(m + 1)) - 1/16*x*e^(-4*b*
log(x^n) + m*log(x) - 4*a)/(4*b*c^(4*b)*n - c^(4*b)*(m + 1)) + 3/8*x^(m + 1
)/(m + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1123 vs. $2(283) = 566$.
time = 0.47, size = 1123, normalized size = 4.22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

```
[Out] 1/8*((m^4 + 4*m^3 - 4*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*co
sh(b*n*log(x) + b*log(c) + a)^4*cosh(m*log(x)) + 4*(m^4 + 4*m^3 - 16*(b^2*m
^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a
)^2*cosh(m*log(x)) + ((m^4 + 4*m^3 - 4*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^
2 + 4*m + 1)*x*cosh(m*log(x)) + (m^4 + 4*m^3 - 4*(b^2*m^2 + 2*b^2*m + b^2)*
n^2 + 6*m^2 + 4*m + 1)*x*sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a)^4
+ 16*((4*(b^3*m + b^3)*n^3 - (b*m^3 + 3*b*m^2 + 3*b*m + b)*n)*x*cosh(b*n*lo
g(x) + b*log(c) + a)*cosh(m*log(x)) + (4*(b^3*m + b^3)*n^3 - (b*m^3 + 3*b*m
^2 + 3*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(m*log(x)))*sinh(b
*n*log(x) + b*log(c) + a)^3 + 3*(64*b^4*n^4 + m^4 + 4*m^3 - 20*(b^2*m^2 + 2
*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(m*log(x)) + 2*(3*(m^4 + 4*m^3 -
 4*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(b*n*log(x) + b*l
og(c) + a)^2*cosh(m*log(x)) + 2*(m^4 + 4*m^3 - 16*(b^2*m^2 + 2*b^2*m + b^2)
*n^2 + 6*m^2 + 4*m + 1)*x*cosh(m*log(x)) + (3*(m^4 + 4*m^3 - 4*(b^2*m^2 + 2
*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2 +
2*(m^4 + 4*m^3 - 16*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x)*sin
h(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a)^2 + 16*((4*(b^3*m + b^3)*n^3 -
 (b*m^3 + 3*b*m^2 + 3*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)^3*cosh(
m*log(x)) + (16*(b^3*m + b^3)*n^3 - (b*m^3 + 3*b*m^2 + 3*b*m + b)*n)*x*cosh
(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + ((4*(b^3*m + b^3)*n^3 - (b*m^3
 + 3*b*m^2 + 3*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)^3 + (16*(b^3*m
 + b^3)*n^3 - (b*m^3 + 3*b*m^2 + 3*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c)
 + a))*sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a) + ((m^4 + 4*m^3 - 4*
(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(b*n*log(x) + b*log(
c) + a)^4 + 4*(m^4 + 4*m^3 - 16*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m
 + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2 + 3*(64*b^4*n^4 + m^4 + 4*m^3 - 2
0*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x)*sinh(m*log(x)))/(m^5
```

```
+ 64*(b^4*m + b^4)*n^4 + 5*m^4 + 10*m^3 - 20*(b^2*m^3 + 3*b^2*m^2 + 3*b^2*m
 + b^2)*n^2 + 10*m^2 + 5*m + 1)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(a+b*ln(c*x**n))**4,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 6880 vs. 2(283) = 566.
time = 0.55, size = 6880, normalized size = 25.86

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(a+b*log(c*x^n))^4,x, algorithm="giac")
```

```
[Out] b^3*c^(4*b)*m*n^3*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b
^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*
m^3 + 10*m^2 + 5*m + 1) + 8*b^3*c^(2*b)*m*n^3*x*x^(2*b*n)*x^m*e^(2*a)/(64*b
^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^
2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1/4*b^2*c^(4*b)*m^2*n
^2*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60
*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 +
5*m + 1) + b^3*c^(4*b)*n^3*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n
^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*
n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 4*b^2*c^(2*b)*m^2*n^2*x*x^(2*b*n)*x^m*e^
(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 -
60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 8*b^3*c^(2
*b)*n^3*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2
 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m
^2 + 5*m + 1) + 24*b^4*n^4*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^
2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*
m^2 + 5*m + 1) - 1/4*b*c^(4*b)*m^3*n*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4
+ 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4
 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1/2*b^2*c^(4*b)*m*n^2*x*x^(4*b
*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^
2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) -
1/2*b*c^(2*b)*m^3*n*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20
*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 1
0*m^3 + 10*m^2 + 5*m + 1) - 8*b^2*c^(2*b)*m*n^2*x*x^(2*b*n)*x^m*e^(2*a)/(64
```

```
*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*
n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 1/16*c^(4*b)*m^4*x*
x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*
m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m +
 1) - 3/4*b*c^(4*b)*m^2*n*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^
4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n
^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1/4*b^2*c^(4*b)*n^2*x*x^(4*b*n)*x^m*e^(4*
a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*
b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 1/4*c^(2*b)*m
^4*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60
*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 +
5*m + 1) - 3/2*b*c^(2*b)*m^2*n*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b
^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*
b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 4*b^2*c^(2*b)*n^2*x*x^(2*b*n)*x^m*e^
(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 -
60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 15/2*b^2*m
^2*n^2*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 +
 m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 1/4
*c^(4*b)*m^3*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^
3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 +
 10*m^2 + 5*m + 1) - 3/4*b*c^(4*b)*m*n*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^
4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m
^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + c^(2*b)*m^3*x*x^(2*b*n)*x^m*
e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5
- 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 3/2*b*c^
(2*b)*m*n*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n
^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10
*m^2 + 5*m + 1) - 8*b^3*m*n^3*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 -
20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 +
 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) - b^3*m*n^3*x*x^m*e^(-4*a)/(
(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2
*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n))
 - 15*b^2*m*n^2*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*
m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m +
 1) + 3/8*c^(4*b)*m^2*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 -
20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 +
 10*m^3 + 10*m^2 + 5*m + 1) - 1/4*b*c^(4*b)*n*x*x^(4*b*n)*x^m*e^(4*a)/(64*b
^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^
2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 3/2*c^(2*b)*m^2*x*x^(
2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2
*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)
 - 1/2*b*c^(2*b)*n*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*
b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n...
```

Mupad [B]

time = 1.19, size = 134, normalized size = 0.50

$$\frac{3\,x\,x^m}{8\,m+8}+\frac{x\,x^m\,{\mathrm{e}}^{-2\,a}}{{\left(c\,x^n\right)}^{2\,b}\,\left(4\,m-8\,b\,n+4\right)}+\frac{x\,x^m\,{\mathrm{e}}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}}{4\,m+8\,b\,n+4}+\frac{x\,x^m\,{\mathrm{e}}^{-4\,a}}{{\left(c\,x^n\right)}^{4\,b}\,\left(16\,m-64\,b\,n+16\right)}+\frac{x\,x^m\,{\mathrm{e}}^{4\,a}\,{\left(c\,x^n\right)}^{4\,b}}{16\,m+64\,b\,n+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(a + b*log(c*x^n))^4,x)
```

```
[Out] (3*x*x^m)/(8*m + 8) + (x*x^m*exp(-2*a))/((c*x^n)^(2*b)*(4*m - 8*b*n + 4)) +
 (x*x^m*exp(2*a)*(c*x^n)^(2*b))/(4*m + 8*b*n + 4) + (x*x^m*exp(-4*a))/((c*x
^n)^(4*b)*(16*m - 64*b*n + 16)) + (x*x^m*exp(4*a)*(c*x^n)^(4*b))/(16*m + 64
*b*n + 16)
```

3.247 $\int \frac{\cosh(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=18

$$\frac{\sinh\left(a+b\log\left(cx^n\right)\right)}{bn}$$

```
[Out] sinh(a+b*ln(c*x^n))/b/n
```

Rubi [A]
time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2717}

$$\frac{\sinh\left(a+b\log\left(cx^n\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*Log[c*x^n]]/x,x]
```

```
[Out] Sinh[a + b*Log[c*x^n]]/(b*n)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh\left(a+b\log\left(cx^n\right)\right)}{x}\,dx &= \frac{\text{Subst}\left(\int \cosh(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n}\\ &= \frac{\sinh\left(a+b\log\left(cx^n\right)\right)}{bn}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 37 vs. 2(18) = 36.
time = 0.01, size = 37, normalized size = 2.06

$$\frac{\cosh\left(b\log\left(cx^n\right)\right)\sinh(a)}{bn}+\frac{\cosh(a)\sinh\left(b\log\left(cx^n\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*Log[c*x^n]]/x,x]
```

```
[Out] (Cosh[b*Log[c*x^n]]*Sinh[a])/(b*n) + (Cosh[a]*Sinh[b*Log[c*x^n]])/(b*n)
```

Maple [A]
time = 2.00, size = 19, normalized size = 1.06

method	result	size
derivativedivides	$\frac{\sinh(a+b\ln(c\,x^{n}))}{bn}$	19
default	$\frac{\sinh(a+b\ln(c\,x^{n}))}{bn}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*ln(c*x^n))/x,x,method=_RETURNVERBOSE)
[Out] sinh(a+b*ln(c*x^n))/b/n
```

Maxima [A]
time = 0.26, size = 18, normalized size = 1.00

$$\frac{\sinh\left(b\log\left(cx^{n}\right)+a\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))/x,x, algorithm="maxima")
[Out] sinh(b*log(c*x^n) + a)/(b*n)
```

Fricas [A]
time = 0.35, size = 19, normalized size = 1.06

$$\frac{\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))/x,x, algorithm="fricas")
[Out] sinh(b*n*log(x) + b*log(c) + a)/(b*n)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(14) = 28$.
time = 0.28, size = 37, normalized size = 2.06

$$\begin{cases} \log\left(x\right)\cosh\left(a\right) & \text{for}\ b=0\wedge n=0 \\ \log\left(x\right)\cosh\left(a+b\log\left(c\right)\right) & \text{for}\ n=0 \\ \log\left(x\right)\cosh\left(a\right) & \text{for}\ b=0 \\ \frac{\sinh\left(a+b\log\left(cx^{n}\right)\right)}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*ln(c*x**n))/x,x)
```

```
[Out] Piecewise((log(x)*cosh(a), Eq(b, 0) & Eq(n, 0)), (log(x)*cosh(a + b*log(c))
, Eq(n, 0)), (log(x)*cosh(a), Eq(b, 0)), (sinh(a + b*log(c*x**n))/(b*n), Tr
ue))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(18) = 36$.
time = 0.42, size = 42, normalized size = 2.33

$$\frac{\left(c^{2\,b}x^{bn}e^{(2\,a)} - \frac{1}{x^{bn}}\right)e^{(-a)}}{2\,bc^{b}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))/x,x, algorithm="giac")
```

```
[Out] 1/2*(c^(2*b)*x^(b*n)*e^(2*a) - 1/x^(b*n))*e^(-a)/(b*c^b*n)
```

Mupad [B]
time = 1.03, size = 18, normalized size = 1.00

$$\frac{\sinh(a + b\,\ln\left(c\,x^{n}\right))}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*log(c*x^n))/x,x)
```

```
[Out] sinh(a + b*log(c*x^n))/(b*n)
```

3.248 $\int \frac{\cosh^2(a+b\log(cx^n))}{x} \, dx$

Optimal. Leaf size=39

$$\frac{\log(x)}{2}+\frac{\cosh\left(a+b\log\left(cx^n\right)\right)\sinh\left(a+b\log\left(cx^n\right)\right)}{2bn}$$

```
[Out] 1/2*ln(x)+1/2*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))/b/n
```

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2715, 8}

$$\frac{\sinh\left(a+b\log\left(cx^n\right)\right)\cosh\left(a+b\log\left(cx^n\right)\right)}{2bn}+\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*Log[c*x^n]]^2/x,x]
```

```
[Out] Log[x]/2 + (Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]])/(2*b*n)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^2\left(a+b\log\left(cx^n\right)\right)}{x} \, dx &= \frac{\text{Subst}\left(\int \cosh^2(a+bx) \, dx,x,\log\left(cx^n\right)\right)}{n}\\
&= \frac{\cosh\left(a+b\log\left(cx^n\right)\right)\sinh\left(a+b\log\left(cx^n\right)\right)}{2bn}+\frac{\text{Subst}\left(\int 1 \, dx,x,\log\left(cx^n\right)\right)}{2n}\\
&= \frac{\log(x)}{2}+\frac{\cosh\left(a+b\log\left(cx^n\right)\right)\sinh\left(a+b\log\left(cx^n\right)\right)}{2bn}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 36, normalized size = 0.92

$$\frac{2\left(a+b\log\left(cx^n\right)\right)+\sinh\left(2\left(a+b\log\left(cx^n\right)\right)\right)}{4bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*Log[c*x^n]]^2/x,x]
```

```
[Out] (2*(a + b*Log[c*x^n]) + Sinh[2*(a + b*Log[c*x^n])])/(4*b*n)
```

Maple [A]
time = 2.59, size = 45, normalized size = 1.15

method	result	size
derivativedivides	$\frac{\frac{\cosh(a+b\ln(c\,x^{n}))\sinh(a+b\ln(c\,x^{n}))}{2}+\frac{b\ln(c\,x^{n})}{2}+\frac{a}{2}}{nb}$	45
default	$\frac{\frac{\cosh(a+b\ln(c\,x^{n}))\sinh(a+b\ln(c\,x^{n}))}{2}+\frac{b\ln(c\,x^{n})}{2}+\frac{a}{2}}{nb}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*ln(c*x^n))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b*(1/2*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))+1/2*b*ln(c*x^n)+1/2*a)
```

Maxima [A]
time = 0.28, size = 49, normalized size = 1.26

$$\frac{e^{(2\,b\log(cx^n)+2\,a)}}{8\,bn}-\frac{e^{(-2\,b\log(cx^n)-2\,a)}}{8\,bn}+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*e^(2*b*log(c*x^n) + 2*a)/(b*n) - 1/8*e^(-2*b*log(c*x^n) - 2*a)/(b*n) +
1/2*log(x)
```

Fricas [A]
time = 0.55, size = 39, normalized size = 1.00

$$\frac{bn\log(x)+\cosh(bn\log(x)+b\log(c)+a)\sinh(bn\log(x)+b\log(c)+a)}{2\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^2/x,x, algorithm="fricas")
```

```
[Out] 1/2*(b*n*log(x) + cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c
) + a))/(b*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cosh^2(a+b\log(cx^n))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*ln(c*x**n))**2/x,x)
```

```
[Out] Integral(cosh(a + b*log(c*x**n))**2/x, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(35) = 70$.
time = 0.42, size = 80, normalized size = 2.05

$$\frac{\left(4\,bc^{2\,b}ne^{(2\,a)}\log\left(x\right) + c^{4\,b}x^{2\,bn}e^{(4\,a)} - \frac{2\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+1}{x^{2\,bn}}\right)e^{(-2\,a)}}{8\,bc^{2\,b}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^2/x,x, algorithm="giac")
```

```
[Out] 1/8*(4*b*c^(2*b)*n*e^(2*a)*log(x) + c^(4*b)*x^(2*b*n)*e^(4*a) - (2*c^(2*b)*
x^(2*b*n)*e^(2*a) + 1)/x^(2*b*n))*e^(-2*a)/(b*c^(2*b)*n)
```

Mupad [B]
time = 1.05, size = 32, normalized size = 0.82

$$\frac{\ln\left(x^n\right)}{2\,n}+\frac{\sinh(2\,a+2\,b\,\ln\left(c\,x^n\right))}{4\,b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*log(c*x^n))^2/x,x)
```

```
[Out] log(x^n)/(2*n) + sinh(2*a + 2*b*log(c*x^n))/(4*b*n)
```

3.249 $\int \frac{\cosh^3(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=42

$$\frac{\sinh\left(a+b\log\left(cx^n\right)\right)}{bn}+\frac{\sinh^3\left(a+b\log\left(cx^n\right)\right)}{3bn}$$

```
[Out] sinh(a+b*ln(c*x^n))/b/n+1/3*sinh(a+b*ln(c*x^n))^3/b/n
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2713}

$$\frac{\sinh^3\left(a+b\log\left(cx^n\right)\right)}{3bn}+\frac{\sinh\left(a+b\log\left(cx^n\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] Sinh[a + b*Log[c*x^n]]/(b*n) + Sinh[a + b*Log[c*x^n]]^3/(3*b*n)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^3\left(a+b\log\left(cx^n\right)\right)}{x}\,dx &= \frac{\text{Subst}\left(\int \cosh^3(a+bx)\,dx,x,\log\left(cx^n\right)\right)}{n}\\ &= \frac{i\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-i\sinh\left(a+b\log\left(cx^n\right)\right)\right)}{bn}\\ &= \frac{\sinh\left(a+b\log\left(cx^n\right)\right)}{bn}+\frac{\sinh^3\left(a+b\log\left(cx^n\right)\right)}{3bn}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 42, normalized size = 1.00

$$\frac{\sinh\left(a+b\log\left(cx^n\right)\right)}{bn}+\frac{\sinh^3\left(a+b\log\left(cx^n\right)\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] Sinh[a + b*Log[c*x^n]]/(b*n) + Sinh[a + b*Log[c*x^n]]^3/(3*b*n)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3\left(a + b\ln\left(c\,x^n\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*ln(c*x^n))^3/x,x)
```

```
[Out] int(cosh(a+b*ln(c*x^n))^3/x,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(40) = 80.
time = 0.29, size = 86, normalized size = 2.05

$$\frac{e^{(3\,b\log(cx^n)+3\,a)}}{24\,bn} + \frac{3\,e^{(b\log(cx^n)+a)}}{8\,bn} - \frac{3\,e^{(-b\log(cx^n)-a)}}{8\,bn} - \frac{e^{(-3\,b\log(cx^n)-3\,a)}}{24\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^3/x,x, algorithm="maxima")
```

```
[Out] 1/24*e^(3*b*log(c*x^n) + 3*a)/(b*n) + 3/8*e^(b*log(c*x^n) + a)/(b*n) - 3/8*
e^(-b*log(c*x^n) - a)/(b*n) - 1/24*e^(-3*b*log(c*x^n) - 3*a)/(b*n)
```

Fricas [A]
time = 0.39, size = 53, normalized size = 1.26

$$\frac{\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^3+3\left(\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2+3\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{12\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^3/x,x, algorithm="fricas")
```

```
[Out] 1/12*(sinh(b*n*log(x) + b*log(c) + a)^3 + 3*(cosh(b*n*log(x) + b*log(c) + a
)^2 + 3)*sinh(b*n*log(x) + b*log(c) + a))/(b*n)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(32) = 64.
time = 1.71, size = 76, normalized size = 1.81

$$\begin{cases} \log\left(x\right)\cosh^3\left(a\right) & \text{for}\ b = 0 \wedge n = 0 \\ \log\left(x\right)\cosh^3\left(a + b\log\left(c\right)\right) & \text{for}\ n = 0 \\ \log\left(x\right)\cosh^3\left(a\right) & \text{for}\ b = 0 \\ -\frac{2\sinh^3\left(a+b\log\left(cx^n\right)\right)}{3bn} + \frac{\sinh\left(a+b\log\left(cx^n\right)\right)\cosh^2\left(a+b\log\left(cx^n\right)\right)}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*ln(c*x**n))**3/x,x)
```

```
[Out] Piecewise((log(x)*cosh(a)**3, Eq(b, 0) & Eq(n, 0)), (log(x)*cosh(a + b*log(
c))**3, Eq(n, 0)), (log(x)*cosh(a)**3, Eq(b, 0)), (-2*sinh(a + b*log(c*x**n
))**3/(3*b*n) + sinh(a + b*log(c*x**n))*cosh(a + b*log(c*x**n))**2/(b*n), T
rue))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(40) = 80$.
time = 0.43, size = 81, normalized size = 1.93

$$\frac{\left(c^{6\,b}x^{3\,bn}e^{(6\,a)}+9\,c^{4\,b}x^{bn}e^{(4\,a)}-\frac{9\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+1}{x^{3\,bn}}\right)e^{(-3\,a)}}{24\,bc^{3\,b}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^3/x,x, algorithm="giac")
```

```
[Out] 1/24*(c^(6*b)*x^(3*b*n)*e^(6*a) + 9*c^(4*b)*x^(b*n)*e^(4*a) - (9*c^(2*b)*x^
(2*b*n)*e^(2*a) + 1)/x^(3*b*n))*e^(-3*a)/(b*c^(3*b)*n)
```

Mupad [B]
time = 1.05, size = 35, normalized size = 0.83

$$\frac{\sinh(a+b\,\ln\left(c\,x^n\right))^3+3\sinh(a+b\,\ln\left(c\,x^n\right))}{3\,b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*log(c*x^n))^3/x,x)
```

```
[Out] (3*sinh(a + b*log(c*x^n)) + sinh(a + b*log(c*x^n))^3)/(3*b*n)
```

3.250 $\int \frac{\cosh^4(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=73

$$\frac{3\log(x)}{8}+\frac{3\cosh\left(a+b\log\left(cx^n\right)\right)\sinh\left(a+b\log\left(cx^n\right)\right)}{8bn}+\frac{\cosh^3\left(a+b\log\left(cx^n\right)\right)\sinh\left(a+b\log\left(cx^n\right)\right)}{4bn}$$

```
[Out] 3/8*ln(x)+3/8*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))/b/n+1/4*cosh(a+b*ln(c
*x^n))^3*sinh(a+b*ln(c*x^n))/b/n
```

Rubi [A]
time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2715, 8}

$$\frac{\sinh\left(a+b\log\left(cx^n\right)\right)\cosh^3\left(a+b\log\left(cx^n\right)\right)}{4bn}+\frac{3\sinh\left(a+b\log\left(cx^n\right)\right)\cosh\left(a+b\log\left(cx^n\right)\right)}{8bn}+\frac{3\log(x)}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] (3*Log[x])/8 + (3*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]])/(8*b*n) +
(Cosh[a + b*Log[c*x^n]]^3*Sinh[a + b*Log[c*x^n]])/(4*b*n)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^4\left(a+b\log\left(cx^n\right)\right)}{x}\,dx &= \frac{\text{Subst}\left(\int \cosh^4(a+bx)\,dx,x,\log\left(cx^n\right)\right)}{n}\\ &= \frac{\cosh^3\left(a+b\log\left(cx^n\right)\right)\sinh\left(a+b\log\left(cx^n\right)\right)}{4bn}+\frac{3\text{Subst}\left(\int \cosh^2(a+bx)\,dx,x\right.}{4n}\\ &= \frac{3\cosh\left(a+b\log\left(cx^n\right)\right)\sinh\left(a+b\log\left(cx^n\right)\right)}{8bn}+\frac{\cosh^3\left(a+b\log\left(cx^n\right)\right)\sinh\left(a\right.}{4bn}\\ &= \frac{3\log(x)}{8}+\frac{3\cosh\left(a+b\log\left(cx^n\right)\right)\sinh\left(a+b\log\left(cx^n\right)\right)}{8bn}+\frac{\cosh^3\left(a+b\log\left(a\right.\right.}{}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 51, normalized size = 0.70

$$\frac{12(a + b\log(cx^n)) + 8\sinh(2(a + b\log(cx^n))) + \sinh(4(a + b\log(cx^n)))}{32bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] (12*(a + b*Log[c*x^n]) + 8*Sinh[2*(a + b*Log[c*x^n])] + Sinh[4*(a + b*Log[c
*x^n])])/(32*b*n)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^4(a + b\ln(c\,x^n))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*ln(c*x^n))^4/x,x)
```

```
[Out] int(cosh(a+b*ln(c*x^n))^4/x,x)
```

Maxima [A]
time = 0.27, size = 93, normalized size = 1.27

$$\frac{e^{(4\,b\log(cx^n)+4\,a)}}{64\,bn} + \frac{e^{(2\,b\log(cx^n)+2\,a)}}{8\,bn} - \frac{e^{(-2\,b\log(cx^n)-2\,a)}}{8\,bn} - \frac{e^{(-4\,b\log(cx^n)-4\,a)}}{64\,bn} + \frac{3}{8}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^4/x,x, algorithm="maxima")
```

```
[Out] 1/64*e^(4*b*log(c*x^n) + 4*a)/(b*n) + 1/8*e^(2*b*log(c*x^n) + 2*a)/(b*n) -
1/8*e^(-2*b*log(c*x^n) - 2*a)/(b*n) - 1/64*e^(-4*b*log(c*x^n) - 4*a)/(b*n)
+ 3/8*log(x)
```

Fricas [A]
time = 0.54, size = 84, normalized size = 1.15

$$\frac{\cosh(bn\log(x) + b\log(c) + a)\sinh(bn\log(x) + b\log(c) + a)^3 + 3\,bn\log(x) + (\cosh(bn\log(x) + b\log(c) + a)^3 + 4\,\cosh(bn\log(x) + b\log(c) + a))\sinh(bn\log(x) + b\log(c) + a)}{8\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^4/x,x, algorithm="fricas")
```

```
[Out] 1/8*(cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3 + 3*
b*n*log(x) + (cosh(b*n*log(x) + b*log(c) + a)^3 + 4*cosh(b*n*log(x) + b*log
(c) + a))*sinh(b*n*log(x) + b*log(c) + a))/(b*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^4\left(a + b\log\left(cx^n\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*ln(c*x**n))**4/x,x)
```

```
[Out] Integral(cosh(a + b*log(c*x**n))**4/x, x)
```

Giac [A]
time = 0.43, size = 114, normalized size = 1.56

$$\frac{\left(24\,bc^{4\,b}ne^{(4\,a)}\log\left(x\right) + c^{8\,b}x^{4\,bn}e^{(8\,a)} + 8\,c^{6\,b}x^{2\,bn}e^{(6\,a)} - \frac{18\,c^{4\,b}x^{4\,bn}e^{(4\,a)} + 8\,c^{2\,b}x^{2\,bn}e^{(2\,a)} + 1}{x^{4\,bn}}\right)e^{(-4\,a)}}{64\,bc^{4\,b}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^4/x,x, algorithm="giac")
```

```
[Out] 1/64*(24*b*c^(4*b)*n*e^(4*a)*log(x) + c^(8*b)*x^(4*b*n)*e^(8*a) + 8*c^(6*b)
*x^(2*b*n)*e^(6*a) - (18*c^(4*b)*x^(4*b*n)*e^(4*a) + 8*c^(2*b)*x^(2*b*n)*e^
(2*a) + 1)/x^(4*b*n))*e^(-4*a)/(b*c^(4*b)*n)
```

Mupad [B]
time = 1.11, size = 50, normalized size = 0.68

$$\frac{3\,\ln\left(x^n\right)}{8\,n} + \frac{\frac{\sinh(2\,a+2\,b\,\ln(c\,x^n))}{4} + \frac{\sinh(4\,a+4\,b\,\ln(c\,x^n))}{32}}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*log(c*x^n))^4/x,x)
```

```
[Out] (3*log(x^n))/(8*n) + (sinh(2*a + 2*b*log(c*x^n))/4 + sinh(4*a + 4*b*log(c*x
^n))/32)/(b*n)
```

3.251 $\int \frac{\cosh^5(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=65

$$\frac{\sinh(a+b\log(cx^n))}{bn}+\frac{2\sinh^3(a+b\log(cx^n))}{3bn}+\frac{\sinh^5(a+b\log(cx^n))}{5bn}$$

```
[Out] sinh(a+b*ln(c*x^n))/b/n+2/3*sinh(a+b*ln(c*x^n))^3/b/n+1/5*sinh(a+b*ln(c*x^n
))^5/b/n
```

Rubi [A]
time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{2713\}$

$$\frac{\sinh^5(a+b\log(cx^n))}{5bn}+\frac{2\sinh^3(a+b\log(cx^n))}{3bn}+\frac{\sinh(a+b\log(cx^n))}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*Log[c*x^n]]^5/x,x]
```

```
[Out] Sinh[a + b*Log[c*x^n]]/(b*n) + (2*Sinh[a + b*Log[c*x^n]]^3)/(3*b*n) + Sinh[
a + b*Log[c*x^n]]^5/(5*b*n)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^5(a+b\log(cx^n))}{x}\,dx &= \frac{\text{Subst}\left(\int \cosh^5(a+bx)\,dx, x, \log(cx^n)\right)}{n}\\ &= \frac{i\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, -i\sinh(a+b\log(cx^n))\right)}{bn}\\ &= \frac{\sinh(a+b\log(cx^n))}{bn}+\frac{2\sinh^3(a+b\log(cx^n))}{3bn}+\frac{\sinh^5(a+b\log(cx^n))}{5bn}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 68, normalized size = 1.05

$$\frac{5\sinh(a+b\log(cx^n))}{8bn}+\frac{5\sinh(3(a+b\log(cx^n)))}{48bn}+\frac{\sinh(5(a+b\log(cx^n)))}{80bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*Log[c*x^n]]^5/x,x]
```

```
[Out] (5*Sinh[a + b*Log[c*x^n]])/(8*b*n) + (5*Sinh[3*(a + b*Log[c*x^n])])/(48*b*n
) + Sinh[5*(a + b*Log[c*x^n])]/(80*b*n)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^{5}\left(a + b\ln\left(c\, x^{n}\right)\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*ln(c*x^n))^5/x,x)
```

```
[Out] int(cosh(a+b*ln(c*x^n))^5/x,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(61) = 122$.
time = 0.26, size = 130, normalized size = 2.00

$$\frac{e^{\left(5\, b \log\left(cx^{n}\right) + 5\, a\right)}}{160\, bn} + \frac{5\, e^{\left(3\, b \log\left(cx^{n}\right) + 3\, a\right)}}{96\, bn} + \frac{5\, e^{\left(b \log\left(cx^{n}\right) + a\right)}}{16\, bn} - \frac{5\, e^{\left(-b \log\left(cx^{n}\right) - a\right)}}{16\, bn} - \frac{5\, e^{\left(-3\, b \log\left(cx^{n}\right) - 3\, a\right)}}{96\, bn} - \frac{e^{\left(-5\, b \log\left(cx^{n}\right) - 5\, a\right)}}{160\, bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^5/x,x, algorithm="maxima")
```

```
[Out] 1/160*e^(5*b*log(c*x^n) + 5*a)/(b*n) + 5/96*e^(3*b*log(c*x^n) + 3*a)/(b*n)
+ 5/16*e^(b*log(c*x^n) + a)/(b*n) - 5/16*e^(-b*log(c*x^n) - a)/(b*n) - 5/96
*e^(-3*b*log(c*x^n) - 3*a)/(b*n) - 1/160*e^(-5*b*log(c*x^n) - 5*a)/(b*n)
```

Fricas [A]
time = 0.46, size = 105, normalized size = 1.62

$$\frac{3\, \sinh\left(bn \log\left(x\right) + b \log\left(c\right) + a\right)^{5} + 5\, {\left(6\, \cosh\left(bn \log\left(x\right) + b \log\left(c\right) + a\right)^{2} + 5\right)} \sinh\left(bn \log\left(x\right) + b \log\left(c\right) + a\right)^{3} + 15\, {\left(\cosh\left(bn \log\left(x\right) + b \log\left(c\right) + a\right)^{4} + 5\, \cosh\left(bn \log\left(x\right) + b \log\left(c\right) + a\right)^{2} + 10\right)} \sinh\left(bn \log\left(x\right) + b \log\left(c\right) + a\right)}{240\, bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^5/x,x, algorithm="fricas")
```

```
[Out] 1/240*(3*sinh(b*n*log(x) + b*log(c) + a)^5 + 5*(6*cosh(b*n*log(x) + b*log(c
) + a)^2 + 5)*sinh(b*n*log(x) + b*log(c) + a)^3 + 15*(cosh(b*n*log(x) + b*l
og(c) + a)^4 + 5*cosh(b*n*log(x) + b*log(c) + a)^2 + 10)*sinh(b*n*log(x) +
b*log(c) + a))/(b*n)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(53) = 106$.

time = 10.33, size = 110, normalized size = 1.69

$$\begin{cases} \log(x)\cosh^5(a) & \text{for } b = 0 \wedge n = 0 \\ \log(x)\cosh^5(a + b\log(c)) & \text{for } n = 0 \\ \log(x)\cosh^5(a) & \text{for } b = 0 \\ \frac{8\sinh^5(a+b\log(cx^n))}{15bn} - \frac{4\sinh^3(a+b\log(cx^n))\cosh^2(a+b\log(cx^n))}{3bn} + \frac{\sinh(a+b\log(cx^n))\cosh^4(a+b\log(cx^n))}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*ln(c*x**n))**5/x,x)
```

```
[Out] Piecewise((log(x)*cosh(a)**5, Eq(b, 0) & Eq(n, 0)), (log(x)*cosh(a + b*log(
c))**5, Eq(n, 0)), (log(x)*cosh(a)**5, Eq(b, 0)), (8*sinh(a + b*log(c*x**n)
)**5/(15*b*n) - 4*sinh(a + b*log(c*x**n))**3*cosh(a + b*log(c*x**n))**2/(3*
b*n) + sinh(a + b*log(c*x**n))*cosh(a + b*log(c*x**n))**4/(b*n), True))
```

Giac [A]
time = 0.42, size = 116, normalized size = 1.78

$$\frac{\left(3\,c^{10\,b}x^{5\,bn}e^{(10\,a)} + 25\,c^{8\,b}x^{3\,bn}e^{(8\,a)} + 150\,c^{6\,b}x^{bn}e^{(6\,a)} - \frac{150\,c^{4\,b}x^{4\,bn}e^{(4\,a)} + 25\,c^{2\,b}x^{2\,bn}e^{(2\,a)} + 3}{x^{5\,bn}}\right)e^{(-5\,a)}}{480\,bc^{5\,b}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^5/x,x, algorithm="giac")
```

```
[Out] 1/480*(3*c^(10*b)*x^(5*b*n)*e^(10*a) + 25*c^(8*b)*x^(3*b*n)*e^(8*a) + 150*c
^(6*b)*x^(b*n)*e^(6*a) - (150*c^(4*b)*x^(4*b*n)*e^(4*a) + 25*c^(2*b)*x^(2*b
*n)*e^(2*a) + 3)/x^(5*b*n))*e^(-5*a)/(b*c^(5*b)*n)
```

Mupad [B]
time = 1.17, size = 49, normalized size = 0.75

$$\frac{\frac{\sinh(a+b\,\ln(c\,x^n))^5}{5} + \frac{2\sinh(a+b\,\ln(c\,x^n))^3}{3} + \sinh(a + b\,\ln(c\,x^n))}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*log(c*x^n))^5/x,x)
```

```
[Out] (sinh(a + b*log(c*x^n)) + (2*sinh(a + b*log(c*x^n))^3)/3 + sinh(a + b*log(c
*x^n))^5/5)/(b*n)
```

3.252 $\int \frac{\cosh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=67

$$-\frac{6iE\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{5bn}+\frac{2\cosh^{\frac{3}{2}}(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{5bn}$$

```
[Out] -6/5*I*(cosh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/cosh(1/2*a+1/2*b*ln(c*x^n))*El
lipticE(I*sinh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/b/n+2/5*cosh(a+b*ln(c*x^n))^
(3/2)*sinh(a+b*ln(c*x^n))/b/n
```

Rubi [A]
time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2715, 2719}

$$\frac{2\sinh(a+b\log(cx^n))\cosh^{\frac{3}{2}}(a+b\log(cx^n))}{5bn}-\frac{6iE\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{5bn}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] (((-6*I)/5)*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2])/(b*n) + (2*Cosh[a + b*L
og[c*x^n]]^(3/2)*Sinh[a + b*Log[c*x^n]])/(5*b*n)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx &= \frac{\text{Subst}\left(\int \cosh^{\frac{5}{2}}(a+bx)\,dx,x,\log(cx^n)\right)}{n}\\ &= \frac{2\cosh^{\frac{3}{2}}(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{5bn}+\frac{3\text{Subst}\left(\int \sqrt{\cosh(a+bx)}\right.}{5n}\\ &= -\frac{6iE\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{5bn}+\frac{2\cosh^{\frac{3}{2}}(a+b\log(cx^n))\sinh(a+b\log(cx^n}{5bn}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 62, normalized size = 0.93

$$\frac{-6iE\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)+\sqrt{\cosh(a+b\log(cx^n))}\sinh(2(a+b\log(cx^n)))}{5bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] ((-6*I)*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2] + Sqrt[Cosh[a + b*Log[c*x^n]
]]*Sinh[2*(a + b*Log[c*x^n])])/(5*b*n)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(97) = 194$.
time = 4.87, size = 256, normalized size = 3.82

method	result
derivativedivides	$\frac{2\sqrt{\left(2\left(\cosh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)}\left(8\left(\cosh^7\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)-16\left(\cosh^5\right.\right.}{5n\sqrt{2\left(\sinh^4\left(\frac{a}{2}+\frac{b\ln\right.\right.}}}$
default	$\frac{2\sqrt{\left(2\left(\cosh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)}\left(8\left(\cosh^7\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)-16\left(\cosh^5\right.\right.}{5n\sqrt{2\left(\sinh^4\left(\frac{a}{2}+\frac{b\ln\right.\right.}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*ln(c*x^n))^(5/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/5/n*((2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(
1/2)*(8*cosh(1/2*a+1/2*b*ln(c*x^n))^7-16*cosh(1/2*a+1/2*b*ln(c*x^n))^5+10*c
osh(1/2*a+1/2*b*ln(c*x^n))^3-3*(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*c
osh(1/2*a+1/2*b*ln(c*x^n))^2+1)^(1/2)*EllipticE(cosh(1/2*a+1/2*b*ln(c*x^n))
,2^(1/2))-2*cosh(1/2*a+1/2*b*ln(c*x^n)))/(2*sinh(1/2*a+1/2*b*ln(c*x^n))^4+s
inh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/sinh(1/2*a+1/2*b*ln(c*x^n))/(2*cosh(1/2
*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^(5/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(cosh(b*log(c*x^n) + a)^(5/2)/x, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 332, normalized size = 4.96

$$-\frac{12\left(\sqrt{2}\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2\sqrt{2}\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sqrt{2}\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)\right)-\left(\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{4}+4\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{4}+6\left(\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}-2\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}-12\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+4\left(\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}-6\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-1\right)\sqrt{\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}}{10\left(bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2\,bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+bn\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")
```

```
[Out] -1/10*(12*(sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*sqrt(2)*cosh(b*n*l
og(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sqrt(2)*sinh(b*n*lo
g(x) + b*log(c) + a)^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, c
osh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a))) - (cosh(
b*n*log(x) + b*log(c) + a)^4 + 4*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*l
og(x) + b*log(c) + a)^3 + sinh(b*n*log(x) + b*log(c) + a)^4 + 6*(cosh(b*n*l
og(x) + b*log(c) + a)^2 - 2)*sinh(b*n*log(x) + b*log(c) + a)^2 - 12*cosh(b*
n*log(x) + b*log(c) + a)^2 + 4*(cosh(b*n*log(x) + b*log(c) + a)^3 - 6*cosh(
b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a) - 1)*sqrt(cosh(
b*n*log(x) + b*log(c) + a)))/(b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*b*n
*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + b*n*sinh
(b*n*log(x) + b*log(c) + a)^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*ln(c*x**n))**(5/2)/x,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^(5/2)/x,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*log(c*x^n) + a)^(5/2)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\mathrm{cosh}\left(a+b\,\ln\left(c\,x^n\right)\right)}^{5/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*log(c*x^n))^(5/2)/x,x)
[Out] int(cosh(a + b*log(c*x^n))^(5/2)/x, x)
```

3.253 $\int \frac{\cosh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=67

$$-\frac{2iF\left(\frac{1}{2}i(a+b\log\left(cx^n\right))\middle|2\right)}{3bn}+\frac{2\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}\,\sinh\left(a+b\log\left(cx^n\right)\right)}{3bn}$$

```
[Out] -2/3*I*(cosh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/cosh(1/2*a+1/2*b*ln(c*x^n))*El
lipticF(I*sinh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/b/n+2/3*sinh(a+b*ln(c*x^n))*
cosh(a+b*ln(c*x^n))^(1/2)/b/n
```

Rubi [A]
time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2715, 2720}

$$\frac{2\sinh\left(a+b\log\left(cx^n\right)\right)\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}}{3bn}-\frac{2iF\left(\frac{1}{2}i(a+b\log\left(cx^n\right))\middle|2\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] (((-2*I)/3)*EllipticF[(I/2)*(a + b*Log[c*x^n]), 2])/(b*n) + (2*Sqrt[Cosh[a
+ b*Log[c*x^n]]]*Sinh[a + b*Log[c*x^n]])/(3*b*n)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cosh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{\text{Subst}\left(\int \cosh^{\frac{3}{2}}(a+bx)\,dx, x, \log(cx^n)\right)}{n}$$

$$= \frac{2\sqrt{\cosh(a+b\log(cx^n))}\sinh(a+b\log(cx^n))}{3bn} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\cosh(a+bx)}}\right.}{3n}$$

$$= -\frac{2iF\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{3bn} + \frac{2\sqrt{\cosh(a+b\log(cx^n))}\sinh(a+b\log(cx^n}{3bn}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.09, size = 114, normalized size = 1.70

$$\frac{\sinh(2(a+b\log(cx^n))) + 2\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\cosh(2(a+b\log(cx^n)))-\sinh(2(a+b\log(cx^n)))\right)\sqrt{1+\cosh(2(a+b\log(cx^n)))+\sinh(2(a+b\log(cx^n)))}}{3bn\sqrt{\cosh(a+b\log(cx^n))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] (Sinh[2*(a + b*Log[c*x^n])] + 2*Hypergeometric2F1[1/4, 1/2, 5/4, -Cosh[2*(a
 + b*Log[c*x^n])] - Sinh[2*(a + b*Log[c*x^n])]]*Sqrt[1 + Cosh[2*(a + b*Log[
c*x^n])] + Sinh[2*(a + b*Log[c*x^n])]])/(3*b*n*Sqrt[Cosh[a + b*Log[c*x^n]]]
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(97) = 194.
time = 3.60, size = 237, normalized size = 3.54

method	result
derivativedivides	$\frac{2\sqrt{\left(2\left(\cosh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)}\left(4\left(\cosh^5\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)-6\left(\cosh^3\right.\right.}{3n\sqrt{2\left(\sinh^4\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)+\text{s}}}$
default	$\frac{2\sqrt{\left(2\left(\cosh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)}\left(4\left(\cosh^5\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)-6\left(\cosh^3\right.\right.}{3n\sqrt{2\left(\sinh^4\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)+\text{s}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*ln(c*x^n))^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/n*((2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(
1/2)*(4*cosh(1/2*a+1/2*b*ln(c*x^n))^5-6*cosh(1/2*a+1/2*b*ln(c*x^n))^3+(-sin
h(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*cosh(1/2*a+1/2*b*ln(c*x^n))^2+1)^(1/2
)*EllipticF(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))+2*cosh(1/2*a+1/2*b*ln(c*x^
n)))/(2*sinh(1/2*a+1/2*b*ln(c*x^n))^4+sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/
sinh(1/2*a+1/2*b*ln(c*x^n))/(2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(cosh(b*log(c*x^n) + a)^(3/2)/x, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.19, size = 170, normalized size = 2.54

$$\frac{2\left(\sqrt{2}\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sqrt{2}\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right){\rm weierstrassPInverse}\left(-4,0,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)+\left(\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}-1\right)\sqrt{\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}}{3\left(bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+bn\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 1/3*(2*(sqrt(2)*cosh(b*n*log(x) + b*log(c) + a) + sqrt(2)*sinh(b*n*log(x) +
 b*log(c) + a))*weierstrassPInverse(-4, 0, cosh(b*n*log(x) + b*log(c) + a)
+ sinh(b*n*log(x) + b*log(c) + a)) + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2
*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n
*log(x) + b*log(c) + a)^2 - 1)*sqrt(cosh(b*n*log(x) + b*log(c) + a)))/(b*n*
cosh(b*n*log(x) + b*log(c) + a) + b*n*sinh(b*n*log(x) + b*log(c) + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^{\frac{3}{2}}\left(a+b\log\left(cx^{n}\right)\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*ln(c*x**n))**(3/2)/x,x)
```

```
[Out] Integral(cosh(a + b*log(c*x**n))**(3/2)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*log(c*x^n) + a)^(3/2)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(a + b\ln(c\,x^n))^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*log(c*x^n))^(3/2)/x,x)
```

```
[Out] int(cosh(a + b*log(c*x^n))^(3/2)/x, x)
```

3.254 $\int \frac{\sqrt{\cosh(a+b\log(cx^n))}}{x}\,dx$

Optimal. Leaf size=28

$$-\frac{2iE\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{bn}$$

```
[Out] -2*I*(cosh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/cosh(1/2*a+1/2*b*ln(c*x^n))*Elli
pticE(I*sinh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/b/n
```

Rubi [A]
time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{2719\}$

$$-\frac{2iE\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cosh[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] ((-2*I)*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2])/(b*n)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\cosh(a+b\log(cx^n))}}{x}\,dx &= \frac{\text{Subst}\left(\int \sqrt{\cosh(a+bx)}\,dx, x, \log(cx^n)\right)}{n}\\ &= -\frac{2iE\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{bn}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 28, normalized size = 1.00

$$-\frac{2iE\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cosh[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] ((-2*I)*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2])/(b*n)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(64) = 128$.
time = 3.39, size = 183, normalized size = 6.54

method	result
derivativedivides	$-\frac{2\sqrt{\left(2\left(\cosh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)}\sqrt{-\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right.}}{n\sqrt{2\left(\sinh^4\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)+\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)}\sinh\left(\frac{a}{2}+\frac{b}{}\right.}$
default	$-\frac{2\sqrt{\left(2\left(\cosh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)}\sqrt{-\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right.}}{n\sqrt{2\left(\sinh^4\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)+\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)}\sinh\left(\frac{a}{2}+\frac{b}{}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*ln(c*x^n))^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -2/n*((2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1
/2)*(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*cosh(1/2*a+1/2*b*ln(c*x^n))^
2+1)^(1/2)*EllipticE(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/(2*sinh(1/2*a+1/2
*b*ln(c*x^n))^4+sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/sinh(1/2*a+1/2*b*ln(c*
x^n))/(2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cosh(b*log(c*x^n) + a))/x, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 58, normalized size = 2.07

$$-\frac{2\left(\sqrt{2}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)))+\sqrt{\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] -2*(sqrt(2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cosh(b*n*log(
x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a))) + sqrt(cosh(b*n*log(
x) + b*log(c) + a)))/(b*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cosh\left(a+b\log\left(cx^{n}\right)\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*ln(c*x**n))**(1/2)/x,x)
[Out] Integral(sqrt(cosh(a + b*log(c*x**n)))/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^(1/2)/x,x, algorithm="giac")
[Out] integrate(sqrt(cosh(b*log(c*x^n) + a))/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{\cosh\left(a+b\,\ln\left(c\,x^{n}\right)\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*log(c*x^n))^(1/2)/x,x)
[Out] int(cosh(a + b*log(c*x^n))^(1/2)/x, x)
```

3.255 $\int \frac{1}{x\sqrt{\cosh(a+b\log(cx^n))}}\,dx$

Optimal. Leaf size=28

$$-\frac{2iF\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{bn}$$

```
[Out] -2*I*(cosh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/cosh(1/2*a+1/2*b*ln(c*x^n))*Elli
pticF(I*sinh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/b/n
```

Rubi [A]
time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2720}

$$-\frac{2iF\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[Cosh[a + b*Log[c*x^n]]]),x]
```

```
[Out] ((-2*I)*EllipticF[(I/2)*(a + b*Log[c*x^n]), 2])/(b*n)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{\cosh(a+b\log(cx^n))}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx, x, \log(cx^n)\right)}{n}\\ &= -\frac{2iF\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{bn}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 28, normalized size = 1.00

$$-\frac{2iF\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[Cosh[a + b*Log[c*x^n]]]),x]
```

```
[Out] ((-2*I)*EllipticF[(I/2)*(a + b*Log[c*x^n]), 2])/(b*n)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(64) = 128$.
time = 3.16, size = 183, normalized size = 6.54

method	result
derivativedivides	$\frac{2\sqrt{\left(2\left(\cosh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)}\ \sqrt{-\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right.}}{n\sqrt{2\left(\sinh^4\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)+\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)}\ \sinh\left(\frac{a}{2}+\frac{b\,l}{}\right.}$
default	$\frac{2\sqrt{\left(2\left(\cosh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)}\ \sqrt{-\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right.}}{n\sqrt{2\left(\sinh^4\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)+\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)}\ \sinh\left(\frac{a}{2}+\frac{b\,l}{}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/cosh(a+b*ln(c*x^n))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/n*((2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/
2)*(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*cosh(1/2*a+1/2*b*ln(c*x^n))^2
+1)^(1/2)/(2*sinh(1/2*a+1/2*b*ln(c*x^n))^4+sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(
1/2)*EllipticF(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/sinh(1/2*a+1/2*b*ln(c*x
^n))/(2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cosh(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*sqrt(cosh(b*log(c*x^n) + a))), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 39, normalized size = 1.39

$$\frac{2\sqrt{2}\ \mathrm{weierstrassPInverse}(-4,0,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right))}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cosh(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*weierstrassPInverse(-4, 0, cosh(b*n*log(x) + b*log(c) + a) + sinh
(b*n*log(x) + b*log(c) + a))/(b*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cosh(a+b*ln(c*x**n))**(1/2),x)
[Out] Integral(1/(x*sqrt(cosh(a + b*log(c*x**n)))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cosh(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
[Out] integrate(1/(x*sqrt(cosh(b*log(c*x^n) + a))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x\,\sqrt{\cosh\left(a+b\,\ln\left(c\,x^n\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*cosh(a + b*log(c*x^n))^(1/2)),x)
[Out] int(1/(x*cosh(a + b*log(c*x^n))^(1/2)), x)
```

3.256 $\int \frac{1}{x\cosh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=63

$$\frac{2iE\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{bn}+\frac{2\sinh(a+b\log(cx^n))}{bn\sqrt{\cosh(a+b\log(cx^n))}}$$

```
[Out] 2*I*(cosh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/cosh(1/2*a+1/2*b*ln(c*x^n))*EllipticE(I*sinh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/b/n+2*sinh(a+b*ln(c*x^n))/b/n/cosh(a+b*ln(c*x^n))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2716, 2719}

$$\frac{2\sinh(a+b\log(cx^n))}{bn\sqrt{\cosh(a+b\log(cx^n))}}+\frac{2iE\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Cosh[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] ((2*I)*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2])/(b*n) + (2*Sinh[a + b*Log[c*x^n]])/(b*n*Sqrt[Cosh[a + b*Log[c*x^n]]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{x\cosh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{\cosh^{\frac{3}{2}}(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$
$$= \frac{2\sinh(a+b\log(cx^n))}{bn\sqrt{\cosh(a+b\log(cx^n))}} - \frac{\operatorname{Subst}\left(\int \sqrt{\cosh(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$
$$= \frac{2iE\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{bn} + \frac{2\sinh(a+b\log(cx^n))}{bn\sqrt{\cosh(a+b\log(cx^n))}}$$

Mathematica [A]
time = 0.05, size = 58, normalized size = 0.92

$$\frac{2\left(iE\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)+\frac{\sinh(a+b\log(cx^n))}{\sqrt{\cosh(a+b\log(cx^n))}}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Cosh[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] (2*(I*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2] + Sinh[a + b*Log[c*x^n]]/Sqrt[
Cosh[a + b*Log[c*x^n]]]))/(b*n)
```

Maple [A]
time = 3.63, size = 141, normalized size = 2.24

method	result
derivativedivides	$\frac{4\cosh\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)+2\,\mathrm{EllipticE}\left(\cosh\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right),\sqrt{2}\right)\sqrt{-2\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx}{2}\right.\right.}}{n\sinh\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\sqrt{2\left(\cosh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)-1}\;b}$
default	$\frac{4\cosh\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)+2\,\mathrm{EllipticE}\left(\cosh\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right),\sqrt{2}\right)\sqrt{-2\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx}{2}\right.\right.}}{n\sinh\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\sqrt{2\left(\cosh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)-1}\;b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/cosh(a+b*ln(c*x^n))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/n*(2*cosh(1/2*a+1/2*b*ln(c*x^n))*sinh(1/2*a+1/2*b*ln(c*x^n))^2+EllipticE(
cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))*(-2*sinh(1/2*a+1/2*b*ln(c*x^n))^2-1)^(
```

```
1/2)*(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2))/sinh(1/2*a+1/2*b*ln(c*x^n))/(2
*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cosh(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
[Out] integrate(1/(x*cosh(b*log(c*x^n) + a)^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 243, normalized size = 3.86

$$\frac{2\left(\left(\sqrt{2}\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2\sqrt{2}\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sqrt{2}\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+\sqrt{2}\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)))+2\left(\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}\right)\sqrt{\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}\right)}{bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2\,bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+bn\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cosh(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
[Out] 2*((sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*sqrt(2)*cosh(b*n*log(x) +
 b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sqrt(2)*sinh(b*n*log(x) +
b*log(c) + a)^2 + sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a))) + 2*(
cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(
b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2)*sqrt(cosh(b
*n*log(x) + b*log(c) + a)))/(b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*b*n*
cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + b*n*sinh(
b*n*log(x) + b*log(c) + a)^2 + b*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\cosh^{\frac{3}{2}}\left(a+b\log\left(cx^{n}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cosh(a+b*ln(c*x**n))**(3/2),x)
[Out] Integral(1/(x*cosh(a + b*log(c*x**n))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cosh(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*cosh(b*log(c*x^n) + a)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\cosh\left(a+b\,\ln\left(c\,x^n\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*cosh(a + b*log(c*x^n))^(3/2)),x)
```

```
[Out] int(1/(x*cosh(a + b*log(c*x^n))^(3/2)), x)
```

3.257 $\int \frac{1}{x \cosh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=67

$$-\frac{2iF\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{3bn}+\frac{2\sinh(a+b\log(cx^n))}{3bn\cosh^{\frac{3}{2}}(a+b\log(cx^n))}$$

```
[Out] -2/3*I*(cosh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/cosh(1/2*a+1/2*b*ln(c*x^n))*El
lipticF(I*sinh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/b/n+2/3*sinh(a+b*ln(c*x^n))/
b/n/cosh(a+b*ln(c*x^n))^(3/2)
```

Rubi [A]

time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2716, 2720}

$$\frac{2\sinh(a+b\log(cx^n))}{3bn\cosh^{\frac{3}{2}}(a+b\log(cx^n))}-\frac{2iF\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Cosh[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] (((-2*I)/3)*EllipticF[(I/2)*(a + b*Log[c*x^n]), 2])/(b*n) + (2*Sinh[a + b*L
og[c*x^n]])/(3*b*n*Cosh[a + b*Log[c*x^n]]^(3/2))
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{x\cosh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\cosh^{\frac{5}{2}}(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$
$$= \frac{2\sinh(a+b\log(cx^n))}{3bn\cosh^{\frac{3}{2}}(a+b\log(cx^n))} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx, x, \log(cx^n)\right)}{3n}$$
$$= -\frac{2iF\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{3bn} + \frac{2\sinh(a+b\log(cx^n))}{3bn\cosh^{\frac{3}{2}}(a+b\log(cx^n))}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.06, size = 122, normalized size = 1.82

$$\frac{2\left(\sinh(a+b\log(cx^n))+\cosh(a+b\log(cx^n))\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\cosh(2(a+b\log(cx^n)))-\sinh(2(a+b\log(cx^n)))\right)\sqrt{1+\cosh(2(a+b\log(cx^n)))+\sinh(2(a+b\log(cx^n)))}\right)}{3bn\cosh^{\frac{3}{2}}(a+b\log(cx^n))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Cosh[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] (2*(Sinh[a + b*Log[c*x^n]] + Cosh[a + b*Log[c*x^n]]*Hypergeometric2F1[1/4,
1/2, 5/4, -Cosh[2*(a + b*Log[c*x^n])] - Sinh[2*(a + b*Log[c*x^n])]]*Sqrt[1
+ Cosh[2*(a + b*Log[c*x^n])] + Sinh[2*(a + b*Log[c*x^n])]]))/(3*b*n*Cosh[a
+ b*Log[c*x^n]]^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(97) = 194.
time = 3.49, size = 295, normalized size = 4.40

method	result
derivativedivides	$\frac{2\left(2\sqrt{-\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)}\sqrt{-2\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)-1}\ \text{EllipticF}\left(\cosh\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right.\right.\right.}{}$
default	$\frac{2\left(2\sqrt{-\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)}\sqrt{-2\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)\right)-1}\ \text{EllipticF}\left(\cosh\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/cosh(a+b*ln(c*x^n))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/n*(2*(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*sinh(1/2*a+1/2*b*ln(c*x
^n))^2-1)^(1/2)*EllipticF(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))*sinh(1/2*a+1
/2*b*ln(c*x^n))^2+2*cosh(1/2*a+1/2*b*ln(c*x^n))*sinh(1/2*a+1/2*b*ln(c*x^n))
^2+(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*sinh(1/2*a+1/2*b*ln(c*x^n))^2
-1)^(1/2)*EllipticF(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2)))*((2*cosh(1/2*a+1/
2*b*ln(c*x^n))^2-1)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/(2*sinh(1/2*a+1/2*
b*ln(c*x^n))^4+sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/(2*cosh(1/2*a+1/2*b*ln(
c*x^n))^2-1)^(3/2)/sinh(1/2*a+1/2*b*ln(c*x^n))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cosh(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*cosh(b*log(c*x^n) + a)^(5/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.16, size = 501, normalized size = 7.48

$$\frac{2\left(\left(\sqrt{2}\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{4}+4\sqrt{2}\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}+\sqrt{2}\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{4}+2\left(3\sqrt{2}\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+\sqrt{2}\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2\sqrt{2}\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+4\left(\sqrt{2}\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}+\sqrt{2}\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sqrt{2}\right){\rm weierstrassPInverse}\left(-4,0,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)+2\left(\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}+3\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}+\left(3\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}-1\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)\sqrt{\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}\right)}{3\left(bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{4}+4bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}+bn\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{4}+2bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2\left(3bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+bn\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+bn+4\left(bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}+bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cosh(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*((sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)^4 + 4*sqrt(2)*cosh(b*n*log(x)
 + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3 + sqrt(2)*sinh(b*n*log(x
) + b*log(c) + a)^4 + 2*(3*sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)^2 + sqrt
(2))*sinh(b*n*log(x) + b*log(c) + a)^2 + 2*sqrt(2)*cosh(b*n*log(x) + b*log(
c) + a)^2 + 4*(sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)^3 + sqrt(2)*cosh(b*n
*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a) + sqrt(2))*weierst
rassPInverse(-4, 0, cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*l
og(c) + a)) + 2*(cosh(b*n*log(x) + b*log(c) + a)^3 + 3*cosh(b*n*log(x) + b*
log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^2 + sinh(b*n*log(x) + b*log(c)
+ a)^3 + (3*cosh(b*n*log(x) + b*log(c) + a)^2 - 1)*sinh(b*n*log(x) + b*log(
c) + a) - cosh(b*n*log(x) + b*log(c) + a))*sqrt(cosh(b*n*log(x) + b*log(c)
+ a)))/(b*n*cosh(b*n*log(x) + b*log(c) + a)^4 + 4*b*n*cosh(b*n*log(x) + b*l
og(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3 + b*n*sinh(b*n*log(x) + b*log(
c) + a)^4 + 2*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*(3*b*n*cosh(b*n*log
(x) + b*log(c) + a)^2 + b*n)*sinh(b*n*log(x) + b*log(c) + a)^2 + b*n + 4*(b
*n*cosh(b*n*log(x) + b*log(c) + a)^3 + b*n*cosh(b*n*log(x) + b*log(c) + a))
*sinh(b*n*log(x) + b*log(c) + a))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cosh(a+b*ln(c*x**n))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5007 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cosh(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
[Out] integrate(1/(x*cosh(b*log(c*x^n) + a)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\cosh\left(a+b\,\ln\left(c\,x^n\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*cosh(a + b*log(c*x^n))^(5/2)),x)
[Out] int(1/(x*cosh(a + b*log(c*x^n))^(5/2)), x)
```

3.258 $\int \cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\,dx$

Optimal. Leaf size=206

$$-\frac{1}{4}x\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)+\frac{5e^{-2a}x(cx^n)^{-4/n}\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{4\left(1+e^{-2a}(cx^n)^{-4/n}\right)^2}+\frac{5x\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{12\left(1+e^{-2a}(cx^n)^{-4/n}\right)}-\frac{5e^{-3a}x(c$$

```
[Out] -1/4*x*cosh(a+2*ln(c*x^n)/n)^(5/2)+5/4*x*cosh(a+2*ln(c*x^n)/n)^(5/2)/exp(2*
a)/((c*x^n)^(4/n))/(1+1/exp(2*a)/((c*x^n)^(4/n)))^2+5/12*x*cosh(a+2*ln(c*x^
n)/n)^(5/2)/(1+1/exp(2*a)/((c*x^n)^(4/n)))-5/4*x*arccsch(exp(a)*(c*x^n)^(2/
n))*cosh(a+2*ln(c*x^n)/n)^(5/2)/exp(3*a)/((c*x^n)^(6/n))/(1+1/exp(2*a)/((c*
x^n)^(4/n)))^(5/2)
```

Rubi [A]
time = 0.11, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {5637, 5645, 360, 356, 352, 248, 283, 221}

$$\frac{5e^{-2a}x(cx^n)^{-4/n}\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{4\left(e^{-2a}(cx^n)^{-4/n}+1\right)^2}-\frac{1}{4}x\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)+\frac{5x\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{12\left(e^{-2a}(cx^n)^{-4/n}+1\right)}-\frac{5e^{-3a}x(cx^n)^{-6/n}\operatorname{csch}^{-1}\left(e^a(cx^n)^{2/n}\right)\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{4\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + (2*Log[c*x^n])/n]^(5/2),x]
```

```
[Out] -1/4*(x*Cosh[a + (2*Log[c*x^n])/n]^(5/2)) + (5*x*Cosh[a + (2*Log[c*x^n])/n]
^(5/2))/(4*E^(2*a)*(c*x^n)^(4/n)*(1 + 1/(E^(2*a)*(c*x^n)^(4/n)))^2) + (5*x*
Cosh[a + (2*Log[c*x^n])/n]^(5/2))/(12*(1 + 1/(E^(2*a)*(c*x^n)^(4/n)))) - (5
*x*ArcCsch[E^a*(c*x^n)^(2/n)]*Cosh[a + (2*Log[c*x^n])/n]^(5/2))/(4*E^(3*a)*
(c*x^n)^(6/n)*(1 + 1/(E^(2*a)*(c*x^n)^(4/n)))^(5/2))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
```

```
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 352

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 356

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*(
(a + b*x^n)^p/(m + 1)), x] - Dist[b*n*(p/(m + 1)), Int[x^(m + n)*(a + b*x^n
)^(p - 1), x], x] /; FreeQ[{a, b, m, n}, x] && EqQ[(m + 1)/n + p, 0] && GtQ
[p, 0]
```

Rule 360

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m, n}, x] &&
 IntegerQ[p + Simplify[(m + 1)/n]] && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 5637

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Cosh[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5645

```
Int[Cosh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[Cosh[d*(a + b*Log[x])]^p/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p
), Int[(e*x)^m*x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p, x], x] /; FreeQ[{
a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)dx = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\cosh^{\frac{5}{2}}\left(a+\frac{2\log(x)}{n}\right)dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x(cx^n)^{-6/n}\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\right)\text{Subst}\left(\int x^{-1+\frac{6}{n}}\left(1+e^{-2a}x^{-4/n}\right)^{5/2} dx\right.}{n\left(1+e^{-2a}(cx^n)^{-4/n}\right)^{5/2}}$$

$$= -\frac{1}{4}x\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right) + \frac{\left(5x(cx^n)^{-6/n}\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\right)\text{Sub}}{2n\left(1+e^{-2a}\right.}$$

$$= -\frac{1}{4}x\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right) + \frac{5x\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{12\left(1+e^{-2a}(cx^n)^{-4/n}\right)} + \frac{\left(5e^{-2a}x(cx^n\right.}{}$$

$$= -\frac{1}{4}x\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right) + \frac{5x\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{12\left(1+e^{-2a}(cx^n)^{-4/n}\right)} + \frac{\left(5e^{-2a}x(cx^n\right.}{}$$

$$= -\frac{1}{4}x\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right) + \frac{5x\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{12\left(1+e^{-2a}(cx^n)^{-4/n}\right)} - \frac{\left(5e^{-2a}x(cx^n\right.}{}$$

$$= -\frac{1}{4}x\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right) + \frac{5e^{-2a}x(cx^n)^{-4/n}\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{4\left(1+e^{-2a}(cx^n)^{-4/n}\right)^2} +$$

$$= -\frac{1}{4}x\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right) + \frac{5e^{-2a}x(cx^n)^{-4/n}\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{4\left(1+e^{-2a}(cx^n)^{-4/n}\right)^2} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.24, size = 85, normalized size = 0.41

$$\frac{1}{14}e^{2a}x(cx^n)^{4/n}\left(1+e^{2a}(cx^n)^{4/n}\right)\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right){}_2F_1\left(2,\frac{7}{2};\frac{9}{2};1+e^{2a}(cx^n)^{4/n}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + (2*Log[c*x^n])/n]^(5/2),x]
```

```
[Out] (E^(2*a)*x*(c*x^n)^(4/n)*(1 + E^(2*a)*(c*x^n)^(4/n))*Cosh[a + (2*Log[c*x^n]
)/n]^(5/2)*Hypergeometric2F1[2, 7/2, 9/2, 1 + E^(2*a)*(c*x^n)^(4/n)])/14
```

Maple [F]
time = 2.58, size = 0, normalized size = 0.00

$$\int \cosh^{\frac{5}{2}}\left(a+\frac{2\ln\left(c\,x^{n}\right)}{n}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+2*ln(c*x^n)/n)^(5/2),x)
```

```
[Out] int(cosh(a+2*ln(c*x^n)/n)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+2*log(c*x^n)/n)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(a + 2*log(c*x^n)/n)^(5/2), x)
```

Fricas [A]
time = 0.47, size = 187, normalized size = 0.91

$$\frac{\left(15\,\sqrt{2}\,x^{3}e^{\left(\frac{3\,(an+2\,\log(c))}{2\,n}\right)}\log\left(\frac{x^{4}e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}-2\,\sqrt{2}\,\sqrt{\frac{1}{2}}\,x\sqrt{\frac{x^{4}e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}+1}{x^{2}}}+2}{x^{4}}\right)+4\,\sqrt{\frac{1}{2}}\left(2\,x^{8}e^{\left(\frac{4\,(an+2\,\log(c))}{n}\right)}+14\,x^{4}e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}-3\right)\sqrt{\frac{x^{4}e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}+1}{x^{2}}}\,e^{\left(-\frac{an+2\,\log(c)}{2\,n}\right)}\right)e^{\left(-\frac{2\,(an+2\,\log(c))}{n}\right)}}{192\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+2*log(c*x^n)/n)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/192*(15*sqrt(2)*x^3*e^(3/2*(a*n + 2*log(c))/n)*log((x^4*e^(2*(a*n + 2*log
(c))/n) - 2*sqrt(2)*sqrt(1/2)*x*sqrt((x^4*e^(2*(a*n + 2*log(c))/n) + 1)/x^2
) + 2)/x^4) + 4*sqrt(1/2)*(2*x^8*e^(4*(a*n + 2*log(c))/n) + 14*x^4*e^(2*(a*
n + 2*log(c))/n) - 3)*sqrt((x^4*e^(2*(a*n + 2*log(c))/n) + 1)/x^2)*e^(-1/2*
(a*n + 2*log(c))/n))*e^(-2*(a*n + 2*log(c))/n)/x^3
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+2*ln(c*x**n)/n)**(5/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+2*log(c*x^n)/n)^(5/2),x, algorithm="giac")
[Out] integrate(cosh(a + 2*log(c*x^n)/n)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cosh\left(a+\frac{2\,\ln\left(c\,x^n\right)}{n}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + (2*log(c*x^n))/n)^(5/2),x)
[Out] int(cosh(a + (2*log(c*x^n))/n)^(5/2), x)
```

3.259 $\int \sqrt{\cosh \left(a+\frac{2 \log(cx^n)}{n}\right)}\, dx$

Optimal. Leaf size=102

$$\frac{1}{2}x\sqrt{\cosh \left(a+\frac{2 \log \left(cx^n\right)}{n}\right)}-\frac{e^{-a}x(cx^n)^{-2/n}\operatorname{csch}^{-1}\left(e^a(cx^n)^{2/n}\right)\sqrt{\cosh \left(a+\frac{2 \log \left(cx^n\right)}{n}\right)}}{2\sqrt{1+e^{-2a}\left(cx^n\right)^{-4/n}}}$$

[Out] 1/2*x*cosh(a+2*ln(c*x^n)/n)^(1/2)-1/2*x*arccsch(exp(a)*(c*x^n)^(2/n))*cosh(
a+2*ln(c*x^n)/n)^(1/2)/exp(a)/((c*x^n)^(2/n))/(1+1/exp(2*a)/((c*x^n)^(4/n))
)^(1/2)

Rubi [A]
time = 0.06, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5637, 5645, 352, 248, 283, 221}

$$\frac{1}{2}x\sqrt{\cosh \left(a+\frac{2 \log \left(cx^n\right)}{n}\right)}-\frac{e^{-a}x(cx^n)^{-2/n}\operatorname{csch}^{-1}\left(e^a(cx^n)^{2/n}\right)\sqrt{\cosh \left(a+\frac{2 \log \left(cx^n\right)}{n}\right)}}{2\sqrt{e^{-2a}\left(cx^n\right)^{-4/n}+1}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Cosh[a + (2*Log[c*x^n])/n]],x]

[Out] (x*Sqrt[Cosh[a + (2*Log[c*x^n])/n]])/2 - (x*ArcCsch[E^a*(c*x^n)^(2/n)]*Sqrt
[Cosh[a + (2*Log[c*x^n])/n]])/(2*E^a*(c*x^n)^(2/n)*Sqrt[1 + 1/(E^(2*a)*(c*x
^n)^(4/n))])

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
```

```
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 352

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 5637

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Cosh[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5645

```
Int[Cosh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[Cosh[d*(a + b*Log[x])]^p/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p
), Int[(e*x)^m*x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p, x], x] /; FreeQ[{
a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sqrt{\cosh \left(a+\frac{2 \log \left(c x^n\right)}{n}\right)} \, dx = \frac{\left(x (c x^n)^{-1/n}\right) \operatorname{Subst}\left(\int x^{-1+\frac{1}{n}} \sqrt{\cosh \left(a+\frac{2 \log(x)}{n}\right)} \, dx, x, c x^n\right)}{n}$$

$$= \frac{\left(x (c x^n)^{-2/n} \sqrt{\cosh \left(a+\frac{2 \log \left(c x^n\right)}{n}\right)}\right) \operatorname{Subst}\left(\int x^{-1+\frac{2}{n}} \sqrt{1+e^{-2a} x^{-4/}\right.}{n \sqrt{1+e^{-2a} \left(c x^n\right)^{-4/n}}}$$

$$= \frac{\left(x (c x^n)^{-2/n} \sqrt{\cosh \left(a+\frac{2 \log \left(c x^n\right)}{n}\right)}\right) \operatorname{Subst}\left(\int \sqrt{1+\frac{e^{-2a}}{x^2}} \, dx, x, (c\right.}{2 \sqrt{1+e^{-2a} \left(c x^n\right)^{-4/n}}}$$

$$= -\frac{\left(x (c x^n)^{-2/n} \sqrt{\cosh \left(a+\frac{2 \log \left(c x^n\right)}{n}\right)}\right) \operatorname{Subst}\left(\int \frac{\sqrt{1+e^{-2a} x^2}}{x^2} \, dx, x\right.}{2 \sqrt{1+e^{-2a} \left(c x^n\right)^{-4/n}}}$$

$$= \frac{1}{2} x \sqrt{\cosh \left(a+\frac{2 \log \left(c x^n\right)}{n}\right)} - \frac{\left(e^{-2a} x (c x^n)^{-2/n} \sqrt{\cosh \left(a+\frac{2 \log \left(c x^n\right.}{n}\right.}\right.}{2 \sqrt{1+}}$$

$$= \frac{1}{2} x \sqrt{\cosh \left(a+\frac{2 \log \left(c x^n\right)}{n}\right)} - \frac{e^{-a} x (c x^n)^{-2/n} \sinh^{-1}\left(e^{-a} (c x^n)^{-2/n}\right) \sqrt{}}{2 \sqrt{1+e^{-2a} \left(c x^n\right)^{-}}}$$

Mathematica [A]
time = 0.22, size = 74, normalized size = 0.73

$$\frac{1}{2} x \left(1-\frac{\tanh^{-1}\left(\sqrt{1+e^{2a} \left(c x^n\right)^{4/n}}\right)}{\sqrt{1+e^{2a} \left(c x^n\right)^{4/n}}}\right) \sqrt{\cosh \left(a+\frac{2 \log \left(c x^n\right)}{n}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cosh[a + (2*Log[c*x^n])/n]],x]
```

```
[Out] (x*(1 - ArcTanh[Sqrt[1 + E^(2*a)*(c*x^n)^(4/n)]]/Sqrt[1 + E^(2*a)*(c*x^n)^(
4/n)])*Sqrt[Cosh[a + (2*Log[c*x^n])/n]])/2
```

Maple [F]
time = 2.48, size = 0, normalized size = 0.00

$$\int \sqrt{\cosh}\left(a+\frac{2\ln \left(c\,x^{n}\right)}{n}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+2*ln(c*x^n)/n)^(1/2),x)
[Out] int(cosh(a+2*ln(c*x^n)/n)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+2*log(c*x^n)/n)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(cosh(a + 2*log(c*x^n)/n)), x)
```

Fricas [A]
time = 0.41, size = 141, normalized size = 1.38

$$\frac{1}{8}\left(4\sqrt{\frac{1}{2}}\,x\sqrt{\frac{x^{4}e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}+1}{x^{2}}}\,e^{\left(\frac{an+2\,\log(c)}{2\,n}\right)}+\sqrt{2}\,e^{\left(\frac{an+2\,\log(c)}{2\,n}\right)}\log\left(\frac{x^{4}e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}-2\,\sqrt{2}\,\sqrt{\frac{1}{2}}\,x\sqrt{\frac{x^{4}e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}+1}{x^{2}}}+2}{x^{4}}\right)\right)e^{\left(-\frac{an+2\,\log(c)}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+2*log(c*x^n)/n)^(1/2),x, algorithm="fricas")
[Out] 1/8*(4*sqrt(1/2)*x*sqrt((x^4*e^(2*(a*n + 2*log(c))/n) + 1)/x^2)*e^(1/2*(a*n
 + 2*log(c))/n) + sqrt(2)*e^(1/2*(a*n + 2*log(c))/n)*log((x^4*e^(2*(a*n + 2
*log(c))/n) - 2*sqrt(2)*sqrt(1/2)*x*sqrt((x^4*e^(2*(a*n + 2*log(c))/n) + 1)
/x^2) + 2)/x^4))*e^(-(a*n + 2*log(c))/n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\cosh\left(a+\frac{2\log\left(cx^{n}\right)}{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+2*ln(c*x**n)/n)**(1/2),x)
```

```
[Out] Integral(sqrt(cosh(a + 2*log(c*x**n)/n)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+2*log(c*x^n)/n)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\cosh\left(a+\frac{2\,\ln\left(c\,x^n\right)}{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + (2*log(c*x^n))/n)^(1/2),x)
```

```
[Out] int(cosh(a + (2*log(c*x^n))/n)^(1/2), x)
```

3.260 $\int \frac{1}{\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx$

Optimal. Leaf size=42

$$-\frac{x\left(1+e^{-2a}(cx^n)^{-4/n}\right)}{2\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

[Out] -1/2*x*(1+1/exp(2*a)/((c*x^n)^(4/n)))/cosh(a+2*ln(c*x^n)/n)^(3/2)

Rubi [A]
time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5637, 5645, 270}

$$-\frac{x\left(e^{-2a}(cx^n)^{-4/n}+1\right)}{2\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

Antiderivative was successfully verified.

[In] Int[Cosh[a + (2*Log[c*x^n])/n]^(-3/2),x]

[Out] -1/2*(x*(1 + 1/(E^(2*a)*(c*x^n)^(4/n))))/Cosh[a + (2*Log[c*x^n])/n]^(3/2)

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5637

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Cosh[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5645

```
Int[Cosh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[Cosh[d*(a + b*Log[x])]^p/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p
), Int[(e*x)^m*x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p, x], x] /; FreeQ[{
a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}}{\cosh^{\frac{3}{2}}\left(a+\frac{2\log(x)}{n}\right)}\,dx,x,cx^n\right)}{n}$$

$$= \frac{\left(x(cx^n)^{2/n}\left(1+e^{-2a}(cx^n)^{-4/n}\right)^{3/2}\right)\operatorname{Subst}\left(\int \frac{x^{-1-\frac{2}{n}}}{(1+e^{-2a}x^{-4/n})^{3/2}}\,dx,x,cx^n\right)}{n\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

$$= -\frac{x\left(1+e^{-2a}(cx^n)^{-4/n}\right)}{2\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

Mathematica [A]
time = 0.08, size = 61, normalized size = 1.45

$$\frac{-\cosh\left(a-2\log(x)+\frac{2\log(cx^n)}{n}\right)+\sinh\left(a-2\log(x)+\frac{2\log(cx^n)}{n}\right)}{x\sqrt{\cosh\left(a+\frac{2\log(cx^n)}{n}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + (2*Log[c*x^n])/n]^(-3/2),x]
```

```
[Out] (-Cosh[a - 2*Log[x] + (2*Log[c*x^n])/n] + Sinh[a - 2*Log[x] + (2*Log[c*x^n]
)/n])/(x*Sqrt[Cosh[a + (2*Log[c*x^n])/n]])
```

Maple [F]
time = 2.38, size = 0, normalized size = 0.00

$$\int \frac{1}{\cosh\left(a+\frac{2\ln(cx^n)}{n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(a+2*ln(c*x^n)/n)^(3/2),x)
```

```
[Out] int(1/cosh(a+2*ln(c*x^n)/n)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(a+2*log(c*x^n)/n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(a + 2*log(c*x^n)/n)^(-3/2), x)
```

Fricas [A]
time = 0.46, size = 68, normalized size = 1.62

$$-\frac{2\sqrt{\frac{1}{2}}\,x\sqrt{\frac{x^4e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}+1}{x^2}}\,e^{\left(-\frac{an+2\,\log(c)}{2\,n}\right)}}{x^4e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(a+2*log(c*x^n)/n)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(1/2)*x*sqrt((x^4*e^(2*(a*n + 2*log(c))/n) + 1)/x^2)*e^(-1/2*(a*n +
2*log(c))/n)/(x^4*e^(2*(a*n + 2*log(c))/n) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\cosh^{\frac{3}{2}}\left(a+\frac{2\log\left(cx^n\right)}{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(a+2*ln(c*x**n)/n)**(3/2),x)
```

```
[Out] Integral(cosh(a + 2*log(c*x**n)/n)**(-3/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(a+2*log(c*x^n)/n)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\cosh\left(a+\frac{2\,\ln\left(c\,x^n\right)}{n}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(a + (2*log(c*x^n))/n)^(3/2),x)
```

```
[Out] int(1/cosh(a + (2*log(c*x^n))/n)^(3/2), x)
```

3.261 $\int \frac{1}{\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx$

Optimal. Leaf size=101

$$-\frac{x\left(1+e^{-2a}(cx^n)^{-4/n}\right)}{6\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}-\frac{e^{-2a}x(cx^n)^{-4/n}\left(1+e^{-2a}(cx^n)^{-4/n}\right)}{15\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

```
[Out] -1/6*x*(1+1/exp(2*a)/((c*x^n)^(4/n)))/cosh(a+2*ln(c*x^n)/n)^(7/2)-1/15*x*(1
+1/exp(2*a)/((c*x^n)^(4/n)))/exp(2*a)/((c*x^n)^(4/n))/cosh(a+2*ln(c*x^n)/n)
^(7/2)
```

Rubi [A]
time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5637, 5645, 277, 270}

$$-\frac{e^{-2a}x(cx^n)^{-4/n}\left(e^{-2a}(cx^n)^{-4/n}+1\right)}{15\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}-\frac{x\left(e^{-2a}(cx^n)^{-4/n}+1\right)}{6\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + (2*Log[c*x^n])/n]^(-7/2),x]
```

```
[Out] -1/6*(x*(1 + 1/(E^(2*a)*(c*x^n)^(4/n))))/Cosh[a + (2*Log[c*x^n])/n]^(7/2) -
 (x*(1 + 1/(E^(2*a)*(c*x^n)^(4/n))))/(15*E^(2*a)*(c*x^n)^(4/n)*Cosh[a + (2*
Log[c*x^n])/n]^(7/2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 5637

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Cosh[d*(a + b*Log[x])]^p, x]
```

```
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5645

```
Int[Cosh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[Cosh[d*(a + b*Log[x])]^p/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p
), Int[(e*x)^m*x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p, x], x] /; FreeQ[{
a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}}{\cosh^{\frac{7}{2}}\left(a+\frac{2\log(x)}{n}\right)}\,dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x(cx^n)^{6/n}\left(1+e^{-2a}(cx^n)^{-4/n}\right)^{7/2}\right)\text{Subst}\left(\int \frac{x^{-1-\frac{6}{n}}}{\left(1+e^{-2a}x^{-4/n}\right)^{7/2}}\,dx, x, cx^n\right)}{n\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

$$= -\frac{x\left(1+e^{-2a}(cx^n)^{-4/n}\right)}{6\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)} + \frac{\left(2e^{-2a}x(cx^n)^{6/n}\left(1+e^{-2a}(cx^n)^{-4/n}\right)^{7/2}\right)\text{Subst}}{3n\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

$$= -\frac{x\left(1+e^{-2a}(cx^n)^{-4/n}\right)}{6\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)} - \frac{e^{-2a}x(cx^n)^{-4/n}\left(1+e^{-2a}(cx^n)^{-4/n}\right)}{15\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

Mathematica [A]
time = 0.15, size = 121, normalized size = 1.20

$$\frac{\left((2+5x^4)\cosh\left(a-2\log(x)+\frac{2\log(cx^n)}{n}\right)+(-2+5x^4)\sinh\left(a-2\log(x)+\frac{2\log(cx^n)}{n}\right)\right)\left(-\cosh\left(2a-4\log(x)+\frac{4\log(cx^n)}{n}\right)+\sinh\left(2a-4\log(x)+\frac{4\log(cx^n)}{n}\right)\right)}{15x^5\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + (2*Log[c*x^n])/n]^(-7/2),x]
```

```
[Out] (((2 + 5*x^4)*Cosh[a - 2*Log[x] + (2*Log[c*x^n])/n] + (-2 + 5*x^4)*Sinh[a -
 2*Log[x] + (2*Log[c*x^n])/n])*(-Cosh[2*a - 4*Log[x] + (4*Log[c*x^n])/n] +
Sinh[2*a - 4*Log[x] + (4*Log[c*x^n])/n]))/(15*x^5*Cosh[a + (2*Log[c*x^n])/n
]^(5/2))
```

Maple [F]
time = 2.42, size = 0, normalized size = 0.00

$$\int \frac{1}{\cosh\left(a+\frac{2\ln(c\,x^n)}{n}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(a+2*ln(c*x^n)/n)^(7/2),x)
[Out] int(1/cosh(a+2*ln(c*x^n)/n)^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(a+2*log(c*x^n)/n)^(7/2),x, algorithm="maxima")
[Out] integrate(cosh(a + 2*log(c*x^n)/n)^(-7/2), x)
```

Fricas [A]
time = 0.40, size = 128, normalized size = 1.27

$$-\frac{8\sqrt{\frac{1}{2}}\left(5\,x^{5}e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}+2\,x\right)\sqrt{\frac{x^{4}e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}+1}{x^{2}}}\,e^{\left(-\frac{an+2\,\log(c)}{2\,n}\right)}}{15\left(x^{12}e^{\left(\frac{6\,(an+2\,\log(c))}{n}\right)}+3\,x^{8}e^{\left(\frac{4\,(an+2\,\log(c))}{n}\right)}+3\,x^{4}e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(a+2*log(c*x^n)/n)^(7/2),x, algorithm="fricas")
[Out] -8/15*sqrt(1/2)*(5*x^5*e^(2*(a*n + 2*log(c))/n) + 2*x)*sqrt((x^4*e^(2*(a*n
+ 2*log(c))/n) + 1)/x^2)*e^(-1/2*(a*n + 2*log(c))/n)/(x^12*e^(6*(a*n + 2*lo
g(c))/n) + 3*x^8*e^(4*(a*n + 2*log(c))/n) + 3*x^4*e^(2*(a*n + 2*log(c))/n)
+ 1)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(a+2*ln(c*x**n)/n)**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(a+2*log(c*x^n)/n)^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(a+\frac{2\,\ln\left(c\,x^n\right)}{n}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(a + (2*log(c*x^n))/n)^(7/2),x)
```

```
[Out] int(1/cosh(a + (2*log(c*x^n))/n)^(7/2), x)
```

3.262 $\int \cosh\left(\frac{a+bx}{c+dx}\right) dx$

Optimal. Leaf size=101

$$\frac{(c+dx)\cosh\left(\frac{a+bx}{c+dx}\right)}{d}+\frac{(bc-ad)\mathrm{Chi}\left(\frac{bc-ad}{d(c+dx)}\right)\sinh\left(\frac{b}{d}\right)}{d^2}-\frac{(bc-ad)\cosh\left(\frac{b}{d}\right)\mathrm{Shi}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}$$

```
[Out] (d*x+c)*cosh((b*x+a)/(d*x+c))/d-(-a*d+b*c)*cosh(b/d)*Shi((-a*d+b*c)/d/(d*x+
c))/d^2+(-a*d+b*c)*Chi((-a*d+b*c)/d/(d*x+c))*sinh(b/d)/d^2
```

Rubi [A]
time = 0.13, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5727, 3378, 3384, 3379, 3382}

$$\frac{\sinh\left(\frac{b}{d}\right)(bc-ad)\mathrm{Chi}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}-\frac{\cosh\left(\frac{b}{d}\right)(bc-ad)\mathrm{Shi}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\cosh\left(\frac{a+bx}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[(a + b*x)/(c + d*x)],x]
```

```
[Out] ((c + d*x)*Cosh[(a + b*x)/(c + d*x)])/d + ((b*c - a*d)*CoshIntegral[(b*c -
a*d)/(d*(c + d*x))]*Sinh[b/d])/d^2 - ((b*c - a*d)*Cosh[b/d]*SinhIntegral[(b
*c - a*d)/(d*(c + d*x))])/d^2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5727

```
Int[Cosh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> Dist[-d^(-1), Subst[Int[Cosh[b*(e/d) - e*(b*c - a*d)*(x/d)]^n/x^2, x],
 x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a
*d, 0]
```

Rubi steps

$$\int \cosh\left(\frac{a+bx}{c+dx}\right) dx = -\frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x^2} dx, x, \frac{1}{c+dx}\right)}{d}$$

$$= \frac{(c+dx)\cosh\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(bc-ad)\text{Subst}\left(\int \frac{\sinh\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x} dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\cosh\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{\left((bc-ad)\cosh\left(\frac{b}{d}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{(bc-ad)x}{d}\right)}{x} dx, x, \frac{1}{c+dx}\right)}{d^2} + \cdots$$

$$= \frac{(c+dx)\cosh\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(bc-ad)\text{Chi}\left(\frac{bc-ad}{d(c+dx)}\right)\sinh\left(\frac{b}{d}\right)}{d^2} - \frac{(bc-ad)\cosh\left(\frac{b}{d}\right)\text{Shi}\left(\cdots\right.}{d^2}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 373 vs. 2(101) = 202.
time = 0.25, size = 373, normalized size = 3.69

$$\frac{2cd\cosh\left(\frac{a+bx}{c+dx}\right) + 2d^2x\cosh\left(\frac{a+bx}{c+dx}\right) + (bc-ad)\text{Chi}\left(\frac{bc-ad}{cd+d^2x}\right)\left(-\cosh\left(\frac{b}{d}\right)+\sinh\left(\frac{b}{d}\right)\right) + (bc-ad)\text{Chi}\left(\frac{-bc+ad}{d(c+dx)}\right)\left(\cosh\left(\frac{b}{d}\right)+\sinh\left(\frac{b}{d}\right)\right) + bc\cosh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{-bc+ad}{d(c+dx)}\right) - ad\cosh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{-bc+ad}{d(c+dx)}\right) + bc\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{-bc+ad}{d(c+dx)}\right) - ad\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{-bc+ad}{d(c+dx)}\right) - bc\cosh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc-ad}{cd+d^2x}\right) + ad\cosh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc-ad}{cd+d^2x}\right) + bc\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc-ad}{cd+d^2x}\right) - ad\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc-ad}{cd+d^2x}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[(a + b*x)/(c + d*x)],x]
```

```
[Out] (2*c*d*Cosh[(a + b*x)/(c + d*x)] + 2*d^2*x*Cosh[(a + b*x)/(c + d*x)] + (b*c
 - a*d)*CoshIntegral[(b*c - a*d)/(c*d + d^2*x)]*(-Cosh[b/d] + Sinh[b/d]) +
(b*c - a*d)*CoshIntegral[(-(b*c) + a*d)/(d*(c + d*x))]*(Cosh[b/d] + Sinh[b/
d]) + b*c*Cosh[b/d]*SinhIntegral[(-(b*c) + a*d)/(d*(c + d*x))] - a*d*Cosh[b
/d]*SinhIntegral[(-(b*c) + a*d)/(d*(c + d*x))] + b*c*Sinh[b/d]*SinhIntegral
[(-(b*c) + a*d)/(d*(c + d*x))] - a*d*Sinh[b/d]*SinhIntegral[(-(b*c) + a*d)/
```

```
(d*(c + d*x))] - b*c*Cosh[b/d]*SinhIntegral[(b*c - a*d)/(c*d + d^2*x)] + a*
d*Cosh[b/d]*SinhIntegral[(b*c - a*d)/(c*d + d^2*x)] + b*c*Sinh[b/d]*SinhInt
egral[(b*c - a*d)/(c*d + d^2*x)] - a*d*Sinh[b/d]*SinhIntegral[(b*c - a*d)/(
c*d + d^2*x)])/(2*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 346 vs. $2(101) = 202$.
time = 1.91, size = 347, normalized size = 3.44

method	result
risch	$\frac{e^{-\frac{bx+a}{dx+c}}a}{\frac{2da}{dx+c}-\frac{2bc}{dx+c}} - \frac{e^{-\frac{bx+a}{dx+c}}bc}{2d\left(\frac{da}{dx+c}-\frac{bc}{dx+c}\right)} - \frac{e^{-\frac{b}{d}}\,\mathrm{expIntegral}\left(1,\frac{ad-bc}{d(dx+c)}\right)a}{2d} + \frac{e^{-\frac{b}{d}}\,\mathrm{expIntegral}\left(1,\frac{ad-bc}{d(dx+c)}\right)bc}{2d^2} + \frac{d\,e^{\frac{bx+a}{dx+c}}xa}{2ad-2bc} - \frac{e^{\frac{bx+a}{dx+c}}x\cdots}{2(ad-b\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh((b*x+a)/(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*exp(-(b*x+a)/(d*x+c))/(d*a/(d*x+c)-b*c/(d*x+c))*a-1/2/d*exp(-(b*x+a)/(d
*x+c))/(d*a/(d*x+c)-b*c/(d*x+c))*b*c-1/2/d*exp(-b/d)*Ei(1,(a*d-b*c)/d/(d*x+
c))*a+1/2/d^2*exp(-b/d)*Ei(1,(a*d-b*c)/d/(d*x+c))*b*c+1/2*d*exp((b*x+a)/(d*
x+c))/(a*d-b*c)*x*a-1/2*exp((b*x+a)/(d*x+c))/(a*d-b*c)*x*b*c+1/2*exp((b*x+a
)/(d*x+c))/(a*d-b*c)*c*a-1/2/d*exp((b*x+a)/(d*x+c))/(a*d-b*c)*c^2*b+1/2/d*e
xp(b/d)*Ei(1,-(a*d-b*c)/d/(d*x+c))*a-1/2/d^2*exp(b/d)*Ei(1,-(a*d-b*c)/d/(d*
x+c))*b*c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((b*x+a)/(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(cosh((b*x + a)/(d*x + c)), x)
```

Fricas [A]
time = 0.46, size = 171, normalized size = 1.69

$$\frac{2\,(d^2x+cd)\cosh\left(\frac{bx+a}{dx+c}\right) - \left((bc-ad)\mathrm{Ei}\left(\frac{bc-ad}{d^2x+cd}\right) - (bc-ad)\mathrm{Ei}\left(-\frac{bc-ad}{d^2x+cd}\right)\right)\cosh\left(\frac{b}{d}\right) + \left((bc-ad)\mathrm{Ei}\left(\frac{bc-ad}{d^2x+cd}\right) + (bc-ad)\mathrm{Ei}\left(-\frac{bc-ad}{d^2x+cd}\right)\right)\sinh\left(\frac{b}{d}\right)}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((b*x+a)/(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(d^2*x + c*d)*cosh((b*x + a)/(d*x + c)) - ((b*c - a*d)*Ei((b*c - a*d
)/(d^2*x + c*d)) - (b*c - a*d)*Ei(-(b*c - a*d)/(d^2*x + c*d)))*cosh(b/d) +
```

```
((b*c - a*d)*Ei((b*c - a*d)/(d^2*x + c*d)) + (b*c - a*d)*Ei(-(b*c - a*d)/(d
^2*x + c*d)))*sinh(b/d))/d^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh\left(\frac{a+bx}{c+dx}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((b*x+a)/(d*x+c)),x)
```

```
[Out] Integral(cosh((a + b*x)/(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 764 vs. 2(101) = 202.
time = 1.92, size = 764, normalized size = 7.56

$$\frac{\left(b^3c^2\mathrm{Ei}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)e^{\frac{b}{d}}-2\,ab^2cd\mathrm{Ei}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)e^{\frac{b}{d}}-\frac{(bx+a)b^2c^2d\mathrm{Ei}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)e^{\frac{b}{d}}}{dx+c}+a^2bd^2\mathrm{Ei}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)e^{\frac{b}{d}}+\frac{2\,(bx+a)abcd^2\mathrm{Ei}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)e^{\frac{b}{d}}}{dx+c}-\frac{(bx+a)a^2d^3\mathrm{Ei}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)e^{\frac{b}{d}}}{dx+c}+b^2c^2de^{\left(\frac{bx+a}{dx+c}\right)}-2\,abcd^2e^{\left(\frac{bx+a}{dx+c}\right)}+a^2d^3e^{\left(\frac{bx+a}{dx+c}\right)}\right)\left(\frac{bc}{(bc-ad)^2}-\frac{ad}{(bc-ad)^2}\right)}{2\left(bd^2-\frac{(bx+a)d^3}{dx+c}\right)}-\frac{\left(b^3c^2\mathrm{Ei}\left(\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)e^{\left(-\frac{b}{d}\right)}-2\,ab^2cd\mathrm{Ei}\left(\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)e^{\left(-\frac{b}{d}\right)}-\frac{(bx+a)b^2c^2d\mathrm{Ei}\left(\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)e^{\left(-\frac{b}{d}\right)}}{dx+c}+a^2bd^2\mathrm{Ei}\left(\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)e^{\left(-\frac{b}{d}\right)}+\frac{2\,(bx+a)abcd^2\mathrm{Ei}\left(\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)e^{\left(-\frac{b}{d}\right)}}{dx+c}-\frac{(bx+a)a^2d^3\mathrm{Ei}\left(\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)e^{\left(-\frac{b}{d}\right)}}{dx+c}-b^2c^2de^{\left(-\frac{bx+a}{dx+c}\right)}+2\,abcd^2e^{\left(-\frac{bx+a}{dx+c}\right)}-a^2d^3e^{\left(-\frac{bx+a}{dx+c}\right)}\right)\left(\frac{bc}{(bc-ad)^2}-\frac{ad}{(bc-ad)^2}\right)}{2\left(bd^2-\frac{(bx+a)d^3}{dx+c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((b*x+a)/(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c^2*Ei(-(b - (b*x + a)*d/(d*x + c))/d)*e^(b/d) - 2*a*b^2*c*d*Ei(-(
b - (b*x + a)*d/(d*x + c))/d)*e^(b/d) - (b*x + a)*b^2*c^2*d*Ei(-(b - (b*x +
 a)*d/(d*x + c))/d)*e^(b/d)/(d*x + c) + a^2*b*d^2*Ei(-(b - (b*x + a)*d/(d*x
 + c))/d)*e^(b/d) + 2*(b*x + a)*a*b*c*d^2*Ei(-(b - (b*x + a)*d/(d*x + c))/d
)*e^(b/d)/(d*x + c) - (b*x + a)*a^2*d^3*Ei(-(b - (b*x + a)*d/(d*x + c))/d)*
e^(b/d)/(d*x + c) + b^2*c^2*d*e^((b*x + a)/(d*x + c)) - 2*a*b*c*d^2*e^((b*x
 + a)/(d*x + c)) + a^2*d^3*e^((b*x + a)/(d*x + c)))*(b*c/(b*c - a*d)^2 - a*
d/(b*c - a*d)^2)/(b*d^2 - (b*x + a)*d^3/(d*x + c)) - 1/2*(b^3*c^2*Ei((b - (
b*x + a)*d/(d*x + c))/d)*e^(-b/d) - 2*a*b^2*c*d*Ei((b - (b*x + a)*d/(d*x +
c))/d)*e^(-b/d) - (b*x + a)*b^2*c^2*d*Ei((b - (b*x + a)*d/(d*x + c))/d)*e^(
-b/d)/(d*x + c) + a^2*b*d^2*Ei((b - (b*x + a)*d/(d*x + c))/d)*e^(-b/d) + 2*
(b*x + a)*a*b*c*d^2*Ei((b - (b*x + a)*d/(d*x + c))/d)*e^(-b/d)/(d*x + c) -
(b*x + a)*a^2*d^3*Ei((b - (b*x + a)*d/(d*x + c))/d)*e^(-b/d)/(d*x + c) - b^
2*c^2*d*e^(-(b*x + a)/(d*x + c)) + 2*a*b*c*d^2*e^(-(b*x + a)/(d*x + c)) - a
^2*d^3*e^(-(b*x + a)/(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)/(b
*d^2 - (b*x + a)*d^3/(d*x + c))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cosh\left(\frac{a+b\,x}{c+d\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh((a + b*x)/(c + d*x)),x)
```

```
[Out] int(cosh((a + b*x)/(c + d*x)), x)
```

3.263 $\int \cosh^2\left(\frac{a+bx}{c+dx}\right)\,dx$

Optimal. Leaf size=107

$$\frac{(c+dx)\cosh^2\left(\frac{a+bx}{c+dx}\right)}{d}+\frac{(bc-ad)\text{Chi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)\sinh\left(\frac{2b}{d}\right)}{d^2}-\frac{(bc-ad)\cosh\left(\frac{2b}{d}\right)\text{Shi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}$$

```
[Out] (d*x+c)*cosh((b*x+a)/(d*x+c))^2/d-(-a*d+b*c)*cosh(2*b/d)*Shi(2*(-a*d+b*c)/d
/(d*x+c))/d^2+(-a*d+b*c)*Chi(2*(-a*d+b*c)/d/(d*x+c))*sinh(2*b/d)/d^2
```

Rubi [A]
time = 0.15, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5727, 3394, 12, 3384, 3379, 3382}

$$\frac{\sinh\left(\frac{2b}{d}\right)(bc-ad)\text{Chi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}-\frac{\cosh\left(\frac{2b}{d}\right)(bc-ad)\text{Shi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\cosh^2\left(\frac{a+bx}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[(a + b*x)/(c + d*x)]^2,x]
```

```
[Out] ((c + d*x)*Cosh[(a + b*x)/(c + d*x)]^2)/d + ((b*c - a*d)*CoshIntegral[(2*(b
*c - a*d))/(d*(c + d*x))]*Sinh[(2*b)/d])/d^2 - ((b*c - a*d)*Cosh[(2*b)/d]*S
inhIntegral[(2*(b*c - a*d))/(d*(c + d*x))])/d^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 5727

```
Int[Cosh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> Dist[-d^(-1), Subst[Int[Cosh[b*(e/d) - e*(b*c - a*d)*(x/d)]^n/x^2, x],
 x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a
*d, 0]
```

Rubi steps

$$\int \cosh^2\left(\frac{a+bx}{c+dx}\right) dx = -\frac{\text{Subst}\left(\int \frac{\cosh^2\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x^2}\,dx, x, \frac{1}{c+dx}\right)}{d}$$

$$= \frac{(c+dx)\cosh^2\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(2i(bc-ad))\text{Subst}\left(\int -\frac{i\sinh\left(\frac{2b}{d}-\frac{2(bc-ad)x}{d}\right)}{2x}\,dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\cosh^2\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(bc-ad)\text{Subst}\left(\int \frac{\sinh\left(\frac{2b}{d}-\frac{2(bc-ad)x}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\cosh^2\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{\left((bc-ad)\cosh\left(\frac{2b}{d}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2(bc-ad)x}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\cosh^2\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(bc-ad)\text{Chi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)\sinh\left(\frac{2b}{d}\right)}{d^2} - \frac{(bc-ad)\cosh\left(\frac{2b}{d}\right)\text{Sh}}{d^2}$$

Mathematica [A]
time = 0.20, size = 111, normalized size = 1.04

$$\frac{d\left(dx+(c+dx)\cosh\left(\frac{2(a+bx)}{c+dx}\right)\right)+2(bc-ad)\text{Chi}\left(\frac{2(-bc+ad)}{d(c+dx)}\right)\sinh\left(\frac{2b}{d}\right)+2(bc-ad)\cosh\left(\frac{2b}{d}\right)\text{Shi}\left(\frac{2(-bc+ad)}{d(c+dx)}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[(a + b*x)/(c + d*x)]^2,x]
```

```
[Out] (d*(d*x + (c + d*x)*Cosh[(2*(a + b*x))/(c + d*x)]) + 2*(b*c - a*d)*CoshInte
gral[(2*(-(b*c) + a*d))/(d*(c + d*x))]*Sinh[(2*b)/d] + 2*(b*c - a*d)*Cosh[(
2*b)/d]*SinhIntegral[(2*(-(b*c) + a*d))/(d*(c + d*x))])/(2*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(107) = 214$.
time = 6.86, size = 358, normalized size = 3.35

method	result
risch	$\frac{x}{2}+\frac{e^{-\frac{2(bx+a)}{dx+c}}a}{\frac{4da}{dx+c}-\frac{4bc}{dx+c}}-\frac{e^{-\frac{2(bx+a)}{dx+c}}bc}{4d\left(\frac{da}{dx+c}-\frac{bc}{dx+c}\right)}-\frac{e^{-\frac{2b}{d}}\,\mathrm{expIntegral}\left(1,\frac{2ad-2bc}{(dx+c)d}\right)a}{2d}+\frac{e^{-\frac{2b}{d}}\,\mathrm{expIntegral}\left(1,\frac{2ad-2bc}{(dx+c)d}\right)bc}{2d^2}+\frac{d\,e^{\frac{2bx+2a}{dx+c}}xa}{4ad-4bc}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh((b*x+a)/(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x+1/4*exp(-2*(b*x+a)/(d*x+c))/(d*a/(d*x+c)-b*c/(d*x+c))*a-1/4/d*exp(-2*
(b*x+a)/(d*x+c))/(d*a/(d*x+c)-b*c/(d*x+c))*b*c-1/2/d*exp(-2*b/d)*Ei(1,2*(a*
d-b*c)/d/(d*x+c))*a+1/2/d^2*exp(-2*b/d)*Ei(1,2*(a*d-b*c)/d/(d*x+c))*b*c+1/4
*d*exp(2*(b*x+a)/(d*x+c))/(a*d-b*c)*x*a-1/4*exp(2*(b*x+a)/(d*x+c))/(a*d-b*c
)*x*b*c+1/4*exp(2*(b*x+a)/(d*x+c))/(a*d-b*c)*c*a-1/4/d*exp(2*(b*x+a)/(d*x+c
))/(a*d-b*c)*c^2*b+1/2/d*exp(2*b/d)*Ei(1,-2*(a*d-b*c)/d/(d*x+c))*a-1/2/d^2*
exp(2*b/d)*Ei(1,-2*(a*d-b*c)/d/(d*x+c))*b*c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((b*x+a)/(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/4*integrate(e^(2*b*c/(d^2*x + c*d) - 2*a/(d*x + c) - 2*b/d), x) +
 1/4*integrate(e^(-2*b*c/(d^2*x + c*d) + 2*a/(d*x + c) + 2*b/d), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(107) = 214$.
time = 0.42, size = 366, normalized size = 3.42

$$\frac{d^2x+(d^2x+cd)\cosh\left(\frac{bx+a}{dx+c}\right)^2+\left(d^2x-(bc-ad)\mathrm{Ei}\left(-\frac{2(bc-ad)}{d^2x+cd}\right)\cosh\left(\frac{2b}{d}\right)+cd\right)\sinh\left(\frac{bx+a}{dx+c}\right)^2+\left((bc-ad)\mathrm{Ei}\left(-\frac{2(bc-ad)}{d^2x+cd}\right)\cosh\left(\frac{bx+a}{dx+c}\right)^2-(bc-ad)\mathrm{Ei}\left(\frac{2(bc-ad)}{d^2x+cd}\right)\right)\cosh\left(\frac{2b}{d}\right)+\left((bc-ad)\mathrm{Ei}\left(-\frac{2(bc-ad)}{d^2x+cd}\right)\cosh\left(\frac{bx+a}{dx+c}\right)^2-(bc-ad)\mathrm{Ei}\left(-\frac{2(bc-ad)}{d^2x+cd}\right)\sinh\left(\frac{bx+a}{dx+c}\right)^2+(bc-ad)\mathrm{Ei}\left(\frac{2(bc-ad)}{d^2x+cd}\right)\right)\sinh\left(\frac{2b}{d}\right)}{2\left(d^2\cosh\left(\frac{bx+a}{dx+c}\right)^2-d^2\sinh\left(\frac{bx+a}{dx+c}\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((b*x+a)/(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(d^2*x + (d^2*x + c*d)*cosh((b*x + a)/(d*x + c))^2 + (d^2*x - (b*c - a*
d)*Ei(-2*(b*c - a*d)/(d^2*x + c*d))*cosh(2*b/d) + c*d)*sinh((b*x + a)/(d*x
+ c))^2 + ((b*c - a*d)*Ei(-2*(b*c - a*d)/(d^2*x + c*d))*cosh((b*x + a)/(d*x
 + c))^2 - (b*c - a*d)*Ei(2*(b*c - a*d)/(d^2*x + c*d)))*cosh(2*b/d) + ((b*c
 - a*d)*Ei(-2*(b*c - a*d)/(d^2*x + c*d))*cosh((b*x + a)/(d*x + c))^2 - (b*c
 - a*d)*Ei(-2*(b*c - a*d)/(d^2*x + c*d))*sinh((b*x + a)/(d*x + c))^2 + (b*c
 - a*d)*Ei(2*(b*c - a*d)/(d^2*x + c*d)))*sinh(2*b/d))/(d^2*cosh((b*x + a)/(
d*x + c))^2 - d^2*sinh((b*x + a)/(d*x + c))^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((b*x+a)/(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 749 vs. 2(107) = 214.
time = 6.48, size = 749, normalized size = 7.00

$$\frac{\left(2\,b^{3}c^{2}\mathrm{Ei}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)e^{\left(\frac{2\,b}{d}\right)}-4\,ab^{2}cd\mathrm{Ei}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)e^{\left(\frac{2\,b}{d}\right)}-\frac{2\,(bx+a)b^{2}c^{2}d\mathrm{Ei}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)e^{\left(\frac{2\,b}{d}\right)}}{dx+c}+2\,a^{2}bd^{2}\mathrm{Ei}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)e^{\left(\frac{2\,b}{d}\right)}+\frac{4\,(bx+a)abcd^{2}\mathrm{Ei}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)e^{\left(\frac{2\,b}{d}\right)}}{dx+c}-\frac{2\,(bx+a)a^{2}d^{3}\mathrm{Ei}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)e^{\left(\frac{2\,b}{d}\right)}}{dx+c}-2\,b^{3}c^{2}\mathrm{Ei}\left(\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)e^{\left(-\frac{2\,b}{d}\right)}+4\,ab^{2}cd\mathrm{Ei}\left(\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)e^{\left(-\frac{2\,b}{d}\right)}+\frac{2\,(bx+a)b^{2}c^{2}d\mathrm{Ei}\left(\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)e^{\left(-\frac{2\,b}{d}\right)}}{dx+c}-2\,a^{2}bd^{2}\mathrm{Ei}\left(\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)e^{\left(-\frac{2\,b}{d}\right)}-\frac{4\,(bx+a)abcd^{2}\mathrm{Ei}\left(\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)e^{\left(-\frac{2\,b}{d}\right)}}{dx+c}+\frac{2\,(bx+a)a^{2}d^{3}\mathrm{Ei}\left(\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)e^{\left(-\frac{2\,b}{d}\right)}}{dx+c}+b^{2}c^{2}de^{\left(\frac{2\,(bx+a)}{dx+c}\right)}-2\,abcd^{2}e^{\left(\frac{2\,(bx+a)}{dx+c}\right)}+a^{2}d^{3}e^{\left(\frac{2\,(bx+a)}{dx+c}\right)}+b^{2}c^{2}de^{\left(-\frac{2\,(bx+a)}{dx+c}\right)}-2\,abcd^{2}e^{\left(-\frac{2\,(bx+a)}{dx+c}\right)}+a^{2}d^{3}e^{\left(-\frac{2\,(bx+a)}{dx+c}\right)}+2\,b^{2}c^{2}d-4\,abcd^{2}+2\,a^{2}d^{3}\right)\left(\frac{bc}{(bc-ad)^{2}}-\frac{ad}{(bc-ad)^{2}}\right)}{4\left(bd^{2}-\frac{(bx+a)d^{3}}{dx+c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((b*x+a)/(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*b^3*c^2*Ei(-2*(b - (b*x + a)*d/(d*x + c))/d)*e^(2*b/d) - 4*a*b^2*c*d
*Ei(-2*(b - (b*x + a)*d/(d*x + c))/d)*e^(2*b/d) - 2*(b*x + a)*b^2*c^2*d*Ei(
-2*(b - (b*x + a)*d/(d*x + c))/d)*e^(2*b/d)/(d*x + c) + 2*a^2*b*d^2*Ei(-2*(
b - (b*x + a)*d/(d*x + c))/d)*e^(2*b/d) + 4*(b*x + a)*a*b*c*d^2*Ei(-2*(b -
(b*x + a)*d/(d*x + c))/d)*e^(2*b/d)/(d*x + c) - 2*(b*x + a)*a^2*d^3*Ei(-2*(
b - (b*x + a)*d/(d*x + c))/d)*e^(2*b/d)/(d*x + c) - 2*b^3*c^2*Ei(2*(b - (b*
x + a)*d/(d*x + c))/d)*e^(-2*b/d) + 4*a*b^2*c*d*Ei(2*(b - (b*x + a)*d/(d*x
+ c))/d)*e^(-2*b/d) + 2*(b*x + a)*b^2*c^2*d*Ei(2*(b - (b*x + a)*d/(d*x + c)
)/d)*e^(-2*b/d)/(d*x + c) - 2*a^2*b*d^2*Ei(2*(b - (b*x + a)*d/(d*x + c))/d)
*e^(-2*b/d) - 4*(b*x + a)*a*b*c*d^2*Ei(2*(b - (b*x + a)*d/(d*x + c))/d)*e^(
-2*b/d)/(d*x + c) + 2*(b*x + a)*a^2*d^3*Ei(2*(b - (b*x + a)*d/(d*x + c))/d)
*e^(-2*b/d)/(d*x + c) + b^2*c^2*d*e^(2*(b*x + a)/(d*x + c)) - 2*a*b*c*d^2*e
^(2*(b*x + a)/(d*x + c)) + a^2*d^3*e^(2*(b*x + a)/(d*x + c)) + b^2*c^2*d*e^
(-2*(b*x + a)/(d*x + c)) - 2*a*b*c*d^2*e^(-2*(b*x + a)/(d*x + c)) + a^2*d^3
*e^(-2*(b*x + a)/(d*x + c)) + 2*b^2*c^2*d - 4*a*b*c*d^2 + 2*a^2*d^3)*(b*c/(
b*c - a*d)^2 - a*d/(b*c - a*d)^2)/(b*d^2 - (b*x + a)*d^3/(d*x + c))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cosh\left(\frac{a+b\,x}{c+d\,x}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh((a + b*x)/(c + d*x))^2,x)
[Out] int(cosh((a + b*x)/(c + d*x))^2, x)
```

3.264 $\int e^{a+bx}\cosh^4(a+bx)\,dx$

Optimal. Leaf size=83

$$-\frac{e^{-3a-3bx}}{48b}-\frac{e^{-a-bx}}{4b}+\frac{3e^{a+bx}}{8b}+\frac{e^{3a+3bx}}{12b}+\frac{e^{5a+5bx}}{80b}$$

```
[Out] -1/48*exp(-3*b*x-3*a)/b-1/4*exp(-b*x-a)/b+3/8*exp(b*x+a)/b+1/12*exp(3*b*x+3
*a)/b+1/80*exp(5*b*x+5*a)/b
```

Rubi [A]
time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.188$, Rules used = {2320, 12, 276}

$$-\frac{e^{-3a-3bx}}{48b}-\frac{e^{-a-bx}}{4b}+\frac{3e^{a+bx}}{8b}+\frac{e^{3a+3bx}}{12b}+\frac{e^{5a+5bx}}{80b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cosh[a + b*x]^4,x]
```

```
[Out] -1/48*E^(-3*a - 3*b*x)/b - E^(-a - b*x)/(4*b) + (3*E^(a + b*x))/(8*b) + E^(
3*a + 3*b*x)/(12*b) + E^(5*a + 5*b*x)/(80*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^{a+bx}\cosh^4(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^4}{16x^4}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1+x^2)^4}{x^4}\,dx, x, e^{a+bx}\right)}{16b}$$
$$= \frac{\text{Subst}\left(\int \left(6+\frac{1}{x^4}+\frac{4}{x^2}+4x^2+x^4\right)\,dx, x, e^{a+bx}\right)}{16b}$$
$$= -\frac{e^{-3a-3bx}}{48b} - \frac{e^{-a-bx}}{4b} + \frac{3e^{a+bx}}{8b} + \frac{e^{3a+3bx}}{12b} + \frac{e^{5a+5bx}}{80b}$$

Mathematica [A]
time = 0.03, size = 62, normalized size = 0.75

$$\frac{e^{-3(a+bx)}\left(-5-60e^{2(a+bx)}+90e^{4(a+bx)}+20e^{6(a+bx)}+3e^{8(a+bx)}\right)}{240b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x]^4,x]
```

```
[Out] (-5 - 60*E^(2*(a + b*x)) + 90*E^(4*(a + b*x)) + 20*E^(6*(a + b*x)) + 3*E^(8
*(a + b*x)))/(240*b*E^(3*(a + b*x)))
```

Maple [A]
time = 1.30, size = 80, normalized size = 0.96

method	result	size
risch	$-\frac{e^{-3bx-3a}}{48b}-\frac{e^{-bx-a}}{4b}+\frac{3\,e^{bx+a}}{8b}+\frac{e^{3bx+3a}}{12b}+\frac{e^{5bx+5a}}{80b}$	69
default	$\frac{5\sinh(bx+a)}{8b}+\frac{5\sinh(3bx+3a)}{48b}+\frac{\sinh(5bx+5a)}{80b}+\frac{\cosh(bx+a)}{8b}+\frac{\cosh(3bx+3a)}{16b}+\frac{\cosh(5bx+5a)}{80b}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 5/8*sinh(b*x+a)/b+5/48/b*sinh(3*b*x+3*a)+1/80/b*sinh(5*b*x+5*a)+1/8*cosh(b*
x+a)/b+1/16/b*cosh(3*b*x+3*a)+1/80*cosh(5*b*x+5*a)/b
```

Maxima [A]
time = 0.26, size = 68, normalized size = 0.82

$$\frac{e^{(5\,bx+5\,a)}}{80\,b}+\frac{e^{(3\,bx+3\,a)}}{12\,b}+\frac{3\,e^{(bx+a)}}{8\,b}-\frac{e^{(-bx-a)}}{4\,b}-\frac{e^{(-3\,bx-3\,a)}}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/80*e^(5*b*x + 5*a)/b + 1/12*e^(3*b*x + 3*a)/b + 3/8*e^(b*x + a)/b - 1/4*e
^(-b*x - a)/b - 1/48*e^(-3*b*x - 3*a)/b
```

Fricas [A]
time = 0.43, size = 113, normalized size = 1.36

$$-\frac{\cosh(bx+a)^4 - 16\cosh(bx+a)\sinh(bx+a)^3 + \sinh(bx+a)^4 + 2\left(3\cosh(bx+a)^2 + 10\right)\sinh(bx+a)^2 + 20\cosh(bx+a)^2 - 16\left(\cosh(bx+a)^3 + 5\cosh(bx+a)\right)\sinh(bx+a) - 45}{120\left(b\cosh(bx+a) - b\sinh(bx+a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/120*(cosh(b*x + a)^4 - 16*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^
4 + 2*(3*cosh(b*x + a)^2 + 10)*sinh(b*x + a)^2 + 20*cosh(b*x + a)^2 - 16*(c
osh(b*x + a)^3 + 5*cosh(b*x + a))*sinh(b*x + a) - 45)/(b*cosh(b*x + a) - b*
sinh(b*x + a))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(65) = 130$.
time = 3.06, size = 139, normalized size = 1.67

$$\begin{cases} \frac{8e^a e^{bx}\sinh^4(a+bx)}{15b} - \frac{8e^a e^{bx}\sinh^3(a+bx)\cosh(a+bx)}{15b} - \frac{4e^a e^{bx}\sinh^2(a+bx)\cosh^2(a+bx)}{5b} + \frac{4e^a e^{bx}\sinh(a+bx)\cosh^3(a+bx)}{5b} + \frac{e^a e^{bx}\cosh^4(a+bx)}{5b} & \text{for } b \neq 0 \\ xe^a\cosh^4(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)**4,x)
```

```
[Out] Piecewise((8*exp(a)*exp(b*x)*sinh(a + b*x)**4/(15*b) - 8*exp(a)*exp(b*x)*si
nh(a + b*x)**3*cosh(a + b*x)/(15*b) - 4*exp(a)*exp(b*x)*sinh(a + b*x)**2*co
sh(a + b*x)**2/(5*b) + 4*exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)**3/(5*
b) + exp(a)*exp(b*x)*cosh(a + b*x)**4/(5*b), Ne(b, 0)), (x*exp(a)*cosh(a)**
4, True))
```

Giac [A]
time = 0.39, size = 60, normalized size = 0.72

$$-\frac{5\left(12e^{(2bx+2a)}+1\right)e^{(-3bx-3a)} - 3e^{(5bx+5a)} - 20e^{(3bx+3a)} - 90e^{(bx+a)}}{240b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/240*(5*(12*e^(2*b*x + 2*a) + 1)*e^(-3*b*x - 3*a) - 3*e^(5*b*x + 5*a) - 2
0*e^(3*b*x + 3*a) - 90*e^(b*x + a))/b
```

Mupad [B]
time = 0.50, size = 58, normalized size = 0.70

$$\frac{90\,\mathrm{e}^{a+b\,x}-60\,\mathrm{e}^{-a-b\,x}-5\,\mathrm{e}^{-3\,a-3\,b\,x}+20\,\mathrm{e}^{3\,a+3\,b\,x}+3\,\mathrm{e}^{5\,a+5\,b\,x}}{240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^4*exp(a + b*x),x)
```

```
[Out] (90*exp(a + b*x) - 60*exp(- a - b*x) - 5*exp(- 3*a - 3*b*x) + 20*exp(3*a +
3*b*x) + 3*exp(5*a + 5*b*x))/(240*b)
```

3.265 $\int e^{a+bx}\cosh^3(a+bx)\,dx$

Optimal. Leaf size=57

$$-\frac{e^{-2a-2bx}}{16b}+\frac{3e^{2a+2bx}}{16b}+\frac{e^{4a+4bx}}{32b}+\frac{3x}{8}$$

```
[Out] -1/16*exp(-2*b*x-2*a)/b+3/16*exp(2*b*x+2*a)/b+1/32*exp(4*b*x+4*a)/b+3/8*x
```

Rubi [A]
time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2320, 12, 272, 45}

$$-\frac{e^{-2a-2bx}}{16b}+\frac{3e^{2a+2bx}}{16b}+\frac{e^{4a+4bx}}{32b}+\frac{3x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cosh[a + b*x]^3,x]
```

```
[Out] -1/16*E^(-2*a - 2*b*x)/b + (3*E^(2*a + 2*b*x))/(16*b) + E^(4*a + 4*b*x)/(32
*b) + (3*x)/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\int e^{a+bx}\cosh^3(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{8x^3}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{x^3}\,dx, x, e^{a+bx}\right)}{8b}\\ &= \frac{\text{Subst}\left(\int \frac{(1+x)^3}{x^2}\,dx, x, e^{2a+2bx}\right)}{16b}\\ &= \frac{\text{Subst}\left(\int \left(3+\frac{1}{x^2}+\frac{3}{x}+x\right)\,dx, x, e^{2a+2bx}\right)}{16b}\\ &= -\frac{e^{-2a-2bx}}{16b}+\frac{3e^{2a+2bx}}{16b}+\frac{e^{4a+4bx}}{32b}+\frac{3x}{8}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 47, normalized size = 0.82

$$\frac{-e^{-2(a+bx)}+3e^{2(a+bx)}+\frac{1}{2}e^{4(a+bx)}+6bx}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x]^3,x]
[Out] (-E^(-2*(a + b*x)) + 3*E^(2*(a + b*x)) + E^(4*(a + b*x))/2 + 6*b*x)/(16*b)
```

Maple [A]
time = 1.14, size = 61, normalized size = 1.07

method	result	size
risch	$-\frac{e^{-2bx-2a}}{16b}+\frac{3\,e^{2bx+2a}}{16b}+\frac{e^{4bx+4a}}{32b}+\frac{3x}{8}$	47
default	$\frac{3x}{8}+\frac{\sinh(2bx+2a)}{4b}+\frac{\sinh(4bx+4a)}{32b}+\frac{\cosh(2bx+2a)}{8b}+\frac{\cosh(4bx+4a)}{32b}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 3/8*x+1/4/b*sinh(2*b*x+2*a)+1/32/b*sinh(4*b*x+4*a)+1/8*cosh(2*b*x+2*a)/b+1/
32*cosh(4*b*x+4*a)/b
```

Maxima [A]
time = 0.27, size = 53, normalized size = 0.93

$$\frac{3\,(bx+a)}{8\,b}+\frac{e^{(4\,bx+4\,a)}}{32\,b}+\frac{3\,e^{(2\,bx+2\,a)}}{16\,b}-\frac{e^{(-2\,bx-2\,a)}}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 3/8*(b*x + a)/b + 1/32*e^(4*b*x + 4*a)/b + 3/16*e^(2*b*x + 2*a)/b - 1/16*e^
(-2*b*x - 2*a)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(46) = 92$.
time = 0.36, size = 95, normalized size = 1.67

$$-\frac{\cosh(bx+a)^3+3\,\cosh(bx+a)\sinh(bx+a)^2-3\,\sinh(bx+a)^3-6\,(2\,bx+1)\cosh(bx+a)+3\,\left(4\,bx-3\,\cosh(bx+a)^2-2\right)\sinh(bx+a)}{32\,(b\cosh(bx+a)-b\sinh(bx+a))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/32*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 - 3*sinh(b*x + a)^
3 - 6*(2*b*x + 1)*cosh(b*x + a) + 3*(4*b*x - 3*cosh(b*x + a)^2 - 2)*sinh(b*
x + a))/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(48) = 96$.
time = 1.24, size = 182, normalized size = 3.19

$$\begin{cases} \frac{3xe^{a}e^{bx}\sinh^{3}(a+bx)}{8}-\frac{3xe^{a}e^{bx}\sinh^{2}(a+bx)\cosh(a+bx)}{8}-\frac{3xe^{a}e^{bx}\sinh(a+bx)\cosh^{2}(a+bx)}{8}+\frac{3xe^{a}e^{bx}\cosh^{3}(a+bx)}{8}-\frac{3e^{a}e^{bx}\sinh^{3}(a+bx)}{8b}+\frac{3e^{a}e^{bx}\sinh(a+bx)\cosh^{2}(a+bx)}{4b}-\frac{e^{a}e^{bx}\cosh^{3}(a+bx)}{8b} & \text{for}\ b\neq 0 \\ xe^{a}\cosh^{3}(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)**3,x)
```

```
[Out] Piecewise((3*x*exp(a)*exp(b*x)*sinh(a + b*x)**3/8 - 3*x*exp(a)*exp(b*x)*sin
h(a + b*x)**2*cosh(a + b*x)/8 - 3*x*exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a +
b*x)**2/8 + 3*x*exp(a)*exp(b*x)*cosh(a + b*x)**3/8 - 3*exp(a)*exp(b*x)*sinh
(a + b*x)**3/(8*b) + 3*exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)**2/(4*b)
 - exp(a)*exp(b*x)*cosh(a + b*x)**3/(8*b), Ne(b, 0)), (x*exp(a)*cosh(a)**3,
 True))
```

Giac [A]
time = 0.40, size = 57, normalized size = 1.00

$$\frac{12\,bx-2\,\left(3\,e^{(2\,bx+2\,a)}+1\right)e^{(-2\,bx-2\,a)}+12\,a+e^{(4\,bx+4\,a)}+6\,e^{(2\,bx+2\,a)}}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/32*(12*b*x - 2*(3*e^(2*b*x + 2*a) + 1)*e^(-2*b*x - 2*a) + 12*a + e^(4*b*x
 + 4*a) + 6*e^(2*b*x + 2*a))/b
```

Mupad [B]
time = 0.26, size = 42, normalized size = 0.74

$$\frac{3\,x}{8}+\frac{\frac{3\,{\mathrm{e}}^{2\,a+2\,b\,x}}{16}-\frac{{\mathrm{e}}^{-2\,a-2\,b\,x}}{16}+\frac{{\mathrm{e}}^{4\,a+4\,b\,x}}{32}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3*exp(a + b*x),x)
```

```
[Out] (3*x)/8 + ((3*exp(2*a + 2*b*x))/16 - exp(- 2*a - 2*b*x)/16 + exp(4*a + 4*b*
x)/32)/b
```

3.266 $\int e^{a+bx}\cosh^2(a+bx)\,dx$

Optimal. Leaf size=49

$$-\frac{e^{-a-bx}}{4b}+\frac{e^{a+bx}}{2b}+\frac{e^{3a+3bx}}{12b}$$

```
[Out] -1/4*exp(-b*x-a)/b+1/2*exp(b*x+a)/b+1/12*exp(3*b*x+3*a)/b
```

Rubi [A]

time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.188$, Rules used = {2320, 12, 276}

$$-\frac{e^{-a-bx}}{4b}+\frac{e^{a+bx}}{2b}+\frac{e^{3a+3bx}}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cosh[a + b*x]^2,x]
```

```
[Out] -1/4*E^(-a - b*x)/b + E^(a + b*x)/(2*b) + E^(3*a + 3*b*x)/(12*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^{a+bx}\cosh^2(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{4x^2}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^2}\,dx, x, e^{a+bx}\right)}{4b}$$
$$= \frac{\text{Subst}\left(\int \left(2+\frac{1}{x^2}+x^2\right)\,dx, x, e^{a+bx}\right)}{4b}$$
$$= -\frac{e^{-a-bx}}{4b}+\frac{e^{a+bx}}{2b}+\frac{e^{3a+3bx}}{12b}$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 0.80

$$\frac{e^{-a-bx}\left(-3+6e^{2(a+bx)}+e^{4(a+bx)}\right)}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x]^2,x]
[Out] (E^(-a - b*x)*(-3 + 6*E^(2*(a + b*x)) + E^(4*(a + b*x))))/(12*b)
```

Maple [A]
time = 1.00, size = 52, normalized size = 1.06

method	result	size
risch	$-\frac{e^{-bx-a}}{4b}+\frac{e^{bx+a}}{2b}+\frac{e^{3bx+3a}}{12b}$	41
default	$\frac{3\sinh(bx+a)}{4b}+\frac{\sinh(3bx+3a)}{12b}+\frac{\cosh(bx+a)}{4b}+\frac{\cosh(3bx+3a)}{12b}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 3/4*sinh(b*x+a)/b+1/12/b*sinh(3*b*x+3*a)+1/4*cosh(b*x+a)/b+1/12/b*cosh(3*b*
x+3*a)
```

Maxima [A]
time = 0.27, size = 40, normalized size = 0.82

$$\frac{e^{(3\,bx+3\,a)}}{12\,b}+\frac{e^{(bx+a)}}{2\,b}-\frac{e^{(-bx-a)}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/12*e^(3*b*x + 3*a)/b + 1/2*e^(b*x + a)/b - 1/4*e^(-b*x - a)/b
```

Fricas [A]
time = 0.39, size = 54, normalized size = 1.10

$$-\frac{\cosh\left(bx+a\right)^{2}-4\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^{2}-3}{6\left(b\cosh\left(bx+a\right)-b\sinh\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(cosh(b*x + a)^2 - 4*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 3
)/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(34) = 68.
time = 0.50, size = 78, normalized size = 1.59

$$\begin{cases} -\frac{2e^{a}e^{bx}\sinh^{2}\left(a+bx\right)}{3b}+\frac{2e^{a}e^{bx}\sinh\left(a+bx\right)\cosh\left(a+bx\right)}{3b}+\frac{e^{a}e^{bx}\cosh^{2}\left(a+bx\right)}{3b} & \text{for}\ b\neq 0\\ xe^{a}\cosh^{2}\left(a\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)**2,x)
```

```
[Out] Piecewise((-2*exp(a)*exp(b*x)*sinh(a + b*x)**2/(3*b) + 2*exp(a)*exp(b*x)*si
nh(a + b*x)*cosh(a + b*x)/(3*b) + exp(a)*exp(b*x)*cosh(a + b*x)**2/(3*b), N
e(b, 0)), (x*exp(a)*cosh(a)**2, True))
```

Giac [A]
time = 0.39, size = 34, normalized size = 0.69

$$\frac{e^{\left(3\,bx+3\,a\right)}+6\,e^{\left(bx+a\right)}-3\,e^{\left(-bx-a\right)}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/12*(e^(3*b*x + 3*a) + 6*e^(b*x + a) - 3*e^(-b*x - a))/b
```

Mupad [B]
time = 0.97, size = 34, normalized size = 0.69

$$\frac{6\,{\mathrm{e}}^{a+b\,x}-3\,{\mathrm{e}}^{-a-b\,x}+{\mathrm{e}}^{3\,a+3\,b\,x}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2*exp(a + b*x),x)
```

```
[Out] (6*exp(a + b*x) - 3*exp(- a - b*x) + exp(3*a + 3*b*x))/(12*b)
```

3.267 $\int e^{a+bx} \cosh(a+bx)\, dx$

Optimal. Leaf size=23

$$\frac{e^{2a+2bx}}{4b} + \frac{x}{2}$$

[Out] 1/4*exp(2*b*x+2*a)/b+1/2*x

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2320, 12, 14}

$$\frac{e^{2a+2bx}}{4b} + \frac{x}{2}$$

Antiderivative was successfully verified.

[In] Int[E^(a + b*x)*Cosh[a + b*x],x]

[Out] E^(2*a + 2*b*x)/(4*b) + x/2

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^{a+bx}\cosh(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{2x}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{1+x^2}{x}\,dx, x, e^{a+bx}\right)}{2b}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{x}+x\right)\,dx, x, e^{a+bx}\right)}{2b}$$
$$= \frac{e^{2a+2bx}}{4b}+\frac{x}{2}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 1.00

$$\frac{e^{2a+2bx}}{4b}+\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x],x]
```

```
[Out] E^(2*a + 2*b*x)/(4*b) + x/2
```

Maple [A]
time = 0.66, size = 37, normalized size = 1.61

method	result	size
risch	$\frac{e^{2bx+2a}}{4b}+\frac{x}{2}$	19
derivativedivides	$\frac{\frac{\left(\cosh^2(bx+a)\right)}{2}+\frac{\cosh(bx+a)\sinh(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}}{b}$	37
default	$\frac{\frac{\left(\cosh^2(bx+a)\right)}{2}+\frac{\cosh(bx+a)\sinh(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}}{b}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/2*cosh(b*x+a)^2+1/2*cosh(b*x+a)*sinh(b*x+a)+1/2*b*x+1/2*a)
```

Maxima [A]
time = 0.27, size = 24, normalized size = 1.04

$$\frac{1}{2}\,x+\frac{a}{2\,b}+\frac{e^{(2\,bx+2\,a)}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/2*a/b + 1/4*e^(2*b*x + 2*a)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(18) = 36$.
time = 0.42, size = 50, normalized size = 2.17

$$\frac{(2\,bx+1)\cosh\left(bx+a\right)-(2\,bx-1)\sinh\left(bx+a\right)}{4\left(b\cosh\left(bx+a\right)-b\sinh\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*((2*b*x + 1)*cosh(b*x + a) - (2*b*x - 1)*sinh(b*x + a))/(b*cosh(b*x + a
) - b*sinh(b*x + a))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(15) = 30$.
time = 0.21, size = 80, normalized size = 3.48

$$\begin{cases} -\frac{xe^{a}e^{bx}\sinh\left(a+bx\right)}{2}+\frac{xe^{a}e^{bx}\cosh\left(a+bx\right)}{2}+\frac{e^{a}e^{bx}\sinh\left(a+bx\right)}{b}-\frac{e^{a}e^{bx}\cosh\left(a+bx\right)}{2b} & \text{for}\ b\neq 0 \\ xe^{a}\cosh\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a),x)
```

```
[Out] Piecewise((-x*exp(a)*exp(b*x)*sinh(a + b*x)/2 + x*exp(a)*exp(b*x)*cosh(a +
b*x)/2 + exp(a)*exp(b*x)*sinh(a + b*x)/b - exp(a)*exp(b*x)*cosh(a + b*x)/(2
*b), Ne(b, 0)), (x*exp(a)*cosh(a), True))
```

Giac [A]
time = 0.38, size = 22, normalized size = 0.96

$$\frac{2\,bx+2\,a+e^{(2\,bx+2\,a)}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*(2*b*x + 2*a + e^(2*b*x + 2*a))/b
```

Mupad [B]
time = 0.92, size = 18, normalized size = 0.78

$$\frac{x}{2}+\frac{\mathrm{e}^{2\,a+2\,b\,x}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*exp(a + b*x),x)
[Out] x/2 + exp(2*a + 2*b*x)/(4*b)
```

3.268 $\int e^{a+bx}\mathbf{sech}(a+bx)\,dx$

Optimal. Leaf size=17

$$\frac{\log\left(1+e^{2a+2bx}\right)}{b}$$

```
[Out] ln(1+exp(2*b*x+2*a))/b
```

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2320, 12, 266}

$$\frac{\log\left(e^{2a+2bx}+1\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Sech[a + b*x],x]
```

```
[Out] Log[1 + E^(2*a + 2*b*x)]/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^{a+bx}\text{sech}(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{2x}{1+x^2}\,dx,x,e^{a+bx}\right)}{b}$$
$$= \frac{2\text{Subst}\left(\int \frac{x}{1+x^2}\,dx,x,e^{a+bx}\right)}{b}$$
$$= \frac{\log\left(1+e^{2a+2bx}\right)}{b}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 1.00

$$\frac{\log\left(1+e^{2a+2bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Sech[a + b*x],x]
[Out] Log[1 + E^(2*a + 2*b*x)]/b
```

Maple [A]
time = 0.37, size = 17, normalized size = 1.00

method	result	size
derivativedivides	$\frac{\ln(\cosh(bx+a))+bx+a}{b}$	17
default	$\frac{\ln(\cosh(bx+a))+bx+a}{b}$	17
risch	$-\frac{2a}{b}+\frac{\ln\left(e^{2bx+2a}+1\right)}{b}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*sech(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b*(ln(cosh(b*x+a))+b*x+a)
```

Maxima [A]
time = 0.48, size = 16, normalized size = 0.94

$$\frac{\log\left(e^{(2\,bx+2\,a)}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a),x, algorithm="maxima")
[Out] log(e^(2*b*x + 2*a) + 1)/b
```

Fricas [A]
time = 0.41, size = 30, normalized size = 1.76

$$\frac{\log\left(\frac{2\,\cosh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a),x, algorithm="fricas")
[Out] log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a)))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{a}\int e^{bx}\,\mathrm{sech}\,(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a),x)
[Out] exp(a)*Integral(exp(b*x)*sech(a + b*x), x)
```

Giac [A]
time = 0.38, size = 16, normalized size = 0.94

$$\frac{\log\left(e^{(2\,bx+2\,a)}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a),x, algorithm="giac")
[Out] log(e^(2*b*x + 2*a) + 1)/b
```

Mupad [B]
time = 0.92, size = 16, normalized size = 0.94

$$\frac{\ln\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a + b*x)/cosh(a + b*x),x)
[Out] log(exp(2*a + 2*b*x) + 1)/b
```

3.269 $\int e^{a+bx}\text{sech}^2(a+bx)\,dx$

Optimal. Leaf size=40

$$-\frac{2e^{a+bx}}{b\left(1+e^{2a+2bx}\right)}+\frac{2\text{ArcTan}\left(e^{a+bx}\right)}{b}$$

```
[Out] -2*exp(b*x+a)/b/(1+exp(2*b*x+2*a))+2*arctan(exp(b*x+a))/b
```

Rubi [A]
time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2320, 12, 294, 209}

$$\frac{2\text{ArcTan}\left(e^{a+bx}\right)}{b}-\frac{2e^{a+bx}}{b\left(e^{2a+2bx}+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Sech[a + b*x]^2,x]
[Out] (-2*E^(a + b*x))/(b*(1 + E^(2*a + 2*b*x))) + (2*ArcTan[E^(a + b*x)])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\int e^{a+bx}\operatorname{sech}^2(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{4x^2}{(1+x^2)^2}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{4\operatorname{Subst}\left(\int \frac{x^2}{(1+x^2)^2}\,dx, x, e^{a+bx}\right)}{b}\\ &= -\frac{2e^{a+bx}}{b\left(1+e^{2a+2bx}\right)}+\frac{2\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^{a+bx}\right)}{b}\\ &= -\frac{2e^{a+bx}}{b\left(1+e^{2a+2bx}\right)}+\frac{2\tan^{-1}\left(e^{a+bx}\right)}{b}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 36, normalized size = 0.90

$$\frac{2\left(-\frac{e^{a+bx}}{1+e^{2(a+bx)}}+\operatorname{ArcTan}\left(e^{a+bx}\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Sech[a + b*x]^2,x]
[Out] (2*(-(E^(a + b*x)/(1 + E^(2*(a + b*x)))) + ArcTan[E^(a + b*x)]))/b
```

Maple [A]
time = 0.63, size = 25, normalized size = 0.62

method	result	size
derivativedivides	$\frac{-\frac{1}{\cosh(bx+a)}+2\arctan\left(e^{bx+a}\right)}{b}$	25
default	$\frac{-\frac{1}{\cosh(bx+a)}+2\arctan\left(e^{bx+a}\right)}{b}$	25
risch	$-\frac{2\,e^{bx+a}}{b\left(e^{2bx+2a}+1\right)}+\frac{i\ln\left(e^{bx+a}+i\right)}{b}-\frac{i\ln\left(e^{bx+a}-i\right)}{b}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*sech(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*(-1/cosh(b*x+a)+2*arctan(exp(b*x+a)))
```

Maxima [A]
time = 0.47, size = 37, normalized size = 0.92

$$\frac{2\,\arctan\left(e^{(bx+a)}\right)}{b}-\frac{2\,e^{(bx+a)}}{b(e^{(2\,bx+2\,a)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 2*arctan(e^(b*x + a))/b - 2*e^(b*x + a)/(b*(e^(2*b*x + 2*a) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(37) = 74$.
time = 0.43, size = 105, normalized size = 2.62

$$\frac{2\left(\left(\cosh\left(bx+a\right)^{2}+2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^{2}+1\right)\arctan\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)-\cosh\left(bx+a\right)-\sinh\left(bx+a\right)\right)}{b\cosh\left(bx+a\right)^{2}+2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^{2}+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 2*((cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*
arctan(cosh(b*x + a) + sinh(b*x + a)) - cosh(b*x + a) - sinh(b*x + a))/(b*c
osh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 + b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{a}\int e^{bx}\operatorname{sech}^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)**2,x)
```

```
[Out] exp(a)*Integral(exp(b*x)*sech(a + b*x)**2, x)
```

Giac [A]
time = 0.39, size = 35, normalized size = 0.88

$$-\frac{2\left(\frac{e^{(bx+a)}}{e^{(2\,bx+2\,a)}+1}-\arctan\left(e^{(bx+a)}\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -2*(e^(b*x + a)/(e^(2*b*x + 2*a) + 1) - arctan(e^(b*x + a)))/b
```

Mupad [B]
time = 0.08, size = 48, normalized size = 1.20

$$\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}}-\frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a + b*x)/cosh(a + b*x)^2,x)
```

```
[Out] (2*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2) - (2*exp(a + b*x))/(b
*(exp(2*a + 2*b*x) + 1))
```

3.270 $\int e^{a+bx}\operatorname{sech}^3(a+bx)\,dx$

Optimal. Leaf size=29

$$\frac{2e^{4a+4bx}}{b\,(1+e^{2a+2bx})^2}$$

[Out] 2*exp(4*b*x+4*a)/b/(1+exp(2*b*x+2*a))^2

Rubi [A]
time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2320, 12, 270}

$$\frac{2e^{4a+4bx}}{b\,(e^{2a+2bx}+1)^2}$$

Antiderivative was successfully verified.

[In] Int[E^(a + b*x)*Sech[a + b*x]^3,x]

[Out] (2*E^(4*a + 4*b*x))/(b*(1 + E^(2*a + 2*b*x))^2)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^{a+bx}\operatorname{sech}^3(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{8x^3}{(1+x^2)^3}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{8\text{Subst}\left(\int \frac{x^3}{(1+x^2)^3}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{2e^{4a+4bx}}{b\left(1+e^{2a+2bx}\right)^2}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 1.00

$$\frac{2e^{4a+4bx}}{b\left(1+e^{2a+2bx}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Sech[a + b*x]^3,x]
```

```
[Out] (2*E^(4*a + 4*b*x))/(b*(1 + E^(2*a + 2*b*x))^2)
```

Maple [A]
time = 1.42, size = 32, normalized size = 1.10

method	result	size
risch	$-\frac{2\left(2\,\mathrm{e}^{2bx+2a}+1\right)}{b\left(\mathrm{e}^{2bx+2a}+1\right)^{2}}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*sech(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(2*exp(2*b*x+2*a)+1)/b/(exp(2*b*x+2*a)+1)^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(27) = 54.
time = 0.26, size = 68, normalized size = 2.34

$$-\frac{4\,e^{(2\,bx+2\,a)}}{b\left(e^{(4\,bx+4\,a)}+2\,e^{(2\,bx+2\,a)}+1\right)}-\frac{2}{b\left(e^{(4\,bx+4\,a)}+2\,e^{(2\,bx+2\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -4*e^(2*b*x + 2*a)/(b*(e^(4*b*x + 4*a) + 2*e^(2*b*x + 2*a) + 1)) - 2/(b*(e^
(4*b*x + 4*a) + 2*e^(2*b*x + 2*a) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(27) = 54$.
time = 0.39, size = 86, normalized size = 2.97

$$-\frac{2\left(3\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)}{b\cosh\left(bx+a\right)^{3}+3\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}+b\sinh\left(bx+a\right)^{3}+3\,b\cosh\left(bx+a\right)+\left(3\,b\cosh\left(bx+a\right)^{2}+b\right)\sinh\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)^3,x, algorithm="fricas")
[Out] -2*(3*cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)
*sinh(b*x + a)^2 + b*sinh(b*x + a)^3 + 3*b*cosh(b*x + a) + (3*b*cosh(b*x +
a)^2 + b)*sinh(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{a}\int e^{bx}\,\mathrm{sech}^{3}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)**3,x)
[Out] exp(a)*Integral(exp(b*x)*sech(a + b*x)**3, x)
```

Giac [A]
time = 0.41, size = 31, normalized size = 1.07

$$-\frac{2\left(2\,e^{(2\,bx+2\,a)}+1\right)}{b(e^{(2\,bx+2\,a)}+1)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)^3,x, algorithm="giac")
[Out] -2*(2*e^(2*b*x + 2*a) + 1)/(b*(e^(2*b*x + 2*a) + 1)^2)
```

Mupad [B]
time = 0.93, size = 31, normalized size = 1.07

$$-\frac{2\left(2\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)}{b\,{\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a + b*x)/cosh(a + b*x)^3,x)
[Out] -(2*(2*exp(2*a + 2*b*x) + 1))/(b*(exp(2*a + 2*b*x) + 1)^2)
```

3.271 $\int e^{a+bx}\text{sech}^4(a+bx)\,dx$

Optimal. Leaf size=95

$$-\frac{8e^{3a+3bx}}{3b\left(1+e^{2a+2bx}\right)^3}-\frac{2e^{a+bx}}{b\left(1+e^{2a+2bx}\right)^2}+\frac{e^{a+bx}}{b\left(1+e^{2a+2bx}\right)}+\frac{\text{ArcTan}\left(e^{a+bx}\right)}{b}$$

```
[Out] -8/3*exp(3*b*x+3*a)/b/(1+exp(2*b*x+2*a))^3-2*exp(b*x+a)/b/(1+exp(2*b*x+2*a)
)^2+exp(b*x+a)/b/(1+exp(2*b*x+2*a))+arctan(exp(b*x+a))/b
```

Rubi [A]
time = 0.03, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2320, 12, 294, 205, 209}

$$\frac{\text{ArcTan}\left(e^{a+bx}\right)}{b}+\frac{e^{a+bx}}{b\left(e^{2a+2bx}+1\right)}-\frac{2e^{a+bx}}{b\left(e^{2a+2bx}+1\right)^2}-\frac{8e^{3a+3bx}}{3b\left(e^{2a+2bx}+1\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Sech[a + b*x]^4,x]
```

```
[Out] (-8*E^(3*a + 3*b*x))/(3*b*(1 + E^(2*a + 2*b*x))^3) - (2*E^(a + b*x))/(b*(1
+ E^(2*a + 2*b*x))^2) + E^(a + b*x)/(b*(1 + E^(2*a + 2*b*x))) + ArcTan[E^(a
 + b*x)]/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int e^{a+bx}\operatorname{sech}^4(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{16x^4}{(1+x^2)^4}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{16\text{Subst}\left(\int \frac{x^4}{(1+x^2)^4}\,dx, x, e^{a+bx}\right)}{b}\\
&= -\frac{8e^{3a+3bx}}{3b\left(1+e^{2a+2bx}\right)^3} + \frac{8\text{Subst}\left(\int \frac{x^2}{(1+x^2)^3}\,dx, x, e^{a+bx}\right)}{b}\\
&= -\frac{8e^{3a+3bx}}{3b\left(1+e^{2a+2bx}\right)^3} - \frac{2e^{a+bx}}{b\left(1+e^{2a+2bx}\right)^2} + \frac{2\text{Subst}\left(\int \frac{1}{(1+x^2)^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= -\frac{8e^{3a+3bx}}{3b\left(1+e^{2a+2bx}\right)^3} - \frac{2e^{a+bx}}{b\left(1+e^{2a+2bx}\right)^2} + \frac{e^{a+bx}}{b\left(1+e^{2a+2bx}\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^{a}\right.}{b}\\
&= -\frac{8e^{3a+3bx}}{3b\left(1+e^{2a+2bx}\right)^3} - \frac{2e^{a+bx}}{b\left(1+e^{2a+2bx}\right)^2} + \frac{e^{a+bx}}{b\left(1+e^{2a+2bx}\right)} + \frac{\tan^{-1}\left(e^{a+bx}\right)}{b}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 64, normalized size = 0.67

$$\frac{e^{a+bx}\left(-3-8e^{2(a+bx)}+3e^{4(a+bx)}\right)}{3b\left(1+e^{2(a+bx)}\right)^3}+\frac{\text{ArcTan}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Sech[a + b*x]^4,x]
```

```
[Out] (E^(a + b*x)*(-3 - 8*E^(2*(a + b*x)) + 3*E^(4*(a + b*x))))/(3*b*(1 + E^(2*(
a + b*x)))^3) + ArcTan[E^(a + b*x)]/b
```

Maple [C] Result contains complex when optimal does not.
time = 1.54, size = 82, normalized size = 0.86

method	result	size
risch	$\frac{e^{bx+a}\left(3\,e^{4bx+4a}-8\,e^{2bx+2a}-3\right)}{3b\left(e^{2bx+2a}+1\right)^{3}}+\frac{i\ln\left(e^{bx+a}+i\right)}{2b}-\frac{i\ln\left(e^{bx+a}-i\right)}{2b}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*sech(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*exp(b*x+a)*(3*exp(4*b*x+4*a)-8*exp(2*b*x+2*a)-3)/b/(exp(2*b*x+2*a)+1)^3
+1/2*I/b*ln(exp(b*x+a)+I)-1/2*I/b*ln(exp(b*x+a)-I)
```

Maxima [A]
time = 0.51, size = 83, normalized size = 0.87

$$\frac{\arctan\left(e^{(bx+a)}\right)}{b}+\frac{3\,e^{(5\,bx+5\,a)}-8\,e^{(3\,bx+3\,a)}-3\,e^{(bx+a)}}{3\,b(e^{(6\,bx+6\,a)}+3\,e^{(4\,bx+4\,a)}+3\,e^{(2\,bx+2\,a)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] arctan(e^(b*x + a))/b + 1/3*(3*e^(5*b*x + 5*a) - 8*e^(3*b*x + 3*a) - 3*e^(b
*x + a))/(b*(e^(6*b*x + 6*a) + 3*e^(4*b*x + 4*a) + 3*e^(2*b*x + 2*a) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 513 vs. $2(86) = 172$.
time = 0.39, size = 513, normalized size = 5.40

$$\frac{3\cosh(bx+a)^5+15\cosh(bx+a)\sinh(bx+a)^4+3\sinh(bx+a)^5+2\left(15\cosh(bx+a)^2-4\right)\sinh(bx+a)^3-8\cosh(bx+a)^3+6\left(5\cosh(bx+a)^3-4\cosh(bx+a)\right)\sinh(bx+a)^2+3\left(\cosh(bx+a)^6+6\cosh(bx+a)\sinh(bx+a)^5+\sinh(bx+a)^6+3\left(5\cosh(bx+a)^2+1\right)\sinh(bx+a)^4+3\cosh(bx+a)^4+4\left(5\cosh(bx+a)^3+3\cosh(bx+a)\right)\sinh(bx+a)^3+3\left(5\cosh(bx+a)^4+6\cosh(bx+a)^2+1\right)\sinh(bx+a)^2+3\cosh(bx+a)^2+6\left(\cosh(bx+a)^5+2\cosh(bx+a)^3+\cosh(bx+a)\right)\sinh(bx+a)+1\right)\arctan\left(\cosh(bx+a)+\sinh(bx+a)\right)+3\left(5\cosh(bx+a)^4-8\cosh(bx+a)^2-1\right)\sinh(bx+a)-3\cosh(bx+a)}{3\left(b\cosh(bx+a)^6+6\,b\cosh(bx+a)\sinh(bx+a)^5+b\sinh(bx+a)^6+3\,b\cosh(bx+a)^4+3\left(5\,b\cosh(bx+a)^2+b\right)\sinh(bx+a)^4+4\left(5\,b\cosh(bx+a)^3+3\,b\cosh(bx+a)\right)\sinh(bx+a)^3+3\,b\cosh(bx+a)^2+3\left(5\,b\cosh(bx+a)^4+6\,b\cosh(bx+a)^2+b\right)\sinh(bx+a)^2+6\left(b\cosh(bx+a)^5+2\,b\cosh(bx+a)^3+b\cosh(bx+a)\right)\sinh(bx+a)+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*cosh(b*x + a)^5 + 15*cosh(b*x + a)*sinh(b*x + a)^4 + 3*sinh(b*x + a)
^5 + 2*(15*cosh(b*x + a)^2 - 4)*sinh(b*x + a)^3 - 8*cosh(b*x + a)^3 + 6*(5*
cosh(b*x + a)^3 - 4*cosh(b*x + a))*sinh(b*x + a)^2 + 3*(cosh(b*x + a)^6 + 6
*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*cosh(b*x + a)^2 + 1
)*sinh(b*x + a)^4 + 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 + 3*cosh(b*x +
 a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 + 6*cosh(b*x + a)^2 + 1)*sinh(b
*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 + 2*cosh(b*x + a)^3 + co
sh(b*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) + 3*(
5*cosh(b*x + a)^4 - 8*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - 3*cosh(b*x + a))
```

```
/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b*x + a)^6
 + 3*b*cosh(b*x + a)^4 + 3*(5*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^4 + 4*(5
*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^3 + 3*b*cosh(b*x + a)
^2 + 3*(5*b*cosh(b*x + a)^4 + 6*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 6*
(b*cosh(b*x + a)^5 + 2*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a) +
 b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{a}\int e^{bx}\operatorname{sech}^{4}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)**4,x)
```

```
[Out] exp(a)*Integral(exp(b*x)*sech(a + b*x)**4, x)
```

Giac [A]
time = 0.41, size = 60, normalized size = 0.63

$$\frac{\frac{3\,e^{(5\,bx+5\,a)}-8\,e^{(3\,bx+3\,a)}-3\,e^{(bx+a)}}{\left(e^{(2\,bx+2\,a)}+1\right)^{3}}+3\,\arctan\left(e^{(bx+a)}\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/3*((3*e^(5*b*x + 5*a) - 8*e^(3*b*x + 3*a) - 3*e^(b*x + a))/(e^(2*b*x + 2*
a) + 1)^3 + 3*arctan(e^(b*x + a)))/b
```

Mupad [B]
time = 0.96, size = 130, normalized size = 1.37

$$\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}}-\frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(2\,\mathrm{e}^{2\,a+2\,b\,x}+\mathrm{e}^{4\,a+4\,b\,x}+1\right)}-\frac{8\,\mathrm{e}^{3\,a+3\,b\,x}}{3\,b\,\left(3\,\mathrm{e}^{2\,a+2\,b\,x}+3\,\mathrm{e}^{4\,a+4\,b\,x}+\mathrm{e}^{6\,a+6\,b\,x}+1\right)}+\frac{\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a + b*x)/cosh(a + b*x)^4,x)
```

```
[Out] atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b)/(b^2)^(1/2) - (2*exp(a + b*x))/(b*(2*
exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - (8*exp(3*a + 3*b*x))/(3*b*(3*ex
p(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1)) + exp(a + b*x)
/(b*(exp(2*a + 2*b*x) + 1))
```

3.272 $\int e^{a+bx}\operatorname{sech}^5(a+bx)\,dx$

Optimal. Leaf size=60

$$-\frac{4}{b\left(1+e^{2a+2bx}\right)^4}+\frac{32}{3b\left(1+e^{2a+2bx}\right)^3}-\frac{8}{b\left(1+e^{2a+2bx}\right)^2}$$

```
[Out] -4/b/(1+exp(2*b*x+2*a))^4+32/3/b/(1+exp(2*b*x+2*a))^3-8/b/(1+exp(2*b*x+2*a)
)^2
```

Rubi [A]
time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = {2320, 12, 272, 45}

$$-\frac{8}{b\left(e^{2a+2bx}+1\right)^2}+\frac{32}{3b\left(e^{2a+2bx}+1\right)^3}-\frac{4}{b\left(e^{2a+2bx}+1\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Sech[a + b*x]^5,x]
```

```
[Out] -4/(b*(1 + E^(2*a + 2*b*x))^4) + 32/(3*b*(1 + E^(2*a + 2*b*x))^3) - 8/(b*(1
 + E^(2*a + 2*b*x))^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\int e^{a+bx}\operatorname{sech}^5(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{32x^5}{(1+x^2)^5}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{32\operatorname{Subst}\left(\int \frac{x^5}{(1+x^2)^5}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{16\operatorname{Subst}\left(\int \frac{x^2}{(1+x)^5}\,dx, x, e^{2a+2bx}\right)}{b}\\ &= \frac{16\operatorname{Subst}\left(\int \left(\frac{1}{(1+x)^5}-\frac{2}{(1+x)^4}+\frac{1}{(1+x)^3}\right)dx, x, e^{2a+2bx}\right)}{b}\\ &= -\frac{4}{b\left(1+e^{2a+2bx}\right)^4}+\frac{32}{3b\left(1+e^{2a+2bx}\right)^3}-\frac{8}{b\left(1+e^{2a+2bx}\right)^2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 44, normalized size = 0.73

$$-\frac{4\left(1+4e^{2(a+bx)}+6e^{4(a+bx)}\right)}{3b\left(1+e^{2(a+bx)}\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Sech[a + b*x]^5,x]
```

```
[Out] (-4*(1 + 4*E^(2*(a + b*x)) + 6*E^(4*(a + b*x))))/(3*b*(1 + E^(2*(a + b*x)))
^4)
```

Maple [A]
time = 1.55, size = 43, normalized size = 0.72

method	result	size
risch	$-\frac{4\left(6\,\mathrm{e}^{4bx+4a}+4\,\mathrm{e}^{2bx+2a}+1\right)}{3b\left(\mathrm{e}^{2bx+2a}+1\right)^{4}}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*sech(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -4/3*(6*exp(4*b*x+4*a)+4*exp(2*b*x+2*a)+1)/b/(exp(2*b*x+2*a)+1)^4
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(55) = 110$.
time = 0.28, size = 172, normalized size = 2.87

$$-\frac{8\,e^{(4\,bx+4\,a)}}{b(e^{(8\,bx+8\,a)}+4\,e^{(6\,bx+6\,a)}+6\,e^{(4\,bx+4\,a)}+4\,e^{(2\,bx+2\,a)}+1)}-\frac{16\,e^{(2\,bx+2\,a)}}{3\,b(e^{(8\,bx+8\,a)}+4\,e^{(6\,bx+6\,a)}+6\,e^{(4\,bx+4\,a)}+4\,e^{(2\,bx+2\,a)}+1)}-\frac{4}{3\,b(e^{(8\,bx+8\,a)}+4\,e^{(6\,bx+6\,a)}+6\,e^{(4\,bx+4\,a)}+4\,e^{(2\,bx+2\,a)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -8*e^(4*b*x + 4*a)/(b*(e^(8*b*x + 8*a) + 4*e^(6*b*x + 6*a) + 6*e^(4*b*x + 4
*a) + 4*e^(2*b*x + 2*a) + 1)) - 16/3*e^(2*b*x + 2*a)/(b*(e^(8*b*x + 8*a) +
4*e^(6*b*x + 6*a) + 6*e^(4*b*x + 4*a) + 4*e^(2*b*x + 2*a) + 1)) - 4/3/(b*(e
^(8*b*x + 8*a) + 4*e^(6*b*x + 6*a) + 6*e^(4*b*x + 4*a) + 4*e^(2*b*x + 2*a)
+ 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(55) = 110$.
time = 0.41, size = 233, normalized size = 3.88

$$-\frac{4\,\left(7\,\cosh\left(bx+a\right)^{2}+10\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+7\,\sinh\left(bx+a\right)^{2}+4\right)}{3\,\left(b\cosh\left(bx+a\right)^{6}+6\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{5}+b\sinh\left(bx+a\right)^{6}+4\,b\cosh\left(bx+a\right)^{4}+\left(15\,b\cosh\left(bx+a\right)^{2}+4\,b\right)\sinh\left(bx+a\right)^{4}+4\,\left(5\,b\cosh\left(bx+a\right)^{3}+4\,b\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right)^{3}+7\,b\cosh\left(bx+a\right)^{2}+\left(15\,b\cosh\left(bx+a\right)^{4}+24\,b\cosh\left(bx+a\right)^{2}+7\,b\right)\sinh\left(bx+a\right)^{2}+2\,\left(3\,b\cosh\left(bx+a\right)^{5}+8\,b\cosh\left(bx+a\right)^{3}+5\,b\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right)+4\,b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -4/3*(7*cosh(b*x + a)^2 + 10*cosh(b*x + a)*sinh(b*x + a) + 7*sinh(b*x + a)^
2 + 4)/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b*x
+ a)^6 + 4*b*cosh(b*x + a)^4 + (15*b*cosh(b*x + a)^2 + 4*b)*sinh(b*x + a)^4
 + 4*(5*b*cosh(b*x + a)^3 + 4*b*cosh(b*x + a))*sinh(b*x + a)^3 + 7*b*cosh(b
*x + a)^2 + (15*b*cosh(b*x + a)^4 + 24*b*cosh(b*x + a)^2 + 7*b)*sinh(b*x +
a)^2 + 2*(3*b*cosh(b*x + a)^5 + 8*b*cosh(b*x + a)^3 + 5*b*cosh(b*x + a))*si
nh(b*x + a) + 4*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{a}\int e^{bx}\operatorname{sech}^{5}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)**5,x)
```

```
[Out] exp(a)*Integral(exp(b*x)*sech(a + b*x)**5, x)
```

Giac [A]
time = 0.39, size = 42, normalized size = 0.70

$$-\frac{4\,\left(6\,e^{(4\,bx+4\,a)}+4\,e^{(2\,bx+2\,a)}+1\right)}{3\,b(e^{(2\,bx+2\,a)}+1)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -4/3*(6*e^(4*b*x + 4*a) + 4*e^(2*b*x + 2*a) + 1)/(b*(e^(2*b*x + 2*a) + 1)^4
)
```

Mupad [B]
time = 0.95, size = 42, normalized size = 0.70

$$-\frac{4\left(4\,{\mathrm{e}}^{2\,a+2\,b\,x}+6\,{\mathrm{e}}^{4\,a+4\,b\,x}+1\right)}{3\,b\,{\left({\mathrm{e}}^{2\,a+2\,b\,x}+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a + b*x)/cosh(a + b*x)^5,x)
```

```
[Out] -(4*(4*exp(2*a + 2*b*x) + 6*exp(4*a + 4*b*x) + 1))/(3*b*(exp(2*a + 2*b*x) +
 1)^4)
```

3.273 $\int e^x \cosh^2(2x)\, dx$

Optimal. Leaf size=26

$$-\frac{1}{12}e^{-3x} + \frac{e^x}{2} + \frac{e^{5x}}{20}$$

```
[Out] -1/12/exp(3*x)+1/2*exp(x)+1/20*exp(5*x)
```

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2320, 12, 276}

$$-\frac{1}{12}e^{-3x} + \frac{e^x}{2} + \frac{e^{5x}}{20}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Cosh[2*x]^2,x]
```

```
[Out] -1/12*1/E^(3*x) + E^x/2 + E^(5*x)/20
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^x \cosh^2(2x)\,dx = \text{Subst}\left(\int \frac{(1+x^4)^2}{4x^4}\,dx, x, e^x\right)$$
$$= \frac{1}{4}\text{Subst}\left(\int \frac{(1+x^4)^2}{x^4}\,dx, x, e^x\right)$$
$$= \frac{1}{4}\text{Subst}\left(\int \left(2+\frac{1}{x^4}+x^4\right)\,dx, x, e^x\right)$$
$$= -\frac{1}{12}e^{-3x}+\frac{e^x}{2}+\frac{e^{5x}}{20}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 1.00

$$-\frac{1}{12}e^{-3x}+\frac{e^x}{2}+\frac{e^{5x}}{20}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cosh[2*x]^2,x]
```

```
[Out] -1/12*1/E^(3*x) + E^x/2 + E^(5*x)/20
```

Maple [A]
time = 0.54, size = 34, normalized size = 1.31

method	result	size
risch	$\frac{e^{5x}}{20}+\frac{e^x}{2}-\frac{e^{-3x}}{12}$	18
default	$\frac{\sinh(x)}{2}+\frac{\sinh(3x)}{12}+\frac{\sinh(5x)}{20}+\frac{\cosh(x)}{2}-\frac{\cosh(3x)}{12}+\frac{\cosh(5x)}{20}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cosh(2*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sinh(x)+1/12*sinh(3*x)+1/20*sinh(5*x)+1/2*cosh(x)-1/12*cosh(3*x)+1/20*c
osh(5*x)
```

Maxima [A]
time = 0.26, size = 17, normalized size = 0.65

$$\frac{1}{20}\,e^{(5\,x)}-\frac{1}{12}\,e^{(-3\,x)}+\frac{1}{2}\,e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(2*x)^2,x, algorithm="maxima")
```

```
[Out] 1/20*e^(5*x) - 1/12*e^(-3*x) + 1/2*e^x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(17) = 34$.
time = 0.43, size = 47, normalized size = 1.81

$$-\frac{\cosh(x)^4 - 16\cosh(x)^3\sinh(x) + 6\cosh(x)^2\sinh(x)^2 - 16\cosh(x)\sinh(x)^3 + \sinh(x)^4 - 15}{30(\cosh(x) - \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(2*x)^2,x, algorithm="fricas")
```

```
[Out] -1/30*(cosh(x)^4 - 16*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 16*cosh(x
)*sinh(x)^3 + sinh(x)^4 - 15)/(cosh(x) - sinh(x))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(19) = 38$.
time = 0.17, size = 42, normalized size = 1.62

$$-\frac{8e^x\sinh^2(2x)}{15} + \frac{4e^x\sinh(2x)\cosh(2x)}{15} + \frac{7e^x\cosh^2(2x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(2*x)**2,x)
```

```
[Out] -8*exp(x)*sinh(2*x)**2/15 + 4*exp(x)*sinh(2*x)*cosh(2*x)/15 + 7*exp(x)*cosh
(2*x)**2/15
```

Giac [A]
time = 0.39, size = 17, normalized size = 0.65

$$\frac{1}{20}e^{(5x)} - \frac{1}{12}e^{(-3x)} + \frac{1}{2}e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(2*x)^2,x, algorithm="giac")
```

```
[Out] 1/20*e^(5*x) - 1/12*e^(-3*x) + 1/2*e^x
```

Mupad [B]
time = 0.08, size = 17, normalized size = 0.65

$$\frac{e^{5x}}{20} - \frac{e^{-3x}}{12} + \frac{e^x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(2*x)^2*exp(x),x)
```

```
[Out] exp(5*x)/20 - exp(-3*x)/12 + exp(x)/2
```

3.274 $\int e^x \cosh(2x)\,dx$

Optimal. Leaf size=19

$$-\frac{e^{-x}}{2}+\frac{e^{3x}}{6}$$

```
[Out] -1/2/exp(x)+1/6*exp(3*x)
```

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2320, 12, 14}

$$\frac{e^{3x}}{6}-\frac{e^{-x}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Cosh[2*x],x]
```

```
[Out] -1/2*1/E^x + E^(3*x)/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$
\begin{aligned}
\int e^x \cosh(2x)\,dx &= \text{Subst}\left(\int \frac{1+x^4}{2x^2}\,dx, x, e^x\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1+x^4}{x^2}\,dx, x, e^x\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x^2}+x^2\right)\,dx, x, e^x\right) \\
&= -\frac{e^{-x}}{2}+\frac{e^{3x}}{6}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 0.84

$$\frac{1}{6}e^{-x}\left(-3+e^{4x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cosh[2*x],x]
[Out] (-3 + E^(4*x))/(6*E^x)
```

Maple [A]
time = 0.36, size = 22, normalized size = 1.16

method	result	size
risch	$\frac{e^{3x}}{6}-\frac{e^{-x}}{2}$	14
default	$\frac{\sinh(x)}{2}+\frac{\sinh(3x)}{6}-\frac{\cosh(x)}{2}+\frac{\cosh(3x)}{6}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cosh(2*x),x,method=_RETURNVERBOSE)
[Out] 1/2*sinh(x)+1/6*sinh(3*x)-1/2*cosh(x)+1/6*cosh(3*x)
```

Maxima [A]
time = 0.27, size = 13, normalized size = 0.68

$$\frac{1}{6}\,e^{(3\,x)}-\frac{1}{2}\,e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(2*x),x, algorithm="maxima")
[Out] 1/6*e^(3*x) - 1/2*e^(-x)
```

Fricas [A]
time = 0.35, size = 26, normalized size = 1.37

$$-\frac{\cosh(x)^2-4\cosh(x)\sinh(x)+\sinh(x)^2}{3(\cosh(x)-\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(2*x),x, algorithm="fricas")
[Out] -1/3*(cosh(x)^2 - 4*cosh(x)*sinh(x) + sinh(x)^2)/(cosh(x) - sinh(x))
```

Sympy [A]
time = 0.10, size = 20, normalized size = 1.05

$$\frac{2e^x\sinh(2x)}{3}-\frac{e^x\cosh(2x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(2*x),x)
[Out] 2*exp(x)*sinh(2*x)/3 - exp(x)*cosh(2*x)/3
```

Giac [A]
time = 0.39, size = 13, normalized size = 0.68

$$\frac{1}{6}e^{(3x)}-\frac{1}{2}e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(2*x),x, algorithm="giac")
[Out] 1/6*e^(3*x) - 1/2*e^(-x)
```

Mupad [B]
time = 0.05, size = 12, normalized size = 0.63

$$\frac{\mathrm{e}^{-x}\left(\mathrm{e}^{4x}-3\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(2*x)*exp(x),x)
[Out] (exp(-x)*(exp(4*x) - 3))/6
```

3.275 $\int e^x \operatorname{sech}(2x)\,dx$

Optimal. Leaf size=92

$$-\frac{\operatorname{ArcTan}\left(1-\sqrt{2}\,e^x\right)}{\sqrt{2}}+\frac{\operatorname{ArcTan}\left(1+\sqrt{2}\,e^x\right)}{\sqrt{2}}+\frac{\log\left(1-\sqrt{2}\,e^x+e^{2x}\right)}{2\sqrt{2}}-\frac{\log\left(1+\sqrt{2}\,e^x+e^{2x}\right)}{2\sqrt{2}}$$

```
[Out] 1/2*arctan(-1+exp(x)*2^(1/2))*2^(1/2)+1/2*arctan(1+exp(x)*2^(1/2))*2^(1/2)+
1/4*ln(1+exp(2*x)-exp(x)*2^(1/2))*2^(1/2)-1/4*ln(1+exp(2*x)+exp(x)*2^(1/2))
*2^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {2320, 12, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\operatorname{ArcTan}\left(1-\sqrt{2}\,e^x\right)}{\sqrt{2}}+\frac{\operatorname{ArcTan}\left(\sqrt{2}\,e^x+1\right)}{\sqrt{2}}+\frac{\log\left(-\sqrt{2}\,e^x+e^{2x}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{2}\,e^x+e^{2x}+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sech[2*x],x]
```

```
[Out] -(ArcTan[1 - Sqrt[2]*E^x]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*E^x]/Sqrt[2] + Log[
1 - Sqrt[2]*E^x + E^(2*x)]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*E^x + E^(2*x)]/(2*
Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^x \text{sech}(2x)\,dx = \text{Subst}\left(\int \frac{2x^2}{1+x^4}\,dx, x, e^x\right)$$
$$= 2\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, e^x\right)$$
$$= -\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, e^x\right) + \text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, e^x\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, e^x\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx, x, e^x\right) + \frac{\text{Subst}\left(\right.}{}$$
$$= \frac{\log\left(1-\sqrt{2}\,e^x+e^{2x}\right)}{2\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\,e^x+e^{2x}\right)}{2\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\,e^x\right)}{\sqrt{2}} -$$
$$= -\frac{\tan^{-1}\left(1-\sqrt{2}\,e^x\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}\,e^x\right)}{\sqrt{2}} + \frac{\log\left(1-\sqrt{2}\,e^x+e^{2x}\right)}{2\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\,e^x\right.}{2\sqrt{2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.01, size = 24, normalized size = 0.26

$$\frac{2}{3}e^{3x}\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-e^{4x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sech[2*x],x]
```

```
[Out] (2*E^(3*x)*Hypergeometric2F1[3/4, 1, 7/4, -E^(4*x)])/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.87, size = 25, normalized size = 0.27

method	result	size
risch	$2\left(\sum_{\_R=\text{RootOf}(256\_Z^4+1)} \_R\ln\left(64\_R^3+e^x\right)\right)$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sech(2*x),x,method=_RETURNVERBOSE)
```

```
[Out] 2*sum(_R*ln(64*_R^3+exp(x)),_R=RootOf(256*_Z^4+1))
```

Maxima [A]
time = 0.47, size = 76, normalized size = 0.83

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{x}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{x}\right)\right)-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}e^{x}+e^{(2x)}+1\right)+\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}e^{x}+e^{(2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/2*sqrt(2)*arctan(-1/2
*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/4*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) +
1/4*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1)
```

Fricas [A]
time = 0.43, size = 113, normalized size = 1.23

$$-\sqrt{2}\arctan\left(-\sqrt{2}e^{x}+\sqrt{2}\sqrt{\sqrt{2}e^{x}+e^{(2x)}+1}-1\right)-\sqrt{2}\arctan\left(-\sqrt{2}e^{x}+\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}e^{x}+4e^{(2x)}+4}+1\right)-\frac{1}{4}\sqrt{2}\log\left(4\sqrt{2}e^{x}+4e^{(2x)}+4\right)+\frac{1}{4}\sqrt{2}\log\left(-4\sqrt{2}e^{x}+4e^{(2x)}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x),x, algorithm="fricas")
```

```
[Out] -sqrt(2)*arctan(-sqrt(2)*e^x + sqrt(2)*sqrt(sqrt(2)*e^x + e^(2*x) + 1) - 1)
 - sqrt(2)*arctan(-sqrt(2)*e^x + 1/2*sqrt(2)*sqrt(-4*sqrt(2)*e^x + 4*e^(2*x
) + 4) + 1) - 1/4*sqrt(2)*log(4*sqrt(2)*e^x + 4*e^(2*x) + 4) + 1/4*sqrt(2)*
log(-4*sqrt(2)*e^x + 4*e^(2*x) + 4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x}\operatorname{sech}(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x),x)
```

```
[Out] Integral(exp(x)*sech(2*x), x)
```

Giac [A]
time = 0.40, size = 76, normalized size = 0.83

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{x}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{x}\right)\right)-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}e^{x}+e^{(2x)}+1\right)+\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}e^{x}+e^{(2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/2*sqrt(2)*arctan(-1/2
*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/4*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) +
1/4*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1)
```

Mupad [B]
time = 1.11, size = 77, normalized size = 0.84

$$\sqrt{2}\,\ln\left(4+\sqrt{2}\,e^x\,(-2-2i)\right)\left(\frac{1}{4}+\frac{1}{4}i\right)+\sqrt{2}\,\ln\left(4+\sqrt{2}\,e^x\,(-2+2i)\right)\left(\frac{1}{4}-\frac{1}{4}i\right)+\sqrt{2}\,\ln\left(4+\sqrt{2}\,e^x\,(2-2i)\right)\left(-\frac{1}{4}+\frac{1}{4}i\right)+\sqrt{2}\,\ln\left(4+\sqrt{2}\,e^x\,(2+2i)\right)\left(-\frac{1}{4}-\frac{1}{4}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/cosh(2*x),x)
```

```
[Out] 2^(1/2)*log(4 - 2^(1/2)*exp(x)*(2 + 2i))*(1/4 + 1i/4) + 2^(1/2)*log(4 - 2^(
1/2)*exp(x)*(2 - 2i))*(1/4 - 1i/4) - 2^(1/2)*log(2^(1/2)*exp(x)*(2 - 2i) +
4)*(1/4 - 1i/4) - 2^(1/2)*log(2^(1/2)*exp(x)*(2 + 2i) + 4)*(1/4 + 1i/4)
```

3.276 $\int e^x \mathbf{sech}^2(2x)\,dx$

Optimal. Leaf size=111

$$-\frac{e^x}{1+e^{4x}}-\frac{\text{ArcTan}\left(1-\sqrt{2}\,e^x\right)}{2\sqrt{2}}+\frac{\text{ArcTan}\left(1+\sqrt{2}\,e^x\right)}{2\sqrt{2}}-\frac{\log\left(1-\sqrt{2}\,e^x+e^{2x}\right)}{4\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\,e^x+e^{2x}\right)}{4\sqrt{2}}$$

```
[Out] -exp(x)/(1+exp(4*x))+1/4*arctan(-1+exp(x)*2^(1/2))*2^(1/2)+1/4*arctan(1+exp
(x)*2^(1/2))*2^(1/2)-1/8*ln(1+exp(2*x)-exp(x)*2^(1/2))*2^(1/2)+1/8*ln(1+exp
(2*x)+exp(x)*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {2320, 12, 294, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\text{ArcTan}\left(1-\sqrt{2}\,e^x\right)}{2\sqrt{2}}+\frac{\text{ArcTan}\left(\sqrt{2}\,e^x+1\right)}{2\sqrt{2}}-\frac{e^x}{e^{4x}+1}-\frac{\log\left(-\sqrt{2}\,e^x+e^{2x}+1\right)}{4\sqrt{2}}+\frac{\log\left(\sqrt{2}\,e^x+e^{2x}+1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sech[2*x]^2,x]
```

```
[Out] -(E^x/(1 + E^(4*x))) - ArcTan[1 - Sqrt[2]*E^x]/(2*Sqrt[2]) + ArcTan[1 + Sqr
t[2]*E^x]/(2*Sqrt[2]) - Log[1 - Sqrt[2]*E^x + E^(2*x)]/(4*Sqrt[2]) + Log[1
+ Sqrt[2]*E^x + E^(2*x)]/(4*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^x \operatorname{sech}^2(2x)\,dx = \text{Subst}\left(\int \frac{4x^4}{(1+x^4)^2}\,dx, x, e^x\right)$$
$$= 4\text{Subst}\left(\int \frac{x^4}{(1+x^4)^2}\,dx, x, e^x\right)$$
$$= -\frac{e^x}{1+e^{4x}} + \text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, e^x\right)$$
$$= -\frac{e^x}{1+e^{4x}} + \frac{1}{2}\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, e^x\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, e^x\right)$$
$$= -\frac{e^x}{1+e^{4x}} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, e^x\right) + \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx, x,\right.$$
$$= -\frac{e^x}{1+e^{4x}} - \frac{\log\left(1-\sqrt{2}\,e^x+e^{2x}\right)}{4\sqrt{2}} + \frac{\log\left(1+\sqrt{2}\,e^x+e^{2x}\right)}{4\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1\right.}{2\sqrt{2}}$$
$$= -\frac{e^x}{1+e^{4x}} - \frac{\tan^{-1}\left(1-\sqrt{2}\,e^x\right)}{2\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}\,e^x\right)}{2\sqrt{2}} - \frac{\log\left(1-\sqrt{2}\,e^x+e^{2x}\right)}{4\sqrt{2}} + \frac{\log}{}$$

Mathematica [**A**]
time = 0.07, size = 106, normalized size = 0.95

$$\frac{1}{8}\left(-\frac{8e^x}{1+e^{4x}} - 2\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\,e^x\right) + 2\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\,e^x\right) - \sqrt{2}\,\log\left(1-\sqrt{2}\,e^x+e^{2x}\right) + \sqrt{2}\,\log\left(1+\sqrt{2}\,e^x+e^{2x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sech[2*x]^2,x]
```

```
[Out] ((-8*E^x)/(1 + E^(4*x)) - 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*E^x] + 2*Sqrt[2]*Arc
Tan[1 + Sqrt[2]*E^x] - Sqrt[2]*Log[1 - Sqrt[2]*E^x + E^(2*x)] + Sqrt[2]*Log
[1 + Sqrt[2]*E^x + E^(2*x)])/8
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.92, size = 36, normalized size = 0.32

method	result	size
risch	$-\frac{e^x}{1+e^{4x}} + 4\left(\sum_{\_R=\text{RootOf}(65536\_Z^4+1)} \_R\ln\left(e^x+16\_R\right)\right)$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sech(2*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -exp(x)/(1+exp(4*x))+4*sum(_R*ln(exp(x)+16*_R),_R=RootOf(65536*_Z^4+1))
```

Maxima [A]
time = 0.46, size = 88, normalized size = 0.79

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{x}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{x}\right)\right)+\frac{1}{8}\sqrt{2}\log\left(\sqrt{2}e^{x}+e^{(2x)}+1\right)-\frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}e^{x}+e^{(2x)}+1\right)-\frac{e^{x}}{e^{(4x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/4*sqrt(2)*arctan(-1/2
*sqrt(2)*(sqrt(2) - 2*e^x)) + 1/8*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) -
1/8*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - e^x/(e^(4*x) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 162 vs. 2(78) = 156.
time = 0.48, size = 162, normalized size = 1.46

$$-\frac{4\left(\sqrt{2}e^{(4x)}+\sqrt{2}\right)\arctan\left(-\sqrt{2}e^{x}+\sqrt{2}\sqrt{\sqrt{2}e^{x}+e^{(2x)}+1}-1\right)+4\left(\sqrt{2}e^{(4x)}+\sqrt{2}\right)\arctan\left(-\sqrt{2}e^{x}+\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}e^{x}+4e^{(2x)}+4}+1\right)-\left(\sqrt{2}e^{(4x)}+\sqrt{2}\right)\log\left(4\sqrt{2}e^{x}+4e^{(2x)}+4\right)+\left(\sqrt{2}e^{(4x)}+\sqrt{2}\right)\log\left(-4\sqrt{2}e^{x}+4e^{(2x)}+4\right)+8e^{x}}{8\left(e^{(4x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*(sqrt(2)*e^(4*x) + sqrt(2))*arctan(-sqrt(2)*e^x + sqrt(2)*sqrt(sqrt
(2)*e^x + e^(2*x) + 1) - 1) + 4*(sqrt(2)*e^(4*x) + sqrt(2))*arctan(-sqrt(2)
*e^x + 1/2*sqrt(2)*sqrt(-4*sqrt(2)*e^x + 4*e^(2*x) + 4) + 1) - (sqrt(2)*e^(
4*x) + sqrt(2))*log(4*sqrt(2)*e^x + 4*e^(2*x) + 4) + (sqrt(2)*e^(4*x) + sqr
t(2))*log(-4*sqrt(2)*e^x + 4*e^(2*x) + 4) + 8*e^x)/(e^(4*x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x}\operatorname{sech}^{2}(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x)**2,x)
```

```
[Out] Integral(exp(x)*sech(2*x)**2, x)
```

Giac [A]
time = 0.38, size = 88, normalized size = 0.79

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{x}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{x}\right)\right)+\frac{1}{8}\sqrt{2}\log\left(\sqrt{2}e^{x}+e^{(2x)}+1\right)-\frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}e^{x}+e^{(2x)}+1\right)-\frac{e^{x}}{e^{(4x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x)^2,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/4*sqrt(2)*arctan(-1/2
*sqrt(2)*(sqrt(2) - 2*e^x)) + 1/8*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) -
1/8*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - e^x/(e^(4*x) + 1)
```

Mupad [B]
time = 1.09, size = 85, normalized size = 0.77

$$\frac{\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}\ \left(\mathrm{e}^x-\frac{\sqrt{2}}{2}\right)\right)}{4}-\frac{\mathrm{e}^x}{\mathrm{e}^{4\,x}+1}+\frac{\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}\ \left(\mathrm{e}^x+\frac{\sqrt{2}}{2}\right)\right)}{4}-\frac{\sqrt{2}\ \ln\left(\left(\mathrm{e}^x-\frac{\sqrt{2}}{2}\right)^2+\frac{1}{2}\right)}{8}+\frac{\sqrt{2}\ \ln\left(\left(\mathrm{e}^x+\frac{\sqrt{2}}{2}\right)^2+\frac{1}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/cosh(2*x)^2,x)
```

```
[Out] (2^(1/2)*atan(2^(1/2)*(exp(x) - 2^(1/2)/2)))/4 - exp(x)/(exp(4*x) + 1) + (2
^(1/2)*atan(2^(1/2)*(exp(x) + 2^(1/2)/2)))/4 - (2^(1/2)*log((exp(x) - 2^(1/
2)/2)^2 + 1/2))/8 + (2^(1/2)*log((exp(x) + 2^(1/2)/2)^2 + 1/2))/8
```

3.277 $\int e^x \cosh^2(3x)\, dx$

Optimal. Leaf size=26

$$-\frac{1}{20}e^{-5x} + \frac{e^x}{2} + \frac{e^{7x}}{28}$$

[Out] -1/20/exp(5*x)+1/2*exp(x)+1/28*exp(7*x)

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2320, 12, 276}

$$-\frac{1}{20}e^{-5x} + \frac{e^x}{2} + \frac{e^{7x}}{28}$$

Antiderivative was successfully verified.

[In] Int[E^x*Cosh[3*x]^2,x]

[Out] -1/20*1/E^(5*x) + E^x/2 + E^(7*x)/28

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^x \cosh^2(3x)\,dx = \text{Subst}\left(\int \frac{(1+x^6)^2}{4x^6}\,dx, x, e^x\right)$$
$$= \frac{1}{4}\text{Subst}\left(\int \frac{(1+x^6)^2}{x^6}\,dx, x, e^x\right)$$
$$= \frac{1}{4}\text{Subst}\left(\int \left(2+\frac{1}{x^6}+x^6\right)\,dx, x, e^x\right)$$
$$= -\frac{1}{20}e^{-5x}+\frac{e^x}{2}+\frac{e^{7x}}{28}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 1.00

$$-\frac{1}{20}e^{-5x}+\frac{e^x}{2}+\frac{e^{7x}}{28}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cosh[3*x]^2,x]
```

```
[Out] -1/20*1/E^(5*x) + E^x/2 + E^(7*x)/28
```

Maple [A]
time = 0.48, size = 34, normalized size = 1.31

method	result	size
risch	$\frac{e^{7x}}{28}+\frac{e^{x}}{2}-\frac{e^{-5x}}{20}$	18
default	$\frac{\sinh(x)}{2}+\frac{\sinh(5x)}{20}+\frac{\sinh(7x)}{28}+\frac{\cosh(x)}{2}-\frac{\cosh(5x)}{20}+\frac{\cosh(7x)}{28}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cosh(3*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sinh(x)+1/20*sinh(5*x)+1/28*sinh(7*x)+1/2*cosh(x)-1/20*cosh(5*x)+1/28*c
osh(7*x)
```

Maxima [A]
time = 0.26, size = 17, normalized size = 0.65

$$\frac{1}{28}\,e^{(7\,x)}-\frac{1}{20}\,e^{(-5\,x)}+\frac{1}{2}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(3*x)^2,x, algorithm="maxima")
```

```
[Out] 1/28*e^(7*x) - 1/20*e^(-5*x) + 1/2*e^x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(17) = 34$.
time = 0.34, size = 67, normalized size = 2.58

$$-\frac{\cosh(x)^6 - 36\cosh(x)^5\sinh(x) + 15\cosh(x)^4\sinh(x)^2 - 120\cosh(x)^3\sinh(x)^3 + 15\cosh(x)^2\sinh(x)^4 - 36\cosh(x)\sinh(x)^5 + \sinh(x)^6 - 35}{70(\cosh(x) - \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(3*x)^2,x, algorithm="fricas")
```

```
[Out] -1/70*(cosh(x)^6 - 36*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 - 120*cosh
(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 - 36*cosh(x)*sinh(x)^5 + sinh(x)^6
 - 35)/(cosh(x) - sinh(x))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(19) = 38$.
time = 0.17, size = 42, normalized size = 1.62

$$-\frac{18e^x\sinh^2(3x)}{35} + \frac{6e^x\sinh(3x)\cosh(3x)}{35} + \frac{17e^x\cosh^2(3x)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(3*x)**2,x)
```

```
[Out] -18*exp(x)*sinh(3*x)**2/35 + 6*exp(x)*sinh(3*x)*cosh(3*x)/35 + 17*exp(x)*co
sh(3*x)**2/35
```

Giac [A]
time = 0.40, size = 17, normalized size = 0.65

$$\frac{1}{28}e^{(7x)} - \frac{1}{20}e^{(-5x)} + \frac{1}{2}e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(3*x)^2,x, algorithm="giac")
```

```
[Out] 1/28*e^(7*x) - 1/20*e^(-5*x) + 1/2*e^x
```

Mupad [B]
time = 0.08, size = 17, normalized size = 0.65

$$\frac{e^{7x}}{28} - \frac{e^{-5x}}{20} + \frac{e^x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(3*x)^2*exp(x),x)
```

```
[Out] exp(7*x)/28 - exp(-5*x)/20 + exp(x)/2
```

3.278 $\int e^x \cosh(3x)\, dx$

Optimal. Leaf size=19

$$-\frac{1}{4}e^{-2x}+\frac{e^{4x}}{8}$$

```
[Out] -1/4/exp(2*x)+1/8*exp(4*x)
```

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2320, 12, 14}

$$\frac{e^{4x}}{8}-\frac{1}{4}e^{-2x}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Cosh[3*x],x]
```

```
[Out] -1/4*1/E^(2*x) + E^(4*x)/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$
\begin{aligned}
\int e^x \cosh(3x)\,dx &= \text{Subst}\left(\int \frac{1+x^6}{2x^3}\,dx, x, e^x\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1+x^6}{x^3}\,dx, x, e^x\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x^3}+x^3\right)\,dx, x, e^x\right)\\
&= -\frac{1}{4}e^{-2x}+\frac{e^{4x}}{8}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 0.84

$$\frac{1}{8}e^{-2x}\left(-2+e^{6x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cosh[3*x],x]
[Out] (-2 + E^(6*x))/(8*E^(2*x))
```

Maple [A]
time = 0.63, size = 26, normalized size = 1.37

method	result	size
risch	$\frac{e^{4x}}{8}-\frac{e^{-2x}}{4}$	14
default	$\frac{\sinh(2x)}{4}+\frac{\sinh(4x)}{8}-\frac{\cosh(2x)}{4}+\frac{\cosh(4x)}{8}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cosh(3*x),x,method=_RETURNVERBOSE)
[Out] 1/4*sinh(2*x)+1/8*sinh(4*x)-1/4*cosh(2*x)+1/8*cosh(4*x)
```

Maxima [A]
time = 0.27, size = 13, normalized size = 0.68

$$\frac{1}{8}\,e^{(4\,x)}-\frac{1}{4}\,e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(3*x),x, algorithm="maxima")
[Out] 1/8*e^(4*x) - 1/4*e^(-2*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(13) = 26$.
time = 0.36, size = 38, normalized size = 2.00

$$-\frac{\cosh(x)^3 - 9\cosh(x)^2\sinh(x) + 3\cosh(x)\sinh(x)^2 - 3\sinh(x)^3}{8(\cosh(x) - \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(3*x),x, algorithm="fricas")
```

```
[Out] -1/8*(cosh(x)^3 - 9*cosh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2 - 3*sinh(x)^3)/
(cosh(x) - sinh(x))
```

Sympy [A]
time = 0.10, size = 20, normalized size = 1.05

$$\frac{3e^x \sinh(3x)}{8} - \frac{e^x \cosh(3x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(3*x),x)
```

```
[Out] 3*exp(x)*sinh(3*x)/8 - exp(x)*cosh(3*x)/8
```

Giac [A]
time = 0.41, size = 13, normalized size = 0.68

$$\frac{1}{8}e^{(4x)} - \frac{1}{4}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(3*x),x, algorithm="giac")
```

```
[Out] 1/8*e^(4*x) - 1/4*e^(-2*x)
```

Mupad [B]
time = 0.93, size = 12, normalized size = 0.63

$$\frac{e^{-2x}(e^{6x} - 2)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(3*x)*exp(x),x)
```

```
[Out] (exp(-2*x)*(exp(6*x) - 2))/8
```

3.279 $\int e^x \operatorname{sech}(3x)\,dx$

Optimal. Leaf size=55

$$-\frac{\operatorname{ArcTan}\left(\frac{1-2e^{2x}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{3}\log\left(1+e^{2x}\right)+\frac{1}{6}\log\left(1-e^{2x}+e^{4x}\right)$$

[Out] -1/3*ln(1+exp(2*x))+1/6*ln(1-exp(2*x)+exp(4*x))-1/3*arctan(1/3*(1-2*exp(2*x))*3^(1/2))*3^(1/2)

Rubi [A]
time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125, Rules used = {2320, 12, 281, 298, 31, 648, 632, 210, 642}

$$-\frac{\operatorname{ArcTan}\left(\frac{1-2e^{2x}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{3}\log\left(e^{2x}+1\right)+\frac{1}{6}\log\left(-e^{2x}+e^{4x}+1\right)$$

Antiderivative was successfully verified.

[In] Int[E^x*Sech[3*x],x]

[Out] -(ArcTan[(1 - 2*E^(2*x))/Sqrt[3]]/Sqrt[3]) - Log[1 + E^(2*x)]/3 + Log[1 - E^(2*x) + E^(4*x)]/6

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\int e^x \operatorname{sech}(3x)\,dx &= \text{Subst}\left(\int \frac{2x^3}{1+x^6}\,dx,x,e^x\right)\\ &= 2\text{Subst}\left(\int \frac{x^3}{1+x^6}\,dx,x,e^x\right)\\ &= \text{Subst}\left(\int \frac{x}{1+x^3}\,dx,x,e^{2x}\right)\\ &= -\left(\frac{1}{3}\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,e^{2x}\right)\right)+\frac{1}{3}\text{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx,x,e^{2x}\right)\\ &= -\frac{1}{3}\log\left(1+e^{2x}\right)+\frac{1}{6}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,e^{2x}\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,e^{2x}\right)\\ &= -\frac{1}{3}\log\left(1+e^{2x}\right)+\frac{1}{6}\log\left(1-e^{2x}+e^{4x}\right)-\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2e^{2x}\right)\\ &= \frac{\tan^{-1}\left(\frac{-1+2e^{2x}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{3}\log\left(1+e^{2x}\right)+\frac{1}{6}\log\left(1-e^{2x}+e^{4x}\right)\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.01, size = 24, normalized size = 0.44

$$\frac{1}{2}e^{4x}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-e^{6x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sech[3*x],x]
[Out] (E^(4*x)*Hypergeometric2F1[2/3, 1, 5/3, -E^(6*x)])/2
```

Maple [C] Result contains complex when optimal does not.
time = 0.76, size = 79, normalized size = 1.44

method	result
risch	$-\frac{\ln(1+e^{2x})}{3}+\frac{\ln\left(e^{2x}-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{6}+\frac{i\ln\left(e^{2x}-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{6}+\frac{\ln\left(e^{2x}-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{6}-\frac{i\ln\left(e^{2x}-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sech(3*x),x,method=_RETURNVERBOSE)
[Out] -1/3*ln(1+exp(2*x))+1/6*ln(exp(2*x)-1/2+1/2*I*3^(1/2))+1/6*I*ln(exp(2*x)-1/
2+1/2*I*3^(1/2))*3^(1/2)+1/6*ln(exp(2*x)-1/2-1/2*I*3^(1/2))-1/6*I*ln(exp(2*
x)-1/2-1/2*I*3^(1/2))*3^(1/2)
```

Maxima [A]
time = 0.47, size = 71, normalized size = 1.29

$$-\frac{1}{3}\sqrt{3}\arctan\left(\sqrt{3}+2e^{x}\right)+\frac{1}{3}\sqrt{3}\arctan\left(-\sqrt{3}+2e^{x}\right)+\frac{1}{6}\log\left(\sqrt{3}e^{x}+e^{(2x)}+1\right)+\frac{1}{6}\log\left(-\sqrt{3}e^{x}+e^{(2x)}+1\right)-\frac{1}{3}\log\left(e^{(2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(3*x),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(sqrt(3) + 2*e^x) + 1/3*sqrt(3)*arctan(-sqrt(3) + 2*e^x)
 + 1/6*log(sqrt(3)*e^x + e^(2*x) + 1) + 1/6*log(-sqrt(3)*e^x + e^(2*x) + 1)
 - 1/3*log(e^(2*x) + 1)
```

Fricas [A]
time = 0.39, size = 83, normalized size = 1.51

$$-\frac{1}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}\cosh(x)+3\sqrt{3}\sinh(x)}{3(\cosh(x)-\sinh(x))}\right)+\frac{1}{6}\log\left(\frac{2\cosh(x)^{2}+2\sinh(x)^{2}-1}{\cosh(x)^{2}-2\cosh(x)\sinh(x)+\sinh(x)^{2}}\right)-\frac{1}{3}\log\left(\frac{2\cosh(x)}{\cosh(x)-\sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(3*x),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(-1/3*(sqrt(3)*cosh(x) + 3*sqrt(3)*sinh(x))/(cosh(x) - s
inh(x))) + 1/6*log((2*cosh(x)^2 + 2*sinh(x)^2 - 1)/(cosh(x)^2 - 2*cosh(x)*s
inh(x) + sinh(x)^2)) - 1/3*log(2*cosh(x)/(cosh(x) - sinh(x)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x}\operatorname{sech}(3x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(3*x),x)
```

```
[Out] Integral(exp(x)*sech(3*x), x)
```

Giac [A]
time = 0.39, size = 44, normalized size = 0.80

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{(2x)}-1\right)\right)+\frac{1}{6}\log\left(e^{(4x)}-e^{(2x)}+1\right)-\frac{1}{3}\log\left(e^{(2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(3*x),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(2*x) - 1)) + 1/6*log(e^(4*x) - e^(2*x)
 + 1) - 1/3*log(e^(2*x) + 1)
```

Mupad [B]
time = 1.04, size = 65, normalized size = 1.18

$$-\frac{\ln\left(8\,\mathrm{e}^{2\,x}+8\right)}{3}-\ln\left(24\,\mathrm{e}^{2\,x}\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+8\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+\ln\left(8-24\,\mathrm{e}^{2\,x}\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\right)\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/cosh(3*x),x)
```

```
[Out] log(8 - 24*exp(2*x)*((3^(1/2)*1i)/6 + 1/6))*((3^(1/2)*1i)/6 + 1/6) - log(24
*exp(2*x)*((3^(1/2)*1i)/6 - 1/6) + 8)*((3^(1/2)*1i)/6 - 1/6) - log(8*exp(2*
x) + 8)/3
```

3.280 $\int e^x \text{sech}^2(3x)\,dx$

Optimal. Leaf size=110

$$-\frac{2e^x}{3(1+e^{6x})}+\frac{2\text{ArcTan}(e^x)}{9}-\frac{1}{9}\text{ArcTan}\left(\sqrt{3}-2e^x\right)+\frac{1}{9}\text{ArcTan}\left(\sqrt{3}+2e^x\right)-\frac{\log\left(1-\sqrt{3}\,e^x+e^{2x}\right)}{6\sqrt{3}}+\frac{\log}{}$$

```
[Out] -2/3*exp(x)/(1+exp(6*x))+2/9*arctan(exp(x))+1/9*arctan(2*exp(x)-3^(1/2))+1/
9*arctan(2*exp(x)+3^(1/2))-1/18*ln(1+exp(2*x)-exp(x)*3^(1/2))*3^(1/2)+1/18*
ln(1+exp(2*x)+exp(x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {2320, 12, 294, 215, 648, 632, 210, 642, 209}

$$\frac{2\text{ArcTan}(e^x)}{9}-\frac{1}{9}\text{ArcTan}\left(\sqrt{3}-2e^x\right)+\frac{1}{9}\text{ArcTan}\left(2e^x+\sqrt{3}\right)-\frac{2e^x}{3(e^{6x}+1)}-\frac{\log\left(-\sqrt{3}\,e^x+e^{2x}+1\right)}{6\sqrt{3}}+\frac{\log\left(\sqrt{3}\,e^x+e^{2x}+1\right)}{6\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sech[3*x]^2,x]
```

```
[Out] (-2*E^x)/(3*(1 + E^(6*x))) + (2*ArcTan[E^x])/9 - ArcTan[Sqrt[3] - 2*E^x]/9
+ ArcTan[Sqrt[3] + 2*E^x]/9 - Log[1 - Sqrt[3]*E^x + E^(2*x)]/(6*Sqrt[3]) +
Log[1 + Sqrt[3]*E^x + E^(2*x)]/(6*Sqrt[3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
```

```
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^x \operatorname{sech}^2(3x)\,dx = \operatorname{Subst}\left(\int \frac{4x^6}{(1+x^6)^2}\,dx, x, e^x\right)$$
$$= 4\operatorname{Subst}\left(\int \frac{x^6}{(1+x^6)^2}\,dx, x, e^x\right)$$
$$= -\frac{2e^x}{3(1+e^{6x})} + \frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{1+x^6}\,dx, x, e^x\right)$$
$$= -\frac{2e^x}{3(1+e^{6x})} + \frac{2}{9}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^x\right) + \frac{2}{9}\operatorname{Subst}\left(\int \frac{1-\frac{\sqrt{3}x}{2}}{1-\sqrt{3}x+x^2}\,dx, x, e^x\right)$$
$$= -\frac{2e^x}{3(1+e^{6x})} + \frac{2}{9}\tan^{-1}(e^x) + \frac{1}{18}\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx, x, e^x\right) + \frac{1}{18}\operatorname{Subst}\left(\int \frac{}{1}\right.$$
$$= -\frac{2e^x}{3(1+e^{6x})} + \frac{2}{9}\tan^{-1}(e^x) - \frac{\log\left(1-\sqrt{3}e^x+e^{2x}\right)}{6\sqrt{3}} + \frac{\log\left(1+\sqrt{3}e^x+e^{2x}\right)}{6\sqrt{3}} - \frac{1}{9}\operatorname{Su}$$
$$= -\frac{2e^x}{3(1+e^{6x})} + \frac{2}{9}\tan^{-1}(e^x) - \frac{1}{9}\tan^{-1}\left(\sqrt{3}-2e^x\right) + \frac{1}{9}\tan^{-1}\left(\sqrt{3}+2e^x\right) - \frac{\log\left(1\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.02, size = 34, normalized size = 0.31

$$\frac{2}{3}e^x\left(-\frac{1}{1+e^{6x}} + {}_2F_1\left(\frac{1}{6},1;\frac{7}{6};-e^{6x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sech[3*x]^2,x]
```

```
[Out] (2*E^x*(-(1 + E^(6*x))^(-1) + Hypergeometric2F1[1/6, 1, 7/6, -E^(6*x)]))/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.01, size = 59, normalized size = 0.54

method	result	size
risch	$-\frac{2\,\mathrm{e}^x}{3(1+\mathrm{e}^{6x})}+4\left(\sum_{\_R=\mathrm{RootOf}(1679616\_Z^4-1296\_Z^2+1)}\_R\ln\left(\mathrm{e}^x+36\_R\right)\right)+\frac{i\ln(\mathrm{e}^x+i)}{9}-\frac{i\ln(\mathrm{e}^x-i)}{9}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sech(3*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*exp(x)/(1+exp(6*x))+4*sum(_R*ln(exp(x)+36*_R),_R=RootOf(1679616*_Z^4-1
296*_Z^2+1))+1/9*I*ln(exp(x)+I)-1/9*I*ln(exp(x)-I)
```

Maxima [A]
time = 0.50, size = 79, normalized size = 0.72

$$\frac{1}{18}\sqrt{3}\log\left(\sqrt{3}e^{x}+e^{(2x)}+1\right)-\frac{1}{18}\sqrt{3}\log\left(-\sqrt{3}e^{x}+e^{(2x)}+1\right)-\frac{2e^{x}}{3\left(e^{(6x)}+1\right)}+\frac{1}{9}\arctan\left(\sqrt{3}+2e^{x}\right)+\frac{1}{9}\arctan\left(-\sqrt{3}+2e^{x}\right)+\frac{2}{9}\arctan\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(3*x)^2,x, algorithm="maxima")
```

```
[Out] 1/18*sqrt(3)*log(sqrt(3)*e^x + e^(2*x) + 1) - 1/18*sqrt(3)*log(-sqrt(3)*e^x
 + e^(2*x) + 1) - 2/3*e^x/(e^(6*x) + 1) + 1/9*arctan(sqrt(3) + 2*e^x) + 1/9
*arctan(-sqrt(3) + 2*e^x) + 2/9*arctan(e^x)
```

Fricas [A]
time = 0.38, size = 154, normalized size = 1.40

$$-\frac{4\left(e^{(6x)}+1\right)\arctan\left(\sqrt{3}+\sqrt{-4\sqrt{3}e^{x}+4e^{(2x)}+4}-2e^{x}\right)+4\left(e^{(6x)}+1\right)\arctan\left(-\sqrt{3}+2\sqrt{\sqrt{3}e^{x}+e^{(2x)}+1}-2e^{x}\right)-4\left(e^{(6x)}+1\right)\arctan\left(e^{x}\right)-\left(\sqrt{3}e^{(6x)}+\sqrt{3}\right)\log\left(4\sqrt{3}e^{x}+4e^{(2x)}+4\right)+\left(\sqrt{3}e^{(6x)}+\sqrt{3}\right)\log\left(-4\sqrt{3}e^{x}+4e^{(2x)}+4\right)+12e^{x}}{18\left(e^{(6x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(3*x)^2,x, algorithm="fricas")
```

```
[Out] -1/18*(4*(e^(6*x) + 1)*arctan(sqrt(3) + sqrt(-4*sqrt(3)*e^x + 4*e^(2*x) + 4
) - 2*e^x) + 4*(e^(6*x) + 1)*arctan(-sqrt(3) + 2*sqrt(sqrt(3)*e^x + e^(2*x)
 + 1) - 2*e^x) - 4*(e^(6*x) + 1)*arctan(e^x) - (sqrt(3)*e^(6*x) + sqrt(3))*
log(4*sqrt(3)*e^x + 4*e^(2*x) + 4) + (sqrt(3)*e^(6*x) + sqrt(3))*log(-4*sqr
t(3)*e^x + 4*e^(2*x) + 4) + 12*e^x)/(e^(6*x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x}\operatorname{sech}^{2}\left(3x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(3*x)**2,x)
```

```
[Out] Integral(exp(x)*sech(3*x)**2, x)
```

Giac [A]
time = 0.40, size = 79, normalized size = 0.72

$$\frac{1}{18}\sqrt{3}\log\left(\sqrt{3}e^{x}+e^{(2x)}+1\right)-\frac{1}{18}\sqrt{3}\log\left(-\sqrt{3}e^{x}+e^{(2x)}+1\right)-\frac{2e^{x}}{3\left(e^{(6x)}+1\right)}+\frac{1}{9}\arctan\left(\sqrt{3}+2e^{x}\right)+\frac{1}{9}\arctan\left(-\sqrt{3}+2e^{x}\right)+\frac{2}{9}\arctan\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(3*x)^2,x, algorithm="giac")
```

```
[Out] 1/18*sqrt(3)*log(sqrt(3)*e^x + e^(2*x) + 1) - 1/18*sqrt(3)*log(-sqrt(3)*e^x
 + e^(2*x) + 1) - 2/3*e^x/(e^(6*x) + 1) + 1/9*arctan(sqrt(3) + 2*e^x) + 1/9
*arctan(-sqrt(3) + 2*e^x) + 2/9*arctan(e^x)
```

Mupad [B]
time = 0.31, size = 84, normalized size = 0.76

$$\frac{2\,\mathrm{atan}\left(\mathrm{e}^x\right)}{9}+\frac{\mathrm{atan}\left(2\,\mathrm{e}^x+\sqrt{3}\right)}{9}+\frac{\mathrm{atan}\left(2\,\mathrm{e}^x-\sqrt{3}\right)}{9}-\frac{2\,\mathrm{e}^x}{3\,\left(\mathrm{e}^{6\,x}+1\right)}-\frac{\sqrt{3}\,\ln\left({\left(\frac{2\,\mathrm{e}^x}{3}-\frac{\sqrt{3}}{3}\right)}^2+\frac{1}{9}\right)}{18}+\frac{\sqrt{3}\,\ln\left({\left(\frac{2\,\mathrm{e}^x}{3}+\frac{\sqrt{3}}{3}\right)}^2+\frac{1}{9}\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/cosh(3*x)^2,x)
```

```
[Out] (2*atan(exp(x)))/9 + atan(2*exp(x) + 3^(1/2))/9 + atan(2*exp(x) - 3^(1/2))/
9 - (2*exp(x))/(3*(exp(6*x) + 1)) - (3^(1/2)*log(((2*exp(x))/3 - 3^(1/2)/3)
^2 + 1/9))/18 + (3^(1/2)*log(((2*exp(x))/3 + 3^(1/2)/3)^2 + 1/9))/18
```

3.281 $\int e^x \cosh^2(4x)\, dx$

Optimal. Leaf size=26

$$-\frac{1}{28}e^{-7x}+\frac{e^x}{2}+\frac{e^{9x}}{36}$$

[Out] -1/28/exp(7*x)+1/2*exp(x)+1/36*exp(9*x)

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2320, 12, 276}

$$-\frac{1}{28}e^{-7x}+\frac{e^x}{2}+\frac{e^{9x}}{36}$$

Antiderivative was successfully verified.

[In] Int[E^x*Cosh[4*x]^2,x]

[Out] -1/28*1/E^(7*x) + E^x/2 + E^(9*x)/36

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$
\begin{aligned}
\int e^x \cosh^2(4x)\,dx &= \text{Subst}\left(\int \frac{(1+x^8)^2}{4x^8}\,dx, x, e^x\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \frac{(1+x^8)^2}{x^8}\,dx, x, e^x\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \left(2+\frac{1}{x^8}+x^8\right)\,dx, x, e^x\right)\\
&= -\frac{1}{28}e^{-7x}+\frac{e^x}{2}+\frac{e^{9x}}{36}
\end{aligned}
$$

Mathematica [**A**]
time = 0.01, size = 26, normalized size = 1.00

$$
-\frac{1}{28}e^{-7x}+\frac{e^x}{2}+\frac{e^{9x}}{36}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cosh[4*x]^2,x]
```

```
[Out] -1/28*1/E^(7*x) + E^x/2 + E^(9*x)/36
```

Maple [**A**]
time = 0.60, size = 34, normalized size = 1.31

method	result	size
risch	$\frac{e^{9x}}{36}+\frac{e^{x}}{2}-\frac{e^{-7x}}{28}$	18
default	$\frac{\sinh(x)}{2}+\frac{\sinh(7x)}{28}+\frac{\sinh(9x)}{36}+\frac{\cosh(x)}{2}-\frac{\cosh(7x)}{28}+\frac{\cosh(9x)}{36}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cosh(4*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sinh(x)+1/28*sinh(7*x)+1/36*sinh(9*x)+1/2*cosh(x)-1/28*cosh(7*x)+1/36*c
osh(9*x)
```

Maxima [**A**]
time = 0.27, size = 17, normalized size = 0.65

$$
\frac{1}{36}\,e^{(9\,x)}-\frac{1}{28}\,e^{(-7\,x)}+\frac{1}{2}\,e^{x}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(4*x)^2,x, algorithm="maxima")
```

```
[Out] 1/36*e^(9*x) - 1/28*e^(-7*x) + 1/2*e^x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(17) = 34$.
time = 0.47, size = 87, normalized size = 3.35

$$-\frac{\cosh(x)^8 - 64\cosh(x)^7\sinh(x) + 28\cosh(x)^6\sinh(x)^2 - 448\cosh(x)^5\sinh(x)^3 + 70\cosh(x)^4\sinh(x)^4 - 448\cosh(x)^3\sinh(x)^5 + 28\cosh(x)^2\sinh(x)^6 - 64\cosh(x)\sinh(x)^7 + \sinh(x)^8 - 63}{126(\cosh(x) - \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(4*x)^2,x, algorithm="fricas")
```

```
[Out] -1/126*(cosh(x)^8 - 64*cosh(x)^7*sinh(x) + 28*cosh(x)^6*sinh(x)^2 - 448*cos
h(x)^5*sinh(x)^3 + 70*cosh(x)^4*sinh(x)^4 - 448*cosh(x)^3*sinh(x)^5 + 28*co
sh(x)^2*sinh(x)^6 - 64*cosh(x)*sinh(x)^7 + sinh(x)^8 - 63)/(cosh(x) - sinh(
x))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(19) = 38$.
time = 0.17, size = 42, normalized size = 1.62

$$-\frac{32e^x\sinh^2(4x)}{63} + \frac{8e^x\sinh(4x)\cosh(4x)}{63} + \frac{31e^x\cosh^2(4x)}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(4*x)**2,x)
```

```
[Out] -32*exp(x)*sinh(4*x)**2/63 + 8*exp(x)*sinh(4*x)*cosh(4*x)/63 + 31*exp(x)*co
sh(4*x)**2/63
```

Giac [A]
time = 0.40, size = 17, normalized size = 0.65

$$\frac{1}{36}e^{(9x)} - \frac{1}{28}e^{(-7x)} + \frac{1}{2}e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(4*x)^2,x, algorithm="giac")
```

```
[Out] 1/36*e^(9*x) - 1/28*e^(-7*x) + 1/2*e^x
```

Mupad [B]
time = 0.96, size = 17, normalized size = 0.65

$$\frac{e^{9x}}{36} - \frac{e^{-7x}}{28} + \frac{e^x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(4*x)^2*exp(x),x)
```

```
[Out] exp(9*x)/36 - exp(-7*x)/28 + exp(x)/2
```

3.282 $\int e^x \cosh(4x)\, dx$

Optimal. Leaf size=19

$$-\frac{1}{6}e^{-3x}+\frac{e^{5x}}{10}$$

```
[Out] -1/6/exp(3*x)+1/10*exp(5*x)
```

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2320, 12, 14}

$$\frac{e^{5x}}{10}-\frac{1}{6}e^{-3x}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Cosh[4*x],x]
```

```
[Out] -1/6*1/E^(3*x) + E^(5*x)/10
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$
\begin{aligned}
\int e^x \cosh(4x)\,dx &= \text{Subst}\left(\int \frac{1+x^8}{2x^4}\,dx, x, e^x\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1+x^8}{x^4}\,dx, x, e^x\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x^4}+x^4\right)\,dx, x, e^x\right)\\
&= -\frac{1}{6}e^{-3x}+\frac{e^{5x}}{10}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 1.00

$$
-\frac{1}{6}e^{-3x}+\frac{e^{5x}}{10}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cosh[4*x],x]
```

```
[Out] -1/6*1/E^(3*x) + E^(5*x)/10
```

Maple [A]
time = 0.57, size = 26, normalized size = 1.37

method	result	size
risch	$\frac{e^{5x}}{10}-\frac{e^{-3x}}{6}$	14
default	$\frac{\sinh(3x)}{6}+\frac{\sinh(5x)}{10}-\frac{\cosh(3x)}{6}+\frac{\cosh(5x)}{10}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cosh(4*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*sinh(3*x)+1/10*sinh(5*x)-1/6*cosh(3*x)+1/10*cosh(5*x)
```

Maxima [A]
time = 0.26, size = 13, normalized size = 0.68

$$
\frac{1}{10}\,e^{(5\,x)}-\frac{1}{6}\,e^{(-3\,x)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(4*x),x, algorithm="maxima")
```

```
[Out] 1/10*e^(5*x) - 1/6*e^(-3*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(13) = 26$.
time = 0.36, size = 46, normalized size = 2.42

$$-\frac{\cosh(x)^4 - 16\cosh(x)^3\sinh(x) + 6\cosh(x)^2\sinh(x)^2 - 16\cosh(x)\sinh(x)^3 + \sinh(x)^4}{15(\cosh(x) - \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(4*x),x, algorithm="fricas")
```

```
[Out] -1/15*(cosh(x)^4 - 16*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 16*cosh(x
)*sinh(x)^3 + sinh(x)^4)/(cosh(x) - sinh(x))
```

Sympy [A]
time = 0.09, size = 20, normalized size = 1.05

$$\frac{4e^x\sinh(4x)}{15} - \frac{e^x\cosh(4x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(4*x),x)
```

```
[Out] 4*exp(x)*sinh(4*x)/15 - exp(x)*cosh(4*x)/15
```

Giac [A]
time = 0.40, size = 13, normalized size = 0.68

$$\frac{1}{10}e^{(5x)} - \frac{1}{6}e^{(-3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(4*x),x, algorithm="giac")
```

```
[Out] 1/10*e^(5*x) - 1/6*e^(-3*x)
```

Mupad [B]
time = 0.05, size = 14, normalized size = 0.74

$$\frac{e^{-3x}(3e^{8x} - 5)}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(4*x)*exp(x),x)
```

```
[Out] (exp(-3*x)*(3*exp(8*x) - 5))/30
```

3.283 $\int e^x \mathbf{sech}(4x)\, dx$

Optimal. Leaf size=371

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2-\sqrt{2}}-2e^x}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}-\frac{\text{ArcTan}\left(\frac{\sqrt{2+\sqrt{2}}-2e^x}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\text{ArcTan}\left(\frac{\sqrt{2-\sqrt{2}}+2e^x}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2+\sqrt{2}}+2e^x}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}$$

```
[Out] -1/2*arctan((-2*exp(x)+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))/(4-2*2^(1/2))^
(1/2)+1/2*arctan((2*exp(x)+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))/(4-2*2^(1/
2))^(1/2)-1/4*ln(1+exp(2*x)-exp(x)*(2-2^(1/2))^(1/2))/(4-2*2^(1/2))^(1/2)+1
/4*ln(1+exp(2*x)+exp(x)*(2-2^(1/2))^(1/2))/(4-2*2^(1/2))^(1/2)+1/2*arctan((
-2*exp(x)+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)-1/2*arc
tan((2*exp(x)+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)+1/4
*ln(1+exp(2*x)-exp(x)*(2+2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)-1/4*ln(1+exp(2
*x)+exp(x)*(2+2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)
```

Rubi [A]

time = 0.23, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 9, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 1.125$, Rules used = {2320, 12, 305, 1141, 1175, 632, 210, 1178, 642}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2-\sqrt{2}}-2e^x}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}-\frac{\text{ArcTan}\left(\frac{\sqrt{2+\sqrt{2}}-2e^x}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\text{ArcTan}\left(\frac{2e^x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}+\frac{\text{ArcTan}\left(\frac{2e^x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\log\left(-\sqrt{2-\sqrt{2}}\,e^x+e^{2x}+1\right)}{4\sqrt{2\left(2-\sqrt{2}\right)}}+\frac{\log\left(\sqrt{2-\sqrt{2}}\,e^x+e^{2x}+1\right)}{4\sqrt{2\left(2-\sqrt{2}\right)}}+\frac{\log\left(-\sqrt{2+\sqrt{2}}\,e^x+e^{2x}+1\right)}{4\sqrt{2\left(2+\sqrt{2}\right)}}-\frac{\log\left(\sqrt{2+\sqrt{2}}\,e^x+e^{2x}+1\right)}{4\sqrt{2\left(2+\sqrt{2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sech[4*x],x]
```

```
[Out] ArcTan[(Sqrt[2 - Sqrt[2]] - 2*E^x)/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2*(2 + Sqrt[2
])]) - ArcTan[(Sqrt[2 + Sqrt[2]] - 2*E^x)/Sqrt[2 - Sqrt[2]]]/(2*Sqrt[2*(2 -
 Sqrt[2])]) - ArcTan[(Sqrt[2 - Sqrt[2]] + 2*E^x)/Sqrt[2 + Sqrt[2]]]/(2*Sqrt
[2*(2 + Sqrt[2])]) + ArcTan[(Sqrt[2 + Sqrt[2]] + 2*E^x)/Sqrt[2 - Sqrt[2]]]/
(2*Sqrt[2*(2 - Sqrt[2])]) - Log[1 - Sqrt[2 - Sqrt[2]]*E^x + E^(2*x)]/(4*Sqr
t[2*(2 - Sqrt[2])]) + Log[1 + Sqrt[2 - Sqrt[2]]*E^x + E^(2*x)]/(4*Sqrt[2*(2
 - Sqrt[2])]) + Log[1 - Sqrt[2 + Sqrt[2]]*E^x + E^(2*x)]/(4*Sqrt[2*(2 + Sqr
t[2])]) - Log[1 + Sqrt[2 + Sqrt[2]]*E^x + E^(2*x)]/(4*Sqrt[2*(2 + Sqrt[2])]
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 305

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Dist[s^3/(2*Sqrt[2]*b*r), Int[x^(
m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Dist[s^3/(2*Sq
rt[2]*b*r), Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x],
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && G
tQ[a/b, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1141

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
```

```
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^x \operatorname{sech}(4x)\,dx = \text{Subst}\left(\int \frac{2x^4}{1+x^8}\,dx, x, e^x\right)$$

$$= 2\text{Subst}\left(\int \frac{x^4}{1+x^8}\,dx, x, e^x\right)$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{1-\sqrt{2}\,x^2+x^4}\,dx, x, e^x\right)}{\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{x^2}{1+\sqrt{2}\,x^2+x^4}\,dx, x, e^x\right)}{\sqrt{2}}$$

$$= -\frac{\text{Subst}\left(\int \frac{1-x^2}{1-\sqrt{2}\,x^2+x^4}\,dx, x, e^x\right)}{2\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1+x^2}{1-\sqrt{2}\,x^2+x^4}\,dx, x, e^x\right)}{2\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1-x^2}{1+\sqrt{2}\,x \cdots}\right.}{2\sqrt{2}} \cdots$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2-\sqrt{2}}\,x+x^2}\,dx, x, e^x\right)}{4\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2-\sqrt{2}}\,x+x^2}\,dx, x, e^x\right)}{4\sqrt{2}} + \cdots$$

$$= -\frac{\log\left(1-\sqrt{2-\sqrt{2}}\,e^x+e^{2x}\right)}{4\sqrt{2\left(2-\sqrt{2}\right)}} + \frac{\log\left(1+\sqrt{2-\sqrt{2}}\,e^x+e^{2x}\right)}{4\sqrt{2\left(2-\sqrt{2}\right)}} + \frac{\log\left(1-\sqrt{2+\sqrt{2}} \cdots\right.}{4\sqrt{2\left(2+\cdots\right.}} \cdots$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2e^x}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2e^x}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}} - \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}+2e^x}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}} + \cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.01, size = 24, normalized size = 0.06

$$\frac{2}{5}e^{5x}\,{}_2F_1\left(\frac{5}{8},1;\frac{13}{8};-e^{8x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sech[4*x],x]
```

```
[Out] (2*E^(5*x)*Hypergeometric2F1[5/8, 1, 13/8, -E^(8*x)])/5
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.88, size = 25, normalized size = 0.07

method	result	size
risch	$2\left(\sum_{\_R=\mathrm{RootOf}\left(16777216\_Z^8+1\right)}\_R\ln\left(-32768\_R^5+\mathrm{e}^x\right)\right)$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sech(4*x),x,method=_RETURNVERBOSE)
```

```
[Out] 2*sum(_R*ln(-32768*_R^5+exp(x)),_R=RootOf(16777216*_Z^8+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(4*x),x, algorithm="maxima")
```

```
[Out] integrate(e^x*sech(4*x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1087 vs. 2(255) = 510.
time = 0.52, size = 1087, normalized size = 2.93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(4*x),x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*arctan(-(2*sqr
t(2)*e^x - sqrt(2)*sqrt(2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x - 2*sqrt(2)*sqrt(-s
qrt(2) + 2)*e^x + 4*e^(2*x) + 4) + sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/
(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))) + 1/8*(sqrt(2)*sqrt(sqrt(2) + 2)
```

```
- sqrt(2)*sqrt(-sqrt(2) + 2))*arctan(-(2*sqrt(2)*e^x - sqrt(2)*sqrt(-2*sqrt
(2)*sqrt(sqrt(2) + 2)*e^x + 2*sqrt(2)*sqrt(-sqrt(2) + 2)*e^x + 4*e^(2*x) +
4) - sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) + sqrt(-sqr
t(2) + 2))) + 1/8*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*
arctan((2*sqrt(2)*e^x - sqrt(2)*sqrt(2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x + 2*sq
rt(2)*sqrt(-sqrt(2) + 2)*e^x + 4*e^(2*x) + 4) + sqrt(sqrt(2) + 2) + sqrt(-s
qrt(2) + 2))/(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))) + 1/8*(sqrt(2)*sqrt(
sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*arctan((2*sqrt(2)*e^x - sqrt(2)*
sqrt(-2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x - 2*sqrt(2)*sqrt(-sqrt(2) + 2)*e^x +
4*e^(2*x) + 4) - sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2)
 - sqrt(-sqrt(2) + 2))) - 1/32*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)*sqrt(-s
qrt(2) + 2))*log(2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x + 2*sqrt(2)*sqrt(-sqrt(2)
+ 2)*e^x + 4*e^(2*x) + 4) + 1/32*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(
-sqrt(2) + 2))*log(2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x - 2*sqrt(2)*sqrt(-sqrt(2
) + 2)*e^x + 4*e^(2*x) + 4) - 1/32*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqr
t(-sqrt(2) + 2))*log(-2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x + 2*sqrt(2)*sqrt(-sqr
t(2) + 2)*e^x + 4*e^(2*x) + 4) + 1/32*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)*
sqrt(-sqrt(2) + 2))*log(-2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x - 2*sqrt(2)*sqrt(-
sqrt(2) + 2)*e^x + 4*e^(2*x) + 4) - 1/4*sqrt(sqrt(2) + 2)*arctan((2*sqrt(sq
rt(sqrt(2) + 2)*e^x + e^(2*x) + 1) - sqrt(sqrt(2) + 2) - 2*e^x)/sqrt(-sqrt(
2) + 2)) - 1/4*sqrt(sqrt(2) + 2)*arctan((2*sqrt(-sqrt(sqrt(2) + 2)*e^x + e^
(2*x) + 1) + sqrt(sqrt(2) + 2) - 2*e^x)/sqrt(-sqrt(2) + 2)) + 1/4*sqrt(-sqr
t(2) + 2)*arctan((2*sqrt(sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1) - sqrt(-sqrt
(2) + 2) - 2*e^x)/sqrt(sqrt(2) + 2)) + 1/4*sqrt(-sqrt(2) + 2)*arctan((2*sqr
t(-sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1) + sqrt(-sqrt(2) + 2) - 2*e^x)/sqrt
(sqrt(2) + 2)) - 1/16*sqrt(-sqrt(2) + 2)*log(sqrt(sqrt(2) + 2)*e^x + e^(2*x
) + 1) + 1/16*sqrt(-sqrt(2) + 2)*log(-sqrt(sqrt(2) + 2)*e^x + e^(2*x) + 1)
+ 1/16*sqrt(sqrt(2) + 2)*log(sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1) - 1/16*s
qrt(sqrt(2) + 2)*log(-sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x}\operatorname{sech}(4x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(4*x),x)
[Out] Integral(exp(x)*sech(4*x), x)
```

Giac [A]

time = 0.46, size = 249, normalized size = 0.67

$$\frac{1}{4}\sqrt{\sqrt{2}+2}\arctan\left(\frac{\sqrt{\sqrt{2}+2}+2e^{x}}{\sqrt{-\sqrt{2}+2}}\right)+\frac{1}{4}\sqrt{\sqrt{2}+2}\arctan\left(-\frac{\sqrt{\sqrt{2}+2}-2e^{x}}{\sqrt{-\sqrt{2}+2}}\right)-\frac{1}{4}\sqrt{-\sqrt{2}+2}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}+2e^{x}}{\sqrt{\sqrt{2}+2}}\right)-\frac{1}{4}\sqrt{-\sqrt{2}+2}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}-2e^{x}}{\sqrt{\sqrt{2}+2}}\right)-\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}\,e^{x}+e^{(2x)}+1\right)+\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(-\sqrt{\sqrt{2}+2}\,e^{x}+e^{(2x)}+1\right)+\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{-\sqrt{2}+2}\,e^{x}+e^{(2x)}+1\right)-\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(-\sqrt{-\sqrt{2}+2}\,e^{x}+e^{(2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(4*x),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(sqrt(2) + 2)*arctan((sqrt(sqrt(2) + 2) + 2*e^x)/sqrt(-sqrt(2) + 2)
) + 1/4*sqrt(sqrt(2) + 2)*arctan(-(sqrt(sqrt(2) + 2) - 2*e^x)/sqrt(-sqrt(2)
 + 2)) - 1/4*sqrt(-sqrt(2) + 2)*arctan((sqrt(-sqrt(2) + 2) + 2*e^x)/sqrt(sq
rt(2) + 2)) - 1/4*sqrt(-sqrt(2) + 2)*arctan(-(sqrt(-sqrt(2) + 2) - 2*e^x)/s
qrt(sqrt(2) + 2)) - 1/8*sqrt(-sqrt(2) + 2)*log(sqrt(sqrt(2) + 2)*e^x + e^(2
*x) + 1) + 1/8*sqrt(-sqrt(2) + 2)*log(-sqrt(sqrt(2) + 2)*e^x + e^(2*x) + 1)
 + 1/8*sqrt(sqrt(2) + 2)*log(sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1) - 1/8*sq
rt(sqrt(2) + 2)*log(-sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1)
```

Mupad [B]
time = 4.56, size = 479, normalized size = 1.29

$$-\ln\left(32768\,e^x\left(\frac{\sqrt{\sqrt{2}+2}}{8}+\frac{\sqrt{2-\sqrt{2}}\,1i}{8}\right)^3-512\right)\left(\frac{\sqrt{\sqrt{2}+2}}{8}+\frac{\sqrt{2-\sqrt{2}}\,1i}{8}\right)+\ln\left(32768\,e^x\left(\frac{\sqrt{\sqrt{2}+2}}{8}+\frac{\sqrt{2-\sqrt{2}}\,1i}{8}\right)^3+512\right)\left(\frac{\sqrt{\sqrt{2}+2}}{8}+\frac{\sqrt{2-\sqrt{2}}\,1i}{8}\right)-\ln\left(32768\,e^x\left(-\frac{\sqrt{2-\sqrt{2}}}{8}+\frac{\sqrt{\sqrt{2}+2}\,1i}{8}\right)^3-512\right)\left(-\frac{\sqrt{2-\sqrt{2}}}{8}+\frac{\sqrt{\sqrt{2}+2}\,1i}{8}\right)+\ln\left(32768\,e^x\left(-\frac{\sqrt{2-\sqrt{2}}}{8}+\frac{\sqrt{\sqrt{2}+2}\,1i}{8}\right)^3+512\right)\left(-\frac{\sqrt{2-\sqrt{2}}}{8}+\frac{\sqrt{\sqrt{2}+2}\,1i}{8}\right)+\sqrt{2}\,\ln\left(-512+\sqrt{2}\,e^x\left(\frac{\sqrt{\sqrt{2}+2}}{8}+\frac{\sqrt{2-\sqrt{2}}\,1i}{8}\right)^3(16384-16384i)\right)\left(\frac{\sqrt{\sqrt{2}+2}}{8}+\frac{\sqrt{2-\sqrt{2}}\,1i}{8}\right)\left(\frac{1}{2}+\frac{1}{2}i\right)+\sqrt{2}\,\ln\left(512+\sqrt{2}\,e^x\left(\frac{\sqrt{\sqrt{2}+2}}{8}+\frac{\sqrt{2-\sqrt{2}}\,1i}{8}\right)^3(16384-16384i)\right)\left(\frac{\sqrt{\sqrt{2}+2}}{8}+\frac{\sqrt{2-\sqrt{2}}\,1i}{8}\right)\left(-\frac{1}{2}-\frac{1}{2}i\right)+\sqrt{2}\,\ln\left(-512+\sqrt{2}\,e^x\left(\frac{\sqrt{\sqrt{2}+2}}{8}+\frac{\sqrt{2-\sqrt{2}}\,1i}{8}\right)^3(16384+16384i)\right)\left(\frac{\sqrt{\sqrt{2}+2}}{8}+\frac{\sqrt{2-\sqrt{2}}\,1i}{8}\right)\left(\frac{1}{2}-\frac{1}{2}i\right)+\sqrt{2}\,\ln\left(512+\sqrt{2}\,e^x\left(\frac{\sqrt{\sqrt{2}+2}}{8}+\frac{\sqrt{2-\sqrt{2}}\,1i}{8}\right)^3(16384+16384i)\right)\left(\frac{\sqrt{\sqrt{2}+2}}{8}+\frac{\sqrt{2-\sqrt{2}}\,1i}{8}\right)\left(-\frac{1}{2}+\frac{1}{2}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/cosh(4*x),x)
```

```
[Out] log(32768*exp(x)*((2^(1/2) + 2)^(1/2)/8 + ((2 - 2^(1/2))^(1/2)*1i)/8)^3 + 5
12)*((2^(1/2) + 2)^(1/2)/8 + ((2 - 2^(1/2))^(1/2)*1i)/8) - log(32768*exp(x)
*((2^(1/2) + 2)^(1/2)/8 + ((2 - 2^(1/2))^(1/2)*1i)/8)^3 - 512)*((2^(1/2) +
2)^(1/2)/8 + ((2 - 2^(1/2))^(1/2)*1i)/8) - log(32768*exp(x)*(((2^(1/2) + 2)
^(1/2)*1i)/8 - (2 - 2^(1/2))^(1/2)/8)^3 - 512)*(((2^(1/2) + 2)^(1/2)*1i)/8
- (2 - 2^(1/2))^(1/2)/8) + log(32768*exp(x)*(((2^(1/2) + 2)^(1/2)*1i)/8 - (
2 - 2^(1/2))^(1/2)/8)^3 + 512)*(((2^(1/2) + 2)^(1/2)*1i)/8 - (2 - 2^(1/2))^
(1/2)/8) + 2^(1/2)*log(2^(1/2)*exp(x)*((2^(1/2) + 2)^(1/2)/8 + ((2 - 2^(1/2
))^(1/2)*1i)/8)^3*(16384 - 16384i) - 512)*((2^(1/2) + 2)^(1/2)/8 + ((2 - 2^
(1/2))^(1/2)*1i)/8)*(1/2 + 1i/2) - 2^(1/2)*log(2^(1/2)*exp(x)*((2^(1/2) + 2
)^(1/2)/8 + ((2 - 2^(1/2))^(1/2)*1i)/8)^3*(16384 - 16384i) + 512)*((2^(1/2)
 + 2)^(1/2)/8 + ((2 - 2^(1/2))^(1/2)*1i)/8)*(1/2 + 1i/2) + 2^(1/2)*log(2^(1
/2)*exp(x)*((2^(1/2) + 2)^(1/2)/8 + ((2 - 2^(1/2))^(1/2)*1i)/8)^3*(16384 +
16384i) - 512)*((2^(1/2) + 2)^(1/2)/8 + ((2 - 2^(1/2))^(1/2)*1i)/8)*(1/2 -
1i/2) - 2^(1/2)*log(2^(1/2)*exp(x)*((2^(1/2) + 2)^(1/2)/8 + ((2 - 2^(1/2))^
(1/2)*1i)/8)^3*(16384 + 16384i) + 512)*((2^(1/2) + 2)^(1/2)/8 + ((2 - 2^(1/
2))^(1/2)*1i)/8)*(1/2 - 1i/2)
```

3.284 $\int e^x \operatorname{sech}^2(4x) \, dx$

Optimal. Leaf size=379

$$-\frac{e^x}{2\left(1+e^{8x}\right)}-\frac{\text{ArcTan}\left(\frac{\sqrt{2-\sqrt{2}}-2e^x}{\sqrt{2+\sqrt{2}}}\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\text{ArcTan}\left(\frac{\sqrt{2+\sqrt{2}}-2e^x}{\sqrt{2-\sqrt{2}}}\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2-\sqrt{2}}+2e^x}{\sqrt{2+\sqrt{2}}}\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}}+\text{Arc}$$

```
[Out] -1/2*exp(x)/(1+exp(8*x))-1/32*ln(1+exp(2*x)-exp(x)*(2-2^(1/2))^(1/2))*(2-2^
(1/2))^(1/2)+1/32*ln(1+exp(2*x)+exp(x)*(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)
-1/8*arctan((-2*exp(x)+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))/(4-2*2^(1/2))^
(1/2)+1/8*arctan((2*exp(x)+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))/(4-2*2^(1/
2))^(1/2)-1/32*ln(1+exp(2*x)-exp(x)*(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)+1/
32*ln(1+exp(2*x)+exp(x)*(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)-1/8*arctan((-2
*exp(x)+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)+1/8*arcta
n((2*exp(x)+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)
```

Rubi [A]

time = 0.23, antiderivative size = 379, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {2320, 12, 294, 219, 1183, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{2-\sqrt{2}}-2e^x}{\sqrt{2+\sqrt{2}}}\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\text{ArcTan}\left(\frac{\sqrt{2+\sqrt{2}}-2e^x}{\sqrt{2-\sqrt{2}}}\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}}+\frac{\text{ArcTan}\left(\frac{2e^x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}}+\frac{\text{ArcTan}\left(\frac{2e^x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}}-\frac{e^x}{2\left(e^{8x}+1\right)}-\frac{1}{32}\sqrt{2-\sqrt{2}}\log\left(-\sqrt{2-\sqrt{2}}e^x+e^{2x}+1\right)+\frac{1}{32}\sqrt{2-\sqrt{2}}\log\left(\sqrt{2-\sqrt{2}}e^x+e^{2x}+1\right)-\frac{1}{32}\sqrt{2+\sqrt{2}}\log\left(-\sqrt{2+\sqrt{2}}e^x+e^{2x}+1\right)+\frac{1}{32}\sqrt{2+\sqrt{2}}\log\left(\sqrt{2+\sqrt{2}}e^x+e^{2x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sech[4*x]^2,x]
```

```
[Out] -1/2*E^x/(1 + E^(8*x)) - ArcTan[(Sqrt[2 - Sqrt[2]] - 2*E^x)/Sqrt[2 + Sqrt[2
]]]/(8*Sqrt[2*(2 - Sqrt[2])]) - ArcTan[(Sqrt[2 + Sqrt[2]] - 2*E^x)/Sqrt[2 -
 Sqrt[2]]]/(8*Sqrt[2*(2 + Sqrt[2])]) + ArcTan[(Sqrt[2 - Sqrt[2]] + 2*E^x)/S
qrt[2 + Sqrt[2]]]/(8*Sqrt[2*(2 - Sqrt[2])]) + ArcTan[(Sqrt[2 + Sqrt[2]] + 2
*E^x)/Sqrt[2 - Sqrt[2]]]/(8*Sqrt[2*(2 + Sqrt[2])]) - (Sqrt[2 - Sqrt[2]]*Log
[1 - Sqrt[2 - Sqrt[2]]*E^x + E^(2*x)])/32 + (Sqrt[2 - Sqrt[2]]*Log[1 + Sqrt
[2 - Sqrt[2]]*E^x + E^(2*x)])/32 - (Sqrt[2 + Sqrt[2]]*Log[1 - Sqrt[2 + Sqrt
[2]]*E^x + E^(2*x)])/32 + (Sqrt[2 + Sqrt[2]]*Log[1 + Sqrt[2 + Sqrt[2]]*E^x
+ E^(2*x)])/32
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 219

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b,
 4]], s = Denominator[Rt[a/b, 4]]}, Dist[r/(2*Sqrt[2]*a), Int[(Sqrt[2]*r -
s*x^(n/4))/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] + Dist[r/(2*Sq
rt[2]*a), Int[(Sqrt[2]*r + s*x^(n/4))/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n
/2)), x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 1] && GtQ[a/b, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int e^x \operatorname{sech}^2(4x)\,dx &= \text{Subst}\left(\int \frac{4x^8}{(1+x^8)^2}\,dx, x, e^x\right)\\
&= 4\text{Subst}\left(\int \frac{x^8}{(1+x^8)^2}\,dx, x, e^x\right)\\
&= -\frac{e^x}{2(1+e^{8x})} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^8}\,dx, x, e^x\right)\\
&= -\frac{e^x}{2(1+e^{8x})} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}-x^2}{1-\sqrt{2}x^2+x^4}\,dx, x, e^x\right)}{4\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+x^2}{1+\sqrt{2}x^2+x^4}\,dx, x, e^x\right)}{4\sqrt{2}}\\
&= -\frac{e^x}{2(1+e^{8x})} + \frac{\text{Subst}\left(\int \frac{\sqrt{2\left(2-\sqrt{2}\right)}-\left(-1+\sqrt{2}\right)x}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, e^x\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2\left(2-\right.}}{1+\sqrt{}}\right.}{8\sqrt{}}\\
&= -\frac{e^x}{2(1+e^{8x})} + \frac{1}{16}\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, e^x\right) + \frac{1}{16}\sqrt{}\\
&= -\frac{e^x}{2(1+e^{8x})} - \frac{1}{32}\sqrt{2-\sqrt{2}}\log\left(1-\sqrt{2-\sqrt{2}}e^x+e^{2x}\right) + \frac{1}{32}\sqrt{2-\sqrt{2}}\log\left(1+\right.\\
&= -\frac{e^x}{2(1+e^{8x})} - \frac{1}{16}\sqrt{2+\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2e^x}{\sqrt{2+\sqrt{2}}}\right) - \frac{1}{16}\sqrt{2-\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{}}{}\right.
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.02, size = 34, normalized size = 0.09

$$\frac{1}{2}e^x\left(-\frac{1}{1+e^{8x}} + {}_2F_1\left(\frac{1}{8}, 1; \frac{9}{8}; -e^{8x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sech[4*x]^2,x]
```

```
[Out] (E^x*(-(1 + E^(8*x))^(-1) + Hypergeometric2F1[1/8, 1, 9/8, -E^(8*x)]))/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.01, size = 36, normalized size = 0.09

method	result	size
risch	$-\frac{e^{x}}{2(1+e^{8x})}+4\left(\sum_{\_R=\mathrm{RootOf}(281474976710656\_Z^{8}+1)}\_R\ln\left(e^{x}+64\_R\right)\right)$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sech(4*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*exp(x)/(1+exp(8*x))+4*sum(_R*ln(exp(x)+64*_R),_R=RootOf(28147497671065
6*_Z^8+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(4*x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*e^x/(e^(8*x) + 1) + 4*integrate(1/8*e^x/(e^(8*x) + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1367 vs. 2(263) = 526.
time = 0.48, size = 1367, normalized size = 3.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(4*x)^2,x, algorithm="fricas")
```

```
[Out] -1/128*(8*sqrt(-sqrt(2) + 2)*(e^(8*x) + 1)*arctan((2*sqrt(sqrt(sqrt(2) + 2)
*e^x + e^(2*x) + 1) - sqrt(sqrt(2) + 2) - 2*e^x)/sqrt(-sqrt(2) + 2)) + 8*sq
rt(-sqrt(2) + 2)*(e^(8*x) + 1)*arctan((2*sqrt(-sqrt(sqrt(2) + 2)*e^x + e^(2
*x) + 1) + sqrt(sqrt(2) + 2) - 2*e^x)/sqrt(-sqrt(2) + 2)) - 2*sqrt(-sqrt(2)
 + 2)*(e^(8*x) + 1)*log(sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1) + 2*sqrt(-sqr
t(2) + 2)*(e^(8*x) + 1)*log(-sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1) + 8*(sqr
t(sqrt(2) + 2)*e^(8*x) + sqrt(sqrt(2) + 2))*arctan((2*sqrt(sqrt(-sqrt(2) +
```

```
2)*e^x + e^(2*x) + 1) - sqrt(-sqrt(2) + 2) - 2*e^x)/sqrt(sqrt(2) + 2)) + 8*
(sqrt(sqrt(2) + 2)*e^(8*x) + sqrt(sqrt(2) + 2))*arctan((2*sqrt(-sqrt(-sqrt(
2) + 2)*e^x + e^(2*x) + 1) + sqrt(-sqrt(2) + 2) - 2*e^x)/sqrt(sqrt(2) + 2))
 + 4*(sqrt(2)*sqrt(sqrt(2) + 2)*e^(8*x) + (sqrt(2)*e^(8*x) + sqrt(2))*sqrt(
-sqrt(2) + 2) + sqrt(2)*sqrt(sqrt(2) + 2))*arctan(-(2*sqrt(2)*e^x - sqrt(2)
*sqrt(2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x - 2*sqrt(2)*sqrt(-sqrt(2) + 2)*e^x +
4*e^(2*x) + 4) + sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2)
 + sqrt(-sqrt(2) + 2))) + 4*(sqrt(2)*sqrt(sqrt(2) + 2)*e^(8*x) + (sqrt(2)*e
^(8*x) + sqrt(2))*sqrt(-sqrt(2) + 2) + sqrt(2)*sqrt(sqrt(2) + 2))*arctan(-(
2*sqrt(2)*e^x - sqrt(2)*sqrt(-2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x + 2*sqrt(2)*s
qrt(-sqrt(2) + 2)*e^x + 4*e^(2*x) + 4) - sqrt(sqrt(2) + 2) + sqrt(-sqrt(2)
+ 2))/(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))) - 4*(sqrt(2)*sqrt(sqrt(2) +
 2)*e^(8*x) - (sqrt(2)*e^(8*x) + sqrt(2))*sqrt(-sqrt(2) + 2) + sqrt(2)*sqrt
(sqrt(2) + 2))*arctan((2*sqrt(2)*e^x - sqrt(2)*sqrt(2*sqrt(2)*sqrt(sqrt(2)
+ 2)*e^x + 2*sqrt(2)*sqrt(-sqrt(2) + 2)*e^x + 4*e^(2*x) + 4) + sqrt(sqrt(2)
 + 2) + sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))) - 4*(
sqrt(2)*sqrt(sqrt(2) + 2)*e^(8*x) - (sqrt(2)*e^(8*x) + sqrt(2))*sqrt(-sqrt(
2) + 2) + sqrt(2)*sqrt(sqrt(2) + 2))*arctan((2*sqrt(2)*e^x - sqrt(2)*sqrt(-
2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x - 2*sqrt(2)*sqrt(-sqrt(2) + 2)*e^x + 4*e^(2
*x) + 4) - sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) - sqr
t(-sqrt(2) + 2))) - (sqrt(2)*sqrt(sqrt(2) + 2)*e^(8*x) + (sqrt(2)*e^(8*x) +
 sqrt(2))*sqrt(-sqrt(2) + 2) + sqrt(2)*sqrt(sqrt(2) + 2))*log(2*sqrt(2)*sqr
t(sqrt(2) + 2)*e^x + 2*sqrt(2)*sqrt(-sqrt(2) + 2)*e^x + 4*e^(2*x) + 4) - (s
qrt(2)*sqrt(sqrt(2) + 2)*e^(8*x) - (sqrt(2)*e^(8*x) + sqrt(2))*sqrt(-sqrt(2
) + 2) + sqrt(2)*sqrt(sqrt(2) + 2))*log(2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x - 2
*sqrt(2)*sqrt(-sqrt(2) + 2)*e^x + 4*e^(2*x) + 4) + (sqrt(2)*sqrt(sqrt(2) +
2)*e^(8*x) - (sqrt(2)*e^(8*x) + sqrt(2))*sqrt(-sqrt(2) + 2) + sqrt(2)*sqrt(
sqrt(2) + 2))*log(-2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x + 2*sqrt(2)*sqrt(-sqrt(2
) + 2)*e^x + 4*e^(2*x) + 4) + (sqrt(2)*sqrt(sqrt(2) + 2)*e^(8*x) + (sqrt(2)
*e^(8*x) + sqrt(2))*sqrt(-sqrt(2) + 2) + sqrt(2)*sqrt(sqrt(2) + 2))*log(-2*
sqrt(2)*sqrt(sqrt(2) + 2)*e^x - 2*sqrt(2)*sqrt(-sqrt(2) + 2)*e^x + 4*e^(2*x
) + 4) - 2*(sqrt(sqrt(2) + 2)*e^(8*x) + sqrt(sqrt(2) + 2))*log(sqrt(sqrt(2)
 + 2)*e^x + e^(2*x) + 1) + 2*(sqrt(sqrt(2) + 2)*e^(8*x) + sqrt(sqrt(2) + 2)
)*log(-sqrt(sqrt(2) + 2)*e^x + e^(2*x) + 1) + 64*e^x)/(e^(8*x) + 1)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x}\operatorname{sech}^{2}(4x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(4*x)**2,x)
```

```
[Out] Integral(exp(x)*sech(4*x)**2, x)
```

Giac [A]
time = 0.40, size = 261, normalized size = 0.69

$$\frac{1}{16}\sqrt{-\sqrt{2}+2}\arctan\left(\frac{\sqrt{\sqrt{2}+2}+2e^{x}}{\sqrt{-\sqrt{2}+2}}\right)+\frac{1}{16}\sqrt{-\sqrt{2}+2}\arctan\left(-\frac{\sqrt{\sqrt{2}+2}-2e^{x}}{\sqrt{-\sqrt{2}+2}}\right)+\frac{1}{16}\sqrt{\sqrt{2}+2}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}+2e^{x}}{\sqrt{\sqrt{2}+2}}\right)+\frac{1}{16}\sqrt{\sqrt{2}+2}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}-2e^{x}}{\sqrt{\sqrt{2}+2}}\right)+\frac{1}{32}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}e^{x}+e^{(2x)}+1\right)-\frac{1}{32}\sqrt{\sqrt{2}+2}\log\left(-\sqrt{\sqrt{2}+2}e^{x}+e^{(2x)}+1\right)+\frac{1}{32}\sqrt{-\sqrt{2}+2}\log\left(\sqrt{-\sqrt{2}+2}e^{x}+e^{(2x)}+1\right)-\frac{1}{32}\sqrt{-\sqrt{2}+2}\log\left(-\sqrt{-\sqrt{2}+2}e^{x}+e^{(2x)}+1\right)-\frac{e^{x}}{2\left(e^{(8x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(4*x)^2,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(-sqrt(2) + 2)*arctan((sqrt(sqrt(2) + 2) + 2*e^x)/sqrt(-sqrt(2) +
2)) + 1/16*sqrt(-sqrt(2) + 2)*arctan(-(sqrt(sqrt(2) + 2) - 2*e^x)/sqrt(-sqr
t(2) + 2)) + 1/16*sqrt(sqrt(2) + 2)*arctan((sqrt(-sqrt(2) + 2) + 2*e^x)/sqr
t(sqrt(2) + 2)) + 1/16*sqrt(sqrt(2) + 2)*arctan(-(sqrt(-sqrt(2) + 2) - 2*e^
x)/sqrt(sqrt(2) + 2)) + 1/32*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2)*e^x +
e^(2*x) + 1) - 1/32*sqrt(sqrt(2) + 2)*log(-sqrt(sqrt(2) + 2)*e^x + e^(2*x)
+ 1) + 1/32*sqrt(-sqrt(2) + 2)*log(sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1) -
1/32*sqrt(-sqrt(2) + 2)*log(-sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1) - 1/2*e^
x/(e^(8*x) + 1)
```

Mupad [B]
time = 3.47, size = 473, normalized size = 1.25

$$-\frac{e^{x}}{2\left(e^{8x}+1\right)}+\ln\left(-\frac{e^{x}}{2}-\frac{\sqrt{\sqrt{2}+2}}{4}-\frac{\sqrt{2-\sqrt{2}}\,1i}{4}\right)\left(\frac{\sqrt{\sqrt{2}+2}}{32}+\frac{\sqrt{2-\sqrt{2}}\,1i}{32}\right)-\ln\left(\frac{\sqrt{\sqrt{2}+2}}{4}-\frac{e^{x}}{2}+\frac{\sqrt{2-\sqrt{2}}\,1i}{4}\right)\left(\frac{\sqrt{\sqrt{2}+2}}{32}+\frac{\sqrt{2-\sqrt{2}}\,1i}{32}\right)+\ln\left(\frac{\sqrt{2-\sqrt{2}}}{4}-\frac{e^{x}}{2}-\frac{\sqrt{\sqrt{2}+2}\,1i}{4}\right)\left(-\frac{\sqrt{2-\sqrt{2}}}{32}+\frac{\sqrt{\sqrt{2}+2}\,1i}{32}\right)-\ln\left(-\frac{e^{x}}{2}-\frac{\sqrt{2-\sqrt{2}}}{4}+\frac{\sqrt{\sqrt{2}+2}\,1i}{4}\right)\left(-\frac{\sqrt{2-\sqrt{2}}}{32}+\frac{\sqrt{\sqrt{2}+2}\,1i}{32}\right)+\sqrt{2}\ln\left(-\frac{e^{x}}{2}+\sqrt{2}\left(\frac{\sqrt{\sqrt{2}+2}}{32}+\frac{\sqrt{2-\sqrt{2}}\,1i}{32}\right)(-4-4i)\right)\left(\frac{\sqrt{\sqrt{2}+2}}{32}+\frac{\sqrt{2-\sqrt{2}}\,1i}{32}\right)\left(\frac{1}{2}+\frac{1}{2}i\right)+\sqrt{2}\ln\left(-\frac{e^{x}}{2}+\sqrt{2}\left(\frac{\sqrt{\sqrt{2}+2}}{32}+\frac{\sqrt{2-\sqrt{2}}\,1i}{32}\right)(-4+4i)\right)\left(\frac{\sqrt{\sqrt{2}+2}}{32}+\frac{\sqrt{2-\sqrt{2}}\,1i}{32}\right)\left(\frac{1}{2}-\frac{1}{2}i\right)+\sqrt{2}\ln\left(-\frac{e^{x}}{2}+\sqrt{2}\left(\frac{\sqrt{\sqrt{2}+2}}{32}+\frac{\sqrt{2-\sqrt{2}}\,1i}{32}\right)(4-4i)\right)\left(\frac{\sqrt{\sqrt{2}+2}}{32}+\frac{\sqrt{2-\sqrt{2}}\,1i}{32}\right)\left(-\frac{1}{2}+\frac{1}{2}i\right)+\sqrt{2}\ln\left(-\frac{e^{x}}{2}+\sqrt{2}\left(\frac{\sqrt{\sqrt{2}+2}}{32}+\frac{\sqrt{2-\sqrt{2}}\,1i}{32}\right)(4+4i)\right)\left(\frac{\sqrt{\sqrt{2}+2}}{32}+\frac{\sqrt{2-\sqrt{2}}\,1i}{32}\right)\left(-\frac{1}{2}-\frac{1}{2}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/cosh(4*x)^2,x)
```

```
[Out] log(- exp(x)/2 - (2^(1/2) + 2)^(1/2)/4 - ((2 - 2^(1/2))^(1/2)*1i)/4)*((2^(1
/2) + 2)^(1/2)/32 + ((2 - 2^(1/2))^(1/2)*1i)/32) - exp(x)/(2*(exp(8*x) + 1)
) - log((2^(1/2) + 2)^(1/2)/4 - exp(x)/2 + ((2 - 2^(1/2))^(1/2)*1i)/4)*((2^
(1/2) + 2)^(1/2)/32 + ((2 - 2^(1/2))^(1/2)*1i)/32) + log((2 - 2^(1/2))^(1/2
)/4 - ((2^(1/2) + 2)^(1/2)*1i)/4 - exp(x)/2)*(((2^(1/2) + 2)^(1/2)*1i)/32 -
 (2 - 2^(1/2))^(1/2)/32) - log(((2^(1/2) + 2)^(1/2)*1i)/4 - exp(x)/2 - (2 -
 2^(1/2))^(1/2)/4)*(((2^(1/2) + 2)^(1/2)*1i)/32 - (2 - 2^(1/2))^(1/2)/32) +
 2^(1/2)*log(- exp(x)/2 - 2^(1/2)*((2^(1/2) + 2)^(1/2)/32 + ((2 - 2^(1/2))^
(1/2)*1i)/32)*(4 + 4i))*((2^(1/2) + 2)^(1/2)/32 + ((2 - 2^(1/2))^(1/2)*1i)/
32)*(1/2 + 1i/2) + 2^(1/2)*log(- exp(x)/2 - 2^(1/2)*((2^(1/2) + 2)^(1/2)/32
 + ((2 - 2^(1/2))^(1/2)*1i)/32)*(4 - 4i))*((2^(1/2) + 2)^(1/2)/32 + ((2 - 2
^(1/2))^(1/2)*1i)/32)*(1/2 - 1i/2) - 2^(1/2)*log(2^(1/2)*((2^(1/2) + 2)^(1/
2)/32 + ((2 - 2^(1/2))^(1/2)*1i)/32)*(4 - 4i) - exp(x)/2)*((2^(1/2) + 2)^(1
/2)/32 + ((2 - 2^(1/2))^(1/2)*1i)/32)*(1/2 - 1i/2) - 2^(1/2)*log(2^(1/2)*((
2^(1/2) + 2)^(1/2)/32 + ((2 - 2^(1/2))^(1/2)*1i)/32)*(4 + 4i) - exp(x)/2)*(
(2^(1/2) + 2)^(1/2)/32 + ((2 - 2^(1/2))^(1/2)*1i)/32)*(1/2 + 1i/2)
```

3.285 $\int F^{c(a+bx)} \cosh^3(d+ex)\,dx$

Optimal. Leaf size=202

$$-\frac{bcF^{c(a+bx)}\cosh^3(d+ex)\log(F)}{9e^2-b^2c^2\log^2(F)}-\frac{6bce^2F^{c(a+bx)}\cosh(d+ex)\log(F)}{9e^4-10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)}+\frac{3eF^{c(a+bx)}\cosh^2(d+ex)\sinh(d+}{9e^2-b^2c^2\log^2(F)}$$

```
[Out] -b*c*F^(c*(b*x+a))*cosh(e*x+d)^3*ln(F)/(9*e^2-b^2*c^2*ln(F)^2)-6*b*c*e^2*F^
(c*(b*x+a))*cosh(e*x+d)*ln(F)/(9*e^4-10*b^2*c^2*e^2*ln(F)^2+b^4*c^4*ln(F)^4
)+3*e*F^(c*(b*x+a))*cosh(e*x+d)^2*sinh(e*x+d)/(9*e^2-b^2*c^2*ln(F)^2)+6*e^3
*F^(c*(b*x+a))*sinh(e*x+d)/(9*e^4-10*b^2*c^2*e^2*ln(F)^2+b^4*c^4*ln(F)^4)
```

Rubi [A]
time = 0.06, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5585, 5583}

$$-\frac{bc\log(F)\cosh^3(d+ex)F^{c(a+bx)}}{9e^2-b^2c^2\log^2(F)}+\frac{3e\sinh(d+ex)\cosh^2(d+ex)F^{c(a+bx)}}{9e^2-b^2c^2\log^2(F)}-\frac{6bce^2\log(F)\cosh(d+ex)F^{c(a+bx)}}{b^4c^4\log^4(F)-10b^2c^2e^2\log^2(F)+9e^4}+\frac{6e^3\sinh(d+ex)F^{c(a+bx)}}{b^4c^4\log^4(F)-10b^2c^2e^2\log^2(F)+9e^4}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Cosh[d + e*x]^3,x]
```

```
[Out] -((b*c*F^(c*(a + b*x))*Cosh[d + e*x]^3*Log[F])/(9*e^2 - b^2*c^2*Log[F]^2))
- (6*b*c*e^2*F^(c*(a + b*x))*Cosh[d + e*x]*Log[F])/(9*e^4 - 10*b^2*c^2*e^2*
Log[F]^2 + b^4*c^4*Log[F]^4) + (3*e*F^(c*(a + b*x))*Cosh[d + e*x]^2*Sinh[d
+ e*x])/(9*e^2 - b^2*c^2*Log[F]^2) + (6*e^3*F^(c*(a + b*x))*Sinh[d + e*x])/
(9*e^4 - 10*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4)
```

Rule 5583

```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rule 5585

```
Int[Cosh[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]^n/(e^2*n^2 - b^2*c
^2*Log[F]^2)), x] + (Dist[n*(n - 1)*(e^2/(e^2*n^2 - b^2*c^2*Log[F]^2)), Int
[F^(c*(a + b*x))*Cosh[d + e*x]^(n - 2), x], x] + Simp[e*n*F^(c*(a + b*x))*S
inh[d + e*x]*(Cosh[d + e*x]^(n - 1)/(e^2*n^2 - b^2*c^2*Log[F]^2)), x]) /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n,
 1]
```

Rubi steps

$$\int F^{c(a+bx)}\cosh^3(d+ex)\,dx = -\frac{bcF^{c(a+bx)}\cosh^3(d+ex)\log(F)}{9e^2-b^2c^2\log^2(F)} + \frac{3eF^{c(a+bx)}\cosh^2(d+ex)\sinh(d+ex)}{9e^2-b^2c^2\log^2(F)} +$$
$$= -\frac{bcF^{c(a+bx)}\cosh^3(d+ex)\log(F)}{9e^2-b^2c^2\log^2(F)} - \frac{6bce^2F^{c(a+bx)}\cosh(d+ex)\log(F)}{9e^4-10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)} +$$

Mathematica [A]
time = 0.44, size = 159, normalized size = 0.79

$$\frac{F^{c(a+bx)}\left(3\cosh(d+ex)\left(-9bce^2\log(F)+b^3c^3\log^3(F)\right)+\cosh(3(d+ex))\left(-bce^2\log(F)+b^3c^3\log^3(F)\right)+6e\left(5e^2-b^2c^2\log^2(F)+\cosh(2(d+ex))\left(e^2-b^2c^2\log^2(F)\right)\right)\sinh(d+ex)\right)}{4\left(9e^4-10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Cosh[d + e*x]^3,x]
```

```
[Out] (F^(c*(a + b*x))*(3*Cosh[d + e*x]*(-9*b*c*e^2*Log[F] + b^3*c^3*Log[F]^3) +
Cosh[3*(d + e*x)]*(-(b*c*e^2*Log[F]) + b^3*c^3*Log[F]^3) + 6*e*(5*e^2 - b^2
*c^2*Log[F]^2 + Cosh[2*(d + e*x)]*(e^2 - b^2*c^2*Log[F]^2))*Sinh[d + e*x]))
/(4*(9*e^4 - 10*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4))
```

Maple [A]
time = 2.29, size = 326, normalized size = 1.61

method	result
risch	$\frac{\left(\ln(F)^3b^3c^3e^{6ex+6d}+3\ln(F)^3b^3c^3e^{4ex+4d}-3\ln(F)^2b^2c^2e\,e^{6ex+6d}+3\ln(F)^3b^3c^3e^{2ex+2d}-3\ln(F)^2b^2c^2e\,e^{4ex+4d}-\ln(F)bc\,e^2e^{6ex+6}}{8(bc}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*cosh(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(ln(F)^3*b^3*c^3*exp(6*e*x+6*d)+3*ln(F)^3*b^3*c^3*exp(4*e*x+4*d)-3*ln(F
)^2*b^2*c^2*e*exp(6*e*x+6*d)+3*ln(F)^3*b^3*c^3*exp(2*e*x+2*d)-3*ln(F)^2*b^2
*c^2*e*exp(4*e*x+4*d)-ln(F)*b*c*e^2*exp(6*e*x+6*d)+ln(F)^3*b^3*c^3+3*ln(F)^
2*b^2*c^2*e*exp(2*e*x+2*d)-27*ln(F)*b*c*e^2*exp(4*e*x+4*d)+3*e^3*exp(6*e*x+
6*d)+3*ln(F)^2*b^2*c^2*e-27*ln(F)*b*c*e^2*exp(2*e*x+2*d)+27*e^3*exp(4*e*x+4
*d)-ln(F)*b*c*e^2-27*e^3*exp(2*e*x+2*d)-3*e^3)/(b*c*ln(F)-e)*exp(-3*e*x-3*d
)/(b*c*ln(F)-3*e)/(e+b*c*ln(F))/(b*c*ln(F)+3*e)*F^(c*(b*x+a))
```

Maxima [A]
time = 0.28, size = 142, normalized size = 0.70

$$\frac{F^{ac}e^{(bcx\log(F)+3\,xe+3\,d)}}{8\,(bc\log(F)+3\,e)} + \frac{3\,F^{ac}e^{(bcx\log(F)+xe+d)}}{8\,(bc\log(F)+e)} + \frac{3\,F^{ac}e^{(bcx\log(F)-xe)}}{8\,(bce^d\log(F)-e^{(d+1)})} + \frac{F^{ac}e^{(bcx\log(F)-3\,xe)}}{8\,(bce^{(3\,d)}\log(F)-3\,e^{(3\,d+1)})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cosh(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*F^(a*c)*e^(b*c*x*log(F) + 3*x*e + 3*d)/(b*c*log(F) + 3*e) + 3/8*F^(a*c)
*e^(b*c*x*log(F) + x*e + d)/(b*c*log(F) + e) + 3/8*F^(a*c)*e^(b*c*x*log(F)
- x*e)/(b*c*e^d*log(F) - e^(d + 1)) + 1/8*F^(a*c)*e^(b*c*x*log(F) - 3*x*e)/
(b*c*e^(3*d)*log(F) - 3*e^(3*d + 1))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4701 vs. $2(197) = 394$.
time = 0.43, size = 4701, normalized size = 23.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cosh(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/8*((3*(cosh(1)^3 + 3*cosh(1)^2*sinh(1) + 3*cosh(1)*sinh(1)^2 + sinh(1)^3)
*cosh(x*cosh(1) + x*sinh(1) + d)^6 + (b^3*c^3*log(F)^3 + 3*cosh(1)^3 - 3*(b
^2*c^2*cosh(1) + b^2*c^2*sinh(1))*log(F)^2 + 9*cosh(1)^2*sinh(1) + 9*cosh(1
)*sinh(1)^2 + 3*sinh(1)^3 - (b*c*cosh(1)^2 + 2*b*c*cosh(1)*sinh(1) + b*c*si
nh(1)^2)*log(F))*sinh(x*cosh(1) + x*sinh(1) + d)^6 + 6*(b^3*c^3*cosh(x*cosh
(1) + x*sinh(1) + d)*log(F)^3 - 3*(b^2*c^2*cosh(1) + b^2*c^2*sinh(1))*cosh(
x*cosh(1) + x*sinh(1) + d)*log(F)^2 - (b*c*cosh(1)^2 + 2*b*c*cosh(1)*sinh(1
) + b*c*sinh(1)^2)*cosh(x*cosh(1) + x*sinh(1) + d)*log(F) + 3*(cosh(1)^3 +
3*cosh(1)^2*sinh(1) + 3*cosh(1)*sinh(1)^2 + sinh(1)^3)*cosh(x*cosh(1) + x*s
inh(1) + d))*sinh(x*cosh(1) + x*sinh(1) + d)^5 + 27*(cosh(1)^3 + 3*cosh(1)^
2*sinh(1) + 3*cosh(1)*sinh(1)^2 + sinh(1)^3)*cosh(x*cosh(1) + x*sinh(1) + d
)^4 + 3*((5*b^3*c^3*cosh(x*cosh(1) + x*sinh(1) + d)^2 + b^3*c^3)*log(F)^3 +
 9*cosh(1)^3 + 15*(cosh(1)^3 + 3*cosh(1)^2*sinh(1) + 3*cosh(1)*sinh(1)^2 +
sinh(1)^3)*cosh(x*cosh(1) + x*sinh(1) + d)^2 - (b^2*c^2*cosh(1) + b^2*c^2*s
inh(1) + 15*(b^2*c^2*cosh(1) + b^2*c^2*sinh(1))*cosh(x*cosh(1) + x*sinh(1)
+ d)^2)*log(F)^2 + 27*cosh(1)^2*sinh(1) + 27*cosh(1)*sinh(1)^2 + 9*sinh(1)^
3 - (9*b*c*cosh(1)^2 + 18*b*c*cosh(1)*sinh(1) + 9*b*c*sinh(1)^2 + 5*(b*c*co
sh(1)^2 + 2*b*c*cosh(1)*sinh(1) + b*c*sinh(1)^2)*cosh(x*cosh(1) + x*sinh(1)
 + d)^2)*log(F))*sinh(x*cosh(1) + x*sinh(1) + d)^4 + (b^3*c^3*cosh(x*cosh(1
) + x*sinh(1) + d)^6 + 3*b^3*c^3*cosh(x*cosh(1) + x*sinh(1) + d)^4 + 3*b^3*
c^3*cosh(x*cosh(1) + x*sinh(1) + d)^2 + b^3*c^3)*log(F)^3 + 4*(15*(cosh(1)^
3 + 3*cosh(1)^2*sinh(1) + 3*cosh(1)*sinh(1)^2 + sinh(1)^3)*cosh(x*cosh(1) +
 x*sinh(1) + d)^3 + (5*b^3*c^3*cosh(x*cosh(1) + x*sinh(1) + d)^3 + 3*b^3*c^
3*cosh(x*cosh(1) + x*sinh(1) + d))*log(F)^3 - 3*(5*(b^2*c^2*cosh(1) + b^2*c
^2*sinh(1))*cosh(x*cosh(1) + x*sinh(1) + d)^3 + (b^2*c^2*cosh(1) + b^2*c^2*
sinh(1))*cosh(x*cosh(1) + x*sinh(1) + d))*log(F)^2 + 27*(cosh(1)^3 + 3*cosh
(1)^2*sinh(1) + 3*cosh(1)*sinh(1)^2 + sinh(1)^3)*cosh(x*cosh(1) + x*sinh(1)
 + d) - (5*(b*c*cosh(1)^2 + 2*b*c*cosh(1)*sinh(1) + b*c*sinh(1)^2)*cosh(x*c
```

```
osh(1) + x*sinh(1) + d)^3 + 27*(b*c*cosh(1)^2 + 2*b*c*cosh(1)*sinh(1) + b*c
*sinh(1)^2)*cosh(x*cosh(1) + x*sinh(1) + d))*log(F))*sinh(x*cosh(1) + x*sin
h(1) + d)^3 - 3*cosh(1)^3 - 27*(cosh(1)^3 + 3*cosh(1)^2*sinh(1) + 3*cosh(1)
*sinh(1)^2 + sinh(1)^3)*cosh(x*cosh(1) + x*sinh(1) + d)^2 - 3*((b^2*c^2*cos
h(1) + b^2*c^2*sinh(1))*cosh(x*cosh(1) + x*sinh(1) + d)^6 - b^2*c^2*cosh(1)
 + (b^2*c^2*cosh(1) + b^2*c^2*sinh(1))*cosh(x*cosh(1) + x*sinh(1) + d)^4 -
b^2*c^2*sinh(1) - (b^2*c^2*cosh(1) + b^2*c^2*sinh(1))*cosh(x*cosh(1) + x*si
nh(1) + d)^2)*log(F)^2 - 9*cosh(1)^2*sinh(1) - 9*cosh(1)*sinh(1)^2 - 3*sinh
(1)^3 + 3*(15*(cosh(1)^3 + 3*cosh(1)^2*sinh(1) + 3*cosh(1)*sinh(1)^2 + sinh
(1)^3)*cosh(x*cosh(1) + x*sinh(1) + d)^4 + (5*b^3*c^3*cosh(x*cosh(1) + x*si
nh(1) + d)^4 + 6*b^3*c^3*cosh(x*cosh(1) + x*sinh(1) + d)^2 + b^3*c^3)*log(F
)^3 - 9*cosh(1)^3 + 54*(cosh(1)^3 + 3*cosh(1)^2*sinh(1) + 3*cosh(1)*sinh(1)
^2 + sinh(1)^3)*cosh(x*cosh(1) + x*sinh(1) + d)^2 + (b^2*c^2*cosh(1) - 15*(
b^2*c^2*cosh(1) + b^2*c^2*sinh(1))*cosh(x*cosh(1) + x*sinh(1) + d)^4 + b^2*
c^2*sinh(1) - 6*(b^2*c^2*cosh(1) + b^2*c^2*sinh(1))*cosh(x*cosh(1) + x*sinh
(1) + d)^2)*log(F)^2 - 27*cosh(1)^2*sinh(1) - 27*cosh(1)*sinh(1)^2 - 9*sinh
(1)^3 - (5*(b*c*cosh(1)^2 + 2*b*c*cosh(1)*sinh(1) + b*c*sinh(1)^2)*cosh(x*c
osh(1) + x*sinh(1) + d)^4 + 9*b*c*cosh(1)^2 + 18*b*c*cosh(1)*sinh(1) + 9*b*
c*sinh(1)^2 + 54*(b*c*cosh(1)^2 + 2*b*c*cosh(1)*sinh(1) + b*c*sinh(1)^2)*co
sh(x*cosh(1) + x*sinh(1) + d)^2)*log(F))*sinh(x*cosh(1) + x*sinh(1) + d)^2
- ((b*c*cosh(1)^2 + 2*b*c*cosh(1)*sinh(1) + b*c*sinh(1)^2)*cosh(x*cosh(1) +
 x*sinh(1) + d)^6 + 27*(b*c*cosh(1)^2 + 2*b*c*cosh(1)*sinh(1) + b*c*sinh(1)
^2)*cosh(x*cosh(1) + x*sinh(1) + d)^4 + b*c*cosh(1)^2 + 2*b*c*cosh(1)*sinh(
1) + b*c*sinh(1)^2 + 27*(b*c*cosh(1)^2 + 2*b*c*cosh(1)*sinh(1) + b*c*sinh(1
)^2)*cosh(x*cosh(1) + x*sinh(1) + d)^2)*log(F) + 6*(3*(cosh(1)^3 + 3*cosh(1
)^2*sinh(1) + 3*cosh(1)*sinh(1)^2 + sinh(1)^3)*cosh(x*cosh(1) + x*sinh(1) +
 d)^5 + 18*(cosh(1)^3 + 3*cosh(1)^2*sinh(1) + 3*cosh(1)*sinh(1)^2 + sinh(1)
^3)*cosh(x*cosh(1) + x*sinh(1) + d)^3 + (b^3*c^3*cosh(x*cosh(1) + x*sinh(1)
 + d)^5 + 2*b^3*c^3*cosh(x*cosh(1) + x*sinh(1) + d)^3 + b^3*c^3*cosh(x*cosh
(1) + x*sinh(1) + d))*log(F)^3 - (3*(b^2*c^2*cosh(1) + b^2*c^2*sinh(1))*cos
h(x*cosh(1) + x*sinh(1) + d)^5 + 2*(b^2*c^2*cosh(1) + b^2*c^2*sinh(1))*cosh
(x*cosh(1) + x*sinh(1) + d)^3 - (b^2*c^2*cosh(1) + b^2*c^2*sinh(1))*cosh(x*
cosh(1) + x*sinh(1) + d))*log(F)^2 - 9*(cosh(1)^3 + 3*cosh(1)^2*sinh(1) + 3
*cosh(1)*sinh(1)^2 + sinh(1)^3)*cosh(x*cosh(1) + x*sinh(1) + d) - ((b*c*cos
h(1)^2 + 2*b*c*cosh(1)*sinh(1) + b*c*sinh(1)^2)*cosh(x*cosh(1) + x*sinh(1)
+ d)^5 + 18*(b*c*cosh(1)^2 + 2*b*c*cosh(1)*sinh(1) + b*c*sinh(1)^2)*cosh(x*
cosh(1) + x*sinh(1) + d)^3 + 9*(b*c*cosh(1)^2 + 2*b*c*cosh(1)*sinh(1) + b*c
*sinh(1)^2)*cosh(x*cosh(1) + x*sinh(1) + d))*log(F))*sinh(x*cosh(1) + x*sin
h(1) + d))*cosh((b*c*x + a*c)*log(F)) + (3*(cos...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*cosh(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [C] Result contains complex when optimal does not.
time = 0.41, size = 1211, normalized size = 6.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cosh(e*x+d)^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*(b*c*log(abs(F)) + 3*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/
2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(
F)) + 3*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*
b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*
c*log(abs(F)) + 3*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 3*e)*x + 3
*d) + I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F)
- 1/2*I*pi*a*c)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F)) + 48*e
) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/
2*I*pi*a*c)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*log(abs(F)) + 48*e))*
e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 3*e)*x + 3*d) + 3/4*(2*(b*c*log(abs
(F)) + e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2
*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) + e)^2) - (pi*b*c
*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(
F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) + e)^2))*
e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) + 3*I*(I*e^(1/2*I*pi*b*c*
x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(8*I*pi*b*c
*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F)) + 16*e) - I*e^(-1/2*I*pi*b*c*x*sg
n(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-8*I*pi*b*c*sg
n(F) + 8*I*pi*b*c + 16*b*c*log(abs(F)) + 16*e))*e^(a*c*log(abs(F)) + (b*c*l
og(abs(F)) + e)*x + d) + 3/4*(2*(b*c*log(abs(F)) - e)*cos(-1/2*pi*b*c*x*sgn
(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b
*c)^2 + 4*(b*c*log(abs(F)) - e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b
*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F
) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2))*e^(a*c*log(abs(F)) + (b*c*log(a
bs(F)) - e)*x - d) + 3*I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2
*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*l
og(abs(F)) - 16*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*p
i*a*c*sgn(F) + 1/2*I*pi*a*c)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*log(
abs(F)) - 16*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - e)*x - d) + 1/4*(2
*(b*c*log(abs(F)) - 3*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a
*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) -
3*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x
- 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(
abs(F)) - 3*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - 3*e)*x - 3*d) +
```

```
I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*
I*pi*a*c)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F)) - 48*e) - I*
e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi
*a*c)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*log(abs(F)) - 48*e))*e^(a*c
*log(abs(F)) + (b*c*log(abs(F)) - 3*e)*x - 3*d)
```

Mupad [B]
time = 1.81, size = 154, normalized size = 0.76

$$\frac{F^{a\,c+b\,c\,x}\,\left(6\,e^3\,\mathrm{sinh}\left(d+e\,x\right)+3\,e^3\,{\mathrm{cosh}\left(d+e\,x\right)}^2\,\mathrm{sinh}\left(d+e\,x\right)+b^3\,c^3\,{\mathrm{cosh}\left(d+e\,x\right)}^3\,{\ln\left(F\right)}^3-b\,c\,e^2\,{\mathrm{cosh}\left(d+e\,x\right)}^3\,\ln\left(F\right)-6\,b\,c\,e^2\,\mathrm{cosh}\left(d+e\,x\right)\,\ln\left(F\right)-3\,b^2\,c^2\,e\,{\mathrm{cosh}\left(d+e\,x\right)}^2\,\mathrm{sinh}\left(d+e\,x\right)\,{\ln\left(F\right)}^2\right)}{b^4\,c^4\,{\ln\left(F\right)}^4-10\,b^2\,c^2\,e^2\,{\ln\left(F\right)}^2+9\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))*cosh(d + e*x)^3,x)
```

```
[Out] (F^(a*c + b*c*x)*(6*e^3*sinh(d + e*x) + 3*e^3*cosh(d + e*x)^2*sinh(d + e*x)
 + b^3*c^3*cosh(d + e*x)^3*log(F)^3 - b*c*e^2*cosh(d + e*x)^3*log(F) - 6*b*
c*e^2*cosh(d + e*x)*log(F) - 3*b^2*c^2*e*cosh(d + e*x)^2*sinh(d + e*x)*log(
F)^2))/(9*e^4 + b^4*c^4*log(F)^4 - 10*b^2*c^2*e^2*log(F)^2)
```

3.286 $\int F^{c(a+bx)} \cosh^2(d+ex)\,dx$

Optimal. Leaf size=132

$$\frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(4e^2-b^2c^2\log^2(F)\right)}-\frac{bcF^{c(a+bx)}\cosh^2(d+ex)\log(F)}{4e^2-b^2c^2\log^2(F)}+\frac{2eF^{c(a+bx)}\cosh(d+ex)\sinh(d+ex)}{4e^2-b^2c^2\log^2(F)}$$

```
[Out] 2*e^2*F^(c*(b*x+a))/b/c/ln(F)/(4*e^2-b^2*c^2*ln(F)^2)-b*c*F^(c*(b*x+a))*cos
h(e*x+d)^2*ln(F)/(4*e^2-b^2*c^2*ln(F)^2)+2*e*F^(c*(b*x+a))*cosh(e*x+d)*sinh
(e*x+d)/(4*e^2-b^2*c^2*ln(F)^2)
```

Rubi [A]
time = 0.04, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5585, 2225}

$$-\frac{bc\log(F)\cosh^2(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}+\frac{2e\sinh(d+ex)\cosh(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}+\frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(4e^2-b^2c^2\log^2(F)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Cosh[d + e*x]^2,x]
```

```
[Out] (2*e^2*F^(c*(a + b*x)))/(b*c*Log[F]*(4*e^2 - b^2*c^2*Log[F]^2)) - (b*c*F^(c
*(a + b*x))*Cosh[d + e*x]^2*Log[F])/(4*e^2 - b^2*c^2*Log[F]^2) + (2*e*F^(c*
(a + b*x))*Cosh[d + e*x]*Sinh[d + e*x])/(4*e^2 - b^2*c^2*Log[F]^2)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 5585

```
Int[Cosh[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]^n/(e^2*n^2 - b^2*c
^2*Log[F]^2)), x] + (Dist[n*(n - 1)*(e^2/(e^2*n^2 - b^2*c^2*Log[F]^2)), Int
[F^(c*(a + b*x))*Cosh[d + e*x]^(n - 2), x], x] + Simp[e*n*F^(c*(a + b*x))*S
inh[d + e*x]*(Cosh[d + e*x]^(n - 1)/(e^2*n^2 - b^2*c^2*Log[F]^2)), x]) /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n,
 1]
```

Rubi steps

$$\int F^{c(a+bx)}\cosh^2(d+ex)\,dx = -\frac{bcF^{c(a+bx)}\cosh^2(d+ex)\log(F)}{4e^2-b^2c^2\log^2(F)} + \frac{2eF^{c(a+bx)}\cosh(d+ex)\sinh(d+ex)}{4e^2-b^2c^2\log^2(F)} +$$
$$= \frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(4e^2-b^2c^2\log^2(F)\right)} - \frac{bcF^{c(a+bx)}\cosh^2(d+ex)\log(F)}{4e^2-b^2c^2\log^2(F)} + \frac{2eF^{c(a+}}{}$$

Mathematica [A]
time = 0.14, size = 85, normalized size = 0.64

$$\frac{F^{c(a+bx)}\left(-4e^2+b^2c^2\log^2(F)+b^2c^2\cosh(2(d+ex))\log^2(F)-2bce\log(F)\sinh(2(d+ex))\right)}{-8bce^2\log(F)+2b^3c^3\log^3(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Cosh[d + e*x]^2,x]
```

```
[Out] (F^(c*(a + b*x))*(-4*e^2 + b^2*c^2*Log[F]^2 + b^2*c^2*Cosh[2*(d + e*x)]*Log
[F]^2 - 2*b*c*e*Log[F]*Sinh[2*(d + e*x)]))/(-8*b*c*e^2*Log[F] + 2*b^3*c^3*L
og[F]^3)
```

Maple [A]
time = 1.50, size = 143, normalized size = 1.08

method	result	size
risch	$\frac{\left(\ln(F)^2b^2c^2e^{4ex+4d}+2\ln(F)^2b^2c^2e^{2ex+2d}-2\ln(F)bce\,e^{4ex+4d}+b^2c^2\ln(F)^2+2\ln(F)bce-8e^2e^{2ex+2d}\right)e^{-2ex-2d}F^{c(bx+a)}}{4bc\ln(F)(bc\ln(F)-2e)(bc\ln(F)+2e)}$	143

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*cosh(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(ln(F)^2*b^2*c^2*exp(4*e*x+4*d)+2*ln(F)^2*b^2*c^2*exp(2*e*x+2*d)-2*ln(F
)*b*c*e*exp(4*e*x+4*d)+b^2*c^2*ln(F)^2+2*ln(F)*b*c*e-8*e^2*exp(2*e*x+2*d))/
b/c/ln(F)/(b*c*ln(F)-2*e)*exp(-2*e*x-2*d)/(b*c*ln(F)+2*e)*F^(c*(b*x+a))
```

Maxima [A]
time = 0.28, size = 98, normalized size = 0.74

$$\frac{F^{ac}e^{(bcx\log(F)+2\,xe+2\,d)}}{4\,(bc\log(F)+2\,e)} + \frac{F^{ac}e^{(bcx\log(F)-2\,xe)}}{4\,(bce^{(2\,d)}\log(F)-2\,e^{(2\,d+1)})} + \frac{F^{bcx+ac}}{2\,bc\log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cosh(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/4*F^(a*c)*e^(b*c*x*log(F) + 2*x*e + 2*d)/(b*c*log(F) + 2*e) + 1/4*F^(a*c)
*e^(b*c*x*log(F) - 2*x*e)/(b*c*e^(2*d)*log(F) - 2*e^(2*d + 1)) + 1/2*F^(b*c
*x + a*c)/(b*c*log(F))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1111 vs. $2(128) = 256$.
time = 0.39, size = 1111, normalized size = 8.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cosh(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(((b^2*c^2*log(F)^2 - 2*(b*c*cosh(1) + b*c*sinh(1))*log(F))*sinh(x*cosh
(1) + x*sinh(1) + d)^4 + 4*(b^2*c^2*cosh(x*cosh(1) + x*sinh(1) + d)*log(F)^
2 - 2*(b*c*cosh(1) + b*c*sinh(1))*cosh(x*cosh(1) + x*sinh(1) + d)*log(F))*s
inh(x*cosh(1) + x*sinh(1) + d)^3 - 8*(cosh(1)^2 + 2*cosh(1)*sinh(1) + sinh(
1)^2)*cosh(x*cosh(1) + x*sinh(1) + d)^2 + (b^2*c^2*cosh(x*cosh(1) + x*sinh(
1) + d)^4 + 2*b^2*c^2*cosh(x*cosh(1) + x*sinh(1) + d)^2 + b^2*c^2)*log(F)^2
 - 2*(6*(b*c*cosh(1) + b*c*sinh(1))*cosh(x*cosh(1) + x*sinh(1) + d)^2*log(F
) - (3*b^2*c^2*cosh(x*cosh(1) + x*sinh(1) + d)^2 + b^2*c^2)*log(F)^2 + 4*co
sh(1)^2 + 8*cosh(1)*sinh(1) + 4*sinh(1)^2)*sinh(x*cosh(1) + x*sinh(1) + d)^
2 - 2*((b*c*cosh(1) + b*c*sinh(1))*cosh(x*cosh(1) + x*sinh(1) + d)^4 - b*c*
cosh(1) - b*c*sinh(1))*log(F) - 4*(2*(b*c*cosh(1) + b*c*sinh(1))*cosh(x*cos
h(1) + x*sinh(1) + d)^3*log(F) - (b^2*c^2*cosh(x*cosh(1) + x*sinh(1) + d)^3
 + b^2*c^2*cosh(x*cosh(1) + x*sinh(1) + d))*log(F)^2 + 4*(cosh(1)^2 + 2*cos
h(1)*sinh(1) + sinh(1)^2)*cosh(x*cosh(1) + x*sinh(1) + d))*sinh(x*cosh(1) +
 x*sinh(1) + d))*cosh((b*c*x + a*c)*log(F)) + ((b^2*c^2*log(F)^2 - 2*(b*c*c
osh(1) + b*c*sinh(1))*log(F))*sinh(x*cosh(1) + x*sinh(1) + d)^4 + 4*(b^2*c^
2*cosh(x*cosh(1) + x*sinh(1) + d)*log(F)^2 - 2*(b*c*cosh(1) + b*c*sinh(1))*
cosh(x*cosh(1) + x*sinh(1) + d)*log(F))*sinh(x*cosh(1) + x*sinh(1) + d)^3 -
 8*(cosh(1)^2 + 2*cosh(1)*sinh(1) + sinh(1)^2)*cosh(x*cosh(1) + x*sinh(1) +
 d)^2 + (b^2*c^2*cosh(x*cosh(1) + x*sinh(1) + d)^4 + 2*b^2*c^2*cosh(x*cosh(
1) + x*sinh(1) + d)^2 + b^2*c^2)*log(F)^2 - 2*(6*(b*c*cosh(1) + b*c*sinh(1)
)*cosh(x*cosh(1) + x*sinh(1) + d)^2*log(F) - (3*b^2*c^2*cosh(x*cosh(1) + x*
sinh(1) + d)^2 + b^2*c^2)*log(F)^2 + 4*cosh(1)^2 + 8*cosh(1)*sinh(1) + 4*si
nh(1)^2)*sinh(x*cosh(1) + x*sinh(1) + d)^2 - 2*((b*c*cosh(1) + b*c*sinh(1))
*cosh(x*cosh(1) + x*sinh(1) + d)^4 - b*c*cosh(1) - b*c*sinh(1))*log(F) - 4*
(2*(b*c*cosh(1) + b*c*sinh(1))*cosh(x*cosh(1) + x*sinh(1) + d)^3*log(F) - (
b^2*c^2*cosh(x*cosh(1) + x*sinh(1) + d)^3 + b^2*c^2*cosh(x*cosh(1) + x*sinh
(1) + d))*log(F)^2 + 4*(cosh(1)^2 + 2*cosh(1)*sinh(1) + sinh(1)^2)*cosh(x*c
osh(1) + x*sinh(1) + d))*sinh(x*cosh(1) + x*sinh(1) + d))*sinh((b*c*x + a*c
)*log(F)))/(b^3*c^3*cosh(x*cosh(1) + x*sinh(1) + d)^2*log(F)^3 - 4*(b*c*cos
h(1)^2 + 2*b*c*cosh(1)*sinh(1) + b*c*sinh(1)^2)*cosh(x*cosh(1) + x*sinh(1)
+ d)^2*log(F) + (b^3*c^3*log(F)^3 - 4*(b*c*cosh(1)^2 + 2*b*c*cosh(1)*sinh(1
```

```
) + b*c*sinh(1)^2)*log(F))*sinh(x*cosh(1) + x*sinh(1) + d)^2 + 2*(b^3*c^3*c
osh(x*cosh(1) + x*sinh(1) + d)*log(F)^3 - 4*(b*c*cosh(1)^2 + 2*b*c*cosh(1)*
sinh(1) + b*c*sinh(1)^2)*cosh(x*cosh(1) + x*sinh(1) + d)*log(F))*sinh(x*cos
h(1) + x*sinh(1) + d))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1052 vs. $2(119) = 238$.
time = 13.86, size = 1052, normalized size = 7.97

$$\begin{cases} -\frac{x\sinh^2(d+ex)}{2}+\frac{x\cosh^2(d+ex)}{2}+\frac{\sinh(d+ex)\cosh(d+ex)}{2e} & \text{for } F=1 \\ \frac{b^2c^2\left(e^{-\frac{2e}{bc}}\right)^{ac}\left(e^{-\frac{2e}{bc}}\right)^{bcx}\log\left(e^{-\frac{2e}{bc}}\right)^2\cosh^2(d+ex)}{b^3c^3\log\left(e^{-\frac{2e}{bc}}\right)^3-4bce^2\log\left(e^{-\frac{2e}{bc}}\right)}-\frac{2bce\left(e^{-\frac{2e}{bc}}\right)^{ac}\left(e^{-\frac{2e}{bc}}\right)^{bcx}\log\left(e^{-\frac{2e}{bc}}\right)\sinh(d+ex)\cosh(d+ex)}{b^3c^3\log\left(e^{-\frac{2e}{bc}}\right)^3-4bce^2\log\left(e^{-\frac{2e}{bc}}\right)}+\frac{2e^2\left(e^{-\frac{2e}{bc}}\right)^{ac}\left(e^{-\frac{2e}{bc}}\right)^{bcx}\sinh^2(d+ex)}{b^3c^3\log\left(e^{-\frac{2e}{bc}}\right)^3-4bce^2\log\left(e^{-\frac{2e}{bc}}\right)}-\frac{2e^2\left(e^{-\frac{2e}{bc}}\right)^{ac}\left(e^{-\frac{2e}{bc}}\right)^{bcx}\cosh^2(d+ex)}{b^3c^3\log\left(e^{-\frac{2e}{bc}}\right)^3-4bce^2\log\left(e^{-\frac{2e}{bc}}\right)} & \text{for } F=e^{-\frac{2e}{bc}} \\ \frac{b^2c^2\left(e^{\frac{2e}{bc}}\right)^{ac}\left(e^{\frac{2e}{bc}}\right)^{bcx}\log\left(e^{\frac{2e}{bc}}\right)^2\cosh^2(d+ex)}{b^3c^3\log\left(e^{\frac{2e}{bc}}\right)^3-4bce^2\log\left(e^{\frac{2e}{bc}}\right)}-\frac{2bce\left(e^{\frac{2e}{bc}}\right)^{ac}\left(e^{\frac{2e}{bc}}\right)^{bcx}\log\left(e^{\frac{2e}{bc}}\right)\sinh(d+ex)\cosh(d+ex)}{b^3c^3\log\left(e^{\frac{2e}{bc}}\right)^3-4bce^2\log\left(e^{\frac{2e}{bc}}\right)}+\frac{2e^2\left(e^{\frac{2e}{bc}}\right)^{ac}\left(e^{\frac{2e}{bc}}\right)^{bcx}\sinh^2(d+ex)}{b^3c^3\log\left(e^{\frac{2e}{bc}}\right)^3-4bce^2\log\left(e^{\frac{2e}{bc}}\right)}-\frac{2e^2\left(e^{\frac{2e}{bc}}\right)^{ac}\left(e^{\frac{2e}{bc}}\right)^{bcx}\cosh^2(d+ex)}{b^3c^3\log\left(e^{\frac{2e}{bc}}\right)^3-4bce^2\log\left(e^{\frac{2e}{bc}}\right)} & \text{for } F=e^{\frac{2e}{bc}} \\ F^{ac}\left(-\frac{x\sinh^2(d+ex)}{2}+\frac{x\cosh^2(d+ex)}{2}+\frac{\sinh(d+ex)\cosh(d+ex)}{2e}\right) & \text{for } b=0 \\ -\frac{x\sinh^2(d+ex)}{2}+\frac{x\cosh^2(d+ex)}{2}+\frac{\sinh(d+ex)\cosh(d+ex)}{2e} & \text{for } c=0 \\ \frac{F^{ac}F^{bcx}b^2c^2\log(F)^2\cosh^2(d+ex)}{b^3c^3\log(F)^3-4bce^2\log(F)}-\frac{2F^{ac}F^{bcx}bce\log(F)\sinh(d+ex)\cosh(d+ex)}{b^3c^3\log(F)^3-4bce^2\log(F)}+\frac{2F^{ac}F^{bcx}e^2\sinh^2(d+ex)}{b^3c^3\log(F)^3-4bce^2\log(F)}-\frac{2F^{ac}F^{bcx}e^2\cosh^2(d+ex)}{b^3c^3\log(F)^3-4bce^2\log(F)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*cosh(e*x+d)**2,x)
```

```
[Out] Piecewise((-x*sinh(d + e*x)**2/2 + x*cosh(d + e*x)**2/2 + sinh(d + e*x)*cos
h(d + e*x)/(2*e), Eq(F, 1)), (b**2*c**2*exp(-2*e/(b*c))**(a*c)*exp(-2*e/(b*
c))**(b*c*x)*log(exp(-2*e/(b*c)))**2*cosh(d + e*x)**2/(b**3*c**3*log(exp(-2
*e/(b*c)))**3 - 4*b*c*e**2*log(exp(-2*e/(b*c)))) - 2*b*c*e*exp(-2*e/(b*c))*
*(a*c)*exp(-2*e/(b*c))**(b*c*x)*log(exp(-2*e/(b*c)))*sinh(d + e*x)*cosh(d +
 e*x)/(b**3*c**3*log(exp(-2*e/(b*c)))**3 - 4*b*c*e**2*log(exp(-2*e/(b*c))))
 + 2*e**2*exp(-2*e/(b*c))**(a*c)*exp(-2*e/(b*c))**(b*c*x)*sinh(d + e*x)**2/
(b**3*c**3*log(exp(-2*e/(b*c)))**3 - 4*b*c*e**2*log(exp(-2*e/(b*c)))) - 2*e
**2*exp(-2*e/(b*c))**(a*c)*exp(-2*e/(b*c))**(b*c*x)*cosh(d + e*x)**2/(b**3*
c**3*log(exp(-2*e/(b*c)))**3 - 4*b*c*e**2*log(exp(-2*e/(b*c)))), Eq(F, exp(
-2*e/(b*c)))), (b**2*c**2*exp(2*e/(b*c))**(a*c)*exp(2*e/(b*c))**(b*c*x)*log
(exp(2*e/(b*c)))**2*cosh(d + e*x)**2/(b**3*c**3*log(exp(2*e/(b*c)))**3 - 4*
b*c*e**2*log(exp(2*e/(b*c)))) - 2*b*c*e*exp(2*e/(b*c))**(a*c)*exp(2*e/(b*c)
)**(b*c*x)*log(exp(2*e/(b*c)))*sinh(d + e*x)*cosh(d + e*x)/(b**3*c**3*log(e
xp(2*e/(b*c)))**3 - 4*b*c*e**2*log(exp(2*e/(b*c)))) + 2*e**2*exp(2*e/(b*c))
**(a*c)*exp(2*e/(b*c))**(b*c*x)*sinh(d + e*x)**2/(b**3*c**3*log(exp(2*e/(b*
c)))**3 - 4*b*c*e**2*log(exp(2*e/(b*c)))) - 2*e**2*exp(2*e/(b*c))**(a*c)*ex
p(2*e/(b*c))**(b*c*x)*cosh(d + e*x)**2/(b**3*c**3*log(exp(2*e/(b*c)))**3 -
4*b*c*e**2*log(exp(2*e/(b*c)))), Eq(F, exp(2*e/(b*c)))), (F**(a*c)*(-x*sinh
(d + e*x)**2/2 + x*cosh(d + e*x)**2/2 + sinh(d + e*x)*cosh(d + e*x)/(2*e)),
 Eq(b, 0)), (-x*sinh(d + e*x)**2/2 + x*cosh(d + e*x)**2/2 + sinh(d + e*x)*c
osh(d + e*x)/(2*e), Eq(c, 0)), (F**(a*c)*F**(b*c*x)*b**2*c**2*log(F)**2*cos
h(d + e*x)**2/(b**3*c**3*log(F)**3 - 4*b*c*e**2*log(F)) - 2*F**(a*c)*F**(b*
c*x)*b*c*e*log(F)*sinh(d + e*x)*cosh(d + e*x)/(b**3*c**3*log(F)**3 - 4*b*c*
e**2*log(F)) + 2*F**(a*c)*F**(b*c*x)*e**2*sinh(d + e*x)**2/(b**3*c**3*log(F
```

```
)**3 - 4*b*c*e**2*log(F)) - 2*F**(a*c)*F**(b*c*x)*e**2*cosh(d + e*x)**2/(b*
*3*c**3*log(F)**3 - 4*b*c*e**2*log(F)), True))
```

Giac [C] Result contains complex when optimal does not.
time = 0.41, size = 889, normalized size = 6.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cosh(e*x+d)^2,x, algorithm="giac")
```

```
[Out] (2*b*c*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi
*a*c)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2) -
(pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a
*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)
^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + I*(I*e^(1/2*I*pi*b*c*x*sgn(F
) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(2*I*pi*b*c*sgn(F)
 - 2*I*pi*b*c + 4*b*c*log(abs(F))) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi
*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-2*I*pi*b*c*sgn(F) + 2*I*pi*b
*c + 4*b*c*log(abs(F))))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 1/2*(2*(
b*c*log(abs(F)) + 2*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c
*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) + 2*
e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x -
1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(ab
s(F)) + 2*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 2*e)*x + 2*d) + I*
(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*
pi*a*c)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c + 8*b*c*log(abs(F)) + 16*e) - I*e^(
-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*
c)/(-4*I*pi*b*c*sgn(F) + 4*I*pi*b*c + 8*b*c*log(abs(F)) + 16*e))*e^(a*c*log
(abs(F)) + (b*c*log(abs(F)) + 2*e)*x + 2*d) + 1/2*(2*(b*c*log(abs(F)) - 2*e
)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)
/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - 2*e)^2) - (pi*b*c*sgn(F
) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1
/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - 2*e)^2))*e^(a
*c*log(abs(F)) + (b*c*log(abs(F)) - 2*e)*x - 2*d) + I*(I*e^(1/2*I*pi*b*c*x*
sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(4*I*pi*b*c*s
gn(F) - 4*I*pi*b*c + 8*b*c*log(abs(F)) - 16*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F
) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-4*I*pi*b*c*sgn(F
) + 4*I*pi*b*c + 8*b*c*log(abs(F)) - 16*e))*e^(a*c*log(abs(F)) + (b*c*log(a
bs(F)) - 2*e)*x - 2*d)
```

Mupad [B]
time = 1.25, size = 100, normalized size = 0.76

$$-\frac{2\,F^{a\,c+b\,c\,x}\,e^2-F^{a\,c+b\,c\,x}\,b^2\,c^2\,{\cosh\left(d+e\,x\right)}^2\,{\ln\left(F\right)}^2+2\,F^{a\,c+b\,c\,x}\,b\,c\,e\,\cosh\left(d+e\,x\right)\,\sinh\left(d+e\,x\right)\,\ln\left(F\right)}{b^3\,c^3\,{\ln\left(F\right)}^3-4\,b\,c\,e^2\,\ln\left(F\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))*cosh(d + e*x)^2,x)
```

```
[Out] -(2*F^(a*c + b*c*x)*e^2 - F^(a*c + b*c*x)*b^2*c^2*cosh(d + e*x)^2*log(F)^2
+ 2*F^(a*c + b*c*x)*b*c*e*cosh(d + e*x)*sinh(d + e*x)*log(F))/(b^3*c^3*log(
F)^3 - 4*b*c*e^2*log(F))
```

3.287 $\int F^{c(a+bx)} \cosh(d+ex)\,dx$

Optimal. Leaf size=75

$$-\frac{bcF^{c(a+bx)}\cosh(d+ex)\log(F)}{e^2-b^2c^2\log^2(F)}+\frac{eF^{c(a+bx)}\sinh(d+ex)}{e^2-b^2c^2\log^2(F)}$$

```
[Out] -b*c*F^(c*(b*x+a))*cosh(e*x+d)*ln(F)/(e^2-b^2*c^2*ln(F)^2)+e*F^(c*(b*x+a))*
sinh(e*x+d)/(e^2-b^2*c^2*ln(F)^2)
```

Rubi [A]
time = 0.01, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {5583}

$$\frac{e\sinh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}-\frac{bc\log(F)\cosh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Cosh[d + e*x],x]
```

```
[Out] -((b*c*F^(c*(a + b*x))*Cosh[d + e*x]*Log[F])/(e^2 - b^2*c^2*Log[F]^2)) + (e
*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
```

Rule 5583

```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int F^{c(a+bx)}\cosh(d+ex)\,dx=-\frac{bcF^{c(a+bx)}\cosh(d+ex)\log(F)}{e^2-b^2c^2\log^2(F)}+\frac{eF^{c(a+bx)}\sinh(d+ex)}{e^2-b^2c^2\log^2(F)}$$

Mathematica [A]
time = 0.07, size = 50, normalized size = 0.67

$$\frac{F^{c(a+bx)}(-bc\cosh(d+ex)\log(F)+e\sinh(d+ex))}{(e-bc\log(F))(e+bc\log(F))}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Cosh[d + e*x],x]
```

```
[Out] (F^(c*(a + b*x))*(-(b*c*Cosh[d + e*x]*Log[F]) + e*Sinh[d + e*x]))/((e - b*c
*Log[F])*(e + b*c*Log[F]))
```

Maple [A]
time = 0.46, size = 74, normalized size = 0.99

method	result	size
risch	$\frac{\left(\ln \left(F \right) b c \,{\mathrm e}^{2 e x +2 d}+b c \ln \left(F \right)-e \,{\mathrm e}^{2 e x +2 d}+e \right) {\mathrm e}^{-e x -d} F^{c \left(b x +a \right)}}{2 \left(b c \ln \left(F \right)-e \right) \left(e +b c \ln \left(F \right)\right)}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*cosh(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(ln(F)*b*c*exp(2*e*x+2*d)+b*c*ln(F)-e*exp(2*e*x+2*d)+e)/(b*c*ln(F)-e)*e
xp(-e*x-d)/(e+b*c*ln(F))*F^(c*(b*x+a))
```

Maxima [A]
time = 0.27, size = 67, normalized size = 0.89

$$\frac{F^{ac}e^{\left(bcx\log\left(F\right)+xe+d\right)}}{2\left(bc\log\left(F\right)+e\right)}+\frac{F^{ac}e^{\left(bcx\log\left(F\right)-xe\right)}}{2\left(bce^{d}\log\left(F\right)-e^{\left(d+1\right)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cosh(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*F^(a*c)*e^(b*c*x*log(F) + x*e + d)/(b*c*log(F) + e) + 1/2*F^(a*c)*e^(b*
c*x*log(F) - x*e)/(b*c*e^d*log(F) - e^(d + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(78) = 156$.
time = 0.45, size = 382, normalized size = 5.09

$$-\frac{\left(\left(\cosh\left(1\right)+\sinh\left(1\right)\right)\cosh\left(x\cosh\left(1\right)+x\sinh\left(1\right)+d\right)^{2}-\left(bc\log\left(F\right)-\cosh\left(1\right)-\sinh\left(1\right)\right)\sinh\left(x\cosh\left(1\right)+x\sinh\left(1\right)+d\right)^{2}-\left(bc\cosh\left(x\cosh\left(1\right)+x\sinh\left(1\right)+d\right)^{2}+bc\right)\log\left(F\right)-2\left(bc\cosh\left(x\cosh\left(1\right)+x\sinh\left(1\right)+d\right)\log\left(F\right)-\left(\cosh\left(1\right)+\sinh\left(1\right)\right)\cosh\left(x\cosh\left(1\right)+x\sinh\left(1\right)+d\right)\right)\sinh\left(x\cosh\left(1\right)+x\sinh\left(1\right)+d\right)-\cosh\left(1\right)-\sinh\left(1\right)\right)\cosh\left(\left(bcx+ac\right)\log\left(F\right)\right)+\left(\left(\cosh\left(1\right)+\sinh\left(1\right)\right)\cosh\left(x\cosh\left(1\right)+x\sinh\left(1\right)+d\right)^{2}-\left(bc\log\left(F\right)-\cosh\left(1\right)-\sinh\left(1\right)\right)\sinh\left(x\cosh\left(1\right)+x\sinh\left(1\right)+d\right)^{2}-\left(bc\cosh\left(x\cosh\left(1\right)+x\sinh\left(1\right)+d\right)^{2}+bc\right)\log\left(F\right)-2\left(bc\cosh\left(x\cosh\left(1\right)+x\sinh\left(1\right)+d\right)\log\left(F\right)-\left(\cosh\left(1\right)+\sinh\left(1\right)\right)\cosh\left(x\cosh\left(1\right)+x\sinh\left(1\right)+d\right)\right)\sinh\left(x\cosh\left(1\right)+x\sinh\left(1\right)+d\right)-\cosh\left(1\right)-\sinh\left(1\right)\right)\sinh\left(\left(bcx+ac\right)\log\left(F\right)\right)}{2\left(b^{2}c^{2}\cosh\left(x\cosh\left(1\right)+x\sinh\left(1\right)+d\right)\log\left(F\right)^{2}-\left(\cosh\left(1\right)^{2}+2\cosh\left(1\right)\sinh\left(1\right)+\sinh\left(1\right)^{2}\right)\cosh\left(x\cosh\left(1\right)+x\sinh\left(1\right)+d\right)+\left(b^{2}c^{2}\log\left(F\right)^{2}-\cosh\left(1\right)^{2}-2\cosh\left(1\right)\sinh\left(1\right)-\sinh\left(1\right)^{2}\right)\sinh\left(x\cosh\left(1\right)+x\sinh\left(1\right)+d\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cosh(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/2*(((cosh(1) + sinh(1))*cosh(x*cosh(1) + x*sinh(1) + d)^2 - (b*c*log(F)
- cosh(1) - sinh(1))*sinh(x*cosh(1) + x*sinh(1) + d)^2 - (b*c*cosh(x*cosh(1
) + x*sinh(1) + d)^2 + b*c)*log(F) - 2*(b*c*cosh(x*cosh(1) + x*sinh(1) + d)
*log(F) - (cosh(1) + sinh(1))*cosh(x*cosh(1) + x*sinh(1) + d))*sinh(x*cosh(
1) + x*sinh(1) + d) - cosh(1) - sinh(1))*cosh((b*c*x + a*c)*log(F)) + ((cos
h(1) + sinh(1))*cosh(x*cosh(1) + x*sinh(1) + d)^2 - (b*c*log(F) - cosh(1) -
 sinh(1))*sinh(x*cosh(1) + x*sinh(1) + d)^2 - (b*c*cosh(x*cosh(1) + x*sinh(
1) + d)^2 + b*c)*log(F) - 2*(b*c*cosh(x*cosh(1) + x*sinh(1) + d)*log(F) - (
```

```
cosh(1) + sinh(1))*cosh(x*cosh(1) + x*sinh(1) + d))*sinh(x*cosh(1) + x*sinh
(1) + d) - cosh(1) - sinh(1))*sinh((b*c*x + a*c)*log(F)))/(b^2*c^2*cosh(x*c
osh(1) + x*sinh(1) + d)*log(F)^2 - (cosh(1)^2 + 2*cosh(1)*sinh(1) + sinh(1)
^2)*cosh(x*cosh(1) + x*sinh(1) + d) + (b^2*c^2*log(F)^2 - cosh(1)^2 - 2*cos
h(1)*sinh(1) - sinh(1)^2)*sinh(x*cosh(1) + x*sinh(1) + d))
```

Sympy [C] Result contains complex when optimal does not.
time = 2.39, size = 442, normalized size = 5.89

$$\begin{cases} -\frac{(-1)^{ac}(-1)^{-\frac{iex}{\pi}}x\sinh{(d+ex)}}{2}+\frac{(-1)^{ac}(-1)^{-\frac{iex}{\pi}}x\cosh{(d+ex)}}{2}+\frac{(-1)^{ac}(-1)^{-\frac{iex}{\pi}}\sinh{(d+ex)}}{e}-\frac{(-1)^{ac}(-1)^{-\frac{iex}{\pi}}\cosh{(d+ex)}}{2e} & \text{for}\ F=-1\wedge b=-\frac{ie}{\pi c} \\ x\cosh{(d)} & \text{for}\ F=1\wedge e=0 \\ \frac{bc\left(e^{-\frac{e}{bc}}\right)^{ac}\left(e^{-\frac{e}{bc}}\right)^{bcx}\log{\left(e^{-\frac{e}{bc}}\right)}\cosh{(d+ex)}}{b^2c^2\log{\left(e^{-\frac{e}{bc}}\right)}^2-e^2}-\frac{e\left(e^{-\frac{e}{bc}}\right)^{ac}\left(e^{-\frac{e}{bc}}\right)^{bcx}\sinh{(d+ex)}}{b^2c^2\log{\left(e^{-\frac{e}{bc}}\right)}^2-e^2} & \text{for}\ F=e^{-\frac{e}{bc}} \\ \frac{bc\left(e^{\frac{e}{bc}}\right)^{ac}\left(e^{\frac{e}{bc}}\right)^{bcx}\log{\left(e^{\frac{e}{bc}}\right)}\cosh{(d+ex)}}{b^2c^2\log{\left(e^{\frac{e}{bc}}\right)}^2-e^2}-\frac{e\left(e^{\frac{e}{bc}}\right)^{ac}\left(e^{\frac{e}{bc}}\right)^{bcx}\sinh{(d+ex)}}{b^2c^2\log{\left(e^{\frac{e}{bc}}\right)}^2-e^2} & \text{for}\ F=e^{\frac{e}{bc}} \\ \frac{F^{ac}F^{bcx}bc\log{(F)}\cosh{(d+ex)}}{b^2c^2\log{(F)}^2-e^2}-\frac{F^{ac}F^{bcx}e\sinh{(d+ex)}}{b^2c^2\log{(F)}^2-e^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*cosh(e*x+d),x)
```

```
[Out] Piecewise((-(-1)**(a*c)*x*sinh(d + e*x)/(2*(-1)**(I*e*x/pi)) + (-1)**(a*c)*
x*cosh(d + e*x)/(2*(-1)**(I*e*x/pi)) + (-1)**(a*c)*sinh(d + e*x)/((-1)**(I*
e*x/pi)*e) - (-1)**(a*c)*cosh(d + e*x)/(2*(-1)**(I*e*x/pi)*e), Eq(F, -1) &
Eq(b, -I*e/(pi*c))), (x*cosh(d), Eq(F, 1) & Eq(e, 0)), (b*c*exp(-e/(b*c))**
(a*c)*exp(-e/(b*c))**(b*c*x)*log(exp(-e/(b*c)))*cosh(d + e*x)/(b**2*c**2*lo
g(exp(-e/(b*c)))**2 - e**2) - e*exp(-e/(b*c))**(a*c)*exp(-e/(b*c))**(b*c*x)
*sinh(d + e*x)/(b**2*c**2*log(exp(-e/(b*c)))**2 - e**2), Eq(F, exp(-e/(b*c)
))), (b*c*exp(e/(b*c))**(a*c)*exp(e/(b*c))**(b*c*x)*log(exp(e/(b*c)))*cosh(
d + e*x)/(b**2*c**2*log(exp(e/(b*c)))**2 - e**2) - e*exp(e/(b*c))**(a*c)*ex
p(e/(b*c))**(b*c*x)*sinh(d + e*x)/(b**2*c**2*log(exp(e/(b*c)))**2 - e**2),
Eq(F, exp(e/(b*c)))), (F**(a*c)*F**(b*c*x)*b*c*log(F)*cosh(d + e*x)/(b**2*c
**2*log(F)**2 - e**2) - F**(a*c)*F**(b*c*x)*e*sinh(d + e*x)/(b**2*c**2*log(
F)**2 - e**2), True))
```

Giac [C] Result contains complex when optimal does not.
time = 0.40, size = 597, normalized size = 7.96

$$\left(\frac{2\,(bc\log(|F|)+e)\cos\left(-\frac{1}{2}\,\pi bcx\mathrm{sgn}(F)+\frac{1}{2}\,\pi bcx-\frac{1}{2}\,\pi ac\mathrm{sgn}(F)+\frac{1}{2}\,\pi ac\right)}{(\pi bc\mathrm{sgn}(F)-\pi bc)^2+4\,(bc\log(|F|)+e)^2}-\frac{(\pi bc\mathrm{sgn}(F)-\pi bc)\sin\left(-\frac{1}{2}\,\pi bcx\mathrm{sgn}(F)+\frac{1}{2}\,\pi bcx-\frac{1}{2}\,\pi ac\mathrm{sgn}(F)+\frac{1}{2}\,\pi ac\right)}{(\pi bc\mathrm{sgn}(F)-\pi bc)^2+4\,(bc\log(|F|)+e)^2}\right)e^{(ac\log(|F|)+(bc\log(|F|)+e)x+d)}+\frac{1}{2}i\left(\frac{i\,e^{\left(\frac{1}{2}i\,\pi bcx\mathrm{sgn}(F)-\frac{1}{2}i\,\pi bcx+\frac{1}{2}i\,\pi ac\mathrm{sgn}(F)-\frac{1}{2}i\,\pi ac\right)}}{i\,\pi bc\mathrm{sgn}(F)-i\,\pi bc+2\,bc\log(|F|)+2\,e}-\frac{i\,e^{\left(-\frac{1}{2}i\,\pi bcx\mathrm{sgn}(F)+\frac{1}{2}i\,\pi bcx-\frac{1}{2}i\,\pi ac\mathrm{sgn}(F)+\frac{1}{2}i\,\pi ac\right)}}{-i\,\pi bc\mathrm{sgn}(F)+i\,\pi bc+2\,bc\log(|F|)+2\,e}\right)e^{(ac\log(|F|)+(bc\log(|F|)+e)x+d)}+\left(\frac{2\,(bc\log(|F|)-e)\cos\left(-\frac{1}{2}\,\pi bcx\mathrm{sgn}(F)+\frac{1}{2}\,\pi bcx-\frac{1}{2}\,\pi ac\mathrm{sgn}(F)+\frac{1}{2}\,\pi ac\right)}{(\pi bc\mathrm{sgn}(F)-\pi bc)^2+4\,(bc\log(|F|)-e)^2}-\frac{(\pi bc\mathrm{sgn}(F)-\pi bc)\sin\left(-\frac{1}{2}\,\pi bcx\mathrm{sgn}(F)+\frac{1}{2}\,\pi bcx-\frac{1}{2}\,\pi ac\mathrm{sgn}(F)+\frac{1}{2}\,\pi ac\right)}{(\pi bc\mathrm{sgn}(F)-\pi bc)^2+4\,(bc\log(|F|)-e)^2}\right)e^{(ac\log(|F|)+(bc\log(|F|)-e)x-d)}+\frac{1}{2}i\left(\frac{i\,e^{\left(\frac{1}{2}i\,\pi bcx\mathrm{sgn}(F)-\frac{1}{2}i\,\pi bcx+\frac{1}{2}i\,\pi ac\mathrm{sgn}(F)-\frac{1}{2}i\,\pi ac\right)}}{i\,\pi bc\mathrm{sgn}(F)-i\,\pi bc+2\,bc\log(|F|)-2\,e}-\frac{i\,e^{\left(-\frac{1}{2}i\,\pi bcx\mathrm{sgn}(F)+\frac{1}{2}i\,\pi bcx-\frac{1}{2}i\,\pi ac\mathrm{sgn}(F)+\frac{1}{2}i\,\pi ac\right)}}{-i\,\pi bc\mathrm{sgn}(F)+i\,\pi bc+2\,bc\log(|F|)-2\,e}\right)e^{(ac\log(|F|)+(bc\log(|F|)-e)x-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cosh(e*x+d),x, algorithm="giac")
```

```
[Out] (2*(b*c*log(abs(F)) + e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a
*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) +
e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x -
```

```
1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(ab
s(F)) + e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) + 1/2*I*(I
*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi
*a*c)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(abs(F)) + 2*e) - I*e^(-1/2*I*
pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-I*
pi*b*c*sgn(F) + I*pi*b*c + 2*b*c*log(abs(F)) + 2*e))*e^(a*c*log(abs(F)) + (
b*c*log(abs(F)) + e)*x + d) + (2*(b*c*log(abs(F)) - e)*cos(-1/2*pi*b*c*x*sg
n(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*
b*c)^2 + 4*(b*c*log(abs(F)) - e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*
b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(
F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2))*e^(a*c*log(abs(F)) + (b*c*log(
abs(F)) - e)*x - d) + 1/2*I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x +
1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log
(abs(F)) - 2*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a
*c*sgn(F) + 1/2*I*pi*a*c)/(-I*pi*b*c*sgn(F) + I*pi*b*c + 2*b*c*log(abs(F))
- 2*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - e)*x - d)
```

Mupad [B]
time = 1.01, size = 74, normalized size = 0.99

$$-\frac{F^{a\,c+b\,c\,x}\,\mathrm{e}^{-d-e\,x}\,\left(e-e\,\mathrm{e}^{2\,d+2\,e\,x}+b\,c\,\ln\left(F\right)+b\,c\,\mathrm{e}^{2\,d+2\,e\,x}\,\ln\left(F\right)\right)}{2\,\left(e^2-b^2\,c^2\,\ln\left(F\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))*cosh(d + e*x),x)
```

```
[Out] -(F^(a*c + b*c*x)*exp(- d - e*x)*(e - e*exp(2*d + 2*e*x) + b*c*log(F) + b*c
*exp(2*d + 2*e*x)*log(F)))/(2*(e^2 - b^2*c^2*log(F)^2))
```

3.288 $\int F^{c(a+bx)}\text{sech}(d+ex)\,dx$

Optimal. Leaf size=68

$$\frac{2e^{d+ex}F^{c(a+bx)}\,{}_2F_1\left(1,\frac{e+bc\log(F)}{2e};\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right);-e^{2(d+ex)}\right)}{e+bc\log(F)}$$

```
[Out] 2*exp(e*x+d)*F^(c*(b*x+a))*hypergeom([1, 1/2*(e+b*c*ln(F))/e],[3/2+1/2*b*c*
ln(F)/e],-exp(2*e*x+2*d))/(e+b*c*ln(F))
```

Rubi [A]
time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{5600\}$

$$\frac{2e^{d+ex}F^{c(a+bx)}\,{}_2F_1\left(1,\frac{e+bc\log(F)}{2e};\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right);-e^{2(d+ex)}\right)}{bc\log(F)+e}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Sech[d + e*x],x]
```

```
[Out] (2*E^(d + e*x)*F^(c*(a + b*x))*Hypergeometric2F1[1, (e + b*c*Log[F])/(2*e),
 (3 + (b*c*Log[F])/e)/2, -E^(2*(d + e*x))])/(e + b*c*Log[F])
```

Rule 5600

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[2^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hyper
geometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)), -E^
(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int F^{c(a+bx)}\text{sech}(d+ex)\,dx=\frac{2e^{d+ex}F^{c(a+bx)}\,{}_2F_1\left(1,\frac{e+bc\log(F)}{2e};\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right);-e^{2(d+ex)}\right)}{e+bc\log(F)}$$

Mathematica [A]
time = 0.02, size = 70, normalized size = 1.03

$$\frac{2e^{d+ex}F^{c(a+bx)}\,{}_2F_1\left(1,\frac{1}{2}+\frac{bc\log(F)}{2e};\frac{3}{2}+\frac{bc\log(F)}{2e};-e^{2(d+ex)}\right)}{e+bc\log(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Sech[d + e*x],x]
```

```
[Out] (2*E^(d + e*x)*F^(c*(a + b*x))*Hypergeometric2F1[1, 1/2 + (b*c*Log[F])/(2*e
), 3/2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(e + b*c*Log[F])
```

Maple [F]
time = 0.28, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)}\mathrm{sech}(ex+d)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*sech(e*x+d),x)
```

```
[Out] int(F^(c*(b*x+a))*sech(e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sech(e*x+d),x, algorithm="maxima")
```

```
[Out] 4*F^(a*c)*integrate(-e^(b*c*x*log(F) + x*e + d + 1)/(b*c*log(F) + (b*c*e^(4
*d)*log(F) - e^(4*d + 1))*e^(4*x*e) + 2*(b*c*e^(2*d)*log(F) - e^(2*d + 1))*
e^(2*x*e) - e), x) + 2*F^(a*c)*e^(b*c*x*log(F) + x*e + d)/(b*c*log(F) + (b*
c*e^(2*d)*log(F) - e^(2*d + 1))*e^(2*x*e) - e)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sech(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*sech(x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int F^{c(a+bx)}\operatorname{sech}\left(d+ex\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*sech(e*x+d),x)
```

```
[Out] Integral(F**(c*(a + b*x))*sech(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sech(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*sech(e*x + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{c\,(a+b\,x)}}{\cosh\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))/cosh(d + e*x),x)
```

```
[Out] int(F^(c*(a + b*x))/cosh(d + e*x), x)
```

3.289 $\int F^{c(a+bx)}\operatorname{sech}^2(d+ex)\,dx$

Optimal. Leaf size=70

$$\frac{4e^{2(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1+\frac{bc\log(F)}{2e};2+\frac{bc\log(F)}{2e};-e^{2(d+ex)}\right)}{2e+bc\log(F)}$$

```
[Out] 4*exp(2*e*x+2*d)*F^(c*(b*x+a))*hypergeom([2, 1+1/2*b*c*ln(F)/e],[2+1/2*b*c*
ln(F)/e],-exp(2*e*x+2*d))/(b*c*ln(F)+2*e)
```

Rubi [A]
time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = $\{5600\}$

$$\frac{4e^{2(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,\frac{bc\log(F)}{2e}+1;\frac{bc\log(F)}{2e}+2;-e^{2(d+ex)}\right)}{bc\log(F)+2e}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Sech[d + e*x]^2,x]
```

```
[Out] (4*E^(2*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/(2
*e), 2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(2*e + b*c*Log[F])
```

Rule 5600

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[2^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hyper
geometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)), -E^
(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int F^{c(a+bx)}\operatorname{sech}^2(d+ex)\,dx = \frac{4e^{2(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1+\frac{bc\log(F)}{2e};2+\frac{bc\log(F)}{2e};-e^{2(d+ex)}\right)}{2e+bc\log(F)}$$

Mathematica [A]
time = 0.02, size = 70, normalized size = 1.00

$$\frac{4e^{2(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1+\frac{bc\log(F)}{2e};2+\frac{bc\log(F)}{2e};-e^{2(d+ex)}\right)}{2e+bc\log(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Sech[d + e*x]^2,x]
```

```
[Out] (4*E^(2*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/(2
*e), 2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(2*e + b*c*Log[F])
```

Maple [F]
time = 0.40, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)}\mathrm{sech}(ex+d)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*sech(e*x+d)^2,x)
```

```
[Out] int(F^(c*(b*x+a))*sech(e*x+d)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sech(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 16*F^(a*c)*b*c*integrate(e^(b*c*x*log(F) + 1)/(b^2*c^2*log(F)^2 - 6*b*c*e*l
og(F) + (b^2*c^2*e^(6*d)*log(F)^2 - 6*b*c*e^(6*d + 1)*log(F) + 8*e^(6*d + 2
))*e^(6*x*e) + 3*(b^2*c^2*e^(4*d)*log(F)^2 - 6*b*c*e^(4*d + 1)*log(F) + 8*e
^(4*d + 2))*e^(4*x*e) + 3*(b^2*c^2*e^(2*d)*log(F)^2 - 6*b*c*e^(2*d + 1)*log
(F) + 8*e^(2*d + 2))*e^(2*x*e) + 8*e^2), x)*log(F) - 4*(4*F^(a*c)*e - (F^(a
*c)*b*c*e^(2*d)*log(F) - 4*F^(a*c)*e^(2*d + 1))*e^(2*x*e))*F^(b*c*x)/(b^2*c
^2*log(F)^2 - 6*b*c*e*log(F) + (b^2*c^2*e^(4*d)*log(F)^2 - 6*b*c*e^(4*d + 1
)*log(F) + 8*e^(4*d + 2))*e^(4*x*e) + 2*(b^2*c^2*e^(2*d)*log(F)^2 - 6*b*c*e
^(2*d + 1)*log(F) + 8*e^(2*d + 2))*e^(2*x*e) + 8*e^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sech(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*sech(x*e + d)^2, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int F^{c(a+bx)}\operatorname{sech}^2(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*sech(e*x+d)**2,x)
[Out] Integral(F**(c*(a + b*x))*sech(d + e*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sech(e*x+d)^2,x, algorithm="giac")
[Out] integrate(F^((b*x + a)*c)*sech(e*x + d)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{c\,(a+b\,x)}}{{\cosh\left(d+e\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))/cosh(d + e*x)^2,x)
[Out] int(F^(c*(a + b*x))/cosh(d + e*x)^2, x)
```

3.290 $\int F^{c(a+bx)}\mathbf{sech}^3(d+ex)\,dx$

Optimal. Leaf size=124

$$\frac{e^{d+ex}F^{c(a+bx)}\,{}_2F_1\left(1,\frac{e+bc\log(F)}{2e};\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right);-e^{2(d+ex)}\right)(e-bc\log(F))}{e^2}+\frac{bcF^{c(a+bx)}\log(F)\mathrm{sech}(d+ex)}{2e^2}+$$

```
[Out] exp(e*x+d)*F^(c*(b*x+a))*hypergeom([1, 1/2*(e+b*c*ln(F))/e],[3/2+1/2*b*c*ln
(F)/e],-exp(2*e*x+2*d))*(e-b*c*ln(F))/e^2+1/2*b*c*F^(c*(b*x+a))*ln(F)*sech(
e*x+d)/e^2+1/2*F^(c*(b*x+a))*sech(e*x+d)*tanh(e*x+d)/e
```

Rubi [A]
time = 0.04, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5598, 5600}

$$\frac{e^{d+ex}F^{c(a+bx)}(e-bc\log(F))\,{}_2F_1\left(1,\frac{e+bc\log(F)}{2e};\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right);-e^{2(d+ex)}\right)}{e^2}+\frac{bc\log(F)\mathrm{sech}(d+ex)F^{c(a+bx)}}{2e^2}+\frac{\tanh(d+ex)\mathrm{sech}(d+ex)F^{c(a+bx)}}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Sech[d + e*x]^3,x]
```

```
[Out] (E^(d + e*x)*F^(c*(a + b*x))*Hypergeometric2F1[1, (e + b*c*Log[F])/(2*e), (
3 + (b*c*Log[F])/e)/2, -E^(2*(d + e*x))]*(e - b*c*Log[F]))/e^2 + (b*c*F^(c*
(a + b*x))*Log[F]*Sech[d + e*x])/(2*e^2) + (F^(c*(a + b*x))*Sech[d + e*x]*T
anh[d + e*x])/(2*e)
```

Rule 5598

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sech[d + e*x]^(n - 2)/(e^2*(n - 1)*
(n - 2))), x] + (Dist[(e^2*(n - 2)^2 - b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n -
2)), Int[F^(c*(a + b*x))*Sech[d + e*x]^(n - 2), x], x] + Simp[F^(c*(a + b*x
))*Sech[d + e*x]^(n - 1)*(Sinh[d + e*x]/(e*(n - 1))), x]) /; FreeQ[{F, a, b
, c, d, e}, x] && NeQ[e^2*(n - 2)^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1] &&
NeQ[n, 2]
```

Rule 5600

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[2^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hyper
geometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)), -E^
(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int F^{c(a+bx)}\operatorname{sech}^3(d+ex)\,dx = \frac{bcF^{c(a+bx)}\log(F)\operatorname{sech}(d+ex)}{2e^2}+\frac{F^{c(a+bx)}\operatorname{sech}(d+ex)\tanh(d+ex)}{2e}+\frac{1}{2}\Big(1-$$

$$=\frac{e^{d+ex}F^{c(a+bx)}\,{}_2F_1\left(1,\frac{e+bc\log(F)}{2e};\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right);-e^{2(d+ex)}\right)(e-bc\log(F))}{e^2}+$$

Mathematica [A]
time = 0.18, size = 96, normalized size = 0.77

$$\frac{F^{c(a+bx)}\left(2e^{d+ex}\,{}_2F_1\left(1,\frac{e+bc\log(F)}{2e};\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right);-e^{2(d+ex)}\right)(e-bc\log(F))+\operatorname{sech}(d+ex)(bc\log(F)+e\tanh(d+ex))\right)}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Sech[d + e*x]^3,x]
```

```
[Out] (F^(c*(a + b*x))*(2*E^(d + e*x)*Hypergeometric2F1[1, (e + b*c*Log[F])/(2*e)
, (3 + (b*c*Log[F])/e)/2, -E^(2*(d + e*x))]*(e - b*c*Log[F]) + Sech[d + e*x
]*(b*c*Log[F] + e*Tanh[d + e*x])))/(2*e^2)
```

Maple [F]
time = 0.51, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)}\operatorname{sech}(ex+d)^3\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*sech(e*x+d)^3,x)
```

```
[Out] int(F^(c*(b*x+a))*sech(e*x+d)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sech(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 48*(F^(a*c)*b*c*e^(d + 1)*log(F) + F^(a*c)*e^(d + 2))*integrate(e^(b*c*x*lo
g(F) + x*e)/(b^2*c^2*log(F)^2 - 8*b*c*e*log(F) + (b^2*c^2*e^(8*d)*log(F)^2
- 8*b*c*e^(8*d + 1)*log(F) + 15*e^(8*d + 2))*e^(8*x*e) + 4*(b^2*c^2*e^(6*d)
*log(F)^2 - 8*b*c*e^(6*d + 1)*log(F) + 15*e^(6*d + 2))*e^(6*x*e) + 6*(b^2*c
^2*e^(4*d)*log(F)^2 - 8*b*c*e^(4*d + 1)*log(F) + 15*e^(4*d + 2))*e^(4*x*e)
```

```
+ 4*(b^2*c^2*e^(2*d)*log(F)^2 - 8*b*c*e^(2*d + 1)*log(F) + 15*e^(2*d + 2))*
e^(2*x*e) + 15*e^2), x) + 8*((F^(a*c)*b*c*e^(3*d)*log(F) - 5*F^(a*c)*e^(3*d
 + 1))*e^(3*x*e) - 6*F^(a*c)*e^(x*e + d + 1))*F^(b*c*x)/(b^2*c^2*log(F)^2 -
 8*b*c*e*log(F) + (b^2*c^2*e^(6*d)*log(F)^2 - 8*b*c*e^(6*d + 1)*log(F) + 15
*e^(6*d + 2))*e^(6*x*e) + 3*(b^2*c^2*e^(4*d)*log(F)^2 - 8*b*c*e^(4*d + 1)*l
og(F) + 15*e^(4*d + 2))*e^(4*x*e) + 3*(b^2*c^2*e^(2*d)*log(F)^2 - 8*b*c*e^(
2*d + 1)*log(F) + 15*e^(2*d + 2))*e^(2*x*e) + 15*e^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sech(e*x+d)^3,x, algorithm="fricas")
[Out] integral(F^(b*c*x + a*c)*sech(x*e + d)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int F^{c(a+bx)}\operatorname{sech}^3(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*sech(e*x+d)**3,x)
[Out] Integral(F**(c*(a + b*x))*sech(d + e*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sech(e*x+d)^3,x, algorithm="giac")
[Out] integrate(F^((b*x + a)*c)*sech(e*x + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{c\,(a+b\,x)}}{{\cosh\left(d+e\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))/cosh(d + e*x)^3,x)
[Out] int(F^(c*(a + b*x))/cosh(d + e*x)^3, x)
```

3.291 $\int F^{c(a+bx)}\mathrm{sech}^4(d+ex)\,dx$

Optimal. Leaf size=133

$$\frac{2e^{2(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1+\frac{bc\log(F)}{2e};2+\frac{bc\log(F)}{2e};-e^{2(d+ex)}\right)(2e-bc\log(F))}{3e^2}+\frac{bcF^{c(a+bx)}\log(F)\mathrm{sech}^2(d+ex}{6e^2}$$

```
[Out] 2/3*exp(2*e*x+2*d)*F^(c*(b*x+a))*hypergeom([2, 1+1/2*b*c*ln(F)/e],[2+1/2*b*
c*ln(F)/e],-exp(2*e*x+2*d))*(2*e-b*c*ln(F))/e^2+1/6*b*c*F^(c*(b*x+a))*ln(F)
*sech(e*x+d)^2/e^2+1/3*F^(c*(b*x+a))*sech(e*x+d)^2*tanh(e*x+d)/e
```

Rubi [A]
time = 0.04, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5598, 5600}

$$\frac{2e^{2(d+ex)}F^{c(a+bx)}(2e-bc\log(F))\,{}_2F_1\left(2,\frac{bc\log(F)}{2e}+1;\frac{bc\log(F)}{2e}+2;-e^{2(d+ex)}\right)}{3e^2}+\frac{bc\log(F)\mathrm{sech}^2(d+ex)F^{c(a+bx)}}{6e^2}+\frac{\tanh(d+ex)\mathrm{sech}^2(d+ex)F^{c(a+bx)}}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Sech[d + e*x]^4,x]
```

```
[Out] (2*E^(2*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/(2
*e), 2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))]*(2*e - b*c*Log[F]))/(3*e^2)
+ (b*c*F^(c*(a + b*x))*Log[F]*Sech[d + e*x]^2)/(6*e^2) + (F^(c*(a + b*x))*S
ech[d + e*x]^2*Tanh[d + e*x])/(3*e)
```

Rule 5598

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sech[d + e*x]^(n - 2)/(e^2*(n - 1)*
(n - 2))), x] + (Dist[(e^2*(n - 2)^2 - b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n -
2)), Int[F^(c*(a + b*x))*Sech[d + e*x]^(n - 2), x], x] + Simp[F^(c*(a + b*x
))*Sech[d + e*x]^(n - 1)*(Sinh[d + e*x]/(e*(n - 1))), x]) /; FreeQ[{F, a, b
, c, d, e}, x] && NeQ[e^2*(n - 2)^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1] &&
NeQ[n, 2]
```

Rule 5600

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[2^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hyper
geometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)), -E^
(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int F^{c(a+bx)}\text{sech}^4(d+ex)\,dx = \frac{bcF^{c(a+bx)}\log(F)\text{sech}^2(d+ex)}{6e^2} + \frac{F^{c(a+bx)}\text{sech}^2(d+ex)\tanh(d+ex)}{3e} + \frac{1}{6}\Big(4$$

$$= \frac{2e^{2(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1+\frac{bc\log(F)}{2e};2+\frac{bc\log(F)}{2e};-e^{2(d+ex)}\right)(2e-bc\log(F))}{3e^2} +$$

Mathematica [A]
time = 0.14, size = 101, normalized size = 0.76

$$\frac{F^{c(a+bx)}\left(4e^{2(d+ex)}\,{}_2F_1\left(2,1+\frac{bc\log(F)}{2e};2+\frac{bc\log(F)}{2e};-e^{2(d+ex)}\right)(2e-bc\log(F))+\text{sech}^2(d+ex)(bc\log(F)+2e\tanh(d+ex))\right)}{6e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Sech[d + e*x]^4,x]
```

```
[Out] (F^(c*(a + b*x))*(4*E^(2*(d + e*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/(
2*e), 2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))]*(2*e - b*c*Log[F]) + Sech[d
 + e*x]^2*(b*c*Log[F] + 2*e*Tanh[d + e*x])))/(6*e^2)
```

Maple [F]
time = 0.52, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)}\text{sech}(ex+d)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*sech(e*x+d)^4,x)
```

```
[Out] int(F^(c*(b*x+a))*sech(e*x+d)^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sech(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] 128*(F^(a*c)*b^2*c^2*e*log(F)^2 + 2*F^(a*c)*b*c*e^2*log(F))*integrate(-F^(b
*c*x)/(b^3*c^3*log(F)^3 - 18*b^2*c^2*e*log(F)^2 + 104*b*c*e^2*log(F) + (b^3
*c^3*e^(10*d)*log(F)^3 - 18*b^2*c^2*e^(10*d + 1)*log(F)^2 + 104*b*c*e^(10*d
 + 2)*log(F) - 192*e^(10*d + 3))*e^(10*x*e) + 5*(b^3*c^3*e^(8*d)*log(F)^3 -
 18*b^2*c^2*e^(8*d + 1)*log(F)^2 + 104*b*c*e^(8*d + 2)*log(F) - 192*e^(8*d
```

```
+ 3))*e^(8*x*e) + 10*(b^3*c^3*e^(6*d)*log(F)^3 - 18*b^2*c^2*e^(6*d + 1)*log
(F)^2 + 104*b*c*e^(6*d + 2)*log(F) - 192*e^(6*d + 3))*e^(6*x*e) + 10*(b^3*c
^3*e^(4*d)*log(F)^3 - 18*b^2*c^2*e^(4*d + 1)*log(F)^2 + 104*b*c*e^(4*d + 2)
*log(F) - 192*e^(4*d + 3))*e^(4*x*e) + 5*(b^3*c^3*e^(2*d)*log(F)^3 - 18*b^2
*c^2*e^(2*d + 1)*log(F)^2 + 104*b*c*e^(2*d + 2)*log(F) - 192*e^(2*d + 3))*e
^(2*x*e) - 192*e^3), x) + 16*(8*F^(a*c)*b*c*e*log(F) + 16*F^(a*c)*e^2 + (F^
(a*c)*b^2*c^2*e^(4*d)*log(F)^2 - 14*F^(a*c)*b*c*e^(4*d + 1)*log(F) + 48*F^(
a*c)*e^(4*d + 2))*e^(4*x*e) - 8*(F^(a*c)*b*c*e^(2*d + 1)*log(F) - 8*F^(a*c)
*e^(2*d + 2))*e^(2*x*e))*F^(b*c*x)/(b^3*c^3*log(F)^3 - 18*b^2*c^2*e*log(F)^
2 + 104*b*c*e^2*log(F) + (b^3*c^3*e^(8*d)*log(F)^3 - 18*b^2*c^2*e^(8*d + 1)
*log(F)^2 + 104*b*c*e^(8*d + 2)*log(F) - 192*e^(8*d + 3))*e^(8*x*e) + 4*(b^
3*c^3*e^(6*d)*log(F)^3 - 18*b^2*c^2*e^(6*d + 1)*log(F)^2 + 104*b*c*e^(6*d +
 2)*log(F) - 192*e^(6*d + 3))*e^(6*x*e) + 6*(b^3*c^3*e^(4*d)*log(F)^3 - 18*
b^2*c^2*e^(4*d + 1)*log(F)^2 + 104*b*c*e^(4*d + 2)*log(F) - 192*e^(4*d + 3)
)*e^(4*x*e) + 4*(b^3*c^3*e^(2*d)*log(F)^3 - 18*b^2*c^2*e^(2*d + 1)*log(F)^2
 + 104*b*c*e^(2*d + 2)*log(F) - 192*e^(2*d + 3))*e^(2*x*e) - 192*e^3)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sech(e*x+d)^4,x, algorithm="fricas")
[Out] integral(F^(b*c*x + a*c)*sech(x*e + d)^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int F^{c(a+bx)}\operatorname{sech}^4(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*sech(e*x+d)**4,x)
[Out] Integral(F**(c*(a + b*x))*sech(d + e*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sech(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*sech(e*x + d)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{c\,(a+b\,x)}}{\cosh\left(d+e\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))/cosh(d + e*x)^4,x)
[Out] int(F^(c*(a + b*x))/cosh(d + e*x)^4, x)
```

3.292 $\int e^{c(a+bx)} \cosh^2(ac+bcx)^{5/2}\,dx$

Optimal. Leaf size=250

$$-\frac{e^{-4c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,\operatorname{sech}(ac+bcx)}{128bc}-\frac{5e^{-2c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,\operatorname{sech}(ac+bcx)}{64bc}+\frac{5e^{2c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,\operatorname{sech}(ac+bcx)}{32bc}+\frac{5e^{4c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,\operatorname{sech}(ac+bcx)}{128bc}+\frac{e^{6c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,\operatorname{sech}(ac+bcx)}{192bc}+\frac{5}{16}x\sqrt{\cosh^2(ac+bcx)}\,\operatorname{sech}(ac+bcx)$$

```
[Out] -1/128*sech(b*c*x+a*c)*(cosh(b*c*x+a*c)^2)^(1/2)/b/c/exp(4*c*(b*x+a))-5/64*
sech(b*c*x+a*c)*(cosh(b*c*x+a*c)^2)^(1/2)/b/c/exp(2*c*(b*x+a))+5/32*exp(2*c
*(b*x+a))*sech(b*c*x+a*c)*(cosh(b*c*x+a*c)^2)^(1/2)/b/c+5/128*exp(4*c*(b*x+
a))*sech(b*c*x+a*c)*(cosh(b*c*x+a*c)^2)^(1/2)/b/c+1/192*exp(6*c*(b*x+a))*se
ch(b*c*x+a*c)*(cosh(b*c*x+a*c)^2)^(1/2)/b/c+5/16*x*sech(b*c*x+a*c)*(cosh(b*
c*x+a*c)^2)^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6852, 2320, 12, 272, 45}

$$-\frac{e^{-4c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,\operatorname{sech}(ac+bcx)}{128bc}-\frac{5e^{-2c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,\operatorname{sech}(ac+bcx)}{64bc}+\frac{5e^{2c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,\operatorname{sech}(ac+bcx)}{32bc}+\frac{5e^{4c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,\operatorname{sech}(ac+bcx)}{128bc}+\frac{e^{6c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,\operatorname{sech}(ac+bcx)}{192bc}+\frac{5}{16}x\sqrt{\cosh^2(ac+bcx)}\,\operatorname{sech}(ac+bcx)$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*(Cosh[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] -1/128*(Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/(b*c*E^(4*c*(a + b*x))
) - (5*Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/(64*b*c*E^(2*c*(a + b*x
))) + (5*E^(2*c*(a + b*x))*Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/(32
*b*c) + (5*E^(4*c*(a + b*x))*Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/(
128*b*c) + (E^(6*c*(a + b*x))*Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/
(192*b*c) + (5*x*Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/16
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int e^{c(a+bx)}\cosh^2(ac+bcx)^{5/2}\,dx &= \left(\sqrt{\cosh^2(ac+bcx)}\,\operatorname{sech}(ac+bcx)\right)\int e^{c(a+bx)}\cosh^5(ac+bcx)\,dx\\
&= \frac{\left(\sqrt{\cosh^2(ac+bcx)}\,\operatorname{sech}(ac+bcx)\right)\operatorname{Subst}\left(\int \frac{(1+x^2)^5}{32x^5}\,dx,x,e^{c(a+bx)}\right)}{bc}\\
&= \frac{\left(\sqrt{\cosh^2(ac+bcx)}\,\operatorname{sech}(ac+bcx)\right)\operatorname{Subst}\left(\int \frac{(1+x^2)^5}{x^5}\,dx,x,e^{c(a+bx)}\right)}{32bc}\\
&= \frac{\left(\sqrt{\cosh^2(ac+bcx)}\,\operatorname{sech}(ac+bcx)\right)\operatorname{Subst}\left(\int \frac{(1+x)^5}{x^3}\,dx,x,e^{2c(a+bx)}\right)}{64bc}\\
&= \frac{\left(\sqrt{\cosh^2(ac+bcx)}\,\operatorname{sech}(ac+bcx)\right)\operatorname{Subst}\left(\int \left(10+\frac{1}{x^3}+\frac{5}{x^2}+\frac{10}{x}+5x+\right.\right.}{64bc}\\
&= -\frac{e^{-4c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,\operatorname{sech}(ac+bcx)}{128bc}-\frac{5e^{-2c(a+bx)}\sqrt{\cosh^2(ac+b}}{64bc}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 106, normalized size = 0.42

$$\frac{\left(-\frac{1}{2}e^{-4c(a+bx)}-5e^{-2c(a+bx)}+10e^{2c(a+bx)}+\frac{5}{2}e^{4c(a+bx)}+\frac{1}{3}e^{6c(a+bx)}+20bcx\right)\cosh^2(c(a+bx))^{5/2}\operatorname{sech}^5(c(a+bx))}{64bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*(Cosh[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] ((-1/2*1/E^(4*c*(a + b*x)) - 5/E^(2*c*(a + b*x)) + 10*E^(2*c*(a + b*x)) + (
5*E^(4*c*(a + b*x)))/2 + E^(6*c*(a + b*x))/3 + 20*b*c*x)*(Cosh[c*(a + b*x)]
^2)^(5/2)*Sech[c*(a + b*x)]^5)/(64*b*c)
```

Maple [A]
time = 7.30, size = 326, normalized size = 1.30

method	result
risch	$\frac{5x\sqrt{\left(1+e^{2c(bx+a)}\right)^{2} e^{-2c(bx+a)}}\, e^{c(bx+a)}}{16\left(1+e^{2c(bx+a)}\right)}+\frac{\sqrt{\left(1+e^{2c(bx+a)}\right)^{2} e^{-2c(bx+a)}}\, e^{7c(bx+a)}}{192cb\left(1+e^{2c(bx+a)}\right)}+\frac{5\sqrt{\left(1+e^{2c(bx+a)}\right)^{2}}}{128cb(1+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 5/16*x*((1+exp(2*c*(b*x+a)))^2*exp(-2*c*(b*x+a)))^(1/2)/(1+exp(2*c*(b*x+a))
)*exp(c*(b*x+a))+1/192/c/b*((1+exp(2*c*(b*x+a)))^2*exp(-2*c*(b*x+a)))^(1/2)
/(1+exp(2*c*(b*x+a)))*exp(7*c*(b*x+a))+5/128/c/b*((1+exp(2*c*(b*x+a)))^2*ex
p(-2*c*(b*x+a)))^(1/2)/(1+exp(2*c*(b*x+a)))*exp(5*c*(b*x+a))+5/32/c/b*((1+e
xp(2*c*(b*x+a)))^2*exp(-2*c*(b*x+a)))^(1/2)/(1+exp(2*c*(b*x+a)))*exp(3*c*(b
*x+a))-5/64/c/b*((1+exp(2*c*(b*x+a)))^2*exp(-2*c*(b*x+a)))^(1/2)/(1+exp(2*c
*(b*x+a)))*exp(-c*(b*x+a))-1/128/c/b*((1+exp(2*c*(b*x+a)))^2*exp(-2*c*(b*x+
a)))^(1/2)/(1+exp(2*c*(b*x+a)))*exp(-3*c*(b*x+a))
```

Maxima [A]
time = 0.27, size = 112, normalized size = 0.45

$$\frac{5\,(bcx+ac)}{16\,bc}+\frac{e^{(6\,bcx+6\,ac)}}{192\,bc}+\frac{5\,e^{(4\,bcx+4\,ac)}}{128\,bc}+\frac{5\,e^{(2\,bcx+2\,ac)}}{32\,bc}-\frac{5\,e^{(-2\,bcx-2\,ac)}}{64\,bc}-\frac{e^{(-4\,bcx-4\,ac)}}{128\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 5/16*(b*c*x + a*c)/(b*c) + 1/192*e^(6*b*c*x + 6*a*c)/(b*c) + 5/128*e^(4*b*c
*x + 4*a*c)/(b*c) + 5/32*e^(2*b*c*x + 2*a*c)/(b*c) - 5/64*e^(-2*b*c*x - 2*a
*c)/(b*c) - 1/128*e^(-4*b*c*x - 4*a*c)/(b*c)
```

Fricas [A]
time = 0.43, size = 218, normalized size = 0.87

$$-\frac{\cosh(bcx+ac)^{5}+5\,\cosh(bcx+ac)\sinh(bcx+ac)^{4}-5\,\sinh(bcx+ac)^{5}-5\left(10\,\cosh(bcx+ac)^{2}+9\right)\sinh(bcx+ac)^{3}+15\,\cosh(bcx+ac)^{3}+5\left(2\,\cosh(bcx+ac)^{3}+9\,\cosh(bcx+ac)\right)\sinh(bcx+ac)^{2}-60\,(2\,bcx+1)\cosh(bcx+ac)-5\left(5\,\cosh(bcx+ac)^{4}-24\,bcx+27\,\cosh(bcx+ac)^{2}+12\right)\sinh(bcx+ac)}{384\,(bc\cosh(bcx+ac)-bc\sinh(bcx+ac))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/384*(cosh(b*c*x + a*c)^5 + 5*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^4 - 5*s
inh(b*c*x + a*c)^5 - 5*(10*cosh(b*c*x + a*c)^2 + 9)*sinh(b*c*x + a*c)^3 + 1
5*cosh(b*c*x + a*c)^3 + 5*(2*cosh(b*c*x + a*c)^3 + 9*cosh(b*c*x + a*c))*sin
h(b*c*x + a*c)^2 - 60*(2*b*c*x + 1)*cosh(b*c*x + a*c) - 5*(5*cosh(b*c*x + a
*c)^4 - 24*b*c*x + 27*cosh(b*c*x + a*c)^2 + 12)*sinh(b*c*x + a*c))/(b*c*cos
h(b*c*x + a*c) - b*c*sinh(b*c*x + a*c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.39, size = 101, normalized size = 0.40

$$\frac{120\,bcx-3\left(30\,e^{(4\,bcx+4\,ac)}+10\,e^{(2\,bcx+2\,ac)}+1\right)e^{(-4\,bcx-4\,ac)}+\left(2\,e^{(6\,bcx+18\,ac)}+15\,e^{(4\,bcx+16\,ac)}+60\,e^{(2\,bcx+14\,ac)}\right)e^{(-12\,ac)}}{384\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/384*(120*b*c*x - 3*(30*e^(4*b*c*x + 4*a*c) + 10*e^(2*b*c*x + 2*a*c) + 1)*
e^(-4*b*c*x - 4*a*c) + (2*e^(6*b*c*x + 18*a*c) + 15*e^(4*b*c*x + 16*a*c) +
60*e^(2*b*c*x + 14*a*c))*e^(-12*a*c))/(b*c)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{e}^{c\,(a+b\,x)}\,{\left({\mathrm{cosh}\left(a\,c+b\,c\,x\right)}^2\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*(cosh(a*c + b*c*x)^2)^(5/2),x)
```

```
[Out] int(exp(c*(a + b*x))*(cosh(a*c + b*c*x)^2)^(5/2), x)
```

3.293 $\int e^{c(a+bx)} \cosh^2(ac+bcx)^{3/2} \, dx$

Optimal. Leaf size=162

$$-\frac{e^{-2c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,\mathrm{sech}(ac+bcx)}{16bc}+\frac{3e^{2c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,\mathrm{sech}(ac+bcx)}{16bc}+\frac{e^{4c(a+bx)}\sqrt{\cosh$$

```
[Out] -1/16*sech(b*c*x+a*c)*(cosh(b*c*x+a*c)^2)^(1/2)/b/c/exp(2*c*(b*x+a))+3/16*e
xp(2*c*(b*x+a))*sech(b*c*x+a*c)*(cosh(b*c*x+a*c)^2)^(1/2)/b/c+1/32*exp(4*c*
(b*x+a))*sech(b*c*x+a*c)*(cosh(b*c*x+a*c)^2)^(1/2)/b/c+3/8*x*sech(b*c*x+a*c
)*(cosh(b*c*x+a*c)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6852, 2320, 12, 272, 45}

$$-\frac{e^{-2c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,\mathrm{sech}(ac+bcx)}{16bc}+\frac{3e^{2c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,\mathrm{sech}(ac+bcx)}{16bc}+\frac{e^{4c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,\mathrm{sech}(ac+bcx)}{32bc}+\frac{3}{8}x\sqrt{\cosh^2(ac+bcx)}\,\mathrm{sech}(ac+bcx)$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*(Cosh[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] -1/16*(Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/(b*c*E^(2*c*(a + b*x)))
 + (3*E^(2*c*(a + b*x))*Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/(16*b*
c) + (E^(4*c*(a + b*x))*Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/(32*b*
c) + (3*x*Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int e^{c(a+bx)}\cosh^2(ac+bcx)^{3/2}\,dx &= \left(\sqrt{\cosh^2(ac+bcx)}\,\operatorname{sech}(ac+bcx)\right)\int e^{c(a+bx)}\cosh^3(ac+bcx)\,dx\\
&= \frac{\left(\sqrt{\cosh^2(ac+bcx)}\,\operatorname{sech}(ac+bcx)\right)\operatorname{Subst}\left(\int \frac{(1+x^2)^3}{8x^3}\,dx, x, e^{c(a+bx)}\right)}{bc}\\
&= \frac{\left(\sqrt{\cosh^2(ac+bcx)}\,\operatorname{sech}(ac+bcx)\right)\operatorname{Subst}\left(\int \frac{(1+x^2)^3}{x^3}\,dx, x, e^{c(a+bx)}\right)}{8bc}\\
&= \frac{\left(\sqrt{\cosh^2(ac+bcx)}\,\operatorname{sech}(ac+bcx)\right)\operatorname{Subst}\left(\int \frac{(1+x)^3}{x^2}\,dx, x, e^{2c(a+bx)}\right)}{16bc}\\
&= \frac{\left(\sqrt{\cosh^2(ac+bcx)}\,\operatorname{sech}(ac+bcx)\right)\operatorname{Subst}\left(\int \left(3+\frac{1}{x^2}+\frac{3}{x}+x\right)\,dx, x, e^{2c}\right.}{16bc}\\
&= -\frac{e^{-2c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,\operatorname{sech}(ac+bcx)}{16bc}+\frac{3e^{2c(a+bx)}\sqrt{\cosh^2(ac+bc}}{16bc}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 78, normalized size = 0.48

$$\frac{\left(-e^{-2c(a+bx)}+3e^{2c(a+bx)}+\frac{1}{2}e^{4c(a+bx)}+6bcx\right)\cosh^2(c(a+bx))^{3/2}\operatorname{sech}^3(c(a+bx))}{16bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*(Cosh[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] ((-E^(-2*c*(a + b*x)) + 3*E^(2*c*(a + b*x)) + E^(4*c*(a + b*x))/2 + 6*b*c*x
)*(Cosh[c*(a + b*x)]^2)^(3/2)*Sech[c*(a + b*x)]^3)/(16*b*c)
```

Maple [A]
time = 6.76, size = 216, normalized size = 1.33

method	result
risch	$\frac{3x\sqrt{\left(1+{\mathrm e}^{2c(bx+a)}\right)^{2}{\mathrm e}^{-2c(bx+a)}}\,{\mathrm e}^{c(bx+a)}}{8\left(1+{\mathrm e}^{2c(bx+a)}\right)}+\frac{\sqrt{\left(1+{\mathrm e}^{2c(bx+a)}\right)^{2}{\mathrm e}^{-2c(bx+a)}}\,{\mathrm e}^{5c(bx+a)}}{32cb\left(1+{\mathrm e}^{2c(bx+a)}\right)}+\frac{3\sqrt{\left(1+{\mathrm e}^{2c(bx+a)}\right)^{2}}}{16cb(1+\mathrm{e}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*x*((1+exp(2*c*(b*x+a)))^2*exp(-2*c*(b*x+a)))^(1/2)/(1+exp(2*c*(b*x+a)))
*exp(c*(b*x+a))+1/32/c/b*((1+exp(2*c*(b*x+a)))^2*exp(-2*c*(b*x+a)))^(1/2)/(
1+exp(2*c*(b*x+a)))*exp(5*c*(b*x+a))+3/16/c/b*((1+exp(2*c*(b*x+a)))^2*exp(-
2*c*(b*x+a)))^(1/2)/(1+exp(2*c*(b*x+a)))*exp(3*c*(b*x+a))-1/16/c/b*((1+exp(
2*c*(b*x+a)))^2*exp(-2*c*(b*x+a)))^(1/2)/(1+exp(2*c*(b*x+a)))*exp(-c*(b*x+a
))
```

Maxima [A]
time = 0.27, size = 74, normalized size = 0.46

$$\frac{3\,(bcx+ac)}{8\,bc}+\frac{e^{(4\,bcx+4\,ac)}}{32\,bc}+\frac{3\,e^{(2\,bcx+2\,ac)}}{16\,bc}-\frac{e^{(-2\,bcx-2\,ac)}}{16\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/8*(b*c*x + a*c)/(b*c) + 1/32*e^(4*b*c*x + 4*a*c)/(b*c) + 3/16*e^(2*b*c*x
+ 2*a*c)/(b*c) - 1/16*e^(-2*b*c*x - 2*a*c)/(b*c)
```

Fricas [A]
time = 0.40, size = 126, normalized size = 0.78

$$-\frac{\cosh\left(bcx+ac\right)^{3}+3\,\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^{2}-3\,\sinh\left(bcx+ac\right)^{3}-6\,(2\,bcx+1)\cosh\left(bcx+ac\right)+3\,\left(4\,bcx-3\,\cosh\left(bcx+ac\right)^{2}-2\right)\sinh\left(bcx+ac\right)}{32\,(bc\cosh\left(bcx+ac\right)-bc\sinh\left(bcx+ac\right))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/32*(cosh(b*c*x + a*c)^3 + 3*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 - 3*si
nh(b*c*x + a*c)^3 - 6*(2*b*c*x + 1)*cosh(b*c*x + a*c) + 3*(4*b*c*x - 3*cosh
(b*c*x + a*c)^2 - 2)*sinh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c) - b*c*sinh(b
*c*x + a*c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)**2)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [A]
time = 0.40, size = 73, normalized size = 0.45

$$\frac{12\,bcx-2\,\left(3\,e^{(2\,bcx+2\,ac)}+1\right)e^{(-2\,bcx-2\,ac)}+\left(e^{(4\,bcx+8\,ac)}+6\,e^{(2\,bcx+6\,ac)}\right)e^{(-4\,ac)}}{32\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/32*(12*b*c*x - 2*(3*e^(2*b*c*x + 2*a*c) + 1)*e^(-2*b*c*x - 2*a*c) + (e^(4
*b*c*x + 8*a*c) + 6*e^(2*b*c*x + 6*a*c))*e^(-4*a*c))/(b*c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{c\,(a+b\,x)}\,\left({\cosh\left(a\,c+b\,c\,x\right)}^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*(cosh(a*c + b*c*x)^2)^(3/2),x)
```

```
[Out] int(exp(c*(a + b*x))*(cosh(a*c + b*c*x)^2)^(3/2), x)
```

3.294 $\int e^{c(a+bx)} \sqrt{\cosh^2(ac+bcx)}\, dx$

Optimal. Leaf size=74

$$\frac{e^{2c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,\mathrm{sech}(ac+bcx)}{4bc}+\frac{1}{2}x\sqrt{\cosh^2(ac+bcx)}\,\mathrm{sech}(ac+bcx)$$

```
[Out] 1/4*exp(2*c*(b*x+a))*sech(b*c*x+a*c)*(cosh(b*c*x+a*c)^2)^(1/2)/b/c+1/2*x*se
ch(b*c*x+a*c)*(cosh(b*c*x+a*c)^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6852, 2320, 12, 14}

$$\frac{e^{2c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,\mathrm{sech}(ac+bcx)}{4bc}+\frac{1}{2}x\sqrt{\cosh^2(ac+bcx)}\,\mathrm{sech}(ac+bcx)$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*Sqrt[Cosh[a*c + b*c*x]^2],x]
```

```
[Out] (E^(2*c*(a + b*x))*Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/(4*b*c) + (
x*Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
```

```
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int e^{c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,dx &= \left(\sqrt{\cosh^2(ac+bcx)}\,\operatorname{sech}(ac+bcx)\right)\int e^{c(a+bx)}\cosh(ac+bcx)\,dx\\ &= \frac{\left(\sqrt{\cosh^2(ac+bcx)}\,\operatorname{sech}(ac+bcx)\right)\operatorname{Subst}\left(\int \frac{1+x^2}{2x}\,dx,x,e^{c(a+bx)}\right)}{bc}\\ &= \frac{\left(\sqrt{\cosh^2(ac+bcx)}\,\operatorname{sech}(ac+bcx)\right)\operatorname{Subst}\left(\int \frac{1+x^2}{x}\,dx,x,e^{c(a+bx)}\right)}{2bc}\\ &= \frac{\left(\sqrt{\cosh^2(ac+bcx)}\,\operatorname{sech}(ac+bcx)\right)\operatorname{Subst}\left(\int \left(\frac{1}{x}+x\right)\,dx,x,e^{c(a+bx)}\right)}{2bc}\\ &= \frac{e^{2c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,\operatorname{sech}(ac+bcx)}{4bc}+\frac{1}{2}x\sqrt{\cosh^2(ac+bcx)}\,\operatorname{sech}(a\end{aligned}$$

Mathematica [A]
time = 0.03, size = 48, normalized size = 0.65

$$\frac{\left(e^{2c(a+bx)}+2bcx\right)\sqrt{\cosh^2(c(a+bx))}\,\operatorname{sech}(c(a+bx))}{4bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*Sqrt[Cosh[a*c + b*c*x]^2],x]
```

```
[Out] ((E^(2*c*(a + b*x)) + 2*b*c*x)*Sqrt[Cosh[c*(a + b*x)]^2]*Sech[c*(a + b*x)])
/(4*b*c)
```

Maple [A]
time = 6.81, size = 106, normalized size = 1.43

method	result	size
risch	$\frac{x\sqrt{\left(1+e^{2c(bx+a)}\right)^2 e^{-2c(bx+a)}}\; e^{c(bx+a)}}{2+2\,e^{2c(bx+a)}}+\frac{\sqrt{\left(1+e^{2c(bx+a)}\right)^2 e^{-2c(bx+a)}}\; e^{3c(bx+a)}}{4cb\left(1+e^{2c(bx+a)}\right)}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*((1+exp(2*c*(b*x+a)))^2*exp(-2*c*(b*x+a)))^(1/2)/(1+exp(2*c*(b*x+a)))
*exp(c*(b*x+a))+1/4/c/b*((1+exp(2*c*(b*x+a)))^2*exp(-2*c*(b*x+a)))^(1/2)/(1
+exp(2*c*(b*x+a)))*exp(3*c*(b*x+a))
```

Maxima [A]
time = 0.27, size = 29, normalized size = 0.39

$$\frac{1}{2}\,x+\frac{a}{2\,b}+\frac{e^{(2\,bcx+2\,ac)}}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/2*a/b + 1/4*e^(2*b*c*x + 2*a*c)/(b*c)
```

Fricas [A]
time = 0.36, size = 66, normalized size = 0.89

$$\frac{(2\,bcx+1)\cosh\left(bcx+ac\right)-(2\,bcx-1)\sinh\left(bcx+ac\right)}{4\,(bc\cosh\left(bcx+ac\right)-bc\sinh\left(bcx+ac\right))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*((2*b*c*x + 1)*cosh(b*c*x + a*c) - (2*b*c*x - 1)*sinh(b*c*x + a*c))/(b*
c*cosh(b*c*x + a*c) - b*c*sinh(b*c*x + a*c))
```

Sympy [C] Result contains complex when optimal does not.
time = 2.49, size = 235, normalized size = 3.18

$$\begin{cases} x & \text{for}\: c = 0 \\ \frac{i\sqrt{\cosh^{2}\left(bcx+\log\left(-ie^{-bcx}\right)\right)}\log\left(-ie^{-bcx}\right)}{bc} & \text{for}\: a = \frac{\log\left(-ie^{-bcx}\right)}{c} \\ ix\sqrt{\cosh^{2}\left(bcx+\log\left(ie^{-bcx}\right)\right)} & \text{for}\: a = \frac{\log\left(ie^{-bcx}\right)}{c} \\ x\sqrt{\cosh^{2}\left(ac\right)}e^{ac} & \text{for}\: b = 0 \\ -\frac{x\sqrt{\cosh^{2}\left(ac+bcx\right)}e^{ac}e^{bcx}\sinh\left(ac+bcx\right)}{2\cosh\left(ac+bcx\right)}+\frac{x\sqrt{\cosh^{2}\left(ac+bcx\right)}e^{ac}e^{bcx}}{2}+\frac{\sqrt{\cosh^{2}\left(ac+bcx\right)}e^{ac}e^{bcx}\sinh\left(ac+bcx\right)}{2bc\cosh\left(ac+bcx\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)**2)**(1/2),x)
```

```
[Out] Piecewise((x, Eq(c, 0)), (I*sqrt(cosh(b*c*x + log(-I*exp(-b*c*x)))**2)*log(
-I*exp(-b*c*x))/(b*c), Eq(a, log(-I*exp(-b*c*x))/c)), (I*x*sqrt(cosh(b*c*x
+ log(I*exp(-b*c*x)))**2), Eq(a, log(I*exp(-b*c*x))/c)), (x*sqrt(cosh(a*c)*
*2)*exp(a*c), Eq(b, 0)), (-x*sqrt(cosh(a*c + b*c*x)**2)*exp(a*c)*exp(b*c*x)
*sinh(a*c + b*c*x)/(2*cosh(a*c + b*c*x)) + x*sqrt(cosh(a*c + b*c*x)**2)*exp
```

```
(a*c)*exp(b*c*x)/2 + sqrt(cosh(a*c + b*c*x)**2)*exp(a*c)*exp(b*c*x)*sinh(a*
c + b*c*x)/(2*b*c*cosh(a*c + b*c*x)), True))
```

Giac [A]
time = 0.40, size = 23, normalized size = 0.31

$$\frac{1}{2}\,x + \frac{e^{\left(2\,bcx + 2\,ac\right)}}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*x + 1/4*e^(2*b*c*x + 2*a*c)/(b*c)
```

Mupad [B]
time = 0.12, size = 76, normalized size = 1.03

$$\frac{\left(x\,{\mathrm{e}}^{a\,c+b\,c\,x}+\frac{{\mathrm{e}}^{3\,a\,c+3\,b\,c\,x}}{2\,b\,c}\right)\,\sqrt{{\left(\frac{{\mathrm{e}}^{a\,c+b\,c\,x}}{2}+\frac{{\mathrm{e}}^{-a\,c-b\,c\,x}}{2}\right)}^2}}{{\mathrm{e}}^{2\,a\,c+2\,b\,c\,x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*(cosh(a*c + b*c*x)^2)^(1/2),x)
```

```
[Out] ((x*exp(a*c + b*c*x) + exp(3*a*c + 3*b*c*x)/(2*b*c))*((exp(a*c + b*c*x)/2 +
 exp(- a*c - b*c*x)/2)^2)^(1/2))/(exp(2*a*c + 2*b*c*x) + 1)
```

3.295 $\int \frac{e^{c(a+bx)}}{\sqrt{\cosh^2(ac+bcx)}}\,dx$

Optimal. Leaf size=44

$$\frac{\cosh(ac+bcx)\log\left(1+e^{2c(a+bx)}\right)}{bc\sqrt{\cosh^2(ac+bcx)}}$$

```
[Out] cosh(b*c*x+a*c)*ln(1+exp(2*c*(b*x+a)))/b/c/(cosh(b*c*x+a*c)^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6852, 2320, 12, 266}

$$\frac{\log\left(e^{2c(a+bx)}+1\right)\cosh(ac+bcx)}{bc\sqrt{\cosh^2(ac+bcx)}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/Sqrt[Cosh[a*c + b*c*x]^2],x]
```

```
[Out] (Cosh[a*c + b*c*x]*Log[1 + E^(2*c*(a + b*x))])/(b*c*Sqrt[Cosh[a*c + b*c*x]^
2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
```

```
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{c(a+bx)}}{\sqrt{\cosh^2(ac+bcx)}}\,dx &= \frac{\cosh(ac+bcx)\int e^{c(a+bx)}\operatorname{sech}(ac+bcx)\,dx}{\sqrt{\cosh^2(ac+bcx)}}\\
&= \frac{\cosh(ac+bcx)\operatorname{Subst}\left(\int \frac{2x}{1+x^2}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\cosh^2(ac+bcx)}}\\
&= \frac{(2\cosh(ac+bcx))\operatorname{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\cosh^2(ac+bcx)}}\\
&= \frac{\cosh(ac+bcx)\log\left(1+e^{2c(a+bx)}\right)}{bc\sqrt{\cosh^2(ac+bcx)}}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 42, normalized size = 0.95

$$\frac{\cosh(c(a+bx))\log\left(1+e^{2c(a+bx)}\right)}{bc\sqrt{\cosh^2(c(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/Sqrt[Cosh[a*c + b*c*x]^2],x]
[Out] (Cosh[c*(a + b*x)]*Log[1 + E^(2*c*(a + b*x))])/(b*c*Sqrt[Cosh[c*(a + b*x)]^
2])
```

Maple [A]
time = 6.47, size = 66, normalized size = 1.50

method	result	size
risch	$\frac{\ln\left(e^{2bcx}+e^{-2ac}\right)\left(1+e^{2c(bx+a)}\right)e^{-c(bx+a)}}{cb\sqrt{\left(1+e^{2c(bx+a)}\right)^2 e^{-2c(bx+a)}}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ln(exp(2*b*c*x)+exp(-2*a*c))/c/b*(1+exp(2*c*(b*x+a)))/((1+exp(2*c*(b*x+a)))
^2*exp(-2*c*(b*x+a)))^(1/2)*exp(-c*(b*x+a))
```

Maxima [A]
time = 0.49, size = 21, normalized size = 0.48

$$\frac{\log\left(e^{(2\,bcx+2\,ac)}+1\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(e^(2*b*c*x + 2*a*c) + 1)/(b*c)
```

Fricas [A]
time = 0.41, size = 42, normalized size = 0.95

$$\frac{\log\left(\frac{2\,\cosh\left(bcx+ac\right)}{\cosh\left(bcx+ac\right)-\sinh\left(bcx+ac\right)}\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] log(2*cosh(b*c*x + a*c)/(cosh(b*c*x + a*c) - sinh(b*c*x + a*c)))/(b*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int\frac{e^{bcx}}{\sqrt{\cosh^2\left(ac+bcx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)**2)**(1/2),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)/sqrt(cosh(a*c + b*c*x)**2), x)
```

Giac [A]
time = 0.39, size = 20, normalized size = 0.45

$$\frac{\log\left(e^{(2\,bcx)}+e^{(-2\,ac)}\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] log(e^(2*b*c*x) + e^(-2*a*c))/(b*c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{e^{c(a+bx)}}{\sqrt{\cosh(ac+bcx)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))/(cosh(a*c + b*c*x)^2)^(1/2),x)
[Out] int(exp(c*(a + b*x))/(cosh(a*c + b*c*x)^2)^(1/2), x)
```

3.296 $\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{3/2}}\,dx$

Optimal. Leaf size=56

$$\frac{2e^{4c(a+bx)}\cosh(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^2\sqrt{\cosh^2(ac+bcx)}}$$

```
[Out] 2*exp(4*c*(b*x+a))*cosh(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^2/(cosh(b*c*x+a
*c)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6852, 2320, 12, 270}

$$\frac{2e^{4c(a+bx)}\cosh(ac+bcx)}{bc\left(e^{2c(a+bx)}+1\right)^2\sqrt{\cosh^2(ac+bcx)}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/(Cosh[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] (2*E^(4*c*(a + b*x))*Cosh[a*c + b*c*x])/(b*c*(1 + E^(2*c*(a + b*x)))^2*Sqrt
[Cosh[a*c + b*c*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{3/2}}\,dx &= \frac{\cosh(ac+bcx)\int e^{c(a+bx)}\text{sech}^3(ac+bcx)\,dx}{\sqrt{\cosh^2(ac+bcx)}} \\
&= \frac{\cosh(ac+bcx)\text{Subst}\left(\int \frac{8x^3}{(1+x^2)^3}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\cosh^2(ac+bcx)}} \\
&= \frac{(8\cosh(ac+bcx))\text{Subst}\left(\int \frac{x^3}{(1+x^2)^3}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\cosh^2(ac+bcx)}} \\
&= \frac{2e^{4c(a+bx)}\cosh(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^2\sqrt{\cosh^2(ac+bcx)}}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 46, normalized size = 0.82

$$\frac{4e^{5c(a+bx)}\sqrt{\cosh^2(c(a+bx))}}{bc\left(1+e^{2c(a+bx)}\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/(Cosh[a*c + b*c*x]^2)^(3/2),x]
[Out] (4*E^(5*c*(a + b*x))*Sqrt[Cosh[c*(a + b*x)]^2])/(b*c*(1 + E^(2*c*(a + b*x))
)^3)
```

Maple [A]
time = 6.51, size = 69, normalized size = 1.23

method	result	size
risch	$-\frac{2\left(2\,\mathrm{e}^{2c(bx+a)}+1\right)\mathrm{e}^{-c(bx+a)}}{cb\sqrt{\left(1+\mathrm{e}^{2c(bx+a)}\right)^2\mathrm{e}^{-2c(bx+a)}}\,\left(1+\mathrm{e}^{2c(bx+a)}\right)}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/c/b*(2*exp(2*c*(b*x+a))+1)/((1+exp(2*c*(b*x+a)))^2*exp(-2*c*(b*x+a)))^(1
/2)/(1+exp(2*c*(b*x+a)))*exp(-c*(b*x+a))
```

Maxima [A]
time = 0.30, size = 84, normalized size = 1.50

$$-\frac{4\,e^{(2\,bcx+2\,ac)}}{bc(e^{(4\,bcx+4\,ac)}+2\,e^{(2\,bcx+2\,ac)}+1)}-\frac{2}{bc(e^{(4\,bcx+4\,ac)}+2\,e^{(2\,bcx+2\,ac)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -4*e^(2*b*c*x + 2*a*c)/(b*c*(e^(4*b*c*x + 4*a*c) + 2*e^(2*b*c*x + 2*a*c) +
1)) - 2/(b*c*(e^(4*b*c*x + 4*a*c) + 2*e^(2*b*c*x + 2*a*c) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(52) = 104$.
time = 0.40, size = 120, normalized size = 2.14

$$-\frac{2\,(3\,\cosh(bcx+ac)+\sinh(bcx+ac))}{bc\cosh(bcx+ac)^3+3\,bc\cosh(bcx+ac)\sinh(bcx+ac)^2+bc\sinh(bcx+ac)^3+3\,bc\cosh(bcx+ac)+\left(3\,bc\cosh(bcx+ac)^2+bc\right)\sinh(bcx+ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(3*cosh(b*c*x + a*c) + sinh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^3 + 3*b
*c*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 + b*c*sinh(b*c*x + a*c)^3 + 3*b*c*
cosh(b*c*x + a*c) + (3*b*c*cosh(b*c*x + a*c)^2 + b*c)*sinh(b*c*x + a*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int\frac{e^{bcx}}{\left(\cosh^2\left(ac+bcx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)**2)**(3/2),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)/(cosh(a*c + b*c*x)**2)**(3/2), x)
```

Giac [A]
time = 0.39, size = 38, normalized size = 0.68

$$-\frac{2\left(2\,e^{(2\,bcx+2\,ac)}+1\right)}{bc(e^{(2\,bcx+2\,ac)}+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(2*e^(2*b*c*x + 2*a*c) + 1)/(b*c*(e^(2*b*c*x + 2*a*c) + 1)^2)
```

Mupad [B]
time = 0.94, size = 76, normalized size = 1.36

$$-\frac{4\,{\mathrm{e}}^{a\,c+b\,c\,x}\,\left(2\,{\mathrm{e}}^{2\,a\,c+2\,b\,c\,x}+1\right)\,\sqrt{{\left(\frac{{\mathrm{e}}^{a\,c+b\,c\,x}}{2}+\frac{{\mathrm{e}}^{-a\,c-b\,c\,x}}{2}\right)}^2}}{b\,c\,{\left({\mathrm{e}}^{2\,a\,c+2\,b\,c\,x}+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))/(cosh(a*c + b*c*x)^2)^(3/2),x)
```

```
[Out] -(4*exp(a*c + b*c*x)*(2*exp(2*a*c + 2*b*c*x) + 1)*((exp(a*c + b*c*x)/2 + ex
p(- a*c - b*c*x)/2)^2)^(1/2))/(b*c*(exp(2*a*c + 2*b*c*x) + 1)^3)
```

3.297 $\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{5/2}}\,dx$

Optimal. Leaf size=141

$$-\frac{4\cosh(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^4\sqrt{\cosh^2(ac+bcx)}}+\frac{32\cosh(ac+bcx)}{3bc\left(1+e^{2c(a+bx)}\right)^3\sqrt{\cosh^2(ac+bcx)}}-\frac{8\cosh(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^2\sqrt{\cosh^2(ac+bcx)}}$$

```
[Out] -4*cosh(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^4/(cosh(b*c*x+a*c)^2)^(1/2)+32/
3*cosh(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^3/(cosh(b*c*x+a*c)^2)^(1/2)-8*co
sh(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^2/(cosh(b*c*x+a*c)^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6852, 2320, 12, 272, 45}

$$-\frac{8\cosh(ac+bcx)}{bc\left(e^{2c(a+bx)}+1\right)^2\sqrt{\cosh^2(ac+bcx)}}+\frac{32\cosh(ac+bcx)}{3bc\left(e^{2c(a+bx)}+1\right)^3\sqrt{\cosh^2(ac+bcx)}}-\frac{4\cosh(ac+bcx)}{bc\left(e^{2c(a+bx)}+1\right)^4\sqrt{\cosh^2(ac+bcx)}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/(Cosh[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] (-4*Cosh[a*c + b*c*x])/(b*c*(1 + E^(2*c*(a + b*x)))^4*Sqrt[Cosh[a*c + b*c*x
]^2]) + (32*Cosh[a*c + b*c*x])/(3*b*c*(1 + E^(2*c*(a + b*x)))^3*Sqrt[Cosh[a
*c + b*c*x]^2]) - (8*Cosh[a*c + b*c*x])/(b*c*(1 + E^(2*c*(a + b*x)))^2*Sqrt
[Cosh[a*c + b*c*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{5/2}}\,dx &= \frac{\cosh(ac+bcx)\int e^{c(a+bx)}\operatorname{sech}^5(ac+bcx)\,dx}{\sqrt{\cosh^2(ac+bcx)}}\\
&= \frac{\cosh(ac+bcx)\operatorname{Subst}\left(\int \frac{32x^5}{(1+x^2)^5}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\cosh^2(ac+bcx)}}\\
&= \frac{(32\cosh(ac+bcx))\operatorname{Subst}\left(\int \frac{x^5}{(1+x^2)^5}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\cosh^2(ac+bcx)}}\\
&= \frac{(16\cosh(ac+bcx))\operatorname{Subst}\left(\int \frac{x^2}{(1+x)^5}\,dx, x, e^{2c(a+bx)}\right)}{bc\sqrt{\cosh^2(ac+bcx)}}\\
&= \frac{(16\cosh(ac+bcx))\operatorname{Subst}\left(\int \left(\frac{1}{(1+x)^5}-\frac{2}{(1+x)^4}+\frac{1}{(1+x)^3}\right)dx, x, e^{2c(a+bx)}\right)}{bc\sqrt{\cosh^2(ac+bcx)}}\\
&= -\frac{4\cosh(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^4\sqrt{\cosh^2(ac+bcx)}}+\frac{32\cosh(ac+bcx)}{3bc\left(1+e^{2c(a+bx)}\right)^3\sqrt{\cosh^2(ac+bcx)}}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 72, normalized size = 0.51

$$-\frac{4\left(1+4e^{2c(a+bx)}+6e^{4c(a+bx)}\right)\cosh(c(a+bx))}{3bc\left(1+e^{2c(a+bx)}\right)^4\sqrt{\cosh^2(c(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/(Cosh[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] (-4*(1 + 4*E^(2*c*(a + b*x)) + 6*E^(4*c*(a + b*x)))*Cosh[c*(a + b*x)])/(3*b
*c*(1 + E^(2*c*(a + b*x)))^4*Sqrt[Cosh[c*(a + b*x)]^2])
```

Maple [A]
time = 6.89, size = 80, normalized size = 0.57

method	result	size
risch	$-\frac{4\left(6\,\mathrm{e}^{4c(bx+a)}+4\,\mathrm{e}^{2c(bx+a)}+1\right)\mathrm{e}^{-c(bx+a)}}{3cb\sqrt{\left(1+\mathrm{e}^{2c(bx+a)}\right)^{2}\mathrm{e}^{-2c(bx+a)}}\,\left(1+\mathrm{e}^{2c(bx+a)}\right)^{3}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/3/c/b*(6*exp(4*c*(b*x+a))+4*exp(2*c*(b*x+a))+1)/((1+exp(2*c*(b*x+a)))^2*
exp(-2*c*(b*x+a)))^(1/2)/(1+exp(2*c*(b*x+a)))^3*exp(-c*(b*x+a))
```

Maxima [A]
time = 0.29, size = 209, normalized size = 1.48

$$-\frac{8\,e^{(4\,bcx+4\,ac)}}{bc\left(e^{(8\,bcx+8\,ac)}+4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}+4\,e^{(2\,bcx+2\,ac)}+1\right)}-\frac{16\,e^{(2\,bcx+2\,ac)}}{3\,bc\left(e^{(8\,bcx+8\,ac)}+4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}+4\,e^{(2\,bcx+2\,ac)}+1\right)}-\frac{4}{3\,bc\left(e^{(8\,bcx+8\,ac)}+4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}+4\,e^{(2\,bcx+2\,ac)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -8*e^(4*b*c*x + 4*a*c)/(b*c*(e^(8*b*c*x + 8*a*c) + 4*e^(6*b*c*x + 6*a*c) +
6*e^(4*b*c*x + 4*a*c) + 4*e^(2*b*c*x + 2*a*c) + 1)) - 16/3*e^(2*b*c*x + 2*a
*c)/(b*c*(e^(8*b*c*x + 8*a*c) + 4*e^(6*b*c*x + 6*a*c) + 6*e^(4*b*c*x + 4*a*
c) + 4*e^(2*b*c*x + 2*a*c) + 1)) - 4/3/(b*c*(e^(8*b*c*x + 8*a*c) + 4*e^(6*b
*c*x + 6*a*c) + 6*e^(4*b*c*x + 4*a*c) + 4*e^(2*b*c*x + 2*a*c) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 315 vs. 2(130) = 260.
time = 0.36, size = 315, normalized size = 2.23

$$-\frac{4\left(7\,\cosh\left(bcx+ac\right)^{2}+10\,\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)+7\,\sinh\left(bcx+ac\right)^{2}+4\right)}{3\left(bc\cosh\left(bcx+ac\right)^{6}+6\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^{5}+bc\sinh\left(bcx+ac\right)^{6}+4\,bc\cosh\left(bcx+ac\right)^{4}+\left(15\,bc\cosh\left(bcx+ac\right)^{2}+4\,bc\right)\sinh\left(bcx+ac\right)^{4}+7\,bc\cosh\left(bcx+ac\right)^{2}+4\left(5\,bc\cosh\left(bcx+ac\right)^{3}+4\,bc\cosh\left(bcx+ac\right)\right)\sinh\left(bcx+ac\right)^{3}+\left(15\,bc\cosh\left(bcx+ac\right)^{4}+24\,bc\cosh\left(bcx+ac\right)^{2}+7\,bc\right)\sinh\left(bcx+ac\right)^{2}+4\,bc+2\left(3\,bc\cosh\left(bcx+ac\right)^{5}+8\,bc\cosh\left(bcx+ac\right)^{3}+5\,bc\cosh\left(bcx+ac\right)\right)\sinh\left(bcx+ac\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -4/3*(7*cosh(b*c*x + a*c)^2 + 10*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + 7*si
nh(b*c*x + a*c)^2 + 4)/(b*c*cosh(b*c*x + a*c)^6 + 6*b*c*cosh(b*c*x + a*c)*s
inh(b*c*x + a*c)^5 + b*c*sinh(b*c*x + a*c)^6 + 4*b*c*cosh(b*c*x + a*c)^4 +
```

```
(15*b*c*cosh(b*c*x + a*c)^2 + 4*b*c)*sinh(b*c*x + a*c)^4 + 7*b*c*cosh(b*c*x
 + a*c)^2 + 4*(5*b*c*cosh(b*c*x + a*c)^3 + 4*b*c*cosh(b*c*x + a*c))*sinh(b*
c*x + a*c)^3 + (15*b*c*cosh(b*c*x + a*c)^4 + 24*b*c*cosh(b*c*x + a*c)^2 + 7
*b*c)*sinh(b*c*x + a*c)^2 + 4*b*c + 2*(3*b*c*cosh(b*c*x + a*c)^5 + 8*b*c*co
sh(b*c*x + a*c)^3 + 5*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)**2)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 0.40, size = 51, normalized size = 0.36

$$-\frac{4\left(6\,e^{(4\,bcx+4\,ac)}+4\,e^{(2\,bcx+2\,ac)}+1\right)}{3\,bc(e^{(2\,bcx+2\,ac)}+1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -4/3*(6*e^(4*b*c*x + 4*a*c) + 4*e^(2*b*c*x + 2*a*c) + 1)/(b*c*(e^(2*b*c*x +
 2*a*c) + 1)^4)
```

Mupad [B]
time = 0.10, size = 89, normalized size = 0.63

$$-\frac{8\,\mathrm{e}^{a\,c+b\,c\,x}\,\sqrt{{\left(\frac{\mathrm{e}^{a\,c+b\,c\,x}}{2}+\frac{\mathrm{e}^{-a\,c-b\,c\,x}}{2}\right)}^2}\,\left(4\,\mathrm{e}^{2\,a\,c+2\,b\,c\,x}+6\,\mathrm{e}^{4\,a\,c+4\,b\,c\,x}+1\right)}{3\,b\,c\,{\left(\mathrm{e}^{2\,a\,c+2\,b\,c\,x}+1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))/(cosh(a*c + b*c*x)^2)^(5/2),x)
```

```
[Out] -(8*exp(a*c + b*c*x)*((exp(a*c + b*c*x)/2 + exp(- a*c - b*c*x)/2)^2)^(1/2)*
(4*exp(2*a*c + 2*b*c*x) + 6*exp(4*a*c + 4*b*c*x) + 1))/(3*b*c*(exp(2*a*c +
2*b*c*x) + 1)^5)
```

3.298 $\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{7/2}}\,dx$

Optimal. Leaf size=191

$$\frac{32\cosh(ac+bcx)}{3bc\left(1+e^{2c(a+bx)}\right)^6\sqrt{\cosh^2(ac+bcx)}}-\frac{192\cosh(ac+bcx)}{5bc\left(1+e^{2c(a+bx)}\right)^5\sqrt{\cosh^2(ac+bcx)}}+\frac{48\cosh(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^4\sqrt{\cosh^2(ac+bcx)}}$$

```
[Out] 32/3*cosh(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^6/(cosh(b*c*x+a*c)^2)^(1/2)-1
92/5*cosh(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^5/(cosh(b*c*x+a*c)^2)^(1/2)+4
8*cosh(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^4/(cosh(b*c*x+a*c)^2)^(1/2)-64/3
*cosh(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^3/(cosh(b*c*x+a*c)^2)^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6852, 2320, 12, 272, 45}

$$-\frac{64\cosh(ac+bcx)}{3bc\left(e^{2c(a+bx)}+1\right)^3\sqrt{\cosh^2(ac+bcx)}}+\frac{48\cosh(ac+bcx)}{bc\left(e^{2c(a+bx)}+1\right)^4\sqrt{\cosh^2(ac+bcx)}}-\frac{192\cosh(ac+bcx)}{5bc\left(e^{2c(a+bx)}+1\right)^5\sqrt{\cosh^2(ac+bcx)}}+\frac{32\cosh(ac+bcx)}{3bc\left(e^{2c(a+bx)}+1\right)^6\sqrt{\cosh^2(ac+bcx)}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/(Cosh[a*c + b*c*x]^2)^(7/2),x]
```

```
[Out] (32*Cosh[a*c + b*c*x])/(3*b*c*(1 + E^(2*c*(a + b*x)))^6*Sqrt[Cosh[a*c + b*c
*x]^2]) - (192*Cosh[a*c + b*c*x])/(5*b*c*(1 + E^(2*c*(a + b*x)))^5*Sqrt[Cos
h[a*c + b*c*x]^2]) + (48*Cosh[a*c + b*c*x])/(b*c*(1 + E^(2*c*(a + b*x)))^4*
Sqrt[Cosh[a*c + b*c*x]^2]) - (64*Cosh[a*c + b*c*x])/(3*b*c*(1 + E^(2*c*(a +
 b*x)))^3*Sqrt[Cosh[a*c + b*c*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{7/2}}\,dx &= \frac{\cosh(ac+bcx)\int e^{c(a+bx)}\operatorname{sech}^7(ac+bcx)\,dx}{\sqrt{\cosh^2(ac+bcx)}}\\
&= \frac{\cosh(ac+bcx)\operatorname{Subst}\left(\int \frac{128x^7}{(1+x^2)^7}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\cosh^2(ac+bcx)}}\\
&= \frac{(128\cosh(ac+bcx))\operatorname{Subst}\left(\int \frac{x^7}{(1+x^2)^7}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\cosh^2(ac+bcx)}}\\
&= \frac{(64\cosh(ac+bcx))\operatorname{Subst}\left(\int \frac{x^3}{(1+x)^7}\,dx, x, e^{2c(a+bx)}\right)}{bc\sqrt{\cosh^2(ac+bcx)}}\\
&= \frac{(64\cosh(ac+bcx))\operatorname{Subst}\left(\int \left(-\frac{1}{(1+x)^7}+\frac{3}{(1+x)^6}-\frac{3}{(1+x)^5}+\frac{1}{(1+x)^4}\right)dx, x, e^{2c(a+bx)}\right)}{bc\sqrt{\cosh^2(ac+bcx)}}\\
&= \frac{32\cosh(ac+bcx)}{3bc\left(1+e^{2c(a+bx)}\right)^6\sqrt{\cosh^2(ac+bcx)}} - \frac{192\cosh(ac+bcx)}{5bc\left(1+e^{2c(a+bx)}\right)^5\sqrt{\cosh^2(ac+bcx)}}
\end{aligned}
$$

Mathematica [A]

time = 0.06, size = 84, normalized size = 0.44

$$-\frac{16\left(1+6e^{2c(a+bx)}+15e^{4c(a+bx)}+20e^{6c(a+bx)}\right)\cosh(c(a+bx))}{15bc\left(1+e^{2c(a+bx)}\right)^{6}\sqrt{\cosh^{2}(c(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/(Cosh[a*c + b*c*x]^2)^(7/2),x]
```

```
[Out] (-16*(1 + 6*E^(2*c*(a + b*x)) + 15*E^(4*c*(a + b*x)) + 20*E^(6*c*(a + b*x))
)*Cosh[c*(a + b*x)])/(15*b*c*(1 + E^(2*c*(a + b*x)))^6*Sqrt[Cosh[c*(a + b*x
)]^2])
```

Maple [A]
time = 7.02, size = 91, normalized size = 0.48

method	result	size
risch	$-\frac{16\left(20\,\mathrm{e}^{6c(bx+a)}+15\,\mathrm{e}^{4c(bx+a)}+6\,\mathrm{e}^{2c(bx+a)}+1\right)\mathrm{e}^{-c(bx+a)}}{15cb\sqrt{\left(1+\mathrm{e}^{2c(bx+a)}\right)^{2}\mathrm{e}^{-2c(bx+a)}}\,\left(1+\mathrm{e}^{2c(bx+a)}\right)^{5}}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -16/15/c/b*(20*exp(6*c*(b*x+a))+15*exp(4*c*(b*x+a))+6*exp(2*c*(b*x+a))+1)/(
(1+exp(2*c*(b*x+a)))^2*exp(-2*c*(b*x+a)))^(1/2)/(1+exp(2*c*(b*x+a)))^5*exp(
-c*(b*x+a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 386 vs. 2(173) = 346.
time = 0.29, size = 386, normalized size = 2.02

$$-\frac{64\,e^{(6\,bcx+6\,ac)}}{3\,bc(e^{(12\,bcx+12\,ac)}+6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}+20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}+1)}-\frac{16\,e^{(4\,bcx+4\,ac)}}{bc(e^{(12\,bcx+12\,ac)}+6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}+20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}+1)}-\frac{32\,e^{(2\,bcx+2\,ac)}}{5\,bc(e^{(12\,bcx+12\,ac)}+6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}+20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}+1)}-\frac{16}{15\,bc(e^{(12\,bcx+12\,ac)}+6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}+20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -64/3*e^(6*b*c*x + 6*a*c)/(b*c*(e^(12*b*c*x + 12*a*c) + 6*e^(10*b*c*x + 10*
a*c) + 15*e^(8*b*c*x + 8*a*c) + 20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*
a*c) + 6*e^(2*b*c*x + 2*a*c) + 1)) - 16*e^(4*b*c*x + 4*a*c)/(b*c*(e^(12*b*c
*x + 12*a*c) + 6*e^(10*b*c*x + 10*a*c) + 15*e^(8*b*c*x + 8*a*c) + 20*e^(6*b
*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*a*c) + 1)) - 32/5
*e^(2*b*c*x + 2*a*c)/(b*c*(e^(12*b*c*x + 12*a*c) + 6*e^(10*b*c*x + 10*a*c)
+ 15*e^(8*b*c*x + 8*a*c) + 20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*a*c)
+ 6*e^(2*b*c*x + 2*a*c) + 1)) - 16/15/(b*c*(e^(12*b*c*x + 12*a*c) + 6*e^(10
```

```
*b*c*x + 10*a*c) + 15*e^(8*b*c*x + 8*a*c) + 20*e^(6*b*c*x + 6*a*c) + 15*e^(
4*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*a*c) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 589 vs. $2(173) = 346$.
time = 0.35, size = 589, normalized size = 3.08

$$-\frac{16\left(21\cosh\left(bcx+ac\right)^{3}+63\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^{2}+19\sinh\left(bcx+ac\right)^{3}+3\left(19\cosh\left(bcx+ac\right)^{2}+3\right)\sinh\left(bcx+ac\right)+21\cosh\left(bcx+ac\right)\right)}{15\left(bc\cosh\left(bcx+ac\right)^{9}+9bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^{8}+bc\sinh\left(bcx+ac\right)^{9}+6bc\cosh\left(bcx+ac\right)^{7}+6\left(6bc\cosh\left(bcx+ac\right)^{2}+bc\right)\sinh\left(bcx+ac\right)^{7}+15bc\cosh\left(bcx+ac\right)^{5}+42\left(2bc\cosh\left(bcx+ac\right)^{3}+bc\cosh\left(bcx+ac\right)\right)\sinh\left(bcx+ac\right)^{6}+3\left(42bc\cosh\left(bcx+ac\right)^{4}+42bc\cosh\left(bcx+ac\right)^{2}+5bc\right)\sinh\left(bcx+ac\right)^{5}+21bc\cosh\left(bcx+ac\right)^{3}+3\left(42bc\cosh\left(bcx+ac\right)^{5}+70bc\cosh\left(bcx+ac\right)^{3}+25bc\cosh\left(bcx+ac\right)\right)\sinh\left(bcx+ac\right)^{4}+\left(84bc\cosh\left(bcx+ac\right)^{6}+210bc\cosh\left(bcx+ac\right)^{4}+150bc\cosh\left(bcx+ac\right)^{2}+19bc\right)\sinh\left(bcx+ac\right)^{3}+21bc\cosh\left(bcx+ac\right)+3\left(12bc\cosh\left(bcx+ac\right)^{7}+42bc\cosh\left(bcx+ac\right)^{5}+50bc\cosh\left(bcx+ac\right)^{3}+21bc\cosh\left(bcx+ac\right)\right)\sinh\left(bcx+ac\right)^{2}+3\left(3bc\cosh\left(bcx+ac\right)^{8}+14bc\cosh\left(bcx+ac\right)^{6}+25bc\cosh\left(bcx+ac\right)^{4}+19bc\cosh\left(bcx+ac\right)^{2}+3bc\right)\sinh\left(bcx+ac\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -16/15*(21*cosh(b*c*x + a*c)^3 + 63*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 +
 19*sinh(b*c*x + a*c)^3 + 3*(19*cosh(b*c*x + a*c)^2 + 3)*sinh(b*c*x + a*c)
+ 21*cosh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^9 + 9*b*c*cosh(b*c*x + a*c)*
sinh(b*c*x + a*c)^8 + b*c*sinh(b*c*x + a*c)^9 + 6*b*c*cosh(b*c*x + a*c)^7 +
 6*(6*b*c*cosh(b*c*x + a*c)^2 + b*c)*sinh(b*c*x + a*c)^7 + 15*b*c*cosh(b*c*
x + a*c)^5 + 42*(2*b*c*cosh(b*c*x + a*c)^3 + b*c*cosh(b*c*x + a*c))*sinh(b*
c*x + a*c)^6 + 3*(42*b*c*cosh(b*c*x + a*c)^4 + 42*b*c*cosh(b*c*x + a*c)^2 +
 5*b*c)*sinh(b*c*x + a*c)^5 + 21*b*c*cosh(b*c*x + a*c)^3 + 3*(42*b*c*cosh(b
*c*x + a*c)^5 + 70*b*c*cosh(b*c*x + a*c)^3 + 25*b*c*cosh(b*c*x + a*c))*sinh
(b*c*x + a*c)^4 + (84*b*c*cosh(b*c*x + a*c)^6 + 210*b*c*cosh(b*c*x + a*c)^4
 + 150*b*c*cosh(b*c*x + a*c)^2 + 19*b*c)*sinh(b*c*x + a*c)^3 + 21*b*c*cosh(
b*c*x + a*c) + 3*(12*b*c*cosh(b*c*x + a*c)^7 + 42*b*c*cosh(b*c*x + a*c)^5 +
 50*b*c*cosh(b*c*x + a*c)^3 + 21*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^2
 + 3*(3*b*c*cosh(b*c*x + a*c)^8 + 14*b*c*cosh(b*c*x + a*c)^6 + 25*b*c*cosh(
b*c*x + a*c)^4 + 19*b*c*cosh(b*c*x + a*c)^2 + 3*b*c)*sinh(b*c*x + a*c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)**2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.44, size = 64, normalized size = 0.34

$$-\frac{16\left(20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}+1\right)}{15\,bc\left(e^{(2\,bcx+2\,ac)}+1\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(7/2),x, algorithm="giac")
```

```
[Out] -16/15*(20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*
a*c) + 1)/(b*c*(e^(2*b*c*x + 2*a*c) + 1)^6)
```

Mupad [B]
time = 0.11, size = 345, normalized size = 1.81

$$\frac{96\,{\mathrm{e}}^{2\,a\,c+2\,b\,c\,x}\,\sqrt{{\left(\frac{{\mathrm{e}}^{a\,c+b\,c\,x}}{2}+\frac{{\mathrm{e}}^{-a\,c-b\,c\,x}}{2}\right)}^2}}{b\,c\,\left({\mathrm{e}}^{a\,c+b\,c\,x}+{\mathrm{e}}^{3\,a\,c+3\,b\,c\,x}\right)\,{\left({\mathrm{e}}^{2\,a\,c+2\,b\,c\,x}+1\right)}^4}-\frac{128\,{\mathrm{e}}^{2\,a\,c+2\,b\,c\,x}\,\sqrt{{\left(\frac{{\mathrm{e}}^{a\,c+b\,c\,x}}{2}+\frac{{\mathrm{e}}^{-a\,c-b\,c\,x}}{2}\right)}^2}}{3\,b\,c\,\left({\mathrm{e}}^{a\,c+b\,c\,x}+{\mathrm{e}}^{3\,a\,c+3\,b\,c\,x}\right)\,{\left({\mathrm{e}}^{2\,a\,c+2\,b\,c\,x}+1\right)}^3}-\frac{384\,{\mathrm{e}}^{2\,a\,c+2\,b\,c\,x}\,\sqrt{{\left(\frac{{\mathrm{e}}^{a\,c+b\,c\,x}}{2}+\frac{{\mathrm{e}}^{-a\,c-b\,c\,x}}{2}\right)}^2}}{5\,b\,c\,\left({\mathrm{e}}^{a\,c+b\,c\,x}+{\mathrm{e}}^{3\,a\,c+3\,b\,c\,x}\right)\,{\left({\mathrm{e}}^{2\,a\,c+2\,b\,c\,x}+1\right)}^5}+\frac{64\,{\mathrm{e}}^{2\,a\,c+2\,b\,c\,x}\,\sqrt{{\left(\frac{{\mathrm{e}}^{a\,c+b\,c\,x}}{2}+\frac{{\mathrm{e}}^{-a\,c-b\,c\,x}}{2}\right)}^2}}{3\,b\,c\,\left({\mathrm{e}}^{a\,c+b\,c\,x}+{\mathrm{e}}^{3\,a\,c+3\,b\,c\,x}\right)\,{\left({\mathrm{e}}^{2\,a\,c+2\,b\,c\,x}+1\right)}^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))/(cosh(a*c + b*c*x)^2)^(7/2),x)
```

```
[Out] (96*exp(2*a*c + 2*b*c*x)*((exp(a*c + b*c*x)/2 + exp(- a*c - b*c*x)/2)^2)^(1
/2))/(b*c*(exp(a*c + b*c*x) + exp(3*a*c + 3*b*c*x))*(exp(2*a*c + 2*b*c*x) +
 1)^4) - (128*exp(2*a*c + 2*b*c*x)*((exp(a*c + b*c*x)/2 + exp(- a*c - b*c*x
)/2)^2)^(1/2))/(3*b*c*(exp(a*c + b*c*x) + exp(3*a*c + 3*b*c*x))*(exp(2*a*c
+ 2*b*c*x) + 1)^3) - (384*exp(2*a*c + 2*b*c*x)*((exp(a*c + b*c*x)/2 + exp(-
 a*c - b*c*x)/2)^2)^(1/2))/(5*b*c*(exp(a*c + b*c*x) + exp(3*a*c + 3*b*c*x))
*(exp(2*a*c + 2*b*c*x) + 1)^5) + (64*exp(2*a*c + 2*b*c*x)*((exp(a*c + b*c*x
)/2 + exp(- a*c - b*c*x)/2)^2)^(1/2))/(3*b*c*(exp(a*c + b*c*x) + exp(3*a*c
+ 3*b*c*x))*(exp(2*a*c + 2*b*c*x) + 1)^6)
```

3.299 $\int e^x \cosh(a+bx)\,dx$

Optimal. Leaf size=41

$$\frac{e^x \cosh(a+bx)}{1-b^2}-\frac{be^x \sinh(a+bx)}{1-b^2}$$

```
[Out] exp(x)*cosh(b*x+a)/(-b^2+1)-b*exp(x)*sinh(b*x+a)/(-b^2+1)
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5583}

$$\frac{e^x \cosh(a+bx)}{1-b^2}-\frac{be^x \sinh(a+bx)}{1-b^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Cosh[a + b*x],x]
```

```
[Out] (E^x*Cosh[a + b*x])/(1 - b^2) - (b*E^x*Sinh[a + b*x])/(1 - b^2)
```

Rule 5583

```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^x \cosh(a+bx)\,dx=\frac{e^x \cosh(a+bx)}{1-b^2}-\frac{be^x \sinh(a+bx)}{1-b^2}$$

Mathematica [A]
time = 0.04, size = 28, normalized size = 0.68

$$\frac{e^x(-\cosh(a+bx)+b\sinh(a+bx))}{-1+b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cosh[a + b*x],x]
```

```
[Out] (E^x*(-Cosh[a + b*x] + b*Sinh[a + b*x]))/(-1 + b^2)
```

Maple [A]
time = 0.86, size = 62, normalized size = 1.51

method	result	size
risch	$\frac{e^{bx+a+x}}{2+2b}-\frac{e^{-bx-a+x}}{2(b-1)}$	33
default	$\frac{\sinh((b-1)x+a)}{2b-2}+\frac{\sinh((1+b)x+a)}{2+2b}-\frac{\cosh((b-1)x+a)}{2(b-1)}+\frac{\cosh((1+b)x+a)}{2+2b}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cosh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(b-1)*sinh((b-1)*x+a)+1/2/(1+b)*sinh((1+b)*x+a)-1/2*cosh((b-1)*x+a)/(b-
1)+1/2*cosh((1+b)*x+a)/(1+b)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-b>0)', see `assume?` for more deta
ils)Is
```

Fricas [A]
time = 0.46, size = 45, normalized size = 1.10

$$-\frac{\cosh(bx+a)\cosh(x)-(b\cosh(x)+b\sinh(x))\sinh(bx+a)+\cosh(bx+a)\sinh(x)}{b^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] -(cosh(b*x + a)*cosh(x) - (b*cosh(x) + b*sinh(x))*sinh(b*x + a) + cosh(b*x
+ a)*sinh(x))/(b^2 - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(31) = 62$.
time = 0.17, size = 99, normalized size = 2.41

$$\begin{cases} \frac{xe^x\sinh(a-x)}{2}+\frac{xe^x\cosh(a-x)}{2}-\frac{e^x\sinh(a-x)}{2} & \text{for } b=-1 \\ -\frac{xe^x\sinh(a+x)}{2}+\frac{xe^x\cosh(a+x)}{2}+\frac{e^x\cosh(a+x)}{2} & \text{for } b=1 \\ \frac{be^x\sinh(a+bx)}{b^2-1}-\frac{e^x\cosh(a+bx)}{b^2-1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(b*x+a),x)
```

```
[Out] Piecewise((x*exp(x)*sinh(a - x)/2 + x*exp(x)*cosh(a - x)/2 - exp(x)*sinh(a
- x)/2, Eq(b, -1)), (-x*exp(x)*sinh(a + x)/2 + x*exp(x)*cosh(a + x)/2 + exp
(x)*cosh(a + x)/2, Eq(b, 1)), (b*exp(x)*sinh(a + b*x)/(b**2 - 1) - exp(x)*c
osh(a + b*x)/(b**2 - 1), True))
```

Giac [**A**]
time = 0.40, size = 32, normalized size = 0.78

$$\frac{e^{(bx+a+x)}}{2\,(b+1)} - \frac{e^{(-bx-a+x)}}{2\,(b-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*e^(b*x + a + x)/(b + 1) - 1/2*e^(-b*x - a + x)/(b - 1)
```

Mupad [**B**]
time = 0.09, size = 45, normalized size = 1.10

$$-\frac{e^{x-a-b\,x}\,\left(b+e^{2\,a+2\,b\,x}-b\,e^{2\,a+2\,b\,x}+1\right)}{2\,\left(b^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*exp(x),x)
```

```
[Out] -(exp(x - a - b*x)*(b + exp(2*a + 2*b*x) - b*exp(2*a + 2*b*x) + 1))/(2*(b^2
 - 1))
```

3.300 $\int e^x \cosh(a + cx^2)\,dx$

Optimal. Leaf size=85

$$-\frac{e^{-a+\frac{1}{4c}}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{1-2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{e^{a-\frac{1}{4c}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{1+2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

```
[Out] -1/4*exp(-a+1/4/c)*erf(1/2*(-2*c*x+1)/c^(1/2))*Pi^(1/2)/c^(1/2)+1/4*exp(a-1
/4/c)*erfi(1/2*(2*c*x+1)/c^(1/2))*Pi^(1/2)/c^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5624, 2266, 2236, 2235}

$$\frac{\sqrt{\pi}\,e^{a-\frac{1}{4c}}\mathrm{Erfi}\left(\frac{2cx+1}{2\sqrt{c}}\right)}{4\sqrt{c}}-\frac{\sqrt{\pi}\,e^{\frac{1}{4c}-a}\mathrm{Erf}\left(\frac{1-2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Cosh[a + c*x^2],x]
```

```
[Out] -1/4*(E^(-a + 1/(4*c))*Sqrt[Pi]*Erf[(1 - 2*c*x)/(2*Sqrt[c])])/Sqrt[c] + (E^
(a - 1/(4*c))*Sqrt[Pi]*Erfi[(1 + 2*c*x)/(2*Sqrt[c])])/(4*Sqrt[c])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 5624

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
```

```
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int e^x \cosh\left(a+cx^2\right)\,dx &= \int \left(\frac{1}{2}e^{-a+x-cx^2}+\frac{1}{2}e^{a+x+cx^2}\right)dx \\
&= \frac{1}{2}\int e^{-a+x-cx^2}\,dx+\frac{1}{2}\int e^{a+x+cx^2}\,dx \\
&= \frac{1}{2}e^{a-\frac{1}{4c}}\int e^{\frac{(1+2cx)^2}{4c}}\,dx+\frac{1}{2}e^{-a+\frac{1}{4c}}\int e^{-\frac{(1-2cx)^2}{4c}}\,dx \\
&= -\frac{e^{-a+\frac{1}{4c}}\sqrt{\pi}\,\operatorname{erf}\left(\frac{1-2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{e^{a-\frac{1}{4c}}\sqrt{\pi}\,\operatorname{erfi}\left(\frac{1+2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 79, normalized size = 0.93

$$\frac{e^{-\frac{1}{4}/c}\sqrt{\pi}\left(e^{\frac{1}{2}/c}\operatorname{Erf}\left(\frac{-1+2cx}{2\sqrt{c}}\right)(\cosh(a)-\sinh(a))+\operatorname{Erfi}\left(\frac{1+2cx}{2\sqrt{c}}\right)(\cosh(a)+\sinh(a))\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cosh[a + c*x^2],x]
```

```
[Out] (Sqrt[Pi]*(E^(1/(2*c))*Erf[(-1 + 2*c*x)/(2*Sqrt[c])]*(Cosh[a] - Sinh[a]) +
Erfi[(1 + 2*c*x)/(2*Sqrt[c])]*(Cosh[a] + Sinh[a])))/(4*Sqrt[c]*E^(1/(4*c)))
```

Maple [A]
time = 1.45, size = 72, normalized size = 0.85

method	result	size
risch	$\frac{\sqrt{\pi}\,\mathrm{e}^{-\frac{4ac-1}{4c}}\operatorname{erf}\left(\sqrt{c}\,x-\frac{1}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{4ac-1}{4c}}\operatorname{erf}\left(x\sqrt{-c}-\frac{1}{2\sqrt{-c}}\right)}{4\sqrt{-c}}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cosh(c*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*Pi^(1/2)*exp(-1/4*(4*a*c-1)/c)/c^(1/2)*erf(c^(1/2)*x-1/2/c^(1/2))+1/4*P
i^(1/2)*exp(1/4*(4*a*c-1)/c)/(-c)^(1/2)*erf(x*(-c)^(1/2)-1/2/(-c)^(1/2))
```

Maxima [A]
time = 0.30, size = 65, normalized size = 0.76

$$\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-c}\,x-\frac{1}{2\sqrt{-c}}\right)e^{\left(a-\frac{1}{4c}\right)}}{4\sqrt{-c}}+\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{c}\,x-\frac{1}{2\sqrt{c}}\right)e^{\left(-a+\frac{1}{4c}\right)}}{4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(c*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*erf(sqrt(-c)*x - 1/2/sqrt(-c))*e^(a - 1/4/c)/sqrt(-c) + 1/4*sq
rt(pi)*erf(sqrt(c)*x - 1/2/sqrt(c))*e^(-a + 1/4/c)/sqrt(c)
```

Fricas [A]
time = 0.38, size = 104, normalized size = 1.22

$$-\frac{\sqrt{\pi}\,\sqrt{-c}\,\left(\cosh\left(\frac{4\,ac-1}{4\,c}\right)+\sinh\left(\frac{4\,ac-1}{4\,c}\right)\right)\mathrm{erf}\left(\frac{(2\,cx+1)\sqrt{-c}}{2\,c}\right)-\sqrt{\pi}\,\sqrt{c}\,\left(\cosh\left(\frac{4\,ac-1}{4\,c}\right)-\sinh\left(\frac{4\,ac-1}{4\,c}\right)\right)\mathrm{erf}\left(\frac{2\,cx-1}{2\sqrt{c}}\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(c*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*sqrt(-c)*(cosh(1/4*(4*a*c - 1)/c) + sinh(1/4*(4*a*c - 1)/c))
*erf(1/2*(2*c*x + 1)*sqrt(-c)/c) - sqrt(pi)*sqrt(c)*(cosh(1/4*(4*a*c - 1)/c
) - sinh(1/4*(4*a*c - 1)/c))*erf(1/2*(2*c*x - 1)/sqrt(c)))/c
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x}\cosh\left(a+cx^{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(c*x**2+a),x)
```

```
[Out] Integral(exp(x)*cosh(a + c*x**2), x)
```

Giac [A]
time = 0.42, size = 73, normalized size = 0.86

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c}\,\left(2\,x+\frac{1}{c}\right)\right)e^{\left(\frac{4\,ac-1}{4\,c}\right)}}{4\sqrt{-c}}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{c}\,\left(2\,x-\frac{1}{c}\right)\right)e^{\left(-\frac{4\,ac-1}{4\,c}\right)}}{4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(c*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c)*(2*x + 1/c))*e^(1/4*(4*a*c - 1)/c)/sqrt(-c)
 - 1/4*sqrt(pi)*erf(-1/2*sqrt(c)*(2*x - 1/c))*e^(-1/4*(4*a*c - 1)/c)/sqrt(c
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int e^x \cosh\left(c\,x^2+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cosh(a + c*x^2),x)
```

```
[Out] int(exp(x)*cosh(a + c*x^2), x)
```

3.301 $\int e^x \cosh(a+bx+cx^2)\,dx$

Optimal. Leaf size=101

$$-\frac{e^{-a+\frac{(1-b)^2}{4c}}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{1-b-2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{e^{a-\frac{(1+b)^2}{4c}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{1+b+2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

```
[Out] -1/4*exp(-a+1/4*(1-b)^2/c)*erf(1/2*(-2*c*x-b+1)/c^(1/2))*Pi^(1/2)/c^(1/2)+1
/4*exp(a-1/4*(1+b)^2/c)*erfi(1/2*(2*c*x+b+1)/c^(1/2))*Pi^(1/2)/c^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {5624, 2266, 2236, 2235}

$$\frac{\sqrt{\pi}\,e^{a-\frac{(b+1)^2}{4c}}\mathrm{Erfi}\left(\frac{b+2cx+1}{2\sqrt{c}}\right)}{4\sqrt{c}}-\frac{\sqrt{\pi}\,e^{\frac{(1-b)^2}{4c}-a}\mathrm{Erf}\left(\frac{-b-2cx+1}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Cosh[a + b*x + c*x^2],x]
```

```
[Out] -1/4*(E^(-a + (1 - b)^2/(4*c))*Sqrt[Pi]*Erf[(1 - b - 2*c*x)/(2*Sqrt[c])])/S
qrt[c] + (E^(a - (1 + b)^2/(4*c))*Sqrt[Pi]*Erfi[(1 + b + 2*c*x)/(2*Sqrt[c])
])/(4*Sqrt[c])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 5624

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int e^x \cosh\left(a+bx+cx^2\right)\,dx &= \int \left(\frac{1}{2}e^{-a+(1-b)x-cx^2}+\frac{1}{2}e^{a+(1+b)x+cx^2}\right)dx \\
&= \frac{1}{2}\int e^{-a+(1-b)x-cx^2}\,dx+\frac{1}{2}\int e^{a+(1+b)x+cx^2}\,dx \\
&= \frac{1}{2}e^{-a+\frac{(1-b)^2}{4c}}\int e^{-\frac{(1-b-2cx)^2}{4c}}\,dx+\frac{1}{2}e^{a-\frac{(1+b)^2}{4c}}\int e^{\frac{(1+b+2cx)^2}{4c}}\,dx \\
&= -\frac{e^{-a+\frac{(1-b)^2}{4c}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{1-b-2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{e^{a-\frac{(1+b)^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1+b+2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}
\end{aligned}
$$

Mathematica [A]
time = 0.11, size = 91, normalized size = 0.90

$$\frac{e^{-\frac{(1+b)^2}{4c}}\sqrt{\pi}\left(e^{\frac{1+b^2}{2c}}\mathrm{Erf}\left(\frac{-1+b+2cx}{2\sqrt{c}}\right)(\cosh(a)-\sinh(a))+\mathrm{Erfi}\left(\frac{1+b+2cx}{2\sqrt{c}}\right)(\cosh(a)+\sinh(a))\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cosh[a + b*x + c*x^2],x]
[Out] (Sqrt[Pi]*(E^((1 + b^2)/(2*c))*Erf[(-1 + b + 2*c*x)/(2*Sqrt[c])]*(Cosh[a] -
 Sinh[a]) + Erfi[(1 + b + 2*c*x)/(2*Sqrt[c])]*(Cosh[a] + Sinh[a])))/(4*Sqrt
[c]*E^((1 + b)^2/(4*c)))
```

Maple [A]
time = 3.36, size = 97, normalized size = 0.96

method	result	size
risch	$\frac{\sqrt{\pi}\,\mathrm{e}^{-\frac{4ac-b^2+2b-1}{4c}}\,\mathrm{erf}\left(\sqrt{c}\,x-\frac{1-b}{2\sqrt{c}}\right)}{4\sqrt{c}}-\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{4ac-b^2-2b-1}{4c}}\,\mathrm{erf}\left(-x\sqrt{-c}+\frac{1+b}{2\sqrt{-c}}\right)}{4\sqrt{-c}}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cosh(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*Pi^(1/2)*exp(-1/4*(4*a*c-b^2+2*b-1)/c)/c^(1/2)*erf(c^(1/2)*x-1/2*(1-b)/
c^(1/2))-1/4*Pi^(1/2)*exp(1/4*(4*a*c-b^2-2*b-1)/c)/(-c)^(1/2)*erf(-x*(-c)^(
1/2)+1/2*(1+b)/(-c)^(1/2))
```

Maxima [A]
time = 0.30, size = 81, normalized size = 0.80

$$\frac{\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-c}\,x-\frac{b+1}{2\sqrt{-c}}\right)e^{\left(a-\frac{(b+1)^2}{4c}\right)}}{4\sqrt{-c}}+\frac{\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{c}\,x+\frac{b-1}{2\sqrt{c}}\right)e^{\left(-a+\frac{(b-1)^2}{4c}\right)}}{4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*erf(sqrt(-c)*x - 1/2*(b + 1)/sqrt(-c))*e^(a - 1/4*(b + 1)^2/c)
/sqrt(-c) + 1/4*sqrt(pi)*erf(sqrt(c)*x + 1/2*(b - 1)/sqrt(c))*e^(-a + 1/4*(
b - 1)^2/c)/sqrt(c)
```

Fricas [A]
time = 0.38, size = 130, normalized size = 1.29

$$-\frac{\sqrt{\pi}\,\sqrt{-c}\left(\cosh\left(-\frac{b^2-4ac+2b+1}{4c}\right)+\sinh\left(-\frac{b^2-4ac+2b+1}{4c}\right)\right)\operatorname{erf}\left(\frac{(2cx+b+1)\sqrt{-c}}{2c}\right)-\sqrt{\pi}\,\sqrt{c}\left(\cosh\left(-\frac{b^2-4ac-2b+1}{4c}\right)-\sinh\left(-\frac{b^2-4ac-2b+1}{4c}\right)\right)\operatorname{erf}\left(\frac{2cx+b-1}{2\sqrt{c}}\right)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*sqrt(-c)*(cosh(-1/4*(b^2 - 4*a*c + 2*b + 1)/c) + sinh(-1/4*(
b^2 - 4*a*c + 2*b + 1)/c))*erf(1/2*(2*c*x + b + 1)*sqrt(-c)/c) - sqrt(pi)*s
qrt(c)*(cosh(-1/4*(b^2 - 4*a*c - 2*b + 1)/c) - sinh(-1/4*(b^2 - 4*a*c - 2*b
 + 1)/c))*erf(1/2*(2*c*x + b - 1)/sqrt(c)))/c
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x}\cosh\left(a+bx+cx^{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(c*x**2+b*x+a),x)
```

```
[Out] Integral(exp(x)*cosh(a + b*x + c*x**2), x)
```

Giac [A]
time = 0.41, size = 91, normalized size = 0.90

$$-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c}\left(2x+\frac{b+1}{c}\right)\right)e^{\left(-\frac{b^2-4ac+2b+1}{4c}\right)}}{4\sqrt{-c}}-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\sqrt{c}\left(2x+\frac{b-1}{c}\right)\right)e^{\left(\frac{b^2-4ac-2b+1}{4c}\right)}}{4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c)*(2*x + (b + 1)/c))*e^(-1/4*(b^2 - 4*a*c + 2
*b + 1)/c)/sqrt(-c) - 1/4*sqrt(pi)*erf(-1/2*sqrt(c)*(2*x + (b - 1)/c))*e^(1
/4*(b^2 - 4*a*c - 2*b + 1)/c)/sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int e^x \cosh\left(c\,x^2+b\,x+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cosh(a + b*x + c*x^2),x)
```

```
[Out] int(exp(x)*cosh(a + b*x + c*x^2), x)
```

3.302 $\int e^{x^2} \cosh(a+bx)\,dx$

Optimal. Leaf size=65

$$\frac{1}{4}e^{-a-\frac{b^2}{4}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{1}{2}(-b+2x)\right)+\frac{1}{4}e^{a-\frac{b^2}{4}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{1}{2}(b+2x)\right)$$

```
[Out] -1/4*exp(-a-1/4*b^2)*erfi(1/2*b-x)*Pi^(1/2)+1/4*exp(a-1/4*b^2)*erfi(1/2*b+x
)*Pi^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5624, 2266, 2235}

$$\frac{1}{4}\sqrt{\pi}\,e^{-a-\frac{b^2}{4}}\mathrm{Erfi}\left(\frac{1}{2}(2x-b)\right)+\frac{1}{4}\sqrt{\pi}\,e^{a-\frac{b^2}{4}}\mathrm{Erfi}\left(\frac{1}{2}(b+2x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*Cosh[a + b*x],x]
```

```
[Out] (E^(-a - b^2/4)*Sqrt[Pi]*Erfi[(-b + 2*x)/2])/4 + (E^(a - b^2/4)*Sqrt[Pi]*Er
fi[(b + 2*x)/2])/4
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 5624

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int e^{x^2}\cosh(a+bx)\,dx=\int\left(\frac{1}{2}e^{-a-bx+x^2}+\frac{1}{2}e^{a+bx+x^2}\right)dx$$
$$=\frac{1}{2}\int e^{-a-bx+x^2}\,dx+\frac{1}{2}\int e^{a+bx+x^2}\,dx$$
$$=\frac{1}{2}e^{-a-\frac{b^2}{4}}\int e^{\frac{1}{4}(-b+2x)^2}\,dx+\frac{1}{2}e^{a-\frac{b^2}{4}}\int e^{\frac{1}{4}(b+2x)^2}\,dx$$
$$=\frac{1}{4}e^{-a-\frac{b^2}{4}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1}{2}(-b+2x)\right)+\frac{1}{4}e^{a-\frac{b^2}{4}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1}{2}(b+2x)\right)$$

Mathematica [A]
time = 0.05, size = 51, normalized size = 0.78

$$\frac{1}{4}e^{-\frac{b^2}{4}}\sqrt{\pi}\left(\mathrm{Erfi}\left(\frac{b}{2}-x\right)(-\cosh(a)+\sinh(a))+\mathrm{Erfi}\left(\frac{b}{2}+x\right)(\cosh(a)+\sinh(a))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*Cosh[a + b*x],x]
```

```
[Out] (Sqrt[Pi]*(Erfi[b/2 - x]*(-Cosh[a] + Sinh[a]) + Erfi[b/2 + x]*(Cosh[a] + Si
nh[a])))/(4*E^(b^2/4))
```

Maple [C] Result contains complex when optimal does not.
time = 1.45, size = 52, normalized size = 0.80

method	result	size
risch	$\frac{i\sqrt{\pi}\,\mathrm{e}^{-a-\frac{b^2}{4}}\,\mathrm{erf}\left(-ix+\frac{1}{2}ib\right)}{4}-\frac{i\sqrt{\pi}\,\mathrm{e}^{a-\frac{b^2}{4}}\,\mathrm{erf}\left(ix+\frac{1}{2}ib\right)}{4}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*cosh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I*Pi^(1/2)*exp(-a-1/4*b^2)*erf(-I*x+1/2*I*b)-1/4*I*Pi^(1/2)*exp(a-1/4*b
^2)*erf(I*x+1/2*I*b)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.30, size = 45, normalized size = 0.69

$$-\frac{1}{4}i\sqrt{\pi}\,\mathrm{erf}\left(\frac{1}{2}i\,b+i\,x\right)e^{\left(-\frac{1}{4}\,b^2+a\right)}-\frac{1}{4}i\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}i\,b+i\,x\right)e^{\left(-\frac{1}{4}\,b^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cosh(b*x+a),x, algorithm="maxima")
```

[Out] -1/4*I*sqrt(pi)*erf(1/2*I*b + I*x)*e^(-1/4*b^2 + a) - 1/4*I*sqrt(pi)*erf(-1/2*I*b + I*x)*e^(-1/4*b^2 - a)

Fricas [**A**]
time = 0.52, size = 44, normalized size = 0.68

$$\frac{1}{4}\sqrt{\pi}\left(\operatorname{erfi}\left(\frac{1}{2}b+x\right)e^{\left(\frac{1}{4}b^2+a\right)}+\operatorname{erfi}\left(-\frac{1}{2}b+x\right)e^{\left(\frac{1}{4}b^2-a\right)}\right)e^{\left(-\frac{1}{2}b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(pi)*(erfi(1/2*b + x)*e^(1/4*b^2 + a) + erfi(-1/2*b + x)*e^(1/4*b^2 - a))*e^(-1/2*b^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x^2}\cosh(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*cosh(b*x+a),x)
```

```
[Out] Integral(exp(x**2)*cosh(a + b*x), x)
```

Giac [**C**] Result contains complex when optimal does not.
time = 0.42, size = 45, normalized size = 0.69

$$\frac{1}{4}i\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}ib-ix\right)e^{\left(-\frac{1}{4}b^2+a\right)}+\frac{1}{4}i\sqrt{\pi}\operatorname{erf}\left(\frac{1}{2}ib-ix\right)e^{\left(-\frac{1}{4}b^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cosh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*I*sqrt(pi)*erf(-1/2*I*b - I*x)*e^(-1/4*b^2 + a) + 1/4*I*sqrt(pi)*erf(1/2*I*b - I*x)*e^(-1/4*b^2 - a)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cosh(a+b\,x)\,\mathrm{e}^{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*exp(x^2),x)
```

```
[Out] int(cosh(a + b*x)*exp(x^2), x)
```

3.303 $\int e^{x^2} \cosh\left(a+cx^2\right) dx$

Optimal. Leaf size=65

$$\frac{e^{-a}\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{1-c}\,x\right)}{4\sqrt{1-c}}+\frac{e^{a}\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{1+c}\,x\right)}{4\sqrt{1+c}}$$

```
[Out] 1/4*erfi(x*(1-c)^(1/2))*Pi^(1/2)/exp(a)/(1-c)^(1/2)+1/4*exp(a)*erfi(x*(1+c)
^(1/2))*Pi^(1/2)/(1+c)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5624, 2235}

$$\frac{\sqrt{\pi}\,e^{-a}\mathrm{Erfi}\left(\sqrt{1-c}\,x\right)}{4\sqrt{1-c}}+\frac{\sqrt{\pi}\,e^{a}\mathrm{Erfi}\left(\sqrt{c+1}\,x\right)}{4\sqrt{c+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*Cosh[a + c*x^2],x]
```

```
[Out] (Sqrt[Pi]*Erfi[Sqrt[1 - c]*x])/(4*Sqrt[1 - c]*E^a) + (E^a*Sqrt[Pi]*Erfi[Sqr
t[1 + c]*x])/(4*Sqrt[1 + c])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 5624

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int e^{x^2}\cosh\left(a+cx^2\right)\,dx &= \int\left(\frac{1}{2}e^{-a+(1-c)x^2}+\frac{1}{2}e^{a+(1+c)x^2}\right)\,dx\\
&= \frac{1}{2}\int e^{-a+(1-c)x^2}\,dx+\frac{1}{2}\int e^{a+(1+c)x^2}\,dx\\
&= \frac{e^{-a}\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{1-c}\,x\right)}{4\sqrt{1-c}}+\frac{e^{a}\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{1+c}\,x\right)}{4\sqrt{1+c}}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 71, normalized size = 1.09

$$\frac{\sqrt{\pi}\left(\sqrt{-1+c}\,(1+c)\mathrm{Erf}\left(\sqrt{-1+c}\,x\right)(\cosh(a)-\sinh(a))+(-1+c)\sqrt{1+c}\,\mathrm{Erfi}\left(\sqrt{1+c}\,x\right)(\cosh(a)+\sinh(a))\right)}{4\left(-1+c^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*Cosh[a + c*x^2],x]
```

```
[Out] (Sqrt[Pi]*(Sqrt[-1 + c]*(1 + c)*Erf[Sqrt[-1 + c]*x]*(Cosh[a] - Sinh[a]) + (
-1 + c)*Sqrt[1 + c]*Erfi[Sqrt[1 + c]*x]*(Cosh[a] + Sinh[a])))/(4*(-1 + c^2)
)
```

Maple [A]
time = 5.96, size = 48, normalized size = 0.74

method	result	size
risch	$\frac{\sqrt{\pi}\,\mathrm{e}^{-a}\,\mathrm{erf}\left(\sqrt{c-1}\,x\right)}{4\sqrt{c-1}}+\frac{\sqrt{\pi}\,\mathrm{e}^{a}\,\mathrm{erf}\left(\sqrt{-c-1}\,x\right)}{4\sqrt{-c-1}}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*cosh(c*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*Pi^(1/2)*exp(-a)/(c-1)^(1/2)*erf((c-1)^(1/2)*x)+1/4*Pi^(1/2)*exp(a)/(-c
-1)^(1/2)*erf((-c-1)^(1/2)*x)
```

Maxima [A]
time = 0.29, size = 47, normalized size = 0.72

$$\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{c-1}\,x\right)e^{(-a)}}{4\sqrt{c-1}}+\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-c-1}\,x\right)e^{a}}{4\sqrt{-c-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cosh(c*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*erf(sqrt(c - 1)*x)*e^(-a)/sqrt(c - 1) + 1/4*sqrt(pi)*erf(sqrt(
-c - 1)*x)*e^a/sqrt(-c - 1)
```

Fricas [A]
time = 0.37, size = 76, normalized size = 1.17

$$\frac{\sqrt{\pi}\left((c+1)\cosh(a)-(c+1)\sinh(a)\right)\sqrt{c-1}\,\mathrm{erf}\left(\sqrt{c-1}\,x\right)-\sqrt{\pi}\left((c-1)\cosh(a)+(c-1)\sinh(a)\right)\sqrt{-c-1}\,\mathrm{erf}\left(\sqrt{-c-1}\,x\right)}{4\left(c^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cosh(c*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(pi)*((c + 1)*cosh(a) - (c + 1)*sinh(a))*sqrt(c - 1)*erf(sqrt(c -
1)*x) - sqrt(pi)*((c - 1)*cosh(a) + (c - 1)*sinh(a))*sqrt(-c - 1)*erf(sqrt(
-c - 1)*x))/(c^2 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x^2}\cosh\left(a+cx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*cosh(c*x**2+a),x)
```

```
[Out] Integral(exp(x**2)*cosh(a + c*x**2), x)
```

Giac [A]
time = 0.40, size = 49, normalized size = 0.75

$$-\frac{\sqrt{\pi}\ \operatorname{erf}\left(-\sqrt{c-1}\,x\right)e^{(-a)}}{4\sqrt{c-1}}-\frac{\sqrt{\pi}\ \operatorname{erf}\left(-\sqrt{-c-1}\,x\right)e^{a}}{4\sqrt{-c-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cosh(c*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-sqrt(c - 1)*x)*e^(-a)/sqrt(c - 1) - 1/4*sqrt(pi)*erf(-sq
rt(-c - 1)*x)*e^a/sqrt(-c - 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{e}^{x^2}\,\mathrm{cosh}\left(c\,x^2+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*cosh(a + c*x^2),x)
```

```
[Out] int(exp(x^2)*cosh(a + c*x^2), x)
```

3.304 $\int e^{x^2} \cosh\left(a + bx + cx^2\right) dx$

Optimal. Leaf size=115

$$-\frac{e^{-a-\frac{b^2}{4(1-c)}}\sqrt{\pi}\,\text{Erfi}\left(\frac{b-2(1-c)x}{2\sqrt{1-c}}\right)}{4\sqrt{1-c}}+\frac{e^{a-\frac{b^2}{4(1+c)}}\sqrt{\pi}\,\text{Erfi}\left(\frac{b+2(1+c)x}{2\sqrt{1+c}}\right)}{4\sqrt{1+c}}$$

[Out] -1/4*exp(-a-1/4*b^2/(1-c))*erfi(1/2*(b-2*(1-c)*x)/(1-c)^(1/2))*Pi^(1/2)/(1-c)^(1/2)+1/4*exp(a-1/4*b^2/(1+c))*erfi(1/2*(b+2*(1+c)*x)/(1+c)^(1/2))*Pi^(1/2)/(1+c)^(1/2)

Rubi [A]
time = 0.13, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {5624, 2266, 2235}

$$\frac{\sqrt{\pi}\,e^{a-\frac{b^2}{4(c+1)}}\,\text{Erfi}\left(\frac{b+2(c+1)x}{2\sqrt{c+1}}\right)}{4\sqrt{c+1}}-\frac{\sqrt{\pi}\,e^{-a-\frac{b^2}{4(1-c)}}\,\text{Erfi}\left(\frac{b-2(1-c)x}{2\sqrt{1-c}}\right)}{4\sqrt{1-c}}$$

Antiderivative was successfully verified.

[In] Int[E^x^2*Cosh[a + b*x + c*x^2],x]

[Out] -1/4*(E^(-a - b^2/(4*(1 - c)))*Sqrt[Pi]*Erfi[(b - 2*(1 - c)*x)/(2*Sqrt[1 - c])])/Sqrt[1 - c] + (E^(a - b^2/(4*(1 + c)))*Sqrt[Pi]*Erfi[(b + 2*(1 + c)*x)/(2*Sqrt[1 + c])])/(4*Sqrt[1 + c])

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 5624

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int e^{x^2}\cosh\left(a+bx+cx^2\right)\,dx=\int\left(\frac{1}{2}e^{-a-bx+(1-c)x^2}+\frac{1}{2}e^{a+bx+(1+c)x^2}\right)dx$$
$$=\frac{1}{2}\int e^{-a-bx+(1-c)x^2}\,dx+\frac{1}{2}\int e^{a+bx+(1+c)x^2}\,dx$$
$$=\frac{1}{2}e^{-a-\frac{b^2}{4(1-c)}}\int e^{\frac{(-b+2(1-c)x)^2}{4(1-c)}}\,dx+\frac{1}{2}e^{a-\frac{b^2}{4(1+c)}}\int e^{\frac{(b+2(1+c)x)^2}{4(1+c)}}\,dx$$
$$=-\frac{e^{-a-\frac{b^2}{4(1-c)}}\sqrt{\pi}\,\text{erfi}\left(\frac{b-2(1-c)x}{2\sqrt{1-c}}\right)}{4\sqrt{1-c}}+\frac{e^{a-\frac{b^2}{4(1+c)}}\sqrt{\pi}\,\text{erfi}\left(\frac{b+2(1+c)x}{2\sqrt{1+c}}\right)}{4\sqrt{1+c}}$$

Mathematica [A]
time = 0.25, size = 122, normalized size = 1.06

$$\frac{e^{-\frac{b^2}{4+4c}}\sqrt{\pi}\left(\sqrt{-1+c}\,(1+c)e^{\frac{b^2c}{2(-1+c^2)}}\text{Erf}\left(\frac{b+2(-1+c)x}{2\sqrt{-1+c}}\right)(\cosh(a)-\sinh(a))+(-1+c)\sqrt{1+c}\,\text{Erfi}\left(\frac{b+2(1+c)x}{2\sqrt{1+c}}\right)(\cosh(a)+\sinh(a))\right)}{4\left(-1+c^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*Cosh[a + b*x + c*x^2],x]
[Out] (Sqrt[Pi]*(Sqrt[-1 + c]*(1 + c)*E^((b^2*c)/(2*(-1 + c^2)))*Erf[(b + 2*(-1 +
 c)*x)/(2*Sqrt[-1 + c])]*(Cosh[a] - Sinh[a]) + (-1 + c)*Sqrt[1 + c]*Erfi[(b
 + 2*(1 + c)*x)/(2*Sqrt[1 + c])]*(Cosh[a] + Sinh[a])))/(4*(-1 + c^2)*E^(b^2
/(4 + 4*c)))
```

Maple [A]
time = 5.89, size = 105, normalized size = 0.91

method	result	size
risch	$\frac{\sqrt{\pi}\,\mathrm{e}^{-\frac{4ac-b^2-4a}{4(c-1)}}\,\mathrm{erf}\left(\sqrt{c-1}\,x+\frac{b}{2\sqrt{c-1}}\right)}{4\sqrt{c-1}}-\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{4ac-b^2+4a}{4+4c}}\,\mathrm{erf}\left(-\sqrt{-c-1}\,x+\frac{b}{2\sqrt{-c-1}}\right)}{4\sqrt{-c-1}}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*cosh(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/4*Pi^(1/2)*exp(-1/4*(4*a*c-b^2-4*a)/(c-1))/(c-1)^(1/2)*erf((c-1)^(1/2)*x+
1/2*b/(c-1)^(1/2))-1/4*Pi^(1/2)*exp(1/4*(4*a*c-b^2+4*a)/(1+c))/(-c-1)^(1/2)
*erf(-(-c-1)^(1/2)*x+1/2*b/(-c-1)^(1/2))
```

Maxima [A]
time = 0.28, size = 89, normalized size = 0.77

$$\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-c-1}\,x-\frac{b}{2\sqrt{-c-1}}\right)e^{\left(a-\frac{b^2}{4\,(c+1)}\right)}}{4\,\sqrt{-c-1}}+\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{c-1}\,x+\frac{b}{2\sqrt{c-1}}\right)e^{\left(-a+\frac{b^2}{4\,(c-1)}\right)}}{4\,\sqrt{c-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cosh(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*erf(sqrt(-c - 1)*x - 1/2*b/sqrt(-c - 1))*e^(a - 1/4*b^2/(c + 1
))/sqrt(-c - 1) + 1/4*sqrt(pi)*erf(sqrt(c - 1)*x + 1/2*b/sqrt(c - 1))*e^(-a
 + 1/4*b^2/(c - 1))/sqrt(c - 1)
```

Fricas [A]
time = 0.38, size = 165, normalized size = 1.43

$$\frac{\sqrt{\pi}\left((c+1)\cosh\left(-\frac{b^2-4\,ac+4\,a}{4\,(c-1)}\right)-(c+1)\sinh\left(-\frac{b^2-4\,ac+4\,a}{4\,(c-1)}\right)\right)\sqrt{c-1}\,\operatorname{erf}\left(\frac{2\,(c-1)x+b}{2\,\sqrt{c-1}}\right)-\sqrt{\pi}\left((c-1)\cosh\left(-\frac{b^2-4\,ac-4\,a}{4\,(c+1)}\right)+(c-1)\sinh\left(-\frac{b^2-4\,ac-4\,a}{4\,(c+1)}\right)\right)\sqrt{-c-1}\,\operatorname{erf}\left(\frac{(2\,(c+1)x+b)\sqrt{-c-1}}{2\,(c+1)}\right)}{4\,(c^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cosh(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(pi)*((c + 1)*cosh(-1/4*(b^2 - 4*a*c + 4*a)/(c - 1)) - (c + 1)*sin
h(-1/4*(b^2 - 4*a*c + 4*a)/(c - 1)))*sqrt(c - 1)*erf(1/2*(2*(c - 1)*x + b)/
sqrt(c - 1)) - sqrt(pi)*((c - 1)*cosh(-1/4*(b^2 - 4*a*c - 4*a)/(c + 1)) + (
c - 1)*sinh(-1/4*(b^2 - 4*a*c - 4*a)/(c + 1)))*sqrt(-c - 1)*erf(1/2*(2*(c +
 1)*x + b)*sqrt(-c - 1)/(c + 1)))/(c^2 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x^2}\cosh\left(a+bx+cx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*cosh(c*x**2+b*x+a),x)
```

```
[Out] Integral(exp(x**2)*cosh(a + b*x + c*x**2), x)
```

Giac [A]
time = 0.43, size = 101, normalized size = 0.88

$$-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-c-1}\left(2\,x+\frac{b}{c+1}\right)\right)e^{\left(-\frac{b^2-4\,ac-4\,a}{4\,(c+1)}\right)}}{4\,\sqrt{-c-1}}-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{c-1}\left(2\,x+\frac{b}{c-1}\right)\right)e^{\left(\frac{b^2-4\,ac+4\,a}{4\,(c-1)}\right)}}{4\,\sqrt{c-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cosh(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c - 1)*(2*x + b/(c + 1)))*e^(-1/4*(b^2 - 4*a*c
 - 4*a)/(c + 1))/sqrt(-c - 1) - 1/4*sqrt(pi)*erf(-1/2*sqrt(c - 1)*(2*x + b/
(c - 1)))*e^(1/4*(b^2 - 4*a*c + 4*a)/(c - 1))/sqrt(c - 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int e^{x^2} \cosh\left(c\,x^2+b\,x+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*cosh(a + b*x + c*x^2),x)
[Out] int(exp(x^2)*cosh(a + b*x + c*x^2), x)
```

3.305 $\int f^{a+bx} \cosh(d+fx^2)\,dx$

Optimal. Leaf size=110

$$\frac{1}{4}e^{-d+\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)+\frac{1}{4}e^{d-\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{2fx+b\log(f)}{2\sqrt{f}}\right)$$

```
[Out] 1/4*exp(-d+1/4*b^2*ln(f)^2/f)*f^(-1/2+a)*erf(1/2*(2*f*x-b*ln(f))/f^(1/2))*P
i^(1/2)+1/4*exp(d-1/4*b^2*ln(f)^2/f)*f^(-1/2+a)*erfi(1/2*(2*f*x+b*ln(f))/f^
(1/2))*Pi^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5624, 2325, 2266, 2236, 2235}

$$\frac{1}{4}\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{4f}-d}\mathrm{Erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)+\frac{1}{4}\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{d-\frac{b^2\log^2(f)}{4f}}\mathrm{Erfi}\left(\frac{b\log(f)+2fx}{2\sqrt{f}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Cosh[d + f*x^2],x]
```

```
[Out] (E^(-d + (b^2*Log[f]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(2*f*x - b*Log[f])
/(2*Sqrt[f])])/4 + (E^(d - (b^2*Log[f]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erfi
[(2*f*x + b*Log[f])/(2*Sqrt[f])])/4
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
```

```
x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5624

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx}\cosh\left(d+fx^2\right)\,dx &= \int\left(\frac{1}{2}e^{-d-fx^2}f^{a+bx}+\frac{1}{2}e^{d+fx^2}f^{a+bx}\right)dx\\ &= \frac{1}{2}\int e^{-d-fx^2}f^{a+bx}\,dx+\frac{1}{2}\int e^{d+fx^2}f^{a+bx}\,dx\\ &= \frac{1}{2}\int e^{-d-fx^2+a\log(f)+bx\log(f)}\,dx+\frac{1}{2}\int e^{d+fx^2+a\log(f)+bx\log(f)}\,dx\\ &= \frac{1}{2}\left(e^{d-\frac{b^2\log^2(f)}{4f}}f^a\right)\int e^{\frac{(2fx+b\log(f))^2}{4f}}\,dx+\frac{1}{2}\left(e^{-d+\frac{b^2\log^2(f)}{4f}}f^a\right)\int e^{-\frac{(-2fx+b\log(f))^2}{4f}}\,dx\\ &= \frac{1}{4}e^{-d+\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)+\frac{1}{4}e^{d-\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{2f
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 102, normalized size = 0.93

$$\frac{1}{4}e^{-\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\left(e^{\frac{b^2\log^2(f)}{2f}}\mathrm{Erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)(\cosh(d)-\sinh(d))+\mathrm{Erfi}\left(\frac{2fx+b\log(f)}{2\sqrt{f}}\right)(\cosh(d)+\sinh(d))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Cosh[d + f*x^2],x]
```

```
[Out] (f^(-1/2 + a)*Sqrt[Pi]*(E^((b^2*Log[f]^2)/(2*f))*Erf[(2*f*x - b*Log[f])/(2*
Sqrt[f])]*(Cosh[d] - Sinh[d]) + Erfi[(2*f*x + b*Log[f])/(2*Sqrt[f])]*(Cosh[
d] + Sinh[d])))/(4*E^((b^2*Log[f]^2)/(4*f)))
```

Maple [A]
time = 0.73, size = 100, normalized size = 0.91

method	result	size
risch	$-\frac{\sqrt{\pi}\, f^{a}\mathrm{e}^{\frac{b^{2}\ln(f)^{2}-4df}{4f}}\mathrm{erf}\left(-\sqrt{f}\, x+\frac{\ln(f)b}{2\sqrt{f}}\right)}{4\sqrt{f}}-\frac{\sqrt{\pi}\, f^{a}\mathrm{e}^{-\frac{b^{2}\ln(f)^{2}-4df}{4f}}\mathrm{erf}\left(-\sqrt{-f}\, x+\frac{\ln(f)b}{2\sqrt{-f}}\right)}{4\sqrt{-f}}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*cosh(f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(1/4*(b^2*ln(f)^2-4*d*f)/f)/f^(1/2)*erf(-f^(1/2)*x+1/2
*ln(f)*b/f^(1/2))-1/4*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-4*d*f)/f)/(-f)^(1/
2)*erf(-(-f)^(1/2)*x+1/2*ln(f)*b/(-f)^(1/2))
```

Maxima [A]
time = 0.29, size = 90, normalized size = 0.82

$$\frac{1}{4}\sqrt{\pi}f^{a-\frac{1}{2}}\operatorname{erf}\left(\sqrt{f}x-\frac{b\log(f)}{2\sqrt{f}}\right)e^{\left(\frac{b^2\log(f)^2}{4f}-d\right)}+\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-f}x-\frac{b\log(f)}{2\sqrt{-f}}\right)e^{\left(-\frac{b^2\log(f)^2}{4f}+d\right)}}{4\sqrt{-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*f^(a - 1/2)*erf(sqrt(f)*x - 1/2*b*log(f)/sqrt(f))*e^(1/4*b^2*l
og(f)^2/f - d) + 1/4*sqrt(pi)*f^a*erf(sqrt(-f)*x - 1/2*b*log(f)/sqrt(-f))*e
^(-1/4*b^2*log(f)^2/f + d)/sqrt(-f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(84) = 168$.
time = 0.37, size = 211, normalized size = 1.92

$$-\frac{\sqrt{\pi}\sqrt{-f}\cosh\left(\frac{b^2\log(f)^2-4af\log(f)-4df}{4f}\right)\operatorname{erf}\left(\frac{(2fx+b\log(f))\sqrt{-f}}{2f}\right)+\sqrt{\pi}\sqrt{f}\cosh\left(\frac{b^2\log(f)^2+4af\log(f)-4df}{4f}\right)\operatorname{erf}\left(-\frac{2fx-b\log(f)}{2\sqrt{f}}\right)+\sqrt{\pi}\sqrt{f}\operatorname{erf}\left(-\frac{2fx-b\log(f)}{2\sqrt{f}}\right)\sinh\left(\frac{b^2\log(f)^2+4af\log(f)-4df}{4f}\right)-\sqrt{\pi}\sqrt{-f}\operatorname{erf}\left(\frac{(2fx+b\log(f))\sqrt{-f}}{2f}\right)\sinh\left(\frac{b^2\log(f)^2-4af\log(f)-4df}{4f}\right)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+d),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*sqrt(-f)*cosh(1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f)*e
rf(1/2*(2*f*x + b*log(f))*sqrt(-f)/f) + sqrt(pi)*sqrt(f)*cosh(1/4*(b^2*log(
f)^2 + 4*a*f*log(f) - 4*d*f)/f)*erf(-1/2*(2*f*x - b*log(f))/sqrt(f)) + sqrt
(pi)*sqrt(f)*erf(-1/2*(2*f*x - b*log(f))/sqrt(f))*sinh(1/4*(b^2*log(f)^2 +
4*a*f*log(f) - 4*d*f)/f) - sqrt(pi)*sqrt(-f)*erf(1/2*(2*f*x + b*log(f))*sqr
t(-f)/f)*sinh(1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f))/f
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx}\cosh\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*cosh(f*x**2+d),x)
```

```
[Out] Integral(f**(a + b*x)*cosh(d + f*x**2), x)
```

Giac [A]
time = 0.41, size = 106, normalized size = 0.96

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{f}\,\left(2\,x-\frac{b\log(f)}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2+4\,af\log(f)-4\,df}{4\,f}\right)}}{4\,\sqrt{f}}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-f}\,\left(2\,x+\frac{b\log(f)}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,af\log(f)-4\,df}{4\,f}\right)}}{4\,\sqrt{-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(f)*(2*x - b*log(f)/f))*e^(1/4*(b^2*log(f)^2 + 4
*a*f*log(f) - 4*d*f)/f)/sqrt(f) - 1/4*sqrt(pi)*erf(-1/2*sqrt(-f)*(2*x + b*l
og(f)/f))*e^(-1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f)/sqrt(-f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{a+b\,x}\,\cosh\left(f\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x)*cosh(d + f*x^2),x)
```

```
[Out] int(f^(a + b*x)*cosh(d + f*x^2), x)
```

3.306 $\int f^{a+bx}\cosh^2\left(d+fx^2\right)\,dx$

Optimal. Leaf size=148

$$\frac{1}{8}e^{-2d+\frac{b^2\log^2(f)}{8f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\frac{4fx-b\log(f)}{2\sqrt{2}\,\sqrt{f}}\right)+\frac{1}{8}e^{2d-\frac{b^2\log^2(f)}{8f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\,\mathrm{Erfi}\left(\frac{4fx+b\log(f)}{2\sqrt{2}\,\sqrt{f}}\right)+\frac{f^{a+bx}}{2b\log(f)}$$

```
[Out] 1/2*f^(b*x+a)/b/ln(f)+1/16*exp(-2*d+1/8*b^2*ln(f)^2/f)*f^(-1/2+a)*erf(1/4*(
4*f*x-b*ln(f))*2^(1/2)/f^(1/2))*2^(1/2)*Pi^(1/2)+1/16*exp(2*d-1/8*b^2*ln(f)
^2/f)*f^(-1/2+a)*erfi(1/4*(4*f*x+b*ln(f))*2^(1/2)/f^(1/2))*2^(1/2)*Pi^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5624, 2225, 2325, 2266, 2236, 2235}

$$\frac{1}{8}\sqrt{\frac{\pi}{2}}\,f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{8f}-2d}\mathrm{Erf}\left(\frac{4fx-b\log(f)}{2\sqrt{2}\,\sqrt{f}}\right)+\frac{1}{8}\sqrt{\frac{\pi}{2}}\,f^{a-\frac{1}{2}}e^{2d-\frac{b^2\log^2(f)}{8f}}\mathrm{Erfi}\left(\frac{b\log(f)+4fx}{2\sqrt{2}\,\sqrt{f}}\right)+\frac{f^{a+bx}}{2b\log(f)}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Cosh[d + f*x^2]^2,x]
```

```
[Out] (E^(-2*d + (b^2*Log[f]^2)/(8*f))*f^(-1/2 + a)*Sqrt[Pi/2]*Erf[(4*f*x - b*Log
[f])/(2*Sqrt[2]*Sqrt[f])])/8 + (E^(2*d - (b^2*Log[f]^2)/(8*f))*f^(-1/2 + a)
*Sqrt[Pi/2]*Erfi[(4*f*x + b*Log[f])/(2*Sqrt[2]*Sqrt[f])])/8 + f^(a + b*x)/(
2*b*Log[f])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5624

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx}\cosh^2\left(d+fx^2\right)\,dx &= \int \left(\frac{1}{2}f^{a+bx}+\frac{1}{4}e^{-2d-2fx^2}f^{a+bx}+\frac{1}{4}e^{2d+2fx^2}f^{a+bx}\right)dx\\ &= \frac{1}{4}\int e^{-2d-2fx^2}f^{a+bx}\,dx+\frac{1}{4}\int e^{2d+2fx^2}f^{a+bx}\,dx+\frac{1}{2}\int f^{a+bx}\,dx\\ &= \frac{f^{a+bx}}{2b\log(f)}+\frac{1}{4}\int e^{-2d-2fx^2+a\log(f)+bx\log(f)}\,dx+\frac{1}{4}\int e^{2d+2fx^2+a\log(f)+bx\log(f)}\,dx\\ &= \frac{f^{a+bx}}{2b\log(f)}+\frac{1}{4}\left(e^{2d-\frac{b^2\log^2(f)}{8f}}f^a\right)\int e^{\frac{(4fx+b\log(f))^2}{8f}}\,dx+\frac{1}{4}\left(e^{-2d+\frac{b^2\log^2(f)}{8f}}f^a\right)\int e^{-}\\ &= \frac{1}{8}e^{-2d+\frac{b^2\log^2(f)}{8f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\frac{4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right)+\frac{1}{8}e^{2d-\frac{b^2\log^2(f)}{8f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\,\mathrm{erfi}\end{aligned}$$

Mathematica [A]
time = 0.52, size = 149, normalized size = 1.01

$$\frac{1}{16}f^a\left(\frac{8f^{bx}}{b\log(f)}+\frac{e^{\frac{b^2\log^2(f)}{8f}}\sqrt{2\pi}\,\mathrm{Erf}\left(\frac{4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right)(\cosh(2d)-\sinh(2d))}{\sqrt{f}}+\frac{e^{-\frac{b^2\log^2(f)}{8f}}\sqrt{2\pi}\,\mathrm{Erfi}\left(\frac{4fx+b\log(f)}{2\sqrt{2}\sqrt{f}}\right)(\cosh(2d)+\sinh(2d))}{\sqrt{f}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Cosh[d + f*x^2]^2,x]
```

```
[Out] (f^a*((8*f^(b*x))/(b*Log[f]) + (E^((b^2*Log[f]^2)/(8*f))*Sqrt[2*Pi]*Erf[(4*
f*x - b*Log[f])/(2*Sqrt[2]*Sqrt[f])]*(Cosh[2*d] - Sinh[2*d]))/Sqrt[f] + (Sq
```

```
rt[2*Pi]*Erfi[(4*f*x + b*Log[f])/(2*Sqrt[2]*Sqrt[f])]*(Cosh[2*d] + Sinh[2*d
]))/(E^((b^2*Log[f]^2)/(8*f))*Sqrt[f])))/16
```

Maple [A]
time = 2.02, size = 126, normalized size = 0.85

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{b^{2} \ln(f)^{2}-16 d f}{8 f}} \sqrt{2}\, \mathrm{erf}\left(-\sqrt{2}\, \sqrt{f}\, x+\frac{b \ln(f) \sqrt{2}}{4 \sqrt{f}}\right)}{16 \sqrt{f}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2} \ln(f)^{2}-16 d f}{8 f}} \mathrm{erf}\left(-\sqrt{-2 f}\, x+\frac{b \ln(f)}{2 \sqrt{-2 f}}\right.}{8 \sqrt{-2 f}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*cosh(f*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(1/8*(b^2*ln(f)^2-16*d*f)/f)*2^(1/2)/f^(1/2)*erf(-2^(
1/2)*f^(1/2)*x+1/4*b*ln(f)*2^(1/2)/f^(1/2))-1/8*Pi^(1/2)*f^a*exp(-1/8*(b^2*
ln(f)^2-16*d*f)/f)/(-2*f)^(1/2)*erf(-(-2*f)^(1/2)*x+1/2*b*ln(f)/(-2*f)^(1/2
))+1/2*f^a*f^(b*x)/b/ln(f)
```

Maxima [A]
time = 0.53, size = 127, normalized size = 0.86

$$\frac{\sqrt{2}\,\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{2}\,\sqrt{f}\, x - \frac{\sqrt{2}\, b \log(f)}{4\,\sqrt{f}}\right) e^{\left(\frac{b^{2} \log(f)^{2}}{8\, f} - 2\, d\right)}}{16\,\sqrt{f}} + \frac{\sqrt{2}\,\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{2}\,\sqrt{-f}\, x - \frac{\sqrt{2}\, b \log(f)}{4\,\sqrt{-f}}\right) e^{\left(-\frac{b^{2} \log(f)^{2}}{8\, f} + 2\, d\right)}}{16\,\sqrt{-f}} + \frac{f^{bx+a}}{2\, b \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*sqrt(pi)*f^a*erf(sqrt(2)*sqrt(f)*x - 1/4*sqrt(2)*b*log(f)/sqrt
(f))*e^(1/8*b^2*log(f)^2/f - 2*d)/sqrt(f) + 1/16*sqrt(2)*sqrt(pi)*f^a*erf(s
qrt(2)*sqrt(-f)*x - 1/4*sqrt(2)*b*log(f)/sqrt(-f))*e^(-1/8*b^2*log(f)^2/f +
 2*d)/sqrt(-f) + 1/2*f^(b*x + a)/(b*log(f))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(114) = 228$.
time = 0.44, size = 278, normalized size = 1.88

$$-\frac{\sqrt{2}\,\sqrt{\pi}\, b\sqrt{-f}\, \cosh\left(\frac{b^{2} \log(f)^{2} - 8\, a f \log(f) - 16\, d f}{8\, f}\right) \operatorname{erf}\left(\frac{\sqrt{2}\,(4\, f x + b \log(f))\,\sqrt{-f}}{4\, f}\right) \log(f) + \sqrt{2}\,\sqrt{\pi}\, b\sqrt{f}\, \cosh\left(\frac{b^{2} \log(f)^{2} + 8\, a f \log(f) - 16\, d f}{8\, f}\right) \operatorname{erf}\left(-\frac{\sqrt{2}\,(4\, f x - b \log(f))}{4\,\sqrt{f}}\right) \log(f) + \sqrt{2}\,\sqrt{\pi}\, b\sqrt{f}\, \operatorname{erf}\left(-\frac{\sqrt{2}\,(4\, f x - b \log(f))}{4\,\sqrt{f}}\right) \log(f) \sinh\left(\frac{b^{2} \log(f)^{2} + 8\, a f \log(f) - 16\, d f}{8\, f}\right) - \sqrt{2}\,\sqrt{\pi}\, b\sqrt{-f}\, \operatorname{erf}\left(\frac{\sqrt{2}\,(4\, f x + b \log(f))\,\sqrt{-f}}{4\, f}\right) \log(f) \sinh\left(\frac{b^{2} \log(f)^{2} - 8\, a f \log(f) - 16\, d f}{8\, f}\right) - 8\, f \cosh\left((bx + a) \log(f)\right) - 8\, f \sinh\left((bx + a) \log(f)\right)}{16\, b f \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(2)*sqrt(pi)*b*sqrt(-f)*cosh(1/8*(b^2*log(f)^2 - 8*a*f*log(f) -
16*d*f)/f)*erf(1/4*sqrt(2)*(4*f*x + b*log(f))*sqrt(-f)/f)*log(f) + sqrt(2)*
sqrt(pi)*b*sqrt(f)*cosh(1/8*(b^2*log(f)^2 + 8*a*f*log(f) - 16*d*f)/f)*erf(-
```

```
1/4*sqrt(2)*(4*f*x - b*log(f))/sqrt(f))*log(f) + sqrt(2)*sqrt(pi)*b*sqrt(f)
*erf(-1/4*sqrt(2)*(4*f*x - b*log(f))/sqrt(f))*log(f)*sinh(1/8*(b^2*log(f)^2
 + 8*a*f*log(f) - 16*d*f)/f) - sqrt(2)*sqrt(pi)*b*sqrt(-f)*erf(1/4*sqrt(2)*
(4*f*x + b*log(f))*sqrt(-f)/f)*log(f)*sinh(1/8*(b^2*log(f)^2 - 8*a*f*log(f)
 - 16*d*f)/f) - 8*f*cosh((b*x + a)*log(f)) - 8*f*sinh((b*x + a)*log(f)))/(b
*f*log(f))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx} \cosh^2\left(d+fx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*cosh(f*x**2+d)**2,x)
```

```
[Out] Integral(f**(a + b*x)*cosh(d + f*x**2)**2, x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.45, size = 355, normalized size = 2.40

$$-\frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{4}\sqrt{2}\sqrt{f}\left(4x-\frac{b\log(f)}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2+8af\log(f)-16df}{8f}\right)}}{16\sqrt{f}}-\frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{4}\sqrt{2}\sqrt{-f}\left(4x+\frac{b\log(f)}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-8af\log(f)-16df}{8f}\right)}}{16\sqrt{-f}}+\left(\frac{2b\cos\left(-\frac{1}{2}\pi bx\operatorname{sgn}(f)+\frac{1}{2}\pi bx-\frac{1}{2}\pi a\operatorname{sgn}(f)+\frac{1}{2}\pi a\right)\log(|f|)}{4b^2\log(|f|)^2+(\pi b\operatorname{sgn}(f)-\pi b)^2}-\frac{(\pi b\operatorname{sgn}(f)-\pi b)\sin\left(-\frac{1}{2}\pi bx\operatorname{sgn}(f)+\frac{1}{2}\pi bx-\frac{1}{2}\pi a\operatorname{sgn}(f)+\frac{1}{2}\pi a\right)}{4b^2\log(|f|)^2+(\pi b\operatorname{sgn}(f)-\pi b)^2}\right)e^{(bx\log(|f|)+a\log(|f|))}+i\left(\frac{ie^{\left(\frac{1}{2}i\pi bx\operatorname{sgn}(f)-\frac{1}{2}i\pi bx+\frac{1}{2}i\pi a\operatorname{sgn}(f)-\frac{1}{2}i\pi a\right)}}{2i\pi b\operatorname{sgn}(f)-2i\pi b+4b\log(|f|)}-\frac{ie^{\left(-\frac{1}{2}i\pi bx\operatorname{sgn}(f)+\frac{1}{2}i\pi bx-\frac{1}{2}i\pi a\operatorname{sgn}(f)+\frac{1}{2}i\pi a\right)}}{-2i\pi b\operatorname{sgn}(f)+2i\pi b+4b\log(|f|)}\right)e^{(bx\log(|f|)+a\log(|f|))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+d)^2,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*sqrt(f)*(4*x - b*log(f)/f))*e^(1/8*
(b^2*log(f)^2 + 8*a*f*log(f) - 16*d*f)/f)/sqrt(f) - 1/16*sqrt(2)*sqrt(pi)*e
rf(-1/4*sqrt(2)*sqrt(-f)*(4*x + b*log(f)/f))*e^(-1/8*(b^2*log(f)^2 - 8*a*f*
log(f) - 16*d*f)/f)/sqrt(-f) + (2*b*cos(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1
/2*pi*a*sgn(f) + 1/2*pi*a)*log(abs(f))/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f)
- pi*b)^2) - (pi*b*sgn(f) - pi*b)*sin(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2
*pi*a*sgn(f) + 1/2*pi*a)/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2))*e^
(b*x*log(abs(f)) + a*log(abs(f))) + I*(I*e^(1/2*I*pi*b*x*sgn(f) - 1/2*I*pi*
b*x + 1/2*I*pi*a*sgn(f) - 1/2*I*pi*a)/(2*I*pi*b*sgn(f) - 2*I*pi*b + 4*b*log
(abs(f))) - I*e^(-1/2*I*pi*b*x*sgn(f) + 1/2*I*pi*b*x - 1/2*I*pi*a*sgn(f) +
1/2*I*pi*a)/(-2*I*pi*b*sgn(f) + 2*I*pi*b + 4*b*log(abs(f))))*e^(b*x*log(abs
(f)) + a*log(abs(f)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{a+b\,x}\,\cosh\left(f\,x^2+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x)*cosh(d + f*x^2)^2,x)
```

```
[Out] int(f^(a + b*x)*cosh(d + f*x^2)^2, x)
```

3.307 $\int f^{a+bx}\cosh^3\left(d+fx^2\right)dx$

Optimal. Leaf size=239

$$\frac{3}{16}e^{-d+\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\text{Erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)+\frac{1}{16}e^{-3d+\frac{b^2\log^2(f)}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{3}}\,\text{Erf}\left(\frac{6fx-b\log(f)}{2\sqrt{3}\sqrt{f}}\right)+\frac{3}{16}e^{d-b}$$

```
[Out] 1/48*exp(-3*d+1/12*b^2*ln(f)^2/f)*f^(-1/2+a)*erf(1/6*(6*f*x-b*ln(f))*3^(1/2
)/f^(1/2))*3^(1/2)*Pi^(1/2)+1/48*exp(3*d-1/12*b^2*ln(f)^2/f)*f^(-1/2+a)*erf
i(1/6*(6*f*x+b*ln(f))*3^(1/2)/f^(1/2))*3^(1/2)*Pi^(1/2)+3/16*exp(-d+1/4*b^2
*ln(f)^2/f)*f^(-1/2+a)*erf(1/2*(2*f*x-b*ln(f))/f^(1/2))*Pi^(1/2)+3/16*exp(d
-1/4*b^2*ln(f)^2/f)*f^(-1/2+a)*erfi(1/2*(2*f*x+b*ln(f))/f^(1/2))*Pi^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {5624, 2325, 2266, 2236, 2235}

$$\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{4f}-d}\text{Erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)+\frac{1}{16}\sqrt{\frac{\pi}{3}}f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{12f}-3d}\text{Erf}\left(\frac{6fx-b\log(f)}{2\sqrt{3}\sqrt{f}}\right)+\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}e^{d-\frac{b^2\log^2(f)}{4f}}\text{Erfi}\left(\frac{b\log(f)+2fx}{2\sqrt{f}}\right)+\frac{1}{16}\sqrt{\frac{\pi}{3}}f^{a-\frac{1}{2}}e^{3d-\frac{b^2\log^2(f)}{12f}}\text{Erfi}\left(\frac{b\log(f)+6fx}{2\sqrt{3}\sqrt{f}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Cosh[d + f*x^2]^3,x]
```

```
[Out] (3*E^(-d + (b^2*Log[f]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(2*f*x - b*Log[f
])/(2*Sqrt[f])])/16 + (E^(-3*d + (b^2*Log[f]^2)/(12*f))*f^(-1/2 + a)*Sqrt[P
i/3]*Erf[(6*f*x - b*Log[f])/(2*Sqrt[3]*Sqrt[f])])/16 + (3*E^(d - (b^2*Log[f
]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erfi[(2*f*x + b*Log[f])/(2*Sqrt[f])])/16
 + (E^(3*d - (b^2*Log[f]^2)/(12*f))*f^(-1/2 + a)*Sqrt[Pi/3]*Erfi[(6*f*x + b*
Log[f])/(2*Sqrt[3]*Sqrt[f])])/16
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5624

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx}\cosh^3\left(d+fx^2\right)\,dx &= \int\left(\frac{1}{8}e^{-3d-3fx^2}f^{a+bx}+\frac{3}{8}e^{-d-fx^2}f^{a+bx}+\frac{3}{8}e^{d+fx^2}f^{a+bx}+\frac{1}{8}e^{3d+3fx^2}f^{a+bx}\right)dx\\ &= \frac{1}{8}\int e^{-3d-3fx^2}f^{a+bx}\,dx+\frac{1}{8}\int e^{3d+3fx^2}f^{a+bx}\,dx+\frac{3}{8}\int e^{-d-fx^2}f^{a+bx}\,dx+\frac{3}{8}\int\\ &= \frac{1}{8}\int e^{-3d-3fx^2+a\log(f)+bx\log(f)}\,dx+\frac{1}{8}\int e^{3d+3fx^2+a\log(f)+bx\log(f)}\,dx+\frac{3}{8}\int e^{-d-}\\ &= \frac{1}{8}\left(3e^{d-\frac{b^2\log^2(f)}{4f}}f^a\right)\int e^{\frac{(2fx+b\log(f))^2}{4f}}\,dx+\frac{1}{8}\left(e^{3d-\frac{b^2\log^2(f)}{12f}}f^a\right)\int e^{\frac{(6fx+b\log(f))^2}{12f}}\,dx\\ &= \frac{3}{16}e^{-d+\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)+\frac{1}{16}e^{-3d+\frac{b^2\log^2(f)}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{3}}\,\mathrm{e}\end{aligned}$$

Mathematica [A]
time = 0.29, size = 286, normalized size = 1.20

$$\frac{1}{16}e^{-\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{3}}\left(3\sqrt{3}\cosh(d)\mathrm{Erfi}\left(\frac{2fx+b\log(f)}{2\sqrt{f}}\right)+e^{\frac{b^2\log^2(f)}{6f}}\cosh(3d)\mathrm{Erfi}\left(\frac{6fx+b\log(f)}{2\sqrt{3}\sqrt{f}}\right)+3\sqrt{3}e^{\frac{b^2\log^2(f)}{2f}}\mathrm{Erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)(\cosh(d)-\sinh(d))+3\sqrt{3}\,\mathrm{Erfi}\left(\frac{2fx+b\log(f)}{2\sqrt{f}}\right)\sinh(d)+e^{\frac{b^2\log^2(f)}{3f}}\mathrm{Erf}\left(\frac{6fx-b\log(f)}{2\sqrt{3}\sqrt{f}}\right)(\cosh(3d)-\sinh(3d))+e^{\frac{b^2\log^2(f)}{6f}}\mathrm{Erfi}\left(\frac{6fx+b\log(f)}{2\sqrt{3}\sqrt{f}}\right)\sinh(3d)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Cosh[d + f*x^2]^3,x]
```

```
[Out] (f^(-1/2 + a)*Sqrt[Pi/3]*(3*Sqrt[3]*Cosh[d]*Erfi[(2*f*x + b*Log[f])/(2*Sqrt
[f])] + E^((b^2*Log[f]^2)/(6*f))*Cosh[3*d]*Erfi[(6*f*x + b*Log[f])/(2*Sqrt[
3]*Sqrt[f])] + 3*Sqrt[3]*E^((b^2*Log[f]^2)/(2*f))*Erf[(2*f*x - b*Log[f])/(2
*Sqrt[f])]*(Cosh[d] - Sinh[d]) + 3*Sqrt[3]*Erfi[(2*f*x + b*Log[f])/(2*Sqrt[
f])]*Sinh[d] + E^((b^2*Log[f]^2)/(3*f))*Erf[(6*f*x - b*Log[f])/(2*Sqrt[3]*S
qrt[f])]*(Cosh[3*d] - Sinh[3*d]) + E^((b^2*Log[f]^2)/(6*f))*Erfi[(6*f*x + b
*Log[f])/(2*Sqrt[3]*Sqrt[f])]*Sinh[3*d]))/(16*E^((b^2*Log[f]^2)/(4*f)))
```

Maple [A]
time = 2.64, size = 207, normalized size = 0.87

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{b^{2} \ln(f)^{2}-36 d f}{12 f}} \sqrt{3}\, \mathrm{erf}\left(-\sqrt{3}\, \sqrt{f}\, x+\frac{\ln(f) b \sqrt{3}}{6 \sqrt{f}}\right)}{48 \sqrt{f}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2} \ln(f)^{2}-36 d f}{12 f}} \mathrm{erf}\left(-\sqrt{-3 f}\, x+\frac{\ln(f) b}{2 \sqrt{-3 f}}\right.}{16 \sqrt{-3 f}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*cosh(f*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*Pi^(1/2)*f^a*exp(1/12*(b^2*ln(f)^2-36*d*f)/f)*3^(1/2)/f^(1/2)*erf(-3^
(1/2)*f^(1/2)*x+1/6*ln(f)*b*3^(1/2)/f^(1/2))-1/16*Pi^(1/2)*f^a*exp(-1/12*(b
^2*ln(f)^2-36*d*f)/f)/(-3*f)^(1/2)*erf(-(-3*f)^(1/2)*x+1/2*ln(f)*b/(-3*f)^(
1/2))-3/16*Pi^(1/2)*f^a*exp(1/4*(b^2*ln(f)^2-4*d*f)/f)/f^(1/2)*erf(-f^(1/2)
*x+1/2*ln(f)*b/f^(1/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-4*d*f)/f)/(
-f)^(1/2)*erf(-(-f)^(1/2)*x+1/2*ln(f)*b/(-f)^(1/2))
```

Maxima [A]
time = 0.50, size = 200, normalized size = 0.84

$$\frac{3}{16}\,\sqrt{\pi}\, f^{a-\frac{1}{2}}\, \mathrm{erf}\left(\sqrt{f}\, x-\frac{b \log\left(f\right)}{2 \, \sqrt{f}}\right) e^{\left(\frac{b^{2} \log\left(f\right)^{2}}{4 \, f}-d\right)}+\frac{\sqrt{3}\, \sqrt{\pi}\, f^{a}\, \mathrm{erf}\left(\sqrt{3}\, \sqrt{f}\, x-\frac{\sqrt{3}\, b \log\left(f\right)}{6 \, \sqrt{f}}\right) e^{\left(\frac{b^{2} \log\left(f\right)^{2}}{12 \, f}-3 \, d\right)}}{48 \, \sqrt{f}}+\frac{\sqrt{3}\, \sqrt{\pi}\, f^{a}\, \mathrm{erf}\left(\sqrt{3}\, \sqrt{-f}\, x-\frac{\sqrt{3}\, b \log\left(f\right)}{6 \, \sqrt{-f}}\right) e^{\left(-\frac{b^{2} \log\left(f\right)^{2}}{12 \, f}+3 \, d\right)}}{48 \, \sqrt{-f}}+\frac{3 \, \sqrt{\pi}\, f^{a}\, \mathrm{erf}\left(\sqrt{-f}\, x-\frac{b \log\left(f\right)}{2 \, \sqrt{-f}}\right) e^{\left(-\frac{b^{2} \log\left(f\right)^{2}}{4 \, f}+d\right)}}{16 \, \sqrt{-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 3/16*sqrt(pi)*f^(a - 1/2)*erf(sqrt(f)*x - 1/2*b*log(f)/sqrt(f))*e^(1/4*b^2*
log(f)^2/f - d) + 1/48*sqrt(3)*sqrt(pi)*f^a*erf(sqrt(3)*sqrt(f)*x - 1/6*sqr
t(3)*b*log(f)/sqrt(f))*e^(1/12*b^2*log(f)^2/f - 3*d)/sqrt(f) + 1/48*sqrt(3)
*sqrt(pi)*f^a*erf(sqrt(3)*sqrt(-f)*x - 1/6*sqrt(3)*b*log(f)/sqrt(-f))*e^(-1
/12*b^2*log(f)^2/f + 3*d)/sqrt(-f) + 3/16*sqrt(pi)*f^a*erf(sqrt(-f)*x - 1/2
*b*log(f)/sqrt(-f))*e^(-1/4*b^2*log(f)^2/f + d)/sqrt(-f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 443 vs. $2(181) = 362$.
time = 0.39, size = 443, normalized size = 1.85

$$-\frac{\sqrt{3}\, \sqrt{\pi}\, \sqrt{-f} \cosh\left(\frac{b^{2} \log\left(f\right)^{2}-12 \, a f \log\left(f\right)-36 \, d f}{12 \, f}\right) \mathrm{erf}\left(\frac{\sqrt{3}\, {\left(6 \, f x+b \log\left(f\right)\right)} \sqrt{-f}}{6 \, f}\right)+\sqrt{3}\, \sqrt{\pi}\, \sqrt{f} \cosh\left(\frac{b^{2} \log\left(f\right)^{2}+12 \, a f \log\left(f\right)-36 \, d f}{12 \, f}\right) \mathrm{erf}\left(-\frac{\sqrt{3}\, {\left(6 \, f x-b \log\left(f\right)\right)}}{6 \, \sqrt{f}}\right)+\sqrt{3}\, \sqrt{\pi}\, \sqrt{f}\, \mathrm{erf}\left(-\frac{\sqrt{3}\, {\left(6 \, f x-b \log\left(f\right)\right)}}{6 \, \sqrt{f}}\right) \sinh\left(\frac{b^{2} \log\left(f\right)^{2}+12 \, a f \log\left(f\right)-36 \, d f}{12 \, f}\right)-\sqrt{3}\, \sqrt{\pi}\, \sqrt{-f}\, \mathrm{erf}\left(\frac{\sqrt{3}\, {\left(6 \, f x+b \log\left(f\right)\right)} \sqrt{-f}}{6 \, f}\right) \sinh\left(\frac{b^{2} \log\left(f\right)^{2}-12 \, a f \log\left(f\right)-36 \, d f}{12 \, f}\right)+9 \, \sqrt{\pi}\, \sqrt{-f} \cosh\left(\frac{b^{2} \log\left(f\right)^{2}-4 \, a f \log\left(f\right)-4 \, d f}{4 \, f}\right) \mathrm{erf}\left(\frac{{\left(2 \, f x+b \log\left(f\right)\right)} \sqrt{-f}}{2 \, f}\right)+9 \, \sqrt{\pi}\, \sqrt{f} \cosh\left(\frac{b^{2} \log\left(f\right)^{2}+4 \, a f \log\left(f\right)-4 \, d f}{4 \, f}\right) \mathrm{erf}\left(-\frac{2 \, f x-b \log\left(f\right)}{2 \, \sqrt{f}}\right)+9 \, \sqrt{\pi}\, \sqrt{f}\, \mathrm{erf}\left(-\frac{2 \, f x-b \log\left(f\right)}{2 \, \sqrt{f}}\right) \sinh\left(\frac{b^{2} \log\left(f\right)^{2}+4 \, a f \log\left(f\right)-4 \, d f}{4 \, f}\right)-9 \, \sqrt{\pi}\, \sqrt{-f}\, \mathrm{erf}\left(\frac{{\left(2 \, f x+b \log\left(f\right)\right)} \sqrt{-f}}{2 \, f}\right) \sinh\left(\frac{b^{2} \log\left(f\right)^{2}-4 \, a f \log\left(f\right)-4 \, d f}{4 \, f}\right)}{48 \, f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] -1/48*(sqrt(3)*sqrt(pi)*sqrt(-f)*cosh(1/12*(b^2*log(f)^2 - 12*a*f*log(f) -
36*d*f)/f)*erf(1/6*sqrt(3)*(6*f*x + b*log(f))*sqrt(-f)/f) + sqrt(3)*sqrt(pi
)*sqrt(f)*cosh(1/12*(b^2*log(f)^2 + 12*a*f*log(f) - 36*d*f)/f)*erf(-1/6*sqr
t(3)*(6*f*x - b*log(f))/sqrt(f)) + sqrt(3)*sqrt(pi)*sqrt(f)*erf(-1/6*sqrt(3
)*(6*f*x - b*log(f))/sqrt(f))*sinh(1/12*(b^2*log(f)^2 + 12*a*f*log(f) - 36*
```

```
d*f)/f) - sqrt(3)*sqrt(pi)*sqrt(-f)*erf(1/6*sqrt(3)*(6*f*x + b*log(f))*sqrt
(-f)/f)*sinh(1/12*(b^2*log(f)^2 - 12*a*f*log(f) - 36*d*f)/f) + 9*sqrt(pi)*s
qrt(-f)*cosh(1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f)*erf(1/2*(2*f*x +
b*log(f))*sqrt(-f)/f) + 9*sqrt(pi)*sqrt(f)*cosh(1/4*(b^2*log(f)^2 + 4*a*f*l
og(f) - 4*d*f)/f)*erf(-1/2*(2*f*x - b*log(f))/sqrt(f)) + 9*sqrt(pi)*sqrt(f)
*erf(-1/2*(2*f*x - b*log(f))/sqrt(f))*sinh(1/4*(b^2*log(f)^2 + 4*a*f*log(f)
 - 4*d*f)/f) - 9*sqrt(pi)*sqrt(-f)*erf(1/2*(2*f*x + b*log(f))*sqrt(-f)/f)*s
inh(1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f))/f
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx} \cosh^3\left(d+fx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*cosh(f*x**2+d)**3,x)
```

```
[Out] Integral(f**(a + b*x)*cosh(d + f*x**2)**3, x)
```

Giac [**A**]
time = 0.42, size = 223, normalized size = 0.93

$$-\frac{\sqrt{3}\,\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{6}\,\sqrt{3}\,\sqrt{f}\left(6\,x-\frac{b\log(f)}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2+12\,af\log(f)-36\,df}{12\,f}\right)}}{48\,\sqrt{f}}-\frac{\sqrt{3}\,\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{6}\,\sqrt{3}\,\sqrt{-f}\left(6\,x+\frac{b\log(f)}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-12\,af\log(f)-36\,df}{12\,f}\right)}}{48\,\sqrt{-f}}-\frac{3\,\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{f}\left(2\,x-\frac{b\log(f)}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2+4\,af\log(f)-4\,df}{4\,f}\right)}}{16\,\sqrt{f}}-\frac{3\,\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-f}\left(2\,x+\frac{b\log(f)}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,af\log(f)-4\,df}{4\,f}\right)}}{16\,\sqrt{-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+d)^3,x, algorithm="giac")
```

```
[Out] -1/48*sqrt(3)*sqrt(pi)*erf(-1/6*sqrt(3)*sqrt(f)*(6*x - b*log(f)/f))*e^(1/12
*(b^2*log(f)^2 + 12*a*f*log(f) - 36*d*f)/f)/sqrt(f) - 1/48*sqrt(3)*sqrt(pi)
*erf(-1/6*sqrt(3)*sqrt(-f)*(6*x + b*log(f)/f))*e^(-1/12*(b^2*log(f)^2 - 12*
a*f*log(f) - 36*d*f)/f)/sqrt(-f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(f)*(2*x - b*
log(f)/f))*e^(1/4*(b^2*log(f)^2 + 4*a*f*log(f) - 4*d*f)/f)/sqrt(f) - 3/16*s
qrt(pi)*erf(-1/2*sqrt(-f)*(2*x + b*log(f)/f))*e^(-1/4*(b^2*log(f)^2 - 4*a*f
*log(f) - 4*d*f)/f)/sqrt(-f)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int f^{a+b\,x}\,{\mathrm{cosh}\left(f\,x^2+d\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x)*cosh(d + f*x^2)^3,x)
```

```
[Out] int(f^(a + b*x)*cosh(d + f*x^2)^3, x)
```

3.308 $\int f^{a+bx} \cosh(d+ex+fx^2)\,dx$

Optimal. Leaf size=115

$$\frac{1}{4}e^{-d+\frac{(e-b\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{e+2fx-b\log(f)}{2\sqrt{f}}\right)+\frac{1}{4}e^{d-\frac{(e+b\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{e+2fx+b\log(f)}{2\sqrt{f}}\right)$$

```
[Out] 1/4*exp(-d+1/4*(e-b*ln(f))^2/f)*f^(-1/2+a)*erf(1/2*(e+2*f*x-b*ln(f))/f^(1/2
))*Pi^(1/2)+1/4*exp(d-1/4*(e+b*ln(f))^2/f)*f^(-1/2+a)*erfi(1/2*(e+2*f*x+b*l
n(f))/f^(1/2))*Pi^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5624, 2325, 2266, 2236, 2235}

$$\frac{1}{4}\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{\frac{(e-b\log(f))^2}{4f}-d}\mathrm{Erf}\left(\frac{-b\log(f)+e+2fx}{2\sqrt{f}}\right)+\frac{1}{4}\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{d-\frac{(b\log(f)+e)^2}{4f}}\mathrm{Erfi}\left(\frac{b\log(f)+e+2fx}{2\sqrt{f}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Cosh[d + e*x + f*x^2],x]
```

```
[Out] (E^(-d + (e - b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(e + 2*f*x - b*L
og[f])/(2*Sqrt[f])])/4 + (E^(d - (e + b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sqrt[
Pi]*Erfi[(e + 2*f*x + b*Log[f])/(2*Sqrt[f])])/4
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
```

```
x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5624

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx}\cosh\left(d+ex+fx^2\right)\,dx &= \int\left(\frac{1}{2}e^{-d-ex-fx^2}f^{a+bx}+\frac{1}{2}e^{d+ex+fx^2}f^{a+bx}\right)dx\\ &= \frac{1}{2}\int e^{-d-ex-fx^2}f^{a+bx}\,dx+\frac{1}{2}\int e^{d+ex+fx^2}f^{a+bx}\,dx\\ &= \frac{1}{2}\int e^{-d-fx^2+a\log(f)-x(e-b\log(f))}\,dx+\frac{1}{2}\int e^{d+fx^2+a\log(f)+x(e+b\log(f))}\,dx\\ &= \frac{1}{2}\left(e^{-d+\frac{(e-b\log(f))^2}{4f}}f^a\right)\int e^{-\frac{(-e-2fx+b\log(f))^2}{4f}}\,dx+\frac{1}{2}\left(e^{d-\frac{(e+b\log(f))^2}{4f}}f^a\right)\int e^{\frac{(e\cdots}{}}\\ &= \frac{1}{4}e^{-d+\frac{(e-b\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{e+2fx-b\log(f)}{2\sqrt{f}}\right)+\frac{1}{4}e^{d-\frac{(e+b\log(f))^2}{4f}}f^{-\frac{1}{2}\cdots}\end{aligned}$$

Mathematica [A]
time = 0.19, size = 123, normalized size = 1.07

$$\frac{1}{4}e^{-\frac{e^2+b^2\log^2(f)}{4f}}f^{a-\frac{be+f}{2f}}\sqrt{\pi}\left(e^{\frac{e^2+b^2\log^2(f)}{2f}}\mathrm{Erf}\left(\frac{e+2fx-b\log(f)}{2\sqrt{f}}\right)(\cosh(d)-\sinh(d))+\mathrm{Erfi}\left(\frac{e+2fx+b\log(f)}{2\sqrt{f}}\right)(\cosh(d)+\sinh(d))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Cosh[d + e*x + f*x^2],x]
[Out] (f^(a - (b*e + f)/(2*f))*Sqrt[Pi]*(E^((e^2 + b^2*Log[f]^2)/(2*f))*Erf[(e +
2*f*x - b*Log[f])/(2*Sqrt[f])]*(Cosh[d] - Sinh[d]) + Erfi[(e + 2*f*x + b*Lo
g[f])/(2*Sqrt[f])]*(Cosh[d] + Sinh[d])))/(4*E^((e^2 + b^2*Log[f]^2)/(4*f)))
```

Maple [A]
time = 0.76, size = 126, normalized size = 1.10

method	result
risch	$-\frac{\sqrt{\pi}\,f^a\mathrm{e}^{\frac{b^2\ln(f)^2-2\ln(f)be-4df+e^2}{4f}}\mathrm{erf}\left(-\sqrt{f}\,x+\frac{b\ln(f)-e}{2\sqrt{f}}\right)}{4\sqrt{f}}-\frac{\sqrt{\pi}\,f^a\mathrm{e}^{-\frac{b^2\ln(f)^2+2\ln(f)be-4df+e^2}{4f}}\mathrm{erf}\left(-\sqrt{-f}\,x+\frac{e+b\ln\cdots}{2\sqrt{-\cdots}}\right.}{4\sqrt{-f}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*cosh(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(1/4*(b^2*ln(f)^2-2*ln(f)*b*e-4*d*f+e^2)/f)/f^(1/2)*er
f(-f^(1/2)*x+1/2*(b*ln(f)-e)/f^(1/2))-1/4*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^
2+2*ln(f)*b*e-4*d*f+e^2)/f)/(-f)^(1/2)*erf(-(-f)^(1/2)*x+1/2*(e+b*ln(f))/(-
f)^(1/2))
```

Maxima [A]
time = 0.30, size = 106, normalized size = 0.92

$$\frac{1}{4}\sqrt{\pi}\,f^{a-\frac{1}{2}}\operatorname{erf}\left(\sqrt{f}\,x-\frac{b\log\left(f\right)-e}{2\sqrt{f}}\right)e^{\left(-d+\frac{(b\log(f)-e)^2}{4f}\right)}+\frac{\sqrt{\pi}\,f^a\operatorname{erf}\left(\sqrt{-f}\,x-\frac{b\log(f)+e}{2\sqrt{-f}}\right)e^{\left(d-\frac{(b\log(f)+e)^2}{4f}\right)}}{4\sqrt{-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*f^(a - 1/2)*erf(sqrt(f)*x - 1/2*(b*log(f) - e)/sqrt(f))*e^(-d
+ 1/4*(b*log(f) - e)^2/f) + 1/4*sqrt(pi)*f^a*erf(sqrt(-f)*x - 1/2*(b*log(f)
 + e)/sqrt(-f))*e^(d - 1/4*(b*log(f) + e)^2/f)/sqrt(-f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 335 vs. 2(94) = 188.
time = 0.38, size = 335, normalized size = 2.91

$$-\frac{\sqrt{\pi}\sqrt{-f}\cosh\left(\frac{b^2\log(f)^2-4df+\cosh(1)^2-2(2af-b\cosh(1)-b\sinh(1))\log(f)+2\cosh(1)\sinh(1)+\sinh(1)^2}{4f}\right)\operatorname{erf}\left(\frac{(2fx+b\log(f)+\cosh(1)+\sinh(1))\sqrt{-f}}{2f}\right)+\sqrt{\pi}\sqrt{f}\cosh\left(\frac{b^2\log(f)^2-4df+\cosh(1)^2+2(2af-b\cosh(1)-b\sinh(1))\log(f)+2\cosh(1)\sinh(1)+\sinh(1)^2}{4f}\right)\operatorname{erf}\left(-\frac{2fx-b\log(f)+\cosh(1)+\sinh(1)}{2\sqrt{f}}\right)+\sqrt{\pi}\sqrt{f}\operatorname{erf}\left(-\frac{2fx-b\log(f)+\cosh(1)+\sinh(1)}{2\sqrt{f}}\right)\sinh\left(\frac{b^2\log(f)^2-4df+\cosh(1)^2+2(2af-b\cosh(1)-b\sinh(1))\log(f)+2\cosh(1)\sinh(1)+\sinh(1)^2}{4f}\right)-\sqrt{\pi}\sqrt{-f}\operatorname{erf}\left(\frac{(2fx+b\log(f)+\cosh(1)+\sinh(1))\sqrt{-f}}{2f}\right)\sinh\left(\frac{b^2\log(f)^2-4df+\cosh(1)^2-2(2af-b\cosh(1)-b\sinh(1))\log(f)+2\cosh(1)\sinh(1)+\sinh(1)^2}{4f}\right)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*sqrt(-f)*cosh(1/4*(b^2*log(f)^2 - 4*d*f + cosh(1)^2 - 2*(2*a
*f - b*cosh(1) - b*sinh(1))*log(f) + 2*cosh(1)*sinh(1) + sinh(1)^2)/f)*erf(
1/2*(2*f*x + b*log(f) + cosh(1) + sinh(1))*sqrt(-f)/f) + sqrt(pi)*sqrt(f)*c
osh(1/4*(b^2*log(f)^2 - 4*d*f + cosh(1)^2 + 2*(2*a*f - b*cosh(1) - b*sinh(1
))*log(f) + 2*cosh(1)*sinh(1) + sinh(1)^2)/f)*erf(-1/2*(2*f*x - b*log(f) +
cosh(1) + sinh(1))/sqrt(f)) + sqrt(pi)*sqrt(f)*erf(-1/2*(2*f*x - b*log(f) +
 cosh(1) + sinh(1))/sqrt(f))*sinh(1/4*(b^2*log(f)^2 - 4*d*f + cosh(1)^2 + 2
*(2*a*f - b*cosh(1) - b*sinh(1))*log(f) + 2*cosh(1)*sinh(1) + sinh(1)^2)/f)
 - sqrt(pi)*sqrt(-f)*erf(1/2*(2*f*x + b*log(f) + cosh(1) + sinh(1))*sqrt(-f
)/f)*sinh(1/4*(b^2*log(f)^2 - 4*d*f + cosh(1)^2 - 2*(2*a*f - b*cosh(1) - b*
sinh(1))*log(f) + 2*cosh(1)*sinh(1) + sinh(1)^2)/f))/f
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx}\cosh\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*cosh(f*x**2+e*x+d),x)
```

```
[Out] Integral(f**(a + b*x)*cosh(d + e*x + f*x**2), x)
```

Giac [A]
time = 0.41, size = 132, normalized size = 1.15

$$-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\sqrt{-f}\left(2x+\frac{b\log(f)+e}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2+2be\log(f)-4af\log(f)+e^2-4df}{4f}\right)}}{4\sqrt{-f}}-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\sqrt{f}\left(2x-\frac{b\log(f)-e}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2-2be\log(f)+4af\log(f)+e^2-4df}{4f}\right)}}{4\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-f)*(2*x + (b*log(f) + e)/f))*e^(-1/4*(b^2*log(
f)^2 + 2*b*e*log(f) - 4*a*f*log(f) + e^2 - 4*d*f)/f)/sqrt(-f) - 1/4*sqrt(pi
)*erf(-1/2*sqrt(f)*(2*x - (b*log(f) - e)/f))*e^(1/4*(b^2*log(f)^2 - 2*b*e*l
og(f) + 4*a*f*log(f) + e^2 - 4*d*f)/f)/sqrt(f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{a+b\,x}\,\cosh\left(f\,x^2+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x)*cosh(d + e*x + f*x^2),x)
```

```
[Out] int(f^(a + b*x)*cosh(d + e*x + f*x^2), x)
```

3.309 $\int f^{a+bx}\cosh^2(d+ex+fx^2)\,dx$

Optimal. Leaf size=161

$$\frac{1}{8}e^{-2d+\frac{(2e-b\log(f))^2}{8f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\frac{2e+4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right)+\frac{1}{8}e^{2d-\frac{(2e+b\log(f))^2}{8f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\,\mathrm{Erfi}\left(\frac{2e+4fx+b\log}{2\sqrt{2}\sqrt{f}}\right.$$

```
[Out] 1/2*f^(b*x+a)/b/ln(f)+1/16*exp(-2*d+1/8*(2*e-b*ln(f))^2/f)*f^(-1/2+a)*erf(1
/4*(2*e+4*f*x-b*ln(f))*2^(1/2)/f^(1/2))*2^(1/2)*Pi^(1/2)+1/16*exp(2*d-1/8*(
2*e+b*ln(f))^2/f)*f^(-1/2+a)*erfi(1/4*(2*e+4*f*x+b*ln(f))*2^(1/2)/f^(1/2))*
2^(1/2)*Pi^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5624, 2225, 2325, 2266, 2236, 2235}

$$\frac{1}{8}\sqrt{\frac{\pi}{2}}f^{a-\frac{1}{2}}e^{\frac{(2e-b\log(f))^2}{8f}-2d}\mathrm{Erf}\left(\frac{-b\log(f)+2e+4fx}{2\sqrt{2}\sqrt{f}}\right)+\frac{1}{8}\sqrt{\frac{\pi}{2}}f^{a-\frac{1}{2}}e^{2d-\frac{(b\log(f)+2e)^2}{8f}}\mathrm{Erfi}\left(\frac{b\log(f)+2e+4fx}{2\sqrt{2}\sqrt{f}}\right)+\frac{f^{a+bx}}{2b\log(f)}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Cosh[d + e*x + f*x^2]^2,x]
```

```
[Out] (E^(-2*d + (2*e - b*Log[f])^2/(8*f))*f^(-1/2 + a)*Sqrt[Pi/2]*Erf[(2*e + 4*f
*x - b*Log[f])/(2*Sqrt[2]*Sqrt[f])])/8 + (E^(2*d - (2*e + b*Log[f])^2/(8*f)
)*f^(-1/2 + a)*Sqrt[Pi/2]*Erfi[(2*e + 4*f*x + b*Log[f])/(2*Sqrt[2]*Sqrt[f])
])/8 + f^(a + b*x)/(2*b*Log[f])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5624

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int f^{a+bx}\cosh^2\left(d+ex+fx^2\right)\,dx &= \int\left(\frac{1}{2}f^{a+bx}+\frac{1}{4}e^{-2d-2ex-2fx^2}f^{a+bx}+\frac{1}{4}e^{2d+2ex+2fx^2}f^{a+bx}\right)dx \\
&= \frac{1}{4}\int e^{-2d-2ex-2fx^2}f^{a+bx}\,dx+\frac{1}{4}\int e^{2d+2ex+2fx^2}f^{a+bx}\,dx+\frac{1}{2}\int f^{a+bx}\,dx \\
&= \frac{f^{a+bx}}{2b\log(f)}+\frac{1}{4}\int\exp\left(-2d-2fx^2+a\log(f)-x(2e-b\log(f))\right)dx+\frac{1}{4} \\
&= \frac{f^{a+bx}}{2b\log(f)}+\frac{1}{4}\left(e^{-2d+\frac{(2e-b\log(f))^2}{8f}}f^a\right)\int e^{-\frac{(-2e-4fx+b\log(f))^2}{8f}}\,dx+\frac{1}{4}\left(e^{2d-\frac{(2e+}{}}\right. \\
&= \frac{1}{8}e^{-2d+\frac{(2e-b\log(f))^2}{8f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{2e+4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right)+\frac{1}{8}e^{2d-\frac{(2e+b\log(f)}{8f}}
\end{aligned}$$

Mathematica [A]
time = 0.45, size = 220, normalized size = 1.37

$$\frac{e^{-\frac{4e^2+b^2\log^2(f)}{8f}}f^{a-\frac{be+f}{2f}}\left(4\sqrt{2}\,e^{\frac{4e^2+b^2\log^2(f)}{8f}}f^{\frac{1}{2}+b\left(\frac{e}{2f}+x\right)}+be^{\frac{4e^2+b^2\log^2(f)}{4f}}\sqrt{\pi}\operatorname{Erf}\left(\frac{2e+4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right)\log(f)(\cosh(2d)-\sinh(2d))+b\sqrt{\pi}\operatorname{Erfi}\left(\frac{2e+4fx+b\log(f)}{2\sqrt{2}\sqrt{f}}\right)\log(f)(\cosh(2d)+\sinh(2d))\right)}{8\sqrt{2}\,b\log(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Cosh[d + e*x + f*x^2]^2,x]
```

```
[Out] (f^(a - (b*e + f)/(2*f))*(4*Sqrt[2]*E^((4*e^2 + b^2*Log[f]^2)/(8*f))*f^(1/2
 + b*(e/(2*f) + x)) + b*E^((4*e^2 + b^2*Log[f]^2)/(4*f))*Sqrt[Pi]*Erf[(2*e
+ 4*f*x - b*Log[f])/(2*Sqrt[2]*Sqrt[f])]*Log[f]*(Cosh[2*d] - Sinh[2*d]) + b
*Sqrt[Pi]*Erfi[(2*e + 4*f*x + b*Log[f])/(2*Sqrt[2]*Sqrt[f])]*Log[f]*(Cosh[2
*d] + Sinh[2*d])))/(8*Sqrt[2]*b*E^((4*e^2 + b^2*Log[f]^2)/(8*f))*Log[f])
```

Maple [A]
time = 2.15, size = 158, normalized size = 0.98

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{b^{2} \ln(f)^{2}-4 \ln(f) b e-16 d f+4 e^{2}}{8 f}} \sqrt{2}\, \operatorname{erf}\left(-\sqrt{2}\, \sqrt{f}\, x+\frac{(b \ln(f)-2 e) \sqrt{2}}{4 \sqrt{f}}\right)}{16 \sqrt{f}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2} \ln(f)^{2}+4 \ln(f) b e-16 d f+4 e}{8 f}}}{8 \sqrt{-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*cosh(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(1/8*(b^2*ln(f)^2-4*ln(f)*b*e-16*d*f+4*e^2)/f)*2^(1/2
)/f^(1/2)*erf(-2^(1/2)*f^(1/2)*x+1/4*(b*ln(f)-2*e)*2^(1/2)/f^(1/2))-1/8*Pi^
(1/2)*f^a*exp(-1/8*(b^2*ln(f)^2+4*ln(f)*b*e-16*d*f+4*e^2)/f)/(-2*f)^(1/2)*e
rf(-(-2*f)^(1/2)*x+1/2*(2*e+b*ln(f))/(-2*f)^(1/2))+1/2*f^a*f^(b*x)/b/ln(f)
```

Maxima [A]
time = 0.50, size = 147, normalized size = 0.91

$$\frac{\sqrt{2}\,\sqrt{\pi}\,f^{a}\operatorname{erf}\left(\sqrt{2}\,\sqrt{-f}\,x-\frac{\sqrt{2}\,(b\log(f)+2\,e)}{4\,\sqrt{-f}}\right)e^{\left(2\,d-\frac{(b\log(f)+2\,e)^{2}}{8\,f}\right)}}{16\,\sqrt{-f}}+\frac{\sqrt{2}\,\sqrt{\pi}\,f^{a}\operatorname{erf}\left(\sqrt{2}\,\sqrt{f}\,x-\frac{\sqrt{2}\,(b\log(f)-2\,e)}{4\,\sqrt{f}}\right)e^{\left(-2\,d+\frac{(b\log(f)-2\,e)^{2}}{8\,f}\right)}}{16\,\sqrt{f}}+\frac{f^{bx+a}}{2\,b\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*sqrt(pi)*f^a*erf(sqrt(2)*sqrt(-f)*x - 1/4*sqrt(2)*(b*log(f) +
2*e)/sqrt(-f))*e^(2*d - 1/8*(b*log(f) + 2*e)^2/f)/sqrt(-f) + 1/16*sqrt(2)*s
qrt(pi)*f^a*erf(sqrt(2)*sqrt(f)*x - 1/4*sqrt(2)*(b*log(f) - 2*e)/sqrt(f))*e
^(-2*d + 1/8*(b*log(f) - 2*e)^2/f)/sqrt(f) + 1/2*f^(b*x + a)/(b*log(f))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 434 vs. 2(130) = 260.
time = 0.39, size = 434, normalized size = 2.70

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(2)*sqrt(pi)*b*sqrt(-f)*cosh(1/8*(b^2*log(f)^2 - 16*d*f + 4*cosh
(1)^2 - 4*(2*a*f - b*cosh(1) - b*sinh(1))*log(f) + 8*cosh(1)*sinh(1) + 4*si
nh(1)^2)/f)*erf(1/4*sqrt(2)*(4*f*x + b*log(f) + 2*cosh(1) + 2*sinh(1))*sqrt
(-f)/f)*log(f) + sqrt(2)*sqrt(pi)*b*sqrt(f)*cosh(1/8*(b^2*log(f)^2 - 16*d*f
 + 4*cosh(1)^2 + 4*(2*a*f - b*cosh(1) - b*sinh(1))*log(f) + 8*cosh(1)*sinh(
1) + 4*sinh(1)^2)/f)*erf(-1/4*sqrt(2)*(4*f*x - b*log(f) + 2*cosh(1) + 2*sin
```

```
h(1))/sqrt(f))*log(f) + sqrt(2)*sqrt(pi)*b*sqrt(f)*erf(-1/4*sqrt(2)*(4*f*x
- b*log(f) + 2*cosh(1) + 2*sinh(1))/sqrt(f))*log(f)*sinh(1/8*(b^2*log(f)^2
- 16*d*f + 4*cosh(1)^2 + 4*(2*a*f - b*cosh(1) - b*sinh(1))*log(f) + 8*cosh(
1)*sinh(1) + 4*sinh(1)^2)/f) - sqrt(2)*sqrt(pi)*b*sqrt(-f)*erf(1/4*sqrt(2)*
(4*f*x + b*log(f) + 2*cosh(1) + 2*sinh(1))*sqrt(-f)/f)*log(f)*sinh(1/8*(b^2
*log(f)^2 - 16*d*f + 4*cosh(1)^2 - 4*(2*a*f - b*cosh(1) - b*sinh(1))*log(f)
 + 8*cosh(1)*sinh(1) + 4*sinh(1)^2)/f) - 8*f*cosh((b*x + a)*log(f)) - 8*f*s
inh((b*x + a)*log(f)))/(b*f*log(f))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx}\cosh^2\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*cosh(f*x**2+e*x+d)**2,x)
```

```
[Out] Integral(f**(a + b*x)*cosh(d + e*x + f*x**2)**2, x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.42, size = 387, normalized size = 2.40

$$-\frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{4}\sqrt{2}\sqrt{-f}\left(4x+\frac{b\log(f)+2e}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2+4be\log(f)-8af\log(f)+4e^2-16df}{8f}\right)}}{16\sqrt{-f}}-\frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{4}\sqrt{2}\sqrt{f}\left(4x-\frac{b\log(f)-2e}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2-4be\log(f)+8af\log(f)+4e^2-16df}{8f}\right)}}{16\sqrt{f}}+\left(\frac{2b\cos\left(-\frac{1}{2}\pi bx\operatorname{sgn}(f)+\frac{1}{2}\pi bx-\frac{1}{2}\pi a\operatorname{sgn}(f)+\frac{1}{2}\pi a\right)\log\left(|f|\right)}{4b^2\log\left(|f|\right)^2+\left(\pi b\operatorname{sgn}\left(f\right)-\pi b\right)^2}-\frac{\left(\pi b\operatorname{sgn}(f)-\pi b\right)\sin\left(-\frac{1}{2}\pi bx\operatorname{sgn}(f)+\frac{1}{2}\pi bx-\frac{1}{2}\pi a\operatorname{sgn}(f)+\frac{1}{2}\pi a\right)}{4b^2\log\left(|f|\right)^2+\left(\pi b\operatorname{sgn}\left(f\right)-\pi b\right)^2}\right)e^{\left(bx\log(|f|)+a\log(|f|)\right)}+i\left(\frac{ie^{\left(\frac{1}{2}i\pi bx\operatorname{sgn}(f)-\frac{1}{2}i\pi bx+\frac{1}{2}i\pi a\operatorname{sgn}(f)-\frac{1}{2}i\pi a\right)}}{2i\pi b\operatorname{sgn}\left(f\right)-2i\pi b+4b\log\left(|f|\right)}-\frac{ie^{\left(-\frac{1}{2}i\pi bx\operatorname{sgn}(f)+\frac{1}{2}i\pi bx-\frac{1}{2}i\pi a\operatorname{sgn}(f)+\frac{1}{2}i\pi a\right)}}{-2i\pi b\operatorname{sgn}\left(f\right)+2i\pi b+4b\log\left(|f|\right)}\right)e^{\left(bx\log(|f|)+a\log(|f|)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*sqrt(-f)*(4*x + (b*log(f) + 2*e)/f)
)*e^(-1/8*(b^2*log(f)^2 + 4*b*e*log(f) - 8*a*f*log(f) + 4*e^2 - 16*d*f)/f)/
sqrt(-f) - 1/16*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*sqrt(f)*(4*x - (b*log(f)
- 2*e)/f))*e^(1/8*(b^2*log(f)^2 - 4*b*e*log(f) + 8*a*f*log(f) + 4*e^2 - 16*
d*f)/f)/sqrt(f) + (2*b*cos(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f
) + 1/2*pi*a)*log(abs(f))/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2) -
(pi*b*sgn(f) - pi*b)*sin(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f)
+ 1/2*pi*a)/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2))*e^(b*x*log(abs(
f)) + a*log(abs(f))) + I*(I*e^(1/2*I*pi*b*x*sgn(f) - 1/2*I*pi*b*x + 1/2*I*p
i*a*sgn(f) - 1/2*I*pi*a)/(2*I*pi*b*sgn(f) - 2*I*pi*b + 4*b*log(abs(f))) - I
*e^(-1/2*I*pi*b*x*sgn(f) + 1/2*I*pi*b*x - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a)/(
-2*I*pi*b*sgn(f) + 2*I*pi*b + 4*b*log(abs(f))))*e^(b*x*log(abs(f)) + a*log(
abs(f)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{a+b\,x}\,{\mathrm{cosh}\left(f\,x^2+e\,x+d\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x)*cosh(d + e*x + f*x^2)^2,x)
```

```
[Out] int(f^(a + b*x)*cosh(d + e*x + f*x^2)^2, x)
```

3.310 $\int f^{a+bx}\cosh^3\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=257

$$\frac{3}{16}e^{-d+\frac{(e-b\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{e+2fx-b\log(f)}{2\sqrt{f}}\right)+\frac{1}{16}e^{-3d+\frac{(3e-b\log(f))^2}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{3}}\,\mathrm{Erf}\left(\frac{3e+6fx-b\log(}{2\sqrt{3}\sqrt{f}}\right.$$

```
[Out] 1/48*exp(-3*d+1/12*(3*e-b*ln(f))^2/f)*f^(-1/2+a)*erf(1/6*(3*e+6*f*x-b*ln(f)
)*3^(1/2)/f^(1/2))*3^(1/2)*Pi^(1/2)+1/48*exp(3*d-1/12*(3*e+b*ln(f))^2/f)*f^
(-1/2+a)*erfi(1/6*(3*e+6*f*x+b*ln(f))*3^(1/2)/f^(1/2))*3^(1/2)*Pi^(1/2)+3/1
6*exp(-d+1/4*(e-b*ln(f))^2/f)*f^(-1/2+a)*erf(1/2*(e+2*f*x-b*ln(f))/f^(1/2))
*Pi^(1/2)+3/16*exp(d-1/4*(e+b*ln(f))^2/f)*f^(-1/2+a)*erfi(1/2*(e+2*f*x+b*ln
(f))/f^(1/2))*Pi^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.238$, Rules used = {5624, 2325, 2266, 2236, 2235}

$$\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{(e-b\log(f))^2}{4f}-d}\mathrm{Erf}\left(\frac{-b\log(f)+e+2fx}{2\sqrt{f}}\right)+\frac{1}{16}\sqrt{\frac{\pi}{3}}f^{a-\frac{1}{2}}e^{\frac{(3e-b\log(f))^2}{12f}-3d}\mathrm{Erf}\left(\frac{-b\log(f)+3e+6fx}{2\sqrt{3}\sqrt{f}}\right)+\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}e^{d-\frac{(b\log(f)+e)^2}{4f}}\mathrm{Erfi}\left(\frac{b\log(f)+e+2fx}{2\sqrt{f}}\right)+\frac{1}{16}\sqrt{\frac{\pi}{3}}f^{a-\frac{1}{2}}e^{3d-\frac{(b\log(f)+3e)^2}{12f}}\mathrm{Erfi}\left(\frac{b\log(f)+3e+6fx}{2\sqrt{3}\sqrt{f}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Cosh[d + e*x + f*x^2]^3,x]
```

```
[Out] (3*E^(-d + (e - b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(e + 2*f*x - b
*Log[f])/(2*Sqrt[f])])/16 + (E^(-3*d + (3*e - b*Log[f])^2/(12*f))*f^(-1/2 +
 a)*Sqrt[Pi/3]*Erf[(3*e + 6*f*x - b*Log[f])/(2*Sqrt[3]*Sqrt[f])])/16 + (3*E
^(d - (e + b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erfi[(e + 2*f*x + b*Log
[f])/(2*Sqrt[f])])/16 + (E^(3*d - (3*e + b*Log[f])^2/(12*f))*f^(-1/2 + a)*S
qrt[Pi/3]*Erfi[(3*e + 6*f*x + b*Log[f])/(2*Sqrt[3]*Sqrt[f])])/16
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5624

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int f^{a+bx}\cosh^3\left(d+ex+fx^2\right)\,dx &= \int\left(\frac{1}{8}e^{-3\left(d+ex+fx^2\right)}f^{a+bx}+\frac{3}{8}\exp\left(2d+2ex+2fx^2-3\left(d+ex+fx^2\right)\right.\right.\\
&= \frac{1}{8}\int e^{-3\left(d+ex+fx^2\right)}f^{a+bx}\,dx+\frac{1}{8}\int\exp\left(6d+6ex+6fx^2-3\left(d+ex+\right.\right.\\
&= \frac{1}{8}\int\exp\left(-3d-3fx^2+a\log(f)-x(3e-b\log(f))\right)\,dx+\frac{1}{8}\int\exp\left(3\right.\\
&= \frac{1}{8}\left(3e^{-d+\frac{(e-b\log(f))^2}{4f}}f^a\right)\int e^{-\frac{(-e-2fx+b\log(f))^2}{4f}}\,dx+\frac{1}{8}\left(e^{-3d+\frac{(3e-b\log(f))^2}{12f}}f^a\right)\\
&= \frac{3}{16}e^{-d+\frac{(e-b\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\text{erf}\left(\frac{e+2fx-b\log(f)}{2\sqrt{f}}\right)+\frac{1}{16}e^{-3d+\frac{(3e-b\log(}{12f}}
\end{aligned}$$

Mathematica [A]

time = 0.62, size = 353, normalized size = 1.37

$$\frac{1}{16}e^{-\frac{3e^2+b^2\log^2(f)}{4f}}f^{a-\frac{be+f}{2f}}\sqrt{\frac{\pi}{3}}\left(3\sqrt{3}\,e^{\frac{e^2}{2f}}\cosh(d)\text{Erfi}\left(\frac{e+2fx+b\log(f)}{2\sqrt{f}}\right)+e^{\frac{b^2\log^2(f)}{6f}}\cosh(3d)\text{Erfi}\left(\frac{3e+6fx+b\log(f)}{2\sqrt{3}\sqrt{f}}\right)+3\sqrt{3}\,e^{\frac{2e^2+b^2\log^2(f)}{2f}}\text{Erf}\left(\frac{e+2fx-b\log(f)}{2\sqrt{f}}\right)(\cosh(d)-\sinh(d))+3\sqrt{3}\,e^{\frac{e^2}{2f}}\text{Erfi}\left(\frac{e+2fx+b\log(f)}{2\sqrt{f}}\right)\sinh(d)+e^{\frac{9e^2+2b^2\log^2(f)}{6f}}\text{Erf}\left(\frac{3e+6fx-b\log(f)}{2\sqrt{3}\sqrt{f}}\right)(\cosh(3d)-\sinh(3d))+e^{\frac{b^2\log^2(f)}{6f}}\text{Erfi}\left(\frac{3e+6fx+b\log(f)}{2\sqrt{3}\sqrt{f}}\right)\sinh(3d)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Cosh[d + e*x + f*x^2]^3,x]
```

```
[Out] (f^(a - (b*e + f)/(2*f))*Sqrt[Pi/3]*(3*Sqrt[3]*E^(e^2/(2*f))*Cosh[d]*Erfi[(
e + 2*f*x + b*Log[f])/(2*Sqrt[f])] + E^((b^2*Log[f]^2)/(6*f))*Cosh[3*d]*Erf
i[(3*e + 6*f*x + b*Log[f])/(2*Sqrt[3]*Sqrt[f])] + 3*Sqrt[3]*E^((2*e^2 + b^2
*Log[f]^2)/(2*f))*Erf[(e + 2*f*x - b*Log[f])/(2*Sqrt[f])]*(Cosh[d] - Sinh[d
]) + 3*Sqrt[3]*E^(e^2/(2*f))*Erfi[(e + 2*f*x + b*Log[f])/(2*Sqrt[f])]*Sinh[
```

```
d] + E^((9*e^2 + 2*b^2*Log[f]^2)/(6*f))*Erf[(3*e + 6*f*x - b*Log[f])/(2*Sqr
t[3]*Sqrt[f])]*(Cosh[3*d] - Sinh[3*d]) + E^((b^2*Log[f]^2)/(6*f))*Erfi[(3*e
 + 6*f*x + b*Log[f])/(2*Sqrt[3]*Sqrt[f])]*Sinh[3*d]))/(16*E^((3*e^2 + b^2*L
og[f]^2)/(4*f)))
```

Maple [A]
time = 2.85, size = 265, normalized size = 1.03

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{b^{2} \ln(f)^{2}-6 \ln(f) b e-36 d f+9 e^{2}}{12 f}} \sqrt{3}\, \mathrm{erf}\left(-\sqrt{3}\, \sqrt{f}\, x+\frac{(b \ln(f)-3 e) \sqrt{3}}{6 \sqrt{f}}\right)}{48 \sqrt{f}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2} \ln(f)^{2}+6 \ln(f) b e-36 d f+9 e^{2}}{12 f}}}{16 \sqrt{-3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*cosh(f*x^2+e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*Pi^(1/2)*f^a*exp(1/12*(b^2*ln(f)^2-6*ln(f)*b*e-36*d*f+9*e^2)/f)*3^(1/
2)/f^(1/2)*erf(-3^(1/2)*f^(1/2)*x+1/6*(b*ln(f)-3*e)*3^(1/2)/f^(1/2))-1/16*P
i^(1/2)*f^a*exp(-1/12*(b^2*ln(f)^2+6*ln(f)*b*e-36*d*f+9*e^2)/f)/(-3*f)^(1/2
)*erf(-(-3*f)^(1/2)*x+1/2*(3*e+b*ln(f))/(-3*f)^(1/2))-3/16*Pi^(1/2)*f^a*exp
(1/4*(b^2*ln(f)^2-2*ln(f)*b*e-4*d*f+e^2)/f)/f^(1/2)*erf(-f^(1/2)*x+1/2*(b*l
n(f)-e)/f^(1/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+2*ln(f)*b*e-4*d*f+
e^2)/f)/(-f)^(1/2)*erf(-(-f)^(1/2)*x+1/2*(e+b*ln(f))/(-f)^(1/2))
```

Maxima [A]
time = 0.51, size = 236, normalized size = 0.92

$$\frac{\sqrt{3}\,\sqrt{\pi}\,f^{a}\,\mathrm{erf}\left(\sqrt{3}\,\sqrt{-f}\,x-\frac{\sqrt{3}\,(b\log(f)+3\,e)}{6\,\sqrt{-f}}\right)e^{\left(3\,d-\frac{(b\log(f)+3\,e)^{2}}{12\,f}\right)}}{48\,\sqrt{-f}}+\frac{3}{16}\,\sqrt{\pi}\,f^{a-\frac{1}{2}}\,\mathrm{erf}\left(\sqrt{f}\,x-\frac{b\log(f)-e}{2\,\sqrt{f}}\right)e^{\left(-d+\frac{(b\log(f)-e)^{2}}{4\,f}\right)}+\frac{\sqrt{3}\,\sqrt{\pi}\,f^{a}\,\mathrm{erf}\left(\sqrt{3}\,\sqrt{f}\,x-\frac{\sqrt{3}\,(b\log(f)-3\,e)}{6\,\sqrt{f}}\right)e^{\left(-3\,d+\frac{(b\log(f)-3\,e)^{2}}{12\,f}\right)}}{48\,\sqrt{f}}+\frac{3\,\sqrt{\pi}\,f^{a}\,\mathrm{erf}\left(\sqrt{-f}\,x-\frac{b\log(f)+e}{2\,\sqrt{-f}}\right)e^{\left(d-\frac{(b\log(f)+e)^{2}}{4\,f}\right)}}{16\,\sqrt{-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/48*sqrt(3)*sqrt(pi)*f^a*erf(sqrt(3)*sqrt(-f)*x - 1/6*sqrt(3)*(b*log(f) +
3*e)/sqrt(-f))*e^(3*d - 1/12*(b*log(f) + 3*e)^2/f)/sqrt(-f) + 3/16*sqrt(pi)
*f^(a - 1/2)*erf(sqrt(f)*x - 1/2*(b*log(f) - e)/sqrt(f))*e^(-d + 1/4*(b*log
(f) - e)^2/f) + 1/48*sqrt(3)*sqrt(pi)*f^a*erf(sqrt(3)*sqrt(f)*x - 1/6*sqrt(
3)*(b*log(f) - 3*e)/sqrt(f))*e^(-3*d + 1/12*(b*log(f) - 3*e)^2/f)/sqrt(f) +
 3/16*sqrt(pi)*f^a*erf(sqrt(-f)*x - 1/2*(b*log(f) + e)/sqrt(-f))*e^(d - 1/4
*(b*log(f) + e)^2/f)/sqrt(-f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 723 vs. $2(207) = 414$.
time = 0.47, size = 723, normalized size = 2.81

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/48*(sqrt(3)*sqrt(pi)*sqrt(-f)*cosh(1/12*(b^2*log(f)^2 - 36*d*f + 9*cosh(
1)^2 - 6*(2*a*f - b*cosh(1) - b*sinh(1))*log(f) + 18*cosh(1)*sinh(1) + 9*si
nh(1)^2)/f)*erf(1/6*sqrt(3)*(6*f*x + b*log(f) + 3*cosh(1) + 3*sinh(1))*sqrt
(-f)/f) + sqrt(3)*sqrt(pi)*sqrt(f)*cosh(1/12*(b^2*log(f)^2 - 36*d*f + 9*cos
h(1)^2 + 6*(2*a*f - b*cosh(1) - b*sinh(1))*log(f) + 18*cosh(1)*sinh(1) + 9*
sinh(1)^2)/f)*erf(-1/6*sqrt(3)*(6*f*x - b*log(f) + 3*cosh(1) + 3*sinh(1))/s
qrt(f)) + sqrt(3)*sqrt(pi)*sqrt(f)*erf(-1/6*sqrt(3)*(6*f*x - b*log(f) + 3*c
osh(1) + 3*sinh(1))/sqrt(f))*sinh(1/12*(b^2*log(f)^2 - 36*d*f + 9*cosh(1)^2
 + 6*(2*a*f - b*cosh(1) - b*sinh(1))*log(f) + 18*cosh(1)*sinh(1) + 9*sinh(1
)^2)/f) - sqrt(3)*sqrt(pi)*sqrt(-f)*erf(1/6*sqrt(3)*(6*f*x + b*log(f) + 3*c
osh(1) + 3*sinh(1))*sqrt(-f)/f)*sinh(1/12*(b^2*log(f)^2 - 36*d*f + 9*cosh(1
)^2 - 6*(2*a*f - b*cosh(1) - b*sinh(1))*log(f) + 18*cosh(1)*sinh(1) + 9*sin
h(1)^2)/f) + 9*sqrt(pi)*sqrt(-f)*cosh(1/4*(b^2*log(f)^2 - 4*d*f + cosh(1)^2
 - 2*(2*a*f - b*cosh(1) - b*sinh(1))*log(f) + 2*cosh(1)*sinh(1) + sinh(1)^2
)/f)*erf(1/2*(2*f*x + b*log(f) + cosh(1) + sinh(1))*sqrt(-f)/f) + 9*sqrt(pi
)*sqrt(f)*cosh(1/4*(b^2*log(f)^2 - 4*d*f + cosh(1)^2 + 2*(2*a*f - b*cosh(1)
 - b*sinh(1))*log(f) + 2*cosh(1)*sinh(1) + sinh(1)^2)/f)*erf(-1/2*(2*f*x -
b*log(f) + cosh(1) + sinh(1))/sqrt(f)) + 9*sqrt(pi)*sqrt(f)*erf(-1/2*(2*f*x
 - b*log(f) + cosh(1) + sinh(1))/sqrt(f))*sinh(1/4*(b^2*log(f)^2 - 4*d*f +
cosh(1)^2 + 2*(2*a*f - b*cosh(1) - b*sinh(1))*log(f) + 2*cosh(1)*sinh(1) +
sinh(1)^2)/f) - 9*sqrt(pi)*sqrt(-f)*erf(1/2*(2*f*x + b*log(f) + cosh(1) + s
inh(1))*sqrt(-f)/f)*sinh(1/4*(b^2*log(f)^2 - 4*d*f + cosh(1)^2 - 2*(2*a*f -
 b*cosh(1) - b*sinh(1))*log(f) + 2*cosh(1)*sinh(1) + sinh(1)^2)/f))/f
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx}\cosh^{3}\left(d+ex+fx^{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*cosh(f*x**2+e*x+d)**3,x)
```

```
[Out] Integral(f**(a + b*x)*cosh(d + e*x + f*x**2)**3, x)
```

Giac [A]
time = 0.42, size = 281, normalized size = 1.09

$$-\frac{\sqrt{3}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{6}\sqrt{3}\sqrt{-f}\left(6x+\frac{b\log(f)+3e}{f}\right)\right)e^{\left(-\frac{b^{2}\log(f)^{2}+6be\log(f)-12af\log(f)+9e^{2}-36df}{12f}\right)}}{48\sqrt{-f}}-\frac{\sqrt{3}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{6}\sqrt{3}\sqrt{f}\left(6x-\frac{b\log(f)-3e}{f}\right)\right)e^{\left(\frac{b^{2}\log(f)^{2}-6be\log(f)+12af\log(f)+9e^{2}-36df}{12f}\right)}}{48\sqrt{f}}-\frac{3\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-f}\left(2x+\frac{b\log(f)+e}{f}\right)\right)e^{\left(-\frac{b^{2}\log(f)^{2}+2be\log(f)-4af\log(f)+e^{2}-4df}{4f}\right)}}{16\sqrt{-f}}-\frac{3\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{f}\left(2x-\frac{b\log(f)-e}{f}\right)\right)e^{\left(\frac{b^{2}\log(f)^{2}-2be\log(f)+4af\log(f)+e^{2}-4df}{4f}\right)}}{16\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+e*x+d)^3,x, algorithm="giac")
```

```
[Out] -1/48*sqrt(3)*sqrt(pi)*erf(-1/6*sqrt(3)*sqrt(-f)*(6*x + (b*log(f) + 3*e)/f)
)*e^(-1/12*(b^2*log(f)^2 + 6*b*e*log(f) - 12*a*f*log(f) + 9*e^2 - 36*d*f)/f
)/sqrt(-f) - 1/48*sqrt(3)*sqrt(pi)*erf(-1/6*sqrt(3)*sqrt(f)*(6*x - (b*log(f
) - 3*e)/f))*e^(1/12*(b^2*log(f)^2 - 6*b*e*log(f) + 12*a*f*log(f) + 9*e^2 -
 36*d*f)/f)/sqrt(f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(-f)*(2*x + (b*log(f) + e)
/f))*e^(-1/4*(b^2*log(f)^2 + 2*b*e*log(f) - 4*a*f*log(f) + e^2 - 4*d*f)/f)/
sqrt(-f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(f)*(2*x - (b*log(f) - e)/f))*e^(1/4*
(b^2*log(f)^2 - 2*b*e*log(f) + 4*a*f*log(f) + e^2 - 4*d*f)/f)/sqrt(f)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int f^{a+b\,x}\,\cosh\left(f\,x^2+e\,x+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x)*cosh(d + e*x + f*x^2)^3,x)
```

```
[Out] int(f^(a + b*x)*cosh(d + e*x + f*x^2)^3, x)
```

3.311 $\int f^{a+cx^2}\cosh(d+ex)\,dx$

Optimal. Leaf size=133

$$-\frac{e^{-d-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{e^{d-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{e+2cx\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}$$

```
[Out] 1/4*exp(-d-1/4*e^2/c/ln(f))*f^a*erfi(1/2*(-e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/
2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/4*exp(d-1/4*e^2/c/ln(f))*f^a*erfi(1/2*(e
+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5624, 2325, 2266, 2235}

$$\frac{\sqrt{\pi}\,f^a e^{d-\frac{e^2}{4c\log(f)}}\mathrm{Erfi}\left(\frac{2cx\log(f)+e}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}-\frac{\sqrt{\pi}\,f^a e^{-\frac{e^2}{4c\log(f)}-d}\mathrm{Erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Cosh[d + e*x],x]
```

```
[Out] -1/4*(E^(-d - e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - 2*c*x*Log[f])/(2*Sqr
t[c]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt[Log[f]]) + (E^(d - e^2/(4*c*Log[f]))*f^a
*Sqrt[Pi]*Erfi[(e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(4*Sqrt[c]*Sqr
t[Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5624

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int f^{a+cx^2}\cosh(d+ex)\,dx &= \int\left(\frac{1}{2}e^{-d-ex}f^{a+cx^2}+\frac{1}{2}e^{d+ex}f^{a+cx^2}\right)dx\\
&= \frac{1}{2}\int e^{-d-ex}f^{a+cx^2}\,dx+\frac{1}{2}\int e^{d+ex}f^{a+cx^2}\,dx\\
&= \frac{1}{2}\int e^{-d-ex+a\log(f)+cx^2\log(f)}\,dx+\frac{1}{2}\int e^{d+ex+a\log(f)+cx^2\log(f)}\,dx\\
&= \frac{1}{2}\left(e^{-d-\frac{e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(-e+2cx\log(f))^2}{4c\log(f)}}\,dx+\frac{1}{2}\left(e^{d-\frac{e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(e+2cx\log(f))^2}{4c\log(f)}}\,dx\\
&= -\frac{e^{-d-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{d-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}
\end{aligned}$$

Mathematica [A]
time = 0.11, size = 104, normalized size = 0.78

$$\frac{e^{-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\left(\mathrm{Erfi}\left(\frac{-e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cosh(d)-\sinh(d))+\mathrm{Erfi}\left(\frac{e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cosh(d)+\sinh(d))\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Cosh[d + e*x],x]
[Out] (f^a*Sqrt[Pi]*(Erfi[(-e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[d]
- Sinh[d]) + Erfi[(e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[d] + S
inh[d])))/(4*Sqrt[c]*E^(e^2/(4*c*Log[f]))*Sqrt[Log[f]])
```

Maple [A]
time = 0.77, size = 117, normalized size = 0.88

method	result
risch	$\frac{\sqrt{\pi}\,f^a\mathrm{e}^{-\frac{4d\ln(f)c+e^2}{4\ln(f)c}}\,\mathrm{erf}\left(\sqrt{-c\ln(f)}\,x+\frac{e}{2\sqrt{-c\ln(f)}}\right)}{4\sqrt{-c\ln(f)}}-\frac{\sqrt{\pi}\,f^a\mathrm{e}^{\frac{4d\ln(f)c-e^2}{4\ln(f)c}}\,\mathrm{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{}{2\sqrt{-c}}\right.}{4\sqrt{-c\ln(f)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*cosh(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*Pi^(1/2)*f^a*exp(-1/4*(4*d*ln(f)*c+e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf((
-c*ln(f))^(1/2)*x+1/2*e/(-c*ln(f))^(1/2))-1/4*Pi^(1/2)*f^a*exp(1/4*(4*d*ln(
f)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*e/(-c*ln(f)
)^(1/2))
```

Maxima [A]
time = 0.28, size = 105, normalized size = 0.79

$$\frac{\sqrt{\pi}\, f^a \operatorname{erf}\left(\sqrt{-c\log\left(f\right)}\, x - \frac{e}{2\,\sqrt{-c\log\left(f\right)}}\right) e^{\left(d - \frac{e^2}{4\,c\log\left(f\right)}\right)}}{4\,\sqrt{-c\log\left(f\right)}} + \frac{\sqrt{\pi}\, f^a \operatorname{erf}\left(\sqrt{-c\log\left(f\right)}\, x + \frac{e}{2\,\sqrt{-c\log\left(f\right)}}\right) e^{\left(-d - \frac{e^2}{4\,c\log\left(f\right)}\right)}}{4\,\sqrt{-c\log\left(f\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*e/sqrt(-c*log(f)))*e^(d - 1/4*
e^2/(c*log(f)))/sqrt(-c*log(f)) + 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x +
1/2*e/sqrt(-c*log(f)))*e^(-d - 1/4*e^2/(c*log(f)))/sqrt(-c*log(f))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(101) = 202$.
time = 0.37, size = 276, normalized size = 2.08

$$-\frac{\sqrt{-c\log\left(f\right)}\left(\sqrt{\pi}\cosh\left(\frac{4\,ac\log\left(f\right)^2+4\,cd\log\left(f\right)-\cosh\left(1\right)^2-2\,\cosh\left(1\right)\sinh\left(1\right)-\sinh\left(1\right)^2}{4\,c\log\left(f\right)}\right)+\sqrt{\pi}\sinh\left(\frac{4\,ac\log\left(f\right)^2+4\,cd\log\left(f\right)-\cosh\left(1\right)^2-2\,\cosh\left(1\right)\sinh\left(1\right)-\sinh\left(1\right)^2}{4\,c\log\left(f\right)}\right)\right)\operatorname{erf}\left(\frac{\left(2\,cx\log\left(f\right)+\cosh\left(1\right)+\sinh\left(1\right)\right)\sqrt{-c\log\left(f\right)}}{2\,c\log\left(f\right)}\right)+\sqrt{-c\log\left(f\right)}\left(\sqrt{\pi}\cosh\left(\frac{4\,ac\log\left(f\right)^2-4\,cd\log\left(f\right)-\cosh\left(1\right)^2-2\,\cosh\left(1\right)\sinh\left(1\right)-\sinh\left(1\right)^2}{4\,c\log\left(f\right)}\right)+\sqrt{\pi}\sinh\left(\frac{4\,ac\log\left(f\right)^2-4\,cd\log\left(f\right)-\cosh\left(1\right)^2-2\,\cosh\left(1\right)\sinh\left(1\right)-\sinh\left(1\right)^2}{4\,c\log\left(f\right)}\right)\right)\operatorname{erf}\left(\frac{\left(2\,cx\log\left(f\right)-\cosh\left(1\right)-\sinh\left(1\right)\right)\sqrt{-c\log\left(f\right)}}{2\,c\log\left(f\right)}\right)}{4\,c\log\left(f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(-c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f) - c
osh(1)^2 - 2*cosh(1)*sinh(1) - sinh(1)^2)/(c*log(f))) + sqrt(pi)*sinh(1/4*(
4*a*c*log(f)^2 + 4*c*d*log(f) - cosh(1)^2 - 2*cosh(1)*sinh(1) - sinh(1)^2)/
(c*log(f))))*erf(1/2*(2*c*x*log(f) + cosh(1) + sinh(1))*sqrt(-c*log(f))/(c*
log(f))) + sqrt(-c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f
) - cosh(1)^2 - 2*cosh(1)*sinh(1) - sinh(1)^2)/(c*log(f))) + sqrt(pi)*sinh(
1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - cosh(1)^2 - 2*cosh(1)*sinh(1) - sinh(1
)^2)/(c*log(f))))*erf(1/2*(2*c*x*log(f) - cosh(1) - sinh(1))*sqrt(-c*log(f)
)/(c*log(f))))/(c*log(f))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^2}\cosh\left(d+ex\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f**(c*x**2+a)*cosh(e*x+d),x)

[Out] Integral(f**(a + c*x**2)*cosh(d + e*x), x)

Giac [A]
time = 0.41, size = 132, normalized size = 0.99

$$-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)}\,\left(2\,x+\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{4\,ac\log(f)^{2}+4\,cd\log(f)-e^{2}}{4\,c\log(f)}\right)}}{4\,\sqrt{-c\log\left(f\right)}}-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)}\,\left(2\,x-\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{4\,ac\log(f)^{2}-4\,cd\log(f)-e^{2}}{4\,c\log(f)}\right)}}{4\,\sqrt{-c\log\left(f\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c*x^2+a)*cosh(e*x+d),x, algorithm="giac")

[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + e/(c*log(f))))*e^(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f) - e^2)/(c*log(f)))/sqrt(-c*log(f)) - 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x - e/(c*log(f))))*e^(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - e^2)/(c*log(f)))/sqrt(-c*log(f))

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+a}\,\cosh(d+e\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a + c*x^2)*cosh(d + e*x),x)

[Out] int(f^(a + c*x^2)*cosh(d + e*x), x)

3.312 $\int f^{a+cx^2} \cosh^2(d+ex)\,dx$

Optimal. Leaf size=161

$$\frac{f^a\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}-\frac{e^{-2d-\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{e-cx\log(f)}{\sqrt{c}\,\sqrt{\log(f)}}\right)}{8\sqrt{c}\,\sqrt{\log(f)}}+\frac{e^{2d-\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{e+cx\log(f)}{\sqrt{c}\,\sqrt{\log(f)}}\right)}{8\sqrt{c}\,\sqrt{\log(f)}}$$

```
[Out] 1/8*exp(-2*d-e^2/c/ln(f))*f^a*erfi((-e+c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(
1/2)/c^(1/2)/ln(f)^(1/2)+1/8*exp(2*d-e^2/c/ln(f))*f^a*erfi((e+c*x*ln(f))/c^
(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/4*f^a*erfi(x*c^(1/2)*ln(f
)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5624, 2235, 2325, 2266}

$$-\frac{\sqrt{\pi}\,f^a e^{-\frac{e^2}{c\log(f)}-2d}\mathrm{Erfi}\left(\frac{e-cx\log(f)}{\sqrt{c}\,\sqrt{\log(f)}}\right)}{8\sqrt{c}\,\sqrt{\log(f)}}+\frac{\sqrt{\pi}\,f^a e^{2d-\frac{e^2}{c\log(f)}}\mathrm{Erfi}\left(\frac{cx\log(f)+e}{\sqrt{c}\,\sqrt{\log(f)}}\right)}{8\sqrt{c}\,\sqrt{\log(f)}}+\frac{\sqrt{\pi}\,f^a\mathrm{Erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Cosh[d + e*x]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) - (E^(
-2*d - e^2/(c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - c*x*Log[f])/(Sqrt[c]*Sqrt[Log
[f]])])/(8*Sqrt[c]*Sqrt[Log[f]]) + (E^(2*d - e^2/(c*Log[f]))*f^a*Sqrt[Pi]*E
rfi[(e + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])])/(8*Sqrt[c]*Sqrt[Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5624

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int f^{a+cx^2}\cosh^2(d+ex)\,dx = \int \left(\frac{1}{2}f^{a+cx^2}+\frac{1}{4}e^{-2d-2ex}f^{a+cx^2}+\frac{1}{4}e^{2d+2ex}f^{a+cx^2}\right)dx$$

$$= \frac{1}{4}\int e^{-2d-2ex}f^{a+cx^2}\,dx+\frac{1}{4}\int e^{2d+2ex}f^{a+cx^2}\,dx+\frac{1}{2}\int f^{a+cx^2}\,dx$$

$$= \frac{f^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{1}{4}\int e^{-2d-2ex+a\log(f)+cx^2\log(f)}\,dx+\frac{1}{4}\int e^{2d+2ex+}$$

$$= \frac{f^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{1}{4}\left(e^{-2d-\frac{e^2}{c\log(f)}}f^a\right)\int e^{\frac{(-2e+2cx\log(f))^2}{4c\log(f)}}\,dx+\frac{1}{4}\left(e^{2d}\right.$$

$$= \frac{f^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}-\frac{e^{-2d-\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e-cx\log(f)}{\sqrt{c}\,\sqrt{\log(f)}}\right)}{8\sqrt{c}\,\sqrt{\log(f)}}+\frac{e^{2d-\frac{}{c\,\mathrm{l}}}}{}$$

Mathematica [A]
time = 0.16, size = 131, normalized size = 0.81

$$\frac{e^{-\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\left(2e^{\frac{e^2}{c\log(f)}}\mathrm{Erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)+\mathrm{Erfi}\left(\frac{-e+cx\log(f)}{\sqrt{c}\,\sqrt{\log(f)}}\right)(\cosh(2d)-\sinh(2d))+\mathrm{Erfi}\left(\frac{e+cx\log(f)}{\sqrt{c}\,\sqrt{\log(f)}}\right)(\cosh(2d)+\sinh(2d))\right)}{8\sqrt{c}\,\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Cosh[d + e*x]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*(2*E^(e^2/(c*Log[f]))*Erfi[Sqrt[c]*x*Sqrt[Log[f]]] + Erfi[(-e
 + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] - Sinh[2*d]) + Erfi[(e +
c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] + Sinh[2*d])))/(8*Sqrt[c]*E^
(e^2/(c*Log[f]))*Sqrt[Log[f]])
```

Maple [A]
time = 1.34, size = 139, normalized size = 0.86

method	result

risch	$\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{2 d \ln(f) c+e^{2}}{\ln(f) c}} \operatorname{erf}\left(\sqrt{-c \ln(f)}\, x+\frac{e}{\sqrt{-c \ln(f)}}\right)}{8 \sqrt{-c \ln(f)}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{2 d \ln(f) c-e^{2}}{\ln(f) c}} \operatorname{erf}\left(-\sqrt{-c \ln(f)}\, x+\frac{}{\sqrt{-c}}\right.}{8 \sqrt{-c \ln(f)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*cosh(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*Pi^(1/2)*f^a*exp(-(2*d*ln(f)*c+e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf((-c*l
n(f))^(1/2)*x+e/(-c*ln(f))^(1/2))-1/8*Pi^(1/2)*f^a*exp((2*d*ln(f)*c-e^2)/ln
(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+e/(-c*ln(f))^(1/2))+1/4*f^a
*Pi^(1/2)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x)
```

Maxima [A]
time = 0.30, size = 131, normalized size = 0.81

$$\frac{\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x-\frac{e}{\sqrt{-c \log(f)}}\right) e^{\left(2 d-\frac{e^{2}}{c \log(f)}\right)}}{8 \sqrt{-c \log(f)}}+\frac{\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x+\frac{e}{\sqrt{-c \log(f)}}\right) e^{\left(-2 d-\frac{e^{2}}{c \log(f)}\right)}}{8 \sqrt{-c \log(f)}}+\frac{\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x\right)}{4 \sqrt{-c \log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - e/sqrt(-c*log(f)))*e^(2*d - e^2/(c
*log(f)))/sqrt(-c*log(f)) + 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x + e/sqrt
(-c*log(f)))*e^(-2*d - e^2/(c*log(f)))/sqrt(-c*log(f)) + 1/4*sqrt(pi)*f^a*e
rf(sqrt(-c*log(f))*x)/sqrt(-c*log(f))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 302 vs. $2(124) = 248$.
time = 0.45, size = 302, normalized size = 1.88

$$-\frac{2 \sqrt{-c \log(f)}\left(\sqrt{\pi} \cosh(a \log(f))+\sqrt{\pi} \sinh(a \log(f))\right) \operatorname{erf}\left(\sqrt{-c \log(f)}\, x\right)+\sqrt{-c \log(f)}\left(\sqrt{\pi} \cosh\left(\frac{a c \log(f)^{2}+2 c d \log(f)-\cosh(1)^{2}-2 \cosh(1) \sinh(1)-\sinh(1)^{2}}{c \log(f)}\right)+\sqrt{\pi} \sinh\left(\frac{a c \log(f)^{2}+2 c d \log(f)-\cosh(1)^{2}-2 \cosh(1) \sinh(1)-\sinh(1)^{2}}{c \log(f)}\right)\right) \operatorname{erf}\left(\frac{(c x \log(f)+\cosh(1)+\sinh(1)) \sqrt{-c \log(f)}}{c \log(f)}\right)+\sqrt{-c \log(f)}\left(\sqrt{\pi} \cosh\left(\frac{a c \log(f)^{2}-2 c d \log(f)-\cosh(1)^{2}-2 \cosh(1) \sinh(1)-\sinh(1)^{2}}{c \log(f)}\right)+\sqrt{\pi} \sinh\left(\frac{a c \log(f)^{2}-2 c d \log(f)-\cosh(1)^{2}-2 \cosh(1) \sinh(1)-\sinh(1)^{2}}{c \log(f)}\right)\right) \operatorname{erf}\left(\frac{(c x \log(f)-\cosh(1)-\sinh(1)) \sqrt{-c \log(f)}}{c \log(f)}\right)}{8 c \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*sqrt(-c*log(f))*(sqrt(pi)*cosh(a*log(f)) + sqrt(pi)*sinh(a*log(f)))
*erf(sqrt(-c*log(f))*x) + sqrt(-c*log(f))*(sqrt(pi)*cosh((a*c*log(f)^2 + 2*
c*d*log(f) - cosh(1)^2 - 2*cosh(1)*sinh(1) - sinh(1)^2)/(c*log(f))) + sqrt(
pi)*sinh((a*c*log(f)^2 + 2*c*d*log(f) - cosh(1)^2 - 2*cosh(1)*sinh(1) - sin
h(1)^2)/(c*log(f))))*erf((c*x*log(f) + cosh(1) + sinh(1))*sqrt(-c*log(f))/(
c*log(f))) + sqrt(-c*log(f))*(sqrt(pi)*cosh((a*c*log(f)^2 - 2*c*d*log(f) -
cosh(1)^2 - 2*cosh(1)*sinh(1) - sinh(1)^2)/(c*log(f))) + sqrt(pi)*sinh((a*c
*log(f)^2 - 2*c*d*log(f) - cosh(1)^2 - 2*cosh(1)*sinh(1) - sinh(1)^2)/(c*lo
```

```
g(f))))*erf((c*x*log(f) - cosh(1) - sinh(1))*sqrt(-c*log(f))/(c*log(f))))/(
c*log(f))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^2}\cosh^2(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*cosh(e*x+d)**2,x)
```

```
[Out] Integral(f**(a + c*x**2)*cosh(d + e*x)**2, x)
```

Giac [A]
time = 0.41, size = 150, normalized size = 0.93

$$-\frac{\sqrt{\pi}\,f^a\operatorname{erf}\left(-\sqrt{-c\log(f)}\,x\right)}{4\sqrt{-c\log(f)}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log(f)}\left(x+\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{ac\log(f)^2+2\,cd\log(f)-e^2}{c\log(f)}\right)}}{8\sqrt{-c\log(f)}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log(f)}\left(x-\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{ac\log(f)^2-2\,cd\log(f)-e^2}{c\log(f)}\right)}}{8\sqrt{-c\log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*f^a*erf(-sqrt(-c*log(f))*x)/sqrt(-c*log(f)) - 1/8*sqrt(pi)*er
f(-sqrt(-c*log(f))*(x + e/(c*log(f))))*e^((a*c*log(f)^2 + 2*c*d*log(f) - e^
2)/(c*log(f)))/sqrt(-c*log(f)) - 1/8*sqrt(pi)*erf(-sqrt(-c*log(f))*(x - e/(
c*log(f))))*e^((a*c*log(f)^2 - 2*c*d*log(f) - e^2)/(c*log(f)))/sqrt(-c*log(
f))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+a}\,{\mathrm{cosh}\left(d+e\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*cosh(d + e*x)^2,x)
```

```
[Out] int(f^(a + c*x^2)*cosh(d + e*x)^2, x)
```

3.313 $\int f^{a+cx^2} \cosh^3(d+ex)\,dx$

Optimal. Leaf size=271

$$-\frac{3e^{-d-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{16\sqrt{c}\,\sqrt{\log(f)}}-\frac{e^{-3d-\frac{9e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{3e-2cx\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{16\sqrt{c}\,\sqrt{\log(f)}}+\frac{3e^{d-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{E}}{16\sqrt{c}}$$

```
[Out] 3/16*exp(-d-1/4*e^2/c/ln(f))*f^a*erfi(1/2*(-e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1
/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/16*exp(-3*d-9/4*e^2/c/ln(f))*f^a*erfi(1
/2*(-3*e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+3/1
6*exp(d-1/4*e^2/c/ln(f))*f^a*erfi(1/2*(e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*
Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/16*exp(3*d-9/4*e^2/c/ln(f))*f^a*erfi(1/2*(3*
e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5624, 2325, 2266, 2235}

$$-\frac{3\sqrt{\pi}\,f^a e^{-\frac{e^2}{4c\log(f)}-d}\mathrm{Erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{16\sqrt{c}\,\sqrt{\log(f)}}-\frac{\sqrt{\pi}\,f^a e^{-\frac{9e^2}{4c\log(f)}-3d}\mathrm{Erfi}\left(\frac{3e-2cx\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{16\sqrt{c}\,\sqrt{\log(f)}}+\frac{3\sqrt{\pi}\,f^a e^{d-\frac{e^2}{4c\log(f)}}\mathrm{Erfi}\left(\frac{2cx\log(f)+e}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{16\sqrt{c}\,\sqrt{\log(f)}}+\frac{\sqrt{\pi}\,f^a e^{3d-\frac{9e^2}{4c\log(f)}}\mathrm{Erfi}\left(\frac{2cx\log(f)+3e}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{16\sqrt{c}\,\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Cosh[d + e*x]^3,x]
```

```
[Out] (-3*E^(-d - e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - 2*c*x*Log[f])/(2*Sqrt[
c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) - (E^(-3*d - (9*e^2)/(4*c*Log[
f]))*f^a*Sqrt[Pi]*Erfi[(3*e - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*
Sqrt[c]*Sqrt[Log[f]]) + (3*E^(d - e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e +
2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (E^(3*
d - (9*e^2)/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(3*e + 2*c*x*Log[f])/(2*Sqrt[c]
*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5624

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+cx^2}\cosh^3(d+ex)\,dx &= \int\left(\frac{1}{8}e^{-3d-3ex}f^{a+cx^2}+\frac{3}{8}e^{-d-ex}f^{a+cx^2}+\frac{3}{8}e^{d+ex}f^{a+cx^2}+\frac{1}{8}e^{3d+3ex}f^{a+cx^2}\right)dx\\
&= \frac{1}{8}\int e^{-3d-3ex}f^{a+cx^2}\,dx+\frac{1}{8}\int e^{3d+3ex}f^{a+cx^2}\,dx+\frac{3}{8}\int e^{-d-ex}f^{a+cx^2}\,dx+\frac{3}{8}\int e^{d}\\
&= \frac{1}{8}\int e^{-3d-3ex+a\log(f)+cx^2\log(f)}\,dx+\frac{1}{8}\int e^{3d+3ex+a\log(f)+cx^2\log(f)}\,dx+\frac{3}{8}\int e^{-d-ex}\\
&= \frac{1}{8}\left(e^{-3d-\frac{9e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(-3e+2cx\log(f))^2}{4c\log(f)}}\,dx+\frac{1}{8}\left(e^{3d-\frac{9e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(3e+2cx\log(f))^2}{4c\log(f)}}\,dx\\
&= -\frac{3e^{-d-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-3d-\frac{9e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{3e-2cx\log}{2\sqrt{c}\sqrt{\log}}\right.}{16\sqrt{c}\sqrt{\log(f)}}\end{aligned}$$

Mathematica [A]
time = 0.34, size = 214, normalized size = 0.79

$$\frac{e^{-\frac{9e^2}{4c\log(f)}}f^a\sqrt{\pi}\left((\cosh(d)+\sinh(d))\left(3e^{\frac{2e^2}{c\log(f)}}\mathrm{Erfi}\left(\frac{e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)+3e^{\frac{2e^2}{c\log(f)}}\mathrm{Erfi}\left(\frac{-e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cosh(2d)-\sinh(2d))+\mathrm{Erfi}\left(\frac{3e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cosh(2d)+\sinh(2d))\right)+\mathrm{Erfi}\left(\frac{-3e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cosh(3d)-\sinh(3d))\right)}{16\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Cosh[d + e*x]^3,x]
```

```
[Out] (f^a*Sqrt[Pi]*((Cosh[d] + Sinh[d])*(3*E^((2*e^2)/(c*Log[f]))*Erfi[(e + 2*c*
x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])] + 3*E^((2*e^2)/(c*Log[f]))*Erfi[(-e + 2
*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] - Sinh[2*d]) + Erfi[(3*e
+ 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] + Sinh[2*d])) + Erfi[(
-3*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[3*d] - Sinh[3*d])))/(1
6*Sqrt[c]*E^((9*e^2)/(4*c*Log[f]))*Sqrt[Log[f]])
```

Maple [A]
time = 2.27, size = 234, normalized size = 0.86

method	result
risch	$\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{3\left(4 d \ln(f) c+3 e^{2}\right)}{4 \ln(f) c}} \operatorname{erf}\left(\sqrt{-c \ln(f)}\, x+\frac{3 e}{2 \sqrt{-c \ln(f)}}\right)}{16 \sqrt{-c \ln(f)}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{3 d \ln(f) c-\frac{9 e^{2}}{4}}{c \ln(f)}} \operatorname{erf}\left(-\sqrt{-c \ln(f)}\, x+\right.}{16 \sqrt{-c \ln(f)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*cosh(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*Pi^(1/2)*f^a*exp(-3/4*(4*d*ln(f)*c+3*e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*er
f((-c*ln(f))^(1/2)*x+3/2*e/(-c*ln(f))^(1/2))-1/16*Pi^(1/2)*f^a*exp(3/4*(4*d
*ln(f)*c-3*e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+3/2*e/(-c
*ln(f))^(1/2))+3/16*Pi^(1/2)*f^a*exp(-1/4*(4*d*ln(f)*c+e^2)/ln(f)/c)/(-c*ln
(f))^(1/2)*erf((-c*ln(f))^(1/2)*x+1/2*e/(-c*ln(f))^(1/2))-3/16*Pi^(1/2)*f^a
*exp(1/4*(4*d*ln(f)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*
x+1/2*e/(-c*ln(f))^(1/2))
```

Maxima [A]
time = 0.30, size = 211, normalized size = 0.78

$$\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x-\frac{3 e}{2 \sqrt{-c \log(f)}}\right) e^{\left(3 d-\frac{9 e^{2}}{4 c \log(f)}\right)}}{16 \sqrt{-c \log(f)}}+\frac{3 \sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x-\frac{e}{2 \sqrt{-c \log(f)}}\right) e^{\left(d-\frac{e^{2}}{4 c \log(f)}\right)}}{16 \sqrt{-c \log(f)}}+\frac{3 \sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x+\frac{e}{2 \sqrt{-c \log(f)}}\right) e^{\left(-d-\frac{e^{2}}{4 c \log(f)}\right)}}{16 \sqrt{-c \log(f)}}+\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x+\frac{3 e}{2 \sqrt{-c \log(f)}}\right) e^{\left(-3 d-\frac{9 e^{2}}{4 c \log(f)}\right)}}{16 \sqrt{-c \log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 3/2*e/sqrt(-c*log(f)))*e^(3*d - 9
/4*e^2/(c*log(f)))/sqrt(-c*log(f)) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*
x - 1/2*e/sqrt(-c*log(f)))*e^(d - 1/4*e^2/(c*log(f)))/sqrt(-c*log(f)) + 3/1
6*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x + 1/2*e/sqrt(-c*log(f)))*e^(-d - 1/4*e
^2/(c*log(f)))/sqrt(-c*log(f)) + 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x +
3/2*e/sqrt(-c*log(f)))*e^(-3*d - 9/4*e^2/(c*log(f)))/sqrt(-c*log(f))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 548 vs. $2(205) = 410$.
time = 0.41, size = 548, normalized size = 2.02

$$-\frac{\sqrt{-c \log(f)}\left(\sqrt{\pi} \cosh\left(\frac{4 a c \log(f)^{2}+12 c d \log(f)-9 \cosh(1)^{2}-18 \cosh(1) \sinh(1)-9 \sinh(1)^{2}}{4 c \log(f)}\right)+\sqrt{\pi} \sinh\left(\frac{4 a c \log(f)^{2}+12 c d \log(f)-9 \cosh(1)^{2}-18 \cosh(1) \sinh(1)-9 \sinh(1)^{2}}{4 c \log(f)}\right)\right) \operatorname{erf}\left(\frac{(2 c x \log(f)+3 \cosh(1)+3 \sinh(1)) \sqrt{-c \log(f)}}{2 c \log(f)}\right)+3 \sqrt{-c \log(f)}\left(\sqrt{\pi} \cosh\left(\frac{4 a c \log(f)^{2}+4 c d \log(f)-\cosh(1)^{2}-2 \cosh(1) \sinh(1)-\sinh(1)^{2}}{4 c \log(f)}\right)+\sqrt{\pi} \sinh\left(\frac{4 a c \log(f)^{2}+4 c d \log(f)-\cosh(1)^{2}-2 \cosh(1) \sinh(1)-\sinh(1)^{2}}{4 c \log(f)}\right)\right) \operatorname{erf}\left(\frac{(2 c x \log(f)+\cosh(1)+\sinh(1)) \sqrt{-c \log(f)}}{2 c \log(f)}\right)+3 \sqrt{-c \log(f)}\left(\sqrt{\pi} \cosh\left(\frac{4 a c \log(f)^{2}-4 c d \log(f)-\cosh(1)^{2}-2 \cosh(1) \sinh(1)-\sinh(1)^{2}}{4 c \log(f)}\right)+\sqrt{\pi} \sinh\left(\frac{4 a c \log(f)^{2}-4 c d \log(f)-\cosh(1)^{2}-2 \cosh(1) \sinh(1)-\sinh(1)^{2}}{4 c \log(f)}\right)\right) \operatorname{erf}\left(\frac{(2 c x \log(f)-\cosh(1)-\sinh(1)) \sqrt{-c \log(f)}}{2 c \log(f)}\right)+\sqrt{-c \log(f)}\left(\sqrt{\pi} \cosh\left(\frac{4 a c \log(f)^{2}-12 c d \log(f)-9 \cosh(1)^{2}-18 \cosh(1) \sinh(1)-9 \sinh(1)^{2}}{4 c \log(f)}\right)+\sqrt{\pi} \sinh\left(\frac{4 a c \log(f)^{2}-12 c d \log(f)-9 \cosh(1)^{2}-18 \cosh(1) \sinh(1)-9 \sinh(1)^{2}}{4 c \log(f)}\right)\right) \operatorname{erf}\left(\frac{(2 c x \log(f)-3 \cosh(1)-3 \sinh(1)) \sqrt{-c \log(f)}}{2 c \log(f)}\right)}{16 c \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(-c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 + 12*c*d*log(f) -
 9*cosh(1)^2 - 18*cosh(1)*sinh(1) - 9*sinh(1)^2)/(c*log(f))) + sqrt(pi)*sin
h(1/4*(4*a*c*log(f)^2 + 12*c*d*log(f) - 9*cosh(1)^2 - 18*cosh(1)*sinh(1) -
```

```
9*sinh(1)^2)/(c*log(f))))*erf(1/2*(2*c*x*log(f) + 3*cosh(1) + 3*sinh(1))*sq
rt(-c*log(f))/(c*log(f))) + 3*sqrt(-c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log
(f)^2 + 4*c*d*log(f) - cosh(1)^2 - 2*cosh(1)*sinh(1) - sinh(1)^2)/(c*log(f)
)) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f) - cosh(1)^2 - 2*cosh(
1)*sinh(1) - sinh(1)^2)/(c*log(f))))*erf(1/2*(2*c*x*log(f) + cosh(1) + sinh
(1))*sqrt(-c*log(f))/(c*log(f))) + 3*sqrt(-c*log(f))*(sqrt(pi)*cosh(1/4*(4*
a*c*log(f)^2 - 4*c*d*log(f) - cosh(1)^2 - 2*cosh(1)*sinh(1) - sinh(1)^2)/(c
*log(f))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - cosh(1)^2 -
2*cosh(1)*sinh(1) - sinh(1)^2)/(c*log(f))))*erf(1/2*(2*c*x*log(f) - cosh(1)
 - sinh(1))*sqrt(-c*log(f))/(c*log(f))) + sqrt(-c*log(f))*(sqrt(pi)*cosh(1/
4*(4*a*c*log(f)^2 - 12*c*d*log(f) - 9*cosh(1)^2 - 18*cosh(1)*sinh(1) - 9*si
nh(1)^2)/(c*log(f))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 - 12*c*d*log(f) -
9*cosh(1)^2 - 18*cosh(1)*sinh(1) - 9*sinh(1)^2)/(c*log(f))))*erf(1/2*(2*c*x
*log(f) - 3*cosh(1) - 3*sinh(1))*sqrt(-c*log(f))/(c*log(f))))/(c*log(f))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^2}\cosh^3(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*cosh(e*x+d)**3,x)
```

```
[Out] Integral(f**(a + c*x**2)*cosh(d + e*x)**3, x)
```

Giac [A]
time = 0.41, size = 264, normalized size = 0.97

$$-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{3e}{c\log(f)}\right)\right)e^{\left(\frac{4ac\log(f)^2+12cd\log(f)-9e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}-\frac{3\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{4ac\log(f)^2+4cd\log(f)-e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}-\frac{3\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x-\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{4ac\log(f)^2-4cd\log(f)-e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x-\frac{3e}{c\log(f)}\right)\right)e^{\left(\frac{4ac\log(f)^2-12cd\log(f)-9e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(e*x+d)^3,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + 3*e/(c*log(f))))*e^(1/4*(4*a
*c*log(f)^2 + 12*c*d*log(f) - 9*e^2)/(c*log(f)))/sqrt(-c*log(f)) - 3/16*sqr
t(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + e/(c*log(f))))*e^(1/4*(4*a*c*log(f)^2
 + 4*c*d*log(f) - e^2)/(c*log(f)))/sqrt(-c*log(f)) - 3/16*sqrt(pi)*erf(-1/2
*sqrt(-c*log(f))*(2*x - e/(c*log(f))))*e^(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f
) - e^2)/(c*log(f)))/sqrt(-c*log(f)) - 1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f
))*(2*x - 3*e/(c*log(f))))*e^(1/4*(4*a*c*log(f)^2 - 12*c*d*log(f) - 9*e^2)/
(c*log(f)))/sqrt(-c*log(f))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+a}\,{\cosh\left(d+e\,x\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*cosh(d + e*x)^3,x)
[Out] int(f^(a + c*x^2)*cosh(d + e*x)^3, x)
```

3.314 $\int f^{a+cx^2} \cosh\left(d+fx^2\right)\,dx$

Optimal. Leaf size=81

$$\frac{e^{-d}f^a\sqrt{\pi}\,\mathrm{Erf}\left(x\sqrt{f-c\log(f)}\right)}{4\sqrt{f-c\log(f)}}+\frac{e^d f^a\sqrt{\pi}\,\mathrm{Erfi}\left(x\sqrt{f+c\log(f)}\right)}{4\sqrt{f+c\log(f)}}$$

```
[Out] 1/4*f^a*erf(x*(f-c*ln(f))^(1/2))*Pi^(1/2)/exp(d)/(f-c*ln(f))^(1/2)+1/4*exp(
d)*f^a*erfi(x*(f+c*ln(f))^(1/2))*Pi^(1/2)/(f+c*ln(f))^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5624, 2325, 2236, 2235}

$$\frac{\sqrt{\pi}\,e^{-d}f^a\mathrm{Erf}\left(x\sqrt{f-c\log(f)}\right)}{4\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}\,e^d f^a\mathrm{Erfi}\left(x\sqrt{c\log(f)+f}\right)}{4\sqrt{c\log(f)+f}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Cosh[d + f*x^2],x]
```

```
[Out] (f^a*Sqrt[Pi]*Erf[x*Sqrt[f - c*Log[f]]])/(4*E^d*Sqrt[f - c*Log[f]]) + (E^d*
f^a*Sqrt[Pi]*Erfi[x*Sqrt[f + c*Log[f]]])/(4*Sqrt[f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5624

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
```

```
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int f^{a+cx^2}\cosh\left(d+fx^2\right)\,dx &= \int\left(\frac{1}{2}e^{-d-fx^2}f^{a+cx^2}+\frac{1}{2}e^{d+fx^2}f^{a+cx^2}\right)dx\\
&= \frac{1}{2}\int e^{-d-fx^2}f^{a+cx^2}\,dx+\frac{1}{2}\int e^{d+fx^2}f^{a+cx^2}\,dx\\
&= \frac{1}{2}\int e^{-d+a\log(f)-x^2(f-c\log(f))}\,dx+\frac{1}{2}\int e^{d+a\log(f)+x^2(f+c\log(f))}\,dx\\
&= \frac{e^{-d}f^a\sqrt{\pi}\,\mathrm{erf}\left(x\sqrt{f-c\log(f)}\right)}{4\sqrt{f-c\log(f)}}+\frac{e^{d}f^a\sqrt{\pi}\,\mathrm{erfi}\left(x\sqrt{f+c\log(f)}\right)}{4\sqrt{f+c\log(f)}}
\end{aligned}$$

Mathematica [A]
time = 0.22, size = 75, normalized size = 0.93

$$\frac{1}{4}f^a\sqrt{\pi}\left(\frac{\mathrm{Erf}\left(x\sqrt{f-c\log(f)}\right)(\cosh(d)-\sinh(d))}{\sqrt{f-c\log(f)}}+\frac{\mathrm{Erfi}\left(x\sqrt{f+c\log(f)}\right)(\cosh(d)+\sinh(d))}{\sqrt{f+c\log(f)}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Cosh[d + f*x^2],x]
```

```
[Out] (f^a*Sqrt[Pi]*((Erf[x*Sqrt[f - c*Log[f]]]*(Cosh[d] - Sinh[d]))/Sqrt[f - c*L
og[f]] + (Erfi[x*Sqrt[f + c*Log[f]]]*(Cosh[d] + Sinh[d]))/Sqrt[f + c*Log[f]
]))/4
```

Maple [A]
time = 0.72, size = 70, normalized size = 0.86

method	result	size
risch	$\frac{\sqrt{\pi}\,f^a e^{-d}\,\mathrm{erf}\left(x\sqrt{f-c\ln(f)}\right)}{4\sqrt{f-c\ln(f)}}+\frac{\sqrt{\pi}\,f^a e^{d}\,\mathrm{erf}\left(\sqrt{-c\ln(f)-f}\,x\right)}{4\sqrt{-c\ln(f)-f}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*cosh(f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*Pi^(1/2)*f^a*exp(-d)/(f-c*ln(f))^(1/2)*erf(x*(f-c*ln(f))^(1/2))+1/4*Pi^
(1/2)*f^a*exp(d)/(-c*ln(f)-f)^(1/2)*erf((-c*ln(f)-f)^(1/2)*x)
```

Maxima [A]
time = 0.29, size = 69, normalized size = 0.85

$$\frac{\sqrt{\pi} f^a \operatorname{erf}\left(\sqrt{-c\log\left(f\right)+f}\, x\right) e^{(-d)}}{4\sqrt{-c\log\left(f\right)+f}}+\frac{\sqrt{\pi} f^a \operatorname{erf}\left(\sqrt{-c\log\left(f\right)-f}\, x\right) e^{d}}{4\sqrt{-c\log\left(f\right)-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x)*e^(-d)/sqrt(-c*log(f) + f) + 1/
4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x)*e^d/sqrt(-c*log(f) - f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 145 vs. 2(63) = 126.
time = 0.40, size = 145, normalized size = 1.79

$$-\frac{\left(\sqrt{\pi}\left(c\log\left(f\right)+f\right)\cosh\left(a\log\left(f\right)-d\right)+\sqrt{\pi}\left(c\log\left(f\right)+f\right)\sinh\left(a\log\left(f\right)-d\right)\right)\sqrt{-c\log\left(f\right)+f}\operatorname{erf}\left(\sqrt{-c\log\left(f\right)+f}\, x\right)+\left(\sqrt{\pi}\left(c\log\left(f\right)-f\right)\cosh\left(a\log\left(f\right)+d\right)+\sqrt{\pi}\left(c\log\left(f\right)-f\right)\sinh\left(a\log\left(f\right)+d\right)\right)\sqrt{-c\log\left(f\right)-f}\operatorname{erf}\left(\sqrt{-c\log\left(f\right)-f}\, x\right)}{4\left(c^2\log\left(f\right)^2-f^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+d),x, algorithm="fricas")
```

```
[Out] -1/4*((sqrt(pi)*(c*log(f) + f)*cosh(a*log(f) - d) + sqrt(pi)*(c*log(f) + f)
*sinh(a*log(f) - d))*sqrt(-c*log(f) + f)*erf(sqrt(-c*log(f) + f)*x) + (sqrt
(pi)*(c*log(f) - f)*cosh(a*log(f) + d) + sqrt(pi)*(c*log(f) - f)*sinh(a*log
(f) + d))*sqrt(-c*log(f) - f)*erf(sqrt(-c*log(f) - f)*x))/(c^2*log(f)^2 - f
^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^2}\cosh\left(d+fx^2\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*cosh(f*x**2+d),x)
```

```
[Out] Integral(f**(a + c*x**2)*cosh(d + f*x**2), x)
```

Giac [A]
time = 0.42, size = 75, normalized size = 0.93

$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)-f}\, x\right) e^{\left(a\log\left(f\right)+d\right)}}{4\sqrt{-c\log\left(f\right)-f}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)+f}\, x\right) e^{\left(a\log\left(f\right)-d\right)}}{4\sqrt{-c\log\left(f\right)+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-sqrt(-c*log(f) - f)*x)*e^(a*log(f) + d)/sqrt(-c*log(f) -
 f) - 1/4*sqrt(pi)*erf(-sqrt(-c*log(f) + f)*x)*e^(a*log(f) - d)/sqrt(-c*log
(f) + f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+a}\cosh\left(f\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*cosh(d + f*x^2),x)
```

```
[Out] int(f^(a + c*x^2)*cosh(d + f*x^2), x)
```

3.315 $\int f^{a+cx^2} \cosh^2(d+fx^2)\, dx$

Optimal. Leaf size=128

$$\frac{f^a\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{e^{-2d}f^a\sqrt{\pi}\,\mathrm{Erf}\left(x\sqrt{2f-c\log(f)}\right)}{8\sqrt{2f-c\log(f)}}+\frac{e^{2d}f^a\sqrt{\pi}\,\mathrm{Erfi}\left(x\sqrt{2f+c\log(f)}\right)}{8\sqrt{2f+c\log(f)}}$$

```
[Out] 1/4*f^a*erfi(x*c^(1/2)*ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/8*f^a*er
f(x*(2*f-c*ln(f))^(1/2))*Pi^(1/2)/exp(2*d)/(2*f-c*ln(f))^(1/2)+1/8*exp(2*d)
*f^a*erfi(x*(2*f+c*ln(f))^(1/2))*Pi^(1/2)/(2*f+c*ln(f))^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5624, 2235, 2325, 2236}

$$\frac{\sqrt{\pi}\,e^{-2d}f^a\mathrm{Erf}\left(x\sqrt{2f-c\log(f)}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi}\,e^{2d}f^a\mathrm{Erfi}\left(x\sqrt{c\log(f)+2f}\right)}{8\sqrt{c\log(f)+2f}}+\frac{\sqrt{\pi}\,f^a\mathrm{Erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Cosh[d + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) + (f^a
*Sqrt[Pi]*Erf[x*Sqrt[2*f - c*Log[f]]])/(8*E^(2*d)*Sqrt[2*f - c*Log[f]]) + (
E^(2*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[2*f + c*Log[f]]])/(8*Sqrt[2*f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5624

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+cx^2}\cosh^2\left(d+fx^2\right)\,dx &= \int \left(\frac{1}{2}f^{a+cx^2}+\frac{1}{4}e^{-2d-2fx^2}f^{a+cx^2}+\frac{1}{4}e^{2d+2fx^2}f^{a+cx^2}\right)dx\\ &= \frac{1}{4}\int e^{-2d-2fx^2}f^{a+cx^2}\,dx+\frac{1}{4}\int e^{2d+2fx^2}f^{a+cx^2}\,dx+\frac{1}{2}\int f^{a+cx^2}\,dx\\ &= \frac{f^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{1}{4}\int e^{-2d+a\log(f)-x^2(2f-c\log(f))}\,dx+\frac{1}{4}\int e^{2d+\cdots}\\ &= \frac{f^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{e^{-2d}f^a\sqrt{\pi}\,\mathrm{erf}\left(x\sqrt{2f-c\log(f)}\right)}{8\sqrt{2f-c\log(f)}}+\frac{e^{2d}f^a\cdots}{\cdots}\end{aligned}$$

Mathematica [A]
time = 0.39, size = 179, normalized size = 1.40

$$\frac{f^a\sqrt{\pi}\left(\mathrm{Erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)\left(-8f^2+2c^2\log^2(f)\right)+\sqrt{c}\,\sqrt{\log(f)}\left(\mathrm{Erf}\left(x\sqrt{2f-c\log(f)}\right)\sqrt{2f-c\log(f)}\,(2f+c\log(f))(-\cosh(2d)+\sinh(2d))-\mathrm{Erfi}\left(x\sqrt{2f+c\log(f)}\right)(2f-c\log(f))\sqrt{2f+c\log(f)}\,(\cosh(2d)+\sinh(2d))\right)\right)}{8\sqrt{c}\,\sqrt{\log(f)}\left(-4f^2+c^2\log^2(f)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Cosh[d + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*(Erfi[Sqrt[c]*x*Sqrt[Log[f]]]*(-8*f^2 + 2*c^2*Log[f]^2) + Sqr
t[c]*Sqrt[Log[f]]*(Erf[x*Sqrt[2*f - c*Log[f]]]*Sqrt[2*f - c*Log[f]]*(2*f +
c*Log[f])*(-Cosh[2*d] + Sinh[2*d]) - Erfi[x*Sqrt[2*f + c*Log[f]]]*(2*f - c*
Log[f])*Sqrt[2*f + c*Log[f]]*(Cosh[2*d] + Sinh[2*d]))))/(8*Sqrt[c]*Sqrt[Log
[f]]*(-4*f^2 + c^2*Log[f]^2))
```

Maple [A]
time = 1.60, size = 101, normalized size = 0.79

method	result
risch	$\frac{\sqrt{\pi}\,f^a e^{-2d}\,\mathrm{erf}\left(x\sqrt{2f-c\ln(f)}\right)}{8\sqrt{2f-c\ln(f)}}+\frac{\sqrt{\pi}\,f^a e^{2d}\,\mathrm{erf}\left(\sqrt{-c\ln(f)-2f}\,x\right)}{8\sqrt{-c\ln(f)-2f}}+\frac{f^a\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-c\ln(f)}\,x\right)}{4\sqrt{-c\ln(f)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*cosh(f*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*Pi^(1/2)*f^a*exp(-2*d)/(2*f-c*ln(f))^(1/2)*erf(x*(2*f-c*ln(f))^(1/2))+1
/8*Pi^(1/2)*f^a*exp(2*d)/(-c*ln(f)-2*f)^(1/2)*erf((-c*ln(f)-2*f)^(1/2)*x)+1
/4*f^a*Pi^(1/2)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x)
```

Maxima [A]
time = 0.30, size = 100, normalized size = 0.78

$$\frac{\sqrt{\pi} f^a \operatorname{erf}\left(\sqrt{-c\log(f) - 2f}\, x\right) e^{(2d)}}{8\sqrt{-c\log(f) - 2f}} + \frac{\sqrt{\pi} f^a \operatorname{erf}\left(\sqrt{-c\log(f) + 2f}\, x\right) e^{(-2d)}}{8\sqrt{-c\log(f) + 2f}} + \frac{\sqrt{\pi} f^a \operatorname{erf}\left(\sqrt{-c\log(f)}\, x\right)}{4\sqrt{-c\log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 2*f)*x)*e^(2*d)/sqrt(-c*log(f) - 2*f)
 + 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 2*f)*x)*e^(-2*d)/sqrt(-c*log(f) +
2*f) + 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x)/sqrt(-c*log(f))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(98) = 196.
time = 0.55, size = 254, normalized size = 1.98

$$-\frac{\left(\sqrt{\pi}\left(c^2\log(f)^2 + 2cf\log(f)\right)\cosh\left(a\log(f) - 2d\right) + \sqrt{\pi}\left(c^2\log(f)^2 + 2cf\log(f)\right)\sinh\left(a\log(f) - 2d\right)\right)\sqrt{-c\log(f) + 2f}\operatorname{erf}\left(\sqrt{-c\log(f) + 2f}\, x\right) + \left(\sqrt{\pi}\left(c^2\log(f)^2 - 2cf\log(f)\right)\cosh\left(a\log(f) + 2d\right) + \sqrt{\pi}\left(c^2\log(f)^2 - 2cf\log(f)\right)\sinh\left(a\log(f) + 2d\right)\right)\sqrt{-c\log(f) - 2f}\operatorname{erf}\left(\sqrt{-c\log(f) - 2f}\, x\right) + 2\left(\sqrt{\pi}\left(c^2\log(f)^2 - 4f^2\right)\cosh\left(a\log(f)\right) + \sqrt{\pi}\left(c^2\log(f)^2 - 4f^2\right)\sinh\left(a\log(f)\right)\right)\sqrt{-c\log(f)}\operatorname{erf}\left(\sqrt{-c\log(f)}\, x\right)}{8\left(c^3\log(f)^3 - 4cf^2\log(f)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*((sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*cosh(a*log(f) - 2*d) + sqrt(p
i)*(c^2*log(f)^2 + 2*c*f*log(f))*sinh(a*log(f) - 2*d))*sqrt(-c*log(f) + 2*f
)*erf(sqrt(-c*log(f) + 2*f)*x) + (sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))*co
sh(a*log(f) + 2*d) + sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))*sinh(a*log(f) +
 2*d))*sqrt(-c*log(f) - 2*f)*erf(sqrt(-c*log(f) - 2*f)*x) + 2*(sqrt(pi)*(c^
2*log(f)^2 - 4*f^2)*cosh(a*log(f)) + sqrt(pi)*(c^2*log(f)^2 - 4*f^2)*sinh(a
*log(f)))*sqrt(-c*log(f))*erf(sqrt(-c*log(f))*x))/(c^3*log(f)^3 - 4*c*f^2*l
og(f))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^2}\cosh^2\left(d + fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*cosh(f*x**2+d)**2,x)
```

```
[Out] Integral(f**(a + c*x**2)*cosh(d + f*x**2)**2, x)
```

Giac [A]
time = 0.42, size = 107, normalized size = 0.84

$$-\frac{\sqrt{\pi}f^{a}\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)}\,x\right)}{4\,\sqrt{-c\log\left(f\right)}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)-2\,f}\,x\right)e^{\left(a\log\left(f\right)+2\,d\right)}}{8\,\sqrt{-c\log\left(f\right)-2\,f}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)+2\,f}\,x\right)e^{\left(a\log\left(f\right)-2\,d\right)}}{8\,\sqrt{-c\log\left(f\right)+2\,f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+d)^2,x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*f^a*erf(-sqrt(-c*log(f))*x)/sqrt(-c*log(f)) - 1/8*sqrt(pi)*er
f(-sqrt(-c*log(f) - 2*f)*x)*e^(a*log(f) + 2*d)/sqrt(-c*log(f) - 2*f) - 1/8*
sqrt(pi)*erf(-sqrt(-c*log(f) + 2*f)*x)*e^(a*log(f) - 2*d)/sqrt(-c*log(f) +
2*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+a}\,\mathrm{cosh}\left(f\,x^2+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*cosh(d + f*x^2)^2,x)
```

```
[Out] int(f^(a + c*x^2)*cosh(d + f*x^2)^2, x)
```

3.316 $\int f^{a+cx^2} \cosh^3(d+fx^2)\,dx$

Optimal. Leaf size=171

$$\frac{3e^{-d}f^a\sqrt{\pi}\,\mathrm{Erf}\left(x\sqrt{f-c\log(f)}\right)}{16\sqrt{f-c\log(f)}}+\frac{e^{-3d}f^a\sqrt{\pi}\,\mathrm{Erf}\left(x\sqrt{3f-c\log(f)}\right)}{16\sqrt{3f-c\log(f)}}+\frac{3e^{d}f^a\sqrt{\pi}\,\mathrm{Erfi}\left(x\sqrt{f+c\log(f)}\right)}{16\sqrt{f+c\log(f)}}+$$

```
[Out] 3/16*f^a*erf(x*(f-c*ln(f))^(1/2))*Pi^(1/2)/exp(d)/(f-c*ln(f))^(1/2)+1/16*f^
a*erf(x*(3*f-c*ln(f))^(1/2))*Pi^(1/2)/exp(3*d)/(3*f-c*ln(f))^(1/2)+3/16*exp
(d)*f^a*erfi(x*(f+c*ln(f))^(1/2))*Pi^(1/2)/(f+c*ln(f))^(1/2)+1/16*exp(3*d)*
f^a*erfi(x*(3*f+c*ln(f))^(1/2))*Pi^(1/2)/(3*f+c*ln(f))^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5624, 2325, 2236, 2235}

$$\frac{3\sqrt{\pi}\,e^{-d}f^a\mathrm{Erf}\left(x\sqrt{f-c\log(f)}\right)}{16\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}\,e^{-3d}f^a\mathrm{Erf}\left(x\sqrt{3f-c\log(f)}\right)}{16\sqrt{3f-c\log(f)}}+\frac{3\sqrt{\pi}\,e^{d}f^a\mathrm{Erfi}\left(x\sqrt{c\log(f)+f}\right)}{16\sqrt{c\log(f)+f}}+\frac{\sqrt{\pi}\,e^{3d}f^a\mathrm{Erfi}\left(x\sqrt{c\log(f)+3f}\right)}{16\sqrt{c\log(f)+3f}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Cosh[d + f*x^2]^3,x]
```

```
[Out] (3*f^a*Sqrt[Pi]*Erf[x*Sqrt[f - c*Log[f]]])/(16*E^d*Sqrt[f - c*Log[f]]) + (f
^a*Sqrt[Pi]*Erf[x*Sqrt[3*f - c*Log[f]]])/(16*E^(3*d)*Sqrt[3*f - c*Log[f]])
+ (3*E^d*f^a*Sqrt[Pi]*Erfi[x*Sqrt[f + c*Log[f]]])/(16*Sqrt[f + c*Log[f]]) +
 (E^(3*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[3*f + c*Log[f]]])/(16*Sqrt[3*f + c*Log[f
]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5624

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int f^{a+cx^2}\cosh^3\left(d+fx^2\right)\,dx &= \int\left(\frac{1}{8}e^{-3d-3fx^2}f^{a+cx^2}+\frac{3}{8}e^{-d-fx^2}f^{a+cx^2}+\frac{3}{8}e^{d+fx^2}f^{a+cx^2}+\frac{1}{8}e^{3d+3fx^2}f^{a+cx^2}\right.\\
&= \frac{1}{8}\int e^{-3d-3fx^2}f^{a+cx^2}\,dx+\frac{1}{8}\int e^{3d+3fx^2}f^{a+cx^2}\,dx+\frac{3}{8}\int e^{-d-fx^2}f^{a+cx^2}\,dx+\\
&= \frac{1}{8}\int e^{-3d+a\log(f)-x^2(3f-c\log(f))}\,dx+\frac{1}{8}\int e^{3d+a\log(f)+x^2(3f+c\log(f))}\,dx+\frac{3}{8}\int e\\
&= \frac{3e^{-d}f^a\sqrt{\pi}\,\mathrm{erf}\left(x\sqrt{f-c\log(f)}\right)}{16\sqrt{f-c\log(f)}}+\frac{e^{-3d}f^a\sqrt{\pi}\,\mathrm{erf}\left(x\sqrt{3f-c\log(f)}\right)}{16\sqrt{3f-c\log(f)}}+
\end{aligned}$$

Mathematica [A]
time = 0.88, size = 270, normalized size = 1.58

$$\frac{f^a\sqrt{\pi}\left(3\mathrm{Erf}\left(x\sqrt{f-c\log(f)}\right)\sqrt{f-c\log(f)}\left(9f^3+9cf^2\log(f)-c^2f\log^2(f)-c^3\log^3(f)\right)(\cosh(d)-\sinh(d))+(f-c\log(f))\left(\mathrm{Erf}\left(x\sqrt{3f-c\log(f)}\right)\sqrt{3f-c\log(f)}\left(3f^2+4cf\log(f)+c^2\log^2(f)\right)(\cosh(3d)-\sinh(3d))+(3f-c\log(f))\left(3\mathrm{Erfi}\left(x\sqrt{f+c\log(f)}\right)\sqrt{f+c\log(f)}(3f+c\log(f))(\cosh(d)+\sinh(d))+\mathrm{Erfi}\left(x\sqrt{3f+c\log(f)}\right)(f+c\log(f))\sqrt{3f+c\log(f)}(\cosh(3d)+\sinh(3d))\right)\right)\right)}{16\left(9f^4-10c^2f^2\log^2(f)+c^4\log^4(f)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Cosh[d + f*x^2]^3,x]
```

```
[Out] (f^a*Sqrt[Pi]*(3*Erf[x*Sqrt[f - c*Log[f]]]*Sqrt[f - c*Log[f]]*(9*f^3 + 9*c*
f^2*Log[f] - c^2*f*Log[f]^2 - c^3*Log[f]^3)*(Cosh[d] - Sinh[d]) + (f - c*Lo
g[f])*(Erf[x*Sqrt[3*f - c*Log[f]]]*Sqrt[3*f - c*Log[f]]*(3*f^2 + 4*c*f*Log[
f] + c^2*Log[f]^2)*(Cosh[3*d] - Sinh[3*d]) + (3*f - c*Log[f])*(3*Erfi[x*Sqr
t[f + c*Log[f]]]*Sqrt[f + c*Log[f]]*(3*f + c*Log[f])*(Cosh[d] + Sinh[d]) +
Erfi[x*Sqrt[3*f + c*Log[f]]]*(f + c*Log[f])*Sqrt[3*f + c*Log[f]]*(Cosh[3*d]
 + Sinh[3*d])))))/(16*(9*f^4 - 10*c^2*f^2*Log[f]^2 + c^4*Log[f]^4))
```

Maple [A]
time = 2.46, size = 144, normalized size = 0.84

method	result
risch	$\frac{\sqrt{\pi}\,f^a\mathrm{e}^{-3d}\,\mathrm{erf}\left(x\sqrt{3f-c\ln(f)}\right)}{16\sqrt{3f-c\ln(f)}}+\frac{\sqrt{\pi}\,f^a\mathrm{e}^{3d}\,\mathrm{erf}\left(\sqrt{-c\ln(f)-3f}\,x\right)}{16\sqrt{-c\ln(f)-3f}}+\frac{3\sqrt{\pi}\,f^a\mathrm{e}^{-d}\,\mathrm{erf}\left(x\sqrt{f-c\ln(f)}\right)}{16\sqrt{f-c\ln(f)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*cosh(f*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*Pi^(1/2)*f^a*exp(-3*d)/(3*f-c*ln(f))^(1/2)*erf(x*(3*f-c*ln(f))^(1/2))+
1/16*Pi^(1/2)*f^a*exp(3*d)/(-c*ln(f)-3*f)^(1/2)*erf((-c*ln(f)-3*f)^(1/2)*x)
+3/16*Pi^(1/2)*f^a*exp(-d)/(f-c*ln(f))^(1/2)*erf(x*(f-c*ln(f))^(1/2))+3/16*
Pi^(1/2)*f^a*exp(d)/(-c*ln(f)-f)^(1/2)*erf((-c*ln(f)-f)^(1/2)*x)
```

Maxima [**A**]
time = 0.30, size = 143, normalized size = 0.84

$$\frac{\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c \log\left(f\right) - 3 \, f}\, x\right) e^{\left(3 \, d\right)}}{16 \, \sqrt{-c \log\left(f\right) - 3 \, f}} + \frac{3 \, \sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c \log\left(f\right) + f}\, x\right) e^{\left(-d\right)}}{16 \, \sqrt{-c \log\left(f\right) + f}} + \frac{\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c \log\left(f\right) + 3 \, f}\, x\right) e^{\left(-3 \, d\right)}}{16 \, \sqrt{-c \log\left(f\right) + 3 \, f}} + \frac{3 \, \sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c \log\left(f\right) - f}\, x\right) e^{d}}{16 \, \sqrt{-c \log\left(f\right) - f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 3*f)*x)*e^(3*d)/sqrt(-c*log(f) - 3*f
) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x)*e^(-d)/sqrt(-c*log(f) + f)
 + 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 3*f)*x)*e^(-3*d)/sqrt(-c*log(f) +
 3*f) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x)*e^d/sqrt(-c*log(f) - f
)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 491 vs. 2(135) = 270.
time = 0.53, size = 491, normalized size = 2.87

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*((sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3)*c
osh(a*log(f) - 3*d) + sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log
(f) - 3*f^3)*sinh(a*log(f) - 3*d))*sqrt(-c*log(f) + 3*f)*erf(sqrt(-c*log(f)
 + 3*f)*x) + 3*(sqrt(pi)*(c^3*log(f)^3 + c^2*f*log(f)^2 - 9*c*f^2*log(f) -
9*f^3)*cosh(a*log(f) - d) + sqrt(pi)*(c^3*log(f)^3 + c^2*f*log(f)^2 - 9*c*f
^2*log(f) - 9*f^3)*sinh(a*log(f) - d))*sqrt(-c*log(f) + f)*erf(sqrt(-c*log(
f) + f)*x) + 3*(sqrt(pi)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) +
9*f^3)*cosh(a*log(f) + d) + sqrt(pi)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f
^2*log(f) + 9*f^3)*sinh(a*log(f) + d))*sqrt(-c*log(f) - f)*erf(sqrt(-c*log(
f) - f)*x) + (sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) + 3*
f^3)*cosh(a*log(f) + 3*d) + sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f
```

```
^2*log(f) + 3*f^3)*sinh(a*log(f) + 3*d))*sqrt(-c*log(f) - 3*f)*erf(sqrt(-c*
log(f) - 3*f)*x))/(c^4*log(f)^4 - 10*c^2*f^2*log(f)^2 + 9*f^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^2}\cosh^3\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*cosh(f*x**2+d)**3,x)
```

```
[Out] Integral(f**(a + c*x**2)*cosh(d + f*x**2)**3, x)
```

Giac [A]
time = 0.41, size = 155, normalized size = 0.91

$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)-3\,f}\,x\right)e^{\left(a\log\left(f\right)+3\,d\right)}}{16\,\sqrt{-c\log\left(f\right)-3\,f}}-\frac{3\,\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)-f}\,x\right)e^{\left(a\log\left(f\right)+d\right)}}{16\,\sqrt{-c\log\left(f\right)-f}}-\frac{3\,\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)+f}\,x\right)e^{\left(a\log\left(f\right)-d\right)}}{16\,\sqrt{-c\log\left(f\right)+f}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)+3\,f}\,x\right)e^{\left(a\log\left(f\right)-3\,d\right)}}{16\,\sqrt{-c\log\left(f\right)+3\,f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+d)^3,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(pi)*erf(-sqrt(-c*log(f) - 3*f)*x)*e^(a*log(f) + 3*d)/sqrt(-c*log
(f) - 3*f) - 3/16*sqrt(pi)*erf(-sqrt(-c*log(f) - f)*x)*e^(a*log(f) + d)/sqr
t(-c*log(f) - f) - 3/16*sqrt(pi)*erf(-sqrt(-c*log(f) + f)*x)*e^(a*log(f) -
d)/sqrt(-c*log(f) + f) - 1/16*sqrt(pi)*erf(-sqrt(-c*log(f) + 3*f)*x)*e^(a*l
og(f) - 3*d)/sqrt(-c*log(f) + 3*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+a}\,{\mathrm{cosh}\left(f\,x^2+d\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*cosh(d + f*x^2)^3,x)
```

```
[Out] int(f^(a + c*x^2)*cosh(d + f*x^2)^3, x)
```

3.317 $\int f^{a+cx^2} \cosh\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=140

$$\frac{e^{-d+\frac{e^2}{4f-4c\log(f)}}f^a\sqrt{\pi}\,\text{Erf}\left(\frac{e+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{e^{d-\frac{e^2}{4(f+c\log(f))}}f^a\sqrt{\pi}\,\text{Erfi}\left(\frac{e+2x(f+c\log(f))}{2\sqrt{f+c\log(f)}}\right)}{4\sqrt{f+c\log(f)}}$$

```
[Out] 1/4*exp(-d+e^2/(4*f-4*c*ln(f)))*f^a*erf(1/2*(e+2*x*(f-c*ln(f)))/(f-c*ln(f))
^(1/2))*Pi^(1/2)/(f-c*ln(f))^(1/2)+1/4*exp(d-1/4*e^2/(f+c*ln(f)))*f^a*erfi(
1/2*(e+2*x*(f+c*ln(f)))/(f+c*ln(f))^(1/2))*Pi^(1/2)/(f+c*ln(f))^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5624, 2325, 2266, 2236, 2235}

$$\frac{\sqrt{\pi}\,f^a e^{\frac{e^2}{4f-4c\log(f)}-d}\text{Erf}\left(\frac{2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}\,f^a e^{d-\frac{e^2}{4(c\log(f)+f)}}\text{Erfi}\left(\frac{2x(c\log(f)+f)+e}{2\sqrt{c\log(f)+f}}\right)}{4\sqrt{c\log(f)+f}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Cosh[d + e*x + f*x^2],x]
```

```
[Out] (E^(-d + e^2/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(e + 2*x*(f - c*Log[f]))/
(2*Sqrt[f - c*Log[f]])])/(4*Sqrt[f - c*Log[f]]) + (E^(d - e^2/(4*(f + c*Log
[f])))*f^a*Sqrt[Pi]*Erfi[(e + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/
(4*Sqrt[f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5624

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int f^{a+cx^2}\cosh\left(d+ex+fx^2\right)\,dx &= \int\left(\frac{1}{2}e^{-d-ex-fx^2}f^{a+cx^2}+\frac{1}{2}e^{d+ex+fx^2}f^{a+cx^2}\right)\,dx\\
&= \frac{1}{2}\int e^{-d-ex-fx^2}f^{a+cx^2}\,dx+\frac{1}{2}\int e^{d+ex+fx^2}f^{a+cx^2}\,dx\\
&= \frac{1}{2}\int e^{-d-ex+a\log(f)-x^2(f-c\log(f))}\,dx+\frac{1}{2}\int e^{d+ex+a\log(f)+x^2(f+c\log(f))}\,dx\\
&= \frac{1}{2}\left(e^{-d+\frac{e^2}{4f-4c\log(f)}}f^a\right)\int\exp\left(\frac{(-e+2x(-f+c\log(f)))^2}{4(-f+c\log(f))}\right)\,dx+\frac{1}{2}\Bigg(e^{d}\\
&= \frac{e^{-d+\frac{e^2}{4f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{e+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{e^{d-\frac{e^2}{4(f+c\log(f))}}f^a\sqrt{\pi}\,\mathrm{erfi}\Bigg(}{4\sqrt{f+c\,\mathrm{lo}}}
\end{aligned}$$

Mathematica [A]
time = 0.47, size = 165, normalized size = 1.18

$$\frac{e^{-\frac{e^2}{4(f+c\log(f))}}f^a\sqrt{\pi}\left(e^{\frac{e^2f}{2f^2-2c^2\log^2(f)}}\mathrm{Erf}\left(\frac{e+2fx-2cx\log(f)}{2\sqrt{f-c\log(f)}}\right)\sqrt{f+c\log(f)}\,(\cosh(d)-\sinh(d))+\mathrm{Erfi}\left(\frac{e+2fx+2cx\log(f)}{2\sqrt{f+c\log(f)}}\right)\sqrt{f-c\log(f)}\,(\cosh(d)+\sinh(d))\right)}{4\sqrt{f-c\log(f)}\,\sqrt{f+c\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Cosh[d + e*x + f*x^2],x]
```

```
[Out] (f^a*Sqrt[Pi]*(E^((e^2*f)/(2*f^2 - 2*c^2*Log[f]^2))*Erf[(e + 2*f*x - 2*c*x*
Log[f])/(2*Sqrt[f - c*Log[f]])]*Sqrt[f + c*Log[f]]*(Cosh[d] - Sinh[d]) + Er
fi[(e + 2*f*x + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Sqrt[f - c*Log[f]]*(C
osh[d] + Sinh[d])))/(4*E^(e^2/(4*(f + c*Log[f])))*Sqrt[f - c*Log[f]]*Sqrt[f
 + c*Log[f]])
```

Maple [A]
time = 0.79, size = 147, normalized size = 1.05

method	result
risch	$\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{4 d \ln(f) c-4 d f+e^{2}}{4(-f+c \ln(f))}} \operatorname{erf}\left(x \sqrt{f-c \ln(f)}+\frac{e}{2 \sqrt{f-c \ln(f)}}\right)}{4 \sqrt{f-c \ln(f)}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{4 d \ln(f) c+4 d f-e^{2}}{4 c \ln(f)+4 f}} \operatorname{erf}\left(-\sqrt{-c \ln(f}\right.}{4 \sqrt{-c \ln(f)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*cosh(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*Pi^(1/2)*f^a*exp(-1/4*(4*d*ln(f)*c-4*d*f+e^2)/(-f+c*ln(f)))/(f-c*ln(f))
^(1/2)*erf(x*(f-c*ln(f))^(1/2)+1/2*e/(f-c*ln(f))^(1/2))-1/4*Pi^(1/2)*f^a*ex
p(1/4*(4*d*ln(f)*c+4*d*f-e^2)/(f+c*ln(f)))/(-c*ln(f)-f)^(1/2)*erf(-(-c*ln(f
)-f)^(1/2)*x+1/2*e/(-c*ln(f)-f)^(1/2))
```

Maxima [A]
time = 0.31, size = 127, normalized size = 0.91

$$\frac{\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)-f}\, x-\frac{e}{2 \sqrt{-c \log(f)-f}}\right) e^{\left(d-\frac{e^{2}}{4(c \log(f)+f)}\right)}}{4 \sqrt{-c \log(f)-f}}+\frac{\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)+f}\, x+\frac{e}{2 \sqrt{-c \log(f)+f}}\right) e^{\left(-d-\frac{e^{2}}{4(c \log(f)-f)}\right)}}{4 \sqrt{-c \log(f)+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*e/sqrt(-c*log(f) - f))*e^(
d - 1/4*e^2/(c*log(f) + f))/sqrt(-c*log(f) - f) + 1/4*sqrt(pi)*f^a*erf(sqrt
(-c*log(f) + f)*x + 1/2*e/sqrt(-c*log(f) + f))*e^(-d - 1/4*e^2/(c*log(f) -
f))/sqrt(-c*log(f) + f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 381 vs. 2(119) = 238.
time = 0.50, size = 381, normalized size = 2.72

$$-\frac{\left(\sqrt{\pi}\,(c \log(f)+f) \cosh\left(\frac{4 a c \log(f)^{2}+4 d f-\cosh(1)^{2}-4(c d+a f) \log(f)-2 \cosh(1) \sinh(1)-\sinh(1)^{2}}{4(c \log(f)-f)}\right)+\sqrt{\pi}\,(c \log(f)+f) \sinh\left(\frac{4 a c \log(f)^{2}+4 d f-\cosh(1)^{2}-4(c d+a f) \log(f)-2 \cosh(1) \sinh(1)-\sinh(1)^{2}}{4(c \log(f)-f)}\right)\right) \sqrt{-c \log(f)+f} \operatorname{erf}\left(\frac{(2 c x \log(f)-2 f x-\cosh(1)-\sinh(1)) \sqrt{-c \log(f)+f}}{2(c \log(f)-f)}\right)+\left(\sqrt{\pi}\,(c \log(f)-f) \cosh\left(\frac{4 a c \log(f)^{2}+4 d f-\cosh(1)^{2}+4(c d+a f) \log(f)-2 \cosh(1) \sinh(1)-\sinh(1)^{2}}{4(c \log(f)+f)}\right)+\sqrt{\pi}\,(c \log(f)-f) \sinh\left(\frac{4 a c \log(f)^{2}+4 d f-\cosh(1)^{2}+4(c d+a f) \log(f)-2 \cosh(1) \sinh(1)-\sinh(1)^{2}}{4(c \log(f)+f)}\right)\right) \sqrt{-c \log(f)-f} \operatorname{erf}\left(\frac{(2 c x \log(f)+2 f x+\cosh(1)+\sinh(1)) \sqrt{-c \log(f)-f}}{2(c \log(f)+f)}\right)}{4\left(c^{2} \log(f)^{2}-f^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] -1/4*((sqrt(pi)*(c*log(f) + f)*cosh(1/4*(4*a*c*log(f)^2 + 4*d*f - cosh(1)^2
 - 4*(c*d + a*f)*log(f) - 2*cosh(1)*sinh(1) - sinh(1)^2)/(c*log(f) - f)) +
sqrt(pi)*(c*log(f) + f)*sinh(1/4*(4*a*c*log(f)^2 + 4*d*f - cosh(1)^2 - 4*(c
*d + a*f)*log(f) - 2*cosh(1)*sinh(1) - sinh(1)^2)/(c*log(f) - f)))*sqrt(-c*
log(f) + f)*erf(1/2*(2*c*x*log(f) - 2*f*x - cosh(1) - sinh(1))*sqrt(-c*log(
f) + f)/(c*log(f) - f)) + (sqrt(pi)*(c*log(f) - f)*cosh(1/4*(4*a*c*log(f)^2
 + 4*d*f - cosh(1)^2 + 4*(c*d + a*f)*log(f) - 2*cosh(1)*sinh(1) - sinh(1)^2
```

```
)/(c*log(f) + f)) + sqrt(pi)*(c*log(f) - f)*sinh(1/4*(4*a*c*log(f)^2 + 4*d*
f - cosh(1)^2 + 4*(c*d + a*f)*log(f) - 2*cosh(1)*sinh(1) - sinh(1)^2)/(c*lo
g(f) + f)))*sqrt(-c*log(f) - f)*erf(1/2*(2*c*x*log(f) + 2*f*x + cosh(1) + s
inh(1))*sqrt(-c*log(f) - f)/(c*log(f) + f)))/(c^2*log(f)^2 - f^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^2}\cosh\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*cosh(f*x**2+e*x+d),x)
```

```
[Out] Integral(f**(a + c*x**2)*cosh(d + e*x + f*x**2), x)
```

Giac [A]
time = 0.43, size = 172, normalized size = 1.23

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)-f}\,\left(2\,x+\frac{e}{c\log(f)+f}\right)\right)e^{\left(\frac{4\,ac\log(f)^2+4\,cd\log(f)+4\,af\log(f)-e^2+4\,df}{4\,(c\log(f)+f)}\right)}}{4\,\sqrt{-c\log\left(f\right)-f}}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)+f}\,\left(2\,x-\frac{e}{c\log(f)-f}\right)\right)e^{\left(\frac{4\,ac\log(f)^2-4\,cd\log(f)-4\,af\log(f)-e^2+4\,df}{4\,(c\log(f)-f)}\right)}}{4\,\sqrt{-c\log\left(f\right)+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - f)*(2*x + e/(c*log(f) + f)))*e^(1/4
*(4*a*c*log(f)^2 + 4*c*d*log(f) + 4*a*f*log(f) - e^2 + 4*d*f)/(c*log(f) + f
))/sqrt(-c*log(f) - f) - 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x - e
/(c*log(f) - f)))*e^(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - 4*a*f*log(f) - e^
2 + 4*d*f)/(c*log(f) - f))/sqrt(-c*log(f) + f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+a}\,\mathrm{cosh}\left(f\,x^2+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*cosh(d + e*x + f*x^2),x)
```

```
[Out] int(f^(a + c*x^2)*cosh(d + e*x + f*x^2), x)
```

3.318 $\int f^{a+cx^2}\cosh^2\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=183

$$\frac{f^a\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{e^{-2d+\frac{e^2}{2f-c\log(f)}}f^a\sqrt{\pi}\,\mathrm{Erf}\left(\frac{e+x(2f-c\log(f))}{\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{e^{2d-\frac{e^2}{2f+c\log(f)}}f^a\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{e+x(2\cdots}{\sqrt{2f\cdots}}\right.}{8\sqrt{2f+c\log(f)}}$$

```
[Out] 1/4*f^a*erfi(x*c^(1/2)*ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/8*exp(-2
*d+e^2/(2*f-c*ln(f)))*f^a*erf((e+x*(2*f-c*ln(f)))/(2*f-c*ln(f))^(1/2))*Pi^(
1/2)/(2*f-c*ln(f))^(1/2)+1/8*exp(2*d-e^2/(2*f+c*ln(f)))*f^a*erfi((e+x*(2*f+
c*ln(f)))/(2*f+c*ln(f))^(1/2))*Pi^(1/2)/(2*f+c*ln(f))^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {5624, 2235, 2325, 2266, 2236}

$$\frac{\sqrt{\pi}\,f^a e^{\frac{e^2}{2f-c\log(f)}-2d}\mathrm{Erf}\left(\frac{x(2f-c\log(f))+e}{\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi}\,f^a e^{2d-\frac{e^2}{c\log(f)+2f}}\mathrm{Erfi}\left(\frac{x(c\log(f)+2f)+e}{\sqrt{c\log(f)+2f}}\right)}{8\sqrt{c\log(f)+2f}}+\frac{\sqrt{\pi}\,f^a\mathrm{Erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Cosh[d + e*x + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) + (E^(
-2*d + e^2/(2*f - c*Log[f]))*f^a*Sqrt[Pi]*Erf[(e + x*(2*f - c*Log[f]))/Sqrt
[2*f - c*Log[f]]])/(8*Sqrt[2*f - c*Log[f]]) + (E^(2*d - e^2/(2*f + c*Log[f]
))*f^a*Sqrt[Pi]*Erfi[(e + x*(2*f + c*Log[f]))/Sqrt[2*f + c*Log[f]]])/(8*Sqr
t[2*f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5624

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int f^{a+cx^2}\cosh^2\left(d+ex+fx^2\right)\,dx &= \int\left(\frac{1}{2}f^{a+cx^2}+\frac{1}{4}e^{-2d-2ex-2fx^2}f^{a+cx^2}+\frac{1}{4}e^{2d+2ex+2fx^2}f^{a+cx^2}\right)\,dx\\
&= \frac{1}{4}\int e^{-2d-2ex-2fx^2}f^{a+cx^2}\,dx+\frac{1}{4}\int e^{2d+2ex+2fx^2}f^{a+cx^2}\,dx+\frac{1}{2}\int f^{a+cx^2}\\
&= \frac{f^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{1}{4}\int\exp\left(-2d-2ex+a\log(f)-x^2(2\right.\\
&= \frac{f^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{1}{4}\left(e^{-2d+\frac{e^2}{2f-c\log(f)}}f^a\right)\int\exp\left(\frac{(-2e+}{4(}\right.\\
&= \frac{f^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{e^{-2d+\frac{e^2}{2f-c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{e+x(2f-c\log(f}{\sqrt{2f-c\log(}}\right.}{8\sqrt{2f-c\log(f)}}
\end{aligned}$$

Mathematica [A]
time = 1.01, size = 258, normalized size = 1.41

$$\frac{e^{\frac{e^2}{2f-c\log(f)}}f^a\sqrt{\pi}\left(-2e^{\frac{e^2}{-2f+c\log(f)}}\mathrm{Erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)\left(4f^2-c^2\log^2(f)\right)-\sqrt{c}\,\sqrt{\log(f)}\left(\mathrm{Erf}\left(\frac{e+2fx-cx\log(f)}{\sqrt{2f-c\log(f)}}\right)\sqrt{2f-c\log(f)}\,(2f+c\log(f))(\cosh(2d)-\sinh(2d))+e^{\frac{4e^2f}{-4f^2+c^2\log^2(f)}}\mathrm{Erfi}\left(\frac{e+2fx+cx\log(f)}{\sqrt{2f+c\log(f)}}\right)(2f-c\log(f))\sqrt{2f+c\log(f)}\,(\cosh(2d)+\sinh(2d))\right)\right)}{8\sqrt{c}\,\sqrt{\log(f)}\left(-4f^2+c^2\log^2(f)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Cosh[d + e*x + f*x^2]^2,x]
```

```
[Out] (E^(e^2/(2*f - c*Log[f]))*f^a*Sqrt[Pi]*(-2*E^(e^2/(-2*f + c*Log[f]))*Erfi[S
qrt[c]*x*Sqrt[Log[f]]]*(4*f^2 - c^2*Log[f]^2) - Sqrt[c]*Sqrt[Log[f]]*(Erf[(
e + 2*f*x - c*x*Log[f])/Sqrt[2*f - c*Log[f]]]*Sqrt[2*f - c*Log[f]]*(2*f + c
*Log[f])*(Cosh[2*d] - Sinh[2*d]) + E^((4*e^2*f)/(-4*f^2 + c^2*Log[f]^2))*Er
fi[(e + 2*f*x + c*x*Log[f])/Sqrt[2*f + c*Log[f]]]*(2*f - c*Log[f])*Sqrt[2*f
```

```
+ c*Log[f]]*(Cosh[2*d] + Sinh[2*d]))))/(8*Sqrt[c]*Sqrt[Log[f]]*(-4*f^2 + c
^2*Log[f]^2))
```

Maple [A]
time = 1.67, size = 177, normalized size = 0.97

method	result
risch	$\frac{\sqrt{\pi}\, f^{a} {\mathrm e}^{-\frac{2 d \ln \left(f \right) c -4 d f +e^{2}}{-2 f +c \ln \left(f \right)}} \operatorname{erf}\left(x \sqrt{2 f -c \ln \left(f \right)}+\frac{e}{\sqrt{2 f -c \ln \left(f \right)}}\right)}{8 \sqrt{2 f -c \ln \left(f \right)}}-\frac{\sqrt{\pi}\, f^{a} {\mathrm e}^{\frac{2 d \ln \left(f \right) c +4 d f -e^{2}}{2 f +c \ln \left(f \right)}} \operatorname{erf}\left(-\sqrt{-c \ln}\right.}{8 \sqrt{-c \ln (}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*cosh(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*Pi^(1/2)*f^a*exp(-(2*d*ln(f)*c-4*d*f+e^2)/(-2*f+c*ln(f)))/(2*f-c*ln(f))
^(1/2)*erf(x*(2*f-c*ln(f))^(1/2)+e/(2*f-c*ln(f))^(1/2))-1/8*Pi^(1/2)*f^a*ex
p((2*d*ln(f)*c+4*d*f-e^2)/(2*f+c*ln(f)))/(-c*ln(f)-2*f)^(1/2)*erf(-(-c*ln(f
)-2*f)^(1/2)*x+e/(-c*ln(f)-2*f)^(1/2))+1/4*f^a*Pi^(1/2)/(-c*ln(f))^(1/2)*er
f((-c*ln(f))^(1/2)*x)
```

Maxima [A]
time = 0.31, size = 161, normalized size = 0.88

$$\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log\left(f\right) - 2 \, f} x - \frac{e}{\sqrt{-c \log\left(f\right) - 2 \, f}}\right) e^{\left(2 \, d - \frac{e^{2}}{c \log\left(f\right) + 2 \, f}\right)}}{8 \, \sqrt{-c \log\left(f\right) - 2 \, f}} + \frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log\left(f\right) + 2 \, f} x + \frac{e}{\sqrt{-c \log\left(f\right) + 2 \, f}}\right) e^{\left(-2 \, d - \frac{e^{2}}{c \log\left(f\right) - 2 \, f}\right)}}{8 \, \sqrt{-c \log\left(f\right) + 2 \, f}} + \frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log\left(f\right)} x\right)}{4 \, \sqrt{-c \log\left(f\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 2*f)*x - e/sqrt(-c*log(f) - 2*f))*e^(
2*d - e^2/(c*log(f) + 2*f))/sqrt(-c*log(f) - 2*f) + 1/8*sqrt(pi)*f^a*erf(sq
rt(-c*log(f) + 2*f)*x + e/sqrt(-c*log(f) + 2*f))*e^(-2*d - e^2/(c*log(f) -
2*f))/sqrt(-c*log(f) + 2*f) + 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x)/sqrt(
-c*log(f))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 480 vs. 2(155) = 310.
time = 0.43, size = 480, normalized size = 2.62

$$-\frac{2 \left(\sqrt{\pi} \left(c^{2} \log\left(f\right)^{2} - 4 \, f^{2}\right) \cosh\left(a \log\left(f\right)\right) + \sqrt{\pi} \left(c^{2} \log\left(f\right)^{2} - 4 \, f^{2}\right) \sinh\left(a \log\left(f\right)\right)\right) \sqrt{-c \log\left(f\right)} \operatorname{erf}\left(\sqrt{-c \log\left(f\right)} x\right) + \left(\sqrt{\pi} \left(c^{2} \log\left(f\right)^{2} + 2 \, c f \log\left(f\right)\right) \cosh\left(\frac{a c \log\left(f\right)^{2} + 4 \, d f - \cosh\left(1\right)^{2} - 2 \, {\left(c d + a f\right)} \log\left(f\right) - 2 \, \cosh\left(1\right) \sinh\left(1\right) - \sinh\left(1\right)^{2}}{c \log\left(f\right) - 2 \, f}\right) + \sqrt{\pi} \left(c^{2} \log\left(f\right)^{2} + 2 \, c f \log\left(f\right)\right) \sinh\left(\frac{a c \log\left(f\right)^{2} + 4 \, d f - \cosh\left(1\right)^{2} - 2 \, {\left(c d + a f\right)} \log\left(f\right) - 2 \, \cosh\left(1\right) \sinh\left(1\right) - \sinh\left(1\right)^{2}}{c \log\left(f\right) - 2 \, f}\right)\right) \sqrt{-c \log\left(f\right) + 2 \, f} \operatorname{erf}\left(\frac{{\left(c x \log\left(f\right) - 2 \, f x - \cosh\left(1\right) - \sinh\left(1\right)\right)} \sqrt{-c \log\left(f\right) + 2 \, f}}{c \log\left(f\right) - 2 \, f}\right) + \left(\sqrt{\pi} \left(c^{2} \log\left(f\right)^{2} - 2 \, c f \log\left(f\right)\right) \cosh\left(\frac{a c \log\left(f\right)^{2} + 4 \, d f - \cosh\left(1\right)^{2} + 2 \, {\left(c d + a f\right)} \log\left(f\right) - 2 \, \cosh\left(1\right) \sinh\left(1\right) - \sinh\left(1\right)^{2}}{c \log\left(f\right) + 2 \, f}\right) + \sqrt{\pi} \left(c^{2} \log\left(f\right)^{2} - 2 \, c f \log\left(f\right)\right) \sinh\left(\frac{a c \log\left(f\right)^{2} + 4 \, d f - \cosh\left(1\right)^{2} + 2 \, {\left(c d + a f\right)} \log\left(f\right) - 2 \, \cosh\left(1\right) \sinh\left(1\right) - \sinh\left(1\right)^{2}}{c \log\left(f\right) + 2 \, f}\right)\right) \sqrt{-c \log\left(f\right) - 2 \, f} \operatorname{erf}\left(\frac{{\left(c x \log\left(f\right) + 2 \, f x + \cosh\left(1\right) + \sinh\left(1\right)\right)} \sqrt{-c \log\left(f\right) - 2 \, f}}{c \log\left(f\right) + 2 \, f}\right)}{8 \, {\left(c^{3} \log\left(f\right)^{3} - 4 \, c f^{2} \log\left(f\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*(sqrt(pi)*(c^2*log(f)^2 - 4*f^2)*cosh(a*log(f)) + sqrt(pi)*(c^2*log
(f)^2 - 4*f^2)*sinh(a*log(f)))*sqrt(-c*log(f))*erf(sqrt(-c*log(f))*x) + (sq
```

```
rt(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*cosh((a*c*log(f)^2 + 4*d*f - cosh(1)^2
 - 2*(c*d + a*f)*log(f) - 2*cosh(1)*sinh(1) - sinh(1)^2)/(c*log(f) - 2*f))
+ sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*sinh((a*c*log(f)^2 + 4*d*f - cosh(
1)^2 - 2*(c*d + a*f)*log(f) - 2*cosh(1)*sinh(1) - sinh(1)^2)/(c*log(f) - 2*
f)))*sqrt(-c*log(f) + 2*f)*erf((c*x*log(f) - 2*f*x - cosh(1) - sinh(1))*sqr
t(-c*log(f) + 2*f)/(c*log(f) - 2*f)) + (sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(
f))*cosh((a*c*log(f)^2 + 4*d*f - cosh(1)^2 + 2*(c*d + a*f)*log(f) - 2*cosh(
1)*sinh(1) - sinh(1)^2)/(c*log(f) + 2*f)) + sqrt(pi)*(c^2*log(f)^2 - 2*c*f*
log(f))*sinh((a*c*log(f)^2 + 4*d*f - cosh(1)^2 + 2*(c*d + a*f)*log(f) - 2*c
osh(1)*sinh(1) - sinh(1)^2)/(c*log(f) + 2*f)))*sqrt(-c*log(f) - 2*f)*erf((c
*x*log(f) + 2*f*x + cosh(1) + sinh(1))*sqrt(-c*log(f) - 2*f)/(c*log(f) + 2*
f)))/(c^3*log(f)^3 - 4*c*f^2*log(f))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^2}\cosh^2\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*cosh(f*x**2+e*x+d)**2,x)
```

```
[Out] Integral(f**(a + c*x**2)*cosh(d + e*x + f*x**2)**2, x)
```

Giac [A]
time = 0.43, size = 198, normalized size = 1.08

$$-\frac{\sqrt{\pi}\,f^{a}\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)}\,x\right)}{4\,\sqrt{-c\log\left(f\right)}}-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)-2\,f}\,\left(x+\frac{e}{c\log(f)+2\,f}\right)\right)e^{\left(\frac{ac\log(f)^2+2\,cd\log(f)+2\,af\log(f)-e^2+4\,df}{c\log(f)+2\,f}\right)}}{8\,\sqrt{-c\log\left(f\right)-2\,f}}-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)+2\,f}\,\left(x-\frac{e}{c\log(f)-2\,f}\right)\right)e^{\left(\frac{ac\log(f)^2-2\,cd\log(f)-2\,af\log(f)-e^2+4\,df}{c\log(f)-2\,f}\right)}}{8\,\sqrt{-c\log\left(f\right)+2\,f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*f^a*erf(-sqrt(-c*log(f))*x)/sqrt(-c*log(f)) - 1/8*sqrt(pi)*er
f(-sqrt(-c*log(f) - 2*f)*(x + e/(c*log(f) + 2*f)))*e^((a*c*log(f)^2 + 2*c*d
*log(f) + 2*a*f*log(f) - e^2 + 4*d*f)/(c*log(f) + 2*f))/sqrt(-c*log(f) - 2*
f) - 1/8*sqrt(pi)*erf(-sqrt(-c*log(f) + 2*f)*(x - e/(c*log(f) - 2*f)))*e^((
a*c*log(f)^2 - 2*c*d*log(f) - 2*a*f*log(f) - e^2 + 4*d*f)/(c*log(f) - 2*f))
/sqrt(-c*log(f) + 2*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+a}\,{\mathrm{cosh}\left(f\,x^2+e\,x+d\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*cosh(d + e*x + f*x^2)^2,x)
```

```
[Out] int(f^(a + c*x^2)*cosh(d + e*x + f*x^2)^2, x)
```

3.319 $\int f^{a+cx^2} \cosh^3(d+ex+fx^2)\,dx$

Optimal. Leaf size=300

$$\frac{3e^{-d+\frac{e^2}{4f-4c\log(f)}}f^a\sqrt{\pi}\,\text{Erf}\left(\frac{e+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}+\frac{e^{-3d+\frac{9e^2}{12f-4c\log(f)}}f^a\sqrt{\pi}\,\text{Erf}\left(\frac{3e+2x(3f-c\log(f))}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}+\frac{3e^{d-\frac{e^2}{4(f+c\log(f}}}{}$$

```
[Out] 3/16*exp(-d+e^2/(4*f-4*c*ln(f)))*f^a*erf(1/2*(e+2*x*(f-c*ln(f)))/(f-c*ln(f)
)^(1/2))*Pi^(1/2)/(f-c*ln(f))^(1/2)+1/16*exp(-3*d+9*e^2/(12*f-4*c*ln(f)))*f
^a*erf(1/2*(3*e+2*x*(3*f-c*ln(f)))/(3*f-c*ln(f))^(1/2))*Pi^(1/2)/(3*f-c*ln(
f))^(1/2)+3/16*exp(d-1/4*e^2/(f+c*ln(f)))*f^a*erfi(1/2*(e+2*x*(f+c*ln(f)))/
(f+c*ln(f))^(1/2))*Pi^(1/2)/(f+c*ln(f))^(1/2)+1/16*exp(3*d-9/4*e^2/(3*f+c*l
n(f)))*f^a*erfi(1/2*(3*e+2*x*(3*f+c*ln(f)))/(3*f+c*ln(f))^(1/2))*Pi^(1/2)/(
3*f+c*ln(f))^(1/2)
```

Rubi [A]
time = 0.48, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {5624, 2325, 2266, 2236, 2235}

$$\frac{3\sqrt{\pi}f^ae^{\frac{e^2}{4f-4c\log(f)}-d}\text{Erf}\left(\frac{2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}f^ae^{\frac{9e^2}{12f-4c\log(f)}-3d}\text{Erf}\left(\frac{2x(3f-c\log(f))+3e}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}+\frac{3\sqrt{\pi}f^ae^{d-\frac{e^2}{4(c\log(f)+f)}}\text{Erfi}\left(\frac{2x(c\log(f)+f)+e}{2\sqrt{c\log(f)+f}}\right)}{16\sqrt{c\log(f)+f}}+\frac{\sqrt{\pi}f^ae^{3d-\frac{9e^2}{4(c\log(f)+3f)}}\text{Erfi}\left(\frac{2x(c\log(f)+3f)+3e}{2\sqrt{c\log(f)+3f}}\right)}{16\sqrt{c\log(f)+3f}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Cosh[d + e*x + f*x^2]^3,x]
```

```
[Out] (3*E^(-d + e^2/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(e + 2*x*(f - c*Log[f])
)/(2*Sqrt[f - c*Log[f]])])/(16*Sqrt[f - c*Log[f]]) + (E^(-3*d + (9*e^2)/(12
*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(3*e + 2*x*(3*f - c*Log[f]))/(2*Sqrt[3*f
 - c*Log[f]])])/(16*Sqrt[3*f - c*Log[f]]) + (3*E^(d - e^2/(4*(f + c*Log[f])
))*f^a*Sqrt[Pi]*Erfi[(e + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(16*
Sqrt[f + c*Log[f]]) + (E^(3*d - (9*e^2)/(4*(3*f + c*Log[f]))))*f^a*Sqrt[Pi]*
Erfi[(3*e + 2*x*(3*f + c*Log[f]))/(2*Sqrt[3*f + c*Log[f]])])/(16*Sqrt[3*f +
 c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5624

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int f^{a+cx^2}\cosh^3\left(d+ex+fx^2\right)\,dx = \int\left(\frac{1}{8}e^{-3\left(d+ex+fx^2\right)}f^{a+cx^2}+\frac{3}{8}\exp\left(2d+2ex+2fx^2-3\left(d+ex+fx^2\right.\right.\right.$$

$$= \frac{1}{8}\int e^{-3\left(d+ex+fx^2\right)}f^{a+cx^2}\,dx+\frac{1}{8}\int\exp\left(6d+6ex+6fx^2-3\left(d+ex+\right.\right.$$

$$= \frac{1}{8}\int\exp\left(-3d-3ex+a\log(f)-x^2(3f-c\log(f))\right)\,dx+\frac{1}{8}\int\exp\left(3\right.$$

$$= \frac{1}{8}\left(3e^{-d+\frac{e^2}{4f-4c\log(f)}}f^a\right)\int\exp\left(\frac{(-e+2x(-f+c\log(f)))^2}{4(-f+c\log(f))}\right)\,dx+\frac{1}{8}\left(\right.$$

$$= \frac{3e^{-d+\frac{e^2}{4f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{e+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}+\frac{e^{-3d+\frac{9e^2}{12f-4c\log(f)}}f^a\sqrt{\pi}}{16\sqrt{3f}}$$

Mathematica [A]
time = 4.06, size = 478, normalized size = 1.59

$$\frac{e^{-\frac{1}{4}e^2\left(\frac{1}{f+c\log(f)}+\frac{9}{3f+c\log(f)}\right)}f^a\sqrt{\pi}\left(3e^{\frac{1}{4}e^2\left(\frac{1}{f-c\log(f)}+\frac{1}{f+c\log(f)}+\frac{9}{3f+c\log(f)}\right)}\mathrm{Erf}\left(\frac{e+2fx-2cx\log(f)}{2\sqrt{f-c\log(f)}}\right)\sqrt{f-c\log(f)}\left(9f^3+9cf^2\log(f)-c^2f\log^2(f)-c^3\log^3(f)\right)(\cosh(d)-\sinh(d))+(f-c\log(f))\left(e^{\frac{1}{4}e^2\left(\frac{9}{3f-c\log(f)}+\frac{1}{f+c\log(f)}+\frac{9}{3f+c\log(f)}\right)}\mathrm{Erf}\left(\frac{3e+6fx-2cx\log(f)}{2\sqrt{3f-c\log(f)}}\right)\sqrt{3f-c\log(f)}\left(3f^2+4cf\log(f)+c^2\log^2(f)\right)(\cosh(3d)-\sinh(3d))+(3f-c\log(f))\left(3e^{\frac{9e^2}{4(3f+c\log(f))}}\mathrm{Erfi}\left(\frac{e+2fx+2cx\log(f)}{2\sqrt{f+c\log(f)}}\right)\sqrt{f+c\log(f)}(3f+c\log(f))(\cosh(d)+\sinh(d))+e^{\frac{e^2}{4(f+c\log(f))}}\mathrm{Erfi}\left(\frac{3e+6fx+2cx\log(f)}{2\sqrt{3f+c\log(f)}}\right)(f+c\log(f))\sqrt{3f+c\log(f)}(\cosh(3d)+\sinh(3d))\right)\right)\right)}{16\left(9f^4-10c^2f^2\log^2(f)+c^4\log^4(f)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + c*x^2)*Cosh[d + e*x + f*x^2]^3,x]
```

```
[Out] (f^a*Sqrt[Pi]*(3*E^((e^2*((f - c*Log[f])^(-1) + (f + c*Log[f])^(-1) + 9/(3*
f + c*Log[f])))/4)*Erf[(e + 2*f*x - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*S
qrt[f - c*Log[f]]*(9*f^3 + 9*c*f^2*Log[f] - c^2*f*Log[f]^2 - c^3*Log[f]^3)*
(Cosh[d] - Sinh[d]) + (f - c*Log[f])*(E^((e^2*(9/(3*f - c*Log[f]) + (f + c*
Log[f])^(-1) + 9/(3*f + c*Log[f])))/4)*Erf[(3*e + 6*f*x - 2*c*x*Log[f])/(2*
Sqrt[3*f - c*Log[f]])]*Sqrt[3*f - c*Log[f]]*(3*f^2 + 4*c*f*Log[f] + c^2*Log
[f]^2)*(Cosh[3*d] - Sinh[3*d]) + (3*f - c*Log[f])*(3*E^((9*e^2)/(4*(3*f + c
*Log[f])))*Erfi[(e + 2*f*x + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Sqrt[f +
 c*Log[f]]*(3*f + c*Log[f])*(Cosh[d] + Sinh[d]) + E^(e^2/(4*(f + c*Log[f]))
)*Erfi[(3*e + 6*f*x + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*(f + c*Log[f]
)*Sqrt[3*f + c*Log[f]]*(Cosh[3*d] + Sinh[3*d])))))/(16*E^((e^2*((f + c*Log[
f])^(-1) + 9/(3*f + c*Log[f])))/4)*(9*f^4 - 10*c^2*f^2*Log[f]^2 + c^4*Log[f
]^4))
```

Maple [A]
time = 2.70, size = 302, normalized size = 1.01

method	result
risch	$\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{3\left(4 d \ln\left(f\right) c-12 d f+3 e^{2}\right)}{4\left(-3 f+c \ln\left(f\right)\right)}} \mathrm{erf}\left(x \sqrt{3 f-c \ln\left(f\right)}+\frac{3 e}{2 \sqrt{3 f-c \ln\left(f\right)}}\right)}{16 \sqrt{3 f-c \ln\left(f\right)}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{3 d \ln\left(f\right) c+9 d f-\frac{9 e^{2}}{4}}{3 f+c \ln\left(f\right)}} \mathrm{erf}\left(-\sqrt{\ldots}\right.}{16 \sqrt{\ldots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*cosh(f*x^2+e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*Pi^(1/2)*f^a*exp(-3/4*(4*d*ln(f)*c-12*d*f+3*e^2)/(-3*f+c*ln(f)))/(3*f-
c*ln(f))^(1/2)*erf(x*(3*f-c*ln(f))^(1/2)+3/2*e/(3*f-c*ln(f))^(1/2))-1/16*Pi
^(1/2)*f^a*exp(3/4*(4*d*ln(f)*c+12*d*f-3*e^2)/(3*f+c*ln(f)))/(-c*ln(f)-3*f)
^(1/2)*erf(-(-c*ln(f)-3*f)^(1/2)*x+3/2*e/(-c*ln(f)-3*f)^(1/2))+3/16*Pi^(1/2
)*f^a*exp(-1/4*(4*d*ln(f)*c-4*d*f+e^2)/(-f+c*ln(f)))/(f-c*ln(f))^(1/2)*erf(
x*(f-c*ln(f))^(1/2)+1/2*e/(f-c*ln(f))^(1/2))-3/16*Pi^(1/2)*f^a*exp(1/4*(4*d
*ln(f)*c+4*d*f-e^2)/(f+c*ln(f)))/(-c*ln(f)-f)^(1/2)*erf(-(-c*ln(f)-f)^(1/2)
*x+1/2*e/(-c*ln(f)-f)^(1/2))
```

Maxima [A]
time = 0.30, size = 263, normalized size = 0.88

$$\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log\left(f\right) - 3 \, f} x - \frac{3 \, e}{2 \, \sqrt{-c \log\left(f\right) - 3 \, f}}\right) e^{\left(3 \, d - \frac{9 \, e^{2}}{4 \, {\left(c \log\left(f\right) + 3 \, f\right)}}\right)}}{16 \, \sqrt{-c \log\left(f\right) - 3 \, f}} + \frac{3 \, \sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log\left(f\right) - f} x - \frac{e}{2 \, \sqrt{-c \log\left(f\right) - f}}\right) e^{\left(d - \frac{e^{2}}{4 \, {\left(c \log\left(f\right) + f\right)}}\right)}}{16 \, \sqrt{-c \log\left(f\right) - f}} + \frac{3 \, \sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log\left(f\right) + f} x + \frac{e}{2 \, \sqrt{-c \log\left(f\right) + f}}\right) e^{\left(-d - \frac{e^{2}}{4 \, {\left(c \log\left(f\right) - f\right)}}\right)}}{16 \, \sqrt{-c \log\left(f\right) + f}} + \frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log\left(f\right) + 3 \, f} x + \frac{3 \, e}{2 \, \sqrt{-c \log\left(f\right) + 3 \, f}}\right) e^{\left(-3 \, d - \frac{9 \, e^{2}}{4 \, {\left(c \log\left(f\right) - 3 \, f\right)}}\right)}}{16 \, \sqrt{-c \log\left(f\right) + 3 \, f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 3*f)*x - 3/2*e/sqrt(-c*log(f) - 3*f)
)*e^(3*d - 9/4*e^2/(c*log(f) + 3*f))/sqrt(-c*log(f) - 3*f) + 3/16*sqrt(pi)*
f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*e/sqrt(-c*log(f) - f))*e^(d - 1/4*e^2/(
```

```
c*log(f) + f))/sqrt(-c*log(f) - f) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) +
 f)*x + 1/2*e/sqrt(-c*log(f) + f))*e^(-d - 1/4*e^2/(c*log(f) - f))/sqrt(-c*
log(f) + f) + 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 3*f)*x + 3/2*e/sqrt(-c
*log(f) + 3*f))*e^(-3*d - 9/4*e^2/(c*log(f) - 3*f))/sqrt(-c*log(f) + 3*f)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 969 vs. $2(253) = 506$.
time = 0.50, size = 969, normalized size = 3.23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*((sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3)*c
osh(1/4*(4*a*c*log(f)^2 + 36*d*f - 9*cosh(1)^2 - 12*(c*d + a*f)*log(f) - 18
*cosh(1)*sinh(1) - 9*sinh(1)^2)/(c*log(f) - 3*f)) + sqrt(pi)*(c^3*log(f)^3
+ 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3)*sinh(1/4*(4*a*c*log(f)^2 + 36*d*
f - 9*cosh(1)^2 - 12*(c*d + a*f)*log(f) - 18*cosh(1)*sinh(1) - 9*sinh(1)^2)
/(c*log(f) - 3*f)))*sqrt(-c*log(f) + 3*f)*erf(1/2*(2*c*x*log(f) - 6*f*x - 3
*cosh(1) - 3*sinh(1))*sqrt(-c*log(f) + 3*f)/(c*log(f) - 3*f)) + 3*(sqrt(pi)
*(c^3*log(f)^3 + c^2*f*log(f)^2 - 9*c*f^2*log(f) - 9*f^3)*cosh(1/4*(4*a*c*l
og(f)^2 + 4*d*f - cosh(1)^2 - 4*(c*d + a*f)*log(f) - 2*cosh(1)*sinh(1) - si
nh(1)^2)/(c*log(f) - f)) + sqrt(pi)*(c^3*log(f)^3 + c^2*f*log(f)^2 - 9*c*f^
2*log(f) - 9*f^3)*sinh(1/4*(4*a*c*log(f)^2 + 4*d*f - cosh(1)^2 - 4*(c*d + a
*f)*log(f) - 2*cosh(1)*sinh(1) - sinh(1)^2)/(c*log(f) - f)))*sqrt(-c*log(f)
 + f)*erf(1/2*(2*c*x*log(f) - 2*f*x - cosh(1) - sinh(1))*sqrt(-c*log(f) + f
)/(c*log(f) - f)) + 3*(sqrt(pi)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*lo
g(f) + 9*f^3)*cosh(1/4*(4*a*c*log(f)^2 + 4*d*f - cosh(1)^2 + 4*(c*d + a*f)*
log(f) - 2*cosh(1)*sinh(1) - sinh(1)^2)/(c*log(f) + f)) + sqrt(pi)*(c^3*log
(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*sinh(1/4*(4*a*c*log(f)^2 +
 4*d*f - cosh(1)^2 + 4*(c*d + a*f)*log(f) - 2*cosh(1)*sinh(1) - sinh(1)^2)/
(c*log(f) + f)))*sqrt(-c*log(f) - f)*erf(1/2*(2*c*x*log(f) + 2*f*x + cosh(1
) + sinh(1))*sqrt(-c*log(f) - f)/(c*log(f) + f)) + (sqrt(pi)*(c^3*log(f)^3
- 3*c^2*f*log(f)^2 - c*f^2*log(f) + 3*f^3)*cosh(1/4*(4*a*c*log(f)^2 + 36*d*
f - 9*cosh(1)^2 + 12*(c*d + a*f)*log(f) - 18*cosh(1)*sinh(1) - 9*sinh(1)^2)
/(c*log(f) + 3*f)) + sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(
f) + 3*f^3)*sinh(1/4*(4*a*c*log(f)^2 + 36*d*f - 9*cosh(1)^2 + 12*(c*d + a*f
)*log(f) - 18*cosh(1)*sinh(1) - 9*sinh(1)^2)/(c*log(f) + 3*f)))*sqrt(-c*log
(f) - 3*f)*erf(1/2*(2*c*x*log(f) + 6*f*x + 3*cosh(1) + 3*sinh(1))*sqrt(-c*l
og(f) - 3*f)/(c*log(f) + 3*f)))/(c^4*log(f)^4 - 10*c^2*f^2*log(f)^2 + 9*f^4
)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*cosh(f*x**2+e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.43, size = 352, normalized size = 1.17

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)-3\,f}\left(2\,x+\frac{3\,e}{c\log(f)+3\,f}\right)\right)e^{\left(\frac{4\,ac\log(f)^2+12\,cd\log(f)+12\,af\log(f)-9\,e^2+36\,df}{4\,(c\log(f)+3\,f)}\right)}}{16\,\sqrt{-c\log\left(f\right)-3\,f}}-\frac{3\,\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)-f}\left(2\,x+\frac{e}{c\log(f)+f}\right)\right)e^{\left(\frac{4\,ac\log(f)^2+4\,cd\log(f)+4\,af\log(f)-e^2+4\,df}{4\,(c\log(f)+f)}\right)}}{16\,\sqrt{-c\log\left(f\right)-f}}-\frac{3\,\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)+f}\left(2\,x-\frac{e}{c\log(f)-f}\right)\right)e^{\left(\frac{4\,ac\log(f)^2-4\,cd\log(f)-4\,af\log(f)-e^2+4\,df}{4\,(c\log(f)-f)}\right)}}{16\,\sqrt{-c\log\left(f\right)+f}}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)+3\,f}\left(2\,x-\frac{3\,e}{c\log(f)-3\,f}\right)\right)e^{\left(\frac{4\,ac\log(f)^2-12\,cd\log(f)-12\,af\log(f)-9\,e^2+36\,df}{4\,(c\log(f)-3\,f)}\right)}}{16\,\sqrt{-c\log\left(f\right)+3\,f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+e*x+d)^3,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 3*f)*(2*x + 3*e/(c*log(f) + 3*f)))
*e^(1/4*(4*a*c*log(f)^2 + 12*c*d*log(f) + 12*a*f*log(f) - 9*e^2 + 36*d*f)/(
c*log(f) + 3*f))/sqrt(-c*log(f) - 3*f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log
(f) - f)*(2*x + e/(c*log(f) + f)))*e^(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f) +
4*a*f*log(f) - e^2 + 4*d*f)/(c*log(f) + f))/sqrt(-c*log(f) - f) - 3/16*sqrt
(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x - e/(c*log(f) - f)))*e^(1/4*(4*a*c*l
og(f)^2 - 4*c*d*log(f) - 4*a*f*log(f) - e^2 + 4*d*f)/(c*log(f) - f))/sqrt(-
c*log(f) + f) - 1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + 3*f)*(2*x - 3*e/(c*
log(f) - 3*f)))*e^(1/4*(4*a*c*log(f)^2 - 12*c*d*log(f) - 12*a*f*log(f) - 9*
e^2 + 36*d*f)/(c*log(f) - 3*f))/sqrt(-c*log(f) + 3*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+a}\,{\mathrm{cosh}\left(f\,x^2+e\,x+d\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*cosh(d + e*x + f*x^2)^3,x)
```

```
[Out] int(f^(a + c*x^2)*cosh(d + e*x + f*x^2)^3, x)
```

3.320 $\int f^{a+bx+cx^2}\cosh(d+ex)\,dx$

Optimal. Leaf size=153

$$-\frac{e^{-d-\frac{(e-b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\text{Erfi}\left(\frac{e-b\log(f)-2cx\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{e^{d-\frac{(e+b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\text{Erfi}\left(\frac{e+b\log(f)+2cx\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}$$

```
[Out] 1/4*exp(-d-1/4*(e-b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-e+b*ln(f)+2*c*x*ln(f))
/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/4*exp(d-1/4*(e+b*ln(f)
)^2/c/ln(f))*f^a*erfi(1/2*(e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(
1/2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {5624, 2325, 2266, 2235}

$$\frac{\sqrt{\pi}\,f^a e^{d-\frac{(b\log(f)+e)^2}{4c\log(f)}}\text{Erfi}\left(\frac{b\log(f)+2cx\log(f)+e}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}-\frac{\sqrt{\pi}\,f^a e^{-\frac{(e-b\log(f))^2}{4c\log(f)}-d}\text{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+e}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Cosh[d + e*x],x]
```

```
[Out] -1/4*(E^(-d - (e - b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - b*Log[f
] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt[Log[f]]) + (E^(d
 - (e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e + b*Log[f] + 2*c*x*L
og[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
```

```
x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5624

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int f^{a+bx+cx^2}\cosh(d+ex)\,dx &= \int\left(\frac{1}{2}e^{-d-ex}f^{a+bx+cx^2}+\frac{1}{2}e^{d+ex}f^{a+bx+cx^2}\right)dx\\
&= \frac{1}{2}\int e^{-d-ex}f^{a+bx+cx^2}\,dx+\frac{1}{2}\int e^{d+ex}f^{a+bx+cx^2}\,dx\\
&= \frac{1}{2}\int \exp\left(-d+a\log(f)+cx^2\log(f)-x(e-b\log(f))\right)dx+\frac{1}{2}\int \exp\left(d+a\right.\\
&= \frac{1}{2}\left(e^{-d-\frac{(e-b\log(f))^2}{4c\log(f)}}f^a\right)\int \exp\left(\frac{(-e+b\log(f)+2cx\log(f))^2}{4c\log(f)}\right)dx+\frac{1}{2}\left(e^{d-\frac{(e-}{}}\right.\\
&= -\frac{e^{-d-\frac{(e-b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{d-\frac{(e+b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e+b}{2\sqrt{}}\right.}{4\sqrt{c}\sqrt{\log(f}}
\end{aligned}$$

Mathematica [A]
time = 0.19, size = 134, normalized size = 0.88

$$\frac{e^{-\frac{e(e+2b\log(f))}{4c\log(f)}}f^{a-\frac{b^2}{4c}}\sqrt{\pi}\left(e^{\frac{be}{c}}\mathrm{Erfi}\left(\frac{-e+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cosh(d)-\sinh(d))+\mathrm{Erfi}\left(\frac{e+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cosh(d)+\sinh(d))\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Cosh[d + e*x],x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*(E^((b*e)/c)*Erfi[(-e + (b + 2*c*x)*Log[f])/(2*
Sqrt[c]*Sqrt[Log[f]])]*(Cosh[d] - Sinh[d]) + Erfi[(e + (b + 2*c*x)*Log[f])/
(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[d] + Sinh[d])))/(4*Sqrt[c]*E^((e*(e + 2*b*L
og[f]))/(4*c*Log[f]))*Sqrt[Log[f]])
```

Maple [A]
time = 0.85, size = 156, normalized size = 1.02

method	result

risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2} \ln(f)^{2}-2 \ln(f) b e+4 d \ln(f) c+e^{2}}{4 \ln(f) c}} \operatorname{erf}\left(-\sqrt{-c \ln(f)}\, x+\frac{b \ln(f)-e}{2 \sqrt{-c \ln(f)}}\right)}{4 \sqrt{-c \ln(f)}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2} \ln(f)^{2}+2 \ln(f) b e-4 d}{4 \ln(f) c}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cosh(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-2*ln(f)*b*e+4*d*ln(f)*c+e^2)/ln(f)/
c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-e)/(-c*ln(f))^(1/2
))-1/4*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+2*ln(f)*b*e-4*d*ln(f)*c+e^2)/ln(f
)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(e+b*ln(f))/(-c*ln(f))^(1
/2))
```

Maxima [A]
time = 0.33, size = 133, normalized size = 0.87

$$\frac{\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x-\frac{b \log(f)+e}{2 \sqrt{-c \log(f)}}\right) e^{\left(d-\frac{(b \log(f)+e)^{2}}{4 c \log(f)}\right)}}{4 \sqrt{-c \log(f)}}+\frac{\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x-\frac{b \log(f)-e}{2 \sqrt{-c \log(f)}}\right) e^{\left(-d-\frac{(b \log(f)-e)^{2}}{4 c \log(f)}\right)}}{4 \sqrt{-c \log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) + e)/sqrt(-c*log(f))
)*e^(d - 1/4*(b*log(f) + e)^2/(c*log(f)))/sqrt(-c*log(f)) + 1/4*sqrt(pi)*f^
a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) - e)/sqrt(-c*log(f)))*e^(-d - 1/4*(
b*log(f) - e)^2/(c*log(f)))/sqrt(-c*log(f))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 338 vs. 2(125) = 250.
time = 0.49, size = 338, normalized size = 2.21

$$-\frac{\sqrt{-c \log(f)}\left(\sqrt{\pi} \cosh\left(-\frac{(b^{2}-4 a c) \log(f)^{2}+\cosh(1)^{2}-2 (2 c d-b \cosh(1)-b \sinh(1)) \log(f)+2 \cosh(1) \sinh(1)+\sinh(1)^{2}}{4 c \log(f)}\right)+\sqrt{\pi} \sinh\left(-\frac{(b^{2}-4 a c) \log(f)^{2}+\cosh(1)^{2}-2 (2 c d-b \cosh(1)-b \sinh(1)) \log(f)+2 \cosh(1) \sinh(1)+\sinh(1)^{2}}{4 c \log(f)}\right)\right) \operatorname{erf}\left(\frac{((2 c x+b) \log(f)+\cosh(1)+\sinh(1)) \sqrt{-c \log(f)}}{2 c \log(f)}\right)+\sqrt{-c \log(f)}\left(\sqrt{\pi} \cosh\left(-\frac{(b^{2}-4 a c) \log(f)^{2}+\cosh(1)^{2}+2 (2 c d-b \cosh(1)-b \sinh(1)) \log(f)+2 \cosh(1) \sinh(1)+\sinh(1)^{2}}{4 c \log(f)}\right)+\sqrt{\pi} \sinh\left(-\frac{(b^{2}-4 a c) \log(f)^{2}+\cosh(1)^{2}+2 (2 c d-b \cosh(1)-b \sinh(1)) \log(f)+2 \cosh(1) \sinh(1)+\sinh(1)^{2}}{4 c \log(f)}\right)\right) \operatorname{erf}\left(\frac{((2 c x+b) \log(f)-\cosh(1)-\sinh(1)) \sqrt{-c \log(f)}}{2 c \log(f)}\right)}{4 c \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + cosh(1)
^2 - 2*(2*c*d - b*cosh(1) - b*sinh(1))*log(f) + 2*cosh(1)*sinh(1) + sinh(1)
^2)/(c*log(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + cosh(1)^2 -
2*(2*c*d - b*cosh(1) - b*sinh(1))*log(f) + 2*cosh(1)*sinh(1) + sinh(1)^2)/(
c*log(f))))*erf(1/2*((2*c*x + b)*log(f) + cosh(1) + sinh(1))*sqrt(-c*log(f)
)/(c*log(f))) + sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2
 + cosh(1)^2 + 2*(2*c*d - b*cosh(1) - b*sinh(1))*log(f) + 2*cosh(1)*sinh(1)
```

```
 + sinh(1)^2)/(c*log(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + co
sh(1)^2 + 2*(2*c*d - b*cosh(1) - b*sinh(1))*log(f) + 2*cosh(1)*sinh(1) + si
nh(1)^2)/(c*log(f))))*erf(1/2*((2*c*x + b)*log(f) - cosh(1) - sinh(1))*sqrt
(-c*log(f))/(c*log(f))))/(c*log(f))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx+cx^2}\cosh\left(d+ex\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cosh(e*x+d),x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*cosh(d + e*x), x)
```

Giac [A]
time = 0.40, size = 167, normalized size = 1.09

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)}\left(2\,x+\frac{b\log(f)-e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2+4\,cd\log(f)-2\,be\log(f)+e^2}{4\,c\log(f)}\right)}}{4\,\sqrt{-c\log\left(f\right)}}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)}\left(2\,x+\frac{b\log(f)+e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2-4\,cd\log(f)+2\,be\log(f)+e^2}{4\,c\log(f)}\right)}}{4\,\sqrt{-c\log\left(f\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(e*x+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f) - e)/(c*log(f))))*e
^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 4*c*d*log(f) - 2*b*e*log(f) + e^2)/
(c*log(f)))/sqrt(-c*log(f)) - 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x +
(b*log(f) + e)/(c*log(f))))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*
log(f) + 2*b*e*log(f) + e^2)/(c*log(f)))/sqrt(-c*log(f))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+b\,x+a}\,\cosh\left(d+e\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*cosh(d + e*x),x)
```

```
[Out] int(f^(a + b*x + c*x^2)*cosh(d + e*x), x)
```

3.321 $\int f^{a+bx+cx^2}\cosh^2(d+ex)\,dx$

Optimal. Leaf size=219

$$\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}-\frac{e^{-2d-\frac{(2e-b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{2e-b\log(f)-2cx\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{8\sqrt{c}\,\sqrt{\log(f)}}+\frac{e^{2d-\frac{(2e+b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{E}}{8\sqrt{c}}$$

```
[Out] 1/8*exp(-2*d-1/4*(2*e-b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-2*e+b*ln(f)+2*c*x*
ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/8*exp(2*d-1/4*(2
*e+b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(2*e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/ln(f)
^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/4*f^(a-1/4*b^2/c)*erfi(1/2*(2*c*x+b)
*ln(f)^(1/2)/c^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5624, 2266, 2235, 2325}

$$\frac{\sqrt{\pi}\,f^{a-\frac{b^2}{4c}}\mathrm{Erfi}\left(\frac{\sqrt{\log(f)}\,(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}-\frac{\sqrt{\pi}\,f^a e^{-\frac{(2e-b\log(f))^2}{4c\log(f)}-2d}\mathrm{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+2e}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{8\sqrt{c}\,\sqrt{\log(f)}}+\frac{\sqrt{\pi}\,f^a e^{2d-\frac{(b\log(f)+2e)^2}{4c\log(f)}}\mathrm{Erfi}\left(\frac{b\log(f)+2cx\log(f)+2e}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{8\sqrt{c}\,\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Cosh[d + e*x]^2,x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(
4*Sqrt[c]*Sqrt[Log[f]]) - (E^(-2*d - (2*e - b*Log[f])^2/(4*c*Log[f]))*f^a*S
qrt[Pi]*Erfi[(2*e - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(8*
Sqrt[c]*Sqrt[Log[f]]) + (E^(2*d - (2*e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt
[Pi]*Erfi[(2*e + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(8*Sqr
t[c]*Sqrt[Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
```

```
x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5624

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx+cx^2}\cosh^2(d+ex)\,dx &= \int \left(\frac{1}{2}f^{a+bx+cx^2}+\frac{1}{4}e^{-2d-2ex}f^{a+bx+cx^2}+\frac{1}{4}e^{2d+2ex}f^{a+bx+cx^2}\right)dx\\ &= \frac{1}{4}\int e^{-2d-2ex}f^{a+bx+cx^2}\,dx+\frac{1}{4}\int e^{2d+2ex}f^{a+bx+cx^2}\,dx+\frac{1}{2}\int f^{a+bx+cx^2}\,dx\\ &= \frac{1}{4}\int \exp\left(-2d+a\log(f)+cx^2\log(f)-x(2e-b\log(f))\right)dx+\frac{1}{4}\int \exp\left(2d\right.\\ &= \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{1}{4}\left(e^{-2d-\frac{(2e-b\log(f))^2}{4c\log(f)}}f^a\right)\int \exp\left(\frac{(-2e+\right.\\ &= \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}-\frac{e^{-2d-\frac{(2e-b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{2e-b\log(f)-2cx}{2\sqrt{c}\,\sqrt{\log}}\right.}{8\sqrt{c}\,\sqrt{\log(f)}}\end{aligned}$$

Mathematica [A]
time = 0.33, size = 183, normalized size = 0.84

$$\frac{e^{-\frac{e(e+b\log(f))}{c\log(f)}}f^{a-\frac{b^2}{4c}}\sqrt{\pi}\left(2e^{\frac{e(e+b\log(f))}{c\log(f)}}\mathrm{Erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)+e^{\frac{2be}{c}}\mathrm{Erfi}\left(\frac{-2e+(b+2cx)\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)(\cosh(2d)-\sinh(2d))+\mathrm{Erfi}\left(\frac{2e+(b+2cx)\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)(\cosh(2d)+\sinh(2d))\right)}{8\sqrt{c}\,\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Cosh[d + e*x]^2,x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*(2*E^((e*(e + b*Log[f]))/(c*Log[f]))*Erfi[((b +
 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])] + E^((2*b*e)/c)*Erfi[(-2*e + (b + 2*c*x)
*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] - Sinh[2*d]) + Erfi[(2*e + (b
 + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] + Sinh[2*d])))/(8*Sq
rt[c]*E^((e*(e + b*Log[f]))/(c*Log[f]))*Sqrt[Log[f]])
```

Maple [A]
time = 1.34, size = 211, normalized size = 0.96

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2} \ln(f)^{2}-4 \ln(f) b e+8 d \ln(f) c+4 e^{2}}{4 \ln(f) c}} \operatorname{erf}\left(-\sqrt{-c \ln(f)}\, x+\frac{b \ln(f)-2 e}{2 \sqrt{-c \ln(f)}}\right)}{8 \sqrt{-c \ln(f)}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2} \ln(f)^{2}+4 \ln(f) b e-8 d}{4 \ln(f) c}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cosh(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-4*ln(f)*b*e+8*d*ln(f)*c+4*e^2)/ln(f
)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-2*e)/(-c*ln(f))^
(1/2))-1/8*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+4*ln(f)*b*e-8*d*ln(f)*c+4*e^2
)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(2*e+b*ln(f))/(-c*l
n(f))^(1/2))-1/4*Pi^(1/2)*f^a*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f
))^(1/2)*x+1/2*b*ln(f)/(-c*ln(f))^(1/2))
```

Maxima [A]
time = 0.28, size = 189, normalized size = 0.86

$$\frac{\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x-\frac{b \log(f)+2 e}{2 \sqrt{-c \log(f)}}\right) e^{\left(2 d-\frac{(b \log(f)+2 e)^{2}}{4 c \log(f)}\right)}}{8 \sqrt{-c \log(f)}}+\frac{\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x-\frac{b \log(f)-2 e}{2 \sqrt{-c \log(f)}}\right) e^{\left(-2 d-\frac{(b \log(f)-2 e)^{2}}{4 c \log(f)}\right)}}{8 \sqrt{-c \log(f)}}+\frac{\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x-\frac{b \log(f)}{2 \sqrt{-c \log(f)}}\right)}{4 \sqrt{-c \log(f)}\, f^{\frac{b^{2}}{4 c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) + 2*e)/sqrt(-c*log(f
)))*e^(2*d - 1/4*(b*log(f) + 2*e)^2/(c*log(f)))/sqrt(-c*log(f)) + 1/8*sqrt(
pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) - 2*e)/sqrt(-c*log(f)))*e^(-2
*d - 1/4*(b*log(f) - 2*e)^2/(c*log(f)))/sqrt(-c*log(f)) + 1/4*sqrt(pi)*f^a*
erf(sqrt(-c*log(f))*x - 1/2*b*log(f)/sqrt(-c*log(f)))/(sqrt(-c*log(f))*f^(1
/4*b^2/c))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 427 vs. 2(171) = 342.
time = 0.52, size = 427, normalized size = 1.95

$$-\frac{2 \sqrt{-c \log(f)} \left(\sqrt{\pi} \cosh\left(-\frac{(b^{2}-4 a c) \log(f)}{4 c}\right)+\sqrt{\pi} \sinh\left(-\frac{(b^{2}-4 a c) \log(f)}{4 c}\right)\right) \operatorname{erf}\left(\frac{(2 c x+b) \sqrt{-c \log(f)}}{2 c}\right)+\sqrt{-c \log(f)} \left(\sqrt{\pi} \cosh\left(-\frac{(b^{2}-4 a c) \log(f)^{2}+4 \cosh(1)^{2}-4 (2 c d-b \cosh(1)-b \sinh(1)) \log(f)+8 \cosh(1) \sinh(1)+4 \sinh(1)^{2}}{4 c \log(f)}\right)+\sqrt{\pi} \sinh\left(-\frac{(b^{2}-4 a c) \log(f)^{2}+4 \cosh(1)^{2}-4 (2 c d-b \cosh(1)-b \sinh(1)) \log(f)+8 \cosh(1) \sinh(1)+4 \sinh(1)^{2}}{4 c \log(f)}\right)\right) \operatorname{erf}\left(\frac{((2 c x+b) \log(f)+2 \cosh(1)+2 \sinh(1)) \sqrt{-c \log(f)}}{2 c \log(f)}\right)+\sqrt{-c \log(f)} \left(\sqrt{\pi} \cosh\left(-\frac{(b^{2}-4 a c) \log(f)^{2}+4 \cosh(1)^{2}+4 (2 c d-b \cosh(1)-b \sinh(1)) \log(f)+8 \cosh(1) \sinh(1)+4 \sinh(1)^{2}}{4 c \log(f)}\right)+\sqrt{\pi} \sinh\left(-\frac{(b^{2}-4 a c) \log(f)^{2}+4 \cosh(1)^{2}+4 (2 c d-b \cosh(1)-b \sinh(1)) \log(f)+8 \cosh(1) \sinh(1)+4 \sinh(1)^{2}}{4 c \log(f)}\right)\right) \operatorname{erf}\left(\frac{((2 c x+b) \log(f)-2 \cosh(1)-2 \sinh(1)) \sqrt{-c \log(f)}}{2 c \log(f)}\right)}{8 c \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*(b^2 - 4*a*c)*log(f)/c) + sqrt(
pi)*sinh(-1/4*(b^2 - 4*a*c)*log(f)/c))*erf(1/2*(2*c*x + b)*sqrt(-c*log(f))/
c) + sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 4*cosh(1
)^2 - 4*(2*c*d - b*cosh(1) - b*sinh(1))*log(f) + 8*cosh(1)*sinh(1) + 4*sinh
```

```
(1)^2)/(c*log(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 4*cosh(1)
^2 - 4*(2*c*d - b*cosh(1) - b*sinh(1))*log(f) + 8*cosh(1)*sinh(1) + 4*sinh(
1)^2)/(c*log(f))))*erf(1/2*((2*c*x + b)*log(f) + 2*cosh(1) + 2*sinh(1))*sqr
t(-c*log(f))/(c*log(f))) + sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*((b^2 - 4*a*
c)*log(f)^2 + 4*cosh(1)^2 + 4*(2*c*d - b*cosh(1) - b*sinh(1))*log(f) + 8*co
sh(1)*sinh(1) + 4*sinh(1)^2)/(c*log(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c
)*log(f)^2 + 4*cosh(1)^2 + 4*(2*c*d - b*cosh(1) - b*sinh(1))*log(f) + 8*cos
h(1)*sinh(1) + 4*sinh(1)^2)/(c*log(f))))*erf(1/2*((2*c*x + b)*log(f) - 2*co
sh(1) - 2*sinh(1))*sqrt(-c*log(f))/(c*log(f))))/(c*log(f))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx+cx^2}\cosh^2\left(d+ex\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cosh(e*x+d)**2,x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*cosh(d + e*x)**2, x)
```

Giac [A]
time = 0.42, size = 223, normalized size = 1.02

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)}\,\left(2\,x+\frac{b}{c}\right)\right)e^{\left(-\frac{b^2\log(f)-4\,ac\log(f)}{4\,c}\right)}}{4\,\sqrt{-c\log\left(f\right)}}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)}\,\left(2\,x+\frac{b\log(f)-2\,e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2+8\,cd\log(f)-4\,be\log(f)+4\,e^2}{4\,c\log(f)}\right)}}{8\,\sqrt{-c\log\left(f\right)}}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)}\,\left(2\,x+\frac{b\log(f)+2\,e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2-8\,cd\log(f)+4\,be\log(f)+4\,e^2}{4\,c\log(f)}\right)}}{8\,\sqrt{-c\log\left(f\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + b/c))*e^(-1/4*(b^2*log(f) - 4
*a*c*log(f))/c)/sqrt(-c*log(f)) - 1/8*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*
x + (b*log(f) - 2*e)/(c*log(f))))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 +
8*c*d*log(f) - 4*b*e*log(f) + 4*e^2)/(c*log(f)))/sqrt(-c*log(f)) - 1/8*sqrt
(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f) + 2*e)/(c*log(f))))*e^(-1/4*
(b^2*log(f)^2 - 4*a*c*log(f)^2 - 8*c*d*log(f) + 4*b*e*log(f) + 4*e^2)/(c*lo
g(f)))/sqrt(-c*log(f))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+b\,x+a}\,{\mathrm{cosh}\left(d+e\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*cosh(d + e*x)^2,x)
```

```
[Out] int(f^(a + b*x + c*x^2)*cosh(d + e*x)^2, x)
```

3.322 $\int f^{a+bx+cx^2}\cosh^3(d+ex)\,dx$

Optimal. Leaf size=315

$$-\frac{3e^{-d-\frac{(e-b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\text{Erfi}\left(\frac{e-b\log(f)-2cx\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{16\sqrt{c}\,\sqrt{\log(f)}}-\frac{e^{-3d-\frac{(3e-b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\text{Erfi}\left(\frac{3e-b\log(f)-2cx\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{16\sqrt{c}\,\sqrt{\log(f)}}+\frac{3e^{d-\frac{(e+}{4}}}{}$$

```
[Out] 3/16*exp(-d-1/4*(e-b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-e+b*ln(f)+2*c*x*ln(f)
)/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/16*exp(-3*d-1/4*(3*e-
b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-3*e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/ln(f)^(
1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+3/16*exp(d-1/4*(e+b*ln(f))^2/c/ln(f))*f^
a*erfi(1/2*(e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln
(f)^(1/2)+1/16*exp(3*d-1/4*(3*e+b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(3*e+b*ln(
f)+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A]

time = 0.37, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5624, 2325, 2266, 2235}

$$-\frac{3\sqrt{\pi}\,f^a e^{-\frac{(e-b\log(f))^2}{4c\log(f)}-d}\text{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+e}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{16\sqrt{c}\,\sqrt{\log(f)}}-\frac{\sqrt{\pi}\,f^a e^{-\frac{(3e-b\log(f))^2}{4c\log(f)}-3d}\text{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+3e}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{16\sqrt{c}\,\sqrt{\log(f)}}+\frac{3\sqrt{\pi}\,f^a e^{d-\frac{(b\log(f)+e)^2}{4c\log(f)}}\text{Erfi}\left(\frac{b\log(f)+2cx\log(f)+e}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{16\sqrt{c}\,\sqrt{\log(f)}}+\frac{\sqrt{\pi}\,f^a e^{3d-\frac{(b\log(f)+3e)^2}{4c\log(f)}}\text{Erfi}\left(\frac{b\log(f)+2cx\log(f)+3e}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{16\sqrt{c}\,\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Cosh[d + e*x]^3,x]
```

```
[Out] (-3*E^(-d - (e - b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - b*Log[f]
- 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) - (E^(
-3*d - (3*e - b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(3*e - b*Log[f] -
 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (3*E^
(d - (e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e + b*Log[f] + 2*c*x
*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (E^(3*d - (
3*e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(3*e + b*Log[f] + 2*c*x*L
og[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5624

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int f^{a+bx+cx^2}\cosh^3(d+ex)\,dx &= \int\left(\frac{1}{8}e^{-3d-3ex}f^{a+bx+cx^2}+\frac{3}{8}e^{-d-ex}f^{a+bx+cx^2}+\frac{3}{8}e^{d+ex}f^{a+bx+cx^2}+\frac{1}{8}e^{3d+3ex}f^{a\cdots}\right.\\
&=\frac{1}{8}\int e^{-3d-3ex}f^{a+bx+cx^2}\,dx+\frac{1}{8}\int e^{3d+3ex}f^{a+bx+cx^2}\,dx+\frac{3}{8}\int e^{-d-ex}f^{a+bx+cx^2}\cdots\\
&=\frac{1}{8}\int\exp\left(-3d+a\log(f)+cx^2\log(f)-x(3e-b\log(f))\right)\,dx+\frac{1}{8}\int\exp\left(3\cdots\right.\\
&=\frac{1}{8}\left(3e^{-d-\frac{(e-b\log(f))^2}{4c\log(f)}}f^a\right)\int\exp\left(\frac{(-e+b\log(f)+2cx\log(f))^2}{4c\log(f)}\right)\,dx+\frac{1}{8}\left(e^{-3\cdots}\right.\\
&=-\frac{3e^{-d-\frac{(e-b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e-b\log(f)-2cx\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{16\sqrt{c}\,\sqrt{\log(f)}}-\frac{e^{-3d-\frac{(3e-b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\cdots}{16\sqrt{c}\,\sqrt{l\cdots}}
\end{aligned}$$

Mathematica [A]
time = 0.61, size = 262, normalized size = 0.83

$$\frac{e^{-\frac{3e(3e+2b\log(f))}{4c\log(f)}}f^{a-\frac{b^2}{4c}}\sqrt{\pi}\left((\cosh(d)+\sinh(d))\left(3e^{\frac{e(2e+b\log(f))}{c\log(f)}}\mathrm{Erfi}\left(\frac{e+(b+2cx)\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)+3e^{\frac{2e(e+b\log(f))}{c\log(f)}}\mathrm{Erfi}\left(\frac{-e+(b+2cx)\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)(\cosh(2d)-\sinh(2d))+\mathrm{Erfi}\left(\frac{3e+(b+2cx)\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)(\cosh(2d)+\sinh(2d))\right)+e^{\frac{3be}{c}}\mathrm{Erfi}\left(\frac{-3e+(b+2cx)\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)(\cosh(3d)-\sinh(3d))\right)}{16\sqrt{c}\,\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Cosh[d + e*x]^3,x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*((Cosh[d] + Sinh[d])*(3*E^((e*(2*e + b*Log[f]))
/(c*Log[f]))*Erfi[(e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])] + 3*E^
((2*e*(e + b*Log[f]))/(c*Log[f]))*Erfi[(-e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]
*Sqrt[Log[f]])]*(Cosh[2*d] - Sinh[2*d]) + Erfi[(3*e + (b + 2*c*x)*Log[f])/(
2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] + Sinh[2*d])) + E^((3*b*e)/c)*Erfi[(-3*
e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[3*d] - Sinh[3*d])))
/(16*Sqrt[c]*E^((3*e*(3*e + 2*b*Log[f]))/(4*c*Log[f]))*Sqrt[Log[f]])
```

Maple [A]
time = 2.36, size = 316, normalized size = 1.00

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2} \ln(f)^{2}-6 \ln(f) b e+12 d \ln(f) c+9 e^{2}}{4 \ln(f) c}} \operatorname{erf}\left(-\sqrt{-c \ln(f)}\, x+\frac{b \ln(f)-3 e}{2 \sqrt{-c \ln(f)}}\right)}{16 \sqrt{-c \ln(f)}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2} \ln(f)^{2}+6 \ln(f) b e-1}{4 \ln(f) c}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cosh(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-6*ln(f)*b*e+12*d*ln(f)*c+9*e^2)/ln
(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-3*e)/(-c*ln(f)
)^(1/2))-1/16*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+6*ln(f)*b*e-12*d*ln(f)*c+9
*e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(3*e+b*ln(f))/(
-c*ln(f))^(1/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-2*ln(f)*b*e+4*d*ln
(f)*c+e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-e
)/(-c*ln(f))^(1/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+2*ln(f)*b*e-4*d
*ln(f)*c+e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(e+b*ln
(f))/(-c*ln(f))^(1/2))
```

Maxima [A]
time = 0.29, size = 271, normalized size = 0.86

$$\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x-\frac{b \log(f)+3 e}{2 \sqrt{-c \log(f)}}\right) e^{\left(3 d-\frac{(b \log(f)+3 e)^{2}}{4 c \log(f)}\right)}}{16 \sqrt{-c \log(f)}}+\frac{3 \sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x-\frac{b \log(f)+e}{2 \sqrt{-c \log(f)}}\right) e^{\left(d-\frac{(b \log(f)+e)^{2}}{4 c \log(f)}\right)}}{16 \sqrt{-c \log(f)}}+\frac{3 \sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x-\frac{b \log(f)-e}{2 \sqrt{-c \log(f)}}\right) e^{\left(-d-\frac{(b \log(f)-e)^{2}}{4 c \log(f)}\right)}}{16 \sqrt{-c \log(f)}}+\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x-\frac{b \log(f)-3 e}{2 \sqrt{-c \log(f)}}\right) e^{\left(-3 d-\frac{(b \log(f)-3 e)^{2}}{4 c \log(f)}\right)}}{16 \sqrt{-c \log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) + 3*e)/sqrt(-c*log(
f)))*e^(3*d - 1/4*(b*log(f) + 3*e)^2/(c*log(f)))/sqrt(-c*log(f)) + 3/16*sqr
t(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) + e)/sqrt(-c*log(f)))*e^(d
- 1/4*(b*log(f) + e)^2/(c*log(f)))/sqrt(-c*log(f)) + 3/16*sqrt(pi)*f^a*erf(
sqrt(-c*log(f))*x - 1/2*(b*log(f) - e)/sqrt(-c*log(f)))*e^(-d - 1/4*(b*log(
f) - e)^2/(c*log(f)))/sqrt(-c*log(f)) + 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f
))*x - 1/2*(b*log(f) - 3*e)/sqrt(-c*log(f)))*e^(-3*d - 1/4*(b*log(f) - 3*e)
^2/(c*log(f)))/sqrt(-c*log(f))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 688 vs. $2(255) = 510$.
time = 0.47, size = 688, normalized size = 2.18

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 9*cosh
(1)^2 - 6*(2*c*d - b*cosh(1) - b*sinh(1))*log(f) + 18*cosh(1)*sinh(1) + 9*s
inh(1)^2)/(c*log(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 9*cosh
(1)^2 - 6*(2*c*d - b*cosh(1) - b*sinh(1))*log(f) + 18*cosh(1)*sinh(1) + 9*s
inh(1)^2)/(c*log(f))))*erf(1/2*((2*c*x + b)*log(f) + 3*cosh(1) + 3*sinh(1))
*sqrt(-c*log(f))/(c*log(f))) + 3*sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*((b^2
- 4*a*c)*log(f)^2 + cosh(1)^2 - 2*(2*c*d - b*cosh(1) - b*sinh(1))*log(f) +
2*cosh(1)*sinh(1) + sinh(1)^2)/(c*log(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a
*c)*log(f)^2 + cosh(1)^2 - 2*(2*c*d - b*cosh(1) - b*sinh(1))*log(f) + 2*cos
h(1)*sinh(1) + sinh(1)^2)/(c*log(f))))*erf(1/2*((2*c*x + b)*log(f) + cosh(1
) + sinh(1))*sqrt(-c*log(f))/(c*log(f))) + 3*sqrt(-c*log(f))*(sqrt(pi)*cosh
(-1/4*((b^2 - 4*a*c)*log(f)^2 + cosh(1)^2 + 2*(2*c*d - b*cosh(1) - b*sinh(1
))*log(f) + 2*cosh(1)*sinh(1) + sinh(1)^2)/(c*log(f))) + sqrt(pi)*sinh(-1/4
*((b^2 - 4*a*c)*log(f)^2 + cosh(1)^2 + 2*(2*c*d - b*cosh(1) - b*sinh(1))*lo
g(f) + 2*cosh(1)*sinh(1) + sinh(1)^2)/(c*log(f))))*erf(1/2*((2*c*x + b)*log
(f) - cosh(1) - sinh(1))*sqrt(-c*log(f))/(c*log(f))) + sqrt(-c*log(f))*(sqr
t(pi)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 9*cosh(1)^2 + 6*(2*c*d - b*cosh(1
) - b*sinh(1))*log(f) + 18*cosh(1)*sinh(1) + 9*sinh(1)^2)/(c*log(f))) + sqr
t(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 9*cosh(1)^2 + 6*(2*c*d - b*cosh(1
) - b*sinh(1))*log(f) + 18*cosh(1)*sinh(1) + 9*sinh(1)^2)/(c*log(f))))*erf(
1/2*((2*c*x + b)*log(f) - 3*cosh(1) - 3*sinh(1))*sqrt(-c*log(f))/(c*log(f))
))/(c*log(f))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx+cx^2}\cosh^3\left(d+ex\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cosh(e*x+d)**3,x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*cosh(d + e*x)**3, x)
```

Giac [**A**]
time = 0.42, size = 339, normalized size = 1.08

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)}\left(2\,x+\frac{b\log(f)-3\,e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2+12\,cd\log(f)-6\,be\log(f)+9\,e^2}{4\,c\log(f)}\right)}}{16\,\sqrt{-c\log\left(f\right)}}-\frac{3\,\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)}\left(2\,x+\frac{b\log(f)-e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2+4\,cd\log(f)-2\,be\log(f)+e^2}{4\,c\log(f)}\right)}}{16\,\sqrt{-c\log\left(f\right)}}-\frac{3\,\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)}\left(2\,x+\frac{b\log(f)+e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2-4\,cd\log(f)+2\,be\log(f)+e^2}{4\,c\log(f)}\right)}}{16\,\sqrt{-c\log\left(f\right)}}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)}\left(2\,x+\frac{b\log(f)+3\,e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2-12\,cd\log(f)+6\,be\log(f)+9\,e^2}{4\,c\log(f)}\right)}}{16\,\sqrt{-c\log\left(f\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(e*x+d)^3,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f) - 3*e)/(c*log(f)))
)*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 12*c*d*log(f) - 6*b*e*log(f) + 9
```

```
*e^2)/(c*log(f)))/sqrt(-c*log(f)) - 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*
(2*x + (b*log(f) - e)/(c*log(f))))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 +
 4*c*d*log(f) - 2*b*e*log(f) + e^2)/(c*log(f)))/sqrt(-c*log(f)) - 3/16*sqrt
(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f) + e)/(c*log(f))))*e^(-1/4*(b
^2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*log(f) + 2*b*e*log(f) + e^2)/(c*log(f)
))/sqrt(-c*log(f)) - 1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f
) + 3*e)/(c*log(f))))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 12*c*d*log(f
) + 6*b*e*log(f) + 9*e^2)/(c*log(f)))/sqrt(-c*log(f))
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+b\,x+a}\,{\cosh\left(d+e\,x\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*cosh(d + e*x)^3,x)
[Out] int(f^(a + b*x + c*x^2)*cosh(d + e*x)^3, x)
```

3.323 $\int f^{a+bx+cx^2}\cosh\left(d+fx^2\right)\,dx$

Optimal. Leaf size=154

$$-\frac{e^{-d+\frac{b^2\log^2(f)}{4f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{Erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{e^{d-\frac{b^2\log^2(f)}{4(f+c\log(f))}}f^a\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{b\log(f)+2x(f+c\log(f))}{2\sqrt{f+c\log(f)}}\right)}{4\sqrt{f+c\log(f)}}$$

```
[Out] -1/4*exp(-d+b^2*ln(f)^2/(4*f-4*c*ln(f)))*f^a*erf(1/2*(b*ln(f)-2*x*(f-c*ln(f
)))/(f-c*ln(f))^(1/2))*Pi^(1/2)/(f-c*ln(f))^(1/2)+1/4*exp(d-1/4*b^2*ln(f)^2
/(f+c*ln(f)))*f^a*erfi(1/2*(b*ln(f)+2*x*(f+c*ln(f)))/(f+c*ln(f))^(1/2))*Pi^
(1/2)/(f+c*ln(f))^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.238$, Rules used = {5624, 2325, 2266, 2236, 2235}

$$\frac{\sqrt{\pi}\,f^a e^{d-\frac{b^2\log^2(f)}{4(c\log(f)+f)}}\mathrm{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+f)}{2\sqrt{c\log(f)+f}}\right)}{4\sqrt{c\log(f)+f}}-\frac{\sqrt{\pi}\,f^a e^{\frac{b^2\log^2(f)}{4f-4c\log(f)}-d}\mathrm{Erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Cosh[d + f*x^2],x]
```

```
[Out] -1/4*(E^(-d + (b^2*Log[f]^2)/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f]
 - 2*x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/Sqrt[f - c*Log[f]] + (E^(d
- (b^2*Log[f]^2)/(4*(f + c*Log[f]))))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*(f +
 c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(4*Sqrt[f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5624

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int f^{a+bx+cx^2}\cosh\left(d+fx^2\right)\,dx &= \int\left(\frac{1}{2}e^{-d-fx^2}f^{a+bx+cx^2}+\frac{1}{2}e^{d+fx^2}f^{a+bx+cx^2}\right)dx\\
&= \frac{1}{2}\int e^{-d-fx^2}f^{a+bx+cx^2}\,dx+\frac{1}{2}\int e^{d+fx^2}f^{a+bx+cx^2}\,dx\\
&= \frac{1}{2}\int\exp\left(-d+a\log(f)+bx\log(f)-x^2(f-c\log(f))\right)dx+\frac{1}{2}\int\exp\left(d\right.\\
&= \frac{1}{2}\left(e^{-d+\frac{b^2\log^2(f)}{4f-4c\log(f)}}f^a\right)\int\exp\left(\frac{(b\log(f)+2x(-f+c\log(f)))^2}{4(-f+c\log(f))}\right)dx+\frac{1}{2}\left(\right.\\
&= -\frac{e^{-d+\frac{b^2\log^2(f)}{4f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{e^{d-\frac{b^2\log^2(f)}{4(f+c\log(f))}}f^a\sqrt{\pi}\,\mathrm{erfi}}{4\sqrt{f+c}}
\end{aligned}$$

Mathematica [A]
time = 0.47, size = 185, normalized size = 1.20

$$\frac{e^{-\frac{b^2\log^2(f)}{4(f+c\log(f))}}f^a\sqrt{\pi}\left(e^{\frac{b^2f\log^2(f)}{2f^2-2c^2\log^2(f)}}\mathrm{Erf}\left(\frac{2fx-(b+2cx)\log(f)}{2\sqrt{f-c\log(f)}}\right)\sqrt{f-c\log(f)}\,(f+c\log(f))(\cosh(d)-\sinh(d))+\mathrm{Erfi}\left(\frac{2fx+(b+2cx)\log(f)}{2\sqrt{f+c\log(f)}}\right)(f-c\log(f))\sqrt{f+c\log(f)}\,(\cosh(d)+\sinh(d))\right)}{4\left(f^2-c^2\log^2(f)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Cosh[d + f*x^2],x]
```

```
[Out] (f^a*Sqrt[Pi]*(E^((b^2*f*Log[f]^2)/(2*f^2 - 2*c^2*Log[f]^2))*Erf[(2*f*x - (
b + 2*c*x)*Log[f])/(2*Sqrt[f - c*Log[f]])]*Sqrt[f - c*Log[f]]*(f + c*Log[f]
)*(Cosh[d] - Sinh[d]) + Erfi[(2*f*x + (b + 2*c*x)*Log[f])/(2*Sqrt[f + c*Log
[f]])]*(f - c*Log[f])*Sqrt[f + c*Log[f]]*(Cosh[d] + Sinh[d])))/(4*E^((b^2*L
og[f]^2)/(4*(f + c*Log[f])))*(f^2 - c^2*Log[f]^2))
```

Maple [A]
time = 0.88, size = 160, normalized size = 1.04

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2} \ln(f)^{2}+4 d \ln(f) c-4 d f}{4(-f+c \ln(f))}} \operatorname{erf}\left(-x \sqrt{f-c \ln(f)}+\frac{\ln(f) b}{2 \sqrt{f-c \ln(f)}}\right)}{4 \sqrt{f-c \ln(f)}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2} \ln(f)^{2}-4 d \ln(f) c-4 d f}{4(f+c \ln(f))}} \operatorname{erf}}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cosh(f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+4*d*ln(f)*c-4*d*f)/(-f+c*ln(f)))/(f
-c*ln(f))^(1/2)*erf(-x*(f-c*ln(f))^(1/2)+1/2*ln(f)*b/(f-c*ln(f))^(1/2))-1/4
*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-4*d*ln(f)*c-4*d*f)/(f+c*ln(f)))/(-c*ln(
f)-f)^(1/2)*erf(-(-c*ln(f)-f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(f)-f)^(1/2))
```

Maxima [A]
time = 0.27, size = 139, normalized size = 0.90

$$\frac{\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)-f}\, x-\frac{b \log(f)}{2 \sqrt{-c \log(f)-f}}\right) e^{\left(-\frac{b^{2} \log(f)^{2}}{4\,(c \log(f)+f)}+d\right)}}{4 \sqrt{-c \log(f)-f}}+\frac{\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)+f}\, x-\frac{b \log(f)}{2 \sqrt{-c \log(f)+f}}\right) e^{\left(-\frac{b^{2} \log(f)^{2}}{4\,(c \log(f)-f)}-d\right)}}{4 \sqrt{-c \log(f)+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*b*log(f)/sqrt(-c*log(f) -
f))*e^(-1/4*b^2*log(f)^2/(c*log(f) + f) + d)/sqrt(-c*log(f) - f) + 1/4*sqrt
(pi)*f^a*erf(sqrt(-c*log(f) + f)*x - 1/2*b*log(f)/sqrt(-c*log(f) + f))*e^(-
1/4*b^2*log(f)^2/(c*log(f) - f) - d)/sqrt(-c*log(f) + f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 324 vs. 2(131) = 262.
time = 0.56, size = 324, normalized size = 2.10

$$-\frac{\left(\sqrt{\pi}\,(c \log(f)+f) \cosh\left(-\frac{(b^{2}-4 a c) \log(f)^{2}-4 d f+4 (c d+a f) \log(f)}{4 (c \log(f)-f)}\right)+\sqrt{\pi}\,(c \log(f)+f) \sinh\left(-\frac{(b^{2}-4 a c) \log(f)^{2}-4 d f+4 (c d+a f) \log(f)}{4 (c \log(f)-f)}\right)\right) \sqrt{-c \log(f)+f}\, \operatorname{erf}\left(-\frac{(2 f x-(2 c x+b) \log(f)) \sqrt{-c \log(f)+f}}{2 (c \log(f)-f)}\right)+\left(\sqrt{\pi}\,(c \log(f)-f) \cosh\left(-\frac{(b^{2}-4 a c) \log(f)^{2}-4 d f-4 (c d+a f) \log(f)}{4 (c \log(f)+f)}\right)+\sqrt{\pi}\,(c \log(f)-f) \sinh\left(-\frac{(b^{2}-4 a c) \log(f)^{2}-4 d f-4 (c d+a f) \log(f)}{4 (c \log(f)+f)}\right)\right) \sqrt{-c \log(f)-f}\, \operatorname{erf}\left(\frac{(2 f x+(2 c x+b) \log(f)) \sqrt{-c \log(f)-f}}{2 (c \log(f)+f)}\right)}{4 \left(c^{2} \log(f)^{2}-f^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+d),x, algorithm="fricas")
```

```
[Out] -1/4*((sqrt(pi)*(c*log(f) + f)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d*f +
4*(c*d + a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c*log(f) + f)*sinh(-1/4*(
(b^2 - 4*a*c)*log(f)^2 - 4*d*f + 4*(c*d + a*f)*log(f))/(c*log(f) - f)))*sqr
t(-c*log(f) + f)*erf(-1/2*(2*f*x - (2*c*x + b)*log(f))*sqrt(-c*log(f) + f)/
(c*log(f) - f)) + (sqrt(pi)*(c*log(f) - f)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^
2 - 4*d*f - 4*(c*d + a*f)*log(f))/(c*log(f) + f)) + sqrt(pi)*(c*log(f) - f)
*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d*f - 4*(c*d + a*f)*log(f))/(c*log(f
```

```
) + f)))*sqrt(-c*log(f) - f)*erf(1/2*(2*f*x + (2*c*x + b)*log(f))*sqrt(-c*l
og(f) - f)/(c*log(f) + f)))/(c^2*log(f)^2 - f^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx+cx^2}\cosh\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cosh(f*x**2+d),x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*cosh(d + f*x**2), x)
```

Giac [**A**]
time = 0.42, size = 181, normalized size = 1.18

$$-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)-f}\,\left(2\,x+\frac{b\log(f)}{c\log(f)+f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2-4\,cd\log(f)-4\,af\log(f)-4\,df}{4\,(c\log(f)+f)}\right)}}{4\,\sqrt{-c\log\left(f\right)-f}}-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)+f}\,\left(2\,x+\frac{b\log(f)}{c\log(f)-f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2+4\,cd\log(f)+4\,af\log(f)-4\,df}{4\,(c\log(f)-f)}\right)}}{4\,\sqrt{-c\log\left(f\right)+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - f)*(2*x + b*log(f)/(c*log(f) + f)))
*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*log(f) - 4*a*f*log(f) - 4*d
*f)/(c*log(f) + f))/sqrt(-c*log(f) - f) - 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log
(f) + f)*(2*x + b*log(f)/(c*log(f) - f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log
(f)^2 + 4*c*d*log(f) + 4*a*f*log(f) - 4*d*f)/(c*log(f) - f))/sqrt(-c*log(f)
 + f)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+b\,x+a}\,\mathrm{cosh}\left(f\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*cosh(d + f*x^2),x)
```

```
[Out] int(f^(a + b*x + c*x^2)*cosh(d + f*x^2), x)
```

3.324 $\int f^{a+bx+cx^2} \cosh^2\left(d+fx^2\right)\,dx$

Optimal. Leaf size=225

$$\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\text{Erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}-\frac{e^{-2d+\frac{b^2\log^2(f)}{8f-4c\log(f)}}f^a\sqrt{\pi}\,\text{Erf}\left(\frac{b\log(f)-2x(2f-c\log(f))}{2\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{e^{2d-\frac{b^2\log^2(f)}{8f+4c\log(f)}}f^a\sqrt{\pi}\,\text{Erf}}{8\sqrt{2f+}}$$

```
[Out] 1/4*f^(a-1/4*b^2/c)*erfi(1/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2))*Pi^(1/2)/c^(1/2
)/ln(f)^(1/2)-1/8*exp(-2*d+b^2*ln(f)^2/(8*f-4*c*ln(f)))*f^a*erf(1/2*(b*ln(f
)-2*x*(2*f-c*ln(f)))/(2*f-c*ln(f))^(1/2))*Pi^(1/2)/(2*f-c*ln(f))^(1/2)+1/8*
exp(2*d-b^2*ln(f)^2/(8*f+4*c*ln(f)))*f^a*erfi(1/2*(b*ln(f)+2*x*(2*f+c*ln(f)
))/(2*f+c*ln(f))^(1/2))*Pi^(1/2)/(2*f+c*ln(f))^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {5624, 2266, 2235, 2325, 2236}

$$-\frac{\sqrt{\pi}\,f^a e^{\frac{b^2\log^2(f)}{8f-4c\log(f)}-2d}\text{Erf}\left(\frac{b\log(f)-2x(2f-c\log(f))}{2\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi}\,f^a e^{2d-\frac{b^2\log^2(f)}{4c\log(f)+8f}}\text{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+2f)}{2\sqrt{c\log(f)+2f}}\right)}{8\sqrt{c\log(f)+2f}}+\frac{\sqrt{\pi}\,f^{a-\frac{b^2}{4c}}\text{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Cosh[d + f*x^2]^2,x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(
4*Sqrt[c]*Sqrt[Log[f]]) - (E^(-2*d + (b^2*Log[f]^2)/(8*f - 4*c*Log[f]))*f^a
*Sqrt[Pi]*Erf[(b*Log[f] - 2*x*(2*f - c*Log[f]))/(2*Sqrt[2*f - c*Log[f]])])/
(8*Sqrt[2*f - c*Log[f]]) + (E^(2*d - (b^2*Log[f]^2)/(8*f + 4*c*Log[f]))*f^a
*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*(2*f + c*Log[f]))/(2*Sqrt[2*f + c*Log[f]])])
/(8*Sqrt[2*f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5624

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int f^{a+bx+cx^2}\cosh^2\left(d+fx^2\right)\,dx = \int \left(\frac{1}{2}f^{a+bx+cx^2}+\frac{1}{4}e^{-2d-2fx^2}f^{a+bx+cx^2}+\frac{1}{4}e^{2d+2fx^2}f^{a+bx+cx^2}\right)dx$$

$$= \frac{1}{4}\int e^{-2d-2fx^2}f^{a+bx+cx^2}\,dx+\frac{1}{4}\int e^{2d+2fx^2}f^{a+bx+cx^2}\,dx+\frac{1}{2}\int f^{a+bx+cx^2}$$

$$= \frac{1}{4}\int \exp\left(-2d+a\log(f)+bx\log(f)-x^2(2f-c\log(f))\right)dx+\frac{1}{4}\int \exp$$

$$= \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{1}{4}\left(e^{-2d+\frac{b^2\log^2(f)}{8f-4c\log(f)}}f^a\right)\int \exp\left(\frac{(b\log\right.$$

$$= \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-2d+\frac{b^2\log^2(f)}{8f-4c\log(f)}}f^a\sqrt{\pi}\,\text{erf}\left(\frac{b\log(f)-2x(2}{2\sqrt{2f-}}\right.}{8\sqrt{2f-c\log(f)}}$$

Mathematica [A]
time = 1.52, size = 257, normalized size = 1.14

$$\frac{1}{8}f^a\sqrt{\pi}\left(\frac{2f^{-\frac{b^2}{4c}}\text{Erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-\frac{b^2\log^2(f)}{8f+4c\log(f)}}\left(e^{\frac{b^2f\log^2(f)}{4f^2-c^2\log^2(f)}}\text{Erf}\left(\frac{4fx-(b+2cx)\log(f)}{2\sqrt{2f-c\log(f)}}\right)\sqrt{2f-c\log(f)}\,(2f+c\log(f))(\cosh(2d)-\sinh(2d))+\text{Erfi}\left(\frac{4fx+(b+2cx)\log(f)}{2\sqrt{2f+c\log(f)}}\right)(2f-c\log(f))\sqrt{2f+c\log(f)}\,(\cosh(2d)+\sinh(2d))\right)}{-4f^2+c^2\log^2(f)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Cosh[d + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*((2*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(Sqrt[c]*f^
(b^2/(4*c))*Sqrt[Log[f]]) - (E^((b^2*f*Log[f]^2)/(4*f^2 - c^2*Log[f]^2))*Er
```

```
f[(4*f*x - (b + 2*c*x)*Log[f])/(2*Sqrt[2*f - c*Log[f]])]*Sqrt[2*f - c*Log[f
]]*(2*f + c*Log[f])*(Cosh[2*d] - Sinh[2*d]) + Erfi[(4*f*x + (b + 2*c*x)*Log
[f])/(2*Sqrt[2*f + c*Log[f]])]*(2*f - c*Log[f])*Sqrt[2*f + c*Log[f]]*(Cosh[
2*d] + Sinh[2*d]))/(E^((b^2*Log[f]^2)/(8*f + 4*c*Log[f]))*(-4*f^2 + c^2*Log
[f]^2))))/8
```

Maple [A]
time = 1.73, size = 217, normalized size = 0.96

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2} \ln(f)^{2}+8 d \ln(f) c-16 d f}{4(-2 f+c \ln(f))}} \operatorname{erf}\left(-x \sqrt{2 f-c \ln(f)}+\frac{\ln(f) b}{2 \sqrt{2 f-c \ln(f)}}\right)}{8 \sqrt{2 f-c \ln(f)}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2} \ln(f)^{2}-8 d \ln(f) c-16}{4(2 f+c \ln(f))}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cosh(f*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+8*d*ln(f)*c-16*d*f)/(-2*f+c*ln(f)))
/(2*f-c*ln(f))^(1/2)*erf(-x*(2*f-c*ln(f))^(1/2)+1/2*ln(f)*b/(2*f-c*ln(f))^(
1/2))-1/8*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-8*d*ln(f)*c-16*d*f)/(2*f+c*ln(
f)))/(-c*ln(f)-2*f)^(1/2)*erf(-(-c*ln(f)-2*f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(f)
-2*f)^(1/2))-1/4*Pi^(1/2)*f^a*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f
))^(1/2)*x+1/2*b*ln(f)/(-c*ln(f))^(1/2))
```

Maxima [A]
time = 0.27, size = 199, normalized size = 0.88

$$\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)-2 f}\, x-\frac{b \log(f)}{2 \sqrt{-c \log(f)-2 f}}\right) e^{\left(-\frac{b^{2} \log(f)^{2}}{4(c \log(f)+2 f)}+2 d\right)}}{8 \sqrt{-c \log(f)-2 f}}+\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)+2 f}\, x-\frac{b \log(f)}{2 \sqrt{-c \log(f)+2 f}}\right) e^{\left(-\frac{b^{2} \log(f)^{2}}{4(c \log(f)-2 f)}-2 d\right)}}{8 \sqrt{-c \log(f)+2 f}}+\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x-\frac{b \log(f)}{2 \sqrt{-c \log(f)}}\right)}{4 \sqrt{-c \log(f)}\, f^{\frac{b^{2}}{4 c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 2*f)*x - 1/2*b*log(f)/sqrt(-c*log(f)
- 2*f))*e^(-1/4*b^2*log(f)^2/(c*log(f) + 2*f) + 2*d)/sqrt(-c*log(f) - 2*f)
+ 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 2*f)*x - 1/2*b*log(f)/sqrt(-c*log(f
) + 2*f))*e^(-1/4*b^2*log(f)^2/(c*log(f) - 2*f) - 2*d)/sqrt(-c*log(f) + 2*f
) + 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*b*log(f)/sqrt(-c*log(f)))/
(sqrt(-c*log(f))*f^(1/4*b^2/c))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 466 vs. 2(185) = 370.
time = 0.43, size = 466, normalized size = 2.07

$$-\frac{\left(\sqrt{\pi}\left(c^{2} \log(f)^{2}+2 c f \log(f)\right) \cosh\left(-\frac{(b^{2}-4 a c) \log(f)^{2}-16 d f+8(c d+a f) \log(f)}{4(c \log(f)-2 f)}\right)+\sqrt{\pi}\left(c^{2} \log(f)^{2}+2 c f \log(f)\right) \sinh\left(-\frac{(b^{2}-4 a c) \log(f)^{2}-16 d f+8(c d+a f) \log(f)}{4(c \log(f)-2 f)}\right)\right) \sqrt{-c \log(f)+2 f} \operatorname{erf}\left(-\frac{(4 f x-(2 c x+b) \log(f)) \sqrt{-c \log(f)+2 f}}{2(c \log(f)-2 f)}\right)+\left(\sqrt{\pi}\left(c^{2} \log(f)^{2}-2 c f \log(f)\right) \cosh\left(-\frac{(b^{2}-4 a c) \log(f)^{2}-16 d f-8(c d+a f) \log(f)}{4(c \log(f)+2 f)}\right)+\sqrt{\pi}\left(c^{2} \log(f)^{2}-2 c f \log(f)\right) \sinh\left(-\frac{(b^{2}-4 a c) \log(f)^{2}-16 d f-8(c d+a f) \log(f)}{4(c \log(f)+2 f)}\right)\right) \sqrt{-c \log(f)-2 f} \operatorname{erf}\left(\frac{(4 f x+(2 c x+b) \log(f)) \sqrt{-c \log(f)-2 f}}{2(c \log(f)+2 f)}\right)+2\left(\sqrt{\pi}\left(c^{2} \log(f)^{2}-4 f^{2}\right) \cosh\left(-\frac{(b^{2}-4 a c) \log(f)}{4 c}\right)+\sqrt{\pi}\left(c^{2} \log(f)^{2}-4 f^{2}\right) \sinh\left(-\frac{(b^{2}-4 a c) \log(f)}{4 c}\right)\right) \sqrt{-c \log(f)} \operatorname{erf}\left(\frac{(2 c x+b) \sqrt{-c \log(f)}}{2 c}\right)}{8\left(c^{3} \log(f)^{3}-4 c f^{2} \log(f)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*((sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*cosh(-1/4*((b^2 - 4*a*c)*log(
f)^2 - 16*d*f + 8*(c*d + a*f)*log(f))/(c*log(f) - 2*f)) + sqrt(pi)*(c^2*log
(f)^2 + 2*c*f*log(f))*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 16*d*f + 8*(c*d +
 a*f)*log(f))/(c*log(f) - 2*f)))*sqrt(-c*log(f) + 2*f)*erf(-1/2*(4*f*x - (2
*c*x + b)*log(f))*sqrt(-c*log(f) + 2*f)/(c*log(f) - 2*f)) + (sqrt(pi)*(c^2*
log(f)^2 - 2*c*f*log(f))*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 16*d*f - 8*(c*
d + a*f)*log(f))/(c*log(f) + 2*f)) + sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))
*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 16*d*f - 8*(c*d + a*f)*log(f))/(c*log(
f) + 2*f)))*sqrt(-c*log(f) - 2*f)*erf(1/2*(4*f*x + (2*c*x + b)*log(f))*sqrt
(-c*log(f) - 2*f)/(c*log(f) + 2*f)) + 2*(sqrt(pi)*(c^2*log(f)^2 - 4*f^2)*co
sh(-1/4*(b^2 - 4*a*c)*log(f)/c) + sqrt(pi)*(c^2*log(f)^2 - 4*f^2)*sinh(-1/4
*(b^2 - 4*a*c)*log(f)/c))*sqrt(-c*log(f))*erf(1/2*(2*c*x + b)*sqrt(-c*log(f
))/c))/(c^3*log(f)^3 - 4*c*f^2*log(f))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx+cx^2} \cosh^2\left(d + fx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cosh(f*x**2+d)**2,x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*cosh(d + f*x**2)**2, x)
```

Giac [A]
time = 0.42, size = 239, normalized size = 1.06

$$-\frac{\sqrt{\pi} \operatorname{erf}\left(-\frac{1}{2} \sqrt{-c \log\left(f\right) - 2 f} \left(2 x + \frac{b \log(f)}{c \log(f) + 2 f}\right)\right) e^{\left(-\frac{b^2 \log(f)^2 - 4 ac \log(f)^2 - 8 cd \log(f) - 8 af \log(f) - 16 df}{4 (c \log(f) + 2 f)}\right)}}{8 \sqrt{-c \log\left(f\right) - 2 f}} - \frac{\sqrt{\pi} \operatorname{erf}\left(-\frac{1}{2} \sqrt{-c \log\left(f\right) + 2 f} \left(2 x + \frac{b \log(f)}{c \log(f) - 2 f}\right)\right) e^{\left(-\frac{b^2 \log(f)^2 - 4 ac \log(f)^2 + 8 cd \log(f) + 8 af \log(f) - 16 df}{4 (c \log(f) - 2 f)}\right)}}{8 \sqrt{-c \log\left(f\right) + 2 f}} - \frac{\sqrt{\pi} \operatorname{erf}\left(-\frac{1}{2} \sqrt{-c \log\left(f\right)} \left(2 x + \frac{b}{c}\right)\right) e^{\left(-\frac{b^2 \log(f) - 4 ac \log(f)}{4 c}\right)}}{4 \sqrt{-c \log\left(f\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+d)^2,x, algorithm="giac")
```

```
[Out] -1/8*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 2*f)*(2*x + b*log(f)/(c*log(f) + 2*
f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 8*c*d*log(f) - 8*a*f*log(f) -
 16*d*f)/(c*log(f) + 2*f))/sqrt(-c*log(f) - 2*f) - 1/8*sqrt(pi)*erf(-1/2*sq
rt(-c*log(f) + 2*f)*(2*x + b*log(f)/(c*log(f) - 2*f)))*e^(-1/4*(b^2*log(f)^
2 - 4*a*c*log(f)^2 + 8*c*d*log(f) + 8*a*f*log(f) - 16*d*f)/(c*log(f) - 2*f)
)/sqrt(-c*log(f) + 2*f) - 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + b/c)
)*e^(-1/4*(b^2*log(f) - 4*a*c*log(f))/c)/sqrt(-c*log(f))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+b\,x+a}\,{\cosh\left(f\,x^2+d\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*cosh(d + f*x^2)^2,x)
[Out] int(f^(a + b*x + c*x^2)*cosh(d + f*x^2)^2, x)
```

3.325 $\int f^{a+bx+cx^2} \cosh^3(d+fx^2)\,dx$

Optimal. Leaf size=323

$$-\frac{3e^{-d+\frac{b^2\log^2(f)}{4f-4c\log(f)}}f^a\sqrt{\pi}\,\text{Erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}-\frac{e^{-3d+\frac{b^2\log^2(f)}{12f-4c\log(f)}}f^a\sqrt{\pi}\,\text{Erf}\left(\frac{b\log(f)-2x(3f-c\log(f))}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}+\frac{3e^{d-\ldots}}{}$$

```
[Out] -3/16*exp(-d+b^2*ln(f)^2/(4*f-4*c*ln(f)))*f^a*erf(1/2*(b*ln(f)-2*x*(f-c*ln(
f)))/(f-c*ln(f))^(1/2))*Pi^(1/2)/(f-c*ln(f))^(1/2)-1/16*exp(-3*d+b^2*ln(f)^
2/(12*f-4*c*ln(f)))*f^a*erf(1/2*(b*ln(f)-2*x*(3*f-c*ln(f)))/(3*f-c*ln(f))^(
1/2))*Pi^(1/2)/(3*f-c*ln(f))^(1/2)+3/16*exp(d-1/4*b^2*ln(f)^2/(f+c*ln(f)))*
f^a*erfi(1/2*(b*ln(f)+2*x*(f+c*ln(f)))/(f+c*ln(f))^(1/2))*Pi^(1/2)/(f+c*ln(
f))^(1/2)+1/16*exp(3*d-1/4*b^2*ln(f)^2/(3*f+c*ln(f)))*f^a*erfi(1/2*(b*ln(f)
+2*x*(3*f+c*ln(f)))/(3*f+c*ln(f))^(1/2))*Pi^(1/2)/(3*f+c*ln(f))^(1/2)
```

Rubi [A]
time = 0.43, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {5624, 2325, 2266, 2236, 2235}

$$-\frac{3\sqrt{\pi}f^a e^{\frac{b^2\log^2(f)}{4f-4c\log(f)}-d}\text{Erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}-\frac{\sqrt{\pi}f^a e^{\frac{b^2\log^2(f)}{12f-4c\log(f)}-3d}\text{Erf}\left(\frac{b\log(f)-2x(3f-c\log(f))}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}+\frac{3\sqrt{\pi}f^a e^{d-\frac{b^2\log^2(f)}{4(c\log(f)+f)}}\text{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+f)}{2\sqrt{c\log(f)+f}}\right)}{16\sqrt{c\log(f)+f}}+\frac{\sqrt{\pi}f^a e^{3d-\frac{b^2\log^2(f)}{4(c\log(f)+3f)}}\text{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+3f)}{2\sqrt{c\log(f)+3f}}\right)}{16\sqrt{c\log(f)+3f}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Cosh[d + f*x^2]^3,x]
```

```
[Out] (-3*E^(-d + (b^2*Log[f]^2)/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f] -
 2*x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/(16*Sqrt[f - c*Log[f]]) - (E^
(-3*d + (b^2*Log[f]^2)/(12*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f] - 2*
x*(3*f - c*Log[f]))/(2*Sqrt[3*f - c*Log[f]])])/(16*Sqrt[3*f - c*Log[f]]) +
(3*E^(d - (b^2*Log[f]^2)/(4*(f + c*Log[f]))))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] +
2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(16*Sqrt[f + c*Log[f]]) + (E^(
3*d - (b^2*Log[f]^2)/(4*(3*f + c*Log[f]))))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*
x*(3*f + c*Log[f]))/(2*Sqrt[3*f + c*Log[f]])])/(16*Sqrt[3*f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5624

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int f^{a+bx+cx^2}\cosh^3\left(d+fx^2\right)\,dx = \int\left(\frac{1}{8}e^{-3d-3fx^2}f^{a+bx+cx^2}+\frac{3}{8}e^{-d-fx^2}f^{a+bx+cx^2}+\frac{3}{8}e^{d+fx^2}f^{a+bx+cx^2}+\frac{1}{8}e^{3d\cdots}\right.$$

$$=\frac{1}{8}\int e^{-3d-3fx^2}f^{a+bx+cx^2}\,dx+\frac{1}{8}\int e^{3d+3fx^2}f^{a+bx+cx^2}\,dx+\frac{3}{8}\int e^{-d-fx^2}f^{a+b\cdots}$$

$$=\frac{1}{8}\int\exp\left(-3d+a\log(f)+bx\log(f)-x^2(3f-c\log(f))\right)\,dx+\frac{1}{8}\int\exp\cdots$$

$$=\frac{1}{8}\left(3e^{-d+\frac{b^2\log^2(f)}{4f-4c\log(f)}}f^a\right)\int\exp\left(\frac{(b\log(f)+2x(-f+c\log(f)))^2}{4(-f+c\log(f))}\right)\,dx+\frac{1}{8}\Big(\cdots$$

$$=-\frac{3e^{-d+\frac{b^2\log^2(f)}{4f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}-\frac{e^{-3d+\frac{b^2\log^2(f)}{12f-4c\log(f)}}f^a\sqrt{\pi}\cdots}{16\sqrt{3f\cdots}}$$

Mathematica [A]
time = 4.72, size = 501, normalized size = 1.55

$$\frac{e^{-\frac{b^2\log^2(f)(2f+c\log(f))}{2(f+c\log(f))(3f+c\log(f))}}f^a\sqrt{\pi}\left(3e^{\frac{1}{4}b^2\log^2(f)\left(\frac{1}{f-c\log(f)}+\frac{1}{f+c\log(f)}+\frac{1}{3f+c\log(f)}\right)}\mathrm{Erf}\left(\frac{2fx-(b+2cx)\log(f)}{2\sqrt{f-c\log(f)}}\right)\sqrt{f-c\log(f)}\,(9f^3+9cf^2\log(f)-c^2f\log^2(f)-c^3\log^3(f))(\cosh(d)-\sinh(d))+(f-c\log(f))\left(e^{\frac{1}{4}b^2\log^2(f)\left(\frac{1}{3f-c\log(f)}+\frac{1}{f+c\log(f)}+\frac{1}{3f+c\log(f)}\right)}\mathrm{Erf}\left(\frac{6fx-(b+2cx)\log(f)}{2\sqrt{3f-c\log(f)}}\right)\sqrt{3f-c\log(f)}\,(3f^2+4cf\log(f)+c^2\log^2(f))(\cosh(3d)-\sinh(3d))+(3f-c\log(f))\left(3e^{\frac{b^2\log^2(f)}{12f+4c\log(f)}}\mathrm{Erfi}\left(\frac{2fx+(b+2cx)\log(f)}{2\sqrt{f+c\log(f)}}\right)\sqrt{f+c\log(f)}\,(3f+c\log(f))(\cosh(d)+\sinh(d))+e^{\frac{b^2\log^2(f)}{4(f+c\log(f))}}\mathrm{Erfi}\left(\frac{6fx+(b+2cx)\log(f)}{2\sqrt{3f+c\log(f)}}\right)(f+c\log(f))\sqrt{3f+c\log(f)}\,(\cosh(3d)+\sinh(3d))\right)\right)\right)}{16\left(9f^4-10c^2f^2\log^2(f)+c^4\log^4(f)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + b*x + c*x^2)*Cosh[d + f*x^2]^3,x]
```

```
[Out] (f^a*Sqrt[Pi]*(3*E^((b^2*Log[f]^2*((f - c*Log[f])^(-1) + (f + c*Log[f])^(-1
) + (3*f + c*Log[f])^(-1)))/4)*Erf[(2*f*x - (b + 2*c*x)*Log[f])/(2*Sqrt[f -
 c*Log[f]])]*Sqrt[f - c*Log[f]]*(9*f^3 + 9*c*f^2*Log[f] - c^2*f*Log[f]^2 -
c^3*Log[f]^3)*(Cosh[d] - Sinh[d]) + (f - c*Log[f])*(E^((b^2*Log[f]^2*((3*f
- c*Log[f])^(-1) + (f + c*Log[f])^(-1) + (3*f + c*Log[f])^(-1)))/4)*Erf[(6*
f*x - (b + 2*c*x)*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Sqrt[3*f - c*Log[f]]*(3
*f^2 + 4*c*f*Log[f] + c^2*Log[f]^2)*(Cosh[3*d] - Sinh[3*d]) + (3*f - c*Log[
f])*(3*E^((b^2*Log[f]^2)/(12*f + 4*c*Log[f]))*Erfi[(2*f*x + (b + 2*c*x)*Log
[f])/(2*Sqrt[f + c*Log[f]])]*Sqrt[f + c*Log[f]]*(3*f + c*Log[f])*(Cosh[d] +
 Sinh[d]) + E^((b^2*Log[f]^2)/(4*(f + c*Log[f])))*Erfi[(6*f*x + (b + 2*c*x)
*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*(f + c*Log[f])*Sqrt[3*f + c*Log[f]]*(Cos
h[3*d] + Sinh[3*d])))))/(16*E^((b^2*Log[f]^2*(2*f + c*Log[f]))/(2*(f + c*Lo
g[f])*(3*f + c*Log[f])))*(9*f^4 - 10*c^2*f^2*Log[f]^2 + c^4*Log[f]^4))
```

Maple [A]
time = 2.49, size = 326, normalized size = 1.01

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2} \ln(f)^{2}+12 d \ln(f) c-36 d f}{4(-3 f+c \ln(f))}} \operatorname{erf}\left(-x \sqrt{3 f-c \ln(f)}+\frac{\ln(f) b}{2 \sqrt{3 f-c \ln(f)}}\right)}{16 \sqrt{3 f-c \ln(f)}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2} \ln(f)^{2}-12 d \ln(f) c}{4(3 f+c \ln(f))}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cosh(f*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+12*d*ln(f)*c-36*d*f)/(-3*f+c*ln(f)
))/(3*f-c*ln(f))^(1/2)*erf(-x*(3*f-c*ln(f))^(1/2)+1/2*ln(f)*b/(3*f-c*ln(f))
^(1/2))-1/16*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-12*d*ln(f)*c-36*d*f)/(3*f+c
*ln(f)))/(-c*ln(f)-3*f)^(1/2)*erf(-(-c*ln(f)-3*f)^(1/2)*x+1/2*ln(f)*b/(-c*l
n(f)-3*f)^(1/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+4*d*ln(f)*c-4*d*f)
/(-f+c*ln(f)))/(f-c*ln(f))^(1/2)*erf(-x*(f-c*ln(f))^(1/2)+1/2*ln(f)*b/(f-c*
ln(f))^(1/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-4*d*ln(f)*c-4*d*f)/(f
+c*ln(f)))/(-c*ln(f)-f)^(1/2)*erf(-(-c*ln(f)-f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(
f)-f)^(1/2))
```

Maxima [A]
time = 0.28, size = 287, normalized size = 0.89

$$\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log\left(f\right) - 3 \, f} x - \frac{b \log\left(f\right)}{2 \, \sqrt{-c \log\left(f\right) - 3 \, f}}\right) e^{\left(-\frac{b^{2} \log\left(f\right)^{2}}{4 \, {\left(c \log\left(f\right) + 3 \, f\right)}} + 3 \, d\right)}}{16 \, \sqrt{-c \log\left(f\right) - 3 \, f}} + \frac{3 \, \sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log\left(f\right) - f} x - \frac{b \log\left(f\right)}{2 \, \sqrt{-c \log\left(f\right) - f}}\right) e^{\left(-\frac{b^{2} \log\left(f\right)^{2}}{4 \, {\left(c \log\left(f\right) + f\right)}} + d\right)}}{16 \, \sqrt{-c \log\left(f\right) - f}} + \frac{3 \, \sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log\left(f\right) + f} x - \frac{b \log\left(f\right)}{2 \, \sqrt{-c \log\left(f\right) + f}}\right) e^{\left(-\frac{b^{2} \log\left(f\right)^{2}}{4 \, {\left(c \log\left(f\right) - f\right)}} - d\right)}}{16 \, \sqrt{-c \log\left(f\right) + f}} + \frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log\left(f\right) + 3 \, f} x - \frac{b \log\left(f\right)}{2 \, \sqrt{-c \log\left(f\right) + 3 \, f}}\right) e^{\left(-\frac{b^{2} \log\left(f\right)^{2}}{4 \, {\left(c \log\left(f\right) - 3 \, f\right)}} - 3 \, d\right)}}{16 \, \sqrt{-c \log\left(f\right) + 3 \, f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 3*f)*x - 1/2*b*log(f)/sqrt(-c*log(f)
 - 3*f))*e^(-1/4*b^2*log(f)^2/(c*log(f) + 3*f) + 3*d)/sqrt(-c*log(f) - 3*f)
```

```
 + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*b*log(f)/sqrt(-c*log(f
) - f))*e^(-1/4*b^2*log(f)^2/(c*log(f) + f) + d)/sqrt(-c*log(f) - f) + 3/16
*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x - 1/2*b*log(f)/sqrt(-c*log(f) + f))
*e^(-1/4*b^2*log(f)^2/(c*log(f) - f) - d)/sqrt(-c*log(f) + f) + 1/16*sqrt(p
i)*f^a*erf(sqrt(-c*log(f) + 3*f)*x - 1/2*b*log(f)/sqrt(-c*log(f) + 3*f))*e^
(-1/4*b^2*log(f)^2/(c*log(f) - 3*f) - 3*d)/sqrt(-c*log(f) + 3*f)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 851 vs. $2(275) = 550$.
time = 0.67, size = 851, normalized size = 2.63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*((sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3)*c
osh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 36*d*f + 12*(c*d + a*f)*log(f))/(c*log(f
) - 3*f)) + sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^
3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 36*d*f + 12*(c*d + a*f)*log(f))/(c*l
og(f) - 3*f)))*sqrt(-c*log(f) + 3*f)*erf(-1/2*(6*f*x - (2*c*x + b)*log(f))*
sqrt(-c*log(f) + 3*f)/(c*log(f) - 3*f)) + 3*(sqrt(pi)*(c^3*log(f)^3 + c^2*f
*log(f)^2 - 9*c*f^2*log(f) - 9*f^3)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d
*f + 4*(c*d + a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c^3*log(f)^3 + c^2*f
*log(f)^2 - 9*c*f^2*log(f) - 9*f^3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d
*f + 4*(c*d + a*f)*log(f))/(c*log(f) - f)))*sqrt(-c*log(f) + f)*erf(-1/2*(2
*f*x - (2*c*x + b)*log(f))*sqrt(-c*log(f) + f)/(c*log(f) - f)) + 3*(sqrt(pi
)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*cosh(-1/4*((b^2
- 4*a*c)*log(f)^2 - 4*d*f - 4*(c*d + a*f)*log(f))/(c*log(f) + f)) + sqrt(pi
)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*sinh(-1/4*((b^2
- 4*a*c)*log(f)^2 - 4*d*f - 4*(c*d + a*f)*log(f))/(c*log(f) + f)))*sqrt(-c*
log(f) - f)*erf(1/2*(2*f*x + (2*c*x + b)*log(f))*sqrt(-c*log(f) - f)/(c*log
(f) + f)) + (sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) + 3*f
^3)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 36*d*f - 12*(c*d + a*f)*log(f))/(c*
log(f) + 3*f)) + sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) +
 3*f^3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 36*d*f - 12*(c*d + a*f)*log(f))
/(c*log(f) + 3*f)))*sqrt(-c*log(f) - 3*f)*erf(1/2*(6*f*x + (2*c*x + b)*log(
f))*sqrt(-c*log(f) - 3*f)/(c*log(f) + 3*f)))/(c^4*log(f)^4 - 10*c^2*f^2*log
(f)^2 + 9*f^4)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx+cx^2}\cosh^3\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cosh(f*x**2+d)**3,x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*cosh(d + f*x**2)**3, x)
```

Giac [A]
time = 0.41, size = 369, normalized size = 1.14

$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)-3f}\left(2x+\frac{b\log(f)}{c\log(f)+3f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2-12cd\log(f)-12af\log(f)-36df}{4(c\log(f)+3f)}\right)}}{16\sqrt{-c\log\left(f\right)-3f}}-\frac{3\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)-f}\left(2x+\frac{b\log(f)}{c\log(f)+f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2-4cd\log(f)-4af\log(f)-4df}{4(c\log(f)+f)}\right)}}{16\sqrt{-c\log\left(f\right)-f}}-\frac{3\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)+f}\left(2x+\frac{b\log(f)}{c\log(f)-f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2+4cd\log(f)+4af\log(f)-4df}{4(c\log(f)-f)}\right)}}{16\sqrt{-c\log\left(f\right)+f}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)+3f}\left(2x+\frac{b\log(f)}{c\log(f)-3f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2+12cd\log(f)+12af\log(f)-36df}{4(c\log(f)-3f)}\right)}}{16\sqrt{-c\log\left(f\right)+3f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+d)^3,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 3*f)*(2*x + b*log(f)/(c*log(f) + 3
*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 12*c*d*log(f) - 12*a*f*log(f
) - 36*d*f)/(c*log(f) + 3*f))/sqrt(-c*log(f) - 3*f) - 3/16*sqrt(pi)*erf(-1/
2*sqrt(-c*log(f) - f)*(2*x + b*log(f)/(c*log(f) + f)))*e^(-1/4*(b^2*log(f)^
2 - 4*a*c*log(f)^2 - 4*c*d*log(f) - 4*a*f*log(f) - 4*d*f)/(c*log(f) + f))/s
qrt(-c*log(f) - f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x + b*lo
g(f)/(c*log(f) - f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 4*c*d*log(f)
 + 4*a*f*log(f) - 4*d*f)/(c*log(f) - f))/sqrt(-c*log(f) + f) - 1/16*sqrt(pi
)*erf(-1/2*sqrt(-c*log(f) + 3*f)*(2*x + b*log(f)/(c*log(f) - 3*f)))*e^(-1/4
*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 12*c*d*log(f) + 12*a*f*log(f) - 36*d*f)/(
c*log(f) - 3*f))/sqrt(-c*log(f) + 3*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+b\,x+a}\,{\mathrm{cosh}\left(f\,x^2+d\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*cosh(d + f*x^2)^3,x)
```

```
[Out] int(f^(a + b*x + c*x^2)*cosh(d + f*x^2)^3, x)
```

3.326 $\int f^{a+bx+cx^2} \cosh\left(d+ex+fx^2\right) dx$

Optimal. Leaf size=161

$$\frac{e^{-d+\frac{(e-b\log(f))^2}{4(f-c\log(f))}} f^a \sqrt{\pi}\, \mathrm{Erf}\left(\frac{e-b\log(f)+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{e^{d-\frac{(e+b\log(f))^2}{4(f+c\log(f))}} f^a \sqrt{\pi}\, \mathrm{Erfi}\left(\frac{e+b\log(f)+2x(f+c\log(f))}{2\sqrt{f+c\log(f)}}\right)}{4\sqrt{f+c\log(f)}}$$

```
[Out] 1/4*exp(-d+1/4*(e-b*ln(f))^2/(f-c*ln(f)))*f^a*erf(1/2*(e-b*ln(f)+2*x*(f-c*l
n(f)))/(f-c*ln(f))^(1/2))*Pi^(1/2)/(f-c*ln(f))^(1/2)+1/4*exp(d-1/4*(e+b*ln(
f))^2/(f+c*ln(f)))*f^a*erfi(1/2*(e+b*ln(f)+2*x*(f+c*ln(f)))/(f+c*ln(f))^(1/
2))*Pi^(1/2)/(f+c*ln(f))^(1/2)
```

Rubi [A]

time = 0.35, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {5624, 2325, 2266, 2236, 2235}

$$\frac{\sqrt{\pi}\, f^a e^{\frac{(e-b\log(f))^2}{4(f-c\log(f))}-d}\mathrm{Erf}\left(\frac{-b\log(f)+2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}\, f^a e^{d-\frac{(b\log(f)+e)^2}{4(c\log(f)+f)}}\mathrm{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+f)+e}{2\sqrt{c\log(f)+f}}\right)}{4\sqrt{c\log(f)+f}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Cosh[d + e*x + f*x^2],x]
```

```
[Out] (E^(-d + (e - b*Log[f])^2/(4*(f - c*Log[f])))*f^a*Sqrt[Pi]*Erf[(e - b*Log[f
] + 2*x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/(4*Sqrt[f - c*Log[f]]) + (
E^(d - (e + b*Log[f])^2/(4*(f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(e + b*Log[f]
 + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(4*Sqrt[f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5624

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int f^{a+bx+cx^2}\cosh\left(d+ex+fx^2\right)\,dx &= \int \left(\frac{1}{2}e^{-d-ex-fx^2}f^{a+bx+cx^2}+\frac{1}{2}e^{d+ex+fx^2}f^{a+bx+cx^2}\right)dx\\
&= \frac{1}{2}\int e^{-d-ex-fx^2}f^{a+bx+cx^2}\,dx+\frac{1}{2}\int e^{d+ex+fx^2}f^{a+bx+cx^2}\,dx\\
&= \frac{1}{2}\int \exp\left(-d+a\log(f)-x(e-b\log(f))-x^2(f-c\log(f))\right)dx+\\
&= \frac{1}{2}\left(e^{-d+\frac{(e-b\log(f))^2}{4(f-c\log(f))}}f^a\right)\int \exp\left(\frac{(-e+b\log(f)+2x(-f+c\log(f)))^2}{4(-f+c\log(f))}\right.\\
&= \frac{e^{-d+\frac{(e-b\log(f))^2}{4(f-c\log(f))}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{e-b\log(f)+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{e^{d-\frac{(e+b\log(f))^2}{4(f+c\log(f))}}f^a\sqrt{}}{4}
\end{aligned}$$

Mathematica [A]
time = 1.04, size = 251, normalized size = 1.56

$$\frac{e^{-\frac{e^2+b^2\log^2(f)}{4(f+c\log(f))}}f^{a+\frac{bef}{-f^2+c^2\log^2(f)}}\sqrt{\pi}\left(e^{\frac{f\left(e^2+b^2\log^2(f)\right)}{2\left(f^2-c^2\log^2(f)\right)}}f^{\frac{be}{2(f+c\log(f))}}\mathrm{Erf}\left(\frac{e+2fx-(b+2cx)\log(f)}{2\sqrt{f-c\log(f)}}\right)\sqrt{f-c\log(f)}\,(f+c\log(f))(\cosh(d)-\sinh(d))+f^{\frac{be}{2f-2c\log(f)}}\mathrm{Erfi}\left(\frac{e+2fx+(b+2cx)\log(f)}{2\sqrt{f+c\log(f)}}\right)(f-c\log(f))\sqrt{f+c\log(f)}\,(\cosh(d)+\sinh(d))\right)}{4\left(f^2-c^2\log^2(f)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + b*x + c*x^2)*Cosh[d + e*x + f*x^2],x]
```

```
[Out] (f^(a + (b*e*f)/(-f^2 + c^2*Log[f]^2))*Sqrt[Pi]*(E^((f*(e^2 + b^2*Log[f]^2)
)/(2*(f^2 - c^2*Log[f]^2)))*f^((b*e)/(2*(f + c*Log[f])))*Erf[(e + 2*f*x - (
b + 2*c*x)*Log[f])/(2*Sqrt[f - c*Log[f]])]*Sqrt[f - c*Log[f]]*(f + c*Log[f]
)*(Cosh[d] - Sinh[d]) + f^((b*e)/(2*f - 2*c*Log[f]))*Erfi[(e + 2*f*x + (b +
 2*c*x)*Log[f])/(2*Sqrt[f + c*Log[f]])]*(f - c*Log[f])*Sqrt[f + c*Log[f]]*(
Cosh[d] + Sinh[d])))/(4*E^((e^2 + b^2*Log[f]^2)/(4*(f + c*Log[f])))*(f^2 -
c^2*Log[f]^2))
```

Maple [A]
time = 0.88, size = 186, normalized size = 1.16

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2} \ln(f)^{2}-2 \ln(f) b e+4 d \ln(f) c-4 d f+e^{2}}{4(-f+c \ln(f))}} \operatorname{erf}\left(-x \sqrt{f-c \ln(f)}+\frac{b \ln(f)-e}{2 \sqrt{f-c \ln(f)}}\right)}{4 \sqrt{f-c \ln(f)}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2} \ln(f)^{2}+2 \ln}{4}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-2*ln(f)*b*e+4*d*ln(f)*c-4*d*f+e^2)/
(-f+c*ln(f)))/(f-c*ln(f))^(1/2)*erf(-x*(f-c*ln(f))^(1/2)+1/2*(b*ln(f)-e)/(f
-c*ln(f))^(1/2))-1/4*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+2*ln(f)*b*e-4*d*ln(
f)*c-4*d*f+e^2)/(f+c*ln(f)))/(-c*ln(f)-f)^(1/2)*erf(-(-c*ln(f)-f)^(1/2)*x+1
/2*(e+b*ln(f))/(-c*ln(f)-f)^(1/2))
```

Maxima [A]
time = 0.27, size = 155, normalized size = 0.96

$$\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)-f}\, x-\frac{b \log(f)+e}{2 \sqrt{-c \log(f)-f}}\right) e^{\left(-\frac{(b \log(f)+e)^{2}}{4(c \log(f)+f)}+d\right)}}{4 \sqrt{-c \log(f)-f}}+\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)+f}\, x-\frac{b \log(f)-e}{2 \sqrt{-c \log(f)+f}}\right) e^{\left(-\frac{(b \log(f)-e)^{2}}{4(c \log(f)-f)}-d\right)}}{4 \sqrt{-c \log(f)+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*(b*log(f) + e)/sqrt(-c*log
(f) - f))*e^(-1/4*(b*log(f) + e)^2/(c*log(f) + f) + d)/sqrt(-c*log(f) - f)
+ 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x - 1/2*(b*log(f) - e)/sqrt(-c*l
og(f) + f))*e^(-1/4*(b*log(f) - e)^2/(c*log(f) - f) - d)/sqrt(-c*log(f) + f
)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 436 vs. 2(143) = 286.
time = 0.52, size = 436, normalized size = 2.71

$$-\frac{\left(\sqrt{\pi}(c \log(f)+f) \cosh\left(-\frac{(b^{2}-4 a c) \log(f)^{2}-4 d f+\cosh(1)^{2}+2(2 c d+2 a f-b \cosh(1)-b \sinh(1)) \log(f)+2 \cosh(1) \sinh(1)+\sinh(1)^{2}}{4(c \log(f)-f)}\right)+\sqrt{\pi}(c \log(f)+f) \sinh\left(-\frac{(b^{2}-4 a c) \log(f)^{2}-4 d f+\cosh(1)^{2}+2(2 c d+2 a f-b \cosh(1)-b \sinh(1)) \log(f)+2 \cosh(1) \sinh(1)+\sinh(1)^{2}}{4(c \log(f)-f)}\right)\right) \sqrt{-c \log(f)+f} \operatorname{erf}\left(-\frac{(2 f x-(2 c x+b) \log(f)+\cosh(1)+\sinh(1)) \sqrt{-c \log(f)+f}}{2(c \log(f)-f)}\right)+\left(\sqrt{\pi}(c \log(f)-f) \cosh\left(-\frac{(b^{2}-4 a c) \log(f)^{2}-4 d f+\cosh(1)^{2}-2(2 c d+2 a f-b \cosh(1)-b \sinh(1)) \log(f)+2 \cosh(1) \sinh(1)+\sinh(1)^{2}}{4(c \log(f)+f)}\right)+\sqrt{\pi}(c \log(f)-f) \sinh\left(-\frac{(b^{2}-4 a c) \log(f)^{2}-4 d f+\cosh(1)^{2}-2(2 c d+2 a f-b \cosh(1)-b \sinh(1)) \log(f)+2 \cosh(1) \sinh(1)+\sinh(1)^{2}}{4(c \log(f)+f)}\right)\right) \sqrt{-c \log(f)-f} \operatorname{erf}\left(\frac{(2 f x+(2 c x+b) \log(f)+\cosh(1)+\sinh(1)) \sqrt{-c \log(f)-f}}{2(c \log(f)+f)}\right)}{4\left(c^{2} \log(f)^{2}-f^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] -1/4*((sqrt(pi)*(c*log(f) + f)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d*f +
cosh(1)^2 + 2*(2*c*d + 2*a*f - b*cosh(1) - b*sinh(1))*log(f) + 2*cosh(1)*si
nh(1) + sinh(1)^2)/(c*log(f) - f)) + sqrt(pi)*(c*log(f) + f)*sinh(-1/4*((b^
```

```
2 - 4*a*c)*log(f)^2 - 4*d*f + cosh(1)^2 + 2*(2*c*d + 2*a*f - b*cosh(1) - b*
sinh(1))*log(f) + 2*cosh(1)*sinh(1) + sinh(1)^2)/(c*log(f) - f)))*sqrt(-c*l
og(f) + f)*erf(-1/2*(2*f*x - (2*c*x + b)*log(f) + cosh(1) + sinh(1))*sqrt(-
c*log(f) + f)/(c*log(f) - f)) + (sqrt(pi)*(c*log(f) - f)*cosh(-1/4*((b^2 -
4*a*c)*log(f)^2 - 4*d*f + cosh(1)^2 - 2*(2*c*d + 2*a*f - b*cosh(1) - b*sinh
(1))*log(f) + 2*cosh(1)*sinh(1) + sinh(1)^2)/(c*log(f) + f)) + sqrt(pi)*(c*
log(f) - f)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d*f + cosh(1)^2 - 2*(2*c*
d + 2*a*f - b*cosh(1) - b*sinh(1))*log(f) + 2*cosh(1)*sinh(1) + sinh(1)^2)/
(c*log(f) + f)))*sqrt(-c*log(f) - f)*erf(1/2*(2*f*x + (2*c*x + b)*log(f) +
cosh(1) + sinh(1))*sqrt(-c*log(f) - f)/(c*log(f) + f)))/(c^2*log(f)^2 - f^2
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx+cx^2}\cosh\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cosh(f*x**2+e*x+d),x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*cosh(d + e*x + f*x**2), x)
```

Giac [A]
time = 0.42, size = 207, normalized size = 1.29

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)-f}\,\left(2\,x+\frac{b\log(f)+e}{c\log(f)+f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2-4\,cd\log(f)+2\,be\log(f)-4\,af\log(f)+e^2-4\,df}{4\,(c\log(f)+f)}\right)}}{4\,\sqrt{-c\log\left(f\right)-f}}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)+f}\,\left(2\,x+\frac{b\log(f)-e}{c\log(f)-f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2+4\,cd\log(f)-2\,be\log(f)+4\,af\log(f)+e^2-4\,df}{4\,(c\log(f)-f)}\right)}}{4\,\sqrt{-c\log\left(f\right)+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - f)*(2*x + (b*log(f) + e)/(c*log(f)
+ f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*log(f) + 2*b*e*log(f)
 - 4*a*f*log(f) + e^2 - 4*d*f)/(c*log(f) + f))/sqrt(-c*log(f) - f) - 1/4*sq
rt(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x + (b*log(f) - e)/(c*log(f) - f)))*
e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 4*c*d*log(f) - 2*b*e*log(f) + 4*a*
f*log(f) + e^2 - 4*d*f)/(c*log(f) - f))/sqrt(-c*log(f) + f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+b\,x+a}\,\mathrm{cosh}\left(f\,x^2+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*cosh(d + e*x + f*x^2),x)
```

```
[Out] int(f^(a + b*x + c*x^2)*cosh(d + e*x + f*x^2), x)
```

3.327 $\int f^{a+bx+cx^2}\cosh^2\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=239

$$\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{e^{-2d+\frac{(2e-b\log(f))^2}{8f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{Erf}\left(\frac{2e-b\log(f)+2x(2f-c\log(f))}{2\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{e^{2d-\frac{(2e+b\log(f))^2}{8f+4c\log(f)}}f^a\sqrt{\ }}{8\sqrt{\ }}$$

```
[Out] 1/4*f^(a-1/4*b^2/c)*erfi(1/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2))*Pi^(1/2)/c^(1/2
)/ln(f)^(1/2)+1/8*exp(-2*d+(2*e-b*ln(f))^2/(8*f-4*c*ln(f)))*f^a*erf(1/2*(2*
e-b*ln(f)+2*x*(2*f-c*ln(f)))/(2*f-c*ln(f))^(1/2))*Pi^(1/2)/(2*f-c*ln(f))^(1
/2)+1/8*exp(2*d-(2*e+b*ln(f))^2/(8*f+4*c*ln(f)))*f^a*erfi(1/2*(2*e+b*ln(f)+
2*x*(2*f+c*ln(f)))/(2*f+c*ln(f))^(1/2))*Pi^(1/2)/(2*f+c*ln(f))^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.192$, Rules used = {5624, 2266, 2235, 2325, 2236}

$$\frac{\sqrt{\pi}\,f^{a-\frac{b^2}{4c}}\mathrm{Erfi}\left(\frac{\sqrt{\log(f)}\,(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{\sqrt{\pi}\,f^a\exp\left(\frac{(2e-b\log(f))^2}{8f-4c\log(f)}-2d\right)\mathrm{Erf}\left(\frac{-b\log(f)+2x(2f-c\log(f))+2e}{2\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi}\,f^a\exp\left(2d-\frac{(b\log(f)+2e)^2}{4c\log(f)+8f}\right)\mathrm{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+2f)+2e}{2\sqrt{c\log(f)+2f}}\right)}{8\sqrt{c\log(f)+2f}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Cosh[d + e*x + f*x^2]^2,x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(
4*Sqrt[c]*Sqrt[Log[f]]) + (E^(-2*d + (2*e - b*Log[f])^2/(8*f - 4*c*Log[f]))
*f^a*Sqrt[Pi]*Erf[(2*e - b*Log[f] + 2*x*(2*f - c*Log[f]))/(2*Sqrt[2*f - c*L
og[f]])])/(8*Sqrt[2*f - c*Log[f]]) + (E^(2*d - (2*e + b*Log[f])^2/(8*f + 4*
c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(2*e + b*Log[f] + 2*x*(2*f + c*Log[f]))/(2*Sqr
t[2*f + c*Log[f]])])/(8*Sqrt[2*f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5624

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx+cx^2}\cosh^2\left(d+ex+fx^2\right)dx &= \int \left(\frac{1}{2}f^{a+bx+cx^2}+\frac{1}{4}e^{-2d-2ex-2fx^2}f^{a+bx+cx^2}+\frac{1}{4}e^{2d+2ex+2fx^2}f^{a+bx+cx}\right.\\ &= \frac{1}{4}\int e^{-2d-2ex-2fx^2}f^{a+bx+cx^2}\,dx+\frac{1}{4}\int e^{2d+2ex+2fx^2}f^{a+bx+cx^2}\,dx+\frac{1}{2}\\ &= \frac{1}{4}\int \exp\left(-2d+a\log(f)-x(2e-b\log(f))-x^2(2f-c\log(f))\right)\,d\\ &= \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{1}{4}\left(\exp\left(-2d+\frac{(2e-b\log(f)}{8f-4c\log(f}\right.\right.\\ &= \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{\exp\left(-2d+\frac{(2e-b\log(f))^2}{8f-4c\log(f)}\right)f^a\sqrt{}}{8\sqrt{2f-}}\end{aligned}$$

Mathematica [A]
time = 4.26, size = 339, normalized size = 1.42

$$\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-\frac{4e^2+b^2\log^2(f)}{8f+4c\log(f)}}f^{a+\frac{4bef}{-4f^2+c^2\log^2(f)}}\sqrt{\pi}\left(e^{\frac{f\left(4e^2+b^2\log^2(f)\right)}{4f^2-c^2\log^2(f)}}f^{\frac{be}{2f+c\log(f)}}\mathrm{Erf}\left(\frac{2(e+2fx)-(b+2cx)\log(f)}{2\sqrt{2f-c\log(f)}}\right)\sqrt{2f-c\log(f)}\,(2f+c\log(f))(\cosh(2d)-\sinh(2d))+f^{\frac{be}{2f-c\log(f)}}\mathrm{Erfi}\left(\frac{2(e+2fx)+(b+2cx)\log(f)}{2\sqrt{2f+c\log(f)}}\right)(2f-c\log(f))\sqrt{2f+c\log(f)}\,(\cosh(2d)+\sinh(2d))\right)}{8\left(-4f^2+c^2\log^2(f)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + b*x + c*x^2)*Cosh[d + e*x + f*x^2]^2,x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(
4*Sqrt[c]*Sqrt[Log[f]]) - (f^(a + (4*b*e*f)/(-4*f^2 + c^2*Log[f]^2))*Sqrt[P
i]*(E^((f*(4*e^2 + b^2*Log[f]^2))/(4*f^2 - c^2*Log[f]^2))*f^((b*e)/(2*f + c
```

```
*Log[f]))*Erf[(2*(e + 2*f*x) - (b + 2*c*x)*Log[f])/(2*Sqrt[2*f - c*Log[f]])
]*Sqrt[2*f - c*Log[f]]*(2*f + c*Log[f])*(Cosh[2*d] - Sinh[2*d]) + f^((b*e)/
(2*f - c*Log[f]))*Erfi[(2*(e + 2*f*x) + (b + 2*c*x)*Log[f])/(2*Sqrt[2*f + c
*Log[f]])]*(2*f - c*Log[f])*Sqrt[2*f + c*Log[f]]*(Cosh[2*d] + Sinh[2*d])))/
(8*E^((4*e^2 + b^2*Log[f]^2)/(8*f + 4*c*Log[f]))*(-4*f^2 + c^2*Log[f]^2))
```

Maple [A]
time = 1.81, size = 249, normalized size = 1.04

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2} \ln(f)^{2}-4 \ln(f) b e+8 d \ln(f) c-16 d f+4 e^{2}}{4(-2 f+c \ln(f))}} \operatorname{erf}\left(-x \sqrt{2 f-c \ln(f)}+\frac{b \ln(f)-2 e}{2 \sqrt{2 f-c \ln(f)}}\right)}{8 \sqrt{2 f-c \ln(f)}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2} \ln(f)^{2}}{}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-4*ln(f)*b*e+8*d*ln(f)*c-16*d*f+4*e^
2)/(-2*f+c*ln(f)))/(2*f-c*ln(f))^(1/2)*erf(-x*(2*f-c*ln(f))^(1/2)+1/2*(b*ln
(f)-2*e)/(2*f-c*ln(f))^(1/2))-1/8*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+4*ln(f
)*b*e-8*d*ln(f)*c-16*d*f+4*e^2)/(2*f+c*ln(f)))/(-c*ln(f)-2*f)^(1/2)*erf(-(-
c*ln(f)-2*f)^(1/2)*x+1/2*(2*e+b*ln(f))/(-c*ln(f)-2*f)^(1/2))-1/4*Pi^(1/2)*f
^a*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*b*ln(f)/(-c*
ln(f))^(1/2))
```

Maxima [A]
time = 0.28, size = 219, normalized size = 0.92

$$\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)-2 f}\, x-\frac{b \log(f)+2 e}{2 \sqrt{-c \log(f)-2 f}}\right) e^{\left(-\frac{(b \log(f)+2 e)^{2}}{4(c \log(f)+2 f)}+2 d\right)}}{8 \sqrt{-c \log(f)-2 f}}+\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)+2 f}\, x-\frac{b \log(f)-2 e}{2 \sqrt{-c \log(f)+2 f}}\right) e^{\left(-\frac{(b \log(f)-2 e)^{2}}{4(c \log(f)-2 f)}-2 d\right)}}{8 \sqrt{-c \log(f)+2 f}}+\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x-\frac{b \log(f)}{2 \sqrt{-c \log(f)}}\right)}{4 \sqrt{-c \log(f)}\, f^{\frac{b^{2}}{4 c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 2*f)*x - 1/2*(b*log(f) + 2*e)/sqrt(-c
*log(f) - 2*f))*e^(-1/4*(b*log(f) + 2*e)^2/(c*log(f) + 2*f) + 2*d)/sqrt(-c*
log(f) - 2*f) + 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 2*f)*x - 1/2*(b*log(f
) - 2*e)/sqrt(-c*log(f) + 2*f))*e^(-1/4*(b*log(f) - 2*e)^2/(c*log(f) - 2*f)
 - 2*d)/sqrt(-c*log(f) + 2*f) + 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/
2*b*log(f)/sqrt(-c*log(f)))/(sqrt(-c*log(f))*f^(1/4*b^2/c))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 602 vs. 2(201) = 402.
time = 0.46, size = 602, normalized size = 2.52

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*((sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*cosh(-1/4*((b^2 - 4*a*c)*log(
f)^2 - 16*d*f + 4*cosh(1)^2 + 4*(2*c*d + 2*a*f - b*cosh(1) - b*sinh(1))*log
(f) + 8*cosh(1)*sinh(1) + 4*sinh(1)^2)/(c*log(f) - 2*f)) + sqrt(pi)*(c^2*lo
g(f)^2 + 2*c*f*log(f))*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 16*d*f + 4*cosh(
1)^2 + 4*(2*c*d + 2*a*f - b*cosh(1) - b*sinh(1))*log(f) + 8*cosh(1)*sinh(1)
 + 4*sinh(1)^2)/(c*log(f) - 2*f)))*sqrt(-c*log(f) + 2*f)*erf(-1/2*(4*f*x -
(2*c*x + b)*log(f) + 2*cosh(1) + 2*sinh(1))*sqrt(-c*log(f) + 2*f)/(c*log(f)
 - 2*f)) + (sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))*cosh(-1/4*((b^2 - 4*a*c)
*log(f)^2 - 16*d*f + 4*cosh(1)^2 - 4*(2*c*d + 2*a*f - b*cosh(1) - b*sinh(1)
)*log(f) + 8*cosh(1)*sinh(1) + 4*sinh(1)^2)/(c*log(f) + 2*f)) + sqrt(pi)*(c
^2*log(f)^2 - 2*c*f*log(f))*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 16*d*f + 4*
cosh(1)^2 - 4*(2*c*d + 2*a*f - b*cosh(1) - b*sinh(1))*log(f) + 8*cosh(1)*si
nh(1) + 4*sinh(1)^2)/(c*log(f) + 2*f)))*sqrt(-c*log(f) - 2*f)*erf(1/2*(4*f*
x + (2*c*x + b)*log(f) + 2*cosh(1) + 2*sinh(1))*sqrt(-c*log(f) - 2*f)/(c*lo
g(f) + 2*f)) + 2*(sqrt(pi)*(c^2*log(f)^2 - 4*f^2)*cosh(-1/4*(b^2 - 4*a*c)*l
og(f)/c) + sqrt(pi)*(c^2*log(f)^2 - 4*f^2)*sinh(-1/4*(b^2 - 4*a*c)*log(f)/c
))*sqrt(-c*log(f))*erf(1/2*(2*c*x + b)*sqrt(-c*log(f))/c))/(c^3*log(f)^3 -
4*c*f^2*log(f))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx+cx^2}\cosh^2\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cosh(f*x**2+e*x+d)**2,x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*cosh(d + e*x + f*x**2)**2, x)
```

Giac [A]
time = 0.43, size = 271, normalized size = 1.13

$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)-2f}\left(2x+\frac{b\log(f)+2e}{c\log(f)+2f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2-8cd\log(f)+4be\log(f)-8af\log(f)+4e^2-16df}{4(c\log(f)+2f)}\right)}}{8\sqrt{-c\log\left(f\right)-2f}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)+2f}\left(2x+\frac{b\log(f)-2e}{c\log(f)-2f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2+8cd\log(f)-4be\log(f)+8af\log(f)+4e^2-16df}{4(c\log(f)-2f)}\right)}}{8\sqrt{-c\log\left(f\right)+2f}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)}\left(2x+\frac{b}{c}\right)\right)e^{\left(-\frac{b^2\log(f)-4ac\log(f)}{4c}\right)}}{4\sqrt{-c\log\left(f\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] -1/8*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 2*f)*(2*x + (b*log(f) + 2*e)/(c*log
(f) + 2*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 8*c*d*log(f) + 4*b*e*
log(f) - 8*a*f*log(f) + 4*e^2 - 16*d*f)/(c*log(f) + 2*f))/sqrt(-c*log(f) -
2*f) - 1/8*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + 2*f)*(2*x + (b*log(f) - 2*e)/
(c*log(f) - 2*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 8*c*d*log(f) -
```

```
4*b*e*log(f) + 8*a*f*log(f) + 4*e^2 - 16*d*f)/(c*log(f) - 2*f))/sqrt(-c*log
(f) + 2*f) - 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + b/c))*e^(-1/4*(b^
2*log(f) - 4*a*c*log(f))/c)/sqrt(-c*log(f))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+b\,x+a}\,\cosh\left(f\,x^2+e\,x+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*cosh(d + e*x + f*x^2)^2,x)
[Out] int(f^(a + b*x + c*x^2)*cosh(d + e*x + f*x^2)^2, x)
```

3.328 $\int f^{a+bx+cx^2} \cosh^3\left(d+ex+fx^2\right) dx$

Optimal. Leaf size=344

$$\frac{3e^{-d+\frac{(e-b\log(f))^2}{4(f-c\log(f))}} f^a \sqrt{\pi}\, \text{Erf}\left(\frac{e-b\log(f)+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}+\frac{e^{-3d+\frac{(3e-b\log(f))^2}{12f-4c\log(f)}} f^a \sqrt{\pi}\, \text{Erf}\left(\frac{3e-b\log(f)+2x(3f-c\log(f))}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}+\frac{3e}{}$$

[Out] 3/16*exp(-d+1/4*(e-b*ln(f))^2/(f-c*ln(f)))*f^a*erf(1/2*(e-b*ln(f)+2*x*(f-c*ln(f)))/(f-c*ln(f))^(1/2))*Pi^(1/2)/(f-c*ln(f))^(1/2)+1/16*exp(-3*d+(3*e-b*ln(f))^2/(12*f-4*c*ln(f)))*f^a*erf(1/2*(3*e-b*ln(f)+2*x*(3*f-c*ln(f)))/(3*f-c*ln(f))^(1/2))*Pi^(1/2)/(3*f-c*ln(f))^(1/2)+3/16*exp(d-1/4*(e+b*ln(f))^2/(f+c*ln(f)))*f^a*erfi(1/2*(e+b*ln(f)+2*x*(f+c*ln(f)))/(f+c*ln(f))^(1/2))*Pi^(1/2)/(f+c*ln(f))^(1/2)+1/16*exp(3*d-1/4*(3*e+b*ln(f))^2/(3*f+c*ln(f)))*f^a*erfi(1/2*(3*e+b*ln(f)+2*x*(3*f+c*ln(f)))/(3*f+c*ln(f))^(1/2))*Pi^(1/2)/(3*f+c*ln(f))^(1/2)

Rubi [A]

time = 0.60, antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {5624, 2325, 2266, 2236, 2235}

$$\frac{\sqrt{\pi}\, f^a \exp\left(\frac{(3e-b\log(f))^2}{12f-4c\log(f)}-3d\right) \text{Erf}\left(\frac{-b\log(f)+2x(3f-c\log(f))+3e}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}+\frac{3\sqrt{\pi}\, f^a e^{\frac{(e-b\log(f))^2}{4(f-c\log(f))}-d} \text{Erf}\left(\frac{-b\log(f)+2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}\, f^a \exp\left(3d-\frac{(b\log(f)+3e)^2}{4(c\log(f)+3f)}\right) \text{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+3f)+3e}{2\sqrt{c\log(f)+3f}}\right)}{16\sqrt{c\log(f)+3f}}+\frac{3\sqrt{\pi}\, f^a e^{d-\frac{(b\log(f)+e)^2}{4(c\log(f)+f)}} \text{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+f)+e}{2\sqrt{c\log(f)+f}}\right)}{16\sqrt{c\log(f)+f}}$$

Antiderivative was successfully verified.

[In] Int[f^(a + b*x + c*x^2)*Cosh[d + e*x + f*x^2]^3,x]

[Out] (3*E^(-d + (e - b*Log[f])^2/(4*(f - c*Log[f])))*f^a*Sqrt[Pi]*Erf[(e - b*Log[f] + 2*x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/(16*Sqrt[f - c*Log[f]]) + (E^(-3*d + (3*e - b*Log[f])^2/(12*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(3*e - b*Log[f] + 2*x*(3*f - c*Log[f]))/(2*Sqrt[3*f - c*Log[f]])])/(16*Sqrt[3*f - c*Log[f]]) + (3*E^(d - (e + b*Log[f])^2/(4*(f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(e + b*Log[f] + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(16*Sqrt[f + c*Log[f]]) + (E^(3*d - (3*e + b*Log[f])^2/(4*(3*f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(3*e + b*Log[f] + 2*x*(3*f + c*Log[f]))/(2*Sqrt[3*f + c*Log[f]])])/(16*Sqrt[3*f + c*Log[f]])

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5624

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx+cx^2}\cosh^3\left(d+ex+fx^2\right)\,dx &= \int\left(\frac{1}{8}e^{-3\left(d+ex+fx^2\right)}f^{a+bx+cx^2}+\frac{3}{8}\exp\left(2d+2ex+2fx^2-3\big(d+ex\right.\right.\\ &= \frac{1}{8}\int e^{-3\left(d+ex+fx^2\right)}f^{a+bx+cx^2}\,dx+\frac{1}{8}\int\exp\left(6d+6ex+6fx^2-3\big(d+\right.\\ &= \frac{1}{8}\int\exp\left(-3d+a\log(f)-x(3e-b\log(f))-x^2(3f-c\log(f))\right)\,dx\\ &= \frac{1}{8}\left(\exp\left(-3d+\frac{(3e-b\log(f))^2}{12f-4c\log(f)}\right)f^a\right)\int\exp\left(\frac{(-3e+b\log(f)+2x}{4(-3f+}\right.\\ &= \frac{3e^{-d+\frac{(e-b\log(f))^2}{4(f-c\log(f))}}f^a\sqrt{\pi}\,\text{erf}\left(\frac{e-b\log(f)+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}+\frac{\exp\left(-3d+\frac{(3e-}{12f-}\right.}{}\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 2991 vs. $2(344) = 688$.
time = 6.44, size = 2991, normalized size = 8.69

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[f^(a + b*x + c*x^2)*Cosh[d + e*x + f*x^2]^3,x]

[Out] (f^a*Sqrt[Pi]*((27*f^3*Cosh[d]*Erf[(e + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(2
*Sqrt[f - c*Log[f]])]*Sqrt[f - c*Log[f]])/E^((-e^2 + 2*b*e*Log[f] - b^2*Log
[f]^2)/(4*(f - c*Log[f]))) + (27*c*f^2*Cosh[d]*Erf[(e + 2*f*x - b*Log[f] -
2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]*Sqrt[f - c*Log[f]])/E^((-e^2 +
 2*b*e*Log[f] - b^2*Log[f]^2)/(4*(f - c*Log[f]))) - (3*c^2*f*Cosh[d]*Erf[(e
 + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]^2*Sqrt[f
 - c*Log[f]])/E^((-e^2 + 2*b*e*Log[f] - b^2*Log[f]^2)/(4*(f - c*Log[f]))) -
 (3*c^3*Cosh[d]*Erf[(e + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log
[f]])]*Log[f]^3*Sqrt[f - c*Log[f]])/E^((-e^2 + 2*b*e*Log[f] - b^2*Log[f]^2)
/(4*(f - c*Log[f]))) + (3*f^3*Cosh[3*d]*Erf[(3*e + 6*f*x - b*Log[f] - 2*c*x
*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Sqrt[3*f - c*Log[f]])/E^((-9*e^2 + 6*b*e
*Log[f] - b^2*Log[f]^2)/(4*(3*f - c*Log[f]))) + (c*f^2*Cosh[3*d]*Erf[(3*e +
 6*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Log[f]*Sqrt[3*f
 - c*Log[f]])/E^((-9*e^2 + 6*b*e*Log[f] - b^2*Log[f]^2)/(4*(3*f - c*Log[f])
)) - (3*c^2*f*Cosh[3*d]*Erf[(3*e + 6*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt
[3*f - c*Log[f]])]*Log[f]^2*Sqrt[3*f - c*Log[f]])/E^((-9*e^2 + 6*b*e*Log[f]
 - b^2*Log[f]^2)/(4*(3*f - c*Log[f]))) - (c^3*Cosh[3*d]*Erf[(3*e + 6*f*x -
b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Log[f]^3*Sqrt[3*f - c*Lo
g[f]])/E^((-9*e^2 + 6*b*e*Log[f] - b^2*Log[f]^2)/(4*(3*f - c*Log[f]))) + (2
7*f^3*Cosh[d]*Erfi[(e + 2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[
f]])]*Sqrt[f + c*Log[f]])/E^((e^2 + 2*b*e*Log[f] + b^2*Log[f]^2)/(4*(f + c*
Log[f]))) - (27*c*f^2*Cosh[d]*Erfi[(e + 2*f*x + b*Log[f] + 2*c*x*Log[f])/(2
*Sqrt[f + c*Log[f]])]*Log[f]*Sqrt[f + c*Log[f]])/E^((e^2 + 2*b*e*Log[f] + b
^2*Log[f]^2)/(4*(f + c*Log[f]))) - (3*c^2*f*Cosh[d]*Erfi[(e + 2*f*x + b*Log
[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Log[f]^2*Sqrt[f + c*Log[f]])/E^
((e^2 + 2*b*e*Log[f] + b^2*Log[f]^2)/(4*(f + c*Log[f]))) + (3*c^3*Cosh[d]*E
rfi[(e + 2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Log[f]^3*
Sqrt[f + c*Log[f]])/E^((e^2 + 2*b*e*Log[f] + b^2*Log[f]^2)/(4*(f + c*Log[f]
))) + (3*f^3*Cosh[3*d]*Erfi[(3*e + 6*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt
[3*f + c*Log[f]])]*Sqrt[3*f + c*Log[f]])/E^((9*e^2 + 6*b*e*Log[f] + b^2*Log
[f]^2)/(4*(3*f + c*Log[f]))) - (c*f^2*Cosh[3*d]*Erfi[(3*e + 6*f*x + b*Log[f
] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Log[f]*Sqrt[3*f + c*Log[f]])/E^
((9*e^2 + 6*b*e*Log[f] + b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) - (3*c^2*f*Cos
h[3*d]*Erfi[(3*e + 6*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]]
)]*Log[f]^2*Sqrt[3*f + c*Log[f]])/E^((9*e^2 + 6*b*e*Log[f] + b^2*Log[f]^2)/
(4*(3*f + c*Log[f]))) + (c^3*Cosh[3*d]*Erfi[(3*e + 6*f*x + b*Log[f] + 2*c*x
*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Log[f]^3*Sqrt[3*f + c*Log[f]])/E^((9*e^2
 + 6*b*e*Log[f] + b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) - (27*f^3*Erf[(e + 2*
f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Sqrt[f - c*Log[f]]*S
inh[d])/E^((-e^2 + 2*b*e*Log[f] - b^2*Log[f]^2)/(4*(f - c*Log[f]))) - (27*c
*f^2*Erf[(e + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[
f]*Sqrt[f - c*Log[f]]*Sinh[d])/E^((-e^2 + 2*b*e*Log[f] - b^2*Log[f]^2)/(4*(
f - c*Log[f]))) + (3*c^2*f*Erf[(e + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqr

```
t[f - c*Log[f]])]*Log[f]^2*Sqrt[f - c*Log[f]]*Sinh[d])/E^((-e^2 + 2*b*e*Log
[f] - b^2*Log[f]^2)/(4*(f - c*Log[f]))) + (3*c^3*Erf[(e + 2*f*x - b*Log[f]
- 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]^3*Sqrt[f - c*Log[f]]*Sinh[d]
)/E^((-e^2 + 2*b*e*Log[f] - b^2*Log[f]^2)/(4*(f - c*Log[f]))) + (27*f^3*Erf
i[(e + 2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Sqrt[f + c*
Log[f]]*Sinh[d])/E^((e^2 + 2*b*e*Log[f] + b^2*Log[f]^2)/(4*(f + c*Log[f])))
 - (27*c*f^2*Erfi[(e + 2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f
]])]*Log[f]*Sqrt[f + c*Log[f]]*Sinh[d])/E^((e^2 + 2*b*e*Log[f] + b^2*Log[f]
^2)/(4*(f + c*Log[f]))) - (3*c^2*f*Erfi[(e + 2*f*x + b*Log[f] + 2*c*x*Log[f
])/(2*Sqrt[f + c*Log[f]])]*Log[f]^2*Sqrt[f + c*Log[f]]*Sinh[d])/E^((e^2 + 2
*b*e*Log[f] + b^2*Log[f]^2)/(4*(f + c*Log[f]))) + (3*c^3*Erfi[(e + 2*f*x +
b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Log[f]^3*Sqrt[f + c*Log[f]
]*Sinh[d])/E^((e^2 + 2*b*e*Log[f] + b^2*Log[f]^2)/(4*(f + c*Log[f]))) - (3*
f^3*Erf[(3*e + 6*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*S
qrt[3*f - c*Log[f]]*Sinh[3*d])/E^((-9*e^2 + 6*b*e*Log[f] - b^2*Log[f]^2)/(4
*(3*f - c*Log[f]))) - (c*f^2*Erf[(3*e + 6*f*x - b*Log[f] - 2*c*x*Log[f])/(2
*Sqrt[3*f - c*Log[f]])]*Log[f]*Sqrt[3*f - c*Log[f]]*Sinh[3*d])/E^((-9*e^2 +
 6*b*e*Log[f] - b^2*Log[f]^2)/(4*(3*f - c*Log[f]))) + (3*c^2*f*Erf[(3*e + 6
*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Log[f]^2*Sqrt[3*f
 - c*Log[f]]*Sinh[3*d])/E^((-9*e^2 + 6*b*e*Log[f] - b^2*Log[f]^2)/(4*(3*f -
 c*Log[f]))) + (c^3*Erf[(3*e + 6*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[3*f
 - c*Log[f]])]*Log[f]^3*Sqrt[3*f - c*Log[f]]*Sinh[3*d])/E^((-9*e^2 + 6*b*e*
Log[f] - b^2*Log[f]^2)/(4*(3*f - c*Log[f]))) + (3*f^3*Erfi[(3*e + 6*f*x + b
*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]]...
```

Maple [A]
time = 2.78, size = 384, normalized size = 1.12

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2} \ln(f)^{2}-6 \ln(f) b e+12 d \ln(f) c-36 d f+9 e^{2}}{4(-3 f+c \ln(f))}} \operatorname{erf}\left(-x \sqrt{3 f-c \ln(f)}+\frac{b \ln(f)-3 e}{2 \sqrt{3 f-c \ln(f)}}\right)}{16 \sqrt{3 f-c \ln(f)}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2} \ln(f}{}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-6*ln(f)*b*e+12*d*ln(f)*c-36*d*f+9*
e^2)/(-3*f+c*ln(f)))/(3*f-c*ln(f))^(1/2)*erf(-x*(3*f-c*ln(f))^(1/2)+1/2*(b*
ln(f)-3*e)/(3*f-c*ln(f))^(1/2))-1/16*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+6*l
n(f)*b*e-12*d*ln(f)*c-36*d*f+9*e^2)/(3*f+c*ln(f)))/(-c*ln(f)-3*f)^(1/2)*erf
(-(-c*ln(f)-3*f)^(1/2)*x+1/2*(3*e+b*ln(f))/(-c*ln(f)-3*f)^(1/2))-3/16*Pi^(1
/2)*f^a*exp(-1/4*(b^2*ln(f)^2-2*ln(f)*b*e+4*d*ln(f)*c-4*d*f+e^2)/(-f+c*ln(f
)))/(f-c*ln(f))^(1/2)*erf(-x*(f-c*ln(f))^(1/2)+1/2*(b*ln(f)-e)/(f-c*ln(f))^
(1/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+2*ln(f)*b*e-4*d*ln(f)*c-4*d*
```

```
f+e^2)/(f+c*ln(f)))/(-c*ln(f)-f)^(1/2)*erf(-(-c*ln(f)-f)^(1/2)*x+1/2*(e+b*l
n(f))/(-c*ln(f)-f)^(1/2))
```

Maxima [**A**]
time = 0.29, size = 323, normalized size = 0.94

$$\frac{\sqrt{\pi} f^a \operatorname{erf}\left(\sqrt{-c\log(f) - 3f}\, x - \frac{b\log(f)+3e}{2\sqrt{-c\log(f) - 3f}}\right) e^{\left(-\frac{(b\log(f)+3e)^2}{4(c\log(f)+3f)}+3d\right)}}{16\sqrt{-c\log(f) - 3f}} + \frac{3\sqrt{\pi} f^a \operatorname{erf}\left(\sqrt{-c\log(f) - f}\, x - \frac{b\log(f)+e}{2\sqrt{-c\log(f) - f}}\right) e^{\left(-\frac{(b\log(f)+e)^2}{4(c\log(f)+f)}+d\right)}}{16\sqrt{-c\log(f) - f}} + \frac{3\sqrt{\pi} f^a \operatorname{erf}\left(\sqrt{-c\log(f) + f}\, x - \frac{b\log(f)-e}{2\sqrt{-c\log(f) + f}}\right) e^{\left(-\frac{(b\log(f)-e)^2}{4(c\log(f)-f)}-d\right)}}{16\sqrt{-c\log(f) + f}} + \frac{\sqrt{\pi} f^a \operatorname{erf}\left(\sqrt{-c\log(f) + 3f}\, x - \frac{b\log(f)-3e}{2\sqrt{-c\log(f) + 3f}}\right) e^{\left(-\frac{(b\log(f)-3e)^2}{4(c\log(f)-3f)}-3d\right)}}{16\sqrt{-c\log(f) + 3f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d)^3,x, algorithm="maxima")

[Out] 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 3*f)*x - 1/2*(b*log(f) + 3*e)/sqrt(-
c*log(f) - 3*f))*e^(-1/4*(b*log(f) + 3*e)^2/(c*log(f) + 3*f) + 3*d)/sqrt(-c
*log(f) - 3*f) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*(b*log(f
) + e)/sqrt(-c*log(f) - f))*e^(-1/4*(b*log(f) + e)^2/(c*log(f) + f) + d)/sq
rt(-c*log(f) - f) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x - 1/2*(b*lo
g(f) - e)/sqrt(-c*log(f) + f))*e^(-1/4*(b*log(f) - e)^2/(c*log(f) - f) - d)
/sqrt(-c*log(f) + f) + 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 3*f)*x - 1/2*
(b*log(f) - 3*e)/sqrt(-c*log(f) + 3*f))*e^(-1/4*(b*log(f) - 3*e)^2/(c*log(f
) - 3*f) - 3*d)/sqrt(-c*log(f) + 3*f)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1099 vs. $2(303) = 606$.
time = 0.54, size = 1099, normalized size = 3.19

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d)^3,x, algorithm="fricas")

[Out] -1/16*((sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3)*c
osh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 36*d*f + 9*cosh(1)^2 + 6*(2*c*d + 2*a*f
- b*cosh(1) - b*sinh(1))*log(f) + 18*cosh(1)*sinh(1) + 9*sinh(1)^2)/(c*log(
f) - 3*f)) + sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f
^3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 36*d*f + 9*cosh(1)^2 + 6*(2*c*d + 2
*a*f - b*cosh(1) - b*sinh(1))*log(f) + 18*cosh(1)*sinh(1) + 9*sinh(1)^2)/(c
*log(f) - 3*f)))*sqrt(-c*log(f) + 3*f)*erf(-1/2*(6*f*x - (2*c*x + b)*log(f)
 + 3*cosh(1) + 3*sinh(1))*sqrt(-c*log(f) + 3*f)/(c*log(f) - 3*f)) + 3*(sqrt
(pi)*(c^3*log(f)^3 + c^2*f*log(f)^2 - 9*c*f^2*log(f) - 9*f^3)*cosh(-1/4*((b
^2 - 4*a*c)*log(f)^2 - 4*d*f + cosh(1)^2 + 2*(2*c*d + 2*a*f - b*cosh(1) - b
*sinh(1))*log(f) + 2*cosh(1)*sinh(1) + sinh(1)^2)/(c*log(f) - f)) + sqrt(pi
)*(c^3*log(f)^3 + c^2*f*log(f)^2 - 9*c*f^2*log(f) - 9*f^3)*sinh(-1/4*((b^2
- 4*a*c)*log(f)^2 - 4*d*f + cosh(1)^2 + 2*(2*c*d + 2*a*f - b*cosh(1) - b*si
nh(1))*log(f) + 2*cosh(1)*sinh(1) + sinh(1)^2)/(c*log(f) - f)))*sqrt(-c*log
(f) + f)*erf(-1/2*(2*f*x - (2*c*x + b)*log(f) + cosh(1) + sinh(1))*sqrt(-c*
log(f) + f)/(c*log(f) - f)) + 3*(sqrt(pi)*(c^3*log(f)^3 - c^2*f*log(f)^2 -
```

```
9*c*f^2*log(f) + 9*f^3)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d*f + cosh(1)
^2 - 2*(2*c*d + 2*a*f - b*cosh(1) - b*sinh(1))*log(f) + 2*cosh(1)*sinh(1) +
 sinh(1)^2)/(c*log(f) + f)) + sqrt(pi)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c
*f^2*log(f) + 9*f^3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d*f + cosh(1)^2
- 2*(2*c*d + 2*a*f - b*cosh(1) - b*sinh(1))*log(f) + 2*cosh(1)*sinh(1) + si
nh(1)^2)/(c*log(f) + f)))*sqrt(-c*log(f) - f)*erf(1/2*(2*f*x + (2*c*x + b)*
log(f) + cosh(1) + sinh(1))*sqrt(-c*log(f) - f)/(c*log(f) + f)) + (sqrt(pi)
*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) + 3*f^3)*cosh(-1/4*((b^2 -
 4*a*c)*log(f)^2 - 36*d*f + 9*cosh(1)^2 - 6*(2*c*d + 2*a*f - b*cosh(1) - b*
sinh(1))*log(f) + 18*cosh(1)*sinh(1) + 9*sinh(1)^2)/(c*log(f) + 3*f)) + sqr
t(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) + 3*f^3)*sinh(-1/4*((
b^2 - 4*a*c)*log(f)^2 - 36*d*f + 9*cosh(1)^2 - 6*(2*c*d + 2*a*f - b*cosh(1)
 - b*sinh(1))*log(f) + 18*cosh(1)*sinh(1) + 9*sinh(1)^2)/(c*log(f) + 3*f)))
*sqrt(-c*log(f) - 3*f)*erf(1/2*(6*f*x + (2*c*x + b)*log(f) + 3*cosh(1) + 3*
sinh(1))*sqrt(-c*log(f) - 3*f)/(c*log(f) + 3*f)))/(c^4*log(f)^4 - 10*c^2*f^
2*log(f)^2 + 9*f^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cosh(f*x**2+e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.44, size = 427, normalized size = 1.24

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)-3\,f}\left(2\,x+\frac{b\log(f)+3\,e}{c\log(f)+3\,f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2-12\,cd\log(f)+6\,be\log(f)-12\,af\log(f)+9\,e^2-36\,df}{4\,(c\log(f)+3\,f)}\right)}}{16\,\sqrt{-c\log\left(f\right)-3\,f}}-\frac{3\,\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)-f}\left(2\,x+\frac{b\log(f)+e}{c\log(f)+f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2-4\,cd\log(f)+2\,be\log(f)-4\,af\log(f)+e^2-4\,df}{4\,(c\log(f)+f)}\right)}}{16\,\sqrt{-c\log\left(f\right)-f}}-\frac{3\,\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)+f}\left(2\,x+\frac{b\log(f)-e}{c\log(f)-f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2+4\,cd\log(f)-2\,be\log(f)+4\,af\log(f)+e^2-4\,df}{4\,(c\log(f)-f)}\right)}}{16\,\sqrt{-c\log\left(f\right)+f}}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)+3\,f}\left(2\,x+\frac{b\log(f)-3\,e}{c\log(f)-3\,f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2+12\,cd\log(f)-6\,be\log(f)+12\,af\log(f)+9\,e^2-36\,df}{4\,(c\log(f)-3\,f)}\right)}}{16\,\sqrt{-c\log\left(f\right)+3\,f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d)^3,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 3*f)*(2*x + (b*log(f) + 3*e)/(c*lo
g(f) + 3*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 12*c*d*log(f) + 6*b*
e*log(f) - 12*a*f*log(f) + 9*e^2 - 36*d*f)/(c*log(f) + 3*f))/sqrt(-c*log(f)
 - 3*f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - f)*(2*x + (b*log(f) + e)/
(c*log(f) + f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*log(f) + 2*
b*e*log(f) - 4*a*f*log(f) + e^2 - 4*d*f)/(c*log(f) + f))/sqrt(-c*log(f) - f
) - 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x + (b*log(f) - e)/(c*log
(f) - f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 4*c*d*log(f) - 2*b*e*lo
g(f) + 4*a*f*log(f) + e^2 - 4*d*f)/(c*log(f) - f))/sqrt(-c*log(f) + f) - 1/
16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + 3*f)*(2*x + (b*log(f) - 3*e)/(c*log(f
) - 3*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 12*c*d*log(f) - 6*b*e*l
```

```
og(f) + 12*a*f*log(f) + 9*e^2 - 36*d*f)/(c*log(f) - 3*f))/sqrt(-c*log(f) +
3*f)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+b\,x+a}\,\cosh\left(f\,x^2+e\,x+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*cosh(d + e*x + f*x^2)^3,x)
[Out] int(f^(a + b*x + c*x^2)*cosh(d + e*x + f*x^2)^3, x)
```

3.329 $\int \left(\frac{x}{\cosh^{\frac{3}{2}}(x)} + x\sqrt{\cosh(x)}\right) dx$

Optimal. Leaf size=20

$$-4\sqrt{\cosh(x)} + \frac{2x\sinh(x)}{\sqrt{\cosh(x)}}$$

```
[Out] 2*x*sinh(x)/cosh(x)^(1/2)-4*cosh(x)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{3396\}$

$$\frac{2x\sinh(x)}{\sqrt{\cosh(x)}} - 4\sqrt{\cosh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[x/Cosh[x]^(3/2) + x*Sqrt[Cosh[x]],x]
```

```
[Out] -4*Sqrt[Cosh[x]] + (2*x*Sinh[x])/Sqrt[Cosh[x]]
```

Rule 3396

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(c + d*x)*Cos[e + f*x]*((b*Sin[e + f*x])^(n + 1)/(b*f*(n + 1))), x] +
 (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)*(b*Sin[e + f*x])^(n + 2), x], x
] - Simp[d*((b*Sin[e + f*x])^(n + 2)/(b^2*f^2*(n + 1)*(n + 2))), x]) /; Fre
eQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}
\int \left(\frac{x}{\cosh^{\frac{3}{2}}(x)} + x\sqrt{\cosh(x)}\right) dx &= \int \frac{x}{\cosh^{\frac{3}{2}}(x)}\,dx + \int x\sqrt{\cosh(x)}\,dx \\
&= -4\sqrt{\cosh(x)} + \frac{2x\sinh(x)}{\sqrt{\cosh(x)}}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 46 vs. 2(20) = 40.

time = 0.25, size = 46, normalized size = 2.30

$$\frac{2\sinh(x)\left(x-\frac{2\cosh(x)\sinh(x)\sqrt{\tanh^2\left(\frac{x}{2}\right)}}{(-1+\cosh(x))^{3/2}\sqrt{1+\cosh(x)}}\right)}{\sqrt{\cosh(x)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/Cosh[x]^(3/2) + x*Sqrt[Cosh[x]],x]
```

```
[Out] (2*Sinh[x]*(x - (2*Cosh[x]*Sinh[x]*Sqrt[Tanh[x/2]^2])/((-1 + Cosh[x])^(3/2)
*Sqrt[1 + Cosh[x]])))/Sqrt[Cosh[x]]
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x}{\cosh(x)^{\frac{3}{2}}}+x\left(\sqrt{\cosh}\,(x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x)
```

```
[Out] int(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sqrt(cosh(x)) + x/cosh(x)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(\cosh^{2}(x)+1\right)}{\cosh^{\frac{3}{2}}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)**(3/2)+x*cosh(x)**(1/2),x)
```

```
[Out] Integral(x*(cosh(x)**2 + 1)/cosh(x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*sqrt(cosh(x)) + x/cosh(x)^(3/2), x)
```

Mupad [B]
time = 0.15, size = 39, normalized size = 1.95

$$-\frac{2\sqrt{\frac{e^{-x}}{2}+\frac{e^{x}}{2}}\,\left(x+2\,e^{2\,x}-x\,e^{2\,x}+2\right)}{e^{2\,x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(x)^(1/2) + x/cosh(x)^(3/2),x)
```

```
[Out] -(2*(exp(-x)/2 + exp(x)/2)^(1/2)*(x + 2*exp(2*x) - x*exp(2*x) + 2))/(exp(2*
x) + 1)
```

3.330 $\int \left(\frac{x}{\cosh^{\frac{5}{2}}(x)}-\frac{x}{3\sqrt{\cosh(x)}}\right)\,dx$

Optimal. Leaf size=24

$$\frac{4}{3\sqrt{\cosh(x)}}+\frac{2x\sinh(x)}{3\cosh^{\frac{3}{2}}(x)}$$

```
[Out] 2/3*x*sinh(x)/cosh(x)^(3/2)+4/3/cosh(x)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {3396}

$$\frac{4}{3\sqrt{\cosh(x)}}+\frac{2x\sinh(x)}{3\cosh^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[x/Cosh[x]^(5/2) - x/(3*Sqrt[Cosh[x]]),x]
```

```
[Out] 4/(3*Sqrt[Cosh[x]]) + (2*x*Sinh[x])/(3*Cosh[x]^(3/2))
```

Rule 3396

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(c + d*x)*Cos[e + f*x]*((b*Sin[e + f*x])^(n + 1)/(b*f*(n + 1))), x] +
 (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)*(b*Sin[e + f*x])^(n + 2), x], x
] - Simp[d*((b*Sin[e + f*x])^(n + 2)/(b^2*f^2*(n + 1)*(n + 2))), x]) /; Fre
eQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{x}{\cosh^{\frac{5}{2}}(x)}-\frac{x}{3\sqrt{\cosh(x)}}\right)dx &= -\left(\frac{1}{3}\int \frac{x}{\sqrt{\cosh(x)}}\,dx\right)+\int \frac{x}{\cosh^{\frac{5}{2}}(x)}\,dx\\ &= \frac{4}{3\sqrt{\cosh(x)}}+\frac{2x\sinh(x)}{3\cosh^{\frac{3}{2}}(x)}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 16, normalized size = 0.67

$$\frac{2(2+x\tanh(x))}{3\sqrt{\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Cosh[x]^(5/2) - x/(3*Sqrt[Cosh[x]]),x]
```

```
[Out] (2*(2 + x*Tanh[x]))/(3*Sqrt[Cosh[x]])
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x}{\cosh(x)^{\frac{5}{2}}}-\frac{x}{3\sqrt{\cosh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cosh(x)^(5/2)-1/3*x/cosh(x)^(1/2),x)
```

```
[Out] int(x/cosh(x)^(5/2)-1/3*x/cosh(x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(5/2)-1/3*x/cosh(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(-1/3*x/sqrt(cosh(x)) + x/cosh(x)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(16) = 32.
time = 0.52, size = 109, normalized size = 4.54

$$\frac{4\left((x+2)\cosh(x)^{3}+3(x+2)\cosh(x)\sinh(x)^{2}+(x+2)\sinh(x)^{3}-(x-2)\cosh(x)+\left(3(x+2)\cosh(x)^{2}-x+2\right)\sinh(x)\right)\sqrt{\cosh(x)}}{3\left(\cosh(x)^{4}+4\cosh(x)\sinh(x)^{3}+\sinh(x)^{4}+2\left(3\cosh(x)^{2}+1\right)\sinh(x)^{2}+2\cosh(x)^{2}+4\left(\cosh(x)^{3}+\cosh(x)\right)\sinh(x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(5/2)-1/3*x/cosh(x)^(1/2),x, algorithm="fricas")
```

```
[Out] 4/3*((x + 2)*cosh(x)^3 + 3*(x + 2)*cosh(x)*sinh(x)^2 + (x + 2)*sinh(x)^3 -
(x - 2)*cosh(x) + (3*(x + 2)*cosh(x)^2 - x + 2)*sinh(x))*sqrt(cosh(x))/(cos
h(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 +
2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\left(-\frac{3x}{\cosh^{\frac{5}{2}}(x)}\right)dx+\int\frac{x}{\sqrt{\cosh(x)}}\,dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)**(5/2)-1/3*x/cosh(x)**(1/2),x)
```

```
[Out] -(Integral(-3*x/cosh(x)**(5/2), x) + Integral(x/sqrt(cosh(x)), x))/3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(5/2)-1/3*x/cosh(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-1/3*x/sqrt(cosh(x)) + x/cosh(x)^(5/2), x)
```

Mupad [B]
time = 0.97, size = 42, normalized size = 1.75

$$\frac{4\,{\mathrm{e}}^x\,\sqrt{\frac{{\mathrm{e}}^{-x}}{2}+\frac{{\mathrm{e}}^x}{2}}\,\left(2\,{\mathrm{e}}^{2\,x}-x+x\,{\mathrm{e}}^{2\,x}+2\right)}{3\,{\left({\mathrm{e}}^{2\,x}+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cosh(x)^(5/2) - x/(3*cosh(x)^(1/2)),x)
```

```
[Out] (4*exp(x)*(exp(-x)/2 + exp(x)/2)^(1/2)*(2*exp(2*x) - x + x*exp(2*x) + 2))/(
3*(exp(2*x) + 1)^2)
```

3.331 $\int \left(\frac{x}{\cosh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\cosh(x)}\right)\,dx$

Optimal. Leaf size=47

$$\frac{4}{15\cosh^{\frac{3}{2}}(x)}-\frac{12\sqrt{\cosh(x)}}{5}+\frac{2x\sinh(x)}{5\cosh^{\frac{5}{2}}(x)}+\frac{6x\sinh(x)}{5\sqrt{\cosh(x)}}$$

```
[Out] 4/15/cosh(x)^(3/2)+2/5*x*sinh(x)/cosh(x)^(5/2)+6/5*x*sinh(x)/cosh(x)^(1/2)-
12/5*cosh(x)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.050$, Rules used = {3396}

$$\frac{4}{15\cosh^{\frac{3}{2}}(x)}-\frac{12\sqrt{\cosh(x)}}{5}+\frac{2x\sinh(x)}{5\cosh^{\frac{5}{2}}(x)}+\frac{6x\sinh(x)}{5\sqrt{\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Cosh[x]^(7/2) + (3*x*Sqrt[Cosh[x]])/5,x]
```

```
[Out] 4/(15*Cosh[x]^(3/2)) - (12*Sqrt[Cosh[x]])/5 + (2*x*Sinh[x])/(5*Cosh[x]^(5/2
)) + (6*x*Sinh[x])/(5*Sqrt[Cosh[x]])
```

Rule 3396

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(c + d*x)*Cos[e + f*x]*((b*Sin[e + f*x])^(n + 1)/(b*f*(n + 1))), x] +
 (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)*(b*Sin[e + f*x])^(n + 2), x], x
] - Simp[d*((b*Sin[e + f*x])^(n + 2)/(b^2*f^2*(n + 1)*(n + 2))), x]) /; Fre
eQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{x}{\cosh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\cosh(x)}\right)\,dx &= \frac{3}{5}\int x\sqrt{\cosh(x)}\,dx+\int \frac{x}{\cosh^{\frac{7}{2}}(x)}\,dx\\ &= \frac{4}{15\cosh^{\frac{3}{2}}(x)}+\frac{2x\sinh(x)}{5\cosh^{\frac{5}{2}}(x)}+\frac{3}{5}\int \frac{x}{\cosh^{\frac{3}{2}}(x)}\,dx+\frac{3}{5}\int x\sqrt{\cosh(x)}\,dx\\ &= \frac{4}{15\cosh^{\frac{3}{2}}(x)}-\frac{12\sqrt{\cosh(x)}}{5}+\frac{2x\sinh(x)}{5\cosh^{\frac{5}{2}}(x)}+\frac{6x\sinh(x)}{5\sqrt{\cosh(x)}}\end{aligned}$$

Mathematica [A]
time = 0.41, size = 64, normalized size = 1.36

$$\frac{1}{5}\sqrt{\cosh(x)}\left(-\frac{12\sinh^2(x)}{\sqrt{-1+\cosh(x)}\,(1+\cosh(x))^{3/2}\sqrt{\tanh^2\left(\frac{x}{2}\right)}}+6x\tanh(x)+\operatorname{sech}^2(x)\left(\frac{4}{3}+2x\tanh(x)\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/Cosh[x]^(7/2) + (3*x*Sqrt[Cosh[x]])/5,x]
```

```
[Out] (Sqrt[Cosh[x]]*((-12*Sinh[x]^2)/(Sqrt[-1 + Cosh[x]]*(1 + Cosh[x])^(3/2)*Sqr
t[Tanh[x/2]^2]) + 6*x*Tanh[x] + Sech[x]^2*(4/3 + 2*x*Tanh[x])))/5
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x}{\cosh\left(x\right)^{\frac{7}{2}}}+\frac{3x\left(\sqrt{\cosh}\left(x\right)\right)}{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cosh(x)^(7/2)+3/5*x*cosh(x)^(1/2),x)
```

```
[Out] int(x/cosh(x)^(7/2)+3/5*x*cosh(x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(7/2)+3/5*x*cosh(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(3/5*x*sqrt(cosh(x)) + x/cosh(x)^(7/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(7/2)+3/5*x*cosh(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)**(7/2)+3/5*x*cosh(x)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3880 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(7/2)+3/5*x*cosh(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(3/5*x*sqrt(cosh(x)) + x/cosh(x)^(7/2), x)
```

Mupad [**B**]
time = 1.11, size = 110, normalized size = 2.34

$$\frac{e^{2x}\left(\frac{8x}{5}+\frac{16}{15}\right)\sqrt{\frac{e^{-x}}{2}+\frac{e^{x}}{2}}}{\left(e^{2x}+1\right)^2}-\left(\frac{6x}{5}+\frac{12}{5}\right)\sqrt{\frac{e^{-x}}{2}+\frac{e^{x}}{2}}+\frac{12\,x\,e^{2x}\sqrt{\frac{e^{-x}}{2}+\frac{e^{x}}{2}}}{5\left(e^{2x}+1\right)}-\frac{16\,x\,e^{2x}\sqrt{\frac{e^{-x}}{2}+\frac{e^{x}}{2}}}{5\left(e^{2x}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x*cosh(x)^(1/2))/5 + x/cosh(x)^(7/2),x)
```

```
[Out] (exp(2*x)*((8*x)/5 + 16/15)*(exp(-x)/2 + exp(x)/2)^(1/2))/(exp(2*x) + 1)^2
- ((6*x)/5 + 12/5)*(exp(-x)/2 + exp(x)/2)^(1/2) + (12*x*exp(2*x)*(exp(-x)/2
 + exp(x)/2)^(1/2))/(5*(exp(2*x) + 1)) - (16*x*exp(2*x)*(exp(-x)/2 + exp(x)
/2)^(1/2))/(5*(exp(2*x) + 1)^3)
```

3.332 $\int \left(\frac{x^2}{\cosh^{\frac{3}{2}}(x)}+x^2\sqrt{\cosh(x)}\right)\,dx$

Optimal. Leaf size=36

$$-8x\sqrt{\cosh(x)}-16iE\left(\frac{ix}{2}\middle|2\right)+\frac{2x^2\sinh(x)}{\sqrt{\cosh(x)}}$$

```
[Out] -16*I*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*sinh(1/2*x),2^(1/2))+2*
x^2*sinh(x)/cosh(x)^(1/2)-8*x*cosh(x)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {3397, 2719}

$$\frac{2x^2\sinh(x)}{\sqrt{\cosh(x)}}-8x\sqrt{\cosh(x)}-16iE\left(\frac{ix}{2}\middle|2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Cosh[x]^(3/2) + x^2*Sqrt[Cosh[x]],x]
```

```
[Out] -8*x*Sqrt[Cosh[x]] - (16*I)*EllipticE[(I/2)*x, 2] + (2*x^2*Sinh[x])/Sqrt[Co
sh[x]]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3397

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n + 1)/(b*f*(n + 1)
)), x] + (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n +
 2), x], x] + Dist[d^2*m*((m - 1)/(b^2*f^2*(n + 1)*(n + 2))), Int[(c + d*x)
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] - Simp[d*m*(c + d*x)^(m - 1)*((b*
Sin[e + f*x])^(n + 2)/(b^2*f^2*(n + 1)*(n + 2))), x]) /; FreeQ[{b, c, d, e,
 f}, x] && LtQ[n, -1] && NeQ[n, -2] && GtQ[m, 1]
```

Rubi steps

$$\int \left(\frac{x^2}{\cosh^{\frac{3}{2}}(x)}+x^2\sqrt{\cosh(x)}\right)\,dx=\int \frac{x^2}{\cosh^{\frac{3}{2}}(x)}\,dx+\int x^2\sqrt{\cosh(x)}\,dx$$
$$=-8x\sqrt{\cosh(x)}+\frac{2x^2\sinh(x)}{\sqrt{\cosh(x)}}+8\int \sqrt{\cosh(x)}\,dx$$
$$=-8x\sqrt{\cosh(x)}-16iE\left(\frac{ix}{2}\middle|2\right)+\frac{2x^2\sinh(x)}{\sqrt{\cosh(x)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.12, size = 76, normalized size = 2.11

$$\frac{4\sqrt{\cosh(x)}\,(\cosh(x)+\sinh(x))\left(-4(-2+x)\cosh(x)+x^2\sinh(x)+8\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2x}\right)(-\cosh(x)+\sinh(x))\sqrt{1+\cosh(2x)+\sinh(2x)}\right)}{1+e^{2x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Cosh[x]^(3/2) + x^2*Sqrt[Cosh[x]],x]
```

```
[Out] (4*Sqrt[Cosh[x]]*(Cosh[x] + Sinh[x])*(-4*(-2 + x)*Cosh[x] + x^2*Sinh[x] + 8
*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^(2*x)]*(-Cosh[x] + Sinh[x])*Sqrt[1 +
Cosh[2*x] + Sinh[2*x]]))/(1 + E^(2*x))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\cosh(x)^{\frac{3}{2}}}+x^2\left(\sqrt{\cosh}\,(x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/cosh(x)^(3/2)+x^2*cosh(x)^(1/2),x)
```

```
[Out] int(x^2/cosh(x)^(3/2)+x^2*cosh(x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/cosh(x)^(3/2)+x^2*cosh(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*sqrt(cosh(x)) + x^2/cosh(x)^(3/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/cosh(x)^(3/2)+x^2*cosh(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(\cosh^2(x)+1\right)}{\cosh^{\frac{3}{2}}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/cosh(x)**(3/2)+x**2*cosh(x)**(1/2),x)
```

```
[Out] Integral(x**2*(cosh(x)**2 + 1)/cosh(x)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/cosh(x)^(3/2)+x^2*cosh(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*sqrt(cosh(x)) + x^2/cosh(x)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int x^2\,\sqrt{\cosh(x)}+\frac{x^2}{{\cosh(x)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(x)^(1/2) + x^2/cosh(x)^(3/2),x)
```

```
[Out] int(x^2*cosh(x)^(1/2) + x^2/cosh(x)^(3/2), x)
```

3.333 $\int (x+\cosh(x))^2\,dx$

Optimal. Leaf size=30

$$\frac{x}{2}+\frac{x^3}{3}-2\cosh(x)+2x\sinh(x)+\frac{1}{2}\cosh(x)\sinh(x)$$

```
[Out] 1/2*x+1/3*x^3-2*cosh(x)+2*x*sinh(x)+1/2*cosh(x)*sinh(x)
```

Rubi [A]
time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}=0.833$, Rules used = {6874, 3377, 2718, 2715, 8}

$$\frac{x^3}{3}+\frac{x}{2}+2x\sinh(x)-2\cosh(x)+\frac{1}{2}\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Cosh[x])^2,x]
```

```
[Out] x/2 + x^3/3 - 2*Cosh[x] + 2*x*Sinh[x] + (Cosh[x]*Sinh[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int (x+\cosh(x))^2\,dx &= \int \left(x^2+2x\cosh(x)+\cosh^2(x)\right)\,dx\\
&= \frac{x^3}{3}+2\int x\cosh(x)\,dx+\int \cosh^2(x)\,dx\\
&= \frac{x^3}{3}+2x\sinh(x)+\frac{1}{2}\cosh(x)\sinh(x)+\frac{\int 1\,dx}{2}-2\int \sinh(x)\,dx\\
&= \frac{x}{2}+\frac{x^3}{3}-2\cosh(x)+2x\sinh(x)+\frac{1}{2}\cosh(x)\sinh(x)
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 26, normalized size = 0.87

$$\frac{1}{6}\left(3\cosh(x)(-4+\sinh(x))+x\left(3+2x^2+12\sinh(x)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Cosh[x])^2,x]
[Out] (3*Cosh[x]*(-4 + Sinh[x]) + x*(3 + 2*x^2 + 12*Sinh[x]))/6
```

Maple [A]
time = 0.25, size = 25, normalized size = 0.83

method	result	size
default	$\frac{x}{2}+\frac{x^3}{3}-2\cosh\left(x\right)+2x\sinh\left(x\right)+\frac{\cosh(x)\sinh(x)}{2}$	25
risch	$\frac{x^3}{3}+\frac{x}{2}+\frac{e^{2x}}{8}+\left(x-1\right)e^{x}+\left(-1-x\right)e^{-x}-\frac{e^{-2x}}{8}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+cosh(x))^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x+1/3*x^3-2*cosh(x)+2*x*sinh(x)+1/2*cosh(x)*sinh(x)
```

Maxima [A]
time = 0.27, size = 36, normalized size = 1.20

$$\frac{1}{3}x^3-(x+1)e^{(-x)}+(x-1)e^{x}+\frac{1}{2}x+\frac{1}{8}e^{(2x)}-\frac{1}{8}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cosh(x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3 - (x + 1)*e^(-x) + (x - 1)*e^x + 1/2*x + 1/8*e^(2*x) - 1/8*e^(-2*x)
```

Fricas [A]
time = 0.39, size = 23, normalized size = 0.77

$$\frac{1}{3}x^3 + \frac{1}{2}(4x + \cosh(x))\sinh(x) + \frac{1}{2}x - 2\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cosh(x))^2,x, algorithm="fricas")
[Out] 1/3*x^3 + 1/2*(4*x + cosh(x))*sinh(x) + 1/2*x - 2*cosh(x)
```

Sympy [A]
time = 0.06, size = 41, normalized size = 1.37

$$\frac{x^3}{3} - \frac{x\sinh^2(x)}{2} + 2x\sinh(x) + \frac{x\cosh^2(x)}{2} + \frac{\sinh(x)\cosh(x)}{2} - 2\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cosh(x))**2,x)
[Out] x**3/3 - x*sinh(x)**2/2 + 2*x*sinh(x) + x*cosh(x)**2/2 + sinh(x)*cosh(x)/2
- 2*cosh(x)
```

Giac [A]
time = 0.41, size = 36, normalized size = 1.20

$$\frac{1}{3}x^3 - (x+1)e^{(-x)} + (x-1)e^x + \frac{1}{2}x + \frac{1}{8}e^{(2x)} - \frac{1}{8}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cosh(x))^2,x, algorithm="giac")
[Out] 1/3*x^3 - (x + 1)*e^(-x) + (x - 1)*e^x + 1/2*x + 1/8*e^(2*x) - 1/8*e^(-2*x)
```

Mupad [B]
time = 0.05, size = 24, normalized size = 0.80

$$\frac{x}{2} - 2\cosh(x) + \frac{\cosh(x)\sinh(x)}{2} + 2x\sinh(x) + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + cosh(x))^2,x)
[Out] x/2 - 2*cosh(x) + (cosh(x)*sinh(x))/2 + 2*x*sinh(x) + x^3/3
```

3.334 $\int (x+\cosh(x))^3\,dx$

Optimal. Leaf size=56

$$\frac{3x^2}{4}+\frac{x^4}{4}-6x\cosh(x)-\frac{3\cosh^2(x)}{4}+7\sinh(x)+3x^2\sinh(x)+\frac{3}{2}x\cosh(x)\sinh(x)+\frac{\sinh^3(x)}{3}$$

```
[Out] 3/4*x^2+1/4*x^4-6*x*cosh(x)-3/4*cosh(x)^2+7*sinh(x)+3*x^2*sinh(x)+3/2*x*cos
h(x)*sinh(x)+1/3*sinh(x)^3
```

Rubi [A]
time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}=1.000$, Rules used = {6874, 3377, 2717, 3391, 30, 2713}

$$\frac{x^4}{4}+\frac{3x^2}{4}+3x^2\sinh(x)+\frac{\sinh^3(x)}{3}+7\sinh(x)-\frac{3\cosh^2(x)}{4}-6x\cosh(x)+\frac{3}{2}x\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Cosh[x])^3,x]
```

```
[Out] (3*x^2)/4 + x^4/4 - 6*x*Cosh[x] - (3*Cosh[x]^2)/4 + 7*Sinh[x] + 3*x^2*Sinh[
x] + (3*x*Cosh[x]*Sinh[x])/2 + Sinh[x]^3/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int (x+\cosh(x))^3\,dx &= \int \left(x^3+3x^2\cosh(x)+3x\cosh^2(x)+\cosh^3(x)\right)\,dx\\ &= \frac{x^4}{4}+3\int x^2\cosh(x)\,dx+3\int x\cosh^2(x)\,dx+\int \cosh^3(x)\,dx\\ &= \frac{x^4}{4}-\frac{3\cosh^2(x)}{4}+3x^2\sinh(x)+\frac{3}{2}x\cosh(x)\sinh(x)+i\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-i\,\text{s}\right.\\ &= \frac{3x^2}{4}+\frac{x^4}{4}-6x\cosh(x)-\frac{3\cosh^2(x)}{4}+\sinh(x)+3x^2\sinh(x)+\frac{3}{2}x\cosh(x)\sinh(x)+\\ &= \frac{3x^2}{4}+\frac{x^4}{4}-6x\cosh(x)-\frac{3\cosh^2(x)}{4}+7\sinh(x)+3x^2\sinh(x)+\frac{3}{2}x\cosh(x)\sinh(x)\end{aligned}$$

Mathematica [A]
time = 0.05, size = 51, normalized size = 0.91

$$-6x\cosh(x)-\frac{3}{8}\cosh(2x)+\frac{1}{12}\left(9x^2+3x^4+9\left(9+4x^2\right)\sinh(x)+9x\sinh(2x)+\sinh(3x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Cosh[x])^3,x]
```

```
[Out] -6*x*Cosh[x] - (3*Cosh[2*x])/8 + (9*x^2 + 3*x^4 + 9*(9 + 4*x^2)*Sinh[x] + 9
*x*Sinh[2*x] + Sinh[3*x])/12
```

Maple [A]
time = 0.39, size = 73, normalized size = 1.30

method	result
risch	$\frac{x^4}{4}+\frac{3x^2}{4}+\frac{9}{16}+\frac{e^{3x}}{24}+\left(-\frac{3}{16}+\frac{3x}{8}\right)e^{2x}+\left(\frac{27}{8}-3x+\frac{3}{2}x^2\right)e^{x}+\left(-\frac{27}{8}-3x-\frac{3}{2}x^2\right)e^{-x}+\left(-\frac{3}{16}-\frac{3x}{8}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+cosh(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4+3/4*x^2+9/16+1/24*exp(3*x)+(-3/16+3/8*x)*exp(2*x)+(27/8-3*x+3/2*x^2
)*exp(x)+(-27/8-3*x-3/2*x^2)*exp(-x)+(-3/16-3/8*x)*exp(-2*x)-1/24*exp(-3*x)
```

Maxima [A]
time = 0.27, size = 81, normalized size = 1.45

$$\frac{1}{4}x^4+\frac{3}{4}x^2+\frac{3}{16}(2x-1)e^{(2x)}-\frac{3}{2}(x^2+2x+2)e^{(-x)}-\frac{3}{16}(2x+1)e^{(-2x)}+\frac{3}{2}(x^2-2x+2)e^x+\frac{1}{24}e^{(3x)}-\frac{3}{8}e^{(-x)}-\frac{1}{24}e^{(-3x)}+\frac{3}{8}e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cosh(x))^3,x, algorithm="maxima")
```

```
[Out] 1/4*x^4 + 3/4*x^2 + 3/16*(2*x - 1)*e^(2*x) - 3/2*(x^2 + 2*x + 2)*e^(-x) - 3
/16*(2*x + 1)*e^(-2*x) + 3/2*(x^2 - 2*x + 2)*e^x + 1/24*e^(3*x) - 3/8*e^(-x
) - 1/24*e^(-3*x) + 3/8*e^x
```

Fricas [A]
time = 0.41, size = 54, normalized size = 0.96

$$\frac{1}{4}x^4+\frac{1}{12}\sinh(x)^3+\frac{3}{4}x^2-6x\cosh(x)-\frac{3}{8}\cosh(x)^2+\frac{1}{4}\left(12x^2+6x\cosh(x)+\cosh(x)^2+27\right)\sinh(x)-\frac{3}{8}\sinh(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cosh(x))^3,x, algorithm="fricas")
```

```
[Out] 1/4*x^4 + 1/12*sinh(x)^3 + 3/4*x^2 - 6*x*cosh(x) - 3/8*cosh(x)^2 + 1/4*(12*
x^2 + 6*x*cosh(x) + cosh(x)^2 + 27)*sinh(x) - 3/8*sinh(x)^2
```

Sympy [A]
time = 0.09, size = 85, normalized size = 1.52

$$\frac{x^4}{4}-\frac{3x^2\sinh^2(x)}{4}+3x^2\sinh(x)+\frac{3x^2\cosh^2(x)}{4}+\frac{3x\sinh(x)\cosh(x)}{2}-6x\cosh(x)-\frac{2\sinh^3(x)}{3}+\sinh(x)\cosh^2(x)+6\sinh(x)-\frac{3\cosh^2(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cosh(x))**3,x)
```

```
[Out] x**4/4 - 3*x**2*sinh(x)**2/4 + 3*x**2*sinh(x) + 3*x**2*cosh(x)**2/4 + 3*x*s
inh(x)*cosh(x)/2 - 6*x*cosh(x) - 2*sinh(x)**3/3 + sinh(x)*cosh(x)**2 + 6*si
nh(x) - 3*cosh(x)**2/4
```

Giac [A]
time = 0.43, size = 75, normalized size = 1.34

$$\frac{1}{4}x^4+\frac{3}{4}x^2+\frac{3}{16}(2x-1)e^{(2x)}-\frac{3}{8}\left(4x^2+8x+9\right)e^{(-x)}-\frac{3}{16}(2x+1)e^{(-2x)}+\frac{3}{8}\left(4x^2-8x+9\right)e^x+\frac{1}{24}e^{(3x)}-\frac{1}{24}e^{(-3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cosh(x))^3,x, algorithm="giac")
```

```
[Out] 1/4*x^4 + 3/4*x^2 + 3/16*(2*x - 1)*e^(2*x) - 3/8*(4*x^2 + 8*x + 9)*e^(-x) -
 3/16*(2*x + 1)*e^(-2*x) + 3/8*(4*x^2 - 8*x + 9)*e^x + 1/24*e^(3*x) - 1/24*
e^(-3*x)
```

Mupad [B]
time = 0.07, size = 48, normalized size = 0.86

$$\frac{20\sinh(x)}{3}+3\,x^2\sinh(x)-\frac{3\cosh(x)^2}{4}+\frac{\cosh(x)^2\sinh(x)}{3}-6\,x\cosh(x)+\frac{3\,x^2}{4}+\frac{x^4}{4}+\frac{3\,x\cosh(x)\,\sinh(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + cosh(x))^3,x)
```

```
[Out] (20*sinh(x))/3 + 3*x^2*sinh(x) - (3*cosh(x)^2)/4 + (cosh(x)^2*sinh(x))/3 -
6*x*cosh(x) + (3*x^2)/4 + x^4/4 + (3*x*cosh(x)*sinh(x))/2
```

3.335 $\int \frac{\cosh(a+bx)}{c+dx^2}\,dx$

Optimal. Leaf size=213

$$\frac{\cosh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\mathrm{Chi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\,\sqrt{d}}-\frac{\cosh\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\mathrm{Chi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)}{2\sqrt{-c}\,\sqrt{d}}-\frac{\sinh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\mathrm{Shi}\left(\right.}{2\sqrt{-c}\,\sqrt{d}}$$

```
[Out] -1/2*Chi(b*x+b*(-c)^(1/2)/d^(1/2))*cosh(a-b*(-c)^(1/2)/d^(1/2))/(-c)^(1/2)/
d^(1/2)+1/2*Chi(-b*x+b*(-c)^(1/2)/d^(1/2))*cosh(a+b*(-c)^(1/2)/d^(1/2))/(-c
)^(1/2)/d^(1/2)-1/2*Shi(b*x+b*(-c)^(1/2)/d^(1/2))*sinh(a-b*(-c)^(1/2)/d^(1/
2))/(-c)^(1/2)/d^(1/2)+1/2*Shi(b*x-b*(-c)^(1/2)/d^(1/2))*sinh(a+b*(-c)^(1/2
)/d^(1/2))/(-c)^(1/2)/d^(1/2)
```

Rubi [A]

time = 0.40, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5389, 3384, 3379, 3382}

$$\frac{\cosh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\mathrm{Chi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\,\sqrt{d}}-\frac{\cosh\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\mathrm{Chi}\left(xb+\frac{\sqrt{-c}\,b}{\sqrt{d}}\right)}{2\sqrt{-c}\,\sqrt{d}}-\frac{\sinh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\mathrm{Shi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\,\sqrt{d}}-\frac{\sinh\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\mathrm{Shi}\left(xb+\frac{\sqrt{-c}\,b}{\sqrt{d}}\right)}{2\sqrt{-c}\,\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]/(c + d*x^2),x]
```

```
[Out] (Cosh[a + (b*Sqrt[-c])/Sqrt[d]]*CoshIntegral[(b*Sqrt[-c])/Sqrt[d] - b*x])/(
2*Sqrt[-c]*Sqrt[d]) - (Cosh[a - (b*Sqrt[-c])/Sqrt[d]]*CoshIntegral[(b*Sqrt[
-c])/Sqrt[d] + b*x])/(2*Sqrt[-c]*Sqrt[d]) - (Sinh[a + (b*Sqrt[-c])/Sqrt[d]]
*SinhIntegral[(b*Sqrt[-c])/Sqrt[d] - b*x])/(2*Sqrt[-c]*Sqrt[d]) - (Sinh[a -
 (b*Sqrt[-c])/Sqrt[d]]*SinhIntegral[(b*Sqrt[-c])/Sqrt[d] + b*x])/(2*Sqrt[-c
]*Sqrt[d])
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5389

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(a+bx)}{c+dx^2}\,dx &= \int \left(\frac{\sqrt{-c}\,\cosh(a+bx)}{2c\left(\sqrt{-c}-\sqrt{d}\,x\right)}+\frac{\sqrt{-c}\,\cosh(a+bx)}{2c\left(\sqrt{-c}+\sqrt{d}\,x\right)}\right)dx\\
&= -\frac{\int \frac{\cosh(a+bx)}{\sqrt{-c}-\sqrt{d}\,x}\,dx}{2\sqrt{-c}}-\frac{\int \frac{\cosh(a+bx)}{\sqrt{-c}+\sqrt{d}\,x}\,dx}{2\sqrt{-c}}\\
&= -\frac{\cosh\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\int \frac{\cosh\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)}{\sqrt{-c}+\sqrt{d}\,x}\,dx}{2\sqrt{-c}}-\frac{\cosh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\int \frac{\cosh\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{\sqrt{-c}-\sqrt{d}\,x}\,d\ldots}{2\sqrt{-c}}\\
&= \frac{\cosh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\mathrm{Chi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\,\sqrt{d}}-\frac{\cosh\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\mathrm{Chi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)}{2\sqrt{-c}\,\sqrt{d}}-\ldots\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.21, size = 180, normalized size = 0.85

$$\frac{i\left(\cosh\left(a-\frac{ib\sqrt{c}}{\sqrt{d}}\right)\mathrm{CosIntegral}\left(-\frac{b\sqrt{c}}{\sqrt{d}}+ibx\right)-\cosh\left(a+\frac{ib\sqrt{c}}{\sqrt{d}}\right)\mathrm{CosIntegral}\left(\frac{b\sqrt{c}}{\sqrt{d}}+ibx\right)+i\left(\sinh\left(a-\frac{ib\sqrt{c}}{\sqrt{d}}\right)\mathrm{Si}\left(\frac{b\sqrt{c}}{\sqrt{d}}-ibx\right)+\sinh\left(a+\frac{ib\sqrt{c}}{\sqrt{d}}\right)\mathrm{Si}\left(\frac{b\sqrt{c}}{\sqrt{d}}+ibx\right)\right)\right)}{2\sqrt{c}\,\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]/(c + d*x^2),x]
```

```
[Out] ((I/2)*(Cosh[a - (I*b*Sqrt[c])/Sqrt[d]]*CosIntegral[-((b*Sqrt[c])/Sqrt[d])
+ I*b*x] - Cosh[a + (I*b*Sqrt[c])/Sqrt[d]]*CosIntegral[(b*Sqrt[c])/Sqrt[d]
+ I*b*x] + I*(Sinh[a - (I*b*Sqrt[c])/Sqrt[d]]*SinIntegral[(b*Sqrt[c])/Sqrt[
d] - I*b*x] + Sinh[a + (I*b*Sqrt[c])/Sqrt[d]]*SinIntegral[(b*Sqrt[c])/Sqrt[
d] + I*b*x])))/(Sqrt[c]*Sqrt[d])
```

Maple [A]
time = 0.81, size = 212, normalized size = 1.00

method	result
risch	$-\frac{e^{-\frac{b\sqrt{-cd}+ad}{d}}\operatorname{expIntegral}\left(1,-\frac{b\sqrt{-cd}-d(bx+a)+ad}{d}\right)}{4\sqrt{-cd}}+\frac{e^{-\frac{-b\sqrt{-cd}+ad}{d}}\operatorname{expIntegral}\left(1,\frac{b\sqrt{-cd}+d(bx+a)-ad}{d}\right)}{4\sqrt{-cd}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/(-c*d)^(1/2)*exp(-(b*(-c*d)^(1/2)+a*d)/d)*Ei(1,-(b*(-c*d)^(1/2)-d*(b*x
+a)+a*d)/d)+1/4/(-c*d)^(1/2)*exp(-(-b*(-c*d)^(1/2)+a*d)/d)*Ei(1,(b*(-c*d)^(
1/2)+d*(b*x+a)-a*d)/d)-1/4/(-c*d)^(1/2)*exp((b*(-c*d)^(1/2)+a*d)/d)*Ei(1,(b
*(-c*d)^(1/2)-d*(b*x+a)+a*d)/d)+1/4/(-c*d)^(1/2)*exp((-b*(-c*d)^(1/2)+a*d)/
d)*Ei(1,-(b*(-c*d)^(1/2)+d*(b*x+a)-a*d)/d)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(cosh(b*x + a)/(d*x^2 + c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(157) = 314.
time = 0.40, size = 316, normalized size = 1.48

$$-\frac{\left(\sqrt{-\frac{b^2c}{d}}\operatorname{Ei}\left(bx-\sqrt{-\frac{b^2c}{d}}\right)+\sqrt{-\frac{b^2c}{d}}\operatorname{Ei}\left(-bx+\sqrt{-\frac{b^2c}{d}}\right)\right)\cosh\left(a+\sqrt{-\frac{b^2c}{d}}\right)-\left(\sqrt{-\frac{b^2c}{d}}\operatorname{Ei}\left(bx+\sqrt{-\frac{b^2c}{d}}\right)+\sqrt{-\frac{b^2c}{d}}\operatorname{Ei}\left(-bx-\sqrt{-\frac{b^2c}{d}}\right)\right)\cosh\left(-a+\sqrt{-\frac{b^2c}{d}}\right)+\left(\sqrt{-\frac{b^2c}{d}}\operatorname{Ei}\left(bx-\sqrt{-\frac{b^2c}{d}}\right)-\sqrt{-\frac{b^2c}{d}}\operatorname{Ei}\left(-bx+\sqrt{-\frac{b^2c}{d}}\right)\right)\sinh\left(a+\sqrt{-\frac{b^2c}{d}}\right)+\left(\sqrt{-\frac{b^2c}{d}}\operatorname{Ei}\left(bx+\sqrt{-\frac{b^2c}{d}}\right)-\sqrt{-\frac{b^2c}{d}}\operatorname{Ei}\left(-bx-\sqrt{-\frac{b^2c}{d}}\right)\right)\sinh\left(-a+\sqrt{-\frac{b^2c}{d}}\right)}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/4*((sqrt(-b^2*c/d)*Ei(b*x - sqrt(-b^2*c/d)) + sqrt(-b^2*c/d)*Ei(-b*x + s
qrt(-b^2*c/d)))*cosh(a + sqrt(-b^2*c/d)) - (sqrt(-b^2*c/d)*Ei(b*x + sqrt(-b
^2*c/d)) + sqrt(-b^2*c/d)*Ei(-b*x - sqrt(-b^2*c/d)))*cosh(-a + sqrt(-b^2*c/
d)) + (sqrt(-b^2*c/d)*Ei(b*x - sqrt(-b^2*c/d)) - sqrt(-b^2*c/d)*Ei(-b*x + s
qrt(-b^2*c/d)))*sinh(a + sqrt(-b^2*c/d)) + (sqrt(-b^2*c/d)*Ei(b*x + sqrt(-b
^2*c/d)) - sqrt(-b^2*c/d)*Ei(-b*x - sqrt(-b^2*c/d)))*sinh(-a + sqrt(-b^2*c/
d)))/(b*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(a + bx)}{c + dx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x**2+c),x)
```

```
[Out] Integral(cosh(a + b*x)/(c + d*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)/(d*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(a + b\,x)}{d\,x^2 + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)/(c + d*x^2),x)
```

```
[Out] int(cosh(a + b*x)/(c + d*x^2), x)
```

3.336 $\int \frac{\cosh(a+bx)}{c+dx+ex^2} \, dx$

Optimal. Leaf size=271

$$\frac{\cosh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\text{Chi}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}-\frac{\cosh\left(a-\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}\right)\text{Chi}\left(\frac{b\left(d+\sqrt{d^2 \cdots\right.\right.}{\sqrt{d^2-4ce}}$$

```
[Out] Chi(b*x+1/2*b*(d-(-4*c*e+d^2)^(1/2))/e)*cosh(a-1/2*b*(d-(-4*c*e+d^2)^(1/2))
/e)/(-4*c*e+d^2)^(1/2)-Chi(b*x+1/2*b*(d+(-4*c*e+d^2)^(1/2))/e)*cosh(a-1/2*b
*(d+(-4*c*e+d^2)^(1/2))/e)/(-4*c*e+d^2)^(1/2)+Shi(b*x+1/2*b*(d-(-4*c*e+d^2)
^(1/2))/e)*sinh(a-1/2*b*(d-(-4*c*e+d^2)^(1/2))/e)/(-4*c*e+d^2)^(1/2)-Shi(b*
x+1/2*b*(d+(-4*c*e+d^2)^(1/2))/e)*sinh(a-1/2*b*(d+(-4*c*e+d^2)^(1/2))/e)/(-
4*c*e+d^2)^(1/2)
```

Rubi [A]
time = 0.55, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {6860, 3384, 3379, 3382}

$$\frac{\cosh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\text{Chi}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}-\frac{\cosh\left(a-\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}\right)\text{Chi}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}+\frac{\sinh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\text{Shi}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}-\frac{\sinh\left(a-\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}\right)\text{Shi}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]/(c + d*x + e*x^2),x]
```

```
[Out] (Cosh[a - (b*(d - Sqrt[d^2 - 4*c*e]))/(2*e)]*CoshIntegral[(b*(d - Sqrt[d^2
- 4*c*e]))/(2*e) + b*x])/Sqrt[d^2 - 4*c*e] - (Cosh[a - (b*(d + Sqrt[d^2 - 4
*c*e]))/(2*e)]*CoshIntegral[(b*(d + Sqrt[d^2 - 4*c*e]))/(2*e) + b*x])/Sqrt[
d^2 - 4*c*e] + (Sinh[a - (b*(d - Sqrt[d^2 - 4*c*e]))/(2*e)]*SinhIntegral[(b
*(d - Sqrt[d^2 - 4*c*e]))/(2*e) + b*x])/Sqrt[d^2 - 4*c*e] - (Sinh[a - (b*(d
 + Sqrt[d^2 - 4*c*e]))/(2*e)]*SinhIntegral[(b*(d + Sqrt[d^2 - 4*c*e]))/(2*e
) + b*x])/Sqrt[d^2 - 4*c*e]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\cosh(a+bx)}{c+dx+ex^2}\,dx = \int \left(\frac{2e\cosh(a+bx)}{\sqrt{d^2-4ce}\left(d-\sqrt{d^2-4ce}+2ex\right)} - \frac{2e\cosh(a+bx)}{\sqrt{d^2-4ce}\left(d+\sqrt{d^2-4ce}+2ex\right)}\right) d$$

$$= \frac{(2e)\int \frac{\cosh(a+bx)}{d-\sqrt{d^2-4ce}+2ex}\,dx}{\sqrt{d^2-4ce}} - \frac{(2e)\int \frac{\cosh(a+bx)}{d+\sqrt{d^2-4ce}+2ex}\,dx}{\sqrt{d^2-4ce}}$$

$$= \frac{\left(2e\cosh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\right)\int \frac{\cosh\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{d-\sqrt{d^2-4ce}+2ex}\,dx}{\sqrt{d^2-4ce}} - \frac{\left(2e\cosh\left(a-\right.\right.}{}$$

$$= \frac{\cosh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\mathrm{Chi}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}} - \frac{\cosh\left(a-\frac{b\left(d+\sqrt{d^2-\right.}{2e}\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.24, size = 248, normalized size = 0.92

$$\frac{\cosh\left(a+\frac{b\left(-d+\sqrt{d^2-4ce}\right)}{2e}\right)\mathrm{CosIntegral}\left(\frac{ib\left(d-\sqrt{d^2-4ce}+2ex\right)}{2e}\right)-\cosh\left(a-\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}\right)\mathrm{CosIntegral}\left(\frac{ib\left(d+\sqrt{d^2-4ce}+2ex\right)}{2e}\right)-\sinh\left(a-\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}\right)\mathrm{Shi}\left(\frac{b\left(d+\sqrt{d^2-4ce}+2ex\right)}{2e}\right)+i\sinh\left(a+\frac{b\left(-d+\sqrt{d^2-4ce}\right)}{2e}\right)\mathrm{Si}\left(\frac{ib\left(-d+\sqrt{d^2-4ce}\right)}{2e}-ibx\right)}{\sqrt{d^2-4ce}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]/(c + d*x + e*x^2),x]
```

```
[Out] (Cosh[a + (b*(-d + Sqrt[d^2 - 4*c*e]))/(2*e)]*CosIntegral[((I/2)*b*(d - Sqr
t[d^2 - 4*c*e] + 2*e*x))/e] - Cosh[a - (b*(d + Sqrt[d^2 - 4*c*e]))/(2*e)]*C
osIntegral[((I/2)*b*(d + Sqrt[d^2 - 4*c*e] + 2*e*x))/e] - Sinh[a - (b*(d +
```

```
Sqrt[d^2 - 4*c*e]))/(2*e)]*SinhIntegral[(b*(d + Sqrt[d^2 - 4*c*e] + 2*e*x))
/(2*e)] + I*Sinh[a + (b*(-d + Sqrt[d^2 - 4*c*e]))/(2*e)]*SinIntegral[((I/2)
*b*(-d + Sqrt[d^2 - 4*c*e]))/e - I*b*x])/Sqrt[d^2 - 4*c*e]
```

Maple [A]
time = 0.83, size = 376, normalized size = 1.39

method	result
risch	$\frac{b\,\mathrm{e}^{-\frac{2ea-bd-\sqrt{-4b^2ce+b^2d^2}}{2e}}\,\mathrm{expIntegral}\left(1,\frac{2(bx+a)e-2ea+bd+\sqrt{-4b^2ce+b^2d^2}}{2e}\right)}{2\sqrt{-4b^2ce+b^2d^2}}-\frac{b\,\mathrm{e}^{-\frac{2ea-bd+\sqrt{-4b^2ce+b^2d^2}}{2e}}\,\mathrm{expInte}}{2\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)/(e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b/(-4*b^2*c*e+b^2*d^2)^(1/2)*exp(-1/2/e*(2*e*a-b*d-(-4*b^2*c*e+b^2*d^2)
^(1/2)))*Ei(1,1/2*(2*(b*x+a)*e-2*e*a+b*d+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)-1/2
*b/(-4*b^2*c*e+b^2*d^2)^(1/2)*exp(-1/2/e*(2*e*a-b*d+(-4*b^2*c*e+b^2*d^2)^(1
/2)))*Ei(1,-1/2*(-2*(b*x+a)*e+2*e*a-b*d+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)-1/2*
b/(-4*b^2*c*e+b^2*d^2)^(1/2)*exp(1/2/e*(2*e*a-b*d+(-4*b^2*c*e+b^2*d^2)^(1/2
)))*Ei(1,1/2*(-2*(b*x+a)*e+2*e*a-b*d+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)+1/2*b/(
-4*b^2*c*e+b^2*d^2)^(1/2)*exp(1/2/e*(2*e*a-b*d-(-4*b^2*c*e+b^2*d^2)^(1/2)))
*Ei(1,-1/2*(2*(b*x+a)*e-2*e*a+b*d+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(e*x^2+d*x+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d^2-4*%e*c>0)', see `assume?` for m
ore det
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1346 vs. 2(235) = 470.
time = 0.47, size = 1346, normalized size = 4.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(e*x^2+d*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*((sqrt((b^2*d^2 - 4*b^2*c*cosh(1) - 4*b^2*c*sinh(1))/(cosh(1)^2 + 2*co
sh(1)*sinh(1) + sinh(1)^2))*(cosh(1) + sinh(1))*Ei(1/2*(2*b*x*cosh(1) + 2*b
*x*sinh(1) + b*d + sqrt((b^2*d^2 - 4*b^2*c*cosh(1) - 4*b^2*c*sinh(1))/(cosh
(1)^2 + 2*cosh(1)*sinh(1) + sinh(1)^2))*(cosh(1) + sinh(1)))/(cosh(1) + sin
h(1))) + sqrt((b^2*d^2 - 4*b^2*c*cosh(1) - 4*b^2*c*sinh(1))/(cosh(1)^2 + 2*
cosh(1)*sinh(1) + sinh(1)^2))*(cosh(1) + sinh(1))*Ei(-1/2*(2*b*x*cosh(1) +
2*b*x*sinh(1) + b*d + sqrt((b^2*d^2 - 4*b^2*c*cosh(1) - 4*b^2*c*sinh(1))/(c
osh(1)^2 + 2*cosh(1)*sinh(1) + sinh(1)^2))*(cosh(1) + sinh(1)))/(cosh(1) +
sinh(1))))*cosh(1/2*(b*d - 2*a*cosh(1) - 2*a*sinh(1) + sqrt((b^2*d^2 - 4*b^
2*c*cosh(1) - 4*b^2*c*sinh(1))/(cosh(1)^2 + 2*cosh(1)*sinh(1) + sinh(1)^2))
*(cosh(1) + sinh(1)))/(cosh(1) + sinh(1))) - (sqrt((b^2*d^2 - 4*b^2*c*cosh(
1) - 4*b^2*c*sinh(1))/(cosh(1)^2 + 2*cosh(1)*sinh(1) + sinh(1)^2))*(cosh(1)
 + sinh(1))*Ei(1/2*(2*b*x*cosh(1) + 2*b*x*sinh(1) + b*d - sqrt((b^2*d^2 - 4
*b^2*c*cosh(1) - 4*b^2*c*sinh(1))/(cosh(1)^2 + 2*cosh(1)*sinh(1) + sinh(1)^
2))*(cosh(1) + sinh(1)))/(cosh(1) + sinh(1))) + sqrt((b^2*d^2 - 4*b^2*c*cos
h(1) - 4*b^2*c*sinh(1))/(cosh(1)^2 + 2*cosh(1)*sinh(1) + sinh(1)^2))*(cosh(
1) + sinh(1))*Ei(-1/2*(2*b*x*cosh(1) + 2*b*x*sinh(1) + b*d - sqrt((b^2*d^2
- 4*b^2*c*cosh(1) - 4*b^2*c*sinh(1))/(cosh(1)^2 + 2*cosh(1)*sinh(1) + sinh(
1)^2))*(cosh(1) + sinh(1)))/(cosh(1) + sinh(1))))*cosh(-1/2*(b*d - 2*a*cosh
(1) - 2*a*sinh(1) - sqrt((b^2*d^2 - 4*b^2*c*cosh(1) - 4*b^2*c*sinh(1))/(cos
h(1)^2 + 2*cosh(1)*sinh(1) + sinh(1)^2))*(cosh(1) + sinh(1)))/(cosh(1) + si
nh(1))) - (sqrt((b^2*d^2 - 4*b^2*c*cosh(1) - 4*b^2*c*sinh(1))/(cosh(1)^2 +
2*cosh(1)*sinh(1) + sinh(1)^2))*(cosh(1) + sinh(1))*Ei(1/2*(2*b*x*cosh(1) +
 2*b*x*sinh(1) + b*d + sqrt((b^2*d^2 - 4*b^2*c*cosh(1) - 4*b^2*c*sinh(1))/(
cosh(1)^2 + 2*cosh(1)*sinh(1) + sinh(1)^2))*(cosh(1) + sinh(1)))/(cosh(1) +
 sinh(1))) - sqrt((b^2*d^2 - 4*b^2*c*cosh(1) - 4*b^2*c*sinh(1))/(cosh(1)^2
+ 2*cosh(1)*sinh(1) + sinh(1)^2))*(cosh(1) + sinh(1))*Ei(-1/2*(2*b*x*cosh(1
) + 2*b*x*sinh(1) + b*d + sqrt((b^2*d^2 - 4*b^2*c*cosh(1) - 4*b^2*c*sinh(1)
)/(cosh(1)^2 + 2*cosh(1)*sinh(1) + sinh(1)^2))*(cosh(1) + sinh(1)))/(cosh(1
) + sinh(1))))*sinh(1/2*(b*d - 2*a*cosh(1) - 2*a*sinh(1) + sqrt((b^2*d^2 -
4*b^2*c*cosh(1) - 4*b^2*c*sinh(1))/(cosh(1)^2 + 2*cosh(1)*sinh(1) + sinh(1)
^2))*(cosh(1) + sinh(1)))/(cosh(1) + sinh(1))) - (sqrt((b^2*d^2 - 4*b^2*c*c
osh(1) - 4*b^2*c*sinh(1))/(cosh(1)^2 + 2*cosh(1)*sinh(1) + sinh(1)^2))*(cos
h(1) + sinh(1))*Ei(1/2*(2*b*x*cosh(1) + 2*b*x*sinh(1) + b*d - sqrt((b^2*d^2
 - 4*b^2*c*cosh(1) - 4*b^2*c*sinh(1))/(cosh(1)^2 + 2*cosh(1)*sinh(1) + sinh
(1)^2))*(cosh(1) + sinh(1)))/(cosh(1) + sinh(1))) - sqrt((b^2*d^2 - 4*b^2*c
*cosh(1) - 4*b^2*c*sinh(1))/(cosh(1)^2 + 2*cosh(1)*sinh(1) + sinh(1)^2))*(c
osh(1) + sinh(1))*Ei(-1/2*(2*b*x*cosh(1) + 2*b*x*sinh(1) + b*d - sqrt((b^2*
d^2 - 4*b^2*c*cosh(1) - 4*b^2*c*sinh(1))/(cosh(1)^2 + 2*cosh(1)*sinh(1) + s
inh(1)^2))*(cosh(1) + sinh(1)))/(cosh(1) + sinh(1))))*sinh(-1/2*(b*d - 2*a*
cosh(1) - 2*a*sinh(1) - sqrt((b^2*d^2 - 4*b^2*c*cosh(1) - 4*b^2*c*sinh(1))/
(cosh(1)^2 + 2*cosh(1)*sinh(1) + sinh(1)^2))*(cosh(1) + sinh(1)))/(cosh(1)
+ sinh(1))))/(b*d^2 - 4*b*c*cosh(1) - 4*b*c*sinh(1))
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh{(a+bx)}}{c+dx+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(e*x**2+d*x+c),x)
[Out] Integral(cosh(a + b*x)/(c + d*x + e*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(e*x^2+d*x+c),x, algorithm="giac")
[Out] integrate(cosh(b*x + a)/(e*x^2 + d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(a+b\,x)}{e\,x^2+d\,x+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)/(c + d*x + e*x^2),x)
[Out] int(cosh(a + b*x)/(c + d*x + e*x^2), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#            Port of original Maple grading function by
#            Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#            added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
      ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation ="none"
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


    print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```